Effects of inclination and rotation on the convective heat transfer characteristics of discrete heat sources

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Effects of Inclination and Rotation on the Convective Heat Transfer Characteristics of Discrete Heat Sources

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ABSTRACT

The dissertation presents a three-dimensional (3-D) steady computational model and a two-dimensional (2-D) unsteady model using developed programs, together with experimental studies, to investigate effects of inclination and rotation on natural convection characteristic from an array of discrete heat sources in a closed enclosure.

Effects of inclination on natural convection in a rectangular cavity with a 3x3 array of discrete heaters on one wall are presented firstly. Generally, the heaters in the same row have almost the same temperature in the experiments, due to weak velocity in the horizontal direction. In-depth 2-D and 3-D numerical simulations for laminar flow are implemented and validated by experimental results. Complex flow pattern evolution, flow field, and temperature fields are observed due to interactions of buoyancy forces tangential and normal to the heater surfaces. With increasing Ra^* , the flow pattern evolutions with three main characteristics are identified in the horizontal orientation (heated from below): toroidal convection, bimodal convection and R-B convection. The horizontal orientation is the most desirable orientation for heat transfer. When the enclosure is inclined toward this orientation, the flow and temperature fields become complex and distorted, which greatly enhance heat transfer for the top row of heaters. Edge effect increases heat transfer greatly. In the situation that the effects of edge can be negligible or small, the results of 2-D model are close to those of 3-D model. The effect of Prandtl number in the range 0.7 to 120, to the overall Nusselt number, is small. Optimal locations of the discrete heat sources in the vertical orientation are not uniform after the onset of convection. Increasing Ra^* value has the effect of pushing the heat sources towards the starting cold current of the clockwise circulation.

ABSTRACT

Flow pattern evolution and periodic heat transfer characteristics for air in an enclosure rotating about its horizontal axis are studied and explained by deeper and clearer understanding the effects of various forces. A straightforward numerical method is firstly used to study various forces with an in-house 2-D program. The physically realizable periodic oscillation solutions are identified with reported experimental results. At low rotational speeds, buoyancy force causes clockwise and counterclockwise circulations, and correspondingly there are two or three local peak Nusselt numbers in each periodic oscillation. With increasing rotation and increasing effect of inertial force, clockwise circulation is enlarged and tends to overcome the counterclockwise circulation.. When the rotation speed is high enough and the effects of inertial force overweight the effects of buoyancy force, the second and third peak Nusselt numbers disappear. In general, rotation increases flow stability, results in imbalance of clockwise and counterclockwise circulations, improves heat transfer in the weak stages and reduces the oscillation of Nusselt number. It may increase or reduce average heat transfer performance. These phenomena are determined by the relative effect of thermal buoyancy force and inertial force. In present study ranges, the effects of Coriolis and centrifugal force to heat transfer are small and differentiated from that of other forces. In the rotation situations, heat transfer behavior for the two side rows of heaters is not symmetrical in each cycle due to asymmetrical behavior of clockwise and counterclockwise motions. The optimal distribution of discrete heat sources in rotating fluid is close to results in the stationary situation if their circulations are same.

All present simulation results are consistent with existing reported work, as well as present experimental results. This gives confidence to all the new simulation results.

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Nomenclatures

a_x, a_y, a_z	Heater aspect ratio in x, y, z direction
a_s	Clearance aspect ratio
A_h	Heater source area (m^2)
A_H, A_W	Enclosure aspect ratio along x and y direction
A_{har}	Heater aspect ratio (l_x/l_z)
A_x, A_y, A_z	Enclosure aspect ratio in x, y, z direction
C_p	Specific heat capacity of the fluid ($J/kg \cdot K$)
d, d'	Distance between side heater and center heater (m)
D	Enclosure width (m)
f	Function
F	Mass force (N)
g	Function or gravitational acceleration (m/s^2)
Gr	Grashof number
h	Heat transfer coefficient ($W/m^2 \cdot K$)
H	Enclosure dimension along the z axis (m)
H_i	Enclosure heater location
i	Row i
K	Thermal conductivity ($W/m \cdot K$)
l_x	Heater length in x direction (m)
l_y	Heater length in y direction (m)
l_z	Heater length in z direction (m)
L	Enclosure along the x direction (m)
n	Rotation number for one periodic cycle
Nu	Nusselt number
P_x, P_z	Heater pitch in the x, z direction (m)
p	Pressure (N/m^2)
P_m	Motion pressure (N/m^2)
P	Dimensionless pressure
Pr	Prandtl number
q''	Heat flux (W/m^2)
q'''	Heat generation (W/m^3)

Q	Total heat input (W)
$\vec{r}$	Radius vector of the fluid particle from the rotation axis
Ra	Rayleigh number
Ra_c	Rayleigh number for onset of convection
Ra^*	Modified Rayleigh number
Ra_ω	Rotational Rayleigh number
t	Time (s)
T	Temperature (K)
Ta	Taylor number
u	Velocity in x direction (m/s)
U	Dimensionless velocity in X direction
v	Velocity in y direction (m/s)
V	Dimensionless velocity in Y direction
$\vec{V}$	The fluid particle velocity vector (m/s)
w	Velocity in z direction (m/s)
W	Dimensionless velocity in Z direction or enclosure along the y direction (m)
x	x coordinate
X	Dimensionless coordinate in x direction
y	y coordinate
Y	Dimensionless coordinate in y direction
z	z coordinate
Z	Dimensionless coordinate in z direction

Greek

α	Thermal diffusivity (m ² /s)
β	Volumetric thermal expansion coefficient (1/K)
δ	Velocity boundary layer thickness (m)
δ_T	Thermal boundary layer thickness (m)
η	Conjugate correction factor
θ	Dimensionless temperature
Ω	Magnitude of angular rotation rate (rpm, rad/s)
ρ	Density of the fluid (kg/m ³)

ρ_0	Density at the mean temperature (kg/m ³)
τ	Dimensionless time
τ_0	Dimensionless time for one rotation
$\Delta\tau$	Dimensionless time step (variable)
ϕ	General variable in discretization equation or angle position of the cavity (°)
ϕ_1, ϕ_2, ϕ_3	Angular position of the first, second, and third inflexion (°)

Superscripts and Subscripts

$\overrightarrow{(\quad)}$	Vector
$\overline{(\quad)}$	Space average
$\overline{\overline{(\quad)}}$	Time and space average
$(\quad)_c$	Cold wall or critical inclined angle
$(\quad)_h$	Heater
$(\quad)_H$	Based on the character length of H or hot wall
$(\quad)_i$	Row i
$(\quad)_{Lz}$	Based on the character length of L_z
(i, j)	Heater (i, j)

