Spectral effects of phosphor-based white light-emitting-diode (LED) in coherence scanning interferometry

Chong, Wee Keat

2014

Chong, W. K. (2014). Spectral effects of phosphor-based white light-emitting-diode (LED) in coherence scanning interferometry. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/55377>

<https://doi.org/10.32657/10356/55377>

*SPECTRAL EFFECTS OF PHOSPHOR-
BASED WHITE LIGHT-EMITTING-
DIODE (LED) IN COHERENCE
SCANNING INTERFEROMETRY*

CHONG WEE KEAT

School of Electrical & Electronics Engineering

A thesis submitted to Nanyang Technological University

in partial fulfillment of the requirement for the degree of

Doctor of Philosophy

2014

Acknowledgements

I would like to thank my supervisors, Prof Soh Yeng Chai (NTU), Dr Li Xiang Leon (SIMTech) and the late Dr Sardha WIJESOMA (NTU), whose help, stimulating suggestions and encouragements have helped me in my research.

I would also like to thank my colleagues from the Singapore Institute of Manufacturing Technology (SIMTech), especially Dr Zhang Ying and Dr Xu Jian for their support, guidance and valuable advice.

Table of Content

Acknowledgements	i
Table of Content	ii
Summary.....	vii
List of Figures.....	ix
List of Tables	xxi
List of Acronyms	xxii
Author's Publications.....	xxiii
1. Introduction.....	1
1.1. Motivation	2
1.2. Objectives	2
1.3. Contributions	3
1.4. Organization of the report.....	4
2. Literature Review	7
2.1. Introduction.....	7
2.1.1. Phase shifting interferometry.....	10
2.1.2. Coherence scanning interferometry.....	13
2.2. Literature Review	15
2.2.1. Reconstruction algorithm for coherence scanning interferometry	16

2.2.2.	Hardware related research	27
2.3.	Conclusion.....	40
3.	Signal Modeling for Coherence Scanning Interferometry	42
3.1.	Motivation	42
3.2.	Signal Modeling for coherence scanning interferometry in elementary form.....	43
3.2.1.	Physical model and related work.....	43
3.2.2.	Proposed model and approach.....	50
3.2.3.	Modeling Gaussian as sum of piecewise cosine functions	51
3.2.4.	Derivation to elementary form	57
3.2.5.	Computational efficiency	58
3.2.6.	Verification.....	59
3.3.	Discussion	64
3.3.1.	Effects of spectral sensitivity of the imaging sensor.....	64
3.3.2.	Effects of numerical aperture of objective lens.....	67
3.3.3.	Plausible cause for the disagreement at high frequency	73
3.4.	Conclusion.....	76
4.	Phosphor-based White LED in Coherence Scanning Interferometry.....	77
4.1.	Motivation	77
4.2.	Effects of phosphor-based white LED in coherence scanning interferometry	78

4.2.1.	Spectral property of phosphor-based white LED	78
4.2.2.	Spectral factor in coherence scanning interferometry	80
4.2.3.	Effects on phosphor-based white LED on correlogram	82
4.2.4.	Effects of phosphor-based white LED on reconstructed height.....	83
4.2.5.	Modification for phosphor-based white LED	86
4.2.6.	Summary.....	90
4.3.	Effects of spectral variation of phosphor-based white LED in coherence scanning interferometry.....	90
4.3.1.	Motivation	90
4.3.2.	Causes of spectral variation of white LED	91
4.3.3.	Modeling spectral variation of white LED	94
4.3.4.	Effects on correlogram	96
4.3.5.	Effects on reconstructed height	101
4.3.6.	Experimental verification	108
4.4.	Spectral effects of dual wavelength low coherence light source.....	110
4.4.1.	Introduction.....	110
4.4.2.	The proposed theory	113
4.4.3.	Fringe contrast function manipulation via spectral shaping	117
4.5.	Other types of white LED.....	120

4.5.1.	Multi-color white LED	121
4.5.2.	Phosphor-free white LED	123
4.6.	Conclusion.....	124
5.	Harnessing the Spectral Property of Phosphor-based White LED in Coherence Scanning Interferometry	126
5.1.	Identifying a suitable height reconstruction algorithm	126
5.2.	Simulation verification	131
5.3.	Experimental verification.....	134
5.4.	Conclusion.....	135
6.	Video-based Interferogram Analysis of Vibration	136
6.1.	Introduction	136
6.2.	Vibration in coherence scanning interferometry	137
6.2.1.	Cause of vibration	137
6.2.2.	How vibration affects coherence scanning interferometry.....	138
6.3.	Video-based interferogram analysis to quantify vibration.....	142
6.3.1.	Experimental verification.....	147
6.4.	Harnessing the spectral effect	148
6.5.	Summary.....	153
7.	Conclusions and Future Works	154

7.1. Conclusions.....	154
7.2. Future works.....	156
Reference.....	158

Summary

A surface profilometer measures the surface topography of an object, and it is an enabling technology in many research areas such as material science and surface finishing. In MEMS applications where the dynamic characterization becomes increasingly important, high precision surface profilometer is required to be operated in stroboscopic mode in which the light source has to be modulated at high speed and there are vibration being induced by the sample.

Among all the surface profilometers, the coherence scanning interferometric profilometer has the advantages of being 1) scalable in accuracy and measurement range, 2) fast measurement as it is a whole field type of measurement, 3) compact in size, and 4) affordable cost.

Unlike phase shifting interferometry, which uses coherent light source (such as laser), coherence scanning interferometry uses low coherence broadband light source and does not have the ambiguity issue in the phase unwrapping process. So it has no problem when measuring rough surfaces.

With recent advancement in lighting technology, high power phosphor-based LED is fast replacing conventional low coherence light source (such as tungsten lamp) in coherence scanning interferometry. Phosphor-based white LED is made out of blue LED and yellow phosphor, so there are two peaks in its spectrum. Compared to prior works that considered the effective spectrum of white light source as a single Gaussian function, the spectral effect (due to the use of unconventional light source) has been either neglected or suppressed. Hence this research investigates the spectral effects of phosphor-based white light emitting diode (LED) in coherence scanning interferometry.

Instead of suppressing or neglecting the spectral effect of phosphor-based white LED, this research work investigates the spectral effects of phosphor-based white LED with the aim to improve the usability and performance of coherence scanning interferometry: First, a signal modeling for coherence scanning interferometry which expresses the signal in elementary form (an elementary form/function is a mathematically defined term, the details of which will be discussed in Chapter 3) is proposed. It is then followed by applying the proposed signal model to investigate and give a theoretical explanation of the spectral effects of phosphor-based white LED in coherence scanning interferometry. With the proposed theoretical explanation, the spectral effects are harnessed to improve the usability and performance of coherence scanning interferometry.

Each of the above mentioned contributions is supported by simulations and experimental results. We believe that results reported in this thesis, particularly the results on the spectral effects of phosphor-based white LED in coherence scanning interferometry, will draw further research in the spectral effects in coherence scanning interferometry and the associated spectral manipulations.

List of Figures

Figure 2-1: Schematic diagram of interferometric profilometer	8
Figure 2-2: Example of interferogram of coherence scanning interferometry: Fringe on a tilted flat surface captured using a CCD camera	9
Figure 2-3: Example of correlogram and corresponding fringe contrast function in coherence scanning interferometry	9
Figure 2-4: Graphical illustration of phase ambiguity in phase shifting interferometry (PSI): (a) original phase profile of a spherical surface (b) recovered phase profile of the spherical surface.....	12
Figure 2-5: Correlogram of coherence scanning interferometry, the maximum of it corresponds to height.....	14
Figure 2-6: Flow chart of Ai and Novak's centroid approach [42] in estimating the peak location	17
Figure 2-7: The centroid approach and prior art for handling multi-peak coherence peak function in multi-layer sample (image is extracted from Ai and Novak [42]).....	19
Figure 2-8: Fringe contrast function, $A(k)$, estimated using Kalman Filter: The frame number of the x-axis corresponds to the change in optical path difference and the y-axis records the interference signal (image is extracted from Gurov [43])	21
Figure 2-9: Illustration of Li et al.'s work [40] of applying filtering through frequency domain followed by Gaussian fitting for estimating peak	22
Figure 2-10: Graphical illustration of Groot's frequency domain analysis.	22

Figure 2-11: Three color fringe signals detected at an arbitrarily selected point on the object; (a), (b), and (c) correspond to the signals from red, green and blue channels, respectively. The wrapped phase distribution obtained for the red and green channels are presented in (d) for the region of interest shown magnified in (a) and (b); the location of phase crossing is indicated by an arrow (image is extracted from Pawlowski [47])	24
Figure 2-12: Graphical illustration of pre-processor, main processor and post-processor (image is extracted from Kim et al. [50])	25
Figure 2-13: Due to imperfections in scanning mechanism, correlograms captured at different scanning ranges are different (image is extracted from Kim et al [51])	26
Figure 2-14: Comparison of measurement result on a standard height before and after compensating the scanning error (image is extracted from Kim et al. [51])	27
Figure 2-15: With additional reference surface ("Tilted Mirror" in (a)) in sample arm, the actual phase difference between interferograms can be identified (image is extracted from Lai and Yatagai [71]).	30
Figure 2-16: Illustration of Zhao and Burge's idea of vibration-compensation (image is extracted from Zhao and Burge [72])	31
Figure 2-17: Graphical illustration on the use of electro-optic modulator (EOM) as phase shifter (image is extracted from Zhao and Burge [72])	32
Figure 2-18: Schematic diagram of scanning interferometer with reference signal: S is the light source; L is the lens; B is the beam splitter; M is the mirror and D is the reference signal detector (image is extracted from Olszak and Schmit [73])	33

Figure 2-19: Schematic setup of Kassamakov et al's super continuum interferometer (image is extracted from Kassamakov et al. [80])	34
Figure 2-20: Comparing effective spectrum of Kassamakov et al's supercontinuum source with some common light sources (image is extracted from Kassamakov et al. [80])	35
Figure 2-21: Schematic view of Heikkinen et al.'s coherence scanning interferometry with multi LED light sources (image is extracted from Heikkinen et al. [61]).....	36
Figure 2-22: Spectrum at 4 different pulsing frequencies. The yellow peak of the non-phosphor LED is lower compared to blue peak at higher frequencies (image is extracted from Heikkinen et al. [61])	36
Figure 2-23: Hanhijärvi et al. found that the spectrum of LED is a function of duty cycle (image is extracted from Hanhijärvi et al. [60])	37
Figure 2-24: Schematic diagram of Molnar and Tutsch's vertical scanning interferometry with a mixed coherence light source (image is extracted from Molnar and Tutsch [62]).....	38
Figure 2-25: Correlograms generated by Molnar and Tutsch's mixed-coherence light source which is a combination of a broadband light source and a laser (image is extracted from Molnar and Tutsch [62])	39
Figure 2-26: Schematic diagram of Heikkinen et al.'s scanning white light interferometry with hybrid light source (image is extracted from Heikkinen et al. [63])	39
Figure 2-27: Schematic diagram of Yang et al.'s interferometric configuration which use a pair of harmonically related light sources for long optical distance with sub-nanometer precision (image is extracted from Yang et al. [64])	40
Figure 3-1: Physical model of Mirau-based interferometry	44

Figure 3-2: Illustration of numerical aperture (NA) of objective lens	46
Figure 3-3: Comparing spectral energy from some common source of white light: tungsten lamp, mercury vapor lamp, noon sunlight and white light emitting diode (LED) [95].....	47
Figure 3-4: Graphical Illustration of a single piecewise cosine function	52
Figure 3-5: A single Gaussian with zero mean and sigma of 0.6 is fitted to a sum of one, two, three or four piecewise cosine functions	53
Figure 3-6: Number of piecewise cosine functions used to represent a single Gaussian function versus the goodness of fit, R^2	55
Figure 3-7: Graphical illustration of transform pairs listed in Table 3-1.	56
Figure 3-8: Comparison of computational time required for simulating a single value of correlogram	58
Figure 3-9: Intensity spectrum of phosphor-based LED, LXHL-LW6C by Lumileds	59
Figure 3-10: Comparison of the goodness of fit for the spectrum of phosphor-based LED represented by Gaussian functions and the proposed model.	60
Figure 3-11: Graphical comparison of the correlogram simulated by direct numerical evaluation on the generalized model and the proposed model.	61
Figure 3-12: Comparison of the correlograms by experiment, the proposed model and existing computationally intensive model.....	62
Figure 3-13: Comparison of the correlograms by experiment data and the proposed model in frequency domain. The magnitudes of the two correlograms are normalized for comparison purpose.....	63

Figure 3-14: Graphical comparison of spectral responses of some scientific CCD sensors: SITe ST001 back illuminated, Sony ICX 285, Kodak 1401e and Sony ICX 061 [103].	65
Figure 3-15: Spectral response of IDS uEYE UI2220M CCD camera [104].	65
Figure 3-16: Comparison of the nominal spectrum and the effective spectrum of the phosphor-based LED with IDS uEYE UI2220M CCD camera.	66
Figure 3-17: Comparison of the correlograms simulated based on the effective (IDS uEYE UI2220M CCD camera) and nominal spectra of the phosphor-based LED.	67
Figure 3-18: Graphical illustration of (a) Michelson interferometric objective and (b) Mirau interferometric objective.	69
Figure 3-19: Picture of Nikon interferometry objective: (a) 2.5x objective in Michelson configuration (b) 20x objective in Mirau configuration.	69
Figure 3-20: Simulated correlograms based on numerical apertures of 0.13, 0.3 and 0.4.	71
Figure 3-21: Simulated fringe contrast functions (envelope of correlogram) based on 5x, 10x and 20x objectives.	71
Figure 3-22: Simulated correlograms based on numerical apertures of 0.13, 0.3 and 0.4.	72
Figure 3-23: Simulated fringe contrast functions (envelope of correlogram) based on 5x, 10x and 20x objectives.	72
Figure 3-24: (a) Fringe spacing in normal incidence illumination interference microscope (Fizeau) = $\lambda/2$ (b) Fringe spacing in oblique incidence illumination interference microscope (Mirau) = $\lambda/2 \times \text{obliquity_factor}$	74
Figure 3-25: Graphical illustration of the geometric effects due to the object tilt and the terms in the revised generalized model.	74

Figure 4-1: Additive color mixing showing combinations to generate white light.	78
Figure 4-2: Intensity spectra of conventional light source and phosphor-based white LED.....	79
Figure 4-3: Intensity spectra of commercially available phosphor-based white LED with different correlated color temperature (CCT): (a) warm white (b) nature white (c) cool white.....	80
Figure 4-4: (a) Effective spectrum of a conventional light source (b) Correlogram based on spectrum of Figure 4-4 (a)	82
Figure 4-5: (a) Spectrum of a phosphor-based LED, LXHL-LW6C by LumiLEDs; (b) Simulated correlogram based on spectrum of (a).....	82
Figure 4-6: Experiment data: Intensity response based on light source of phosphor-based white LED (LXHL-LW6C by LumiLEDs).....	83
Figure 4-7: Reconstructed $1\mu\text{m}$ step height using (a) Gaussian Fitting by Li et al. [40] (b) Centroid approach by Ai and Novak [42] (c) Frequency domain analysis by Groot and Deck [45] for phosphor-based white LED.....	85
Figure 4-8: Reconstructed $1\mu\text{m}$ step height using (a) Gaussian Fitting by Li et al. [40] (b) Centroid approach by Ai and Novak [42] (c) Frequency domain analysis by Groot and Deck [45] for a light source of Gaussian spectrum.....	85
Figure 4-9: Performance comparison of different algorithms with different light sources: (a) standard deviation of a perfectly flat surface (ideal value is zero) (b) height measurement (ideal value is $1\mu\text{m}$).	85
Figure 4-10: Fitting the fringe contrast function with a single Gaussian by selecting a subset of data (such as $-0.5 \leq \text{defocus position} \leq 0.5$).	87

Figure 4-11: Effects of data selection in spatial frequency domain (a) 80 data (b) 20 data from the correlogram.	88
Figure 4-12: Simulation verification: Comparing standard deviation of measuring perfectly flat surface reproduced by proposed modification and original algorithm.....	88
Figure 4-13: Experimental verification of the proposed modification for phosphor-based LED: Comparison of the standard deviation in measuring an optically flat surface reconstructed with and without proposed modification.	89
Figure 4-14: The correlated color temperature of some commonly used light sources.....	91
Figure 4-15: Example of spectral variation: Intensity spectra of three commercially available white LEDs with different correlated color temperature (CCT).....	92
Figure 4-16: Example of spectral variation of white LED: the intensity spectrum of non-phosphor white LED varies depending on the pulsing frequencies (image is extracted from Heikkinen et al. [61]).....	93
Figure 4-17: Example of spectral variation of white LED: the intensity spectrum varies depending on the duty cycle of pulsing (image is extracted from Hanhijarvi et al. [60])	93
Figure 4-18: Comparison of the intensity spectrum of simulated and commercially available phosphor-based white LED with different correlated color temperature of (a) warm white, BY ratio = 0.59 (b) daylight white, BY ratio = 1.14 (c) cool white, BY ratio = 1.6...	96
Figure 4-19: Effects of changing the blue to yellow ratio (BY ratio) on correlogram (Numerical Aperture of objective is assumed to be 0.4).....	97
Figure 4-20: Illustration of the feature extraction process which transforms the distinctive features due to phosphor-based white LED into two features (highlighted in red).	98

Figure 4-21: The effects of changing the BY ratio on the correlogram: (a) the peak (b) the valley as indicated in Figure 4-20 (Numerical aperture of objective is assumed to be 0.4)	99
Figure 4-22: Graphical representation of the extended simulation result showing the relationship of the amplitude of the peak with different numerical aperture setting against the BY ratio.....	99
Figure 4-23: Transform pairs listed in Table 4-1.....	100
Figure 4-24: Scatter plot showing the measurement repeatability of Ai and Novak's centroid approach and the BY ratio.....	103
Figure 4-25: Scatter plots showing the relationship between the BY ratio and the measurement repeatability of coherence scanning interferometric reconstruction algorithms optimized for (a) phosphor-based warm white LED (BY ratio < 1.0) (b) phosphor-based daylight and cool white LED (BY ratio ≥ 1.0)	104
Figure 4-26: Comparison of the value of Pearson's r of the BY ratio and the measurement repeatability (in terms of standard deviation of measurement) with different reconstruction algorithms	106
Figure 4-27: Experimental verification on the measurement repeatability of Ai and Novak's centroid approach with phosphor-based white LED with different BY ratio	109
Figure 4-28: Experimental verification on the measurement repeatability of coherence scanning interferometric reconstruction algorithms optimized for (a) phosphor-based warm white LED (BY ratio < 1.0), and (b) phosphor-based daylight and cool white LED (BY ratio ≥ 1.0)	110
Figure 4-29: Spectra of some modern light sources used in the prior works [61], [63], [109]	111

Figure 4-30: Graphical illustration of the distinctive feature: (a) spectrum of phosphor-based white LED, LXHL-LW6C (b) corresponding interference signals (by simulation and experiment) with the distinctive features highlighted in yellow box.....	112
Figure 4-31: Illustration that the distinctive feature is the result of destructive interference between two colors (which are blue and yellow lights for phosphor-based white LED)	117
Figure 4-32: Effective spectrum of the hybrid light source designed to shift the location of the distinctive feature away from the zero OPD location	119
Figure 4-33: Evaluating the accuracy of the proposed explanation in controlling/predicting the location of the distinctive feature: By simulation and experimental verification, the location of the feature is around $0.8\mu\text{m}$ (with respect to the zero OPD location) while the location estimated by the proposed explanation is $0.88\mu\text{m}$	120
Figure 4-34: Picture of RGB white LED, LATB T66B by OSRAM.....	121
Figure 4-35: Intensity Spectrum of each LED in RGB white LED (LATB T66B by OSRAM).....	122
Figure 4-36: The relative luminous intensity of blue, amber and green colored LED chip under different forward current.	123
Figure 4-37: Example of intensity spectra of phosphor free white LED (a) monolithic white LED (type-1) by Yamada et al. [127], (b) L915NPWC by American Opto Plus LED	124
Figure 5-1: Illustration of the phase crossing algorithm by Pawlowski et al. [128] with the conventional white light source: (a) a region of the correlogram is selected for further processing, (b) the selected region is Fourier transformed and two filter windows are applied to extract the interference signals contributed by two narrow band signals, and (c) the phase information of the extracted interference signals are recovered and the phase crossing point is identified	129

Figure 5-2: Illustration of the phase crossing algorithm by Pawlowski et al. [128] with the phosphor-based white LED: (a) a region of the correlogram is selected for further processing, (b) the selected region is Fourier transformed and two filter windows are applied to extract the interference signals contributed by two narrow band signals, and (c) the phase information of the extracted interference signals is recovered and the phase crossing point is identified131

Figure 5-3: Comparison of correlograms using the conventional white light source and the phosphor-based white LED (LXHL-LW6C by LumiLEDs).....132

Figure 5-4: 1 μ m step height reconstructed by the phase crossing algorithm using the conventional white light source132

Figure 5-5: 1 μ m step height reconstructed by the phase crossing algorithm using the phosphor-based white LED (LXHL-LW6C by LumiLEDs).....133

Figure 5-6: Comparing the measurement repeatability (in terms of the standard deviation) of the phase crossing algorithm using different light sources, the standard deviation of a perfectly flat surface is zero133

Figure 5-7: Comparing the measurement accuracy of the phase crossing algorithm using different light sources, the ideal value is 1 μ m134

Figure 6-1: Schematic diagram showing the measurement principle of interferometry: A computer captures and analyzes interference pattern with known optical path difference (between D_{BS-ref} and $D_{BS-sample}$).140

Figure 6-2: Illustration of how vibration affects interferometer. Vibration transmitted through vibration isolation system introduces relative motion, a change in $D_{BS-sample}$ and/or D_{BS-ref} . Vibration isolation system consists of mechanical structure of interferometer and optical

table, these components act like a filter reducing the effect of vibration to interferometer 144

Figure 6-3: Measurement Principle: Assuming D_{BS-ref} (in Figure 6-2) is constant, a small change in $\Delta D_{BS-sample}$ (in Figure 6-2) will cause a corresponding shift in interference pattern in interferogram, $\Delta D_{BS-sample}$ is proportional to S_{pixel} . If the sample surface is flat and tilted at an angle, $\Delta D_{BS-sample}$ is linearly proportional to S_{pixel} , $\Delta D_{BS-sample} = A \times S_{pixel}$ where A is the gradient of the slope, in the unit of height per pixel..... 144

Figure 6-4: Graphical illustration of the aperture problem: (a) Observing a moving fringe though finite field of view poses ambiguity in direction, V_1 and V_2 are possible. (b) As the block is moving in the direction V_1 in constant speed, tracked motion without considering aperture would be non-linear. 146

Figure 6-5: Measuring the undesired optical path difference change due to the vibration simulated by high precision piezoelectric stage: The presented method measure a vibration with a square pattern (amplitude of 200nm and period of 1000millisecond) generated by high precision piezoelectric stage. 147

Figure 6-6: Measurement by the presented method: the undesired OPD change induced by the cooling fan in our coherence scanning interferometer..... 147

Figure 6-7: Measurement by the presented method: the undesired OPD change induced by a gentle tap on the optical table (where our coherence scanning interferometer is mounted) 148

Figure 6-8: Interferogram of coherence scanning interferometry using the conventional white light source with a tilted flat sample surface..... 150

Figure 6-9: Interferogram of coherence scanning interferometry using phosphor-based white LED with a tilted flat sample surface	150
Figure 6-10: Highlighting the distinctive feature in the interferogram of coherence scanning interferometry using phosphor-based white LED with a tilt flat sample surface	152

List of Tables

Table 2-1: List of phase measurement algorithms used in phase shifting interferometry	12
Table 3-1: Selected transform pairs from a single Gaussian to a sum of two cosine terms.....	55
Table 3-2 : Summary of interferometric objective (by Nikon) used for evaluation	68
Table 4-1: Extended simulation in which the amplitude of the peak is represented in power function.....	100
Table 4-2: Guideline for the interpretation of the Pearson's r coefficient.....	106
Table 4-3: Specification of individual LED in RGB LED (LATB T66B by OSRAM).....	122

List of Acronyms

CSI	Coherence scanning interferometry
VSI	Vertical scanning interferometry
PSI	Phase Shifting Interferometry
WLI	White Light Interferometry
OPD	Optical Path Difference
NA	Numerical Aperture
CCD	Charged-couple Device
LED	Light emitting diode
RGB	Red-Green-Blue

Author's Publications

1. W. K. Chong, X. Li, and S. Wijesoma, "Computationally efficient signal modeling for vertical scanning interferometry," *Appl. Opt.*, vol. 49, no. 26, pp. 4990–4994, 2010.
2. W. K. Chong, X. Li, and S. Wijesoma, "Effects of phosphor-based LEDs on vertical scanning interferometry," *Opt. Lett.*, vol. 35, no. 17, pp. 2946–2948, 2010.
3. W. K. Chong, X. Li, and S. Yeng Chai, "Harnessing Spectral Property of Dual Wavelength White LED to improve Vertical Scanning Interferometry," *Appl. Opt.*, vol. 52, no. 19, pp. 4652–4662, Jul. 2013.
4. W. K. Chong, X. Li, and S. Yeng Chai, "Spectral effects of dual wavelength low coherence light source in white light interferometry," *Opt Laser Eng.*, vol. 51, no. 6, pp. 651–655, Jun. 2013.
5. W. K. Chong, X. Li, and Y. C. Soh, "Phosphor-Based White Light Emitting Diode (LED) for Vertical Scanning Interferometry (VSI)," in *Interferometry - Research and Applications in Science and Technology*, InTech, 2012.
6. W. K. Chong, X. Li, and S. Wijesoma, "Video-Based Interferogram Analysis for Measuring Influence of Vibration to White Light Interferometry," *KEM*, vol. 447–448, pp. 599–603, Sep. 2010.
7. W. K. Chong, X. Li, and S. Yeng Chai, "Effects of the spectral variation of phosphor-based white LED on vertical scanning interferometry," (in preparation)

1. Introduction

A surface profilometer is an equipment that measures the surface topography. It is an important enabling and supporting technology in the fields of surface finishing, machining and material sciences.

Surface profilometers can be classified into two primary categories: (1) Contact profilometer, and (2) Optical profilometer (non-contact profilometer).

A contact profilometer uses a probe to scan along the surface of an object. The advantage of contact profilometer is its excellent lateral resolution beyond optical resolution of optical microscopes [1]. The atomic force microscope is an example of contact profilometer.

An optical profilometer has the advantage of non-contact measurement at high accuracy and dynamic range in the axial axis, but a lower lateral resolution due to optical diffraction. Optical profilometer can be further categorized into: (1) area-based measurement (also known as whole field measurement) which normally has greater speed, and (2) point-based measurement which requires scanning mechanism in lateral direction and normally has scalable lateral range. The coherence scanning interferometer (CSI) is an example of area-based, non contact profilometer.

