Numerical study on reinforced concrete beam-column frames in progressive collapse

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**NUMERICAL STUDY ON
REINFORCED CONCRETE BEAM-COLUMN FRAMES
IN PROGRESSIVE COLLAPSE**

**LONG XU
SCHOOL OF CIVIL & ENVIRONMENTAL ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY**

2013

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REINFORCED CONCRETE BEAM-COLUMN FRAMES
IN PROGRESSIVE COLLAPSE**

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Summary

The objective of current research is to numerically investigate the deformation behaviour of reinforced concrete (RC) beam-column framed structures subjected to destructive external loading. Firstly, besides the conventional uniaxial concrete models to predict flexural failures, a unified plasticity concrete model is proposed to accurately simulate shear deformations of beams.

Secondly, a three dimensional co-rotational beam finite element is formulated with considerations of material nonlinearities for both steel and concrete. The proposed co-rotational beam formulation is shown to be capable of predicting steel and reinforced concrete framed structures with satisfactory accuracy and efficiency.

Thirdly, a component-based mechanical model is proposed to simplify two dimensional RC beam-column joints, where three types of components are considered, viz., the bond-slip component, shear-panel component and interfacial shear component. Analytical models are respectively proposed to reasonably calibrate the bond-slip component and the shear-panel component, and an empirical model is summarized for the interfacial shear component based on extensive experimental results and design regulations.

Fourthly, as an integrated system, the proposed concrete models, the co-rotational beam element and the component-based joint model are studied at the system level to show the prediction accuracy, computational efficiency and robustness in numerical algorithms. Advantages and disadvantages of different concrete models are also discussed.

Finally, a superelement concept is proposed for structural analysis of large-scale structures. Compared with models without superelement, significant saving in computational cost and satisfactory prediction accuracy can be obtained without any loss in critical information of structural responses. This aspect is particularly crucial for progressive collapse analysis of structures subjected to localized damage.

Statement of Originality

The author of this thesis would like to make an original statement about the content of the thesis as follows.

The plasticity-based concrete model for three dimensional concrete behaviour under compression in Section 3.3.1.1 and Section 3.1.1.3 is originally proposed by Dr. Bao Jinjing and going to be published in a journal, in which the author, as the second author of the publication, proposed an incremental algorithm to realize the proposed flow rule in the finite element analysis. Moreover, the author also assisted in drafting and amending of the journal paper, error-proofing of formulation derivation and debugging of program implementation. As for the unified plasticity concrete model for 3D fibre beam element in Section 3.3.2, the content is completely done by the author.

The basic superelement concept in this study was initially proposed by the author and his colleagues, which is going to be published in a journal. It should be noted that Professor Yuan Weifeng, as the second author of the paper, came up with the original idea for this superelement technique as discussed in Section 7.2. However, only the deformation of the nonlinear zone is of concern in the original idea. The author of this thesis proposed an algorithm to obtain both the deformations of nonlinear and linear zones, with a significant saving in computational cost.

For the other chapters, all the studies are conducted by the author. When others' works and published results are referred to, the corresponding papers and technical reports are quoted and summarized in the References attached at the end of this thesis.

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List of Symbols

Scalars:

a	Interested column length
a_b	Depth of the compression zone in the beam
a_c	Depth of the compression zone in the column
a_s	Width of the concrete strut
b	Width of a cross-section
b_b	Joint thickness
b_c	Width of the column cross-section
b_s	Width of the concrete strut
d	Depth of a cross-section
d_b	Bar diameter
dx	An infinitesimal element along steel reinforcement
$e_{iy,n_i}, e_{iy,m_i}, e_{iz,n_i}$	Global vectorial rotational variables of Node i
f_0	Bar stress at the joint centre acting as a boundary
f_c	Concrete compressive cylinder strength
f_{cx} and f_{cy}	Average horizontal and vertical stresses of concrete
f_{c1} and f_{c2}	Principle tensile and compressive stresses
f_s	Bar stress at the point of interest
$\tilde{f}_s$	Bar stress at the joint perimeter
f_t	Concrete tensile strength in the cylinder splitting test
f_u	Ultimate tensile stress
f_{yh}	Yield strength of stirrups
f_y	Yield strength of steel
h_b	Depth of the beam cross-section
h_c	Height of the column cross-section
h_i	Lagrangian interpolation function
h_{core}	Width of concrete core
k_0	Shear factor of cross-sections
l_e	Length of elastic steel reinforcement

l_{fs}	Propagation length of the bar stress
l_y	Length of plastic steel reinforcement
m	User-defined iteration number
$r_{iy,n_i}, r_{iy,m_i}, r_{iz,n_i}$	Local vectorial rotational variables of Node i
s	Stirrup spacing
u_i, v_i, w_i	Local displacements of Node i
x, y, z	Local coordinate system
y_l, z_l	Relative coordinates of a point to the central line
A	Cross-sectional area
A_b	Cross-sectional area of steel reinforcement
A_g	Gross cross-sectional area
A_h	Joint horizontal cross-sectional area
A_t	Transverse reinforcement area
A_v	Joint transverse cross-sectional area
A_{strut}	Effective area of the concrete strut
D	Idealized forces for the diagonal mechanism
DI	A non-dimensional damage parameter
E, E_s	Elastic modulus of steel
E_c	Elastic modulus of concrete
E_h, H_s	Steel hardening modulus
F	Yield function of steel
F_d	Idealized forces for the diagonal mechanism
F_h	Idealized forces for the horizontal mechanism
F_v	Idealized forces for the vertical mechanism
G	Shear modulus of steel
H	Strain hardening parameter of steel
K	Confinement factor in concrete model
L_b	Length of bond deterioration zone
L_c	Length of curvature influence zone
L_{c0}	Length of initial curvature influence zone

L_{eq}	Equivalent embedment length for a bent bar
L_J	Joint width
I_b	Cross-sectional moment of inertia of steel reinforcement
M	Internal nodal moment at connecting nodes
N	Axial load
R	Ratio of the part outside the yield surface to the whole stress increment
R_d, R_h and R_v	Coefficients in load transfer mechanism
S_h	Centre-to-centre spacing of stirrups or hoop sets
V	Volume of a CR beam element
V_{jh}	Horizontal joint shear force
V_{jv}	Vertical joint shear force
U_i, V_i, W_i	Global displacements of Node i
X_1, X_2, X_3	Global coordinate system
Z	Strain softening slope in concrete model
σ_c	Concrete compressive stress
σ_h and σ_v	Average horizontal and vertical stresses
σ_r and σ_d	Tensile and compressive stresses of the concrete strut
$\bar{\sigma}_s$	Average bar stress
σ_{si}	Local bar stress at each steel fibre
σ_y	Yield strength of steel
ε_0	Concrete strain at the maximum compressive stress
ε_c	Concrete compressive strain
ε_{cr}	Concrete strain at the maximum tensile stress
ε_{cu}	Concrete ultimate strain in tension
ε_h and ε_v	Average horizontal and vertical strains
ε_r and ε_d	Principle tensile and compressive strains
ε_r	Unloading start point

ε_p	Unloading end point the strain axis
$\bar{\varepsilon}_s$	Average bar strain
ε_{si}	Local bar strain at each steel fibre
ε_u	Concrete ultimate strain in compression
ρ_s	Ratio of the volume of hoop reinforcement to that of concrete core
ρ_v	Transverse reinforcement ratio
ζ	Natural coordinate system along the beam centre line
τ_E	Mechanical bond stress of reinforcement in concrete
τ_Y	Frictional bond stress of reinforcement in concrete
δ_b	Transverse shear displacement
Φ_{max}	Maximum curvature
α	Strut area reduction coefficient
γ_{hv}	Shear strain at the joint panel
γ_h, γ_v	Coefficients in the reduced statically indeterminate mechanisms
θ	Direction of principle stress
$\theta_1, \theta_2, \theta_3$	Rotational variables of the connecting node
τ_{cxy}	Joint shear stress
λ	Arbitrary nonzero factor

Vectors:

$\mathbf{a}$	Flow vector
$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$	Local nodal vectors of the connecting node expressed in the global system
$\mathbf{e}_{iy}, \mathbf{e}_{iz}$	Direction vectors in the global system of Node i
$\mathbf{f}_G$	Internal force vector in the global system
$\mathbf{f}_L$	Internal force vector in the local system
$\mathbf{r}_{iy}, \mathbf{r}_{iz}$	Direction vectors in the local system of Node i
$\mathbf{t}_i$	Displacement vector at any point in the beam element
$\mathbf{u}_1$	Predictive displacement
$\mathbf{u}_2$	Corrective displacement
$\mathbf{u}_G$	Degrees of freedom in the global system
$\mathbf{u}_L$	Degrees of freedom in the local system
$\mathbf{P}$	External load
$\mathbf{P}_s$	External load within the superelement zone
$\mathbf{R}$	Out-of-balance force
${}^1\mathbf{U} \sim {}^6\mathbf{U}$	Components of the deformation when forming stiffness matrix of superelement
$d\boldsymbol{\sigma}$	Incremental stress vector
$d\boldsymbol{\sigma}_e$	Whole stress increment of steel
$d\boldsymbol{\sigma}^{(0)} \sim d\boldsymbol{\sigma}^{(5)}$	Components of incremental stress
$d\boldsymbol{\varepsilon}$	Incremental strain vector
$\boldsymbol{\varepsilon}$	Green strain
$\boldsymbol{\varepsilon}^{(0)} \sim \boldsymbol{\varepsilon}^{(5)}$	Components of Green strain
	Parameter to control the load increment

Matrices:

B	Geometric matrix
D	Material matrix
D_{ep}	Equivalent material matrix
K_G	Stiffness matrix in the global system
K_L	Stiffness matrix in the local system
K_s	Stiffness matrix of the superelement
K_{ns}	Non-superelement stiffness matrix
R	Orthogonal rotation matrix
R_i	Rigid-body rotation matrix at connecting node <i>i</i>
S	Skew-symmetric matrix
T	Transformation matrix from the local to the global system
ω	Pseudo-vector of the natural rotation

Chapter 1 Introduction

1.1 General

Progressive collapse is defined as the collapse of all or a large part of a structure caused by the failure or damage of a relatively small part of the structure. A specific description of the phenomenon provided by General Services Administration (2003) is “Progressive collapse is a situation where local failure of a primary structural component leads to the collapse of adjoining members which, in turn, leads to additional collapse.”

The research regarding progressive collapse was initiated by the well-known collapse of Ronan Point apartment tower on May 16, 1968 as shown in **Fig. 1.1** (Wikipedia 2012), due to an internal gas explosion which knocked out load-bearing precast concrete panels near one corner of the building.

Fig. 1.1 Ronan Point building after the May 1968 partial collapse (Wikipedia 2012)

Another progressive collapse tragedy is the collapse of the Alfred P. Murrah Federal Office Building in Oklahoma City on April 19, 1995 which was damaged by a bomb (**Fig. 1.2**) (Lew 2002). A 4000lb TNT bomb, kept in a truck 15.6 feet away from the base of the building, destroyed or badly damaged three columns G16, G20 and G24 as shown in **Fig. 1.2**. The collapse of Murrah Federal Office Building was initiated from the loss of support from these columns leading to the catastrophic failure of a transfer girder between G16 and G24.

Fig. 1.2 Alfred P. Murrah Federal Office Building after the 1995 explosion and partial collapse (Lew 2002)

The most notable recent progressive collapse is the collapse of the twin towers of World Trade Centre (WTC) 1 and 2 on September 11, 2001 resulting from the impact of two Boeing 767 jetliners at a high speed as depicted in **Fig. 1.3** (Thierry 2002). The crash caused structural damage at and near the point of impact and set off an intense fire within the building. The structure near the impact zone lost its ability to support the load above it. As a result of the

combination of impact and subsequent fire damage, the weight and impact of the collapsing upper part of the tower caused further propagations of failures extending downwards all the way to the ground.

Fig. 1.3 Impact zone on the north face of WTC 1 (Thierry 2002)

In these three serious tragedies, the similarity is that internal or external abnormal loading destroyed critical structural members and the capacity of the whole structure is reduced to a level at which the structure cannot support its own weight. In fact, the progressive collapses occurred before September 2001 did not attract so much attention in the academic research community on the need to evaluate the progressive collapse potential of existing buildings. There was very little research conducted except some technical reports until the collapse of the twin towers of World Trade Centre in September 2001.

Recently, the General Services Administration (GSA) (2003) and the Department of Defense (DoD) (2009) in the United States stipulated more

detailed guidelines to resist progressive collapse of building structures. Meanwhile, GSA (2003) and DoD (2009) recommended step-by-step procedures for linear static, nonlinear static and nonlinear dynamic analyses for analysing the potential of progressive collapse of reinforced concrete (RC) structures. It is worth noting that they recommended using a nonlinear analysis procedure when a primary vertical element (basically a column) is removed from large-scale buildings.

The instant column removal scenario is simplified from the complete failure of a column in practice due to an external abnormal loading. In order to simulate the instant removal of a column, alternate load path approach is conventionally recommended due to its simplicity in implementation, directness in interpretation and independence of threat scenarios. As a standard scenario to simulate the commencement of progressive collapse, artificial column removal is employed in both numerical and experimental studies on progressive collapse. For beam-column framed structures, a column removal scenario is taken as the direct design procedure for evaluating the progressive collapse potential. Nonetheless, only one column removed at one time is of concern in these regulations (GSA (2003) and DoD (2009)), which may not be the most critical column removal in reality. Therefore, a robust and efficient finite element program to perform nonlinear analyses for reinforced concrete framed structures is urgently needed by practicing engineers.

Nevertheless, such a finite element program has to be well formulated to take account of structural behaviour as realistic as possible. Of course, extensive validations against published experimental studies need to be conducted to ensure accuracy and reliability in predictions. Moreover, computational efficiency is of concern as well, since numerical simulations of large-scale structures take a long time and may impede the design progress in practice.

1.2 Numerical Simulations

From numerical simulations, before the complete failure of a critical structural member with sufficient ductility, under increasing load, the member goes through the stages of small deformation, material failure and large deformation. This means that the deformation of the structural member should include material nonlinearity, geometric nonlinearity and their coupled effects. Additionally, the joints in framed structures are not rigid joints as are usually assumed in a conventional analysis. A joint element is necessary to simulate the joint rigidity and strength. Besides, a suitable evaluation of actual joint behaviour can significantly improve the accuracy of framed analysis.

1.2.1 *Beam/column member simulations*

For simulating reinforced concrete framed structures undergoing progressive collapse, geometric nonlinearity implies the large displacements and large rotations of beam-column structural members, while material nonlinearity means the cracking and crushing of concrete and yielding and fracturing of steel reinforcement. Therefore, in order to study the structural behaviour of reinforced concrete framed structures under progressive collapse, efficient beam element approaches are conventionally preferred for large deformation and large rotation analysis of framed structures. Generally, the beam element formulations can be classified into three types, that is, total Lagrangian formulation (Bathe and Bolourchi 1979; Schulz and Filippou 2001; Nanakorn and Vu 2006), updated Lagrangian formulation (Bathe and Bolourchi 1979; Cardona and Geradin 1988; Teh and Clarke 1999), and co-rotational formulation (Hsiao et al. 1987; Crisfield 1990; Felippa and Haugen 2005; Li 2007; Battini 2008). For material nonlinearity, there have been a few publications that include both numerical and experimental aspects. There is a comprehensive review collection from Hinton and Owen (1984) for nonlinear steel behaviour. As for concrete materials, there are different types of concrete models to emphasize different failure modes in reinforced concrete beam members, such as flexural failure and shear failure. In order to obtain an accurate stress and strain prediction around the beam cross-

section, an efficient approach is to use fibre models. Finally, it should be pointed out that at the material level, the constitutive laws of both steel and concrete should be formulated so that they are suitable for fibre beam element formulations.

It is obvious that a good number of approaches are available to simulate the behaviour of reinforced concrete framed structures. Different concrete models and beam formulations have to be reviewed and carefully selected to accurately predict the nonlinear structural behaviour of reinforced concrete framed structures subjected to progressive collapse, which is the main objective of the present study.

1.2.2 Two dimensional beam-column joint simulations

When numerically analysing the behaviour of beam-column reinforced concrete joints, component-based mechanical method is a standard method to determine the rigidity and strength of joints subjected to bending moment, shear and tension or compression axial force. That is, several components in a joint can be artificially separated in terms of failure modes and load resistance mechanisms. As widely acknowledged (Alath and Kunnath 1995; Youssef and Ghobarah 2001; Lowes and Altoontash 2003; Altoontash 2004; Bao et al. 2008), the failures of bond-slip, shear-panel and the shear transfer capacity between the beam-joint and column-joint interfaces dominate the failure modes of 2D beam-column reinforced concrete joints. The bar-slip components are employed to simulate the stiffness and strength loss due to the anchorage failure of beam and column longitudinal reinforcement embedded in the joint region, whereas the shear-panel component is to simulate the strength and stiffness loss due to shear failure of the joint panel. The interface-shear components are applied to simulate the loss of shear-transfer capacity due to shear transfer failure at the beam-joint and the column-joint interfaces.

To calibrate the three types of components in the joint model, appropriate analytical models are preferred but have to be validated against experimental results. Alternatively, empirical models based on a comprehensive series of

experimental studies can also be employed. Extensive researches have been conducted on the bar-slip component and the shear-panel component. However, only a few studies are applicable to reinforced concrete beam-column joints due to the joint characteristics in terms of typical dimension, material properties and reinforcement detailing. For the bar-slip component, previously published bond stress-slip relationships obtained from pure axial pullout tests need to be modified and adapted for the reinforcement slip in beam-column joints. Moreover, according to the best knowledge of the author, to date, there has not been any publication of an analytical model on the shear-panel component of reinforced concrete beam-column joints that represents well a complete failure response and takes into account all important structural parameters. Thus, the structural performance of beam-column reinforced concrete joints under the scenario of progressive collapse remains an area that requires further intensive research studies.

1.3 Scope and Layout of the Thesis

According to the numerical requirements to simulate the deformation behaviour of reinforced concrete framed structures under progressive collapse, the scope and the layout of the present work are summarized as follows:

- In **Chapter 2**, the previous works on concrete models, beam finite element formulations, component-based mechanical joint models and superelement formulations will be reviewed. The shortcomings of the previous works will be discussed in the context of structural behaviour predictions of reinforced concrete framed structures. To address the shortcomings in the previous works, the areas of necessary improvements will be pointed out.
- In **Chapter 3**, concrete models suitable for beam finite element formulations will be studied. In addition to the uniaxial concrete models for the prediction of flexural failures, concrete models based on plasticity theory will also be reviewed and consequently, a concise and convenient plasticity-based concrete model will be proposed to predict shear failures.

- In **Chapter 4**, a three-node co-rotational beam formulation will be derived and modified to meet the requirements of geometric nonlinearity. Nonlinear material model for elasto-plastic material and fibre model will be formulated and implemented to accurately predict the stress-strain state and the failure propagation across the beam or column section. Additionally, two different beam formulations, viz. total Lagrangian and co-rotational framework, will be compared in terms of prediction accuracy and computational efficiency.
- In **Chapter 5**, as the most crucial foundation of numerical stability and computation accuracy in the joint simulation, analytical models will be proposed for the bar-slip component and the shear-panel component, respectively. Accordingly, the validations at the component level will be conducted with comparisons of published experimental results in the literature. Besides, an empirical model will be proposed for the interface-shear component. This will also be added into the joint model. At last, in order to take account of unloading and reloading for the joint due to load redistribution, different resistance-deformation states for each component will be described in detail and incorporated into the numerical algorithm.
- In **Chapter 6**, with the proposed concrete models, co-rotational beam formulation, component-based mechanical joint models integrated as a complete system, reinforced concrete beam-column framed structures are studied at the system level. The examples include beam-column subassemblages with knee joints, exterior joints and interior joints, two three-storey framed structures and a five-storey framed structure with different column-removal scenarios. The aim of this chapter is to validate the proposed simulation approach as an integrated system in terms of prediction accuracy, computational efficiency and numerical robustness. Meanwhile, the advantages of the proposed simulation approach are illustrated and its conditions or assumptions for applications are also discussed.

- In **Chapter 7**, in order to further improve computational efficiency without any loss of accuracy, a new superelement approach will be formulated. The proposed superelement formulation will be validated against several examples of both steel and RC structures with comparisons of prediction accuracy and CPU time for numerical models with and without superelement applications.
- Finally, in **Chapter 8** all the meaningful conclusions in this thesis are summarized. Also, several promising future research projects are highlighted and discussed at the end.

In summary, the originality of the present work is to propose an integrated and robust numerical approach to accurately, reliably and efficiently analyse the structural performance of reinforced concrete framed structures under different scenarios of progressive collapse.

Chapter 2 Literature Review

This chapter reviews previous works on concrete models, beam element formulations and joint models for RC framed structures. **Section 2.1** focuses on existing models which describe the concrete nonlinear behaviour. **Section 2.2** reviews existing beam element formulations to simulate geometric nonlinearity due to large displacements and rotations. **Section 2.3** highlights the component-based joint model and calibration studies for the incorporated components. The last section reviews the superelement approach to significantly improve the computational efficiency of 3D multi-storey frames. Meanwhile, based on the shortcomings found in the previous studies, the motivations and novelties of the present study are highlighted and the scope of the present thesis is summarized at the end of this chapter.

2.1 Concrete Models

Theoretically speaking, numerical simulations using 3D solid elements are capable of accurately predicting the deformation and failure behaviour of RC beam-column members, provided that an appropriate plastic-fracture model is employed to describe the concrete compressive and tensile behaviour. In reality, the stress-strain state at any material point is three dimensional. Nevertheless, for simplicity and efficiency in numerical simulations, various simplified formulations have been proposed to deal with specific stress or strain states in certain geometric configurations. For example, the beam element is derived from the idealization that one dimension (length) is much larger than the cross-sectional dimensions (width and depth), while the shell element is similarly derived from the mathematical idealization that one typical dimension (thickness) is much smaller compared to its longitudinal and transverse dimensions. Consequently, the stress and strain states in the idealized elements are simplified as well based on the mathematical assumptions. For instance, the stress state in a beam element consists of three components, that is, one normal stress component along the beam longitudinal axis and two orthogonal shear stress components as shown in **Fig. 2.1**.

Fig. 2.1 Three stress components in a beam element

In practice, in order to simulate the behaviour of RC beam-column members, a 3D fibre beam element formulation is preferred with satisfactory accuracy and acceptable computational cost (de Felice 2009; Shi et al. 2012). Fibre integration scheme is usually employed at the cross-section level so as to simulate more accurately stress and strain details. At each fibre cross-section, there are one normal stress component along the fibre longitudinal axis and two orthogonal shear stress components along strong and weak axes. For simplicity, the constitutive laws for the normal stress and shear stresses are usually independent of each other. Most of the widely applied and validated concrete constitutive laws were published for concrete under uniaxial compression, such as the Modified Kent and Park model (Park et al. 1982), the Mander's model (Chang and Mander 1994; Waugh 2009) and some other recent models (Binici 2005; Samani and Attard 2012). This type of 1D concrete models are often termed as *uniaxial concrete models*. Nevertheless, there are a few publications on the shear stress components and consequently, concrete shear behaviour is numerically approximated to be elastic, or follows empirical shear models obtained from experimental studies (Patwardhan 2005). It is noteworthy that the concrete models in the nonlinear finite element software Engineer's Studio developed by Tokyo University (Engineer's Studio User's Help 1.00.01 2009) and OpenSees developed by the University of California, Berkeley (Mazzoni et al. 2009) (both of which are widely used for academic research), only consider *uniaxial concrete models* in fibre beam elements.

It should be noted that for beam elements that are based on uniaxial concrete models, the interaction between the normal stress component and the shear

stress components is neglected, even though the combined contributions from all stress components are taken into account in the failure criterion. Therefore, such formulations can only predict well the flexural behaviour of beams with high shear span-to-depth ratio. In the present study, in order to emphasize the possible shear failure zone, the concept of shear span-to-depth ratio is defined as the ratio of the distance between a transverse concentrated load and its adjacent boundary to the beam cross-sectional depth. However, in RC beams with small to medium shear span-to-depth ratios, the shear behaviour will dramatically influence the deformation history and failure mode (Park and Paulay 1975; Imam et al. 1997; Xia et al. 2011). Thus, a concrete model based on the plasticity theory is more advantageous for beam elements to simultaneously update both the normal and the shear stress components when calculating structural deformations of members with dominant shear behaviour.

There are some published plasticity-based concrete models, such as those proposed by Grassl et al. (2002) and Papanikolaou and Kappos (2007). Their plastic potential functions are shown in **Eqs. (2.1)** and **(2.2)**, respectively.

$$g = -A \left(\frac{\rho}{\sqrt{q}} \right)^2 - B \frac{\rho}{\sqrt{q}} + \frac{\xi}{\sqrt{q}} \quad (2.1)$$

$$g = -A \left(\frac{\rho}{k\sqrt{c}f_c} \right)^n + \left[C + \frac{1}{2}(B-C)(1-\cos 3\theta) \frac{\rho}{k\sqrt{c}f_c} \right] + \frac{\xi}{k\sqrt{c}f_c} - a \quad (2.2)$$

where g is the plastic potential function, ρ , θ and ξ are the coordinates of the Haigh-Westergaard stress space, f_c is the uniaxial concrete strength, q is a function of k (hardening function) and c (softening function), a is the attraction parameter of the plastic potential function and n is the order of the plastic potential function.

As shown in **Eqs. (2.1)** and **(2.2)**, there are two (A and B) and three (A , B and C) parameters in their plastic functions (Grassl et al. 2002; Papanikolaou and Kappos 2007) and also in their flow rules ($\partial g / \partial \sigma$, where σ is the stress vector), respectively, which do not have clear physical meanings. Besides, these

parameters have to be calibrated in the uniaxial, biaxial and triaxial compressive stress states, which are challenging to be conducted experimentally. In fact, there are no highly credible test results available for numerical implementations. Therefore, compared with the previous complicated concrete models, a promising concrete model based on the plasticity theory should have as few as possible parameters for which a convenient calibration approach should be suggested as well. To achieve this aim, a one-parameter flow rule (Bao et al. 2012) was proposed with a concise formulation and a convenient calibration approach, compared with the previous flow rules.

Besides the concrete model based on plasticity theory to simulate the concrete deformation in compression, a fracture model should also be incorporated to simulate the concrete tensile behaviour in RC beam members (Cervenka et al. 1998). To avoid complicating the concrete constitutive law in the beam element formulation, the classical concrete fracture model by Hinton and Owen (1983; 1984) can be employed and modified considering crack opening and closing rules.

It should be noted that the attained plastic-fracture concrete model is formulated to describe the compressive and tensile behaviour in the 3D stress space. However, one of the main objectives of the present study is to simulate the structural behaviour of RC frames under progressive collapse by employing an efficient beam finite element formulation. As discussed in the beginning of this section, only one normal stress component and two accompanying shear stress components are assumed to be important in a beam element formulation, with the other stress components to be zero. In the previously proposed plastic-fracture concrete models (Grassl et al. 2002; Papanikolaou and Kappos 2007), only concrete material deformation behaviour in the 3D stress space was of interest and none of them has incorporated the model into a beam element formulation. Therefore, in the plastic-fracture concrete model proposed in **Chapter 3** of the present study, this deficiency in terms of stress component has to be rectified in a certain way and zero stress components stemming from the beam element stress simplification should be taken into account.

2.2 Beam Finite Element Formulation

For framed structures subjected to extreme loading such as an earthquake or a blast event, collapse takes place initially from the failures of some critical structural members, such as beams, columns and joints, before a localized damage spreads to the entire structure. At the member level, it is important for finite element analysis to capture the characteristics of material yielding and large geometric deformation, so as to simulate the process of progressive collapse. Additionally, the analysis should also model hardening property of materials as well as overall softening of structural response, as the structure is on the verge of collapse.

In terms of geometric nonlinearity, the efficient approaches of beam elements for large deformation analysis of framed structures can be generally classified into three types, that is, total Lagrangian (TL) formulation (Bathe and Bolourchi 1979; Schulz and Filippou 2001; Nanakorn and Vu 2006), updated Lagrangian (UL) formulation (Bathe and Bolourchi 1979; Cardona and Geradin 1988; Teh and Clarke 1999), and co-rotational (CR) formulation (Hsiao et al. 1987; Crisfield 1990; Felippa and Haugen 2005; Li 2007; Battini 2008).

In the context of progressive collapse, it is challenging to simulate the coupled effects of large deformation and material failure due to the computation accuracy of strain and stress in the deformed configuration. However, based on a CR framework, all the information necessary to determine the material stress state can be derived in the local system with only pure deformation excluding the rigid-body movement. This is the most appealing advantage of CR formulations, which will result in a more accurate and efficient computational scheme on strain and stress compared with other formulations. Therefore, CR formulations are employed for the beam formulation in the present study. Nevertheless, the superiority of CR formulations was only discussed at a theoretical level (Hsiao et al. 1987; Felippa and Haugen 2005; Li 2007) and needs to be numerically confirmed in the present study by comparison with other formulations, such as TL formulation (Dvorkin et al. 1988).

There are various formulations (Hsiao et al. 1987; Crisfield 1990; Felippa and Haugen 2005; Li 2007; Battini 2008) proposed in the literature, even though they share the common characteristics in terms of CR framework. Compared to the other CR formulations, there are two main advantages of the CR formulation proposed by Li (2007). In his approach, a set of vectorial rotational variables, which are three orthogonal components of normal vectors, is defined to describe spatial rotations. Through the judicious selection of vectorial rotational variables, all variables in the incremental solution process can be treated as vectors subjected to the usual rules of commutative addition. This results in a symmetric geometric stiffness matrix both in the local and the global systems. Furthermore, updating of vectorial rotational variables in incremental loading is much simpler compared to the conventional definitions of rotational degrees of freedom using absolute rotations about coordinate axes. Thus, the general idea of the CR beam formulation by Li (2007) is advantageous over the other CR formulations.

However, this CR formulation was initially derived for linear elastic material, which is not suitable for simulating structures undergoing large deformation with material yielding and approaching failure. For material nonlinearity, there are substantial publications concerning numerical and experimental aspects. There is a comprehensive review collection on material nonlinearity from Hinton and Owen (1984), but none of them can be directly applied to the CR framework by Li (2007). Therefore, if the CR beam formulation by Li (2007) were employed to predict the RC beam geometric nonlinearity, derivations for material nonlinearity at the fibre level for both steel and concrete have to be conducted in the framework of this CR formulation. This work is described in greater detail in **Chapter 4**.

In order to accurately simulate structures subjected to large deformation and material nonlinearity, the coupled effects of geometric and material nonlinearities of steel and RC structures should be incorporated for the proposed three dimensional CR beam. This is fundamental to the study of deformation behaviour of structural members at the ultimate limit state. Therefore, validations for the proposed derivations incorporating material nonlinearity

should be conducted through examples including isolated RC structural members and RC beam-column subassemblages.

In addition to the formulation derivations and material properties, computational efficiency of beam elements is also of interest when conducting finite element analysis for large-scale structures. Theoretically, CR formulations have an intrinsic advantage compared with TL and UL formulations. That is, CR formulations decompose the deformation into a rigid-body movement and a pure deformation. Moreover, the pure deformation and the corresponding strain depicted in local coordinate system are assumed to be small and can be efficiently calculated. Nevertheless, it should be pointed out that even though the CR beam formulation was proposed by Li (2007) a few years ago and since then, the efficiency advantage has been claimed, there is no direct comparison published in terms of computational time. The computational efficiency has to be evaluated based on the same material model and solution technique to be absolutely fair. This requires a lot of work to be done. For example, the TL beam element proposed by Dvorkin et al. (1988) can be utilized to represent a typical TL formulation, and a benchmark in terms of computational accuracy and efficiency can be conducted with comparison of the CR formulation. With the main objective of this thesis to simulate the nonlinear behaviour of RC framed structures during progressive collapse, the examples to conduct the benchmark tests should be based on structural problems involving geometric and material nonlinearities.

2.3 Component-Based Mechanical Model for RC Beam-Column Joints

Framed structures with rigid joints cannot consider the finite rotation capacity of joints, which is not in accord with reality. Component-based method is a good approach to determine the rigidity and strength of joints which is subjected to bending moment, shear force, tensile or compressive axial force. The idea of the component-based mechanical joint model is to identify critical regions of a joint in terms of failure mode and treat each one as an independent and functional component. The ductility of components is important so as to describe realistic

joint rotation capacity, which can then be compared with experimental moment-rotation curves. Therefore, a suitable evaluation of joint behaviour can substantially enhance the analysis accuracy of framed structures and provide crucial information of the joint deformation.

The component-based approach was originally proposed for beam-column steel connections. A condensed model of the right side connection (Bayo et al. 2006) is shown in **Fig. 2.2**, where K_{eq2} represents the resultant of the stiffness based on interaction of columns and beams, whereas K_{cws} and K_{cwc} are the stiffness of panel zone under shear and compression, respectively. These axial springs are assembled to form a single elasto-plastic rotational spring to model the beam-column connection in the structural analysis. The calculation of the moment-rotation curve can also be conducted. In the simulation of framed structures, there are different types of joints, viz. interior, exterior and knee joints. For 2D cases, a four-node element is introduced for the beam-column joint. Different types of joint elements (Bayo et al. 2006) are shown in **Fig. 2.3** and **Fig. 2.4**.

Fig. 2.2 Component-based model of the right side of a semi-rigid steel connection

Fig. 2.3 Degrees of freedom of (a) complete structure including the interior element and (b) interior element

Fig. 2.4 Degrees of freedom of (a) exterior element and (b) knee element

For a 2D RC beam-column joint, various component-based approaches had been proposed to predict the joint deformation behaviour (Alath and Kunnath 1995; Youssef and Ghobarah 2001; Lowes and Altoontash 2003; Altoontash 2004;

Bao et al. 2008). The general idea of the 2D beam-column joint model proposed by Lowes and Altoontash (2003; 2004) is widely adopted in the numerical study. As shown in **Fig. 2.5**, this joint model includes four external nodes (indicated by solid circles at the rigid plates) and four internal nodes (indicated by unshaded circles around the shear panel). In terms of components in the joint model as depicted in **Fig. 2.5**, 8 bar-slip components are employed to simulate the stiffness and strength loss due to the potential anchorage failure of beam and column longitudinal reinforcement embedded into the joint. One shear-panel component is employed to simulate the strength and stiffness loss due to shear failure of the joint core. Besides, 4 interface-shear components are employed to simulate the loss of shear-transfer capacity due to shear failure at the beam-joint and the column-joint interfaces. This represents the shear resistance due to aggregate interlock. It is noteworthy that the internal and external planes and nodes as shown in **Fig. 2.5** are actually coincident at the same physical position, which means the initial dimension of the bar-slip components is zero.

Fig. 2.5 Components of the 2D beam-column joint model

Before conducting a finite element analysis, joint design details are required to set up a numerical model incorporating the component-based mechanical joint. It is an inevitable and critical step to transform the joint design information into stiffness coefficients of various components in the component-based mechanical joint. The calibration procedure and corresponding results will bring about important effects on numerical stability and prediction accuracy. The

significance and the previous work on the component calibrations are reviewed as follows.

2.3.1 Bar-slip component

Previous experimental studies (Eligehausen et al. 1983; Shima et al. 1987; Russo et al. 1990; Sezen and Moehle 2003) under generalized excitations showed that besides flexural deformations, significant additional deformations (**Fig. 2.6**) were caused by the fixed end rotations due to slippage of longitudinal steel reinforcement at the beam-column junctions. Other than the total deformations, validated numerical simulations by Shima et al. (1987) and Lykidis and Spiliopoulos (2008) showed that the behaviour of RC members with and without bond action is quite different in terms of predicted structural ductility and stiffness due to the effect of tension stiffening. In the present study, tension stiffening implies the concrete residual tension resistance after cracking due to bonding with reinforcement, which contributes to the overall stiffness of the structure and is usually identified as the post-peak descending branch in a tensile stress-strain relationship. Besides, the bar slip behaviour causes significant stiffness degradation in the load-deformation relationships of moment-resisting frames (Eligehausen et al. 1983). In some extreme situations, brittle failure due to sudden loss of bond action between reinforcing bars and concrete in anchorage zones may cause severe local damage, leading to partial or total collapse of structures (Eligehausen et al. 1983). Therefore, bar slip behaviour should be incorporated when accurately analysing progressive collapse resistance of RC beam-column structures.

Fig. 2.6 Additional deformation resulting from local bar slip at the “fixed end condition”

(a) Model by Lowes and Altoontash (2003) and Sezen and Moehle (2003)

(b) Model by Alsiwat and Saatcioglu (1992)

Fig. 2.7 Bond stress distribution in the piecewise uniform form **Fig. 2.8** Different piecewise nonuniform forms of bond stress distributions

In the previous studies, several local bond stress-slip relationships between steel reinforcement and concrete subjected to axial pullout have been proposed and can generally be classified in terms of bond stress distribution. One of them is a piecewise uniform distribution (Alsiwat and Saatcioglu 1992; Lowes and

Altoontash 2003; Sezen and Moehle 2003), that is, bond stress distribution is idealized as two segments of uniform bond stress along the whole embedment length (**Fig. 2.7**). The piecewise uniform bond stresses represent mechanical bond and frictional bond, denoted as τ_E and τ_Y , respectively. The mechanical bond is induced by mechanical interlocking between lugs of reinforcing bars and surrounding concrete. After the concrete keys between the lugs have been sheared off, the frictional resistance between the rough concrete surfaces is the only remaining mechanism. Alternatively, some other types of piecewise nonuniform distributions as shown in **Fig. 2.8** are found in the literature, such as the multi-linear distribution (Yankelevsky 1985; Ueda et al. 1986; Kwak and Filippou 1990; Kwak and Filippou 1997; Khalfallah and Ouchenane 2008) (**Fig. 2.8 (a)**), logarithmic distribution (Shima et al. 1987) (**Fig. 2.8 (b)**), exponential ascending and linear descending distribution (Eligehausen et al. 1983; Russo et al. 1990; Noh 2009) (**Fig. 2.8 (c)**) and nonlinear ascending and linear descending distribution (Ožbolt et al. 2002; Lettow et al. 2004; Lowes et al. 2004; Eligehausen et al. 2006) (**Fig. 2.8 (d)**) in which the ascending branch is controlled by the Menegotto-Pinto equation (Menegotto and Pinto 1973).

In the analytical model based on a piecewise uniform distribution proposed by Lowes and Altoontash (2003) and Sezen and Moehle (2003), the bond stress at the elastic segment of steel reinforcement is larger than that at the plastic segment as shown in **Fig. 2.7 (a)**. In contrast, as shown in **Fig. 2.7 (b)**, Alsiwat and Saatcioglu (1992) proposed an analytical model in which the bond stress at the plastic segment of steel reinforcement is a summation of elastic bond stress and frictional bond stress. As a matter of fact, the bond stress at the plastic segment along the steel reinforcement (where large local straining and bar slip occur and probably, concrete keys between lugs have been sheared off) should be only the frictional bond stress rather than the summation of the mechanical bond stress and frictional bond stress as proposed by Alsiwat and Saatcioglu (1992).

As for the two simple bi-uniform analytical models proposed by Sezen and Moehle (2003) and Lowes and Altoontash (2003) as shown in **Fig. 2.7 (a)**,

different values were utilized for mechanical bond and frictional bond. Even though the failure mode and resisting mechanisms were greatly simplified, the predictions by these models were still satisfactory. Nevertheless, there are some shortcomings (Lowes and Altoontash 2003; Sezen and Moehle 2003) which need to be addressed as follows. Firstly, the boundary conditions considered in these models are limited to a few cases. Consequently, these models are only capable of predicting some of the failure modes, such as either fracturing of steel reinforcement with a sufficient embedment length, or bond slip failure of steel reinforcement with an insufficient embedment length. Other possible failure modes associated with certain embedment lengths and steel mechanical properties are not considered, such as fracturing of steel reinforcement with an insufficient embedment length. Secondly, these analytical models for bond-slip relationship were originally proposed for seismic loading. To take account of the influence of load cycles, bond stress deterioration is usually proposed in terms of predefined unloading and reloading paths from backbone envelopes. In seismic analysis, the damage accumulation due to repeated cycles of unloading and reloading should have been considered when proposing these models to quantify bar slippage. Therefore, the backbone envelopes are not simply identical to the monotonic loading envelopes. Since the focus in the present study is on progressive collapse analysis where only monotonic loading condition is considered, instant column removals are assumed at the beginning of analysis and the initial reaction forces at locations of removed columns are treated as the applied quasi-static loads; alternatively, a more generalized element removal technique proposed by Talaat and Mosalam (2009; 2009) can be employed. Therefore, the proposed bond stress distributions in these analytical models need to be validated against detailed bar-slip experiments subjected to monotonic loading.

It is noteworthy that each of the models in **Fig. 2.8** has its own shortcomings. The multi-linear distribution (**Fig. 2.8 (a)**) which is derived based on a non-yielding bar, is too simplistic to represent the complicated bar-slip behaviour (Ueda et al. 1986). Likewise, although the logarithmic distribution (**Fig. 2.8 (b)**) proposed by Shima et al. (1987) describes not only the ascending branch but

also the post-yielding range of steel reinforcement; there is no descending branch to reflect bond deterioration even when local bar slip is sufficiently large. Another demerit of the model (Shima et al. 1987) is that since the variation of strain is represented by the bond stress distribution in the form of logarithmic function starting from the unloaded end (**Fig. 2.8 (b)**), an extremely steep bar strain variation and consequently, an extremely large bond stress near the unloaded bar end are obtained in the short pullout tests which is not observed in actual tests. The remaining two models shown in **Fig. 2.8 (c)** and **(d)**, viz., the exponential ascending and linear descending distribution (Eligehausen et al. 1983; Russo et al. 1990; Noh 2009) and Menegotto-Pinto equation controlled ascending and linear descending distribution (Ožbolt et al. 2002; Lettow et al. 2004; Eligehausen et al. 2006) are similar in form. In fact, both of them (**Fig. 2.8 (c)** and **(d)**) were originally proposed by Eligehausen et al. (1983) and his colleagues in the University of California, Berkeley. The Menegotto-Pinto equation controlled ascending and linear descending distribution (Ožbolt et al. 2002; Lettow et al. 2004; Eligehausen et al. 2006) was validated by Lettow et al. (2004) to be suitable for both short and long embedment lengths subjected to monotonic loading. Additionally, both analytical models consider bond deterioration for large local slip. However, one common demerit of these two models is that the embedment length has to be divided into many segments, upon which iterative calculations have to be performed to satisfy the steel stress-strain relationship, the equilibrium between bond force and bar force, and the boundary conditions for different segments of embedment lengths. Consequently, when analysing large-scale RC framed structures, this approach requires far too much computational effort for each steel reinforcing bar of RC beam-column joints. In conclusion, the analytical model based on piecewise nonuniform bond stress distributions is far too complex, even though some of them are capable of predicting the test results well.

In addition, it should also be noted that the measured bond strains and associated stresses in experimental studies scattered considerably, even for tests performed at the same laboratory (Eligehausen et al. 1983; CEB 2010). For a given value of slip, the coefficient of variation of bond stress may be as large as 30% (CEB

2010). Therefore, the conventional practice is to assume some average values for bond stresses in the respective elastic and plastic segments of steel reinforcement (Alsiwat and Saatcioglu 1992).

Based on the above discussions on the merits and demerits of previous analytical models (**Figs. 2.7** and **2.8**) for axial pullout tests, the ideal bond stress-slip relationship for large-scale structures should be simple and reliable, incorporating the important factors associated with (a) nonlinearity of steel materials and (b) different embedment lengths and boundary conditions. Therefore, an analytical model is proposed later in the present study, based on a piecewise uniform bond stress distribution, to overcome the demerits of previous models (**Figs. 2.7** and **2.8**). This is the main novelty of the present study.

Another common demerit of previous analytical models on bond stress-slip relationship is that, only pullout action is accounted for and dowel action of reinforcing bars is ignored. In fact, with sufficient restraint from surrounding structures at the early stage of loading history, beam-column joints will undergo compressive arch action and flexural action with small deformations. But with increasing loads, concrete will crack at the tension region and bar-slip behaviour will commence with reinforcing bars subjected to pullout action from the RC joints. Moreover, with increasing deformation, the pullout behaviour at the joint becomes more significant due to mobilization of catenary action in the beam bridging over the removed column. Consequently, dowel action at the bottom steel reinforcement of the joint region commences due to opening of cracks at the beam soffit, as shown in **Fig. 2.6**. This joint is located above the missing column and would experience a reversal in bending moment. Based on the observations in the test series on RC beam-column subassemblages conducted by Yu and Tan (2010; 2011), the inclination angle θ of beams with respect to the horizontal axis can be up to 15° when catenary action is mobilized. Additionally, experimental studies (Maekawa and Qureshi 1996a; Soltani et al. 2005) showed that the steel reinforcement under the combined axial pullout and transverse dowel actions will yield earlier than the steel reinforcement subjected

to only pullout. Therefore, where dowel action is significant, the proposed bar-slip behaviour should incorporate not only the pullout mechanism but also dowel action to resist transverse shear.

As a consequence, in the present study of component-based RC beam-column joints, the proposed simple analytical model based on a bi-uniform bond stress distribution should be capable of predicting the bar-slip behaviour in the RC beam-column joints subject to not only axial pullout but also dowel action. The important factors, including bond deterioration, pullout failure, post-yielding range of steel, steel fracturing, various bar embedment lengths and boundary conditions which have not been systematically incorporated in the previous analytical models, will also be considered in the new analytical model. For this purpose, a simple and reliable analytical model will be proposed and validated in **Chapter 5** of this thesis.

2.3.2 Shear-panel component

In the relatively small volume inside the beam-column joints, there is a highly nonlinear region due to the composite action of steel reinforcement and concrete and local stress variations within the RC beam-column joints, which brings about difficulties when analysing the behaviour of RC beam-column joints. In practice, component-based joint models are usually employed as an approximation to model the complex deformation behaviour. For 2D RC beam-column joints, various component-based approaches have been proposed to simplify the joint deformation behaviour (Alath and Kunnath 1995; Youssef and Ghobarah 2001; Lowes and Altoontash 2003; Altoontash 2004; Bao et al. 2008; Birely et al. 2012). As an example of a typical interior joint (Lowes and Altoontash 2003), shown in **Fig. 2.5**, the idea of component-based beam-column joint model is to differentiate the characteristics of critical regions in the joint and treat each spring as an independent and functional component. The shear-panel component is employed to simulate the strength and stiffness loss due to shear failure of the joint core.

To predict the shear strength of the shear-panel component in the RC joint (**Fig. 2.5**), the modified compression field theory (MCFT) (Vecchio and Collins 1986; 1993) has been widely employed (Youssef and Ghobarah 2001; Lowes and Altoontash 2003; Altoontash 2004; Shin and Lafave 2004; Mitra and Lowes 2007; Bao et al. 2008), even though it was reported (Shin and Lafave 2004; Mitra and Lowes 2007) that the analytical form of MCFT is not appropriate to predict the RC joint shear behaviour with low joint transverse reinforcement ratios. In fact, the comparisons (Kim and LaFave 2009) based on an extensive database of RC joints clearly demonstrated that the MCFT is incapable of predicting the shear strength accurately, even if the joint panel is effectively confined as stipulated in the ACI 352R-02 (2002). The MCFT model was originally proposed to predicting the in-plane shear behaviour of 2D concrete elements reinforced with uniform transverse and longitudinal steel. This behaviour is markedly different from that of the beam-column joints. An experimental study (Wong et al. 1990) showed that distributed longitudinal steel reinforcement in the beam-column joints cannot represent conventional horizontal joint core hoops as shear reinforcement, because the latter in the form of hoops can provide more efficient and dependable diagonal compression struts to resist the horizontal shear force acting on the joint. In general, it is well accepted that the MCFT model is not suitable for the shear strength predictions of RC joints due to different steel reinforcement detailing. Therefore, the MCFT model is excluded in the present study.