Chapter 1

Introduction

1.1 Background

Many areas of engineering and science involve the rotation of various objects; in science the object sometimes is the earth. In fact, note that all experiments on earth are carried out in a rotating coordinate system; in engineering it might be a rotor or a space vehicle. In most cases the object also involves the rotational of internal or external fluids. Vanyo (1993) showed a communication satellite with a value of \$100,000,000 in the early 1980s, manufactured by Ford Aerospace for the India Space Agency and placed in orbit by a Delta rocket. During transition from its low circular orbit to its elliptical transfer orbit, it was to be spun at 40 rpm about its long axis (minimum axis of inertia) to achieve gyroscopic attitude stability. At that time, the vehicle was up to be 55% liquid by mass. Katz and Plotkin (2001) discussed an aircraft model rotating about its centre of gravity at a fairly low rate, but representative of possible aircraft flight conditions. In some situations, rotating fluid is the principal phenomena of interest. Success or failure of the analysis can depend critically on understanding and predicting rotating fluid phenomena. Unfortunately, for natural convection, the velocity distribution is governed by the temperature distribution, but the temperature distribution also depends on the advection of heat on the velocity distribution. There is no possibility of determining one independently of the other, as one could be done in forced

convection (Tritton, 1991). Therefore, understanding of rotating flow is prerequisite for studying the rotating convection. However, due to the engineering applications and its intricacy, the rotating flow itself becomes one of the most challenging research fields of fluid mechanics.

Actually, as pointed out by Boubnov and Golitsyn (1995), convection in rotating fluids up to quite recently was a neglected subject. Most of the early works are experimental study and based on the simplest cases, such as a plane horizontal rotating layer, heated rotating annulus, heated rotating cubic enclosure. In some technical and technological applications one may encounter with sufficiently fast rotation and here many interesting phenomena may arise which are still explored only very poorly, such as periodical natural convection rotating flow when the enclosure is rotating about its longitudinal horizontal axis, but which may have potentially important applications.

While sharing many of the generic problems in rotating fluids, electronics cooling also poses very unique challenges with the background rotation. In thermal management of electronics package, temperature is a key factor in the operation of the electronic equipment. The evidence of the importance of the thermal consideration, as shown in Fig. 1.1, is due to the survey by the U.S. Air Force indicating that more than fifty percent of all electronic failures are temperature related. The reliability of a silicon chip is decreased by about ten percent for every 2°C temperature rise (Yeh, 1995). As reported by Bobco (1981), free convection has been applied to the design of a vented Gallileo mission descent module parachuting into the Jupiter atmosphere. Actually, in engineering, most rotating objects, such as satellite, aircraft and vented Gallileo mission descent module, consist of electronic devices and thermal management with the background rotation become a special and important sub-field of electronic cooling. However, until now, there is no systemic and comprehensive study available in the literature. Thermal management in the unsteady and severe environment will

be the subject of interest in research with the development of mobile electronics devices and spaced-based manufacturing processes. This study will present a basic study on rotating convection with the discrete heat sources. It extends research fields from the traditional inertial system to the non-inertial system.

Fig. 1.1 : Major causes of electronic failures (Yeh, 1995)

Traditional broad classifications of electronics cooling used in industry include air-cooling systems, liquid cooling systems, boiling liquid systems, as shown in Fig. 1.2. Despite many recent rapid enhancing breakthroughs in such areas as super-conductivity, micro-channel cooling and boiling cooling, direct air cooling is still the most common method of removing heat from the electronic devices. Air has been the preferred fluid for cooling electronic packages ranging from printed circuit boards (PCBs) to chips and multichip modules due to its simplicity, low cost, ease of maintenance, and high reliability. Free convection for air is a cost effective and attractive technique. This technique is easy to implement because no auxiliary equipment is required. Heat removal by this technique, in which heat fluxes are limited to roughly 0.1 W/cm^2 , may be sufficient to keep moderately heated electronic chips below the allowable device temperatures of 85°C (Mudawar 1992); otherwise, more effective cooling techniques such as forced convection or liquid cooling must be used. Despite its poor thermo-physical properties as a coolant, research has been ongoing to extend the limits of air cooling capabilities. On the other hand, wherever possible, the

passive cooling system, employing natural convection, is still the first choice and always be favored over other electronics cooling techniques due to noise, vibration, electromagnetic interference free and low maintenance requirement.

Fig. 1.2: Heat transfer coefficients for various coolants and cooling modes

(Bar-Cohen, 1993b)

The literature survey shows that steady natural convection with the discrete heat sources under the gravity force field has been studied by many researchers. But with the background rotation and under the centrifugal force and Coriolis force field there are only limited studies. Furthermore, it has hardly been studied for periodic heat transfer when the enclosure rotates about its horizontal axis. This study is motivated by the need to give a basic understanding for the phenomena found in this periodic rotating fluid, such as flow pattern evolution and natural convection heat transfer characteristics with the increase of rotation speed gradually. At the same time, it is meaningful to explain these phenomena by studying effects of coupled body forces. However, due to complex couple of interacting body forces, it is important to sort out them clearly, especially the effects of Coriolis force and centrifugal force. In previous studies, a vast amount of work, both theoretical and experimental, exists in the literature for forced convection, such as rotating ducts, disks, turbine blade cooling and so on. Unfortunately, natural convection in a closed system is very different to them and will cause higher challenges in numerical investigations. Based on these acknowledge, the present project presents an attempt to explore this challenging subject.

1.2 Objective

The objective of this project **is** to study the effects of inclination and rotation on natural convection flow pattern evolution, flow structure, periodic heat transfer characteristics from an array of discrete heat sources in a closed enclosure and effects of various forces by numerical simulations and experimental studies together.

1.3 Scope of Investigation

The scopes of this project involve:

- (1) To develop the governing equations for natural convection cooling from an array of discrete heat sources flush-mounted in a shallow enclosure for stationary fluids in the inclined gravity force orientation and roll rotating fluids. It should be pointed out that this periodic rotating heat transfer problem cannot be solved by present commercial softwares, such as FLUENT, ANSYS, FLOTHERM, ICEPAK.
- (2) To design and construct test cells and test bed and to carry out experimental study so as to validate the numerical code with published results and the experimental data.
- (3) To perform the numerical investigations on the thermal and hydraulic characteristics of the proposed cooling structure. To analysis temperature, velocity and local/overall heat transfer rate for various inclined orientations and rotation speed.
- (4) To investigate the effect of Prandtl number on the stationary single-phase heat transfer performances.
- (5) To investigate the effects of Rayleigh number, Rotating Rayleigh number, and Taylor number and effects of thermal buoyancy force, inertial force, Coriolis force and centrifugal force systematically and comprehensively for the first time.
- (6) To identify the trends that affect the heat transfer performance and give some optimal and meaningful advices from the opinion of thermal management in industry.

