Sensing, allocation and trading of spectral resources in cognitive radio networks

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**SENSING, ALLOCATION AND TRADING OF
SPECTRAL RESOURCES IN COGNITIVE
RADIO NETWORKS**

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2013

SENSING, ALLOCATION AND TRADING OF SPECTRAL RESOURCES IN COGNITIVE RADIO NETWORKS

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Abstract

The invention of cognitive radio concept is to overcome the spectral scarcity issues of emerging radio systems. In cognitive radio, the *secondary users* can either opportunistically access the licensed spectrum in the absence of their *primary users*, or buy/rent the spectrum access rights from the *primary users*. For the former case, spectrum sensing and resource allocation are highly essential. While spectrum sensing aims to identify the unused spectrum through detecting the presence of primary users' signal, resource allocation exploits information obtained from spectrum sensing to schedule the unused spectrum among the secondary users. For the latter case, spectrum trading is emphasized. The problems of spectrum sensing, resource allocation and spectrum trading in cognitive radio networks are focused on this thesis.

For spectrum sensing, due to its sensing accuracy, the cooperative spectrum sensing rather than stand-alone spectrum sensing is studied. The challenges arising when secondary users have no *a priori* knowledge of primary users' bands are discussed. These challenges are to dynamically detect the primary user's bands and to reduce the overheads required to facilitate fusion center operation. A dynamic band clustering algorithm that uses K-means clustering technique, and a novel entropy-based decision-fusion technique are proposed to address these challenges, respectively. The proposed decision-fusion technique is shown to be comparable to conventional information-fusion techniques while reducing significantly the bandwidth overheads.

For resource allocation problem, two non-cooperative games named *Interference Minimization* game and *Capacity Maximization* game which reflect the targets of *data radios* and *voice radios* are proposed, respectively. From the extensive simulation, it is shown that these proposed games help improve the average SINRs and capacities of all secondary users.

Finally, the problem of spectrum trading in a cognitive radio network with multiple primary users competing to sell spectrum to a secondary user is addressed. It is assumed that the spectrum requirements for the primary users' services are time varying, and the spectrum trading process is carried out before the realization of these values. Both cases when the spectrum demand of the secondary user is a deterministic function of price, and when it is uncertain and may vary from time to time, are considered. In the first case, a *Cournot game model* of competition is proposed to model the spectrum trading. In this model, the primary users compete with each other by setting the size of spectrum to sell. The *Nash equilibrium* for a static game when the players (i.e., the primary users) have complete information on other players is studied first. The proofs for existence and uniqueness of Nash equilibrium are also provided. Next, a dynamic game, in which the players adaptively change their strategies to reach the Nash equilibrium, is also discussed. Furthermore, the trading problem is extended to a scenario which involves multiple secondary user networks. In the second case, a *Supply Function Equilibrium* model is proposed. In this model, the primary users compete to sell spectrum by submitting their supply functions to the secondary user. Similar to the first case, the Nash equilibrium is studied for the static game and an iterative algorithm to reach the Nash equilibrium is proposed for the dynamic game. Detailed simulation results are provided in both cases to prove the robustness of the theoretic analyses.

In summary, the novel technique and algorithms for an accurate and low spectrum over-head cooperative spectrum sensing system are proposed. The sensing system can be deployed in an extreme scenario where no *a priori* knowledge of primary users' bands is given. Furthermore, the innovative methods to allocate unused spectrum among the secondary users who have different objectives are also proposed. Finally, the frameworks to analyze the spectrum trading among multiple primary and secondary users under realistic network scenarios are also proposed.

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List of Abbreviations

AWGN	Additive White Gaussian Noise
BDE	Boundary Detection Error
BER	Bit Error Rate
CAF	Cyclic Auto-correlation Function
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
CFBD	Canny Frequency Boundary Detector
CFBD-DBC	Canny Frequency Boundary Detector with Dynamic Band Clustering
CPUTD	Cooperative Primary User Transmission Detector
CR	Cognitive Radio
CSCG	Circularly Symmetric Complex Gaussian
CSD	Cyclic Spectral Density
CSS	Cooperative Spectrum Sensing
DBC	Dynamic Band Clustering
DF	Decision Fusion
DVB-SH	Digital Video Broadcasting - Satellite Services to Handhelds
EGC	Equal Gain Combining
EPG	Exact Potential Game
FBD	Frequency Boundary Detection
FC	Fusion Center
FDMA	Frequency Division Multiple Access
FIR	Finite Impulse Response
IF	Information Fusion
i.i.d	Independent and Identically Distributed
IIR	Infinite Impulse Response
IWFA	Iterative Water-filling Algorithm
KKT	Karush-Kuhn-Tucker
LOS	Line-of-Sight
MRC	Maximal Ratio Combining
NE	Nash Equilibrium
NCSS	Non-Cooperative Spectrum Sensing
NLOS	Non-Line-of-Sight
OFDMA	Orthogonal Frequency-Division Multiple Access

OPG	Ordinal Potential Game
OSC	Optimal Soft Combining
PDF	Probability Density Function
PoD	Probability of Detection
PU	Primary User
PUTD	Primary User Transmission Detector
PSD	Power Spectral Density
QoS	Quality of Service
ROC	Receiver Operating Characteristics
RRC	Raised-root Cosine
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SU	Secondary User
TB	Tolerance Bandwidth
TDMA	Time Division Multiple Access
VCG	Vickrey-Clarke-Groves
WSS	Wide-sense Stationary

Notations

x	Variable name of type constant
$\mathbf{x}$	Variable of type vector
$\mathcal{S}$	Set of elements
$ x $	Absolute value of x
$ \mathcal{S} $	Cardinality of set $\mathcal{S}$
$Re(x)$	Real part of x
$Pr[.]$	Probability of the event specified in the argument
$E[.]$	Expectation (statistical average) operation
$Var[.]$	Variance operation
$[.]^*$	Conjugate operation
$[.]^T$	Transpose operation
$\otimes$	Convolution operation
$f_X(x)$	Probability Density Function of random variable X evaluated at x
$F_X(x)$	Cumulative Distribution Function of random variable X evaluated at x

List of Publications

International Conferences

1. N. Duong, A. Madhukumar, A. Premkumar, and N. Chong, “A novel architecture for distributed spectrum sensing for cognitive radio applications,” in *Proc. IEEE TENCON*, pp. 1–5, Jan. 2009. Published.
2. N. Duong, A. Madhukumar, S. Dharma, and B. Ng, “Cognitive radios in cooperative environment: Detection, sensing and clustering of spectral bands,” in *Proc. IEEE Vehicular Technology Conference (VTC)*, pp. 1–5, Sep. 2010. Published.
3. N. Duong, A. Madhukumar, and A. Premkumar, “A game theoretic approach for power control and spectrum allocation for cognitive radio networks,” in *Proc. IEEE Midwest Symposium on Circuits and Systems (MWSCAS)*, pp. 1-4, Aug. 2011. Published.

International Journals

1. N. Duong, S. Tio, and A. Madhukumar, “A cooperative spectrum sensing technique with dynamic frequency boundary detection and information-entropy-fusion for primary user detection,” *Circuits, Systems, and Signal Processing*, pp. 823–845, Aug. 2011. SP Birkhauser Boston. Published.
2. N. Duong and A. Madhukumar, “Non-cooperative power control and spectrum allocation in cognitive radio networks: a game theoretic perspective,” *Wireless Communications and Mobile Computing*, 2012. John Wiley & Sons, Ltd. Published.

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3. N. Duong and A. Madhukumar, “Spectrum trading in cognitive radio networks with uncertainty in primary service requirements,” *IET Signal Processing*, 2013. Accepted.
 4. N. Duong and A. Madhukumar, “Spectrum trading with primary service requirement and secondary user demand uncertainty,” *IEEE Trans. Veh. Technol.*, 2013. Under review.

Chapter 1

Introduction

1.1 Background

The last three decades have witnessed a great revolution in wireless technology. Starting with the liberation of spectrum access through the creation of license-exempt bands including ISM (2.4 GHz), UNII (5 – 6 GHz) and microwave (57 – 64 GHz) bands [1], many wireless technologies such as IEEE802.11 [2], Bluetooth [3] and Cordless telephony have been invented. Nowadays, wireless technologies have profoundly affected every facet of modern life from the business transaction to classroom teachings, from file-sharing to interactive gaming. As the global population of wireless devices is continuously growing, the free and unlicensed spectrum is expected to be overcrowded. Due to its success, there has been effort to allocate more spectrum for license-exempt bands. Unfortunately, the implementation of the traditional spectrum management techniques based on command-and-control model makes most of the usable frequency bands have either been licensed to the authorized users by government or reserved for mission-critical or military communication purposes. While debates between license-exempt and license-based advocates have been rigorous and are still on-going, it is becoming clear that both approaches are likely to stay. Therefore, to solve the problem of spectrum scarcity, a better spectrum management technique to achieve better spectrum utilization is highly demanded. This view is supported by a study from the Spectrum Task Force of Federal Communication

Commission (FCC) in 2002, that highlights the poor (or low) utilization level of large part of the licensed spectrum during a significant percentage of time [4]. This revelation led to the concept of *cognitive radio*. A *cognitive radio* (CR) as defined in [5] is an intelligent wireless communication system that is aware of its surrounding environment. This system has the capability to learn from and adapt to the ambient radio environment by altering its transmission parameters such as transmit power, frequency band, modulation mode, etc. The key approach in cognitive radio is *dynamic spectrum access* (DSA), through which the unlicensed users, hereinafter referred to as *secondary users*, are allowed to dynamically access the frequency bands which are under-utilized by the licensed users (hereinafter referred to as *primary users*). Similar to [6], DSA techniques for cognitive radio can be categorized into two basic models including *exclusive-use model* and *shared-use model*.

In *shared-use model*, a secondary user (SU) can opportunistically access a licensed spectrum if it does not cause any harmful interference to a primary user (PU). The spectrum can be shared between primary and secondary users by using two different approaches, namely *spectrum overlay* and *spectrum underlay* as illustrated in Figure 1.1. In *spectrum underlay* approach, a SU can transmit along with a PU at the same time by limiting the transmit power of the SU to maintain the interference at the receiver of the PU below an acceptable interference limit. This approach is already being implemented in radio systems using Ultra-wide band (UWB) and Spread Spectrum [7] technologies. However, due to the extremely low transmit power, this approach is only suitable for short range and indoor applications. Unlike *spectrum underlay* approach, *spectrum overlay* approach imposes no constraint on the transmit power of the SU, and hence can be used for medium and long range applications. This advantage makes *spectrum overlay* approach become the most commonly studied approach in literature, and can be considered as a de-facto standard in DSA for cognitive radios. In this approach, a SU is allowed

(a) Spectrum Underlay

(b) Spectrum Overlay

Figure 1.1: Shared-use Model: Spectrum Underlay and Overlay Approaches

to opportunistically access a licensed frequency band if it is not being utilized by its licensed user (i.e., the primary user) at a particular time and a specific geographical area. This licensed frequency band is referred to as an *unused frequency band* or a *spectrum hole*. Whenever the PU resumes transmitting, the SU must vacate the frequency band immediately to avoid interference to the PU. As a result, in order to access a frequency band, the SU has to continuously perform *spectrum sensing* (SS) to detect the activity of a PU within that band. Once the unused frequency bands are determined, a cardinal problem is to assign these unused frequency bands among SUs in such a way that the SUs can meet their Quality of Service (QoS) requirements. This problem is known as the

resource allocation problem. Efficient *spectrum sensing* (SS) and *resource allocation* form the core of the communication architecture for cognitive radio networks using *spectrum overlay shared-use* model.

In *exclusive-use model*, the basic structure of the current spectrum management policies by which the radio spectrum is licensed to users/services for exclusive use is maintained. However, in order to improve the spectrum utilization, the license users (i.e., the PUs) are provided with some levels of flexibility in the way they can use their licensed spectrum. As the PUs may not fully utilize the allocated spectrum incessantly in all the geographical areas, they may sell/grant the spectrum access rights to the SUs. In this model, the spectrum is considered as a valuable asset that can be traded among the primary and secondary users. *Spectrum trading*, which is defined as the process of selling and buying spectral resources including the frequency bands, transmit power, time slots, channel access codes, etc., plays an important role in this model. Through *spectrum trading*, the PUs and SUs may compete or cooperate with one another to maximize the revenue of the PUs as well as the satisfaction of the SUs.

In *shared-use model*, the *technical perspective* of cognitive radio networks is concentrated specifically on the problems of *spectrum sensing* and *resource allocation*. However, in *exclusive-use model*, the *economic perspective* of cognitive radio networks, particularly the problem of *spectrum trading* is emphasized. New and innovative methods for *spectrum sensing*, *resource allocation* and *spectrum trading* in cognitive radio networks are studied in this thesis.

1.2 Motivation and Objectives

Most of the present literature in spectrum sensing assumes the availability of complete knowledge of PUs' frequency bands to the SUs. In such cases, the task of spectrum sensing is to identify possible transmission within those predefined PUs' bands. However, such

an assumption is idealistic and imposes strict limitation on the practical deployment scenarios. For example, the frequencies allocated for today's 2G and 3G networks may differ among countries and so do the frequencies dedicated for military purposes. In another extreme case, it may be assumed that SUs are not equipped with any knowledge of the PUs and their band locations. In this case, the task of spectrum sensing is made complicated by the additional processing required to identify the location of the PUs' bands. Detection for any sign of PU transmission is then performed on those bands that have been identified. As standardization efforts are still on-going, it is not yet clear which of the scenarios will eventually be adopted. Nonetheless, the present work assumes the latter scenario, being the more general of the two.

Previously, the task of SS was assumed to be performed individually by each SU and each SU tries to apprehend the spectrum utilization pattern based on its own sensing statistics. This is referred to as *stand-alone spectrum sensing* or *non-cooperative spectrum sensing* (NCSS). One obvious advantage of NCSS is its simplicity. However, the reliability of NCSS is sensitive to the dynamic nature of the wireless links. For example, the presence of a brick wall (or other shadowing objects) between a PU and a SU would lead to Non-Line-of-Sight (NLOS) propagation condition, resulting in poor reception of PU signal by the SU. As such, based on the sensing statistics, the particular SU would think that the spectrum is vacant and proceed to operate on the PU's operating band. This could then lead to a destructive effect as signal from the SU would interfere with that of PU and cause deterioration to PU's performance. In a more complex scenario where *a priori* knowledge of PUs' band locations is unavailable, NCSS is prone to other problems. One major problem is false detection of PUs' bands due to impairments such as spurious noise. The presence of many false and narrow bands would result in longer processing delay as the detection needs to be performed on each of those bands. In a cognitive radio network, the utilization of non-vacant PUs' bands (due to inaccurate SS results) by SUs

constitutes a serious breach of the basic principle of cognitive radio, while long sensing delay reduces overall spectral efficiency. In order to improve the reliability of SS results, *cooperative spectrum sensing* (CSS) has been proposed. As decision in CSS is based on the sensing statistics of multiple SUs, the geospatial diversity of SUs reduces the sensitivity of SS results to fluctuations in signal quality. In addition, the noise averaging is likely to reduce the number of false primary bands and hence the sensing delay incurred. Because of its accurate performance, CSS rather than NCSS is focused in the present research. However, one major drawback associated with CSS is the overhead involved to exchange or spread the sensing decision or statistics among the multiple SUs. The present work is motivated to address the challenges arising when CSS is deployed in a cognitive radio environment without *a priori* knowledge of PUs' band locations. The first challenge is to make a collective decision about the PUs' band locations based on the results of SUs, with the aim of minimizing false PUs' bands. The second challenge of CSS is to make a collective decision about the presence or absence of PU based on the detection results from SUs, with the aim of minimizing the overhead incurred to exchange the decision results of multiple SUs.

Spectrum sensing in a cognitive radio environment should be followed by the appropriate allocation of spectral resources among SUs. The game theory, which provides the mathematical tools for modeling and analyzing the competition or cooperation among intelligent rational decision makers, appears as a perfect match for the resource allocation problem. In a game, the decision makers (the players) may make their decisions independently to maximize their own benefits (i.e., *non-cooperative game*) or form coalitions to maximize common goals (i.e., *cooperative game*). While cooperation among the SUs in cognitive radio networks is not always realizable and requires additional enforcement measures to prevent the misbehaviors of users, it is more natural to formulate the resource allocation problem using *non-cooperative* games. In addition, because different

radios may have different QoS requirements, they may have different objectives. For example, in *voice communication*, high speed transmission is the most important factor, and hence voice cognitive radios always try to maximize their transmission rate. In *data communication*, the most important factor is the BER. The data cognitive radios, hence always try to maximize their SINRs. As a result, this work is motivated to study non-cooperative game theoretic frameworks which can reflect the objectives of different SUs.

Besides technical issues, economic issues in cognitive radio networks, especially the problem of spectrum trading, have gained more and more attention nowadays. Various spectrum trading models have been studied for different cognitive radio network configurations, especially for the *multiple primary-single secondary user* and *single primary-multiple secondary user* scenarios. In the first scenario, multiple PUs compete to sell spectrum to a single SU. Inversely, in the second scenario, multiple SUs compete to buy spectrum from a single PU. These scenarios can be formulated as *oligopoly markets* where a few firms compete with one another in terms of price (i.e., Bertrand oligopoly markets [8]) or in terms of quantity of goods brought to the market (i.e., Cournot oligopoly markets [9]) to earn the highest profit. Another approach is to formulate these spectrum trading scenarios as *single-sided auction* models [10], in which multiple bidders bid for goods or services provided by an auctioneer [11]. However, in the existing spectrum trading research, the authors often assume that the size of spectrum required for the PU's service is fixed and the PU knows this value exactly. This assumption is not practical due to the volatile characteristics of radio networks. This work is motivated to study a spectrum trading scenario in which the spectrum requirements for the PUs' services are not fixed and are time varying, and the PUs are unable to judge these values exactly at the time the trade agreement is made. In addition, the spectrum demands of the SUs are supposed to change over the time. This work is motivated to further extend the

problem to a scenario in which both the spectrum requirements for PUs' services and the spectrum demands of the SUs are uncertain.

1.3 Research Contributions

The main contributions of this thesis to dynamic spectrum access techniques in cognitive radio in both *shared-use* and *exclusive-use* models are briefly presented in Figure 1.2, and the details are provided as follows.

Figure 1.2: Contributions to Dynamic Spectrum Access Techniques in Cognitive Radio

Starting with the *shared-use* model, a cooperative spectrum sensing system to detect the spectrum holes when the SUs have no *a priori* knowledge of PUs' band locations is proposed. In this context, the following techniques and algorithm are proposed:

- A frequency boundary detection technique based on *Canny Edge Detection* technique to detect the PUs' band locations.
- A *Dynamic Band Clustering* (DBC) algorithm to make a collective decision about the primary band locations based on the results of individual SUs. The proposed DBC algorithm applies the *K-means Clustering* technique [12], a technique that is popularly used to solve similar problems in image processing.
- A novel *decision-fusion* technique that makes use of information-entropy (which measures the uncertainty of the information received by a SU) to make a collective decision about the presence or absence of PU based on the detection results of individual SUs. The proposed technique aims to minimize the contribution of the highly uncertain SUs and maximize the contribution of less uncertain SUs to the cooperative decision. Based on extensive simulation studies, the proposed fusion technique is shown to be comparable to conventional *information-fusion* techniques. The performance offered by the proposed technique is achieved with significant reduction in the bandwidth overheads.

Next, two different non-cooperative games are proposed to schedule the spectrum holes detected in spectrum sensing among the SUs:

- An *Interference Minimization* game to reflect the interference minimization objective of data radio systems. In this game, each player (i.e., SU) aims to maximize its utility function which is determined as the difference between the benefit that the player earns and its cost for choosing a specific strategy. The benefit of a player is defined as the total received power at the player's receiver. While the cost is defined as the total interference that other players generate on this player and the interference of the player itself to other players.

- A *Capacity Maximization* game to reflect the capacity maximization objective of voice radio systems. In this game, the utility function of a player is determined as the total weighted capacity that the player earns by allocating power on channels.

The contributions of this thesis are also on the *exclusive-use* models of dynamic spectrum access, in particular on the problem of spectrum trading. Different from the other works in the literature, this thesis considers a scenario in which the spectrum requirements for the PUs' services are not fixed but time varying. These values are not observable when the spectrum trading occurs, and only their probability distributions are available to the PUs. The considered cognitive radio network consists of multiple PUs and a single SU. Two trading models are proposed:

- When the spectrum demand of the SU is a deterministic function of price, a *Cournot game model* of competition is proposed to analyze the behaviors of the PUs. In this model, the PUs compete by submitting the size of spectrum they want to sell to the SU. The NE for a static game, in which the PUs have complete knowledge on the utility functions of other players, is studied first. The proofs on existence and uniqueness of NE are also provided. It is followed by a discussion on a dynamic game in which the players (i.e., the PUs) adaptively change their strategies to reach the NE. The trading problem is then extended to a scenario which involves multiple primary and secondary users.
- When both the spectrum requirements for PUs' services and the spectrum demand of the SU are uncertain, a *Supply Function Equilibrium* model [13] is proposed to model the spectrum trading problem. Different from the *Cournot game model*, in this model the PUs compete by submitting supply functions to the SU, which state the size of spectrum that the PUs are willing to sell at each price. A non-cooperative game theoretic framework is formulated to study the behaviors of the

PUs. The NE of a static game, in which players have the complete information on other players, is studied first. Next, an iterative algorithm which allows the players to adaptively change their strategies to reach the NE is proposed for the case of incomplete information. The stability of the proposed algorithm is also studied.

1.4 Potential Applications

The proposed cooperative spectrum sensing and resource allocation techniques can be applied in several practical applications.

- They can be used in an Ad-Hoc mobile network to sense a wide-band spectrum for spectrum holes and to distributedly allocate the spectrum holes among the users. Such an Ad-Hoc mobile network usually do not possess any information on the PUs' bands.
- The cooperative spectrum sensing can also be used in signal transmission detection systems, which are usually used to jam/interpret the signal from the enemies for military purposes.

Furthermore, the models for spectrum trading proposed in this thesis can be used as frameworks for the secondary spectrum market among the primary and secondary users. Corresponding to the market situation and the information available, the PUs can apply either the models proposed in the thesis to determine the optimal strategy to maximize their profits.

1.5 Thesis Organization

The remaining part of this thesis is organized as follows.

Chapter 2 provides a literature review on dynamic spectrum access for cognitive radio networks, in particular the problems of spectrum sensing, resource allocation and spectrum trading.

Chapter 3 addresses challenges that arise when cooperative spectrum sensing is deployed in a cognitive radio environment where SUs have no *a priori* knowledge of PUs' band locations. A frequency boundary detection technique is proposed first to detect PUs' band locations. A dynamic band clustering algorithm to combine the primary band locations detected by different SUs is proposed next. Various information-fusion techniques are studied subsequently to make a collective decision about the presence or absence of PU. Furthermore, a novel decision-fusion technique that makes use of information-entropy is proposed. Performances of the proposed algorithm and techniques under different deployment configurations of cognitive radio network are analyzed.

Chapter 4 addresses the problem of power control and frequency channel allocation among SUs, who have different objectives in spectrum usage. Two different non-cooperative games namely *Interference Minimization* game and *Capacity Maximization* game are proposed. The numerical results are also provided to prove the superiority of the proposed games.

Chapter 5 addresses the problem of spectrum trading in a cognitive radio network which involves multiple PUs competing with each other to sell spectrum to a SU when the spectrum requirements for the PUs' services are uncertain. Here the spectrum trading scenario is modelled as a *Cournot game model* of competition by which the PUs set the size of spectrum to sell.

In **Chapter 6**, a spectrum trading problem is addressed for cognitive radio networks when both the spectrum requirements for the PUs' services and the spectrum demand of the SU are uncertain. Similar to **Chapter 5**, the spectrum trading scenario considered in **Chapter 6** involves multiple PUs and a single SU. However, instead of competing on

the size of spectrum offered to the SU, the PUs compete by submitting supply functions, which state the size of spectrum that they are willing to sell at each price, to the SU.

Finally, conclusions and contributions of this work are summarized in **Chapter 7**. In addition, some possible research areas which serve as future directions are also provided.

Chapter 2

Dynamic spectrum access techniques for cognitive radios: A review

2.1 Introduction

The concept of *cognitive radio* appears as one promising solution to the paradoxical situation of spectral scarcity for emerging radio systems against the high level of under-utilization of licensed spectrum. The key approach in cognitive radio is *dynamic spectrum access*, through which the *secondary users* (i.e., the unlicensed users) are allowed to dynamically access the frequency bands which are under-utilized by the *primary users* (i.e., the licensed users). Dynamic spectrum access techniques for cognitive radio can be categorized into two basic models namely *shared-use model* and *exclusive-use model*.

In *shared-use model*, secondary users (SUs) can opportunistically access the licensed spectrum if they do not cause any harmful interference to primary users (PUs). Two major functions that allow dynamic spectrum access in this model is *spectrum sensing* and *resource allocation*. The task of spectrum sensing function is to detect the presence of PUs through which to identify the unused spectrum (or the spectrum holes) within the operating range of SUs. If the location of licensed spectrum bands are clearly defined and known to SUs, a number of methods can be used to perform spectrum sensing, such as *energy detection*, *waveform based-sensing*, *cyclostationarity-based sensing*, and

matched-filtering. In all these methods, each SU would make its own decision on whether a PU is present or not. This is referred to as stand-alone (non-cooperative) approach. However, as decision is made based on the statistics of a single node, its accuracy is sensitive to physical impairments such as noise and shadowing propagation environment. This motivates the study of cooperative spectrum sensing, which exploits the geospatial diversity of multiple SUs to improve the accuracy of spectrum sensing. In cooperative spectrum sensing, the decision on the presence of PU is made by combining the sensing information from multiple SUs.

Exploiting information obtained from spectrum sensing function, resource allocation function aims to schedule the unused spectrum among SUs. Through this function, the radio resources including transmit power, frequency channels, time slots, channel access codes, etc., are allocated among SUs in such a way that the radio resources can be exploited optimally and each SU can acquire some radio resources for its transmission. Resource allocation function can be carried out by several techniques, such as formulating as a centralized optimization problem, or using graph coloring algorithm. However, due to the distributed characteristic of radio networks as well as the diversity in the objectives of SUs, these centralized techniques may not be suitable for several cases. The game theory appears as a perfect match for this problem.

Different from *shared-use model*, where the *technical perspective* of cognitive radio networks, in particular *spectrum sensing* and *resource allocation* to avoid the interference to PUs and among SUs, is concentrated, in *exclusive-use model*, the *economic perspective* of cognitive radio networks is emphasized. In this model, PUs are allowed to sell/grant the access rights of their licensed spectrum which are not fully utilized incessantly in all the geographical areas to SUs. *Spectrum trading*, which is defined as the process of selling and buying spectral resources including the frequency bands, transmit power, time slots, channel access codes, etc., plays an important role in this model. In general, there are

two commonly used models for the spectrum trading namely *pricing models* and *auction models*. In *auction models*, the primary and secondary users take part in an auction to sell/buy spectrum. Whereas, in *pricing models*, they compete by stating the price or the size of spectrum to sell/buy.

In this chapter, a literature review on the current works on *spectrum sensing*, *resource allocation*, and *spectrum trading* is presented. Section 2.2 discusses into details some of the existing approaches in spectrum sensing including both stand-alone (non-cooperative) spectrum sensing and cooperative spectrum sensing. Discussion on resource allocation is presented in Section 2.3. An overview on some basic concepts of game theory, which is used as the basis for further analysis in this area, is provided first. It is followed by a discussion on the applications of game theory and some special games such as super-modular games, potential games, and Stackleberg games for resource allocation in a cognitive radio environment. Furthermore, some of the most typical works on spectrum trading are introduced in Section 2.4. Finally, Section 2.5 ends this chapter with a summary.

2.2 Spectrum Sensing

In general, spectrum sensing methods are classified into stand-alone (non-cooperative) spectrum sensing and cooperative spectrum sensing. A spectrum sensing approach is referred to as non-cooperative spectrum sensing when the decision on the presence of PU is made by a stand-alone SU. In cooperative spectrum sensing approach, the decision on the presence of PU is made by combining the sensing information from multiple SUs who are geographically separated. A brief review on the most common spectrum sensing methods are provided in this section.

2.2.1 Non-Cooperative Spectrum Sensing

Among the non-cooperative scenario, *energy detection*, *waveform based-sensing*, *cyclostationarity-based sensing*, and *matched-filtering* are the most commonly used sensing methods. These methods are explained here.

2.2.1.1 Energy detection

Energy detection also known as radiometer or periodogram is the most popular spectrum sensing method [14–18]. The details of this sensing method is provided as follows.

Let the received signal at a SU is sampled at sampling frequency f_s . The discrete received signal at the SU which follows a binary hypothesis $\mathcal{H}_0$ (when the PU is inactive) and $\mathcal{H}_1$ (when the PU is active), can be expressed as:

$$\begin{aligned}\mathcal{H}_0 : \text{No Primary User} &\rightarrow y(n) = v(n) \\ \mathcal{H}_1 : \text{Primary User is Present} &\rightarrow y(n) = x(n) + v(n),\end{aligned}\tag{2.1}$$

where $x(n)$ is the primary signal and $v(n)$ is the additive white Gaussian noise (AWGN) samples. The test statistics of energy detection is defined as:

$$Y = \sum_{n=1}^M |y(n)|^2, \tag{2.2}$$

where M is the number of samples used to generate the test statistics. Consider the noise $v(n)$ to be circularly symmetric complex Gaussian (CSCG) noise and the primary signal $x(n)$ as also a CSCG signal. For a simplified analysis, the following assumptions are made:

- The noise $v(n)$ is independent and identically distributed (i.i.d) with zero-mean and variance σ_v^2 .
- The primary signal $x(n)$ is also i.i.d with zero-mean and variance σ_x^2 .

- The primary signal $x(n)$ and the noise $v(n)$ are independent.

Because of the above assumptions, the test statistics Y defined in equation (2.2) is a random variable whose probability density function (PDF) is a Chi-square distribution with $2M$ degrees of freedom.

$$Y \sim \begin{cases} \frac{\sigma_v^2}{2} \chi_{2M}^2, & \mathcal{H}_0 \\ \frac{\sigma_x^2 + \sigma_v^2}{2} \chi_{2M}^2, & \mathcal{H}_1. \end{cases} \quad (2.3)$$

Based on the PDF of the test statistics under $\mathcal{H}_0$ and $\mathcal{H}_1$, the probability of false alarm $P_F(\epsilon)$ and the probability of detection $P_D(\epsilon)$ over AWGN channels are respectively, given as:

$$\begin{aligned} P_F(\epsilon) &= Pr(Y > \epsilon | \mathcal{H}_0) = \int_{\epsilon}^{\infty} p_0(Y) dY \\ &= 1 - \Gamma\left(M, \frac{\epsilon}{\sigma_v^2}\right), \end{aligned} \quad (2.4)$$

$$\begin{aligned} P_D(\epsilon) &= Pr(Y > \epsilon | \mathcal{H}_1) = \int_{\epsilon}^{\infty} p_1(Y) dY \\ &= 1 - \Gamma\left(M, \frac{\epsilon}{\sigma_x^2 + \sigma_v^2}\right), \end{aligned} \quad (2.5)$$

where ϵ is the decision threshold, and $\Gamma(\cdot, \cdot)$ is the incomplete gamma function as given in [19].

