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EXPERIMENTAL STUDIES OF THE VORTICITY FIELD IN THE FAR REGION OF A CYLINDER WAKE

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**Experimental Studies of the Vorticity Field
in the Far Region of a Cylinder Wake**

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Summary

The main objective of the present study is to examine the characteristics of the large and small-scale turbulence structures in the far region of a cylinder wake. Effort is first focused on the spatial criteria of wire separation for velocity (or temperature) derivative measurements. The effect of wire separation on velocity (or temperature) derivative is examined by using two parallel hot (or cold) wires with changeable separations. The results show that the spectra and corresponding moments of velocity (or temperature) derivatives are significantly affected by the wire separation. However, reliable measurements of velocity (or temperature) derivatives are available by using finite differences from two hot (or cold) wires if the separation between the wires is in the range of $3\sim 5\eta$, where η is the Kolmogorov length scale.

Based on this spatial criterion, a probe consisting of six wires was fabricated to measure simultaneously the turbulent energy and temperature dissipation rates. The performance of this probe had been checked properly. Local isotropy of the small-scale turbulence structures across the wake was checked. The results show that in the region of $x_2/L \leq 1$, local isotropy is approximately satisfied. When x_2/L increases beyond 1, the departure from local isotropy becomes apparent due to the influence of large-scale turbulence structures at the edge of the wake.

The correlations among energy dissipation rate, temperature dissipation rate and the one-dimensional enstrophy (or vorticity) are examined using the measurements of the six-wire probe. While spanwise vorticity is highly correlated with the energy dissipation rate, the correlation between temperature dissipation rate and the vorticity field is small, in accordance with investigations based on direct

dissipation is further supported by the three-dimensional vorticity data measured using an eight-wire probe.

The scaling exponents of the longitudinal and transverse velocity structure functions obtained both spatially and temporally using two X-wire probes and their possible dependence on Reynolds numbers are examined in the intermediate wake. The scaling exponents of the transverse velocity increments are smaller than the longitudinal one for all moment orders ($n=1-8$) with the temporal one being even smaller than the spatial one. While the scaling exponents of the longitudinal velocity increments agree well with refined similarity hypothesis (RSH) and do not depend on R_λ , the Taylor microscale Reynolds number, the scaling exponents of the transverse velocity increments increase with R_λ . It is conjectured that when R_λ is high enough, the scaling exponents of the longitudinal and transverse velocity increments may be equivalent. The association of the transverse velocity increments with the one-dimensional enstrophy ($\Omega = 3\nu\omega_z^2$) verifies that the refined similarity hypothesis for transverse velocity structure functions (RSHT) is valid for the spatial transverse velocity increments, but is only partially valid for the temporal ones. The latter result is further supported by the measurements of the full enstrophy using an eight-wire probe.

The appropriateness of using the one-dimensional surrogate for energy dissipation rate is checked by comparing with the data measured by the six-wire probe and the eight-wire probe. It is found that the use of the one-dimensional surrogate, as usually used in experimental studies, may cause significant errors, especially for the instantaneous values even though its mean values agree well with other approximations.

The characteristics of the vortical structures were investigated using the

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wavelet multi-resolution technique by decomposing the vorticity components into a number of orthogonal wavelet components based on different central frequencies. The contributions from the wavelet components to the time-averaged velocity variances, Reynolds shear stress and vorticity variances were examined. The Reynolds number effects on the contributions from large-, intermediate and small-scale structures to the three-dimensional vorticity were also studied. It is found that the dominant contributions to the vorticity variances are from the intermediate and relatively small-scale structures. The contributions from the small-scale structures to all three vorticity variances jump significantly when Reynolds number is changed from 2000 to 4000, which is connected to previous observations in the near wake that there is a significant increase in the generation of small-scale structures once the Reynolds number reaches about 5000. This result reinforces the conception that turbulence 'remembers' its origin.

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NOMENCLATURES

A	calibration constant
B	calibration constant
c_ψ	constant, depending on the choice of wavelet
d	diameter of the cylinder
d_w	diameter of hot wire
$\overrightarrow{Dv_{m;n_1,n_2}}$	two dimensional discrete vector wavelet transform
e_1, e_2	output voltages from hot (cold) wires
E, E_c	mean voltage
$E(k)$	three-dimensional energy spectrum
f_0	central frequency of turbulent structures
f_c	cutoff frequency
f_{noise}	frequency at which the aliasing of high-frequency noise occurs
f_s	sampling frequency
f_K	Kolmogorov frequency, $\equiv U_1 / 2\pi\eta$
F_α	flatness factor of α , $\equiv \langle \alpha^4 \rangle / \langle \alpha^2 \rangle^2$
G	gain set of the CCA
I	current set of the CCA
k	wavenumber vector
k_1, k_2, k_3	wavenumbers in the x_1, x_2 and x_3 directions
l	length of hot (cold) wires

l_n	scale of eddy
L	velocity half width in x_2 -direction
L_S	integral length scale
n_1, n_2	uncorrelated noise fluctuations
$p(\beta \alpha)$	PDF of β conditioned on α , $= p_{\alpha,\beta} / p_\alpha$
Pr	molecular Prandtl number
Pr_T	turbulent Prandtl number
P_s	static pressure
P_T	total pressure
ΔP	pressure difference
r	length scale
r_{ij}	correction ratio for $\phi_{u_i,j}$
r_{ijji}	correction ratio for $Co_{u_i,j}u_{j,i}$
R	resistance of unheated cold wire
Re_L	Reynolds number, $= U_0 L / \nu$
Re_d	Reynolds number, $= U_1 d / \nu$
R_λ	Taylor microscale Reynolds number, $= u_1' \lambda / \nu$
S_α	skewness factor of α , $\equiv \langle \alpha^3 \rangle / \langle \alpha^2 \rangle^{3/2}$
δS_L	longitudinal velocity increments
δS_T	transverse velocity increments
T_0	temperature increase relative to ambient on the centerline of the wake

$s_{i,j}$	rate of strain, $\equiv (u_{i,j} + u_{j,i}) / 2$
$\langle T \rangle$	mean temperature
T_w	temperature of the heated wire
T_a	temperature of the ambient fluids
Δt	sampling time interval, $\equiv 1 / f_s$
u_i	velocity fluctuation in the x_i direction ($i=1,2,3$)
$u_{k,i}$	wavelet component of u_i at k^{th} wavelet level
$u_{i,j}$	derivative of u_i in the x_j direction, $\equiv \partial u_i / \partial x_j$
$\langle U \rangle$	mean velocity in the streamwise direction
U_1	free stream velocity
U_0	maximum velocity deficit of the wake
u_K	Kolmogorov velocity scale, $= \nu / \eta$
Δu_i	velocity difference
$\Delta u_i / \Delta x_j$	finite difference approximation for $\partial u_i / \partial x_j$
x_i	co-ordinate in streamwise ($i=1$), transverse ($i=2$) and spanwise ($i=3$) directions
Δx_i	hot or cold wire separations in the i direction ($i=1,2,3$)

Greek Symbols

α_T	thermal diffusivity
β	expansion coefficient
$\langle \beta \gamma \rangle_i$	wavelet component at i^{th} level

$\langle \beta \alpha \rangle$	conditional expectation, $= \int_{-\infty}^{+\infty} \beta p(\beta \alpha) d\beta$
$\Gamma(k)$	three-dimensional temperature spectrum
ε	instantaneous energy dissipation rate $\equiv 2\nu s_{ij}s_{ij}$
δ_{ij}	Kronecker delta, $=1$ when $i = j$; $=0$ when $i \neq j$
$\delta\alpha$	velocity increment, α represents u_1 , u_2 or u_3
$\Delta\theta$ or $\delta\theta$	temperature difference
$\Delta\theta/\Delta x_j$	finite difference approximation for $\partial\theta/\partial x_j$
$\zeta_\alpha(n)$	scaling exponent
η	Kolmogorov length scale, $\equiv (\nu^{3/4}/\langle \varepsilon \rangle^{1/4})$
θ	temperature fluctuation
$\theta_{,j}$	temperature derivative in the x_j direction, $\equiv \partial\theta/\partial x_j$
θ_K	Kolmogorov temperature scale, $\equiv (\chi\eta/U_K)^{1/2}$
κ	thermal diffusivity
λ_c, λ_K	wavelength of vortex street (Chapter 1)
λ	longitudinal Taylor microscale, $= u'_1/u'_{1,1}$
ν	kinematic viscosity
ν_T	eddy momentum diffusivity
ρ	air density
$\rho_{\alpha,\beta}$	correlation coefficient, $= \langle (\alpha - \langle \alpha \rangle)(\beta - \langle \beta \rangle) \rangle / \sigma_\alpha \sigma_\beta$
ρ_{u_1}	correlation coefficient between $u_1(x_2)$ and $u_1(x_2 + \Delta x_2)$
ρ_θ	correlation coefficient between $\theta(x_2)$ and $\theta(x_2 + \Delta x_2)$

σ_α	standard deviation of α
τ	temporal delay
υ	non-dimensional stochastic variable
$\phi_{i,j}(\mathbf{k})$	three-dimensional spectral density tensor of velocity
$\phi_\alpha(k_1)$	one-dimensional spectrum of α , defined as $\int_0^\infty \phi_\alpha(k_1) dk_1 = \langle \alpha^2 \rangle$
$\phi_{u_i}(k_1)$	one-dimensional spectrum of u_i
$\phi_{u_{i,j}}(k_1)$	one-dimensional spectrum of $\partial u_i / \partial x_j$
$\phi_\theta(k_1)$	one-dimensional spectrum of θ
$\phi_{\theta,j}(k_1)$	one-dimensional spectrum of $\partial \theta / \partial x_j$
$\phi_{\omega_i}(k_1)$	one-dimensional spectrum of ω_i
χ	instantaneous temperature dissipation rate, $\equiv \kappa \theta_{,i} \theta_{,i}$
ω	vorticity magnitude
ω_i	vorticity component in the x_i direction
$\psi(t)$	mother wavelet
$\bar{\psi}(\frac{t-\tau}{\lambda})$	complex conjugate of $\psi(\frac{t-\tau}{\lambda})$, where λ is scaling coefficients
$\Psi(\omega)$	Fourier spectrum of the wavelet
Ω	enstrophy, $\equiv \omega_i \omega_i$

Superscripts

c	corrected value
cal	calculation

m	measured value
$\langle \rangle$	time average
$'$	root-mean-square value
$+$	normalization by L , U_0 (T_0) and R_L^* (Eq. 3.22) or L , U_0 (T_0)
$\dagger$	normalization of conditional expectation by mean value
$*$	normalization by Kolmogorov scales: u_K , θ_K , f_K or η

Subscripts

ap	approximation
f	full turbulent energy or temperature dissipation rate
iso	isotropy
r	locally averaged value

Chapter 1 Introduction

1.1 Introduction and Literature Review

The study of the turbulent wake flow behind cylinder structures is important to understand the dynamics and the transfer processes of momentum, heat and mass transfer in most flows of engineering interest. The identification of the structures present in such flows could be used to control turbulence, which is important for many practical applications, such as the pollutant transport and dispersion in the atmosphere and in the ocean downstream of a wastewater discharge pipeline. The findings and outcomes can also be employed in many important engineering applications, such as heat exchangers, offshore pilings, tower of post-tensioning bridges, chimneys and piping in power plants.

In a physical domain, turbulent flows contain eddies of various sizes. Energy is injected into the flow through the biggest eddies. These energy containing eddies break down into smaller ones in the flow. An ideal model of the eddy and energy cascade is demonstrated in Figure 1.1, where the eddies of various sizes are represented as blobs stacked in decreasing sizes. The uppermost eddies have a scale of l_0 . The successive generations of eddies have scales $l_n = l_0 r^n$, ($n = 0, 1, 2, \dots$), where $0 < r < 1$. The smallest eddies have scales comparable with η , the Kolmogorov dissipation scale. The number of eddies per unit volume is assumed to grow with n as r^{-3n} to ensure that small eddies are as space filling as large ones. Energy, introduced at the top at a rate of ε , is 'cascading' down to the hierarchy of eddies, and is eventually dissipated to the bottom of the 'cascade' at the same rate ε . This is normally known as the energy cascade based on Kolmogorov (1941)'s theory.

Figure 1.1 The cascade according to the Kolmogorov 1941 theory.

The energy transport from large to small scales within a turbulent flow can also be examined in a spectral domain. Figure 1.2 shows energy spectrum of streamwise component of the velocity fluctuation in an axisymmetric jet (Frisch, 1995), corresponding to a Taylor microscale Reynolds number $R_\lambda (\equiv u'_1 \lambda / \nu$, where $\lambda = u'_1 / u'_{1,1}$ is Taylor microscale) of about 626. Three regions can be distinguished in the spectrum. For $k_1^* < 0.002$ (k_1^* is the wavenumber in the x_1 direction and normalized by Kolmogorov frequency), there is significant contribution of energy to the spectrum and this region is normally referred to as large-scale structures. The region of $k_1^* = 0.002 - 0.09$ is normally called inertial range and the region of $k_1^* > 0.25$ is called the dissipative range. The dissipative range and the inertial range are normally referred to as small scales. According to Kolmogorov (1941)'s similarity hypotheses, in the case of turbulence with sufficiently high Reynolds number, the statistics of the velocity increments for a given separation r within the

dissipative range are unambiguously defined by the average energy dissipation rate $\langle \varepsilon \rangle$ and the fluid kinematic viscosity ν . When r falls within the inertial range, the statistics of the velocity increments are unambiguously determined only by $\langle \varepsilon \rangle$ and are independent of ν . Small-scale turbulence at sufficiently high Reynolds numbers is statistically independent of the large scale, and is therefore locally isotropic. It is now well known that Kolmogorov (1941)'s theory cannot be perfect since many facets of turbulence, in particular the strong variability of the energy dissipation rate, are ignored. Kolmogorov (1962) refined his original theory to take into account the spatial fluctuation in the turbulent energy dissipation rate ε at scale r . This is normally known as the refined similarity hypothesis (RSH). RSH states that the non-dimensional stochastic variable

$$\nu = \frac{\delta u_1}{(r\varepsilon_r)^{1/3}}, \quad (1.1)$$

where $\delta u_1 [\equiv u_1(x_1 + r) - u_1(x_1)]$ is a universal function of the Reynolds number $\text{Re}_r [\equiv r(r\varepsilon_r)^{1/3} / \nu]$.

Figure 1.2 Energy spectrum of the longitudinal velocity at $R_\lambda = 626$ in a jet flow (Frisch, 1995).

Recently, the refined theory of Kolmogorov (1962) has been extended to passive scalar fields (RSHP) (e.g. Stolovitzky and Sreenivasan, 1995). The refined similarity hypothesis for passive scalars states that the n^{th} -order moment of the temperature increment $\delta\theta = \theta(x_1 + r) - \theta(x_1)$ can be expressed as

$$\langle (\delta\theta)^n \rangle \sim \langle \chi_r^{n/2} \varepsilon_r^{-n/6} \rangle r^{n/3} \sim r^{\zeta_\theta(n)}, \quad (1.2)$$

where χ is the temperature dissipation rate and ε is the energy dissipation rate. Therefore, the small-scale scalar characteristics are influenced not only by ε and χ but also by their correlations. The correlation between ε_r and χ_r is important in quantifying $\langle (\delta\theta)^n \rangle$ and perhaps in a more physical sense, ascertaining the interdependence between the two dissipative fields through their relation to the vortical structure of the flow.

Bluff body wake flows have direct engineering significance. The alternate shedding of vortices in the near wake (e.g. $x_1/d < 40$) in the classical vortex street configuration leads to large fluctuating pressure forces in a direction transverse to the flow and may cause structural vibrations, acoustic noise, or resonance, which in some cases can trigger failure. Therefore, the study of bluff-body flows has been a subject of interest to engineers and scientists for many decades. However, bluff body wakes are complex; they involve the interactions of three shear layers in the same problem, namely a boundary layer, a separating free shear layer, and a wake. As has been recently remarked by Roshko (1993), "the problem of bluff body flow remains almost entirely in the empirical, descriptive realm of knowledge," although our knowledge of this flow is extensive. It is relatively well known that the fundamental wake instability results in the formation of the classical von Kármán street configuration for $Re_d > 49$, which is normally considered as two-dimensional. The two-dimensional aspects of this wake instability are well understood. However, it is

only recently that a large number of investigations have been concerned with the development of three-dimensional structure in the near region of cylinder wakes at intermediate and high Reynolds numbers (e.g. Hussain and Hayakawa, 1987; Kiya and Matsumura, 1988; Wu et al. 1996, Zhang et al., 2000; Rinoshika and Zhou, 2004; Rinoshika and Zhou, 2005). In an early study, Hama (1957) noted that the near wake of cylinders was three-dimensional based on the finding that a transverse waviness appeared in his flow visualization at Reynolds numbers higher than 150. Using hot-wire anemometry and cross-correlation techniques, Grant (1958) studied the wake of circular cylinders. He found that when Reynolds number exceeds certain value, the Kármán vortices develop three-dimensionalities composed of counter-rotating streamwise vortices. Gerrard (1978) conducted a flow visualization study of cylinder wakes in which he observed what he termed 'fingers' of dye forming along the span of his cylinder at Reynolds numbers above 140. With hindsight, it seems reasonable to conclude that the transverse waviness of Hama (1957) and the fingers of Gerrard (1978) resulted from the same phenomenon, i.e. the presence of longitudinal vortices inclined at an angle to the streamwise direction. In recent years, direct evidence of the existence of the three-dimensional vortices using flow visualization in the near or intermediate wake has been provided. It is generally accepted, based on flow visualization, that there are two major types of vortical structures in the near wake, that is, a quasi-two-dimensional spanwise structure or the Kármán vortex, with predominantly spanwise vorticity and quasi-longitudinal structure in the (x_1, x_2) -plane with x_1 - and x_2 -components of vorticity. The latter is a rib-like structure, wrapping around and connecting consecutive spanwise structures (e.g. Hussain and Hayakawa, 1987; Kiya and Matsumura, 1988; Chyu and Rockwell, 2002; Rinoshika and Zhou, 2005). A schematic of these structures (Hussain and Hayakawa, 1987) are presented in Figure 1.3. It is of fundamental interest to

understand how the two types of vortical structures are related and how they contribute to the three vorticity components.

Figure 1.3 Schematic of spanwise and rib-like structures in the near wake (Hussain and Hayakawa, 1987).

Flow visualizations (e.g. Taneda, 1959; Cimbala, 1985) and hot-wire measurements (e.g. Grant, 1958; Townsend, 1979) in the turbulent far wake of a circular cylinder have also highlighted the existence of large-scale organized motions alternating around the centerline, at sufficiently large values of the Reynolds number, typically $Re_d > 300$ for the flow to be considered turbulent. The distance from the cylinder was also sufficiently large (typically in the range $x_1/d = 100 \sim 1000$) for far wake conditions to prevail. In the far wake, the alternating arrangement seems to prevail, but the motion is much less regular than in the near wake. Taneda (1959), using flow visualization in a water tunnel, found a far-wake structure somewhat reminiscent of the Kármán vortex street. He reported the appearance of a "new Kármán vortex street" for $Re_d \approx 300$ and a distance as large as $x_1/d = 1600$. He also stated that the wavelength λ_c of this street could not be determined precisely in view of the large scatter in the photographic data but it was significantly larger, by about 8.5, than the wavelength λ_K of the periodic "primary" Kármán vortex street observed

in the near-wake of the cylinder. Subsequent investigations confirmed the quasi-periodic nature of the "secondary" Kármán-like vortex street. Keffer (1965) found that a wake, which was subjected to a uniform and constant rate of strain exhibited eddies that were more regular and intense than for the unstrained wake. Keffer's observations indicated a value for the ratio λ_c/λ_K in the range of 3-4. Papailiou and Lykoudis (1974), who established the persistence of the secondary vortex street at very large Reynolds numbers, emphasized that the effect of the strain acted to amplify an already existing set of relatively well organized structures. Their measurements of λ_c for relatively small values of x_1/d ($=20-135$) suggested a value of λ_c/λ_K which was only slightly larger than unity and which increased slowly with x_1/d . Cimbala (1985), using smoke-wire flow visualization and hot-wire anemometry, showed that the far-wake structure does not depend directly on the scale λ_K or frequency f_K of the Kármán street shed by the cylinder. He also found that the growth of this structure resulted from a hydrodynamic instability of the mean wake profile; the characteristic frequency of this structure decreasing with x_1 in agreement with an inviscid linear stability theory that takes into account the streamwise growth of the wake. This is in agreement with the suggestion of Taneda (1959). The experiments of Cimbala (1985) also ruled out the conclusion by Matsui and Okude (1981) that the far-wake organized structures were caused by the amalgamation or pairing of the Kármán vortices. Experiments with different initial conditions, e.g. porous flat plates (Cimbala, 1985), have indicated that, even when the initial Kármán street is absent or eradicated, the secondary vortex-street is observed sufficiently far downstream of the wake-generating bodies. The latter observations do not preclude the possibility that in the case of a stationary cylinder, the Kármán vortex streets may play a part in establishing the formation of "the secondary street". Cimbala (1985) ($Re_d \approx 100$)

reported strong experimental evidence in support of Taneda's (1959) stability hypothesis. They employed the smoke-wire flow-visualization technique and hot-wire anemometry to show that concentrated vortices of the Kármán vortex street decay exponentially. As the very rapid decay of the Kármán vortex street has been known for quite some time (see for example Roshko, 1993), this fact has tended to be obscured, however, owing to misinterpretation of streakline flow visualization. Cimbala (1985) showed that at $x_1/d=100\sim150$, the initial vortex street can no longer be found. Thus, they concluded that the development of any subsequent structure cannot be directly dependent upon the original vortex street (i.e. amalgamation) but rather must be the result of local wake instability of the developing mean velocity profile.

There is flow visualization evidence (Keffer, 1965; Antonia and Fulachier, 1986), however, to suggest that the organized motion in the far wake is sometimes symmetrical, with vortices opposite each other across the centerline. Keffer (1965) attributed the lack of correlation between bulges on opposite sides of the centerline to the fact that they could be either alternately spaced or arranged symmetrically. This observation is also supported by the photographs of dye streaklines (Antonia and Fulachier, 1986) in a water tunnel, where streaklines introduced on opposite sides of the centerline in the far wake of a cylinder were usually deformed alternately around the centerline but symmetrical displacements of the streaklines were also observed to occur.

Zdravkovich (1968) studied the development of the wake behind a set of three circular cylinders in close proximity. He found that the shed vortices from the three cylinders rapidly decayed as they interacted with each other. Subsequently, a new vortex street of larger scale appeared downstream. Zdravkovich (1968) explained the formation of the secondary vortex street as a coupling of two

mechanisms: shear-layer instability and a rolling-up process induced by the distributed vorticity in the wake.

Using multiple hot-wire arrays, Townsend (1979) found periodic flow patterns resembling Kármán vortex streets for a Reynolds number $Re_d=8000$ at $x_1/d=170$. 'Groups' of three to five vortical structures were observed, the passage frequency within a group being uniform but varying considerably from one group to the next. Regions of random fluctuations seem to fill the gaps between successive groups. Long-time-averaged spectra therefore do not show a peak at any particular frequency. Townsend (1979) estimated that these eddy groups contribute about 15~20% of the total turbulent energy. Such grouping of structures had previously been suggested by Gupta et al. (1971) for the turbulent boundary layer. Matsui and Okude (1981) later abandoned Taneda's (1959) original conjecture that the secondary street results from hydrodynamic instability, following the 'breakdown' of the primary street. Instead they proposed that pairing is the mechanism for the change in scale. Matsui and Okude's (1981) conclusion, however, was based partly on flow visualization, where the flow tracer was introduced upstream of the cylinder. However, this type of visualization can be very misleading (Cimbala et al., 1988). Furthermore, with a pairing mechanism the frequency of the secondary street ought to be half that of the primary street, but this is not the case for all Reynolds numbers.

Matsui and Okude (1981) again studied the wake at $Re_d = 140$. Using simultaneous hot-wire measurement and smoke visualization at $x_1/d = 75$, they showed that the smoke lumps are indeed vortices at this location. Therefore, they concluded, based on flow visualization of these lumps, that pairing is the predominant mechanism for the change of scale. The frequency ratio between the primary Kármán vortex and the secondary vortex is not always exactly one-half because not all of the vortices pair up; a few are left stranded, causing some

irregularity. However, when Matsui and Okude (1983) acoustically excited the wake at $1/2$ or $1/3$ of the Kármán vortex frequency, they observed regular pairing or tripling, respectively.

In contrast to the conclusions of Matsui and Okude (1983), Desruelle (1983) confirmed the findings of Taneda (1959) in a different flow facility. In addition they used acoustic excitation to experimentally derive the stability characteristics of the wake. As expected, the band of amplified frequencies shifts to lower values and broadens as downstream distance is increased. Later, Wygnanski et al. (1986) introduced disturbances at the trailing edge of a flat plate, and investigated the spatial growth rate of the vortex. They found that the turbulent wake response corresponds surprisingly well to that calculated from linear stability theory applied to the mean flow and with its growth taken into account.

Recently, Vernet et al. (1999, 2002) argued that Townsend's (1956) and Grant's (1958) double roller and mixing jet structures might possibly be the two different aspects of a single horseshoe-shaped structure in the turbulent far wake. Using wavelet multi-resolution techniques, Rinoshika and Zhou (2005), in the near wake of a circular cylinder, found the spanwise vorticity contours of the wavelet component at the averaging frequency of Kármán vortices display a secondary spanwise structure near the saddle point, whose vorticity is opposite-signed to that of the Kármán vortices. This structure is observed for the first time and its occurrence is consistent with the streamwise decay in the vorticity strength of the spanwise structures.

Hussain and Hayakawa (1987) pointed that vorticity is the least ambiguous quantity to use for identifying organized structures. Ideally, to study the spatially three-dimensional vortical structures, the three components of the vorticity vector

$$\omega_1 = \frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}, \quad (1.3)$$

$$\omega_2 = \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}, \text{ and} \quad (1.4)$$

$$\omega_3 = \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}, \quad (1.5)$$

where u_1 , u_2 and u_3 are the velocity fluctuations in the x_1 (streamwise), x_2 (vertical) and x_3 (spanwise) directions, respectively, should be measured simultaneously in the whole flow field. This is, however, impractical in the context of the hot-wire measurement because of the requirements of a large number of probes and a possible blockage to the flow. Therefore, one has to settle a simpler and less rigorous measurement scheme, often relaxing to a limited number of probes. Hussain and Hayakawa (1987) and also Zhou and Antonia (1993) studied the topology of large-scale structures based on the measurement from an array of eight X-wire probes. Zhou and Antonia (1994) investigated the critical points associated with large-scale vortical structures in a velocity field measured from sixteen X-wire probes. Kiya & Matsumura (1985) developed a phase-average technique to study the topology and momentum transport of coherent and incoherent structures in a cylinder wake. A similar technique was employed by Zhou et al. (2002) to investigate the wake of two side-by-side cylinders. This technique requires only one moving measurement probe and one fixed single or X-wire (the latter probe serves the need to provide a reference signal only), thus providing a good compromise between the sophistication of the vorticity probe and the need to measure the flow field for the study of the large-scale vortical structures. However, the phase-average technique is performed based on the vortex shedding frequency only. It cannot provide any information on the contribution from the various turbulence structures to the

turbulent energy, Reynolds stress and vorticity variances. Recently, the wavelet analysis has widely been used to analyze turbulence structures (e.g. Bonnet, 1998; Hangan, 2001; Zhou and Li, 2003; Schram 2004; Rinoshika and Zhou 2004; Rinoshika and Zhou 2005). This technique enables the examination of the turbulent structures in terms of time and scale by separating the turbulent structures into a number of subsets based on their central (or characteristic) frequencies which are representative of the turbulent structures of different scales.

The good estimate of velocity derivatives is crucial for the measurements of vorticity (Eqs. 1.3-1.5). The simplest way to measure velocity derivatives using hot wires is to approximate them by finite differences, i.e. $\partial\alpha/\partial\beta \approx \Delta\alpha/\Delta\beta$ (where α represents u_1 , u_2 or u_3 and β represents x_1 , x_2 or x_3). The measured values obtained from the finite differences may depend on the wire separation. Generally, too large a wire separation will cause spectral attenuation of the velocity derivatives and result in underestimation of $\partial\alpha/\partial\beta$, while too small a separation may cause noise contamination between the wires, leading to the overestimate of the derivatives. Therefore, it is desirable to find an optimal wire separation at which velocity derivatives can be approximated properly by finite differences. Klewicki and Falco (1990) proposed a wire separation of $1-3.33\eta$ for velocity derivative measurements, in line with that proposed by Wyngaard (1969). The latter suggested a separation of less than 3η between a pair of parallel hot wires for the accurate measurements of $\partial u_1/\partial x_2$. Based on the assumption of local isotropy, Antonia and Mi (1993) extended the analysis of Wyngaard (1969) on spectrum correction to parallel wires. Extensive measurements (e.g. Antonia et al., 1993b; Shafi et al., 1997) on $\partial u_1/\partial x_2$ or $\Delta u_1/\Delta x_2$ using parallel wires had been carried out on the centerline of a circular jet, a plane jet, in a turbulent channel flow and in a rough wall turbulent

boundary layer. These results, even though obtained in different flows and at different Reynolds numbers, have shown an optimum wire separation of $3-5\eta$ for the accurate measurement of $\partial u_1 / \partial x_2$. This separation is slightly larger than that suggested by Wyngaard (1969) and Klewicki and Falco (1990). While the above measurements were mainly done at locations where local isotropy is approximately satisfied, the influences of anisotropy and mean shear on velocity derivative measurements have not been examined.

The small-scale properties of turbulence can best be investigated by vorticity (or velocity derivatives), as pointed out by Corrsin and Kistler (1955), in the sense that vorticity (or velocity derivatives) mainly contains information about small scales of turbulence. The characteristics of vorticity statistics should be close to local isotropy, irrespective of the values of Reynolds number and initial conditions of the flow (Antonia et al., 2002). Therefore, the measurements of vorticity are important in the viewpoints of examining the characteristics of both large-scale and small-scale turbulence structures.

In the previous few decades, there had been many attempts to study the vorticity characteristics of turbulent flows either by experiments or numerical simulations (Tsinober et al., 1992; Marasli et al., 1993; Wallace and Foss, 1995 and references therein). The simultaneous measurement of all three components of the vorticity fluctuations was first attempted by Wassman and Wallace (1979) using a nine-sensor probe, arranged in three arrays. A common central prong for each array was used to make the probe as small as possible. Various modifications have been made (e.g. Balint et al., 1987) to the nine-sensor probe to improve its performance either by separating the wire prongs or arranging the wires in different configurations. To eliminate electronic cross-talking between the wires caused by the use of a common prong, Tsinober et al. (1992) further modified the nine-sensor

probe of Balint et al. (1987) by adding an additional sensor to each array (e.g. a total of 12 sensors) and using separate prongs for each sensor. Recently, more complicated probes have been developed to measure the three vorticity components simultaneously (e.g. Lemonis, 1995). Statistics of vorticity components have been obtained in several flows such as a mixing layer (Balint et al., 1987), grid and boundary layer turbulence (Tsinober et al., 1992; Lemonis, 1995) and the wake of a circular cylinder (Marasli et al., 1993). Using a simpler configuration, Zhu and Antonia (1996) employed 4 X-wires, with two aligned in the x_1 - x_2 plane and the others in the x_2 - x_3 plane, to measure all three components of the vorticity vector simultaneously in a turbulent far-wake. This probe has the advantages of ease in making wires, ease in calibrating the probe and the possibility for correcting the effect of spatial resolution.

Apart from hot wire techniques, several optical techniques have been developed to measure one or more components of the vorticity vector. These techniques include laser Doppler anemometry (LDA) (e.g. Wallace and Foss, 1995 and references therein), particle imaging velocimetry (PIV) and scalar imaging (e.g. Su and Dahm, 1996a, b; Li et al., 2002).

1.2 Objectives and Scope

Due to the importance of the vorticity in studying large-scale and small-scale turbulence, efforts have been made to measure one, two and all three components of the vorticity vector. Before the vorticity measurement is carried out, efforts should be focused on the spatial criteria for velocity derivative measurements and the fabrication of the vorticity probe. Therefore, the first objective of the present work is to get the optimum separation between two wires by using two parallel wires (hot and cold) with changeable separations. Since vorticity dominated phenomena must be associated with the small-scale structure of turbulence, it is necessary to check how closely the statistics of the vorticity fluctuations and their components conform to local isotropy. Consequently, the second objective of the present work is to study the small-scale structure of vorticity and its agreement with local isotropy. Since the energy and temperature dissipation rates are important quantities of the small-scale properties of turbulence, the third objective of the present work is to measure these quantities simultaneously with multi-hot (cold) wire probes. Kolmogorov's scaling phenomena is crucial to understand the small-scale turbulence. For this reason, the fourth objective of the present work is to study the scaling exponents of the velocity increments (both longitudinal and transverse) and their possible Reynolds numbers dependence by using two X-wire probes. Although the general picture of energy cascade from the larger scale to smaller scale structures in turbulent flow was accepted, the relationship between the multi-scale vortical structures, dynamics and statistics of the large scale motions and small scale motions are not yet clear. Therefore, the last objective of the present work is to examine the turbulent structures using wavelet multi-resolution analysis based on the three-dimensional vorticity measurements. The scope of the present work can be summarized as follows:

1. To provide spatial criteria for the fabrication of vorticity probe by using two parallel wires (hot and cold) with changeable separations. The effect of wire separation on the measurements of velocity and temperature derivatives will be examined.
2. To check the extent to which local isotropy is satisfied using data of velocity (and temperature) derivatives and vorticity fluctuations obtained in the far region of a cylinder wake.
3. To validate the performance of the one- and three-dimensional vorticity probes.
4. To examine the correlations between vorticity, energy dissipation rate and temperature dissipation rate using data obtained from simultaneous measurements of one- and three-component vorticity and temperature dissipation rate.
5. To investigate the scaling exponents of the longitudinal and transverse velocity structure functions and their dependence on Reynolds number at the centerline of a turbulent wake flow using data obtained from two X-wire probes.
6. To examine the effects of the different intermittencies associated with the locally averaged energy dissipation rate and enstrophy fluctuations on the scaling of longitudinal and transverse velocity increments.
7. To study the contributions to Reynolds shear stress, velocity and vorticity variance from different wavelet components based on their central frequencies using the wavelet multi-resolution analysis.

Chapter 2

Experimental Setup and Instrumentation

In this chapter, only basic experimental setup and measurement techniques are provided. Experimental details for different studies will be given in the respective chapters.

2.1 Wind Tunnel

The experiments were conducted in a closed loop wind tunnel with dimensions of 1.2m (width) $\times$ 0.8m (height) and 2.4m long. The schematic diagram of the test rig is given in Figure 2.1, showing the location of and orientation of the test specimen, the location of the fan, and the various parts of the wind tunnel. The wind tunnel consists of two test sections, namely the high speed and the low speed sections. The present measurements are conducted in the high speed test section. By using the high speed test section, the speed of the motor can be reduced, and as a consequence the noise generated by the motor can be lower. The technical specifications of the two sections of the tunnel are given in Table 2.1

Figure 2.1 The schematic diagram of the test rig.

Table 2.1 Technical specifications of the wind tunnel.

Test Sections	High Speed Section	Low Speed Section
Dimension	W1.2m * H0.8m * L2.4m	L3m*W2.4m*H1.6m
Max. Air Flow	72m/s	18m/s
Velocity Drift	1.5%	2%
Turbulence Level	0.6%	1%
Flow Uniformity	1%	1.5%
Contraction Ratio	4	Nil
Type of Fan	3 Blades Propeller, Cessna Aircraft	
Size of Motor	260 hp 460V 450A DC	

The schematic diagram of the test specimen is given in Figure 2.2, showing the spanwise, transverse and longitudinal directions together with various regions behind the cylinder. The measurement location is indicated by x_1/d , where d is the diameter of the circular cylinder (stainless steel). U_1 is the free stream velocity and U_0 is the maximum velocity deficit. L is the half-width of the wake.

Figure 2.2 Definition of the coordinate system and the schematic diagram of test specimen.

2.2 Measuring System

2.2.1 Hot-Wire and Cold-Wire Anemometers

Hot-wire anemometer (HWA) is one of the most popular methods for velocity measurement in air flows because of its high temporal resolution. It is based on the principle that a transducer element with a temperature dependent resistance is heated

by an electric current and placed in the flow. The electronic circuit measures the cooling of the sensor caused by the flowing medium. In the constant temperature anemometer (CTA), the cooling is measured by the power needed to maintain the sensor at a constant temperature. To measure temperature fluctuation in a turbulent flow, constant current anemometer (CCA) is widely used. In the CCA, the cooling is measured by the power dissipated in the sensor when a constant current is maintained.

The hot wire is heated by passing an electrical current through the wire exposed to a fluid flow under investigation. Heat transfer then takes place from the filament wire to the flow. The transfer of thermal energy increases with increasing flow rate in the neighborhood of the filament. This heat is given up to the flow into which the wire is inserted by conduction, free and forced convection, and by radiation, the last component, however, is relatively small in magnitude and can be neglected.

The principle of a CTA circuit is illustrated in Figure 2.2. The hot wire probe is placed in a wheatstone bridge, of which the hot wire forms one of the arms.

Figure 2.3 Wheatstone bridge in CTA

When the fluid passing through a hot wire, the velocity fluctuation of the fluid causes a change in the convection coefficient of heat transfer of the sensor. The resulting cooling of the wire element is recorded by the sensitive Wheatstone bridge. Variation in resistance is monitored by various electronic methods which give signals related to the flow variation. The hot wire method can therefore be used for measuring instantaneous flow at a particular point.

Before the system is placed in operation, the adjustable resistor is set to a value larger than that is required to balance the bridge. When power is applied, the feedback amplifier increases the sensor resistance until the bridge becomes balanced. An increase in flow velocity cools the sensor and unbalances the bridge. A differential feedback amplifier detects the bridge unbalance and automatically adjusts the sensor current to maintain bridge balance, thus holding the wire temperature remains virtually constant as the velocity changes. The voltage difference of the bridge is proportional to the fluid velocity and hence used as a measurement input to determine the velocity at that point in the flow.

In contrast to the CTA, the CCA supplies constant heating to the sensor at low overheat ratio. It is termed 'cold' as the current carried is too small to heat the sensing element appreciably and it thus operates as a resistance thermometer. As shown in Figure 2.3, the CCA contains both a wheatstone bridge circuit and an amplifier, but without the use of feedback technique. Instead, the bridge current is provided by a constant power supply.

In this application, the wheatstone bridge is balanced only at one flow and becomes unbalanced as the flow changes. The voltage difference of the CCA bridge is a function of flow temperature.

Figure 2.4 Wheatstone bridge in CCA

2.2.2 Pitot-Static Tube

Pitot-static tube is placed inside the closed wind tunnel to measure the difference between the static pressure (P_s) and total pressure (P_t). The pressure difference is indicated by the manometer and will be used to calculate the mean velocity of the flow by Bernoulli's equation. Bernoulli's equation, when applied to the Pitot-static tube, can be written as:

$$\frac{P_s}{\rho} + \frac{U^2}{2} = \frac{P_t}{\rho}, \quad (2.1)$$

where U is the mean velocity of the flow and ρ ($\approx 1.18 \text{ kg/m}^3$) is air density. The mean velocity can then be obtained via.

$$U = \sqrt{2(P_t - P_s) / \rho} = \sqrt{2 \cdot \Delta P / \rho}, \quad (2.2)$$

where Δp is the measured pressure difference.

In the present study, the inclined manometer (Type5, Airflow Development LTD., England) together with a Pitot-static tube were used to calibrate the hot-wire probes. The red manometer fluid with a constant density of 0.784Sg (Specific gravity) was used to indicate the manometer readings (or pressure differences, ΔP). The close

fitting of the scale to the glass tube minimizes parallax and the selected manometer fluid ensures a crisp meniscus for accurate reading. Before calibrating the hot-wire probes, the horizontal level of the base of the manometer was adjusted to be horizontal by using the attached spirit level. After that, the initial reading of the manometer fluid was adjusted to zero at zero velocity. In addition, the connection tubing was checked to be in good condition, without damage or leakage. Finally, during calibration the reading of the manometer was recorded only after the level of the manometer fluid was stable. Following these procedures, the errors of the manometer readings could be minimized. Therefore, the manometer was not checked for its calibration.

2.2.3 Calibration of the Hot and Cold Wires

(A) Calibration of the Hot Wire

The hot-wire probe needs to be calibrated to obtain a relationship between the mean velocity measured by the Pitot-static tube and the mean voltage, the output of the CTA. From the modified King's Law, the relationship can be presented as (Bruun, 1995):

$$E^2 = A + B \cdot U^n, \quad (2.3)$$

where A , B and n are the calibration constant; E is the mean voltage. The relationship between E and U can also be expressed using a polynomial relation:

$$U = \sum_{i=0}^k a_i \cdot E^i. \quad (2.4)$$

The CTA's (voltage) resolution cannot be simply dictated by the A - D board. The express of U in a polynomial form of E in Eq. (2.4), compared with Eq. (2.3), has computational advantage with a good accuracy. In the calibration of present study, $k = 3$ is chosen. That is

$$U = a_0 + a_1 \cdot E + a_2 \cdot E^2 + a_3 \cdot E^3. \quad (2.5)$$

By applying polynomial fit to the measured data, coefficient a_i can be obtained. These coefficients can be used as input to convert voltage signals to velocity using MATLAB[®] programs.

(B) Calibration of the Cold Wire

The cold wire also needs to be calibrated to evaluate its temperature coefficient α . The output of a CCA (voltage) is linearly related with the temperature above ambients (Bruun, 1995):

$$T = \frac{E_c}{\alpha G I R} + C \quad (2.6)$$

where α is the cold-wire temperature coefficient,

E_c is the output voltage,

G is the gain set of the CCA,

I is the current set of the CCA,

R is the resistance of unheated cold wire,

T is the fluid temperature, and

C is the calibration constant.

The temperature sensitivity of a cold wire varies with its resistance. It is therefore necessary to calibrate every cold-wire probe to obtain the temperature coefficient after it has been fabricated. After the temperature coefficient α is obtained, Eq. (2.6) is used to convert voltage signals from a CCA to temperature signals. The mean temperature T of the flow is measured by a thermocouple (type K) with HP-34970A data acquisition system.

