Predictive modeling for composites properties based on reinforcing fibre architecture

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**PREDICTIVE MODELING FOR COMPOSITES
PROPERTIES BASED ON REINFORCING FIBRE
ARCHITECTURE**

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List of Symbols

English symbols:

A	Area
AR	Yarn aspect ratio
B	Power term
C	Elasticity compliance matrix
d	Fiber diameter
E	Young's (Elastic) Modulus
E	Elasticity stiffness matrix
F	Force
G	Shear modulus
h	Height of yarn
h	Hypotenuse length of the interface element
k	Bulk modulus
l	Length of the element
L	Length
M_r	Material modulus ratio
N	Number of divisions
p	Pitch
$Powers$	$Powers$ variable in the elliptical scaling factor equation
R	Radius of reinforcement
r	Ratio of vertical half-gap to horizontal half-gap
S	Length of square unit cell
t	Thickness
u	Undulation
V	Volume fraction

Greek symbols:

α	Ratio of elliptical to circular transverse Young's modulus
β	Ratio of elliptical to circular transverse Poisson's ratio
γ	Fiber separation parameter, shear force deformation
δx	Horizontal edge length of the interface element
δy	Vertical edge length of the interface element
Δ	Displacement
ε	Tensile strain
ϵ	Porosity
η	Modulus ratio
θ	Angle
ξ	Empirical parameter
ρ	Half-gap between reinforcements
ν	Poisson ratio
σ	Normal stress
τ	Shear stress

Subscripts:

a	Semi-major axis
b	Semi-minor axis
cir	circular
ell	elliptical
eq	equivalent
f	Fibers
$fill$	Fill

g	Gap section
L	Longitudinal
m	Matrix
N	Normal
PS	Parallel-Series
$reinf$	Reinforcement
rot	rotational
s	Shear
st	Straight section
SP	Series-Parallel
T	Transverse
u	Undulation section
w	Weft
$woven$	Woven
x	X-directional
y	Y-directional
$yarn$	Yarn

Abstract

Use of structurally reinforced composites is widespread in many industries and applications due to their high strength-to-weight ratio. For textile fiber architectures, their entwined nature ensures that multi-directional integrity is maintained without being affected much by climate and other wear-and-tear. Different types of fiber reinforcement architectures impart different properties to the corresponding composites.

This study seeks to compare the mechanical properties of differently structured performs – namely, unidirectional, woven and braided textiles. Novel expressions for the transverse Young's modulus and Poisson's ratio of unidirectional fibers were derived to provide improvements over the contemporary computational and classical models. These new expressions make use of shear effects at the yarn-matrix interface and are adapted for non-circular yarns as well.

Consequently, mechanical property predictors for woven and braided textile composites are created. Geometric and mechanical stiffness models in MATLAB® using Classical Lamination Theory (CLT) are developed. These models make use of contemporary as well as the new-improved yarn formulations. As envisaged, improvements over the existing textile modeling work are observed. A general modeling methodology that can be used for implementing CLT for predicting properties of other textiles (such as knitted) is also established.

Chapter 1: Introduction

There has been a phenomenal rise in the use of man-made composites in the past fifty years. The ability to combine two materials at the macro-level in order to create a composite that has enhanced properties is of particular interest to the aerospace industry with its emphasis on superior strength-to-weight and stiffness-to-weight ratios as well economy of cost.

Within the domain of composites research itself, textile pre-forms are beginning to demonstrate their significant advantage. The superior in-plane and off-axis properties of planar textile reinforced composites enables the design of sturdier and balanced structures without the defects that are routinely encountered in layered laminates of directional fiber-reinforced composites. Additionally, the use of textile manufacturing techniques leads to shorter build times and better large-scale manufacturability.

1.1 Background & Motivation

There is a constant need for strong lightweight composite materials within the aerospace sector, where weight penalties can substantially limit capabilities and progress. The characteristics of composites themselves are greatly affected by the quality of fiber pre-forming. This study aims at exploring the underlying theory and patterns of fiber preform composites, so as to provide a means to accurately predict the mechanical properties of composites (manufactured using unidirectional fibers, fiber weaving and braiding) and enable appropriate design guidelines and benchmarks to be established. This ability to design stronger, lighter and economical products will have spillover effects into the defense, automobile and consumer industries.

Additionally, in cases where there are gaps in the underlying material and composites research (leading to inadequacies in building up the prediction models), a micromechanical bottoms-up approach is desired to establish and validate mathematical and empirical solutions, and then extend these to more complicated preform structures as well. This requirement to have a consistent micromechanical approach, in predicting the material composite properties at both the yarn-level as well as textile-level, drives the motivation of this study. A consistent easy-to-apply micromechanical approach for composites would also inspire research work into other complicated preforms, such as knitted textiles.

1.2 Project Methodology

Initially, a literature review was carried out to build up the sufficient domain knowledge and determine the geometrical and physical parameters that affect the design of fiber pre-forms. Since weaving and braiding methods have its base in textiles, a comprehensive study of the techniques and nomenclature used for textile structures was also done.

During the course of this study, different types of reinforcement structures were looked at. Firstly, in the case of long fiber composites, the lack of a comprehensive and accurate formulation for the micromechanics-based transverse direction Young's modulus and Poisson ratio in available literature led to the development of a wholly independent formulation for the same. Also, an FEA-based semi-empirical scaling factor system was developed to convert the transverse elastic properties of circular yarn (a bundle of fibers used in forming reinforcement in composites) properties to their elliptical equivalents. This was further used to develop and compare geometrical structural models in MATLAB[®] for simulating woven and braided textile composites using Classical Lamination Theory (CLT).

1.3 Objectives

This project seeks to:

- (1) Gain understanding of the different types of fiber pre-forms.
- (2) Develop novel formulations for transverse mechanical elastic properties for resin-infused yarns and assess their performance in long fiber-reinforced and textile-reinforced composites.
- (3) Derive semi-empirical FEA-based mathematical models to convert available elastic transverse mechanical properties of circular in cross-sections, resin-infused yarns to equivalent properties of elliptical in cross-sections, resin-infused yarns
- (4) Determine the relative merits and de-merits of weaving and braiding of pre-forms in terms of geometric and mechanical characteristics.

- (5) Formulate user-friendly predictive models, where appropriate, for mechanical properties of the resulting composites.
- (6) Deduce guidelines for design of textile composites in terms of ready-made programming-based formulations that can accurately predict composite properties

1.4 Scope

This study seeks to simulate fiber textile pre-forms for use as reinforcements in composites. Therefore, the textile geometries will be created and the mechanical properties obtained using programming language MATLAB[®]. Novel formulations of the transverse Young's Modulus and transverse Poisson's ratio for unidirectional fibers have been created. Additionally, appropriate mathematical means of converting these properties from circular to elliptical models have been derived using Finite Element Analysis (FEA) software Abaqus and data-analysis techniques.

Consequently, the geometric and mechanical stiffness models (using CLT) for the properties of the one-directional planar as well as two-directional 3D woven unit cells have been developed. A similar model is created for two-directional 3D braided unit cell models, involving an adaptation of the 3D woven model (with transformations of geometry and material orientations and geometries).

Throughout the textile model developmental work, studies are carried out to compare the effects of CLT summation techniques, yarn input models and textile structure variations on mechanical properties. It must be noted that only the final composite structures are analysed; the manufacturing process that results in the creation of these composites is not looked at. Also, the mechanical properties that will be investigated and contrasted are the elastic properties; damage and non-elastic properties are not looked at within this study.

1.5 Structure of this report

Based on the project methodology and objectives discussed, the remainder of this report is structured as thus:

Chapter 2: Literature Review – This section looks at previous work in the field of fiber architectures for composites, dealing with comparisons of textile structures, elastic properties of unidirectional long fiber-reinforced composites, applying unit cell analysis to adapt circular yarn properties to elliptical yarn properties, and models on woven and braided textile composites. Regions of improvement are identified as motivations for this study.

Chapter 3: Shear-Inclusive Transverse Young's Modulus – This section focusses on conceptualizing a square unit cell with quarter circle reinforcement, looking at deformations at the reinforcement-matrix interface to normal and shear interactions and deriving a novel formulation for the transverse direction Young's modulus. This formulation is then validated by comparing to other unit cell models, existing classical & computational models and available experimental data.

Chapter 4: Shear-Inclusive Transverse Poisson Ratio – Using the same unit cell concept discussed in the previous chapter, this section focusses on looking at Poisson effects at the reinforcement-matrix interface to derive a novel formulation for the transverse Young's modulus. Validation studies using FEA software Abaqus is carried out to test the derived expression.

Chapter 5: Shear-Inclusive Elliptical (SIE) Yarn Properties – Having obtained the shear-inclusive transverse properties for circular yarn inputs, mathematical scaling factors were determined using FEA-based data modeling to convert these circular yarn transverse properties to elliptical yarn transverse properties. These scaling factors are then applied to independent FEA results to assess their performance.

Chapter 6: Woven Textile Composites Modeling – With improved yarn transverse property input models obtained for unidirectional reinforcements, a woven textile geometry modeler and final composite elasticity prediction program is coded in MATLAB. This is then used to not only compare the model's fidelity to experimental results, but also to provide a means of comparisons between the SIE, SIC and other prevalent classical models.

Chapter 7: Braided Textile Composites Modeling – The woven textile composite code from the previous section is now adapted using equivalent geometry considerations and yarn property orientation transformations to predict elastic properties of braided textile reinforced composites. 1-by-1 diamond braids are thus converted to plain weaves for analysis. The

predicted properties are then compared to experimental results on diamond braids with the use of various yarn input models.

Chapter 8: Conclusions and Recommendations – Having carried out study of the material property inputs and geometry features of fiber structures in composites, a summary of results and observations is presented. Future work suggestions are also provided.

Chapter 2: Literature Review

In order to effectively compare the properties of different types of textile performs, it is important to develop sufficient knowledge on textile performs as well as the mechanical properties of composite materials. This led to the literature review to be split into four major parts:

- i. A study of structural fiber composites in general, to understand the varying features that exist in terms of structural geometry and mechanical properties. Uni-directional long-fiber reinforced and planar woven & braided fiber structures are investigated.
- ii. An exploration into the transverse properties of fiber-reinforced composites to look at advantages and disadvantages of existent prediction methods; to provide a sound theoretical basis for a new formulation of the transverse direction Young's modulus & transverse direction Poisson ratio and a means of comparison with the classical theory and numerical estimations.
- iii. A brief look into the factors that affect the mechanical properties of long-fiber reinforced composites with non-circular yarns, especially those with elliptical cross-sections;
- iv. An in-depth analysis of woven and braided fabric textile composites, to understand its characteristics, as well setting up of woven and braided textile composite model codes using MATLAB. This enables the group to appreciate the intricacies of textile composites and exploit their similarities in building the geometry-driven structural models and property prediction methodology.

2.1 Structural Fiber Composites

Composites are now being used in aerospace, automotive, marine and other industries. Compared to conventional structural materials, composites may be relatively new; however, they bring marked improvements in strength, modulus, toughness and wear resistance compared to conventional materials [1]. The growth and confidence in composite usage has been spurred by its successful applications in the aircraft industry. The ability to build

composite structures confidently not only led to an increase in popularity of use, but also greater economy as the costs of composite raw materials fell [2, 3].

Long fiber reinforced composites make use of continuous fiber impregnated with resin and have been used extensively as stacks of unidirectional reinforced laminae [1]. The unidirectional lamina within a multidirectional laminate composite can be stacked in various directions as shown in Figure 2.1(a) to provide planar stiffness and strength to the composite.

Figure 2.1(a): Stack of Long Fiber-Reinforced Laminae [www.appleinsider.com] (b) Material orientations within a unidirectional composites

Figure 2.1(b) shows the material orientation within a single layer of unidirectional composite. The reinforcement can be fibers or an agglomeration of fibers (known as yarn), surrounded by the matrix. Within this structure, the reinforcements (due to their stronger properties) are the principal load-bearing members, while the matrix provides continuity within the composite lamina and bridging between the reinforcements.

The entire lamina therefore exhibits stiffer properties in the natural direction of the reinforcements (longitudinal direction) and weaker properties in the orthogonal plane perpendicular to the direction of the reinforcements (transverse direction). Thus, the well-defined structural features of the unidirectional composite enable it to be treated as an orthotropic composite and in particular, as a transversely isotropic composite, where the transverse planar properties are isotropic in nature and follow their corresponding

conventions [4]. There exist classical and computational techniques to obtain the consolidated composite properties of the longitudinal and transverse directions of this layer, and these techniques and their shortcomings are discussed in detail later on. As mentioned before, stacking many laminae layers with different orientations produces a laminate whose consolidated planar mechanical properties are better than those of a unidirectional lamina; these can be calculated via Classical Lamination Theory (CLT) [4].

On the other hand, fiber textile composites are materials that use textile fabrics as reinforcements. These textile fabrics follow a periodic pattern and can be designed to provide superior mechanical properties in more than one direction within the same lamina. This is due to the interweaving fibers or yarns that reinforce the lamina along the plane [5].

As a result, a single textile reinforcement lamina has better in-plane properties than a single unidirectional reinforcement lamina. Additionally, a single textile lamina does not suffer much from delamination effects as compared to multiple unidirectional laminae stacked together to improve in-plane properties. Hence, it has become increasingly common to make use of textile composites for producing strong and stiff lightweight structures. The commonly found textile structures are shown in Table 2.1.

Table 2.1: (a) Woven carbon textile (b) Braided textile lamina (c) Knitted glass fabric

(a)	(b)	(c)
<p>Woven structure [6]</p>	<p>Braided structure [7]</p>	<p>Knitted structure [8]</p>
<p>Woven textile [9]</p>	<p>Braided textile [10]</p>	<p>Knitted textile [11]</p>

Woven textiles consist of yarns that are intertwined at orthogonal angles ($0/90$ orientation) to each other as shown in Table 2.1(a). As a result, the corresponding composite is expected to exhibit the best properties along these two directions. Braided textiles are created by intertwining yarns at braiding bias angles ($+0/-0$) which are non-orthogonal to each other as shown in Table 2.1(b). Thus, braided textiles can be designed to directionally exhibit certain characteristics, by altering the braiding bias angles [12]. Lastly, knitted textiles follow a convoluted intertwining as shown in Table 1(c), resulting in a reinforced composite that is less stiff than woven or braided, but more resistant to damage [13].

Due to the periodic and highly regular structure of textile composites, the properties of the geometry can be studied by making use of the textile unit cell. The unit cell comprises of the smallest repeatable unit of the textile geometry, such that the geometrical and mechanical properties of the unit cell represent the same as that of the entire textile. The unit cells are shown above as being extracted from the textile pattern.

Textile composites were introduced as an improvement to long fiber-reinforced composites (which exhibit desired properties along a single principal direction only). Additionally, the reinforcement for textile composites is less labor-intensive as compared to fibrous laminated composites, due to the widespread textile manufacturing processes that can prepare large volumes of yarns into dry preforms [14, 15]. This also results in shorter build time, complex shape formability and the ease of mold impregnation [16] due to the intertwined structures. Woven textile composites are useful for desired properties in orthogonal directions. The yarns in the two orthogonal planar directions can thus be chosen, so as to optimize the target material requirement. Ishikawa & Chou [17] explored the mechanical stiffness and compliance properties of woven fabric composites by use of Classical Laminate Theory

(CLT) over the entire unit cell. Ishikawa [18] has done similar work related to satin weave composites. Ju & Zhang [19] looked at local pair-wise fiber interactions with the use of ensemble-averaged field equations. Ishikawa & Chou [20] have focused on the elastic stiffness and strength of woven textile reinforced composites at different lengths of periodicity, in particular making use of mosaic models, fiber undulation and bridging models. Also, Hill [21] provides a very detailed mathematical approach to composite materials.

However, woven textiles lose some of their efficiency for composite property requirements that may not be in two mutually perpendicular directions. In fact, for creating composites with properties that are appropriate in two directions not perpendicular to each other, braided composites are better suited. With the ability to align the fibers in non-orthogonal directions, the material properties can be actively enhanced. Additionally, due to their oblique arrangement, there is greater interaction between yarns and thus, braided composites exhibit higher resistances to twisting, shearing and impact [5].

Tsai *et al.* [7] utilized a parallelogram spring-like model for 2D braided composites. Finite Element Method (FEM) was used to compare with theoretical and experimental data from literature. Zhang & Harding [22] provided a Strain Energy Equivalence Principle method used for woven fabric to be used effectively for braided fabric as well. A fabric geometry model for predicting elastic constants of warp knit composites has been provided by Ko *et al.* [23].

Since the project is focused on comparing woven and braided composites, it is imperative to have a common methodology to determine the mechanical properties for all. The simplicity of application of Classical Lamination Theory [4] makes it an attractive option to be applied in creation of the material models. However, to ensure that sufficiently accurate tow properties are input into a woven or braided model, unidirectional tow or lamina properties have to be studied in detail.

2.2 Transverse Elastic Properties of Unidirectional Composites

Unidirectional fiber-reinforced lamina forms the basic composite block for modeling laminates as well as woven and textile composites. Many classical methods are available to determine the longitudinal and transverse elastic properties [24, 25, 27-30]. Some of these methods are relatively straightforward to use (e.g. Rule of Mixture, Halpin-Tsai, etc.), while

others require a rigorous set of calculations before they can be used to predict final composite properties (e.g. Hashin and Rosen).

2.2.1 Classical mathematical models

For composites, the rule of mixtures is commonly used to determine properties. Its basis lies in obtaining a weighted average of the final composite properties from the properties of the constituent fibers and matrix and their corresponding volume fractions [24]. The derivation of the simple and inverse rule of mixture formulations for the longitudinal and transverse Young's modulus is shown below.

Table 2.2: Derivation of Young's moduli in longitudinal and transverse directions [24]

<p>Consider a laminate with three regions layered with unique thicknesses, longitudinal Young's moduli (E_{LL}) and transverse Young's moduli (E_{TT}). The properties can be determined by applying loads in the longitudinal and transverse directions and determining the mechanical response of the laminate.</p>	
Longitudinal Load (Iso-strain condition)	Transverse Load (Iso-stress condition)

$F_{total} = F_1 + F_2 + F_3$ $\Rightarrow \sigma_{eq} A_{eq} = \sigma_1 A_1 + \sigma_2 A_2 + \sigma_3 A_3$ <p style="text-align: center;">Substituting $\sigma = E_L \varepsilon$:</p>	$\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ $\Rightarrow \varepsilon_{eq} t = \varepsilon_1 t_1 + \varepsilon_2 t_2 + \varepsilon_3 t_3$ <p style="text-align: center;">Substituting $\varepsilon = \sigma / E_T$:</p>

$\Rightarrow E_{Leq} \varepsilon_{eq} A_{eq} = E_{L1} \varepsilon_1 A_1 + E_{L2} \varepsilon_2 A_3 + E_{L3} \varepsilon_3 A_3$ <p>Since $\varepsilon_{eq} = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \frac{\Delta L}{L}$ (Isostrain condition)</p> $\Rightarrow E_{Leq} \left(\frac{\Delta L}{L} \right) A_{eq} = E_{L1} \left(\frac{\Delta L}{L} \right) A_1 + E_{L2} \left(\frac{\Delta L}{L} \right) A_3 + E_{L3} \left(\frac{\Delta L}{L} \right) A_3$ $\Rightarrow E_{Leq} \left(\frac{\Delta L}{L} \right) (t_1 + t_2 + t_3) w = E_{L1} \left(\frac{\Delta L}{L} \right) (t_1) w + E_{L2} \left(\frac{\Delta L}{L} \right) (t_2) w + E_{L3} \left(\frac{\Delta L}{L} \right) (t_3) w$ <p>Dividing throughout by $\left(\frac{\Delta L}{L} \right)$ and simplifying:</p> $\Rightarrow E_{Leq} = E_{L1} (V_{f1}) + E_{L2} (V_{f2}) + E_{L3} (V_{f3})$ <p>where $V_{f1} + V_{f2} + V_{f3} = 1$</p>	$\Rightarrow \frac{\sigma_{eq}}{E_{Teq}} t = \frac{\sigma_1}{E_{T1}} t_1 + \frac{\sigma_2}{E_{T2}} t_2 + \frac{\sigma_3}{E_{T3}} t_3$ <p>Since $\sigma_{eq} = \sigma_1 = \sigma_2 = \sigma_3$ (Isostress condition)</p> $\Rightarrow \frac{1}{E_{Teq}} = \frac{\frac{t_1}{E_{T1}} + \frac{t_2}{E_{T2}} + \frac{t_3}{E_{T3}}}{(t_1 + t_2 + t_3)}$ $\Rightarrow \frac{1}{E_{Teq}} = \frac{V_{f1}}{E_{T1}} + \frac{V_{f2}}{E_{T2}} + \frac{V_{f3}}{E_{T3}}$ <p>where $V_{f1} + V_{f2} + V_{f3} = 1$</p>
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The equations thus derived are:

Iso-strain condition : $E_{Leq} = E_{L1} (V_{f1}) + E_{L2} (V_{f2}) + E_{L3} (V_{f3}) \quad \dots(2.1a)$

Iso-stress condition : $\frac{1}{E_{Teq}} = \frac{V_{f1}}{E_{T1}} + \frac{V_{f2}}{E_{T2}} + \frac{V_{f3}}{E_{T3}} \quad \dots(2.1b)$

Similar equations are used to determine the overall composite Poisson ratio as shown in Table 2.3.

Table 2.3: Derivation of Poisson's ratio in longitudinal and transverse directions [24]

Longitudinal Load (Iso-strain condition)	Transverse Load (Iso-stress condition)
<p>The Poisson ratio here refers to ν_{xy} i.e. the Poisson contraction of y-direction from elongation in x-direction</p>	<p>The Poisson ratio here refers to ν_{yx} i.e. the Poisson contraction of x-direction from elongation in y-direction</p>
$t = t_1 + t_2 + t_3$ $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ $\Rightarrow \varepsilon_{y,eq} t = \varepsilon_{y,1} t_1 + \varepsilon_{y,2} t_2 + \varepsilon_{y,3} t_3$ $\Rightarrow -\nu_{Leq} \varepsilon_{x,eq} t$ $= -\nu_{L1} \varepsilon_{x,1} t_1 - \nu_{L2} \varepsilon_{x,2} t_2 - \nu_{L3} \varepsilon_{x,3} t_3$ <p>Since $\varepsilon_{x,eq} = \varepsilon_{x,1} = \varepsilon_{x,2} = \varepsilon_{x,3} = \left(\frac{\Delta L}{L}\right)$ (Isostrain condition)</p> $\Rightarrow -\nu_{Leq} \left(\frac{\Delta L}{L}\right) t = -\nu_{L1} \left(\frac{\Delta L}{L}\right) t_1 - \nu_{L2} \left(\frac{\Delta L}{L}\right) t_2 - \nu_{L3} \left(\frac{\Delta L}{L}\right) t_3$ <p>Dividing throughout by $\left(\frac{\Delta L}{L}\right)$</p> $\Rightarrow \nu_{Leq} = \nu_{L1} \frac{t_1}{t} + \nu_{L2} \frac{t_2}{t} + \nu_{L3} \frac{t_3}{t}$	$t = t_1 + t_2 + t_3$ $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ $\Rightarrow \varepsilon_{eq} t = \varepsilon_{y,1} t_1 + \varepsilon_{y,2} t_2 + \varepsilon_{y,3} t_3$ $\Rightarrow -\frac{\varepsilon_{x,eq} t}{\nu_{Teq}} = -\frac{\varepsilon_{x,1} t_1}{\nu_{T1}} - \frac{\varepsilon_{x,2} t_2}{\nu_{T2}} - \frac{\varepsilon_{x,3} t_3}{\nu_{T3}}$ <p>Since $\varepsilon_{x,eq} = \varepsilon_{x,1} = \varepsilon_{x,2} = \varepsilon_{x,3} = \left(\frac{\Delta L}{L}\right)$ and $\Delta L = \Delta L_1 = \Delta L_2 = \Delta L_3$</p> $\Rightarrow -\left(\frac{\Delta L}{L}\right) \frac{t}{\nu_{Teq}} = -\left(\frac{\Delta L}{L}\right) \frac{t_1}{\nu_{T1}} - \left(\frac{\Delta L}{L}\right) \frac{t_2}{\nu_{T2}} - \left(\frac{\Delta L}{L}\right) \frac{t_3}{\nu_{T3}}$

$\Rightarrow \nu_{Leq} = \nu_{L1} V_{f1} + \nu_{L2} V_{f2} + \nu_{L3} V_{f3}$ <p style="text-align: center;">where</p> $V_{f1} + V_{f2} + V_{f3} = 1$	<p style="text-align: center;">Dividing throughout by $\left(\frac{\Delta L}{L}\right)$</p> $\Rightarrow \frac{1}{\nu_{Teq}} = \frac{1}{\nu_{T1}} \frac{t_1}{t} + \frac{1}{\nu_{T2}} \frac{t_2}{t} + \frac{1}{\nu_{T3}} \frac{t_3}{t}$ $\Rightarrow \frac{1}{\nu_{Teq}} = \frac{V_{f1}}{\nu_{T1}} + \frac{V_{f2}}{\nu_{T2}} + \frac{V_{f3}}{\nu_{T3}}$ <p style="text-align: center;">where</p> $V_{f1} + V_{f2} + V_{f3} = 1$
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The equations thus derived are:

Iso-strain condition : $\nu_{Leq} = \nu_{L1}(V_{f1}) + \nu_{L2}(V_{f2}) + \nu_{L3}(V_{f3}) \quad \dots(2.2a)$

Iso-stress condition : $\frac{1}{\nu_{Teq}} = \frac{V_{f1}}{\nu_{T1}} + \frac{V_{f2}}{\nu_{T2}} + \frac{V_{f3}}{\nu_{T3}} \quad \dots(2.2b)$

Though this derivation has been carried out for laminates, the iso-strain condition is valid and can be safely used for long-fiber reinforced composites as well. However, the iso-stress condition requires special cases of load carrying and is highly unrealistic except in the case of perfectly bonded laminates.

In fact, while the regular rule of mixtures equations for longitudinal elastic properties correlate well with experiments, the inverse rule of mixtures used for the transverse Young's modulus has poor correlation with experimental data [24]. The transverse modulus formulation does not take into consideration the transverse plane geometry of composite; hence, it's considered inadequate. A sufficiently accurate model is desired since the properties in the transverse direction are quite important for the long-fiber reinforced composites [25]; therefore, it is thus imperative to study other classical models that show better correlation to experimental data.

Amongst classical models for the transverse Young's modulus for fiber-reinforced yarns, the Halpin-Tsai model is taken as the industry standard [26]. The formulation in the case of Transverse Young's Modulus for this model is shown in Equations 2.3a and 2.3b .

$$E_{eq} = E_m \frac{1 + \xi_E \eta_E V_f}{1 - \eta_E V_f} \quad \dots(2.3a)$$

$$\eta_E = \frac{\frac{E_f}{E_m} - 1}{\frac{E_f}{E_m} + \xi_E} \quad \dots(2.3b)$$

The term ξ_E is an empirical parameter based on the aspect ratio of the reinforcement and takes a value of 2 in the case of square and circular fibers according to Halpin & Kardos [26]. Besides being a self-consistent micromechanical model, Halpin-Tsai is also very versatile; in fact, this formulation can be used to determine shear modulus (or other elasticity constants) as shown in Equations 2.4a and 2.4b .

$$G_{eq} = G_m \frac{1 + \xi_G \eta_G V_f}{1 - \eta_G V_f} \quad \dots(2.4a)$$

$$\eta_G = \frac{\frac{G_f}{G_m} - 1}{\frac{G_f}{G_m} + \xi_G} \quad \dots(2.4b)$$

In this case, ξ_G takes a value of 1 for square and circular fibers [25]. However, this model does not provide a direct equation for the transverse Poisson ratio. Hence, it is suggested to use the transverse isotropic relation to obtain this indirectly from the shear modulus [27].

$$\nu_{eq} = \frac{E_{eq}}{2G_{eq}} - 1 \quad \dots(2.5)$$

However, in the case of very high fiber volume fractions, the value of 2 for ξ_E causes an under-estimation of the transverse Young's modulus. Hence, Hewitt & de Malherbe [28] have suggested an empirical correction for ξ represented by the term in Equation 3 .

$$\xi_E = 2 + 40V_f^{10} \quad \dots(2.6)$$

The presence of the empirical parameter ξ disadvantages the Halpin-Tsai model as being semi-empirical. As such, Spencer [29] has tried to improve the Halpin-Tsai Transverse Young's modulus estimation by eliminating the empirical parameter and instead, taking into consideration the fiber packing characteristics. The model presented by Equations 2.7a, 2.7b and 2.7c focus on the parts of minimum clearance between adjacent fibers.

$$E_{eq} = E_m \left(\frac{\gamma-1}{\gamma} + \frac{1}{k} \left(-\frac{\pi}{2} + \frac{2\gamma}{\sqrt{\gamma^2-k^2}} \tan^{-1} \sqrt{\frac{\gamma+k}{\gamma-k}} \right) \right) \quad \dots(2.7a)$$

$$k = 1 - \frac{E_m}{E_f} \quad \dots(2.7b)$$

$$\gamma = \frac{1}{\sqrt{(1.1V_f^2 - 2.1V_f + 2.2)V_f}} \quad \dots(2.7c)$$

The term γ is the fiber separation parameter used to represent square and triangular arrays based on the value of volume-fraction of fibers as shown in Table 2.4.

Table 2.4: Variation of packing array with fiber volume fraction

Triangular packing	Square packing

$\gamma = \frac{p}{d};$ $V_f = \frac{1}{\gamma^2} \frac{\pi}{2\sqrt{3}}$ $\gamma^2 = \frac{1}{1.103V_f}$	$\gamma = \frac{p}{d};$ $V_f = \frac{1}{\gamma^2} \frac{\pi}{4}$ $\gamma^2 = \frac{1}{1.272V_f}$

Thus, Equation 2.7c is an all-encompassing equation that is used to represent both the triangular and square cases shown in Table 2.4.

Also, Hashin & Rosen [30] have provided the highly sophisticated Composite Cylinder Assemblage (CCA) model that can be used to determine a wide range of elasticity parameters. The CCA model is derived by a variational energy method and makes use of representative volume elements (RVEs) using hexagonal arrays. The model provides good correlations with experimental data for overall performance, but its usability is impaired by its complicated formulation and the consideration of three-dimensional effects.

The de Morais model [31] is a micromechanics model that makes use of square unit cells and a relatively simple mechanics of materials analysis. The single equation model is given by Equation 5 .

$$\frac{1}{E_{eq}} = \frac{\sqrt{V_f}}{\frac{\sqrt{V_f}}{E_f} + (1 - \sqrt{V_f})\frac{1 - v_m^2}{E_m}} + (1 - \sqrt{V_f})\frac{E_m}{1 - v_m^2} \quad \dots(2.8)$$

This model provided good agreement with a three-dimensional finite element cell model in the shape of a hexagon, as well as Aboudi's method of cells approach [32]. Paley & Aboudi [33] expanded the method of cells approach to an arbitrary number of sub-cells, more than the original four.

It is difficult to find direct formulations for transverse Poisson ratio for unidirectional composites. Some cases focus on obtaining reliable means of estimating the various moduli of stiffness and indirectly work out the corresponding Poisson ratio (as shown by Equation 2.5). However, it is felt to be fundamental to use the same micromechanical theories that give Young's modulus to obtain direct Poisson's ratio formulations, as they provide an important re-verification of the underlying concept. Additionally, Lair and Kennedy [34] have demonstrated that Poisson ratio affect stress concentrations in the finite element analysis of composite unit cells. Hsueh [35] also carried out studies on the Poisson's effect for with stress analysis methods.

Hence, it is desired to obtain expressions for the transverse Young's modulus and transverse Poisson's ratio for unidirectional composites, making use of yarn-packing characteristics as discussed in this chapter. The formulations that are derived later on in this report seek to be compared against the some of the above classical models since they are similar in form and function and do not require intensive number-crunching.

2.2.2 Computational models

This section looks at algorithm level models that make use of number-crunching in order to estimate the transverse properties of fiber composites. While mathematical models represent neat and efficient formulations for the properties, computational and numerical models are able to apply theory of elasticity principles on an element-level and thus, are able to incorporate assumptions and simplifications into their models. As a result, they are an invaluable standard of comparison for the formulations that are derived.

Shan & Chou [36] make use of an elastic contact model to determine the transverse properties of fiber-reinforced composites. The theory of elasticity is incorporated by matrix algebra in order to simulate voids and imperfect fiber-matrix interfaces. Theocaris & Varias [37] modify Kerner's model with a mesophase between fiber and matrix. Good comparisons with Hashin and Rosen have been reported. Li *et al.* [38] have carried out FEM analyses for reduced size plain weave textile by making use of symmetries, reflection and rotation operations. Wang *et al.* [39] used complex Fourier series numerical modeling and a multi-phase approach to obtain micromechanical behavior as well as macroscopic properties of fiber reinforced composites. Mahiou and Beakou [40] apply homogenization techniques for elastic constants with the study of interphase properties on the fiber-matrix interface. This homogenization is also carried out in our work to use porosity to obtain effective yarn (agglomeration of fibers) properties and then homogenize once again to obtain final composite properties.

For metal matrix composites, fiber wetting is difficult; this results in debonded fibers and voids at the interface. Takahashi & Chou [41] have thus created mathematical models, using constitutive stress-strain relations, which are solved numerically. The data of perfectly bonded interfaces is useful for comparison with our model.

Gosz *et al.* [42] take great pains to model the viscoelastic interfacial zone using hexagonal arrays by finite element analysis. Our model simulates the interface region as well; but without any special properties. Jacquet *et al.* [43] seek to improve the rule of mixtures with rectangular fibers. A comparison with FEM shows satisfactory correlations. Zhang *et al.* [44] study the influence of fiber packing geometry using FEM analyses on a 3D model. Stagni made use of self-consistent models to look at unidirectional composite materials with identical multi-layered fibers [45]

It is seen that in general, the computation models provide closer matches to experimental data, since complicated phenomena in the composites can be readily modelled.

2.2.3 Other similar models

Zhang and Evans [46] made use of an Energy Equivalence method to determine the properties of fiber-reinforced composites (FRCs). FEM is carried out using AbaqusTM for anisotropic but homogeneous fiber matrix. Lin *et al.* [47] utilizes square and hexagonal array

FEM models to solve for the transverse strength characteristics of FRCs using the von-Mises Hencky criterion. The inter-plane characteristics of the transverse Young's modulus on glass and epoxy composites using FEM is explored by Wacker *et al.* [48]. The data provides a computational and experimental means of comparison for our work.

The consideration of porosity effects are carried out in the work of Madsen & Lilholt [49] and Madsen *et al.* [50]. The comparison with the Halpin-Tsai model for off-axis properties yields favourable results. Wu *et al.* [51] makes use of a slightly unconventional statistically designed method to determine the effect of metal volume fraction in the prediction of mechanical properties in metal fiber laminates.

Miyagawa *et al.* [52] determine the theoretical and experimental transverse moduli of fibers by use of FEM, spectroscopy and strain measurements.

Thus it is seen that the formulations of the transverse elastic properties desired should be of similar form to Halpin-Tsai or the Morais model, but should be devoid of experimentally determined parameters and not assuming simplistic shape of yarns. The available computational models provide fair estimates, although not very accurate, however, they suffer from being computationally intensive and do not provide quick solutions.

It was therefore taken as the first task in this study to propound a more inclusive model for the transverse elastic properties. Focus was to be placed on the reinforcement-matrix interface region, especially to take into account both normal and shear effects with yarn and fiber arrangement patterns playing a role.

2.3 Scalability factors for accommodating elliptical yarns

For textile reinforced composites, the textile manufacturing and composite fabrication process results in the reinforcing yarn of fibers assuming a non-circular shape. Many works on textile composite modeling focus on using this non-circular shape and even approximate it with an elliptical shape [20, 53-56]. Thus elliptical yarns can be seen as “flattened” circular yarns that exhibit curved non-sharp shapes, yet are better able to fit into the available space within a textile arrangement as shown in Figure 2.2(b).

Figure 2.2: Quarter unit cells for (a) circular and (b) elliptical yarns

The geometrical similarities between elliptical and circular shapes thus provide the theoretical possibility to take the features and techniques applied to circular yarns and extend them to elliptical yarns. While the derived micromechanical transverse modulus formulations (using an elliptical yarn unit cell model) had many incompatibility issues for applying to composite geometry, finite element analysis (FEA) seemed to suggest an obvious alternative means of estimating elliptical yarn properties. FEA has been used to study various aspects of composite properties all the way from fiber and yarn properties of unidirectional composites [46-48, 57-58] to studies on properties of textile structures [59-62].

The technique of using a scalability factor (dependent on geometry, material and arrangement properties) to convert existing circular yarn composite properties to their corresponding elliptical yarn composite properties was thus utilized as a way to eliminate the rigidity phenomenon seen in finite element analysis [56, 63]. Literature review and initial FEM studies were carried out to assess which parameters may affect the composite properties.

Using FEM, Tarfaoui and Akesbi [60] studied the effect of using meshes of different yarn shapes on the mechanical behavior of composites. FE studies cited yarn cross-section as a crucial factor that influences composite properties. Additionally, factors such as properties of constituent materials, fiber volume fractions and array periodicity were found to affect elastic behavior of fiber-reinforced composites with transversely isotropic constituents [64]. Other than changing yarn aspect ratio (AR), parameters such as yarn volume fraction (V_{yarn}), yarn porosity (ϵ) and fiber-to-matrix modulus ratio (M_r) are to be focused upon. This list isn't

exhaustive, but provides a good starting point keeping in mind that these parameters play a role.

Therefore, to maintain the link between circular and elliptical yarn properties, their FEA results are used to construct scalability factors to obtain transverse elastic properties. These new models will utilize regression models [65] to study the effects of yarn aspect ratio, porosity, fiber volume fraction and the ratio of fiber-to-matrix modulus to obtain the elliptical yarn elastic properties.

2.4 Woven Textile Composites

Due to the wide range of textile fabric structures (woven, braided or knitted) available, it was desired to develop material property expertise in one structure, before applying it to other fabric structures. This strategy was adopted by Gommers *et al.* [66] combining the Mori-Tanaka method with orientation tensors in woven, braided and knitted textiles to predict damage and strength behavior. Tang & Postle [67-68] also discuss predicting general mechanical behavior of textile composites with consideration to geometry and fiber volume fractions for tensile and shear moduli by numerical simulations and mathematical modeling, with a focus on braided structures.

As mentioned before, Ishikawa & Chou's work [17] provided a sound theoretical basis on which to create suitable woven mathematical models. The use of mosaic models (assemblages of asymmetrical cross-ply laminates) for woven composites by Ishikawa & Chou [20] are crucial as well. Hewitt *et al.* [61] developed a finite element analysis computational model using the manufacturing considerations for modeling a single layer 3D reproduction of the woven reinforcement. They later expanded this into a generic system for designing multiple layers of woven structure and converted this to loom code for use on a jacquard loom [62]. Thus modeling of the textile structure provides important guidelines to determining final composite properties.

Vandeurzen *et al.* [54, 55] looked at large range of 2D woven architectures and used a wide range of macro and micro unit cells designs to create the geometry of woven textile composites and obtain their 3D shear and elastic properties using finite element analysis. De

Oliveira and Avila [69] created single layer three dimensional CAD/CAE models to visualize and study elastic properties of textile composites.

Woven textile architecture is chosen, not only due to its established literature and practicality, but also due to its inherent simplicity. The geometry creation and property determination design methodologies that are obtained thus can be applied to other textile structures. Application of these concepts to the entire woven unit cell required a sturdy suitable methodology. This was provided by Naik & Shembekar's work [70] dealing with complete two-dimensional models for elastic analysis. Of particular attention within these models is the emphasis on fiber undulation and yarn continuity. Figure 2.3 shows a fundamental unit cell for a woven textile with the intertwining fill and weft yarns surrounded by matrix [70].

Figure 2.3: The unit cell model adapted from Naik & Shembekar's work [70]

With the geometry clearly defined and quantified, this will enable the exploration of both one-directional (x-direction and y-direction) planar and two-directional 3D weaves (x-y plane). CLT can be used to combine all planar strip elements for one-directional and all 3D vertical block elements (shown in Figure 2.4) for two-directional woven composite property derivation in MATLAB. The yarn cross-section as well individual yarn properties within the model can be manipulated based on user requirements.

Figure 2.4: MATLAB woven unit cell 3D geometry and a single vertical block element

This unit cell development has thus provided an important blueprint for a micromechanical analysis platform which could be used to simulate comparative models for braided textile structures as well. Shembekar and Nair's work [71] on the analysis of fiber textile laminates is the next step in development of design methodologies. Their work has carried out comparative analyses between unidirectional laminated symmetric cross-ply laminates and plain weave fiber laminates.

Aitharaju and Averill [59] made use of computational models to break up the woven fabric composites geometry into regions of cross-ply and fiber waviness in both direction and implemented a 3D finite element analysis using average stiffness properties. Additionally, Sankar and Marrey [15] carried out extensive discretization of 3D and plate elements on textiles and analytically predicted the thermo-elastic properties using a Selective Averaging Method (SAM).

Both the extensive discretization of geometry and homogenization summation of average stiffness properties along with suggested micromechanical methodologies [18, 20, 22, 70] are used to create a plain weave textile composite model capable of taking in different geometrical parameters and enabling comparisons of different yarn input models.

2.5 Braided Textile Composites

As mentioned before, the freedom of having yarns in braided textiles obliquely angled to each other (not restricted to diagonal) brings in a new design parameter, in addition to the typical factors such as fiber volume fraction, waviness of EEE and thickness of preforms [19, 72]. Figure 2.5 shows how the geometry of a unit cell changes when the braiding angle is changed, while keeping the yarn dimension and spacing the same.

Figure 2.5: Changes in Braiding Angle

Based on the conventional braiding axis shown in Figure 2.5, the machine direction (MD) is usually the important direction of consideration when designing and evaluating braided structures. The bias yarns are aligned at angle θ from the machine direction; the braided textile properties are generally obtained with the machine direction in consideration (and not along the yarn directions as is commonly done in woven textiles). The cross direction (CD), by convention, is orientated perpendicular to the machine direction (within the plane of the unit cell).

The change in geometrical shape and yarn orientations contributes to changes in overall braided textile composite elastic properties. These are discussed in detail in Chapter 7. Recent

work with complex 3D geometries creation and structural properties modeling for braided textiles has also been carried out [56, 73]. Ivanov and Tabiei [74] explored the behavior of non-orthogonal yarns in textile composites with micromechanical techniques and a general homogenization procedure in MATLAB as a foundation to studying braided textile composites. This work suggested a pathway for both the woven model discussed earlier and the braided textile models developed in this study.

Work by Roberts et al. [75] and review studies by Ayranci and Carey [76] looked into fabrication process and impregnation techniques that affect elastic property predictions for stiffness critical applications. Also, Kelkar and Whitcomb [77] performed evaluation, modeling, fabrication and testing on biaxially braided composites fabricated using vacuum-assisted resin transfer molding (VARTM).

Just like different weaving patterns in woven textiles, braided textiles can be categorized by the braid manufacture technique. Different braiding patterns result in different mechanical properties. For the same yarn dimensions, spacing and braiding angle, the 2-by-2 regular braid has demonstrated higher planar stiffness properties compared to the diamond braid [78 - 81]. This is expected since the 2-by-2 regular braid has more sections of the yarn orientated within the plane of the composite. Since this study seeks to link the braided composite model properties to the woven composite model properties, the 1-by-1 plain weave is directly adapted to the 1-by-1 diamond braid [78].

With the advent of sophisticated solid modeling software (like SolidWorks, AutoCAD, etc.) and powerful finite element platforms (like Abaqus, ANSYS, etc.) the use of 3D models for design, analysis and applications for use in composites has increased greatly [15, 59, 61, 69]. The ability of these programs to create complicated geometry and have it analysed thoroughly is highly sought after. However, there is a time-penalty involved, since changing solid geometries and then analyzing them in finite element software can take time.

This kind of detailing and tediousness may be undesired at a preliminary design stage where a simple estimate of composite property is required. A numerical micromechanical approach may be better suited where dual geometric modelers-and-solvers within the same programs create the new geometry and analyse them instantly, providing final composite properties within seconds. There will be no geometry compatibility issues that may crop when exporting from modeler to solver as compared to FEA. Additionally, micromechanical models can represent the given textile adequately. They will not suffer from the boundary condition

effects (caused by local deformations in FEA) that are resolved only by adding more unit cells to the existing ones [56, 73].

2.6 Concluding Remarks

Having discussed the shortcomings and gaps that exist in the literature, two broad areas of fiber composites will be addressed in this study.

The first will focus on improving transverse direction elastic properties of unidirectional composites. A shear-inclusive model that is able to capture the normal and shear interactions of the reinforcement-matrix interface would be espoused. Additionally, this model would take into consideration different patterns of fiber arrangement. In case of an aggregation of fibers, the effects of changing unit cell and yarn shapes to other forms will also be explored. This work will help improve local yarn input properties for use in complex textile structures.