Chapter 2

Literature Review

Firstly, the thermal model for present study of electronics cooling is provided. Then, the literature survey is carried out from three aspects: numerical study on natural convection in vertical/horizontal orientation, study on natural convection in the inclined orientation; and study on rotating enclosures. Finally, the conclusions are shown.

2.1 Introduction

The general study of natural convection heat transfer can be seen in the book of Jaluria (1980), Bejan (1995). With the advent of electronics cooling, new study involving discrete heat sources have emerged. In most of the recent literature, an array of heated square blocks deployed along one wall of a parallel plate duct is used to model the circuit board. It's a preferred mode of cooling in many telecommunication devices such as telephone switching units, avionics packages, computers, and many other electronic systems. It allows greater variability in electronic packaging at the board, component and system levels. In many applications, particularly when the electronic devices are operated in non-benign environments, the packages are sealed, and may be modeled as a vertical or tilted enclosure with discrete heat sources on one side wall. The contents of this study focus on such a situation, with the nominal configuration corresponding to a 3×3 or 3×1 array of chips mounted to one of the walls of a rectangular enclosure. The formulation is made consistent

with the studies of Poleniti *et al.* (1993), Heindel *et al.* (1995a, b), Tou *et al.* (1999), Tou and Zhang (2003).

2.2 Numerical study on natural convection in verticalhorizontal orientation

Review of the passive cooling systems, employing natural convection in vertical orientation is presented by Incropera (1988), Bar-Cohen (1991, 1993) and Incopera (1999). Obviously, the majority of the early studies focus on single multiple strip heat sources and neglect three-dimensionaleffects, which can be significant for small heat sources arranged in closely spaced arrays. In 1990s, three-dimension numerical simulation received a great deal of attention.

Kuhn and Oosthuizen (1986) simulated three-dimensional flow and heat transfer for one or two discrete heat sources mounted to one wall of a vertical cavity with major and minor aspect ratios of $A_x=5$ and $A_y=3$ respectively and for $0 \leq Ra \leq 10^5$. The opposing wall is chilled, while the top and bottom surfaces are assumed adiabatic. The effect of the heater location on heat transfer was shown to be small if the heaters were located at least two elemental heights away from the top or bottom walls. Outside this range, heat transfer was strongly influenced by location. When the element was near the cavity ceiling, fluid motion was inhibited and heat transfer was dominated by conduction. In studying element size effects, the height of the discrete heat source was shown to be more important than its width, unless the element's width was much smaller than its height. Three-dimensional flow increased local heat transfer near the heater edges, causing the average Nusselt number to exceed results obtained from the corresponding two-dimensional predictions.

Afrid and Zebib (1991) studied one of the most common equipment cooling configuration, namely, free convection in vertical parallel plate channels which were open to ambience. Laminar and turbulent conjugate conduction-natural convection air cooling of BO

uniformly heated cubic blocks were mounted to an insulated plate. The side of the cubes was 1.27 cm and the channel width was 4 times that of the side of the cubes. The distance between centres of consecutive cubes was 3 times the size of a cube side. The authors used the standard two-equation k - ϵ model to solve the turbulent natural convection problem. A control volume based on the finite difference method was used to discretize the conservative differential equations. The grids of $22 \times 84 \times 17$ and $38 \times 84 \times 31$ were selected for the three-dimensional laminar and turbulent flow respectively.

Wroblewski and Joshi (1992) considered a square, protruding heat source mounted at the center of a conducting substrate and an isothermal cold plate for the opposing wall. The conjugate heat conduction in the substrate and the package was solved simultaneously with the natural convection within the liquid. The study covered heat transfer cases under steady and unsteady state. It was found that steady state conditions were characterized by thin boundary layers along the hot and cold surfaces. There was a well-stratified core and a strong plume being formed above the package. The temperature field depicted oval-shaped isotherms along the substrate surface. With FC-75 as the coolant, a large fraction of the heat generated in the chip was conducted to the substrate relative to that of the fluid, due to the high thermal conductivity of the substrate relative to that of the fluid. Even when the upper lid of the package was removed, thus exposing the chip directly to the fluid, the chip temperature dropped only by 5%. This indicated the importance of the substrate conduction in problem of this kind. Furthermore, a chip that has twice the cross sectional area as the base case, has resulted in 25% drop in chip temperature, with little effect on the flow fields. Another numerical study by Wroblewski and Joshi (1993) was focused on conjugate heat transfer from a single protruding substrate-mounted chip or package attached to one vertical wall of a liquid-filled cubical enclosure. Conduction through the substrate played a dominant role in the cooling of the chip for values of R_s , ratio of protrusion thermal conductivity to fluid thermal

conductivity, greater than 10. Wroblewski and Joshi (1994) continued to investigate the effects of heater geometry for a constant protrusion height. For a square heat source, two-dimensional calculations yielded a maximum heater temperature 2.8 times higher than that predicted from the three-dimensional model. If the heater aspect ratio was greater than 4, the two and three-dimensional predictions agreed to within 10 percent.

Mukutmoni *et al.* (1993a) carried out a comprehensive numerical study for a three by three array of chip protrusions in a vertical substrate that was being cooled by the dielectric liquid FC-71 in an enclosure. Also the top and bottom enclosure walls were assumed isothermal, while the remaining boundaries, including the rear of the substrate containing the protrusions, were insulated. The package was modelled as sources of constant heat **flux** at some portion of the wall, and the viscosity of FC-71 was treated as temperature dependent. The only geometrical parameter that was varied was the enclosure width, from 13 to 30 mm. A conjugate heat transfer problem was examined where conduction was solved for in the package and substrate, and natural convection flow occurs over the rest of the enclosure. The effect of varying the width of the enclosure was investigated and the results showed that chip temperatures increased marginally as the width reduced. The percentage of heat dispersing directly to the substrate from the chips registered a slight increase as the width was decreased. This study, which used the same geometry and configuration, was continued by Mukutmoni *et al.* (1993b) to investigate the effect of substrate thermal conductivity. The results indicated that component temperatures decreased significantly as the conductivity was increased and the percentage of heat going directly into the substrate from the protrusions increased.

Also, Mukutmoni and Yang (1993) studied the Rayleigh-Benard (R-B) convection in a small aspect ratio enclosure. The vertical walls are adiabatic. The bottom wall is heated and the top wall is cooled, both isothermally. The Boussinesq fluid is water (average temperature of 70°C) with a Prandtl number of 2.5. The results showed that the R-B convection in a small

aspect ratio box undergo a sequence of bifurcation from steady to oscillatory convection. The steady sub-critical R-B convection consisted of two symmetric counter rotating rolls. The flow field was found to be completely three-dimensional, especially near the end-walls. A weak spiralling motion was observed along the rolls axis. In the time-dependent domain, the qualitative behavior of oscillation amplitudes near the critical Rayleigh number (from steady to unsteady convection) was identical to results from stability theory. The nature of the oscillations was determined and found to consist of an approximate standing wave pattern. The maximum temperature oscillations were found to occur around the symmetry line separating the two rolls. The four-fold symmetry in the horizontal plane of the flow and temperature fields was preserved for the temperature oscillation amplitudes as well.