2.3 Hot and Cold Wire Probe

In the present study, five hot and cold wire probes have been used:

1. Two-wire probe consisting of two parallel hot wires to measure velocity derivatives.
2. Two-wire probe consisting of two parallel cold wires to measure temperature derivative.
3. Four-wire probe consisting of two X-wires to investigate the scaling exponents of the longitudinal and transverse velocity structure functions.
4. Six-wire probe consisting of four hot wires and two cold wires for the approximation to energy and temperature dissipation rates simultaneously.
5. Eight-wire probe consisting of four X-wires for three-dimensional vorticity measurements.

Details of the probe geometries and the related specifications can be found in individual chapters.

2.4 Experimental Uncertainty Analysis

The systematic and random errors of hot- or cold-wire anemometry are caused by many factors such as frequency response, signal processing, noise contamination, spatial resolution and possible disturbance caused by the probe. Most of these errors have been discussed in Bruun (1995).

In the present study, experimental uncertainties are inferred from errors in the hot- and cold-wire calibration data as well as from the scatter (20 to 1 odds) observed in repeating the experiment a number of times.

For measurements involving two parallel hot (cold) wires, the uncertainties in U_1 and T are about $\pm 1\%$. The uncertainties in u_1' and θ' are about $\pm 2\%$. Using

the method of propagation of errors (e.g. Taylor, 2002) and taking into account the errors in measuring the separation between the two parallel wires ($\pm 1\%$), the uncertainty in velocity and temperature derivative measurements is estimated to be around $\pm 3\%$.

For measurements involving X-wire(s), it is assumed that each X-wire measures the two velocity components at the center of the probe. While the measured velocity components can be significantly in error when the velocity gradients are large (e.g. Park and Wallace 1993; Petrovic et al. 2003), the mean velocity gradient is not significant in the present wake flow so that the effect of the mean velocity gradient within each X-wire is negligible. The cooling effect due to the binormal velocity component on each X-wire has been neglected, which only gives rise to an error of about 1-3% for u_i' when the local turbulence intensity is about 10% (Tutu and Chevray 1975). The present local turbulent intensity is about 2% and thus the errors in u_i' neglecting the binormal cooling effect is estimated to be smaller than $\pm 2\%$. Using the error propagation analysis and taking into account the errors in measuring the distance between wires (about $\pm 3\%$), the maximum uncertainty in velocity derivative measurements is estimated to be around $\pm 5\%$. Therefore, the influence of the fluctuating binormal velocity component can be neglected in the present study.

Using the previous estimates, uncertainties for other quantities can be estimated by using the method of propagation of errors (e.g. Taylor, 2002). The resulting experimental uncertainties for the three vorticity components are around $\pm 7\%$. It should be noted that the vorticity uncertainty does not include the errors due to the spatial resolution of the specific probe. Errors caused by spatial resolution will be discussed and corrected in individual chapters.

Chapter 3

Transverse Velocity and Temperature Derivative Measurements

3.1 Introduction

In turbulent flows, the small-scale motion depends only on the rate at which it is supplied with energy by the large-scale motion and on the kinematic viscosity. The rate of energy supply should be equal to the rate of dissipation. With this conception, viscous dissipation of energy in small-scale can be estimated from the large-scale dynamics, which do not involve viscosity. This estimate is one of the cornerstone assumptions in turbulence theory (e.g. Tennekes and Lumley, 1972).

The turbulent energy dissipation rate $\varepsilon \equiv 2\nu s_{ij}s_{ij}$ and the temperature dissipation rate $\chi \equiv \kappa \theta_{,i}\theta_{,i}$ are important quantities of the small scale properties of turbulence, where $s_{ij} = (u_{i,j} + u_{j,i})/2$ is the rate of strain, ν is the kinematic viscosity, $u_{i,j}$ represents velocity derivative $\partial u_i / \partial x_j$; $\theta_{,i}$ represents temperature (or passive scalar) derivative $\partial \theta / \partial x_i$ and κ is the thermal diffusivity of the fluid. Although it is now not difficult to determine these quantities simultaneously using direct numerical simulations (DNS) with the help of supercomputers, experimentally, the task is fraught with difficulty since twelve velocity derivative correlations and three temperature derivative correlations need to be resolved. This will require multi-hot (cold) wire measurements (e.g. Tsinober et al., 1992; Wallace and Foss, 1995; Zhu and Antonia, 1996 for vorticity measurements; Sreenivasan et al., 1977; Anselmet and Antonia, 1985 and Danaila et al., 2000 for temperature dissipation rate measurements). The simplest way to estimate the full energy and temperature dissipation rates is by assuming local isotropy, viz.

$$\varepsilon_{iso} = 15\nu u_{1,1}^2 \quad (3.1)$$

$$\chi_{iso} = 3\kappa\theta_{,1}^2 \quad (3.2)$$

where u_1 is the velocity fluctuation in the streamwise (x_1) direction and θ is the temperature fluctuation. In turbulent flow, as the eddy size becomes smaller the permanent anisotropy decreases so that at small scales the strain-rate field itself may be expected to be isotropic in the mean, even though the mean flow may be evolving (Tennekes and Lumley, 1972). In other words, turbulence is increasingly “scrambled” at small scales, and any permanent sense of direction is lost. This concept is called local isotropic (Kolmogorov, 1941). The adjective “local” refers to small scales (high wave numbers in spectral analysis). On the other hand, it is generally accepted that velocity derivatives are more related with small-scale properties (e.g. Corrsin and Kistler, 1955; Antonia and Kim, 1994), i.e., the velocity gradients, unlike the turbulent velocities themselves, are dominated by the small scale. This is related to the importance of the small scales in dissipation, because the dissipation is determined by the mean-squared velocity gradients. Therefore, Eqs. (3.1) and (3.2) can be used to estimate the mean energy and temperature dissipation rates.

Experimentally, measurement of flow velocities at several points simultaneously is far from easy (Mathieu and Scott, 2000) while single point measurements are much easier (and thus employed in the present study). However, restrictive assumptions about the flow have to be made before using the temporal data from a single point to obtain information about the spatial structure of turbulence. Specifically, it is assumed that the interested spatial structures do not change significantly during the time taken for their passage across the probe, i.e., the intrinsic time scale for turbulence dynamics is long compared with this passage time (Eulerian time scale). If the mean flow $\langle U_i \rangle$ is dominantly in the x_1 -direction, this

assumption can be expressed as $u_i = u_i(x_1 - \langle U_1 \rangle t, x_2, x_3)$, that is, the velocity fluctuations are merely convected at a constant speed $\langle U_1 \rangle$. This assumption is called the Taylor's hypothesis. Such special behavior of the turbulence is never exactly realized since real turbulence evolves as it is convected. Errors resulting from Taylor's hypothesis and the assumption of local isotropy are relatively well known, especially in flows, e.g. jet turbulence, where local turbulence intensity is relatively high, and in the vicinity of wall, where the mean shear is significant. Nevertheless, sufficiently far away from solid boundaries so that the mean shear and the turbulence intensity are not too high, Taylor's hypothesis does lead, on average, to good approximation of the streamwise derivatives of the velocity components (Bernard and Wallace, 2002). Therefore, for single point measurements (e.g. present study), Eqs. (3.1) and (3.2) should provide good approximations to the mean energy and temperature dissipation rates with the help of Taylor's hypothesis when the turbulence intensity and mean shear are not significant. Using Taylor's hypothesis, $\partial u_1 / \partial x_1$ and $\partial \theta / \partial x_1$ can be estimated from $\Delta u_1 / \Delta t$ and $\Delta \theta / \Delta t$, i.e. $\partial u_1 / \partial x_1 = -U_1^{-1} \Delta u_1 / \Delta t$ and $\partial \theta / \partial x_1 = -U_1^{-1} \cdot \Delta \theta / \Delta t$, where Δu_1 or $\Delta \theta$ represents the difference between values of u_1 or θ at the same point in space but separated by one sampling time interval $\Delta t (\equiv 1/f_s)$ and U_1 is the free stream velocity. This means that only one single hot wire or cold wire is sufficient for determining ε_{iso} and χ_{iso} experimentally.

However, the use of Eqs. (3.1) and (3.2) as adequate substitutes for the instantaneous values of ε and χ has been questioned (e.g. Hosokawa et al., 1996; Chen et al., 1997; Zhou and Antonia, 2000a). To measure all three temperature derivative correlations in the expression of χ , two orthogonal pairs of parallel cold

wires are required (e.g. Sreenivasan et al., 1977; Anselmet et al., 1994; Danaila et al., 2000). To measure all twelve velocity gradient correlations involved in ε simultaneously, at least nine hot wires are needed (e.g. Wallace and Foss, 1995). As the number of hot and cold wires increases, problems related with spatial resolutions, possible flow blockage, electronic noise between wires and aerodynamics interference constraints may become important (e.g. Mestayer and Chambaud, 1979; Wyngaard, 1969; Zhu and Antonia, 1996). Generally, too large a measurement volume (and therefore too large a spatial resolution) is likely to cause attenuation of derivative spectra, whereas too small measurement volume will result in noise contamination and possible aerodynamic interference between the wires. Therefore, it is very important to select an appropriate wire separation for the multi-hot (cold) wire configuration when velocity (temperature) derivatives are needed.

These conflicting difficulties are best investigated in the context of simple probe geometry. The simplest configuration is provided by a pair of parallel wires (hot or cold). This arrangement has been analyzed by many investigators (e.g. Wyngaard, 1969; Antonia et al., 1993b; Anselmet et al., 1994; Lasserre et al., 2000) although, by and large, most of the attention has focused on the correcting for the attenuation when Δx_2 becomes large compared with $\eta[\equiv (\nu^{3/4} / \langle \varepsilon \rangle^{1/4})]$, the Kolmogorov length scale, where angular brackets denote time averaging. Recently, Lasserre et al. (2000) examined a wide range of possible causes for the spurious increase in the measured values of $\langle (\Delta u_1 / \Delta x_2)^2 \rangle$ as Δx_2 decreases. Effects such as thermal and electronic cross-talk between the wires, differences between velocity sensitivities of the wires and the uncertainty in estimating Δx_2 were readily dismissed. Electronic noise is deemed to be important whilst, aerodynamic disturbance due to the probe geometry could not be altogether eliminated. Indeed, a

significant aerodynamic effect was observed on the mean velocity measured by two wires (such an effect was first reported in the context of both single and crossed wires by Comte-Bellot and Corrsin, 1971 and Strohl and Comte-Bellot, 1973) though not on the rms values of velocity fluctuations.

The main objective of this chapter is to get the optimum separation by using two parallel wires (hot and cold) with changeable separations. This optimum separation is an important criterion for the fabrication of vorticity probes which will be used in the later part of the project. The effect of wire separation on the measured and corrected spectra of velocity and temperature derivatives, and the effect on moments of velocity and temperature derivatives are investigated in the far region of a circular cylinder wake both on the centerline and at the half-width of the wake. Further local isotropy and its dependence on transverse location are also checked.

3.2 Experimental Details

The experiment was conducted in the wind tunnel described in Chapter 2. The free stream velocity U_1 was about 5m/s. The measurement location is at $x_1/d=240$, where $d(=6.35\text{mm})$ is the diameter of the circular cylinder (stainless steel). At this location, the Kolmogorov length scale η is about 0.50mm (The rms value of $\partial u_1 / \partial x_1$ estimated from Taylor hypotheses is 15.8/s. According to $\langle \varepsilon \rangle_{iso} = 15\nu \langle (\partial u_1 / \partial x_1)^2 \rangle$, the value of $\langle \varepsilon \rangle_{iso}$ is $0.058\text{m}^2/\text{s}^3$. Following $\eta = (\nu^3 / \langle \varepsilon \rangle)^{1/4}$, the value of η is 0.50mm). The local turbulence intensity u'_i / U_1 is about 2% [where i represents 1, 2 or 3 and a prime denotes the root mean square (rms) value], favoring the use of Taylor's hypothesis. The stainless steel tube was heated electrically. The mean temperature increase T_0 on the centerline of the wake at the measurement location, relatively to ambient, was about 0.5°C . This value is small

enough to avoid any buoyancy effect and large enough to assure high signal-to-noise ratio of the temperature signals.

A probe consisting of two parallel wires (Figure 3.1) was used to measure either velocity or temperature derivatives on the centerline ($x_2/L=0$) and the wake half-width ($x_2/L=1$) separately, where x_2 is the vertical coordinate and L is the half-width of the wake (Figure 3.2). U_0 is the maximum velocity deficit. In order to examine the effect of wire separation on the statistics of velocity and temperature derivatives, the experiment was carried out by varying the separation Δx_2 . To ensure that the measurement location was either on the centerline or at the wake half-width, the midpoint between the two wires is remained at the same location by moving the two wires with an equal distance in the opposite direction when the separation needs to be changed.

Figure 3.1 Sketches of the parallel wire probe

Figure 3.2 Definition sketches of the flow and coordinate system

The wires with diameter d_w of $5\mu\text{m}$ were used for measurement of velocity (act as hot wires), while the diameter of the cold wires for temperature measurements was $0.64\mu\text{m}$. All the wires were etched from Wollaston Pt-10% Rh wires. The active lengths of the hot and cold wires were about $200d_w$ and $800d_w$, respectively. The whole length of each hot or cold wire is about 3mm, including both etched (active) (around 0.5-1mm) and unetched parts. Each wire was supported by two prongs (which are about 0.1mm in diameter at the tip), i.e. totally 4 prongs for two wires. The four prongs have the same length of about 18mm. The prongs are arranged along the flow direction to reduce any possible disturbance to the flow. The related perturbation due to the probe and its support can be minimized by the calibration before experiments were conducted. The hot wires were operated with in-house constant temperature anemometers (CTA) at an overheat ratio of 1.5. The cold wires were operated with in-house constant current (0.05mA) circuits (CCA). The hot wires were calibrated at the centerline of the tunnel based on the measurements of a Pitot-static tube. The output signals from the anemometers were passed through buck

and gain circuits and low-pass filtered at the cut-off frequency $f_c \equiv 1600$ Hz, which is close to f_K , where $f_K \equiv U_1 / 2\pi\eta$ is the Kolmogorov frequency. The filtered signals are sampled at a frequency $f_s = 2f_c$ into a PC using a 16-bit A/D converter (National Instrument). The sampling period is about 30 μ s for both hot- and cold-wires. Further analysis of the data was carried out using the MATLAB[®] programs.

Several precautions were taken to ensure high quality of the data as much as possible. Firstly, the wires were etched to the same length and checked for straightness. Secondly, the etched portions of the wires were parallel to each other and were in the same vertical plane. Finally, the wires were operated for about six hours prior to collecting data in order to minimize any possible voltage drift of the wires.

Buoyancy effect is an important issue in the hot-wire measurements. The heat transfer of hot-wire is by both nature and forced convection. On the other hand, the air density will change with temperature. The buoyancy effect on the present measurements is discussed as following:

a) The buoyancy effect on the hot-wire:

The buoyancy effect on the hot-wire measurements depends on Grashof number defined as (Bruun, 1995) $Gr = g\beta(T_w - T_a)d^3 / \nu^2$, where g is acceleration of gravity,

β is the expansion coefficient ($\beta = 1/T$, where T is the absolute temperature),

T_w is the temperature of the heated wire,

T_a is the temperature of the ambient fluids,

d is the diameter of a cylindrical element,

ν is the kinematic viscosity.

In the present study, the temperature of the heated wire is around 350°C and the temperature of the heated air is around 28.5°C. Then the temperature difference

$\Delta\theta$ between the wire and the heated air is 321.5°C . The diameter of the present hot-wire is $5\mu\text{m}$. Therefore, the Grashof number is $4.65\text{E-}06$ and $Gr^{1/3}$ is $1.77\text{E-}02$. The Reynolds number for the hot-wire is 1.62 . According to Collis and William (1959), the buoyancy effect can be neglected when $Re > Gr^{1/3}$ with large values of length-to-diameter ratio. The present length-to-diameter ratio of the hot-wire is around 200 , which is high enough to neglect the buoyancy effect.

b) Buoyancy effect on the flow field:

In the present study, the mean temperature increase T_0 on the centerline of the wake at the measurement location, relatively to ambient, was about 0.5°C . According to Table B.2 (Potter and Wiggert, 2001), the change of the air density caused by the present mean temperature increase is very small (around 0.0067%). Therefore, the buoyancy effect on the present study is negligible.

3.3 Results

3.3.1 Effect of Wire Separation on Mean and RMS Velocity and Temperature Measurements

The effect of Δx_2 on the mean velocity $\langle U \rangle$ is shown in Figure 3.3 (The data shown here were obtained at the centerline in the far field of a cylinder wake). For $\Delta x_2^* \geq 3$ (Hereafter the asterisk denotes that the values are normalized by Kolmogorov length scale η , the velocity scale $u_K \equiv \nu/\eta$ and/or the temperature scale $\theta_K \equiv (\langle \chi \rangle \eta / u_K)^{1/2}$ correspondingly), the mean velocities from wire 1 and wire 2 are within $\pm 1\%$ of the averaged values of the two wires (the averaging was taken over $\Delta x_2^* \geq 1$). It seems that there is little aerodynamic interference effect on $\langle U \rangle$ for this Δx_2^* range. When $\Delta x_2^* < 1$, $\langle U \rangle$ from wire 1 increases, whereas $\langle U \rangle$ from wire 2 decreases. The effect of Δx_2 on the mean temperature $\langle T \rangle$ of the two wires is shown in Figure 3.4. The values of mean temperature measured from wire 1 and wire 2 follow each other closely and remain within $\pm 1\%$ of their averaged value (the average was taken over the range $\Delta x_2^* > 1.4$). Given that the experimental uncertainty in measuring $\langle T \rangle$ is about $\pm 1\%$, it could be reasonable to conclude that aerodynamic interference due to the geometry of the probe does not affect the mean temperature measured by the two wires in this range. When $\Delta x_2^* < 1.4$, $\langle T \rangle$ of wire 1 remains within $\pm 1\%$ of the averaged value, whereas $\langle T \rangle$ of wire 2 increases sharply as Δx_2^* approaches zero.

Figure 3.3 Effect of Δx_2 on the mean values of velocity at $x_2 / L = 0$.
▲, from wire 1; □, from wire 2; ---, range corresponding to $\pm 1\%$ of the averaged value.

Figure 3.4 Effect of Δx_2 on the mean values of temperature $x_2 / L = 0$.
▲, from wire 1; □, from wire 2; ---, range corresponding to $\pm 1\%$ of the averaged value.

The effects of Δx_2 on rms values u_1' and θ' are shown in Figures 3.5 and 3.6, respectively. Considering the experimental uncertainty ($\pm 2\%$ in measuring the rms value), the measured values of u_1' and θ' can be considered to be independent of Δx_2^* for $\Delta x_2^* > 0.8$, implying the negligible aerodynamic effect on fluctuating signals in this range. Further corroboration of this is provided by the spectra of u_1 and θ . The spectrum $\phi_\alpha(k_1)$ is defined such that $\int_0^\infty \phi_\alpha(k_1) dk_1 = \langle \alpha'^2 \rangle$ (where α represents either u_1 or θ , and k_1 is the wavenumber in the x_1 direction). To calculate the spectrum ϕ_α , the auto-correlation $R_\alpha(r)$ of the signal α is first calculated (where r is the separation between two points in the signals). By using Fast Fourier Transform (FFT) method, the spectrum can then be obtained via

$$\phi_\alpha(k_1) = 2 \int_{-\infty}^{\infty} R_\alpha(r) e^{-i2\pi k_1 r} dk_1. \quad (3.3)$$

As shown in Figures 3.7 and 3.8, there is nearly no difference among $\phi_{u_1}(k_1)$ [or $\phi_\theta(k_1)$] of different separations for the whole range of wave number. The spectra were normalized such that $\int_0^\infty \phi_{u_1} dk_1^* = \langle u_1'^2 \rangle$ and $\int_0^\infty \phi_\theta dk_1^* = \langle \theta'^2 \rangle$ (Hereafter, k^* denotes the wave number normalized by Kolmogorov frequency f_k). To avoid crowding, only spectra corresponding to a few wire separations are shown here.

Figure 3.5 Effect of Δx_2 on rms values of velocity fluctuations. $\square$, from wire 1; $\blacktriangle$, from wire 2; ---, range corresponding to $\pm 2\%$ of the averaged value.

Figure 3.6 Effect of Δx_2 on rms values of temperature fluctuations. $\square$, from wire 1; $\blacktriangle$, from wire 2; ---, range corresponding to $\pm 2\%$ of the averaged value.

Figure 3.7 Longitudinal velocity spectra for different wire separations.
—, $\Delta x_2^* = 1.40$; --, $\Delta x_2^* = 3.59$; ---, $\Delta x_2^* = 8.56$; ----, $\Delta x_2^* = 16.50$;

Figure 3.8 Temperature spectra for different wire separations.
— $\Delta x_2^* = 1.40$; --, $\Delta x_2^* = 3.59$; ---, $\Delta x_2^* = 8.56$; ----, $\Delta x_2^* = 16.50$.

3.3.2 Effect of Wire Separation on the Second-order Moment of Velocity and Temperature Derivatives

The measured values of the finite differences $\langle(\Delta u_1^* / \Delta x_2^*)^2\rangle$ and $\langle(\Delta \theta^* / \Delta x_2^*)^2\rangle$ for different separations Δx_2^* are shown in Figures 3.9 and 3.10, respectively. The isotropic relations, Eqs. (3.1) and (3.2) are used to calculate η , u_K and θ_K . For isotropic turbulence, the values of $\langle(\partial u_1^* / \partial x_2^*)^2\rangle$ and $\langle(\partial \theta^* / \partial x_2^*)^2\rangle$ should be $2/15$ and $Pr/3(=0.24)$, respectively. It is expected that $\langle(\Delta u_1^* / \Delta x_2^*)^2\rangle$ and $\langle(\Delta \theta^* / \Delta x_2^*)^2\rangle$ should approach $\langle(\partial u_1^* / \partial x_2^*)^2\rangle$ and $\langle(\partial \theta^* / \partial x_2^*)^2\rangle$ monotonously as the wire separations Δx_2 approaches zero. However, the trends shown in Figures 3.9 and 3.10 indicate that $\langle(\Delta u_1^* / \Delta x_2^*)^2\rangle$ and $\langle(\Delta \theta^* / \Delta x_2^*)^2\rangle$ increase systematically in a rapid and spurious manner as Δx_2 approaches zero. The sharp increase of $\langle(\Delta u_1^* / \Delta x_2^*)^2\rangle$ and $\langle(\Delta \theta^* / \Delta x_2^*)^2\rangle$ at small Δx_2^* mainly reflects the noise contamination (e.g. Anselmet et al., 1994); but Mestayer and Chambaud (1979) also considered the contribution from errors in wire calibration. The decrease of $\langle(\Delta u_1^* / \Delta x_2^*)^2\rangle$ and $\langle(\Delta \theta^* / \Delta x_2^*)^2\rangle$ at large Δx_2^* reflects the attenuation of finite spatial resolution. The effects of wire contamination and spectral attenuation can both be corrected. Details of the corrections are given in the following discussion. Figures 3.9 and 3.10 show that there is only a small range of Δx_2^* (e.g. $\Delta x_2^* = 3 - 5$) over which the measured values of $\langle(\Delta u_1^* / \Delta x_2^*)^2\rangle$ and $\langle(\Delta \theta^* / \Delta x_2^*)^2\rangle$ agree favorably with their corresponding isotropic values.

Figure 3.9 Effect of Δx_2^* on the measured values of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$. $\square$, measured values before correction; $\blacktriangledown$, measured values after spectral correction; Δ , measured values after noise correction; $\blacksquare$, values inferred from the correlation coefficient between $u_1(x_2)$ and $u_1(x_2 + \Delta x_2)$. Note that details of the corrections are given in the following discussion.

Figure 3.10 Effect of Δx_2^* on the measured values of $\langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$. $\square$, measured values before correction; $\blacktriangledown$, measured values after spectral correction; Δ , measured values after noise correction; $\blacksquare$, values inferred from the correlation coefficient between $\theta(x_2)$ and $\theta(x_2 + \Delta x_2)$. Note that details of the corrections are given in the following discussion.

Corrections for noise contamination can be made using method proposed by Anselmet et al. (1994). Suppose that the output voltage e_1 and e_2 of the two hot wires are given by:

$$\left. \begin{aligned} e_1 &= u_1(1) + n_1 \\ e_2 &= u_1(2) + n_2, \end{aligned} \right\} \quad (3.4)$$

where n_1 and n_2 represent the uncorrelated noise fluctuations, then

$$\langle (e_1 - e_2)^2 \rangle = \langle (\Delta u_1)^2 \rangle + \langle n_1^2 \rangle + \langle n_2^2 \rangle. \quad (3.5)$$

When $\Delta x_2^* \rightarrow 0$, $\langle (e_1 - e_2)^2 \rangle \rightarrow \langle n_1^2 \rangle + \langle n_2^2 \rangle$. The sum of $(\langle n_1^2 \rangle + \langle n_2^2 \rangle)$ was determined using a linear extrapolation (see Anselmet et al., 1994) to $\langle (e_1 - e_2)^2 \rangle$ when $\Delta x_2^* \rightarrow 0$. The corrected values of $\langle (\Delta u_1)^2 \rangle$ were then inferred from $\langle (e_1 - e_2)^2 \rangle$. A similar approach was applied to correct $\langle (\Delta \theta)^2 \rangle$. These noise corrected values are also included in Figures 3.9 and 3.10. The resulting variations of $\langle (\partial u_1^* / \partial x_2^*)^2 \rangle$ and $\langle (\partial \theta^* / \partial x_2^*)^2 \rangle$ with respect to Δx_2^* are clearly smaller than those of the original (noise contamination) values. Figures 3.11 and 3.12 show the way to get the estimation of noise level.

Figure 3.11 Evolution of the velocity difference $[u_1(x_2 + \Delta x_2) - u_1(x_2)]^2$ for the estimation of the noise level. •, raw data; ---, curve fit for noise level.

Figure 3.12 Evolution of the temperature difference $[\theta(x_2 + \Delta x_2) - \theta(x_2)]^2$ for the estimation of the noise level. •, raw data; ---, curve fit for noise level.

The values of the derivative variances $\langle (\partial u_1^* / \partial x_2^*)^2 \rangle$ and $\langle (\partial \theta^* / \partial x_2^*)^2 \rangle$ can also be inferred from the correlations between $u_1(x_2)$ and $u_1(x_2 + \Delta x_2)$ or between $\theta(x_2)$ and $\theta(x_2 + \Delta x_2)$. Using Taylor series expansion to order $(\Delta x_2)^3$,

$$u_1(x_2 + \Delta x_2) = u_1(x_2) + \Delta x_2 \left(\frac{\partial u_1}{\partial x_2} \right) + \frac{(\Delta x_2)^2}{2} \left(\frac{\partial^2 u_1}{\partial x_2^2} \right) \quad (3.6)$$

$$\theta(x_2 + \Delta x_2) = \theta(x_2) + \Delta x_2 \left(\frac{\partial \theta}{\partial x_2} \right) + \frac{(\Delta x_2)^2}{2} \left(\frac{\partial^2 \theta}{\partial x_2^2} \right). \quad (3.7)$$

The correlation coefficients can then be written as

$$\rho_{u_1} = \langle u_1(x_2) u_1(x_2 + \Delta x_2) \rangle / \langle u_1^2(x_2) \rangle^{1/2} \langle u_1^2(x_2 + \Delta x_2) \rangle^{1/2} \quad (3.8)$$

and
$$\rho_{\theta} = \langle \theta(x_2) \theta(x_2 + \Delta x_2) \rangle / \langle \theta^2(x_2) \rangle^{1/2} \langle \theta^2(x_2 + \Delta x_2) \rangle^{1/2}. \quad (3.9)$$

This can further be simplified as (after noting that the variances $\langle u^2 \rangle$ and $\langle \theta^2 \rangle$ do not depend on Δx_2)

$$\rho_{u_1} = 1 - \frac{(\Delta x_2)^2}{2} \left(\frac{(\partial u_1 / \partial x_2)^2}{\langle u_1^2 \rangle} \right) \quad (3.10)$$

and
$$\rho_\theta = 1 - \frac{(\Delta x_2)^2}{2} \frac{(\partial \theta / \partial x_2)^2}{\langle \theta \rangle^2} \quad (3.11)$$

Parabolic fits to the distributions of ρ_{u_1} (Figure 3.13) and ρ_θ (Figure 3.14) yielded the values of $\langle (\partial u_1^* / \partial x_2^*)^2 \rangle$ and $\langle (\partial \theta^* / \partial x_2^*)^2 \rangle$ which were also included in Figures 3.9 and 3.10. These values are in close agreement with the peaks of the noise corrected distributions of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ and $\langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$.

Figure 3.13 Parabolic fit to the distribution of ρ_{u_1} for small separations.

•, raw data; ---, parabolic fit.

Figure 3.14 Parabolic fit to the distribution of ρ_θ for small separations.

•, raw data; ---, parabolic fit.

The measured spectra of $(\Delta u_1 / \Delta x_2)$ and $(\Delta \theta / \Delta x_2)$ for different wire separations are shown in Figures 3.15 and 3.16. The spectra were normalized such that $\int_0^\infty \phi_{u_{1,2}}^* dk_1^* = \langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ and $\int_0^\infty \phi_{\theta_2}^* dk_1^* = \langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$. In Figures 3.15 and 3.16, it can be seen that the measured spectra for various wire separations depart significantly from each other. This is due to the spectral attenuation caused by finite wire separation. This attenuation becomes more apparent as Δx_2^* increases, especially for high wavenumbers.

Figure 3.15 Measured (uncorrected) spectra of $(\Delta u_1 / \Delta x_2)$ for different separations.

—, $\Delta x_2^* = 1.40$; ---, $\Delta x_2^* = 3.59$; ---, $\Delta x_2^* = 8.56$; -.-.-, $\Delta x_2^* = 16.50$.

Figure 3.16 Measured (uncorrected) spectra of $(\Delta\theta/\Delta x_2)$ for different separations.

—, $\Delta x_2^*=1.40$; ---, $\Delta x_2^*=3.59$; ---, $\Delta x_2^*=8.56$; -.-, $\Delta x_2^*=16.50$.

By assuming local isotropy, the attenuation effect caused by the finite values of Δx_2 and wire length l on the spectra of $(\Delta u_1/\Delta x_2)$ and $(\Delta\theta/\Delta x_2)$ can be corrected for the high wavenumber attenuation. The correction factors are given by (e.g. Antonia et al., 1993b)

$$\frac{\phi_{\Delta u_1/\Delta x_2}(k_1)}{\phi_{\partial u_1/\partial x_2}(k_1)} = \frac{\sin^2(\frac{lk_3}{2})}{(\frac{lk_3}{2})^2} \frac{4 \int \int_{-\infty}^{\infty} \sin^2(\frac{k_2 \Delta x_2}{2}) \frac{E(k)}{4\pi k^4} (k^2 - k_1^2) dk_2 dk_3}{(\Delta x_2)^2 \int \int_{-\infty}^{\infty} k_2^2 \frac{E(k)}{4\pi k^4} (k^2 - k_1^2) dk_2 dk_3} \quad (3.12)$$

and

$$\frac{\phi_{\Delta\theta/\Delta x_2}(k_1)}{\phi_{\partial\theta/\partial x_2}(k_1)} = \frac{\sin^2(\frac{lk_3}{2})}{(\frac{lk_3}{2})^2} \frac{4 \int \int_{-\infty}^{\infty} \sin^2(\frac{k_2 \Delta x_2}{2}) \frac{\Gamma(k)}{4\pi k^4} dk_2 dk_3}{(\Delta x_2)^2 \int \int_{-\infty}^{\infty} k_2^2 \frac{\Gamma(k)}{4\pi k^4} dk_2 dk_3} \quad (3.13)$$

where k is the magnitude of the wavenumber vector (with components k_1 , k_2 , and k_3 in the x_1 , x_2 and x_3 directions, respectively), $E(k)$ and $\Gamma(k)$ are the three dimensional energy and temperature spectra. There are many attempts to determine

the form of $E(k)$ and $\Gamma(k)$ (e.g. Monin and Yaglom, 1975) and several analytical expressions have been proposed. Pao (1965) used the relation:

$$E(k) = \alpha k^{-5/3} \exp\left(-\frac{3}{2} \alpha k^{4/3}\right) \quad (3.14)$$

with $\alpha=1.7$. Qian (1984) used

$$E(k) = 1.19(1 + 5.3k^{2/3}) \exp(-5.4k^{4/3}) \quad (3.15)$$

which takes into the account the pre-dissipative "bump". Corrsin (1964) and Pao (1965) used the following form for $\Gamma(k)$

$$\Gamma(k) = nk^{-3/5} \exp\left(-\frac{3}{2} \frac{n}{Pr} k^{4/3}\right). \quad (3.16)$$

Relations (3.14-3.16) are assumed to be universal, i.e. they are independent of Reynolds number and the nature of the flow. The distributions of $E(k)$ and $\Gamma(k)$ can also be inferred from the measured one-dimensional spectra of u and θ via isotropic relations (e.g. Monin and Yaglom, 1975)

$$E(k) = k^2 \left(\frac{\partial^2 \phi_u}{\partial k^2} \right)_{k_1=k} - k \left(\frac{\partial \phi_u}{\partial k} \right)_{k_1=k} \quad (3.17)$$

$$\text{and} \quad \Gamma(k) = -k \left(\frac{\partial \phi_\theta}{\partial k} \right)_{k_1=k}. \quad (3.18)$$

The main advantage of using Eqs. (3.17) and (3.18) is that they take into account the effect of the Reynolds number and the initial conditions of the flow on $E(k)$ and $\Gamma(k)$. The distributions of $E(k)$ and $\Gamma(k)$ obtained with present measured $\phi_{u_1}(k_1^*)$ and $\phi_\theta(k_1^*)$ are shown in Figures 3.17 and 3.18, respectively. As a comparison, the distributions of $E(k)$ (or $\Gamma(k)$) using Eqs. (3.14) and (3.15) (or Eq. 3.16) are also given in Figure 3.17 (or Figure 3.18). The large departure between the measurements and the theoretical relations at lower wavenumber range is due to the small value of R_λ (≈ 40) in the present study, where R_λ is the Taylor

microscale Reynolds number defined as $R_\lambda = u'_1 \lambda / \nu$ [$\lambda \equiv u'_1 / (\partial u'_1 / \partial x_1)$ is the longitudinal Taylor microscale).

Figure 3.17 Comparison of the three-dimensional energy spectrum $E(k^*)$ obtained using different relations. ---, Eq. (3.14); -.-, Eq. (3.15); —, Eq.(3.17).

Figure 3.18 Comparison of the three-dimensional temperature spectrum $\Gamma(k^*)$ obtained using different relations. ---, Eq. (3.16) (with $n=1.5$); —, Eq.(3.18)).

After the correction factors, i.e. Eqs. (3.12) and (3.13), are applied, Eqs. (3.17) and (3.18) was used for the calculations of $E(k)$ and $\Gamma(k)$. The spectra of $(\Delta u_1 / \Delta x_2)$ and $(\Delta \theta / \Delta x_2)$ (Figures 3.19 and 3.20) agree quite well with each other for $\Delta x_2^* \geq 3$, except at the lowest wavenumbers. The corrected values of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ and $\langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$, which are also included in Figures 3.9 and 3.10, , respectively, were estimated by integrating over the corrected spectra of $(\Delta u_1^* / \Delta x_2^*)$ and $(\Delta \theta^* / \Delta x_2^*)$. For $\Delta x_2^* \geq 5$, the corrected values of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ and $\langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$ (Figures 3.9 and 3.10) are essentially constant, and agree to within $\pm 5\%$ with the corresponding isotropic values $2/15$ and $Pr/3$. When Δx_2^* decreases, the corrected values of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ and $\langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$ increase slightly. At $\Delta x_2^* = 3$, the magnitude of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ is about 6.3% larger than the isotropic values. It is likely that this discrepancy may reflect the fact that the spectra of $(\Delta u_1 / \Delta x_2)$ have not been corrected for noise contamination.

Figure 3.19 Corrected spectra of $(\Delta u_1 / \Delta x_2)$ for different wire separations.

—, $\Delta x_2^* = 1.40$; --, $\Delta x_2^* = 3.59$; ---, $\Delta x_2^* = 8.56$; ----, $\Delta x_2^* = 16.50$.

Figure 3.20 Corrected spectra of $(\Delta\theta/\Delta x_2)$ for different wire separations.

—, $\Delta x_2^* = 1.40$; --, $\Delta x_2^* = 3.59$; ---, $\Delta x_2^* = 8.56$; -.-.-, $\Delta x_2^* = 16.50$.

With the assumption of local isotropy, the mean squared velocity or temperature derivatives are related as follows

$$2 \left\langle \left(\frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle = \left\langle \left(\frac{\partial u_1}{\partial x_2} \right)^2 \right\rangle = \left\langle \left(\frac{\partial u_1}{\partial x_3} \right)^2 \right\rangle \quad (3.19)$$

and

$$\left\langle \left(\frac{\partial \theta}{\partial x_1} \right)^2 \right\rangle = \left\langle \left(\frac{\partial \theta}{\partial x_2} \right)^2 \right\rangle = \left\langle \left(\frac{\partial \theta}{\partial x_3} \right)^2 \right\rangle. \quad (3.20)$$

There is a large body of evidence to suggest that Eq. (3.19) is not generally satisfied, especially when the mean rate of strain is significant (e.g. Browne et al., 1987; George and Hussein, 1991). However, when the mean rate of strain is sufficiently small, there is evidence (e.g. Champagne, 1978; Antonia et al., 1986; Zhou and Antonia, 2000b) to indicate that Eq. (3.19) is satisfied to a good approximation, even at small R_λ . Eq. (3.20) was verified by Antonia and Mi (1993) on the centerline of a circular jet flow and by Danaila et al. (2000) in heated grid flow turbulence. When

there is a mean temperature gradient, the departure from Eq. (3.20) is significant (e.g. Tong and Warhaft, 1994; Pumir, 1994; Mydlarski and Warhaft, 1998). And therefore, Eq. (3.20) is not hold in flows with a mean scalar gradient such as the present flow. However, on the wake centerline, the influence of scalar gradient may be small. In Figures 3.9 and 3.10, isotropic values $\langle(\partial u_1^* / \partial x_2^*)^2\rangle$ and $\langle(\partial \theta^* / \partial x_2^*)^2\rangle$ (i.e. $2/15$ and $Pr/3$) are also included. They agree to within $\pm 5\%$ with the values inferred from the correlation method. The result indicates that Eqs. (3.19) and (3.20) are satisfied on the centerline of a cylinder wake.

3.3.3 Future Checks for Local Isotropy

According to Kolmogorov (1941), the dissipative range scales are isotropic and their statistics are characterized uniquely by dissipation rate ε and by kinematic viscosity ν whereas the inertial range scales are determined only by ε . Obukhov (1949) and Corrsin (1951) extended this notion later to passive scalars (such as temperature). Many tests can be used to check local isotropy, each with a different level of sensitivity (e.g. Antonia et al., 1986). It is evident for example that isotropic relations between different velocity (or temperature) derivative variances would provide less information about which length scales satisfy isotropy than isotropic relations between either different velocity (or temperature) derivative spectra or different velocity (or temperature) structure functions.

A measure of departure of velocity and temperature derivatives from isotropy is provided by comparing the directly measured spectra with those calculated using the following isotropic relations (e.g. Monin and Yaglom, 1975; Browne et al., 1983)

$$\phi_{\partial u_1 / \partial x_2}^{cal}(k_1) = 2 \int_{k_1}^{\infty} \frac{1}{k} \phi_{\partial u_1 / \partial x_1}(k_1) dk \quad (3.21)$$

and
$$\phi_{\partial\theta/\partial x_2}^{cal}(k_1) = 2 \int_{k_1}^{\infty} \frac{1}{k} \phi_{\partial\theta/\partial x_1}(k) dk, \quad (3.22)$$

where *cal* represents “calculation”. The distributions of $\phi_{\partial u_1/\partial x_1}(k_1)$ and $\phi_{\partial\theta/\partial x_1}(k_1)$ which appear in the integrands of Eqs. 3.21 and 3.22 were corrected both for the effect of wire length and the finite sampling interval. The correction ratios for $\phi_{\partial u_1/\partial x_1}(k_1)$ and $\phi_{\partial\theta/\partial x_1}(k_1)$ have the same form and can be written as (e.g. Antonia et al., 1993b)

$$R_{\partial u_1/\partial x_1}(k_1^*) \left[\equiv R_{\partial\theta/\partial x_1}(k_1^*) \right] = \frac{\sin^2(\frac{k_1 l}{2}) \sin^2(\frac{k_1 \Delta x_1}{2})}{(\frac{k_1 l}{2})^2 (\frac{k_1 \Delta x_1}{2})^2}, \quad (3.23)$$

where $\Delta x_1 (\equiv -\langle U \rangle / f_s)$ is the streamwise separation which corresponds to the time delay between consecutive samples in the digital time series. The measured spectra can then be corrected via

$$\phi_{\partial u_1/\partial x_1}^{*c}(k_1^*) = \phi_{\partial u_1/\partial x_1}^{*m}(k_1^*) / R_{\partial u_1/\partial x_1}(k_1^*) \quad (3.24)$$

and
$$\phi_{\partial\theta/\partial x_1}^{*c}(k_1^*) = \phi_{\partial\theta/\partial x_1}^{*m}(k_1^*) / R_{\partial\theta/\partial x_1}(k_1^*), \quad (3.25)$$

where the superscript *c* represents “corrected” while *m* represents “measured”.

Comparisons between measured ($\Delta x_2^* = 3.78$) and calculated distributions of $\phi_{\partial u_1/\partial x_2}(k_1)$ and $\phi_{\partial\theta/\partial x_2}(k_1)$ are given in Figures 3.21 and 3.22, respectively. It has been assumed here that the measured distribution $\phi_{\partial\alpha/\partial x_2}$ can be identified with that of $\phi_{\Delta\alpha/\Delta x_2}$ when $\Delta x_2^* \geq 3$. In each case, there is a significant wavenumber range over which the agreement between the two distributions may be tolerable, indicating that local isotropy is satisfied over this range.

Figure 3.21 Comparison of calculated and measured (after correction) spectra $\phi_{\partial u_i / \partial x_2}$.
—, calculated; ---, measured (after correction).

Figure 3.22 Comparison of calculated and measured (after correction) spectra $\phi_{\partial \theta / \partial x_2}$.
—, calculated; ---, measured (after correction).