Fig. 2.9 Strut configurations used in previous researches on the SAT model

As an alternative to the MCFT model, the strut and tie (SAT) concept is widely utilized in the design of deep beams, shear walls and beam-column joints where there are clear force paths or discrete struts joining the loading point to the support. However, the behaviour of beam-column joints is too complex to be modelled realistically with simple strut and tie models based on plasticity theory and, thus, empirical approaches were proposed to develop an essentially descriptive strut and tie model for beam-column joints (Vollum and Newman 1999). Fundamentally, the proposed SAT model consists of the strut configurations and the corresponding load transfer mechanisms. In many previous research works on SAT models, several strut configurations have been proposed as shown in **Fig. 2.9** (the arrow indicates the concrete strut), such as one direct and two horizontally indirect struts (Vollum and Newman 1999) (**Fig. 2.9 (a)**), one direct and four indirect struts (Hwang and Lee 1999; 2000) (**Fig. 2.9 (b)**) and one direct and one indirect struts (Park and Mosalam 2012a; 2012b) (**Fig. 2.9 (c)**). The model with one direct and one indirect struts (Park and Mosalam 2012a; 2012b) is proposed for exterior beam-column joints without transverse reinforcement, while the first two models with appropriate load transfer mechanisms (Hwang and Lee 1999; Vollum and Newman 1999; Hwang and Lee 2000) are general and can be applied to many types of 2D beam-column joints. The concept of softened strut-and-tie model (Hwang and Lee 1999; 2000) has been adopted by other researchers (Favvata et al. 2008) to predict the shear strengths of exterior RC beam-column joints under seismic loading.

It should be noted that in the previous studies on beam-column joints using the SAT concept (Hwang and Lee 1999; Vollum and Newman 1999; Hwang and Lee 2000; Favvata et al. 2008; Park and Mosalam 2009; 2012a; 2012b; 2013a; 2013b), researchers were only interested in the predictions of joint shear strengths under seismic loading. However, prior to the crushing of a concrete strut, concrete will crack and transverse reinforcement may also yield. Therefore, if one wishes to simulate realistic joint shear behaviour, besides the prediction of ultimate shear strength, it is important for the analytical model to simulate the critical stages of development of concrete cracking and transverse reinforcement yielding. In the publication on the shear strength predictions of beam-column

joints to date, only the analytical model recently developed by Park and Mosalam (2012a) estimated the joint shear stress-strain relationship based on SAT model. However, the proposed relationship is oversimplified and exclusively focused on the predictions of shear strengths for exterior beam-column joints without transverse reinforcement.

In order to eliminate the limitations of the previous analytical models, such as the MCFT model and the SAT model, a new analytical model for RC shear panels is proposed. The most attractive feature is that, the proposed analytical model is capable of predicting the critical stages for concrete cracking, transverse reinforcement yielding, ultimate shear strength and subsequent strength descending for 2D RC beam-column joints subjected to monotonic loading. It should be mentioned that throughout all the stages in the proposed analytical model, equilibrium, compatibility and constitutive laws for concrete and steel reinforcement are satisfied in terms of average stress and strain criteria, which makes the proposed model considerably rational when compared to other existing research findings.

In the derivation of the proposed analytical model, important structural effects due to RC joint characteristics should be incorporated as follows. Due to the presence of tensile strain perpendicular to the strut direction, the compressive behaviour of concrete in the joint region is different from that in the standard cylinder test under uniaxial compression (Kashiwazaki and Noguchi 1996). This is known as concrete compression softening phenomenon and has been observed in deep beams (Arabzadeh et al. 2009; Hong and Ha 2012) and shear walls (Vecchio and Collins 1986; 1993; 1998). In the study of RC joints, a similar concept should be taken into account for beam-column joints as shown in **Fig. 2.10**. Besides, the confinement effect from the joint transverse reinforcement (Scott et al. 1982; Foster and Gilbert 1996; Tsonos 2007) should also be accounted for in the new analytical model. Due to the presence of transverse reinforcement, the concrete compressive strength and the maximum compressive strain are enhanced, which will influence the ductility of the beam-column joints. In addition, as reported in many previous studies (Bakir and

Boduroğlu 2002; Park and Mosalam 2012b), there are many geometric, material and loading parameters to be considered, which will complicate the analytical model study. Hence, several important parameters will be identified as the dominant parameters to shear strength predictions of RC beam-column joints in the new analytical model, which will be highlighted in **Chapter 5** of this thesis.

Fig. 2.10 Concrete compression softening phenomenon in beam-column joints

2.3.3 Interface-shear component

For the calibration of interface-shear component at the joint perimeter between joint and beam or column, there are two available alternatives to obtain the shear response of the joint.

The first approach is the analytical method (Walraven 1981; 1994) based on the assumption that all the shear force is transferred through the aggregate fraction and aggregate interlocking phenomena from a statistical point of view. Dowel action from steel reinforcement is assumed to be of minor importance and consequently can be neglected. The advantage of the analytical method (Walraven 1981; 1994) is that the shear displacement response for a certain cross-section can be attained, thus the location of the interface-shear component will be exactly at the interface between the joint and the beam or the column. However, the shear displacement is a function of both shear stress and crack width (Walraven 1981; 1994). This means besides the shear stress, crack width has to be a known variable prior to the calculation of shear displacement.

Another shortcoming for this analytical model in the finite element implementation is that the response curve of shear displacement is not so smooth when shear deformation is relatively small. This shortcoming will result in unstable numerical problems when the stiffness coefficient of the interface-shear component is calculated.

The second option is based on an empirical method. Based on the testing data and curve fitting, the shear response for the interface-shear component can be obtained. Nevertheless, the tests conducted exclusively for the calibration of interface-shear component are rare. Therefore, one assumption has to be made. A short column is treated as a region of interface-shear component and its length can be determined according to the ratio of the interested column length (a) and the column cross-sectional depth (d) as shown in **Fig. 2.11** (Patwardhan 2005). **Fig. 2.11** shows that in the range of aspect ratios (a/d) between 2 and 4, the values of maximum shear strength and shear demand are very close. Therefore, the failure mode within this range is difficult to be differentiated, which is usually defined as shear-flexure failure. When the aspect ratio is less than 2, the failure mode for the column belongs to shear, while the failure mode for the column with an aspect ratio larger than 4 is flexural. As a result, the length of the short column suitably representing the interface-shear component is chosen as 2 times the column cross-sectional depth.

After attaining the length of interest for the interfacial-shear component, the shear resistance has to be empirically determined based on experimental results. Among the published experimental studies, Patwardhan (2005) conducted a series of RC column specimens subjected to shear failure. Therefore, an extensive collection of shear specimens is employed to calibrate the interfacial-shear component.

Fig. 2.11 The relationship of failure type and aspect ratio (Patwardhan 2005)

2.4 Superelement

The concept of superelement has been widely applied in efficient numerical simulations using finite element analysis. A superelement consists of a finite number of elements and is termed as a substructure as well. The idea was invented by aerospace engineers in the early 1960s to carry out a first-level breakdown of complex systems such as a complete airplane (Przemieniecki 1968). In the 1970s, the superelement technique as a new structural analysis method was incorporated in NASTRAN. Zemer (1979) conducted quite extensive comparisons and proved that the superelement technique can result in substantial cost benefits for large-scale structure analysis. Jacobsen (1983) concluded the advantages through the use of fully integrated superelements in basic finite element programs in terms of replication, reusability, matrix bandwidths, data generation, load condition, reanalysis and computational cost.

Due to the attractive advantage of significantly improving computational efficiency, superelement method has been employed when solving various problems in recent years. The characteristics suitable for superelement applications can be summarized into three distinguishing features, that is,

iterative computational tasks, localized nonlinearity and a large number of finite elements in the numerical models for applications in structural dynamics.

Firstly, as for the iterative analysis and design in the topology optimization, superelement method can be adopted when a structure is locally designed for the topology optimization. Hence in a structure, the part outside the zone which is supposed to be locally optimized can be defined as a superelement and the corresponding stiffness matrix remains the same in the later analysis, while only the stiffness matrix of the part within the optimized zone needs to be reformulated iteratively (Qiu et al. 2009).

Secondly, in multibody problems, the large relative rotation between individual bodies introduces geometric nonlinearity in the computational model, while the deformations inside each body are small enough to be considered as elastic and, therefore, suitable for the superelement application (Cardona and Geradin 1991; Cardona 2000). Similarly, in the vehicle joints modelling (Maressa et al. 2011) the coupling interface between vehicle parts is the crucial region and can be defined as the connection between superelements.

Thirdly, for dynamic analysis, a large number of elements are needed to obtain accurate simulation results for the eigenvalue and frequency response. Thus, the idea of superelement can be utilized to decrease the model evaluation time by using a reduced model (Belyi 1993; Agrawal et al. 1994; De Gersem et al. 2007). Besides, the concept of superelement can also be easily implemented to condense a group of members or components into a superelement for large-scale structure analysis (Ju and Choo 2005; Steenbergen 2007; Belesis and Labeas 2010; Huang et al. 2010).

In addition to decreasing computational cost, the application of superelement provides the opportunity to overcome some modelling difficulties when tedious geometric details are involved in the models. For instance, in the three-dimensional seismic analysis of a high-rise building, due to the fine mesh around the shear wall with various types of openings, the degrees of freedom from the nodes which are not connected to beams and columns of interest result

in unnecessary computational cost and they can be eliminated via the implementation of superelement before structural analysis commences (Kim et al. 2005).

Similar problems are also encountered in the present study on structural analysis for progressive collapse of buildings. To check for progressive collapse, the computational cost for nonlinear analysis of a large structure such as a multi-storey reinforced concrete building is always exorbitant since a large number of finite elements are needed to discretise the structure in order to obtain reasonably accurate predictions of the structural response (Hartmann et al. 2008). This is because although a relatively small portion of the building is initially subjected to extreme loads such as a blast event, it is required to simulate the response of the entire building with a time-consuming nonlinear model to ascertain whether the initial damage has spread to the other parts of the structure. Therefore, in practice, structural engineers often need to strike a balance between the computational cost employed and the accuracy in structural response predictions.

It should be noted that modelling geometric and material nonlinearities using the finite element method almost always requires repeating updates of tangent stiffness matrix and solutions of the corresponding linear equation systems, which is the most time-consuming step in the iterative numerical scheme. However, in many practical problems involving nonlinear analysis of large-scale structures, the material nonlinearity phenomena are usually localized in certain critical structural members (Department of Defense (DoD) 2005) and may not dramatically spread throughout the whole structure. Thus, at each load increment, the computational effort spent on assembling the tangent stiffness matrix calculation and updating of structural members properties could be avoided using the concept of *superelement*.

Conventionally, the superelement formulation is derived based on static condensation (Wilson 1974) which is also known as Guyan reduction (Chen and Pan 1988). However, there are several significant shortcomings in this method.

Firstly, in the conventional superelement formulation based on static condensation, many nodes in the finite element mesh have to be renumbered, or the rows and columns in the stiffness matrix have to be swapped to make the degrees of freedom associated with the superelement to lie within the upper left sub-matrix in the system stiffness matrix. This will increase the computational time to certain extent and, thus, negate the objective to achieve efficiency.

Secondly, material nonlinearity is the only concerned time-consuming part in finite element analysis of conventional superelement formulations based on static condensation. However, besides material nonlinearity, geometric nonlinearity can also be involved due to large rigid-body rotations when analysing progressive collapse of structures. It should be noted that the structural regimes suitable for superelement applications are not always fixed on the foundation (or other essential boundary conditions) and do not always undergo small deformation. In general, there are two types of superelement configurations based on whether the superelement is directly fixed onto the foundation. In many simulations regarding structural failures in the seismic events or terrorist bombing accidents, material nonlinearity is localized at the first storey of structures and the storeys above only undergo small deformations. Thus, the deformations of the storeys above the first are accompanied by large rigid-body rotations as shown in **Fig. 2.12**, which should be considered in the superelement formulation. However, in the conventional superelement formulations based on static condensation, the effect of superelement zone on the nonlinear zone is considered in the condensed stiffness matrix of the whole structure, and there is no independent explicit expression for the superelement stiffness matrix after the formation of superelement. Therefore, once the geometric nonlinearity is taken into account for the superelement zone, the stiffness matrix of the whole structure has to be updated and this will considerably increase the computational cost.

In order to overcome the shortcomings of the conventional superelement formulations as discussed above, a new superelement approach needs to be

formulated in a new way so as to take full advantage of the computational time when analysing structural behaviour under progressive collapse.

Fig. 2.12 A multi-storey building with a column removal at the first storey

2.5 Closure

Since the main objective of the thesis is to simulate the structural behaviour of reinforced concrete (RC) framed structures for different column-removal scenarios, concrete models, beam element formulations and joint models are the fundamental and essential components, on which the previous works are reviewed in this chapter. A brief summary for concrete models and beam element formulations is given as follows.

- Uniaxial concrete models are usually adopted in the finite element analysis of RC framed structures because of their simplicity and efficiency. Nonetheless, only the structural behaviour of RC beam-column members dominated by flexural failure can be accurately predicted. In order to enrich the simulation capability for shear failure predictions of RC beam members, a concrete model based on plasticity theory should be employed. Nevertheless, the review on the previous works shows that more than one parameter without clear physical meanings were proposed in the previous plasticity-based concrete models and they are usually difficult to be experimentally calibrated.

Thus, if a new plasticity-based concrete model is going to be proposed in the present study to predict shear failures of RC beam members, the following properties are preferred: (1) the model should have a minimum number of parameters; (2) the parameters should have clear physical meanings and can be conveniently calibrated according to uniaxial compression cylinder tests.

- In the context of progressive collapse, co-rotational (CR) formulations have significant advantages over total Lagrangian (TL) formulation and updated Lagrangian (UL) formulation, due to the intrinsic characteristics of CR formulations which contain only pure deformations in the local system. The separation of pure deformation from rigid-body movement will result in a more accurate and efficient computational scheme on strain and stress. Compared with previously proposed CR beam element formulations, the advantages of CR formulations proposed by Li (2007) are (a) symmetric stiffness matrix and (b) updating of rotational variables based on vectorial rotational variables. Computational efficiency of beam elements is also of interest when conducting finite element analysis for large-scale structures. Theoretically, the CR beam element formulations should be more efficient. However, a benchmark test in terms of computational accuracy and efficiency needs to be conducted and compared with TL formulation (Dvorkin et al. 1988).

As for the 2D RC beam-column joint, the component-based mechanical joint model is usually employed incorporating the bar-slip component, the shear-panel component and the interface-shear component. In order to obtain accurate and reliable predictions, the calibrations for all types of components are critical. The shortcomings in the previous analytical models on the bar-slip component and the shear-panel component are reviewed and the findings are summarized as follows.

- To calibrate the bar-slip component, one type of analytical models employs piecewise nonuniform distributions obtained from experimental studies. Nonetheless, there are different limitations for different

analytical models. One common demerit of these analytical models is high computational cost. As for the other type of analytical models with bi-uniform bond stresses, the advantage is that even though the failure mode and resisting mechanisms are greatly simplified, the predictions are still satisfactory. However, they do not take account of all possibilities for embedment lengths and steel properties. Furthermore, previous analytical models were usually proposed for seismic loading. As the main objective of this thesis is to study the structural behaviour under progressive collapse, the proposed uniform bond stress distribution has to be validated against detailed bar-slip experiments subjected to monotonic loading. As a balanced choice, the bond stress-slip relationship suitable for large-scale structures should be simple and reliable with considerations of nonlinearity of steel constitutive model and different embedment lengths. In addition, experimental studies showed that if transverse deformation is evident, the steel reinforcement under the combined axial pullout and transverse dowel action will yield earlier than the steel reinforcement under the axial pullout only. An analytical model for the bond stress-slip relationship should also account for the effect of the dowel action incorporated with axial pullout.

- To calibrate the shear-panel component, the modified compression field theory (MCFT) and the strut and tie (SAT) models have been widely employed. However, as reviewed in the experimental studies, it is found that the MCFT model is not appropriate to predict the RC joint shear behaviour due to the reinforcement detailing. As for SAT models, the simplified load transfer mechanisms are meaningful for 2D beam-column joints, provided that reasonable strut configurations are taken into consideration. However, most publications only focus on the shear strengths of joints under seismic loading. In order to eliminate the limitations of previous analytical models, a new analytical model is proposed to obtain a complete shear force-deformation response including the critical stages for concrete cracking and transverse reinforcement yielding.

Lastly, in order to perform efficient analyses without loss in accuracy, conventional superelement formulations based on static condensation are reviewed and their limitations in practice are discussed from the point of view of RC structure simulations subjected to progressive collapse. One disadvantage is that the nodal degrees-of-freedom have to be renumbered or swapped. Another disadvantage is that there is no independent explicit expression for the superelement stiffness matrix after the formation of superelement. Therefore, the update of stiffness matrix of the whole structure will be time-consuming. In order to overcome these shortcomings, a new superelement approach has to be reformulated to take full advantage of computational time when analysing structural behaviour under progressive collapse.

Chapter 3 Concrete Models for RC Beam Members

3.1 Introduction

The outline of this chapter is summarized as follows. In **Section 3.2**, in order to predict flexural failures in RC beam members, two of the most widely employed uniaxial concrete models are briefly introduced and modified for the finite element implementation, which will be incorporated into the proposed beam element formulation in **Chapter 4**.

However, uniaxial concrete models have limitations in predicting shear failures for RC beam members with short and medium shear span-to-depth ratios, due to the stress simplification associated with beam element formulation. Therefore, a concrete model based on plasticity theory is used to predict shear failures of RC beam members. In **Section 3.3**, a general compressive concrete constitutive law is proposed based on a three-parameter failure function and a concise one-parameter potential function, while a fixed crack approach to model the concrete tensile behaviour is adopted. Besides, the proposed material properties are calibrated for concrete plasticity and fracture models. Based on the concrete plastic-fracture model originally proposed for a 3D stress state, the *unified plasticity concrete model* is proposed for fibre beam element formulations with corresponding constraint equations due to simplifications from a 3D solid element to a beam element.

3.2 Uniaxial Concrete Models

To describe the uniaxial compressive behaviour of concrete, the modified Kent and Park model (Park et al. 1982) and the Mander's model (Chang and Mander 1994; Waugh 2009) are briefly introduced herein and modified to consider the uniaxial tensile behaviour of concrete. Both of the uniaxial concrete models have been implemented in a self-developed finite element package FEMFAN3D in NTU, Singapore.

3.2.1 The modified Kent and Park model

The modified Kent and Park model (Park et al. 1982) is illustrated in **Fig. 3.1**, showing that the monotonic concrete stress-strain relationship in compression is described by three regions:

$$\sigma_c = Kf_c \left[2 \left(\frac{\epsilon_c}{\epsilon_0} \right) - \left(\frac{\epsilon_c}{\epsilon_0} \right)^2 \right], \quad \text{if } \epsilon_c \leq \epsilon_0 \quad (3.1)$$

$$\sigma_c = Kf_c [1 - Z(\epsilon_c - \epsilon_0)] \geq 0.2Kf_c, \quad \text{if } \epsilon_0 \leq \epsilon_c \leq \epsilon_u \quad (3.2)$$

where

$$\epsilon_0 = 0.002K \quad (3.3)$$

$$K = 1 + \frac{\rho_s f_{yh}}{f_c} \quad (3.4)$$

$$Z = \frac{0.5}{\frac{3 + 0.29f_c}{145f_c - 1000} + 0.75\rho_s \sqrt{\frac{h_{core}}{S_h}} - 0.002K} \quad (3.5)$$

In the above formulae, ϵ_0 is the concrete strain at the maximum stress, K is a factor which accounts for the strength increase due to confinement, Z is the strain softening slope, f_c is the concrete compressive cylinder strength in MPa, f_{yh} is the yield strength of stirrups in MPa, ρ_s is the ratio of the volume of hoop reinforcement to the volume of concrete core measured to the external dimensions of stirrups, h_{core} is the width of concrete core measured to the external dimensions of stirrups, and S_h is the centre-to-centre spacing of stirrups or hoop sets. In case of concrete confined by stirrups, it is suggested that ϵ_u is determined conservatively by **Eq. (3.6)**.

$$\epsilon_u = 0.004 + 0.9\rho_s (f_{yh}/300) \quad (3.6)$$

It should be noted that the tensile strength of concrete was ignored in the original Kent and Park model. To take the concrete tensile strength into account, the bilinear stress-strain relationship suggested by Rots et al. (1984) as illustrated in **Fig. 3.2** is adopted with $\epsilon_{cu} = \alpha_1 \epsilon_{cr}$, $\alpha_1 = 10 \sim 25$ and $\epsilon_{cr} = f_t / E_c$ where f_t is the concrete tensile strength in MPa and E_c is the elastic modulus of concrete in MPa (Barzegar-Jamshidi 1987).

Fig. 3.1 Stress-strain relationship for confined and unconfined concrete under compression

Fig. 3.2 Stress-strain relationship for concrete under tension

The loading and unloading rules of stress-strain relationship for both confined and unconfined concrete materials are also considered. The governing rules are illustrated in **Fig. 3.3**. According to **Fig. 3.3**, unloading from a point on the envelop curve takes place along a straight line connecting the point ϵ_r at which unloading starts to a point ϵ_p on the strain axis given by **Eqs. (3.7)** and **(3.8)**.

$$\frac{\epsilon_p}{\epsilon_0} = 0.145 \left(\frac{\epsilon_r}{\epsilon_0} \right)^2 + 0.13 \left(\frac{\epsilon_r}{\epsilon_0} \right) \quad \text{for } \left(\frac{\epsilon_r}{\epsilon_0} \right) < 2 \quad (\text{Karsan and Jirsa 1969}) \quad (3.7)$$

$$\frac{\epsilon_p}{\epsilon_0} = 0.707 \left(\frac{\epsilon_r}{\epsilon_0} - 2 \right) + 0.834 \quad \text{for } \left(\frac{\epsilon_r}{\epsilon_0} \right) \geq 2 \quad (\text{Taucer et al. 1991}) \quad (3.8)$$

where ϵ_0 is the strain level corresponding to the maximum compressive stress. It should also be noted that the above loading and unloading rules of concrete in compression does not account for the cyclic damage of concrete.

Fig. 3.3 Loading and unloading rules of concrete under compression

3.2.2 The Mander's model

The stress-strain envelope of the Mander's model proposed by Chang and Mander (1994) includes two parts as shown in **Fig. 3.4**. The first part is a nonlinear curve depicted by Tsai's equations, and the second part is a straight line. The detailed explanations of these equations are given by Waugh (2009).

Fig. 3.4 Stress-strain envelop of the Mander's Model

The compression envelope of the Mander's model is defined by the initial slope (E_c), the peak stress (f_c) and its corresponding strain (ϵ_c), parameters (r and n) in Tsai's equation and the critical strain (ϵ_{cr-c}) to define the spalling strain of concrete. As given in **Eq. (3.9)**, Tsai's equation can be written in a non-dimensional form.

$$y(x) = \frac{nx}{D(x)}, \quad z(x) = \frac{(1-x^r)}{[D(x)]^2} \quad (3.9)$$

where

$$D(x) = \begin{cases} 1 + (n - \frac{r}{r-1})x + \frac{x^r}{r-1} & (r \neq 1) \\ 1 + (n-1 + \ln x)x & (r = 1) \end{cases} \quad (3.10)$$

In **Eqs. (3.9)** and **(3.10)**, the non-dimensional variables x and n on the compression envelope which are denoted as x^- and n^- , respectively, are calculated by

$$x^- = |\epsilon / \epsilon_c|, \quad n^- = |(E_c \epsilon_c) / f_c| \quad (3.11)$$

The non-dimensional spalling strain x_{sp}^- on the compression envelope can be calculated by using **Eq. (3.12)**.

$$x_{sp}^- = x_{cr-c} - y(x_{cr-c}) / [n^- z(x_{cr-c})] \quad \text{where} \quad (3.12)$$

$$x_{cr-c} = \epsilon_{cr-c} / \epsilon_c$$

The stress σ^- and the tangent Young's modulus E_t^- for any strain on the compression envelope are given by three piecewise **Eqs. (3.13)** through **(3.15)**.

(a) For $x^- < x_{cr-c}$ (Nonlinear curve)

$$\sigma^- = f_c y(x^-), \quad E_t^- = E_c z(x_{cr-c}) \quad (3.13)$$

(b) For $x_{sp}^- \leq x^- \leq x_{cr-c}$ (Straight line)

$$\sigma^- = f_c [y(x_{cr-c}) + n^- z(x_{cr-c})(x^- - x_{cr-c})], E_t^- = E_c z(x_{cr-c}) \quad (3.14)$$

(c) For $x^- > x_{sp}^-$ (Spalling)

$$\sigma^- = E_t^- = 0 \quad (3.15)$$

In **Eqs. (3.13)** and **(3.14)**, f_c is the cylinder compression strength of concrete.

As for the tension envelope, the origin of the tension part is shifted by a parameter ε_0 as shown in **Fig. 3.4**. Nevertheless, the reason for such a shift was not explicitly given by Chang and Mander (1994). Usually, this shift is ignored, i.e. $\varepsilon_0 = 0$, when implementing the Mander's model into finite element analysis packages, such as OpenSees (Mazzoni et al. 2009). Similar to the compression envelope, the tension envelope of the Mander's model is defined by the initial slope (E_t), the peak stress (f_t) and its corresponding strain (ε_t), parameters (r and n) in Tsai's equation (Waugh 2009) and the critical strain (ε_{cr-t}) to define the cracking strain of concrete (Chang and Mander 1994). Besides, the non-dimensional variables x and n , denoted as x^+ and n^+ , respectively, in the tension envelope can be obtained in **Eq. (3.16)**.

$$x^+ = (\varepsilon - \varepsilon_0) / \varepsilon_t, n^+ = |(E_t \varepsilon_t) / f_t| \quad (3.16)$$

where the term f_t is the tensile strength of concrete and the term ε_t is the corresponding strain.

The non-dimensional cracking strain x_{crk}^+ on the tension envelope is calculated from the positive non-dimensional critical strain x_{cr-t} according to **Eq. (3.17)**.

$$x_{crk}^+ = x_{cr-t} - y(x_{cr-t}) / [n^+ z(x_{cr-t})], x_{cr-t} = (\varepsilon_{cr-t} - \varepsilon_0) / \varepsilon_t \quad (3.17)$$

On the tension envelope, the stress σ^+ and the tangent Young's modulus E_t^+ at

any strain are obtained by three piecewise **Eqs. (3.18)** through **(3.20)**.

(a) For $x^+ < x_{cr-t}$ (Nonlinear curve)

$$\sigma^+ = f_t y(x), E_t^+ = E_t z(x_{cr-t}) \quad (3.18)$$

(b) For $x_{cr-t} \leq x^+ \leq x_{crk}^+$ (Straight line)

$$\sigma^+ = f_t [y(x_{cr-t}) + n^+ z(x_{cr-t})(x - x_{cr-t})], E_t^+ = E_t z(x_{cr-t}) \quad (3.19)$$

(c) For $x^+ > x_{crk}^+$ (Cracked)

$$\sigma^+ = E_t^+ = 0 \quad (3.20)$$

The parameter r in Tsai's equation controls the nonlinear descending part of the nonlinear curve and the values of r are different for compression and tension envelopes. The different values of r for compression and tension envelopes can be empirically determined respectively by quantifying the combined contributions of stirrups and longitudinal reinforcement to concrete confinement.

In order to consider the loading and unloading rules of concrete in Mander's model, the studies by Chang and Mander (1994) and Waugh (2009) can be referred to. Again, the loading and unloading of concrete in compression does not account for the cyclic damage of concrete, even though the unloading and reloading scenarios are taken into account in the model.

3.2.3 Shear model

Apart from the uniaxial concrete models to describe the concrete compressive and tensile behaviour, the shear stress in a beam member should also be considered when conducting a finite element analysis. There are a few publications for the shear components, which are approximated to be elastic or following empirical models from experimental studies on shear behaviour of RC beams.

Fig. 3.5 Stress-strain envelop of the shear model

Table 3.1 Calibration equations for the critical points in the shear model

	τ	γ
A	$3.5\sqrt{f_c} + 0.3\frac{N}{bd}$ (ACI 426 (1973))	τ_A / G
B	$2(1 + \frac{N}{2000A_g})\sqrt{f_c} + \frac{A_t f_{yh}}{bs}$ (ACI 318 (2002))	$\frac{\tau_B}{E_s}(\frac{1}{\rho_v} + 4\frac{E_s}{E_c})$ (CEB (1985))
C	$2(1 + \frac{N}{2000A_g})\sqrt{f_c} + \frac{A_t f_{yh}}{bs}$ (ACI 318 (2002))	$(4 - 12\frac{\tau_B}{f_c})\gamma_B$ (Patwardhan (2005))
D	0	$\gamma_C + 0.5\gamma_B$

In this thesis, a multi-linear shear stress-strain relationship as shown in **Fig. 3.5** is proposed following the empirical models obtained from experimental studies on shear behaviour of RC beams (Patwardhan 2005) (mainly for critical deformations) and design guidelines (ASCE-ACI Task Committee 426 1973; Comit Euro-International du Beton (CEB) 1985; American Concrete Institute (ACI) 318-02 2002) (mainly for critical strengths). The calibration equations for the critical points are given in **Table 3.1**. It is assumed that the shear stress-strain relationship is symmetrical for both strong and weak axes of beam cross-section, respectively.

3.3 Unified Plasticity Concrete Model

Even though uniaxial concrete models have an obvious advantage in terms of simplicity when predicting flexural failure of concrete, they have limitations

when predicting the shear failures for RC beams with short and medium shear span-to-depth ratios. Therefore, a concrete model based on plasticity theory can be a promising alternative to predict shear failures of RC beam members.

In the present section, a concrete plasticity model (Bao et al. 2012) originally proposed for 3D compressive concrete behaviour is applied in conjunction with 3D fibre beam elements to accurately simulate the compressive deformations of reinforced concrete (RC) beam-column members. Different from the conventional uniaxial concrete models, the proposed unified plasticity concrete model deals with normal stresses and shear stresses of concrete fibres simultaneously when calculating the compressive deformations. Therefore, the application of the proposed unified plasticity concrete model is more general and can be employed to predict shear failures for RC beam members with short and medium shear span-to-depth ratios.

3.3.1 Concrete model for a 3D solid element

To simulate the concrete behaviour under complex 3D stress states in a solid element, the constitutive law of concrete should cover the compression-shear interaction behaviour and tension-shear interaction behaviour. As for the compression-shear constitutive law to describe the 3D concrete behaviour under compression, a three-parameter model is employed as the failure surface to predict the concrete strength and a concise one-parameter flow rule proposed by Bao et al. (2012) is adopted to predict the concrete structural deformations.

Volumetric component strain is used as the hardening parameter, as initially proposed by Grassl et al. (2002) and adopted by Papanikolaou and Kappos (2007). However, there are two and three parameters in the flow rules (Grassl et al. 2002; Papanikolaou and Kappos 2007), respectively, which do not have clear physical meanings. Besides, these parameters have to be respectively calibrated in the uniaxial, biaxial and triaxial compressive stress states. Therefore, compared with the previous flow rules, the one-parameter flow rule (Bao et al. 2012) is concise and convenient. Moreover, the only needed material parameter,

termed as *brittleness index of concrete*, has a very clear physical meaning and its value could be calibrated by conventional uniaxial compression tests.

As the main purpose of the present study is to show the advantages of the proposed unified plasticity concrete model for concrete compressive behaviour, a simple fixed smeared crack model (Owen et al. 1983; Hinton and Owen 1984) is adopted and modified as tension-shear constitutive law in the 3D solid element to describe the concrete tensile behaviour, rather than the more advanced fracture models, such as fixed-angle softened-truss model (Wang and Hsu 2001), microplane model (Bazant et al. 2000; Bazant and Caner 2005) and damage plasticity model as used in Abaqus (2009). The combined concrete constitutive relationships have been successfully implemented and validated at the material level (Bao et al. 2012).

3.3.1.1 Compression-shear constitutive law

To predict the concrete strength and deformation, the failure surface ($f = 0$) and the potential surface ($g = 0$) of plasticity can be conveniently formulated in the Haigh-Westergaard stress space which are defined by the cylindrical coordinates of hydrostatic length (ρ), deviatoric length (ξ) and Lode angle (θ) as shown in **Fig. 3.6**. Since it is the simplest way to deal with the stress state with the three principle stresses $\sigma_1, \sigma_2, \sigma_3$ ($\sigma_1 > \sigma_2 > \sigma_3$), these coordinates ρ, ξ and θ in the Haigh-Westergaard stress space are functions of the invariants (I_1, J_2, J_3) and are defined according to **Eqs. (3.21)**, **(3.22)** and **(3.23)**.

$$\xi = I_1 / \sqrt{3} \text{ where } I_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad (3.21)$$

$$\rho = \sqrt{2J_2} \text{ where } J_2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] / 6 \quad (3.22)$$

$$\theta = \frac{1}{3} \cos^{-1} \left(\frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}} \right) \text{ where } J_3 = (\sigma_1 - I_1/3)(\sigma_2 - I_1/3)(\sigma_3 - I_1/3) \quad (3.23)$$

Fig. 3.6 Coordinates in the Haigh-Westergaard stress space

The three-parameter failure surface proposed by Panpanikolaou and Kappos (2007) is adopted and described by **Eq. (3.24)** in terms of Haigh-Westergaard coordinates.

$$f(\xi, \rho, \theta) = \left(\sqrt{1.5} \frac{\rho}{k \cdot f_c} \right)^2 + m \left(\frac{\rho}{\sqrt{6} \cdot k \cdot f_c} r(\theta, e) + \frac{\xi}{\sqrt{3} \cdot k \cdot f_c} \right) - c = 0 \quad (3.24)$$

where k and c are the concrete hardening and softening functions, respectively. The term m is the friction parameter and r is the elliptic function, both of which are defined in **Eqs. (3.25)** and **(3.26)**.

$$m = 3 \frac{(k \cdot f_c)^2 - (\lambda_t \cdot f_t)^2}{k \cdot f_c \cdot \lambda_t \cdot f_t} \cdot \frac{e}{e + 1} \quad (3.25)$$

where λ_t is a scaling factor for the tensile concrete strength in order to provide intersection between the failure and potential surfaces.

$$r(\theta, e) = \frac{4(1 - e^2) \cos^2 \theta + (2e - 1)^2}{2(1 - e^2) \cos \theta + (2e - 1) [4(1 - e^2) \cos^2 \theta + 5e^2 - 4e]^{1/2}} \quad (3.26)$$

The three parameters in **Eq. (3.24)** that define the shape and size of the loading surface in the stress space are the mean uniaxial concrete compressive strength (f_c), the mean uniaxial concrete tensile strength (f_t) and the eccentricity parameter of out-of-roundness (e). As shown in **Fig. 3.7**, concrete hardening and softening are controlled by functions $k(\varepsilon_v^p)$ and $c(\varepsilon_v^p)$, respectively, where the

parameter ε_v^p is the plastic volume strain defined (Grassl et al. 2002) as the component summation of the plastic strain vector corresponding to the three normal stresses, of which the incremental form is given in **Eq. (3.43)**. The hardening function has the same form (Cervenka et al. 1998) as given in **Eq. (3.27)**.

$$k(\varepsilon_v^p) = \begin{cases} k_0 + (1 - k_0) \sqrt{1 - \left(\frac{\varepsilon_v^p - \varepsilon_{v,t}^p}{\varepsilon_{v,t}^p} \right)^2} & (\varepsilon_v^p < \varepsilon_{v,t}^p) \\ 1 & (\varepsilon_v^p \geq \varepsilon_{v,t}^p) \end{cases} \quad (3.27)$$

where k_0 is defined as f_{c0}/f_c , f_{c0} is the uniaxial concrete stress defining the onset of plastic flow and $\varepsilon_{v,t}^p$ is the plastic volumetric strain at uniaxial concrete strength. The softening function has the same form (Van Gysel and Taerwe 1996) as shown in **Eq. (3.28)**.

$$c(\varepsilon_v^p) = \begin{cases} 1 & (\varepsilon_v^p < \varepsilon_{v,t}^p) \\ \left[\frac{1}{1 + \left(\frac{\varepsilon_v^p - \varepsilon_{v,t}^p}{t} \right)^2} \right]^2 & (\varepsilon_v^p \geq \varepsilon_{v,t}^p) \end{cases} \quad (3.28)$$

where material parameter t controls the slope of the softening function.

Fig. 3.7 Evolution of concrete hardening function k and softening function c

To simplify the calibration of the concrete properties, as the derivative of a certain plastic function g with respect to the stress vector σ , a one-parameter flow rule $\mathbf{h}$ proposed by Bao et al. (2012) is employed herein as

$$\mathbf{h} = \frac{\partial g}{\partial \boldsymbol{\sigma}} = \frac{\partial (\sqrt{2J_2})}{\partial \boldsymbol{\sigma}} + \alpha'_{p0} \frac{\partial I_1}{\partial \boldsymbol{\sigma}} \quad (3.29)$$

where

$$\alpha'_{p0} = \begin{cases} \alpha_{p0}(1 + \sigma_1 / f_t) & (\sigma_1 \geq 0) \\ \alpha_{p0} & (\sigma_1 < 0, \sigma_2 = \sigma_3 = 0) \\ \alpha_{p0} / 2 & (\sigma_1 \leq 0, \sigma_2 < 0, \sigma_3 = 0) \\ \alpha_{p0} \left[(0.45e^{8\sigma_1 / f_c}) + 0.05 \right] & (\sigma_1 \leq 0, \sigma_2 < 0, \sigma_3 < 0) \end{cases} \quad (3.30)$$

It should be noted that the symbol e in **Eq. (3.30)** represents exponential function and is different from the eccentricity used in **Eq. (3.24)-(3.26)**.

The only material parameter α_{p0} , termed as brittleness index of concrete in **Eq. (3.30)**, reflects the ductility and post-peak stress-strain relationship and can be easily calibrated with only uniaxial compression test results. In general, the higher the value of α_{p0} , the more brittle the concrete is (Bao et al. 2012).

For the integration of constitutive equations, an implicit backward-Euler return-mapping algorithm (Macari et al. 1997) is conducted. The suggested algorithm (Cervenka and Papanikolaou 2008) is numerically stable with a fast convergence rate, independent of load step size and does not require differentiation of the failure surface. A detailed flowchart of the backward-Euler return-mapping algorithm is explained by Bao et al. (2012).

3.3.1.2 Tension-shear constitutive law

In the employed fixed smeared crack model (Hinton and Owen 1984), it is assumed that (1) the first crack forms in the plane perpendicular to the direction of the maximum principal tensile stress when the latter attains the tensile strength, (2) the angle of the crack is fixed once it has been determined and (3) the cracks are perpendicular to each other. The loading, unloading and reloading paths of cracked concrete are shown in **Fig. 3.8**, where ϵ_m is the ultimate tensile strain with strength where the descending and the residual branches intersect, α

is usually taken as 0.5-0.7 (Hinton and Owen 1984) and ε^p is the normal plastic strain perpendicular to the crack.

Fig. 3.8 The loading, unloading and reloading of the cracked concrete

As shown in **Fig. 3.8**, the tension stiffening is accounted for in the fracture model and a small residual tensile stress after ε_m is assumed (e.g. $10^{-4} f_t$ in the present study) to ensure numerical stability. It should be noted that the unloading path does not return to the origin but the strain-stress state $(\varepsilon^p, 0)$, so the residual plastic strain upon unloading is considered.

The cracked shear modulus G^c is assumed to be a function of the current tensile strain. Taking 1, 2 and 3 as the three principle directions of the stress state in a 3D solid element (**Fig. 3.9**), when the concrete cracks in the 1-direction, the incremental shear stress-strain relationship is expressed as

$$\Delta \tau_{12} = G_{12}^c \Delta \varepsilon_{12}, \quad \Delta \tau_{13} = G_{13}^c \Delta \varepsilon_{13} \quad (3.31)$$

where the cracked shear moduli at the crack plane (Hinton and Owen 1984) are given as

$$G_{12}^c = G_{13}^c = \max\{0.0, \frac{1}{4} G [1 - 250(\varepsilon_1 - \varepsilon_1^p)]\} \quad (3.32)$$

where G is the shear modulus of intact concrete.

Fig. 3.9 Three dimensional strain state in a 3D solid element

Besides, once the tensile stress in the 2-direction reaches the tensile strength, a second crack plane perpendicular to the first one is formed and the incremental shear stress-strain relationship in the 1-direction can be expressed in **Eq. (3.31)**. The new cracked shear moduli are given (Hinton and Owen 1984) as follows.

$$G_{12}^c = \max \left\{ 0.0, \frac{1}{2} G_{12}' \right\} \quad (3.33 \text{ a})$$

$$G_{13}^c = \max \left\{ 0.0, \frac{1}{4} G[1 - 250(\epsilon_1 - \epsilon_1^p)] \right\} \quad (3.33 \text{ b})$$

$$G_{12}' = \min \left\{ \frac{1}{4} G[1 - 250(\epsilon_1 - \epsilon_1^p)], \frac{1}{4} G[1 - 250(\epsilon_2 - \epsilon_2^p)] \right\} \quad (3.33 \text{ c})$$

It should be noted that G_{12}' in **Eq. (3.33 c)** is used in **Eq. (3.33 a)**. Once the normal strain at a certain crack plane (e.g. 1-direction) is negative, the crack is deemed to have closed and the corresponding cracked shear moduli (e.g. G_{12}^c and G_{13}^c) will revert to the intact shear modulus G .

3.3.1.3 Suggested material parameters for the proposed plasticity and fracture models

With concrete cylinder strength as the only required input material parameter, all the parameters with suggested values related to the proposed constitutive laws are summarized in **Table 3.2** for concrete cylinder strengths ranging from 20 MPa to 100 MPa. For a particular concrete cylinder strength, the relevant material parameters can be obtained by linear interpolation from **Table 3.2**.

It should be pointed out that the calibration of α_{p0} is based on experimental results with different loading scenarios (Cervenka and Papanikolaou 2008) by Bao et al. (2012). The suggested values of brittleness index of concrete α_{p0} follow the trend that the post-peak behaviour is more ductile (smaller value of α_{p0}) with decreasing concrete cylinder strength. To accurately describe the ductility and post-peak stress-strain relationship of concrete used in the model, α_{p0} can also be specified by users when the concrete properties can be obtained from uniaxial compression tests.

Table 3.2 Suggested parameters for the proposed fracture and plasticity models

f_c (MPa)	20	30	40	50	60	70	80	90	100
E_c (MPa) ^(a)	24377	27530	30011	32089	33893	35497	36948	38277	39506
ν ^(a)	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
f_t (MPa) ^(a)	1.917	2.446	2.906	3.323	3.707	4.066	4.405	4.728	5.036
λ_t ^(a)	1.043	1.227	1.376	1.505	1.619	1.722	1.816	1.904	1.986
e ^(a)	0.5281	0.5232	0.5198	0.5172	0.5151	0.5133	0.5117	0.5104	0.5092
f_{c0} (MPa) ^(a)	-4.32	-9.16	-15.62	-23.63	-33.14	-44.11	-56.50	-70.3	-85.48
$\mathcal{E}_{v,t}^p$ ^(a)	$4.92 \cdot 10^{-4}$	$6.54 \cdot 10^{-4}$	$8.00 \cdot 10^{-4}$	$9.35 \cdot 10^{-4}$	$1.06 \cdot 10^{-4}$	$1.18 \cdot 10^{-4}$	$1.30 \cdot 10^{-4}$	$1.41 \cdot 10^{-4}$	$1.52 \cdot 10^{-4}$
t ^(a)	$1.33 \cdot 10^{-3}$	$2.00 \cdot 10^{-3}$	$2.67 \cdot 10^{-3}$	$3.33 \cdot 10^{-3}$	$4.00 \cdot 10^{-3}$	$4.67 \cdot 10^{-3}$	$5.33 \cdot 10^{-3}$	$6.00 \cdot 10^{-3}$	$6.67 \cdot 10^{-3}$
α ^(b)	0.5-0.7	0.5-0.7	0.5-0.7	0.5-0.7	0.5-0.7	0.5-0.7	0.5-0.7	0.5-0.7	0.5-0.7
$\mathcal{E}_m$ ^(b)	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
α_{p0}	0.26	0.4016	0.5760	0.6893	0.8070	1.0691	1.2153	1.2153	1.3673

(a): data from Cervenka and Papanikolaou (2008); (b): data from Hinton and Owen (1984)

3.3.2 Unified plasticity concrete model for a 3D fibre beam element

In a 3D solid element formulation, the incremental concrete stress vector $\Delta\sigma_6$ can be obtained in Voigt's notation by multiplying the incremental concrete strain vector $\Delta\epsilon_6$ and tangential material matrix D_6 as shown in **Eq. (3.34)**.

$$\Delta\sigma_6 = \begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{yz} \\ \Delta\tau_{xz} \\ \Delta\tau_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\epsilon_x \\ \Delta\epsilon_y \\ \Delta\epsilon_z \\ \Delta\epsilon_{yz} \\ \Delta\epsilon_{xz} \\ \Delta\epsilon_{xy} \end{Bmatrix} = D_6 \Delta\epsilon_6 \quad (3.34)$$

With concrete assumed as an isotropic material, the tangential material matrix D_6 can also be expressed as a positive-definite fourth-order tensor D (Belytschko et al. 2000) with the operator $\otimes$ to define the dyadic product of two vectors and I to define the identity matrix or unit matrix.

$$D_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \text{ or } D = \lambda I \otimes I + 2\mu I \quad (3.35)$$

with the so-called minor and major symmetries

$$D_{ijkl} = D_{jikl} = D_{ijlk} = D_{klij} \quad (3.36)$$

and the conventional mapping of the first and second pairs of indices

$$\begin{array}{lll} 11 \rightarrow x & 22 \rightarrow y & 33 \rightarrow z \\ 23 \rightarrow yz & 13 \rightarrow xz & 12 \rightarrow xy \end{array} \quad (3.37)$$

Therefore, the entries in the tangential material matrix D_6 are explicitly given as follows:

$$\mathbf{D}_6 = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu \end{bmatrix} \quad (3.38)$$

In **Eqs. (3.35)** through **(3.38)**, the two independent material constants λ and μ are called the Lamé constants and are given as

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)} \quad (3.39)$$

where E is the Young's modulus and ν is the Poisson's ratio. The Lamé's second constant μ is also known as the shear modulus and is usually denoted as G .

Fig. 3.10 Definition of the three known strain components in a beam element

In a displacement-based beam formulation, the incremental strain components $\Delta \varepsilon_x$, $\Delta \varepsilon_{xy}$ and $\Delta \varepsilon_{xz}$ are known (**Fig. 3.10**) according to the incremental deformations induced by a load increment. On the other hand, the incremental stress components $\Delta \sigma_y$, $\Delta \sigma_z$ and $\Delta \tau_{yz}$ are equal to zero at all the load increments and iterations due to the beam idealization from the stress state of a solid element. Thus, based on the constitutive law of a solid element formulation given in **Eq. (3.34)**, the other three unknown incremental strain components

$\Delta \varepsilon_y$, $\Delta \varepsilon_z$ and $\Delta \varepsilon_{yz}$ can be calculated by solving the constraint equations given in **Eqs. (3.40)** and **(3.41)**.

$$\Delta \boldsymbol{\sigma}^{un} = \mathbf{D}_3 \Delta \boldsymbol{\varepsilon}^{un} + \mathbf{M} = \mathbf{0} \quad (3.40)$$

$$\mathbf{M} = \bar{\mathbf{D}}_3 \bar{\Delta \boldsymbol{\varepsilon}}_3 \quad (3.41)$$

where the superscript “un” represents that the associated vector is unknown and the vector terms are defined as follows.

$$\Delta \boldsymbol{\sigma}^{un} = \begin{Bmatrix} \Delta \sigma_y \\ \Delta \sigma_z \\ \Delta \tau_{yz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \quad \Delta \boldsymbol{\varepsilon}^{un} = \begin{Bmatrix} \Delta \varepsilon_y \\ \Delta \varepsilon_z \\ \Delta \varepsilon_{yz} \end{Bmatrix}, \quad \bar{\Delta \boldsymbol{\varepsilon}}_3 = \begin{Bmatrix} \Delta \varepsilon_x \\ \Delta \varepsilon_{xz} \\ \Delta \varepsilon_{xy} \end{Bmatrix}, \quad \mathbf{D}_3 = \begin{bmatrix} D_{22} & D_{23} & D_{24} \\ D_{32} & D_{33} & D_{34} \\ D_{42} & D_{43} & D_{44} \end{bmatrix},$$

and

$$\bar{\mathbf{D}}_3 = \begin{bmatrix} D_{21} & D_{25} & D_{26} \\ D_{31} & D_{35} & D_{36} \\ D_{41} & D_{45} & D_{46} \end{bmatrix}.$$

It is seen from **Eqs. (3.40)** and **(3.41)** that zero stress components are taken as a set of constraint equations to calculate the non-zero strain components and provide the unknown strain vector $\Delta \boldsymbol{\varepsilon}^{un}$. Besides, the residual stress vector $\mathbf{M}$ is also taken into account, which results from the known incremental strain vector $\bar{\Delta \boldsymbol{\varepsilon}}_3$ and the material sub-matrix $\bar{\mathbf{D}}_3$ associated with the unknown incremental strain components $\Delta \varepsilon_y$, $\Delta \varepsilon_z$ and $\Delta \varepsilon_{yz}$. The entries of material sub-matrix $\mathbf{D}_3$ and $\bar{\mathbf{D}}_3$ can be obtained from those of tangential material matrix $\mathbf{D}_6$ with an array size of 6×6 as shown in **Eq. (3.38)**.

