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PERFORMANCE ANALYSIS ON APPLICATIONS OF LV DISTRIBUTION

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Summary

Time-frequency analysis is widely used in revealing the joint time-frequency characteristics of non-stationary signals which have been used in many fields, such as sonar, wireless communication, radar target recognition and imaging. Many methods have been developed for processing these signals, such as the short-time Fourier transform (STFT), Wigner-Ville distribution (WVD), local polynomial periodogram (LPP), polynomial Fourier transform (PFT), fractional Fourier transform (FrFT), etc. Each of these methods has inherent merits and shortcomings, which lead to limited applications. For example, bilinear transforms, e.g., the WVD, provide high energy concentration but suffer from cross-terms. Linear transforms including STFT and PFT are free of cross-terms but can not achieve good signal concentration. Many efforts have been devoted to find methods having both high concentration and negligible cross-terms, usually by using various time-frequency windows, or kernels. However, there always exists a trade-off between auto-terms and cross-terms in kernel based distributions, which often limits further improvement on signal concentration in the time-frequency domain.

Recently, a new time-frequency analysis method named Lv distribution (LVD) was proposed for multi-component linear frequency modulated (LFM) signals. The LVD is able to provide high signal concentration with negligible cross-terms. Different from the FrFT which indirectly provides estimations of frequency and chirp rate by searching rotation angles, the LVD directly represents the LFM signals in the frequency-chirp rate domain mainly by using a scaling operation on the time and lag variables. In principle,

the LVD is bilinear, but simultaneously has the property of linear transforms based on coherent integration. Therefore it is considered to have advantages of both bilinear and linear transforms. However, since the LVD is very new, there are many challenges remaining. This thesis focuses on various related issues of LVD, including performance analysis, signal reconstruction, extensions and applications, to provide thorough analysis and performance of the LVD.

First, some important performances of LVD including frequency-chirp rate resolution, signal energy concentration, detection, representation errors and computational complexity are discussed theoretically and experimentally. An efficient method is supposed to have the following characteristics: high resolution to distinguish signals closely located in the transform domain; high energy concentration to achieve high signal detection probability; low representation errors to estimate parameters of the signal accurately and low computational complexity to save costs of time and resources. Based on theoretical analysis and experimental results, it is shown that the LVD achieves desirable performances on these issues. Furthermore, comparison with other methods, such as FrFT, shows that LVD is preferred for LFM signal processing.

Second, since the LVD has such impressive merits over other approaches theoretically, we will verify these advantages with applications. Three applications based on the LVD are presented. The first one is to use LVD to estimate chirp rates for the time-frequency representation (TFR) based on LPP. The parameter estimation is a key step for LPP to achieve a high quality TFR. In previous work, the PFT has been employed to estimate the chirp rates, resulting poor performance in heavy noise environments, because of the low accuracy in the parameter estimation. We aim to improve the LPP by using LVD instead of PFT. In the second application, we propose short-time Lv transform (STLVT) for processing long signal sequence. The STLVT is designed by applying the inverse Lv distribution (ILVD) to process consecutive segments of a long data sequence, where the

ILVD is a transform to represent the signal in the time-frequency domain. Furthermore, it can reveal non-linear frequency modulated signals while the LVD can not. For an application on the spectrum sensing for cognitive radio, we propose a method to jointly use the STLVT and Hough transform. The third application is to employ the LVD to suppress LFM interference for spread spectrum communications. To effectively eliminate intentional multiple component interference in communication systems is challenging, this is mainly due to the difficulties in accurately estimating the parameters of interference components, particularly in low signal-to-noise ratio (SNR) environments. Based on two concepts of interference cancellation and phase cancellation plus filtering, we propose adaptive interference suppression methods by using the LVD to estimate parameters of the jammer. To measure the concentration degree of LVD, a high-order statistics named skewness is also used. Simulation results based on Monte Carlo experiments show the proposed methods suppressing the interference effectively.

Third, we investigate the signal reconstruction from the LVD domain to the time domain, and propose two signal recovery methods, named the LVD based signal reconstruction (LSR) and the LSR with suppressed cross-terms (LSRC), where the latter is an improvement of the former. Both of them can accurately recover the signal with small errors. Based on LSR and LSRC, applications including signal decomposition, de-noising and re-sampling are presented and verified, showing that they can effectively remove the noise in negative SNR environments and re-sample the signal to an arbitrary new sampling frequency with trivial errors.

Finally, we extend LVD to a more general case, proposing the two-dimensional (2D) LVD for high dimensional applications (e.g., image processing). The 2D LVD, which is extended from the 1D LVD to a 2D version, inherits the properties from the 1D LVD so that it has both advantages of bilinear and linear methods to achieve high concentration and trivial cross-terms. At last, an application of the new 2D technique

on image watermarking by using 2D LFM signals is presented. Simulations show that the 2D LVD works well against various attacks.

The LVD has been considered to be a better analysis method for LFM signals. However, it has some limitations. It is not proper to be used for signals with arbitrary order of polynomial phase since the LVD is particularly designed for signals with the 2nd order polynomial phase. The LVD could not provide high resolution of chirp rate for signals whose time duration is short due to the uncertainty principle.

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List of Abbreviation

AISL	adaptive interference suppression based on Lv distribution
CR	cognitive radio
CRLB	Cramer-Rao low bound
DCR	distribution concentration ratio
FT	Fourier transform
FCR	frequency-chirp rate representation
FFT	fast Fourier transform
FM	frequency modulated
FrFT	fractional Fourier transform
GT	Gabor transform
HT	Hough transform
IF	instantaneous frequency
ILVD	inverse Lv distribution
LFM	linear frequency modulated
LHT	local polynomial periodogram-Hough transform
LPP	local polynomial periodogram
LSR	Lv distribution based signal reconstruction
LSRC	LSR with suppressed cross-terms
LVD	Lv distribution
MSE	mean square error
PFT	polynomial Fourier transform
PPS	polynomial phase signal
PSIAF	parametric symmetric instantaneous auto-correlation function
SLHT	short-time Lv transform-Hough transform
SNR	signal-to-noise ratio
SPWVD	smoothed pseudo Wigner-Ville distribution
STFT	short-time Fourier transform
STLVT	short-time Lv transform
TFR	time-frequency representation
WHT	Wigner-Hough transform
WM	wireless microphone
WSR	Wigner-Ville distribution based signal reconstruction
WVD	Wigner-Ville distribution

Chapter 1

Introduction

1.1 Background and Motivation

Time-frequency analysis has been widely recognized to be a useful tool to reveal the characteristics of signal frequencies varying with time for many practical applications, such as music, speech, geology, wireless communication and radar [1–3]. The chirp waveform, also called the linear frequency modulated (LFM) signal, is ubiquitously present in both natural and artificial signals. For instance, natural signals including seismic signals [4, 5] and sounds of birds, bats, whales, frogs and whistling [6–8], as well as man-made signals included in radar [3, 9–13], sonar [14], biomedical engineering [15, 16], communications [17–23] and imaging processing [24], typically have the characteristics of LFM signals. The following are some examples.

- In the direct sequence spread spectrum (DSSS) communications, the LFM interference may jam the transmission band and significantly degrade the communication performance. A pre-processing before the decoding to suppress the interference is indispensable for improving the performance of bit error rate. Therefore detection/estimation of the LFM interference plays an important role in the DSSS communication system.

- In biomedical engineering, the electroencephalogram (EEG) is a popular method to measure the electrical activity of human brain. The EEG signals have been reported to be non-stationary and exhibit features related to the LFM signals [25]. In literature, many time-frequency methods were proposed for the analysis of EEG signals to characterize their chirp-like features.
- In radar systems, many architectures are based on the LFM signals. A classic type of pulse compression radars employs the LFM signals. In cases of synthetic aperture radar (SAR), inverse synthetic aperture radar (ISAR), moving target indicator (MTI) radar and continuous-wave (CW) radar, the signals can also be modeled as chirps.

The instantaneous frequency (IF) of an LFM signal, defined as the derivative of phase, varies linearly with time. It is not proper to use the Fourier transform (FT) for representing chirps since the FT can not show when the frequencies occur. It is widely recognized that to characterize these signals, time-frequency analysis methods, such as the short-time Fourier transform (STFT) and Wigner-Ville distribution (WVD), are much more suitable than the simple FT [26]. They provide details of the joint information of time and frequency, to show how the frequency varies with time. Based on this advantage over the FT, time-frequency analysis has been employed in various applications, such as image processing [27], radar [28, 29], communications [30], music and audio processing [1, 2, 31].

Many approaches for the time-frequency analysis have been reported in literature for the LFM signals. Basically they could be classified into the categories of time-frequency representation (TFR) and frequency-chirp rate representation (FCR). The TFR, such as the STFT, local polynomial periodogram (LPP) [32, 33] and Cohen's class [26], produces a distribution of time-frequency to display the frequency variation with time. Another

category, the FCR, provides a distribution of parameters of the LFM signal, i.e., the frequency and chirp rate. For example, the polynomial FT (PFT) [34, 35] and fractional FT (FrFT) [36–40] are able to produce the FCR plane containing all the information of a signal. Furthermore, some combination uses of TFR and integral algorithm such as LPP and Hough transform (HT) [41, 42] have also been reported to obtain parameter estimation. Each approach has advantages and shortcomings, for instance, the STFT is simple and linear but with low signal concentration, and the WVD is optimal for a mono-LFM signal but suffers from significant cross-terms for a multi-component input. Generally, a good method has the following merits:

- **High resolution.** If two windowed signals are closely located in the time and/or frequency domain, we can separate them only with a high resolution analysis method. The resolution is closely related to the main-lobe width of window, usually not smaller than a half of this width.
- **High concentration.** Signals are usually concentrated in a transform domain so their features can be recognized. Generally speaking, high concentration is preferred to perform easy detection and parameter estimation of the signal from noise and interference. To improve the signal concentration, minimizations of the side-lobes energy, the main-lobe bandwidth, and the cross-terms energy are needed for multi-component sinusoidal waves.
- **Low representation errors.** Representation errors are inevitably present in noisy environments. It is known that the Cramer-Rao low bound (CRLB) [43, 44] gives a theoretical lower bound for the representation errors of estimated parameters. There is a demand for analysis methods with performance close to the CRLB, to provide accurate estimation of parameters of noise corrupted signals.

- **Low cross-terms.** Some methods such as the WVD and LPP may suffer from cross-terms when dealing with a multi-component signal, because they make the auto-terms of the signal difficult to be interpreted. Therefore suppression of the cross-terms is greatly desired. For the WVD based bilinear methods, it is known that there is a trade-off between the auto-terms and cross-terms.
- **Low computational complexity.** To save the costs of time and resources, a method with low computational complexity is preferred for most practical applications.

Various approaches of time-frequency analysis have both advantages and shortcomings, e.g., linear methods including the STFT and PFT [34] reveal the LFM signals having no cross-term but low resolution and concentration; bilinear methods such as the WVD provide high energy concentration but suffers from cross-terms; some combinations using TFR and integral algorithm such as the LPP [32, 33] and HT [41, 42] perform well but have high computational complexity. To improve the performance of WVD, many methods such as the smoothed pseudo WVD (SPWVD) [45] and S-method [46], have been proposed to reduce the effects of cross-terms. Although some improvements have been achieved, there is a trade-off between auto-terms and cross-terms resulting that this kind of approaches is limited for applications which deal with multi-component LFM signals.

The Lv distribution (LVD) is a new tool for time-frequency analysis recently reported in [47], which aims at processing multi-component LFM signals. It belongs to the category of FCR, providing accurate representation with centroid frequency and chirp rate. Different from the FrFT that searches rotation angles to obtain the chirp rate indirectly, the LVD directly produces the FCR by applying instantaneous auto-correlation, scaling operation and FT. Due to the introduction of instantaneous auto-correlation, the

LVD achieves high energy concentration as the WVD. Furthermore, it is asymptotically linear so that a signal consisting of multiple LFM components can be represented without much undesirable cross-terms. These two merits lead to the better performance of LVD than many widely used approaches such as the Radon-Wigner transform [48, 49], Radon-ambiguity transform [50] and FrFT.

Since the LVD is very new, its capability and merits for LFM signal processing have not been fully investigated, especially in issues of frequency-chirp rate resolution, energy concentration, signal detection, representation errors, etc. Also, based on LVD, it is interesting to investigate various applications, such as on interference suppression for wireless communication. Furthermore, the LVD does not have reconstruction transform from the LVD domain back to the time domain. Although the LVD is able to process the LFM signals in many scenarios, there are still a lot of areas it cannot be adopted, such as continuous real-time signal processing and two-dimensional (2D) processing. In this thesis, we focus on the following topics.

1.1.1 Performance Analysis of LVD

Although the performances of LVD have been partially analyzed in [47], its capability of representing LFM signals has not been fully investigated and its merits have not been compared with these of other commonly used methods. For example, the performances of LVD on concentration and detection in heavy noise are unclear. Signal distribution concentration is a key performance in time-frequency analysis, and high concentration is always pursued by all methods for desirable performance in signal parameter estimation and detection [51]. However, a comprehensive research of LVD on this topic has not been reported. Consequently, signal detection and estimation including theoretical benchmarks and experimental performances are also crucial issues to be fully addressed. Therefore, it is necessary to analyze the performances of LVD on various issues and make comparison with other time-frequency approaches.

1.1.2 Applications of LVD

1.1.2.1 Application on Parameter Estimation for LPP

As a relatively new TFR, the LPP [32, 33] provides much better time-frequency resolution than the STFT, because LPP uses the estimated chirp rate in the computation. Therefore the chirp rate estimation is a key step for the LPP to obtain a desirable signal concentration in the time-frequency domain. In literature, the PFT [34] has been used for chirp rate estimation [33, 35, 52]. Although the PFT-LPP has made substantial improvements on the energy concentration in the time-frequency domain, it has been found that PFT-LPP may not be good enough for applications in low signal-to-noise ratio (SNR) environments due to the large bandwidth of main-lobe when estimating chirp rate by the PFT [33, 52]. Therefore, it is needed to find a method with more accurate parameter estimation for the applications of LPP. Since the LVD has better concentration comparing with the PFT, it is expected the LVD will be employed on the parameter estimation for LPP in order to improve the performance of LPP.

1.1.2.2 Application on Short-Time Processing: Short-Time Lv Transform

The inverse LVD (ILVD) has been proposed in [47] to generate the TFR of an LFM signal, showing how the frequency varies with time. Compared with other TFR methods such as the STFT and LPP, it has a distinct capability of dealing with multiple LFM components with high concentration without cross-term in the time-frequency domain. However, the ILVD has difficulties in dealing with long data sequence due to the limited memory size of computing devices. Therefore, it cannot be directly used to deal with continuous data streams. Furthermore, the ILVD is not appropriate to process nonlinear FM signals. Then developing an improvement of the ILVD based on the concept of short-time processing is required.

1.1.2.3 Application on Interference Suppression for DSSS Communication

Direct sequence spread spectrum (DSSS) communication is widely used in many applications for the low probability of intercept communications, by transmitting signals using a frequency spectrum which is much wider than the information bandwidth [53]. Since the spread spectrum signals are transmitted within a broadband-width that may also contain narrow- and wide-band jamming signals, the interference should be suppressed before the decision of bit information. Therefore the interference suppression plays an important role in the DSSS communication receiver.

Extensive research efforts have been made on the detection and rejection of interference in the literature. The transform domain filtering based on the FT and further works for suppressing narrow-band interference have been proposed, for example, in [54] and [17]. Although these methods are effective on suppressing various interference, further improvements are still needed for frequency modulated (FM) interference because the FT based methods are not appropriate to deal with the interference with time varying frequencies. Since time-frequency analysis has desirable advantages in revealing the time-varying frequency characteristics, extensive investigations have been reported on removing FM interference by excision methods such as the STFT [55], WVD [56] and local polynomial FT [57].

Existing suppression methods still have challenges. For instance, reported methods may not achieve satisfied excision performance for multi-component interference. This may be due to the difficulties in minimizing cross-terms between components in the time-frequency domain. For example, the WVD has strong cross-terms that may be mistakenly considered to be the interference components. Therefore it is desired to use a signal analysis method that has linear property. The LVD is a good candidate for a new interference suppression method for performance improvement.

1.1.3 LVD Based Signal Reconstruction Methods

Signal transformations, such as Fourier and time-frequency transforms, are very important tools for signal processing, and reconstructing a signal from a transform domain is also used widely. People can obtain parameter information of a received signal by an analysis method, e.g., the frequency content of a single-tone sinusoid signal from the FT by mapping the time-domain signal to the frequency-domain. Then they may need to convert it back to the time domain after processing by using a reconstruction transform. In fact, there are some applications that are exactly based on forward and recovery transforms, such as the transform domain communication system [58, 59]. Moreover, from the perspective of compressed sensing [60, 61], we can explore the inherent sparsity of signal in its transformed domain to process the signal more efficiently with reduced cost, i.e., to sample the signal with sub-Nyquist sampling rate and then recover the time samples by using reconstruction algorithms.

Some applications may perform the LVD to obtain signal information in the first place, and further reconstruct the signal to the time domain for subsequent processing. Or some just employ the reconstruction technique for signal separation, noise suppression, interference rejection, re-sampling, and so on. For instance, a user may receive an LFM signal jammed by various interference from other users, so it needs to remove the interference in the LVD domain and recover the desired signal in time domain. Therefore an effective recovery method for the LVD is required. However, it has not been studied yet. The ILVD proposed in [47] is a method to transform signal from the LVD domain to the time-frequency domain, not to the time domain. This topic needs to be investigated.

1.1.4 Extension of LVD: Two-Dimensional Lv Distribution

Techniques based on low dimensional processing are insufficient for multi-dimensional scenarios because our world does not only consist of single dimensional objects. Therefore developing higher dimensional techniques is necessary for many applications such as

image, holographic imaging, video, antenna array, etc. Signals in these high-dimensional fields obviously have more than 1D information. To deal with this kind of signals, corresponding processing methods are developed, for instance, the 2D FT [62]. Techniques based on the 2D FT are widely used in image processing [63], magnetic resonance imaging in medical study [64], Lamb wave based measurement [65] and so on.

The 2D time-frequency analysis has been employed in many applications such as Lamb wave based measurement [66], textured images segmentation [27, 67, 68] and digital image watermarking [69]. Particularly, 2D LFM signals have been used in these fields such as image watermarking [24]. Some well-known 1D time-frequency distributions which provide effective performance on estimating IF and frequency & chirp rate for 1D LFM signals, can be developed to 2D distributions that still work well for 2D signals, such as the 2D WVD. Basically, 2D distributions inherit the properties of their 1D forms so that they still have drawbacks. For instance, the 2D WVD suffers from strong cross-terms for multi-component LFM signals, similarly to what its 1D version does. For this reason, developing a new 2D time-frequency distribution to improve performances of estimation and detection for 2D multi-LFM signals is required. Since the 1D LVD is a good approach having valuable merits, the 2D LVD is to be proposed.

1.2 Objectives

As the LVD is a new analysis tool lacking of comprehensive understanding, this thesis aims at theoretical analysis, experimental demonstration and comparison with existing widely used methods. The objectives are as follows.

- *To investigate the inherent performances of LVD and provide benchmarks.* The performances are mainly focused on issues of resolution, concentration, detection, representation and complexity. Theoretical benchmarks and experimental comparisons with other methods are to be presented.

- *To apply the LVD on parameter estimation for the LPP, short-time processing and interference suppression for the DSSS communication.* It is expected that the LVD can provide satisfied behaviors on practical applications. For improving the performances of LPP, short-time processing and DSSS system, experiments and comparisons are to be implemented.
- *To recover the signal from the LVD domain to the time domain by proposing two reconstruction approaches.* Furthermore, both of them are also to be employed on applications such as signal separation, de-noising and re-sampling.
- *To develop extension of the LVD for 2D signal processing.* Theoretical analysis, experimental demonstrations and application are to be shown.

1.3 Major Contributions

The original contributions of this thesis are listed as follows.

- Extensive performance analysis of LVD is presented to give benchmarks of various issues including representation resolution, signal concentration, detection in heavy noise, representation errors and computational complexity. Furthermore, comparisons with other methods, such as the PFT and FrFT, give a better understanding of the merits of LVD. Based on theoretical analysis and experimental results, the LVD is considered to have both advantages of linear and bilinear transforms without increasing complexity, providing high resolutions of centroid frequency and chirp rate, high signal energy concentration and negligible cross-terms. This analysis is helpful in selecting the LVD for some particular applications, since LVD may be the best candidate for multi-component LFM signal processing.

- Applying the LVD on three applications: parameter estimation for the LPP, short-time processing and interference suppression for the DSSS communication. Due to the merits of LVD, i.e., high concentration and trivial cross-terms, remarkable performance improvements have been achieved. For the short-time Lv transform (STLVT), signal energy concentration and computational complexity are discussed, and an application on the spectrum sensing of wireless microphone (WM) signals for cognitive radio (CR) communication is presented. For the DSSS communication, our proposed schemes have the capability to effectively suppress the interference probably for all environments corrupted by jammer, no matter how large the energy of interference is. This demonstrates the LVD is highly useful in practice involving LFM signals.
- The LVD based signal reconstruction (LSR) is proposed to accurately recover the LFM signal from the LVD domain to the time domain with negligible errors. Furthermore, an improved methods, named as the LSR with suppressed cross-terms (LSRC), is developed. The LSRC can perform better than the LSR, with the inherent strategy of reducing the cross-term effects. Both of them are applied on three applications such as signal decomposition, de-noising and re-sampling, providing effective performances on reconstruction. Comparing with other approaches such as the FrFT based de-noising and re-sampling, the LSR and LSRC achieve better performance.
- An extension of the LVD, 2D LVD, is proposed for high dimensional signal processing. Its performances including signal energy concentration and computational complexity are analyzed and discussed. To demonstrate the usefulness of 2D LVD, an application on watermarking for image copyright authorization is implemented.

1.4 Organization of the Thesis

The rest of this thesis is organized as follows:

Chapter 2 reviews the background of time-frequency analysis methods including the STFT, LPP, WVD, PFT, FrFT, and the LVD. Their representations, properties, advantages and disadvantages are presented, compared and discussed. This chapter provides an overall view of existing techniques in the time-frequency processing field.

Chapter 3 analyzes performance of the LVD theoretically and experimentally, including frequency-chirp rate resolution, signal energy concentration, signal detection, representation error and computational complexity. Comparisons with the PFT and FrFT are also presented.

In Chapter 4, two applications based on the LVD, one is employed on parameter estimation for the LPP, and another is on short-time processing, are implemented to verify that the LVD can achieve performance improvements in practical applications.

The third application on wide-band interference suppression for the DSSS communications is presented in Chapter 5, proposing two adaptive schemes to suppress the LFM interference.

Chapter 6 proposes the LSR and LSRC to recover the signal from the LVD domain to the time domain. Several applications based on the LSR and LSRC on signal decomposition, de-noising and re-sampling are presented.

Chapter 7 develops an extension of the LVD, i.e., 2D LVD. Performances such as the energy concentration and computational complexity are also evaluated. Consequently, it is employed on an application on the image watermarking.

Finally, some conclusions and future works are presented in Chapter 8.

Chapter 2

Background Review

This chapter provides a review on some widely used approaches for time-frequency analysis, including their definitions, performances, advantages and shortcomings. Firstly some time-frequency representation (TFR) based methods such as the short-time Fourier transform (STFT) and Wigner-Ville distribution (WVD) are reviewed. Next several frequency-chirp rate representation (FCR) based methods are shown, such as the fractional Fourier transform (FrFT). Finally the Lv distribution (LVD) is introduced as an FCR based approach for multi-component linearly frequency modulated (LFM) signals, with mathematical definition, implementation, properties and an inverse form for TFR.

2.1 Introduction

The widely used LFM signals can be defined as

$$s(t) = \sum_{k=1}^K s_k(t) = \sum_{k=1}^K A_k \exp(j\theta_k + j2\pi f_k t + j\pi\gamma_k t^2), \quad (2.1)$$

where K is the number of LFM components, A_k , f_k , γ_k and θ_k are their real constant amplitudes, centroid frequencies, chirp rates and random initial phases, respectively. In literature, LFM signal revealing methods could be mainly classified into the categories of TFR and FCR. The TFR, such as the STFT [26], produces spectrograms to display the frequency variation with time. As a generalized form of the STFT, the local

polynomial periodogram (LPP) is able to more accurately describe the time-frequency characteristics by using estimated parameters [32, 33]. To improve signal distribution concentration, various transforms in the Cohen's class have been proposed. The most well-known method, WVD, is able to achieve the best signal concentration if the input is a single-component LFM signal. As the WVD generally suffers from cross-terms for multi-component signals, many methods, such as smoothed pseudo WVD (SPWVD) [45], Choi-Williams distribution [70], S-method [46] and scaled L-Wigner distribution [71], have been reported to minimize the undesirable effects of cross-terms. Although some desirable improvements on this issue have been made, the trade-off between auto-terms and cross-terms always exists, which often limits further improvement on signal concentration in the time-frequency domain.

Another category of the methods dealing with LFM signals provides parameter estimation instead of the TFR, i.e., the FCR. For example, the polynomial Fourier transform (PFT) [34, 35] and FrFT [36–40] are able to provide information of signal frequencies and chirp rates. Since the LFM signal is distributed into a line on the TFR, some integral-based methods, including the Radon-Wigner transform [48, 49], Radon-ambiguity transform [50], Wigner-Hough transform [72] and LPP-Hough transform [42] have been used in parameter estimation. In general, better processing performance can be achieved by using much higher computational complexity such as the Radon and Hough transform (HT). Among these methods, the FrFT has been receiving increasing attention because of its linearity, i.e., no cross-terms, and relatively high signal concentration.

The recently reported LVD [47] belongs to the category of FCR. Different from the FrFT that uses rotation angles to indirectly obtain the chirp rate, the LVD directly provides accurate representations of LFM signals with centroid frequencies and chirp rates by applying instantaneous auto-correlation, scaling operation and Fourier transform (FT). Furthermore, the LVD is asymptotic linear so that a signal consisting of multiple

LFM components can be represented without much undesirable cross-terms. It is also shown that the LVD is able to achieve a good estimation performance in low signal-to-noise ratio (SNR) environments.

The rest of this chapter is organized as follows. First, Section 2.2 reviews the background of TFRs, including the introductions of STFT, LPP, WVD, S-method, etc. The background of FCR based methods such as the PFT and FrFT is presented in Section 2.3. Next in Section 2.4, the LVD is introduced by mathematical definition, implementation, properties and its inverse form. Conclusions are given in Section 2.5.

2.2 Background of Time-Frequency Representations

The TFRs can be classified into two categories as linear and bilinear representations. We first introduce some useful linear TFRs including the STFT, Gabor transform (GT) and LPP, then the WVD based bilinear ones are reviewed.

2.2.1 Linear TFRs

2.2.1.1 The STFT

The STFT is a classical and popular method to represent joint time-frequency characteristics of non-stationary signals. It performs the transformation from the time domain to frequency domain on small segments of a signal by using the FT method [73]. For a signal $s(t)$, the STFT is defined as,

$$S_{STFT}(t, f) = \int_{-\infty}^{\infty} s(\tau) w(\tau - t) e^{-j2\pi f\tau} d\tau, \quad (2.2)$$

where $w(t)$ is a window function in $[-T_w/2, T_w/2]$. It assumes that the input signal $s(t)$ is stationary in the window. Also the spectrogram of signal is defined as an energy form of the STFT

$$S_{spec}(t, f) = |S_{STFT}(t, f)|^2. \quad (2.3)$$

Obviously, the time-frequency resolution of STFT is closely related to the window. It is known that a longer window length leads to a better frequency resolution but worse capability of frequency tracking in time, and a shorter window length leads to a higher time resolution. An example of a three-component LFM signal represented by the STFT using a triangular window is depicted in Figure 2.1(a), showing low concentration and resolution.

2.2.1.2 The GT

The GT is a special case of the STFT using a Gaussian window, which is defined as [74, 75]

$$S_{GT}(t, f) = \int_{-\infty}^{\infty} s(\tau) e^{-\pi(\tau-t)^2} e^{-j2\pi f\tau} d\tau. \quad (2.4)$$

It is widely used in time-frequency analysis as the Gaussian window has good concentration, which means the signal near the time t is analyzed with higher weight. Figure 2.1(b) gives an example of the GT based TFR, and we can see that it performs better than the STFT using triangular window. Certainly, the TFRs based on STFT and GT could be improved by using longer window length.

2.2.1.3 The LPP

The LPP is a relatively new time-frequency transform proposed in [32] and recently elaborated in [33]. It can be considered as a generalized form of the STFT with estimated parameters. It is a transform for arbitrary order FM signals, called the polynomial phase signals (PPSs), by using the estimation of polynomial phase coefficients. Since the LFM signals studied in this thesis are the 2nd order PPS, the 2nd order local polynomial time frequency transform (LPTFT) is introduced as,

$$S_{LPTFT}(t, f) = \int_{-\infty}^{\infty} s(t + \tau) w(\tau) e^{-j2\pi f\tau} e^{-j\pi\tilde{\gamma}\tau^2} d\tau, \quad (2.5)$$

Figure 2.1: TFRs of a three-component LFM signal (sampling frequency $f_s = 128$ Hz, sample length $N_s = 512$) represented by (a) $|\text{STFT}|^2$, (b) GT and (c) LPP.

where $\tilde{\gamma} = [\tilde{\gamma}_1, \tilde{\gamma}_2, \dots, \tilde{\gamma}_k, \dots, \tilde{\gamma}_K]$ is a vector of estimated chirp rates which are needed to be estimated firstly, in which K is the number of components. Also, $w(t)$ is a window function with length of T_w to divide the input sequence. For energy representation, we define the LPP following the concept of spectrogram as

$$S_{LPP}(t, f) = |S_{LPTFT}(t, f)|^2. \quad (2.6)$$

Figure 2.1(c) presents an LPP based TFR by using a rectangular window. As observed in the figure, the LPP provides better time-frequency concentration and resolution than the spectrogram and GT, if the parameters are estimated accurately. Due to this good property, the LPP has been used in radar for improving the inverse synthetic aperture radar (ISAR) imaging of fast maneuvering targets [52, 76]. It is also applied to improve the performance of interference suppression in wireless communication systems, comparing with the STFT based suppression systems [57, 77]. However, from Figure 2.1(c) it presents cross-terms caused by the compensation phase $-\pi\tilde{\gamma}\tau^2$, i.e., the effects of one component compensation phase on other components [78]. If the chirp rates of components do not differ much, the cross-terms will be small enough to be ignored. Otherwise they are evident and may disturb the auto-terms, such as this example in Figure 2.1(c).

2.2.2 Bilinear TFRs

2.2.2.1 The WVD

The linear TFRs present a common shortcoming that they do not have good signal concentration since they do not use the high correlation property of signal. Therefore, many representations based on the energy content of signal have been developed. Some widely used distributions are included in the Cohen's bilinear class. As a particular case in the Cohen's class, the WVD is a popular nonlinear time-frequency transform defined as

$$S_{WVD}(t, f) = \int_{-\infty}^{\infty} s\left(t + \frac{\tau}{2}\right) s^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau. \quad (2.7)$$

It is known the WVD has optimum concentration in the time-frequency plane of a complex signal with a linear instantaneous frequency. But it suffers from cross-terms when dealing with multi-component signals, as shown in Figure 2.2(a), which greatly limits its use in most practical applications. Therefore many efforts are reported to improve the performance of WVD for multi-component signals and two of them will be introduced below.

2.2.2.2 The SPWVD

In dealing with multi-component signals, most of WVD based methods suppress cross-terms by window functions. The SPWVD uses two windows with respect to time and frequency domains to suppress cross-terms in the WVD plane, which is defined as [79]

$$S_{SPWVD}(t, f) = \int_{-\infty}^{\infty} w_f(\tau) \int_{-\infty}^{\infty} w_t(u - t) s\left(u + \frac{\tau}{2}\right) s^*\left(u - \frac{\tau}{2}\right) du e^{-j2\pi f\tau} d\tau, \quad (2.8)$$

where $w_t(u - t)$ and $w_f(\tau)$ are smoothing windows in time and frequency domains, respectively. These two windows employed together can suppress cross-terms in both time and frequency directions, but the TFR is much affected by window effects. Figure 2.2(b) presents the TFR based on SPWVD, which suppresses the cross-terms observed in Figure 2.2(a). At the same time, the auto-terms are also reduced because of the window effects. The common problem of the window based WVDs is the trade-off between auto-terms and cross-terms.