One important problem with energy detection is to determine the threshold value. In typical deployment scenarios, the threshold value ϵ is determined from the desired detection performance level. The threshold value can also be selected to obtain a certain false alarm rate as in [20]. In [21], the authors tried to determine the threshold value in order to minimize the probability of error P_E , which is defined as the sum of probability of false alarm P_F and the probability of miss-detection $(1 - P_D)$:

$$P_E = P_F + (1 - P_D). \quad (2.6)$$

The performance of energy detection over various fading channels was investigated in [22]. The closed form expressions for the probability of detection under Rayleigh, Nakagami and Ricean fading channels were also derived in [22]. As the probability of false alarm is independent of SNR, its value under fading channels remains the same as in AWGN channel. In [23], the performance of energy detection under log-normal shadowing and Rayleigh fading were studied. In [24], the closed form expressions for the probability of energy detection of a received signal with various random (Rayleigh, Rice, Nakagami, and other) amplitudes were provided. In addition, the real performance of energy detection in multipath fading environments, where the noise power is uncertain, was examined in [25].

Energy detection is simple to implement and has low computational complexity along with a short sensing time. This method is quite generic as it does not require the knowledge of the PU signaling features, and hence can be deployed in many scenarios. However, energy detection is unable to discriminate interference from PUs and noise [26]. It also has poor performance at low SNR [15, 27]. As shown in [28] and [29], in the presence of noise uncertainty, this method fails to detect the presence of a PU signal if the SNR is below a certain value. In addition, it can be inferred from the results provided in [22, 23] that the performance of energy detection degrades in shadowing/fading environment. Furthermore, energy detection does not work properly for detecting spread spectrum signals [30, 31].

2.2.1.2 Waveform-based sensing

In waveform-based sensing [15, 32, 33], SUs must know the signal pattern of the PU. This sensing method is carried out by correlating the received signal with a known copy of itself [34]. Comparing with energy detection, waveform-based sensing outperforms in terms of reliability and sensing sensitivity [15]. It is also shown that a longer known signal pattern in the PU signal will result in a better sensing performance [15, 35].

Let the received signal at a SU be given as in equation (2.1). The waveform-based sensing metric is defined as:

$$Y = Re \left[\sum_{n=1}^M y(n)x^*(n) \right], \quad (2.7)$$

where M is the length of known pattern, and $*$ is the conjugate operation. Under hypothesis $\mathcal{H}_0$ (when the PU is absent), the sensing metric becomes:

$$Y = Re \left[\sum_{n=1}^M v(n)x^*(n) \right]. \quad (2.8)$$

Under hypothesis $\mathcal{H}_1$ (when the PU is present), the sensing metric is:

$$Y = \sum_{n=1}^M |x(n)|^2 + Re \left[\sum_{n=1}^M v(n)x^*(n) \right]. \quad (2.9)$$

The presence or absence of the PU signal can be determined by comparing the sensing metric with a certain threshold value ϵ_W which is determined by a certain rate of false alarm.

2.2.1.3 Cyclostationarity-based sensing

Cyclostationarity-based sensing [36–42] exploits the cyclostationarity features of the received signal, which are caused by the periodicity in the signal or in its statistics such as mean and auto-correlation, for detecting PU transmissions. The cyclic spectral density (CSD) function of the received signal at a SU given as in equation (2.1) can be determined as [43]:

$$S(f, \alpha) = \sum_{\tau=-\infty}^{\infty} R_y^\alpha(\tau) e^{-j2\pi f\tau}, \quad (2.10)$$

where α is the cyclic frequency, $R_y^\alpha(\tau)$ is the cyclic auto-correlation function (CAF). The CAF is defined as the auto-correlation function multiplied with the cyclic component and is given below:

$$R_y^\alpha(\tau) = E \left[y(n+\tau)y^*(n-\tau)e^{j2\pi\alpha n} \right]. \quad (2.11)$$

The CSD function reaches the peak values when the cyclic frequency is equal to the fundamental frequency of the primary signal $x(n)$ [34].

Cyclostationarity-based sensing is very useful to classify different types of modulated signals (such as BPSK, AM, FSK, MSK, QAM, PAM), because these modulated signals have significantly distinct CSDs even when they have overlapped power spectral densities [44]. Cyclostationarity-based methods were employed in [45] to detect the presence of the spectrum owner signal (GMSK modulated GSM signal) under severe interference from the SU signal (OFDM based WLAN signal). In addition, cyclostationarity-based sensing can also discriminate noise from the PU signals based on the fact that noise is uncorrelated wide-sense stationary (WSS), while modulated signals are cyclostationary with spectral correlation [46]. In [47], a cyclostationarity-based sensing algorithm was developed, and was applied to perform sensing in IEEE 802.22 wireless regional area network (WRAN). However, the disadvantages of the cyclostationarity-based sensing are the long sensing time, and computational and implementation complexities.

2.2.1.4 Matched-filtering

Matched-filtering [48] is a coherent detection method, which is only applicable when SUs have the full knowledge about the PU signal [49]. As a receiver is required for each PU signal, matched-filtering suffers from high implementation complexity when the sensing involves multiple types of primary signals. The performance of matched-filtering was studied in [50].

2.2.1.5 Other sensing methods

In [51, 52], a spectrum sensing method based on the sample covariance matrix of the received signal was proposed. In this method, the maximum eigenvalue of the sample covariance matrix is used as a test statistic. The threshold is set to meet a pre-defined probability of false alarm by using the random matrix theory [53]. As shown in [51], this

method outperforms energy detection for highly correlated signals and approaches energy detection for i.i.d signals. In [54], a blind sensing algorithm based on oversampling of the received signal or by employing multiple receive antennas was proposed. This algorithm requires no *a priori* knowledge of the channel and the noise power. Two signal statistics are computed in [54] by combining the linear prediction and QR decomposition of the oversampled received signal matrix. The ratio of these two statistics is used to determine whether the primary signal is present or not.

It is important to note that for all the spectrum sensing methods mentioned above, the frequency bands belonging to PUs are assumed to be fully known to all SUs. Under this assumption, the task of spectrum sensing is to identify possible transmission within those predefined PUs' bands. Different from these sensing methods, the present work considers an extreme case in which SUs are not equipped with any knowledge of PUs. Consequently, this necessitates SUs to conduct an additional task of primary band detection before the PU transmission detection can be carried out.

2.2.2 Cooperative Spectrum Sensing

In general, NCSS strategy performs well in situations where strong Line-of-Sight (LOS) paths exist between PUs and SUs with easily distinguishable signal transmissions by PUs. However, NCSS strategy performs less reliably in Non-LOS (NLOS) and high noise environment. In NLOS environment, the ability of the classic detection techniques to detect PU's presence weakens. Particularly, in the case of energy detection, reducing the threshold level might improve the detection level but at the same time it might increase the false alarm rate. The inherent spatial diversity in a network of wireless SUs motivates the use of Cooperative Spectrum Sensing (CSS) techniques.

In CSS approach, a node that collects and makes decision based on the sensing results from all the cooperating nodes is commonly referred to as the fusion center. The fusion

center selection can be based on centralized [55–58] or distributed strategy [15, 59]. In the centralized approach, only one SU is selected as the fusion center (FC). Since there is only one FC, a single decision is made, after which the decision will be shared with or distributed to all the cooperating SUs. With the distributed approach, multiple or entire SUs will act as FCs. These multiple FCs arrive at a common decision after a series of negotiation at the cost of potentially high overhead. As in any distributed-based algorithm, this overhead usually reduces the practical feasibility of the distributed FC option. However, issues related to the centralized or distributed approach is beyond the scope of this thesis and interested readers may refer to [60] for further details. Discussions presented in the rest of this section are generic and can be applied to both the centralized or distributed strategy.

The techniques for cooperative primary user detection can be broadly classified into two categories. The first category involves the use of pre-processed sensing data for decision making at the FC. One form of the pre-processed sensing data is the decision made by each SU. The first category is hence commonly referred to as the *decision-fusion* technique. The second category involves the use of raw sensing data for decision making at the FC. This class of technique is commonly referred to as the *information-fusion* technique. Some of the available CSS techniques are discussed in the following section. This discussion assumes the presence of I independent SUs in the CSS environment.

2.2.2.1 Cooperative Spectrum Sensing using Decision-Fusion

(i) *OR-Rule* [22, 61]

In a cooperative spectrum sensing system using OR-Rule, the PU is considered to be present if any of the SUs detects the presence of the PU. Let $P_{D,i}$ and $P_{F,i}$ are respectively the probability of detection and probability of false alarm of SU i in the CSS system ($i \in \{1, 2, \dots, I\}$). Assuming that all decisions made by SUs

are independent, the cooperative probability of detection $P_{D(OR)}$ and probability of false alarm $P_{F(OR)}$ using OR-Rule are given by [22]:

$$P_{D(OR)} = 1 - \prod_{i=1}^I (1 - P_{D,i}), \quad (2.12)$$

$$P_{F(OR)} = 1 - \prod_{i=1}^I (1 - P_{F,i}). \quad (2.13)$$

Even though the CSS system using OR-rule has a good probability of detection, this scheme usually suffers from a significant probability of false alarm.

(ii) *AND-Rule* [62, 63]

In a cooperative spectrum sensing system using AND-Rule, the PU is considered to be present if only all of the SUs detect the presence of the PU. Same as before, it is assumed that all the decisions made by SUs are independent. The cooperative probability of detection $P_{D(AND)}$ and probability of false alarm $P_{F(AND)}$ using AND-Rule are respectively given as [63]:

$$P_{D(AND)} = \prod_{i=1}^I P_{D,i}, \quad (2.14)$$

$$P_{F(AND)} = \prod_{i=1}^I P_{F,i}. \quad (2.15)$$

Unlike OR-Rule, AND-Rule has a low probability of detection and a low probability of false alarm.

(iii) *Majority-Rule* [61–63]

In a cooperative spectrum sensing system using Majority-Rule, the PU is considered to be present if half of the SUs or more, detect the presence of the PU. Assuming that SUs make their decisions independently and that $P_{D,1} = P_{D,2} = \dots = P_{D,I} = P_D$ and $P_{F,1} = P_{F,2} = \dots = P_{F,I} = P_F$, the probability of detection and probability

of false alarm of the CSS system using Majority-Rule are given by [63]:

$$P_{D(Majority)} = \sum_{i=0}^{I-\lceil \frac{I}{2} \rceil} \binom{I}{\lceil \frac{I}{2} \rceil + i} (1 - P_D)^{I-\lceil \frac{I}{2} \rceil - i} P_D^{\lceil \frac{I}{2} \rceil + i}, \quad (2.16)$$

$$P_{F(Majority)} = \sum_{i=0}^{I-\lceil \frac{I}{2} \rceil} \binom{I}{\lceil \frac{I}{2} \rceil + i} (1 - P_F)^{I-\lceil \frac{I}{2} \rceil - i} P_F^{\lceil \frac{I}{2} \rceil + i}, \quad (2.17)$$

where $\lceil \frac{I}{2} \rceil$ denotes the smallest integer not less than $\frac{I}{2}$, and $\binom{c}{k} = \frac{c!}{k!(c-k)!}$.

(iv) *k-of-I Rule* [64, 65]

In this technique, the PU is considered to be present if k or more SUs among I SUs in the cooperative spectrum sensing system detect the presence of the PU (k is an integer between 1 and I). The probability of detection and probability of false alarm of the CSS system using *k-of-I Rule* are given by [64]:

$$P_{D(k-of-I)} = \sum_{i=k}^I \binom{I}{i} (1 - P_D)^{I-i} P_D^i, \quad (2.18)$$

$$P_{F(k-of-I)} = \sum_{i=k}^I \binom{I}{i} (1 - P_F)^{I-i} P_F^i. \quad (2.19)$$

(v) *Bayesian Detection* [66, 67]

In this technique, the *a priori* probabilities and conditional densities of the observations for each hypothesis are given i.e., $P(\mathcal{H}_0)$ and $P(\mathcal{H}_1)$ are known. Fixed costs are assigned to each possible decision. Let C_{ij} be the cost on deciding hypothesis $\mathcal{H}_i$ when hypothesis $\mathcal{H}_j$ is present for $i, j \in \{0, 1\}$. Optimum decision rules are then derived to minimize the average cost i.e., the *Bayes risk*. The *Bayes risk* function is defined as [67]:

$$R_B = \sum_{i=0}^1 \sum_{j=0}^1 C_{ij} P(\mathcal{H}_j) P(\text{deciding } \mathcal{H}_i | \mathcal{H}_j \text{ is present}). \quad (2.20)$$

2.2.2.2 Cooperative Spectrum Sensing using Information-Fusion

In the CSS system using information-fusion, SUs send their original sensing information to the FC without any local processing and the decision is made at the FC by combining them appropriately. Let Y_i be the raw sensing information observed by SU i for $i \in \{1, 2, \dots, I\}$. Assuming that the communication channels between every SU and the FC are noise-free and distortionless, the cooperative decision statistic is defined as:

$$Y_{COOP} = \sum_{i=0}^I w_i Y_i, \quad (2.21)$$

where w_i is the weight corresponding to SU i and $w_i > 0$.

(i) *Optimal Soft Combining (OSC)*

Based on the Neyman-Pearson criterion [67], the Optimal Soft Combining (OSC) technique aims to maximize the detection probability for a given false alarm probability. If the test statistics Y_i of SU i is defined as in equation (2.2), the weight corresponding to SU i is given as [68]:

$$w_{i,OSC} = \frac{\gamma_i}{1 + \gamma_i}, \quad (2.22)$$

where γ_i is the instantaneous SNR at SU i .

(ii) *Maximal Ratio Combining (MRC)*

In the Maximal Ratio Combining (MRC) technique, each SU is assigned with a weight which is directly proportional to its instantaneous SNR γ_i :

$$w_{i,MRC} \propto \gamma_i. \quad (2.23)$$

(iii) *Equal Gain Combining (EGC)*

In the Equal Gain Combining (EGC) technique, each SU is assigned with the same weight:

$$w_{1,EGC} = w_{2,EGC} = \dots = w_{I,EGC} = 1. \quad (2.24)$$

The information-fusion approaches are widely recognized to be superior to the decision-fusion approaches in terms of detection accuracy. However, they suffer from the high overhead due to the need to exchange and/or distribute the raw sensing information. This necessitates the requirement of a fusion technique which can improve the accuracy of the decision-fusion techniques, while reducing the overhead problem of the information-fusion approaches. The present thesis proposes a novel decision-fusion technique that makes use of information-entropy to meet this requirement. More details of these algorithms will be discussed in **Chapter 3** of the thesis.

2.3 Resource Allocation

The unused frequency bands specified through spectrum sensing should be allocated among SUs in such a way that the spectral resources can be exploited optimally and each SU can get some spectral resources for its transmission. This problem is known as the resource allocation problem, and a review on the resource allocation approaches is presented in this section.

Traditionally, the problem of resource allocation can be modelled as an optimization problem in which a single-user objective or a system-wise objective is maximized or minimized in a centralized manner for a given set of constraints. In [69], a power control mechanism was proposed in order to maximize the total capacity of a cognitive radio network while allowing each SU to contract or expand its bandwidth. In [70], a three-step cross-layer optimization for sub-carrier, time slot and power allocations for OFDMA based cognitive radio system was studied. In [71], a near optimal scheme for jointly allocating channels and power levels among SUs who compete for a channel access in a CR network was studied. This scheme aims to maximize the total capacity of the system while respecting total power constraints per individual SU. Another approach that was commonly used for the resource allocation problem is the graph-coloring algorithm. In

[72], the authors proposed a graph coloring algorithm for the channel/slot assignment problem in which all possible collisions were avoided for a given network topology. Here, each channel/slot is mapped to a color and each vertex corresponds to a user and an algorithm is proposed to minimize the color usage where each vertex is assigned to one color. In [73], a graph-theoretical model was proposed to characterize the spectrum access problem under a number of different optimization functions. Rules for users to utilize available spectrum while avoiding interference with the neighbors was also devised in this work. The available bandwidth is divided into a set of spectrum bands, and these bands differ from each other in bandwidth and transmission range. The user can utilize any number of spectrum bands at a time. Unlike the traditional graph-coloring proposed in [72], in [73], the authors tried to maximize the utility of the system such as Max-Sum-Bandwidth, Max-Min-Bandwidth, and Max-Proportional-Fair. However, one of the limitations of the above resource allocation approaches is their complexity. As the optimization problems formulated by these approaches are NP-hard problems, they cannot be solved in polynomial time. In addition, in most of the cases in wireless networking, the users have to take actions/decisions in a distributed way. They may have mutual or possibly conflicting objectives, and hence they may act selfishly or even maliciously in order to achieve their objectives. The game theory, which provides a mathematical tool for modeling and analyzing the competition or cooperation among intelligent rational decision makers (i.e., players), appears as a perfect match for the resource allocation problem. The introduction about game theory and the applications of game theory for resource allocation are provided in the following sections.

2.3.1 Introduction to Game Theory

A game is made up of three basic components:

- The set of players.

- The strategies associated with the players: The actions are the alternatives available to each player. In dynamic or extensive form games, the set of actions might change over time.
- The utilities (payoffs) for the players: A function that governs the payoff that a player will be awarded for taking a certain move given the other players' moves.

In game theory, it is common to assume that the players always try to maximize their utilities. As a result, the probabilistic result of every action (strategy) can be perfectly calculated.

Based on the objectives of the players, games can be generally classified into cooperative and non-cooperative games. The definitions of these games are provided as follows.

Definition 2.1 *A **non-cooperative game** is the one in which the players make their decisions independently to maximize their own benefits.*

Definition 2.2 *A **cooperative game** is the one in which coalitions of players are formed, and the players aim to maximize common goals.*

For the scope of this thesis, only the non-cooperative games are considered. The non-cooperative games are further classified into static games and dynamic games.

Definition 2.3 *A **static game** is the one in which the interaction among the players occurs only once. All players make decisions (or select strategy) simultaneously without the knowledge of the strategies that are being chosen by other players [49].*

Definition 2.4 *A **dynamic game** is the one in which each move or action that a player takes at a given time will affect the moves of other players at a later time. In a dynamic game, the players interact in a certain pre-defined order (i.e., sequential game) or in a repeated fashion (i.e., repeated game).*

Based on the information available to a player when he makes decision about a specific move, dynamic games can be further divided into perfect information games and imperfect information games.

Definition 2.5 *A game is referred to as one of perfect information, if the players move alternatively and they are fully aware of all previous moves whenever they make a new move. Otherwise, the game is referred to as one of imperfect information.*

In addition, based on the information that the players have, games can be categorized into complete information games and incomplete information games.

Definition 2.6 *A game is referred to as one of complete information if all elements of the game including the sets of players, strategies and utility functions are common knowledge to all players. Otherwise, it is referred to as one of incomplete information.*

The non-cooperative static and dynamic games will be discussed further as follows.

2.3.1.1 Non-cooperative Static Games

In this section, the definition of static games in strategic form is provided first. Next the definitions of dominant strategy and dominated strategy are given. The Nash Equilibrium, which is considered as the game outcome is introduced subsequently. Finally, one very important concept in game theory, the Pareto optimality is provided.

Static Games in Strategic Form

Static games are usually represented in the strategic (normal) form. The definition of a static game in strategic form is given as follows.

Definition 2.7 *In strategic (normal) form, a game $G = (\mathcal{N}, \mathcal{S}, \{u_i\}_{i \in \mathcal{N}})$ has three elements. $\mathcal{N} = \{1, 2, \dots, N\}$ is a finite set of players (decision makers). $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \times \mathcal{S}_N$ is the joint set of the pure strategy spaces of all players where $\mathcal{S}_i$ is the pure strategy*

space associated with player i . The pure strategy space of the opponents of player i is represented as $\mathcal{S}_{-i} = \mathcal{S} \setminus \mathcal{S}_i$. $\{u_i\}_{i \in \mathcal{N}} = \{u_1, u_2, \dots, u_N\}$ is the set of utility functions that the players want to maximize. The utility function u_i measures the game outcome for player i given the particular strategy s_i ($s_i \in \mathcal{S}_i$) chosen by player i and the particular strategies $\mathbf{s}_{-i}$ chosen by all other players in the game (i.e., $\mathbf{s}_{-i} = \{s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N\}$). The set of chosen strategies constitutes a strategy profile $\mathbf{s} = \{s_i, \mathbf{s}_{-i}\}$.

In the strategy space of a player, there may be strategies which are always better than the other strategies for the player's interest. These strategies are known as dominant strategies, which are defined below.

Definition 2.8 *A dominant strategy is defined as one which earns a player a larger payoff than any other strategy no matter how that player's opponents may do.*

In contrast to dominant strategies, dominated strategies are defined as follows.

Definition 2.9 *A dominated strategy is defined as one which earns a player a smaller payoff than some other strategies regardless of what the player's opponents may do.*

It is important to note that strategies can be classified into *pure strategies* and *mixed strategies*. While a *pure strategy* defines a specific move (or action) that a player will make for any situation he could face in a game, a *mixed strategy* of a player is a probability distribution over the player's *pure strategies*. Only the pure strategy is within the scope of this thesis and will be discussed later in this chapter.

Nash equilibrium and Pareto optimality

The most commonly used solution concept in game theory to predict the outcome of a game is the Nash equilibrium. A Nash equilibrium is a strategy profile where no player can increase his utility by unilaterally deviating [49]. The concept of Nash equilibrium is formally defined as follows.

Definition 2.10 *Nash equilibrium (NE)*: *The pure strategy profile $\mathbf{s}^*$ constitutes a Nash equilibrium if, for each player i ,*

$$u_i(s_i^*, \mathbf{s}_{-i}^*) \geq u_i(s_i, \mathbf{s}_{-i}^*), \forall s_i \in \mathcal{S}_i.$$

A Nash equilibrium is said to be strict if:

$$u_i(s_i^*, \mathbf{s}_{-i}^*) > u_i(s_i, \mathbf{s}_{-i}^*), \forall s_i \in \mathcal{S}_i.$$

Alternatively, a Nash equilibrium is a strategy profile such that each player's strategy is a best response to the other players' strategies. The concept of best response is provided as follows.

Definition 2.11 *Best response*: *The best response of player i to a set of strategies $\mathbf{s}_{-i}$ chosen by other players is defined as:*

$$BR_i(\mathbf{s}_{-i}) = \arg \max_{s_i \in \mathcal{S}_i} u_i(s_i, \mathbf{s}_{-i}). \quad (2.25)$$

As a result, the strategy profile $\mathbf{s}^*$ is a Nash equilibrium if:

$$s_i^* = BR_i(\mathbf{s}_{-i}^*), \quad \forall i \in \mathcal{N}. \quad (2.26)$$

As a game may have multiple Nash equilibria, the optimal equilibrium should be selected among these equilibria. The Pareto-optimality is an important criterion for specifying the optimal Nash equilibrium.

Definition 2.12 *Pareto-optimality*: *A strategy profile is Pareto-optimal (or Pareto-efficient) if the payoff of any player cannot be increased without reducing the payoff of at least one other player [74].*

2.3.1.2 Non-cooperative Dynamic Games

In this section, two subsets of dynamic games including *sequential games* and *repeated games* are discussed. The definition of *sequential games* is provided as follows.

Definition 2.13 A ***sequential game*** is the one in which players make their moves in a certain predetermined order and at least some players are aware of the moves of their previous players.

It is important to note that every game can be described by two ways: the strategic form and the extensive form, which are nearly equivalent. While static games are usually represented in strategic form, it is more convenient to represent sequential games in extensive form. In extensive form, a game is represented as a tree where the root (the initial node) of the tree is the starting point of the game. Each level of the tree is defined as a stage of the game. The nodes of the tree are the decision points at which players can take actions. At a given node, only one player can take the action. The branches coming out of a node represent the possible actions which may be taken at that node. Each terminal node is a potential outcome of the game and it is labeled with the payoffs that the players would receive if the game terminates at that node [74].

Conventionally, a sequential game can be solved by using the process of *backward induction* whose details are provided in [74–76]. This technique however, can be neither applied to games of imperfect, incomplete information nor games of infinite length [77]. An alternative way to solve a sequential game is to use the concept of *subgame perfect equilibrium* (SPE) whose details are given in [49, 78].

Next, the definition of *repeated games*, another subset of dynamic games, is provided as follow.

Definition 2.14 A ***repeated game*** is the one in which players interact by playing a similar game many number of times. In each interaction (which is called a stage), the players make their moves simultaneously.

A repeated game is referred to as a *finitely-repeated game* if the number of stages is finite [74]. Otherwise, it is referred to as an *infinitely-repeated game*. The payoff of player i ($i \in \mathcal{N}$) in a T -stage repeated game with a discount factor δ ($0 < \delta < 1$) is given by

$$u_i = \sum_{t=1}^T \delta^{t-1} u_i^t, \quad (2.27)$$

where T is the number of stages, u_i^t denotes the payoff that player i receives in stage t . One very important notion in repeated games is the *Folk theorem* [79]. In game theory context, the term *Folk theorem* refers to any theorem establishing feasible payoffs for repeated games. Each *Folk theorem*, which is proved by different authors, considers a subclass of games and identifies a set of payoffs that are feasible under some equilibrium strategy profile. The details of these theorems are provided in [79]. In addition, there are some repeated games where players may try to maximize their payoffs only for the next stage. These repeated games are known as *myopic games*.

2.3.2 Resource Allocation using Game Theory

The applications of game theory for resource allocation problems in cognitive radio networks have been widely studied. In [80], a rate maximization power allocation game in frequency-selective Gaussian interference channel using iterative water-filling algorithm (IWFA) was proposed. In [81], a linear convergence of the distributed IWFA for arbitrary symmetric interference environment and for certain asymmetric channel conditions with any number of users was established. In [82], a sequential IWFA, in which the players make their moves one by one in order to maximize their own information rates, was proposed for the power allocation problem. Due to its sequential updating strategy, the sequential IWFA may suffer from slow convergence when the number of users is large. In [83, 84], a simultaneous IWFA, in which users make their moves simultaneously, was proposed to overcome the slow convergence problem of the sequential IWFA. In [85],

a self-enforcing non-cooperative game theoretic framework for power allocation over an unlicensed band was studied. The behaviors of players in one shot game (static game) and repeated game were studied in this paper. In [86], a competitive channel allocation among wireless users, each consisting of multiple radio transmitters, was studied. In [87], a time domain spectrum allocation was studied using game theory. Game theory was also applied in [88–90] for the resource allocation problem in MIMO cognitive radio networks. In addition, evolutionary games and genetic algorithms [91] were used in [92–95] to study the dynamic behaviors of wireless users. Furthermore, the applications of special games such as super-modular games, potential games, and Stackleberg games for resource allocation problems were also studied extensively. A brief review on these special games and their applications are discussed below.

2.3.2.1 Super-modular Game

Super-modular game is a special class of games that possesses several remarkable properties like the existence of pure strategy Nash equilibrium, dominance solvability, identical bounds on joint strategy space [96–98]. In [97], the application of super-modular games to distributed power control was examined. This paper showed that the distributed power control scenarios considered by Yates in [99] can be modelled using super-modular game. Other super-modular power control algorithms were also discussed in [98]. In addition, a super-modular game was formulated in [100] to model the power and channel allocation problem among multiple SUs.

Definition 2.15 *The game $G = (\mathcal{N}, \mathcal{S}, \{u_i\}_{i \in \mathcal{N}})$ is a **super-modular game** if, $\forall i \in \mathcal{N}$ [49]:*

- $\mathcal{S}_i$ is a compact subset of $\mathbb{R}$.
- u_i is upper semi-continuous in $(s_i, \mathbf{s}_{-i})$.

- u_i has increasing differences in $(s_i, \mathbf{s}_{-i})$.

In a simpler identification, a game can be identified as a super-modular game if the strategy space is compact and the following relationships are satisfied:

$$\frac{\partial^2 u_i(\mathbf{s})}{\partial s_i \partial s_j} \geq 0, \quad \forall i, j \in \mathcal{N}, j \neq i. \quad (2.28)$$

2.3.2.2 Potential Game

Potential game [98, 101–103] is a special type of games, which is characterized by a potential function $P : \mathcal{S} \rightarrow \mathbb{R}$ whose change in value is related to the change in the utility function of any player if he unilaterally deviates in his strategy. There are various kinds of potential games. Among them, exact potential games and ordinal potential games are the most well-known games.

Definition 2.16 A game $G = (\mathcal{N}, \mathcal{S}, \{u_i\}_{i \in \mathcal{N}})$ is an **exact potential game (EPG)** if there exists an exact potential function $P : \mathcal{S} \rightarrow \mathbb{R}$ which satisfies [78]:

$$u_i(s_i, \mathbf{s}_{-i}) - u_i(s'_i, \mathbf{s}_{-i}) = P(s_i, \mathbf{s}_{-i}) - P(s'_i, \mathbf{s}_{-i}), \quad \forall i \in \mathcal{N}, \forall \mathbf{s} \in \mathcal{S}. \quad (2.29)$$

In [102], the necessary and sufficient condition for a game to be an exact potential game was given and this condition is stated as follows:

$$\frac{\partial^2 u_i(\mathbf{s})}{\partial s_i \partial s_j} = \frac{\partial^2 u_j(\mathbf{s})}{\partial s_j \partial s_i}, \quad \forall i, j \in \mathcal{N}, \mathbf{s} \in \mathcal{S}. \quad (2.30)$$

Definition 2.17 A game $G = (\mathcal{N}, \mathcal{S}, \{u_i\}_{i \in \mathcal{N}})$ is an **ordinal potential game (OPG)** if there exists an ordinal potential function $P : \mathcal{S} \rightarrow \mathbb{R}$ which satisfies [78]:

$$u_i(s_i, \mathbf{s}_{-i}) > u_i(s'_i, \mathbf{s}_{-i}) \Leftrightarrow P(s_i, \mathbf{s}_{-i}) > P(s'_i, \mathbf{s}_{-i}), \quad \forall i \in \mathcal{N}, \forall \mathbf{s} \in \mathcal{S}. \quad (2.31)$$

The condition for the existence and uniqueness of Nash Equilibrium of potential games was provided in [104] and this condition is described by the following theorem.

Theorem 2.1 *Let $G = (\mathcal{N}, \mathcal{S}, \{u_i\}_{i \in \mathcal{N}})$ be a potential game, with a potential function P , and $\mathcal{P}_{max}$ denotes the set of maxima of P on $\mathcal{S}$ (assumed non-empty). Then, G admits at least one NE. In addition, if $\mathcal{S}$ is a compact and convex set, and P is a continuously differentiable function on the interior of $\mathcal{S}$ and strictly concave on $\mathcal{S}$, then the NE of G is unique.*

It can be inferred from the above theorem that:

- Every finite potential game admits at least one NE [102].
- For infinite potential games, sufficient conditions for the existence of NE (non-emptiness of $\mathcal{P}_{max}$) are:
 - $\mathcal{S}$ is a compact set.
 - P is upper semi-continuous on $\mathcal{S}$.