Making use of the extensive literature on woven textiles composites, the second part of this study will seek to exploit the similarities in geometrical modeling and property determination techniques to bridge woven and braided textile composites properties. Common grounds of understanding textile composite behavior and focusing on composite property trends will be explored. A general methodology of property determination will also be implemented, such that this technique is the same when applied to woven or braided textiles and can possibly be applied to other types of weaves (satin, etc.), braids (regular, Hercules, etc.) and even new textile structures like knits. It is hoped that with our work, explorations at designing and combining different textiles with the aim of capitalizing on their individual strengths and negating their individual weaknesses, within the same structure becomes possible.

Chapter 3: SIC Transverse Young's Modulus

Classical composite theory makes use of rule-of-mixture expressions to obtain the longitudinal and transverse elastic moduli of fiber-reinforced composites as shown in Equations 3.1a and 3.1b respectively (Jones [24]) in terms of composite fibers and matrix properties.

$$E_{eq} = V_f E_f + (1 - V_f) E_m \quad \dots(3.1a)$$

$$\frac{1}{E_{eq}} = \frac{V_f}{E_f} + \frac{(1-V_f)}{E_m} \quad \dots(3.1b)$$

Equation 3.1a is obtained by making use of the iso-strain conditions in long-fiber reinforced composites. This equation is widely used to obtain fairly accurate estimates of the longitudinal Young' modulus of composites. On the other hand, Equation 3.1b is obtained from the iso-stress conditions, and is not reliable for predicting the transverse Young's moduli of composites. It is expected that shear effects are also involved in the transverse direction; however, this is not reflected in Equation 3.1b, which may be noted from its derivation shown in Chapter 2.

As a result, other sophisticated and semi-empirical models such as Halpin-Tsai [26], Spencer [29] and Hashin-Rosen [30] are utilized instead to predict the transverse direction elastic moduli of long fiber-reinforced composites. They are able to correlate better with experimental values and traditionally have been used as reliable estimates. However, these models are inherently complicated due to the consideration of experimental data or empirical parameters (in case of Halpin-Tsai) or the consideration of three-dimensional effects (in case of Hashin-Rosen). Thus, it is desired to have an expression that is based on the inclusion of shear effects, simpler to use and of similar format as the Equations 3.1a and 3.1b.

In this chapter, the detailed derivation of an expression for transverse direction Young's modulus of a long fiber-reinforced composite is introduced and validated with comparisons to experiments and computational results.

3.1 Model Definition

Long fiber-reinforced composites can have different arrangements. For the current derivation, the arrangements are idealized into a square lattice formation, commonly used for most formulations, as shown in Figure 3.1. From this arrangement, it is possible to isolate and derive a unit-cell for our work. The unit cell derived consists of a quarter-circle reinforcement surrounded by matrix within a square unit cell, extruded along the reinforcement direction.

Figure 3.1: A square prism unit cell extracted from a long reinforcement composite

However, the objective of this derivation is to obtain the transverse elastic modulus of the long reinforced composite, this three-dimensional unit cell can be further simplified by considering it as a square two-dimensional unit cell as shown in Figure 3.2.

Figure 3.2: The two dimensional unit cell

Instead of a square lattice arrangement, it is also possible to have other arrangements (triangular, hexagonal, etc.) as is demonstrated by other work in long fiber composites [58, 81]; however, it is simplest within the square unit cell to demonstrate the method of

derivation for the transverse Young's modulus. Comparable derived formulations for other lattice arrangements will be provided using the same method used for the square lattice arrangement.

It is also important to note that in this formulation, the term "reinforcement" is used instead of "yarn" or "fiber". This is primarily to indicate that the planar elastic properties of each of the two phases, reinforcement and matrix are constant throughout their respective phases. If the reinforcement is substituted with a single fiber or a "yarn" of fibers, then the corresponding equivalent properties will have to be used for the formulation.

The unit cell shown in Figure 3.2 consists of a quarter circle of reinforcement of radius denoted by R and an extension denoted by ρ . Therefore, the side of the square unit cell has a length of $\rho + R$. Based on this, the volume fraction of reinforcement, V_{reinf} can be represented in terms of the values of ρ and R ; and the ratio ρ/R can be written as a function of volume fraction, as shown in Equations 3.2a and 3.2b.

$$V_{\text{reinf}} = \frac{\frac{\pi R^2}{4}}{(\rho + R)^2} \quad \dots(3.2a)$$

$$\frac{\rho}{R} + 1 = \sqrt{\frac{\pi}{4V_{\text{reinf}}}} \quad \dots(3.2b)$$

As a direct consequence of the equations above, a maximum value of reinforcement volume fraction can be determined. This limit is given by $\rho \rightarrow 0$; consequently, the V_{reinf} tends to $\frac{\pi}{4}$ which is approximately equal to 0.785 or 78.5% fraction by volume.

3.2 Reinforcement-Matrix Interface

A significant drawback of the iso-stress condition represented in Equation 3.2b is the absence of a means to model the reinforcement-matrix interface, represented by the curved quarter-perimeter portion of the circle as in Figure 3.3. Since the iso-stress condition looks only at the normal force interaction between reinforcement and matrix, the contribution to elastic modulus by shear is completely ignored.

In order to rectify this anomaly, this formulation proposes infinitesimally small triangular interface elements at the reinforcement-matrix interface as shown in Figure 3.3.

Figure 3.3(a): Locating interface cells at angle θ (b) zooming in to an interface element (c) displaying the interface element as two triangular individual reinforcement & matrix elements

Having isolated these triangular elements, it is now required to resolve the forces and stresses acting on each element individually to determine their micromechanical responses. The responses of these interface elements, along with the constituent reinforcement and matrix regions will be aggregated to provide a combined response for the entire unit cell.

Figure 3.4: The two obtained interfacial elements

It can be seen from Figure 3.4 that both these interface elements are identical triangles. The vertical edge of the elements is designated as δy . Therefore, using Trigonometric relations, the following relations can be determined:

$$\frac{l}{\delta y} = \tan \theta \Rightarrow l = \delta y \cdot \tan \theta \quad \dots(3.3a)$$

$$\frac{\delta y}{h} = \cos \theta \Rightarrow h = \frac{\delta y}{\cos \theta} \quad \dots(3.3b)$$

It must be noted that while the interface elements at a particular value of θ are identical, the interface elements change shape at different points on the interface (at different values of θ).

3.3 Resolution of Applied Forces

The force applied on the side of unit cell is a distributed load. This load transmits itself through the material and across the interface. The load applied on the interface elements is shown in Figure 3.5. Even though the load is applied on both the slanted and the vertical edge, for our analysis purposes, it will be taken to be applied along the slanted edge only.

Figure 3.5: Equal and opposite forces acting on both elements

This applied force can be resolved into a normal force component, F_N and a shear force component, F_S as shown by Equations 3.4a and 3.4b.

$$F_N = F \cos \theta \quad \dots(3.4a)$$

$$F_S = F \sin \theta \quad \dots(3.4b)$$

The deformation of the triangular elements is then determined as an effect of these forces individually and then vectorially added to determine total deformation of the element.

It is important to note that only the derivation of the mechanical deformation of reinforcement interface elements are shown in the next subsection. However, for the matrix interface elements, the derivation is identical with matrix elastic properties substituting of reinforcement.

3.3.1 Computation of Normal Force Deformation

The normal force on the element is again resolved in the principal x- and y- directions (Figure 3.6) as shown in Equations 3.5a and 3.5b and then the applied stress on the element is calculated as (shown in Equations 3.6a and 3.6b).

Figure 3.6: Resolution of normal force into x- and y- components

$$F_x = F_N \cos \theta \quad \dots(3.5a)$$

$$F_y = F_N \sin \theta \quad \dots(3.5b)$$

$$\sigma_x = \frac{F_x}{A_x} = \frac{F_N \cos \theta}{\delta y \cdot 1} = \frac{F_N \cos \theta}{\delta y} \quad \dots(3.6a)$$

$$\sigma_y = \frac{F_y}{A_y} = \frac{F_N \sin \theta}{l \cdot 1} = \frac{F_N \sin \theta}{\delta y \cdot \tan \theta \cdot 1} = \frac{F_N \cos \theta}{\delta y} \quad \dots(3.6b)$$

The strains in the x- and y- direction are computed using the constitutive equations for plane stress.

Constitutive Equation for Plane Stress in x-direction: $\epsilon_x = \frac{\sigma_x}{E_x} - \nu \frac{\sigma_y}{E_y}$

substituting,

$$\begin{aligned} &= \frac{F_N \cos \theta}{\delta y \cdot E_{reinf}} - \nu_{reinf} \frac{F_N \cos \theta}{\delta y \cdot E_{reinf}} \\ &= \frac{F_N \cos \theta}{\delta y \cdot E_{reinf}} (1 - \nu_{reinf}) \quad \dots(3.7a) \end{aligned}$$

Constitutive Equation for Plane Stress in y-direction: $\epsilon_y = \frac{\sigma_y}{E_y} - \nu \frac{\sigma_x}{E_x}$ similarly,

$$= \frac{F_N \cos \theta}{\delta y \cdot E_{reinf}} (1 - \nu_{reinf}) \quad \dots(3.7b)$$

The corresponding deformations are then computed as shown by Equations 3.8a and 3.8b.

$$\Delta x_{normal} = \varepsilon_x \cdot l = \frac{F_N \cos \theta}{\delta y \cdot E_{reinf}} (1 - \nu_{reinf}) \cdot \delta y \cdot \tan \theta$$

$$= \frac{F_N \sin \theta}{E_{reinf}} (1 - \nu_{reinf}) \quad \dots(3.8a) \text{ and}$$

$$\Delta y_{normal} = \varepsilon_y \cdot \delta y = \frac{F_N \cos \theta}{\delta y \cdot E_{reinf}} (1 - \nu_{reinf}) \cdot \delta y$$

$$= \frac{F_N \cos \theta}{E_{reinf}} (1 - \nu_{reinf}) \quad \dots(3.8b)$$

3.3.2 Computation of Shear force deformation

In case of the shear force deformation, there is no need to resolve the force into mutually perpendicular directions. The shear stress and the subsequent shear strain, γ is calculated (Equations 3.9 and 3.10) as:

$$\tau_s = \frac{F_s}{A_s} = \frac{F_s}{h \cdot 1} = \frac{F_s \cos \theta}{\delta y} \quad \dots(3.9)$$

$$\gamma = \frac{\tau_s}{G_{reinf}} = \frac{F_s \cos \theta}{G_{reinf} \cdot \delta y} \quad \dots(3.10)$$

The shear deformation is then computed as shown in Equations 3.11a and 3.11b.

$$\Delta x_{shear} = \gamma \cdot \delta y = \frac{F_s \cos \theta}{G_{reinf} \cdot \delta y} \delta y = \frac{F_s \cos \theta}{G_{reinf}} \quad \dots(3.11a)$$

$$\Delta y_{shear} = -\gamma \cdot l = \frac{F_s \cos \theta}{G_{reinf} \cdot \delta y} \delta y \cdot \tan \theta = -\frac{F_s \sin \theta}{G_{reinf}} \quad \dots(3.11b)$$

3.3.3 Vectorial Addition of deformation

Having determined the normal and shear components of deformation individually, they are now combined to obtain the total deformations.

$$\begin{aligned}\Delta x_{total} &= \Delta x_{normal} + \Delta x_{shear} \\ &= \frac{F_N \sin \theta}{E_{rein f}} (1 - \nu_{rein f}) + \frac{F_s \cos \theta}{G_{rein f}}\end{aligned}$$

Since $F_N = F \cos \theta$ and $F_s = F \sin \theta$,

$$\therefore \Delta x_{total,f} = \frac{F \sin \theta \cos \theta}{E_{rein f}} (1 - \nu_{rein f}) + \frac{F \sin \theta \cos \theta}{G_{rein f}} \quad \dots(3.12a)$$

Similarly for the y-direction,

$$\begin{aligned}\Delta y_{total} &= \Delta y_{normal} + \Delta y_{shear} \\ \Delta y_{total,f} &= \frac{F \cos^2 \theta}{E_{rein f}} (1 - \nu_{rein f}) - \frac{F \sin^2 \theta}{G_{rein f}} \quad \dots(3.12b)\end{aligned}$$

It is important to note that rather than spatial movement of the two triangular interface pieces, their relative size-change (elongation or contraction) is of primary interest. Therefore, in the case of the matrix interface piece, the deformation will be summarized in similar fashion to the reinforcement piece as shown in Equations 3.13a and 3.13b.

$$\Delta x_{total,m} = \frac{F \sin \theta \cos \theta}{E_m} (1 - \nu_m) + \frac{F \sin \theta \cos \theta}{G_m} \quad \dots(3.13a)$$

$$\Delta y_{total,m} = \frac{F \cos^2 \theta}{E_m} (1 - \nu_m) - \frac{F \sin^2 \theta}{G_m} \quad \dots(3.13b)$$

In the global reference frame, these values are negative. However, since it is desired to utilize the size-change of this piece, the values are taken as positive as shown in Equations 3.13.

3.4 Consolidation and Integration of all terms of deformation

Having worked out the deformation of the interface elements of thickness δy , it is now required to determine the total deformation of an infinitesimal length element of thickness, δy . This consists of an aggregation of the parts I, II, III and IV as shown in Figure 3.7(a) below.

Figure 3.7: The location of all the interface elements

Parts I and IV signify the rectangular elements of the reinforcement and matrix respectively; II and III are the triangular interface reinforcement and matrix elements as shown in Figure 3.7(b). Part V consists of the additional matrix that is not covered by the thin strip of thickness δy and is included to completely occupy the area of the unit cell under investigation.

Since an area summation will be carried out, the change in the total area of an interface unit cell must be determined. The original dimensions of the interface unit cell are taken as δx and δy , and the extensions are Δx and Δy as shown in Figure 3.8.

Figure 3.8: Extension in both directions

Therefore, the change in total area, ΔA of the extended triangular portion is

$$\begin{aligned}
 \Delta A &= \frac{1}{2}(\delta x + \Delta x)(\delta y + \Delta y) - \frac{1}{2}\delta x \delta y \\
 &= \frac{1}{2}(\delta x \delta y + \Delta x \delta y + \Delta y \delta x + \Delta x \Delta y) - \frac{1}{2}\delta x \delta y \\
 &= \frac{1}{2}(\Delta x \delta y + \Delta y \delta x + \Delta x \Delta y)
 \end{aligned}$$

Since $\Delta x \Delta y$ term has a very small value, it can be ignored; therefore,

$$\Delta A = \frac{1}{2}(\Delta x \delta y + \Delta y \delta x) \quad \dots(3.14)$$

Summarizing for the entire unit cell area

Total extended area, A_{final}

$$\begin{aligned}
 &= \int_0^R (\text{Extended length})_{reinf} \cdot dy + \int_0^R \Delta A_{reinf} + \int_0^R \Delta A_m + \int_0^{\rho+R} (\text{Extended length})_m \cdot dy \\
 &= \int_0^R [l_{reinf,final}] \cdot dy + \int_0^R \left[\frac{1}{2}(\Delta x_{total,reinf}) \right] \cdot dy + \int_0^R \left[\frac{1}{2}(\Delta y_{total,reinf}) \right] \cdot dx \\
 &\quad + \int_0^R \left[\frac{1}{2}(\Delta x_{total,m}) \right] \cdot dy + \int_0^R \left[\frac{1}{2}(\Delta y_{total,m}) \right] \cdot dx + \int_0^R [l_{matrix,final}] \cdot dy \\
 &\quad + \int_R^{\rho+R} [l_{matrix,final}] \cdot dy
 \end{aligned}$$

$$\begin{aligned}
&= \int_0^R (R \cos \theta)(1 + \varepsilon_{rein}) dy + \int_0^R \left[\frac{1}{2}(\Delta x_{total, rein}) \right] \cdot dy + \int_0^R \left[\frac{1}{2}(\Delta y_{total, rein}) \right] \cdot dx \\
&\quad + \int_0^R \left[\frac{1}{2}(\Delta x_{total, m}) \right] \cdot dy + \int_0^R \left[\frac{1}{2}(\Delta y_{total, m}) \right] \cdot dx \\
&\quad + \int_0^R (\rho + R - R \cos \theta)(1 + \varepsilon_m) \cdot dy + \int_R^{\rho+R} (\rho + R)(1 + \varepsilon_m) dy
\end{aligned}$$

Before proceeding, it is easier to isolate and solve each of these integral terms individually, therefore,

$$\text{Let } I = \int_0^R (R \cos \theta)(1 + \varepsilon_{rein}) dy$$

$$\text{Let } II = \int_0^R \frac{1}{2} [(\Delta x_{total, rein})] \cdot dy + \int_0^R \frac{1}{2} [(\Delta y_{total, rein})] \cdot dx$$

$$\text{Let } III = \int_0^R \frac{1}{2} [(\Delta x_{total, m})] \cdot dy + \int_0^R \frac{1}{2} [(\Delta y_{total, m})] \cdot dx$$

$$\text{Let } IV = \int_0^R (\rho + R - R \cos \theta)(1 + \varepsilon_m) \cdot dy$$

$$\text{Let } V = \int_R^{\rho+R} (\rho + R)(1 + \varepsilon_m) dy$$

Each of the above integrals corresponds to the same numbered area in Figure 3.7. These integrals are solved one-by-one by making use of the relations given below:

$$\text{Let } y = R \sin \theta \Rightarrow dy = R \cos \theta d\theta$$

$$\text{Also, } dx = dy \tan \theta = R \cos \theta d\theta \cdot \tan \theta = R \sin \theta d\theta$$

$$\varepsilon_{rein} = \frac{\sigma_x}{E_{rein}} = \frac{F}{(\rho + R)E_{rein}}$$

$$\varepsilon_m = \frac{\sigma_x}{E_m} = \frac{F}{(\rho + R)E_m}$$

The evaluated integrals take the form as shown in Table 3.1 below.

Table 3.1: Integrals and their evaluated forms

Integral	Evaluates to
$I = \int_0^R (R \cos \theta)(1 + \varepsilon_{reinf}) dy$	$\frac{\pi R^2}{4} \left(1 + \frac{F}{(\rho + R)E_{reinf}} \right)$
$II = \int_0^R \frac{1}{2} [(\Delta x_{total, reinf})] \cdot dy + \int_0^R \frac{1}{2} [(\Delta y_{total, reinf})] \cdot dx$	$FR \left[\frac{(1 - \nu_f)}{3E_f} \right] - FR \left[\frac{1}{6G_f} \right]$
$III = \int_0^R \frac{1}{2} [(\Delta x_{total, m})] \cdot dy + \int_0^R \frac{1}{2} [(\Delta y_{total, m})] \cdot dx$	$FR \left[\frac{(1 - \nu_m)}{3E_m} \right] - FR \left[\frac{1}{6G_m} \right]$
$IV = \int_0^R (\rho + R - R \cos \theta)(1 + \varepsilon_m) \cdot dy$	$\left(1 + \frac{F}{(\rho + R)E_m} \right) \left((\rho + R)R - \frac{\pi R^2}{4} \right)$
$V = \int_R^{\rho+R} (\rho + R)(1 + \varepsilon_m) \cdot dy$	$\rho(\rho + R) \left(1 + \frac{F}{(\rho + R)E_m} \right)$

The detailed proof for the evaluation of these integrals is given in Appendix A.

3.5 Formulation of final transverse modulus expression

Having evaluated the integrals, the total final area can be obtained:

Combining I + II + III + IV + V, we get:

$$\begin{aligned}
&= \frac{\pi R^2}{4} \left(1 + \frac{F}{(\rho + R)E_{reinf}} \right) + \frac{FR}{6} \left[\frac{2(1 - \nu_{reinf})}{E_{reinf}} - \frac{1}{G_{reinf}} \right] + \frac{FR}{6} \left[\frac{2(1 - \nu_m)}{E_m} - \frac{1}{G_m} \right] \\
&\quad + \left(1 + \frac{F}{(\rho + R)E_m} \right) \left((\rho + R)R - \frac{\pi R^2}{4} \right) + \rho(\rho + R) \left(1 + \frac{F}{(\rho + R)E_m} \right) \\
\therefore A_{final} &= \frac{\pi R^2}{4} \left(1 + \frac{F}{(\rho + R)E_{reinf}} \right) + \frac{FR}{6} \left[\frac{2(1 - \nu_{reinf})}{E_{reinf}} + \frac{2(1 - \nu_m)}{E_m} - \frac{1}{G_{reinf}} - \frac{1}{G_m} \right]
\end{aligned}$$

$$+ \left((\rho + R)^2 - \frac{\pi R^2}{4} \right) \left(1 + \frac{F_T}{(\rho + R)E_m} \right) \quad \dots(3.15)$$

By making use of the definition of strain, the expression of total equivalent strain for the unit cell is given by

$$\therefore \text{Total equivalent strain, } \varepsilon_{eq} = \frac{A_{final} - A_{initial}}{A_{initial}} \quad \dots(3.16)$$

The expression for area for the initial square unit cell and the applied strain is given by:

$$\text{Area for the initially square unit cell, } A_{initial} = (\rho + R)^2$$

$$\varepsilon_{eq} = \frac{\sigma_{applied}}{E_{eq}} = \frac{F}{(\rho + R)E_{eq}} \quad \dots(3.17)$$

Making use of Equations 3.15, 3.16 and 3.17 gives us,

$$\begin{aligned} \therefore \text{Total equivalent strain, } \varepsilon_{eq} &= \frac{A_{final} - A_{initial}}{A_{initial}} \\ \Rightarrow \frac{F}{(\rho + R)E_{eq}} &= \frac{A_{final} - A_{initial}}{A_{initial}} \\ \Rightarrow \frac{F}{(\rho + R)E_{eq}} &= \frac{\left(\frac{\pi R^2}{4} \right)}{(\rho + R)^2} \left(\frac{F}{(\rho + R)E_{reinf}} \right) \\ &+ \frac{FR}{6(\rho + R)^2} \left[\frac{2(1 - \nu_{reinf})}{E_{reinf}} + \frac{2(1 - \nu_m)}{E_m} - \frac{1}{G_{reinf}} - \frac{1}{G_m} \right] \\ &+ \frac{\left((\rho + R)^2 - \frac{\pi R^2}{4} \right)}{(\rho + R)^2} \left(\frac{F}{(\rho + R)E_m} \right) \end{aligned}$$

Since $\frac{\left(\frac{\pi R^2}{4} \right)}{(\rho + R)^2} = V_{reinf}$, we can simplify to get:

$$\frac{1}{E_{eq}} = \frac{V_{reinf}}{E_{reinf}} + \frac{R}{6(\rho + R)} \left[\frac{2(1 - \nu_{reinf})}{E_{reinf}} + \frac{2(1 - \nu_m)}{E_m} - \frac{1}{G_{reinf}} - \frac{1}{G_m} \right] + \frac{(1 - V_{reinf})}{E_m} \quad \dots(3.18)$$

If the reinforcement is taken as an agglomeration of yarns, instead of a single homogeneous body, we get

$$\frac{1}{E_{eq}} = \frac{V_{yarn}}{E_{yarn}} + \frac{1}{6(\frac{\rho}{R}+1)} \left[\frac{2(1-\nu_{yarn})}{E_{yarn}} + \frac{2(1-\nu_m)}{E_m} - \frac{1}{G_{yarn}} - \frac{1}{G_m} \right] + \frac{(1-V_{yarn})}{E_m} \quad \dots(3.19)$$

By making use of $\frac{\rho}{R} + 1 = \sqrt{\frac{\pi}{4V_{yarn}}}$, this can be rewritten as

$$\frac{1}{E_{eq}} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1-V_{yarn})}{E_m} + \frac{1}{3} \sqrt{\frac{V_{yarn}}{\pi}} \left[\frac{2(1-\nu_{yarn})}{E_{yarn}} + \frac{2(1-\nu_m)}{E_m} - \frac{1}{G_{yarn}} - \frac{1}{G_m} \right] \quad \dots(3.20)$$

As mentioned earlier, this equation has a similar form to the iso-stress volume of mixture expression (Equation 3.1b), along with an additional term to account for shear effects.

This formulation for the transverse Young' modulus is validated with other mathematical formulations and estimates as well as experimental data in the next section. Alternative equations of transverse elastic modulus of fiber-reinforced composite, derived using the same methodology, are also presented.

3.6 Alternative formulations using interface elements

After having explained the working of the dual reinforcement matrix interface elements, it is thus possible to formulate alternative expressions for the transverse Young's modulus, as shown in Table 3.2. These expressions utilize different lattice structures leading to a different fiber packing and consequently, a different transverse modulus expression. The reinforcement here is once again taken as a yarn having homogenized material properties.

Table 3.2: A summary of the various formulations obtained by considering different unit cells

Single quarter yarn - Squared (SQS)
$V_{yarn} = \frac{\frac{\pi}{4}}{\left(\frac{\rho}{R}+1\right)^2}; \quad \frac{\rho}{R} + 1 = \sqrt{\frac{\pi}{4V_{yarn}}}; \quad V_{yarn} \leq \frac{\pi}{4}$ $\frac{1}{E_{eq}} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} +$ $\frac{1}{6\left(\frac{\rho}{R} + 1\right)} \left[\frac{2(1 - \nu_{yarn})}{E_{yarn}} - \frac{1}{G_{yarn}} + \frac{2(1 - \nu_m)}{E_m} - \frac{1}{G_m} \right]$
Double quarter yarn - Squared (DQS)
$V_{yarn} = \frac{\frac{\pi}{2}}{\left(\frac{\rho}{R}+1\right)^2}; \quad \frac{\rho}{R} + 1 = \sqrt{\frac{\pi}{2V_{yarn}}}; \quad V_{yarn} \leq \frac{\pi}{4}$ $\frac{1}{E_{eq}} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} +$ $\frac{1}{3\left(\frac{\rho}{R} + 1\right)} \left[\frac{2(1 - \nu_{yarn})}{E_{yarn}} - \frac{1}{G_{yarn}} + \frac{2(1 - \nu_m)}{E_m} - \frac{1}{G_m} \right]$
Single quarter yarn - Horizontal Triangled (SQHT)	
	$V_{yarn} = \frac{\frac{\pi}{2\sqrt{3}}}{\left(\frac{\rho}{R}+1\right)^2}; \quad \frac{\rho}{R} + 1 = \sqrt{\frac{\pi}{2\sqrt{3}V_{yarn}}}; \quad V_{yarn} \leq \frac{\pi}{2\sqrt{3}}$

$\frac{1}{E_{eq}} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} +$ $\frac{1}{3\sqrt{3}(\frac{\rho}{R} + 1)} \left[\frac{2(1 - v_{yarn})}{E_{yarn}} + \frac{2(1 - v_m)}{E_m} - \frac{1}{G_{yarn}} - \frac{1}{G_m} \right]$
Single quarter yarn - Vertical Triangled (SQVT)
$V_{yarn} = \frac{\frac{\pi}{2\sqrt{3}}}{(\frac{\rho}{R} + 1)^2}; \quad \frac{\rho}{R} + 1 = \sqrt{\frac{\pi}{2\sqrt{3}V_{yarn}}}; \quad V_{yarn} \leq \frac{\pi}{2\sqrt{3}}$ $\frac{1}{E_{eq}} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} +$ $\frac{1}{3(\frac{\rho}{R} + 1)} \left[\frac{2(1 - v_{yarn})}{E_{yarn}} + \frac{2(1 - v_m)}{E_m} - \frac{1}{G_{yarn}} - \frac{1}{G_m} \right]$
Double quarter yarn - Rectangled (DQR)
$V_{yarn} = \frac{\frac{\pi}{2}}{(\frac{\rho}{R} + 1)}; \quad \frac{\rho}{R} + 1 = \frac{\pi}{2V_{yarn}}; \quad V_{yarn} \leq \frac{\pi}{2\sqrt{3}}$ $\frac{1}{E_{eq}} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} +$ $\frac{1}{3} \left[\frac{2(1 - v_{yarn})}{E_{yarn}} + \frac{2(1 - v_m)}{E_m} - \frac{1}{G_{yarn}} - \frac{1}{G_m} \right]$

It may be noted that the yarns are not solid fibers. The presence of porosity, ϵ , in them alters not only the mechanical properties of the yarns, but also the volume fraction of the yarn (as shown in Equation 3.21 below).

$$V_{yarn} = \frac{V_{fibers}}{1-\epsilon} \quad \dots(3.21)$$

It should be noted that the expression in Equation 3.21 is a universal expression independent of the yarn arrangement or geometry. It is a means to size up the unit cell models (in Table 3.2) to enable adaptation of the macro-scale fiber composites volume fraction to the individual unit cell geometry. Hence, fiber volume fraction (V_{fibers}) and porosity (ϵ) is used to obtain the yarn volume fraction (V_{yarn}). Then by considering the yarn arrangement or geometry as shown in Table 3.2, the yarn volume fraction is used to obtain the corresponding $\frac{\rho}{R}$ ratio for the each formulation in Table 3.2. An application of this is shown in Table 3.4.

These pores eventually get filled with matrix. Thus, it is essential to use the properties of these fibers surrounded by matrix in the yarn to get the accurate expression. The yarn material properties that are being used are derived from the iso-stress relations of yarn-fiber interactions.

$$\frac{1}{E_{yarn}} = \frac{1-\epsilon}{E_f} + \frac{\epsilon}{E_m} \quad \dots(3.22a)$$

$$\frac{1}{\nu_{yarn}} = \frac{1-\epsilon}{\nu_f} + \frac{\epsilon}{\nu_m} \quad \dots(3.22b)$$

$$G_{yarn} = (1 - \epsilon)G_f + \epsilon G_m \quad \dots(3.22c)$$

It is expected that the fibers within a yarn are so densely packed that the yarn, along with the matrix within pores acts as a single entity, as considered in the iso-stress formulations.

Experimental data for a glass/epoxy composite taken from Tsai & Hahn [82] is then compared with the models shown in Table 3.2. The input values for the model are shown in Table 3.3.

Table 3.3: Input values for Tsai & Hahn's experimental data

Fiber properties (Glass)	$E_f = 73.1 \text{ GPa}$ $\nu_f = 0.22$
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	$G_f = \frac{E_f}{2(1+\nu_f)} = \mathbf{30 \text{ GPa}}$ (Glass fiber is isotropic)
Matrix properties (Epoxy)	$E_m = \mathbf{3.45 \text{ GPa}}$ $\nu_m = \mathbf{0.35}$ $G_m = \frac{E_m}{2(1+\nu_m)} = \mathbf{1.28 \text{ GPa}}$ (Epoxy is isotropic)
Yarn Porosity	$\epsilon = \mathbf{0.25}$

A sample calculation for prediction of the transverse Young's modulus at $V_{\text{fibers}} = 0.40$ is shown in Table 3.4.

Table 3.4: Computation of transverse Young's modulus based on $V_{\text{fibers}} = 0.40$

Step I: Initial Input Values for the fibers	Taken from Table 3.3
Step II: Obtain Yarn property values	$V_{\text{yarn}} = \frac{V_{\text{fibers}}}{1-\epsilon} = \mathbf{0.53}$ (from Equation 3.21) $E_{\text{yarn}} = \mathbf{12.09 \text{ GPa}}$ (from Equation 3.22a) $\nu_{\text{yarn}} = \mathbf{0.24}$ (from Equation 3.22b) $G_{\text{yarn}} = \mathbf{22.79 \text{ GPa}}$ (from Equation 3.22c)
Step III: Obtain the ρ/R ratio for the particular model	ρ/R for SQS formulation = 0.21

	ρ/R for DQS formulation = 0.72 ρ/R for SQHT formulation = 0.3 ρ/R for SQVT formulation = 0.3 ρ/R for DQR formulation = 1.95
Step IV: Input the V_{yarn}, E_{yarn}, ν_{yarn}, G_{yarn} and ρ/R ratio into the model's E_{eq} equation to obtain the transverse Young's modulus.	E_{eq} for SQS formulation = 7.42 GPa E_{eq} for DQS formulation = 8.59 GPa E_{eq} for SQHT formulation = 7.60 GPa E_{eq} for SQVT formulation = 10.37 GPa E_{eq} for DQR formulation = 14.03 GPa

The calculation shown above is carried out for all values of $V_{fibers} < 0.65$ and the results are shown in Figure 3.9. Due to the unit cell model, each model has a limitation of the maximum V_{yarn} . This in turn places a maximum restriction on V_{fibers} due to Equation 3.21. As a result, the data shown below seeks to provide a means of comparison of the different variations. Only the fiber volume fraction of experimental data at V_{fibers} from 0.4 to 0.65 is used since volume fractions outside this range were non-feasible and impractical for composites fabrication. The V_{fibers} value below 0.4 will lead to very weak composites with matrix-dominated behaviour. Additionally, the $V_{fibers} > 0.65$ is hard to achieve without making it matrix-starved.

Figure 3.9: Different variations based on Table 3.2

Figure 3.9 shows the different variations of the model, the experimental results taken from Tsai and Hahn [82] and the predictions of the other transverse models, namely, original Halpin-Tsai [26], adjusted Halpin-Tsai (based on Hewitt and de Malherbe [28] work), Spencer [29] and de Morias [31]. The presence of the other comparative models gives a general idea as to what the ideal predicted properties should be and thus, reduces the vagueness that is inherent with experimental data scatter.

It is seen that the Double Quarter Yarn Rectangled (DQR) variation is unsuitable for use. The y-intercept for all data points must converge at $E_m = 3.45$ GPa (Pure matrix). However, the DQR model inaccurately over-estimates the transverse Young's modulus and must be discarded.

The Double Quarter Yarn Squared model (DQS) and the Single Quarter Vertical Triangled model (SQVT) also overestimate the predicted value of the transverse Young's modulus. This is due to a very pronounced effect of the shear interface contribution to the formulation.

It is also seen that the Single Quarter Yarn Squared model (SQS) and the Single Quarter Horizontal Triangled (SQHT) are able to predict the value with a reasonable amount of accuracy; this is observable with the proximity of these predictions to the other mathematical

models as well as the experimental results. This points out to the fact that in unidirectional composites, the fiber arrangement mostly remains in square lattice form.

Therefore, the SQS model will be used for predictions in the next few sections since it provides a more conservative estimate of the transverse direction Young's modulus and falls evenly in the zone of the other classical mathematical models (Halpin-Tsai [26], Spencer [29], de Morais [31]). Additionally, the original derivation was carried out for the SQS model and this model is able to correlate well with other estimations and experiments. Hence, it is quite a safe choice for use in further analyses.

3.7 Effect of Yarn Porosity, ϵ

Based on the same case in the previous section, the effect of yarn porosity, ϵ on the SQS formulation is investigated. The results are shown in Figure 3.10 below. The legend signifies the value of yarn porosity (0.15, 0.20, 0.2, 0.30 & 0.35).

Figure 3.10: Effect of yarn porosity on the SQS formulation

As expected it is seen that as the yarn porosity increases, the transverse Young's modulus falls. However, the drop is not as large due to the reinforcing effect of the yarn interface. The increase in yarn porosity results in a higher ρ/R ratio. This results in increasing the shear effect of the yarn perimeter, thus compensating for the drop.

The yarn porosity values (0.15 - 0.35) will be used for comparative analysis in the next section as well since it gives an idea of the stability of the formulation, especially since yarn porosity can assume a large range of values (according to Madsen & Lilholt [49]).

It is important to note that the maximum value of V_{yarn} is limited to $\frac{\pi}{4}$. This imposes a limit on the largest yarn porosity that the model can predict for (if the value of V_{fibers} is fixed). This is shown in Equation 3.23a and 3.23b.

$$V_{yarn} = \frac{V_{fibers}}{1-\epsilon} < \frac{\pi}{4} \quad \dots(3.23a)$$

$$\epsilon < 1 - \frac{4V_{fibers}}{\pi} \quad \dots(3.23b)$$

Alternatively, a limit can be set on V_{fibers} if the yarn porosity is fixed as shown 3.23c.

$$V_{fibers} < \frac{\pi(1-\epsilon)}{4} \quad \dots(3.23c)$$

Beyond this value, the ρ/R ratio will become negative and the prediction inaccurate.

3.8 Comparison with various classical mathematical models

Figure 3.11 shows the same data as before, but compared against only the Halpin-Tsai models (original and adjusted) and the iso-stress model.

Figure 3.11: Comparison of formulations with Halpin-Tsai and Iso-stress models

The iso-stress model is defined by Equation 3.1b presented earlier. The Halpin-Tsai model is shown as steadily increasing. The adjustment made for the value of ζ is visible only at higher values of V_{fibers} , where the model diverges from the original. The Halpin-Tsai model does not take fiber packing into account; as a result, its all-encompassing nature might be responsible for the model forming a relative upper-bound for all the other classical models.

On the other hand, the iso-stress model is considered inadequate for fiber-reinforced yarns, due to its non-consideration of shear effects. Therefore, it can form the lower bound for the transverse Young's modulus.

Our formulation lies between these two models and is very well-correlated to the experimental data from Tsai & Hahn [82].

The comparison with the Spencer and de Moraes model is shown in Figure 3.12.

Figure 3.12: Comparison of formulations with Spencer and de Morais models

These two models are closer in results to our formulation. The de Morais model has been designed for square-shaped fibers. Hence, its accuracy may be expected to reduce at high values of V_{fibers} , as the role of circular fibers increases and the interface region's effect becomes more pronounced.

Spencer's model takes into consideration fiber-packing array, and is therefore exhibits slightly better correlations to the SQS formulation than the de Morais model.

To quantify the accuracy of the model, error estimates of the data points to its corresponding prediction on each model is studied. This enables a fair comparison of the degree to which the model sufficiently represents the experimental data. The error estimates are summarized in Table 3.5; the E_T values can be seen Figure 3.12.

Table 3.5: Error estimates of prediction models with comparison to Tsai & Hahn [82]

Model	All 10 points		Best 6 points		Best 8 points	
	RMS Error (in GPa)	R ²	RMS Error (in GPa)	R ²	RMS Error (in GPa)	R ²
SQS with $\epsilon = 0.15$	2.566	0.582	1.021	0.795	1.928	0.539
SQS with $\epsilon = 0.20$	2.444	0.621	0.954	0.821	2.047	0.480
SQS with $\epsilon = 0.25$	2.353	0.649	0.938	0.827	2.005	0.502
SQS with $\epsilon = 0.3$	2.289	0.667	0.961	0.819	1.989	0.510
SQS with $\epsilon = 0.35$	2.249	0.679	1.006	0.801	1.910	0.548
Iso-stress	5.615	-1.001	4.050	-2.220	5.076	-2.194
Halpin-Tsai Original	2.059	0.731	1.041	0.787	1.676	0.652
Halpin-Tsai Adjusted	2.483	0.609	1.246	0.695	1.981	0.513
Spencer	2.185	0.697	1.060	0.779	1.665	0.656
Morias	2.400	0.634	1.090	0.767	1.830	0.585

The Root Mean Square (RMS) error is the square root of mean of the square of the error estimate. It can also be addressed as the standard deviation of the error estimate. The R² is the Regression Correlation coefficient. It is a "goodness-of-fit" parameter reflecting well the prediction model correlates with the experimental data. It is desirable for the R² to have a value close to 1.

In Table 3.5, we have looked at the error data of all 10 points as well as the "Best 6" and "Best 8" data points that match with each model. This is because the experimental data itself has considerable scatter, which is common in composites given the possible processing variations. The five SQS models are shown as a function of their yarn porosity; the five

mathematical models are also shown. Also, in bold text is highlighted the best SQS estimate as well as the best mathematical model estimate.

It can be seen that when all 10 points are considered, the $\varepsilon = 0.35$ provides the least error-prone model with a RMS error of 2.249 and a R^2 value of 0.679. However, it can be seen that this model is still inferior to the Original Halpin-Tsai formulation which exhibits lesser error and a better fit.

However, it must be noted that the above numbers are greatly influenced by experimental outliers, as their error estimate has a stronger effect on the RMS error and the R^2 value. If the outlier points are excluded by taking the "Best 6" data points, this reasoning becomes clear. Not only is the $\varepsilon = 0.25$ better than the classical model estimates, but also each and every case of the SQS formulations are better than the best classical model estimate (Halpin-Tsai Original, in this case).

However, it is not wise to consider just these six points, since the errors increase significantly as more points are included. For the "Best 8" case, the estimate of the SQS model ($\varepsilon = 0.35$) still underperforms compared to the Spencer model.

3.9 Comparison of SQS model with Wacker *et al.* data and classical mathematical models

The SQS model is also compared to the work of Wacker *et al.* [48]. In this work, the interphase layer between fibers and matrix is investigated by FEM analysis and compared to experimental data as well as other computational and mathematical formulations.

3.9.1 Comparison with anhydride-cured epoxy resin and PU-treated glass experimental data

For comparison with the experimental data of Wacker *et al.*, the initial material properties are shown in Table 3.6.

Table 3.6: Initial material properties for comparison with the experimental data of Wacker *et al.* [48]

Fiber properties (PU-treated glass)	$E_f = 77 \text{ GPa}$ $\nu_f = 0.2$ $G_f = \frac{E_f}{2(1+\nu_f)} = 32.08 \text{ GPa (Isotropic)}$
Matrix properties (Anhydride-cured epoxy resin)	$E_m = 4 \text{ GPa}$ $\nu_m = 0.35$ $G_m = \frac{E_m}{2(1+\nu_m)} = 1.48 \text{ GPa (Isotropic)}$

Based on the data in Table 3.6, the curves for the SQS formulation, as well as the other mathematical models are graphed, as shown in Figure 3.13.

Figure 3.13: Comparison of formulations with mathematical models for Wacker *et al.* [48]

It can be seen that the SQS formulation correlates well with the experimental points. The formulations are stable with respect to the yarn porosity as well, showing little divergence.

Amongst the other mathematical models, de Morais' predictions are the best-fitting next to the current SQS model. This is also reflected quantitatively by Table 3.7.

Table 3.7: Error estimates of prediction models with comparison to experimental data of Wacker *et al.* [48]

Model	All 4 points		Best 3 points	
	RMS Error (in GPa)	R ²	RMS Error (in GPa)	R ²
SQS with $\varepsilon = 0.15$	0.706	0.941	0.384	0.975
SQS with $\varepsilon = 0.20$	0.639	0.951	0.284	0.986
SQS with $\varepsilon = 0.25$	0.586	0.959	0.198	0.993
SQS with $\varepsilon = 0.3$	0.545	0.964	0.136	0.997
SQS with $\varepsilon = 0.35$	0.518	0.968	0.117	0.998*
Iso-stress	2.308	0.372	1.313	0.706
Halpin-Tsai Original	2.087	0.487	1.722	0.494
Halpin-Tsai Adjusted	2.136	0.462	1.797	0.449
Spencer	1.216	0.825	0.712	0.913
Morias	0.812	0.922	0.464	0.963

* In the case of the "Best 3" points, all the SQS formulations lie within the error range of the data points

It is thus seen that the SQS formulation provides a high correlation and least RMS error. In fact, all the SQS formulations ($\varepsilon = 0.15$ to 0.35) provide better results than the best mathematical formulation (by de Morais). All comparisons with experiments are discussed in publications [83].

3.9.2 Comparison with boron aluminum composite computational data

For comparison with the computational data obtained from the experimental formulation of Wacker *et al.*, the initial material properties are shown in Table 3.8 .

Table 3.8: Initial material properties for comparison with the Case 1 computational data of Wacker *et al.* [48]

Fiber properties (Boron fibers)	$E_f = \mathbf{379.3 \text{ GPa}}$ $\nu_f = \mathbf{0.1}$ $G_f = \frac{E_f}{2(1+\nu_f)} = \mathbf{172.41 \text{ GPa}}$ (Isotropic)
Matrix properties (Aluminum matrix)	$E_m = \mathbf{68.3 \text{ GPa}}$ $\nu_m = \mathbf{0.3}$ $G_m = \frac{E_m}{2(1+\nu_m)} = \mathbf{26.27 \text{ GPa}}$ (Isotropic)
Fiber Volume Fraction, V_{fibers}	$V_{\text{fibers}} = \mathbf{0.47}$

This is the case used in Example 1 of the work of Wacker *et al.* This case doesn't have an interphase region and carries out FEM computation to obtain the transverse Young's modulus for only a single value of fiber volume fraction ($V_{\text{fibers}} = 0.47$). The results of our model and comparisons with other computational models are shown in Table 3.9.

Table 3.9: Transverse Young's Modulus predictions based Case 1 of Wacker *et al.* [48]

Model	E_T (in GPa)
SQS with $\varepsilon = 0.15$	148.90
SQS with $\varepsilon = 0.20$	148.27
SQS with $\varepsilon = 0.25$	147.82
SQS with $\varepsilon = 0.3$	147.60
SQS with $\varepsilon = 0.35$	147.66
Wacker <i>et al.</i> [48] FEM	140.4
Sun & Vaidya's [84] Square Array FEM	144
Sun & Vaidya's [84] Hexagonal Array FEM	136.5
Sun & Chen [85]	135
Hashin & Rosen [30]	139.1
Experimental observation of Kenaga <i>et al.</i> [86]	140

As can be seen from Table 3.9, when the SQS formulation is compared to the computational models, there are mixed results. Sun and Vaidya's square array FEM model [84] gives values quite close to the SQS formulations. However, the hexagonal array FEM model gives a lower value. This is expected since square arrays give higher values for the transverse elastic modulus of unidirectional composites than hexagonal arrays [84].

The work of Wacker *et al.* [48] and Hashin & Rosen [30] is close to the Kenaga experimental value [86] (which lies in an acceptable range; within 10% of the SQS formulations).