2.2.2.3 The S-Method

Another improvement of the WVD was reported as the S-method [46, 69], which belongs to the general class of SPWVD. It reduces cross-terms of the WVD based TFR by a frequency smoothing window $w_f(\theta)$. We give the relationship of STFT and WVD as

$$S_{WVD}(t, f) = \int_{-\infty}^{\infty} S_{STFT}(t, f + \theta) S_{STFT}^*(t, f - \theta) d\theta, \quad (2.9)$$

Figure 2.2: TFRs of a three-component LFM signal ($f_s = 128$ Hz, $N_s = 512$) represented by (a) WVD, (b) SPWVD and (c) S-method.

and the S-method is defined by

$$S_{SM}(t, f) = \int_{-\infty}^{\infty} w_f(\theta) S_{STFT}(t, f + \theta) S_{STFT}^*(t, f - \theta) d\theta, \quad (2.10)$$

where the window $w_f(\theta)$ has a finite length in the frequency domain. It is easy to find that for $w_f(\theta) = \delta(\theta)$,

$$S_{SM}(t, f) = |S_{STFT}(t, f)|^2 = S_{spec}(t, f);$$

and for $w_f(\theta) = 1$,

$$S_{SM}(t, f) = S_{WVD}(t, f).$$

From Figure 2.2(c) it is seen that the S-method provides a better TFR comparing to those by using the WVD and SPWVD. Although it does not provide better signal concentration than the LPP in Figure 2.1(c), it performs better in suppressing cross-terms.

2.3 Background of Frequency-Chirp Rate Representations

In previous section, the background of TFR based methods which represent the signal in time-frequency domain is reviewed. Another category of time-frequency analysis is based on the FCR, where the signal is revealed as peaks located according to the information of frequencies and chirp rates, instead of frequency lines on the TFR. It is usually used for signal detection and parameter estimation since detecting the signal by several points is much more convenient than by frequency lines containing numerous points. Next we will review some useful FCR methods.

2.3.1 The PFT

Similar to the LPP, the PFT can be used for arbitrary order PPSs. We only review the 2nd order PFT which is suitable for LFM signals. In FCR domain, the 2nd order PFT

Figure 2.3: FCRs of a three-component LFM signal ($f_s = 128$ Hz, $N_s = 512$) represented by the (a, b) PFT; (c, d) FrFT.

models an LFM signal as peaks by searching the chirp rates, defined as [34, 35, 37],

$$S_{PFT}(f, \gamma) = F_t \{ \hat{s}(t) \} = \int_{-\infty}^{\infty} s(t) e^{-j2\pi(ft + \mathbf{r}t^2/2)} dt, \quad (2.11)$$

where $F_t\{\cdot\}$ means the FT operation with respect to t and $\hat{s}(t) = s(t)e^{-j\pi\mathbf{r}t^2}$, in which $\mathbf{r}$ denotes a searching vector for chirp rate in a proper scope. For instance, usually we can set the searching scope as $\mathbf{r} \in (-f_s/T_d, f_s/T_d)$, where f_s is the sampling frequency and T_d means the total time duration. At the output of PFT, there will be several peaks in the FCR whose coordinates correspond to the frequency and chirp rate information of input LFM components. The PFT could be seen as a de-chirp method since it compensates the phase terms of chirp rates first. An example of the FCR based on PFT is shown in Figures 2.3(a) and (b) for a three-component LFM signal. The information of frequencies and chirp rates of input signal is directly obtained by the locations of peaks in the FCR plane, which is more convenient than the TFR based methods. It is also seen that the PFT produces wide width of main-lobe and high amplitudes of side-lobes, implying its concentration performance is poor. This may degrade detection and estimation when the number of signal components is increased or distorted by heavy noise.

2.3.2 The FrFT

The FrFT is a valuable and widely used approach on signal processing. A large number of achievements and applications based on it were reported due to its good representation especially for LFM signals. It is a generalized form of the FT obtained by introducing a rotation angle α , defined as [38, 40, 80–82],

$$S_{FrFT}(\alpha, u) = \int_{-\infty}^{\infty} s(t) K_{\alpha}(t, u) dt, \quad (2.12)$$

where the kernel function $K_{\alpha}(t, u)$ is presented as

$$K_{\alpha}(t, u) = \begin{cases} \delta(t - u), & \alpha = 2n\pi \\ \delta(t + u), & \alpha + \pi = 2n\pi \\ \sqrt{\frac{1-j\cot\alpha}{2\pi}} e^{j\frac{u^2+t^2}{2}\cot\alpha - jut\csc\alpha}, & \text{otherwise} \end{cases} \quad (2.13)$$

Figure 2.4: HT planes of a three-component LFM signal ($f_s = 128$ Hz, $N_s = 512$) obtained by the (a) LHT and (b) S-method-HT.

where n is an integer. When the rotation angle $\alpha = \pi/2$, the FrFT turns to be the standard FT, and when $\alpha = 2n\pi$, it is an identity operator as the outcome is the input signal itself. Similar to the chirp rate searched in (2.11), this angle is also a searching quantity in $[0, \pi)$. After the angle searching procedure, the FrFT produces peaks whose coordinates correspond to the parameters of input, as shown in Figures 2.3(c) and (d). It is clear to see three peaks on the FCR plane with narrower main-lobe width and lower side-lobes than those on the PFT based FCR plane. As it is a linear transform, there is no cross-term. Due to its good concentration property, the FrFT attracts much interest and is widely applied in many fields, such as filtering [83, 84], SAR/ISAR image in radar [85, 86], communication [21, 87], compression [88] and pattern recognition [89].

2.3.3 Integral Based Methods

Another way to produce the FCR is to accumulate the energy along the frequency lines in the TFR based on some integral methods, such as the Radon transform [90, 91] and HT [41, 92, 93]. By combining a TFR method and an integral algorithm, some integral

based FCR methods were reported, such as the Radon-Wigner transform [48, 49], Radon-ambiguity transform [50], Wigner-HT [72] and LPP-HT (LHT) [42]. Figure 2.4 presents two FCRs based on the LHT and S-method-HT, where the corresponding TFRs are shown in Figures 2.1(c) and 2.2(c) respectively. Three peaks are clearly shown in the FCR planes, indicating that the integral based FCR methods are proper to detect and estimate LFM signals. However, high computational complexity is usually needed for this type FCR in the accumulation procedure.

2.4 Lv Distribution

Some commonly used approaches for time-frequency analysis have been reviewed in previous sections. In general, these approaches encounter drawbacks, such as poor concentration, high cross-terms and high computation complexity. It is needed to find a method that makes improvements on such shortcomings, such as the LVD to be discussed in this thesis. It is a new time-frequency analysis method reported recently in [47] to provide accurate estimation of centroid frequency and chirp rate. In this section, we will review it mathematically including its definition, implementation, properties and inverse form with respect to the time-frequency domain.

2.4.1 Definition of LVD

Let us consider a noise corrupted multi-component LFM signal expressed as

$$\begin{aligned} x(t) &= s(t) + v(t) = \sum_{k=1}^K s_k(t) + v(t) \\ &= \sum_{k=1}^K A_k \exp(j2\pi f_k t + j\pi\gamma_k t^2 + j\theta_k) + v(t), \end{aligned} \quad (2.14)$$

where K , A_k , f_k , γ_k and θ_k are defined in (2.1), and $v(t) \sim N(0, \sigma_v^2)$ is additive white Gaussian noise. Its parametric symmetric instantaneous auto-correlation function

(a): LVD

(b): LVD

Figure 2.5: FCR of a three-component LFM signal ($f_s = 128$ Hz, $N_s = 512$) represented by the LVD.

(PSIAF) is defined as

$$\begin{aligned}
 R_x^C(t, \tau) &= x\left(t + \frac{\tau + a}{2}\right) x^*\left(t - \frac{\tau + a}{2}\right) \\
 &= \sum_{k=1}^K A_k^2 \exp(j2\pi f_k(\tau + a) + j2\pi\gamma_k(\tau + a)t) \\
 &\quad + \sum_{i=1}^{K-1} \sum_{k=i+1}^K (R_{s_i s_k}^C(t, \tau) + R_{s_k s_i}^C(t, \tau)) \\
 &\quad + \sum_{k=1}^K (R_{s_k v}^C(t, \tau) + R_{v s_k}^C(t, \tau)) + R_v^C(t, \tau), \tag{2.15}
 \end{aligned}$$

where a denotes a constant time-delay, $R_{s_i s_k}^C$ represents the cross-terms between different LFM components, $R_{s_k v}^C$ and $R_{v s_k}^C$ denote the cross-terms between signal components and noise, and R_v^C represents the noise auto-term. Please be noted that the initial phases θ_k are lost in the auto-terms. From the auto-terms in (2.15), it is seen that t and τ couple together. If they are decoupled, the auto-terms will have a form like 2D single-tone signals, and can be well represented by simply using FT. To decouple them, we employ the concept of keystone transform which can remove a similar coupling of range frequency and slow time [94], to define a scaling operator Γ of a phase function G with respect to (t, τ) as

$$\Gamma[G(t, \tau)] \rightarrow G\left(\frac{t_n}{h(\tau + a)}, \tau\right), \tag{2.16}$$

where h is a scaling factor and t_n is the scaled time expressed as $t_n = (\tau + a)ht$. Let us perform the scaling operation Γ on the PSIAF in (2.15) to obtain

$$\begin{aligned}
 \Gamma[R_x^C(t, \tau)] &= \sum_{k=1}^K A_k^2 \exp(j2\pi f_k(\tau + a) + j2\pi\gamma_k t_n/h) \\
 &\quad + \sum_{i=1}^{K-1} \sum_{k=i+1}^K \Gamma[R_{s_i s_k}^C(t, \tau) + R_{s_k s_i}^C(t, \tau)] \\
 &\quad + \sum_{k=1}^K \Gamma[R_{s_k v}^C(t, \tau) + R_{v s_k}^C(t, \tau)] + \Gamma[R_v^C(t, \tau)]. \tag{2.17}
 \end{aligned}$$

It is seen that in auto-terms, the new time variable t_n and lag variable τ are not coupled. Then we take the FTs of (2.17) in terms of τ and t_n to obtain the LVD as

$$\begin{aligned}
 L_x(f, \gamma) &= F_\tau \{ F_{t_n} \{ \Gamma [R_x^C(t, \tau)] \} \} \\
 &= \sum_{k=1}^K L_{s_k}(f, \gamma) + \sum_{i=1}^{K-1} \sum_{k=i+1}^K (L_{s_i s_k}(f, \gamma) + L_{s_k s_i}(f, \gamma)) \\
 &\quad + \sum_{k=1}^K (L_{s_k v}(f, \gamma) + L_{v s_k}(f, \gamma)) + L_v(f, \gamma), \tag{2.18}
 \end{aligned}$$

where $F\{\cdot\}$ means the FT and its subscript indicates the variable associated with the operation. The auto-term $L_{s_k}(f, \gamma)$ of the k th component is expressed as

$$L_{s_k}(f, \gamma) = A_k^2 e^{j2\pi a f} \delta(f - f_k) \delta(\gamma - \gamma_k/h). \tag{2.19}$$

The second term of (2.18) is associated to the cross-terms among signal components and the third term is the cross-terms among signal components and noise. The LVD has a property of asymptotic linearity that results in very small cross-terms among different components comparing to the auto-terms in (2.19). If $v(t) = 0$ in (2.14), we therefore have

$$L_s(f, \gamma) \approx \sum_{k=1}^K L_{s_k}(f, \gamma). \tag{2.20}$$

From the LVD domain, all the parameters of LFM components, i.e., amplitudes, centroid frequencies and chirp rates, are obtained by the peaks (f_k, γ_k) of $L_x(f, \gamma)$ in (2.18).

According to [47], we generally use the parameters $a = 1$ and $h = 1$ for obtaining a desirable FCR in the thesis. For a signal containing three LFM components, Figure 2.5 shows three sharp peaks to represent the signal components and very small values that are related to the cross-terms. Comparing to other FCR methods shown in Figures 2.3 and 2.4, it is clear that the LVD provides a better representation that has higher concentration and less cross-terms. In addition, a more detailed comparison with various

Table 2.1: Comparison with Various Time-Frequency Approaches for LFM Signals

<i>Method (TFR)</i>	<i>Resolution</i>	<i>Concentration</i>	<i>Cross-terms</i>	<i>Complexity</i>
STFT	Very low	Very low	None	Low ($O(N_s N_w \log_2 N_w)$)
GT	Low	Low	None	Low ($O(N_s N_w \log_2 N_w)$)
LPP	High	High	Medium	Low ($O(N_s N_w \log_2 N_w)$)
WVD	Very high	Very high	High	Medium ($O(N_s^2 \log_2 N_s)$)
SPWVD	Medium	Medium	Medium	Medium ($O(N_s N_{w_t}^2)$)
S-Method	Medium	Medium	Low	Medium ($O(N_s N_{w_t} N_{w_f})$)
ILVD	Very high	High	None	Medium ($O(N_s^2 \log_2 N_s)$)
<i>Method (FCR)</i>	<i>Resolution</i>	<i>Concentration</i>	<i>Cross-terms</i>	<i>Complexity</i>
PFT	Medium	Medium	None	Medium ($O(N_s^2 \log_2 N_s)$)
FrFT	High	High	None	Medium ($O(N_s^2 \log_2 N_s)$)
LHT	Very high	Very high	Low	High ($O(N_s N_w N_{HT}^2)$)
S-Method-HT	Medium	High	Low	High ($O(N_s N_{w_f} N_{HT}^2)$)
LVD	Very high	Very high	Negligible	Medium ($O(N_s^2 \log_2 N_s)$)

methods on issues of resolution, signal concentration, cross-terms and computational complexity is shown in Table 2.1, where N_s denotes the input length, N_w means the window length, N_{HT}^2 is the size of HT plane, N_{w_t} and N_{w_f} denotes the window lengths in time domain and frequency domain, respectively. Advantages and disadvantages of the methods reviewed in Sections 2.2 and 2.3 are listed in this table, indicating that the LVD may be the best candidate for LFM signal processing. In this table, the inverse LVD (ILVD) will be introduced in Section 2.4.4 as a TFR based method.

2.4.2 Implementation of LVD

By observing the definition of LVD, it is known that the key step of implementing LVD is the scaling operation Γ in (2.17), which makes the PSIAF to be a 2D single-tone signal where τ and t_n are no longer coupled with each other. In this way, the parameters f_k and γ_k can be estimated by applying FTs simply. Moreover, since the scaling operation makes the LVD to be a coherent accumulation of signal energy, the cross-terms between

different components are much suppressed. Since the principle of the scaling is similar to that of the keystone transform [94, 95], the procedure of Γ can be implemented by using the scaled FT (SFT) and inverse FT (IFT) [96], denoted as SFT-IFT. The SFT is given by [47]

$$S_{SFT}(\eta, f) = \int s(t) \exp(-j2\pi\eta ft) dt, \quad (2.21)$$

in which $\eta = (2mT_s + a)h/N$, where T_s is the sampling interval and N is the length of PSIAF. The scaled PSIAF shown in (2.17) can be accomplished by first using the SFT, and then using the IFT to convert back to the time domain. Because the approach of SFT-IFT is interpolation free by using only multiplications and FT, it is efficient and easy to be implemented for modern digital signal processing [47, 97].

2.4.3 Properties of LVD

Some important properties of the LVD are reviewed here. First we denote $L[s(t)]$ to be the LVD of $s(t)$, $\Gamma_R[s(t)]$ to be the scaling transform on the PSIAF of $s(t)$, and $L_s(f, \gamma)$ as the result of LVD in terms of FCR. The LVD has such properties as following:

P1: Asymptotic linearity.

Let $s(t)$ and $g(t)$ be two LFM signals, c_1 and c_2 be two scalars, then we have

$$L[c_1s(t) + c_2g(t)] \approx |c_1|^2 L_s(f, \gamma) + |c_2|^2 L_g(f, \gamma). \quad (2.22)$$

We have already introduced this property in (2.20). It is a key property of LVD making the cross-terms to be negligible so that the auto-terms are well concentrated. The main reason is that the auto-terms are coherently accumulated, while the cross-terms are not accumulated since their phases are not coherent. With this property, many good performances of LVD can be achieved.

P2: Frequency and chirp rate shift.

Let $g(t) = s(t - t_0) \exp(j2\pi f_0 t) \exp(j\pi\gamma_0 t^2)$, where t_0 , f_0 and γ_0 are real, then we have

$$\begin{aligned} L[g(t)] &= F_{t_n} \left\{ F_\tau \left\{ \Gamma \left[R_x^C(t, \tau) \right] e^{-j2\pi\gamma(\tau+a)t_0} e^{j2\pi f_0(\tau+a)} e^{j2\pi\gamma_0 \frac{t_n}{h}} \right\} \right\} \\ &= L_s(f + \gamma t_0 - f_0, \gamma - \gamma_0/h) \exp(-j2\pi\gamma a t_0) \exp(j2\pi f_0 a). \end{aligned} \quad (2.23)$$

This property shows that the translation in time domain and the 1st order frequency modulation will cause the frequency shift, and the chirp rate shift is produced by the 2nd order modulation.

P3: Scaling.

For a nonzero real scalar c_3 , if $g(t) = s(c_3 t)$, we have

$$L[g(t)] = \frac{1}{c_3^2} L_s\left(\frac{1}{c_3} f, \frac{1}{c_3} \gamma\right). \quad (2.24)$$

P4: Convolution.

Referring to the 2D convolution property of FT, easy to know

$$\Gamma_R[s(t)] \star \Gamma_R[g(t)] \Rightarrow L_s(f, \gamma) L_g(f, \gamma), \quad (2.25)$$

$$L_s(f, \gamma) \star L_g(f, \gamma) \Rightarrow \Gamma_R[s(t)] \Gamma_R[g(t)], \quad (2.26)$$

where $\star$ represents the operator of 2D convolution.

P5: Frequency and chirp rate resolution.

For a continues signal $s(t)$ with time interval T_d , the resolution of centroid frequency and chirp rate in the LVD domain are

$$\delta f = \frac{1}{T_d}, \quad \delta \gamma = \frac{2}{h T_d}. \quad (2.27)$$

More details about these properties can be found in [47].

Figure 2.6: The TFR of a three-component LFM signal ($f_s = 128$ Hz, $N_s = 512$) by using ILVD.

2.4.4 The Inverse LVD

By masking the LVD based FCR plane, a TFR method based on inverse operations, called the ILVD, was also proposed in [47] as

$$S_{ILVD}(t, f) = F_\tau \left\{ \Gamma^{-1} \left[F_f^{-1} \left\{ F_\gamma^{-1} \left\{ M[L_s(f, \gamma)] \right\} \right\} \right] \right\}, \quad (2.28)$$

where $F^{-1}\{\cdot\}$, $\Gamma^{-1}[\cdot]$ and $M[\cdot]$ represent the inverse FT, inverse scaling transform and masking operation, respectively. The masking operator is defined as

$$M[L_s(f, \gamma)] = \begin{cases} L_s(f, \gamma), & \text{for } (f, \gamma) \in S, \\ 0, & \text{for } (f, \gamma) \notin S, \end{cases}$$

where S denotes the support region of signal auto-terms specified by points (f_k, γ_k) in the LVD domain. Figure 2.6 presents the ILVD based TFR for the three-component LFM signal as used in Figures 2.1 and 2.2. Referring to Table 2.1 which compares the ILVD with the methods reviewed in Section 2.2, the signal in this TFR is best concentrated with narrowest main-lobe bandwidth and no cross-term, implying that the ILVD may break through the compromise between resolution and cross-terms [47].

2.5 Summary

This chapter reviews several commonly used methods for time-frequency analysis such as the STFT, LPP, WVD and FrFT. By investigating the performances in their transform domains, advantages and disadvantages are discussed. Then the LVD is introduced, including its definition, implementation, properties and inverse form with respect to the TFR. The merits of the LVD, such as trivial cross-terms if the input is multi-component, are discussed and shown in its FCR domain. Comparing to other methods, it seems to be a preferred candidate for LFM signal processing.

Chapter 3

Performance Analysis of Lv Distribution

This chapter compares performances of the LVD in terms of frequency-chirp rate resolution, signal distribution concentration, detection probability, representation errors and computational complexity in the frequency-chirp rate representation (FCR) for LFM signals, with those achieved by the FrFT [40] and the 2nd order PFT [34, 35], which have been reported to be suitable for revealing the frequencies and chirp rates of multiple LFM component signals. In this chapter, we assume that the dimension sizes of representing the frequency and chirp rate by PFT and the number of angles used by FrFT are $N_s/2$, where N_s is the number of input signal samples, to compare the representation performances obtained by the PFT, FrFT and LVD on the same basis.

3.1 Introduction

In Chapter 2, the LVD has been introduced by mathematical definition and important properties. Because of the property of asymptotic linearity, the LVD provides a bilinear distribution without much undesirable cross-terms for a signal consisting of multiple LFM components. Although the performances of LVD have been partially analyzed in [47], its desirable capability of representing LFM signals has not been fully investigated and

its merits have not been compared with other commonly used methods. This chapter presents performance analysis in terms of some important issues, such as frequency-chirp rate resolution, signal concentration, detection, representation errors and computational complexity of the LVD, which are essential for practical applications. For example, the comparisons with FrFT and PFT are particularly relevant to the selection of a suitable signal representation for a particular application. The results presented in this chapter show that the LVD can achieve better performance compared to those by the FrFT and PFT. Furthermore, since the LVD is asymptotic linear, similar to the STFT/FrFT, and deals with instantaneous auto-correlation of signals, similar to the WVD, it can be considered as a desirable combination of linear and bilinear methods to make use of both advantages of trivial cross-terms and high signal concentration.

The rest of this chapter is organized as follows. In Section 3.2, the parameter selection is discussed. Section 3.3 presents the frequency-chirp rate resolution of the LVD. Section 3.4 discusses the energy distribution concentration mathematically and experimentally. Performance on LFM signal detection in low SNR environment is investigated in Section 3.5. Representation errors and computational complexity are discussed in Section 3.6 and Section 3.7, respectively. Some conclusions are presented in Section 3.8 and mathematic derivations are shown in Section 3.9.

3.2 Parameter Selection

Referring to the definition of LVD in Section 2.4.1, there are two parameters to be selected, the constant time-delay a and the scaling factor h . The work in [47] has claimed that the optimal value of ah is 1 to guarantee the precision of interpolation which is required by the scaling operation in (2.16). Although $a = h = 1$ has been selected in [47], it may be not proper for some particular applications. In the computation of LVD,

(a): $h = 0.2$

(b): $h = 10$

(c): $h = 1$

Figure 3.1: FCRs of a two-component LFM signal specified in Example 3.1 based on the LVD by using: (a) $h = 0.2$; (b) $h = 10$; (c) $h = 1$.

h is closely related to the resolution of chirp rate as mentioned in (2.27), showing a high resolution with a large h . We use an example to show this.

Example 3.1: Following the definition in (2.14), the signal is with $K = 2$, $A_k = 1$ for all k , centroid frequencies $f_1 = f_2 = 0$ Hz, chirp rates $\gamma_1 = -1$ Hz/s and $\gamma_2 = 1$ Hz/s, sampling frequency $f_s = 64$ Hz and signal length $N_s = 512$. In this example, the noise term, $v(t)$, in (2.14) is assumed to be absent. This signal has chirp rates located closely in the LVD domain. The centroid frequencies are set to be 0 since they are not related to the parameters of a and h .

Figure 3.1 shows the LVD based FCRs of this example by using different values of h . It is seen that when $h = 0.2$, the two components are difficult to be separated, while they can be distinguished when h becomes larger. This indicates that to reach an adequate resolution, h is required to be large enough, e.g., $h \geq 1$ in this example. For applications which require higher resolution of chirp rate, the value of h would be increased. In this chapter, to directly read the chirp rate from the FCR plane, we choose $h = 1$, as noted in [47]. Therefore in the rest of this chapter, we adopt $a = 1$ and $h = 1$ in the computation of LVD.

3.3 Frequency-Chirp Rate Resolution

According to the property P5 in (2.27), the resolutions of centroid frequency and chirp rate achieved by the LVD are $\delta f = 1/T_d$ and $\delta \gamma = 2/T_d$. Next we use an example to show the performances of PFT, FrFT and LVD.

Example 3.2: This signal has the form as in (2.14), with $K = 3$, $A_k = 1$ for all k , centroid frequencies $f_1 = f_2 = 25.5$ Hz and $f_3 = 27.5$ Hz, chirp rates $\gamma_1 = \gamma_3 = 6.5$ Hz/s and $\gamma_2 = 8.5$ Hz/s, sampling frequency $f_s = 256$ Hz and signal length $N_s = 1024$. It has three LFM components with largest differences in centroid frequencies by 2 Hz and in chirp rates by 2 Hz/s. In this example, we also assume $v(t) = 0$.

Figure 3.2 shows the PFT, FrFT and LVD of the signal specified in Example 3.2, respectively. In this figure, the LFM components are clearly represented with three peaks by the LVD only. In contrast, both the PFT and FrFT cannot provide enough resolution to deal with small differences in frequencies and chirp rates. It is also observed in Figure 3.2(a) that the interference around three signal peaks makes the detection too difficult. The interference is mainly due to the finite length of signal sequence so that the window effects may result in non-zero components between peaks of the signal components.

3.4 Signal Concentration

Signal distribution concentration is a key performance in time-frequency analysis, and high concentration is always pursued by all methods for desirable performance in signal parameter estimation and detection in noise [51]. In this section, we evaluate the concentration performance in the LVD domain by using the distribution concentration ratio (DCR). Following the same concept of distribution concentration in [98, 99], the DCR is defined to measure signal-term concentration in the FCR domain as follows,

$$DCR_{(f,\gamma)} = 10 \log_{10} \frac{\text{ave} \left[\left| FCR(f, \gamma)_{(f,\gamma) \in S} \right|^2 \right]}{\text{ave} \left[\left| FCR(f, \gamma)_{(f,\gamma) \notin S} \right|^2 \right]}, \quad (3.1)$$

where $\text{ave} [|\cdot|^2]$ measures average energy in a region and S is the region specified by points (f_k, γ_k) in the FCR. A higher DCR value indicates a better signal concentration.

3.4.1 Distribution Concentration Ratios for LVD

Now we consider the computation of DCR in the LVD based FCR domain mathematically. Firstly we consider the DCR of a signal containing a single LFM component without any noise. Then the DCR of signal having multiple LFM components is presented in the environments with and without noise.

(a): PFT

(b): FrFT

(c): LVD

Figure 3.2: FCRs of a three-component LFM signal specified in Example 3.2 obtained by: (a) PFT; (b) FrFT; (c) LVD.

3.4.1.1 DCR for Mono-Component LFM Signals

An input signal of finite length is obtained by a window function. In Section 3.9.1, the DCR expression for LVD of a finite length signal containing a single LFM component, segmented by a rectangular window $w(t)$, is given as

$$DCR_1 \approx 20 \log_{10} \frac{|W(0)|^2}{ave \left[|W(f)|_{f \neq 0}^2 \right]}, \quad (3.2)$$

where $W(f)$ is the FT of $w(t)$ and has a form of sinc function. The sinc function in discrete time FT domain is defined by [100],

$$\text{sinc}(\omega) = \begin{cases} \frac{\sin(\omega N/2)}{\sin(\omega/2)} e^{-j\omega(N-1)/2}, & 0 < |\omega| \leq \pi \\ N, & \omega = 0 \end{cases} \quad (3.3)$$

where N is the number of sampling points of the PSIAF (Equation (2.15)) and also the size of centroid frequency with $N = N_s/2$ in the LVD domain [47]. According to (3.2), the DCR of this discrete sequence is expressed as

$$\begin{aligned} DCR_1 &\approx 20 \log_{10} \frac{\text{sinc}^2(0)}{ave \left[|\text{sinc}(\omega)|_{\omega \neq 0}^2 \right]} \\ &= 20 \log_{10} \frac{N^2}{a_s} \\ &= 20 \log_{10} N, \end{aligned} \quad (3.4)$$

where

$$a_s = \frac{1}{\pi} \int_0^\pi |\text{sinc}(\omega)|^2 d\omega = N. \quad (3.5)$$

It is noted that the DCR increases as N increases, indicating that longer length of the input signal effectively enhances signal concentration, which is directly related to the performance of signal detection.

3.4.1.2 DCR for Multi-Component LFM Signals

For a signal containing multiple LFM components, all the values of sinc functions, except the maximum ones, associated with different components, are added together due to the asymptotic linearity property of LVD. Therefore, the output of average operation in the denominator of (3.1) is increased as the number of components increases, which leads to the decrease of DCR. We rewrite the DCR of a mono-component signal according to the definition in (3.1) as,

$$DCR_1 = 10 \log_{10} \left(\frac{B_1^2}{I_1^2 / (N^2 - 1)} \right), \quad (3.6)$$

where $B_1 = A_1^2 N^2 / f_s^2$ is the amplitude of auto-term in the FCR domain [47] and I_1^2 is the total energy of sinc function except its maximum value, and N^2 is the total number of points in the LVD domain. Using the expression in (3.28) and (3.33) we have $I_1 = A_1^2 \int_{f \neq f_1} |W(f - f_1)|^2 df$ related to the first signal component, and $I_k = A_k^2 \int_{f \neq f_k} |W(f - f_k)|^2 df$ related to the k th component. Since the LVD is asymptotic linear, a K -component signal has K sinc functions added together. Therefore the DCR for a signal having K LFM components is derived as,

$$\begin{aligned} DCR_K &= 10 \log_{10} \frac{\frac{1}{K} \sum_{k=1}^K B_k^2}{\frac{1}{N^2 - K} \left(\int_{f \neq f_k} \sum_{k=1}^K A_k^2 |W(f - f_k)|^2 df \right)^2} \\ &\approx 10 \log_{10} \frac{B_1^2}{\frac{1}{N^2 - 1} \left(\sum_{k=1}^K I_k \right)^2} \\ &= DCR_1 - 20 \log_{10}(K), \end{aligned} \quad (3.7)$$

where the amplitudes B_k are assumed to be equal to B_1 and I_k to be equal to I_1 for simplicity. If the input length $N_s = 512$ is used as an example, the theoretical values of DCR_K for $K = 1 \sim 4$ can be calculated according to the value of DCR_1 in (3.4) and

are listed below,

$$\begin{aligned}
 DCR_1 &= 48.1648 \text{ dB}, \\
 DCR_2 &= 42.1442 \text{ dB}, \\
 DCR_3 &= 38.6224 \text{ dB}, \\
 DCR_4 &= 36.1236 \text{ dB}.
 \end{aligned} \tag{3.8}$$

It should be pointed out that (3.7) is valid only when the number of signal components K is far smaller than the length of signal.

3.4.1.3 DCR for Noisy Signals

For noise corrupted signals, the noise energy and cross-terms between signal and noise are spread in the entire LVD domain. We set

$$D_K = 10^{DCR_K/10},$$

which can be obtained from (3.7), and the DCR for noise corrupted signal can be approximated as (see Section 3.9.2),

$$DCR_{K,SNR} = 10 \log_{10} \frac{1 + 2m_r/B_1 + E[|r|^2]/B_1^2}{1/D_K + 2Km_r/(N^2B_1) + E[|r|^2]/B_1^2}, \tag{3.9}$$

where $m_r = N\sigma_v^2$ and $E[|r|^2] = 4N^2\sigma_v^2 \sum_{k=1}^K A_k^2 + 3N^2\sigma_v^4/2$, given in (3.34) and (3.35), respectively. For further analysis of this equation, we define the input SNR to be $10 \log_{10} (E[|s(t)|^2]/\sigma_v^2)$. It can be easily proven that

$$\frac{2m_r}{B_1} \rightarrow 0, \quad \frac{E[|r|^2]}{B_1^2} \rightarrow 0, \quad \text{when} \quad \frac{E[|s(t)|^2]}{\sigma_v^2} \rightarrow \infty;$$

and

$$\frac{2m_r}{B_1} \rightarrow \infty, \quad \frac{E[|r|^2]}{B_1^2} \rightarrow \infty, \quad \text{when} \quad \frac{E[|s(t)|^2]}{\sigma_v^2} \rightarrow 0.$$

Therefore, (3.9) becomes

$$\begin{aligned}
 \lim_{SNR \rightarrow +\infty} DCR_{K,SNR} &= DCR_K, \\
 \lim_{SNR \rightarrow -\infty} DCR_{K,SNR} &= 0.
 \end{aligned} \tag{3.10}$$

3.4.2 Experimental DCRs

Next the results of some experiments on the PFT, FrFT and LVD are presented. To compare these three methods fairly, we use the distributions of magnitude squared FCRs obtained by the PFT and FrFT to compute the DCRs since their computation does not have any correlation operation in time domain. The simulation signals are generated by the following example:

Example 3.3: Following the same expression as in (2.14), the parameters of LFM components are: $K = 3$, $A_k = 1$ for all k , centroid frequencies $f_1 = 16.5$ Hz, $f_2 = -6.5$ Hz, $f_3 = 25.5$ Hz, chirp rates $\gamma_1 = 1.5$ Hz/s, $\gamma_2 = 6.5$ Hz/s, $\gamma_3 = -6.5$ Hz/s, sampling frequency $f_s = 128$ Hz and signal length $N_s = 512$.

The theoretical DCR analysis for multi-component signals in (3.7) is based on the assumption of linearity, since the LVD has the property of asymptotic linearity. For the input interval $T_d \rightarrow \infty$ (see Theorem 1 in [47]),

$$L_s(f, \gamma) \rightarrow e^{j2\pi a f} \sum_{k=1}^K \delta(f - f_k) \delta(\gamma - \gamma_k),$$

which means for longer input, the effects of cross-terms are more negligible relative to the auto-terms, which is demonstrated in Table 3.1. Here it uses a two-component LFM signal whose parameters are the same as those of the first two components in Example 3.3. The 2nd and 3rd columns show the DCR values obtained by theoretical analysis and experimental simulation, respectively. It is seen that for longer input interval, the differences between the DCRs obtained by theoretical calculation and simulated experiments are decreased, showing that the experimental result is more close to the theoretical one. This implies that the LVD is approaching to be a linear transform as the input length is increased.