Coherence scanning interferometry (CSI) is also known as white-light interferometry (WLI) and vertical scanning interferometry (VSI). It is a non-contact three-dimensional surface measurement technique that provides an accuracy of up to nanometer level and a measurement range of up to a few hundred micrometers.

The principle of coherence scanning interferometry can be briefly summarized as follows: First, the light from a single source is split and channeled into the two arms of interferometer, where the test sample is placed at the end of one arm and a reference mirror is placed at the

end of the other arm. The reflected lights, from the sample surface and reference mirror, combine and interfere with each other at the imaging part. Then a scanning mechanism introduces an optical path difference between the arms and the interference signal is recorded as a function of optical path difference. Lastly, the surface height profile is reconstructed by analyzing the function of interference signal against the optical path difference (OPD). This technology is considered as established and mature, and there is no major change in its hardware in the past three decades.

1.1. Motivation

Other than being a broadband low coherence light, there is no specific requirement on the light source for coherence scanning interferometry, so most prior arts considered the light source as a very broad spectrum white light with a single Gaussian distribution and neglected the spectral effects due to the light source.

As high power white LED promises longer lifetime, low heat dissipation and compactness, it is fast replacing conventional source of white light for coherence scanning interferometry. Most commercially available high power white LED has two peaks in its intensity spectrum while conventional source of white light has only one peak that is in Gaussian distribution.

This research will focus on the spectral effects of phosphor-based white LED in coherence scanning interferometry by investigating the neglected spectral effects and the use of new coherence light source.

1.2. Objectives

The key objective of this research is to investigate the spectral effects of high power phosphor-based white LED in coherence scanning interferometry with the aim to improve the

usability and performance of coherence scanning interferometry. Unlike previous works that either neglected or suppressed the spectral effects of low coherence light source, this research investigates the spectral factor in coherence scanning interferometry by: (1) Deriving a new signal model for coherence scanning interferometry to resolve the computational issue of prior models and to facilitate fundamental study; (2) Identifying the spectral features of phosphor-based white LED which has two peaks in its spectrum; (3) Modeling the spectral variations of phosphor-based white LED; (4) Investigating the effects of phosphor-based white LED in coherence scanning interferometry in term of interference signal and reconstructed height data; (5) Investigating the mechanism of how the spectral properties of phosphor-based white LED affects the coherence scanning interferometry; and (6) Harnessing the identified spectral properties to improve the usability and performance of coherence scanning interferometry.

1.3. Contributions

The major contributions of this research are as follows:

1. The derivation of an interferometric signal model which is an elementary function that incorporates both the geometric and spectral effects. As the proposed model is an elementary function, it not only resolves the computational load problem of prior models, but also facilitates the fundamental study in coherence scanning interferometry such as understanding the spectral effects in interference signal. The accuracy of this model is benchmarked against computationally intensive counterparts and verified by experiments.
2. The investigation of the spectral properties of high power phosphor-based white LED and their effects in coherence scanning interferometry. We showed that phosphor-based white LED degrades the performance of coherence scanning interferometry.

We also proposed a method to compensate for the spectral effects of phosphor-based white LED so as to improve the performance of coherence scanning interferometry with phosphor-based white LED. While the research focuses on the spectral properties of high power phosphor-based white LED which has two peaks in its spectrum, and that this investigation is done with phosphor-based white LED, the research findings are not limited to phosphor-based white LED.

3. The modeling of the spectral variations of high power phosphor-based white LED and the investigation into its effects in coherence scanning interferometry. The spectral variation of phosphor-based white LED is modeled as a change in the intensity ratio between blue and yellow light. Such spectral variations could be the result of either engineered variation to achieve particular correlated color temperature (CCT) or undesired variations that are influenced by operating conditions such as degradation, pulsing frequency and duty cycle. We showed that, regardless of the spectral variations, the spectral properties of high power white LED affects the interference signal and the result of the corresponding height measurement.
4. The investigation into the mechanism of how the spectral properties of phosphor-based white LED affects the interference signal of coherence scanning interferometry. With the understanding of the mechanism, we demonstrated that (1) the interference signal of coherence scanning interferometry can be manipulated by knowledgeable spectrum shaping, and (2) the spectral effects of phosphor-based white LED can be harnessed to improve the usability and performance of coherence scanning interferometry.

1.4. Organization of the report

The rest of this thesis is organized as follows:

Chapter 2 first introduces the fundamental of interferometric principle for surface height measurement, followed by a literature review on the reconstruction algorithms and hardware-related research activities in phase shifting interferometry and coherence scanning interferometry. The issues and challenges posed by hardware-related stroboscopic interferometry, especially in the light source selection, are discussed in detail.

Chapter 3 presents a novel signal modeling for coherence scanning interferometry. By modeling Gaussian function as a sum of piecewise cosine functions, the presented model expresses the interference signal of coherence scanning interferometry in elementary form and it significantly improves the computational efficiency by up to 256800 times as compared to existing direct calculation approaches. The performance of this novel model is benchmarked against existing prior works in terms of accuracy and computational cost, followed by experimental verifications.

Chapter 4 presents a comprehensive investigation on the spectral effects of phosphor-based white LED in coherence scanning interferometry. We identify and model the spectral properties of phosphor-based white LED. With that, we investigate its spectral effects in coherence scanning interferometry and discover that phosphor-based white LED may affect the performance of coherence scanning interferometry negatively if the spectral effects are not compensated. As such, we propose a method to compensate the spectral effects. Other than these, we also present a theoretical explanation of the distinctive feature which can be used to predict and manipulate the interference signal (and its fringe contrast function) by spectral manipulation.

Chapter 5 presents an implementation study where the spectral effects of phosphor-based white LED are harnessed, instead of being compensated, to improve the performance of a phase-based height reconstruction algorithm. We analyze how the phase-based reconstruction algorithm takes advantages of the spectral property of phosphor-based white

LED to improve its reconstruction performance. Experimental and simulation verifications are conducted to support the findings.

Chapter 6 presents video-based interferogram analysis to quantify the effects of vibration on coherence scanning interferometry. The advantages of the presented method are (1) it does not require additional hardware, (2) the properties of interferometer, such as the bandwidth of sensor and the performance of vibration isolation design, are considered, and (3) the effect of vibration is objectively quantified and measured. In addition to these, we also explore the possibilities of applying the spectral properties of phosphor-based white LED to improve the presented method.

Chapter 7 summaries the contributions of this thesis and recommends several possible research directions as future work.

2. Literature Review

2.1. Introduction

The phenomenon of interference of light wave was first discovered by Thomas Young in his famous double slit experiment [2]. It was first used to prove the wave-particle duality of light, then to measure the wavelength of light and subsequent expansion to applications in refractive index measurement, displacement sensing and etc. In this thesis, the primary interest is in the use of the interferometric method for surface height profile measurement. In general, there are two types of interferometer for surface profile measurement, namely coherence scanning interferometry (CSI) and phase shifting interferometry (PSI). In terms of hardware, these two interferometries are the same except for the light source. Phase shifting interferometry uses coherence light source such as laser or filtered white light, while coherence scanning interferometry uses low coherence light source such as white light.

Figure 2-1 shows the schematic diagram of an interferometer which consists of two parts. The first part is for the generation of interference pattern which consists of the light source, a beam splitter (the bottom one) and a reference mirror. The light is split by the beam splitter and then channeled to the reference surface and the sample surface individually. When the reflected lights meet, an interference pattern is produced. The second part is for the imaging and capturing of interference pattern produced. Normally, it consists of a beam splitter to channel the light onto a photo-detector such as CCD camera. Data collected at this point are not yet the surface height profile, but an image of the interference pattern. It has to be further processed by height profile reconstruction algorithm to obtain the required height information.

Figure 2-1: Schematic diagram of interferometric profilometer

Before we look into the interferometry for surface height profile measurement, some basic terms commonly used in interferometry field are summarized below:

1. Interferogram: A two-dimensional data of interference pattern captured using area-based photo detector array [1], as shown in Figure 2-2.
2. Correlogram: A one-dimensional tracing of the light intensity level of a particular point in the interferogram over the optical path difference [3], as shown in Figure 2-3.
3. Fringe contrast function: As shown in Figure 2-3, this term is applicable to coherence scanning interferometry only. It is defined as the envelope function of the correlogram, and it is also known as the coherence peak function [4].

Figure 2-2: Example of interferogram of coherence scanning interferometry: Fringe on a tilted flat surface captured using a CCD camera

Figure 2-3: Example of correlogram and corresponding fringe contrast function in coherence scanning interferometry

Due to the lack of high precision positioning technologies, the earliest use of interference pattern for surface quality measurement was done by analyzing a single interferogram of the coherence light - the straighter the lines, the flatter the surface. Such an approach [5] has the advantages of ease of use, fast (only one interferogram is required, no complex computation) and affordable. But this approach has the following disadvantages: (1) low lateral resolution;

(2) unable to measure step height; (3) result cannot be related to physical meaning such as surface roughness; (4) ambiguity in interpreting the result, for example both concave and convex mirrors would give the same interferogram. However this approach is still widely used in surface quality measurement, especially for flatness inspection of optical components.

For a detailed quantitative analysis, the interferogram can be further analyzed (and such process is also known as fringe pattern analysis), and many methods [6]–[17] have been proposed.

2.1.1. Phase shifting interferometry

As precision engineering progresses to the micrometer and sub-micrometer levels, the earlier mentioned approach was improved and termed as phase shifting interferometry. The distinctive features of phase shifting interferometry are (1) the use of laser or coherence light source with known wavelength; (2) the use of wavelength information in retrieving the surface profile; and (3) analyzing a number of interferograms of coherence light with known optical path difference among them. In phase shifting interferometry, the intensity of interference signal can be formulated as:

$$I = I_{dc} + I' \cos(\phi + \alpha) \quad (2.1)$$

where

I_{dc} is a constant signal which is independent of interference,

ϕ is the phase shift with respect to α ,

I' is the amplitude of the interference signal, and

α is the phase change due to height profile.

To obtain the height information, α has to be recovered. There are three unknowns (I_{dc} , ϕ and α) in the problem, so a minimum of 3 independent sets of data are required to find α . For the purpose of simplicity in derivation and computation, 4 sets of data are normally collected in the following settings:

$$\begin{aligned}
 I_1 &= I_{dc} + I' \cos(\phi_1 + \alpha); & \phi_1 &= 0 \\
 I_2 &= I_{dc} + I' \cos(\phi_2 + \alpha); & \phi_2 &= \pi/2 \\
 I_3 &= I_{dc} + I' \cos(\phi_3 + \alpha); & \phi_3 &= \pi \\
 I_4 &= I_{dc} + I' \cos(\phi_4 + \alpha); & \phi_4 &= 3\pi/2
 \end{aligned} \tag{2.2}$$

which can be rearranged into the following forms:

$$\begin{aligned}
 I_1 &= I_{dc} + I' \cos(\alpha); & \phi_1 &= 0 \\
 I_2 &= I_{dc} + I' \sin(\alpha); & \phi_2 &= \pi/2 \\
 I_3 &= I_{dc} - I' \cos(\alpha); & \phi_3 &= \pi \\
 I_4 &= I_{dc} - I' \sin(\alpha); & \phi_4 &= 3\pi/2
 \end{aligned} \tag{2.3}$$

These are solved by grouping all the sine and cosine terms together, then with $\tan(\alpha) = \sin(\alpha)/\cos(\alpha)$, the phase that corresponds to the height is recovered. The height information is then retrieved by using the prior knowledge of the wavelength of the light.

As shown in Table 2-1, there are a number of phase retrieval algorithms that can be used to process the interferograms with pre-determined phase difference among them. Although the phase difference between the interferograms is not necessarily known or linear [18], [19], all the phase retrieval algorithms suffer from ambiguity in retrieving the height information from

just the phase information. This is because the phase retrieval algorithm uses the arc tangent function which returns a value from $-\pi$ to π only.

Table 2-1: List of phase measurement algorithms used in phase shifting interferometry

Name	Number of Measurement	Formula
n/a	3	$\alpha = \tan^{-1} \left(\frac{I_3 - I_2}{I_1 - I_2} \right)$
n/a	4	$\alpha = \tan^{-1} \left(\frac{I_4 - I_2}{I_1 - I_3} \right)$
Carré[20]	4	$\alpha = \tan^{-1} \left(\frac{\sqrt{[3(I_2 - I_3) - (I_1 - I_4)][(I_2 - I_3) - (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)} \right)$
Hariharan [21]	5	$\alpha = \tan^{-1} \left(\frac{2(I_2 - I_4)}{2I_3 - I_5 - I_1} \right)$

Figure 2-4: Graphical illustration of phase ambiguity in phase shifting interferometry (PSI): (a) original phase profile of a spherical surface (b) recovered phase profile of the spherical surface.

As shown in Figure 2-4, the spherical surface is not recovered correctly – there is a discontinuity in the recovered phase. The discontinuity is due to the fact that the arc tangent function always returns a value between $-\pi$ to π . This contributes to the ambiguity in

recovering the height information from just the phase information. For example, as $\tan(\pi/3)=\tan(2\pi+\pi/3)=\tan(4\pi+\pi/3)$, a recovered phase information of $\pi/3$ could be due to a phase value of $2\pi+\pi/3$ or $4\pi+\pi/3$.

The dual and multiple wavelength phase shifting interferometer [22]–[25] were invented to increase the equivalent wavelength and to cater to rougher surface profiling. However, this approach does not resolve the phase ambiguity issue.

A possible solution to this ambiguity is phase unwrapping which involves making certain assumptions such as assuming the height difference between neighboring points is less than $1/2$ wavelength of the coherence light source [25]. However, the assumption that the neighboring pixels do not have a height difference of more than $1/2$ wavelength of the coherence light source is not realistic, as most manufacturing processes give roughness profile in term of micrometer [26] and $1/2$ wavelength of a commonly used red laser is around 330nm. Therefore, the assumption is not valid for most samples.

As a result, this has become one of the biggest issues in phase shifting interferometry. If the assumption on the surface profile/finishing is not valid, an inaccurate surface profile will be presented to the users without any warning. For these reasons, phase shifting interferometry is not applicable for very rough surfaces [22] and it is primarily used to measure samples with prior knowledge of the surface quality, i.e. when the assumption of height difference between neighboring pixels holds.

2.1.2. Coherence scanning interferometry

Coherence scanning interferometry [27] is an established optical non-contact measurement method in optical and high precision engineering[28]–[30]. Unlike phase shifting interferometry, it uses low coherence light, which consists of a broad spectrum of light, and it has no problem in measuring rough surfaces [31], but it requires scanning in the axial

direction (which is normal to the sample's surface). So it takes a longer time to measure one surface.

Optical Path Difference between Reference and Sample Beams

Figure 2-5: Correlogram of coherence scanning interferometry, the maximum of it corresponds to height.

Due to the property of low coherence light, the phenomenon of interference occurs only when the optical path difference between the reference and the sample beams is close to zero, and the interferometric signal fades as optical path difference increases. The correlogram of coherence scanning interferometry, as shown in Figure 2-5, explains why coherence scanning interferometry does not have any issue of ambiguity.

Coherence scanning interferometry measures the surface height profile by mechanically scanning the sample's surface in the axial direction (i.e. perpendicular to the sample surface), the interference pattern for each point in the field of view is recorded as a function of the optical path difference, which is referred to as the correlogram. As the interference pattern by white light (low coherence light) is highly localized (as shown in Figure 2-5), the location of the interference pattern corresponds to the surface height. For example, two points with a height difference of 10 μ m will have a shift of 10 μ m in the location of interference pattern in their correlograms.

As a result of the scanning process, there is a huge increase in data and scanning time as compared to phase shifting interferometry which normally takes 3-5 interferograms. An increase in scanning time makes the result vulnerable to perturbation by vibrations [32].

In summary, the major differences between phase shifting interferometry and coherence scanning interferometry are: (1) the light source, and (2) reconstruction algorithm. For phase shifting interferometry, laser or coherence light is used; for coherence scanning interferometry, white light or low coherence light is used. The difference in the light source used results in a change in the interference pattern which requires different algorithms to analyze and extract the height information.

In the next section, existing research results and activities in coherence scanning interferometry will be reviewed.

2.2. Literature Review

Ever since the invention of coherence scanning interferometry, the development of algorithms for fringe analysis has been a key concern. However, the research focus has been changing over time. At first, the primary focus was to design algorithms that are computationally efficient in terms of computer resource allocation such as the implementation of circular buffer scheme [33], and improve the speed of reconstruction by introducing hardware-assisted acceleration to deal with Fourier Transform and Hilbert Transform, and the least squares fitting process for the reconstruction algorithm [34], [35].

Then, the research activities on the reconstruction algorithms were shifted to achieving high accuracy and precision, by assuming that there is either no noise or just one uncontrollable noise. These research activities have helped to build a strong fundamental knowledge on the interferometric test methods.

In recent decades, the research focuses are mainly on: (1) improving the resilience against error, (2) expanding the use of interferometry to other purposes such as refractive index measurement, and (3) dynamic characterization of MEMS. The demand for greater accuracy makes the effects of noise more significant. Other than the well known errors such as imperfections in high precision motion stage, errors arising from vibration [36]–[39] and surface coating [40] have also been studied. Meanwhile, the application of the interferometric test method to other samples such as biological tissues has helped to highlight previously neglected issues such as dispersion [41] as it has become more significant. For the dynamic characterization of MEMS, stroboscopic interferometric microscope is adopted and it raises the challenges in modulating the light source at high frequency and the spectral variation due to the modulation.

As there are similarities between phase shifting interferometry and coherence scanning interferometry, some phase shifting interferometry research results that are relevant to coherence scanning interferometry are also covered here.

2.2.1. Reconstruction algorithm for coherence scanning interferometry

The reconstruction algorithms for coherence scanning interferometry can be classified into two approaches: (1) fringe contrast approach, and (2) phase-based approach.

2.2.1.a) Fringe contrast approach

Centroid method

Due to the sampling process and noise, the peak of the coherence peak function cannot be determined directly. Against conventional solutions of using as many data as possible for least squares fitting or iterative method, Ai and Novak [42] found that the effective peak of

the coherence peak function is equivalent to the centroid of the square of the first order derivative of the correlogram.

Other than improvement in computational efficiency and resource allocation, their work also aimed to resolve the ambiguities in multi layer sample, which generates multiple peaks in the correlogram function. It has the advantage of not requiring dedicated hardware for high speed processing.

Figure 2-6 shows the flow chart of Ai and Novak's centroid approach [42] in estimating the peak location.

Figure 2-6: Flow chart of Ai and Novak's centroid approach [42] in estimating the peak location

The idea of the centroid approach is that the correlogram can be modeled as

$$I(z) = I_o + m(z)\cos(wz + \phi) \quad (2.4)$$

where

I_o is the constant bias component of the signal,

z is the scanner position which corresponds to the OPD,

$m(z)$ is the fringe contrast function,

w is the wavenumber, and

$\emptyset$ is the initial phase, which is assumed to be constant.

The coherence peak function is reasonably assumed to be a Gaussian function. Other than that, the coherence peak function is symmetric along the peak; all these properties of coherence peak function were used to conclude that the peak location can be determined by finding the centroid of the n-th order derivatives of the correlogram.

Unlike prior work that derived the peak location based on a common assumption that the envelope function of the correlogram has only one peak, the centroid approach does not assume as such and it can give consistent result in the multi-peak case. Figure 2-7 shows the comparison between the centroid approach and prior work. To date, the centroid approach is still considered as one of the most practical algorithms that promises both speed and quality at the same time.

Despite the advantages of high speed and capability in handling complex scenarios, this approach is not considered as the most accurate one for the following reasons:

1. It is not based on sound optical fundamental, and the accuracy of the algorithm depends on the sampling process

2. It blindly processes all sampled data without indicators for notifying possible multi-peak and improper sampling process.

Figure 2-7: The centroid approach and prior art for handling multi-peak coherence peak function in multi-layer sample (image is extracted from Ai and Novak [42])

These two factors make this approach less reliable, highly influenced by user skill and prior knowledge on sample. However, this method pointed out a significant idea that it is not necessarily to find the exact true peak of the correlogram, as long as the identified peak holds consistent relationship with the exact true peak for all measurement points.

Hilbert Transform

Based on the phase shifting algorithm, Larkin [34] worked out an approximated Hilbert transform for envelope detection. His work was primarily focused on computational efficiency and accuracy in the presence of mis-calibration (in high precision scanning mechanism). By the use of approximated Hilbert transform, the computational efficiency is improved significantly. For accuracy, Larkin introduced least squares fitting on filtered data to estimate the peak position.

In summary, the computationally efficiency of this approach is about two to three times faster than the real-space Hilbert transform approach and it produces highly repeatable results. However, his research was primarily based on simulations.

Kalman Filter

As the requirement of the precision level in mechanical part and computer technologies increases, the performance expectation on interferometry also increased from the micrometer level to the nanometer level. Many researchers have realized that some of the variations in measurement results are contributed by the data processing and height reconstruction process. Thus, some researchers have looked beyond existing approaches and explored other techniques such as the Kalman Filter based method [43].

The Kalman Filter is a recursive filter that estimates the state of a system. It is designed to work with noisy measurement and widely used in machine vision and control systems [44]. The idea of applying Kalman filter in the height reconstruction algorithm is that by modeling the correlogram as a system, the location of the peak (which corresponds to the height information) can be detected and estimated.

In Gurov's work [43], the possibilities of solving the coherence peak sensing problem using stochastic difference equations has been demonstrated and it can be an alternative approach to coherence scanning interferometry. The advantages of this approach are its precise estimation and high speed processing. However, as the basis of this approach relies on the assumption that the process of finding the peak in the correlogram can be represented as a system, the performance of the algorithms will be influenced by the model built. Figure 2-8 shows the estimation result generated by using the Kalman Filter. However, no benchmarking in repeatability and reliability have been conducted to evaluate the performance of this algorithm.

Figure 2-8: Fringe contrast function, $A(k)$, estimated using Kalman Filter: The frame number of the x-axis corresponds to the change in optical path difference and the y-axis records the interference signal (image is extracted from Gurov [43])

As the demand for precision increases, the complexity of the reconstruction algorithms increases. Fourier transformation has been used to extract more information from the correlogram. There are two immediate advantages of using the Fourier transform: First, it provides more information, such as the coefficients in spatial frequency domain, which correspond to the wavelengths of light; Second, it may be faster as Fourier transform is widely used in various applications and there are a number of optimized algorithms that can be integrated into computer.

Taking advantages of the availability of fast Fourier transform, Li et al. [40] have worked on transparent coating measurement using coherence scanning interferometry. The height reconstruction process was done with fast Fourier transform followed by a Gaussian fitting process. The proposed system first captures the interference signal and generates the correlogram, then applies fast Fourier transform operator to it. Next, depending on prior knowledge on the sample, such as how many layers are contributing to the correlogram, a Gaussian fitting operator is applied to estimate the location of the peak. Figure 2-9 illustrates Li et al.'s work [40].

Figure 2-9: Illustration of Li et al.'s work [40] of applying filtering through frequency domain followed by Gaussian fitting for estimating peak

2.2.1.b) Phase-based approach

Color insensitive approach

Figure 2-10: Graphical illustration of Groot's frequency domain analysis.

A few researchers, including Groot and Deck [45], [46], have exploited the advantage of increased information through Fourier transformation. In spatial frequency domain as shown in Figure 2-10, the magnitude versus frequency chart is related to the intensity spectrum of the light source and the phase versus frequency chart contains the information about the phase of different wavelengths. In Groot's work [46], the extraction of the surface

topography without relying on fringe contrast function was demonstrated and the implementation was also documented in detail. The way to extract the surface topography without relying on the fringe contrast function is to transform the correlogram into the spatial frequency domain, then select a series of Fourier coefficients with corresponding wavenumbers, and apply linear fitting to the selected data and finally compute the rate of change of phase with respect to the wavenumber. It is claimed that the mean phase value together with the rate of change of phase with wavenumber can be combined to reconstruct the height information with the same accuracy as phase shifting interferometry [46].

Color sensitive approach

The availability of more information through spatial frequency transformation has attracted significant research attention, and one of the latest works was by Pawlowski [47]. Pawlowski suggested that by recording the correlogram with color sensitive image sensor, the location of the peak can be identified by a phase-crossing algorithm.

In the demonstration of his idea, Pawlowski used 3-CCD color camera to capture three correlograms for the measured point, as shown in Figure 2-11, and three series of Fourier coefficients.

Figure 2-11: Three color fringe signals detected at an arbitrarily selected point on the object; (a), (b), and (c) correspond to the signals from red, green and blue channels, respectively. The wrapped phase distribution obtained for the red and green channels are presented in (d) for the region of interest shown magnified in (a) and (b); the location of phase crossing is indicated by an arrow (image is extracted from Pawlowski [47])

The key idea of the phase-crossing algorithm is that the peak location corresponds to the location where all three channels have zero phase difference, which is shown by the arrow in Figure 2-11(d).

Although this approach uses similar principles as in Groot's work [45], [46], the use of multiple photo-detectors has converted Groot's idea from one of virtually extracting the information contributed by multiple wavelengths to one of physically extracting them.

However, the performance of this approach has not been benchmarked against other existing techniques.

White Light Phase Shifting Interferometry

The white light phase shifting interferometry, which is also known as high definition vertical scanning interferometry or improved coherence scanning interferometry, has been proposed to achieve measurement accuracy and resolution of the phase shifting interferometry without the phase ambiguity [3], [4], [48], [49]. The working principle is that the correlogram of low coherence is first processed to identify the peak of the fringe contrast function (which is equivalent to fringe contrast approach for a coarse measurement). It is then followed by a five-frame phase retrieval algorithm for fine and accurate measurement. Lastly, a phase correction process is applied to remove reconstruction artifacts.

Kim et al. [50] further improved the white light phase shifting interferometry approach by applying numerical phase error correction schemes of pre-processor, main processor and post processor, as shown in Figure 2-12.

Figure 2-12: Graphical illustration of pre-processor, main processor and post-processor (image is extracted from Kim et al. [50])

Kim et al. [50]'s work have successfully minimized the measurement error without additional equipment, and achieved repeatability of 0.2nm. However, this method is computationally intensive and its improvement is primarily based on iterative or least squares method without utilizing knowledge on vibration or hardware configuration.

Instead of solely relying on intensive processing in the reconstruction stage, Kim et al. [51] proposed a calibration method which is capable of identifying the actual scanning error by analyzing the spectral distribution of the captured correlograms.

According to Kim et al [51], correlograms captured at different operating ranges are different, as shown in Figure 2-13. This is due to imperfections in the scanning mechanism. By analyzing the spectral distribution of the captured correlograms and the corresponding input (control) signal, the response curve of scanning mechanism is obtained and used to compensate for errors in the scanning process. Figure 2-14 shows the performance comparison between one with and one without compensation the calibration method.

