Probabilistic risk assessment of underground rock caverns

Zhang, Wengang

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**NANYANG
TECHNOLOGICAL
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**PROBABILISTIC RISK ASSESSMENT OF
UNDERGROUND ROCK CAVERNS**

ZHANG WENGANG

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

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Probabilistic Risk Assessment of Underground Rock Caverns

Zhang Wengang

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Abstract

There have been significant advances in the use of computational methods for the design of underground rock caverns over the last thirty to forty years. One limitation of the conventional deterministic design approach is that the stochastic nature of the rock mass properties, the in situ stress fields and the geometrical complexities cannot be taken into account explicitly. In contrast, these uncertainties can be accounted for by adopting probabilistic methods. The objective of this study is to develop some practical approaches for reliability assessment of single and twin rock caverns and to propose some simple yet potentially useful charts for preliminary design and checking.

The global factor of safety, determined by the shear strength reduction technique, is adopted as the criterion for evaluating the ultimate limit state for underground rock cavern(s). Based on the Finite difference (FDM) simulation results, predictive models using Logarithmic Regression (LR), Back-propagation neural network (BPNN) and Multivariate adaptive regression splines (MARS) are developed respectively to estimate the global factor of safety FS for single and twin rock caverns. The influence of the ratio of the cavern span to the height of the side wall (B/H) on FS is investigated for single caverns. Charts relating FS to various parameters including the tunneling quality index Q , B and the pillar width S_c are also established for twin caverns.

The numerical outputs for serviceability assessment of underground rock cavern(s) include the maximum key point displacement $u_{\max}$ and the percent strain ε . Models to estimate $u_{\max}$ can be used to provide preliminary estimates of the expected maximum cavern displacements during construction of underground caverns. The percent strain is adopted as the criterion for evaluating the serviceability limit state in this thesis as the threshold strain value can be determined through the concept of critical strain or the design line methodology.

MARS and BPNN algorithms are compared with regard to estimating FS , $u_{\max}$ and ε for the single cavern case in terms of the prediction accuracy, the model interpretability, and the computational efficiency. In addition, for each of the built

MARS models, parameter sensitivity analysis is performed and the most critical parameters are identified. Knot locations of the built MARS models for the single cavern case are plotted and response surfaces relating the percent strain of the twin caverns to Q and in situ stress ratio K_0 , Q and S_c/B , and Q and B are established, respectively.

The twin cavern stability is investigated from the perspective of the pillar performances, with respect to the maximum principal stress in the pillar, the vertical stress and the principal stress difference in the pillar core, and the average pillar strength and average stress, in order to examine the failure mechanisms and to identify situations in which there is substantial cavern interaction and the overlap of the plastic zones induced by cavern excavation. In addition, the relationship between the pillar factor of safety FS_p and the global factor of safety FS is also proposed to avoid overlap of the plastic zones. An additional ANN model is built to determine overlap of the plastic zones using the basic design input parameters, and charts relating the plastic zone distribution function $H(X)$ to various design parameters have been developed. The charts can be used to assess if there is overlap of the plastic zones.

Reliability analyses using the First-order reliability method (FORM) and Monte Carlo simulations (MCS) are performed on ultimate and serviceability limit states for both single cavern and twin cavern cases. System reliability for both single and twin caverns can be ensured by performing probabilistic assessment using FORM on the percent strain ε and concurrently satisfying the global FS . Optimal design in terms of avoiding over-design of rock mass is achievable if a certain level of risk/expected performance level is acceptable.

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List of Symbols and Abbreviations

The following list of symbols and abbreviations is provided for ease of reference for those symbols and abbreviations that are most frequently used in this thesis. It is therefore not exhaustive. Nonetheless all symbols and abbreviations used in this thesis are defined at their first mention in the main text.

Symbols:

FS	=	global factor of safety
Q	=	tunneling quality index
FS_{g_s}	=	global factor of safety for a single cavern
u_{max_s}	=	maximum key point displacement of a single cavern
ε_s	=	percent strain of a single cavern
FS_{g_t}	=	global factor of safety for twin caverns
B	=	cavern width/span
S_c, W, W_p	=	pillar width
K_0	=	in situ stress ratio
u_{max_t}	=	maximum key point displacement of twin caverns
ε_t	=	percent strain of twin caverns
ε_c	=	threshold/critical strain
ε_e	=	elastic strain
ε_d	=	strains of the design line
β	=	reliability index
RMR	=	rock mass rating
RQD	=	rock-quality designation
GSI	=	geological strength index
E_m	=	rock mass deformation modulus
σ_{cm}	=	uniaxial compressive strength of rock mass
E_i	=	deformation modulus of intact rock
H	=	height of the sidewall
D	=	damage index
σ_{ci}	=	uniaxial compressive strength of intact rock
γ	=	rock density
JP	=	joint persistence
s	=	the fracturing degree
c	=	cohesion
ϕ	=	friction angle
σ_t	=	tensile strength
J_r	=	joint roughness number
J_a	=	joint alteration number
J_c	=	joint condition factor
COV	=	coefficient of variation
R_N	=	nominal resistance
S_N	=	nominal load
m_S	=	mean value of load
m_R	=	mean value of resistance

P_f	=	probability of failure
$\mathbf{R}, [\mathbf{R}]$	=	correlation matrix
$\mathbf{x}$	=	original correlated basic random vector
$\mathbf{u}$	=	independent standard normal vector
$\mathbf{u}^*, \mathbf{x}^*$	=	design point
μ	=	the mean
σ^2	=	the variance
ν	=	the skewness
τ_f	=	shear strength
σ_n	=	normal stress
σ_v	=	vertical in situ stress
σ_h	=	horizontal in situ stress
μ	=	Poisson's ratio
$FS_{g_s_LR}$	=	predictions of FS_{g_s} from LR model
$FS_{g_s_FDM}$	=	target/actual FS_{g_s} from FDM
$FS_{g_s_BPNN}$	=	predictions of FS_{g_s} from BPNN model
R^2	=	coefficient of determination
$FS_{g_s_MARS}$	=	predictions of FS_{g_s} from MARS model
ε_s	=	percent strain of a single cavern
$u_{max_s_FDM}$	=	target/actual u_{max_s} from FDM
$u_{max_s_LR}$	=	predictions of u_{max_s} from LR model
$u_{max_s_BP}$	=	predictions of u_{max_s} from BPNN model
e	=	relative error
$u_{max_s_MARS}$	=	predictions of u_{max_s} from MARS model
ε_{s_LR}	=	predictions of ε_s from LR model
ε_{s_FDM}	=	target/actual ε_s from FDM
ε_{s_BP}	=	predictions of ε_s from BPNN model
ε_{s_MARS}	=	predictions of ε_s from MARS model
β_{SLS}	=	reliability index of serviceability limit state
$FS_{g_t_LR}$	=	predictions of FS_{g_t} from LR model
$FS_{g_t_FDM}$	=	target/actual FS_{g_t} from FDM
$FS_{g_t_MARS}$	=	predictions of FS_{g_t} from MARS model
h	=	height of pillar
$\sigma_1 - \sigma_3$	=	principal stress difference
$H(X)$	=	function of distribution of plastic zones
$H(X)_{BPNN}$	=	BPNN model of distribution of plastic zones
S	=	pillar strength
H_c	=	height of roof arch
σ_p	=	average stress in the pillar
FS_p	=	factor of safety for pillar
$FS_{g_s_cr}$	=	threshold global factor of safety for a single cavern
$FS_{g_t_cr}$	=	threshold global factor of safety for twin caverns
$u_{max_t_FDM}$	=	target/actual u_{max_t} from FDM
$u_{max_t_MARS}$	=	predictions of u_{max_t} from MARS model
ε_{t_FDM}	=	target/actual ε_t from FDM
ε_{t_MARS}	=	predictions of ε_t from MARS model

Abbreviations:

ANN	=	Artificial neural network
ANOVA	=	analysis of variance
BEM	=	Boundary Element Method
BF	=	Basis function
BPNN	=	Back-propagation neural network
CDF	=	cumulative distribution function
DDA	=	Discontinuous Deformation Analysis
DEM	=	Distinct Element Method
DP	=	Design point
ESR	=	excavation support ratio
FDM	=	Finite difference method
FEM	=	Finite Element Method
FORM	=	First-order reliability method
FS	=	factor of safety
GCV	=	Generalized cross-validation
GSI	=	Geological Strength Index
LR	=	logarithmic regression
LRFD	=	Load and Resistance Factor Design
LSS	=	limit state surface
MARS	=	Multivariate adaptive regression splines
MCS	=	Monte Carlo simulation
NGI	=	Norwegian Geotechnical Institute
pdf	=	probability density function
PEM	=	Point estimate method
RMC	=	rock mass classification
RMR	=	Rock Mass Rating
RSM	=	Response surface method
SLS	=	Serviceability limit state
SORM	=	Second-order reliability methods
SRF	=	stress reduction factor
STD	=	standard deviation
UCS	=	uniaxial compressive strength
ULS	=	Ultimate limit state
WSD	=	Working stress design

Chapter 1

INTRODUCTION

1.1 Background

The steadily growing demand of modern society for infrastructure facilities in congested urban areas has encouraged innovations in underground construction techniques so as to free up land on the surface for other purposes such as for new recreational facilities and residential areas. Over the last thirty to forty years the number of underground rock caverns has steadily increased, primarily due to improvements in construction technology. It is believed that this trend will continue to grow in the future since underground construction provides a solution for the need of space in densely populated urban areas.

However, current rock cavern design methods which rely on empirical and numerical methods are deterministic approaches that do not deal with the stochastic nature of the complicated environment. There are many uncertainties associated with any design calculation of the behavior of a single rock cavern or twin caverns. The principal sources of uncertainty affecting the cavern design include the mechanical and geometrical properties of the cavern(s), the applied loadings, the engineering and geometrical properties of the rock bolt system, and the analytical models used to predict performance.

Due to these uncertainties, the performance of the cavern cannot be determined precisely, and the conventional lumped factor of safety or the computed displacement does not reflect the degree of uncertainty of the underlying random variables. Another complication arises because failure occurs due to the fact that the initial state of stresses is altered by excavation, generating a new state of equilibrium and mobilizing the shear strength of the rock mass in the near field around the excavated cavern. In such a complex material as rock mass, different failure modes including squeezing, rock burst, slabbing, cave-in and block falls may occur. Consequently, failure should be interpreted in the most general sense, i.e., the cavern performance is unsatisfactory as a result of instability (collapse) or excessive

movements. Collapse refers to ultimate limit state (ULS) failure, in which the stresses exceed the capacity of the rock or structural elements. Excessive movements, which will cause difficulties during excavation such as lining placement and reinforcement installation, refer to the failure of the serviceability limit state (SLS). These two distinct failures are illustrated in Figs. 1.1a and 1.1b, respectively. Both these two failures should be prevented in the design.

(a) (photo from <http://www.uwec.edu>)

(b) (photo from <http://blog.geotechnica.com>)

Fig. 1.1 Illustration of ULS and SLS failures: (a) ULS failure; (b) SLS failure

Conventional deterministic evaluation of stability of geotechnical structures and underground openings involves the calculation of the global factor of safety FS .

This single *FS* value can neither quantify the degree of safety with absolute certainty nor meet the serviceability requirements. For the serviceability limit requirement, numerically this is assessed either by the calculated displacements or strains, the plastic zone volume or the yielding zone radius. Thus adopting probabilistic design approaches to assess the ultimate and serviceability limit state functions under uncertainties and complexities mentioned above is more rational and of vital importance.

The complexity to the probabilistic analysis for underground rock cavern(s) is brought to another level because the limit state function, either ultimate or serviceability, is often not available as an explicit, closed-form function of the input variables. Such difficulties can be dealt with using numerical approaches, such as the response surface method (RSM), the artificial neural networks (ANN) and the nonparametric regression methods. After obtaining the limit states functions, given the statistics of the input variables, probabilistic assessment approaches such as First-order reliability method (FORM), Point estimate method (PEM) and the Monte Carlo simulation (MCS) can be used to derive the probability that certain threshold value has been exceeded.

While a sensitivity analysis provides valuable information, more insight is desirable. Statistics, probability, and reliability analysis can assist engineers to quantify and assess the risk of failure systematically, and the formulation of design methods for controlling the safety of rock caverns. Reliability analysis will enable engineers to better understand the relationship between cavern behavior and the design parameters and to develop a more rational approach to risk assessment.

1.2 Objectives and scope of research

The objectives of this study are as follows:

- (1) To develop a deterministic framework of combining numerical and empirical design approaches for analyzing the performance of the cavern(s) in terms of the global factor of safety and the deformations (including the maximum key point displacement and the percent strain).

- (2) To build predictive models for cavern(s) performance estimation, based on which a series of potentially useful charts for preliminary design and checking are established.
- (3) To develop a probabilistic framework incorporating the limit state functions into reliability assessment methods such as FORM or MCS to perform reliability analysis.
- (4) To investigate different approaches of developing surrogate models for implicit performance functions, and demonstrate the use of these surrogate models coupled with reliability methods. The derived probability of failure values from different reliability approaches are also compared.
- (5) To illustrate a rational and practical approach to assess the system reliability for cavern(s) design.

Firstly, the global stability for a single cavern is studied followed by investigations into the failure of the serviceability limit state. Subsequently, the ultimate and serviceability limit states of twin caverns were investigated.

1.3 Outline of thesis

The present thesis is intended to contribute to the probabilistic assessment of cavern(s) limit states in the following aspects: a) to combine numerical and empirical approaches to analyze cavern(s) performances; b) assess the overall performance of cavern(s) in terms of global factor of safety, maximum key point displacements and percent strains; c) develop surrogate models creatively utilizing nonlinear regression, back-propagation neural networks and multivariate adaptive regression splines approaches for comparison; d) propose probabilistic framework incorporating surrogate models into reliability assessment methods; and e) propose a novel and practical approach to ensure system reliability. The study will demonstrate that determination of the reliability index provides a systematic method for evaluating the influences of the uncertainties in the various parameters affecting the ultimate and serviceability limit states of rock caverns and can be used to assist design engineers in assessing the acceptable level of risk. This dissertation consists of 7 chapters.

Chapter 1 presents the background, objectives and scope of the study, and an outline of this thesis.

Chapter 2 gives a condensed review of existing tools and design methods for underground rock caverns and a summary of some stochastic properties of rocks and rock failure criteria. In addition, it introduces some basics of probabilistic analysis, presents the common approaches utilized to derive the limit state functions, and reviews the widely used probabilistic assessment methods.

Chapter 3 describes the finite difference modeling (FDM) for a single underground rock cavern to derive the global factor of safety. Global stability is quantified by the global factor of safety FS_{g_s} through the shear strength reduction technique. Based on the numerical results, models are built to relate FS_{g_s} to the cavern dimensions as well as the tunneling quality index Q . Charts, potentially useful for preliminary design and checking of single cavern stability are proposed. The merits and demerits of the surrogate models are compared. These models can be implemented into the probabilistic assessment methods to calculate the probability of failure for cavern stability.

Chapter 4 considers the serviceability limit state of a single cavern case. The numerical outputs for serviceability assessment include the maximum key point displacement u_{max_s} and the percent strain ε_s . Based on the results, models for u_{max_s} and ε_s estimations are built respectively. The merits and demerits of the surrogate models developed using Back-propagation neural network (BPNN) and Multivariate adaptive regression splines (MARS) are compared. Percent strain is adopted as the criterion for evaluating the serviceability limit state, the threshold value of which is determined from the concept of critical strain. With determination of the performance functions, reliability assessment of serviceability limit state can be carried out using MCS and FORM. The system reliability is ensured by performing probabilistic assessment on the percent strain ε_s while concurrently satisfying the FS_{g_s} during the search for the design point. Tables and the charts are constructed which can be used as a preliminary design tool when selecting a

required FS_{g_s} provided the quality of rock mass, the cavern span, the shape of the opening and the in-situ stress are known.

Chapter 5 presents numerical simulations to investigate the interaction between two adjacent horse-shoe shaped caverns in terms of the global factor of safety for twin caverns FS_{g_t} . The pillar performances with respect to the maximum principal stress in the pillar, the vertical stress and the principal stress difference in the pillar core, and the average pillar strength and average stress are studied in order to examine the failure mechanisms and to identify situations in which there is substantial cavern interaction and the overlap of the plastic zones. For the global stability, models were developed to predict FS_{g_t} in terms of Q , cavern width B and ratio of pillar width to cavern span S_c/B . A series of charts relating FS_{g_t} to Q , B and S_c have been developed, which are potentially useful for preliminary design and checking of twin cavern stability. Using MCS, probabilistic assessments on ULS are carried out on the derived limit state functions.

Chapter 6 focuses on the serviceability limit state of twin caverns. The dependent responses include the maximum key point displacement u_{max_t} and the percent strain ε_t . MARS models predicting u_{max_t} and ε_t are built respectively. The response surfaces relating percent strain ε_t to Q and in situ stress ratio K_0 , Q and S_c/B , Q and B are established, respectively. Percent strain is adopted as the serviceability criterion. The threshold strain values ε_c are determined through the elastic and design line methodology. Probabilistic assessments on SLS were performed on the MARS percent strain model using MCS and FORM. The system reliability can be ensured by performing probabilistic assessment on the percent strain ε_t and concurrently satisfying the FS_{g_t} during the search for the design point.

Chapter 7 contains a summary of the findings of this study, with recommendations for future studies.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

This chapter is divided into three parts. The first part, consisting of section 2.2, 2.3, and 2.4, is devoted to the review of existing tools and design methods for underground rock caverns, a summary of some stochastic properties of rocks, and some basic concepts of probabilistic analysis, respectively. The second part, which comprises sections 2.5, 2.6 and 2.7, introduces a summary of some rock failure criteria, presents the common approaches utilized to derive the limit state functions, and reviews the widely used probabilistic assessment methods. The last part contains concluding remarks of this chapter.

2.2 Tools and design methods for underground rock caverns

The tools and design methods available for assessing the performances of underground rock caverns can be categorized as follows:

- (1) Empirical methods.
- (2) Observational methods.
- (3) Physical modeling.
- (4) Analytical solutions.
- (5) Numerical modeling.

2.2.1 Empirical methods

The empirical methods which are denoted as the rock mass classification (RMC) systems in this thesis, are widely used as they rely on simple measurements and observations to provide a quantitative index of rock mass quality. RMC systems have been in use for more than 40 years. Some of the developed systems are directed to characterize the rock mass in general or to give an input to the definition of the rock mass properties. The most commonly used empirical design systems are

- a) Q system. On the basis of an evaluation of a large number of case histories of underground excavations, Barton et al. (1974) of the Norwegian Geotechnical Institute (NGI) proposed a tunneling quality index (Q) for the determination of rock mass characteristics and tunnel support requirements. The numerical value of the index Q varies on a logarithmic scale from 0.001 to a maximum of 1,000 and is defined by:

$$Q = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF} \quad (2.1)$$

where *RQD* is the Rock quality designation, J_n is the joint set number, J_r is the joint roughness number, J_a is the joint alteration number, J_w is the joint water reduction factor, and *SRF* is the stress reduction factor.

- b) RMR system. Bieniawski (1973, 1974) published the details of a rock mass classification called the Geomechanics Classification or the Rock Mass Rating (RMR) system. *RMR* is defined as

$$RMR = A1 + A2 + A3 + A4 + A5 + B \quad (2.2)$$

where *A1* = uniaxial compressive strength of rock material; *A2* = *RQD*; *A3* = spacing of discontinuities; *A4* = condition of discontinuities; *A5* = groundwater conditions and *B* = orientation of discontinuities.

- c) GSI system. Hoek and Brown (1997) introduced the Geological Strength Index (GSI) as an input parameter for their empirical failure criterion, which provides a system for estimating the reduction in rock mass strength for different geological conditions as identified by field observations. The GSI chart (Hoek 1999) can be considered as a sufficient tool for the use of engineers with experience in quantifying rock masses.

Each of the three systems mentioned above has its merits and disadvantages. While other RMC systems exist, they are less frequently adopted in engineering practice, and therefore are not elaborated herein. As the various RMC systems were being developed, a question arose: if two classification systems are applied to two different sites, how can the rock masses in the two sites be compared? The answer is to establish a correlation between the systems to calculate one from the other. Various researchers have tried to correlate the systems. It should also be noted that

the available systems are most appropriate only for the determination of structural resistance.

In general, the aims of rock mass classification systems are given by Milne et al. (1998):

- a) to identify zones of material of similar geo-mechanical characteristics,
- b) to provide an indication of predicted stability for excavations of a given size,
- c) to aid in the selection of an appropriate support strategy,
- d) to provide an indication of in-situ rock mass strength, modulus of deformability, etc.

All of these aims are applicable to the design process, for which (a) and (b) are the most relevant, while aim (d) is related to finding parameters for calculations. Some of the common correlations between Q and RMR are listed in Table 2.1. Apart from this, empirical relations have also been proposed to calculate the rock mass deformation modulus E_m and the Uniaxial compressive strength (UCS) σ_{cm} . Some common relationships are listed in Table 2.2 and Table 2.3. Equations in Table 2.1-2.3 are commonly used to determine the rock mass properties, which will be later used as inputs for the numerical analyses in Chapter 3.

Table 2.1 Comparison of various correlations between Q and RMR

Researcher(s)	Correlations
Bieniawski (1984)	$RMR = 9 \ln Q + 44$
Rutledge and Preston (1978)	$RMR = 5.9 \ln Q + 43$
Cameron-Clarke and Budavari (1981)	$RMR = 5 \ln Q + 60.8$
Abad et al. (1984)	$RMR = 10.5 \ln Q + 41.8$
Kaiser and Gale (1985)	$RMR = 8.7 \ln Q + 38$
Al-Harhi (1993)	$RMR = 9 \ln Q + 49$
Barton (1995)	$RMR = 15 \ln Q + 50$
Tugrul (1998)	$RMR = 7 \ln Q + 36$
Kumar et al. (2004)	$RMR = 6.4 \ln Q + 49.6$

Table 2.2 Empirical equations for estimation of E_m

Researcher(s)	Equation (units GPa)
Bieniawski (1978)	$E_m = 2RMR - 100 \quad (RMR > 50)$
Serafim and Pereira (1983)	$E_m = 10^{(RMR-10)/40} \quad (RMR \leq 50)$
Nicholson and Bieniawski (1990)	$E_m = E_i[0.0028 RMR^2 + 0.9\exp(RMR/22.821)]/100$ (E_i = deformation modulus of intact rock)
Verman (1993), Verman et al. (1997)	$E_m = 0.3(H)^\alpha 10^{(RMR-20)/38a} \quad (H > 50 \text{ m})$
Mitri et al. (1994)	$E_m = E_i[0.5(1 - \cos(\pi \times RMR/100))]$
Singh (1997)	$E_m = H^{0.2} \cdot Q^{0.36} \quad (Q < 10)$
Barton (2002)	$E_m = 10Q_c^{1/3} \quad (Q_c = Q(\sigma_{ci}/100))$ $E_m = (1-D/2) (\sigma_{ci}/100)^{1/2} \cdot 10^{(GSI-10)/40} \quad (\sigma_{ci} \leq 100 \text{ MPa})$
Hoek et al. (2002)	$E_m = (1-D/2) 10^{(GSI-10)/40b} \quad (\sigma_{ci} > 100 \text{ MPa})$ (σ_{ci} = uniaxial compressive strength of intact rock)
Ramamurthy (2004)	$E_m = E_i \exp[(RMR-100)/17.4]$
Zhang and Einstein (2004)	$E_m = E_i (10^{0.0186RQD-1.91})$
Hoek and Diederichs (2006)	$E_m = E_i[0.02 + (1-D/2) / (1 + e^{(60+15D-GSI)/11})]$

Table 2.3 Empirical equations for estimation of σ_{cm} (Aksoy et al. 2010)

Researcher(s)	Equation (units MPa)
Hoek and Brown (1980)	$\sigma_{cm} = \sigma_{ci} \exp[(RMR-100)/18]$
Yudhbir et al. (1983)	$\sigma_{cm} = \sigma_{ci} \exp[7.65(RMR-100)/100]$
Ramamurthy (1986)	$\sigma_{cm} = \sigma_{ci} \exp[(RMR-100)/18.75]$
Kalamaris and Bieniawski (1995)	$\sigma_{cm} = \sigma_{ci} \exp[(RMR-100)/24]$
Bhasin and Grimstad (1996)	$\sigma_{cm} = (\sigma_{ci}/100) 7\gamma Q^{1/3} \quad (\gamma = \text{rock density})$
Sheorey (1997)	$\sigma_{cm} = \sigma_{ci} \exp[(RMR-100)/20]$
Trueman (1998)	$\sigma_{cm} = 0.5 \exp(0.06RMR)$
Aydan and Dalgic (1998)	$\sigma_{cm} = RMR \times \sigma_{ci} / [RMR + \beta(100-RMR)]$
Palmstrom (2000)	$\sigma_{cm} = RMR = \sigma_{ci} JP \quad (JP = \text{joint persistence})$
Barton (2002)	$\sigma_{cm} = 5\gamma(Q\sigma_{ci}/100)^{1/3}$
Hoek et al. (2002)	$\sigma_{cm} = \sigma_{ci} s^a \quad (s = \text{the fracturing degree})$

Aim (b) targets to provide an indication of predicted stability for excavations of a given size. Rock mass classification methods have been correlated with stability of underground openings. For example, the development of the Q system was based on the availability of about 200 case studies of underground openings, and this database has grown to about 1200 cases (Stacey 2003). In addition, Barton et al. (1980) proposed that the maximum unsupported span can be estimated from

$$B_{\text{max, unsupported}} = 2ESR \cdot Q^{0.4} \quad (2.3)$$

in which *ESR* is the excavation support ratio, the value of which is related to the intended use of the excavation and the degree of security, which has influence on the support system to be installed to maintain the stability of the excavation.

Similarly, correlations between stability and opening size have been established from case study data for the GSI and RMR systems. Representative graphical relationships for these three methods are shown in Figs. 2.1 to 2.3. It should be noted that in Fig. 2.3, the size of the opening is represented by the hydraulic radius, defined as the plan area of the opening divided by its perimeter. It is clear from Figs. 2.1, 2.2 and 2.3 that each approach will predict different stable sizes of openings.

Fig. 2.1 Unsupported span vs. Q (after Houghton and Stacey 1980)

Fig. 2.2 Unsupported span vs. *RMR* (after Hutchinson and Diederichs 1996)

Fig. 2.3 Adjusted *RMR* vs. hydraulic radius (after Laubscher 2001)

The stability of non-stope excavations can be estimated in terms of stand-up time from the *RMR* value using the graph in Fig. 2.4 (Bieniawski 1993). This graph is based on the original concept of stand-up time by Lauffer (1958). The accuracy of the stand-up time is doubtful since it is influenced by effects such as the excavation methods, durability and in situ stress which the classification system does not take into account. Therefore, it should be used for comparative purposes only.

Fig. 2.4 Unsupported span, stand-up time and *RMR* (after Bieniawski 1989, 1993)

Aim (c) tends to aid in the selection of an appropriate support strategy. Fig. 2.5 is the Q support chart for various sizes of underground excavations, correlating rock mass quality, span and recommended support. It proved satisfactory for the design of support for the 62 m span underground ice hockey cavern used for the 1994 Winter Olympic Games (Chryssanthakis and Barton 1996). The average rock mass quality Q was 8 and the permanent support consisted of 6 m long rockbolts and 12 m long twin strand cables on a 2.5 m $\times$ 2.5 m grid, and 100 mm of steel fibre reinforced shotcrete. From the value of RMR in the actual excavation, the rock support can be estimated from a special excavation and support table (for tunnels of 10m span), as shown in Table 2.4.

Fig. 2.5 Q support chart (after Grimstad and Barton 1993)

Table 2.4 Guidelines for excavation and support of 10 m span rock tunnels in accordance with the *RMR* system (Bieniawski 1989)

Ground mass class	Excavation	Support		
		Rock bolts	Shotcrete	Steel sets
1. Very good RMR:81-100	Full face: 3 m advance	Generally no support required except for occasional spot bolting		
2. Good RMR:61-80	Full face:1.0-1.5 m advance; complete support 20 m from face	Locally bolts in crown, 3 m long, spaced 2.5 m with occasional wire mesh	50 mm in crown where required	None
3. Fair RMR: 41-60	Top heading and bench: 1.5-3 m advance in top heading; commence support after each blast;commence support 10 m from face	Systematic bolts 4 m long, spaced 1.5-2 m in crown and walls with wire mesh in crown	50-100 mm in crown and 30 mm in sides	None
4. Poor RMR: 21-40	Top heading and bench: 1.0-1.5 m advance in top heading; install support concurrently with excavation- 10 m from face	Systematic bolts 4-5 m long, spaced 1-1.5 m in crown and walls with wire mesh	100-150 mm in crown and 100 mm in sides	Light ribs spaced 1.5 m where required
5. Very poor RMR < 21	Multiple drifts: 0.5-1.5 m advance in top heading; install support concurrently with excavation; shotcrete as soon as possible	Systematic bolts 5-6 m long, spaced 1-1.5 m in crown and walls with wire mesh. Bolt invert	150-200 mm in crown, 150 mm in sides, and 50 mm on face	Medium to heavy ribs spaced 0.75 m with steel lagging and forepoling. Close invert

2.2.2 Observational methods

There has been considerable discussion of what the observational method should include. At its simplest, it has been characterized as Terzaghi's 'learn-as you-go' method. However, this approach often leads to time delays and claims related to unspecified and unforeseeable conditions. As complexity and unpredictability exist in nature, careful observations and interpretations of what can be measured in the field become prerequisites for geotechnical engineers to conduct safe and economical construction work. Observational methods have evolved from basic visual procedures, constructed on site, to sets of sophisticated procedures using modernized measuring instruments and computer-based back analysis techniques (Sakurai et al. 2003).

The back analysis procedure that was introduced in tunneling engineering by Gioda and Maier (1980), Cividini et al. (1981) and Gioda and Sakurai (1987) constitutes

an essential tool for assessing design parameters in underground structure. Recent publications of back analysis in underground cavern/tunnel engineering include Miranda et al. (2011), Yazdani et al. (2012) and Vardakos et al. (2012). Advances in numerical modeling methods, high performance computers, and automated continuous field measurement techniques are all supporting factors for the successful employment of back analysis within the framework of the observational method. As back analysis bridges field measurements, design, and construction, it is a promising practical engineering tool.

2.2.3 Physical modeling

The complexities of the physics involved in complex processes such as rockbursting or splitting or spalling are difficult to accommodate with confidence. Physical modeling can provide useful information particularly when examining the modes of failure as well designed experiments may yield important insights into behavior of rock structures by constructing and testing physical models that represent several aspects of the mechanical behavior of the prototype.

Physical model tests offer a powerful method to explore the influence of material nonlinearities, plastic flow, and spalling failure in structures constructed in discontinuous rocks. These methods include a variety of experiments on caverns or tunnels. Heuer and Hendron (1971) constructed geomechanical model tests on the excavation of underground caverns under static loading. Li et al. (2005) incorporated some of the basic principles of numerical simulation into the 3-D physical model test for underground caverns, and improved the simulation fidelity of the physical model test. Shin et al. (2008) performed a large-scale model test for pipe-reinforced tunnel headings in granular soils to understand reinforcing mechanism. Meguid et al. (2008) reviewed some physical modeling studies related to tunnel excavation in soft ground. Taking the Shuangjiangkou hydropower station in the Dadu River in China as the prototype for physical model tests, Zhu et al. (2011) concluded that the physical model test has more advantages than numerical methods in studying the reinforcement effect of the surrounding rock mass. Tunsakul et al. (2013) performed a series of physical and numerical model tests to investigate the failure behavior of a continuous rock mass surrounding a silo-shaped

cavern under high internal pressure. The resulting fracture patterns indicate that the lateral earth pressure coefficient at rest K_0 has a strong influence on the position of crack initiation and the propagation direction of the failure path.

2.2.4 Analytical solutions

One commonly used analytical solution is the Kirsch equations (Kirsch 1898) which were proposed for evaluating the stress and displacement distributions around a circular opening. Closed-form solutions that indicate the relationship between stress and displacement around underground openings are available for simple excavation shapes in homogeneous ground conditions. These solutions have considerable value in providing a conceptual understanding of rock mass behavior and in testing numerical models (An 2010, Yuval 2012). Solutions for circular and elliptical excavations can be found in Bray (1987). Design of large rectangular-like underground caverns in relatively unfractured rock is dealt with using exact elastic solutions by Cristescu and Paraschiv (1995) in pursuit of the optimal shape of cavern cross section. Kooi and Verruijt (2001) calculated the interaction effect between twin caverns using a relatively simple iteration procedure, which is an analytical solution for an infinite elastic medium. Li and Low (2010) calculated the plastic zone radius and the inward displacement of the tunnel wall through an analytical approach, in which it is assumed that a circular tunnel is excavated in a continuous, homogeneous, isotropic, initially elastic rock mass following the Mohr-Coulomb criterion, subjected to a hydrostatic far field stress and uniform support pressure.

2.2.5 Numerical modeling

In this context, the term ‘numerical modeling’ is used for all types of calculations that are based on numerical solutions of the complex equations encountered in underground excavations. Common numerical methods used to evaluate cavern performances can be categorized as continuum method such as the Finite Element Method (FEM) (Zienkiewicz 1977), Finite Difference Method (FDM) (Cundall 1976) and Boundary Element Method (BEM) (Brady and Bray 1978), and the discontinuum methods such as the Distinct Element Method (DEM) (Cundall 1971)

and the Discontinuous Deformation Analysis (DDA) (Shi 1988). In many cases, especially for highly fractured and massive ground, continuum models with appropriate material properties will be more suitable; however the number of discontinuities that can be handled is limited. In addition, it fails to model accurately the kinematics of collapse. In discontinuum-based methods, the discontinuities presented in the rock mass are explicitly modeled. However, they have difficulties in simulating failure through intact rock. In fact, the selection of a continuum or discontinuum approach depends on the size or the scale of the problem to be solved. There are no universal quantitative guidelines to determine when one method should be used instead of the other (Bobet et al. 2009).

Hybrid models are frequently used in rock engineering, basically for flow and stress/deformation problems of fractured rocks. The main types of hybrid models are the hybrid BEM/FEM (Zienkiewicz et al. 1977), and the DEM/BEM models (Lorig and Brady 1982). The main advantage of the BEM is the reduction of the model dimension by one, with much simpler mesh generation and therefore input data preparation, compared with the FEM and FDM (Jing and Hudson 2002). The hybrid DEM/FEM models have also been developed (Pan and Reed 1991, Stead et al. 2006). The numerical manifold method NMM (Shi 1996) is another combined continuum-discontinuum method, integrating FEM and DDA. Jing (2003) concluded her review article by saying that today's numerical modeling capability can handle very large-scale and complex equations systems and more attention should be focused on the quantitative representation of the physics of fractured rocks. Table 2.5 is a comparison table to show the limitations of each numerical method mentioned above.

Table 2.5 Limitations of each numerical method (adapted from Jing 2003)

Numerical methods	Limitations
FDM	Inflexibility in dealing with complex boundary conditions and material heterogeneity; Inability to incorporate explicit representation of fractures.
FEM	Large-scale opening, sliding, and complete detachment of elements are not permitted; The global stiffness matrix tends to be ill-conditioned when many fracture elements are incorporated; Requirement of small element size, continuous re-meshing with fracture growth, and conformable fracture path and element edges.
BEM	Not as efficient as the FEM in dealing with material heterogeneity and in simulating non-linear material behavior.
DEM, DDA	The general application is limited by lack of knowledge about the geometry of the rock fractures; The adequacy is highly dependent on the interpretation of the in situ fracture system geometry.
Hybrid models	Special attention needs to be paid to the continuity or compatibility conditions at the interfaces between regions of different models, particularly when different material assumptions are involved, such as rigid and deformable block-region interfaces.

2.2.6 A short critical review

The different deterministic design tools and approaches for underground rock caverns are each valuable in its own right. No single method is perfect for all problems; each method has its strengths and limitations. The *Q* system, for example, works best in ground conditions where block falls are likely to occur. However, it is pointed out that the application of *SRF* is unclear for buckling, rock-bursting, and/or squeezing conditions, or for zones of weakness. In addition, *RQD* has several limitations in characterizing the degree of jointing. As the *RMR* system is primarily developed for mining, i.e., for tunnels of limited size, hence the effect of cover size is not considered. The empirical methods are mainly developed to assess structural resistance, which is only one of the design issues to be accounted for in the design. Furthermore, they do not allow the user to quantify the degree of safety achieved by the design. In applying the observational methods, several requirements need to be met before construction. These include assessing the range of possible behavior and showing that there is an acceptable probability that the actual performance will be within acceptable limits. It is not directly stated what ‘an acceptable probability’ is, but it is obvious that it is asking for a probabilistic approach. Physical modeling

tests were mainly conducted for visualization and observation of the progressive development of the excavation damage zones. Modeling tests carried out in the laboratory with a small-scale model are also limited by the fact that in situ stresses are not realistically simulated, and there is inconsistency of scaling factors for different measured quantities, e.g. length, inertia force, and creep (Meguid et al. 2008). Analytical solutions of stress and displacement around underground openings only exist for simple excavation shapes. Numerical modeling allows one to conduct more realistic analyses that take into account factors such as the geometrical complexities, the in situ stress fields, and the excavation sequences. However, numerical modeling, especially a full three-dimensional analysis, typically takes greater time and effort to prepare, run and interpret. It is one tool that can be used in the design process and, generally, is more applicable to parametric studies than to exact calculations to provide definite answers.

2.3 Variability of geotechnical parameters in cavern design

Common sources of uncertainties in rock engineering include the spatial and temporal variability of the rock mass properties; random and systematic errors in data mapping, logging, testing, and monitoring; analytical and numerical simplifications; human omissions and errors. This thesis focuses on the variability of the in situ stress field and the mechanical properties of the rock mass, which are mainly due to the inherent randomness or natural variation. The following sections summarize the commonly assumed distributions types of in situ stress field and the mechanical properties of the rock mass, and provide a review of uncertainty quantization in terms of the coefficient of variation.

2.3.1 Distribution types

Table 2.6 is a summary of the commonly used distributions types of in situ stress field and the mechanical properties of the rock mass. These distributions are widely adopted in the literature. Table 2.7 discusses situations in which various distributions are more commonly used.

Table 2.6 Distributions types of geotechnical parameters in cavern design

Category	Parameters	Distribution type	References
Mechanical	E_m (deformation modulus)	Normal	Li and Low (2010); Cai (2011); Lü et al. (2011); Zhang and Goh (2012); Lü and Low (2011);
		Log-normal	Schweiger et al. (2001); Li and Low (2010); Park et al. (2012);
		Symmetric Beta	Li and Low (2010)
	σ_{cm} (UCS)	Normal	Tang and Kaiser (1998); Martin et al. (2003); Cai (2011)
		Log-normal	Lü et al. (2012a); Lü et al. (2012b);
	c (cohesion)	Normal	Li and Low (2010); Cai (2011); Lü and Low (2011); Zhang and Goh (2012)
		Log-normal	Schweiger et al. (2001); Bai et al. (2004); Li and Low (2010); Park et al. (2012)
		Symmetric Beta	Li and Low (2010)
		Uniform	Christian and Baecher (2002)
	ϕ (friction angle)	Normal	Bai et al. (2004); Li and Low (2010); Cai (2011); Lü and Low (2011); Zhang and Goh (2012);
		Log-normal	Schweiger et al. (2001); Li and Low (2010); Park et al. (2012)
		Symmetric Beta	Li and Low (2010);
		Tri-angular	Christian and Baecher (2002)
	σ_t (tensile strength)	Normal	Cai (2011)
In situ stress field	In situ stress ratio	Normal	Schweiger et al. (2001); Brady and Brown (2004); Park et al. (2012); Zhang and Goh (2012)
		Weibull	Martin et al. (2003)
Rock mass classification systems	RMR	Normal	Stavropoulou et al. (2007);
	Q	Normal	Stypulkowski (2004); Goh and Zhang (2012)
	GSI	Normal	Cai (2011); Lü et al. (2012a); Lü et al. (2012b);

Table 2.6 Distributions types of geotechnical parameters in cavern design contd.

Category	Parameters	Distribution type	References
Joint conditions	Joint spacing	Negative exponential	Priest and Hudson (1981)
		Log-normal	Song and Lee (2001)
	J_r (roughness number)	Logistic	Stypulkowski (2004)
	J_a (alteration number)	Pareto	Stypulkowski (2004)
		Normal	Cai (2011)
	Joint continuity	Tri-angular	Christian and Baecher (2002)
	J_c (condition factor)	Normal	Cai (2011)

Table 2.7 Some common distribution types and uses

Distribution type	Applications
Exponential	to describe “time to failure” problems
Uniform	to represent variables which have known upper and lower bounds and which have equal likelihood of occurring anywhere between these bounds
Weibull	to describe problems of the strength properties of materials and the lifetime of manufactured devices
Normal	to describe many natural “additive” type phenomena, or phenomena involving many accumulating factors. As an approximation, the normal is nevertheless often used to represent material properties
Lognormal	to avoid the disadvantage of allowing negative values in modeling material properties and loads in engineering
Triangular	to represent cases in which lower and upper limits and the mode are known, especially used in cases where the relationship between variables is known, but data is scarce
Beta	to model the behavior of random variables limited to intervals of finite length in a wide variety of disciplines, such as, variability of soil properties, proportions of the minerals in rocks in stratigraphy. Symmetric beta is often used in lieu of the normal distribution
Pareto	a power law probability distribution used in description of social, scientific, geophysical and many other types of observable phenomena
Logistic	to resemble the normal distribution in shape but has heavier tails

2.3.2 Quantifying uncertainties in rock properties

The uncertainties in the estimated value of rock properties needs to be quantitatively expressed using mathematical formulation of probability theory before they could be further incorporated into any engineering analysis or practical design. This section reviews the fundamental and essential concepts in statistics and summaries the typical coefficient of variation values of the in situ stress field and the rock mechanical properties.

If a rock property is represented by a random variable X , the mean (average) or expected value of X calculated based on N observations or data points is:

$$\text{Mean} = E[X] = \mu_X = \frac{1}{N} \sum_{i=1}^N x_i \quad (2.4)$$

The dispersion in the data about the mean is measured by the *variance* of X :

$$\text{Variance} = \text{Var}[X] = \sigma_X^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \mu_X)^2 \quad (2.5)$$

in which σ_X is known as the standard deviation of X . The mean and variance are also called the first and second moments of X , respectively. Another useful statistic is the coefficient of variation, the degree of dispersion relative to the mean:

$$\text{COV}[X] = \sigma_X / \mu_X \quad (2.6)$$

COV is dimensionless and it is a particularly useful measure of uncertainty. Some *COV* values commonly adopted are listed in Table 2.8.

Table 2.8 Typical COV values of geotechnical parameters in cavern design

Rock property or stress ratio	COV	References
σ_c (compressive strength)	0.25 - 0.30	Martin et al. (2003)
	0.39	Cai (2011)
	0.46, 0.28	Stavropoulou et al. (2007)
ϕ (friction angle)	0.1	Park et al. (2012); Lilly and Li (2000)
	0.06	Li and Low (2010)
	0.22	Bai et al. (2004)
	0.07, 0.05	Schweiger et al. (2001)
	0.16	Stavropoulou et al. (2007)
	0.05	Cai (2011)
c (cohesion)	0.3	Li and Low (2010); Lilly and Li (2000)
	0.54	Bai et al. (2004)
	0.1	Park et al. (2012)
	0.19, 0.17	Schweiger et al. (2001)
	0.25	Cai (2011)
E_m (deformation modulus)	0.13	Li and Low (2010)
	0.21	Barton et al. (1994)
	0.04	Park et al. (2012)
	0.27, 0.45	Stavropoulou et al. (2007)
	0.16, 0.17	Schweiger et al. (2001)
K_0 (in-situ stress ratio)	0.16	Cai (2011)
	0.7	Martin et al. (2003)
	0.14	Schweiger et al. (2001)
	0.15	Cai (2011)
σ_t (tensile strength)	0.08	Park et al. (2012)
	0.42	Cai (2011)
Note: COV values represent the degree of uncertainty. A small uncertainty would typically be represented by a COV=0.05 while considerable uncertainty would be indicated by a COV=0.25. Various COV values are adopted in different literatures. The variability in rock property encountered on any point is inextricably related to the particular site and to a specific regional geology.		

2.4 Basic probabilistic analysis concepts

Geotechnical engineers try to deal with the uncertainties in the design parameters (e.g., strength of the rock) by choosing reasonably conservative parameters for the deterministic stability evaluation. This approach, however, fails to address the problem of properly and consistently dealing with uncertainties. In the following discussions, the limitations of use of deterministic factor of safety, the design limit states and a brief introduction of probabilistic approaches are emphasized.

2.4.1 Factor of safety and limitations

Working stress design (WSD) essentially ensures that the characteristic load acting on the structure does not exceed some allowable limit. Characteristic values of either loads or rock properties are referred to as nominal, working, or design values. The design safety is ensured in a deterministic approach by requiring that the nominal resistance R_N be greater than the nominal load S_N with a specified factor of safety FS defined as

$$FS = R_N / S_N \quad (2.7)$$

The nominal resistance R_N is usually a conservative value, perhaps 1, 2, or 3 standard deviations below the mean value. The nominal load S_N is also a conservative value, but it is generally several standard deviations above the mean value. Thus, FS depends on many other factors, namely the uncertainty in the load and resistance and how conservatively the nominal load and resistance values are selected. Consequently FS may fail to convey the actual margin of safety in a design.

2.4.2 Limit states

Reliability is defined as covering the stability, serviceability and durability of a structure. In the limit state method, the performance of a whole structure or a part of it is described with reference to a set of limit states beyond which the structure fails to satisfy the fundamental requirements of:

- a) having a negligible risk of collapsing or endangering life
- b) having a negligible risk of deforming excessively, and
- c) being sufficiently durable.

The term “failure” in this thesis is interpreted in the most general sense, i.e., the cavern performance is unsatisfactory as a result of instability (collapse) or excessive movements. Collapse refers to ultimate limit state (ULS) failure, in which the stresses exceed the capacities of the rock or structural elements. Excessive movements, which will cause difficulties during excavation such as lining

placement and reinforcement installation, refer to the failure of the serviceability limit state (SLS). Both these two distinct failures should be prevented in the design.

2.4.3 Probabilistic approaches

A probabilistic approach provides an organized and systematic means of considering uncertainty and its influences on the response. Clear expositions of the probabilistic approach are found in various publications (Ang and Tang 1984; Lacasse 1999; Shinozuka 1983; U. S. Army Corps of Engineers 1997). Fig. 2.6 illustrates a 3-D joint probability distribution of the load S and the resistance R , with mean values denoted by m_S and m_R , respectively. The boundary separating the safe and 'failure' domains is the limit state surface (boundary) defined by

$$g(X) = R - S = 0 \quad (2.8)$$

in which X denotes the vector of the random variables. Mathematically, $R > S$ or $g(X) > 0$ would denote a 'safe' domain. An unsatisfactory or 'failure' domain occurs when $R < S$ or $g(X) < 0$. The calculation of the probability of failure P_f involves the determination of the joint probability distribution of R and S and the integration of the probability density function (*pdf*) over the failure domain as indicated by the volume $abcd$ in Fig. 2.6.

Fig. 2.6 Joint *pdf* for two variables

For a problem with multiple n random variables denoted by the vector X , the calculation of P_f involves the determination of a multidimensional joint *pdf* of the

random variables and the integration of the *pdf* over the failure domain $g(X) < 0$. The hyper-surface $g(X) = 0$ that separates the ‘failure’ and ‘non-failure’ region in n -dimensional space is generally referred to as the limit state surface.

2.5 Rock failure criteria

The failure criteria of rocks are important components of constitutive stress-strain relationships and are usually incorporated as yield surfaces or/and plastic potential functions in a plasticity model. The three most widely used yield (failure) criteria are:

- 1) Mohr-Coulomb failure criterion
- 2) Drucker–Prager yield criterion
- 3) Hoek-Brown failure criterion

As a first order approximation to the behavior of rock, the Mohr–Coulomb failure criterion is recommended when the principal stresses are compressive and when considering a limited range of mean stress. Hence it is suited to analyses for deep tunnels, caverns and rock foundations within block jointed rock masses. The main advantage of the Mohr-Coulomb failure criterion over other failure criteria is its mathematical simplicity and the physical meaning of the material parameters are well-understood and these parameters can be readily obtained from laboratory/field tests. For these reasons, Mohr-Coulomb failure criterion is adopted for the following Finite Difference analyses.

Two major limitations of this criterion are: (1) it considers the strength response to be linear, and (2) the effect of the intermediate principal stress on the strength behavior is ignored. However, a modified non-linear form of Mohr-Coulomb strength criterion has been suggested by Singh and Singh (2012) to overcome these limitations.

The Drucker-Prager yield criterion (Drucker and Prager, 1952) is a pressure-dependent model for determining whether a material has failed or undergone plastic yielding. This criterion and its many variants have been applied to rock, concrete, polymers, foams, and other pressure-dependent materials. The Drucker–Prager

yield surface is a smooth version of the Mohr–Coulomb yield surface. Like the Mohr-Coulomb failure criterion, the Drucker-Prager failure criterion receives attention and usage, although not to as great an extent.

The Hoek-Brown failure criterion was proposed by Hoek and Brown (1980, 1988, 1997) and Hoek (1983, 1994). An update of the criterion was presented in 2002 that included improvements in the correlation between the model parameters and the geological strength index (GSI). The basic idea of the Hoek-Brown criterion was to start with the intact rock properties and to add factors to reduce those properties because of the existence of joints in the rock. Together with the Mohr-Coulomb criterion, the Hoek-Brown criterion enjoys wide applications, especially for fractured hard rocks, for both numerical modeling and rock classification (correlation between the model parameters and GSI). The two criteria (MC and HB) can be transformed from one to another (Hoek and Brown, 1997). However, the criterion requires some subjective assessment on the disturbance factor with regard to the existence of the joints.