3.10 Effect of rectangularisation of the SQS unit cell

Having worked on the quarter yarn square unit cell for experimental and computation validation for our formulation, it is quite impractical to expect that the yarn arrangements will always remain perfectly square. Under compression, the fiber yarn array may deform changing the unit cell geometry into rectangular shape. While the area around a yarn can be idealized to a square, it is inappropriate to assume that the unit cell will behave as such. Hence it is necessary to briefly investigate the effects of rectangularising the square unit cell.

Table 3.10: Formulation of the Single Quarter Yarn - Uneven Rectangled model (SQUR)

Single Quarter Yarn - Uneven Rectangled (SQUR)
$V_{yarn} = \frac{\frac{\pi}{4}}{\left(\frac{\rho_h}{R}+1\right)\left(\frac{\rho_v}{R}+1\right)}; \quad r = \frac{\rho_v}{\rho_h};$ $\frac{\rho_h}{R} = \frac{\sqrt{(r+1)^2 - 4r\left(1 - \frac{\pi}{4V_{yarn}}\right) - (1+r)}}{2r}; \quad V_{yarn} \leq \frac{\pi}{4}$ $\frac{1}{E_{eq}} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} +$ $\frac{1}{6\left(\frac{\rho_h}{R} + 1\right)} \left[\frac{2(1 - v_{yarn})}{E_{yarn}} - \frac{1}{G_{yarn}} + \frac{2(1 - v_m)}{E_m} - \frac{1}{G_m} \right]$

Table 3.10 shows the formulation of the SQUR model. It can be seen that the spacing parameter ρ from previous formulations has been replaced with two new parameters, the horizontal spacing, ρ_h and the vertical spacing, ρ_v . This gives rise to a parameter, r which is the ratio ρ_v/ρ_h . Based on r , the entire formulation is created. It can be noted that if r is forced to take a value of 1, the entire formulation will reduce to the SQS formulation.

The formulation is once again compared to the data obtained from Tsai and Hahn [82]. The initial values for the formulation are taken from Table 3.3. Additionally, the yarn porosity is held at a constant value of $\epsilon = 0.2$, while the value of r is changed from 0.25 to 4 at a multiple of 2. Each value of r is shown as a different curve in Figure 3.14.

Figure 3.14: Stability of the SQR formulation with changes to r

It can be seen that the formulation diverges at $V_{\text{fibers}} = 0$ and then converges again at a particular value. This is due to the fact that the value of ρ_h/R is the same at these two points. This ensures that the ratio r is rendered inconsequential.

The reason for the divergence is due to the fact that the ρ_h/R affects the shear component of the formulation. If this value is affected, the model will change its behaviour accordingly.

It is also seen that divergence of the formulation is not significant. At the point of maximum divergence, there is a difference of only 0.66 GPa between the values of $r = 0.25$ and $r = 4$ (multiple change by 16 times). Hence the formulation is quite stable for changes in r .

3.11 Concluding Remarks

This chapter has thus focused on the use, features improvements of the SQS model. Based on the findings, the following conclusions can be drawn:

- i. Despite significant scatter in the experimental data seen in Figures 3.12-3.14, the current SIC formulation provides a matching trend at most of the fiber volume fractions. In addition, the SIC concept covers all the essential load transfer mechanisms from phase-to-phase.
- ii. Both the Single Quarter Yarn - Squared (SQS) model and the Single Quarter Yarn - Horizontal Triangled (SQHT) model provide good estimates of the transverse Young's modulus. However, due to its conservative nature it is better to use the SQS model for further analyses.
- iii. The model can be used when the reinforcement is either a single homogeneous body or a cluster of fibers. The SQS model takes into account fiber packing in the unit cell; something that is not completely addressed in the Halpin-Tsai model.
- iv. Due to the nature of the unit cell shape, the maximum fiber volume fraction that can be investigated is limited to $\frac{\pi}{4}$ or 0.785 (when the reinforcement is a single homogeneous body). This value is limited further when yarn porosity is introduced.
- v. The increase of yarn porosity has a dual effect on the transverse Young's modulus; while causing a decrease in the normal component of elasticity, it causes an increase in the shear component of elasticity since yarn perimeter has increased. As a result, the transverse Young's modulus is quite stable to changes in yarn porosity.
- vi. The SQS model is capable of competing effectively with the classical models and it provides a reliable means of predicting the transverse Young's modulus. The correlation with experiments and computational data is within acceptable range, despite the fact that the experimental points are already scattered.

- vii. It is also possible to alter the shape of the unit cell from square (in the case of SQS) to rectangle (in the case of SQUR), without greatly affecting the transverse Young's modulus. This provides greater impetus for the practical application of this model, since real-life unit cells are likelier to be uneven rectangles than squares.
- viii. The final form of the SQS equation appears quite similar to the original iso-stress equation for composites (Equation 3.3a), with the addition of a shear component. This renders the equation user-friendly and easy to understand as an extension of the original iso-stress theory.

The SQS model will thus be used as yarn property input for the woven and braided textile MATLAB[®] model discussed later on. Due to its consideration of shear reinforcing effects in square circular yarn unit cells, it is henceforth denoted a part of the Shear-Inclusive Circular (SIC) yarn input model. The Poisson ratio formulation for this SIC model is discussed in the next chapter.

Chapter 4: SIC Transverse Poisson's Ratio

Classical composite theory makes use of rule-of-mixture expressions to obtain the longitudinal and transverse Poisson ratio of fiber-reinforced composites as shown in Equations 4.1a and 4.1b (Jones [24]) in terms of composite fibers and matrix properties.

$$\nu_{eqLT} = V_f \nu_f + (1 - V_f) \nu_m \quad \dots(4.1a)$$

$$\frac{1}{\nu_{eqTT}} = \frac{V_f}{\nu_f} + \frac{(1-V_f)}{\nu_m} \quad \dots(4.1b)$$

As was the case of the transverse Young's modulus in the previous chapter, Equation 4.1a is obtained by making use of the iso-strain conditions in long-fiber reinforced composites. This equation is widely used to obtain fairly accurate estimates of the longitudinal Poisson's ratio of composites. On the other hand, Equation 4.1b is obtained from the iso-stress conditions, and is not reliable for predicting the transverse Poisson ratio of composites [24]. It is expected that shear effects are also involved in the transverse direction; however, these are not reflected in Equation 4.1b, which may be noted from its derivation shown in Chapter 2. As said before, fiber packing arrangement is also not considered.

Additionally after having obtained the transverse Young's modulus, it is desirable to obtain other relations that complement and reinforce the interfacial complementary triangular reinforcement-matrix elements method postulated in the previous chapter. Hence, using these same elements, a novel derivation of a transverse Poisson's ratio is considered. This would complete the yarn transverse-direction isotropic SIC theory and enable these formulations to be used in further applications as a set.

In this chapter, the detailed derivation of an expression for Transverse Poisson's Ratio of a long fiber-reinforced composite is introduced and validated with comparisons to classical and FEA results.

4.1 Obtaining local elemental normal and shear strains

For the transverse Poisson ratio, it is important to consider the strains experienced by the triangular reinforcement matrix elements (Figure 4.1). This entails determining the local elemental strain and transforming these to the global unit cell level (refer to Figure 3.3).

Figure 4.1: Equal and opposite forces acting on both elements

As shown in Section 3.3, the normal and shear forces acting on the hypotenuse section of the triangular element is given by Equations 4.2a and 4.2b.

$$F_N = F \cos \theta \quad \dots(4.2a)$$

$$F_S = F \sin \theta \quad \dots(4.2b)$$

These forces can be converted to normal and stress components by dividing by the length of the hypotenuse (given by the length h in the previous chapter) and a unit length.

$$\sigma_N = \frac{F_N}{h \cdot 1} = \frac{F_N}{\left(\frac{dy}{\cos \theta}\right)} = \frac{F \cos^2 \theta}{dy} \quad \dots(4.3a)$$

$$\tau_S = \frac{F_S}{h \cdot 1} = \frac{F_S}{\left(\frac{dy}{\cos \theta}\right)} = \frac{F \cos \theta \sin \theta}{dy} \quad \dots(4.3b)$$

Dividing Equations 4.3a and 4.3b by the Young's and shear moduli of the constituent material respectively provides the corresponding local normal and shear strains.

$$\epsilon_N = \frac{\sigma_N}{E} = \frac{F \cos^2 \theta}{E dy} \quad \dots(4.4a)$$

$$\gamma_S = \frac{\tau_S}{G} = \frac{F \cos \theta \sin \theta}{G dy} \quad \dots(4.4b)$$

It should be noted that, resulting from the normal force extension, there is a transverse normal strain on the hypotenuse of the element, resulting in a contraction of length as shown in Figure 4.2.

Figure 4.2: Strains acting on the triangular element

This transverse strain, ε_T is quantified as a relation to normal strain, ε_N (using the Poisson deformation effect) shown by Equation 4.4c.

$$\varepsilon_T = -\nu \varepsilon_N = -\nu \frac{F}{E dy} \cos^2 \theta \quad \dots(4.4c)$$

4.2 Converting local strains to global unit cell level strains

These local element strain values can now be transformed to the global unit cell-level x and y coordinates using Mohr's Circle expressions for strain [87] adapted as shown by Equations 4.5a and 4.5b.

$$\varepsilon_x = \frac{\varepsilon_N + \varepsilon_T}{2} + \frac{\varepsilon_T - \varepsilon_N}{2} \cos 2\theta_{rot} + \frac{\gamma_S}{2} \sin 2\theta_{rot} \quad \dots(4.5a)$$

$$\varepsilon_y = \frac{\varepsilon_N + \varepsilon_T}{2} - \frac{\varepsilon_T - \varepsilon_N}{2} \cos 2\theta_{rot} - \frac{\gamma_S}{2} \sin 2\theta_{rot} \quad \dots(4.5b)$$

$$\text{where } \theta_{rot} = 90^\circ - \theta$$

The angle of rotation is not the same as θ , but instead holds a value of $(90^\circ - \theta)$. This results in the set of Equations 4.6a and 4.6b, which are formed by taking into consideration all the normal and shear strains of the interface element (Equations 4.4a, 4.4b and 4.4c).

$$\varepsilon_x = \frac{(1-\nu)}{2} \frac{F \cos^2 \theta}{E dy} + \frac{(1+\nu)}{2} \frac{F \cos^2 \theta}{E dy} \cos 2\theta + \frac{F \sin \theta \cos \theta}{2G dy} \sin 2\theta \quad \dots(4.6a)$$

$$\varepsilon_y = \frac{(1-\nu)}{2} \frac{F \cos^2 \theta}{E dy} - \frac{(1+\nu)}{2} \frac{F \cos^2 \theta}{E dy} \cos 2\theta - \frac{F \sin \theta \cos \theta}{2G dy} \sin 2\theta \quad \dots(4.6b)$$

Simplifying, we get

$$\varepsilon_x = \frac{(1-\nu)}{2} \frac{F \cos^2 \theta}{E dy} + \frac{(1+\nu)}{2} \frac{F \cos^2 \theta \cos 2\theta}{E dy} + \frac{F \sin^2 \theta \cos^2 \theta}{G dy} \quad \dots(4.7a)$$

$$\varepsilon_y = \frac{(1-\nu)}{2} \frac{F \cos^2 \theta}{E dy} - \frac{(1+\nu)}{2} \frac{F \cos^2 \theta \cos 2\theta}{E dy} - \frac{F \sin^2 \theta \cos^2 \theta}{G dy} \quad \dots(4.7b)$$

4.3 Aggregating all unit cell sections

For obtaining the Poisson ratio, a summation over the entire unit cell is to be carried out. This is done by obtaining the ratio of equivalent y-direction strain, ε_y to the equivalent x-direction strain, ε_x as shown in Equation 4.8.

$$\nu_{eqT} = -\frac{(\varepsilon_y)_{eq}}{(\varepsilon_x)_{eq}} = \frac{[\iint \Delta y dx]_{Unitcell}/(\rho+R)^2}{[\iint \Delta x dy]_{Unitcell}/(\rho+R)^2} = -\frac{[\iint \Delta y dx]_{Unitcell}}{[\iint \Delta x dy]_{Unitcell}} \quad \dots(4.8)$$

$$\begin{aligned}
&= - \frac{\int_{A_{rein}} (\Delta y)_{rein} dx + \int_{A_m} (\Delta y)_m dx + \left[\iint_{Triangle} (\varepsilon_y) dA \right]_{rein} + \left[\iint_{Triangle} (\varepsilon_y) dA \right]_m}{\int_{A_{rein}} (\Delta x)_{rein} dy + \int_{A_m} (\Delta x)_m dy + \left[\iint_{Triangle} (\varepsilon_x) dA \right]_{rein} + \left[\iint_{Triangle} (\varepsilon_x) dA \right]_m} \\
\Rightarrow v_{eqT} &= - \frac{(\varepsilon_y)_{rein} \left[\frac{\pi R^2}{4} \right] + (\varepsilon_y)_m \left[(\rho+R)^2 - \frac{\pi R^2}{4} \right] + \left[\frac{1}{2} \int \Delta y dx \right]_{rein} + \left[\frac{1}{2} \int \Delta y dx \right]_m}{(\varepsilon_x)_{rein} \left[\frac{\pi R^2}{4} \right] + (\varepsilon_x)_m \left[(\rho+R)^2 - \frac{\pi R^2}{4} \right] + \left[\frac{1}{2} \int \Delta x dy \right]_{rein} + \left[\frac{1}{2} \int \Delta x dy \right]_m} \dots(4.9)
\end{aligned}$$

The integrals for the first and second terms in both numerator and denominator in Equation 4.9 above are the areas of the pure reinforcement and pure matrix regions respectively, as shown in the previous chapter. The unique point of interest here is determining the interface region triangular terms.

Using the relation, $y = R \sin \theta$, we differentiate with respect to θ to get,

$$dy = R \cos \theta d\theta$$

$$dx = dy \tan \theta = R \sin \theta d\theta$$

Using the terms in Equation 4.7a,

$$\begin{aligned}
&\int \Delta x dy = \int (\varepsilon_x dx) \cdot dy \\
&= \int_0^{\frac{\pi}{2}} \left[\frac{(1-\nu)}{2} \frac{F \cos \theta \sin \theta}{E} + \frac{(1+\nu)}{2} \frac{F \cos \theta \sin \theta}{E} \cos 2\theta + \frac{F \sin^3 \theta \cos \theta}{G} \right] \cdot R \cos \theta d\theta \\
&= FR \int_0^{\frac{\pi}{2}} \left[\frac{(1-\nu)}{2E} \cos^2 \theta \sin \theta + \frac{(1+\nu)}{2} \cos^2 \theta \sin \theta \cos 2\theta + \frac{1}{G} \sin^3 \theta \cos^2 \theta \right] \cdot d\theta \\
&= \frac{FR}{2} \left[\frac{(1-\nu)}{E} VII + \frac{(1+\nu)}{E} VIII + \frac{1}{G} IX \right]
\end{aligned}$$

where the values for the integrals VII, VIII and IX are as shown in Table 4.1 that give Equation 4.10.

Table 4.1: Integrals and their evaluated forms

Integral	Evaluates to
$VII = \int_0^{\frac{\pi}{2}} \cos^2 \theta \sin \theta d\theta$	$\frac{1}{3}$

$VIII = \int_0^{\frac{\pi}{2}} \cos^2 \theta \sin \theta \cos 2\theta d\theta$	$\frac{1}{15}$
$IX = \int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^2 \theta d\theta$	$\frac{2}{15}$

Note: Appendix A provides the integration details for the terms *VII*, *VIII* and *IX*.

$$= \frac{FR}{2} \left[\frac{(1-\nu)}{E} \left(\frac{1}{3} \right) + \frac{(1+\nu)}{E} \left(\frac{1}{15} \right) + \frac{1}{G} \left(\frac{2}{15} \right) \right] = \frac{FR}{2} \left[\frac{(6-4\nu)}{15E} + \frac{2}{15G} \right]$$

$$\text{Therefore} \quad \int \Delta x dy = \frac{FR}{15} \left[\frac{(3-2\nu)}{E} + \frac{1}{G} \right] \quad \dots(4.10)$$

Using the terms in Equation 4.7b,

$$\begin{aligned} \int \Delta y dx &= \int (\varepsilon_y dy) \cdot dx \\ &= \int_0^{\frac{\pi}{2}} \left[\frac{(1-\nu)}{2} \frac{F \cos^2 \theta}{E} - \frac{(1+\nu)}{2} \frac{F \cos^2 \theta}{E} \cos 2\theta - \frac{F \sin^2 \theta \cos^2 \theta}{G} \right] \cdot R \sin \theta d\theta \\ &= FR \int_0^{\frac{\pi}{2}} \left[\frac{(1-\nu)}{2E} \cos^2 \theta \sin \theta - \frac{(1+\nu)}{2E} \cos^2 \theta \sin \theta \cos 2\theta - \frac{1}{G} \sin^3 \theta \cos^2 \theta \right] \cdot d\theta \\ &= \frac{FR}{2} \left[\frac{(1-\nu)}{E} VII - \frac{(1+\nu)}{E} VIII - \frac{1}{G} IX \right] \\ &= \frac{FR}{2} \left[\frac{(1-\nu)}{E} \left(\frac{1}{3} \right) - \frac{(1+\nu)}{E} \left(\frac{1}{15} \right) - \frac{1}{G} \left(\frac{2}{15} \right) \right] = \frac{FR}{2} \left[\frac{(4-6\nu)}{15E} - \frac{2}{15G} \right] \\ \text{Therefore} \quad \int \Delta y dx &= \frac{FR}{15} \left[\frac{(2-3\nu)}{E} - \frac{1}{G} \right] \quad \dots(4.11) \end{aligned}$$

For the regions of pure reinforcement and pure matrix,

$$\varepsilon_x = \frac{F}{(\rho+R) \cdot 1 \cdot E} = \frac{F}{(\rho+R)E} \quad \dots(4.12a)$$

$$\varepsilon_y = -\nu \varepsilon_x = -\nu \frac{F}{(\rho+R)E} \quad \dots(4.12b)$$

Using the results obtained in Equations 4.10, 4.11, 4.12a and 4.12b and applying them to Equation 4.9, we get,

$$(4.9) \Rightarrow -\frac{\frac{-\nu_{reinf} F}{(\rho+R)E_{reinf}} \left[\frac{\pi R^2}{4} \right] + \frac{-\nu_m F}{(\rho+R)E_m} \left[(\rho+R)^2 - \frac{\pi R^2}{4} \right] + \left[\frac{FR}{30} \left[\frac{2-3\nu}{E} - \frac{1}{G} \right] \right]_{reinf} + \left[\frac{FR}{30} \left[\frac{2-3\nu}{E} - \frac{1}{G} \right] \right]_m}{\frac{F}{(\rho+R)E_{reinf}} \left[\frac{\pi R^2}{4} \right] + \frac{F}{(\rho+R)E_m} \left[(\rho+R)^2 - \frac{\pi R^2}{4} \right] + \left[\frac{FR}{30} \left[\frac{3-2\nu}{E} + \frac{1}{G} \right] \right]_{reinf} + \left[\frac{FR}{30} \left[\frac{3-2\nu}{E} + \frac{1}{G} \right] \right]_m}$$

Dividing throughout by $F(\rho + R)$, we get,

$$\begin{aligned} & \frac{\nu_{reinf}}{E_{reinf}} \left[\frac{(\pi R^2)/4}{(\rho+R)^2} \right] + \frac{\nu_m}{E_m} \left[\frac{(\rho+R)^2 - \frac{\pi R^2}{4}}{(\rho+R)^2} \right] - \frac{R}{30(\rho+R)} \left[\frac{2-3\nu}{E_{reinf}} - \frac{1}{G_{reinf}} + \frac{2-3\nu_m}{E_m} - \frac{1}{G_m} \right] \\ &= \frac{\frac{1}{E_{reinf}} \left[\frac{(\pi R^2)/4}{(\rho+R)^2} \right] + \frac{1}{E_m} \left[\frac{(\rho+R)^2 - \frac{\pi R^2}{4}}{(\rho+R)^2} \right] + \frac{R}{30(\rho+R)} \left[\frac{3-2\nu}{E_{reinf}} + \frac{1}{G_{reinf}} + \frac{3-2\nu_m}{E_m} + \frac{1}{G_m} \right]}{\frac{1}{E_{reinf}} \left[\frac{(\pi R^2)/4}{(\rho+R)^2} \right] + \frac{1}{E_m} \left[\frac{(\rho+R)^2 - \frac{\pi R^2}{4}}{(\rho+R)^2} \right] + \frac{R}{30(\rho+R)} \left[\frac{3-2\nu}{E_{reinf}} + \frac{1}{G_{reinf}} + \frac{3-2\nu_m}{E_m} + \frac{1}{G_m} \right]} \end{aligned}$$

Since $\frac{(\pi R^2/4)}{(\rho+R)^2} = V_{reinf}$, we can simplify to get:

$$\begin{aligned} v_{eqT} = & \frac{\frac{\nu_{reinf} V_{reinf}}{E_{reinf}} + \frac{\nu_m (1 - V_{reinf})}{E_m} - \frac{1}{30 \left(\frac{\rho}{R} + 1 \right)} \left[\frac{2-3\nu_{reinf}}{E_{reinf}} - \frac{1}{G_{reinf}} + \frac{2-3\nu_m}{E_m} - \frac{1}{G_m} \right]}{\frac{V_{reinf}}{E_{reinf}} + \frac{(1 - V_{reinf})}{E_m} + \frac{1}{30 \left(\frac{\rho}{R} + 1 \right)} \left[\frac{3-2\nu_{reinf}}{E_{reinf}} + \frac{1}{G_{reinf}} + \frac{3-2\nu_m}{E_m} + \frac{1}{G_m} \right]} \end{aligned}$$

If the reinforcement is taken as an agglomeration of yarns, instead of a single homogeneous body, we get

$$\begin{aligned} v_{eqT} = & \frac{\frac{\nu_{yarn} V_{yarn}}{E_{yarn}} + \frac{\nu_m (1 - V_{yarn})}{E_m} - \frac{1}{30 \left(\frac{\rho}{R} + 1 \right)} \left[\frac{2-3\nu_{yarn}}{E_{yarn}} - \frac{1}{G_{yarn}} + \frac{2-3\nu_m}{E_m} - \frac{1}{G_m} \right]}{\frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} + \frac{1}{30 \left(\frac{\rho}{R} + 1 \right)} \left[\frac{3-2\nu_{yarn}}{E_{yarn}} + \frac{1}{G_{yarn}} + \frac{3-2\nu_m}{E_m} + \frac{1}{G_m} \right]} \end{aligned}$$

By making use of $\frac{\rho}{R} + 1 = \sqrt{\frac{\pi}{4V_{yarn}}}$, the equation can be rewritten as

$$\begin{aligned} v_{eqT} = & \frac{\frac{\nu_{yarn} V_{yarn}}{E_{yarn}} + \frac{\nu_m (1 - V_{yarn})}{E_m} - \frac{1}{15 \sqrt{\frac{V_{yarn}}{\pi}}} \left[\frac{2-3\nu_{yarn}}{E_{yarn}} - \frac{1}{G_{yarn}} + \frac{2-3\nu_m}{E_m} - \frac{1}{G_m} \right]}{\frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} + \frac{1}{15 \sqrt{\frac{V_{yarn}}{\pi}}} \left[\frac{3-2\nu_{yarn}}{E_{yarn}} + \frac{1}{G_{yarn}} + \frac{3-2\nu_m}{E_m} + \frac{1}{G_m} \right]} \dots(4.13) \end{aligned}$$

This constitutes the transverse Poisson ratio formulation within the SIC yarn model. Even though testing of different formulations of transverse Young's modulus, contingent on the shape and dimensions of the unit cell was done in Chapter 3, similar testing for different

formulations of Poisson ratio was not considered necessary. This is because the idea was to build a consistent model for fiber reinforced composites, utilizing the same unit cell model for transverse Poisson ratio as was used for the transverse elastic modulus in the previous chapter. This, therefore, completes the objective of having a consistent formulation of the transversely isotropic SIC yarn model.

Unlike transverse Young's modulus, it is difficult to validate the transverse Poisson's ratio of long fiber reinforced composites by experiments due to practical design concerns. Hence, comparisons with other classical models (as discussed in Chapter 2) are carried out. Additionally, the versatility and flexibility of finite element analysis to study the properties of composites made it another viable means to compare and assess the formulation.

4.4 Comparison with classical formulations

In order to assess how the transverse Poisson's ratio formulation (Equation 4.14) relates to contemporary mathematical formulations, two comparisons were carried out. The use of equivalent yarn properties is required in order to consider the effects of porosity (denoted by ϵ). For the same fiber volume fractions in the unit cell, the presence of porosity affects the yarn radius and thus, the size of the interface region. As was the case in the previous chapter, the porosity sets a limit on the maximum fiber volume fraction of the composite (based on Equations 3.23).

The first comparison made use of boron fibers in aluminum matrix composite. The isotropic boron fiber properties used are: $E_f = 379.3$ GPa, $\nu_f = 0.1$, $G_f = 172.4$ GPa; and the isotropic aluminum matrix properties are: $E_m = 68.3$ GPa, $\nu_m = 0.3$, $G_m = 26.3$ GPa [48]. The second comparison looked at glass fibers-epoxy matrix composites. The isotropic glass fiber properties used are: $E_f = 73.1$ GPa, $\nu_f = 0.22$, $G_f = 30.0$ GPa; and the isotropic epoxy matrix properties are $E_m = 3.45$ GPa, $\nu_m = 0.35$, $G_f = 1.28$ GPa [82].

The performance of Tsai-Halpin's model for transverse Young's modulus when compared to experiments is quite good and has been discussed in the previous chapter. However, the Tsai-Halpin model doesn't provide a direct equation for the transverse Poisson ratio; hence, the transverse isotropy Equation 2.5 is used as an indirect method [27] to obtain Poisson ratio. Additionally, the inverse rule-of-mixture (Equation 4.1b) is also compared against. The

results for the first case study (boron fibers in aluminum matrix composite) along with of the current formulation (Equation 4.13) with yarn porosities of 0.3, 0.35, 0.4 and 0.45 (filled with matrix) are shown in Figure 4.3.

Figure 4.3: Comparison of derived formulation with classical models for boron fiber-aluminum composite [48]

It may be seen that the derived formulation starts at the point of pure matrix (i.e. $v_m = 0.3$) and gradually falls as the fiber volume fraction increases. This trend is expected as the composites will begin to assume fiber characteristics as the fiber volume fraction increases [88]. Note that as the porosity increases, even though the fiber volume fraction remains constant, the radius of the yarn increases. This results in an increase of the circumference and a more pronounced interface region effects. This is expected since a larger yarn within the unit cell will provide greater resistance to deformation. Hence, the Poisson ratio lowers.

It can also be seen that all the curves lie in the region between the inverse rule-of-mixture and the Tsai-Halpin model. The inverse rule-of-mixture, though not an accurate estimator of composites transverse Poisson's ratio, it still provides a classical lower bound to this mathematical formulation. The indirect Halpin-Tsai formulation proves to be quite erroneous

when compared to the other classical formulations. Within the region of $V_f = 0$ to $V_f = 0.6$, the value of the transverse Poisson ratio is shown to be greater than that of either of the constituent materials (boron and aluminum). This abnormality is due to the shear modulus predictions from Tsai-Halpin being underestimated, resulting in higher values than expected for the indirect Poisson's ratio [27]. Therefore, it is not accurate to use the Tsai-Halpin model to indirectly determine transverse Poisson ratio.

The results for the second case (glass fibers in epoxy matrix composite) along with of the current formulation (Equation 4.13) with yarn porosities of 0.3, 0.35, 0.4 and 0.45 are shown in Figure 4.4.

Figure 4.4: Comparison of derived formulation with classical models for glass fiber-epoxy composite [82]

The second set of comparisons yield similar results. It is seen that the derived formulation also starts at the point of pure matrix (i.e. $v_m = 0.35$) and lies in the region bounded by the inverse rule of mixtures. The Tsai-Halpin model is again unable to provide meaningful comparisons.

Another feature of interest is the closeness of the glass-epoxy case curves to their inverse rule of mixture formulation as compared to the boron-aluminum case curves and its

corresponding inverse rule of mixture. This could be due to the relative ratio of fiber modulus to matrix modulus (both Young's and shear moduli). The first case has fiber-to-matrix moduli ratio of only about 5.55 whereas the second case has fiber-to-matrix moduli ratio greater than 21. This means that starting from the pure matrix case, the introduction of fibers greatly influences the overall resistance of the composites to deformation in the glass-epoxy case than in the boron-aluminum case. Also, the use of high modulus matrix (e.g. aluminium) may also enhance the fiber-matrix interface shear contribution to the composite properties.

4.5 Comparison with finite element analysis

The quarter circle enclosed in a square unit cell as shown in Figure 3.2 is a representative geometry that can be replicated to form a wholesome reinforced composites. Thus, the properties of this single unit cell should be sufficient for modeling the entire composite. However, the physical boundary limitations and discretization of the unit cell into elements normally prevalent in FEA cause variations in the obtained properties. To safeguard against these computational errors, some convergence tests have to be carried out. This is primarily to ensure that localized effects of boundary conditions and discretization errors do not affect the FEA results.

4.5.1 Bidirectional boundary conditions

A sixteen (4×4) unit cell arrangement with 0.3 unit mesh size is used for this FEA work. The reason for using this arrangement and mesh size is explained in the next section on convergence tests.

Figure 4.5: Applied boundary conditions

The unit cell arrangement is thus subjected to the roller boundary conditions on two adjacent edges ($x = 0$; $y = 0$) and normal displacement boundary conditions on the two opposite edges ($x = 4(\rho + R)$; $y = 4(\rho + R)$) shown in Figure 4.5. The displacement boundary conditions are chosen such that unequal strains can be applied at the two edges. The FEA model with the von Mises stress plot for unidirectional displacement boundary condition is shown in Figure 4.6. As expected, it is seen that the stress along the faces is transmitted through the matrix and fibers, rather than only through the matrix.

Figure 4.6: FEA model for 16 unit cells arrangement showing von Mises stress

Global mesh sizes are taken at 0.3 units and local area mesh sizes are scaled down accordingly. The mesh is thus refined finely along the yarn-matrix interface in order to capture the interface effects and enable appropriate comparisons with the SIC Poisson's Ratio formulation.

After simulation, the nodal reaction forces are aggregated at each edge and divided by the edge length to obtain the average stresses acting on the edge. Taking advantage of the transverse isotropy of the composite, the transverse Poisson ratio and Young's moduli are solved for simultaneously by the use of the 2D constitutive stress-strain relation in Equation 4.14 [89].

$$\begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} \\ -\frac{\nu}{E} & \frac{1}{E} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \end{bmatrix} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} \quad \dots(4.14)$$

where E and ν are the transverse Young's modulus and Poisson ratio respectively. σ and ε are the applied stress and strain in corresponding directions respectively. This system of equations is manipulated to obtain the following expression for ν :

$$\nu = \frac{\sigma_y \varepsilon_x - \sigma_x \varepsilon_y}{\sigma_y \varepsilon_y - \sigma_x \varepsilon_x} \quad \dots(4.15)$$

4.5.2 FE convergence tests

The element size is an important aspect for FEA, since a coarser mesh can provide results quicker than a finer mesh for the same solver parameters. However, due to roughly cut elements, the coarser mesh may provide results that are worse than finer meshes. Hence a balance must be reached between the two. Since element size is a gauge of the mesh density, a convergence test of simulation precision (up to 4 decimal places) against mesh density was carried out as shown in Figure 4.7.

Figure 4.7: Precision values for changing element sizes

The element size was progressively reduced from 0.7 unit to 0.15 unit and the corresponding changes in the transverse Poisson ratio mapped. Assuming the precision value at 0.15 unit to be 100%, the precision value at element size of 0.3 unit was 99.99%. Since, the duration of simulations increased greatly from 0.3 to 0.15 unit element size, a minimum precision of 99.99% at element size of 0.3 was deemed satisfactory for the FEA work.

Additionally, the application of displacement boundary conditions to the edges of the quarter yarn unit cell led to local stress build-ups. This was seen to affect the unit cell properties, since the smaller size of the unit cell would not allow the stress buildups to be distributed over the entire textile. Hence, an investigation was necessary to evaluate whether adding more unit cells would improve the FEA results. The simulation of multiple unit cells was done in arrangements of four (2×2), sixteen (4×4) (as shown in Figure 7) and thirty-six (6×6) unit cells to depict the actual composite pattern (Figure 4.8).

Figure 4.8: Arrangement of four (2×2), sixteen (4×4) and thirty six (6×6) unit cells for FEA

Assuming the values at thirty-six unit cells have 100% precision, the use of multiple unit cells (rather than just one) greatly improves the simulation precision. In fact, the values of single unit cells are only 88% precise for transverse Poisson ratio, compared to values of the 16 unit cells arrangement (with precisions over 99.5%). Hence, the (4×4) unit cell arrangement was chosen for the FEA.

Figure 4.7: Precision values for changing unit cells arrangement

It should be noted that the 16 unit cells (4×4) required more than 70,000 elements in total. Even though using the 36 unit cells (6×6) would improve precision, the increase would only be about 0.27%; however, the accompanying geometrical construction and FEA preprocessing would be greatly increased by more than twofold. The FEA models for different unit cells arrangements are shown in Appendix E.

4.5.3 FEA results

A comparison of the first case formulations (boron-aluminum) presented with FEA at different porosities of 0.3, 0.35, 0.4 and 0.45 was carried out; this is shown in Figure 4.8. The derived mathematical formulations are shown by the smooth lines, whereas the FEA simulation results are depicted by data points connected by lines.

Figure 4.8: Comparison of derived formulation with FEA for boron-aluminum composites [48]

Similarly, the second case glass-epoxy composites are also compared with FEA simulation results at porosities of 0.3, 0.35 & 0.4 as shown in Figure 4.9.

Figure 4.9: Comparison of derived formulation with FEA for glass-epoxy composites [82]

In both cases, it is seen that the FEA simulations follow the same trend as the derived formulation, causing a drop in the Poisson ratio as the porosity increased. The results match

quite well at low fiber volume fractions. However, the FEA results start to deviate to lower values than the transverse Poisson ratios estimated using the proposed formulation. This could be due to the fact that the FE model became more rigid due to higher content of stiffer fibers than the matrix, thereby reducing overall deformations seen by the unit cells in FEA [56].

Additionally, it must be noted that the SIC model considers porosity within both the geometry and the material properties; the mathematical effects are thus conserved and manifested in the final formulation. However, the FE model only considers it within the material properties, since it is difficult to distribute porosity between fibers and the overall composite. This may have led to errors within the FE model. Nonetheless, the SIC model shows good correlations with FEA, better than the other classical models available (e.g. Inverse Rule of Mixtures [24] & Tsai-Halpin [26]).

While it is possible to alter the shape of the unit cell from square to rectangular as was carried out in the previous chapter (Section 3.10 of the previous chapter), the effect of the change is expected to be insignificant to the transverse Poisson ratio as the reinforcement – matrix interface region does not change in dimensions (or integration parameters). However, the change of the interface region (i.e. the change of the reinforcement shape from circular) is expected to have an effect on the transverse properties of the composite. These changes are discussed in detail in the next chapter.

4.6 Concluding Remarks

This chapter has thus focused on the derivation and validation of the Poisson ratio formulation based on the SIC model. Based on the findings presented in this chapter, the following conclusions can be drawn:

- i. The final form of the SIC Poisson ratio formulation has a form that shows a similarity to the classical definition of Poisson ratio (Equation 4.7), where the numerator terms track the Poisson deformation and the denominator terms track the original deformation. This renders the equation easy to understand and implement for the SIC transverse properties model.
- ii. The Poisson ratio formulation can be used when the reinforcement is either a single homogeneous body or a cluster of fibers.

- iii. Due to the nature of the unit cell shape, the maximum fiber volume fraction that can be investigated is limited to $\frac{\pi}{4}$ or 0.785 (when the reinforcement is a single homogeneous body). This value is limited further when yarn porosity is introduced.
- iv. The derived formulation starts at pure matrix Poisson ratio value at zero fiber volume fraction and begins to grow stiffer as the strong fiber component is introduced. A reduction in deformation will thus lead to a fall in the overall Poisson ratio.
- v. The increase of yarn porosity has a direct effect on the transverse Poisson ratio. As the yarn size increases, it exhibits a greater resistance to deformation within the unit cell and therefore, has a lower composite Poisson ratio.
- vi. The ratio of fiber modulus to matrix modulus appears to be affecting the trend of the Poisson ratio formulation, resulting in lower values of Poisson ratio for high fiber modulus to matrix modulus ratio. The presence of the reinforcement-matrix interface interactions brings in shear effects that are not captured in other classical formulations.
- vii. The SIC Poisson ratio formulation fares better than the other classical models when compared to the FEA results, not only in overall trends but also in predicting comparable values of transverse Poisson ratio.

The obtained Shear-Inclusive Circular (SIC) yarn model expressions for transverse Young's modulus and transverse Poisson ratio are thus -

$$\frac{1}{E_T} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1-V_{yarn})}{E_m} + \frac{1}{3} \sqrt{\frac{V_{yarn}}{\pi}} \left[\frac{2(1-\nu_{yarn})}{E_{yarn}} + \frac{2(1-\nu_m)}{E_m} - \frac{1}{G_{yarn}} - \frac{1}{G_m} \right] \quad \dots(4.16a)$$

$$\nu_{TT} = \frac{\frac{\nu_{yarn} V_{yarn}}{E_{yarn}} + \frac{\nu_m (1 - V_{yarn})}{E_m} - \frac{1}{15} \sqrt{\frac{V_{yarn}}{\pi}} \left[\frac{2 - 3\nu_{yarn}}{E_{yarn}} - \frac{1}{G_{yarn}} + \frac{2 - 3\nu_m}{E_m} - \frac{1}{G_m} \right]}{\frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} + \frac{1}{15} \sqrt{\frac{V_{yarn}}{\pi}} \left[\frac{3 - 2\nu_{yarn}}{E_{yarn}} + \frac{1}{G_{yarn}} + \frac{3 - 2\nu_m}{E_m} + \frac{1}{G_m} \right]} \dots (4.16b)$$

This system of equations are used to consider shear reinforcing effects in square unit cells with circular reinforcements and form a fundamental means of comparisons for yarn input properties against the Tsai-Halpin [26] & Composite Cylinder Assemblage (CCA) [30] models as well as the Shear-Inclusive Elliptical (SIE) model developed in Chapter 5.

Chapter 5: Scalability of SIC Model to Elliptical Yarns

The derivations of the formulations of the transverse Young's modulus (E_T) and Poisson's ratio (ν_{TT}) formulations for matrix-infused circular yarns and their subsequent validation using experimental, computational and FEA techniques are reported in the previous chapters. This can be applied to unidirectional long fiber reinforced composites as well as multi-directional laminates with different unidirectional layers stacked up. However, to use for more complicated structures like textiles requires looking at non-circular yarns and evaluating how its properties would differ from circular yarns. This can be argued to have an impact on the properties of textile reinforced composites, especially since the yarns are largely non-circular (even if the fibers are assumed to be circular) as a result of textile creation and composite manufacturing techniques [20, 53-56].

5.1 Modeling for elliptical yarns in rectangular unit cell

The derivation work carried out in the previous chapters formed the foundation for developing a novel formulation for the transverse direction Young's modulus of an elliptical yarn with aspect ratio, AR , being the ratio of semi-minor axis to semi major axis. This, along with the final derived expression for the equivalent unit cell Young's modulus is shown in Table 5.1.

Table 5.1: Geometry and formulation of the Single Quarter Elliptical Yarn-Rectangled model

Single Quarter Elliptical Yarn - Rectangled (SQER)
$AR = \frac{R_b}{R_a}; \quad V_{yarn} = \frac{\frac{\pi}{4}}{\left(\frac{\rho}{R_b}+1\right)\left(\frac{\rho}{R_a}+1\right)};$ $\frac{\rho}{R_a} = \frac{\sqrt{(AR-1)^2 + \frac{\pi AR}{V_{yarn}}} - (AR+1)}{2}; \quad V_{yarn} \leq \frac{\pi}{4}$ $\frac{1}{E_T} = \frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m}$ $+ \frac{1}{2 \left(\frac{1}{AR} \frac{\rho}{R_a} + 1 \right)} \left[X \left(\frac{(1 - \nu_{yarn})}{E_{yarn}} + \frac{(1 - \nu_m)}{E_m} \right) + XI \left(\frac{1}{G_{yarn}} + \frac{1}{G_m} \right) \right]$

	<p style="text-align: center;">where</p> $X = \left(\frac{AR}{AR - 1} \right) - \left(\frac{AR(AR^2 - 1) + AR^2}{(AR^2 - 1)^{\frac{3}{2}}} \right) \tan^{-1} \left[\sqrt{(AR^2 - 1)} \right]$ $XI = \left(\frac{1}{AR - 1} \right) - \left(\frac{AR(AR^2 - 1) + AR^2}{(AR^2 - 1)^{\frac{3}{2}}} \right) \tan^{-1} \left[\sqrt{(AR^2 - 1)} \right]$ <p>Note: The formulation is only valid for $AR > 1$</p>
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The SQER expression was derived for the case of $AR > 1$. Even though the yarns and matrix properties are obtained and input into the model in the same way as in Chapter 3, the integration of the normal and shear deformation components at the interface region was not straightforward. A specialized mathematical analysis software called Maple 14 [90] had to be utilized to obtain the final integrated forms of the terms X and XI .

Since $AR = 1$ represents the circular yarn case and is an integral part of representing elliptical yarn behavior, the SQER formulations should be able to reduce to the SQS formulation derived in Chapter 3. However, it can be seen from the finalized terms X and XI that singularities exist for the case of $AR = 1$; therefore, the formulation in Table 5.1 provides indeterminate values of E_T for $AR = 1$. A discontinuous range of E_T values for changing aspect ratios (stemming from real and imaginary mathematical solutions) makes it unreliable as a system for predicting composite properties.

Figure 5.1 shows the variation of transverse direction Young' modulus as a function of fiber volume fraction at various aspect ratios and a porosity value of 0.25. The constituent material is glass fibers reinforced epoxy matrix taken from Tsai and Hahn [82]. Aspect ratio values of 1.25, 1.40, 1.50 and 1.65 are examined.

Figure 5.1: Transverse Young's moduli as a function of fiber volume fraction for the SQER model

When these values are compared against the original experimental values and the SQS formulation (introduced and validated in Chapter 3), the error due to formulation is obviously seen. At $AR = 1.25$, the formulation does not behave like a typical fiber reinforced composite, showing a decrease in transverse Young's modulus as fiber volume fraction increases. At higher values of aspect ratio ($AR = 1.5$ and $AR = 1.65$), the value of transverse Young's modulus increases abnormally (and is even noted to exceed fiber modulus values), once again violating the general principles of material behavior. Additionally, it should be expected that when the AR moves closer to 1, the curves would move closer to the dotted SQS line. However, there is no trend seen in Figure 5.1 that suggests this behavior. Therefore, it is not reasonable to continue with the SQER formulation or methodologies to obtain predictable estimates of elliptical yarn unit cell behavior.

5.2 FEA modeling for scaling circular yarns to elliptical shapes

Work has been previously carried out in FEA to study elliptical yarn behavior in composites [56, 60]. The versatility of FEA makes it possible to vary many aspects of geometry and

material behavior and assess its effects on composite properties. Therefore, it is possible to obtain elliptical yarn composite properties by carrying out FEA on elliptical yarn geometries. However, as shown in the previous chapter, there exists a difference in transverse properties of circular yarns obtained by mathematical formulations to that obtained by FEA. The finite element models exhibit rigidity that is not encountered within the analytical formulations. It is expected that the same rigidity will extend in comparisons between transverse properties of elliptical yarns obtained by mathematical formulations to that obtained by FEA.

Hence, the methodology proposed is to adopt a scaling factor to implement on the properties of the matrix infused circular yarn. This scaling factor would ideally take into consideration constituent material properties, yarn aspect ratio and other relevant properties. Since the aim of the work of this chapter is to obtain scalability factors linking elliptical yarn properties of composites to their corresponding circular ones, it is hoped that the rigidity of finite elements mentioned before will be negated. Thus, the scalability factor obtained for FEA linking circular and elliptical yarn properties can also be used with analytical or other models.

The representation of the unit cell is shown in Figure 5.2. It can be seen that the elliptical yarn cross-section is quantified by the semi-major axis, R_a and semi-minor axis, R_b .

Figure 5.2: Quarter unit cells for circular and elliptical yarns

In order to have a coherent comparison strategy, the yarn volume fraction, constituent materials and the porosity, ϵ of both the circular and non-circular unit cells was designed to be the same. The values of R_a and R_b would be determined by the Equations 5.1.

$$R_a = \frac{R}{\sqrt{AR}} \quad \dots(5.1a)$$

$$R_b = R\sqrt{AR} \dots(5.1b)$$

where R is the radius of the basis circular yarn model, and AR is the aspect ratio of the elliptical yarn. For example, to design a unit cell with a yarn aspect ratio of 1.5, the R_a and R_b values would be

$$R_a = \frac{R}{\sqrt{1.5}}; \quad R_b = R\sqrt{1.5}$$

Verifying the aspect ratio of this yarn,

$$AR = \frac{R_b}{R_a} = \frac{R\sqrt{1.5}}{\frac{R}{\sqrt{1.5}}} = 1.5$$

It should be noted that when yarns no longer maintaining their circular shape, the long fiber reinforced composite does not exhibit transverse isotropy. The composite now consists of the longitudinal direction and the a- and b- directions in the transverse plane of the composite. It is thus to be treated as an orthotropic composite [4].