Figure 2-13: Due to imperfections in scanning mechanism, correlograms captured at different scanning ranges are different (image is extracted from Kim et al [51])

Figure 2-14: Comparison of measurement result on a standard height before and after compensating the scanning error (image is extracted from Kim et al. [51])

Similar to Kim et al.'s concept [51], Kiyono et al. proposed a self-calibration technique for coherence scanning interferometry, which can minimize the effects of imperfections in the scanning mechanism [52].

2.2.2. Hardware related research

Other than improvements to computational resource allocation and the reconstruction algorithm, there are existing research results on the hardware of the interferometry. These are essentially to address the following problems:

(1) Error due to vibration

The vulnerability of interferometry to vibrations arises from the measurement principle that it takes several interferograms for analysis to produce the required surface height profile. Normally, these interferograms are captured at different times which make them susceptible to erroneous and differing optical path difference caused by vibrations.

The most common method to deal with vibration is to use an optical table (vibration isolation system). An optical table is a platform engineered to isolate environmental vibration by being rigid, stiff, and having a high natural frequency. The optical table can remove virtually all the vibrations above 50Hz. But many environmental vibrations occur at below 30Hz. For example, the vertical vibration of 10 to 30Hz from people, traffic and construction, and the horizontal vibration of 1 to 10Hz in tall buildings [53]. Other than the limitation of vibrations at low frequency, the optical table cannot handle vibrations induced by devices operating on it. Meanwhile, the cost and the size of vibration isolation system also increase as the performance requirement of vibration isolation increases. For more information about optical table and vibration in precision applications, please refer to [53]–[55].

(2) Stroboscopic interferometry for dynamic characterization of MEMS

The MEMS technology industry has matured and the manufacturing of MEMS devices is gaining momentum. There is now a need to characterize MEMS devices while they are in motion [56]. To measure the MEMS in motion, the data acquisition process has to be synchronized with the motion of MEMS, and each interferogram has to be captured with a short exposure time to avoid effects such as motion blur [57]–[59].

Unlike the phase shifting interferometry, a conventional source of white light cannot be modulated to synchronize with the MEMS's motion and/or to deliver high brightness in a short time. Researchers in coherence scanning interferometry have been looking for a new source of low coherence light and resolving issues related to high modulating frequency and short duty cycle of the light source [60], [61].

(3) Enhance interferometric test method

A relatively small number of researchers have also looked into the design of the hardware of the interferometry to improve the measurement accuracy and to solve the reflectivity issue [62]–[64].

2.2.2.a) Vibration resistant interferometry

Groot [36] has studied the influence of vibration on phase shifting interferometry and analyzed the performance of the various reconstruction algorithms against vibrations. Phillion [65], Kim et al [51], Guo et al [66], Kong et al. [67], Deck [37] and Honda et al. [39] have proposed to estimate and compensate for the error in the surface height reconstruction process by post processing methods, such as the iterative least squares fitting methods. However, these methods are computationally intensive and applicable only to certain vibration patterns.

An obvious solution to vibration resistant interferometry is to capture all the interferograms at the same time, but this will take a lot more research and development. For the phase shifting interferometry, Wyant [38], Millerd et al. [68], [69] and Koliopoulos [70] have developed simultaneous phase shifting interferometers which acquire four interferograms with controlled optical path difference at the same times. This approach is radical and revolutionary. The method has been proven to be efficient against vibration but currently the method is applicable to phase shifting interferometry that requires only four interferograms. However, coherence scanning interferometry requires hundred of interferograms for measurement, and a similar radical approach of redesigning for instant interferograms capture is not yet feasible, at least in the near future.

Lai and Yatagai [71] introduced a reference mirror in the sample arm, as shown in Figure 2-15. The reference surface chosen is a tilted flat mirror which produces straight fringes, as the wavelength of the light and the profile of the mirror are known, the phase difference between interferograms can be identified. Next, the identified phase information is used to

determine the unknown profile of the test surface. The additional reference surface in the sample arm improves the repeatability by identifying and compensating for the height error (between interferograms) due to high precision scanner and vibration. However, this approach sacrifices the lateral resolution and imposes extra effort in positioning the sample. These factors are believed to have constrained it from being widely adopted.

(a) Top view of sample holder (b) Interferogram captured
Figure 2-15: With additional reference surface (“Tilted Mirror” in (a)) in sample arm, the actual phase difference between interferograms can be identified (image is extracted from Lai and Yatagai [71]).

Similar to Lai and Yatagai’s approach, Zhao and Burge [72] used an active control to compensate for the effects of vibrations. This mechanism consists of a digital signal processor and a high speed phase control from electro-optic modulator (EOM) and a sensor for phase measurement at 4000 Hz. These three components form a high speed close-loop control system with electro-optics modulator driver, where the phase measurement sensor captures the influence of vibration and the digital signal processor computes and feeds the EOM driver with a control signal that is vibration-compensated.

Figure 2-16: Illustration of Zhao and Burge's idea of vibration-compensation (image is extracted from Zhao and Burge [72])

Figure 2-16 shows the graphical illustration of Zhao and Burge's [72] method, where the electro-optic modulator (EOM) is used as a phase shifter. The "Photodiode", "Fiber", "half-wavelength plate" and "PBS" blocks form a vibration measurement system that feeds the "vibration compensation" DSP. Lastly, the vibration compensated control signal is sent to the "EOM driver". Figure 2-17 illustrates how the electro-optic modulator (EOM) is used as a phase shifting device.

The advantages of this approach are that (1) it can be readily applied to interferometer with a minimum change in the reconstruction algorithm, and (2) it does not affect the way interferometer acquires the required information. However, the performance of this approach hinges on the performance of the close-loop control system, especially the sensor for phase measurement. Other than this, the additional hardware required also increases the size and cost of the interferometer.

Figure 2-17: Graphical illustration on the use of electro-optic modulator (EOM) as phase shifter (image is extracted from Zhao and Burge [72])

Olszak and Schmit [73], [74] proposed the addition of a reference signal module to correct the imperfections in the scanning mechanism and other sources such as vibrations. An additional hardware module named “reference signal” is added to measure the displacement of the scanner, as shown in the schematic diagram of Figure 2-18. It has the advantages of (1) being independent of interferometers’ setting such as the light source, and (2) the extracted information can be used to identify and compensate for error, phase unwrapping in phase shifting interferometry.

The concept seems to be similar to Zhao and Burge’s proposal [72], but it is actually more similar to Lai and Yatagai’s work [71] in the sense that they do not try to compensate for or remove the influence of error, they merely record the errors so that these errors can be considered in the reconstruction process. However, this approach is similar to other hardware-based methods in terms of additional hardware requirement which increases the cost, and the performance will be affected by the performance of additional hardware introduced.

Figure 2-18: Schematic diagram of scanning interferometer with reference signal: S is the light source; L is the lens; B is the beam splitter; M is the mirror and D is the reference signal detector (image is extracted from Olszak and Schmit [73])

2.2.2.b) Stroboscopic interferometry for dynamic characterization of MEMS

Inspired by the feature of stroboscopic phase shifting interferometry [57], [59], [75]–[77], Novak et al. proposed a stroboscopic interferometric system for dynamic MEMS measurement [78]. Novak et al.'s work identified the following challenges in stroboscopic interferometry:

1. Optical design of LED: Unlike conventional light source, LED comes with different lens configuration. The viewing angle of LED must be considered and to match the optical design of interferometry
2. Dynamic spectral variation of LED: Although the center wavelength of LED is relatively stable, it shifts with operating temperature and current. Undesired shift in the center wavelength degrades the measurement. For example, a few nanometer shift in the center wavelength could introduce an error of several hundred nanometer in the measurement of a 100 μ m step height.

3. Mechanism to deliver intense pulse light: While an efficient optical design can help to deliver more intense light to the test sample, the required light intensity is still much higher than continuous lighting. An obvious solution is to combine multiple LEDs into one chip. Unfortunately, doing so will increase the operating current/temperature and also pulse spreading. Overdriving an LED might cause overheating (which affects LED's properties such as spectrum and efficiency), but overdriving an LED in pulse mode is considered to be a better solution.

Novak et al. [78] achieved the stroboscopic interferometric system (which combines coherence scanning interferometry and phase shifting interferometry) and demonstrated dynamic measurement of a MEMS device at the 22kHz resonant frequency.

By changing the light source of a commercial coherence scanning interferometric system to LED, Davis et al. [79] managed to achieve high modulation frequency of 2MHz and visualize the acoustic displacements of capacitive micro-machined transducers.

Figure 2-19: Schematic setup of Kassamakov et al's super continuum interferometer (image is extracted from Kassamakov et al. [80])

The modulation frequency of LED is limited by LED's junction capacitance, quantum efficiency and mean wavelength-duty cycle [80]. As laser is capable of delivering high power at high modulation frequency, a low coherence light generated by a supercontinuum source (which is a low coherence light generated by laser) is an obvious candidate and is being applied for stroboscopic interferometric use. The supercontinuum source is a new type of white light source. It generates low coherence light by launching laser pulses into micro structured optical fiber (MOF), and new wavelengths are created by a series of nonlinear optical processes [81]–[84]. With the advantages of the supercontinuum source, Reolon et al. [85] demonstrated a broadband supercontinuum interferometer with improved measurement resolution. Separately, Kassamakov et al [80] harnessed the supercontinuum source to realize modulation at frequency that exceeds 10MHz for stroboscopic interferometry. Figure 2-19 shows the schematic diagram of Kassamakov et al's setup, and Figure 2-20 shows the effective spectrum of the supercontinuum source with respect to the spectral sensitivity of camera.

Figure 2-20: Comparing effective spectrum of Kassamakov et al's supercontinuum source with some common light sources (image is extracted from Kassamakov et al. [80])

With the improvement in the modulation frequency of stroboscopic interferometry, several researchers have started to look into the various issues due to higher modulation frequency.

By using a hybrid light source, which is a combination of a non phosphor-based white LED and a single color LED, Heikkinen et al. [61] have demonstrated stroboscopic coherence scanning interferometry measurement at 5.4 MHz. The schematic diagram is shown in Figure 2-21. The intensity spectrum of a non-phosphor-based white LED is similar to phosphor-based white LED, however it does not use phosphor to generate yellow white. There is also no issue with phosphor degradation [86]–[88] and is expected to last longer [89], [90].

Figure 2-21: Schematic view of Heikkinen et al.'s coherence scanning interferometry with multi LED light sources (image is extracted from Heikkinen et al. [61])

Figure 2-22: Spectrum at 4 different pulsing frequencies. The yellow peak of the non-phosphor LED is lower compared to blue peak at higher frequencies (image is extracted from Heikkinen et al. [61])

Other than achieving high modulation frequency, Heikkinen et al. [61] also found that there is spectral variation at different pulsing frequency. As shown in Figure 2-22, it is found that the ratio between the blue and yellow lights of the non phosphor-based white LED changes with pulsing frequency and the wavelength of the yellow light also increases as the pulsing frequency increases. However, the effects of these spectral changes were not further investigated.

Figure 2-23: Hanhijärvi et al. found that the spectrum of LED is a function of duty cycle (image is extracted from Hanhijärvi et al. [60])

Hanhijärvi et al. [60] also identified spectral variation under different pulsing condition. Figure 2-23 shows the spectrum shifts when the duty cycle of pulse operation changes. The effects of such spectral shift are studied and it is found that the spectral shift caused by the duty cycle does not affect the measurement uncertainty of the coherence scanning interferometry.

2.2.2.c) Others

Other than spectral manipulation for stroboscopic applications, a relatively small number of researchers have manipulated the spectrum of the light source to improve the performance or the usability of coherence scanning interferometry.

Molnar and Tutsch proposed a mixed-coherence light source to improve the measurement accuracy of the coherence scanning interferometry [62]. Figure 2-24 shows the schematic diagram of Molnar and Tutsch's setup, and the mixed coherence light source is a combination of broadband light source and a laser. By mixing broadband and coherence light source into one, the correlogram collected is unique as shown in Figure 2-25. This method delivers the accuracy of phase shifting interferometry without phase ambiguity issue.

Figure 2-24: Schematic diagram of Molnar and Tutsch's vertical scanning interferometry with a mixed coherence light source (image is extracted from Molnar and Tutsch [62])

Heikkinen et al. [63] have also proposed a hybrid light source which is a combination of multiple LEDs and a laser to expand the use of coherence scanning interferometry for samples which cannot be earlier measured because of the reflectivity issue. At the same time, the proposed light source can also be modulated at high frequency for dynamic

characterization of MEMS. Figure 2-26 shows the schematic diagram of Heikkinen et al.'s proposed method [63].

Figure 2-25: Correlograms generated by Molnar and Tutsch's mixed-coherence light source which is a combination of a broadband light source and a laser (image is extracted from Molnar and Tutsch [62])

Figure 2-26: Schematic diagram of Heikkinen et al.'s scanning white light interferometry with hybrid light source (image is extracted from Heikkinen et al. [63])

Figure 2-27: Schematic diagram of Yang et al.'s interferometric configuration which use a pair of harmonically related light sources for long optical distance with sub-nanometer precision (image is extracted from Yang et al. [64])

To achieve a high precision over a long measurement range, Yang et al. [64] combined the advantages of phase shifting interferometry and coherence scanning interferometry by using a pair of harmonically related light sources (one continuous wave and one low coherence) as light source and a phase crossing algorithm. With these, Yang et al achieved distance measurement of sub-nanometer precision over an arbitrarily long range [64].

2.3. Conclusion

This chapter has presented a literature review on interferometry and the research trend of coherence scanning interferometry in the past three decades. We notice that in the most recent decade, the research focuses have shifted to addressing issues which are specific to applications such as dynamic characterization of MEMS, film thickness/refractive index measurement, etc. Although the spectral approach, in which researchers manipulated the light source in coherence scanning interferometry, is one of the most commonly used approaches, these works have been focused on addressing application specific issues. The spectral effects in coherence scanning interferometry have not been investigated in detail. Among many light sources, such as supercontinuum, phosphor-based white LED, phosphor-

free white LED, combinations of LEDs and lasers, etc, phosphor-based white LED will be the focus of this thesis for the following reasons:

- Phosphor-based white LED is the most commonly used high power white LED [91], and it is replacing conventional white light source in various applications including the light source in coherence scanning interferometry
- The intensity spectrum of phosphor-based white LED is significantly different from the conventional white light source and its spectral effects in coherence scanning interferometry can no longer be neglected
- The spectral property of phosphor-based white LED is similar to phosphor-free white LED (a new type of high power white LED), so the study on phosphor-based white LED can be readily applied to phosphor-free white LED

Hence, the following chapters will investigate the spectral effects of phosphor-based white LED in coherence scanning interferometry with the aim to improve the usability and performance of coherence scanning interferometry.

3. Signal Modeling for Coherence Scanning Interferometry

This chapter presents a new coherence scanning interferometric signal model. It incorporates both the geometric and spectral effects, in elementary form. An elementary function (or form) is mathematically defined as a real function which is made out of elementary operations and a collection of selected functions. As the presented model is an elementary function, it not only resolves the computational issue of prior models, but also facilitates a fundamental study on coherence scanning interferometry, such as a detail understanding of the spectral effects in the interference signal. As the model is in elementary form, no numerical integration or transformation is involved, so the computational time is improved by 256800 times. The accuracy of the presented model will be verified by widely accepted method (e.g. direct numerical integration on the generalized model) and experimental data [92].

With the presented model, the effects of spectral sensitivity of the photo-detector and the numerical aperture of the objective lens on the coherence scanning interferometry are studied in detail.

3.1. Motivation

The working principle of the coherence scanning interferometry is characterized by analyzing a series of interference patterns of low coherence light with known optical path difference among them. The recorded interference signal with changing optical path difference is known as the correlogram, and it is the raw data used for measurement. As such, retrieving the high quality interference signal is an important issue in coherence scanning interferometric applications. The quality of the correlogram is a function of the noise level in the photo detector and the accuracy in determining the optical path difference.

Other than the optical path difference, the interference signal is also affected by aberration and geometric form of optical components, imperfections in piezoelectric scanning mechanism, spectrum of the light source, the response of light receiving sensors, vibrations, materials of the sample, etc. While some of these factors such as aberrations in optical components and imperfections in piezoelectric scanning mechanism can be compensated for by calibration, other factors such as sensor noise and vibrations are not repeatable and cannot be compensated by calibration.

So the retrieval of high quality correlogram remains a challenge until today, and it is not uncommon that researchers evaluated their works based on certain models [34], [40], [93], then followed by experimental verification.

3.2. Signal Modeling for coherence scanning interferometry in elementary form

3.2.1. Physical model and related work

Figure 2-1 shows the physical model of Mirau-based interferometry, where the dotted box is Mirau interferometry objective lens which is commonly used for high magnification (10x magnification or higher) applications. In Mirau interferometry objective lens, the light entering the objective lens is split by the half mirror (which serves as a beam splitter) into two arms: one to the reference mirror and the other to the sample surface. Then the reflected lights meet and interfere with each other.

A full physical model has to incorporate factors such as (1) partial coherence of the light source, (2) polarization of the light source, (3) imaging properties of optical components (such as tube lens and objective lens), (4) the interaction of light with matter, such as diffraction, and (5) artifacts in the presence of discontinuous sample surface.

Figure 3-1: Physical model of Mirau-based interferometry

For interferometric surface height measurement, a full physical model is not necessary. So, the proposed model is a simplified full physical model based on the following assumptions: (1) the light source is a randomly polarized low coherence extended source; (2) the sample surface is smooth, and (3) there is reflection light only. These assumptions are commonly adopted, for example in works by Davidson et al. [28], Kino and Chim [35], Sheppard and Larkin [93], and Groot and Colonna de Lega [94].

Based on the two-wave interference analysis, the interference signal of two rays (with the same wavelength) is expressed as

$$f(z) = A_1^2 + A_2^2 + 2A_1A_2 \cos[2kz + (\phi_1 - \phi_2)] \quad (3.1)$$

where

k is the wavenumber ($=2\pi/\lambda$),

A_i is the amplitude of ray (i), $i=1, 2$,

ϕ_i is the phase of ray (i), $i=1, 2$, and

z is an independent variable.

However, in implementation, instead of a single ray bundle, each point on the interferogram is contributed by a collection of ray bundles. The number of ray bundle is linked to the numerical aperture of the objective lens in use: the bigger the numerical aperture, the larger the number of ray bundles. The effect of this summation is known as the geometric effect.

The numerical aperture of a microscope objective lens is to quantify the resolving power of the objective lens and the amount of light it can collect. Mathematically, the numerical aperture is defined as

$$\text{Numerical Aperture (NA)} = n \sin(\theta_0) \quad (3.2)$$

where

n is the refractive index of the medium in which the lens is working, and

θ_0 is the half-angle of the maximum cone of the light captured by the objective lens.

Figure 3-2 graphically illustrates the definition of numerical aperture. In general, the numerical aperture of an objective lens is proportional to its magnification, and inversely proportional to its working distance.

Figure 3-2: Illustration of numerical aperture (NA) of objective lens

Other than the numerical aperture of the objective lens, the intensity spectrum of the light source also affects the interference signal. As the light source of coherence scanning interferometry has many wavelengths, the resultant interference signal is the summation of the interference signals contributed by these wavelengths and the effect of this summation is known as the spectral effect.

While the spectral effect is linked to the spectrum of the light source, the spectrum of the light source will vary depending on the lighting technology. Figure 3-3 shows the spectral variation among some common sources of white light such as tungsten lamp, mercury vapor lamp, noon sunlight and white LED.

Both the geometric and spectral effects are applicable to phase shifting interferometry and coherence scanning interferometry, however, the effects are much more significant and complex in coherence scanning interferometry due to the use of low coherence light.

Figure 3-3: Comparing spectral energy from some common source of white light: tungsten lamp, mercury vapor lamp, noon sunlight and white light emitting diode (LED) [95]

Currently, there are two major and commonly adopted models. The first is the generalized model [35], [94] that expresses the correlogram as

$$I_{\text{interference}}(z) = C_1 \int_{\text{bandwidth}} \int_0^{\theta_0} \{k^2 \times \cos[2k(z - z_0) \cos \theta + \phi] \times \sin \theta \cos \theta d\theta\} F(k) dk \quad (3.3)$$

where

z is the defocus position (related to optical path difference),

z_0 is related to the profile of the sample surface,

C_1 is a constant,

k is the angular wavenumber ($k=2\pi/\lambda$),

$\sin \theta_0$ is the numerical aperture of the objective lens (assuming $n=1$),

$F(k)$ is the effective intensity spectrum of the light source, and

ϕ is the phase offset.

The strengths of the generalized model are that: (1) it can simulate the effects of changing the spectrum of light source and the numerical aperture of objective lens, and (2) it has strong correlation between the model parameters and the physical setting of the system. For example, the term $F(k)$ in Equation (3.3) is equal to the intensity spectrum which can be measured by an optical spectrum analyzer.

The drawback of the generalized model is its intensive computational load. As the intensity spectrum of the light source is either difficult to be expressed in symbolic form or modeled as Gaussian, Equation (3.3) cannot be expressed as a finite number of elementary functions. So Equation (3.3) has to be solved by numerical integration, which is a computationally intensive and time consuming process.

The second model is a simplified version of the first one, and it is expressed as

$$I(z) = I_{dc} + I_{amplitude} \exp\left(\frac{-(z - z_0)^2}{\sigma^2}\right) \cos\left(\frac{4\pi(z - z_0)}{\lambda_m} + \phi\right) \quad (3.4)$$

where

z is the defocus position (related to optical path difference),

z_0 is related to the profile of the sample surface,

I_{dc} is a constant value and not related to the interference,

$I_{amplitude}$ is the amplitude of the interference signal,

λ_m is the equivalent wavelength of light,

σ is related to the coherence length of the light source, and

ϕ is the phase offset.

The second model is a simplified model of the first model, based on assumptions that the numerical aperture of the objective lens is small and the intensity spectrum of the light source, $F(k)$ in Equation (3.3), is a single Gaussian function. The advantage of such simplifications is computational efficiency, but the drawback of this model is its poor correlation between the model parameters and the theoretical parameters. For example, the parameter σ in Equation (3.4) does not have a clear physical link to the spectrum of the light source. As such, the parameters of the second model have to be determined empirically, and it is impossible to simulate the correlogram based on specified spectra and numerical aperture.

To cater to a variety of objectives and spectra of the light source, there is a need to have a computationally efficient method to compute the generalized model (i.e. Equation (3.3)). Groot et al. [94] proposed a simplification in the frequency domain: Equation (3.3) is first transformed into the frequency domain, then only the non-zero frequency components are selected for numerical integration. An inverse Fourier transform is then applied to get back to the original domain. By removing the zero frequency components, this approach makes the computational load more feasible for simulating the correlograms. It can reduce the computational load by 200 times.

However, due to the use of numerical integration, it is still a computationally intensive process, and the performance is subjected to complexity of the intensity spectrum. In the worst case that all frequency components have to be selected for numerical integration, this approach can only halve the computational load by removing the negative frequency components.

3.2.2. Proposed model and approach

The objective of the proposed signal modeling for coherence scanning interferometry is to address the issue of intensive computational load of the generalized model by: (1) eliminating the need for numerical integration in the calculation of the generalized model, and (2) representing the generalized model as elementary functions.

As the intensity spectrum term, $F(k)$ in the generalized model (Equation 3.3) is either difficult to be expressed in symbolic form or modeled as Gaussian, so computationally intensive numerical integration process is necessary for the generalized model. Numerical integration is a process that calculates the approximated value of a definite integral, and it which can be expressed as

$$\int_a^b f(x)dx \approx \frac{(b-a)}{n} \left(\frac{f(a)+f(b)}{2} + \sum_{k=1}^{n-1} f\left(a+k\frac{b-a}{n}\right) \right) \quad (3.5)$$

where

n is the number of intervals.

The accuracy of the numerical integration process is proportional to the parameter n in Equation (3.5). However, as n increases, the computational load also increases. This is identified as the root cause of intensive computational load in the generalized model.

Mathematically, there are two reasons for carrying out the numerical integration: (1) the integrand may be known for certain region only; (2) the anti-derivative of the integrand does not exist [96]. In general, the intensity spectrum, $F(k)$ in Equation (3.3), is either sampled by spectrum analyzer or modeled as Gaussian (which has no explicit integral). As such, the generalized model, i.e. Equation (3.3), has to be solved by numerical integration [97].

To eliminate the use of numerical integration, it is proposed that we represent the generalized model as a finite number of elementary functions. Mathematically, a function is elementary if it is built from a finite number of exponentials, logarithms, constant, and n-th roots of polynomial equations through composition and combination using the four elementary operations of addition, subtraction, multiplication and division [98]. As these functions and constants are allowed to be complex number, trigonometric function (such as cosine and sine) and its inverses are considered as elementary functions too. In this way, the computation load for evaluating the elementary function is low.

An example of an elementary function is

$$f(x) = \frac{\exp(\sin(x))}{1+x^2} \tan\left[\sqrt{1+\log(x)}\right] \quad (3.6)$$

While an example of non-elementary function is

$$f(x) = \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt \quad (3.7)$$

The challenge to develop the proposed signal model is that one needs a model to accurately represent a Gaussian function (which is commonly used to represent intensity spectrum of the light source) yet ensuring that Equation (3.3) has an explicit integral.

3.2.3. Modeling Gaussian as sum of piecewise cosine functions

As mentioned earlier, a Gaussian function is not an elementary function and it does not have an explicit integral. For the proposed model, the Gaussian term has to be represented by

another form which is an elementary function with explicit integral. Trigonometric functions (such as sine, cosine and tangent) and polynomial functions are two potential candidates.

Compared to the polynomial function, trigonometric function is preferred as it is tidier and simpler in mathematical manipulations. So it is proposed to represent a single Gaussian function with a sum of piecewise cosine functions which is defined as follows:

$$f(x) = \sum_{j=1}^{j=n} a_j \cos\left(\frac{2\pi(x-\mu)}{b_j}\right); (\mu-c) \leq x \leq (\mu+c) \quad (3.8)$$

Graphically, a single piecewise cosine function is as shown in Figure 3-4.

Figure 3-4: Graphical Illustration of a single piecewise cosine function

Modeling a single Gaussian function as a sum of piecewise cosine functions is equivalent to non-linear fitting of cosine functions into a Gaussian function, and the trust-region approach [99] is adopted for non-linear least squares fitting. The trust-region approach has a region around its current search point, and the size and the step are modified during the search process, depending on how well the model fits the actual model.

The number of piecewise cosine functions required to represent a Gaussian term needs to be investigated. By the trust region approach, a Gaussian function can be represented by the sum of one, two, three or four piecewise cosine functions. The data selection prior to the non-linear fitting process is based on the amplitude of the signal, with all data above the 1%

signal level being fed to the non linear fitting processing. The fitting results are shown in Figure 3-5. Visually, Figure 3-5 shows that there is no significant improvement in the quality of fitting with three or more piecewise cosine terms.