Three-dimensional natural convection from a 3x3 array of heat sources flush-mounted to one vertical wall of a rectangular enclosure, with one opposing wall isothermally cooled has been considered comprehensively by Heindel *et al.* (1995a, 1995b). Flow patterns, temperature field and local Nusselt number distribution of the wall mounted with heaters are shown in order to emphasize the three-dimensional effect. Overall, variations between the two and three-dimensional velocity predictions were small and were confined to the adiabatic regions between heater columns. However, the three-dimensional model captured edge effects that could not be predicted by the two-dimensional model. These edge effects, which included local variations in the velocity and temperature fields, did influence local and overall heat transfer rates. For $A_{har}=1$, local Nusselt number (Nu) predicted by three-dimensional model was 5% higher at the heater mid-plane and 10–15% higher at lateral edges than that predicted by the two-dimensional model. For $A_{har} \geq 3.0$, the difference of row-averaged Nu between the two model are less than 5%. Heindel *et al.* (1995c, 1995d) continued to study the effects of the substrate conjugate heat transfer. With increasing Modified Rayleigh number, ($10^4 \leq Ra^* \leq 10^9$), the enclosure flow pattern becomes more boundary layer-like along the vertical walls,

and multiple fluid cells develop in the center region. Thermal spreading in the substrate increases with decreasing Ra^* and with increasing values of the substrate/fluid thermal conductivity ratio ($10^{-1} \leq Rs \leq 10^3$). Thermal spreading in the substrate also has a significant effect on circulation in the enclosure.

Tou *et al.* (1999) carried on the numerical study using the same parameters but with various liquids ($Pr=5, 9, 25, 130$). The enclosure aspect ratio varies from 1 to 20. The row-average Nu increases with enclosure aspect ratio in the range of 1-3 and attains the maximum when aspect ratio is greater than 3. The heat surface temperature is the highest at the top row of heaters regardless of the Ra^* and enclosure aspect ratio. The effects of Prandtl number are negligible in the range from 5 to 130.

Sezai and Mohamad (2000) studied steady natural convection from a discrete flush-mounted rectangular heat source on the bottom of a horizontal enclosure numerically. Ra (based on the enclosure height) varies from 10^3 until unstable flow is predicted for a fixed Prandtl number of 0.71. The enclosure is cooled from above and insulated from the bottom. The flow pattern, consists of a toroidal roll cell in the cavity for a square heater with aspect ratio, $a_x=1$ and $a_y=1$, for $Ra=10^3 \sim 10^5$. The toroidal roll cell is destroyed at a Ra between 10^5 and 10^6 and a completely different flow pattern is formed having two vortices near the top corners of the cavity. Although the heater cross section is square, the buoyant jet diffuses into a circular form above the bottom plate. It is found that the rate of heat transfer is not so sensitive to the vertical wall boundary conditions. The limit of maximum Ra to obtain a convergent solution decreases as the aspect ratio of the enclosure is increased.

Bejan (1996) developed constructal-theory network of conducting paths for cooling a heat generating volume. Based on constructal method of Bejan (2000) and maximum global performance from 2-D numerical simulations, Silva *et al.* (2004) studied optimal distribution of discrete heat sources on a wall with natural convection. The results show that the optimal

distribution is not uniform (the sources are not equidistant), and that as the Ra increases the heat sources placed near the tip of a boundary layer should have zero spacing.

2.3 Study on natural convection in the inclined orientation

In a variety of engineering applications enclosures are inclined to the direction of gravity which results in the need to consider the effects of both the tangential and normal components of the buoyancy force relative to the differentially heated walls. At large inclination angles the effect of the component of buoyancy normal to the heated surface becomes very important in modifying both the flow structure and heat transfer.

The importance of the inclined convection problem is pointed out in the review papers by Ostrach (1972), Hamady *et al.* (1989). Until about 1970 there had been little consideration given to the effects of inclined boundaries on thermal convection or to the effect of inclusion of parameters such as the enclosure aspect ratio or the angle of inclination. Comprehensive work was conducted by Hart (1976), who used both analytical and experimental procedures to reveal the significant features of the effect of enclosure inclination between two limiting cases, R-B convection and convection in a vertical slot.

Actually, most of present knowledge about R-B convection is primarily acquired from experiments. The reason that it has received so much attention is not that it is of practical importance, but rather it has become a context for the development of ideas about the consequences of instability and evolution towards turbulent motion. Rayleigh formulated the theory of convective instability of a layer of fluid between horizontal plates. He derived the linear equations and showed that instability would occur only when the adverse temperature gradient was so large that the dimensionless parameter Ra exceeded a certain critical value, i.e. the critical Rayleigh number (Ra_c), which is around 1708 for the Benard convection. Since then, researchers began to use the linear stability theory to predict Ra , for the onset of R-B

convection in a closed box. However, due to complex of Benard convection in nature, linear stability theory does not in principle predict the type of motion above Ra . Yang *et al.* (1988) pointed out the great error when studying three-dimensional bimodal buoyant flow transition in an inclined and small aspect ratio enclosure with the adiabatic side walls. The bottom wall is heated and the upside wall is cooled, both isothermally. From the linear stability analysis of Davis (1963, the model should have three cells in each direction (four cells in the numerical prediction). However, the symmetry of the geometry and initial condition can not produce this symmetrical model unless appropriate disturbances are introduced at the beginning. Furthermore, Lin (2001) also pointed out that the quantitative agreement between theory and experiment for the onset of vortex instability of the flow under R-B convection in the open surface is still unsatisfactory. The discrepancy between the theoretical critical Grashof number (Ga_c) and these experimental data were one or two orders in the literature.

Maveety and Leith (1998) presented a class of problems in thermal convection, in which the mechanism that were responsible for the evolution of a dynamical system to different states were explored. Experimental investigations and heat flux characteristics for R-B convection in moderate size containers with air exhibited a secondary bifurcation structure which can be understood, in a hierarchical sense, in term of the container size. It is concluded that the heat transport is strongly dependent on both size and shape of the container.