Table 3.2 shows the experimental DCR values obtained for LFM signals in Example 3.3 that have one and three components by using the PFT, FrFT and LVD. Because

Table 3.1: Theoretical and Experimental DCRs of the LVD (no noise and $K = 2$)

Input sample length	Theoretical (dB)	Experimental (dB)
$N_s = 256$	36.1236	33.8260
$N_s = 384$	39.6454	38.3791
$N_s = 512$	42.1442	41.0905
$N_s = 1024$	48.1648	47.1435

Table 3.2: Experimental DCRs for Signals in Example 3.3 Without Noise

Method	DCR (dB, $K = 1$)	DCR (dB, $K = 3$)
$ \text{PFT} ^2$	32.3121	26.1502
$ \text{FrFT} ^2$	38.5978	30.0791
LVD	47.1831	37.7133

Figure 3.3: DCRs of a mono-component LFM signal specified as the first component of Example 3.3.

these DCR values are for signals without noise, they approach to the theoretical results given in (3.8). It is seen that the DCRs for these two signals based on the LVD are much larger than those obtained by the PFT and FrFT. This table also indicates that the DCR decreases when the number of LFM components increases.

Figures 3.3 and 3.4 show the DCR performance in noise environments based on the results of Monte Carlo trials for the signals having one and three components specified in Example 3.3. Generally, high DCR value means better detection performance, which will be seen in Section 3.5. For the LVD, the differences between the theoretical DCR curves obtained from (3.9) and those from simulated experiments are smaller than 0.8 dB. This is because the theoretical DCR curves are obtained from an ideal situation, i.e., the signal energy is best concentrated, while the experimental ones are obtained from a non-ideal environment with some trivial errors due to the representation resolution in the LVD domain. Furthermore, the theoretical DCR in Figure 3.4 does not include the effects of cross-terms among the LFM components. It is also seen that the DCR values obtained by the LVD are approaching to those listed in Table 3.2 when SNR is larger than 10 dB. In both Figures 3.3 and 3.4, DCRs of the LVD are always larger than those from the PFT and FrFT. The main reason is that the PFT or FrFT achieves the signal distribution by a wider main-lobe in the FCR domain, which leads to a lower signal distribution concentration.

Figure 3.5 gives the DCRs based on LVD for different numbers of LFM components in noise corrupted signals. It is shown that the DCR values decrease as the number of components increases, as indicated in (3.7). Two possible reasons for this performance degradation are as follows. The first one is that for a K -component signal, the average energy of non-peak points in the LVD domain, calculated by the denominator in the first line of (3.7), increases while the numerator does not. Another reason is the effects of cross-terms between components and noise. The other two methods also have similar

Figure 3.4: DCRs of a signal having three LFM components specified in Example 3.3.

Figure 3.5: DCRs of noise corrupted signals having multiple LFM components with $K = 1 \sim 4, 20$ obtained by the LVD.

degradation as mentioned above. When SNR is large enough, the curves in Figure 3.5 approach to the theoretical values in (3.8). With $K = 20$, its largest DCR is also close to the theoretical value according to (3.7), implying this equation is still valid when $K = 20$ for $N_s = 512$.

The computation of LVD with an instantaneous auto-correlation operation is similar to the concept of Cohen's class based methods, such as WVD. However, the WVD generally suffers from the effects of cross-terms among signal components. Although some methods, such as the S-method [46], are able to make improvement, a trade-off between the auto-terms and cross-terms is still needed. In comparison, the LVD performance is close to the theoretical DCR values, indicating that it has both advantages of the WVD and linear methods (e.g., the FrFT), to achieve high signal concentration with significantly reduced effects of cross-terms.

3.5 Detection Performance

Now we consider the detection performance based on the LVD for LFM signals. Firstly the theoretical detection methods based on the PFT, FrFT and LVD are provided to obtain the statistic distributions and their associated thresholds. Then results from experiments are presented in terms of detection probability and effects of signal sequence length. In this section, we model the detection problem as a typical binary hypothesis testing having two hypotheses as,

$$\begin{aligned} H_0 : x(t) &= v(t), \\ H_1 : x(t) &= s(t) + v(t), \end{aligned} \tag{3.11}$$

where $v(t) \sim N(0, \sigma_v^2)$ is Gaussian noise and $s(t)$ is the signal defined in (2.14).

3.5.1 Detection Analysis of the PFT and FrFT

Because both the PFT and FrFT are considered as the FT with different kernels, they have the same type of statistics in their FCR domains with different parameters. In the following, we present the statistical analysis of detection using PFT for illustration. Also, the distributions of magnitude squared PFT and FrFT are computed to perform the detection. For hypothesis H_0 , the type of statistics of Gaussian noise remains the same in the FT domain since the FT is a linear transform [101]. Referring to Section 3.9.3, the point values p in the normalized $|\text{PFT}|^2$ domain follow a chi-square distribution with two degrees of freedom. For a given false alarm probability P_{fa} , the threshold in the normalized $|\text{PFT}|^2$ domain is

$$th_p = -2 \ln(P_{fa}), \quad (3.12)$$

and the corresponding theoretical detection probability is

$$P_d = \int_{th_p}^{\infty} \frac{1}{2} I_0(\sqrt{m_\chi p}) e^{-\frac{p+m_\chi}{2}} dp, \quad (3.13)$$

where I_0 denotes the Bessel function and m_χ is given in (3.47).

3.5.2 Detection Analysis of the LVD

According to Section 2.4.1, the LVD of a one-component signal $x(t)$ is expressed as,

$$\begin{aligned} L_x &= F \left\{ F \left\{ \Gamma \left[x \left(t + \frac{\tau+1}{2} \right) x^* \left(t - \frac{\tau+1}{2} \right) \right] \right\} \right\} \\ &= F \{ F \{ \Gamma [ss^* + sv^* + s^*v + vv^*] \} \} \\ &= L_s + L_{sv^*} + L_{s^*v} + L_v, \end{aligned} \quad (3.14)$$

where L_s and L_v denote the signal and noise components after the LVD transformation, respectively, L_{sv^*} and L_{s^*v} are the cross-terms between signal and noise. For H_0 ,

$L_x(f, \gamma) = L_v$ is a sum of many independent variables. Therefore, L_v is assumed to follow the Gaussian distribution according to the Central Limit Theorem as

$$H_0 : l = L_x(f, \gamma) \sim N(m_l, \sigma_l^2),$$

where l is the amplitude of point in the LVD domain, $m_l = N\sigma_v^2$ and $\sigma_l^2 = N^2\sigma_v^4/2$, which are given by [47]. The probability density function (pdf) of l for H_0 is,

$$pdf_l(l|H_0) = \frac{1}{\sqrt{2\pi\sigma_l^2}} \exp\left(-\frac{(l - m_l)^2}{2\sigma_l^2}\right), \quad (3.15)$$

and its false alarm probability can be derived as,

$$P_{fa} = \int_{th_l}^{\infty} pdf_l(l|H_0) dl = 1 - \Phi\left(\frac{th_l - m_l}{\sigma_l}\right), \quad (3.16)$$

where $\Phi(x)$ is the cumulative distribution function of a standard normal distribution. Consequently, the threshold in LVD domain is given by

$$th_l = m_l + \sigma_l \Phi^{-1}(1 - P_{fa}). \quad (3.17)$$

For hypothesis H_1 , it is difficult to express the statistic of points in the LVD domain since the cross-terms between signal and noise are not independent. However, due to the asymptotic linear property, we know that the energy of these cross-terms, i.e., $L_{sv^*} + L_{s^*v}$, is much smaller than L_s , or much smaller than L_v . Therefore, it is appropriate to derive an approximation of the detection probability for H_1 , which only considers the auto-terms as $L_{Au}(f, \gamma) = L_s + L_v$, where L_s and L_v are independent. Similar to the situation of H_0 , we have

$$H_1 : L_{Au}(f, \gamma) \sim N(m_{ls}, \sigma_l^2),$$

where $m_{ls} = m_l + L_s$. Consequently it is obtained the approximate theoretical detection probability as,

$$P_d^{apx} = 1 - \Phi\left(\frac{th_l - m_{ls}}{\sigma_l}\right). \quad (3.18)$$

(a): $|\text{FrFT}|^2$

(b): LVD

Figure 3.6: FCRs of the mono-component signal defined as the first component in Example 3.3 based on (a) $|\text{FrFT}|^2$ and (b) LVD with $\text{SNR} = -13$ dB.

Figure 3.7: Detection probabilities of the mono-component LFM signal defined as the first component in Example 3.3 with $N_s = 512$ and $P_{fa} = 0.01$.

3.5.3 Experimental Results

The signal used in our simulations has been specified as the first component in Example 3.3, and its FrFT and LVD are given in Figure 3.6. From this figure, it is seen that the LVD obtains a much larger noise margin than that obtained from the FrFT, which implies that a better detection performance should be achieved. The PFT has been found to be worse than the FrFT, which is not shown here.

Here we present the detection performance obtained by simulations. Figure 3.7 gives the detection probability with $P_{fa} = 0.01$ obtained by the above mentioned three methods. The theoretical curves are obtained from (3.13) and (3.18), and the corresponding experimental results fit them with differences up to 2 dB in SNR when $P_d \rightarrow 1$. For LVD, the gap in this figure is mainly because the effects of cross-terms between signal and noise, i.e., $L_{sv*} + L_{s*v}$, are not included in (3.18). From this figure, it is seen that

Figure 3.8: Detection probabilities of the LFM signal defined as the first component in Example 3.3 with false alarm probability $P_{fa} = 0.01$.

$P_d \rightarrow 1$ is achieved when $\text{SNR} = -8, -11$ or -14 dB obtained by the PFT, FrFT, or LVD, respectively. The LVD achieves a detection performance better than the FrFT and PFT by 3 and 6 dB in terms of input SNR. This conclusion verifies the supposition principle from Figure 3.3 in Section 3.4.1. However, it is different from the result in [47] that the output SNR of LVD is slightly lower than that of the FrFT. The reason is as follows. In [47] the energies of signal and noise located at the point (f_0, γ_0) are taken into account only, instead of considering the noise in the entire LVD domain. Obviously, the performance of LVD is better than the FrFT when dealing with the entire noise, because the LVD has a better capability of signal concentration.

Figure 3.8 presents the effects of different lengths of input sequences on detection performance based on the LVD with $\text{SNR} = -18, -20$ and -22 dB. In general, P_d increases as the signal length increases because a wider window results in a better signal concentration. The simulation results also provide information on the minimum number

of signal samples needed to achieve a required P_d with a given SNR value. For example, 2500 samples are required to reach $P_d \geq 0.98$ when $\text{SNR} = -20$ dB.

3.6 Representation Errors

Signal representation errors are generally unavoidable due to the effects of noise. The performance of parameter estimation is usually evaluated in terms of mean square errors (MSEs) of the parameters, defined as

$$\begin{aligned} \text{MSE}_f &= E[|\tilde{f} - f|^2], \\ \text{MSE}_\gamma &= E[|\tilde{\gamma} - \gamma|^2], \end{aligned} \tag{3.19}$$

where $\tilde{f}$ and $\tilde{\gamma}$ denote the estimates of f and γ , respectively. Although it is very difficult to derive a closed mathematical expression of the MSEs based on LVD, two upper bounds of the MSEs have been reported in [47] as

$$\text{MSE}_{\text{LVD},f} < \frac{147N^3 + 36f_s^2 N^2}{\pi^2 (98N^4 + 72f_s^4) \text{SNR}}, \tag{3.20}$$

$$\text{MSE}_{\text{LVD},\gamma} < \frac{294}{\pi^2 N \cdot \text{SNR}}. \tag{3.21}$$

The MSEs associated with the PFT, FrFT and LVD obtained from simulations are presented in Figure 3.9 in which the corresponding Cramer-Rao lower bounds (CRLBs) are obtained as [43, 44],

$$\begin{aligned} \text{CRLB}_f &\approx \frac{6f_s^2}{N_s^3 \text{SNR}}, \\ \text{CRLB}_\gamma &\approx \frac{90f_s^4}{N_s^5 \text{SNR}}, \end{aligned} \tag{3.22}$$

where $\text{SNR} = E[|s(t)|^2]/\sigma_v^2$. In this experiment, the signal peaks in $|\text{PFT}|^2$, $|\text{FrFT}|^2$ and LVD planes are obtained by using a 2D searching. This figure indicates that the LVD produces smaller representation errors in both frequency and chirp rate compared to the

 (a): Frequency f_1

 (b): Chirp rate γ_1

Figure 3.9: MSEs of a mono-component LFM signal specified in Example 3.3 (the first component, $N_s = 512$) by using the $|PFT|^2$, $|FrFT|^2$ and LVD on: (a) frequency f_1 ; (b) chirp rate γ_1 .

PFT and FrFT, in low SNR environments. The MSEs of estimated frequency and chirp rate obtained by the LVD approach to the CRLBs when SNR is larger than -4 and -5 dB, respectively. In comparison, the MSEs obtained by FrFT reach to the CRLBs when $\text{SNR} = -2$ dB. The PFT requires even higher SNRs by 0 dB to reach the CRLBs. Please be note that there is a difference comparing with [47] in which the MSEs of LVD approach to the CRLBs at $\text{SNR} \geq -1$ and -2 dB. This is because [47] simulated a two-component signal while we use a single-component signal, therefore better performance shows here.

3.7 Computational Complexity

The LVD requires about the same computational complexity as those needed by the PFT and FrFT. From the definition of PFT in [33], it is known that the required number of complex multiplications is $R_1 (N_s + \frac{1}{2}N_s \log_2 N_s)$ when the fast FT (FFT) with a complex multiplication complexity of $\frac{1}{2}N_s \log_2 N_s$ is employed, where R_1 represents the size of chirp rate dimension. In [40], it shows that the number of multiplications required by FrFT is $R_2 (2N_s + \frac{N_s}{2} \log_2 N_s)$, where R_2 represents the number of searching angles. The LVD mainly performs an instantaneous auto-correlation, a scaling operation and a 2D FFT, according to the definition from (2.15) to (2.18), requiring

- a). $N_s(N_s + 2)/4$ complex multiplications in (2.15);
- b). $N_s/2 \times N_s/2$ complex multiplications in (2.17);
- c). $\frac{N_s^2}{4} \log_2 \frac{N_s}{2}$ complex multiplications for the 2D FT operation, defined in (2.18), in terms of τ and t_n , respectively.

Therefore, the total number of complex multiplications needed by the LVD is $\frac{1}{4}N_s^2 + \frac{1}{4}N_s^2 \log_2 N_s + \frac{1}{2}N_s$. To obtain the same resolution in FCR domains achieved by these three methods, we set $R_1 = R_2 = N_s/2$, and the numbers of multiplications required by

these methods are listed as follows,

$$\begin{aligned}
 PFT : \quad & \frac{1}{2}N_s^2 + \frac{1}{4}N_s^2 \log_2 N_s, \\
 FrFT : \quad & N_s^2 + \frac{1}{4}N_s^2 \log_2 N_s, \\
 LVD : \quad & \frac{1}{4}N_s^2 + \frac{1}{4}N_s^2 \log_2 N_s + \frac{1}{2}N_s,
 \end{aligned} \tag{3.23}$$

which shows that they are all in the same order of $O(N_s^2 \log_2 N_s)$.

3.8 Summary

This chapter provides detailed investigation on various processing performances obtained by the recently reported LVD. It has the property of having negligible cross-terms when dealing with multiple signal components and high signal energy concentration. Comparisons on these issues show that the LVD is able to obtain better performances than the PFT and FrFT on signal concentration in the FCR domain without requiring more computational complexity.

3.9 Appendix

3.9.1 The DCR in LVD Domain

We consider a finite-duration signal of a single LFM component as

$$s_1(t)w(t) = A_1 \exp(j\theta_1 + j2\pi f_1 t + j\pi\gamma_1 t^2)w(t),$$

where $w(t)$ is assumed to be a rectangular, real and symmetric window function. According to (2.15), its PSIAF becomes

$$R(t, \tau') = \left(s_1 \left(t + \frac{\tau'}{2} \right) w \left(t + \frac{\tau'}{2} \right) \right) \cdot \left(s_1^* \left(t - \frac{\tau'}{2} \right) w^* \left(t - \frac{\tau'}{2} \right) \right),$$

where $\tau' = \tau + 1$. After rearranging the terms in the above equation and applying the scaling operation Γ defined in (2.16), we obtain

$$\begin{aligned} K(t_n, \tau') &= \Gamma[R(t, \tau')] \\ &= \left(s_1 \left(\frac{t_n}{\tau'} + \frac{\tau'}{2} \right) s_1^* \left(\frac{t_n}{\tau'} - \frac{\tau'}{2} \right) \right) \left(w \left(\frac{t_n}{\tau'} + \frac{\tau'}{2} \right) w \left(\frac{t_n}{\tau'} - \frac{\tau'}{2} \right) \right) \\ &= R_s(t_n, \tau') R_w(t_n, \tau'), \end{aligned} \quad (3.24)$$

where $R_s(t_n, \tau')$ and $R_w(t_n, \tau')$ are the scaled PSIAFs of un-windowed input signal and window function, respectively. Then the LVD of the windowed signal is given by

$$\begin{aligned} L(f, \gamma) &= F_\tau \{ F_{t_n} \{ K(t_n, \tau + 1) \} \} \\ &= e^{j2\pi f} F_\tau \{ F_{t_n} \{ K(t_n, \tau) \} \} \\ &= e^{j2\pi f} F_\tau \{ F_{t_n} \{ R_s(t_n, \tau) R_w(t_n, \tau) \} \} \\ &= e^{j2\pi f} RS(f, \gamma) \star RW(f, \gamma), \end{aligned} \quad (3.25)$$

where $\star$ represents the operator of a 2D convolution in the domain of (f, γ) , $RS(f, \gamma)$ and $RW(f, \gamma)$ are the FTs of $R_s(t_n, \tau)$ and $R_w(t_n, \tau)$, respectively, given as

$$\begin{aligned} RS(f, \gamma) &= F_\tau \{ F_{t_n} \{ R_s(t_n, \tau) \} \} \\ &= A_1^2 \delta(f - f_1) \delta(\gamma - \gamma_1), \\ RW(f, \gamma) &= F_\tau \{ F_{t_n} \{ R_w(t_n, \tau) \} \}. \end{aligned} \quad (3.26)$$

According to the definition in (3.1), the DCR in LVD domain for one-component LFM signal is given by

$$DCR_1 = 10 \log_{10} \frac{|L(f, \gamma)|_{(f_1, \gamma_1)}^2}{ave \left[|L(f, \gamma)|_{(f, \gamma) \neq (f_1, \gamma_1)}^2 \right]}, \quad (3.27)$$

i.e., a ratio between the peak point energy and the average energy of other points in the LVD domain. We can express the DCR as

$$DCR_1 = 10 \log_{10} \frac{E_{(f_1, \gamma_1)}}{\frac{1}{N^2 - 1} E_{(f, \gamma) \neq (f_1, \gamma_1)}}, \quad (3.28)$$

where N^2 is the number of points in the LVD domain, and $E_{(f,\gamma)}$ denotes the total energy of $L(f, \gamma)$ defined as

$$E_{(f,\gamma)} = \int_f \int_\gamma |L(f, \gamma)|^2 d\gamma df, \quad (3.29)$$

and consequently we have

$$E_{(f_1, \gamma_1)} = |L(f_1, \gamma_1)|^2,$$

$$E_{(f,\gamma) \neq (f_1, \gamma_1)} = \int_{f|f \neq f_1} \int_{\gamma| \gamma \neq \gamma_1} |L(f, \gamma)|^2 d\gamma df.$$

Obviously,

$$E_{(f,\gamma)} = E_{(f_1, \gamma_1)} + E_{(f,\gamma) \neq (f_1, \gamma_1)}.$$

Following (3.25) and (3.29), $E_{(f,\gamma)}$ becomes

$$\begin{aligned} E_{(f,\gamma)} &= A_1^4 \int_f \int_\gamma \delta(f - f_1) \delta(\gamma - \gamma_1) d\gamma df \cdot \int_f \int_\gamma |RW(f, \gamma)|^2 d\gamma df \\ &= A_1^4 E_n \int_f \int_\gamma \delta(f - f_1) \delta(\gamma - \gamma_1) d\gamma df, \end{aligned} \quad (3.30)$$

where

$$E_n = \int_f \int_\gamma |RW(f, \gamma)|^2 d\gamma df.$$

With the Parseval's theorem, E_n can be expressed into

$$\begin{aligned}
 E_n &= \int_{\tau} \int_{-\frac{T_n}{2}}^{\frac{T_n}{2}} |R_w(t_n, \tau)|^2 dt_n d\tau \\
 &= \int_{\tau} \int_{-\frac{T_n}{2}}^{\frac{T_n}{2}} w^2\left(\frac{t_n}{\tau} + \frac{\tau}{2}\right) w^2\left(\frac{t_n}{\tau} - \frac{\tau}{2}\right) dt_n d\tau \\
 &= \int_{\tau} \int_{-\frac{T_n}{2\tau}}^{\frac{T_n}{2\tau}} w^2\left(t + \frac{\tau}{2}\right) w^2\left(t - \frac{\tau}{2}\right) \tau dt d\tau \\
 &= \int_{\tau} \int_{-\frac{T_n}{2}}^{\frac{T_n}{2}} w^2\left(t + \frac{\tau}{2}\right) w^2\left(t - \frac{\tau}{2}\right) dt d\tau \\
 &= \int_{\tau} w^2(\tau) \otimes w^2(-\tau) d\tau \\
 &= \left(\int_{\tau} w^2(\tau) d\tau \right)^2 \\
 &= \left(\int_f |W(f)|^2 df \right)^2,
 \end{aligned} \tag{3.31}$$

where $(-\frac{T_n}{2}, \frac{T_n}{2})$ is the integration interval of t_n , $\otimes$ means the 1D convolution respect to τ , and $W(f)$ is the FT of rectangular window $w(t)$ and has a form of sinc. Substituting E_n into (3.30) we get

$$\begin{aligned}
 E_{(f, \gamma)} &= A_1^4 \left(\int_f |W(f)|^2 df \right)^2 \cdot \left(\int_f \int_{\gamma} \delta(f - f_1) \delta(\gamma - \gamma_1) d\gamma df \right)^2 \\
 &= A_1^4 \left(\int_f \delta(f - f_1) \otimes |W(f)|^2 df \right)^2 \\
 &= A_1^4 \left(\int_f |W(f - f_1)|^2 df \right)^2.
 \end{aligned} \tag{3.32}$$

From this result, it is easily to verify that

$$\begin{aligned}
 E_{(f_1, \gamma_1)} &= A_1^4 |W(0)|^4, \\
 E_{(f, \gamma) \neq (f_1, \gamma_1)} &= A_1^4 \left(\int_{f|f \neq f_1} |W(f - f_1)|^2 df \right)^2 \\
 &= A_1^4 \left(\int_{f|f \neq 0} |W(f)|^2 df \right)^2,
 \end{aligned} \tag{3.33}$$

and the DCR in (3.28) for a single component LFM signal segmented by a rectangular window is expressed as

$$\begin{aligned}
 DCR_1 &= 10 \log_{10} \frac{|W(0)|^4}{\frac{1}{N^2-1} \left(\int_{f|f \neq 0} |W(f)|^2 df \right)^2} \\
 &\approx 20 \log_{10} \frac{|W(0)|^2}{\frac{1}{N} \int_{f|f \neq 0} |W(f)|^2 df}, \\
 &= 20 \log_{10} \frac{|W(0)|^2}{\text{ave} \left[|W(f)|^2_{f \neq 0} \right]},
 \end{aligned}$$

which is (3.2), and here we assume $N \gg 1$.

3.9.2 DCR for Noise Corrupted Signal

According to (2.18), the LVD is expressed as

$$L_x = L_s + L_{sv^*} + L_{s^*v} + L_v,$$

where L_s means the auto-terms of signal $s(t)$, L_{sv^*} and L_{s^*v} denotes the cross-terms between signal and noise, L_v is the auto-term of noise $v(t)$, and the terms of cross-terms between signal components are ignored based on the asymptotic linearity. We re-write this equation as

$$L_x = L_s + r, \quad r = L_{sv^*} + L_{s^*v} + L_v,$$

and compute the mean and mean square of r as

$$m_r = E[r] = E[r^*] = E[L_v] = N\sigma_v^2, \quad (3.34)$$

$$\begin{aligned}
 E[|r|^2] &= E[|L_{sv^*}|^2 + L_{sv^*}L_{s^*v}^* + L_{sv^*}L_v^* + L_{s^*v}L_{sv^*}^* + |L_{s^*v}|^2 \\
 &\quad + L_{s^*v}L_v^* + L_{sv^*}^*L_v + L_{s^*v}^*L_v + |L_v|^2] \\
 &= 4N^2\sigma_v^2 \sum_{k=1}^K A_k^2 + E[|L_v|^2] \\
 &= 4N^2\sigma_v^2 \sum_{k=1}^K A_k^2 + \frac{3}{2}N^2\sigma_v^4,
 \end{aligned} \quad (3.35)$$

which is based on

$$\begin{aligned} E[|L_{sv^*}|^2] &= E[|DFT_{t_n, \tau}\{sv^*\}|^2] = N^2 E[|sv^*|^2] \\ &= N^2 E[|s|^2] E[|v^*|^2] = N^2 \sigma_v^2 \sum_{k=1}^K A_k^2, \end{aligned} \quad (3.36)$$

$$\begin{aligned} E[L_{sv^*} L_{s^*v}^*] &= E[L_{s^*v} L_{sv^*}^*] = E[|L_{s^*v}|^2] \\ &= E[|L_{sv^*}|^2] = N^2 \sigma_v^2 \sum_{k=1}^K A_k^2, \end{aligned} \quad (3.37)$$

$$\begin{aligned} E[L_{sv^*}] &= E[L_{s^*v}] = E[L_{sv^*} L_v^*] = E[L_{s^*v} L_v^*] \\ &= E[L_{sv^*}^* L_v] = E[L_{s^*v}^* L_v] = 0, \end{aligned} \quad (3.38)$$

and in [47]

$$E[L_v] = N\sigma_v^2, \quad E[|L_v|^2] = 3N^2\sigma_v^4/2.$$

Then the DCR of noise corrupted signal in the LVD domain is expressed as

$$\begin{aligned} DCR_{K, SNR} &= 10 \log_{10} \frac{E[|B_1 + r|^2]}{E\left[\left|KA_1^2 RW(f, \gamma) \Big|_{\substack{f \neq f_k \\ \gamma \neq \gamma_k}} + r\right|^2\right]} \\ &= 10 \log_{10} \frac{B_1^2 + 2B_1 m_r + E[|r|^2]}{\frac{I_K^2}{N^2 - 1} + u + E[|r|^2]}, \end{aligned}$$

where $I_K^2 = \left(KA_1^2 \int_{f \neq 0} |W(f)|^2 df\right)^2$ and

$$\begin{aligned} u &= \frac{2KA_1^2}{N^2 - 1} \int_{f \neq 0} |W(f)|^2 r df \\ &\approx \frac{2KA_1^2}{N} E[|W(f)|_{f \neq 0}^2 r]. \end{aligned}$$

Since the term of $W(f)$ is deterministic and r is a random variable, we have

$$\begin{aligned} u &= \frac{2KA_1^2}{N} \text{ave}[|W(f)|_{f \neq 0}^2] E[r] \\ &= \frac{2KA_1^2 m_r}{N} \text{ave}[|W(f)|_{f \neq 0}^2] \\ &= \frac{2KA_1^2 m_r}{N} c, \end{aligned} \quad (3.39)$$

where $c = ave \left[|W(f)|_{f \neq 0}^2 \right]$. $W(f)$ is the FT of a rectangular window with a width T so that $W(0) = T$ where $T = N/f_s$ [102]. Based on (3.2) and (3.4) we have $c = T^2/N$, therefore

$$u = 2KA_1^2 T^2 m_r / N^2 = 2KB_1 m_r / N^2,$$

and $DCR_{K,SNR}$ is simplified into

$$\begin{aligned} DCR_{K,SNR} &= 10 \log_{10} \frac{B_1^2 + 2B_1 m_r + E[|r|^2]}{\frac{I_K^2}{N^2-1} + \frac{2K}{N^2} B_1 m_r + E[|r|^2]} \\ &= 10 \log_{10} \frac{1 + 2m_r/B_1 + E[|r|^2]/B_1^2}{1/D_K + 2K m_r / (N^2 B_1) + E[|r|^2]/B_1^2}, \end{aligned}$$

which is (3.9).

3.9.3 The Statistics in PFT domain

Following the same concept of statistics in the FT/LPP domain in [101, 103], we derive the statistics in $|\text{PFT}|^2$ domain in this appendix. According to Section 2.3.1, the PFT of noise-corrupted signal $x(t)$ is defined as

$$\begin{aligned} P_x(f, \gamma) &= \int_{-\infty}^{\infty} (s(t) + v(t)) e^{-j2\pi f t - j\pi \gamma t^2} dt \\ &= P_s(f, \gamma) + P_v(f, \gamma), \end{aligned} \tag{3.40}$$

where $P_s(f, \gamma)$ and $P_v(f, \gamma)$ denote the PFTs of signal $s(t)$ and noise $v(t)$, respectively.

We present the real and imaginary parts of $P_s(f, \gamma)$ as

$$P_s(f, \gamma) = m_R - jm_I,$$

where m_R and m_I are both deterministic terms. $P_v(f, \gamma)$ can be also represented by real and imaginary parts as

$$P_v(f, \gamma) = P_{vR}(f, \gamma) - jP_{vI}(f, \gamma),$$

where

$$P_{vR}(f, \gamma) = \int_{-\infty}^{\infty} v(t) \cos(2\pi ft + \pi\gamma t^2) dt,$$

$$P_{vI}(f, \gamma) = \int_{-\infty}^{\infty} v(t) \sin(2\pi ft + \pi\gamma t^2) dt.$$

The means and variances of $P_{vR}(f, \gamma)$ and $P_{vI}(f, \gamma)$ are

$$E[P_{vR}(f, \gamma)] = E[P_{vI}(f, \gamma)] = 0,$$

$$\sigma_R^2 = \text{Var}[P_{vR}(f, \gamma)] = \sigma_v^2 \int_{-\infty}^{\infty} \cos^2(2\pi ft + \pi\gamma t^2) dt, \quad (3.41)$$

$$\sigma_I^2 = \text{Var}[P_{vI}(f, \gamma)] = \sigma_v^2 \int_{-\infty}^{\infty} \sin^2(2\pi ft + \pi\gamma t^2) dt. \quad (3.42)$$

By normalizing the PFT plane as

$$P_{xR_n}(f, \gamma) = \frac{P_{xR}(f, \gamma)}{\sigma_R}, \quad P_{xI_n}(f, \gamma) = \frac{P_{xI}(f, \gamma)}{\sigma_I}, \quad (3.43)$$

it is obtained the real and imaginary parts follow the normal distribution as

$$P_{xR_n} \sim N\left(\frac{m_R}{\sigma_R}, 1\right), \quad P_{xI_n} \sim N\left(\frac{m_I}{\sigma_I}, 1\right). \quad (3.44)$$

The square of values in the normalized PFT domain is computed as

$$p = |P_{x_n}(f, \gamma)|^2 = P_{xR_n}^2(f, \gamma) + P_{xI_n}^2(f, \gamma), \quad (3.45)$$

and p follows a noncentral chi-square distribution with two degrees of freedom as

$$p \sim \chi_2'^2(m_\chi, 1), \quad (3.46)$$

where

$$m_\chi = (m_R/\sigma_R)^2 + (m_I/\sigma_I)^2. \quad (3.47)$$

For hypothesis H_0 , we have $p \sim \chi_2^2(0, 1)$ and the pdf as

$$pdf_p(p|H_0) = \frac{1}{2}e^{-\frac{p}{2}}. \quad (3.48)$$

The false alarm probability is given by

$$P_{fa} = \int_{th_p}^{\infty} pdf_p(p|H_0) dp = e^{-\frac{th_p}{2}}, \quad (3.49)$$

and the threshold th_p can be calculated in (3.12).

For hypothesis H_1 , the pdf is presented as

$$pdf_p(p|H_1) = \frac{1}{2}I_0(\sqrt{m_\chi p}) e^{-\frac{p+m_\chi}{2}}, \quad (3.50)$$

then the theoretical detection probability is computed in (3.13).

Chapter 4

Applications of the Lv Distribution on TFR

In the previous chapter, some important and useful performances of the LVD have been shown, such as on the signal detection and estimation in heavy noise. Comparing to other methods, the LVD is a preferred approach for LFM signal processing. Consequently people may be interested in what application could employ it and how is its performance. Therefore in this chapter we show two applications of the LVD on TFR, one is to estimate chirp rates for the LPP in Section 4.1, and the other is to develop its short-time transform to process consecutive segments of a long data sequence in Section 4.2. At last, Section 4.3 presents comparison of these two TFR methods and concludes this chapter.