Various other failure (or strength) criteria have been proposed for rock masses over the years. Some of them are listed in the Table 2.9:

Table 2.9 Rock failure criteria

Failure (strength) criteria	References
Anisotropy of jointed rock mass strength	Amadei and Savage (1989)
Time-dependent tensile strength of saturated granite	Sun and Hu (1997)
Failure criteria for anisotropic rocks	Duvean et al. (1998)
Compressive failure of rocks	Gupta and Bergström (1997), Tharp (1997)
Testing factors for establishing rock strength	Hawkins (1998)
Shear failure envelope for modified Hoek–Brown criterion	Kumar (1998)
Effect of intermediate principal stress on strength of anisotropic rocks	Singh et al. (1998)
Modified Mohr–Coulomb failure criterion for layered rocks	Lai et al. (1999)
Short- and long-term strength of isotropic rocks	Aubertin et al. (2000)
Relation between failure criterion and deformability of rock	Hibino et al. (2000)
Water effects on rock strength	Masuda (2001)
Failure criterion for transversely isotropic rocks	Tien and Kuo (2001)

2.6 Review of approaches to obtain limit state functions

One critical aspect before performing probabilistic analysis is the determination of the limit state surface numerically represented by limit state functions. In many complicated and nonlinear problems including underground cavern or tunnel where the analyses involve the use of numerical procedures such as the finite element method, this surface may be difficult to determine explicitly in terms of the random variables, and therefore the limit state function can only be expressed implicitly rather than in a closed-form solution.

2.6.1 Polynomial regression or logarithmic regression (LR)

Polynomial and logarithmic regression models are commonly adopted regression methods to relate the dependent variable to the independent variables and to explore the forms of these relationships. Zhu et al. (2008) predicted the displacements of key points on the side walls of underground openings by using the square and cubic polynomial regressions. Basarir (2008) used LR models to assess the relationship between support pressure, depth, and tunnel deformation for different rock conditions and constructed the response surfaces. Siahmansouri et al. (2012) derived a logarithmic scale formula to predict the ratio of width to height rock pillar in twin circular tunnels. Goh and Zhang (2012) proposed a LR equation to relate the global factor of safety values to tunneling quality index Q and the cavern geometries B and H .

2.6.2 Response surface method (RSM)

The basic concept of RSM is to fit the actual performance function $g(X)$ by a closed-form polynomial function with/without cross terms, via selected deterministic analyses and an iterative algorithm. Once the meta-model is obtained, reliability-based methods can then be performed. RSM bridges current deterministic methods and the reliability approaches without having to make any changes to the existing methods and corresponding stand-alone program codes. RSM used in underground excavations include Mollon et al. (2009) and Lü and Low (2011).

2.6.3 Artificial neural network (ANN)

An ANN has a parallel-distributed architecture with a number of interconnected nodes, commonly referred to as neurons. The neurons interact with each other via weighted connections. Each neuron is connected to all the neurons in the next layer. By far the most commonly used ANN model is known as the Back-propagation (BPNN) algorithm (Rumelhart et al. 1986). In the BPNN algorithm, the ANN “learns” the complicated model relationship from examples of input and output patterns through modifying the connection weights to reduce the errors between the actual output values and the target output values. This is carried out by minimizing the defined error function (e.g., sum squared error) using the gradient descent approach. Validation of neural network performance is carried out by “testing” with a separate set of data that was never used in training process, to assess the generalization capability of the trained neural network model to produce the correct input-output mapping. Applications of ANN in underground excavation have been reported (e.g., Shi et al. (1998), Goh and Kulhawy (2003), Lü et al. (2012a), Goh and Zhang (2012), Zhang and Goh (2013)).

2.6.4 Multivariate adaptive regression splines (MARS)

As with neural networks to model nonlinear and multidimensional relationships, MARS (Friedman 1991) requires no prior knowledge of the form of the function. The main advantages of MARS are its capacity to find the complex data mapping in high-dimensional data and produce simple, easier to interpret models, and its ability to estimate the contributions of the input variables. Previous applications of MARS algorithm in civil engineering can be found in Atttoh-Okine et al. (2009), Lashkari (2012), Mirzahosseinia et al. (2011), Zarnani et al. (2011), Samui (2011), Samui and Karup (2011), Zhang and Goh (2013) and Goh et al. (2013). However, use of MARS in underground excavations is limited. Thus before adopting the MARS method in assessing the underground cavern performance, it is necessary to provide details of the MARS methodology.

MARS is a nonlinear and nonparametric regression method that models the nonlinear responses between the inputs and the output of a system by a series of

piecewise linear segments (splines) of differing gradients. No specific assumption about the underlying functional relationship between the input variables and the output is required. The end points of the segments are called knots. A knot marks the end of one region of data and the beginning of another. The resulting piecewise curves (known as basis functions), give greater flexibility to the model, allowing for bends, thresholds, and other departures from linear functions.

MARS generates basis functions by searching in a stepwise manner. An adaptive regression algorithm is used for selecting the knot locations. MARS models are constructed in a two-phase procedure. The forward phase adds functions and finds potential knots to improve the performance, resulting in an overfit model. The backward phase involves pruning the least effective terms. An open source code on MARS from Jekabsons (2010) is used in carrying out the analyses presented in the following chapters.

Let y be the target output and $\mathbf{X} = (X_1, \dots, X_P)$ be a matrix of P input variables. Then it is assumed that the data are generated from an unknown “true” model. In case of a continuous response this would be

$$y = f(X_1, \dots, X_p) + e = f(\mathbf{X}) + e \quad (2.9)$$

in which e is the distribution of the error. MARS approximates the function f by applying basis functions (BFs). BFs are splines (smooth polynomials), including piecewise linear and piecewise cubic functions. For simplicity, only the piecewise linear function is expressed. Piecewise linear functions are of the form $(x-t)_+$ with a knot occurring at value t . The equation $(.)_+$ means that only the positive part of $(.)$ is used; otherwise it is given a zero value. Formally,

$$(x-t)_+ = \begin{cases} x-t, & \text{if } x \geq t \\ 0, & \text{otherwise} \end{cases} \quad (2.10)$$

The MARS model $f(\mathbf{X})$, is constructed as a linear combination of BFs and their interactions, and is expressed as

$$f(X) = \beta_0 + \sum_{m=1}^M \beta_m \lambda_m(X) \quad (2.11)$$

where each λ_m is a basis function. It can be a spline function, or the product of two or more spline functions already contained in the model (higher orders can be used when the data warrants it; for simplicity, at most second-order is assumed in this thesis). The coefficients β are constants, estimated using the least-squares method.

Fig. 2.7 shows a simple example of how MARS would use piecewise linear spline functions to attempt to fit data. The MARS mathematical equation is expressed as

$$y = 4.4668 + 1.1038 \times \text{BF1} - 3.997 \times \text{BF2} + 1.967 \times \text{BF3} \quad (2.12)$$

where $\text{BF1} = \max(0, x - 16)$, $\text{BF2} = \max(0, 16 - x)$ and $\text{BF3} = \max(0, 25 - x)$ and $\max(a, b)$ is equal to a if $a > b$, else b . The knots are located at $x = 16$ and 25 . They delimit three intervals where different linear relationships are identified.

Fig. 2.7 Knots and linear splines for a simple MARS example

The MARS modeling is a data-driven process. To fit the model in Eq. (2.11), first a forward selection procedure is performed on the training data. A model is constructed with only the intercept, β_0 , and the basis pair that produces the largest decrease in the training error is added. Considering a current model with M basis functions, the next pair is added to the model in the form

$$\hat{\beta}_{M+1} \lambda_l(X)(X_j - t)_+ + \hat{\beta}_{M+2} \lambda_l(X)(t - X_j)_+ \quad (2.13)$$

with each β being estimated by the method of least squares. As a basis function is added to the model space, interactions between BFs that are already in the model are also considered. BFs are added until the model reaches some maximum specified number of terms leading to a purposely overfit model.

To reduce the number of terms, a backward deletion sequence follows. The aim of the backward deletion procedure is to find a close to optimal model by removing extraneous variables. The backward pass prunes the model by removing the basis functions with the lowest contribution to the model until it finds the best sub-model. Thus, the basis functions maintained in the final optimal model are selected from the set of all candidate basis functions, used in the forward selection step. Model subsets are compared using the less computationally expensive method of Generalized Cross-Validation (GCV). The GCV equation is a goodness of fit test that penalizes large numbers of BFs and serves to reduce the chance of overfitting. For the training data with N observations, GCV for a model is calculated as follows (Hastie et al. 2009)

$$GCV = \frac{\frac{1}{N} \sum_{i=1}^N [y_i - f(x_i)]^2}{\left[1 - \frac{M + d \times (M - 1) / 2}{N}\right]^2} \quad (2.14)$$

in which M is the number of BFs, d is the penalizing parameter, N is the number of data sets, and $f(x_i)$ denotes the predicted values of the MARS model. The numerator is the mean squared error of the evaluated model in the training data, penalized by the denominator. The denominator accounts for the increasing variance in the case of increasing model complexity. Note that $(M - 1) / 2$ is the number of hinge function knots. The GCV penalizes not only the number of the model's basis functions but also the number of knots. A default value of 3 is assigned to penalizing parameter d (Friedman 1991). At each deletion step a basis function is removed to minimize Eq. (2.11), until an adequately fitted model is

found. MARS is an adaptive procedure because the selection of BFs and the variable knot locations are data-based and specific to the problem at hand.

After the optimal MARS model is determined, by grouping together all the BFs that involve one variable and another grouping of BFs that involve pairwise interactions (and even higher level interactions when applicable), the procedure known as analysis of variance (ANOVA) decomposition (Friedman 1991) can be used to assess the contributions from the input variables and the BFs. This will be discussed further in Chapter 3.

2.6.5 A short critical review

For problems involving several design (input) variables and nonlinear responses, particularly with statistically dependent input variables, regression methods (polynomial or LR) are usually adopted. However, regression models become computationally impractical for problems involving a large number of design variables, particularly when mixed variables are involved. Another criticism of regression methods lies in their strong model assumptions.

RSM suffers from the same problems as LR, including limited number of design variables and strong model assumption. In addition, the computational effort is large even for a modest number of factors.

Generalization of BPNN is influenced by factors such as the size of the training data, how representative the data is of the problem to be considered, and the physical complexity of the problem. The BP algorithm has been criticized for its computational inefficiency i.e. long iterative process to determine the optimal network configuration since this is not known a priori. Too few hidden neurons may mean that the network is unable to model the nonlinear problem correctly. An excessive number of neurons will result in unnecessary arithmetic calculations and high computational cost and may cause a phenomenon called “overfitting”. Determining the optimal number of hidden neurons is commonly carried out by a trial-and-error approach through repeatedly increasing the number of hidden neurons till no further improvement in the network performance is obtained.

Besides finding the optimal number of hidden neurons and the number of hidden layers, finding the optimal BP architecture also involves determining the optimal transfer function and learning rate, as well as the maximum number of training cycles (epochs), all of which require considerable computational effort.

MARS is a nonlinear and nonparametric regression technique which uses piecewise linear segments (splines) to represent nonlinear behavior between input and the output variables of a system. Section 2.5.4 lists some recent successful applications of MARS. It is anticipated that this methodology can be useful in determining both the ultimate and the serviceability limit state functions of underground cavern(s), since MARS has the capacity to map complex data in high-dimensional problems, produce simple, easy to interpret models, and to estimate the contributions of the input variables (Zhang and Goh, 2013). Last but not the least, since MARS explicitly defines the intervals (boundaries) for the input variables, this model enables engineers to have an insight and understanding of where significant changes in the data may occur.

2.7 Review of probabilistic assessment methods

Probabilistic assessment method, also referred to as reliability-based method, is typically performed at one of the following three levels (Fenton and Griffiths 2008):

- (1) Level I: A semi-probabilistic approach where design is based on factored load and resistance values. This level is the basis of the Load and Resistance Factor Design (LRFD) method commonly used in USA. This level of analysis is simple. But it is difficult to choose a proper partial factor.
- (2) Level II: An approximate probabilistic analysis, which takes into account the probability distributions of loads and resistances but with the following simplifying assumptions: (a) loads and resistances are assumed to be independent, and (b) the components of the load and the resistance are assumed to be captured by single random variables, rather than by spatial and/or tempo random fields.
- (3) Level III: A fully probabilistic analysis in which resistance is modeled using cross-correlated space-time varying random fields. This level of analysis is

typically very complex and usually only solvable via Monte Carlo Simulation (MCS) combined with sophisticated nonlinear multidimensional finite element models. It is computationally demanding, though accurate.

The following sections review the commonly used reliability assessment methods and describe their applications in probabilistic analyses on cavern stability.

2.7.1 First-order reliability methods (FORM)

Hasofer and Lind (1974) proposed an invariant definition for the reliability index. The approach is referred to as the First-order reliability method (FORM). In classical FORM, the original correlated basic random vector $\mathbf{x}$ in X-space are transformed into independent standard normal vector $\mathbf{u}$ in U-space. The reliability index β is defined as:

$$\beta = \min_{\mathbf{u} \in F} \sqrt{\mathbf{u}^T \mathbf{u}} \quad (2.15)$$

where F denotes the failure domain in which the performance function $g(\mathbf{u}) \leq 0$. The point denoted by $\mathbf{u}^*$ which minimizes Eq. (2.15) and satisfies $\mathbf{u} \in F$ is the design point (DP), and is also the point on the limit state surface (LSS) ($g(\mathbf{u}) = 0$) that is closest to the origin of the U-space. An iterative algorithm may be used to search for $\mathbf{u}^*$, and has been well-documented (Ang and Tang 1984, Ditlevsen and Madsen 1996, Melchers 1999, Baecher and Christian 2003, Choi et al. 2006). A new algorithm for FORM was proposed by Low and Tang (2007), which interpreted the reliability index based on the perspective of an expanding ellipsoid in the original X-space of the basic random variables, and the reliability index β can be computed as:

$$\beta = \min_{\mathbf{x} \in F} \sqrt{\left(\frac{x_i - \mu_i}{\sigma_i}\right)^T [\mathbf{R}]^{-1} \left(\frac{x_i - \mu_i}{\sigma_i}\right)} \quad (2.16)$$

where x_i is the set of n random variables, μ_i is the set of mean values, $\mathbf{R}$ is the correlation matrix and F is the failure region. The minimization in Eq. (2.16) is performed over F corresponding to the region $g(X)=0$. Low and Tang (2007) had shown that an EXCEL (Microsoft) spreadsheet environment can be used to perform

the minimization and determine β . If the random variables have probability distributions close to normal, then P_f can be obtained from the expression:

$$P_f \approx 1 - \Phi(\beta) \quad (2.17)$$

in which $\Phi(\beta)$ is the value of the cumulative probability. This value can be obtained from tables of the standard cumulative normal distribution function found in many textbooks or from built-in functions in most spreadsheets. Lü et al. (2012b) and Zhang and Goh (2012) used the algorithm of Low and Tang (2007) to compute the DP and the probability of failure P_f .

2.7.2 Second-order reliability methods (SORM)

In the Second-order reliability method (SORM), the limit state function is approximated by a second-order function, which is used to enhance the accuracy of the results obtained from FORM, by fitting a quadratic surface to the actual limit state surface (LSS) in the vicinity of the DP to capture the non-linearity of LSS. The key concept of SORM is to evaluate the multidimensional probability integral over this approximate surface. One approach for fitting such a surface is to obtain the principal curvature of LSS at DP. This can be done through a procedure of coordinate rotation and matrix diagonalization (e.g., Choi et al. (2006), Lü and Low (2011), Lü et al. (2011)). Another approach for SORM is to adopt a point fitting strategy (Der Kiureghian et al. 1987).

Lü et al. (2012b) calculated the P_f based on SORM and then compared it with results from FORM. Comparisons indicate that SORM outperforms FORM in terms of accuracy. However, it should be noted that FORM is a necessary step in SORM, which builds on the DP and the reliability index of FORM.

2.7.3 Monte Carlo simulation (MCS)

A Monte Carlo Simulation (MCS) is a procedure, which seeks to simulate stochastic processes by random selection of input values to an analytical model in proportion to their joint probability density function. The basis of MCS is the use of random numbers which were originally either manually or mechanically generated

by using such techniques as spinning wheels, dice rolling or card shuffling. It is a powerful technique that is applicable to both linear and non-linear problems, but can require a large number of simulations to provide a reliable distribution of the response. In general, if the maximum error of P_f is e at confidence $1-\alpha$, then the required number of realizations n is

$$n = \hat{p}_f \hat{q}_f \left(\frac{z_{\alpha/2}}{e} \right)^2 \quad (2.18)$$

in which $\hat{p}_f$ is the estimated value of P_f , $\hat{q}_f = 1 - \hat{p}_f$, and $z_{\alpha/2}$ is the point on the standard normal distribution satisfying $P[Z > z_{\alpha/2}] = \alpha/2$. For example, if the estimated P_f is 0.1% and the desired maximum error on P_f is 0.0001 at confidence 90% ($\alpha=0.10$), then the required n is

$$n = \hat{p}_f \hat{q}_f \left(\frac{z_{\alpha/2}}{e} \right)^2 = 0.001(0.999) \left(\frac{1.645}{0.0001} \right)^2 \approx 270000 \quad (2.19)$$

Oreste (2005) presented a probabilistic numerical approach for the design of primary tunnel supports, according to the hyperstatic reaction method. The approach considered the probabilistic distributions of the geomechanical index of the rock mass and of the mechanical parameters of the support material, using MCS. From the calculation results it is possible to obtain the probabilistic distribution of the support safety factors and then to design the supports on the basis of the acquired probabilistic curve.

2.7.4 Point estimate method (PEM)

The Point Estimate Method (PEM) (Rosenblueth 1975, 1981) is a simple, approximate way of determining the first three moments (the mean μ , the variance σ^2 and the skewness ν) of a variable that depends on one or more random input variables. Unlike FORM or SORM, PEM does not require knowledge of the particular form of the probability density function of the input, nor does it typically explicitly account for spatial correlation. The PEM is essentially a weighted average method reminiscent of numerical integration formula involving ‘sampling points’

and ‘weighting parameters’. The PEM seeks to replace a continuous probability density function with a discrete function having the same first three central moments.

Lilly and Li (2000) demonstrated the PEM application in a 5m wide conveyor drive excavated in a rock mass with Mohr-Coulomb parameters, by using it to calculate the moments of the probability distribution for the displacement field. Park et al. (2012) proposed a probabilistic numerical approach for assessing the reliability of underground structures, combining a finite difference code and PEM.

2.7.5 A short critical review

PEM is a simple, direct, and effective method of computing the low-order moments of functions of random variables. This method is reasonably robust and is reasonably accurate for a range of practical problems, though computational requirements increase rapidly with the number of uncertain quantities of interest. Caution should be taken in applying the method to cases in which the transformation of uncertain quantities severely changes the distribution form, or to moments higher than second order.

The popularity of FORM method is due to its simplicity, since only second moment information and the probability distribution type of the random variables are required. According to Lü et al. (2011), for some cases involving highly non-linear limit state functions, the probability of failure estimated by FORM may give inaccurate results.

SORM enhances the accuracy of the results obtained from FORM, by fitting a quadratic surface to the actual limit state surface in the vicinity of the design point to capture the non-linearity. However, it should be noted that FORM is a necessary step in SORM, which builds on the design point and the reliability index of FORM. Its accuracy largely depends on whether FORM is able to locate the design point exactly and the shape of the limit state surface.

MCS uses randomly generated points to cover the range of values that enter into a calculation. This technique has the advantage that it is relatively easy to implement

on a computer and can deal with a wide range of functions, including those that cannot be expressed conveniently in explicit form. The main disadvantages are its large computational time and slow convergence.

2.8 Concluding remarks

In this chapter, literature review was presented for: (1) tools and design methods for underground rock caverns, (2) variability of geotechnical parameters in cavern design, (3) commonly used rock failure criteria, (4) approaches to obtain limit state functions, and (5) commonly used probabilistic methods.

The RMC systems available were developed to assess structural resistance, which is only one of the design issues to be accounted for in modern cavern design. It fails to address serviceability considerations and to allow the user to quantify the degree of safety. The conventional lumped factor of safety approach for cavern design based on deterministic analysis cannot explicitly account for the uncertainties of the underlying parameters. By using probabilistic concepts that reflect the degree of uncertainty of the underlying random variables, engineers will have an increased awareness of the uncertainties and their effects on the probability of failure. As suggested by Duncan (2000) the probability of failure should not be viewed as a replacement of the factor of safety, but as a supplement. The determination of the probability of failure provides a systematic method for evaluating the influences of the uncertainties in the various parameters affecting FS and can be used to assist design engineers in assessing the acceptable level of risk.

Chapter 3

RELIABILITY ASSESSMENT ON ULTIMATE LIMIT STATE OF A SINGLE CAVERN

3.1 Introduction

One of the major considerations in the design of an underground rock cavern is the evaluation of its stability since the excavation of the rock causes a redistribution of the stresses in the proximity of the underground opening. Various methods have been proposed to assess the cavern stability, and to assess the necessary support system to maintain the stability of the excavation. Conventional deterministic evaluation of stability of underground openings involves the use of global factor of safety FS which considers the relationship between the resistance R and the load (stress) S . However, the ultimate limit state function $g(X)$ is not known explicitly. Instead, it may be known only implicitly through a numerical procedure. Therefore, the failure domain only can be found through repeated point-by-point numerical analyses with different input values. A closed-form limit state surface then is constructed artificially using numerical techniques such as LR, BPNN and MARS. The details of the BPNN and MARS algorithms (programmed in MATLAB) adopted in this thesis are included in Appendix A and B, respectively. Even after obtaining the limit state surface, due to the uncertainties in the design parameters in the limit state surface, it is impossible to predict the state of the system with accuracy. The determination of the probability of failure provides a systematic method for evaluating the influences of the uncertainties in the various parameters affecting FS and can be used to assist design engineers in assessing the acceptable level of risk.

In this chapter, 120 numerical cases are analyzed, using the rock mass classification correlations and the shear strength reduction technique to solve for the global factor of safety FS . The ultimate limit state failure refers to the stress-induced instability

measured by FS . Based on the 120 results, prediction models utilizing LR, BPNN, and MARS are built. Charts relating FS to influencing parameters including the cavern width B , the ratio of cavern width to side wall height B/H , as well as the rock mass quality Q are proposed. Once the ultimate limit state functions are determined through the above three methods, given the statistics of the design variables, probabilistic assessment can be performed by MCS and FORM to calculate the probability that the predicted FS is smaller than a certain threshold value.

3.2 Previous studies

In this thesis, the global stability FS for the rock caverns are assessed using the shear strength reduction technique. This technique has been carried out by various authors including Matsui and San (1992), Dawson et al. (1999) and Dawson et al. (2000), and is now available in many commercial finite element and finite difference programs. This procedure essentially involves repeated analyses by progressively reducing the shear strength properties until collapse occurs. For a Mohr-Coulomb material, by reducing the shear strength by a factor F the shear strength equation becomes:

$$\frac{\tau_f}{F} = \frac{c}{F} + \sigma_n \frac{\tan \phi}{F} \quad (3.1)$$

$$F = \frac{\tau_f}{c^* + \sigma_n \tan \phi^*} \quad (3.2)$$

where τ_f is the shear strength, σ_n is the normal stress, and $c^* = c/F$ and $\phi^* = \arctan(\tan \phi / F)$ are the new Mohr-Coulomb shear strength parameters. Systematic increments of F are performed until the finite element or finite difference model does not converge to a solution (i.e. failure occurs). The critical strength reduction value which corresponds to non-convergence is taken to be the global factor of safety FS .

The technique has been applied to a number of underground excavation problems including rock caverns (Hammah et al. 2007, Zhang and Goh 2012, Goh and Zhang 2012) and circular tunnels (Vermeer et al. 2002).

3.3 Numerical modeling

This section describes the basic assumptions of the numerical analysis, the cross-section layout, and the basic design parameters which influence the FS results. To differentiate the global FS values for twin caverns in Chapter 5, the global factor of safety for the single cavern case is denoted as FS_{g_s} .

3.3.1 Assumptions

The basic assumptions of the numerical analyses are:

- a) the study was a two-dimensional plane strain problem;
- b) the rock material obeyed Mohr-Coulomb failure criterion that follows the elastic perfectly-plastic stress-strain relationship;
- c) only stress-induced failure was considered, the discontinuous nature of the rock mass is incorporated implicitly in the Mohr-Coulomb constitutive relationship used to represent the mass as an equivalent continuum;
- d) caverns are unsupported, and creep not considered;
- e) full-face excavation.

3.3.2 Cross-section and boundary conditions

The FLAC^{3D} code (Itasca 2005) was utilized for the numerical experiments. The cross-section of the cavern, the side view and boundary conditions in red are shown in Fig. 3.1. The cavern roof arc is semi-circular and the overburden height D from the ground surface to the top of the side wall is 100 m. The plane strain conditions are enforced by including a thin 1 m slice of material in the longitudinal direction and imposing boundary conditions on the two off-plane surfaces that allow movement vertically but are restrained against displacements normal to these planes. Outer boundaries are located far from the cavern to minimize the boundary effects. No surface loading above-ground surface is considered. The initial vertical in situ

stress σ_v is induced by self-weight of the rock. The horizontal stress σ_h is calculated using $K_0 \times \sigma_v$.

Fig. 3.1 Cavern configuration, boundary fixity and side view

3.3.3 Ranges of design parameters and mechanical inputs

In the FDM analyses, the four variables and their respective ranges are listed in Table 3.1. Q cannot be directly used in the FLAC^{3D} calculations, though it is a commonly used quality index representing rock mass competence. The rock mass properties are indirectly (through RMR) determined from the Q value by means of empirical equation listed in Table 3.2 and previously in Table 2.1. It should be pointed out that while the K_0 was found to significantly influence the state of stress and magnitude of displacements in proximity to the cavern, it had minimal influence on the FS_{g-s} with regard to stress-induced instability. Considering this, K_0 is omitted in the ULS analysis. However, it plays a significant role in determining the SLS, as will be discussed in Chapter 4.

Table 3.1 Factors and their values used in parametric studies

Parameter	Description	Values
K_0	In situ stress ratio	0.5, 1, 2
B	Cavern span (m)	5, 10, 15, 20, 25, 30
B/H	Ratio of cavern width to side wall height	0.25, 0.5, 1, 2
Q	Tunneling quality index	0.4, 1, 4, 10, 40

In the preliminary stage of an engineering design, the need for an approximate estimate of the rock mass parameters frequently arises (Bieniawski 1978, Barton et. al. 1980, Hoek and Brown 1997, Palmstrom and Broch 2006, Basarir 2006, Aksoy et al. 2010). For the finite difference analyses carried out in this study, the

following equations in Table 3.2 are adopted. It should be noted that these relationships are intended to provide the initial estimates of the rock mass properties and should be used with caution in engineering design.

Table 3.2 Empirical equations relating Q with rock mass properties

Properties	Equations	References
RMR	$RMR = 7 \ln Q + 36$	(Tugrul 1998)
c (MPa)	$c(MPa) = 0.005(RMR - 1)$	(Bieniawski 1989)
ϕ (°)	$\phi(^{\circ}) = 0.5RMR + 4.5$	(Bieniawski 1989)
σ_{cm} (MPa)	$\sigma_{cm}(MPa) = RMR$	(Palmstrom 2000)
E_m (GPa)	$E_m(GPa) = 2RMR - 100$ ($RMR > 50$)	(Bieniawski 1978)
	$E_m(GPa) = 10^{(RMR-10)/40}$ ($RMR \leq 50$)	(Serafim and Pereira 1983)
σ_t (MPa)	$\sigma_t(MPa) = \sigma_{cm}/15$	assumed

Note: E_m is the deformation modulus of rock mass, c is the cohesive strength, ϕ is the friction angle, σ_{cm} is the UCS and σ_t is the tensile strength of the rock mass.

Adopting the empirical equations listed in Table 3.2, the Q value of each category and its corresponding Mohr-Coulomb rock properties to be used in the numerical calculations are shown in Table 3.3. In Table 3.3, the Poisson's ratio μ values are assumed.

Table 3.3 Rock mass properties with different Q values

Q	c (MPa)	ϕ (°)	E_m (GPa)	μ	σ_t (MPa)
0.4	0.14	19.3	3.09	0.35	1.97
1	0.18	22.5	4.47	0.35	2.40
4	0.22	27.4	7.81	0.20	3.05
10	0.26	30.6	11.30	0.20	3.47
40	0.30	35.4	19.75	0.16	4.12

3.4 Modeling results and analyses of FS_{g_s}

Based on the parameter combinations, a total of 120 cases are analyzed. The results are presented in the following sections.

3.4.1 Failure mechanism

Fig. 3.2a and 3.2b show typical plots of distribution of plastic zones for $Q=1$ and $Q=4$ respectively, with $B = 20$ m, $B/H=1$, and $K_0=1$. Fig. 3.3a and 3.3b show the distribution of plastic zones for $Q=0.4$ and $Q=4$ respectively, with $B = 20$ m, $B/H=2$,

and $K_0=1$. It is obvious from Fig. 3.2 that with increase of Q , and the corresponding increases to FS_{g_s} from 0.92 to 1.14, the extent of the plastic zones significantly decreases. A considerably large area of plastic zones that extended to the ground surface has been reduced to a moderate zone of yielding peripheral to the opening. An even more obvious contrast can be observed in Fig. 3.3. The extent of plastic zones has been significantly reduced for case $Q=4$, with $FS_{g_s} = 1.26$. Figs. 3.2a and 3.3a also indicate that for a case of $FS_{g_s} < 1$, total collapse occurs.

(a)

(b)

Fig. 3.2 Distribution of plastic zones ($B=20$ m, $B/H=1$ and $K_0=1$): (a) $Q=1$ ($FS_{g_s}=0.92$), (b) $Q=4$ ($FS_{g_s}=1.14$)

(a)

(b)

Fig. 3.3 Distribution of plastic zones ($B=20$ m, $B/H=2$ and $K_0=1$): (a) $Q=0.4$ ($FS_{g_s}=0.82$), (b) $Q=4$ ($FS_{g_s}=1.26$)

3.4.2 Modeling results of FS_{g-s}

The FS_{g-s} values for various combinations of Q , B and B/H are shown in Table 3.4. Influences of B/H on FS_{g-s} are shown in Fig. 3.4. It is obvious that for the same Q and B , FS_{g-s} increases with the increase of B/H , which indicates that as the cavern height increases, it becomes less stable. For the same Q and B , the increase of FS_{g-s} is marginal for $B/H \geq 1.0$. The flattening of the curves indicates that B/H ratio has minor effect on global factor of safety when B is greater than H .

Table 3.4 Results from numerical experiments for FS_{g-s}

Q	$B=5$ (m)				$B=10$ (m)				$B=15$ (m)			
	$B/H=0.25$	0.5	1	2	0.25	0.5	1	2	0.25	0.5	1	2
0.4	0.93	1.12	1.29	1.38	0.69	0.86	1.00	1.08	0.58	0.74	0.85	0.92
1	1.13	1.35	1.56	1.64	0.84	1.04	1.21	1.32	0.71	0.90	1.03	1.13
4	1.42	1.69	1.95	2.02	1.05	1.31	1.51	1.65	0.88	1.12	1.29	1.42
10	1.65	1.97	2.27	2.35	1.23	1.52	1.77	1.93	1.03	1.31	1.51	1.64
40	1.95	2.32	2.67	2.76	1.46	1.79	2.08	2.27	1.22	1.55	1.78	1.94
Q	$B=20$ (m)				$B=25$ (m)				$B=30$ (m)			
	0.25	0.5	1	2	0.25	0.5	1	2	0.25	0.5	1	2
0.4	0.49	0.65	0.76	0.82	0.44	0.58	0.69	0.75	0.39	0.52	0.64	0.69
1	0.60	0.79	0.92	1.01	0.53	0.70	0.84	0.92	0.48	0.64	0.77	0.85
4	0.75	0.99	1.14	1.26	0.66	0.88	1.04	1.15	0.60	0.80	0.97	1.06
10	0.87	1.15	1.34	1.47	0.78	1.03	1.12	1.33	0.70	0.93	1.13	1.23
40	1.04	1.36	1.58	1.73	0.94	1.22	1.44	1.58	0.85	1.12	1.33	1.46

Fig. 3.4 Influence of B/H on $FS_{g,s}$: (a) $B=5$ m, (b) $B=10$ m, (c) $B=15$ m, (d) $B=20$ m, (e) $B=25$ m, and (f) $B=30$ m

3.4.3 Determination of FS_{g_s} using LR

Based on the above results, the LR model was developed for predicting FS_{g_s} in terms of Q , B and B/H , as shown in Eq. (3.3)

$$FS_{g_s_LR} = 2.9562Q^{0.1605}B^{-0.4127}(B/H)^{0.2314} \quad (3.3)$$

A comparison between $FS_{g_s_LR}$ and $FS_{g_s_FDM}$ (FS_{g_s} obtained from FLAC^{3D}) is shown in Fig. 3.5. The high coefficient of determination R^2 of 0.975 indicates that the predictions are in good agreement with the target FS_{g_s} values.

Fig. 3.5 Comparison between $FS_{g_s_FDM}$ and $FS_{g_s_LR}$

3.4.4 Design curves of FS_{g_s}

Based on the mathematical equation Eq. (3.3), a series of charts relating FS_{g_s} to Q , B and B/H have been developed as shown in Fig. 3.6. The proposed charts are potentially useful for preliminary design and checking of single cavern stability. It should also be noted that since no support is considered, the analysis results are more applicable to caverns in fair or good rock quality massed (Q value ranging from 4 to 40). For Q values of 0.4 and 1, the design curves should be used with caution.

Fig. 3.6 Design curves of FS_{g_s} : (a) $B/H=0.25$, (b) $B/H=0.5$, (c) $B/H=1$, and (d) $B/H=2$

3.4.5 Predictions of FS_{g_s} using BPNN

Utilizing Matlab codes adapted from Matlab toolbox, a BPNN model using the observations in Table 3.4 was developed for FS_{g_s} predictions. Three inputs that represented Q , B and B/H are the neurons in the input layer. Of the 120 observations, 90 data samples were randomly selected as the training data and the remaining 30 data sets were used for testing the validity of the developed neural network. The optimal BPNN model consisted of five hidden neurons. The testing data sets together with the BPNN predictions are listed in Table 3.5. For comparative purpose, training data results for BPNN and MARS are also included and are shown in Appendix C. A plot of $FS_{g_s_BPNN}$ versus $FS_{g_s_FDM}$ for the training and testing samples is shown in Fig. 3.7. The high R^2 indicate that BPNN predictions are reasonably accurate.

Table 3.5 Testing data sets for FS_{g_s}

Q	B (m)	B/H	$FS_{g_s_FDM}$	$FS_{g_s_BPNN}$	$FS_{g_s_MARS}$
0.4	5	0.5	1.12	1.17	1.21
0.4	10	1	1.00	1.05	1.02
0.4	15	0.25	0.58	0.52	0.53
0.4	20	2	0.82	0.81	0.85
0.4	25	0.5	0.58	0.58	0.56
0.4	30	1	0.64	0.67	0.60
4	5	0.5	1.69	1.68	1.69
4	10	1	1.51	1.54	1.51
4	15	0.25	0.88	0.88	0.88
4	20	2	1.26	1.25	1.26
4	25	0.5	0.88	0.87	0.88
4	30	1	0.97	0.94	0.97
40	5	0.5	2.32	2.29	2.39
40	10	1	2.08	2.15	2.07
40	15	0.25	1.22	1.20	1.22
1	5	0.5	1.35	1.38	1.40
1	10	1	1.21	1.25	1.19
1	15	0.25	0.71	0.67	0.69
1	20	2	1.01	0.99	1.03
1	25	0.5	0.70	0.70	0.72
1	30	1	0.77	0.78	0.76
10	5	0.5	1.97	1.94	2.06
10	10	1	1.77	1.80	1.69
10	15	0.25	1.03	1.04	1.05
10	20	2	1.47	1.47	1.51
10	25	0.5	1.03	0.99	1.03
10	30	1	1.13	1.07	1.02
40	20	2	1.73	1.71	1.70
40	25	0.5	1.22	1.20	1.20
40	30	1	1.33	1.34	1.35

Fig. 3.7 Comparison between $FS_{g_s_FDM}$ and $FS_{g_s_BPNN}$

3.4.6 Predictions of FS_{g_s} using MARS

Using the same training and testing datasets, this problem is reanalyzed by means of MARS. The MARS model adopted 19 BF's of linear spline functions with second-order interaction. The MARS predictions of the testing data sets are listed in Table 3.5, together with the BPNN predictions and the target FS_{g_s} values. A plot of MARS predicted FS_{g_s} versus the $FS_{g_s_FDM}$ values is shown in Fig. 3.8. The high R^2 indicate that MARS predictions are almost as accurate as the BPNN estimations. A comparison with Fig. 3.5 shows that the LR predictions give slightly more scatter compared with BPNN and MARS.

Fig. 3.8 Comparison between $FS_{g_s_FDM}$ and $FS_{g_s_MARS}$

3.4.7 Comparisons between BPNN and MARS

It is clear that the accuracy of the MARS model is almost as good as that of the BPNN model. In addition, to compare the performances of MARS and BPNN, the processing speed (CPU time) and the model interpretability are also considered. Using a PC with 3.0 GHz Intel Core2Quad Q9650 processor and 4GB RAM, the execution time of BPNN algorithm is 19.453 s, while the processing time of the MARS algorithm is 0.72 s. The above comparison indicates that MARS is computationally more efficient.

For the built optimal BPNN model, the procedure of partitioning of weights is described in Appendix D. The parameter relative importance comparison indicates that Q is the most important variable in determining FS_{g_s} , followed by B/H and B .

The mathematical expression of this model is shown in Appendix E. It can be seen from Appendix E that the built BPNN model is hard to interpret, as the weights and the bias values give no physical indication for engineers to have an insight and understanding of where significant changes in the data may occur.

The MARS model is much easier to interpret. Table 3.6 displays the ANOVA decomposition of the developed MARS model. The first column in Table 3.6 lists the ANOVA function number. The second column gives an indication of the importance of the corresponding ANOVA function, by listing the GCV score for a model with all BFs corresponding to that particular ANOVA function removed. This GCV score can be used to evaluate whether the ANOVA function is making an important contribution to the model, or whether it just marginally improves the global GCV score. The third column provides the standard deviation of this function. It should be noted that the STD value can also be used to evaluate the relative importance of parameters. The fourth column gives the number of BFs comprising the ANOVA function. The last column gives the particular input variables associated with the ANOVA function. Parameter relative importance is evaluated by the increase in the GCV value or STD value caused by removing the considered variables from the developed MARS model. Both the GCV and STD values in Table 3.6 consistently indicate that Q is the most significant parameter in determining FS_{g-s} , followed by B and B/H .

Table 3.6 ANOVA decomposition of the developed MARS model for FS_{g-s}

Functions	GCV	STD	#basis	variable(s)
1	0.731	0.378	4	Q
2	0.659	0.373	2	B
3	0.321	0.288	2	B/H
4	0.042	0.097	4	Q, B
5	0.077	0.129	5	$Q, B/H$
6	0.008	0.035	2	$B, B/H$

Table 3.7 lists the BFs of the MARS model and their corresponding equations. The interpretable MARS model to predict FS_{g-s} is given by

$$\begin{aligned}
 FS_{g_s_MARS} = & 1.472 - 0.033 \times BF1 + 0.097 \times BF2 - 0.039 \times BF3 + 0.28 \times BF4 \\
 & - 1.347 \times BF5 - 0.0095 \times BF6 + 0.008 \times BF7 + 0.046 \times BF8 + 0.019 \times BF9 \\
 & - 0.061 \times BF10 + 0.0043 \times BF11 - 0.21 \times BF12 - 0.057 \times BF13 - 0.01 \times BF14 \\
 & + 0.0003 \times BF15 + 0.0004 \times BF16 + 0.002 \times BF17 - 0.066 \times BF18 + 0.048 \times BF19
 \end{aligned} \tag{3.4}$$

Table 3.7 Basis functions and their corresponding equations for FS_{g_s}

BFs	Equation	BFs	Equation
BF1	$\max(0, B - 10)$	BF11	$\max(0, Q - 1)$
BF2	$\max(0, 10 - B)$	BF12	$\max(0, 1 - Q)$
BF3	$\max(0, 4 - Q)$	BF13	$\max(0, 10 - Q)$
BF4	$\max(0, B/H - 0.5)$	BF14	$BF11 \times \max(0, B/H - 1)$
BF5	$\max(0, 0.5 - B/H)$	BF15	$BF11 \times \max(0, B - 20)$
BF6	$BF3 \times \max(0, 10 - B)$	BF16	$BF11 \times \max(0, 20 - B)$
BF7	$BF4 \times \max(0, Q - 10)$	BF17	$BF13 \times \max(0, B - 15)$
BF8	$BF4 \times \max(0, 10 - Q)$	BF18	$BF13 \times \max(0, B/H - 1)$
BF9	$BF5 \times \max(0, B - 10)$	BF19	$BF13 \times \max(0, 1 - B/H)$
BF10	$BF5 \times \max(0, 10 - B)$		

Fig. 3.9a-c plots the knot locations for Q , B and B/H respectively. It is obvious from Eq. (3.4), Tables 3.7 and Fig. 3.9 that MARS essentially builds flexible models using linear regression and approximates the model by segmenting separate slopes in distinct intervals of the input variables. The variables to use and the knot locations of the intervals for each variable are determined via a fast but intensive search procedure. The forward selection and backward deletion also ensures that an optimal model can be found. In addition, the resulting MARS model can be easily interpreted. Since MARS explicitly defined the intervals (boundaries) for the input variables, the model enables engineers to have an insight and understanding of where significant changes in the data may occur.

(a)

(b)

(c)

Fig. 3.9 Knot locations for: (a) Q , (b) B , and (c) B/H

3.5 Probabilistic assessments on ULS

Before describing the probabilistic assessments on ULS, it should be mentioned that for the FORM and MCS analyses in the present and the following chapters, the

parameters are assumed to be uncorrelated. With the determination of the ultimate limit state functions Eqs. (3.3), (3.4) and (E-17) in Appendix E, reliability assessment of the FS_{g_s} can be carried out using either MCS or FORM which is computationally less intensive. Failure in terms of global stability occurs if the predicted FS_{g_s} is smaller than a threshold factor of safety $FS_{g_s,cr}$ ($FS_{g_s,cr}$ is assumed as unity in this chapter). Probabilistic analysis is performed by calculating the reliability index β , to which the probability of failure is directly related. The reliability index β can be used to quantify the expected performance level. Typical values for β and probability of failure P_f for representative geotechnical components and systems and their expected performance levels have been proposed (US Army Corps of Engineers 1997), as shown in Table 3.8.

Table 3.8 Target reliability indices (US Army Corps of Engineers 1997)

β	$P_f \approx 1 - \Phi(\beta)$	Expected performance level
1.0	0.16	Hazardous
1.5	0.07	Unsatisfactory
2.0	0.023	Poor
2.5	0.006	Below average
3.0	0.001	Above average
4.0	0.00003	Good
5.0	0.0000003	High

EN 1990 proposed a table relating the target reliability index and probability of failure to different limit state and length of reference periods as in Table 9 below:

Table 3.9 Target reliability index, β , and target probability of failure, P_f , values

Limit state	Target Reliability Index, β		Target Probability of failure, P_f	
	1 year	50 years	1 year	50 years
ULS	4.7	3.8	1×10^{-6}	7.2×10^{-5}
SLS	2.9	1.5	2×10^{-3}	6.7×10^{-2}

3.5.1 MCS_LR, MCS_BPNN and MCS_MARS

One of the most useful software packages for MCS probability analysis is a Microsoft Excel add-in program called @RISK (<http://www.palisade.com>) which can be used for risk evaluations using sampling techniques including Latin Hypercube and Monte Carlo samplings. In this section, the derived LR, BPNN and MARS models are incorporated into @RISK cells respectively for FS_{g_s} estimation.

For each MCS simulation, the number of iterations is 1 000 000 and Latin Hypercube sampling is adopted. The MCS procedures and the calculation of P_f are illustrated in Fig. 3.10.

(a)

(b)

(c)

Fig. 3.10 MCS with @RISK: (a) LR, (b) BPNN, and (c) MARS

3.5.2 FORM_LR, FORM_BPNN and FORM_MARS

In the FORM procedure, the performance functions derived through LR, BPNN and MARS models are incorporated into an EXCEL spreadsheet environment based on the approach by Low and Tang (2007) from which β and P_f can be determined. The typical FORM procedures and the calculation of P_f are illustrated in Fig. 3.11. Take Fig. 3.11a for example. Fig. 3.11a shows a sample spreadsheet for computing β where Q , B and B/H are assumed to be normally distributed. The spreadsheet cells A2:A4 allows the selection of distribution types for the input variables, including Normal, Lognormal, Triangular etc as explained in Low and Tang (2007). For nonnormals, the nonnormal distribution types are replaced by an equivalent Normal ellipsoid, centered at the equivalent Normal mean. Cells C2:F4 are parameters which are set corresponding to the variable distribution types. For Normal distributions, cells C2:C4 correspond to the mean values and cells D2:D4 correspond to the standard deviations. The correlation matrix $[\mathbf{R}]$ cells H2:J4 are used to define the correlations between Q , B and B/H . The n_i vector in cells K2:K4 contains equations for $(x_i - \mu_i^N) / \sigma_i^N$. Cell L3 contains the ULS function $g(\mathbf{x}) = FS_{g_s_LR} - FS_{g_s_cr}$. The DP (x^* values) was obtained by using the spreadsheet's built-in optimization routine SOLVER to minimize the cell, by changing the x^* values, under the constraint that the ULS function $g(x^*) = 0$. Prior to invoking the SOLVER search algorithm, the x^* values in cells G2:G4 were set equal to the mean values (4, 15, 1) of the original random variables. Iterative numerical derivatives and directional search for DP x^* were automatically carried out in the spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Distribution	Variables	Para1	Para2	Para3	Para4	x_i^*	Correlation matrix [R]			n_i	$g(\underline{x})$	β	$P_f(\%)$
2	Normal	Q	4	0.8			2.535	1	0	0	-1.831			
3	Normal	B (m)	15	1.5			18.56	0	1	0	2.372	-5E-03	3.278	0.052
4	Normal	B/H	1	0.1			0.867	0	0	1	-1.33			

$$g(\underline{x}) = FS_{g_{s,LR}} - FS_{g_{s,LR}} = FS_{g_{s,LR}} - 1 = 2.9562Q^{0.1685}B^{-0.4127}(B/H)^{0.2314} - 1$$

(a)

Distribution	Variables	Para1	Para2	Para3	Para4	x_i^*	BPNN Process			Correlation matrix [R]			n_i	$g(\underline{x})$	β	$P_f(\%)$
Normal	Q	4	0.8			2.505	-0.894	1.847	-0.146	1	0	0	-1.869			
Normal	B (m)	10	1			10.32	-0.574	-1.813	0.682	0	1	0	0.32	5E-01	1.897	2.894
Normal	B/H	1	0.1			0.999	-0.144	-2.850	-0.037	0	0	1	-0.012			

$$g(\underline{x}) = FS_{g_{s,BPNN}} - 1$$

(b)

Distribution	Variables	Para1	Para2	Para3	Para4	x_i^*	MARS PROCESS			Correlation matrix [R]			n_i	$g(\underline{x})$	β	$P_f(\%)$
Normal	Q	4	0.8			1.633	BF1=max(0, B - 10)	7.122	1	0	0	-3			3.51	0.022
Normal	B (m)	15	1.5			17.122	BF2=max(0, 10 - B)	0.000	0	1	0	1.4	1E-10			
Normal	B/H	1	0.1			0.874	BF3=max(0, 4 - Q)	2.367	0	0	1	-1.3				

$$g(\underline{x}) = FS_{g_{s,MARS}} - 1$$

(c)

Fig. 3.11 FORM incorporated with: (a) LR, (b) BPNN, and (c) MARS

3.5.3 Comparisons between probabilistic assessment methods

P_f values of eight cases are calculated using the six reliability assessment methods proposed in the previous section for comparison. The assumed statistical information of the input variables for the eight cases is listed in Table 3.10.

Fig. 3.12 shows a comparison of the P_f for Lognormal and Normal distributions for the three inputs using MCS_MARS and FORM_LR. It is obvious that the differences between Normal and Lognormal are significant. For all the cases, since the P_f values obtained from Normal distributions are larger than those obtained based on Lognormal distributions when the COV of Q is large (i.e., the normal

assumptions gives relatively more conservative results), the results presented subsequently in Fig. 3.11 are based on Normal distributions.

Table 3.10 Statistical information of the input variables for reliability assessment on ULS

Case No.	Variables	Distribution	Statistics
1	Q	Normal	Mean = 4, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 10, $COV=0.1$
	B/H	Normal	Mean = 0.5, $COV=0.1$
2	Q	Normal	Mean = 4, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 10, $COV=0.1$
	B/H	Normal	Mean = 1, $COV=0.1$
3	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 10, $COV=0.1$
	B/H	Normal	Mean = 0.5, $COV=0.1$
4	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 15, $COV=0.1$
	B/H	Normal	Mean = 1, $COV=0.1$
5	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 10, $COV=0.1$
	B/H	Normal	Mean = 0.25, $COV=0.1$
6	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 20, $COV=0.1$
	B/H	Normal	Mean = 1, $COV=0.1$
7	Q	Normal	Mean = 40, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 15, $COV=0.1$
	B/H	Normal	Mean = 0.5, $COV=0.1$
8	Q	Normal	Mean = 40, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 20, $COV=0.1$
	B/H	Normal	Mean = 1, $COV=0.1$

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig. 3.12 P_f derived based on Normal/Lognormal distributions: (a) case 1, (b) case 2, (c) case 3, (d) case 4, (e) case 5, (f) case 6, (g) case 7, and (h) case 8

The values of P_f derived using the six probabilistic methods for the eight cases are shown in Fig. 3.13 for comparison. The relationship between FS_{g-s} and P_f depends on the uncertainties and variability of the system under consideration. It is clear that P_f increases with the increase of COV of Q , demonstrating that a single FS_{g-s} value which is determined by a combination of mean values of Q , B and B/H may not always provide an adequate quantification of safety. It is obvious that for most of the cases considered, the differences between MCS and FORM for the same response surface (LR, BPNN or MARS) are marginal. Generally LR tends to give higher P_f than the other two methods. BPNN and MARS methods should be more accurate as they give higher R^2 values when comparing Figs. 3.5, 3.7 and 3.8. A comparison of case 4 and 6 shows that for the same Q and B/H statistics, increase of the cavern span B causes significant increase of P_f .