The scalability factors for the transverse Young's modulus and the Poisson ratio that are derived and validated by the end of this chapter are shown in Equations 5.2a and 5.2b respectively.

$$\alpha = \frac{E_{ell}}{E_{cir}} = (AR)^{\ln[1.014809 - 0.01034(Powers_E)]} \dots(5.2a)$$

$$\beta = \frac{\nu_{ell}}{\nu_{cir}} = (AR)^{-[\ln(0.9446) + 0.0556 \ln(Powers_\nu)]} \dots(5.2b)$$

where $Powers_\nu = (1 - 1.87836\epsilon)(1 - 9.1356V_{yarn})(1 - 5.76812 \ln M_r)$,

$Powers_E = (1 - 1.898\epsilon)(1 - 9.167V_{yarn})(1 - 4.70588 \ln M_r)$,

R_a is designated as the loading direction elliptical yarn axis, ϵ is the yarn porosity, V_{yarn} is the yarn volume fraction and M_r is the ratio of fiber-to-matrix modulus.

The FEA data collection, analysis, mathematical modeling and validation carried out till the end of the chapter discusses in detail the process of obtaining Equations 5.2a and 5.2b.

5.3 FEA Simulations for Transverse Young's Modulus

Using the sixteen (4×4) quarter unit cell geometry and inputting the constituent material properties, simple roller and displacement boundary conditions were applied and the corresponding nodal forces aggregated to obtain the reaction force as shown in Figure 5.3. This reaction force is converted to unidirectional stress to obtain the Young's modulus in that particular direction.

Figure 5.3: Boundary conditions for Young's modulus in the x-direction

The 4×4 quarter unit cell arrangement is used to mitigate the boundary conditions effect that was seen to affect the FEA result in the case of circular yarns (in the previous chapter). The FEA model for elliptical yarns of $AR = 0.75$ is shown in Figure 5.4.

Figure 5.4: FEA model for $AR = 0.75$

Additionally, the same element size of 0.3 unit, used in Chapter 4 for validation of circular yarn Poisson's ratio models, is used throughout this chapter as well.

It should also be noted that a unit cell of any aspect ratio other than 1 gives rise to a converse unit cell with an inverse aspect ratio.

Figure 5.5: Using the same geometry to obtain inverse yarn aspect ratio properties

For example, as shown in Figure 5.5, the yarn has an aspect ratio of 0.5. By applying boundary conditions as shown on the left side of the figure, the corresponding direction's Young's modulus can be obtained. This will give one set of results for $AR = 0.5$. On the other hand, applying the boundary conditions on the right side of the figure, it is possible to obtain results for an aspect ratio of 2 (inverse of 0.5) while holding the constituent material properties and geometry fixed. This means that it is possible to obtain results for two sets of yarn aspect ratios by just interchanging the boundary conditions.

This technique is important as it will be used in later sections to obtain Poisson ratio values using FEA (i.e. v_{xy} & v_{yx}) simultaneously for the cross-sectional plane as shown in Figure 5.5.

5.3.1 Obtaining Regression Power trend

Using the same constituent material properties as Tsai and Hahn's glass fiber-epoxy matrix composites [82], some preliminary FEA investigations were carried. The yarn porosity fraction used for these unit cells was 0.3. The variation of transverse Young's modulus as a function of yarn aspect ratio is shown in Figures 5.6 and 5.7. Figure 5.6 depicts results for fiber volume fractions of 0.198 (with associated circular yarn radius $R = 12$).

Figure 5.6: Transverse modulus as a function of yarn aspect ratio for fiber volume fraction of 0.198

Figure 5.7 shows the same relationship for fiber volume fractions of 0.309 (with associated circular yarn radius, $R=15$).

Figure 5.7: Transverse modulus as a function of yarn aspect ratio for fiber volume fraction of 0.309

Figure 5.8: Elliptical yarn as expressed in terms of aspect ratio

From a physics point of a view, it is seen that the Young's modulus is influenced by the amount of the stiffer yarn being aligned in the direction of the applied force. For the case of $AR < 1$, (Figure 5.8a), the yarn (stronger than the matrix) is aligned more in the direction of the applied force, leading to a higher modulus; however, for $AR > 1$ (Figure 5.8c), the yarn is aligned in a more perpendicular orientation to the direction of the applied force, leading to a lower modulus. As a result, it is seen that the data points follow a decaying trend in both cases as the yarn aspect ratio increases.

Additionally, when regression curves for logarithmic and power relations were plotted against these points, there was a good correlation as shown in Figures 5.6 and 5.7. The equations associated with the regression relations are shown in the figures' legends.

However, in order to decide on the better fit amongst these data points, the regression correlation coefficient (R^2) was calculated as shown in Table 4.1.

Table 5.2: Correlation Coefficients for the logarithmic and power relations

Correlation Coefficient (Equation)	Volume Fraction	
	$V_f = 0.198$	$V_f = 0.309$
Logarithmic relation	0.9259 ($y = 4.55E9 - 3.15E8 \log x$)	0.9719 ($y = 5.43E9 - 8.07E8 \log x$)
Power relation	0.9328 ($y = 4.46E9x^{-0.069}$)	0.9771 ($y = 5.42E9x^{-0.144}$)

The associated relation (in equation form) is also shown in brackets below the correlation coefficient. It is thus seen that in both cases the power relation gives a better fit (R^2 closer to

1) compared to the logarithmic relation, and is thus chosen as the basis of formulation for the further work discussed in this chapter. It can be noted that the correlations for Figure 5.6 are worse off than Figure 5.7 for both logarithmic and power relations. This is due to the fact that Figure 5.6 has nine data points as compared to Figure 5.7 with only seven data points; this leads to a poor fit for Figure 5.6, since the regression relation must try to appease more points.

5.3.2 Conjecturing provisional Power relationship

Thus the relation that is sought out for further study can now be hypothesized to take the form as shown in Equation 5.3.

$$\alpha = \frac{E_{ell}}{E_{cir}} = A \left(\frac{R_b}{R_a} \right)^B = A(AR)^B \quad \dots(5.3)$$

where A and B are constants which will be determined by regression analyses.

It can be seen that the relation has been written as a ratio of the elliptical modulus over the circular modulus. This is to provide a valid means of comparison when material property values change, since the material modulus ratio will be an independent factor (Chapter 4). Additionally, this is expected to negate the effect of artificial stiffening that routinely occurs in FEA. The next stage of this work is to determine what parameters affect the values of A and B .

5.4 Factors affecting Transverse Young's Modulus

After extensively varying the materials and geometries and carrying out their FEA simulations, it is observed that three parameters seem to have significant effects on the values of A and B . These are the yarn porosity ϵ , the yarn volume fraction of the unit cell V_{yarn} and the Material Modulus ratio M_r . It should be noted that the fiber volume fraction is linked to the yarn volume fraction and porosity by the Equation 3.21 introduced before.

5.4.1 Yarn porosity, ϵ

This set of FEA simulations involve holding M_r and V_{yarn} constant and altering the value of ϵ to assess its effects on the constants A and B . Following Tsai & Hahn's work, the M_r is defined the ratio of the constituent glass fiber modulus to the epoxy matrix modulus.

$$M_r = \frac{E_{fibers}}{E_m} = \frac{73.1 \text{ GPa}}{3.45 \text{ GPa}} = 21.18$$

The value of V_{yarn} is determined by the size of the square unit cell, S and the radius of the circular yarn, R . In these experiments, $R = 12$ units and $S = 20$ units.

$$V_{yarn} = \frac{\pi R^2}{4S^2} = \frac{\pi(12)^2}{4(20)^2} = 0.2827$$

Thus, the fiber volume fraction based on the changing value of porosity is as shown in Table 5.3 (obtained using Equation 3.21).

Table 5.3: Fiber volume fraction as a function of porosity for fixed yarn volume fraction

ϵ	V_{fibers}
0.2	$V_{fibers} = 0.2827 (1 - 0.2) = \mathbf{0.2262}$
0.3	0.1979
0.35	0.1838
0.4	0.1696

The values of porosity investigated are 0.2, 0.3, 0.35 and 0.4. The range is similar to what has been used in Chapter 2 earlier [49]. For each set of ϵ at the given M_r and V_{yarn} , the values of A and B are obtained by the principles of Power Regression as shown in Appendix B. These values of A and B will then be compared with ϵ to determine meaningful relationships. However, the value of fiber volume fraction being derived from yarn volume fraction has no bearing on the analysis carried out in this chapter. The V_{yarn} is better employed as an indicator of the unit cell geometry, since it represents the geometrical yarn consideration.

As has been discussed before, the simulations are carried out using different aspect ratios. The chosen aspect ratios as well as the values of the corresponding geometries and their volume fractions are shown in Table 5.4.

Table 5.4: Deriving yarn geometry using fixed circular yarn parameters

AR	R_a(Equation 5.1a)	R_b(Equation 5.1b)	V_{yarn}
0.5	$R_a = \frac{12}{\sqrt{0.5}} = 16.97$	$R_b = 12\sqrt{0.5} = 8.485$	$\frac{\pi(16.97)(8.485)}{4(20)^2} = 0.2827$
0.6	15.492	9.295	0.2827
0.75	13.856	10.392	0.2827
0.9025	12.632	11.4	0.2827
1	12	12	0.2827
1.1083	11.4	12.632	0.2827
1.3333	10.392	13.856	0.2827
1.6667	9.295	15.492	0.2827
2	8.485	16.97	0.2827

* Values of R_a and R_b are obtained using circular radius, $R = 12$ and Equations 5.1

This method ensures that the aspect ratio can be altered without affecting the V_{yarn} or other parameters in the simulation. Additionally, it is observed that there lies a symmetry in the values of R_a and R_b . For example, the value of R_a in $AR = 0.5$ is 16.97, which is the same as the value of R_b in $AR = 2$, and so on and so forth.

It should also be noted that each porosity case has a different set of material property inputs as the FEA input for yarn property must be calculated and then input into the yarn material property in Abaqus. Using the values of AR introduced above, the FEA simulations to determine the transverse Young's Modulus based on changing porosity is carried out and the results obtained are shown in Table 5.5.

Table 5.5: Transverse Young's moduli for changing porosities

Aspect Ratio, AR	Transverse Young's Modulus for changing porosity, E_{ell} (/Pa)			
	$\epsilon = 0.2$	$\epsilon = 0.3$	$\epsilon = 0.35$	$\epsilon = 0.4$
0.5	5.36E+09	4.85E+09	4.65E+09	4.48E+09
0.6	5.15E+09	4.73E+09	4.56E+09	4.41E+09
0.75	4.96E+09	4.62E+09	4.48E+09	4.35E+09
0.9025	4.85E+09	4.55E+09	4.43E+09	4.31E+09
1	4.8E+09	4.52E+09	4.4E+09	4.29E+09
1.1083	4.75E+09	4.49E+09	4.38E+09	4.27E+09
1.3333	4.69E+09	4.45E+09	4.35E+09	4.23E+09
1.6667	4.63E+09	4.42E+09	4.32E+09	4.22E+09
2	4.6E+09	4.4E+09	4.3E+09	4.21E+09

Since it has been discussed earlier that, in the spirit of non-dimensionality, the ratio of elliptical modulus to circular modulus is of interest, the values in each column of Table 5.5 are then divided by their corresponding value at $AR = 1$ (as shaded in yellow) in the same column to obtain Table 5.6. This will convert all the values to the corresponding α value as shown in Equation 5.3 before.

Table 5.6: Summary of α for changing yarn aspect ratios and porosities

Aspect Ratio, AR	Ratio of elliptical modulus to circular modulus, α			
	$\epsilon = 0.2$	$\epsilon = 0.3$	$\epsilon = 0.35$	$\epsilon = 0.4$
0.5	1.118	1.072	1.056	1.043
0.6	1.073	1.046	1.036	1.028
0.75	1.034	1.022	1.018	1.014
0.9025	1.011	1.007	1.006	1.004
1	1.000	1.000	1.000	1.000
1.1083	0.991	0.994	0.995	0.996
1.3333	0.977	0.985	0.988	0.990
1.6667	0.966	0.977	0.981	0.985
2	0.960	0.972	0.977	0.982

This data can also be represented in graphical form as in Figure 5.9.

Figure 5.9: α as a function of yarn aspect ratio (at different porosities)

The porosity seems to have an effect and to look at the effect in a quantitative sense, a Power relation for each value of porosity was determined using Regression Analysis as explained in Appendix B. The results are shown in Table 5.7.

Table 5.7: α relation using Power regression (on yarn AR) for changing porosities

	Power relation, $\alpha = \frac{E_{ell}}{E_{cir}} = (AR)^B = A \left(\frac{R_b}{R_a} \right)^B$			
	$\epsilon = 0.2$	$\epsilon = 0.3$	$\epsilon = 0.35$	$\epsilon = 0.4$
$\alpha =$	$1.01322 \left(\frac{R_b}{R_a} \right)^{-0.10617}$	$1.00784 \left(\frac{R_b}{R_a} \right)^{-0.06868}$	$1.00601 \left(\frac{R_b}{R_a} \right)^{-0.05476}$	$1.00457 \left(\frac{R_b}{R_a} \right)^{-0.04327}$
A	1.01322	1.00784	1.00601	1.00457
B	-0.10617	-0.06868	-0.05476	-0.04327

It is thus seen that the values of both A and B are affected by the change in porosity. However, while the value of A changes insignificantly (0.86% maximum) and seems to be quite stable, there is a substantial change (at least 59.2%) in the value of B . If the value of B is plotted against porosity as in Figure 5.10, the effective trend of porosity vs. B is quite obvious.

Figure 5.10: Value of B as a function of porosity

Using linear regression on these four points, as shown in Appendix B, it is possible to obtain not only the equation of the line, but also the R^2 , which is 0.9876.

Even though A changes with respect to porosity, its value must be looked at in the physical sense. For Equation 5.3, when the yarn $AR = 1$ (i.e. circular yarn, with $R_a = R_b$),

$$\Rightarrow \alpha = \frac{E_{ell}}{E_{cir}} = A(1)^B = A$$

This means that for the value of E_{ell} to be equal to E_{cir} for the circular yarn (Figure 5.7b), A must be equal to 1. This is also proved using FEA by the closeness of all values of A in Table 5.7. Hence, it is better to revise expression in Equation 5.3 to the form as shown below.

$$\alpha = \frac{E_{ell}}{E_{cir}} = \left(\frac{R_b}{R_a}\right)^B \quad \dots(5.4)$$

It should be noted that since the relation of the other parameters to B are not known, it is not wise to directly adopt the linear formula (shown in Figure 5.9) as the fundamental definition of B . It is important to look at all the other parameters affecting B and devise a mathematical definition that encompasses all these parameters.

5.4.2 Yarn Volume Fraction, V_{yarn}

This section will explore the relationship of B to the yarn volume fraction, V_{yarn} . The value of material modulus ratio will be fixed at 21.18 (using Tsai and Hahn's values [82], as in the previous section) and porosity will be fixed at 0.3.

The value of the baseline case circular yarn radius, R will then be varied to change the yarn volume fraction as shown in Table 5.8 below. The same square unit cell at $S = 20$ will be used.

Table 5.8: Effect of changing radius on yarn & fiber volume fractions

Radius of circular case, R	Corresponding yarn volume fraction, V_{yarn}	Corresponding fiber volume fraction, V_{fibers}
10	$\frac{\pi(10)^2}{4(20)^2} = 0.1964$	$0.1964(1 - 0.3) = 0.1374$
12	0.2827	0.1979
13	0.3318	0.2323
15	0.4418	0.3093

The simulations are carried out as before and the results obtained are shown in Table 5.9 below.

Table 5.9: Transverse Young's moduli for changing yarn volume fractions

Aspect Ratio, AR	Transverse Young's Modulus for changing porosity, E_{ell}		
	$V_{yarn} = 0.1964$	$V_{yarn} = 0.2827$	$V_{yarn} = 0.3318$
0.5	4.3E+09	4.85E+09	5.24E+09
0.6	4.25E+09	4.73E+09	5.06E+09
0.75	4.19E+09	4.62E+09	4.9E+09
0.9025	4.16E+09	4.55E+09	4.81E+09
1	4.14E+09	4.52E+09	4.76E+09
1.1083	4.12E+09	4.49E+09	4.73E+09
1.3333	4.1E+09	4.45E+09	4.67E+09

1.6667	4.08E+09	4.42E+09	4.62E+09
2.0000	4.07E+09	4.4E+09	4.6E+09

It is seen that the case of $V_{yarn} = 0.4418$ is not seen in the results in Table 5.9. This was because the geometry for the AR cases greater than for 0.6667 and 1.5 was not possible to be generated. Hence, a new set of AR cases are generated for the $V_{yarn} = 0.4418$ case.

As before, this data is non-dimensionalized to the corresponding α by dividing by the shaded $AR = 1$ case and is shown in Table 5.10.

Table 5.10: Summary of α for changing yarn aspect ratios & yarn volume fractions

Aspect Ratio, AR	Ratio of elliptical modulus to circular modulus, α		
	$V_{yarn} = 0.1964$	$V_{yarn} = 0.2827$	$V_{yarn} = 0.3318$
0.5	1.039	1.072	1.100
0.6	1.026	1.046	1.062
0.75	1.012	1.022	1.029
0.9025	1.004	1.007	1.009
1	1	1	1
1.1083	0.996	0.994	0.992
1.3333	0.991	0.985	0.981
1.6667	0.986	0.977	0.971
2	0.983	0.972	0.965

For the $V_{yarn} = 0.4418$ case, a special set of yarn aspects ratios ranging between 0.667 and 1.5 (due to geometrical concerns) were chosen and the corresponding transverse Modulus and corresponding α values generated as shown in Table 5.11.

Table 5.11: Summary of α for changing yarn aspect ratios & $V_{yarn} = 0.4418$

$V_{yarn} = 0.4418$		
Aspect Ratio, AR	E_{ell}	α
0.7	5.74E+09	1.066741
0.75	5.65E+09	1.050082
0.87	5.49E+09	1.021108
0.95	5.42E+09	1.007202
1	5.38E+09	1
1.052632	5.34E+09	0.993384
1.149425	5.29E+09	0.983237
1.333333	5.21E+09	0.968955
1.428571	5.18E+09	0.963339

Again as before, the Power relation regression is carried out on each column of the data points for all four V_{yarn} cases and the relations obtained.

Table 5.12: α relation using Power regression (on yarn AR) for changing yarn volume fraction

	Power relation, $\alpha = \frac{E_{ell}}{E_{cir}} = (AR)^B = \left(\frac{R_b}{R_a}\right)^B$			
	$V_{yarn} = 0.1964$	$V_{yarn} = 0.2827$	$V_{yarn} = 0.3318$	$V_{yarn} = 0.4418$
$\alpha =$	$\left(\frac{R_b}{R_a}\right)^{-0.03969}$	$\left(\frac{R_b}{R_a}\right)^{-0.06868}$	$\left(\frac{R_b}{R_a}\right)^{-0.09113}$	$\left(\frac{R_b}{R_a}\right)^{-0.14109}$
B	-0.03969	-0.06868	-0.09113	-0.14109

This data when represented by a graph (as in Figure 5.11) shows a strong linear correlation between B and V_{yarn} .

Figure 5.11: B as a function of changing yarn volume fraction

Therefore, this linear relationship, just like in the previous case of porosity, will be kept in mind for further exploration in the chapter.

5.4.3 Material Modulus Ratio, M_r

The final parameter that is expected to affect the transverse Young's modulus is the difference that exists between the fiber properties and the matrix properties within the composite. Since most constituent material Poisson ratios lie between 0.1 to 0.5 (without much variation), it may be more meaningful to consider the effect the Young's or shear moduli have on the properties of elliptical yarn composites. This has also been discussed earlier in Chapters 3 and 4. Hence, in this study, the focus will be on the material modulus ratio, M_r .

The porosity will be fixed at 0.2 and the V_{yarn} will be fixed at 0.2827. The M_r will vary from 2 to a value of 21.19. A summary of the materials is given in Table 5.13.

Table 5.13: Summary of material properties used to vary the material modulus ratios

Material	Source	Fiber Properties	Matrix Properties	Material Modulus Ratio, M_r
Glass fiber - epoxy	Tsai & Hahn [82]	$E_f = 73.1$ GPa $\nu_f = 0.22$ $G_f = 30$ GPa	$E_f = 3.45$ GPa $\nu_f = 0.35$ $G_f = 1.28$ GPa	$\frac{73.1 \text{ GPa}}{3.45 \text{ GPa}} = 21.18$
Boron fiber - Aluminum	Wacker, Bledzki, Chate [48]	$E_f = 379.3$ GPa $\nu_f = 0.1$ $G_f = 172.41$ GPa	$E_f = 68.3$ GPa $\nu_f = 0.3$ $G_f = 26.27$ GPa	$\frac{379.3 \text{ GPa}}{68.3 \text{ GPa}} = 5.55$
H-1	Hypothetical	$E_f = 48$ GPa $\nu_f = 0.2$ $G_f = 20$ GPa	$E_f = 4$ GPa $\nu_f = 0.33$ $G_f = 1.5$ GPa	$\frac{48 \text{ GPa}}{4 \text{ GPa}} = 12$
H-2	Hypothetical	$E_f = 8$ GPa $\nu_f = 0.2$ $G_f = 3.33$ GPa	$E_f = 4$ GPa $\nu_f = 0.33$ $G_f = 1.5$ GPa	$\frac{8 \text{ GPa}}{4 \text{ GPa}} = 2$

Materials H-1 and H-2 are hypothetically created in order to provide varied data points and enable a better analysis of the relation of material modulus ratio to the properties of elliptical yarns. All the materials in Table 5.13 are then input into the FEA simulations and their corresponding transverse Young's moduli obtained are shown in Table 5.14.

Table 5.14: Transverse Young's moduli for changing material modulus ratios

Aspect Ratio, AR	Transverse Young's Modulus for changing porosity, E_{ell}			
	$M_r = 2$	$M_r = 5.55$	$M_r = 12$	$M_r = 21.18$
0.5	4.67E+09	9.5E+10	6.01E+09	5.36E+09
0.6	4.65E+09	9.28E+10	5.8E+09	5.15E+09
0.75	4.62E+09	9.08E+10	5.62E+09	4.96E+09
0.9025	4.61E+09	8.95E+10	5.51E+09	4.85E+09
1	4.6E+09	8.89E+10	5.45E+09	4.8E+09
1.1083	4.6E+09	8.84E+10	5.41E+09	4.75E+09
1.3333	4.59E+09	8.76E+10	5.34E+09	4.69E+09
1.6667	4.58E+09	8.69E+10	5.28E+09	4.63E+09

2	4.58E+09	8.66E+10	5.25E+09	4.6E+09
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The case $M_r = 5.55$ seems to have very high transverse modulus values (on a different magnitude than the rest); however, it should be remembered that this case makes use of boron fibers in aluminium matrix. Hence, when these values are scaled down to their corresponding α as shown in Table 5.15, a comparison becomes easier.

Table 5.15: Summary of α for changing yarn aspect ratios & material modulus ratios

Aspect Ratio, AR	Ratio of elliptical modulus to circular modulus, α			
	$M_r = 2$	$M_r = 5.55$	$M_r = 12$	$M_r = 21.18$
0.5	1.015	1.068	1.102	1.118
0.6	1.010	1.044	1.064	1.073
0.75	1.005	1.021	1.030	1.034
0.9025	1.002	1.007	1.010	1.011
1	1.000	1.000	1.000	1.000
1.1083	0.999	0.994	0.992	0.991
1.3333	0.997	0.985	0.979	0.977
1.6667	0.995	0.978	0.969	0.966
2	0.994	0.973	0.963	0.960

Consequently, the Power relation is then applied using a regression analysis to these data points as shown in Table 5.16.

Table 5.16: α relation using Power regression (on yarn AR) for changing material modulus ratio

	Power relation, $\alpha = \frac{E_{ell}}{E_{cir}} = (AR)^B = \left(\frac{R_b}{R_a}\right)^B$			
	$M_r = 2$	$M_r = 5.55$	$M_r = 12$	$M_r = 21.18$
$\alpha =$	$\left(\frac{R_b}{R_a}\right)^{-0.01438}$	$\left(\frac{R_b}{R_a}\right)^{-0.06548}$	$\left(\frac{R_b}{R_a}\right)^{-0.09464}$	$\left(\frac{R_b}{R_a}\right)^{-0.10617}$
B	-0.01438	-0.06548	-0.09464	-0.10617

Though there seems to be a decreasing trend in the value of B as M_r increases, it is better to present these points graphically to have a better idea. This is shown in Figure 5.12.

Figure 5.12: B as a function of material modulus ratio

The data points seem to suggest a logarithmic correlation. The logarithmic regression formulation in Appendix B can thus be used to determine the corresponding correlation as well as the correlation coefficient (R^2). The value of R^2 at 0.9765 for M_r is less than that in the case of V_{yarn} and ϵ (both above 0.985). However, the correlation is still very strong and will be used later on.

5.5 Combining effects of factors influencing α

Having worked out the relationship of the parameters that affect the value of α , this can be summarized by the same Equation 5.4 as -

$$= \frac{E_{ell}}{E_{cir}} = \left(\frac{R_b}{R_a} \right)^B$$

where B is correlated to V_{yarn} , M_r and ϵ by the relations shown in Table 5.17.

Table 5.17: Relations on how V_{yarn} , M_r and ϵ affect the power B

Parameter	Relationship with B	Regression Equation	Correlation Coefficient (R^2)
ϵ	Linear	$B = -0.1677 + 0.3183\epsilon$ $= -\mathbf{0.1677(1 - 1.898\epsilon)}$	0.9876
V_{yarn}	Linear	$B = 0.0455 - 0.4171V_{yarn}$ $= \mathbf{0.0455(1 - 9.167V_{yarn})}$	0.9939
M_r	Logarithmic	$B = 0.0085 - 0.04 \ln M_r$ $= \mathbf{0.0085(1 - 4.706 \ln M_r)}$	0.9765

It can be seen in the regression equation column, that the formula has been factorized into “(1 - ...)” form (as shown in bold). This will be used to compound the effect of the all three parameters, into a single expression.

The possible expression (using the forms in Table 5.17) is thus as Equation 5.5.

$$B = J(1 + K\epsilon)(1 + LV_{yarn})(1 + M \ln M_r) \quad \dots(5.5)$$

where the values of J , K , L and M are constants with values determined.

The beauty of this expression lies in its simplicity. If any two of V_{yarn} , M_r or ϵ are held constant, the expression mimics the relation between B and the third parameter as summarized in Table 5.17. Thus, by direct comparison between Equation 5.5 and Table 5.17, the values of the constants K , L & M are taken to be -1.898, -9.167 and -4.706 respectively. To obtain the value of the constant J is more complicated. Since its value cannot be reliably determined by just measuring variation over one parameter, a combined study of variation of

all three parameters in the form of ten sets of conditions were looked at. This is summarized in Table 5.18.

Table 5.18: Range of input parameters used to obtain the relations in Table 5.17

	$M_r = 2$	$M_r = 5.55$	$M_r = 21.18$	$M_r = 12$
$V_{yarn} = 0.1964$			$\epsilon = 0.3$	
$V_{yarn} = 0.2827$	$\epsilon = 0.2$	$\epsilon = 0.2$	$\epsilon = 0.2, \epsilon = 0.3,$ $\epsilon = 0.35, \epsilon = 0.4$	$\epsilon = 0.2$
$V_{yarn} = 0.3318$			$\epsilon = 0.3$	
$V_{yarn} = 0.4418$			$\epsilon = 0.3$	

It was felt that an overarching relation in these ten cases would provide a suitable estimate of the value of the constant J . Using the value of B obtained by power regression, the corresponding value of J was obtained using a variant of Equation 5.5 as -

$$J = \frac{B_{regression}}{Powers_E} \quad \dots(5.6)$$

$$Powers_E = (1 - 1.898\epsilon)(1 - 9.167V_{yarn})(1 - 4.706 \ln M_r) \quad \dots(5.7)$$

where the “ $Powers$ ” name denotes that these terms fall into the power relation and the “ E ” subscript refers to the fact that this formulation is for the Young’s moduli.

Table 5.19: Values of $Powers_E$, B & J obtained from inputs in Table 5.18

Case	Parameter Values			$Powers_E$ obtained using Equation 5.7	Value of B obtained by Power Regression	Value of J obtained using Equation 5.6
	Material Ratio, M_r	Porosity, ϵ	Yarn volume fraction, V_{yarn}			
1	2	0.2	0.2827	2.233	-0.01438	-0.00644
2	5.55	0.2	0.2827	6.975	-0.06548	-0.00939
3	12	0.2	0.2827	10.559	-0.09464	-0.00896
4	21.18	0.2	0.2827	13.200	-0.10617	-0.00804
5	21.18	0.35	0.2827	7.143	-0.05476	-0.00767
6	21.18	0.4	0.2827	5.123	-0.04327	-0.00845
7	21.18	0.3	0.2827	9.162	-0.06868	-0.00750
8	21.18	0.3	0.1964	4.605	-0.03969	-0.00862
9	21.18	0.3	0.3318	11.75	-0.09113	-0.00775
10	21.18	0.3	0.4418	17.55	-0.14109	-0.00804

Note: Input values for Case 4 and 7 have been made bold to show that these form part of initial analyses of two sets of parameters. For example, Case 4 occurs commonly where Cases 1-4 are used for the analysis of M_r and Cases 4-7 are used for the analysis of ϵ . Similarly, Case 7 occurs commonly where Cases 4-7 are used for the analysis of ϵ and Cases 7-10 are used for the analysis of V_{yarn} .

It is seen that the values of J (obtained in the last column of Table 5.19) are all not the same constant value. In fact, J seems to drift from a value of -0.00644 to a value of -0.00939. This seems to suggest one of two things. Either the relationship of the power term B is not a simple linear or logarithmic relation with the parameters of V_{yarn} & ϵ and M_r respectively, but a more complicated formulation; or that there are more parameters (other than V_{yarn} , ϵ and M_r) that affect the value of B .

Assuming that the power term B is not a simple relation of the parameters V_{yarn} , ϵ and M_r , further investigation of the nature of the J term was desired. If it was possible to obtain some relation of the J term to the parameters, it might still be possible to formulate an independent

expression that would predict transverse elastic moduli of elliptical yarns. After intensive trial-and-error processes are carried out to link these values together, it was noticed that when the exponential of B (i.e. e^B) is plotted against $Powers_E$ as shown in Figure 5.13, a strong linear correlation emerges.

Figure 5.13: Exponential of B as a function of $Powers_E$

The value of correlation ($R^2 = 0.9785$) appears to be quite high and similar to that shown in Table 5.17 earlier for the individual parameters (V_{yarn} , ϵ and M_r). Hence, a linear trend is taken to be the best fit possible and used for validation as shown in the next section.

5.6 Validation studies for the α expression

Even though a correlation existing between B and $Powers_E$ has been obtained, it is important to try out a few test cases at various other parameter and yarn aspect ratio input to determine how robust the overall power formulation is. This involves different inputs from those used in Table 5.18 earlier.

Table 5.20: Summary of new input parameters tried (shown in yellow)

	$M_r = 2$	$M_r = 5.55$	$M_r = 12$	$M_r = 15$	$M_r = 21.18$
$V_{yarn} = 0.1964$		$\epsilon = 0.2$	$\epsilon = 0.4$		$\epsilon = 0.3$
$V_{yarn} = 0.2827$	$\epsilon = 0.2$	$\epsilon = 0.2$	$\epsilon = 0.2$		$\epsilon = 0.2$ $\epsilon = 0.3$ $\epsilon = 0.35$ $\epsilon = 0.4$
$V_{yarn} = 0.3318$				$\epsilon = 0$ $\epsilon = 0.4^*$	$\epsilon = 0.3$
$V_{yarn} = 0.4418$					$\epsilon = 0.3$

Table 5.20 shows the value of the previous parameters in grey. These grey values were used to determine the linear and logarithmic relations for the parameters. The values in yellow are the new parameter values that are tested to see how durable the formulation is.

The special case (“ $\epsilon = 0.4$ ” asterisked in row “ $V_{yarn} = 0.3318$ ”, column “ $M_r = 15$ ”) is to show that two sets of yarn aspect ratios were considered for this case. The first set were the usual aspect ratios ($AR = 0.5, 0.6, 0.75, 0.9025, 1$ and their inverses) where the second set of aspect ratios were wider spread ($AR = 0.3, 0.5, 0.7, 0.9, 1$ and their inverses) such that the maximum yarn aspect ratio touched 3.33 (inverse of 0.3).

Table 5.21: Values of $Powers_E$, B & J obtained from inputs in Table 5.20

Case	Parameter Values			$Powers_E$	B
	Material Ratio, M_r	Porosity, ϵ	Yarn volume fraction, V_{yarn}		
1	2	0.2	0.2827	2.233	-0.01438
2	5.55	0.2	0.2827	6.975	-0.06548
3	12	0.2	0.2827	10.559	-0.09464
4	21.18	0.2	0.2827	13.200	-0.10617
5	21.18	0.35	0.2827	7.143	-0.05476
6	21.18	0.4	0.2827	5.123	-0.04327
7	21.18	0.3	0.2827	9.162	-0.06868
8	21.18	0.3	0.1964	4.605	-0.03969
9	21.18	0.3	0.3318	11.75	-0.09113
10	21.18	0.3	0.4418	17.55	-0.14109
T-1	5.55	0.2	0.1964	3.485	-0.03812
T-2	12	0.4	0.1964	2.060	-0.02266
T-3	15	0	0.3318	23.98	-0.3303
T-4	15	0.4	0.3318	5.773	-0.04893
T-5*	15	0.4	0.3318	5.773	-0.03886

Note: The wider set is given by case T-5. The exponential of B is taken and its value is then plotted against the $Powers_E$ parameter as shown in Figure 5.14.

Figure 5.14: Validation values of exponential of B against $Powers_E$

The values for cases 1-10 are shown by the blue markers, whereas the test values T-1 to T-5 are shown in maroon markers. Additionally, the relation along with the associated R^2 value is shown. Even though this R^2 value of 0.9113 is lower than the earlier value (considering only the cases 1 to 10), it should be noted that these values are broader in their scope and are not linked to the derivation carried out earlier, i.e. they are unique. Hence, it is better to include them in and use the linear relation thus defined.

The linear relation shown in Figure 5.14 is as follows:

$$y = 1.014809 - 0.01034x \quad \text{or}$$

$$\Rightarrow e^B = 1.014809 - 0.01034(Powers_E) \quad \text{or}$$

$$\Rightarrow B = \ln[1.014809 - 0.01034(Powers_E)]$$

However, since J is defined by B and $Powers_E$ by the relation:

$$B = J \times Powers_E$$

We get,

$$J = \frac{\ln[1.014809 - 0.01034(Powers_E)]}{Powers_E}$$

Thus it is seen that the J is a complicated formulation that makes use of the parameter $Powers_E$ in the numerator and denominator. If this is combined wholly with the expression shown in Equation 5.4 shown earlier, we get:

$$\alpha = \frac{E_{ell}}{E_{cir}} = \left(\frac{R_b}{R_a}\right)^{\ln[1.014809-0.01034(Powers_E)]}$$

where a is the loading direction. This can be written in the final form:

$$\alpha = \frac{E_{ell}}{E_{cir}} = \left(\frac{R_b}{R_a}\right)^{\ln[1.014809-0.01034(1-9.167V_{yarn})(1-1.898\epsilon)(1-4.70588 \ln M_r)]} \dots(5.8)$$

Having derived this final expression, it is important to try it on the five test cases (T-1 to T-5) to see how they perform for varying yarn aspect ratio values. They are compared to the additional FEA computations carried out and the associated largest absolute error value is presented.

Table 5.22: Correlation and error values of the five test cases T-1 to T-5 for α

Case	Yarn Aspect Ratio values (AR)	Parameter Values			Correlation Coefficient (R ²)	Largest Absolute Error
		Material Ratio, M_r	Porosity, ϵ	Yarn volume fraction, V_{yarn}		
T-1	0.5, 0.6, 0.75, 0.9025, 1 & inverses	5.55	0.2	0.1964	0.9799	-2.14%
T-2	0.5, 0.6, 0.75, 0.9025, 1 & inverses	12	0.4	0.1964	0.9747	-1.67%
T-3	0.5, 0.6, 0.75, 0.9025, 1 &	15	0	0.3318	0.8882	-19.86%

	inverses					
T-4	0.5, 0.6, 0.75, 0.9025, 1 & inverses	15	0.4	0.3318	0.9880	-1.95%
T-5	0.3, 0.5, 0.7, 0.9, 1 & inverses	15	0.4	0.3318	0.9747	-3.96%

A comparison of the ratio α for case T-2 is shown in Figure 5.15 for both FEA and the prediction from the formula.

Figure 5.15: Comparison of FEA against the prediction of Equation 5.8 for T-2

Even though there appears to be a divergence between FEA and predicted values, it should be reminded that the associated correlation coefficient is 0.9747. This is a high degree of correlation between the values. Also, it can be seen in this case that the points swivel around the $AR = 1$ point. This means that if the exponential relation is improved by adding in other parameters or moving on to higher order relations, it might be possible to reduce the error (currently at 1.67% maximum).

However, the swivel nature of the points is not a given. In the case of the sample T-3 as shown in Figure 5.16, the points come closest at the $AR = 1$ data point and seem to diverge from it, remaining consistently below the FEA values. The associated correlation coefficient is 0.8882 with a maximum absolute error of 19.86% occurring at the $AR = 0.5$ point (as shown in Table 5.22).

Figure 5.16: Comparison of FEA against the prediction of Equation 4.7 for T-3

However, if the single offending point at $AR = 0.5$ is removed, the predictive ability improves with the correlation coefficient climbing to 0.9108 and the maximum error falling to 8.51% (for the $AR = 2$ case). It is thus also seen that the further the yarn aspect ratio lies from the $AR = 1$ data point, the worse the α ratio estimation will be.

When compared with all the results, it is observed that using low values of porosity seems to adversely affect the estimation. As shown earlier, the T-3 case with the porosity of 0 had the worst value. Also, the sample cases 1-4 (as shown in Table 5.22) with $\epsilon = 0.2$ exhibit a worse performance than the higher porosity cases of 0.3 and 0.4.

It is expected that a deeper foray into the relationship of porosity (and other parameters) with α may yield better predictive abilities. However due to resource and time constraints, it was not possible to investigate further into the relations of the parameters. FEA simulations with

135 unique combinations of material and geometry inputs were used to provisionally derive and validate the expression in Equation 5.8. The primary objective of the work in this chapter is to provide a roadmap to carry out semi-empirical FEA data-mining to formulate relations that can be used for quick first estimations of transverse modulus values without the need for complicated classical modeling or time-consuming finite element analysis for a preliminary analysis.

5.7 Carrying out FE modeling and analysis for Poisson's Ratio

Having obtained reliable estimates for the Young's modulus, similar work can be carried out to determine the Poisson's ratio for the two directions (i.e. ν_{xy} and ν_{yx}). Even though the Poisson ratio is being looked at in this section, the data-mining analysis method that has been established in the earlier part of this chapter for scalability in transverse Young's modulus is again applied to the modeling expressions for scalability of Poisson ratio from elliptical to circular yarns.

In terms of the finite element simulations required, there is a slight adjustment. Each test case will utilize a single simulation with roller and bidirectional displacement boundary conditions applied to adjacent faces on the unit cell as shown in Figure 5.17.

Figure 5.17: Bidirectional displacement boundary conditions to obtain transverse Poisson's ratios

This will provide nodal reaction force values which can be converted to stress values and solved using the constitutive equation as shown below [4].

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \end{pmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} \end{bmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \end{pmatrix} \quad \dots(5.9)$$

The values of E_x and E_y are the same FEA values obtained for transverse Young's modulus in the previous section. Equation 5.9 is thus manipulated to obtain Equations 5.10a and 5.10b.

$$\nu_{yx} = \frac{E_y}{\sigma_y} \left(\frac{\sigma_x}{E_x} - \varepsilon_x \right) \quad \dots(5.10a)$$

$$\nu_{xy} = \frac{E_x}{\sigma_x} \left(\frac{\sigma_y}{E_y} - \varepsilon_y \right) \quad \dots(5.10b)$$

After each set of values is obtained, it is also verified using the relation shown in Equation 5.11 below.

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad \dots(5.11)$$

This relation holds true for orthotropic elasticity and ensures that the values follow the fundamental tenets of material elasticity [91].

5.7.1 Obtaining the relations of the parameters V_{yarn} , M_r and ϵ

As was done previously for transverse Young's modulus, a provisional relation to scale elliptical Poisson ratio against circular Poisson ratio is given by Equation 5.12.

$$\beta = \frac{\nu_{ell}}{\nu_{cir}} = \left(\frac{R_b}{R_a} \right)^B \quad \dots(5.12)$$

Taking the material modulus ratio (M_r) and yarn volume fractions (V_{yarn}) at values of 21.18 and 0.2827 respectively, with constituent material property values the same as Section 5.3.1, the transverse Poisson ratio for changing values of porosity are obtained as shown in Table 5.23.

Table 5.23: Summary of transverse Poisson's ratio for changing yarn aspect ratios and porosity values

Aspect Ratio, <i>AR</i>	Transverse Poisson ratio for changing porosity, ν_{ell}			
	$\epsilon = 0.2$	$\epsilon = 0.3$	$\epsilon = 0.35$	$\epsilon = 0.4$
0.5	0.280602	0.29602	0.302378	0.308017
0.6	0.294424	0.305088	0.309619	0.313761
0.75	0.307322	0.313736	0.316649	0.319401
0.9025	0.315622	0.319414	0.321273	0.323139
1	0.318027	0.32104	0.322597	0.324199
1.1083	0.319696	0.322138	0.323482	0.324903
1.3333	0.324168	0.325153	0.325924	0.326867
1.6667	0.327693	0.327438	0.327766	0.328335
2	0.328992	0.328225	0.328363	0.328796

As was done in the case of the transverse elastic modulus, since the ratio of elliptical Poisson ratio to circular Poisson ratio is of interest, the values in each column of Table 5.23 are then divided by their corresponding value at $AR = 1$ (as shaded in yellow) to obtain Table 5.24. This will convert all the values to the corresponding β value.

Table 5.24: Summary of β for changing yarn aspect ratios & porosities

Aspect Ratio, <i>AR</i>	Ratio of elliptical Poisson ratio to circular Poisson ratio, β			
	$\epsilon = 0.2$	$\epsilon = 0.3$	$\epsilon = 0.35$	$\epsilon = 0.4$
0.5	0.88232	0.922066	0.937324	0.950084
0.6	0.925782	0.950311	0.959768	0.967803
0.75	0.96634	0.977249	0.981561	0.9852
0.9025	0.992439	0.994936	0.995894	0.996728
1	1	1	1	1
1.1083	1.005247	1.003421	1.002744	1.002169
1.3333	1.01931	1.012812	1.010311	1.008227
1.6667	1.030394	1.019928	1.016021	1.012756
2	1.034479	1.022381	1.017872	1.014179

Using the power regression shown in the previous section of the chapter, it is possible to obtain the values of the power B .

Table 5.25: β relation using power regression (on yarn AR) for changing porosity

	Power relation, $\beta = \frac{v_{ell}}{v_{cir}} = (AR)^B = \left(\frac{R_b}{R_a}\right)^B$			
	$\epsilon = 0.2$	$\epsilon = 0.3$	$\epsilon = 0.35$	$\epsilon = 0.4$
B	0.10881	0.07120	0.05706	0.04532

This value of B is seen to correlate linearly to the porosity as was the case for the transverse Young's moduli and the relation is shown in Figure 5.18.

Figure 5.18: Value of B as a function of porosity

As before, the regression correlation coefficient is high (0.9885) and the equation is noted for further study.

Similarly, by fixing M_r at 22.18 and ϵ at 0.2, it is possible to obtain the transverse Poisson ratios and their corresponding β values for changing V_{yarn} values of 0.1964, 0.2827, 0.3318 and 0.4418. Applying a Power regression analysis gives us the expressions as shown in Table 5.26.

Table 5.26: β relation using power regression (on yarn AR) for changing yarn volume fractions

	Power relation, $\beta = \frac{v_{ell}}{v_{cir}} = (AR)^B = \left(\frac{R_b}{R_a}\right)^B$			
	$V_{yarn} = 0.1964$	$V_{yarn} = 0.2827$	$V_{yarn} = 0.3318$	$V_{yarn} = 0.4418$
B	0.03953	0.07120	0.09588	0.14446

As before, the values of the power B change linearly with respect to yarn volume fraction and exhibit high correlation ($R^2 = 0.9968$), as is shown in Figure 5.19.

Figure 5.19: Value of B as a function of yarn volume fraction

Finally, similar Power relations for changing material modulus ratios of 2, 5.55, 12 and 21.18 are determined by regression analyses. The porosity, ϵ is taken at 0.2 and yarn volume fraction V_{yarn} at 0.2827. These regression analysis values are shown in Table 5.27.