Due to its properties, potential games have been employed widely in the resource allocation problem. In [103], the potential games were employed to formulate the power control, call admission control and interference avoidance problems for cognitive radio networks. In [105], a distributed adaptive channel allocation problem for cognitive radio networks using a potential game framework was studied. The considered cognitive radio network consists of a set of N transmitting-receiving pairs of nodes competing for a single channel among K available channels. The utility function for each player is defined as the total interference caused by other players to this player and by this player to other players if they access the same channel. In [106], a potential game framework was proposed to perform power allocation in an OFDM cognitive radio network consisting of a primary system and a secondary system. Unlike [105], the cognitive terminals compete for K available subcarriers in [106]. Two constraints are imposed on each subcarrier to respect the target BER of the secondary system as well as to protect the primary system. The

utility function for each player is defined as the sum of capacity values acquired over K subcarriers after subtracting the interference to the PU. The potential games were also studied in [104, 107] for the power control problem in CDMA system and in [108] to model an interference avoidance scheme. In addition, potential game frameworks encompassing both power allocation and channel selection in cognitive radio networks were studied in [109, 110].

2.3.2.3 Stackleberg game

Stackleberg games are usually used to model the resource allocation problem in cognitive radio networks where SUs are allowed to access the frequency bands licensed to a PU as long as the minimum SINR requirement is ensured for that PU. The definition of a Stackleberg game is given as follows.

Definition 2.18 *A **Stackleberg game** is a strategic game in which one player acts as a leader and other players act as followers. The leader declares his strategy before followers and then enforces it.*

In cognitive radio networks, the PU, who has the priority over SUs, is usually assigned as a Stackleberg leader, and SUs are assigned as the followers. In [109], a Stackleberg game model was proposed for the power control and channel allocation problem in which the leader (the PU) charges the followers (SUs) a virtual price for accessing its licensed frequency bands. For a given virtual price, the followers compete in an exact potential game to maximize their utilities. However, there is a chance that the virtual price is too high for SUs to access the channels. In [111], a Stackleberg game was proposed for the power allocation problem in which the PU (the leader) announces prices for the available tones through the maximization of its own utility function. Based on the announced prices, SUs (the followers) compete for the available bandwidth to maximize their own utilities while guaranteeing that their interference to the PU does not exceed

an interference temperature constraint. The other works for power allocation problems using Stackleberg games are [112–114]. The Stackleberg games were also applied in [115–119] to control the behavior of radios. In addition, a multi-leader multi-follower game, a general case of Stackelberg games, was studied in [120] for spectrum sharing problem.

In the present thesis, the problem power control and channel allocation in a cognitive radio network where N SUs compete to access K frequency channels is considered. The diverse QoS requirements of different radio systems are considered to set their respective targets in the game formulation. We have considered two typical radio systems corresponding to voice and data in this context. Voice radio systems always try to maximize their transmission rates, whereas data radio system always try to maximize the SINRs. This necessitates the development of two different games to reflect the targets of data and voice radio systems. In this context, the present work proposes two different non-cooperative games, namely *Interference Minimization* game and *Capacity Maximization* game. More details of these games will be discussed in **Chapter 4** of the thesis.

2.4 Spectrum Trading

The spectrum sensing and resource allocation concentrate on the *technical perspective* of cognitive radio networks. The spectrum can be traded between PUs and SUs and can be considered as the *economic perspective* of cognitive radio networks. In this section, a survey on some of the most typical works on the spectrum trading problem is discussed. There are two commonly used models to formulate the spectrum trading problem namely *pricing model* and *auction model*. The details of these models are provided as follows.

2.4.1 Pricing Models

In spectrum trading problem using pricing models, PUs play the role of sellers, and SUs play the role of the buyers. Based on structure, the pricing models are generally divided into *single seller* and *multiple seller* models.

In a *single seller* model, the seller has the right to change the quantity (or the size) and the price of the radio resources to maximize his profit. This model hence is also known as *monopoly* model. A monopoly spectrum trading model was proposed in [121] for a cognitive radio network which consists of a single PU and a single SU. A pricing model was also studied in [122] for uplink transmit power control.

Different from *single seller* models, in *multiple seller* models, multiple sellers compete to sell radio resources to the market. These models are usually referred to as *oligopoly* models. A spectrum trading model which consists of multiple competing PUs and a single SU was proposed in [8, 123, 124]. This model was formulated by using the *Bertrand game model* [77], whereby PUs compete by setting the price of spectral resources offered to the SU. In order to prevent the trivial solution of a Bertrand game, the spectral resources offered by different PUs are assumed to be different. Based on the set of chosen prices, the SU will then determine the size of spectrum that he wants to buy from each PU. A commonly used quadratic utility function was adopted in these works as the utility function for the SU. In [125], a spectrum trading problem consisting of multiple competing PUs and a single SU was studied using a *Cournot game model* [77]. In contrast to a Bertrand game model, in a Cournot game model, PUs compete by setting the size of spectrum offered to the SU rather than prices. In [9, 126], a spectrum trading model consisting of multiple buyers (SUs) who compete for spectrum offered by a single seller (PU) was proposed. This problem was formulated as a Cournot game model, however the players in the game are buyers instead of sellers in the original setting. The major limitation of this approach is the absence of discussion on the spectrum availability of the PU i.e., the PU is assumed to have an unlimited spectrum available to sell. A spectrum trading problem similar to that proposed in [9, 126] was also studied in [127] by taking into consideration the difference in the spectrum.

Furthermore, the problem of spectrum trading which involves multiple PUs selling spectrum opportunities to multiple SUs was proposed in [128]. In [128], PUs compete

with each other in terms of the size of spectrum to be shared and the spectrum price. Given the prices and the size of spectrum offered by PUs, SUs play in an evolutionary game to determine PUs to buy the spectrum. One of the limitations of [128] is that SUs cannot specify the amount of spectral resources to buy. The SUs, who buy spectrum opportunities from the same PU, get the same size of spectrum and have to pay the same price.

2.4.2 Auction Models

Beside *pricing models*, the problem of spectrum trading was also studied widely by using *auction theory* [11, 129–131]. Based on the characteristics of the participants, auctions can be classified into *supply auctions*, *demand auctions*, and *double auctions*. In *supply auctions*, multiple potential sellers offer their goods in response to requests from a buyer. In *demand auctions*, multiple potential buyers bid for goods being sold by a seller. As there is only one group of participants (either sellers or buyers), *supply auctions* and *demand auctions* can be referred to as *single-sided auctions*. Unlike *single-sided auctions*, in *double auctions*, multiple potential buyers bid for goods which are offered by multiple potential sellers.

Single-sided auction

Single-sided auctions include *supply auctions* and *demand auctions*. In a *demand auction*, the bidders (the potential buyers) submit their bids to an auctioneer (a seller) who then determines the one who wins the auction. In a *supply auction*, the auctioning process is the inverse of that in a *demand auction*.

Traditionally, only one unit of goods is offered for sale in an auction; this kind of auction is referred to as a *single-unit auction*. There are many types of *single-unit auction*, and some typical types are introduced as follows [10]:

- (i) *Ascending-bid auction*, also known as English auction. It is an open auction, in which the bidders can observe the bids from other bidders. Starting with a minimum bid price, the bidders compete against each others by bidding higher than the previous bid. The auction terminates when no bidder is willing to bid further. The bidder with highest bid wins the auction and pays the value of his bid.
- (ii) *Descending-bid auction*, also known as Dutch auction. In this kind of auction, starting with a high asking price, the auctioneer will gradually lower the price. The first bidder who accepts the price wins the auction.
- (iii) *First price sealed-bid auction*. In this kind of auction, the bidders simultaneously submit sealed-bids to the auctioneer. The highest bidder wins the auction and pays the value of his bid.
- (iv) *Second price sealed-bid auction*. Similar to the first price sealed-bid auction, but in this kind of auction, the highest bidder wins the auction and pays the second highest bid rather than his bid.

In contrast to a *single-unit auction*, in a *multi-unit auction*, multiple identical goods are sold at the same time. Multi-unit auctions can be further divided into uniform price auctions and discriminator price auctions [10].

- (i) *Uniform price auction*. In a uniform price auction, all winning bidders pay the same per unit price regardless of their actual bids.
- (ii) *Discriminator price auction*. In a discriminator price auction, the winning bidders pay different per unit prices depending on their actual bids.

A single-sided auction-based algorithm was proposed in [130] to allow users to fairly compete for a wireless fading channel. A second-price auction mechanism is employed in

[130] whereby user bids for the channel during each time slot, based on the fading state of the channel. The user that makes the highest bid wins use of the channel by paying the second highest bid. In [131], a credit token-based rental protocol for dynamic spectrum access using auctioning was proposed to support negotiations between the primary and secondary users. In [132], a competitive spectrum allocation scheme based on the auction theory, which allows each PU to take precautions to avoid the degradation of its own performance, was proposed. The method is proposed for a simple network with one PU and multiple SUs, and then extended to a network with multiple PUs. For the multiple PU case, SUs submit their bids to the *optimal* PU for the channel that SUs can achieve the highest spectral efficiency in transmission. However, a SU may earn a lower revenue by choosing its optimal PU if many SUs bid the same PU. In [133], a real time multi-unit spectrum auction framework which allocates spectrum among multiple wireless users under interference constraints was proposed. The impact of two pricing models (including a simple uniform pricing model and a discriminator pricing model) and bidding behaviors on spectrum utilization and revenue was studied in [133]. A truthful and computationally efficient multi-unit spectrum auction model was proposed in [134] based on the classical Vickrey-Clarke-Groves (VCG) auction [135, 136]. In [137], a truthful spectrum auction, which aims to maximize the expected revenue of the PU, was studied. In addition, auction mechanisms were also studied in [138–140] for frequency band trading problem, and in [141, 142] for transmit power trading problem.

Double auction

In a double auction, the potential buyers and the potential sellers simultaneously submit their bids and their asking prices to an auctioneer. After that, the auctioneer determines a price to clear the market: all the sellers who ask less than this price and all the buyers who bid more than this price will sell and buy the goods at this price. Similar to

single-sided auctions, double auctions can be further classified into single-unit [143] and multi-unit [144] double auctions.

A double auction based spectrum trading scheme which resolves the spectrum access between multiple primary and secondary networks was studied in [145]. In [146], a truthful double spectrum auction (TRUST) framework, in which each seller (spectrum owners) offers one distinct channel and each buyer (spectrum users) requests only one channel, was proposed. The spectrum reusability was also considered in this paper to maximize the spectrum utilization. A multi-unit double auction mechanism was proposed in [147] for a secondary spectrum market where SUs play the roles of both sellers and buyers. A spectrum trading model with multiple primary and secondary users through a spectrum broker, who plays a similar role of an auctioneer, was also studied in [148]. In [148], the unit price of spectrum is initially announced by the spectrum broker. PUs bid for the amount of spectrum that they are willing to sell, and SUs bid for the amount that they are willing to buy at that price. However, the market supply and demand may not be equal. In order to solve this problem, the authors applied a proportional rule, whereby a user is allocated a volume proportional to its bidding strategy. As shown in [148], the traded volume is dependent on the price announced by the broker and it is very difficult to directly get the optimal price to maximize the traded volume. In [149], a joint spectrum bidding and pricing model was proposed for spectrum trading in an IEEE802.22 network which involves multiple TV broadcasters, IEEE802.22 service providers and WRAN users. In this model, the TV broadcasters offer available TV bands to WRAN service providers, which again sell spectrum to WRAN users. As there are multiple buyers and sellers, a double auction model was employed to model the spectrum trading among TV broadcasters and WRAN service providers. Once the service providers obtain spectrum, they compete with each other in terms of price charged to WRAN users. The WRAN service providers, hence must competitively select their bidding and pricing strategies to maximize their profits.

Most of the studies in this direction did not consider the fluctuation on the spectrum requirements for PUs' services and the variation on the spectrum demands of SUs. However, due to the volatile characteristics of radio networks, the spectrum requirements for PUs' services and the spectrum demands of SUs are subject to change over time. In this context, the present thesis investigates a spectrum trading scenario, in which the spectrum requirements for PUs' services are uncertain. Subsequently, a more practical scenario, in which both the spectrum requirements for the primary services and the spectrum demands of SUs are uncertain, is studied. More details of these studies will be provided in **Chapters 5** and **6** of the thesis.

2.5 Summary

A concise review of spectrum sensing, resource allocation and spectrum trading in cognitive radio networks is presented in this chapter.

- Starting with spectrum sensing, various spectrum sensing methods, which are generally classified into stand-alone (non-cooperative) spectrum sensing and cooperative spectrum sensing, are presented. A brief introduction on non-cooperative spectrum sensing methods such as energy detection, waveform based sensing, cyclostationarity based sensing, and matched-filtering is provided. As decision is made based on the statistics of a single radio, non-cooperative spectrum sensing systems often suffer from noise uncertainty, fading, shadowing and hidden primary node problems. By combining multiple SUs from different geographic locations, cooperative spectrum sensing can mitigate these problems. An overview on some of the available cooperative spectrum sensing techniques using decision-fusion and information-fusion is also presented.
- Various resource allocation approaches, in particular resource allocation approaches using game theory, are discussed in detail. The basic concepts of game theory

are introduced within the context of resource allocation for cognitive radios. In general, games are classified into cooperative and non-cooperative games. The present thesis is focussing on non-cooperative games which are further divided into static games and dynamic games, and their details are discussed. In addition, several applications of game theory for strategic allocation of resources in cognitive radio networks are provided.

- Finally, some typical works on spectrum trading problem are also reviewed. Details of two commonly used approaches to model the spectrum trading problem in cognitive radio networks, namely *pricing models* and *auction models* are provided.

Chapter 3

Cooperative spectrum sensing with dynamic frequency boundary detection and information entropy fusion

3.1 Introduction

Spectrum sensing is one of the most important functions of cognitive radio that enable dynamic spectrum access. As discussed in **Chapter 2**, cooperative spectrum sensing (CSS) tries to smooth out noise uncertainties and exploit geospatial diversity of multiple SUs to enhance the robustness of sensing accuracy. One critical issue in CSS is the detection of PUs' bands. Most of the available literature assume static locations for PUs' bands and each SU is equipped with perfect knowledge of these locations. However, such an assumption is idealistic and puts a limit on the efficient spectral usage of SUs. It is in this context, the present research assumes the absence of *a priori* knowledge of PUs' band locations for SUs. The proposed system uses a non-cooperative spectrum sensing technique to perform the task of primary band detection. This task is conducted through a module named *Frequency Boundary Detector*. It is followed by a discussion on two specific challenges, that arise when CSS is deployed in CR environment where SUs do not have *a priori* knowledge of PUs' band locations. The first challenge is

to make a collective decision about the primary band locations based on the results of individual SUs with the aim of minimizing false primary bands. In this context, a dynamic band clustering (DBC) algorithm that uses K-means clustering technique is proposed. The second challenge is to derive a cooperative decision on the presence of PU based on the detection results of individual SUs. The conventional *information-fusion* approaches using Equal Gain Combining and Maximal Ratio Combining are studied in this context. While the *information-fusion* approach yields the most reliable PU detection performance, the high number of bits required to represent the information (e.g. signal energy) makes such an approach practically less appealing. The *decision-fusion* approach, which offers reduced overhead compared to the *information-fusion* approach, is a tempting alternative. It is in this context, this chapter proposes a decision-fusion technique that makes use of information-entropy (which measures the uncertainty of the information received by a SU). The proposed approach aims to minimize the contribution of the highly uncertain SUs and maximize the contribution of more certain SUs in the cooperative decision making process.

The rest of this chapter is organized as follows: In Section 3.2, a non-cooperative spectrum sensing technique is discussed. This section presents a Frequency Boundary Detection approach based on Canny Edge Detection technique. A quick review on Primary User Transmission Detector using classic energy detection technique is also provided in this section. A novel cooperative spectrum sensing method is discussed in Section 3.3. In this section, a Dynamic Band Clustering algorithm is presented. In addition, various *information-fusion* and *decision-fusion* approaches for the proposed distributed spectrum sensing system are also provided in this section. In Section 3.4, the results of the simulation studies are discussed. Finally, Section 3.5 summarizes this chapter.

3.2 Non-Cooperative Spectrum Sensing

In CR paradigm, a network of SUs does not have a static frequency allocated for its use. Instead, a much wider band (compared to its required operating bandwidth) known as *spectrum pool*, is defined as the possible operating frequency range. For example, in IEEE802.22, the operating bandwidth is defined as 6, 7 or 8 MHz while the spectrum pool is defined from 54 – 862 MHz. In this pioneering standard for CR paradigm, the uncertainty factor is minimized as the spectrum pool mainly belongs to TV bands. The primary band characteristics (width and location) are hence fully known and static in nature. However, the deployment environment in future CR standards can be expected to be loosely constrained where the spectrum pool could be much larger and the primary band characteristics are unknown to the SUs. This necessitates the additional task of primary band detection before carrying out primary user transmission detection. This section presents a review of a non-cooperative spectrum sensing (NCSS) technique that can be deployed in such a challenging environment. The NCSS technique comprises of two modules, namely the *Frequency Boundary Detector* and *Primary User Transmission Detector*.

3.2.1 Frequency Boundary Detection Technique

The goal of the frequency boundary detection (FBD) process is to break down the spectrum pool into a train of non-overlapping spectrum bands. Similar to [150], it is assumed that the power spectral characteristic is smooth within each band but exhibits distinct characteristic difference between adjacent bands. This assumption comes from the fact that in the realistic scenario, in order to minimize the inter-symbol-interference (ISI) problem, the transmitted signal is often filtered with the pulse shaping filter such as boxcar filter, raised-root cosine filter or square-root-raised-cosine filter. After filtering, the power spectral density of the signal will be nearly smooth in the transmission band.

A wavelet-based technique, which is commonly used to analyze the irregularities in time series and images [151–154], was proposed in [150] for FBD problem. However, in [150], the power spectral density (PSD) within each band is assumed to be smooth and almost flat, and the edge discontinuities are distinctly sharp. In wireless communication system, the use of rectangular filter would be capable of producing such sharp transition at band edges. However, rectangular filter is characterized by infinite impulse response (IIR) that is practically not implementable. Finite impulse response (FIR) filters such as raised-root cosine filter (RRC) are commonly used instead, in which the steepness of the edges are defined by the filter's roll-off factor. The FBD technique proposed in this chapter can be employed in such cases. Detailed operation of the proposed technique is presented below. Let the information symbol transmitted by the PU be $x(n)$, where $x(n)$ is defined as:

$$x(n) = \begin{cases} 0 & \text{for } n \leq 0 \\ x(n) & \text{for } n > 0. \end{cases} \quad (3.1)$$

The signal received at SU i can be expressed as:

$$y_i(n) = h_i x(n) + v_i(n), \quad (3.2)$$

where h_i is the channel between SU i and the PU, and $v_i(n)$ corresponds to the additive white Gaussian noise (AWGN) component at SU i with the variance denoted by $\sigma_{i,v}^2$.

The auto-correlation of the received signal r_i can be defined as:

$$r_i(n) = \frac{1}{N} \sum_{n_1=n}^{N-1} y_i(n_1) y_i^*(n_1 - n), \quad \text{for } n = 0 \text{ to } N - 1, \quad (3.3)$$

where N is the number of time-domain samples that are used to generate the auto-correlation value. The corresponding PSD of the received signal ($\phi_i(n)$) can be expressed as:

$$\phi_i(n) = \sum_{n_1=-(N-1)}^{N-1} r(n_1) e^{-\frac{j2\pi n n_1}{N}}. \quad (3.4)$$

An innovative FBD technique based on *Canny Edge Detection* technique (a technique popularly used to solve edge detection problem in image processing [155]) is proposed to detect the band edge within the wireless spectrum pool. Apart from Canny edge detector, there are several edge detection techniques such Sobel edge detector [156], PreWitt edge detector [157], Robert edge detector [158] or Laplacian of Gaussian edge detector [159]. However, from the simulation studies provided in [160], Canny edge detector has better performance than all other techniques under almost all scenarios, especially under high level of noise. It is because of this reason, this method is employed in this thesis. The proposed *Canny Frequency Boundary Detector* (CFBD) comprises of four sub-modules as shown in Figure 3.1. As shown in the figure, the first step in CFBD algorithm is to perform smoothing on the estimated PSD. The purpose of the smoothing operation is to reduce the fluctuations in PSD characteristics caused by the noise. After the PSD of the signal has been smoothened out, the first order derivative of the smoothened signal is computed. The presence of frequency boundary is characterized by a spike (or irregularity) in the first order derivative product. Therefore, the next step is to identify such irregularities and generate the Local Maxima set. *Thresholding* is performed to remove spikes attributed to noise fluctuations. More details of the algorithm are summarized below.

Figure 3.1: Processing modules for the proposed frequency boundary detection technique

Canny Frequency Boundary Detector (CFBD) Algorithm

- (i) Perform smoothing to the estimated PSD $\bar{\phi}_i(n)$

$$\bar{z}_i(n) = \bar{\phi}_i(n) \otimes \Psi(n) = \sum_{n_1=1}^N \bar{\phi}_i(n_1) \Psi(n - n_1), \quad (3.5)$$

where $\Psi(n)$ is the impulse response of the smoothing filter, and $\otimes$ is the convolution operation.

- (ii) Compute the first order derivative of $\bar{z}_i(n)$

$$\begin{aligned}\bar{z}'_i(n) &= \frac{d}{dn} \bar{z}_i(n) \\ &= \sum_{n_1=1}^N \bar{\phi}_i(n_1) \Psi'(n - n_1).\end{aligned}\quad (3.6)$$

- (iii) Generate the Local Maxima Set ξ_i

$$\xi_i = \left\{ \max(\bar{\mathbf{z}}'_i(p)), \text{ for } p = 0 \text{ to } \frac{N}{N_{MAX}} - 1 \right\}, \quad (3.7)$$

where $\bar{\mathbf{z}}'_i(p)$ is a sub-set of $\bar{z}'_i(n)$, $\bar{\mathbf{z}}'_i(p) = \{\bar{z}'_i(pN_{MAX} + 1), \dots, \bar{z}'_i((p+1)N_{MAX})\}$, and N_{MAX} is the cardinality of the local set $\bar{\mathbf{z}}'_i(p)$, $N_{MAX} < N$.

- (iv) Generate the frequency boundary set Φ_i by performing *Thresholding* operation

$$\Phi_i = \{\xi_i(p) \mid \xi_i(p) \in \xi_i \text{ and } \xi_i(p) > \rho\}, \quad (3.8)$$

where ρ is the threshold value, and $\xi_i(p)$ is the p^{th} element of set ξ_i , i.e., the maximum values of sub-set $\bar{\mathbf{z}}'_i(p)$ for $p = \left\{0, 1, \dots, \frac{N}{N_{MAX}} - 1\right\}$.

The threshold ρ is determined as follows:

$$\rho = \frac{\max(\bar{z}'_i(n))}{\max(\bar{z}_i(n)) - N_0} \times (PSD_{min} - N_0), \quad (3.9)$$

where $\max(\bar{z}_i(n))$ is the maximum value of $\bar{z}_i(n)$ which is defined in equation (3.5), and $\max(\bar{z}'_i(n))$ is the maximum value of $\bar{z}'_i(n)$ over n which is defined in equation (3.6). PSD_{min} is the predefined minimum peak PSD level. A PU signal should have the peak PSD level greater than PSD_{min} . N_0 is the average noise PSD level.

Derivation for threshold expression: After performing a smoothing to the estimated PSD, the expression for the threshold value given in equation (3.9) is derived from the following relation:

- The minimum peak PSD level of a PU signal should be PSD_{min} .
- The maximum peak PSD level is $\max(\bar{z}_i(n))$.

The first order derivative of $\bar{z}_i(n)$ can be determined as:

$$\bar{z}'_i(n) = \frac{d}{dn}\bar{z}_i(n) \approx \frac{\Delta\bar{z}_i(n)}{\Delta n}.$$

Thus,

$$\max(\bar{z}'_i(n)) \approx \frac{\max(\Delta\bar{z}_i(n))}{\Delta n} \approx \frac{\max(\bar{z}_i(n)) - N_0}{\Delta n}. \quad (3.10)$$

The threshold ρ is determined from the value of $\bar{z}'_i(n)$ generated by the minimum peak PSD level of a PU signal (PSD_{min}), and is given as:

$$\rho \approx \frac{PSD_{min} - N_0}{\Delta n}. \quad (3.11)$$

From equations (3.10) and (3.11),

$$\frac{\rho}{\max(\bar{z}'_i(n))} \approx \frac{PSD_{min} - N_0}{\max(\bar{z}_i(n)) - N_0}.$$

As a result, the threshold value can be approximated as in equation (3.9).

The complexity of the Canny Frequency Boundary Detector is given below.

- In the first step to perform smoothing to the estimated PSD, it requires N multiply operations for each $\bar{z}_i(n)$ (for $n = 1, 2, \dots, N$). Hence, the complexity for this step is $\mathbf{O}(N^2)$.
- In the second step to compute the first order derivative of $\bar{z}_i(n)$ (for $n = 1, 2, \dots, N$), computing the derivative of $\bar{z}_i(n)$ needs N subtractions (as $\bar{z}_i(n)$ are discrete in n). Consequently, this step requires $\mathbf{O}(N)$.
- In the third step to generate the local maxima set, it requires N comparisons. Therefore, the complexity of this step is $\mathbf{O}(N)$.

- In the fourth step, it needs N comparisons for thresholding. The complexity of this step hence, is $\mathbf{O}(N)$.

Overall, the number of operations of the Canny Frequency Boundary Detector is $N^2 + 3N$. The complexity of this technique is $\mathbf{O}(N^2)$.

3.2.2 Primary User Transmission Detector

In order to perform detection of PU's transmission, information about the location of PUs' bands is required. When such information is assumed to be unknown, it can be obtained using the CFBD proposed in the previous section. Once the primary bands are identified, detection of any transmission on those bands can be performed. Many methods are suggested for the signal detection in a selected band. A brief introduction of these methods is given in Section 2.2.1. Due to its simplicity as well as its ability to be deployed in most scenarios without any knowledge of the PU signaling features, the energy detection is employed in the present analysis. A brief review of the technique is given below.

Let M be the number of complex-valued samples used to generate the test statistics. The test statistics Y_i for SU i using equation (3.2) is defined as:

$$Y_i = \frac{1}{M} \sum_{m=1}^M |y_i(m)|^2. \quad (3.12)$$

The problem of energy detection is one that chooses between two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, defined respectively as:

$$\begin{aligned} \mathcal{H}_0 : \text{ No Primary User} &\rightarrow y_i(m) = v_i(m) \\ \mathcal{H}_1 : \text{ Primary User is Present} &\rightarrow y_i(m) = h_i x(m) + v_i(m). \end{aligned} \quad (3.13)$$

The parameter M must be chosen to be sufficiently large for an accurate detection (e.g., $M = 40$). As stated in [14], the larger the number of samples, the more accurate is

the detection. However, a larger number of samples requires a longer detection time and generates more complexity. As M is sufficiently large, the Central Limit Theorem can be applied to estimate the probability density function (PDF) of the test statistics. Under hypothesis $\mathcal{H}_0$, the PDF of the test statistics denoted by $p_0(Y_i)$ can be approximated by a Gaussian distribution with mean $\mu_{i,0} = \sigma_{i,v}^2$, and variance $\sigma_{i,0}^2 = \frac{1}{M} \sigma_{i,v}^4$ [63]. Under hypothesis $\mathcal{H}_1$, the PDF of the test statistics denoted by $p_1(Y_i)$ can be approximated by a Gaussian distribution with mean $\mu_{i,1} = (1 + \gamma_i) \sigma_{i,v}^2$, and variance $\sigma_{i,1}^2 = \frac{1}{M} (1 + \gamma_i)^2 \sigma_{i,v}^4$. Here, γ_i is defined as the Signal-to-Noise Ratio (SNR) of the PU signal observed at SU i , i.e., $\gamma_i = \frac{|h_i|^2 \sigma_{i,x}^2}{\sigma_{i,v}^2}$, where $\sigma_{i,x}^2 = E[|x|^2]$ corresponds to the transmission power of the PU. Based on the PDF of the test statistics under $\mathcal{H}_0$ and $\mathcal{H}_1$, the probability of false alarm $P_{F,i}(\epsilon)$, and the probability of detection $P_{D,i}(\epsilon)$ are given respectively as:

$$\begin{aligned} P_{F,i}(\epsilon) &= Pr(Y_i > \epsilon | \mathcal{H}_0) = \int_{\epsilon}^{\infty} p_0(Y_i) dY_i \\ &\approx \mathcal{Q}\left(\left(\frac{\epsilon}{\sigma_{i,v}^2} - 1\right) \sqrt{M}\right), \end{aligned} \quad (3.14)$$

$$\begin{aligned} P_{D,i}(\epsilon) &= Pr(Y_i > \epsilon | \mathcal{H}_1) = \int_{\epsilon}^{\infty} p_1(Y_i) dY_i \\ &\approx \mathcal{Q}\left(\left(\frac{\epsilon}{(1 + \gamma_i) \sigma_{i,v}^2} - 1\right) \sqrt{M}\right), \end{aligned} \quad (3.15)$$

where ϵ refers to the threshold value of the detector. This threshold can be determined by fixing the probability of false alarm (or the probability of detection) to a predefined value. For example, the probability of false alarm can be set to 1%. Using inverse marcum-Q table, the value of threshold can be determined as:

$$\epsilon = \left(\frac{\mathcal{Q}^{-1}(P_{F,i})}{\sqrt{M}} + 1\right) \sigma_{i,v}^2, \quad (3.16)$$

where $\mathcal{Q}^{-1}(\cdot)$ is the inverse marcum-Q function.

3.3 Cooperative Spectrum Sensing

In general, NCSS strategy performs well in situations where strong Line-of-Sight (LOS) paths exist between the PUs and SUs with easily distinguishable signal transmissions from PUs. However, NCSS strategy performs less reliably in Non-LOS (NLOS) and high-noise environments. In NLOS environment, the ability of the classical detection techniques to detect PU's presence weakens. In the case of energy detection, reducing the threshold level might improve the detection level, but at the same time it might increase the false alarm rate.

The inherent spatial diversity in a network of wireless SUs motivates the use of cooperative spectrum sensing (CSS) technique. Most of the current cooperative spectrum sensing (CSS) approaches assume the knowledge of primary band locations. Without such an assumption, each SU is first required to perform frequency boundary detection followed by the primary user detection. Due to the spatial diversity of the nodes and other physical impairments such as noise fluctuations, local oscillators' mismatch, etc., different SUs are likely to produce different sets of boundaries. One simple way is to combine the boundary detection results without any pre-processing. However, as illustrated in Figure 3.2, the main drawback for such methods is the high number of narrow bands due to non-synchronized results. As detection is performed on each such band, a large sensing time would be incurred. Another problem associated with CSS is the bandwidth penalty incurred as each SU has to report its sensing results to the fusion center. The following sections propose innovative solutions to address these difficulties.