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig. 3.13 P_f derived for case: (a) case 1, (b) case 2, (c) case 3, (d) case 4, (e) case 5, (f) case 6, (g) case 7, and (h) case 8

3.6 Concluding remarks

Utilizing the rock mass classification correlations and the numerical software $FLAC^{3D}$, numerical analyses are carried out to determine the global stability of rock caverns of various geometries. The global factor of safety FS_{g_s} for stress-induced instability is determined using the shear strength reduction technique. Based on the numerical results, LR, BPNN and MARS models are built to relate FS_{g_s} to the cavern dimensions B and B/H , as well as the rock mass quality Q . Charts potentially useful for preliminary design and checking of single cavern stability are proposed.

Compared to the BPNN model, the built MARS model is much easier to interpret. In addition, the MARS is computationally more efficient and is almost as accurate as BPNN. These models can be incorporated into the probabilistic assessment methods MCS and FORM to perform reliability assessment of cavern stability.

In summary, this study of probabilistic assessment on ultimate limit state of a single cavern illustrated that:

- (1) Combined use of empirical and numerical methods for underground rock cavern stability analysis is reasonable.
- (2) The shear strength reduction technique can be used to derive the global factor of safety.
- (3) Q is the most important variable in determining FS_{g-s} . FS_{g-s} increases as the increase of B/H . For $B/H \geq 1.0$, the increase of FS_{g-s} is marginal.
- (4) LR model for predicting FS_{g-s} is simple. The design curves are potentially useful for preliminary design and checking of single cavern stability.
- (5) MARS model is almost as accurate as BPNN. However, MARS is much easier to interpret and computationally more efficient.
- (6) The built LR, BPNN, and MARS models can be incorporated into the probabilistic assessment methods MCS and FORM to calculate the probability of failure for cavern stability.
- (7) It is obvious that P_f increases with the increase of COV of Q , demonstrating that a single FS_{g-s} value which is deterministically determined may not always provide an adequate quantification of stability.

Chapter 4

RELIABILITY ASSESSMENT ON SERVICEABILITY LIMIT STATE AND SYSTEM RELIABILITY OF A SINGLE CAVERN

4.1 Introduction

The ultimate limit state failure for stress-induced instability was assessed in terms of global factor of safety in Chapter 3. However, the serviceability limit state, which denotes failure due to excessive movements, should also be considered. In defining the serviceability criterion, two approaches namely the maximum key point displacement and the percent strain are considered. For each approach, performance functions are built through LR, BPNN, and MARS models for comparison. Probabilistic assessments on serviceability limit state are performed using FORM incorporated with the three built models. To ensure the system reliability, the SLS and ULS should be evaluated concurrently, through satisfying both the minimum FS_{g-t} value and the target performance levels of serviceability limit state.

4.2 Previous studies on cavern deformations

Bhasin et al. (1996) carried out detailed investigation and performance monitoring studies at the site of an underground powerhouse cavern in the Himalayan region. They obtained the instrumentation data from multi-point borehole extensometers and updated the plot of the data measured during excavation stage for $Q/SPAN$ versus the measured deformation. They also concluded that the existing simplified relation provides a reasonable range of deformation values for a given rock mass quality and opening span.

Hao and Azzam (2005) carried out parametric studies and revealed that the fault parameters affect the stability of underground structure by the tendency of

increasing the plastic zones and displacements and causing both to be asymmetrically distributed. The relationship that the induced plastic zones and the maximum displacements vary with the fault parameters was established. The distribution of plastic zone and displacements was also graphically presented.

Meguid and Rowe (2006) investigated the behavior of the Darlington D-shaped tunnel constructed in rock mass subjected to high in situ horizontal stresses. The computed stresses and displacements were in good agreement with the field measurements taken at different locations across the tunnel. They proposed the use of a design line to limit the induced strains to within tolerable limits. The analyses suggested that strains between the limits bounded by the elastic and design lines would not cause instability.

Zhu et al. (2008) studied the side wall displacement of underground openings under different conditions and derived the best-fit formulae to predict the displacement of key points by using the dimensionless technique. The formulations can describe the effects of rock quality, overburden thickness, horizontal in situ stress, the height of the cavern and the spacing between openings. Prediction values based on these formulations were in close agreement with those obtained by field measurements and back analysis.

Basarir (2008) conducted extensive numerical simulations to investigate the stresses, displacements, and yielded zones around a circular opening excavated at different depths and under different rock conditions. Multiple regression models were built to assess the relationship between support pressure, depth, and tunnel deformation for different rock conditions.

Li and Low (2010) probabilistically investigated the stress-controlled instability of a circular tunnel excavated in a continuous, homogeneous, isotropic, initially elastic rock mass subjected to a hydrostatic far field stress and uniform support pressure. Unsatisfactory performance is defined as the situation when the plastic zone extent exceeds the permissible threshold ratio or the ratio of the inward displacement to tunnel radius is greater than the limiting threshold.

Cai (2011) proposed an approach for tunnel and cavern design utilizing quantitative risk assessment with the use of the GSI system to determine the required rock support and to quantify uncertainty in the analysis. The mechanical properties were derived using Monte Carlo method in @RISK and FEM stress analyses were performed applying PEM. He then applied the method to characterize the joint masses at the Kazunogawa powerhouse and obtained the yielding zone depth and displacement distributions. The probability that the anchorage portions of the anchors fall within the yielding zone or the possibility that the result displacements exceed the limiting threshold can then be estimated.

Lü et al. (2012a) performed probabilistic evaluation of ground-support interaction for deep rock excavation using artificial neural network and uniform design. They developed the deterministic numerical model using FLAC^{3D}, with which the performance functions are evaluated. The probabilistic assessment is conducted with respect to two performance functions. The serviceability limit state is based on the cavern displacement criterion and becomes unsatisfactory when the computed maximum horizontal displacement exceeds a threshold of 150 mm, considering that the cross-section is large and the in situ stresses are high, and the corresponding convergence limit is about 1.3%, within a reasonable range of 1.0–5.0 %.

4.3 Numerical modeling and displacement

The basic assumptions, cross-section and boundary conditions are the same as those used for the ultimate limit state calculations in Chapter 3. Apart from taking into account the parameter K_0 , which was found to significantly influence the deformations, the cases with FS_{g-s} less than unity in Table 3.4 were omitted since FS_{g-s} less than unity denotes collapse and consequently the nodal displacements are large.

The numerical study focused on determining the maximum key point displacement and the percent strain. The key points including crown C, springline S, middle sidewall M, and invert I are illustrated in Fig. 4.1. Also displayed are the more peripheral nodes around the opening, which are used to derive the percent strain.

Fig. 4.1 Details of cavern periphery

The maximum key point displacement of a single cavern $u_{\max_s}$ is defined as $\max(u_C, u_S, u_M, u_I)$, in which u_C , u_S , u_M , and u_I are the displacements of peripheral nodes C, S, M and I, respectively. The percent strain ε_s is the maximum value among the values of average strain ε_{Xx} of peripheral elements around the cavern. Strain ε_{Xx} is calculated as follows (Sakurai, 1997):

$$\varepsilon_{Xx}(\%) = \frac{u_X - u_x}{l_{Xx}} \times 100 \quad (4.1)$$

and the percent strain is:

$$\varepsilon_s = \max(\varepsilon_{Xx}) \quad (4.2)$$

where X is C, S, M or I and x is c, s, m or i; u_x is the displacement of the corresponding inner node x and l_{Xx} is the length between nodes X and x (Fig. 4.1). It should be noted that the selected length of l_{Xx} is typical of many extensometers and the locations of inner nodes such as x are chosen at the grid of the third layer elements around the opening. Numerical trials carried out on different locations of inner nodes and different mesh densities indicate that the choice of the reference length and the density of the mesh at the opening have minimal influence on the magnitude of the maximum strain. For brevity, these results have been omitted.

4.4 Modeling results and analyses of $u_{\max_s}$

Based on the parameter combinations, a total of 219 cases are analyzed using FDM. The results are presented in the following sections.

4.4.1 Modeling results of $u_{\max_s}$

$u_{\max_s}$ is influenced by Q , B , B/H and K_0 , as indicated in Table 4.1.

Table 4.1 Summary of results for $u_{\max_s_FDM}$

Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)	Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)
1	5	0.25	2	0.208	4	15	1	1	0.046
4	5	0.25	2	0.063	4	20	1	1	0.064
4	10	0.25	2	0.167	4	25	1	1	0.086
10	5	0.25	2	0.025	10	5	1	1	0.005
10	10	0.25	2	0.065	10	10	1	1	0.011
10	15	0.25	2	0.125	10	15	1	1	0.017
40	5	0.25	2	0.011	10	20	1	1	0.024
40	10	0.25	2	0.026	10	25	1	1	0.032
40	15	0.25	2	0.046	10	30	1	1	0.040
40	20	0.25	2	0.087	40	5	1	1	0.002
1	5	0.25	1	0.163	40	10	1	1	0.004
4	5	0.25	1	0.045	40	15	1	1	0.006
4	10	0.25	1	0.131	40	20	1	1	0.009
10	5	0.25	1	0.018	40	25	1	1	0.011
10	10	0.25	1	0.048	40	30	1	1	0.014
10	15	0.25	1	0.098	0.4	5	1	0.5	0.130
40	5	0.25	1	0.007	0.4	10	1	0.5	0.408
40	10	0.25	1	0.018	1	5	1	0.5	0.033
40	15	0.25	1	0.033	1	10	1	0.5	0.083
40	20	0.25	1	0.062	1	15	1	0.5	0.144
1	5	0.25	0.5	0.137	4	5	1	0.5	0.009
4	5	0.25	0.5	0.035	4	10	1	0.5	0.022
4	10	0.25	0.5	0.106	4	15	1	0.5	0.036
10	5	0.25	0.5	0.014	4	20	1	0.5	0.051
10	10	0.25	0.5	0.038	4	25	1	0.5	0.069
10	15	0.25	0.5	0.085	10	5	1	0.5	0.003
40	5	0.25	0.5	0.006	10	10	1	0.5	0.009
40	10	0.25	0.5	0.015	10	15	1	0.5	0.014
40	15	0.25	0.5	0.028	10	20	1	0.5	0.019
40	20	0.25	0.5	0.058	10	25	1	0.5	0.025
0.4	5	0.5	2	0.402	10	30	1	0.5	0.032
1	5	0.5	2	0.117	40	5	1	0.5	0.001
1	10	0.5	2	0.237	40	10	1	0.5	0.003
4	5	0.5	2	0.034	40	15	1	0.5	0.005
4	10	0.5	2	0.074	40	20	1	0.5	0.007
4	15	0.5	2	0.123	40	25	1	0.5	0.009

Table 4.1 Summary of results for $u_{\max_s_FDM}$ contd.

Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)	Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)
10	5	0.5	2	0.012	40	30	1	0.5	0.012
10	10	0.5	2	0.030	0.4	5	2	2	0.208
10	15	0.5	2	0.049	0.4	10	2	2	0.411
10	20	0.5	2	0.076	1	5	2	2	0.066
10	25	0.5	2	0.113	1	10	2	2	0.115
40	5	0.5	2	0.006	1	15	2	2	0.172
40	10	0.5	2	0.012	1	20	2	2	0.268
40	15	0.5	2	0.020	4	5	2	2	0.020
40	20	0.5	2	0.029	4	10	2	2	0.038
40	25	0.5	2	0.039	4	15	2	2	0.053
40	30	0.5	2	0.052	4	20	2	2	0.065
0.4	5	0.5	1	0.293	4	25	2	2	0.079
1	5	0.5	1	0.086	4	30	2	2	0.097
1	10	0.5	1	0.197	10	5	2	2	0.005
4	5	0.5	1	0.023	10	10	2	2	0.013
4	10	0.5	1	0.055	10	15	2	2	0.020
4	15	0.5	1	0.093	10	20	2	2	0.026
10	5	0.5	1	0.008	10	25	2	2	0.031
10	10	0.5	1	0.021	10	30	2	2	0.036
10	15	0.5	1	0.036	40	5	2	2	0.002
10	20	0.5	1	0.058	40	10	2	2	0.005
10	25	0.5	1	0.088	40	15	2	2	0.007
40	5	0.5	1	0.003	40	20	2	2	0.010
40	10	0.5	1	0.008	40	25	2	2	0.012
40	15	0.5	1	0.014	40	30	2	2	0.014
40	20	0.5	1	0.020	0.4	5	2	1	0.126
40	25	0.5	1	0.039	0.4	10	2	1	0.289
40	30	0.5	1	0.041	1	5	2	1	0.043
0.4	5	0.5	0.5	0.244	1	10	2	1	0.071
1	5	0.5	0.5	0.066	1	15	2	1	0.118
1	10	0.5	0.5	0.172	1	20	2	1	0.194
4	5	0.5	0.5	0.016	4	5	2	1	0.012
4	10	0.5	0.5	0.043	4	10	2	1	0.023
4	15	0.5	0.5	0.075	4	15	2	1	0.033
10	5	0.5	0.5	0.007	4	20	2	1	0.042
10	10	0.5	0.5	0.017	4	25	2	1	0.053
10	15	0.5	0.5	0.029	4	30	2	1	0.067
10	20	0.5	0.5	0.047	10	5	2	1	0.003
10	25	0.5	0.5	0.075	10	10	2	1	0.007
40	5	0.5	0.5	0.003	10	15	2	1	0.012
40	10	0.5	0.5	0.006	10	20	2	1	0.016
40	15	0.5	0.5	0.011	10	25	2	1	0.019
40	20	0.5	0.5	0.016	10	30	2	1	0.023
40	25	0.5	0.5	0.024	40	5	2	1	0.001
40	30	0.5	0.5	0.035	40	10	2	1	0.003
0.4	5	1	2	0.252	40	15	2	1	0.004
0.4	10	1	2	0.553	40	20	2	1	0.006
1	5	1	2	0.078	40	25	2	1	0.007
1	10	1	2	0.140	40	30	2	1	0.009

Table 4.1 Summary of results for $u_{\max_s_FDM}$ contd.

Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)	Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)
1	15	1	2	0.220	0.4	5	2	0.5	0.126
4	5	1	2	0.024	0.4	10	2	0.5	0.289
4	10	1	2	0.048	1	5	2	0.5	0.043
4	15	1	2	0.067	1	10	2	0.5	0.071
4	20	1	2	0.085	1	15	2	0.5	0.118
4	25	1	2	0.105	1	20	2	0.5	0.194
10	5	1	2	0.008	4	5	2	0.5	0.012
10	10	1	2	0.017	4	10	2	0.5	0.023
10	15	1	2	0.025	4	15	2	0.5	0.033
10	20	1	2	0.033	4	20	2	0.5	0.042
10	25	1	2	0.042	4	25	2	0.5	0.053
10	30	1	2	0.050	4	30	2	0.5	0.067
40	5	1	2	0.003	10	5	2	0.5	0.003
40	10	1	2	0.007	10	10	2	0.5	0.007
40	15	1	2	0.011	10	15	2	0.5	0.012
40	20	1	2	0.014	10	20	2	0.5	0.016
40	25	1	2	0.017	10	25	2	0.5	0.019
40	30	1	2	0.021	10	30	2	0.5	0.023
0.4	5	1	1	0.159	40	5	2	0.5	0.001
0.4	10	1	1	0.421	40	10	2	0.5	0.003
1	5	1	1	0.050	40	15	2	0.5	0.004
1	10	1	1	0.102	40	20	2	0.5	0.006
1	15	1	1	0.166	40	25	2	0.5	0.007
4	5	1	1	0.015	40	30	2	0.5	0.009
4	10	1	1	0.029					

Influences of B/H , Q and K_0 are shown in Fig. 4.2. It is obvious that for the same B/H , B and K_0 , $u_{\max_s}$ decreases with the increase of Q . In addition, $u_{\max_s}$ increases with the increase of K_0 , due to higher horizontal stress release during excavation.

(a)

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(b)

(c)

(d)

(e)

(f)

Fig. 4.2 Influence of B/H , Q and K_0 on $u_{\max_s}$: (a) $B=5$ m, (b) $B=10$ m, (c) $B=15$ m, (d) $B=20$ m, (e) $B=25$ m, and (f) $B=30$ m

4.4.2 Determination of $u_{\max_s}$ using LR

Based on the above results, a LR model was developed for predicting $u_{\max_s}$ in terms of Q , B , B/H and K_0 , as shown in Eq. (4.3)

$$u_{\max_s_LR}(m) = 0.01Q^{-0.9081}B^{1.1288}(B/H)^{-0.7939}K_0^{0.3908} \quad (4.3)$$

A comparison of $u_{\max_s_LR}$ and $u_{\max_s_FDM}$ is shown in Fig. 4.3. The high R^2 of 0.867 indicates that the predictions are in good agreement with the $u_{\max_s_FDM}$ values.

Fig. 4.3 Comparison between $u_{\max_s_FDM}$ and $u_{\max_s_LR}$

4.4.3 Determination of $u_{\max_s}$ using BPNN

Using the data from Table 4.1, a BPNN model was also developed to predict $u_{\max_s}$. Four inputs that represented Q , B , B/H and K_0 are the neurons in the input layer. Of the 219 observations, 156 data samples were randomly selected as the training data and the remaining 63 data sets were used for testing the validity of the developed neural network. The optimal BPNN model consisted of eight hidden neurons. The testing data sets together with the BPNN predictions are listed in Table 4.2. The training data sets together with the BPNN predictions are shown in Appendix F.

A plot of $u_{\max_s_FDM}$ versus $u_{\max_s_BP}$ is shown in Fig. 4.4. The high R^2 indicate that BPNN predictions are reasonably accurate and show less scatter than the LR model.

Table 4.2 Testing data sets of $u_{\max_s_BP}$

Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)	$u_{\max_s_BP}$ (m)	Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)	$u_{\max_s_BP}$ (m)
0.4	5	0.5	1	0.293	0.273	10	10	1	2	0.017	0.014
0.4	10	1	1	0.421	0.413	10	15	1	2	0.025	0.025
0.4	5	1	0.5	0.130	0.145	10	25	1	1	0.032	0.034
0.4	5	2	1	0.126	0.116	10	10	1	0.5	0.009	0.007
1	5	0.25	1	0.163	0.162	10	25	1	0.5	0.025	0.029
1	5	0.5	1	0.086	0.089	10	10	2	2	0.013	0.011
1	15	2	0.5	0.118	0.112	10	15	2	2	0.020	0.016
1	10	2	1	0.071	0.070	10	10	2	1	0.007	0.008
1	10	2	2	0.115	0.105	10	15	2	1	0.012	0.010
1	10	1	0.5	0.083	0.083	10	10	2	0.5	0.007	0.007
1	10	1	1	0.102	0.091	10	25	2	0.5	0.019	0.013
1	10	1	2	0.140	0.126	40	15	0.25	2	0.046	0.045
1	5	0.5	2	0.117	0.133	40	10	0.25	1	0.018	0.015
4	10	0.25	2	0.167	0.171	40	15	0.25	0.5	0.028	0.027
4	10	0.25	1	0.131	0.128	40	10	0.5	1	0.008	0.006
4	10	0.5	2	0.074	0.063	40	30	0.5	1	0.041	0.052
4	15	0.5	1	0.093	0.087	40	10	0.5	0.5	0.006	0.005
4	10	1	2	0.048	0.045	40	20	0.5	0.5	0.016	0.019
4	10	1	1	0.029	0.029	40	15	0.5	2	0.020	0.022
4	10	1	0.5	0.022	0.024	40	25	0.5	2	0.039	0.038
4	10	2	2	0.038	0.040	40	15	1	2	0.011	0.009
4	30	2	2	0.097	0.099	40	25	1	2	0.017	0.022
4	10	2	1	0.023	0.027	40	10	1	1	0.004	0.004
4	25	2	1	0.053	0.059	40	30	1	1	0.014	0.025
4	10	2	0.5	0.023	0.022	40	10	2	2	0.005	0.006
4	30	2	0.5	0.067	0.063	40	25	2	2	0.012	0.007
10	10	0.25	2	0.065	0.071	40	10	2	1	0.003	0.004
10	15	0.25	1	0.098	0.098	40	15	2	1	0.004	0.004
10	20	0.5	2	0.076	0.067	40	15	1	0.5	0.005	0.004
10	15	0.25	0.5	0.085	0.086	40	30	1	0.5	0.012	0.019
10	10	0.5	1	0.021	0.019	40	25	2	0.5	0.007	0.004
10	20	0.5	0.5	0.047	0.043

Fig. 4.4 Comparison between $u_{\max_s_FDM}$ and $u_{\max_s_BP}$

4.4.4 Determination of $u_{\max_s}$ using MARS

Using the same training and testing datasets, this problem is also reanalyzed by means of MARS. The MARS model adopted 21 BF's of linear spline functions with second-order interaction. MARS predictions of the testing data sets and the relative error e in percentage are listed in Table 4.3. Comparing Figs. 4.4 and 4.5, the MARS training results as well as the relative error e

$$(e(\%)) = \text{abs}\left(\frac{u_{\max_s_MARS} - u_{\max_s_FDM}}{u_{\max_s_FDM}}\right) \times 100, \text{ abs}() \text{ is absolute value function) are}$$

shown in Appendix G.

Table 4.3 Testing data sets of $u_{\max_s_MARS}$

Q	B (m)	B/H	K_0	$u_{\max_s_MARS}$ (m)	e (%)	Q	B (m)	B/H	K_0	$u_{\max_s_MARS}$ (m)	e (%)
0.4	5	0.5	1	0.253	13.9	10	10	1	2	0.018	8.9
0.4	10	1	1	0.402	4.5	10	15	1	2	0.029	12.9
0.4	5	1	0.5	0.171	31.1	10	25	1	1	0.036	15.2
0.4	5	2	1	0.089	29.4	10	10	1	0.5	0.004	53.3
1	5	0.25	1	0.151	7.4	10	25	1	0.5	0.036	44.3
1	5	0.5	1	0.094	9.8	10	10	2	2	0.004	69.8
1	15	2	0.5	0.139	17.9	10	15	2	2	0.015	25.3
1	10	2	1	0.082	14.3	10	10	2	1	-0.010	234.0
1	10	2	2	0.136	17.7	10	15	2	1	0.001	93.1
1	10	1	0.5	0.096	15.7	10	10	2	0.5	-0.010	234.0
1	10	1	1	0.096	5.8	10	25	2	0.5	0.022	16.6
1	10	1	2	0.150	7.1	40	15	0.25	2	0.053	15.7
1	5	0.5	2	0.148	27.4	40	10	0.25	1	0.020	10.8
4	10	0.25	2	0.105	37.3	40	15	0.25	0.5	0.039	40.3
4	10	0.25	1	0.091	30.9	40	10	0.5	1	0.000	101.2
4	10	0.5	2	0.073	1.9	40	30	0.5	1	0.040	0.4
4	15	0.5	1	0.082	12.1	40	10	0.5	0.5	0.000	101.6
4	10	1	2	0.052	9.8	40	20	0.5	0.5	0.027	63.2
4	10	1	1	0.038	30.5	40	15	0.5	2	0.027	33.5
4	10	1	0.5	0.038	73.7	40	25	0.5	2	0.054	38.8
4	10	2	2	0.038	0.1	40	15	1	2	0.021	92.9
4	30	2	2	0.075	22.8	40	25	1	2	0.026	52.2
4	10	2	1	0.024	7.0	40	10	1	1	0.004	2.2
4	25	2	1	0.061	15.4	40	30	1	1	0.012	12.7
4	10	2	0.5	0.024	7.0	40	10	2	2	0.004	20.8
4	30	2	0.5	0.061	9.0	40	25	2	2	0.012	1.4
10	10	0.25	2	0.064	1.3	40	10	2	1	-0.010	454.6
10	15	0.25	1	0.077	21.7	40	15	2	1	-0.007	263.3
10	20	0.5	2	0.077	0.8	40	15	1	0.5	0.007	33.1
10	15	0.25	0.5	0.077	9.2	40	30	1	0.5	0.012	4.4
10	10	0.5	1	0.020	5.0	40	25	2	0.5	-0.002	121.5
10	20	0.5	0.5	0.063	34.1						

A plot of $u_{\max_s_MARS}$ versus $u_{\max_s_FDM}$ is shown in Fig. 4.5. It can be seen from the R^2 that the MARS predictions are only slightly less accurate than the BPNN results.

Fig. 4.5 Comparison between $u_{\max_s_FDM}$ and $u_{\max_s_MARS}$

4.4.5 Comparisons between BPNN and MARS

With regard to the predictive ability, the BPNN model is only slightly more accurate than the MARS model. However, the execution time of 5.469 s for the BPNN algorithm, compared with 0.554 s for the MARS model, indicates that MARS is computationally more efficient than BPNN.

For the built BPNN model of $u_{\max_s}$ prediction, the procedure of partitioning of weights is described in Appendix H. The parameter relative importance comparison indicates that Q is the most important variable in determining $u_{\max_s}$, followed by B and K_0 . The mathematical expression of this model is shown in Appendix I.

The MARS model is much easier to interpret. Table 4.4 displays the ANOVA decomposition of the developed MARS model. It can be observed that Q is the most significant parameter in determining $u_{\max_s}$, followed by B and K_0 .

Table 4.4 ANOVA decomposition of the developed MARS model for $u_{\max_s}$

Functions	GCV	STD	#basis	variable(s)
1	0.103	0.221	4	Q
2	0.004	0.040	1	B
3	0.003	0.020	3	B/H
4	0.004	0.037	1	K_0
5	0.046	0.154	5	Q, B
6	0.002	0.025	3	$Q, B/H$
7	0.001	0.021	2	Q, K_0
8	0.002	0.023	2	$B, B/H$

Table 4.5 lists the BFs of the MARS model and their corresponding equations. The interpretable MARS model to predict $u_{\max_s}$ is given by

$$\begin{aligned}
 u_{\max_s_MARS} = & 0.05 - 0.001 \times BF1 + 1.25 \times BF2 + 0.013 \times BF3 + 0.003 \times BF4 \\
 & - 0.002 \times BF5 + 0.013 \times BF6 - 0.003 \times BF7 + 0.1 \times BF8 + 5 \times 10^{-5} \times BF9 \\
 & - 0.05 \times BF10 + 0.006 \times BF11 - 0.2 \times BF12 - 0.004 \times BF13 + 0.1 \times BF14 \\
 & - 0.001 \times BF15 + 0.04 \times BF16 + 0.014 \times BF17 - 0.014 \times BF18 + 0.1 \times BF19 \\
 & - 0.0006 \times BF20 + 0.004 \times BF21
 \end{aligned} \tag{4.4}$$

Table 4.5 Basis functions and corresponding equations for $u_{\max_s}$

BFs	Equation	BFs	Equation
BF1	$\max(0, Q - 1)$	BF12	$BF2 \times \max(0, B/H - 1)$
BF2	$\max(0, 1 - Q)$	BF13	$BF7 \times \max(0, 1 - B/H)$
BF3	$\max(0, 4 - Q)$	BF14	$BF2 \times \max(0, K_0 - 1)$
BF4	$BF3 \times \max(0, B - 10)$	BF15	$BF8 \times \max(0, Q - 4)$
BF5	$BF3 \times \max(0, 10 - B)$	BF16	$BF8 \times \max(0, 4 - Q)$
BF6	$BF3 \times \max(0, K_0 - 1)$	BF17	$\max(0, K_0 - 1)$
BF7	$\max(0, 25 - B)$	BF18	$\max(0, B/H - 0.5)$
BF8	$\max(0, 1 - B/H)$	BF19	$\max(0, 0.5 - B/H)$
BF9	$BF7 \times \max(0, Q - 1)$	BF20	$BF11 \times \max(0, 10 - B)$
BF10	$BF7 \times \max(0, 1 - Q)$	BF21	$\max(0, B/H - 1) \times \max(0, 10 - B)$
BF11	$\max(0, 10 - Q)$		

This $u_{\max_s}$ equation can be used to give preliminary estimates of the expected maximum cavern displacements during construction of a single cavern.

Fig. 4.6a-4.6d plots the knot locations of Q , B , B/H , and K_0 respectively.

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(a)

(b)

(c)

Fig. 4.6 Knot locations of: (a) Q , (b) B , (c) B/H , and (d) K_0

4.5 Modeling results and analyses of the percent strain ε_s

The displacements of the key points together with their corresponding inner nodes illustrated in Fig. 4.1 were extracted for each of the 219 observations previously described. Based on Eqs. (4.1) and (4.2), percent strain values are calculated.

4.5.1 Modeling results of ε_s

The percent strain value of a single cavern ε_s is influenced by Q , B , B/H and K_0 , as indicated in Table 4.6.

Table 4.6 Summary of results for ε_s

Q	B (m)	B/H	K_0	ε_s (%)	Q	B (m)	B/H	K_0	ε_s (%)
1	5	0.25	2	2.800	4	15	1	1	0.541
4	5	0.25	2	1.260	4	20	1	1	0.652
4	10	0.25	2	0.988	4	25	1	1	0.397
10	5	0.25	2	0.581	10	5	1	1	0.110
10	10	0.25	2	0.544	10	10	1	1	0.167
10	15	0.25	2	0.409	10	15	1	1	0.219
40	5	0.25	2	0.191	10	20	1	1	0.260
40	10	0.25	2	0.256	10	25	1	1	0.155
40	15	0.25	2	0.209	10	30	1	1	0.180
40	20	0.25	2	0.256	40	5	1	1	0.046
1	5	0.25	1	1.954	40	10	1	1	0.064
4	5	0.25	1	0.702	40	15	1	1	0.085
4	10	0.25	1	0.743	40	20	1	1	0.101
10	5	0.25	1	0.295	40	25	1	1	0.062

Table 4.6 Summary of results for ε_s contd.

Q	B (m)	B/H	K_0	ε_s (%)	Q	B (m)	B/H	K_0	ε_s (%)
10	10	0.25	1	0.358	40	30	1	1	0.071
10	15	0.25	1	0.316	0.4	5	1	0.5	2.335
40	5	0.25	1	0.100	0.4	10	1	0.5	4.795
40	10	0.25	1	0.143	1	5	1	0.5	0.555
40	15	0.25	1	0.141	1	10	1	0.5	0.999
40	20	0.25	1	0.194	1	15	1	0.5	1.435
1	5	0.25	0.5	1.610	4	5	1	0.5	0.178
4	5	0.25	0.5	0.535	4	10	1	0.5	0.298
4	10	0.25	0.5	0.552	4	15	1	0.5	0.397
10	5	0.25	0.5	0.221	4	20	1	0.5	0.494
10	10	0.25	0.5	0.271	4	25	1	0.5	0.310
10	15	0.25	0.5	0.253	10	5	1	0.5	0.088
40	5	0.25	0.5	0.072	10	10	1	0.5	0.127
40	10	0.25	0.5	0.114	10	15	1	0.5	0.160
40	15	0.25	0.5	0.117	10	20	1	0.5	0.189
40	20	0.25	0.5	0.169	10	25	1	0.5	0.116
0.4	5	0.5	2	4.970	10	30	1	0.5	0.137
1	5	0.5	2	1.680	40	5	1	0.5	0.040
1	10	0.5	2	1.935	40	10	1	0.5	0.051
4	5	0.5	2	0.666	40	15	1	0.5	0.061
4	10	0.5	2	0.882	40	20	1	0.5	0.077
4	15	0.5	2	0.851	40	25	1	0.5	0.050
10	5	0.5	2	0.319	40	30	1	0.5	0.053
10	10	0.5	2	0.495	0.4	5	2	2	3.737
10	15	0.5	2	0.390	0.4	10	2	2	5.490
10	20	0.5	2	0.379	1	5	2	2	0.984
10	25	0.5	2	0.570	1	10	2	2	1.590
40	5	0.5	2	0.133	1	15	2	2	1.900
40	10	0.5	2	0.253	1	20	2	2	2.330
40	15	0.5	2	0.201	4	5	2	2	0.377
40	20	0.5	2	0.157	4	10	2	2	0.617
40	25	0.5	2	0.227	4	15	2	2	0.732
40	30	0.5	2	0.215	4	20	2	2	0.830
0.4	5	0.5	1	3.485	4	25	2	2	0.474
1	5	0.5	1	0.958	4	30	2	2	0.528
1	10	0.5	1	1.515	10	5	2	2	0.187
4	5	0.5	1	0.339	10	10	2	2	0.295
4	10	0.5	1	0.556	10	15	2	2	0.371
4	15	0.5	1	0.544	10	20	2	2	0.426
10	5	0.5	1	0.139	10	25	2	2	0.242
10	10	0.5	1	0.269	10	30	2	2	0.267
10	15	0.5	1	0.236	40	5	2	2	0.079
10	20	0.5	1	0.227	40	10	2	2	0.127
10	25	0.5	1	0.339	40	15	2	2	0.165
40	5	0.5	1	0.054	40	20	2	2	0.195
40	10	0.5	1	0.109	40	25	2	2	0.115
40	15	0.5	1	0.110	40	30	2	2	0.128
40	20	0.5	1	0.099	0.4	5	2	1	2.253
40	25	0.5	1	0.227	0.4	10	2	1	3.810
40	30	0.5	1	0.120	1	5	2	1	0.587

Table 4.6 Summary of results for ε_s contd.

Q	B (m)	B/H	K_0	ε_s (%)	Q	B (m)	B/H	K_0	ε_s (%)
0.4	5	0.5	0.5	2.840	1	10	2	1	1.019
1	5	0.5	0.5	0.758	1	15	2	1	1.284
1	10	0.5	0.5	1.255	1	20	2	1	1.720
4	5	0.5	0.5	0.252	4	5	2	1	0.187
4	10	0.5	0.5	0.412	4	10	2	1	0.316
4	15	0.5	0.5	0.401	4	15	2	1	0.417
10	5	0.5	0.5	0.103	4	20	2	1	0.503
10	10	0.5	0.5	0.200	4	25	2	1	0.295
10	15	0.5	0.5	0.184	4	30	2	1	0.359
10	20	0.5	0.5	0.174	10	5	2	1	0.088
10	25	0.5	0.5	0.254	10	10	2	1	0.134
40	5	0.5	0.5	0.038	10	15	2	1	0.170
40	10	0.5	0.5	0.079	10	20	2	1	0.206
40	15	0.5	0.5	0.083	10	25	2	1	0.121
40	20	0.5	0.5	0.080	10	30	2	1	0.138
40	25	0.5	0.5	0.101	40	5	2	1	0.039
40	30	0.5	0.5	0.095	40	10	2	1	0.057
0.4	5	1	2	4.590	40	15	2	1	0.072
0.4	10	1	2	6.810	40	20	2	1	0.086
1	5	1	2	1.207	40	25	2	1	0.052
1	10	1	2	1.894	40	30	2	1	0.059
1	15	1	2	2.295	0.4	5	2	0.5	2.253
4	5	1	2	0.477	0.4	10	2	0.5	3.810
4	10	1	2	0.732	1	5	2	0.5	0.587
4	15	1	2	0.908	1	10	2	0.5	1.019
4	20	1	2	1.017	1	15	2	0.5	1.284
4	25	1	2	0.566	1	20	2	0.5	1.720
10	5	1	2	0.231	4	5	2	0.5	0.187
10	10	1	2	0.354	4	10	2	0.5	0.316
10	15	1	2	0.443	4	15	2	0.5	0.417
10	20	1	2	0.502	4	20	2	0.5	0.503
10	25	1	2	0.282	4	25	2	0.5	0.295
10	30	1	2	0.301	4	30	2	0.5	0.359
40	5	1	2	0.098	10	5	2	0.5	0.088
40	10	1	2	0.149	10	10	2	0.5	0.134
40	15	1	2	0.192	10	15	2	0.5	0.170
40	20	1	2	0.223	10	20	2	0.5	0.206
40	25	1	2	0.129	10	25	2	0.5	0.121
40	30	1	2	0.142	10	30	2	0.5	0.138
0.4	5	1	1	2.973	40	5	2	0.5	0.039
0.4	10	1	1	5.145	40	10	2	0.5	0.057
1	5	1	1	0.701	40	15	2	0.5	0.072
1	10	1	1	1.274	40	20	2	0.5	0.086
1	15	1	1	1.700	40	25	2	0.5	0.052
4	5	1	1	0.238	40	30	2	0.5	0.059
4	10	1	1	0.406					

Note: for all the numerical cases considered in this thesis, the strain direction is inward, that is, displacing into the opening.

Influences of B/H , Q and K_0 on ε_s are shown in Fig. 4.7. It is obvious that for the same B/H , B and K_0 , ε_s decreases as Q increases. It is logical that as Q increases (and there is a corresponding increase in the deformation modulus), the percent strain reduces. It can also be observed that for the same Q , B , and K_0 , ε_s increases as B/H decreases. As the height of the side wall increases, the cavern is less stable and larger deformations occur at the middle side wall. In addition, the influence of K_0 on ε_s is significant as the high in situ stress field results in significantly larger ε_s values.

(a)

(b)

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(c)

(d)

(e)

Fig. 4.7 Influence of B/H , Q and K_0 on ε_s : (a) $B=5$ m, (b) $B=10$ m, (c) $B=15$ m, (d) $B=20$ m, (e) $B=25$ m, and (f) $B=30$ m

4.5.2 Determination of ε_s using LR

Based on the above results, the LR model was developed for predicting ε_s in terms of Q , B , B/H and K_0 , as shown in Eq. (4.5)

$$\varepsilon_{s_LR}(\%) = 0.749Q^{-0.8088}B^{0.2842}(B/H)^{-0.3476}K_0^{0.5421} \quad (4.5)$$

A plot of ε_{s_LR} versus ε_{s_FDM} shown in Fig. 4.8 indicates that the predictions are generally in agreement with the target ε_s , particularly for ε_s less than 3%.

Fig. 4.8 Comparison between ε_{s_FDM} and ε_{s_LR}

4.5.3 Determination of ε_s using BPNN

The BPNN model using the observations in Table 4.6 was developed to predict ε_s . As with the BPNN model for $u_{\max_s}$, there are four inputs that represented Q , B , B/H and K_0 in the input layer. The datasets for training and testing are also the same as those used for the $u_{\max_s}$ predictions. The optimal BPNN model consisted of six hidden neurons. The testing data sets together with the BPNN predictions ε_{s_BP} are listed in Table 4.7. The training data sets together with the BPNN predictions are shown in Appendix J.

Table 4.7 Testing data sets for ε_{s_BP}

Q	B (m)	B/H	K_0	ε_{s_FDM} (%)	ε_{s_BP} (%)	Q	B (m)	B/H	K_0	ε_{s_FDM} (%)	ε_{s_BP} (%)
0.4	5	0.5	1	3.485	3.516	10	10	1	2	0.354	0.388
0.4	10	1	1	5.145	5.248	10	15	1	2	0.443	0.425
0.4	5	1	0.5	2.335	2.798	10	25	1	1	0.155	0.195
0.4	5	2	1	2.253	2.549	10	10	1	0.5	0.127	0.199
1	5	0.25	1	1.954	1.948	10	25	1	0.5	0.116	0.172
1	5	0.5	1	0.958	1.029	10	10	2	2	0.295	0.378
1	15	2	0.5	1.284	1.215	10	15	2	2	0.371	0.403
1	10	2	1	1.019	1.050	10	10	2	1	0.134	0.216
1	10	2	2	1.590	1.525	10	15	2	1	0.170	0.229
1	10	1	0.5	0.999	1.043	10	10	2	0.5	0.134	0.193
1	10	1	1	1.274	1.206	10	25	2	0.5	0.121	0.159
1	10	1	2	1.894	1.747	40	15	0.25	2	0.209	0.167
1	5	0.5	2	1.680	1.665	40	10	0.25	1	0.143	0.166
4	10	0.25	2	0.988	1.200	40	15	0.25	0.5	0.117	0.162
4	10	0.25	1	0.743	0.713	40	10	0.5	1	0.109	0.166
4	10	0.5	2	0.882	0.830	40	30	0.5	1	0.120	0.165
4	15	0.5	1	0.544	0.691	40	10	0.5	0.5	0.079	0.062
4	10	1	2	0.732	0.718	40	20	0.5	0.5	0.080	0.061
4	10	1	1	0.406	0.449	40	15	0.5	2	0.201	0.166
4	10	1	0.5	0.298	0.388	40	25	0.5	2	0.227	0.166
4	10	2	2	0.617	0.691	40	15	1	2	0.149	0.167
4	30	2	2	0.528	0.525	40	25	1	2	0.129	0.166
4	10	2	1	0.316	0.434	40	10	1	1	0.064	0.062
4	25	2	1	0.295	0.387	40	30	1	1	0.071	0.061
4	10	2	0.5	0.316	0.377	40	10	2	2	0.127	0.098
4	30	2	0.5	0.359	0.268	40	25	2	2	0.115	0.165
10	10	0.25	2	0.544	0.523	40	10	2	1	0.057	0.062
10	15	0.25	1	0.316	0.268	40	15	2	1	0.072	0.062
10	20	0.5	2	0.379	0.422	40	15	1	0.5	0.061	0.062
10	15	0.25	0.5	0.253	0.206	40	30	1	0.5	0.053	0.061
10	10	0.5	1	0.269	0.343	40	25	2	0.5	0.052	0.061
10	20	0.5	0.5	0.174	0.211						

A plot of ε_{s_BP} versus the actual ε_s values shown in Fig. 4.9 indicates that the BPNN predictions are more accurate than the LR model.

Fig. 4.9 Comparison between ε_{s_FDM} and ε_{s_BP}

4.5.4 Determination of ε_s using MARS

Using the same training and testing datasets, this problem is reanalyzed by means of MARS. The MARS model adopted 22 BF's of linear spline functions with second-order interaction. A plot of ε_{s_MARS} versus the actual ε_s values is shown in Fig. 4.10. The high R^2 indicate that the MARS predictions are almost as accurate as the BPNN results.

Fig. 4.10 Comparison between ε_{s_FDM} and ε_{s_MARS}

MARS predictions of the testing data sets and the relative error e in percentage are listed in Table 4.8. The MARS training results together with the relative error e are shown in Appendix K.

Table 4.8 Testing data sets for ε_{s_MARS}

Q	B (m)	B/H	K_0	ε_{s_MARS} (%)	e (%)	Q	B (m)	B/H	K_0	ε_{s_MARS} (%)	e (%)
0.4	5	0.5	1	3.219	7.6	10	10	1	2	0.368	4.0
0.4	10	1	1	4.911	4.5	10	15	1	2	0.391	11.6
0.4	5	1	0.5	2.819	20.7	10	25	1	1	0.195	26.4
0.4	5	2	1	2.045	9.2	10	10	1	0.5	0.149	17.0
1	5	0.25	1	1.981	1.4	10	25	1	0.5	0.195	68.4
1	5	0.5	1	0.946	1.2	10	10	2	2	0.276	6.5
1	15	2	0.5	1.352	5.4	10	15	2	2	0.299	19.3
1	10	2	1	0.979	3.9	10	10	2	1	0.056	58.0
1	10	2	2	1.519	4.5	10	15	2	1	0.080	53.1
1	10	1	0.5	1.031	3.2	10	10	2	0.5	0.175	30.3
1	10	1	1	1.223	4.0	10	25	2	0.5	0.222	83.1
1	10	1	2	1.762	6.9	40	15	0.25	2	0.337	61.2
1	5	0.5	2	1.486	11.6	40	10	0.25	1	0.098	31.3
4	10	0.25	2	1.160	17.4	40	15	0.25	0.5	0.122	4.5
4	10	0.25	1	0.810	9.0	40	10	0.5	1	0.098	10.1
4	10	0.5	2	0.696	21.1	40	30	0.5	1	0.145	21.3
4	15	0.5	1	0.586	7.8	40	10	0.5	0.5	0.098	24.4
4	10	1	2	0.649	11.2	40	20	0.5	0.5	0.145	81.4
4	10	1	1	0.430	6.1	40	15	0.5	2	0.207	2.9
4	10	1	0.5	0.302	1.6	40	25	0.5	2	0.230	1.5
4	10	2	2	0.557	9.7	40	15	1	2	0.137	7.6
4	30	2	2	0.394	25.3	40	25	1	2	0.184	42.7
4	10	2	1	0.338	6.9	40	10	1	1	0.052	18.0
4	25	2	1	0.367	24.4	40	30	1	1	0.099	39.8
4	10	2	0.5	0.329	3.9	40	10	2	2	0.045	64.6
4	30	2	0.5	0.166	53.8	40	25	2	2	0.092	20.0
10	10	0.25	2	0.544	0.0	40	10	2	1	-0.040	170.3
10	15	0.25	1	0.218	30.9	40	15	2	1	-0.017	123.2
10	20	0.5	2	0.461	21.5	40	15	1	0.5	0.076	23.5
10	15	0.25	0.5	0.218	13.6	40	30	1	0.5	0.099	86.8
10	10	0.5	1	0.195	26.4	40	25	2	0.5	0.125	142.4
10	20	0.5	0.5	0.241	39.1						

4.5.5 Comparisons between BPNN and MARS

The MARS model performs almost as well as the BPNN model in terms of predictive accuracy. However, MARS is computationally more efficient than BPNN as the execution time is 1.20 s, compared with 13.906 s for the BPNN model.

For the built BPNN model of ε_s prediction, the procedure of partitioning of weights is described in Appendix L. The parameter relative importance comparison indicates that Q is the most important variable in determining ε_s , followed by B/H . The mathematical expression of this model is shown in Appendix M.

The MARS model is much easier to interpret. Table 4.9 displays the ANOVA decomposition of the developed MARS model. It can be observed that Q is the most significant parameter in determining ε_s , followed by K_0 and B/H .

Table 4.9 ANOVA decomposition of the developed MARS model for ε_s

Functions	GCV	STD	#basis	variable(s)
1	7.536	1.777	3	Q
2	0.019	0.030	1	B
3	0.030	0.058	1	B/H
4	0.052	0.104	1	K_0
5	2.885	1.153	5	Q, B
6	0.427	0.421	5	$Q, B/H$
7	0.174	0.283	4	Q, K_0
8	0.019	0.045	2	$B/H, K_0$

Table 4.10 lists the BFs of the MARS model and their corresponding equations. The interpretable MARS model to predict ε_s is given by

$$\begin{aligned}
 \varepsilon_{s_MARS} (\%) = & 0.24 + 10 \times BF1 + 0.1 \times BF2 + 0.6 \times BF3 + 1.8 \times BF4 - 1.2 \times BF5 \\
 & + 0.2 \times BF6 - 0.003 \times BF7 + 0.076 \times BF8 - 0.09 \times BF9 - 0.006 \times BF10 \\
 & - 0.003 \times BF11 + 0.22 \times BF12 - 0.005 \times BF13 + 0.52 \times BF14 + 0.013 \times BF15 \\
 & - 0.02 \times BF16 - 0.68 \times BF17 + 0.7 \times BF18 - 0.005 \times BF19 - 0.46 \times BF20 \\
 & + 0.24 \times BF21 - 0.043 \times BF22
 \end{aligned} \quad (4.6)$$

Table 4.10 Basis functions and their corresponding equations for ε_{MARS}

BFs	Equation	BFs	Equation
BF1	$\max(0, 1 - Q)$	BF12	$\max(0, 0.5 - B/H) \times \max(0, 10 - Q)$
BF2	$\max(0, 4 - Q) \times \max(0, K_0 - 1)$	BF13	$BF6 \times \max(0, Q - 10)$
BF3	$\max(0, 4 - Q) \times \max(0, B/H - 0.5)$	BF14	$BF6 \times \max(0, 0.5 - B/H)$
BF4	$BF1 \times \max(0, K_0 - 1)$	BF15	$\max(0, 4 - Q) \times \max(0, B - 15)$
BF5	$BF1 \times \max(0, B/H - 1)$	BF16	$\max(0, 4 - Q) \times \max(0, 15 - B)$
BF6	$\max(0, K_0 - 1)$	BF17	$\max(0, 4 - Q) \times \max(0, B/H - 1)$
BF7	$\max(0, Q - 10)$	BF18	$\max(0, 4 - Q) \times \max(0, 1 - B/H)$
BF8	$\max(0, 10 - Q)$	BF19	$\max(0, 20 - B)$
BF9	$\max(0, B/H - 0.5)$	BF20	$BF19 \times \max(0, 1 - Q)$
BF10	$BF8 \times \max(0, B - 20)$	BF21	$\max(0, 1 - K_0) \times \max(0, B/H - 1)$
BF11	$BF8 \times \max(0, 20 - B)$	BF22	$\max(0, 1 - K_0) \times \max(0, 10 - Q)$

Fig. 4.11a-d plots the knot locations for Q , B , B/H , and K_0 respectively.

(a)

(b)

(c)

Fig. 4.11 Knot locations for: (a) Q , (b) B , (c) B/H , and (d) K_0

4.6 Probabilistic assessments on SLS

The cavern serviceability limit state can be assessed either by the calculated (or measured) maximum key point displacements or the derived percent strains.

4.6.1 Serviceability criterion

Eqs. (4.3), (4.4) or (I-24) in Appendix I can be used to predict $u_{\max_s}$. However, when it comes to the choice of a threshold displacement that can be utilized to limit the induced deformations within tolerable ranges, there is very limited guidance from the literature. As a matter of fact, the choice of such a threshold value is specific to the size and shape of the openings and it does not provide information about the degree of stability. Consequently, Eqs. (4.5) or (4.6), in terms of percent strain values ε_s , is logically adopted as the criterion for assessing the serviceability limit state as explained in the following section.

4.6.2 Threshold strain values

An appropriate value can be assigned to the percent strain threshold through the concept of critical strain. Sakurai (1986, 1997) proposed a relationship between critical strain and Young's modulus E_m and Uniaxial compressive strength σ_{cm} , and suggested Fig. 4.12 and Fig. 4.13 as representing the limiting bounds for these

relationships. These relationships are based on various rock tests and field measurements. However, Sakurai (1997) showed that critical strain results for in situ rock masses also fall within these bounding lines. Hazard warning levels for stability assessment of tunnels have been established by Sakurai on the basis of the relationship between critical strain ε_c and σ_{cm} in laboratory specimens and then subsequently confirmed through in situ displacement monitoring. Hazard warning level II (Fig. 4.13) is coincident with the central line of the critical strain versus σ_{cm} limit zone, and represents the transition from a stable to unstable tunnel.

Fig.4.12 Sakurai's relationship between ε_c and E_m (adapted from Sakurai 1997)

Fig.4.13 Sakurai's relationship between ε_c and σ_{cm} , also showing hazard warning level II for assessing the stability of tunnels (adapted from Sakurai 1986, 1997)

Table 4.11 shows the range of critical strain values obtained from the upper and lower bound curves in Figs. 4.12 and 4.13, and from the hazard warning level II in Fig. 4.13. The method proposed by Sakurai relates the critical strain values to strength and stiffness parameters of the rock mass.