The extent to which local isotropy is satisfied can also be examined by the skewness and flatness factors of the velocity and temperature derivatives. The skewness is defined as $S_\alpha = \langle \alpha^3 \rangle / \langle \alpha^2 \rangle^{3/2}$. As shown in Figure 3.23, the magnitude of $S_{\partial u_i / \partial x_i}$ ($= -0.43$) is independent of Δx_2^* . The values of $S_{\Delta u_i / \Delta x_2}$ and $S_{\Delta \theta / \Delta x_2}$ are nearly zero and independent of Δx_2^* when $\Delta x_2^* \geq 3$. The above results are consistent with local isotropy and symmetry. $S_{\partial \theta / \partial x_i}$ should also be zero in isotropic turbulence. However, in the present study, its magnitude is 0.64. This value is too big to be attributed to experimental uncertainty. The non-zero value of $S_{\partial \theta / \partial x_i}$ in Figure 3.23 is almost certainly due to the means scalar gradient, $\partial T / \partial x_i$, and not by off-centerline positions. It may also be caused, in part, by the noise effect for temperature measurements (The signal-to-noise ratio is about 80).

Figure 3.23 Effect of Δx_2 on the values of skewness factors.

▼, $S_{\Delta u / \Delta y}$; ▲, $S_{\Delta u / \Delta x}$; ○, $S_{\Delta \theta / \Delta y}$; ●, $S_{\partial \theta / \partial x}$.

The flatness factor $F_\alpha = \langle \alpha^4 \rangle / \langle \alpha^2 \rangle^2$ can be used to examine the intermittency of α . For a Gaussian distribution, F_α is 3. As shown in Figure 3.24, when $\Delta x_2^* \geq 3$, $F_{\Delta u_1 / \Delta x_2}$ decreases monotonically with Δx_2^* and is expected to approach Gaussian value when Δx_2^* reaches the integral length scale (≈ 50). For $\Delta x_2^* < 3$, $F_{\Delta u_1 / \Delta x_2}$ decreases, which is consistent with the increase of $\langle (\Delta u_1 / \Delta x_2)^2 \rangle$ over this range. The measured decrease of $F_{\Delta u_1 / \Delta x_2}$ at small wire separations is most likely caused by noise effect. Shafi et al. (1997) also noticed this in a turbulent wake. The values of $F_{\partial u_1 / \partial x_1}$ are almost constant for different wire separations and much smaller than those of $F_{\Delta u_1 / \Delta x_2}$ over the range where $\Delta x_2^* > 1.5$ and $\Delta x_2^* < 7$. This is consistent with Sreenivasan and Antonia (1997) that $(\Delta u_1 / \Delta x_2)$ is more intermittent than $(\partial u_1 / \partial x_1)$. For local isotropic turbulence, the value of $F_{\Delta u_1 / \Delta x_2}$ should be equal to the value of $F_{\partial u_1 / \partial x_1}$ when $R_\lambda \rightarrow \infty$. The inequality between $F_{\Delta u_1 / \Delta x_2}$ and $F_{\partial u_1 / \partial x_1}$ indicated in Figure 3.24 is mainly caused by the small value of R_λ in the present experiment ($R_\lambda \approx 40$).

The flatness factors of the temperature derivatives should be equal in isotropic turbulence, i.e. $F_{\partial \theta / \partial x_1} = F_{\partial \theta / \partial x_2} = F_{\partial \theta / \partial x_3}$. The values of $F_{\partial \theta / \partial x_1}$ and $F_{\Delta \theta / \Delta x_2}$ are also shown in Figure 3.24. The magnitude of $F_{\partial \theta / \partial x_1}$ is nearly equal to the peak value of $F_{\Delta \theta / \Delta x_2}$ around $\Delta x_2^* = 4$. For $\Delta x_2^* > 10$, the values of $F_{\Delta \theta / \Delta x_2}$ is about 50% larger than those of $F_{\Delta u_1 / \Delta x_2}$, indicating that the small-scale temperature field is more intermittent than the small-scale velocity field. This result is consistent with other investigations, e.g. Sreenivasan and Antonia (1997). It seems that $F_{\partial \theta / \partial x_1} = F_{\Delta \theta / \Delta x_2}$

is satisfied over the range of $3 \leq \Delta x_2^* \leq 5$. This range is consistent with that inferred from Figures 3.9 and 3.10.

Figure 3.24 Effect of Δx_2 on the values of flatness factors.

$\nabla, F_{\Delta u_1 / \Delta x_2}$; $\blacktriangle, F_{\Delta u_1 / \Delta x_1}$; $\circ, F_{\Delta \theta / \Delta x_2}$; $\bullet, F_{\Delta \theta / \Delta x_1}$.

3.3.4 The Measurement of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ and $\langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$ at $x_2/L=1$

In order to check the possible influence of mean shear on transverse velocity derivative measurements, experiments using the same parallel hot-wire probe were also conducted at the wake half width ($x_2/L=1$).

Figure 3.25 shows that, at $x_2/L=1$ the range ($3 \leq \Delta x_2^* \leq 5$) over which the measured values of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ agree favorably with their corresponding isotropic values is almost the same as for $x_2/L=0$. This agreement indicates that there is no significant effect from mean shear on the determination of the optimum separations, at least for $x_2/L \leq 1$. The range over which the measured spectrum of $(\Delta u_1 / \Delta x_2)$ agrees favorably with isotropic calculation (Eq. 3.21) reduced only for $k_1^* > 0.1$ (Figure 3.26). This result further corroborates that isotropic relations

between different velocity derivative variances would provide less information about which length scales satisfy isotropy than isotropic relations between different velocity derivative spectra. A more detailed check of local isotropy at different transverse locations will be given in Chapter 4.

Figure 3.25 Measured and corrected values of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ at $x_2/L=0$ and $x_2/L=1$.
▲, measured, $x_2/L=0$; ⊕, measured, $x_2/L=1$; ▼ corrected, $x_2/L=0$; ○, corrected, $x_2/L=1$;

Figure 3.26 Comparison of calculated and measured (after correction) spectra $\phi_{\partial u_1 / \partial x_2}^*$ at the wake half width. —, calculated; ---, measured (after correction).

The measured and corrected values of $\langle(\Delta\theta^*/\Delta x_2^*)^2\rangle$ at $x_2/L = 0$ and $x_2/L=1$ is shown in Figure 3.27. At $x_2/L=1$, the range ($3 \leq \Delta x_2^* \leq 5$) over which the measured values of $\langle(\Delta u_1^*/\Delta x_2^*)^2\rangle$ agree favorably with their corresponding isotropic values is almost the same as for $x_2/L=0$. This agreement indicates that temperature derivatives satisfy local isotropy reasonably in the region of $x_2/L \leq 1$ and there is no significant effect from mean shear on the determination of the optimum separations for the measurements of temperature derivative, at least for $x_2/L \leq 1$.

Figure 3.27 Measured and corrected values of $\langle(\Delta\theta^*/\Delta x_2^*)^2\rangle$ at $x_2/L=0$ and $x_2/L=1$.

□, measured, $x_2/L=0$; ⊕, measured, $x_2/L=1$; ▼ corrected, $x_2/L=0$; ○, corrected, $x_2/L=1$;

3.4 Concluding Remarks

Transverse velocity and temperature derivatives have been measured using two parallel wires (hot and cold). The separation between the parallel wires was varied to investigate its effect on the derivative measurements. The results indicate that when $\Delta x_2^* \geq 1$, the mean velocity measured by the two wires is not influenced by the transverse separation between the wires. When $\Delta x_2^* < 1$, the increased aerodynamic interference as Δx_2^* decreases results in $\langle U \rangle$ being overestimated by one wire and underestimated by the other. The interference influences the mean temperature $\langle T \rangle$ when $\Delta x_2^* < 1.4$. There is no significant effect of Δx_2^* on rms values of velocity and temperature when $\Delta x_2^* > 1.4$. The different order moments of velocity and temperature derivatives are affected significantly and similarly when $\Delta x_2^* < 3$, indicating that the contamination was due to electronic noise but not aerodynamic interference. After noise correction was applied, the peaks of the noise corrected distributions of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ and $\langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$ agree favorably with their isotropic values, i.e. $2/15$ and $Pr/3$, respectively. The values also agree favorably with the derivative variances inferred from the two-point correlation functions of u_1 and θ , i.e. ρ_{u_1} and ρ_θ , with respect to Δx_2^* .

The present results indicate that the transverse velocity and temperature derivatives satisfy local isotropy reasonably in the region of $x_2/L \leq 1$. The range (e.g. $\Delta x_2^* = 3 - 5$) over which the measured values of $\langle (\Delta u_1^* / \Delta x_2^*)^2 \rangle$ and $\langle (\Delta \theta^* / \Delta x_2^*)^2 \rangle$ agree favorably with their corresponding isotropic values is independent of the transverse locations with wake halfwidth. Therefore, reasonable statistics of $(\partial u_1 / \partial x_2)$ and $(\partial \theta / \partial x_2)$ can be obtained if the separation between the two parallel (hot and cold) wires is in the range of $3-5\eta$.

Chapter 4

Approximations to Energy and Temperature Dissipation Rates Using a Six-wire Probe

4.1 Introduction

The turbulent energy dissipation rate ε and the temperature dissipation rate χ are important characteristics of the small-scale properties of turbulence. For example, the combination with some power of ε_r and χ_r , the locally averaged values of ε and χ , features in the refined similarity hypothesis for passive scalars (RSHP) (e.g. Stolovitzky and Sreenivasan 1995). The RSHP states that the n^{th} -order moment of the temperature increment $\delta\theta = \theta(x_1 + r) - \theta(x_1)$ can be expressed as

$$\langle (\delta\theta)^n \rangle \sim \langle \chi_r^{n/2} \varepsilon_r^{-n/6} \rangle r^{n/3} \sim r^{\zeta_\theta(n)}, \quad (4.1)$$

when the separation r is in the inertial range, i.e. $\eta \ll r \ll L_s$, and L_s is the integral length scale, $\zeta_\theta(n)$ is the scaling exponent of $\langle (\delta\theta)^n \rangle$. Van Atta (1971) assumed that the probability density functions (pdfs) of ε_r and χ_r for the inertial range scales are lognormally distributed and their joint pdf is bivariate lognormal with a correlation coefficient $\rho_{\alpha,\beta}$ defined as

$$\rho_{\alpha,\beta} = \frac{\langle (\alpha - \langle \alpha \rangle)(\beta - \langle \beta \rangle) \rangle}{\sigma_\alpha \sigma_\beta}, \quad (4.2)$$

where α and β represent $\ln \varepsilon_r$ and $\ln \chi_r$ respectively, σ_α and σ_β are the standard deviations of α and β . Therefore, the small-scale scalar characteristics are influenced not only by ε and χ but also by their correlations.

Measurements of either the energy dissipation rate ε or the temperature dissipation rate χ have been made by a number of researchers, e.g. Wallace and Foss

(1995) and references therein for the former and Sreenivasan et al. (1977), Anselmet et al. (1994) and Danaila et al. (2000) for the later. While it is difficult to measure simultaneously the above two dissipation fields, direct numerical simulations (DNS) complement experiments in some respects and provide new information on both statistical and structural aspects of the small-scale structures since they can resolve complete information on three-dimensional vorticity and full energy and temperature dissipation fields. The correlation of the above two dissipation fields can therefore be analyzed using DNS results (e.g. Hosokawa et al., 1996; Chen et al., 1997). Differences of the correlations are compared when either the full expressions for ε and χ or their isotropic counterparts ε_{iso} and χ_{iso} are used. The spatial relationship between the fields of vorticity, energy and temperature dissipation has been reviewed by Sreenivasan and Antonia (1997). The numerical study of Ruetsch and Maxey (1991) also showed that intense ω^2 occurs in vorticity tubes and ε is strongly correlated with these tubes. The regions where χ is largest are dissociated from the vortex tubes and occur as large flat sheets (see also Pumir, 1994). There is general agreement that the highest vorticity appears to reside in tubes while the moderate vorticity resides in sheets with large concentrations of strain rate and scalar gradient nearby. The moderate values of the energy dissipation rate surround the tubes while the large values of the scalar gradient are wrapped in sheets around the tubes and this gradient is in the direction of the compressive rate of strain. This suggests a relatively high correlation between ω^2 and ε but a poor correlation between χ and ω^2 or ε . Results obtained by simply approximating ε and χ by ε_{iso} and χ_{iso} (i.e. Eqs. 3.1 and 3.2) based on the isotropic assumption have yielded rather disparate values for the correlation between ε and χ (e.g. Antonia and Van Atta, 1975; Meneveau et al., 1990).

The first objective of this chapter is to measure the energy and temperature dissipation rates simultaneously in a cylinder wake using more reasonable approximations to ε and χ than ε_{iso} and χ_{iso} (Eqs. 3.1 and 3.2). Due to the complication of the probe and the possible interference between hot and cold wires, no previous measurement has been tried to measure ε and χ simultaneously in the far region of a cylinder wake. The present study is the first attempt to measure the two quantities simultaneously. On the basis of these measurements, the correlations between the two dissipation fields and the vorticity are further examined. The correlations between vorticity and temperature fluctuations are also analyzed. As ω^2 , ε and χ mainly represent the small-scale properties of turbulence, the second objective of this chapter is to provide some further checks on local isotropy based on these quantities. The turbulent Prandtl number is an important numerical parameter for turbulent heat mass transfer models. Therefore, the third objective of the present study is to measure the turbulent Prandtl number in the wake flow to gain an insight into the turbulent transport phenomena.

4.2 Experimental Details

The experiment was conducted in the wind tunnel as described in Chapter 2. The free stream velocity U_1 is about 5m/s, corresponding to a Taylor microscale Reynolds number $R_\lambda (\equiv u'_1 \lambda / \nu, \text{ where } \lambda = u'_1 / u'_{1,1} \text{ is Taylor microscale})$ of about 45 (or $\text{Re}_d \equiv U_1 d / \nu = 2000, d \equiv 6.35\text{mm}$ is the diameter of the stainless steel circular cylinder). The measurement location is at $x_1 / d = 240$. At this location, the Kolmogorov length scale η is about 0.50mm. The local turbulence intensity u'_i / U_1 ($i=1, 2$ or 3) is about 2%, favoring the use of Taylor's hypothesis. The stainless steel

tube was heated electrically. The mean temperature increase T_0 on the centerline of the wake at the measurement location, relative to ambient, was about 0.5°C . This value is small enough to avoid any buoyancy effect and large enough to assure high signal-to-noise ratio of the temperature signals.

A probe consisting of four hot wires and a pair of parallel cold wires (Figure 4.1) was used to measure approximately the energy and temperature dissipation rates simultaneously. The four hot wires also allow the calculation of the spanwise component ω_3 of the vorticity vector (or the transverse component ω_2 after a 90° rotation) using the measured velocity signals u_1 and u_2 . With the pair of parallel cold wires, two components, θ_1 and θ_2 (or after a 90° rotation, θ_1 and θ_3) of χ were measured simultaneously. The included angle of the X-wire is about 100° (the X-wire in the six-wire probe is fabricated based on the criterion of Zhu and Antonia, 1995). The separations Δx_2 between the two parallel hot wires c and d and Δx_3 between the two inclined hot wires a and b are about 1.5mm. The separation Δx_c between the two parallel cold wires is about 2.4mm. These separations correspond to $\Delta x_2^* = \Delta x_3^* = 3$ and $\Delta x_c^* = 4.8$ in the central part of the wake, which are quite close to the optimum wire separations for the hot and cold wires (Chapter 3) for the measurements of velocity and temperature derivatives.

The hot and cold wires were etched from Wollaston (Pt-10%Rh) wires. The active lengths are about $200d_w$ and $800d_w$ for the hot and cold wires respectively (where d_w is the diameter of the wires and equals to $5\mu\text{m}$ for the hot wires and $0.64\mu\text{m}$ for the cold wires). The hot wires were operated with in-house constant temperature circuits at an overheat ratio of 1.5. The cold wires were operated with constant current (0.05 mA) circuits. The probe was calibrated at the centerline of the tunnel against a Pitot-static tube. The yaw calibration was performed over $\pm 20^\circ$. The

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output signals from the anemometers were passed through buck and gain circuits and low-pass filtered at the cut-off frequency $f_c \approx 1600$ Hz, which is close to f_K , where $f_K \equiv U_1 / 2\pi\eta$ is the Kolmogorov frequency. The filtered signals are sampled at a frequency $f_s = 2f_c$ into a PC using a 16-bit A/D converter (National Instrument). The sampling period is about 30s. Further analyse of the data were carried out using the MATLAB[®] programs.

Figure 4.1 Sketches of the six-wire probe

Using the present probe, three of the nine velocity derivatives that make up ε are measured simultaneously. These derivatives are $u_{1,1}$, $u_{1,2}$ and $u_{2,1}$ or after a 90° rotation, $u_{1,1}$, $u_{1,3}$ and $u_{3,1}$. Assuming local isotropy and homogeneity, simultaneous approximations to ε can be made based on continuity (Zhou and Antonia, 2000a), viz.

$$\varepsilon_{ap} \approx \nu(6u_{1,1}^2 + 3u_{1,2}^2 + 2u_{2,1}^2 + 2u_{1,2}u_{2,1}) \quad (4.3)$$

$$\text{or} \quad \varepsilon_{ap} \approx \nu(6u_{1,1}^2 + 3u_{1,3}^2 + 2u_{3,1}^2 + 2u_{1,3}u_{3,1}), \quad (4.4)$$

where the subscript *ap* represents "approximation". The two parallel cold wires are

capable of measuring the temperature derivatives $\theta_{,1}$ and $\theta_{,2}$ or $\theta_{,1}$ and $\theta_{,3}$ after a 90° rotation. The temperature dissipation rate χ can then be approximated as (Anselmet and Antonia, 1985)

$$\chi_{ap} \approx \kappa(\theta_{,1}^2 + 2\theta_{,2}^2) \quad (4.5)$$

or
$$\chi_{ap} \approx \kappa(\theta_{,1}^2 + 2\theta_{,3}^2). \quad (4.6)$$

Relations (4.3) and (4.5) will be used in the present study as approximations to ε and χ . Since more derivative correlations are included in Eqs. (4.3) and (4.4) or Eqs. (4.5) and (4.6), it is expected that they should provide more reasonable approximations to ε or χ than when ε_{iso} or χ_{iso} (Eqs. 3.1 or 3.2) is used. With the present four hot-wire probe, the transverse vorticity component ω_3 , or ω_2 after a 90° rotation, can be obtained viz.

$$\omega_3 = u_{2,1} - u_{1,2} \approx \frac{\Delta u_2}{\Delta x_1} - \frac{\Delta u_1}{\Delta x_2} \quad (4.7)$$

or
$$\omega_2 = u_{1,3} - u_{3,1} \approx \frac{\Delta u_1}{\Delta x_3} - \frac{\Delta u_3}{\Delta x_1}, \quad (4.8)$$

where Δu_1 in (4.7) and (4.8) is velocity difference between the longitudinal velocity fluctuations from the two parallel wires c and d which are separated either in the x_2 or x_3 direction (Figure 4.1). Δu_2 or Δu_3 represents the difference between values of u_2 or u_3 at the same point in space but separated by one sampling time interval $\Delta t (= 1/f_s)$. In (4.7) and (4.8), Taylor's hypothesis will be used to estimate the velocity derivatives along streamwise direction, i.e. $\partial u_i / \partial x_i = -U_1^{-1} \Delta u_i / \Delta t$ ($i = 2, 3$).

4.3 Basic Performance Checks of the Probe

Since the energy and temperature dissipation rates are approximated using the current probe, it is important to ensure the performance of the probe before any further analysis can be conducted.

Figures 4.2 and 4.3 show the distributions of $u_1^{'+}$, $u_2^{'+}$ and $u_3^{'+}$ across the wake, where the superscript “+” denotes normalization by the maximum velocity deficit U_0 , i.e. $u_i^{'+} = u_i' / U_0$. The transverse location is normalized by wake half-width L , i.e. $x_2^{+} = x_2 / L$. The solid lines represent best fits to the corresponding data of Browne et al. (1987), which were obtained in the wake of a circular cylinder at $x_1 / d = 420$ using standard single hot wire, single X-probe and a pair of X-probes. Since the data of Browne et al. (1987) and that in the present study were both obtained in the self-preserving region of the wake, the velocity profiles should follow each other. This is the case in Figures 4.2 and 4.3. Also, the rms values of the three velocity components from different wires of the vorticity probe agree to within $\pm 4\%$ with each other, which should be within the experimental uncertainty, thus indicating the internal consistency check of the data. Further corroboration of this is provided by the spectra of u_1 in Figure 4.4 for ω_3 measurement on the wake centerline (result from ω_2 measurement is the same and therefore is not shown here). There is nearly no difference among distributions of $\phi_{u_1}^{*}(k_1^{*})$ from different wires.

Figure 4.2 RMS values of streamwise velocity fluctuations. —, Browne et al. (1987).
For ω_2 measurement: $\circ$, wire c; $\square$, wire d; ∇ , X-wire.
For ω_3 measurement: $\oplus$, wire c; $\boxplus$, wire d; $\triangle$, X-wire.

Figure 4.3 RMS values of transverse and spanwise velocity fluctuations.
 $+$, $i = 2$; $\circ$, $i = 3$; —, Browne et al. (1987).

Figure 4.4 Spectra of streamwise velocity u_1 for ω_3 measurement.
 ---, X-wire; --- wire c; —, wire d.

Figure 4.5 shows the distributions of θ^+ across the wake, where the superscript “+” denotes normalization by the maximum temperature increase above ambient T_0 , i.e. $\theta^+ = \theta' / T_0$. The temperature data for ω_2 and ω_3 measurements are both shown here. Since the data were both obtained in the self-preserving region of the wake, the temperature profiles should also follow each other like the way of velocity. This is the case in Figure 4.5. There is a reasonable agreement between the two sets of data. Also, the departure of the rms values of temperature measured from the two different cold wires is quite small, which should be within the experimental uncertainty (2%), both for ω_2 and ω_3 measurements. Further corroboration of this is provided by the spectra of θ in Figure 4.6 for ω_3 measurement on the wake centerline (result from ω_2 measurement is the same and therefore is not shown here). There is nearly no difference between $\phi_\theta^*(k_1^*)$ from the two parallel cold wires.

Figure 4.5 RMS values of temperature fluctuations.
For ω_2 measurement: $\circ$, cold wire 1; $+$, cold wire 2;
For ω_3 measurement: ∇ , cold wire 1; Δ , cold wire 2.

Figure 4.6 Spectra of temperature fluctuations for ω_3 measurement. ---, cold wire1; ---, cold wire2

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Before presenting the statistics of ω_i ($i=2$ or 3), it is relevant to address the effect of imperfect spatial resolution of the probe has on the measurement of ω_3 (or ω_2) and its components. The high wavenumber part of the spectra of the velocity derivatives and the vorticity will be attenuated due to the finite wire length and wire separation. Details of the correction procedures can be found in Antonia et al. (1996). Only a brief description of the method is given here. Since the variance of ω_3 is

$$\langle \omega_3^2 \rangle = \langle u_{2,1}^2 \rangle + \langle u_{1,2}^2 \rangle - 2\langle u_{2,1}u_{1,2} \rangle, \quad (4.9)$$

the accuracy of $\langle \omega_3^2 \rangle$ will depend on the measurement accuracy of $\langle u_{2,1}^2 \rangle$, $\langle u_{1,2}^2 \rangle$ and $\langle u_{2,1}u_{1,2} \rangle$. Alternatively, the accuracy of ϕ_{ω_3} , the spectrum of ω_3 , will depend on the accuracy of $\phi_{u_{2,1}}$, $\phi_{u_{1,2}}$ and the cospectrum $Co_{u_{2,1}u_{1,2}}$. The correction procedure of the latter three spectra can be written as

$$r_{21} = \frac{\phi_{u_{2,1}}^m(k_1)}{\phi_{u_{2,1}}(k_1)} = \frac{\int \int_{-\infty}^{\infty} \phi_{u_{2,1}}^m(k_1) dk_2 dk_3}{\int \int_{-\infty}^{\infty} \phi_{u_{2,1}}(k_1) dk_2 dk_3}, \quad (4.10)$$

$$r_{12} = \frac{\phi_{u_{1,2}}^m(k_1)}{\phi_{u_{1,2}}(k_1)} = \frac{\int \int_{-\infty}^{\infty} \phi_{u_{1,2}}^m(k_1) dk_2 dk_3}{\int \int_{-\infty}^{\infty} \phi_{u_{1,2}}(k_1) dk_2 dk_3} \quad (4.11)$$

$$\text{and} \quad r_{1221} = \frac{Co_{u_{1,2}u_{2,1}}^m(k_1)}{Co_{u_{1,2}u_{2,1}}(k_1)} = \frac{\int \int_{-\infty}^{\infty} Co_{u_{1,2}u_{2,1}}^m(k_1) dk_2 dk_3}{\int \int_{-\infty}^{\infty} Co_{u_{1,2}u_{2,1}}(k_1) dk_2 dk_3}, \quad (4.12)$$

where the numerators represent the measured spectra for terms on the RHS of Eq. (4.9), which can be written as (Antonia et al., 1996)

$$\phi_{u_{2,1}}^m(\mathbf{k}) = \frac{1 - \cos(\mathbf{k} \cdot \Delta \mathbf{x}_1)}{2\Delta x_1^2} \left\{ \left[A_a^2 + 2\cos(\mathbf{k} \cdot \Delta \mathbf{x}_3)A_aA_b + A_b^2 \right] \phi_{22}(\mathbf{k}) + \tan^2 \beta \left[A_a^2 - 2\cos(\mathbf{k} \cdot \Delta \mathbf{x}_3)A_aA_b + A_b^2 \right] \phi_{11}(\mathbf{k}) + 2\tan \beta (A_b^2 - A_a^2) \phi_{12}(\mathbf{k}) \right\}, \quad (3.13)$$

$$\phi_{u_{1,2}}^m(\mathbf{k}) = 4A^2 \sin^2 \frac{\mathbf{k} \cdot \Delta \mathbf{x}_2}{2} \phi_{11}(\mathbf{k}) / \Delta x_2^2 \quad (4.14)$$

$$\begin{aligned}
 Co_{u_{1,2}u_{2,1}}^m(\mathbf{k}) = & 2A \tan \beta \sin \frac{\mathbf{k} \cdot \Delta \mathbf{x}_2}{2} \left\{ \left[\sin(\mathbf{k} \cdot \Delta \mathbf{x}_1 + \frac{\mathbf{k} \cdot \Delta \mathbf{x}_3}{2}) - \sin \frac{\mathbf{k} \cdot \Delta \mathbf{x}_3}{2} \right] A_a \right. \\
 & \left. - \left[\sin(\mathbf{k} \cdot \Delta \mathbf{x}_1 - \frac{\mathbf{k} \cdot \Delta \mathbf{x}_3}{2}) + \sin \frac{\mathbf{k} \cdot \Delta \mathbf{x}_3}{2} \right] A_b \right\} \phi_{11}(\mathbf{k}) / \Delta x_1 \Delta x_2 \\
 & - 2A \sin \frac{\mathbf{k} \cdot \Delta \mathbf{x}_2}{2} \left\{ \left[\sin(\mathbf{k} \cdot \Delta \mathbf{x}_1 + \frac{\mathbf{k} \cdot \Delta \mathbf{x}_3}{2}) - \sin \frac{\mathbf{k} \cdot \Delta \mathbf{x}_3}{2} \right] A_a \right. \\
 & \left. + \left[\sin(\mathbf{k} \cdot \Delta \mathbf{x}_1 - \frac{\mathbf{k} \cdot \Delta \mathbf{x}_3}{2}) - \sin \frac{\mathbf{k} \cdot \Delta \mathbf{x}_3}{2} \right] A_b \right\} \phi_{12}(\mathbf{k}) / \Delta x_1 \Delta x_2. \quad (4.15)
 \end{aligned}$$

The denominators in Eqs. (4.10) - (4.12) represent the “true” spectra, which can further be expressed as:

$$\phi_{u_{2,1}}(\mathbf{k}) = k_1^2 \phi_{22}(\mathbf{k}), \quad (4.16)$$

$$\phi_{u_{1,2}}(\mathbf{k}) = k_2^2 \phi_{11}(\mathbf{k}) \quad (4.17)$$

and $Co_{u_{1,2}u_{2,1}}(\mathbf{k}) = k_1 k_2 \phi_{12}(\mathbf{k}), \quad (4.18)$

where $\phi_{i,j}(\mathbf{k})$ is the three dimensional spectral density tensor of velocity. To allow integrals in Eqs. (4.10) - (4.12) to be evaluated, relations ϕ_{ij} should be provided.

For isotropic turbulence (Batchelor, 1953):

$$\phi_{ij}(\mathbf{k}) = \phi_{ij}(k) = \frac{E(k)}{4\pi k^4} (k^2 \delta_{ij} - k_i k_j), \quad (4.19)$$

where $\delta_{i,j}$ is Kronecker delta (=1 when $i=j$; =0 when $i \neq j$). Various expressions for $E(k)$ can be found in Chapter 3.

After the spectra of $\phi_{u_{2,1}}$, $\phi_{u_{1,2}}$ and $Co_{u_{2,1}u_{1,2}}$ are corrected, the vorticity spectrum $\phi_{\omega_3}(k_1)$ can then be corrected, viz.,

$$\phi_{\omega_3}(k_1) = \phi_{u_{2,1}}(k_1) + \phi_{u_{1,2}}(k_1) - 2Co_{u_{2,1}u_{1,2}}(k_1). \quad (4.20)$$

For isotropic turbulence, the second order moments of ω_i are equal to each other and are related to $\langle u_{1,1}^2 \rangle$, viz.

$$\langle \omega_1^2 \rangle = \langle \omega_2^2 \rangle = \langle \omega_3^2 \rangle = 5 \langle u_{1,1}^2 \rangle. \quad (4.21)$$

The scaling of rms values of ω_i is based on L , U_0 and $R_L (\equiv LU_0/\nu)$ as suggested by Antonia et al. (1996) to account for the Reynolds number effect, i.e.

$$\omega_i^+ = \omega_i \frac{L}{U_0} R_L^{-1/2}, \quad (4.22)$$

where ω_i^+ denotes the normalized rms values of ω_i .

In Figs. 4.7 and 4.8, due to the attenuation caused by the finite wire length and wire separation, large differences between the measured and corrected values of rms values of ω_i at different transverse locations are observed.

The data of Antonia et al. (1988), Zhu and Antonia (1999) and the present study are all measured in the self-preserving region of the wake. It is therefore expected that all the three data sets should follow each other closely near the wake centerline. This is the case in Fig. 4.8. After the correction to account for the effect of spatial resolution, the present measured rms values of ω_3 agree well with those obtained by Antonia et al. (1988) and Zhu and Antonia (1999). In Fig. 4.7, the present data agree well with those of Zhu & Antonia (1999). However, significant discrepancy was observed between the present data and those of Antonia et al. (1988). Similar probe was employed by Zhu and Antonia (1999) and therefore the agreement is better. While for Antonia et al. (1988), the data were measured using two X-wires separated in the lateral direction. Furthermore, the measured data of Antonia et al. (1988) were not corrected to account for the effect of spatial resolution. And hence, some differences between the present data and those of Antonia et al. (1988) were observed near the wake centerline.

Figs 4.7 and 4.8 also show that the present measured rms values of ω_2 and ω_3 are in satisfactory agreement with the isotropic calculations, indicating local isotropy of small-scale of turbulence around the centerline region of the wake.

Figure 4.7 Distributions of rms values of ω_2^+ . +, Antonia et al. (1988); $\square$, Zhu and Antonia, (1999). $\blacktriangle$, present measurement (uncorrected); $\triangle$, present measurement (corrected); $\circ$, isotropic values.

Figure 4.8 Distributions of rms values of ω_3^+ . +, Antonia et al. (1988); $\square$, Zhu and Antonia (1999). $\blacktriangle$, present measurement (uncorrected); $\triangle$, present measurement (corrected); $\circ$, isotropic values.

4.4 Local Isotropy Checks

4.4.1 Local Isotropy Checks on the Centerline of the Wake

Several methods could be used to check whether the measured velocity and vorticity (and its components) agree with isotropy. The idea of using spectra analysis is to decompose the turbulent fluctuations into sinusoidal components and to study the distribution of turbulent energy among the different wavelength, representing different scales of turbulence, and its evolution with time (Mathieu and Scott, 2000). For homogenous turbulence, spectra analysis allows the examination of the dynamic process governing the evolution of energy density. This gives the information about how energy is produced, dissipated and exchanged between different scales within a turbulent flow.

In the part of the spectrum in which local isotropy prevails, time scales are short compared with those of mean flow (Tennekes and Lumley, 1972). This means that the small scales, i.e. the high wavenumber motions, tend to be approximately homogeneous and isotropic, even if large scales are not. The range of wave numbers exhibiting local isotropy is called the equilibrium range which includes inertial range and dissipation range. Therefore, the behavior of the spectra of the turbulence quantities in the equilibrium range can be used to check local isotropy.

For isotropic turbulence, all off-diagonal terms of $\langle u_i u_j \rangle$ are zero while the diagonal terms are all equal to $u'^2 = \langle u_i u_i \rangle / 3$. Hence, the cospectrum of u_1 and u_2 (or u_3) should be zero in isotropic turbulence. This is the case on the wake centerline as shown in Figure 4.9 for $k_1^* > 0.2$. The large departure of the cospectrum from zero at low wavenumbers reflects the influence from the large-scale turbulent

structures in the far wake. The zero value of the cospectrum of u_1 and u_3 is due to the requirement of both isotropy and symmetry.

Figure 4.9 Measured cospectra $Co_{u_1 u_2}$ and $Co_{u_1 u_3}$ at $x_2^+ = 0$. —, $Co_{u_1 u_2}^*(k_1^*)$; ---, $Co_{u_1 u_3}^*(k_1^*)$.

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Since velocity derivatives give more weight to the high wavenumber part of the spectra than velocity fluctuations (Corrsin and Kistler, 1955; Antonia et al., 1986; Balint et al., 1991), theoretically, velocity derivatives, vorticity and energy dissipation rates should provide a better test for local isotropy than u_i (Antonia et al., 2002). Therefore, it is important to check local isotropy using the isotropic relations among the spectra of these turbulence quantities. For isotropic turbulence, the spectra $\phi_{\omega_i}(k_1)$ can be calculated from $\phi_{u_{i,1}}$ (Van Atta, 1991; Kim and Antonia, 1993):

$$\phi_{\omega_1}^{cal}(k_1) = \phi_{u_{1,1}}(k_1) + 4 \int_{k_1}^{\infty} \frac{\phi_{u_{1,1}}(k)}{k} dk \quad (4.23)$$

and

$$\phi_{\omega_2}^{cal}(k_1) = \phi_{\omega_3}^{cal}(k_1) = \frac{5}{2} \phi_{u_{1,1}}(k_1) - \frac{k_1}{2} \frac{\partial \phi_{u_{1,1}}(k_1)}{\partial k_1} + 2 \int_{k_1}^{\infty} \frac{\phi_{u_{1,1}}(k)}{k} dk. \quad (4.24)$$

The components of $\phi_{\omega_2}(k_1)$ and $\phi_{\omega_3}(k_1)$ can also be calculated using the following isotropic relations:

$$\phi_{u_{2,1}}^{cal}(k_1) = \phi_{u_{3,1}}^{cal}(k_1) = \frac{3}{2} \phi_{u_{1,1}}(k_1) - \frac{1}{2} k_1 \frac{\partial \phi_{u_{1,1}}}{\partial k_1}, \quad (4.25)$$

$$\phi_{u_{1,2}}^{cal}(k_1) = \phi_{u_{1,3}}^{cal}(k_1) = 2 \int_{k_1}^{\infty} \frac{\phi_{u_{1,1}}(k)}{k} dk, \quad (4.26)$$

$$Co_{u_{2,1}u_{1,2}}^{cal}(k_1) = Co_{u_{3,1}u_{1,3}}^{cal}(k_1) = -\frac{1}{2} \phi_{u_{1,1}}(k_1), \quad (4.27)$$

where $\phi_{u_{i,1}}$, the streamwise velocity derivative spectrum, was determined by first differentiating the time series of u_i using Taylor's hypothesis. The spectra of ω_i ($i=2$ or 3) and their components are compared with the corresponding isotropic calculations in Figures 4.10-4.13. Figure 4.10 shows that the measured spectra $\phi_{u_{2,1}}$ and $\phi_{u_{3,1}}$ are almost the same and agree well with the isotropic values for $k_1^* > 0.05$, indicating local isotropy of the small structures. Figure 4.11 shows that

the measured spectra $\phi_{u_{1,2}}$ and $\phi_{u_{1,3}}$ are almost the same and agree well with the isotropic values over the entire range even for lower wavenumbers. This is in contrast to the velocity spectra (which will be shown later in Figure 4.15), where good agreement between measurement and calculation is found only for $k_1^* > 0.05$. This result seems to suggest that transverse velocity derivatives are more related with small scale structures and therefore satisfy local isotropy at all wavenumbers even R_λ is low (≈ 45 in the present study). The measured cospectra $Co_{u_2, u_{1,2}}(k_1)$ and $Co_{u_3, u_{1,3}}(k_1)$ are shown and compared with isotropic calculations in Figure 4.12. The measured ones agree reasonably with the calculated distributions nearly over the entire wavenumber range. The upturn of the measured distributions at high wavenumbers is due to noise contamination.

Since the spectra of the velocity derivatives involved in the expressions of ω_2 and ω_3 (Eq. 4.25) are nearly identical, it is expected that ϕ_{ω_2} and ϕ_{ω_3} should also be the same. This is the case as shown in Figure 4.13. The measured distributions ϕ_{ω_2} and ϕ_{ω_3} are almost the same except at low wavenumbers. The agreement between the two distributions is always better than 10%, and both of them agree favorably with the calculated spectra. All the results shown here imply that local isotropy is approximately satisfied on the centerline of the wake, even the Taylor microscale Reynolds number R_λ is low (≈ 45), reflecting the fact that vorticity dominated phenomena are associated with the small-scale structure of turbulence (Corrsin and Kistler, 1955).

Figure 4.10 Comparison of measured $\phi_{u_{2,1}}^*(k_1^*)$ and $\phi_{u_{3,1}}^*(k_1^*)$ with isotropic calculations.
—, calculated spectrum, Eq. (4.25); ---, $\phi_{u_{2,1}}^*(k_1^*)$; - · -, $\phi_{u_{3,1}}^*(k_1^*)$.

Figure 4.11 Comparison of measured $\phi_{u_{1,2}}^*(k_1^*)$ and $\phi_{u_{1,3}}^*(k_1^*)$ with isotropic calculations.
—, calculated spectrum, Eq. (4.26); ---, $\phi_{u_{1,2}}^*(k_1^*)$; - · -, $\phi_{u_{1,3}}^*(k_1^*)$.

Figure 4.12 Comparison of measured $Co_{u_2, u_1, 2}^*(k_1^*)$ and $Co_{u_3, u_1, 3}^*(k_1^*)$ with isotropic calculations. —, calculated spectrum, Eq. (4.27); ---, $Co_{u_2, u_1, 2}^*(k_1^*)$; -.-, $Co_{u_3, u_1, 3}^*(k_1^*)$.

Figure 4.13 Corrected spectra of vorticity and comparison with isotropy. —, calculated spectrum, Eq. (4.24); ---, $\phi_{\omega_2}^*(k_1^*)$; -.-, $\phi_{\omega_3}^*(k_1^*)$.

The same level of agreement between isotropic calculation and the measured temperature derivative spectra can also be observed for $k_1^* > 0.01$ (Figure 4.14). The spectrum of temperature derivative can be calculated using the isotropic relation given in Chapter 3 for parallel two cold wires, i.e. Eq. (3.22). Figure 4.14 shows that the measured temperature derivative spectrum ϕ_{θ_2} (after correction) is in satisfactory agreement with the isotropic calculations on the centerline region of the wake. The departure of measurement from isotropic calculation at the low wavenumber region is caused by low frequency noise contamination reflected by the low signal-to-noise ratio (≈ 80) in the experiment.

Figure 4.14 Comparison of calculated and measured (corrected) spectra of $\phi_{\partial\theta/\partial x_2}$.
—, calculated; --- measured (after correction).

4.4.2 Local Isotropy Checks off Centerline

A measure of anisotropy of the velocity field is provided by comparing the measured spectrum of transverse or spanwise velocity component with that calculated based on the isotropic assumption. For isotropic turbulence, the relationship between streamwise and transverse (or spanwise) velocity spectra is (e.g. Batchelor 1953; Monin and Yaglom 1975; Antonia and Kim 1994):

$$\phi_{u_2}^{cal}(k_1) = \phi_{u_3}^{cal}(k_1) = \frac{1}{2}(\phi_{u_1} - k_1 \frac{\partial \phi_{u_1}}{\partial k_1}) \quad (4.29)$$

where the superscript *cal* represents “calculation”. In Eq. (4.29), the measured longitudinal velocity spectrum ϕ_{u_1} is used as input for the calculation of ϕ_{u_2} or ϕ_{u_3} . The distributions of the ratio $\phi_{u_2}^{cal}/\phi_{u_2}^m$ at different transverse locations are shown in Figure 4.15. When isotropy is satisfied, $\phi_{u_2}^{cal}/\phi_{u_2}^m$ should be 1 at all scales. At $x_2/L=0$, the ratio is less than 1.1 for $k_1^* > 0.05$. The apparent departure from 1 for $k_1^* \leq 0.05$ reflects strong anisotropy of the large scale structures. Indeed, the present values of $\langle u_2^2 \rangle$ and $\langle u_3^2 \rangle$, which are mainly related with the large scale structures, show large departures from isotropy; the ratios $\langle u_1^2 \rangle / \langle u_2^2 \rangle$ and $\langle u_1^2 \rangle / \langle u_3^2 \rangle$ being about 1.4 (isotropy requires the above ratios to be 1). At around the half-width ($x_2/L=1$), the departure from isotropy is more apparent in the sense that the ratio $\phi_{u_2}^{cal}/\phi_{u_2}^m$ is 1.1 only for $k_1^* > 0.2$. At the edge of the wake ($x_2/L=1.6$ and 2), only the very small structures (e.g. $k_1^* > 0.4$) satisfy isotropy. The distributions of the ratio $\phi_{u_3}^{cal}/\phi_{u_3}^m$ at different transverse locations show the same results (and hence not shown here).

Figure 4.15 Ratios of the calculated and measured ϕ_{u_2} at different transverse locations
 ---, $x_2^+ = 0$; - · -, $x_2^+ = 1$; · · · ·, $x_2^+ = 1.6$; - - - -, $x_2^+ = 2$.