Lir and Lin (2001) carried out the flow visualization to study the vortex flow patterns associated with the R-B convection in a horizontal shallow cavity with air. The cavity is a rectangular enclosure characterized by the aspect ratios $A_x=16$ and $A_z=20$. Attention is focused on the convection rolls driven at slightly supercritical and subcritical buoyancies. The results show that, at slightly subcritical Ra , the induced vortex flow is in the form of rectangular rolls along the cavity sides and short straight parallel rolls in the cavity core. At slightly higher Ra (near $Ra_c=1788$) more rectangular rolls appear and the short straight rolls in the cavity core

merge together to form a serpentine roll. At slightly supercritical buoyancy with $2000 \leq Ra \leq 3000$ the entire cavity is filled with the straight rolls all parallel to the short side of the cavity. At an even higher Ra of 4000, the vortex become irregular and time dependent. Moreover, the processes through which various vortex flow structure evolve during the transient stage are shown to be rather complicated and the vortex flow patterns during the flow formation are significantly affected by the heating rate in raising buoyancy force.

Steady natural convection induced by multiple discrete heat sources in two-dimensional horizontal enclosure is numerically investigated by Deng et al. (2002). A general combined temperature scale method for constant wall temperature and uniform heat flux is first implemented. Computational results demonstrated that the combined temperature scale method and the unified heat transfer characteristics analysis are convenient and efficient to evaluate the complex interaction between discrete heat sources and its effect on the fluid flow and heat transfer structures in horizontal natural convection enclosures.

Some of the important contributions of analysis of natural convection in inclined enclosures were the introduction of variable properties in the numerical methods and establishment of the limits of validity of the Boussinesq approximation. Zhong *et al.* (1985) and Yang *et al.* (1987) have applied their numerical analysis to study of the effects of variable thermal properties, thermal conductivity, viscosity and heat capacity. In their work, flow patterns and heat transfer were analyzed for a complete set of inclination angles. Conductive heat transfer was found to dominate the heat transfer for inclination angles ranging from 0° (heated from above) to 45° . For large angles, convection was the prevailing heat transfer mode, due to the apparent increase of the gravitational effect along the differentially heated walls. Transitions from uni-cell to three-dimensional oblique rolls have been seen at critical angles between 150° and 180° , and are accompanied by a local minimum in the heat transfer

rate. Moreover, their correlation for the mean Nu based on fully variable properties compared favorably with the experimental results.

One of the features as observed in experiments and verified in numerical studies is associated with enclosures where, as the differently-heated side walls are tilted from horizontal to vertical position, there exist a critical angle corresponding to flow pattern change and the minimum heat transfer rate. Yang *et al.* (1988) carried out a numerical study to track laminar flow transitions and heat transfer characteristics in tilted three-dimensional rectangular enclosures with square differentially heated side surfaces. The bimodal flow pattern for the heated-from-below case are found to be very stable and insensitive to changes in the aspect ratios and Ra . Tilt of the enclosure produces a complicated three-dimensional transition that can be viewed as the combination of transitions found in transverse and longitudinal enclosures. A local minimum heat transfer has been found at a critical angle that corresponds to the transition of three-dimensional complicated flow to an unicell.

A few studies of heat transfer from arrays of discrete square or rectangular heat sources have been reported. Liu *et al.* (1988) presented a numerical simulation study for a 3x3 array of equally heated rectangular protrusions mounted on one wall of an enclosure filled with FC-75 and cooled from the top and bottom. The horizontal spacing between the sources was comparable to their longest dimension and the protrusion from the wall was sizable. Although the computations were restricted to low Ra , the computed flow patterns were found to be extremely complex, with substantial flow penetration into the gaps between adjacent heat sources.

The effect of inclination on the steady natural convection local heat transfer characteristics in an air-filled differentially heated enclosure with cross-sectional aspect ratio of one is studied experimentally and numerically by Hamady *et al.* (1989). The results showed that even for a gravitationally stable conduction at 0° (heated from above) the local

values of the heat transfer rate are different from the unity near the corners due to the existence of convective flows, which come from the insulated side walls. The heat flux at the hot and cold boundaries show strong dependence on the angle of inclination and Ra

Poleniti et al. (1993) performed an extensive experimental study for a 3x3 heater array flush-mounted on one wall of an enclosure, with the opposing wall isothermally cooled. Heat transfer was independent of the enclosure aspect ratio ($2.5 \leq A_z \leq 7.5$) but increased with increasing angle of inclination, where the cold plate was always oriented above the heat sources. The increased heat transfer was due to increased turbulent mixing within the enclosure and decreased thermal stratification. Prandtl number (Pr) effects were negligible for the two fluids used in this study, water ($Pr=5$) and FC-77 ($Pr=25$). The effect of heat location and spacing was studied by powering various combinations of one or two heater rows. With only one row powered, the average Nu was highest (lowest) when the individual row was located near the cavity floor (ceiling). With two rows energized concurrently, the average Nu , based on a local length scale extending from 6.4 mm below the lowest energized row to the centre of each row, increased slightly with decreasing element distance from the bottom wall and increasing row spacing.

Lin (2001) presents a numerical study of the formation of longitudinal vortices in natural convection flow over horizontal and inclined plates. The criterion on the position marking on the onset of longitudinal vortices is defined. The results show that the onset position characterized by the Grashof number (Gr) depends on the Pr , wave number, and the inclined angle ϕ from the horizontal. The flow is found to become more stable at the vortex mode of instability as the value of inclined angle increases, owing to a decrease in buoyancy force in the normal direction. However, Pr has a destabilizing effect on the flow and the critical values of Gr decrease with an increase in Pr .

According to the study of Tou and Zhang (2003), the interactions of flow tangential and normal to the heater surfaces in an inclined enclosure cause a slight oscillation in Nusselt number. Inclination has little effects on the average heat transfer characteristics except configuration close to the conduction case with heaters on top leading to a sharp decrease in Nusselt number. The critical Rayleigh number of 1708 is also observed regardless of inclinations.

The present survey of numerical simulations with the discrete heat sources is summarized in Table 2.1.