3.3.1 Canny Frequency Boundary Detector with Dynamic Band Clustering

First, let the set of frequency boundaries of the primary bands within the targeted spectrum pool be denoted by:

$$\psi_0 = \{f_{0,1} \cdots f_{0,B_0}\}, \quad (3.17)$$

Figure 3.2: An illustration of Canny Frequency Boundary Detector (CFBD) with and without dynamic band clustering for 2 cooperating secondary users

where $f_{0,b}$ is the b^{th} boundary, and B_0 is the number of boundaries in the spectrum pool. Now, let the frequency boundaries identified by SU i be contained in the set ψ_i , defined as:

$$\psi_i = \{f_{i,1} \cdots f_{i,B_i}\} \text{ for } i = 1 \text{ to } I, \quad (3.18)$$

where I is the number of SUs in the CSS system, $f_{i,b}$ is the b^{th} estimated boundary, and B_i is the number of boundaries identified by SU i . Here, it is assumed that after finishing the FBD operation, all cooperating SUs will share their result sets with the sink node, where dynamic band clustering (DBC) is to be carried out.

In brief, the goal of the Canny Frequency Boundary Detector with dynamic band clustering (CFBD-DBC) algorithm is to synchronize the result sets from the different SUs and merge similar frequency boundary points into a single point. As a result, the number of frequency boundary points is minimized, and hence reduces the complexity of PU detection. The idea behind the proposed CFBD-DBC algorithm is borrowed from the field of Image Processing, where similar problem in the area of edge detection has been

Figure 3.3: Flowchart diagram of dynamic band clustering (DBC) algorithm

solved by using the *K-means clustering* algorithm [12]. Figure 3.3 depicts the flowchart of the CFBD-DBC algorithm. As shown in Figure 3.3, the proposed technique comprises of three stages, namely the *initialization*, *group growing*, and *group splitting with mean updating* stage.

- During the *initialization* stage, the result set with the minimum number of elements (or boundary points) is selected. In the flowchart, the SU with the smallest result set is represented by i_0 while the size of the smallest result set is $\hat{B}_0$. A cluster bin F_{b_1} is then prepared for each element in the set, and the corresponding mean for each bin is denoted by $\hat{f}_{0,b_1}$.
- During the second (*group growing*) stage, each frequency boundary point will be assigned to the cluster bin whose mean is closest to it.

In the mathematical representation, define $\mathbf{F} = \{F_1, F_2, \dots, F_{\hat{B}}\}$ be the set of cluster bins determined prior to this stage, and $\hat{\mathbf{f}}_0 = \{\hat{f}_{0,1}, \hat{f}_{0,2}, \dots, \hat{f}_{0,\hat{B}}\}$ be the set of corresponding means of these cluster bins. A frequency boundary point $f_{i,b}$ ($f_{i,b} \in \{f_{i,1}, f_{i,2}, \dots, f_{i,B_i}\}$, $i \in \{1, 2, \dots, I\}$) will be assigned to a cluster bin F_{b_2} ($F_{b_2} \in \mathbf{F}$) whose mean is $\hat{f}_{0,b_2}$, such that:

$$\left| f_{i,b} - \hat{f}_{0,b_2} \right| \leq \left| f_{i,b} - \hat{f}_{0,b_1} \right|, \quad \forall \hat{f}_{0,b_1} \in \hat{\mathbf{f}}_0.$$

- During the third stage (*group splitting with mean updating*), a check is performed on each bin such that its dynamic range (i.e., the difference between the smallest and largest elements in the same bin) does not exceed a predefined threshold ξ . If the dynamic range of a bin is below the threshold level, then the bin does not need to be split and the modified mean can be computed. If the dynamic range exceeds the threshold value, a new bin will be created. The respective elements will be split with half of them being placed into the newly created bin. The corresponding means for both the bins are then updated.

The second and third stages of the algorithm will be iteratively performed until no new bin is created during the third stage.

The complexity of this algorithm is provided as follows. Let B be the number of frequency boundary points detected by all SUs, i.e.,

$$B = \sum_{i=1}^I B_i.$$

where B_i is the number of frequency boundary points detected by SU i . Define k be number of cluster bins (this number is equal to the number of frequency boundary points detected after running the algorithm), and t be the number of iterations required for the algorithm to converge. In each iteration, it requires:

- $B \times k$ operations for distance calculation and comparison in the *group growing* stage, and
- k operations for mean calculation in the *group splitting with mean updating* stage.

Consequently, the overall complexity of the proposed algorithm is $\mathbf{O}(t \times B \times k)$.

3.3.2 Cooperative Primary User Transmission Detection

After the CFBD-DBC operation is completed, the next step in the spectrum sensing task is to sense the presence of PUs within the bands that have been detected as primary bands. As mentioned in **Chapter 2**, in cooperative primary user detection, techniques can be generally classified into two categories namely *information-fusion* and *decision-fusion*. Different combining techniques for the two categories are presented in this section.

3.3.2.1 Information-Fusion Technique

As discussed earlier, the detection of PU's transmission can be based on cyclostationary features, energy detection or other techniques. The proposed approach uses energy detection for the detection of PU's transmission. The energy or test statistics generated are

essentially the raw sensing data that can be referred to as the *information* to be fused. Figure 3.4 illustrates a pictorial view of the generalized information-fusion process.

Figure 3.4: Block diagram for Cooperative Primary User Transmission Detector (CPUTD) with information-fusion approach

As shown in Figure 3.4, SU i transmits its test statistics, Y_i on the channel that is orthogonal to all other channels used by other SUs. The orthogonal channel can be implemented using conventional multiple access methods such as frequency-division multiple access (FDMA), time-division multiple access (TDMA) or code-division multiple access (CDMA). The signal received at the FC from SU i can be expressed as:

$$Z_i = \alpha_i Y_i + \omega_i, \quad (3.19)$$

where α_i is the channel coefficient between SU i and the FC, and ω_i is the zero mean AWGN component with variance δ_i^2 encountered during the orthogonal period assigned to SU i . The information combining is performed by first stacking the I test statistics into a vector $\mathbf{Z}$ of size $I \times 1$ and pre-multiplying it with the combining weight vector. This can be mathematically expressed as:

$$\dot{Y} = \mathbf{G}^T \mathbf{Z} = [G_1 \cdots G_I] [Z_1 \cdots Z_I]^T, \quad (3.20)$$

where $\mathbf{G} = [G_1 \cdots G_I]^T$ and $\mathbf{Z} = [Z_1 \cdots Z_I]^T$. G_i represents the weight assigned to the information generated by SU i . Alternatively, G_i can be viewed as the impact factor of

the information generated by SU i on the collective decision to be made at the FC. It is important to note that for centralized cooperative spectrum sensing (i.e., only one SU is selected as the FC), if some spectrum sensing information from a SU cannot reach the FC, the contribution of this SU on the cooperative decision will be discarded. In the case of distributed cooperative spectrum sensing, where each SU acts as a FC, the possibility that the spectrum sensing sample from a SU cannot reach all other SUs is extremely low due to the geospatial diversity of the SUs. Consequently, though the sensing result from this SU will be discarded in some SUs, it will be considered in other SUs. Two classical combining techniques can be used for this purpose. The first technique is *Equal Gain Combining* (EGC), where the i^{th} weight is given by:

$$G_{i,EGC} = \frac{1}{\alpha_i}, \quad (3.21)$$

and the weighted sum of contribution from the I CRs is given by:

$$\dot{Y}_{EGC} = \sum_{i=1}^I Y_i + \sum_{i=1}^I \frac{\omega_i}{\alpha_i}, \quad (3.22)$$

where Y_i is the test statistics of SU i given as in equation (3.12). $\dot{Y}_{EGC}$ can be rewritten as:

$$\dot{Y}_{EGC} = \frac{1}{M} \sum_{i=1}^I \sum_{m=1}^M |y_i(m)|^2 + \sum_{i=1}^I \frac{\omega_i}{\alpha_i}, \quad (3.23)$$

where $y_i(m)$ is given as in equation (3.13). The hypothesis testing for EGC, hence is provided as:

$$\begin{aligned} \mathcal{H}_0 : \text{No PU} &\rightarrow \dot{Y}_{EGC} = \frac{1}{M} \sum_{i=1}^I \sum_{m=1}^M |v_i(m)|^2 + \sum_{i=1}^I \frac{\omega_i}{\alpha_i} \\ \mathcal{H}_1 : \text{PU is Present} &\rightarrow \dot{Y}_{EGC} = \frac{1}{M} \sum_{i=1}^I \sum_{m=1}^M |h_i x(m) + v_i(m)|^2 + \sum_{i=1}^I \frac{\omega_i}{\alpha_i}. \end{aligned} \quad (3.24)$$

As Y_i , ω_i are independent random variables that are normally distributed, $\dot{Y}_{EGC}$ is also normally distributed. The mean and variance of $\dot{Y}_{EGC}$ can be determined from equation

(3.22) as follows:

$$\begin{aligned} E[\dot{Y}_{EGC}] &= \sum_{i=1}^I E[Y_i] + \sum_{i=1}^I \frac{E[\omega_i]}{\alpha_i} \\ &= \sum_{i=1}^I E[Y_i], \end{aligned} \quad (3.25)$$

$$\begin{aligned} \text{Var}[\dot{Y}_{EGC}] &= \sum_{i=1}^I \text{Var}[Y_i] + \sum_{i=1}^I \frac{\text{Var}[\omega_i]}{\alpha_i^2} \\ &= \sum_{i=1}^I \text{Var}[Y_i] + \sum_{i=1}^I \frac{\delta_i^2}{\alpha_i^2}, \end{aligned} \quad (3.26)$$

where $E[\cdot]$ and $\text{Var}[\cdot]$ are the expectation and variance operations, respectively. Consequently, the mean $\mu_{\dot{Y}_{EGC}|\mathcal{H}_0}$ and variance $\sigma_{\dot{Y}_{EGC}|\mathcal{H}_0}$ of the test statistics under hypothesis $\mathcal{H}_0$ are given respectively as:

$$\begin{aligned} \mu_{\dot{Y}_{EGC}|\mathcal{H}_0} &= E[\dot{Y}_{EGC}|\mathcal{H}_0] = \sum_{i=1}^I E[Y_i|\mathcal{H}_0] \\ &= \sum_{i=1}^I \sigma_{i,v}^2, \end{aligned} \quad (3.27)$$

$$\begin{aligned} \sigma_{\dot{Y}_{EGC}|\mathcal{H}_0} &= \text{Var}[\dot{Y}_{EGC}|\mathcal{H}_0] = \sum_{i=1}^I \text{Var}[Y_i|\mathcal{H}_0] + \sum_{i=1}^I \frac{\delta_i^2}{\alpha_i^2} \\ &= \sum_{i=1}^I \left(\frac{1}{M} \sigma_{i,v}^4 + \frac{\delta_i^2}{\alpha_i^2} \right). \end{aligned} \quad (3.28)$$

Under hypothesis $\mathcal{H}_1$, the mean $\mu_{\dot{Y}_{EGC}|\mathcal{H}_1}$ and variance $\sigma_{\dot{Y}_{EGC}|\mathcal{H}_1}$ of the test statistics are given respectively as:

$$\begin{aligned} \mu_{\dot{Y}_{EGC}|\mathcal{H}_1} &= E[\dot{Y}_{EGC}|\mathcal{H}_1] = \sum_{i=1}^I E[Y_i|\mathcal{H}_1] \\ &= \sum_{i=1}^I (1 + \gamma_i) \sigma_{i,v}^2, \end{aligned} \quad (3.29)$$

$$\begin{aligned} \sigma_{\dot{Y}_{EGC}|\mathcal{H}_1} &= \text{Var}[\dot{Y}_{EGC}|\mathcal{H}_1] = \sum_{i=1}^I \text{Var}[Y_i|\mathcal{H}_1] + \sum_{i=1}^I \frac{\delta_i^2}{\alpha_i^2} \\ &= \sum_{i=1}^I \left(\frac{1}{M} (1 + \gamma_i)^2 \sigma_{i,v}^4 + \frac{\delta_i^2}{\alpha_i^2} \right). \end{aligned} \quad (3.30)$$

The cooperative decision on the presence or absence of PU signal can be derived by comparing the test statistics $\dot{Y}_{EGC}$ with a threshold value ϵ . The probability of false alarm $\dot{P}_{F,EGC}$ and probability of detection $\dot{P}_{D,EGC}$, can derived respectively as:

$$\dot{P}_{F,EGC} = Pr \left(\dot{Y}_{EGC} > \epsilon | \mathcal{H}_0 \right) = \mathcal{Q} \left(\frac{\epsilon - \mu_{\dot{Y}_{EGC} | \mathcal{H}_0}}{\sigma_{\dot{Y}_{EGC} | \mathcal{H}_0}} \right), \quad (3.31)$$

$$\dot{P}_{D,EGC} = Pr \left(\dot{Y}_{EGC} > \epsilon | \mathcal{H}_1 \right) = \mathcal{Q} \left(\frac{\epsilon - \mu_{\dot{Y}_{EGC} | \mathcal{H}_1}}{\sigma_{\dot{Y}_{EGC} | \mathcal{H}_1}} \right). \quad (3.32)$$

The value of ϵ can be determined by fixing the probability of false alarm $\dot{P}_{F,EGC}$ (or the probability of detection $\dot{P}_{D,EGC}$) to a predefined value. For example, set $\dot{P}_{F,EGC} = 1\%$. Using inverse marcum-Q table, the value of threshold can be determined as:

$$\epsilon = \mathcal{Q}^{-1} \left(\dot{P}_{F,EGC} \right) \sigma_{\dot{Y}_{EGC} | \mathcal{H}_0} + \mu_{\dot{Y}_{EGC} | \mathcal{H}_0}, \quad (3.33)$$

where $\mathcal{Q}^{-1}(\cdot)$ is the inverse marcum-Q function.

The second technique is *Maximal Ratio Combining* (MRC), where the i^{th} weight and the weighted sum of contribution from the I CRs is given respectively by:

$$G_{i,MRC} = \frac{\gamma_i}{\alpha_i \sum_{j=1}^I \gamma_j}, \quad (3.34)$$

$$\dot{Y}_{MRC} = \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) Y_i + \sum_{i=1}^I \frac{\gamma_i \omega_i}{\alpha_i \sum_{j=1}^I \gamma_j}. \quad (3.35)$$

Similar to the EGC, the $\dot{Y}_{MRC}$ can be rewritten as:

$$\dot{Y}_{MRC} = \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) \sum_{m=1}^M |y_i(m)|^2 + \sum_{i=1}^I \frac{\gamma_i \omega_i}{\alpha_i \sum_{j=1}^I \gamma_j}. \quad (3.36)$$

The hypothesis testing for MRC, hence is represented as:

$$\begin{aligned} \mathcal{H}_0 : \dot{Y}_{MRC} &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) \left(\frac{1}{M} \sum_{m=1}^M |v_i(m)|^2 \right) + \sum_{i=1}^I \frac{\gamma_i \omega_i}{\alpha_i \sum_{j=1}^I \gamma_j} \\ \mathcal{H}_1 : \dot{Y}_{MRC} &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) \left(\frac{1}{M} \sum_{m=1}^M |h_i x(m) + v_i(m)|^2 \right) + \sum_{i=1}^I \frac{\gamma_i \omega_i}{\alpha_i \sum_{j=1}^I \gamma_j}. \end{aligned} \quad (3.37)$$

Similar to the case of EGC, the mean and variance of $\dot{Y}_{MRC}$ can be determined from equation (3.35) as follows:

$$\begin{aligned} E[\dot{Y}_{MRC}] &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) E[Y_i] + \sum_{i=1}^I \frac{\gamma_i E[\omega_i]}{\alpha_i \sum_{j=1}^I \gamma_j} \\ &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) E[Y_i], \end{aligned} \quad (3.38)$$

$$\begin{aligned} \text{Var}[\dot{Y}_{MRC}] &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right)^2 \text{Var}[Y_i] + \sum_{i=1}^I \left(\frac{\gamma_i}{\alpha_i \sum_{j=1}^I \gamma_j} \right)^2 \text{Var}[\omega_i] \\ &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right)^2 \left[\text{Var}[Y_i] + \frac{\delta_i^2}{\alpha_i^2} \right]. \end{aligned} \quad (3.39)$$

The mean $\mu_{\dot{Y}_{MRC}|\mathcal{H}_0}$ and variance $\sigma_{\dot{Y}_{MRC}|\mathcal{H}_0}$ of the test statistics $\dot{Y}_{MRC}$ under hypothesis $\mathcal{H}_0$ are given respectively as:

$$\begin{aligned} \mu_{\dot{Y}_{MRC}|\mathcal{H}_0} &= E[\dot{Y}_{MRC}|\mathcal{H}_0] = \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) E[Y_i|\mathcal{H}_0] \\ &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) \sigma_{i,v}^2, \end{aligned} \quad (3.40)$$

$$\begin{aligned} \sigma_{\dot{Y}_{MRC}|\mathcal{H}_0} &= \text{Var}[\dot{Y}_{MRC}|\mathcal{H}_0] = \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right)^2 \left[\text{Var}[Y_i|\mathcal{H}_0] + \frac{\delta_i^2}{\alpha_i^2} \right] \\ &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right)^2 \left(\frac{1}{M} \sigma_{i,v}^4 + \frac{\delta_i^2}{\alpha_i^2} \right). \end{aligned} \quad (3.41)$$

Under $\mathcal{H}_1$, the mean $\mu_{\dot{Y}_{MRC}|\mathcal{H}_1}$, and variance $\sigma_{\dot{Y}_{MRC}|\mathcal{H}_1}$ of the test statistics $\dot{Y}_{MRC}$ are

given respectively as:

$$\begin{aligned}\mu_{\dot{Y}_{MRC}|\mathcal{H}_1} &= E[\dot{Y}_{MRC}|\mathcal{H}_1] = \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) E[Y_i|\mathcal{H}_1] \\ &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right) (1 + \gamma_i) \sigma_{i,v}^2,\end{aligned}\tag{3.42}$$

$$\begin{aligned}\sigma_{\dot{Y}_{MRC}|\mathcal{H}_1} &= \text{Var}[\dot{Y}_{MRC}|\mathcal{H}_1] = \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right)^2 \left[\text{Var}[Y_i|\mathcal{H}_1] + \frac{\delta_i^2}{\alpha_i^2} \right] \\ &= \sum_{i=1}^I \left(\frac{\gamma_i}{\sum_{j=1}^I \gamma_j} \right)^2 \left(\frac{1}{M} (1 + \gamma_i)^2 \sigma_{i,v}^4 + \frac{\delta_i^2}{\alpha_i^2} \right).\end{aligned}\tag{3.43}$$

Similar to the case of EGC, the probability of false alarm $\dot{P}_{F,MRC}$ and probability of detection $\dot{P}_{D,MRC}$ of MRC, can derived respectively as:

$$\dot{P}_{F,MRC} = Pr(\dot{Y}_{MRC} > \epsilon|\mathcal{H}_0) = \mathcal{Q}\left(\frac{\epsilon - \mu_{\dot{Y}_{MRC}|\mathcal{H}_0}}{\sigma_{\dot{Y}_{MRC}|\mathcal{H}_0}}\right),\tag{3.44}$$

$$\dot{P}_{D,MRC} = Pr(\dot{Y}_{MRC} > \epsilon|\mathcal{H}_1) = \mathcal{Q}\left(\frac{\epsilon - \mu_{\dot{Y}_{MRC}|\mathcal{H}_1}}{\sigma_{\dot{Y}_{MRC}|\mathcal{H}_1}}\right),\tag{3.45}$$

where ϵ is a predefined threshold, whose value can be determined by fixing the probability of false alarm $\dot{P}_{F,MRC}$ (or the probability of detection $\dot{P}_{D,MRC}$) to a predefined value. The value of threshold can be determined as:

$$\epsilon = \mathcal{Q}^{-1}(\dot{P}_{F,MRC}) \sigma_{\dot{Y}_{MRC}|\mathcal{H}_0} + \mu_{\dot{Y}_{MRC}|\mathcal{H}_0},\tag{3.46}$$

where $\mathcal{Q}^{-1}(\cdot)$ is the inverse marcum-Q function.

As shown in Simulation Results (Section 3.4.3), MRC technique has better performance in terms of probability of detection and probability of false alarm than EGC technique. However, this combining technique requires signal-to-noise ratio (SNR) information from SUs. Consequently, this technique incurs higher overhead and longer processing time. The EGC does not require SNR information.

3.3.2.2 Decision-Fusion Approach

It is widely recognized that the information-fusion approach produces superior detection quality. However, the need to exchange and/or distribute the raw sensing information incurs high overhead. In the specific case of energy detection, the raw information is essentially a measure of the spectrum energy. Due to fading and shadowing, the spectrum energy may fluctuate over large dynamic range. This in turn requires large number of bits to represent such information. The spectral inefficiency is hence a major practicality constraint of the information-fusion approach.

One attractive alternative to information-fusion approach is the *decision-fusion* approach. The decision-fusion approach has been proposed with the aim of reducing the overhead required to facilitate collaboration among the multiple cooperating SUs. Instead of using raw information, each SU first conducts its own hypothesis testing and makes an individual decision on the presence or absence of PU. The individual decisions are collected at the FC, and then a combined decision is made. As decision simply indicates the presence or absence of PU, it can be represented using a single bit and hence, can reduce the amount of information exchanges.

One simple way of generating combined decision is to use a simple OR-rule. The hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are selected based on the following

$$\begin{aligned}\mathcal{H}_0 : \sum_{i=1}^I d_i &= 0 \\ \mathcal{H}_1 : \sum_{i=1}^I d_i &\geq 1,\end{aligned}\tag{3.47}$$

where d_i is the decision made by SU i . Based on equation (3.47), it can be deduced that hypothesis $\mathcal{H}_1$ is chosen as long as there is one SU selecting $\mathcal{H}_1$. One advantage of the simple OR-rule is its implementation simplicity. However, by assigning uniform weights to all the individual decisions, the simple OR-rule disregards the inherently

diverse operating environment during which the individual decisions are made. While the probability of detection is certainly improved as there are now multiple nodes sensing a common spectrum band, the probability of false alarm is also likely to increase as the occurrence of false alarm at a single SU would be propagated to all the other cooperating SUs.

In this context, a novel dynamic weight assignment algorithm is proposed to reduce the false alarm vulnerability of the cooperative detector. The proposed weight assignment technique makes use of the entropy value during which the decision is made. From information-theory perspective, entropy value indicates the uncertainty of the information being received. Therefore, higher entropy value means that there is a higher probability that the decision is wrong and vice-versa. Based on this property, the inverse of the entropy value is used as the confidentiality factor of the decision, and is defined as

$$\zeta_i = (-Pr(\mathcal{H}_0|Y_i) \log_2 Pr(\mathcal{H}_0|Y_i) - Pr(\mathcal{H}_1|Y_i) \log_2 Pr(\mathcal{H}_1|Y_i))^{-1}, \quad (3.48)$$

where ζ_i is the inverse of the entropy value at SU i . Based on equation (3.48), computation of this value requires information on the posteriori probabilities of hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$. Applying *Bayes'* theorem, the posteriori probabilities of $\mathcal{H}_0$ and $\mathcal{H}_1$ are given respectively as

$$Pr(\mathcal{H}_0|Y_i) = \frac{Pr(Y_i|\mathcal{H}_0) Pr(\mathcal{H}_0)}{Pr(Y_i)}, \quad (3.49)$$

$$Pr(\mathcal{H}_1|Y_i) = \frac{Pr(Y_i|\mathcal{H}_1) Pr(\mathcal{H}_1)}{Pr(Y_i)}, \quad (3.50)$$

where $Pr(Y_i|\mathcal{H}_0)$ and $Pr(Y_i|\mathcal{H}_1)$ can be found using the statistical properties of Y_i . As discussed in Section 3.2.2, under hypothesis $\mathcal{H}_0$, Y_i can be approximated by a Gaussian random variable with mean $\mu_{i,0} = \sigma_{i,v}^2$, and variance $\sigma_{i,0}^2 = \frac{1}{M}\sigma_{i,v}^4$. Under hypothesis $\mathcal{H}_1$, Y_i can be approximated by a Gaussian random variable with mean $\mu_{i,1} = (1 + \gamma_i)\sigma_{i,v}^2$,

and variance $\sigma_{i,1}^2 = \frac{1}{M}(\gamma_i + 1)^2 \sigma_{i,v}^4$. $Pr(Y_i|\mathcal{H}_0)$ and $Pr(Y_i|\mathcal{H}_1)$ are respectively given as:

$$\begin{aligned} Pr(Y_i|\mathcal{H}_0) &= \frac{1}{\mu_{i,0}\sqrt{2\pi}} \exp\left(-\frac{(Y_i - \mu_{i,0})^2}{2\sigma_{i,0}^2}\right) \\ &= \frac{1}{\sigma_{i,v}^2 \sqrt{2\pi} \exp\left(-\frac{(Y_i - \sigma_{i,v}^2)^2}{\frac{2}{M}\sigma_{i,v}^4}\right)}, \end{aligned} \quad (3.51)$$

$$\begin{aligned} Pr(Y_i|\mathcal{H}_1) &= \frac{1}{\mu_{i,1}\sqrt{2\pi}} \exp\left(-\frac{(Y_i - \mu_{i,1})^2}{2\sigma_{i,1}^2}\right) \\ &= \frac{1}{(1 + \gamma_i)\sigma_{i,v}^2 \sqrt{2\pi}} \exp\left(-\frac{(Y_i - (1 + \gamma_i)\sigma_{i,v}^2)^2}{\frac{2}{M}(\gamma_i + 1)^2 \sigma_{i,v}^4}\right). \end{aligned} \quad (3.52)$$

Meanwhile, by definition, $Pr(Y_i)$ is given as

$$Pr(Y_i) = Pr(Y_i|\mathcal{H}_0) Pr(\mathcal{H}_0) + Pr(Y_i|\mathcal{H}_1) Pr(\mathcal{H}_1). \quad (3.53)$$

For each SU, the prior probabilities of hypothesis $\mathcal{H}_0$ ($Pr(\mathcal{H}_0)$) and hypothesis $\mathcal{H}_1$ ($Pr(\mathcal{H}_1)$) can be estimated by observing the past cooperative decisions on the presence of PUs. These probability values are updated after each sensing. The computed entropy value is then quantized and shared/distributed together with the decision d_i . Assuming the decision d_i , and the inverse of the entropy value ζ_i are correctly received at the FC, the cooperative decision test statistics is computed as

$$\acute{d} = \sum_{i=1}^I \left(\frac{\zeta_i}{\sum_{j=1}^I \zeta_j} \right) d_i. \quad (3.54)$$

The global hypothesis testing problem is then given by

$$\begin{aligned} \mathcal{H}_0 : \acute{d} &< 0.5 \\ \mathcal{H}_1 : \acute{d} &\geq 0.5. \end{aligned} \quad (3.55)$$

3.4 Simulation Results

This section presents the results and findings obtained from extensive simulation exercises to validate the theoretical analysis presented in this chapter. The performance of the

proposed scheme is investigated under different deployment configurations of cognitive radio networks, and the results are gained from simulation by using Monte Carlo method. The performance of the proposed CFBD algorithm in NCSS is presented first, followed by the performance of CFBD-DBC and entropy-based fusion algorithm in CSS environment.

3.4.1 Performance of Canny Frequency Boundary Detection (CFBD) algorithm

The performance of the FBD algorithm is evaluated in a system with spectrum pool that ranges from 0 to 1000 MHz. For the studies conducted in this section, the parameters are given as in Table 3.1. The number of time-domain samples used to generate the auto-correlation value is chosen as 1000 and 2000 ($N = 1000$ and 2000). The detection resolution can be derived as 1 MHz and 0.5 MHz, respectively. In order to measure the accuracy of the FBD algorithm, the *Boundary Detection Error* (BDE), defined as the root mean square of the error between the real and estimated frequency boundaries, is used.

Table 3.1: Parameters for Studies on Canny Frequency Boundary Detection Algorithm

Parameters	Values
Spectrum pool range	0 – 1000 MHz
Number of time-domain samples (N)	1000 or 2000 samples
Tolerance bandwidth (TB)	1 or 2 MHz

Figure 3.5 shows the BDE performance obtained using the FBD method presented earlier. From Figure 3.5, it can be seen that the proposed FBD algorithm performs accurately when the number of time-domain samples is larger ($N = 2000$). The proposed detection algorithm provides small BDE even at the low SNRs. For instance, at 4 dB, the BDE is only around 0.76 MHz. The system's performance will be decreased for a smaller number of samples (i.e., lower resolution). For example, when the number of

Figure 3.5: Boundary Detection Error (BDE) of the proposed Frequency Boundary Detection (FBD) approach for different SNR values for different numbers of samples

samples is 1000, at 4 dB, the BDE is around 2.3 MHz. This value is around 1.2 MHz for high SNRs.

The *Probability of Detection* (PoD) of the proposed FBD approach is also studied. A band is assumed to be detected if the difference between the estimated frequency boundaries and the real frequency boundaries is within a *tolerance bandwidth* (TB). Figure 3.6 shows the probability of detection of the proposed FBD approach for different values of SNR. The TB is chosen as 1 MHz and 2 MHz. As seen from the figure, when N is 2000 and the TB is 2 MHz, the band is always correctly detected when the SNR is above 5 dB. The corresponding value is 10 dB when the tolerance bandwidth is 1 MHz. The PoD is worse for the lower resolution detection. When $N = 1000$ and the TB is 2 MHz, the PoD approaches 100% for high SNR (SNR is above 12 dB). This value approaches 50% at high SNRs when $N = 1000$ and the TB is 1 MHz.

Figure 3.6: Probability of Detection (PoD) of the proposed Frequency Boundary Detection (FBD) approach for different SNR values with different numbers of samples and tolerance bandwidths

3.4.2 Performance of Canny Frequency Boundary Detector with Dynamic Band Clustering (CFBD-DBC)

The performance of the proposed DBC algorithm is investigated in an environment with multiple cooperating SUs. The parameters for this study are provided as in Table 3.2. The present simulation considers the presence of 5 SUs. Each SU is assumed to be equipped without any prior knowledge of the PUs' bands. After the FBD is performed at each SU individually, the result from each user is shared with every other user. The DBC algorithm is then applied to synchronize all the results. For this example, the number of samples is selected to be 1000 ($N = 1000$). For analytical brevity, it is assumed that only a single PU's band is present. As a result, there are only 2 frequency boundary points.

Figure 3.7 shows the average number of boundary points obtained using different edge detection schemes. Three different edge detection schemes namely non-cooperative (stand-alone) boundary detection, cooperative boundary detection with brute-force combining and cooperative boundary detection with DBC are analyzed. For this study, the

Table 3.2: Parameters for the Study on Canny Frequency Boundary Detector with Dynamic Band Clustering

Parameters	Values
Spectrum pool range	0 – 1000 MHz
Number of time-domain samples (N)	1000 samples
Number of SUs	5
SNR at SUs	4 – 16 dB
The number of PUs' bands	1

SNR at each SU is randomly selected around a mean SNR value, which varies from 4 dB to 16 dB, in order to model the spatial diversity of individual SUs in the realistic environment. From the figure, it can be seen that at low SNRs, a brute force combining of the results from multiple SUs yields large number of boundary points. At 4 dB, the number of boundary points detected using the brute-force technique is 9.5 points. On the other hand, the proposed DBC technique produces significantly smaller number of boundary points during both low and high SNR values. Starting from SNR = 12 dB, the number of boundary points obtained using DBC technique is about 2, which is the real number of boundary points.