Figures 4.16 and 4.17 show the corrected spectra of ω_3 and its components and the corresponding comparisons with isotropy at different transverse locations. (Figure 4.16 for $x_2^+ = 1$; Figure. 4.17 for $x_2^+ = 1.6$; data from spectra of ω_2 lead the same results and are therefore not shown here). Fig 4.14 shows that at $x_2^+ = 1$, reasonable agreement with isotropy are still observed, especially for ϕ_{ω_3} over the entire wavenumber range. However, the discrepancy from isotropic calculation of the measured spectra becomes larger as x_2^+ further increases, especially for lower wavenumbers (Figure 4.17). For example, the range over which isotropy is satisfied by the vorticity spectrum reduced significantly, from the entire wavenumber range as shown in Figure 4.13, reduced to $k_1^* > 0.1$ in Figure 4.17. The different levels of agreement of the measured spectra with isotropic calculations are mainly due to the departures from local isotropy when the location is away from the centerline. Among

all velocity derivatives, $\phi_{u_{2,1}}^m$ and $Co_{u_{2,1}u_{1,2}}^*(k_1^*)$ show the largest departure from local isotropy. This departure is mainly caused by the anisotropy of the large-scale structures at the edge of the wake.

Figure 4.16 Corrected spectra of spanwise vorticity and its components and comparison with isotropy at $x_2^+ = 1$. —, calculated values based on isotropy; ---, $\phi_{w_3}^*$; - · -, $\phi_{u_{1,2}}^*$; - - -, $\phi_{u_{2,1}}^*$; - - - -, $-Co_{u_{1,2}u_{2,1}}^*$.

Figure 4.17 Corrected spectra of spanwise vorticity and its components and comparison with isotropy at $x_2^+ = 1.6$. —, calculated values based on isotropy; ---, $\phi_{w_3}^*$; - · -, $\phi_{u_{1,2}}^*$; - - -, $\phi_{u_{2,1}}^*$; - - - -, $-Co_{u_{1,2}u_{2,1}}^*$.

To check the departure of the measured vorticity spectra from local isotropy in a systematic way, Figure 4.18 compares the ratio $\phi_{\omega_3}^{cal} / \phi_{\omega_3}^m$ at different transverse locations, where the vorticity spectra have been corrected for the effect of special resolution. When local isotropy is perfectly satisfied, this ratio should be equal to 1 at all scales, as the solid horizontal line shown in Figure 4.18. Unlike the ratio $\phi_{u_2}^{cal} / \phi_{u_2}^m$ shown in Figure 4.15, which is close to 1 only for $k_1^* > 0.05$ at $x_2 / L = 0$, the ratio $\phi_{\omega_3}^{cal} / \phi_{\omega_3}^m$ on the centerline is close to 1 (within $\pm 5\%$) over the entire wavenumber range, independently of the small value of R_λ in the present study (≈ 45). This ratio is 0.9 for $k_1^* > 0.02$ when x_2 / L increases to 1, indicating a good agreement with local isotropy. The slight departure from isotropy is observed when $k_1^* < 0.06$ due to the anisotropy of the large scale structures when $x_2 / L = 1$. The departure of the measured spectrum from the isotropic calculation becomes apparent at $x_2 / L = 1.6$, where only the small scale structures, e.g. $k_1^* > 0.1$, satisfy isotropy. This value increases to $k_1^* > 0.65$ at the edge of the wake ($x_2 / L = 2$). The results shown in Figure 4.18 indicate that local isotropy is valid only in the range of $x_2 / L \leq 1$. Same level of agreement between the measured temperature derivative spectra and isotropic calculations can also be obtained (results are not shown here).

Figure 4.18 Ratios of the calculated and measured ϕ_{ω_3} at different transverse locations

---, $x_2^+ = 0$; —, $x_2^+ = 1$; - · -, $x_2^+ = 1.6$; - - - , $x_2^+ = 2$.

Local isotropy across the wake can be further checked by examining how closely the statistics of ω_i confirm with isotropy. The relations for the third-, and fourth-order moments of vorticity can be written as (Antonia et al., 1996):

$$\langle \omega_1^3 \rangle = \langle \omega_2^3 \rangle = \langle \omega_3^3 \rangle = 0 \quad (4.29)$$

and

$$\langle \omega_1^4 \rangle = \langle \omega_2^4 \rangle = \langle \omega_3^4 \rangle = 25 \langle u_{1,1}^4 \rangle. \quad (4.30)$$

For vorticity components, the isotropic relations are:

$$\langle u_{1,2}^2 \rangle = \langle u_{1,3}^2 \rangle = \langle u_{2,1}^2 \rangle = \langle u_{3,1}^2 \rangle = 2 \langle u_{1,1}^2 \rangle, \quad (4.31)$$

$$-\langle u_{1,2} u_{2,1} \rangle = -\langle u_{1,3} u_{3,1} \rangle = \frac{1}{2} \langle u_{1,1}^2 \rangle. \quad (4.32)$$

Due to the symmetry of vorticity probability density function $p(\omega_i)$, the isotropic value of $\langle \omega_i^3 \rangle$ should be zero (Champagne, 1978). As a result, the skewness

S_{ω_i} ($S_{\omega_i} = \langle \omega_i^3 \rangle / \langle \omega_i^2 \rangle^{3/2}$) should be zero, i.e.

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$$S_{\omega_1} = S_{\omega_2} = S_{\omega_3} = 0, \quad (4.33)$$

and the skewness factors of vorticity components are also equal to zero,

$$S_{u_{1,2}} = S_{u_{2,1}} = S_{u_{1,3}} = S_{u_{3,1}} = 0. \quad (4.34)$$

The flatness factors F_{ω_i} ($F_{\omega_i} = \langle \omega_i^4 \rangle / \langle \omega_i^2 \rangle^2$) are all equal and with the same magnitude as the flatness factor of $u_{1,1}$, namely

$$F_{\omega_1} = F_{\omega_2} = F_{\omega_3} = F_{u_{1,1}}. \quad (4.35)$$

The flatness factors of the components of ω_2 and ω_3 are also equal to $F_{u_{1,1}}$:

$$F_{u_{1,2}} = F_{u_{2,1}} = F_{u_{1,3}} = F_{u_{3,1}} = F_{u_{1,1}}. \quad (4.36)$$

The skewness factors of ω_2 and ω_3 , and their components are plotted in Figures 4.19 and 4.20, respectively. In Figures 4.19 and 4.20, except for $u_{1,1}$, all the skewness are close to zero in the centerline region ($x_2^+ \leq 1$). The results are consistent with local isotropy. The departure from zero becomes larger as x_2^+ further increases. This trend is in agreement with that revealed by the velocity derivatives (Figures 4.16 and 4.17). The very large value of the skewness factors at the outer of the wake is caused by the high intermittency of the flow.

Figure 4.19 Skewness factors of velocity derivatives across the wake.
 Δ , $u_{1,1}$; $\circ$, $u_{3,1}$; $\square$, $u_{1,3}$; $\bullet$, $u_{2,1}$; $\blacksquare$, $u_{1,2}$.

Figure 4.20 Skewness factors of vorticity components across the wake. $\blacktriangle$, ω_2 ; $\blacktriangledown$, ω_3 .

For local isotropic turbulence, the flatness factors of ω_i and $u_{i,j}$ are all equal. This is the case as shown in Figs 4.21 (a, b) and 4.22 (a, b) near the wake centerline except for the flatness of $u_{1,1}$. It is observed in the enlarged figures that the flatness of $u_{1,1}$ is smaller than other velocity derivatives and vorticity, indicating a less intermittence than other quantities. As the edge of the wake is approached, F_α becomes very large, due mainly to the high intermittency effect of the flow. The present data are in good agreement with those reported by Zhu and Antonia (1999) in the same flow at a comparable turbulence Reynolds number.

Figure 4.21 Flatness factors of velocity derivatives. Δ , $u_{1,1}$; $+$, $u_{3,1}$; $\square$, $u_{1,3}$; $\circ$, $u_{2,1}$; ∇ , $u_{1,2}$.
(a) $x_2^+ < 3$; (b) Enlarged figure for $x_2^+ < 1$.

Figure 4.22 Flatness factors of vorticity. Δ , $u_{1,1}$; $+$, ω_2 ; ∇ , ω_3 .
(a) $x_2^+ < 3$; (b) Enlarged figure for $x_2^+ < 1$.

Local isotropy can also be examined by skewness and flatness factors of the temperature derivatives. In Figure 4.23, the values of $S_{\theta,3}$ are nearly zero for the range of $x_2^+ < 1.4$. This result is consistent with local isotropy and symmetry. The

values of S_{θ_1} and S_{θ_2} should also be zero in isotropic flow without a mean temperature gradient (e.g. Danaila et al., 2000). However, there is a large departure between S_{θ_1} (and S_{θ_2}) and the isotropic value, as shown in Figure 4.23. This result is consistent with values reported in turbulence shear flow (e.g. Sreenivasan et al., 1977; Mestayer, 1982), where S_{θ_1} (and S_{θ_2}) is of order 1, and in grid turbulence with a mean cross-stream temperature gradient (Mydlarski and Warhaft, 1998), where S_{θ_1} (and S_{θ_2}) ≈ 2 . These non-zero value of S_{θ_1} (and S_{θ_2}) most likely reflect a departure from local isotropy associated with the ramp-like temperature signatures of large scale motion (Sreenivasan, 1991). The flatness factors of the temperature derivatives should be equal in isotropic turbulence, i.e. $F_{\theta_1} = F_{\theta_2} = F_{\theta_3}$. This is the case for the range of $x_2^+ < 1.2$ in Figures 4.24 (a, b). In the enlarged Figure 4.24b, F_{θ_2} and F_{θ_3} are both smaller than F_{θ_1} , indicating that transverse and spanwise temperature derivatives are less intermittent than the streamwise temperature derivative.

Figure 4.23 Skewness factors of temperature derivatives.

○, $S_{\theta_1}(\omega_2)$; +, $S_{\theta_1}(\omega_3)$; ▽, S_{θ_3} ; △, S_{θ_2} .

Figure 4.24 Flatness factors of temperature derivatives.
 $\circ$, $F_{\theta_1}(\omega_2)$; $+$, $F_{\theta_1}(\omega_3)$; ∇ , F_{θ_3} ; Δ , F_{θ_2} .
(a) $x_2^+ < 3$; (b) Enlarged figure for $x_2^+ < 1$.

4.5 Statistics of Energy and Temperature Dissipation Rates

4.5.1 Mean Energy and Temperature Dissipation rates

Simple approximation to energy dissipation rate ε and temperature dissipation rate χ can be obtained based on the isotropic assumption, i.e. Eqs. (3.1) and (3.2). However, the use of Eqs. (3.1) and (3.2) as appropriate substitutes of ε and χ has been questioned (e.g. Hosokawa et al., 1996; Chen et al., 1997). Using the present probe, three of the nine velocity derivatives that make up ε are measured simultaneously. These derivatives are $u_{1,1}$, $u_{1,2}$ and $u_{2,1}$ or after a 90° rotation, $u_{1,1}$, $u_{1,3}$ and $u_{3,1}$. Assuming local isotropy and homogeneity, simultaneous approximations to ε can be made based on continuity using Eqs. (4.3) and (4.4). The two parallel cold wires are capable of measuring the temperature derivatives $\theta_{,1}$ and $\theta_{,2}$ or $\theta_{,1}$ and $\theta_{,3}$ after a 90° rotation. The temperature dissipation rate χ can then be approximated using Eqs. (4.5) and (4.6).

The mean energy and temperature dissipation rates using different methods are compared in Figures 4.25 and 4.26. Since more derivative correlations are included in Eqs. (4.3) or Eq. (4.5), it is expected that they should provide more reasonable approximations to ε or χ than when ε_{iso} or χ_{iso} (Eqs. 3.1 or 3.2) is used. The basis of comparison lies on the fact that the approximated quantities and spectra should approach to the isotropic values when local isotropy is satisfied. To account for the effect of the spatial resolution, all velocity derivatives have been corrected using method outlined in this chapter. Also shown are results by Browne et al. (1987) and Zhu and Antonia (1999). The scaling of $\langle \varepsilon \rangle$ is based on L and U_0 , i.e. $\langle \varepsilon \rangle^+ = \langle \varepsilon \rangle L / U_0^3$, and the scaling of $\langle \chi \rangle$ is based on L , U_0 and T_0 , i.e. $\langle \chi \rangle^+ = \langle \chi \rangle L / T_0^2 U_0$.

Figure 4.25 shows that the magnitude of $\langle \varepsilon_{ap} \rangle$ is about 20% larger than that of $\langle \varepsilon_{iso} \rangle$ across the wake. Significant differences between ε_{iso} and ε_{ap} indicates that the use of the one-dimensional surrogate, as usually used in experimental studies, may cause significant errors. While the present measured data agree well with those of Zhu and Antonia (1999) before the corrections of the effect of finite spatial resolution, significant discrepancy was observed between the present corrected values and those of Zhu and Antonia (1999). It is therefore suspected that the obtained energy dissipation rates of Zhu and Antonia (1999) were not corrected to account for the effect of spatial resolution. It is also observed that the present corrected values agree favorably with those of Browne et al. (1987) over the wake.

In Fig. 4.26, there is no apparent difference between χ_{iso} and χ_{ap} near the wake centerline. The result seems to suggest that Eq. (3.2) provides a good approximation to the averaged temperature dissipation rate in the present turbulent wake. This is, however, not applied to the instantaneous values as discussed in other parts of this chapter.

Figure 4.25 Comparison of the mean turbulent energy dissipation rate $\langle \varepsilon \rangle$. $\blacktriangle, \langle \varepsilon_{iso} \rangle$; $\nabla, \langle \varepsilon_{ap} \rangle$; —, Browne et al., 1987; $\circ$, Zhu and Antonia (1999).

Figure 4.26 Comparison of the mean temperature dissipation rate $\langle \chi \rangle$. $\blacktriangle, \langle \chi_{iso} \rangle$; $\nabla, \langle \chi_{ap} \rangle$.

As a further check of the validity of Eqs. (4.3) and (4.5), the spectra corresponding to energy and temperature dissipation rates obtained on the centerline using different methods are compared in Figures 4.27 and 4.28. The spectrum $\phi_{\varepsilon_f}(k_1)$ (Figure 4.27) measured using an eight-wire vorticity probe is also shown here, where the subscript f represents the full energy dissipation rate. Details of the measurement using the eight-wire and the expressions for ε_{ful} can be found in Chapter 6. Since the spectra are normalized by the Kolmogorov scales, the areas under the various distributions should be equal to 1. The distribution of $\phi_{\varepsilon_{ap}}(k_1)$ agrees well with that of $\phi_{\varepsilon_f}(k_1)$. In contrast, $\phi_{\varepsilon_{iso}}(k_1)$ departs significantly from both $\phi_{\varepsilon_f}(k_1)$ and $\phi_{\varepsilon_{ap}}(k_1)$, except for $k_1^* > 0.15$. This result indicates that the use of ε_{iso} as a substitute of ε , as usually used in experimental studies, may cause significant errors, at least in the spectral domain. In Figure 4.28, there is a good agreement between the spectra $\phi_{\chi_f}(k_1) [\equiv \phi_{\theta_1}(k_1) + \phi_{\theta_2}(k_1) + \phi_{\theta_3}(k_1)]$ and $\phi_{\chi_{ap}}(k_1)$. This is mainly because of the good agreement between the measured distributions $\phi_{\theta_2}(k_1)$ (Figure 4.14) and $\phi_{\theta_3}(k_1)$ [not shown, almost identical with $\phi_{\theta_2}(k_1)$]. The measured distribution $\phi_{\chi_{iso}}(k_1)$ departs significantly from both $\phi_{\chi_f}(k_1)$ and $\phi_{\chi_{ap}}(k_1)$ for the whole wavenumbers range, indicating the inappropriateness of χ_{iso} as a substitute of χ , at least in the spectral domain.

Figure 4.27 Spectra of energy dissipation rate obtained using various methods.
---, $\phi_{\epsilon_f}^*(k_1^*)$; - · -, $\phi_{\epsilon_{ap}}^*(k_1^*)$; · · ·, $\phi_{\epsilon_{iso}}^*(k_1^*)$

Figure 4.28 Spectra of temperature dissipation rate obtained using various methods.
---, $\phi_{\chi_f}^*(k_1^*)$; - · -, $\phi_{\chi_{ap}}^*(k_1^*)$; · · ·, $\phi_{\chi_{iso}}^*(k_1^*)$

4.5.2 Correlation between Turbulent Energy and Temperature Fields

The correlation coefficients between vorticity and temperature fluctuations are calculated. The correlation coefficient $\rho_{\alpha,\beta}$ is defined in Eq. (4.2). Symmetry in the x_3 -direction requires that the correlation between θ and ω_2 be zero. This is confirmed by the measured values, as shown in Figure 4.29. The correlation coefficient between θ and ω_3 has negative values for $x_2^+ > 0$. The largest negative values occur near the edge of the wake around $x_2^+ = 2$. The negative values of ρ_{θ,ω_3} in the upper part of the wake indicate that spanwise vorticity mainly resides in relatively warm fluid near the wake edge. Observations of dye in the far wake of a cylinder (Antonia et al., 1987) suggested a reasonable high correlation between dye concentrations in the outer wake and the spanwise vorticity, as inferred from the path of the dye streaklines introduced just outside the edge of the wake. ρ_{θ,ω_3} becomes smaller and finally disappears as further approaching to the edge of the wake due to the decrease of heat effect.

Figure 4.29 Correlation coefficients between θ and ω_2 , θ and ω_3 . $\square$, ρ_{θ,ω_2} ; Δ , ρ_{θ,ω_3} .

To gain a better understanding on the correlations between temperature and vorticity, the cospectra $Co_{\theta\omega_2}$ and $-Co_{\theta\omega_3}$ are shown in Figures 4.30 and 4.31, respectively. As it can be seen in Figure 4.30, the cospectra $Co_{\theta\omega_2}$ are zero at different transverse locations ($x_2^+ = 0, 1$ and 2) for $k_1^* > 0.2$, indicating that there is nearly no correlation between θ and ω_2 as required by symmetry with respect to x_3 . This result is consistent with the correlation coefficients shown in Figure 4.29 in which the values of ρ_{θ,ω_2} are approximately equal to zero. The slight departure of the cospectra from zero at low wavenumbers ($k_1^* < 0.2$) may reflect the influence from the large-scale vortical structures in the far wake. This influence seems to follow a similar pattern at different transverse locations ($x_2^+ = 0, 1$ and 2) since there is nearly no difference among the three cospectra. In contrast, the cospectra $-Co_{\theta\omega_3}$ at three different transverse locations show large difference (Figure 4.31). On the wake centerline ($x_2^+ = 0$), the cospectrum $-Co_{\theta\omega_3}$ is similar to $-Co_{\theta\omega_2}$ as shown in Figure 4.30. This is because $-Co_{\theta\omega_3}$ should be zero on the centerline as required by the antisymmetry. The departure of the cospectrum $-Co_{\theta\omega_3}$ from zero at low wavenumbers increases as the wake edge is approached; for example, at $x_2^+ = 1$ and $x_2^+ = 2$, the upper range over which the cospectra are zero shifts to higher k_1^* , i.e. for $x_2^+ = 1$, the cospectrum is zero when $k_1^* > 0.3$ and $k_1^* > 0.5$ for $x_2^+ = 2$, indicating that the spanwise vorticity mainly resides in relatively warm fluid near the wake edge. This is also consistent with negative mean temperature gradient in this region.

Figure 4.30 Measured cospectra $Co_{\theta\omega_2}$ at different transverse locations.
—, $x_2^+ = 0$; ---, $x_2^+ = 1$; -.-, $x_2^+ = 2$.

Figure 4.31 Measured cospectra $-Co_{\theta\omega_3}$ at different transverse locations.
—, $x_2^+ = 0$; ---, $x_2^+ = 1$; -.-, $x_2^+ = 2$.

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The correlation between $\ln \varepsilon_r$ and $\ln \chi_r$ is important in quantifying the behavior of the temperature structure functions $\langle (\delta\theta)^n \rangle$ in the inertial range, where r is a certain length scale (longitudinal separation). Van Atta (1971) noted that $\rho_{\ln \varepsilon_r, \ln \chi_r}$ should vary only weakly with r across the inertial range. Previous experimental results obtained by simply replacing ε and χ using ε_{iso} and χ_{iso} have yielded apparently disparate values for $\rho_{\ln \varepsilon_r, \ln \chi_r}$ in different flows (e.g. Antonia and Van Atta, 1975; Antonia and Chambers, 1980; Meneveau et al., 1990). However, the validity of ε_{iso} and χ_{iso} as substitutes for ε and χ has been questioned. The magnitude of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ may depend on a number of factors, e.g. the Reynolds number, the separation between the hot and cold wires, the type of flow as well as the type of approximation used for ε and χ . Meneveau et al. (1990) noted that the apparent discrepancy of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ may be due to the influence of the hot wire on the cold wire signals; this effect may be significant in a turbulent jet because of the high turbulent intensity of this flow. However, this explanation may not be adequate, especially when the separation between the hot and cold wires is sufficient to avoid any interference from the hot wire signals. The values of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ calculated using either $(\varepsilon_{ap}, \chi_{ap})$ or $(\varepsilon_{iso}, \chi_{iso})$ in the present wake centerline are shown in Figure 4.32. The magnitude of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ obtained using $(\varepsilon_{ap}, \chi_{ap})$ is about 20% larger in the range of $r^* \leq 300$ than when $(\varepsilon_{iso}, \chi_{iso})$ are used. This finding suggests that for the present flow, the isotropic substitutes $(\varepsilon_{iso}, \chi_{iso})$ cannot represent (ε, χ) properly. The magnitude of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ in the present wake is about 2-3 times as large as that in the grid flow (Mydlarski and Warhaft, 1998; with $R_\lambda = 582$), where $(\varepsilon_{iso}, \chi_{iso})$ were used to represent (ε, χ) . It

is also larger than that reported by Meneveau et al. (1990) in a heated wake at higher values of R_λ . This may suggest that the correlation coefficient $\rho_{\ln \varepsilon_r, \ln \chi_r}$ decreases with the increase of R_λ in the same flow, which is in contrast with the R_λ dependence of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ indicated by the DNS data of Wang et al. (1999) in a forced box turbulence. The present measured values of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ differ significantly from those for a heated turbulent jet (Antonia and Van Atta, 1975) and the atmospheric surface layer (Antonia and Chambers, 1980). The large scatter in Figure 4.32 indicates that the magnitude of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ may depend on the relations used to approximate ε and χ . A more likely possibility is the effect of R_λ . Further, the dependence of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ on the nature of the flow or the initial conditions cannot be dismissed.

Figure 4.32 Comparison of correlation coefficient $\rho_{\ln \varepsilon_r, \ln \chi_r}$ with previous experimental values. Present: Δ , $\varepsilon = \varepsilon_{ap}$, $\chi = \chi_{ap}$; ∇ , $\varepsilon = \varepsilon_{iso}$, $\chi = \chi_{iso}$. $\square$, Meneveau et al. (1990); $\bullet$, Antonia and Van Atta (1975); $\times$, Antonia and Chambers (1980); $\circ$, Wang et al. (1999).

The correlations between ω_3^2 , ε and χ can be quantified by examining $\langle \beta | \alpha \rangle$, the expectations of β conditioned on particular values of α (where α represents ω_3^2 , β represents ε or χ). This conditional expectation is defined as:

$$\langle \beta | \alpha \rangle = \int_{-\infty}^{+\infty} \beta p(\beta | \alpha) d\beta, \quad (4.37)$$

where $p(\beta | \alpha) \equiv p_{\alpha, \beta} / p_{\alpha}$ is the PDF of β conditioned on α . The expectations of ε_{ap} and χ_{ap} conditioned on ω_3^2 , measured on the centerline of the wake and normalized by their mean values, are shown in Figure 4.33. The values $\langle \varepsilon_{ap} | \omega_3^2 \rangle / \langle \varepsilon_{ap} \rangle$ increase linearly with an overall gradient of 0.75, reflecting an observable dependence of ε_{ap} on ω_3^2 . The dependence of $\langle \varepsilon_{ap} \rangle$ on $\langle \omega_3^2 \rangle$, as shown in Figure 4.33, is consistent with that reported by Chen et al. (1997) using direct numerical simulations. The values of $\langle \chi_{ap} | \omega_3^2 \rangle / \langle \chi_{ap} \rangle$ increase slowly with $\omega_3^2 / (\omega_3^2)'$ (with a gradient close to 0.52) when the latter is smaller than 0.5, indicating a small correlation between χ and ω_3^2 . When $\omega_3^2 / (\omega_3^2)'$ is greater than 0.5, $\langle \chi_{ap} | \omega_3^2 \rangle / \langle \chi_{ap} \rangle$ keeps approximately constant with a magnitude close to 1, indicating that there is nearly no correlation between χ and ω_3^2 for large fluctuations. The expectations of ε_{iso} and χ_{iso} conditioned on ω_3^2 are also shown in Figure 4.33. The dependence of $\langle \varepsilon_{iso} \rangle$ on $\langle \omega_3^2 \rangle$, as compared with $\langle \varepsilon_{ap} \rangle$ is very weak, in the sense that $\langle \varepsilon_{iso} | \omega_3^2 \rangle / \langle \varepsilon_{iso} \rangle$ increases linearly with $\omega_3^2 / (\omega_3^2)'$ at a gradient of 0.42 for $\omega_3^2 / (\omega_3^2)' < 1$ (the gradient decreases as $\omega_3^2 / (\omega_3^2)'$ further increases). χ_{iso} depends on $\langle \omega_3^2 \rangle$ weakly only when $\omega_3^2 / (\omega_3^2)'$ is smaller than 0.5. When $\omega_3^2 / (\omega_3^2)'$ is larger than 0.5, $\langle \chi_{iso} | \omega_3^2 \rangle / \langle \chi_{iso} \rangle$ is nearly equal to zero,

indicating that there is no correlation between χ_{iso} and $\langle \omega_3^2 \rangle$. The results suggest that the use of ε_{iso} and χ_{iso} as substitutes of ε and χ may possibly lead to incorrect conclusions.

Figure 4.33 Expectations of ε_{ap} and χ_{ap} conditioned on particular values of ω_3^2 .
 $\circ, \beta$ represents ε_{iso} ; $\square, \varepsilon_{ap}$; ∇, χ_{iso} ; Δ, χ_{ap} .

To further check the spatial correlation between ω_3^2 , ε and χ , and to check the appropriateness of using ε_{iso} and χ_{iso} to represent ε and χ , respectively, the conditional expectations of the three quantities obtained using the six-wire probe and the eight-wire probe (Chapter 6) are given in Figures 4.34 - 4.39. In Figures 4.34 - 4.35, $\langle (\omega_3^2)_r^\dagger | \varepsilon_r^\dagger \rangle$ and $\langle (\omega_3^2)_r^{2\dagger} | \varepsilon_r^\dagger \rangle$ are shown as a function of $\varepsilon_r^\dagger$ for r in the inertial range, where $\dagger$ means that the quantities are normalized by their mean

values, i.e. $(\omega_3^2)_r^\dagger = (\omega_3^2)_r / \langle (\omega_3^2) \rangle$ and $\varepsilon_r^\dagger = \varepsilon_r / \langle \varepsilon \rangle$. The energy dissipation rate ε is obtained either by the approximation relation Eq. 4.3 (Figure 4.34) or by the isotropic relation of Eq. 3.1 (Figure 4.35). The functions $y = cx$ and $y = cx^2$ are also shown in the figures for comparison, where c is a constant determined by the curve fit. In Figure 4.34, it shows clearly that the first and second order moments of $(\omega_3^2)_r^\dagger$ increase with $\varepsilon_r^\dagger$. $\langle (\omega_3^2)_r^\dagger | \varepsilon_r^\dagger \rangle$ grows linearly with $\varepsilon_r^\dagger$ over the range of $\varepsilon_r^\dagger > 0.1$ and $\langle (\omega_3^2)_r^{2\dagger} | \varepsilon_r^\dagger \rangle$ also agrees quite well with $y = cx^2$ in this range. The results imply a strong spatial correlation between the spanwise vorticity and the energy dissipation rate. The present results are consistent with the DNS study by Chen et al. (1997) where the full energy dissipation rate and enstrophy $\Omega (= \nu \omega_i \omega_i)$ were used. When ε_{iso} is used (Figure 4.35), even though the first and second order moments of $(\omega_3^2)_r^\dagger$ increase with $\varepsilon_r^\dagger$, the gradients become much smaller than that in Figure 4.34. The results seem to suggest that the use of the one-dimensional energy dissipation rate to substitute the full energy dissipation rate may lead to erroneous conclusions. To validate the present six-wire results and to clarify the different behaviors between ε_{iso} and ε_{ful} , the conditional expectations of ω_3^2 and ε obtained from the three-dimensional vorticity measurements using an eight-wire probe are also presented in this section. Details of the measurement using an eight-wire probe and the expression for ε_{ful} can be found in Chapter 6. Figures 4.36 and 4.37 shows the variations of $\langle (\omega_3^2)_r^\dagger | \varepsilon_r^\dagger \rangle$ and $\langle (\omega_3^2)_r^{2\dagger} | \varepsilon_r^\dagger \rangle$ against $\varepsilon_r^\dagger$. The energy dissipation rate ε is obtained either by the full expression (Eq. 6.8) or by the one-dimensional isotropic substitute (Eq. 3.1). It is expected that, with more velocity

derivative terms involved, Eq. 6.8 should be more accurate than Eq. (4.3). In Figure 4.36, $\langle (\omega_3^2)_r^\dagger | \varepsilon_r^\dagger \rangle$ and $\langle (\omega_3^2)_r^{2\dagger} | \varepsilon_r^\dagger \rangle$ reveal almost the identical trends as those shown in Figure 4.34 and both of them agree well with functions $y=cx$ and $y=cx^2$, respectively, thus proving the strong correlation between the spanwise vorticity and the energy dissipation rate. The results also support the use of Eq. (4.3) as a proper substitute to the full energy dissipation rate. In Figure 4.37, the conditional expectations obtained using ε_{iso} show large discrepancies with those obtained using ε_{ful} (Eq. 6.8) as shown in Figure 4.36. The discrepancies further imply that it is inappropriate to use ε_{iso} as proper substitute of ε .

Figure 4.34 Conditional expectations $\langle (\omega_3^2)_r^\dagger | (\varepsilon_{ap})_r^\dagger \rangle$ as a function of $\varepsilon_{ap}^\dagger$ using six-wire measurements. $\circ$, $n=1$; $\square$, $n=2$; ---, $y=cx$; - - -, $y=cx^2$.

derivative terms involved, Eq. 6.8 should be more accurate than Eq. (4.3). In Figure 4.36, $\langle (\omega_3^2)_r^\dagger | \varepsilon_r^\dagger \rangle$ and $\langle (\omega_3^2)_r^{2\dagger} | \varepsilon_r^\dagger \rangle$ reveal almost the identical trends as those shown in Figure 4.34 and both of them agree well with functions $y = cx$ and $y = cx^2$, respectively, thus proving the strong correlation between the spanwise vorticity and the energy dissipation rate. The results also support the use of Eq. (4.3) as a proper substitute to the full energy dissipation rate. In Figure 4.37, the conditional expectations obtained using ε_{iso} show large discrepancies with those obtained using ε_{ful} (Eq. 6.8) as shown in Figure 4.36. The discrepancies further imply that it is inappropriate to use ε_{iso} as proper substitute of ε .

Figure 4.34 Conditional expectations $\langle (\omega_3^2)_r^\dagger | (\varepsilon_{ap})_r^\dagger \rangle$ as a function of $\varepsilon_{ap}^\dagger$ using six-wire measurements. $\circ$, $n=1$; $\square$, $n=2$; ---, $y=cx$; - · -, $y=cx^2$.

Figure 4.35 Conditional expectations $\langle (\omega_3^2)_r^{\dagger} | (\epsilon_{iso})_r^{\dagger} \rangle$ as a function of $\epsilon_{iso}^{\dagger}$ using six-wire measurements. $\circ$, $n=1$; $\square$, $n=2$; ---, $y=cx$; - - -, $y = cx^2$.

Figure 4.36 Conditional expectations $\langle (\omega_3^2)_r^{\dagger} | (\epsilon_{ful})_r^{\dagger} \rangle$ as a function of $\epsilon_{ful}^{\dagger}$ using eight-wire measurements. $\circ$, $n=1$; $\square$, $n=2$; ---, $y=cx$; - - -, $y = cx^2$.

Figure 4.37 Conditional expectations $\langle (\omega_3^2)_r^\dagger | (\varepsilon_{iso})_r^\dagger \rangle$ as a function of $\varepsilon_{iso}^\dagger$ using eight-wire measurements. $\circ$, $n=1$; $\square$, $n=2$; ---, $y=cx$; - - -, $y=cx^2$.

The correlation between temperature dissipation rate and spanwise vorticity ω_3^2 can be checked by the conditional expectations $\langle (\omega_3^2)_r^\dagger | \chi_r^\dagger \rangle$ and $\langle (\omega_3^2)_r^{2\dagger} | \chi_r^\dagger \rangle$ as shown in Figures 4.38 and 4.39. The temperature dissipation rates are substituted either by Eq. (4.5) or by the isotropic relation Eq (3.2). The two figures suggest that there are nearly no correlations between $(\omega_3^2)_r^\dagger$, $(\omega_3^2)_r^{2\dagger}$ and χ , both for χ_{ap} and χ_{iso} . However, care should be taken to make any comments on the similar trends obtained using χ_{ap} and χ_{iso} . The similarity as shown in Figures 4.38 and 4.39 may not mean that χ_{iso} is an appropriate substitute to χ_{ful} . It may rather suggest the low correlation between χ and ω_3^2 as already shown in Figure 4.33 than the appropriateness of using χ_{iso} as the substitute of χ .

Figure 4.38 Conditional expectations of $\langle (\omega_3^2)_r^\dagger | (\chi_{ap})_r^\dagger \rangle$ as function of $\chi_{ap}^\dagger$ using six-wire measurements. $\circ$, $n=1$; $\square$, $n=2$;

Figure 4.39 Conditional expectations of $\langle (\omega_3^2)_r^\dagger | (\chi_{iso})_r^\dagger \rangle$ as function of $\chi_{iso}^\dagger$ using six-wire measurements. $\circ$, $n=1$; $\square$, $n=2$;

4.6 Turbulent Prandtl Number Estimation

The turbulent Prandtl number Pr_T is crucial in developing turbulent heat and mass transfer models (Bejan, 2004). The turbulent Prandtl number reflects the similarity between turbulent momentum and heat transfer, viz.

$$Pr_T = \frac{\nu_T}{\alpha_T}, \quad (4.38)$$

where ν_T is the eddy momentum diffusivity and α_T is the thermal diffusivity. ν_T and α_T can be obtained by the following equations:

$$\nu_T = -\frac{\langle u_1 u_2 \rangle}{(\partial U / \partial x_2)} \quad (4.39)$$

and
$$\alpha_T = -\frac{\langle u_2 \theta \rangle}{(\partial T / \partial x_2)}, \quad (4.40)$$

where $\partial U / \partial x_2$ and $\partial T / \partial x_2$ are the mean velocity and temperature gradients, $\langle u_1 u_2 \rangle$ and $\langle u_2 \theta \rangle$ are Reynolds shear stress and heat flux, respectively. The accurate measurements of all four quantities on the right sides of Eqs. (4.39) and (4.40) are extremely difficult, and for this reason reliable data on local values of Pr_T are relatively rare (Kays and Crawford, 1993). It is generally accepted that $Pr_T \cong 0.7$ in free turbulent flows of air (Launder, 1976). Antonia et al (1993a) found that the average value of Pr_T is close to 1 at $x_1/d=10$ across a heated turbulent cylinder wake but decreases significantly at $x_1/d=20$. The investigation of Kays (1994) showed that, for wall turbulence, Pr_T is 0.8-1.0 in the full turbulent region and 0.5-0.7 in the wake region. Recently, Tagawa et al. (2001) measured the velocity and temperature fields in the turbulent wake behind a heated cylinder using LDV and a three-wire temperature probe. They show that the turbulent Prandtl number is

principally constant (around 0.7). In their DNS studies, Debusschere and Rutland (2004) found that for plane channel and Couette flows, Pr_T has a value of 1 near the wall but decreases to 0.8 at the centerline. They also found that streamwise vortices play a significant role in the near wall heat transfer for both flows.

Using the present six-wire probe, the four quantities on the right-hand sides of Eqs. (4.39) and (4.40) can be measured simultaneously to provide a relatively accurate estimation of Pr_T . The Reynolds shear stress is shown in Figure 4.40 after normalization by the local wake centerline velocity deficit U_0 . As shown in Figure 4.40, the magnitude of the maximum Reynolds shear stress appears at around $x_2^+ = 1$ with an asymptotic peak value of 0.0095. It passes the origin as requested by the antisymmetry. The distribution of the heat flux is shown in Figure 4.41. The maximum heat flux also appears around $x_2^+ = 1$, with an asymptotic peak value of 0.0055.

Figure 4.40 Distribution of the Reynolds shear stress across the wake.

$\circ$, $\langle u_1 u_2 \rangle^+$; ---, 4th-order polynomial curve fit.

Figure 4.41 Distribution of the heat flux across the wake.

$\circ$, $\langle u_2 \theta \rangle^+$; ---, 4th order polynomial curve fit.

The distributions of the turbulent eddy momentum diffusivity ν_T and thermal diffusivity α_T are shown in Figure 4.42. They are calculated based on the distributions of $\partial U / \partial x_2$, $\partial T / \partial x_2$, $\langle u_1 u_2 \rangle$ and $\langle u_2 \theta \rangle$ by using the 4th order polynomial fits to the distributions of U , T , $\langle u_1 u_2 \rangle$ and $\langle u_2 \theta \rangle$. For $\partial U / \partial x_2$ and $\partial T / \partial x_2$, the limiting values at $x_2^+ = 0$ were obtained using L'Hopital's rule. A reasonable definition of the wake edge in the present study should be the range where $x_2^+ > 2$. It can be seen that the turbulent eddy momentum diffusivity keeps approximately constant for $x_2^+ < 1.5$ and increases as the edge of the wake is approached. On the other hand, the turbulent thermal diffusivity has a peak value around $x_2^+ = 0.5$ and decreases thereafter till $x_2^+ = 1.5$. It then slowly increases as the edge of the wake is further approached. The values of Pr_T are shown in Figure 4.43. Pr_T is roughly a constant with a magnitude of 0.76 in the region of $x_2^+ \leq 1$.

For $x_2^+ > 1$, the values of Pr_T increase monotonously ($Pr_T=1.5$ around $x_2^+ = 2$) due to the increased v_T and decreased α_T in the outer range of the wake. The present results imply that the turbulent Prandtl number Pr_T , which is an important parameter used in computational fluid mechanics and heat transfer, should not be considered as a constant value across the wake. It should be taken as a function of x_2/L .

Figure 4.42 Distributions of v_T and α_T across the wake. $\circ$, v_T ; $\triangle$, α_T .

Figure 4.43 Distribution of the turbulent Prandtl number across the wake.

4.7 Concluding Remarks

A six-wire probe was used to determine energy and temperature dissipation rates in the far field of a cylinder wake. Local isotropy across the wake was evaluated by examining the ratios $\phi_{u_2}^{cal} / \phi_{u_2}^m$ and $\phi_{\omega_3}^{cal} / \phi_{\omega_3}^m$ at different transverse locations. At $x_2^+ = 0$, $\phi_{u_2}^{cal} / \phi_{u_2}^m$ is less than 1.1 only for $k_1^* > 0.05$. The wavenumber at which local isotropy is satisfied increases as the wake edge is approached, indicating a departure from local isotropy at these locations. The ratio $\phi_{\omega_3}^{cal} / \phi_{\omega_3}^m$ on the centerline is close to 1 (within $\pm 5\%$) over the entire wavenumber range, independently of the small value of R_λ . This ratio is relatively small (0.9) only for $k_1^* > 0.02$ when x_2^+ increases to 1.2. Skewness and flatness factors of ω_2 and ω_3 and their components also suggest that local isotropy is approximately satisfied for $x_2^+ \leq 1$, which is consistent with that revealed by the flatness factors of the temperature gradients. The non-zero values of $S_{\theta,1}$ and $S_{\theta,2}$ away from the centerline may be associated with the ramp-like temperature signatures of the large scale motion. They may also be due to the effect of the mean shear.

The spectra corresponding to energy and temperature dissipation rates obtained using different methods are compared. The result shows that the use of ε_{iso} and χ_{iso} as substitutes of ε and χ , as usually used in experimental studies, may cause significant errors, at least in the spectral domain. Previous experimental results obtained using ε_{iso} and χ_{iso} yield rather disparate values for $\rho_{\ln \varepsilon_r, \ln \chi_r}$ in different flows and at different R_λ . The present results show that the use of ε_{iso} and χ_{iso} underestimates $\rho_{\ln \varepsilon_r, \ln \chi_r}$ by about 20% in the scaling range. The large scatter of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ in the literature indicates that the correlation coefficient may depend on

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the magnitude of R_λ as well as the nature or the initial conditions of the flow. The correlation between both ε and χ with ω_3^2 are also discussed for different estimations to ε and χ using conditional analysis. Strong correlations between either ε_{ful} or ε_{ap} with ω_3^2 are observed, in accordance with DNS-based investigations. This result also indicates that the instantaneous values of ε can be properly represented by ε_{ap} . In contrast, the correlation between ε_{iso} and ω_3^2 is weak, indicating that ε_{iso} may not be a good substitute to ε_{ful} . The correlation between either χ_{ap} or χ_{iso} with ω_3^2 is low, no matter which estimations are used. The negative values of ρ_{θ, ω_3} in the upper part of the wake indicate that spanwise vorticity mainly resides in the relatively warm fluids near the edge of the wake. This is also consistent with the negative mean temperature gradient in this region.

The present measurements using a six-wire probe enable the evaluation of the turbulent Prandtl number across the wake. The results show that the turbulent Prandtl number is not a constant across the wake; it changes from 0.76 for $x_2^+ \leq 1$ to 1.5 at $x_2^+ = 2$. This result indicates that it might incur errors in computational fluid mechanics and heat transfer if the turbulent Prandtl number is treated as a constant value across the wake.