Figure 3-5: A single Gaussian with zero mean and sigma of 0.6 is fitted to a sum of one, two, three or four piecewise cosine functions

For an objective measure, the goodness of fit, R^2 , is used to measure how well a model fits the actual model. Mathematically, the goodness of fit, R^2 , is expressed as:

$$R^2 = 1 - \frac{\sum_i^n (y_i - f_i)^2}{\sum_i^n \left(y_i - \frac{1}{n} \sum_i^n y_i \right)^2} \quad (3.9)$$

where

n is the number of observations,

y_i is the i^{th} observed value, and

f_i is the i^{th} modeled/predicted value.

If a model represents the observation perfectly, the value of R^2 will be one. A larger deviation of R^2 from one will mean a poor fit for the actual model [100], [101]. Figure 3-6 analyzed the number of piecewise cosine functions used to model a single Gaussian function versus the goodness of fit (R^2). It showed that a sum of two piecewise cosine functions is good enough to represent a single Gaussian function. Therefore it is determined that a single Gaussian function can be sufficiently represented by the sum of two piecewise cosine functions, and it can be expressed as

$$ae^{\left(\frac{-(x-x_m)^2}{\sigma^2}\right)} = \begin{cases} a_1 \cos\left(\frac{2\pi(x-x_m)}{b_1}\right) + a_2 \cos\left(\frac{2\pi(x-x_m)}{b_2}\right) & ; (x_m - c) \leq x \leq (x_m + c) \\ 0 & \text{else} \end{cases} \quad (3.10)$$

The transformation from a single Gaussian (with parameters of a , x_m and σ) to the sum of two piecewise cosine functions (with parameters of a_1 , a_2 , b_1 , b_2 , x_m , c , and C_{range}) can be represented as a linear transformation, as follow:

$$\begin{bmatrix} a_1 \\ a_2 \\ x_m \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 & 0 & 0 \\ A_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & B_{11} & B_{12} \\ 0 & 0 & B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} a \\ x_m \\ \sigma \\ 1 \end{bmatrix} \quad (3.11)$$

$$c = C_{range} \min(|b_1|, |b_2|)$$

Figure 3-6: Number of piecewise cosine functions used to represent a single Gaussian function versus the goodness of fit, R^2

Table 3-1: Selected transform pairs from a single Gaussian to a sum of two cosine terms

ID	Gaussian	Sum of two cosine functions			
	$e^{\left[-\left(\frac{x}{\sigma}\right)^2\right]}$	$a_1 \cos\left(\frac{2\pi x}{b_1}\right) + a_2 \cos\left(\frac{2\pi x}{b_2}\right)$			
	Value of σ	Value of			
		a_1	b_1	a_2	b_2
<i>gauss00</i>	0.2	0.7353	1.684	0.2440	0.548
<i>gauss01</i>	0.4	0.7367	3.376	0.2449	1.099
<i>gauss02</i>	0.6	0.7367	5.052	0.2440	1.644
<i>gauss03</i>	0.8	0.7360	6.744	0.2445	2.195
<i>gauss04</i>	1.0	0.7367	8.420	0.2440	2.740
<i>gauss05</i>	1.2	0.7362	10.11	0.2443	3.292
<i>gauss06</i>	1.4	0.7366	11.79	0.2440	3.837
<i>gauss07</i>	1.6	0.7363	13.48	0.2443	4.388
<i>gauss08</i>	1.8	0.7360	15.17	0.2444	4.939

To solve Equation (3.11), a selected transform pairs as shown in Table 3-1 can be used, in which a series of Gaussian functions with different σ with corresponding parameters of the two cosine functions (fitted by the trust region method) are tabulated. The relationship can also be visualized as in Figure 3-7 and it confirms that the linear model (shown in Equation (3.11)) proposed earlier is good for our application. The unknown 5x4 matrix in Equation (3.11) is solved by linear regression, and C_{range} can be solved by minimizing the fitting error.

Figure 3-7: Graphical illustration of transform pairs listed in Table 3-1.

Solving the Equation (3.11), we get the following equations:

$$\begin{cases} a_1 = 0.7888a \\ a_2 = 0.2049a \\ x_m = x_m \\ b_1 = 7.6777\sigma - 0.0078 \\ b_2 = -2.4769\sigma + 0.0024 \\ c = 0.852 \min(|b_1|, |b_2|) \end{cases} \quad (3.12)$$

With Equation (3.10) and Equation (3.11), the challenge of modeling a Gaussian function as a sum of piecewise cosine functions can be readily is solved. In principle, the correlogram can now be expressed in terms of a finite number of elementary functions and computed efficiently. The next section will present the derivation from Equation (3.3) to elementary form.

3.2.4. Derivation to elementary form

As Equation (3.3) is a generalized model which incorporates both the geometric and spectral effects on the coherence scanning interferometry, no further simplification is introduced. For simplicity and tidiness of derivation, only one piecewise cosine term is used to represent the intensity spectrum, $F(k)$. First, Equation (3.3) is integrated with respect to θ , followed by applying the proposed modeling, and the elementary form can be expressed as

$$\begin{aligned}
 I_{\text{interference}}(z) &= C \int_{\text{bandwidth}} k^2 F(k) \int_0^{\theta_0} \cos[2k(z - z_0) \cos \theta + \phi] \sin \theta \cos \theta d\theta dk \\
 &= C \int_{\text{bandwidth}} k^2 F(k) \left[\frac{(2kz \cos \theta - 2kz_0 \cos \theta) \sin(2kz \cos \theta - 2kz_0 \cos \theta + \phi) + \cos(2kz \cos \theta - 2kz_0 \cos \theta + \phi)}{4k^2 z^2 - 8k^2 z_0 z + 4k^2 z_0^2} \right]_{\theta=0}^{\theta_0} dk \\
 &= C \int_{k_{\text{ll}}}^{k_{\text{ul}}} \frac{a \cos\left(2\pi \frac{k - k_m}{b}\right)}{8z_0 z - 4z^2 - 4z_0^2} [(2kz \cos \theta_0 - 2kz_0 \cos \theta_0) \sin(2kz \cos \theta_0 - 2kz_0 \cos \theta_0 + \phi) + \cos(2kz \cos \theta_0 - 2kz_0 \cos \theta_0 + \phi) - \\
 &\quad (2kz - 2kz_0) \sin(2kz - 2kz_0 + \phi) - \cos(2kz - 2kz_0 + \phi)] dk \\
 &= C \int_{k_{\text{ll}}}^{k_{\text{ul}}} \frac{a \cos\left(2\pi \frac{k - k_m}{b}\right)}{8z_0 z - 4z^2 - 4z_0^2} [2k \cos \theta_0 (z - z_0) \sin(2k \cos \theta_0 (z - z_0) + \phi) + \\
 &\quad \cos(2k \cos \theta_0 (z - z_0) + \phi) - 2k(z - z_0) \sin(2k(z - z_0) + \phi) - \cos(2k(z - z_0) + \phi)] dk \quad (3.13) \\
 &= C [(g(z, z_0, \theta_0, k_{\text{ul}}, b, \phi) - g(z, z_0, 0, k_{\text{ul}}, b, \phi)) - \\
 &\quad - (g(z, z_0, \theta_0, k_{\text{ll}}, b, \phi) - g(z, z_0, 0, k_{\text{ll}}, b, \phi))]
 \end{aligned}$$

where $g(z, z_0, \theta, k, b, \phi)$ is defined as

$$\begin{aligned} \frac{D}{4} & \left(bU \left(\left(2(\pi - bU)k \cos\left(\frac{(b\phi + 2k_m\pi - 2\pi k + 2bUk)}{b}\right) + b \sin\left(\frac{b\phi + 2k_m\pi - 2\pi k + 2bUk}{b}\right) \right) \right) / (\pi - bU)^2 + \right. \\ & \left(-2(\pi + bU)k \cos\left(\frac{b\phi - 2k_m\pi + 2\pi k + 2bUk}{b}\right) + b \sin\left(\frac{b\phi - 2k_m\pi + 2\pi k + 2bUk}{b}\right) \right) / (\pi + bU)^2 + \\ & \left. \left(b \left(\sin\left(\frac{b\phi + 2k_m\pi - 2\pi k + 2bUk}{b}\right) / (-\pi + bU) + \sin\left(\frac{b\phi - 2k_m\pi + 2\pi k + 2bUk}{b}\right) / (\pi + bU) \right) \right) \right) \end{aligned}$$

with

$$D = 1 / (8zz_0 - 4z^2 - 4z_0^2), \text{ and } U = (z - z_0) \cos \theta$$

3.2.5. Computational efficiency

Figure 3-8: Comparison of computational time required for simulating a single value of correlogram

To measure the performance of the proposed model, we shall evaluate the computational time, t_s , for simulating a single value in correlogram, i.e. the value of $I(z)$ as $z=z_1$.

The testing platform is a system with Pentium 4 3.2GHz processor and 4GB RAM, and the computational software is MATLAB 6.5 under Microsoft Windows XP 32-bit. The same

platform is used to simulate a single value of correlogram (i.e. $I(z)$ as $z=z_1$) and is timed using *tic* and *toc* command in MATLAB.

The time required for simulating a single value of correlogram by numerical integration of Equation (3.3) and the proposed method are 133.31second and 0.000519second respectively. Clearly, the proposed model is 256800 times faster than the numerical integration approach, and it is also 1284 times faster than Groot et al.'s approach [94].

3.2.6. Verification

For the purpose of verification, the configuration of a solid flat surface as specimen, the spectrum of phosphor-based LED, LXHL- by LumiLEDs LW6C (as shown in Figure 3-9) and 20x Michelson interferometry objective lens by Nikon (Numerical aperture = 0.40) is used. For simplicity, it is assumed that the reflectivities of the reference and the specimen surface are the same.

Figure 3-9: Intensity spectrum of phosphor-based LED, LXHL-LW6C by LumiLEDs

To apply the proposed model, the intensity spectrum of the phosphor-based LED (as shown in Figure 3-9) is first fitted with a sum of three Gaussian functions by the trust-region method, followed by transforming each Gaussian function into a sum of two piecewise cosine functions according to Equation (3.8).

To evaluate the goodness of fit, Figure 3-10 shows the overlaps of the actual spectrum with the Gaussian fitting and the proposed model. When compared to the actual spectrum, the R^2 values of the proposed model and the Gaussian fitting are 0.9968 and 0.9972, respectively. Both the R^2 value and a visual inspection of Figure 3-10 show that the proposed model fits the Gaussian function well.

Figure 3-10: Comparison of the goodness of fit for the spectrum of phosphor-based LED represented by Gaussian functions and the proposed model.

3.2.6.a) Simulation verification

With the intensity spectrum represented by the proposed model, the interference signal contributed by each cosine term is computed by Equation (3.12). Lastly, the resultant interference signal is the sum of the interference signal contributed by all three pairs of cosine terms.

Figure 3-11 shows a comparison of the correlograms simulated by direct numerical evaluation on the generalized model and the proposed model. When compared to a direct numerical evaluation of the generalized model, the R^2 value for the correlogram simulated by the proposed model is 0.9974.

Figure 3-11: Graphical comparison of the correlogram simulated by direct numerical evaluation on the generalized model and the proposed model.

Both the R^2 value and a visual inspection of Figure 3-11 support that the proposed model can accurately represent, and hence replace, the computationally intensive direct numerical evaluation approach.

3.2.6.b) Experimental verification

Figure 3-12 compares the correlograms simulated by the proposed model and direct numerical evaluation on the generalized model to the correlogram obtained by physical experiment. Clearly, the proposed model agrees well with the experimental data and the correlogram simulated by the existing computationally intensive approach. The proposed model has successfully yielded the unique envelope of the correlogram. Previously, this

unique envelope function of the correlogram can only be obtained by computationally intensive approach.

Figure 3-12: Comparison of the correlograms by experiment, the proposed model and existing computationally intensive model.

However, there are some disagreement between the simulation models (both the computationally intensive model and the proposed model) and the experiment data at high frequency. Figure 3-13 highlights this disagreement in the frequency domain of the correlograms between the proposed model and the experiment data. As shown in Figure 3-13, both correlograms have two main frequency components of $3.5\mu\text{m}^{-1}$ and $4.5\mu\text{m}^{-1}$ but with a different weightage. As shown in Figure 3-9, the phosphor-based LED has more blue light than yellow light, and the correlogram simulated by the proposed model follows this relationship. However the experimental data shown in Figure 3-13 shows that there is more yellow light than blue light.

Since the effective spectrum of the light source is subject to spectral sensitivity of the imaging sensor, the spectral response of the imaging sensor can change the weightage of yellow and

blue lights in the correlogram. So, the effects of spectral sensitivity of imaging sensor will be investigated in the next section.

Figure 3-13: Comparison of the correlograms by experiment data and the proposed model in frequency domain. The magnitudes of the two correlograms are normalized for comparison purpose.

Other than spectral sensitivity of the imaging sensor, this disagreement could also be the result of systematic noise in the experimental setup, discrepancy between the actual and manufacturer's claimed spectrum of the light source, and/or the limitations of the physics model.

However, by repeating the experiments with different sets of piezoelectric scanning mechanism and LED, the possibilities of systematic noise in the experimental setup and discrepancy between the actual and the manufacturer's claimed spectrum of the light source were ruled out.

On the validity of the physical model (Equation 3.3), we note that this model was initiated by Kino and Chim [35] and Sheppard and Larkin [93], and it has been widely used in Groot and

Colonna Lega [94] and Kim and Kim [102]. Based on this model, a simplified model (assuming there is only a single Gaussian peak in the light spectrum and the effects of numerical aperture is negligible) is introduced by Larkin [34] and then used in Li et al. [40], Gurov et al. [43] Larkin [34], and Kim et al [50], 's works. As such, this model is widely accepted for interferometry related work, but this model has never been applied to phosphor-based LED (or any other light source with more than one peaks in its spectrum). However, an investigation into the limitation of this physical model is beyond the scope of this research.

In the next section, we shall investigate the effects of spectral sensitivity of the imaging sensor (which is a monochrome CCD camera in our research) and the effects of numerical aperture of the interferometry objective lens.

3.3. Discussion

3.3.1. Effects of spectral sensitivity of the imaging sensor

In earlier simulations, it was assumed that the imaging sensor responds to all the wavelengths uniformly, i.e. uniform spectral sensitivity. This assumption is hardly true in practice, but it is a common practice to neglect the effects of spectral sensitivity. This is because most white lights such as halogen and incandescent light have a very broad spectrum and the spectral sensitivity of light receiving sensor acts like a band pass filter which makes the effective spectrum of the light source more Gaussian like. Hence, researchers tend to associate the effective spectrum with the spectrum of the light source, and neglect the spectral sensitivity of the sensor. Figure 3-14 compares the spectral responses of some scientific CCD sensors. The variations of the spectral responses are obvious.

Figure 3-14: Graphical comparison of spectral responses of some scientific CCD sensors: SiTe ST001 back illuminated, Sony ICX 285, Kodak 1401e and Sony ICX 061 [103].

In the experimental verification, the imaging sensor used is UI2220M by IDS uEYE. It is a 8-bit monochrome CCD camera with a sensor size of $\frac{1}{2}$ " and a resolution of 768x576. As shown in Figure 3-15, this camera model is for visible wavelength in the range of 400nm to 800nm.

Figure 3-15: Spectral response of IDS uEYE UI2220M CCD camera [104].

With the spectral sensitivity of the imaging sensor known, the nominal and effective spectrums of the phosphor-based LED are shown in Figure 3-16. It is noticeable that the

effective spectrum has more yellow component than the nominal one, while the location of the peaks in the intensity spectrum remains unchanged. Although there is an increase in the weightage of yellow light, the blue light is still much stronger than yellow light.

So, for this particular phosphor-based white LED and the camera model, the spectral sensitivity of the camera has significantly affected the weightages of the blue and yellow light components, but the mean wavelengths of blue and yellow lights remain unchanged.

Figure 3-16: Comparison of the nominal spectrum and the effective spectrum of the phosphor-based LED with IDS uEYE UI2220M CCD camera.

Figure 3-17 shows the correlograms simulated based on the effective (IDS uEYE UI2220M CCD camera) and nominal spectra of the phosphor-based LED. The difference between these two correlograms is minor: for high frequency components, the signal amplitude of the correlogram based on the effective spectrum is slightly smaller than the nominal one; for fringe contrast function (envelope function of the correlogram), there is virtually no change. For an objective measure of how these two correlograms are similar, the objective measure, R^2 , is used and the value of 0.9877 suggests the difference is small. So the disagreement in experimental verification (as shown in Figure 3-12) cannot be explained by the effects of

spectral sensitivity of imaging sensor, and it is believed that the disagreement is due to limitation of the generalized model (Equation (3.3)).

Figure 3-17: Comparison of the correlograms simulated based on the effective (IDS uEYE UI2220M CCD camera) and nominal spectra of the phosphor-based LED.

3.3.2. Effects of numerical aperture of objective lens

The objective lens is characterized by magnification and numerical aperture, and it is a common practice to change the objective lens to adjust the lateral resolution of measurement. The magnification determines the lateral measurement resolution, while the numerical aperture determines the amount of light entering the objective lens. However, the effects of changing the objective lens (which corresponds to numerical aperture) to correlogram of coherence scanning interferometry are not addressed. In this section, the presented signal model is applied to study the effects of numerical aperture of objective. The following commercially available interferometric objectives by Nikon (as shown in Table 3-2) are included in this study.

Table 3-2 : Summary of interferometric objective (by Nikon) used for evaluation

Magnification	2.5X	5X	10X	20X	50X
Numerical Aperture	0.075	0.13	0.3	0.4	0.55
Working Distance (mm)	10.3	9.3	7.4	4.7	3.4
Focal length (mm)	80	40	19.8	9.96	4
Depth of Focus(μm)	± 24.3	± 8.1	± 1.5	± 0.85	± 0.45

Depending on applications, the interference pattern can be generated in a number of configurations such as Michelson, Mirau, Linnik, Mach-Zehnder, etc. Figure 3-18 (a) and Figure 3-18 (b) show the schematic diagram of Michelson and Mirau interferometric configurations respectively. For the Michelson interferometer, a beam splitter is placed before an objective lens, then the light beam (from the source) is split into the reference beam and the sample beam. The angle between these two beams is 90° . After these two beams are reflected and meet, interference occurs. For the Mirau interferometer, a reference mirror is at the center of objective lens. A half mirror (which serves as a beam splitter) is placed between the specimen and the reference mirror to generate the interference pattern.

In general, the Michelson configuration has a lower numerical aperture value and is usually used for low magnification while the Mirau configuration can achieve a higher numerical aperture and is used for high magnification objective.

Figure 3-18: Graphical illustration of (a) Michelson interferometric objective and (b) Mirau interferometric objective

Figure 3-19: Picture of Nikon interferometry objective: (a) 2.5x objective in Michelson configuration (b) 20x objective in Mirau configuration

For the commercially available interferometry objective lens by Nikon, both the 2.5x and 5.0x objectives lens adopted the Michelson configuration and the 10x and above objectives adopted the Mirau configuration. Figure 3-19 shows Nikon interferometry objectives with 2.5x magnification (Michelson configuration) and 20x magnification (Mirau configuration). The structural difference between the Michelson and the Mirau configurations is pretty obvious.

However, the difference in the interference configuration does not have influence on the geometric effects due to numerical aperture. Michelson-based and Mirau-based interferometry objectives with identical numerical aperture value are considered to be identical in terms of geometric effects.

We investigated the effects of numerical aperture on the coherence scanning interferometry using conventional light source where its intensity spectrum can be expressed as a single Gaussian function. Figure 3-20 shows the correlograms based on Nikon 5x, 10x and 20x interferometry objectives, the difference is hardly visible due to the presence of high frequency interference pattern. So a low pass filter is applied to show the fringe contrast function (envelope of correlogram), and the result is shown in Figure 3-21. The spread of the fringe contrast function is reduced as the numerical aperture of objective is increased. This is because a larger numerical aperture value will mean a greater amount of light that enters the objective. However these lights are not identical, their optical paths are all slightly different from each other. As a result, the interference pattern occurs within a smaller region in the vertical direction.

The disadvantage of having the interference occurring within a small window is that a user would have to spend more effort in aligning the sample for measurement. However, as the interference occurs within a small window, the global maximum of the fringe contrast function becomes more significant and it might lead to performance improvement in the measurement result.

Figure 3-20: Simulated correlograms based on numerical apertures of 0.13, 0.3 and 0.4.

Figure 3-21: Simulated fringe contrast functions (envelope of correlogram) based on 5x, 10x and 20x objectives.

Other than potential performance improvement, a high numerical aperture may increase the measurability of thin film/coating thickness. For example, the correlogram for a $0.2\mu\text{m}$ -thick film with refractive index of 2 deposited on solid reflective substrate would have two set of interference patterns (contributed by the interfaces between (1) air and film, and (2) film and substrate). These two set of interference patterns would be separated by $2 \times 0.2\mu\text{m}$. If the spread of each interference pattern is too big, these two interference patterns would not be separable.

We then investigated the effects of numerical aperture on the coherence scanning interferometry using phosphor-based white LED. The simulation configuration is as follows: Solid flat surface is used as specimen, the same phosphor-based LED (LXHL- by LumiLEDs LW6C) is used as the light source, and IDS uEYE UI2220M camera as the imaging sensor. Figure 3-22 shows the correlograms based on Nikon 5x, 10x and 20x interferometry objectives, Figure 3-23 shows the corresponding fringe contrast functions.

Figure 3-22: Simulated correlograms based on numerical apertures of 0.13, 0.3 and 0.4.

Figure 3-23: Simulated fringe contrast functions (envelope of correlogram) based on 5x, 10x and 20x objectives.

As shown in Figure 3-23, the spread of the fringe contrast function is reduced as the numerical aperture of the objective increases. However, unlike the conventional light source, the sharpness of the global peak is not affected. This result suggests that the spectrum of phosphor-based white LED can have a significant effect on the correlogram of coherence scanning interferometry.

3.3.3. Plausible cause for the disagreement at high frequency

On a closer examination, the disagreement between the proposed model and the experimental data (as shown in Figure 3-13) is that the high frequency component in the experimental data has a longer wavelength as compared to the simulation results, and this is similar to the effects of numerical aperture shown in Figure 3-20 and Figure 3-22. These lead us to assume that the geometric effects contributed by the objective lens are the cause of the disagreement. With reference to the relevant prior works in phase shifting interferometry [105]–[107], the high frequency component with which the disagreement occurs is known as the fringe spacing and it can be expressed as follows:

$$\text{Fringe Spacing} = \frac{\lambda}{2} \chi \quad (3.14)$$

where

χ is the obliquity factor, and

λ is the mean wavelength of the light source with respect to the detector.

While the obliquity factor is always one for normal incidence illumination interference microscope (for example, the Fizeau configuration), its value for oblique incidence illumination interference microscope (for example, Mirau, Michelson, and Linnik

configurations) is always greater than one. The actual value is a function of the numerical aperture of the objective lens and the angle between the test surface and the reference mirror in the objective lens. Figure 3-24 illustrates the difference between the normal and the oblique incidence illumination microscopes, and the objective lens selected in our experimental and simulation verifications is Mirau-based.

Figure 3-24: (a) Fringe spacing in normal incidence illumination interference microscope (Fizeau) = $\lambda/2$ (b) Fringe spacing in oblique incidence illumination interference microscope (Mirau) = $\lambda/2 \times obliquity_factor$

Based on these two factors and Schmit et al. [108]'s work, which demonstrated that the fringe spacing of coherence scanning interferometry varies with the tilt of the object, we believe that the disagreement is caused by the geometric factor contributed by the tilt between the test surface and the Mirau objective lens.

Figure 3-25: Graphical illustration of the geometric effects due to the object tilt and the terms in the revised generalized model

Theoretically, the generalized model is capable of handling the geometric effects due to the numerical aperture of the objective lens and the object tilt. The reason why the simulation (both the computationally intensive model and the proposed model) does not agree with the experiment data in terms of fringe spacing is due to a simplification which assumes that there is no object tilt. Figure 3-25 illustrates how the amount of light collected by objectives lens varies with the tilting, and the revised generalized model without the assumption that $\theta_2 - \theta_1 = 2\theta_0$ is given below:

$$I_{\text{interference}}(z) = C_2 \int_{\text{bandwidth}} \int_{\theta_1}^{\theta_2} \{k^2 \times \cos[2k(z - z_0) \cos \theta + \phi] \times \sin \theta \cos \theta d\theta\} F(k) dk \quad (3.15)$$

where

z is the defocus position (related to optical path difference),

z_0 is related to the profile of the sample surface,

C_2 is a constant,

k is the angular wavenumber ($k=2\pi/\lambda$),

θ_1, θ_2 are the angles of reflected ray,

$F(k)$ is the effective intensity spectrum of the light source, and

ϕ is the phase offset.

Based on our data and prior works, we have identified the object tilt as the plausible cause for the disagreement (between the proposed model and the experiment data) and proposed a revision as future work for the generalized model to address the disagreement.

3.4. Conclusion

We have presented a coherence scanning interferometric model by incorporating both the geometric and spectral effects on the correlogram. By modeling the intensity spectrum of the light source as a sum of piecewise cosine functions, the resultant signal model can be represented by an elementary function which not only resolves the computational load problem of prior models, it also facilitates a detail study on the spectral effects of phosphor-based white LED in coherence scanning interferometry. In terms of computational load, the new model is 256800 times faster than prior model, and the model's accuracy has been validated by comparison with the computationally intensive counterpart and the experimental data. The main constraint of this work is the assumption that the intensity spectrum of the light source can be represented by a sum of Gaussian functions.

With the new model, the effects of spectral sensitivity of the imaging sensor and the numerical aperture of objective on the correlogram of the coherence scanning interferometry have been investigated in detail. The results suggest that the spectrum of phosphor-based white LED can have a significant effect on the correlogram of the coherence scanning interferometry.

4. Phosphor-based White LED in Coherence Scanning Interferometry

In this chapter, we shall investigate the spectral factor in coherence scanning interferometry which can no longer be neglected as the previous chapter has suggested the spectrum of phosphor-based white LED can have a significant effect on the correlogram of the coherence scanning interferometry. Phosphor-based white LED is the most commonly used high power white LED [91], and the spectral properties of the light source has two peaks in its spectrum. First, we will identify the spectral features of typical phosphor-based white LED and investigate its effects in coherence scanning interferometry [109]. Then, we will model the spectral variation of phosphor-based white LED and investigate its effects in coherence scanning interferometry [110]. Lastly, we shall present a theoretical explanation for the spectral effects due to the use of phosphor-based white LED [111].

4.1. Motivation

Apart from the light source, the basic structure of coherence scanning interferometry has not changed much since its invention. Due to its advantages of longer lifetime, low heat dissipation and compactness, high power phosphor-based white LED is fast replacing the conventional light source [112] in coherence scanning interferometry. Conventional low coherence light source is defined as a light source in which the effective spectrum has only one peak with a Gaussian distribution.

Phosphor-based white LED consists of a single color (normally blue) LED and phosphor of different color (normally yellow) to produce the white light [113], [114]. So, there are two peaks in its spectrum.

In most prior works [34], [40], [41], [43], one assumed the use of conventional light source with Gaussian spectrum and so the spectral factors in coherence scanning interferometry are neglected. Here, the spectral feature of phosphor-based white LED and its effects in coherence scanning interferometry will be studied in detail.