Table 4.11 Critical strains derived from Sakurai's relationships

Q	E_m (GPa)	σ_{cm} (MPa)	Critical strain ε_c (%)		
			From Fig. 4.12	From Fig. 4.13	Hazard warning level II (From Fig. 4.13)
0.4	3.1	29.6	1.482-10.476	0.145-1.447	0.427
1	4.5	36.0	1.461-10.256	0.140-1.397	0.413
4	7.8	45.7	1.424-10.026	0.128-1.284	0.381
10	11.3	52.1	1.386-9.782	0.121-1.209	0.360
40	19.7	61.8	1.292-9.197	0.110-1.096	0.327

4.6.3 Comparisons between probabilistic assessment methods

With the determination of the percent strains in Eqs. (4.5) and (4.6), reliability assessment of serviceability limit state can be carried out using MCS and FORM.

The typical MCS procedures of @RISK and the calculation of P_f are illustrated in Fig. 4.14. As with the ULS assessment in the previous chapter, for each MCS simulation, the number of iterations is 1 000 000 and the sampling technique is Latin Hypercube.

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Parameters	Distribution	Statistics	Output $\varepsilon_c - \varepsilon_s_{LR} > 0$
Q	Normal	mean=10, s.d.=2	10
B	Normal	mean=10, s.d.=1	10
B/H	Normal	mean=1, s.d.=0.1	1
K_0	Normal	mean=1, s.d.=0.2	1

$$\varepsilon_{s,LR}(\%) = 0.749 Q^{-0.8088} B^{0.2842} (B/H)^{-0.3476} K_0^{0.5421}$$

a)

(b)

Fig. 4.14 MCS with @RISK: (a) LR and (b) MARS

The performance functions derived through LR and MARS models are incorporated into a FORM spreadsheet and the reliability index $\beta(P_f)$ can be then determined. As the MARS model is almost as accurate as the BPNN model, and is simpler to interpret, the BPNN model has been omitted for brevity. The typical FORM procedures and the calculation of P_f are illustrated in Fig. 4.15. Take Fig. 4.15b for example. Fig. 4.15b shows a sample spreadsheet for computing the reliability index where Q , B , B/H and K_0 are assumed to be normally distributed. The spreadsheet cells A2:A5 allows the selection of various distribution types for the input variables. Cells C2:F5 are parameters which are set corresponding to the variable distribution

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types. The correlation matrix **[R]** cells K2:N5 are used to define the correlations between Q , B , B/H and K_0 . The n_i vector in cells O2:O5 contains equations for $(x_i - \mu_i^N) / \sigma_i^N$. Cells I2:J23 contain the mathematical expressions of basis functions and algorithms obtained from the developed MARS model. Cells P2:P4 contain the serviceability limit state function $g(\mathbf{x}) = \varepsilon_c - \varepsilon_{\text{MARS}}$. The DP (x^* values) was obtained by using the spreadsheet's built-in optimization routine SOLVER to minimize the cell, by changing the x^* values, under the constraint that the ultimate limit state function $g(x^*) = 0$. Prior to invoking the SOLVER search algorithm, the x^* values were set equal to the mean values (10, 10, 1, 1) of the original random variables. Iterative numerical derivatives and directional search for the DP x^* were automatically carried out in the spreadsheet environment.

P_f values of eight cases are calculated using the four reliability assessment methods for comparison. The assumed statistical information of the input variables for the six cases is listed in Table 4.12.

Distribution	Variables	Para1	Para2	Para3	Para4	x_i^*	Correlation matrix [R]	n_i	$g(\mathbf{x})$	β	$P_f(\%)$
Normal	Q	10	2			5.664	1 0 0 0	-2.168	0.000	2.195	1.407
Normal	B (m)	10	1			10.21	0 1 0 0	0.211			
Normal	B/H	1	0.1			0.973	0 0 1 0	-0.271			
Normal	K_0	1	0.2			1	0 0 0 1	0			

$$g(\mathbf{x}) = \varepsilon_c - \varepsilon_{s_LR} = \varepsilon_c - 0.749Q^{-0.8088} B^{0.2842} (B/H)^{-0.3476} K_0^{0.5421}$$

(a)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Distribution	Variables	Para1	Para2	Para3	Para4	x_i^*	MARS PROCESS			Correlation matrix [R]			n_i	$g(\mathbf{x})$	β	$P_f(\%)$	
2	Normal	Q	10	2			6.394	BF1=	max(0, 1 - Q)	0.000	1	0	0	0	-1.8	2E-09	2.01	2.22
3	Normal	B	10	1			10.284	BF2=	max(0, 4 - Q) * max(0, K0 -1)	0.000	0	1	0	0	0.28			
4	Normal	B/H	1	0.1			0.983	BF3=	max(0, 4 - Q) * max(0, B/H -0.5)	0	0	0	1	0	-0.2			
5	Normal	K0	1	0.2			1.165	BF4=	BF1 * max(0, K0 -1)	0.000	0	0	0	1	0.83			
6								BF5=	BF1 * max(0, B/H -1)	0						$g(\mathbf{x}) = \varepsilon_c - \varepsilon_{s_MARS}$		
7								BF6=	max(0, K0 -1)	0.165								
8								BF7=	max(0, Q -10)	0.000								
9								BF8=	max(0, 10 - Q)	3.606								
10								BF9=	max(0, B/H -0.5)	0.483								
11								BF10=	BF8 * max(0, B -20)	0.000								
12								BF11=	BF8 * max(0, 20 - B)	35.03								
13								BF12=	max(0, 0.5 - B/H) * max(0, 10 - Q)	0								
14								BF13=	BF6 * max(0, Q -10)	0								
15								BF14=	BF6 * max(0, 0.5 - B/H)	0								
16								BF15=	max(0, 4 - Q) * max(0, B -15)	0								
17								BF16=	max(0, 4 - Q) * max(0, 15 - B)	0								
18								BF17=	max(0, 4 - Q) * max(0, B/H -1)	0								
19								BF18=	max(0, 4 - Q) * max(0, 1 - B/H)	0								
20								BF19=	max(0, 20 - B)	9.716								
21								BF20=	BF19 * max(0, 1 - Q)	0								
22								BF21=	max(0, 1 - K0) * max(0, B/H -1)	0								
23								BF22=	max(0, 1 - K0) * max(0, 10 - Q)	0								

(b)

Fig. 4.15 FORM incorporated with: (a) LR and (b) MARS

**Table 4.12 Statistical information of the input variables for reliability
assessment on SLS**

Case No.	Variables	Distribution	Statistics
1	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 10, $COV=0.1$
	B/H	Normal	Mean = 2, $COV=0.1$
	K_0	Normal	Mean = 1, $COV=0.2$
2	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 10, $COV=0.1$
	B/H	Normal	Mean = 1, $COV=0.1$
	K_0	Normal	Mean = 1, $COV=0.2$
3	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 15, $COV=0.1$
	B/H	Normal	Mean = 2, $COV=0.1$
	K_0	Normal	Mean = 1, $COV=0.2$
4	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 10, $COV=0.1$
	B/H	Normal	Mean = 1, $COV=0.1$
	K_0	Normal	Mean = 0.5, $COV=0.2$
5	Q	Normal	Mean = 10, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 15, $COV=0.1$
	B/H	Normal	Mean = 1, $COV=0.1$
	K_0	Normal	Mean = 0.5, $COV=0.2$
6	Q	Normal	Mean = 40, $COV=0.1, 0.2, 0.3, 0.4$
	B	Normal	Mean = 15, $COV=0.1$
	B/H	Normal	Mean = 0.5, $COV=0.1$
	K_0	Normal	Mean = 1, $COV=0.2$

The values of P_f derived using the four probabilistic methods for the six cases are shown in Fig. 4.16 for comparison. It is clear that P_f increases with the increase of COV of Q , demonstrating that a single ε_s value which is deterministically determined may not always provide an adequate quantification of serviceability. The differences between MCS and FORM for the same response surface (LR or MARS) are marginal for the six cases. For the same Q and B/H values, increase of the cavern span B results in significant increase of P_f .

(a)

(b)

Fig. 4.16 P_f of SLS derived for case: (a) case 1, (b) case 2, (c) case 3, (d) case 4, (e) case 5, and (f) case 6

4.7 System reliability

Ultimate limit states generally pertain to total collapse while serviceability limit states pertain to conditions under which performance requirements under normal service loads are exceeded such as excessive movements. Generally, serviceability limit states have a higher probability of failure than ultimate limit states, principally because they occur at lower absolute ‘loads’. However, when probabilistic assessments on the serviceability limit state are performed, the ultimate limit state should also be checked to ensure the system reliability. Thus it is proposed that the reliability assessment is performed on the percent strain ε_s while at the same time ensuring that the FS_{g_s} is satisfied during the search for the design point. Fig. 4.17 shows a typical spreadsheet for assessing the system reliability. Cell P2 contains the mathematical expression for the limit state function $g(\mathbf{x}) = \varepsilon_c - \varepsilon_{s_MARS}$, which is a function of Q , B , B/H and K_0 . Among the developed three FS_{g_s} models, the LR model is selected to demonstrate the system reliability analysis due to the simplicity of the mathematical expression compared with the MARS or BPNN models. The LR model for FS_{g_s} is implemented into cell F16 representing the constraint function from FS_{g_s} , in which the mathematical expression of $FS_{g_s} - FS_{g_s_cr} \geq 0$ must be satisfied for each combination of parameters Q , B , and B/H . The design point (x_i^* values in cells G2:G5) was obtained by using the EXCEL spreadsheet’s built-in optimization routine SOLVER to minimize the cell, by changing the x_i^* values, under the constraints that the performance function $g(x_i^*) = 0$ (cell P2) and that $FS_{g_s} - FS_{g_s_cr} \geq 0$ (cell F16). Prior to invoking the SOLVER search algorithm, the x_i^* values were set equal to the mean values of the original four random variables. In this illustrative example, the values are (4, 5, 0.5, 0.5). Iterative numerical derivatives and directional search for the design point x_i^* are automatically carried out in the spreadsheet environment. Reliability indices β_{SLS} corresponding to different FS_{g_s} values can be calculated based on the FORM spreadsheet method described in Fig. 4.17. Table 4.13-4.15 list the calculation results of β_{SLS} corresponding to the required minimum FS_{g_s} under various combinations of B , K_0 , and B/H for $Q = 4, 10$ and 40 respectively. It is obvious that to obtain the same target performance level, the required FS_{g_s} increases as K_0

increases if B/H is constant. It is also clear that to obtain the same β_{SLS} , FS_{g-s} increases as B/H increases if K_0 is constant. For the same Q , K_0 and B/H , the required FS_{g-s} to satisfy the target performance level decreases as B increases. It is logical that the larger the cavern size, the smaller the required FS_{g-s} . Fig. 4.18 illustrates the relationship between FS_{g-s} and β_{SLS} of serviceability limit for caverns with $Q=4, 10$ and 40 for some typical cavern spans. In Fig. 4.18, the various classes denoted as unsatisfactory, poor, below average, etc are based on Table 3.8.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	
1	Distribution	Variables	Para1	Para2	Para3	Para4	x_i^*	MARS PROCESS				Correlation matrix [R]				n_i	$g(\underline{x})$	β	$P_f(\%)$
2	Normal	Q	4	0.8			3.191	BF1= $\max(0, 1-Q)$		0.000	1	0	0	0	-1	-1E-11	1.08	13.92	
3	Normal	B	5	0.5			4.897	BF2= $\max(0, 4-Q) * \max(0, K0-1)$		0.000	0	1	0	0	-0.2				
4	Normal	B/H	0.5	0.05			0.507	BF3= $\max(0, 4-Q) * \max(0, B/H-0.5)$		0.006	0	0	1	0	0.14				
5	Normal	K0	0.5	0.1			0.530	BF4=BF1 * $\max(0, K0-1)$		0.000	0	0	0	1	0.3				
6								BF5=BF1 * $\max(0, B/H-1)$		0									
7								BF6= $\max(0, K0-1)$		0.000									
8								BF7= $\max(0, Q-10)$		0.000									
9								BF8= $\max(0, 10-Q)$		6.809									
10	$FS_{g-s, LR} = 2.9562Q^{0.1605} B^{-0.4127} (B/H)^{0.2314}$								BF9= $\max(0, B/H-0.5)$		0.007								
11									BF10=BF8 * $\max(0, B-20)$		0.000								
12	FS_{g-s} constraint ($FS_{g-s, LR}-FS_{g-s, cr} \geq 0$)								BF11=BF8 * $\max(0, 20-B)$		102.8								
13									BF12= $\max(0, 0.5-B/H) * \max(0, 10-Q)$		0								
14									BF13=BF6 * $\max(0, Q-10)$		0								
15									BF14=BF6 * $\max(0, 0.5-B/H)$		0								
16									BF15= $\max(0, 4-Q) * \max(0, B-15)$		0								
17									BF16= $\max(0, 4-Q) * \max(0, 15-B)$		8.171								
18									BF17= $\max(0, 4-Q) * \max(0, B/H-1)$		0								
19									BF18= $\max(0, 4-Q) * \max(0, 1-B/H)$		0.399								
20									BF19= $\max(0, 20-B)$		15.103								
21									BF20=BF19 * $\max(0, 1-Q)$		0								
22									BF21= $\max(0, 1-K0) * \max(0, B/H-1)$		0								
23									BF22= $\max(0, 1-K0) * \max(0, 10-Q)$		3.2								

Fig. 4.17 Calculation on the system reliability using FORM_MARS

Table 4.13 Calculation results of β_{SLS} corresponding to required FS_{g-s} under combinations of B , K_0 , and B/H for $Q = 4$

K_0	β_{SLS}	required FS_{g-s}			
		$B=5$ m		$B=10$ m	
		$B/H=0.5$	$B/H=1$	$B/H=0.5$	$B/H=1$
0.5	1.5	1.58	1.81	1.30	1.51
	2	1.69	1.87	1.33	1.54
	2.5	1.73	1.91	1.37	1.58
	3	1.78	1.97	1.40	1.62
	4	1.88	2.05	1.48	1.70
1	1.5	1.74	2.00	1.32	1.57
	2	1.78	2.07	1.38	2.07
	2.5	1.83	2.11	1.42	1.65
	3	1.89	2.17	1.47	1.67
	4	2.01	2.31	1.56	1.78
2	1.5	—	1.98	—	—
	2	1.69	2.10	—	—
	2.5	1.82	2.19	—	1.51
	3	1.90	2.27	1.32	1.65
	4	2.06	2.45	1.52	1.81

Table 4.14 Calculation results of β_{SLS} corresponding to required $FS_{g,s}$ under combinations of B , K_0 , and B/H for $Q=10$

K_0	β_{SLS}	required $FS_{g,s}$					
		$B=10$ m		$B=15$ m		$B=20$ m	
		$B/H=0.5$	$B/H=1$	$B/H=0.5$	$B/H=1$	$B/H=0.5$	$B/H=1$
0.5	1.5	–	–	1.13	–	1.07	1.21
	2	1.21	–	1.16	–	1.09	1.24
	2.5	1.25	1.43	1.19	–	1.12	1.27
	3	1.32	1.52	1.22	1.32	1.14	1.30
	4	1.42	1.63	1.27	1.43	1.19	1.35
1	1.5	1.31	1.53	1.21	1.38	1.10	1.26
	2	1.37	1.59	1.25	1.42	1.13	1.30
	2.5	2.11	1.65	1.28	1.46	1.16	1.33
	3	1.46	1.71	1.32	1.50	1.19	1.37
	4	1.59	1.82	1.40	1.59	1.26	1.45
2	1.5	1.54	1.82	1.30	1.54	1.14	1.36
	2	1.60	1.87	1.35	1.59	1.19	1.41
	2.5	1.65	1.93	1.39	1.65	1.23	1.45
	3	1.70	1.99	1.44	1.69	1.28	1.50
	4	1.82	2.13	1.54	1.81	1.37	1.61

Table 4.15 Calculation results of β_{SLS} corresponding to required $FS_{g,s}$ under combinations of B , K_0 , and B/H for $Q=40$

K_0	β_{SLS}	required $FS_{g,s}$							
		$B=15$ m		$B=20$ m		$B=25$ m		$B=30$ m	
		$B/H=0.5$	$B/H=1$	$B/H=0.5$	$B/H=1$	$B/H=0.5$	$B/H=1$	$B/H=0.5$	$B/H=1$
0.5	3	1.14	1.30	1.03	1.19	1.01	1.10	–	–
	4	1.23	1.40	1.14	1.31	1.07	1.23	1.01	1.06
1	3	1.17	1.35	1.05	1.22	1.02	1.12	–	1.03
	4	1.30	1.49	1.18	1.35	1.09	1.25	1.03	1.15
2	1.5	1.47	1.63	1.33	1.47	1.23	1.34	1.15	1.25
	2	1.52	1.71	1.37	1.55	1.27	1.43	1.18	1.33
	2.5	1.57	1.78	1.41	1.61	1.32	1.49	1.22	1.38
	3	1.62	1.85	1.45	1.66	1.36	1.54	1.26	1.43
	4	1.72	1.98	1.54	1.78	1.45	1.64	1.34	1.53

(a)

(b)

(c)

(d)

(e)

(f)

Fig. 4.18 Relationship between β_{SLS} and required FS_{g_s} for $K_0=1$ cases: (a) $Q=4$ $B=5$ m, (b) $Q=4$ $B=10$ m, (c) $Q=10$ $B=10$ m, (d) $Q=10$ $B=15$ m, (e) $Q=10$ $B=20$ m, (f) $Q=40$ $B=15$ m, (g) $Q=40$ $B=20$ m, (h) $Q=40$ $B=25$ m, and (i) $Q=40$ $B=30$ m

Comparison between Fig. 4.18b and 4.18c shows that a target β_{SLS} of 3.0 (Target performance level: Above average) is achievable for FS_{g_s} larger than 1.67 for $Q=4$, $B=10$ m, $B/H=1$ and $K_0=1$. For the same geometries with a larger Q value of 10, an above average performance level is achievable for FS_{g_s} larger than 1.71. It is logical that the higher the quality of the rock mass, the larger the required FS_{g_s} . Previous results have shown that FS_{g_s} is not influenced by the in situ stress ratio K_0 . However, the serviceability assessment is performed on the percent strain value, which is significantly influenced by K_0 . The result is that to achieve the same performance level under the same Q , B , and B/H combination, the required FS_{g_s} for $K_0 = 0.5, 1$ and 2 are quite different. Generally the required FS_{g_s} for $K_0 = 2$ is much higher than that for the other two lower K_0 cases, as a result of higher horizontal stress release during excavation. The reference FS_{g_s} values are proposed. The lines constructed in Fig. 4.18 can be used as a preliminary design tool when selecting a required FS_{g_s} provided the quality of rock mass, the cavern span, the shape of the opening and the in situ stress are known.

It is obvious that optimal design in terms of avoiding over-design of rock mass is achievable if a certain level of risk is acceptable. Thus the procedures outlined in this chapter can be used to obtain a rational optimal design of underground single rock cavern. A FS_{g-s} no less than the values proposed in Tables 4.12-4.14 can be used to satisfy the target performance levels for caverns with different spans and shapes under different ground conditions. These FS_{g-s} values are also recommended to meet the performance level requirement under similar ground conditions. They can also be used as guidance with regard to the choice of a proper factor of safety for underground single rock cavern problems.

4.8 Concluding remarks

This chapter considered the serviceability limit state of a single cavern, i.e. the failure due to excessive movements. The numerical outputs for serviceability assessment were the maximum key point displacement u_{max-s} and the percent strain ε_s . Based on the results, LR, BPNN and MARS models for u_{max-s} and ε_s were developed. Comparisons between BPNN and MARS models indicate that MARS algorithm performs almost as well as BPNN in terms of predictive accuracy. However, with regard to the computational efficiency and model interpretability, MARS outperforms BPNN.

Percent strain was adopted as the criterion for assessing the serviceability limit state, with the threshold value determined from the concept of critical strain. With determination of the performance functions, reliability assessment of serviceability limit state can be carried out using either MCS or FORM. The system reliability is ensured by performing probabilistic assessment on the percent strain ε_s and concurrently satisfying the FS_{g-s} during the search for the design point. The proposed Tables 4.13-4.15 and the charts constructed in Fig. 4.18 can be used as a preliminary design tool when selecting a required FS_{g-s} provided the quality of rock mass, the cavern span, the shape of the opening and the in situ stress are known. Procedures outlined in this chapter can be used to obtain a rational optimal design of underground single rock cavern.

In summary, this study of probabilistic assessment on SLS and system reliability of a single cavern illustrated that:

- (1) The maximum key point displacement and the percent strain can be used as the criterion for assessing the serviceability limit state. The percent strain was adopted as the serviceability limit state criterion in this study as the threshold value can be determined through the concept of critical strain.
- (2) For both the maximum key point displacement and the percent strain, LR, BPNN and MARS models were built. The LR model is simple but not as accurate as MARS or BPNN. The MARS model is almost as accurate as BPNN. However, MARS outperforms BPNN in terms of model interpretability and computational efficiency.
- (3) Q is the most significant parameter in determining $u_{\max_s}$, followed by B and K_0 . For the same B/H , B and K_0 , $u_{\max_s}$ decreases with the increase of Q . Based on this, it is logical that ε_s decreases with the increase of Q . In addition, ε_s is primarily influenced by Q . The built $u_{\max_s}$ MARS model can be used to give preliminary estimates of the expected maximum cavern displacements during construction of a single cavern.
- (4) Adopting the built models for percent strain ε_s , reliability assessment of serviceability limit state can be carried out using either MCS or FORM. It is obvious that P_f increases with the increase of COV of Q , demonstrating that a single ε_s value which is deterministically determined may not always provide an adequate quantification of serviceability.
- (5) System reliability can be ensured by performing probabilistic assessment on the percent strain ε_s and concurrently satisfying the FS_{g_s} during the search for the design point. This is a rational approach for determining a proper factor of safety for underground single rock cavern.
- (6) Optimal design in terms of avoiding over-design of rock mass is achievable if a certain level of risk is acceptable. The proposed Tables 4.13-4.15 and the charts constructed in Fig. 4.18 can be used as a preliminary design tool to select a required FS_{g_s} provided the quality of rock mass, the cavern span, the shape of the opening and the in situ stress are known.

Chapter 5

RELIABILITY ASSESSMENT ON ULTIMATE LIMIT STATE OF TWIN CAVERNS

5.1 Introduction

The construction of a new cavern modifies the state of stresses and movements in a zone around the existing cavern. For multiple caverns, the size of this influence zone depends on the ground type, the in situ stress, the cavern span, the width of the pillar separating the caverns, and the excavation sequence. If two adjacent excavations are constructed far apart such that their influence zones do not overlap, then the individual cavern can be considered separately as single caverns and analyzed as such. However, if the influence zones of the two caverns do overlap, some degree of interaction between the two caverns will take place. Interaction of the two caverns will affect the global stability, the state of stress and the deformations around the caverns.

This chapter presents a study of the interaction of two parallel caverns and the influence of such interaction on stress-induced global stability in terms of a global *FS*. A series of finite difference analyses were performed to derive the global *FS* of a system of two parallel and adjacent caverns. The sequential excavation of the two caverns has been simulated in the analyses. Influences of various parameters on the behavior of the two parallel caverns have been investigated with focus on the global factor of safety and the effect of cavern interaction on the stresses and plastic zones. These important parameters have been identified as: the cavern span B ; the width of the pillar separating the two caverns S_c ; sequence of excavation; the in situ stress ratio K_0 ; and the tunneling quality index Q . The effect of cavern interaction on displacement is discussed in Chapter 6.

5.2 Previous studies

The subject of interaction between parallel caverns/tunnels has been studied by several authors who have reported the results of field measurements or analytical studies of the problem. This brief discussion of some of the literature on the subject in Table 5.1 is not intended as a comprehensive review and must be considered incomplete.

Table 5.1 Literature on the interaction between parallel caverns/tunnels

References	Type of analysis	Excavations	Geometric parameters	Interaction factors
Barla and Ottoviani (1974)	FE linear elastic and non-linear	Excavated at the same time in rock	Tunnel diameters: 6 and 12 m; Depth to diameter ratio (H/D): 0.75 to 3; Pillar width to diameter ratio (S_c/D): 0.25 to 1.	Interactions become negligible for $S_c/D=1$ or greater
Ghaboussi and Ranken (1977)	Mainly FE linear elastic, some selected cases were elasto-plastic	Sequenced excavations as well as excavations at the same time	Circular tunnels; H/D : 1.5 and 5.5; S_c/D : 0.25, 0.5 and 1.0.	Interaction effects are small at $S_c/D=1$, and that at a $S_c/D=2$, the two tunnels behave independently of each other
Gercek (2005)	FE elasto-plastic	Parallel excavations in rock	Conventional shape having an ached roof and a flat floor; Alignment: diagonal, vertical and parallel; Alignment angle: $\pi/2$, $\pi/4$, 0 and $-\pi/2$; Pillar width to tunnel span (S_c/B): 0.5, 1 and 1.5.	no interaction exists between the parallel openings when $S_c/B > 1.5$

Cehade and Shahour (2008)	FE elastic perfectly plastic	Sequenced excavations in soil	Circular twin tunnel of three configurations: aligned-horizontally, vertically and inclined; H/D : 8; Ratio spacing (S_x/D): 2 to 5; Inclination angle: 39° and 45° . (S_x : horizontal spacing)	Beyond $S_x/D=2$, the construction of the first tunnel does not affect the second one.
Liu et al. (2008)	3D FE elasto-plastic	Sequenced excavations in rock	Polycentric cross-section and the diameter is 10 m; Relative positions: horizontal, vertical and staggered tunnels; S_x/D : 2 and 3; S_y/D : 1.5. (S_y : vertical spacing).	Horizontally parallel tunnels with $S_x/D=3$ have the minimum adverse effects on the existing support system, compared with the other cases.
Zhao and Ma (2009)	FE static and dynamic nonlinear	Excavation of cavern groups in rock	Conventional shapes for hydraulic power stations; Cavern A: height of 74.4 m, width of 30 m, Cavern B: height of 30.5 m, width of 20 m; Spacing between the two caverns is selected as 10, 25, 50, 75 and 90 m, respectively.	Critical cavern spacing for the adjacent caverns located in soft, medium hard and hard rock, is twice, less than twice, and one time the characteristic length (taken to the biggest width of a cavern in the group), respectively
Chen et al. (2009)	FE elasto-plastic	three-parallel-tube and four-parallel-tube tunnels excavated in rock adopting drill and blast and TBM methods	Main tunnels: horse-shoe shaped and pilot tunnel: circular; Diameters for the main and pilot tunnels were 12 m and 5 m, respectively; The ratio of net width of the rock pillar S_c to the sum of the radii of the main and the pilot tunnels B (S_c/B): 0.41 to 9;	For S_c/B less than 2.5, the impact of the interaction increased significantly.

Mortazavi et al. (2009)	FD elasto-plastic	Consecutive mining in host rock around the pillar	Rectangular pillar with height (h) of 5 m while pillar width (S_c) changes; S_c/h : 0.5, 1, 1.5, 2, and 2.5;	As S_c/h ratio reduces, pillar load-bearing capacity drops and at low S_c/h ratios, it is the inherent material strength that governs the failure. At high S_c/h ratios, pillar behaves in a very stiff manner in elastic range demonstrating a high load-bearing capacity.
Karademir (2010)	3D FE elasto-plastic	Sequenced excavations in weak rock	Two parallel NATM tunnels; Ratio of pillar width S_c to tunnel diameter D : 0.5, 1, 1.5, and 2	Interaction effects may be eliminated by increasing S_c/D ratio equal to or larger than an approximate value of 2.5-3.0
Esterhuizen et al. (2011a)	Based on observations of stable and failed cases, supplemented by numerical models	Room-and-pillar mining in stone mines	Pillar width S_c : 4.6 to 21.5 m; Pillar height h : 4.8 to 40 m; S_c/h ratio from ≤ 0.5 to > 1.2	Pillars having a S_c/h ratio of less than 0.8 should be avoided

5.3 Numerical modeling

This chapter focuses on investigating the interaction between two adjacent horse-shoe shaped caverns in terms of the global stability. The approach adopted is similar to the single cavern case (Chapter 3) and FLAC^{3D} numerical analysis tool was adopted. The basic assumptions of the numerical analysis, the cross-section layout of the twin caverns, and the basic design parameters used are described herein.

5.3.1 Assumptions of numerical analysis

The basic assumptions of numerical analyses in this case are:

- the study was a two-dimensional plane strain problem;

- b) the rock material obeyed Mohr-Coulomb failure criterion that follows the elastic perfectly-plastic stress-strain relationship;
- c) the caverns are unsupported;
- d) the twin caverns are of equal size, both horse-shoe shaped, with semi-circular roof, the span-to-side wall height ratio (B/H) is 2, and horizontally aligned;
- e) the excavation involves six stages: heading, first benching, second benching of the right cavern, followed by heading, first benching, second benching of the left cavern;
- f) the effect of creep was not considered in the analysis.

5.3.2 Cross-section layout

One significant parameter influencing the interaction is the cavern span B . In this study, cavern span values of 10, 20 and 30 m are considered. In the numerical models, the cavern crown is 65 m below the ground surface. The initial vertical in situ stress is induced by self-weight of the rock. The physical geometric model including the twin caverns and the design variables considered are shown in Fig. 5.1.

Fig. 5.1 Geometrical model and basic design parameters

5.3.3 Ranges of design parameters

The main factors affecting interaction between the twin caverns are found to be the cavern geometrical characteristics, the rock strength properties, the in situ stress field, and the excavation sequence. While the effects of some of the factors such as

the strength properties and the in situ stresses are measurable, the effects of the others such as excavation sequence cannot be quantified. For simplicity, excavation in six stages as described in Fig. 5.1 is regarded as deterministic. The other important factors are shown in Table 5.2.

Table 5.2 Factors and their values used in parametric studies

Parameter	Description	Values
K_0	In situ stress ratio	0.5, 1, 2
B	Cavern span (m)	10, 20, 30
S_c/B	Ratio of pillar width to cavern span	1, 1.5, 2, 2.5
Q	Tunneling quality index	1, 4, 10, 40, 100

For the FDM analyses, the rock mechanical properties are indirectly (through RMR) determined from the Q system by means of the same empirical equations used in the single cavern case (Chapter 3).

5.4 Modeling results and analyses

Based on the parameter combinations, a total of 180 cases are simulated. The global factor of safety of the twin caverns FS_{g_t} is obtained through the use of the shear strength reduction technique. The results are presented in the following sections.

5.4.1 Modeling results of FS_{g_t}

As for the single cavern case in Chapter 3, for the twin caverns, the in situ stress ratio K_0 was found to have no influence on FS_{g_t} . FS_{g_t} is influenced by Q , B and S_c/B , as indicated in Table 5.3. Also shown in Table 5.3 for comparison are the global factors of safety FS_{g_s} for a single cavern. The FS_{g_t} values decrease as the S_c/B ratio decrease.

As for the single cavern case, for the twin caverns, both B and Q significantly affect the FS_{g_s} . Fig. 5.2 shows that for the same Q and B , FS_{g_t} increases with the increase of the pillar width. Fig. 5.2 also plots the single cavern FS_{g_s} values (dots with hollow symbol). It is obvious that as the pillar width S_c gradually increases, FS_{g_t} converges to the FS_{g_s} . The difference between FS_{g_t} and FS_{g_s} is generally less than 7% for $S_c/B \geq 2.0$.

Table 5.3 Results from numerical experiments for FS_{g-t}

B (m)	Q	FS_{g-t}				FS_{g-s}
		$S_c/B=1$	$S_c/B=1.5$	$S_c/B=2$	$S_c/B=2.5$	
30	100	1.38	1.47	1.59	1.66	1.67
	40	1.24	1.31	1.42	1.48	1.48
	10	1.04	1.10	1.19	1.25	1.25
	4	0.90	0.96	1.03	1.08	1.09
	1	0.72	0.77	0.83	0.87	0.88
20	100	1.55	1.64	1.72	1.79	1.82
	40	1.38	1.46	1.53	1.61	1.63
	10	1.17	1.23	1.29	1.35	1.36
	4	1.01	1.06	1.12	1.17	1.19
	1	0.81	0.85	0.90	0.94	0.96
10	100	2.13	2.15	2.20	2.24	2.25
	40	1.90	1.92	1.96	2.00	2.02
	10	1.61	1.62	1.65	1.68	1.69
	4	1.39	1.40	1.43	1.46	1.47
	1	1.11	1.12	1.15	1.17	1.19

Fig. 5.2 Influence of S_c on the FS_{g-t} : (a) $B=10$ m, (b) $B=20$ m, and (c) $B=30$ m

As mentioned in section 5.3.1, in this thesis, it has been assumed that the right cavern was excavated first (in 3 stages) before the left cavern was excavated. However, some additional numerical analyses were also performed to check whether $FS_{g,t}$ will be different if both caverns are excavated at the same time. That is, the heading of the right and left caverns are concurrently excavated first, followed by the first benching of the twin caverns, and finally the second benching of the two caverns are excavated. Fig. 5.3 plots the $FS_{g,t}$ for the two different construction sequences for Q values of 4, 10 and 40. While the $FS_{g,t}$ values differ during the intermediate stages (2 to 5) for the different construction sequences, the final $FS_{g,t}$ values were the same at the end of the final excavation.

Fig. 5.3 Influences of excavation sequences on $FS_{g,t}$: (a), $Q=4$, (b) $Q=10$, and (c) $Q=40$ ($B = 20$ m)

5.4.2 Determination of FS_{g_t} using LR

Based on the above results, a LR model was developed for predicting FS_{g_t} in terms of Q , B and S_c/B , as shown in Eq. (5.1)

$$FS_{g_t_LR} = 2.3044Q^{0.1404}B^{-0.3331}(S_c/B)^{0.1393} \quad (5.1)$$

A comparison of FS_{g_t} between the predictions from Eq. (5.1) and the $FS_{g_t_FDM}$ as shown in Fig. 5.4 indicates that the predictions are in good agreement with the target FS_{g_t} values.

Fig. 5.4 Comparison between $FS_{g_t_FDM}$ and $FS_{g_t_LR}$

5.4.3 Determination of FS_{g_t} using MARS

Using the same observations, this problem is reanalyzed by means of MARS. Of the 60 observations, 48 patterns were selected as the training data and the remaining 12 data were used for testing. The testing data sets are listed in Table 5.4. The MARS training results together with the relative error e are shown in Appendix N.

Table 5.4 Testing data sets for FS_{g_t}

Q	B (m)	S_c/B	$FS_{g_t_FDM}$	$FS_{g_t_MARS}$	Q	B (m)	S_c/B	$FS_{g_t_FDM}$	$FS_{g_t_MARS}$
4	30	1	0.90	0.888	4	20	2	1.12	1.140
10	30	1.5	1.10	1.144	40	20	2.5	1.61	1.551
40	30	2	1.42	1.384	4	10	1	1.39	1.365
4	30	2.5	1.08	1.081	10	10	1.5	1.62	1.569
40	20	1	1.38	1.358	40	10	2	1.96	1.980
10	20	1.5	1.23	1.257	4	10	2.5	1.46	1.471

The MARS model adopted 10 BFs of linear spline functions with second-order interaction. A plot of $FS_{g_t_MARS}$ versus $FS_{g_t_FDM}$ as shown in Fig. 5.5 indicates that the MARS model is slightly more accurate than the LR model.

Fig. 5.5 Comparison between $FS_{g_t_FDM}$ and $FS_{g_t_MARS}$

Table 5.5 displays the ANOVA decomposition of the developed MARS model. Table 5.6 gives the parameter relative importance. It can be observed that B is the most important parameter in determining FS_{g_t} , followed by Q .

Table 5.5 ANOVA decomposition of the developed MARS model for FS_{g_t}

Functions	GCV	STD	#basis	variable(s)
1	0.205	0.297	4	Q
2	0.262	0.261	2	B
3	0.02	0.072	2	S_c/B
4	0.033	0.08	1	Q, B
5	0.005	0.024	1	$B, S_c/B$

Table 5.6 Relative importance of the parameters

Parameters	Relative importance (%)
Q	78.0
B	100
S_c/B	6.6

Table 5.7 lists the BFs of the MARS model for FS_{g_t} and their corresponding equations. The interpretable MARS model to predict FS_{g_t} is given by

$$\begin{aligned}
 FS_{g_t_MARS} = & 1.322 - 0.0224 \times BF1 - 0.0113 \times BF2 + 0.0484 \times BF3 \\
 & + 0.1082 \times BF4 - 0.17 \times BF5 - 0.000573 \times BF6 + 0.00335 \times BF7 \\
 & - 0.0496 \times BF8 + 0.0173 \times BF9 - 0.0028 \times BF10
 \end{aligned} \tag{5.2}$$

Table 5.7 BFs and corresponding equations of MARS model for FS_{g_t}

BFs	Equation	BFs	Equation
BF1	$\max(0, 10 - Q)$	BF6	$BF3 \times \max(0, 40 - Q)$
BF2	$\max(0, B - 20)$	BF7	$\max(0, Q - 4)$
BF3	$\max(0, 20 - B)$	BF8	$\max(0, 4 - Q)$
BF4	$\max(0, S_c/B - 1.5)$	BF9	$BF3 \times \max(0, 1.5 - S_c/B)$
BF5	$\max(0, 1.5 - S_c/B)$	BF10	$\max(0, 40 - Q)$

5.4.4 Design curves of FS_{g_t}

Based on Eq. (5.1) and (5.2), charts relating FS_{g_t} to Q , B and S_c have been developed as shown in Fig. 5.6. It can be observed that the MARS predictions exhibit similar trends as the curves developed using LR. The proposed charts are potentially useful for preliminary design and checking of twin cavern stability. By specifying the pillar geometry S_c/B for a given Q as well as the cavern span B , the expected global factor of safety can be estimated. Alternatively, by assuming the required global factor of safety for a given Q and cavern span, the appropriate pillar width can be determined.

(a)

(b)

(c)

Fig. 5.6 Design curves for FS_{g_t} : (a) $FS_{g_t}=1.0$, (b) $FS_{g_t}=1.2$, and (c) $FS_{g_t}=1.5$

5.5 Failure mechanism of twin caverns

The term “pillars” is widely accepted to refer to the in situ rock remnants left between adjacent underground openings. Rock pillars serve as an underground support element, without which it is difficult to sustain the weight of the overburden materials. If the strength of a pillar is exceeded, it will fail, and the load that it carried will be transferred, and thereby contribute to the collapse of the twin caverns. The lack of confinement in slender pillars (width-to-height ratio S_c/h less than or equal to 1, while wide pillars are those with S_c/h greater than 2) also contributes to the complete collapse of pillars at relatively low stress magnitudes. From a design point of view, understanding the pillar failure mechanism and the interaction effect between twin caverns is essential. The focus of this section is to study the influence of Q , B , S_c and K_0 on the pillar performance, with focus on the principal stress difference and the vertical stress in the pillar core (the central core elements of the pillars), the maximum principal stress along pillar elements and the average pillar strength and stress.

5.5.1 Failure mechanisms of pillars

Pillar failure occurs when a pillar is compressed beyond its peak resistance and load shedding or yielding occurs (Brady and Brown 1985). From a theoretical point of view, the loaded pillars will undergo a transition through the following phases (Mortazavi et al. 2009):

- a) initial virgin conditions;
- b) stresses are increasing at pillar sides with core stress relatively unchanged;
- c) sides failed and core stress increased;
- d) side yielding zones grow larger while core elements approach yield conditions;
- e) convergence of yielding zones;
- f) ground distress around failed pillar.

The above process has been demonstrated conceptually and experimentally by various authors (e.g., Salamon 1970, Esterhuizen et al. 2011b, Wang et al. 2011). In general, failure of pillar is mainly governed by confinement of the pillar core, the

strength of the pillar rock mass, the mechanical behavior of the rock mass and the pillar end conditions (Ozbay et al. 2001). Empirical evidence and theoretical studies suggest that as S_c/h of pillars is reduced, the failure potential increases as a result of the rapid decrease in strength of a slender pillar after it has reached its peak resistance (Salamon 1970). Research of Esterhuizen et al. (2011b) implies that if slender pillars show signs of spalling, they may be loaded at or near their ultimate strength, and failure may be imminent. The slender pillars also display a rapid drop in strength after reaching the peak strength, while the wider pillars lose their strength more gradually.

5.5.2 Maximum principal stress in pillar

In order to analyze the pillar maximum principal stress σ_1 , a series of analyses was carried out. For each analysis the stresses at various locations within the pillar were monitored during various stages of excavation. Fig. 5.7 shows the approximate location of the equally spaced monitoring points.

Fig. 5.7 Location of stress monitoring points

In order to simulate more realistically the construction of the caverns and the gradual loading on the pillar, six excavation stages as illustrated in Fig. 5.1 were considered. Stage 0 represents the initial virgin conditions (before excavation commenced). Fig. 5.8 displays the σ_1 changes of the monitoring points for the case of $Q=4$, $B=20$ m, $S_c/B=1$, and $K_0=1.0$, in which the derived FS_{g-t} is 1.01 and the single cavern $FS_{g-s}=1.19$. In Fig. 5.8, the horizontal axis indicates the locations of the measuring points along the pillar. 0 denotes the location of the mid-point of the

pillar core. Positive coordinate values refer to the points near the first excavated cavern (the right cavern) while points with negative coordinate values are near the second excavated cavern. The points with coordinates 10 and -10 are close to the side walls of the twin caverns.

As can be seen from Fig. 5.8, before excavation starts, the elements are at the initial virgin (at-rest) state. After the first heading (stage 1), the pillar loads (stresses) redistribute and σ_1 values of the measuring points increase to varying degrees. The excavation of the first benching (right cavern stage 2) results in the major principal stress decreasing noticeably for elements close to the excavated (right) cavern and major principal stress increasing away from the cavern. The excavation of the second benching (stage 3) leads to a more obvious stress concentration shifting toward the pillar elements near the unexcavated (left) cavern. In subsequent stages there is no further σ_1 change of the peripheral elements near the right cavern, indicating that these elements have yielded. The excavation of the left cavern heading introduces an even more obvious stress concentration for the left section of the pillar, with minimal changes to the stresses in the right section of the pillar. The σ_1 values of many monitoring points are below the initial (at-rest) level after stage 5. It can also be observed that the peak σ_1 value has decreased. A further decrease occurs after the excavation of second benching of the left cavern (stage 6) and the peripheral elements near the left cavern have yielded. The decreased peak σ_1 exists in the pillar core, indicating that the pillar core is at the plastic state and thus there is overlap of the plastic zones around the twin caverns, as displayed in Fig. 5.9.

The maximum displacement induced by the overlap of plastic zones around the twin caverns is as large as 0.714 m, compared with the displacement of 0.010 m for the single cavern case. This can be explained as follows: as the peak major principal stress of the pillar core keeps increasing during excavation, this implies that the pillar core is in the elastic state and that the strength has not been fully mobilized, indicating that there is still a certain safety reserve against failure or yielding. In other words, as excavation proceeds, any decrease of the peak σ_1 indicates that the state of the pillar core is in the yielding state (colored in yellow in Fig. 5.9). In

addition, there is overlap of the plastic zones in the pillar, which means that there is substantial cavern interaction.

Figs. 5.8b plots the displacement curves of grid nodes along pillar. It can be observed that the excavation of the right cavern introduce gradual increase of displacements of the measuring points. A sudden increase after the excavation stage 6 (second benching of the left cavern), indicates that twin cavern interaction has occurred.

(a)

(b)

Fig. 5.8 Case $Q=4$, $B=20$ m, $S_c/B=1$, and $K_0=1.0$: (a) σ_1 changes in pillar; (b) displacement curves

Fig. 5.9 Distribution of plastic zones after stage 6 excavation ($Q=4$, $B=20$ m, $S_c/B=1$, and $K_0=1.0$)

Plots of Fig. 5.10a and Fig. 5.12a show the σ_1 changes of slender pillars for Q values of 10 and 40, respectively. In Fig. 5.10a, the decreased peak major principal stress occurs in the pillar core, indicating substantial yielding and cavern interaction as also reflected in the plastic zone distribution in Fig. 5.11. For this case FS_{g_t} is 1.17 while $FS_{g_s}=1.36$. In Fig. 5.11a, as excavation proceeds, σ_1 increases even after stage 6, which indicates a certain degree of strength reserve against pillar failure. Fig. 5.13 indicates that while there is no overlap of the plastic zones, the FS_g has reduced from $FS_{g_s} = 1.63$ to $FS_{g_t} = 1.38$, indicating that there is still some interaction between the twin caverns for this case with S_c/B of 1.

Figs. 5.10b and 5.12b plot the displacement curves of grid points along pillar for Q values of 10 and 40 respectively. As excavation proceeds, the displacement of the measuring points increase gradually. Excavation of the left cavern cause a further increase of displacement of the measuring points near the right cavern, indicating a certain degree of interaction exists.

(a)

(b)

Fig. 5.10 Case $Q=10$, $B=20$ m, $S_c/B=1$, and $K_0=1.0$: (a) σ_1 changes in pillar, (b) displacement curves

Fig. 5.11 Distribution of plastic zones after stage 6 excavation ($Q=10$, $B=20$ m, $S_c/B=1$, and $K_0=1.0$)

(a)

(b)

Fig. 5.12 Case $Q=40$, $B=20$ m, $S_c/B=1$, and $K_0=1.0$: (a) σ_1 changes in pillar, (b) displacement curves

Fig. 5.13 Distribution of plastic zones after stage 6 excavation ($Q=40$, $B=20$ m, $S_c/B=1$, and $K_0=1.0$)

Failure of the rock surrounding underground excavations tends to initiate by a process of spalling in which slabs of rock are formed parallel to the excavation surfaces. Spalling is a process that occurs when the confining stress is low and the rock splits in a direction parallel to the major compressive stress and forms slabs which can dislodge and fall (Stacey 1981). Assessment of the spalling mode of failure shows that extension fractures develop at low confinement, which can be seen as a cohesion weakening process (Hajiabdolmajid et al. 2000). As the confining stress increases, the frictional properties of the rock are mobilized resulting in resistance to shearing. Spalling can initiate at a stress that is much lower than σ_{cm} (Kaiser et al. 2000; Diederichs 2002; Stacey and Yathavan 2003).

Slender pillars behave differently from wide pillars because of the absence of a confined core. In wide pillars, the central core of the pillar is confined by the perimeter material which results in an increase in the overall strength of the pillar. When pillars are slender, this confinement is absent or may be insignificant, resulting in lower pillar strength. Numerical analyses performed by Esterhuizen (2006) indicate that there is little change in pillar strength when S_c/h is below 1.0. As S_c/h increases beyond 1.0, the pillar strength will increase rapidly as confinement is generated (Lunder 1994). This is confirmed by the plots in Figs. 5.14-5.16, in which the distributions of σ_1 after final excavation and the plastic zones for various S_c/B values are plotted for $Q = 4, 10$ and 40 , respectively. It is obvious that when pillars are wide, failure typically starts at the perimeter of the pillar while the central core remains intact and provides resistance to the imposed stress. Pillar stress increases as excavation proceeds on. A wide pillar will progressively fail from the outside inwards. The failed perimeter of the pillar provides confinement to the core, allowing the rock strength to develop. Fig. 5.14 shows that for $K_0 = 1$, with Q value of 4 with cavern span of 20 m and S_c/B values of 1 and 1.5 there is substantial yielding and cavern interaction while a wider pillar of S_c/B value of 2 ensures a certain degree of strength reserve against pillar failure and no overlap of plastic zones. As Q increases (Figs. 5.15 and 5.16), there is minimum cavern interaction with reduction in pillar width. It can be concluded that the ultimate pillar strength and overlap of plastic zones will depend on both pillar geometry S_c/B and rock quality Q .

(a)

(b)

(c)

(d)

Fig. 5.14 σ_1 distribution and plastic zones after stage 6 excavation for $Q=4$: (a) σ_1 distribution, (b) plastic zones for $S_c/B=1$, and (c) plastic zones for $S_c/B=1.5$, and (d) plastic zones for $S_c/B=2$

(a)

(b)

(c)

(d)

Fig. 5.15 σ_1 distribution and plastic zones after stage 6 excavation for $Q=10$: (a) σ_1 distribution, (b) plastic zones for $S_c/B=1$, and (c) plastic zones for $S_c/B=1.5$, and (d) plastic zones for $S_c/B=2$

(a)

(b)

(c)

(d)

Fig. 5.16 σ_1 distribution and plastic zones after stage 6 excavation for $Q=40$: (a) σ_1 distribution, (b) plastic zones for $S_c/B=1$, and (c) plastic zones for $S_c/B=1.5$, and (d) plastic zones for $S_c/B=2$

5.5.3 Vertical stress in pillar core

The analyses indicate that the vertical stress in the pillar core σ_v changes as excavation proceeds. A typical case with $B=10$ m is shown in Fig. 5.17 a-c where σ_v with excavation stages is plotted. It can be observed that, from stage 4 onwards (with excavation on both sides of the pillar), there is greater incremental increase in σ_v . Examination of the plastic zones for these cases indicated that where the maximum peak σ_v occurs at stage 6, there is no overlap of the plastic zones. For the cases in which the peak σ_v occurs before stage 6 and noticeably drops at the subsequent stage, there is substantial cavern interaction and overlap of the plastic

zones. For brevity, only the 6 cases where there is substantial cavern interaction and overlap of the plastic zones are shown in Fig. 5.18.

Fig. 5.17 Change of vertical stress in pillar core ($B=10$ m): (a) $K_0=0.5$, (b) $K_0=1.0$, and (c) $K_0=2$

Fig. 5.18 Distributions of plastic zones for: (a) $K_0=0.5$, $S_c/B=1.0$, $Q=4$, (b) $K_0=0.5$, $S_c/B=1.0$, $Q=10$, (c) $K_0=0.5$, $S_c/B=1.5$, $Q=4$, (d) $K_0=1$, $S_c/B=1.0$, $Q=4$, (e) $K_0=1$, $S_c/B=1.0$, $Q=10$, and (f) $K_0=2$, $S_c/B=1.0$, $Q=4$

5.5.4 Principal stress difference in pillar core

As excavation proceeds, pillar elements are subjected to more compressive loading. Shear failure may occur in the pillar due to the gradual loss in confinement. Thus it is reasonable to assess the twin cavern interaction through investigating the principal stress difference in the pillar core ($\sigma_1 - \sigma_3$) during the various excavation stages. Fig. 5.19 *a-i* plots ($\sigma_1 - \sigma_3$) for each excavation stage.