Figure 3.7: Average number of detected boundary points for different edge detection methods in an environment with multiple secondary users

The impact of the number of SUs on the performance of the DBC algorithm is also investigated. For this purpose, the SNR at each SU is randomly selected but the mean SNR is set as 10 dB. Figure 3.8 shows the average number of boundary points that are produced using different detection schemes. From the figure, it can be seen that the average number of frequency boundary points resulting from non-cooperative detection is not influenced by the number of SUs. This is expected as each SU performs independent detection and does not influence any other SU. When the brute-force combining technique is used, the resulting average number of points increases dramatically with respect to the increase in the number of SUs present in the system. For the proposed DBC algorithm, though the average number of points increases with the increase in the number of SUs, it is not substantial. In particular, the average number of points produced by using the proposed DBC algorithm with 10 cooperating SUs remains below 3.

Figure 3.8: Average number of detected boundary points for different edge detection methods with different numbers of secondary users (SNR = 10 dB)

3.4.3 Performance of Cooperative Primary User Transmission Detection (CPUTD) with Different Fusion Strategies

The performance of Cooperative Primary User Transmission Detection techniques are investigated and are compared against that of stand-alone sensing. The two fusion techniques: information-fusion (IF) and decision-fusion (DF) are simulated. For IF-based technique, the performance of EGC and MRC are studied and included in the analysis. For DF-based technique, the performance of simple OR-rule and the proposed entropy-based weighting technique are included in the analysis. Between the primary and each secondary user, the channel is assumed to be independent and identically distributed according to *Rayleigh* distribution. Between SUs, the channels are assumed to be ideal. The parameters for the studies conducted in this section are given as in Table 3.3. The number of cooperating SUs is varied from 2 to 10 in order to simulate different cooperation levels. The number of samples used for primary user transmission detection (PUTD) employing classic energy detection is selected as $M = 40$.

Table 3.3: Parameters for the Study on Cooperative Primary User Transmission Detection

Parameters	Values
Number of SUs	2 – 10
Number of samples used for PUTD using energy detection (M)	40
Number of PUs	1

Figure 3.9 shows the receiver operating characteristic of the proposed entropy-fusion PUTD for different number of quantizing bits. For the results shown, the mean SNR from all SUs is chosen as 9 dB while the SNR at each SU is randomly varied. From the figure, the improvement in performance with the increase in the number of quantizing bits from one to three is clearly visible. With 3, 4, and 5 quantizing bits, the performances are more or less similar. These results conclude that generally 3 bits are sufficient to

represent the uncertainty factor of the SUs and hence the subsequent discussions on the proposed algorithm assume the use of 3 quantizing bits.

Figure 3.9: Probability of detection versus probability of false alarm for cooperative cognitive systems using Entropy-fusion algorithm when the average SNR is 9 dB

Next, the performance of the entropy-fusion strategy is compared with that of four other strategies, namely the NCSS (stand-alone), CSS using simple OR-rule, CSS using MRC and CSS using EGC. Figure 3.10 shows the performance comparison for a mean SNR of 10 dB. Based on the figure, the performance of the NCSS is considerably inferior to all the CSS-based techniques. Given 10% as the desired probability of FA, the NCSS can only achieve about 55% probability of detection. Meanwhile, CSS with DF detects PU's presence 85% of the time. For the proposed entropy-fusion strategy, the probability of detection is slightly less than 95%. This is only about 5% lower than the detection performance obtained using the CSS with IF approaches. Given the significantly small number of bits required to represent the entropy as compared to raw sensing statistics, the tradeoff in the detection performance is not significant.

The above schemes are also compared based on their probability of detection against SNR values, and the results are shown in Figure 3.11. For this test case, the false alarm level of 10% is selected. From Figure 3.11, it can be seen that the performance of NCSS

Figure 3.10: Probability of detection versus probability of false alarm for different fusion methods when the average SNR is 10 dB

is considerably inferior to the other CSS-based techniques. To achieve probability of detection of 90%, NCSS performance is about 4 dB worse compared to the proposed technique. Compared to DF-based CSS, the proposed system delivers probability of detection of 90% at a SNR value that is 2 dB lower. However, when compared to the IF-based CSS schemes, the proposed scheme is slightly worse.

Finally, the impact of the number of cooperating SUs on the PoD is investigated. For this case, false alarm level of 10% is selected while the mean SNR among the SUs is fixed as 10 dB. Figure 3.12 shows the comparison obtained using the 5 different schemes. Based on the figure, it is observed that the PoD obtained using the NCSS is only about 57%. For the DF-based CSS, the PoD improves as the number of SUs increases. However, the PoD performance reaches the upper limit of around 91% with 9 SUs. When the number of cooperating SUs is lesser (i.e., 2), the PoD performance of the proposed method is only slightly superior to the DF-based CSS (70% against 68%). However, the performance of the proposed scheme improves at a much faster rate and reaches around 99% with 10 SUs. Based on the observation in Figure 3.12, the performance gap between the

Figure 3.11: Probability of detection of different fusion methods compared against different SNR values when the probability of false alarm is 10%

Figure 3.12: Probability of detection for non-cooperative, cooperative using OR-Rule combining algorithm, maximal ratio combining algorithm, equal gain combining algorithm and using Entropy-fusion algorithm with respect to different number of secondary users

proposed scheme and the IF-based schemes diminish as the number of SUs increases. This means that when the number of cooperating SUs is high, the proposed scheme is the more attractive option as the overhead incurred with the entropy-fusion technique is

considerably lower than that incurred in IF-based techniques.

3.5 Summary

This chapter addresses the challenges that arise when cooperative spectrum sensing is deployed in a cognitive radio environment where no *a priori* knowledge of PUs' band locations is available for SUs. The main contributions of this chapter are as follows:

- A frequency boundary detection technique based on *Canny Edge Detection* technique is proposed to detect the PUs' bands. From the simulation, it can be seen that when the number of time-domain samples is large (e.g., $N = 2000$), the proposed FBD technique produces very low boundary detection error even at low SNRs.
- A dynamic band clustering algorithm is proposed to synchronize the result sets detected by the different SUs about the PUs' bands. The performance of DBC algorithm is presented and is shown to reduce the number of non-existent narrow spectrum bands due to the identification mismatch among SUs.
- The conventional *information-fusion* approaches including *Equal Gain Combining* and *Maximal Ratio Combining* are studied to derive the cooperative decision on the presence of PU based on the detection results of individual SUs. From the simulation results, it can be seen that MRC technique has better performance in terms of probability of detection and probability of false alarm than EGC technique. However, this combining technique requires signal-to-noise ratio (SNR) information from SUs, and hence incurs large spectrum overhead.
- A novel weighting technique that can be coupled with the decision-fusion strategy is also proposed. The proposed technique makes use of entropy information as the weighting coefficient and has been shown to be effective in improving the performance of DF approach while avoiding the overhead in IF approach.

Chapter 4

A game theoretic framework for power control and channel allocation

4.1 Introduction

Once the unused spectrum is determined, it has to be allocated among SUs in cognitive radio networks. This step is known as resource allocation, which provides the methods to schedule radio resources including transmit power, frequency channels, time slots, channel access codes, etc., among SUs. The transmit power and frequency channels are the resources considered within the scope of the present work. Because different SUs may have different QoS requirements, they may have different objectives. Consider the transmission of different types of communication signals such as voice signals and data through wireless channels. In *voice communication*, high speed transmission is the most important factor, and hence voice radio systems always try to maximize their transmission rates (i.e., maximize their capacities). On the other hand, the most important factor in *data communication* is low BER. Thus, the data radio systems try to maximize their SINRs (i.e., minimize the interference). Correspondingly, we can have radio systems focussing on the respective quality-of-service parameters. It is in this context, this chapter examines two different non-cooperative games named *Interference Minimization* game and *Capacity Maximization* game depending on the type of radio transmissions. These

games reflect the interference minimization target of data radio systems and the capacity maximization target of voice radio systems, respectively.

The rest of this chapter is organized as follows: Section 4.2 provides the system description. Section 4.3 provides the way to formulate the channel allocation problem as a non-cooperative game. The details of *Interference Minimization* and *Capacity Maximization* games are also provided in this section. Section 4.4 provides the numerical results and the discussion. Finally, the summary of this chapter is provided in Section 4.5.

4.2 System Model

A cognitive radio network consisting of a set of N SU systems, each formed by a single transmitter-receiver pair of nodes which are uniformly distributed in a geographical area, is considered. The nodes are assumed to be fixed or slowly moving in order to achieve the steady state solution. It is also assumed that there are K frequency channels available for data transmission. For non-trivial solutions, only a case where $K < N$ is considered. The following assumptions are also made for the analysis:

- Multiple transmitter-receiver pairs are allowed to transmit at the same time over a shared channel.
- Access to all channels are equally likely for all transceivers.
- Complete channel information is available to all transceivers.
- All channels have the same bandwidth.

Let p_{ik} be the transmit power of transmitter i over transmitting channel k , and g_{ij}^k be the channel gain between transmitter i and receiver j when the transmission is made

through channel k . Under these assumptions, the SINR at the receiver i via transmitting channel k is given by

$$SINR(i, k) = \frac{p_{ik}g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk}g_{ji}^k + \sigma_0}, \quad (4.1)$$

where σ_0 is the average noise level and is assumed to be the same for all receivers.

The total transmission capacity of transmitter-receiver pair i is defined as

$$C(i) = \sum_{k=1}^K W_k \log \left(1 + \frac{p_{ik}g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk}g_{ji}^k + \sigma_0} \right), \quad (4.2)$$

where W_k is the bandwidth of the transmitting channel k . As assumed above, $W_1 = W_2 = \dots = W_K = W$. It is also assumed that the total transmit power of each transmitter is constrained by a maximum power constraint though different SUs may have different power constraints. Let $P_{i,max}$ be the power constraint of transmitter i , the transmit power p_{ik} must satisfy the following condition

$$\sum_{k=1}^K p_{ik} \leq P_{i,max}. \quad (4.3)$$

Based on these assumptions, the power control and channel allocation problem can be modelled as the outcome of a game in which the players are the transmitter-receiver pairs, their actions (their strategies) are the choices of transmitting channels and the transmit power on these channels. The details of the game formulation are presented in the next section.

4.3 A Game Theoretic Framework

A game theoretic framework to allocate K channels to N transmitter-receiver pairs of SUs (with $K < N$) is proposed in this section. The set of players $\mathcal{N}$ consists of N transmitter-receiver pairs of SUs, i.e., $\mathcal{N} = \{1, 2, \dots, N\}$. For player i ($i \in \mathcal{N}$), the strategy s_i is defined as a K -dimension vector representing the transmit power of player i over transmitting channels, and is represented as:

$$s_i = [p_{i1}, p_{i2}, \dots, p_{iK}],$$

where p_{ik} is the transmit power of player i over transmitting channel k . Let $\mathcal{K}$ denote the set of K channels, i.e., $\mathcal{K} = \{1, 2, \dots, K\}$.

Let $\mathbf{s}_{-i}$ be the strategies chosen by all the players in the game except player i , i.e., $\mathbf{s}_{-i} = \{s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N\}$. The set of strategies chosen by all the players constitutes a strategy profile which is denoted as $\mathbf{s} = \{s_i, \mathbf{s}_{-i}\}$. The last and also the most crucial task to formulate a game is to choose the utility function (payoff) which measures the outcome of the game for a player given a particular strategy profile. The present work suggests two utility functions which reflect different objectives, including minimizing the interference and maximizing capacity. Their details will be presented as follows.

4.3.1 Interference Minimization Game

The game proposed in this section aims to reflect the objective of radios in *data communication*. In *data communication*, the radios try to maximize their SINRs (i.e., to reduce BERs). However, as the radios may access the same channels, maximizing SINR of a radio will decrease the SINRs of other radios. A utility function rather than SINR or transmission rate, which takes into account both the benefit of channel accessing and the impact on other players is required. In this context, the utility function u_i of player i ($i \in \mathcal{N}$) is defined as:

$$u_i(s_i, \mathbf{s}_{-i}) = \sum_{k=1}^K \left\{ p_{ik} g_{ii}^k - \sum_{j=1}^N \delta_{ji}^k p_{jk} g_{ji}^k - \sum_{j=1}^N \delta_{ij}^k p_{ik} g_{ij}^k \right\}, \quad (4.4)$$

where $\delta_{ij}^k = 1$ if player i and player j ($i \neq j$) both transmit via channel k ($p_{ik} \times p_{jk} \neq 0$) and $\delta_{ij}^k = 0$ otherwise. The first component in the utility function represents the benefit that a player earns for allocating power on channels. The second and the third components represent the cost of a player: the interference caused by other players to this player and the interference caused by this player to other players if they share the common

channels. Next, the Nash equilibrium, which is considered as the solution of this game, is determined. The set $\mathbf{s}^* = \{s_1^*, \dots, s_N^*\}$ is the Nash equilibrium if and only if

$$s_i^* = BR_i(\mathbf{s}_{-i}^*), \quad \forall i \in \mathcal{N}, \quad (4.5)$$

where BR_i is the best response function of player i to the set of strategies taken by other players, and $\mathbf{s}_{-i}^*$ is the set of best response strategies for player j with $j \in \mathcal{N}$, $j \neq i$. $BR_i(\mathbf{s}_{-i})$ is defined as

$$BR_i(\mathbf{s}_{-i}) = \arg \max_{s_i} u_i(s_i, \mathbf{s}_{-i}). \quad (4.6)$$

A method to determine the $BR_i(\mathbf{s}_{-i})$ of player i is provided as follows. The utility (payoff) that player i earns by transmitting signal on frequency channel k ($k \in \mathcal{K}$) with transmit power p_{ik} is given as

$$\begin{aligned} u_i^k(p_{ik}) &= p_{ik}g_{ii}^k - \sum_{j=1}^N \delta_{ji}^k p_{jk}g_{ji}^k - \sum_{j=1}^N \delta_{ij}^k p_{ik}g_{ij}^k \\ &= \alpha_i^k p_{ik} - \beta_i^k, \end{aligned} \quad (4.7)$$

where $\alpha_i^k = g_{ii}^k - \sum_{j=1}^N \delta_{ij}^k g_{ij}^k$ and $\beta_i^k = \sum_{j=1}^N \delta_{ji}^k p_{jk}g_{ji}^k$. It is easy to see that the values of α_i^k and β_i^k depend only on the strategies of other players. Therefore, for a given $\mathbf{s}_{-i}$, these values are already known.

When $\alpha_i^k \leq 0$, $u_i^k(p_{ik})$ is always smaller than or equal to 0, for all $p_{ik} > 0$. As a result, the Proposition 4.1 is derived as follows.

Proposition 4.1 *The strategies by which a player i allocates power on a channel k , where $\alpha_i^k \leq 0$, are always dominated by strategies whereby the player does not occupy that channel.*

In addition, the following proposition and corollary are devised for the proposed game.

Proposition 4.2 *The strategies by which a player allocates power on a group of channels are always dominated by the strategies whereby the player allocates all of its power available for transmission (maximum power constraint) on a single channel.*

Proof: Assuming that s_i is a strategy by which player i ($i \in \mathcal{N}$) allocates power on a group of channels $\mathcal{K}_1$ where $\mathcal{K}_1 \subset \mathcal{K}$. The utility of that player for choosing strategy s_i is provided as:

$$u_i(s_i, \mathbf{s}_{-i}) = \sum_{k \in \mathcal{K}_1} u_i^k(p_{ik}) = \sum_{k \in \mathcal{K}_1} \{\alpha_i^k p_{ik}\} - \sum_{k \in \mathcal{K}_1} \{\beta_i^k\}. \quad (4.8)$$

Let $m = \arg \max_{k \in \mathcal{K}_1} \{\alpha_i^k\}$, and $\alpha_{i,max} = \alpha_i^m$. Because the overall transmit power available for player i is constrained by a maximum power constraint as in equation (4.3), the following inequality is given:

$$u_i(s_i, \mathbf{s}_{-i}) < \alpha_{i,max} P_{i,max} - \sum_{k \in \mathcal{K}_1} \beta_i^k < \alpha_i^m P_{i,max} - \beta_i^m. \quad (4.9)$$

Let s'_i be the strategy by which player i allocates a transmit power of $P_{i,max}$ on channel m

$$u_i(s'_i, \mathbf{s}_{-i}) = \alpha_i^m P_{i,max} - \beta_i^m. \quad (4.10)$$

From inequality (4.9) and equation (4.10),

$$u_i(s_i, \mathbf{s}_{-i}) < u_i(s'_i, \mathbf{s}_{-i}). \quad (4.11)$$

Consequently, strategy s_i is said to be dominated by strategy s'_i . ■

Corollary 4.1 *The best response of player i ($i \in \mathcal{N}$) should be a strategy by which either only one channel is accessed (single-channel strategy) or none at all. In the case of one channel access (i.e., channel k , $k \in \mathcal{K}$), the following conditions must be satisfied:*

- $\alpha_i^k > 0$,

- the transmit power, $p_{ik} = P_{i,max}$,
- the utility function, $u_i > 0$.

As a result, the best response of a player with respect to the opponent strategies can be determined as a strategy which generates the largest utility function among strategies satisfying the conditions stated in Corollary 4.1. If there is no single-channel strategy generating a positive utility, the best response strategy is not to access any channel. As there are at most K single-channel strategies satisfying conditions stated in Corollary 4.1, the complexity to find the best response is $O(K)$.

Similar to the games proposed in [105, 161], the proposed game is an exact potential game. The exact potential function of the proposed game is given as follows:

$$\begin{aligned} \mathcal{P}(\mathbf{s}) &= \frac{1}{2} \sum_{i=1}^N u_i(\mathbf{s}) + \frac{1}{2} \sum_{i=1}^N \sum_{k=1}^K p_{ik} g_{ii}^k \\ &= \sum_{i=1}^N \sum_{k=1}^K p_{ik} g_{ii}^k - \frac{1}{2} \sum_{i=1}^N \left\{ \sum_{k=1}^K \sum_{j=1}^N \delta_{ji}^k p_{jk} g_{ji}^k + \sum_{k=1}^K \sum_{j=1}^N \delta_{ij}^k p_{ik} g_{ij}^k \right\}. \end{aligned} \quad (4.12)$$

Proving the proposed game as an exact potential game is similar to what is presented in [161]. As in [98], one important property of an exact potential game is that there is at least one pure-strategy Nash equilibrium. This equilibrium can be achieved by letting players iteratively play in a myopic game [98]. In each *iteration*, the players make their moves in a predefined order, and they will select the best response strategies which maximize their utility functions based on the observed opponent strategies. This algorithm is known as best response dynamics [98].

4.3.2 Capacity Maximization Game

For the capacity maximization target, the utility function $u_i(s_i, \mathbf{s}_{-i})$ of player i ($i \in \mathcal{N}$) given a strategy profile $\mathbf{s} = \{s_i, \mathbf{s}_{-i}\}$ is defined as

$$u_i(s_i, \mathbf{s}_{-i}) = \sum_{k=1}^K \frac{g_{ii}^k}{\sum_{j=1, j \neq i}^N g_{ij}^k a_{jk}} \log \left(1 + \frac{p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0} \right), \quad (4.13)$$

where $a_{jk} = 1$ if player j communicates over transmitting channel k (i.e., $p_{jk} > 0$), and $a_{jk} = 0$ otherwise (i.e., $p_{jk} = 0$), for $k \in \mathcal{K}$. The above utility function is derived from the formula for the total transmission capacity of a player which is defined as in equation (4.2). The difference is the introduction of weighting factors. The weighting factor that player i assigns to channel k is determined as the ratio of the channel gain between player i 's transmitter and receiver (i.e., g_{ii}^k) to the sum of the channel gains between player i 's transmitter and other players' receivers who access this channel (i.e., $\sum_{j \neq i} g_{ij}^k a_{jk}$). By choosing this utility, the players will allocate more power to channels which bring them larger transmission capacity and generate less interference to other players.

The best response strategy BR_i of player i to the set of strategies $\mathbf{s}_{-i}$ taken by other players is studied next. $BR_i(\mathbf{s}_{-i})$, as defined in equation (4.6), can be determined as the solution of the following optimization problem

$$\max_{s_i} u_i(s_i, \mathbf{s}_{-i})$$

such that

$$\begin{cases} p_{ik} \geq 0, & \forall k \in \mathcal{K} \\ \sum_{k=1}^K p_{ik} \leq P_{i,max}. \end{cases}$$

The Lagrangian $\mathcal{L}$ associated with the above optimization problem is defined as follows

$$\begin{aligned} \mathcal{L}(s_i, \lambda) &= (-u_i(s_i, \mathbf{s}_{-i})) + \lambda \left[\sum_{k=1}^K p_{ik} - P_{i,max} \right] \\ &= - \sum_{k=1}^K \frac{g_{ii}^k}{\sum_{j=1, j \neq i}^N g_{ij}^k a_{jk}} \log \left(1 + \frac{p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0} \right) + \lambda \left[\sum_{k=1}^K p_{ik} - P_{i,max} \right], \end{aligned} \quad (4.14)$$

where λ is the Lagrange multiplier, $\lambda \geq 0$. From the *Karush-Kuhn-Tucker* (KKT) conditions [162], the strategy $\hat{s}_i = [\hat{p}_{i1}, \dots, \hat{p}_{iK}]$ is the best response of player i if and only if the following conditions are satisfied

$$\begin{aligned} \frac{\partial \mathcal{L}(\hat{s}_i, \lambda)}{\partial p_{ik}} &= 0, \quad \forall k \in \mathcal{K}, \\ \lambda \left[\sum_{k=1}^K \hat{p}_{ik} - P_{i,max} \right] &= 0, \\ \sum_{k=1}^K \hat{p}_{ik} - P_{i,max} &\leq 0, \quad \lambda \geq 0, \\ \hat{p}_{ik} &\geq 0, \quad \forall k \in \mathcal{K}, \end{aligned} \quad (4.15)$$

where $\frac{\partial \mathcal{L}(s_i, \lambda)}{\partial p_{ik}}$ is the partial derivative of $\mathcal{L}(s_i, \lambda)$ with respect to p_{ik} ($\forall k \in \mathcal{K}$), and is given as

$$\frac{\partial \mathcal{L}(s_i, \lambda)}{\partial p_{ik}} = - \frac{g_{ii}^k}{\ln(2) \sum_{j=1, j \neq i}^N g_{ij}^k a_{jk}} \left[\frac{g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0 + p_{ik} g_{ii}^k} \right] + \lambda. \quad (4.16)$$

Similar to the *Interference Minimization* game, in this game, the players also iteratively play a myopic game in order to achieve the steady state (i.e., the Nash equilibrium). In each *iteration*, players select the best response strategies (determined by condition (4.15)), which maximizes their own utility, based on the observed opponent strategies.

4.4 Numerical Results

In this section, the performances of the proposed games are provided and discussed. The results are gained from simulation by using Monte Carlo method. The simulation parameters are given in Table 4.1. The simulations consider 20 transmitter-receiver cognitive radio pairs (players) and 5 frequency channels. The transmitters and receivers are randomly located in a square region of dimension $L \times L$ where L is selected as 1 km as shown in Figure 4.1. The power constraints of the players are identical and equal to 40 mW. The channel gains are determined based on the path-loss model. The path-loss exponent is selected as 3. It is assumed that the players have complete knowledge about the channel gains between transmitters and receivers, i.e., $\forall i, j \in \{1, \dots, 20\}$ and $\forall k \in \{1, \dots, 5\}$, g_{ij}^k is known.

Table 4.1: Parameters for Simulations on Power Control and Frequency Channel Allocation Approaches

Parameters	Values
Number of SUs (N)	20
Number of frequency channels (K)	5
Transmit power constraints of SUs	40 mW
Area of the distributed region of SUs	1 km ²
Path-loss exponent	3
Noise power	10^{-10} mW

4.4.1 Numerical Results of Interference Minimization Game

Let us analyze the convergence characteristics of the players' strategies in *Interference Minimization* game. During initialization, each player randomly allocates its power on a transmitting channel such that its maximum power constraint is satisfied. After that, the players iteratively play the game until a steady state is reached. In each *iteration*, the players play sequentially and they will choose the best response strategies based on the observed opponent strategies. Each time when a player makes a move is defined as a

Figure 4.1: An example of transmitter-receiver allocation

step. The transmit powers of the players over different transmitting channels in each step are shown in Figure 4.2. As seen from the figure, the game takes maximum of 60 steps to reach a steady state. In this steady state, the strategy taken by each player is the best response to the other players' strategies. By definition, this steady state is also a Nash Equilibrium of the proposed game. The strategies of the players during initialization and at NE are shown in Table 4.2. In addition, the value of the potential function of the proposed game after each step is shown in Figure 4.3. It is also an indication of the convergence characteristics of the proposed game. As seen from the figure, during initialization, the potential function value is 0.0021. After 60 steps, the potential function is stabilized and reaches the maximum value of 0.0024.

The SINRs of the players before and after applying *Interference Minimization* game are also investigated. Figure 4.4 shows the average SINRs of the players during initialization (before applying the proposed game) and after applying *Interference Minimization* game. It can be seen that the average SINR of each player increases significantly after applying the proposed game. In particular, the average SINR of each player during initialization is around 7 dB. After applying the proposed game, this value improves to 11

Figure 4.2: Convergence characteristics of player's strategy in *Interference Minimization* game

Figure 4.3: Potential function of *Interference Minimization* game

Table 4.2: Strategies of players during initialization, and at Nash equilibrium in *Interference Minimization* game

Player	At initialization	At NE	Player	At initialization	At NE
1	[0, 0, 0, 40, 0]	[0, 0, 40, 0, 0]	11	[0, 0, 0, 0, 40]	[0, 40, 0, 0, 0]
2	[0, 0, 0, 40, 0]	[40, 0, 0, 0, 0]	12	[40, 0, 0, 0, 0]	[0, 0, 0, 0, 40]
3	[0, 0, 40, 0, 0]	[0, 0, 40, 0, 0]	13	[0, 0, 0, 40, 0]	[0, 0, 40, 0, 0]
4	[0, 0, 0, 40, 0]	[40, 0, 0, 0, 0]	14	[40, 0, 0, 0, 0]	[0, 0, 0, 0, 40]
5	[0, 0, 0, 0, 40]	[0, 0, 0, 40, 0]	15	[0, 0, 40, 0, 0]	[0, 0, 40, 0, 0]
6	[0, 40, 0, 0, 0]	[0, 40, 0, 0, 0]	16	[40, 0, 0, 0, 0]	[40, 0, 0, 0, 0]
7	[0, 0, 40, 0, 0]	[0, 0, 40, 0, 0]	17	[0, 0, 40, 0, 0]	[0, 0, 0, 40, 0]
8	[0, 0, 0, 40, 0]	[40, 0, 0, 0, 0]	18	[0, 0, 0, 40, 0]	[0, 40, 0, 0, 0]
9	[0, 0, 0, 40, 0]	[0, 40, 0, 0, 0]	19	[40, 0, 0, 0, 0]	[40, 0, 0, 0, 0]
10	[0, 0, 40, 0, 0]	[0, 0, 40, 0, 0]	20	[0, 0, 0, 40, 0]	[0, 0, 0, 0, 40]

dB due to the reduction in interference.

 Figure 4.4: Average SINRs of players before and after applying *Interference Minimization* game

In addition, the average capacities of players before and after applying the proposed game are also studied and the results are shown in Figure 4.5. As seen from the figure, the average capacity per unit bandwidth of each player is around 3.23 bits/s/Hz during initialization. The corresponding value after applying the proposed game is around 3.86 bits/s/Hz.

Figure 4.5: Average capacities per unit bandwidth of players before and after applying *Interference Minimization* game

4.4.2 Numerical Results of Capacity Maximization Game

The convergence characteristics of players' strategies in *Capacity Maximization* game is shown in Figure 4.6. During initialization, N players randomly allocate their transmit powers on K transmitting channels such that their maximum power constraints are satisfied. As seen from the figure, the players in *Capacity Maximization* game take maximum of 110 steps to reach a steady state, which is also a NE. The initial strategies and the NE strategies of the players in *Capacity Maximization* game are shown in Table 4.3. The convergence of *Capacity Maximization* game is also shown in Figure 4.7, which plots the total capacity per unit bandwidth of players in *Capacity Maximization* game after each step.

The performance of *Capacity Maximization* game is also studied in terms of SINR. The average SINRs of the players in different channels before and after applying *Capacity Maximization* game are shown in Figure 4.8. As seen from the figure, the *Capacity Maximization* game increases the average SINR of each player in each channel significantly. In particular, during initialization (by using random allocation), the average SINR of each

Figure 4.6: Convergence characteristics of players' strategies in *Capacity Maximization* game

Figure 4.7: Total capacity per unit bandwidth of players in *Capacity Maximization* game

Table 4.3: Strategies of players during initialization, and at Nash equilibrium in *Capacity Maximization* game

Player	At initialization	At NE
1	[9.59, 12.78, 4.98, 1.42, 11.23]	[10.38, 0, 9.07, 10.90, 9.64]
2	[6.42, 9.42, 9.36, 5.77, 9.03]	[2.57, 6.01, 17.29, 14.13, 0]
3	[9.71, 8.05, 2.93, 7.85, 11.47]	[9.64, 9.30, 1.87, 9.51, 9.68]
4	[5.40, 9.49, 8.84, 12.30, 3.96]	[8.68, 0, 17.78, 13.55, 0]
5	[7.01, 7.92, 0.54, 12.97, 11.57]	[0, 0, 0, 0, 40]
6	[8.17, 9.42, 10.33, 7.03, 5.04]	[10.05, 0, 10.52, 10.39, 9.04]
7	[1.90, 2.20, 15.54, 8.08, 12.28]	[0, 0, 40, 0, 0]
8	[6.41, 0.16, 12.69, 7.37, 13.37]	[0, 38.05, 0, 0, 1.95]
9	[12.82, 1.54, 8.91, 8.82, 7.91]	[8.29, 0.65, 16.54, 6.86, 7.66]
10	[6.52, 13.86, 2.61, 8.02, 8.98]	[8.24, 0, 0, 0, 31.7600]
11	[0.51, 10.96, 14.31, 6.75, 7.47]	[0, 40, 0, 0, 0]
12	[1.85, 12.64, 18.96, 4.50, 2.06]	[7.72, 4.75, 0, 1.98, 25.55]
13	[2.80, 9.94, 10.90, 9.39, 6.97]	[7.97, 7.99, 8.02, 8.01, 8.01]
14	[5.45, 11.12, 9.75, 11.51, 2.17]	[10, 0, 14.70, 15.30, 0]
15	[8.19, 8.27, 11.30, 10.07, 2.16]	[8.19, 8.27, 11.30, 10.07, 2.16]
16	[22.66, 4.77, 5.20, 0.20, 7.18]	[8.52, 7.42, 7.07, 8.65, 8.33]
17	[5.98, 12.77, 8.64, 4.06, 8.55]	[10.04, 9.50, 0, 10.64, 9.82]
18	[2.41, 12.97, 2.33, 10.59, 11.69]	[0, 40, 0, 0, 0]
19	[0.57, 5.45, 1.17, 31.00, 1.81]	[11.24, 0, 8.36, 8.60, 11.81]
20	[11.54, 7.90, 13.91, 6.21, 0.44]	[11.29, 7.09, 0, 10.91, 10.71]

player in each channel is around -6 dB. This value is around -1.5 dB after applying the proposed *Capacity Maximization* game.