It is noteworthy that the unknown incremental strain vector $\Delta \boldsymbol{\varepsilon}^{un}$ can be calculated when the element is in the elastic state because the material sub-matrix $\mathbf{D}_3$ can be directly derived from the initial elastic material matrix $\mathbf{D}_6$.

However, when the element is in the plastic state, the tangential material matrix D_6 has to be updated according to the derivation as follows.

Firstly, the new stress state in the plastic model is computed by using a predictor-corrector formula (Cervenka and Papanikolaou 2008) and the stress increment $d\sigma$ is expressed in a tensor form as

$$d\sigma = D : (d\epsilon - d\epsilon^p) \quad (3.42)$$

With the operator $:$ to define the double-dot product and

$$d\epsilon^p = d\lambda \frac{\partial g}{\partial \sigma} = d\lambda h \quad (3.43)$$

where g is the potential function, h is the flow rule discussed in **Section 3.3.1.1** and plastic multiplier $d\lambda$ is given in **Eqs. (3.44)** and **(3.45)**.

$$d\lambda = \frac{\frac{\partial f}{\partial \sigma} : D : d\epsilon}{\frac{\partial f}{\partial \sigma} : D : \frac{\partial g}{\partial \sigma} - \frac{\partial f}{\partial \alpha} \left(\delta : \frac{\partial g}{\partial \sigma} \right)} = \frac{\frac{\partial f}{\partial \sigma} : D : d\epsilon}{\frac{\partial f}{\partial \sigma} : D : h - \frac{\partial f}{\partial \alpha} (\delta : h)} \quad (3.44)$$

$$d\alpha = \delta : d\epsilon^p \quad (3.45)$$

where f is the failure function. Therefore, the equivalent stress-strain relationship in the plastic state can be obtained in **Eq. (3.46)**.

$$d\sigma = D^{ep} : d\epsilon \quad (3.46)$$

where

$$D^{ep} = D : \left(I - \frac{\frac{\partial g}{\partial \sigma} \otimes \left(\frac{\partial f}{\partial \sigma} : D \right)}{\frac{\partial f}{\partial \sigma} : D : \frac{\partial g}{\partial \sigma} - \frac{\partial f}{\partial \alpha} \left(\delta : \frac{\partial g}{\partial \sigma} \right)} \right) = D : \left(I - \frac{h \otimes \left(\frac{\partial f}{\partial \sigma} : D \right)}{\frac{\partial f}{\partial \sigma} : D : h - \frac{\partial f}{\partial \alpha} (\delta : h)} \right)$$

Based on the above discussion, the constitutive laws for 3D solid elements can be applied to the fibre beam element in the plastic state. With the zero stress vector $\Delta\sigma^{im}$ taken as constraint equations, the incremental strain vector $\Delta\epsilon^{im}$ can be obtained in an iterative approach as shown in **Fig. 3.11**, where all the symbols and entry sequence are the same with those in **Eqs. (3.34)** through **(3.41)**. It should be noted that the subscript 2:4 in the second box indicates a vector formed by extracting the 2nd to 4th entries of the original vector to conform to the requirement of matrix multiplication. In a general case for a displacement-based beam formulation, the available information at a certain load increment is the equilibrium stress vector σ_3^{eq} and elastic and plastic strain vectors $\epsilon_3^{eq,e}$ and $\epsilon_3^{eq,p}$, respectively, at the last load increment.

The basic idea of the iterative approach shown in **Fig. 3.11** is to adjust the magnitude of strain components $\Delta\epsilon^{im}$ to make the corresponding stress vector $\Delta\sigma^{im}$ equal to zero to meet the beam element simplification at each iteration and load increment. If the predicted stress components of $\Delta\sigma^{im}$ do not satisfy the predefined tolerance (*Tol.*), then the stress vector $\Delta\sigma^{im}$ has to be utilized to correct the prediction of the strain vector $\Delta\epsilon^{im}$ until the tolerance is satisfied. When specifying the constraint equations, the error tolerance (*Tol.* in **Fig. 3.11**) is predefined to be 10^{-6} in the thesis. To ensure the accuracy of stress-strain relationship, the material matrix D_6 should be updated when the concrete becomes plastic or when concrete cracks at a certain fibre as discussed in **Section 3.3.1.2**.

It should be noted that the calculation above is to obtain the stress and strain vectors at equilibrium after certain strain increments. Therefore, the calculation can be employed to determine the stress-strain state for a fibre at a Gaussian point in a fibre beam element as discussed in **Chapter 4**.

Fig. 3.11 Flow chart to determine the stress-strain relationship for a fibre at a Gaussian point in a fibre beam element

3.4 Closure

In this chapter, two types of concrete models are covered. One is the uniaxial concrete model which is commonly used in finite element analysis. The other is the unified plasticity concrete model which takes all the three stress components into consideration by enforcing the beam simplification when updating the stress and strain states in finite element analysis. Comparing these two types of

concrete models, it is obvious that the simulation with uniaxial concrete model is much more simple and stable. However, only flexural failure of concrete along the beam longitudinal direction can be predicted.

The unified plasticity concrete model is proposed for 3D fibre beam elements to accurately simulate the compressive deformation behaviour of reinforced concrete (RC) beam-column members. Based on the failure and potential functions in terms of plasticity theory, the unified plasticity concrete model can deal with complex stress state calculations and is capable of predicting the shear behaviour of beam-column members with small and medium shear span-to-depth ratios. To extend the use of proposed unified plasticity concrete model for more complex examples, the classical Hinton concrete fracture model is modified to consider opening and closing of cracks.

The adoption of the unified plasticity concrete model in a fibre beam element is an application of the unified plasticity concrete model with some assumptions for certain degrees of freedom in the finite element. Obviously, the proposed unified plasticity concrete model can be further applied to other types of elements, such as shell elements, in a similar approach as discussed in **Section 3.2.2**.

Chapter 4 A 3D Co-Rotational Beam Element Formulation

4.1 Introduction

In this chapter, a 3-node three-dimensional (3D) co-rotational beam element using vectorial rotational variables is employed to consider the geometric nonlinearity in the 3D space. To account for different shapes and reinforced concrete cross-sections, fibre model is derived and implemented into FEMFAN3D. Numerical integrations over the cross-section are performed, considering both normal and shear stresses. In addition, the derivations associated with material nonlinearity are given in terms of elasto-plastic incremental stress-strain relationships for both steel and concrete materials. Steel reinforcement is treated as an elasto-plastic material with Von Mises yield criterion. Compressive concrete behaviour is described by the concrete models discussed in **Chapter 3**, while tensile stiffening effect is taken into account as well.

To validate the proposed 3D co-rotational beam element with fibre model, examples involving steel beams are employed to eliminate the effect due to the more complex concrete material nonlinearity. After validating the proposed 3D co-rotational beam element formulation, several numerical examples, including one-element RC member tests, RC column tests and a series of RC shear beams, are presented to validate the proposed unified plasticity concrete model combined with the fracture model for concrete. Uniaxial concrete models (Kent and Park model, Mander's model) and unified plasticity concrete model are incorporated into co-rotational fibre beam formulations. Through the discussion on the simulation results, the advantages and disadvantages of different types of concrete models are highlighted and the proposed 3D co-rotational beam element with fibre model is shown to be capable of simulating steel and reinforced concrete framed structures with satisfactory accuracy and efficiency.

4.2 Co-Rotational (CR) Beam Formulation

The greatest challenge for a 3D beam element formulation is to simulate spatial rotations. A three-node CR beam formulation is employed to simulate the geometric nonlinearity of three dimensional deformations. In the CR beam formulation, rotational variables in spatial rotations are defined by vectorial rotational variables. The details of the 3D beam formulations can be found in the study by Li (2007).

Some assumptions are made in the CR beam formulations. (1) In the local coordinate system the strain is small. (2) Normal vectors to the neutral axis before deformation remain straight but not necessarily normal to the neutral axis after deformation. (3) The shape of the CR beam cross-section does not warp. (4) For large-deformation problems, the incremental load factor should be small enough to ensure the existence of vectorial rotational variables.

Fig. 4.1 Undeformed and deformed configurations of a CR beam

The numbering sequence of a three-node CR beam is shown in **Fig. 4.1** with the end nodes tagged as 1 and 2, and the middle node as 3. As shown in **Fig. 4.1**, both local and global coordinates systems are created in order to describe the local and global displacements and rotations. The local system $\{x, y, z\}$ remains fixed with the middle node (Node 3) and does not deform with the movement of the element. The local x axis is set to be tangential to the beam longitudinal axis by default. Therefore, there are only two active end nodes (Nodes 1 and 2) for the CR beam element in the local coordinate system. Directional changes of the local y and z axes imply the local deformation of the beam element. The degrees of freedom in the local and global systems are

$$\mathbf{u}_L = \left\{ u_1 \ v_1 \ w_1 \ r_{1y,n_1} \ r_{1y,m_1} \ r_{1z,n_1} \ u_2 \ v_2 \ w_2 \ r_{2y,n_2} \ r_{2y,m_2} \ r_{2z,n_2} \right\}^T$$

$$\text{and } \mathbf{u}_G = \left\{ U_1 \ V_1 \ W_1 \ e_{1y,n_1} \ e_{1y,m_1} \ e_{1z,n_1} \ U_2 \ V_2 \ W_2 \right.$$

$$\left. e_{2y,n_2} \ e_{2y,m_2} \ e_{2z,n_2} \ U_3 \ V_3 \ W_3 \ e_{3y,n_3} \ e_{3y,m_3} \ e_{3z,n_3} \right\}^T$$

where u_i , v_i , w_i are the local displacements of Node i ($i=1,2$), r_{iy,n_i} , r_{iy,m_i} , r_{iz,n_i} are the *local* vectorial rotational variables representing the rotation of Node i ($i=1,2$). The terms U_i , V_i , W_i are the global displacements of Node i ($i=1,2,3$) and e_{iy,n_i} , e_{iy,m_i} , e_{iz,n_i} are the *global* vectorial rotational variables representing the rotation of Node i ($i=1,2,3$).

The subscripts n_i and m_i indicate the n th and m th components of the direction vectors of Node i . The vectorial rotational variables r_{iy,n_i} , r_{iy,m_i} , r_{iz,n_i} in the local coordinate system and e_{iy,n_i} , e_{iy,m_i} , e_{iz,n_i} in the global coordinate system are, respectively, defined according to the relative quantities and permutation sequence of all three components for direction vectors $\mathbf{r}_{iy}$ and $\mathbf{r}_{iz}$ of Node i in the local system (see **Fig. 4.1**) and direction vectors $\mathbf{e}_{iy}$ and $\mathbf{e}_{iz}$ of Node i in the global system as discussed by Li (2007). For example, assuming $|r_{iy,l_i}| > |r_{iy,n_i}|$

and $|r_{iy,l_i}| > |r_{iy,m_i}|$ ($n, m, l \in \{1, 2, 3\}$ and $n \neq m \neq l$), if $|r_{iz,l_i}| > |r_{iz,n_i}|$ and $|r_{iz,l_i}| > |r_{iz,m_i}|$ are satisfied, then the values of n , m and l follow a cyclic permutation of $\{1, 2, 3\}$. In the case of a beam bending slightly in the local x-y plane, the direction vectors $\mathbf{r}_{1y}$ and $\mathbf{r}_{2y}$ at Nodes 1 and 2 slightly rotate about the local z axis, then the component with the maximum value among all three components should be the one along the local y axis, that is, l should be equal to 2 with n equal to 3 and m equal to 1. After each load increment or iteration, the vectorial rotational variables should be updated based on the orthogonality conditions $\mathbf{r}_{iy}^T \mathbf{r}_{iz} = 0$ and $\mathbf{e}_{iy}^T \mathbf{e}_{iz} = 0$. In addition, the definition of the local system $\{x, y, z\}$ as shown in **Fig. 4.1** indicates the cross-sectional orientation of the CR beam element. The direction of local y axis is taken as the weak axis direction, whereas local z axis is the stronger axis direction. Both the local system $\{x, y, z\}$ and the global system $\{X_1, X_2, X_3\}$ follow the right-hand rule.

Since the CR formulation decomposes the incremental deformations into a rigid-body movement and pure deformations, the deformations in the local coordinate system is assumed to be small. The deformation at any point of the element can be obtained based on nodal deformations by means of quadratic Lagrangian interpolation functions.

$$\mathbf{u} = \sum_{i=1}^3 h_i(\zeta) \left[\mathbf{t}_i + y(\mathbf{r}_{iy} - \mathbf{r}_{iy0}) + z(\mathbf{r}_{iz} - \mathbf{r}_{iz0}) \right] \quad (4.1)$$

where $\mathbf{t}_i = \{u_i \ v_i \ w_i\}^T$ consists of the local nodal translational displacements; $\mathbf{r}_{iy0}$ and $\mathbf{r}_{iy}$ are direction vectors along the cross-sectional weak axis (local y axis) at Node i before and after deformation, respectively (see **Fig. 4.1**); $\mathbf{r}_{iz0}$ and $\mathbf{r}_{iz}$ are direction vectors along the cross-sectional strong axis (local z axis) at Node i before and after deformation, respectively (see **Fig. 4.1**); y and z are the local coordinates along the cross-sectional weak and strong axes of the

beam element; h_i is the Lagrangian interpolation function; ζ is one-dimensional natural coordinate along the centre line of the beam element.

As the first derivative of displacement with respect to local degrees of freedom $\mathbf{u}_L$, the corresponding strain in the local coordinate system is based on Green strain. In a compact form (Li 2007), Green strain can be written as

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{(0)} + y\boldsymbol{\varepsilon}^{(1)} + z\boldsymbol{\varepsilon}^{(2)} + yz\boldsymbol{\varepsilon}^{(3)} + y^2\boldsymbol{\varepsilon}^{(4)} + z^2\boldsymbol{\varepsilon}^{(5)} \quad (4.2)$$

The six coefficients of $\boldsymbol{\varepsilon}$ are derived by the author of this thesis and are listed below for completeness.

$$\begin{aligned} \boldsymbol{\varepsilon}^{(0)} &= \begin{Bmatrix} \frac{1}{2} \frac{\partial \mathbf{u}_t}{\partial x} \frac{\partial \mathbf{u}_t}{\partial x} + \frac{\partial \mathbf{u}_t}{\partial x} \frac{\partial \mathbf{x}_0}{\partial x} \\ \frac{\partial \mathbf{u}_t}{\partial x} \mathbf{r}_y + \frac{\partial \mathbf{x}_0}{\partial x} (\mathbf{r}_y - \mathbf{r}_{y0}) \\ \frac{\partial \mathbf{u}_t}{\partial x} \mathbf{r}_z + \frac{\partial \mathbf{x}_0}{\partial x} (\mathbf{r}_z - \mathbf{r}_{z0}) \end{Bmatrix} \quad \boldsymbol{\varepsilon}^{(1)} = \begin{Bmatrix} \frac{\partial \mathbf{u}_t}{\partial x} \frac{\partial \mathbf{r}_y}{\partial x} + \frac{\partial \mathbf{x}_0}{\partial x} \frac{\partial (\mathbf{r}_y - \mathbf{r}_{y0})}{\partial x} \\ \frac{\partial \mathbf{r}_y}{\partial x} \mathbf{r}_y - \frac{\partial \mathbf{r}_{y0}}{\partial x} \mathbf{r}_{y0} \\ \frac{\partial \mathbf{r}_y}{\partial x} \mathbf{r}_z - \frac{\partial \mathbf{r}_{y0}}{\partial x} \mathbf{r}_{z0} \end{Bmatrix} \quad \boldsymbol{\varepsilon}^{(2)} = \begin{Bmatrix} \frac{\partial \mathbf{u}_t}{\partial x} \frac{\partial \mathbf{r}_z}{\partial x} + \frac{\partial \mathbf{x}_0}{\partial x} \frac{\partial (\mathbf{r}_z - \mathbf{r}_{z0})}{\partial x} \\ \frac{\partial \mathbf{r}_z}{\partial x} \mathbf{r}_y - \frac{\partial \mathbf{r}_{z0}}{\partial x} \mathbf{r}_{y0} \\ \frac{\partial \mathbf{r}_z}{\partial x} \mathbf{r}_z - \frac{\partial \mathbf{r}_{z0}}{\partial x} \mathbf{r}_{z0} \end{Bmatrix} \\ \boldsymbol{\varepsilon}^{(3)} &= \begin{Bmatrix} \frac{\partial \mathbf{r}_y}{\partial x} \frac{\partial \mathbf{r}_z}{\partial x} - \frac{\partial \mathbf{r}_{y0}}{\partial x} \frac{\partial \mathbf{r}_{z0}}{\partial x} \\ 0 \\ 0 \end{Bmatrix} \quad \boldsymbol{\varepsilon}^{(4)} = \begin{Bmatrix} \frac{1}{2} \left(\frac{\partial \mathbf{r}_y}{\partial x} \frac{\partial \mathbf{r}_y}{\partial x} - \frac{\partial \mathbf{r}_{y0}}{\partial x} \frac{\partial \mathbf{r}_{y0}}{\partial x} \right) \\ 0 \\ 0 \end{Bmatrix} \quad \boldsymbol{\varepsilon}^{(5)} = \begin{Bmatrix} \frac{1}{2} \left(\frac{\partial \mathbf{r}_z}{\partial x} \frac{\partial \mathbf{r}_z}{\partial x} - \frac{\partial \mathbf{r}_{z0}}{\partial x} \frac{\partial \mathbf{r}_{z0}}{\partial x} \right) \\ 0 \\ 0 \end{Bmatrix} \end{aligned}$$

where $\mathbf{x}_0 = \{x_0 \ y_0 \ z_0\}^T$ is the local coordinate at any point in the beam element;

$\mathbf{r}_{y0} = \sum_{i=1}^3 h_i(\zeta) \mathbf{r}_{iy0}$ is the initial direction vector along the cross-sectional weak

axis at any point; $\mathbf{r}_y = \sum_{i=1}^3 h_i(\zeta) \mathbf{r}_{iy}$ is the current direction vector along the cross-

sectional weak axis at any point after deformation; $\mathbf{r}_{z0} = \sum_{i=1}^3 h_i(\zeta) \mathbf{r}_{iz0}$ is the initial

direction vector along the cross-sectional strong axis at any point; $\mathbf{r}_z = \sum_{i=1}^3 h_i(\zeta) \mathbf{r}_{iz}$

is the current direction vector along the cross-sectional strong axis at any point

after deformation; $\mathbf{u}_i = \sum_{i=1}^3 h_i(\zeta) \mathbf{t}_i$ is the translational displacements at any point.

The subscript i indicates the corresponding function at node i ; the subscript 0 indicates the function in the state *before deformation* and if there is no 0, the function is in the *current deformed state* as shown in **Fig. 4.1**. Jacobian matrix is calculated as the relationship between the natural coordinate system and the local coordinate system.

With respect to the local unknown variables $\mathbf{u}_L$, the geometric matrix $\mathbf{B}$ can be expressed in a compact form as

$$\begin{aligned} \mathbf{B} &= \frac{\partial \boldsymbol{\varepsilon}}{\partial \mathbf{u}_L} = \frac{\partial \boldsymbol{\varepsilon}^{(0)}}{\partial \mathbf{u}_L} + y_l \frac{\partial \boldsymbol{\varepsilon}^{(1)}}{\partial \mathbf{u}_L} + z_l \frac{\partial \boldsymbol{\varepsilon}^{(2)}}{\partial \mathbf{u}_L} + y_l z_l \frac{\partial \boldsymbol{\varepsilon}^{(3)}}{\partial \mathbf{u}_L} + y_l^2 \frac{\partial \boldsymbol{\varepsilon}^{(4)}}{\partial \mathbf{u}_L} + z_l^2 \frac{\partial \boldsymbol{\varepsilon}^{(5)}}{\partial \mathbf{u}_L} \\ &= \mathbf{B}^{(0)} + y_l \mathbf{B}^{(1)} + z_l \mathbf{B}^{(2)} + y_l z_l \mathbf{B}^{(3)} + y_l^2 \mathbf{B}^{(4)} + z_l^2 \mathbf{B}^{(5)} \end{aligned} \quad (4.3)$$

where $\mathbf{B}$ is a 3×12 matrix relating element strains and local displacements and y_l and z_l are the local coordinates which are defined by the cross-sectional strong and weak axes.

With the definition of Green Strain $\boldsymbol{\varepsilon}$ and geometric matrix $\mathbf{B}$, it is straightforward to obtain the expression of internal force vector $\mathbf{f}_L$ and stiffness matrix $\mathbf{K}_L$ in the local coordinate system for CR beam. The strain energy of the CR beam element can be expressed as

$$U = \int_V \frac{1}{2} \boldsymbol{\varepsilon}^T \mathbf{D} \boldsymbol{\varepsilon} dV \quad (4.4)$$

where

$$\mathbf{D} = \begin{bmatrix} E & 0 & 0 \\ 0 & k_0 G & 0 \\ 0 & 0 & k_0 G \end{bmatrix} \text{ is the elastic matrix to represent the material property; } E$$

and G are the elastic modulus and shear modulus, respectively; k_0 is the shear

factor depending on the shape of the employed cross-section and is equal to 5/6 for a rectangular cross-section; V is the volume of a CR beam element.

The first derivative of strain energy with respect to unknown variables $\mathbf{u}_L$ in the local coordinate system leads to local internal force vector $\mathbf{f}_L$.

$$\mathbf{f}_L = \frac{\partial U}{\partial \mathbf{u}_L} = \int_V \frac{1}{2} \frac{\partial \boldsymbol{\epsilon}^T}{\partial \mathbf{u}_L} \mathbf{D} \boldsymbol{\epsilon} dV + \int_V \frac{1}{2} \boldsymbol{\epsilon}^T \mathbf{D} \frac{\partial \boldsymbol{\epsilon}}{\partial \mathbf{u}_L} dV = \int_V \mathbf{B}^T \mathbf{D} \boldsymbol{\epsilon} dV \quad (4.5)$$

The first derivative of out-of-balance force (internal force $\mathbf{f}_L$ minus external load $\mathbf{P}$) with respect to local unknown variables can be used to calculate the local stiffness matrix $\mathbf{K}_L$.

$$\begin{aligned} \mathbf{K}_L &= \frac{\partial (\mathbf{f}_L - \mathbf{P})}{\partial \mathbf{u}_L} = \frac{\partial}{\partial \mathbf{u}_L} \left[\int_V \mathbf{B}^T \mathbf{D} \boldsymbol{\epsilon} dV \right] \\ &= \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV + \int_V \boldsymbol{\epsilon}^T \mathbf{D} \frac{\partial \mathbf{B}}{\partial \mathbf{u}_L} dV \end{aligned} \quad (4.6)$$

where it is assumed that $\partial \mathbf{P} / \partial \mathbf{u}_L = 0$.

Substituting the compact form of Green strain and geometric matrix into the local internal force vector $\mathbf{f}_L$, **Eq. (4.5)** can be rewritten as

$$\begin{aligned} \mathbf{f}_L &= \int_V \mathbf{B}^T \mathbf{D} \boldsymbol{\epsilon} dV = \int_L \int_A \left[\mathbf{B}^T \mathbf{D} \boldsymbol{\epsilon} \right] dA dx \\ &= \int_L \int_A \left\{ \left[\mathbf{B}^{(0)} + y_l \mathbf{B}^{(1)} + z_l \mathbf{B}^{(2)} + y_l z_l \mathbf{B}^{(3)} + y_l^2 \mathbf{B}^{(4)} + z_l^2 \mathbf{B}^{(5)} \right]^T \cdot \right. \\ &\quad \left. \mathbf{D} \left[\boldsymbol{\epsilon}^{(0)} + y_l \boldsymbol{\epsilon}^{(1)} + z_l \boldsymbol{\epsilon}^{(2)} + y_l z_l \boldsymbol{\epsilon}^{(3)} + y_l^2 \boldsymbol{\epsilon}^{(4)} + z_l^2 \boldsymbol{\epsilon}^{(5)} \right] \right\} dA dx \end{aligned} \quad (4.7)$$

where the scalar terms A and L are the cross-sectional area and the length of CR beam, respectively and y_l and z_l are the local coordinates which are defined by the cross-sectional strong and weak axes.

When incorporating the fibre model into the CR beam formulation, for the convenience of programming, local internal force vector $\mathbf{f}_L$ can also be rewritten in the form of **Eq. (4.8)** by expanding **Eq. (4.7)**.

$$\mathbf{f}_L = \int_L \left[\sum_{i=1}^{14} C_i \mathbf{f}_i \right] dx \quad (4.8)$$

where the coefficients C_i ($i=1, \dots, 14$) are derived as follows.

$$\begin{aligned} C_0 &= \int_A dA, \quad C_1 = \int_A y_l dA, \quad C_2 = \int_A z_l dA, \quad C_3 = \int_A y_l z_l dA, \quad C_4 = \int_A y_l^2 dA, \quad C_5 = \int_A z_l^2 dA, \\ C_6 &= \int_A y_l^2 z_l dA, \quad C_7 = \int_A y_l z_l^2 dA, \quad C_8 = \int_A y_l^2 z_l^2 dA, \quad C_9 = \int_A y_l^4 dA, \quad C_{10} = \int_A z_l^4 dA, \\ C_{11} &= \int_A y_l^3 dA, \quad C_{12} = \int_A z_l^3 dA, \quad C_{13} = \int_A y_l^3 z_l dA, \quad C_{14} = \int_A y_l z_l^3 dA. \end{aligned}$$

Details of the vectors $\mathbf{f}_i$ ($i=1, \dots, 14$) can be found in **Appendix A**. Similarly, after substituting the Green strain and geometric matrix into the local stiffness matrix $\mathbf{K}_L$, the tangential stiffness matrix in the local coordinate system can be written as

$$\begin{aligned} \mathbf{K}_L &= \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV + \int_V \boldsymbol{\varepsilon}^T \mathbf{D} \frac{\partial \mathbf{B}}{\partial \mathbf{u}_L} dV = \int_L \int_A \left[\mathbf{B}^T \mathbf{D} \mathbf{B} + \boldsymbol{\varepsilon}^T \mathbf{D} \frac{\partial \mathbf{B}}{\partial \mathbf{u}_L} \right] dA dx \\ &= \int_L \int_A \left\{ \left[\mathbf{B}^{(0)} + y_l \mathbf{B}^{(1)} + z_l \mathbf{B}^{(2)} + y_l z_l \mathbf{B}^{(3)} + y_l^2 \mathbf{B}^{(4)} + z_l^2 \mathbf{B}^{(5)} \right]^T \right. \\ &\quad \mathbf{D} \left[\mathbf{B}^{(0)} + y_l \mathbf{B}^{(1)} + z_l \mathbf{B}^{(2)} + y_l z_l \mathbf{B}^{(3)} + y_l^2 \mathbf{B}^{(4)} + z_l^2 \mathbf{B}^{(5)} \right] \\ &\quad + \left[\boldsymbol{\varepsilon}^{(0)} + y_l \boldsymbol{\varepsilon}^{(1)} + z_l \boldsymbol{\varepsilon}^{(2)} + y_l z_l \boldsymbol{\varepsilon}^{(3)} + y_l^2 \boldsymbol{\varepsilon}^{(4)} + z_l^2 \boldsymbol{\varepsilon}^{(5)} \right]^T \\ &\quad \left. \mathbf{D} \left[\frac{\partial \mathbf{B}^{(0)}}{\partial \mathbf{u}_L} + y_l \frac{\partial \mathbf{B}^{(1)}}{\partial \mathbf{u}_L} + z_l \frac{\partial \mathbf{B}^{(2)}}{\partial \mathbf{u}_L} + y_l z_l \frac{\partial \mathbf{B}^{(3)}}{\partial \mathbf{u}_L} + y_l^2 \frac{\partial \mathbf{B}^{(4)}}{\partial \mathbf{u}_L} + z_l^2 \frac{\partial \mathbf{B}^{(5)}}{\partial \mathbf{u}_L} \right] \right\} dA dx \end{aligned} \quad (4.9)$$

The tangential stiffness matrix can also be expressed in the form of **Eq. (4.10)** by expanding **Eq. (4.9)**.

$$\mathbf{K}_L = \int_L \left[\sum_{i=1}^{14} C_i \mathbf{K}_i \right] dx \quad (4.10)$$

where the coefficients C_i are the same as those in **Eq. (4.8)** and the details of the sub-matrices $\mathbf{K}_i$ ($i=1,\dots,14$) are given in **Appendix B**. These are derived by the author of this thesis. According to the transformation matrix $\mathbf{T}$ from the local to the global coordinate system (Li 2007), global internal force vector $\mathbf{f}_G$ and tangential stiffness matrix $\mathbf{K}_G$ can be derived, respectively.

$$\mathbf{f}_G = \mathbf{T}^T \mathbf{f}_L \quad \text{and} \quad \mathbf{K}_G = \mathbf{T}^T \mathbf{K}_L \mathbf{T} + \frac{\partial \mathbf{T}^T}{\partial \mathbf{u}_G} \mathbf{f}_L \quad (4.11)$$

It should be noted that the present derivations of local internal force vector and stiffness matrix are for general beam cross-sections including non-symmetric sections, so that non-symmetric steel reinforcement for a concrete beam section can also be modelled and this will be discussed in the next section.

4.3 The Fibre Model

After the internal force vector $\mathbf{f}_L$ and stiffness matrix $\mathbf{K}_L$ in the local coordinate for the CR beam have been computed as given in **Eq. (4.7)** and **Eq. (4.9)**.

Conventionally, in the process of three dimensional integration, both the local internal force vector $\mathbf{f}_L$ and local stiffness matrix $\mathbf{K}_L$ can be obtained by integrating certain functions at the cross-section A and then integrating them along the element length L . That is, the integration can be treated as an integration of a known function $\mathbf{X}$ with respect to the cross-section A and element length L to obtain $\mathbf{F}$, as expressed in **Eq. (4.12)**.

$$\mathbf{F} = \int_L \int_A [\mathbf{X}] dA dx \quad (4.12)$$

Since the integration of $\mathbf{X}$ can be performed at the cross-section of each fibre first and then summed up together to obtain the integrated value around the entire cross-section, the material properties and the cross-sectional shape can be implemented at the fibre level. The cross-section may contain fibres with different material properties (or even voids) or different shapes. **Fig. 4.2 (a)**

shows a non-symmetric section. However, the assumption of “plane sections remain plane” has to be kept, so that the studied beam element is assumed to be laterally restrained and no warping effect is considered.

Fig. 4.2 The fibre model of the proposed CR beam element

To simulate RC members, steel reinforcement and concrete are assigned to different fibres. Fibre model assumes perfect bond between concrete and reinforcement. Therefore, the integration process provides the opportunity to employ fibre model to represent the CR beam cross-section and to simulate more accurately the mechanical behaviour and the stress and strain constitutive relations at specified ‘cells’ around the CR beam cross-sections. By employing the fibre model, the local internal force vector $\mathbf{f}_L$ and local stiffness matrix $\mathbf{K}_L$ can be expressed as

$$\mathbf{f}_L = \int_L \sum_{NF} \left[\sum_{i=1}^{14} C_i \mathbf{f}_i \right] dx \quad \text{and} \quad \mathbf{K}_L = \int_L \sum_{NF} \left[\sum_{i=1}^{14} C_i \mathbf{K}_i \right] dx \quad (4.13)$$

where NF is the number of fibres of the cross-section at the Gaussian point along the longitudinal axis of the CR beam element (**Fig. 4.2 (b)**); the coefficients C_i ($i=1,\dots,14$) at the fibre cross-sections are first computed by

using **Eq. (4.8)** and $\sum_{i=1}^{14} C_i \mathbf{f}_i$ at the fibre level can be computed conveniently.

The summation of $\sum_{i=1}^{14} C_i \mathbf{f}_i$ from all of the fibres, which is the integrated value for the whole cross-sectional area at a Gaussian point along a CR beam element, can then be obtained. The calculation procedure of local stiffness matrix $\mathbf{K}_L$ follows the same way.

It should be noted that reduced integration with two Gaussian points along the longitudinal axis of the proposed three-node CR beam element is adopted as explicitly described in **Fig. 4.2 (b)**. However, a lower order integration scheme (i.e. single-point integration) for each fibre around the cross-section is utilized with the assumption of uniform stress for each fibre area. In all of the examples employed in the present thesis, single-point integration is applied for fibre model by default.

4.4 Material Nonlinearity

When handling material nonlinearity for RC structures, steel reinforcement is treated as an elasto-plastic material with Von Mises yield criterion which conforms to associated flow rule, plastic potential and normality condition. On the other hand, the constitutive relationship of concrete is assumed to follow empirical formulae mentioned in **Section 3.2** (Karsan and Jirsa 1969; Rots et al. 1984; Barzegar-Jamshidi 1987; Dvorkin et al. 1988; Taucer et al. 1991) which have been widely employed and verified to be suitable for numerical computation.

4.4.1 Steel reinforcement

From Hinton and Owen (1984), the elasto-plastic incremental stress-strain relation is given as:

$$d\boldsymbol{\sigma} = \mathbf{D}_{ep} d\boldsymbol{\varepsilon} \quad (4.14)$$

where the equivalent material matrix $\mathbf{D}_{ep} = \mathbf{D} - \mathbf{D}\mathbf{a}\mathbf{a}^T\mathbf{D}/(H + \mathbf{a}^T\mathbf{D}\mathbf{a})$, $\mathbf{D}$ is the elastic material matrix, flow vector $\mathbf{a} = \partial F / \partial \boldsymbol{\sigma}$ and F is the yield function indicating the plastic state of the material in yield criteria. Therefore,

$$d\boldsymbol{\sigma} = \mathbf{D}_{ep} d\boldsymbol{\varepsilon} = \mathbf{D} d\boldsymbol{\varepsilon} - \frac{\mathbf{D}\mathbf{a}\mathbf{a}^T\mathbf{D}}{H + \mathbf{a}^T\mathbf{D}\mathbf{a}} d\boldsymbol{\varepsilon} = d\boldsymbol{\sigma}_e - \mathbf{D}\mathbf{a} \frac{\mathbf{a}^T\mathbf{D}d\boldsymbol{\varepsilon}}{H + \mathbf{a}^T\mathbf{D}\mathbf{a}} = d\boldsymbol{\sigma}_e - d\lambda \mathbf{D}\mathbf{a} \quad (4.15)$$

where $d\lambda = \mathbf{a}^T\mathbf{D}d\boldsymbol{\varepsilon}/(H + \mathbf{a}^T\mathbf{D}\mathbf{a})$ and H is the hardening modulus.

With a common procedure to handle problems including elasto-plastic and strain hardening behaviour, the stress increments can be divided into one part inside the yield surface and another part outside the yield surface, with R as the ratio of the part outside the yield surface to the whole stress increment, as shown in detail in Fig. 4.3.

$$d\boldsymbol{\sigma}_e = (1-R)d\boldsymbol{\sigma}_e + R d\boldsymbol{\sigma}_e \quad (4.16)$$

Fig. 4.3 Incremental stress changes in an elasto-plastic material

Substituting the part outside the yield surface $Rd\sigma_e$ back to the elasto-plastic incremental stress-strain relation $d\sigma = d\sigma_e - d\lambda \mathbf{D}\mathbf{a}$ and considering the contribution from the part inside the yield surface, the whole incremental stress can be obtained as

$$d\sigma = (1-R)d\sigma_e + Rd\sigma_e - d\lambda \mathbf{D}\mathbf{a} \quad (4.17)$$

The plastic part outside the yield surface $Rd\sigma_e - d\lambda \mathbf{D}\mathbf{a}$ will be eliminated through several iterations. After that, the elasto-plastic incremental stress-strain relation for the first iteration can be expressed as

$$d\sigma = (1-R)d\sigma_e + \frac{Rd\sigma_e}{m} - \frac{d\lambda \mathbf{D}\mathbf{a}}{m} \quad (4.18)$$

where m is the user-defined iteration number and is suggested by Hinton and Owen (1984) to be the nearest integer which is less than $8(Rd\sigma_e/\sigma_{Y0})+1$, where σ_{Y0} is the initial uniaxial yield strength.

In the present CR beam formulation, the strain is calculated from **Eq. (4.2)**. The incremental strain can be then written in the form:

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^{(0)} + y_l d\boldsymbol{\varepsilon}^{(1)} + z_l d\boldsymbol{\varepsilon}^{(2)} + y_l z_l d\boldsymbol{\varepsilon}^{(3)} + y_l^2 d\boldsymbol{\varepsilon}^{(4)} + z_l^2 d\boldsymbol{\varepsilon}^{(5)} \quad (4.19)$$

So the incremental stress is given as

$$\begin{aligned} d\sigma &= (1-R)\mathbf{D}d\boldsymbol{\varepsilon} + \frac{R\mathbf{D}d\boldsymbol{\varepsilon}}{m} - \frac{1}{m} \left(\frac{\mathbf{a}^T \mathbf{D} R d\boldsymbol{\varepsilon}}{H + \mathbf{a}^T \mathbf{D}\mathbf{a}} \right) \mathbf{D}\mathbf{a} \\ &= \left[(1-R)\mathbf{D} + \frac{R\mathbf{D}}{m} \right] d\boldsymbol{\varepsilon} - \frac{1}{m} \left(\frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D}\mathbf{a}} \cdot R d\boldsymbol{\varepsilon} \right) \mathbf{D}\mathbf{a} \\ &= \left[(1-R)\mathbf{D} + \frac{R\mathbf{D}}{m} \right] \left(d\boldsymbol{\varepsilon}^{(0)} + y_l d\boldsymbol{\varepsilon}^{(1)} + z_l d\boldsymbol{\varepsilon}^{(2)} + y_l z_l d\boldsymbol{\varepsilon}^{(3)} + y_l^2 d\boldsymbol{\varepsilon}^{(4)} + z_l^2 d\boldsymbol{\varepsilon}^{(5)} \right) \\ &\quad - \frac{1}{m} \left[\frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D}\mathbf{a}} \cdot R \left(d\boldsymbol{\varepsilon}^{(0)} + y_l d\boldsymbol{\varepsilon}^{(1)} + z_l d\boldsymbol{\varepsilon}^{(2)} + y_l z_l d\boldsymbol{\varepsilon}^{(3)} + y_l^2 d\boldsymbol{\varepsilon}^{(4)} + z_l^2 d\boldsymbol{\varepsilon}^{(5)} \right) \right] \mathbf{D}\mathbf{a} \end{aligned} \quad (4.20)$$

In general, assuming that vector $\mathbf{x}$ can be expressed as $\alpha_1 \mathbf{x}_1 + \alpha_2 \mathbf{x}_2$ where α_1 and α_2 are scalar quantities, the matrix-vector computation can be performed as follows.

$$\begin{aligned} (\mathbf{a} \cdot \mathbf{x}) \mathbf{c} &= [\mathbf{a} \cdot (\alpha_1 \mathbf{x}_1 + \alpha_2 \mathbf{x}_2)] \mathbf{c} \\ &= (\mathbf{a} \cdot \alpha_1 \mathbf{x}_1) \mathbf{c} + (\mathbf{a} \cdot \alpha_2 \mathbf{x}_2) \mathbf{c} \\ &= \alpha_1 (\mathbf{a} \cdot \mathbf{x}_1) \mathbf{c} + \alpha_2 (\mathbf{a} \cdot \mathbf{x}_2) \mathbf{c} \end{aligned} \quad (4.21)$$

From **Eq. (4.21)**, the expression $(\mathbf{a} \cdot \mathbf{x}) \mathbf{c}$ can be calculated by superposition of vector components. So the incremental stress in **Eq. (4.20)** can be rewritten as

$$\begin{aligned} d\boldsymbol{\sigma} &= \left[(1-R) \mathbf{D} + \frac{R\mathbf{D}}{m} \right] (d\boldsymbol{\varepsilon}^{(0)} + y_l d\boldsymbol{\varepsilon}^{(1)} + z_l d\boldsymbol{\varepsilon}^{(2)} + y_l z_l d\boldsymbol{\varepsilon}^{(3)} + y_l^2 d\boldsymbol{\varepsilon}^{(4)} + z_l^2 d\boldsymbol{\varepsilon}^{(5)}) \\ &\quad - \frac{1}{m} \left[\frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D} \mathbf{a}} \cdot R d\boldsymbol{\varepsilon}^{(0)} \right] \mathbf{D} \mathbf{a} - y_l \frac{1}{m} \left[\frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D} \mathbf{a}} \cdot R d\boldsymbol{\varepsilon}^{(1)} \right] \mathbf{D} \mathbf{a} \\ &\quad - z_l \frac{1}{m} \left[\frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D} \mathbf{a}} \cdot R d\boldsymbol{\varepsilon}^{(2)} \right] \mathbf{D} \mathbf{a} - y_l z_l \frac{1}{m} \left[\frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D} \mathbf{a}} \cdot R d\boldsymbol{\varepsilon}^{(3)} \right] \mathbf{D} \mathbf{a} \\ &\quad - y_l^2 \frac{1}{m} \left[\frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D} \mathbf{a}} \cdot R d\boldsymbol{\varepsilon}^{(4)} \right] \mathbf{D} \mathbf{a} - z_l^2 \frac{1}{m} \left[\frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D} \mathbf{a}} \cdot R d\boldsymbol{\varepsilon}^{(5)} \right] \mathbf{D} \mathbf{a} \end{aligned} \quad (4.22)$$

Defining the term $d\lambda_i$ as follows,

$$d\lambda_i = \frac{1}{m} \frac{\mathbf{a}^T \mathbf{D}}{H + \mathbf{a}^T \mathbf{D} \mathbf{a}} \cdot R d\boldsymbol{\varepsilon}^{(i)}, \quad i = 0, 1, \dots, 5 \quad (4.23)$$

the incremental stress and the components cast in the CR framework can be specified as

$$d\boldsymbol{\sigma} = d\boldsymbol{\sigma}^{(0)} + y_l d\boldsymbol{\sigma}^{(1)} + z_l d\boldsymbol{\sigma}^{(2)} + y_l z_l d\boldsymbol{\sigma}^{(3)} + y_l^2 d\boldsymbol{\sigma}^{(4)} + z_l^2 d\boldsymbol{\sigma}^{(5)} \quad (4.24)$$

where

$$d\boldsymbol{\sigma}^{(i)} = (1-R) \mathbf{D} d\boldsymbol{\varepsilon}^{(i)} + \frac{R\mathbf{D}}{m} d\boldsymbol{\varepsilon}^{(i)} - d\lambda_i \mathbf{D} \mathbf{a}.$$

4.4.2 Concrete

Similarly, the elasto-plastic incremental stress-strain relation for concrete materials is written in the form of

$$d\boldsymbol{\sigma} = \mathbf{D}_{ep} d\boldsymbol{\varepsilon} \quad (4.25)$$

where $\mathbf{D}_{ep} = \begin{bmatrix} E & 0 & 0 \\ 0 & k_0 G & 0 \\ 0 & 0 & k_0 G \end{bmatrix}$ is the material matrix of concrete.

E and G are the elastic modulus and shear modulus, respectively. In the present study, the shear stress components are assumed to be elastic or following empirical shear models obtained from experimental studies as discussed in **Section 3.2.3**, which is reasonable for most applications when failure of concrete is due to cracking or crushing at the fibre level. The compressive and tensile behaviour of normal concrete stresses has been highlighted in **Chapter 3**.

4.5 Solution Strategy

Due to the stiffening and softening characteristics of structural deformations, direct displacement-control or load-control method cannot by themselves guarantee numerical convergence in all cases with critical points in the load-displacement curves, e.g. limit points, snap-through points and snap-back points as shown in **Fig. 4.4**.

The challenging difficulties can be summarized into two points. One of them is to appropriately adjust the step sizes near the critical points. The other one is to change the loading directions when stiffening and softening of the structures occur. Direct displacement control or load control method cannot trace the total load-deflection curves with these critical points.

Fig. 4.4 Limit points, snap-through points and snap-back points in the load-displacement curve

To achieve greater numerical robustness, generalized displacement control method proposed by Yang and Shieh (1990) is employed to ensure stability for iteration at regions near the critical points by adaptive incremental load steps and loading directions.

The equilibrium equation of a nonlinear structure with certain degrees of freedom can be expressed as

$$[\mathbf{K}]_{j-1}^i \{\mathbf{u}\}_j^i = \lambda_j^i \{\mathbf{P}\} + \{\mathbf{R}\}_{j-1}^i \quad (4.26)$$

where $\mathbf{K}$ is the stiffness matrix, $\mathbf{u}$ is the displacement and rotational variables, λ is a unknown parameter to control the load increment, $\mathbf{P}$ is the external loading and $\mathbf{R}$ is the out-of-balance force. Since the λ is also a unknown variables, another constraint equation is needed to compute the value of λ . The superscript represents the loading increment and the subscript represents the number of iterations in the current loading increment.

The equilibrium equation can be rewritten conveniently as

$$[\mathbf{K}]_{j-1}^i \{\mathbf{u}_1\}_j^i = \{\mathbf{P}\} \quad (4.27a)$$

$$[\mathbf{K}]_{j-1}^i \{\mathbf{u}_2\}_j^i = \{\mathbf{R}\}_{j-1}^i \quad (4.27b)$$

$$\{\mathbf{u}\}_j^i = \lambda_j^i \{\mathbf{u}_1\}_j^i + \{\mathbf{u}_2\}_j^i \quad (4.27c)$$

where $\mathbf{u}_1$ is the predictive displacement resulting from the total load and the stiffness matrix from the last iteration. $\mathbf{u}_2$ is the corrective displacement from the out-of-balance force and the stiffness matrix from the last iteration.

In the generalized displacement control method proposed by Yang and Shieh (1990), λ is computed by

$$\lambda_1^i = \lambda_1^1 \left(\frac{\left(\{\mathbf{u}_1\}_1^1 \right)^T \{\mathbf{u}_1\}_1^1}{\left(\{\mathbf{u}_1\}_1^{i-1} \right)^T \{\mathbf{u}_1\}_1^i} \right)^{\frac{1}{2}} = \lambda_1^1 (\text{GSP})^{\frac{1}{2}} \text{ for increment steps} \quad (4.28a)$$

$$\lambda_j^i = - \frac{\left(\{\mathbf{u}_1\}_1^{i-1} \right)^T \{\mathbf{u}_2\}_j^i}{\left(\{\mathbf{u}_1\}_1^{i-1} \right)^T \{\mathbf{u}_1\}_j^i} \quad \text{for iterative steps} \quad (4.28b)$$

where λ_1^1 is a prescribed value for the first load step and GSP is defined to indicate the change of loading direction.

The superior advantage of this solution method is that the values of λ_j^i and $\{\mathbf{u}\}_j^i$ are bounded. That is, the solution method can ensure numerical stability in the region near critical points by self-adapting step sizes.

4.6 Validations

Several examples including steel and RC beams and frames are modelled to test the capabilities of the proposed 3D CR beam elements of simulating structural deformations involving geometric and material nonlinearities and the versatility of the developed fibre model.

To demonstrate the computational accuracy and efficiency of the proposed CR formulation, the total Lagrangian (TL) beam element developed according to Dvorkin et al. (1988) is utilized to compare the predictions by CR formulation for an isolated steel beam with large deformation and a spatial steel frame with material nonlinearity. When simulating large-scale structures, computation cost has to be balanced between modelling accuracy and processing time. It is ideal

to have fewer elements in structural modelling and yet achieving acceptable accuracy with a dominant failure mode and distinct deformation behaviour. Therefore, using TL formulations as a benchmark, a case study is conducted for CR beams on the minimum number of elements used and the CPU time required for both large deformation and elasto-plastic problems. Besides, I-shaped cross-sections with appropriate fibre schemes are employed to test the capabilities of the proposed CR formulation to predict large deformation and material nonlinear behaviour of steel structures with non-rectangular cross-sections.