Table 5.27: β relation using power regression (on yarn AR) for changing material modulus ratio

	Power relation, $\beta = \frac{v_{ell}}{v_{cir}} = (AR)^B = \left(\frac{R_b}{R_a}\right)^B$			
	$M_r = 2$	$M_r = 5.55$	$M_r = 12$	$M_r = 21.18$
B	0.01607	0.06740	0.09617	0.10881

As before, the values of B follow a logarithmic pattern as a function of the material modulus ratio and correlates well as shown in Figure 5.20.

Figure 5.20: Value of B as a function of material modulus ratio

5.7.2 Combining effects of parameters affecting transverse Poisson's ratio

The data obtained from all the previous relations is collated and presented in Table 5.28.

Table 5.28: Relations on how V_{yarn} , M_r and ϵ affect the power B

Parameter	Relationship with B	Correlation Coefficient (R^2)	Regression Equation
ϵ	Linear	0.9885	$B = 0.171 - 0.3212 \epsilon$ $= \mathbf{0.171(1 - 1.878\epsilon)}$
V_{yarn}	Linear	0.9968	$B = -0.0472 + 0.4312V_{yarn}$ $= \mathbf{-0.0472(1 - 9.136V_{yarn})}$
M_r	Logarithmic	0.9776	$B = -0.0069 + 0.0398 \ln M_r$ $= \mathbf{-0.0069(1 - 5.768 \ln M_r)}$

As was done before for the transverse modulus case, it is thus proposed that the function B will be an aggregated relation combining all the individual parameter relations as is shown by Equation 5.13.

$$B = K(1 - 1.878\epsilon)(1 - 9.136V_{yarn})(1 - 5.768 \ln M_r) \quad \dots(5.13)$$

Additionally, the terms consisting of the parameters will be placed under a $Powers_v$ variable as is shown in Equation 5.14.

$$Powers_v = (1 - 1.878\epsilon)(1 - 9.136V_{yarn})(1 - 5.768 \ln M_r) \quad \dots(5.14)$$

In order to obtain reliable estimates of the constant K shown in Equation 5.13, five new data sample points are also obtained. These points are in addition to the original ten data points used to obtain the values listed in Table 5.28. The fifteen sets of inputs are shown in Table 5.29.

Table 5.29: Summary of input parameters for transverse Poisson ratio

	$M_r = 2$	$M_r = 5.55$	$M_r = 12$	$M_r = 21.18$	$M_r = 15$
$V_{yarn} = 0.1964$		$\epsilon = 0.3$		$\epsilon = 0.2,$ $\epsilon = 0.3$	
$V_{yarn} = 0.2827$	$\epsilon = 0.2$	$\epsilon = 0.2$	$\epsilon = 0.2$	$\epsilon = 0.2,$ $\epsilon = 0.3,$ $\epsilon = 0.35,$ $\epsilon = 0.4$	
$V_{yarn} = 0.3318$				$\epsilon = 0.2,$ $\epsilon = 0.25,$ $\epsilon = 0.35$	$\epsilon = 0.4$
$V_{yarn} = 0.4418$				$\epsilon = 0.2$	

The original points are shown in grey while the new data points are highlighted in yellow. It can be seen that learning from the previous Young's modulus elliptical formulations, where low values of porosities are impractical [49] as well as erroneous for our model, the new data points are chosen to have high porosities.

5.7.3 Creating the final expression for β

Taking hints from the previous section 5.6, the exponential of B as a function of $Powers_v$ is plotted for the original ten points as shown in Figure 5.21.

Figure 5.21: Exponential of B as a function of $Powers_v$

As before, the points exhibit an exponential relation, as shown by the trend line. Therefore, continuing with this methodology, the values of B for the ten data points and the five new cases are calculated as shown in Table 5.30.

Table 5.30: Validation studies by inclusions of new input parameters T-1 to T-5

Case	Parameter Values			$Powers_v$	B
	Material Ratio, M_r	Porosity, ϵ	Yarn volume fraction, V_{yarn}		
1	2	0.2	0.2827	2.962	0.01607
2	5.55	0.2	0.2827	8.728	0.06740
3	12	0.2	0.2827	13.17	0.09617
4	21.18	0.2	0.2827	16.41	0.10880
5	21.18	0.35	0.2827	9.006	0.05706
6	21.18	0.4	0.2827	6.537	0.04532
7	21.18	0.2	0.2827	11.47	0.07120
8	21.18	0.2	0.1964	8.199	0.03953
9	21.18	0.2	0.3318	21.06	0.09588
10	21.18	0.2	0.4418	31.48	0.14446

T-1	15	0.4	0.3318	7.384	0.05408
T-2	21.18	0.3	0.1964	5.732	0.03953
T-3	5.55	0.3	0.1964	3.066	0.01671
T-4	21.18	0.25	0.3318	17.90	0.12001
T-5	21.18	0.35	0.3318	11.56	0.07620

It should be noted that the five cases T-1 to T-5 in this section (pertaining to transverse Poisson's ratio) are not the same as those obtained in the previous section (pertaining to transverse Young's modulus). To better illustrate these points, they are graphed along with the five test points marked in brown as shown in Figure 5.22.

Figure 5.22: Validation values of exponential of B against $Powers_v$

The R^2 value of 0.9 shows a high degree of correlation for the 15 points and the accompanying power regression relation is shown by Equation 5.15.

$$y = 0.9446x^{0.0556} \quad \dots(5.15)$$

$$\Rightarrow e^B = 0.9446(Powers_v)^{0.0556}$$

$$\Rightarrow B = \ln 0.9446 + 0.0556 \ln(Powers_v)$$

However, since K is defined by B and $Powers_v$ by the relation:

$$B = K \times Powers_v$$

We get,

$$K = \frac{[\ln 0.9446 + 0.0556 \ln(Powers_v)]}{Powers_v}$$

Thus, it is seen that the K is a complicated formulation that makes use of the parameter $Powers_v$ twice. If this is combined wholly with the expression shown in Equation 4.10 shown earlier, we get:

$$\begin{aligned} \beta &= \frac{v_{ell}}{v_{cir}} = \left(\frac{R_a}{R_b}\right)^{(\ln 0.9446 + 0.0556 \ln(Powers_v))} \\ &= \left(\frac{R_b}{R_a}\right)^{-(\ln 0.9446 + 0.0556 \ln(Powers_v))} \end{aligned}$$

where a is the loading direction. This can be written in the final form:

$$\beta = \left(\frac{R_b}{R_a}\right)^{-[\ln 0.9446 + 0.0556 \ln\{(1 - 9.136V_{yarn})(1 - 1.878\epsilon)(1 - 5.768 \ln M_r)\}]} \quad \dots(5.16)$$

5.7.4 Validating the scalability factor, β for transverse Poisson's ratio

Having defined this final expression, it is important to try it on the five test cases to see how they perform, as compared to carrying out the tedious FEA calculations for varying yarn aspect ratio values.

Table 5.31: Correlation and error values of the five test cases T-1 to T-5 for β

Case	Yarn Aspect Ratio values	Parameter Values			R^2	Largest Absolute Error
		Material Ratio, M_r	Porosity, ϵ	Yarn volume fraction, V_{yarn}		
T-1	0.5, 0.6, 0.75, 0.9025, 1 and inverses	15	0.4	0.3318	0.587	3.47%
T-2	0.5, 0.6,	21.18	0.3	0.1964	0.867	1.28%

	0.75, 0.9025, 1 and inverses					
T-3	0.5, 0.6, 0.75, 0.9025, 1 and inverses	5.55	0.3	0.1964	0.425	1.41%
T-4	0.5, 0.6, 0.75, 0.9025, 1 and inverses	21.18	0.25	0.3318	0.707	7.38%
T-5	0.5, 0.6, 0.75, 0.9025, 1 and inverses	21.18	0.35	0.3318	0.717	3.48%

The results of the five test cases along with the maximum absolute error and the correlation coefficient are shown in Table 5.31. It can be seen the results are mixed. The absolute error in the case of the transverse Poisson ratio formulation is much lower than that in the case of the transverse Young's modulus as shown in Table 5.22 earlier.

However, the correlation values seem to be weaker than expected. They range from 0.425 to 0.867, compared to the earlier Table 5.22 for the transverse Young's modulus where the values are all above 0.88 (despite having higher absolute error). This means that the current formulation for transverse Poisson ratio as shown in Equation 5.16 may be inadequate and a more sophisticated overall expression might be desired.

This gives a contrast between accuracy and precision. The transverse Young's modulus expression α with its higher correlation coefficient gives an accurate trend to the points but displays a wider spread of errors (not precise). On the other hands, the transverse Poisson ratio expression the β exhibits lower correlation to the trend (less accurate), but the points lying closer to the expected values seem more precise with lower errors. The incorporation of the ratio of fiber-to-matrix Poisson ratios (i.e. $\frac{\nu_{fiber}}{\nu_{matrix}}$) might aid in improving the overall values as well.

5.8 Forming Shear-Inclusive Elliptical (SIE) yarn input models

The two FEA-based semi-empirical formulations for obtaining the transverse Young's modulus and transverse Poisson's ratio of elliptical yarns as a scaling of their circular yarn values is given as shown below in Equations 5.17.

$$\alpha = \frac{E_{ell}}{E_{cir}} = \left(\frac{R_b}{R_a}\right)^{\ln[1.014809-0.01034(Powers_E)]} \quad \dots(5.17a)$$

$$\beta = \frac{\nu_{ell}}{\nu_{cir}} = \left(\frac{R_b}{R_a}\right)^{-(\ln 0.9446+0.0556 \ln(Powers_\nu))} \quad \dots(5.17b)$$

where $Powers_E = (1 - 1.898\epsilon)(1 - 9.167V_{yarn})(1 - 4.706 \ln M_r)$,

$Powers_\nu = (1 - 1.878\epsilon)(1 - 9.136V_{yarn})(1 - 5.768 \ln M_r)$,

and a is the loading direction.

Table 5.23: Value of parameter coefficient for the $Powers_E$ and $Powers_\nu$ expressions

	$Powers_E$	$Powers_\nu$
Porosity (ϵ)	-1.898	-1.878
Yarn volume fraction (V_{yarn})	-9.167	-9.136
Material modulus ratio (M_r)	-4.706	-5.768

It is interesting to note that the coefficients of porosity and yarn volume fraction (as shown in Table 5.23) in the $Powers_E$ and $Powers_\nu$ expressions are similar in value, even though their values have been separately determined. This might suggest some deeper link existing in the nature of the transverse properties of elliptical yarns. This warrants a further exploration which, due to resource and time constraints, falls out of the scope of this report. The provisional relationships produced in Equations 5.17a and 5.17b have acceptable error value tolerances (below 8% for all cases, except the special case T-3 in Table 5.22).

A large number of FEA simulations (135 simulations for transverse Young's modulus and 120 simulations for transverse Poisson's ratio) by altering the yarn aspect ratios, porosities, yarn volume fractions and material properties were carried out to obtain the basic values of

all the coefficients obtained in Equation 4.15. These formulations are expected to be useful in providing material property inputs for yarns for textile reinforced composites modeling later on in this study.

Using Equations 5.17a and 5.17b is fairly straightforward. To obtain the Young's modulus and Poisson ratio in the transverse direction properties of a unidirectional elliptical yarn composite, firstly obtain the corresponding properties of the circular yarn composite using the same material properties, porosity and yarn volume fraction. This can be obtained by experimentation, FEA, SIC, Halpin-Tsai, CCA or other classical models. Consequently, determine the α and β scalability factors using the same yarn inputs and then multiply with the corresponding circular yarn composite properties to obtain E_{ell} and ν_{ell} .

It should be noted, however, that these expressions cannot be used in all cases. Due to the need for $Powers_E$ and $Powers_\nu$ to have positive values (due to application of logarithmic and exponential functions), there exist mathematical limits to the values of ϵ , V_{yarn} and M_r . These are shown in Table 5.24.

Table 5.24: Mathematical limitations to the values of the parameters

Parameter	Limitation
Porosity	$\epsilon < 0.5269$
Yarn volume fraction	$V_{yarn} > 0.1091$
Material modulus ratio	$M_r > 1.23677$

If these limits are not adhered to, it is possible that the formulas in Equations 5.17a and 5.17b may give rise to complex number values for the scalability factor. However, if the terms fall within the acceptable range, there will be no issues. Due to a lack of time, it was not possible to improve on the expressions to eliminate or minimize these limitations.

These expressions can be further incorporated by making use of the SIC values obtained from Chapters 3 and 4 provides a basis of consideration of shear-inclusive properties for these elliptical yarn composites. Hence, the process of combining SIC models along with Equations

5.17a and 5.17b is designated as the Shear-Inclusive Elliptical (SIE) yarn model (shown in Equations 5.18a and 5.18b).

$$E_{ell} = E_{cir} \alpha = \frac{\alpha}{\left(\frac{1}{E_{cir}}\right)}$$

$$\Rightarrow E_{ell} = \frac{(AR)^{\ln[1.014809-0.01034(Powers_E)]}}{\left(\frac{V_{yarn}}{E_{yarn}} + \frac{(1-V_{yarn})}{E_m} + \frac{1}{3} \sqrt{\frac{V_{yarn}}{\pi}} \left[\frac{2(1-\nu_{yarn})}{E_{yarn}} + \frac{2(1-\nu_m)}{E_m} - \frac{1}{G_{yarn}} - \frac{1}{G_m} \right] \right)}$$

...(5.18a)

$$\nu_{ell} = \nu_{cir} \beta$$

$$\Rightarrow \nu_{ell} = \frac{\left(\frac{\nu_{yarn} V_{yarn}}{E_{yarn}} + \frac{\nu_m (1 - V_{yarn})}{E_m} - \frac{1}{15} \sqrt{\frac{V_{yarn}}{\pi}} \left[\frac{2 - 3\nu_{yarn}}{E_{yarn}} - \frac{1}{G_{yarn}} + \frac{2 - 3\nu_m}{E_m} - \frac{1}{G_m} \right] \right)}{\left(\frac{V_{yarn}}{E_{yarn}} + \frac{(1 - V_{yarn})}{E_m} + \frac{1}{15} \sqrt{\frac{V_{yarn}}{\pi}} \left[\frac{3 - 2\nu_{yarn}}{E_{yarn}} + \frac{1}{G_{yarn}} + \frac{3 - 2\nu_m}{E_m} + \frac{1}{G_m} \right] \right)}$$

$$(AR)^{(\ln 0.9446 + 0.0556 \ln(Powers_v))}$$

...(5.17b)

Where $Powers_E = (1 - 1.898\epsilon)(1 - 9.167V_{yarn})(1 - 4.706 \ln M_r)$, $Powers_v = (1 - 1.878\epsilon)(1 - 9.136V_{yarn})(1 - 5.768 \ln M_r)$ and a is the loading direction.

The SIE model is used in Chapters 6 and 7 to apply these woven textile and braided textile composite models respectively.

5.9 Concluding Remarks

This chapter has thus focused on the analysis and conception of scalability factors for the transverse Young's modulus and Poisson ratio of elliptical yarns composites as a relation to circular yarn composites using FEA results. Based on the findings presented in this chapter, the following conclusions can be drawn:

- i. It is difficult to obtain a unified micromechanical formulation that considers shear-interface interactions for elliptical yarns and is able to deal with different yarn aspect ratios. Hence, there is a need to use FEA data-analysis techniques to provide a quick means of obtaining transverse properties of elliptical yarns.

- ii. The scalability factors obtained in this chapter are shown to follow a power relationship with the yarn aspect ratio. This power is also seen to vary linearly with yarn volume fraction and porosity and logarithmically with the material modulus ratio.
- iii. Consolidated scalability expressions for the transverse Youngs' modulus and Poisson ratio are derived and validated with new FEA simulations. The difficulty of carrying out experiments on the transverse properties of long fiber reinforced composites, while maintaining the elliptical yarn shape and structural integrity (within the elastic region) prompts the use of reliable FEA simulations instead.
- iv. The Young's modulus expression shows a good correlation (but greater individual errors) with FEA simulations, while the Poisson ratio expression has lower errors (but poorer correlation) with the FEA results.
- v. In the case of the transverse Poisson ratio scalability factor, it might be prudent to bring in the ratio of the fiber-to-matrix Poisson ratios as an addition to the usual Material Modulus ratio.
- vi. The final expressions are provisionally-structured mathematical equations that intend to demonstrate the fundamentals of the FEA data-analysis and modeling methodology. They can be further fine-tuned with more FEA simulation testing and more sophisticated models linking the parameters to the output variables.
- vii. The combination of the scalability factors with the SIC formulations derived in the previous chapters gives rise to the SIE formulations that tackle elliptical yarns. These can be considered as property inputs for long-fiber as well as textile composite yarns.

This chapter will mark the end of the study of unidirectional long fiber composites. The next two chapters on Woven and Braided textile composites will focus on using some of the concepts and theories (especially yarn input models) previously obtained to assess their effect and suitability to textile reinforced composites.

Chapter 6: Woven Fabric Unit Cell

Since the next stage of this study is to compare the key aspects of fiber pre-forming in textile composites, especially woven and braided textile composites, it was imperative to develop technical know-how along with the detailed literature study that was being carried out. With this in mind, it was proposed to simulate a woven textile composite structure by replicating the work done previously [17, 20, 70] and adding in new concepts [15, 59, 71] using MATLAB[®] software. Woven textile models were explored where the work is simplistically explained and the necessary theoretical background and supporting equations are provided to enable comprehensive coding into MATLAB[®]. This would not only give a better understanding of the underlying principles of textile composite structures and materials micromechanics, but also act as a suitable technical springboard for future study in comparing with braided textile structures.

6.1 Two dimensional woven structure

This simple two-dimensional (2D) study has been chosen to aid simplicity and reinforce the model-building concepts before the more complicated three-dimensional (3D) woven structure is dwelt upon. The 2D woven structure is perceived as a planar arrangement of orthogonal fill and weft yarns. In textile manufacturing terminology, the weft yarn is along the weaving direction, with the fill yarn passing through perpendicular to the weft yarn [92]. However, when the textile is ready, the weft and fill yarns are similar in structure. The side face of the unit cell in Figure 6.1 below shows the direction of propagation of the 2D woven structure.

Figure 6.1: One directional weave is shown in planar space

Thus, there is only a single direction of propagation of the woven arrangement. The geometry that is used to represent this is shown in Figure 6.2(a). Its equivalent geometrical model that has been built in MATLAB[®] is shown in Figure 6.2(b).

Figure 6.2: (a) The woven geometry [70] (b) The MATLAB[®] model equivalent

This geometry is considered infinitely long in the direction perpendicular to the page. Hence, only the planar properties of the yarn can be obtained. The distinct regions in the woven geometry are shown in Figure 6.3. The fill yarn flows from the right to the left of the geometry. The two weft yarns flow in the plane perpendicular to the page; as a result, their cross-sectional view is seen. The matrix region surrounds the fill and weft yarns.

Figure 6.3: Different regions shown in the yarn

6.1.1 Parameterization of the 2D model

The geometry of the yarns uses the same geometrical parameters as [70]. These parameters are shown in Figure 6.4. The width is denoted by ' a ' and the heights by ' h_f ' and ' h_w ' for the fill and weft yarns respectively. The gap between consecutive weft yarns is denoted by ' g '. The parameter ' u ' provides the length of undulation i.e. the length within which the yarn slopes and intertwines. The undulation is characterized by a sinusoidal relation that is tangent at the point at which the straight portion of the yarn resumes. In case the parameters u and g are reduced to a very small value, the unit cell approximates to the classical mosaic woven model [20].

Figure 6.4: Geometrical parameters used for coding

Furthermore, material properties are assigned for each of the distinct regions. The fill and weft yarns are transversely isotropic, while the matrix region is considered isotropic. The parametric inputs for these properties are shown in Table 6.1.

Table 6.1: Mechanical material properties being used for the woven model

Fill & Weft Yarns	Transversely Isotropic - Single longitudinal and two identical transverse directions	Young's Modulus - Longitudinal (E_L) Young's Modulus - Transverse (E_T) Poisson ratio - Longitudinal-transverse (ν_{LT})* Poisson ratio - Transverse-transverse (ν_{TT}) Shear modulus - Longitudinal-transverse (G_{LT}) Shear modulus - Transverse-transverse (G_{TT})
Matrix	Isotropic - non-directional	Young's Modulus - Longitudinal (E_m) Poisson ratio - Longitudinal-transverse (ν_m) Shear modulus - Longitudinal-transverse (G_m)

* ν_{TL} can easily be obtained from E_L , E_T & ν_{LT} using the relation of the theory of elasticity shown by Equation 6.1 [91].

$$\frac{\nu_{TL}}{E_T} = \frac{\nu_{LT}}{E_L} \quad \dots(6.1)$$

In addition to these geometrical parameters, discretization parameters are also used to sufficiently divide the continuous woven model into finite sections as shown below in Figure 6.5. The parameters N_{st} , N_u and N_g are the number of divisions for the straight, undulated and gap sections of the yarn respectively.

Figure 6.5: The number of divisions of the unit cell sections

Also, it is illustrated how the width of a single strip within the undulating section is the section length, $\frac{u}{2}$, divided by the corresponding number of divisions, N_u . The width of the vertical strips in the other sections is given by Table 6.2.

Table 6.2: Discretization parameters of the unit cell

Region	Length	Number of divisions	Width of a single division
Straight region	$\frac{(a-u)}{2}$	N_{st}	$\frac{(a-u)}{2N_{st}}$
Undulating region	$\frac{u}{2}$	N_u	$\frac{u}{2N_u}$
Gap region	$\frac{g}{2}$	N_g	$\frac{g}{2N_g}$

6.1.2 Computational method of determining mechanical properties of the unit cell

Each vertical strip is then treated as a composite lamina, consisting of two layers of matrix (top & bottom) separated by one or two layers of yarns. Each yarn has its own elasticity stiffness matrix as shown in Equation 6.2 [93].

$$[E_{ij}] = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad \dots(6.2)$$

For the undulating part of the fill yarn, the slope of the yarn is calculated using the sinusoidal equations and the corresponding properties in the x direction are obtained using compliance matrix transformations [70].

The mechanical properties of each vertical strip area are calculated using Classical Lamination Theory (CLT) and the elastic stiffness matrices of the fill yarn region ($E_{i,fill}$), the weft yarn region ($E_{i,weft}$) and the matrix region ($E_{i,matrix}$) as shown by Equation 6.3 with the all-encompassing weighted stiffness $E_{w,i}$ term.

$$[E_{w,i}(i)] = h_{i,matrix}[E_{i,matrix}] + h_{i,fill}[E_{i,fill}] + h_{i,weft}[E_{i,weft}] \quad \dots(6.3)$$

The elasticity matrices are thus multiplied by the heights of the respective vertical regions of the fill yarn ($h_{i,fill}$), the weft yarn ($h_{i,weft}$) and the matrix region ($h_{i,matrix}$) in order to obtain an averaged representation for the properties of the strip at the i^{th} location. This can be inverted to obtain the corresponding weighted compliance matrix for the single strip.

$$[C_{w,i}(i)] = [E_{w,i}(i)]^{-1}$$

Matrix inversions procedures are shown in Annex C. The compliance matrices of all the strips are then aggregated [94] to obtain an overall unit cell weighted compliance matrix as shown in Equation 6.4.

$$[\overline{C}_w^s] = \frac{1}{a+g} \int_0^{a+g} [C_{w,i}(i)] dx$$

$$= \frac{1}{a+g} \sum_{i=1}^{2*(N_{st}+N_u+N_g)} [C_{w,i}(i)] \Delta x_i$$

where Δx_i is the width of the i^{th} strip

The overall compliance matrix for the entire unit is obtained by multiplying the weighted compliance matrix by the height of the unit cell. Thus,

$$[\overline{C}_{eq}] = [\overline{C}_w^S] (h_A + h_B)$$

The mechanical properties of the unit cell are then extracted from this overall matrix as -

$$[\overline{C}_{eq}] = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xz}}{E_x} & 0 \\ -\frac{\nu_{xz}}{E_x} & \frac{1}{E_z} & 0 \\ 0 & 0 & \frac{1}{G_{xz}} \end{bmatrix} \quad \dots(6.4)$$

The aggregation is carried out using the iso-stress condition. Hence, this produces a lower bound to the stiffness predictions. The iso-strain condition, on the other hand, produces an upper-bound to the stiffness predictions [95].

6.1.3 Convergence testing of the woven model

It is important to evaluate the effect of the discretization parameters N_{st} , N_u and N_g on the results in order to determine their optimal values that strike a balance between accuracy and computational speed.

The convergence test seeks to ensure that any discrepancies in results due to region discretization are effectively eliminated by using a sufficiently large number of divisions. For result convergence, the first term of the final compliance matrix, $\bar{C}_{11}$ is chosen. The value of $\bar{C}_{11}$ is given by Equation 6.5.

$$\bar{C}_{11} = \frac{1}{E_x} \quad \dots(6.5)$$

This term is monitored to determine convergence of the parameters. Shown in Figure 6.6 are the convergence test results for N_{st} and N_g . Also, shown is the convergence test for N_u in Figure 6.7. The initial conditions used were $a = 3.15$ mm, $u = 1.8$ mm and $g = 0$ mm.

Figure 6.6(a): Convergence of divisions of straight region (b) Convergence of divisions of gapped region

Figure 6.7: Convergence of divisions of the undulating region

From Figures 6.6(a) and 6.6(b), it can be seen that the effect of N_{st} and N_g parameters is quite stable, while the N_u parameter in Figure 6.7 exhibits a typical convergence behaviour. In the case of N_{st} , the yarn follows the horizontal direction and hence the material property is oriented to the on-axis properties. The undulation region has material behaviour that is

oriented off-axis, and hence, when varying N_u , $\bar{C}_{11}$ is expected to fluctuate initially before stabilizing at the given final value. In the case of N_g , this parameter is seen to have no effect due to the fact that g has a value of 0 mm (indicating no gap). Thus, the precision values for all the final divisions are up to four decimal places.

Hence, as a result of these convergence tests, the final values of N_{st} , N_u and N_g taken for this work are 9, 192 & 1 respectively. The value of 9 for N_{st} (despite being invariant) was taken to ensure similar element size to the 192 divisions within the undulating region.

6.1.4 Results

Having setup and carried out the convergence tests on the woven textile yarn model, the sample cases shown in Tables 7 & 8 of Naik & Shembekar's work [70] were simulated. The Young's modulus values E_x are computed (by inverting Equation 6.5). Not only are the results of the in-house self-coded woven model compared against the literature woven model, but also a brief comparison of the various input models is carried out.

Table 6.3: Comparison of in-house woven model and Naik & Shembekar's woven model Young's modulus values with experimental results

Material	Dimensions	Exp. [70]	Naik & Shembekar Woven model [70]		In-house woven model*	
			CCA	diff	CCA	diff
T-300 Carbon/ Epoxy	Warp direction – a = 0.96 mm; g = 0.151 mm	60.3	67.8	12.44%	56.878	-5.67%
	Fill direction – a = 1.10 mm; g = 0.011 mm	49.3	54.1	9.73%	51.343	4.14%
E-glass/ Epoxy a	Warp direction – a = 0.52 mm; g = 0.25 mm	18.1	28.6	58.01%	21.471	18.62%
E-glass/ Epoxy b	Warp direction – a = 1.12 mm; g = 0.6 mm	14.8	29.5	99.32%	18.08	22.16%

	Fill direction – a = 1.44 mm; g = 0.12 mm	13.8	<i>22.1</i>	<i>60.14%</i>	15.155	9.81%
E-glass/ Epoxy c	Warp & fill direction – a = 0.5 mm; g = 0.23 mm	14.5	<i>21.1</i>	<i>45.52%</i>	16.134	11.27%

* In house woven model simulates a woven textile composite structure by replicating the work done previously [17, 20, 70] and adding in new concepts [15, 59, 71] using MATLAB[®] software.

It may be seen that the in-house developed computational woven model leads to values closer to the experimental results than the computational model from the literature [70]. For different values of unit cell dimensions and constituent material properties, our parallel woven model is able to consistently outperform the Naik & Shembekar's model as shown by the lesser % deviation for every experimental data point.

It is mentioned that Composite Cylinder Assemblage (CCA) model was used to determine the transverse Young's modulus of the yarn; however, due to the complexity of the CCA model as well as the inapplicability of its range of values, alternative models were sought after. As part of the studies carried out in earlier chapters, newer SIC and SIE formulations were also incorporated into our parallel woven model. A comparison of the corresponding results is shown in Table 6.4.

Table 6.4: Comparison of in-house woven model (utilizing CCA, SIC and SIE yarn inputs) Young's modulus computations with experimental results

Material	Dimensions	Experiment [70]	Parallel Woven Model		
			CCA	SIC	SIE
T-300 Carbon/ Epoxy	Warp direction – a = 0.96 mm; g = 0.151 mm	60.3	56.878	52.427	54.19
	Fill direction –	49.3	51.343	46.892	47.836

	a = 1.10 mm; g = 0.011 mm				
E-glass/ Epoxy a	Warp direction – a = 0.52 mm; g = 0.25 mm	18.1	21.471	20.796	21.751
E-glass/ Epoxy b	Warp direction – a = 1.12 mm; g = 0.6 mm	14.8	18.08	16.406	16.473
	Fill direction – a = 1.44 mm; g = 0.12 mm	13.8	15.155	13.609	13.742
E-glass/ Epoxy c	Warp & fill direction – a = 0.5 mm; g = 0.23 mm	14.5	16.134	16.424	16.838

It can be seen that the results are mixed. In the case of the epoxy glass samples, the SIC model generally performs equivalent or better than the CCA model. This might be due to the fact that the SIC incorporates improved transverse properties as compared to the general CCA model with lower differences in glass and epoxy constituent material properties. However, the results in the case of carbon-epoxy composite perform worse than the CCA model. This could be due to the incorporation of higher difference between constituent material properties (carbon and epoxy).

It is also seen that the SIE model predicts a higher elastic modulus (E_x) than the SIC. This is primarily due to the strengthening effect of considering flattened elliptical yarns in the plane of the composite. The α factors are less than 1 and the β factors more than one (as is shown in the previous sections). However, it is not possible to use the SIE model in all cases due to the model parameter limitations (Table 5.25). This is discussed in Section 6.4.3 in detail.

6.2 Three dimensional woven structure

During the process of debugging and reconfirmation of the woven planar MATLAB[®] model, the geometry for the 3D model was created. The same geometric conventions are used, with two sets of yarn geometry specifications; one for the fill yarn and another for the weft yarn. The fill and weft yarns run along two orthogonal directions. As a result, the entire textile woven forms a 3D structure, as shown in Figure 6.8.

Figure 6.8: Two directional weave exists in 3D space

It can be easily seen that the previous 2D woven structure consisted of yarn flowing only in the x-direction. The 3D weave has yarn flowing in both x- and y- directions. Within MATLAB[®], the geometry in both the x- and y- directions have been input as is shown in Figure 6.9(a) and 6.9(b).

Figure 6.9: (a) Cross-sectional geometry in the x-direction (b) Cross-sectional geometry in the y-direction

6.2.1 Parameterization of the 3D model

Using these two yarn shapes, the internal geometry is approximated by linear interpolation and thus the entire 3D unit cell yarn model is generated as shown in Figure 6.10.

Figure 6.10: Three dimensional unit cell model for disparate yarns

Figure 6.11: Unit cell dimensions

It is thus seen that it is possible to design varying yarns within the same unit cell with the aid of unique inputs in the x- and y- directions of yarn propagation. As a result, the unit cell may no longer be limited to square shape (with unique a_x , g_x , a_y & g_y input values) and the yarn heights (h_w and h_f) enable accounting for yarns of different geometries and material properties.

Work carried out in Naik & Shembekar [70] investigated the effect of undulation on the mechanical properties of the yarn. The undulation is the “curviness” of the yarn over the extent of the unit cell. Table 6.5 shows how the undulation can be altered to create unit cells of varying geometries. It is shown as a fraction of the total length of the unit cell ($a+g$) in both directions.

Table 6.5: Effect of undulation on 2D and 3D woven geometry

It can be seen that at 10% undulation, the unit cell approaches the mosaic model where the curvature of the yarn is non-existent and the composite begins to resemble bricks of unidirectional fiber composites patched together. As the undulation increases, the geometry begins to approach natural textile patterns. The results of the change in unit cell properties as a function of undulation are discussed later on.

Thus, it can be seen that it is possible to generate varying geometries within our three dimensional woven yarn model based on the given requirements. Additionally, this complete

unit cell yarn shape consolidation enables the yarn heights to be calculated for application of the CLT model [20, 70].

As before, the geometry is again divided into straight, undulating and gap regions in both the x- and y- directions for a comprehensive discretization of the unit cell as shown in Tables 6.6 and 6.7. These follow the same format of the 2D case presented earlier (along with a directional connotation).

Table 6.6: X-direction discretization parameters of the unit cell

Region	Length	Number of divisions	Width of a single division
Straight region	$\frac{(a_x - u_x)}{2}$	N_{st_x}	$\frac{(a_x - u_x)}{2N_{st_x}}$
Undulating region	$\frac{u_x}{2}$	N_{u_x}	$\frac{u_x}{2N_{u_x}}$
Gap region	$\frac{g_x}{2}$	N_{g_x}	$\frac{g_x}{2N_{g_x}}$

Table 6.7: Y-direction discretization parameters of the unit cell

Region	Length	Number of divisions	Width of a single division
Straight region	$\frac{(a_y - u_y)}{2}$	N_{st_y}	$\frac{(a_y - u_y)}{2N_{st_y}}$
Undulating region	$\frac{u_y}{2}$	N_{u_y}	$\frac{u_y}{2N_{u_y}}$
Gap region	$\frac{g_y}{2}$	N_{g_y}	$\frac{g_y}{2N_{g_y}}$

6.2.2 Method of determining mechanical properties of a single 3D element

The vertical strip shown for the one dimensional case earlier is now converted into a cuboidal block which has components of fill yarn, weft yarn and matrix above and below as shown in Figure 6.12.

Figure 6.12: Single vertical block of composite consisting of fill yarn, warp yarn and matrix

The height of the block is determined by the sizes of the fill yarn, weft yarn and matrix components as shown in Figure 6.12 (b), with the rectangular base of the block measured in dx and dy shown by the widths in Tables 6.6 and 6.7 respectively.

This entire cuboidal block is then treated as a laminate, consisting of two layers of matrix (top & bottom) separated by one or two layers of yarns. Each yarn has its own elasticity stiffness matrix as shown in Equation 6.6 below [93].

$$E_{ij} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad \dots(6.6)$$

For the undulating parts of both the fill and weft yarns, the slopes of the yarn were calculated using the sinusoidal equations as before and the corresponding properties in the x- and y- directions are obtained.

The mechanical properties of each vertical block volume are calculated using CLT and the elastic stiffness matrix of the fill yarn region ($E_{ij,fill}$), the weft yarn region ($E_{ij,weft}$) and the

matrix region ($E_{ij,matrix}$) as shown by Equation 8 with the all-encompassing weighted elasticity stiffness $E_{w,ij}$ term.

$$\begin{aligned} [E_{w,ij}(i,j)] = & h_{ij,matrix}[E_{ij,matrix}] + h_{ij,fill}[E_{ij,fill}] + \\ & h_{ij,weft}[E_{ij,weft}] \quad \dots(6.7) \end{aligned}$$

The elasticity stiffness matrices are thus multiplied by the heights of the respective vertical regions of the fill yarn ($h_{ij,fill}$), the weft yarn ($h_{ij,weft}$) and the matrix region ($h_{ij,matrix}$) in order to obtain an averaged representation for the properties of the block at the $i^{th} - j^{th}$ location.

This can be inverted to obtain the corresponding weighted elasticity compliance matrix for the single i -th and j -th block.

$$[C_{w,ij}(i,j)] = [E_{w,ij}(i,j)]^{-1}$$

6.3 Aggregating the vertical blocks in the 2D woven composite

In the one directional modeling in the previous section, the series method was used for aggregating all the vertical strips along the x-direction. This involved obtaining a weighted average of the compliance matrices of each vertical strip [17]. In the two directional modeling, there exist $i \times j$ vertical blocks, each one with its unique property matrices (stiffness or compliance). Thus, all the techniques discussed later involve summing the property matrices first in the x-direction and then in the y-direction, or vice versa. A few approaches are possible. The simplest approaches would be a straightforward weighted average of, either all the stiffness matrices, or all the compliance matrices of all the unit cells. These are denoted as Area-Averaged Stiffness or Area-Averaged Compliance additions and shown below.

6.3.1 Area-Averaged Stiffness (AAS) addition

As the name implies, this method takes into consideration all the stiffness matrices of all the vertical blocks (where each block consists of fill yarn, weft yarn & matrix as shown in Figure 6.12 earlier) and uses a simple averaging as -

$$[\bar{E}^{avg}_w] = \left(\frac{1}{a_x + g_x} \right) \left(\frac{1}{a_y + g_y} \right) \int_0^{a_x + g_x} \int_0^{a_y + g_y} [E_{w,ij}(i,j)] dy dx$$

Upon discretization, the integrals along the x- and y-directions are rewritten as a dual summation of i- and j-elements to obtain the final value (to be divided by the area of the unit cell).

$$[\bar{E}^{avg}_w] = \left(\frac{1}{a_x + g_x}\right)\left(\frac{1}{a_y + g_y}\right) \sum_{i=1}^{2(N_{st,x}+N_{u,x}+N_{g,x})} \left\{ \sum_{j=1}^{2(N_{st,y}+N_{u,y}+N_{g,y})} [E_{w,ij}(i,j)] \Delta y_j \right\} \Delta x_i$$

where $\Delta x_i \Delta y_j$ is the rectangular area of the i-th and j-th element

6.3.2 Area-Averaged Compliance (AAC) addition

This method is identical to that in 6.3.1, with the exception that, in this case, the compliance matrices of all the vertical blocks are used instead.

$$\begin{aligned} [\bar{C}^{avg}_w] &= \left(\frac{1}{a_x + g_x}\right)\left(\frac{1}{a_y + g_y}\right) \int_0^{a_x+g_x} \int_0^{a_y+g_y} [C_{w,ij}(i,j)] dy dx \\ &= \left(\frac{1}{a_x + g_x}\right)\left(\frac{1}{a_y + g_y}\right) \sum_{i=1}^{2(N_{st,x}+N_{u,x}+N_{g,x})} \left\{ \sum_{j=1}^{2(N_{st,y}+N_{u,y}+N_{g,y})} [C_{w,ij}(i,j)] \Delta y_j \right\} \Delta x_i \end{aligned}$$

where $\Delta x \Delta y$ is the rectangular area of the i-th and j-th element

Thus one method focusses on the stiffness matrices, while the other on the compliance matrices of the woven model. However for both the AAS and AAC additions demonstrated, it must be noted that each vertical block within the composite arrangement is treated in isolation. If the above expressions are examined carefully, it is seen that there is no combined effect of having all the blocks oriented along a certain i-th line or j-th line. There is no “cohesive” effect, linking them together; it is as though any vertical block behaves independently of its neighbour, just as it would if it were removed from its current i-j position and placed in another location within the composite.

Thus, a more sophisticated method propounded and implemented involves making use of the micromechanical relations of all the vertical blocks within the composite [20, 70]. In this case, all the blocks are aggregated using applicable iso-stress or iso-strain relations along their i-th and j-th lines to “intelligently” obtain the unit cell properties. This gives rise to the Series-Parallel and Parallel-Series formulations.

6.3.3 Series-Parallel and Parallel-Series addition

The x-direction is taken as the direction of primary property determination. Therefore, all blocks along all j-th lines (x-direction) will be aggregated using iso-stress relations, since they are in series; whereas, all blocks along i-th lines (y-direction) will be aggregated using iso-strain, since they are in parallel. This is illustrated by Figure 6.13 .

Figure 6.13: Top view of the vertical blocks along their i-th and j-th lines

Thus, every j-line (i.e. $j = 1, j = 2$ and so on) will sum up their i-blocks (in series) using an iso-stress relation; this will entail a length-averaged summation of the respective j-line compliance matrices. On the other hand, every i-line (i.e. $i = 1, i = 2$ and so on) will sum up their j-blocks (in parallel) using an iso-strain relation and thus carry out a width-averaged summation of the respective i-line stiffness matrices. The external force applied on the composite follows the x-direction as the principal property determination direction. Hence, both the Series-Parallel and Parallel-Series carry out iso-strain and iso-stress additions, but in different orders.

6.3.3.1 The Series-Parallel (SP) Model

As the name suggests, a series addition is carried out before the parallel addition. Every j-line is summed up by length-averaging their height-weighted compliance matrices, using iso-stress relations (since they are all in series). The summation takes place along x-direction.

$$\left[\bar{c}_{w,j}^s(j) \right] = \frac{1}{a_x + g_x} \int_0^{a_x + g_x} [C_{w,ij}(i, j)] dx$$

$$\Rightarrow [\bar{C}_{w,j}^S(j)] = \frac{1}{a_x + g_x} \sum_{i=1}^{2*(N_{stx}+N_{ux}+N_{gx})} [C_{w,ij}(i,j)] \Delta x_i$$

where Δx_i is the length of the i-th block

Now, the resulting compliance matrix is inverted to obtain a stiffness matrix.

$$[\bar{E}_{w,j}^S(j)] = [\bar{C}_{w,j}^S(j)]^{-1}$$

This stiffness matrix contains information of every j-row. It is summed by width-averaging using iso-strain relations. The summation takes place along the y-direction.

$$[\bar{E}_w^{SP}] = \frac{1}{a_y + g_y} \int_0^{a_y+g_y} [\bar{E}_{w,j}^S(j)] dy$$

$$\Rightarrow [\bar{E}_w^{SP}] = \frac{1}{a_y + g_y} \sum_{j=1}^{2*(N_{sty}+N_{uy}+N_{gy})} [\bar{E}_{w,j}^S(j)] \Delta y_j$$

where Δy_j is the width of the j-th block

Thus, the final $\bar{E}_w^{SP}$ weighted stiffness matrix contains the overall woven unit cell properties.

6.3.3.2 The Parallel-Series (PS) Model

The Parallel-Series model carries out a parallel (iso-strain) summation of the weighted stiffness matrices along the i-th line (all j values) first.

$$[\bar{E}_{w,i}^P(i)] = \frac{1}{a_y + g_y} \int_0^{a_y+g_y} [E_{w,ij}(i,j)] dy$$

$$= \frac{1}{a_y + g_y} \sum_{j=1}^{2*(N_{sty}+N_{uy}+N_{gy})} [E_{w,ij}(i,j)] \Delta y_j$$

where Δy_j is the length of the j-th block

The resulting stiffness matrix is inverted to provide a compliance matrix.

$$\left[\bar{C}_{w,i}^P(i) \right] = \left[\bar{E}_{w,i}^P(i) \right]^{-1}$$

This stiffness matrix contains information of every i-row. It is summed by length-averaging using iso-stress relations. The summation takes place along the x-direction.

$$\begin{aligned} [\bar{C}^{PS}_w] &= \frac{1}{a_x + g_x} \int_0^{a_x + g_x} \left[\bar{C}_{w,i}^P(i) \right] dx \\ \Rightarrow [\bar{C}^{PS}_w] &= \frac{1}{a_x + g_x} \sum_{i=1}^{2*(N_{stx} + N_{ux} + N_{gx})} \left[\bar{C}_{w,i}^P(i) \right] \Delta x_i \end{aligned}$$

where Δx_i is the length of the i-th block

Thus, at the end of the PS summation, the $\bar{a}^{PS}$ matrix holds the material property values of the woven unit cell.

Note that Equation 6.3 carries out a height-weighted summation of the vertical block properties. Therefore, to obtain the elasticity matrix of woven unit cell, either the stiffness matrix must be divided by the height of the total unit cell (Equation 6.8a), or the compliance matrix must be multiplied by height of the total unit cell (Equation 6.8b).

$$[E]_{woven} = \begin{bmatrix} \frac{E_x}{1-\nu_{xy}\nu_{yx}} & \frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ \frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & \frac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} = \frac{[\bar{E}_w]}{(h_A + h_B)} \quad \dots(6.8a)$$

$$[C]_{woven} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} = [\bar{C}_w](h_A + h_B) \quad \dots(6.8b)$$

where $[E]_{woven}$ and $[C]_{woven}$ are the equivalent elasticity and compliance matrices of the woven unit cell respectively (and matrix inversions of each other), and $\bar{A}$ or $\bar{a}$ are the final obtained matrices in Sections 6.3.1 to 6.3.4.

Thus, the properties of the final woven composite like Young's moduli in the x- and y-directions (E_x & E_y), Poisson ratio (ν_{xy}) and shear modulus (G_{xy}) can be easily obtained. All these data storage, computations and calculations are carried out by the MATLAB® code developed in-house for this study. The user needs to only provide the material (yarn and matrix) properties and geometry, and the MATLAB® program will automatically calculate the final woven composite properties. The finalized codes that were used are shown in Appendix F.

The next section will show the results of the modeling.

6.4 Results of 3D woven composite modeling

As discussed in Section 6.2, the effects of undulation on the final woven composite properties are investigated using the MATLAB® code developed in this study. The AAS and AAC summations as well as the SP and PS methods are compared. The composite consists of carbon fibers arranged in fill and weft yarns and epoxy matrix impregnated within the fibers and the surrounding matrix area. The material properties are taken from Naik & Shembekar's work [70].