Finally, the average capacities per unit bandwidth of players by using random allocation, traditional *Capacity Maximization* approach using water-filling algorithm and the proposed *Capacity Maximization* game are compared. The results are shown in Figure 4.9. It can be seen that the proposed *Capacity Maximization* game provides the best performance in terms of its capacity. The overall capacities per unit bandwidth of a SU network by using random allocation, traditional *Capacity Maximization* approach and the proposed *Capacity Maximization* game are 83.34, 95.80, and 99.65 bits/s/Hz, respectively.

It is important to note that the average SINR of each player after applying *Capacity*

(a) During initialization by using random allocation

 (b) After applying *Capacity Maximization* game

 Figure 4.8: Average SINRs of the players before and after applying *Capacity Maximization* game

Maximization game (shown in Figure 4.8(b)) is smaller than the average SINR after applying *Interference Minimization* game (shown in Figure 4.4). However, in comparison

Figure 4.9: Average capacities per unit bandwidth of players by using random allocation, traditional *Capacity Maximization* approach and proposed *Capacity Maximization* game

to *Interference Minimization* game, the average capacity of each player after applying *Capacity Maximization* game is larger.

4.5 Summary

In this chapter, a non-cooperative game theoretic framework is studied to jointly allocate transmit power and frequency channels among SUs who have different objectives. The main contributions of this chapter are as follows:

- A *Interference Minimization* game is proposed to meet the objectives of the *data transmission radios*. In this game, the utility function of a player is determined as the benefit that the player earns minus its cost for choosing a specific strategy. The benefit of a player is defined as the total received power, while the cost is defined as the total interference that other players generate on this player and the interference of this player to other players. This game is an exact potential game which will finally converge to a Nash equilibrium by following the best response dynamics.

- A *Capacity Maximization* game is proposed to meet the objectives of the *voice radios*. In this game, the utility function of a player is determined as the total weighted capacity.
- The methods to determine the best responses strategy for a player based on the observed opponent strategies are provided for both proposed games.
- From the numerical results, after applying *Interference Minimization* game, SINR of each player in each channel improves due to the reduction in interference. While after applying *Capacity Maximization* game, the players acquire very high average capacities (higher than those after applying *Interference Minimization* game and traditional *Capacity Maximization* approach using water-filling algorithm).

Chapter 5

Spectrum Trading with Uncertain Primary Service Requirements

5.1 Introduction

The *technical perspective* of cognitive radio networks, specifically on the problems of *spectrum sensing* and *resource allocation* has been studied in the previous chapters. In contrast, this chapter addresses the *economic perspective* of cognitive radio networks, particularly the problem of *spectrum trading*. By definition, spectrum trading is the process of selling and buying spectral resources such as frequency bands, transmit power, time slots, or channel access codes among the primary and secondary users.

The present work introduces a game theoretic framework for the spectrum trading problem in a cognitive radio network with multiple primary users (PUs) who compete with each other to sell the unused spectrum to a secondary user (SU). In contrast to the other works in the literature, the present work assumes variable spectrum requirements for the PUs' services from time to time. Furthermore, the trade agreement is assumed to be made before the realization of these values. As a result, there is a case that the spectrum retained for a PU after the spectrum trading is smaller than that required for its primary service. In such a case, a cost will be charged to the PU due to the primary service degradation. The trading process is assumed to follow the Cournot

oligopoly model of competition [77] by which the sellers (i.e., the PUs) determine the size of spectrum that they want to sell. The market price is then determined by the buyer (i.e., the SU) as a function of the total size of spectrum offered by the sellers. The spectrum trading problem is then extended to a scenario which involves the trading among multiple PUs and multiple SUs. As different SUs have different demands on the spectrum, they might be willing to pay different prices for the same size of spectrum. Similar to the single SU scenario, only the competition among the PUs is considered in this scenario.

The rest of this chapter is organized as follows: The system model and the related assumptions are discussed in Section 5.2. Section 5.3 provides a game theoretic framework for the spectrum trading problem. The extended scenario with multiple SUs is studied in Section 5.4. Section 5.5 provides the numerical results and discussion. Finally, Section 5.6 summarizes this chapter.

5.2 System Model

In this chapter, a spectrum trading model consisting of N ($N \geq 2$) PUs competing with each other to sell spectrum to a SU is studied. The spectrum offered by PUs are assumed to be similar and completely substitutable (i.e., the spectrum are homogeneous). Each PU has a fixed size of available spectrum. At each time period, only a part of the available spectrum is required for the PU's service, and the unused spectrum can be sold to the SU for an extra revenue. In addition, the spectrum requirement for a PU's service is not fixed and is time varying due to the volatility of radio networks. Also, the spectrum requirement for a PU's service is assumed to be a random variable whose value is not observable at the time the spectrum trading occurs. Only its probability distribution is available to the PU.

Figure 5.1: Multiple primary user, single secondary user trading model

The *Cournot game model* of competition [77] is employed in this chapter to model the spectrum trading process. In this model, the PUs compete with each other in terms of the size of spectrum offered to the SU as illustrated in Figure 5.1. The price is then determined by the SU as a function of the total size of spectrum offered by these PUs. Similar to [124, 163], the utility of the SU can be represented as:

$$U_s = \sum_{i=1}^N q_i \alpha_i - \frac{1}{2} \left(\sum_{i=1}^N \beta_i q_i^2 + 2\nu \sum_{i \neq j} q_i q_j \right) - \sum_{i=1}^N p_i q_i, \quad (5.1)$$

where q_i is the size of spectrum bought by the SU from PU i and p_i is the corresponding price, α_i is the earning parameter and is defined as $\alpha_i = k_i^{(s)} \times \ell^{(s)}$ with $k_i^{(s)}$ is the spectral efficiency that the SU gains from the spectrum bought from PU i (and is defined as in [124]), and $\ell^{(s)}$ is the secondary user service earning rate ($\ell^{(s)} > 0$), β_i is the cost parameter ($\beta_i > 0$), and ν is the spectrum substitutability parameter. According to whether ν is greater, equal, or smaller than 0, the spectrum from different PUs are substitutes, independent, or complements. For the spectrum trading scenario considered in this thesis, the spectrum offered by different PUs are completely substitutable. Consequently, as stated in [163], $\alpha_1 = \alpha_2 = \dots = \alpha_N = \alpha$, $\beta_1 = \beta_2 = \dots = \beta_N = \beta$, $p_1 = p_2 = \dots =$

$p_N = p$, and $\nu = \beta$. The utility function of the SU in equation (5.1) can be re-written as:

$$\begin{aligned} U_s &= \alpha \sum_{i=1}^N q_i - \frac{1}{2} \beta \left(\sum_{i=1}^N q_i \right)^2 - p \sum_{i=1}^N q_i \\ &= \alpha q - \frac{1}{2} \beta q^2 - pq, \end{aligned} \quad (5.2)$$

where $q = \sum_{i=1}^N q_i$ is the total size of spectrum bought by the SU. The 1st order derivative of the utility function U_s of the SU with respect to the size of spectrum q is given as

$$\frac{\partial U_s}{\partial q} = \alpha - \beta q - p.$$

At a given price p , the optimal size of spectrum that the SU should buy in order to maximize its utility function is

$$q = \frac{\alpha}{\beta} - \frac{1}{\beta} p = \epsilon - \gamma p, \quad (5.3)$$

where $\epsilon = \frac{\alpha}{\beta}$, $\gamma = \frac{1}{\beta}$. The above equation is known as the spectrum demand of the SU. The price at which the SU is willing to buy a total size of spectrum q offered by PUs is given as:

$$p(q) = \alpha - \beta q = \alpha - \beta \sum_{i=1}^N q_i. \quad (5.4)$$

This price function is assumed to be fixed and is of common knowledge to all PUs.

5.3 Spectrum Trading Competition

5.3.1 Primary User Profit Function

The profit function of a PU for selling spectrum to the SU is studied in this section. Let W_i be the spectrum available for PU i , and B_i be the spectrum requirement for PU i 's service. The revenue that PU i earns for selling a size of spectrum q_i ($W_i \geq q_i \geq 0$) at price p is determined as:

$$R_i = p \times q_i, \quad (5.5)$$

where the price p is determined as in equation (5.4). The revenue of PU i , hence can be rewritten as:

$$R_i = \left[\alpha - \beta \left(q_i + \sum_{j=1, j \neq i}^N q_j \right) \right] q_i. \quad (5.6)$$

The benefit that PU i can earn from its primary service if the spectrum requirement B_i is satisfied is:

$$R_i^{(1)} = \ell_i^{(p)} \times k_i^{(p)} \times B_i, \quad (5.7)$$

where $k_i^{(p)}$ is the spectral efficiency of PU i (which is defined in [124]), and $\ell_i^{(p)}$ is the primary service earning rate ($\ell_i^{(p)} > 0$). After selling spectrum, the size of spectrum retained for PU i is only $W_i - q_i$. As PU i sells the spectrum before observing the actual value of the spectrum requirement for its primary service, there is a case that the size of spectrum available after the spectrum trading is smaller than that required for its primary service, i.e., $W_i - q_i < B_i$. Therefore, the actual benefit that PU i can earn from its primary service is:

$$R_i^{(2)} = \ell_i^{(p)} \times k_i^{(p)} \times \min(B_i, W_i - q_i). \quad (5.8)$$

The difference between $R_i^{(1)}$ and $R_i^{(2)}$ is considered as the cost of spectrum trading, and is represented as:

$$\begin{aligned} C_i(q_i) &= R_i^{(1)} - R_i^{(2)} \\ &= \ell_i^{(p)} \times k_i^{(p)} \times (B_i - \min(B_i, W_i - q_i)) = r_i \times d_i, \end{aligned} \quad (5.9)$$

where $r_i = \ell_i^{(p)} \times k_i^{(p)}$ is defined as the penalty rate, and $d_i = (B_i - \min(B_i, W_i - q_i))$ is defined as the violated spectrum. If the size of spectrum retained for a PU after spectrum trading is larger than or equal to the spectrum required for the PU's service, the violated spectrum d_i is equal to zero. Otherwise, the violated spectrum is determined

as the difference between the spectrum requirement for the PU's service and the size of spectrum available after selling. d_i can be expressed as:

$$d_i = \begin{cases} q_i + B_i - W_i & \text{if } W_i - q_i < B_i \\ 0 & \text{if } W_i - q_i \geq B_i. \end{cases} \quad (5.10)$$

As B_i is a random variable, d_i is also a random variable, with the expected value (mean) given as:

$$\begin{aligned} E[d_i] &= \int_0^{W_i} d_i f_{B_i}(b_i) db_i \\ &= \int_{W_i - q_i}^{W_i} (q_i + b_i - W_i) f_{B_i}(b_i) db_i, \end{aligned} \quad (5.11)$$

where $E[\cdot]$ is the expectation operation, $f_{B_i}(b_i)$ is the probability density function (PDF) of random variable B_i evaluated at b_i ($0 \leq b_i \leq W_i$).

The profit of PU i for selling a size of spectrum q_i at price p is defined as the difference between its revenue and cost, and is given below:

$$\pi_i = R_i - C_i(q_i) = \left[\alpha - \beta \left(q_i + \sum_{j=1, j \neq i}^N q_j \right) \right] q_i - r_i d_i. \quad (5.12)$$

5.3.2 Cournot Game Model

The proposed spectrum trading problem is modelled as a non-cooperative game in which the set of players $\mathcal{N}$ is the set of N PUs ($\mathcal{N} = \{1, 2, \dots, N\}$). The strategies of PUs are the size of spectrum sold to the SU. Let q_i be the strategy taken by player i ($i \in \mathcal{N}$), $q_i \geq 0$. As PU i cannot sell more than its total available spectrum, $q_i \leq W_i$. The set of strategies taken by all other players in the game except player i is denoted as $\mathbf{q}_{-i} = \{q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_N\}$.

The PUs should select strategies in order to maximize their profits. However, the spectrum requirements for the primary services are unknown when the trade agreement is made and the only available parameters for the PUs are the probability distributions.

As a result, instead of maximizing the profit functions, the targets of the PUs are to maximize their expected profits. Hence, the expected profits are selected as the utility functions of the players in the proposed game. The utility function of player i is defined as:

$$\begin{aligned} u_i(q_i, \mathbf{q}_{-i}) &= E[\pi_i] \\ &= \left[\alpha - \beta \left(q_i + \sum_{j=1, j \neq i}^N q_j \right) \right] q_i - r_i E[d_i], \end{aligned} \quad (5.13)$$

where π_i is the profit function of player i and is given as in equation (5.12), and $E[d_i]$ is the expected size of violated spectrum and is defined as in equation (5.11).

The best response function $BR_i(\mathbf{q}_{-i})$ of player i to a set of strategies taken by other players $\mathbf{q}_{-i}$ is defined as:

$$BR_i(\mathbf{q}_{-i}) = \arg \max_{0 \leq q_i \leq W_i} u_i(q_i, \mathbf{q}_{-i}). \quad (5.14)$$

Determining $BR_i(\mathbf{q}_{-i})$ is presented as follows. The 1st order derivative of $u_i(q_i, \mathbf{q}_{-i})$ with respect to q_i is determined as:

$$\frac{\partial u_i(q_i, \mathbf{q}_{-i})}{\partial q_i} = -2\beta q_i + (\alpha - \beta \sum_{j=1, j \neq i}^N q_j) - r_i \frac{\partial E[d_i]}{\partial q_i}, \quad (5.15)$$

where

$$\begin{aligned} \frac{\partial E[d_i]}{\partial q_i} &= (q_i + (W_i - q_i) - W_i) f_{B_i}(W_i - q_i) + \int_{W_i - q_i}^{W_i} f_{B_i}(b_i) db_i \\ &= 0 + \int_{W_i - q_i}^{W_i} f_{B_i}(b_i) db_i = 1 - F_{B_i}(W_i - q_i), \end{aligned} \quad (5.16)$$

where $F_{B_i}(b_i)$ is the cumulative distribution function (CDF) of random variable B_i evaluated at b_i .

In order to ease the computational complexity, assuming that B_i is uniformly distributed over the range $[0, W_i]$. Its probability density function $f_{B_i}(b_i)$ and cumulative

distribution function $F_{B_i}(b_i)$ are given as:

$$f_{B_i}(b_i) = \frac{1}{W_i}, \quad (5.17)$$

$$F_{B_i}(b_i) = \frac{b_i}{W_i}. \quad (5.18)$$

Replacing equation (5.18) into equation (5.16),

$$\frac{\partial E[d_i]}{\partial q_i} = 1 - \frac{W_i - q_i}{W_i} = \frac{q_i}{W_i}. \quad (5.19)$$

Replacing equation (5.19) into equation (5.15),

$$\frac{\partial u_i(q_i, \mathbf{q}_{-i})}{\partial q_i} = -2\beta q_i + (\alpha - \beta \sum_{j=1, j \neq i}^N q_j) - r_i \frac{q_i}{W_i}. \quad (5.20)$$

The optimal size of spectrum to maximize player i 's utility function can be determined by setting 1st order derivative of $u_i(q_i, \mathbf{q}_{-i})$ to 0, and is given as:

$$q_{i,opt} = \frac{\alpha - \beta \sum_{j=1, j \neq i}^N q_j}{2\beta + \frac{r_i}{W_i}}. \quad (5.21)$$

It is important to note that the second order derivative of $u_i(q_i, \mathbf{q}_{-i})$ is always negative.

That is,

$$\frac{\partial^2 u_i(q_i, \mathbf{q}_{-i})}{\partial q_i^2} = -2\beta - \frac{r_i}{W_i}. \quad (5.22)$$

Thus, $u_i(q_i, \mathbf{q}_{-i})$ is a concave function of q_i , and $u_i(q_i, \mathbf{q}_{-i})$ will increase at any $q_i < q_{i,opt}$.

It means that player i should sell a size of spectrum equal to his total size of spectrum W_i , when $W_i < q_{i,opt}$. Consequently, the best response function of player i given the strategies taken by other players can be determined as:

$$BR_i(\mathbf{q}_{-i}) = \min \{W_i, q_{i,opt}\}. \quad (5.23)$$

Next, the Nash equilibrium of the proposed game is studied. By definition, *Nash equilibrium* (NE) of a game is a strategy profile where no player can increase his utility

by changing his strategy unilaterally [77]. In other words, in NE, every player plays the best response against other players. The set $\mathbf{q}^* = \{q_1^*, \dots, q_N^*\}$ denotes the NE of this game if and only if

$$q_i^* = BR_i(\mathbf{q}_{-i}^*), \quad \forall i \in \mathcal{N}, \quad (5.24)$$

where $\mathbf{q}_{-i}^*$ is the set of best responses for other players in the game except player i .

Proposition 5.1 *The proposed game has at least one NE.*

Proof: For every player i ($i \in \mathcal{N}$),

- Its set of strategies is a compact and convex set (as $0 \leq q_i \leq W_i$).
- Its utility function $u_i(q_i, \mathbf{q}_{-i})$ is a concave function of q_i .

Consequently, the proposed game has at least one NE [164]. ■

Proposition 5.2 *The proposed game has a unique NE.*

Proof: A weighted non-negative sum of the utility function $u_i(q_i, \mathbf{q}_{-i})$ of player i is defined as:

$$\delta(\mathbf{q}, \mathbf{s}) = \sum_{i=1}^N s_i u_i(q_i, \mathbf{q}_{-i}), \quad (5.25)$$

where $\mathbf{s} = [s_1, s_2, \dots, s_N]^T$ is a non-negative vector, $\mathbf{s} \in \mathbb{R}^N$. As stated in [164], if the function $\delta(\mathbf{q}, \mathbf{s})$ is *diagonally strictly concave* for some $\mathbf{s} = \bar{\mathbf{s}} > 0$, then the proposed game has a unique NE. The function $\delta(\mathbf{q}, \mathbf{s})$ is said to be diagonally strictly concave when for every $\mathbf{q}^0, \mathbf{q}^1 \in \mathbb{R}^N$, $\mathbf{q}^0 \neq \mathbf{q}^1$ then:

$$\chi = (\mathbf{q}^1 - \mathbf{q}^0)^T (g(\mathbf{q}^0, \mathbf{s}) - g(\mathbf{q}^1, \mathbf{s})) > 0, \quad (5.26)$$

where $g(\mathbf{q}, \mathbf{s})$ is the pseudogradient of $\delta(\mathbf{q}, \mathbf{s})$, and is determined as:

$$g(\mathbf{q}, \mathbf{s}) = \begin{bmatrix} s_1 \frac{\partial u_1(\mathbf{q})}{\partial q_1} \\ s_2 \frac{\partial u_2(\mathbf{q})}{\partial q_2} \\ \vdots \\ s_N \frac{\partial u_N(\mathbf{q})}{\partial q_N} \end{bmatrix}. \quad (5.27)$$

χ defined in equation (5.26) can be re-written as:

$$\chi = \sum_{i=1}^N (q_i^1 - q_i^0) \left(s_i \frac{\partial u_i(\mathbf{q})}{\partial q_i^0} - s_i \frac{\partial u_i(\mathbf{q})}{\partial q_i^1} \right) \quad (5.28)$$

$$\begin{aligned} &= \sum_{i=1}^N s_i (q_i^1 - q_i^0) \left\{ \left[-2\beta q_i^0 + \left(\alpha - \beta \sum_{j=1, j \neq i}^N q_j^0 \right) - \frac{r_i}{W_i} q_i^0 \right] \right. \\ &\quad \left. - \left[-2\beta q_i^1 + \left(\alpha - \beta \sum_{j=1, j \neq i}^N q_j^1 \right) - \frac{r_i}{W_i} q_i^1 \right] \right\} \end{aligned} \quad (5.29)$$

$$= \sum_{i=1}^N s_i (q_i^1 - q_i^0) \left\{ \beta \left(\sum_{j=1}^N q_j^1 - \sum_{j=1}^N q_j^0 \right) + \left(\beta + \frac{r_i}{W_i} \right) (q_i^1 - q_i^0) \right\} \quad (5.30)$$

$$= \sum_{i=1}^N s_i (q_i^1 - q_i^0)^2 \left(\beta + \frac{r_i}{W_i} \right) + \sum_{i=1}^N s_i (q_i^1 - q_i^0) \beta \sum_{j=1}^N (q_j^1 - q_j^0). \quad (5.31)$$

As a result, when $s_1 = s_2 = \dots = s_N = \bar{s} > 0$,

$$\chi = \bar{s} \sum_{i=1}^N (q_i^1 - q_i^0)^2 \left(\beta + \frac{r_i}{W_i} \right) + \beta \bar{s} \sum_{i=1}^N (q_i^1 - q_i^0)^2 > 0. \quad (5.32)$$

■

For a static game, when the players have complete knowledge on the utility functions and sets of strategies of other players, the NE can be obtained by solving the set of equations (5.24).

5.3.3 Dynamic Cournot Game

The Nash equilibrium in the previous section can be obtained only if all players have complete information about the utility functions of other players. However, this condition is quite infeasible in practical cognitive radio networks. This necessitates the requirement of an iterative algorithm, which allows the players to learn from the past and adaptively change their strategies to reach the NE. The details of such an algorithm is discussed in the following section.

Let $q_i(t)$ and $u_i(t)$ respectively be the strategy taken by player i and its utility at iteration t . It is assumed that the players can observe the strategies adopted by other

players in the previous iteration. The strategy of player i at *iteration* $(t + 1)$ can be obtained as:

$$q_i(t + 1) = \frac{\partial u_i(t)}{\partial q_i(t)} \theta + q_i(t), \quad (5.33)$$

where θ is the learning rate ($\theta > 0$), and $\frac{\partial u_i(t)}{\partial q_i(t)}$ is defined as in equation (5.20), and is given as:

$$\frac{\partial u_i(t)}{\partial q_i(t)} = -2\beta q_i(t) + \left[\alpha - \beta \sum_{j=1, j \neq i}^N q_j(t) \right] - \frac{r_i}{W_i} q_i(t). \quad (5.34)$$

When $q_i(t + 1) > W_i$, $q_i(t + 1)$ is set to W_i , and when $q_i(t + 1) < 0$, $q_i(t + 1)$ is set to 0.

The stability of the proposed iterative algorithm will be examined subsequently. The ability of the proposed algorithm to reach its Nash equilibrium is determined by this analysis. As in [165], this algorithm is stable if and only if all eigenvalues of the following Jacobian matrix have the absolute value smaller than 1.

$$J = \begin{bmatrix} \frac{\partial q_1(t+1)}{\partial q_1(t)} & \frac{\partial q_1(t+1)}{\partial q_2(t)} & \cdots & \frac{\partial q_1(t+1)}{\partial q_N(t)} \\ \frac{\partial q_2(t+1)}{\partial q_1(t)} & \frac{\partial q_2(t+1)}{\partial q_2(t)} & \cdots & \frac{\partial q_2(t+1)}{\partial q_N(t)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial q_N(t+1)}{\partial q_1(t)} & \frac{\partial q_N(t+1)}{\partial q_2(t)} & \cdots & \frac{\partial q_N(t+1)}{\partial q_N(t)} \end{bmatrix}, \quad (5.35)$$

where,

$$\frac{\partial q_i(t + 1)}{\partial q_j(t)} = \begin{cases} 1 - \left(2\beta + \frac{r_i}{W_i} \right) \theta & \text{for } i = j \\ -\beta \theta & \text{for } i \neq j, \end{cases} \quad (5.36)$$

$\forall i, j \in \mathcal{N}$.

The proposed iterative algorithm can also be applied to the case where only the local information is available for the players. In such a case, the players cannot observe the strategies of the opponents, but only the market price in the previous *iteration*. Let the price at *iteration* t be $p(t)$, it can be represented as:

$$p(t) = \alpha - \beta \left[q_i(t) + \sum_{j=1, j \neq i}^N q_j(t) \right]. \quad (5.37)$$

As a result,

$$\alpha - \beta \sum_{j=1, j \neq i}^N q_j(t) = p(t) + \beta q_i(t). \quad (5.38)$$

From equation (5.33), the strategy of player i at iteration $(t + 1)$ can be determined as:

$$q_i(t + 1) = \left[-2\beta q_i(t) + p(t) + \beta q_i(t) - \frac{r_i}{W_i} q_i(t) \right] \theta + q_i(t), \quad (5.39)$$

which is dependent only on the local information of player i .

5.3.4 Example

In this section, a special case of two PUs ($N = 2$) is considered as an example. First, the NE for the static game is studied. The set of equations in (5.24) can be expressed as follows:

$$q_1^* = \min \left\{ W_1, \frac{\alpha - \beta q_2^*}{2\beta + \frac{r_1}{W_1}} \right\}, \quad (5.40)$$

$$q_2^* = \min \left\{ W_2, \frac{\alpha - \beta q_1^*}{2\beta + \frac{r_2}{W_2}} \right\}. \quad (5.41)$$

The NE of this game can be determined by solving equations (5.40) and (5.41). There are four possible NEs corresponding to four possible cases whose details are given as follows.

Case 1: when W_1 and W_2 are large enough,

$$q_1^* = \frac{\left(\beta + \frac{r_2}{W_2} \right) \alpha}{\left(2\beta + \frac{r_1}{W_1} \right) \left(2\beta + \frac{r_2}{W_2} \right) - \beta^2}, \quad (5.42)$$

$$q_2^* = \frac{\left(\beta + \frac{r_1}{W_1} \right) \alpha}{\left(2\beta + \frac{r_1}{W_1} \right) \left(2\beta + \frac{r_2}{W_2} \right) - \beta^2}. \quad (5.43)$$

Case 2: when q_1^* found in equation (5.42) is larger than W_1 , and W_2 is large enough,

$$q_1^* = W_1, \quad (5.44)$$

$$q_2^* = \frac{\alpha - \beta W_1}{2\beta + \frac{r_2}{W_2}}, \quad (5.45)$$

with the constraint

$$\frac{\alpha - \beta W_1}{2\beta + \frac{r_2}{W_2}} \leq W_2.$$

Case 3: when W_1 is large enough, and q_2^* found in equation (5.43) is larger than W_2 ,

$$q_1^* = \frac{\alpha - \beta W_2}{2\beta + \frac{r_1}{W_1}}, \quad (5.46)$$

$$q_2^* = W_2, \quad (5.47)$$

with the constraint

$$\frac{\alpha - \beta W_2}{2\beta + \frac{r_1}{W_1}} \leq W_1.$$

Case 4: when the constraint on *Case 2* and/or the constraint on *Case 3* are/is not met,

$$q_1^* = W_1, \quad (5.48)$$

$$q_2^* = W_2. \quad (5.49)$$

Next, the dynamic game, when the players have incomplete information about the utility functions of other players, is considered. The strategies of player 1 and player 2 at *iteration* $(t + 1)$ can be respectively determined as:

$$q_1(t + 1) = \frac{\partial u_1(t)}{\partial q_1(t)} \theta + q_1(t), \quad (5.50)$$

$$q_2(t + 1) = \frac{\partial u_2(t)}{\partial q_2(t)} \theta + q_2(t), \quad (5.51)$$

where

$$\frac{\partial u_1(t)}{\partial q_1(t)} = -2\beta q_1(t) + [\alpha - \beta q_2(t)] - \frac{r_1}{W_1} q_1(t), \quad (5.52)$$

$$\frac{\partial u_2(t)}{\partial q_2(t)} = -2\beta q_2(t) + [\alpha - \beta q_1(t)] - \frac{r_2}{W_1} q_2(t). \quad (5.53)$$

The Jacobian matrix of the proposed iterative algorithm (as defined in equation (5.35)) is given as:

$$J = \begin{bmatrix} 1 - \left(2\beta + \frac{r_1}{W_1}\right) \theta & -\beta \theta \\ -\beta \theta & 1 - \left(2\beta + \frac{r_2}{W_2}\right) \theta \end{bmatrix}. \quad (5.54)$$

The eigenvalues of the Jacobian matrix in equation (5.54) are the solutions of the following equation:

$$\lambda^2 + B\lambda + C = 0, \quad (5.55)$$

and are given as

$$\lambda_{1,2} = \frac{-B \pm \sqrt{\Delta}}{2}, \quad (5.56)$$

where

$$\begin{aligned} B &= \left[4\beta + \frac{r_1}{W_1} + \frac{r_2}{W_2} \right] \theta - 2, \\ C &= \left[1 - \left(2\beta + \frac{r_1}{W_1} \right) \theta \right] \left[1 - \left(2\beta + \frac{r_2}{W_2} \right) \theta \right] - \beta^2 \theta^2, \\ \Delta &= B^2 - 4C = \left[4\beta^2 + \left(\frac{r_1}{W_1} - \frac{r_2}{W_2} \right)^2 \right] \theta^2. \end{aligned}$$

$$\begin{aligned} \lambda_{1,2} &= \frac{2 - \theta \left[4\beta + \frac{r_1}{W_1} + \frac{r_2}{W_2} \right] \pm \theta \sqrt{4\beta^2 + \left(\frac{r_1}{W_1} - \frac{r_2}{W_2} \right)^2}}{2} \\ &= 1 - \frac{\theta}{2} \left\{ \left[4\beta + \frac{r_1}{W_1} + \frac{r_2}{W_2} \right] \pm \sqrt{4\beta^2 + \left(\frac{r_1}{W_1} - \frac{r_2}{W_2} \right)^2} \right\}. \end{aligned}$$

Because $\beta, r_1, r_2, W_1, W_2 > 0$, the following inequality is always true

$$\left[4\beta + \frac{r_1}{W_1} + \frac{r_2}{W_2} \right] > \sqrt{4\beta^2 + \left(\frac{r_1}{W_1} - \frac{r_2}{W_2} \right)^2}. \quad (5.57)$$

As a result, the proposed iterative algorithm is stable (i.e., $|\lambda_{1,2}| < 1$) when the learning rate θ is small enough, i.e., when

$$\theta < \frac{4}{\left[4\beta + \frac{r_1}{W_1} + \frac{r_2}{W_2} \right] + \sqrt{4\beta^2 + \left(\frac{r_1}{W_1} - \frac{r_2}{W_2} \right)^2}}. \quad (5.58)$$

5.4 Multiple Secondary Users

The spectrum trading problem is extended to a scenario with N PUs and M SUs, for $N, M > 1$. The sellers (PUs) and the buyers (SUs) directly negotiate for the prices and the size of spectrum to sell as illustrated in Figure 5.2. Furthermore, different SUs may have different demands on spectrum, and hence they might be willing to pay different prices for the same size of spectrum.

Figure 5.2: Multiple primary user, multiple secondary user trading model

Let q_{ik} be the size of spectrum that PU i ($i \in \{1, \dots, N\}$) sells to SU k ($k \in \{1, \dots, M\}$). The total size of spectrum q_k that SU k buys from N PUs is given as:

$$q_k = \sum_{i=1}^N q_{ik}$$

Similar to equation (5.4), the price that SU k pays for a total size of spectrum q_k is determined as:

$$p_k = \alpha_k - \beta_k q_k = \alpha_k - \beta_k \sum_{i=1}^N q_{ik}. \quad (5.59)$$

For simplicity, assume that $\beta_1 = \beta_2 = \dots = \beta_M = \beta$. The above price functions of the SUs are assumed to be of common knowledge to all PUs.