Chapter 5

Scaling of Longitudinal and Transverse Velocity Increments in a Cylinder Wake

5.1 Introduction

The similarity hypotheses proposed by Kolmogorov (1941, hereafter, K41), and the later revision (Kolmogorov, 1962 or K62) are important for the study of small-scale turbulence. The main assumptions used in these hypotheses are the local isotropy and very large Reynolds numbers both in the dissipative range ($r \ll L_s$, where L_s is the integral length scale of turbulence) and in the inertial range ($\eta \ll r \ll L_s$, where $\eta \equiv \nu^{3/4} / \langle \varepsilon \rangle^{1/4}$ is the Kolmogorov length scale; $\langle \varepsilon \rangle$ and ν are the mean energy dissipation rate and kinematic viscosity, respectively; the angular brackets denote time averaging). According to K41, the moments of the velocity increment $\delta\alpha \equiv \alpha(\bar{\mathbf{x}} + \mathbf{r}) - \alpha(\bar{\mathbf{x}})$, the so-called structure function $(\delta\alpha)^n$, depends only on $\langle \varepsilon \rangle$ and ν when the separation r is sufficiently small, i.e. within the dissipative range. In the definition of $\delta\alpha$, α represents any component (u_1 , u_2 and u_3) of the velocity fluctuations in the longitudinal (streamwise), transverse and spanwise directions, respectively, and r is the separation between the two points. If the velocity component is in the same direction of the separation of the two points, the velocity increments are defined as longitudinal one. In contrast, if the velocity component is perpendicular to the separation r , $\delta\alpha$ is referred to as the transverse velocity increment. It is assumed that in dissipative range,

$$\langle (\delta u_1)^n \rangle \sim r^n \langle (\partial u_1 / \partial x_1)^n \rangle. \quad (5.1)$$

When r is in the inertial range, the effect of ν can be neglected and

$$\langle (\delta u_1)^n \rangle \sim (r \langle \varepsilon \rangle)^{\frac{n}{3}}. \quad (5.2)$$

For isotropic turbulence, Kolmogorov also derived an equation, starting from the Karman-Howarth (1938) equation, to relate $\langle (\delta u_1)^2 \rangle$ and $\langle (\delta u_1)^3 \rangle$ viz.

$$-\langle (\delta u_1)^3 \rangle + 6\nu \frac{\partial}{\partial r} \langle (\delta u_1)^2 \rangle = \frac{4}{5} \langle \varepsilon \rangle r. \quad (5.3)$$

For inertial range values of r , the second term in Eq. (5.3) is negligible and Eq. (5.3) is simplified to the well-known “4/5” law, i.e.

$$-\langle (\delta u_1)^3 \rangle = \frac{4}{5} \langle \varepsilon \rangle r. \quad (5.4)$$

Over the past years, extensive experimental and numerical investigations have highlighted that the scaling exponents of $\langle (\delta \alpha)^n \rangle$ deviate from $n/3$, known as the anomalous scaling. The departure of the scaling exponents from $n/3$, or the so-called anomalous scaling, increases nonlinearly with the increase of n . This anomalous scaling is normally attributed to the so-called small-scale intermittency. To account for the spatial and temporal intermittency effect of the dissipative field, Kolmogorov (1962) introduced the refined similarity hypothesis (RSH), normally referred to as K62. One consequence in K62 is that the velocity structure functions $\langle (\delta \alpha)^n \rangle$ are scaled as

$$\langle (\delta \alpha)^n \rangle \sim r^{\zeta_\alpha(n)} \quad (5.5)$$

in the inertial range, where $\zeta_\alpha(n)$ are the scaling exponents of $\langle (\delta \alpha)^n \rangle$ and may not be equal to $n/3$.

The similarity hypotheses K62, or K41, did not distinguish between n^{th} -order moments of the longitudinal velocity increments, δS_L , where the velocity is parallel

to the displacement vector, and the transverse velocity increments, δS_T , where the velocity vector is normal to the displacement vector. The most common method to measure the longitudinal velocity increment δS_L is temporally sampling the velocity fluctuations u_1 along the mean flow direction. Taylor's hypothesis is used to convert the temporal delay τ to a spatial increment r in the streamwise direction, i.e.

$$\delta S_{L,1} = \delta u_1(r) = u_1(x_1 + r) - u_1(x_1) \quad (5.6)$$

with $r \equiv \tau U$, where U is the mean velocity in the streamwise direction. Note that the subscript number "1" next to L has been used here to distinguish it from that introduced later.

Several methods can be used to obtain δS_T . The most commonly used one in experimental studies is to sample the transverse velocity component u_2 or u_3 using an X-wire probe at a fixed location of the flow. The transverse velocity increment can then be defined as

$$\delta S_{T,1} = u_3(x_1 + r) - u_3(r) \quad [\text{or } u_2(x_1 + r) - u_2(r)], \quad (5.7)$$

where u_3 (or u_2) is the velocity fluctuations in the spanwise (or transverse) direction and r is the separation of the two points in the streamwise direction. Taylor's hypothesis is again used to convert the temporal delay to a spatial separation r . The subscript number next to T distinguishes between different definitions used in this report. Another definition for δS_T is the spatial transverse velocity increment,

$$\delta S_{T,2} \equiv u_1(x_3 + \Delta x_3) - u_1(x_3), \quad (5.8)$$

where x_3 is in the spanwise direction and Δx_3 is the separation of the two points in this direction. With only a few exceptions (e.g. Kahalerras et al., 1996; Camussi et al., 1996; Noullez et al., 1997), recent experiments (e.g. Herweijer and van de Water, 1995; Camussi and Benzi, 1997; Sreenivasan and Antonia, 1997; van de Water and

Herweijer, 1999; Zhou et al. 2001; Antonia et al. 2002; Bi and Wei, 2003), and direct numerical simulation (e.g. Chen et al., 1997; Grossman et al., 1997; Boratav and Pelz, 1997; Chen et al. 2004) have shown that the scaling exponents $\zeta_T(n)$, associated with the moments of δS_T , viz. $\langle (\delta S_T)^n \rangle \sim r^{\zeta_T(n)}$, are slightly smaller than those of δS_L (where $\langle (\delta S_L)^n \rangle \sim r^{\zeta_L(n)}$). Other experiments (e.g. Antonia et al., 1998; Boratav and Pelz, 1997; Pearson and Antonia, 2001) have shown that $\zeta_T(n)$ are significantly smaller than $\zeta_L(n)$. To explain the difference between $\zeta_T(n)$ and $\zeta_L(n)$, several studies have been performed both numerically and experimentally. Boratav and Pelz (1997) suggested that the difference between $\zeta_T(n)$ and $\zeta_L(n)$ is the result of the different contributions from the strain-dominated structures and the enstrophy-dominated structures to the longitudinal and transverse velocity increments. Chen et al. (1997) proposed and verified numerically a modified model to the RSH, which they called the refined similarity hypothesis for transverse velocity structure functions (RSHT). RSHT has been tested by Antonia et al. (1998) and Zhou and Antonia (2000b) in a decaying grid turbulence. These authors found that RSHT can only partially account for the difference of the scaling exponents between the longitudinal and transverse velocity structure functions. They argued that the anisotropy of turbulent structures in the scaling range, which reflects the small values of R_λ (< 100) in the grid turbulence, is more likely to account for most of the difference, where R_λ is the Taylor microscale Reynolds number defined as $R_\lambda = u_1' \lambda / \nu$ [$\lambda \equiv u_1' / (\partial u_1 / \partial x_1)$ is the longitudinal Taylor microscale, the superscript represents the root-mean-square (rms) values]. The significant difference between the longitudinal and transverse scaling exponents was also explained (e.g. Pearson, 1999; Zhou et al. 2001) by the different definitions used to obtain δS_T and

the method for obtaining $\zeta_T(n)$, especially for shear flows with small to moderate values of R_λ . It needs to be noted that in the study of Antonia et al. (1998) and Zhou and Antonia (2000b), the transverse velocity structure functions were obtained from measurements using an X-wire probe (i.e. Eq. 5.7). RSHT was verified satisfactorily in the study by Bi and Wei (2003), where the transverse velocity increments were obtained using both an X-wire probe and a rake of single hot wires. In their study, the instantaneous energy dissipation rate was substituted by the isotropic counterpart, i.e.

$$\varepsilon = 15\nu(\partial u_1 / \partial x_1)^2, \quad (5.9)$$

and the enstrophy field was represented by the longitudinal vorticity component only, i.e.

$$\Omega = 3\nu\omega_1^2. \quad (5.10)$$

Due to the limited values of R_λ in experimental studies, the scaling exponents for the longitudinal velocity structure functions were mainly estimated using the extended self-similarity (ESS) method (e.g. Benzi et al., 1993) by plotting the n^{th} -order velocity increments against the third-order moment of $|\delta S_L|$, i.e.

$$\langle |\delta S_L|^n \rangle \sim \langle |\delta S_L|^3 \rangle^{\zeta_L(n)}. \text{ Several methods have been used for estimating } \zeta_T(n) \text{ using}$$

variants of ESS. Dhruva et al. (1997) and Antonia & Pearson (1997) defined $\zeta_T(n)$

relative to the third-order moment of $|\delta S_L|$, i.e. $\langle |\delta S_{T,1}|^n \rangle \sim \langle |\delta S_L|^3 \rangle^{\zeta_{T,1}(n)}$. There is

evidence (e.g. Dhruva et al., 1997; Pearson, 1999) to show that the magnitude of

$\zeta_{T,1}(n)$ increases with R_λ and asymptotically approaches that of $\zeta_L(n)$ at large

R_λ . Also, $\zeta_T(n)$ can be defined relative to the third-order moment of $|\delta S_T|$, such as

$$\langle |\delta S_{T,2}|^n \rangle \sim \langle |\delta S_{T,2}|^3 \rangle^{\zeta_{T,2}(n)}.$$

In the present study, the main aim is to investigate the scaling exponents of the longitudinal and transverse velocity structure functions and their dependence on Reynolds number at the centerline of a turbulent wake flow using data obtained from two X-wire probes. The longitudinal velocity increments are obtained using both Eq. (5.6) and the following relation:

$$\delta S_{L,2} = \delta u_3(r) = u_3(x_3 + \Delta x_3) - u_3(x_3), \quad (5.11)$$

where u_3 is the velocity fluctuation in the spanwise (or x_3) direction measured by two X-wire probes separated by a spanwise distance Δx_3 . The definition of the spatial longitudinal velocity structure function has never been used before. Therefore, no such measurements have been done before. The transverse velocity increments are obtained using Eqs. (5.7) and (5.8). Using these two X-wire probes, the transverse vorticity component can also be measured, allowing a direct check of RSHT at different Reynolds numbers. The definition for longitudinal velocity increment using Eq.(5.11) and the dependence on R_λ of the scaling exponents of the locally averaged energy dissipation rate and enstrophy have not been addressed properly in the literature.

5.2 Experimental Details

Experiments were conducted in the wind tunnel as described in Chapter 2. Velocity fluctuations u_1 and u_3 at the downstream location of $75d$ ($d \equiv 25.4$ mm is the diameter of the cylinder) were measured using two X-wire probes on the centerline of the cylinder wake. The cylinder was located vertically 20cm downstream of the entrance of the test section. The two X-wire probes were aligned in the x_1 - x_3 plane so that u_1 and u_3 can be measured simultaneously. The co-ordinate system, the cylinder and the arrangement of the two X-wire probes are shown in Figure 5.1. The upper X-wire probe is attached to a traversing mechanism and is able to move along the spanwise direction so as to achieve the desired separation (Δx_3) while the lower X-wire probe was kept stable 20cm above the bottom wall of the tunnel. The flow conditions of the free stream have been checked to be uniform in the wind tunnel with the turbulent intensity being less than 0.5%. The measurements were conducted under four free stream velocities, i.e. 5 m/s, 10 m/s, 15 m/s and 20 m/s, corresponding to R_λ of 100, 200, 250, and 300, respectively.

The transverse vorticity component can be approximated by

$$\omega_2 = \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \approx \frac{\Delta u_1}{\Delta x_3} - \frac{\Delta u_3}{\Delta x_1} \quad (5.12)$$

where Δu_1 is the difference between u_1 from the two X-wires; Δu_3 is the difference between values of u_3 at the same point in space but separated in time by two sampling time intervals (where central difference is used). As the turbulence intensity is relatively small ($u'_1/U \leq 6\%$), the use of Taylor's hypothesis, i.e. $\Delta/\Delta x_1 = -U^{-1}\Delta/\Delta t$ in Eq. (5.7) should be satisfactory.

The hot wires were operated with constant temperature circuits in-house build at an over heat ratio of 1.5. Each of the two wires had a diameter of $2.5\mu\text{m}$. The wire

separation was about 1mm. The output signals were low-pass filtered through the buck and gain circuits at a cutoff frequency $f_c = 1.6 \text{ kHz}$, 5.2 kHz , 9.2 kHz , 9.2 kHz , respectively, for the four freestream velocities, which are close to the Kolmogorov frequency $f_K = U_1 / 2\pi\eta$ except at the highest R_λ . The filtered signals were sampled at a frequency $f_s = 2f_c$ into a PC using a 16-bit A/D converter (National Instrument). The sampling period was 120s for all velocities. Further analyse of the data were carried out using the MATLAB® programs.

Figure 5.1 Co-ordinate system and probe arrangements.

5.3 R_λ Dependence of Second and Third Order Moments of Longitudinal Velocity Structure Functions

In the present study, the energy dissipation rate used to normalize the structure functions is estimated based on the isotropic assumption, i.e. Eq. (5.9). After normalization by $u_K (\equiv \nu^{1/4} \langle \varepsilon \rangle^{1/4})$ and/or η , Eq. (5.1) can be rewritten as:

$$\left\langle (\delta S_{L,1}^*)^2 \right\rangle r^{*-2} = \frac{1}{15}, \quad (5.13)$$

where the asterisk denotes normalization by u_K and/or η . The variance of $\delta S_{L,1}^*$ is shown in Figure 5.2 in terms of r^* for four values of Reynolds numbers. The variance has been multiplied by r^{*-2} to check local isotropy over the dissipative range using Eq. (5.13). The data approach a constant value of $1/15$ as $r^* \rightarrow 1$, regardless of R_λ , indicating that isotropy is approximately satisfied at small scales.

The constancy of $\left\langle (\delta S_{L,1}^*)^2 \right\rangle r^{*-2}$ shown in this figure implies that the effect of noise at different Reynolds numbers on $\left\langle (\delta S_{L,1}^*)^2 \right\rangle$ is unlikely to be important for the whole range of separations. Indeed, Eq. (5.13) cannot be used to check the local isotropy as the agreement with isotropy in the dissipative range is forced, as required by the use Eq. (5.9) for the estimation of the energy dissipation rate.

Figure 5.2 Dependence of $\langle (\delta S_L^*)^2 \rangle r^{*-2}$ on R_λ . $\circ$, $R_\lambda = 100$; $\square$, 200; ∇ , 250; $\bullet$, 300.

In the inertial range, Eq. (5.4) can be rewritten after normalization by u_K and η as:

$$\langle (\delta S_{L,1}^*)^2 \rangle r^{*-2/3} = C, \quad (5.14)$$

where C is the Kolmogorov constant. Eq. (5.14) implies that in the inertial range, the second-order velocity structure function, weighted by $r^{*-2/3}$, is a constant. According to Yaglom (1981), $C = 2.0$. The present results for $\langle (\delta S_{L,1}^*)^2 \rangle r^{*-2/3}$ are shown in Figure 5.3. There is a very narrow plateau when $R_\lambda = 100$, which corresponds to a value of C that is about 20% smaller than 2. When R_λ increases, the distributions $\langle (\delta S_{L,1}^*)^2 \rangle r^{*-2/3}$ approach to 2 and the plateau becomes wider. When R_λ increases to 300, the value of C ($\equiv 2.2$) is slightly higher than the theoretical value. This may be because of the use of $\langle \varepsilon \rangle_{iso}$ as a substitute of $\langle \varepsilon \rangle$. Browne et al. (1987) found that

Figure 5.2 Dependence of $\langle (\delta S_L^*)^2 \rangle r^{*-2}$ on R_λ . $\circ$, $R_\lambda = 100$; $\square$, 200; ∇ , 250; $\bullet$, 300.

In the inertial range, Eq. (5.4) can be rewritten after normalization by u_K and η as:

$$\langle (\delta S_{L,l}^*)^2 \rangle r^{*-2/3} = C, \quad (5.14)$$

where C is the Kolmogorov constant. Eq. (5.14) implies that in the inertial range, the second-order velocity structure function, weighted by $r^{*-2/3}$, is a constant. According to Yaglom (1981), $C = 2.0$. The present results for $\langle (\delta S_{L,l}^*)^2 \rangle r^{*-2/3}$ are shown in Figure 5.3. There is a very narrow plateau when $R_\lambda = 100$, which corresponds to a value of C that is about 20% smaller than 2. When R_λ increases, the distributions $\langle (\delta S_{L,l}^*)^2 \rangle r^{*-2/3}$ approach to 2 and the plateau becomes wider. When R_λ increases to 300, the value of C ($\equiv 2.2$) is slightly higher than the theoretical value. This may be because of the use of $\langle \varepsilon \rangle_{iso}$ as a substitute of $\langle \varepsilon \rangle$. Browne et al. (1987) found that

$\langle \varepsilon \rangle \approx 1.4 \langle \varepsilon \rangle_{iso}$ on the centerline of a far wake. At the location of $x/d = 75$ in the present study, due to the limit of the spatial resolution in the streamwise direction, the spectrum of $\partial u_1 / \partial x_1$ is attenuated, resulting in the magnitudes of $\langle \varepsilon \rangle_{iso}$ being underestimated. A method proposed by Antonia and Mi (1993) (or as shown in Chapter 3) can be used to correct the attenuation effect of the spectrum of $\partial u_1 / \partial x_1$. The use of the correction requires only the assumption of local isotropy. Details of the correction method can be found in Antonia and Mi (1993). Table 5.1 shows the correction ratios of $\partial u_1 / \partial x_1$ ($R_{\partial u_1 / \partial x_1}$) and the correction ratios for $\langle \varepsilon \rangle$ (R_ε) at various Reynolds numbers. At $R_\lambda = 300$, the correction ratio of $\langle \varepsilon \rangle$ is as high as 25%, i.e. $\langle \varepsilon \rangle$ is underestimated by about 25%, which is slightly lower than that reported by Browne et al. (1987). The distributions of the second-order velocity structure functions, normalized by the corrected energy dissipation rates and weighted by $r^{*-2/3}$ at various R_λ , are shown in Figure 5.4. The peak values, as compared with those shown in Figure 5.3, are reduced, but still follow an increasing trend with the increase in R_λ , approaching the asymptotic value of 2.

Table 5.1 Correction ratios of the energy dissipation rate at various Reynolds numbers.

R_λ	100	200	250	300
$\Delta x_1 / \eta$	4.0	4.0	4.3	6.9
$R_{\partial u_1 / \partial x_1}$	1.05	1.05	1.07	1.12
R_ε	1.10	1.10	1.10	1.25

Figure 5.3 Dependence of $\langle (\delta S_{L,1}^*)^2 \rangle r^{*-2/3}$ on R_λ . $\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $\bullet$, (300).

Figure 5.4 Dependence of $\langle (\delta S_{L,1}^*)^2 \rangle r^{*-2/3}$ on R_λ (normalized by corrected energy dissipation rate). $\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $\bullet$, (300).

Figure 5.3 Dependence of $\langle (\delta S_{L,1}^*)^2 \rangle r^{*-2/3}$ on R_λ . $\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $\bullet$, (300).

Figure 5.4 Dependence of $\langle (\delta S_{L,1}^*)^2 \rangle r^{*-2/3}$ on R_λ (normalized by corrected energy dissipation rate). $\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $\bullet$, (300).

The region where $\langle (\delta S_{L,1}^*)^3 \rangle$ varies linearly with r is often identified as the inertial range. It has been suggested that Eq. (5.4), i.e. the so-called “4/5” law, is the most sensitive and most appropriate test for local isotropy (e.g. Lindborg, 1996). After normalization by Kolmogorov scales, Eq. (5.4) is reduced to $-\langle (\delta S_{L,1}^*)^3 \rangle = \frac{4}{5} r^*$. The present distributions of $\langle (\delta S_{L,1}^*)^3 \rangle$ at various R_λ , weighted by r^{*-1} , are shown in Figure 5.5. The peak value of the distribution of $\langle (\delta S_{L,1}^*)^3 \rangle r^{*-1}$ at $R_\lambda = 100$ departs significantly from 4/5, indicating the lack of isotropy due to the small value of R_λ . The peak value increases systematically with R_λ and approaches “4/5” asymptotically. At the same time the plateau becomes wider as R_λ increases, indicating the improvement of local isotropy in the inertial range. Note that in Figure 5.2, $\langle (\delta S_{L,1}^*)^2 \rangle r^{*-2}$ satisfy Eq. (5.13) at small scales, independently of R_λ , indicating that the dissipation range scales approach isotropy more rapidly than the inertial range scales. It can also be argued here that the agreement with isotropy in the dissipative range is forced, as required by the use Eq. (5.9) for the estimation of the energy dissipation rate.

Figure 5.5 Third-order velocity structure functions multiplied by r^{*-1} .
○, ($R_\lambda = 100$); □, (200); ▽, (250); ●, (300).

Figure 5.5 Third-order velocity structure functions multiplied by r^{*-1} .
○, ($R_\lambda = 100$); □, (200); ▽, (250); ●, (300).

5.4 Verification of the Kolmogorov Equation at Different Reynolds Numbers

After normalization by u_K and η , Eq. (5.3) becomes

$$-\left\langle (\delta S_{L,1}^*)^3 \right\rangle + 6 \frac{\partial}{\partial r^*} \left\langle (\delta S_{L,1}^*)^2 \right\rangle = \frac{4}{5} r^*. \quad (5.15)$$

All terms in Eq. (5.15), measured for $R_\lambda = 100, 200, 250$ and 300 are plotted in Figures 5.6(a - d). It shows that the above equation is only satisfied for small scales where the major contribution to Eq (5.15) is provided by $6 \frac{\partial}{\partial r^*} \left\langle (\delta S_{L,1}^*)^2 \right\rangle$. For the values of r^* in the inertial range, where most of the contribution to Eq (5.15) comes from $-\left\langle (\delta S_{L,1}^*)^3 \right\rangle$, the imbalance is significant, which is consistent with the results shown in Figure 5.5. The imbalance is attributed to the anisotropy in the inertial range because of the small values of R_λ in the experiments. With the increase of R_λ , the departure from the straight line in the inertial range becomes smaller, indicating the improvement of local isotropy as R_λ increases.

Figure 5.6 Terms in Eq. (5.15). ---, $-\langle(\delta S_{L,1}^*)^2\rangle$; -.-, $6 \frac{\partial}{\partial r^*} \langle(\delta S_{L,1}^*)^2\rangle$;
 $\circ$, sum of terms on left-hand side of Eq. (5.15); —, $\frac{4}{5} r^*$.
 (a) $R_\lambda = 100$; (b) 200; (c) 250; (d) 300;

5.5 Relationship between the Second-order Longitudinal and Transverse Velocity Structure Functions

For isotropic turbulence, the second-order longitudinal and transverse velocity structure functions are related (e.g. Monin and Yaglom, 1975) via.

$$\left\langle (\delta S_T)^2 \right\rangle_{iso} = \left(1 + \frac{r}{2} \frac{d}{dr} \right) \left\langle (\delta S_L)^2 \right\rangle. \quad (5.16)$$

The subscript “*iso*” means that the values of $\left\langle (\delta S_T^*)^2 \right\rangle$ are calculated from Eq. (5.16)

using the measured values of $\left\langle (\delta S_L^*)^2 \right\rangle$. In the present study, the longitudinal and

transverse velocity increments have been defined by Eqs. (5.6 - 5.8) and (5.11). The

calculated values of $\left\langle (\delta S_T^*)^2 \right\rangle_{iso}$ are compared with $\left\langle (\delta S_{T,1}^*)^2 \right\rangle$ and $\left\langle (\delta S_{T,2}^*)^2 \right\rangle$ in

Figures 5.7 (a - d). For the four different Reynolds numbers, $\left\langle (\delta S_{L,1}^*)^2 \right\rangle$ agree

favorably with $\left\langle (\delta S_{L,2}^*)^2 \right\rangle$ over the range of $20 \leq r^* \leq 500$ (For $R_\lambda = 100$, the

range is $20 \leq r^* \leq 200$). For scales $r^* < 20$, the departure of $\left\langle (\delta S_{L,2}^*)^2 \right\rangle$ from

$\left\langle (\delta S_{L,1}^*)^2 \right\rangle$ may be caused by the noise effect of the probes. The errors in measuring

the initial separation between the two X-wire probes may not be completely

dismissed. For large scales, the departure may be caused by the global anisotropy,

which is reflected by the ratio $u_3'/u_1' \cong 0.85$ (Note that for global isotropy, the ratio

is equal to 1). There exists good agreement between $\left\langle (\delta S_T^*)^2 \right\rangle_{iso}$ and $\left\langle (\delta S_{T,2}^*)^2 \right\rangle$

for $r^* < 300$, verifying the local isotropy in this region. The agreement between

$\left\langle (\delta S_T^*)^2 \right\rangle_{iso}$ and $\left\langle (\delta S_{T,1}^*)^2 \right\rangle$ is poor except over a very narrow region ($r^* < 50$). The

results shown in Figures 5.7 (a - d) seem to suggest that local isotropy [Eq. (5.16)] is better satisfied by the spatial transverse structure functions than the temporal ones in low to moderate Reynolds number flows. While for the longitudinal structure functions, there is always satisfactory agreement between spatial and temporal methods.

Figure 5.7 Comparison of measured second-order transverse velocity structure functions with isotropic calculation. $-\cdots-$, $\langle (\delta S_{L,1}^*)^2 \rangle$; $\square$, $\langle (\delta S_{L,2}^*)^2 \rangle$; $---$, $\langle (\delta S_{T,1}^*)^2 \rangle$; $\circ$, $\langle (\delta S_{T,2}^*)^2 \rangle$; $---$, $\langle (\delta S_T^*)^2 \rangle_{iso}$. (a) $R_\lambda = 100$; (b) 200; (c) 250; (d) 300;

5.6 Scaling Exponents of the Longitudinal and Transverse Velocity Structure Functions

Due to the small values of R_λ in the present study, an apparent inertial range, over which $\langle |\delta S_L^*|^3 \rangle$ depends linearly on r^* , cannot be defined unambiguously, even at $R_\lambda = 300$ (see Figure 5.5). Therefore, the extended self-similarity (ESS) method (Benzi et al. 1993) will be used to estimate the scaling exponents of the longitudinal and transverse velocity structure functions with a slight modification - the region where $\langle |\delta S_L^*|^3 \rangle$ is approximately linear to r is used to define the scaling range. This method has been widely used for laboratory turbulence (see for example Stolovitzky and Sreenivasan 1993; Arneodo et al., 1996; Sreenivasan and Antonia, 1997; Sreenivasan and Dhruva, 1998; van de Water and Herweijer, 1999; Zhou and Antonia, 2000b; Kurien and Sreenivasan, 2000; Bi and Wei, 2003). The corresponding scaling exponents are known as the values relative to the third-order velocity increments. When estimating the scaling exponents using the ESS method, self-cross plotting has the advantage of minimizing the effects due to the anisotropy (van de Water and Herweijer, 1999), although actually ESS has no rigorous theoretical basis. The present distributions of $\langle |\delta S_L^*|^3 \rangle r^{*-1}$ for different values of R_λ are shown in Figure 5.8. There exist apparent scaling ranges for all the four values of R_λ . The peak value and width of the scaling range increase with R_λ . For instance, when $R_\lambda = 100$, the scaling range is estimated to be $30 \leq r^* \leq 50$; when $R_\lambda = 200$, the scaling range is $40 \leq r^* \leq 80$; for $R_\lambda = 250$, the scaling range is in the range of $50 \leq r^* \leq 120$ and for $R_\lambda = 300$, the scaling range is in the range of $60 \leq r^* \leq 160$.

Figure 5.8 Kolmogorov normalized 3rd-order longitudinal structure functions multiplied by r^{*-1} .
 $\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $\bullet$, (300).

The longitudinal scaling exponents $\zeta_L(n)$ can be obtained based on the following relation:

$$\langle |\delta S_{L,1}^*|^n \rangle \sim \langle |\delta S_{L,1}^*|^3 \rangle^{\zeta_{L,1}(n)}. \quad (5.17)$$

From the plot of $\log \langle |\delta S_{L,1}^*|^n \rangle$ vs. $\log \langle |\delta S_{L,1}^*|^3 \rangle$ at $R_\lambda = 250$ (Figure 5.9) (distributions of $\log \langle |\delta S_{L,1}^*|^n \rangle$ vs. $\log \langle |\delta S_{L,1}^*|^3 \rangle$ for other values of R_λ are not shown here), scaling exponents $\zeta_{L,1}(n)$ were estimated from the least squares linear regressions over the scaling range obtained from Figure 5.8. It can be seen from Figure 5.9 that the experimental data show excellent linearity over the range much wider than that shown by Figure 5.8.

Figure 5.9 Moments of order n (1 to 8) of $\left\langle \left| \delta S_{L,1}^* \right|^n \right\rangle$ as a function of $\left\langle \left| \delta S_{L,1}^* \right|^3 \right\rangle$ for $R_\lambda = 250$.

The scaling exponents of the n^{th} -order moments of the longitudinal structure functions for different values of R_λ are shown in Figure 5.10. It can be seen that the scaling exponents $\zeta_{L,1}(n)$ do not depend on R_λ . The very small difference of $\zeta_{L,1}(n)$ for different values of R_λ may be due to the limited sampling points in the present experiments. The values of $\zeta_{L,1}(n)$ depart significantly from the prediction of K41 (i.e. $n/3$) for $n > 4$. This may indicate the increased effect of the intermittency for higher orders of longitudinal velocity structure functions. The predictions of $\zeta_L(n)$ using the lognormal model (K62) and the She-Leveque (SL) model (She and Leveque, 1994) are also shown for comparison in Figure 5.10. For K62,

$$\zeta_L(n) = \frac{n}{3} - \frac{\mu}{18} n(n-3), \quad (5.18)$$

where μ is the intermittency parameter with a magnitude of 0.2 - 0.3. The values of

μ can be obtained using the 6th-order velocity structure function since $\langle (\delta S_L)^6 \rangle \sim r^{2-\mu}$ [i.e. $\mu = 2 - \zeta_L(6)$] in the inertial range. For the She-Leveque (1994) model,

$$\zeta_L(n) = \frac{n}{9} + 2 \left[1 - \left(\frac{2}{3} \right)^{n/3} \right]. \quad (5.19)$$

The measured values of $\zeta_{L,1}(n)$ show satisfactory agreement with those predicted by K62 and the SL model for $n \leq 8$.

Figure 5.10 Scaling exponents $\zeta_{L,1}(n)$ as a function of n at different values of R_λ .
 —, K41; ---, Lognormal model (K62); —, model of She-Leveque (1994).
 ○, ($R_\lambda = 100$); □, (200); ▽, (250); +, (300);

From the definition of the longitudinal velocity increment, another way to measure the longitudinal velocity increment can be represented by Eq. (5.11), where u_3 is the velocity component in the spanwise (x_3) direction and was measured using two X-wire probes separated by a distance Δx_3 . The scaling exponents can then be obtained from the plot

$$\log \left\langle |\delta S_{L,2}^*|^n \right\rangle \sim \log \left\langle |\delta S_{L,2}^*|^3 \right\rangle^{\zeta_{L,2}(n)}. \quad (5.20)$$

The scaling range over which $\log \left\langle |\delta S_{L,2}^*|^n \right\rangle$ depends linearly on $\log \left\langle |\delta S_{L,2}^*|^3 \right\rangle$ is well defined, as shown in Figure 5.11. The scaling exponents defined by Eq. (5.20) are shown in Figure 5.12. They agree satisfactorily with the predictions of K62 and the SL model. The present results seem to suggest that the scaling exponents of the longitudinal velocity structure functions, no matter which methods are used, support the K62 and SL models favorably. A comparison of the scaling exponents (already shown in Figures 5.10 and 5.12) for the longitudinal velocity increments obtained using different methods are given in Tables 5.1 and 5.2.

Figure 5.11 Moments of order n (1 to 8) of $\left\langle |\delta S_{L,2}^*|^n \right\rangle$ as a function of $\left\langle |\delta S_{L,2}^*|^3 \right\rangle$ for $R_\lambda = 250$.

Figure 5.12 Scaling exponents $\zeta_{L,2}(n)$ as a function of n at different R_λ . —, K41; — — —, Lognormal model (K62); — · —, model of She-Leveque (1994). $\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $+$, (300);

Table 5.2 Longitudinal scaling exponents $\zeta_{L,1}(n)$ (Eq. 17) for different magnitudes of R_λ .

n	$R_\lambda = 100$	150	250	300
1	0.346	0.347	0.346	0.346
2	0.692	0.694	0.692	0.691
3	1	1	1	1
4	1.293	1.284	1.281	1.285
5	1.559	1.548	1.571	1.548
6	1.783	1.776	1.793	1.788
7	2.024	1.991	2.014	2.004
8	2.221	2.194	2.217	2.188

Table 5.3 Longitudinal scaling exponents $\zeta_{L,2}(n)$ (Eq. 20) for different magnitudes of R_λ .

n	$R_\lambda = 100$	150	250	300
1	0.381	0.382	0.38	0.366
2	0.729	0.726	0.721	0.712
3	1.052	1.044	1.037	1.013
4	1.331	1.325	1.311	1.295
5	1.593	1.573	1.562	1.543
6	1.824	1.791	1.791	1.772
7	2.041	1.982	2.015	1.973
8	2.227	2.153	2.181	2.155

In the present study, the scaling exponents of the transverse velocity structure functions are also estimated using the extended self-similarity (ESS) method over the same scaling range as for the longitudinal ones (Figure 5.8). As there are a few methods to define the transverse velocity structure functions, the scaling exponents can be obtained as follows:

$$\langle |\delta S_{T,1}|^n \rangle \sim \langle |\delta S_{L,1}|^3 \rangle^{\zeta_{T,1}(n)} \quad (5.21)$$

and
$$\langle |\delta S_{T,2}|^n \rangle \sim \langle |\delta S_{L,2}|^3 \rangle^{\zeta_{T,2}(n)}. \quad (5.22)$$

The scaling exponents are obtained over the scaling range by plotting $\log \langle |\delta S_T^*|^n \rangle$ vs. $\log \langle |\delta S_L^*|^3 \rangle$. It can be seen that the claim by Benzi *et al.* (1993) that the scaling extends into the dissipative range becomes tenuous for $\langle |\delta S_{T,1}^*|^n \rangle$ vs. $\langle |\delta S_{L,1}^*|^3 \rangle$ (Figure 5.13), especially when the magnitude of n increases.

Figure 5.13 Moments of order n (1 to 8) of $\langle |\delta S_{T,1}^*|^n \rangle$ as a function of $\langle |\delta S_{L,1}^*|^3 \rangle$ for $R_\lambda = 250$.

The transverse scaling exponents $\zeta_{T,1}(n)$ obtained from Eq. (5.21) for the four different Reynolds numbers are shown in Figure 5.14. The longitudinal scaling exponents $\zeta_{L,1}(n)$ and the predictions using the lognormal model (K62) with $\mu = 0.2$, the K41 and the SL model are also shown for comparison in the figure. Obviously, $\zeta_{T,1}(n)$ are significantly smaller than $\zeta_{L,1}(n)$ for all orders, independent of R_λ . It seems that with the increase of R_λ , $\zeta_{T,1}(n)$ also increases, especially for high orders, indicating a weak dependence of $\zeta_{T,1}(n)$ on R_λ . This result is in consistence with that reported by Zhou and Antonia (2000b). It is conjectured that the difference between $\zeta_{T,1}(n)$ and $\zeta_{L,1}(n)$ may diminish when R_λ goes to infinity because the separation between the integral length scales and the IR scales increases with R_λ , thus reducing the influence from the large-scale anisotropy (Pearson, 1999). Another explanation for the difference between $\zeta_{T,1}(n)$ and $\zeta_{L,1}(n)$ is due to the global anisotropy of the flow field, as revealed by the ratio u'_3/u'_1 ($\cong 0.85$ in the present wake flows). However, it is suspected here that for the above difference between $\zeta_{T,1}(n)$ and $\zeta_{L,1}(n)$, the use of Taylor's hypothesis for the calculation of the transverse velocity structure functions may not be dismissed completely. This effect is especially significant for flows of high turbulence intensity.

The scaling exponents $\zeta_{T,2}(n)$ obtained using Eq. (5.22) are shown in Figure 5.15. For $n \leq 4$, $\zeta_{T,2}(n)$ agree favorably with $\zeta_{L,1}(n)$ and the K62 and SL models. When $n \geq 5$, the departure of $\zeta_{T,2}(n)$ from the above two model predictions is apparent. The magnitude of $\zeta_{T,2}(n)$ also reveals a weak dependence on R_λ , with the increase in R_λ , $\zeta_{T,2}(n)$ increases slightly. The differences between $\zeta_{L,1}(n)$ and

$\zeta_{T,2}(n)$ as shown in Figure 5.15 is much smaller than that between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ as shown in Figure 5.14. The difference between the longitudinal scaling exponents $\zeta_{L,1}(n)$ [and $\zeta_{L,2}(n)$] and the transverse scaling exponents $\zeta_{T,1}(n)$ [and $\zeta_{T,2}(n)$] may reflect the different intermittency of the velocity increments. This can be illustrated by the probability density function (PDF) of the velocity increments as shown in Figure 5.16, where the PDF of the longitudinal and transverse velocity increments for separation $r^* = 38$ in the scaling range are plotted and compared. Clearly, the distributions of PDFs of $\delta S_{T,1}$ and $\delta S_{T,2}$ are more stretched and more symmetric for large fluctuations than those of $\delta S_{L,1}$ and $\delta S_{L,2}$.

For the convenience of comparison of the transverse scaling exponents obtained using different definitions, the values of $\zeta_{T,1}(n)$ and $\zeta_{T,2}(n)$ at different values of R_λ , which are already shown in Figures 5.14 and 5.15, are listed in Tables 5.4 and 5.5.

Figure 5.14 Scaling exponents $\zeta_{T,1}(n)$ as a function of n at different values of R_λ .
 Δ , $\zeta_{L,1}(n)$; —, K41; ---, Lognormal model (K62); — · —, model of She-Leveque (1994).
 $\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $+$, (300);

Figure 5.15 Scaling exponents $\zeta_{T,2}(n)$ as a function of n at different values of R_λ .
 Δ , $\zeta_{L,1}(n)$; —, K41; ---, Lognormal model (K62); —·—, model of She-Leveque (1994).
 $\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $+$, (300);

Figure 5.16 PDFs of the velocity increments for inertial range separation $r^* = 38$ at $R_\lambda = 250$.
---, $\delta S_{L,1}$; -·-, $\delta S_{L,2}$; —, $\delta S_{T,1}$; —·—, $\delta S_{T,2}$.

Table 5.4 Transverse scaling exponents $\zeta_{T,1}(n)$ (Eq. 5.21) for different magnitudes of R_λ .

n	$R_\lambda = 100$	150	250	300
1	0.279	0.292	0.282	0.298
2	0.560	0.582	0.563	0.595
3	0.726	0.816	0.795	0.838
4	0.958	1.021	0.996	1.049
5	1.111	1.195	1.175	1.233
6	1.245	1.344	1.338	1.394
7	1.365	1.47	1.495	1.539
8	1.472	1.576	1.651	1.672

Table 5.5 Transverse scaling exponents $\zeta_{T,2}(n)$ (Eq. 5.21) for different magnitudes of R_λ .

n	$R_\lambda = 100$	150	250	300
1	0.394	0.386	0.385	0.383
2	0.723	0.719	0.715	0.713
3	1	1	1	1
4	1.233	1.244	1.245	1.248
5	1.429	1.452	1.456	1.462
6	1.594	1.627	1.637	1.646
7	1.733	1.774	1.791	1.806
8	1.853	1.896	1.922	1.945

5.7 Refined Similarity Hypothesis for Transverse Velocity Structure Functions

The larger departure of the scaling exponents of the transverse velocity structure functions from K41 than those of the longitudinal ones implies that the former are more intermittent than the latter. Several models have been proposed to explain the above inequality. Boratav and Pelz (1997) suggested that the inequality mainly reflects the greater contribution to intermittency from the enstrophy-dominated structures than from the strain-dominated structures. Chen et al. (1997) proposed a modification to the refined similarity hypothesis (RSH) (K62) for the transverse velocity structure functions, which they called RSHT. For RSHT, it is supposed that the longitudinal velocity structure functions are associated with the locally averaged energy dissipation rate, whereas the transverse velocity structure functions are associated with the locally averaged enstrophy:

$$\langle (\delta S_L)^n \rangle \sim D_L^n \langle \varepsilon_r^{n/3} \rangle r^{n/3} \quad (\text{RSH}) \quad (5.23)$$

$$\text{and} \quad \langle (\delta S_T)^n \rangle \sim D_T^n \langle \Omega_r^{n/3} \rangle r^{n/3} \quad (\text{RSHT}), \quad (5.24)$$

where D_L^n and D_T^n are constants, $\Omega_r (\equiv \nu \omega_i \omega_i)$ and ε_r are the locally averaged enstrophy and energy dissipation rate, respectively, which are obtained using

$$\varepsilon_r = \frac{1}{r} \int_{x_0-r/2}^{x_0+r/2} \varepsilon(r) dx \quad (5.25)$$

$$\text{and} \quad \Omega_r = \frac{1}{r} \int_{x_0-r/2}^{x_0+r/2} \Omega(r) dx. \quad (5.26)$$

The spatial integration on x is converted from the temporal one by applying Taylor's hypothesis. It is assumed that $\langle \varepsilon_r^n \rangle$ and $\langle \Omega_r^n \rangle$ scales as $r^{\tau^d(n)}$ and $r^{\tau^o(n)}$, respectively. Therefore, in RSH and RSHT, the scaling of the longitudinal and

transverse velocity structure functions can be expressed as:

$$\zeta_L(n) = \frac{n}{3} + \tau^d(n/3) \quad (5.27)$$

$$\text{and} \quad \zeta_T(n) = \frac{n}{3} + \tau^o(n/3). \quad (5.28)$$

If RSH and RSHT work satisfactorily, $\tau^d(n/3)$ and $\tau^o(n/3)$ should be both negative and the magnitudes of $\zeta_L(n) - \tau^d(n/3)$ and $\zeta_T(n) - \tau^o(n/3)$ should be equal to $n/3$.

In the present study, the longitudinal and spanwise velocity components u_1 and u_3 are measured using two X-wire probes (Figure 5.1). Three velocity gradients i.e. $\partial u_1 / \partial x_1$, $\partial u_1 / \partial x_3$ and $\partial u_3 / \partial x_1$ can be measured simultaneously. The energy dissipation rate ε can then be approximated using the relation:

$$\varepsilon_{ap} \approx \nu (6u_{1,1}^2 + 3u_{1,3}^2 + 2u_{3,1}^2 + 2u_{1,3}u_{3,1}). \quad (5.29)$$

This relation has been verified to be more reliable than the isotropic relation, i.e.