4.1 Application on Parameter Estimation for LPP

With LVD, the chirp rates of LFM signals can be more accurately estimated for the LPP. Comparing to the estimation methods based on PFT and FrFT, it is shown that LPP with the chirp rate estimated by LVD effectively obtains better performance on energy concentration in the time frequency domain.

Figure 4.1: The FCRs of an multi-component LFM signal without noise obtained by (a) PFT and (b) LVD.

4.1.1 Introduction

The LPP, a relatively new time-frequency representation (TFR) [32, 33], which has been introduced in Section 2.2.1.3, belongs to the linear class of TFRs. When dealing with an LFM signal, it provides much better time-frequency resolution than the STFT because the use of estimated chirp rate in computation. Therefore the chirp rate estimation is a key step for the LPP to obtain a desirable signal concentration in the time-frequency domain. In literature the PFT [34] was used for chirp rate estimation, therefore we call the LPP based on this estimation method as PFT-LPP [33, 35, 52]. Although the PFT-LPP has made substantial improvements on the energy concentration in the time-frequency domain, it has been found that the PFT-LPP may not be good enough for applications in low SNR environments, due to the large bandwidth of main-lobe when estimating chirp rate by the PFT. Therefore, finding a method of accurate estimation of chirp rates becomes important for the applications of LPP.

From the discussions and conclusions in Chapter 3, the LVD can provide a more accurate FCR with a narrower main-lobe width compared to the widely known FrFT

and PFT. In particular, the LVD has a better representation performance for signals having multiple LFM components. This section employs the LVD to be the chirp rate estimation method for LPP, denoted as LVD-LPP, to obtain better performance on TFR.

4.1.2 Review of the PFT-LPP

The LPP is introduced in Section 2.2.1.3. It can be considered as a generalized form of the STFT having the properties of simplicity and linearity. With the estimated chirp rate, it can provide a much better TFR than the STFT. However, if the chirp rate is not estimated properly, the quality of TFR becomes significantly deteriorated. Therefore the estimation of chirp rate is a critical operation for the LPP, requiring a method to provide an accurate estimation.

Previous work reported in [33, 35, 52] has used the PFT (reviewed in Section 2.3.1) as the chirp rate estimation method, i.e., the parameter $\tilde{\gamma}$ in (2.5) is estimated by searching the peak location in the PFT domain of an LFM signal. In this section, the dimensional size of chirp rate is assumed to be $N_s/2$, where N_s is input sample length, as the same as in Chapter 3. And for the computation of LPP, a rectangular window is employed.

Because the PFT has a wide main-lobe bandwidth, as shown in Figure 4.1(a), it unavoidably produces large estimation errors of the chirp rate when the signal of interest is heavily corrupted by noise. To obtain an adequate estimation accuracy, it usually needs a high resolution of the chirp rate with a heavy computational complexity [33, 52]. These problems have greatly limited the LPP used for practical applications. In contrast, the LVD does not require much computational complexity, provides accurate estimations and is particularly advantageous for signals containing multiple LFM components, as seen in Figure 4.1(b).

(a): PFT-LPP

(b): LVD-LPP

Figure 4.2: TFRs of the two-component LFM signal at $\text{SNR} = -8$ dB obtained by (a) PFT-LPP; (b) LVD-LPP.

4.1.3 The LVD-LPP

Because the PFT achieves poorer performance on signal representation in the FCR domain, as shown in Figure 3.9(b), the LVD becomes a preferred choice of estimating chirp rate for the LPP. The main computation steps of the LPP based on LVD estimation (LVD-LPP) are as follows:

- (i) Compute the LVD of an input signal to obtain $L_x(f, \gamma)$ according to (2.18);
- (ii) Search the peaks in $L_x(f, \gamma)$ to obtain their location coordinates, i.e., the estimate of chirp rate, $\tilde{\gamma}$;
- (iii) Compute (2.5) and (2.6) by using $\tilde{\gamma}$.

4.1.4 Performance

Performance comparisons among the pseudo WVD (PWVD), inverse LVD (ILVD), PFT-LPP and LVD-LPP are made in terms of the TFR and corresponding distribution concentration rates (DCRs) of an LFM signal. The PWVD is a windowed version of the WVD, defined as [73]

$$S_{PWVD}(t, f) = \int_{-\infty}^{\infty} w(\tau) s\left(t + \frac{\tau}{2}\right) s^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau, \quad (4.1)$$

where $w(\tau)$ is a smoothing window. The ILVD was presented in Section 2.4.4. First we show the TFRs of PFT-LPP and LVD-LPP, then performances based on the DCR will be presented and discussed. A two-component LFM signal is simulated here, as defined in (2.14), having the following parameters: $K = 2$, centroid frequencies = 32 and 19.5 Hz, chirp rates = 16 and 6.5 Hz/s, sampling frequency $f_s = 128$ Hz and signal length $N_s = 512$. The signal is corrupted by additive white Gaussian noise. It is also assumed that a normalized rectangular window is used by the PWVD, PFT-LPP and LVD-LPP.

Figure 4.2 shows the TFRs of this signal obtained by using the PFT-LPP and LVD-LPP when $\text{SNR} = -8$ dB. It is seen that the LVD-LPP provides a representation with

(a): $K = 1$

(b): $K = 2$

Figure 4.3: DCRs of the noisy two-component signal obtained by different methods with (a) $K = 1$ and (b) $K = 2$.

higher amplitude than that obtained by the PFT-LPP because the PFT used in the latter is less accurate than LVD.

Similar to the definition in (3.1), the DCR in TFR is defined to measure the signal concentration as follows:

$$DCR_{(t,f)} = 10 \log_{10} \frac{\text{ave} \left[\left| TFR(t, f)_{(t,f) \in S'} \right|^2 \right]}{\text{ave} \left[\left| TFR(t, f)_{(t,f) \notin S'} \right|^2 \right]}, \quad (4.2)$$

where S' denotes the signal domain, i.e., the lines of LFM components in the TFR. In general, higher value of DCR means better performance for signal concentration in the time-frequency domain. Figure 4.3 presents the DCR curves obtained by these methods in noisy environments based on results of Monte Carlo trials. In Figure 4.3(a), the LVD-LPP has obtained higher DCR values than those achieved by the PWVD and ILVD, and about the same values as those obtained by the PFT-LPP when $\text{SNR} \geq -1$ dB for $K = 1$. In particular, the LVD-LPP obtains higher DCR than all other methods when SNR value is negative, mainly because of the better capability of signal concentration by LPP and the accurate estimation of chirp rate by LVD. Figure 4.3(b) dealing with the two-component LFM signal shows that the DCRs obtained by all these methods are decreased compared to those in Figure 4.3(a). It is seen again that the LVD-LPP achieves the best signal distribution concentration in the TFR.

4.2 Short-Time Lv Transform

A new time-frequency transform, known as the STLVT, is proposed by applying the ILVD to process consecutive segments of a long data sequence. Compared to other TFRs, the STLVT can achieve better energy concentration in the time-frequency domain for signals containing multiple linear and/or nonlinear FM components. The merits of STLVT are discussed in terms of the effects of window length and overlap length between adjacent

segments on signal energy concentration in the time-frequency domain, and the required computational complexity. For an application on the spectrum sensing of cognitive ratio (CR) communications, we propose a method to jointly use STLVT and HT.

4.2.1 Introduction

The ILVD has been introduced in Section 2.4.4 to generate the TFRs of LFM signals. Compared with the TFRs reviewed in Section 2.2, it has a distinct capability of revealing multiple LFM components with no cross-term among signal components in the time-frequency domain. However, the ILVD has difficulties in dealing with long data sequences due to the limited memory size of computing devices. Therefore, it cannot be directly employed to deal with continuous data streams. Furthermore, the LVD and ILVD are not appropriate to process nonlinear FM signals, since the computation in (2.15) and (2.17) cannot decouple the time variables t and τ of nonlinear FM signals.

Based on the concept that has been used in STFT, we propose the STLVT in this section by cascading the ILVD of consecutive segments of a long data sequence. In this way, the STLVT is able to reveal the time-frequency characteristics of both linear and nonlinear FM signals with reduced computational complexity. After providing the detailed procedures of segmenting the input data sequence, the performances in terms of signal concentration in the time-frequency domain achieved by the smoothed pseudo WVD (SPWVD) [79], LPP [32, 33] and the proposed STLVT are compared based on Monte Carlo simulation results. Other issues such as the effects on signal concentration due to the type and length of segmentation window and the overlap length between adjacent segments are also discussed. Computational complexities required by the above mentioned three methods are compared.

At the end of this section, we propose a joint use of STLVT and HT and apply it on spectrum sensing for the CR communications. Dealing with the wireless microphone

Figure 4.4: A segmentation example for the STLVT ($L < N/2$): (a) input segmentation; (b) output cascading.

(WM) signal, which is a non-linear FM signal, the effective sensing scheme based on STLVT-HT (SLHT) first produces the TFR by using STLVT, accumulates energy along the sinusoid curves by using the generalized HT, and then detects peaks on the Hough plane. Discussion and comparison will be also presented.

4.2.2 Definition

To avoid these limitations of the ILVD for practical applications, the similar concept to that used in the STFT is applied to cascade segments to formulate the time-frequency distribution of signal. In particular, this procedure allows us to process nonlinear FM signals by assuming the chirp rates of these FM components in each short segment are constant. With this simple arrangement, we can obtain an approximated TFR of the

input signal by setting appropriate lengths of the data segments and overlapping. We name this processing method as the short-time Lv transform (STLVT).

Assuming each segment contains N samples, there are $N/2$ valid output samples produced by the PSIAF defined in (2.15) for each data segment. The PSIAF requires delay operation of the input segments. As the delay, τ , increases, $s(t + (\tau + a)/2)$ and $s^*(t - (\tau + a)/2)$ are shifted into opposite directions, respectively. Therefore, the overlap length between two adjacent segments should be $N - L$, where $L \leq N/2$ is the number of valid output samples used to form the final TFR. Therefore, this segmentation process is described by

$$s_q[n] = s[qL + n]w[n], \quad q \geq 0, \quad (4.3)$$

where the data stream s has an increasing index starting from 0, s_q is the q th segment, $w[n]$, $0 \leq n < N$, is the window function. Figure 4.4 shows the input segmentation process and the output cascading process of the STLVT. The selection of L and N depends on the time and frequency resolutions required by the applications.

The computational complexity required by the LVD was reported to be in the order of $O(N^2 \log_2 N)$ [47] by assuming the FFT requires computation complexity of $O(N \log_2 N)$ [104]. Now we consider the numbers of complex multiplications required by the LVD and ILVD based on the FFT requiring $\frac{N}{2} \log_2 N$ complex multiplications. Section 3.7 shows the number of multiplications needed by the LVD is $\frac{N}{2} + \frac{N^2}{4}(1 + \log_2 N)$. According to the definition in (2.28), the ILVD requires two inverse FFTs, one inverse scaling operation and one FFT. Thus, the total number of complex multiplications required by the STLVT processing each N -sample segment is

$$\frac{N}{2} + \frac{N^2}{8}(1 + 5 \log_2 N).$$

Figure 4.5: The TFRs of the multi-component signals based on the STLVT with different segment lengths $N =$: (a) 128, (b) 512, (c) 1024.

Figure 4.6: TFR of a multi-component signal containing LFM and sinusoid based on the STLVT with $N = 512$.

4.2.3 Experimental Results

In this section, experimental results are presented and discussed. The simulated LFM signal follows the form in (2.14) and is denoted as $s_{LFM}(t)$ with $K = 3$, $A_k = 1$ for all k , centroid frequencies $f_1 = -2$ Hz, $f_2 = 10$ Hz and $f_3 = 0$ Hz, chirp rates $\gamma_1 = 6.5$ Hz/s, $\gamma_2 = 5$ Hz/s and $\gamma_3 = -6.25$ Hz/s and sampling frequency $f_s = 128$ Hz.

Figure 4.5 presents the TFRs of $s_{LFM}(t)$ obtained by the proposed STLVT with $N = 128$, 512 and 1024, respectively. It is seen that the STLVT provides a TFR with excellent signal concentration without cross-terms among different components. Due to the window effects, a shorter window length ($N = 128$) leads to poorer signal concentration compared to that with longer length ($N = 1024$).

The proposed STLVT is also applied to deal with signals that have nonlinear FM components. As an example, we consider the sinusoidal FM signal defined by,

$$s_{sin}(t) = \exp(j2\pi f_c t + j2\pi k_f \sin(2\pi f_m t)), \quad (4.4)$$

where the centroid frequency $f_c = 4$ Hz, frequency deviation constant $k_f = 28.6479$ Hz, modulating frequency $f_m = 0.1$ Hz, and $f_s = 128$ Hz. Figure 4.6 presents a well

concentrated TFR of the signal containing an LFM and a sinusoidal FM component. In general, the faster the IF of signal changes, the shorter window length is used to meet the assumption that the chirp rates of segments are constant.

Next we compare the performances of signal concentration in the time-frequency domain obtained by the STLVT, SPWVD [79] and LPP [32]. The last two methods have been reported in the literature to obtain high energy concentration of multi-component signals in the time-frequency domain. Because other well known methods, such as the STFT and pseudo WVD [33, 42] have obvious problems for dealing with multi-component signals in low SNR environments, performance comparisons with these methods are not presented.

Following the same concept used in (4.2), the DCR is employed to measure the signal-term energy concentration in the time-frequency domain. Figure 4.7 presents DCR values of the multi-component LFM signal $s_{LFM}(t)$ achieved by the SPWVD, LPP and the proposed STLVT. In this simulation, the STLVT and LPP methods use the normalized rectangular window for segmenting the input data stream, and the SPWVD uses the time window with length 41 and the frequency window with length 165 to obtain the highest DCR value. Figure 4.7 shows that the DCRs obtained by STLVT with $N = 256$ and 512 are larger than those achieved by both LPP and SPWVD. The STLVT is particularly suited to work in low SNR environments. To obtain the DCR of 12 dB, for example, the STLVT with $N = 512$ requires $\text{SNR} = -11$ dB compared to that the LPP with $N = 512$ needs $\text{SNR} = -5$ dB and the SPWVD needs 1 dB. Figure 4.7 also shows that the use of longer window achieves higher DCRs for our simulated signal.

Another issue is the effects of window types on the DCR values. Figure 4.8 shows the DCR values obtained by the STLVT using different types of windows, such as Gaussian, Hamming and rectangular windows of the same length, i.e., $N = 512$ and $L = N/2$. It is observed that the DCR values are related to the main-lobe widths of windows.

Figure 4.7: DCRs of the noise-corrupted $s_{LFM}(t)$ obtained by different methods with different window lengths.

For example, the Gaussian window with $\alpha = 2.5$ has the widest main-lobe width in the frequency domain among the four windows used in our simulation, and achieves the lowest DCR. While the rectangular window has the most narrow main-lobe width and obtains the highest DCR. Therefore, the rectangular window is generally preferred to obtain the best possible DCRs without requiring any computation for the segmentation process.

The last important issue to be considered is the length of overlapping between the input segments. It is generally true that the signal concentration in the time-frequency domain can be improved substantially by using those output samples containing the information most relevant to the input signal, which can be done by increasing the overlapping length $N - L$. Figure 4.8 also compares the DCRs obtained by using the rectangular windows with different lengths of overlapping. It is seen that substantial increase in DCRs is obtained when L is deducted from $N/2$ to $N/4$ (the overlap length is

Figure 4.8: DCRs of the noise-corrupted $s_{LFM}(t)$ obtained by the STLVT with different window types and overlap lengths.

equivalently increased from $N/2$ to $3N/4$). However, marginal DCR increase is achieved if L value is from $3N/8$ to $N/4$. It means that using too much overlap between segments is not necessary to achieve higher DCRs.

It is shown in Section 4.2.2 that the STLVT requires $\frac{N}{2} + \frac{N^2}{8} + \frac{5}{8}N^2 \log_2 N$ complex multiplications. The number of complex multiplications required by the LPP is $N^2 + \frac{1}{2}N^2 \log_2 N$ [33]. In addition, the LPP needs parameter estimation that substantially increases the computational complexity. In general, the computational complexities required by the LPP and STLVT are in the same order, i.e., $O(N^2 \log_2 N)$. According to [79], the number of complex multiplications required by the SPWVD is $N_1 N_2^2 \log_2 N_2$, where N_1 and N_2 denote the time and frequency smoothing window lengths, respectively. In the simulations for Figure 4.7, the STLVT requires less computational cost by a few times compared to those needed by the LPP and SPWVD.

4.2.4 Application on Spectrum Sensing for CR

As the STLVT has good ability to reveal a linear or non-linear FM signal containing multiple components in time-frequency domain with high energy concentration, this section will employ it on spectrum sensing of WM signals for the CR system. In [103], we have proposed an effective method to sense the WM signals by using a combination scheme of the LPP and HT, or called LHT. It performs very well in sensing single-component WM signals in negative SNR environments, better than some existing methods such as the cyclostationary based sensing. However, as we discussed in Section 2.2.1.3, the LPP would produce cross-terms that degrade the detection of a multi-component signal. The method based on the LHT may fail to sense the WM signals when several microphones work at the same time. This case probably happens because there exist several addressors or performers in many scenarios.

Referring to the comparison in Figure 4.7, the STLVT concentrates better than the LPP mainly because of no cross-term, so we employ it instead of the LPP to sense multi-component WM signals. Since detecting the signal directly in the time-frequency domain is complex and ineffective, a preferred method is detecting it in an integrated domain, such as the domain of HT. Many examples have been reported in literature such as [42, 72, 103, 105], and show that frequency varying signals can be detected easily in the HT domain. Therefore here we adopt this concept to detect the WM signals by a joint use of STLVT and HT (SLHT), i.e., we use the STLVT to produce the TFR of received WM signals, and then perform the HT before the detection is implemented. The following contents will firstly introduce the background of CR, spectrum sensing, WM signals and generalized HT, then detection based on the SLHT is presented and simulated.

4.2.4.1 Background

Spectrum sensing is a crucial step in the CR systems to find out available spectrum holes, which mean the absence of primary signals, for the CR communications by using effective signal detection methods [106–109]. Since the primary users are licensed users while the CR users are not, the CR system should be capable to decide if the primary signals exist in very low SNRs, such as < -10 dB. Obviously, signal detection in heavily noised environments has challenges for spectrum sensing.

The WM signal is one of the licensed signals in the locally TV bands, where the CR users have been permitted to communicate in these bands on a basis of IEEE 802.22 [110]. The WM signal can be described as

$$s_{WM}(t) = \cos \left(2\pi f_c t + 2\pi k_f \int_0^t \cos(2\pi f_m \tau) d\tau \right), \quad (4.5)$$

where the term of $\cos(2\pi f_m \tau)$, as a modulating signal, represents the voice signal. The frequency deviation k_f is the frequency sensitivity of modulator and f_c is the carrier frequency. In this chapter, $f_c = 40$ MHz which may be allocated for TV services. The WM signals belong to FM signals and have time-frequency features, resulting that many existing sensing approaches such as the cyclostationary feature based method may not be able to detect them when SNR being smaller than -10 dB [111, 112]. As their frequencies change with time, a time-frequency technique could reveal and detect them effectively [32, 113]. Here we will propose a new spectrum sensing method for the WM signals based on the joint use of STLVT and HT, which is known as SLHT.

As a well-known pattern detector in image processing, the HT is a one-to-many mapping method from a data plane to a parameter plane, and converts the difficult problem of global detection in the data space to the easy problem of local peak detection in the parameter space [41, 92, 93]. The concept of line detection by the HT has been applied to find the LFM signals from the TFR produced by the STFT [105], WVD [72]

and LPP [42]. It is shown that the HT is able to further accumulate the signal energy by the line integration operation along the direction of LFM signal in the time-frequency domain. For the non-linear FM signals, such as the WM signals, a generalized HT is employed. It is able to detect any parameterizable curves such as circles and ellipse [114]. In the time-frequency domain, from (4.5) it is known the time-frequency curve of WM signal can be parameterized as

$$f = f_c + k_f \cos(2\pi f_m t), \quad (4.6)$$

where the parameters are k_f and f_m . Therefore we can construct a 2D accumulator $A(k_f, f_m)$. For each value of f_m , the corresponding k_f is computed by using the above equation, and matrix A is generated. Then a searching operation in A is made to find the peaks. Therefore, any curve can be detected based on the HT as long as the equation describing the curve is obtained.

4.2.4.2 Detection Procedure

The WM signals are detected based on the SLHT by the following steps:

- (a) Compute the representation of WM signal in the time-frequency domain by using the STLVT;
- (b) Convert the representation obtained by the previous step into the parameter domain obtained by using the generalized HT;
- (c) Detect the peaks in the parameter domain (or Hough plane), obtain the coordinates of selected peaks, compare with the parameters of WM signals and make the sensing decision.

It is worth to point out the detection and estimation provided by the LVD for each segment in the first place can not directly tell the presence and parameters of the WM signal containing all segments. This is mainly due to each segment is assumed to be an

LFM signal, while the full input is a non-linear FM signal having different parameters. Therefore it is necessary to reveal the WM signal in the time-frequency domain and accumulate its energy with all segments, then detect in the HT domain.

Literature [103] detects the WM signals based on the constant false alarm rate (CFAR) [115] principle that it first computes the threshold according to a required false alarm probability, then compares with the peaks in Hough plane and makes final decision. The CFAR approach is a commonly used scheme in many applications such as in radar detection [116, 117]. However, it may be not appropriate for the CR application. Different from radars, the CR system must guarantee the communication of primary users. It means a CR user should make the detection probability as high as possible, with very small false alarm probability. In this section, we use a more effective method instead of the CFAR detection, which first searches the peaks in entire Hough plane, obtains their coordinates information, then compares with the parameters of WM signals (usually the CR users are assumed to have *a priori* knowledge of the primary signals when they are attempting to access the channels), at last makes the decision of “primary user exists” if the information of peaks fits that of WM signals.

4.2.4.3 Simulations

Experimental results will be presented in this section. Basically there are three types of WM signals reported in [110] as follows:

- (a) Silent: The modulating frequency $f_m = 32$ KHz and k_f is ± 5 KHz;
- (b) Soft-speaker: $f_m = 3.9$ KHz, $k_f = \pm 15$ KHz;
- (c) Loud-speaker: $f_m = 13.4$ KHz, $k_f = \pm 32.6$ KHz.

As seen in Figure 4.6, the STLVT can be applied to reveal the TFR of WM signal by assuming it is LFM in each short segment.

Figure 4.9 shows the STLVT and LPP of a two-component WM signal containing silent and loud situations by using segment length $N = 512$, and their HT results. It

(a): LPP

(b): STLVT

(c): LHT

(d): SLHT

Figure 4.9: Transform results for the two-component WM signal containing silent and loud situations without noise based on: (a) LPP with $N = 512$; (b) STLVT with $N = 512$; (c) LHT; (d) SLHT.

Table 4.1: Comparison of DCRs based on STLVT and LPP for WM signals

WM signal	SNR (dB)	DCR _{LPP} (dB)	DCR _{STLVT} (dB)
	no noise	19.7963	22.1532
Silent + loud	-10	9.5181	18.2502
	-15	3.2164	5.1575
	no noise	15.5337	19.4442
Silent + soft + loud	-10	5.4861	11.4950
	-15	1.0126	3.8909

is clear to see cross-terms in the LPP plane, while none in the STLVT plane, resulting that the two peaks in SLHT domain are more concentrated and easily to be detected than those in the LHT domain. Noise-corrupted results are further presented in Figure 4.10 at SNR = -14 dB, indicating that the SLHT based method can sense these two components while the LHT based approach fails in such low SNR situation. A DCR comparison of the STLVT and LPP is shown in Table 4.1. For the two-component WM signal, the STLVT concentrates better than the LPP, which is implied in the TFRs in Figures 4.9 and 4.10. Except the two-component one, the DCRs for signal with three components also have same performance, indicating that the SLHT based sensing method would perform better than the LHT for more WM signals.

Next a detection simulation based on the Monte Carlo trials will be implemented. Figure 4.11 presents the detection probability of the two-component signal having silent and loud cases by using the energy detection (ED), LHT and SLHT. The ED approach [118] is introduced as a basic sensing method, which is used as a comparison reference. It is seen that the SLHT can sense the WM signal in heavier noise than the LHT and ED, due to the higher performance on signal concentration of STLVT when dealing with multi-component signal. In [103], the LHT was verified to be a better method than other sensing approach such as based on the STFT-HT, pseudo WVD-HT, cyclostationary feature [111, 112], singular value decomposition [119] and energy with minimum eigenvalue [120].

Figure 4.10: Transform results for the two-component WM signal containing silent and loud situations at $\text{SNR} = -14$ dB based on: (a) LPP with $N = 512$; (b) STLVT with $N = 512$; (c) LHT; (d) SLHT.

Figure 4.11: Detection probability of the two WM components signal (silent + loud) based on the ED, LHT and SLHT.

As the proposed SLHT performs even better, it is considered as a preferred sensing method in strong noise.

4.3 Summary

Two applications based on the LVD are presented in this Chapter. The First part introduces a TFR method, known as the LVD-LPP, for processing time-varying signals. Compared to the previously reported method based on the PFT, the LVD-LPP provides better estimation of chirp rates and time-frequency energy concentration particularly in low SNR environments. The second part is STLVT based on the concept of segmentation for continuous signal processing. Performance on energy concentration and computational complexity are presented, and an application of the STLVT on spectrum sensing for WM signals is simulated.

When producing TFRs, the LVD-LPP and STLVT have their own advantages. The former can provide TFRs with higher concentration for LFM signals whose chirp rates of components do not differ much, otherwise the latter takes the advantage. This is because the LPP suffers from cross-terms as said in Section 2.2.1.3 while the STLVT does not.

Chapter 5

Applications of the Lv Distribution on Interference Suppression

Chapter 4 has introduced two applications of the LVD on time-frequency representing, and desirable performances are achieved. This chapter will apply the LVD to suppress wide-band interference for the direct sequence spread spectrum (DSSS) communications. It is generally difficult to effectively eliminate intentional multiple component interference in any communication systems, mainly because of the difficulties in accurately estimating the parameters of interference components particularly in low SNR environments. This chapter proposes effective methods of adaptive interference suppression based on LVD (AISL). Since the recently reported LVD provides better concentration of signal energy in the domain of FCR than other methods reported in the literature, it achieves performance improvement on estimating the parameters for multi-LFM signals. With the desirable properties of LVD, the proposed interference suppression methods are able to excise interference of multiple sinusoidal components. The proposed methods firstly examine if the received signal is interfered by using a high-order statistics, known as skewness, to measure the distribution concentration of received signal in the LVD domain. Then the interference excision process is employed only when the interference is present. Two methods, i.e., AISL-I based on the concept of interference cancellation and AISL-II based on phase cancellation & filtering, are investigated. The merits of these methods are

analyzed and compared in terms of bit error rates (BERs). Results of Monte Carlo experiments are presented to show the effectiveness of suppressing interference by the proposed methods.

5.1 Introduction

Spread spectrum communications transmit signals using a frequency spectrum which is much wider than the information bandwidth [53]. Different spreading techniques were reported, such as direct sequence and frequency hopping. Among them, the DSSS modulation system is widely used in many applications for the low probability of intercept communications, and considered in this chapter. For all types of spread spectrum systems, the interference, or jammer, should be suppressed before the decision of bit information. That is because the spread spectrum signals are transmitted at broadband existing narrow- and wide-band jamming signals which may make the receiver fail to demodulate the signal containing bit information perfectly. Therefore the interference suppression plays an important role in the DSSS communication receiver.

Extensive research efforts have been made on the detection and rejection of interference in the literature. The transform domain filtering based on FT for suppressing narrow-band interference was proposed in [54]. Further work for performance improvement on narrow-band interference excision has been also reported, for example, in [17, 121, 122]. Although it appears that these reported methods are effective on suppressing various interference, further improvements are still needed for FM interference because the FT based methods are not appropriate to deal with the interference whose frequencies are time varying. Since time-frequency analysis has desirable advantages in revealing the time-varying frequency characteristics, extensive investigations have been reported on removing FM interference by the excision methods based on adaptive filters

[19, 123], STFT [55, 124], WVD [18, 56], local polynomial FT [57], product high-order ambiguity function [125], Radon-Wigner transform [126], FrFT [127, 128] and so on.

Two categories of approaches based on time-frequency analysis have been studied in [56]. One is to directly remove interference in the transform domain and then the desired time domain signal is obtained by the inverse transform before detecting the transmitted bits. In literature, the STFT and FrFT have been used for this type of interference suppression [55, 128]. The other one is based on interference cancellation by using the synthesized interference. The approaches in the first category usually work poorly when strong interference is encountered, because the side-lobes of interference in the time-frequency domain still have undesirable effects on the detection process [55, 127]. Those in the second category may avoid this problem and work for strong interference if its parameters can be estimated accurately. When the characteristics of interference are not obvious, for example, the energy of interference being comparable to that of received signal, it is possible to incorrectly synthesize the interference due to the errors on estimating the parameters of interference. Therefore, the interference cancellation process may undesirably increase the BERs because the synthesized one actually becomes additional interference [56]. In this chapter, we propose the suppression methods based on the concept of interference cancellation and the above mentioned problem is avoided by using a skewness parameter to detect the presence of interference.

Another difficult issue is to deal with multi-component interference that exists in some applications [129–131]. Most reported methods in the literature achieve excellent excision performance for mono-component interference only. This may be due to the difficulties in minimizing cross-terms between components in the time-frequency domain. For example, it is well known that the WVD has strong cross-terms that may be mistakenly considered to be the interference components. Therefore it is desired to use a signal analysis method that has linear property to accurately reveal the characteristics

Figure 5.1: The representation of a 3-component LFM signal based on (a) FrFT and (b) LVD.

of interference components with cross-terms as small as possible. The LVD is a good candidate.

Different from the FrFT which is based on searching a suitable angle in the frequency domain, the LVD directly obtains the information of amplitudes, centroid frequencies and chirp rates of signals containing multiple LFM components. Figure 5.1(b) presents a three-component LFM signal in the LVD domain in which the centroid frequencies and chirp rates of these components are the coordinates of peaks, with higher concentration than the FrFT in Figure 5.1(a).

Due to the advantages discussed in Chapter 3, we employ the LVD for interference suppression in DSSS communication systems to reject both the narrow- and wide-band multiple interference. The suppression process is able to be adaptively enforced according to the skewness value of received signal's LVD. Thus the proposed methods are known as adaptive interference suppression based on LVD (AISL).

In this chapter, we propose two interference suppression methods which employ the skewness parameter to detect the presence of sinusoidal-type interference. The AISL-I

based on the concept of cancellation can achieve good performance when the interference is synthesized correctly, and the performance becomes degraded when errors of estimating the parameters of interference occur. This is particularly true when the characteristics of interference are not obvious so that correctly estimating these parameters becomes difficult. Therefore a mechanism minimizing these undesirable effects is necessary. The second method, AISL-II, uses a two-step processing scheme. The first step is to remove the interference by multiplying the received signal with a synthesized phase of the interference component. The second step is to reduce any possible residue of imperfect phase cancellation by a simple high-pass (HP) filter. The concept of filter based suppression is first proposed in [18] for mono-component interference with the known information of channel noise energy, and then extended in [125] for multi-component interference. Our proposed method, AISL-II, improves the suppression procedure in [125] by introducing the skewness and LVD. Different from the filter used in [125] only filters out three frequency bins containing the direct current (DC) component and two bins around the DC component, AISL-II employs the filter with a pre-set cut-off frequency to effectively remove the possible estimation errors. Furthermore, AISL-II is proved to be robust to the effects of random phases of the interference. Our theoretical analysis and simulation results show that the two proposed methods achieve performance improvements on rejecting multiple component LFM and/or single-tone sinusoidal interference compared to other previously reported ones. It is worth to point out that our methods could be also employed on interference suppression for global positioning system (GPS) [130, 131] and noise radar system [132–134], since they use the transmission techniques similar to the spread spectrum communications.

The rest of this chapter is organized as follows. Section 5.2 reviews the DSSS communication receiver. In Section 5.3, the proposed interference suppression methods are introduced. Theoretical analysis is presented in Section 5.4 for skewness threshold setting, detection performance and error analysis. To support the results of theoretical

analysis, experiments based on Monte Carlo simulations are presented and performance comparison with other reported methods are made in Section 5.5. At last, Section 5.6 concludes this chapter and Section 5.7 presents some theoretical derivations.