For RC structures, the tensile stiffening, compressive softening and loading and unloading rules of concrete model in CR formulation are taken into consideration, along with the yielding and fracturing behaviour of steel reinforcement. Firstly, one CR beam element is employed to verify the combined constitutive relationships in the context of member behaviour subjected to compression, tension and shear. Later, two RC columns are validated to demonstrate the numerical accuracy and stability of the proposed CR formulation with different types of concrete models as discussed in **Chapter 3**. Finally, one series of beam members with shear failures are simulated by the proposed CR beam formulations with different types of concrete models and the predictions are compared with reliable experimental results.

4.6.1 A cantilever beam with an end point load

A cantilever beam with an end point load as shown in **Fig. 4.5** is employed to demonstrate the computation accuracy of the proposed CR formulation to simulate problems with large displacements and large rotations. The discretisation schemes for the cantilever are two, three and four CR and TL beam elements. The beam length is 3.0 m and the concentrated load P is 3.11×10^6 N. The material is linear elastic with Young's modulus of 2.1×10^{11} N/m². Sixteen fibres are employed to discretise the beam cross-section for both CR and TL elements. Numerical evaluations of elliptic integral solutions of some large deflection problems have been conducted by Mattiasson (1981). As for geometric nonlinearity in the problems with framed structures, elliptic

integral solutions offer exact solutions. The result is utilized to verify the numerical solution from CR beam formulation.

Fig. 4.5 A cantilever beam with an end point load

In the comparison with theoretical results as shown in **Fig. 4.6**, there is good agreement with predictions when three and more CR elements are used to mesh the cantilever equally, while the predictions by four TL beam elements are not sufficiently accurate. In the legends of **Fig. 4.6**, the combination of an Arabic number and ‘e’ means the number of elements used to mesh the studied beam. For instance, ‘CR (2e)’ indicates that two CR elements are used to mesh the cantilever. It should be noted that when the cantilever is divided into two elements equally, while the CR predictions are still reasonably acceptable, the TL results are crude.

Fig. 4.6 The comparison of results for a cantilever beam with an end point load (Mattiasson 1981)

4.6.2 An I-shaped cross-section beam with both ends clamped

To illustrate the versatility of fibre model in CR beam formulation for a different cross-sectional shape than rectangular, a numerical example of an I-shaped beam with both ends clamped from Hinton and Owen (1984) is shown in

Fig. 4.7. The I-shaped cross-section is discretised into six fibres as shown in **Fig. 4.7** and five CR beam elements are employed to mesh the beam span as the same mesh employed by Hinton and Owen (1984). The material properties are Young's modulus $E = 210 \text{ kN/mm}^2$, Poisson ratio $\nu = 0.3$, yield strength $\sigma_0 = 0.25 \text{ kN/mm}^2$ and strain hardening parameter $H = 0.0$.

Fig. 4.7 Finite element idealization and fibre scheme for the I-shaped cross-section

The applied load versus central deflection graph is shown in **Fig. 4.8**. It should be noted that the applied load in **Fig. 4.8** represents one arrow in **Fig. 4.7**, that is, $1/9^{\text{th}}$ of the total load. Yielding of the cross-section at both clamped ends initiates from flange fibres 1 and 6 and spreads to web fibres 2, 3, 4 and 5. Similarly, yielding of fibres of the cross-section at the mid-span follows the same sequence. As shown in **Fig. 4.8**, in both the elastic deformation and yielding stages, numerical predictions obtained by CR beam elements agree well with the results from Hinton and Owen (1984).

A simulation using TL beam elements with the same number of elements and fibre scheme is conducted as well. To achieve the same 25mm mid-span deflection, the computation time for TL formulation is 7.031s and the average iteration for each load increment is 9 to 11 in the elasto-plastic stage, while CR formulation requires 4.656s and in the elasto-plastic stage the average iteration is only 7 to 8. This represents a computational saving by the proposed CR formulation of more than 30% for elasto-plastic problems. Moreover, with the

same mesh and fibre scheme the prediction accuracy of CR formulation is better than those from the TL formulation as shown in **Fig. 4.8**.

Fig. 4.8 The relationship of the applied load and the deflection at the mid-span
(Hinton and Owen 1984)

4.6.3 A space frame with an elasto-perfectly plastic material and different cross-sectional shapes

To demonstrate the capability of CR formulation to simulate 3D structures using an elasto-perfectly plastic material with different cross-sectional shapes, a space frame with eight members is employed and shown in **Fig. 4.9 (a)**, which have been analysed by Marino (1970), Yang and Fan (1988) and Gendy and Saleeb (1993) based on different approaches. The columns and beams are made of W10×60 and W18×60 sections, respectively. The material properties are $E = 30,000$ ksi (206.9 GPa), $G = 11,500$ ksi (79.3 GPa), and $\sigma_y = 34$ ksi (234.48 MPa). Each member is of length $L = 144$ in (3.655 m) with warping restrained at both ends. Each member is idealized using two CR beam elements and the cross-section orientations and fibre discretisation are illustrated in **Fig. 4.9 (b)**.

The prediction by 16 CR beam elements is shown in **Fig. 4.10** with the comparison of numerical results (Marino 1970; Yang and Fan 1988; Gendy and

Saleeb 1993). To demonstrate the advantage in terms of the minimum number of elements for the proposed CR formulation, the comparisons based on three sets of simulations with each structural member idealized by one, two and three CR and TL elements are conducted. In the legends of **Fig. 4.10**, the combination of an Arabic number and 'e' means the number of elements used to mesh the studied frame. Good agreement is achieved by the proposed CR formulation with a small number of elements as shown in **Fig. 4.10**. On the other hand, with the same number of beam elements, the simulations by TL formulation demonstrate lower accuracy compared with the results by the proposed CR formulation.

As shown in **Fig. 4.10**, the predictions by different numbers of CR elements are quite close, while there is an apparent discrepancy between the predictions by 8, 16 and 32 TL elements. Obviously, compared with TL formulation, there is a clear advantage using CR formulation when solving an elasto-plastic problem as the latter requires fewer CR beam elements to produce the same or better level of accuracy.

Fig. 4.9 A space frame with different cross-sectional shapes

Fig. 4.10 Curves of Load-horizonal displacement u at the loaded point
(Marino 1970; Yang and Fan 1988; Gendy and Saleeb 1993)

Notes: 1 kips=4.448 kN; 1 in.=25.38 mm

4.6.4 Material level test for unified plasticity concrete model

The combined constitutive relationships of the proposed unified plasticity concrete model and the fracture model for concrete as discussed in **Chapter 3** have been successfully validated at the material level (Bao et al. 2012). To verify the combined constitutive relationships in the context of member behaviour simulations, one CR beam element is employed to simulate a concrete member which is subjected to compression, tension and shear. The beam dimensions are given in **Fig. 4.11 (a)** and the concrete cylinder strength is 25.3 MPa. The cross-section is discretised into 4 concrete fibres.

Fig. 4.11 Dimension and loading cases of a one-element concrete member

For pure compression case (**Fig. 4.11 (b)**), the simulation result based on unified plasticity concrete model is compared with the calculation at the material level from a separated program for material model development and the predictions based on uniaxial concrete models as shown in **Fig. 4.12**. **Fig. 4.12 (a)** shows that the prediction results based on unified plasticity concrete model is identical to the calculation at the material level. According to the comparisons of the predicted stress-strain response with the two uniaxial concrete models (e.g. Kent and Park model and Mander's model) for the concrete cylinder subjected to pure compression, these two models are basically the same for the ascending part but there are some differences for the descending part. It is evident that the numerical stability of Kent and Park model will be more robust due to its constant concrete descending stiffness. In fact, the difference of these two uniaxial models in the descending part is negligible, since similar predictions can be obtained for structural deformation or load capacity. In terms of computational time (all examples are simulated on the same computer with 2.66 GHz processor and 3.25 GB RAM), the comparison in **Fig. 4.12 (b)**

demonstrates that the unified plasticity concrete model is much more expensive especially at large plastic strain. The reason is that when applying unified plasticity concrete model, once the plastic strain is large (e.g. in this problem the strain state in the post-peak stage), a great number of iterations are needed to maintain the beam simplification in uniaxial strain and stress states, so as to satisfy equilibrium by using the proposed flow rule between failure surface and potential surface as discussed in **Chapter 3**. Compared with uniaxial concrete models in this problem, there is no apparent improvement for the prediction accuracy when using unified plasticity concrete model. Therefore, uniaxial concrete models is obviously more suitable to simulate problems with flexural failures accompanied by severe concrete crushing.

For pure tension case (**Fig. 4.11 (c)**), since the fracture models for all of the proposed uniaxial and plasticity concrete models are essentially the same, only the prediction based on unified plasticity concrete model is compared with the calculation result at the material level in **Fig. 4.13**. Good agreement can be achieved to prove that the implementation of the fracture model is successful.

Fig. 4.12 Comparisons of a one-element concrete member subjected to uniaxial compression

Fig. 4.13 Comparisons of a one-element concrete member subjected to uniaxial tension

Fig. 4.14 Crack pattern of a one-element concrete member subjected to shear

Unlike the compression and tension cases, the load-deformation response under shear force is strongly related to the dimensions of the model and, therefore, the simulation result based on a one-element concrete member cannot be directly compared with the calculation at the material level for the shear case. As a matter of fact, compared with the load-deformation response, the crack pattern is more meaningful and emphasized herein. For the numerical model with one beam element based on unified plasticity concrete model, the concrete member shown in **Fig. 4.11** is used again and the cross-section is discretised into 100 concrete fibres to obtain a smoother crack direction. As for the boundary conditions, one end is clamped, while the other is under shear force and

restrained by vertical rollers as shown in **Fig. 4.14**. Since there are two Gaussian points along the beam element, the crack direction for each fibre around the beam cross-section is plotted in **Fig. 4.14**. The crack pattern demonstrated in **Fig. 4.14** is compatible with the stress state corresponding to the applied shear force and the induced bending moment. Due to uniform shear distribution and linear normal stress distribution about the neutral axis, the normal stress component at the extreme top and bottom fibres is dominant compared with shear stress components. Therefore, the crack direction is almost perpendicular to the fibre cross-section at the extreme top and bottom fibres. However, the shear stress components dominate the stress state at fibres adjacent to the neutral axis and the crack direction is almost 45° with respect to the axial direction.

4.6.5 RC columns with concentric or eccentric axial loads

Normal-strength concrete columns subjected to short-term concentric or eccentric axial loads are simulated and validated against the test results reported by Mander et al. (1988) and Kim and Yang (1995). The columns are modelled with six 3-node CR beam elements. The cross-section is discretised into 100 concrete fibres. The number of steel fibres is equal to the number of steel bars in the column cross-section. Transverse reinforcement is also considered through confined concrete model. The reinforcement is shown in **Fig. 4.15**. The column properties are listed in **Table 4.1**. As discussed in the **Section 4.6.4**, compared with uniaxial concrete models, the unified plasticity concrete model is much more expensive for large plastic strain. Comparatively speaking, uniaxial concrete models are more suitable to simulate problems with severe concrete crushing. Additionally, the two models (Kent and Park model and Mander's model) are basically the same for the ascending part and the predictions for the descending part are quite close. Therefore, only a uniaxial concrete model (Kent and Park model) is employed for RC columns predictions.

Fig. 4.15 Reinforced concrete columns**Table 4.1** Properties of the RC columns

Column properties	Mander et al. (1988)	Kim and Yang (1995)
Column label in the original paper	C6	10M2
Length (mm)	1200	240
Load type (eccentricity)	Concentric	Eccentric (24mm)
Cylinder compressive strength (MPa)	25.3	63.5
Crushing strain of plain concrete	0.002	0.002
Concrete elastic modulus (GPa)	23	33.356
Longitudinal steel ratio (%)	1.79	1.98
Yield strength of longitudinal steel (MPa)	394	387
Yield strength of stirrup (MPa)	309	300
Stirrup transverse volumetric ratio (%)	0.883	0.3
Concrete core width measured to the centreline of stirrup (mm)	410	62
Stirrup spacing (mm)	72	60

Based on experimental results, the predictions of the proposed CR formulation for columns subjected to concentric or eccentric axial loads are shown in **Fig. 4.16** and **Fig. 4.17**, respectively. Excellent agreement for the initial elastic deformation stage is achieved in predicting both of RC columns. However, as shown in **Fig. 4.16**, without any calibration, the agreement in the descending part for the RC column by Mander et al. (1988) is not as good as that for the

ascending part due to the approximation in post-peak descending curves according to the Kent and Park concrete model as discussed in **Chapter 3**.

As discussed in **Section 3.2.1**, the strain softening slope of the descending part is controlled by Z defined in **Eq. (3.5)**. In order to conduct a more accurate prediction, different coefficients are multiplied by Z to calibrate the descending part of the load-strain response. As shown in **Fig. 4.16**, with different coefficients, the effect of calibration is significant and the trend of the descending parts of the load-strain response is reasonable. It is obviously seen in **Fig. 4.16** that when the coefficient is equal to 0.25, the predicted result is fairly close with the experimental results, which means that the actual confinement applied in the test is much more effective than that the empirical Kent and Park concrete model assumes. Therefore, if the descending slope of the concrete model is calibrated according to the reinforced concrete utilized in the experiment, the proposed CR formulation can provide a closer trend to test results.

Fig. 4.16 Result comparisons for an RC column in the test by Mander et al. (1988)

Fig. 4.17 Result comparisons for an RC column in the test by Kim and Yang (1995)

4.6.6 RC shear beams series by Bresler and Scordelis

A classical set of RC beams with variations in simple span length, concrete strength, beam width and stirrup details were tested by Bresler and Scordelis (1963). To validate the application of the proposed constitutive laws in predicting shear failure behaviour of beam members, all the 12 simply-supported beams subjected to a concentrated load at the mid-span are simulated by the proposed CR fibre beam elements.

The geometry, loading, boundary condition and steel reinforcement details are illustrated in **Fig. 4.18**. The depth of all the specimens is approximated to 560 mm. Five fibre beam elements are utilized to mesh the beam. In addition, equivalent steel fibres are assigned to the location of steel reinforcement as shown in **Fig. 4.18**, that is, each bar is modelled by one longitudinal fibre. The properties of concrete and steel reinforcement and geometric dimension are listed in **Table 4.2**.

Both uniaxial concrete model (Kent and Park model) and unified plasticity model are employed in the comparison study. The comparisons of experimental studies and numerical simulations by the proposed unified plasticity concrete model and the uniaxial concrete model are given in **Fig. 4.19**. Based on the values of shear span-to-depth ratios, all the 12 specimens are accordingly

grouped. In general, there is good agreement between the predictions of the proposed concrete constitutive laws and experimental results for all the specimens with different reinforcement details and shear span-to-depth ratios. Compared with the predictions by the uniaxial concrete model, the predictions by the proposed unified plasticity concrete model are generally better and shear failure can even be captured for specimens with shear span-to-depth ratios of 4.0 and 5.0 as shown in **Fig. 4.19 (a)** and **(b)**.

As for the third series with a shear span-to-depth ratio of 7.0 as shown in **Fig. 4.19 (c)**, the predictions by the uniaxial concrete model and the proposed unified plasticity concrete model are similar and no sudden shear failure is predicted as that in **Fig. 4.19 (a)** and **(b)**. The reason is that the shear span-to-depth ratio of this series is 7.0 and the shear behaviour is not so dominant compared with the first two series with smaller ratios. Therefore, the proposed unified plasticity concrete model is capable of predicting shear failure with small shear span-to-depth ratios.

Fig. 4.18 Geometry and steel reinforcement details of shear beams by Bresler and Scordelis (1963)

Table 4.2 Material properties of RC beams tested by Bresler and Scordelis (1963)

Specimen	OA1	OA2	OA3	A1	A2	A3	B1	B2	B3	C1	C2	C3
Span length L (mm)	3658	4572	6400	3658	4572	6400	3658	4572	6400	3658	4572	6400
Width b (mm)	310.0	305.0	307.0	307.0	305.0	305.0	228.4	228.4	228.4	154.8	154.8	154.8
Depth h (mm)	556.0	560.1	556.0	560.0	558.0	558.0	552.0	552.0	552.0	553.3	553.3	553.3
Effective depth d (mm)	460.6	465.7	461.1	465.7	463.7	465.7	460.6	465.2	460.1	463.1	463.9	458.3
Shear span to depth Ratio $L/(2d)$	3.97	4.90	6.94	3.92	4.93	6.91	3.95	4.91	6.95	3.95	4.93	6.98
Cylinder strength f_c (MPa)	22.54	23.72	37.59	24.06	24.27	35.04	24.76	23.17	38.75	29.59	23.79	35.04
Yield strength of bottom rebars f_y (MPa)	555.2	555.2	552.4	555.2	555.2	552.4	555.2	555.2	552.4	555.2	555.2	552.4
Yield strength of top rebars f_y (MPa)	-	-	-	345.5	345.5	345.5	345.5	345.5	345.5	345.5	345.5	345.5

A statistical analysis of the prediction results of ultimate shear strength for each series of the RC beams is given in **Table 4.3**. The overall mean of the ratios of the peak values from predictions and experiments for all the 12 specimens is 81.6% with a standard deviation of 0.126. If the OA series without transverse reinforcement is not included in the statistical analysis, the overall mean of the ratios is 86.5% with a standard deviation of 0.099. The reason for the discrepancy is that the rigid-plane assumption in the fibre beam element results in additional pseudo lateral constraint along the perimeter of the beam cross-section, which bears greater resemblance in behaviour to RC beams with transverse reinforcement. Therefore, the fibre beam formulation with the proposed unified plasticity concrete model is more applicable to shear failure simulation of RC members with transverse reinforcement.

(a) First series with shear span-to-depth ratio of approximately 4.0

(b) Second series with shear span-to-depth ratio of approximately 5.0

(c) Third series with shear span-to-depth ratio of approximately 7.0

Fig. 4.19 Comparisons of load-displacement responses of the shear beam tests
(Bresler and Scordelis 1963)

Table 4.3 Statistical analysis of the prediction results of RC beams

Specimen	Experiment	Prediction	Specimen	Experiment	Prediction
OA1	327.7	547.8	A1	467.3	630.1
OA2	354.2	492.8	A2	462.2	559.8
OA3	353.7	514.2	A3	468.4	554.4
Mean of ratios		66.8%	Mean of ratios		80.4%
B1	423.2	526.6	C1	317.8	318.1
B2	378.0	414.2	C2	308.9	311.0
B3	329.1	446.2	C3	272.8	293.2
Mean of ratios		81.8%	Mean of ratios		97.4%
Overall mean of ratios and standard derivation				81.6%, 0.126	
Overall mean of ratios and standard derivation (exclusive of OA series)				86.5%, 0.099	

$$* \text{ ratio} = \frac{V_{\text{Experiment}}}{V_{\text{Prediction}}}$$

As a summary of the numerical validations, all the examples are compared with experimental studies. Based on two types of concrete models, the proposed 3D fibre beam element is capable of predicting flexural and shear failures of RC beam members. The uniaxial concrete models are shown to be efficient and accurate for predictions of flexural failures, while the unified plasticity model has the additional advantage to predict shear failures of RC beams with short and medium shear span-to-depth ratios.

However, there are two minor disadvantages for the 3D fibre beam element when predicting shear failures. Firstly, the rigid-plane assumption for the beam element cross-section results in fictitious continuities between fibres, which in reality should be discontinuous after the occurrence of concrete cracking. This will make the predictions of shear strength by fibre beam elements larger than the experimental results.

Secondly, the boundary and loading conditions in a 3D fibre beam element are applied at the centroid of the beam cross-section, as the beam element is still a line element. However, in laboratories, the loading point and boundary condition

are mostly applied at the top and bottom surfaces in shear beam tests. This will result in discrepancy between experimental studies and numerical predictions for shear failure, particularly for beams of large depth. Nevertheless, based on the comparisons for all the examples, the predictions by the 3D fibre beam element along with the proposed unified plasticity concrete model and a simple fracture model are reliable and reasonably accurate to predict the shear failure of RC beam members with short and medium shear span-to-depth ratios.

4.7 Closure

Based on 3-node 3D CR beam elements using vectorial rotational variables, fibre model and material nonlinearity in terms of elasto-plastic incremental stress-strain relationship for both steel and concrete are derived and conducted. By using the proposed fibre model, the derivations based on local internal force vector and stiffness matrix are generalized into cross-sections without symmetry. Different cross-sectional shapes and steel reinforcement detailing can be conveniently discretised into a combination of fibres with various areas and material properties for steel and concrete regions.

With the advantages of proposed CR formulation along with fibre model, the calculations for stress and strain of steel and concrete fibres and the element pure deformation decomposed from a rigid-body movement are conducted in the local coordinate system. The predictions by the proposed CR formulation for steel framed structures are validated to be accurate and efficient for large displacement and large rotation problems and elasto-plastic problems based on the comparison with TL formulation. For RC framed structures, compressive concrete behaviour can be described by both the unified plasticity concrete model and uniaxial concrete models, such as the Kent and Park model and the Mander's model, while tensile stiffening effect is also taken into account. The prediction capabilities of the proposed CR beam element formulation and the concrete models have been validated against experimental studies on RC columns and beams.

In order to identify the advantages and disadvantages of the two types of concrete models, the failure mode of RC members should be identified first. As for the unified plasticity concrete model, it is originally proposed for the prediction of three dimensional concrete material stress state subjected to three dimensional loading. As shown in the validation section of the present chapter, the proposed unified plasticity concrete model is capable of predicting shear failures of RC beams with small and medium shear span-to-depth ratios. Therefore, compared with uniaxial concrete models, the proposed unified plasticity concrete model is more accurate when shear failure is dominant. All the current examples show that the predictions by the 3D fibre beam element with unified plasticity concrete model and modified fracture model are reliable and satisfactorily accurate, even though there are some assumptions in the beam element formulation which result in inaccuracy compared with experimental studies.

Nevertheless, in order to find out the equilibrium stress state by using flow rule between the failure surface and the potential surface, a great number of iterations are necessary when applying unified plasticity concrete model, especially for large-strain problems. So, the computational efficiency and numerical stability of the numerical models with uniaxial concrete model is suitable when severe concrete crushing occurs associated with flexural failures, even though unified plasticity concrete model may be slightly more accurate in terms of predicted load capacity.

Thus, the choice of the concrete models depends on the failure mode of specimens. If the shear span-to-depth ratios are not so small, then flexural failure is dominant and uniaxial concrete model is more suitable. For beams with short and medium shear span-to-depth ratios, unified plasticity concrete model should be employed to identify possible shear failures.

In conclusion, through several numerical examples and validations with test results, the proposed co-rotational 3D beam element demonstrates satisfactory numerical capability when analysing both steel and RC structures with arbitrary

cross-sectional shapes undergoing geometric and material nonlinearities. The proposed CR beam formulation is shown to be an effective approach to simulate the deformations of steel and RC framed structures.

Chapter 5 Component-Based Model for Beam-Column RC Joints

5.1 Introduction

Both experimental research work and failures of RC structures after earthquakes indicate that the loss of stiffness and strength in beam-column joints is crucial, in that joints are the most critical region for the forces transferred between beam/column members within the whole structure. Failures of joints are also important in terms of structural continuity as they affect the boundary condition of beam/column members.

It should be noted that conventional frame analysis is limited to RC frames with rigid joints. Nevertheless, this does not represent the actual situation for framed structures. In fact, joint deformation behaviour is especially critical when analysing progressive collapse potential for framed structures due to the loss in stiffness and strength (Park and Mosalam 2013b). Therefore, the research on the joint element formulation with inelastic deformation capacity is fundamental and meaningful to simulate structural behaviour. In this chapter, component-based mechanical method will be utilized to determine the global behaviour of joints in terms of the deformation capacity and strength.

Based on the idea of component method in **Section 5.2**, a beam-column joint model is presented and will be employed in the RC joint modelling. The theory of the joint model is derived and implemented for finite element analysis. The calibration for each type of components in the beam-column joint model is the most crucial for numerical stability and computation accuracy of the joint simulation. In **Section 5.3**, the bar-slip component, interface-shear component and shear-panel component are calibrated under certain assumptions for each component. Furthermore, the proposed analytical models for the bar-slip component and shear-panel component are validated against some experimental results from the literature.

In reality, load transfer through beam-column joints is the most commonly encountered. Meanwhile, the scenarios of unloading and reloading for the joint may occur when simulating a structure is subjected to a redistribution of internal forces. Therefore, different resistance-deformation states for each type of components are necessary for the simulation of RC joints and will be described in detail in **Section 5.4**.

5.2 Beam-Column Joint for RC Joint Simulation

Lowes and Altoontash (2003) proposed a beam-column joint model as shown in **Fig. 5.1**, which includes four external nodes (denoted as solid circles) with a total of 12 external degrees of freedom and four internal nodes (denoted as hollow circles) with a total of four additional internal degrees of freedom. In terms of components in the joint model, eight bar-slip components are employed to simulate the stiffness and strength loss due to anchorage failure of beam and column longitudinal reinforcement embedded within the joint. One shear-panel component is employed to simulate the strength and stiffness loss due to shear failure of the joint core, and four interface-shear components are employed to simulate the loss of shear-transfer capacity due to shear transfer failure at the beam-joint and the column-joint interfaces. All of the component details are depicted in **Fig. 5.1**. It is noteworthy that the interior and exterior planes and nodes are coincident at the same physical position. This means the initial deformation and dimension of bar-slip components and interface-shear components are zero, thus, the dimension of shear panel component characterizes the dimension of the beam-column joint.

Overall, the most critical foundation in the component-based method is the load-deformation relationship of each component, which will be discussed in detail in the later sections. In addition, to be consistent to assemble the joint elements to form the global stiffness matrix, static condensation is employed.

Fig. 5.1 Components of the beam-column joint model

Fig. 5.2 Joint element deformation, resistance distribution and degrees of freedom

Joint element deformation, resistance distributions and degrees of freedom are illustrated in **Fig. 5.2**. The geometric relationship of the component deformations and independent degrees-of-freedom in the joint model can be described by

$$\Delta = \mathbf{A} \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} \quad (5.1)$$

where vector Δ denotes the 13 component deformations, vector $\mathbf{u}$ represents the 12 external degrees of freedom of the four external nodes (two translations and one rotation for each external node), vector $\mathbf{v}$ represents the four internal degrees of freedom of the four internal nodes (one shear displacement at each internal node) and based on the geometric relationship of deformations, the matrix $\mathbf{A}$ can be expressed in an explicit form as follows.

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & w/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & -w/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & h/2 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -h/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -w/2 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & w/2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -h/2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & h/2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/h & 1/w & 1/h & -1/w \end{bmatrix}$$

where w and h are the width and the height of a beam-column joint, respectively, as shown in **Fig. 5.1**.

When formulating the internal load vector and the corresponding stiffness matrix, the system strain energy in terms of the component stiffness k_{ii} and the component deformation Δ_i can be expressed in **Eq. (5.2)**.

$$\Pi = \frac{1}{2} k_{im} \Delta_i \Delta_m \delta_{im} = \frac{1}{2} k_{ii} \Delta_i^2 \quad (5.2)$$

where δ is the Kronecker operator, i.e. $\delta_{im} = 1$ when $i = m$ and $\delta_{im} = 0$ when $i \neq m$ ($i, m = 1, 2, \dots, 13$).

The internal load vector and the corresponding stiffness matrix are derived based on the derivatives of the system strain energy with respect to the 16 independent degrees-of-freedom of the joint model.

$$\mathbf{F} = \frac{\partial \Pi}{\partial \mathbf{u}_j} = k_{ii} \Delta_i \frac{\partial \Delta_i}{\partial \mathbf{u}_j} = \mathbf{A}^T \mathbf{k} \Delta = \mathbf{A}^T \mathbf{f} \quad (5.3)$$

$$\mathbf{K} = \frac{\partial^2 \Pi}{\partial \mathbf{u}_j \partial \mathbf{u}_l} = k_{ii} \frac{\partial \Delta_i}{\partial \mathbf{u}_j} \frac{\partial \Delta_i}{\partial \mathbf{u}_l} + k_{ii} \Delta_i \frac{\partial^2 \Delta_i}{\partial \mathbf{u}_j \partial \mathbf{u}_l} = \mathbf{A}^T \mathbf{k} \mathbf{A} \quad (5.4)$$

where $A_{ij} = \frac{\partial \Delta_i}{\partial \mathbf{u}_j}$, $\frac{\partial A_{ij}}{\partial \mathbf{u}_l} = \frac{\partial^2 \Delta_i}{\partial \mathbf{u}_j \partial \mathbf{u}_l} = 0$, $i, j, l = 1, 2, \dots, 16$. The terms $\mathbf{f}$ and $\mathbf{k}$ are the component force vector and the component tangent matrix, respectively.

However, the effective internal load vector and the corresponding stiffness matrix should only be relevant to the 12 external degrees-of-freedom. The internal load vector for the four additional internal degrees-of-freedom should be zero when the joint deformation satisfies internal equilibrium (**Eq. (5.5)**). Newton-Raphson algorithm is used to achieve the values of internal nodal displacements.

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \tilde{\mathbf{A}}^T \begin{Bmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_{13} \end{Bmatrix} \quad (5.5)$$

where $\tilde{\mathbf{A}}$ refers to columns 13 to 16 of $\mathbf{A}$. However, it is found that the contribution of the external nodal displacements has not been taken into account by Lowes and Altoontash (2003), when the iterations are utilized to eliminate the internal load vector for the four additional internal degrees-of-freedom. To account for the contribution of external nodal displacements, the internal nodal displacements should be calculated by $\tilde{\mathbf{A}}^T \mathbf{k} \mathbf{A} \{\mathbf{u}^T \mathbf{v}^T\}^T$ rather than $\tilde{\mathbf{A}}^T \mathbf{k} \tilde{\mathbf{A}} \mathbf{v}$ as reported by Lowes and Altoontash (2003).

An efficient approach for the Newton-Raphson algorithm for solving nonlinear equations is used herein in **Eq. (5.6)**.

$$f_i(x_1, x_2, \dots, x_N) = 0 \quad (i = 1, 2, \dots, N) \quad (5.6)$$

Denoting $\mathbf{x} = (x_1, x_2, \dots, x_N)^T$ and ignoring the second and higher order terms, the Taylor expansion for the function about $\mathbf{x}$ can be written as

$$f_i(\mathbf{x}^{(0)} + \delta \mathbf{x}^{(0)}) \approx f_i(\mathbf{x}^{(0)}) + \sum_{j=1}^N \frac{\partial f_i}{\partial x_j} \delta x_j^{(0)} \quad (5.7)$$

If the matrix formed by $\partial f_i / \partial x_j$ is not singular, then the iteration equation can be obtained as

$$x_i^{new} = x_i^{old} + \delta x_i \quad (i = 1, 2, \dots, N) \quad (5.8)$$

where δx_i can be obtained by solving the linear equations $f_i(x) + \sum_{j=1}^N \frac{\partial f_i}{\partial x_j} \delta x_j = 0$ by LU decomposition method. If the values of

$\sum_{i=1}^N |f_i(x^{(k)})|$ is less than the predefined error tolerance, e.g. 10^{-10} in the present study, the iteration can be terminated and the equivalent state is achieved. In practice, the number of iterations to achieve this tolerance is usually 2 or 3.

When solving the internal nodal displacement based on the self-equilibrium state of the joint element, the component stiffness linking the component deformation and component force is temporarily assumed to be linear in the term $\tilde{\mathbf{A}}^T \mathbf{k} \mathbf{A} \{\mathbf{u}^T \mathbf{v}^T\}^T$, where $\mathbf{A} \{\mathbf{u}^T \mathbf{v}^T\}^T = \Delta$ represents the component deformations, and $\mathbf{k} \mathbf{A} \{\mathbf{u}^T \mathbf{v}^T\}^T = \mathbf{k} \Delta = \mathbf{f}$ represents the component forces. Therefore, in the scenario of taking the nonlinear component constitutive relationship into account, the component stiffness as pointed out above should be an equivalent secant stiffness for the component to satisfy the linear calculation from the component deformations to the component forces as shown in the **Fig. 5.3**.

Fig. 5.3 Linearized relation of component deformations and component forces

Based on the nodal displacements and component forces in the equilibrium state, the effective internal load vector and the corresponding stiffness matrix relevant to the 12 external degrees-of-freedom can be expressed in **Eqs. (5.9)** and **(5.10)**.

$$\mathbf{F}^e = \hat{\mathbf{A}}^T \mathbf{f} \quad (5.9)$$

$$\mathbf{K}^e = \mathbf{K}_{ee} - \mathbf{K}_{ei} \mathbf{K}_{ii}^{-1} \mathbf{K}_{ie} \quad (5.10)$$

where $\mathbf{K} = \begin{bmatrix} \mathbf{K}_{ee} & \mathbf{K}_{ei} \\ \mathbf{K}_{ie} & \mathbf{K}_{ii} \end{bmatrix}$ and $\hat{\mathbf{A}}$ refers to columns 1 to 12 of $\mathbf{A}$.

Since the joint model is two-dimensional, the out-of-plane degrees-of-freedom should be restrained when assembling the joint stiffness matrix. Therefore, the singularity of the joint stiffness matrix can be avoided when solving the joint deformation.

5.3 Calibration of Components in the Joint Model

As shown in **Fig. 5.1**, the joint model consists of three component types, that is, the bar-slip component, the interface-shear component and the shear-panel component, which represent the anchorage failure of longitudinal reinforcement, shear transfer failure at the joint perimeter and shear failure of the joint core. Since only joint design detailing is available before finite element analysis, the calibration to convert the joint design information into usable stiffness coefficients of corresponding components in the component-based joint is a critical step.

5.3.1 Bar-slip component

There are no conventionally accepted failure criteria for determining the state of “progressive collapse” for structures, and often times, deflections of affected beams over the “missing column” are often used as performance criteria. However, when simulating the deformation behaviour and the strength of reinforced concrete (RC) framed structures for progressive collapse analysis, besides the flexural deformations, the so-called “fixed end” rotations induced by longitudinal bar slips at the beam-column ends connected to the joints can be significant and may result in additional vertical deformations not accounted for in the initial analysis. Hence, it is important to quantify the deformations arising from fixed end rotations to arrive at a more reliable quantitative deflection criterion for progressive collapse. Several bond stress-slip relationships between steel reinforcement and concrete were previously proposed in the literature. In the present work, their merits and demerits are discussed in terms of application and prediction accuracy in **Chapter 2**. To address the limitations of previous bond-slip models, a new analytical model based on the bond stress integration along the bar stress propagation length is proposed to predict the bar-slip behaviour in RC beam-column joints. Besides, the phenomena of combined axial pullout and transverse dowel action at the joints are considered through incorporating the concepts of bond deterioration zone and curvature influence zone into the proposed model. The proposed analytical model on the bond stress-slip relationship is validated against experimental studies from the literature and is shown to be simple and reliable for predicting structural performance associated with progressive collapse.

5.3.1.1 Analytical model on the bond stress-slip relationship under axial pullout action

In this chapter, the term *slip* is defined as the relative displacement between the main steel reinforcement and the surrounding concrete. Only the relative deformation along the longitudinal direction of the steel reinforcement is considered, while the contact of steel reinforcement with concrete in the

transverse direction is assumed to be perfect. For the surrounding concrete, it is assumed to be well confined by sufficient steel reinforcement or with sufficient cover (concrete cover $\geq 5d_b$ and clear spacing between bars $\geq 10d_b$ as stipulated in CEB (2010), where d_b is the bar diameter). Thus, no splitting failure is considered in the proposed analytical model. In fact, Alsiwat and Saatcioglu (1992) reported that pullout cone failure does not occur at the beam-column joints with transverse reinforcement. Moreover, compared with concrete, the area of steel reinforcement is small and the steel strain is sufficiently large, so that it is commonly assumed that there is negligible influence of concrete deformation on slip.

Fig. 5.4 Resisting mechanisms and failure modes in the bond stress-slip relationship

Based on the experimental studies on the relationship between bond stress and slip, the resisting mechanisms and failure modes can be well described in **Figs. 5.4** and **5.5**. The initial bond resistance is attributed to adhesion between concrete and steel reinforcement. After the formation of internal inclined cracks (Goto 1971) as shown in **Fig. 5.5 (b)**, the mechanical interlocking mechanism commences due to lugs at the bar surface. This mechanism is terminated by shear failure of concrete keys in between the lugs as shown in **Fig. 5.5 (c)**. The bond stress decreases gradually with increasing local slip until the concrete keys are completely sheared off. After this, the only mechanism left is the frictional resistance between the rough concrete and the steel reinforcement. As shown in **Fig. 5.4**, the area encompassed by the descending branch is equal to the

interfacial fracture energy G_f which characterizes the debonding resistance (Haskett et al. 2008; Muhamad et al. 2011).

(a) Adhesion at the intact interface

Primary crack

(b) Internal inclined cracks and the corresponding compressive force (Goto 1971)

Primary crack

(c) Shear-off of concrete keys

Fig. 5.5 The forces action on concrete after the formation of internal inclined cracks

Based on the equivalence of energy dissipation, the nonuniform local bond stress-slip relationship can be expressed by an equivalent constant value with the same dissipated energy along the effective steel reinforcement length as shown in **Fig. 5.6**. In the equivalent bi-uniform distribution of bond stress with τ_E and τ_Y , the average respective values for bond stresses are $1.8\sqrt{f_c}$ and $0.4\sqrt{f_c}$ for tension and $2.2\sqrt{f_c}$ and $3.6\sqrt{f_c}$ for compression (f_c , τ_E and τ_Y in

MPa), as proposed by Lowes and Altoontash (2003). It should be noted that the bi-uniform distribution of bond stress proposed by Sezen and Moehle (2003) is $1.0\sqrt{f_c}$ for τ_E and $0.5\sqrt{f_c}$ for τ_Y in tension. In fact, compressive bond-slip behaviour is more relevant to surrounding concrete in RC joints (Lowes et al. 2003), therefore, it is not meaningful to analytically study the compressive bar-slip behaviour by only considering the bond resistance. Therefore, in the present study, only the tensile bond-slip behaviour is of interest, while the bond-slip behaviour under compression is calibrated according to design regulations as suggested by Lowes and Altoontash (2003). For reinforcement with sufficient embedment, bar fracturing failure occurs, and the value of $1.4\sqrt{f_c}$ (the average of suggested values by Lowes and Altoontash (2003) and Sezen and Moehle (2003) and shortcomings of these two models have been clarified in **Section 2.3.1**)) is taken for τ_E , and the value of τ_Y is conservatively taken as $0.4\sqrt{f_c}$. For reinforcement with insufficient embedment, pullout failure dominates and the value of $2.5\sqrt{f_c}$ is selected for τ_E , which is the maximum bond stress proposed by Eligehausen et al. (1983) based on their experimental study and has been adopted by the CEB-FIP Model Code (2010). As for τ_Y , a relatively larger value of $0.8\sqrt{f_c}$ is taken to reflect an increase of embedment length due to the penetration at the unloaded end under a large strain, because the point of steel reinforcement, which is initially located at the interface of beam/column and joint, will move inwards to the centre of joint.

Fig. 5.6 Equivalent bond stress in bond stress-slip relationship

However, it should be noted that the effective embedment length of a steel reinforcement is not necessarily taken as the actual embedment length of steel reinforcement to resist slip. In reality, the effective length of steel reinforcement

to resist a bar slip is dependent on the magnitude of the applied tension and the surrounding bond condition. Therefore, a more realistic concept termed as “*stress propagation length*” is proposed in this study to describe the propagation of bar stress along the steel reinforcement subject to variations of applied load and the current state of bond deterioration.

Stress propagation length can be smaller than the actual embedment length for reinforcement with sufficient embedment. On the other hand, stress propagation length can also be greater than the actual embedment length for reinforcement with insufficient embedment, in which certain boundary condition contributes equivalently to the fictitiously additional propagation length. One commonly encountered example of the latter case is that the midpoint of a continuous reinforcement in a joint can provide symmetric boundary forces for the stress propagations at both sides. By using the proposed concept of stress propagation length, the bar-slip resistance can be conveniently obtained by integrating the bond stress over the circumferential area and also along the effective length of steel reinforcement. Such a calculation approach is able to overcome the disadvantages of previous analytical models (Lowes and Altoontash 2003; Sezen and Moehle 2003), such as the predictions of bar-slip behaviour with an insufficient embedment length of steel reinforcement.

(I) Steel reinforcement with a sufficient embedment length

For steel reinforcement with sufficient embedment length, three assumptions are made prior to the derivation of bar-slip behaviour as shown in **Fig. 5.7**. Firstly, the bond stress along the anchored length of a reinforcing bar is bi-uniform, that is, the bond stress distributions are uniform for both elastic and plastic segments. Secondly, the slip of steel reinforcement along the anchored length is a function of bar strain distribution. Thirdly, bar-slip is zero at the point of zero bar stress, provided there is a sufficient embedment length of steel reinforcement.

Firstly, it is assumed that the bond force and the bar force for an infinitesimal length dx are in equilibrium, that is,

$$df_s \cdot A_b = (\tau_E \pi d_b)dx \text{ or } (\tau_Y \pi d_b)dx \quad (5.11)$$

where df_s is the bar stress increment along an infinitesimal length dx .

In addition, with an assumed bi-linear stress-strain relationship, the steel reinforcement strain is given in **Eq. (5.12)**.

$$\varepsilon_s = \begin{cases} f_s / E_s & \text{when } f_s \leq f_y \\ f_y / E_s + (f_s - f_y) / E_h & \text{when } f_s > f_y \end{cases} \quad (5.12)$$

where f_s is the bar stress at the point of interest, f_y is the steel yield strength, E_s is the steel Young's modulus, E_h is the hardening modulus, A_b and d_b are the cross-sectional area and diameter of steel reinforcement, respectively.

Based on equilibrium and bilinear constitutive model for steel bars, the overall relationship of bar slip and bond stress can be obtained for any magnitude of the applied load.

Fig. 5.7 Assumed bond stress and bar stress distribution for a reinforcing bar anchored in a joint

(a) Elastic state

When the absolute value of the applied stress ($\tilde{f}_s$) at the loaded end is less than the yield strength (f_y) of steel reinforcement, that is, $\tilde{f}_s \leq f_y$, the induced slip can be obtained from **Eq. (5.13)**.

$$\begin{aligned} Slip &= \int_0^{l_{fs}} \varepsilon_E dx = \int_0^{l_{fs}} \frac{f_s}{E_s} dx \\ &= \int_0^{l_{fs}} \frac{\tau_E \pi d_b}{E_s A_b} x dx = \int_0^{l_{fs}} \frac{4 \tau_E}{E_s d_b} x dx \end{aligned} \quad (5.13)$$

in which the term $l_{fs} = \frac{\tilde{f}_s A_b}{\tau_E \pi d_b}$ is denoted as the stress propagation length, the range of x starts from the zero-stress point to the loaded end, ε_E is the elastic strain over the stress propagation length, and $\tilde{f}_s$ is the applied bar stress at the joint perimeter.

It should be noted that the stress propagation length l_{fs} is the summation of the elastic segment length l_e and the inelastic segment length l_y . But l_y is equal to zero in the present case with only elastic state.

(b) Elasto-plastic state

When the absolute value of the applied stress at the joint perimeter is larger than the yield strength of steel reinforcement, that is, $\tilde{f}_s > f_y$, the induced slip can be obtained from **Eq. (5.14)**.

$$\begin{aligned} Slip &= \int_0^{l_e} \varepsilon_E dx + \int_{l_e}^{l_y+l_e} \left(\frac{f_y}{E_s} + \frac{d\varepsilon_Y}{dx} (x - l_e) \right) dx \\ &= \int_0^{l_e} \frac{4 \tau_E}{E_s d_b} x dx + \frac{f_y l_y}{E_s} + \int_{l_e}^{l_y+l_e} \left(\frac{4 \tau_Y}{E_h d_b} (x - l_e) \right) dx \end{aligned} \quad (5.14)$$

with the terms $l_e = \frac{f_y}{\tau_E} \frac{A_b}{\pi d_b}$ and $l_y = \frac{\bar{f}_s - f_y}{\tau_Y} \frac{A_b}{\pi d_b}$ as indicated in **Fig. 5.7**, while the terms ε_E and ε_Y are the elastic strain and plastic strain over the stress propagation length ($l_{fs} = l_e + l_y$).

(II) Steel reinforcement with an insufficient embedment length

For interior joints with continuous steel reinforcement and subject to the same moment, bar stress is not always zero at the point of zero bar slip (Yu and Tan 2012a). When the applied load at the end of the rebar is too large or the embedment length is inadequate at the midpoint of the steel embedment length, it is assumed to have zero slip but not zero strain as shown in **Fig. 5.8**. Therefore, the third assumption “the bar-slip is zero at the point of zero bar stress” made when deriving the bond stress-slip relationship for the steel reinforcement with a sufficient anchorage length is invalid in some cases for interior joints. Thus, the effect of a limited embedment length for bond stress should be taken into account when integrating the bond stress along the stress propagation length.

Fig. 5.8 Insufficient anchorage length in the interior joints

For simplicity of derivation, assuming that the load transfer along each steel reinforcing bar throughout the interior joint region is symmetric, then the point with zero slip is located at the middle point of the steel embedment length within the joint region. Thus, the anchorage length for the bond-slip behaviour is limited to one-half of the joint width L_j and the bar stress f_0 at the joint centre is taken as the boundary condition to balance the applied pullout force. Since the bar stress propagates from the loaded end of the steel reinforcement with

increasing load, there are five possible distributions of the bar stress and the associated bond stress as shown in **Fig. 5.9** for different boundary conditions.

Fig. 5.9 Stress propagation of the steel reinforcement and the corresponding bond stress

(a) Elastic state

Fig. 5.9 (a) applies to the case, when the applied pullout load is not so large and there is a sufficient length for propagation of bar stress, that is, $\tilde{f}_s \leq f_y$ and $l_{fs} \leq L_J/2$, where L_J is the width of the interior joint and l_{fs} is the propagation length of bar stress. The term $l_{fs} = \frac{\tilde{f}_s A_b}{\tau_E \pi d_b}$ is the same as that indicated in **Fig. 5.7** and the slip at the loaded end can be obtained from **Eq. (5.15)**. The integration variable x is measured from the zero-stress point to the loaded end, which is also illustrated in **Fig. 5.9**.

$$\begin{aligned} Slip &= \int_0^{l_{fs}} \varepsilon_E x dx = \int_0^{l_{fs}} \frac{f_s}{E_s} x dx \\ &= \int_0^{l_{fs}} \frac{\tau_E \pi d_b}{E_s A_b} x dx = \frac{2 \tau_E}{E_s d_b} (l_{fs})^2 \end{aligned} \quad (5.15)$$

(b) Elastic state with non-zero stress boundary

With increasing applied bar stress $\tilde{f}_s$ at the joint perimeter, bar stress will propagate towards the joint centre along the steel reinforcement. If the joint width is insufficient and the yield strength is relatively large, then the distributions of bond stress and bar stress as shown in **Fig. 5.9 (b)** are mobilized with $\tilde{f}_s \leq f_y$ and $l_{fs} > L_J/2$. The integrated slip at the loaded end can be obtained from **Eq. (5.16)**.

$$\begin{aligned} Slip &= \int_0^{L_J/2} \varepsilon_E x dx = \int_0^{L_J/2} \left[\frac{f_0 + \frac{2x(f_s - f_0)}{L_J}}{E_s} \right] dx \\ &= \int_0^{L_J/2} \left(\frac{f_0 + \frac{\tau_E \pi d_b}{A_b} x}{E_s} \right) dx = \frac{f_0}{E_s} \frac{L_J}{2} + \frac{2 \tau_E}{E_s d_b} \left(\frac{L_J}{2} \right)^2 \end{aligned} \quad (5.16)$$

with $f_0 = \tilde{f}_s - \frac{\tau_E \pi d_b}{A_b} \frac{L_J}{2}$.