$\varepsilon_{iso} = 15\nu (\partial u / \partial x)^2$ (See Figure 4.28). From the measured velocity fluctuations u_1 and u_3 , only the transverse vorticity component

$$\omega_2 = \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \quad (5.30)$$

can be measured using the present probe (Figure 5.1). By assuming isotropy, the enstrophy can then be approximated by

$$\Omega = 3\nu\omega_2^2. \quad (5.31)$$

The scaling exponents $\tau^d(n/3)$ and $\tau^o(n/3)$ are inferred from the distributions of $\log \langle \varepsilon_r^{n/3} \rangle$ and $\log \langle \Omega_r^{n/3} \rangle$ (Figures. 5.17a,b) (note that only results for $R_\lambda = 250$ are shown here) over the same scaling range of r^* as that used to estimate $\zeta_{L,1}(n)$

and $\zeta_{L,2}(n)$. The least square linear fits to the measured data in Figures 5.17(a,b) are reliable with correlation coefficient being always larger than 0.98 over the scaling range. The values of $\tau^d(n/3)$ and $\tau^o(n/3)$ are listed in Table 5.6 for the four Reynolds numbers covered in the present study. The numbers inside the brackets represent the corresponding standard deviations of the fits. It seems that the values of $\tau^o(n/3)$ are always smaller than that of $\tau^d(n/3)$, indicating that $\langle(\nu\Omega_r)^{n/3}\rangle$ is more intermittent than $\langle\varepsilon_r^{n/3}\rangle$. This result is consistent with that reported by Chen et al. 1997 using direct numerical simulation. The magnitudes of $\tau^d(n/3)$ keep approximately constant with the increase of R_λ , resulting in constant values of $\zeta_L(n)$ at different R_λ . This result is consistent with that shown in Figures 5.10 and 5.12. In contrast, the magnitudes of $\tau^o(n/3)$ decrease with the increase in R_λ , indicating the reduced intermittency effect with the increase in R_λ . It is conjectured that the scaling exponents of $\langle\varepsilon_r^{n/3}\rangle$ and $\langle(\nu\Omega_r)^{n/3}\rangle$ may be equivalent in the case when R_λ is extremely large. This trend is consistent with that reported by Nelkin (1999). The values of $\zeta_L(n) - \tau^d(n/3)$ and $\zeta_T(n) - \tau^o(n/3)$, with $\zeta_L(n)$ representing $\zeta_{L,1}(n)$ and $\zeta_{L,2}(n)$, and $\zeta_T(n)$ representing $\zeta_{T,1}(n)$ and $\zeta_{T,2}(n)$, respectively, are compared with $n/3$ in Figures 5.18(a,b) and Figures 5.19(a,b). There should be satisfactory agreements between $\zeta_L(n) - \tau^d(n/3)$ and $\zeta_T(n) - \tau^o(n/3)$ with $n/3$ if RSH and RSHT work well. This is indeed the case for $\zeta_L(n) - \tau^d(n/3)$ [Figures 5.18(a,b)], no matter which definitions are used for the longitudinal velocity structure functions. However, the agreement between $\zeta_T(n) - \tau^o(n/3)$ and $n/3$

depends on the definition of the transverse velocity increments (Figures 5.19 a, b). The departure of $\zeta_{T,1}(n) - \tau^o(n/3)$ from $n/3$ (Figure 5.19a) is significant, and cannot be explained by the experimental uncertainty, even though there is a trend that $\zeta_{T,1}(n) - \tau^o(n/3)$ approaches $n/3$ slowly when R_λ increases. By contrast, there is satisfactory agreement between $\zeta_{T,2}(n) - \tau^o(n/3)$ and $n/3$ (Figure 5.19b), independent of R_λ . This result indicates that RSHT works well to explain the departure of $\zeta_{T,2}(n)$ from K62 and the SL model, supporting the experimental results of Bi and Wei (2003). It seems that RSHT can only partially account for the inequality between $\zeta_{T,1}(n)$ and $\zeta_{L,1}(n)$ or $\zeta_{L,2}(n)$, at least for the present Reynolds numbers. The discrepancy between $\zeta_{T,1}(n)$ and $\zeta_{T,2}(n)$ may reflect the influence of global isotropy or the use of Taylor's hypothesis when calculating $\langle |\delta S_{T,1}|^n \rangle$. It needs to be noted that the verification of RSHT using $\zeta_{T,2}(n)$ (Figure 5.19b) is based on the assumption that $\Omega = 3\nu\omega_2^2$. However, the intermittency characteristics of the enstrophy $\Omega (= \nu\omega_i\omega_i)$ may not be perfectly represented by $3\omega_2^2$. In this case, satisfactory agreement between $\zeta_{T,2}(n) - \tau^o(n/3)$ and $n/3$ may not be obtained. The discrepancy of the present results (Figure 5.19a) from that reported by Antonia et al. (1998) and Zhou and Antonia (2000b) is due to the different definitions of the transverse velocity increments, where only $\zeta_{T,1}(n)$ estimated from the temporal transverse velocity increments were reported.

Figure 5.17 Distributions of $\log \langle \epsilon_r^{n/3} \rangle$ and $\log \langle \Omega_r^{n/3} \rangle$ vs. $\log r^*$.

(a) $\log \langle \epsilon_r^{n/3} \rangle$; (b) $\log \langle \Omega_r^{n/3} \rangle$.

Figure 5.17 Distributions of $\log \langle \epsilon_r^{n/3} \rangle$ and $\log \langle \Omega_r^{n/3} \rangle$ vs. $\log r^*$.
(a) $\log \langle \epsilon_r^{n/3} \rangle$; (b) $\log \langle \Omega_r^{n/3} \rangle$.

Table 5.6 Scaling exponents of locally averaged energy dissipation rate and enstrophy at various R_λ .

R_λ	100		200		250		300	
n	$\tau^o(n/3)$	$\tau^d(n/3)$	$\tau^o(n/3)$	$\tau^d(n/3)$	$\tau^o(n/3)$	$\tau^d(n/3)$	$\tau^o(n/3)$	$\tau^d(n/3)$
1	0.0488 (± 0.0016)	0.0193 (± 0.0013)	0.052 (± 0.0018)	0.0226 (± 0.0015)	0.101 (± 0.0068)	0.014 (± 0.0006)	0.064 (± 0.0038)	0.04 (± 0.0043)
2	0.0388 (± 0.0016)	0.1412 (± 0.0017)	0.037 (± 0.0017)	0.0135 (± 0.0018)	0.05 (± 0.0066)	0.01 (± 0.0008)	0.037 (± 0.0038)	0.018 (± 0.0051)
3	-0.0125 (± 0.0012)	-0.0134 (± 0.002)	-0.0156 (± 0.0014)	-0.0161 (± 0.0022)	-0.0174 (± 0.0064)	-0.016 (± 0.0001)	-0.0138 (± 0.0038)	-0.016 (± 0.006)
4	-0.102 (± 0.0013)	-0.063 (± 0.0024)	-0.103 (± 0.0015)	-0.0665 (± 0.0025)	-0.107 (± 0.0061)	-0.063 (± 0.001)	-0.09 (± 0.004)	-0.063 (± 0.0064)
5	-0.226 (± 0.0016)	-0.134 (± 0.0026)	-0.223 (± 0.0016)	-0.137 (± 0.0027)	-0.22 (± 0.0056)	-0.13 (± 0.001)	-0.192 (± 0.0042)	-0.127 (± 0.0068)
6	-0.383 (± 0.0018)	-0.227 (± 0.0028)	-0.371 (± 0.0017)	-0.228 (± 0.0029)	-0.356 (± 0.0053)	-0.216 (± 0.0011)	-0.318 (± 0.0044)	-0.213 (± 0.007)
7	-0.569 (± 0.0022)	-0.34 (± 0.003)	-0.545 (± 0.0019)	-0.337 (± 0.0031)	-0.517 (± 0.0055)	-0.321 (± 0.0012)	-0.47 (± 0.0046)	-0.342 (± 0.007)
8	-0.781 (± 0.0025)	-0.457 (± 0.0032)	-0.74 (± 0.002)	-0.464 (± 0.0032)	-0.7 (± 0.0058)	-0.445 (± 0.0014)	-0.645 (± 0.0047)	-0.555 (± 0.0065)

Figure 5.18 Verification of RSH for the longitudinal scaling exponents at different values of R_λ .

$\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $+$, (300); $---$, $n/3$.

(a), $\zeta_{L,1}(n) - \tau^d(n/3)$; (b), $\zeta_{L,2}(n) - \tau^d(n/3)$.

Figure 5.19 Verification of RSHT for the transverse scaling exponents at different values of R_λ .

$\circ$, ($R_\lambda = 100$); $\square$, (200); ∇ , (250); $+$, (300); —, $n/3$.

(a), $\zeta_{T,1}(n) - \tau^o(n/3)$; (b), $\zeta_{T,2}(n) - \tau^o(n/3)$.

5.8 CONCLUDING REMARKS

Longitudinal and transverse velocity structure functions are measured both temporally and spatially using two X-wire probes separated in the spanwise direction in the intermediate region of a cylinder wake over Taylor microscale Reynolds number in the range of 100-300. The two X-wire probes also allow the measurements of the transverse vorticity component when the separation is small enough. The scaling exponents of the longitudinal velocity structure functions for both temporal and spatial measurements agree favorably with the predictions of K62 and the She-Leveque (SL) model. The scaling exponents of the transverse velocity increments are considerably smaller than those of the longitudinal ones, with the spatial values being slightly larger than the temporal ones. Both of them increase slowly with R_λ . The association of the scaling exponents of the longitudinal and transverse velocity structure functions with the locally averaged energy dissipation rate (RSH) and enstrophy (RSHT) are also examined. While the scaling exponents of the locally averaged dissipation rate does not depend on R_λ , the scaling exponents of the locally averaged enstrophy decrease with R_λ , a trend that is consistent with that reported by Nelkin (1999). The present results show that RSH is valid for longitudinal velocity structure functions. In contrast, RSHT is verified for the spatial transverse velocity increments but only partially valid for the temporal transverse velocity structure functions.

Chapter 6

Three-component Vorticity Measurements Using an Eight-wire Probe

6.1 Introduction

As have been mentioned in Chapter 1, vorticity is an important flow characteristic that can provide valuable information on the nature of turbulence. The vorticity vector is one of the defining properties of turbulence, whether in development or equilibrium. An accurate measurement of vorticity vector ω_i ($i=1, 2$ and 3) is crucial to investigate both small- and large- scale structures of turbulence.

The most commonly used method to measure vorticity is based on thermal anemometry technique using multiple hot-wire probe to measure velocity components at a closely spaced location in the flow field to obtain the appropriate velocity gradients. Kovasznay (1954) was the first attempt to measure the longitudinal vorticity by using a four-wire probe. Davies (1976) developed an X-wire probe to determine the spanwise vorticity in the wake flow behind a D-shaped single cylinder. Cantwell and Coles (1983) used X-array hot-wire probes mounted on a pair of whirling arms to measure the spanwise vorticity to study the entrainment and transport phenomena in the turbulent near wake of a cylinder. After that, many researchers have attempted to measure the spanwise vorticity in the cylinder wake using X-wire probes (e.g. Hussain, 1986; Kiya and Matsumura 1988; Bisset et al., 1990; Matsumura et al. 1991). Mi and Antonia (1996) obtained separately the lateral and spanwise vorticity in the turbulent intermediate wake of a single cylinder by using two X-wire probes. Wassman and Wallace (1979), Vukoslavcevic et al (1991) and Andreopoulos and Honkan (1996), among others, attempted three-component simultaneous measurements of the vorticity vector using various configurations of

clusters of hot-wire probes, the latter using nine independent wires and invoke Taylor's hypothesis. By using a simpler configuration with four X-wires (totally of eight wires), Zhu and Antonia (1996) measured all three components of the vorticity vector in a turbulent far-wake. This probe has the advantages of ease in making wires, ease in calibrating the probe and the possibility for correcting the effect of spatial resolution, and is therefore used in the present study.

As the number of wire increases, the interference between wires may cause significant errors to the measurements. To ensure the high quality of the experimental data, the performance of the probe has to be checked before any further analysis is carried out. Therefore, the first objective of this chapter is to study the properties of the small-scale turbulent structures to validate the performance of the probe. Based on the measured velocity gradients, the full energy dissipation rate can be calculated. The appropriateness of using isotropic substitute ε_{iso} to represent ε will be examined. The correlation between vorticity and energy dissipation rate will be quantified based on the simultaneous measurements of these quantities. Even though there were some measurements done for the three-dimensional vorticity in the literature, there are no reports about the correlations of the energy dissipation rate and the enstrophy. Previous investigations on this problem used mainly the direct numerical simulations (DNS).

As stated in Chapter 5, various explanations have been proposed for the difference in the scaling exponents between the longitudinal and transverse velocity structure functions. Among them, Chen et al. (1997) suggested that while the longitudinal velocity structure functions scale with the energy dissipation rate ε (known as RSH), the transverse velocity structure functions are associated with the enstrophy $\Omega(\equiv \nu \omega_i \omega_i)$, which they called as RSHT. Based on the measured energy

dissipation rate and enstrophy, the suggestions made by Chen et al. (1997) can be further examined in this chapter.

6.2 Experimental Details

The experiment was conducted in the wind tunnel as described in Chapter 2. The free stream velocity U_1 is about 10 m/s, corresponding to a Taylor microscale Reynolds number R_λ of about 65 (or $Re = 4000$). The measurement location is at $x_1'/d = 240$, where $d = 6.35$ mm is the diameter of the stainless steel circular cylinder. At this location, the Kolmogorov length scale η is about 0.3 mm. The local turbulence intensity u_i'/U_1 (where a prime denotes rms value) is about 2%, favoring the use of Taylor's hypothesis.

The three-component vorticity probe consists of four X-wires (Figure 6.1). Two are in the x_1 -, x_2 -plane and separated in the x_3 -direction; the other two are in the x_1 -, x_3 -plane and separated in the x_2 -direction. It is assumed that each X-wire measures two velocity components at the center of the probe. The separation between two inclined wires in each pair of X-wire is about 1 mm. The separations between the centers of the two X-probes in x_2 - and x_3 -directions are about 2.7 mm. The effective angle for each X-wire is about 40° . The hot wires were etched from Wollaston (Pt-10%Rh) wires. The active lengths are about $200d_w$ where d_w is the diameter of the wires and equals to $2.5\mu\text{m}$. The hot wires are operated with in-house constant temperature circuits at an overheat ratio of 1.5. The probe was calibrated at the centerline of the tunnel against a Pitot-static tube. The yaw calibration was performed over $\pm 20^\circ$. The output signals from the anemometers were passed through buck and gain circuits and low-pass filtered at the cut-off frequency $f_c \equiv 5200$ Hz, which is close to f_K , where $f_K \equiv U_1/2\pi\eta$ is the Kolmogorov frequency. The

filtered signals were sampled at a frequency $f_s = 2f_c$ into a PC using a 16-bit A/D converter. The sampling period was about 60s. Further analyse of the data were carried using the MATLAB® programs.

Figure 6.1 Sketches of the eight-wire vorticity probe

The three vorticity components are obtained from the measured u_i of the four X-wires using the following finite difference method, i.e. $\partial u_i / \partial x_j = \Delta u_i / \Delta x_j$. The streamwise derivatives are calculated using Taylor's hypothesis, i.e. $\partial u_i / \partial x_1 = -U_1^{-1} \Delta u_i / 2\Delta t$ ($i = 2, 3$), where U_1 is the local mean velocity and $\Delta t (\equiv 1/f_s)$ is one sampling time interval. This method would have the advantage of keeping $\Delta x_1 \approx \Delta x_2 \approx \Delta x_3$ to avoid phase shift between the velocity gradients in the expressions of vorticity components. The three vorticity components can be obtained viz.

$$\omega_1 = u_{3,2} - u_{2,3} \approx \frac{\Delta u_3}{\Delta x_2} - \frac{\Delta u_2}{\Delta x_3}, \quad (6.1)$$

$$\omega_2 = u_{1,3} - u_{3,1} \approx \frac{\Delta u_1}{\Delta x_3} - \frac{\Delta u_3}{\Delta x_1} \quad (6.2)$$

and

$$\omega_3 = u_{2,1} - u_{1,2} \approx \frac{\Delta u_2}{\Delta x_1} - \frac{\Delta u_1}{\Delta x_2}. \quad (6.3)$$

The full expression for the mean energy dissipation rate $\langle \varepsilon \rangle \equiv 2\nu \langle s_{i,j} s_{i,j} \rangle$ can be written as

$$\begin{aligned} \langle \varepsilon_{ful} \rangle \equiv \nu \{ & 2\langle u_{1,1}^2 \rangle + 2\langle u_{2,2}^2 \rangle + 2\langle u_{3,3}^2 \rangle + \langle u_{1,2}^2 \rangle + \langle u_{2,1}^2 \rangle + \langle u_{1,3}^2 \rangle + \langle u_{3,1}^2 \rangle \\ & + \langle u_{2,3}^2 \rangle + \langle u_{3,2}^2 \rangle + 2\langle u_{1,2} u_{2,1} \rangle + 2\langle u_{1,3} u_{3,1} \rangle + 2\langle u_{2,3} u_{3,2} \rangle \} \end{aligned} \quad (6.4)$$

where the subscript “ful” represents the full energy dissipation rate. Seven of the above nine velocity derivatives can be obtained with the present probe. By assuming incompressibility, the quantities $u_{2,2}^2$ and $u_{3,3}^2$ can be estimated by the following equation:

$$u_{1,1} + u_{2,2} + u_{3,3} = 0 \quad (6.5)$$

or

$$2\langle u_{2,2}^2 \rangle + 2\langle u_{3,3}^2 \rangle = 2\langle u_{1,1}^2 \rangle - 4\langle u_{2,2} u_{3,3} \rangle \quad (6.6)$$

By assuming homogeneity, the last term on the right of Eq. (6.6) can be replaced by $-4\langle u_{2,3} u_{3,2} \rangle$ since

$$\langle u_{2,2} u_{3,3} \rangle = \langle u_{2,3} u_{3,2} \rangle \quad (6.7)$$

Substitution of Eqs. (6.6) and (6.7) into Eq. (6.4) yields

$$\begin{aligned} \langle \varepsilon_{ful} \rangle \equiv \nu \{ & 4\langle u_{1,1}^2 \rangle + \langle u_{1,2}^2 \rangle + \langle u_{2,1}^2 \rangle + \langle u_{1,3}^2 \rangle + \langle u_{3,1}^2 \rangle \\ & + \langle u_{2,3}^2 \rangle + \langle u_{3,2}^2 \rangle + 2\langle u_{1,2} u_{2,1} \rangle + 2\langle u_{1,3} u_{3,1} \rangle - 2\langle u_{2,3} u_{3,2} \rangle \}. \end{aligned} \quad (6.8)$$

The reasons for performing the experiments in the far wake region need to be clarified. In the present study, one of the objectives is to examine the correlation between the vorticity field and the enstrophy. For this purpose, to make the analysis more reliable, we need to make sure that the vorticity and enstrophy are measured

reliably. We chose the far wake because (1): in this region, Kolmogorov length scale is large enough to be resolved by the present probes, and it is also easy to achieve the optimum wire separation of $3\sim 5\eta$ for velocity derivative measurements; (2) based on our studies, local isotropy is approximately satisfied near the centerline region of $x_2^+ \leq 1$. Therefore, it is convenient to compare the present experimental results with those of the theoretical and numerical studies.

It should be noted that the present vorticity probe can also be used in the near field. Compared with far wake, the turbulent intensity in the near wake is much higher and the Kolmogorov length scale is smaller. Therefore, the spatial resolution of the probe for vorticity measurements will become worse. What is more, the bi-normal cooling effect from the third component may not be negligible for the measurement using X-wire probes. Therefore, the accuracy may not be as high as in the far wake. In the near wake, as local isotropy is not valid, the measured values of the energy dissipation rate and vorticity cannot be correct. Therefore, more care should be taken when this probe is used in the near wake.

6.3 Basic Performance Checks of the Vorticity Probe

Figures 6.2 and 6.3 show the distributions of u_1^+ , u_2^+ and u_3^+ across the wake, where the superscript + denotes normalization by the maximum velocity deficit U_0 , and the wake half-width L as described in Chapter 4. The solid lines represent best fits to the corresponding data of Browne et al. (1987). The results from the six-wire probe presented in Chapter 4 (Section 4.3) are also shown. Since the data obtained by Browne et al. (1987) and those from the six-wire probe and the eight-wire probe are all in the self-preserving region of the wake, the velocity profiles should follow each

other. This is the case as shown in Figures 6.2 and 6.3. Also, the rms values of the three velocity components from different wires of the vorticity probe agree to within $\pm 4\%$ with each other across the wake, which is within the experimental uncertainty, thus indicating the internal consistency of the measurements. Further corroboration of this is provided by the spectra of u_1 in Figure 6.4 for the measurement on the wake centerline. There is nearly no difference among distributions of $\phi_{u_1}^*(k_1^*)$ from different wires. These results indicate that the present eight-wire vorticity probe is reliable for three-component velocity measurements. This is an important prerequisite, especially when all the three vorticity components need to be calculated from the velocity signals.

Figure 6.2 RMS values of streamwise velocity fluctuations. —, Browne et al. (1987).

○, X-wire 1; □, X-wire 2; ▽, X-wire 3; ●, X-wire 4; ⊠, six-wire.

Figure 6.3 RMS values of transverse and spanwise velocity fluctuations u_i .
 ∇ , $i = 2$, X-wire 2; $\bullet$, $i = 2$, X-wire 4; Δ , $i = 3$, X-wire 1;
 $\blacksquare$, $i = 3$, X-wire 3; $+$, $i = 2$ six-wire; $\circ$, $i = 3$ six-wire;
—, Browne et al. (1987).

Figure 6.4 Spectra of streamwise velocity u_1 on the wake centerline.
--- X-wire 1; -.-, X-wire 2; .-. X-wire 3; - - - X-wire 4.

6.4 Correction for Spatial Resolution and Local Isotropy Checks

For the present eight-wire probe, the separation between the two inclined wires in each X-wire probe is about 1mm. The separations between centers of the two X-probes in x_2 - and x_3 -directions are 2.7 mm. Under such conditions, the wire separation scaled by Kolmogorov length scale η varies from about 9.6 (near the wake centerline) to 2.5 (near the wake edge).

Based on the optimum wire separation ($3-5\eta$) presented in Chapter 3, the spatial resolution of the present eight-wire vorticity probe is inadequate. Therefore the effect of imperfect spatial resolution needs to be corrected. Details of the correction procedure can be found in Zhu and Antonia (1996). A brief description of the method is given here. The high wavenumber part of the vorticity spectrum are expected to be attenuated due to the imperfect resolution is given by the ratio

$$R_{\omega_i} = \frac{\phi_{\omega_i}''(k_1)}{\phi_{\omega_i}(k_1)} = \frac{\int \int_{-\infty}^{\infty} \phi_{\omega_i}''(\mathbf{k}) d\mathbf{k}_2 d\mathbf{k}_3}{\int \int_{-\infty}^{\infty} \phi_{\omega_i}(\mathbf{k}) d\mathbf{k}_2 d\mathbf{k}_3} \quad (6.9)$$

where $\mathbf{k}$ is the wavenumber vector, ϕ_{ω_i}'' is the measured spectrum and ϕ_{ω_i} is the “corrected” spectrum. For each value of i , $\phi_{\omega_i}(\mathbf{k})$ can be expressed in term of $\phi_{ij}(\mathbf{k})$, the energy spectrum tensor. For instance, for $i=2$,

$$\phi_{\omega_2}(\mathbf{k}) = \phi_{u_{1,3}}(\mathbf{k}) + \phi_{u_{3,1}}(\mathbf{k}) - 2Co_{u_{1,3}u_{3,1}}(\mathbf{k}) \quad (6.10)$$

$$\phi_{u_{i,m}}(\mathbf{k}) = k_m^2 \phi_{ii}(\mathbf{k}) \quad (6.11)$$

and
$$Co_{u_{i,m}u_{j,n}}(\mathbf{k}) = k_m k_n \phi_{ij}(\mathbf{k}). \quad (6.12)$$

The components $\phi_{u_{1,3}}''(\mathbf{k})$, $\phi_{u_{3,1}}''(\mathbf{k})$ and $Co_{u_{1,3}u_{3,1}}''(\mathbf{k})$ can also be written in terms of $\phi_{ij}(\mathbf{k})$ and all the geometrical parameters of the probe (Zhu and Antonia,

1996). By assuming isotropy, Eq. (4.19) will be used to evaluate the integrals in Eq. 6.9.

In Chapter 4, the results obtained using the six-wire probe indicate that the lateral vorticity component approximately satisfies local isotropy near the wake centerline. Local isotropy should also be observed for the three-dimensional vorticity measurements since the present eight-wire probe is reliable for three-component velocity measurements.

The uncorrected and corrected spectra of ω_i measured on the wake centerline are shown in Figures 6.5-6.7. It can be seen that the vorticity spectra are attenuated at all wavenumbers. The corrections yield around 10% increase for $\langle \omega_1^{*2} \rangle$, 30% for both $\langle \omega_2^{*2} \rangle$ and $\langle \omega_3^{*2} \rangle$ due to the spectral attenuation of the present probe. In order to check local isotropy of the vorticity field on the wake centerline, the calculated vorticity spectra using Eqs. (4.23) and (4.24) are also shown for comparison. There is a good agreement between the corrected spectrum and the corresponding isotropic calculation for each vorticity component despite the small R_λ (≈ 65) in the present study, indicating the satisfactory performance of the probe in vorticity measurements. The upturns of the measured spectra for large wavenumbers ($k_1^* \approx 1$) are due to noise contaminations.

Figure 6.5 Uncorrected and corrected $\phi_{\alpha_1}^*(k_1^*)$ and comparison with isotropic calculation on the wake centerline. —, isotropic spectrum; ---, measured spectrum; - - -, corrected spectrum.

Figure 6.6 Uncorrected and corrected $\phi_{\alpha_2}^*(k_1^*)$ and comparison with isotropic calculation on the wake centerline. —, isotropic spectrum; ---, measured spectrum; - - -, corrected spectrum.

Figure 6.7 Uncorrected and corrected $\phi_{\omega_3}^*(k_1^*)$ and comparison with isotropic calculation on the wake centerline. —, isotropic spectrum; ---, measured spectrum; - - -, corrected spectrum.

Isotropy can also be checked by comparing the measured spectra of the velocity derivatives with calculations based on isotropy. The isotropic relations for lateral vorticity ω_2 and ω_3 (and their components) have been given in Eqs. (4.26 - 4.28). The spectra and cospectrum of velocity derivatives involved in the calculation of ω_1 , $\phi_{u_{2,3}}(k_1)$ and $\phi_{u_{3,2}}(k_1)$, can be calculated by the following equations (Antonia et al., 1996):

$$\phi_{u_{2,3}}^{cal}(k_1) = \phi_{u_{3,2}}^{cal}(k_1) = \frac{1}{2}\phi_{u_{1,1}}(k) + \frac{3}{2}\int_{k_1}^{\infty} \frac{\phi_{u_{1,1}}(k)}{k} dk \quad (6.13)$$

and
$$Co_{u_{2,3}u_{3,2}}(k) = k_2 k_3 \phi_{23}(k). \quad (6.14)$$

Figures 6.8-6.12 show the comparisons between the spectra of measured velocity derivative and the corresponding isotropic calculations on the wake centerline. At low wavenumbers, there are some discrepancies between the measured and calculated spectra, especially for $-Co_{u_{2,3}u_{3,2}}^*(k_1^*)$ when $k_1^* < 0.2$ in Figure 6.12. For high wave numbers, all the spectra and cospectra agree well with the corresponding isotropic calculations. In Figures 6.9 and 6.10, the distributions of $\phi_{u_{1,2}}^*(k_1^*)$, $\phi_{u_{1,3}}^*(k_1^*)$, $\phi_{u_{2,3}}^*(k_1^*)$ and $\phi_{u_{3,2}}^*(k_1^*)$ show that the measured spectra agree well with the calculated spectra over the entire range even for lower wavenumbers. The results indicate that local isotropy is approximately satisfied on the wake centerline, thus confirming the satisfactory performance of the present probe.

Figure 6.8 Comparison of measured spectra $\phi_{u_{2,1}}^*(k_1^*)$ and $\phi_{u_{3,1}}^*(k_1^*)$ with isotropic calculations. —, calculated spectrum; ---, $\phi_{u_{2,1}}^*(k_1^*)$; -·-, $\phi_{u_{3,1}}^*(k_1^*)$.

Figure 6.9 Comparison of measured spectra $\phi_{u_{1,2}}^*(k_1^*)$ and $\phi_{u_{1,3}}^*(k_1^*)$ with isotropic calculations. —, calculated spectrum; ---, $\phi_{u_{1,2}}^*(k_1^*)$; -·-, $\phi_{u_{1,3}}^*(k_1^*)$.

Figure 6.10 Comparison of measured spectra $\phi_{u_{2,3}}^*(k_1^*)$ and $\phi_{u_{3,2}}^*(k_1^*)$ with isotropic calculations. —, calculated spectrum; ---, $\phi_{u_{2,3}}^*(k_1^*)$; - · -, $\phi_{u_{3,2}}^*(k_1^*)$.

Figure 6.11 Comparison of measured cospectra $-Co_{u_{2,3}u_{1,2}}^*(k_1^*)$ and $-Co_{u_{3,1}u_{1,3}}^*(k_1^*)$ with isotropic calculations. —, calculated spectrum; ---, $Co_{u_{2,3}u_{1,2}}^*(k_1^*)$; - · -, $Co_{u_{3,1}u_{1,3}}^*(k_1^*)$.

Figure 6.12 Comparison of measured cospectra $-Co_{\mu_{2,3}\mu_{3,2}}^*(k_1^*)$ with isotropic calculations.
—, calculated spectrum; ---, measured $-Co_{\mu_{2,3}\mu_{3,2}}^*(k_1^*)$.

6.5 Statistics of Vorticity and their Components

The performance of the present probe can be further checked by comparing the vorticity data with those obtained using other probe configurations and by examining how closely the statistics of the vorticity ω_i ($i=1, 2$ and 3) and their components confirm with local isotropy.

The rms values of the three vorticity components are compared in Figures 6.13-6.15 with those obtained by the six-wire probe (Chapter 4), Antonia et al. (1988) and Zhu and Antonia (1999). In Figure 6.13, in the central region of the wake, the present measured rms values of ω_1 are smaller than those of Antonia et al. (1988), which are approximated by $\omega_1 = \left(\langle u_{2,3}^2 \rangle + \langle u_{3,2}^2 \rangle \right)^{1/2}$, but larger than those of Zhu and Antonia (1999). After correction for spatial resolution, the rms values of ω_1 agree

well (within $\pm 4\%$) with those of Antonia et al. (1988) and Zhu and Antonia (1999). In Figure 6.14, the present measured rms values of ω_2 agree well with those obtained by the six-wire probe (within $\pm 3\%$), but smaller than Zhu and Antonia (1999). The measured values are larger than Antonia et al. (1988) near the wake centerline. After correction, the agreement between the present values and Zhu and Antonia (1999) becomes much better near the wake centerline. In Figure 6.15, the present measured rms values of ω_3 are smaller than those obtained using other probe configurations near the wake centerline. After correction, the present measured values agree well with Zhu and Antonia (1999) (within $\pm 4\%$) but slightly larger than Antonia et al. (1988) and the six-wire probe. The above figures also show that the agreement between the present eight-wire and six-wire measurements for the rms values of ω_2 and ω_3 is good (within $\pm 3\%$) across the wake.

Figure 6.13 Distributions of rms values of ω_1 . +, Antonia et al., 1988; □, Zhu and Antonia, 1999; ▲, present measurement (uncorrected); △, present measurement (corrected);

Figure 6.14 Distributions of rms values of ω_2 . +, Antonia et al., 1988; $\square$, Zhu and Antonia, 1999; $\blacktriangle$, present measurement (uncorrected); $\triangle$, present measurement (corrected); $\circ$, six-wire.

Figure 6.15 Distributions of rms values of ω_3 . +, Antonia et al. 1988; $\square$, Zhu and Antonia 1999; $\blacktriangle$, present measurement (uncorrected); $\triangle$, present measurement (corrected); $\circ$, six-wire.

The skewness of ω_i and their components is shown in Figures 6.16 and 6.17. All the skewness of these curves is close to zero around the centerline region ($x_2^+ \leq 1$) except for $u_{1,1}$. The results are consistent with local isotropy. As the local isotropy is not valid away from the centerline, the departure from zero of the skewness factors becomes larger as x_2^+ increases. The results agree well with those obtained from the six-wire probe and those obtained by Zhu and Antonia (1999). The very large value of the skewness factors at the edge of the wake is caused by the high intermittency of the flow. The fact that the skewness of $u_{1,1}$ is larger than other velocity derivatives (for $x_2^+ \leq 1$ in Figures 4.19 and 6.16) may indicate that $u_{1,1}$ is more intermittent than other quantities.

Figure 6.16 Skewness factors of velocity derivatives across the wake.

Δ , $u_{1,1}$; $\circ$, $u_{3,1}$; $\square$, $u_{1,3}$; $\bullet$, $u_{2,1}$; ∇ , $u_{1,2}$; $\oplus$, $u_{2,3}$; $\boxtimes$, $u_{3,2}$;

Figure 6.17 Skewness factors of vorticity components across the wake. O, ω_1 ; Δ , ω_2 ; +, ω_3 .

For local isotropic turbulence, the flatness factors of ω_i and $u_{i,j}$ should be equal to each other near the wake center. In Figures 6.18 (a, b), this is the case for the flatness of $u_{2,1}$ and $u_{3,1}$, $u_{1,2}$ and $u_{3,2}$, and $u_{1,3}$ and $u_{2,3}$, individually, indicating the similar intermittence for the velocity derivatives along the same direction.. As the wake edge is approached, all the flatness factors F_α increase sharply, indicating strong intermittency effect towards the free stream. In Figures 6.19 (a, b), the flatness of the three vorticity components follows each other closely for the region of $x_2^+ \leq 1$, indicating satisfactory agreement with the isotropy for vorticity data and further affirming the concept that vorticity are more related with small scale structures (Antonia et al., 2002).

Figure 6.18 Flatness factors of velocity derivatives.

Δ , $u_{1,1}$; O , $u_{3,1}$; $\square$, $u_{1,3}$; $\times$, $u_{2,1}$; ∇ , $u_{1,2}$; $\oplus$, $u_{2,3}$; $\boxtimes$, $u_{3,2}$;
(a) $x_2^+ < 3$; (b) Enlarged figure for $x_2^+ < 1$.

Figure 6.19 Flatness factors of vorticity components across the wake. O, ω_1 ; $\times$, ω_2 ; ∇ , ω_3 . (a) $x_2^+ < 3$; (b) Enlarged figure for $x_2^+ < 1$.

6.6 Statistics of Energy Dissipation Rates

6.6.1 Mean Energy Dissipation Rates

As mentioned in Chapter 4, simple approximation to energy dissipation rate ε can be obtained based on the isotropic assumption, i.e. Eq. (3.1). However, the use of Eq. (3.1) as an appropriate substitute of ε has been questioned (e.g. Chen, 1997; Hosokawa, 1997). Using the present probe, seven of the nine velocity derivatives that make up ε are measured simultaneously. Assuming local isotropy and homogeneity, approximations to ε can be obtained based on continuity using Eq. (6.8).

The mean energy dissipation rate ε obtained by Eqs. (6.8), (4.3) and (3.1) are compared in Figure 6.20. Since more derivative correlations are included in Eqs. (4.3) and (6.8), it is expected that they should provide more reasonable approximations to ε than when ε_{iso} (Eqs. 3.1) is used. To account for the effect of the spatial resolution, all velocity derivatives have been corrected. The results obtained by Browne et al. (1987) and Zhu and Antonia (1999) are also shown here. The normalization of $\langle \varepsilon \rangle$ is based on L and U_0 , i.e. $\langle \varepsilon \rangle^+ = \langle \varepsilon \rangle L / U_0^3$. Figure 6.20 shows that the magnitude of $\langle \varepsilon_{ful} \rangle$ is about 10%-20% larger than that of $\langle \varepsilon_{iso} \rangle$ (Eq 3.1) across the wake, indicating that the use of the one-dimensional surrogate, as usually used in experimental studies, may cause significant errors. The present results agree favorably with that obtained using the six-wire probe (Eq. 4.3) and Browne et al. (1987), indicating a better approximation of ε when Eqs. (4.3) or (6.8) used. However, the results obtained by Zhu and Antonia (1999) are about 20% smaller than the present results in the central region of the wake. Please also refer to Section 4.5.1 for more discussions about Zhu and Antonia's (1999) data and the future interpretation of the differences observed.

Figure 6.20 Comparison of the mean turbulent energy dissipation rate $\langle \varepsilon \rangle$ obtained using different probes. $\blacktriangle$, present $\langle \varepsilon_{iso} \rangle$; ∇ , present $\langle \varepsilon_{ful} \rangle$ (Eq. 6.8) ; $\bullet$, present six-wire in Chapter 4; —, Browne et al., 1987 ; $\circ$, Zhu and Antonia (1999).

6.6.2 Correlation between Energy Dissipation Rates and Enstrophy

The definition of conditional expectation has been given in Eq. (4.37). Figure 6.21 shows the conditional expectations $\langle \varepsilon | \omega_3^2 \rangle / \langle \varepsilon \rangle$ measured on the wake centerline. The present values $\langle \varepsilon_{ful} | \omega_3^2 \rangle / \langle \varepsilon_{ful} \rangle$ increase linearly with ω_3^2 at an overall gradient of 0.81, reflecting a strong dependence of ε on ω_3^2 . This gradient is about 0.75 from the previous six-wire measurement (Chapter 4), in which the energy dissipation rate is approximated using Eq. (4.3). The increase of the gradient from 0.75 to 0.81 may suggest a higher correlation between ε with ω_3^2 as the full energy dissipation rate is used. Compared with $\langle \varepsilon_{ful} | \omega_3^2 \rangle / \langle \varepsilon_{ful} \rangle$, the values of $\langle \varepsilon_{iso} | \omega_3^2 \rangle / \langle \varepsilon_{iso} \rangle$ increase slowly with $\omega_3^2 / (\omega_3^2)'$, indicating a much weak correlation between ε and ω_3^2

when using ε_{iso} to represent ε .

Figure 6.21 Expectations of ε conditioned on particular values of ω_3^2 .

$\Delta, \langle \varepsilon_{ful} | \omega_3^2 \rangle$ Eq. (6.7); $\circ, \langle \varepsilon_{iso} | \omega_3^2 \rangle$ Eq. (3.1); $\nabla, \langle \varepsilon_{ap} | \omega_3^2 \rangle$ Eq. (4.3).

For isotropic turbulence, the globe averages of energy dissipation rate ε and enstrophy $\Omega (= \omega_i \omega_i)$ are related with equation $\langle \varepsilon \rangle = \nu \langle \Omega \rangle$, but this does not mean that their local averages scale identically. If they do, however, the scaling exponents for small-scale turbulence should be unique (Chen et al. 1997). Existing experimental data, surveyed by Sreenivasan and Antonia (1997), suggest that the scaling exponents for dissipation and enstrophy may indeed not be the same, but the conclusion was not definitive as the experimental results were affected by the use of Taylor's hypothesis and the use of one-dimensional surrogates of dissipation and vorticity. It is therefore important to study the scaling of enstrophy and dissipation to verify the local isotropy and to identify the small scale quantities which have the same scaling properties.

The spatial correlation between enstrophy Ω and energy dissipation rate can be quantified by means of the conditional expectation $\langle \Omega_r^{n\uparrow} | \varepsilon_r^\uparrow \rangle$ or its reverse conditional expectation $\langle \varepsilon_r^{n\uparrow} | \Omega_r^\uparrow \rangle$, where $\uparrow$ means that the quantities are normalized by their mean values. The conditional expectations $\langle \Omega_r^{n\uparrow} | (\varepsilon_{ful})_r^\uparrow \rangle$ and the reverse conditional expectations $\langle (\varepsilon_{ful})_r^{n\uparrow} | \Omega_r^\uparrow \rangle$ for different orders ($n=1, 2, 4, 6, 8$) are shown in Figures 6.22 (a-e) to check the spatial correlation between the two quantities. It is found that while $\langle \Omega_r^{n\uparrow} | (\varepsilon_{ful})_r^\uparrow \rangle \sim (\varepsilon_{ful})_r^{\alpha n\uparrow}$ for large $(\varepsilon_{ful})_r^\uparrow$, the reversed conditional expectations $\langle (\varepsilon_{ful})_r^{n\uparrow} | \Omega_r^\uparrow \rangle$ seem to scale with $\Omega_r^{\beta n\uparrow}$ for large $\Omega_r^\uparrow$, with $\beta < \alpha \leq 1$. For example when $n=1$ (Figure 6.22a), the values of $\langle \Omega_r^\uparrow | (\varepsilon_{ful})_r^\uparrow \rangle$ grow linearly with $(\varepsilon_{ful})_r^\uparrow$ over the range of $(\varepsilon_{ful})_r^\uparrow > 0.6$ ($\alpha = 1$ in this case), reflecting a strong correlation between enstrophy and energy dissipation rate

for high amplitude of $(\varepsilon_{ful})_r^\dagger$. However, the distribution of $\langle (\varepsilon_{ful})_r^\dagger | \Omega_r^\dagger \rangle$ grows with $\Omega_r^\dagger$ by a power law of 0.7 ($\beta = 0.7$ in this case) for $\Omega_r^\dagger > 0.6$. While for $n=6$ (Figure 6.22d), the dependence of $\langle \Omega_r^{6\dagger} | (\varepsilon_{ful})_r^\dagger \rangle$ on $(\varepsilon_{ful})_r^\dagger$ over the range of $(\varepsilon_{ful})_r^\dagger > 0.6$ shows an approximate power law of 4.3 (or $\alpha=0.72$); the power is 3 ($\beta=0.5$ in this case) for distribution of $\langle (\varepsilon_{ful})_r^{6\dagger} | \Omega_r^\dagger \rangle$ over the range of $\Omega_r^\dagger > 0.6$. The values of α and β for different values of n are listed in Table 6.1. It can be seen that while both α and β decrease with the increase of n , the value of α is always larger than the corresponding value of β . The fact $\alpha > \beta$ suggests that the spatial growth of $\varepsilon_r^\dagger$ in statistical sense is slower than that of $\Omega_r^\dagger$. The present results are consistent with the DNS study by Chen et al. (1997) in which the authors found that, though strong domain correlation exists, energy dissipation and enstrophy are not correlated point by point.