4.2. Effects of phosphor-based white LED in coherence scanning interferometry

4.2.1. Spectral property of phosphor-based white LED

A white light is defined as the color that is perceived when three types of color sensitive cone cells in the human eye are subjected to equal amount/intensity of stimulation. Figure 4-1 illustrates the additive color mixing of visible light. For example, the mixing of red, green and blue lights produces white light.

Figure 4-1: Additive color mixing showing combinations to generate white light.

However the color, white, depends on perception, and there is an infinite number of combinations of the wavelengths of light that can produce a white light. Conventional white light source such as sun light and incandescence light have a very broad spectrum which may

range from ultra violet to far infra red region. However, due to the spectral sensitivity of the light receiving sensor, the effective spectrum of conventional light source has only one peak with a Gaussian distribution. An example of the spectrum is shown in Figure 4-2.

Unlike a conventional white light source, the modern light source does not have a very broad spectrum and there can be multiple peaks in its spectrum. An example of modern light source is the white LED which is a solid state lighting that promises longer lifetime, low heat dissipation and compactness, and it is commercially available. It generates white light by either (1) using individual LED that emit red, green and blue light or (2) using blue LED and phosphor material which converts light (from blue LED) to yellow light. A white LED using the second approach is known as phosphor-based white LED, and it is the most commonly used high power LED.

Figure 4-2: Intensity spectra of conventional light source and phosphor-based white LED.

Phosphor-based white LED consists of a monochromatic blue LED and yellow phosphor to produce white light, so there are two peaks in its spectrum [115]. Figure 4-2 shows the intensity spectrum of a phosphor-based LED, LXHL-LW6C by LumiLEDs. There are two peaks,

contributed respectively by a blue LED and yellow light (emission by phosphor material), in its spectrum. Such spectrum is typical for phosphor-based white LED. Normally, the mean wavelengths of the blue and yellow lights do not change, and the optical powers of these two lights are adjusted for different correlated color temperature (CCT). Figure 4-3 shows the intensity spectra of three commercially available phosphor-based white LED with different correlated color temperature (CCT).

Figure 4-3: Intensity spectra of commercially available phosphor-based white LED with different correlated color temperature (CCT): (a) warm white (b) nature white (c) cool white.

In summary, the spectral feature of phosphor-based white LED is that there are two peaks in its intensity spectrum: one at the blue light wavelength and the other at the yellow light wavelength.

4.2.2. Spectral factor in coherence scanning interferometry

As discussed in Chapter 3, the correlogram is affected by the following factors: (1) the optical path difference, (2) spectrum of the light source, (3) numerical aperture of the objective lens, (4) optical transfer function of the imaging system, (5) reflectivities of the reference and sample surfaces, and (6) phase change at the reference and sample surfaces. The correlation term of the intensity response (also known as correlogram) can be formulated as

$$I_{\text{interference}}(z) = C_1 \int_{\text{bandwidth}} \int_0^{\theta_0} \{k^2 \times \cos[2k(z - z_0) \cos \theta + \phi] \times \sin \theta \cos \theta d\theta\} F(k) dk d\theta \quad (4.1)$$

where

z is the defocus position (related to the optical path difference),

z_0 is related to the profile of the sample surface,

C_1 is a constant,

k is the angular wavenumber ($k=2\pi/\lambda$),

$\sin \theta_0$ is the numerical aperture of the objective lens (assuming $n=1$),

$F(k)$ is the effective intensity spectrum of the light source, and

ϕ is the phase offset.

In general, the spectral bandwidth of the light source will affect the spread of the fringe contrast function, while the wavelength of the light source affects the high frequency components of the correlogram. Therefore, despite the spectral variation in a conventional light source, the correlogram always has only one high frequency component and the fringe contrast function (the envelope of correlogram) can be reasonably modeled as single Gaussian function. Figure 4-4 shows the effective intensity spectrum and the corresponding correlogram of a conventional light source.

Figure 4-4: (a) Effective spectrum of a conventional light source (b) Correlogram based on spectrum of Figure 4-4 (a)

For a light source with multiple peaks in its spectrum, there are multiple high frequency components in its corresponding intensity response. As interference also occurs among these high frequency components, the fringe contrast function may not be modeled as a single Gaussian function.

4.2.3. Effects on phosphor-based white LED on correlogram

Figure 4-5: (a) Spectrum of a phosphor-based LED, LXHL-LW6C by LumiLEDs; (b) Simulated correlogram based on spectrum of (a)

Based on the intensity spectrum of a phosphor-based LED (as shown in Figure 4-5(a)) and a numerical aperture of 0.4, the corresponding intensity is simulated and shown in Figure 4-5(b). The distinctive feature highlighted in Figure 4-5(b) is the result of having two peaks in

the spectrum of phosphor-based white LED, and it is consistent with the intensity response obtained by experiment (shown in Figure 4-6).

Figure 4-6: Experiment data: Intensity response based on light source of phosphor-based white LED (LXHL-LW6C by LumiLEDs).

In summary, there are significant spectral effects of phosphor-based white LED on the correlogram of coherence scanning interferometry.. Due to the use of phosphor-based white LED, there is a discontinuity in the correlogram (highlighted in Figure 4-5(b)), and the fringe contrast function can no longer be assumed as a single Gaussian function.

4.2.4. Effects of phosphor-based white LED on reconstructed height

In this section, the three reconstruction algorithms proposed by Li et al. [40], Ai and Novak [42], and Groot and Deck [45] will be selected to investigate the effects of white LED on the reconstruction results.

Among these three algorithms, only the Groot and Deck's approach [45] is a phase-based approach. The other two are based on the fringe contrast approach. Among these two methods, the major difference is that Li et al's method [40] assumes that the fringe contrast function is a Gaussian signal while the centroid approach [42] does not. Groot and Deck's method represents the white light as a combination of multiple single wavelength components, then applies the phase signal analysis method that is similar to phase shifting interferometry. The details of these construction algorithms have been reviewed in Chapter 2.

Our simulation is formulated to investigate the effects of white LED on the height reconstructed profile. Two sets of data are simulated: one based on a light source with Gaussian spectrum, and one based on a phosphor-based white LED (LXHL-LW6C by LumiLEDs). A line profile with a $1\mu\text{m}$ step height is used. The line profile consists of 256 surface points and each surface point has a corresponding intensity response. The sampling interval of the intensity response is 50nm, and each intensity response is corrupted by Gaussian white noise (with zero mean, and a variance of 0.05) to simulate sensor noise.

The sampling interval of the correlogram is set at 50nm for fairness of comparison. We note that the phase-based approach is more resilient to coarse sampling interval, while the fringe contrast approach is relatively more sensitive to influence by the choice of sampling interval.

With the three reconstruction algorithms mentioned earlier, the height profile is reconstructed and shown in Figure 4-7 and Figure 4-8. The repeatability (in terms of standard deviation) and the accuracy of reconstructed profiles are also analyzed for an objective measure of performance.

Figure 4-7: Reconstructed 1 μ m step height using (a) Gaussian Fitting by Li et al. [40] (b) Centroid approach by Ai and Novak [42] (c) Frequency domain analysis by Groot and Deck [45] for phosphor-based white LED

Figure 4-8: Reconstructed 1 μ m step height using (a) Gaussian Fitting by Li et al. [40] (b) Centroid approach by Ai and Novak [42] (c) Frequency domain analysis by Groot and Deck [45] for a light source of Gaussian spectrum

Figure 4-9: Performance comparison of different algorithms with different light sources: (a) standard deviation of a perfectly flat surface (ideal value is zero) (b) height measurement (ideal value is 1 μ m).

For phosphor-based white LED, Figure 4-9(a) shows that the use of phosphor-based LED decreases the repeatability of measurement differently. We note that the Gaussian fitting

method by Li et al. [40] suffers the most, followed by the frequency domain analysis approach by Groot and Deck [45], and then the centroid approach by Ai and Novak [42].

The centroid approach by Ai and Novak [42] does not make any assumption on the fringe contrast function, so a change in the fringe contrast function has relatively little effect on its reconstruction.

The Gaussian fitting method of Li et al. [40] performs the worst in repeatability because the algorithm assumes that the fringe contrast function is a Gaussian function. Since the assumption on the fringe contrast function is not valid, the fitting process is unable to produce a good result.

Although the frequency domain analysis by Groot and Deck [45] is performed in the frequency domain, a change in the fringe contrast function does affect the amount and the quality of information selected for frequency domain analysis.

4.2.5. Modification for phosphor-based white LED

With the use of phosphor-based LED, the usual assumption adopted by the reconstruction algorithm no longer hold. It is proposed that a constraint on the input to existing reconstruction algorithm be imposed to make sure that the required assumptions are valid. We will determine how the constraint (on the data selected for reconstruction) should be applied to meet the assumptions required by the Gaussian fitting and the frequency domain analysis approaches.

For the Gaussian fitting approach, the envelope of correlogram between two valleys (distinctive features highlighted in Figure 4-5(b) and Figure 4-6) can be reasonably modeled as a single Gaussian, setting the constraint of selecting only these data for single Gaussian fitting would meet the assumption of the original Gaussian fitting approach proposed by

Mingzhou et al. Figure 4-10 illustrates how well the fringe contrast function is fitted to a single Gaussian with and without data selection. So, instead of fitting the whole correlogram to a Gaussian, the Gaussian fitting approach is modified such that it fits a subset of the correlogram to a Gaussian.

Figure 4-10: Fitting the fringe contrast function with a single Gaussian by selecting a subset of data (such as $-0.5 \leq \text{defocus position} \leq 0.5$).

Based on the work on frequency domain analysis by Groot and Deck [45], there should be only one peak in the spatial frequency range of $20 \text{ rad}/\mu\text{m}$ to $30 \text{ rad}/\mu\text{m}$, which corresponds to wavelengths of 628nm and 419nm. However, for phosphor-based LED, there are two peaks in the frequency ranges of interest (as shown in Figure 4-11(a)), so a constraint is applied on the input for frequency domain analysis such that it would not have two peaks in the spatial frequency domain analysis. Figure 4-11(b) shows that the assumption of frequency domain analysis is met by reducing the amount of data for the frequency domain analysis, since there is only one distinct peak in the spatial frequency domain.

Figure 4-11: Effects of data selection in spatial frequency domain (a) 80 data (b) 20 data from the correlogram.

Figure 4-12: Simulation verification: Comparing standard deviation of measuring perfectly flat surface reproduced by proposed modification and original algorithm.

Figure 4-12 shows that the proposed modification to the Gaussian fitting approach and the frequency domain analysis approach have significantly improved the repeatability of measurement. Indeed, the standard deviation for measuring a flat surface is improved from 0.0723 μm to 0.0215 μm for the Gaussian fitting approach and from 0.1735 μm to 0.0612 μm for the frequency domain analysis approach.

For the experimental verification, the configuration used in the earlier simulation in Section 4.2.4 is adopted but the test sample is changed to a $10\mu\text{m} \pm 80\text{nm}$ standard step height. 256

correlograms are collected at a sampling interval of 50nm. The repeatability of measuring an optically flat surface is used to quantify the performance of modified Gaussian fitting, modified frequency domain analysis, original Gaussian fitting and original frequency domain analysis approaches.

Figure 4-13: Experimental verification of the proposed modification for phosphor-based LED: Comparison of the standard deviation in measuring an optically flat surface reconstructed with and without proposed modification.

As shown in Figure 4-13, the repeatability of the modified reconstruction algorithms have been improved from 0.090um to 0.021um for the Gaussian fitting approach and from 0.050um to 0.025um for the frequency domain analysis approach. This result agrees well with the simulation result, but it is against to the common norm that more data leads to a better result.

However, the result shows that for the surface reconstruction of coherence scanning interferometry, it is more important to satisfy the assumptions of the reconstruction algorithm rather than fitting as many data as possible. The modification of inputting a subset of correlograms improves the performance of the reconstruction algorithm for phosphor-based LED, and it is applicable to the reconstruction algorithms of both the fringe contrast approach and the phase-based approach.

4.2.6. Summary

We have shown that the use of phosphor-based white LED (which is the most commonly used high power white LED) in coherence scanning interferometry affects the repeatability and accuracy of coherence scanning interferometry, especially repeatability. With phosphor-based white LED, the fringe contrast function can no longer be modeled as single Gaussian function, while the effect on the reconstructed height profile varies depending on the assumption adopted in the reconstruction algorithm. However it has been demonstrated that by applying a constraint on the input to existing reconstruction algorithm, phosphor-based white LED can improve the performance of vertical scanning interferometer

4.3. Effects of spectral variation of phosphor-based white LED in coherence scanning interferometry

We had earlier shown that the spectrum of phosphor-based white LED can affect the coherence scanning interferometry. We shall now investigate the effects of spectral variation of phosphor-based white LED on the coherence scanning interferometry by: (1) modeling of the spectral variation of high power white LED which could be the result of either engineered variation to achieve particular correlated color temperature (CCT) or undesired variation due to the operating conditions such as degradation, pulsing frequency and duty cycle, and (2) investigating its effects on the coherence scanning interferometry in term of interference signal and reconstructed height result. By doing so, a guideline/reference is generated for reconstruction algorithm selection and spectral consideration in coherence scanning interferometry.

4.3.1. Motivation

First of all, no all white LED is created equally in terms of electrical and optical properties such as lifetime, reliability, intensity spectrum, etc. Some of these variations are introduced in a

controlled manner for specific application such as achieving the desired correlated color temperature (CCT), and some are uncontrolled variations due to manufacturing process, aging, overdriving, and etc.

Since the issue of spectral influence in coherence scanning interferometry is relatively unexplored and we have shown in section 4.2 that the intensity spectrum of phosphor-based white LED does affect the coherence scanning interferometry significantly. It is therefore of theoretical and practical interest to investigate the effects of spectral variation of phosphor-based white LED in coherence scanning interferometry.

4.3.2. Causes of spectral variation of white LED

In addition to the categorization in terms of optical and electrical powers, commercially available phosphor-based white LED is categorized by its correlated color temperature (CCT). Correlated color temperature is the temperature with which the perceived color of black body radiator can best approximate, but this is meaningful only if the light source is nearly white [116]. Figure 4-14 illustrates the correlated color temperature of some commonly used light source.

Figure 4-14: The correlated color temperature of some commonly used light sources

Figure 4-15: Example of spectral variation: Intensity spectra of three commercially available white LEDs with different correlated color temperature (CCT)

An example application of correlated color temperature (CCT) for interior illumination is the use of warmer white light (lower CCT) is used to promote relaxation while a cooler white light (higher CCT) is used in office to enhance concentration [117].

In general, the intensity spectrum of white LED will depend on the desired CCT. For example, Figure 4-15 shows the intensity spectra of three commercially available phosphor-based white LED with different CCT.

Other than the designed variation for achieving the desired CCT, the manufacturing process can also introduce some undesired variations to the intensity spectrum. The undesired spectral variation due to manufacturing process is of significant concern so much so that the US-based National Electrical Manufacturers Association (NEMA) has published SSL 3-2010 "High-Power White LED Binning for General Illumination" to manage [118], [119] and/or to address it.

Figure 4-16: Example of spectral variation of white LED: the intensity spectrum of non-phosphor white LED varies depending on the pulsing frequencies (image is extracted from Heikkinen et al. [61])

Figure 4-17: Example of spectral variation of white LED: the intensity spectrum varies depending on the duty cycle of pulsing (image is extracted from Hanhijarvi et al. [60])

Other than the designed and the undesired spectral variations mentioned above, the intensity spectrum of white LED can vary depending on its operating condition. For example, Figure 4-16 shows Heikkinen et al.'s finding [61] that when the LED is being pulsed at high frequency, the relative intensity of the blue and yellow light components changes, and Figure 4-17 shows Hanhijarvi et al.'s finding [60] that the center wavelength of blue light shifts by 7nm when the duty cycle in pulsing mode is changed from 1% to 10%.

As the intensity spectrum of the light source affects the contribution of individual wavelengths to the white light interference signal, the spectral variation of phosphor-based white LED will affect the correlogram, fringe contrast function, and the measurement performance of coherence scanning interferometry.

4.3.3. Modeling spectral variation of white LED

With reference to prior works [60], [61], [120]–[122], the spectral variation of phosphor-based white LED can be categorized into two groups: (1) spectral shift, and (2) variation in the ratio of blue and yellow lights in the spectral domain. As coherence scanning interferometry does not rely on the information of wavelength in the reconstruction, the spectral shift is considered to be less significant when compared to the variation in the intensity ratio of blue and yellow lights. As such, the variation in the intensity ratio of blue and yellow lights is defined as the primary focus of our investigation into the spectral variation.

To investigate the effects of spectral variation of white LED in coherence scanning interferometry, two assumptions are made:

Assumption 4.1. The intensity spectrum of white LED has two Gaussian functions and it can be modeled as follows:

$$f(k) = BYratio \times e^{-\left(\frac{k-k_{blue}}{\sigma_{blue}}\right)^2} + e^{-\left(\frac{k-k_{yellow}}{\sigma_{yellow}}\right)^2} \quad (4.2)$$

where

k is the angular wavenumber ($=2\pi/\lambda$),

k_{blue} indicates the peak angular wavenumber of blue light,

k_{yellow} indicates the peak angular wavenumber of yellow light,

σ_{blue} indicates the spread of blue light in the spectral domain, and

σ_{yellow} indicates the spread of yellow light in the spectral domain.

In general, the wavelengths of blue and yellow lights are 450 nm (13.96 rad/nm) and 570nm (11.02 rad/nm) respectively, and the spread of blue and yellow lights in spectral domain are 0.4941 rad/nm and 1.439 rad/nm. These values vary slightly among manufacturers/model.

Assumption 4.2. The spectral variation of white LED is the result of the variation in correlated color temperature (CCT) and/or degradation [86], [88], [120]. These variations are represented in terms of the blue to yellow ratio (*BYratio* in Equation (4.2)): When *BYratio* is approximately 1, the intensities of blue and yellow lights are approximately equal, and the white light is daylight white and the CCT is around 4,000K – 5,000K; when the yellow light is stronger than blue light (*BYratio* is less than 1), the white light become warmer and the CCT is around 2,000K - 4,000K; when the blue light is stronger than yellow light (*BYratio* is larger than 1), the white light become cooler and the CCT is around 5,500K - 10,000K.

Figure 4-18 validates these two assumptions by comparing three commercially available phosphor-based white LED with different CCT with their corresponding simulated counterparts. The simulated intensity spectra are close to the commercially available products.

Figure 4-18: Comparison of the intensity spectrum of simulated and commercially available phosphor-based white LED with different correlated color temperature of (a) warm white, BY ratio = 0.59 (b) daylight white, BY ratio = 1.14 (c) cool white, BY ratio = 1.6.

This model is applicable to both phosphor-based white LED (which is the commonly used high power LED) and phosphor-free white LED (do refer to section 4.5.2 for detail).

It should be noted that the proposed spectral variation model is also applicable to the spectral variation reported in optical degradation [121], [122] and stroboscopic coherence scanning interferometry [61].

4.3.4. Effects on correlogram

With assumptions (4.1) and (4.2), a collection of intensity spectra with increasing BY ratio from 0.5 to 1.7 (from warm white light to cool white light) at a step increment of 0.1 is selected for our study. For each BY ratio, the corresponding correlogram is simulated based on the computationally efficient signal model presented in Chapter 3. When compared to existing direct calculation approaches, the presented signal model is 256800 times faster and it reduces the computation time from weeks to hours.

For ease of visual comparison, Figure 4-19 shows the correlograms of three different BY ratios. It shows that the existence of distinctive features (which are highlighted in yellow) is insensitive to the correlated color temperature and the BY ratio of phosphor-based white LED.

Figure 4-19: Effects of changing the blue to yellow ratio (BY ratio) on correlogram (Numerical Aperture of objective is assumed to be 0.4).

To quantify the effects of CCT (in terms of changing BY ratio) on the correlogram, the correlogram is further processed by extracting the envelope of correlogram (also known as fringe contrast function), and then followed by applying the peak and valley detection algorithm. The feature extraction process is graphically illustrated in Figure 4-20. The distinctive feature (highlighted in Figure 4-19) is transformed into two features: the valley and the peak. The positions of the peak and valley (x_{peak} and x_{valley} in Figure 4-20) are measured with respect to the global peak of the fringe contrast function (which is 0 in Figure 4-20) while the amplitudes of the peak and valley (y_{peak} and y_{valley} in Figure 4-20) are measured with respect to the constant signal (which is 0.5 in Figure 4-20). These four variables are used to quantify the effects of changing BY ratio on the correlogram, and the changes of these two features against the BY ratio are shown in Figure 4-21.

Figure 4-20: Illustration of the feature extraction process which transforms the distinctive features due to phosphor-based white LED into two features (highlighted in red).

Figure 4-21 (a) shows that the position of the peak is independent of the BY ratio, and the amplitude of the peak increases with the BY ratio. On the other hand, Figure 4-21 (b) shows that the position of the valley decreases rapidly with the BY ratio for BY ratio ≤ 1 and decreases at a much slower pace for BY ratio > 1 . The amplitude of the valley is at the minimum when BY ratio is approximately 1.2.

We noticed that among these four parameters, the amplitude of the peak varies to the change of the BY ratio significantly and it can be represented by a power function which can be represented as follows:

$$y = ax^b \quad (4.3)$$

where

a and b are constant real numbers.

Figure 4-21: The effects of changing the BY ratio on the correlogram: (a) the peak (b) the valley as indicated in Figure 4-20 (Numerical aperture of objective is assumed to be 0.4)

So we have extended our simulation to different numerical aperture settings (Numerical aperture of 0.13, 0.2, 0.3, and 0.4), and represented the amplitude of the peak (at each numerical aperture setting) as a power function. Figure 4-22 shows the result of the extended simulation with different numerical aperture settings, and Table 4-1 tabulates the result in which the amplitude of the peak is represented in power function form.

Figure 4-22: Graphical representation of the extended simulation result showing the relationship of the amplitude of the peak with different numerical aperture setting against the BY ratio

Table 4-1: Extended simulation in which the amplitude of the peak is represented in power function

Numerical Aperture (NA)	$y = ax^b$ where x is the BY ratio, and y is the amplitude of peak.	
	a	b
0.13	0.4044	0.3351
0.2	0.4013	0.3328
0.3	0.3889	0.3233
0.4	0.3474	0.3004

Figure 4-23 visualizes the relationship of the amplitude of the peak, the numerical aperture of objective and the BY ratio, and it shows that the relationship can be represented by 2nd order polynomial.

Figure 4-23: Transform pairs listed in Table 4-1

After solving the 2nd order polynomial functions with the transform pairs listed in Table 4-1 by least squares approach, a generalized model which predicts the amplitude of the peak against the BY ratio is derived as follows:

$$y = \left(-1.07 (NA)^2 + 0.36 (NA) + 0.37 \right) x^{(-0.55(NA)^2 + 0.16(NA)^2 + 0.32)} \quad (4.4)$$

where

y is the amplitude of the peak,

x is the BY ratio, and

NA is the numerical aperture of the objective lens.

Equation (4.4) can be used to trace change in the BY ratio of phosphor-based LED.

We have shown that the distinctive feature in the correlogram of coherence scanning interferometry exists regardless of the correlated color temperature (CCT) of phosphor-based white LED. However, the magnitude and position of the distinctive feature are affected, these changes can be used to trace in the BY ratio of phosphor-based white LED.

4.3.5. Effects on reconstructed height

In this section, the effects of correlated color temperature (CCT) of phosphor-based white LED on three reconstruction algorithms, namely the centroid approach by Ai and Novak [42], the Gaussian fitting approach by Li et al. [40] and the frequency domain analysis (FDA) by de Groot and Deck [45], are investigated. Among these three algorithms, Groot and Deck's approach is phase-based approach; the other two are fringe contrast based approach.

As fringe contrast based approach, Mingzhou et al's method makes the assumption that the fringe contrast function is a Gaussian signal, so the height is recovered by applying low pass filter to extract fringe contrast function, followed by Gaussian fitting to determine the maximum of the fringe contrast function. For Ai and Novak's centroid approach, it does not

assume anything on the correlogram and the height is recovered by finding the centroid of the correlogram. As a phase-based approach, Groot and Deck's FDA approach assumes that the white light can be represented by an equivalent wavelength and breaks it into multiple single wavelength components and applies phase signal analysis similar to phase shifting interferometry.

However as discussed in Section 4.2, it is more important to fulfill the assumption(s) adopted by each individual surface reconstruction algorithm than inputting more data to the reconstruction algorithm [109]: For phosphor-based white LED, a constraint is required to select the data between two valleys (as shown in Figure 4-19) for surface height reconstruction [109]. Based on the position of the valley (as shown in Figure 4-21(b)), each reconstruction algorithm is optimized for: (1) cool and daylight white phosphor-based white LED (BY ratio ≥ 1), and (2) warm phosphor-based white LED (BY ratio < 1).

For each BY ratio, a line profile of $1\mu\text{m}$ step height and 256 surface points is simulated. Each surface point has a corresponding correlogram which is sampled at 50nm and is corrupted by a Gaussian noise (with zero mean, and a standard deviation of 0.05). The corresponding correlogram is then reconstructed by (1) the centroid approach, (2) the Gaussian fitting approach optimized for cool and daylight white phosphor-based white LED, (3) the Gaussian fitting approach optimized for warm white phosphor-based white LED, (4) the FDA approach optimized for cool and daylight white phosphor-based white LED, and (5) FDA approach optimized for warm white phosphor-based white LED.

The measurement repeatability (in terms of the standard deviation, where standard deviation is inversely proportional to repeatability) is selected to quantify the effects on the reconstructed height profile quantitatively. Scatter plot and Pearson's r (also known as Pearson product moment correlation coefficient) are adopted to identify and quantify the type of relationship between the BY ratio and the measurement repeatability.

In the form of scatter plot, Figure 4-24 visualizes the relationship between the BY ratio and the measurement repeatability using the centroid approach, while Figure 4-25(a) and Figure 4-25(b) visualize the relationship between the BY ratio and the measurement repeatability using the other two reconstruction algorithms optimized for warm white LED (BY ratio < 1) and daylight/cool white LED (BY ratio ≥ 1), respectively. These figures suggest that the measurement repeatability of these reconstruction algorithms exhibits some trend with the BY ratio. However this observation is subjective based on the scatter plots. So Pearson's r (also known as Pearson product moment correlation coefficient) is adopted to assess these two observations.

Figure 4-24: Scatter plot showing the measurement repeatability of Ai and Novak's centroid approach and the BY ratio

Figure 4-25: Scatter plots showing the relationship between the BY ratio and the measurement repeatability of coherence scanning interferometric reconstruction algorithms optimized for (a) phosphor-based warm white LED (BY ratio < 1.0) (b) phosphor-based daylight and cool white LED (BY ratio ≥ 1.0)

Pearson's r quantifies the correlation (in linear relationship) between the measurement repeatability and the BY ratio, and It is widely in statistical analysis. Mathematically, it can be represented as follows:

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} \quad (4.5)$$

where

n is the number of observations,

x_i is the i^{th} observed data (1),

$\bar{x}$ is the average of the observed data (1),

y_i is the i^{th} observed data (2), and

$\bar{y}$ is the average of the observed data (2).