5.2 Review of the DSSS Receiver

Referring to [135], the received signal is expressed as

$$x_m(t) = b_m p(t) + i_m(t) + v_m(t), \quad (5.1)$$

where m is the index of data, $b_m \in \{-1, 1\}$ for all m are the data bits, $p(t) \in \{-1, 1\}$ is a pseudo noise (PN) spreading code, $v_m(t)$ is additive white Gaussian noise with $v_m(t) \sim N(0, \sigma_{v_m}^2)$ and $i_m(t)$ is the intentional interference. Here we consider the interference $i(t)$ containing multiple LFM components, which is generally expressed as

$$i(t) = \sum_{k=1}^K i_k(t) = \sum_{k=1}^K A_k \exp(j\theta_k + j2\pi f_k t + j\pi \gamma_k t^2), \quad (5.2)$$

where K , A_k , f_k , γ_k and θ_k are defined same as in (2.1). Usually $x_m(t)$ is expressed as a vector form as

$$\vec{x}_m = b_m \vec{p} + \vec{i}_m + \vec{v}_m, \quad (5.3)$$

where the PN sequence $\vec{p}$ has a length of L , and $\vec{x}_m$, $\vec{p}$, $\vec{i}_m$ and $\vec{v}_m$ are $(L \times 1)$ vectors. Without the interference $\vec{i}_m$, the received signal generally has a flat spectrum due to the effect of sequence spreading. The receiver makes decision to recover the data bit based on the correlation between received signal $\vec{x}_m$ and PN sequence $\vec{p}$. It is known that the PN code has an energy of

$$\vec{p}^T \vec{p} = \sum_{l=1}^L p^2(l) = L,$$

Figure 5.2: (a) The LVD of received signal without LFM jammer when $\text{SNR} = 0$ dB. (b) The LVD of received signal with 2-component LFM interference when $\text{SNR} = 0$ dB and $\text{SIR} = -10$ dB.

where $\vec{p}^T$ is the transposed $\vec{p}$. Therefore the decision variable (or the correlation output) is obtained as,

$$D_m = \vec{p}^T \vec{x}_m = Lb_m + \vec{p}^T \vec{i}_m + \vec{p}^T \vec{v}_m, \quad (5.4)$$

and b_m is decided to be 1 when $D_m \geq 0$ or -1 when $D_m < 0$.

Detection result is much related to the length of PN code, L . The performance gain obtained by the spreading process is roughly $10 \log L$. In this chapter, L is assumed to be 16 to obtain a spreading gain of $10 \log 16 \approx 12$ dB. In general, the reception performance is improved when the value of L is increased.

5.3 New Schemes for Interference Suppression

In this section we present two interference suppression methods, termed as the AISL-I and AISL-II, based on the estimated parameters of LFM interference from the LVD of received signal.

Figure 5.3: The block diagrams of the interference suppression algorithms: (a) AISL-I and (b) AISL-II.

The interference suppression method based on cancellation reported in [56] works satisfactorily for a large range of signal-to-interference ratio (SIR). However, the suppression process becomes ineffective due to the errors of estimated parameters when the interference becomes weak relative to the signal component. In general, when the interference is not strong enough, the DSSS system is quite robust due to the gain obtained from the spreading process. Therefore, one good strategy is not to use the suppression process which may otherwise bring in more harmful disturbance. However, it is necessary to monitor the presence of interference in received signal so that the interference suppression methods are used only when necessary.

As introduced in Section 2.4, the distribution of received signal in the LVD domain is concentrated only when the sinusoidal-type interference is present. Otherwise, the LVD of noise plus transmitted bits is relatively flat, as shown in Figure 5.2. The flatness of the LVD of received signal can be measured by a high-order statistics, known as the skewness, defined by [136],

$$r = \frac{E[(l - \mu)^3]}{\sigma^3}, \quad (5.5)$$

where l denotes the statistical variable, i.e., the absolute amplitudes of points in the LVD domain, μ and σ are the mean and standard variance of l , respectively. For the additive white Gaussian noise corrupted LVD plane, since the absolute value of l is considered, the skewness value is positive. In general, r becomes a minimum value when the distribution of received signal in the LVD domain is flat, or a large value when the distribution is highly concentrated in some region of the LVD domain.

Since the LVD of transmitted data bits, $b_m \vec{p}$, is noise-like and has a flat distribution, the skewness value of received signal $\vec{x}_m$ is mainly determined by the LFM interference. Therefore, it is possible to classify $\vec{x}_m$ into two categories of flat and non-flat distributions in the LVD domain as follows,

$$\begin{aligned} H_1 : r &\leq r_0, \rightarrow \text{LFM interference absent;} \\ H_2 : r &> r_0, \rightarrow \text{LFM interference present;} \end{aligned} \quad (5.6)$$

where r_0 is a skewness based threshold. It is noted that the single-tone interference is a special case of LFM interference with a zero chirp rate and can be classified into the category of H_2 . With the above classification, two interference suppression methods are shown in Figure 5.3 to perform the following steps:

(i) Compute the LVD of received signal and calculate its skewness value r .

(ii) Compare r with the threshold r_0 to classify the received signal into H_1 or H_2 .

When H_1 is classified, the received signal does not contain any interference and therefore is directly sent to the correlator for detection. When the received signal is classified into H_2 , the interference suppression process is applied on the received signal.

(iii) Performing the de-spreading process on the interference suppressed sequence (in case of H_2) or on the received signal (in case of H_1).

Based on the estimated parameters of $i_m(t)$, two suppression methods, as shown in Figure 5.3, are proposed. The first one, known as AISL-I, is based on the concept of interference cancellation, which has been implemented by using the WVD for rejecting the single-component LFM interference [56]. Figure 5.3(a) shows the main processing steps of this suppression method. In case of H_2 , i.e., the received signal containing sufficiently large LFM interference, θ_k is estimated by a phase estimation method and A_k , f_k & γ_k are estimated according to (2.19) for all components. Then the synthesized interference is used to cancel the interference in received signal. In this chapter, the maximum likelihood (ML) phase estimation [137] is employed for estimating the initial phase, and the details can be found in Section 5.7.1. For strong LFM interference, it is known that the ML method and LVD can provide accurate representation of these interference components. However, estimation errors of these associated parameters may occur because of the effects of noise and the information bits in received signal. Therefore the interference residues in the signal after suppression still have effects to degrade the detection performance of receiver.

To minimize the estimation error effects, we consider another interference suppression method, known as AISL-II, based on the concept of phase cancellation and filtering interference residues, as shown in Figure 5.3(b). This concept, proposed in [18] and [125], is combined with the LVD to deal with multiple interference components. Moreover, the AISL-II does not require the estimation of initial phases. Assuming these HP filters are able to completely suppress the phase residues without distorting the information bits and channel noise, we prove that all the K interference components are eliminated by the above described process. When the case of H_2 is classified, centroid frequencies f_k and chirp rates γ_k are estimated to synthesize the phases as

$$\tilde{\phi}_k(t) = 2\pi\tilde{f}_k t + \pi\tilde{\gamma}_k t^2 = 2\pi(f_k + \Delta f_k)t + \pi(\gamma_k + \Delta\gamma_k)t^2, \quad (5.7)$$

where Δf_k and $\Delta\gamma_k$ are estimation errors. Referring to Figure 5.3(b), u_n is used to represent the input of the n th HP filter and u'_n to represent the output of same HP filter. We have

$$\begin{aligned} u_n(t) &= u'_{n-1}(t) e^{j(\tilde{\phi}_{n-1}(t) - \tilde{\phi}_n(t))} \\ &= [b_m p(t) + v_m(t)] e^{-j\tilde{\phi}_n(t)} + A_n e^{j(\theta_n - 2\pi\Delta f_n t - \pi\Delta\gamma_n t^2)} \\ &\quad + \sum_{k=n+1}^K A_k e^{j(\theta_k + 2\pi f_k t + \pi\gamma_k t^2)} e^{-j\tilde{\phi}_n(t)}, \end{aligned} \quad (5.8)$$

where the second term corresponds to the errors associated with the n th interference component. In the above equation, $u'_0 = x_m(t)$ and $\tilde{\phi}_0(t) = 0$ for $n = 1$. Because Δf_n and $\Delta\gamma_n$ are assumed to be small, the second term should be in low frequency band in the frequency domain and can be removed by an HP filter to obtain u'_n , expressed as,

$$u'_n(t) = [b_m p(t) + v_m(t)] e^{-j\tilde{\phi}_n(t)} + \sum_{k=n+1}^K A_k e^{j(\theta_k + 2\pi f_k t + \pi\gamma_k t^2)} e^{-j\tilde{\phi}_n(t)}, \quad (5.9)$$

which does not contain the first n interference components and all other components remain. Consequently, the output of HP filter used for the last interference suppression

step is

$$u'_K(t) = [b_m p(t) + v_m(t)] e^{-j\tilde{\phi}_K(t)}, \quad (5.10)$$

whose phase term $e^{-j\tilde{\phi}_K(t)}$ can be removed by multiplying $e^{j\tilde{\phi}_K(t)}$ to produce the final output of entire interference migration process

$$y_2(t) = u'_K(t) e^{j\tilde{\phi}_K(t)} = b_m p(t) + v_m(t). \quad (5.11)$$

In general, the AISL-II is able to provide better performance of interference migration because the suppression process does not directly suffer from the errors of estimating the amplitudes and initial phases of interference components. Furthermore, the effects of residue from each step of phase cancellation, due to the estimation errors of f_k and γ_k , can be minimized by the HP filters. Because the successive use of HP filters may also remove the useful information of transmitted bits, the cut-off frequency of HP filters must be small enough. To ensure the second term in (5.8) is effectively removed, it is assumed that the deviation of instantaneous frequency, $|\Delta f + \Delta \gamma t|$, can be removed by the HP filter with a loss probability smaller than 10^{-3} . When the length of input sequence N_s is large enough, i.e., > 100 , it is appropriate to set the normalized cut-off frequency f_c to be 0.05. In this way, our proposed method does not need any *a priori* information on the noise energy and is able to deal with the multiple component interference.

5.4 Theoretical Analysis

5.4.1 Threshold Setting

Now we discuss the setting of threshold r_0 used in Figure 5.3 to detect the presence of interference in received signal. The flatness of signal distribution in the LVD domain is measured by the skewness defined in (5.5).

Figure 5.4: Skewness values in the LVD domain with different number of interference components.

In the case of H_1 , the distribution of $\vec{x}_m$ in the LVD domain can be modeled as Gaussian noise $N(0, \sigma_l)$ according to the central limit theorem. In terms of absolute value of the LVD domain, the probability density function (pdf) is expressed as,

$$p_{|l|} = \frac{2}{\sqrt{2\pi}\sigma_l} e^{-\frac{l^2}{2\sigma_l^2}}, \quad l \in (0, \infty) \quad (5.12)$$

and its mean and variance are

$$\begin{aligned} \mu &= E[l] = \int_0^\infty l \frac{2}{\sqrt{2\pi}\sigma_l} e^{-\frac{l^2}{2\sigma_l^2}} dl \\ &\approx 0.798\sigma_l, \end{aligned} \quad (5.13)$$

$$\begin{aligned} \sigma^2 &= E[(l - \mu)^2] = \int_0^\infty (l - \mu)^2 \frac{2}{\sqrt{2\pi}\sigma_l} e^{-\frac{l^2}{2\sigma_l^2}} dl \\ &\approx 0.363\sigma_l^2. \end{aligned} \quad (5.14)$$

Then the skewness value is calculated as

$$\begin{aligned}
 r &= \frac{E[(l - \mu)^3]}{\sigma^3} \\
 &= \frac{1}{\sigma^3} \int_0^\infty (l - \mu)^3 \frac{2}{\sqrt{2\pi}\sigma_l} e^{-\frac{l^2}{2\sigma_l^2}} dl \\
 &\approx 0.995.
 \end{aligned} \tag{5.15}$$

Because the max value of $|l|$ is generally smaller than 3σ , it is obtained $r \leq 0.995$ for H_1 .

The case of H_2 represents the existence of interference that is concentrated in the LVD domain, as shown in Figure 5.1(b). Figure 5.4 shows the skewness values with different numbers of interfering components in noisy environments based on Monte Carlo experiments. The simulated LFM interference is in the form described in (2.14) and the first three components have following parameters: $A_1 = A_2 = A_3 = 1$, $f_1 = 0$ Hz, $f_2 = 19.2$ Hz, $f_3 = 15.36$ Hz, $\gamma_1 = 0.6388$ Hz/s, $\gamma_2 = -5.7612$ Hz/s, $\gamma_3 = -14.7212$ Hz/s, sampling frequency $f_s = 128$ Hz and the sequence length $N_s = 512$. The parameters of other 7 components are similarly defined, which is not listed here for simplicity. In Figure 5.4, the horizontal axis denotes the interference-to-signal and noise ratio (ISNR). When $\text{ISNR} > 12$ dB, the skewness becomes a constant maximum. It is also observed that the skewness value decreases when the number of interference components is increased because more peaks mean the LVD of received signal becomes less concentrated. However, the skewness value is still larger than 0.995. For example, when the number of components is $K = 10$, r is in the range of $[2.5424, 6.5212]$ for $\text{ISNR} \in [0, 20]$ dB. We have also simulated an interference containing three LFM components with different amplitudes, i.e., $A_1 = 1$, $A_2 = 0.5$ and $A_3 = 0.7$, as shown in Figure 5.4. The skewness value for this case becomes larger compared to that for the interference having components of equal amplitudes, because the signal energy in the latter case is more concentrated in the LVD domain. Therefore it is concluded that the threshold $r_0 = 0.995$ is a proper choice for our purpose.

5.4.2 Detection of Transmitted Bit

When the interference $\vec{i}_m$ presents, the decision signal D_m after correlation with local PN sequence is expressed in (5.4). It has the same type of statistics as noise $\vec{v}_m$, i.e., the Gaussian distribution. With $b_m = \pm 1$, D_m has a mean value of $\Re\{\vec{p}^T \vec{i}_m\} \pm L$, where $\Re\{\cdot\}$ is an operator to extract the real part from a complex quantity. The variance of D_m is

$$\sigma_{D_m}^2 = \sigma_{v_m}^2 \vec{p}^T \vec{p} = \sigma_{v_m}^2 L,$$

where $\sigma_{v_m}^2$ is the variance of noise $\vec{v}_m$. Therefore we have the BER expression as

$$P_e = \frac{1}{2}Q\left(\frac{L + \Re\{\vec{p}^T \vec{i}_m\}}{\sigma_{v_m} \sqrt{L}}\right) + \frac{1}{2}Q\left(\frac{-L + \Re\{\vec{p}^T \vec{i}_m\}}{\sigma_{v_m} \sqrt{L}}\right), \quad (5.16)$$

which is derived by assuming all the signal processing operations for recovering the information bits are perfectly performed, and can be used as a reference to observe the degradations due to the interference. For convenience, we rewrite the interference to be $\vec{i}_m = A\vec{i}_a$, where A is the amplitude and $\vec{i}_a$ is the interference with a normalized average energy, indicating the interference $\vec{i}_m$ has an average energy

$$A^2 = E[|\vec{i}_m|^2] = \sum_{k=1}^K A_k^2.$$

We also define the input SNR and SIR to be

$$SNR_{in} = \frac{|b_m|^2}{\sigma_{v_m}^2} = \frac{1}{\sigma_{v_m}^2}, \quad SIR_{in} = \frac{1}{A^2}, \quad (5.17)$$

to express (5.16) into [18]

$$\begin{aligned} P_e = & \frac{1}{2}Q\left(\sqrt{SNR_{in}L}\left(1 + \frac{\Re\{\vec{p}^T \vec{i}_a\}}{L\sqrt{SIR_{in}}}\right)\right) \\ & + \frac{1}{2}Q\left(\sqrt{SNR_{in}L}\left(1 - \frac{\Re\{\vec{p}^T \vec{i}_a\}}{L\sqrt{SIR_{in}}}\right)\right). \end{aligned} \quad (5.18)$$

When the interference $\vec{i}_m$ is absent, we have

$$P_e = Q\left(\sqrt{SNR_{in}L}\right). \quad (5.19)$$

5.4.3 Error Analysis

For the event H_2 , the two methods shown in Figure 5.3 would have a detection performance expressed by (5.19) if the interference is completely removed. With the errors from estimating the amplitude, initial phase, frequency and chirp rate of the interference component, expressed as ΔA_k , $\Delta \theta_k$, Δf_k and $\Delta \gamma_k$, respectively, the interference suppressed output y_1 of AISL-I, shown in Figure 5.3(a), is given by

$$y_1(t) = b_m p(t) + v_m(t) + \Delta i_m(t),$$

where

$$\begin{aligned} \Delta i_m(t) &= i_m(t) - \tilde{i}_m(t) = \Delta A \Delta i_a(t) \\ &= \sum_{k=1}^K (A_k e^{j\theta_k + j2\pi f_k t + j\pi \gamma_k t^2} (1 - e^{j\Delta \theta_k + j2\pi \Delta f_k t + j\pi \Delta \gamma_k t^2}) \\ &\quad - \Delta A_k e^{j(\theta_k + \Delta \theta_k) + j2\pi(f_k + \Delta f_k)t + j\pi(\gamma_k + \Delta \gamma_k)t^2}), \end{aligned} \quad (5.20)$$

in which

$$\tilde{i}_m(t) = \sum_{k=1}^K (A_k + \Delta A_k) \exp [j(\theta_k + \Delta \theta_k) + j2\pi(f_k + \Delta f_k)t + j\pi(\gamma_k + \Delta \gamma_k)t^2],$$

is the synthesized interference, ΔA^2 is the average energy of $\Delta i_m(t)$, and $\Delta i_a(t)$ is with the normalized average energy. Similar to the concept of BER in (5.18), the BER with estimation errors ΔA_k , $\Delta \theta_k$, Δf_k and $\Delta \gamma_k$ is expressed as

$$\begin{aligned} P_e &= \frac{1}{2} Q \left(\sqrt{SNR_{in} L} \left(1 + \frac{\Re\{\vec{p}^T \Delta \vec{i}_a\}}{L \sqrt{\Delta SIR}} \right) \right) \\ &\quad + \frac{1}{2} Q \left(\sqrt{SNR_{in} L} \left(1 - \frac{\Re\{\vec{p}^T \Delta \vec{i}_a\}}{L \sqrt{\Delta SIR}} \right) \right), \end{aligned} \quad (5.21)$$

where $\Delta SIR = 1/\Delta A^2$. Our intention is to minimize ΔA^2 so that the BER, obtained after suppressing the interference, is close to that in (5.19).

Next we consider two cases in deducting ΔA^2 . The first one is for small estimation errors, which is for the case of strong interference. The details of deriving the presented results are given in Section 5.7.2. When ΔA_k , $\Delta \theta_k$, Δf_k and $\Delta \gamma_k$ are small enough to be close to the CRLB, we have the following results.

(a) $\Delta A_k \neq 0$, $\Delta \theta_k = \Delta f_k = \Delta \gamma_k = 0$,

$$\Delta A^2 \approx \frac{(1 + \sigma_{v_m}^2)K}{2N_s}, \quad (5.22)$$

where N_s is the length of input samples and K is the number of interference components.

(b) $\Delta \theta_k \neq 0$, $\Delta A_k = \Delta f_k = \Delta \gamma_k = 0$,

$$\Delta A^2 \approx \frac{9(1 + \sigma_{v_m}^2)K}{8N_s} - \sum_{k=1}^K \frac{27(1 + \sigma_{v_m}^2)^2}{256N_s^2 A_k^2}. \quad (5.23)$$

(c) $\Delta f_k \neq 0$, $\Delta A_k = \Delta \theta_k = \Delta \gamma_k = 0$,

$$\Delta A^2 \approx \frac{2\pi^2 L^2 K (1 + \sigma_{v_m}^2)}{N_s^3}. \quad (5.24)$$

(d) $\Delta \gamma_k \neq 0$, $\Delta A_k = \Delta \theta_k = \Delta f_k = 0$,

$$\Delta A^2 \approx \frac{9\pi^2 L^4 K (1 + \sigma_{v_m}^2)}{8N_s^5}. \quad (5.25)$$

(e) $\Delta A_k \neq 0$, $\Delta \theta_k \neq 0$, $\Delta f_k \neq 0$ and $\Delta \gamma_k \neq 0$,

$$\begin{aligned} \Delta A^2 \approx & \frac{(1 + \sigma_{v_m}^2)K}{2N_s} + \left(\frac{9(1 + \sigma_{v_m}^2)}{8N_s} - \frac{27(1 + \sigma_{v_m}^2)^2}{256N_s^2 A_k^2} + \frac{2\pi^2 L^2 (1 + \sigma_{v_m}^2)}{N_s^3} \right. \\ & \left. - \frac{9\pi^2 L^2 (1 + \sigma_{v_m}^2)^2}{8N_s^4 A_k^2} \right) \sum_{k=1}^K \left(1 + \sqrt{\frac{1 + \sigma_{v_m}^2}{2N_s A_k^2}} \right). \end{aligned} \quad (5.26)$$

From (5.22) to (5.26), it is concluded that the average energy of $\Delta i_m(t)$ is very small, i.e., $\Delta A^2 \ll 1$, indicating that $\Delta i_m(t)$ may have trivial effects on the BERs. Therefore, the suppression method AISL-I is able to provide desirable performance of interference suppression.

If the errors ΔA_k , $\Delta \theta_k$, Δf_k and $\Delta \gamma_k$ are sufficiently large, i.e., away from the CRLB with obvious distances, the average energy of $\Delta i_m(t)$ are given as follows.

(a) $\Delta A_k \neq 0$, $\Delta \theta_k = \Delta f_k = \Delta \gamma_k = 0$,

$$\Delta A^2 = \sum_{k=1}^K \Delta A_k^2. \quad (5.27)$$

(b) $\Delta \theta_k \neq 0$, $\Delta A_k = \Delta f_k = \Delta \gamma_k = 0$,

$$\Delta A^2 = \sum_{k=1}^K 2A_k^2 (1 - \cos \Delta \theta_k). \quad (5.28)$$

(c) $\Delta f_k \neq 0$, $\Delta A_k = \Delta \theta_k = \Delta \gamma_k = 0$,

$$\Delta A^2 = 2A^2. \quad (5.29)$$

(d) $\Delta \gamma_k \neq 0$, $\Delta A_k = \Delta \theta_k = \Delta f_k = 0$,

$$\Delta A^2 = 2A^2. \quad (5.30)$$

(e) $\Delta A_k \neq 0$, $\Delta \theta_k \neq 0$, $\Delta f_k \neq 0$ and $\Delta \gamma_k \neq 0$,

$$\Delta A^2 = \sum_{k=1}^K (\Delta A_k^2 + 2A_k^2 + 2A_k \Delta A_k). \quad (5.31)$$

In this case, ΔA^2 is much larger than those given in (5.22) $\sim$ (5.26), implying that $\Delta i_m(t)$ may significantly affect the BER performance. In general, the capability of AISL-I may not be sufficient to suppress the interference. Fortunately, the method of AISL-II is able to minimize these errors to meet the detection requirements.

It should be noted that ΔA_k and $\Delta \theta_k$ always exist for noise corrupted interference while the other two may be absent because the LVD may give accurate estimation of (f, γ) . It means that the interference can not be rejected completely by AISL-I even if the frequencies and chirp rates are estimated accurately. In contrast, the method of AISL-II is not directly affected by ΔA_k & $\Delta \theta_k$ and the effects of phase cancellation errors are minimized by the HP filters.

(a): BERs versus SNR

(b): BERs versus SIR

 Figure 5.5: BERs with $L = 16$ and a mono-component LFM interference, (a) BERs versus SNR and (b) BERs versus SIRs.

Figure 5.6: The WHT/SPWHT based framework for interference suppression.

5.5 Experimental Results

This section presents experimental results from Monte Carlo simulations of 10^4 trials. We assume that the length of PN sequence is $L = 16$, the simulation signal has a length of 512 and the sampling frequency is 128 Hz. The interference consists of LFM, single-tone sinusoidal and impulse signals.

5.5.1 BERs with Mono-Component LFM Interference

The mono-component LFM interference has a centroid frequency 14 Hz and a chirp rate 2 Hz/s. Figure 5.5 presents the BERs obtained by using suppression methods AISL-I and AISL-II in terms of SNR and SIR. The theoretical curve when the interference is absent is plotted in Figure 5.5(a) based on (5.19), and the curve obtained by using the WVD-HT (WHT) [72, 92] in Figure 5.5(b) is based on the suppression scheme shown in Figure 5.6 [56]. In Figure 5.5(a), the receivers using both AISL-I and AISL-II achieve about the same BER performance as the theoretical one, indicating that the LFM interference is removed so that the BER performance is not affected. This is because for such strong LFM interference, both the ML phase estimation and LVD are able to provide accurate estimations of parameters. In Figure 5.5(b), comparing to the WHT based method, our proposed methods achieve about the same BER performance when the interference is strong enough, i.e., $\text{SIR} < -10$ dB. When the energies of signal and interference tend to be similar, for example, $|\text{SIR}| < 10$ dB, the proposed AISLs are able to make improvement.

(a): BERs versus SNR

(b): BERs versus SIR

 Figure 5.7: BERs with $L = 16$ and a two-LFM interference, (a) BERs versus SNRs and (b) BERs versus SIRs.

Table 5.1: BERs with Suppressed Mono-Component LFM interference

SNR (dB)	SIR (dB)	L	BER (AISL-I)	BER (AISL-II)
0	-50	16	3.76×10^{-5}	4.55×10^{-5}
0	-100	16	3.54×10^{-5}	3.35×10^{-5}
0	-150	16	4.01×10^{-5}	2.73×10^{-5}
-10	-50	128	2.41×10^{-4}	2.51×10^{-4}
-10	-100	128	1.20×10^{-4}	2.31×10^{-4}
-10	-150	128	2.18×10^{-4}	1.69×10^{-4}

The reason is that when interference is comparable to the signal, the WHT may not correctly estimate the parameters of interference so that more errors are produced by the suppression process, which is a common problem of these methods requiring synthesizing the interference. It is noted that in this situation, the interference may not have much impacts on signal detection and the suppression method is not necessarily used. Our proposed methods use the skewness value to avoid the interference synthesis when the transmitted signal is strong enough. The AISLs are also valid to deal with the narrow band single-tone interference because it is a special case of LFM waveform with zero chirp rate. Therefore, the same BER performance as shown in Figure 5.5 can be achieved in this case.

To show the performance obtained by our proposed methods in very low SIR and SNR environments, the simulation results are presented in Table 5.1. For $L = 16$, BERs obtained by the receivers using both AISL-I and AISL-II are smaller than 10^{-4} , which are similar to those in Figure 5.5(b). These results indicate that our proposed methods are effective for a wide range of SIRs. In general, low SIR values mean strong LFM interference that is easily detected and synthesized for interference suppression. When SNR is decreased, i.e., -10 dB, high performance of BER can be obtained by increasing the length of spreading sequence, i.e., $L = 128$ in Table 5.1.

Figure 5.8: BERs with $L = 16$ and a four-component interference having three LFM and one single-tone.

Figure 5.9: BERs with $L = 16$ and a three-component interference having single-tone, LFM and impulse components.

5.5.2 BERs with Multiple Component Interference

In this section, we use three signals containing multiple LFM component interference. The first one has two LFM components with $f_1 = 14$ Hz, $f_2 = 26$ Hz and $\gamma_1 = 2$ Hz/s, $\gamma_2 = 3.5$ Hz/s. Figure 5.7 shows the BERs obtained by using different interference suppression methods. For comparison, an interference suppression method based on the joint use of SPWVD and HT, known as the SPWHT, is simulated because the SPWVD is more suitable for multi-component signals than the WVD. Its principle of interference detection and suppression is similar to the WHT based method, as shown in Figure 5.6. It is seen from Figure 5.7 that the BER performance is about the same as those in Figure 5.5, indicating that the effects of multi-component interference has been effectively mitigated. Comparing to the SPWHT, our methods give better results when $\text{SIR} \geq -10$ dB since the suppression method based on SPWHT has the same problem as the WHT in Figure 5.5(b). That is, when interference is weak, the SPWHT may not correctly estimate the parameters of jammer, resulting more errors are introduced. When $\text{SIR} > 0$ dB, the SPWHT based BER approaches to the BER of regular decision with no excision, since the spreading process makes enough gain when the bit signal is such strong.

The second simulation uses a four-component interference with $f_1 = 12$ Hz, $f_2 = 13$ Hz, $f_3 = 30$ Hz, $f_4 = 10$ Hz, $\gamma_1 = 0.5$ Hz/s, $\gamma_2 = 2$ Hz/s, $\gamma_3 = 10$ Hz/s and $\gamma_4 = 0$ Hz/s. This jammer has one single-tone and three LFM components. It should be noted that for more components, although the ML method estimates the initial phases with more errors, these errors are still very small and have limited effects on interference suppression when the LFM jammer is strong. Figure 5.8 shows the BER performance based on the AISL-I and -II. It is also indicated our proposed methods are able to provide suppression performance approaching to the theoretical one with no interference. Comparing to less components in Figures 5.5(a) and 5.7(a), the AISL based BERs for four-component

jammer are slightly larger. This is because for AISL-I, more errors of initial phases and amplitudes are involved; and for AISL-II, more information of bit signal is filtered.

The third experiment is to use an interference composed of impulse, LFM and single-tone components. The interference specifications are as follows: the LFM component is the same as the one in Section 5.5.1, the frequency of single-tone component is 15 Hz, and the impulse component has two impulses distributed randomly in the time domain. All the three components have the same average energy. Figure 5.9 shows that the BER performances achieved by the two proposed methods are worse than those shown in Figures 5.5 and 5.7, because our proposed methods do not take any measure on minimizing the effects of impulse noise. However, it also shows that the receiver using AISL-II achieves lower BERs than that using AISL-I.

5.5.3 BERs of LFM Interference with Estimated Errors

Here we consider the impacts of parameter estimation errors on the detection performance by using the same interference specified for Figure 5.7. We quantify the errors by $e_{A_k} = \frac{\Delta A_k}{A_k} \times 100\%$, $e_{\theta_k} = \frac{\Delta \theta_k}{\theta_k} \times 100\%$, $e_{f_k} = \frac{\Delta f_k}{f_k} \times 100\%$ and $e_{\gamma_k} = \frac{\Delta \gamma_k}{\gamma_k} \times 100\%$. In the experiments for Figure 5.10(a), $e_{A_1} = e_{A_2} = e_{\theta_1} = e_{\theta_2} = e_{f_1} = e_{f_2} = e_{\gamma_1} = e_{\gamma_2} = 5\%$ and 10% are considered, respectively, and $\text{SIR} = -25$ dB. Based on the error statistics, it should be emphasized that the probability of 10% errors in our setting is smaller than 10^{-3} . It is seen from Figure 5.10(a) that the BERs obtained by using AISL-II are much lower and about the same as the theoretical curve without any interference in (5.19), while the receiver with AISL-I has a poor robustness against estimation errors.

For more information about the effects of estimation errors on the BERs, Figure 5.10(b) presents the BER curves associated with errors on a particular parameter. For example, the BER curve with black square symbol is obtained by using a two-component LFM interference with $e_{A_1} = e_{A_2} = 10\%$ and $e_{\theta_1} = e_{\theta_2} = e_{f_1} = e_{f_2} = e_{\gamma_1} = e_{\gamma_2} = 0$ based

(a): SIR = -25 dB

(b): SNR = -2.5 dB

 Figure 5.10: Detection results when $L = 16$ and the two-component LFM interference is with errors when: (a) SIR = -25 dB; (b) SNR = -2.5 dB.

on the AISL-I. In this figure, the errors of estimated initial phases are not considered since they are estimated by the ML method, which has been known as an accurate estimation if the LFM interference is strong. It is observed that Δf_k and $\Delta \gamma_k$ have more effects than ΔA_k on the BERs. In fact, the same observations can be made by noting that $2A_k^2$ in (5.29) and (5.30) is larger than ΔA_k^2 in (5.27). Because the residues of phase cancellation is removed by the HP filter, the receiver using AISL-II achieves the BERs that are not disturbed by the interference.

5.5.4 Comparison with Other Methods

Many LFM interference suppression methods for the DSSS communication have been proposed in the literature. As shown in Table 5.2, we have compared the proposed methods with other ones, such as those based on the discrete polynomial phase transform (DPPT) [138], WVD [56], matched signal transform (MST) [139], FrFT [127], STFT [55] and modified discrete chirp Fourier transform (MDCFT) [140]. The one based on the STFT employs impulse, single-tone and LFM interference randomly in each simulation, while other methods deal with LFM interference. The last column lists the conditions in terms of SIR to achieve $\text{BER} < 10^{-3}$. The MST based method fails to achieve $\text{BER} < 10^{-3}$ for all SIR range. Our proposed methods with $L = 16$ performs better according to Figure 5.5 and Table 5.2 because it is able to effectively deal with the LFM interference with a large SIR range. Comparing to the DPPT, Table 5.2 shows that under the same condition, the proposed methods also obtain better results.

5.6 Summary

New interference suppression methods, known as AISL-I and -II, are introduced for the DSSS communication systems. The improved performance of interference suppression has been achieved by the proposed methods from the following three aspects. The first

Table 5.2: Performance Comparison with Methods in Literature

Method	K	L	SNR	BER $< 10^{-3}$
DPPT [138]	1	128	-10 dB	SIR > -10 dB
WVD [56]	1	32	0 dB	SIR < -5 dB
MST [139]	1	31	3 dB	none
FrFT [127]	1	16	0 dB	SIR > -21 dB
STFT [55]	1	16	0 dB	SIR > -45 dB
MDCFT [140]	1	16	0 dB	SIR > -75 dB
MDCFT [140]	2	16	0 dB	SIR > -35 dB
Proposed	1	16	0 dB	all SIR range
Proposed	2	16	0 dB	all SIR range
Proposed	1	128	-10 dB	all SIR range

one is that the LVD is able to better represent multi-component LFM signals, which has built a basic foundation on correctly synthesizing the interference. The second one is to implement a mechanism to detect the presence of interference that requires effective suppression process. The last aspect is that the initial phase of interference can be dealt with by AISL-II without requiring any additional computation resources. Our theoretical analysis and simulation results have shown that for more flexibility of dealing with the interference in various environments, AISL-II is a promising method to achieve nearly optimal suppression performance.