Figs. 5.19 *a-c* are for the cases of $K_0 = 2$. It can be observed that excavation of the right cavern results in a reduction of $(\sigma_1 - \sigma_3)$. This is because the minor principal stress σ_3 (close to vertical direction) slightly decreases with this excavation. But the major principal stress σ_1 (close to horizontal direction) is subjected to an even greater reduction in magnitude caused by stress distribution under high horizontal in situ stress. Consequently, $(\sigma_1 - \sigma_3)$ gradually decreases. However, $(\sigma_1 - \sigma_3)$ began to increase after excavation stage 3 mainly as a result of stress concentration in pillar. For the cases where the maximum stress difference occurs at stage 6 it was found that there is no overlap of the plastic zones. For the cases in which peak stress difference occurs before stage 6 and either decreases or remains constant subsequently, it was found that there is significant cavern interaction and overlap of the plastic zones. For $K_0=2$, overlap of plastic zones occurred for the following: case $S_c/B=1.0$, $Q=10$, $B=30$ m with $FS_{g-t}=1.04$, case $S_c/B=1.0$, $Q=4$, $B=20$ m with $FS_{g-t}=1.01$, case $S_c/B=1.0$, $Q=10$, $B=20$ m with $FS_{g-t}=1.17$, case $S_c/B=1.5$, $Q=4$, $B=20$ m with $FS_{g-t}=1.06$ and case $S_c/B=1.0$, $Q=4$, $B=10$ m with $FS_{g-t}=1.39$.

Figs. 5.19 *d-f* are for the cases of $K_0=1$. Excavation of the first cavern results in a reduction of σ_3 but an increase of σ_1 . Thus $(\sigma_1 - \sigma_3)$ gradually increases. For $K_0=1$, case $S_c/B=1.0$, $Q=10$, $B=30$ m, case $S_c/B=1.0$, $Q=4$, $B=20$ m, case $S_c/B=1.0$, $Q=10$, $B=20$ m, case $S_c/B=1.0$, $Q=4$, $B=10$ m, and case $S_c/B=1.0$, $Q=10$, $B=10$ m all showed overlap of the plastic zones.

Figs. 5.19 *g-i* are for the cases of $K_0=0.5$. Excavations of twin caverns result in reduction of σ_3 (close to horizontal direction) but a considerable increase of σ_1 (close to vertical direction). Thus $(\sigma_1 - \sigma_3)$ gradually increases. For $K_0=0.5$, case $S_c/B=1.0$, $Q=10$, $B=30$ m, case $S_c/B=1.0$, $Q=4$, $B=20$ m, case $S_c/B=1.0$, $Q=10$, $B=20$ m, case $S_c/B=1.5$, $Q=4$, $B=20$ m, case $S_c/B=1.0$, $Q=4$, $B=10$ m, case $S_c/B=1.0$, $Q=10$, $B=10$ m, and case $S_c/B=1.5$, $Q=4$, $B=10$ m all showed overlap of the plastic zones.

All the cases mentioned previously where there is overlap of the plastic zones have been plotted in Figs. 5.20-5.22.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

Fig. 5.19 ($\sigma_1 - \sigma_3$) change in pillar core: (a) $K_0 = 2, B = 30$ m, (b) $K_0 = 2, B = 20$ m, (c) $K_0 = 2, B = 10$ m, (d) $K_0 = 1, B = 30$ m, (e) $K_0 = 1, B = 20$ m, (f) $K_0 = 1, B = 10$ m, (g) $K_0 = 0.5, B = 30$ m, (h) $K_0 = 0.5, B = 20$ m, (i) $K_0 = 0.5, B = 10$ m

Fig. 5.20 Distributions of plastic zones after stage 6 excavation for $K_0 = 2$: (a) $S_c/B=1.0$, $Q=10$, $B=30$ m, (b) $S_c/B=1.0$, $Q=4$, $B=20$ m, (c) $S_c/B=1.0$, $Q=10$, $B=20$ m, (d) $S_c/B=1.5$, $Q=4$, $B=20$ m, (e) $S_c/B=1$, $Q=4$, $B=10$ m

Fig. 5.21 Distributions of plastic zones after stage 6 excavation for $K_0 = 1$: (a) $S_c/B=1.0$, $Q=10$, $B=30$ m, (b) $S_c/B=1$, $Q=4$, $B=20$ m, (c) $S_c/B=1.0$, $Q=10$, $B=20$ m, (d) $S_c/B=1.0$, $Q=4$, $B=10$ m, and (e) $S_c/B=1.0$, $Q=10$, $B=10$ m

Fig. 5.22 Distributions of plastic zones after stage 6 excavation for $K_0 = 0.5$: (a) $S_c/B=1.0$, $Q=10$, $B=30$ m, (b) $S_c/B=1.0$, $Q=4$, $B=20$ m, (c) $S_c/B=1.0$, $Q=10$, $B=20$ m, (d) $S_c/B=1.5$, $Q=4$, $B=20$ m, (e) $S_c/B=1$, $Q=4$, $B=10$ m, (f) $S_c/B=1$, $Q=10$, $B=10$ m, and (g) $S_c/B=1.5$, $Q=4$, $B=10$ m

5.5.5 ANN Model for determining overlap of the plastic zones

As overlap of plastic zones results in substantial cavern interaction, this section describes the procedure used to develop simple charts to assess if overlap of plastic zones will occur, through the use of an ANN model. For the case in which there is overlap of the plastic zones, the dependent response $H(X)$ is assigned a value of 1, while for the case without overlap of the yielding zones, a value of 0 is assigned to $H(X)$. Tables 5.8-5.10 list these numerical observations on distribution of the plastic zones for $K_0=0.5$, 1, and 2, respectively.

Using the data from Tables 5.8-5.10, a BPNN model was developed to model $H(X)$ as a function of B , Q , S_c/B , and K_0 . Four inputs that represented B , Q , S_c/B , and K_0 are the neurons in the input layer. Of the 150 observations, 113 data samples were randomly selected as the training data and the remaining 37 data sets were used for

test the validity of the developed neural network. The optimal BPNN model consisted of ten hidden neurons. The transfer functions adopted are ‘*logsig*’ transfer function for hidden layer to output layer and ‘*tansig*’ transfer function for output layer to target. The testing data sets together with the BPNN predictions are listed in Table 5.11. The training data sets together with the BPNN predictions are shown in Appendix O.

Table 5.8 Observations on distribution of the plastic zones for $K_0=0.5$

B (m)	Q	$H(X)$ (1: overlap, 0: none)			
		$S_c/B=1$	$S_c/B=1.5$	$S_c/B=2$	$S_c/B=2.5$
30	100	0	0	0	0
	40	0	0	0	0
	10	1	1	0	0
	4	–	–	1	0
20	100	0	0	0	0
	40	1	0	0	0
	10	1	1	0	0
	4	1	1	0	0
10	100	0	0	0	0
	40	0	0	0	0
	10	1	0	0	0
	4	1	1	0	0
	1	1	1	1	1

Table 5.9 Observations on distribution of the plastic zones for $K_0=1$

B (m)	Q	Distribution of plastic zones (1: overlap, 0: none)			
		$S_c/B=1$	$S_c/B=1.5$	$S_c/B=2$	$S_c/B=2.5$
30	100	0	0	0	0
	40	0	0	0	0
	10	1	0	0	0
	4	–	–	0	0
20	100	0	0	0	0
	40	0	0	0	0
	10	1	0	0	0
	4	1	1	0	0
10	100	0	0	0	0
	40	0	0	0	0
	10	1	0	0	0
	4	1	1	0	0
	1	1	1	1	1

Table 5.10 Observations on distribution of the plastic zones for $K_0=2$

B (m)	Q	Distribution of plastic zones (1: overlap, 0: none)			
		$S_c/B=1$	$S_c/B=1.5$	$S_c/B=2$	$S_c/B=2.5$
30	100	0	0	0	0
	40	0	0	0	0
	10	1	0	0	0
	4	–	–	1	0
20	100	0	0	0	0
	40	0	0	0	0
	10	1	0	0	0
	4	1	1	0	0
10	100	0	0	0	0
	40	0	0	0	0
	10	1	0	0	0
	4	1	0	0	0
	1	1	1	1	0

Table 5.11 Testing data sets of $H(X)$

B (m)	Q	S_c/B	K_0	Target $H(X)$	$H(X)_{BPNN}$	B (m)	Q	S_c/B	K_0	Target $H(X)$	$H(X)_{BPNN}$
30	100	1.5	0.5	0	0	20	4	2	2	0	0
30	40	2	0.5	0	0	10	100	1.5	2	0	0
20	100	1.5	0.5	0	0	10	40	2	2	0	0
20	4	2	0.5	0	0	10	10	1.5	2	0	0
10	100	1.5	0.5	0	0	10	1	2	2	1	1
10	40	2	0.5	0	0	30	10	2	0.5	0	0
10	10	1.5	0.5	0	0	30	10	1.5	1	0	0
10	1	2	0.5	1	1	30	10	1.5	2	0	1
30	100	1.5	1	0	0	30	4	2.5	1	0	0
30	40	2	1	0	0	20	40	1.5	0.5	0	0
20	100	1.5	1	0	0	20	40	2	1	0	0
20	4	2	1	0	0	20	40	1.5	2	0	0
10	100	1.5	1	0	0	20	10	2	0.5	0	0
10	40	2	1	0	0	20	10	1.5	1	0	0
10	10	1.5	1	0	0	20	10	2	2	0	0
10	1	2	1	1	1	10	4	1.5	0.5	1	1
30	100	1.5	2	0	0	10	4	2	1	0	0
30	40	2	2	0	0	10	4	1.5	2	0	0
20	100	1.5	2	0	0						

The results in Table 5.11 and Appendix O show that the built BPNN model had an overall accuracy of 99.3%, with no error in the training patterns and one in the testing patterns. The partitioning of BPNN weights for $H(X)$ is shown in Appendix P and the mathematical expression of this model is shown in Appendix Q. Based on this BPNN model, a series of simple charts have been developed as shown in Fig. 5.23. These proposed charts are potentially useful for preliminary design to assess if substantial cavern interaction will occur.

(a)

(b)

(c)

Fig. 5.23 Design charts to assess no overlap of plastic zones: (a) $B=30$ m, (b) $B=20$ m, and (c) $B=10$ m

5.5.6 Ratio of shear strength to shear stress τ_f/τ

In order to better illustrate the influence of B , Q , S_c/B , and K_0 on the relationship between the shear strength and mobilized shear stress, the ratio of shear strength to mobilized shear stress τ_f/τ is presented in this section. τ_f/τ is defined as

$$\frac{\tau_f}{\tau} = \frac{c + \sigma \tan \phi}{\frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\alpha_f} \quad (5.3)$$

in which c is the cohesion; ϕ is the friction angle (both are determined by Q through empirical equations) α_f is defined as $(90^\circ + \phi)/2$ and the normal stress σ is calculated as

$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3) \cos 2\alpha_f \quad (5.4)$$

σ_1 and σ_3 are the principal stresses after excavation stage 6. Fig. 5.24 *a-c* plot τ_f/τ of pillar core (point 0 in Fig. 5.8) for $B=10, 20$ and 30 m respectively. It can be observed that τ_f/τ is significantly influenced by S_c/B as the shear stress of pillar core will be lower for pillars of greater width. A significant increase of τ_f/τ occurs when S_c/B is further increased to 2.0 or 2.5. The τ_f/τ is also influenced by Q as a pillar with large Q value is more competent, indicating a higher resistance to shear loading. Influences of K_0 on τ_f/τ are complicated, and specific to the cavern geometries.

Fig. 5.25 *a-c* plot the normalized strength τ/τ_f of pillar core for $B=10, 20$ and 30 respectively. It is obvious that τ/τ_f is essentially influenced by S_c/B . A τ/τ_f value of 0.7 is observed for $S_c/B=1.0$ for case $B=30$ m, $K_0=2$, and $Q=40$. A considerable decrease of τ/τ_f occurs when S_c/B value is increased to 2.0 or 2.5.

(a)

(b)

(c)

Fig. 5.24 τ_f / τ of pillar core: (a) $B=10$ m, (b) $B=20$ m, (c) $B=30$ m

(a)

(b)

(c)

Fig. 5.25 Normalized strength: (a) $B=10$ m, (b) $B=20$ m, (c) $B=30$ m

5.5.7 Average pillar strength and pillar stress calculation

For comparison, pillar strength concepts based on research from the mining industry were also considered. Estimating pillar strength is more difficult and has been the subject of much research in the mining industry (Martin and Maybee 2000, Hustrulid 1976, Esterhuizen 2006). Due to the complexity of pillar mechanics, empirically established pillar strength equations, which are based on the statistics of failed and stable pillar systems, have found wide acceptance (Mark 1999). Very few pillar failures have occurred in stone mines, and consequently the field data alone are inadequate to develop a statistically based pillar strength equation. An empirically developed pillar strength estimation method that was originally developed by Roberts et al. (2007) was used as a starting point in this thesis. The rock strength, rock mass rating and mining dimensions are similar to those found in stone mines (Esterhuizen 2008). In the method by Roberts et al. (2007) the pillar strength (S) is expressed as a power equation incorporating the UCS of the rock, the pillar width (W) and the pillar height (h) as follows:

$$S = 0.65 \times UCS \times \frac{W^{0.30}}{h^{0.59}} \quad (5.5)$$

Eq. (5.5) was adopted to calculate the average strength of pillar between twin caverns of horse-shoe shape. The UCS values are related to Q through the empirical equations in Table 3.2. W has the same meaning as S_c . h is the height of rectangular or square pillars. For the irregular shape cavern in this study, lower and upper bound values are utilized. The lower bound of h is the height of the side wall, which is the half of the cavern span. The upper bound equals to the sum of the heights of side wall H and the roof arch H_c . The results based on Eq. (5.5) are presented in Fig. 5.26, which shows the average pillar strength increases with Q . For the same Q , the average pillar strength increases as the pillar broadens.

(a)

(b)

(c)

Fig. 5.26 Average pillar strength: (a) $B=10$ m, (b) $B=20$ m, (c) $B=30$ m

The average stress (σ_p) on a square post pillar, using the tributary area theory (Hoek and Brown 1980), can be calculated as

$$\sigma_p = \gamma z \left(1 + \frac{W_0}{W_p}\right)^2 \quad (5.6)$$

in which γ is the unit weight of the overlying rock, z is the depth below ground surface, W_0 is the excavation width and W_p is the pillar width.

Eq. (5.6) was adopted to calculate the pillar stress, assuming that W_0 equals to cavern span B and W_p equals to S_c . The results based on Eq. (5.6) are displayed in Fig. 5.27, which shows the pillar stress decreases as pillar width broadens.

Fig. 5.27 The average stress in pillar

Traditional strength-based pillar design methods applicable to hard-rock mines uses a factor of safety (FS_p) defined as average pillar strength (S) (average of the upper and lower bounds) divided by average pillar stress (σ_p):

$$FS_p = S / \sigma_p \quad (5.7)$$

Based on Eq. (5.7), FS_p values were calculated for each numerical data set with $FS_{g,t}$ greater than 1, and are listed in Table 5.12. Fig. 5.28 displays the plot of FS_p against S_c/B . Esterhuizen et al. (2011a) suggested that to ensure that the pillar is stable, a pillar factor of safety of at least 1.8 is recommended. It is obvious that pillars having a S_c/B of less than or equal to 1 should be strictly avoided for $Q < 10$.

Table 5.12 FS_p values based on Eq. (5.7)

B (m)	Q	FS_p			
		$S_c/B=1$	$S_c/B=1.5$	$S_c/B=2$	$S_c/B=2.5$
30	100	2.35	3.82	5.14	6.31
	40	2.13	3.46	4.66	5.72
	10	1.8	2.92	3.93	4.82
	4	—	—	3.45	4.23
20	100	2.73	4.43	5.97	7.32
	40	2.47	4.02	5.41	6.64
	10	2.08	3.39	4.56	5.59
	4	1.83	2.97	4	4.91
10	100	3.44	5.60	7.53	9.24
	40	3.12	5.07	6.82	8.38
	10	2.63	4.27	5.75	7.06
	4	2.31	3.75	5.05	6.19
	1	1.82	2.95	3.98	4.88

Fig. 5.28 The calculated FS_p values using Eqs. (5.5)-(5.7): (a) $B=10$ m, (b) $B=20$ m, (c) $B=30$ m

Based on Tables 5.3, 5.8-5.10, and 5.12, simplified charts relating the overlap of the plastic zones to FS_{g_t} and FS_p are developed, as shown in Fig. 5.29, to enable a quick and simple way of estimating when overlap of plastic zones will occur (FS_{g_t} can be estimated using Eq. (5.1) and FS_p is calculated using Eq. (5.7)). For $B=10$ m, when FS_{g_t} is less than about 1.62, the FS_p value should be greater than 5.0 to avoid overlap of the plastic zones. For $B=20$ m, when FS_{g_t} is less than 1.45, a FS_p value of at least 3.5 is needed. For $B=30$ m, when FS_{g_t} is less than 1.15, a FS_p value of at least 3.5 should be provided to ensure that no overlap of the plastic zones occur. These charts are an alternative to Fig. 5.23.

Fig. 5.29 Design charts utilizing FS_{g_t} and FS_p : (a) $B=10$ m, (b) $B=20$ m, (c) $B=30$ m

5.6 Probabilistic assessments on ULS

Two expressions of the global FS : Eqs. (5.1) and (5.2) are available. Assuming the critical FS_{g_t} value is $FS_{g_t_cr}$, the ultimate limit state function based on LR model is then given by

$$g(\mathbf{x}) = FS_{g_t_LR} - FS_{g_t_cr} = 2.3044Q^{0.1404}B^{-0.3331}(S_c / B)^{0.1393} - FS_{g_t_cr} \quad (5.8)$$

Alternatively, the ultimate limit state function of the MARS model is:

$$\begin{aligned} g(\mathbf{x}) = FS_{g_t_MARS} - FS_{g_t_cr} = & 1.322 - 0.022 * BF1 - 0.011 * BF2 + \\ & 0.048 * BF3 + 0.108 * BF4 - 0.17 * BF5 - 0.0006 * BF6 + 0.003 * BF7 - \\ & 0.05 * BF8 + 0.017 * BF9 - 0.003 * BF10 - FS_{g_t_cr} \end{aligned} \quad (5.9)$$

The expressions of BFs in this equation can be found in Table 5.7. With the determination of the limit state functions - Eqs. (5.8) and (5.9), reliability assessment of the FS_{g_t} can be carried out using MCS, as shown in Fig. 5.30 ($FS_{g_t_cr}$ is assumed as 1). Failure in terms of global stability occurs if the predicted FS_{g_t} is smaller than unity. For the illustrative purpose, the assumed statistical information of the input variables is listed in Table 5.13.

(a)

(b)

Fig. 5.30 Implementation of models into MCS: (a) LR, and (b) MARS

Table 5.13 Statistical information of the input variables for reliability assessment on ULS

Variable	Distribution type	Mean	COV
Q	Normal	4, 10	0.1, 0.2, 0.3, 0.4
B	Normal	10, 20, 30	0.1
S_c/B	Normal	1, 1.5, 2, 2.5	0.1

Fig. 5.31a and 5.31b plot the influences of the choice of $FS_{g_t_cr}$ on the calculated P_f using the LR and MARS models, respectively. It is obvious that $FS_{g_t_cr}$ substantially influences P_f . Fig. 5.31c and 5.31d plot the influences of design parameter statistics on P_f , assuming $FS_{g_t_cr}$ value of 1. The results indicate that both

the COV and the average value of Q significantly influence P_f . It is also clear that P_f of a cavern of large span is much higher than that of a small one. In addition, P_f decreases with the increase of S_c/B .

(a)

(b)

(c)

(d)

Fig. 5.31 Influence of design parameters on P_t : (a) the choice of $FS_{g_t_cr}$ using LR, (b) the choice of $FS_{g_t_cr}$ using MARS, (c) COV of Q , mean values of B and Q and (d) S_c/B and mean values of B and Q

5.7 Concluding remarks

Numerical simulations are performed to investigate the interaction between two adjacent horse-shoe shaped caverns in terms of the global FS . The pillar performances with respect to the maximum principal stress in pillar, the vertical

stress and the principal stress difference in pillar core, and the average pillar strength and average stress are studied in order to examine the failure mechanisms and to identify the situations in which there is substantial cavern interaction and the overlap of the plastic zones.

For the global stability, LR and MARS models were developed to predict FS_{g-t} in terms of Q , B and S_c/B . A series of charts relating FS_{g-t} to Q , B and S_c have been developed, which are potentially useful for preliminary design and checking of twin cavern stability. Probabilistic assessments on ULS can be performed once the limit state functions are determined and statistics of the design parameters are given. Probabilistic assessments on ULS are carried out based on MCS and the limit state functions derived by LR and MARS model are used as a comparison.

In summary, this study of probabilistic assessment on ultimate limit state of twin caverns illustrated that:

- (1) For the same Q and B , FS_{g-t} increases with the broadening of the pillar width.
- (2) LR and MARS models are built to predict the global FS for twin caverns. Curves of design charts based on MARS model exhibits similar trends as the curves developed using LR. The proposed charts are potentially useful for preliminary design and checking of twin cavern stability.
- (3) It can be observed that B is the most important parameter in determining the global FS of twin caverns, followed by Q .
- (4) Pillar performances with respect to the changes of maximum principal stress in pillar, the vertical stress and the principal stress difference in pillar core, and the average pillar strength and average stress are studied in order to examine the failure mechanisms and to identify situations in which there is substantial cavern interaction and the overlap of the plastic zones.
- (5) An ANN model was built to determine overlap of the plastic zones using the basic input parameters, and simple charts relating the plastic zone distribution function $H(X)$ to Q , B , S_c/B and K_0 have been developed. The charts can be used to assess if there is overlap of the plastic zones.

- (6) The safety of factor for pillar FS_p derived from Eq. (5.7) is consistent with the FS_{g_t} obtained through FDM. To avoid overlap of the plastic zones, the relationship between FS_p and FS_{g_t} is proposed.
- (7) The built LR and MARS models can be implemented into MCS to calculate the probability of failure for twin cavern stability.
- (8) The choice of $FS_{g_t_cr}$, and the average value and COV of Q significantly influence P_f . It is logical that P_f of a cavern of large span is much higher than that of a small one. In addition, P_f decreases with the increase of S/B .

Chapter 6

RELIABILITY ASSESSMENT ON SERVICEABILITY LIMIT STATE AND SYSTEM RELIABILITY OF TWIN CAVERNS

6.1 Introduction

The ultimate limit state failure of twin caverns and overlap of plastic zones was considered in Chapter 5. The serviceability limit state, which usually involves excessive movements, should also be considered. For the serviceability limit criterion, the maximum key point displacement and the percent strain approach adopted in Chapter 4 are used. For each criterion, a MARS model was built. Probabilistic assessments on serviceability limit state are performed using FORM and MCS incorporated with the built percent strain MARS model. To ensure the system reliability, the SLS and ULS should be evaluated concurrently, through satisfying the minimum FS_{g_t} value and at the same time the target performance levels of serviceability limit state.

6.2 Numerical modeling and the dependent responses

The basic assumptions, cross-section and boundary conditions for twin cavern analysis are the same as those used for the ultimate limit state considerations in Chapter 5. The cases with FS_{g_t} less than the unity in Table 5.3 are not considered as collapse (failure of ULS) occurs when FS_{g_t} is less than 1.0.

From the numerical results, the maximum key point displacement of the twin caverns u_{max_t} and the percent strain ε_i are obtained. u_{max_t} is defined as $max(u_{CR}, u_{SR}, u_{MR}, u_{IR}, u_{CL}, u_{SL}, u_{ML}, u_{IL})$, in which u_{CR} , u_{SR} , u_{MR} , and u_{IR} are the displacements of peripheral nodes C, S, M and I of the right cavern (firstly excavated), respectively, while u_{CL} , u_{SL} , u_{ML} , and u_{IL} are the displacements of peripheral nodes C, S, M and I

of the left cavern. The percent strain is derived the same way as the single cavern case in Chapter 4. The only difference is that more peripheral nodes around the twin caverns are involved, as shown in Fig. 6.1.

Fig. 6.1 Details of cavern peripheral nodes between twin caverns

As illustrated in Fig. 6.1, the displacements of 32 nodes are obtained from the numerical analyses from which the percent strain is determined.

6.3 Modeling results and analyses of $u_{\max_t}$

During construction, the displacements of the key points around the cavern like the crown or the invert are usually continuously monitored. Based on the magnitude and the changing trend of these key point displacements, the stability of excavations is assessed. In this section, the influences of S_c/B , Q , B and K_0 on key point displacements of the twin caverns are investigated. These key points include the crown, the springline, the middle side wall, and the invert of the right cavern and of the left cavern, denoted as CR, SR, MR, IR, CL, SL, ML and IL respectively.

6.3.1 Modeling results of $u_{\max_t}$

Fig. 6.2 *a-c* shows the key point displacement with the excavation stages for the case of $S_c/B=1.5$, $Q=4$, $B=20$ m under $K_0 = 2$, 1, and 0.5 respectively. It can be observed that under high in situ stress ratio ($K_0 = 2$), the key point displacements are relatively larger than those for $K_0 = 1$ and 0.5. The largest key point displacement is about 0.04 m and occurs at the crown of the firstly excavated

cavern. It is also clear that large displacements of the key points around the right cavern have already occurred before the start of the left cavern excavation (stage 4) as a result of a high horizontal stress release during excavation. For the cases of $K_0 = 1$ and 0.5, displacements of the key points around the left cavern are small before stage 4 and for these two cases, the largest displacement occurs at the springline of the firstly excavated cavern.

Fig. 6.2 Key point displacements with excavation stage for $S_c/B=1.5$, $Q=4$, $B=20$ m: (a) $K_0 = 2$, (b) $K_0 = 1$, and (c) $K_0 = 0.5$

Tables 6.1-6.3 list the key point displacements of the numerical cases with FS_{g_t} greater than 1.0 for $B = 10, 20$, and 30 m, respectively. Since the node displacements for cases with FS_{g_t} less than the unity are large as failure of ULS has occurred, these cases have been omitted. The last column in Tables 6.1-6.3 includes $u_{\max_t}$ for each case.

Table 6.1 Key point displacements and $u_{\max_t}$ for $B = 10$ m

Q	K_0	S_c (m)	Displacement of key points (m)								$u_{\max_t}$ (m)
			CR	SR	MR	IR	CL	SL	ML	IL	
1	0.5	10	0.023	0.030	0.025	0.011	0.022	0.009	0.010	0.010	0.030
		15	0.022	0.031	0.027	0.010	0.021	0.017	0.014	0.009	0.031
		20	0.018	0.026	0.023	0.009	0.018	0.019	0.016	0.008	0.026
		25	0.015	0.020	0.017	0.017	0.008	0.015	0.014	0.008	0.020
	1	10	0.025	0.030	0.025	0.016	0.022	0.006	0.009	0.013	0.030
		15	0.028	0.028	0.025	0.015	0.020	0.014	0.010	0.012	0.028
		20	0.025	0.025	0.022	0.014	0.019	0.019	0.014	0.012	0.025
		25	0.019	0.019	0.017	0.013	0.016	0.017	0.014	0.011	0.019
	2	10	0.093	0.087	0.076	0.068	0.058	0.010	0.022	0.042	0.093
		15	0.078	0.068	0.063	0.060	0.051	0.022	0.018	0.040	0.078
		20	0.067	0.048	0.046	0.054	0.045	0.025	0.018	0.038	0.067
		25	0.061	0.035	0.034	0.052	0.040	0.016	0.013	0.036	0.061
4	0.5	10	0.007	0.009	0.008	0.005	0.007	0.006	0.005	0.005	0.009
		15	0.005	0.006	0.005	0.004	0.005	0.005	0.005	0.004	0.006
		20	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.004	0.004
		25	0.004	0.003	0.003	0.004	0.004	0.003	0.003	0.004	0.004
	1	10	0.007	0.009	0.008	0.007	0.007	0.005	0.005	0.006	0.009
		15	0.006	0.006	0.005	0.006	0.005	0.005	0.004	0.005	0.006
		20	0.005	0.004	0.004	0.005	0.005	0.004	0.004	0.005	0.005
		25	0.005	0.004	0.004	0.005	0.004	0.004	0.004	0.005	0.005
	2	10	0.018	0.016	0.014	0.019	0.014	0.004	0.005	0.014	0.019
		15	0.016	0.009	0.010	0.018	0.012	0.004	0.003	0.013	0.018
		20	0.015	0.008	0.009	0.017	0.011	0.004	0.004	0.013	0.017
		25	0.015	0.008	0.009	0.016	0.011	0.005	0.005	0.013	0.016
10	0.5	10	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
		15	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
		20	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.002
		25	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
	1	10	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.003	0.003
		15	0.003	0.002	0.002	0.003	0.002	0.002	0.002	0.003	0.003
		20	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.003	0.003
		25	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.002	0.003
	2	10	0.007	0.005	0.005	0.009	0.005	0.001	0.002	0.006	0.009
		15	0.007	0.004	0.004	0.008	0.005	0.002	0.002	0.006	0.008
		20	0.006	0.004	0.004	0.007	0.005	0.002	0.002	0.006	0.007
		25	0.005	0.004	0.004	0.007	0.004	0.003	0.003	0.006	0.007
40	0.5	10	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		15	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		20	0.001	0.001	0.000	0.001	0.001	0.001	0.000	0.001	0.001
		25	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
	1	10	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		15	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		20	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		25	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	2	10	0.002	0.002	0.002	0.003	0.002	0.001	0.001	0.002	0.003
		15	0.002	0.002	0.002	0.003	0.002	0.001	0.001	0.002	0.003
		20	0.002	0.002	0.002	0.003	0.001	0.001	0.001	0.002	0.003
		25	0.002	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.002

Table 6.1 Key point displacements and $u_{\max_t}$ for $B = 10$ m contd.

Q	K_0	S_c (m)	Displacement of key points (m)								$u_{\max_t}$ (m)
			CR	SR	MR	IR	CL	SL	ML	IL	
100	0.5	10	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
		15	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
		20	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
		25	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
	1	10	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
		15	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
		20	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
		25	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
	2	10	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.002
		15	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		20	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		25	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Table 6.2 Key point displacements and $u_{\max_t}$ for $B = 20$ m

Q	K_0	S_c (m)	Displacement of key points (m)								$u_{\max_t}$ (m)
			CR	SR	MR	IR	CL	SL	ML	IL	
4	0.5	20	0.657	0.711	0.634	0.010	0.658	0.685	0.583	0.010	0.711
		30	0.022	0.028	0.024	0.008	0.022	0.023	0.016	0.008	0.028
		40	0.013	0.013	0.010	0.008	0.013	0.011	0.008	0.008	0.013
		50	0.011	0.009	0.007	0.007	0.011	0.009	0.006	0.007	0.011
	1	20	0.666	0.714	0.638	0.012	0.664	0.695	0.615	0.011	0.714
		30	0.019	0.023	0.020	0.011	0.018	0.019	0.012	0.010	0.023
		40	0.013	0.012	0.010	0.010	0.012	0.011	0.008	0.009	0.013
		50	0.012	0.011	0.009	0.010	0.011	0.010	0.007	0.009	0.012
	2	20	0.673	0.711	0.638	0.037	0.662	0.677	0.594	0.027	0.711
		30	0.039	0.028	0.027	0.034	0.028	0.015	0.010	0.025	0.039
		40	0.036	0.021	0.021	0.032	0.024	0.010	0.008	0.024	0.036
		50	0.034	0.021	0.020	0.031	0.024	0.012	0.010	0.024	0.034
10	0.5	20	0.010	0.012	0.010	0.006	0.010	0.009	0.006	0.006	0.012
		30	0.007	0.005	0.004	0.005	0.007	0.005	0.004	0.005	0.007
		40	0.006	0.004	0.003	0.005	0.006	0.004	0.003	0.005	0.006
		50	0.005	0.004	0.003	0.005	0.005	0.001	0.003	0.005	0.005
	1	20	0.010	0.012	0.010	0.007	0.009	0.009	0.005	0.006	0.012
		30	0.007	0.006	0.005	0.006	0.006	0.005	0.004	0.005	0.007
		40	0.006	0.005	0.004	0.005	0.006	0.005	0.004	0.005	0.006
		50	0.006	0.005	0.004	0.005	0.005	0.005	0.004	0.005	0.006
	2	20	0.017	0.015	0.014	0.017	0.014	0.006	0.004	0.013	0.017
		30	0.015	0.010	0.009	0.016	0.010	0.005	0.004	0.012	0.016
		40	0.013	0.009	0.009	0.015	0.010	0.006	0.005	0.012	0.015
		50	0.012	0.009	0.009	0.014	0.010	0.007	0.006	0.012	0.014
40	0.5	20	0.003	0.002	0.002	0.003	0.003	0.002	0.002	0.003	0.003
		30	0.003	0.002	0.001	0.002	0.003	0.001	0.001	0.002	0.003
		40	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
		50	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
	1	20	0.003	0.002	0.002	0.003	0.003	0.002	0.002	0.003	0.003
		30	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.003	0.003
		40	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
		50	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	2	20	0.005	0.004	0.004	0.007	0.004	0.002	0.002	0.005	0.007
		30	0.004	0.004	0.004	0.006	0.003	0.002	0.002	0.005	0.006
		40	0.004	0.004	0.004	0.006	0.003	0.003	0.003	0.005	0.006
		50	0.004	0.004	0.004	0.005	0.003	0.003	0.003	0.005	0.005
100	0.5	20	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
		30	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
		40	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
		50	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.002
	1	20	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
		30	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.002
		40	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.002
		50	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.002
	2	20	0.002	0.002	0.002	0.003	0.002	0.001	0.001	0.003	0.003
		30	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.003	0.003
		40	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.003	0.003
		50	0.002	0.003	0.002	0.003	0.002	0.002	0.002	0.003	0.003

Table 6.3 Key point displacements and $u_{\max_t}$ for $B = 30$ m

Q	K_0	S_c (m)	Displacement of key points (m)								$u_{\max_t}$ (m)
			CR	SR	MR	IR	CL	SL	ML	IL	
4	0.5	60	0.027	0.025	0.018	0.010	0.026	0.024	0.016	0.010	0.027
		75	0.019	0.015	0.010	0.010	0.019	0.015	0.010	0.010	0.019
	1	60	0.025	0.022	0.017	0.013	0.022	0.019	0.013	0.012	0.025
		75	0.021	0.018	0.014	0.013	0.019	0.016	0.012	0.012	0.021
	2	60	0.061	0.039	0.035	0.042	0.037	0.022	0.016	0.031	0.061
		75	0.054	0.034	0.030	0.040	0.033	0.020	0.016	0.031	0.054
10	0.5	30	0.031	0.033	0.020	0.007	0.032	0.028	0.016	0.007	0.033
		45	0.011	0.009	0.006	0.006	0.011	0.008	0.005	0.006	0.011
		60	0.009	0.006	0.005	0.006	0.009	0.006	0.004	0.006	0.009
		75	0.008	0.006	0.004	0.006	0.008	0.005	0.004	0.006	0.008
	1	30	0.016	0.017	0.012	0.008	0.015	0.013	0.007	0.007	0.017
		45	0.011	0.009	0.007	0.008	0.010	0.007	0.005	0.007	0.011
		60	0.009	0.008	0.006	0.007	0.009	0.007	0.005	0.007	0.009
		75	0.009	0.007	0.006	0.007	0.008	0.007	0.005	0.007	0.009
	2	30	0.027	0.022	0.019	0.021	0.021	0.011	0.006	0.016	0.027
		45	0.023	0.015	0.014	0.021	0.016	0.008	0.006	0.016	0.023
		60	0.020	0.014	0.013	0.020	0.014	0.009	0.008	0.016	0.020
		75	0.018	0.014	0.013	0.019	0.014	0.010	0.010	0.015	0.019
40	0.5	30	0.005	0.003	0.002	0.003	0.005	0.003	0.002	0.003	0.005
		45	0.004	0.002	0.002	0.003	0.004	0.002	0.002	0.003	0.004
		60	0.004	0.002	0.001	0.003	0.004	0.002	0.001	0.003	0.004
		75	0.004	0.002	0.001	0.003	0.004	0.002	0.001	0.003	0.004
	1	30	0.004	0.003	0.002	0.004	0.004	0.003	0.002	0.003	0.004
		45	0.004	0.003	0.002	0.004	0.004	0.003	0.002	0.003	0.004
		60	0.003	0.003	0.002	0.003	0.003	0.003	0.002	0.003	0.003
		75	0.003	0.003	0.002	0.003	0.003	0.003	0.002	0.003	0.003
	2	30	0.007	0.006	0.005	0.008	0.005	0.002	0.002	0.006	0.008
		45	0.006	0.006	0.005	0.008	0.005	0.004	0.003	0.007	0.008
		60	0.006	0.006	0.006	0.008	0.005	0.004	0.004	0.006	0.008
		75	0.005	0.006	0.006	0.007	0.004	0.005	0.005	0.006	0.007
100	0.5	30	0.003	0.001	0.001	0.002	0.003	0.001	0.001	0.002	0.003
		45	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
		60	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
		75	0.002	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002
	1	30	0.002	0.002	0.001	0.002	0.002	0.001	0.001	0.002	0.002
		45	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.002
		60	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.002
		75	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.002
	2	30	0.003	0.003	0.003	0.004	0.003	0.002	0.002	0.004	0.004
		45	0.003	0.003	0.003	0.004	0.002	0.002	0.002	0.004	0.004
		60	0.003	0.003	0.003	0.004	0.002	0.003	0.003	0.003	0.004
		75	0.002	0.004	0.004	0.004	0.002	0.003	0.003	0.003	0.004

Among all the cases listed in Tables 6.1-6.3, the key point displacements for cases with $B=20$, $Q=4$, $S_c=20$ ($K_0=0.5$, 1 and 2) are extremely large ($FS_{g_t}=1.01$). The maximum displacements for these three cases are 0.711, 0.714 and 0.711 m respectively. A further look at the distribution of the plastic zones in Chapter 5 (Figs. 5.20b, 5.21b, and 5.22b) indicates extensive zones of yielding, which means that failure is imminent. Based on the numerical results in Tables 6.1-6.3, Figs. 6.3-6.5 show the influence of pillar geometry S_c/B on $u_{\max_t}$, compared with the single cavern case for $B=10$, 20 and 30 m respectively. The greatest difference between excavation of single cavern and the twin caverns lies in the fact that, for the latter, the excavation of the left cavern influences deformation of the right cavern as a result of significant interaction between the twin caverns. It is obvious that generally with increase of S_c/B , $u_{\max_t}$ converges to $u_{\max_s}$, indicating that the interaction attenuates with increasing of spacing. For different cases, interaction diminishes at different S_c/B values. However, in all cases, interaction is considerable for a S_c/B value of 1.0. $u_{\max_t}$ decreases as Q increases since higher Q corresponds to greater deformation modulus and increased strength of the rock mass. In addition, for the same Q value, $u_{\max_t}$ is significantly influenced by K_0 . For each plot, the $u_{\max_t}$ value under $K_0=2$ is much higher than those under $K_0=1$ and $K_0=0.5$ since higher K_0 value results in larger horizontal stress release during excavation.

(a)

(b)

(c)

(d)

(e)

Fig. 6.3 Influence of K_0 , S_c , and Q on $u_{\max,t}$ for $B = 10$ m: (a) $Q=1$, (b) $Q=4$, (c) $Q=10$, (d) $Q=40$, and (e) $Q=100$

(a)

(b)

(c)

(d)

Fig. 6.4 Influence of K_0 , S_c , and Q on u_{max_t} for $B = 20$ m: (a) $Q=4$, (b) $Q=10$, (c) $Q=40$, and (d) $Q=100$

(a)

(b)

(c)

(d)

Fig. 6.5 Influence of K_0 , S_c , and Q on $u_{\max,t}$ for $B = 30$ m: (a) $Q=4$, (b) $Q=10$, (c) $Q=40$, and (d) $Q=100$

It should also be noted from Figs. 6.3-6.5 that for high Q values ($Q > 10$) and $K_0 = 2$, the $u_{\max_t}$ values are generally lower than $u_{\max_s}$. This is likely to be because of rock cavern roof arching effects for excavations carried out in rocks of considerable quality and high in situ horizontal stress. The high in situ stress favours the maintainance of the arch shape by decreasing the roof settlement during excavation. For twin cavern excavation with considerable pillar width, the arching effect is more obvious and favourable, compared with the single cavern case.

6.3.2 Influences of overlap of the plastic zones on cavern displacements

The influence of overlap of the plastic zones on the cavern displacement was investigated in this section. Fig. 6.6 *a-c* show the $u_{\max_t}$ values of a few examples for $K_0=2$, 1 and 0.5, respectively. In Fig. 6.6, cases with overlap of the plastic zones are denoted by the solid symbols, while the hollow symbols represent the cases in which there is no overlap of the yielding zones. For $K_0=0.5$ and 1.0, the cavern displacement is generally substantially larger for the overlap cases compared with no overlap cases. With increase of S_c/B , less and less interaction (and overlapping) takes place resulting in a reduction of $u_{\max_t}$.

Fig. 6.6 Influence of overlap of the plastic zones on cavern displacements: (a) $K_0=2$, (b) $K_0=1$, and (c) $K_0=0.5$

6.3.3 Determination of $u_{\max_t}$ using MARS

Based on the observations listed in Tables 6.1-6.3 except the three cases with extremely large displacements (> 0.7 m) mentioned in section 6.3.1, this problem is analyzed by means of MARS. Of the 147 observations, 109 patterns were randomly selected as the training data and the remaining 38 data were used for testing. The testing data sets are listed in Table 6.4. The MARS training results are shown in Appendix R. To eliminate unnecessary rounding errors, the unit of $u_{\max_t}$ is changed from meters (m) to millimeters (mm). The same units are also used for the MARS model predictions in Eq. (6.1).

Table 6.4 Testing data sets of MARS model for $u_{\max_t}$

B (m)	Q	K_0	S_c	$u_{\max_t_}$ FDM (mm)	$u_{\max_t_}$ MARS (mm)	B (m)	Q	K_0	S_c	$u_{\max_t_}$ FDM (mm)	$u_{\max_t_}$ MARS (mm)
10	1	0.5	15	30.9	27.2	20	10	1	30	6.6	5.2
10	1	1	20	24.9	22.0	20	10	2	40	14.9	9.5
10	1	2	15	78.4	76.5	20	40	0.5	30	2.7	4.5
10	4	0.5	20	4.5	3.0	20	40	1	40	2.5	3.3
10	4	1	15	5.8	4.1	20	40	2	30	6.0	4.5
10	4	2	20	17.1	21.3	20	100	0.5	40	1.6	1.8
10	10	0.5	15	2.5	5.2	20	100	1	30	1.6	3.0
10	10	1	20	2.7	4.1	20	100	2	40	2.9	1.8
10	10	2	15	7.9	10.6	30	4	0.5	60	27.1	25.7
10	40	0.5	20	1.2	3.3	30	4	2	75	53.6	47.2
10	40	1	15	1.2	4.5	30	10	0.5	45	11.1	13.1
10	40	2	20	2.5	3.3	30	10	1	60	9.2	12.0
10	100	0.5	15	0.8	3.0	30	10	2	45	22.7	22.8
10	100	1	20	0.7	1.8	30	40	0.5	60	3.8	3.3
10	100	2	15	1.4	3.0	30	40	1	45	3.8	4.5
20	4	0.5	30	28.2	17.4	30	40	2	60	7.5	7.6
20	4	1	40	13.3	16.2	30	100	0.5	45	2.5	3.0
20	4	2	30	38.7	35.7	30	100	1	60	2.2	1.8
20	10	0.5	40	5.7	4.1	30	100	2	45	4.2	7.3

The optimal MARS model consisted of 13 BF's of linear spline functions with second-order interaction. A plot of $u_{\max_t_MARS}$ versus $u_{\max_t_FDM}$ shown in Fig. 6.7 indicates that the built MARS model is reasonably accurate.

Fig. 6.7 Comparison between $u_{\max_t_MARS}$ and $u_{\max_t_FDM}$

For brevity, the ANOVA decomposition of the developed MARS model and the relative importance of the parameters are not detailed here. But the conclusion is that $u_{\max_t}$ is mainly influenced by Q , followed by S_c/B .

Table 6.5 lists the BFs of the MARS model and their corresponding equations. The interpretable MARS model to predict $u_{\max_t}$ is given by

$$\begin{aligned} u_{\max_t_MARS} (mm) = & 17.35 - 2.023 \times BF1 + 7.193 \times BF2 + 8.151 \times BF3 + \\ & 1.998 \times BF4 + 1.975 \times BF5 + 0.026 \times BF6 + 2.726 \times BF7 - 0.221 \times BF8 + \\ & 0.430 \times BF9 - 2.281 \times BF10 + 0.397 \times BF11 - 0.869 \times BF12 + 0.180 \times BF13 \end{aligned} \quad (6.1)$$

Table 6.5 BFs and corresponding equations of MARS model for $u_{\max_t}$

BFs	Equation
BF1	$\max(0, Q - 4)$
BF2	$\max(0, 4 - Q)$
BF3	$BF2 \times \max(0, K_0 - 1)$
BF4	$\max(0, Q - 10)$
BF5	$\max(0, 10 - Q) \times \max(0, K_0 - 1)$
BF6	$\max(0, B - 20) \times \max(0, 40 - Q)$
BF7	$BF2 \times \max(0, 2.5 - S_c/B)$
BF8	$\max(0, 20 - B) \times \max(0, 10 - Q)$
BF9	$\max(0, B - 20) \times \max(0, K_0 - 1)$
BF10	$\max(0, S_c/B - 1.5)$
BF11	$\max(0, 1.5 - S_c/B) \times \max(0, 40 - Q)$
BF12	$\max(0, 1.5 - S_c/B) \times \max(0, 20 - B)$
BF13	$\max(0, K_0 - 1) \times \max(0, 40 - Q)$

This $u_{\max_t}$ equation can be used to provide preliminary estimates of the expected maximum cavern displacements during construction of the twin caverns.

6.4 Modeling results and analyses of ε_i

The displacements of the 32 nodes illustrated in Fig. 6.1 are evaluated for each case of the 147 observations used in the previous section. The strain in each of the key points, take ε_{XR} (R is for the right cavern) for example, is:

$$\varepsilon_{XR} = \max(\varepsilon_{XR_xjR}) \quad (6.2)$$

where X is C, S, M or I and x is c, s, m and i; ε_{XR_xjR} ($j=1, 2, 3$) is defined as

$$\varepsilon_{XR_xjR}(\%) = \frac{u_{XR} - u_{xjR}}{l_{XR_xjR}} \times 100 \quad (6.3)$$

The percent strain of the twin caverns ε_i is the maximum of the strains of the eight key points:

$$\varepsilon_i = \max(\varepsilon_{XR}, \varepsilon_{XL}) \quad (6.4)$$

6.4.1 Modeling results of ε_i

Table 6.6 lists the percent strains for the 147 observations. Based on the ε_i values in Table 6.6, Fig. 6.8 presents some typical plots of the influence of pillar geometry S_c/B on the percent strains, compared with the single cavern case. Generally ε_i decreases with the increase of S_c/B and converges to the level of a single cavern case, which indicates that the interaction between the twin caverns attenuates with increasing of spacing. For different cases, interaction diminishes at different S_c/B values. ε_i is also significantly influenced by K_0 . For each plot, the ε_i value under $K_0=2$ is much greater than those under $K_0=1$ and $K_0=0.5$, because of the larger horizontal stress release during cavern excavation.

Table 6.6 Percent strains of the 147 observations

B (m)	Q	K_θ	S_c (m)	ε_i (%)	B (m)	Q	K_θ	S_c (m)	ε_i (%)
10	1	0.5	10	0.407	20	4	1	40	0.088
10	1	0.5	15	0.348	20	4	1	50	0.087
10	1	0.5	20	0.286	20	4	2	30	0.358
10	1	0.5	25	0.234	20	4	2	40	0.355
10	1	1	10	0.366	20	4	2	50	0.354
10	1	1	15	0.322	20	10	0.5	20	0.098
10	1	1	20	0.305	20	10	0.5	30	0.047
10	1	1	25	0.271	20	10	0.5	40	0.034
10	1	2	10	1.171	20	10	0.5	50	0.035
10	1	2	15	1.087	20	10	1	20	0.094
10	1	2	20	0.989	20	10	1	30	0.047
10	1	2	25	0.938	20	10	1	40	0.047
10	4	0.5	10	0.164	20	10	1	50	0.046
10	4	0.5	15	0.099	20	10	2	20	0.187
10	4	0.5	20	0.075	20	10	2	30	0.186
10	4	0.5	25	0.080	20	10	2	40	0.183
10	4	1	10	0.160	20	10	2	50	0.180
10	4	1	15	0.095	20	40	0.5	20	0.021
10	4	1	20	0.095	20	40	0.5	30	0.014
10	4	1	25	0.094	20	40	0.5	40	0.013
10	4	2	10	0.380	20	40	0.5	50	0.013
10	4	2	15	0.373	20	40	1	20	0.023
10	4	2	20	0.363	20	40	1	30	0.021
10	4	2	25	0.358	20	40	1	40	0.021
10	10	0.5	10	0.078	20	40	1	50	0.021
10	10	0.5	15	0.043	20	40	2	20	0.080
10	10	0.5	20	0.035	20	40	2	30	0.077
10	10	0.5	25	0.041	20	40	2	40	0.074
10	10	1	10	0.066	20	40	2	50	0.075
10	10	1	15	0.049	20	100	0.5	20	0.008
10	10	1	20	0.049	20	100	0.5	30	0.008
10	10	1	25	0.048	20	100	0.5	40	0.008
10	10	2	10	0.190	20	100	0.5	50	0.008
10	10	2	15	0.185	20	100	1	20	0.013
10	10	2	20	0.184	20	100	1	30	0.012
10	10	2	25	0.183	20	100	1	40	0.012
10	40	0.5	10	0.021	20	100	1	50	0.012
10	40	0.5	15	0.016	20	100	2	20	0.043
10	40	0.5	20	0.014	20	100	2	30	0.042
10	40	0.5	25	0.015	20	100	2	40	0.043
10	40	1	10	0.024	20	100	2	50	0.044
10	40	1	15	0.020	30	4	0.5	60	0.113
10	40	1	20	0.019	30	4	0.5	75	0.119
10	40	1	25	0.019	30	4	1	60	0.106

Table 6.6 Percent strains of the 147 observations contd.