The Pearson's r ranges from -1 to +1, and it can be interpreted in the following manners: (1) a value of 1 means these two dataset are perfectly positively correlated (i.e. x increases as y increases), (2) a value of -1 means these two dataset are perfectly negatively correlated (i.e. x decreases as y increases), and (3) a value of 0 means these two dataset are not correlated. The guideline adopted in our study for the interpretation of the Pearson's r coefficient [123], [124] is tabulated in Table 4-2.

Table 4-2: Guideline for the interpretation of the Pearson's r coefficient

Value of Pearson's r	Correlation / Strength
-1.0 to -0.5	Negative / Strong
-0.5 to -0.3	Negative / Medium
-0.3 to -0.1	Negative / Small
-0.09 to 0.0	None
0.0 to 0.09	None
0.1 to 0.3	Positive / Small
0.3 to 0.5	Positive / Medium
0.5 to 1.0	Positive / Strong

Figure 4-26: Comparison of the value of Pearson's r of the BY ratio and the measurement repeatability (in terms of standard deviation of measurement) with different reconstruction algorithms

For each reconstruction algorithm, the value of Pearson's r between the measurement repeatability (in terms of the standard deviation of measurement) and the spectral variation (in terms of the BY ratio) is calculated and shown in Figure 4-26.

With reference to Figure 4-26 and Table 4-2, the following observations can be made:

- The standard deviation of height profile reconstructed by the centroid approach has the negative positive correlation to the BY ratio. Its measurement repeatability of the centroid approach improves as the BY ratio increases. Although the centroid approach does not make assumption on the shape of the fringe contrast function, it is a fringe contrast based approach which makes sure of the amplitude of the interference signal for height reconstruction. As the BY ratio increases, the amplitude of the interference signal increases with the amplitude of the peak (which is shown in Figure 4-21). Hence its performance improves (which corresponds to smaller standard deviation in measurement result) when the BY ratio increases.
- Among the other 4 reconstructed height profiles, the FDA approach optimized for the BY ratio < 1 (which corresponds to warm white phosphor-based white LED) is the least sensitive to the changing BY ratio, followed by the Gaussian fitting approach optimized for the BY ratio < 1 (warm white LED).

This result is against to the common norm that when the amount and percentage of the useful data are reduced, the reconstruction result would suffer. For the reconstruction algorithms under studied, the useful data is defined as the data point between the valleys (as shown in Figure 4-19). As the reconstruction algorithm is optimized for warm white LED, it accepts and processes more data than it should as the BY ratio increases. It is shown that the effects of changing the BY ratio on the correlogram and the fringe contrast function are not significant enough to affect the

quality of the reconstructed profile when the reconstruction algorithms are optimized for phosphor-based warm white LED.

When optimized for phosphor-based daylight and cool white LED (as shown in Figure 4-25 (b)), the performance of the reconstruction algorithm is poorer when compared to that optimized for phosphor-based warm white LED. This result is not surprising as the reconstruction algorithm tends to accept and process less data than it should be, so the reconstruction algorithm suffers and is sensitive to the changing BY ratio.

4.3.6. Experimental verification

As an experimental verification, three commercially available phosphor-based white LEDs (with intensity spectra as shown in Figure 4-18) and Nikon 20x Mirau interferometric objective with a numerical aperture of 0.4 are used to measure a certified $10 \pm 0.08 \mu\text{m}$ step height. The experimental result is compared with the simulation result in Figure 4-27 and Figure 4-28, and they show that the simulation result agrees well with experimental result.

Figure 4-27: Experimental verification on the measurement repeatability of Ai and Novak's centroid approach with phosphor-based white LED with different BY ratio

(b)
Figure 4-28: Experimental verification on the measurement repeatability of coherence scanning interferometric reconstruction algorithms optimized for (a) phosphor-based warm white LED (BY ratio < 1.0), and (b) phosphor-based daylight and cool white LED (BY ratio ≥ 1.0)

4.4. Spectral effects of dual wavelength low coherence light source

In this section, we shall investigate the spectral effects of dual wavelength low coherence light source (which is a generalized representation of modern/hybrid light sources such as phosphor-based white LED) in coherence scanning interferometry. We shall propose a theoretical explanation of the distinctive feature and demonstrate that the distinctive feature (and the fringe contrast function) can be manipulated by knowledgeable spectrum shaping.

4.4.1. Introduction

The spectral effects in coherence scanning interferometry have been a neglected domain. It is because despite the variety of conventional light sources, the conventional light source has only one Gaussian distribution in its effective spectrum. The corresponding interference

signal always has only one high frequency component, and its fringe contrast function (which is the envelope function of the interference signal) can be reasonably modeled as a single Gaussian function [109]. So, most prior works [40], [42], [45], [125] have neglected the spectral effects of the light source.

Driven by advancement in lighting technology and new applications such as dynamic characterization of MEMS, various modern and hybrid low coherence light sources such as phosphor-based white LED, supercontinuum light and hybrid light source have been used as the low coherence light source in coherence scanning interferometry [58], [61], [63], [80], [109]. As shown in Figure 4-29, these modern light sources [61], [63], [109] are made out of narrow range of wavelengths in the visible range.

Figure 4-29: Spectra of some modern light sources used in the prior works [61], [63], [109]

As these modern light sources [61], [63], [109] are different from the conventional white light sources, a distinctive feature (in the interference signal) has been observed [60], [61], [63], [109]. The distinctive feature means that the fringe contrast function (of the coherence scanning interferometry) cannot be modeled as a single Gaussian function, and it is consistent with the distinctive feature due to the use of phosphor-based white LED (please refer to 4.2 for details).

As a graphical illustration of the distinctive feature, Figure 4-30 shows the intensity spectrum of a phosphor-based white LED which has two peaks in its intensity spectrum and the corresponding correlogram with the distinctive features highlighted in yellow boxes.

Figure 4-30: Graphical illustration of the distinctive feature: (a) spectrum of phosphor-based white LED, LXHL-LW6C (b) corresponding interference signals (by simulation and experiment) with the distinctive features highlighted in yellow box.

With the distinctive features in the interference signal, the envelope of the interference signal can no longer be modeled as a single Gaussian function. The effects of this distinctive feature to the reconstructed height profile will vary depending on the working principle of the reconstruction algorithm. However, it has been shown that the measurement repeatability will degrade if the spectral effect is not treated properly [109] (please refer to 4.2 and 4.3 for details).

Researchers [61]–[63], [109] have studied and shown that the distinctive feature is related to the low coherence light source with two peaks in the spectrum, but there is no theoretical explanation for it. As such, spectral shaping to achieve the distinctive feature has been done based on the trial-and-error (or ad hoc) approach.

4.4.2. The proposed theory

Although the distinctive feature in the interference signal (as shown in Figure 4-30) can be simulated by the generalized physical model, the generalized model does not help in explaining why and how the distinctive feature is generated. This is because the generalized model is hard to be manipulated algebraically. For example, it is difficult to identify and extract the term(s) representing the OPD range where the interference occurs and the fringe contrast function out of the generalized physical model.

4.4.2.a) The proposed theory

First of all, a dual wavelength low coherence light source is not a modern light source. Molnar and Tutsch had created a dual wavelength low coherence light by combining a conventional broadband light source and a He-Ne laser with a wavelength of 632.8nm [62], and the hybrid light source in Heikkinen et al.'s work [63] is made out of two monochromatic LEDs. On the other hand, the intensity spectrum of supercontinuum light source, which is an example of modern low coherence light source, is similar to the conventional light source [58], [80], and there is no distinctive feature in its corresponding interference signal.

Although it has been shown that the distinctive feature is related to the low coherence light source with two peaks in the spectrum [61]–[63], [109], there is no available mathematical model describing the light source. So we shall first model the light source to generate the distinctive feature of a dual wavelength low coherence light source, and it can be represented as follows:

$$f(k) = A_1 e^{-\left(\frac{k-k_1}{\sigma_1}\right)^2} + A_2 e^{-\left(\frac{k-k_2}{\sigma_2}\right)^2} \quad (4.6)$$

where

k is the angular wave number ($=2\pi/\lambda$),

A_1, A_2 are the scaling factors,

k_1, k_2 indicate the peak wave number of each light, and

σ_1, σ_2 indicate the spread of each color in the spectral domain.

Next, we derive the corresponding interference signal by substituting Equation (4.6) into Equation (4.1) to derive the corresponding correlogram:

$$I(z) = g_1(z - z_0) \cos(2k_1(z - z_0) + \phi_1) + g_2(z - z_0) \cos(2k_2(z - z_0) + \phi_2) \quad (4.7)$$

where

z is the defocus position (related to the OPD),

z_0 is related to the profile of the sample surface,

k_1, k_2 are the equivalent wave numbers of light ($k=2\pi/\lambda$),

ϕ_1, ϕ_2 are the phase offsets, and

g_1, g_2 are the fringe contrast functions.

As there are two dominant colours in the spectrum of the light source, there are g_1, g_2, k_1, k_2 and etc. The shape of the fringe contrast function is a Gaussian function, and the spread of the Gaussian function is influenced by the numerical aperture of objective lens and the spread of each color in spectral domain.

With the correlogram of the dual wavelength low coherence light source derived, we found that the correlogram is similar to a beat. A beat is an interference between two waves of very similar frequencies, and mathematically it can be represented as follows:

$$\begin{aligned}
 & \cos(2\pi f_1 t) + \cos(2\pi f_2 t) \\
 &= 2 \cos\left(\frac{2\pi(f_1 - f_2)t}{2}\right) \cos\left(\frac{2\pi(f_1 + f_2)t}{2}\right) \\
 &= 2 \cos\left(2\pi \frac{\Delta f}{2} t\right) \cos(2\pi f_{average} t)
 \end{aligned} \tag{4.8}$$

where

f_1, f_2 are the frequencies,

t is the independent variable,

$f_{average} = (f_1 + f_2)/2$, $\Delta f = (f_1 - f_2)$, and

$\frac{\Delta f}{2}$ is the beat frequency.

Next, we re-arrange and express the corresponding correlogram (Equation(4.7)) in the form of beat. For simplicity and tidiness of derivation, it is assumed that $g_1 = g_2 = g$, $\phi_1 = \phi_2 = 0$, and $z_0 = 0$, the corresponding correlogram is represented as follows

$$\begin{aligned}
 I(z) &= g(z) [\cos(2k_1 z) + \cos(2k_2 z)] \\
 &= g(z) \left[2 \cos\left(\frac{2(k_1 - k_2)z}{2}\right) \cos\left(\frac{2(k_1 + k_2)z}{2}\right) \right] \\
 &= g(z) [2 \cos(\Delta k z) \cos(2k_{average} z)]
 \end{aligned} \tag{4.9}$$

where

k_1, k_2 are the wave numbers ($k=2\pi/\lambda$),

$k_{average} = (k_1+k_2)/2$, and

$\Delta k = (k_1-k_2)$.

As the distinctive feature (shown in Figure 4-30(b)) cannot be explained by the sub fringe contrast functions (g_1 and g_2 in Equation(4.7)) and Equation(4.9) suggests an interference in the form of beat, we propose that the distinctive feature is due to the interference between the high frequency components (k_1 and k_2 in Equation(4.7)). A destructive interference between these two high frequency components results a dip in the effective correlogram, matching the definition of the distinctive feature.

4.4.2.b) Verification of the proposed theory

As a verification of the proposed theoretical explanation, we apply it on the phosphor-based white LED where the distinctive feature was first investigated in [109]. The intensity spectrum of the phosphor-based white LED is shown in Figure 4-30(a), where k_1 and k_2 are 440nm (14.3 rad/ μ m) and 550nm (11.4 rad/ μ m) respectively; σ_1 and σ_2 are 0.4941 rad/ μ m and 1.439 rad/ μ m respectively; and A_1/A_2 is around 1.6. The wrapped phase distribution of these two high frequency components is recovered by Fourier Transform Method [7] and expressed as follows:

$$\begin{aligned}\Phi_1 &= \text{mod}\left(\left(2k_1(z-z_0)+\phi_1\right), 2\pi\right) \\ \Phi_2 &= \text{mod}\left(\left(2k_2(z-z_0)+\phi_2\right), 2\pi\right)\end{aligned}\tag{4.10}$$

where

mod is a modulus operator.

Figure 4-31: Illustration that the distinctive feature is the result of destructive interference between two colors (which are blue and yellow lights for phosphor-based white LED)

As shown in Figure 4-31, the location of the distinctive feature is in the region (the defocus position is between $0.35\mu\text{m}$ to $0.8\mu\text{m}$) where there are destructive interferences. This supports the proposed explanation that the distinctive feature is due to the occurrence of destructive interferences among the high frequency components within the localized window where the OPD is small.

4.4.3. Fringe contrast function manipulation via spectral shaping

With the proposed explanation, it is now feasible to perform spectrum shaping to manipulate the fringe contrast function (and the distinctive feature). With reference to Equation(4.10),

the location of the total destructive interference between two colours can be determined by solving:

$$\Phi_1 = \Phi_2 + \pi \quad (4.11)$$

By assuming $k_1 > k_2$, $z_0 = 0$, $\phi_1 = \phi_2$ in Equation(4.10), the location of the total destructive interference (which corresponds to the distinctive feature) can be expressed as follows:

$$\begin{aligned} 2k_1(z) &= 2k_2(z) + \pi \\ 2k_1(z) - 2k_2(z) &= \pi \\ (k_1 - k_2)z &= \frac{\pi}{2} \\ z &= \frac{\pi}{2(k_1 - k_2)} \end{aligned} \quad (4.12)$$

Based on Equation(4.12), the location of the distinctive feature with respect to the zero OPD location is inversely proportional to the difference between the wave numbers of two colours (which is $(k_1 - k_2)$ in Equation(4.12)). If the difference is too small, the distinctive feature will occur outside the localized region (where interference occurs) and will not be observable. Meanwhile, the difference between the wave numbers of the two colours is limited to the finite spectral sensitivity of detector.

4.4.3.a) Experimental Verification

As an experimental verification, we design a hybrid light source (which consists of two monochromatic LEDs) to shift the location of the distinctive feature away from the zero OPD location.

Figure 4-32: Effective spectrum of the hybrid light source designed to shift the location of the distinctive feature away from the zero OPD location

The hybrid light source is made out of blue (LXHL-LB5C) and green (LXHL-LM5C) LEDs to create an effective spectrum as shown in Figure 4-32. The dominant wavelengths are 528nm (11.9 rad/ μm) and 458nm (13.71rad/ μm) respectively, and the blue-green ratio is 0.78:1.

With the effective spectrum of the hybrid source shown in Figure 4-32, the proposed explanation estimates that the location of the distinctive feature is around 0.88 μm away from the zero OPD location. With simulation and physical experiment, the location of the distinctive feature is found to be around 0.8 μm away from the zero OPD location. The estimation by the proposed explanation (which is 0.88 μm) agrees well with the simulation and experiment results (which are around 0.8 μm). See Figure 4-33.

Figure 4-33: Evaluating the accuracy of the proposed explanation in controlling/predicting the location of the distinctive feature: By simulation and experimental verification, the location of the feature is around $0.8\mu\text{m}$ (with respect to the zero OPD location) while the location estimated by the proposed explanation is $0.88\mu\text{m}$.

It is shown that the proposed explanation is accurate and can be applied to manipulate/predict the location of the distinctive feature.

4.5. Other types of white LED

Other than phosphor-based white LED which dominates high power white LED, there are two other primary white LED technologies: (1) Multi-color white LED white LED, and (2) phosphor-free white LED.

4.5.1. Multi-color white LED

Multi-color white LED generates white light by mixing different colored LED. As the most common color combination is red, green and blue, the multi-color white LED is commonly known as RGB white LED.

Although RGB white LED has an early start in its development, it has some stability issues in that the characteristics of the three LEDs are not the same. For example, the brightness of green LED is much stronger than the blue, and each LED degrades at a different rate. So nowadays, RGB white LED is not primarily used as white LED, but it remains relevant for customized applications in which dynamic color tuning is desirable.

LATB T66B by OSRAM (as shown in Figure 4-34) is an example where the three LEDs is packaged into one. Table 4-3 and Figure 4-35 show the detailed specifications and the spectra of each LED in LATB T6B. By the difference in junction temperature, operating current and power consumption, one can imagine the work required to keep these three LEDs working as expected. Other than this, tuning the weightage to produce a desired white light is a laborious task as the maximum power and efficiency of each LED is different.

Figure 4-34: Picture of RGB white LED, LATB T66B by OSRAM

Table 4-3: Specification of individual LED in RGB LED (LATB T66B by OSRAM)

Parameter	LED		
	Amber	Green	Blue
Junction Temperature (°C)	125	125	110
Forward Current (mA)	50	20	20
Surge Current (mA)	100	400	300
Power Consumption (mW)	130	85	85
Wavelength at peak emission (nm)	628	523	465
Spectral bandwidth at 50% (nm)	16	33	25
Optical efficiency (lm/W)	24	13	3

Figure 4-35: Intensity Spectrum of each LED in RGB white LED (LATB T66B by OSRAM).

Figure 4-36: The relative luminous intensity of blue, amber and green colored LED chip under different forward current.

4.5.2. Phosphor-free white LED

As phosphor-based white LED suffers phosphor degradation over time [86]–[88], phosphor-free white LED is introduced [89], [126], [127] and is expected to have longer lifetime [90].

Compared to phosphor-based white LED, phosphor-free white LED is new and the manufacturing process is not as efficient as phosphor-based white LED yet. So only a small amount of this white LED (compared to phosphor-based white LED) is available commercially.

However, in terms of intensity spectrum (as shown in Figure 4-37), phosphor-free white LED is almost identical to phosphor-based white LED. The white light is generated by blue and yellow lights, and there are two peaks in the spectral domain.

Figure 4-37: Example of intensity spectra of phosphor free white LED (a) monolithic white LED (type-1) by Yamada et al. [127], (b) L915NPWC by American Opto Plus LED

Clearly, the research finding in this chapter is also applicable to phosphor-free white LED.

4.6. Conclusion

The spectral property of phosphor-based white LED has a distinctive feature where the fringe contrast function can no longer be modeled as a single Gaussian function, and it affects the measurement repeatability and accuracy of coherence scanning interferometry if the spectral effects are not compensated. Other than typical phosphor-based white LED, we have modeled the spectral variation of phosphor-based white LED and dual wavelength low coherence light source (which is a generalized representation of phosphor-based white LED) in order to give a theoretical explanation of the distinctive feature due to the use of phosphor-based white LED. In doing so, we have demonstrated that the distinctive feature (and the fringe contrast function) can be manipulated by spectral shaping.

For implementation, two key factors to be considered are: (1) Most phosphor-based white LED is classified as consumer electronic, so the deviation between the actual and nominal spectrums may be significant, and (2) Commercially available coherence scanning

interferometer is likely to consist of proprietary configuration/algorithm which should be understood or isolated before implementing the presented works.

Lastly, this research essentially focuses on the spectral properties of a light source that has two peaks in its spectrum, so the developed results are not limited to phosphor-based white LEDs and they are also readily applicable to other high power white LEDs such as phosphor-free white LEDs.

5. Harnessing the Spectral Property of Phosphor-based White LED in Coherence Scanning Interferometry

In Chapter 4, we had shown that the spectrum of phosphor-based white LED can affect the coherence scanning interferometry negatively if it is not treated properly. We shall now look into harnessing the spectral effects of phosphor-based white LED to improve the height reconstruction in coherence scanning interferometry. Instead of developing a new reconstruction algorithm, we shall focus on identifying an existing height reconstruction algorithm in which its working principle will take advantages of the spectral property of phosphor-based white LED and improve its reconstruction performance.

5.1. Identifying a suitable height reconstruction algorithm

For coherence scanning interferometry, height reconstruction algorithm can be categorized into two approaches: (1) fringe contrast based approach and (2) phase-based approach. The fringe contrast based approach recovers the height information by finding the maximum of the fringe contrast function (which is the envelope of correlogram); while the phase-based approach transforms the correlogram into frequency domain, followed by analyzing the phase information. In general, the phase-based approach is superior to the fringe contrast based approach because (1) based on communication theory, the phase information is more robust to noise as compared to the amplitude information, and (2) the phase-based approach makes use of the prior knowledge such as the wavelength of light source [128]. With this consideration, we have searched for a suitable reconstruction algorithm among those using the phase-based approach and identified the phase crossing algorithm by Pawlowski et al. [128] as the reconstruction algorithm with which its performance may be improved by the spectral property of phosphor-based white LED.

The working principle of Pawlowski et al.'s phase crossing algorithm [47], [128], is that the location of zero optical path difference (which corresponds to the height information) is the singular point at which the phase of the interference signal contributed by different wavelengths of light is equal to each other. In Pawlowski et al.'s phase crossing algorithm, the conventional white light source is used and the correlogram is modeled as follows:

$$g(z) = a + b(z - z_0) \cos(2\bar{k}(z - z_0) + \alpha(\bar{k})) \quad (5.1)$$

where

- z is the defocus position,
- a is the DC component in the interference signal,
- b is the fringe contrast function,
- z_0 is the height of the sample surface,
- $\bar{k}$ is the mean wave number, and
- α is the phase difference between the reference and the sample arms.

Then the correlogram is decomposed into multiple (minimum of 2) interference signals contributed by different wavelengths of light. Lastly, the height information is recovered by finding the singular point at which the phase of all decomposed interference signals is equal.

There are 3 steps in the implementation of the phase crossing algorithm:

- Step (1): a region of the correlogram is selected based the use of a local standard deviation estimator.

- Step (2): the selected region of the correlogram is Fourier transformed, and two filter windows are applied to extract the interference signals contributed by two narrow band signals.
- Step (3): the phase information of the extracted interference signals is recovered by Fourier transform method [7]. Lastly, the location where the extracted interference signals have equal phase (phase crossing point) is determined.

(a)

(b)

Figure 5-1: Illustration of the phase crossing algorithm by Pawlowski et al. [128] with the conventional white light source: (a) a region of the correlogram is selected for further processing, (b) the selected region is Fourier transformed and two filter windows are applied to extract the interference signals contributed by two narrow band signals, and (c) the phase information of the extracted interference signals are recovered and the phase crossing point is identified

Figure 5-1 illustrates the implementation of the phase crossing algorithm with the conventional white light source. As the effective spectrum of the conventional white light has only one dominant wavelength, there is only one peak in the spatial frequency domain of the correlogram. To extract the interference signals contributed by different wavelengths of light, two filter windows (on the left and right of the only peak in the frequency domain) are applied. After extracting the two interference signals contributed by different wavelengths of light, the phase information of each interference signal is recovered and the phase crossing point is determined. Based on its working principle, the quality of the extracted interference signals is critical to its performance.

As phosphor-based white LED is made out of two colors (which are blue and yellow colors), there are two peaks in its intensity spectrum. Together with the spectral property of phosphor-based white LED studied earlier, we propose that the use of phosphor-based white LED will improve the quality of filtering/extracting of interference signal contributed by different wavelengths of light. That is, the phase crossing algorithm may harness the spectral properties of phosphor-based white LED and improve its performance.

With phosphor-based white LED, the correlogram is made out of two high frequency components which correspond to two peaks in its spatial frequency domain. With these two peaks in the spatial frequency domain, the filter windows can extract the interference signal contributed by different wavelengths of light better. Figure 5-2 illustrates the implementation of the phase crossing algorithm with the phosphor-based white LED. As the effective spectrum of the phosphor-based white LED has two dominant wavelengths, there are two peaks in the spatial frequency domain of the correlogram. With two distinct peaks in the spatial frequency domain, the interference signals contributed by different wavelengths of light can be separated and extracted better.

(c)

Figure 5-2: Illustration of the phase crossing algorithm by Pawlowski et al. [128] with the phosphor-based white LED: (a) a region of the correlogram is selected for further processing, (b) the selected region is Fourier transformed and two filter windows are applied to extract the interference signals contributed by two narrow band signals, and (c) the phase information of the extracted interference signals is recovered and the phase crossing point is identified

5.2. Simulation verification

In the simulation verification, two sets of data are simulated: one is based on the conventional white light source; another one is based on the phosphor-based LED, LXHL-LW6C by LumiLEDs. A line profile of $1\mu\text{m}$ step height is selected; the line profile consists of 256 surface points and each surface point has a corresponding correlogram. The sampling interval of the intensity response is 50nm, and each correlogram is corrupted by Gaussian white noise (zero mean, variance of 0.02). Figure 5-3 compares the correlograms using the conventional white light source and the phosphor-based white LED (LXHL-LW6C by LumiLEDs).

Figure 5-3: Comparison of correlograms using the conventional white light source and the phosphor-based white LED (LXHL-LW6C by LumiLEDs)

Figure 5-4: 1 μ m step height reconstructed by the phase crossing algorithm using the conventional white light source

Using the phase crossing algorithm, the height profiles measured with different light sources are reconstructed and shown in Figure 5-4 and Figure 5-5. For an objective assessment, the measurement repeatability (in terms of the standard deviation in measuring the flat surface) and the accuracy of reconstructed profiles are further analyzed.

Figure 5-5: 1 μm step height reconstructed by the phase crossing algorithm using the phosphor-based white LED (LXHL-LW6C by LumiLEDs)

Figure 5-6: Comparing the measurement repeatability (in terms of the standard deviation) of the phase crossing algorithm using different light sources, the standard deviation of a perfectly flat surface is zero

Figure 5-7: Comparing the measurement accuracy of the phase crossing algorithm using different light sources, the ideal value is 1μm

Figure 5-7 shows the comparison of measurement accuracy of the phase crossing algorithm using different lighting, and it shows that the measurement accuracy is improved when the phosphor-based white LED is used.

Figure 5-6 shows that the use of phosphor-based LED improves the measurement repeatability (where standard deviation is inversely proportional to repeatability) of the phase crossing algorithm.

5.3. Experimental verification

For experimental verification, we adopted the configuration similar to the earlier simulation but measured a WYKO $10.02 \pm 0.08 \mu\text{m}$ standard step height. The experimental result shows that the performance of the phase crossing algorithm is improved by using the phosphor-based white LED. In terms of the measurement repeatability, the standard deviation for measuring flat surface is 2.17nm when using the phosphor-based white LED, which is a significant improvement from 26.24nm when using the conventional white light. In terms of

the measurement accuracy, the difference from the nominal value of the standard step height is smaller when using the dual wavelength white LED. The step height value is $10.09\mu\text{m}$ when measured using the phosphor-based white LED as compared to $10.18\mu\text{m}$ when measured using the conventional white light. This result is consistent with the simulation result presented earlier.

5.4. Conclusion

In this chapter, we have demonstrated that the spectral effects of phosphor-based white LED can be harnessed to improve the performance of the phase crossing height reconstruction algorithm [128]. A detailed explanation on how the working principle of the reconstruction algorithm takes advantages of the spectral property of phosphor-based white LED is also presented.