To further check the appropriateness of using ε_{iso} as the substitute of ε , the first and sixth order moments of $\Omega_r^\dagger$ conditioned on both ε_{ful} and ε_{iso} are shown in Figures 6.23 and 6.24, respectively. In Figure 6.23, the distribution of $\langle \Omega_r^\dagger | (\varepsilon_{ful})_r^\dagger \rangle$ shows a strong correlation between enstrophy and energy dissipation rate for high amplitude of $(\varepsilon_{ful})_r^\dagger$ with a power law of 1 for $(\varepsilon_{ful})_r^\dagger > 0.6$, whereas $\langle \Omega_r^\dagger | (\varepsilon_{iso})_r^\dagger \rangle$ shows much less strength of this correlation with $\alpha=0.35$ for $(\varepsilon_{iso})_r^\dagger > 0.6$. An even larger difference of the behavior between ε_{ful} and ε_{iso} is observed in Figure 6.24 where $n=6$. For $(\varepsilon_{ful})_r^\dagger > 0.6$, the $\langle \Omega_r^{6\dagger} | (\varepsilon_{ful})_r^\dagger \rangle \sim (\varepsilon_{ful})_r^{\alpha n}$ with $\alpha=0.72$, indicating a considerable correlation between enstrophy and energy

dissipation rate. In contrast, $\langle \Omega_r^{6\dagger} | (\varepsilon_{iso})_r^\dagger \rangle \sim (\varepsilon_{iso})_r^{\alpha n^\dagger}$ shows a very small α with the value of 0.15 for $(\varepsilon_{iso})_r^\dagger > 0.6$, indicating that there is nearly no correlation between enstrophy and energy dissipation rate. The large discrepancy between $\langle \Omega_r^{n^\dagger} | (\varepsilon_{ful})_r^\dagger \rangle$ and $\langle \Omega_r^{n^\dagger} | (\varepsilon_{iso})_r^\dagger \rangle$ observed in Figures 6.23 ($n=1$) and Figure 6.24 ($n=6$) suggests that using ε_{iso} as a substitute of ε should be re-examined, especially for instantaneous values. The reason for the selecting of the first and sixth order is that the first order is representative for the low-order moments while and the sixth order for the high-order moments. The same conclusions can be obtained when the 2nd and 5th order moments are used.

Figure 6.22 Conditional expectations $\langle \Omega_r^{\dagger} | (\varepsilon_{ful})_r^{\dagger} \rangle$ and $\langle (\varepsilon_{ful})_r^{\dagger} | \Omega_r^{\dagger} \rangle$ as function of $\varepsilon_{ful}^{\dagger}$ and $\Omega_r^{\dagger}$, respectively. $\circ$, $\langle \Omega_r^{\dagger} | (\varepsilon_{ful})_r^{\dagger} \rangle$; $\square$, $\langle (\varepsilon_{ful})_r^{\dagger} | \Omega_r^{\dagger} \rangle$; ---, $y = cx^{\alpha n}$; - - -, $y = cx^{\beta n}$.

(a) $n=1$: $\alpha=1$, $\beta=0.7$; (b) $n=2$: $\alpha=0.85$, $\beta=0.65$; (c) $n=4$: $\alpha=0.76$, $\beta=0.64$; (d) $n=6$: $\alpha=0.72$, $\beta=0.5$;
(e) $n=8$: $\alpha=0.6$, $\beta=0.35$.

Table 6.1 The values of α and β for different values of n .

n	1	2	4	6	8
α	1.00	0.85	0.76	0.72	0.60
β	0.70	0.65	0.64	0.50	0.35

Figure 6.23 Conditional expectations $\langle \Omega_r^\dagger | (\epsilon_{ful})_r^\dagger \rangle$ and $\langle \Omega_r^\dagger | (\epsilon_{iso})_r^\dagger \rangle$ as functions of $\epsilon_{ful}^\dagger$ and $\epsilon_{iso}^\dagger$, respectively. $\circ$, obtained from $\epsilon_{ful}^\dagger$; $\square$, obtained from $\epsilon_{iso}^\dagger$; ---, $y=x$; - - -, $y=x^{0.35}$.

Figure 6.24 Conditional expectations $\langle \Omega_r^{6\dagger} | (\epsilon_{ful})_r^\dagger \rangle$ and $\langle \Omega_r^{6\dagger} | (\epsilon_{iso})_r^\dagger \rangle$ as functions of $\epsilon_{ful}^\dagger$ and $\epsilon_{iso}^\dagger$, respectively. $\circ$, obtained from $\epsilon_{ful}^\dagger$; $\square$, obtained from $\epsilon_{iso}^\dagger$; ---, $y=x^{4.3}$; - - -, $y=x^{0.9}$.

6.7 Scaling of Velocity Structure Functions and Locally Averaged Enstrophy and Energy Dissipation Rates

In Chapter 5 (Section 5.7), it is found that RSH is valid for longitudinal velocity increments which are obtained either spatially or temporally. RSHT is proved to be valid for the spatial transverse velocity increments but only partially valid for the temporal transverse velocity increments. It needs to be emphasized that the verification of RSHT in Chapter 5 is based on the assumption that $\Omega = 3\nu\omega_2^2$. To further clarify this problem, the association of the scaling of the temporal longitudinal and transverse velocity increments with the locally averaged energy dissipation rate (RSH) and enstrophy (RSHT), respectively, will be examined using data obtained from the eight-wire vorticity probe. The main advantage of discussing RSHT using the eight-wire data is that all three vorticity components and hence that the full enstrophy are measured, avoiding the use of replacing the enstrophy by only one vorticity component (e.g. Eq. 5.31).

An indicator of the scaling range is provided by the behavior of $\langle(\delta u_i)^3\rangle$ (Eq. 5.4). After normalization by Kolmogorov scales, Eq. (5.4) becomes

$$-\langle(\delta S_{L,1}^*)^3\rangle = \frac{4}{5}r^*, \quad (6.15)$$

where $\delta S_{L,1}$ is defined by Eq. (5.6). The region where $\langle(\delta S_{L,1}^*)^3\rangle$ varies linearly with r is often identified as the inertial range. The present distribution $\langle(\delta S_{L,1}^*)^3\rangle/r^*$ vs. r^* is shown in Figure 6.25, where there is only a very small plateau around $r^* \approx 22$. The inertial subrange shown in Fig 6.25 is small due to the small R_λ (≈ 65) in the present study. The peak value is smaller than 4/5 due to the same reason. The ESS method (Section 5.6, Chapter 5) is used to estimate the scaling exponents of the longitudinal and transverse velocity structure functions with one modification — the

scaling exponents are estimated over the region where $\langle |\delta S_{L,1}|^3 \rangle$ is approximately linear to r . The distribution of $\langle |\delta S_{L,1}^*|^3 \rangle$, weighted by r^{*-1} , is also shown in Figure 6.25. $r^{*-1} \langle |\delta S_{L,1}^*|^3 \rangle$ is qualitatively similar to that of $r^{*-1} \langle (\delta S_{L,1}^*)^3 \rangle$, with the center of the plateau being slightly shifted to $r^* \approx 33$. The method used to determine the scaling range is different from that of Benzi et al. (1993) since the claim that the scaling of the structure functions is extended into the dissipative range becomes tenuous as the magnitude of n increases, as indicated in Figure 5.13. This can be seen in Figure 6.26 on the plot of $\log \langle |\delta S_{L,1}^*|^n \rangle$ vs. $\log \langle |\delta S_{L,1}^*|^3 \rangle$, and becomes more apparent in Figure 6.27 on the plot of $\log \langle |\delta S_{T,1}^*|^n \rangle$ vs. $\log \langle |\delta S_{L,1}^*|^3 \rangle$, where $\delta S_{T,1}$ is defined by Eq.(5.7). The curvature exhibited by the transverse structure functions (Figure 6.27) represents a departure from the relatively linear behavior exhibited by the longitudinal increments (Figure 6.26). The definitions of $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ are given in Eqs. (5.17) and (5.21), respectively. The scaling exponents $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ for the longitudinal and transverse velocity increments are estimated from least-squares regressions over the region (scaling range) indicated by the arrowed horizontal line in Figures 6.26 and 6.27. The scaling exponents $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ are shown in Figure 6.28. Predictions of K41, K62 and the She-Leveque model are also included. As in Figure 5.14, Figure 6.28 shows that $\zeta_{T,1}(n)$ are significantly smaller than $\zeta_{L,1}(n)$. The difference between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ increases with n . The results for $\delta S_{T,1}$ defined as $u_3(x_1 + r) - u_3(r)$ are not shown here since the results are essentially identical to those obtained from the present definition, i.e.

$\delta S_{T,1} = u_2(x_1 + r) - u_2(r)$. The present values of $\zeta_{L,1}$ show quite good agreement with that of the lognormal estimations (K62) and the SL model for $n \leq 8$. The departure of measured $\zeta_{L,1}$ from K41, increases with n indicating the increased intermittency effect for higher order longitudinal velocity structure functions.

Figure 6.25 Scaling range (SR) estimated using 3rd-order longitudinal structure functions multiplied by r^{*-1} . $\circ$, $-\langle (\delta S_{L,1}^*)^3 \rangle r^{*-1}$; $\square$, $\langle |\delta S_{L,1}^*|^3 \rangle r^{*-1}$.

Figure 6.26 Moments of order 1 to 8 of $\log |\delta S_{L,1}^*|^n$ as a function of $\log |\delta S_{L,1}^*|^3$. $\circ$, $n=1$; $\square$, $n=2$; ∇ , $n=3$; $\bullet$, $n=4$; $\blacksquare$, $n=5$; $\triangle$, $n=6$; $+$, $n=7$; $\oplus$, $n=8$;

Figure 6.27 Moments of order 1 to 8 of $\log \langle |\delta S_{T,1}^*|^n \rangle$ as a function of $\log \langle |\delta S_{L,1}^*|^3 \rangle$.
○, $n=1$; □, $n=2$; ▽, $n=3$; ●, $n=4$; ■, $n=5$; △, $n=6$; +, $n=7$; ⊕, $n=8$;

Figure 6.28 Scaling exponents for $\delta S_{L,1}(n)$ and $\delta S_{T,1}(n)$. ▲, $\delta S_{L,1}(n)$, estimated from the data in Figure 6.26; ●, $\delta S_{T,1}(n)$, estimated from the data in Figure 6.27; —, K41; ---, lognormal model (K62). —, She-Leveque model (1994).

Several suggestions have been proposed to explain the inequality between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$. One possible reason is the small R_λ in laboratory flows which means that anisotropy will tend to affect δS_T more than δS_L (Antonia et al. 1998; Zhou and Antonia, 2000b). It is conjectured that, for sufficiently high Reynolds number, the difference between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ should disappear. Chen et al. (1997) proposed the refined similarity hypothesis for the transverse velocity structure functions (RSHT, Section 5.7). RSHT has been verified satisfactorily for spatial transverse velocity structure functions by using $3\nu\omega_2^2$ to represent Ω . To further verify RSHT using the measured full enstrophy (instead of its one component as used in Chapter 5), the scaling exponents $\tau^d(n/3)$ and $\tau^o(n/3)$, which are inferred from the distributions of $\log\langle\epsilon_r^{n/3}\rangle$ (Figures 6.29) and $\log\langle\Omega_r^{n/3}\rangle$ (Figures 6.30) vs. $\log r^*$, respectively, are given in Table 6.2. The scaling range is the same as that used to estimate $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ (Figures 6.26 & 6.27). The values of $\zeta_L(n)[\equiv \frac{n}{3} + \tau^d(n/3)]$ and $\zeta_T(n)[\equiv \frac{n}{3} + \tau^o(n/3)]$ estimated using Eqs. (5.27) and (5.28) are given in Table 6.2. The values of $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ shown in Figure 6.28 are also given for comparison with $\zeta_L(n)$ and $\zeta_T(n)$, respectively. If RSHT is valid, the difference between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ should be compensated by the difference between $\tau^d(n/3)$ and $\tau^o(n/3)$ or between $\zeta_L(n)$ and $\zeta_T(n)$. In Table 6.2, for $n > 3$, ζ_T is smaller than ζ_L , indicating the more intermittent nature of Ω_r than ϵ_r . It is also observed that while $\zeta_{L,1}(n) \approx \zeta_L(n)$ which verifies the RSH, $\zeta_{T,1}(n)$ are still significantly smaller than $\zeta_T(n)$ for all n , i.e. the difference between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ cannot be compensated completely by that between

$\zeta_L(n)$ and $\zeta_T(n)$. This result suggests that the discrepancy between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ cannot be explained by RSHT, i.e. it is caused by the different degrees of intermittency of ω_r^2 and ε_r considering the small R_λ (≈ 65) in the present study. Therefore, RSHT is only partially valid for the temporal transverse velocity increments. This result is consistent with that obtained from the two X-wire measurements as discussed in Chapter 5. Ideally, RSHT should also be checked using transverse velocity structure functions obtained spatially. However, this experiment is not done in the far wake. Therefore RSHT cannot be checked using a similar way as that used in Chapter 5.

Table 6.2 Scaling exponents of the longitudinal and transverse velocity structure functions.

n	$\tau^d(n/3)$	$\tau^o(n/3)$	ζ_L	$\zeta_{L,1}$	ζ_T	$\zeta_{T,1}$
1	0.020 (0.0015)	0.0321 (0.0017)	0.354	0.356	0.365	0.292
2	0.019 (0.0014)	0.0311 (0.0016)	0.687	0.688	0.698	0.535
3	0.000 (0)	0.0000 (0)	1.000	1.000	1.000	0.736
4	-0.0381 (0.0026)	-0.0585 (0.0027)	1.295	1.293	1.275	0.902
5	-0.0936 (0.0061)	-0.1420 (0.0064)	1.573	1.568	1.525	1.037
6	-0.1653 (0.0104)	-0.2483 (0.0107)	1.835	1.829	1.752	1.144
7	-0.2523 (0.0153)	-0.3753 (0.0155)	2.081	2.078	1.958	1.222
8	-0.3534 (0.0316)	-0.5210 (0.0203)	2.313	2.317	2.146	1.273

Figure 6.29 Moments of locally averaged energy dissipation rate.

Figure 6.30 Moments of locally averaged enstrophy fluctuations.

It needs to be noted that the test of RSHT using $\zeta_T(n)$ (Table 6.2) is based on the full expression for enstrophy, viz.

$$\Omega = \nu(\omega_1^2 + \omega_2^2 + \omega_3^2). \quad (6.16)$$

However, in Chapter 5, RSHT is verified satisfactorily using the spatial transverse velocity structure functions and $\Omega = 3\nu\omega_2^2$. The intermittency characteristics of the full enstrophy Ω ($\equiv \nu\omega_i\omega_i$) may not be perfectly represented by ω_2^2 . Using the three-component vorticity data measured by the eight-wire probe, the appropriateness of using individual vorticity component to represent enstrophy, viz.

$$\Omega = 3\nu\omega_k^2 \quad (k=1, 2 \text{ or } 3), \quad (6.17)$$

can be checked in the context of RSHT. The values of $\zeta_{T,1}(n) - \tau^o(n/3)$ with $\tau^o(n/3)$ estimated using Eqs. (6.16) and (6.17) are shown and compared with $n/3$ in Figure 6.31. It can be seen that when Eq. (6.17) is used to calculate enstrophy fluctuations, the resulting values of $\tau^o(n/3)$ [or indeed $\zeta_{T,1}(n) - \tau^o(n/3)$] agree well with each other for $k=1, 2$ and 3 . The values of $\tau^o(n/3)$ result in $\zeta_{T,1}(n) - \tau^o(n/3)$ significantly smaller than $n/3$, indicating that RSHT is not verified satisfactorily either using vorticity components to represent the full enstrophy field. For $n > 4$, the discrepancy between $\zeta_{T,1}(n) - \tau^o(n/3)$ and $n/3$ estimated using Eq. (6.16) is apparently larger than when Eq. (6.17) is used. This result also implies that the individual locally averaged enstrophy components are more intermittent than the full enstrophy. Overall, the considerable inequality between $\zeta_{T,1}(n) - \tau^o(n/3)$ and $n/3$, either using Eq. (6.16) or (6.17), indicates that satisfactory verification of RSHT cannot be obtained. The values of $\zeta_{T,1}(n) - \tau^o(n/3)$ obtained from the two X-wire probes in Chapter 5 ($R_\lambda = 300$) with the scaling exponents $\zeta_{T,1}(n)$ calculated from

the temporal transverse velocity increments, i.e. Eq. (5.7), are also shown in Figure 6.31. All results shown in Figure 6.31 seem to suggest that RSHT does not work satisfactorily, at least for $\zeta_{T,1}(n)$ (where $\zeta_{T,1}(n)$ are obtained from the temporal transverse velocity structure functions).

Figure 6.31 Verification of RSHT for the temporal transverse velocity increments.

○, from Eq. (6.16); +, from Eq. (6.17), $k=1$; ▽, from Eq. (6.17), $k=2$;

□, from Eq. (6.17), $k=3$; △, 2x-wire ($R_\lambda = 300$), from Eq. (6.17), $k=2$;

—, $n/3$.

In the present study, Table 6.2 indicates that enstrophy are more intermittent than energy dissipation rate. To get a better understanding on the intermittency phenomena and to study the scaling features of enstrophy and energy dissipation, it is necessary to examine the scaling relation between the high-order moment of enstrophy and those of energy dissipation rate. Wang et al. (1996), in their DNS studies, pointed out that, for the energy dissipation, the one-dimensional surrogate field is in general more intermittent than the full field. To study these problems, both isotropic (Eq. 3.1) and the full energy dissipation rate (Eq. 6.8) are used to examine the relative scaling with the enstrophy. In Figures 6.32-6.35, the quantities $\langle \Omega_r^{n\dagger} \rangle$ are plotted against $\langle \varepsilon_r^{n\dagger} \rangle$ for $n=2, 4, 6$ and 8 . The relative power laws can be identified in the scaling range (a region indicated by the double arrowed vertical line) and quantified by the scaling exponent k , which is calculated from $\langle \Omega_r^n \rangle \sim \langle \varepsilon_r^n \rangle^k$. For convenience, the values of k for different moments n are listed in Table 6.3. It can be seen that the values of k are always greater than 1 for $n=2, 4, 6$ and 8 . This result shows that the locally averaged full enstrophy are larger than those of locally averaged full dissipation rate, indicating a stronger intermittency of the former than the latter. The present values of k is qualitatively close to those of the DNS study by Chen et al. (1997), where $k=1.56$ and 1.47 for $n=2$ and 4 , respectively, were obtained. The values of k obtained using $\langle \Omega_r^n \rangle \sim \langle (\varepsilon_{iso})_r^n \rangle^k$ are much smaller than those of $\langle \Omega_r^n \rangle \sim \langle (\varepsilon_{ful})_r^n \rangle^k$ and even smaller than 1, thus leading to a conclusion that the intermittency of the dissipation rate based on isotropic relations is stronger than that of the full enstrophy. The result also indicates that, for the energy dissipation rate, the one-dimensional surrogate field is more intermittent than the full field, which is consistent with Wang et al. (1996). The differences in intermittency between ε_{iso}

and ε_{ful} can also be seen in Figures 6.36 (a-d) where the quantities $\langle (\varepsilon_{iso})_r^n \rangle$ are plotted against $\langle (\varepsilon_{ful})_r^n \rangle$ for $n=2, 4, 6$ and 8 (the scaling ranges are the same as those shown in Figures 6.32-6.35). It can be seen that the values of k obtained using $\langle \varepsilon_{iso}^n \rangle \sim \langle (\varepsilon_{ful})_r^n \rangle^k$ are always greater than 1 for $n=2, 4, 6$ and 8 , indicating that ε_{iso} is more intermittent than ε_{ful} . The differences in intermittency between ε_{iso} and ε_{ful} may be due to the reason that the full energy dissipation rate involves all nine velocity derivatives while ε_{iso} involves only one (i.e. $\partial u_1 / \partial x_1$). The full dissipation can be considered roughly as an average of nine intermittent structures, each of which is like ε_{iso} , or slightly larger than ε_{iso} [based on the discussions on the flatness factors of $u_{i,j}$ by Antonia et al. (1996) and Sreenivasan and Antonia (1997)]. The averaging tends to reduce the degree of intermittency of individual structures (Wang et al, 1996). The present study in turn indicates that the use of the one-dimensional surrogate may lead to fundamental changes in statistics. It needs to be noted that the values of k obtained from $\langle \Omega_r^n \rangle \sim \langle (\varepsilon_{iso})_r^n \rangle^k$ increases with n , whereas the values obtained from $\langle \Omega_r^n \rangle \sim \langle (\varepsilon_{ful})_r^n \rangle^k$ keep approximately constant (Table 6.3).

Figure 6.32 $\langle \Omega_r^{2\dagger} \rangle$ as a function of $\langle \epsilon_r^{2\dagger} \rangle$. Here r is an implicit variable.
○, obtained using ϵ_{ful} (Eq. 6.8); □, obtained using ϵ_{iso} (Eq. 3.1); ---, $y = x$;
Arrowed vertical line indicates points within inertial range.

Figure 6.33 $\langle \Omega_r^{4\dagger} \rangle$ as a function of $\langle \epsilon_r^{4\dagger} \rangle$. Here r is an implicit variable.
○, obtained using ϵ_{ful} (Eq. 6.8); □, obtained using ϵ_{iso} (Eq. 3.1); ---, $y = x$;
Arrowed vertical line indicates points within inertial range.

Figure 6.34 $\langle \Omega_r^{6\dagger} \rangle$ as a function of $\langle \varepsilon_r^{6\dagger} \rangle$. Here r is an implicit variable.

○, obtained using ε_{fil} (Eq. 6.8); □, obtained using ε_{iso} (Eq. 3.1); ---, $y = x$;
Arrowed vertical line indicates points within inertial range.

Figure 6.35 $\langle \Omega_r^{8\dagger} \rangle$ as a function of $\langle \varepsilon_r^{8\dagger} \rangle$. Here r is an implicit variable.

○, obtained using ε_{fil} (Eq. 6.8); □, obtained using ε_{iso} (Eq. 3.1); ---, $y = x$;
Arrowed vertical line indicates points within inertial range.

Table 6.3 Scaling exponents k in $\langle \Omega_r^n \rangle \sim \langle \varepsilon_r^n \rangle^k$ obtained from the full (Eq. 6.8) and one-dimensional isotropic (Eq. 3.1) energy dissipation rate for $n=2, 4, 6, 8$.

n	2	4	6	8
$k(\varepsilon_{ful})$	1.43	1.37	1.35	1.38
$k(\varepsilon_{iso})$	0.77	0.87	0.93	0.97

Figure 6.36 $\langle (\varepsilon_{iso})_r^n \rangle$ as a function of $\langle (\varepsilon_{ful})_r^n \rangle$. Here r is an implicit variable.

(a) $n=2$; (b) $n=4$; (c) $n=6$; (d) $n=8$; ---, $y = x$;

6.8 Concluding Remarks

The three-component vorticity measurements were conducted using an eight-wire probe in the far region of a circular cylinder wake. The results show that the probe performs satisfactorily in terms of both internal consistencies, e.g. the good agreement (4%) among the statistics of velocity fluctuations obtained by the four X-wires, and external consistencies, e.g. the comparisons with data obtained by other researchers. After correction for spatial resolution of the probe, satisfactory agreement between the measured spectra of vorticity (and their components) and those calculated based on isotropy suggests that local isotropy is approximately satisfied near the centerline region of the wake ($x_2^+ \leq 1$). This result is consistent with the previous measurements by the two parallel wires and the six-wire probe. Skewness and flatness factors of vorticity and the components further suggest that local isotropy is approximately satisfied in this region.

The eight-wire probe also enables the measurements of the full energy dissipation rate and the full enstrophy. The results of the conditional expectations $\langle \Omega_r^{n\dagger} | \varepsilon_r^\dagger \rangle$ and $\langle \varepsilon_r^{n\dagger} | \Omega_r^\dagger \rangle$ show that there is a strong correlation between enstrophy and the full energy dissipation rate for large $(\varepsilon_{ful})_r^\dagger$, which is consistent with previous DNS studies. However, conditional expectations obtained using ε_{iso} show that there is nearly no correlation between enstrophy and ε_{iso} , indicating that the one-dimensional substitute cannot represent the energy dissipation rate properly, especially for instantaneous values. The studies on the scaling relation between the high-order moments of enstrophy $\langle \Omega_r^n \rangle$ and those of the energy dissipation rate $\langle \varepsilon_r^n \rangle$ show that the former is more intermittent than the later. Further investigation shows that for energy dissipation rate, the one-dimensional surrogate is more

intermittent than the full field. Since the full energy dissipation rate involves nine different velocity derivatives while ε_{iso} involves only one (i.e. $\partial u_1 / \partial x_1$), the present results may suggest that the averaging of the nine intermittent structures tends to reduce the degree of intermittence of individual structures. This finding in turn indicates that the use of the one-dimensional surrogate energy dissipation rate may lead to fundamental changes in statistics.

Over the scaling range where $\langle |\delta S_{L,1}^*|^3 \rangle$ varies linearly with r , the scaling exponents of the transverse velocity increments are significantly smaller than the longitudinal ones for all moment orders ($n=1-8$). The finding that the full enstrophy ($\Omega = \nu \omega_i \omega_i$) is more intermittent than the energy dissipation rate suggests that the strong intermittency of the transverse velocity increments may be associated with the intermittency of the enstrophy field. The present results show that while $\zeta_{L,1}(n) = \zeta_L(n)$ which verifies the RSH, $\zeta_{T,1}(n)$ is significantly smaller than $\zeta_T(n)$ for all n , i.e. the difference between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ cannot be compensated completely by that between $\zeta_L(n)$ and $\zeta_T(n)$, which are obtained using RSH and RSHT, respectively. This result suggests that the discrepancy between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ cannot be explained by RSHT proposed by Chen et al (1997). Therefore, RSHT is only partially valid for the temporal transverse velocity increments.

Chapter 7

Wavelet Multi-resolution Analysis of the Three-component Vorticity

7.1 Introduction

Vorticity is an important characteristic of turbulence that can provide fundamental information to identify multi-scale turbulent structures. The investigation of the connection between the three-component vorticity and the multi-scale vortical structures is therefore essential to understand turbulence phenomena. However, the investigation of turbulence structures experimentally remains a challenging task, mainly due to the difficulty to define or quantify the vortical structures. Hussain (1986) introduced and defined the conception of coherent structure as “a large-scale turbulent fluid mass with spatially phase-correlated vorticity”. In this definition, coherent structures are linked to a spatial correlation and vorticity. Later, Lesieur (1997) proposed that coherence is related not only to the spatial correlation but also to persistence in time of this correlation for a time significantly larger than a local time scale. Several techniques have been developed to investigate the large-scale structure in the turbulent wake, such as vorticity method (Hussain and Hayakawa, 1987), ensemble-averaged measurements (Kolar et al, 1997), window average gradient method (WAG) (e.g. Bisset et al.,1990; Zhang et al., 2000) and phase-averaging method (e.g. Zhou et al., 2002). However, the above mentioned methods cannot provide information on the contributions from the various scales of vortical structures to the turbulent energy, Reynolds stresses and vorticity variances. To extract features at various scales, wavelet technique has been widely used. Wavelet transformation has the advantage of quasi-locality in both physical-space and Fourier space which allows extracting the time evolution of the various scales in

the sampled signal. The identification of turbulence structures based on wavelet techniques has been studied by many researchers (e.g. Farge 1992; Addison, 2002). The continuous wavelet transform has been applied to analyzing turbulent structures of various scales in terms of time and scale which can extract continuously the characterization of local regularity (e.g. Li, 1998; Schram et al., 2001). However, continuous wavelet transform cannot reconstruct the original function from wavelet composition since no orthonormal mother wavelet function is available. The discrete wavelet transform is invertible and allows an orthonormal projection on a minimal number of independent modes. It can thus be used to produce a multi-resolution representation and to extract various scales in turbulence structures. The one-dimensional discrete wavelet transform was first applied by Meneveau (1991) to obtain local energy spectra and the flux of kinetic energy from experimental and direct numerical simulation data. Staszewski et al. (1997) identified the turbulent structure of atmospheric boundary-layer using the discrete wavelet transform. Later, Farge et al. (1999) proposed a coherent vortex simulation method to decompose turbulent flows into coherent and incoherent structures based on orthonormal wavelets. Li et al. (2002) developed a wavelet-based vector multi-resolution technique which was applied to a high-resolution stereoscopic particle image velocimetry system for studying the three-dimensional multi-scale structure features of the lobed jet mixing flow. Zhou and Li (2003) further investigate the effect of initial conditions on the self-preserving plane wakes generated by a triangular cylinder and a screen by using the wavelet multi-resolution technique. Using this technique, Rinoshika and Zhou (2005) observed for the first time that the spanwise vorticity contours of the wavelet component at the averaging frequency of Kármán vortices display a secondary spanwise structure near the saddle point in the near wake of a circular cylinder.

In the present study, the wavelet-based vector multi-resolution technique is used to decompose the measured three-component vorticity signals into a number of wavelet components based on their central frequencies, which are representative of the turbulent structures of different scales. The contributions from various turbulent structures to the Reynolds stresses and vorticity variances are calculated. Since the Reynolds number, Re ($Re = U_{\infty} d / \nu$), has a significant effect on the formation of vortices, the second objective of this chapter is to study the Re number dependence of the contributions from large- and small-scale structures to the three-dimensional vorticity in the far field of a circular cylinder wake.

In this chapter, only key concepts and equations of wavelet transform and multi-resolution analysis will be given. More rigorous mathematical treatments of these subjects can be found in Daubechies (1992), Strang and Nguyen (1996), Mertins (1999), Percival and Walden (2000), Addison (2002) and many other references.

7.2 Experimental Details

The three-dimensional vorticities were measured under three free stream velocities, i.e. 5m/s, 10m/s and 15m/s, corresponding to Re of 2000, 4000 and 6000, respectively. For 10m/s, the vorticity data used in the present study are the same as that used in Chapter 6 measured by an eight-wire probe. The same probe was employed for 5m/s and 15m/s measurements. The performance of this probe has been examined in Chapter 6. Details of the experimental setup are given in Section 6.2. The sampling frequency f_s for a specific flow condition is determined by the cut-off frequency f_c , i.e. $f_s = 2f_c$. Six cut-off frequencies are available for the filter of present CTA, i.e. 500Hz, 900Hz, 1600Hz, 2800Hz, 5200Hz and 9200Hz.

The cut-off frequency is determined by the flowing procedure:

- (i) Sample the signal using the highest sampling frequency of the CTA, i.e. 18400 Hz;
- (ii) Plot the FFT spectrum of the signal;
- (iii) Find the frequency at which the aliasing of high-frequency noise occurs, i.e. f_{noise} ;
- (iv) f_{noise} will fall in the range between two available cut-off frequencies of the filter, select the smaller one as the cut-off frequency.

If there is no aliasing of high-frequency noise in the FFT spectrum, the sampling frequency 18400Hz should be used. The sampling frequency for the present 5m/s, 10m/s and 15m/s measurements are 3200Hz, 10400Hz and 18400Hz, respectively. The sampling periods were all about 60s. For 15m/s measurements, the Kolmogorov frequency (f_K) is 10180 and the cut-off frequency is 9200Hz (with $f_s=18400$ Hz) which should be very close to f_K . Therefore, the sampling frequency of 18400Hz can resolve the Kolmogorov length scale.

7.3 Wavelet Multi-resolution Analysis

7.3.1 Basic Concepts of Frequency and Scale

Wavelet transforms enable to obtain orthonormal basis expansions of a signal using time-frequency atoms called wavelets that have good properties of localization in both time and frequency domains (e.g. Coifman et al. 1994; Kumar and Fofoula-Georgiou 1997). The basic idea can be understood using a time-frequency plane that indicates the frequency content of a signal at every time (Figure 7.1). In any such decomposition the time-frequency plane is layered with cells, called Heisenberg cells, whose minimum area is determined by the uncertainty principle.

Heisenberg's uncertainty principle dictates that one cannot measure with arbitrarily high resolution in both time and frequency. For the time series, i.e. linear time-invariant transform using Dirac delta functions, the process can be very well localized in the time domain but not at all in the frequency domain. This is schematically depicted by tall thin boxes in Fig 7.1a. In the case of Fourier bases the exact localization in frequency can be obtained but none in time, which is depicted by long horizontal cells in Fig 7.1b. If apply a moving window to the signal and take the Fourier transform at every location in an attempt to localize the presence of a feature, a short or windowed Fourier transform can be obtained. This partitions the entire time-frequency plane with rectangular cells of the same size and aspect ratio (Fig 7.1c). The wavelet transform is based on an octave band decomposition of the time-frequency plane (Fig 7.1d). In this scheme, higher frequencies can be well localized in time, but the uncertainty in frequency localization increases as the frequency increases, which is reflected as taller, thinner cells with increase in frequency. Consequently, the frequency axis is partitioned to finely only near lower frequencies. The implication of this is that the large scale features get well resolved in the frequency domain, but there is a large uncertainty associated with their location. On the other hand, the small-scale features, such as sharp discontinuities, get well resolved in the time (physical) domain, but there is a large uncertainty associated with their frequency content. This trade-off is an inherent limitation due to the Heisenberg's uncertainty principle. To further improve time-frequency analysis, wavelet packets will be used to decouple the scale parameter from the frequency parameter. The time-frequency domain will be layered using rectangular cells of arbitrary aspect ratios, with area greater than the minimum dictated by the uncertainty principle (e.g. Coifman et al. 1994).

Figure 7.1 Schematic of different domains and wavelet analysis.
(a) Time series; (b) Fourier transform; (c) Windowed Fourier transform; (d) Wavelet transform.

7.3.2 Continuous Wavelet Transform (CWT)

A continuous-time wavelet transform (CWT) of $f(t)$ is defined as:

$$CWT(\lambda, \tau) = W_f(\lambda, \tau) = |\lambda|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t) \overline{\psi\left(\frac{t-\tau}{\lambda}\right)} dt, \quad (7.1)$$

where λ and τ are dilating and translating coefficients, respectively. Note that $\lambda \neq 0$ and τ is real number. $\overline{\psi\left(\frac{t-\tau}{\lambda}\right)}$ is the complex conjugate of $\psi\left(\frac{t-\tau}{\lambda}\right)$. The multiplication of $|\lambda|^{-\frac{1}{2}}$ is for energy normalization purposes so that the transformed signal will have the same energy at every scale. The analysis function $\psi(t)$, the so-called mother wavelet, is scaled by λ . Changing the value of λ has the effect of dilating ($\lambda > 1$) or contracting ($\lambda < 1$) the function $\psi(t)$, and changing τ has the effect of analyzing the function $\psi(t)$ around different points τ . When the scale λ increases, the wavelet becomes more spread out and takes only long time behavior of $\psi(t)$ into account and vice versa. Therefore the wavelet transform provides a

flexible time-scale window that narrows when focusing on small-scale features and widens on large-scale features. It is important to note that $\psi(t)$ has the same shape for all values of λ . The inverse wavelet transform is given by

$$f(t) = \frac{1}{c_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_f(\lambda, \tau) \psi\left(\frac{t-\tau}{\lambda}\right) d\lambda d\tau, \quad (7.2)$$

where c_ψ is a constant, depending on the choice of wavelet.

The wavelet transform itself poses no restrictions on the form of the mother wavelet. This is an important difference to other transforms. The wavelet can then be constructed to fit the situation of different applications as long as the mother wavelet $\psi(t)$ satisfies two conditions, the admissibility condition and the regularity condition. The admissibility condition is

$$\int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{\omega} d\omega < \infty, \quad (7.3)$$

where $\Psi(\omega) = \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt$ is the Fourier spectrum of the wavelet. The admissibility condition indicates that the transformation is invertible and the wavelet function must have a value of zero at zero frequency. Therefore, the wavelet itself has an average value of zero. This means that the wavelet describes an oscillatory signal, where the positive and negative values cancel each other. This gives the ‘wave’ in wavelet. Eq. (7.3) suggests that the wavelet is a compact support function that has sufficiently fast decay to obtain localization in space. The regularity condition requires that the mother wavelet has to be locally smooth and concentrated in both the time and frequency domains. From this condition, the concept of vanishing moments is defined as

$$\int_{-\infty}^{\infty} t^m \psi(t) dt = 0, \quad m \in [0, M]. \quad (7.4)$$

Eq. (7.4) states that if the Fourier transform of the wavelet is M times differentiable the wavelet has M vanishing moments.

7.3.3 Discrete Wavelet Transform (DWT)

To apply the wavelet transform on sampled signals, the scale and location parameters need to be discretized. In discrete wavelet transforms, both redundant and non-redundant representations can be obtained using appropriate choices of wavelets and discretization schemes. The original signal can be completely reconstructed by a sample version of $W_f(\lambda, \tau)$ provided that the sampling (λ, τ) is sufficiently dense, i.e.

$$DWT(m, n) = W_f(m, n) = \lambda_0^{-\frac{m}{2}} \int_{-\infty}^{\infty} f(t) \psi(\lambda_0^{-m} t - n\tau_0) dt, \quad (7.5)$$

where the parameters λ and τ are discretized to $\lambda = \lambda_0^{-m}$ and $\tau = n\tau_0 \lambda_0^m$ with λ_0 and τ_0 being the sampling intervals and m and n are integers. The idea is that with a certain value of m , the magnification (λ_0^{-m}) is determined and the process at a particular location ($n\tau_0$) will be studied, and then the next location (keep m and change n). If the magnification is large (i.e. small scale) a small step (i.e. $\lambda_0^{-m} t$) will be moved and vice versa. The discrete wavelet is thus defined as

$$\psi_{m,n}(t) = \lambda_0^{-\frac{m}{2}} \psi(\lambda_0^{-m} t - n\tau_0). \quad (7.6)$$

The discrete wavelets that provide complete representation of the function $f(t)$ are called wavelet frames. To form a frame, $\psi_{m,n}(t)$ must satisfy the reconstruction conditions (Daubechies, 1992), viz.

$$A \|f\|^2 \leq \sum_m \sum_n |W_f(m, n)|^2 \leq B \|f\|^2, \quad (7.7)$$

where $\|f\|^2$ denotes the energy of $f(t)$, and $A > 0$ and $B < \infty$ are constants determined by the choices of λ_0 and τ_0 . A frame is not always orthonormal basis (only the condition $A = B = 1$ gives an orthonormal basis), resulting in a redundant representation of $f(t)$.

For turbulence signals whose frequency content changes with time, often the most desirable basis for decomposition of a signal is one that is orthogonal, local, and universal (Gamage and Blumen, 1993). In a special case when $W_f(\lambda, \tau)$ is sampled in dyadic grid, i.e., $\lambda_0 = 2$, $\tau_0 = 1$, and

$$\psi_{m,n}(t) = 2^{-\frac{m}{2}} \psi(2^{-m}t - n) = \frac{1}{\sqrt{2^m}} \psi\left(\frac{t - n2^m}{2^m}\right) \quad (7.8)$$

the set of functions $\{\psi_{m,n}\}$ for all m and n form an orthonormal basis. The most remarkable property of this basis is that the functions are orthonormal to their dilates and translates (Mallat, 1989). Due to the orthonormal properties, there is no redundancy information in the above discrete wavelets. By a linear combination of wavelets $\psi_{m,n}(t)$, $f(t)$ can be reconstructed via

$$f(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} D_{m,n} \psi_{m,n}(t), \quad (7.9)$$

where the first summation is over scales (m , from small to large) and the second summation is over all translates at each scale (n , time location). $D_{m,n}$ can be obtained by

$$D_{m,n} = \int f(t) \psi_{m,n}(t) dt, \quad (7.10)$$

which shows that $D_{m,n}$ measures the contribution of scale 2^m at location $n2^m$.

The two dimensional discrete vector wavelet transform is defined as

$$\overline{Dv}_{m,n_1,n_2} = \sum_i \sum_j \bar{v}(x_1^i, x_2^j) \psi_{m,n_1,n_2} \bar{v}(x_1^i, x_2^j) \quad (7.11)$$

where $\vec{v}(x_1, x_2)$ is the two-dimensional vector function and $\psi_{m,n_1,n_2}(x_1, x_2)$ is the wavelet basis. As in Eq. (7.6), m is the wavelet level (i.e. the scale), n_1 and n_2 are the locations. Here $\overline{Dv}_{m,n_1,n_2}$ is called discrete vector wavelet transform coefficients. The two-dimensional orthonormal wavelet basis, $\psi_{m,n_1,n_2}(x_1, x_2)$, is simply to take the tensor product functions generated by two-dimensional orthonormal bases, viz.

$$\psi_{m,n_1,n_2}(x_1, x_2) = 2^{-m} \psi(2^{-m}x_1 - n_1) \psi(2^{-m}x_2 - n_2). \quad (7.12)$$

In the basis, the two variables x_1 and x_2 are dilated separately. The discrete vector wavelet transform has inverse transform since it is an orthonormal transform. The original vector function can be reconstructed from the inversion of $\overline{Dv}_{m,n_1,n_2}$, viz.

$$\vec{v}(x_1, x_2) = \sum_m \sum_{n_1} \sum_{n_2} \overline{Dv}_{m,n_1,n_2} \psi_{m,n_1,n_2}(x_1, x_2) \quad (7.13)$$

As a linear transform, the wavelet transform of a vector function is a vector whose elements are the wavelet transforms of the vector components sampled with a scale of 2^m . The high scale level of m indicates high frequency corresponding to the small scales. In general, the fine-scale discrete vector wavelets can capture the local features of the original vector functions and provide a parsimonious representation of the vector function

7.3.4 Multi-resolution Analysis

Decomposition of a time series into various resolutions through an iteration process is termed as multi-resolution. Multi-resolution provides an efficient mechanism for going from one resolution to another. Just as fast Fourier transform algorithms made the Fourier transform a practical tool for spectral analysis, the multi-resolution analysis has made the discrete wavelet transform a viable tool for computational time-scale analysis.