(c) Elasto-plastic state with zero stress boundary

It is evident in **Fig. 5.9 (b)** that at a certain virtual point along the steel reinforcement, as shown by the dash lines, anchorage force f_0 acts as a boundary. Besides the scenario in **Fig. 5.9 (b)**, one possibility is that there is sufficient joint width (L_J) but the stress propagation length is less than $L_J/2$. Then the distributions of bond stress and bar stress are shown in **Fig. 5.9 (c)** with $\tilde{f}_s > f_y$ and $l_{fs} = l_y + l_e < L_J/2$, where l_e and l_y are the respective elastic and plastic steel reinforcement length. The terms $l_e = \frac{f_y}{\tau_E} \frac{A_b}{\pi d_b}$ and $l_y = \frac{\tilde{f}_s - f_y}{\tau_Y} \frac{A_b}{\pi d_b}$ bear the same meanings as those indicated in **Fig. 5.7**. For such a situation, the corresponding slip at the loaded end is given in **Eq. (5.17)**.

$$\begin{aligned}
 \text{Slip} &= \int_0^{l_e} \varepsilon_E x \, dx + \int_{l_e}^{l_y+l_e} \frac{f_y}{E_s} + \Delta \varepsilon_Y (x - l_e) \, dx \\
 &= \int_0^{l_e} \frac{4 \tau_E}{E_s d_b} x \, dx + \frac{f_y l_y}{E_s} + \int_{l_e}^{l_y+l_e} \frac{4 \tau_Y}{E_h d_b} (x - l_e) \, dx \quad (5.17) \\
 &= \frac{2 \tau_E}{E_s d_b} (l_e)^2 + \frac{f_y l_y}{E_s} + \frac{2 \tau_Y}{E_h d_b} (l_y)^2
 \end{aligned}$$

(d) Elasto-plastic state with non-zero stress boundary

No matter whether the state of stress is in **Fig. 5.9 (b)** or **Fig. 5.9 (c)**, with increase stress at the loaded end, the following stage shown in **Fig. 5.9 (d)** will occur with $\tilde{f}_s > f_y$, $l_{fs} = l_y + l_e > L_J/2$ and $l_y < L_J/2$. Yielding occurs for a certain range of steel reinforcement near the loaded end of steel reinforcement. Since the local strain of yielded steel is greater than that of elastic steel, the corresponding bond stress for yielded steel is shown in **Fig. 5.9 (d)**. The integrated slip at the loaded end is given by **Eq. (5.18)**.

$$\begin{aligned}
 Slip &= \int_0^{\frac{L_J}{2}-l_y} \frac{f_0 + \frac{(f_y - f_0)x}{\frac{L_J}{2} - l_y}}{E_s} dx \\
 &+ \int_0^{l_y} \frac{f_y}{E_s} + \frac{(f_s - f_y)x}{l_y E_h} dx \\
 &= \int_0^{\frac{L_J}{2}-l_y} \frac{f_0 + \frac{\tau_E \pi d_b}{A_b} x}{E_s} dx + \int_0^{l_y} \frac{f_y}{E_s} + \frac{4\tau_Y x}{E_h d_b} dx \\
 &= \frac{f_0}{E_s} \left(\frac{L_J}{2} - l_y \right) + \frac{2\tau_E}{E_s d_b} \left(\frac{L_J}{2} - l_y \right)^2 + \frac{f_y l_y}{E_s} \\
 &+ \frac{2\tau_Y}{E_h d_b} (l_y)^2
 \end{aligned} \tag{5.18}$$

$$\text{with } f_0 = f_y - \frac{\tau_E \pi d_b}{A_b} \left(\frac{L_J}{2} - l_y \right).$$

(e) Plastic state

The ultimate stage of the bond stress-slip behaviour is shown in **Fig. 5.9 (e)** in which the whole steel embedment within the joint region has yielded with $\tilde{f}_s > f_y$, $f_0 > f_y$, $l_{fs} = l_e + l_y > L_J/2$ and $l_y > L_J/2$. The slip at the loaded end can be obtained from **Eq. (5.19)**.

$$\begin{aligned}
 Slip &= \int_0^{\frac{L_J}{2}} \frac{f_y}{E_s} + \frac{f_0 - f_y + \frac{(f_s - f_0)x}{\frac{L_J}{2}}}{E_h} dx \\
 &= \frac{f_y}{E_s} \frac{L_J}{2} + \frac{f_0 - f_y}{E_h} \frac{L_J}{2} + \frac{2\tau_Y}{E_h d_b} \left(\frac{L_J}{2} \right)^2
 \end{aligned} \tag{5.19}$$

$$\text{with } f_0 = \tilde{f}_s - \frac{\tau_Y \pi d_b}{A_b} \frac{L_J}{2}.$$

Table 5.1 Slips due to axial pullout in interior joints with different embedment lengths and bar stress boundary conditions

Case No.	Applied bar stress $\tilde{f}_s$	Length of elastic segment l_e within joint	Length of plastic segment l_y within joint	Propagation length of the bar stress l_{fs}	Bar stress at the joint centre f_0	Induced slip at the loaded end
(a)	$\tilde{f}_s \leq f_y$	$\frac{\tilde{f}_s}{\tau_E} \frac{A_b}{\pi d_b}$	0	$l_{fs} = l_e \leq \frac{L_J}{2}$	0	$\frac{2 \tau_E}{E_s d_b} (l_{fs})^2$
(b)	$\tilde{f}_s \leq f_y$	$\frac{L_J}{2}$	0	$l_{fs} = l_e > \frac{L_J}{2}$	$\tilde{f}_s - \frac{\tau_E \pi d_b}{A_b} \frac{L_J}{2}$	$\frac{f_0}{E_s} \frac{L_J}{2} + \frac{2 \tau_E}{E_s d_b} \left(\frac{L_J}{2}\right)^2$
(c)	$\tilde{f}_s > f_y$	$\frac{f_y}{\tau_E} \frac{A_b}{\pi d_b}$	$\frac{\tilde{f}_s - f_y}{\tau_Y} \frac{A_b}{\pi d_b}$	$l_{fs} = l_y + l_e < \frac{L_J}{2}$	0	$\frac{2 \tau_E}{E_s d_b} (l_e)^2 + \frac{f_y l_y}{E_s} + \frac{2 \tau_Y}{E_h d_b} (l_y)^2$
(d)	$\tilde{f}_s > f_y$	$\frac{L_J}{2} - l_y$	$\frac{\tilde{f}_s - f_y}{\tau_Y} \frac{A_b}{\pi d_b}$	$l_{fs} = l_y + l_e > \frac{L_J}{2}$	$f_y - \frac{\tau_E \pi d_b}{A_b} \left(\frac{L_J}{2} - l_y\right)$	$\frac{f_0}{E_s} \left(\frac{L_J}{2} - l_y\right) + \frac{2 \tau_E}{E_s d_b} \left(\frac{L_J}{2} - l_y\right)^2 + \frac{f_y l_y}{E_s} + \frac{2 \tau_Y}{E_h d_b} (l_y)^2$
(e)	$\tilde{f}_s > f_y$	0	$\frac{L_J}{2}$	$l_{fs} = l_y > \frac{L_J}{2}$	$\tilde{f}_s - \frac{\tau_Y \pi d_b}{A_b} \frac{L_J}{2}$	$\frac{f_y}{E_s} \frac{L_J}{2} + \frac{f_0 - f_y}{E_h} \frac{L_J}{2} + \frac{2 \tau_Y}{E_h d_b} \left(\frac{L_J}{2}\right)^2$

Different bar slip conditions with different embedment lengths and bar stress boundary conditions in the interior RC joints due to axial pullout are summarized in **Table 5.1**. Nevertheless, it should be noted that unlike interior joints, for knee joints and exterior joints, the reinforcement embedment length is simply the total joint width. Unlike the continuous steel reinforcement in the interior joints as shown in **Fig. 5.8**, there is no zero-slip point in the knee joints and the exterior joints after the stress propagation length exceeds the embedment length because the reinforcement detailing in the knee and exterior joints (**Fig. 5.10**) is not capable of providing additional anchorage force.

Fig. 5.10 Reinforcement details for the knee joint and the exterior joint

(III) Bent bars in the knee and exterior joints

Due to the bent bars in the knee and exterior joints as shown in **Fig. 5.11**, the anchorage condition is enhanced compared with the straight embedded bars. To consider this effect, a simple equivalent embedment length is employed for bent bars as shown in **Fig. 5.12**. The bend is replaced with a straight bar anchorage as given in **Eq. (5.20)** proposed by Eligehausen et al. (1982) based on an extensive experimental study on the bond behaviour of bent bars in RC joints, which has been adopted by Filippou et al. (1983).

$$L_{eq} = L_{straight} + 5d_b \quad (5.20)$$

where the parameters are shown in **Fig. 5.12** for clarity.

Fig. 5.11 Actual reinforcement arrangement in exterior joints

Fig. 5.12 Equivalent reinforcement arrangement in exterior joints

Based on the derivations above, the bar end load and slip response under axial pullout force can be obtained and compared with the experimental results. Last but not least, even though there is a large scatter in the experimental results of bond stress under the same laboratory conditions (Eligehausen et al. 1983; Alsiwat and Saatcioglu 1992), it is noteworthy that since the formulations of the analytical models are proposed and derived based on the equilibrium, compatibility and steel constitutive law, the predicted relationships of bond stress and slip represent the important failure mode and deformation characteristics of embedded steel reinforcement in concrete.

5.3.1.2 Analytical model on the bond stress-slip relationship under the coupled actions of axial pullout and transverse shear

As discuss in **Section 2.3.1**, with increasing the applied loads, the dowel action in the beam-column joints commences and coexists with the axial pullout. As

shown in **Fig. 2.6**, the dowel action especially at the bottom steel reinforcement is evident and the inclinations of the associated beams with respect to the undeformed beam direction can be up to 15 degrees after the occurrence of catenary action (Yu and Tan 2010; 2011). Therefore, in this case, the bar-slip behaviour is not simply the pullout mechanism but should be coupled with dowel action to resist the transverse shear.

When considering the combination of pullout behaviour and dowel action, the beam-on-elastic foundation (BEF) theory (Hetényi 1946; Dei Poli et al. 1992) can be applied to model dowel action as a beam resting on an elastic and cohesionless foundation. However, experimental studies (Mishima et al. 1995; Maekawa and Qureshi 1996b) indicate that the BEF theory is only applicable when the concrete deformation around reinforcing bars is still linear. Because of bond deterioration (due to concrete cracking and crushing around the reinforcement) and reinforcement curvature (due to transverse shear displacement at the beam/column and joint interface), the assumption of perfect elastic foundation fails and the corresponding prediction should be corrected. On the other hand, due to concrete nonlinearity, the two actions of axial pullout and transverse shear have to be considered simultaneously when modelling the embedded steel reinforcement.

To eliminate the limitation discussed above, by using Shima's model (Shima et al. 1987) with a logarithmic bond distribution, Maekawa and Qureshi (1996a) proposed two empirical concepts, viz., the bond deterioration zone and the curvature influence zone, to simulate the localized damage of concrete, such as splitting and crushing around the reinforcing bars. It was reported by Maekawa and Qureshi (1996a) that as long as a bond deterioration zone of appropriate size is considered, the extent of degradation is not a highly sensitive parameter, which renders the opportunity to adopt the proposed bi-uniform bond-slip model in the present work.

For the curvature influence zone as shown in **Fig. 5.13 (a)**, the length of the initial curvature influence zone L_{c0} is obtained from **Eq. (5.21)** based on the BEF theory.

$$L_{c0} = \frac{3\pi^4}{4} \sqrt{\frac{4E_s I_b}{K}} \quad (5.21)$$

where E_s is the steel Young's modulus, I_b is the moment of inertia of the reinforcing bar cross-section and $K = 150f_c$.

(a) Bar curvature distribution

(b) Bond stress distribution

Fig. 5.13 Bar curvature and bond stress distributions along the embedded bar
(Maekawa and Qureshi 1996a)

With increasing transverse shear displacement δ_b , the curvature influence zone is observed to increase one to two times bar diameter. By defining a non-dimensional damage parameter DI , the length of the curvature influence zone can be empirically expressed in **Eq. (5.22)**.

$$L_c = f(x) = \begin{cases} L_{c0} & DI < 0.02 \\ L_{c0}\{1 + 3(DI - 0.02)^{0.8}\} & DI \geq 0.02 \end{cases} \quad (5.22)$$

where $DI = (1 + 150 \text{ Slip}/d_b)\delta_b/d_b$. It should be noted that the parameter DI has nothing to do with seismic loading. The so-called damage parameter is to represent the damage due to the curvature influence zone.

As shown in **Fig. 5.13 (a)**, the curvature distribution along a reinforcing bar is empirically given in **Eq. (5.23)**.

$$\Phi(x) = \begin{cases} \frac{3\Phi_{\max}(L_c - x)}{L_c^2} & 0 \leq x \leq \frac{L_c}{2} \\ -\frac{3\Phi_{\max}}{L_c^2} \left[3\left(\frac{L_c}{2} - x\right)^2 - L_c\left(\frac{3}{4}L_c - x\right) \right] & \frac{L_c}{2} < x < L_c \end{cases} \quad (5.23)$$

where the maximum curvature $\Phi_{\max} = 64\delta_b/(11L_c^2)$ can be obtained (Soltani and Maekawa 2008) by satisfying the boundary and continuity conditions.

Applying the concept of bond deterioration zone, the region where the bond performance may deteriorate near the interface is empirically taken into account as shown in **Fig. 5.13 (b)**. The length of bond deterioration zone L_b is taken as the greater value of L_c and $5d_b$. Consequently, the bond stress distribution is given in **Eq. (5.24)**.

$$\tau_b = \begin{cases} 0 & 0 \leq x \leq \frac{L_b}{2} \\ \frac{\tau_{\max}}{L_b}x & \frac{L_b}{2} < x \leq L_b \end{cases} \quad (5.24)$$

where τ_{max} is equal to the bond stress with τ_E or τ_Y as proposed in **Section 5.3.1.1**, depending on the stress state near the joint interface.

Thus, the curvature distribution along the reinforcement bar and the bond stress deterioration near the beam-column joint interface can be calculated and both the axial and transverse stresses in the embedded bar can be iteratively computed with the coupled axial pullout behaviour and transverse dowel action. A similar concept has been adopted by other researchers (He and Kwan 2001) in finite element analysis to model the dowel action of reinforcement in RC structures at a structural level. However, such a concept has not ever been considered in the any previous bar-slip analytical model. To the author's knowledge, it is the first time to apply this concept in an analytical model as attempted in the present thesis.

The overall computational procedure is schematically given in **Fig. 5.14**. To accurately describe the stress-strain profiles at the critical zones (bond deterioration zone and curvature influence zone), the whole bar embedment length is discretised to be N segments for the iterative calculations. As a balance of accuracy and computational cost, a value of 10 is employed for N in the present study and a typical case is illustrated in **Fig. 5.15**. As shown in **Fig. 5.14**, two nested iteration loops over the embedment length of bars should be conducted to enforce equilibrium, local bond-slip relationships (as proposed in **Section 5.3.1.1**), constitutive models of steel and compatibility conditions between steel and concrete. Firstly, with the plane-section-remain-plane assumption, the average bar strain $\bar{\epsilon}_s$ is iteratively obtained to satisfy the steel constitutive model with local bar strain ϵ_{si} and bar stress σ_{si} at each steel fibre over the cross-sectional area and the average bar stress $\bar{\sigma}_s$ calculated based on the local bond stress-bar stress equilibrium. Secondly, with the satisfaction of cross-sectional analysis at each segment, the stress propagation length l_{fs} is iteratively determined to satisfy the boundary condition of bar slip at the interface.

Fig. 5.14 Computational procedure of the combination of axial pullout and transverse dowel action

Fig. 5.15 A typical case of the discretised bar embedment length with 10 segments

It should be noted that in order to describe the shear transfer mechanism in a more accurate way, aggregate interlock model proposed by Maekawa and Qureshi (1997) was also unified with the shear transfer by dowel action. Nevertheless, aggregate interlock mechanism does not contribute much to the bar-slip behaviour when analysing a single reinforcing bar at the tensile region in an RC joint because the crack has already propagated throughout the concrete area around the reinforcing bar and there is an interface formed at the crack. Thus, the effect of aggregate interlock mechanism is excluded in the present work.

5.3.1.3 Validations of the proposed bond-slip model

In order to validate the proposed analytical model for the bond stress and slip relationship, the distributions of bond stress, bar stress and bar strain along the steel reinforcement should be examined. Besides, the important relationship between the slip at the loaded end and the applied bar stress should be validated against experimental results, which will directly influence the prediction accuracy of bond-slip behaviour in the RC beam-column joints.

In the present study, in order to validate the prediction accuracy of the proposed analytical model in the axial pullout loading scenario, the experimental studies by Ueda et al. (1986) and Shima et al. (1987) are employed due to their comprehensive descriptions of the test details and well quoted test results.

Firstly, the validation for the bar-slip model is conducted by comparing the computational results with test data from Ueda et al. (1986). Six specimens with different bar sizes, bar yield strengths and concrete strengths are chosen for validation. Specimens S61, S64, S101 and S107 consisted of embedded straight bars, while Specimens B81 and B103 consisted of embedded bent bar. The test details and material properties of steel reinforcement and concrete are listed in **Table 5.2**. Since there is no detailed information available in this series of tests, such as the distributions of bond stress, slip, steel stress and strain along the steel reinforcement, only the relationships between the slip at the loaded end and the applied force were reported. Thus, they are employed to be compared with the predictions by the proposed analytical model.

Table 5.2 Material properties and test details in the test by Ueda et al. (1986)

Specimen name	Concrete strength (MPa)	Bar diameter (mm)	Bar yield strength (MPa)	Bar ultimate strength (MPa)	Elastic modulus (MPa)	Yield plateau
S61	23.79	19.0	438.6	775.9	200000	0.0019
S64	28.76	19.0	438.6	775.9	200000	0.0019
S101	19.93	32.2	414.5	661.4	204138	0.0080
S107	18.21	32.2	331.7	548.3	204138	0.0146
B81	22.62	25.4	469.0	844.8	200000	0.0014
B103	20.55	32.2	414.5	661.4	204138	0.0080

It was reported by Ueda et al. (1986) that the loading on specimens S61, S107 and B103 was reversed at the maximum axial displacement and, therefore, the actual experiment curves should extend further if the loading was applied monotonically until failures occurred. In specimens S64 and B81, there was no failure observed in the experiment. In **Fig. 5.16**, the predictions by the proposed analytical model are compared against the experimental results. The predicted failure mode for all the specimens is pullout failure, which is compatible with the reported fact that no bar fracture failures were observed in the tests. Even though there was the deficiency of applied loading, such as reversion of loading direction and termination of loading before the maximum displacement, the

relationships between the slip at the loaded end and the applied force were measured and reported for all specimens. In the comparisons of the measured relationships between the slip at the loaded end and the applied force, good agreement on the slopes at both the elastic and the plastic ranges of steel reinforcement is attained by the proposed analytical model.

As for the specimen S101, the predicted failure mode is by bar pullout, which is similar with those observed in the actual monotonic loading tests. Nevertheless, the predicted ultimate pullout force is slightly underestimated compared with that in the experimental study. This is because the actual bond condition is slightly better than the empirically assumed bi-uniform bond stress distribution.

(a) S61

(b) S64

(c) S101

(d) S107

Fig. 5.16 Comparisons of numerical and experimental results for the tests by Ueda et al. (1986)

Next, the experimental study conducted by Shima et al. (1987) is employed here to validate the proposed analytical model. Shima et al. (1987) conducted three well designed tests when studying the bond characteristics in post-yield range of steel, that is, specimens SD30, SD50 and SD70. The embedment length of 50 times of the bar diameter is sufficient to provide the boundary condition of zero slip at the unloaded end. To emphasize the effect of steel reinforcement in the post-yield range, three kinds of steel with the same Young's modulus but different yield strengths were used as shown in **Table 5.3**. The stress-strain relationship of the steel bars used in the analysis by Shima et al. (1987) were described by complex equations, which are too complicated to be employed in practice. Instead, a simple bilinear stress-strain relationship is assumed in the present study and the strain-hardening coefficients of steel bars in **Table 5.3** are obtained based on the slope of the stress-strain curves between the yielding initiation and the maximum applied stress (approximately corresponding to 3% strain) as reported by Shima et al. (1987). The compressive strength of concrete f_c is 19.6 MPa for all the three specimens. The steel bars were embedded in concrete with a sufficient cover thickness to avoid splitting cracks.

Table 5.3 Properties of steel bars in the test by Shima et al. (1987)

Specimen name	SD30	SD50	SD70
Bar diameter d_b (mm)	19.5	19.5	19.5
Young's modulus E (GPa)	190	190	190
Yield strength f_y (MPa)	350	610	820
Ultimate stress f_u (MPa)	540	800	910
Strain-hardening coefficient (MPa)	1775.7	3359.5	3166.7

Fig. 5.17 Distributions of bond stress, steel stress, strain and slip along the bar in specimen SD30 (Shima et al. 1987)

Fig. 5.18 Distributions of bond stress, steel stress, strain and slip along the bar in specimen SD50 (Shima et al. 1987)

Fig. 5.19 Distributions of bond stress, steel stress, strain and slip along the bar in specimen SD70 (Shima et al. 1987)

In **Figs. 5.17-5.19**, the distributions of bond stress, slip, steel stress and strain along the bar obtained from the proposed analytical model for all three specimens are compared with the results measured from the experimental study. As shown in **Figs. (a)** of **Figs. 5.17-5.19**, the proposed analytical model is capable of predicting the general trend of the bond stress distribution and the critical point between elastic and plastic ranges of steel reinforcement. Even though the strain variation at the range with yield strength is difficult to be accurately determined, the predictions of steel stress and strain distributions are reasonably acceptable as shown in **Figs. (b)** and **(c)** of **Figs. 5.17-5.19**. In **Figs. (d)** of **Figs. 5.17-5.19**, the predicted slip by the proposed analytical model agrees well with the measured slip, which means that the assumed bilinear bond stress distribution can be considerably accurate in an average sense for both the elastic and the plastic ranges along the steel reinforcement. The relation between the slip at the loaded end and applied bar stress is demonstrated in **Fig. 5.20**, which shows good agreement between the predictions by the proposed analytical model and experimental results. It is found that even though the slip is calculated based on strain integration for each discretised segment along the stress propagation length, the accumulated error is not so significant and, thus, the proposed analytical model is considerably reliable in terms of accuracy. Besides, the predicted failure mode for all the specimens is by fracturing of

rebars as the predicted stress is greater than the ultimate strength of steel reinforcement, which is the same with the experimental results.

Fig. 5.20 Relations between the slip at the loaded end and applied bar stress in the tests by Shima et al. (1987)

It should be clarified that the ultimate tensile strengths of the steel bars, given in the measured stress-strain relationships as listed in **Table 5.3**, are 540 MPa for Specimen SD30, 800 MPa for Specimen SD50 and 910 MPa for Specimen SD70. As shown in **Fig. 5.20**, the experiments had not been conducted until the failure points with ultimate tensile strengths of the steel bars. The reason is that the bars fractured at lower ultimate tensile strengths than that determined from material tests. However, the ultimate tensile strengths are taken as the criterion of steel fracture in the analytical model. Thus, the maximum slips and ultimate applied bar stresses predicted by the proposed analytical model are slightly greater than those obtained from the experimental studies.

In general, the proposed analytical model is capable of predicting the bond-slip behaviour with the failure modes of pullout failure and bar fracturing due to axial pullout action. In the remaining of validation, the bar-slip behaviour under the combined actions of axial pullout and transverse shear will be considered. When validating the proposed analytical model with considerations of the combination of pullout behaviour and dowel action, the two empirical concepts

of the bond deterioration zone and the curvature influence zone will be validated first. The experimental studies conducted by Maekawa and his colleagues (Maekawa and Qureshi 1996a; Soltani et al. 2005; Soltani and Maekawa 2008) are employed herein.

Fig. 5.21 Typical distributions of axial steel stress, strain and curvature along the embedded bar

In order to illustrate the effect of dowel action to the steel reinforcement embedded in concrete, one of the studied specimens (Maekawa and Qureshi 1996a), viz. specimen 4, is analysed with the two empirical concepts of the bond deterioration zone and the curvature influence zone. As shown in **Fig. 5.21**, similar to the distributions shown by Soltani et al. (2005), typical distributions

of axial steel stress, strain and curvature along the embedded bar by the proposed analytical model illustrate that the curvature distribution near the interface does influence the stress and strain distributions and results in localized yielding. On the other hand, the curvature distributions along the bar for different transverse displacements are calculated and compared with the experimental results (Soltani and Maekawa 2008) in **Fig. 5.22**, which shows that the predictions by the analytical model in the present work agree well with the experimental results.

Fig. 5.22 Curvature distributions along the embedded bar with different transverse displacements, validated against experiment results from Soltani and Maekawa (2008)

Finally, a series of tests from Maekawa and Qureshi (1996a) with both axial pullout behaviour and transverse dowel action are employed to validate the proposed analytical model in the presence of both axial pullout and transverse dowel action. The predictions for all the eight specimens are shown in **Fig. 5.23** in terms of the relationship between the slip at the loaded end and the applied bar stress. In general, the analytical model is capable of predicting the coupled actions of axial pullout and transverse shear. As shown in **Fig. 5.23**, the predictions subjected to an axial pullout are denoted as ‘Pullout only’ and the results subjected to coupled axial and transverse actions are denoted as ‘Pullout

and dowel action'. It is apparent that the presence of the transverse dowel action brings about an evident degradation of the pullout resistance due to the localized yielding as previously shown in **Fig. 5.21** and it is important to simultaneously consider axial pullout and transverse shear when modelling the behaviour of embedded steel reinforcement.

It should be noted that there are certain discrepancies for Specimens 3 and 8. The discrepancy for Specimen 3 is due to damage accumulation stemming from the applied cyclic loading even when the specimen was approaching failure in the test. As for Specimen 8, it is evidently found that the ratio of transverse displacement with respect to slip is of the same order with the one for Specimens 3 as reported by Maekawa and Qureshi (1996a), which is much greater than the ratios for the other specimens in the series. Therefore, the proposed analytical model is more suitable for the predictions of bond-slip behaviour subjected to monotonic loading with relatively moderate transverse displacement with respect to axial slip.

It should be noted that there is certain slight discrepancy in predicting Specimen 7. In fact, the gradient of the applied transverse displacement with respect to slip is obviously smaller, especially in the later stage of loading, compared with the other reported specimens. However, the general trend of the relationship between the applied bar stress and the measured slip at the loaded end is similar to those for the other specimens with similar material properties. Therefore, it is believed that the measured transverse displacement should be slightly smaller than the applied transverse displacement. This is the reason that the predicted bar stress based on the measured transverse displacement in Specimen 7 is slightly stiffer.

(a) Specimen 1

(b) Specimen 2

(c) Specimen 3

(d) Specimen 4

(e) Specimen 5

(f) Specimen 6

Fig. 5.23 Relations between the slip at the loaded end and the applied bar stress in the tests by Maekawa and Qureshi (1996a)

5.3.1.4 Summary for the bond-slip model

In the present section, a simple and reliable analytical model based on a bi-uniform bond stress distribution is proposed to predict the relationship between the slip at the loaded end and the applied load in RC joints.

Based on experimental results obtained from the literature, the bi-uniform bond stress distribution is suggested. Due to the insufficient embedment length of steel reinforcement in some cases, different formulations according to the proposed stress propagation length are derived to satisfy the equilibrium and compatibility conditions in the axial pullout loading scenario. Besides axial pullout, transverse dowel action of steel reinforcement at the joint region due to the inclination of the pullout force with respect to the horizontal direction is also incorporated in the proposed analytical model. A computational procedure is proposed schematically to satisfy force equilibrium, local bond-slip relationships, constitutive models of steel and compatibility conditions between steel and concrete.

The proposed bond-slip analytical model is validated against experimental results under loading scenarios of axial pullout with and without transverse dowel action. The validations for axial pullout predictions include not only the comparisons of the relationship between the slip at the loaded end and the

applied load, but also the comparisons of the detailed distributions of bond stress, bar stress and bar strain along the steel reinforcement. At the end, the proposed analytical model is validated against a series tests in the presence of both axial pullout and transverse dowel action. It is shown that the proposed analytical model is considerably reliable in terms of accuracy, even though the slip is calculated based on strain integration for each discretised segment along the stress propagation length.

In conclusion, the proposed simple and reliable analytical model on the bond stress-slip relationship is capable of effectively predicting the bar-slip behaviour under loading scenarios of axial pullout with and without transverse dowel action in the RC beam-column joints.

5.3.2 Shear-panel component

In the previous analytical studies on 2D reinforced concrete (RC) beam-column joint, the modified compression field theory (MCFT) and the strut and tie (SAT) model are usually employed. In **Chapter 2**, the limitations of these analytical models for RC joint applications are reviewed. For predictions of RC joint shear behaviour, essentially the MCFT model is not applicable, whereas the SAT model can only predict the ultimate shear strength. To eliminate these limitations, a new analytical model is derived based on the SAT concept, which is applicable to some commonly encountered 2D joints, viz., interior and exterior joints, subjected to monotonic loading.

The most attracting novelty of the proposed new SAT model is that, it is capable of predicting all the critical stages of the beam-column joint behaviour, including the stages prior to concrete cracking, transverse reinforcement yielding and concrete crushing of shear stress-strain relationships for RC joints. This model satisfies compatibility, equilibrium and constitutive law for both concrete and steel reinforcement. The concrete compression softening phenomenon due to tensile strain and the confinement effect of transverse reinforcement to the concrete core inside the RC joints are taken into account. To validate the model, available experimental studies under monotonic loading in the literature on both

interior and exterior RC beam-column joints are studied. The predicted shear stress-strain relationships are compared with the results from both experimental studies and other widely-used analytical models, such as the MCFT and SAT models. Generally, the agreement is consistently good.

The remaining of this section is organized as follows. In **Section 5.3.2.1**, a new analytical model for predicting the shear stress-strain relationships of 2D beam-column joints subjected to monotonic loading is proposed, which satisfies equilibrium, compatibility and constitutive laws for concrete and steel reinforcement. In **Section 5.3.2.2**, a detailed numerical solution procedure is presented, which is suitable for finite element analysis and has been implemented in a finite element program by the author. Finally, the proposed analytical model is validated in **Section 5.3.2.3** for both interior and exterior 2D RC beam-column joints subjected to monotonic loading. Eight interior joints and nineteen exterior joints are selected from several series of published experimental studies (Taylor 1974; Noguchi and Kurusu 1988; Noguchi and Kashiwazaki 1992) and the predictions based on the proposed analytical model are compared with corresponding experimental results and predicted results by some widely-used analytical models. Clearly, the comparison study shows that the proposed analytical model gives better agreement with all these test results.

5.3.2.1 A new analytical model for shear panels in RC beam-column joints

A new analytical model is proposed to predict not only the ultimate shear strengths but also the complete shear stress-strain responses of RC beam-column joints. The proposed analytical model incorporates average stress and strain fields and load transfer mechanisms to simulate the nonlinear shear deformation behaviour of RC beam-column joints subjected to monotonic loading. In the proposed analytical model, several critical stages have been identified as follows: (a) prior to concrete cracking, (b) prior to stirrup yielding, (c) stirrup has yielded but prior to crushing of concrete strut, and (d) after crushing of concrete strut. Throughout all the stages in the proposed analytical model, equilibrium,

compatibility and constitutive laws for concrete and steel reinforcement are satisfied in terms of average stress and strain criteria.

i. Equilibrium conditions

In the analytical model based on the SAT concept, the effective area of the concrete strut has to be determined before proceeding to subsequent stages.

Fig. 5.24 Effective area of the concrete strut

As shown in **Fig. 5.24**, the width a_s of the diagonal concrete strut can be approximated as

$$a_s = \sqrt{(a_b)^2 + (a_c)^2} \quad (5.25)$$

where a_b and a_c are the depth of the compression zone in the beam and the column cross-sections, respectively. However, due to inevitable concrete crushing at the small beam compression zone, the contribution of a_b to the strut dimension can be neglected. On the other hand, for typical strong-column-and-weak-beam design, the adjacent column of the joint usually does not reach its nominal moment of resistance prior to that of the adjacent beam. Therefore, previous studies (Zhang and Jirsa 1982; Paulay and Priestley 1992) recommended the depth of the compression zone in the adjacent column a_c to be the depth of the flexural compression zone for an elastic column, empirically defined as follows (Hwang and Lee 1999; 2000; Mitra 2007).

$$a_c = \left(0.25 + 0.85 \frac{N}{A_g f_c} \right) h_c \quad (5.26)$$

where N is the applied column axial load, f_c is the concrete cylinder strength, $A_g = b_c h_c$ is the gross cross-sectional area with b_c and h_c as the width and height of the column cross-section, respectively, as shown in **Fig. 5.25**.

Fig. 5.25 Typical dimensions of a 2D beam-column joint

If the width of the concrete strut is taken as the confined thickness b_b (**Fig. 5.25**) inside a beam-column joint, the effective area of the concrete strut is given as

$$A_{strut} = a_c b_b = \left(0.25 + 0.85 \frac{N}{A_g f_c} \right) h_c b_b \quad (5.27)$$

It should be noted that the predictions for the ultimate shear strengths of RC joints in all the proposed SAT models are highly dependent on the dimensions of the concrete struts. According to reported studies (Pantazopoulou and Bonacci 1992; Vollum and Newman 1999; Bakir and Boduroğlu 2002; Park and Mosalam 2012b), the effect of column axial load on the shear strength of RC joints has not been completely understood. In the analytical model by Pantazopoulou and Bonacci (1992), the joint shear strength decreases with increasing column axial load. Vollum and Newman (1999) summarized their known test data and concluded that joint shear strength is reasonably independent of column axial load unless a hinge is formed in the upper column end of the beam-column joint without stirrups. Based on considerable scattered experimental data, Bakir and Boduroğlu (2002) also arrived at a similar conclusion that the column axial load does not influence the joint shear strength of monotonically-loaded exterior beam-column joints. A more balanced conclusion was drawn by Park and Mosalam (2012b) that a high column axial load will actually benefit the joint shear strength for weak-column-and-strong-

beam design. However, for strong-column-and-weak-beam design, the effect of a high column axial load may not be significant. According to the comparison study (Park and Mosalam 2012b), the joint shear strength is not affected by the column axial load up to $0.2A_g f_c$.

In the present analytical model, the effective area of the concrete strut is determined by **Eq. (5.28)**.

$$A_{strut} = \alpha \times a_c b_b \quad (5.28)$$

where the strut area reduction coefficient α is taken as 1.0 in the case of interior joints but $A_{strut} \geq 0.325 h_c b_b$ where the value 0.325 is taken as the average of 0.25 from the references (Zhang and Jirsa 1982; Paulay and Priestley 1992) and 0.40 from reference (Vollum and Newman 1999). The value of reduction coefficient α is assumed to be 0.5 for exterior joints and the value of A_{strut} should be modified as the average of the original A_{strut} (obtained from **Eq. (5.28)**) and $0.325 h_c b_b$, if the original A_{strut} is less than $0.325 h_c b_b$, to reflect the effect of different joint types due to boundary conditions.

Based on the evidence observed in the numerical and experimental studies as reported by other researchers (Bakir and Boduroğlu 2002; Haach et al. 2008), cracks of joint concrete form and propagate along the diagonal direction of the joint region. Therefore, as shown in **Fig. 5.24**, the direction of principle stress can be determined from the joint geometry as

$$\tan \theta = \frac{h_b}{h_c} \quad (5.29)$$

where h_b is the cross-sectional height of adjacent beam and h_c is the cross-sectional height of adjacent column, as shown in **Fig. 5.25**.

Since the joint region is idealized to be subjected to pure shear throughout the loading stage, the vertical joint shear force V_{jv} and horizontal joint shear force V_{jh} can be approximately related (Hwang and Lee 1999; 2000) by

$$\frac{V_{jv}}{V_{jh}} = \frac{h_b}{h_c} = \tan \theta \quad (5.30)$$

where the subscripts h and v indicate the directions of transverse reinforcement and longitudinal column bars, respectively, which will be used in the later derivation. This relationship between horizontal and vertical joint shear forces is kept the same throughout the loading history.

Based on the SAT concept, the load transfer mechanism idealized by Hwang and Lee (1999; 2000; 2002) and Hwang et al. (2000) as shown in **Fig. 2.9 (b)** in **Chapter 2** is adopted in the present analytical model, because this SAT configuration (**Fig. 2.9 (b)**) is the most general in terms of load transfer path and is applicable to both interior and exterior types of beam-column joints. For ease of derivation, the work on the computation of compressive stress of the concrete strut by Hwang and Lee (1999; 2000) is quoted in **Eq. (5.31)** to **Eq. (5.39)**.

The compressive stress of the concrete strut obtained from the load decomposition (Hwang and Lee 1999; 2000) can be written as

$$\sigma_d = \frac{1}{A_{strut}} \left\{ F_d + \frac{\cos \left(\theta - \tan^{-1} \left(\frac{h_b}{2 h_c} \right) \right)}{\cos \left(\tan^{-1} \left(\frac{h_b}{2 h_c} \right) \right)} F_h + \frac{\cos \left(\tan^{-1} \left(\frac{2 h_b}{h_c} \right) - \theta \right)}{\sin \left(\tan^{-1} \left(\frac{2 h_b}{h_c} \right) \right)} F_v \right\} \quad (5.31)$$

where the forces F_d , F_h and F_v are idealized from the diagonal, horizontal and vertical mechanisms, respectively, and can be given as

$$F_d = \frac{1}{\cos \theta} \frac{R_d}{R_d + R_h + R_v} V_{jh} \quad (5.32)$$

$$F_h = \frac{R_h}{R_d + R_h + R_v} V_{jh} \quad (5.33)$$

$$F_v = \frac{1}{\cot \theta} \frac{R_v}{R_d + R_h + R_v} V_{jh} \quad (5.34)$$

The coefficients R_d , R_h and R_v are obtained as follows

$$R_d = \frac{(1 - \gamma_h)(1 - \gamma_v)}{1 - \gamma_h \gamma_v} \quad (5.35)$$

$$R_h = \frac{\gamma_h(1 - \gamma_v)}{1 - \gamma_h \gamma_v} \quad (5.36)$$

$$R_v = \frac{(1 - \gamma_h)\gamma_v}{1 - \gamma_h \gamma_v} \quad (5.37)$$

with the empirical relationships (Jennewein and Schäfer 1992; Schäfer 1996)

$$\gamma_h = \frac{2 \tan \theta - 1}{3} \text{ for } 0 \leq \gamma_h \leq 1 \quad (5.38)$$

$$\gamma_v = \frac{2 \cot \theta - 1}{3} \text{ for } 0 \leq \gamma_v \leq 1 \quad (5.39)$$

Once the yielding of horizontal tie (joint transverse reinforcement) or vertical tie (column longitudinal reinforcement) occurs, the shear resisting mechanism within the RC joint region will be redistributed and the corresponding values of γ_h or γ_v will be assigned as zero in the later stage of analysis.

In addition to the equilibrium of the nodal zone or concrete strut as discussed in **Eq. (5.31)** to **Eq. (5.39)**, equilibrium across the horizontal and vertical cross-sections must be achieved by equilibrating the respective force of steel reinforcement and concrete as shown in **Fig. 5.26**. Similar to the assumptions made by Pantazopoulou and Bonacci (1992) and Bakir and Boduroğlu (2002), in terms of average stress of both steel reinforcement and concrete, equilibrium condition in the respective beam and the column adjacent to the joint can be expressed as

$$f_{cx} = -\frac{F_h}{h_b b_b} \quad (5.40)$$

$$f_{cy} = -\frac{F_v}{h_c b_c} \quad (5.41)$$

where the forces F_h and F_v due to joint transverse reinforcement and column longitudinal reinforcement can be obtained from **Eqs. (5.33)** and **(5.34)**, while f_{cx} and f_{cy} are the average horizontal and vertical stresses of concrete,

respectively. The terms h_b , b_b , h_c and b_c are the dimensions of adjacent beams and columns of a beam-column joint, as shown in **Fig. 5.25**.

Fig. 5.26 Equilibrium across the horizontal and vertical cross-sections

Fig. 5.27 Mohr's circle with the average horizontal and vertical stresses

By using the Mohr's circle in stress (**Fig. 5.27**) with the assumption of continuous stress field, the joint shear stress can be determined as

$$\tau_{cxy} = (f_{c1} - f_{cx}) \tan \theta \quad (5.42)$$

$$\tau_{cxy} = \frac{f_{c1} - f_{cy}}{\tan \theta} \quad (5.43)$$

The principle compressive stress f_{c2} can be given as

$$f_{c2} = f_{c1} - \tau_{cxy} \left(\tan \theta + \frac{1}{\tan \theta} \right) \quad (5.44)$$

Thus, the principle tensile stress f_{c1} can be determined in **Eq. (5.45)** from **Eqs. (5.42)** and **(5.44)**.

$$f_{c1} = \frac{(1 + \tan^2 \theta) f_{cx} - f_{c2}}{\tan^2 \theta} \quad (5.45)$$

From **Eqs. (5.43)** and **(5.44)**, the principle tensile stress f_{c1} can be rewritten as

$$f_{c1} = \left[\left(1 + \frac{1}{\tan^2 \theta} \right) f_{cy} - f_{c2} \right] \tan^2 \theta \quad (5.46)$$

ii. Constitutive law for reinforced concrete

The concrete compressive strain can be calculated with consideration of compression softening effect (Vecchio and Collins 1986; 1993; Zhang and Hsu 1998) and confinement effect due to stirrups in the joint core (Scott et al. 1982; Foster and Gilbert 1996; Tsonos 2007). The Kent and Park model (Park et al. 1972; 1982) is adopted for the stress-strain relationship for confined concrete struts inside the beam-column joints. For the ascending curve prior to attainment of ultimate compressive strength, the compressive stress σ_d is given (Park et al. 1972; 1982) as

$$\sigma_d = f_{d,max} \left[2 \left(\frac{\varepsilon_d}{\varepsilon_0} \right) - \left(\frac{\varepsilon_d}{\varepsilon_0} \right)^2 \right] \quad (5.47)$$

where ε_d is principle compressive strain, $f_{d,max}$ is the modified ultimate compressive strength and ε_0 is the corresponding strain. In addition, $f_{d,max}$ (Vecchio and Collins 1986) and ε_0 (Foster and Gilbert 1996) are given in **Eqs. (5.48)** and **(5.49)**, respectively.

$$f_{d,max} = \frac{f_c}{0.8 - 0.34 \frac{\varepsilon_r}{\varepsilon_0}} \quad (5.48)$$

$$\varepsilon_0 = \begin{cases} -0.002 & (f_c < 20 \text{ MPa}) \\ -0.002 - 0.001 \left(\frac{f_c - 20}{80} \right) & (20 \text{ MPa} \leq f_c \leq 100 \text{ MPa}) \\ -0.003 & (f_c > 100 \text{ MPa}) \end{cases} \quad (5.49)$$

where ε_r is the principle tensile strain.

For the descending portion after the ultimate compressive strength, which will strongly influence ductility of RC beam-column joints, the concrete compressive stress is given (Park et al. 1972; 1982) as

$$\sigma_d = f_{d,max} [1 - Z_m (\varepsilon_d - \varepsilon_0)] \quad (5.50)$$

where the descending gradient Z_m and the ultimate concrete compressive strain ε_u (Scott et al. 1982; Tsonos 2007) are given in **Eqs. (5.51)** and **(5.52)**, respectively.

$$Z_m = \frac{f_c - f_{c,res}}{\varepsilon_0 - \varepsilon_u} \quad (5.51)$$

$$\varepsilon_u = -0.004 - 0.9 \frac{f_{yh}}{300} \quad (5.52)$$

where f_{yh} is the yield strength of transverse reinforcement in MPa and the residual stress $f_{c,res}$ for crushed concrete is taken as $0.2f_c$ (Scott et al. 1982).

On the other hand, the concrete tensile stress (Vecchio and Collins 1986) is empirically given by

$$\sigma_r = \begin{cases} E_c \varepsilon_r & \text{for } \varepsilon_r \leq \varepsilon_{cr} \\ \frac{f_t}{1 + \sqrt{200 \varepsilon_r}} & \text{for } \varepsilon_r > \varepsilon_{cr} \end{cases} \quad (5.53)$$

where E_c is the Young's modulus of concrete and f_t is the concrete tensile strength.

As for steel reinforcement, the stress-strain relationship is assumed to be bi-linear with the stress corresponding to the junction point as the yield strength and the maximum stress as the fracture criterion.

iii. Compatibility conditions

In the first two stages (a) prior to concrete cracking and (b) prior to stirrup yielding, it is reasonable to assume continuous stress and strain fields (Wang et al. 2012) and the joint shear strain can be determined by Mohr's circle. This assumption is similar to the one made in the MCFT model (Vecchio and Collins 1986; 1993) throughout the loading history to attain an arbitrary strain along a certain direction and the joint shear strain. In this study, this assumption holds until the yielding of stirrups or the crushing of concrete struts. As a result, the average horizontal and vertical strain can be given as

$$\varepsilon_h = \frac{\sigma_h}{E_h} = \frac{F_h}{E_h A_h} \quad (5.54)$$

$$\varepsilon_v = \frac{\sigma_v}{E_v} = \frac{F_v}{E_v A_v} \quad (5.55)$$

Fig. 5.28 Mohr's circle with the average horizontal and vertical strains

According to Mohr's circle in strain (**Fig. 5.28**), one obtains

$$\frac{\gamma_{hv}}{2} = \frac{\varepsilon_h - \varepsilon_d}{\tan \theta} = (\varepsilon_r - \varepsilon_h) \tan \theta \quad (5.56)$$

$$\varepsilon_h + \varepsilon_v = \varepsilon_r + \varepsilon_d \quad (5.57)$$

where θ is the direction of the joint diagonal, ε_h is the average horizontal strain, ε_v is the average vertical strain, γ_{hv} is the shear strain at the joint panel, ε_r and ε_d are the principle tensile strain and principle compressive strain along the direction of the joint diagonal, respectively.

After reorganizing the expressions, the tensile strain that lies orthogonal to the diagonal strut in the joint plane can be determined from **Eqs. (5.58)** and **(5.59)**.

$$\varepsilon_r = \varepsilon_h + (\varepsilon_h - \varepsilon_d) \cot^2 \theta \quad (5.58)$$

$$\varepsilon_r = \varepsilon_v + (\varepsilon_v - \varepsilon_d) \tan^2 \theta \quad (5.59)$$

In order to obtain a conservative estimate in the critical stage (c) prior to the crushing of concrete strut with the yielding of transverse reinforcement, the contribution due to transverse reinforcement hardening is neglected and the constitutive relationship of transverse reinforcement is assumed to be elasto-perfectly-plastic. Therefore, the average horizontal strain of the joint stirrup after yielding cannot be accurately calculated based on the stress-strain relationship and has to be determined empirically.

Similar difficulties were encountered by Altoontash (2004) when analysing the beam-column joints without transverse reinforcement. To solve the problem, 45% of the beam or column longitudinal reinforcement at the joint perimeter was taken by Altoontash (2004) as the effective transverse reinforcement, based on a limited calibration with seven specimens to best fit the measured joint shear strength.

In the present analytical model, based on the participation distributions of the transverse reinforcement or the longitudinal column bars from reference (Hwang

and Lee 2000), it is assumed that ties in both horizontal and vertical mechanisms do not fully yield and the remaining elastic proportion of ties will contribute to the post-yielding resistance and, thus, mobilize further shear deformation. As shown in **Fig. 5.29**, the areas $0.5 A_h$ and $0.5 A_v$ in the horizontal and vertical mechanisms are assumed to fully participate in the shear resistance prior to the occurrence of yielding of ties and, therefore, the remaining elastic portion in terms of both area and strength will contribute to the post-yielding shear resistance. Based on average stress and strain, the equivalent hardening modulus H_s after yield strength can be obtained according to **Fig. 5.29** by formulating

$$H_{sh} = \frac{(0.25A_h \times 2) \times E_s(1 - 50\%)}{A_h} \quad (5.60)$$

$$H_{sv} = \frac{(0.25A_v \times 2) \times E_s(1 - 50\%)}{A_v}$$

Thus, $H_s = 0.25 E_s$ and the equivalent stress-strain relationship at the cross-sectional level can be shown in the **Fig. 5.30** based on full A_h and A_v .