6.4.1 Comparison of summation techniques

Figure 6.14 shows the principal direction elastic modulus E_x for the woven model as a function of yarn undulation, by making use of the aggregation techniques discussed in the previous section. The value is expected to initially fall and then rise again as the undulation increases. This initial fall is due to the initial sharp reduction in the number of fiber elements that are orientated along the length of the composite; however, as the undulation increases further, the average angle of deviation of the fiber elements from the composite direction falls and thus a strengthening effect takes place [70].

Figure 6.14: Elastic modulus as a function of unit cell undulation (using various summation techniques)

In our comparison of summation methods, it is seen that the AAS technique is a raw summation of the stiffness properties without micro-mechanical interactions between vertical blocks) and forms an upper bound of the properties. On the other hand, the AAC technique is a raw summation of the inverses of the stiffness properties and forms a lower bound. The PS and SP techniques, being a combination of aggregating compliance and stiffness matrix quantities (based on micromechanical relations), thus lie within the region between the upper- and the lower-bound values. Additionally, when compared against other computational and experimental data, the PS and SP models correlate well in obtaining the woven composite properties. This is discussed in the results section later.

Figure 6.15: Shear modulus as a function of unit cell undulation (using various summation techniques)

The shear modulus property variation of the woven composite is shown in Figure 6.15. It displays the same trends as the elastic modulus, with the PS and SP properties lying within the region between the AAS and AAC. In both cases, PS and SP results are close to each other. In theory, the PS summation is expected to be higher than the SP summation due to the fact that, the 1st set of summations play a more important part in the overall result than the 2nd set [70]. Hence, if observed carefully in Figures 6.14 and 6.15, it is seen that the SP results are either equal to or lower than the corresponding PS results [70].

The reason for the closeness between PS and SP model results in this particular case is the fine level of discretization applied on the woven model. The i- and j- directions of the woven composite have both been divided into 120 sections, the resulting 14400 vertical blocks of the woven model. Due to such fine sectioning, both summation procedures (either PS or SP) lead to similar results. In fact, it is seen that the experimental results (in Section 6.4.3) are very close to the values obtained by our own woven model. Thus, both the PS and SP are considered valid for use in our work.

6.4.2 Comparison of computational convergences with Naik & Shembekar's model

Shown in Tables 6.8 and 6.9 are the results obtained by our computational code compared against those presented in [70]. Of particular interest is the preciseness of the code, subject to the summation technique. It is seen that for every sample, the values of composite Young's modulus obtained by PS and SP techniques are similar in value (as highlighted in yellow) to each other – usually within the range of 1 GPa.

Table 6.8: Unit cell modulus of elasticity E_y (along the warp direction)

Constituent Materials	Our Work		N & S [70]	
	PS	SP	PS	SP
T-300 Carbon/Epoxy	49.083	49.078	58.8	38.2
E-glass/Epoxy (a)	19.046	19.031	21.5	18.4
E-glass/Epoxy (b)	18.144	18.123	21.6	21.5
E-glass/Epoxy (c)	13.828	13.730	14.9	13.9

Table 6.9: Unit cell modulus of elasticity E_x (along the fill direction)

Constituent Materials	Our Work		N & S [70]	
	PS	SP	PS	SP
T-300 Carbon/Epoxy	43.841	43.833	45.8	31.1
E-glass/Epoxy (a)	15.321	15.307	17.1	16.7
E-glass/Epoxy (b)	15.241	15.220	16.1	15.7
E-glass/Epoxy (c)	13.823	13.730	14.9	13.9

On the other hand, the computations carried out in [70] show large variations in the results for PS and SP techniques. In the case of the T-300 Carbon/Epoxy material in the warp direction, the difference between PS and SP computation results is more than 20 GPa. This means that the computation results of [70] are affected by the summation technique and a convergence of values between SP and PS doesn't seem to have taken place.

Once again, the values obtained by our computations are invariant to the summation technique used, and therefore, for the remainder of this report, we will make use of the PS summation as this has been used as a basis in others' work too [20, 70, 96].

6.4.3 Comparison of various yarn input models

The CCA model is a mathematical model for converting fiber and matrix properties into equivalent yarn properties. The Shear-Inclusive Circular (SIC) and Shear-Inclusive Elliptical (SIE) models discussed in the previous two chapters provide a means to bring in shear properties into the conversion to equivalent yarn properties and to account for non-circular yarns respectively. In this case, the SIE model is not used because the porosity limitations for it are violated (Table 5.24). Due the 3D nature of the woven composite, the associated yarn porosity assumes a value more than 0.6, thus causing it to violate the mathematical limits set for the use of the SIE model (Equations 5.18).

A comparison of our in-house woven model with CCA and SIC yarn inputs with experiments [70] for Young's modulus as a function of Yarn undulation are shown in Figure 6.16.

Figure 6.16: Comparison of woven modeling Young's modulus results with CCA and SIC inputs against experiments

Firstly, it is seen that the CCA model coincides almost identically to the experimental results, with the maximum deviation of -1.78% between the results occurring at an undulation of 0.1 mm. This is important since it indicates that the MATLAB® code that has been developed (from scratch) in this project is working properly and the various material-level, yarn-level and vertical block-level considerations and micromechanical features that were described in Naik and Shembekar's work has been successfully recreated in this MATLAB® code.

It is also now possible to see the additional effects that the SIC model brings in. The SIC model affects the transverse properties of the yarn, but has no effect on the longitudinal properties of the matrix-infused yarn. The equivalent yarn formulations for transverse Young's modulus and transverse Poisson's ratio in the CCA model are replaced by their corresponding shear-inclusive values. As seen in Figure 6.16, the SIC model follows closely to the Naik & Shembekar and CCA results for low values of undulation, but begins to deviate at higher values. This is because of the strengthening effect taking place on the highly undulated yarn due to a higher transverse Young's modulus (initially 9 GPa for CCA to 14.4 GPa for SIC). As a result, the value is higher.

This effect, however, is not seen to be dominant when the composite shear modulus is compared as shown in Figure 6.17.

Figure 6.17: Comparison of woven modeling shear modulus results with CCA and SIC against experiments

The shear modulus values computed using the SIC and CCA models are higher (1.14% to 1.51%) than the experimental values. However, due to the overall planar nature of composite, the stiffer modulus in the transverse directions of the yarns may or may not have any significant bearing on shear modulus [97]. The values of longitudinal and transverse Young's modulus of yarns are expected to affect the shear modulus of the planar arrangement incorporating these yarns; however, since the longitudinal modulus (E_L) of the yarn is close to 188 GPa, the difference between having a transverse modulus (E_T) of 9 GPa or 14.4 GPa in the overall configuration is insignificant and does not affect the overall property averaging. Hence, the CCA and SIC values seem close.

Having compared the results of the CCA model for carbon/epoxy composite for changing undulation, it is now desired to place different yarn model inputs (such as SIC and Halpin-Tsai or H-Tsai discussed earlier) into the woven yarn model and observe their corresponding outputs. Table 6.10 compares the experimental ranges with the values of elastic modulus obtained by inputting various yarn models into the code developed for warp and fill directions of the composite.

Table 6.10: Comparing experiment & computational Young's modulus results with various yarn input models

Material	Dimensions	Experimental range [70] (GPa)	PS Woven model (GPa)		
			SIC	CCA	H-Tsai New
T-300 Carbon/ Epoxy	Warp direction – a = 0.96 mm; g = 0.151 mm	56-61	<i>50.625</i>	<i>49.083</i>	<i>49.368</i>
	Fill direction – a = 1.10 mm; g = 0.011 mm	47-50	<i>45.722</i>	<i>43.841</i>	<i>44.226</i>
E-glass/ Epoxy a	Warp direction – a = 0.52 mm; g = 0.25 mm	15-22	<i>19.923</i>	<i>19.046</i>	<i>19.317</i>
E-glass/ Epoxy b	Warp direction – a = 1.12 mm; g = 0.6 mm	14-22	<i>18.849</i>	<i>18.123</i>	<i>18.363</i>
	Fill direction – a = 1.44 mm; g = 0.12 mm	12-16	<i>16.481</i>	<i>15.241</i>	<i>15.717</i>
E-glass/ Epoxy c	Warp & fill direction – a = 0.5 mm; g = 0.23 mm	10-16	<i>15.519</i>	<i>13.823</i>	<i>14.080</i>

It is seen that in all the cases, the SIC model provides higher Young's modulus of the composite as compared to the CCA and the new Halpin-Tsai models. This is primarily due to the incorporation of shear effects in the values of the transverse Young's modulus and the transverse Poisson ratio of the yarn as was discussed in Chapters 2 and 3 earlier. Consequently, the stiffer transverse yarn properties lead to a stronger composite property as is shown in Table 6.10 (when compared to the CCA and the H-Tsai new models).

Additionally, it is seen that the SIC model values lie within the experimental range from [20] as shown above in the case of E-glass/epoxy samples a, b and c. There is a good agreement between the computational values and the experimental values. In the case of T-300 Carbon/Epoxy, the computed values are lower than that found in the experimental range; with the SIC values coming closest to the experimental range as compared to the CCA or the H-Tsai formulation. As was discussed earlier in Section 3.4, the disconnect in the values may be due to the larger disparity in the moduli between the carbon and epoxy than the glass and epoxy in the composites.

6.5 Concluding Remarks

This chapter has examined the methodology of creating one-directional 2D and two-directional 3D woven textile composite models for predicting elastic properties and created an equivalent model in MATLAB[®] programming language. The chapter also focused on comparing the in-house woven model results with experiments and literature, as well as contrasting classical yarn input models such CCA and H-Tsai with the SIC and SIE models developed in previous chapters (evaluating their effects in predicting woven textile composite properties). Based on the findings presented in this chapter, the following conclusions can be drawn:

- i. The woven textile geometry is studied and its important geometric features are observed. Based on these features, the 2D and 3D unit cell geometries and surfaces are created in MATLAB[®]. By applying CLT, the program is then able to determine the elastic mechanical properties of the corresponding composite unit cells.
- ii. Convergence tests carried out on the 2D woven model provide a precision of up to four decimal places.
- iii. The 2D parallel woven model values are closer to experimental results as compared to the literature computational modeling results for the CCA yarn input models.

- iv. It is observed that for epoxy glass samples, the SIC yarn input model is stiffer than the CCA yarn input model; this is expected due the shear-strengthening effect of the SIC model. However, this effect is less obvious in the case of the carbon-epoxy composite. This might be due to the existence of high difference between constituent material properties in carbon-epoxy samples.
- v. Amongst the yarn input models, the SIE model predicts higher elastic modulus than the SIC model. The flattened elliptical yarns (providing a heavier length contribution to the stiffness properties) are considered the strengthening agent in this case.
- vi. In the MATLAB[®] code FOR 3D, various summation techniques were explored to aggregate the vertical blocks and obtain consolidated woven unit cell properties. The AAS and AAC addition techniques provided upper-bound and lower-bound estimates of the moduli along the principal directions.
- vii. The PS and SP summation techniques results are close to each other for the in-house woven model (usually within 1 GPa) and therefore, exhibit better numerical convergence than Naik & Shembekar's work.
- viii. When compared with experiments, the choice of yarn input model (for the same in-house woven textile program) seems to have an effect. The CCA model exhibits the closest correlation to the experiments as undulation changes; the SIC model, however, due to the transverse modulus strengthening effect, shows higher stiffness values than experiments.
- ix. Shear modulus is not seen to be affected by changes in the yarn input model. This might be due to the "averaging" effect that takes place when constituent materials longitudinal, transverse and shear properties are together seen to influence the overall woven composite model.
- x. The methodology for determining the elastic properties of woven textile has been implemented into MATLAB[®]. This type of vertical block creation and summation

technique can be applied braided, knitted or more complicated textile structures in future.

Having created and validated a comprehensive woven textile composite unit cell model, this work can now be used to model braided textile composites as will be shown in the next chapter.

Chapter 7: Braided Fabric Unit Cell

Having created the entire three-dimensional woven structure and instituted a programming code to calculate the elastic mechanical properties of a woven textile composite in the previous chapter, it is not necessary to build from scratch, the geometry and programming code for braided textile composite geometry. The underlying principles of geometry creation, vertical block property assignment and summation techniques would remain the same. It was with this rationale that a conversion method was applied. Most importantly, the contributions by constituents including the yarns can still be accounted for accurately using the transformation of the material properties for the oblique braid angle.

This method would convert a 1-by-1 diamond braided textile unit cell into an equivalent 1-by-1 plain woven textile unit cell (as shown in Figure 7.1) and carry out subsequent computations on this equivalent woven unit cell to determine the elastic properties of the original braided unit cell.

Figure 7.1: Braided unit cell (left) and its equivalent woven unit cell (right)

Therefore, the braided textile constituent geometry and material properties would be input into the program and the MATLAB[®] code would convert these into equivalent woven unit cell constituent geometry and material properties. The vertical block properties are then aggregated (by PS or SP summation techniques) to obtain the consolidated woven composite properties and finally, these woven composite properties are transformed to obtain the braided textile properties along the principal braiding directions (i.e. MD and CD) introduced earlier in Chapter 2.

Since the creation of the vertical blocks and the associated aggregation PS technique is discussed in Chapter 6, it will not be re-explained here. Instead, this chapter will focus on the two additional steps required to obtain the braided textile properties, namely, (1) the

conversion of braided textile geometry into the equivalent woven textile geometry parameters and, (2) the transformation of material properties to account for the oblique angled yarns.

7.1 Conversion of geometry from braided to equivalent woven

The conversion of the complicated braided geometry into the simpler equivalent geometry may seem to be an arduous task. However, it follows the basic principle of conservation of volume. This means that any section or geometry of the braided unit cell must, on conversion to the equivalent woven unit cell, occupy the same volume as before. It is crucial to ensure this, since volume fractions of yarns, fibers, etc. play an important part in determining properties in composites. Therefore, changing the geometry to a more agreeable shape must not have an impact on the volume of that shape.

Consider a braided textile unit cell shown in three views in Figure 7.2.

Figure 7.2: Geometric parameters of the braided textile

Even though fill and weft yarns do not usually apply to braided textiles (usually referred to simply as bias yarns), they are used in this section as denotations to show the similarity between 1×1 plain weaves and 1×1 diamond braids. The fill yarn (in orange shade) orients in the x-direction and its orientation is shown by the “X-dir View”. Since its cross-section

is observed perpendicular to its lengthwise direction, the “Y-dir view” shows the normally projected yarn width (w_{fill}) as well as the fill yarn gap (g_{fill}) that lies between two fill yarns. As seen in the “Y-dir view”, the location of fill yarns dictates how the green weft yarn will orientate.

On the other hand, the green weft yarn is oriented at an angle of 2θ from the x-direction, where θ is the braiding angle associated with the braided textile. The weft yarn width and gap dimensions are w_{weft} and g_{weft} respectively. However, since the “X-dir View” only shows the oblique projected view of the weft yarns, the dimensions are extended by a factor of $\frac{1}{\sin 2\theta}$ to accommodate for the skewed weft yarn spacings. If the value of θ is taken as 45° , the braided textile forms a woven textile with the same fill and weft yarn dimensions as in the previous chapter.

As mentioned before, the process of converting the braided textile to its equivalent woven representation must ensure that overall volumes within the unit cell remain unchanged. Therefore, using the same dimensions as shown in Figure 7.2, the alternative geometry as shown in Figure 7.3 is proposed. It is seen that this new equivalent woven unit cell follows the same “X-dir View” and “Y-dir View” as shown in the braided textile.

Figure 7.3: Geometric parameters of the converted equivalent woven textile

It is seen that while the fill yarn dimensions from the braided textile remain unchanged in the equivalent woven model, the weft yarn dimensions are “scaled up” by a factor of $\frac{1}{\sin 2\theta}$ to obtain the equivalent weft yarn dimensions of the new converted woven textile.

In simple terms, it is the conversion of a parallelogram (shown in Figure 7.4) into a rectangle having the same area and side dimensions.

Figure 7.4: Area of braided textile

Figure 7.1 (Page 23) has the same braided and converted equivalent woven unit cells side-by-side for comparison. This conversion is quantified by the equations.

$$\text{Area of braided textile} = (w_{fill} + g_{fill})(w_{weft} + g_{weft}) \frac{1}{\sin 2\theta}$$

$$= (w_{fill} + g_{fill}) \left(\frac{w_{weft}}{\sin 2\theta} + \frac{g_{weft}}{\sin 2\theta} \right)$$

$$\text{Area of woven textile with} = \left(\frac{w_{fill}}{2} + g_{fill} + \frac{w_{fill}}{2} \right) \left(\frac{w_{weft}}{2 \sin 2\theta} + \frac{g_{weft}}{\sin 2\theta} + \frac{w_{weft}}{2 \sin 2\theta} \right)$$

where

$$\text{height} = \left(\frac{w_{fill}}{2} + g_{fill} + \frac{w_{fill}}{2} \right) \quad \text{and,} \quad \text{base} = \left(\frac{w_{weft}}{2 \sin 2\theta} + \frac{g_{weft}}{\sin 2\theta} + \frac{w_{weft}}{2 \sin 2\theta} \right) \quad \dots(7.1)$$

To ensure that the yarn curvature is translated effectively from the braided to the equivalent woven textile, it is important to discretize the geometry into many vertical parallelogram prismatic blocks, such that each of these blocks are directly converted by area equality into appropriate vertical cuboidal blocks as shown in Section 6.2. For the braided case, using the 120 sections (for numerical convergence) in each of the i- and j-segments were sufficient enough to ensure that the resulting 14400 vertical blocks also transferred geometrically accurate from the braided textile to the equivalent woven textile.

Therefore, considering an example using the yarn dimension input values from Section 6.4.1, a braided textile with a braiding angle of 25° can be easily converted into an equivalent woven textile. Using Equation 7.1, the converted equivalent woven textile takes up dimensions shown in Table 7.1.

Table 7.1: Conversion of sample braid dimensions to weave dimensions using Equation 7.1

Original Braided Textile Parameters		Equivalent Woven Textile Parameters
Fill Bias Yarn (orientated at 0° to X axis) $w_{fill} = 1 \text{ mm}$, $a_{fill} = 1 \text{ mm}$, $u_{fill} = 0.6 \text{ mm}$, $h_{fill} = 0.5 \text{ mm}$	Height $\Rightarrow$	Fill Yarn (orientated at 0° to X axis) $w_{fill} = 1.000 \text{ mm}$, $a_{fill} = 1.000 \text{ mm}$, $u_{fill} = 0.600 \text{ mm}$, $h_{fill} = 0.500 \text{ mm}$
Weft Bias Yarn (orientated at 50° to X axis) $w_{weft} = 1 \text{ mm}$, $a_{weft} = 1 \text{ mm}$, $u_{weft} = 0.6 \text{ mm}$, $h_{weft} = 0.5 \text{ mm}$	Base $\Rightarrow$	Weft Yarn (orientated at 90° to X axis) $w_{weft} = 1.305 \text{ mm}$, $a_{weft} = 1.305 \text{ mm}$, $u_{weft} = 0.783 \text{ mm}$, $h_{weft} = 0.500 \text{ mm}$

The height is unaffected in this conversion. The conversion however enables the woven textile modeler code from the previous chapter to now take over and recreate a new equivalent woven textile composite geometry as shown in Figure 7.5.

Figure 7.5: Equivalent converted woven textile based on Table 7.1

It must be stated that the braided MATLAB[®] code does not create a surface or textile model of the original braided textile, but instead utilizes the given braids' yarn dimensions and spacing to obtain converted parameters to create the equivalent woven surface model. It can

therefore be seen from the axes dimensions of Figure 7.5 that even though the original braided textile has identical dimensions, its equivalent woven textile's yarn dimensions are converted to non-identical ones by scaling up by a factor of $\frac{1}{\sin 2\theta}$ (to ensure equality of areas and volumes in the model). The heights and dimensions of this converted woven textile are used to predict the original braided textile composite's elastic mechanical properties.

7.2 Orientation of material properties from braided to equivalent woven

Having converted the geometry from braided to the equivalent woven textile model, it is now important to look at material orientation within the yarns and ensure that these are transferred appropriately onto the equivalent woven model.

Shown in Figure 7.6 are the original braided textile with the material orientations and other braiding orientations.

Figure 7.6: Material & yarn orientations and principal braiding directions

The fill and weft yarn properties are denoted by 'f' and 'w' respectively. Based on the conventional braiding axis shown in Figure 7.6, the fill yarn direction and weft yarn direction are aligned at an angle of 2θ to each other. The machine direction is thus an angle bisector between the fill and weft yarn directions (aligned at angle θ from both the fill yarn and weft yarn).

In the braided textile unit cell above, the yarn directions are denoted by '1f' and '2f' (for the fills yarns) and '1w' and '2w' (for the weft yarns). This is to signify the principal local directions of the yarns. The '1' direction denotes the primary longitudinal direction of the yarn and is illustrated by the longer arrow, while the '2' direction denotes the secondary

transverse direction of the yarn and therefore, illustrated by the shorter arrow. As expected, the longitudinal or ‘1’ directions of both the fill and weft yarn axes are orientated with the corresponding yarn directions, while the ‘2’ direction arrow lies perpendicular to it. The local yarn properties will therefore be calculated according to this orientation.

However, when the yarn orientations and properties are transferred to the equivalent woven unit cell, the original local yarn orientations are maintained. This is illustrated in Figure 7.7.

Figure 7.7: Transferring yarn orientations to the equivalent woven textile

Even though the geometry changes from braided to woven model, the material orientation does not. As a result, the properties that will be input into the vertical block will consider not only x- and y- direction properties for the fill yarn, but the off-axis properties for the weft yarn as well. The local yarn properties are encapsulated as an orthotropic entity denoted by the compliance matrix $[C]$ as shown in Equation 7.2.

$$[C] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \quad \dots(7.2)$$

It is seen that this matrix takes in yarn the principal directional values of Young's moduli, Poisson ratio and shear modulus. This matrix is symmetric and thus possesses six independent values (with two coefficient values - C_{13} and C_{23} equal to zero).

Since the fill yarn is already orientated to the global X-Y axis, its yarn properties can be directly taken and input into Equation 6.7 in the previous chapter. However, the weft yarn is

orientated off-axis at an angle of 2θ from the global X-Y axis. Hence, its properties must undergo a transformation through an angle -2θ to be orientated to the global X-Y axis and then be input into Equation 6.7. It must be noted that the resulting transformed matrix will still be symmetric; however, the two coefficient values C_{13} and C_{23} may not be zero anymore (as shown in Equation 7.3).

$$[C]_{rotated} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & \dots \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & \dots \\ \dots & \dots & \frac{1}{G_{xy}} \end{bmatrix} \quad \dots(7.3)$$

Since the process of transforming elastic compliance matrices through various angles is required at various points in this chapter and Appendix D covers comprehensively the method on how to transform elastic compliance matrices.

The constituent material properties (with regards to orientation) are input into the model in the same manner as material properties and orientations are input into FEA software like ANSYS or Abaqus [61, 69, 99, 100]. The only difference is that since the woven and braided models are developed using MATLAB[®], a general purpose programming software language, the author of this study has to manually program in the commands to assess yarn directions at various points and transform the material properties from principal to off-axis orientations (whereas this is done automatically by the FEA software).

7.3 Summation of vertical blocks and final composite properties

Having input the constituent material properties into the vertical blocks for the equivalent woven model, the summation techniques as were discussed in the previous chapter are applied. Both the PS and SP summation is shown later (to establish numerical convergence). The summation technique thus results in a combined compliance matrix for the entire equivalent woven unit cell in the global coordinate system X-Y. The various terms within this compliance matrix are comparable to the terms shown in Equation 7.4.

$$[C]_{equivalent\ woven} \equiv \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & \dots \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & \dots \\ \dots & \dots & \frac{1}{G_{xy}} \end{bmatrix} \quad \dots(7.4)$$

This matrix will also be a symmetric matrix with six independent parameters. However, note that the elastic mechanical values obtained from this matrix, i.e. E_x , E_y , ν_{xy} and G_{xy} are all orientated with respect to the global X-Y axes (as shown in Figure 7.7).

However, for braided textile composites, the relevant coordinate axes are the machine and cross directions (MD and CD). Therefore, the elastic compliance matrix obtained above must again be transformed through an angle $+\theta$. This will orientate the x-coordinate and y-coordinate of the equivalent woven textile with the machine direction and cross directions respectively.

7.4 Variation of properties with changes in braid angle

Having adapted the woven textile composite programming code to predict the properties of braided textile composites, it was important to ensure that the predictions of this new code are in line with the typical properties of braided textile composites. This was carried out by inputting in typical fiber and matrix elastic mechanical properties into the code and obtaining the final braided composite properties as a function of the braiding angle. The change in braiding angle alters the orientation of the yarns with respect to MD and CD as shown in Figure 7.8.

Figure 7.8: Changes in braiding angle

Holding the yarn widths and spacing constant, the braiding angle adds an additional design parameter to braided textile composites (missing from woven textile composites). In Figure 7.8, as the braiding angle is increased from θ_1 to θ_2 , the yarns change their orientation from being in line with MD to becoming more in line with the CD. This has an effect on the mechanical properties of the entire braided unit cell as will be discussed later.

The constituent material properties input are taken from [70], consisting of T-300 carbon fibers in epoxy matrix at fiber volume fraction of 0.47. The range of braiding angles is taken 10° to 80° at intervals of 5° . Each value of braiding angle has its own unit cell model automatically created in the MATLAB[®] code to have the input values processed and the corresponding braided textile composite properties in the machine and cross directions determined. The variation of elastic modulus, Poisson's ratio and shear modulus as a function of braiding angle is shown in Figures 7.9, 7.10 and 7.11 respectively. The summation techniques demonstrated are the PS and SP techniques.

Figure 7.9: Elastic modulus as a function of braiding angle

Figure 7.9 shows the variation of the elastic modulus as a function of braiding angle. Note that, even though the properties are obtained by the two different PS and SP summation techniques, there is a complete overlap of PS and SP values for the same property. This means that the MATLAB[®] model developed for braided textiles is fully converged and is invariant in terms of the summation technique used.

It is seen that as the braiding angle increases, the elastic modulus in the machine direction decreases. This occurs primarily due to the yarns aligning themselves away from the machine direction and towards the cross direction. Consequently, the cross direction Young's modulus is seen to increase as the braiding angle increases [99]. Also, since the yarn dimensions and spacing are kept constant, there exists symmetry in the machine and cross direction properties along the braiding angle of 45° (where the yarns are aligned equally away from both the machine and cross directions). This 'symmetry' effect is seen in the Poisson's ratio (Figure 7.10) and shear modulus (figures 7.11) properties as well [56, 73, 101, 102].

The Poisson's ratio of the composite is shown in Figure 7.10. Even though the values of Poisson's ratio higher than 1 are quite unusual for linear elastic materials, it must be remembered that braided textile composites exhibit non-linear elastic behavior. Hence, for

constituent yarns, there is a large difference in the longitudinal and transverse properties (with longitudinal elastic properties being stiffer than transverse elastic properties). Therefore, their changing alignments as braiding angle changes can result in unusual behavior.

Figure 7.10: Poisson's ratio as a function of braiding angle

In the case of Figure 7.10, the value of Poisson's ratio ν_{MC} being greater than 1 (in the range of braiding angle of 25° to 30°) indicates that the cross-directional Poisson deformational strain is more than the machine-directional direct deformational strain due to applied loads. This effect is mirrored for ν_{CM} (in the range of braiding angle of 60° to 65°), where the machine directional Poisson deformational strain is more than the cross directional direct deformational strain due to the applied loads. As was seen earlier, the overlapping of PS and SP values for both ν_{MC} and ν_{CM} indicates convergence in summation techniques.

Finally, the shear modulus (Figure 7.11) is looked at. In textile composites, shear modulus is largely a planar property. Since it provides a measure of the resistance to rotation of the planar composite, its value is affected by the orientations of both yarns and how these yarns influence resistance to rotation.

Figure 7.11: Shear modulus as a function of braiding angle

It is thus seen in Figure 7.11 that at low and high values of braiding angles, the shear modulus is the lowest. This indicates that the similar orientation of both yarns reduces their ability to resist rotational deformation in the plane of the textile composite. Consequently, it is seen that the braiding angles of 45° provide the best shear modulus, since the yarns being at orthogonal angles to each other strengthen the planar resistance of the composite to rotation. This phenomenon has been observed before as well [56, 101, 102]. Finally, the overlap of the values of PS and SP calculations show that both summation techniques match up well; and therefore, for the validation of results as shown in the next section, only one, the PS summation will be used.

It should also be noted that the trends for elastic property changes (as a function of braiding angle) observed in this section are consistent with many other literature models [56, 73, 101] and further reinforce the applicability of determining elastic properties of a braided textile composite unit cell using a converted equivalent woven textile composite.

7.5 Validation of braided textile composite modeling

Having explored and explained the various trends observed in braided textile composites, it is now desired to assess the accuracy of the braided textile composite model that has been developed in this study. While FEA studies of braided textile composites usually make direct use of braided textile geometries to obtain the corresponding elastic mechanical properties, our novel approach of converting braided textile composites geometries into equivalent woven textile geometries and translating off-axis yarn properties directly into the equivalent woven model must be studied in comparison to experiments, FEA and other computational approaches to assess their suitability as a means to predict braided textile composite properties.

Since this braided textile geometry studied is derived directly from a plain weave woven textile geometry, the resulting braid is of diamond configuration, 1-by-1 [78, 98]. The model validation therefore looks at experimental and computational work that caters specifically to the mechanical properties of diamond braids.

7.6.1 Comparison with FEA work of Xu *et al.* [73]

Xu *et al.* [73] has carried out extensive finite element analysis for fiber-to-yarn properties as well as elastic property determination and damage analysis of braided textiles (both regular and diamond braided). Tensile tests were also carried out on diamond braided (1-by-1) composites for glass fiber-epoxy composites. The constituent material properties are given in Table 7.2.

Table 7.2: Glass fiber and epoxy matrix properties [73]

Glass Fiber	Epoxy Matrix
$E_L = 74 \text{ GPa}$, $E_T = 74 \text{ GPa}$, $\nu_{LT} = 0.2$, $\nu_{TT} = 0.2$, $G_{LT} = 30.8 \text{ GPa}$, $G_{TT} = 30.8 \text{ GPa}$	$E_m = 3.31 \text{ GPa}$, $\nu_m = 0.35$

A machine direction Young's modulus comparison of the experimental results, Xu's FEA predictions and the formulation modeled (with CCA and SIC inputs) is carried out as shown in Figure 7.12.

Figure 7.12: Comparison of Young's modulus of our models with Xu et al. [73]

It is thus seen that the braided composite model with CCA inputs is lower than both the experimental points as well as Xu's FEA results. In the case of the SIC input model, it is seen to follow exactly the CCA input model for low values of braiding angles (where longitudinal yarn properties are dominant) and exhibit stronger values for higher braiding angles (where the SIC input's stiffer transverse yarn modulus comes into play). In these cases, Xu's FEA results seem to display the best match to experiments.

However, when the shear modulus properties are looked at (Figure 7.13), a different picture emerges. Xu's work overestimates the experimental results, whereas the CCA model provides a good fit. The SIC input model strengthens transverse yarn properties; hence, its effect comes into play in the range of braiding angles of 45° (where the overall stiffness characteristics of the planar composite is desired), leading to a strengthened shear modulus.

Figure 7.13: Comparison of shear modulus of our models with Xu et al. [73]

To quantify the graphical results shown above, a comparison of the “wellness-of-fit” can be seen by comparing the Correlation Coefficient (R^2) as is shown in Table 7.3.

Table 7.3: Correlation coefficient comparison of Xu’s FEA work against the braided model (with CCA and SIC inputs)

	Young’s Modulus	Shear Modulus
Xu et al. FEA predictions	0.8993	0.6958
Braided Model – CCA Inputs	0.5022	0.9390
Braided Model – SIC Inputs	0.5356	0.2167

Thus, it is observed that, in comparison to Xu’s FEA predictions, the braided model developed in this study seems to have mixed results.

However, FEA requires a lot of yarn geometry creation, finite element modeling preprocessing and simulation time. On the other hand, the braided model developed in this study takes a fraction of the time and effort. In fact, the total run time of the MATLAB[®] braided textile model code to generate the complete set of points (at different braid angles) was about 42 seconds. In terms of modeling input, it is very straightforward to add in new parameters and braiding angles and generate uniquely new data points as those shown in Figure 7.13. Hence, it can be argued that the braided MATLAB[®] code is able to provide reasonably good estimates of braided textile composites properties in a cost-effective manner.

7.6.2 Comparison with Chretien's results [80]

Computational work was carried out by Chretien using micromechanical classical formulations in FORTRAN to determine the properties of carbon-epoxy composites. The properties are given in Table 7.4 below and the comparison of Chretien's results with the braided models with SIC and CCA inputs are given in Figure 7.14.

Table 7.4: Carbon fiber and epoxy matrix properties [79]

Carbon Fiber	Epoxy Matrix
$E_L = 192 \text{ GPa}$, $E_T = 58.65 \text{ GPa}$, $\nu_{LT} = 0.19$, $\nu_{TT} = 0.286$, $G_{LT} = 53 \text{ GPa}$, $G_{TT} = 22 \text{ GPa}$	$E_m = 3.45 \text{ GPa}$, $\nu_m = 0.35$

Figure 7.14: Comparison of Chretien's work with our braided models

From Figure 7.14, it can be seen that Chretien's results seem to lie within the values of the two braided models. There is also an obvious strengthening effect of the SIC model as compared to the CCA model. This seems to be due to the stronger transverse Young's modulus of the SIC yarn input as compared to the CCA yarn input for transverse Young's modulus. In this case, the high value of fiber volume fraction as well as low porosity leads an enhanced reinforcement-interface region effect in the SIC model leading to an increased transverse modulus (Chapter 2).

Table 7.5: Correlation coefficients of braided models with different yarn inputs

	For Young's Modulus
Braided Model – CCA Inputs	0.8667
Braided Model – SIC Inputs	0.9721

As is seen from Table 7.5, the in-house braided model with both SIC and CCA yarn input models show high correlation to Chretien's computational results ($R^2 > 0.85$). This shows that the braided textile composite MATLAB[®] model that has been developed (as a modification of the woven textile composite MATLAB[®] model) is a reasonably accurate tool for obtaining the elastic properties of braided textile composites.

7.6.3 Comparison with Air Traffic Organization (ATO) report

Under the sponsorship of the US Department of Transportation, information sharing was carried out by Kelkar and Whitcomb on the structural properties of braided structures for the the Air Traffic Organization [77]. Carbon-epoxy composite samples were manufactured at braid angles of 25, 30 and 45 and subject to static tensile loading along machine direction to obtain the experimental Young's modulus. The mechanical properties of the carbon fiber and epoxy matrix are given in Table 7.6 and the results are shown in Figure 7.15.

Table 7.6: Carbon fiber and epoxy matrix properties [77]

Carbon Fiber*	Epoxy Matrix
$E_L = 228 \text{ GPa}$, $\nu_{LT} = 0.2$, $G_{LT} = 24.82 \text{ GPa}$, $E_T = 16.54 \text{ GPa}$, $G_{TT} = 6.89 \text{ GPa}$, $\nu_{TT} = 0.25$	$E_m = 3.1 \text{ GPa}$, $\nu_m = 0.35$

* Yarn property estimates are provided in [77]; hence, fiber properties are back-calculated using yarn-packing density of 0.75 and overall composite volume fraction of 0.64 [77]

Figure 7.15: Comparison of braided models with ATO data [77]

Thus, it is observed that the braided models slightly underestimate the experimental values. As is expected, the braided model with shear-inclusive circular yarn (SIC) inputs exhibit higher Young's modulus values at high braiding angles, than the braided model with CCA inputs, due to their improved transverse modulus values. A quantitative comparison of the results in terms of the correlation coefficients (R^2) is shown in Table 7.7.

Table 7.7: Correlation coefficients of braided models with different yarn inputs

	Young's Modulus
Braided Model – CCA Inputs	0.8529
Braided Model – SIC Inputs	0.8933

Therefore, both the braided models (with CCA inputs and SIC inputs) are able to correlate well with the experimental results of Kelkar and Whitcomb [77], with $R^2 > 0.85$. The SIC input model ($R^2 = 0.8933$), by its natural tendency to strengthen yarns, is able to match closer to the experimental results.

Thus, from the validations carried out in this chapter as well as those in the previous chapter, it is demonstrated that not only are the developed MATLAB[®] woven and braided textile models able to predict the corresponding composite properties, but also the theories and program routines used in implementation of Classical Lamination Theory (CLT) can be

applied to other textile composite structures. This study provides a general methodological roadmap for creating computational mechanical prediction models.

7.7 Concluding Remarks

This chapter has expanded the methodology of 3D woven textile composite models for predicting elastic properties from the previous chapter to accommodate for braided textile composites and created an equivalent model in MATLAB[®] programming language. The chapter also focuses on comparing the in-house braided (SIC) model results with experiments and literature results, as well as the CCA classical yarn input model evaluating their effects in predicting braided textile composite properties. Based on the findings presented in this chapter, the following conclusions can be drawn:

- i. The inputs to the two-directional woven textile geometry modeler (previously developed) are adapted to accommodate for braided textiles. This results in the modeler being able to take in braided textile input parameters and generating a converted equivalent woven textile structure with unit cell geometries and surfaces created in MATLAB[®].
- ii. The planar elastic mechanical properties of the original braided composite are determined by applying transformed braided yarn orientations and CLT on the converted equivalent woven textile model.
- iii. Comparisons of the PS and SP summation techniques on the braided textile models confirmed numerical convergence of the model based on 14400 vertical blocks. The elastic property trends generally observed for braided textile composites in literature are also observed in our models in the principal braiding machine and cross directions. These trends are also deemed to be numerically converged due to the overlapping of PS and SP summations.
- iv. As the braiding angle increases, the elastic modulus in the machine direction falls, while the elastic modulus in the cross direction rises. This is due to the alignment

of yarns away from the machine direction towards the cross direction as braid angle rises.

- v. The shear modulus is seen to initially rise and then fall as the braid angle increases, while exhibiting symmetry along the 45° braid angle; this occurs due to the maximum rotational resistance within the composite plane provided when the yarns are aligned orthogonal to each other.
- vi. The consolidated Poisson ratio of the braided textile composite seems to amplify the orthotropic properties of the yarn with an initial increasing and then decreasing trend in value, due to the weakness of yarns in their transverse directions. As a result, for certain braid angles, the Poisson deformational strain is actually greater than the original tensile deformational strain and can reach values greater than 1.
- vii. As was the case in for woven textile composites, the use of different yarn input models for braided textile composites seems to have an impact. The SIC model is seen to have a strengthening effect over the classical CCA yarn input models; in fact, this effect is more pronounced at higher braid angles, when the transverse elastic properties of yarns come into play.
- viii. The braided textile composite MATLAB® models developed in this chapter (with the CCA and SIC) provide good correlation with experimental data for determining braided textile composite properties. The models are also shown to outperform FEA or classical methods.
- ix. Especially the proposed SIC model seems to provide the most optimum performance in terms of accuracy, computation time and efforts.
- x. The ability of the program to successfully process the input textile geometry and material properties to predict the final composite elastic properties provides a strong validation for the implemented code for CLT. This gives a strong methodological framework to implement CLT to obtain mechanical properties of other textile structures as well.

Chapter 8: Conclusions and Recommendations for future work

This thesis has thus focused on fiber textile structures, looking at the composite properties not only at the individual fiber or yarn levels, but also at the level of unidirectional lay-up, woven fabrics and braided textiles. The use of a single programming tool to determine properties for all kinds of fiber textile structures enables a common ground for comparison and analysis of the geometrical and mechanical properties.

Based on the literature review, an improvement into predicting the transverse properties of unidirectional fiber composites was identified as one of the key areas. The SIC model established in this research is thus able to consider fiber arrangement patterns and yarn porosity and account for shear contributions at the reinforcement-matrix interface (all of which wasn't satisfactorily addressed in any previous works).

Scaling factors were then derived using FEA data-fitting and effectively applied to the SIC models to produce the elliptical yarn inclusive SIE models. The SIE models strengthen yarn properties in the direction of yarn cross-section elongation (due to the stiffer material lying along the direction of application of load). Their performance alongside the SIC models was assessed in the in-house created woven and braided textile composite models.

Both the woven and braided MATLAB[®] models are able to show the expected elastic property trends as is observed due to changes in undulation, gaps and braiding angle (in the case of the braided model). The textile models also exhibit favourable comparisons with experimental data and other computational results. When compared with experiments, the choice of yarn input model (for the same in-house textile program – woven or braided) seems to have an effect. The SIC and SIE models are seen to have a strengthening effect over the classical CCA yarn input models.

The methodology for determining the elastic properties of both woven and braided textile has been implemented into MATLAB[®]. The vertical block creation and summation technique is a materials property data-management and manipulation system that can be used for other textiles. The ability of the program to successfully process the input textile geometry and material properties to predict the final composite elastic properties provides a strong

validation for the implemented code for CLT. This gives a strong methodological framework to implement CLT to obtain mechanical properties of other textile structures as well.

In summary, the following contributions were made through this research:

- i. Development of the SIC model for more accurate prediction of transverse Young's modulus for composite laminates.
- ii. Derivation of a transverse Poisson ratio formula consistent with the SIC model to enable comprehensive formulation for the transverse properties of fibers.
- iii. Extension of the SIC model to the SIE model transverse Young's modulus and Poisson ratio for a more realistic representation of the fiber yarn shape.
- iv. Validation of all SIC and SIE models using experimental, computational and FEA data.
- v. Implementation of the SIC and SIE models to woven textile composites.
- vi. Extension of the SIC and SIE models to braided textile composites using geometry and material orientations transformations.
- vii. Establishment and analysis of geometrical and material property trends for textiles along with their associated reasons.
- viii. Validation of the woven and braided textile properties using experimental and computational data.
- ix. Provision of a strong methodological framework to predict elastic properties of other textile structural composites

Recommendations for future work

Over the course of this study, many avenues for improvement and progress in the field of structural textile composites have been identified. Some crucial and interesting topics that warrant further exploration are listed below:

- i. Due to limited time, the formula derivation to include effects of rectangularisation of the square unit cell on the Poisson ratio formulation (as was done in the case of the transverse modulus) wasn't carried out. Even though a large change isn't expected, it would be prudent to have some measure of comparison of the variation. This could be explored further.
- ii. Further sensitivity studies of the ratio of fiber-Poisson-ratio to matrix-Poisson-ratio ($\frac{\nu_f}{\nu_m}$) on the final mathematical SIE formulations can also be investigated.
- iii. More FEA studies can be incorporated into the data modeling for the SIE formulations to improve the model accuracy.
- iv. Due to time constraints, it was only possible to create MATLAB[®] models of 1-by-1 plain woven and 1-by-1 diamond braided textile composites. Using the same vertical block building and summation techniques, formulations and codes for other textiles such as satin weaves (4-by-4 or more), braids (2-by-2 or higher) or knitted textiles could possibly be developed.
- v. Although it will be an involved task, suitable experiments may be designed and conducted in-house to validate the accuracy as well as limitations of the various models formulated and presented in this thesis.