5.4.1 Game Formulation and the Nash Equilibrium

This spectrum trading is also modelled as a non-cooperative game where the set of players $\mathcal{N}$ is the set of N PUs. The strategies of players are the vectors of size equal to the spectrum that they offer to the SUs. For a player i , its strategy $\mathbf{q}_i$ can be represented as:

$$\mathbf{q}_i = [q_{i1}, q_{i2}, \dots, q_{iM}]. \quad (5.60)$$

The total size of the spectrum sold by player i is given as:

$$q_i = \sum_{k=1}^M q_{ik} \quad \text{for } 0 \leq q_i \leq W_i. \quad (5.61)$$

Let $\mathbf{q}_{-i} = \{\mathbf{q}_1, \dots, \mathbf{q}_{i-1}, \mathbf{q}_{i+1}, \dots, \mathbf{q}_N\}$ be the set of strategies taken by all other players in the game except player i . The utility functions of the players are their expected profit functions. The utility function of player i with respect to the strategy profile $\{\mathbf{q}_i, \mathbf{q}_{-i}\}$ is given as:

$$u_i(\mathbf{q}_i, \mathbf{q}_{-i}) = \sum_{k=1}^M p_k q_{ik} - r_i E[d_i], \quad (5.62)$$

where d_i is the size of violated spectrum and is given as in equation (5.10),

$$d_i = \begin{cases} q_i + B_i - W_i & \text{if } W_i - q_i < B_i \\ 0 & \text{if } W_i - q_i \geq B_i. \end{cases}$$

Similar to the previous case with single SU, the set $\mathbf{q}^* = \{\mathbf{q}_1^*, \dots, \mathbf{q}_N^*\}$ is the Nash equilibrium of this game if and only if

$$\mathbf{q}_i^* = BR_i(\mathbf{q}_{-i}^*), \quad \forall i \in \mathcal{N}, \quad (5.63)$$

where $\mathbf{q}_{-i}^*$ is the set of best responses for other players in the game except player i . The best response function $BR_i(\mathbf{q}_{-i})$ of player i to the set of strategies $\mathbf{q}_{-i}$ taken by other players is defined as:

$$BR_i(\mathbf{q}_{-i}) = \arg \max_{\mathbf{q}_i, q_i \leq W_i} u_i(\mathbf{q}_i, \mathbf{q}_{-i}). \quad (5.64)$$

It is important to note that $u_i(\mathbf{q}_i, \mathbf{q}_{-i})$ is a concave function of $\mathbf{q}_i$. The $BR_i(\mathbf{q}_{-i})$ is determined as the solution of an optimization problem whose Lagrangian is defined as:

$$\begin{aligned}\mathcal{L}(\mathbf{q}_i, \mu) &= (-u_i(\mathbf{q}_i, \mathbf{q}_{-i})) + \mu(q_i - W_i) \\ &= -\sum_{k=1}^M p_k q_{ik} + r_i E[d_i] + \mu \left[\sum_{k=1}^M q_{ik} - W_i \right],\end{aligned}\quad (5.65)$$

where μ is the Lagrange multiplier, $\mu \geq 0$. The partial derivative of $\mathcal{L}(\mathbf{q}_i, \mu)$ with respect to q_{ik} for $k = 1, \dots, M$ is given as:

$$\frac{\partial \mathcal{L}(\mathbf{q}_i, \mu)}{\partial q_{ik}} = \left(2\beta + \frac{r_i}{W_i} \right) q_{ik} - \left[\alpha_k - \beta \sum_{n=1, n \neq i}^N q_{nk} \right] + \mu. \quad (5.66)$$

From the Karush-Kuhn-Tucker (KKT) conditions [162], the strategy $\hat{\mathbf{q}}_i = [\hat{q}_{i1}, \dots, \hat{q}_{iM}]$ is the best response strategy of player i with respect to the strategies taken by other players if and only if the following conditions are met:

$$\begin{aligned}\frac{\partial \mathcal{L}(\hat{\mathbf{q}}_i, \mu)}{\partial q_{ik}} &= 0, \quad \forall k \in \{1, \dots, M\}, \\ \mu \left(\sum_{k=1}^M \hat{q}_{ik} - W_i \right) &= 0, \\ \sum_{k=1}^M \hat{q}_{ik} - W_i &\leq 0, \\ \mu \geq 0, \quad \hat{q}_{ik} &\geq 0, \quad \forall k \in \{1, \dots, M\}.\end{aligned}\quad (5.67)$$

In the rest of this section, the method to solve equation (5.67) will be provided.

Firstly, assuming that $\mu = 0$, $\hat{q}_{ik}$ is given as:

$$\hat{q}_{ik} = \frac{\alpha_k - \beta \sum_{n=1, n \neq i}^N q_{nk}}{2\beta + \frac{r_i}{W_i}}. \quad (5.68)$$

If the sum of $\hat{q}_{ik}$ found in equation (5.68) for $k = 1, \dots, M$ is greater than W_i , the above assumption is invalid, and hence $\mu > 0$. Let $\hat{\mathcal{M}} = \{k | k \in \{1, \dots, M\}, \hat{q}_{ik} > 0\}$, and $|\hat{\mathcal{M}}|$

be the cardinality of set $\hat{\mathcal{M}}$. Assuming that $\forall k \in \{1, \dots, M\}$, $\hat{q}_{ik} > 0$, i.e., $|\hat{\mathcal{M}}| = M$,

$$\hat{q}_{ik} = \frac{\alpha_k - \beta \sum_{n=1, n \neq i}^N q_{nk} - \mu}{2\beta + \frac{r_i}{W_i}}, \quad (5.69)$$

where

$$\mu = \frac{\sum_{k \in \hat{\mathcal{M}}} \alpha_k - \beta \sum_{k \in \hat{\mathcal{M}}} \sum_{n=1, n \neq i}^N q_{nk} - (2\beta W_i + r_i)}{|\hat{\mathcal{M}}|}. \quad (5.70)$$

If there is any m ($m \in \hat{\mathcal{M}}$) such that $\hat{q}_{im} < 0$, then $\hat{q}_{im}$ is set to 0 and the set $\hat{\mathcal{M}}$ is updated as:

$$\hat{\mathcal{M}} = \hat{\mathcal{M}} \setminus m.$$

After that, the values of $\hat{q}_{ik}$ are re-calculated (from equation (5.69)) for all k in the newly updated set $\hat{\mathcal{M}}$. The above step will be repeated until $\hat{q}_{ik} \geq 0$, $\forall k \in \hat{\mathcal{M}}$.

5.4.2 Dynamic Game

This section studies a case where the players do not have the complete knowledge about the utility functions of others. In such a case, an iterative algorithm that allows the players to learn from the past and adaptively change their strategies to reach the Nash equilibrium is required. The details of this algorithm are given below.

Let $\mathbf{q}_i(t) = [q_{i1}(t), \dots, q_{iM}(t)]$ be the strategy taken by player i at *iteration* t . Given the strategies taken by other players at *iteration* t , the strategy of player i at *iteration* $(t + 1)$ can be obtained as:

$$q_{ik}(t + 1) = \frac{\partial u_i(t)}{\partial q_{ik}(t)} \theta + q_{ik}(t), \quad k \in \{1, \dots, M\}, \quad (5.71)$$

where

$$\frac{\partial u_i(t)}{\partial q_{ik}(t)} = - \left[2\beta + \frac{r_i}{W_i} \right] q_{ik}(t) + \left[\alpha_k - \beta \sum_{n=1, n \neq i}^N q_{nk}(t) \right]. \quad (5.72)$$

$q_{ik}(t+1)$ is set to 0 for all negative values of $q_{ik}(t+1)$ found in equation (5.71) as q_{ik} must be non-negative, $\forall k \in \{1, \dots, M\}$. In addition, in order to guarantee the constraint $\sum_{k=1}^M q_{ik}(t+1) \leq W_i$, the following adjustment step is required whenever $\sum_{k=1}^M q_{ik}(t+1) > W_i$:

$$q_{ik}(t+1) = \rho q_{ik}(t+1), \quad \forall k \in \{1, \dots, M\}, \quad (5.73)$$

where

$$\rho = \frac{W_i}{\sum_{k=1}^M q_{ik}(t+1)}. \quad (5.74)$$

It is important to note that in the theoretical analysis discussed above, the price function information of the SUs is common knowledge. If the price function information of the SUs is not completely known, there will be two scenarios dependent on the amount of information on the price function available for the PUs.

- In the first scenario, the PUs can observe the past and have the knowledge on the statistical distribution of the price functions, the utility functions of the PUs will be their expected profits over the distribution of the price functions. This scenario can be formulated in the same way as the scenario discussed in this chapter.
- In the second scenario, the PUs have no information on the price functions. In such a scenario, a dynamic interactive game should be considered. In this game, the PUs and SUs will repeatedly play the game until a stable solution is reached (i.e., an NE). Firstly, the PUs will announce the size of spectrum to be sold to the SUs. The SUs will feedback the prices to the PUs. If the prices are low, the PUs will gradually reduce the size of spectrum. If the prices are high, the PUs will increase the size of spectrum in order to earn more profit. A stable solution (i.e., an NE) is achieved when the PUs have no interest to decrease or increase the size of spectrum to be sold.

5.5 Numerical Results

The numerical results on the best response strategies of the players (i.e., the PUs) and the NE for a spectrum trading model which involves multiple PUs and single SU are discussed in this section. It is followed by the numerical results on the stability of the proposed iterative algorithm which allows the players to gradually adjust their strategies to reach the NE. Finally, the numerical analysis on the stability of the proposed iterative algorithm for the extended scenario with multiple SUs is provided. The results present in this section are acquired from simulation by using Monte Carlo method.

5.5.1 Best Response and Nash Equilibrium

The numerical results on the best response strategies of the players and the NE for the single SU scenario are discussed in this section. The parameters for this study are given as:

- The investigated spectrum trading model consists of two PUs ($N = 2$) with each having the same available spectrum, $W_1 = W_2 = 20$ MHz.
- The price function of the SU is $p = 4 - 0.16q$.

This study aims to examine the strategy (i.e., the size of spectrum) that a player (i.e., a PU) should take in response to a strategy taken by the opponent in order to maximize his utility (i.e., the expected profit). For this purpose, the best responses of player 1 ($BR_1(q_2)$) to various strategies taken by player 2 (q_2) are studied, and the results are plotted in Figure 5.3. Similarly, the best responses of player 2 ($BR_2(q_1)$) to various strategies taken by player 1 (q_1) are also shown in Figure 5.3. In addition, the effects of the penalty rates (r_1, r_2) on the decisions of the players are also investigated in this study. In this context, this study is conducted with two different cases, where the penalty rates of player 1 and player 2 are $r_1 = r_2 = 3$, and $r_1 = 5, r_2 = 7$, respectively. As seen

Figure 5.3: Best response strategies of player 1 and player 2 and the Nash equilibrium for different penalty rates

from the figure, the best response function of player 1 is a strictly decreasing function in q_2 . In other words, when the size of spectrum offered by player 2 increases, the best response of player 1 is to decrease its size of spectrum offered to the SU. Similarly, the best response function of player 2 is also a strictly decreasing function in q_1 . In addition, it can be seen from the figure that the players will choose to sell lesser spectrum when their penalty rates are high. In particular, when $r_1 = 3$, the best response of player 1 to a strategy taken by player 2 of $q_2 = 4$ MHz is 7.1489 MHz. The corresponding value is only 5.8947 MHz when $r_1 = 5$. Similarly, the best responses of player 2 to a strategy taken by player 1 of $q_1 = 4$ MHz when $r_2 = 3$ and when $r_2 = 7$ are 7.1489 MHz and 5.0149 MHz, respectively. Furthermore, the Nash equilibrium of the game corresponding to each case is also studied and is shown in Figure 5.3 as a point where the two best response functions intersect. For the first case, the strategies of the players at NE are identical and are given as $q_1^* = q_2^* = 6.3492$ MHz. For the second case, the NE strategies of player 1 and player 2 are $q_1^* = 5.7255$ MHz, and $q_2^* = 4.6029$ MHz, respectively. It is easy to see that, in the second case, $q_2^* < q_1^*$ due to $r_2 < r_1$. Though the given parameters

are selected for demonstration purpose, this behavior can be observed for other values of parameters as well.

5.5.2 Iterative Algorithm

The numerical results on the stability of the proposed iterative algorithm for the single SU scenario are provided in this section. The cognitive radio network under consideration consists of five PUs (i.e., $N = 5$). The spectrum available for these PUs are $W_1 = 20$ MHz, $W_2 = 22$ MHz, $W_3 = 24$ MHz, $W_4 = 26$ MHz, and $W_5 = 28$ MHz, respectively. The penalty rates of these PUs are equal, and are given as $r_1 = r_2 = r_3 = r_4 = r_5 = 5$. The learning rate (θ) is set to 0.5, i.e., $\theta = 0.5$. The price function of the SU is the same as in the previous study, $p = 4 - 0.16q$. The strategies of players after each iteration are studied and are plotted in Figure 5.4. For the initialization, strategies of players

Figure 5.4: Strategies of players at each iteration for single SU scenario

are equally set to 3 MHz. As seen from the figure, the proposed iterative algorithm converges to a steady state after 30 iterations. At this state, the strategies of these players are $q_1 = 3.0832$ MHz, $q_2 = 3.2641$ MHz, $q_3 = 3.4318$ MHz, $q_4 = 3.5877$ MHz, and

$q_5 = 3.7330$ MHz, respectively. It can be derived from equation (5.24) that this steady state is also the NE of the proposed game. The utilities of these players corresponding to their strategies at each iteration are also studied, and are shown in Figure 5.5. From the figure, the utilities of these players are stable after the NE is reached.

Figure 5.5: Utilities of players at each iteration for single SU scenario

In addition, the average number of iterations required for the proposed iterative algorithm to reach the NE is studied for different number of players. The proposed algorithm is assumed to be convergent if the difference in the strategies (i.e., the size of spectrum offered to the SU) taken by each player between two consecutive iterations is smaller than 0.1%. Figure 5.6 plots the average number of iterations when the number of players varies from 2 to 10. In this study, the size of spectrum available for the PUs are randomly selected in the range from 10 MHz to 30 MHz. The other parameters are kept unchanged from the previous study: the penalty rates of the PUs are equal to 5 and the learning rate (θ) is set to 0.5. As seen from the figure, when the number of players varies from 2 to 10, the proposed iterative algorithm takes around 30 iterations to converge. As a result, when the learning rate is small enough, the proposed iterative algorithm always converges for any number of players.

Figure 5.6: Average number of iterations required for the proposed iterative algorithm to converge

5.5.3 Multiple Secondary Users

The stability of the proposed iterative algorithm for the extended scenario with multiple SUs is analyzed for practical scenarios. The parameters for this study are given as:

- The investigated spectrum trading model consists of two PUs ($N = 2$) and two SUs ($M = 2$).
- For the PUs: the spectrum available for PU 1 and PU 2 are $W_1 = 20$ MHz, and $W_2 = 25$ MHz, respectively; the penalty rates of these PUs are $r_1 = r_2 = 5$; and the learning rate is $\theta = 0.5$.
- For the SUs: the price function of SU 1 is $p = 4 - 0.16q$; and the price function of SU 2 is $p = 6 - 0.16q$.

The strategies (the size of spectrum offered by the PUs to the SUs) of players after each iteration are shown in Figure 5.7. From the figure, it can be observed that a steady state is achieved after 30 iterations. At the steady state, the strategies of player 1 and

Figure 5.7: Strategies of players at each iteration for two SU scenario

of player 2 are $\mathbf{q}_1 = [5.31, 7.97]$ (MHz), and $\mathbf{q}_2 = [6.05, 9.08]$ (MHz), respectively. It can easily be derived from equation (5.67) that $\mathbf{q}_1 = [5.31, 7.97]$ is the best response of player 1 to the strategy $\mathbf{q}_2 = [6.05, 9.08]$ taken by player 2. Similarly, $\mathbf{q}_2 = [6.05, 9.08]$ is the best response of player 2 with respect to the strategy $\mathbf{q}_1 = [5.31, 7.97]$ taken by player 1. By definition, the steady state $(\mathbf{q}_1, \mathbf{q}_2)$ is the NE of the proposed game. Finally, the fluctuation of the utilities of these players corresponding to their strategies at each iteration is also studied, and is shown in Figure 5.8. After reaching the NE, the utilities of the players are stable.

5.6 Summary

A game-theoretic framework for spectrum trading problem in a cognitive radio network with multiple PUs and a single SU is studied in this chapter. The study assumes each PU has a fixed size of available spectrum, and only a part of the available spectrum is required for the PU's service at each time period. The unused spectrum can be sold to the SU for an extra revenue. The spectrum requirement for a PU's service, however is

Figure 5.8: Utilities of players at each iteration for two SU scenario

not fixed and is assumed to be a random variable with known probability distribution. In addition, the trading process is carried out before this value is realized. The main contributions of this chapter are as follows:

- A Cournot game model of competition is formulated to analyze the competition among the PUs. In this model, the PUs compete by stating the size of spectrum that they want to sell.
- The Nash equilibrium of a static game where players have the complete knowledge about the utility functions of other players is studied first. In addition, the proofs on the existence and the uniqueness of the Nash equilibrium are also provided.
- Furthermore, an iterative algorithm which allows the players to learn from the past and adjust their strategies to reach the Nash equilibrium is proposed for the practical situation when the players have incomplete information. The stability of this iterative algorithm is also studied.
- The case where two PUs compete to sell spectrum to a SU is given as an example.

- An extended scenario which involves the trading among multiple PUs and multiple SUs is also studied.
- Finally, the numerical results are provided to validate the theoretic analysis.

Chapter 6

Spectrum Trading with Uncertain Spectrum Demand from a Secondary User

6.1 Introduction

The problem of spectrum trading which involves multiple primary users (PUs) competing to sell spectrum to a secondary user (SU) under the uncertainty in the spectrum requirements for the PUs' services was analyzed in the previous chapter. The analysis assumed that the demand from the SU is a deterministic function of price, which is known to all PUs. However, this assumption may not be realistic in practical cognitive radio networks where the spectrum demand of a SU may vary from time to time. In this chapter, a spectrum trading problem, in which both the spectrum requirements for the PUs' services and the spectrum demand of the SU are uncertain, is studied. Instead of competing on the size of spectrum offered to the SU as discussed before, the PUs compete with each other by submitting supply functions to the SU. The supply function states the size of spectrum that a PU is willing to sell at each price. The SU will then determine the market clearing price at which the size of spectrum demanded by the SU exactly matches the aggregate size of spectrum supplied by the PUs. As the spectrum demand is only realized after the trade agreement is made, the PUs must choose the

supply functions to maximize their profits for each realization of spectrum demand by the SU.

The rest of this chapter is organized as follows: In Section 6.2, the system model and assumptions are discussed. Section 6.3 provides a game theoretic framework for spectrum trading problem. Section 6.4 provides the numerical results on the proposed spectrum trading model. Finally, the summary of this chapter is provided in Section 6.5.

6.2 System Model and Assumptions

As mentioned before, a spectrum trading problem with uncertain primary service requirements and secondary user spectrum demand is studied in this chapter. Similar to the previous chapter, a spectrum trading model which consists of N PUs ($N \geq 2$) competing to sell spectrum to a SU is considered. The spectrum offered by PUs is assumed to be similar and completely substitutable. It also assumes that each PU has a fixed size of available spectrum and only a part of the available spectrum is required for the PU's service at each period of time. As in the previous chapter, the spectrum requirements for the PUs' services are not fixed and are time varying due to the volatility of radio networks. In addition, the PUs are unable to observe these values when the spectrum trading occurs. As a result, the size of spectrum available for a PU after the spectrum trading may be smaller than that required for its primary service. A cost must be imposed on the PU in such cases due to the degradation of primary service.

Let W_i , B_i be the available spectrum of PU i ($i = 1, 2, \dots, N$) and the spectrum requirement for PU i 's service ($0 \leq B_i \leq W_i$), respectively. As in the previous chapter, the profit of PU i for selling the spectrum is defined as the difference between its revenue and cost. Let q_i ($0 \leq q_i \leq W_i$) be the size of spectrum that PU i sells at price p ($0 \leq p$), its profit is given as:

$$\pi_i(q_i, p) = R_i(q_i, p) - C_i(q_i), \quad (6.1)$$

where R_i is the spectrum selling revenue (given as in equation (5.5)), and C_i is its cost. The cost of PU i for selling a size of spectrum q_i is defined as in equation (5.9). The expected cost of PU i is given as:

$$\begin{aligned}\overline{C}_i(q_i) &= E[C_i] = r_i E[d_i] = r_i \int_0^{W_i} d_i f_{B_i}(b_i) db_i \\ &= r_i \int_{W_i - q_i}^{W_i} (q_i + b_i - W_i) f_{B_i}(b_i) db_i,\end{aligned}\quad (6.2)$$

where r_i is the penalty rate ($r_i > 0$), d_i is the size of violated spectrum (given as in equation (5.10)), and $f_{B_i}(b_i)$ is the probability density function (PDF) of random variable B_i evaluated at b_i . The expected marginal cost, which is defined as the expected cost for providing an extra unit size of spectrum, is given as:

$$\begin{aligned}\overline{MC}_i(q_i) &= \frac{\partial \overline{C}_i(q_i)}{\partial q_i} = r_i \int_{W_i - q_i}^{W_i} f_{B_i}(b_i) db_i \\ &= r_i (1 - F_{B_i}(W_i - q_i)),\end{aligned}\quad (6.3)$$

where $F_{B_i}(\cdot)$ is the cumulative distribution function (CDF) of B_i . The expected profit of PU i can be determined as:

$$\overline{\pi}_i(q_i, p) = E[\pi_i(q_i, p)] = pq_i - \overline{C}_i(q_i). \quad (6.4)$$

In **Chapter 5** we have discussed the Cournot model for spectrum trading in which the PUs compete by submitting the size of spectrum they want to sell to the SU. However, the outcome of this model is dependent on the spectrum demand of the SU, which is assumed to be a deterministic function of price, and is of common knowledge to all PUs. Instead, here the spectrum demand of the SU is assumed to be uncertain, and is given as

$$D(p, \epsilon) = D_0(p) + \epsilon, \quad (6.5)$$

where ϵ is a random variable which is distributed over a range $(\underline{\epsilon}, \bar{\epsilon})$, and $D_0(p)$ is a deterministic function of price p . Assume that $D_0(p)$ is linear in p with constant slope,

$\gamma = \frac{\partial D_0(p)}{\partial p}$. Resulting from the fact that the spectrum demand of the SU decreases when the price increases, γ must be negative, i.e., $\gamma < 0$. The demand of the SU is assumed to be realized only after the trade agreement is made. As a result, the Cournot model may not be suitable to model this scenario of spectrum trading. The *Supply Function Equilibrium* model, which was initially studied in [13] to formulate the competition among generators in the electricity market, is used to model the spectrum trading process here. In the proposed trading model, the PUs compete by submitting supply functions which state the size of spectrum that they are willing to sell at each price to the SU as illustrated in Figure 6.1.

Figure 6.1: Supply Function Equilibrium Spectrum Trading Model

Once the supply functions from the PUs are received, the SU will determine the market clearing price at a level where its spectrum demand is equal to the aggregate spectrum supply from the PUs. The PUs must sell the size of spectrum determined by their supply functions at the market clearing price. It is important to note the existence of a market clearing price corresponding to each realization of spectrum demand by the SU. The PUs must select the supply functions in order to maximize their expected profits (given as in equation (6.4)) for each realization of demand.

Let $q_i(p)$ be the supply function of PU i ($i \in \mathcal{N}$), $q_i(p)$ is defined as $q_i(p) : (0, \bar{p}) \rightarrow (0, W_i)$, where $\bar{p}$ is the price cap and is defined as the level where the spectrum demand

of the SU is always zero, i.e.,

$$D_0(\bar{p}) + \bar{\epsilon} = 0. \quad (6.6)$$

In addition, $q_i(p)$ must be continuous and non-decreasing in p , i.e.,

$$\frac{\partial q_i(p)}{\partial p} \geq 0. \quad (6.7)$$

Given the set of supply functions from the PUs, for each realization of ϵ , there is a market clearing price p^* , which satisfies

$$D(p^*, \epsilon) = \sum_{i=1}^N q_i(p^*). \quad (6.8)$$

6.3 Resource Trading Competition and Solution

The spectrum trading problem is modelled as a non-cooperative game in which:

- The set of players $\mathcal{N}$ is the set of N PUs ($\mathcal{N} = \{1, 2, \dots, N\}$).
- The strategies of the players are their supply functions, which state the size of spectrum that they are willing to sell at each price. The strategy of player i ($i \in \mathcal{N}$) is represented as $q_i(p) : (0, \bar{p}) \rightarrow (0, W_i)$, which is a continuous and non-decreasing function of price p . The set of strategies taken by all other players in the game except player i is denoted as $\mathbf{q}_{-i}(p) = \{q_1(p), \dots, q_{i-1}(p), q_{i+1}(p), \dots, q_N(p)\}$.
- The utility functions of the players are their expected profit functions, which are defined as in equation (6.4). The utility function of player i is represented as:

$$u_i(q_i(p), \mathbf{q}_{-i}(p)) = \bar{\pi}_i(q_i(p), p) = pq_i(p) - \bar{C}_i(q_i(p)), \quad (6.9)$$

where p is the market clearing price, and is determined by the SU at a level of price where the SU's spectrum demand is equal to the aggregate spectrum supply from the PUs. For each realization of SU's spectrum demand (i.e., realization of

ϵ), p can be determined by using the market clearing condition in equation (6.8) as follows:

$$D(p, \epsilon) = \sum_{i=1}^N q_i(p). \quad (6.10)$$

6.3.1 Nash equilibrium

The players should select strategies (supply functions) to maximize their utility functions (i.e., their expected profits) for each realization of ϵ . In order to analyze the outcome of this game, the concept of Nash equilibrium (NE), which is defined as a strategy profile where no player can increase his utility by changing his strategy unilaterally, is employed. In NE, every player plays the best response against other players.

Let $BR_i(\mathbf{q}_{-i}(p))$ be the best response of player i ($i \in \mathcal{N}$) to a set of strategies taken by other players, $\mathbf{q}_{-i}(p)$. $BR_i(\mathbf{q}_{-i}(p))$ is defined as:

$$BR_i(\mathbf{q}_{-i}(p)) = \arg \max_{q_i(p)} u_i(q_i(p), \mathbf{q}_{-i}(p)), \quad \forall \epsilon \in (\underline{\epsilon}, \bar{\epsilon}). \quad (6.11)$$

Details to determine the best response of player i are provided as follows. For a given set of strategies taken by other players $\mathbf{q}_{-i}(p)$ with a market clearing price of p , player i should supply a size of spectrum which is equal to the residual demand from the SU and is given as in the following equation to meet the market clearing condition:

$$q_i(p) = D(p, \epsilon) - \sum_{j=1, j \neq i}^N q_j(p). \quad (6.12)$$

From equations (6.9) and (6.12), the utility function of player i can be represented as a function of price p as:

$$u_i(p, \mathbf{q}_{-i}(p)) = p \left[D(p, \epsilon) - \sum_{j=1, j \neq i}^N q_j(p) \right] - \bar{C}_i \left(D(p, \epsilon) - \sum_{j=1, j \neq i}^N q_j(p) \right). \quad (6.13)$$

The optimal market clearing price, which provides the highest utility (expected profit) to player i , can be determined by taking the 1st order derivative of $u_i(p, \mathbf{q}_{-i}(p))$ with

respect to price p and setting it to 0. This can be given as:

$$\begin{aligned} \frac{\partial u_i(p, \mathbf{q}_{-i}(p))}{\partial p} &= \left[\frac{\partial D(p, \epsilon)}{\partial p} - \sum_{j=1, j \neq i}^N \frac{\partial q_j(p)}{\partial p} \right] \left[p - \overline{MC}_i \left(D(p, \epsilon) - \sum_{j=1, j \neq i}^N q_j(p) \right) \right] \\ &\quad + \left[D(p, \epsilon) - \sum_{j=1, j \neq i}^N q_j(p) \right] \\ &= 0, \end{aligned} \quad (6.14)$$

where $\overline{MC}_i(\cdot)$ is the expected marginal cost of player i and is defined as in equation (6.3), and

$$\frac{\partial D(p, \epsilon)}{\partial p} = \frac{\partial D_0(p)}{\partial p} = \gamma. \quad (6.15)$$

In addition, by replacing equation (6.12) where $q_i(p) = D(p, \epsilon) - \sum_{j=1, j \neq i}^N q_j(p)$ into equation (6.14), the relation between the optimal market clearing price (p) and the optimal size of spectrum ($q_i(p)$) that player i should sell at that price can be derived as:

$$\left[\gamma - \sum_{j=1, j \neq i}^N \frac{\partial q_j(p)}{\partial p} \right] [p - \overline{MC}_i(q_i(p))] + q_i(p) = 0. \quad (6.16)$$

In order to ease the computational complexity, the spectrum requirement for player i 's service (B_i) is assumed to be uniformly distributed over a range from 0 to W_i , as in the previous chapter. Its PDF ($f_{B_i}(b_i)$) and CDF ($F_{B_i}(b_i)$) are defined as in equations (5.17) and (5.18). Hence, $\overline{MC}_i(q_i)$ can be determined as:

$$\overline{MC}_i(q_i) = r_i(1 - F_{B_i}(W_i - q_i)) = \frac{r_i}{W_i} q_i. \quad (6.17)$$

Replacing equation (6.17) into equation (6.16), the best response of player i to a set of strategies taken by other players can be derived as follows:

$$q_i(p) = \left[\sum_{j=1, j \neq i}^N \frac{\partial q_j(p)}{\partial p} - \gamma \right] \left[p - \frac{r_i}{W_i} q_i(p) \right]. \quad (6.18)$$

Consequently, any set of supply functions $\mathbf{q}^*(p) = \{q_1^*(p), \dots, q_N^*(p)\}$, which satisfies the following conditions:

$$\begin{aligned} q_i^*(p) &: (0, \bar{p}) \rightarrow (0, W_i), \quad \forall i \in \mathcal{N}, \\ \frac{\partial q_i^*(p)}{\partial p} &\geq 0, \quad \forall i \in \mathcal{N}, \\ q_i^*(p) &= \left[\sum_{j=1, j \neq i}^N \frac{\partial q_j^*(p)}{\partial p} - \gamma \right] \left[p - \frac{r_i}{W_i} q_i^*(p) \right], \quad \forall i \in \mathcal{N}, \end{aligned} \quad (6.19)$$

is an NE of this game.