Fig. 5.29 Participation distribution of transverse reinforcement and intermediate column bars

Fig. 5.30 Equivalent average stress-strain relationship of steel at the cross-sectional level

Nevertheless, it should be noted that the occurrence of stirrup yielding is not inevitable, because in some cases, the stirrups do not yield and consequently, there will be no stage (c) at all in the shear deformation history. For instance, for joint specimens with sufficient transverse reinforcement, the deformation of the joint is directly controlled by the crushing of concrete struts as indicated in stage (d). If the criterion $\sigma_d > f_{d,max}$ is satisfied, then the ultimate shear strength can be captured and in the last stage, the evolution of compressive strain of concrete strut takes over in stage (d). The empirical expressions for average horizontal and vertical strains can be written as

$$\varepsilon_h = \frac{\frac{F_h}{A_h} - f_{yh}}{H_{sh}} + \frac{f_{yh}}{E_{sh}} \quad (5.61)$$

$$\varepsilon_v = \frac{\frac{F_v}{A_v} - f_{yv}}{H_{sv}} + \frac{f_{yv}}{E_{sv}} \quad (5.62)$$

where f_y is the yield strength, E_s is the Young's modulus, H_s is the hardening modulus and A is the cross-sectional area of the steel reinforcement. The subscripts h and v indicate the horizontal and vertical directions, respectively.

5.3.2.2 Solution procedure

The aim of the assumptions and the empirical formulae introduced above is to build average stress and strain fields and load transfer mechanisms with satisfying equilibrium, compatibility and constitutive laws for concrete and steel

reinforcement throughout all the critical stages in the shear panels of RC beam-column joints subjected to monotonic loading. To demonstrate how these assumptions and formulae work together, a numerical solution procedure is given as follows.

The solution procedure of the proposed joint analytical model, which has been successfully implemented into a finite element program FEMFAN3D (Long et al. 2012a), is presented here. The solution procedure is separated into 2 parts as shown in **Fig. 5.31 (a)** and **(b)**. The first part describes an equilibrium analysis based on the load transfer path of the SAT model with consideration of yielding of transverse reinforcement and longitudinal column bars. The second part is the average stress and strain analysis based on respective concrete and steel reinforcement constitutive laws and compatibility conditions with consideration of concrete compression softening effect and confinement effect due to transverse reinforcement.

In **Fig. 5.31 (a)** and **(b)**, several indicators are employed to represent the different stages of RC beam-column joints. The “*Type*” indicator is an integer with 0 for stage (a) prior to concrete cracking, 1 for stage (c) with transverse reinforcement yielding and prior to crushing of concrete strut, and 2 for stage (c) with longitudinal column bars yielding and prior to crushing of concrete strut. The “*Sign*” indicator denotes the shear loading direction and a value of 1 represents an increase of the applied shear load prior to concrete strut crushing, while a value of -1 indicates a decrease in the applied shear load in stage (d) after crushing of strut. The “*iLow*” is an indicator to differentiate between different cases of sufficient and insufficient beam longitudinal reinforcement, since the former will enhance the confinement effect of concrete struts and, therefore, weaken the compression softening effect due to existence of tensile strain orthogonal to the predetermined joint region crack.

The crushing criterion of the concrete strut is determined from the condition of $\sigma_d > f_{d,max}$. Once the error term defined by $\|(f_{d,max} - \sigma_d)/f_{d,max}\|$ is less than a given tolerance *Tol*. (which is assigned as 10^{-5} in the present study), the ultimate shear strength is calculated and the compressive strain of concrete strut

becomes the dominant criterion for the joint shear strain in stage (d) after crushing of the concrete strut.

(a) Part 1 of solution procedure: Equilibrium analysis

(b) Part 2 of solution procedure: Average stress and strain analysis

Fig. 5.31 Numerical solution procedure of shear-panel analytical model

5.3.2.3 Validations of the proposed shear-panel model

To validate the proposed analytical model for different types of 2D RC joints subjected to monotonic loading, two series of interior and exterior joints are selected from several series of available experimental studies. The predictions based on the proposed analytical model are compared with corresponding experimental results and other published analytical models (the MCFT model (Vecchio and Collins 1986) and the SAT model (Hwang and Lee 1999; 2000)). In the present study, the implemented original MCFT model (Vecchio and Collins 1986) has been verified against the experimental results on RC shear panels (Vecchio and Collins 1986; Maekawa 2003) as shown in **Fig. 5.32**, while the implemented SAT model has been verified by comparisons with the published shear strength predictions (Hwang and Lee 1999; 2000) on RC beam-column joints (Megget 1974; Lee et al. 1977; Alameddine 1990; Kaku and Asakusa 1991) as shown in **Table 5.4**. Clearly, the implemented MCFT model gives good predictions of RC shear panels with uniform transverse and longitudinal reinforcement, whereas the implemented SAT model in **Table 5.4** gives acceptable results compared with the original model (Hwang and Lee 1999; 2000). Thus, the credibility of MCFT and SAT models as programmed by the author are very reliable. These two models will be used in the following studies.

Fig. 5.32 Verifications of the implemented MCFT model against test results
(Vecchio and Collins 1986; Maekawa 2003)

Table 5.4 Verifications of the implemented SAT model

Specimen	SAT model (Hwang and Lee 1999; 2000) (kN)	Implemented SAT model (kN)
Unit A (Megget 1974)	419	420
6 (Lee et al. 1977)	155	155
LL8 (Alameddine 1990)	724	722
HH11 (Alameddine 1990)	937	937
2 (Kaku and Asakusa 1991)	300	300
4 (Kaku and Asakusa 1991)	349	347
6 (Kaku and Asakusa 1991)	210	209
14 (Kaku and Asakusa 1991)	261	262
15 (Kaku and Asakusa 1991)	233	234

i. Interior joints

There are fairly limited numbers of publications on interior RC beam-column joint tests subjected to monotonic loading. The experimental studies by Noguchi (1988; 1992) are chosen to validate the applications of the proposed analytical model for 2D RC interior joints. The dimensions and reinforcement details of the specimens are shown in **Fig. 5.33**. The material properties of concrete and

Table 5.5 Concrete properties of the series of interior joints

Specimen	OKJ-6	OKJ-1, 2, 4, 5	OKJ-3	No.2, 4
Cylinder strength (MPa)	53.5	70.0	107.0	70.6
Young's modulus (MPa)	28500	35100	40300	35100
Poisson's ratio	0.2	0.2	0.2	0.2
Compressive strain corresponding to compressive strength	0.00223	0.00296	0.00286	0.00296

Table 5.6 Steel reinforcement properties of the series of interior joints

Bar size	D6	D13
Young's modulus	186 GPa	182 GPa
Poisson's ratio	0.3	0.3
Yield strength	718 MPa	955 MPa
Maximum stress	767 MPa	1140 MPa

Fig. 5.34 Experimental result comparison of interior joints OKJ-1 and OKJ-2
(Noguchi and Kashiwazaki 1992)

It was reported that shear failure with or without yielding of beam longitudinal reinforcement in the joint panel was observed in all the specimens. Even though most of the specimens in these series of tests were conducted under cyclic loading, one of the specimens was tested under both cyclic and monotonic loading, indicated as OKJ-1 and OKJ-2, respectively. With the backbone curves

obtained from the published test results, a comparison of experimental load-deformation relationships of these two specimens is given in **Fig. 5.34**.

It is obvious that the moderate effect of loading reversal does not affect the deformation behaviour until at a much later stage after attaining the peak strength. Therefore, the experimental data from these series of tests can be adopted to validate the proposed analytical model under monotonic loading. When validating the proposed analytical model, the MCFT model predictions are also compared with the experimental results as shown in **Figs. 5.35 (a)-(g)**. Both the MCFT and SAT models in the present study have been validated against the RC shear panels (Vecchio and Collins 1986) and the published predictions (Hwang and Lee 1999; 2000). Therefore, it is safely concluded that the MCFT predictions are too conservative for ductility and tend to overestimate the ultimate shear strengths of interior RC beam-column joints, while the SAT model predictions (Hwang and Lee 1999; 2000) underestimate the ultimate shear strengths for OKJ series (**Figs. 5.35 (a)-(e)**) and the performance is satisfactory for specimens No. 2 and No. 4 (**Figs. 5.35 (f)-(g)**). Thus, the proposed analytical model, by contrast, is capable of reasonably predicting both the ductility and the ultimate shear strengths of interior RC beam-column joints.

It is noteworthy that since there was sufficient confinement from transverse reinforcement and longitudinal column bars, no yielding of confining reinforcement occurred and crushing of concrete struts constituted the main shear resisting mechanism. For the descending part of the curve, the post-peak concrete behaviour is fairly accurately reflected, which is governed by the ratio and yield strength of transverse reinforcement, and maximum concrete compressive strain (Scott et al. 1982).

(a) OKJ-1 and OKJ-2

(b) OKJ-3

(c) OKJ-4

(d) OKJ-5

(e) OKJ-6

(f) No.2

(g) No.4

Fig. 5.35 Comparison of shear stress and strain relationships of RC interior beam-column joints (Noguchi and Kurusu 1988; Noguchi and Kashiwazaki 1992)

ii. Exterior joints

A series of 2D RC exterior joints subjected to monotonic loading were tested by Taylor (1974) in 1970s with variations in beam steel reinforcement, column axial load, beam thrust, concrete strength and beam depth. The dimensions and steel reinforcement details are illustrated in **Figs. 5.36 (a) and (b)**, respectively. Concrete cover to the main steel is 22mm. The material properties of concrete and steel reinforcement are given in **Tables 5.7 and 5.8**, respectively. The elastic modulus of concrete is determined by the empirical formula proposed by Pang and Hsu (1996).

(a) Dimensions of the exterior joints

(b) Steel reinforcement details of the exterior joints

Fig. 5.36 Dimensions and reinforcement details of the exterior joints

Table 5.7 Concrete properties of the series of exterior joints

Cylinder strength (MPa)	f_c
Young's modulus (MPa)	$3900\sqrt{f_c}$ in MPa (Pang and Hsu 1996)
Poisson's ratio	0.2

Table 5.8 Steel reinforcement properties of the series of exterior joints

	Series P	Series A to F
Young's modulus	200 GPa	200 GPa
Poisson's ratio	0.3	0.3
Yield strength	410 MPa	460 MPa
Maximum stress	515 MPa	578 MPa

As shown in **Figs. 5.37-5.41**, the predictions by the proposed analytical model on the load-deformation relationships of exterior beam-column joints in all series (P, A, D, E and F) are satisfactory in terms of ductility and ultimate shear strength compared with the predictions by the MCFT and SAT models. Markedly different from the other specimens, there is no yielding of transverse reinforcement in specimen D3/41/06 as shown in **Figs. 5.39 (d)** and the deformation of the joint is directly controlled by the crushing of concrete strut as indicated in stage (d) because of the low concrete cylinder strength.

Similar to the conclusions for interior joint, the MCFT predictions for exterior RC beam-column joints are generally too conservative for ductility. As shown in **Figs. 5.39-5.40** for D and E series, the terminations of the MCFT predictions result from shear failure of RC joints and there is a descending stage in the shear stress-strain response, which, however, is not so significant in **Figs. 5.39-5.40** due to the small ductility. Since the convergence in the post-peak stage is difficult to attain for MCFT, there is no descending stage after the shear capacity for several specimens in F series as shown in **Figs. 5.41**. Nevertheless, it should be noted that the predictions by the MCFT model with only transverse reinforcement is far from the experimental results. Thus, 45% of the beam

longitudinal reinforcement is assumed to contribute to the joint confinement as proposed by Altoontash (2004).

Based on the validations above, the proposed analytical model is generally capable of predicting the critical stages (including the stages prior to concrete cracking, transverse reinforcement yielding and concrete strut crushing) of shear panels in interior and exterior RC beam-column joints. In addition, the shear stress-strain relationships with consideration of concrete compression softening phenomenon and transverse confinement effect can be obtained.

Fig. 5.37 Comparison of shear stress and strain relationships of P series exterior joints (Taylor 1974)

Fig. 5.38 Comparison of shear stress and strain relationships of A series exterior joints (Taylor 1974)

Fig. 5.39 Comparison of shear stress and strain relationships of D series exterior joints (Taylor 1974)

Fig. 5.40 Comparison of shear stress and strain relationships of E series exterior joints (Taylor 1974)

Fig. 5.41 Comparison of shear stress and strain relationships of F series exterior joints (Taylor 1974)

5.3.2.4 Summary for the shear-panel model

Based on the strut-and-tie concept, a new analytical model for 2D reinforced concrete (RC) beam-column joint is proposed and is applicable to interior and exterior types of 2D joints subjected to monotonic loading. The proposed analytical model satisfies the compatibility, equilibrium and constitutive laws for both concrete and steel reinforcement.

The most appealing advantage of the proposed analytical model is its capability of predicting all the critical stages including the stages prior to concrete cracking, transverse reinforcement yielding and concrete strut crushing. The approach also provides shear stress-strain relationships with consideration of concrete compression softening phenomenon due to tensile strain and confinement effect of transverse reinforcement to the concrete core inside the RC joints. According to previous theoretical and experimental studies, several important parameters are taken into account in the proposed RC joint analytical model, such as the joint aspect ratio, joint stirrup details, column reinforcement ratio, beam longitudinal reinforcement ratio, concrete cylinder strength, and column axial stress.

With validations against experimental studies and other available analytical models (the MCFT and SAT models) considering the variations in beam and column longitudinal steel reinforcement, transverse reinforcement, column axial load, concrete strength and joint aspect ratio, the proposed analytical model is capable of providing stable and reliable predictions on the shear stress-strain relationships of 2D RC interior and exterior beam-column joints subjected to monotonic shear loading.

5.3.3 Interface-shear component

The envelope of the relationship of lateral load and shear displacement as shown in **Fig. 5.42** is employed to approximately calibrate the interfacial-shear component in the 2D component-based RC beam-column joints. To determine the critical point A (concrete cracking), B (maximum shear strength) and C

(ultimate shear deformation) in the envelope, two of the shear displacement, the lateral load and the corresponding slope are necessary. Empirical formulae based on an extensive collection of shear specimens (Patwardhan 2005) are employed.

The calculation of critical values in the **Fig. 5.42** is in **Table 5.9**. The used symbols in **Table 5.9** are given in **Fig. 5.42** and under the Section for “**List of Symbols**”. It is noteworthy that the unit conversion from British System of Units to International System of Units should be conducted in the finite element implementation.

Fig. 5.42 The relationship of lateral load and shear displacement

Table 5.9 Critical values in the relationship of lateral load and shear displacement
(Patwardhan 2005) (Unit: kips, ksi, in)

	Shear displacement	Lateral load	Slope
Point A	$\Delta_{cr} = \frac{N}{50000} + 0.0062$		$K_{shear} = GA / L$
Point B	$\Delta_n = \frac{1}{25000} \frac{a/d f_{yh} \rho_v}{\sqrt{A_g f_c}} - 0.0011$	$V_n = V_c + V_s$ $V_c = 2(1 + \frac{N}{2000A_g})\sqrt{f_c}bd$ $V_s = \frac{A_v f_{yh} d}{s}$	
Point C	$\Delta_u = (4 - 12 \frac{V_n}{f_c})\Delta_n$	V_n	

5.4 Joint Resistance-Deformation States for Beam-Column Joints

Due to local material failures, such as concrete crushing and steel reinforcement fracturing, in RC beam/column members, the position of the neutral axis at the cross-section will be changed to mobilize an equilibrium state, therefore, the original tension zone may be converted to the compression zone, and vice versa. In the perspective of the whole simulated structure, the internal forces will be redistributed and the components of the joint model proposed in this chapter will also be subjected to unloading and reloading scenarios at the joint region connected to beam/column members.

There are 12 possible resistance-deformation states for each component to take into account the loading, unloading and reloading scenarios as shown in **Fig. 5.43**. The scenarios 1, 2 and 3 are for the loading scenarios under tension, while the scenarios 7, 8 and 9 are for loading scenarios under compression. The scenarios 4, 5 and 6 are for the unloading and reloading scenarios from the tension part, while scenarios 10, 11 and 12 are for the unloading and reloading scenarios from the compression part. The initiation points for the unloading and reloading scenarios are denoted by (d^r, f^r) , while the destination points for the unloading and reloading scenarios are denoted by (d_{min}, f_{min}) and (d_{max}, f_{max}) for compression and tension, respectively. To depict the unloading and reloading paths which significantly influences the numerical stability of the proposed joint model especially when simulating large-scale structures, the parameters $rDispP$, $rforceP$, $uforceP$, $rforceN$, $uforceN$ and $uforceN$ are utilized and the default values for these parameters are referred to the OpenSees manual (Mazzoni et al. 2009).

All necessary parameters in **Fig. 5.43** to define the unloading and reloading scenarios are explained in **Table 5.10**. The possible loading routes ($\Delta d > 0$) and unloading routes ($\Delta d < 0$) are given in **Tables 5.11** and **5.12**, respectively, where Δd is the increment of component deformation at the current iteration step.

Fig. 5.43 Loading, unloading and reloading scenarios for each component in the joint model

Table 5.10 The physical meanings of all the associated parameters to define the unloading and reloading scenarios in the component-based joint model

Parameter	Physical meaning
f^r	The component force when the unloading occurs
d^r	The corresponding deformation when the unloading occurs
f_c	Compressive yield force of the component
d_c	The corresponding deformation at the compressive yield strength of the component
f_{cu}	Ultimate compressive force of the component
d_{cu}	The corresponding deformation at the ultimate compressive strength of the component
f_t	Tensile yield force of the component
d_t	The corresponding deformation at the tensile yield strength of the component
f_{tu}	Ultimate tensile force of the component
d_{tu}	The corresponding deformation at the ultimate tensile strength of the component
k_c	Initial elastic compressive stiffness of the component
k_t	Initial elastic tensile stiffness of the component
h_c	Compressive hardening parameter of the component
h_t	Tensile hardening parameter of the component
d_{min}	Minimum historic deformation demand and equal to d_c in the initial loading stage
f_{min}	The force corresponding to the minimum historic deformation demand and equal to f_c in the initial loading stage
d_{max}	Maximum historic deformation demand and equal to d_t in the initial loading stage
f_{max}	The force corresponding to the maximum historic deformation demand and equal to f_t in the initial loading stage
$rDispP$	Floating point value defining the ratio of the deformation at which

	reloading occurs to the maximum historic deformation demand (default: 0.5)
$rforceP$	Floating point value defining the ratio of the force at which reloading begins to force corresponding to the maximum historic deformation demand (default: 0.25)
$uforceP$	Floating point value defining the ratio of strength developed upon unloading from negative load to the maximum strength developed under monotonic loading (default: 0.05)
$rDispN$	Floating point value defining the ratio of the deformation at which reloading occurs to the minimum historic deformation demand (optional, default: $rDispP$)
$rforceN$	Floating point value defining the ratio of the force at which reloading begins to the force corresponding to the minimum historic deformation demand (optional, default: $rforceP$)
$uforceN$	Floating point value defining the ratio of the strength developed upon unloading from a positive load to the minimum strength developed under monotonic loading (optional, default: $uforceP$)

Table 5.11 All possible loading routes for a component

Starting state	Potential loading routes	Important items
1	$1 \rightarrow 2 \rightarrow 3$	
2	$2 \rightarrow 3$	Update d_{max} and f_{max}
3	3	Irreversible
4	$4 \rightarrow 2 \rightarrow 3$	Use d^r , f^r
5	$5 \rightarrow 4 \rightarrow 2 \rightarrow 3$	Use d^r , f^r , d_{min} and f_{min}
6	$6 \rightarrow 5 \rightarrow 4 \rightarrow 2 \rightarrow 3$	Use d^r , f^r , d_{min} and f_{min}
7	$7 \rightarrow 1 \rightarrow 2 \rightarrow 3$	
8	$8 \rightarrow 10 \rightarrow 11 \rightarrow 12 \rightarrow 2 \rightarrow 3$	
9	9	Irreversible

10	$10 \rightarrow 11 \rightarrow 12 \rightarrow 2 \rightarrow 3$	Use d^r and f^r
11	$11 \rightarrow 12 \rightarrow 2 \rightarrow 3$	Use d^r , f^r , $d_{\max}$ and $f_{\max}$
12	$12 \rightarrow 2 \rightarrow 3$	Use d^r , f^r , $d_{\max}$ and $f_{\max}$

Table 5.12 All possible unloading routes for a component

Starting state	Potential loading routes	Important items
1	$1 \rightarrow 7 \rightarrow 8 \rightarrow 9$	
2	$2 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 8 \rightarrow 9$	
3	3	Irreversible
4	$4 \rightarrow 5 \rightarrow 6 \rightarrow 8 \rightarrow 9$	Use d^r and f^r
5	$5 \rightarrow 6 \rightarrow 8 \rightarrow 9$	Use d^r , f^r , $d_{\min}$ and $f_{\min}$
6	$6 \rightarrow 8 \rightarrow 9$	Use d^r , f^r , $d_{\min}$ and $f_{\min}$
7	$7 \rightarrow 8 \rightarrow 9$	
8	$8 \rightarrow 9$	Update $d_{\min}$ and $f_{\min}$
9	9	Irreversible
10	$10 \rightarrow 8 \rightarrow 9$	Use d^r and f^r
11	$11 \rightarrow 10 \rightarrow 8 \rightarrow 9$	Use d^r , f^r , $d_{\max}$ and $f_{\max}$
12	$12 \rightarrow 11 \rightarrow 10 \rightarrow 8 \rightarrow 9$	Use d^r , f^r , $d_{\max}$ and $f_{\max}$

5.5 Closure

Based on the concept of component method, a beam-column joint model consisting of bar-slip component, interface-shear component and shear-panel component is studied and implemented in the RC joint finite element analysis. The derivation of joint element based on the geometric relationship of nodal displacements and component deformations is presented and internal force vector and stiffness matrix are obtained. Calibrations for different components

in the beam-column joint model are of vital importance for the numerical stability and computation accuracy of the joint simulation.

With the proposed concept of stress propagation length, different formulations are derived to satisfy both the equilibrium and compatibility conditions in the axial pullout loading scenario. The insufficient embedment length of steel reinforcement is also taken into account. Besides axial pullout, transverse dowel action of steel reinforcement at the joint region due to the inclination of the pullout force with respect to the undeformed beam direction is also incorporated in the proposed analytical model. Based on the validations against experimental results under loading scenarios of axial pullout with and without transverse dowel action, the proposed simple and reliable analytical model is capable of effectively predicting the bar-slip behaviour under loading scenarios of axial pullout with and without transverse dowel action in the RC beam-column joints.

A new analytical model for 2D reinforced concrete (RC) beam-column joint is proposed and applicable to different types of 2D joints subjected to monotonic loading. The proposed analytical model satisfies the compatibility, equilibrium and constitutive laws for both concrete and steel reinforcement. The most appealing advantage of the proposed analytical model is the capability of predicting all the critical stages and providing complete shear stress-strain relationships. An extensive collection of important parameters are taken into account in the proposed RC joint analytical model. With a comprehensive validation against experimental studies and other available analytical models (the MCFT and SAT models), the proposed analytical model is capable of providing stable and reliable predictions on the shear stress-strain relationships of 2D RC interior and exterior beam-column joints subjected to monotonic shear loading.

Considering the scenarios of loading, unloading and reloading for the joint element in the finite element analysis, different resistance-deformation states for each type of components are considered and all possible routes of loading and unloading between different resistance-deformation states are described in detail.

Chapter 6 Study at the System Level of RC Beam-Column Framed Structures with 2D Component-Based Joints

6.1 Introduction

In this chapter, reinforced concrete (RC) beam-column framed structures with consideration of 2D component-based joints are studied at the system level. Firstly, several beam-column subassemblages, including knee joints, exterior joints and interior joints, are employed in **Section 6.2** to validate the proposed co-rotational (CR) beam element formulation (**Chapter 4**) and the calibrated 2D component-based joint model (**Chapter 5**). Based on the comparisons in terms of prediction accuracy, numerical stability and computational time, the advantages and disadvantages of three different concrete models presented in **Chapter 3** are discussed herein.

In **Section 6.3**, under the scenario of a middle-column removal, a three-storey 2D framed structure is simulated using the proposed CR beam element formulation and 2D component-based joint model. With comparisons against the experimental results, the accuracy and reliability of the proposed numerical approach is validated. Moreover, in order to demonstrate the numerical robustness to predict the deformation behaviour of full-scale framed structures for progressive collapse analysis, a three-storey and two-bay framed structure and a five-storey and four-bay framed building are simulated with different column-removal scenarios and the prediction results are discussed in **Section 6.4**. Lastly, a brief conclusion is drawn in **Section 6.5** for the usage of the proposed CR beam element formulation and the 2D component-based joint model in practice.

In this chapter, both “joint model” and “joint element” will be used. The former refers to the proposed component-based concept in **Chapter 5**, while the latter refers to the implemented substance in finite element models.

It should be noted that all the simulations are conducted on the same computer with 2.66 GHz processor and 3.25 GB RAM. Furthermore, the calibration of the

various components of the 2D joint model is conducted before the execution of finite element analysis. For the bar-slip component in the 2D joint model, if dowel action is considered, transverse shear deformation has to be independently determined. While, this can be measured for an isolated structural member, in the analyses of beam-column subassemblages or full-scale frames, it is impractical to estimate the transverse deformation for each reinforcing bar. Therefore, only the pullout axial action is considered in the present study at the system level of RC beam-column framed structures with 2D component-based joints. This is fairly reasonable when dowel action is not so significant. For structures in which catenary action is mobilized, the predictions based on the proposed numerical model may be slightly larger than the actual local pullout resistance as discussed in **Section 5.3.1.3**. Nevertheless, it is found that the predictions of the whole structures are still acceptably accurate as shown later in this chapter.

6.2 Beam-Column Subassemblages

In general, the most commonly encountered 2D joints are the knee joints, exterior joints and interior joints. Furthermore, the classification of joints types not only depends on the joint configurations but also on the column-removal scenarios. Three types of 2D joints are illustrated in **Fig. 6.1** with the knee joint denoted by **K**, exterior joint by **E** and interior joint by **I**.

As shown in **Fig. 6.1**, knee joints are commonly located at the topmost storey of framed structures. However, an exterior joint right above the removed column also becomes a knee joint in a column removal scenario as shown in **Fig. 6.1**. When a column is removed, internal forces will be redistributed throughout the whole structure and the upper knee joints will be subjected to monotonic loading resulting in further deformation for the adjacent beams, columns and joints.

Fig. 6.1 Different types of 2D joints in a framed building under column removal scenario

6.2.1 Knee joint

Limited experimental studies on knee joints were published with specified dimensions, material properties and load-displacement response graphs under monotonic loading scenario. In the present study, one specimen experimentally studied by Peng and Wang (2010) is employed here to validate the CR beam element formulation and the 2D joint model proposed in this study. Since only one knee joint is found and investigated in the present study, the objective is to show the potential by using the proposed joint model to predict the knee joint response. Nevertheless, it should be noted that the capability of the joint model should be further validated against more experimental studies on knee joints in future. The dimensions and reinforcement detailing of the knee joint are shown in **Fig. 6.2**. The bottom end of the column is simply supported and a concentrated load is applied along the horizontal direction at the beam end.

Firstly, the knee joint is simulated by the proposed CR beam elements and the uniaxial Kent and Park concrete model with and without a joint element. As shown in **Fig. 6.3**, there are differences for load-displacement responses based on the numerical models with and without a joint element. The simulation with a joint element can represent well the joint behaviour in terms of both ascending and descending stages, while the simulation without a joint element overestimates the load capacity of this particular knee joint. As a matter of fact, there is a certain discrepancy between the simulations and the experimental study, which may be due to the reported installation error at the simply

supported boundary. As reported by Peng and Wang (2010), a larger stiffness was expected if the supported boundary were perfect. This imperfect boundary condition can also be confirmed by the 3D solid element simulation as discussed later.

Fig. 6.2 Dimension, boundary condition and reinforcement details of the knee joint

Fig. 6.3 Applied load-displacement response for a knee joint with and without a joint element, compared with test results (Peng and Wang 2010)

In addition to uniaxial concrete model, the same numerical model with CR beam elements is also analysed with the proposed unified plasticity concrete model in **Chapter 3**. As shown in **Fig. 6.4**, the load-displacement responses are compared for different concrete models and the effect of the presence of a joint element is

also addressed. For simplicity, the acronym “KP” in this chapter denotes the “Kent and Park” concrete model, while “UP” denotes the “unified plasticity” concrete model.

Fig. 6.4 Applied load-displacement response for a knee joint with different concrete models, compared with test results (Peng and Wang 2010)

Fig. 6.5 Computational time of the knee joint simulations

Compared with KP concrete model, the proposed UP model without a joint element gives a more accurate prediction on the structural load capacity as shown in **Fig. 6.4**. However, the joint strength deterioration can only be captured when a joint element is introduced. It is interesting to see that for numerical simulations without a joint element, the predictions with different

concrete models yield different results; while for numerical simulations with joint elements, the predictions with different concrete models are almost the same. The reason is that, if there is no joint element to connect beams and columns, then there will be concentrated material nonlinearity adjacent to the beam-column connecting node. Compared with KP concrete model, UP model is able to give more accurate predictions. Nevertheless, with joint elements, the joint deformation behaviour is more accurately captured by the proposed joint model and consequently, the material nonlinearity at the adjacent beam/column interface is not so severe. Thus, based on the numerical models with joint elements, the simulation results with both concrete models are almost the same. Moreover, as concentrated material nonlinearity at the joint region is greatly alleviated due to joint elements, numerical simulations with joint elements are more computationally efficient, which is reflected in terms of the required time to complete an analysis as shown in **Fig. 6.5**.

It should be noted that the applied load-displacement response was reported (Peng and Wang 2010) but the shear force-deformation response of the shear panel is not available. In the present study, only the shear force-deformation response of the shear panel is of interest so as to validate the 2D joint shear panels. Thus, a numerical model by using solid elements in Abaqus (2009) is also analysed and the employed concrete and steel properties in Abaqus are listed in **Tables 6.1** and **6.2**, respectively. Consequently, good agreement is obtained for the applied load-displacement response compared with that from the experimental study (Peng and Wang 2010) as shown in **Fig. 6.3**. Based on the same structural response, it is fairly reasonable to employ the Abaqus results to obtain the shear force-deformation response of the shear panel in this knee joint to validate the proposed analytical model on shear panels. Nevertheless, it is worth noting that there are several parameters in Abaqus which have not been well calibrated for RC joints. Therefore, a parametric study is conducted and compared with several series of joint tests to calibrate these material property parameters and as a result, the obtained concrete and steel properties for RC joints in Abaqus are listed in **Tables 6.1** and **6.2**.

As given in the Abaqus manual (2009), the parameters in the 4th to 7th rows in **Table 6.1** are explained briefly as follows. Dilation angle and eccentricity are used to describe the shape of the potential function; f_{bo}/f_c is the ratio of the biaxial compressive strength and uniaxial compressive strength; K_c is the ratio of the second stress invariant on the tensile meridian to that on the compressive meridian. The viscosity parameter in the last row of **Table 6.1** represents the relaxation time of the viscoplastic system.

Table 6.1 Concrete properties of the knee joint

Cylinder strength	26.2 MPa
Young's modulus	30000 MPa
Poisson's ratio	0.2
Dilation angle	40°
Eccentricity	0.1
f_{bo}/f_c	1.16
K_c	0.6667
Viscosity parameter	0.005

Table 6.2 Steel reinforcement properties of the knee joint

	Rebar	Stirrup
Young's modulus	200 GPa	200 GPa
Poisson's ratio	0.3	0.3
Yield strength	360 MPa	331 MPa
Maximum stress	400 MPa	365 MPa

The predicted shear force-deformation response of the shear panel in the knee joint by the proposed analytical model is shown in **Fig. 6.6**. Compared with 3D solid element simulations (the simulation is terminated due to large plastic strain and the corresponding numerical stability), it is evident that better predictions are obtained by the proposed analytical model rather than MCFT (Vecchio and Collins 1986) or SAT model (Hwang and Lee 1999; 2000). Thus, the proposed analytical model is capable of predicting well the shear response of the shear panel in this knee joint.

Fig. 6.6 Comparison of shear stress and strain relationships of the knee joint

6.2.2 Exterior joint

Two exterior RC beam-column joints, namely, Specimens NS03 and LS03, from the experimental study by Yap and Li (2011) are employed here to validate the proposed CR beam element formulation and 2D joint model for exterior joints. The dimensions of these two specimens are identical, as shown in **Fig. 6.7**. The reinforcement detailing of the exterior joints are given in **Fig. 6.8 (a)** and **(b)**, respectively. The two ends of the column are simply supported on rollers so that the vertical direction of the column is free to deform.

As shown in **Fig. 6.8 (a)** and **(b)**, the reinforcement detailing in Specimens NS03 and LS03 are different at the joint regions and, thus, the joint behaviour and the structural responses of these two specimens are different.

Fig. 6.7 Dimension and boundary condition of the exterior joint NS03 and LS03

Fig. 6.8 Steel reinforcement details of the exterior joint NS03 and LS03

The deformation predictions of Specimens NS03 and LS03 based on the numerical models with proposed CR beam elements and 2D joint elements are shown in **Figs. 6.9** and **6.10**, respectively. Comparing the results by the numerical models with and without joint elements, it is obvious that the former model can predict the joint strength deterioration and shear failure of shear panels, whereas the latter cannot do so. Since joint behaviour is highlighted

when designing these two specimens and material nonlinearity at the beam or the column ends is not so significant, different concrete models do not bring about a significant difference in terms of load-displacement response. As shown in **Figs. 6.9** through **6.12**, for numerical models with and without the joint elements, compared with KP model, UP model takes more time to complete the analysis, but the accuracy is similar.

Fig. 6.9 Load-displacement response for exterior joint NS03 (Yap and Li 2011)

Fig. 6.10 Load-displacement response for exterior joint LS03 (Yap and Li 2011)

Fig. 6.11 Computational time for exterior joint NS03

Fig. 6.12 Computational time for exterior joint LS03

To find out the equilibrium stress state by using the proposed flow rule (**Chapter 3**) between the failure surface and the potential surface, a great number of iterations is necessary when applying UP concrete model for large plastic strain problems. So, for both of these specimens, the computational

efficiency of the numerical model with KP concrete model is much better than that with UP model as shown in **Figs. 6.11** and **6.12**. The reason has been discussed in the fourth paragraph of **Section 6.2.1**.

In fact, the tolerance value of 10^{-6} in an energy-based convergence criterion is employed for all the examples in this study. Nevertheless, this tolerance value of convergence is too small for the numerical model with UP model when the concrete is severely crushed in these specimens, and in consequence, the numerical model with UP model cannot converge. Therefore, stability of numerical models with UP model is not so satisfactory as shown later in this chapter. However, it should be noted that all the given results with UP model in the thesis are convergent solutions.

6.2.3 Interior joint

A series of interior RC beam-column subassembly tests were conducted in Nanyang Technological University to study the development of catenary action under large deformation (Yu and Tan 2012b). Seven specimens are numerically analysed in the present work to show the capability of the proposed CR beam element formulation and 2D joint element for interior joints. The dimensions and steel reinforcement details of the RC subassembly are shown in **Fig. 6.13 (a)**, in which the beam section is 250 mm×150 mm and their net spans and other geometric properties are given in **Table 6.3**. Thirty one elements are employed to mesh the subassembly as illustrated in **Fig. 6.13 (b)**. It should be noted that the reinforcement configuration along the beam is symmetric but not uniform.

The material properties of longitudinal steel reinforcement and stirrup are listed in **Table 6.4**. As for concrete, the compressive strength is 38.2 MPa, tensile strength is 3.5 MPa, and initial modulus of elasticity is 29,645 MPa.

(a) Dimensions and reinforcement details of the RC subassembly (unit in mm)

(b) Numerical model

Fig. 6.13 Detailing and boundary conditions of the RC subassembly

Table 6.3 The geometric properties of specimens

Test	L_n (mm)	L/h	Position of rebar curtailment l_{01} (mm)	Longitudinal reinforcement			
				A-A section		B-B section	
				Top	Bottom	Top	Bottom
S1	2750	23	1000	2T10+1T13	2T10	2T10	2T10
S2	2750	23	1000	3T10	2T10	2T10	2T10
S3	2750	23	1000	3T13	2T10	2T13	2T10+1T13
S4	2750	23	1000	3T13	2T13	2T13	2T13
S5	2750	23	1000	3T13	3T13	2T13	3T13
S6	2750	23	1000	3T16	2T13	2T16	2T13
S7	2150	18.2	780	3T13	2T13	2T13	2T13

Table 6.4 Material properties of reinforcement

Rebar type	Nominal diameter (mm)	Yield strength f_y (MPa)	Elastic Modulus E_s (MPa)	Strain at the start of hardening ε_{sh} (%)	Tensile strength f_u (MPa)	Ultimate strain ε_u (%)	Hardening Modulus E_h (MPa)
R6	6	349	199177	--	459	--	--
T10	10	511	211020	2.51	622	11.00	1031
T13	13	494	185873	2.66	593	10.92	929
T16	16	513	184423	2.87	612	13.43	752

Fig. 6.14 Nonlinear behaviour of the proposed spring element to simulate the specimen supports**Table 6.5** Calibration results of boundary conditions of interior joints

Test	Horizontal restraints	Tension stiffness (kN/m)	Compression stiffness (kN/m)	Tension Gap (mm)	Compression Gap (mm)
S1	Top	43234.25	--	1.8	--
	Bottom	42589.05	122601.52	1.2	-4.6
S2	Top	55957.26	--	0.0	--
	Bottom	63942.28	102326.82	1.7	-3.9
S3	Top	62413.11	--	2.4	--
	Bottom	23050.53	146390.7	4.1	-0.7
S4	Top	100571.92	--	1.8	--
	Bottom	49255.37	175277.46	3.5	-2.1
S5	Top	76262.49	--	0.9	--
	Bottom	--	195343.58	--	-1.5
S6	Top	105286.84	--	0.2	--
	Bottom	80139.36	175093.03	1.5	-4.8
S7	Top	108723.92	--	0.3	--
	Bottom	72955.43	157282.14	2.0	-3.8

In order to improve the accuracy of numerical simulations, the stiffness and assembly gap at the specimen supports connected with the end column stubs were measured in the test. The information has been calibrated and converted into equivalent nonlinear spring elements at the compression and tension zones, as shown in **Fig. 6.14** to simulate gaps in the actual boundary conditions. The calibration results are summarized in **Table 6.5**.

Using KP concrete model, the simulation predictions in terms of load-displacement response for the numerical models with and without the joint elements are shown in **Fig. 6.15**. Additionally, the corresponding computational time of all simulations when the maximum vertical displacement achieves 700 mm is summarized in **Fig. 6.16**. In the comparison of numerical models with and without joint elements, some conclusions can be drawn as follows: (1) the initial slopes of the curves coincide well with experimental results, (2) the peak load associated with concrete crushing but prior to the occurrence of catenary action can be accurately captured, (3) the fracturing of bottom reinforcement at the interior joint region is predicted but not at the exact load or deformation, and (4) with the same applied load during the development of catenary action, the middle joint displacement in the numerical model with a joint element is greater than that without a joint element. This means the proposed bar-slip component in the 2D joint model introduces additional deformation. It can also be concluded that, the numerical models without the joint element, i.e. based on fibre model only, can predict the structural response fairly accurately. Nevertheless, it is evident that the predicted ultimate load capacity due to catenary action in the numerical model with a joint model is much more accurate in some cases, especially in Specimen 5. This is due to the bar-slip component calibration of the middle joint where ultimate failure occurs. It is worth noting that compared with other specimens, the predicted bar-slip behaviour in Specimen 5 is more accurate, since the predicted joint displacement where reinforcement fractures numerically is fairly close to experimental observation.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

Fig. 6.15 Load-displacement response of interior joints using the KP model, compared with test results (Yu and Tan 2012b)

Fig. 6.16 Computational time of interior joints

For all the specimens, prior to concrete crushing, the predicted slopes from numerical models with joint elements are slightly *steeper* than those without joint elements. This discrepancy stems from discretisation of beam elements. As shown in **Fig. 6.17 (a)**, a beam-column subassembly is commonly discretised by means of beam elements from the common node of the beam and column members. Nevertheless, in order to consider the typical dimensions of the joint region, the Gaussian points of adjacent beam elements have to be slightly shifted away from the joint region. Therefore, the joint region without a Gaussian point will be numerically treated as a rigid region as shown in **Fig. 6.17 (b)**, which will result in a slightly more rigid behaviour and steeper load-

displacement response. Furthermore, with the same number of beam elements, compared with the model with the joint region replaced by a rigid cross (**Fig. 6.17 (b)**), the numerical models with joint elements (**Fig. 6.17 (c)**) are definitely more flexible. However, the structural flexibility in **Fig. 6.17 (c)** depends not only on the dimensions of the joint region and locations of Gaussian points, but also the stiffness calibration of the joint components. Therefore, it is meaningless to directly compare the flexibilities of the numerical models in **Figs. 6.17 (a)** and **(c)**. In addition, with appropriate calibrations of the joint components as proposed in **Chapter 5**, the predictions by the numerical model in **Fig. 6.17 (c)** will be more meaningful to simulate the actual joint behaviour and allow engineers to check the joint design subjected to complex loading conditions.

Fig. 6.17 Discretisations of a beam-column subassembly

In terms of computational time, the numerical model with a joint element is more efficient than those without a joint element as shown in **Fig. 6.16**. The reason is that the incorporation of a joint element can alleviate the

computational cost due to concentrated material nonlinearity at the joint region, which has been discussed in **Section 6.2.1**. The same specimens have been simulated by using Mander's model and similar predictions and conclusions are obtained.

Since the beams are symmetrical about the interior joints, the shear response of the interior joint is not so significant. Theoretically, the shear-panel component in the joint element does not deform at all. In fact, the bond stress-slip behaviour plays a dominant role in this series of RC beam-column subassemblages. As shown in **Fig. 6.15**, after the crushing of concrete, the beam-column subassemblage can sustain more loading due to catenary action which is accompanied by bond stress-slip behaviour at the bottom reinforcement of the interior joint. Therefore, based on uniaxial concrete models, the proposed CR beam formulation and the analytical models for the 2D joint model are capable of predicting the deformations of RC beams with flexural failures.

It should be clarified that due to severe crushing of concrete at large deformation, the accumulated plastic strain in the extreme fibre is large. When applying UP concrete model, it is difficult to obtain convergence after the peak load capacity but prior to commencement of catenary action as shown in **Fig. 6.18**. The reason is that once the plastic strain is large (especially the strain state in the post-peak stage of the concrete stress-strain relationship), a great number of iterations are needed to maintain the beam simplification in uniaxial strain and stress states, which has been discussed in the second paragraph of **Section 4.5.4**. Even so, the accuracy of predictions by using UP model has not improved in these specimens, because the failure mode in this series of specimens is dominated by flexural failure. Therefore, for simulations of RC beam-column framed structures with severe flexural failures, uniaxial concrete models (such as the KP model) are more suitable for efficient and accurate numerical predictions to achieve a complete load-displacement response. As for the slightly higher capacity yielded by models with joint elements, the reason is the same as explained based on **Fig. 6.17**.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

Fig. 6.18 Load-displacement response of interior joints using UP concrete model, compared with test results (Yu and Tan 2012b)

6.3 A Three-Storey Framed Structure with Experimental Results

In addition to the beam-column subassemblages, the study on a full-scale framed structure is meaningful to engineering practice. In this section, a three-storey framed structure will be simulated and compared with published experimental results to illustrate the capability of the proposed CR beam elements, component-based joint model and different concrete models, namely, the KP model, the Mander's model and the UP model. The experimental study was conducted and published by Yi et al. (2008) and their test results are employed here to validate the numerical prediction. The numerical model for the three-storey framed structure is shown in **Fig. 6.19** in which some elements are labelled for ease of discussion. The beam length is 2667mm, while the column length is 1567 mm for the first floor and 1100 mm for the other floors. The dimensions and the reinforcement details of the columns and beams are given in **Table 6.6**. The material properties of reinforcing steel and concrete are listed in **Table 6.7**. The ratio of elongation represents the ultimate tensile strain of longitudinal reinforcement, and the given value in **Table 6.7** is taken as the average of measured ultimate tensile strains by steel gauges with five times and ten times the bar diameter.

Fig. 6.19 Numerical model for the three-storey framed structure

Table 6.6 Dimension and reinforcement details of the columns and beams

	Dimension (mm×mm)	Longitudinal reinforcement	Lateral reinforcement	
			Diameter (mm)	Spacing (mm)
Column	200×200	4 D12	6	150
Beam	100×200	2D12 (top) 2D12(bottom)	6	150

Table 6.7 Material properties of reinforcing steel and concrete

Material	Property	Measured values
Longitudinal reinforcement	Yield strength, MPa	416
	Ultimate tensile strength, MPa	526
	Ratio of elongation	25%
Lateral reinforcement	Yield strength, MPa	370
Concrete	Cylinder strength in compression, MPa	20

Based on the numerical models with and without joint elements, the predicted load versus (a) vertical displacement at the removed middle column location and (b) horizontal displacement of other columns at the first floor level are compared with experimental results in **Figs. 6.20** and **6.21**, respectively. These concerned displacement directions (with positive magnitudes) are specified in **Fig. 6.19** as well. As shown in **Fig. 6.20**, good agreement for numerical models with all three concrete models is achieved in terms of the initial slope and the

plastic hinge formation. The failure mode of the three-storey framed structure observed in the experimental study is shown in **Fig. 6.22**. It was reported that the steel bars near the end of the first floor beam adjacent to the middle column fractured, which caused a sudden decrease in the load-deformation response, indicating incipient collapse of the frame. The same failure mode is predicted by the proposed approaches with and without the joint elements for KP model and Mander's model, that is, the bottom reinforcing bars at both sides of the middle joint fracture. As shown in **Fig. 6.20 (c)**, the numerical model with UP models cannot predict further due to convergence problems stemming from large plastic strains as discussed in **Section 4.5.4**. Due to the removal of the middle column of the first storey, most of the initial axial forces have been redistributed via adjacent beams to the other columns. This is confirmed by the experimental measurements. This proves that the proposed CR beam elements, joint element and concrete models are capable of predicting well the nonlinear behaviour of RC framed structures.

Compared with numerical models with joint elements, numerical models without joint elements significantly overestimate the load capacity of the three-storey frame as they could not simulate the fracture of the bottom steel reinforcement at the middle joint. In fact, fracturing of longitudinal bottom steel reinforcement is not the only failure occurring in the catenary action state and bar-slip behaviour in the middle joint should also be taken into account.

Fig. 6.20 Predicted load-displacement responses based on the numerical models with and without joint models, compared with test results (Yi et al. 2008)

For the displacement comparisons in **Fig. 6.21**, it should be noted that the locations of these sections are shown in **Fig. 6.19** in which positive displacement implies a downward displacement of the beam section. Due to symmetry of numerical models and applied loading as shown in **Fig. 6.19**, the predicted displacements at Sections 3-1 and 3-4 are identical. So also are the displacements at Sections 3-2 and 3-3. Therefore, only the downward displacements at Section 3-1 and 3-2 are compared with the corresponding experimental results in **Fig. 6.21**. Even though the simulations with UP model cannot converge due to severely crushed concrete, the load capacity predicted by UP model is rather accurate and can be employed in practice provided that

catenary action stage is not taken into account. This phenomenon is similar to that when simulating the interior joints in **Section 6.2.3** and the reason has been elaborated in **Section 4.5.4**.

As shown in **Fig. 6.21**, numerical models with both uniaxial concrete models (KP model and Mander's model) and UP model are capable of accurately predicting the variations of the horizontal displacements for all measured sections. Due to compressive arch action at the initial loading stage (where arching forces, rather than the flexural action, are dominating the deformation behaviour of the corresponding beams), the columns at the first storey are pushed away from the removed middle column. Subsequently, the columns at the first storey are pulled inwards due to tension forces developing in the two-span beams, which indicates the occurrence of catenary action at the middle joint. Therefore, the proposed approach for predicting the nonlinear behaviour of full-scale RC framed structures is validated.

Other than the predictions of nodal displacements by the numerical models with and without joint elements using the same concrete model, it is also meaningful to compare the structural deformations of the three-storey framed structure with different concrete models as given in **Fig. 6.23**. It shows that similar structural deformation at the final stage can be predicted based on KP concrete model and Mander's concrete model. The predicted failure modes are fracturing of the steel bars near the end of the first floor beam adjacent to the middle column, which exactly coincide with what had been observed in the experimental study as shown in **Fig. 6.22**.