5.7 Appendix

5.7.1 The ML Phase Estimation

The ML estimation for initial phases is well known, given by (see Chapter 13 in [137])

$$\tilde{\theta}_k = \tan^{-1} \left(\frac{\Im \left\{ \int x(t) e^{-j2\pi \tilde{f}_k t - j\pi \tilde{\gamma}_k t^2} dt \right\}}{\Re \left\{ \int x(t) e^{-j2\pi \tilde{f}_k t - j\pi \tilde{\gamma}_k t^2} dt \right\}} \right), \quad (5.32)$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ are operators to extract the real and imaginary parts from a complex quantity, respectively, $\tilde{f}_k$ and $\tilde{\gamma}_k$ are estimated from the LVD domain. It is known

that the ML method is able to provide accurate estimation for initial phases if the LFM interference is strong. Since the interference suppression scheme in the AISL-I is implemented when the LFM interference is strong, we employ the ML method to estimate the initial phases.

5.7.2 The Energy of Interference Residue

Assuming the estimation errors are small enough, we consider them into the following cases.

(a) $\Delta A_k \neq 0, \Delta \theta_k = \Delta f_k = \Delta \gamma_k = 0$.

$$\begin{aligned} \Delta A^2 &= E [|\Delta i_m(t)|^2] \\ &= E \left[\sum_{k=1}^K \Delta A_k^2 + \sum_{k=1}^K \sum_{\substack{n=1 \\ n \neq k}}^K \Delta A_k \Delta A_n e^{j(\theta_k - \theta_n) + j2\pi(f_k - f_n)t + j\pi(\gamma_k - \gamma_n)t^2} \right] \\ &= \sum_{k=1}^K \Delta A_k^2. \end{aligned} \quad (5.33)$$

(b) $\Delta \theta_k \neq 0, \Delta A_k = \Delta f_k = \Delta \gamma_k = 0$.

$$\begin{aligned} \Delta A^2 &= E [|\Delta i_m(t)|^2] \\ &= E \left[\sum_{k=1}^K A_k^2 |1 - e^{j\Delta \theta_k}|^2 \right. \\ &\quad \left. + \sum_{k=1}^K \sum_{\substack{n=1 \\ n \neq k}}^K A_k A_n (1 - e^{j\Delta \theta_k}) (1 - e^{-j\Delta \theta_n}) e^{j(\theta_k - \theta_n) + j2\pi(f_k - f_n)t + j\pi(\gamma_k - \gamma_n)t^2} \right] \\ &= \sum_{k=1}^K 2A_k^2 (1 - \cos \Delta \theta_k). \end{aligned} \quad (5.34)$$

(c) $\Delta f_k \neq 0, \Delta A_k = \Delta \theta_k = \Delta \gamma_k = 0$. We have

$$\begin{aligned} \Delta A^2 &= E \left[\sum_{k=1}^K 2A_k^2 (1 - \cos(2\pi \Delta f_k t)) \right] + E_c \\ &= \sum_{k=1}^K 2A_k^2 (1 - E [\cos(2\pi \Delta f_k t)]), \end{aligned} \quad (5.35)$$

where $E[\cos(2\pi\Delta f_k t)]$ is generally non-zero when $\Delta f_k t$ is not an integer, and

$$\begin{aligned} E_c = E[& \sum_{k=1}^K \sum_{n=1, n \neq k}^K A_k A_n e^{j(\theta_k - \theta_n)} (e^{j(2\pi(f_k - f_n)t + \pi(\gamma_k - \gamma_n)t^2)} \\ & - e^{j(2\pi(f_k - f_n - \Delta f_n)t + \pi(\gamma_k - \gamma_n)t^2)} - e^{j(2\pi(f_k + \Delta f_k - f_n)t + \pi(\gamma_k - \gamma_n)t^2)} \\ & + e^{j(2\pi(f_k + \Delta f_k - f_n - \Delta f_n)t + \pi(\gamma_k - \gamma_n)t^2)})] = 0. \end{aligned}$$

(d) $\Delta\gamma_k \neq 0$, $\Delta A_k = \Delta\theta_k = \Delta f_k = 0$,

$$\Delta A^2 = \sum_{k=1}^K 2A_k^2 (1 - E[\cos(\pi\Delta\gamma_k t^2)]). \quad (5.36)$$

(e) $\Delta A_k \neq 0$, $\Delta\theta_k \neq 0$, $\Delta f_k \neq 0$ and $\Delta\gamma_k \neq 0$,

$$\Delta A^2 = \sum_{k=1}^K (\Delta A_k^2 + 2A_k (A_k + \Delta A_k) \times (1 - E[\cos(\Delta\theta_k + 2\pi\Delta f_k t + \pi\Delta\gamma_k t^2)])) \quad (5.37)$$

When the interference is strong enough, these errors are very close to the CRLB [43, 44] as

$$\begin{aligned} E[|\Delta A_k|^2] &\approx \frac{1 + \sigma_{v_m}^2}{2N_s}, \\ E[|\Delta\theta_k|^2] &\approx \frac{9(1 + \sigma_{v_m}^2)}{8N_s A_k^2}, \\ E[|\Delta f_k|^2] &\approx \frac{6(1 + \sigma_{v_m}^2)f_s^2}{N_s^3 A_k^2}, \\ E[|\Delta\gamma_k|^2] &\approx \frac{90(1 + \sigma_{v_m}^2)f_s^4}{N_s^5 A_k^2}, \end{aligned} \quad (5.38)$$

where f_s is the sampling frequency and N_s is the input length. We can calculate the average energy of $\Delta i_m(t)$ as followings.

(a) $\Delta A_k \neq 0$, $\Delta\theta_k = \Delta f_k = \Delta\gamma_k = 0$. We obtain

$$\Delta A^2 = \sum_{k=1}^K \Delta A_k^2 \approx \frac{(1 + \sigma_{v_m}^2)K}{2N_s},$$

which is (5.22), by substituting the CRLB value in (5.38).

(b) $\Delta\theta_k \neq 0$, $\Delta A_k = \Delta f_k = \Delta\gamma_k = 0$. We obtain

$$\Delta A^2 \approx \frac{9(1 + \sigma_{v_m}^2)K}{8N_s} - \sum_{k=1}^K \frac{27(1 + \sigma_{v_m}^2)^2}{256N_s^2 A_k^2},$$

which is (5.23), by using the approximation $\cos x \approx 1 - x^2/2 + x^4/4!$.

(c) $\Delta f_k \neq 0$, $\Delta A_k = \Delta\theta_k = \Delta\gamma_k = 0$. We have

$$\begin{aligned} \Delta A^2 &= \sum_{k=1}^K 2A_k^2 \left(1 - \frac{1}{T_L} \int_{-\frac{T_L}{2}}^{\frac{T_L}{2}} \cos(2\pi \Delta f_k t) dt \right) \\ &\approx \sum_{k=1}^K 2A_k^2 \left(\frac{\pi^2 \Delta f_k^2 T_L^2}{6} - \frac{\pi^4 \Delta f_k^4 T_L^4}{120} \right) \\ &\approx \sum_{k=1}^K \frac{\pi^2 \Delta f_k^2 T_L^2 A_k^2}{3} \\ &\approx \frac{2\pi^2 L^2 K (1 + \sigma_{v_m}^2)}{N_s^3}, \end{aligned} \tag{5.39}$$

which is (5.24), by using $T_L = L/f_s$, which is the time duration of one PN sequence.

(d) $\Delta\gamma_k \neq 0$, $\Delta A_k = \Delta\theta_k = \Delta f_k = 0$. (5.36) becomes

$$\begin{aligned} \Delta A^2 &\approx \sum_{k=1}^K 2A_k^2 \left(\frac{\pi^2 \Delta\gamma_k^2 T_L^4}{160} - \frac{\pi^4 \Delta\gamma_k^4 T_L^8}{55296} \right) \\ &\approx \sum_{k=1}^K \frac{\pi^2 \Delta\gamma_k^2 T_L^4 A_k^2}{80} \\ &\approx \frac{9\pi^2 L^4 K (1 + \sigma_{v_m}^2)}{8N_s^5}, \end{aligned} \tag{5.40}$$

which is (5.25).

(e) $\Delta A_k \neq 0$, $\Delta \theta_k \neq 0$, $\Delta f_k \neq 0$ and $\Delta \gamma_k \neq 0$. (5.37) becomes

$$\begin{aligned}
 \Delta A^2 &\approx \sum_{k=1}^K (\Delta A_k^2 + 2A_k(A_k + \Delta A_k) \\
 &\quad \times (\frac{\Delta \theta_k^2}{2} - \frac{\Delta \theta_k^4}{24} + \frac{\pi^2 \Delta f_k^2 T_L^2}{6} - \frac{\pi^2 \Delta \theta_k^2 \Delta f_k^2 T_L^2}{12} + \frac{\pi^2 \Delta \gamma_k^2 T_L^4}{160} - \frac{\pi^4 \Delta f_k^4 T_L^4}{120} \\
 &\quad - \frac{\pi^2 \Delta \theta_k^2 \Delta \gamma_k^2 T_L^4}{320} - \frac{\pi^4 \Delta f_k^2 \Delta \gamma_k^2 T_L^6}{448} - \frac{\pi^4 \Delta \gamma_k^4 T_L^8}{55296})) \\
 &\approx \sum_{k=1}^K \left(\Delta A_k^2 + A_k(A_k + \Delta A_k) \left(\Delta \theta_k^2 - \frac{\Delta \theta_k^4}{12} + \frac{\pi^2 \Delta f_k^2 T_L^2}{3} - \frac{\pi^2 \Delta \theta_k^2 \Delta f_k^2 T_L^2}{6} \right) \right) \\
 &\approx \frac{(1 + \sigma_{v_m}^2)K}{2N_s} + \left(\frac{9(1 + \sigma_{v_m}^2)}{8N_s} - \frac{27(1 + \sigma_{v_m}^2)^2}{256N_s^2 A_k^2} + \frac{2\pi^2 L^2(1 + \sigma_{v_m}^2)}{N_s^3} \right. \\
 &\quad \left. - \frac{9\pi^2 L^2(1 + \sigma_{v_m}^2)^2}{8N_s^4 A_k^2} \right) \sum_{k=1}^K \left(1 + \sqrt{\frac{1 + \sigma_{v_m}^2}{2N_s A_k^2}} \right),
 \end{aligned}$$

which is (5.26).

The second case is when the estimation errors ΔA_k , $\Delta \theta_k$, Δf_k and $\Delta \gamma_k$ are large enough compared to the CRLB. The average energy of $\Delta i_m(t)$ are deducted as follows.

(a) $\Delta A_k \neq 0$, $\Delta \theta_k = \Delta f_k = \Delta \gamma_k = 0$. (5.33) remains as $\Delta A^2 = \sum_{k=1}^K \Delta A_k^2$, which is (5.27).

(b) $\Delta \theta_k \neq 0$, $\Delta A_k = \Delta f_k = \Delta \gamma_k = 0$. (5.34) remains as $\Delta A^2 = \sum_{k=1}^K 2A_k^2 (1 - \cos \Delta \theta_k)$, which is (5.28).

(c) $\Delta f_k \neq 0$, $\Delta A_k = \Delta \theta_k = \Delta \gamma_k = 0$. (5.35) becomes

$$\Delta A^2 = \sum_{k=1}^K 2A_k^2 = 2A^2,$$

which is (5.29).

(d) $\Delta \gamma_k \neq 0$, $\Delta A_k = \Delta \theta_k = \Delta f_k = 0$. (5.36) becomes

$$\Delta A^2 = 2A^2,$$

which is (5.30).

(e) $\Delta A_k \neq 0$, $\Delta \theta_k \neq 0$, $\Delta f_k \neq 0$ and $\Delta \gamma_k \neq 0$, (5.37) becomes

$$\Delta A^2 = \sum_{k=1}^K (\Delta A_k^2 + 2A_k^2 + 2A_k \Delta A_k),$$

which is (5.31).

Chapter 6

Lv Distribution based Signal Reconstruction Methods

In this chapter, time-domain signal reconstruction methods for multi-component LFM signals based on the LVD are considered, known as the LVD based signal reconstruction (LSR) and LSR with suppressed cross-terms (LSRC). They recover LFM signals from the transform domain to the time domain by using the peaks of auto-terms in the LVD domain. Due to the asymptotic linearity of LVD, in which the auto-terms are almost undisturbed by cross-terms, the LSR is capable to provide accurate reconstruction. The LSRC can perform even better than the LSR, with the inherent strategy of reducing the cross-term effects. Several examples are simulated to show that both the LSR and LSRC are able to recover multi-component LFM signals effectively. Finally, some applications are presented based on the LSR and LSRC, demonstrating their good performance of de-noising in degraded SNR environments and re-sampling to arbitrary sampling rate with trivial errors.

6.1 Introduction

Reconstruction of a time-domain signal from a transformed domain is important for many applications. It is common that a given signal is transformed into another domain for

the convenience of processing. Therefore, properly retrieving the desired time-domain signal from the transformed domain is a critical operation. For example, the inverse FT is needed after a given signal filtered based on the convolution operation performed in the frequency domain.

For processing non-stationary signals, time-domain reconstruction techniques are also necessary in the areas of signal separation, noise suppression, interference rejection and signal re-sampling applications. For instance, a jammed LFM signal can be regenerated after removing the interference in the time-frequency domain. Although the ILVD has been also proposed in [47], however, it is intended to obtain the signal distribution in the time-frequency domain rather than in the time domain. In this chapter, a method known as the LSR is introduced to retrieve the time-domain signal from its LVD domain.

The basic concept of LSR is motivated from a signal synthesis method proposed in [141], named as the WVD based signal reconstruction (WSR), which was reported to reconstruct the time-domain signal by using only the auto-terms of WVD for various applications such as signal de-noising and separation. When dealing with multi-component signals, however, the auto-terms usually affected by the cross-terms in the WVD plane, which results in inaccuracies of the auto-term information. In contrast, due to the property of asymptotic linearity in the LVD domain, the auto-terms are highly concentrated as peaks and only slightly affected by the trivial cross-terms. Therefore the LSR is proposed to reconstruct the time-domain signal by using only the peaks of auto-terms, or called the auto-peaks. It is shown that the LSR performs better than the WSR for multiple LFM component signals due to the inherent advantages of LVD. Comparing to other reconstruction methods reported in the literature, such as the RWT based signal synthesis [142], cross entropy principle based synthesis [143], masked WVD based signal synthesis [144] and array processing based synthesis [145], the LSR recovers the time-domain signal with less mean square errors (MSEs).

It is seen that the LSR suffers from the effects of side-lobes and cross-terms. To further improve the performance, another reconstruction method, named the LSRC, is presented to recover the time-domain signal by using suppressed cross-terms in the LVD domain. It is implemented by subtracting the regenerated cross-terms from the LVD of the signal before the reconstruction process. Since the effects of cross-terms are minimized, the LSRC achieves better accuracy than the LSR. To verify the usefulness of LSR and LSRC, these reconstruction methods are used for de-noising operation and sampling rate conversion of multiple components LFM signals.

The rest of this chapter is organized as follows. In Section 6.2, the WVD and WSR are reviewed. The LSR is presented for multi-component LFM signal recovery in Section 6.3, with definition, implementation and simulations. Section 6.4 presents the LSRC and simulation results on the error performance. Experimental performances of the LSR and LSRC on several issues are discussed in Section 6.5. Section 6.6 discusses the performances of LSR and LSRC when the LFM signal has random initial phases. In Section 6.7, three applications of the LSR and LSRC for decomposition, de-noising and sampling rate conversion are provided. Sections 6.8 and 6.9 present some discussions and conclusions of this chapter. At last, mathematic derivations are shown in Section 6.10.

6.2 Review of WVD and WSR

For a multi-component LFM signal as

$$s(t) = \sum_{k=1}^K s_k(t) = \sum_{k=1}^K A_k \exp(j2\pi f_k t + j\pi \gamma_k t^2), \quad (6.1)$$

its WVD is given by [146, 147]

$$\begin{aligned} W_s(t, f) &= \int_{\tau} s\left(t + \frac{\tau}{2}\right) s^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f \tau} d\tau \\ &= W_{A_u}(t, f) + W_{C_r}(t, f), \end{aligned} \quad (6.2)$$

Figure 6.1: The WVD of Example 6.5 listed in Table 6.1.

where $W_{A_u}(t, f)$ denotes auto-terms as

$$W_{A_u}(t, f) = \sum_{k=1}^K A_k^2 \delta(f - (f_k + \gamma_k t)), \quad (6.3)$$

and $W_{C_r}(t, f)$ represents cross-terms. By observing the expression of auto-terms, the WVD has such property as [146, 147]

$$\int_f W_{A_u}\left(\frac{t}{2}, f\right) e^{j2\pi ft} df = \sum_{k=1}^K A_k^2 e^{j2\pi f_k t + j\pi \gamma_k t^2}. \quad (6.4)$$

Referring to this property, the WSR by using the windowed WVD was reported in [141] to reconstruct the auto-terms, for some applications such as de-noising and signal separation. However, it is known that the WVD usually suffers from the cross-terms resulting errors of detecting the auto-terms. Figure 6.1 shows an example that strong cross terms preventing correct auto-term detection. In Figure 6.1, the auto-term of the 3rd component, $s_3(t)$, is submerged in the cross-terms, therefore, can not be properly detected.

6.3 The LVD Based Signal Reconstruction

Due to the asymptotic linearity, the auto-peaks are highly concentrated in the LVD domain and only slightly disturbed by cross-terms, as shown in Figure 6.2(a). It is expected that using an LVD based signal recovery method would improve the reconstruction performance comparing to the WSR. This section will propose the LSR and present some simulations. Since the information of initial phases are lost in the WVD and not concerned in the WSR, similarly we first propose the LSR without introducing the initial phases to show the inherent performance, then will discuss them in Section 6.6.

6.3.1 Review of LVD

Referring to Section 2.4.1, it is known that the LVD of input signal $s(t)$ is expressed as

$$L_s(f, \gamma) = F_\tau \{ F_{t_n} \{ \Gamma [R_s^C(t, \tau)] \} \}. \quad (6.5)$$

$R_s^C(t, \tau)$ is known as the PSIAF of $s(t)$, defined by

$$\begin{aligned} R_s^C(t, \tau) &= s\left(t + \frac{\tau + a}{2}\right) s^*\left(t - \frac{\tau + a}{2}\right) \\ &= A_u^a(t, \tau) + C_r^a(t, \tau), \end{aligned} \quad (6.6)$$

where $A_u^a(t, \tau)$ and $C_r^a(t, \tau)$ represent the auto-terms and cross-terms of PSIAF, respectively, as following

$$A_u^a(t, \tau) = \sum_{k=1}^K s_k\left(t + \frac{\tau + a}{2}\right) s_k^*\left(t - \frac{\tau + a}{2}\right) \quad (6.7)$$

$$\begin{aligned} &= \sum_{k=1}^K A_k^2 \exp(j2\pi f_k(\tau + a) + j2\pi \gamma_k(\tau + a)t), \\ C_r^a(t, \tau) &= \sum_{k=1}^{K-1} \sum_{l=k+1}^K (R_{s_k s_l}^C(t, \tau) + R_{s_l s_k}^C(t, \tau)). \end{aligned} \quad (6.8)$$

Similar to the WVD, the LVD is bilinear as

$$\begin{aligned} L_s(f, \gamma) &= F_\tau \{F_{t_n} \{\Gamma[A_u^a(t, \tau) + C_r^a(t, \tau)]\}\} \\ &= L_{A_u}(f, \gamma) + L_{C_r}(f, \gamma), \end{aligned} \quad (6.9)$$

where $L_{A_u}(f, \gamma)$ and $L_{C_r}(f, \gamma)$ denotes auto-terms and cross-terms, respectively. According to the Lemma 1 in [47], we have the expression of auto-terms as

$$L_{A_u}(f, \gamma) = \sum_{k=1}^K A_k^2 e^{j2\pi a f} \delta(f - f_k) \delta(\gamma - \gamma_k/h), \quad (6.10)$$

as auto-peaks in the LVD domain.

6.3.2 Definition of LSR

Motivated by the concept of property in (6.4), a similar property of LVD can be derived and leads to the following Theorem.

Theorem 1: For an infinite continuous signal $s(t)$ defined in (6.1), the LVD has such property as

$$\int_f \int_\gamma L_{A_u}(f, \gamma) e^{j\pi\gamma h t^2} e^{j2\pi f(t-a)} d\gamma df = \sum_{k=1}^K A_k^2 e^{j2\pi f_k t + j\pi\gamma_k t^2}. \quad (6.11)$$

By substituting $L_{A_u}(f, \gamma)$ given in (6.10), Theorem 1 can be proven. It should be aware that in (6.11), $L_{A_u}(f, \gamma)$ is assumed to be the ideal one which contains the signal energy only. In practice, however, the value of $L_{A_u}(f, \gamma)$ is obtained from the LVD which has the influence of cross-terms and side-lobes when the length of signal is finite. Therefore, the results from (6.11) are related to the accuracy of auto-peaks.

By observing the right side of (6.11) and the expression of signal in (6.1), it is seen that the Theorem 1 provides a recovered signal having same centroid frequencies and chirp rates as the original signal, but with squared amplitudes. To get the reconstruction

of input signal directly, we propose the LVD based signal reconstruction (LSR) as defined by

$$s_R(t) = \int_f \int_\gamma \sqrt{L_{A_u}(f, \gamma)} e^{j\pi\gamma ht^2} e^{j2\pi f(t-a/2)} d\gamma df. \quad (6.12)$$

By substituting $L_{A_u}(f, \gamma)$ in (6.10), it is easy to deduct the result of LSR for an infinite multi-LFM signal as

$$\begin{aligned} s_R(t) &= \int_f \int_\gamma \sum_{k=1}^K A_k e^{j\pi f a} \delta(f - f_k) \delta(\gamma - \gamma_k/h) e^{j\pi\gamma ht^2} e^{j2\pi f(t-a/2)} d\gamma df \\ &= \sum_{k=1}^K A_k e^{j2\pi f_k t + j\pi\gamma_k t^2} = \sum_{k=1}^K s_k(t) = s(t). \end{aligned} \quad (6.13)$$

This indicates that the LSR has the capability of reconstructing the original signal based on the auto-peaks, $L_{A_u}(f, \gamma)$, defined in (6.10). Therefore in this form, the recovery signal is obtained directly.

6.3.3 Implementation of LSR

The previously presented LSR is based on the assumption that the signal length is infinite and parameters of auto-peaks in (6.10) can be exactly obtained. In practice, we deal with finite-length signals which has the effects of window or side-lobes. Therefore, the parameters of auto-peaks may not be accurately represented because some signal energy is distributed to the side-lobes and the cross-terms influence the peaks in the LVD domain.

To implement the LSR defined in (6.12) and (6.13), the auto-peaks $L_{A_u}(f, \gamma)$ are to be identified firstly. It has been reported that the region of auto-terms in the WVD domain is usually difficult to be specified because the existence of cross-terms. In contrast, the LVD has a desirable property of asymptotic linearity [47] leading to trivial cross-terms compared to the auto-peaks. Therefore, (6.9) can be approximated as

$$L_s(f, \gamma) \approx L_{A_u}(f, \gamma). \quad (6.14)$$

Figure 6.2(a) shows the LVD of a signal having three peaks as the auto-terms and small cross-terms. These auto-peaks may be obtained by a masking operation defined as

$$\tilde{L}_{A_u}(f, \gamma) = \begin{cases} L_s(f, \gamma), & \text{for } (f, \gamma) \in S, \\ 0, & \text{for } (f, \gamma) \notin S, \end{cases} \quad (6.15)$$

where S denotes the region of auto-peaks specified by points (f_k, γ_k) in the LVD domain (In experiment, S is determined by detecting the signal peaks and specifying their locations). By using such estimated auto-peaks, the LSR for finite-length signals is implemented as

$$s_r(t) = \int_f \int_\gamma \sqrt{\tilde{L}_{A_u}(f, \gamma)} e^{j\pi\gamma ht^2} e^{j2\pi f(t-a/2)} d\gamma df. \quad (6.16)$$

One way of obtaining the estimated auto-peaks is to assume

$$\tilde{L}_{A_u}(f, \gamma) = \sum_{k=1}^K \tilde{A}_k^2 e^{j\tilde{\phi}_k} \delta(f - \tilde{f}_k) \delta(\gamma - \tilde{\gamma}_k/h), \quad (6.17)$$

where $\tilde{A}_k$, $\tilde{\phi}_k$, $\tilde{f}_k$ and $\tilde{\gamma}_k$ are estimated parameters from the LVD domain. However, these parameters are estimated with errors because of the effects of window, cross-terms and the limited resolution of the LVD domain. The window effects of the finite sequence length may reduce the value of $\tilde{A}_k$ because a part of signal energy is distributed to side-lobes. The cross-terms may affect the amplitudes and phases of auto-peaks. The limited resolution in the LVD domain may also result in errors in estimating the values of $\tilde{A}_k$, $\tilde{\phi}_k$, $\tilde{f}_k$ and $\tilde{\gamma}_k$.

Based on the complexity of FFT algorithm in the order of $O(N_s \log_2 N_s)$ where N_s is the size of FFT, the number of complex multiplications of LVD is in the order of $O(N_s^2 \log_2 N_s)$ [47, 148]. The LSR takes only K peak values and requires the number of complex multiplications in the order of $O(KN_s + K \log_2 N_s)$.

Table 6.1: Parameters of Signals for Simulations ($f_s = 128$ Hz, $N_s = 256$)

Examples	K	A	f (Hz)	γ (Hz/s)
6.1	1	$A_1 = 1$	$f_1 = -26$	$\gamma_1 = 1$
6.2	2	$A_1 = A_2 = 1$	$f_1 = -26, f_2 = 26$	$\gamma_1 = 1, \gamma_2 = -1$
6.3	2	$A_1 = 1, A_2 = 2$	$f_1 = -26, f_2 = 26$	$\gamma_1 = 1, \gamma_2 = -1$
6.4	3	$A_1 = 1$	$f_1 = -26$	$\gamma_1 = 1$
		$A_2 = 1$	$f_2 = 26$	$\gamma_2 = -1$
		$A_3 = 1$	$f_3 = -6$	$\gamma_3 = 3$
6.5	3	$A_1 = 1$	$f_1 = -26$	$\gamma_1 = 1$
		$A_2 = 2$	$f_2 = 26$	$\gamma_2 = -1$
		$A_3 = 0.7$	$f_3 = -6$	$\gamma_3 = 3$

Figure 6.2: The LVDs of original and recovery signals for an LFM signal having three components with equal amplitudes (Example 6.4, noise-free).

Figure 6.3: Original and recovery signals (real part) in time domain for a three-component LFM signal with equal amplitudes (Example 6.4, noise-free) based on the LSR.

6.3.4 Examples

Now we consider the recovery accuracy achieved by the LSR as follows. Table 6.1 lists the parameters of five examples used for the experiments conducted in this chapter. These signals are expressed in the form of (6.1) with a sampling frequency $f_s = 128$ Hz. Figure 6.2 presents the LVDs of original and recovery signals for Example 6.4 in Table 6.1, showing that these two LVDs have about the same distributions. Figure 6.3 shows the original and recovered signals in time domain based on the LSR by using the signal parameters for Example 6.4. It shows that this three-component signal is well reconstructed by the scheme in (6.16).

The accuracy of LSR is also measured by the MSE between original and recovery signals defined as

$$MSE = E [|s(t) - s_r(t)|^2] ,$$

and experimental MSEs based on the LSR for signals with components number $K = 2 \sim 4$ and different sample lengths are shown in Table 6.2. All the signals simulated in this

Table 6.2: MSEs based on the LSR/LSRC

Input length N_s	$K = 2 (\times 10^{-11})$ (LSR/LSRC)	$K = 3 (\times 10^{-9})$ (LSR/LSRC)	$K = 4 (\times 10^{-7})$ (LSR/LSRC)
256	3.457/1.442	5.014/0.439	14.394/1.021
512	1.282/0.496	1.248/0.228	3.277/0.240
1024	0.422/0.195	0.816/0.105	0.996/0.078
2048	0.162/0.130	0.203/0.133	0.181/0.065

table have LFM components with a same amplitude $A_k = 1$, for all k . The MSEs in the table can be considered negligible so that the corresponding $s_r(t)$ is believed as an effective and accurate reconstruction of the original signal. The errors between $s(t)$ and $s_r(t)$ are mainly caused by the effects of finite sequence length and the cross-terms in the LVD domain. It is shown that the errors increase as the number of signal components increases, which is the consequence of accumulated estimation errors on $\tilde{A}_k$ and $\tilde{\phi}_k$. It is also observed that the MSEs can be effectively reduced as the length of signal sequence is increased. This observation verifies the fact that the estimations of A_k and ϕ_k become more accurate by increasing the signal length, as stated by the property of LVD expressed as [47]

$$\text{For } N_s \rightarrow \infty, \quad L_s(f, \gamma) \rightarrow L_{A_u}(f, \gamma). \quad (6.18)$$

6.4 The LSR with Suppressed Cross-Terms

Because of the effects of both side-lobes and cross-terms, the LSR recovers signals with errors as shown in Table 6.2. In fact, the information of cross-terms can be known when the parameters of signal components are available. Assuming the LVD is able to estimate parameters accurately as $\tilde{f}_k = f_k$, $\tilde{\gamma}_k = \gamma_k$ and $\tilde{A}_k \rightarrow A_k$, the cross-terms can be regenerated as tending to the original cross-terms. This motivates an improvement of

the LSR by obtaining more accurate auto-peaks of the LVD. We call this method LSR with suppressed cross-terms (LSRC). The processing steps are as follows.

- (a). After obtaining the LVD of signal, estimate the parameters of auto-peaks, including amplitudes, centroid frequencies and chirp rates;
- (b). Based on (6.8), produce the estimated cross-terms of PSIAF, $\tilde{C}_r^a(t, \tau)$, by using $\tilde{A}_k$, $\tilde{f}_k$ and $\tilde{\gamma}_k$ obtained from the first step. Then perform the scaling operator Γ and FT on $\tilde{C}_r^a(t, \tau)$ and produce an estimation of cross-terms, $\tilde{L}_{C_r}(f, \gamma)$;
- (c). Subtract $\tilde{L}_{C_r}(f, \gamma)$ from $L_s(f, \gamma)$ to achieve $L_s^{-c}(f, \gamma)$. The auto-peaks are obtained by

$$\tilde{L}_{A_u}^{-c}(f, \gamma) = \begin{cases} L_s^{-c}(f, \gamma), & \text{for } (f, \gamma) \in S, \\ 0, & \text{for } (f, \gamma) \notin S, \end{cases} \quad (6.19)$$

where $\tilde{L}_{A_u}^{-c}(f, \gamma)$ is more accurate than $\tilde{L}_{A_u}(f, \gamma)$ in (6.15) (see Section 6.10);

- (d). Following the concept similar to (6.16), we reconstruct the signal by using $\tilde{L}_{A_u}^{-c}(f, \gamma)$, i.e., the LSRC is defined as

$$s_r^{-c}(t) = \int_f \int_\gamma \sqrt{\tilde{L}_{A_u}^{-c}(f, \gamma)} e^{j\pi\gamma ht^2} e^{j2\pi f(t-a/2)} d\gamma df. \quad (6.20)$$

It is noted that the LSRC performs better than the LSR under the condition of accurate estimated parameters, i.e., $\tilde{f}_k = f_k$, $\tilde{\gamma}_k = \gamma_k$ and $\tilde{A}_k \rightarrow A_k$. This condition is usually satisfied, when the signal is not corrupted by noise and (f, γ) lies on the grid of the LVD plane [47]. For instance, simulation shows $\tilde{f}_k = f_k$, $\tilde{\gamma}_k = \gamma_k$, and $|\tilde{A}_k - A_k|/A_k < 10^{-4}$, $k = 1, \dots, K$, for Example 6.5 in Table 6.1.

It is easy to know that the computational complexity of LSRC is in the order of $O(N_s^2 \log_2 N_s + K N_s + K \log_2 N_s)$ as its computation is mainly for the 2D FT and (6.20). The LSRC costs more computation than the LSR, in the same order of the complexity of LVD.

Table 6.2 presents the MSEs between the signal reconstructed by the LSRC method and the original signal in various situations, indicating that the LSRC achieves more

accurate signal reconstruction than the LSR because the effects of cross-terms on the auto-peaks are minimized. It is difficult to entirely remove the cross-terms, because the parameters used for producing the cross-terms are estimated with errors, as shown in (6.35).