Table 2.1: Survey of Three-dimensional Numerical simulation over arrays of heated blocks

No.	Authors (Year)	Fluid	Flow Regime	Mesh	Definition of Ra	Heaters	Turbulent Model	Geometry
1	Kuhn & Oodthhuizen (1986)	Air	Lam NC $Ra=0-10^5$	Non-uniform $22 \times 15 \times 37$	$g\beta\Delta TL^3/\alpha\nu$	$1 \times 1, F$ $1 \times 2, F$		Ver, $A_x=5, A_y=3, a_x=0.716, 1.286$ $a_y=0.333, 0.667$
2	Afrid & Zebib (1991)	Air	Lam&Tur NC Unknown	Uniform $22 \times 84 \times 17$	$g\beta(qL^2/k)L^3/\alpha\nu$	$1 \times 10, P$	Standard k- ϵ with-out wall function	Ver, $L=1.27\text{cm}, l_{ch}/L=10,$ $W/L=4, D/L=3$
3	Wroblewski & Joshi. (1992)	FC75	Lam NC $Ra=1.15 \times 10^9 \sim 4.75 \times 10^6$	Non-uniform $28 \times 26 \times 28$	$gq\beta L^2/\alpha\nu k$	$1 \times 1, P$		Ver, for chip $1.5 \times 1.5 \times 0.4\text{mm},$ $3 \times 3 \times 0.4\text{mm}$ for package $8.9 \times 8.9 \times 1.9\text{mm}$
4	Mukutmoni <i>et al.</i> (1993a)	FC71	Lam NC $B_0=4.1 \times 10^{12}$	Non-uniform $18 \times 36 \times 63$	$gq\beta L^2/\alpha^2 k$	$3 \times 3, P$		Ver, $W=13-30\text{mm}, H=152\text{mm}$ $Wth=203.2\text{mm}, Wchip=8\text{mm}$ $Hchip=24\text{mm}$
5	Mukutmoni <i>et al.</i> (1993b)	FC71	Lam NC $B_0=3.21 \times 10^{11} \sim 2.28 \times 10^{12}$	Non-uniform $36 \times 58 \times 66$	$gq\beta L^2/\alpha^2 k$	$3 \times 3, P$		Ver, $W=30\text{mm}, H=152\text{mm},$ $Wth=203.2\text{mm}, Wchip=8\text{mm}$ $HCHIP=24\text{mm}, BSUB=19.5\text{mm}$
6	Mukutmoni & Yang (1993)	Water ($Pr=2.5$)	Lam NC $Ra=2 \sim 4 \times 10^4$	Non-uniform $24 \times 22 \times 22$	$g\beta\Delta TL^3/\alpha\nu$	$1 \times 1, F$		Hor, $A_x=3.5, A_y=2.1,$ $a_x=A_x, a_y=A_y$
7	Wroblewski & Joshi (1993)	FC71	Lam NC $Ra=10^3 \sim 10^9$	Non-uniform $80 \times 80 \times 80$	$gq\beta L^2/\alpha\nu k$	$1 \times 1, P$		Ver, for chip $H=0.05 \sim 0.4$ $L=1.0 \sim 4.5$ For package $W=2.55 \sim 7.65$
8	Wroblewski & Joshi (1994)	FC75	Lam NC $Ra=10^8$	Non-uniform $24 \times 22 \times 22$	$gq\beta L^2/\alpha\nu k$	$1 \times 1, P$		Ver, for chip $H=0.05 \sim 0.4$ $L=1.0 \sim 4.5$ For package $W=2.55 \sim 7.65$

Table 2.1: Survey of Three-dimensional Numerical simulation over arrays of heated blocks (continued)

No.	Authors (Year)	Fluid	Flow Regime	Mesh	Definition of Ra	Heaters	Turbulent Model	Geometry
9	Heindel <i>et al.</i> (1995a)	Water FC77	Lam NC $Ra^* = 10^5 \sim 10^8$	Non-uniform $30 \times 30 \times 68$	$gq\beta L^2/\alpha\nu k$	$3 \times 3, F$		Ver, $Ax=7.5, Ay=4.5, ax=1, ay=1, as=0.25$
10	Heindel <i>et al.</i> (1995b)	Water FC77	Lam NC $Ra^* = 10^5 \sim 10^8$	Non-uniform $32 \times 40 \times 80$	$gq\beta L^4/\alpha\nu k$	$3 \times 3, F$		Ver, $Ax=7.5, Ay=4.5, ax=1, ay=1, as=0.25$
11	Tou <i>et al.</i> (1999)	Water FC72 FC75	Lam NC $Ra^* = 10^4 \sim 10^8$	Non-uniform $55 \times 31 \times 91$	$gq\beta L^4/\alpha\nu k$	$3 \times 3, F$		Ver, $Ax=7.5, Ay=4.5, ax=1, ay=1, as=0.25$
12	Sezai & Mohamad (2000)	Air	Lam NC $Ra = 10^3 \sim 10^6$	Non-uniform $66 \times 66 \times 42$	$gq\beta L^4/\alpha\nu k$	$1 \times 1, F$		Hor, $Ax=4, Ay=4, ax=1, ay=1 \sim 4$
13	Tou and Zhang (2003)	FC72	Lam NC $Ra^* = 10^2 \sim 10^8$	Non-uniform $30 \times 60 \times 90$	$gq\beta L^4/\alpha\nu k$	$3 \times 3, F$		Inclined angle $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 135^\circ, 180^\circ, Ax=7.5, Ay=4.5, ax=1, ay=1, as=0.25$
	Note		Lam: Laminar Tur: Turbulent NC: Natural convection		L: Heater Length Q: Total heat q: Heat flux	P: Protruded F: Flushed		Ver: Vertical (90°), Hor: horizontal Ax, Ay: aspect ratio of enclosure in X/Y direction, ax, ay: aspect ratio of heater in X/Y direction, as: aspect ratio of the clearance

2.4 Study on natural convection in the rotating enclosures

Greenspan (1968) described the basic theory for rotating fluids, and other work were presented by Vanyo (1993) and Boubnov and Golitsyn (1995). Fernando and Smith (2001) surveyed a variety of vortex structures arising during buoyant convection in the presence of background stratification and rotation. It should be pointed out that present work is very different to forced convection in rotating ducts and disks, such as research work of Shih, Rigby and Chen (Lin et al. 2001). For one thing, with small density change and low rotation speed, the effect of centrifugal force for natural convection in a closed system can be neglected completely (Greenspan, 1968, Hamady et al. 1994). For another, thermal buoyancy force is the driving force of the fluid flow and causes periodic heat transfer. Only those relevant to the present study are briefly reviewed here.

Morris (1965) investigated the influence of rotation on the laminar asymptotic velocity and temperature profiles obtained when fluid flows through a vertical tube which rotates about a parallel axis with uniform angular velocity, and which is subjected to a uniform temperature gradient. The conservation equations are solved using a series expansion in ascending powers of the rotational Rayleigh number (Ra_ω), the resulting solutions being approximate and validate only for low rates of heating. A simple analysis is presented by Veronis (1965) for the finite-amplitude, steady motions in a rotating layer of fluid that is heated uniformly from below and cooled from above. Later, Veronis (1968) studied large-amplitude R-B convection in a rotating fluid and comparison with some of the experimental features observed and reported by Rossby (1966) was also discussed. Thermally induced convection in a rotating cylinder of fluid heated from above and strongly influenced by centrifugal accelerations is treated using boundary layer methods by Homsy and Hudson (1969). Kuppers and Lortz (1969) studied transition from laminar convection to thermal turbulence in a rotating fluid layer. The convective flow in an infinite horizontal fluid layer

rotating rigidly about a normal axis is investigated for the special case of infinite Pr and free boundary conditions. A stability calculation shows that no stable steady-state convective flow exists if Taylor number (Ta) exceeds the critical value of 2285. Homsy and Hudson (1971a) used boundary-layer methods to study asymptotic stability of a rapid rotating, horizontally bounded fluid, heated from below. Later (1971b), they presented a mathematical analysis of centrifugally driven thermal convection in a cylinder which rotates about its vertical axis and discussed the influence of rotation on laminar heat convection when the fluid flowed through a vertical tube with a uniform heat flux boundary condition that rotated about a parallel axis. Boundary layer solutions were extended to include large diameter/height aspect ratios and a centrifugal acceleration of the same magnitude as that of gravity.