Proposition 6.1 *If $\mathbf{q}^*(p) = \{q_1^*(p), \dots, q_N^*(p)\}$ is an NE of the proposed game, for any market clearing price p ($0 \leq p \leq \bar{p}$), the following condition must be held*

$$p \geq \frac{r_i}{W_i} q_i^*(p), \quad \forall i \in \mathcal{N}. \quad (6.20)$$

Proof: Because $\frac{\partial q_i^*(p)}{\partial p} \geq 0$, $\forall i \in \mathcal{N}$ and $\gamma < 0$, the following inequality is held

$$\frac{\partial q_i^*(p)}{\partial p} - \gamma > 0, \quad \forall i \in \mathcal{N}. \quad (6.21)$$

As a result, from equation (6.19), the condition $q_i^*(p) \geq 0$ is held only if,

$$p - \frac{r_i}{W_i} q_i^*(p) \geq 0.$$

■

It is important to note that the proposed game has several Nash equilibria. However, in the scope of this chapter, only the case where the supply functions of the PUs are linear functions of price is considered. The supply function of PU i ($i \in \mathcal{N}$) is defined as:

$$q_i(p) = a_i p, \quad (6.22)$$

where a_i is a constant, $a_i > 0$. Then, equation (6.19) can be rewritten as:

$$a_i p = \left[\sum_{j=1, j \neq i}^N a_j - \gamma \right] \left[p - \frac{r_i}{W_i} a_i p \right], \quad \forall i \in \mathcal{N}. \quad (6.23)$$

As a result, the condition on the slopes of these supply functions is obtained as:

$$a_i = \left[\sum_{j=1, j \neq i}^N a_j - \gamma \right] \left[1 - \frac{r_i}{W_i} a_i \right], \quad \forall i \in \mathcal{N}. \quad (6.24)$$

The set of linear supply functions $\mathbf{q}^*(p) = \{q_1^*(p), \dots, q_N^*(p)\}$ where $q_i^*(p) = a_i^* p$ with a_i^* obtained from equation (6.24) is an NE of this game.

The proposed *Supply Function Equilibrium* model is strategically equivalent to multi-unit uniform price auctions. However, while in the proposed model, the trading goods (i.e., the spectrum) are divisible, in multi-unit auctions the goods are indivisible. The comparisons on Cournot model, Supply Function Equilibrium model, and multi-unit auction model are provided in Table 6.1.

6.3.2 Iterative algorithm to achieve Nash equilibrium

In the previous section, we have discussed a method to determine NE of a game when the players are assumed to have complete information on utility functions of other players. However, this assumption is not practical in cognitive radio networks. An iterative algorithm which allows the players to learn from the past and adaptively change their strategies to reach the NE is discussed in this section.

Let $a_i(t)$ denote the slope of the supply function taken by player i at *iteration* t . It is assumed that the players can observe the strategies adopted by other players in the previous iteration. The strategy of player i at *iteration* $(t + 1)$ can be obtained as

$$\begin{aligned} a_i(t+1) &= \left[\sum_{j=1, j \neq i}^N a_j(t) - \gamma \right] \left[1 - \frac{r_i}{W_i} a_i(t+1) \right] \\ &= \frac{\sum_{j=1, j \neq i}^N a_j(t) - \gamma}{1 + \frac{r_i}{W_i} \left[\sum_{j=1, j \neq i}^N a_j(t) - \gamma \right]}. \end{aligned} \quad (6.25)$$

The stability of proposed iterative algorithm is studied to check whether the NE can be reached at the steady state or not. As in [165], this algorithm is stable if and only if

Table 6.1: Comparisons on Cournot model, Supply Function Equilibrium model, and Multi-unit auction model

	Cournot model	Supply Function Equilibrium (SFE) model	Multi-unit auction
Characteristics	<p>Sellers (the PUs) compete by stating the quantities of goods (i.e., the size of spectrum) offered to the buyer (the SU).</p> <p>The market price is determined at the level where demand is equal to aggregate quantity of goods offered by sellers.</p>	<p>Sellers compete by submitting the supply functions which state the quantity of goods that the sellers are obligated to sell at any price.</p> <p>The market price is determined by the intersection of the aggregate supply curve and the demand curve.</p> <p>SFE model is strategically equivalent to multi-unit uniform price auctions.</p>	<p>Sellers/bidders bid sequences of prices and the quantities of goods to the buyer/auctioneer.</p> <p>Price can be either uniform price or pay-as-bid (discriminator) price.</p>
Suitability	Applicable when the sellers have common knowledge and when demand is known.	Applicable when the demand is uncertain and the goods are divisible.	Applicable when the demand is uncertain and the goods are indivisible.
Advantages	Easy to formulate and analyze the results.	Results are less sensitive to the demand.	Results are less sensitive to the demand.
Disadvantages	Results are dependent on the demand.	Complex and very hard to find a closed form NE solution for non-linear supply functions.	Complex and very hard (if not impossible) to find a closed form NE solution.

all eigenvalues of the following Jacobian matrix have the absolute value smaller than 1.

$$J = \begin{bmatrix} \frac{\partial a_1(t+1)}{\partial a_1(t)} & \frac{\partial a_1(t+1)}{\partial a_2(t)} & \cdots & \frac{\partial a_1(t+1)}{\partial a_N(t)} \\ \frac{\partial a_2(t+1)}{\partial a_1(t)} & \frac{\partial a_2(t+1)}{\partial a_2(t)} & \cdots & \frac{\partial a_2(t+1)}{\partial a_N(t)} \\ \vdots & \vdots & & \vdots \\ \frac{\partial a_N(t+1)}{\partial a_1(t)} & \frac{\partial a_N(t+1)}{\partial a_2(t)} & \cdots & \frac{\partial a_N(t+1)}{\partial a_N(t)} \end{bmatrix}, \quad (6.26)$$

where

$$\frac{\partial a_i(t+1)}{\partial a_j(t)} = \begin{cases} \frac{1}{\left[1 + \frac{r_i}{W_i} \left(\sum_{j=1, j \neq i}^N a_j(t) - \gamma\right)\right]^2} & \text{for } i \neq j \\ 0 & \text{for } i = j, \end{cases} \quad (6.27)$$

for all $i, j \in \{1, \dots, N\}$.

It is important to note that it is practically very hard to find the eigenvalues for large matrices. There are two alternatives that can be employed to evaluate the stability of the dynamic algorithm. The first is to study the upper bound on the largest eigenvalue of the Jacobian matrix. There are some methods available to determine such upper bound such as [166–168]. However, the upper bound must be as closed as possible so that the stability can be analyzed correctly. The second method is to use the Lyapunov's direct method [165]. If a Lyapunov function can be found for the dynamic algorithm, we can conclude the stability of this algorithm.

6.3.3 Example

In this section, a special case of two PUs ($N = 2$) is studied and is given as an example. First, the NE for the static game is studied. The set of equations in (6.24) can be rewritten as:

$$a_1 = [a_2 - \gamma] \left[1 - \frac{r_1}{W_1} a_1 \right], \quad (6.28)$$

$$a_2 = [a_1 - \gamma] \left[1 - \frac{r_2}{W_2} a_2 \right]. \quad (6.29)$$

From equations (6.28) and (6.29), a_1 is determined as the solution of the following equation

$$A_1 a_1^2 + B_1 a_1 + C_1 = 0, \quad (6.30)$$

where

$$\begin{aligned} A_1 &= \frac{r_1}{W_1} + \frac{r_2}{W_2} - \gamma \left(\frac{r_1}{W_1} \right) \left(\frac{r_2}{W_2} \right), \\ B_1 &= \left(\frac{r_1}{W_1} \right) \left(\frac{r_2}{W_2} \gamma^2 - 2\gamma \right), \\ C_1 &= - \left(\frac{r_2}{W_2} \gamma^2 - 2\gamma \right). \end{aligned}$$

Because $r_1, W_1, r_2, W_2 > 0$ and $\gamma < 0$, the following inequality is held

$$\frac{C_1}{A_1} = - \frac{\left(\frac{r_2}{W_2} \gamma^2 - 2\gamma \right)}{\frac{r_1}{W_1} + \frac{r_2}{W_2} - \gamma \left(\frac{r_1}{W_1} \right) \left(\frac{r_2}{W_2} \right)} < 0. \quad (6.31)$$

Consequently, equation (6.30) has one positive and one negative solutions. As $a_1 > 0$, only the positive solution is selected. Once a_1 is determined, replace a_1 into equation (6.29) to obtain a_2 .

Next, the stability of the proposed iterative algorithm for the dynamic game when the players do not have complete information on the utility functions of other players is examined. The Jacobian matrix of the proposed algorithm (which is defined as in equation (6.26)) is given as

$$J = \begin{bmatrix} 0 & \frac{1}{\left[1 + \frac{r_1}{W_1} (a_2(t) - \gamma) \right]^2} \\ \frac{1}{\left[1 + \frac{r_2}{W_2} (a_1(t) - \gamma) \right]^2} & 0 \end{bmatrix}. \quad (6.32)$$

The eigenvalues of the Jacobian matrix in equation (6.32) are

$$\lambda_{1,2} = \pm \frac{1}{\left[1 + \frac{r_1}{W_1} (a_2(t) - \gamma) \right] \left[1 + \frac{r_2}{W_2} (a_1(t) - \gamma) \right]}. \quad (6.33)$$

As $\frac{r_1}{W_1}, \frac{r_2}{W_2} > 0$ and $a_1(t), a_2(t) > 0$, $\gamma < 0$, the stability condition, $|\lambda_{1,2}| < 1$, is always met. As a result, the proposed iterative algorithm converges to the NE for the special case of two PUs.

6.4 Numerical Results

In this section, the numerical results on the best response strategies of the players, and on the NE are discussed. It is followed by a discussion on the numerical results on the stability of the proposed iterative algorithm which allows the players to gradually adjust their strategies to reach the NE. The results present in this section are acquired from simulation by using Monte Carlo method.

6.4.1 Best Response and Nash Equilibrium

The best response strategies of the PUs and the NE of the proposed spectrum trading model are studied in this section. The strategies of PUs are their supply functions, which are assumed to be linear functions of price, i.e., the strategy of player i ($i \in \mathcal{N}$) is denoted as $q_i(p) = a_i p$ (MHz). Hence, a_i is the only strategic variable of player i . Following are the simulation parameters used for this analysis.

- The investigated spectrum trading model consists of two PUs ($\mathcal{N} = \{1, 2\}$).
- The total size of spectrum available for PU 1 and PU 2 are identical and equal to $W_1 = W_2 = 20MHz$.
- The PUs have equal penalty rates, $r_1 = r_2 = 3$.
- The spectrum demand of the SU is given as $D(p, \epsilon) = D_0(p) + \epsilon$ (MHz), with $D_0(p) = -6.25p$ (MHz), and ϵ is randomly picked over the range from 10 (MHz) to 40 (MHz).

The best response of PU 1 to a strategy taken by PU 2 is studied initially. Given $a_2 = 4$ (i.e., the supply function of PU 2 is $q_2(p) = 4p$), the utility (expected profit) of PU 1 corresponding to different values of a_1 is plotted in Figure 6.2. The study is conducted with four realizations of spectrum demand when ϵ is equal to 15, 20, 25, and

30 MHz. As seen from the figure, for each realization of spectrum demand, the expected profit of PU 1 first increases with an increase in a_1 . Once reaching its maximum value, the expected profit of PU 1 decreases with an increase in a_1 . It can be also inferred from Figure 6.2 that given the supply function of PU 2 is $q_2(p) = 4p$, the expected profit of PU 1 is always maximized when $a_1 = 4.094$ (i.e., when the supply function of PU 1 is $q_1(p) = 4.094p$) regardless of the value of ϵ . As a result, $q_1(p) = 4.094p$ is the best response of PU 1 when PU 2 takes the strategy $q_2(p) = 4p$. This result exactly matches to the theoretical best response strategy determined by equation (6.28).

Figure 6.2: Expected profit of PU 1 with respect to different strategies and different realizations of ϵ given $q_2(p) = 4p$

The best response of PU 2 to a strategy taken by PU 1 is also investigated. In this study, the strategy taken by PU 1 is given as $q_1(p) = 4.094p$ (i.e., $a_1 = 4.094$). The expected profit of PU 2 corresponding to different values of a_2 is shown in Figure 6.3. This study investigates four different realizations of spectrum demand when ϵ is equal to 18, 22, 27, and 29 MHz. Similar to the previous study, for each realization of spectrum demand, the expected profit of PU 2 first increases with an increase in a_2 until reaching the maximum value. After that, the expected profit of PU 2 decreases with an increase

in a_2 . The expected profit of PU 2 reaches its maximum values when $a_2 = 4.0539$ for all realizations of ϵ . Consequently, $q_2(p) = 4.0539p$ is the best response of PU 2 to the strategy $q_1(p) = 4.094p$ taken by PU 1. This best response strategy perfectly matches with the theoretical results obtained from equation (6.29).

Figure 6.3: Expected profit of PU 2 with respect to different strategies and different realizations of ϵ given $q_1(p) = 4.094p$

Next, the best responses of PU 1 with respect to different strategies taken by PU 2 are studied, and the results are plotted in Figure 6.4. The best responses of PU 2 corresponding to different strategies of PU 1 are also shown in this figure. This study involves two different cases: in the first case, the penalty rates of PU 1 and PU 2 are given as $r_1 = r_2 = 3$; in the second case, the corresponding values are $r_1 = 5$, $r_2 = 7$. Other parameters are kept unchanged. As seen from the figure, the best responses of the PUs are dependent on their penalty rates. The PUs will select low slope supply functions (i.e., small a_i for $i = 1, 2$) when their penalty rates are high. Inversely, they will choose high slope supply functions when their penalty rates are small. In other words, the size of spectrum offered by a PU decreases with an increase in his penalty rate at the same level of price. The NE of the proposed spectrum trading game is also plotted in Figure

6.4 as the point where all the best response curves intersect. For the first case (when $r_1 = r_2 = 3$), at NE, both PUs take the same strategy. The slopes of their supply functions are given as $a_1^* = a_2^* = 4.0466$. For the second case (when $r_1 = 5, r_2 = 7$), the slopes of their supply functions are $a_1^* = 2.7114$ and $a_2^* = 2.1664$, respectively. It is easy to see that in this case, $a_2^* < a_1^*$ as a result of $r_2 < r_1$. Furthermore, because the penalty rates in the second case are higher than those in the first case, the slopes of the supply functions of both PUs in the second case are smaller than those in the first case. Though the given parameters are selected for demonstration purpose, similar behavior can be observed for different values of parameters.

Figure 6.4: Best responses of PU 1 and PU 2 and the Nash equilibrium

6.4.2 Iterative Algorithm

The numerical results on the stability of the proposed iterative algorithm are studied in this section. The parameters of this study are set as follows:

- The investigated spectrum trading model consists of five PUs ($\mathcal{N} = \{1, 2, 3, 4, 5\}$).

- The total size of spectrum available for PUs are $W_1 = 20$ MHz, $W_2 = 22$ MHz, $W_3 = 24$ MHz, $W_4 = 26$ MHz, and $W_5 = 28$ MHz, respectively.
- The penalty rates of PUs are equally set to $r_1 = r_2 = r_3 = r_4 = r_5 = 5$.
- The spectrum demand of the SU is given as $D(p, \epsilon) = D_0(p) + \epsilon$ (MHz), with $D_0(p) = -6.25p$ (MHz), and ϵ is randomly picked over the range from 10 (MHz) to 40 (MHz).

The slopes of supply functions of PUs after each iteration are studied and shown in Figure 6.5. For the initialization, the slopes of the supply functions of players are equally set to 2. As seen from the figure, the proposed iterative algorithm converges very fast to a steady state, i.e., an NE, after 5 iterations. At the NE, the slopes of the supply functions of players are $a_1^* = 3.3962$, $a_2^* = 3.6727$, $a_3^* = 3.9388$, $a_4^* = 4.1947$, and $a_5^* = 4.4408$, respectively. It is easy to verify that these values are exactly equal to the theoretical values determined by equation (6.24).

Figure 6.5: Slopes of supply functions of PUs at each iteration

Finally, the average number of iterations required for the iterative algorithm to reach the NE is studied. This algorithm is assumed to be convergent if the difference in the

strategies (i.e., the slopes of the supply functions) taken by each player (PU) between two consecutive iterations is smaller than 0.1%. Figure 6.6 plots the average number of iterations when the number of players varies from 2 to 10. The present study assumes that the size of spectrum available for the PUs are randomly selected over a range from 10 MHz to 30 MHz. The penalty rates of players are randomly chosen over a range from 2 to 8. The other parameters are kept unchanged from the previous studies. As seen from the figure, the proposed iterative algorithm takes around 5 iterations to converge when the number of PUs varies from 2 to 10.

Figure 6.6: Average number of iterations to converge

6.5 Summary

This chapter studies a spectrum trading problem in a cognitive radio network which consists of multiple primary and single secondary users. For the given spectrum trading problem, the spectrum requirements for the PUs' services and the spectrum demand of the SU are uncertain. The main contributions of this chapter are as follows:

- A *Supply Function Equilibrium* model is proposed to analyze the trading process. In this model, the PUs compete with each other by submitting their supply functions to the SU. A market clearing price is then determined by the SU by matching the aggregate spectrum supply function and its spectrum demand.
- The NE of a static game when players (the PUs) have the complete information on other players is studied. The closed form formula of the NE for the case when the supply functions of the PUs are linear functions of price is also given.
- An iterative algorithm is proposed for a game of incomplete information. The proposed algorithm allows the players to learn from the past and dynamically adjust their strategies to reach the NE. The stability of the proposed iterative algorithm is also studied.
- A special case of two PUs is also analyzed for demonstration purpose.
- Finally, the numerical results are provided to validate the theoretic analysis.

Chapter 7

Conclusions and Future Directions

7.1 Contribution to Research

The present research addresses various aspects of cognitive radio, in both technical and economic perspectives. The results and contributions of this work to the research on cognitive radio are summarized in this section.

For the problem of spectrum sensing, most of the available literature assumes static location of PUs' bands and each SU is equipped with perfect knowledge of these locations. However, such an assumption is idealistic and imposes strict limitation on practical deployment scenarios. The present research considers a scenario in which the primary band characteristics (both width and location) are unknown to all SUs. As a result, it is necessary for the SUs to perform an additional task of primary band detection before primary user transmission detection can be carried out. An innovative frequency boundary detection technique based on *Canny Edge Detection* algorithm in image processing is proposed for the primary band detection. In addition, the decision making process in traditional spectrum sensing uses non-cooperative spectrum sensing methods such as *energy detection*, *waveform-based sensing*, and *cyclostationarity-based sensing*. All these methods are based on the statistics of a single node. Therefore, the detection accuracy is sensitive to physical impairments such as noise and shadowing in propagation environment. In order to improve the reliability of spectrum sensing results, cooperative spectrum sensing is

considered. Two crucial challenges arise when cooperative spectrum sensing is deployed in a cognitive radio environment where SUs have no *a priori* knowledge of PUs' bands. The first challenge is to make a collective decision about the primary band locations based on the results of individual SUs with the aim of minimizing false primary bands. A *dynamic band clustering* (DBC) algorithm that uses K-means clustering technique is proposed to address this challenge. It is shown that the proposed DBC algorithm significantly reduces the number of false primary bands during both low and high SNR regions. The second challenge is to derive a cooperative decision on the presence of PU based on the detection results of individual SUs. The conventional *information-fusion* approaches using *Equal Gain Combining* and *Maximal Ratio Combining* are studied first. Even though the *information-fusion* approach produces superior detection quality, it incurs high overhead due to the need for exchanging and/or distributing the raw sensing information. In order to reduce the overhead, a novel weighting technique that can be coupled with the *decision-fusion* strategy is proposed. The proposed technique makes use of entropy information as the weighting coefficient and is shown to be effective in improving the performance of decision-fusion approach while avoiding the overhead in information-fusion approach.

For the problem of resource allocation, two different non-cooperative games namely *Interference Minimization* game and *Capacity Maximization* game are proposed to analyze the behaviors of SUs for distributing transmit power and allocating frequency channels. These games reflect the objectives of data and voice radio systems, respectively. The best response of a player (i.e., a SU) with respect to a set of strategies taken by other players is analyzed for each game. In addition, the NE of the proposed games can be achieved by letting the players follow the best response dynamics. It is demonstrated that the average SINRs and capacities of the SUs improve after the incorporation of these games. The average SINRs of the SUs after applying *Capacity Maximization* game are

smaller than the average SINRs after applying *Interference Minimization* game. However, in comparison to *Interference Minimization* game, the average capacities of the SUs after applying *Capacity Maximization* game are larger.

The issue of spectrum trading in a cognitive radio network which involves multiple PUs competing with each other to sell unused spectrum to a SU is addressed next. Different from the existing spectrum trading approaches where the size of spectrum required for a PU's service is known, a more practical scenario, where the spectrum requirement for a PU's service is not fixed and is time varying, is considered. First, a case where the spectrum demand of a SU is a deterministic function of price is studied. In this case, the spectrum trading process is modelled as a *Cournot game model* of competition in which the PUs set the size of spectrum to sell. The NE for a static game when the PUs have complete knowledge about the other PUs is studied. An iterative algorithm that allows the PUs to gradually adjust their strategies to reach the NE in a dynamic game is provided. The stability of this iterative algorithm, in particular for a special case of two PUs, is also analyzed. Extending the issue of spectrum trading to a cognitive radio network which involves multiple primary and secondary users is also considered. It is also demonstrated that the proposed theoretical analysis is accurate and robust.

A more practical case with uncertain spectrum demand from the SU is also studied. This case is modelled as a *Supply Function Equilibrium* model, in which the PUs compete to sell spectrum by submitting their supply functions to the SU. The SU then decides the market clearing price at which the spectrum demanded and spectrum supplied are equal. A non-cooperative game theoretic framework is formulated to predict the behaviors of the PUs. The NE for a static game is studied. It is shown that, the payoffs (the expected profits) of the PUs are maximized at the NE for each realization of the spectrum demand. An iterative algorithm to reach the NE for a dynamic game is proposed. Moreover, the

stability of this iterative algorithm is analyzed. Through extensive simulation studies, it is also shown that the proposed theoretical analysis is accurate, and the proposed iterative algorithm is solid and converges very fast to the NE.

7.2 Future Directions

The following sections briefly describe some of the possible extensions to the work presented so far, which serve as future research directions.

7.2.1 Throughput Maximization through Sensing Band Selection

The throughput maximization problem for secondary user networks through the appropriate sensing of frequency bands is an open and interesting research direction. It is assumed that there are N SUs, who sense K narrow bands for the presence of PUs. Each SU is allowed to sense only one band. SUs, who sense the same frequency band, will cooperate with each other in order to improve the sensing accuracy. The cooperative decision is derived by using the OR-rule decision-fusion technique. If an unused frequency band is detected (i.e., its PU is absent), the SUs who sense the band are equally likely to access. Thus, it is necessary for SUs to determine which band to sense to maximize their throughput. This problem can be formulated as a non-cooperative evolutionary game. The set of players of the game is the set of N SUs. The strategy of a player is the choice of frequency band to sense. The payoff or utility of a player is defined as the throughput that the player can earn for sensing a frequency band, and is determined as follows.

Supposing that the signal received at SU i ($i \in \{1, 2, \dots, N\}$) on frequency band k ($k \in \{1, 2, \dots, K\}$) is sampled at a sampling frequency of f_s , the discrete signal received

at SU i on frequency band k is expressed as

$$\begin{aligned}\mathcal{H}_{0,k} : y_i^k(n) &= v_i(n) \\ \mathcal{H}_{1,k} : y_i^k(n) &= h_i^k x^k(n) + v_i(n), \quad n = 1, 2, \dots, \tau f_s,\end{aligned}\tag{7.1}$$

where $x^k(n)$ is the primary signal on frequency band k , $v_i(n)$ is the additive white Gaussian noise (AWGN) component at SU i with the variance denoted by $\sigma_{i,v}^2$, h_i^k is the channel gain from the PU's transmitter to the receiver of SU i on frequency band k , τ is the sensing duration. The test statistics Y_i of the energy detection is defined as

$$Y_i = \frac{1}{\tau f_s} \sum_{n=1}^{\tau f_s} |y_i^k(n)|^2.\tag{7.2}$$

Comparing Y_i with a threshold ϵ_i , the probability of false alarm and probability of detection of SU i is given as

$$P_{F,i}(\epsilon) \approx \mathcal{Q}\left(\left(\frac{\epsilon_i}{\sigma_{i,v}^2} - 1\right) \sqrt{\tau f_s}\right),\tag{7.3}$$

$$P_{D,i}(\epsilon) \approx \mathcal{Q}\left(\left(\frac{\epsilon}{(1 + \gamma_i^k) \sigma_{i,v}^2} - 1\right) \sqrt{\tau f_s}\right),\tag{7.4}$$

where γ_i^k is the SNR observed by SU i on frequency band k , $\gamma_i^k = \frac{|h_i^k|^2 \sigma_{x,k}^2}{\sigma_{i,v}^2}$ with $\sigma_{x,k}^2 = E[|x^k(n)|^2]$. Let $\mathcal{N}_k$ denote the set of SUs sensing the same frequency band k . Probability of false alarm and probability of detection on frequency band k by using the OR-rule cooperative decision-fusion technique is given as

$$P_F^k = 1 - \prod_{i \in \mathcal{N}_k} (1 - P_{F,i}),\tag{7.5}$$

$$P_D^k = 1 - \prod_{i \in \mathcal{N}_k} (1 - P_{D,i}).\tag{7.6}$$

Let the probability of absence of PU on frequency band k be $P_{0,k}$. Assuming that the spectrum sensing is conducted every T period of time, the overall throughput that $|\mathcal{N}_k|$ SUs can earn for sensing frequency band k is

$$R_k(\tau) = \frac{T - \tau}{T} P_{0,k} [1 - P_F^k].\tag{7.7}$$

As there are $|\mathcal{N}_k|$ SUs sensing the same frequency band k and each SU has the equal chance to access, the average throughput that SU i ($i \in \mathcal{N}_k$) can acquire for sensing frequency band k is

$$R_{i,k} = \frac{R_k}{|\mathcal{N}_k|} = \frac{1}{|\mathcal{N}_k|} \frac{T - \tau}{T} P_{0,k} [1 - P_F^k]. \quad (7.8)$$

It is important to note that, the accuracy of the spectrum sensing increases with an increase in the number of SUs who sense the same frequency band. As a result, when the number of SUs sensing the same frequency band increases, the overall throughput of the SUs earned from accessing this frequency band increases. However, as the number of SUs is larger, the individual throughput of each SU will be smaller.

Furthermore, as seen from equation (7.8), the average throughput $R_{i,k}$ is an increasing function of sensing period T . Therefore, SUs tend to increase T . However, as SUs only sense the spectrum every T period of time, there are chances that the PU is active during this period without any detection by the SUs (i.e., the PU is active outside of the sensing duration τ). Increasing the value of T leads to an increase in the probability of mis-detection on the presence of PU. Consequently, choosing a proper sensing period T needs to be studied thoroughly. An alternative is to let SUs conduct spectrum sensing continuously. In such a scenario, SUs can alternatively sense the spectrum and transmit their signal. An efficient and fair scheduling algorithm is required in this scenario.

7.2.2 Spectrum Trading with Difference in Spectrum

In the present work, the spectrum offered by different PUs are homogeneous and can be completely substituted. However, due to the channel condition (for examples, fading, noise, etc.) and the preference of SUs on the spectrum to operate, the spectrum offered by different PUs may not be completely substitutable. The spectrum trading scenarios with different spectrum offered by different PUs were investigated in [8, 123, 124] and in [127]. However, these works did not consider the uncertainty in the spectrum requirements

for the primary services as well as the uncertainty in the spectrum demand of the SU. Two spectrum trading cases discussed in the present work can be extended further by considering different available spectrum. In the first case, the spectrum requirements for the primary services are uncertain but the spectrum demand of the secondary is known. This case can be modelled using both *Bertrand game model* and *Cournot game model*. In the second case, both the spectrum requirements for the primary services and the spectrum demand of the SU are uncertain. *Menu auction with demand uncertainty* model, which was proposed in [169], appears as a promising solution for this case.

7.2.3 Spectrum Trading with Multiple Competitive Primary and Secondary Users

Spectrum trading with multiple competitive primary and secondary users is also an interesting problem. This problem was studied in [128]. However, instead of specifying the amount of spectral resources to buy, the SUs in this work are only allowed to choose the PUs to buy spectral resources from. The SUs, who choose the same PU, get the same size of spectrum and have to pay the same price. Hence, it may not be feasible in practical scenarios where different SUs may have different demands on spectrum. A spectrum trading with multiple primary and secondary users was studied in [146] using double auction. However, each PU/SU is allowed to sell/buy only one channel in [146]. In [148], a spectrum trading model through a spectrum broker was studied. In this work, the spectrum broker initially announces a trading price. After that, PUs/SUs bid for the size of spectrum that they are willing to sell/buy at that price. However, it is very difficult to directly get the optimal price to maximize the traded volume. This necessitates a more practical spectrum trading model which can reflect the demands of different SUs and at the same time can maximize the benefits of trading participants (i.e., PUs and SUs).

7.2.4 Spectrum Trading Using Auction Theory

As mentioned in Chapter 2, auction theory is an alternative to the proposed approaches using pricing models to address the spectrum trading problem. An auction can be formulated to analyze the spectrum trading involving multiple SUs competing for the spectrum offered by a single PU. In this auction, the PU plays the role of an auctioneer, whereas the SUs play the roles of bidders. The SUs competitively send bids which state amount of money they would be willing to pay for the spectrum to the PU. The PU will then decide the amount of bandwidth allocated to each SU (e.g., the SUs will be assigned an amount of bandwidth proportional to their bids). Each SU is assumed to have a virtual budget constraint for bidding. As different SUs have different QoS requirements, they should have different budget constraints. A truthful and computationally efficient auction mechanism should be designed for the spectrum auction. In addition, the spatial reuse of spectrum should also be considered. This auction will be further extended to a scenario which involves multiple PUs.

7.2.5 Other Spectrum Trading Problems

The present work assumes that the SUs buy the spectrum for a predefined and identical duration of time. In practice, different SUs may want to buy the spectrum access right for different time duration, based on their needs. In addition, the spectrum buyers (i.e., the SUs) can be discriminated based on their priorities. The higher priority buyers will get a chance to buy the spectral resources first. The remaining spectral resources will be sold to the lower priority buyers. The priority can be assigned to the buyers based on their characteristics. For example, military or public service users have higher priorities than commercial users. It can also be assigned based on loyalty of the buyers, e.g., regular buyers have higher priorities than one-time buyers. The differences in the time duration and in the priority of the SUs can be taken into account for further studies.

7.2.6 Implementation of Proposed Schemes into Actual Cognitive Radio System

Having analyzed the theoretical performance of the proposed schemes, it is essential to apply them into actual cognitive radio systems. Several challenges associated with this implementation need to be addressed. First, a dedicated channel to share the information (e.g., the sensing information, channel gains, etc.) among the SUs and PUs is required. In addition, for the cooperative spectrum sensing, the SUs must be time-synchronized so that the sensing results can be combined correctly. Apart from that, the parameters pertaining to spectrum sensing process such as sensing duration, sensing period, the number of SUs taking part in cooperative spectrum sensing must be studied thoroughly. For the resource allocation, the channel gains between the transceivers of the SUs must be estimated and shared among the SUs in real-time. In addition, the enforcement and punishment measures to prevent the malicious actions from the SUs are required. For the spectrum trading, there should be a third party to control the trading process among the PUs and SUs. This party will impose penalties to those who violate the trading agreement. The other issues also need to be taken in account such as how to make the payment, the duration of spectrum leasing, etc. Furthermore, due to the spatial reuse of spectrum, the SUs' signal may interfere with signal of other PUs and SUs in the neighboring geographical areas. Consequently, the interference management should also be addressed.

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