Mallat (1989) showed that a time series $f(t)$ can be completely decomposed in terms of “approximations” and “details” relative to different time scales, which are provided by the scaling functions and wavelets, respectively. In Eq. (7.8), the orthonormal dyadic discrete wavelets are associated with the scaling functions and their dilation equations. The scaling function is associated with the smoothing of the signal and has the same form as the wavelet, given by

$$\phi_{m,n}(t) = 2^{-\frac{m}{2}} \psi(2^{-m}t - n), \quad (7.14)$$

where $\phi(t)$ is the father scaling function (cf. mother wavelet) and

$$\int_{-\infty}^{\infty} \phi(t) dt = 1. \quad (7.15)$$

The scaling function is orthonormal to translation of itself, but not to dilations of itself. The scaling function can be convolved with the signal to produce approximation coefficients as

$$W_s(m, n) = \int_{-\infty}^{\infty} f(t) \phi_{m,n}(t) dt \quad (7.16)$$

The approximation coefficients at a special scale m are known as discrete approximation of the signal at that scale. A continuous approximation of the signal at scale m can be obtained viz.

$$f_m(t) = \sum_{n=-\infty}^{\infty} W_s(m, n) \phi_{m,n}(t), \quad (7.17)$$

where $f_m(t)$ is a smooth, scaling-function-dependent version of the signal $f(t)$ at the scale m . The scaling equation describes the scaling function $\phi(t)$ in terms of contracted and shifted version of itself as

$$\phi(t) = \sum_k c_k \phi(2t - k), \quad (7.18)$$

where the scaling coefficients must satisfy

$$\sum_k c_k = 2. \quad (7.19)$$

In order to create an orthonormal system, the following condition must be satisfied:

$$\sum_k c_k c_{k+2k'} = \begin{cases} 2 & \text{if } k' = 0 \\ 0 & \text{otherwise} \end{cases} \quad (7.20)$$

The signal $f(t)$ can be represented by the combination of series expansion of both the approximation coefficients and the wavelet (detail) coefficients, viz.

$$f(t) = \sum_{n=-\infty}^{\infty} W_s(m_0, n) \phi_{m_0, n}(t) + \sum_{m=-\infty}^{m_0} \sum_{n=-\infty}^{\infty} W_f(m, n) \psi_{m, n}(t). \quad (7.21)$$

Eq. (7.21) shows that the original signal is expressed as the combination of an approximation of itself at arbitrary scale index m_0 added to a succession of signal details from scales m_0 . The multi-resolution analysis therefore decomposes a function into a low resolution (coarse) part and a sequence of correction (detail) terms at increasing resolutions.

7.3.5 Daubechies Wavelets

The oldest and simplest orthonormal wavelet is the Haar wavelet which is constructed as

$$\psi(t) = \begin{cases} 1 & 0 \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \quad (7.22)$$

The disadvantage of the Haar wavelet is its discontinuity. This limits its ability to approximate continuous functions. For practical applications, it will be advantageous to use smoother basis. In the past decades, various orthonormal wavelet bases have been constructed (e.g. Daubechies, 1988; Meyer 1992 and Chui 1992a,b). The

orthonormal wavelets invented by Daubechies (1988) are the only continuous wavelets with full scaling and translational orthogonality of scaling functions. The remarkable property is that Daubechies wavelets have a compact support, i.e. the basis functions are non-zero only over a finite interval (in contrast, the sinusoidal basis functions of the Fourier expansion are infinite in extent). This allows the wavelet transformation to efficiently represent functions (or signals) which have localized features, such as the coherent structures in turbulence. The structured component of the signal is well represented by a relative few of the wavelet basis functions, whereas the unstructured component (e.g. noise) on the signal projects almost equally onto the entire basis functions. The structured and unstructured parts of the signal are then separated in the wavelet transform domain. In addition, Daubechies requires the wavelets are smooth to certain degree, viz.

$$\sum_{k=0}^{N_k-1} (-1)^k c_k k^m = 0, \quad (7.23)$$

where integers $m=0,1,2, \dots, N_k/2-1$. Eq. (7.23) indicates that Daubechies wavelets have $N_k/2$ vanishing moments. Figure 7.2 shows the Daubechies wavelet which has 20 scaling coefficients, the DB20. In MATLAB®, the “DB” represents a wavelet from the Daubechies family and “20” means the number of non zero scaling coefficients. As can be seen, the wavelet function (Figure 7.2b) is a fairly smooth basis with good localization properties in the Fourier domain. Out of a finite interval, both the scaling and wavelet functions have zero values.

Figure 7.2 Daubechies wavelet: DB20. (a) scaling function; (b) wavelet function.

7.3.6 Vector Wavelet Multi-resolution Analysis

Given that the discrete wavelet transform is an orthonormal and linear transform, from Eq. (7.13), a non-linear vector function can be represented by the summation of vector wavelet components at different central frequencies (from low to high), viz.

$$\vec{v}(x_1, x_2) = \vec{Mv}_1 + \vec{Mv}_2 + \cdots \vec{Mv}_k \quad (7.24)$$

where

$$\vec{Mv}_i = \sum_{n_1} \sum_{n_2} \overline{Df}_{i;n_1,n_2} \psi_{i;n_1,n_2} \vec{v}(x_1, x_2). \quad (7.25)$$

In Eq. (7.24), $\vec{Mv}_1$ and $\vec{Mv}_k$ are the wavelet components at level l (the lowest frequency, coarse information) and level k (the highest frequency, detailed information), respectively. The method described in Eqs. (7.24) and (7.25) are called vector wavelet multi-resolution analysis which is an orthonormal decomposition

processing. The original vector function can be reconstructed by summation of all the frequency components in multi-resolution analysis.

In the present study, the instantaneous velocity component $U_i(x_1, x_2, x_3, t)$ can be written as the sum of a time-averaged component $\bar{U}_i$ and a fluctuation component u_i , viz.

$$U_i = \bar{U}_i + u_i. \quad (7.26)$$

To extract various scales in turbulence structures, the multi-resolution analysis is used to decompose the velocity fluctuation components u_i into a number of orthonormal wavelet components based on wavelet levels, which correspond to the central frequencies and are directly linked to turbulent structure scales (Li et al., 2002; Zhou and Li, 2003). The multi-resolution analysis may process fewer data by selecting relevant details that are essential to perform the extraction of multi-scale flow structures, and decompose the velocity fluctuations both in Fourier and physical spaces. Each wavelet component represents the turbulent structures of a certain range of frequencies (i.e. a non-zero frequency band) so that the information of any scales contained in the original data will not be lost due to a limited number of wavelets level. From Eq. (7.24), the u_i can be written as

$$u_i = \sum_{k=1}^N u_{k,i}, \quad (7.27)$$

where N is the total number of wavelet levels, and $u_{k,i}$ is the wavelet component of u_i at k^{th} wavelet level. Eq. (7.26) can be rewritten as

$$U_i = \bar{U}_i + \sum_{k=1}^N u_{k,i}. \quad (7.28)$$

The wavelet components of the instantaneous vorticity components $\omega_i(x_1, x_2, x_3, t)$ can be written as

$$(\omega_1)_i = (u_{3,2})_i - (u_{2,3})_i \approx \frac{(\Delta u_3)_i}{\Delta x_2} - \frac{(\Delta u_2)_i}{\Delta x_3}, \quad (7.29)$$

$$(\omega_2)_i = (u_{1,3})_i - (u_{3,1})_i \approx \frac{(\Delta u_1)_i}{\Delta x_3} - \frac{(\Delta u_3)_i}{\Delta x_1} \quad (7.30)$$

$$\text{and} \quad (\omega_3)_i = (u_{2,1})_i - (u_{1,2})_i \approx \frac{(\Delta u_2)_i}{\Delta x_1} - \frac{(\Delta u_1)_i}{\Delta x_2}. \quad (7.31)$$

In the present study, the Daubechies family with an order of 20 (i.e. DB20, Figure 7.2) is selected as the wavelet basis. To carry out vector multi-resolution analysis, wavelet coefficients are calculated based on the discrete wavelet transform (Eq. 7.11), the multi-resolution method described in Section 7.3.4 together with the requirement of Daubechies (Eq. 7.23). Then the inverse vector discrete wavelet transform is applied to the wavelet coefficient of each wavelet level and the wavelet components of vector data are constructed (Eqs. 7.13 and 7.24). The turbulence structures are studied by the wavelet decompositions (velocity and vorticity) described in this section.

7.4 Spectra of the Measured Velocity Signals

Figures 7.3 (a-c) shows the spectra ϕ_{u_1} and ϕ_{u_2} on the wake centerline ($x_2^+ = 0$) at $x_1/d = 240$ for the free stream velocities of 5m/s, 10m/s and 15m/s, respectively. In the three figures, there are three pronounced peaks in the distributions of ϕ_{u_2} , at frequencies of 60Hz, 105Hz and 180Hz, respectively. The peaks become more apparent as Re number increases. These frequencies correspond to a Strouhal number St ($\equiv fd/U_\infty$) of about 0.7, which indicates the shedding of the organized vortical structures. These corresponding shedding frequencies will therefore be used as the

basic frequencies f_0 in the wavelet analysis to represent the large-scale vortical structures. While there is a pronounced peak in the distributions of ϕ_{u_1} in Figure 7.3a, there is no such peak in the distributions of ϕ_{u_1} in Figures 7.3 (b, c). In Fig 7.3a, for $Re=2000$, the frequency (20Hz) at which the peak occurs in ϕ_{u_1} is much smaller than the shedding frequency (60Hz), indicating the existence of organized longitudinal vortical structures with lower frequency but larger size than those at the shedding frequency. It should be noted that, despite the existence of organized longitudinal vortical structures, there is no pronounced peak in the distributions of ϕ_{u_1} for $Re=4000$ and 6000. This may be due to the generation of small-scale structures at $Re=4000$.

Figure 7.3 Comparison of measured spectra of u_1 and u_2 on the wake centerline at different Re .
---, ϕ_{u_1} ; —, ϕ_{u_2} . (a) $Re=2000$; (b) $Re=4000$; (c) $Re=6000$;

7.5 Contributions to Reynolds Shear Stress, Velocity and Vorticity Variances from Different Wavelet Components at Re of 4000

In this section, results and discussion will be focused on the measurements at the velocity of 10m/s (corresponding to Re of 4000) to study the contributions to Reynolds shear stress, velocity and vorticity variances from different wavelet components. The Reynolds number effect on vorticity will be investigated in Section 7.6.

In order to examine the turbulent structures of various scales, the wavelet multi-resolution technique is used to decompose the velocity fluctuations $u_i(x_1, x_2, x_3, t)$ into a number of orthonormal wavelet components based on wavelet levels, which correspond to the central frequencies $f_0/4, f_0/2, f_0, 2f_0, 4f_0, 8f_0, 16f_0$ and $32f_0$, and are directly linked to the scales of the turbulent structures. Turbulence structures with frequency smaller and equal to f_0 represent large-scale structures while those at multiple of f_0 represent the intermediate and smaller scale structures.

7.5.1 Velocity Variances and Reynolds Shear Stress

Figures 7.4 (a-d) show the distributions of time-averaged velocity variances and Reynolds shear stress at different transverse locations, obtained from the vector wavelet multi-resolution analysis for different wavelet levels or central frequencies. The values of $\langle \beta\gamma \rangle_i$ are normalized by the maximum value of $\langle \beta\gamma \rangle_{\max}$, where β or γ may each represent u_1 or u_2 and $\langle \beta\gamma \rangle_i$ represents the wavelet component

at i^{th} level, so that the contribution from different central frequencies can be evaluated. The measured values of $\langle \beta \gamma \rangle / \langle \beta \gamma \rangle_{\max}$ are also included in the figures. The negative values of $\langle \beta \gamma \rangle / \langle \beta \gamma \rangle_{\max}$ for higher central frequencies have been removed from the figures when plotting the values in the log-scale. From Figures 7.4 (a-d), it can be seen that the values of $\langle \beta \gamma \rangle_i / \langle \beta \gamma \rangle_{\max}$ decrease as approaching the wake edge for all turbulent structures with different wavelet scales. At the central frequency f_0 , the maximum values of $\langle u_j^2 \rangle_{f_0} / \langle u_j^2 \rangle_{\max}$ (where j represents any one of the three velocity components) are 20.9%, 25.8% and 22.9% for $j=1, 2$ and 3 , respectively, and 30.5% for $\langle u_1 u_2 \rangle_{f_0} / \langle u_1 u_2 \rangle_{\max}$. These results are consistent with those reported by Zhou and Li (2003) that the maximum coherent contribution from the large-scale vortical structures with the central frequency f_0 is between 20% and 30% in the far wake behind a triangular cylinder ($x_1/d = 220$). It is also observed that the values of $\langle \beta \gamma \rangle_i / \langle \beta \gamma \rangle_{\max}$ tend to be largest at f_0 for all locations of x_2^+ and decreases as the frequency increases, indicating that lower frequency eddies contain more energy than the high frequency ones. It should be noted in Figures 7.4 (b) and (c) that, the decreases of $\langle u_2^2 \rangle$ and $\langle u_3^2 \rangle$ are rather small when the central frequency increases from f_0 to $2f_0$, and the wavelet components corresponding to these two frequencies contribute most to $\langle u_2^2 \rangle$ and $\langle u_3^2 \rangle$. This result indicates that there is a significant contribution from the large-scale structures to the total energy distributions of time-averaged transverse and spanwise velocity variance. In Figures 7.4 (a-d), it is also observed that when the central frequency is greater than $4f_0$, the values of $\langle \beta \gamma \rangle_i / \langle \beta \gamma \rangle_{\max}$ drop quickly with the increase of the central frequency.

Figures 7.5 shows the total contributions from the wavelet components of $f_0/4$,

$f_0/2, f_0, 2f_0, 4f_0, 8f_0, 16f_0$ and $32f_0$. At around half-width (where $x_2^+ \approx 1$), the total contributions from these components account for about 82.2% for $\langle u_1^2 \rangle$, 85.1% for $\langle u_2^2 \rangle$, 90.0% for $\langle u_3^2 \rangle$ and 87.7% for $\langle u_1 u_2 \rangle$, indicating that the wavelet components can reasonably represent the flow in term of $\langle \beta \gamma \rangle$ in this region. However, as the centerline is approached ($x_2^+ < 0.8$), while the total contributions to $\langle u_2^2 \rangle$ and $\langle u_3^2 \rangle$ are still representative (account for above 89%), the contributions to $\langle u_1^2 \rangle$ and $\langle u_1 u_2 \rangle$ are not so (below 80%) and decline quite sharply as the centerline is approached. The deficits of the wavelet contributions to $\langle u_1^2 \rangle$ and $\langle u_1 u_2 \rangle$ may be mainly due to neglecting the central frequencies lower than $f_0/4$ since energy contained by the structures with frequencies higher than $16f_0$ are negligible as indicated in Figures 7.4 (a) and (d). Also, in these two figures, it can be seen that the wavelet components corresponding to f_0 and $f_0/2$ contribute most to $\langle u_1^2 \rangle$ and $\langle u_1 u_2 \rangle$. The results suggest that, compared with $\langle u_2^2 \rangle$ and $\langle u_3^2 \rangle$, $\langle u_1^2 \rangle$ and $\langle u_1 u_2 \rangle$ are more influenced by the large-scale turbulent structures. This may imply the existence of organized longitudinal vortical structures as the flow evolves downstream.

Figure 7.4 Velocity variances and Reynolds shear stress of the measured and the wavelet components at various central frequencies. (a) $\langle u_1^2 \rangle / \langle u_1^2 \rangle_{\max}$ and $\langle u_1^2 \rangle_i / \langle u_1^2 \rangle_{\max}$; (b) $\langle u_2^2 \rangle / \langle u_2^2 \rangle_{\max}$ and $\langle u_2^2 \rangle_i / \langle u_2^2 \rangle_{\max}$; (c) $\langle u_3^2 \rangle / \langle u_3^2 \rangle_{\max}$ and $\langle u_3^2 \rangle_i / \langle u_3^2 \rangle_{\max}$; (d) $\langle u_1 u_2 \rangle / \langle u_1 u_2 \rangle_{\max}$ and $\langle u_1 u_2 \rangle_i / \langle u_1 u_2 \rangle_{\max}$.
 ●, measured; □, $f_0/4$; ▽, $f_0/2$; ○, f_0 ; +, $2f_0$; △, $4f_0$; ×, $8f_0$; ⊕, $16f_0$; ■, $32f_0$.

Figure 7.5 The total contributions to velocity variances and Reynolds shear stress from the wavelet components of $f_0/4, f_0/2, f_0, 2f_0, 4f_0, 8f_0, 16f_0$ and $32f_0$. o, $\langle u_1^2 \rangle_i / \langle u_1^2 \rangle_{\max}$; □, $\langle u_2^2 \rangle_i / \langle u_2^2 \rangle_{\max}$; ▽, $\langle u_3^2 \rangle_i / \langle u_3^2 \rangle_{\max}$; •, $\langle u_1 u_2 \rangle_i / \langle u_1 u_2 \rangle_{\max}$.

7.5.2 Vorticity Variances

The distributions of the three vorticity variances, obtained from the vector wavelet multi-resolution analysis at different wavelet levels or central frequencies are shown in Figures 7.6 (a-c). The values of $\langle \omega_j^2 \rangle_i$ are normalized by the maximum value of $\langle \omega_j^2 \rangle_{\max}$ to indicate the contribution from different central frequencies, where j ($\equiv 1, 2$ or 3) represents any component of the vorticity vector and $\langle \omega_j^2 \rangle_i$ represent the wavelet components at i^{th} level. The measured values of $\langle \omega_j^2 \rangle$ are also included in the figure for comparison. It can be seen that in each figure, both $\langle \omega_j^2 \rangle_i / \langle \omega_j^2 \rangle_{\max}$ and $\langle \omega_j^2 \rangle_i / \langle \omega_j^2 \rangle_{\max}$ drop towards the free-stream. It needs to be noted that the maximum

value of $\langle \omega_1^2 \rangle_i / \langle \omega_1^2 \rangle_{\max}$ corresponding to f_0 (Figure 7.6 a) accounts for about 13% of the maximum longitudinal vorticity variance for $x_2^+ < 1$. This value is larger than that for $\langle \omega_2^2 \rangle_i / \langle \omega_2^2 \rangle_{\max}$ (7.4%, Figure 7.6b) and $\langle \omega_3^2 \rangle_i / \langle \omega_3^2 \rangle_{\max}$ (8.9%, Figure 7.6c), indicating a larger contribution to the longitudinal total vorticity variance from the large-scale structures than both to the transverse and to the spanwise vorticity components. This result is consistent with the spectra of the three vorticity components on the wake centerline, as shown in Figure 7.7. The several peaks over the range of 40-220Hz in ϕ_{ω_1} indicating that the spectrum of ω_1 contains more energy than the other two vorticity components for low to moderate frequencies (e.g. $f \leq 300$ Hz). The minor peak occurs around 40Hz in may imply the vision of organized longitudinal vortical structures with lower frequency but larger size than those at the shedding frequency. On the other side, there is no such pronounced peaks occur for ϕ_{ω_2} and ϕ_{ω_3} at frequency lower than f_0 , implying that the large-scale structures contribute more to longitudinal vorticity variance than to either transverse or spanwise vorticity variances. The present result is different from the observation in the near wake behind a circular cylinder ($x_1/d = 10$, Zhou et al. 2004), where a relatively large contribution to the total spanwise vorticity variance from the large-scale structures is found. It seems that as the vortical structures evolve downstream, the spanwise and rib-like structures (Figure 1.3, Chapter 1) show different contributions to the three vorticity components. Compared with Figures 7.4 (a-c), Figures 7.6 (a-c) show that, the turbulent structures with higher central frequencies tend to contribute more to vorticity variance than those of lower ones, since $\langle \omega_1^2 \rangle_i / \langle \omega_1^2 \rangle_{\max}$ is largest at $4f_0$, while the contributions to $\langle \omega_2^2 \rangle_i / \langle \omega_2^2 \rangle_{\max}$ and $\langle \omega_3^2 \rangle_i / \langle \omega_3^2 \rangle_{\max}$ are dominated by structures with central frequency of $8f_0$. In

Figures 7.6 (b) and (c), it can be seen that within the halfwidth, contributions at $2f_0$, $4f_0$, $8f_0$ and $16f_0$ are all higher than that corresponding to f_0 for all three vorticity components. Figure 7.8 shows the total contributions from the intermediate- and relatively small-scale structures with central frequencies of $2f_0$, $4f_0$, $8f_0$, $16f_0$ and $32f_0$. It shows that on the wake centerline, the total contribution from the included subsets ($2 \sim 32f_0$) takes about 84.6% of $\langle \omega_2^2 \rangle_{\max}$ and $\langle \omega_3^2 \rangle_{\max}$, and 72.6% for $\langle \omega_1^2 \rangle_{\max}$, indicating that more transverse and spanwise vorticity resides in the intermediate and small-scale structures than the longitudinal vorticity. In addition, this result is consistent with the previous perception (Antonia et al., 2002) that, compared with the large-scale turbulent structures (with central frequencies lower than $2f_0$), the intermediate and relatively small-scale structures contribute more to vorticity.

Figure 7.6 Vorticity variances of the measured and the wavelet components at various central frequencies. (a) $\langle \omega_1^2 \rangle / \langle \omega_1^2 \rangle_{\max}$ and $\langle \omega_1^2 \rangle_i / \langle \omega_1^2 \rangle_{\max}$; (b) $\langle \omega_2^2 \rangle / \langle \omega_2^2 \rangle_{\max}$ and $\langle \omega_2^2 \rangle_i / \langle \omega_2^2 \rangle_{\max}$; (c) $\langle \omega_3^2 \rangle / \langle \omega_3^2 \rangle_{\max}$ and $\langle \omega_3^2 \rangle_i / \langle \omega_3^2 \rangle_{\max}$. $\bullet$, measured; $\square$, $f_0/4$; ∇ , $f_0/2$; $\circ$, f_0 ; $+$, $2f_0$; Δ , $4f_0$; $\times$, $8f_0$; $\oplus$, $16f_0$; $\blacksquare$, $32f_0$.

Figure 7.7 Comparison of measured spectra of ω_1 , ω_2 and ω_3 on the wake centerline at $Re=4000$.
---, ω_1 ; -.-, ω_2 ; .-. , ω_3 .

Figure 7.8 The total contributions to vorticity variance from the wavelet components of $2f_0$, $4f_0$, $8f_0$, $16f_0$ and $32f_0$. o, $\langle \omega_1^2 \rangle / \langle \omega_1^2 \rangle_{\max}$; $\square$, $\langle \omega_2^2 \rangle / \langle \omega_2^2 \rangle_{\max}$; ∇ , $\langle \omega_3^2 \rangle / \langle \omega_3^2 \rangle_{\max}$.

7.6 Reynolds Number Effect on the Contributions to Vorticity

Variances from Different Wavelet Components

Before presenting the wavelet analysis of Reynolds number effect on vorticity, Figures 7.9 (a-c) show rms values of the three vorticity components across the wake at different Reynolds numbers $Re=2000$, 4000 and 6000 , respectively. Clearly, the rms values of the three vorticity components increase with Reynolds number. On the wake centerline, the increase ratios of ω_1^+ are 7.8% and 1.3% for Re changing from 2000 to 4000 and 4000 to 6000. The corresponding increase ratios are 16.8% and 1.5% for ω_2^+ , and 13.6% and 3.7% for ω_3^+ . The increase ratios of the rms values of the three vorticity components indicate that vorticity may be generated in a suddenly “jump” way for Re changing from 2000 to 4000. On the other hand, the generation ratio declines when Re is changed from 4000 to 6000. The Reynolds number effect on vorticity generation declines significantly when Re is changed from 4000 to 6000. This trend is consistent with that reported by Yiu et al. (2004) in the turbulent near wakes and may indicate the “memory” effect of turbulence in the far wake.

Figures 7.10-7.15 show the vorticity variances of the wavelet components at the central frequencies $f_0/2$, f_0 , $2f_0$, $4f_0$, $8f_0$ and $16f_0$. The structures with frequencies lower than $f_0/2$ and higher than $16f_0$ are not calculated since, as shown in Figures 7.6 (a-c), the structures with central frequencies of $f_0/4$ and $32f_0$ contribute least to the total energy with a percentage being smaller than 3.

Figures 7.10 and 7.11 show the vorticity variances from the wavelet components at the central frequencies of $f_0/2$ and f_0 , which represent large-scale structures. Similar variations with Re are found in the figures for individual vorticity component. In Figures 7.10 ($f_0/2$), when $x_2^+ < 1.5$, the contributions decrease with the increase of Re . For example, $\langle \omega_1^2 \rangle_i / \langle \omega_1^2 \rangle_{\max}$, $\langle \omega_2^2 \rangle_i / \langle \omega_2^2 \rangle_{\max}$ and $\langle \omega_3^2 \rangle_i / \langle \omega_3^2 \rangle_{\max}$ decrease by about 3.4%, 2.4%, and 1.5% when Re is changed from

2000 to 4000. The corresponding decreases of the three vorticity components at f_0 are 4.5%, 3.9% and 2.8%, respectively. The differences of the contributions between $Re=4000$ and 6000 are 2.1%, 0.3% and 0.3% at $f_0/2$ for the three vorticity components, and 2.9%, 0.1% and 0.5% at f_0 . It may be concluded that there is a weak, negative Reynolds number dependence for the contributions from large-scale turbulent structures, i.e. the contributions from the large-scale structures to the total vorticity variances decrease with the increase in Re .

The contributions to vorticity variances with central frequencies of $2f_0$ and $4f_0$, which mainly represent the intermediate scale turbulent structures, are shown in Figures 7.12 and 7.13. It can be seen that, for $x_2^+ < 1.5$, the contributions to $\langle \omega_1^2 \rangle$, $\langle \omega_2^2 \rangle$ and $\langle \omega_3^2 \rangle$ at $2f_0$ decrease about 5.4%, 8.3% and 4.4%, respectively, when Re increases from 2000 to 4000. At frequency $4f_0$, the decrease of contributions is about 7.7% for $\langle \omega_2^2 \rangle$ when Re increases from 2000 to 4000. In contrast to $\langle \omega_2^2 \rangle$, the decreases of $\langle \omega_1^2 \rangle$ and $\langle \omega_3^2 \rangle$ are much smaller, which are -1.6% and 0.9%, respectively. The negative or small decreases of the contributions to the longitudinal and spanwise vorticity variances may imply the appearance of the quasi-two-dimensional longitudinal and spanwise vortex structures as Re increases. This can also be inferred from frequency $2f_0$, at which the decreases of the contributions to longitudinal (5.4%) and spanwise vorticity (4.4%) are both smaller than that to transverse vorticity (8.3%). Zhou and Li (2003), both in triangular cylinder and screen far wakes, observed the occurrence of longitudinal rib-like structures in the saddle regions which occur between successive spanwise structures. As a matter of fact, the generation of the intermediate scale structures is essential to forming the inertial scale subrange of the high Reynolds number flows. The

decreases of the contributions at frequency $4f_0$ from $Re=4000$ to 6000 are 1.9%, 1.0% and 1.9%, respectively, to the three-component vorticity variances.

The contributions to vorticity variances with frequencies of $8f_0$ and $16f_0$, which mainly represent the small-scale structures, are shown in Figures 7.14 and 7.15. For $x_2^+ < 2$, significant increases of contributions to the three vorticity components are observed when Re is changed from 2000 to 4000, indicating a strong positive Re dependence over the present Re range (2000-4000). In Figures 7.14 ($8f_0$), the increases of the contributions to $\langle \omega_1^2 \rangle$, $\langle \omega_2^2 \rangle$ and $\langle \omega_3^2 \rangle$ are 10.1%, 10.1% and 13.5%, respectively. The corresponding increases are more remarkable at $16f_0$, which are 4.9%, 16.3% and 15.0%. With the caveat of the experimental uncertainty associated with vorticity measurements, the differences are still appreciable. The sudden “jump” of the contributions to vorticity variances from the turbulent structures with the central frequencies of $8f_0$ (Fig. 14) and $16f_0$ (Fig. 15) when Re is increased from 2000 to 4000 is probably due to the generation of small-scale structures at $Re \approx 5000$ in the near field (Yiu et al. 2004), which persistent in the far field. This result indicates that the vorticity field in the wake depends not only on the initial conditions (e.g. Antonia et al. 1996), but also on the Reynolds numbers, which reveals the “memory” effects to the Reynolds number change in the near field. The results shown in Figures 14 and 15 confirm the observations from Figures 7.9 (a-c). Furthermore, it can be concluded that the Re effect on vorticity variances is essentially due to relatively small-scale structures.

The present study further reinforces the concept that turbulence ‘remembers’ its origin. On the other hand, it is well known that the flow structure behind a circular cylinder is very sensitive to the end conditions arising from the boundary layers. A problem may be asked: should the present study examine the effect of the boundary

layer on the vorticity measurements? Following are the discussions:

The conditions of the flow at the ends of a slender object (wings, cylinder etc) may affect the flow over the entire span of that object. End disturbance in the spanwise direction could trigger oblique shedding and totally violate the two-dimensional character of the flow (Hammache and Gharib, 1991). Stager and Eckelmann (1991) found that the size of the affected region decreases with the increase of Re , e.g., it is about $10D$ at $Re=100$ and about $5D$ at $Re=300-5000$, where D is the diameter of the cylinder. Szepessy and Bearman (1992), further studied the influence of the length-to-diameter ratio (L/D) of the cylinder on vortex formation and their shedding from the bluff body at different Reynolds numbers in the range of at $Re=8 \times 10^3 - 1.5 \times 10^5$, where L is the length of the cylinder. They found that, when $L/D > 6.7$, the end effects will lose their influence on the flow field.

In the present study, the vorticity measurements were conducted near the centerline of the wake generated by a circular cylinder with a diameter of 6.35mm and length of 0.8m (which is the same as the height of the test section) at $Re=2000-6000$. The length-to-diameter ratio (L/D) is around 126, which is much larger than the L/D of 6.7. In addition, the vorticity measurements were conducted at the midspan of the test section which is away from the wall of the test section with a distance about 0.6m. This distance is much larger than the size of the possible affected region of $5D$ (about 32mm in the present study). Therefore, according to the findings of Stager and Eckelmann (1991) and those of Szepessy and Bearman (1992), the end effect on the vorticity measurements is negligible in the present study. Therefore, the present study did not examine the effect of the boundary layer.

Figure 7.10 Vorticity variances of the wavelet components at the central frequency $f_0/2$.

(a) ω_1 ; (b) ω_2 ; (c) ω_3 .

○, $Re=2000$; □, $Re=4000$; ▽, $Re=6000$.

Figure 7.11 Vorticity variances of the wavelet components at the central frequency f_0 .

(a) ω_1 ; (b) ω_2 ; (c) ω_3 .

○, $Re=2000$; □, $Re=4000$; ▽, $Re=6000$.

Figure 7.12 Vorticity variances of the wavelet components at the central frequency $2f_0$.

(a) ω_1 ; (b) ω_2 ; (c) ω_3 .
 $\circ$, $Re=2000$; $\square$, $Re=4000$; ∇ , $Re=6000$.

Figure 7.13 Vorticity variances of the wavelet components at the central frequency $4f_0$.

(a) ω_1 ; (b) ω_2 ; (c) ω_3 .
 $\circ$, $Re=2000$; $\square$, $Re=4000$; ∇ , $Re=6000$.

Figure 7.14 Vorticity variances of the wavelet components at the central frequency $8f_0$.

(a) ω_1 ; (b) ω_2 ; (c) ω_3 .
 $\bigcirc$, $Re=2000$; $\square$, $Re=4000$; ∇ , $Re=6000$.

Figure 7.15 Vorticity variances of the wavelet components at the central frequency $16f_0$.

(a) ω_1 ; (b) ω_2 ; (c) ω_3 .
 $\bigcirc$, $Re=2000$; $\square$, $Re=4000$; ∇ , $Re=6000$.

7.6 Concluding Remarks

All three components of the vorticity vector in the far region of a circular cylinder wake ($x_1/d = 240$) were measured simultaneously by employing an eight hot-wire vorticity probe. Using the wavelet multi-resolution technique, contributions to Reynolds shear stress, velocity and vorticity variances from different wavelet components are evaluated. The maximum coherent contribution to velocity variances and Reynolds shear stress from the large-scale vortical structures with the central frequency of f_0 is between 20% and 30%, which is consistent with those reported by Zhou and Li (2003).

The present results at $Re = 2000$ show that the large-scale structures (with central frequency f_0) contribute appreciably (13%) to the longitudinal vorticity variance for $x_2^+ < 1$. In contrast, they contribute only about 7.4% to $\langle \omega_2^2 \rangle / \langle \omega_2^2 \rangle_{\max}$ and 8.9% to $\langle \omega_3^2 \rangle / \langle \omega_3^2 \rangle_{\max}$. This result indicates a relatively large contribution to the longitudinal vorticity variance from the large-scale structures than both to the transverse and to the spanwise vorticity components. The dominant contributions to the vorticity variances are from the intermediate and small-scale structures with central frequencies of $2f_0$, $4f_0$, $8f_0$ and $16f_0$, which takes about 84.6% for $\langle \omega_2^2 \rangle_{\max}$ and $\langle \omega_3^2 \rangle_{\max}$, and 72.6% for $\langle \omega_1^2 \rangle_{\max}$, indicating that vorticity resides mainly in the small-scale structures. The contributions from the large and intermediate scale structures to the vorticity variances decrease as Re increases from 2000 to 4000, while the changes of the wavelet contributions are not apparent when Re is increased from 4000 to 6000 (Figures 7.10-7.15). There is a rapid increase in the contributions to the vorticity variances from the turbulent structures of $8f_0$ and $16f_0$, which represent the small scale turbulence structures (Figures 7.14 and 7.15), as

Reynolds number is increased from 2000 to 4000. This change echoes the observation in the near field (Yiu et al., 2004), which was ascribed to the generation of small-scale structures at $Re \approx 5000$. The present result could suggest a persistence of turbulent near-field structures of relatively small scales in the far field. This result reinforces the conception that turbulence ‘remembers’ its origin.

Chapter 8 Conclusions and Future Work

8.1 Conclusions

In the present report, a pair of parallel (hot and cold) wires were used to study the effect of wire separation on transverse velocity and temperature derivatives measurements. Based on this study, a probe consisting of six-wire was fabricated to measure simultaneously the turbulent energy and temperature dissipation rates. The measured values of the energy dissipation rate were further compared with those obtained using isotropic relation and the full dissipation rates by a probe consisting of eight-hot-wire probe. Longitudinal and transverse velocity increments are measured both temporally and spatially using two X-wire probes in the intermediate region of a cylinder wake over Taylor microscale Reynolds numbers in the range of 100-300. The wavelet vector multi-resolution technique is used to decompose the measured vorticity signals into a number of wavelet components based on the central frequencies. Following are the main conclusions:

- Velocity and temperature derivatives can be measured reliably using finite differences method if the wire separation is in the range of $3-5\eta$. For separations smaller than 3η , the derivatives are overestimated due to noise contamination, while for separations larger than 5η , the derivatives are underestimated due to the spectral attenuations of the probe.
- Local isotropy across the wake was evaluated based on the ratios $\phi_{u_2}^{cal} / \phi_{u_2}^m$ and $\phi_{\omega_3}^{cal} / \phi_{\omega_3}^m$ at different transverse locations. The present results suggest that local isotropy is approximately satisfied for $x_2^+ \leq 1$.

- The appropriateness of using ε_{iso} and χ_{iso} to represent the full dissipation rates of energy and temperature is evaluated. The results show that the use of ε_{iso} and χ_{iso} underestimates $\rho_{\ln \varepsilon_r, \ln \chi_r}$ by about 20% in the scaling range. The large scatter of $\rho_{\ln \varepsilon_r, \ln \chi_r}$ in the literature may indicate that the correlation coefficient may depend on the magnitude of R_λ , the method to measure ε as well as the nature or the initial conditions of the flow. The correlation between both ε and χ with ω_3^2 are also discussed for different estimations to ε and χ using conditional analysis. Strong correlations of ε_{ful} and ε_{ap} with ω_3^2 are obtained, in accordance with DNS-based investigations. This result also indicates that the instantaneous values of ε can be properly represented by ε_{ap} . In contrast, the correlation between ε_{iso} and ω_3^2 is weak, indicating that ε_{iso} is not a good substitute to ε_{ful} . The correlation between χ_{ap} and χ_{iso} with ω_3^2 is low, no matter which estimations for χ are used.
- Longitudinal and transverse velocity structure functions are measured both temporally and spatially using two X-wire probes over Taylor microscale Reynolds number in the range of 100-300. The scaling exponents of the longitudinal velocity structure functions for both temporal and spatial measurements agree favorably with the predictions of K62 (RSH) and the SL model. The scaling exponents of the transverse velocity increments are considerably smaller than those of the longitudinal ones, with the spatial values being slightly larger than the temporal ones. Both of them increase

slowly with R_λ . While the scaling exponents of the locally averaged dissipation rate does not depend on R_λ , the scaling exponents of the locally averaged enstrophy decrease with R_λ , a trend that is consistent with that reported by Nelkin (1999). The present results show that RSH is valid for longitudinal velocity structure functions. In contrast, RSHT is verified for the spatial transverse velocity increments but only partially valid for the temporal transverse velocity structure functions.

- The three-component vorticity measurements were conducted using an eight-wire probe in the far region of a circular cylinder wake. The results show that the probe performs satisfactorily in terms of both internal consistencies, e.g. the good agreement (4%) among the statistic of velocity fluctuations obtained by the four X-wires, and external consistencies, e.g. the comparisons with data obtained by other researchers. After corrections for spatial resolution of the probe, satisfactory agreement between the measured spectra of vorticity (and their components) and those calculated based on isotropy suggests that local isotropy is approximately satisfied near the centerline region of the wake ($x_2^+ \leq 1$). This result is consistent with the previous measurements by the two parallel wires and the six-wire probe.
- The eight-wire probe also enables the measurement of the full energy dissipation rate and the full enstrophy. It is observed that there is a strong correlation between enstrophy and the energy dissipation for large $(\varepsilon_{ful})_r^\dagger$. However, conditional expectations obtained using ε_{iso} show that there is nearly no correlation between enstrophy and ε_{iso} . The studies on the scaling

relation between the high-order moments of enstrophy $\langle \Omega_r^n \rangle$ and those of the energy dissipation rate $\langle \varepsilon_r^n \rangle$ show that the former is more intermittent than the later. Further investigation shows that for energy dissipation rate, the one-dimensional surrogate is more intermittent than the full field. These results suggest that the one-dimensional substitute cannot represent the full energy dissipation rate properly, especially for instantaneous values.

- Over the scaling range where $\langle |\delta S_{L,1}^*|^3 \rangle$ varies linearly with r , the scaling exponents of the transverse velocity increments are discernible smaller than the longitudinal ones for all moment orders ($n=1-8$). The finding that the full enstrophy ($\Omega = \nu \omega_i \omega_i$) is more intermittent than the energy dissipation rate suggests that the strong intermittency of the transverse velocity increments may be associated with the intermittency of the enstrophy field. The present results show that while $\zeta_{L,1}(n) \simeq \zeta_L(n)$ which verifies the RSH, $\zeta_{T,1}(n)$ is significantly smaller than $\zeta_T(n)$ for all n i.e. the difference between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ cannot be compensated completely by that between $\zeta_L(n)$ and $\zeta_T(n)$, which are obtained using RSH and RSHT, respectively. This result suggests that the discrepancy between $\zeta_{L,1}(n)$ and $\zeta_{T,1}(n)$ cannot be explained by RSHT proposed by Chen et al (1997). Therefore, RSHT is only partially valid for the temporal transverse velocity increments.
- The maximum coherent contribution to velocity variances and Reynolds shear stress from the large-scale vortical structures with the central frequency f_0 is between 20% and 30%, which is consistent with those reported by

Zhou and Li (2003). The present results also show that the large-scale structures (with central frequency f_0) contribute appreciably (13%) to the longitudinal vorticity variance for $x_2^+ < 1$. In contrast, they contribute only 7.4% to $\langle \omega_2^2 \rangle / \langle \omega_2^2 \rangle_{\max}$ and 8.9% to $\langle \omega_3^2 \rangle / \langle \omega_3^2 \rangle_{\max}$. This result indicates a relatively larger contribution to the longitudinal vorticity variance from the large-scale structures than both to the transverse and to the spanwise vorticity components. The dominant contributions to the vorticity variances are from the intermediate and relatively small-scale structures (with central frequencies of $2f_0$, $4f_0$, $8f_0$, $16f_0$ and $32f_0$), indicating that vorticity is mainly related with small-scale structures.

- The study of the Re effect on the contributions from large-, intermediate and small-scale structures to the three-dimensional vorticity variances shows that the contributions from the large-scale structures to the total vorticity variances decrease with the increase in Re . The contributions from the small-scale structures to all three vorticity variances jump significantly when Reynolds number is changed from 2000 to 4000, which is connected to previous observations in the near wake that there is a significant increase in the generation of small-scale structures once the Reynolds number reaches about 5000. This result reinforces the conception that turbulence ‘remembers’ its origin.

8.2 Recommendation for Future Work

Due to the importance of vorticity and temperature dissipation rate measurements in the investigation of both large- and small-scale structures of turbulence, three-dimensional vorticity and temperature dissipation rate should be measured simultaneously in the far wake of a circular cylinder. To our knowledge, these measurements have never been performed previously in a turbulent wake. The measurements are important in further quantifying the correlation between ε and χ , and the appropriateness χ_{iso} as substitutes of χ_{ful} . Sketches of the probe, which is proposed to be used in future study, are given in Figure 8.1(a, b). Four cold wires are added into the three-dimensional eight-wire probe to measure the temperature derivatives involved in the expression of temperature dissipation rate simultaneously.

Another X-wire probe (Figure 8.2), placed at $x_2 = 4d \sim 7d$, will be used in conjunction with the vorticity probe in order to provide a phase reference for the measured vorticity and temperature dissipation rate signals. The phase-average technique will be used to study the topology and momentum transport of coherent and incoherent structures in the cylinder far wake. This technique requires only one moving measurement probe (the vorticity probe) and one fixed single or X-wire (the reference probe), thus providing a good compromise between the sophistication of the vorticity probe and the need to measure the flow field for the study of the large-scale vortical structures.

The wavelet analysis will also be used to study turbulence structures. The measured velocity and temperature signals will be decomposed into a number of wavelet components based on their central frequencies (the vortex shedding frequency), which are representative of the turbulent structures of different scales.

The contributions that the wavelet components make to the time-averaged root-mean-square vorticity (and temperature dissipation rate) will be calculated.

The main tasks of the future work may include:

- (1) To check the correlation between temperature and the three-dimensional vorticity fluctuations.
- (2) To examine the correlation between vorticity fields and the temperature dissipation fields based on the full measurements of the two dissipation rates.
- (3) To study of large-scale structures of turbulence using the phase-average techniques.
- (4) To qualify the contributions to energy and temperature dissipation rates from various scales of turbulence using wavelet method.

To investigate the turbulent structures behind multiple bluff bodies, another possible future work is to study the turbulent wake after multiple cylinders with different geometry configurations, e.g. two cylinders with different separations, three or four cylinders with different attack angles, etc.

Figure 8.1 Sketches of the vorticity and temperature dissipation rate probe.

Figure 8.2 Coordinate system and probe arrangements.

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