Tang (1975) carried out an experimental study for gravitational instabilities in a vertical rotating cylinder. For a fixed Ra , with increasing rotation speed, measurements with low viscosity oil ($Pr \approx 7$) showed non-monotonic dependence of Nu on Ta . The decrease of Nu with the increase of Ta was attributed to solid body rotation even in low rotation speed (Ω –100 rpm). Abell and Hudson (1975) studied the thermal convection in a vertical rotating cylinder heated from above. It was found that centrifugal acceleration played a critical role in the heat transfer processes. Coriolis acceleration contributed significantly to the induced secondary flow, which also affected the heat transfer characteristics. Hudson *et al.* (1978) and Tang and Hudson (1983) carried out an experimental study on a vertical closed circular cylinder, heated from below and rotating about its axis. The experiments used silicone oil as heat transfer medium. The result showed that Nusselt number increased with rotation speeds. Buhler and Oertel (1982) conducted theoretical and experimental studies of thermal cellular convection in a rotating rectangular box, with their interest centered on the effect of Pr . Linear stability theory employing the Boussinesq approximation was used in the analysis. Their computation showed that the roll cells changed their orientation with increasing Ta , and led to

the conclusion that centrifugal force dominates in high Pr fluids, but Coriolis force is dominant in low Pr fluids. The onset of steady natural convection was shown by Buell and Catton I. (1983) to be rather sensitive to the lateral boundary conditions. Neti et al. (1985) and Levy et al. (1986) carried out numerical studies on laminar heat transfer for air in a rectangle duct rotating about a parallel axis. Their results indicated that the secondary flow induced by the Coriolis force and density gradients produced a significant effect on heat transfer and pressure drop.

Hamady (1987) and Hamady *et al.* (1994) focused on low rotation speed range ($\Omega \leq 18$ rpm or $Ta \leq 4 \times 10^5$) and studied experimentally and numerically the natural convection heat transfer characteristics. Their results showed that Coriolis force arising from rotation has a remarkable influence on the local heat transfer when compared with non-rotating results (Hamady *et al.*, 1989). Another important contribution is to sort out the effect of centrifugal force by theoretical analysis and validated by numerical and experimental results clearly. However, the boundary conditions of their numerical simulations are those experimentally measured, and therefore each numerical result may have different boundary conditions.

Kirdyashin and Distanov (1990) found that a periodically changing rotation speed could result in periodical temperature change throughout the entire liquid layer. The effects of rotation on natural convection in a vertical annulus with differentially heated vertical side walls were examined in detail by Busse and Or (1986, 1987). Using numerical methods, Zhao and Lock (1993) studied the effect of steady three-dimensional free convection inside a long vertical rotating porous box, with the bottom surface heated. The effects of Ra , Ra , and Ta were examined.

Lee and Lin (1996) studied three-dimensional convection of air in a differentially heated vertical rotating cubic cavity. Significant flow modification was noted when $Ra_\omega > Ra$ or $Ta > Ra$, and effects of centrifugal and Coriolis forces on the thermal buoyancy driven flow

were examined in detail. Kim *et al.* (2000) reported numerically buoyant flow and heat transfer in a vertically mounted cup-like cylinder that rotates about the longitudinal axis. When the base surface is insulated, for the Richardson number larger than order one, the meridional flow weakens, the boundary layer-like character in the temperature field diminished, and the azimuthal flow approaches that of a rigid-body rotation. Edo *et al.* (2000) studied experimentally the influence of Coriolis force on heat transfer in a rotating transient boundary layer for a flat plate with nearly uniform heat flux applied to the surface. The results indicate that heat transfer is enhanced when Coriolis force acts towards the wall. The velocity measurements under equivalent conditions show that Coriolis instability induces counter-rotating longitudinal vortices that augment the lateral transport of the fluid on the pressure surface.

A numerical study is made by Yang and Wang (2003), Wang and Yang (2004) on fully developed bifurcation structure and stability of combined free and forced convection in a rotating curved duct of square cross-section. The solution is determined as the variation of a parameter indicating the magnitude of buoyancy force and steady solution is complicated with five types of physically realizable solutions identified numerically.

Zhang (2003) studied the steady rotating heat transfer with discrete heat sources in an enclosure where the rotating axis passes through the enclosure vertical centreline. It is found that centrifugal-buoyancy force has a more significant effect than that of Coriolis-buoyancy at low Ra^* . At high Ra^* , Coriolis force begins to play a part in heat transfer enhancement.

The studies that are directly related to present work and give precious guidance are summarized in Table 2.2.

Table 2.2: Survey of studies on natural convection in a rotating enclosure

No.	Authors (Year)	Aid	Fluid	Steady/unsteady	Heaters	Experimental studies	Numerical studies	Geometry
1	Buhler and Oertel (1982)	Thermal cellular convection, effects of the Coriolis and centrifugal forces	nitrogn, Silicone oil, Air,	<i>Steady convection</i>	None	$10^3 < Ra < 4 \times 10^3$, $10^2 < Ta < 10^3$ for oil $10^3 < Ra < 2 \times 10^4$, $10^2 < Ta < 10^3$ for Nitrogen	$(10^3 < Ra_c < 10^4, 0 < Ta < 2 \times 10^4)$	Rectangular enclosures rotating along vertical axis,
2	Hamady (1987)	Local and mean heat transfer characteristics, effect of various body forces	Air,	<i>Periodic laminar convection</i>	None	$10^4 < Ra < 3 \times 10^5$, $Ta < 10^4$	None	Rectangular enclosure with square cross section rotating along horizontal axis
3	Hamady et al. (1994)		Air	<i>Periodic laminar convection</i>	None	$10^4 < Ra < 3 \times 10^5$, $Ta < 10^4$	$10^4 < Ra < 3 \times 10^5$, $Ta < 4 \times 10^5$	
4	Lee and Lin (1996)	Flow structure, Effects of centrifugal and Coriolis forces	Air	<i>Steady convection</i>	None	None	$10^2 < Ra < 3 \times 10^7$, $10^2 < Ra_w < 3 \times 10^7$, $10^2 < Ta < 3 \times 10^7$	Cubic enclosure rotating along vertical axis
5	Zhang (2003)	Flow structure, effect of various body forces	FC71, water	<i>Steady convection</i>	3 × 3, F	$2 \times 10^5 < Ra^* < 10^6$, $4.5 \times 10^{-4} < E_k < 3.6 \times 10^{-3}$, $0.001 < Rg < 0.1$, $Pr=5$	$10^2 < Ra^* < 10^7$, $0.005 < E_k < 0.5$, $10^{-2} < Rg < 10$, $Pr=9$	Rectangular enclosure rotating along vertical axis
6	Present study	Flow pattern evolution, effect of various body forces	Air	<i>Periodic laminar convection</i>	1 × 1, F	$1 \times 10^3 < Ra^* < 1 \times 10^5$, $3 \times 10^2 < Ta < 4 \times 10^3$	$10^2 < Ra^* < 10^7$, $0 < Ta < 10^4$	Rectangular enclosures rotating along horizontal axis