Fig. 6.21 Downward displacement of the middle column versus horizontal displacement of columns at the first floor level based on the numerical models with and without joint models, compared with test results (Yi et al. 2008)

Fig. 6.22 Failure mode of the three-storey framed structure in the experimental study (Yi et al. 2008)

Fig. 6.23 Structural deformation of the three-storey framed structure with different concrete models

6.4 Robustness Study at the System Level

For the conventional column-removal scenarios, the DOD (2009) only requires removal of a single column at a time at critical locations for analysis purpose. In reality, if the columns are spaced close to each other, depending on the magnitude of the explosives, a blast event may knock out more than one column. In the remaining of this chapter, some unfavourable column removal scenarios are investigated at the system level considering the removal of one, two and even three columns at the ground level, rather than only one column being removed from subassemblages as studied in **Sections 6.2 and 6.3**.

As for simulating the column removal, a general and rigorous element removal technique is proposed by Talaat and Mosalam (2008; 2009), which satisfies dynamic equilibrium before and after column removal in time history analyses. Nevertheless, the focus in the present study is on progressive collapse analysis where only monotonic loading condition is considered. Thus, instant column removals are assumed at the beginning of analysis and the initial reaction forces at locations of removed columns are treated as applied quasi-static loads.

Firstly, a three-storey and two-bay frame is studied with an exterior column removed, as an example of most commonly encountered RC structures. Later on, a five-storey and four-bay frame, representing one example of real-world framed structures, is analysed with different column-removal scenarios. The objective of these two simulations is to demonstrate the robustness of the proposed numerical approach to predict the deformation behaviour of realistic framed structures with the potential of progressive collapse.

Since only the numerical study is conducted for the following two examples and there is no experimental result to validate the predictions, only one uniaxial concrete model (KP model) is employed.

6.4.1 A Three-Storey Framed Structure

The numerical model of the three-storey RC frame with three point loads applied downwards at the top level is given in **Fig. 6.24**, which also shows the

dimensions and reinforcement details of the beams and columns. The material properties for steel and concrete listed in **Table 6.7** are employed in the present example. The missing exterior column of the first floor represents a typical column loss due to a car bombing at a building corner.

After analysing the numerical model as shown in **Fig. 6.24**, final deformation of the 2D three-storey RC frame is given in **Fig. 6.25**, showing that good compatibility between beam elements and joint elements in the proposed approach can be attained. Accordingly, a complete load-vertical displacement response at the top floor is shown in **Fig. 6.26**, in which the sequence of the longitudinal reinforcement fractures in the joints and the adjacent beams/columns is labelled and fractures occurring at close load increments are treated as one fracturing event. Six failures are observed indicated by a sudden decrease of applied load. Even though there are sudden changes when fracturing failures occur, the finite element program with the proposed concrete model, beam element and joint model can still run normally. The locations and sequence of the longitudinal reinforcement fractures are illustrated in detail in **Fig. 6.27**, which is identical to the fracturing sequence labelled in **Fig. 6.26**.

This example shows that the proposed numerical approach is capable of giving robust predictions of structural performance of RC structures with an exterior column removed. It should be noted that as shown in **Fig. 6.27**, material nonlinearity is concentrated at the left bay of the three-storey framed building, while the right bay just undergoes small deformations without obvious material failures.

Fig. 6.24 A 2D three-storey RC frame with an exterior column removed (unit in mm)

Fig. 6.25 Deformation of the 2D three-storey RC frame with an exterior column removed

Fig. 6.26 Load-displacement response of the three-storey RC frame with an exterior column removed

Fig. 6.27 Failure location and sequence in the 2D three-storey RC frame with an exterior column removed

6.4.2 A Five-Storey Framed Structure

A numerical model of a typical five-storey RC frame with applied vertical loads is shown in **Fig. 6.28**. The dimensions and reinforcement details of columns and beams are the same with those in the three-storey framed structure in **Section 6.4.1**. The applied loads are idealized as a combination of live loads and dead loads in the structure. The choice of the five-storey frame is to further demonstrate the robustness of the program. The design of the five-storey and four-bay frame is checked according to both Eurocode 2-2004 and ACI 318-02 in ETABS (2011). Since the applied loads are transferred directly to the ground through the columns, the column axial force plays an important role in the intact frame. However, it should be noted that the member forces in the frame will be redistributed once there are any changes in the geometric configuration, e.g. the removal of one or more columns. Therefore, in order to perform reasonable predictions for the frame deformation, a nonlinear analysis should be conducted rather than relying solely on the linear analysis by ETABS.

Fig. 6.28 Numerical model of the five-storey and four-bay frame

To represent the scenario due to a car bomb event occurring adjacent to one side of the first floor, several column-removal scenarios are studied as shown in **Fig. 6.29**, including the removal of an exterior column, a penultimate column, a

middle column and their combinations. The numerical models for different column-removal scenarios are analysed by the proposed approach of CR beam elements and the component-based joint model. The predictions of numerical models with and without joint elements are compared to show the effect of joint elements, in which the joint component properties are calibrated based on the approaches as proposed in **Chapter 5**. Since numerical robustness in a large-scale framed structure is the major objective of this example, only one uniaxial concrete model (KP model) is employed.

For all the column-removal scenarios, the critical positions with bar fracture failures in the joints and the adjacent beams/columns are labelled in **Fig. 6.29** with the Arabic numbers indicating the failure sequence. To quantitatively analyse the failure due to the column-removal scenarios, the displacements at three nodes in the frame are of interest, that is, the top left corner denoted as Point 'A', the top middle point as 'B' and the top right corner as 'C'. The load-displacement responses under different column-removal scenarios for these three nodes are compared in **Figs. 6.30, 6.31 and 6.32**, respectively, in which the Arabic numbers indicate the failure sequence. The corresponding structural deformations for all the three removal scenarios are plotted in **Fig. 6.33**.

Figs. 6.30, 6.31 and 6.32 show that the predicted deformation based on numerical models without joint elements is slightly greater than that based on numerical models with joint elements. In finite element analyses, in order to consider the physical dimensions of the joint region, Gaussian points of adjacent beam elements have to be slightly shifted away from the joint region. Therefore, it is inappropriate to compare the numerical models with joint element and those without joint element. Instead, it is more objective to compare the models with calibrated joint and rigid joint with the same mesh and adjacent beams/columns. As discussed in **Fig. 6.17**, the numerical models with calibrated joint model are definitely more flexible than those with rigid joint.

Meanwhile, the predicted load capacities for different column-removal scenarios based on both numerical models are almost identical. This means that the

proposed joint model brings about certain fixed end rotations to the local joint behaviour but introduces no reduction for the load capacity. In addition, the numerical models without joint elements will significantly overestimate the structural deformation capacity because of omission of bond-slip behaviour.

It should also be noted that in the numerical models with and without joint elements, there is a discrepancy in the load-displacement response at the concerned nodes when they are not directly located above the removed column(s) as shown in **Figs. 6.30 (b) and (c)** and **Fig. 6.31 (c)**. However, the absolute magnitudes of these displacements are relatively small and the corresponding discrepancy can be fairly tolerated. Furthermore, as shown in **Figs. 6.30 (b) and (c)**, the responses at Points B and C of numerical models with joint elements are stiffer than those without joint elements, while it is the other way around for the response at Point C as shown in **Fig. 6.31 (c)**. The discrepancy is due to the combined effects associated with the beam Gaussian point locations and the joint component calibrations, which has been explained in **Fig. 6.17** of **Section 6.2.3**. Nevertheless, good agreement can be achieved at all the three nodes when large displacements occur under the scenario with exterior, penultimate and middle columns removed for the five-storey frame as shown in **Fig. 6.32**. Thus, the proposed joint model integrates well with the beam elements and is rather robust even for simulating large-scale redistribution of internal forces associated with the removal of three columns.

The structural deformations of the five-storey frame under all the three columns removal scenarios are illustrated in **Fig. 6.33** with comparison of numerical models with and without joint elements. It is shown that the deformations based on the numerical models with and without joint elements are at the same level, which is compatible with the load-deformation response shown in **Figs. 6.30, 6.31 and 6.32** where the differences of the final deformation based on the numerical models with and without joint elements are not so obvious especially when compared with the height and width of the whole structure. However, the local bond-slip behaviour can be obtained in the numerical models with joint

elements, which provides more information on the joint deformations and failure modes and are important when designing a framed structure.

It should be noted that the computation with joint elements (for the removal scenario of exterior and penultimate columns) terminates as shown in **Fig. 6.31** due to a sudden change of joint component states from the state with certain resistance (e.g. State 2) to the state without any resistance (e.g. State 3) (see **Fig. 5.43**) when a dramatic decrease of structural load resistance takes place and consequently, large redistribution of internal forces takes place in remaining columns. This has been confirmed by the results as shown in **Fig. 6.34** obtained from the same numerical model but with all of joint elements strengthened (a large number is multiplied by the calibrated joint component stiffness to achieve the strengthened components and rule out the joint nonlinearity). The simulation with strengthened joint elements yields a stiffer structural response but can perfectly run until the ultimate deformation stage after a significant reduction of load resistance, which proves that the convergence problem is due to significant nonlinearity effect at the joint components. Even though the numerical model with normal joint elements simulating the five-storey frame subjected to exterior and penultimate columns removal scenario shows collapse at a certain load level, the ultimate deformations after collapse are not available for this simulation. Hence, only the final structural deformations before collapse are compared in **Fig. 6.33 (b)**, rather than the ultimate structural deformations after collapse as shown in **Figs. 6.33 (a) and (c)**. Nevertheless, the ultimate structural deformations of the numerical model with strengthened joint elements are compared with the model without joint elements in **Fig. 6.35**, showing good agreement between these two sets of results.

(a) Exterior column removal scenario

(b) Exterior and penultimate columns removal scenario

(c) Exterior, penultimate and middle columns removal scenario

Fig. 6.29 Different column-removal scenarios studied for the five-storey frame

Fig. 6.30 Load-displacement response under an exterior column removal scenario for the five-storey frame

(a) Point A

(b) Point B

(c) Point C

Fig. 6.31 Load-displacement response under exterior and penultimate columns removal scenario for the five-storey frame

(a) Point A

(b) Point B

(c) Point C

Fig. 6.32 Load-displacement response under exterior, penultimate and middle columns removal scenario studied for the five-storey frame

(a) An exterior column removal scenario

(b) Exterior and penultimate columns removal scenario

(c) Exterior, penultimate and middle columns removal scenario

Fig. 6.33 Structural deformation of the five-storey frame under exterior, penultimate and middle columns removal scenarios

(a) Point A

(b) Point B

(c) Point C

Fig. 6.34 Comparisons of load-displacement response with strengthened joints under exterior and penultimate columns removal scenario for the five-storey frame

Fig. 6.35 Structural deformation of the five-storey frame with strengthened joints under exterior and penultimate columns removal scenario

6.5 Closure

In this chapter, the proposed CR beam element, 2D component-based joint model and different concrete models are integrated and studied at the system level of RC beam-column framed structures, including beam-column subassemblages and framed structures. Based on the obtained predictions, numerical models with joint elements are capable of providing more accurate predictions for both local joint failures and global structural failures, compared with numerical models without joint elements. A better computational efficiency for beam-column subassemblages can be achieved by using the proposed simulation approach, because of alleviation of material nonlinearity at the joint region. According to the simulations conducted for the three-storey 2D framed structure with an interior column removed, the proposed simulation approach provides good predictions for both the load-displacement response and the transition stage between compressive arch action and catenary action when compared with experimental results. The robustness of the proposed numerical approach to predict the deformation behaviour of realistic framed structures for progressive collapse analysis is demonstrated by employing a three-storey and two-bay frame with an exterior column-removal scenario and a five-storey and four-bay frame with different column-removal scenarios.

Even though numerical models without joint elements can also predict the deformation behaviour of framed structures, local joint behaviour is not available and, thus, the simulations of the framed structures cannot provide more information on the joint deformation and failure modes. These results are important when designing a framed structure, especially for an important building with a high security classification or subjected to potential terrorist attacks.

For the two types of the proposed concrete models (the KP and Mander's models versus the UP model), the uniaxial concrete models are numerically efficient and capable of predicting flexural failures, while the UP concrete model is capable of predicting shear failures of beam members but difficult to find out the equilibrium stress state for large plastic strain problems. Therefore, it is suggested to use uniaxial concrete models when simulating large-scale framed structures, especially if the beam members are not so short. For those short beam members with significant shear behaviour, one can select the UP model for better accuracy.

Chapter 7 A Superelement Formulation for Efficient Structural Analysis in Progressive Collapse

7.1 Introduction

It is usually time-consuming to analyse fine-mesh finite element models for large-scale structures, such as multi-storey reinforced concrete (RC) buildings. In this chapter, an integrated superelement concept is proposed to improve the computational efficiency when studying structural responses during progressive collapse analyses. While the proposed methodology is straightforward and can be implemented into an existing finite element program with little effort, it is able to significantly reduce the computational cost without any loss of any critical information of structural responses. Compared with the models without superelement, significant saving in computational cost and satisfactory prediction accuracy can be obtained with the proposed approach. Besides, the proposed methodology is independent of element types and material models.

The outline of the present chapter is summarized as follows. In **Section 7.2**, the basic concept of the proposed superelement formulation is presented by using a simple 2D frame example. The deformation of the superelement is discussed in detail for different superelement configurations in **Section 7.3**. In **Section 7.4**, the proposed superelement formulation is firstly validated for 2D RC framed structures modelled by the proposed approach in the previous chapters, such as the concrete model, the co-rotational beam formulation and also the component-based mechanical joint model. An example of a 3-storey frame from **Chapter 6** is studied and an obvious computational efficiency improvement is achieved. At last, the superelement formulation is validated against a 3D framed structure with either reinforced concrete or pure steel material so as to illustrate the advantage of the proposed superelement formulation in terms of computational efficiency improvement under an extreme loading scenario. In all the presented examples, prediction accuracy and CPU time are compared for numerical models with and without superelements.

7.2 Basic Concept of the Proposed Superelement (Long et al. 2012c)

To illustrate the basic approach in this study, a 2D frame example as shown in **Fig. 7.1** is considered (Long et al. 2012c). The frame is uniformly divided into eight three-node 3D beam elements, and three point loads are applied at nodes 3, 7 and 17, respectively. With load P_3 at node 17 as the major load, it is assumed that in this example, attention should be paid on key elements between node 5 and node 17 as nonlinear behaviour is expected to occur in this zone. The zone comprising the key elements is denoted as a *nonlinear zone*. In this case, all the elements between node 1 and node 9 can be merged into one superelement or *linear zone* where only linear analysis is required.

Fig. 7.1 A 2D frame subjected to external forces

Now consider the linear zone shown in **Fig. 7.2**, with the key elements and the forces applied on them removed from the original model. To construct a superelement, its nodes must be determined from the beginning. Theoretically speaking, the superelement deformation should be the combined effect due to the external load in the linear zone and the transferred internal forces from the common nodes shared by the nonlinear and the linear zones. Thus, an additional node is necessary to represent the superelement deformation behaviour if there are external loads applied in the linear zone.

For the numerical model in **Fig. 7.1**, node 5 is the only common node shared by the nonlinear and the linear zones. Therefore, the superelement has only two nodes, viz. common node 5 which has six degrees of freedom and an *additional node*, which is in the linear zone. Without loss of generality, node 6 is selected

to be the *additional node*, in that there are external loads applied in the linear zone. To obtain the stiffness matrix of the superelement, a series of linear analyses has to be conducted based on the configuration shown in **Fig. 7.2**.

Fig. 7.2 Linear zone of the 2D frame under real forces

Firstly, forces λP_1 and λP_2 , where λ is an arbitrary nonzero factor, are applied to the linear zone and the associated deformations at nodes 6 and 5 are calculated. The forces λP_1 and λP_2 can be represented by a unit force “1” when formulating the superelement stiffness matrix. Denoted by ${}^0U$, where superscript ‘0’ indicates the load case number, and the corresponding deformation vector induced by a combination of all scaled forces applied on the linear zones (λP_1 and λP_2 in this example) is expressed as ${}^0U = ({}^0u_{6-x}, {}^0u_{5-x}, {}^0u_{5-y}, {}^0u_{5-z}, {}^0\theta_{5-x}, {}^0\theta_{5-y}, {}^0\theta_{5-z})^T$. Note that the terms u and θ represent the nodal displacements and rotations, respectively. The Arabic number in the subscript denotes the node number, while the Latin letter indicates the respective coordinate axis. It should be mentioned that all the six components of the deformation at node 5 are stored in ${}^0U$ but only one degree of freedom at node 6 needs to be considered. In fact, among the six degrees of freedom at node 6, any nonzero component can be selected to form ${}^0U$. In this method, in node 6, the maximum translation or rotation for load case ‘0’ is selected as the only component at the *additional node*.

Then the applied loads in the linear zone are removed and a unit load is applied sequentially at all degrees of freedom of the common nodes of the linear and nonlinear zones. For this example, in the second step the forces λP_1 and λP_2 are

removed and a unit load P_{5-x} is applied to node 5 in the x -direction as shown in **Fig. 7.3**. Similar to the initial step, the deformations at node 6 and 5 are denoted as ${}^1U = ({}^1u_{6-x}, {}^1u_{5-x}, {}^1u_{5-y}, {}^1u_{5-z}, {}^1\theta_{5-x}, {}^1\theta_{5-y}, {}^1\theta_{5-z})^T$.

Fig. 7.3 Linear zone of the 2D frame under virtual force scenario

The second step is repeated five times at the common node 5 to generate five more load cases, viz. two unit loads (along the y and z -axis) and three unit moments (about x , y and z -axis) are applied in sequence. The corresponding deformation vectors are stored as ${}^2U, {}^3U, {}^4U, {}^5U$ and ${}^6U$. It should be mentioned that the global stiffness matrix of the linear zone only requires to be factorized once during such calculations. Hence, ${}^1U \sim {}^6U$ can be obtained with modest additional computational effort compared with the solution procedure of ${}^0U$.

Based on the properties of linear elasticity, the above three steps can be summarized by the following equation,

$$\mathbf{K}_s \boldsymbol{\psi} = \begin{pmatrix} \mathbf{1} & \mathbf{0}_{1 \times 6} \\ \mathbf{0}_{6 \times 1} & \mathbf{I}_{6 \times 6} \end{pmatrix}_{7 \times 7} = \mathbf{I}_{7 \times 7} \quad (7.1)$$

where $\boldsymbol{\psi} = ({}^0U, {}^1U, {}^2U, {}^3U, {}^4U, {}^5U, {}^6U)$ is obtained by conducting a series of linear analyses with consideration of the external loads in the linear zone (the forces λP_1 and λP_2) and the transferred internal forces from the common nodes (the assumed unit load cases applied to all degrees of freedom of node 5) and $\mathbf{K}_s$ is an unknown matrix and equal to $\boldsymbol{\psi}^{-1}$, i.e., the superelement stiffness matrix.

For this example, the superelement has seven DOFs, including six DOFs at node 5 and only one DOF at node 6.

It should be mentioned that in **Eq. (7.1)**, the load case combination of λP_1 and λP_2 is represented by a unit force “1” acting at node 6 in the direction with the maximum deformation. Nevertheless, without loss of generality, if no load is applied in the linear zone, a set of virtual concentrated loads along *the translational direction* at the additional node will be automatically applied when formulating the superelement stiffness matrix for ease of programming. But, these virtual loads are not taken into consideration when calculating the global deformation behaviour.

Moreover, the superelement stiffness matrix $\mathbf{K}_s$ is not symmetric as the usual element stiffness matrix, which requires that the overall solution cannot be solved by LU decomposition-based methods. In the present study, a solver named PARDISO (Parallel Sparse Direct Solver PARDISO 2011), which is a high-performance and robust parallel sparse direct solver, is utilized to solve the unsymmetric linear systems of equations.

7.3 Superelement Deformation

When analysing the potential of buildings for progressive collapse, based on the structural deformation behaviour and the material states at different parts of the structure, the whole structure can be divided into two regions. The first is the linear elastic region where nonlinear material effect is negligible so that the stiffness matrix does not change due to material nonlinearity and consequently, can be simulated by a superelement. The other is the nonlinear region where significant amount of nonlinear responses occur and nonlinearity has to be solved by iterations in which incremental tangential stiffness matrix for this region will be updated as deformations increase.

In general, two types of superelement can be constructed. The first type of superelement is connected to both the nonlinear zone and the foundation (or where essential boundary conditions are specified). The second type of

superelement is only connected to the nonlinear zone. For the first type of superelement the whole structure is partitioned into superelement and non-superelement zone as shown in **Fig. 7.4**. In this case, a direct amplification of the deformations in the superelement can be applied as follows. Large rigid-body rotation will be prevented by the foundations. The tangential stiffness matrix of this type of superelement will remain the same throughout the loading history. Therefore, the superelement stiffness matrix is only computed once before any iteration starts. To calculate the superelement deformations, after each converged load increment is achieved, the nodal force vector at the common nodes (along the boundary between the superelement and the nonlinear zone) is applied as external loads to the superelement as shown in **Fig. 7.5**. Meanwhile, the computations of deformations in the superelement takes into account the nonlinear effects in the nonlinear zone.

For the second type of superelement which is only connected to the nonlinear zone such as the framed structure shown in **Fig. 7.6**, ‘*weak member method*’, which assigns a relatively smaller value (e.g. 10^{-5} in the present study) to the material properties (i.e. both Young’s modulus and shear modulus) for elements that are in the nonlinear zone, is employed to calculate the stiffness matrix of the superelement without the restraint from the foundation. It should be clarified that the boundary conditions are still applied to the whole structure, therefore, no numerical singularity problem is encountered.

In addition, ‘*strong member method*’, which assigns a relatively larger value (e.g. 10^{20} in the present study) to the material properties for elements that are in the nonlinear zone, is employed to eliminate the nonlinear effect from the nonlinear zone when calculating the relative deformations of the superelement for the initial load factor. With the assumption that the superelement zone behaves elastically, the global deformations can be obtained by combining the relative deformations and the rotation of the nonlinear zone.

Fig. 7.4 Superelement zone (inside dash box) and nonlinear zone (outside dash box)

for the first type of superelement

Fig. 7.5 The equivalent loading for superelement

Fig. 7.6 Superelement zone (inside dash box) and nonlinear zone (outside dash box)

for the second type of superelement

Fig. 7.7 Deformed configuration of a node of the superelement

It should be noted that the relative deformations of superelement zone (with respect to the nonlinear zone) show linear relationship with external loads in the superelement zone. In addition, the nonlinear effects of nonlinear zone on the superelement can be taken into account when calculating the rigid-body rotation. This analysis can be conducted with respect to a selected node as shown in **Fig. 7.7**. In **Fig. 7.7 (a)**, the resultant displacement vector of a certain node within the superelement can be captured by the addition of the spatial vector (pointing from the interested node in the undeformed configuration to the same node in the deformed configuration) and the relative deformation vector. To consider the effect of rigid-body rotation as discussed in **Chapter 2** due to rotational deformation at the connecting node, rotation matrix $\mathbf{R}$ is employed to rotate the resultant vector with respect to the connecting node to the actual position in the deformed configuration, as shown in **Fig. 7.7 (b)**.

A conventional approach to calculate the rotation matrix proposed by Crisfield (1990) (relevant to Argyris' work (1982) dealing with 3D rotations) is to employ pseudo-vectors and skew-symmetric matrices to describe the rotational variables. Incorporating both local and global coordinate systems, the skew-symmetric matrix can be obtained as

$$\mathbf{S}(\boldsymbol{\theta}) = \begin{bmatrix} 0 & -\theta_3 & \theta_2 \\ \theta_3 & 0 & -\theta_1 \\ -\theta_2 & \theta_1 & 0 \end{bmatrix} \quad (7.2)$$

where θ_1 , θ_2 and θ_3 are the rotational variables of the connecting node. The sign convention of rotational variables follows the right-hand rule as shown in **Fig. 7.8**.

The orthogonal rotation matrix $\mathbf{R}$ that rotates a vector into a new position is given by Argyris (1982) and Crisfield (1990).

$$\mathbf{R}(\boldsymbol{\theta}) = \mathbf{I} + \frac{\sin \theta}{\theta} \mathbf{S}(\boldsymbol{\theta}) + \frac{1 - \cos \theta}{\theta^2} \mathbf{S}(\boldsymbol{\theta}) \mathbf{S}(\boldsymbol{\theta}) \quad (7.3)$$

where $\boldsymbol{\theta} = (\theta_1 \ \theta_2 \ \theta_3)^T$ and $\theta = \|\boldsymbol{\theta}\|$. It should be noted that the sign $\|\cdot\|$ represents the L_2 -norm.

Fig. 7.8 Rotation directions defined by the skew-symmetric matrix

Another approach to calculate the rigid-body rotation about a certain node is based on the nodal vector (**Eq. (7.4)**) in the deformed configuration as discussed by Crisfield (1996).

$$\mathbf{R} = [\mathbf{e}_x \ \mathbf{e}_y \ \mathbf{e}_z] \quad (7.4)$$

where $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are the unit local nodal vectors of the connecting node expressed in the global system.

Since it is well known that the definitions of rotation variables depend on large rotational formulation of finite elements employed in the analysis (Dvorkin et al. 1988; Li 2007; Long et al. 2012b), both approaches using **Eqs. (7.3)** and **(7.4)** have been implemented in the current study in order to broaden the application range of the proposed superelement to different finite element formulations, such as total Lagrangian formulation, updated Lagrangian formulation and co-rotational formulation. For example, the first approach using natural rotation to define $\mathbf{R}$ (**Eq. (7.3)**) can be used directly with the 3D total Lagrangian beam element as suggested by Dvorkin et al. (1988). However, if vectorial rotational vectors (utilized by the co-rotational beam element formulation proposed in **Chapter 4**) are employed instead to define the rotational variables (Li 2007; Long et al. 2012b), it will be more convenient to use the second approach (**Eq. (7.4)**).

Rigid-body rotation and scalable relative deformation with respect to the nonlinear zone are computed to obtain the deformed configuration of the superelement in the global coordinate system. The computation procedure with superelement deformation is summarized in **Fig. 7.9**. Firstly, the superelement stiffness matrix $\mathbf{K}_s$ and its inverse matrix $\mathbf{K}_s^{-1}$ are calculated using the ‘weak member method’ and the ‘strong member method’ prior to increase of loading (**Fig. 7.9 (a)**). As shown in **Fig. 7.9 (b)**, the relative deformation $\mathbf{u}_L$ of the superelement with respect to the nonlinear zone due to nodal force F , nodal moment M and external load P_s within the superelement zone can be computed from **Eq. (7.5)**.

Fig. 7.9 Scalable relative deformation and rigid-body rotation with respect to the nonlinear zone

$$\mathbf{u}_L = \mathbf{K}_s^{-1} \begin{pmatrix} P_s \\ F \\ M \end{pmatrix} \quad (7.5)$$

It should be noted that the additional node and its degree of freedom have been considered when calculating the stiffness matrix $\mathbf{K}_s$ in **Eq. (7.5)**, therefore, in the relative deformation vector $\mathbf{u}_L$, there will be a relative deformation of the additional degree of freedom with respect to the nonlinear zone.

Assuming that the natural rotational variable (**Eqs. (7.2) and (7.3)**) are employed for large rotation formulation, then the rotation variables of connecting nodes can form the rigid-body rotation matrix $\mathbf{R}$ so that $\mathbf{R}^T = \mathbf{R}^{-1}$. Hence, the transformation matrix $\mathbf{T}$ can be obtained by assembling $\mathbf{R}^T$ corresponding to both the translational and rotational variables and the update of superelement stiffness matrix $\mathbf{K}_s$ can be calculated as

$$\mathbf{K}_s^{update} = \mathbf{T}^T \mathbf{K}_s \mathbf{T} \quad (7.6)$$

As illustrated in **Fig. 7.9 (c)**, after reaching the convergence of the next load increment, the superelement stiffness matrix $\mathbf{K}_s$ and the nonlinear zone stiffness matrix $\mathbf{K}_{ns}$ are computed, and the rotational variables of connecting nodes can be extracted to form the new rigid-body rotation matrix $\mathbf{R}$. Therefore, the superelement deformation $\mathbf{u}_G$ for the next load increment in the global coordinate system can be calculated as

$$\mathbf{u}_G = \mathbf{R} \mathbf{u}_L \quad (7.7)$$

It should be pointed out that in the case of multi-connecting nodes, the formation of resultant rigid-body rotation matrix $\mathbf{R}$ of the superelement is approximated based on the ‘average’ of rigid-body rotation matrix $\mathbf{R}_i$ at each connecting node. For the first approach using the natural rotation, it is apparent that the rotation matrix formulation in **Eq. (7.3)** is not additive due to non-

vectorial property of the 3D natural rotation. Therefore, the resultant of rotation matrices should be obtained based on compound rotations. As discussed by Crisfield (1990), one should first calculate the pseudo-vector $\boldsymbol{\omega}$ of the natural rotation at each connecting node, as given in **Eq. (7.8)**,

$$\boldsymbol{\omega} = \frac{\tan(\theta/2)}{\theta/2} \boldsymbol{\theta} \quad (7.8)$$

Then the compound pseudo-vector for all connecting nodes can be computed. For example, $\boldsymbol{\omega}_{ij}$ is the compound pseudo-vector for connecting nodes i and j together and can be expressed as

$$\boldsymbol{\omega}_{ij} = \frac{\boldsymbol{\omega}_i + \boldsymbol{\omega}_j - \frac{1}{2} \boldsymbol{\omega}_i \times \boldsymbol{\omega}_j}{1 - \frac{1}{4} \boldsymbol{\omega}_i^T \boldsymbol{\omega}_j} \quad (7.9)$$

Lastly, the resultant rigid-body rotation matrix $\mathbf{R}(\boldsymbol{\omega}_{ij})$ of the superelement can be formed by replacing $\boldsymbol{\theta}$ by $\boldsymbol{\omega}_{ij}$ in **Eq. (7.3)**.

Compared with the first approach, the second approach using local nodal vectors to form the rotation matrix in **Eq. (7.4)** can be implemented more conveniently because of the additive property of the nodal vectors. Thus, the direct average of rigid-body rotation matrices $\mathbf{R}_i$ at connecting nodes can be treated as the resultant rigid-body rotation matrix $\mathbf{R}$ of the superelement.

The calculations for the superelement deformation are summarized for both types of superelement in a flow chart as shown in **Fig. 7.10**. For the situation of a superelement fixed onto the foundation, a direct amplification of the deformation of the superelement is conducted with reference to load factor (the ratio of the current applied load with respect to the maximum load intended to apply). For the situation of a superelement connected to other structural members and not directly fixed to the foundation, the present work proposes the concept of ‘strong member method’ in the superelement zone to recover the deformations of superelement zone. The formulation concerning the

superelement zone with large displacement and large rotation in the 3D space is presented for different definitions of the rotational variables. The stiffness matrix in the superelement zone can be efficiently updated or kept constant as calculated in the undeformed configuration, respectively, depending whether the user chooses accuracy or efficiency as the priority. The calculation of superelement deformation will be conducted with respect to the connecting node(s) and will avoid intensive computations on members which only undergo small linear deformations.

Fig. 7.10 Flow chart of the calculations for the superelement deformation

7.4 Numerical Examples

To validate the accuracy and effectiveness of the proposed superelement formulation, four examples including first and second types of the superelement are employed. Examples involving either reinforced concrete or pure steel material are compared with numerical predictions from the literature or full nonlinear analyses. Firstly, a 2D three-storey two-bay RC frame with an exterior column removed is investigated to show the application of the proposed superelement formulation in 2D RC framed structures. Later on, three examples of 3D beam-column frames with either reinforced concrete or pure steel material are employed to demonstrate the potential of the superelement formulation in analysing 3D progressive collapse. In addition, the CPU time needed to complete the analyses with or without superelements are compared to study the efficiency of the superelement formulation. All the examples presented in this study are simulated on the same computer with 2.66 GHz processor and 3.25 GB RAM.

It should be noted that an accurate and efficient finite element formulation is a prerequisite for the successful application of the proposed superelement formulation for progressive collapse analysis of structures. In the present chapter, 3D three-node beam element with fibre model proposed in **Chapter 4** is employed. For all the examples, the stiffness matrices of the superelement zone are kept constant and calculated based on the original configuration and, thus, the second order effect is neglected in the superelement zone. However, for the examples shown in this study, it should be remarked here that further simulations using updated superelement stiffness matrix yield little changes in the computation of deformations in the nonlinear zone.

As for fibre models of the beam cross-sections, a ten-layer scheme is employed to discretise the concrete cross-section and equivalent steel fibres are assigned according to the reinforcement details of the 2D RC frame in Example 1. More detailed fibre schemes (10×10 fibres) are used for 3D cross-sections in Examples 2, 3 and 4 to capture refined stress and strain states such as yielding of

steel fibres. For all of the examples employed in the present chapter, single-point integration is applied to each fibre and reduced integration scheme with two Gaussian points along the beam axis is selected.

Example 1: A 2D three-storey RC frame with an exterior column removed

As an example from **Chapter 6**, a 2D three-storey two-bay RC frame with an exterior column removed is studied to show the application of the proposed superelement formulation in RC framed structures. As an integrated system, this example consists of all the previously proposed ingredients, that is, uniaxial concrete model (the Kent and Park model), the co-rotational beam elements, and the component-based mechanical joint model. However, it should be noted that the proposed methodology is independent of the element types and material models in the original numerical models. The numerical model of the RC frame without an exterior column at the first storey is given in **Fig. 7.11**, in which the dimensions and reinforcement detailing of the beams and columns and the applied load are given. Additionally, the region of the structure above the removed exterior column is of interest in terms of potential nonlinear behaviour. Therefore, the right half of the structure (also highlighted in purple in **Fig. 7.11**) is treated as the linear zone and defined as the superelement zone. The material properties for steel and concrete are listed in **Table 7.1**. The ratio of elongation represents the ultimate tensile strain of longitudinal reinforcement, and the given value is taken as the average of measured ultimate tensile strains by steel gauges with five times and ten times the bar diameter.

The numerical model with superelement is analysed using the proposed co-rotational beam elements and the corresponding result is compared with the results based on full nonlinear analyses as given in **Chapter 6**. Under the same value of the applied load, final deformations of the 2D three-storey RC frame with and without superelement are given in **Fig. 7.12**, showing that good agreement of the predictions by both models can be obtained. The corresponding complete load-displacement responses of the two numerical models are essentially the same as shown in **Fig. 7.13**. However, it should be

noted that the reason for the discrepancy of the two curves (with and without superelement) in **Fig. 7.13** is that the material nonlinearity may have already propagated to the right half of the structure (highlighted in purple in **Fig. 7.11**), since the bar fracture occurs at the joints at the interface between nonlinear zone and superelement zone. Therefore, the constraint on the nonlinear zone is overestimated, which can be reflected by a smaller displacement in the load-displacement responses. The Arabic numbers in **Fig. 7.13** indicate the failure sequence as also shown in **Fig. 7.14**. No matter a superelement is employed or not, the same failure mode can be captured by the proposed finite element approach. The location and sequence of fractures of the reinforcement in the joints and the adjacent beams/columns are illustrated in detail in **Fig. 7.14**, which is identical to the observation based on the numerical model with full nonlinear analysis in **Chapter 6**.

Fig. 7.11 Example 1: A 2D three-storey RC frame with an exterior column removed (unit in mm)

Table 7.1 Material properties of reinforcing steel and concrete in Example 1

Material	Longitudinal reinforcement	Measured values
Longitudinal reinforcement	Yield strength, MPa	416
	Ultimate tensile strength, MPa	526
	Ratio of elongation	25%
Lateral reinforcement	Yield strength, MPa	370
Concrete	Cylinder strength of compression, MPa	20

Fig. 7.12 Deformation of the 2D three-storey RC frame in Example 1

Fig. 7.13 Load-displacement response of the three-storey RC frame in Example 1

Fig. 7.14 Failure location and sequence of the 2D three-storey RC frame in Example 1

Fig. 7.15 Computational time of the 2D three-storey RC frame in Example 1

Nevertheless, the computational time for the numerical models with and without superelement is significantly different. As shown in **Fig. 7.15**, the computational time for the numerical model without superelement is more than twice as much as that with superelement defined. As a result, an evident saving in computational time can be achieved by using the superelement approach and yet the prediction accuracy is maintained.

As presented in **Sections 7.2** and **7.3**, the proposed methodology is independent of the element types and material models in the original numerical models. Since the main objective of the present chapter is to show the improvement in computational efficiency due to the superelement application, a 3D two-storey steel frame validated against experimental results is employed in Example 2, which also illustrates the capability of the proposed superelement formulation for 3D structures. To discuss the suitable conditions in which the superelement application can significantly improve computational efficiency, a high-rise steel-framed structure and a 3D three-storey three-bay structure will be employed with different superelement zones defined in Examples 3 and 4, respectively.

Example 2: A 3D two-storey steel frame with material yield and failure at the first storey

To demonstrate the capability of superelement to simulate responses of 3D structures consisting of elasto-perfectly plastic material, a two-storey beam-column frame with different cross-sectional shapes is employed as shown in **Fig. 7.16 (a)**. A similar one-storey frame has been analysed in **Section 4.5.3** and also validated against results from Marino (1970), Yang and Fan (1988) and Gendy and Saleeb (1993) based on different approaches. The two-storey frame is validated based on results from (Marino 1970; Yang and Fan 1988; Gendy and Saleeb 1993) because the applied loading and material yielding points are located at the first storey. The second storey only undergoes a rigid-body movement since the plastic hinges are confined to the first storey.

Fig. 7.16 Example 2: A 3D frame with material yield and failure at the first storey

The columns and beams in both storeys are made of W10×60 and W18×60 sections, respectively. The material properties are $E_s = 30,000$ ksi (206.9 GPa), $G_s = 11,500$ ksi (79.3 GPa), and $\sigma_y = 34$ ksi (234.48 MPa). Each member is of

length $L = 144$ in (3.655 m) with warping restrained at both ends. All members are discretised by using eight beam elements and the cross-section orientations and fibre discretisation are illustrated in **Fig. 7.16 (b)**.

Firstly, the numerical analysis without superelement is conducted and the corresponding result is compared with available simulation results (Marino 1970; Yang and Fan 1988; Gendy and Saleeb 1993). As shown in **Fig. 7.17**, the fibre model beam element (Long et al. 2012b) produces satisfactory prediction for the two-storey frame subjected to external loads. Then a numerical model (**Fig. 7.18 (a)**) with all elements at the second storey defined as a superelement is employed and compared with the full model without superelement. The deformation and the displacement versus loading curves are shown in **Figs. 7.18 (b)** and **7.19**, respectively. To achieve an 8-inch deformation in the Z direction at node 2, a comparison of CPU time needed to complete the analysis is presented in **Table 7.2**. To demonstrate the amount of CPU savings, the comparison of CPU time for the analyses with and without superelement plotted against the load increment numbers is shown in **Fig. 7.20**.

As shown in **Fig. 7.19**, the predictions with and without superelement for the ultimate strength of the structure are very close. However, the computational efficiency is not significantly improved at initial load increments, since the nonlinear zone consists of only one-half of the structure. Moreover, after large deformation has occurred at the first storey, additional constraint from the second storey will be applied at the first storey and the second storey will play a critical role to sustain the structure, which results in a small discrepancy between the models with and without superelement. To better demonstrate the efficiency improvement, an eleven-storey steel frame with the same material properties and cross-sectional dimensions as Example 2 will be conducted next.

Fig. 7.17 Result comparisons for Example 2

Fig. 7.18 Numerical model and deformation for Example 2

Fig. 7.19 Displacement versus loading curves for critical points in Example 2

Fig. 7.20 CPU time comparison for Example 2 with and without superelement

Table 7.2 Comparisons of computational cost and CPU time for Example 2

2-storey frame	With superelement	Without superelement	Ratio (%)
No. of nodes	137	252	54.4
Computational time (s)	461.0	689.6	66.9

Example 3: An eleven-storey steel frame with material yield and failure at the first two storeys

To illustrate the advantages of superelement in the analysis of localized material nonlinearity problems, the material properties and cross-sectional dimensions for both beams and columns in Example 2 for a two-storey frame is employed for an eleven-storey frame as shown in **Fig. 7.21 (a)**. The same loading condition shown in **Fig. 7.16** is used again in the first storey. The numerical model and corresponding deformation are shown in **Fig. 7.21 (b)**. In the model with superelement, all elements above the first two storeys are combined as a superelement. The displacement versus loading curves for node 2 and node 1365 are shown in **Fig. 7.22**. The comparisons of the required CPU time for the 3D eleven-storey frame is listed in **Table 7.3**. The comparison of CPU time for the models with and without superelement for different numbers of load increments is shown in **Fig. 7.23**, after the deformation in Z direction of node 2 has achieved 8 inches.

As shown in **Fig. 7.22**, the results of predictions with and without superelement for the ultimate strength of the structure agreed well. In addition, from **Fig. 7.23** and **Table 7.3**, it is very obvious that the model with superelement significantly improved the computational efficiency.

In Examples 2 and 3, both superelements are of the second type, whereby they are not directly fixed onto the foundation. To demonstrate an improvement for the first type of superelement, a 3D three-storey, three-bay steel frame with the same material properties and cross-sectional dimensions are conducted in Example 4. Meanwhile, since reinforced concrete framed structures are the main objective of this thesis, the application of superelement for 3D reinforced concrete structures are also studied in Example 4. Nonetheless, it should be noted that the proposed joint model in **Chapter 5** is two dimensional. Therefore, no beam-column joint element is incorporated in the last example.

Fig. 7.21 Example 3: Numerical model and deformation for a 3D eleven-storey frame

Fig. 7.22 Displacement versus loading curves for critical points in Example 3

Fig. 7.23 CPU time comparison for Example 3 with and without superelement

Table 7.3 Comparisons of computational cost and CPU time for Example 3

11-storey frame	With superelement	Without superelement	Ratio (%)
No. of nodes	253	1368	18.5
Computational time (s)	738.2	3390.5	21.8

Example 4: A 3D three-storey three-bay frame with a horizontal point load at the first storey

The numerical model of the 3D three-storey three-bay frame is shown in **Fig. 7.24**. To avoid too much computational cost to analyse the full model without superelement, only three storeys are made in the present example. Both steel and reinforced concrete materials are assigned and studied for this frame. For the steel framed structure, the cross-sectional dimension, material properties and element discretisation for each beam and column member are identical to that in Examples 2 and 3. Compared with the model of the steel framed structure, only the cross-sectional properties are different for the reinforced concrete structure. For convenience, the cross-section shapes and material properties in Example 1

are employed. The Kent and Park model as one of the uniaxial concrete models is utilized.

To simulate a car bombing accident, a horizontal point load is applied at the middle point of one column at the first storey as shown in **Fig. 7.24 (c)**. With increasing the point load H , steel column will yield at the loading point and at the two ends of the column. Later, the yielding zone will propagate to other elements of the column. Therefore, it is assumed that material nonlinearity is localized within the column and the other structural members still remain elastic and can be defined as a superelement as shown in **Fig. 7.24 (d)**.

Fig. 7.24 Example 4: Numerical model for a 3D three-storey three-bay frame

After analysing the numerical models with and without superelement, displacement versus loading curves at the loading point are compared in **Figs. (a) of Figs. 7.25 and 7.26**, respectively, for both steel and reinforced concrete structures. As shown in **Figs. (a) of Figs. 7.25 and 7.26**, good agreement in terms of displacement response at the loading point is achieved before and after material nonlinearity has occurred. This means the assumption about the superelement zone is reasonable for both steel and reinforced concrete structures. However, it is noteworthy that compared with the reinforced concrete frame, better agreement is obtained for the steel frame. This implies that material nonlinearity may have already propagated to the superelement zone in the reinforced concrete frame and, therefore, the constraint on the nonlinear zone is overestimated which results in a higher load capacity, while material nonlinearity in the model of steel frame is more localized compared to the reinforced concrete frame. Since International System of Units in the empirical formulae of the concrete model in **Chapter 3** is employed, all the dimension and material values are converted correspondingly.

As shown in **Figs. (b) of Fig. 7.25 and 7.26**, even though different load increments in the models with and without superelement are taken to achieve the same deformation at the loading point, the computational time of the model with superelement is significantly decreased. To illustrate the efficiency improvement when using superelement in the numerical model, the computational cost and CPU time are listed in **Table 7.4** as well. It can be easily seen that the efficiency of the simulation with superelement is improved tremendously for both steel and reinforced concrete structures, particularly in the context of 3D three-storey, three-bay steel framed simulations.

(a) Displacement versus loading curves at the loading point

(b) CPU time

Fig. 7.25 Comparisons for the steel frame with and without superelement in Example 4

(a) Displacement versus loading curves at the loading point

(b) CPU time

Fig. 7.26 Comparisons for the reinforced concrete frame with and without superelement in Example 4

Table 7.4 Comparisons of computational cost and CPU time for Example 4

3-bay 3-storey frame	With superelement	Without superelement	Ratio (%)
No. of nodes	18	1864	0.97
Computational time for steel frame (s)	313.5	17621.0	1.78
Computational time for reinforced concrete frame (s)	1653.7	46583.6	3.55

7.5 Closure

In this chapter, a new superelement formulation is proposed and it improves significantly the efficiency of structural deformation analysis and can be directly implemented into an existing finite element program. The proposed methodology is independent of the element types and material models in the original numerical models. The investigated numerical examples in this chapter demonstrate that the proposed superelement formulation can be applied to both 2D and 3D multi-storey steel and RC frames and gives accurate results when compared with full nonlinear analysis. However, it is noteworthy that the definition of superelement in the numerical model should be determined reasonably and can only be applied at the region with elements undergoing elastic and small deformations.

The improvement of computational efficiency with the application of superelement will be much more obvious when a large portion of the structure is converted to the superelement zone. However, due to the preparation steps of superelement prior to the start of incremental-iterative solution, the simulation with the proposed superelement formulation may be relatively slower than the modelling without superelement in the first few increments. Nevertheless, the application of the proposed superelement formulation will be much more

advantageous if a larger number of increments are applied in the nonlinear analysis.

It should be noted that the current formulation of superelement can only be applied to simulations with material nonlinearity localized in certain critical structural members and this region does not dramatically spread throughout the whole domain. However, the obvious limitation can be eliminated by adaptively defining the zone of superelement and such development will be considered in the future work.

Chapter 8 Conclusions and Future Research

8.1 Introduction

The main objective of this thesis is to numerically assess the potential for progressive collapse of reinforced concrete structures. For this purpose, concrete models, a co-rotational beam finite element formulation and a component-based joint model are proposed to facilitate efficient finite element analysis. As an integrated system, the proposed concrete models, beam element formulation and the joint model are implemented into a self-developed finite element package FEMFAN3D in NTU, Singapore. A study is conducted at the system level to validate the prediction capability of the integrated approach for progressive collapse analysis of reinforced concrete structures. As an efficient solution for the finite element structure analysis, a superelement formulation is proposed without significant loss in accuracy.

In the following sections, all the conclusions in this thesis are summarized. In light of the limitations of the proposed approach for assessing the potential for progressive collapse analysis of reinforced concrete structures, several promising ideas for future research projects are drawn up and discussed at the end.

8.2 Concrete Models in the Simulations of Beam-Column Framed Structures

In **Chapter 3**, both uniaxial concrete models and plasticity-based model are proposed for the beam finite element to simulate the behaviour of beam-column members made of concrete and reinforced by steel bars.

As a stable and efficient constitutive model of concrete, uniaxial models predict the cracking and crushing failures fully based on the independent normal and shear stress components in the beam element. Such a kind type of concrete model gives satisfactory deformation predictions of beam-column structures with flexural failures where only the normal stress dominates the failure state.

However, they have intrinsic limitations when dealing with combined stress states, such as shear dominant failure.

However, in general, the material stress state is three dimensional and hence, unified plasticity concrete model is proposed for fibre beam element formulations. This plasticity-based model takes all the three stress components into consideration by appropriately formulating the beam uniaxial strain and stress states from a 3D solid element.

Comparing these two types of concrete models, it is obvious that uniaxial concrete models (the Kent and Park model and the Mander's model) provide a simple, stable and efficient tool to predict flexural failures of concrete along the beam longitudinal direction. On the other hand, unified plasticity concrete model is capable of accurately predicting complicated stress states of shear failures in beam members with short and medium shear span-to-depth ratios.