Appendix A- Evaluation of Integrals

$$\begin{aligned}
 I &= \int_0^R (R \cos \theta)(1 + \varepsilon_f) dy \\
 &= \int_0^{\frac{\pi}{2}} (R \cos \theta) \left(1 + \frac{F}{(a + R)E_f} \right) R \cos \theta d\theta \\
 &= R^2 \left(1 + \frac{F}{(a + R)E_f} \right) \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta \\
 &= \frac{R^2}{2} \left(1 + \frac{F}{(a + R)E_f} \right) \left[\frac{\sin 2\theta}{2} + \theta \right]_0^{\frac{\pi}{2}} \\
 &= \frac{\pi R^2}{4} \left(1 + \frac{F}{(a + R)E_f} \right)
 \end{aligned}$$

$$\begin{aligned}
 II &= \int_0^R \frac{1}{2} [(\Delta x_{total,f})] \cdot dy + \int_0^R \frac{1}{2} [(\Delta y_{total,f})] \cdot dx \\
 &= \frac{1}{2} \int_0^{\frac{\pi}{2}} \left[\frac{F \sin \theta \cos \theta}{E_f} (1 - v_f) + \frac{F \sin \theta \cos \theta}{G_f} \right] \cdot R \cos \theta d\theta \\
 &\quad + \frac{1}{2} \int_0^{\frac{\pi}{2}} \left[\frac{F \cos^2 \theta}{E_f} (1 - v_f) - \frac{F \sin^2 \theta}{G_f} \right] \cdot R \sin \theta d\theta \\
 &= \frac{FR}{2} \left[\frac{(1 - v_f)}{E_f} + \frac{1}{G_f} \right] \int_0^{\frac{\pi}{2}} \sin \theta \cos^2 \theta d\theta + \frac{FR}{2E_f} (1 - v_f) \int_0^{\frac{\pi}{2}} \sin \theta \cos^2 \theta d\theta \\
 &\quad - \frac{FR}{2G_f} \int_0^{\frac{\pi}{2}} \sin^3 \theta d\theta \\
 &= \frac{FR}{2} \left[\frac{(1 - v_f)}{E_f} + \frac{1}{G_f} \right] \left[-\frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}} + \frac{FR}{2E_f} (1 - v_f) \left[-\frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}} \\
 &\quad + \frac{FR}{2G_f} \left[\cos \theta - \frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{FR}{2} \left[\frac{(1 - v_f)}{E_f} + \frac{1}{G_f} \right] \left[0 - \left(-\frac{1}{3} \right) \right] + \frac{FR}{2E_f} (1 - v_f) \left[\frac{1}{3} \right] + \frac{FR}{2G_f} \left[\frac{-2}{3} \right] \\
&= FR \left[\frac{(1 - v_f)}{E_f} \right] \left[\frac{1}{6} + \frac{1}{6} \right] + FR \left[\frac{1}{G_f} \right] \left[\frac{1}{6} - \frac{1}{3} \right] \\
&= FR \left[\frac{(1 - v_f)}{3E_f} \right] - FR \left[\frac{1}{6G_f} \right]
\end{aligned}$$

$$\begin{aligned}
III &= \int_0^R \frac{1}{2} [(\Delta x_{total,m})] \cdot dy + \int_0^R \frac{1}{2} [(\Delta y_{total,m})] \cdot dx \\
&= \frac{1}{2} \int_0^{\frac{\pi}{2}} \left[\frac{F \sin \theta \cos \theta}{E_m} (1 - v_m) + \frac{F \sin \theta \cos \theta}{G_m} \right] \cdot R \cos \theta d\theta \\
&\quad + \frac{1}{2} \int_0^{\frac{\pi}{2}} \left[\frac{F \cos^2 \theta}{E_m} (1 - v_m) - \frac{F \sin^2 \theta}{G_m} \right] \cdot R \sin \theta d\theta \\
&= \frac{FR}{2} \left[\frac{(1 - v_m)}{E_f} + \frac{1}{G_m} \right] \int_0^{\frac{\pi}{2}} \sin \theta \cos^2 \theta d\theta + \frac{FR}{2E_m} (1 - v_m) \int_0^{\frac{\pi}{2}} \sin \theta \cos^2 \theta d\theta \\
&\quad - \frac{FR}{2G_m} \int_0^{\frac{\pi}{2}} \sin^3 \theta d\theta \\
&= \frac{FR}{2} \left[\frac{(1 - v_m)}{E_m} + \frac{1}{G_m} \right] \left[-\frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}} + \frac{FR}{2E_m} (1 - v_m) \left[-\frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}} \\
&\quad + \frac{FR}{2G_m} \left[\cos \theta - \frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}} \\
&= \frac{FR}{2} \left[\frac{(1 - v_m)}{E_m} + \frac{1}{G_m} \right] \left[0 - \left(-\frac{1}{3} \right) \right] + \frac{FR}{2E_m} (1 - v_m) \left[\frac{1}{3} \right] + \frac{FR}{2G_m} \left[\frac{-2}{3} \right] \\
&= FR \left[\frac{(1 - v_m)}{E_m} \right] \left[\frac{1}{6} + \frac{1}{6} \right] + FR \left[\frac{1}{G_m} \right] \left[\frac{1}{6} - \frac{1}{3} \right] \\
&= FR \left[\frac{(1 - v_m)}{3E_m} \right] - FR \left[\frac{1}{6G_m} \right]
\end{aligned}$$

$$IV = \int_0^R (a + R - R \cos \theta)(1 + \varepsilon_m) \cdot dy$$

$$\begin{aligned}
&= \int_0^{\frac{\pi}{2}} (a + R - R \cos \theta) \left(1 + \frac{F}{(a + R)E_m} \right) R \cos \theta \, d\theta \\
&= \left(1 + \frac{F}{(a + R)E_m} \right) \int_0^{\frac{\pi}{2}} [(a + R)R \cos \theta - R^2 \cos^2 \theta] \, d\theta \\
&= \left(1 + \frac{F}{(a + R)E_m} \right) \int_0^{\frac{\pi}{2}} [(a + R)R \cos \theta - R^2 \cos^2 \theta] \, d\theta \\
&= \left(1 + \frac{F}{(a + R)E_m} \right) \left(R(a + R) [\sin \theta]_0^{\frac{\pi}{2}} - \frac{R^2}{2} \left[\frac{\sin 2\theta}{2} + \theta \right]_0^{\frac{\pi}{2}} \right) \\
&= \left(1 + \frac{F}{(a + R)E_m} \right) \left((a + R)R - \frac{\pi R^2}{4} \right)
\end{aligned}$$

$$\begin{aligned}
V &= \int_R^{a+R} (a + R)(1 + \varepsilon_m) \cdot dy \\
&= \int_R^{a+R} (a + R) \left(1 + \frac{F}{(a + R)E_m} \right) dy \\
&= (a + R) \left(1 + \frac{F}{(a + R)E_m} \right) \int_R^{a+R} dy \\
&= (a + R) \left(1 + \frac{F_T}{(a + R)E_m} \right) [a + R - R] \\
&= a(a + R) \left(1 + \frac{F_T}{(a + R)E_m} \right)
\end{aligned}$$

$$\begin{aligned}
VII &= \int_0^{\frac{\pi}{2}} \cos^2 \theta \sin \theta \, d\theta = - \int_0^{\frac{\pi}{2}} \cos^2 \theta \cdot d(\cos \theta) \\
&= \left[-\frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}} = \frac{1}{3}
\end{aligned}$$

$$\begin{aligned}
 VIII &= \int_0^{\frac{\pi}{2}} \cos^2 \theta \sin \theta \cos 2\theta \, d\theta = - \int_0^{\frac{\pi}{2}} \cos^2 \theta [2 \cos^2 \theta - 1] \cdot d(\cos \theta) \\
 &= \int_0^{\frac{\pi}{2}} [\cos^2 \theta - 2 \cos^4 \theta] \cdot d(\cos \theta) \\
 &= \left[\frac{\cos^3 \theta}{3} - \frac{2 \cos^5 \theta}{5} \right]_0^{\frac{\pi}{2}} = \frac{2}{5} - \frac{1}{3} = \frac{\mathbf{1}}{\mathbf{15}}
 \end{aligned}$$

$$\begin{aligned}
 IX &= \int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^2 \theta \, d\theta = - \int_0^{\frac{\pi}{2}} (1 - \cos^2 \theta) \cos^2 \theta \cdot d(\cos \theta) \\
 &= \int_0^{\frac{\pi}{2}} [\cos^4 \theta - \cos^2 \theta] \cdot d(\cos \theta) \\
 &= \left[\frac{\cos^5 \theta}{5} - \frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}} = \frac{\mathbf{2}}{\mathbf{15}}
 \end{aligned}$$

Appendix B - Regression Analyses [65]

B.1 Linear relation

Consider a simple linear function:

$$y = A + Bx$$

The associated error function is given by:

$$\epsilon = y - Bx - A$$

Summing the squares of all the errors:

$$\sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - Bx_i - A)^2$$

Minimizing the sum of squares the errors with respect to A and B:

$$\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dA} = \sum_{i=1}^n -2(y_i - Bx_i - A)$$

$$\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dB} = \sum_{i=1}^n -2x_i(y_i - Bx_i - A)$$

Solving for A:

$$\begin{aligned} \frac{d(\sum_{i=1}^n \epsilon_i^2)}{dA} &= 0 \\ \Rightarrow \sum_{i=1}^n y_i &= \sum_{i=1}^n A + \sum_{i=1}^n Bx_i \\ \Rightarrow \left(\sum_{i=1}^n y_i \right) &= A \left(\sum_{i=1}^n 1 \right) + B \left(\sum_{i=1}^n x_i \right) \end{aligned}$$

Solving for B:

$$\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dB} = 0$$

$$\Rightarrow \sum_{i=1}^n y_i x_i = \sum_{i=1}^n A x_i + \sum_{i=1}^n B x_i^2$$

$$\Rightarrow \left(\sum_{i=1}^n y_i x_i \right) = A \left(\sum_{i=1}^n x_i \right) + B \left(\sum_{i=1}^n x_i^2 \right)$$

Rewriting both the solutions together in matrix form:

$$\Rightarrow \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n 1 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

B.2 Linearized Power relation

Consider a simple power function:

$$y = Ax^B$$

Taking natural logarithm on both sides,

$$\ln y = \ln A + B \ln x$$

The associated error function is given by:

$$\epsilon = \ln y - \ln A - B \ln x$$

$$\epsilon_i = \ln y_i - \ln A - B \ln x_i$$

Summing the squares of all the errors:

$$\sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (\ln y_i - \ln A - B \ln x_i)^2$$

Minimizing the sum of squares the errors with respect to A and B:

$$\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dA} = \sum_{i=1}^n -\frac{2}{A} (\ln y_i - \ln A - B \ln x_i)$$

$$\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dB} = \sum_{i=1}^n -2 \ln x_i (\ln y_i - \ln A - B \ln x_i)$$

Solving for A:

$$\begin{aligned} \frac{d(\sum_{i=1}^n \epsilon_i^2)}{dA} &= 0 \\ \Rightarrow \sum_{i=1}^n \ln y_i &= \sum_{i=1}^n \ln A + \sum_{i=1}^n B \ln x_i \\ \Rightarrow \left(\sum_{i=1}^n \ln y_i \right) &= \ln A \left(\sum_{i=1}^n 1 \right) + B \left(\sum_{i=1}^n \ln x_i \right) \end{aligned}$$

Solving for B:

$$\begin{aligned}\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dB} &= 0 \\ \Rightarrow \sum_{i=1}^n \ln y_i \ln x_i &= \sum_{i=1}^n \ln A \ln x_i + \sum_{i=1}^n B \ln x_i \ln x_i \\ \Rightarrow \left(\sum_{i=1}^n \ln y_i \ln x_i \right) &= \ln A \left(\sum_{i=1}^n \ln x_i \right) + B \left(\sum_{i=1}^n (\ln x_i)^2 \right)\end{aligned}$$

Rewriting both the solutions together in matrix form:

$$\Rightarrow \begin{bmatrix} \sum_{i=1}^n (\ln y_i) \\ \sum_{i=1}^n (\ln y_i \ln x_i) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n 1 & \sum_{i=1}^n (\ln x_i) \\ \sum_{i=1}^n (\ln x_i) & \sum_{i=1}^n (\ln x_i)^2 \end{bmatrix} \begin{bmatrix} \ln A \\ B \end{bmatrix}$$

B.3 Linearized Logarithmic relation

Consider a simple logarithm function:

$$y = A + B \ln x$$

The associated error function is given by:

$$\epsilon = y - B \ln x - A$$

Summing the squares of all the errors:

$$\sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - B \ln x_i - A)^2$$

Minimizing the sum of squares the errors with respect to A and B:

$$\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dA} = \sum_{i=1}^n -2(y_i - B \ln x_i - A)$$

$$\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dB} = \sum_{i=1}^n -2 \ln x_i (y_i - B \ln x_i - A)$$

Solving for A:

$$\begin{aligned} \frac{d(\sum_{i=1}^n \epsilon_i^2)}{dA} &= 0 \\ \Rightarrow \sum_{i=1}^n y_i &= \sum_{i=1}^n A + \sum_{i=1}^n B \ln x_i \\ \Rightarrow \left(\sum_{i=1}^n y_i \right) &= A \left(\sum_{i=1}^n 1 \right) + B \left(\sum_{i=1}^n \ln x_i \right) \end{aligned}$$

Solving for B:

$$\begin{aligned}\frac{d(\sum_{i=1}^n \epsilon_i^2)}{dB} &= 0 \\ \Rightarrow \sum_{i=1}^n y_i \ln x_i &= \sum_{i=1}^n A \ln x_i + \sum_{i=1}^n B \ln x_i \ln x_i \\ \Rightarrow \left(\sum_{i=1}^n y_i \ln x_i \right) &= A \left(\sum_{i=1}^n \ln x_i \right) + B \left(\sum_{i=1}^n (\ln x_i)^2 \right)\end{aligned}$$

Rewriting both the solutions together in matrix form:

$$\Rightarrow \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \ln x_i) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n 1 & \sum_{i=1}^n (\ln x_i) \\ \sum_{i=1}^n (\ln x_i) & \sum_{i=1}^n (\ln x_i)^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

B.4 Regression Correlation Coefficient (R^2)

This coefficient is used as a wellness-of-fit indicator for comparing computed or predicted results with baseline actual or experimental values. The closer the value of R^2 is to 1, the better is the fit of the data.

$$R^2 = 1 - \frac{(\sum_{i=1}^n (\hat{y}_i - y_i)^2)}{(\sum_{i=1}^n \hat{y}_i^2) - \frac{(\sum_{i=1}^n \hat{y}_i)^2}{n}}$$

where y_i is the predicted value (from the regression analyses or other means) and $\hat{y}_i$ is the actual or experimental value

Appendix C - Matrix Inversions

Matrix inversions are carried out to convert elasticity compliance matrices (C_{ij}) into elasticity stiffness matrices (E_{ij}) or vice versa. They can be used interchangeably to represent and quantify the elasticity properties of a material, whether it is constituent or composite.

$$C_{ij} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & \dots \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & \dots \\ \dots & \dots & \frac{1}{G_{12}} \end{bmatrix} \text{ or } E_{ij} = \begin{bmatrix} \frac{E_1}{1 - \nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} & \dots \\ \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} & \frac{E_2}{1 - \nu_{12}\nu_{21}} & \dots \\ \dots & \dots & G_{12} \end{bmatrix}$$

Even though MATLAB has its own matrix operation libraries that are able to calculate matrix inverses, computational power is consumed for the Gaussian Elimination method used within [103]. While it may appear insignificant for a single matrix, the presence of 120×120 vertical blocks in our woven model (each with its own constituent fill yarn, weft yarn and matrix elastic properties) might make it computational intensive to handle formulating and inverting matrices for all 14400 blocks (or their constituent materials).

Therefore, in our MATLAB code, we use the large 120×120 matrix only as a locational indicator (i-j) and never to do any serious matrix operations. Taking into consideration of the symmetry of our matrices, we have developed simple algorithm routines to convert our 14400 sets of material properties using Maple 14's inverse matrix function.

C.1 Symmetrical 3×3 matrix with 4 independent parameters

Consider the following symmetric 3×3 matrix with 5 non-zero elements, but 4 independent parameters (a, b, c & d) that requires inversion.

$$[M] = \begin{bmatrix} a & b & 0 \\ b & c & 0 \\ 0 & 0 & d \end{bmatrix}$$

Therefore the required matrix inversion is:

$$[M]^{-1} = \begin{bmatrix} \frac{c}{ac - b^2} & \frac{-b}{ac - b^2} & 0 \\ \frac{-b}{ac - b^2} & \frac{a}{ac - b^2} & 0 \\ 0 & 0 & \frac{1}{d} \end{bmatrix}$$

The screenshot taken from Maple 14 provides the same solution as shown in Figure C.1 below.

Figure C.1: Screenshot from Maple 14 for a four variable matrix inverse

It is seen that the inverse matrix itself also consists of 4 independent variables and is symmetric. Thus it is possible to re-use the formula for the inversion of any 5 element 4 variable symmetric matrix.

C.2 Symmetrical 3×3 matrix with 6 independent parameters

Consider the following symmetric 3×3 matrix with 9 non-zero elements, but 6 independent parameters (a, b, c, d, e & f) that requires inversion.

$$[N] = \begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix}$$

Therefore the required matrix inversion is:

$$[N]^{-1} = \frac{1}{D} \begin{bmatrix} cf - e^2 & bf - de & be - cd \\ bf - de & af - d^2 & ae - bd \\ be - cd & ae - bd & ac - b^2 \end{bmatrix}$$

$$\text{where } D = 2dbe + acf - a^2c - b^2f - ae^2$$

The screenshot below taken from Maple 14 provides the same solution as shown in Figure C.2.

$$\begin{bmatrix} \frac{cf - e^2}{2dbe - d^2c - b^2f + acf - ae^2} & \frac{bf - de}{2dbe - d^2c - b^2f + acf - ae^2} & \frac{be - cd}{2dbe - d^2c - b^2f + acf - ae^2} \\ \frac{bf - de}{2dbe - d^2c - b^2f + acf - ae^2} & \frac{-d^2 + af}{2dbe - d^2c - b^2f + acf - ae^2} & \frac{ae - bd}{2dbe - d^2c - b^2f + acf - ae^2} \\ \frac{be - cd}{2dbe - d^2c - b^2f + acf - ae^2} & \frac{ae - bd}{2dbe - d^2c - b^2f + acf - ae^2} & \frac{-b^2 + ac}{2dbe - d^2c - b^2f + acf - ae^2} \end{bmatrix}$$

Figure C.2: Screenshot from Maple 14 for a six variable matrix inverse

As before, the inverse matrix is symmetric and itself consists of six independent variables. Thus it is possible to re-use the formula for the inversion of any 9 non-zero element 6 variable symmetric matrix. Also, if the values of d and e in the C.2 formula are put to zero, the entire system will reduce down to the 4 variable formula in Section C.1.

Thus, throughout our MATLAB code, the two formulas as in Sections C.1 and C.2 are used many times to invert the elasticity or stiffness matrices within our programs, without ever resorting to MATLAB's inbuilt library. Not only is the computational power conserved for our code, but also it provides a means of checking for individual properties or entries made from inverting the matrices.

Appendix D – Rotating elasticity compliance matrices

An elasticity compliance matrix with planar properties is a symmetric 3×3 square matrix with material properties encapsulated within as shown as $[C]$ below.

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

The properties change when the material orientation is rotated to a certain angle (such that the local material coordinate system is altered), but the properties are required in the original coordinate system. This is illustrated in Figure X below.

Figure X:

Figure X(a) shows the local elasticity compliance matrix $[C]$ coordinate system x - y orientated to the global coordinate system X - Y . However, as seen in Figure X, when the material is rotated through any given angle θ , it's local coordinate system x' - y' doesn't correspond to the global coordinate system X - Y . Hence, a transformation is required to rewrite $[C]$ (which exists in the local coordinate system) as a new elasticity compliance matrix $[\bar{C}]$ in the global coordinate system (shown in the Equation below).

$$[\bar{C}] = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} \\ \bar{C}_{21} & \bar{C}_{22} & \bar{C}_{23} \\ \bar{C}_{31} & \bar{C}_{32} & \bar{C}_{33} \end{bmatrix}$$

This new matrix compliance $[\bar{C}]$ is also a symmetric square matrix with six independent coefficients. The relationship between $[\bar{C}]$ and $[C]$ is given by Equation X.

$$[\bar{C}] = [T]^T [C] [T] \quad \dots(3.4)$$

where the coordinate transformation matrix $[T]$ is

$$T = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}$$

where $c = \cos \theta$; $s = \sin \theta$

Thus, the original properties would be inserted at $[C]$ and the final properties obtained at $[\bar{C}]$. Equation 3.4 can be rewritten as :

$$\begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} \\ \bar{C}_{21} & \bar{C}_{22} & \bar{C}_{23} \\ \bar{C}_{31} & \bar{C}_{32} & \bar{C}_{33} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & -sc \\ s^2 & c^2 & sc \\ 2sc & -2sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}$$

This form is difficult to use and program in MATLAB. Therefore, after opening the brackets and getting collecting and organizing the six independent coefficients for both compliance matrices, the following formulation is obtained.

$$\begin{bmatrix} \bar{C}_{11} \\ \bar{C}_{12} \\ \bar{C}_{22} \\ \bar{C}_{13} \\ \bar{C}_{23} \\ \bar{C}_{33} \end{bmatrix} = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & -2sc^3 & -2cs^3 & s^2c^2 \\ c^2s^2 & c^4 + s^4 & c^2s^2 & sc(c^2 - s^2) & -sc(c^2 - s^2) & -c^2s^2 \\ s^4 & 2c^2s^2 & c^4 & 2cs^3 & 2sc^3 & s^2c^2 \\ 2sc^3 & 2sc(c^2 - s^2) & -2cs^3 & c^2(c^2 - 3s^2) & s^2(3c^2 - s^2) & -sc(c^2 - s^2) \\ 2cs^3 & 2sc(c^2 - s^2) & -2sc^3 & s^2(3c^2 - s^2) & c^2(c^2 - 3s^2) & sc(c^2 - s^2) \\ 4s^2c^2 & -8s^2c^2 & 4s^2c^2 & 4sc(c^2 - s^2) & -4sc(c^2 - s^2) & (c^2 - s^2)^2 \end{bmatrix} \begin{bmatrix} C_{11} \\ C_{12} \\ C_{22} \\ C_{13} \\ C_{23} \\ C_{33} \end{bmatrix}$$

since $C_{21} = C_{12}$, $C_{31} = C_{13}$ & $C_{32} = C_{23}$ and $\bar{C}_{21} = \bar{C}_{12}$, $\bar{C}_{31} = \bar{C}_{13}$ & $\bar{C}_{32} = \bar{C}_{23}$ due to matrix symmetry

This form makes it easier to utilize the formulation and see the relationship of each of the local coordinate terms with the global coordinate terms. This form has been used in the Braided textile model code to firstly rotate the weft yarn properties from local axes (1w) onto the global axes (X-Y) and then later on, the overall unit cell properties from the global axes (X-Y) to the machine and cross directions (MD-CD) for analysis.

It should be noted that the rotation formulation is reversible. If a matrix $[\bar{C}]$ is obtained by rotating another matrix $[C]$ through any angle $+\theta$, the resulting matrix $[\bar{C}]$ can then be rotated through an angle of $-\theta$ to obtain back the original matrix $[C]$. This is illustrated by the Equations A1.2 and A1.3 below.

Rotating through angle θ

$$[C] \xrightarrow[\text{through}+\theta]{\text{rotate}} [\bar{C}] \xrightarrow[\text{through}-\theta]{\text{rotate}} [C] \quad \dots(\text{A1.2})$$

Or rotating through angle β

$$[C] \xrightarrow[\text{through}+\beta]{\text{rotate}} [\bar{C}] \xrightarrow[\text{through}-\beta]{\text{rotate}} [C] \quad \dots(\text{A1.3})$$

Appendix E – Unit cells used for FEA

<p>Single Quarter Unit Cell (1 by 1)</p>	<p>FEA stress plot of a single quarter unit cell (1 by 1). The plot shows a curved boundary with a color-coded stress distribution. The legend indicates stress values ranging from $+1.121e+08$ (blue) to $+1.558e+08$ (red). The average stress is 75%.</p>
<p>Sixteen Quarter Unit Cells (4 by 4)</p>	<p>FEA stress plot of sixteen quarter unit cells (4 by 4). The plot shows a periodic pattern of circular features with a color-coded stress distribution. The legend indicates stress values ranging from $+1.182e+08$ (blue) to $+1.105e+08$ (red). The average stress is 75%.</p>

*It should be noted that these meshes were initially not refined (at the yarn-matrix interface region, as shown in Figure 4.6) when carrying out convergence tests on the arrangement. Only after choosing the sixteen unit cell arrangement were the density of meshes increased at the yarn-matrix interface to capture its effects.

Appendix F – MATLAB Codes

F.1 Composite Cylinder Assemblage (CCA) Formulation

This is the implementation of the classical CCA formulation discussed in Chapter 2 and used as a measure of comparison for the yarn input models.

```
function [E_L, E_Tmax, E_Tmin, G_LT, G_TTmin, G_TTmax,  
v_LT, v_TT] = CCA(E_Lfibres, E_Tfibres, G_LTfibres,  
G_TTfibres, v_LTfibres, v_TTfibres, E_matrix, G_matrix,  
v_matrix, V_f)
```

```
%Read Input
```

```
E_Lf = E_Lfibres;
```

```
E_Tf = E_Tfibres;
```

```
G_LTf = G_LTfibres;
```

```
G_TTf = G_TTfibres;
```

```
v_LTf = v_LTfibres;
```

```
v_TTf = v_TTfibres;
```

```
E_m = E_matrix;
```

```
G_m = G_matrix;
```

```
v_m = v_matrix;
```

```
V_fibre = V_f;
```

```
V_matrix = 1-V_fibre;
```

```
% Transverse bulk moduli
```

```
k_f = 1/(-1/G_TTf-4*v_LTf*v_LTf/E_Lf+4/E_Tf);
```

```
k_m = 1/(-1/G_m-4*v_m*v_m/E_m+4/E_m);
```

```
k = (k_m*(k_f+G_m) * V_matrix + k_f * (k_m+G_m) *  
V_fibre) / (((k_f+G_m) * V_matrix + (k_m+G_m) * V_fibre));
```

```
%Outputs of Longitudinal Direction
```

```
E_L = E_Lf*V_fibre+E_m*V_matrix+(4*(v_LTf -  
v_m)*(v_LTf - v_m) * V_fibre * V_matrix) /  
(V_matrix/k_f+V_fibre/k_m+1/G_m);
```

```
v_LT = v_LTf * V_fibre + v_m*V_matrix + ((v_LTf -  
v_m)*(1/k_m-1/k_f) * V_fibre*V_matrix) /  
(V_matrix/k_f+V_fibre/k_m+1/G_m);
```

```
G_LT = G_m * (G_m*V_matrix+(1+V_fibre) * G_LTf) /  
(G_m*(1+V_fibre)+V_matrix*G_LTf);
```

```
% Alpha, beta, gamma and rho values
```

```
gamma = G_TTf/G_m;
```

```
beta1 = k_m/(k_m+2*G_m);
```

```
beta2 = k_f/(k_f+2*G_TTf);
```

```
alpha = (beta1-gamma*beta2)/(1+gamma*beta2);
```

```
rho = (gamma + beta1)/(gamma - 1);
```

```
%Output values for G_TT (since G_TTf > G_m)
```

```
G_TTmin = G_m + V_fibre/(1/(G_TTf-  
G_m)+(k_m+2*G_m)*V_matrix/(2*G_m*(k_m+G_m))));
```

```
G_TTmax = G_m*(1+(1+beta1)*V_fibre/(rho -  
V_fibre*(1+(3*(beta1*V_matrix)^2)/(alpha*V_fibre^3+1)))));
```

```
%Output of E_T
```

```
m = 1 + 4*k*v_LT*v_LT/E_L;
```

```
E_Tmax = 4*k*G_TTmax/(k+m*G_TTmax);
```

```
E_Tmin = 4*k*G_TTmin/(k+m*G_TTmin);
```

```
% Final V_TT output
```

```
v_TT = v_TTf*V_fibre+v_m*V_matrix; %Estimation (not part  
of CCA)
```

F.2 Shear Inclusive Circular (SIC) & Elliptical (SIE) Formulation

This is the combined implementation of the Shear Inclusive Circular and Elliptical Yarn models. The input value of aspect ratio (AR) will determine which model will come into effect. Using a value of $AR = 1$ uses the default SIC model, while $AR \neq 1$ activates the SIE model.

```
function [E_L, E_T, G_LT, G_TT, v_LT, v_TT] =
Formula(E_Lfibres, E_Tfibres, G_LTfibres, G_TTfibres,
v_LTfibres, v_TTfibres, E_matrix, G_matrix, v_matrix,
porosity, AR)
```

```
%Read Input
```

```
E_Lf = E_Lfibres;
```

```
E_Tf = E_Tfibres;
```

```
G_LTf = G_LTfibres;
```

```
G_TTf = G_TTfibres;
```

```
v_LTf = v_LTfibres;
```

```
v_TTf = v_TTfibres;
```

```
E_m = E_matrix;
```

```
G_m = G_matrix;
```

```
v_m = v_matrix;
```

```
V_fibre = 1-porosity;
```

```
porosity = 0;
```

```
M_r = E_Lfibres/E_matrix;
```

```
Termsv = (1-9.1356*V_fibre)*(1-1.87836*porosity)*(1-
5.76812*log(M_r));
```

```
TermsE = (1-9.167*V_fibre)*(1-1.898*porosity)*(1-
4.70588*log(M_r));
```

```
M = 0.0072*((Termsv)^0.9278)/Termsv;
```

```
%Outputs of Longitudinal Direction
```

```
E_L = E_Lf*(V_fibre)+E_m*(1-V_fibre);
```

```
v_LT = v_LTf*(V_fibre)+v_m*(1-V_fibre);
```

```
G_LT = G_m*(G_m*(1-
V_fibre)+(1+V_fibre)*G_LTf)/(G_m*(1+V_fibre)+(1-
V_fibre)*G_LTf);
```

```
%Use Special Formula
```

```
E_TT_cir = 1/(V_fibre/E_Tf + (1-V_fibre)/E_m +
(1/3)*sqrt(V_fibre/pi)*(2*(1-v_TTf)/E_Tf-1/G_TTf+2*(1-
v_m)/E_m-1/G_m));
```

```
v_TT_cir = (v_TTf*V_fibre/E_Tf + (1-V_fibre)*v_m/E_m -
(1/15)*sqrt(V_fibre/pi)*((2-3*v_TTf)/E_Tf-1/G_TTf+(2-
3*v_m)/E_m-1/G_m))/(V_fibre/E_Tf + (1-V_fibre)/E_m +
(1/15)*sqrt(V_fibre/pi)*((3-2*v_TTf)/E_Tf+1/G_TTf+(3-
2*v_m)/E_m+1/G_m));
```

```
% Enabling Elliptical
```

```
E_T = E_TT_cir*((AR)^(-log(1.014809-0.01034*TermsE)));
```

```
v_TT = v_TT_cir*((AR)^(M*Termsv));
```

```
G_TT = 0.5*E_T/(1+v_TT);
```

```
E_TT_b = E_TT_cir*(AR)^(log(1.014809-0.01034*TermsE));
```

```
v_TT_b = v_TT_cir*(AR)^(-M*Termsv);
```


F.3 Woven & Braided PS Formulation

This is the combined implementation for the woven and braided textile models. Using a *braidAngle* value of 45° activates the woven model, whereas other values of *braidAngle* are used normally for the braided textile models.

```
function [E_MD, E_CD, Poisson_MC, Poisson_CM,
G_braided] = Braided_ParallelSeries(braidAngle)
```

```
% Fibre Properties Only -----
```

```
%FILL FIBRES T-300 Carbon%
```

```
E_1f_fibres = 230E9; %Pa%
```

```
E_2f_fibres = 40E9; %Pa%
```

```
E_3f_fibres = 40E9; %Pa%
```

```
G_13f_fibres = 24E9; %Pa%
```

```
G_23f_fibres = 14.3E9; %Pa%
```

```
G_12f_fibres = 24E9; %Pa%
```

```
v_13f_fibres = 0.26; %Poisson ratio%
```

```
v_12f_fibres = 0.26; %Poisson ratio%
```

```
v_23f_fibres = 0.30; %Poisson ratio%
```

```
%WEFT FIBRES T-300 Carbon%
```

```
E_1w_fibres = 230E9; %Pa%
```

```
E_2w_fibres = 40E9; %Pa%
```

```
E_3w_fibres = 40E9; %Pa%
```

```
G_13w_fibres = 24E9; %Pa%
```

```
G_23w_fibres = 14.3E9; %Pa%
```

```
G_12w_fibres = 24E9; %Pa%
```

```
v_12w_fibres = 0.26; %Poisson ratio%
```

```
v_13w_fibres = 0.26; %Poisson ratio%
```

```
v_23w_fibres = 0.30; %Poisson ratio%
```

```
%MATRIX Epoxy%
```

```
E_1m = 3.5E9; %Pa%
```

```
E_2m = 3.5E9; %Pa%
```

```
E_3m = 3.5E9; %Pa%
```

```
G_13m = 1.28E9; %Pa%
```

```
G_23m = 1.28E9; %Pa%
```

```
G_12m = 1.28E9; %Pa%
```

```
v_13m = 0.35; %Poisson ratio%
```

```
v_12m = 0.35; %Poisson ratio%
```

```
v_23m = 0.35; %Poisson ratio%
```

```
%-----
```

```
vf_fiber2Lamina = 0.64;
```

```
braidAngleDeg = braidAngle; % in Degrees
```

```
braidAngleRad = braidAngleDeg*pi()/180;
```

```
sin2BraidAngle = sin(2*braidAngleRad);
```

```
cos2BraidAngle = cos(2*braidAngleRad);
```

```
%Fill yarn dimensions
```

```
fill_yarn_width = 1.1E-3;
```

```
fill_yarn_height = 7.5E-5;
```

```
fill_yarn_gap = 0.011E-3;
```

```
%Weft Yarn dimensions
```

```
weft_yarn_width = 0.96E-3;
```

```
weft_yarn_height = 7.5E-5;
```

```
weft_yarn_gap = 0.151E-3;
```

```
%Unit cell height in m%
```

```
H_above = 8E-5;
```

```
H_below = 8E-5;
```

```
h_f = fill_yarn_height;
```

```
h_w = weft_yarn_height;
```



```

%Unit cell dimensions in y-direction in m%

a_y = fill_yarn_width;

g_y = fill_yarn_gap;

u_y = 0.6*a_y;

%Unit cell dimensions in x-direction in m%

a_x = weft_yarn_width/sin2BraidAngle;

g_x = weft_yarn_gap/sin2BraidAngle;

u_x = 0.6*a_x;

a_values_x = zeros(5,1);

a_values_x(1) = 0.5*(a_x-u_x);

a_values_x(2) = 0.5*a_x;

a_values_x(3) = 0.5*(a_x+g_x);

a_values_x(4) = 0.5*a_x + g_x;

a_values_x(5) = g_x + 0.5*(a_x+u_x);

a_values_y = zeros(5,1);

a_values_y(1) = 0.5*(a_y-u_y);

a_values_y(2) = 0.5*a_y;

a_values_y(3) = 0.5*(a_y+g_y);

a_values_y(4) = 0.5*a_y + g_y;

a_values_y(5) = g_y + 0.5*(a_y+u_y);

% No of sections in x-direction%

N_st_x = 30;

N_un_x = 15;

N_gap_x = 30;

N_tot_x = 2*(N_st_x + N_un_x + N_gap_x);

% No of sections in y-direction%

N_st_y = 30;

N_un_y = 15;

N_gap_y = 30;

```

```

N_tot_y = 2*(N_st_y + N_un_y+ N_gap_y);

% Creating vectors in x-direction

x_coord = zeros (N_tot_x,1);

for i = 1:N_tot_x

    if i < N_st_x+1

        x_coord(i) = (i-0.5)*0.5*(a_x-u_x)/N_st_x;

    elseif i < N_un_x+N_st_x+1

        x_coord(i) = a_values_x(1)+ (i-0.5-
N_st_x)*0.5*u_x/N_un_x;

    elseif i < N_un_x+N_st_x+N_gap_x+1

        x_coord(i) = a_values_x(2)+ (i-0.5-N_st_x-
N_un_x)*0.5*g_x/N_gap_x;

    elseif i < N_un_x+N_st_x+2*N_gap_x+1

        x_coord(i) = a_values_x(3)+ (i-0.5-N_st_x-N_un_x-
N_gap_x)*0.5*g_x/N_gap_x;

    elseif i < N_st_x+2*(N_un_x+N_gap_x)+1

        x_coord(i) = a_values_x(4)+ (i-0.5-N_st_x-N_un_x-
2*N_gap_x)*0.5*u_x/N_un_x;

    else x_coord(i) = a_values_x(5)+ (i-0.5-N_st_x-
2*(N_un_x+N_gap_x))*0.5*(a_x-u_x)/N_st_x;

    end

end

x3_coord = zeros( N_st_x + N_un_x, 1);

x4_coord = zeros( N_st_x + N_un_x, 1);

for i = 1: N_st_x + N_un_x

    x3_coord(i) = x_coord(i);

    x4_coord(i) = x_coord(N_tot_x - N_st_x - N_un_x + i);

end

% Creating vectors in y-direction

y_coord = zeros (N_tot_y,1);

for i = 1:N_tot_y

    if i < N_st_y+1

```

```

        y_coord(i) = (i-0.5)*0.5*(a_y-u_y)/N_st_y;

elseif i < N_un_y+N_st_y+1

    y_coord(i) = a_values_y(1)+ (i-0.5-
N_st_y)*0.5*u_y/N_un_y;

    elseif i < N_un_y+N_st_y+N_gap_y+1

        y_coord(i) = a_values_y(2)+ (i-0.5-N_st_y-
N_un_y)*0.5*g_y/N_gap_y;

        elseif i < N_un_y+N_st_y+2*N_gap_y+1

            y_coord(i) = a_values_y(3)+ (i-0.5-N_st_y-N_un_y-
N_gap_y)*0.5*g_y/N_gap_y;

            elseif i < N_st_y+2*(N_un_y+N_gap_y)+1

                y_coord(i) = a_values_y(4)+ (i-0.5-N_st_y-N_un_y-
2*N_gap_y)*0.5*u_y/N_un_y;

                else y_coord(i) = a_values_y(5)+ (i-0.5-N_st_y-
2*(N_un_y+N_gap_y))*0.5*(a_y-u_y)/N_st_y;

            end

        end

end

y3_coord = zeros( N_st_y + N_un_y, 1);

y4_coord = zeros( N_st_y + N_un_y, 1);

for i = 1: N_st_y + N_un_y

    y3_coord(i) = y_coord(i);

    y4_coord(i) = y_coord(N_tot_y - N_st_y - N_un_y + i);

end

% zx_vectors

%-----

zx1_coord = zeros (N_tot_x,1);

for i = 1:N_tot_x

    if i < N_st_x+1

        zx1_coord(i) = h_f;

    elseif i > (N_tot_x - N_st_x)

        zx1_coord(i) = 0;

    else zx1_coord(i) = 0.5*h_f*(1+cos((x_coord(i)-
a_values_x(1))*pi()/(u_x+g_x)));

    end

end

```

```

end

zx2_coord = zeros (N_tot_x,1);

for i = 1:N_tot_x

    if i < N_st_x+1

        zx2_coord(i) = 0;

    elseif i > (N_tot_x - N_st_x)

        zx2_coord(i) = -h_f;

    else zx2_coord(i) = 0.5*h_f*(cos((x_coord(i)-
a_values_x(1))*pi()/(u_x+g_x))-1);

    end

end

zx3_amp = h_w + 0.5*h_f*(cos(0.5*u_x*pi()/(u_x+g_x))-1);

zx4_amp = h_w -
0.5*h_f*(cos((g_x+0.5*u_x)*pi()/(u_x+g_x))+1);

zx3_coord = zeros (N_st_x + N_un_x,1);

for i = 1:N_st_x + N_un_x

    if i < N_st_x+1

        zx3_coord(i) = -h_w;

    else zx3_coord(i) = 0.5*h_f*(cos(0.5*u_x*pi()/(u_x+g_x))-1)
- zx3_amp*cos((x3_coord(i)-a_values_x(1))*pi()/u_x);

    end

end

zx4_coord = zeros (N_st_x + N_un_x,1);

for i = 1:N_st_x + N_un_x

    if i > N_un_x

        zx4_coord(i) = h_w;

    else zx4_coord(i) =
0.5*h_f*(cos((g_x+0.5*u_x)*pi()/(u_x+g_x))+1) +
zx4_amp*sin((x4_coord(i)-a_values_x(4))*pi()/u_x);

    end

end

```

```

H_above_coord_x = zeros (N_tot_x, 1);

H_above_coord_x(:) = H_above;

H_below_coord_x = zeros (N_tot_x, 1);

H_below_coord_x(:) = -H_below;

%-----
%-----

zy1_coord = zeros (N_tot_y,1);

for i = 1:N_tot_y

    if i < N_st_y+1

        zy1_coord(i) = -h_w;

    elseif i > (N_tot_y - N_st_y)

        zy1_coord(i) = 0;

    else zy1_coord(i) = -0.5*h_w*(1+cos((y_coord(i)-
a_values_y(1))*pi()/(u_y+g_y)));

    end

end

zy1_coord;

zy2_coord = zeros (N_tot_y,1);

for i = 1:N_tot_y

    if i < N_st_y+1

        zy2_coord(i) = 0;

    elseif i > (N_tot_y - N_st_y)

        zy2_coord(i) = h_w;

    else zy2_coord(i) = -0.5*h_w*(cos((y_coord(i)-
a_values_y(1))*pi()/(u_y+g_y))-1);

    end

end

zy3_amp = h_f + 0.5*h_w*(cos(0.5*u_y*pi()/(u_y+g_y))-1);

zy4_amp = h_f -
0.5*h_w*(cos((g_y+0.5*u_y)*pi()/(u_y+g_y))+1);

```

```

zy3_coord = zeros (N_st_y + N_un_y,1);

for i = 1:N_st_y + N_un_y

    if i < N_st_y+1

        zy3_coord(i) = h_f;

    else zy3_coord(i) = -0.5*h_w*(cos(0.5*u_y*pi()/(u_y+g_y))-
1) + zy3_amp*cos((y3_coord(i)-a_values_y(1))*pi()/(u_y));

    end

end

zy4_coord = zeros (N_st_y + N_un_y,1);

for i = 1:N_st_y + N_un_y

    if i > N_un_y

        zy4_coord(i) = -h_f;

    else zy4_coord(i) = -
0.5*h_w*(cos((g_y+0.5*u_y)*pi()/(u_y+g_y))+1) -
zy4_amp*sin((y4_coord(i)-a_values_y(4))*pi()/(u_y));

    end

end

H_above_coord_y = zeros (N_tot_y, 1);

H_above_coord_y(:) = H_above;

H_below_coord_y = zeros (N_tot_y, 1);

H_below_coord_y(:) = -H_below;

%-----

%figure(1);

%plot(x_coord, zx1_coord, x_coord, zx2_coord, x3_coord,
zx3_coord, x4_coord, zx4_coord, x_coord, H_above_coord_x,
x_coord, H_below_coord_x);

xlabel('X-direction Yarn Length'); ylabel('Height');

title('Plot of Yarn Shape');

%figure(2);

```

```
%plot(y_coord, zy1_coord, y_coord, zy2_coord, y3_coord,
zy3_coord, y4_coord, zy4_coord, y_coord, H_above_coord_y,
y_coord, H_below_coord_y);
```

```
%xlabel('Y-direction Yarn Length'); ylabel('Height');
```

```
%title('Plot of Yarn Shape');
```

```
surface_top = zeros(N_st_y+N_un_y, N_st_x+N_un_x);
```

```
for i = 1:N_st_y+N_un_y
```

```
    for j = 1:N_st_x+N_un_x
```

```
        surface_top(i,j) = zx1_coord(j)+zy3_coord(i)-h_f;
```

```
    end
```

```
end
```

```
x_surface = zeros(N_st_x+N_un_x,1);
```

```
for i = 1:N_st_x+N_un_x
```

```
    x_surface(i) = x_coord(i);
```

```
end
```

```
y_surface = zeros(N_st_y+N_un_y,1);
```

```
for i = 1:N_st_y+N_un_y
```

```
    y_surface(i) = y_coord(i);
```

```
end
```

```
surface_middle = zeros(N_st_y+N_un_y, N_st_x+N_un_x);
```

```
for i = 1:N_st_y+N_un_y
```

```
    for j = 1:N_st_x+N_un_x
```

```
        surface_middle(i,j) = zx2_coord(j)+zy2_coord(i);
```

```
    end
```

```
end
```

```
surface_bottom = zeros(N_st_y+N_un_y, N_st_x+N_un_x);
```

```
for i = 1:N_st_y+N_un_y
```

```
    for j = 1:N_st_x+N_un_x
```

```
        surface_bottom(i,j) = zx3_coord(j)+zy1_coord(i)+h_w;
```

```
    end
```

```
end
```

```
%figure(3);
```

```
%surf(x_surface,y_surface,surface_top);
```

```
%xlabel('x-direction'); ylabel('y-direction');
```

```
%hold on;
```

```
%surf(x_surface,y_surface,surface_middle);
```

```
%surf(x_surface,y_surface,surface_bottom);
```

```
%-----
```

```
%X-direction gap here
```

```
%-----
```

```
centre_top_y = zeros(N_st_y+N_un_y,1);
```

```
for i = 1:N_st_y+N_un_y
```

```
    if i < N_st_y+1
```

```
        centre_top_y(i) = h_f/2;
```

```
    else centre_top_y(i) = 0.5*h_f*cos((y_coord(i)-0.5*(a_y-
u_y))*pi()/u_y);
```

```
    end
```

```
end
```

```
centre_bot_y = -centre_top_y;
```

```
gap_top_x = zeros(N_st_y+N_un_y, N_gap_x);
```

```
for i = 1:N_st_y+N_un_y
```

```
    for j = 1:N_gap_x
```

```

        gap_top_x(i,j) = surface_top(i, N_st_x+N_un_x)-(j-
0.5)*(surface_top(i, N_st_x+N_un_x)-
centre_top_y(i))/N_gap_x;

    end

end

gap_bot_x = zeros(N_st_y+N_un_y, N_gap_x);

for i = 1:N_st_y+N_un_y

    for j = 1:N_gap_x

        gap_bot_x(i,j) = surface_middle(i, N_st_x+N_un_x)-(j-
0.5)*(surface_middle(i, N_st_x+N_un_x)-
centre_bot_y(i))/N_gap_x;

    end

end

x_gap_x = zeros(N_gap_x,1);

for i = 1:N_gap_x

    x_gap_x(i) = x_coord(N_st_x+N_un_x+i);

end

y_gap_x = zeros(N_st_y+N_un_y,1);

for i = 1:N_st_y+N_un_y

    y_gap_x(i) = y_coord(i);

end

%figure(4);

%surf(x_gap_x,y_gap_x,gap_top_x);

%xlabel('x-direction'); ylabel('y-direction');

%hold on;

%surf(x_gap_x,y_gap_x,gap_bot_x);

%-----

%y-direction gap here

%-----