6. Video-based Interferogram Analysis of Vibration

In Chapter 5, we had shown that the spectrum of phosphor-based white LED can be harnessed to improve the coherence scanning interferometry. We will now continue to explore the use of spectral effects to address the vibration issues in the coherence scanning interferometry. First, we shall propose a video-based interferogram analysis to quantify the effects of vibration in coherence scanning interferometry. The proposed method does not require additional hardware, and it considers the properties of the interferometer such as the bandwidth of the sensor and the performance of vibration isolation design. With the proposed method, the influence of vibration on coherence scanning interferometer can be objectively quantified and measured [32]. Lastly, we shall also look into the possibilities of applying the spectral properties of phosphor-based white LED to improve its performance.

6.1. Introduction

The problem of vibration is a common and significant issue in all high precision measurement equipment. Although the vibration itself can be quantified or measured by equipment such as Laser Doppler Vibrometer, the effects of vibration on the measurement equipment cannot be measured directly. In coherence scanning interferometry, the undesired relative motion affecting the optical path difference between the reference and sample arms is the most critical factor to the performance of coherence scanning interferometry. Hence, in this section, we propose a video-based interferogram analysis that measures the undesired optical path difference in coherence scanning interferometry.

6.2. Vibration in coherence scanning interferometry

6.2.1. Cause of vibration

For high precision measurement equipment, any undesired motion is considered as vibration, and it affects measurement result depending on the measurement principle. Unlike other environmental factors such as temperature fluctuation, humidity, external lighting, etc, vibration isolation remains a challenging and expensive task. For example, to achieve proper vibration isolation, the US National Institute of Standards and Technology (NIST) builds its high precision measurement labs 12 meters below ground, structurally isolated from its main building and positions the measurement instruments on a specially designed, heavy mass isolation slabs supported on pneumatic “air springs” and an isolated, raised floor system spans over the pit containing each isolation slab [129]. These measures are costly and rare in practice.

The most common method to deal with vibration is the use of optical table (vibration isolation system). The optical table is a platform engineered to isolate environmental vibration by being rigid, stiff, having high natural frequency, and it can isolate virtually all vibrations above 50Hz. However, most environmental vibrations are below 30Hz. Some examples are the vertical vibration of 10 to 30Hz from people, traffic and construction work and the horizontal vibration of 1 to 10Hz in tall buildings [53]. Other than its inability to handle vibration at low frequency, the optical table cannot handle vibration induced by devices operating on it, such as by high precision scanner and cooling fan. The significance and technical knowledge of optical table for high precision applications are summarized by Hayes and others [53], [54], [55, p. 101].

6.2.2. How vibration affects coherence scanning interferometry

The vulnerability of interferometry to vibrations comes from the measurement principle in that it requires several interferograms for the analysis to produce the surface height profile. By tracing the light intensity level of a particular point in interferogram over the optical path difference, a one-dimensional data known as correlogram is generated. The correlogram is crucial for coherence scanning interferometry as it is the raw data for further processing such as height reconstruction algorithm. Assuming the use of a conventional light source, where its spectrum has a single Gaussian distribution, the correlogram can be represented as follows:

$$I(z(t)) = I_{\text{amplitude}} \exp\left(\frac{-(z(t) - z_0)^2}{\sigma^2}\right) \cos\left(\frac{4\pi(z(t) - z_0)}{\lambda_m} + \phi\right) \quad (6.1)$$

where

t is the time

$z(t)$ is the defocus position (related to the optical path difference) at time t ,

z_0 is related to the profile of the sample surface,

$I_{\text{amplitude}}$ is the amplitude of the interference signal,

λ_m is the equivalent wavelength of low coherence light,

σ is related to the coherence length of light, and

ϕ is the phase difference between the reference and sample beams.

In general, coherence scanning interferometry introduces the designated optical path difference in a time dependent and linear manner by the high precision scanning mechanism, such as piezoelectric stage. The motion profile can be represented as follows:

$$z = ut \tag{6.2}$$

where

u is the velocity of the scanning mechanism, and

t is the time.

Other than the sensor noise of the light sensitive photo detector, the quality of the correlogram primarily depends on the accuracy and precision in controlling the defocus position. The accuracy in controlling (or determining) the defocus position is subjected to the following factors:

1. Imperfections in the high precision motion stage
2. Environmental vibration

Figure 6-1 graphically illustrates the schematic diagram of coherence scanning interferometry and identifies the optical path difference between D_{BS-ref} and $D_{BS-sample}$ as the path which is sensitive to vibration.

As the imperfection in the high precision stage is a consistent and repeatable error, it can be compensated by calibration. On the other hand, the environmental vibration is a random and non-repeatable error, because the interference signals captured at different times are subject to different erroneous optical path different caused by vibration. So the error due to vibration cannot be compensated by calibration.

Figure 6-1: Schematic diagram showing the measurement principle of interferometry: A computer captures and analyzes interference pattern with known optical path difference (between D_{BS-ref} and $D_{BS-sample}$).

During the measurement process of coherence scanning interferometry, vibration introduces an undesired displacement which affects the designated optical path difference introduced by high precision scanning. The effect of vibration varies depending on the exposure time and the frame rate of the camera and the velocity of the scanning mechanism, so the effective vibration wavelength (with respect to the defocus position in correlogram) is subjected to the speed of scanning mechanism and the frame rate of the camera. The correlogram corrupted by vibration can be represented in terms of time-independent variable as follows:

$$\begin{aligned}
 I'(z) &= I(z + v(z)) \\
 &= I_{\text{amplitude}} \exp\left(\frac{-(z + v(z) - z_0)^2}{\sigma^2}\right) \cos\left(\frac{4\pi(z + v(z) - z_0)}{\lambda_m} + \phi\right)
 \end{aligned} \quad (6.3)$$

where

$v(z)$ is the undesired displacement due to vibration at time $t=z/u$.

Next, Equation (6.3) is further expanded as follows:

$$\begin{aligned}
 I(z + v(z)) &= I_{amp} \exp\left(-\frac{(z + v(z))^2}{\sigma^2}\right) \exp\left(\frac{2(z + v(z))z_0}{\sigma^2}\right) \exp\left(-\frac{z_0^2}{\sigma^2}\right) \\
 &\quad \cos\left(4\pi \frac{((z + v(z)) - z_0)}{\lambda}\right) \\
 &= I_{amp} \exp\left(-\frac{z^2 + 2zv(z) + v(z)^2}{\sigma^2}\right) \exp\left(\frac{2zz_0 + 2v(z)z_0}{\sigma^2}\right) \exp\left(-\frac{z_0^2}{\sigma^2}\right) \\
 &\quad \cos\left(4\pi \frac{(z + v(z) - z_0)}{\lambda}\right) \\
 &= I_{amp} \exp\left(-\frac{z^2 - 2zz_0 + z_0^2}{\sigma^2}\right) \exp\left(\frac{2v(z)z_0 - 2zv(z) + v(z)^2}{\sigma^2}\right) \\
 &\quad \cos\left(4\pi \frac{(z - z_0 + v(z))}{\lambda}\right) \\
 &= I_{amp} \exp\left(-\frac{(z - z_0)^2}{\sigma^2}\right) \exp\left(\frac{2v(z)z_0 - 2zv(z) + v(z)^2}{\sigma^2}\right) \\
 &\quad \cos\left(4\pi \frac{(z - z_0 + v(z))}{\lambda}\right)
 \end{aligned} \tag{6.4}$$

The mathematical model derived so far is applicable to various form of vibrations, such as periodic pattern and impulse. For the purpose of vibration analysis, it is considered that the vibration is of sinusoidal form which can be represented as follows:

$$v(z) = A_{vibration} \cos\left(\frac{2\pi z}{\lambda_{vibration}} + \phi\right) \tag{6.5}$$

where

$A_{vibration}$ is the amplitude of vibration,

$\lambda_{vibration}$ is the wavelength of vibration, and

ϕ is the phase offset of vibration.

By substituting Equation (6.5) into Equation (6.4), one obtain

$$\begin{aligned}
 I(z + v(z)) &= I_{amp} \exp\left(-\frac{(z - z_0)^2}{\sigma^2}\right) \exp\left(\frac{2v(z)z_0 - 2zv(z) + v(z)^2}{\sigma^2}\right) \cos\left(4\pi \frac{(z - z_0 + v(z))}{\lambda}\right) \\
 &= I_{amp} \exp\left(-\frac{(z - z_0)^2}{\sigma^2}\right) \\
 &\quad \exp\left(\frac{2v(z)z_0 - 2A_{vibration}z \cos\left(\frac{2\pi z}{\lambda_{vibration}} + \phi\right) + A_{vibration}^2 \cos^2\left(\frac{2\pi z}{\lambda_{vibration}} + \phi\right)}{\sigma^2}\right) \\
 &\quad \cos\left(4\pi \frac{\left(z - z_0 + A_{vibration} \cos\left(\frac{2\pi z}{\lambda_{vibration}} + \phi\right)\right)}{\lambda}\right)
 \end{aligned} \tag{6.6}$$

Equation (6.6) represents the correlogram of coherence scanning interferometry corrupted by vibration with a periodic sinusoidal pattern.

As shown in the derivation above, the effects of vibration to the interference signal of coherence scanning interferometry are complex and difficult to be interpreted.

6.3. Video-based interferogram analysis to quantify vibration

As discussed in earlier section, the undesired optical path difference between the reference and sample beam of light in interferometer has the most significant effect on the coherence scanning interferometry. So, in this section, we will investigate video-based interferogram

analysis to quantify the vibration in coherence scanning interferometry by measuring the undesired optical path difference (OPD).

Although there are some established vibration measurement technologies such as Laser Doppler Vibrometer (LDV), accelerometer, velocity pickup, etc, these measurement technologies do not measure the undesired optical path difference between the reference and sample light beams directly. Hence, further analysis on the measurement results by these measurement technologies is required in order to interpret the effects of vibration on interferometry. To illustrate this, Figure 6-2 shows how vibration is transmitted through vibration isolation system and mechanical framework, and introduces undesired change in $D_{BS-sample}/D_{BS-ref}$. Although the vibration might introduce a lateral movement, it is assumed negligible in this research work.

As shown in Figure 6-2 , a vibration source generates a vibration which is transmitted through interferometer with built-in vibration isolation system and produces a change in the optical path difference between reference and sample beams. The change in optical path difference due to vibration is reflected in the interferogram and is recorded by the camera. Figure 6-3 shows a graphical illustration of how fringes shift as optical path difference changes when the sample is a tilted flat surface.

Figure 6-2: Illustration of how vibration affects interferometer. Vibration transmitted through vibration isolation system introduces relative motion, a change in $D_{BS-sample}$ and/or D_{BS-ref} . Vibration isolation system consists of mechanical structure of interferometer and optical table, these components act like a filter reducing the effect of vibration to interferometer

Figure 6-3: Measurement Principle: Assuming D_{BS-ref} (in Figure 6-2) is constant, a small change in $\Delta D_{BS-sample}$ (in Figure 6-2) will cause a corresponding shift in interference pattern in interferogram, $\Delta D_{BS-sample}$ is proportional to S_{pixel} . If the sample surface is flat and tilted at an angle, $\Delta D_{BS-sample}$ is linearly proportional to S_{pixel} , $\Delta D_{BS-sample} = A \times S_{pixel}$ where A is the gradient of the slope, in the unit of height per pixel.

The sample need not necessarily be a tilted flat surface, it can be a spherical surface which will produce a circular fringe pattern. Circular fringe patterns do not have the aperture problem (which will be discussed later), but it requires greater effort in tracking the fringe,

and resolving the tracked fringe into optical path difference. Hence a tilted flat is selected for demonstrating our method.

Based on the rate of change among the interferograms, experienced interferometer user can assess the intensity of vibration subjectively. Single-frame based objective method such as single frame interferogram evaluation analysis [10] can measure the surface profile based on single interferogram. Similarly, Millerd et al. [68] can generate the surface height profile but not the height (or OPD) change over time.

Instead of measuring the surface height profile, the focus in this work is to measure the change in optical path difference, which corresponds to the height, in metric length unit. In the proposed method, there are three steps: (1) enhance and analyze the interferogram for interference pattern detection, (2) track the change in the interference pattern over time and (3) transform the tracked change in the interferogram to physical units representing the change in optical path difference. Here are the details of these three steps:

Step (1) - Image analysis/enhancement for interference pattern (which is also known as fringe) detection: The interferogram recorded by the camera is enhanced by gamma correction and contrast enhancement for reliable segmentation, and the result of segmentation is further processed to remove the noise. The output of this step is the interference pattern separated from the background, its motion will be tracked in the next step.

Step (2) - Tracking the motion of the interference pattern over time: The key challenge in this stage is the aperture problem which is the result of ambiguity in motion of a feature-less pattern such as a stripe as shown in Figure 6-4. To solve the ambiguity in motion, a constraint that the fringe moves in a direction tangential to the fringe pattern is applied. The output of this step is the motion of the interference pattern. This gives an objective measure about the

intensity/magnitude of the vibration. However, the tracked motion is expressed in terms of image pixel and it is not measuring the optical path difference.

Step (3) - Transforming the tracked motion of the fringe (in terms of pixel) to the optical path difference change in metric space: To express the change in optical path difference in metric length units, the tilt angle of the flat surface in the sample arm has to be known. This can be recovered by either the single frame interferogram analysis such as [10] or a normal height profile measurement.

Figure 6-4: Graphical illustration of the aperture problem: (a) Observing a moving fringe through finite field of view poses ambiguity in direction, V_1 and V_2 are possible. (b) As the block is moving in the direction V_1 in constant speed, tracked motion without considering aperture would be non-linear.

The proposed method has two major advantages: (1) it does not require additional hardware or hard-wiring, and (2) it measures the actual erroneous optical path difference with factors such as the properties of vibration isolation system and the frame rate of the camera considered. The limitation of this method is that it assumes that there is no lateral motion (due to vibration) and the erroneous optical path difference is the same within one field of view.

6.3.1. Experimental verification

To verify the presented method, a high precision piezoelectric stage is used to simulate vibration with a square pattern (amplitude of 200nm and period of 1000millisecond), Figure 6-5 shows vibration signal measured by the presented method.

As shown in Figure 6-5, the presented method is able to accurately measure the vibration simulated by the high precision piezoelectric stage.

Figure 6-5: Measuring the undesired optical path difference change due to the vibration simulated by high precision piezoelectric stage: The presented method measure a vibration with a square pattern (amplitude of 200nm and period of 1000millisecond) generated by high precision piezoelectric stage.

6.3.1.a) Measurement result

With the experimental verification on the accuracy of the presented method, we applied the presented method to measure the undesired optical path difference (OPD) change due to (1) the cooling fan in our coherence scanning interferometer, and (2) a gentle tap on the optical table where our coherence scanning interferometer is mounted. The measurement results are shown in Figure 6-7 and Figure 6-6.

Figure 6-6: Measurement by the presented method: the undesired OPD change induced by the cooling fan in our coherence scanning interferometer.

As shown in Figure 6-6, the undesired optical path difference change due to the cooling fan in our coherence scanning interferometer is dominated by high frequency components and its amplitude is around 30nm. With the presented method, the effect of vibration to the interferometer is assessed quantitatively. The quantitative output may serve as a feedback for mechanical designer to optimize/improve the mechanical design and/or for user to decide if the measurement result is corrupted by environmental vibration.

Figure 6-7: Measurement by the presented method: the undesired OPD change induced by a gentle tap on the optical table (where our coherence scanning interferometer is mounted)

Figure 6-7 shows the undesired change in optical path difference due to a gentle tap on the optical table where our interferometer is mounted. As shown in Figure 6-7, the undesired optical path difference change is short (in terms of duration) and made out low frequency components. As the presented method factors in the properties of the interferometer (such as the frame rate of the camera) and only measures the effective undesired change in the optical path difference, it is possible that the actual vibration by the tap is at a much higher frequency than the result shown in Figure 6-7.

6.4. Harnessing the spectral effect

As discussed in Chapter 4 and earlier section in this chapter, the spectral property of phosphor-based white LED introduces the distinctive feature to the interference signal of coherence scanning interferometry. The spectral effects of phosphor-based white LED can be harnessed to improve the presented method.

For coherence scanning interferometry using the conventional white light source (where its intensity spectrum is a single Gaussian function), the interferogram is derived and expressed as follows:

$$I(x, y, z) = I_{dc}(x, y) + I_{amplitude} \exp\left(\frac{-(z - z_0(x, y))^2}{\sigma^2}\right) \cos\left(\frac{4\pi(z - z_0(x, y))}{\lambda_m} + \phi\right) \quad (6.7)$$

where

x & y are the image coordinate in pixel,

z is the defocus position (related to the optical path difference),

z_0 is related to the profile of the sample surface,

I_{dc} is a constant value and not related to the interference,

$I_{amplitude}$ is the amplitude of the interference signal,

λ_m is the equivalent wavelength of light,

σ is related to the coherence length of the light source, and

ϕ is the phase offset.

Based on the simulation equation above, Figure 6-8 shows the interferogram with the conventional white light source.

Figure 6-8: Interferogram of coherence scanning interferometry using the conventional white light source with a tilted flat sample surface.

Figure 6-9: Interferogram of coherence scanning interferometry using phosphor-based white LED with a tilted flat sample surface

With a phosphor-based white LED (LXHL-LW6C by LumiLEDs), the corresponding interferogram is derived based on the signal model presented in Chapter 3 and expressed as follows:

$$I_{\text{interference}}(x, y, z) = C \left[\left(g(z, z_0(x, y), \theta_0, k_{ul}, b) - g(z, z_0(x, y), 0, k_{ul}, b) \right) - \left(g(z, z_0(x, y), \theta_0, k_{ll}, b) - g(z, z_0(x, y), 0, k_{ll}, b) \right) \right]$$

where $g(z, z_0(x, y), \theta, k, b)$ is defined as

$$= \frac{D}{4} \left(bU \left(\left(2(\pi - bU)k \cos \left(\frac{b\phi + 2k_m\pi - 2\pi k + 2bUk}{b} \right) + b \sin \left(\frac{b\phi + 2k_m\pi - 2\pi k + 2bUk}{b} \right) \right) \right) / (\pi - bU)^2 + \right. \\ \left. \left(-2(\pi + bU)k \cos \left(\frac{b\phi - 2k_m\pi + 2\pi k + 2bUk}{b} \right) + b \sin \left(\frac{b\phi - 2k_m\pi + 2\pi k + 2bUk}{b} \right) \right) \right) / (\pi + bU)^2 + \\ \left(b \left(\sin \left(\frac{b\phi + 2k_m\pi - 2\pi k + 2bUk}{b} \right) / (-\pi + bU) + \sin \left(\frac{b\phi - 2k_m\pi + 2\pi k + 2bUk}{b} \right) / (\pi + bU) \right) \right) \right)$$

with

$$D = 1 / \left(8z z_0(x, y) - 4z^2 - 4z_0(x, y)^2 \right), \text{ and } U = (z - z_0(x, y)) \cos \theta$$

(6.8)

where

x & y are the image coordinate in pixel

z is the defocus position (related to optical path difference),

z_0 is related to the profile of the sample surface,

C_1 is a constant,

k is the angular wave number ($k = 2\pi/\lambda$),

$\sin \theta_0$ is the numerical aperture of the objective lens (assuming $n=1$),

$F(k)$ is the effective intensity spectrum of the light source, and

ϕ is the phase offset.

Based on the derived corresponding interferogram, the interferogram with the phosphor-based white LED is simulated and shown in Figure 6-9

Figure 6-10: Highlighting the distinctive feature in the interferogram of coherence scanning interferometry using phosphor-based white LED with a tilt flat sample surface

Figure 6-10 highlights the distinctive feature due to the use of phosphor-based white LED in the corresponding interferogram (which is shown in Figure 6-9), and this distinctive feature is the distinctive feature investigated in Chapter 4 and the earlier section in this chapter.

The presented video-based interferogram analysis method may harness this distinctive feature to improve its performance in quantifying the vibration to coherence scanning interferometry through the following approaches:

1. Improving the interference fringe detection and tracking in the step (1-2). As the distinctive feature is unique compared to the interferogram with conventional light source, it reduces the ambiguity in fringe tracking which can lead to performance improvement.
2. Recovering the tilt of the flat sample in the step (3) of the presented method. Instead of relying on single interferogram analysis [10] or height profile measurement, the tilt of the flat sample can be recovered by measuring the separation between the distinctive features in the interferogram. Together with the theoretical explanation to

the distinctive feature presented in the earlier section in this chapter, the separation between the distinctive features can be transformed to the tilt required to convert the fringe motion in image pixel to the optical path difference change.

6.5. Summary

In this chapter, we have presented a video-based interferogram analysis to quantify the effects of vibration to coherence scanning interferometry. The presented method does not require additional hardware and considers the properties of the interferometer, such as the bandwidth of the sensor. With the presented method, the influence of vibration on coherence scanning can be quantitatively measured. In addition to that, we have also discussed how the spectral properties of phosphor-based white LED can be harnessed to improve the presented method.

7. Conclusions and Future Works

7.1. Conclusions

In this thesis, we have investigated of the spectral effects of phosphor-based white LED in coherence scanning interferometry with the aim to improve its usability and performance.

From the modeling perspective, a new interferometric signal model in mathematically elementary form has been derived for coherence scanning interferometry. Prior to the derived signal model, there is no generalized signal model in elementary form, and so the existing results in coherence scanning interferometry have been dominated by numerical approaches. Other than resolving the computational load problem of the numerical approach, the derived signal model may open up a new frontier for symbolic algebraic approach in the research and development of coherence scanning interferometry.

From the perspective of interferometry and its applications, both the mechanism and the influence of the spectral effects on the height measurement using coherence scanning interferometry have been investigated. Although LED has been being used as the light source in coherence scanning interferometry for many years, its spectral effects have been either neglected or compensated with ad-hoc approaches. To the best of our knowledge, we are the first to identify, investigate and harness the spectral effects of phosphor-based white LED in coherence scanning interferometry. Although our investigation work focused on the spectral properties of phosphor-based white LED, it is worth nothing that our research findings are also applicable to other types of dual wavelength white LEDs. Furthermore, with the rapid evolution in the lighting technology, we expect our current work to attract further research and investigation into the spectral effects in coherence scanning interferometry.

Our investigations have successfully addressed a number of outstanding issues in coherence scanning interferometry, and provided a better understanding of its mechanism and operation limitations when using dual wavelength white LEDs. To summarize the main contributions of this research, we would like to highlight the following:

1. We have derived an interferometric signal model which is an elementary function. The derived signal model not only incorporates both the geometric and spectral effects, but also resolves the computational load problem of prior models.
2. We have investigated the spectral effects of phosphor-based white LED in coherence scanning interferometry. While our study has shown that phosphor-based white LED may degrade the performance of coherence scanning interferometry, we have proposed a method to compensate for the spectral effects and improve the performance of coherence scanning interferometry.
3. To generalize our finding on the spectral effects, we have modeled the spectral variations of phosphor-based white LED and investigated its effects in coherence scanning interferometry. We have shown that the spectral effects of phosphor-based LED in coherence scanning interferometry are significant regardless of its spectral variations.
4. To harness the spectral effects, we have investigated and presented a theoretical explanation of the spectral effects of phosphor-based white LED in coherence scanning interferometry. With the presented theoretical explanation, we have demonstrated that (1) the interference signal of coherence scanning interferometry can be manipulated by spectrum shaping, and (2) the spectral effects of phosphor-based white LED can be harnessed to improve the usability and performance of coherence scanning interferometry.

7.2. Future works

While this research has yielded several new results on the spectral effects of phosphor-based white LED in coherence scanning interferometry, there are still many other unique spectral properties that can be further investigated and that the use of the spectral properties of phosphor-based white LED can be further expanded. Below is a list of our proposed future research directions:

1. As discussed in the thesis, most existing height reconstruction algorithms are developed for coherence scanning interferometry using a conventional white light source. Since we have proposed a method to compensate for the spectral effects of phosphor-based white LED on three established reconstruction algorithms, it would be of practical and theoretical interest to investigate the proposed method on white light phase shifting interferometry [3], [48], [50], [51]. White light phase shifting interferometry is a coherence scanning interferometry which integrates some phase shifting interferometry algorithms into its reconstruction process, so its reconstruction process is more complex and computationally intensive. The spectral effects and method to compensate the undesired effects on these reconstruction algorithms will be the focuses of future works.
2. Another interesting research idea is spectral manipulation for interferometric height profile measurement in which the spectral effects of coherence scanning interferometry are harnessed and integrated into the measurement principle of coherence scanning interferometry. As we have already shown that the location of the distinctive feature can be manipulated by spectral mean, this can be used to detect the imperfection or error in the mechanical scanning mechanism by monitoring the deviation between the recorded interference data and the spectral-manipulated interference data. Other potential applications include but not limited

to: (1) increase the measurement speed by increasing the scanning step size, and (2) improve the robustness against vibration.

3. The presented theoretical explanation of how the spectral properties of phosphor-based white LED affects coherence scanning interferometry is currently capable of predicting and manipulating the location of the distinctive features (due to the use of phosphor-based white LED) in the interference signal only. To further harness the spectral effects of coherence scanning interferometry, the presented theory has to be enhanced and be capable of predicting the amplitude of the distinctive features. As the amplitude of the distinctive feature is subject to factors such as spectral bandwidth and power of each dominant wavelength, and the geometric factors contributed by the objective lens, these have to be considered in the derivation of the enhanced theory. It is noted that a comprehensive and solid theoretical explanation of the spectral effects of coherence scanning interferometry is essential to harnessing the spectral effects in interferometry related research such as single interferogram analysis and improving the robustness of interferometric method against noise/vibration.

Reference

- [1] D. Malacara, *Optical Shop Testing*, 3rd ed. Wiley-Interscience, 2007.
- [2] E. S. Barr, "Men and Milestones in Optics. II: Thomas Young," *The Physics Teacher*, vol. 5, no. 2, pp. 53–60, 1967.
- [3] A. Harasaki, J. Schmit, and J. C. Wyant, "Improved Vertical-Scanning Interferometry," *Appl. Opt.*, vol. 39, no. 13, pp. 2107–2115, May 2000.
- [4] P. Hariharan and M. Roy, "White-light Phase-stepping Interferometry for Surface Profiling," *Journal of Modern Optics*, vol. 41, no. 11, pp. 2197–2201, 1994.
- [5] J. P. M. Verbunt, "Simple Optical Interference Method for the Inspection of Solid Surfaces," *Appl. Opt.*, vol. 12, no. 8, pp. 1839–1840, 1973.
- [6] D. J. Bone, H.-A. Bachor, and R. J. Sandeman, "Fringe-pattern analysis using a 2-D Fourier transform," *Appl. Opt.*, vol. 25, no. 10, pp. 1653–1660, May 1986.
- [7] M. Takeda, H. Ina, and S. Kobayashi, "Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry," *J. Opt. Soc. Am.*, vol. 72, no. 1, pp. 156–160, Jan. 1982.
- [8] J. B. Liu and P. D. Ronney, "Modified Fourier transform method for interferogram fringe pattern analysis," *Appl. Opt.*, vol. 36, no. 25, pp. 6231–6241, 1997.
- [9] C. Roddier and F. Roddier, "Interferogram analysis using Fourier transform techniques," *Appl. Opt.*, vol. 26, no. 9, pp. 1668–1673, May 1987.
- [10] E. Garbusi, C. Pruss, and W. Osten, "Single frame interferogram evaluation," *Appl. Opt.*, vol. 47, no. 12, pp. 2046–2052, Apr. 2008.
- [11] M. Takeda, "Spatial-carrier fringe-pattern analysis and its applications to precision interferometry and profilometry: An overview," *Industrial Metrology*, vol. 1, no. 2, pp. 79–99, Jun. 1990.
- [12] J. Macy, "Two-dimensional fringe-pattern analysis," *Appl. Opt.*, vol. 22, no. 23, pp. 3898–3901, Dec. 1983.