B (m)	Q	K_0	S_c (m)	ε_i (%)	B (m)	Q	K_0	S_c (m)	ε_i (%)
10	40	2	10	0.081	30	4	1	75	0.123
10	40	2	15	0.080	30	4	2	60	0.314
10	40	2	20	0.078	30	4	2	75	0.310
10	40	2	25	0.076	30	10	0.5	30	0.160
10	100	0.5	10	0.010	30	10	0.5	45	0.047
10	100	0.5	15	0.008	30	10	0.5	60	0.039
10	100	0.5	20	0.008	30	10	0.5	75	0.049
10	100	0.5	25	0.008	30	10	1	30	0.084
10	100	1	10	0.012	30	10	1	45	0.047
10	100	1	15	0.010	30	10	1	60	0.051
10	100	1	20	0.010	30	10	1	75	0.061
10	100	1	25	0.010	30	10	2	30	0.248
10	100	2	10	0.045	30	10	2	45	0.173
10	100	2	15	0.043	30	10	2	60	0.170
10	100	2	20	0.041	30	10	2	75	0.166
10	100	2	25	0.040	30	40	0.5	30	0.022
20	4	0.5	30	0.168	30	40	0.5	45	0.015
20	4	0.5	40	0.091	30	40	0.5	60	0.015
20	4	0.5	50	0.080	30	40	0.5	75	0.019
20	4	1	30	0.133	30	40	1	30	0.031
30	40	2	60	0.075	30	40	1	45	0.021
30	40	2	75	0.077	30	40	1	60	0.022
30	100	0.5	30	0.014	30	40	1	75	0.028
30	100	0.5	45	0.009	30	40	2	30	0.106
30	100	0.5	60	0.009	30	40	2	45	0.076
30	100	0.5	75	0.010	30	100	2	30	0.062
30	100	1	30	0.017	30	100	2	45	0.043
30	100	1	45	0.012	30	100	2	60	0.044
30	100	1	60	0.013	30	100	2	75	0.045
30	100	1	75	0.017

Fig. 6.8 Influence of S_c/B on ε_t : (a) $B=10$ m, (b) $B=20$ m, and (c) $B=30$ m

6.4.2 Influences of overlap of the plastic zones on the percent strains

Fig. 6.9 shows ε_t values of a few typical examples, in which the overlap of the plastic zones is denoted by solid symbols, while the hollow symbols represent the

cases in which there is no overlap of the yielding zones. Generally the ε_i values are larger for the overlap cases, particularly for $K_0=0.5$ and 1.0.

Fig. 6.9 Influence of overlap of the plastic zones on ε_i : (a) $K_0=2$, (b) $K_0=1$, and (c) $K_0=0.5$

6.4.3 Determination of ε_i using MARS

MARS model was built, using the data sets in Table 6.7. Of the 147 observations, 109 patterns were selected as the training data and the remaining 38 were used for testing. The optimal MARS model adopted 12 BFs of linear spline functions. The testing data sets and the corresponding MARS predictions are listed in Table 6.7. The MARS training results are shown in Appendix S. Plot of ε_{i_MARS} versus ε_{i_FDM} is shown in Fig. 6.10. It is obvious MARS predictions are of high accuracy.

Table 6.7 Testing data sets of MARS predictions of ε_i

B (m)	Q	K_0	S_c	ε_{i_FDM} (%)	ε_{i_MARS} (%)	B (m)	Q	K_0	S_c	ε_{i_FDM} (%)	ε_{i_MARS} (%)
10	1	0.5	15	0.348	0.338	20	10	1	30	0.047	0.063
10	1	1	20	0.305	0.274	20	10	2	40	0.183	0.170
10	1	2	15	1.087	1.067	20	40	0.5	30	0.014	0.018
10	4	0.5	20	0.075	0.099	20	40	1	40	0.021	0.018
10	4	1	15	0.095	0.123	20	40	2	30	0.077	0.078
10	4	2	20	0.363	0.347	20	100	0.5	40	0.008	0.009
10	10	0.5	15	0.043	0.063	20	100	1	30	0.012	0.009
10	10	1	20	0.049	0.044	20	100	2	40	0.043	0.043
10	10	2	15	0.185	0.190	30	4	0.5	60	0.113	0.099
10	40	0.5	20	0.014	0.018	30	4	2	75	0.310	0.347
10	40	1	15	0.020	0.018	30	10	0.5	45	0.047	0.063
10	40	2	20	0.078	0.078	30	10	1	60	0.051	0.044
10	100	0.5	15	0.008	0.009	30	10	2	45	0.173	0.190
10	100	1	20	0.010	0.009	30	40	0.5	60	0.015	0.018
10	100	2	15	0.043	0.043	30	40	1	45	0.021	0.018
20	4	0.5	30	0.168	0.123	30	40	2	60	0.075	0.078
20	4	1	40	0.088	0.099	30	100	0.5	45	0.009	0.009
20	4	2	30	0.358	0.371	30	100	1	60	0.013	0.009
20	10	0.5	40	0.034	0.044	30	100	2	45	0.043	0.043

For brevity, the ANOVA decomposition of the developed MARS model and the relative importance of the parameters are not detailed here. But the conclusion is that ε_i is mainly influenced by Q , followed by K_0 . Table 6.8 lists the BFs of the MARS model for ε_i and their corresponding equations. The interpretable MARS model to predict ε_i is given by

$$\begin{aligned} \varepsilon_{i_MARS} (\%) = & 0.1 - 0.0093 \times BF1 + 0.046 \times BF2 + 0.14 \times BF3 + 0.074 \times BF4 \\ & - 0.0004 \times BF5 + 0.018 \times BF6 + 0.025 \times BF7 + 0.0007 \times BF8 + 0.0013 \times BF9 \\ & + 0.0084 \times BF10 + 0.0018 \times BF11 + 0.0043 \times BF12 \end{aligned} \quad (6.5)$$

Fig. 6.10 Comparison between ε_{t_MARS} and ε_{t_FDM}

Table 6.8 BFs and corresponding equations of MARS model for ε_t

BFs	Equation	BFs	Equation
BF1	$\max(0, Q - 4)$	BF7	$BF2 \times \max(0, 2.5 - S_c/B)$
BF2	$\max(0, 4 - Q)$	BF8	$\max(0, Q - 40)$
BF3	$BF2 \times \max(0, K_0 - 1)$	BF9	$\max(0, 40 - Q) \times \max(0, 2 - S_c/B)$
BF4	$\max(0, K_0 - 1)$	BF10	$\max(0, Q - 10)$
BF5	$BF4 \times \max(0, Q - 10)$	BF11	$\max(0, 40 - Q) \times \max(0, K_0 - 1)$
BF6	$BF4 \times \max(0, 10 - Q)$	BF12	$\max(0, 1.5 - S_c/B) \times \max(0, B - 20)$

The response surfaces relating ε_t to Q and K_0 , Q and S_c/B , Q and B are illustrated in Figs. 6.11a, 6.11b and 6.11c, respectively. These response surfaces provide a practical design tool, especially during early design stages of design and construction. By specifying the pillar geometries S_c/B for a given Q as well as the cavern span, the expected percent strain can be estimated quickly. Or by assuming the threshold of the percent strain for a given Q and the cavern span, the appropriate pillar width can be estimated. Also, the effect of changing Q and in situ stress ratio K_0 can be assessed.

(a)

(b)

(c)

Fig. 6.11 Response surfaces relating ε_t to: (a) Q and K_0 , (b) Q and S_c/B , and (c) Q and B

6.5 Probabilistic assessments on SLS

6.5.1 Serviceability criterion

The serviceability limit state can be assessed either by $u_{\max_t}$ or ε_t . In this thesis, Eq. (6.5) is used to determine ε_t and to assess the cavern serviceability limit state.

6.5.2 Threshold strain values

In Chapter 4, the concept of critical strain from Sakurai (1986, 1997) was adopted to determine the threshold strain values for the single cavern case, in which ε_s is related to Q , B , B/H and K_0 . The relationship between critical strain ε_c and E_m and σ_{cm} is unique, irrespective of B/H which represents the cavern shape. In this section, as the B/H is constant (as mentioned in section 5.3, only $B/H=2$ is considered), another feasible approach for estimating threshold percent strains ε_c is introduced.

Meguid and Rowe (2006) investigated the influence of rock mass strength reduction on the stability of D-shaped tunnels by examining the induced displacements at four different locations including the face centre, springline, crown and invert. They first obtained an elastic line which represents the elastic response of the rock mass. A design line, which is based on an additional 50% of the elastic strain ε_e , is then suggested to limit the induced strains to within tolerable limits. Strains between the limits bounded by the elastic and design lines are considered to be acceptable and will not cause instability at those four locations.

Based on this design methodology, numerical trials are performed to derive the elastic responses under combinations of Q , K_0 and B . The elastic responses are obtained through FDM analyses with the rock mass assumed as linear elastic. Table 6.9 lists the elastic strains and the strain values for the design line. The strain value of the design line ε_d is 1.5 times that of the elastic strain ε_e .

As the strain values for the three cases of $B = 10, 20$, and 30 m under the same K_0 and Q are nearly the same, it can be assumed that ε_d and ε_e are independent of the cavern span. Based on the strain values of Table 6.9, Fig. 6.12 plots the elastic line

and the design line as a function of Q . It is clear that these lines are specific to K_0 , as also indicated in Meguid and Rowe (2006).

Table 6.9 Elastic strains and the strain values of the design line

K_0	Q	B (m)	ε_e (%)	ε_d (%)
0.5	1	10	0.083	0.125
		20	0.084	0.126
		30	0.080	0.120
	4	10	0.050	0.075
		20	0.051	0.077
		30	0.050	0.075
	10	10	0.034	0.052
		20	0.035	0.053
		30	0.034	0.052
	40	10	0.020	0.030
		20	0.020	0.030
		30	0.020	0.030
	100	10	0.014	0.021
		20	0.014	0.021
		30	0.014	0.021
1	1	10	0.081	0.121
		20	0.082	0.123
		30	0.079	0.119
	4	10	0.045	0.068
		20	0.047	0.071
		30	0.046	0.070
	10	10	0.031	0.047
		20	0.033	0.049
		30	0.032	0.048
	40	10	0.018	0.027
		20	0.019	0.028
		30	0.018	0.027
	100	10	0.012	0.018
		20	0.013	0.019
		30	0.013	0.019
2	1	10	0.147	0.221
		20	0.151	0.227
		30	0.154	0.231
	4	10	0.085	0.128
		20	0.087	0.131
		30	0.089	0.133
	10	10	0.059	0.088
		20	0.060	0.090
		30	0.061	0.092
	40	10	0.034	0.050
		20	0.034	0.052
		30	0.035	0.052
	100	10	0.023	0.035
		20	0.024	0.036
		30	0.024	0.036

Fig. 6.12 Elastic and the design lines for $K_0=0.5, 1$ and 2

Based on the results above, the design of underground rock cavern constructed in rock mass represented by Q under anisotropic/isotropic stress conditions represented by K_0 should satisfy the criterion of limiting the induced strain to within tolerable limits. The allowable strains are summarized in Table 6.10 for the cases of $Q=1, 4, 10, 40$, and 100 , respectively.

Table 6.10 Suggested ε_c for various Q values

Q	1	4	10	40	100
ε_c (%)	0.082-0.227	0.047-0.131	0.033-0.090	0.019-0.052	0.013-0.036

6.5.3 Comparisons between probabilistic assessment methods

The serviceability limit state function based on the MARS model is given by

$$g(\mathbf{x}) = \varepsilon_c - \varepsilon_{t_MARS} = \varepsilon_c - 0.099 + 0.0093 * BF1 - 0.0456 * BF2 - 0.1401 * BF3 - 0.0735 * BF4 + 0.0004 * BF5 - 0.0184 * BF6 - 0.0253 * BF7 - 0.0007 * BF8 - 0.0013 * BF9 - 0.0084 * BF10 - 0.0018 * BF11 - 0.0043 * BF12 \quad (6.6)$$

The expressions of BFs in this equation can be found in Table 6.8. Reliability assessment of the percent strain using MCS is shown in Fig. 6.13. For each MCS simulation, the number of iterations is 1,000,000 and the sampling technique is Latin Hypercube. Failure in terms of percent strain occurs if the MARS predicted strain value ε_{t_MARS} is greater than ε_c . It should be highlighted that P_f is significantly influenced by the choice of ε_c and usually when using the lower bound value of ε_c ,

the cavern performance is nearly elastic. Thus, in the following analyses, the upper bound values of ε_c are adopted based on the mean value of Q . The assumed statistical information of the input variables is listed in Table 6.11.

Fig. 6.13 Implementation of MARS model into MCS for reliability analyses

Table 6.11 Statistical information of the input variables for reliability assessment on SLS

Variable	Distribution type	Mean	COV
Q	Normal	4, 10, 40	0.1, 0.2, 0.3, 0.4
B	Normal	10, 20, 30	0.1
S_c/B	Normal	1, 1.5, 2, 2.5	0.1
K_o/B	Normal	0.5, 1, 2	0.1

Fig. 6.14 plots the influence of the various design parameters on P_f of SLS. It is clear that both the COV and the average value of Q significantly influence the P_f . However, the influence of B on P_f is not as significant as that for S_c/B . In addition, from Fig. 6.14c it can be observed that the influence of K_0 on P_f is also significant when $Q=40$ compared with $Q=100$. The P_f under $K_0 = 1$ is generally higher than that under $K_0 = 0.5$ as the ε_t for $K_0 = 1$ is larger than for $K_0 = 0.5$ but ε_c is the same.

Fig. 6.14 Influence of Q , S_c/B , B , and K_0 on P_f of SLS: (a) COV of Q , B and mean Q , (b) S_c/B , COV and mean Q , and (c) S_c/B , K_0 , COV and mean Q

Probabilistic assessment of the percent strain ε_i can also be performed using FORM based on the built MARS model. The serviceability limit state function Eq. (6.6) and its corresponding BF equations in Table 6.8 are incorporated into an EXCEL spreadsheet environment based on the approach by Low and Tang (2007), from which the reliability index can be determined. Fig. 6.15 shows a sample spreadsheet for computing the reliability index β where the statistics of the design parameters are the same as those used in the previous section. The spreadsheet cells A2:A5 allows the selection of various distribution types for the input variables, including normal, lognormal, triangular etc as explained in Low and Tang (2007). For nonnormals, the nonnormal distributions are replaced by an equivalent normal ellipsoid, centered at the equivalent normal mean. Cells C2:F5 are parameters which are set corresponding to the variable distribution types. For lognormals, cells C2:C5 correspond to the mean values while cells D2:D5 correspond to the standard deviations. The correlation matrix $[R]$ in cells K2:N5 are used to define the correlations between Q , B , S_c/B and K_0 . The n_i vector in cells O2:O5 contains equations for $(x_i - \mu_i^N) / \sigma_i^N$. Cells H2:J13 contain the basis function expressions and equations obtained from the MARS model. The design point (x^* values) was obtained by using the spreadsheet's built-in optimization routine SOLVER to minimize the cell, by changing the x^* values, under the constraint that the performance function $g(x^*) = 0$. Prior to invoking the SOLVER search algorithm, the x^* values were set equal to the mean values (10, 20, 1.5, 1) of the original random variables. Iterative numerical derivatives and directional search for the design point x^* were automatically carried out in the spreadsheet environment.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Distribution	Variables	Para1	Para2	Para3	Para4	x_i^*	MARS PROCESS			Correlation matrix [R]				n_i	$g(x)$	β	$P_f(\%)$
2	Normal	Q	10	2			8.353	BF1=	max(0, Q - 4)	4.353	1	0	0	0	-0.8			
3	Normal	B	20	2			20.022	BF2=	max(0, 4 - Q)	0.000	0	1	0	0	0	2E-09	1.04	14.94
4	Normal	Sc / B	1.5	0.15			1.466	BF3=	BF2*max(0, K0 - 1)	0	0	0	1	0	-0.2			
5	Normal	K0	1	0.1			1.059	BF4=	max(0, K0 - 1)	0.059	0	0	0	1	0.6			
6								BF5=	BF4*max(0, Q - 10)	0								
7								BF6=	BF4*max(0, 10 - Q)	0.097								
8								BF7=	BF2*max(0, 2.5 - Sc/B)	0								
9								BF8=	max(0, Q - 40)	0.000								
10								BF9=	max(0, 40-Q)*max(0, 2-Sc/B)	16.9								
11								BF10=	max(0, Q - 10)	0.000								
12								BF11=	max(0, 40 - Q)*max(0, K0-1)	1.872								
13								BF12=	max(0, 1.5-Sc/B)*max(0, B-20)	7E-04								

Diagram 1: Distribution types of design variables (A2:A5) → Mean and standard deviation values (C2:F5) → Basis function expressions and equations (H2:J13)

Diagram 2: Design points (G2:G5) → Basis function expressions and equations (H2:J13)

Equation for n_i : $n_i = (x_i^* - \mu_i) / \sigma_i$

Equation for β : $\beta = \min_{x \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i} \right]^T [R]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i} \right]}$

Mathematical equation for limit state surface using $g(x) = \varepsilon_c - \varepsilon_{i_MARS}$; in this case, as the Q mean value is 10, value of 0.090 is assumed for ε_c .

Equation for P_f : $P_f \approx 1 - \Phi(\beta)$

Fig. 6.15 Calculation of P_f for serviceability limit state using FORM_MARS spreadsheet

A comparison of the P_f obtained using the FORM_MARS and the MCS_MARS methods in Fig. 6.16 shows that the FORM_MARS gives similar and consistent results with MCS_MARS.

Fig. 6.16 Comparison of P_f of SLS from FORM_MARS and MCS_MARS

6.6 System reliability

An appropriate FS_{g-t} for designing twin caverns can be proposed from failure statistics if a sufficient number of failed and stable case histories have been observed (Harr 1987). Unfortunately, in the case of excavation in rock, the few individual cases that have failed are not sufficient to conduct this kind of study. It is proposed in this thesis that the FS_{g-t} be determined from the concept of system reliability, that is, satisfying the serviceability limit state and concurrently ensuring that the ultimate limit state is not exceeded. Thus the reliability is assessed based on the percent strain ε_t and at the same time FS_{g-t} must be satisfied during the search for the design point.

Fig. 6.17 shows a typical spreadsheet setup for assessing the system reliability. Cell P2 contains the mathematical expression (Eq. (6.6)) for the serviceability limit state function $g(\mathbf{x}) = \varepsilon_c - \varepsilon_{t_MARS}$, which is a function of Q , B , S_c/B and K_0 . The LR model for FS_{g-t} is implemented into cell D12 representing the constraint function FS_{g-t} , in which mathematical expression (Eq. (5.8)) $FS_{g-t_LR} - FS_{cr} \geq 0$ must be satisfied for each combination of parameters Q , B , and S_c/B . It should be noted that the LR

model is adopted herein instead of the MARS model, since the mathematical equation for the LR model is simpler and the accuracy is satisfactory. The design point (x_i^* values in cells G2:G5) was obtained by using the EXCEL spreadsheet's built-in optimization routine SOLVER to minimize the cell, by changing the x_i^* values, under the constraints that the performance function $g(x_i^*) = 0$ (cell P2) and that $FS_{g_t_LR} - FS_{cr} \geq 0$ (cell D12). Prior to invoking the SOLVER search algorithm, the x_i^* values were set equal to the mean values (10, 20, 1, 1) of the original four random variables. Iterative numerical derivatives and directional search for the design point x_i^* are automatically carried out in the spreadsheet environment. Reliability indices β corresponding to different FS_{g_t} values can be calculated based on the FORM spreadsheet method described in Fig. 6.15. Tables 6.12 and 6.13 lists the calculation results of β corresponding to the required minimum FS_{g_t} under combinations of Q , B , and S_c/B for $K_0=0.5$ and 1, respectively. It is obvious that for different target performance levels, the differences of the required FS_{g_t} values for $K_0=0.5$ and $K_0=1$ cases are not significant. For the same Q and B , the required FS_{g_t} to satisfy the target performance level increases as the S_c/B increases. It is logical that the wider the pillar, the greater the required FS_{g_t} . Fig. 6.18 illustrates the relationship between the required FS_{g_t} and β_{SLS} under various combinations of Q , B , and S_c/B for $K_0=1$ cases.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Distribution	Variables	Para1	Para2	Para3	Para4	x_i^*	MARS PROCESS			Correlation matrix [R]			n_i	$g(x)$	β	$P_f(\%)$	
2	Normal	Q	10	2			7.935	BF1=	max(0, Q - 4)	3.935	1	0	0	0	-1	-8E-10	1.31	9.45
3	Normal	B	20	2			21.333	BF2=	max(0, 4 - Q)	0.000	0	1	0	0	0.7			
4	Normal	Sc / B	1.5	0.15			1.440	BF3=	BF2*max(0, K0 - 1)	0	0	0	1	0	-0.4			
5	Normal	K0	1	0.1			1.023	BF4=	max(0, K0 - 1)	0.023	0	0	0	1	0.2			
6	<div>FS_{g_tLR}=2.3044Q^{0.1404}B^{-0.3331}(Sc/B)^{0.1393}</div> <div>FS_{g_t} constraint (FS_{g_tLR} - FS_{cr} ≥ 0)</div> <div>FS_{cr}: 1.17</div> <div>FS_{g_tLR} - FS_{cr} = 0.00</div>							BF5=	BF4*max(0, Q - 10)	0	<div>$n_i = (x_i^* - \mu_i) / \sigma_i$</div> <div>$\beta = \min_{ref} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i} \right]^T [R]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i} \right]}$</div> <div>Mathematical equation for limit state surface using g(x)=ε_c-ε_{c_{MARS}}, in this case, as the Q mean value is 10, value of 0.090 is assumed for ε_c</div>							
7								BF6=	BF4*max(0, 10 - Q)	0.047							<div>$P_f \approx 1 - \Phi(\beta)$</div>	
8								BF7=	BF2*max(0, 2.5 - Sc/B)	0								
9								BF8=	max(0, Q - 40)	0.000								
10								BF9=	max(0, 40-Q)*max(0, 2-Sc/B)	17.97								
11								BF10=	max(0, Q - 10)	0.000								
12								BF11=	max(0, 40 - Q)*max(0, K0-1)	0.737								
13								BF12=	max(0, 1.5-Sc/B)*max(0, B-20)	0.081								

Fig. 6.17 Calculation on the system reliability using FORM_MARS

Table 6.12 Calculation results of β_{SLS} corresponding to the required FS_{g-t} under combinations of Q , B , and S_c/B for $K_0=1$

Q	B (m)	S_c / B	required FS_{g-t}				
			$\beta_{SLS}=1.5$	$\beta_{SLS}=2$	$\beta_{SLS}=2.5$	$\beta_{SLS}=3$	$\beta_{SLS}=4$
4	10	1	1.36	1.42	1.46	1.50	1.59
		1.5	1.46	1.50	1.54	1.58	1.67
		2	1.50	1.53	1.57	1.61	1.69
		2.5	1.54	1.58	1.62	1.66	1.75
10	10	1	1.56	1.59	1.65	1.69	1.78
		1.5	1.59	1.63	1.67	1.72	1.81
		2	1.64	1.66	1.72	1.77	1.88
		2.5	1.67	1.70	1.76	1.82	1.93
	20	1	1.24	1.27	1.31	1.34	1.42
		1.5	1.27	1.29	1.32	1.35	1.44
		2	—	1.30	1.35	1.39	1.47
		2.5	—	1.32	1.38	1.42	1.51
40	20	1	1.34	1.40	1.45	1.49	1.58
		1.5	—	1.44	1.47	1.53	1.63
		2	—	—	1.52	1.58	1.72
		2.5	—	—	1.59	1.62	1.79
	30	1	1.26	1.28	1.31	1.33	1.38
		1.5	—	—	1.32	1.34	1.43
		2	—	—	1.35	1.36	1.48
		2.5	—	—	—	1.38	1.53
100	30	1	—	—	—	1.36	1.41
		1.5	—	—	—	1.40	1.48
		2	—	—	—	1.46	1.54
		2.5	—	—	—	1.50	1.56

Note: ‘—’ indicates that the derived β_{SLS} is much higher than the corresponding target level.

Table 6.13 Calculation results of β_{SLS} corresponding to the required FS_{g-t} under combinations of Q , B , and S_c/B for $K_0=0.5$

Q	B (m)	S_c / B	required FS_{g-t}				
			$\beta_{SLS}=1.5$	$\beta_{SLS}=2$	$\beta_{SLS}=2.5$	$\beta_{SLS}=3$	$\beta_{SLS}=4$
4	10	1	1.36	1.42	1.46	1.50	1.59
		1.5	1.45	1.48	1.51	1.55	1.63
		2	1.48	1.51	1.55	1.58	1.67
		2.5	1.51	1.54	1.58	1.62	1.71
10	10	1	1.55	1.58	1.61	1.65	1.74
		1.5	1.56	1.60	1.64	1.68	1.78
		2	—	1.65	1.70	1.76	1.86
		2.5	—	—	1.73	1.78	1.89
	20	1	1.23	1.25	1.28	1.31	1.38
		1.5	1.25	1.27	1.30	1.33	1.40
		2	—	1.29	1.32	1.37	1.45
		2.5	—	—	1.35	1.38	1.47
40	20	1	1.32	1.34	1.40	1.44	1.53
		1.5	—	—	1.45	1.46	1.56
		2	—	—	—	1.55	1.63
		2.5	—	—	—	1.61	1.67
	30	1	1.24	1.26	1.28	1.30	1.34
		1.5	—	—	1.28	1.31	1.37
		2	—	—	—	1.33	1.41
		2.5	—	—	—	1.40	1.45
100	30	1	—	—	—	1.36	1.44
		1.5	—	—	—	1.38	1.44
		2	—	—	—	1.41	1.45
		2.5	—	—	—	1.43	1.47

Note: ‘—’ indicates that the derived β_{SLS} is much higher than the corresponding target level.

(a)

(b)

(c)

(d)

Fig. 6.18 Relationship between β_{SLS} and the required FS_{g-t} for $K_0=1$ cases: (a) $Q=4$ $B=10$ m, (b) $Q=10$ $B=10$ m, (c) $Q=10$ $B=20$ m, (d) $Q=40$ $B=20$ m, (e) $Q=40$ $B=30$ m, and (f) $Q=100$ $B=30$ m

Comparison between Fig. 6.18a and 6.18b shows that a target β_{SLS} of 3.0 is achievable for FS_{g-t} larger than 1.50 for $Q=4$, $B=10$ and $S_c/B=1.0$. For the same geometries with a larger Q value of 10, an above average performance level is achievable for FS_{g-t} larger than 1.69. The same performance level can also be achieved for $Q=10$, $B=20$ and $S_c/B=1.0$ with FS_{g-t} larger than 1.34, compared with

FS_{g-t} larger than 1.49 as Q increase to 40. It is logical that the higher the quality of the rock mass, the larger the required FS_{g-t} . Previous results in Chapter 5 have shown that FS_{g-t} is not influenced by the in situ stress ratio K_0 . However, the serviceability assessment is performed on the percent strain value, which is significantly affected by K_0 . The result is that to achieve the same performance level under the same Q , B , and S_c/B combination, the required FS_{g-t} values for $K_0 = 0.5$, 1 and 2 are different. Generally the required FS_{g-t} for $K_0 = 1$ is slightly higher than that for the other two cases. Thus the FS_{g-t} values for $K_0 = 1$ are proposed as the references for general project use under similar ground conditions. The lines constructed in Fig. 6.18 can be used as a preliminary design tool when selecting a required FS_{g-t} provided the quality of rock mass, the twin cavern spacing, the single cavern span, and the in-situ stress are known.

It is obvious that optimal design of twin caverns in terms of avoiding over-design of rock mass is achievable if a certain level of risk is acceptable. Thus the procedures outlined in this chapter can be used to obtain a rational optimal design of underground twin rock caverns. A FS_{g-t} value no less than the values proposed in Tables 6.12 and 6.13 can be used to satisfy the target performance levels for caverns with different spans and pillar widths under different ground conditions. These FS_{g-t} values are also recommended to meet the performance level requirement under similar ground conditions. They can also be used as guidance with regard to the choice of a proper factor of safety for underground twin rock cavern problems.

6.7 Concluding remarks

This chapter considered the serviceability limit state of twin caverns. The dependent responses considered include the maximum key point displacement u_{max-t} and the percent strain ε_t . MARS models for estimating u_{max-t} and ε_t were developed. The response surfaces relating percent strain ε_t to Q and K_0 , Q and S_c/B , Q and B were established.

Percent strain ε_t is adopted as the serviceability criterion. The threshold strain values are determined through the elastic and design line methodology proposed by Meguid and Rowe (2006). Probabilistic assessments on SLS were performed using

the MARS percent strain model. Both the FORM_MARS and MCS_MARS reliability methods give similar and consistent results.

To ensure system reliability, the reliability assessment is performed on the percent strain ε_t while at the same time ensuring that FS_{g-t} is satisfied. The FORM_MARS method is used to derive the required FS_{g-t} corresponding to the target reliability index β_{SLS} . Calculation results of β_{SLS} corresponding to the required minimum FS_{g-t} under combinations of Q , B , and S_c/B were listed and plotted. They can be used to select an approximate FS_{g-t} provided the quality of rock mass, the twin cavern spacing, the single cavern span, and the in-situ stress are known.

In summary, this study of probabilistic assessment on serviceability limit state of twin caverns illustrated that:

- (1) More key points are required to derive the maximum key point displacement compared with the single cavern case. In addition, displacements of more nodes, peripheral or inner, are required in order to derive the percent strain values.
- (2) Influences of overlap of the plastic zones on the cavern behavior in terms of the maximum key point displacement u_{max-t} and the percent strain ε_t were investigated. It was found that the strain and the cavern displacement are significantly larger for the overlap cases compared with no overlap cases.
- (3) MARS model was built to predict the maximum key point displacement for twin caverns. It can be used to give preliminary estimates of the expected maximum cavern displacements during construction. Parameter relative importance comparison indicates that Q followed by S_c/B are the two most important parameters that influence u_{max-t} .
- (4) MARS model was built to predict the percent strain ε_t . MARS ANOVA decomposition indicates that Q followed by K_0 are the two most important parameters that influence ε_t .
- (5) Response surfaces relating percent strain ε_t to Q and K_0 , Q and S_c/B , Q and B were established.
- (6) Threshold strains can be determined through the elastic and design line methodology proposed by Meguid and Rowe (2006).

- (7) Utilizing the MARS model, reliability assessment of the serviceability limit state can be carried out using MCS and FORM. It is obvious that P_f increases as the COV of Q increases, and P_f decreases with the widening of the pillar.
- (8) System reliability of twin caverns can also be ensured by performing probabilistic assessment on the percent strain ε_t and concurrently satisfying the FS_{g-t} .
- (9) Optimal design in terms of avoiding over-design of rock mass is achievable if a certain level of risk is acceptable. The proposed Tables 6.12 and 6.13 and the charts constructed in Fig. 6.18 can be used as a preliminary design tool to select a required FS_{g-t} provided the quality of rock mass, the twin cavern spacing, the single cavern span, and the in-situ stress are known.

Chapter 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary and conclusions

This study has five aims. The first one is to develop a deterministic framework of combining numerical and empirical design approaches for analyzing the performance of the cavern(s). The second aim is to build predictive models for cavern(s) performance estimation. The third is to develop a probabilistic framework incorporating the limit state functions into reliability assessment methods to perform reliability analysis. The fourth aim is to investigate different approaches of developing surrogate models for implicit performance functions, and demonstrate the use of these surrogate models coupled with reliability methods. The final aim is to illustrate a rational and practical approach to assess the system reliability for cavern(s) design. The main findings are summarized below.

7.1.1 Estimation of Mohr-Coulomb parameters from empirical equations

Several empirical equations for estimation of Mohr-Coulomb parameters have been adopted. Parameters determined using these equations are then used by the Finite Difference program to estimate the cavern performances. In the absence of quality test data of rock mass, these equations can be used to determine the mechanical properties of the rock mass.

7.1.2 Prediction of cavern(s) performances in terms of design limit states

Two kinds of design limit states: the ultimate limit state and the serviceability limit state are taken into consideration in this study. The ultimate limit state is assessed in terms of the global factor of safety, which is derived from the shear strength reduction technique. The numerical outputs for serviceability assessment include the maximum key point displacement and the percent strain. The percent strain is

adopted as the criterion for evaluating the serviceability limit state as the threshold strain value can be determined through the concept of critical strain or the design line methodology. In addition, both these two design limit states should be ensured in the design. The study has demonstrated that determination of the reliability index provides a systematic method for evaluating the influences of the uncertainties in the various parameters affecting the ultimate and serviceability limit states and can be used to assist design engineers in assessing the acceptable level of risk.

7.1.3 Surrogate models and design charts for performance predictions of cavern(s)

Predictive models using Logarithmic Regression (LR), Back-propagation neural network (BPNN) and Multivariate adaptive regression splines (MARS) are developed respectively to estimate the global factor of safety for underground rock cavern(s). Charts relating the global factor of safety to the mechanical and geometrical properties are established. The charts are potentially useful for preliminary design and stability checking of rock cavern(s).

Estimation models using LR, BPNN and MARS are developed for both the maximum key point displacement and the percent strain. Prediction of the maximum key point displacement can give preliminary estimates of the expected cavern deformation during construction. For the twin caverns, by specifying the pillar geometry for a given rock mass property as well as the cavern span, the expected percent strain can be estimated from the design charts. Alternatively, by assuming the threshold of the percent strain for a given rock mass and cavern span, the appropriate pillar width can be estimated.

7.1.4 Approaches of developing surrogate models

A new approach to develop surrogate models using MARS together with the related parameter sensitivity analysis is introduced in this study, in comparison with the commonly used BPNN and LR methods. MARS and BPNN algorithms are compared with regard to estimating the global factor of safety, the maximum key point displacement and the percent strain for the single cavern case in terms of the accuracy, the model interpretability, and the computational efficiency. The

comparisons indicate that the MARS algorithm performs as well as the BPNN in terms of accuracy. However, MARS is easier to interpret and more computationally efficient. The predictive capacity of the optimal LR models is slightly less accurate than MARS and BPNN.

7.1.5 Failure mechanism interpreted in terms of pillar performance

Pillar performance with respect to the maximum principal stress in the pillar, the vertical stress and the principal stress difference in the pillar core, and the average pillar strength and average stress are studied in order to examine the failure mechanisms and to identify situations in which there is substantial cavern interaction. The charts derived from an ANN model can be used to assess if there is overlap of the plastic zones.

7.1.6 Framework incorporating limit state functions into reliability assessment methods to perform reliability analysis

Reliability analyses using First-order reliability method (FORM) and Monte Carlo simulation (MCS) have been performed on ultimate and serviceability limit states of both single cavern and twin cavern cases. The procedures of implementing the built LR, BPNN, and MARS models into Microsoft Excel add-in program @RISK and Excel spreadsheet to calculate the probability of failure have been illustrated.

7.1.7 A rational and practical approach to ensure the system reliability

System reliability for both single and twin caverns can be ensured by performing probabilistic assessment using FORM on the percent strain and concurrently satisfying the global factor of safety. Tables and charts are proposed (for preliminary design of single cavern) for estimating the global factor of safety as a function of the quality of rock mass, the cavern span, the shape of the opening and the in situ stress. Similarly, tables and charts are proposed (for preliminary design of twin caverns) for estimating the global safety factor as a function of the quality of rock mass, the twin cavern spacing, the cavern span, and the in situ stress. Optimal design in terms of avoiding over-design of rock mass is achievable if a certain level of risk/expected performance level is acceptable.

7.2 Recommendations for future work

Some suggestions for future research are given below:

- (1) The analyses presented in this thesis focused only on stress-induced instability and the discontinuous nature of the rock mass was incorporated implicitly in the Mohr-Coulomb constitutive relationship used to represent the mass as an equivalent continuum. Future studies may extend the deterministic and probabilistic assessment methods described herein to include discontinuities such as joint sets, faults and bedding planes, as well as to consider other failure modes such as structurally-controlled instability. Programs like UDEC/3DEC may be used to characterize the joint attributes such as the spacing, the persistence, and the mechanical properties. However, the general application is usually hampered by the limited availability of details of the geometry of the rock fractures. If the uncertainties from the mechanical and geometrical properties of joints are regarded as design variables, more parameters will be involved in the process of both the deterministic and probabilistic analyses. For situations with a major fault near the cavern, the fault parameters including the fault dips, fault shear strength and fault locations relative to the underground cavern must be identified and treated as random design variables.
- (2) The mechanical properties of rock mass are derived from the commonly used empirical equations. It should be remembered that all of these empirical equations are not intended to replace laboratory test. The accuracy and applicability of the findings depend largely on those empirical relationships. Thus expanding the database to include more case studies is important. Since the uncertainty of estimating properties from empirical correlations can be significant at times, the reduction of the uncertainty associated with the correlations is important. The use of a Bayesian approach which is oriented to update empirical predictions with regional information or site-specific observations is promising (Zhang et al. 2004).
- (3) The present study is based on two-dimensional plane strain analysis without considering rock reinforcements. Further 3D analyses accounting for

relevant excavation factors such as excavation rate and rock bolt system can be considered.

- (4) The rock material in this study is assumed to obey the Mohr-Coulomb failure criterion that follows the elastic perfectly-plastic stress-strain relationship. Other constitutive models such as the Hoek-Brown and Drucker-Prager should also be considered.
- (5) For the twin cavern case, only horizontally aligned caverns are analyzed. Numerical models and analyses should also be conducted for vertically aligned and the inclined configurations (Liu et al. 2008, Chehade and Shahour 2008, Gercek 2005). The performances of rock reinforcement under interaction between twin caverns should also be considered.
- (6) Various self-pruning neural network programs can be considered to alleviate the problem of finding the optimal architecture (Williams 1995, Burden and Winkler 2009). These programs automatically add and delete hidden neurons as they learn the mapping function. For the MARS algorithm adopted in this study, the number of basis functions and the choices of basis functions and the knots are determined by Generalized Cross-Validation. Further work may also consider other machine learning approaches such as the genetic algorithms and support vector machines to optimize the knot locations and the basis functions.
- (7) As for the reliability assessment methods, the adopted Monte Carlo method (@RISK package) in this thesis is based on the Latin Hypercube sampling technique. More advanced simulation techniques such as the directional simulation and subset simulation may further improve the efficiency and resolution of probabilistic analysis (Wang et al. 2011). For the reliability analysis problems considered in this thesis, the basic variables are assumed to be single-valued random variables. This assumption is valid for quantities that are concentrated at discrete points in space. A random field approach to model the rock properties can also be considered.

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Appendix A

MATLAB BP algorithm adopted for the BPNN models

```

clear
clc                % clear the command window

rand('seed', 1);
% randn('seed', 1);

hiddenminno = 10
best = 10
for run = 1:1:best
    maxepoch = 500;

% trainset;
Variable1 = [...];
Variable2 = [...];
Dependent = [...];
traininput1 = [Variable1; Variable2];
traintarget1 = Dependent;

% normalize
[traininput, mintraininput1, maxtraininput1, traintarget, mintraintarget1, maxtraintarget1] =
premnmx(traininput1, traintarget1);

% testset;
testVariable1 = [...];
testVariable2 = [...];
testDependent = [...];
testinput1 = [testVariable1; testVariable2];
[testinput] = tramnmx(testinput1, mintraininput1, maxtraininput1);

testtarget = testDependent;
[PN, minp, maxp, TN, mint, maxt] = premnmx(testinput1, testtarget);
[atrno, trainexpno] = size(traininput);          % 'atrno' is the number of attributes,
'trainexpno' is the number of training examples

% rand('state', sum(100*clock));

% train the neural networks
net = newff(MinMax(traininput), [hiddenminno 1], {'logsig' 'tansig'});
net.trainParam.epochs = maxepoch;
net.trainParam.goal = 1e-10;
net.trainParam.max_fail = 100;
net.trainParam.min_grad = 1e-15;
net.trainParam.mu_max = 1e20;
net.trainParam.mu_dec = 0.7;
net.trainParam.mu_inc = 1.03;
net.trainParam.lr = 0.01;
net = train(net, traininput, traintarget);

```

```

% test
[n,testexpno] = size(testinput);           % 'testexpno' is the number of test
examples, 'n' is useless
[m,trexpxno] = size(traininput);
output2 = zeros(1,trexpxno);
output1 = zeros(1,testexpno);
output1 = sim(net,testinput);
[output_te] = postmnmx(output1,mintraintarget1,maxtraintarget1);
output2 = sim(net,traininput);
[output_tr] = postmnmx(output2,mintraintarget1,maxtraintarget1);

figure(run);
[m,b,r]=postreg(output_te,testtarget);
mse_te = mse(output_te - testtarget);      % obtain the mean squared error of
the ensemble
mse_tr = mse(output_tr - traintarget1);
mse_te1 = mse(output1 - TN) ;              % according to the scaled value
mse_tr2 = mse(output2 - traintarget) ;
fprintf('single hidden=%g run=%g r=%-12.5g mse_te=%-12.5g mse_tr=%-12.5g',
hiddenminno, run, r, mse_te, mse_tr);
fprintf('\n\n');
end

% end of function

```

For the mathematical explanations of the MATLAB built-in functions *premnmx*, *tramnmx*, *postmnmx*, *postreg*, ..., please refer to MATLAB Help on Neural Network Toolbox.

After obtaining the optimal BPNN model, to obtain the weights connecting the input layer and the hidden layer, type “w1=net.iw{1,1}” in the MATLAB command window; to obtain the bias values at neurons of the hidden layer, type “theta1=net.b{1}” in the command window; to get the weights connecting the hidden layer and the output layer, type “w2=net.lw{2,1}” in the command window; and to obtain the bias value at the output layer, type “theta2=net.b{2}” in the command window.

Appendix B

MARS algorithms adopted for the built models using MATLAB

```

clc
clear
cd 'C:\ASCEBP'
a=load('pillaraverage.txt');
X1=a(:,1);
X2=a(:,2);
X=[X1, X2];
Y=a(:,3);
params = aresparams(4, [], false, [], [], 2);
model = aresbuild(X, Y, params)
aresanova(model, X, Y)
Yq = arespredict(model, X);
mu=mean(Y);
J=sum((Yq-Y).^2);
S=sum((Y-mu).^2);
tmse=sum((Y-Yq).^2);
mse=tmse/length(Y)
rmse=sqrt(mse);
r2=1-J/S
b=load('pillaraveragetesting.txt');
Xt(:,1)=b(:, 1);
Xt(:,2)=b(:, 2);
Yt=b(:, 3);
[MSE, RMSE, RRMSE, R2] = arestest(model, Xt, Yt)
aresplot(model)

```

aresparams

```

function trainParams = aresparams(maxFuncs, c, cubic, cubicFastLevel, ...selfInteractions,
maxInteractions, threshold, prune, useMinSpan, ...useEndSpan, maxFinalFuncs)

```

```

if (nargin < 1) || (isempty(maxFuncs))
    trainParams.maxFuncs = 21;
else
    trainParams.maxFuncs = maxFuncs;
end

```

```

if (nargin < 2) || (isempty(c))
    trainParams.c = 3;
else
    trainParams.c = c;
end

```

```

if (nargin < 3) || (isempty(cubic))
    trainParams.cubic = true;
else

```

```

    trainParams.cubic = cubic;
end

if (nargin < 4) || (isempty(cubicFastLevel))
    trainParams.cubicFastLevel = 2;
else
    trainParams.cubicFastLevel = cubicFastLevel;
end

if (nargin < 5) || (isempty(selfInteractions))
    trainParams.selfInteractions = 1;
else
    trainParams.selfInteractions = selfInteractions;
end
if (trainParams.cubic) && (trainParams.selfInteractions > 1)
    trainParams.selfInteractions = 1;
    disp('Warning: trainParams.selfInteractions value reverted to 1 due to piecewise-cubic
model setting.');
```

```

end

if (nargin < 6) || (isempty(maxInteractions))
    trainParams.maxInteractions=1;
else
    trainParams.maxInteractions = maxInteractions;
end

if (nargin < 7) || (isempty(threshold))
    trainParams.threshold = 1e-4;
else
    trainParams.threshold = threshold;
end

if (nargin < 8) || (isempty(prune))
    trainParams.prune = true;
else
    trainParams.prune = prune;
end

if (nargin < 9) || (isempty(useMinSpan))
    trainParams.useMinSpan = -1; % default = -1 = automatic
else
    if useMinSpan == 0
        trainParams.useMinSpan = 1; % 1 and 0 is the same here (no endspar)
    else
        trainParams.useMinSpan = useMinSpan;
    end
end

if (nargin < 10) || (isempty(useEndSpan))
    trainParams.useEndSpan = -1;
else
    if useEndSpan == 0
        trainParams.useEndSpan = 1;
    end
end

```

```

    else
        trainParams.useEndSpan = useEndSpan;
    end
end

if (nargin < 11) || (isempty(maxFinalFuncs))
    trainParams.maxFinalFuncs = Inf;
else
    trainParams.maxFinalFuncs = maxFinalFuncs;
end

return

```

aresbuild

```

function [model, time] = aresbuild(Xtr, Ytr, trainParams, weights, modelOld, verbose)

if nargin < 2
    error('Too few input arguments.');
```

end

```

if (isempty(Xtr)) || (isempty(Ytr))
    error('Training data is empty.');
```

end

```

if (~isfloat(Xtr)) || (~isfloat(Ytr))
    error('Data type should be floating-point.');
```

end

```

[n, d] = size(Xtr); % number of data cases and number of input variables
if size(Ytr,1) ~= n
    error('The number of rows in the matrix and the vector should be equal.');
```

end

```

if size(Ytr,2) ~= 1
    error('Ytr should have one column.');
```

end

```

if (nargin < 3) || (isempty(trainParams))
    trainParams = aresparams();
end
if trainParams.maxInteractions >= 2
    trainParams_actual_c = trainParams.c;
else
    trainParams_actual_c = 2*trainParams.c/3; % penalty coefficient for additive modeling
end
if (trainParams.cubic) && (trainParams.selfInteractions > 1)
    trainParams.selfInteractions = 1;
    disp('Warning: trainParams.selfInteractions value reverted to 1 due to piecewise-cubic
model setting.');
```

end

```

if trainParams.cubic
    doCubicFastLevel = trainParams.cubicFastLevel;
    if trainParams.cubicFastLevel > 0
        trainParams.cubic = false;
    end
end

```

```

    end
else
    doCubicFastLevel = -1;
end
if trainParams.useMinSpan == 0
    trainParams.useMinSpan = 1;
end
if trainParams.useEndSpan == 0
    trainParams.useEndSpan = 1;
end

if (nargin < 4)
    weights = [];
else
    if (~isempty(weights)) && ...
        ((size(weights,1) ~= n) || (size(weights,2) ~= 1))
        error('weights vector is of wrong size.');
```

end

```

end
wd = diag(weights);

if nargin < 5
    modelOld = [];
end

if (nargin < 6) || (isempty(verbose))
    verbose = true;
end

if verbose, fprintf('Building ARES model...\n'); end
ws = warning('off');
tic;

maxIters = floor(trainParams.maxFuncs / 2);
YtrMean = mean(Ytr);
YtrSS = sum((Ytr - YtrMean) .^ 2);
minX = min(Xtr);
maxX = max(Xtr);

if trainParams.useEndSpan < 0
    endSpan = getEndSpan(d); % automatic
else
    endSpan = trainParams.useEndSpan;
end

if isempty(modelOld)
    X = ones(n,1);
    err = 1; % normalized error for the constant model
    model.coefs = YtrMean;
    model.knotdims = {};
    model.knotsites = {};
    model.knotdirs = {};
    model.parents = [];
```

```

    model.trainParams = [];
    model.MSE = Inf;
    model.GCV = Inf;
else
    model = modelOld; % modelOld is the initial model
end

if endSpan*2 >= n
    if isempty(modelOld)
        model.MSE = YtrSS / n;
        model.GCV = gcv(model, model.MSE, n, trainParams_actual_c);
        if trainParams.cubic
            model.t1 = [];
            model.t2 = [];
        end
    end
else
    % FORWARD PHASE

    if isempty(modelOld) % no forward phase when modelOld is used

        if verbose, fprintf('Forward phase .'); end

        % create sorted lists of data cases for knot placements
        [sortedXtr sortedXtrInd] = sort(Xtr);
        if trainParams.useEndSpan ~= 1 % throw away data cases at the ends of the intervals
            sortedXtr = sortedXtr(endSpan:end-(endSpan-1),:);
            sortedXtrInd = sortedXtrInd(endSpan:end-(endSpan-1),:);
        end

        if trainParams.cubic
            tmp_t1 = [];
            tmp_t2 = [];
        end
        basisFunctionList = []; % will contain candidate basis functions
        numNewFuncs = 0; % how many basis functions added in the last iteration

        % the main loop of the forward phase
        for depth = 1 : maxIters
            basisFunctionList = createList(basisFunctionList, Xtr, sortedXtr, sortedXtrInd, ...
                n, d, model, numNewFuncs, trainParams, endSpan);

            % stop the forward phase if basisFunctionList is empty
            if isempty(basisFunctionList)
                if trainParams.cubic
                    t1 = tmp_t1;
                    t2 = tmp_t2;
                end
                break;
            end

            tmpErr = inf(1,size(basisFunctionList,2));