6.5 Experimental Performance of LSR/LSRC

In this section, we will investigate the signal reconstruction performance based on the LSR and LSRC experimentally. The performance of reconstruction is affected by the locations of auto-peaks and their side-lobes. Furthermore, these effects are increased if the auto-peaks are close to each other. Here we consider an example dealing with a signal having two components whose parameters are $f_1 = -1.5$ Hz, $f_2 = 1.5$ Hz, $\gamma_1 = 1$ Hz/s and $\gamma_2 = -1$ Hz/s ($A_1 = A_2 = 1$, $N_s = 256$). The MSEs of LSR and LSRC are

$$MSE_{LSR} = 1.698 \times 10^{-3}, \quad MSE_{LSRC} = 6.674 \times 10^{-4}.$$

respectively. It is seen that these MSEs are much larger than those listed in Table 6.2 because of the higher side-lobes and cross-terms. The examples listed in Table 6.1 are used in this chapter for simulations. Since the distances between auto-peaks of these examples are large enough, the effects of side-lobes and cross-terms associated with these signals are limited on the peaks of auto-terms.

In general, the LVD of a finite length signal has finite resolutions of centroid frequency and chirp rate. Consequently, the LSR and LSRC reconstruct the signals with errors due to limited resolution. We can see this in Table 6.3. The resolution errors are defined as

$$e_{f_k} = \frac{|f_k - \tilde{f}_k|}{\delta f}, \quad e_{\gamma_k} = \frac{|\gamma_k - \tilde{\gamma}_k|}{\delta \gamma},$$

where δf and $\delta \gamma$ denote the centroid frequency and chirp rate resolutions, respectively, expressed as [47]

$$\delta f = f_s/N_s, \quad \delta \gamma = 2f_s/(hN_s).$$

Table 6.3: LSR/LSRC based MSEs for Example 6.2 with Resolution Errors of Frequency and Chirp Rate

Resolution errors (for all k)	LSR	LSRC
$e_{f_k} = 0.001, e_{\gamma_k} = 0$	1.367×10^{-5}	1.011×10^{-5}
$e_{f_k} = 0.01, e_{\gamma_k} = 0$	2.680×10^{-3}	2.331×10^{-3}
$e_{f_k} = 0.1, e_{\gamma_k} = 0$	1.150×10^{-1}	1.150×10^{-1}
$e_{f_k} = 0, e_{\gamma_k} = 0.001$	4.154×10^{-6}	4.002×10^{-6}
$e_{f_k} = 0, e_{\gamma_k} = 0.01$	2.268×10^{-4}	2.001×10^{-4}
$e_{f_k} = 0, e_{\gamma_k} = 0.1$	4.021×10^{-2}	3.970×10^{-2}
$e_{f_k} = e_{\gamma_k} = 0.001$	8.703×10^{-5}	8.677×10^{-5}
$e_{f_k} = e_{\gamma_k} = 0.01$	4.054×10^{-3}	4.001×10^{-3}
$e_{f_k} = e_{\gamma_k} = 0.1$	1.583×10^{-1}	1.557×10^{-1}

In this simulation, the signal specified in Example 6.2 is used, and the resolutions in the LVD domain are $\delta f = 0.5$ Hz and $\delta \gamma = 1$ Hz/s. The simulated signal has two components and both these have the resolution errors. Table 6.3 shows that the MSEs based on LSR/LSRC are significantly increased when the resolution errors increase since the errors of estimated parameters are increased accordingly. For LSR and LSRC, spectrum leakage or the side-lobe effect is unavoidable since they only use the peak information of the auto-terms. If we directly apply these two algorithms, it will lose useful information. Therefore the MSEs in Table 6.3 are significantly larger than those in Table 6.2. To account for this phenomenon, refinement technique can be utilized to obtain a better and finer estimation [149, 150]. The basic idea of the refinement procedure is to generate a series of LFM signals with new centroid frequencies and chirp rates in a finer grid around the current estimation. Therefore, centroid frequencies and chirp rates of the most correlated LFM signal with respect to the original signal are used as the estimates.

All the previous simulations in this chapter process the signals containing components with equal amplitudes. More complex situation is a signal consisting of components with unequal amplitudes. For Example 6.5 which contains unequal amplitudes, the

Table 6.4: LSR/LSRC based MSEs for Example 6.2 with Different Amplitudes ($A_1 = 1$)

–	LSR	LSRC
$A_2 = 2$	3.372×10^{-10}	6.342×10^{-11}
$A_2 = 4$	1.195×10^{-8}	7.086×10^{-9}
$A_2 = 6$	8.651×10^{-3}	9.072×10^{-4}

MSEs based on LSR and LSRC are 1.706×10^{-7} and 3.046×10^{-8} , respectively. While the WSR is not able to reconstruct this signal since the third component is submerged from Figure 6.1. Table 6.4 presents MSEs for Example 6.2 with different amplitudes of the components, and indicates that the recovery performance becomes worse when the difference of amplitudes is larger, based on both LSR and LSRC. This is because the effects of side-lobes of strong components and cross-terms are increased on the weak component. Furthermore, large differences between the amplitudes lead to the side-lobes and the cross-terms being higher than the weak components, resulting the weak ones are fully submerged. In this situation, the LSR/LSRC fails to detect and recover the weak ones. In general, the LSRC can provide more accurate reconstruction than the LSR in almost all kinds of environments.

6.6 Performance of LSR/LSRC with Random Initial Phases

The previous sections have introduced the LSR/LSRC and discussed their inherent performance without the effects of initial phases. Now we will discuss the performance when signals have random initial phases, defined in (2.1). Since the initial phases are lost in the computation of LVD, they must be estimated firstly. Equation (6.17) is modified to be

$$\tilde{L}_{A_u}^\theta(f, \gamma) = \sum_{k=1}^K \tilde{A}_k^2 e^{j\tilde{\phi}_k} e^{j2\tilde{\theta}_k} \delta(f - \tilde{f}_k) \delta(\gamma - \tilde{\gamma}_k/h), \quad (6.21)$$

where $\tilde{\theta}_k$ is the k th estimated initial phase, and then the LSR is implemented as in (6.16) by using $\tilde{L}_{A_u}^\theta(f, \gamma)$ instead of $\tilde{L}_{A_u}(f, \gamma)$. Similarly, the LSRC can be implemented by multiplying $e^{j2\tilde{\theta}_k}$ with each component of $\tilde{L}_{A_u}^{-c}(f, \gamma)$ in (6.20). Easy to see that the accuracy of reconstruction is not only affected by $\tilde{A}_k$, $\tilde{\phi}_k$, $\tilde{f}_k$ and $\tilde{\gamma}_k$, but also by the estimated initial phases $\tilde{\theta}_k$ which are not obtained by the LVD. In this chapter, we also use the ML phase estimation, as in Section 5.7.1, to estimate the initial phases.

The ML method produces estimation errors of initial phases, and more components will lead to more estimation errors. Although in the previous chapter, the suppression performance is generally not affected by the estimated errors of initial phases, these errors may degrade the performance of LSR/LSRC. First, we show the performance of ML method by using the MSE of original signal and reconstructed signal which includes only the estimated errors of initial phases, i.e., for Example 6.2 ($K = 2$, noise-free) with $N_s = 256$ and 512, the MSEs are 4.332×10^{-14} and 2.548×10^{-15} , respectively. Second, we consider the impacts of all kinds of errors, i.e., for Example 6.2, the MSEs of LSR and LSRC are:

$$N_s = 256 : \text{MSE}_{LSR} = 3.032 \times 10^{-10}, \text{MSE}_{LSRC} = 1.208 \times 10^{-10}; \quad (6.22)$$

$$N_s = 512 : \text{MSE}_{LSR} = 1.064 \times 10^{-10}, \text{MSE}_{LSRC} = 4.770 \times 10^{-11}. \quad (6.23)$$

Comparing to Table 6.2, the reconstruction performances of both LSR and LSRC are degraded by the estimate errors of initial phases.

6.7 Applications of the LSR/LSRC

The LSR and LSRC have been introduced and their effectiveness has been demonstrated for recovering multi-component LFM signals. In this section we will present three applications based on these two algorithms, i.e., signal decomposition, de-noising and re-sampling. In these applications, we consider the signals have random initial phases and use the ML method to estimate them.

Figure 6.4: Original and decomposed components based on the LSR in time domain for an LFM signal having three components with unequal amplitudes (Example 6.5, noise-free) and random initial phases.

6.7.1 Decomposition Based on the LSR/LSRC

A direct application of the LSR/LSRC is on the signal decomposition, which is usually employed on signal separation for audio, sonar, medical processing and so on [151–156]. Similar to the concept of signal separation based on the WSR in [141], the LSR/LSRC can decompose a multi-component LFM signal by reconstructing the auto-peaks one by one. The pure auto-peaks in LVD domain could be expressed as

$$L_{A_u}(f, \gamma) = \sum_{k=1}^K L_{s_k}(f, \gamma), \quad (6.24)$$

where $L_{s_k}(f, \gamma)$ denotes the LVD of $s_k(t)$. It is easy to know that each component can be reconstructed by

$$s_k^{dc}(t) = \int_f \int_\gamma \sqrt{\tilde{L}_{s_k}(f, \gamma)} e^{j\pi\gamma ht^2} e^{j2\pi f(t-a/2)} d\gamma df, \quad (6.25)$$

where $\tilde{L}_{s_k}(f, \gamma)$ is the estimated auto-peak of the k th component from the LVD domain. By using this procedure the signal is decomposed.

Figure 6.4 presents the LSR based decomposition for Example 6.5, which containing three-components with unequal amplitudes and random initial phases. Each LFM component is separated and recovered with trivial MSE in the order of 10^{-7} , including all kinds of errors. If the estimated errors of initial phases are not included, the MSE of each component is in the order of 10^{-8} . And if we only consider the estimated errors of initial phases, which are estimated by the ML method, the MSE is in the order of 10^{-11} . On this application our LSR is very useful while the WSR does not perform well, as shown in Figure 6.1, where the auto-term of the 3rd component may not to be specified from the WVD plane because of heavy cross-terms. By simulation, the LSRC also can decompose the multi-component signal and provide higher accuracy.

Figure 6.5: Original, noisy and de-noised signals in time domain for a three-component LFM signal (Example 6.4). (a) Real part of the original signal $s(t)$; (b) Real part of noisy signal $s(t)$ at SNR = 10 dB; (c) Real part of the de-noised signal $s_{dn}(t)$ based on the LSR at SNR = 10 dB. (d) Imaginary part of the original signal $s(t)$; (e) Imaginary part of noisy signal $s(t)$ at SNR = 0 dB; (f) Imaginary part of the de-noised signal $s_{dn}(t)$ at SNR = 0 dB.

6.7.2 De-noising Based on the LSR

Because practical signals are usually corrupted by noise, de-noising operation is widely used in many signal processing systems. Although many efforts have been reported [157–160] to suppress noise, however, these methods usually do not work well in negative SNR environments and the de-noised signal still has quite large MSE with respect to the original one. Our proposed LSR described in (6.16) may be used for such kind of applications because the signal in the LVD domain is highly concentrated and the noise is uniformly distributed in the entire LVD domain. The LSR in (6.16) makes use of auto-peaks only, ignores other points in the LVD domain that are closely related to side-lobes, cross-terms and noise, and produces the signal reconstruction that is affected by a small part of noise at the locations of auto-peaks.

In our simulation, Example 6.4 in Table 6.1 is used for Figure 6.5. In this section we denote the output of LSR in (6.16) to be $s_{dn}(t)$ as the de-noised signal. It is seen that $s_{dn}(t)$ has a smooth time representation without noise spikes and matches the original one very well. The noise effects on the amplitudes and initial phases are not evident for such non-negative SNR environments since the noise is not strong.

In the following, we will show the MSEs of the original and de-noised signals based on the LSR and FrFT by using Examples 6.1, 6.2 and 6.3 listed in Table 6.1 with random initial phases. The results based on Monte Carlo trials are plotted in Figure 6.6. The FrFT based de-noising [161] uses the similar concept as the LSR based de-noising method. That is detecting signal in the FrFT domain and then performing the inverse FrFT with the signal peaks only. The MSE of the original and de-noised signals is computed as

$$MSE = E [|s(t) - s_{dn}(t)|^2] .$$

Figure 6.6(a) shows the performances of reconstruction without considering the estimated errors of initial phases, and the simulations in Figure 6.6(b) include the impact of these

(a): MSEs not including the impact of errors of initial phases

(b): MSEs including the impact of errors of initial phases

Figure 6.6: MSEs of the FrFT and LSR based de-noised signals for Examples 6.1, 6.2 and 6.3: (a) without and (b) with random initial phases.

errors. From both figures, it is seen that the LSR based de-noising method achieves better performance than the FrFT based method when SNR is low. The main reason is that the LVD concentrates LFM signals better than the FrFT [148]. Therefore the noise effects on the auto-peaks are relatively weaker in the LVD domain. Since Figure 6.6(b) includes the effects of estimation errors of initial phases, the performances of both LSR and FrFT based de-noising are degraded comparing to Figure 6.6(a).

Our simulation results shows that the LSRC has achieved almost the same performance as the LSR in de-noising. This is mainly because the noise has dominant effects on the reconstruction and the effects of cross-terms become negligible. The technique of LSR based de-noising can be easily adopted for interference rejection [83, 130, 131, 140].

6.7.3 Re-sampling Based on the LSR/LSRC

This section will introduce another application of the LSR/LSRC for signal re-sampling. Re-sampling is widely used in data storage, exchange and transmission [81, 162, 163]. Basically we have two types of re-sampling: increasing the sampling frequency by interpolation and decreasing the sampling frequency by decimation. A rational re-sampling could be implemented by a combination of interpolation and decimation. In literature, many methods, such as filter based methods, have been reported to implement re-sampling. Distortions are usually inevitable due to the imperfection of filter design. The LSR/LSRC based re-sampling methods for multi-component LFM signals achieve the re-sampling operation with a fractional factor.

A discrete signal $s[n]$ sampled by a frequency f_s is represented as

$$s[n] = \sum_{k=1}^K A_k e^{j\theta_k + j2\pi f_k n T_s + j\pi \gamma_k (n T_s)^2}. \quad (6.26)$$

The auto-peaks in the LVD domain are given by

$$L_{Au}[p, q] = \sum_{k=1}^K A_k^2 N^2 e^{j\pi \frac{p a}{N T_s}} \delta \left[p - \frac{2N f_k}{f_s} \right] \delta \left[q - \frac{N \gamma_k}{f_s h} \right], \quad (6.27)$$

Figure 6.7: (a) Frequency representation of the original signal $s[n]$ at $f_s = 128$ Hz; (b) Frequency representation of the LSR based re-sampled signal $s_r[n']$ at new $f'_s = 200$ Hz; (c) Time representation of the signal $s[n']$ using standard-sampling at $f'_s = 200$ Hz; (d) Time representation of the LSR based re-sampled signal $s_r[n']$ at $f'_s = 200$ Hz. The signal used here having unequal amplitudes is specified in Example 6.5.

where $T_s = 1/f_s$ is the sample interval, N denotes the sample length of PSIAF as $N = N_s/2$, n , p , and q are all integers.

Next we consider the LSR based method to re-sample the signal $s[n]$ by a new sampling frequency f'_s . We can accomplish this operation by the LSR directly using the new sampling parameters. The LSR based recovery signal is expressed as

$$\begin{aligned} s_r[n'] &= \frac{1}{N} \sum_{p'} \sum_{q'} \sqrt{\tilde{L}_{A_u}^\theta[p', q']} e^{j \frac{\pi p'}{N'} (n' - \frac{a}{2T'_s})} e^{j \frac{\pi q'}{N'} T'_s h n'^2} \\ &= \sum_{k=1}^K \tilde{A}_k e^{j \Delta \phi_k / 2} e^{j \tilde{\theta}_k + j 2\pi f_k n' T'_s + j \pi \gamma_k (n' T'_s)^2}, \end{aligned} \quad (6.28)$$

where $T'_s = 1/f'_s$ is the new sampling interval, N' is the sample length of PSIAF with

Table 6.5: MSEs of the FrFT and LSR/LSRC based Re-sampling (old $f_s = 128$ Hz)

Method	f'_s (Hz)	$K = 2$ (e.g. 6.2, equal A_k)	$K = 3$ (e.g. 6.4, equal A_k)
	100	2.987×10^{-4}	1.052×10^{-3}
FrFT	160	4.995×10^{-4}	1.856×10^{-3}
	200	7.652×10^{-4}	2.908×10^{-3}
	100	$3.129/1.744 (\times 10^{-10})$	$2.199/0.488 (\times 10^{-8})$
LSR/LSRC	160	$3.831/1.717 (\times 10^{-10})$	$1.942/0.418 (\times 10^{-8})$
	200	$3.576/1.732 (\times 10^{-10})$	$2.210/0.514 (\times 10^{-8})$
Method	f'_s (Hz)	$K = 3$ (e.g. 6.5, unequal A_k)	$K = 5$ (equal A_k)
	100	3.289×10^{-3}	7.123×10^{-3}
FrFT	160	4.034×10^{-3}	7.938×10^{-3}
	200	5.197×10^{-3}	1.182×10^{-2}
	100	$1.335/0.304 (\times 10^{-6})$	$5.260/0.561 (\times 10^{-5})$
LSR/LSRC	160	$1.151/0.457 (\times 10^{-6})$	$5.949/0.604 (\times 10^{-5})$
	200	$2.172/0.319 (\times 10^{-6})$	$5.797/0.560 (\times 10^{-5})$

respect to f'_s that $N' = Nf'_s/f_s$, p' and q' are new indices of frequency and chirp rate as

$$p' = \frac{N'}{N}p, \quad q' = \frac{N'}{N}q.$$

The estimated auto-peaks of LVD with random initial phases in the new indices as

$$\tilde{L}_{A_u}^\theta[p', q'] = \sum_{k=1}^K \tilde{A}_k^2 N^2 e^{j2\tilde{\theta}_k} e^{j\tilde{\phi}_k} \delta\left[p' - \frac{2N'f_k}{f'_s}\right] \delta\left[q' - \frac{N'\gamma_k}{f'_s h}\right], \quad (6.29)$$

in which $\tilde{\phi}_k = \frac{\pi p'_a}{N'T'_s} + \Delta\phi_k$. Here f_k and γ_k are also assumed to be estimated accurately without errors. We denote $s[n']$ to be the signal with f'_s by using standard-sampling method as

$$s[n'] = \sum_{k=1}^K s_k[n'] = \sum_{k=1}^K A_k e^{j\theta_k + j2\pi f_k n'T'_s + j\pi\gamma_k (n'T'_s)^2}. \quad (6.30)$$

Comparing to (6.28), it is easy to see that the LSR based recovery signal $s_r[n']$ equals to the standard-sampled $s[n']$ if $\tilde{\theta}_k = \theta_k$, $\tilde{A}_k = A_k$ and $\Delta\phi_k = 0$.

Similarly, the LSRC based re-sampling is defined as

$$s_r^{-c}[n'] = \frac{1}{N} \sum_{p'} \sum_{q'} \sqrt{\tilde{L}_{A_u}^{-c,\theta}[p', q']} e^{j\frac{\pi p'}{N'}(n' - \frac{a}{2T_s})} e^{j\frac{\pi q'}{N'}T_s h n'^2},$$

where $\tilde{L}_{A_u}^{-c,\theta}[p', q']$ is the auto-peaks with suppressed cross-terms multiplying $e^{j2\tilde{\theta}_k}$ with each component.

Several simulations are presented as follows. Figure 6.7 shows the time and frequency representations of the original, LSR based re-sampled and the standard-sampled signals, using Example 6.5. The LSR based re-sampling for an arbitrary new sampling frequency can be achieved when the new sampling rate is not smaller than the Nyquist rate. The real parts of time representations of $s_r[n']$ and $s[n']$ show no difference in the figure. The MSEs of LSR/LSRC based re-sampled signals and standard-sampled $s[n']$ are shown in Table 6.5, using Examples 6.2, 6.4, 6.5 and another signal with 5 components, having random initial phases. The re-sampling performance is not related to the new sampling frequency since the MSEs for different f'_s are in the same order, implying there is no limit of f'_s for the LSR/LSRC based re-sampling. In addition, the LSRC still provides better performance than the LSR. The MSEs of LSR/LSRC for Examples 6.2 and 6.4 are larger than those for $K = 2, 3$ and $N_s = 256$ in Table 6.2, as the estimation errors of initial phases degrade the re-sampling performance.

A sampling rate converter based on the FrFT for LFM signals has been reported in [81]. With the MATLAB code provided by the authors, Table 6.5 shows the comparisons between the MSEs achieved by our method and the one reported in [81]. It is seen that the re-sampling accuracy achieved by our method is significantly better than the FrFT based method.

6.8 Discussion

As presented above, LSR and LSRC are able to obtain excellent performance on recovering LFM signals since the signals are highly concentrated in the LVD domain. Notably,

single-tone signal which is a special case of LFM signal with zero chirp rate, can of course be reconstructed by our proposed approach. However, polynomial phase signals (PPSs) whose order larger than 3, i.e., the third derivative of instantaneous phase is not zero, the LSR and LSRC fail to work. In order to tailor the approaches to PPSs based applications, the signal can be segmented into small sequences where each segment could be approximately treated as an LFM signal. Therefore the LSR and LSRC can be further used.

6.9 Summary

This chapter introduces two time-domain signal reconstruction methods based on LVD, known as the LSR and LSRC, for multi-component LFM signals. The former is able to provide effective reconstruction by using the peaks of auto-terms in the LVD domain, and the latter achieves with improved performance by suppressing the effects of cross-terms. Based on the LSR and LSRC, applications on de-noising and re-sampling are presented to demonstrate their performances. Numerical result shows that they can work in negative SNR environments and arbitrary new sampling frequencies.

6.10 Appendix

In this appendix, we will demonstrate $\tilde{L}_{A_u}^{-c}(f, \gamma)$ is more accurate than $\tilde{L}_{A_u}(f, \gamma)$. Assuming the centroid frequencies and chirp rates are perfectly obtained from the LVD domain for a noise-free signal, it is only needed to consider the accuracy of estimation of amplitudes and phases. From [47] it is known for finite signals, the auto-peaks in the LVD domain can be expressed as

$$L_{A_u}(f, \gamma) = \sum_{k=1}^K P_k \delta(f - f_k) \delta(\gamma - \gamma_k/h),$$

where

$$P_k = \frac{hN^2 A_k^2}{f_s^2} e^{j2\pi a f}, \quad (6.31)$$

for $k = 1, \dots, K$. Because of the effects of side-lobes of non-self components and cross-terms, we get the estimation of auto-peaks, $\tilde{P}_k$, as following

$$\tilde{P}_k = P_k + D_k + C_k, \quad (6.32)$$

where D_k and C_k represent the side-lobes of non-self components and cross-terms at the points of (f_k, γ_k) , respectively. Referring to the Lemma 2 in [47], the cross-terms at (f_k, γ_k) can be written as

$$C_k = \sum_{\substack{l=1 \\ l \neq k}}^K A_k A_l c_{kl}(f_k, \gamma_k), \quad (6.33)$$

where $c_{kl}(f_k, \gamma_k)$ denotes the coefficient of cross-terms between the k th and l th components at the points (f_k, γ_k) . The phase of $c_{kl}(f_k, \gamma_k)$ is determined only by the locations of the k th and l th components, and the same in $\tilde{L}_{A_u}(f, \gamma)$ and $\tilde{L}_{A_u}^{-c}(f, \gamma)$. The errors of auto-peak estimations are given by

$$\Delta P_k = \tilde{P}_k - P_k = D_k + \sum_{\substack{l=1 \\ l \neq k}}^K A_k A_l c_{kl}(f_k, \gamma_k). \quad (6.34)$$

In fact, ΔP_k represents the difference between $\tilde{L}_{A_u}(f, \gamma)$ and the true one $L_{A_u}(f, \gamma)$ in (6.10). Using $\tilde{P}_k$ obtained from the LVD domain, the estimation of signal component amplitudes is calculated as

$$\tilde{A}_k = \sqrt{\frac{|\tilde{P}_k| f_s^2}{hN^2}},$$

and also the cross-term is reconstructed as

$$\tilde{C}_k = \sum_{\substack{l=1 \\ l \neq k}}^K \tilde{A}_k \tilde{A}_l c_{kl}(f_k, \gamma_k). \quad (6.35)$$

Subtracting it from $\tilde{P}_k$, we obtain the estimation of auto-peaks in the plane of $\tilde{L}_{A_u}^{-c}(f, \gamma)$ as

$$\tilde{P}_k^{-c} = \tilde{P}_k - \tilde{C}_k = P_k + D_k + C_k - \tilde{C}_k. \quad (6.36)$$

Consequently, the errors of auto-peak estimations are given by

$$\begin{aligned} \Delta P_k^{-c} &= \tilde{P}_k^{-c} - P_k \\ &= D_k + \sum_{\substack{l=1 \\ l \neq k}}^K \left(A_k A_l - \tilde{A}_k \tilde{A}_l \right) c_{kl}(f_k, \gamma_k). \end{aligned} \quad (6.37)$$

ΔP_k^{-c} represents the difference between $\tilde{L}_{A_u}^{-c}(f, \gamma)$ and $L_{A_u}(f, \gamma)$. By experimenting it is found that the estimation of amplitudes in a noise-free LVD domain is accurate as

$$\left| A_k - \tilde{A}_k \right| \ll 0.1 A_k,$$

then we can derive that

$$\left| A_k A_l - \tilde{A}_k \tilde{A}_l \right| < |A_k A_l|.$$

This means ΔP_k^{-c} is less effected by the cross-term coefficient $c_{kl}(f_k, \gamma_k)$ than ΔP_k , therefore the estimated peaks in $\tilde{L}_{A_u}^{-c}(f, \gamma)$ is more accurate than those in $\tilde{L}_{A_u}(f, \gamma)$.

Chapter 7

Two-Dimensional Lv Distribution

This chapter will introduce an extension of the LVD, the 2D LVD, by expanding the original LVD to a high dimension for applications such as image processing. In the first part of this chapter, we propose a new 2D time-frequency distribution by extending the 1D LVD to a 2D version. The 2D LVD inherits similar properties from the 1D LVD so that it has both advantages of bilinear and linear methods which are high concentration and trivial cross-terms, since the LVD is based on coherent integration. Performances of signal energy concentration and computational complexity are analyzed and discussed. The second part of this chapter presents an application of the new 2D technique on image watermarking, by using 2D LFM signals to be watermarks. Simulations show that the 2D LVD works well against various attacks.

7.1 Introduction

Techniques based on low dimensional processing are insufficient for multi-dimensional scenarios while our world does not only consist of single dimensional objects. Therefore developing high dimensional techniques is necessary for many applications such as image, holographic imaging, video, antenna array, etc. Signals in these high-dimensional fields obviously have more than 1D information. To deal with this kind of signals, corresponding processing methods are developed, for instance, the 2D FT [62]. Techniques based on the

2D FT are widely used in image processing [63], magnetic resonance imaging in medical study [64], Lamb wave based measurement [65] and so on.

If an FM signal whose frequency varies with time is involved, time-frequency analysis based method is a more suitable candidate than the FT since it is able to reveal the joint information of time and frequency. The 2D time-frequency analysis has been employed in many applications such as Lamb wave based measurement [66], direction of arrival estimation [164, 165], textured images segmentation [27, 67, 68], image quality assessment [166] and digital image watermarking [69]. Particularly, 2D LFM signals have been used in these fields such as image watermarking [24]. Some well-known 1D time-frequency distributions which provide effective performance on estimating IF and frequency & chirp rate for 1D LFM signals, can be developed to 2D distributions that still work well for 2D signals, such as the 2D WVD. Basically, 2D distributions inherit all the properties of their 1D forms so that they still have the same drawbacks. For instance, the 2D WVD suffers from strong cross-terms for multi-component LFM signals, similarly to what its 1D version does. For this reason, we aim to develop a new 2D time-frequency distribution to improve performances of estimation and detection for 2D multi-LFM signals.

The LVD is a good choice as it has been shown valuable merits in previous contents. It concentrates LFM signals highly and produces negligible cross-terms since it has the property of asymptotic linearity. In our opinion, the LVD is a desirable combination of bilinear and linear methods having both advantages of them, i.e., high signal concentration and trivial cross-terms. Due to the distinguished performance for 1D LFM signals, it is expected to develop the 2D LVD for high dimensional applications.

In the following contents, we propose the 2D LVD and give the corresponding FCR in Section 7.2. Next, performances of signal concentration and computational complexity are analyzed and discussed in Section 7.3. We will see that the 2D LVD maintains the merits of 1D LVD with high concentration and negligible cross-terms. To show its ability

practically, Section 7.4 presents an application on image watermarking by using 2D LFM signals. Conclusions and mathematical derivations can be found in Sections 7.5 and 7.6, respectively.

7.2 The 2D LVD

7.2.1 Definition

Following the definition of LVD in Section 2.4.1, the 2D LVD is proposed in this section.

A 2D LFM signal is defined as

$$I_1(x, y) = A_1 e^{j\theta_{x1} + j2\pi f_{x1}x + j\pi\gamma_{x1}x^2 + j\theta_{y1} + j2\pi f_{y1}y + j\pi\gamma_{y1}y^2}, \quad (7.1)$$

where A_1 is the constant amplitude, θ_{x1} and θ_{y1} are random initial phases of dimensions x and y , f_{x1} and f_{y1} are centroid frequencies of these dimensions, γ_{x1} and γ_{y1} are chirp rates of the two dimensions, respectively. Its 2D PSIAF is defined as

$$\begin{aligned} R(x, \tau_x, y, \tau_y) &= I_1\left(x + \frac{\tau_x + a_x}{2}, y + \frac{\tau_y + a_y}{2}\right) I_1^*\left(x - \frac{\tau_x + a_x}{2}, y - \frac{\tau_y + a_y}{2}\right) \\ &= A_1^2 \exp(j2\pi(f_{x1}(\tau_x + a_x) + \gamma_{x1}(\tau_x + a_x)x + f_{y1}(\tau_y + a_y) + \gamma_{y1}(\tau_y + a_y)y)), \end{aligned} \quad (7.2)$$

where a_x and a_y denote constant time-delays. Similar to the 1D LVD, the initial phases are lost in the auto-terms. According to the Section 2.4.1, a scaling operator Γ of a phase function G with respect to (x, τ_x, y, τ_y) is defined as

$$\Gamma[G(x, \tau_x, y, \tau_y)] \rightarrow G\left(\frac{x_n}{h_x(\tau_x + a_x)}, \tau_x, \frac{y_n}{h_y(\tau_y + a_y)}, \tau_y\right),$$

where h_x and h_y are scaling factors, and x_n, y_n are the scaled times expressed as

$$x_n = (\tau_x + a_x) h_x x, \quad y_n = (\tau_y + a_y) h_y y.$$

After performing Γ on the PSIAF in (7.2), the scaled PSIAF is obtained as

$$\Gamma[R] = A_1^2 \exp\left(j2\pi\left(f_{x1}(\tau_x + a_x) + \frac{\gamma_{x1}}{h_x}x_n + f_{y1}(\tau_y + a_y) + \frac{\gamma_{y1}}{h_y}y_n\right)\right). \quad (7.3)$$

By performing FTs of (7.3) in terms of τ_x, τ_y, x_n, y_n , the 2D LVD is generated as

$$\begin{aligned} L_{I_1}(f_x, \gamma_x, f_y, \gamma_y) &= F_{\tau_x \tau_y} \{F_{x_n y_n} \{\Gamma[R]\}\} \\ &= A_1^2 e^{j2\pi(a_x f_x + a_y f_y)} \delta(f_x - f_{x1}) \delta(f_y - f_{y1}) \delta\left(\gamma_x - \frac{\gamma_{x1}}{h_x}\right) \delta\left(\gamma_y - \frac{\gamma_{y1}}{h_y}\right), \end{aligned} \quad (7.4)$$

where $F_{\tau_x \tau_y} \{\cdot\}$ and $F_{x_n y_n} \{\cdot\}$ mean the 2D FTs with respect to $\tau_x \tau_y$ and $x_n y_n$. In implementation, we generally use the optimal parameters $a_x = a_y = 1$ and $h_x = h_y = 1$ for obtaining a desirable distribution since both dimensions are LFM signals [47]. From this FCR plane represented by $L_{I_1}(f_x, \gamma_x, f_y, \gamma_y)$, all the parameters of LFM signal are obtained by extracting amplitude and coordinates of the peak $(f_{x1}, \gamma_{x1}, f_{y1}, \gamma_{y1})$.