-
- [13] K. Larkin, "A self-calibrating phase-shifting algorithm based on the natural demodulation of two-dimensional fringe patterns," *Opt. Express*, vol. 9, no. 5, pp. 236–253, 2001.
- [14] M. Kujawinska and J. Wojak, "Spatial phase-shifting techniques of fringe pattern analysis in photomechanics," in *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, 1991, vol. 1554, p. 503.
- [15] L. Crescentini, "Fringe pattern analysis in low-quality interferograms," *Appl. Opt.*, vol. 28, no. 6, pp. 1231–1234, Mar. 1989.
- [16] U. P. Kumar, W. Haifeng, N. K. Mohan, and M. P. Kothiyal, "White light interferometry for surface profiling with a colour CCD," *Optics and Lasers in Engineering*, no. 0.
- [17] K. A. Nugent, "Interferogram analysis using an accurate fully automatic algorithm," *Appl. Opt.*, vol. 24, no. 18, pp. 3101–3105, 1985.
- [18] C. J. Tay, C. Quan, and L. Chen, "Phase retrieval with a three-frame phase-shifting algorithm with an unknown phase shift," *Appl. Opt.*, vol. 44, no. 8, pp. 1401–1409, Mar. 2005.
- [19] J. Novak, "Five-step phase-shifting algorithms with unknown values of phase shift," *Optik - International Journal for Light and Electron Optics*, vol. 114, no. 2, pp. 63–68, 2003.
- [20] P. Carré, "Installation et utilisation du comparateur photoélectrique et interférentiel du Bureau International des Poids et Mesures," *Metrologia*, vol. 2, no. 1, pp. 13–23, Jan. 1966.
- [21] P. Hariharan, B. F. Oreb, and T. Eiju, "Digital phase-shifting interferometry: a simple error-compensating phase calculation algorithm," *Applied Optics*, vol. 26, no. 13, p. 2504, Jul. 1987.
- [22] Y.-Y. Cheng and J. C. Wyant, "Two-wavelength phase shifting interferometry," *Appl. Opt.*, vol. 23, no. 24, pp. 4539–4543, Dec. 1984.
- [23] Y.-Y. Cheng and J. C. Wyant, "Multiple-wavelength phase-shifting interferometry," *Appl. Opt.*, vol. 24, no. 6, pp. 804–807, Mar. 1985.

-
- [24] C. Polhemus, "Two-Wavelength Interferometry," *Appl. Opt.*, vol. 12, no. 9, pp. 2071–2074, 1973.
- [25] D. Malacara, *Interferogram Analysis for Optical Testing*. CRC Press, 1998.
- [26] T. V. Vorburger and J. Raja, "Surface Finish Metrology Tutorial." National Institute of Standard and Technology, Jun-1990.
- [27] P. de Groot, "Coherence Scanning Interferometry," in *Optical Measurement of Surface Topography*, R. Leach, Ed. Springer Berlin Heidelberg, 2011, pp. 187–208.
- [28] M. Davidson, K. Kaufman, and F. Cohen, "An application of interference microscopy to integrated circuit inspection and metrology," in *Integrated Circuit, Metrology, Inspection and Process Control*, 1987.
- [29] P. A. Flournoy, R. W. McClure, and G. Wyntjes, "White-Light Interferometric Thickness Gauge," *Appl. Opt.*, vol. 11, no. 9, pp. 1907–1915, Sep. 1972.
- [30] T. Tsuruta and Y. Ichihara, "Accurate Measurement of Lens Thickness by Using White-Light Fringes," *Japanese Journal of Applied Physics*, vol. 14S1, no. Supplement 14–1, pp. 369–372, 1975.
- [31] P. J. Caber, "Interferometric profiler for rough surfaces," *Appl. Opt.*, vol. 32, no. 19, pp. 3438–3441, Jul. 1993.
- [32] W. K. Chong, X. Li, and S. Wijesoma, "Video-Based Interferogram Analysis for Measuring Influence of Vibration to White Light Interferometry," *KEM*, vol. 447–448, pp. 599–603, Sep. 2010.
- [33] L. Deck and P. de Groot, "High-speed noncontact profiler based on scanning white-light interferometry," *Appl. Opt.*, vol. 33, no. 31, pp. 7334–7338, Nov. 1994.
- [34] K. G. Larkin, "Efficient nonlinear algorithm for envelope detection in white light interferometry," *J. Opt. Soc. Am. A*, vol. 13, no. 4, pp. 832–843, Apr. 1996.
- [35] G. S. Kino and S. S. C. Chim, "Mirau correlation microscope," *Appl. Opt.*, vol. 29, no. 26, pp. 3775–3783, 1990.
- [36] P. J. de Groot and L. L. Deck, "Numerical simulations of vibration in phase-shifting interferometry," *Appl. Opt.*, vol. 35, no. 13, pp. 2172–2178, May 1996.

-
- [37] L. L. Deck, "Vibration Resistant Interferometry," US20080266571.
- [38] J. C. Wyant, "Vibration insensitive interferometric optical testing," in *Frontiers in Optics*, 2004, p. OTuB2.
- [39] T. Honda and K. Okada, "Vibration-resistant interferometer," 5650853.
- [40] M. Li, C. Quan, C. J. Tay, I. Reading, and S. Wang, "Measurement of transparent coating thickness by the use of white light interferometry," in *Third International Conference on Experimental Mechanics and Third Conference of the Asian Committee on Experimental Mechanics*, SPIE, 2005, vol. 5852, pp. 401–406.
- [41] Pavel Pavlíček and J. Soubusta, "Measurement of the Influence of Dispersion on White-Light Interferometry," *Appl. Opt.*, vol. 43, no. 4, pp. 766–770, Feb. 2004.
- [42] C. Ai and E. K. Novak, "Centroid approach for estimating modulation peak in broad-bandwidth interferometry," 5633715.
- [43] I. Gurov, E. Ermolaeva, and A. Zakharov, "Analysis of low-coherence interference fringes by the Kalman filtering method," *J. Opt. Soc. Am. A*, vol. 21, no. 2, pp. 242–251, Feb. 2004.
- [44] P. S. Maybeck, *Stochastic Models, Estimation, and Control*. Academic Press, 1979.
- [45] P. de Groot and L. Deck, "Surface Profiling by Analysis of White-light Interferograms in the Spatial Frequency Domain," *Journal of Modern Optics*, vol. 42, no. 2, pp. 389–401, Feb. 1995.
- [46] P. J. de Groot and L. L. Deck, "Surface profiling by frequency-domain analysis of white light interferograms," in *Optical Measurements and Sensors for the Process Industries*, 1994, vol. 2248, pp. 101–104.
- [47] M. E. Pawlowski, Y. Sakano, Y. Miyamoto, and M. Takeda, "Phase-Crossing Algorithm for White-Light Fringe Analysis," in *Lasers and Electro-Optics, 2005. CLEO/Pacific Rim 2005. Pacific Rim Conference on*, 2005, pp. 1691–1692.
- [48] D. Chen, "High-definition vertical-scan interferometry," 760592520-Oct-2009.
- [49] T. Dresel, G. Häusler, and H. Venzke, "Three-dimensional sensing of rough surfaces by coherence radar," *Appl. Opt.*, vol. 31, no. 7, pp. 919–925, Mar. 1992.

- [50] J.-H. Kim, S.-W. Yoon, J.-H. Lee, W.-J. Ahn, and H.-J. Pahk, "New algorithm of white-light phase shifting interferometry pursuing higher repeatability by using numerical phase error correction schemes of pre-processor, main processor, and post-processor," *Optics and Lasers in Engineering*, vol. 46, no. 2, pp. 140–148, Feb. 2008.
- [51] S.-W. Kim, M.-G. Kang, and S. Y. Lee, "White light phase-shifting interferometry with self-compensation of PZT scanning errors," in *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, 1999, vol. 3740, pp. 16–19.
- [52] S. Kiyono, W. Gao, S. Zhang, and T. Aramaki, "Self-calibration of a scanning white light interference microscope," *Opt. Eng.*, vol. 39, no. 10, pp. 2720–2725, Oct. 2000.
- [53] J. Hayes, "Dynamic interferometry handles vibration," *Laser Focus World*, vol. 38, no. 3, 01-Mar-2002.
- [54] A. Patel, "Vibration-Free Optical Tabletops Are Critical in the Lab." [Online]. Available: <http://www.mellesgriot.com/bioarticle/vibrationsuppression.htm>. [Accessed: 14-Sep-2009].
- [55] "Optical Table 101.pdf." [Online]. Available: <http://www.analytical-online.com/Application%20Notes/Photonics/Optical%20Table%20101.pdf>. [Accessed: 10-Oct-2009].
- [56] "Measuring Dynamic MEMS Devices." [Online]. Available: www.lot-orient.com/files/downloads/zygo/.../dynamicmems_eu01.pdf. [Accessed: 15-Feb-2012].
- [57] M. R. Hart, R. A. Conant, K. Y. Lau, and R. S. Muller, "Stroboscopic phase-shifting interferometry for dynamic characterization of optical MEMS," in *18th Congress of the International Commission for Optics*, San Francisco, CA, USA, 1999, vol. 3749, pp. 468–469.
- [58] K. Hanhijärvi, I. Kassamakov, V. Heikkinen, J. Aaltonen, L. Sainiemi, K. Grigoras, S. Franssila, and E. Hæggström, "Stroboscopic supercontinuum white-light interferometer for MEMS characterization," *Opt. Lett.*, vol. 37, no. 10, pp. 1703–1705, May 2012.
- [59] Liang-Chia Chen, Huang-Wen Laia, Yao-Ting Huang, Jui-Chin Chang, C. C. Chang, and Jin-Liang Chen, "Dynamic Nano-scale Surface Profilometry Using Stroboscopic Interferometry," in *33rd Annual Conference of the IEEE Industrial Electronics Society, 2007. IECON 2007*, 2007, pp. 3027–3032.

-
- [60] K. Hanhijärvi, J. Aaltonen, I. Kassamakov, K. Grigoras, L. Sainiemi, S. Franssila, and E. Hæggström, "Effect of LED spectral shift on vertical resolution in stroboscopic white light interferometry," *Proceedings of SPIE*, vol. 7003, no. 1, p. 70031S–70031S–11, Apr. 2008.
- [61] V. Heikkinen, J. Aaltonen, I. Kassamakov, B. Wälchli, H. Räikkönen, T. Paulin, and E. Hæggström, "Non-phosphor white LED light source for interferometry," *Proceedings of SPIE*, vol. 8133, no. 1, pp. 813309–813309–8, Sep. 2011.
- [62] G. Molnar and R. Tutsch, "Vertical scanning interferometry with a mixed-coherence light source," in *Optical Measurement Systems for Industrial Inspection V*, Munich, Germany, 2007, vol. 6616, pp. 661608–7.
- [63] V. Heikkinen, K. Hanhijärvi, J. Aaltonen, H. Räikkönen, B. Wälchli, T. Paulin, I. Kassamakov, K. Grigoras, S. Franssila, and E. Hæggström, "Hybrid light source for scanning white light interferometry-based MEMS quality control," *Proceedings of SPIE*, vol. 8082, no. 1, p. 80822O, May 2011.
- [64] C. Yang, A. Wax, R. R. Dasari, and M. S. Feld, "2? ambiguity-free optical distance measurement with subnanometer precision with a novel phase-crossing low-coherence interferometer," *Opt. Lett.*, vol. 27, no. 2, pp. 77–79, Jan. 2002.
- [65] D. W. Phillion, "General methods for generating phase-shifting interferometry algorithms," *Appl. Opt.*, vol. 36, no. 31, pp. 8098–8115, Nov. 1997.
- [66] H. Guo, Z. Zhao, and M. Chen, "Efficient iterative algorithm for phase-shifting interferometry," *Optics and Lasers in Engineering*, vol. 45, no. 2, pp. 281–292, Feb. 2007.
- [67] I.-B. Kong and S.-W. Kim, "General algorithm of phase-shifting interferometry by iterative least-squares fitting," *Opt. Eng.*, vol. 34, no. 1, pp. 183–188, Jan. 1995.
- [68] J. Millerd, N. Brock, J. Hayes, M. North-Morris, B. Kimbrough, and J. Wyant, "Pixelated Phase-Mask Dynamic Interferometers," in *Fringe 2005*, 2006, pp. 640–647.
- [69] J. Millerd, N. Brock,, J. Hayes,, B. Kimbrough,, M. North-Morris, and J. C. Wyant, "Vibration Insensitive Interferometry," in *6th International Conference on Space Optics*, 2006.

-
- [70] C. L. Koliopoulos, "Simultaneous phase-shift interferometer," in *Proceedings of SPIE*, San Diego, CA, USA, 1992, pp. 119–127.
- [71] G. Lai and T. Yatagai, "Generalized phase-shifting interferometry," *J. Opt. Soc. Am. A*, vol. 8, no. 5, pp. 822–827, May 1991.
- [72] C. Zhao and J. H. Burge, "Vibration-Compensated Interferometer for Surface Metrology," *Appl. Opt.*, vol. 40, no. 34, pp. 6215–6222, Dec. 2001.
- [73] A. Olszak and J. Schmit, "Scanning interferometry with reference signal," 6,624,89423-Sep-2003.
- [74] A. Olszak and J. Schmit, "High-stability white-light interferometry with reference signal for real-time correction of scanning errors," *Optical Engineering*, vol. 42, no. 1, pp. 54–59, Jan. 2003.
- [75] J. S. Harris, R. L. Fusek, and J. S. Marcheski, "Stroboscopic interferometer," *Appl. Opt.*, vol. 18, no. 14, pp. 2368–2371, Jul. 1979.
- [76] "Stroboscopic phase-shifting interferometry for dynamic characterization of optical MEMS." [Online]. Available: <http://cat.inist.fr/?aModele=afficheN&cpsidt=17386083>. [Accessed: 12-Nov-2010].
- [77] L.-C. Chen, Y.-T. Huang, X.-L. Nguyen, J.-L. Chen, and C.-C. Chang, "Dynamic out-of-plane profilometry for nano-scale full-field characterization of MEMS using stroboscopic interferometry with novel signal deconvolution algorithm," *Optics and Lasers in Engineering*, vol. 47, no. 2, pp. 237–251, Feb. 2009.
- [78] E. Novak, D.-S. Wan, P. Unruh, and J. Schmit, "Dynamic MEMS measurement using a strobed interferometric system with combined coherence sensing and phase information," in *International Conference on MEMS, NANO and Smart Systems, 2003. Proceedings*, 2003, pp. 285–288.
- [79] L. A. J. Davis, D. R. Billson, D. A. Hutchins, and R. A. Noble, "Visualizing acoustic displacements of capacitive micromachined transducers using an interferometric microscope," *ARLO*, vol. 6, no. 2, pp. 75–79, Apr. 2005.
- [80] I. Kassamakov, K. Hanhijärvi, I. Abbadi, J. Aaltonen, H. Ludvigsen, and E. Hæggröm, "Scanning white-light interferometry with a supercontinuum source," *Opt. Lett.*, vol. 34, no. 10, pp. 1582–1584, May 2009.

-
- [81] J. M. Dudley, L. Provino, N. Grossard, H. Maillotte, R. S. Windeler, B. J. Eggleton, and S. Coen, "Supercontinuum generation in air?silica microstructured fibers with nanosecond and femtosecond pulse pumping," *J. Opt. Soc. Am. B*, vol. 19, no. 4, pp. 765–771, Apr. 2002.
- [82] W. J. Wadsworth, A. Ortigosa-Blanch, J. C. Knight, T. A. Birks, T.-P. M. Man, and P. S. J. Russell, "Supercontinuum generation in photonic crystal fibers and optical fiber tapers: a novel light source," *J. Opt. Soc. Am. B*, vol. 19, no. 9, pp. 2148–2155, 2002.
- [83] S. Leon-Saval, T. Birks, W. Wadsworth, P. St. J. Russell, and M. Mason, "Supercontinuum generation in submicron fibre waveguides," *Opt. Express*, vol. 12, no. 13, pp. 2864–2869, Jun. 2004.
- [84] S. Coen, A. H. L. Chau, R. Leonhardt, J. D. Harvey, J. C. Knight, W. J. Wadsworth, and P. S. J. Russell, "White-light supercontinuum generation with 60-ps pump pulses in a photonic crystal fiber," *Opt. Lett.*, vol. 26, no. 17, pp. 1356–1358, 2001.
- [85] D. Reolon, M. Jacquot, I. Verrier, G. Brun, and C. Veillas, "Broadband supercontinuum interferometer for high-resolution profilometry," *Opt. Express*, vol. 14, no. 1, pp. 128–137, Jan. 2006.
- [86] E. Jung, J. H. Ryu, C. H. Hong, and H. Kim, "Optical Degradation of Phosphor-Converted White GaN-Based Light-Emitting Diodes under Electro-Thermal Stress," *Journal of The Electrochemical Society*, vol. 158, no. 2, pp. H132–H136, 2011.
- [87] M. Meneghini, L.-R. Trevisanello, F. de Zuani, N. Trivellin, G. Meneghesso, and E. Zanoni, "Extensive analysis of the degradation of phosphor-converted LEDs," 2009, p. 74220H–74220H–12.
- [88] "Thermally Activated Degradation of Phosphor-Converted White LEDs — LED professional - LED Lighting Technology, Application Magazine," 01-Feb-2010.
- [89] Chih-Feng Lu, Chi-Feng Huang, Yung-Sheng Chen, Wen-Yu Shiao, Cheng-Yen Chen, Yen-Cheng Lu, and Chih-Chung Yang, "Phosphor-Free Monolithic White-Light LED," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 15, no. 4, pp. 1210–1217, Aug. 2009.
- [90] "Strain-adjusted LED produces phosphor-free white light - Laser Focus World," 02-Jan-2009.

-
- [91] "LED Packaging 2011." Yole Développement, Jul-2011.
- [92] W. K. Chong, X. Li, and S. Wijesoma, "Computationally efficient signal modeling for vertical scanning interferometry," *Appl. Opt.*, vol. 49, no. 26, pp. 4990–4994, 2010.
- [93] C. J. R. Sheppard and K. G. Larkin, "Effect of numerical aperture on interference fringe spacing," *Appl. Opt.*, vol. 34, no. 22, pp. 4731–4734, 1995.
- [94] P. de Groot and X. Colonna de Lega, "Signal Modeling for Low-Coherence Height-Scanning Interference Microscopy," *Appl. Opt.*, vol. 43, no. 25, pp. 4821–4830, 2004.
- [95] "Olympus Microscopy Resource Center | Sources of Visible Light - Introduction." [Online]. Available: <http://www.olympusmicro.com/primer/lightandcolor/lightsourcesintro.html>. [Accessed: 01-Feb-2012].
- [96] WH Press, SA Teukolsky, WT Vetterling, and BP Flannery, "Numerical Recipes: The Art of Scientific Computing," in *Chapter 4. Integration of Functions*, 3rd ed., New York: Cambridge University Press, 2007.
- [97] G. Dahlquist and Å. Björck, *Numerical Methods in Scientific Computing: Volume 1*. Society for Industrial Mathematics, 2008.
- [98] "Elementary Function -- from Wolfram MathWorld." [Online]. Available: <http://mathworld.wolfram.com/ElementaryFunction.html>. [Accessed: 21-Mar-2012].
- [99] M. A. Branch, T. F. Coleman, and Y. Li, "A Subspace, Interior, and Conjugate Gradient Method for Large-Scale Bound-Constrained Minimization Problems," *SIAM J. Sci. Comput.*, vol. 21, no. 1, pp. 1–23, 1999.
- [100] A. Colin Cameron and F. A. G. Windmeijer, "An R-squared measure of goodness of fit for some common nonlinear regression models," *Journal of Econometrics*, vol. 77, no. 2, pp. 329–342, 1997.
- [101] R. G. D. Steel and J. H. Torrie, *Principles and procedures of statistics: with special reference to the biological sciences*. McGraw-Hill, 1960.
- [102] S.-W. Kim and G.-H. Kim, "Thickness-Profile Measurement of Transparent Thin-Film Layers by White-Light Scanning Interferometry," *Appl. Opt.*, vol. 38, no. 28, pp. 5968–5973, Oct. 1999.

- [103] "Nikon MicroscopyU | Fundamentals of Digital Imaging." [Online]. Available: <http://www.microscopyu.com/articles/digitalimaging/ccdintro.html>. [Accessed: 01-Feb-2012].
- [104] "IDS USB 2 UI-2220ME-C / USB 2 UI-2220ME-M." [Online]. Available: http://www.ids-imaging.com/frontend/products.php?cam_id=53. [Accessed: 21-Mar-2012].
- [105] J. F. Biegen, "Calibration requirements for Mirau and Linnik microscope interferometers," *Appl. Opt.*, vol. 28, no. 11, pp. 1972–1974, Jun. 1989.
- [106] K. Creath, "Calibration of numerical aperture effects in interferometric microscope objectives," *Appl. Opt.*, vol. 28, no. 16, pp. 3333–3338, Aug. 1989.
- [107] G. Schulz and K.-E. Elssner, "Errors in phase-measurement interferometry with high numerical apertures," *Appl. Opt.*, vol. 30, no. 31, pp. 4500–4506, Nov. 1991.
- [108] J. Schmit, P. Unruh, and D.-S. Wan, "Fringe spacing in white-light interferometry," 2004, vol. 5531, pp. 228–235.
- [109] W. K. Chong, X. Li, and S. Wijesoma, "Effects of phosphor-based LEDs on vertical scanning interferometry," *Opt. Lett.*, vol. 35, no. 17, pp. 2946–2948, 2010.
- [110] W. K. Chong, X. Li, and S. Yeng Chai, "Harnessing Spectral Property of Dual Wavelength White LED to improve Vertical Scanning Interferometry," *Appl. Opt.*, vol. 52, no. 19, pp. 4652–4662, Jul. 2013.
- [111] W. K. Chong, X. Li, and S. Yeng Chai, "Spectral effects of dual wavelength low coherence light source in white light interferometry," *Opt Laser Eng.*, vol. 51, no. 6, pp. 651–655, Jun. 2013.
- [112] H. Yamamoto, "White LED phosphors: the next step," *Proceedings of SPIE*, vol. 7598, no. 1, pp. 759808–759808–10, Feb. 2010.
- [113] S. Fujita, S. Yoshihara, A. Sakamoto, S. Yamamoto, and S. Tanabe, "YAG glass-ceramic phosphor for white LED (I): background and development," *Proceedings of SPIE*, vol. 5941, no. 1, pp. 594111–594111–7, Aug. 2005.
- [114] S. Tanabe, S. Fujita, S. Yoshihara, A. Sakamoto, and S. Yamamoto, "YAG glass-ceramic phosphor for white LED (II): luminescence characteristics," *Proceedings of SPIE*, vol. 5941, no. 1, pp. 594112–594112–6, Aug. 2005.

-
- [115] "Phosphor-Coated LED Converts Blue Light to White," *Photonics.com*.
- [116] W. Walter, "Determination of correlated color temperature based on a color-appearance model," *Color Research & Application*, vol. 17, no. 1, pp. 24–30, Feb. 1992.
- [117] R. Paschotta, *Encyclopedia of Laser Physics and Technology*. Wiley-VCH, 2008.
- [118] "LEDs Magazine - NEMA publishes white-LED binning standard (SSL 3-2010)," *NEMA publishes white-LED binning standard (SSL 3-2010)*, 15-Apr-2010. [Online]. Available: <http://www.ledsmagazine.com/news/7/4/17>. [Accessed: 04-Oct-2011].
- [119] "NEMA SSL 3 - High-Power White LED Binning for General Illumination." National Electrical Manufacturers Association, Mar-2011.
- [120] M. Meneghini, L. Trevisanello, C. Sanna, G. Mura, M. Vanzi, G. Meneghesso, and E. Zanoni, "High temperature electro-optical degradation of InGaN/GaN HBLEDs," *Microelectronics Reliability*, vol. 47, no. 9–11, pp. 1625–1629, September.
- [121] C. M. Tan, B. K. Eric Chen, G. Xu, and Y. Liu, "Analysis of humidity effects on the degradation of high-power white LEDs," *Microelectronics Reliability*, vol. 49, no. 9–11, pp. 1226–1230, Sep. 2009.
- [122] M. Meneghini, L.-R. Trevisanello, G. Meneghesso, and E. Zanoni, "A Review on the Reliability of GaN-Based LEDs," *Device and Materials Reliability, IEEE Transactions on*, vol. 8, no. 2, pp. 323–331, 2008.
- [123] J. Cohen, *Statistical Power 2nd Ed*. Routledge, 1988.
- [124] A. Buda, "Life time of correlation between stocks prices on established and emerging markets," *arXiv:1105.6272*, May 2011.
- [125] S. S. C. Chim and G. S. Kino, "Correlation microscope," *Opt. Lett.*, vol. 15, no. 10, pp. 579–581, May 1990.
- [126] H. Fang, L. W. Sang, L. B. Zhao, S. L. Qi, Y. Z. Zhang, X. L. Yang, Z. J. Yang, and G. Y. Zhang, "Luminescent properties in the strain adjusted phosphor-free GaN based white light-emitting diode," *Applied Physics Letters*, vol. 93, no. 26, pp. 261117–261117–3, Dec. 2008.

-
- [127] M. Yamada, Y. Narukawa, and T. Mukai, "Phosphor Free High-Luminous-Efficiency White Light-Emitting Diodes Composed of InGaN Multi-Quantum Well," *Jpn. J. Appl. Phys.*, vol. 41, pp. L246–L248, 2002.
- [128] M. E. Pawlowski, Y. Sakano, Y. Miyamoto, and M. Takeda, "Phase-crossing algorithm for white-light fringes analysis," *Optics Communications*, vol. 260, no. 1, pp. 68–72, Apr. 2006.
- [129] "Advanced Measurement Laboratory." [Online]. Available: http://www.nist.gov/public_affairs/factsheet/amlbrochure.cfm. [Accessed: 28-Feb-2012].