```



```

tmpCoefs = inf(length(model.coefs)+2, size(basisFunctionList,2));
Xtmp = zeros(n,size(X,2)+2);
if ~trainParams.cubic
    Xtmp(:,1:end-2) = X;
end

% try all the reflected pairs in the list
for i = 1 : size(basisFunctionList,2)
    if trainParams.cubic
        [t1    t2    dif] = findsideknots(model,    basisFunctionList{1,i},
basisFunctionList{2,i}, ..., minX, maxX, tmp_t1, tmp_t2);
        Xtmp(:,1:end-2) = X;
        % update basis functions with the updated side knots
        for j = 1 : length(model.knotdims)
            if dif(j)
                Xtmp(:,j+1) = createbasisfunction(Xtr, Xtmp, model.knotdims{j},
model.knotsites{j}, ... model.knotdirs{j}, model.parents(j), minX, maxX, t1(j,:), t2(j,:));
            end
        end
        % New basis function
        dirs = basisFunctionList{3,i};
        Xtmp(:,end-1) = createbasisfunction(Xtr, Xtmp, basisFunctionList{1,i},
basisFunctionList{2,i}, ... dirs, basisFunctionList{4,i}, minX, maxX, t1(end,:), t2(end,:));
        if isnan(Xtmp(1,end-1)), Xtmp(:,end-1) = []; end
        % Reflected partner
        dirs(end) = -dirs(end);
        Xtmp(:,end) = createbasisfunction(Xtr, Xtmp, basisFunctionList{1,i},
basisFunctionList{2,i}, ... dirs, basisFunctionList{4,i}, minX, maxX, t1(end,:), t2(end,:));
        if isnan(Xtmp(1,end)), Xtmp(:,end) = []; end
    else
        % New basis function
        dirs = basisFunctionList{3,i};
        Xtmp(:,end-1) = createbasisfunction(Xtr, Xtmp, basisFunctionList{1,i}, ...
            basisFunctionList{2,i}, dirs, basisFunctionList{4,i}, minX, maxX);
        if isnan(Xtmp(1,end-1)), Xtmp(:,end-1) = []; end
        % Reflected partner
        dirs(end) = -dirs(end);
        Xtmp(:,end) = createbasisfunction(Xtr, Xtmp, basisFunctionList{1,i}, ...
            basisFunctionList{2,i}, dirs, basisFunctionList{4,i}, minX, maxX);
        if isnan(Xtmp(1,end)), Xtmp(:,end) = []; end
    end
    if size(Xtmp,2) == size(X,2)+2 % both basis functions created
        [coefs tmpErr(i)] = lreg(Xtmp, Ytr, weights, wd);
        tmpErr(i) = tmpErr(i) / YtrSS;
        tmpCoefs(:,i) = coefs;
    elseif size(Xtmp,2) == size(X,2)+1 % one of the basis functions not created
        [coefs tmpErr(i)] = lreg(Xtmp, Ytr, weights, wd);
        tmpErr(i) = tmpErr(i) / YtrSS;
        tmpCoefs(:,i) = [coefs; NaN];
        Xtmp = [Xtmp zeros(n,1)];
    else
        tmpErr(i) = Inf;
        Xtmp = [Xtmp zeros(n,2)];
    end
end

```

```

    end
end

[newErr, ind] = min(tmpErr); % find out the best modification

% stop the forward phase if no correct model was created or if the decrease in error
is below the threshold
if (isnan(newErr)) || (err(end) - newErr < trainParams.threshold)
    if trainParams.cubic
        t1 = tmp_t1;
        t2 = tmp_t2;
    end
    break;
end

if trainParams.cubic
    [t1, t2, dif] = findsideknots(model, basisFunctionList{1, ind},
basisFunctionList{2, ind}, ... d, minX, maxX, tmp_t1, tmp_t2);
    % update basis functions with the updated side knots
    for j = 1 : length(model.knotdims)
        if dif(j)
            X(:,j+1) = createbasisfunction(Xtr, X, model.knotdims{j},
model.knotsites{j}, ... model.knotdirs{j}, model.parents(j), minX, maxX, t1(j,:), t2(j,:));
            end
        end
        % Add the new basis function
        dirs = basisFunctionList{3, ind};
        Xn = createbasisfunction(Xtr, X, basisFunctionList{1, ind},
basisFunctionList{2, ind}, ... dirs, basisFunctionList{4, ind}, minX, maxX, t1(end,:), t2(end,:));
        if isnan(Xn(1)), Xn = []; end
        % Add the reflected partner
        dirs(end) = -dirs(end);
        Xn2 = createbasisfunction(Xtr, X, basisFunctionList{1, ind},
basisFunctionList{2, ind}, ... dirs, basisFunctionList{4, ind}, minX, maxX, t1(end,:), t2(end,:));
        if isnan(Xn2(1)), Xn2 = []; end
        X = [X Xn Xn2];
        if ~isempty(Xn) && ~isempty(Xn2) % one of the basis functions is not created
            t1(end+1,:) = t1(end,:);
            t2(end+1,:) = t2(end,:);
        end
    end
else
    dirs = basisFunctionList{3, ind};
    % Add the new basis function
    Xn = createbasisfunction(Xtr, X, basisFunctionList{1, ind}, ...
        basisFunctionList{2, ind}, dirs, basisFunctionList{4, ind}, minX, maxX);
    if isnan(Xn(1)), Xn = []; end
    % Add the reflected partner
    dirs(end) = -dirs(end);
    Xn2 = createbasisfunction(Xtr, X, basisFunctionList{1, ind}, ...
        basisFunctionList{2, ind}, dirs, basisFunctionList{4, ind}, minX, maxX);
    if isnan(Xn2(1)), Xn2 = []; end
    X = [X Xn Xn2];
end
end

```

```

model.coefs = tmpCoefs(:,ind);

% add the basis functions to the model
numNewFuncs = 0;
dirs = basisFunctionList{3, ind};
if ~isempty(Xn)
    model.knotdims{end+1,1} = basisFunctionList{1, ind};
    model.knotsites{end+1,1} = basisFunctionList{2, ind};
    model.knotdirs{end+1,1} = dirs;
    model.parents(end+1,1) = basisFunctionList{4, ind};
    numNewFuncs = numNewFuncs + 1;
else
    model.coefs(end) = [];
end
if ~isempty(Xn2)
    dirs(end) = -dirs(end);
    model.knotdims{end+1,1} = basisFunctionList{1, ind};
    model.knotsites{end+1,1} = basisFunctionList{2, ind};
    model.knotdirs{end+1,1} = dirs;
    model.parents(end+1,1) = basisFunctionList{4, ind};
    numNewFuncs = numNewFuncs + 1;
else
    model.coefs(end) = [];
end

if verbose, fprintf('..'); end
err(end+1) = newErr;
if (newErr < trainParams.threshold) || ...
    (length(model.coefs) + 2 > n)
    break;
end

if trainParams.cubic
    tmp_t1 = t1;
    tmp_t2 = t2;
end
basisFunctionList(:,ind) = [];
end % end of the main loop

if verbose, fprintf('\n'); end

end % end of "isempty(modelOld)"

if isempty(modelOld)
    if (doCubicFastLevel == 1) || ...
        ((doCubicFastLevel >= 2) && (~trainParams.prune))
        % turn the cubic modelling on
        trainParams.cubic = true;
        [t1 t2] = findsideknots(model, [], [], d, minX, maxX, [], []);
        % update all the basis functions
        for i = 1 : length(model.knotdims)
            X(:,i+1) = createbasisfunction(Xtr, X, model.knotdims{i}, model.knotsites{i}, ...
                model.knotdirs{i}, model.parents(i), minX, maxX, t1(i,:), t2(i,:));
        end
    end
end

```

```

    end
    [model.coefs model.MSE] = lreg(X, Ytr, weights, wd);
    model.MSE = model.MSE / n;
else
    model.MSE = err(end) * YtrSS / n;
end
model.GCV = gcv(model, model.MSE, n, trainParams_actual_c);
if trainParams.cubic
    model.t1 = t1;
    model.t2 = t2;
end
end
end

% BACKWARD PHASE

if trainParams.prune

    if verbose, fprintf('Backward phase .'); end

    if ~isempty(modelOld) % create basis functions from scratch when modelOld is used
        if (doCubicFastLevel == -1) || (doCubicFastLevel >= 2)
            % create all the basis functions (linear) from scratch
            X = ones(n, length(model.knotdims)+1);
            for i = 1 : length(model.knotdims)
                X(:,i+1)=createbasisfunction(Xtr, X, model.knotdims{i}, model.knotsites{i}, ...
                    model.knotdirs{i}, model.parents(i), minX, maxX);
            end
            [model.coefs model.MSE] = lreg(X, Ytr, weights, wd);
            model.MSE = model.MSE / n;
            model.GCV = gcv(model, model.MSE, n, trainParams_actual_c);
        else
            % create all the basis functions (cubic) from scratch
            t1 = model.t1;
            t2 = model.t2;
            X = ones(n, length(model.knotdims)+1);
            for i = 1 : length(model.knotdims)
                X(:,i+1)=createbasisfunction(Xtr, X, model.knotdims{i}, model.knotsites{i}, ...
                    model.knotdirs{i}, model.parents(i), minX, maxX, t1(i,:), t2(i,:));
            end
        end
    end
end

models = {model};
mses = model.MSE;
gcvs = model.GCV;

% the main loop of the backward phase
for j = 1 : length(model.knotdims)
    tmpErr = inf(1, length(model.knotdims));
    tmpCoefs = inf(length(model.coefs)-1, length(model.knotdims));

    % try to delete model's basis functions one at a time
    for k = 1 : length(model.knotdims)

```

```

Xtmp = X;
Xtmp(:,k+1) = [];
if trainParams.cubic
    % create temporary t1, t2, and model with a deleted basis function
    tmp_t1 = t1;
    tmp_t1(k,:) = [];
    tmp_t2 = t2;
    tmp_t2(k,:) = [];
    tmp_model.knotdims = model.knotdims;
    tmp_model.knotdims(k) = [];
    tmp_model.knotsites = model.knotsites;
    tmp_model.knotsites(k) = [];
    tmp_model.knotdirs = model.knotdirs;
    tmp_model.knotdirs(k) = [];
    tmp_model.parents = model.parents;
    tmp_model.parents(k) = [];
    tmp_model.parents = updateParents(tmp_model.parents, k);
    [tmp_t1 tmp_t2 dif] = findsideknots(tmp_model, [], [], d, minX, maxX, tmp_t1,
tmp_t2);
    % update basis functions with the updated side knots
    for i = 1 : length(tmp_model.knotdims)
        if dif(i)
            Xtmp(:,i+1) = createbasisfunction(Xtr, Xtmp, tmp_model.knotdims{i},
tmp_model.knotsites{i}, ...tmp_model.knotdirs{i}, tmp_model.parents(i), minX, maxX,
tmp_t1(i,:), tmp_t2(i,:));
        end
    end
    end
    [coefs tmpErr(k)] = lreg(Xtmp, Ytr, weights, wd);
    tmpCoefs(:,k) = coefs;
end

[dummy, ind] = min(tmpErr); % find out the best modification
X(:,ind+1) = [];
model.coefs = tmpCoefs(:,ind);
model.knotdims(ind) = [];
model.knotsites(ind) = [];
model.knotdirs(ind) = [];
model.parents(ind) = [];
model.parents = updateParents(model.parents, ind);

if trainParams.cubic
    t1(ind,:) = [];
    t2(ind,:) = [];
    [t1 t2 dif] = findsideknots(model, [], [], d, minX, maxX, t1, t2);
    % update basis functions with the updated side knots
    for i = 1 : length(model.knotdims)
        if dif(i)
            X(:,i+1)=createbasisfunction(Xtr,X,model.knotdims{i}, odel.knotsites{i}, ...
model.knotdirs{i}, model.parents(i), minX, maxX, t1(i,:), t2(i,:));
        end
    end
    end
    model.t1 = t1;

```

```

    model.t2 = t2;
end

models{end+1} = model;
mses(end+1) = tmpErr(ind) / n;
gcvsv(end+1) = gcv(model, mses(end), n, trainParams_actual_c);

if verbose, fprintf('.'); end
end % end of the main loop

if trainParams.maxFinalFuncs >= length(models{1}.coefs)
    [g, ind] = min(gcvsv);
elseif trainParams.maxFinalFuncs > 1
    [g, ind] = min(gcvsv(end-trainParams.maxFinalFuncs+1:end));
    ind = ind + length(gcvsv) - trainParams.maxFinalFuncs;
else
    g = gcvsv(end);
    ind = length(gcvsv);
end
model = models{ind};

if doCubicFastLevel >= 2
    % turn the cubic modeling on
    trainParams.cubic = true;
    [t1 t2] = findsideknots(model, [], [], d, minX, maxX, [], []);
    % update all the basis functions
    X = ones(n, length(model.coefs));
    for i = 1 : length(model.knotdims)
        X(:,i+1) = createbasisfunction(Xtr, X, model.knotdims{i}, model.knotsites{i}, ...
            model.knotdirs{i}, model.parents(i), minX, maxX, t1(i,:), t2(i,:));
    end
    model.t1 = t1;
    model.t2 = t2;
    [model.coefs model.MSE] = lreg(X, Ytr, weights, wd);
    model.MSE = model.MSE / n;
    model.GCV = gcv(model, model.MSE, n, trainParams_actual_c);
else
    model.MSE = mses(ind);
    model.GCV = g;
end

if verbose, fprintf('\n'); end

end % end of "trainParams.prune"

end % end of "if endSpan*2 >= n"

model.trainParams = trainParams;
model.minX = minX;
model.maxX = maxX;
model.endSpan = endSpan;

time = toc;

```

```

if verbose
    fprintf('Number of basis functions in the final model: %d\n', length(model.coefs));
    fprintf('Total effective number of parameters: %0.1f\n', ...
        length(model.coefs) + model.trainParams.c * length(model.knotdims) / 2);
    maxDeg = 0;
    if length(model.knotdims) > 0
        for i = 1 : length(model.knotdims)
            if length(model.knotdims{i}) > maxDeg
                maxDeg = length(model.knotdims{i});
            end
        end
    end
    fprintf('Highest degree of interactions in the final model: %d\n', maxDeg);
    fprintf('Execution time: %0.2f seconds\n', time);
end
warning(ws);
return

```

aresanova

```

function aresanova(model, Xtr, Ytr)

if nargin ~= 3
    error('The number of input arguments should be exactly three.');
```

end

```

if (isempty(Xtr)) || (isempty(Ytr))
    error('Data is empty.');
```

end

```

n = size(Xtr,1);
if size(Ytr,1) ~= n
    error('The number of rows in the matrix and the vector should be equal.');
```

end

```

if size(Ytr,2) ~= 1
    error('Ytr should have one column.');
```

end

```

if model.trainParams.cubic
    fprintf('Type: piecewise-cubic\n');
```

else

```

    fprintf('Type: piecewise-linear\n');
```

end

```

fprintf('GCV: %0.3f\n', model.GCV);
fprintf('Total number of basis functions: %d\n', length(model.coefs));
fprintf('Total effective number of parameters: %0.1f\n', ...
    length(model.coefs) + model.trainParams.c * length(model.knotdims) / 2);
fprintf('ANOVA decomposition:\n');
```

```

fprintf('Func.\tSTD\tGCV\t#basis\t#params\tvariable(s)\n');
```

```

counter = 0;
for i = 1 : model.trainParams.maxInteractions
    combs = nchoosek(1:length(model.minX),i);
    for j = 1 : size(combs,1)
        [modelReduced usedBasis] = aresanovareduce(model, combs(j,:), true);
        if length(usedBasis) > 0

```

```

        counter = counter + 1;
        fprintf('%d\t\t', counter);
        fprintf('%7.3f\t', std(arespredict(modelReduced, Xtr))); % standard deviation of the
ANOVA function
        modelReduced = deleteBasis(model, usedBasis);
        Yq = arespredict(modelReduced, Xtr);
        MSE = mean((Ytr - Yq).^2);
        fprintf('%11.3f\t\t', gcv(modelReduced, MSE, n, model.trainParams.c)); % GCV
when the basis functions are deleted
        fprintf('%6d\t', length(usedBasis)); % the number of basis functions for that ANOVA
function
        fprintf('%7.1f\t\t', length(usedBasis) + model.trainParams.c * length(usedBasis) /
2); % effective parameters
        fprintf('%d ', combs(j,:)); % used variables
        fprintf('\n');
    end
end
end
return

```

```

function modelReduced = deleteBasis(model, nums)
modelReduced = model;
for i = length(nums) : -1 : 1
    modelReduced.coefs(nums(i)+1) = [];
    modelReduced.knotdims(nums(i)) = [];
    modelReduced.knotsites(nums(i)) = [];
    modelReduced.knotdirs(nums(i)) = [];
    modelReduced.parents(nums(i)) = [];
    if modelReduced.trainParams.cubic
        modelReduced.t1(nums(i),:) = [];
        modelReduced.t2(nums(i),:) = [];
    end
end
modelReduced.parents(:) = 0;
if modelReduced.trainParams.cubic
    % correct the side knots for the modified cubic model
    [modelReduced.t1 modelReduced.t2] = ...
        findsideknots(modelReduced, [], [], size(modelReduced.t1,2), modelReduced.minX,
modelReduced.maxX, [], []);
end
return

```

```

function g = gcv(model, MSE, n, c)
% Calculates GCV from model complexity, its Mean Squared Error, number of data cases
n, and penalty coefficient c.
enp = length(model.coefs) + c * length(model.knotdims) / 2; % model's effective number of
parameters
if enp >= n
    g = Inf;
else
    p = 1 - enp / n;
    g = MSE / (p * p);
end

```



```
return
```

```
arespredict —————
```

```
function Yq = arespredict(model, Xq)
if nargin < 2
    error('Too few input arguments.');
```

```
end
X = ones(size(Xq,1),length(model.knotdims)+1);
if model.trainParams.cubic
    for i = 1 : length(model.knotdims)
        X(:,i+1) = createbasisfunction(Xq, X, model.knotdims{i}, model.knotsites{i}, ...
            model.knotdirs{i}, model.parents(i), model.minX, model.maxX, model.t1(i,:),
model.t2(i,:));
    end
else
    for i = 1 : length(model.knotdims)
        X(:,i+1) = createbasisfunction(Xq, X, model.knotdims{i}, model.knotsites{i}, ...
            model.knotdirs{i}, model.parents(i), model.minX, model.maxX);
    end
end
Yq = X * model.coefs;
return
```

```
arestest —————
```

```
function [MSE, RMSE, RRMSE, R2] = arestest(model, Xtst, Ytst)

if nargin < 3
    error('Too few input arguments.');
```

```
end
if (isempty(Xtst)) || (isempty(Ytst))
    error('Data is empty.');
```

```
end
if (size(Xtst, 1) ~= size(Ytst, 1))
    error('The number of rows in the matrix and the vector should be equal.');
```

```
end
if size(Ytst,2) ~= 1
    error('The vector Ytst should have one column.');
```

```
end
MSE = mean((arespredict(model, Xtst) - Ytst) .^ 2);
RMSE = sqrt(MSE);
if size(Ytst, 1) > 1
    RRMSE = RMSE / std(Ytst, 1);
    R2 = 1 - MSE / var(Ytst, 1);
else
    RRMSE = Inf;
    R2 = Inf;
end
return
```

aresplot

```

function aresplot(model, minX, maxX, vals, gridSize)
if nargin < 1
    error('Too few input arguments.');
```

end

```

if (nargin < 2) || (isempty(minX))
    minX = model.minX;
else
    if length(minX) ~= length(model.minX)
        error('Vector minX is of wrong size.');
```

end

```

end
if (nargin < 3) || (isempty(maxX))
    maxX = model.maxX;
else
    if length(maxX) ~= length(model.maxX)
        error('Vector maxX is of wrong size.');
```

end

```

end
if (nargin < 5) || (isempty(gridSize))
    gridSize = 50;
end

if length(model.minX) < 2
    step = (maxX - minX) / gridSize;
    X = [minX:step:maxX]';
    plot(X, arespredict(model, X));
    return
end

if (nargin < 4) || (isempty(vals))
    ind1 = 1; vals(1) = NaN;
    ind2 = 2; vals(2) = NaN;
    if length(minX) > 2
        for i = 3 : length(minX)
            vals(i) = (maxX(i) - minX(i)) / 2;
        end
    end
else
    if length(minX) ~= length(vals)
        error('Vector vals is of wrong size.');
```

end

```

tmp = 0;
for i = 1 : length(vals)
    if isnan(vals(i))
        if tmp == 0
            ind1 = i;
            tmp = 1;
        elseif tmp == 1
            ind2 = i;
            tmp = 2;
        else

```

```

        tmp = 3;
        break;
    end
end
end
if tmp ~= 2
    error('Vector vals should contain NaN exactly two times.');
```

```

end
end

step1 = (maxX(ind1) - minX(ind1)) / gridSize;
step2 = (maxX(ind2) - minX(ind2)) / gridSize;

[X1,X2] = meshgrid(minX(ind1):step1:maxX(ind1), minX(ind2):step2:maxX(ind2));
XX1 = reshape(X1, numel(X1), 1);
XX2 = reshape(X2, numel(X2), 1);

X = zeros(size(XX1,1), length(minX));
X(:,ind1) = XX1;
X(:,ind2) = XX2;
for i = 1 : length(minX)
    if (i ~= ind1) && (i ~= ind2)
        X(:,i) = vals(i);
    end
end

YY = arespredict(model, X);
Y = reshape(YY, size(X1,1), size(X2,2));
surf(X1, X2, Y);
return

```

Appendix C

Table C. 1 Training data results of FS_{g_s} for BPNN and MARS

Q	B (m)	B/H	$FS_{g_s \text{ FDM}}$	$FS_{g_s \text{ BPNN}}$	$FS_{g_s \text{ MARS}}$
0.4	5	0.25	0.93	0.92	0.91
0.4	5	1	1.29	1.34	1.33
0.4	5	2	1.38	1.35	1.42
0.4	10	0.25	0.69	0.67	0.67
0.4	10	0.5	0.86	0.89	0.89
0.4	10	2	1.08	1.08	1.10
0.4	15	0.5	0.74	0.73	0.73
0.4	15	1	0.85	0.87	0.85
0.4	15	2	0.92	0.91	0.94
0.4	20	0.25	0.49	0.44	0.47
0.4	20	0.5	0.65	0.63	0.64
0.4	20	1	0.76	0.76	0.77
0.4	25	0.25	0.44	0.41	0.41
0.4	25	1	0.69	0.69	0.69
0.4	25	2	0.75	0.75	0.77
0.4	30	0.25	0.39	0.40	0.35
0.4	30	0.5	0.52	0.56	0.48
0.4	30	2	0.69	0.72	0.69
1	5	0.25	1.13	1.12	1.10
1	5	1	1.56	1.58	1.53
1	5	2	1.64	1.63	1.63
1	10	0.25	0.84	0.84	0.83
1	10	0.5	1.04	1.08	1.06
1	10	2	1.32	1.32	1.28
1	15	0.5	0.90	0.89	0.90
1	15	1	1.03	1.04	1.02
1	15	2	1.13	1.12	1.12
1	20	0.25	0.60	0.57	0.63
1	20	0.5	0.79	0.77	0.81
1	20	1	0.92	0.91	0.93
1	25	0.25	0.53	0.52	0.56
1	25	1	0.84	0.83	0.84
1	25	2	0.92	0.91	0.94
1	30	0.25	0.48	0.50	0.50
1	30	0.5	0.64	0.67	0.63
1	30	2	0.85	0.86	0.85
4	5	0.25	1.42	1.41	1.45
4	5	1	1.95	1.92	1.93
4	5	2	2.02	2.04	2.05
4	10	0.25	1.05	1.08	1.04
4	10	0.5	1.31	1.34	1.30
4	10	2	1.65	1.67	1.56
4	15	0.5	1.12	1.11	1.13
4	15	1	1.29	1.29	1.26
4	15	2	1.42	1.42	1.39
4	20	0.25	0.75	0.75	0.79
4	20	0.5	0.99	0.96	1.01

Table C. 1 Training data results of FS_{g_s} for BPNN and MARS contd.

Q	B (m)	B/H	$FS_{g_s_FDM}$	$FS_{g_s_BPNN}$	$FS_{g_s_MARS}$
4	20	1	1.14	1.13	1.14
4	25	0.25	0.66	0.67	0.71
4	25	1	1.04	1.01	1.03
4	25	2	1.15	1.13	1.16
4	30	0.25	0.60	0.63	0.62
4	30	0.5	0.80	0.81	0.79
4	30	2	1.06	1.05	1.05
10	5	0.25	1.65	1.64	1.64
10	5	1	2.27	2.21	2.20
10	5	2	2.35	2.35	2.39
10	10	0.25	1.23	1.28	1.21
10	10	0.5	1.52	1.56	1.55
10	10	2	1.93	1.94	1.88
10	15	0.5	1.31	1.29	1.37
10	15	1	1.51	1.51	1.51
10	15	2	1.64	1.67	1.70
10	20	0.25	0.87	0.88	0.89
10	20	0.5	1.15	1.12	1.18
10	20	1	1.34	1.31	1.32
10	25	0.25	0.78	0.78	0.76
10	25	1	1.12	1.17	1.17
10	25	2	1.33	1.33	1.36
10	30	0.25	0.70	0.71	0.64
10	30	0.5	0.93	0.91	0.88
10	30	2	1.23	1.22	1.21
40	5	0.25	1.95	1.93	1.97
40	5	1	2.67	2.65	2.64
40	5	2	2.76	2.78	2.77
40	10	0.25	1.46	1.49	1.48
40	10	0.5	1.79	1.82	1.81
40	10	2	2.27	2.27	2.20
40	15	0.5	1.55	1.52	1.56
40	15	1	1.78	1.81	1.82
40	15	2	1.94	1.93	1.95
40	20	0.25	1.04	1.03	1.02
40	20	0.5	1.36	1.33	1.31
40	20	1	1.58	1.59	1.57
40	25	0.25	0.94	0.92	0.94
40	25	1	1.44	1.44	1.46
40	25	2	1.58	1.57	1.59
40	30	0.25	0.85	0.87	0.86
40	30	0.5	1.12	1.13	1.10
40	30	2	1.46	1.47	1.48

Appendix D

Procedures for partitioning of BPNN weights for FS_{g_s} of single cavern

This appendix details the procedure for partitioning the connection weights to determine the relative importance of the various inputs. The method essentially involves partitioning the hidden-output connection weights of each hidden neuron into components associated with each input neuron.

The neural network works with three input neurons, five hidden neurons and one output neuron with the connection weights as shown below:

Table D.1 BPNN weights and outputs

Hidden neurons	Weights			Output
	Input 1	Input 2	Input 3	
Hidden 1	-5.3695	0.3458	-0.1813	-2.7793
Hidden 2	0.1661	-0.1654	-1.5712	-1.9866
Hidden 3	-0.5772	1.1342	2.7913	-2.3118
Hidden 4	8.9585	0.1639	-0.0765	11.5924
Hidden 5	-0.0846	0.4932	-0.0093	2.8236

The computation process is as follows:

(1) For each hidden neuron i , multiply the absolute value of the hidden-output layer connection weight by the absolute value of the hidden-input layer connection weight. Do this for each input variable j . The following products P_{i-j} are obtained:

Table D.2 P_{i-j} results

Hidden neurons	P_{i-j}		
	Input 1	Input 2	Input 3
Hidden 1	14.9235	0.9611	0.5039
Hidden 2	0.3300	0.3286	3.1213
Hidden 3	1.3344	2.6220	6.4529
Hidden 4	103.8505	1.9000	0.8868
Hidden 5	0.2389	1.3926	0.0263

(2) For each hidden neuron, divide P_{i-j} by the sum for all the input variables to obtain Q_{i-j} . For example for hidden 1, $Q_{1-1} = \frac{P_{1-1}}{P_{1-1} + P_{1-2} + P_{1-3}} = 0.9106$

(3) For each input neuron, sum the product S_j formed from the previous computation of Q_{i-j} . For example, $S_1 = Q_{1-1} + Q_{2-1} + Q_{3-1} + Q_{4-1} + Q_{5-1}$.

Table D.3 Q_{i-j} results

Hidden neurons	Q_{i-j}		
	Input 1	Input 2	Input 3
Hidden 1	0.9106	0.0586	0.0307
Hidden 2	0.0873	0.0869	0.8258
Hidden 3	0.1282	0.2519	0.6199
Hidden 4	0.9739	0.0178	0.0083
Hidden 5	0.1441	0.8401	0.0158
Sum	$S_1=2.2441$	$S_2=1.2553$	$S_3=1.5006$

(4) Divide S_j by $\max(S_j)$ for all the input variables. Expressed as a percentage, this gives the relative importance of all output weights attributable to the given input variable. For example, for the input neuron 1, the relative importance (%) is equal to $(S_1 \times 100) / \max(S_1, S_2, S_3) = 100$.

Table D.4 Relative importance results

Relative importance	Input 1	Input 2	Input 3
(%)	100	55.94	66.87

Appendix E

Calculation of BPNN output FS_{g_s}

The transfer functions used for BP neural network output for FS_{g_s} are ‘*tansig*’ transfer function for hidden layer to output layer and ‘*purelin*’ transfer function for output layer to target. The calculation process of BPNN output for FS_{g_s} is elaborated in detail as follows:

From the connection weights for a trained neuron network, it is possible to develop a mathematical equation relating the input parameters and the single output parameter Y using

$$[E-1] \ Y = f_{sig} \left\{ b_0 + \sum_{k=1}^h \left[w_k f_{sig} \left(b_{hk} + \sum_{i=1}^m w_{ik} X_i \right) \right] \right\}$$

in which b_0 is the bias at the output layer, w_k is the weight connection between neuron k of the hidden layer and the single output neuron, b_{hk} is the bias at neuron k of the hidden layer ($k=1, h$), w_{ik} is the weight connection between input variable i ($i=1, m$) and neuron k of the hidden layer, X_i is the input parameter i , and f_{sig} is the sigmoid (*tansig* & *purelin*) transfer function.

Using the connection weights of the trained neural network, the following steps can be followed to mathematically express the BPNN model:

Step1: Normalize the input values for Q , B and B/H linearly using

$$x_{norm} = 2(x_{actual} - x_{min}) / (x_{max} - x_{min}) - 1$$

Let the actual $Q = X_{1a}$ and the normalized $Q = X_1$

$$[E-2] \ X_1 = -1 + 2 * (X_{1a} - 0.4) / 39.6$$

Let the actual $B = X_{2a}$ and the normalized $B = X_2$

$$[E-3] \ X_2 = -1 + 2 * (X_{2a} - 5) / 25$$

Let the actual $B/H = X_{3a}$ and the normalized $B/H = X_3$

$$[E-4] \ X_3 = -1 + 2 * (X_{3a} - 0.25) / 1.75$$

Step2: Calculate the normalized value (Y_1) using the following expressions:

$$[E-5] \ A_1 = -2.7793 - 5.3695X_1 + 0.3458X_2 - 0.1813X_3$$

$$[E-6] \ A_2 = -1.9866 + 0.1661X_1 - 0.1654X_2 - 1.5712X_3$$

$$[E-7] \ A_3 = -2.3118 - 0.5772X_1 + 1.1342X_2 + 2.7913X_3$$

$$[E-8] \ A_4 = 11.5924 + 8.9585X_1 + 0.1639X_2 - 0.0765X_3$$

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$$[E-9] A_5=2.8236-0.0846X_1+0.4932X_2-0.0093X_3$$

$$[E-10] B_1= -0.1533 \times \tanh(A_1)$$

$$[E-11] B_2= -0.7188 \times \tanh(A_2)$$

$$[E-12] B_3= 0.0377 \times \tanh(A_3)$$

$$[E-13] B_4= 32.8240 \times \tanh(A_4)$$

$$[E-14] B_5= -60.0059 \times \tanh(A_5)$$

$$[E-15] C_1=26.0038+ B_1+ B_2+ B_3+B_4+B_5$$

$$[E-16] Y_1=C_1$$

Step3: De-normalize the output to obtain the factor of safety

$$[E-17] FS_{g_s}=0.39+2.37 \times (Y_1+1)/2$$

Appendix F

Table F. 1 Training data results of $u_{\max_s}$ for BPNN

Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)	$u_{\max_s_BP}$ (m)	Q	B (m)	B/H	K_0	$u_{\max_s_FDM}$ (m)	$u_{\max_s_BP}$ (m)
0.4	5	0.5	2	0.402	0.394	10	15	0.5	1	0.036	0.032
0.4	5	0.5	0.5	0.244	0.244	10	20	0.5	1	0.058	0.049
0.4	5	1	2	0.252	0.265	10	25	0.5	1	0.088	0.085
0.4	10	1	2	0.553	0.555	10	5	0.5	0.5	0.007	0.008
0.4	5	1	1	0.159	0.160	10	10	0.5	0.5	0.017	0.015
0.4	10	1	0.5	0.408	0.402	10	15	0.5	0.5	0.029	0.028
0.4	5	2	2	0.208	0.204	10	25	0.5	0.5	0.075	0.073
0.4	10	2	2	0.411	0.409	10	5	1	2	0.008	0.004
0.4	10	2	1	0.289	0.288	10	20	1	2	0.033	0.037
0.4	5	2	0.5	0.126	0.123	10	25	1	2	0.042	0.045
0.4	10	2	0.5	0.289	0.296	10	30	1	2	0.050	0.054
1	5	0.25	2	0.208	0.203	10	5	1	1	0.005	0.007
1	5	0.25	0.5	0.137	0.136	10	10	1	1	0.011	0.009
1	10	0.5	2	0.237	0.237	10	15	1	1	0.017	0.013
1	10	0.5	1	0.197	0.198	10	20	1	1	0.024	0.024
1	5	0.5	0.5	0.066	0.065	10	30	1	1	0.040	0.042
1	10	0.5	0.5	0.172	0.184	10	5	1	0.5	0.003	0.006
1	5	1	2	0.078	0.074	10	15	1	0.5	0.014	0.010
1	15	1	2	0.220	0.221	10	20	1	0.5	0.019	0.019
1	5	1	1	0.050	0.051	10	30	1	0.5	0.032	0.035
1	15	1	1	0.166	0.162	10	5	2	2	0.005	0.003
1	5	1	0.5	0.033	0.039	10	20	2	2	0.026	0.021
1	15	1	0.5	0.144	0.147	10	25	2	2	0.031	0.027
1	5	2	2	0.066	0.065	10	30	2	2	0.036	0.039
1	15	2	2	0.172	0.163	10	5	2	1	0.003	0.007
1	20	2	2	0.268	0.269	10	20	2	1	0.016	0.012
1	5	2	1	0.043	0.044	10	25	2	1	0.019	0.017
1	15	2	1	0.118	0.114	10	30	2	1	0.023	0.024
1	20	2	1	0.194	0.198	10	5	2	0.5	0.003	0.006
1	5	2	0.5	0.043	0.040	10	30	2	0.5	0.023	0.018
1	10	2	0.5	0.071	0.069	10	15	2	0.5	0.012	0.008
1	20	2	0.5	0.194	0.194	10	20	2	0.5	0.016	0.010
4	5	0.25	2	0.063	0.064	40	5	0.25	2	0.011	0.011
4	5	0.25	1	0.045	0.046	40	10	0.25	2	0.026	0.026
4	5	0.25	0.5	0.035	0.036	40	20	0.25	2	0.087	0.081
4	10	0.25	0.5	0.106	0.109	40	5	0.25	1	0.007	0.006
4	5	0.5	2	0.034	0.036	40	15	0.25	1	0.033	0.032
4	15	0.5	2	0.123	0.122	40	20	0.25	1	0.062	0.061
4	5	0.5	1	0.023	0.026	40	5	0.25	0.5	0.006	0.006
4	10	0.5	1	0.055	0.043	40	10	0.25	0.5	0.015	0.011
4	5	0.5	0.5	0.016	0.020	40	20	0.25	0.5	0.058	0.053
4	10	0.5	0.5	0.043	0.034	40	5	0.5	1	0.003	0.004
4	15	0.5	0.5	0.075	0.073	40	15	0.5	1	0.014	0.012
4	5	1	2	0.024	0.027	40	20	0.5	1	0.020	0.022
4	15	1	2	0.067	0.066	40	25	0.5	1	0.039	0.031
4	20	1	2	0.085	0.085	40	5	0.5	0.5	0.003	0.004

Table F. 1 Training data results of $u_{\max_s}$ for BPNN contd.

Q	B (m)	B/H	K_0	$u_{\max_s, \text{FDM}}$ (m)	$u_{\max_s, \text{BP}}$ (m)	Q	B (m)	B/H	K_0	$u_{\max_s, \text{FDM}}$ (m)	$u_{\max_s, \text{BP}}$ (m)
4	25	1	2	0.105	0.108	40	15	0.5	0.5	0.011	0.009
4	5	1	1	0.015	0.022	40	25	0.5	0.5	0.024	0.028
4	15	1	1	0.046	0.042	40	30	0.5	0.5	0.035	0.043
4	20	1	1	0.064	0.061	40	5	0.5	2	0.006	0.006
4	25	1	1	0.086	0.082	40	10	0.5	2	0.012	0.012
4	5	1	0.5	0.009	0.018	40	20	0.5	2	0.029	0.028
4	15	1	0.5	0.036	0.033	40	30	0.5	2	0.052	0.062
4	20	1	0.5	0.051	0.051	40	5	1	2	0.003	0.004
4	25	1	0.5	0.069	0.071	40	10	1	2	0.007	0.007
4	5	2	2	0.020	0.024	40	20	1	2	0.014	0.014
4	15	2	2	0.053	0.054	40	30	1	2	0.021	0.024
4	20	2	2	0.065	0.067	40	5	1	1	0.002	0.004
4	25	2	2	0.079	0.081	40	15	1	1	0.006	0.005
4	5	2	1	0.012	0.021	40	20	1	1	0.009	0.007
4	15	2	1	0.033	0.036	40	25	1	1	0.011	0.015
4	20	2	1	0.042	0.046	40	5	2	2	0.002	0.003
4	30	2	1	0.067	0.075	40	15	2	2	0.007	0.007
4	5	2	0.5	0.012	0.017	40	20	2	2	0.010	0.007
4	15	2	0.5	0.033	0.029	40	30	2	2	0.014	0.008
4	20	2	0.5	0.042	0.038	40	5	2	1	0.001	0.004
4	25	2	0.5	0.053	0.049	40	20	2	1	0.006	0.004
10	5	0.25	2	0.025	0.032	40	25	2	1	0.007	0.005
10	15	0.25	2	0.125	0.127	40	30	2	1	0.009	0.009
10	5	0.25	1	0.018	0.024	40	5	1	0.5	0.001	0.004
10	10	0.25	1	0.048	0.052	40	10	1	0.5	0.003	0.004
10	5	0.5	2	0.012	0.011	40	20	1	0.5	0.007	0.005
10	10	0.5	2	0.030	0.029	40	25	1	0.5	0.009	0.011
10	15	0.5	2	0.049	0.043	40	5	2	0.5	0.001	0.004
10	25	0.5	2	0.113	0.114	40	10	2	0.5	0.001	0.004
10	5	0.25	0.5	0.014	0.019	40	15	2	0.5	0.004	0.004
10	10	0.25	0.5	0.038	0.044	40	20	2	0.5	0.006	0.004
10	5	0.5	1	0.008	0.009	40	30	2	0.5	0.009	0.004

Appendix G

Table G. 1 Training data results of $u_{\max_s}$ for MARS and relative error

Q	B (m)	B/H	K_0	$u_{\max_s_MARS}$ (m)	e (%)	Q	B (m)	B/H	K_0	$u_{\max_s_MARS}$ (m)	e (%)
0.4	5	0.5	2	0.375	6.6	10	15	0.5	1	0.042	15.0
0.4	5	0.5	0.5	0.253	3.6	10	20	0.5	1	0.063	8.2
0.4	5	1	2	0.293	16.4	10	25	0.5	1	0.085	4.3
0.4	10	1	2	0.525	5.1	10	5	0.5	0.5	-0.001	117.9
0.4	5	1	1	0.171	7.4	10	10	0.5	0.5	0.020	20.3
0.4	10	1	0.5	0.402	1.5	10	15	0.5	0.5	0.042	44.1
0.4	5	2	2	0.211	1.4	10	25	0.5	0.5	0.085	13.2
0.4	10	2	2	0.423	2.7	10	5	1	2	0.007	8.4
0.4	10	2	1	0.300	4.0	10	20	1	2	0.040	18.7
0.4	5	2	0.5	0.089	29.4	10	25	1	2	0.050	21.2
0.4	10	2	0.5	0.300	4.0	10	30	1	2	0.050	0.6
1	5	0.25	2	0.205	1.2	10	5	1	1	-0.007	247.9
1	5	0.25	0.5	0.151	10.5	10	10	1	1	0.004	63.9
1	10	0.5	2	0.230	2.7	10	15	1	1	0.015	15.0
1	10	0.5	1	0.176	10.5	10	20	1	1	0.026	5.3
1	5	0.5	0.5	0.094	43.5	10	30	1	1	0.036	9.1
1	10	0.5	0.5	0.176	2.4	10	5	1	0.5	-0.007	291.5
1	5	1	2	0.079	0.3	10	15	1	0.5	0.015	7.1
1	15	1	2	0.207	5.6	10	20	1	0.5	0.026	32.0
1	5	1	1	0.025	50.9	10	30	1	0.5	0.036	12.1
1	15	1	1	0.153	7.5	10	5	2	2	0.013	141.0
1	5	1	0.5	0.025	25.1	10	20	2	2	0.026	0.5
1	15	1	0.5	0.153	6.5	10	25	2	2	0.036	17.2
1	5	2	2	0.084	26.8	10	30	2	2	0.036	1.5
1	15	2	2	0.193	12.3	10	5	2	1	-0.001	133.8
1	20	2	2	0.250	6.7	10	20	2	1	0.012	25.9
1	5	2	1	0.030	29.0	10	25	2	1	0.022	16.6
1	15	2	1	0.139	17.9	10	30	2	1	0.022	3.0
1	20	2	1	0.196	1.1	10	5	2	0.5	-0.001	133.8
1	5	2	0.5	0.030	29.0	10	30	2	0.5	0.022	3.0
1	10	2	0.5	0.082	14.3	10	15	2	0.5	0.001	93.1
1	20	2	0.5	0.196	1.1	10	20	2	0.5	0.012	25.9
4	5	0.25	2	0.058	7.6	40	5	0.25	2	0.15	30.7
4	5	0.25	1	0.045	1.2	40	10	0.25	2	0.034	30.4
4	5	0.25	0.5	0.045	28.5	40	20	0.25	2	0.072	17.8
4	10	0.25	0.5	0.091	14.6	40	5	0.25	1	0.001	86.2
4	5	0.5	2	0.032	7.9	40	15	0.25	1	0.039	17.9
4	15	0.5	2	0.096	22.3	40	20	0.25	1	0.058	7.3
4	5	0.5	1	0.018	21.6	40	5	0.25	0.5	0.001	82.3
4	10	0.5	1	0.059	6.1	40	10	0.25	0.5	0.020	37.1
4	5	0.5	0.5	0.018	8.5	40	20	0.25	0.5	0.058	0.2
4	10	0.5	0.5	0.059	37.6	40	5	0.5	1	-0.014	502.7
4	15	0.5	0.5	0.082	8.7	40	15	0.5	1	0.013	1.5
4	5	1	2	0.022	8.6	40	20	0.5	1	0.027	31.9
4	15	1	2	0.065	4.0	40	25	0.5	1	0.040	3.2
4	20	1	2	0.077	9.8	40	5	0.5	0.5	-0.014	606.5

Table G. 1 Training data results of $u_{\max_s}$ for MARS and relative error contd.

Q	B (m)	B/H	K_0	$u_{\max_s_MARS}$ (m)	e (%)	Q	B (m)	B/H	K_0	$u_{\max_s_MARS}$ (m)	e (%)
4	25	1	2	0.089	14.7	40	15	0.5	0.5	0.013	23.4
4	5	1	1	0.008	45.2	40	25	0.5	0.5	0.040	68.3
4	15	1	1	0.051	10.3	40	30	0.5	0.5	0.040	16.7
4	20	1	1	0.063	2.0	40	5	0.5	2	0.000	94.9
4	25	1	1	0.076	11.8	40	10	0.5	2	0.014	18.2
4	5	1	0.5	0.008	6.8	40	20	0.5	2	0.041	41.6
4	15	1	0.5	0.051	42.1	40	30	0.5	2	0.054	4.3
4	20	1	0.5	0.063	23.3	40	5	1	2	0.015	348.8
4	25	1	0.5	0.076	8.9	40	10	1	2	0.018	151.7
4	5	2	2	0.028	41.1	40	20	1	2	0.024	67.1
4	15	2	2	0.051	4.8	40	30	1	2	0.026	28.0
4	20	2	2	0.063	3.4	40	5	1	1	0.001	38.9
4	25	2	2	0.075	4.2	40	15	1	1	0.007	4.7
4	5	2	1	0.014	15.2	40	20	1	1	0.010	8.1
4	15	2	1	0.037	9.8	40	25	1	1	0.012	8.1
4	20	2	1	0.049	17.7	40	5	2	2	0.021	837.9
4	30	2	1	0.061	9.0	40	15	2	2	0.007	9.6
4	5	2	0.5	0.014	15.2	40	20	2	2	0.010	3.8
4	15	2	0.5	0.037	9.8	40	30	2	2	0.012	13.4
4	20	2	0.5	0.049	17.7	40	5	2	1	0.007	435.9
4	25	2	0.5	0.061	15.4	40	20	2	1	-0.004	174.0
10	5	0.25	2	0.037	46.8	40	25	2	1	-0.002	121.5
10	15	0.25	2	0.091	27.3	40	30	2	1	-0.002	118.0
10	5	0.25	1	0.024	34.2	40	5	1	0.5	0.001	22.3
10	10	0.25	1	0.050	5.9	40	10	1	0.5	0.004	25.6
10	5	0.5	2	0.013	3.3	40	20	1	0.5	0.010	35.0
10	10	0.5	2	0.034	15.1	40	25	1	0.5	0.012	31.5
10	15	0.5	2	0.056	13.2	40	5	2	0.5	0.007	435.9
10	25	0.5	2	0.099	12.7	40	10	2	0.5	-0.010	454.6
10	5	0.25	0.5	0.024	68.2	40	15	2	0.5	-0.007	263.3
10	10	0.25	0.5	0.050	31.3	40	20	2	0.5	-0.004	174.0
10	5	0.5	1	-0.001	117.9	40	30	2	0.5	-0.002	118.0

Appendix H

Procedures for partitioning of BPNN weights for $u_{\max_s}$

The neural network for $u_{\max_s}$ prediction in this thesis works with four input neurons, eight hidden neurons and one output neuron with the connection weights as shown below:

Table H. 1 BPNN weights and outputs

Hidden neurons	Weights				Output
	Input 1	Input 2	Input 3	Input 4	
Hidden 1	-30.2265	-24.9068	-0.4375	-0.5227	-55.9338
Hidden 2	-114.3282	21.3612	-31.6100	0.2127	-112.222
Hidden 3	0.6585	-1.4923	5.0574	-0.1700	5.3273
Hidden 4	0.8313	-2.7127	3.2400	-0.5334	0.9202
Hidden 5	-50.6780	0.5412	0.3683	77.1775	-24.1557
Hidden 6	3.2117	-0.4915	0.0329	-0.2173	3.0177
Hidden 7	0.2417	1.7824	-0.0303	4.4797	-1.7061
Hidden 8	-37.2940	0.9901	-0.1553	0.0754	-38.7685

The computation process is as follows:

(1) For each hidden neuron i , multiply the absolute value of the hidden-output layer connection weight by the absolute value of the hidden-input layer connection weight. Do this for each input variable j . The following products P_{i-j} are obtained:

Table H. 2 P_{i-j} results

Hidden neurons	P_{i-j}			
	Input 1	Input 2	Input 3	Input 4
Hidden 1	1690.683	1393.132	24.471	29.237
Hidden 2	12830.139	2397.197	3547.337	23.870
Hidden 3	3.508	7.95	26.942	0.906
Hidden 4	0.765	2.496	2.981	0.491
Hidden 5	1224.163	13.073	8.897	1864.277
Hidden 6	9.692	1.483	0.099	0.656
Hidden 7	0.412	3.041	0.052	7.643
Hidden 8	1445.832	38.385	6.021	2.923

(2) For each hidden neuron, divide P_{i-j} by the sum for all the input variables to obtain Q_{i-j} . For example for hidden 1, $Q_{1-1} = \frac{P_{1-1}}{P_{1-1}+P_{1-2}+P_{1-3}+P_{1-4}} = 0.539$

(3) For each input neuron, sum the product S_j formed from the previous computation of Q_{i-j} . For example, $S_1 = Q_{1-1} + Q_{2-1} + Q_{3-1} + Q_{4-1} + Q_{5-1} + Q_{6-1} + Q_{7-1} + Q_{8-1}$.

Table H. 3 Q_{i-j} results

Hidden neurons	Q_{i-j}			
	Input 1	Input 2	Input 3	Input 4
Hidden 1	0.539	0.444	0.008	0.009
Hidden 2	0.683	0.128	0.189	0.001
Hidden 3	0.089	0.202	0.685	0.023
Hidden 4	0.114	0.371	0.443	0.073
Hidden 5	0.394	0.004	0.003	0.599
Hidden 6	0.812	0.124	0.008	0.055
Hidden 7	0.037	0.273	0.005	0.686
Hidden 8	0.968	0.026	0.004	0.002
Sum	$S_1=3.635$	$S_2=1.572$	$S_3=1.345$	$S_4=1.448$

(4) Divide S_j by $\max(S_j)$ for all the input variables. Expressed as a percentage, this gives the relative importance of all output weights attributable to the given input variable. For example, for the input neuron 2, the relative importance (%) is equal to $(S_2 \times 100) / \max(S_1, S_2, S_3, S_4) = 43.22$.

Table H.4 Relative importance results

Relative importance	Input 1	Input 2	Input 3	Input 4
(%)	100	43.22	36.99	39.84

Appendix I

Calculation of BPNN output $u_{\max_s}$

The transfer functions used for BP neural network output for $u_{\max_s}$ predictions are ‘*tansig*’ transfer function for hidden layer to output layer and ‘*purelin*’ transfer function for output layer to target. The calculation process of BPNN output for $u_{\max_s}$ is elaborated in detail as follows:

From the connection weights for a trained neuron network, it is possible to develop a mathematical equation relating the input parameters and the single output parameter Y using

$$[I-1] \ Y = f_{sig} \left\{ b_0 + \sum_{k=1}^h \left[w_k f_{sig} \left(b_{hk} + \sum_{i=1}^m w_{ik} X_i \right) \right] \right\}$$

in which b_0 is the bias at the output layer, ω_k is the weight connection between neuron k of the hidden layer and the single output neuron, b_{hk} is the bias at neuron k of the hidden layer ($k=1, h$), ω_{ik} is the weight connection between input variable i ($i=1, m$) and neuron k of the hidden layer, X_i is the input parameter i , and f_{sig} is the sigmoid (*tansig* & *purelin*) transfer function.