8.3 Simulations of Reinforced Concrete Beam-Column Structural Members

In **Chapter 4**, an elegant co-rotational beam formulation with vectorial rotational vectors to describe the 3D spatial rotation is incorporated with fibre model and uniaxial and plasticity-based concrete models in **Chapter 3**. Detailed strain and stress profiles along the beam cross-sections can be obtained. Furthermore, the fibre model allows different cross-sections and reinforcement detailing. The proposed co-rotational beam formulation is shown to be capable of accurately predicting (a) the beam geometric nonlinearity due to large displacement and rotations, and (b) material nonlinearity due to yielding and fracturing of steel reinforcement and cracking and crushing of concrete. This lays the most critical foundation for a simulation tool for assessing progressive collapse of reinforced concrete structures. Based on the comparison with a total Lagrangian formulation (Dvorkin et al. 1988), the simulations by the proposed co-rotational formulation are more efficient and accurate, and fewer elements are required to produce the same accuracy for elasto-plastic problems.

In order to identify the advantages and disadvantages of two types of concrete models, viz. uniaxial versus plasticity-based concrete models, different concrete models are tested for the same numerical models. Compared with uniaxial concrete models, plasticity-based model is more accurate in cases when shear failure is dominant. Nevertheless, in order to satisfy equilibrium by using the proposed flow rule between failure surface and potential surface, a great number of iterations are needed to maintain the beam simplification in uniaxial strain and stress states, especially when large plastic strain occurs. Therefore, the computational efficiency and stability of numerical models with uniaxial concrete model is better than those with unified plasticity concrete model when severe concrete crushing occurs, even though the latter may yield marginally more accurate predictions of load capacities.

On the whole, along with the concrete models, the proposed co-rotational 3D beam element yields a satisfactory numerical model to model both steel and reinforced concrete structures with arbitrary cross-sectional shapes undergoing geometric and material nonlinearities. Furthermore, this co-rotational beam formulation is an efficient approach to simulate the deformations of steel and reinforced concrete framed structures for resistance to progressive collapse.

8.4 Component Calibrations in the Reinforced Concrete Joint Model

To more realistically simulate the joint behaviour in reinforced concrete framed structures, a component-based mechanical model is employed in **Chapter 5** to consider different types of potential failures in the joint region. To achieve reasonable predictions using the mechanical model, calibrations for different types of components are crucial. In general, the bar-slip component and the shear-panel component are dominant in joint local behaviour.

A simple and yet reliable analytical model based on bi-uniform bond stress distribution is proposed to predict the relationship between slip at the loaded end and applied load in reinforced concrete joints. The proposed model accounts for all the possible cases including insufficient embedment length of steel reinforcement by means of the proposed concept of stress propagation length in

the axial pullout case. Besides axial pullout, transverse dowel action of steel reinforcement at the joint region due to inclination of pullout force with respect to the undeformed beam direction is also incorporated in the proposed analytical model.

For the shear-panel component, a new analytical model is proposed to predict all the critical stages and provide complete shear stress-strain responses of shear panels in 2D reinforced concrete beam-column joints subjected to monotonic shear loading. The proposed model is derived based on average stress and strain fields and load transfer mechanisms, with satisfying compatibility, equilibrium and constitutive law for both concrete and steel reinforcement. Through validations against experimental studies and other analytical models (the modified compression field theory (MCFT) and the strut and tie (SAT) model), the proposed analytical model is shown to be capable of providing stable and reliable predictions on the shear stress-strain relationships of 2D reinforced concrete interior and exterior joints subjected to monotonic shear loading. Theoretically speaking, the proposed analytical model can be applied to knee joints. However, experimental studies on knee joints are limited. Even though the prediction has been validated against the experimental result and solid element simulations on one knee joint so far, it is incorrect to claim that the model can be applicable to knee joints. With more experimental studies for knee joints to validate the mechanical model, then it can be said that the 2D joint model study is completed.

In addition, an empirical model for interface-shear component is also proposed based on design regulations and experimental studies on shear beams in the literature.

8.5 System Level Study of 2D Reinforced Concrete Framed Structures

Even though the individual modules have been validated against experimental studies, research at the system level is crucial to validate the prediction capability of the integrated approach for progressive collapse resistance of reinforced concrete structures. Firstly, through different types of beam-column

subassemblages, prediction accuracy, numerical stability and computational time are discussed in terms of the component-based joint model and different types of concrete models. The findings are concluded as follows:

- Compared with uniaxial concrete models, the simulations with unified plasticity concrete model shows more accurate deformations but the advantage is limited since concentrated material nonlinearity is greatly alleviated by joint element.
- For knee and exterior joints with joint shear failures, no matter which type of concrete model is employed, the numerical models with a joint model provide much more accurate predictions than the numerical models without a joint model. Furthermore, with a joint model, predictions in the form of a complete load-displacement response can be obtained.
- For interior joints studied with flexural failures, bar-slip behaviour and concrete crushing dominate the joint behaviour. The numerical model using the proposed unified plasticity concrete model is difficult to converge due to the severe crushing of concrete after the peak load capacity but prior to catenary action; when uniaxial concrete model is employed, a complete load-displacement response including concrete crushing and catenary action can be observed. Moreover, the proposed bar-slip component can represent well the fixed end rotations and in some specimens, the predicted ultimate load capacity due to catenary action by the numerical model with a joint model is more accurate.
- For all the knee, exterior and interior joints, the numerical model with joint model is more efficient than that without joint model. This is because the incorporation of joint model can alleviate computational difficulties due to material nonlinearity at the joint region.

Secondly, with comparison of the experimental results of a three-storey 2D framed structure, the prediction capability of the proposed approach is validated for a full-scale framed structure. It is found that when compared with the numerical models with joint models for all types of concrete models, numerical

models without joint models significantly overestimate the load capacity of the three-storey frame. This means that besides fracturing failure of steel reinforcement, the bar-slip behaviour in the middle joint should also be taken into account for more realistic simulations, that is, a joint model should be utilized instead of relying only on the fibre model. In addition, good agreement is generally achieved in terms of structural deformation and failure mode. Thus, it is confirmed that the numerical approach is well formulated and the calibration of the component-based joint model is satisfactory.

At last, a three-storey frame and a five-storey frame are analysed under different column-removal scenarios to demonstrate the robustness of the proposed numerical approach to predict the deformation behaviour of full-scale framed structures with the potential of progressive collapse in practice. In order to model severe failure propagation, column removals are studied in the following sequence: an exterior column, a penultimate column, and a middle column. Under all the different column-removal scenarios, the proposed joint model is capable of working integrally with the beam elements to consider internal force redistributions. Meanwhile, the proposed joint model brings about certain fixed end rotations to local joint behaviour but does not reduce the load capacity.

8.6 Efficient Simulation Approach Based on Superelement

It is time-consuming to numerically analyse full-scale structures with nonlinear behaviour. With the aim to significantly reduce the computational cost without loss of any critical information, a superelement formulation is proposed in **Chapter 7** for structures with localized material nonlinearity. The proposed methodology is straightforward and can be implemented into an existing finite element program with little effort. It should be noted that the proposed methodology is independent of the element types and material models in the original numerical models.

With the validations against several examples of both steel and reinforced concrete structures, compared with the simulation without superelement, an evident saving in computational time can be achieved by using the superelement

concept. Accurate results are obtained for both 2D and 3D multi-storey steel and reinforced concrete frames when compared with full nonlinear analysis. To take full advantage of the superelement approach, the portion of the structure that is assigned as the superelement zone should be as large as possible. However, the definition of superelement in the numerical model should be reasonably determined and can only be applied at the region with members undergoing elastic deformation.

8.7 Future Research

When integrating the proposed concrete models, beam elements and component-based joint model to analyse the potential of progressive collapse of reinforced concrete structures, several future research ideas are indicated and discussed herein.

- The proposed reinforced concrete joint model accounts for some important structural parameters. Nevertheless, the effects of certain parameters, such as column axial stress, have not been thoroughly understood. In addition, the component calibration for reinforced concrete joints should be more general and consider additional structural parameters, such as the dimension of joint influence zone in the adjacent beam and column members due to the formation of plastic hinge and also its effect on the joint rotation capacity.
- In reality, all joints are three-dimensional, in which the shear transfer region is a three-dimensional block with the interaction of torsional and bending moments rather than a two-dimensional panel with only in-plane shear as simplified. Therefore, experimental studies on three-dimensional reinforced concrete joints need to be conducted and fundamental structural parameters should be investigated for calibration of the shear block in the three-dimension space.
- In order to understand the dynamic behaviour of reinforced concrete structures for progressive collapse, dynamic amplification factor as a convenient method should be investigated to characterize the dynamic

effect based on the results of static analysis. The value of 2.0 in General Services Administration (GSA) (2003) and the United States Department of Defense (DoD) (2009) is recommended, which is however considered to be highly conservative and more studies need to be conducted.

- Conventional simplified numerical models (GSA (2003) and DoD (2009)) for progressive collapse analysis do not consider the effects of slabs and walls. Such a simplification makes sense for pre-cast reinforced concrete structures with limited integrity between beams and slab members. However, this seriously underestimates the resistance of cast-in-place reinforced concrete structures. To address this effect, the beam cross-section is usually modified and validated to account for the effect of slabs (Sasani 2008). Alternatively, fully 3D structures with slabs and walls are necessary. Consequently, shell and plate elements should be incorporated in the finite element analysis.
- Lastly, the proposed superelement formulation is capable of dramatically improving the computational efficiency of large-scale structure simulations. Current formulation can only be applied to simulations with material nonlinearity localized in certain critical structural members. This limitation can be eliminated by adaptively defining the zone of superelement, that is, the nonlinear zone is allowed to grow with load increments. Consequently, sensible element and member criteria need to be proposed.

References

- Abaqus/Standard user's manual 6.9.1 (2009). Hibbit, Karlsson & Sorensen, Inc., Pawtucket, Rhode Island, U.S.
- ACI-ASCE Committee 352 (2002). "Recommendations for design of beam-column joints in monolithic reinforced concrete structures (ACI 352R-02)." *American Concrete Institute, Farmington Hills, MI*.
- Agrawal, O. P., Danhof, K. J. and Kumar, R. (1994). "A superelement model based parallel algorithm for vehicle dynamics." *Computers & Structures* **51**(4): 411-423.
- Alameddine, F. F. (1990). *Seismic design recommendations for high-strength concrete beam-to-column connections*. PhD thesis, University of Arizona.
- Alath, S. and Kunnath, S. K. (1995). Modeling inelastic shear deformation in RC beam-column joints. *Proceedings of the 10th Conference on Engineering Mechanics. Part 1 (of 2), May 21, 1995 - May 24, 1995*, Boulder, CO, USA, ASCE.
- Alsiwat, J. M. and Saatcioglu, M. (1992). "Reinforcement anchorage slip under monotonic loading." *Journal of Structural Engineering-ASCE* **118**(9): 2421-2438.
- Altoontash, A. (2004). "Simulation and damage models for performance assessment of reinforced concrete beam-column joints." *PhD dissertation, Stanford University*.
- American Concrete Institute (ACI) 318-02 (2002). "Building Code Requirements for Structural Concrete."
- Arabzadeh, A., Rahaie, A. R. and Aghayari, R. (2009). "A simple strut-and-tie model for prediction of ultimate shear strength of RC deep beams." *International Journal of Civil Engineering* **7**(3): 141-153.
- Argyris, J. (1982). "An excursion into large rotations." *Computer Methods in Applied Mechanics and Engineering* **32**(1-3): 85-155.
- ASCE-ACI Task Committee 426 (1973). "The Shear Strength of Reinforced Concrete Members." *Journal of the Structural Division-ASCE* **99**(6): 1091-1187.
- Bakir, P. G. and Boduroğlu, H. M. (2002). "A new design equation for predicting the joint shear strength of monotonically loaded exterior beam-column joints." *Engineering Structures* **24**(8): 1105-1117.
- Bao, J. Q., Long, X., Tan, K. H., et al. (2012). "A generalized Drucker-Prager potential flow for concrete under compression (under revision after review)." *Engineering Structures*.
- Bao, Y. H., Kunnath, S. K., El-Tawil, S., et al. (2008). "Macromodel-based simulation of progressive collapse: RC frame structures." *Journal of Structural Engineering-ASCE* **134**(7): 1079-1091.
- Barzegar-Jamshidi, F. (1987). *Non-linear finite element analysis of re-inforced concrete under short term monotonic loading*. PhD thesis, University of Illinois at Urbana-Champaign.
- Bathe, K. J. and Bolourchi, S. (1979). "Large displacement analysis of three-dimensional beam structures." *International Journal for Numerical Methods in Engineering* **14**(7): 961-986.

- Battini, J. M. (2008). "A non-linear corotational 4-node plane element." *Mechanics Research Communications* **35**(6): 408-413.
- Bayo, E., Cabrero, J. M. and Gil, B. (2006). "An effective component-based method to model semi-rigid connections for the global analysis of steel and composite structures." *Engineering Structures* **28**(1): 97-108.
- Bazant, Z. P. and Caner, F. C. (2005). "Microplane model M5 with kinematic and static constraints for concrete fracture and anelasticity. I: Theory." *Journal of Engineering Mechanics-ASCE* **131**(1): 31-40.
- Bazant, Z. P., Caner, F. C., Carol, I., et al. (2000). "Microplane model M4 for concrete I: Formulation with work-conjugate deviatoric stress." *Journal of Engineering Mechanics* **126**(9): 944-953.
- Belesis, S. and Labeas, G. (2010). "Development of an efficient engineering methodology for non-linear damage and post-buckling analysis of large-scale structures." *International Journal of Structural Integrity* **1**(2): 126-139.
- Belyi, M. V. (1993). "Superelement method for transient dynamic analysis of structural systems." *International Journal for Numerical Methods in Engineering* **36**(13): 2263-2286.
- Belytschko, T., Liu, W. K. and Moran, B. (2000). *Nonlinear finite elements for continua and structures*. New York, U.S.
- Binici, B. (2005). "An analytical model for stress-strain behavior of confined concrete." *Engineering Structures* **27**(7): 1040-1051.
- Birely, A. C., Lowes, L. N. and Lehman, D. E. (2012). "A model for the practical nonlinear analysis of reinforced-concrete frames including joint flexibility." *Engineering Structures* **34**(1): 455-465.
- Bresler, B. and Scordelis, A. C. (1963). "Shear strength of RC beams." *ACI Journal* **60** (1): 51-74.
- Cardona, A. (2000). "Superelements modelling in flexible multibody dynamics." *Multibody System Dynamics* **4**(2-3): 245-266.
- Cardona, A. and Geradin, M. (1988). "A beam finite element non-linear theory with finite rotations." *International Journal for Numerical Methods in Engineering* **26**(11): 2403-2438.
- Cardona, A. and Geradin, M. (1991). "Modeling of superelements in mechanism analysis." *International Journal for Numerical Methods in Engineering* **32**(8): 1565-1593.
- CEB (2010). *CEB-FIP Model Code 2010*. Lausanne, Switzerland.
- Cervenka, J., Cervenka, V. and Eligehausen, R. (1998). Fracture-plastic material model for concrete. Application to analysis of powder actuated anchors. *Proceedings of the 3rd International Conference on Fracture Mechanics of Concrete Structures-FraMCoS 3*, Gifu, Japan.
- Cervenka, J. and Papanikolaou, V. K. (2008). "Three dimensional combined fracture-plastic material model for concrete." *International Journal of Plasticity* **24**(12): 2192-2220.
- Chang, G. A. and Mander, J. B. (1994). Seismic energy based fatigue damage analysis of bridge columns: Part 1 - Evaluation of seismic capacity. Technical Report NCEER-94-0006. National Center for Earthquake Engineering Research, State University of New York, Buffalo, New York.
- Chen, S. H. and Pan, H. H. (1988). "Guyan reduction." *Commun. Appl. Numer. M.* **4**(4): 549-556.

- Comite Euro-International du Beton (CEB) (1985). Design Manual, Cracking and Deformations International system of unified standard codes of practice for structures. **158**.
- Crisfield, M. A. (1990). "A consistent co-rotational formulation for non-linear, three-dimensional, beam-elements." *Computer Methods in Applied Mechanics and Engineering* **81**(2): 131-150.
- Crisfield, M. A. (1996). *Nonlinear finite element analysis of solid and structures*, Wiley, Chinchester.
- de Felice, G. (2009). "Assessment of the load-carrying capacity of multi-span masonry arch bridges using fibre beam elements." *Engineering Structures* **31**(8): 1634-1647.
- De Gerssem, H., Moens, D., Desmet, W., et al. (2007). "Interval and fuzzy dynamic analysis of finite element models with superelements." *Computers & Structures* **85**(5-6): 304-319.
- Dei Poli, S., Di Prisco, M. and Gambarova, P. (1992). "Shear response, deformations, and subgrade stiffness of a dowel bar embedded in concrete." *ACI Structural Journal* **89**(6).
- Department of Defense (DoD) (2005). *Unified Facilities Criteria (UFC): Design of buildings to resist progressive collapse, UFC 4-023-03*. Washington, D.C.
- Department of Defense (DoD) (2009). *Unified Facilities Criteria (UFC): Design of buildings to resist progressive collapse, UFC 4-023-03*. Washington DC.
- Dvorkin, E. N., Onte, E. and Oliver, J. (1988). "On a non-linear formulation for curved Timoshenko beam elements considering large displacement/rotation increments." *International Journal for Numerical Methods in Engineering* **26**(7): 1597-1613.
- Eligehausen, R., Bertero, V. V. and Popov, E. P. (1982). Hysteretic behavior of reinforcing deformed hooked bars in R/C joints. *Proceedings of the Seventh European Conference on Earthquake Engineering*, Athens, Greece.
- Eligehausen, R., Ozbolt, J., Genesio, G., et al. (2006). Three-dimensional Modelling of Poorly Detailed RC Frame Joints. *Proceedings of the New Zealand Society for Earthquake Engineering Conference 2006*, New Zealand.
- Eligehausen, R., Popov, E. P. and Bertero, V. V. (1983). Local bond stress-slip relationships of deformed bars under generalized excitations Report UCB/EERC-83/23, Earthquake Engineering Research Center. University of California, Berkeley, California, U.S.
- Engineer's Studio User's Help 1.00.01 (2009). FORUM8 Co., Ltd., Japan.
- ETABS Version 9.7.4 (2011). Computers and Structures, Inc. Berkeley, California, USA,.
- Favvata, M. J., Izzuddin, B. A. and Karayannis, C. G. (2008). "Modelling exterior beam-column joints for seismic analysis of RC frame structures." *Earthquake Engineering & Structural Dynamics* **37**(13): 1527-1548.
- Felippa, C. A. and Haugen, B. (2005). "A unified formulation of small-strain corotational finite elements: I. Theory." *Computer Methods in Applied Mechanics and Engineering* **194**(21-24): 2285-2335.
- Filippou, F. C., Popov, E. P. and Bertero, V. V. (1983). Effects of bond deterioration on hysteretic behavior of reinforced concrete joints. Report UCB/EERC-83/19. Earthquake Engineering Research Center, University of California, Berkeley, California, U.S.

- Foster, S. J. and Gilbert, R. I. (1996). "The design of nonflexural members with normal and high-strength concretes." *ACI Structural Journal* **93**(1): 3-10.
- Gendy, A. S. and Saleeb, A. F. (1993). "Generalized yield surface representations in the elasto-plastic three-dimensional analysis of frames." *J. Comput. Struct.* **49**(2): 351-362.
- Goto, Y. (1971). "Cracks formed in concrete around deformed tension bars." *ACI Journal* **68**(4): 244-251.
- Grassl, P., Lundgren, K. and Gylltoft, K. (2002). "Concrete in compression: a plasticity theory with a novel hardening law." *International Journal of Solids and Structures* **39**(20): 5205-5223.
- Haach, V. G., Lúcia Homce De Cresce El Debs, A. and Khalil El Debs, M. (2008). "Evaluation of the influence of the column axial load on the behavior of monotonically loaded R/C exterior beam-column joints through numerical simulations." *Engineering Structures* **30**(4): 965-975.
- Hartmann, D., Breidt, M., Nguyen, V. V., et al. (2008). "Structural collapse simulation under consideration of uncertainty - Fundamental concept and results." *Computers & Structures* **86**(21-22): 2064-2078.
- Haskett, M., Oehlers, D. J. and Ali, M. S. M. (2008). "Local and global bond characteristics of steel reinforcing bars." *Engineering Structures* **30**(2): 376-383.
- He, X. G. and Kwan, A. K. H. (2001). "Modeling dowel action of reinforcement bars for finite element analysis of concrete structures." *Computers & Structures* **79**(6): 595-604.
- Hetényi, M. (1946). *Beams on elastic foundation*, The University of Michigan Press, Ann Arbor, MI.
- Hinton, E. and Owen, D. R. J. (1984). *Finite element software for plates and shells*. University College, Swansea, U.K.
- Hong, S. G. and Ha, T. (2012). "Effective capacity of diagonal strut for shear strength of reinforced concrete beams without shear reinforcement." *ACI Structural Journal* **109**(2): 139-148.
- Hsiao, K. M., Horng, H. J. and Chen, Y. R. (1987). "A corotational procedure that handles large rotations of spatial beam structures." *Computers & Structures* **27**(6): 769-781.
- Huang, S. J., Li, P. Z. and Society, I. C. (2010). Superelement method based structure simulation of large pallet structure. *Second International Conference on Computer Engineering and Applications*.
- Hwang, S. J. and Lee, H. J. (1999). "Analytical model for predicting shear strengths of exterior reinforced concrete beam-column joints for seismic resistance." *ACI Structural Journal* **96**(5): 846-857.
- Hwang, S. J. and Lee, H. J. (2000). "Analytical model for predicting shear strengths of interior reinforced concrete beam-column joints for seismic resistance." *ACI Structural Journal* **97**(1): 35-44.
- Hwang, S. J. and Lee, H. J. (2002). "Strength prediction for discontinuity regions by softened strut-and-tie model." *Journal of Structural Engineering-ASCE* **128**(12): 1519-1526.
- Hwang, S. J., Lu, W. Y. and Lee, H. J. (2000). "Shear strength prediction for deep beams." *ACI Structural Journal* **97**(3): 367-376.

- Imam, M., Vandewalle, L., Mortelmans, F., et al. (1997). "Shear domain of fibre-reinforced high-strength concrete beams." *Engineering Structures* **19**(9): 738-747.
- Jacobsen, K. P. (1983). "Fully integrated superelements: a database approach to finite element analysis." *Computers & Structures* **16**(1-4): 307-315.
- Jennwein, M. and Schäfer, K. (1992). *Standardisierte Nachweise von häufigen D-Bereichen (in German)*. DAfStb. Heft No. 430, Beuth-Verlag, Berlin.
- Ju, F. and Choo, Y. S. (2005). "Super element approach to cable passing through multiple pulleys." *International Journal of Solids and Structures* **42**(11-12): 3533-3547.
- Kaku, T. and Asakusa, H. (1991). "Ductility estimation of exterior beam-column subassemblages in reinforced concrete frames." *Design of Beam-Column Joints for Seismic Resistance*: 167-185.
- Karsan, I. D. and Jirsa, J. O. (1969). "Behavior of concrete under compressive loadings." *Journal of the Structural Division, ASCE* **95**(ST12): 2543-2563.
- Kashiwazaki, T. and Noguchi, H. (1996). "Three-dimensional nonlinear finite element analysis on the shear strength of RC interior beam-column joints with ultra high strength materials." *Proceedings of the Eleventh World Conference on Earthquake Engineering* **Paper No. 476**.
- Khalfallah, S. and Ouchenane, M. (2008). "Prediction of bond between steel and concrete by numerical analysis." *The Open Civil Engineering Journal* **2**: 1-8.
- Kim, H. S., Lee, D. G. and Kim, C. K. (2005). "Efficient three-dimensional seismic analysis of a high-rise building structure with shear walls." *Engineering Structures* **27**(6): 963-976.
- Kim, J. and LaFave, J. M. (2009). "Joint shear behavior of reinforced concrete beam-column connections subjected to seismic lateral loading." *NSEL Report Series Report No. NSEL-020*.
- Kim, J. K. and Yang, J. K. (1995). "Buckling behaviour of slender high-strength concrete columns." *Engineering Structures* **17**(1): 39-51.
- Kwak, H. G. and Filippou, F. C. (1990). Finite element analysis of reinforced concrete structures under monotonic loads, Department of Civil Engineering, University of California, Berkeley, California, U.S., Rep. UCB/SESM-90/14.
- Kwak, H. G. and Filippou, F. C. (1997). "Nonlinear FE analysis of R/C structures under monotonic loads." *Computers & Structures* **65**(1): 1-16.
- Lee, D. L. N., Hanson, R. D. and Wight, J. K. (1977). "RC beam-column joints under large load reversals." *Journal of the Structural Division* **103**(12): 2337-2350.
- Lettow, S., Mayer, U., Ozbolt, J., et al. (2004). Bond of RC members using nonlinear 3D FE analysis. *Proceedings of the Fifth International Conference on Fracture Mechanics of Concrete and Concrete Structures. FraMCoS 5*, Colorado, U.S.
- Lew, H. S. (2002). "Case study on Alfred P. Murrah Federal Office Building, Oklahoma City" [doi: www.nist.gov/el/disasterstudies/blast/upload/OklahomaCityLew2002.pdf](http://www.nist.gov/el/disasterstudies/blast/upload/OklahomaCityLew2002.pdf).
- Li, Z. X. (2007). "A co-rotational formulation for 3D beam element using vectorial rotational variables." *Computational Mechanics* **39**(3): 309-322.
- Long, X., Bao, J. Q., Tan, K. H., et al. (2012a). Parametric study on RC beam-column subassemblages under column-removal scenarios by fibre beam

- elements in FEMFAN3D. *The 4th international conference on design and analysis of protective structures*, Jeju, Korea.
- Long, X., Tan, K. H. and Lee, C. K. (2012b). A 3D co-rotational beam element with geometric and material nonlinearities for steel and RC framed structures. *The 4th International Conference on Design and Analysis of Protective Structures*, Jeju, Korea.
- Long, X., Yuan, W. F., Tan, K. H., et al. (2012c). "A superelement formulation for efficient structural analysis in progressive collapse (under review)." *Structural Engineering and Mechanics*.
- Lowes, L. N. and Altoontash, A. (2003). "Modeling reinforced-concrete beam-column joints subjected to cyclic loading." *Journal of Structural Engineering-ASCE* **129**(12): 1686-1697.
- Lowes, L. N., Mitra, N. and Altoontash, A. (2003). A beam-column joint model for simulating the earthquake response of reinforced concrete frames. Pacific Earthquake Engineering Research Center, College of Engineering, University of California.
- Lowes, L. N., Moehle, J. R. and Govindjee, S. (2004). "Concrete-steel bond model for use in finite element modeling of reinforced concrete structures." *ACI Structural Journal* **101**(4): 501-511.
- Lykidis, G. C. and Spiliopoulos, K. (2008). "3D solid finite-element analysis of cyclically loaded RC structures allowing embedded reinforcement slippage." *Journal of Structural Engineering* **134**(4): 629-638.
- Macari, E. J., Weihe, S. and Arduino, P. (1997). "Implicit integration of elastoplastic constitutive models for frictional materials with highly non-linear hardening functions." *Mechanics of Cohesive-frictional Materials* **2**(1): 1-29.
- Maekawa, K. (2003). *Nonlinear mechanics of reinforced concrete*, Taylor & Francis.
- Maekawa, K. and Qureshi, J. (1996a). "Computational model for reinforcing bar embedded in concrete under combined axial pullout and transverse displacement." *Journal of Materials, Concrete Structures and Pavements, JSCE* **31**: 227-239.
- Maekawa, K. and Qureshi, J. (1996b). "Embedded bar behavior in concrete under combined axial pullout and transverse displacement." *Journal of Materials, Concrete Structures and Pavements, JSCE* **30**(532): 183-196.
- Maekawa, K. and Qureshi, J. (1997). "Stress transfer across interfaces in reinforced concrete due to aggregate interlock and dowel action." *Journal of Materials, Concrete Structures and Pavements, JSCE*: 159-172.
- Mander, J. B., Priestley, M. J. N. and Park, R. (1988). "Observed stress-strain behavior of confined concrete." *Journal of Structural Engineering, ASCE* **114**(8): 1827-1849.
- Maressa, A., Mundo, D., Donders, S., et al. (2011). "A wave-based substructuring approach for concept modeling of vehicle joints." *Computers & Structures* **89**(23-24): 2369-2376.
- Marino, S. (1970). *Analysis of space frames*, Thesis presented to Lehigh University, Bethlehem, PA., in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

- Mattiasson, K. (1981). "Numerical results from large deflection beam and frame problems analysed by means of elliptic integrals." *International Journal for Numerical Methods in Engineering* **17**(1): 145-153.
- Mazzoni, S., McKenna, F., Scott, M. H., et al. (2009). *OpenSees Command Language Manual*, Pacific Earthquake Engineering Research Center, University of California, Berkeley, OpenSees version 2.0.
- Megget, L. (1974). "Cyclic behavior of exterior reinforced concrete beam-column joints." *Bulletin of New Zealand National Society for Earthquake Engineering* **7**(1): 22-47.
- Menegotto, M. and Pinto, P. E. (1973). Method of analysis for cyclically loaded R.C. plane frames including changes in geometry and non-elastic behavior of elements under combined normal force and bending. *Proceedings of the International Association for Bridge and Structural Engineering on the resistance and ultimate deformability of structures acted on by well defined repeated loads*, Lisbon, Portugal.
- Mishima, T., Suzuki, A., Shinoda, Y., et al. (1995). "Nonelastic behavior of axial reinforcement subjected to axial and slip deformation at the crack surface." *ACI Structural Journal* **92**(3).
- Mitra, N. (2007). "An analytical study of reinforced concrete beam-column joint behavior under seismic loading." *PhD dissertation, University of Washington*.
- Mitra, N. and Lowes, L. N. (2007). "Evaluation, calibration, and verification of a reinforced concrete beam-column joint model." *Journal of Structural Engineering-ASCE* **133**(1): 105-120.
- Muhamad, R., Ali, M. S. M., Oehlers, D., et al. (2011). "Load-slip relationship of tension reinforcement in reinforced concrete members." *Engineering Structures* **33**(4): 1098-1106.
- Nanakorn, P. and Vu, L. N. (2006). "A 2D field-consistent beam element for large displacement analysis using the total Lagrangian formulation." *Finite Elements in Analysis and Design* **42**(14-15): 1240-1247.
- Noguchi, H. and Kashiwazaki, T. (1992). Experimental studies on shear performances of RC interior column-beam joints with high-strength materials. *Proceedings of the Tenth World Conference on Earthquake Engineering, Vols 1-10*, Rotterdam, A a Balkema.
- Noguchi, H. and Kurusu, K. (1988). Correlation of bond and shear in RC beam-column connections subjected to seismic forces. *Proceedings of the Ninth World Conference on Earthquake Engineering, Vol.IV*, Tokyo-Kyoto.
- Noh, S. Y. (2009). "Tension stiffening model for numerical analysis of RC structures by using bond-slip relationship." *Journal of Advanced Concrete Technology* **7**(1): 61-78.
- Owen, D. R. J., Figueiras, J. A. and Damjanic, F. (1983). "Finite-element analysis of reinforced and prestressed concrete structures including thermal loading." *Computer Methods in Applied Mechanics and Engineering* **41**(3): 323-366.
- Ožbolt, J., Lettow, S. and Kožar, I. (2002). Discrete bond element for 3D finite element analysis of reinforced concrete structures. *Beiträge aus der Befestigungstechnik und dem Stahlbetonbau*, Stuttgart.
- Pang, X. B. D. and Hsu, T. T. C. (1996). "Fixed angle softened truss model for reinforced concrete." *ACI Structural Journal* **93**(2): 197-207.

- Pantazopoulou, S. and Bonacci, J. (1992). "Consideration of questions about beam-column joints." *ACI Structural Journal* **89**(1): 27-36.
- Papanikolaou, V. K. and Kappos, A. J. (2007). "Confinement-sensitive plasticity constitutive model for concrete in triaxial compression." *International Journal of Solids and Structures* **44**(21): 7021-7048.
- Parallel Sparse Direct Solver PARDISO (2011). "Parallel Sparse Direct And Multi-Recursive Iterative Linear Solvers, User Guide Version 4.1.2."
- Park, R., Kent, D. C. and Sampson, R. A. (1972). "Reinforced concrete members with cyclic loading." *Journal of the Structural Division* **98**(7): 1341-1360.
- Park, R. and Paulay, T. (1975). *Ultimate strength design of reinforced concrete structures*. University of Canterbury, Christchurch, New Zealand.
- Park, R., Priestley, M. J. N. and Gill, W. D. (1982). "Ductility of square-confined concrete columns." *Journal of the Structural Division-ASCE* **108**(4): 929-950.
- Park, S. and Mosalam, K. M. (2009). Shear strength models of exterior beam column joints without transverse reinforcement. PEER Technical Report 2009/106, Pacific Earthquake Engineering Research Center, University of California, Berkeley.
- Park, S. and Mosalam, K. M. (2012a). "Analytical model for predicting shear strength of unreinforced exterior beam-column joints." *ACI Structural Journal* **109**(2): 149-160.
- Park, S. and Mosalam, K. M. (2012b). "Parameters for shear strength prediction of exterior beam-column joints without transverse reinforcement." *Engineering Structures* **36**(3): 198-209.
- Park, S. and Mosalam, K. M. (2013a). "Experimental investigation of nonductile RC corner beam-column joints with floor slabs." *Journal of Structural Engineering* **139**(1): 1-14.
- Park, S. and Mosalam, K. M. (2013b). "Simulation of Reinforced Concrete Frames with Nonductile Beam-Column Joints." *Earthquake Spectra* **29**(1): 233-257.
- Patwardhan, C. (2005). *Shear strength and deformation modeling of reinforced concrete columns*. M.S. dissertation, The Ohio State University.
- Paulay, T. and Priestley, M. J. N. (1992). "Seismic design of reinforced concrete and masonry buildings." *John Wiley and Sons, Inc.*
- Peng, Z. Z. and Wang, Z. Q. (2010). "Abaqus FEM analysis on knee joints in reinforced concrete frames." *Journal of Chongqing Jiaotong University (Natural Science)* **29**(5): 677-790.
- Przemieniecki, J. S. (1968). *Theory of matrix structural analysis*. Mc Graw-Hill Publication, New York.
- Qiu, K. P., Zhang, W. H., Domaszewski, M., et al. (2009). "Topology optimization of periodic cellular solids based on a superelement method." *Engineering Optimization* **41**(3): 225-239.
- Rots, J. G., Kusters, G. M. A. and Blaauwendraad, J. (1984). The need for fracture mechanics options in finite element models for concrete structures. *Proceedings of international conference on Computer Aided Analysis and Design of Concrete Structures Part I*, Pineridge Press, Swansea, U.K.
- Russo, G., Zingone, G. and Romano, F. (1990). "Analytical solution for bond-slip of reinforcing bars in R.C. joints." *Journal of Structural Engineering-ASCE* **116**(2): 336-355.

- Samani, A. K. and Attard, M. M. (2012). "A stress-strain model for uniaxial and confined concrete under compression." *Engineering Structures* **41**(8): 335-349.
- Sasani, M. (2008). "Response of a reinforced concrete infilled-frame structure to removal of two adjacent columns." *Engineering Structures* **30**(9): 2478-2491.
- Schäfer, K. (1996). *Strut-and-Tie Models for the Design of Structural Concrete*, Notes of Workshop, Department of Civil Engineering, National Cheng Kung University, Tainan, Taiwan: 140.
- Schulz, M. and Filippou, F. C. (2001). "Non-linear spatial Timoshenko beam element with curvature interpolation." *International Journal for Numerical Methods in Engineering* **50**(4): 761-785.
- Scott, B. D., Park, R. and Priestley, M. J. N. (1982). "Stress-strain behavior of concrete confined by overlapping hoops at low and high-strain rates." *Journal of the American Concrete Institute* **79**(1): 13-27.
- Sezen, H. and Moehle, J. P. (2003). *Bond-slip behavior of reinforced concrete members. fib-Symposium: Concrete Structures in Seismic Regions*, CEB-FIP, Athens, Greece.
- Shi, G., Wang, M., Bai, Y., et al. (2012). "Experimental and modeling study of high-strength structural steel under cyclic loading." *Engineering Structures* **37**(4): 1-13.
- Shima, H., Chou, L. L. and Okamura, H. (1987). "Micro and macro models for bond in reinforced concrete." *Journal of the faculty of engineering, the university of Tokyo* **XXXIX**(2): 133-194.
- Shin, M. and Lafave, J. M. (2004). "Modeling of cyclic joint shear deformation contributions in RC beam-column connections to overall frame behavior." *Structural Engineering and Mechanics* **18**(5): 645-669.
- Soltani, M., An, X. and Maekawa, K. (2005). "Localized nonlinearity and size-dependent mechanics of in-plane RC element in shear." *Engineering Structures* **27**(6): 891-908.
- Soltani, M. and Maekawa, K. (2008). "Path-dependent mechanical model for deformed reinforcing bars at RC interface under coupled cyclic shear and pullout tension." *Engineering Structures* **30**(4): 1079-1091.
- Steenbergen, R. D. J. M. (2007). *Super elements in high-rise buildings under stochastic wind load*. PhD Thesis, Delft University of Technology.
- Talaat, M. and Mosalam, K. M. (2008). *Computational modeling of progressive collapse in reinforced concrete frame structures*. PEER Technical Report 2007/10, Pacific Earthquake Engineering Research Center, University of California, Berkeley.
- Talaat, M. and Mosalam, K. M. (2009). "Modeling progressive collapse in reinforced concrete buildings using direct element removal." *Earthquake Engineering & Structural Dynamics* **38**(5): 609-634.
- Taucer, F. F., Spacone, E. and Filippou, F. C. (1991). *A fiber beam-column element for seismic response analysis of reinforced concrete structures*. UCB/EERC 91/17, Earthquake Engineering Research Center, University of California, Berkeley.
- Taylor, H. P. J. (1974). "The behaviour of in situ concrete beam column joints." *Technical report 42.492, Cement and Concrete Association*.

- Teh, L. and Clarke, M. (1999). "Symmetry of tangent stiffness matrices of 3D elastic frame." *Journal of Engineering Mechanics* **125**(2): 248-251.
- Thierry, M. (2002). "September 11, 2001." doi: [http:// www.e-reading.org.ua/chapter.php/ 38358/ 68/ Meiissan-1Isentyabrya2001.html](http://www.e-reading.org.ua/chapter.php/38358/68/Meiissan-1Isentyabrya2001.html).
- Tsonos, A. G. (2007). "Cyclic load behavior of reinforced concrete beam-column subassemblages of modern structures." *ACI Structural Journal* **104**(4): 468-478.
- Ueda, T., Lin, I. and Hawkins, N. M. (1986). "Beam bar anchorage in exterior column-beam connections." *Journal of the American Concrete Institute* **83**(3): 412-422.
- United States General Services Administration (GSA) (2003). *Progressive collapse analysis and design guidelines for new federal office buildings and major modernization projects*. Washington DC.
- Van Gysel, A. and Taerwe, L. (1996). "Analytical formulation of the complete stress-strain curve for high strength concrete." *Materials and Structures* **29**(193): 529-533.
- Vecchio, F. J. and Collins, M. P. (1986). "The modified compression-field theory for reinforced-concrete elements subjected to shear." *Journal of the American Concrete Institute* **83**(2): 219-231.
- Vecchio, F. J. and Collins, M. P. (1993). "Compression response of cracked reinforced-concrete." *Journal of Structural Engineering-ASCE* **119**(12): 3590-3610.
- Vollum, R. L. and Newman, J. B. (1999). "Strut and tie models for analysis/design of external beam-column joints." *Magazine of Concrete Research* **51**(6): 415-425.
- Walraven, J. (1994). "Rough Cracks Subjected to Earthquake Loading." *Journal of Structural Engineering* **120**(5): 1510-1524.
- Walraven, J. C. (1981). "Fundamental analysis of aggregate interlock." *Journal of the Structural Division, ASCE* **107**(ST11): 2245-2270.
- Wang, G. L., Dai, J. G. and Teng, J. G. (2012). "Shear strength model for RC beam-column joints under seismic loading." *Engineering Structures* **40**(7): 350-360.
- Wang, T. and Hsu, T. T. C. (2001). "Nonlinear finite element analysis of concrete structures using new constitutive models." *Computers & Structures* **79**(32): 2781-2791.
- Waugh, J. D. (2009). *Nonlinear analysis of T-shaped concrete walls subjected to multi-directional displacements*. Ph.D. dissertation, Iowa State University.
- Wikipedia (2012). "Ronan Point." doi: [http:// en.wikipedia.org/ wiki/ Ronan_Point](http://en.wikipedia.org/wiki/Ronan_Point).
- Wilson, E. L. (1974). "The static condensation algorithm." *International Journal for Numerical Methods in Engineering* **8**(1): 198-203.
- Wong, P. K. C., Priestley, M. J. N. and Park, R. (1990). "Seismic resistance of frames with vertically distributed longitudinal reinforcement in beams." *ACI Structural Journal* **87**(4): 488-498.
- Xia, J., Mackie, K. R., Saleem, M. A., et al. (2011). "Shear failure analysis on ultra-high performance concrete beams reinforced with high strength steel." *Engineering Structures* **33**(12): 3597-3609.

- Yang, Y. B. and Fan, H. T. (1988). "Yield surface modeling of I-sections with nonuniform torsion." *Journal of Engineering Mechanics-ASCE* **114**(6): 953-972.
- Yang, Y. B. and Shieh, M. S. (1990). "Solution method for nonlinear problems with multiple critical points." *AIAA Journal* **28**(12): 2110-2116.
- Yankelevsky, D. Z. (1985). "Analytical model for bond-slip behavior under monotonic loading." *Building and Environment* **20**(3): 163-168.
- Yap, S. L. and Li, B. (2011). "Experimental Investigation of Reinforced Concrete Exterior Beam-Column Subassemblages for Progressive Collapse." *ACI Structural Journal* **108**(5): 542-552.
- Yi, W. J., He, Q. F., Xiao, Y., et al. (2008). "Experimental study on progressive collapse-resistant behavior of reinforced concrete frame structures." *ACI Structural Journal* **105**(4): 433-439.
- Youssef, M. and Ghobarah, A. (2001). "Modelling of RC beam-column joints and structural walls." *Journal of Earthquake Engineering* **5**(1): 93-111.
- Yu, J. and Tan, K. H. (2010). Experimental study on catenary action of RC beam-column subassemblages. *3rd fib International Congress*, Singapore.
- Yu, J. and Tan, K. H. (2011). "Experimental and numerical investigation on progressive collapse resistance of reinforced concrete beam column subassemblages." *Engineering Structures* DOI: 10.1016/j.engstruct.2011.08.040.
- Yu, J. and Tan, K. H. (2012a). Bar stress-slip relationship in reinforced concrete joints with large inelastic bar strains. *The 4th International Conference on Design and Analysis of Protective Structures*, Jeju, Korea.
- Yu, J. and Tan, K. H. (2012b). "Structural behavior of reinforced concrete beam column sub-assemblages under a middle column removal scenario." *Journal of the Structural Division, ASCE*
- Zemer, D. T. (1979). Implementation of superelement analysis at the production level. *Proceeding of the MSC/NASTRAN Users' Conference*.
- Zhang, L. and Jirsa, J. O. (1982). "A study of shear behavior for reinforced concrete beam column joints." *Technical Report PMFSEL Report No. 82-1, Department of Civil Engineering, University of Texas at Austin*.
- Zhang, L. X. B. and Hsu, T. T. C. (1998). "Behavior and analysis of 100 MPa concrete membrane elements." *Journal of Structural Engineering-ASCE* **124**(1): 24-34.

Appendix A

$$\mathbf{f}_0 = \mathbf{B}^{(0)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(0)}$$

$$\mathbf{f}_1 = \mathbf{B}^{(1)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(0)} + \mathbf{B}^{(0)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(1)}$$

$$\mathbf{f}_2 = \mathbf{B}^{(2)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(0)} + \mathbf{B}^{(0)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(2)}$$

$$\mathbf{f}_3 = \mathbf{B}^{(3)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(0)} + \mathbf{B}^{(0)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(3)} + \mathbf{B}^{(2)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(1)} + \mathbf{B}^{(1)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(2)}$$

$$\mathbf{f}_4 = \mathbf{B}^{(4)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(0)} + \mathbf{B}^{(1)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(1)} + \mathbf{B}^{(0)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(4)}$$

$$\mathbf{f}_5 = \mathbf{B}^{(5)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(0)} + \mathbf{B}^{(2)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(2)} + \mathbf{B}^{(0)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(5)}$$

$$\mathbf{f}_6 = \mathbf{B}^{(3)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(1)} + \mathbf{B}^{(1)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(3)} + \mathbf{B}^{(2)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(4)} + \mathbf{B}^{(4)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(2)}$$

$$\mathbf{f}_7 = \mathbf{B}^{(5)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(1)} + \mathbf{B}^{(1)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(5)} + \mathbf{B}^{(2)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(3)} + \mathbf{B}^{(3)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(2)}$$

$$\mathbf{f}_8 = \mathbf{B}^{(5)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(4)} + \mathbf{B}^{(3)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(3)} + \mathbf{B}^{(4)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(5)}$$

$$\mathbf{f}_9 = \mathbf{B}^{(4)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(4)}$$

$$\mathbf{f}_{10} = \mathbf{B}^{(5)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(5)}$$

$$\mathbf{f}_{11} = \mathbf{B}^{(4)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(1)} + \mathbf{B}^{(1)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(4)}$$

$$\mathbf{f}_{12} = \mathbf{B}^{(5)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(2)} + \mathbf{B}^{(2)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(5)}$$

$$\mathbf{f}_{13} = \mathbf{B}^{(4)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(3)} + \mathbf{B}^{(3)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(4)}$$

$$\mathbf{f}_{14} = \mathbf{B}^{(5)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(3)} + \mathbf{B}^{(3)\text{T}} \mathbf{D}\boldsymbol{\epsilon}^{(5)}$$

$$\mathbf{K}_{11} = \mathbf{B}^{(4)\text{T}} \mathbf{D} \mathbf{B}^{(1)} + \boldsymbol{\varepsilon}^{(4)\text{T}} \mathbf{D} \frac{\partial \mathbf{B}^{(1)}}{\partial \mathbf{u}_L} + \mathbf{B}^{(1)\text{T}} \mathbf{D} \mathbf{B}^{(4)} + \boldsymbol{\varepsilon}^{(1)\text{T}} \mathbf{D} \frac{\partial \mathbf{B}^{(4)}}{\partial \mathbf{u}_L}$$

$$\mathbf{K}_{12} = \mathbf{B}^{(5)\text{T}} \mathbf{D} \mathbf{B}^{(2)} + \boldsymbol{\varepsilon}^{(5)\text{T}} \mathbf{D} \frac{\partial \mathbf{B}^{(2)}}{\partial \mathbf{u}_L} + \mathbf{B}^{(2)\text{T}} \mathbf{D} \mathbf{B}^{(5)} + \boldsymbol{\varepsilon}^{(2)\text{T}} \mathbf{D} \frac{\partial \mathbf{B}^{(5)}}{\partial \mathbf{u}_L}$$

$$\mathbf{K}_{13} = \mathbf{B}^{(4)\text{T}} \mathbf{D} \mathbf{B}^{(3)} + \boldsymbol{\varepsilon}^{(4)\text{T}} \mathbf{D} \frac{\partial \mathbf{B}^{(3)}}{\partial \mathbf{u}_L} + \mathbf{B}^{(3)\text{T}} \mathbf{D} \mathbf{B}^{(4)} + \boldsymbol{\varepsilon}^{(3)\text{T}} \mathbf{D} \frac{\partial \mathbf{B}^{(4)}}{\partial \mathbf{u}_L}$$

$$\mathbf{K}_{14} = \mathbf{B}^{(5)\text{T}} \mathbf{D} \mathbf{B}^{(3)} + \boldsymbol{\varepsilon}^{(5)\text{T}} \mathbf{D} \frac{\partial \mathbf{B}^{(3)}}{\partial \mathbf{u}_L} + \mathbf{B}^{(3)\text{T}} \mathbf{D} \mathbf{B}^{(5)} + \boldsymbol{\varepsilon}^{(3)\text{T}} \mathbf{D} \frac{\partial \mathbf{B}^{(5)}}{\partial \mathbf{u}_L}$$