Since the 2D LVD is an expansion of the original LVD from 1D to 2D, it is demonstrated that the 2D LVD has same properties as the 1D LVD in Section 2.4.3, such as the asymptotic linearity, frequency-chirp rate shift, scaling and convolution, which are mainly based on the FT. Particularly, due to the property of asymptotic linearity, the 2D LVD does not suffer from the cross-terms because they are negligible comparing to the auto-terms in (7.4). That is, for a multi-component LFM signal as

$$I(x, y) = \sum_{k=1}^K I_k(x, y) = \sum_{k=1}^K A_k e^{j\theta_{xk} + j2\pi f_{xk}x + j\pi\gamma_{xk}x^2 + j\theta_{yk} + j2\pi f_{yk}y + j\pi\gamma_{yk}y^2}, \quad (7.5)$$

the 2D LVD can be approximated as

$$L_I(f_x, \gamma_x, f_y, \gamma_y) \approx \sum_{k=1}^K L_{I_k}(f_x, \gamma_x, f_y, \gamma_y). \quad (7.6)$$

7.2.2 Simulations

Figure 7.1 shows the LVD representations of a 2D signal containing three LFM components denoted as:

Example 7.1: the signal is as the form of (7.5) with $K = 3$, $f_{x1} = -13$ Hz, $f_{x2} = 38$ Hz, $f_{x3} = 25$ Hz, $f_{y1} = 77$ Hz, $f_{y2} = -26$ Hz, $f_{y3} = 51$ Hz, $\gamma_{x1} = -12.8$ Hz/s, $\gamma_{x2} = 38.4$

Figure 7.1: The 2D LVD of Example 7.1 containing 3 LFM components with size of 512×512 : (a) the chirp rates plane; (b) the FCR plane of dimension x ; (c) vertical view of the chirp rates plane; (d) vertical view of the FCR plane.

Hz/s, $\gamma_{x3} = 25.6$ Hz/s, $\gamma_{y1} = 76.8$ Hz/s, $\gamma_{y2} = -25.6$ Hz/s, $\gamma_{y3} = 51.2$ Hz/s, signal size $N_{sx} \times N_{sy} = 512 \times 512$, sampling frequency $f_s = 256$ Hz.

Since the 2D LVD $L_I(f_x, \gamma_x, f_y, \gamma_y)$ is a four-dimensional matrix that cannot be plotted here, we extract 2D representations with $L_I(\gamma_x, \gamma_y)$ and $L_I(f_x, \gamma_x)$ to show the results in the figure. Figure 7.1(a) describes the LVD plane containing two chirp rates dimensions and three components are revealed as three peaks without evident cross-terms. The FCR plane of the first dimension x shown in Figure 7.1(b) gives information of f_{xk} and γ_{xk} . All the parameters of centroid frequencies and chirp rates can be obtained from Figures 7.1(c) and 7.1(d) without disturbance of cross-terms, thanks to the property of asymptotic linearity.

As same as the 1D LVD, the 2D LVD is a method that perfectly combines the advantages of linear and bilinear methods. It concentrates signals as the WVD while nearly has no cross-term as the STFT/FrFT. For parameter estimation of 2D LFM signals, the 2D LVD probably will be the best candidate. Like the 1D LVD, a shortcoming is emerged that it may not be appropriate to deal with very short signals in time domain [47], i.e., the size $N_{sx} \times N_{sy}$ does not satisfy $\min(N_{sx}, N_{sy}) \geq 2f_s$. This is because the LVD needs a constant time-delay of 1 second ($a_x = a_y = 1$) in (7.2) so the signal should last at least 2 seconds in each dimension.

7.3 Performance of the 2D LVD

This section will show the performances of signal concentration based on DCR and computational complexity in terms of the number of complex multiplications.

Table 7.1: The 2D LVD based DCRs for Different Component Signals in Example 7.2 Without Noise

No. of components	Theoretical DCR (dB)	Experimental DCR (dB)
$K = 1$	72.2472	71.2150
$K = 2$	66.2266	65.0667
$K = 3$	62.7048	61.0758
$K = 6$	56.6842	52.9135

7.3.1 Signal Concentration

Similar to (3.1), the DCR is defined to measure the signal-term energy concentration in the 2D LVD domain as follows,

$$DCR^{2D} = 10 \log_{10} \frac{\text{ave} \left[\left| L(f_x, f_y, \gamma_x, \gamma_y)_{(f_x, f_y, \gamma_x, \gamma_y) \in S} \right|^2 \right]}{\text{ave} \left[\left| L(f_x, f_y, \gamma_x, \gamma_y)_{(f_x, f_y, \gamma_x, \gamma_y) \notin S} \right|^2 \right]}. \quad (7.7)$$

Referring to the conclusion of DCR value for mono-component in (3.2), it is obtained for the 2D LVD we have

$$DCR_1^{2D} \approx 20 \log_{10} \frac{|\text{sinc}_2(0, 0)|^2}{\text{ave} \left[|\text{sinc}_2(f_x, f_y)|^2_{f_x \neq 0, f_y \neq 0} \right]}, \quad (7.8)$$

where the 2D sinc function is denoted as

$$\text{sinc}_2(f_x, f_y) = \text{sinc}(f_x) \text{sinc}(f_y),$$

where $\text{sinc}(f_x)$ and $\text{sinc}(f_y)$ are the 1D sinc function [62]. (7.8) can be deducted by using the same concept of derivation in Section 3.9.1. Following the result of DCR in (3.4), the DCR for 2D LVD is obtained as

$$DCR_1^{2D} \approx 20 \log_{10} (N_x N_y), \quad (7.9)$$

where $N_x \times N_y$ is the number of sampling points of the 2D PSIAF with $N_x = N_{sx}/2$ and $N_y = N_{sy}/2$.

Same as the concept in 1D LVD domain given in Section 3.4.1, for K -component 2D LFM signals, the DCR is

$$DCR_K^{2D} \approx DCR_1^{2D} - 20 \log_{10} K. \quad (7.10)$$

From this conclusion, we can obtain theoretical DCR values for different K . For example, DCR values in the second column of Table 7.1 are from (7.9) and (7.10) for $N_{sx} = N_{sy} = 128$. This indicates that the signal concentration is degraded when number of components grows, because the signal with more components results in a flatter distribution. As the same as 1D LVD, (7.10) is valid for K is far smaller than the length of each dimension.

For noise corrupted multi-component signal, also adopting the result in (3.9), the corresponding DCR is expressed as,

$$DCR_{K,SNR}^{2D} = 10 \log_{10} \frac{1 + 2m_r/B_1 + E[|r|^2]/B_1^2}{1/D_K^{2D} + 2Km_r/(N_x^2 N_y^2 B_1) + E[|r|^2]/B_1^2}. \quad (7.11)$$

where $D_K^{2D} = 10^{DCR_K^{2D}/10}$, $B_1 = N_x^2 N_y^2 A_1^2 / f_s^4$ is the auto-term amplitude of noise-free signal in the 2D LVD domain (see Section 7.6.1), $m_r = N_x N_y \sigma_v^2$ and $E[|r|^2] = 4N_x^2 N_y^2 \sigma_v^2 \sum_{k=1}^K A_k^2 + 3N_x^2 N_y^2 \sigma_v^4 / 2$ (see Section 7.6.2). From (7.11), it is easy to see that,

$$\begin{aligned} \lim_{SNR \rightarrow +\infty} DCR_{K,SNR}^{2D} &= DCR_K^{2D}, \\ \lim_{SNR \rightarrow -\infty} DCR_{K,SNR}^{2D} &= 0. \end{aligned}$$

where the SNR means the input SNR in terms of dB.

Next we will show some experimental results for the above conclusions of DCR. The simulated 2D LFM signal here is denoted as,

Example 7.2: following the signal form in (7.5) with $K = 6$, $f_{x1} = 6$ Hz, $f_{x2} = 2$ Hz, $f_{x3} = -6$ Hz, $f_{x4} = -13$ Hz, $f_{x5} = -16$ Hz, $f_{x6} = -10$ Hz, $f_{y1} = 2$ Hz, $f_{y2} = -6$ Hz, $f_{y3} = 6$ Hz, $f_{y4} = 11$ Hz, $f_{y5} = -5$ Hz, $f_{y6} = -13$ Hz, $\gamma_{x1} = 6.5$ Hz/s, $\gamma_{x2} = 2$ Hz/s, $\gamma_{x3} = -6$ Hz/s, $\gamma_{x4} = -13$ Hz/s, $\gamma_{x5} = -16$ Hz/s, $\gamma_{x6} = -9.5$ Hz/s, $\gamma_{y1} = 2$ Hz/s,

Figure 7.2: 2D LVD based DCR values of LFM signals specified in Example 7.2 with components $K = 1 \sim 3, 6$.

$\gamma_{y2} = -6$ Hz/s, $\gamma_{y3} = 6$ Hz/s, $\gamma_{y4} = 11$ Hz/s, $\gamma_{y5} = -5$ Hz/s, $\gamma_{y6} = -13$ Hz/s, signal size 128×128 , sampling frequency $f_s = 64$ Hz.

Table 7.1 presents the experimental DCR values for noise-free signals consisting of different LFM components in the third column. They fit well with their corresponding theoretical values and also decrease when K gets larger. The difference between the theoretical and experimental values are mainly caused by the representation errors and the cross-terms in the LVD domain, similar as the situation in Section 3.4.2. Figure 7.2 shows the DCR values for different components signals in Gaussian noisy environments based on the 2D LVD. The theoretical curve for $K = 1$ is referring to (7.11), and the experimental one fits it well. It is seen that for $\text{SNR} \rightarrow -\infty$, $\text{DCR} \rightarrow 0$, and for SNR large enough, DCR trends to the values for noise-free signals in the third column of Table 7.1.

7.3.2 Computational Complexity

The 2D LVD proposed in Section 7.2.1 is mainly computed by three steps, instantaneous auto-correlation in (7.2), scaling operation in (7.3) and 2D FT in (7.4). By using digital computation based on 1D FFT whose number of complex multiplications is $\frac{1}{2}N \log_2 N$, the multiplications for these three processes are listed as

$$\begin{aligned} \text{Equation (7.2)} & : \frac{(N_{sx} + 2)N_{sx}/2}{2} \times \frac{(N_{sy} + 2)N_{sy}/2}{2}, \\ \text{Equation (7.3)} & : \left(\frac{N_{sx}}{2}\right)^2 \left(\frac{N_{sy}}{2}\right)^2, \\ \text{Equation (7.4)} & : \left(\frac{N_{sx}}{2}\right)^2 \left(\frac{N_{sy}}{2}\right)^2 \left(\log_2 \frac{N_{sx}}{2} + \log_2 \frac{N_{sy}}{2}\right), \end{aligned}$$

and the overall number of multiplications is given by

$$\frac{1}{24}N_{sx}N_{sy} + \frac{1}{23}(N_{sx}^2N_{sy} + N_{sx}N_{sy}^2) + \frac{1}{24}N_{sx}^2N_{sy}^2 \log_2(N_{sx}N_{sy}), \quad (7.12)$$

or the computational complexity of 2D LVD is in the order of $O(N_{sx}^2N_{sy}^2 \log_2(N_{sx}N_{sy}))$.

7.4 Application on Image Watermarking

7.4.1 Background

In recent years, watermarking techniques have attracted much attention due to the need for copyright protection since multimedia files (audio, image and video) can be exchanged easily over the Internet. Watermarks in images could be visible or invisible to the human visual system, and fragile or robust against various attacks such as low/high-pass (LP/HP) filtering, median filtering, JPEG compressing, cropping, scaling and so on [167]. Fragile or semi-fragile watermarking is mainly for tamper detection and reconstruction [168], not for copyright protection, so it is not the issue we discuss here. We focus on the study of robust watermarking techniques using invisible watermarks since a visible one is easy to be detected and removed.

Figure 7.3: The flowchart of image watermarking procedure based on the 2D LVD.

Many efforts have been reported on the study for watermarking techniques in transform domain because watermarks in spatial domain seem to be less robust [167]. In literature, many transform domain based algorithms have been proposed such as the discrete cosine transform [169–171], discrete FT [172–174], discrete wavelet transform [175–177], discrete Hadamard transform [178], discrete evolutionary transform [179], S-method [180] and Radon-Wigner distribution (RWD) [24]. Each of these techniques has its own characteristics and inherent advantages or shortcomings. To understand and use the information in image better, time-frequency analysis is adopted on watermarking because it presents information in spatial/time-frequency domain, not only in one domain. For instance, the 2D RWD based watermarking method [24] using 2D LFM signals works well against various attacks. However, the RWD produces cross-terms when multi-component 2D LFM signals are embedded. Although the parameters of watermark signal can be selected carefully to make the cross-terms to be negligible, it does limit the application for various images. That means, the watermark with trivial cross-terms in the RWD plane still may not be suitable for some images. For instance, the watermark may not be applied on texture image having chirp characteristics because the RWD may produce high cross-terms between the watermark and texture image.

Fortunately, the LVD is investigated as a method with high concentration and asymptotic linearity. Comparing to the RWD, it produces negligible cross-terms for all LFM signals. Therefore we will apply its 2D form on image watermarking to protect the copyright of image against various attacks.

7.4.2 Procedures of Watermarks Embedding and Extracting

A watermarking procedure based on the 2D LVD using LFM watermarks is presented in Figure 7.3. An LFM signal is generated to be the watermark and embedded in an image with suitable energy to satisfy the requirement of invisibility; then the watermarked image is transmitted and may be attacked by several ways such as LP/HP filtering, median filtering, compressing, cropping and scaling; at last we perform the 2D LVD on the received image to detect the LFM watermark and make the decision.

Consider an image $G(x, y)$ adding a watermark $J(x, y)$ as

$$G_J(x, y) = G(x, y) + J(x, y), \quad (7.13)$$

where the watermark is a 2D multi-component LFM signal as

$$J(x, y) = \sum_{k=1}^K J_k(x, y) = \sum_{k=1}^K A_k \cos(\pi\gamma_{xk}x^2 + \pi\gamma_{yk}y^2). \quad (7.14)$$

Here we do not include the centroid frequencies because they are weak against attacks, e.g., LP/HP filtering or cropping can change them. This signal is the real part of (7.5) since the image is real, so the watermark appears pairs of peaks in the LVD domain and we only show one quadrant in following figures.

To guarantee that the watermark is invisible, its energy must be limited when embedding it. Generally, in image we use the peak signal-to-noise ratio (PSNR) to measure energy ratio between image and watermark, which is defined as [181]

$$PSNR = 10 \log_{10} \frac{\max_G^2}{MSE}, \quad (7.15)$$

where $\max_G$ denotes the maximum pixel value of image, and the mean square error (MSE) is given by

$$MSE = \frac{1}{N_x N_y} \sum_{n_x=1}^{N_x} \sum_{n_y=1}^{N_y} (G(n_x, n_y) - G_J(n_x, n_y))^2. \quad (7.16)$$

In this section, an 8-bits image is used so $\max_G = 255$. The PSNR must be high enough to make the watermark is imperceptible for human. In detection part, because the image could be considered as consisting of direct-current (DC), low frequency components and noise in sense of the LVD, the watermark with LFM components can be concentrated and detected in the FCR plane.

7.4.3 Simulation Examples

Now we present some examples by using the Lena image with 256×256 pixels. The watermark embedded in this section is denoted as,

Example 7.3: following the form in (7.14), LFM components $K = 2$, $\gamma_{x1} = 38.5$ Hz/s, $\gamma_{x2} = 10.5$ Hz/s, $\gamma_{y1} = 13$ Hz/s, $\gamma_{y2} = 39$ Hz/s, size 256×256 as that of the image, sampling frequency $f_s = 128$ Hz in both dimensions. The PSNR is selected to be 44.5 dB and the amplitudes of different components is assumed to be equal, so we calculate that $A_1 = A_2 = 1.5189$.

Figure 7.4(a) shows the original image and 7.4(b) presents the watermarked image where the watermark is invisible. Figure 7.4(c) shows the representation of the watermarked image by using the 2D RWD, and Figure 7.4(d) gives the transform result of the watermarked image based on the 2D LVD. It is seen that two LFM components are clearly distinguished in the 2D LVD domain without evident cross-terms as in the 2D RWD domain. From the chirp rates plane in Figure 7.4(d), the parameters of watermark can be estimated more accurately, and then demonstrate our copyright of the image.

Following simulations present performances of the watermarking method against various attacks. Figures 7.5 ~ 7.7 show the watermarked images after LP filtering with cut-off frequency $f_c = 0.3f_s$, 3×3 median filtering and scaling & cropping, as well as their 2D LVD representations, respectively. Obviously, the watermarking against these four attacks is robust as the peaks in LVD domain can be detected. It is noticed that

(a): Original image

(b): Watermarked image

(c): 2D RWD

(d): 2D LVD

Figure 7.4: (a) Original image with size 256×256 ; (b) watermarked image based on Example 7.3; (c) the 2D RWD representation of the watermarked image; (d) the 2D LVD representation of the watermarked image.

(a): Filtered image

(b): 2D LVD

Figure 7.5: (a) Watermarked image filtered by LP filter with cut-off frequency $f_c = 0.3f_s$ and (b) its 2D LVD representation.

(a): Filtered image

(b): 2D LVD

Figure 7.6: (a) Watermarked image filtered by 3×3 median filter and (b) its 2D LVD representation.

(a): Scaled image

(b): 2D LVD

Figure 7.7: (a) Watermarked image attacked by scaling & cropping with scaling ratios $s_{rx} = 1$, $s_{ry} = 1.1228$ and (b) its 2D LVD representation.

after scaling, the locations of peaks have changed. The watermark in scaled image is denoted as $J'(x', y')$ where x' and y' are the scaled indices. We define the scaling ratios $s_{rx} = x'/x$, $s_{ry} = y'/y$ to measure the scaling degree, so the watermark becomes

$$\begin{aligned} J'(x', y') &= \sum_{k=1}^K A_k \cos(\pi \gamma'_{xk} x'^2 + \pi \gamma'_{yk} y'^2) \\ &= \sum_{k=1}^K A_k \cos(\pi \gamma'_{xk} s_{rx}^2 x^2 + \pi \gamma'_{yk} s_{ry}^2 y^2). \end{aligned} \quad (7.17)$$

As the scaling operation does not change the values of each point in $J(x, y)$, $J'(x', y')$ equals to (7.14) and consequently we have

$$\gamma'_{xk} = \gamma_{xk}/s_{rx}^2, \quad \gamma'_{yk} = \gamma_{yk}/s_{ry}^2. \quad (7.18)$$

In Figure 7.7, the scaling ratios are setting as $s_{rx} = 1$, $s_{ry} = 1.1228$, therefore the scaled chirp rates in y dimension change to be $\gamma'_{y1} = 10.3119$ Hz/s and $\gamma'_{y2} = 30.9357$ Hz/s, while those in x dimension unchange.

More decisions for watermark detection against other attacks with different degrees are shown in Table 7.2, where H_1 denotes the watermark detected and H_0 means not. The detection method here is to search the peaks in LVD domain, extract the locations and compare with the true parameters of watermark, then decide H_1 for equal and H_0 for not. We can know that our method works for various situations of attacking, e.g., cropping away no more than 75%. In fact, too badly attacking usually does not happen because the image may become unavailable at the same time. Against the attack of scaling, the watermark is always detected because scaling changes but does not reduce the information of watermark.

7.5 Summary

The 2D LVD is introduced in this chapter for high dimension processing. Performances and discussions on important issues such as energy concentration and computational

Table 7.2: Decisions of Watermarks in Attacked Watermarked Image

Attack	Ratio	Decision
LP filter	$f_c = 0.4f_s$	H_1
	$f_c = 0.225f_s$	H_1
	$f_c < 0.225f_s$	H_0
HP filter	$f_c = 0.05f_s$	H_1
	$f_c = 0.15f_s$	H_1
	$f_c > 0.15f_s$	H_0
Median filter	2×2	H_1
	4×4	H_1
	$> 4 \times 4$	H_0
Scaling	$s_{rx} = 1.2, s_{ry} = 1$	H_1
	$s_{rx} = 1.5, s_{ry} = 1$	H_1
	$s_{rx} = 1, s_{ry} = 1.5$	H_1
JPEG compression	10%	H_1
	50%	H_1
	$> 50\%$	H_0
Cropping	36%	H_1
	75%	H_1
	$> 75\%$	H_0

complexity are presented. Also an applications based on the new extension, i.e., the 2D LVD based image watermarking, are presented and simulated to show that they perform satisfactorily.

7.6 Appendix

7.6.1 The Amplitude of Signal Auto-Term in the 2D LVD Domain

For a finite-length signal which is truncated by a rectangular window $w(x, y)$ with size of $T_x \times T_y$, referring to the Appendix B in [47], we can know the 2D LVD is expressed as

$$\begin{aligned}
 L_{I_1} &= A_1^2 \int \int \int \int w \left(\frac{x_n + \frac{\tau'_x}{2}}{T_x}, \frac{y_n + \frac{\tau'_y}{2}}{T_y} \right) w \left(\frac{x_n - \frac{\tau'_x}{2}}{T_x}, \frac{y_n - \frac{\tau'_y}{2}}{T_y} \right) \\
 &\quad \cdot e^{-j2\pi((f_x - f_{x1})\tau'_x + (f_y - f_{y1})\tau'_y)} e^{-j2\pi((\gamma_x - \gamma_{x1}/h_x)x_n + (\gamma_y - \gamma_{y1}/h_y)y_n)} dx_n dy_n d\tau_x d\tau_y \\
 &= A_1^2 h_x h_y T_x^2 T_y^2 e^{j2\pi(a_x f_x + a_y f_y)} \text{sinc}(T_x(f_x - f_{x1})) \text{sinc}(T_y(f_y - f_{y1})) \\
 &\quad \cdot \text{sinc}\left(\frac{h_x T_x}{2}(\gamma_x - \gamma_{x1}/h_x)\right) \text{sinc}\left(\frac{h_y T_y}{2}(\gamma_y - \gamma_{y1}/h_y)\right), \quad \text{for } |\tau'_x| \leq T_x \text{ \& } |\tau'_y| \leq T_y,
 \end{aligned} \tag{7.19}$$

where $\tau'_x = \tau_x + a_x$, $\tau'_y = \tau_y + a_y$, $T_x = N_x/f_s$, $T_y = N_y/f_s$, and $\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$, therefore $B_1 = |L_{I_1}|_{\max} = N_x^2 N_y^2 A_1^2 / f_s^4$ when $h_x = h_y = 1$.

7.6.2 The Mean and Variance of Noise in the 2D LVD Domain

Here we will derive the statistic properties of Gaussian noise in the 2D LVD domain.

The 2D LVD of noise $v(x, y) \sim N(0, \sigma_v^2)$ is given by,

$$L_v = F_{\tau_x \tau_y} \{F_{x_n y_n} \{\Gamma[R_v(x, \tau_x, y, \tau_y)]\}\}, \tag{7.20}$$

where $R_v(x, \tau_x, y, \tau_y)$ is the PSIAF of noise referring to (7.2). The mean and variance of L_v are

$$\begin{aligned}
 m_l &= E[L_v] = F_{\tau_x \tau_y} \{F_{x_n y_n} \{E[\Gamma[R_v(x, \tau_x, y, \tau_y)]]\}\}, \\
 \sigma_l^2 &= E[(L_v - m_l)^2] = E[|L_v|^2] - m_l^2.
 \end{aligned} \tag{7.21}$$

To calculate mean and variance, we can follow the same concept of derivation for the 1D LVD in [47] by using discrete transform. The discrete form of $\Gamma [R_v (x, \tau_x, y, \tau_y)]$ is given by,

$$\begin{aligned}
 R_v (n'_x, m_x, n'_y, m_y) = & \frac{1}{N_x} \frac{1}{N_y} \sum_{l_x} \sum_{l_y} \sum_{n_x} \sum_{n_y} R_v (n_x, m_x, n_y, m_y) \\
 & \cdot \exp \left(-j2\pi \frac{(2m_x T_s + a_x) h_x}{N_x} n_x l_x - j2\pi \frac{(2m_y T_s + a_y) h_y}{N_y} n_y l_y \right) \\
 & \cdot \exp \left(j \frac{2\pi}{N_x} n'_x l_x + j \frac{2\pi}{N_y} n'_y l_y \right), \tag{7.22}
 \end{aligned}$$

where $T_s = 1/f_s$, $R_v (n_x, m_x, n_y, m_y)$ is the discrete form of PSIAF $R_v (x, \tau_x, y, \tau_y)$, and $R_v (n'_x, m_x, n'_y, m_y)$ is the scaled result. According to Appendix C in [47], its expectation is calculated as

$$E [R_v (n'_x, m_x, n'_y, m_y)] = N_x N_y \sigma_v^2 \delta (2m_x + q_x) \delta (2m_y + q_y) \delta (n'_x) \delta (n'_y),$$

where $q_x = a_x/T_s$, $q_y = a_y/T_s$, and based on

$$\sum_{n=0}^{N-1} \exp \left(-j2\pi \frac{k}{N} n \right) = N \delta (k), \quad 0 \leq k \leq N-1.$$

Then the mean of L_v is

$$\begin{aligned}
 m_l = & E [F_{m_x m_y} \{F_{n'_x n'_y} \{R_v (n'_x, m_x, n'_y, m_y)\}\}] \\
 = & \sum_{m_x} \sum_{m_y} \sum_{n'_x} \sum_{n'_y} E [R_v (n'_x, m_x, n'_y, m_y)] \\
 & \cdot \exp \left(-j2\pi \frac{2m_x + q_x}{N_x} f_x - j2\pi \frac{2m_y + q_y}{N_y} f_y \right) \exp \left(-j2\pi \frac{n'_x \gamma_x}{N_x h_x} - j2\pi \frac{n'_y \gamma_y}{N_y h_y} \right) \\
 = & N_x N_y \sigma_v^2, \tag{7.23}
 \end{aligned}$$

and the mean square is

$$\begin{aligned}
 E[|L_v|^2] &= \sum_{m_{x2}} \sum_{m_{y2}} \sum_{n'_{x2}} \sum_{n'_{y2}} \sum_{m_{x1}} \sum_{m_{y1}} \sum_{n'_{x1}} \sum_{n'_{y1}} \\
 &\quad E[R_v(n'_{x2}, m_{x2}, n'_{y2}, m_{y2}) R_v(n'_{x1}, -m_{x1}, n'_{y1}, -m_{y1})] \\
 &\quad \cdot \exp\left(-j4\pi \frac{m_{x2} - m_{x1}}{N_x} f_x - j4\pi \frac{m_{y2} - m_{y1}}{N_y} f_y\right) \\
 &\quad \cdot \exp\left(-j2\pi \frac{n'_{x2} - n'_{x1}}{N_x h_x} \gamma_x - j2\pi \frac{n'_{y2} - n'_{y1}}{N_y h_y} \gamma_y\right) \\
 &= \frac{3}{2} N_x^2 N_y^2 \sigma_v^4.
 \end{aligned} \tag{7.24}$$

Consequently, we have the variance of L_v is

$$\sigma_l^2 = E[|L_v|^2] - m_l^2 = \frac{1}{2} N_x^2 N_y^2 \sigma_v^4. \tag{7.25}$$

Following the deduction of (3.35) in Section 3.9.2, it is computed

$$\begin{aligned}
 m_r &= N_x N_y \sigma_v^2, \\
 E[|r|^2] &= 4N_x^2 N_y^2 \sigma_v^2 \sum_{k=1}^K A_k^2 + \frac{3}{2} N_x^2 N_y^2 \sigma_v^4,
 \end{aligned} \tag{7.26}$$

by using the results in (7.23) and (7.25).

Chapter 8

Conclusions and Future Works

This chapter concludes the research in this thesis firstly, then proposes some suggestions for future works related to the LVD.

8.1 Conclusions

As the LVD is a very new method proposed recently, many issues have not been investigated before. This thesis extensively and deeply analyzes the LVD, and develops for various applications.

Chapter 3 provides detailed investigation on various processing performances of LVD. The LVD has the property of negligible cross-terms when dealing with multiple components and high signal energy concentration. Comparisons on these issues show that the LVD obtains better performances than the PFT and FrFT on signal concentration in the FCR domain without requiring more computation.

Three applications based on the LVD are presented in Chapters 4 and 5. The First one introduces a TFR method, known as the LVD-LPP, for processing time-varying signals. Compared to the previously reported method based on the PFT, the LVD-LPP provides better estimation of chirp rates and time-frequency energy concentration particularly in low SNR environments. The second one proposes another TFR method, the STLVT, based on the concept of segmentation for continuous signal processing. Its performances

on energy concentration and computational complexity are discussed. In addition, an application of the STLVT on spectrum sensing for WM signals is presented. In the third application of LVD, new interference suppression methods, known as AISL-I and -II, are introduced for DSSS communication systems. The improved performance of interference suppression has been achieved by the proposed methods from the following three aspects. The first one is that the LVD is able to better represent multi-component LFM signals, which has built a basic foundation on correctly synthesizing the interference; The second one is to implement a mechanism to detect the presence of interference that requires effective suppression process; The last aspect is that the initial phase of interference can be dealt with by AISL-II without requiring any additional computation resources. Our theoretical analysis and simulation results have shown that, for more flexibility of dealing with the interference in various environments, AISL-II is a promising method to achieve nearly optimal suppression performance.

Chapter 6 introduces two time-domain signal reconstruction methods based on LVD, known as the LSR and LSRC, for multi-component LFM signals. The former is able to provide effective reconstruction by using the peaks of auto-terms in the LVD domain, and the latter achieves with improved performance by suppressing the effects of cross-terms. Based on the LSR and LSRC, applications on decomposition, de-noising and re-sampling are presented to demonstrate their performances. Numerical result shows that they can work in negative SNR environments and arbitrary new sampling frequencies.

In Chapter 7, an extension of the LVD is introduced, i.e., the 2D LVD, for high dimension processing. Performances and discussions on important issues such as energy concentration and computational complexity are presented. Also an application based on the new extension, i.e., the 2D LVD based image watermarking, is presented and with satisfactory performances.

8.2 Recommendations for Future Works

In the following, there are some recommendations for future research based on the LVD.

8.2.1 High Order LVD

The LVD is especially proper for the LFM signals, whose highest order of the phase is two, since the auto-correlation function used in the LVD is a 2nd order correlation. However, there are still some types of signals whose highest order of phases exceeds two. For example, when dealing with a signal represented by

$$s(t) = A_0 e^{j2\pi f_0 t + j\pi \gamma_0 t^2 + j\pi g_0 t^3/3},$$

the LVD fails to reveal the 3rd order parameter g_0 . Therefore, developing higher order LVD is required.

8.2.2 Cosine Transform of the LVD

In real applications, what we generate, send, receive, and process are all real signals. Since the FT based techniques introducing imaginary domain make the process to be complicated, many real domain based approaches have been proposed, such as the cosine FT and its discrete form, the discrete cosine transform (DCT) [182]. It has been widely used in many areas such as the image processing because the DCT has better compression performance than the FT. Similarly, it is obvious that the LVD involving imaginary domain is not optimal for real LFM signals existing in practice. Therefore, it is expected a cosine form of LVD to be developed for improving the performance of real signals.

8.2.3 LVD Based ISAR Imaging

Inverse synthetic aperture radar (ISAR) is a well-known radar technique which is able to provide high resolution image of a target, and attracts more attentions these years.

In literature, the ISAR model employs wide-band signals such as the LFM signals, so time-frequency analysis is usually adopted [28, 183–186]. As this thesis shows the LVD has impressive merits comparing to other time-frequency methods, it is much interesting to show whether the LVD can improve the ISAR imaging or not. So the LVD based ISAR imaging will be a potential research topic in future.

Author's Publications

- 1) Shan Luo, and Guoan Bi, "Aliased Polyphase Sampling", *Signal Processing*, vol. 90, no. 4, pp. 1323-1326, Apr. 2010.
- 2) Shan Luo, Guoan Bi, Xiaolei Lv, and Fengye Hu, "Performance Analysis on Lv Distribution and Its Application", *Digital Signal Processing*, vol. 23, no. 3, pp. 797-807, May 2013.
- 3) Shan Luo, and Guoan Bi, "Adaptive Interference Suppression based on Lv Distribution for DSSS Communications", *in preparation*.
- 4) Shan Luo, Guoan Bi, Lifan Zhao, and Xiaolei Lv, "Lv Distribution Based Signal Reconstruction Methods and Applications", *in preparation*.
- 5) Shan Luo, and Guoan Bi, "Short-Time Lv Transform and Its Application on Spectrum Sensing", *in preparation*.
- 6) Shan Luo, and Guoan Bi, "Two Dimensional Lv Distribution and Its Application on Image Watermarking", *in preparation*.
- 7) Shan Luo, Xiaolei Lv, and Guoan Bi, "Lv's Distribution for Time-Frequency Analysis", *in Proceedings of the 2nd international conference on Circuits, systems, control, signals (CSCS 2011)*, Sep. 2011, pp. 110-115.

- 8) Shan Luo, and Guoan Bi, “Spectrum Sensing of Wireless Microphone Signals based on LHT”, in *8th International Conference on Information, Communications and Signal Processing (ICICS 2011)*, Dec. 2011.
- 9) Shan Luo, and Guoan Bi, “Performance Improvement on Local Polynomial Periodogram”, in *11th WSEAS International Conference on System Science and Simulation in Engineering (ICOSSSE 2012)*, May, 2012.
- 10) Haixin Sun, Shan Luo, and Yaowu Shi, “Broadband Interference Suppression in Fractional Fourier Domain Communication System”, in *8th International Conference on Information, Communications and Signal Processing (ICICS 2011)*, Dec. 2011.
- 11) Shan Luo, and Guoan Bi, “Suppressing Frequency Modulated Interference Based on Lv Distribution”, *Accepted by IEEE International Conference Information, Communications and Signal Processing (ICICS 2013)*.

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