```

```

centre_top_x = zeros(N_st_x+N_un_x,1);

for i = 1:N_st_x+N_un_x

    if i < N_st_x+1

        centre_top_x(i) = -h_w/2;

        else centre_top_x(i) = -0.5*h_w*cos((x_coord(i)-0.5*(a_x-
u_x))*pi()/u_x);

    end

end

centre_bot_x = -centre_top_x;

gap_top_y = zeros(N_gap_y, N_st_x+N_un_x );

for i = 1:N_st_x+N_un_x

    for j = 1:N_gap_y

        gap_top_y(j,i) = surface_bottom(N_st_y+N_un_y, i)-(j-
0.5)*(surface_bottom(N_st_y+N_un_y, i)-
centre_top_x(i))/N_gap_y;

    end

end

gap_bot_y = zeros(N_gap_y, N_st_x+N_un_x );

for i = 1:N_st_x+N_un_x

    for j = 1:N_gap_y

        gap_bot_y(j,i) = surface_middle(N_st_y+N_un_y, i)-(j-
0.5)*(surface_middle(N_st_y+N_un_y, i)-
centre_bot_x(i))/N_gap_y;

    end

end

y_gap_y = zeros(N_gap_y, 1);

for i = 1:N_gap_y

    y_gap_y(i) = y_coord(N_st_y+N_un_y+i);

```

```

end

x_gap_y = zeros(N_st_x+N_un_x,1);

for i = 1:N_st_x+N_un_x

    x_gap_y(i) = x_coord(i);

end

%figure(4);

%surf(x_gap_y,y_gap_y,gap_top_y);

%xlabel('x-direction'); ylabel('y-direction');

%hold on;

%surf(x_gap_y,y_gap_y,gap_bot_y);

%-----

%Creating complete x-direction half-yarns

fill_top = zeros(N_st_y+N_un_y, N_st_x+N_un_x+N_gap_x);

for i = 1:N_st_y+N_un_y

    for j = 1:N_st_x+N_un_x+N_gap_x

        if j < N_st_x+N_un_x+1

            fill_top(i,j) = surface_top(i,j);

        else fill_top(i,j) = gap_top_x(i,j-N_st_x-N_un_x);

        end

    end

end

fill_bottom = zeros(N_st_y+N_un_y,
N_st_x+N_un_x+N_gap_x);

for i = 1:N_st_y+N_un_y

    for j = 1:N_st_x+N_un_x+N_gap_x

        if j < N_st_x+N_un_x+1

            fill_bottom(i,j) = surface_middle(i,j);

```

```

        else fill_bottom(i,j) = gap_bot_x(i,j-N_st_x-N_un_x);

        end

    end

end

x_full = zeros(1, N_st_x+N_un_x+N_gap_x);

for i = 1:N_st_x+N_un_x+N_gap_x

    if i < N_st_x+N_un_x+1

        x_full(i) = x_surface(i);

    else x_full(i) = x_gap_x(i-N_st_x-N_un_x);

    end

end

%figure(5);

%surf(x_full,y_gap_x,fill_top);

%xlabel('x-direction'); ylabel('y-direction');

%hold on;

%surf(x_full,y_gap_x,fill_bottom);

%-----

%-----

%Creating complete y-direction half-yarns

weft_top = zeros(N_st_y+N_un_y+N_gap_y, N_st_x+N_un_x);

for i = 1:N_st_x+N_un_x

    for j = 1:N_st_y+N_un_y+N_gap_y

        if j < N_st_y+N_un_y+1

            weft_top(j,i) = surface_middle(j,i);

        else weft_top(j,i) = gap_bot_y(j-N_st_y-N_un_y,i);

        end

    end

end

end

```

```
weft_bottom = zeros(N_st_y+N_un_y+N_gap_y,
N_st_x+N_un_x);
```

```
for i = 1:N_st_x+N_un_x
```

```
    for j = 1:N_st_y+N_un_y+N_gap_y
```

```
        if j < N_st_y+N_un_y+1
```

```
            weft_bottom(j,i) = surface_bottom(j,i);
```

```
        else weft_bottom(j,i) = gap_top_y(j-N_st_y-N_un_y,i);
```

```
        end
```

```
    end
```

```
end
```

```
y_full = zeros(1, N_st_y+N_un_y+N_gap_y);
```

```
for i = 1:N_st_y+N_un_y+N_gap_y
```

```
    if i < N_st_y+N_un_y+1
```

```
        y_full(i) = y_surface(i);
```

```
    else y_full(i) = y_gap_y(i-N_st_y-N_un_y);
```

```
    end
```

```
end
```

```
%figure(6);
```

```
%surf(x_gap_y,y_full, weft_top);
```

```
%xlabel('x-direction'); ylabel('y-direction');
```

```
%hold on;
```

```
%surf(x_gap_y,y_full,weft_bottom);
```

```
%-----
```

```
%---- Both yarns together
```

```
%figure(7);
```

```
%surf(x_full,y_gap_x,fill_top);
```

```
%xlabel('x-direction'); ylabel('y-direction');
```

```
%hold on;
```

```
%surf(x_full,y_gap_x,fill_bottom);
```

```
%surf(x_gap_y,y_full, weft_top);
```

```
%surf(x_gap_y,y_full,weft_bottom);
```

```
% --- Replication of yarns to form unit cell
```

```
full_fill_top = zeros(N_st_y+N_un_y,
2*(N_st_x+N_un_x+N_gap_x));
```

```
for i = 1:N_st_y+N_un_y
```

```
    for j = 1:2*(N_st_x+N_un_x+N_gap_x)
```

```
        if j < N_st_x+N_un_x+1
```

```
            full_fill_top(i,j) = surface_top(i,j);
```

```
        elseif j < N_st_x+N_un_x+N_gap_x+1
```

```
            full_fill_top(i,j) = gap_top_x(i,j-N_st_x-N_un_x);
```

```
        elseif j < N_st_x+N_un_x+2*N_gap_x+1
```

```
            full_fill_top(i,j) = -
gap_bot_x(i,N_st_x+N_un_x+2*N_gap_x-j+1);
```

```
        else
```

```
            full_fill_top(i,j) = -surface_middle(i,
2*N_st_x+2*N_un_x+2*N_gap_x-j+1);
```

```
        end
```

```
    end
```

```
end
```

```
full_fill_bottom = zeros(N_st_y+N_un_y,
2*(N_st_x+N_un_x+N_gap_x));
```

```
for i = 1:N_st_y+N_un_y
```

```
    for j = 1:2*(N_st_x+N_un_x+N_gap_x)
```

```
        if j < N_st_x+N_un_x+1
```

```

        full_fill_bottom(i,j) = surface_middle(i,j);

elseif j < N_st_x+N_un_x+N_gap_x+1

        full_fill_bottom(i,j) = gap_bot_x(i,j-N_st_x-N_un_x);

elseif j < N_st_x+N_un_x+2*N_gap_x+1

        full_fill_bottom(i,j) = -
gap_top_x(i,N_st_x+N_un_x+2*N_gap_x-j+1);

        else

                full_fill_bottom(i,j) = -surface_top(i,
2*N_st_x+2*N_un_x+2*N_gap_x-j+1);

        end

end

end

full_weft_top = zeros(2*(N_st_y+N_un_y+N_gap_y),
(N_st_x+N_un_x));

for j = 1:N_st_x+N_un_x

        for i = 1:2*(N_st_y+N_un_y+N_gap_y)

                if i < N_st_y+N_un_y+1

                        full_weft_top(i,j) = surface_middle(i,j);

                elseif i < N_st_y+N_un_y+N_gap_y+1

                        full_weft_top(i,j) = gap_bot_y(i-N_st_y-N_un_y,j);

                elseif i < N_st_y+N_un_y+2*N_gap_y+1

                        full_weft_top(i,j) = -
gap_top_y(N_st_y+N_un_y+2*N_gap_y-i+1,j);

                else

                        full_weft_top(i,j) = -
surface_bottom(2*N_st_y+2*N_un_y+2*N_gap_y-i+1,j);

                end

        end

end

full_weft_bottom = zeros(2*(N_st_y+N_un_y+N_gap_y),
(N_st_x+N_un_x));

for j = 1:N_st_x+N_un_x

        for i = 1:2*(N_st_y+N_un_y+N_gap_y)

```

```

                if i < N_st_y+N_un_y+1

                        full_weft_bottom(i,j) = surface_bottom(i,j);

                elseif i < N_st_y+N_un_y+N_gap_y+1

                        full_weft_bottom(i,j) = gap_top_y(i-N_st_y-N_un_y,j);

                elseif i < N_st_y+N_un_y+2*N_gap_y+1

                        full_weft_bottom(i,j) = -
gap_bot_y(N_st_y+N_un_y+2*N_gap_y-i+1,j);

                else

                        full_weft_bottom(i,j) = -
surface_middle(2*N_st_y+2*N_un_y+2*N_gap_y-i+1,j);

                end

        end

end

%figure(8);

%surf(x_coord,y_gap_x,full_fill_top);

xlabel('x-direction'); ylabel('y-direction');

hold on;

%surf(x_coord,y_gap_x,full_fill_bottom);

%surf(x_gap_y,y_coord,full_weft_top);

%surf(x_gap_y,y_coord,full_weft_bottom);

y_gap_x_back = zeros(N_st_y+N_un_y,1);

for i = 1:N_st_y+N_un_y

        y_gap_x_back(i) = y_coord(i+N_st_y+N_un_y+2*N_gap_y);

end

x_gap_y_back = zeros(N_st_x+N_un_x,1);

for i = 1:N_st_x+N_un_x

        x_gap_y_back(i) = x_coord(i+N_st_x+N_un_x+2*N_gap_x);

end

full_fill_top_back = zeros(N_st_y+N_un_y,
2*(N_st_x+N_un_x+N_gap_x));

```



```

full_fill_bottom_back = zeros(N_st_y+N_un_y,
2*(N_st_x+N_un_x+N_gap_x));

for i=1:N_st_y+N_un_y

    full_fill_top_back(i,:) = -full_fill_top(N_st_y+N_un_y-i+1,:);

    full_fill_bottom_back(i,:) = -
full_fill_bottom(N_st_y+N_un_y-i+1,:);

end

full_weft_top_back = zeros(2*(N_st_y+N_un_y+N_gap_y),
(N_st_x+N_un_x));

full_weft_bottom_back = zeros(2*(N_st_y+N_un_y+N_gap_y),
(N_st_x+N_un_x));

for j=1:N_st_x+N_un_x

    full_weft_top_back(:,j) = -full_weft_top(:,N_st_x+N_un_x-
j+1);

    full_weft_bottom_back(:,j) = -
full_weft_bottom(:,N_st_x+N_un_x-j+1);

end

%surf(x_coord,y_gap_x_back,full_fill_top_back);

%surf(x_coord,y_gap_x_back,full_fill_bottom_back);

%surf(x_gap_y_back,y_coord,full_weft_top_back);

%surf(x_gap_y_back,y_coord,full_weft_bottom_back);

%-----

%Obtaining the heights

fill_front = zeros(N_st_y+N_un_y,
2*(N_st_x+N_un_x+N_gap_x));

fill_back = zeros(N_st_y+N_un_y,
2*(N_st_x+N_un_x+N_gap_x));

fill_front(:) = full_fill_top(:)-full_fill_bottom(:);

fill_back(:) = full_fill_bottom_back(:)-full_fill_top_back(:);

```

```

weft_front= zeros(2*(N_st_y+N_un_y+N_gap_y),
(N_st_x+N_un_x));

weft_back = zeros(2*(N_st_y+N_un_y+N_gap_y),
(N_st_x+N_un_x));

weft_front(:) = full_weft_top(:)-full_weft_bottom(:);

weft_back(:) = full_weft_bottom_back(:)-full_weft_top_back(:);

% Obtaining angles

slope_fill = zeros (N_tot_x,1);

for i = 1:N_tot_x

    if i < N_st_x+1

        slope_fill(i) = 0;

    elseif i > (N_tot_x - N_st_x)

        slope_fill(i) = 0;

    else slope_fill(i) = -0.5*h_f*pi()*sin((x_coord(i)-
a_values_x(1))*pi()/(u_x+g_x))/(u_x+g_x);

    end

end

slope_weft = zeros (N_tot_y,1);

for j = 1:N_tot_y

    if j < N_st_y+1

        slope_weft(j) = 0;

    elseif j > (N_tot_y - N_st_y)

        slope_weft(j) = 0;

    else slope_weft(j) = 0.5*h_w*pi()*sin((y_coord(j)-
a_values_y(1))*pi()/(u_y+g_y))/(u_y+g_y);

    end

end

%Obtaining volumes

volume_fill = 0;

volume_weft = 0;

```

```

volume_matrix = 0;

fill_height = zeros(2*(N_st_y+N_un_y+N_gap_y),
2*(N_st_x+N_un_x+N_gap_x));

weft_height = zeros(2*(N_st_y+N_un_y+N_gap_y),
2*(N_st_x+N_un_x+N_gap_x));

matrix_height = zeros(2*(N_st_y+N_un_y+N_gap_y),
2*(N_st_x+N_un_x+N_gap_x));

for j = 1:2*(N_st_y+N_un_y+N_gap_y)

    for i=1:2*(N_st_x+N_un_x+N_gap_x)

        if j<N_st_y+N_un_y+1

            fill_height(j,i) = fill_front(j,i);

        elseif j>N_st_y+N_un_y+2*N_gap_y

            fill_height(j,i) = fill_back(j-
(N_st_y+N_un_y+2*N_gap_y),i);

        end

    end

end

for i = 1:2*(N_st_x+N_un_x+N_gap_x)

    for j=1:2*(N_st_y+N_un_y+N_gap_y)

        if i<N_st_x+N_un_x+1

            weft_height(j,i) = weft_front(j,i);

        elseif i>N_st_x+N_un_x+2*N_gap_x

            weft_height(j,i) = weft_back(j,i-
(N_st_x+N_un_x+2*N_gap_x));

        end

    end

end

matrix_height(:,i) = H_above+H_below-fill_height(:,i)-
weft_height(:,i);

dx = 0;

```

```

dy = 0;

for j = 1:2*(N_st_y+N_un_y+N_gap_y)

    for i=1:2*(N_st_x+N_un_x+N_gap_x)

        if j<N_st_y+1

            dy = 0.5*(a_y-u_y)/N_st_y;

        elseif j<N_st_y+N_un_y+1

            dy = 0.5*(u_y)/N_un_y;

        elseif j<N_st_y+N_un_y+N_gap_y+1

            dy = 0.5*(g_y)/N_gap_y;

        elseif j<N_st_y+N_un_y+2*N_gap_y+1

            dy = 0.5*(g_y)/N_gap_y;

        elseif j<N_st_y+2*(N_un_y+N_gap_y)+1

            dy = 0.5*(u_y)/N_un_y;

        else

            dy = 0.5*(a_y-u_y)/N_st_y;

        end

    end

    if i<N_st_x+1

        dx = 0.5*(a_x-u_x)/N_st_x;

    elseif i<N_st_x+N_un_x+1

        dx = 0.5*(u_x)/N_un_x;

    elseif i<N_st_x+N_un_x+N_gap_x+1

        dx = 0.5*(g_x)/N_gap_x;

    elseif i<N_st_x+N_un_x+2*N_gap_x+1

        dx = 0.5*(g_x)/N_gap_x;

    elseif i<N_st_x+2*(N_un_x+N_gap_x)+1

        dx = 0.5*(u_x)/N_un_x;

    else

        dx = 0.5*(a_x-u_x)/N_st_x;

    end

    volume_fill = volume_fill+fill_height(j,i)*dx*dy;

```

```

        volume_weft = volume_weft+weft_height(j,i)*dx*dy;

        volume_matrix =
        volume_matrix+matrix_height(j,i)*dx*dy;

    end

end

vf_fillYarn2Lamina =
volume_fill/(volume_fill+volume_weft+volume_matrix);

vf_weftYarn2Lamina =
volume_weft/(volume_fill+volume_weft+volume_matrix);

vf_yarn2Lamina = vf_fillYarn2Lamina+vf_weftYarn2Lamina;

% YARN PROPERTY ESTIMATION -----
-----

% Volume Fractions of fibres to yarns

vf_fill = vf_fiber2Lamina/vf_yarn2Lamina;

vf_weft = vf_fiber2Lamina/vf_yarn2Lamina;

% Running the fibre properties through the CCA to obtain the
yarn properties

[E_Lf, E_Tfmax, E_Tfmin, G_LTf, G_TTfmin, G_TTfmax,
v_LTf, v_TTf] = CCA(E_1f_fibres, E_2f_fibres, G_12f_fibres,
G_23f_fibres, v_12f_fibres, v_23f_fibres, E_1m, G_23m,
v_12m, vf_fill);

[E_Lw, E_Twmax, E_Twmin, G_LTw, G_TTwmin,
G_TTwmax, v_LTw, v_TTw] = CCA(E_1w_fibres,
E_2w_fibres, G_12w_fibres, G_23w_fibres, v_12w_fibres,
v_23w_fibres, E_1m, G_23m, v_12m, vf_weft);

%FILL YARN T-300 Carbon%

E_1f = 144.80E9; %Pa%

E_2f = 11.73E9; %Max in Pa%

E_3f = 11.73E9; %Max in Pa%

G_13f = 5.52E9; %Pa%

G_23f = G_TTfmin; %Max in Pa%

G_12f = 5.52E9; %Pa%

```

```

v_13f = 0.23; %Poisson ratio%

v_12f = 0.23; %Poisson ratio%

v_23f = 0.3; %Estimated Poisson ratio%

%WEFT YARN T-300 Carbon%

E_1w = 144.80E9; %Pa%

E_2w = 11.73E9 ; %Max in Pa%

E_3w = 11.73E9; %Max in Pa%

G_13w = 5.52E9; %Pa%

G_23w = G_TTwmin; %Max in Pa%

G_12w = 5.52E9; %Pa%

v_12w = 0.23; %Poisson ratio%

v_13w = 0.23; %Poisson ratio%

v_23w = 0.3; %Poisson ratio%

% YARN PROPERTY ESTIMATION ENDS HERE-----
-----

angle_fill = zeros (N_tot_x,1);

angle_fill(:) = atan(slope_fill(:));

angle_weft = zeros (N_tot_y,1);

angle_weft(:) = atan(slope_weft(:));

%Fill Q properties

E_fill_x = zeros (N_tot_x,1);

for i = 1:N_tot_x

    E_fill_x(i) = 1/(cos((angle_fill(i)))^4/E_1f +
((sin(angle_fill(i)))^4)/E_3f
+((sin(angle_fill(i))*cos(angle_fill(i)))^2)*(1/G_13f -
2*v_13f/E_1f));

end

E_fill_y = E_2f;

v_fill_yx = zeros (N_tot_x,1);

```

```

for i = 1:N_tot_x

    v_fill_yx(i) = v_12f*E_2f/E_1f*(cos(angle_fill(i)))^2 +
    v_23f*(sin(angle_fill(i)))^2;

end

G_fill_xy = zeros (N_tot_x,1);

for i = 1:N_tot_x

    G_fill_xy(i) = 1/((sin(angle_fill(i))^2)/G_23f +
    (cos(angle_fill(i))^2)/G_12f);

end

v_fill_xy = zeros (N_tot_x,1);

for i = 1:N_tot_x

    v_fill_xy(i) = E_fill_x(i)*v_fill_yx(i)/E_fill_y;

end

D_fill_v = zeros (N_tot_x,1);

for i = 1:N_tot_x

    D_fill_v(i) = 1 - v_fill_xy(i)*v_fill_yx(i);

end

fill_yarn_complianceMatrix = zeros (N_tot_x,6);

for i = 1:N_tot_x

    fill_yarn_complianceMatrix(i,1) = 1/E_fill_x(i);

    fill_yarn_complianceMatrix(i,2) = -v_fill_xy(i)/E_fill_x(i);

    fill_yarn_complianceMatrix(i,3) = 1/E_fill_y;

    fill_yarn_complianceMatrix(i,6) = 1/G_fill_xy(i);

end

fill_yarn_stiffnessMatrix = zeros (N_tot_x,6);

for i = 1:N_tot_x

    fill_yarn_stiffnessMatrix(i,1) =
    fill_yarn_complianceMatrix(i,3)/(fill_yarn_complianceMatrix(i,
    1)*fill_yarn_complianceMatrix(i,3)-
    fill_yarn_complianceMatrix(i,2)*fill_yarn_complianceMatrix(i,
    2));

```

```

    fill_yarn_stiffnessMatrix(i,2) = -
    fill_yarn_complianceMatrix(i,2)/(fill_yarn_complianceMatrix(i,
    1)*fill_yarn_complianceMatrix(i,3)-
    fill_yarn_complianceMatrix(i,2)*fill_yarn_complianceMatrix(i,
    2));

    fill_yarn_stiffnessMatrix(i,3) =
    fill_yarn_complianceMatrix(i,1)/(fill_yarn_complianceMatrix(i,
    1)*fill_yarn_complianceMatrix(i,3)-
    fill_yarn_complianceMatrix(i,2)*fill_yarn_complianceMatrix(i,
    2));

    fill_yarn_stiffnessMatrix(i,6) =
    1/fill_yarn_complianceMatrix(i,6);

end

%fill_yarn_stiffnessMatrix

%Weft Q properties

E_weft_x = E_2w;

E_weft_y = zeros (N_tot_y,1);

for j = 1:N_tot_y

    E_weft_y(j) = 1/(cos((angle_weft(j)))^4/E_1w +
    ((sin(angle_weft(j)))^4)/E_3w
    +(sin(angle_weft(j))*cos(angle_weft(j)))^2*(1/G_13w -
    2*v_13w/E_1w));

end

v_weft_xy = zeros (N_tot_y,1);

for j = 1:N_tot_y

    v_weft_xy(j) = v_12w*E_2w/E_1w*(cos(angle_weft(j)))^2 +
    v_23w*(sin(angle_weft(j)))^2;

end

G_weft_xy = zeros (N_tot_y,1);

for j = 1:N_tot_y

    G_weft_xy(j) = 1/((sin(angle_weft(j))^2)/G_23w +
    (cos(angle_weft(j))^2)/G_12w);

end

v_weft_yx = zeros (N_tot_y,1);

```

```

for j = 1:N_tot_y

    v_weft_yx(j) = E_weft_y(j)*v_weft_xy(j)/E_weft_x;

end

D_weft_v = zeros (N_tot_y,1);

for j = 1:N_tot_y

    D_weft_v(j) = 1 - v_weft_xy(j)*v_weft_yx(j);

end

weft_yarn_complianceMatrix = zeros (N_tot_y,4);

for j = 1:N_tot_y

    weft_yarn_complianceMatrix(j,1) = 1/E_weft_x;

    weft_yarn_complianceMatrix(j,2) = -
v_weft_yx(j)/E_weft_y(j);

    weft_yarn_complianceMatrix(j,3) = 1/E_weft_y(j);

    weft_yarn_complianceMatrix(j,4) = 1/G_weft_xy(j);

end

%Obtaining the weft yarn transformations to the y-axis

s = cos2BraidAngle;

c = sin2BraidAngle;

rotater_matrix = zeros(6,4);

%Row 1 for 1/Ex

rotater_matrix(1,1) = c^4;

rotater_matrix(1,2) = 2*s*s*c*c;

rotater_matrix(1,3) = s^4;

rotater_matrix(1,4) = s*s*c*c;

%Row 2 for v_xy/Ex

rotater_matrix(2,1) = s*s*c*c;

rotater_matrix(2,2) = (s^4+c^4);

rotater_matrix(2,3) = s*s*c*c;

rotater_matrix(2,4) = -s*s*c*c;

```

```

%Row 3 for 1/Ey

rotater_matrix(3,1) = s^4;

rotater_matrix(3,2) = 2*s*s*c*c;

rotater_matrix(3,3) = c^4;

rotater_matrix(3,4) = s*s*c*c;

%Row 4 for G_zone1

rotater_matrix(4,1) = 2*s*c^3;

rotater_matrix(4,2) = 2*s*c*(s*s-c*c);

rotater_matrix(4,3) = -2*c*s^3;

rotater_matrix(4,4) = -s*c*(c*c-s*s);

%Row 5 for G_zone2

rotater_matrix(5,1) = 2*c*s^3;

rotater_matrix(5,2) = 2*s*c*(c*c-s*s);

rotater_matrix(5,3) = -2*s*c^3;

rotater_matrix(5,4) = s*c*(c*c-s*s);

%Row 6 for 1/G_xy

rotater_matrix(6,1) = 4*s*s*c*c;

rotater_matrix(6,2) = -8*s*s*c*c;

rotater_matrix(6,3) = 4*s*s*c*c;

rotater_matrix(6,4) = (c*c-s*s)*(c*c-s*s);

%rotater_matrix

%Rotating the weft yarn by angle (90-2*braidAngle)

weft_yarn_complianceMatrixRot = zeros (N_tot_y,6);

for j = 1:N_tot_y

    weft_yarn_complianceMatrixRot(j,1) =
rotater_matrix(1,1)*weft_yarn_complianceMatrix(j,1)+rotater_
matrix(1,2)*weft_yarn_complianceMatrix(j,2)+rotater_matrix(1,
3)*weft_yarn_complianceMatrix(j,3)+rotater_matrix(1,4)*weft_
yarn_complianceMatrix(j,4);

```

```

    weft_yarn_complianceMatrixRot(j,2) =
    rotater_matrix(2,1)*weft_yarn_complianceMatrix(j,1)+rotater_
    matrix(2,2)*weft_yarn_complianceMatrix(j,2)+rotater_matrix(2,
    3)*weft_yarn_complianceMatrix(j,3)+rotater_matrix(2,4)*weft_
    yarn_complianceMatrix(j,4);

    weft_yarn_complianceMatrixRot(j,3) =
    rotater_matrix(3,1)*weft_yarn_complianceMatrix(j,1)+rotater_
    matrix(3,2)*weft_yarn_complianceMatrix(j,2)+rotater_matrix(3,
    3)*weft_yarn_complianceMatrix(j,3)+rotater_matrix(3,4)*weft_
    yarn_complianceMatrix(j,4);

    weft_yarn_complianceMatrixRot(j,4) =
    rotater_matrix(4,1)*weft_yarn_complianceMatrix(j,1)+rotater_
    matrix(4,2)*weft_yarn_complianceMatrix(j,2)+rotater_matrix(4,
    3)*weft_yarn_complianceMatrix(j,3)+rotater_matrix(4,4)*weft_
    yarn_complianceMatrix(j,4);

    weft_yarn_complianceMatrixRot(j,5) =
    rotater_matrix(5,1)*weft_yarn_complianceMatrix(j,1)+rotater_
    matrix(5,2)*weft_yarn_complianceMatrix(j,2)+rotater_matrix(5,
    3)*weft_yarn_complianceMatrix(j,3)+rotater_matrix(5,4)*weft_
    yarn_complianceMatrix(j,4);

    weft_yarn_complianceMatrixRot(j,6) =
    rotater_matrix(6,1)*weft_yarn_complianceMatrix(j,1)+rotater_
    matrix(6,2)*weft_yarn_complianceMatrix(j,2)+rotater_matrix(6,
    3)*weft_yarn_complianceMatrix(j,3)+rotater_matrix(6,4)*weft_
    yarn_complianceMatrix(j,4);

end

%weft_yarn_complianceMatrixRot

weft_yarn_stiffnessMatrixRot = zeros (N_tot_y,6);

for j = 1:N_tot_y

    determinant =
    weft_yarn_complianceMatrixRot(j,1)*(weft_yarn_complianceM
    atrixRot(j,3)*weft_yarn_complianceMatrixRot(j,6)-
    weft_yarn_complianceMatrixRot(j,5)*weft_yarn_complianceM
    atrixRot(j,5))-
    weft_yarn_complianceMatrixRot(j,2)*(weft_yarn_complianceM
    atrixRot(j,2)*weft_yarn_complianceMatrixRot(j,6)-
    weft_yarn_complianceMatrixRot(j,4)*weft_yarn_complianceM
    atrixRot(j,5))+weft_yarn_complianceMatrixRot(j,4)*(weft_yarn
    _complianceMatrixRot(j,2)*weft_yarn_complianceMatrixRot(j,
    5)-
    weft_yarn_complianceMatrixRot(j,3)*weft_yarn_complianceM
    atrixRot(j,4));

    weft_yarn_stiffnessMatrixRot(j,1) =
    (weft_yarn_complianceMatrixRot(j,3)*weft_yarn_complianceM
    atrixRot(j,6)-
    weft_yarn_complianceMatrixRot(j,5)*weft_yarn_complianceM
    atrixRot(j,5))/determinant;

    weft_yarn_stiffnessMatrixRot(j,2) =
    (weft_yarn_complianceMatrixRot(j,4)*weft_yarn_complianceM
    atrixRot(j,5)-

```

```

    weft_yarn_complianceMatrixRot(j,2)*weft_yarn_complianceM
    atrixRot(j,6))/determinant;

    weft_yarn_stiffnessMatrixRot(j,3) =
    (weft_yarn_complianceMatrixRot(j,1)*weft_yarn_complianceM
    atrixRot(j,6)-
    weft_yarn_complianceMatrixRot(j,4)*weft_yarn_complianceM
    atrixRot(j,4))/determinant;

    weft_yarn_stiffnessMatrixRot(j,4) =
    (weft_yarn_complianceMatrixRot(j,2)*weft_yarn_complianceM
    atrixRot(j,5)-
    weft_yarn_complianceMatrixRot(j,3)*weft_yarn_complianceM
    atrixRot(j,4))/determinant;

    weft_yarn_stiffnessMatrixRot(j,5) =
    (weft_yarn_complianceMatrixRot(j,2)*weft_yarn_complianceM
    atrixRot(j,4)-
    weft_yarn_complianceMatrixRot(j,1)*weft_yarn_complianceM
    atrixRot(j,5))/determinant;

    weft_yarn_stiffnessMatrixRot(j,6) =
    (weft_yarn_complianceMatrixRot(j,1)*weft_yarn_complianceM
    atrixRot(j,3)-
    weft_yarn_complianceMatrixRot(j,2)*weft_yarn_complianceM
    atrixRot(j,2))/determinant;

end

%weft_yarn_stiffnessMatrixRot

%Matrix Q properties

E_matrix_x = E_2m;

E_matrix_y = E_1m;

v_matrix_yx = v_13m*E_matrix_y/E_matrix_x;

D_matrix_v = 1 -
v_matrix_yx*v_matrix_yx*E_matrix_x/E_matrix_y;

G_matrix_xy = G_12m;

Q_matrix = [E_matrix_x/D_matrix_v,
v_matrix_yx*E_matrix_x/D_matrix_v,
E_matrix_y/D_matrix_v, G_matrix_xy];

% Obtaining the A_ij and a_ij matrices

A_ij_one = zeros(N_tot_y, N_tot_x);

A_ij_two = zeros(N_tot_y, N_tot_x);

A_ij_three = zeros(N_tot_y, N_tot_x);

A_ij_four = zeros(N_tot_y, N_tot_x);

```

```

A_ij_five = zeros(N_tot_y, N_tot_x);

A_ij_six = zeros(N_tot_y, N_tot_x);

for j = 1:2*(N_st_y+N_un_y+N_gap_y)

    for i=1:2*(N_st_x+N_un_x+N_gap_x)

        A_ij_one(j,i) =
(matrix_height(j,i)*Q_matrix(1))+(fill_height(j,i)*fill_yarn_stiffnessMatrix(i,1))+(weft_height(j,i)*weft_yarn_stiffnessMatrixRot(j,1));

        A_ij_two(j,i) =
(matrix_height(j,i)*Q_matrix(2))+(fill_height(j,i)*fill_yarn_stiffnessMatrix(i,2))+(weft_height(j,i)*weft_yarn_stiffnessMatrixRot(j,2));

        A_ij_three(j,i) =
(matrix_height(j,i)*Q_matrix(3))+(fill_height(j,i)*fill_yarn_stiffnessMatrix(i,3))+(weft_height(j,i)*weft_yarn_stiffnessMatrixRot(j,3));

        A_ij_four(j,i) =
(fill_height(j,i)*fill_yarn_stiffnessMatrix(i,4))+(weft_height(j,i)*weft_yarn_stiffnessMatrixRot(j,4));

        A_ij_five(j,i) =
(fill_height(j,i)*fill_yarn_stiffnessMatrix(i,5))+(weft_height(j,i)*weft_yarn_stiffnessMatrixRot(j,5));

        A_ij_six(j,i) =
(matrix_height(j,i)*Q_matrix(4))+(fill_height(j,i)*fill_yarn_stiffnessMatrix(i,6))+(weft_height(j,i)*weft_yarn_stiffnessMatrixRot(j,6));

    end

end

%A_ij_one

%A_ij_two

%A_ij_three

%A_ij_four

%A_ij_five

%A_ij_six

a_ij_one = zeros(N_tot_y, N_tot_x);

a_ij_two = zeros(N_tot_y, N_tot_x);

a_ij_three = zeros(N_tot_y, N_tot_x);

a_ij_four = zeros(N_tot_y, N_tot_x);

```

```

a_ij_five = zeros(N_tot_y, N_tot_x);

a_ij_six = zeros(N_tot_y, N_tot_x);

for j = 1:2*(N_st_y+N_un_y+N_gap_y)

    for i=1:2*(N_st_x+N_un_x+N_gap_x)

        determinant = A_ij_one(j,i)*(A_ij_three(j,i)*A_ij_six(j,i)-A_ij_five(j,i)*A_ij_five(j,i))-A_ij_two(j,i)*(A_ij_two(j,i)*A_ij_six(j,i)-A_ij_four(j,i)*A_ij_five(j,i))+A_ij_four(j,i)*(A_ij_two(j,i)*A_ij_five(j,i)-A_ij_three(j,i)*A_ij_four(j,i));

        a_ij_one(j,i) = (A_ij_three(j,i)*A_ij_six(j,i)-A_ij_five(j,i)*A_ij_five(j,i))/determinant;

        a_ij_two(j,i) = (A_ij_four(j,i)*A_ij_five(j,i)-A_ij_two(j,i)*A_ij_six(j,i))/determinant;

        a_ij_three(j,i) = (A_ij_one(j,i)*A_ij_six(j,i)-A_ij_four(j,i)*A_ij_four(j,i))/determinant;

        a_ij_four(j,i) = (A_ij_two(j,i)*A_ij_five(j,i)-A_ij_three(j,i)*A_ij_four(j,i))/determinant;

        a_ij_five(j,i) = (A_ij_two(j,i)*A_ij_four(j,i)-A_ij_five(j,i)*A_ij_one(j,i))/determinant;

        a_ij_six(j,i) = (A_ij_three(j,i)*A_ij_one(j,i)-A_ij_two(j,i)*A_ij_two(j,i))/determinant;

    end

end

%a_ij_one

%a_ij_two

%a_ij_three

%a_ij_four

%a_ij_five

%a_ij_six

% Parallel-Series Additions for Ex

A_ij_SumOfParallel_one= zeros(N_tot_x,1);

A_ij_SumOfParallel_two= zeros(N_tot_x,1);

A_ij_SumOfParallel_three= zeros(N_tot_x,1);

A_ij_SumOfParallel_four= zeros(N_tot_x,1);

```

```

A_ij_SumOfParallel_five= zeros(N_tot_x,1);

A_ij_SumOfParallel_six= zeros(N_tot_x,1);


dy=0;


for i = 1:2*(N_st_x+N_un_x+N_gap_x)


    for j=1:2*(N_st_y+N_un_y+N_gap_y)


        if j<N_st_y+1

            dy = 0.5*(a_y-u_y)/N_st_y;

        elseif j<N_st_y+N_un_y+1

            dy = 0.5*(u_y)/N_un_y;

        elseif j<N_st_y+N_un_y+N_gap_y+1

            dy = 0.5*(g_y)/N_gap_y;

        elseif j<N_st_y+N_un_y+2*N_gap_y+1

            dy = 0.5*(g_y)/N_gap_y;

        elseif j<N_st_y+2*(N_un_y+N_gap_y)+1

            dy = 0.5*(u_y)/N_un_y;

        else

            dy = 0.5*(a_y-u_y)/N_st_y;

        end


        A_ij_SumOfParallel_one(i)=
A_ij_SumOfParallel_one(i)+A_ij_one(j,i)*dy;


        A_ij_SumOfParallel_two(i)=
A_ij_SumOfParallel_two(i)+A_ij_two(j,i)*dy;


        A_ij_SumOfParallel_three(i)=
A_ij_SumOfParallel_three(i)+A_ij_three(j,i)*dy;


        A_ij_SumOfParallel_four(i)=
A_ij_SumOfParallel_four(i)+A_ij_four(j,i)*dy;


        A_ij_SumOfParallel_five(i)=
A_ij_SumOfParallel_five(i)+A_ij_five(j,i)*dy;


        A_ij_SumOfParallel_six(i)=
A_ij_SumOfParallel_six(i)+A_ij_six(j,i)*dy;


    end

```

end

```

A_ij_SumOfParallel_one =
A_ij_SumOfParallel_one/(a_y+g_y);

A_ij_SumOfParallel_two =
A_ij_SumOfParallel_two/(a_y+g_y);

A_ij_SumOfParallel_three =
A_ij_SumOfParallel_three/(a_y+g_y);

A_ij_SumOfParallel_four =
A_ij_SumOfParallel_four/(a_y+g_y);

A_ij_SumOfParallel_five =
A_ij_SumOfParallel_five/(a_y+g_y);

A_ij_SumOfParallel_six = A_ij_SumOfParallel_six/(a_y+g_y);


a_ij_SumOfParallel_one= zeros(N_tot_x,1);

a_ij_SumOfParallel_two= zeros(N_tot_x,1);

a_ij_SumOfParallel_three= zeros(N_tot_x,1);

a_ij_SumOfParallel_four= zeros(N_tot_x,1);

a_ij_SumOfParallel_five= zeros(N_tot_x,1);

a_ij_SumOfParallel_six= zeros(N_tot_x,1);


for i = 1:2*(N_st_x+N_un_x+N_gap_x)


    determinant =
A_ij_SumOfParallel_one(i)*(A_ij_SumOfParallel_three(i)*A_ij_
_SumOfParallel_six(i)-
A_ij_SumOfParallel_five(i)*A_ij_SumOfParallel_five(i))-
A_ij_SumOfParallel_two(i)*(A_ij_SumOfParallel_two(i)*A_ij_
SumOfParallel_six(i)-
A_ij_SumOfParallel_four(i)*A_ij_SumOfParallel_five(i))+A_ij_
_SumOfParallel_four(i)*(A_ij_SumOfParallel_two(i)*A_ij_Su
mOfParallel_five(i)-
A_ij_SumOfParallel_three(i)*A_ij_SumOfParallel_four(i));


    a_ij_SumOfParallel_one(i) =
(A_ij_SumOfParallel_three(i)*A_ij_SumOfParallel_six(i)-
A_ij_SumOfParallel_five(i)*A_ij_SumOfParallel_five(i))/deter
minant;


    a_ij_SumOfParallel_two(i) =
(A_ij_SumOfParallel_four(i)*A_ij_SumOfParallel_five(i)-
A_ij_SumOfParallel_two(i)*A_ij_SumOfParallel_six(i))/determ
inant;


    a_ij_SumOfParallel_three(i)
=(A_ij_SumOfParallel_one(i)*A_ij_SumOfParallel_six(i)-

```



```

A_ij_SumOfParallel_four(i)*A_ij_SumOfParallel_four(i))/deter
minant;

a_ij_SumOfParallel_four(i) =
(A_ij_SumOfParallel_two(i)*A_ij_SumOfParallel_five(i)-
A_ij_SumOfParallel_three(i)*A_ij_SumOfParallel_four(i))/deter
minant;

a_ij_SumOfParallel_five(i) =
(A_ij_SumOfParallel_two(i)*A_ij_SumOfParallel_four(i)-
A_ij_SumOfParallel_five(i)*A_ij_SumOfParallel_one(i))/deter
minant;

a_ij_SumOfParallel_six(i) =
(A_ij_SumOfParallel_three(i)*A_ij_SumOfParallel_one(i)-
A_ij_SumOfParallel_two(i)*A_ij_SumOfParallel_two(i))/deter
minant;

end

%a_ij_SumOfParallel_one

a_ij_ParallelSeries_one = 0;

a_ij_ParallelSeries_two = 0;

a_ij_ParallelSeries_three = 0;

a_ij_ParallelSeries_four = 0;

a_ij_ParallelSeries_five = 0;

a_ij_ParallelSeries_six = 0;

dx = 0;

for i = 1:2*(N_st_x+N_un_x+N_gap_x)

    if i<N_st_x+1

        dx = 0.5*(a_x-u_x)/N_st_x;

    elseif i<N_st_x+N_un_x+1

        dx = 0.5*(u_x)/N_un_x;

    elseif i<N_st_x+N_un_x+N_gap_x+1

        dx = 0.5*(g_x)/N_gap_x;

    elseif i<N_st_x+N_un_x+2*N_gap_x+1

        dx = 0.5*(g_x)/N_gap_x;

    elseif i<N_st_x+2*(N_un_x+N_gap_x)+1

        dx = 0.5*(u_x)/N_un_x;

```

```

else

    dx = 0.5*(a_x-u_x)/N_st_x;

end

a_ij_ParallelSeries_one=
a_ij_ParallelSeries_one+dx*a_ij_SumOfParallel_one(i);

a_ij_ParallelSeries_two=
a_ij_ParallelSeries_two+dx*a_ij_SumOfParallel_two(i);

a_ij_ParallelSeries_three=
a_ij_ParallelSeries_three+dx*a_ij_SumOfParallel_three(i);

a_ij_ParallelSeries_four=
a_ij_ParallelSeries_four+dx*a_ij_SumOfParallel_four(i);

a_ij_ParallelSeries_five=
a_ij_ParallelSeries_five+dx*a_ij_SumOfParallel_five(i);

a_ij_ParallelSeries_six=
a_ij_ParallelSeries_six+dx*a_ij_SumOfParallel_six(i);

end

a_ij_ParallelSeries_one=
a_ij_ParallelSeries_one*(H_above+H_below)/(a_x+g_x);

a_ij_ParallelSeries_two=
a_ij_ParallelSeries_two*(H_above+H_below)/(a_x+g_x);

a_ij_ParallelSeries_three=
a_ij_ParallelSeries_three*(H_above+H_below)/(a_x+g_x);

a_ij_ParallelSeries_four=
a_ij_ParallelSeries_four*(H_above+H_below)/(a_x+g_x);

a_ij_ParallelSeries_five=
a_ij_ParallelSeries_five*(H_above+H_below)/(a_x+g_x);

a_ij_ParallelSeries_six=
a_ij_ParallelSeries_six*(H_above+H_below)/(a_x+g_x);

if braidAngle == 45

    E_x_PS = 1/a_ij_ParallelSeries_one

end

% Rotating onto Machine & Cross directions

c=cos(braidAngleRad);

s=sin(braidAngleRad);

```

```
a_ij_rotated_one=(c^4)*a_ij_ParallelSeries_one+2*(s*s*c*c)*a
_ij_ParallelSeries_two+(s^4)*a_ij_ParallelSeries_three-
2*s*c*(c*c*a_ij_ParallelSeries_four+s*s*a_ij_ParallelSeries_fi
ve)+(s*s*c*c)*a_ij_ParallelSeries_six;
```

```
a_ij_rotated_two=
(s*s*c*c)*(a_ij_ParallelSeries_one+a_ij_ParallelSeries_three)+(
s^4+c^4)*a_ij_ParallelSeries_two+s*c*(c*c-
s*s)*(a_ij_ParallelSeries_four-a_ij_ParallelSeries_five)-
(s*s*c*c)*a_ij_ParallelSeries_six;
```

```
a_ij_rotated_three=
(s^4)*a_ij_ParallelSeries_one+2*(s*s*c*c)*a_ij_ParallelSeries_
two+(c^4)*a_ij_ParallelSeries_three+2*s*c*(s*s*a_ij_ParallelS
eries_four+c*c*a_ij_ParallelSeries_five)+(s*s*c*c)*a_ij_Parall
elSeries_six;
```

```
a_ij_rotated_six=
4*(s*s*c*c)*(a_ij_ParallelSeries_one+a_ij_ParallelSeries_three
)-8*(s*s*c*c)*a_ij_ParallelSeries_two+4*s*c*(c*c-
s*s)*(a_ij_ParallelSeries_four-a_ij_ParallelSeries_five)+(c*c-
s*s)*(c*c-s*s)*a_ij_ParallelSeries_six;
```

```
% Outputs - Series
```

```
Poisson_MC = -a_ij_rotated_two/a_ij_rotated_one;
```

```
Poisson_CM = -a_ij_rotated_two/a_ij_rotated_three;
```

```
D_v = 1-Poisson_MC*Poisson_CM;
```

```
E_MD = 1/a_ij_rotated_one;
```

```
E_CD = 1/a_ij_rotated_three;
```

```
G_braided = 1/a_ij_rotated_six;
```

```
%figure (11);
```

```
%surf(x_coord, y_coord,A_ij_one);
```

```
%xlabel('x-direction'); ylabel('y-direction');
```

```
%figure (12);
```

```
%surf(x_coord, y_coord,A_ij_two);
```

```
%figure (13);
```

```
%surf(x_coord, y_coord,A_ij_three);
```

```
%xlabel('x-direction'); ylabel('y-direction');
```

```
%figure (14);
```

```
%surf(x_coord, y_coord,A_ij_six);
```

```
%figure (15);
```

```
%surf(x_coord, y_coord,A_ij_one);
```

```
%hold;
```

```
%surf(x_coord, y_coord,A_ij_three);
```

```
%xlabel('x-direction'); ylabel('y-direction');
```

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