Using the connection weights of the trained neural network, the following steps can be followed to mathematically express the BPNN model:

Step1: Normalize the input values for Q , B , B/H , and K_0 linearly using

$$x_{\text{norm}} = 2(x_{\text{actual}} - x_{\text{min}}) / (x_{\text{max}} - x_{\text{min}}) - 1$$

Let the actual $Q = X_{1a}$ and the normalized $Q = X_1$

$$[I-2] \ X_1 = -1 + 2 * (X_{1a} - 0.4) / 39.6$$

Let the actual $B = X_{2a}$ and the normalized $B = X_2$

$$[I-3] \ X_2 = -1 + 2 * (X_{2a} - 5) / 25$$

Let the actual $B/H = X_{3a}$ and the normalized $B/H = X_3$

$$[I-4] \ X_3 = -1 + 2 * (X_{3a} - 0.25) / 1.75$$

Let the actual $K_0 = X_{4a}$ and the normalized $K_0 = X_4$

$$[I-5] \ X_4 = -1 + 2 * (X_{4a} - 0.5) / 1.5$$

Step2: Calculate the normalized value (Y_1) using the following expressions:

$$[I-6] \ A_1 = -55.9338 - 30.2265X_1 - 24.9068X_2 - 0.4375X_3 - 0.5227X_4$$

$$[I-7] \ A_2 = -112.222 - 114.3282X_1 + 21.3612X_2 - 31.6100X_3 + 0.2127X_4$$

$$[I-8] \ A_3 = 5.3273 + 0.6585X_1 - 1.4923X_2 + 5.0574X_3 - 0.1700X_4$$

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$$[I-9] A_4=0.9202+0.8313X_1-2.7127X_2+3.2400X_3-0.5334X_4$$

$$[I-10] A_5=-24.1557-50.6780X_1+0.5412X_2+0.3683X_3+77.1775X_4$$

$$[I-11] A_6=-3.0177+3.2117X_1-0.4915X_2+0.0329X_3-0.2173X_4$$

$$[I-12] A_7=-1.7061+0.2417X_1+1.7824X_2-0.0303X_3+4.4797X_4$$

$$[I-13] A_8=-38.7685-37.2940X_1+0.9901X_2-0.1553X_3+0.0754X_4$$

$$[I-14] B_1=-0.2528 \times \tanh(A_1)$$

$$[I-15] B_2= 0.1485 \times \tanh(A_2)$$

$$[I-16] B_3= -0.3552 \times \tanh(A_3)$$

$$[I-17] B_4= -0.0311 \times \tanh(A_4)$$

$$[I-18] B_5= -0.0913 \times \tanh(A_5)$$

$$[I-19] B_6= -0.2434 \times \tanh(A_6)$$

$$[I-20] B_7= -0.0965 \times \tanh(A_7)$$

$$[I-21] B_8= 44.9498 \times \tanh(A_8)$$

$$[I-22] C_1=44.4905+ B_1+ B_2+ B_3+B_4+B_5+B_6+B_7+B_8$$

$$[I-23] Y_1=C_1$$

Step3: De-normalize the output to obtain the maximum key point displacement

$$[I-24] u_{\max_s} (m) = 0.001+0.552 \times (Y_1+1)/2$$

Appendix J

Table J. 1 Training data results of ε_s for BPNN

Q	B (m)	B/H	K_0	ε_{s_FDM} (%)	ε_{s_BP} (%)	Q	B (m)	B/H	K_0	ε_{s_FDM} (%)	ε_{s_BP} (%)
0.4	5	0.5	2	4.970	4.961	10	15	0.5	1	0.236	0.365
0.4	5	0.5	0.5	2.840	2.868	10	20	0.5	1	0.227	0.344
0.4	5	1	2	4.590	4.501	10	25	0.5	1	0.339	0.309
0.4	10	1	2	6.810	6.835	10	5	0.5	0.5	0.103	0.112
0.4	5	1	1	2.973	3.234	10	10	0.5	0.5	0.200	0.212
0.4	10	1	0.5	4.975	4.645	10	15	0.5	0.5	0.184	0.229
0.4	5	2	2	3.737	3.553	10	25	0.5	0.5	0.254	0.181
0.4	10	2	2	5.490	5.434	10	5	1	2	0.231	0.226
0.4	10	2	1	3.810	4.151	10	20	1	2	0.502	0.401
0.4	5	2	0.5	2.253	2.209	10	25	1	2	0.282	0.357
0.4	10	2	0.5	3.810	3.672	10	30	1	2	0.301	0.311
1	5	0.25	2	2.800	2.790	10	5	1	1	0.110	0.108
1	5	0.25	0.5	1.610	1.600	10	10	1	1	0.167	0.223
1	10	0.5	2	1.935	2.027	10	15	1	1	0.219	0.245
1	10	0.5	1	1.515	1.475	10	20	1	1	0.260	0.226
1	5	0.5	0.5	0.758	0.717	10	30	1	1	0.180	0.182
1	10	0.5	0.5	1.255	1.167	10	5	1	0.5	0.088	0.104
1	5	1	2	1.207	1.278	10	15	1	0.5	0.160	0.216
1	15	1	2	2.295	2.269	10	20	1	0.5	0.189	0.199
1	5	1	1	0.701	0.709	10	30	1	0.5	0.137	0.144
1	15	1	1	1.700	1.650	10	5	2	2	0.187	0.210
1	5	1	0.5	0.555	0.562	10	20	2	2	0.426	0.377
1	15	1	0.5	1.435	1.443	10	25	2	2	0.242	0.335
1	5	2	2	0.984	1.082	10	30	2	2	0.267	0.293
1	15	2	2	1.900	1.934	10	5	2	1	0.088	0.126
1	20	2	2	2.330	2.392	10	20	2	1	0.206	0.209
1	5	2	1	0.587	0.618	10	25	2	1	0.121	0.179
1	15	2	1	1.284	1.391	10	30	2	1	0.138	0.149
1	20	2	1	1.720	1.756	10	5	2	0.5	0.088	0.119
1	5	2	0.5	0.587	0.501	10	30	2	0.5	0.170	0.202
1	10	2	0.5	1.019	0.909	10	15	2	0.5	0.206	0.184
1	20	2	0.5	1.720	1.540	10	20	2	0.5	0.138	0.134
4	5	0.25	2	1.260	1.254	40	5	0.25	2	0.191	0.167
4	5	0.25	1	0.702	0.755	40	10	0.25	2	0.256	0.167
4	5	0.25	0.5	0.535	0.538	40	20	0.25	2	0.256	0.166
4	10	0.25	0.5	0.552	0.511	40	5	0.25	1	0.100	0.166
4	5	0.5	2	0.666	0.586	40	15	0.25	1	0.141	0.166
4	15	0.5	2	0.851	0.908	40	20	0.25	1	0.194	0.166
4	5	0.5	1	0.339	0.373	40	5	0.25	0.5	0.072	0.064
4	10	0.5	1	0.556	0.621	40	10	0.25	0.5	0.114	0.104
4	5	0.5	0.5	0.252	0.209	40	20	0.25	0.5	0.169	0.165
4	10	0.5	0.5	0.412	0.441	40	5	0.5	1	0.054	0.166
4	15	0.5	0.5	0.401	0.503	40	15	0.5	1	0.110	0.166
4	5	1	2	0.477	0.423	40	20	0.5	1	0.099	0.166
4	15	1	2	0.908	0.823	40	25	0.5	1	0.227	0.165
4	20	1	2	1.017	0.789	40	5	0.5	0.5	0.038	0.062

Table J. 1 Training data results of ε_s for BPNN contd.

Q	B (m)	B/H	K_0	ε_{s_FDM} (%)	ε_{s_BP} (%)	Q	B (m)	B/H	K_0	ε_{s_FDM} (%)	ε_{s_BP} (%)
4	25	1	2	0.566	0.689	40	15	0.5	0.5	0.083	0.062
4	5	1	1	0.238	0.180	40	25	0.5	0.5	0.101	0.061
4	15	1	1	0.541	0.531	40	30	0.5	0.5	0.095	0.061
4	20	1	1	0.652	0.503	40	5	0.5	2	0.133	0.167
4	25	1	1	0.397	0.430	40	10	0.5	2	0.253	0.167
4	5	1	0.5	0.178	0.144	40	20	0.5	2	0.157	0.166
4	15	1	0.5	0.397	0.458	40	30	0.5	2	0.215	0.165
4	20	1	0.5	0.494	0.432	40	5	1	2	0.098	0.167
4	25	1	0.5	0.310	0.367	40	10	1	2	0.192	0.166
4	5	2	2	0.377	0.403	40	20	1	2	0.223	0.166
4	15	2	2	0.732	0.771	40	30	1	2	0.142	0.165
4	20	2	2	0.830	0.728	40	5	1	1	0.046	0.062
4	25	2	2	0.474	0.631	40	15	1	1	0.085	0.062
4	5	2	1	0.187	0.203	40	20	1	1	0.101	0.062
4	15	2	1	0.417	0.426	40	25	1	1	0.062	0.061
4	20	2	1	0.503	0.458	40	5	2	2	0.079	0.064
4	30	2	1	0.359	0.310	40	15	2	2	0.165	0.161
4	5	2	0.5	0.187	0.171	40	20	2	2	0.195	0.166
4	15	2	0.5	0.417	0.426	40	30	2	2	0.128	0.165
4	20	2	0.5	0.503	0.394	40	5	2	1	0.039	0.062
4	25	2	0.5	0.295	0.333	40	20	2	1	0.086	0.061
10	5	0.25	2	0.581	0.537	40	25	2	1	0.052	0.061
10	15	0.25	2	0.409	0.432	40	30	2	1	0.059	0.061
10	5	0.25	1	0.295	0.341	40	5	1	0.5	0.040	0.062
10	10	0.25	1	0.358	0.332	40	10	1	0.5	0.051	0.062
10	5	0.5	2	0.319	0.247	40	20	1	0.5	0.077	0.061
10	10	0.5	2	0.495	0.413	40	25	1	0.5	0.050	0.061
10	15	0.5	2	0.390	0.449	40	5	2	0.5	0.039	0.062
10	25	0.5	2	0.570	0.373	40	10	2	0.5	0.057	0.062
10	5	0.25	0.5	0.221	0.234	40	15	2	0.5	0.072	0.062
10	10	0.25	0.5	0.271	0.259	40	20	2	0.5	0.086	0.061
10	5	0.5	1	0.139	0.223	40	30	2	0.5	0.059	0.061

Appendix K

Table K. 1 Training data results of ε_s for MARS and relative error

Q	B (m)	B/H	K_0	ε_{s_MARS} (m)	e (%)	Q	B (m)	B/H	K_0	ε_{s_MARS} (m)	e (%)
0.4	5	0.5	2	4.912	1.2	10	15	0.5	1	0.218	7.4
0.4	5	0.5	0.5	3.014	6.1	10	20	0.5	1	0.241	6.3
0.4	5	1	2	4.717	2.8	10	25	0.5	1	0.241	28.7
0.4	10	1	2	6.605	3.0	10	5	0.5	0.5	0.171	66.2
0.4	5	1	1	3.024	1.7	10	10	0.5	0.5	0.195	2.7
0.4	10	1	0.5	4.707	1.8	10	15	0.5	0.5	0.218	18.9
0.4	5	2	2	3.738	0.0	10	25	0.5	0.5	0.241	4.8
0.4	10	2	2	5.626	2.5	10	5	1	2	0.344	49.3
0.4	10	2	1	3.932	3.2	10	20	1	2	0.414	17.5
0.4	5	2	0.5	1.958	13.1	10	25	1	2	0.414	47.0
0.4	10	2	0.5	3.846	0.9	10	30	1	2	0.414	37.7
1	5	0.25	2	2.651	5.3	10	5	1	1	0.125	14.0
1	5	0.25	0.5	1.789	11.2	10	10	1	1	0.149	11.0
1	10	0.5	2	1.932	0.2	10	15	1	1	0.172	21.5
1	10	0.5	1	1.393	8.1	10	20	1	1	0.195	24.9
1	5	0.5	0.5	0.754	0.5	10	30	1	1	0.195	8.8
1	10	0.5	0.5	1.201	4.3	10	5	1	0.5	0.125	42.3
1	5	1	2	1.316	9.1	10	15	1	0.5	0.172	7.7
1	15	1	2	2.208	3.8	10	20	1	0.5	0.195	3.6
1	5	1	1	0.777	10.8	10	30	1	0.5	0.195	43.1
1	15	1	1	1.669	1.8	10	5	2	2	0.252	34.5
1	5	1	0.5	0.585	5.3	10	20	2	2	0.322	24.3
1	15	1	0.5	1.477	2.9	10	25	2	2	0.322	33.4
1	5	2	2	1.072	8.9	10	30	2	2	0.322	20.9
1	15	2	2	1.965	3.4	10	5	2	1	0.033	62.5
1	20	2	2	2.307	1.0	10	20	2	1	0.103	50.0
1	5	2	1	0.533	9.2	10	25	2	1	0.103	14.8
1	15	2	1	1.426	11.1	10	30	2	1	0.103	25.3
1	20	2	1	1.768	2.8	10	5	2	0.5	0.152	71.7
1	5	2	0.5	0.460	21.7	10	30	2	0.5	0.198	16.6
1	10	2	0.5	0.906	11.1	10	15	2	0.5	0.222	7.5
1	20	2	0.5	1.695	1.5	10	20	2	0.5	0.222	60.6
4	5	0.25	2	1.050	16.7	40	5	0.25	2	0.290	52.1
4	5	0.25	1	0.700	0.3	40	10	0.25	2	0.314	22.4
4	5	0.25	0.5	0.572	7.1	40	20	0.25	2	0.360	41.1
4	10	0.25	0.5	0.682	23.7	40	5	0.25	1	0.075	24.8
4	5	0.5	2	0.586	12.1	40	15	0.25	1	0.122	13.4
4	15	0.5	2	0.805	5.3	40	20	0.25	1	0.145	25.1
4	5	0.5	1	0.367	8.1	40	5	0.25	0.5	0.075	4.6
4	10	0.5	1	0.476	14.3	40	10	0.25	0.5	0.098	13.6
4	5	0.5	0.5	0.239	5.5	40	20	0.25	0.5	0.145	13.9
4	10	0.5	0.5	0.348	15.4	40	5	0.5	1	0.075	38.3
4	15	0.5	0.5	0.458	14.4	40	15	0.5	1	0.122	10.6
4	5	1	2	0.540	13.1	40	20	0.5	1	0.145	47.2
4	15	1	2	0.759	16.3	40	25	0.5	1	0.145	36.0
4	20	1	2	0.869	14.5	40	5	0.5	0.5	0.075	98.0

Table K. 1 Training data results of ε_s for MARS and relative error contd.

Q	B (m)	B/H	K_0	ε_{s_MARS} (m)	e (%)	Q	B (m)	B/H	K_0	ε_{s_MARS} (m)	e (%)
4	25	1	2	0.678	19.9	40	15	0.5	0.5	0.122	46.3
4	5	1	1	0.320	34.9	40	25	0.5	0.5	0.145	43.6
4	15	1	1	0.540	0.1	40	30	0.5	0.5	0.145	52.6
4	20	1	1	0.650	0.2	40	5	0.5	2	0.160	20.4
4	25	1	1	0.459	15.5	40	10	0.5	2	0.183	27.5
4	5	1	0.5	0.192	7.8	40	20	0.5	2	0.230	46.6
4	15	1	0.5	0.412	4.0	40	30	0.5	2	0.230	7.2
4	20	1	0.5	0.522	5.7	40	5	1	2	0.114	15.7
4	25	1	0.5	0.331	6.7	40	10	1	2	0.161	16.3
4	5	2	2	0.447	18.6	40	20	1	2	0.184	17.4
4	15	2	2	0.667	8.8	40	30	1	2	0.184	29.1
4	20	2	2	0.777	6.4	40	5	1	1	0.029	37.6
4	25	2	2	0.586	23.7	40	15	1	1	0.076	10.7
4	5	2	1	0.228	22.3	40	20	1	1	0.099	1.7
4	15	2	1	0.448	7.4	40	25	1	1	0.099	59.4
4	20	2	1	0.558	10.9	40	5	2	2	0.022	72.6
4	30	2	1	0.175	51.2	40	15	2	2	0.068	58.5
4	5	2	0.5	0.219	17.2	40	20	2	2	0.092	53.0
4	15	2	0.5	0.439	5.2	40	30	2	2	0.092	28.2
4	20	2	0.5	0.548	9.0	40	5	2	1	-0.063	262.5
4	25	2	0.5	0.357	21.2	40	20	2	1	0.007	92.1
10	5	0.25	2	0.521	10.3	40	25	2	1	0.007	86.8
10	15	0.25	2	0.568	38.8	40	30	2	1	0.007	88.5
10	5	0.25	1	0.171	41.9	40	5	1	0.5	0.029	28.0
10	10	0.25	1	0.195	45.5	40	10	1	0.5	0.052	3.1
10	5	0.5	2	0.390	22.4	40	20	1	0.5	0.099	28.3
10	10	0.5	2	0.414	16.4	40	25	1	0.5	0.099	100.0
10	15	0.5	2	0.437	12.1	40	5	2	0.5	0.055	41.7
10	25	0.5	2	0.461	19.2	40	10	2	0.5	0.079	38.3
10	5	0.25	0.5	0.171	22.5	40	15	2	0.5	0.102	42.5
10	10	0.25	0.5	0.218	7.4	40	20	2	0.5	0.125	46.3
10	5	0.5	1	0.241	6.3	40	30	2	0.5	0.125	111.5

Appendix L

Procedures for partitioning of BPNN weights for ε_s

The neural network for ε_s prediction in this thesis works with four input neurons, six hidden neurons and one output neuron with the connection weights as shown below:

Table L. 1 BPNN weights and outputs

Hidden neurons	Weights				Output
	Input 1	Input 2	Input 3	Input 4	
Hidden 1	-1.8446	-0.6466	-0.0709	0.1480	-3.0155
Hidden 2	-1.8298	-0.6128	-0.0692	0.1470	-3.0152
Hidden 3	-1.5557	4.4566	-52.2698	52.1996	3.9737
Hidden 4	-52.1758	0.0277	-85.9624	-0.0020	-113.186
Hidden 5	2.6121	0.6480	5.2623	-0.1785	8.1279
Hidden 6	-35.9391	0.6161	-0.1193	0.0866	-37.5631

The computation process is as follows:

(1) For each hidden neuron i , multiply the absolute value of the hidden-output layer connection weight by the absolute value of the hidden-input layer connection weight. Do this for each input variable j . The following products P_{i-j} are obtained:

Table L. 2 P_{i-j} results

Hidden neurons	P_{i-j}			
	Input 1	Input 2	Input 3	Input 4
Hidden 1	5.5624	1.9498	0.2138	0.4463
Hidden 2	5.5172	1.8477	0.2087	0.4432
Hidden 3	6.1819	17.7092	207.7045	207.4256
Hidden 4	5905.5701	3.1353	9729.7402	0.2264
Hidden 5	21.2309	5.2669	42.7714	1.4508
Hidden 6	1349.9840	23.1426	4.4813	3.2530

(2) For each hidden neuron, divide P_{i-j} by the sum for all the input variables to obtain Q_{i-j} . For example for hidden 1, $Q_{1-1} = \frac{P_{1-1}}{P_{1-1}+P_{1-2}+P_{1-3}+P_{1-4}} = 0.6806$

(3) For each input neuron, sum the product S_j formed from the previous computation of Q_{i-j} . For example, $S_1 = Q_{1-1} + Q_{2-1} + Q_{3-1} + Q_{4-1} + Q_{5-1} + Q_{6-1}$.

Table L. 3 Q_{i-j} results

Hidden neurons	Q_{i-j}			
	Input 1	Input 2	Input 3	Input 4
Hidden 1	0.6806	0.2386	0.0262	0.0546
Hidden 2	0.6882	0.2305	0.0260	0.0553
Hidden 3	0.0141	0.0403	0.4731	0.4725
Hidden 4	0.3776	0.0002	0.6222	0.0000
Hidden 5	0.3002	0.0745	0.6048	0.0205
Hidden 6	0.9776	0.0168	0.0032	0.0024
Sum	$S_1=3.0384$	$S_2=0.6008$	$S_3=1.7555$	$S_4=0.6053$

(4) Divide S_j by $\max(S_j)$ for all the input variables. Expressed as a percentage, this gives the relative importance of all output weights attributable to the given input variable. For example, for the input neuron 2, the relative importance (%) is equal to $(S_2 \times 100) / \max(S_1, S_2, S_3, S_4) = 19.77$.

Table L. 4 Relative importance results

Relative importance	Input 1	Input 2	Input 3	Input 4
(%)	100	19.77	57.78	19.92

Appendix M

Calculation of BPNN output ε_s

The transfer functions used for BP neural network output for ε_s predictions are ‘*tansig*’ transfer function for hidden layer to output layer and ‘*purelin*’ transfer function for output layer to target. The calculation process of BPNN output for ε is elaborated in detail as follows:

From the connection weights for a trained neuron network, it is possible to develop a mathematical equation relating the input parameters and the single output parameter Y using

$$[M-1] \quad Y = f_{sig} \left\{ b_0 + \sum_{k=1}^h \left[w_k f_{sig} \left(b_{hk} + \sum_{i=1}^m w_{ik} X_i \right) \right] \right\}$$

in which b_0 is the bias at the output layer, ω_k is the weight connection between neuron k of the hidden layer and the single output neuron, b_{hk} is the bias at neuron k of the hidden layer ($k=1, h$), ω_{ik} is the weight connection between input variable i ($i=1, m$) and neuron k of the hidden layer, X_i is the input parameter i , and f_{sig} is the sigmoid (tansig & purelin) transfer function.

Using the connection weights of the trained neural network, the following steps can be followed to mathematically express the BPNN model:

Step1: Normalize the input values for Q , B , Sc/H , and K_0 linearly using

$$X_{norm} = 2(x_{actual} - x_{min}) / (x_{max} - x_{min}) - 1$$

Let the actual $Q = X_{1a}$ and the normalized $Q = X_1$

$$[M-2] \quad X_1 = -1 + 2 * (X_{1a} - 0.4) / 39.6$$

Let the actual $B = X_{2a}$ and the normalized $B = X_2$

$$[M-3] \quad X_2 = -1 + 2 * (X_{2a} - 5) / 25$$

Let the actual $B/H = X_{3a}$ and the normalized $B/H = X_3$

$$[M-4] \quad X_3 = -1 + 2 * (X_{3a} - 0.25) / 1.75$$

Let the actual $K_0 = X_{4a}$ and the normalized $K_0 = X_4$

$$[M-5] \quad X_4 = -1 + 2 * (X_{4a} - 0.5) / 1.5$$

Step2: Calculate the normalized value (Y_l) using the following expressions:

$$[M-6] \quad A_1 = -3.0155 - 1.8446X_1 - 0.6466X_2 - 0.0709X_3 + 0.1480X_4$$

$$[M-7] \quad A_2 = -3.0152 - 1.8298X_1 - 0.6128X_2 - 0.0692X_3 + 0.1470X_4$$

$$[M-8] \quad A_3 = 3.9737 - 1.5557X_1 + 4.4566X_2 - 52.2698X_3 + 52.1996X_4$$

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$$[M-9] A_4 = -113.186 - 52.1758X_1 + 0.0277X_2 - 85.9624X_3 - 0.0020X_4$$

$$[M-10] A_5 = 8.1279 + 2.6121X_1 + 0.6480X_2 + 5.2623X_3 - 0.1785X_4$$

$$[M-11] A_6 = -37.5631 - 35.9391X_1 + 0.6161X_2 - 0.1193X_3 + 0.0866X_4$$

$$[M-12] B_1 = -25.1282 \times \tanh(A_1)$$

$$[M-13] B_2 = 27.3132 \times \tanh(A_2)$$

$$[M-14] B_3 = 0.0153 \times \tanh(A_3)$$

$$[M-15] B_4 = -0.1076 \times \tanh(A_4)$$

$$[M-16] B_5 = -0.4182 \times \tanh(A_5)$$

$$[M-17] B_6 = 37.2583 \times \tanh(A_6)$$

$$[M-18] C_1 = 38.776 + B_1 + B_2 + B_3 + B_4 + B_5 + B_6$$

$$[M-19] Y_1 = C_1$$

Step3: De-normalize the output to obtain the percent strain

$$[M-20] \varepsilon_s (\%) = 0.038 + 6.772 \times (Y_1 + 1) / 2$$

Appendix N

Table N.1 Training data results of FS_{g_t} for MARS and relative error

Q	B (m)	S_c/B	FS_{g_t} _FDM	FS_{g_t} _MARS	e (%)	Q	B (m)	S_c/B	FS_{g_t} _FDM	FS_{g_t} _MARS	e (%)
1	30	1	0.72	0.663	7.9	1	20	2	0.9	0.915	1.7
10	30	1	1.04	1.059	1.9	10	20	2	1.29	1.311	1.7
40	30	1	1.24	1.245	0.4	40	20	2	1.53	1.497	2.2
100	30	1	1.38	1.445	4.7	100	20	2	1.72	1.697	1.3
1	30	1.5	0.77	0.748	2.8	1	20	2.5	0.94	0.969	3.1
4	30	1.5	0.96	0.973	1.3	4	20	2.5	1.17	1.194	2.0
40	30	1.5	1.31	1.330	1.5	10	20	2.5	1.35	1.365	1.1
100	30	1.5	1.47	1.530	4.1	100	20	2.5	1.79	1.751	2.2
1	30	2	0.83	0.802	3.3	1	10	1	1.11	1.123	1.2
4	30	2	1.03	1.027	0.3	10	10	1	1.61	1.571	2.4
10	30	2	1.19	1.198	0.7	40	10	1	1.9	1.928	1.5
100	30	2	1.59	1.584	0.4	100	10	1	2.13	2.129	0.1
1	30	2.5	0.87	0.856	1.6	1	10	1.5	1.12	1.121	0.1
10	30	2.5	1.25	1.252	0.2	4	10	1.5	1.4	1.363	2.6
40	30	2.5	1.48	1.438	2.9	40	10	1.5	1.92	1.926	0.3
100	30	2.5	1.66	1.639	1.3	100	10	1.5	2.15	2.127	1.1
1	20	1	0.81	0.776	4.2	1	10	2	1.15	1.175	2.2
4	20	1	1.01	1.001	0.9	4	10	2	1.43	1.417	0.9
10	20	1	1.17	1.172	0.2	10	10	2	1.65	1.623	1.6
100	20	1	1.55	1.558	0.5	100	10	2	2.2	2.181	0.9
1	20	1.5	0.85	0.861	1.3	1	10	2.5	1.17	1.229	5.1
4	20	1.5	1.06	1.086	2.4	10	10	2.5	1.68	1.677	0.2
40	20	1.5	1.46	1.443	1.2	40	10	2.5	2	2.034	1.7
100	20	1.5	1.64	1.643	0.2	100	10	2.5	2.24	2.235	0.2

Appendix O

Table O. 1 Training data results of $H(X)$ for BPNN

B (m)	Q	S_c/B	K_0	Target $H(X)$	$H(X)_{BPNN}$	B (m)	Q	S_c/B	K_0	Target $H(X)$	$H(X)_{BPNN}$
30	100	1	0.5	0	0	20	4	1	2	1	1
30	100	2	0.5	0	0	20	4	1.5	2	1	1
30	100	2.5	0.5	0	0	20	4	2.5	2	0	0
30	40	1	0.5	0	0	10	100	1	2	0	0
30	40	1.5	0.5	0	0	10	100	2	2	0	0
30	40	2.5	0.5	0	0	10	100	2.5	2	0	0
20	100	1	0.5	0	0	10	40	1	2	0	0
20	100	2	0.5	0	0	10	40	1.5	2	0	0
20	100	2.5	0.5	0	0	10	40	2.5	2	0	0
20	4	1	0.5	1	1	10	10	1	2	1	1
20	4	1.5	0.5	1	1	10	10	2	2	0	0
20	4	2.5	0.5	0	0	10	10	2.5	2	0	0
10	100	1	0.5	0	0	10	1	1	2	1	1
10	100	2	0.5	0	0	10	1	1.5	2	1	1
10	100	2.5	0.5	0	0	10	1	2.5	2	1	1
10	40	1	0.5	0	0	30	10	1	0.5	1	1
10	40	1.5	0.5	0	0	30	10	1.5	0.5	1	1
10	40	2.5	0.5	0	0	30	10	2.5	0.5	0	0
10	10	1	0.5	1	1	30	10	1	1	1	1
10	10	2	0.5	0	0	30	10	2	1	0	0
10	10	2.5	0.5	0	0	30	10	2.5	1	0	0
10	1	1	0.5	1	1	30	10	1	2	1	1
10	1	1.5	0.5	1	1	30	10	2	2	0	0
10	1	2.5	0.5	1	1	30	10	2.5	2	0	0
30	100	1	1	0	0	30	4	2	0.5	1	1
30	100	2	1	0	0	30	4	2.5	0.5	0	0
30	100	2.5	1	0	0	30	4	2	1	0	0
30	40	1	1	0	0	30	4	2	2	1	1
30	40	1.5	1	0	0	30	4	2.5	2	0	0
30	40	2.5	1	0	0	20	40	1	0.5	1	1
20	100	1	1	0	0	20	40	2	0.5	0	0
20	100	2	1	0	0	20	40	2.5	0.5	0	0
20	100	2.5	1	0	0	20	40	1	1	0	0
20	4	1	1	1	1	20	40	1.5	1	0	0
20	4	1.5	1	1	1	20	40	2.5	1	0	0
20	4	2.5	1	0	0	20	40	1	2	0	0
10	100	1	1	0	0	20	40	2	2	0	0
10	100	2	1	0	0	20	40	2.5	2	0	0
10	100	2.5	1	0	0	20	10	1	0.5	1	1
10	40	1	1	0	0	20	10	1.5	0.5	1	1
10	40	1.5	1	0	0	20	10	2.5	0.5	0	0
10	40	2.5	1	0	0	20	10	1	1	1	1
10	10	1	1	1	1	20	10	2	1	0	0
10	10	2	1	0	0	20	10	2.5	1	0	0
10	10	2.5	1	0	0	20	10	1	2	1	1
10	1	1	1	1	1	20	10	1.5	2	0	0

Table O. 1 Training data results of $H(X)$ for BPNN contd.

B (m)	Q	S_c/B	K_0	Target $H(X)$	$H(X)_{\text{BPNN}}$	B (m)	Q	S_c/B	K_0	Target $H(X)$	$H(X)_{\text{BPNN}}$
10	1	1.5	1	1	1	20	10	2.5	2	0	0
10	1	2.5	1	1	1	10	4	1	0.5	1	1
30	100	1	2	0	0	10	4	2	0.5	0	0
30	100	2	2	0	0	10	4	2.5	0.5	0	0
30	100	2.5	2	0	0	10	4	1	1	1	1
30	40	1	2	0	0	10	4	1.5	1	1	1
30	40	1.5	2	0	0	10	4	2.5	1	0	0
30	40	2.5	2	0	0	10	4	1	2	1	1
20	100	1	2	0	0	10	4	2	2	0	0
20	100	2	2	0	0	10	4	2.5	2	0	0
20	100	2.5	2	0	0						

Appendix P

Procedures for partitioning of BPNN weights for $H(X)$

The neural network for $H(X)$ prediction in this thesis works with four input neurons, ten hidden neurons and one output neuron with the connection weights as shown below:

Table P. 1 BPNN weights and outputs

Hidden neurons	Weights				Output
	Input 1	Input 2	Input 3	Input 4	
Hidden 1	9.4161	12.2090	-6.6212	1.9692	-8.8381
Hidden 2	5.3803	-8.4018	1.3520	-8.4882	5.0491
Hidden 3	-4.5059	0.1408	4.0456	8.8500	-0.0305
Hidden 4	0.3604	-3.9866	1.2679	1.5378	8.8865
Hidden 5	-9.9011	-2.7981	3.7899	-8.3560	1.1954
Hidden 6	-7.0706	1.4196	9.3286	-8.1114	10.1642
Hidden 7	-0.3430	32.4180	0.3074	-0.4481	30.7663
Hidden 8	-2.4766	-4.1348	-2.1375	-8.5788	-7.2552
Hidden 9	2.8257	-10.1523	-8.5838	-1.8165	-10.5584
Hidden 10	0.5276	-6.5357	0.1045	-12.8356	-0.6866

The computation process is as follows:

(1) For each hidden neuron i , multiply the absolute value of the hidden-output layer connection weight by the absolute value of the hidden-input layer connection weight. Do this for each input variable j . The following products P_{i-j} are obtained:

Table P. 2 P_{i-j} results

Hidden neurons	P_{i-j}			
	Input 1	Input 2	Input 3	Input 4
Hidden 1	83.2204	107.9044	58.5188	17.4040
Hidden 2	27.1657	42.4215	6.8264	42.8578
Hidden 3	0.1374	0.0043	0.1234	0.2699
Hidden 4	3.2027	35.4269	11.2672	16.6657
Hidden 5	11.8358	3.3448	4.5304	9.9888
Hidden 6	71.8670	14.4291	94.8178	82.4459
Hidden 7	10.5528	997.3819	9.4576	13.7864
Hidden 8	17.9682	29.9988	15.5080	62.2409
Hidden 9	29.8349	107.1920	90.6312	19.1793
Hidden 10	0.3623	4.4874	0.0717	8.8129

(2) For each hidden neuron, divide P_{i-j} by the sum for all the input variables to obtain Q_{i-j} . For example for hidden 1, $Q_{1-1} = \frac{P_{1-1}}{P_{1-1}+P_{1-2}+P_{1-3}+P_{1-4}} = 0.3116$

(3) For each input neuron, sum the product S_j formed from the previous computation of Q_{i-j} . For example, $S_1 = Q_{1-1} + Q_{2-1} + \dots + Q_{10-1}$.

Table P. 3 Q_{i-j} results

Hidden neurons	Q_{i-j}			
	Input 1	Input 2	Input 3	Input 4
Hidden 1	0.3116	0.4041	0.2191	0.0652
Hidden 2	0.2278	0.3557	0.0572	0.3593
Hidden 3	0.2569	0.0080	0.2306	0.5045
Hidden 4	0.0504	0.5574	0.1773	0.2150
Hidden 5	0.3985	0.1126	0.1525	0.3363
Hidden 6	0.2727	0.0547	0.3598	0.3128
Hidden 7	0.0102	0.9672	0.0092	0.0134
Hidden 8	0.1429	0.2386	0.1234	0.4951
Hidden 9	0.1209	0.4343	0.3672	0.0777
Hidden 10	0.0264	0.3267	0.0052	0.6417
Sum	$S_1=1.8182$	$S_2=3.4593$	$S_3=1.7015$	$S_4=3.0210$

(4) Divide S_j by $\max(S_j)$ for all the input variables. Expressed as a percentage, this gives the relative importance of all output weights attributable to the given input variable. For example, for the input neuron 1, the relative importance (%) is equal to $(S_1 \times 100) / \max(S_1, S_2, S_3, S_4) = 52.56$.

Table P. 4 Relative importance results

Relative importance	Input 1	Input 2	Input 3	Input 4
(%)	52.56	100	49.19	87.33

Appendix Q

Calculation of BPNN output $H(X)$

The transfer functions used for BP neural network output for $H(X)$ predictions are ‘*logsig*’ transfer function for hidden layer to output layer and ‘*tansig*’ transfer function for output layer to target. The calculation process of BPNN output for $H(X)$ is elaborated in detail as follows:

From the connection weights for a trained neuron network, it is possible to develop a mathematical equation relating the input parameters and the single output parameter Y using

$$[Q-1] \ Y = f_{sig} \left\{ b_0 + \sum_{k=1}^h \left[w_k f_{sig} \left(b_{hk} + \sum_{i=1}^m w_{ik} X_i \right) \right] \right\}$$

in which b_0 is the bias at the output layer, ω_k is the weight connection between neuron k of the hidden layer and the single output neuron, b_{hk} is the bias at neuron k of the hidden layer ($k=1, h$), ω_{ik} is the weight connection between input variable i ($i=1, m$) and neuron k of the hidden layer, X_i is the input parameter i , and f_{sig} is the sigmoid (logsig & tansig) transfer function.

Using the connection weights of the trained neural network, the following steps can be followed to mathematically express the BPNN model:

Step1: Normalize the input values for B , Q , S_c/B , and K_0 linearly using

$$x_{norm} = 2(x_{actual} - x_{min}) / (x_{max} - x_{min}) - 1$$

Let the actual $B = X_{1a}$ and the normalized $B = X_1$

$$[Q-2] \ X_1 = -1 + 2 * (X_{1a} - 10) / 20$$

Let the actual $Q = X_{2a}$ and the normalized $Q = X_2$

$$[Q-3] \ X_2 = -1 + 2 * (X_{2a} - 1) / 99$$

Let the actual $S_c/B = X_{3a}$ and the normalized $S_c/B = X_3$

$$[Q-4] \ X_3 = -1 + 2 * (X_{3a} - 1) / 1.5$$

Let the actual $K_0 = X_{4a}$ and the normalized $K_0 = X_4$

$$[Q-5] \ X_4 = -1 + 2 * (X_{4a} - 0.5) / 1.5$$

Step2: Calculate the normalized value (Y_l) using the following expressions:

$$[Q-6] \ A_1 = -8.8381 + 9.4161X_1 + 12.2090X_2 - 6.6212X_3 + 1.9692X_4$$

$$[Q-7] \ A_2 = 5.0491 + 5.3803X_1 - 8.4018X_2 + 1.3520X_3 - 8.4882X_4$$

$$[Q-8] \ A_3 = -0.0305 - 4.5059X_1 + 0.1408X_2 + 4.0456X_3 + 8.8500X_4$$

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$$[Q-9] A_4 = 8.8865 + 0.3604X_1 - 3.9866X_2 + 1.2679X_3 + 1.5378X_4$$

$$[Q-10] A_5 = 1.1954 - 9.9011X_1 - 2.7981X_2 + 3.7899X_3 - 8.3560X_4$$

$$[Q-11] A_6 = 10.1642 - 7.0706X_1 + 1.4196X_2 + 9.3286X_3 - 8.1114X_4$$

$$[Q-12] A_7 = 30.7663 - 0.3430X_1 + 32.4180X_2 + 0.3074X_3 - 0.4481X_4$$

$$[Q-13] A_8 = -7.2552 - 2.4766X_1 - 4.1348X_2 - 2.1375X_3 - 8.5788X_4$$

$$[Q-14] A_9 = -10.5584 + 2.8257X_1 - 10.1523X_2 - 8.5838X_3 - 1.8165X_4$$

$$[Q-15] A_{10} = -0.6866 + 0.5276X_1 - 6.5357X_2 + 0.1045X_3 - 12.8356X_4$$

$$[Q-16] B_1 = -15.8245 \times \text{logsig}(A_1)$$

$$[Q-17] B_2 = 7.6055 \times \text{logsig}(A_2)$$

$$[Q-18] B_3 = 6.3377 \times \text{logsig}(A_3)$$

$$[Q-19] B_4 = 4.8325 \times \text{logsig}(A_4)$$

$$[Q-20] B_5 = 10.6261 \times \text{logsig}(A_5)$$

$$[Q-21] B_6 = -16.5208 \times \text{logsig}(A_6)$$

$$[Q-22] B_7 = -42.2178 \times \text{logsig}(A_7)$$

$$[Q-23] B_8 = 12.6148 \times \text{logsig}(A_8)$$

$$[Q-24] B_9 = 33.5369 \times \text{logsig}(A_9)$$

$$[Q-25] B_{10} = -3.5850 \times \text{logsig}(A_{10})$$

$$[Q-26] C_1 = 6.7054 + B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + B_7 + B_8 + B_9 + B_{10}$$

$$[Q-27] Y_1 = \tanh(C_1)$$

Step3: De-normalize the output to obtain $H(X)$

$$[Q-28] H(X) = (Y_1 + 1)/2$$

Appendix R

Table R. 1 Training data results of $u_{\max_t}$ for MARS

B (m)	Q	K_0	S_c	$u_{\max_t}$ _FDM (mm)	$u_{\max_t}$ _MARS (mm)	B (m)	Q	K_0	S_c	$u_{\max_t}$ _FDM (mm)	$u_{\max_t}$ _MARS (mm)
10	1	0.5	1	30.4	34.7	20	10	1	2	5.9	4.1
10	1	0.5	2	25.8	22.0	20	10	1	2.5	5.5	2.9
10	1	0.5	2.5	19.6	16.8	20	10	2	1	17.4	16.6
10	1	1	1	29.6	34.7	20	10	2	1.5	15.9	10.6
10	1	1	1.5	28.2	27.2	20	10	2	2.5	14.1	8.3
10	1	1	2.5	18.6	16.8	20	40	0.5	1	3.2	4.5
10	1	2	1	92.8	84.0	20	40	0.5	2	2.5	3.3
10	1	2	2	67.2	71.2	20	40	0.5	2.5	2.4	2.2
10	1	2	2.5	61.2	66.0	20	40	1	1	2.9	4.5
10	4	0.5	1	9.3	6.9	20	40	1	1.5	2.6	4.5
10	4	0.5	1.5	6.2	4.1	20	40	1	2.5	2.4	2.2
10	4	0.5	2.5	4.1	1.8	20	40	2	1	6.6	4.5
10	4	1	1	9.5	6.9	20	40	2	2	5.7	3.3
10	4	1	2	5.2	3.0	20	40	2	2.5	5.3	2.2
10	4	1	2.5	5.0	1.8	20	100	0.5	1	1.8	3.0
10	4	2	1	19.0	25.2	20	100	0.5	1.5	1.6	3.0
10	4	2	1.5	17.8	22.4	20	100	0.5	2.5	1.6	0.7
10	4	2	2.5	16.2	20.1	20	100	1	1	1.7	3.0
10	10	0.5	1	3.4	6.8	20	100	1	2	1.5	1.8
10	10	0.5	2	2.3	4.1	20	100	1	2.5	1.5	0.7
10	10	0.5	2.5	2.3	2.9	20	100	2	1	3.4	3.0
10	10	1	1	3.2	6.8	20	100	2	1.5	3.1	3.0
10	10	1	1.5	2.8	5.2	20	100	2	2.5	2.8	0.7
10	10	1	2.5	2.5	2.9	30	4	0.5	2.5	19.2	24.5
10	10	2	1	8.5	12.2	30	4	1	2	24.6	25.7
10	10	2	2	7.4	9.5	30	4	1	2.5	21.0	24.5
10	10	2	2.5	7.0	8.3	30	4	2	2	60.9	48.3
10	40	0.5	1	1.3	0.1	30	10	0.5	1	33.2	19.1
10	40	0.5	1.5	1.2	4.5	30	10	0.5	2	9.0	12.0
10	40	0.5	2.5	1.2	2.2	30	10	0.5	2.5	8.3	10.8
10	40	1	1	1.3	0.1	30	10	1	1	16.9	19.1
10	40	1	2	1.2	3.3	30	10	1	1.5	10.7	13.1
10	40	1	2.5	1.1	2.2	30	10	1	2.5	8.5	10.8
10	40	2	1	3.0	0.1	30	10	2	1	27.0	28.8
10	40	2	1.5	2.7	4.5	30	10	2	2	20.3	21.6
10	40	2	2.5	2.3	2.2	30	10	2	2.5	18.8	20.5
10	100	0.5	1	0.9	-1.4	30	40	0.5	1	4.7	4.5
10	100	0.5	2	0.8	1.8	30	40	0.5	1.5	4.2	4.5
10	100	0.5	2.5	0.8	0.7	30	40	0.5	2.5	3.6	2.2
10	100	1	1	0.8	-1.4	30	40	1	1	4.1	4.5
10	100	1	1.5	0.8	3.0	30	40	1	2	3.4	3.3
10	100	1	2.5	0.7	0.7	30	40	1	2.5	3.4	2.2
10	100	2	1	1.5	-1.4	30	40	2	1	7.8	8.8
10	100	2	2	1.3	1.8	30	40	2	1.5	8.1	8.8
10	100	2	2.5	1.2	0.7	30	40	2	2.5	7.1	6.5

Table R. 1 Training data results of $u_{\max_t}$ for MARS contd.

B (m)	Q	K_0	S_c	$u_{\max_t}$ _FDM (mm)	$u_{\max_t}$ _MARS (mm)	B (m)	Q	K_0	S_c	$u_{\max_t}$ FDM (mm)	$u_{\max_t}$ _MARS (mm)
20	4	0.5	2	13.0	16.2	30	100	0.5	1	2.7	3.0
20	4	0.5	2.5	10.8	15.1	30	100	0.5	2	2.3	1.8
20	4	1	1.5	22.9	17.4	30	100	0.5	2.5	2.3	0.7
20	4	1	2.5	12.2	15.1	30	100	1	1	2.3	3.0
20	4	2	2	35.9	34.5	30	100	1	1.5	2.2	3.0
20	4	2	2.5	34.3	33.4	30	100	1	2.5	2.1	0.7
20	10	0.5	1	12.4	11.2	30	100	2	1	4.1	7.3
20	10	0.5	1.5	6.6	5.2	30	100	2	2	3.9	6.1
20	10	0.5	2.5	5.4	2.9	30	100	2	2.5	3.7	5.0
20	10	1	1	12.0	11.2						

Appendix S

Table S. 1 Training data results of ε_t for MARS

B (m)	Q	K_0	S_c	ε_{t_FDM} (%)	ε_{t_MARS} (%)	B (m)	Q	K_0	S_c	ε_{t_FDM} (%)	ε_{t_MARS} (%)
10	1	0.5	1	0.407	0.401	20	10	1	2	0.047	0.044
10	1	0.5	2	0.286	0.274	20	10	1	2.5	0.046	0.044
10	1	0.5	2.5	0.234	0.236	20	10	2	1	0.187	0.210
10	1	1	1	0.366	0.401	20	10	2	1.5	0.186	0.190
10	1	1	1.5	0.322	0.338	20	10	2	2.5	0.180	0.170
10	1	1	2.5	0.271	0.236	20	40	0.5	1	0.021	0.018
10	1	2	1	1.171	1.130	20	40	0.5	2	0.013	0.018
10	1	2	2	0.989	1.003	20	40	0.5	2.5	0.013	0.018
10	1	2	2.5	0.938	0.965	20	40	1	1	0.023	0.018
10	4	0.5	1	0.164	0.147	20	40	1	1.5	0.021	0.018
10	4	0.5	1.5	0.099	0.123	20	40	1	2.5	0.021	0.018
10	4	0.5	2.5	0.080	0.099	20	40	2	1	0.080	0.078
10	4	1	1	0.160	0.147	20	40	2	2	0.074	0.078
10	4	1	2	0.095	0.099	20	40	2	2.5	0.075	0.078
10	4	1	2.5	0.094	0.099	20	100	0.5	1	0.008	0.009
10	4	2	1	0.380	0.395	20	100	0.5	1.5	0.008	0.009
10	4	2	1.5	0.373	0.371	20	100	0.5	2.5	0.008	0.009
10	4	2	2.5	0.358	0.347	20	100	1	1	0.013	0.009
10	10	0.5	1	0.078	0.083	20	100	1	2	0.012	0.009
10	10	0.5	2	0.035	0.044	20	100	1	2.5	0.012	0.009
10	10	0.5	2.5	0.041	0.044	20	100	2	1	0.043	0.043
10	10	1	1	0.066	0.083	20	100	2	1.5	0.042	0.043
10	10	1	1.5	0.049	0.063	20	100	2	2.5	0.044	0.043
10	10	1	2.5	0.048	0.044	30	4	0.5	2.5	0.119	0.099
10	10	2	1	0.190	0.210	30	4	1	2	0.106	0.099
10	10	2	2	0.184	0.170	30	4	1	2.5	0.123	0.099
10	10	2	2.5	0.183	0.170	30	4	2	2	0.314	0.347
10	40	0.5	1	0.021	0.018	30	10	0.5	1	0.160	0.105
10	40	0.5	1.5	0.016	0.018	30	10	0.5	2	0.039	0.044
10	40	0.5	2.5	0.015	0.018	30	10	0.5	2.5	0.049	0.044
10	40	1	1	0.024	0.018	30	10	1	1	0.084	0.105
10	40	1	2	0.019	0.018	30	10	1	1.5	0.047	0.063
10	40	1	2.5	0.019	0.018	30	10	1	2.5	0.061	0.044
10	40	2	1	0.081	0.078	30	10	2	1	0.248	0.232
10	40	2	1.5	0.080	0.078	30	10	2	2	0.170	0.170
10	40	2	2.5	0.076	0.078	30	10	2	2.5	0.166	0.170
10	100	0.5	1	0.010	0.009	30	40	0.5	1	0.022	0.039
10	100	0.5	2	0.008	0.009	30	40	0.5	1.5	0.015	0.018
10	100	0.5	2.5	0.008	0.009	30	40	0.5	2.5	0.019	0.018
10	100	1	1	0.012	0.009	30	40	1	1	0.031	0.039
10	100	1	1.5	0.010	0.009	30	40	1	2	0.022	0.018
10	100	1	2.5	0.010	0.009	30	40	1	2.5	0.028	0.018
10	100	2	1	0.045	0.043	30	40	2	1	0.106	0.100
10	100	2	2	0.041	0.043	30	40	2	1.5	0.076	0.078
10	100	2	2.5	0.040	0.043	30	40	2	2.5	0.077	0.078
20	4	0.5	2	0.091	0.099	30	100	0.5	1	0.014	0.030

Table S. 1 Training data results of ε_i for MARS contd.

B (m)	Q	K_0	S_c	ε_{L_FDM} (%)	ε_{L_MARS} (%)	B (m)	Q	K_0	S_c	ε_{L_FDM} (%)	ε_{L_MARS} (%)
20	4	0.5	2.5	0.080	0.099	30	100	0.5	2	0.009	0.009
20	4	1	1.5	0.133	0.123	30	100	0.5	2.5	0.010	0.009
20	4	1	2.5	0.087	0.099	30	100	1	1	0.017	0.030
20	4	2	2	0.355	0.347	30	100	1	1.5	0.012	0.009
20	4	2	2.5	0.374	0.347	30	100	1	2.5	0.017	0.009
20	10	0.5	1	0.098	0.083	30	100	2	1	0.062	0.064
20	10	0.5	1.5	0.047	0.063	30	100	2	2	0.044	0.043
20	10	0.5	2.5	0.035	0.044	30	100	2	2.5	0.045	0.043
20	10	1	1	0.094	0.083						