2.5 Conclusions

Based on the literature survey, the following conclusions can be drawn concerning the state of knowledge about the study over natural convection from an array of discrete electronic components mounted to one wall of an enclosure:

- (1) Most early work in stationary cases was focused on the vertical orientation, both experimentally and numerically. However, detailed study including both numerical and experimental work is very limited in the horizontal orientation.
- (2) Most of the early studies on inclined enclosure with the simplest configuration used air ($Pr = 0.7$) as fluid. But in electronics cooling, it is necessary to study free convection using liquid coolants, such as water and fluorinert. And the effect of Pr in the inclined orientation has not been investigated comprehensively.
- (3) Even though linear stability theory has successfully predicted the onset of the R-B convection, it does not in principle predict the type of motion after onset, even in the simplest configuration. Numerical simulation is a good method and has been validated in some cases.
- (4) Periodic rotating heat transfer with discrete heat sources in an enclosure where the rotating axis passes through the enclosure horizontal centreline both numerically and experimentally has never been studied. Hence this is the originality of the present study.
- (5) The effects of multiple body forces, such as thermal buoyancy force, Coriolis force, centrifugal force coupled closely together and how much Coriolis and centrifugal forces affect heat transfer *are* not known. And it is the main contribution of the present work.
- (6) Optimal distribution of discrete heat sources for natural convection has been studied in the vertical orientation. Evaluation and optimization for the rotating convection heat transfer has never been studied that has potential industrial application.

Chapter 3

Model Formulation and Numerical Method

This chapter describes the derivation of the general governing equations for present laminar rotating fluid in the non-inertial Cartesian coordinates. The motion and energy transfer are governed by the buoyancy force, inertial force, friction force, centrifugal force, Coriolis force and the body forces are coupled closely. Numerical method is developed to solve the laminar governing equations in primitive variables. In the discretization of the physical domain a three-dimensional, non-uniform and staggered grid is used with a control volume formulation and incorporating the SIMPLE (Semi-Implicit Method for Pressure-Linked Equation) algorithm of Patankar (1980), with power law scheme for the difference of dependent variables in the discrete formulation. The discrete equations are solved by the line-by-line procedure, which is the combination of the Tri-Diagonal Matrix Algorithm (TDMA) and the Gauss-Seidal iteration technique. At last, a straightforward numerical method to sort out the effects of Coriolis force and centrifugal force that is impossible to do clearly in theoretical analysis and experimental studies is proposed firstly in this non-inertial frame.

3.1 Physical Model

Figure 3.1 illustrates a schematic sketch of the simulated rectangular enclosure, where one side wall is composed of an array of discrete, flush-mounted, isoflux heat sources and the opposite wall is treated as an isothermal surface and maintained at a lower temperature

whereas the remaining walls are assumed adiabatic. The enclosure, filled with air or liquid, can be rotated about the horizontal z-axis to alter its configuration set by the angle of inclination.

Figure 3.1: Schematic diagram of physical configuration and the coordinate system

The coordinate system is fixed onto the enclosure and serves as a non-inertia frame of reference. Therefore, the rotational axis passes through the enclosure center and the enclosure rotates along its longitudinal axis.

3.2 Mathematical model

3.2.1 General governing equations

The equations governing the motion and energy transfer in the enclosure are the conservations of mass, momentum and energy. The general descriptions of the rotating fluid

motion without chemical reaction and radiation absorption, are shown below (Boubnov and Golitsyn, 1995):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (3.1)$$

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\nabla \left(\frac{P}{\rho} - \frac{1}{2} [\vec{\Omega} \times \vec{r}]^2 \right) + \nabla \cdot (\nu \nabla \vec{V}) - 2[\vec{\Omega} \times \vec{V}] + \vec{F} \quad (3.2)$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + \vec{V} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) \quad (3.3)$$

where,

$\vec{V}$: the fluid particle velocity vector;	P : pressure;
ρ : density of the fluid;	ν : kinematic viscosity of the fluid;
$\vec{r}$:: the radius-vector of the fluid particle from the rotation axis;	
$\vec{\Omega}$: angular rotation vector;	$\vec{F}$: vector of mass forces;
C_p : specific heat capacity of the fluid	k : heat conductivity;

For natural convection, the Boussinesq approximation is used. The terms representing the thermal and rotational buoyancies and Coriolis force are, respectively, equal to $\rho_0 \vec{g} \beta \Delta T$, $-\rho_0 \beta \Delta T \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$ and $-2\rho_0 [1 - \beta \Delta T] \cdot \vec{\Omega} \times \vec{V}$. There is no clear boundary between turbulent and laminar flow in both numerical simulations and reality. Therefore, strict validations, including direct experimental validation and lots of comparison with literature data, are carried out throughout this thesis to ensure that present study fields are confined in laminar flow. The resulting laminar flow development can be described as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.4)$$

Momentum equations:

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = & -\frac{1}{\rho_0} \frac{\partial P_m}{\partial x} + \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \\ & + 2 \Omega v - 2 \Omega v \beta (T - T_c) - \Omega^2 x \beta (T - T_c) \\ & + g \beta (T - T_c) \sin(\Omega t) \end{aligned} \quad (3.5a)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = & -\frac{1}{\rho_0} \frac{\partial P_m}{\partial z} + \nu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] \\ & - 2 \Omega u + 2 \Omega u \beta (T - T_c) - \Omega^2 y \beta (T - T_c) \\ & + g \beta (T - T_c) \cos(\Omega t) \end{aligned} \quad (3.5b)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho_0} \frac{\partial P_m}{\partial z} + \nu \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] \quad (3.5c)$$

Energy equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \quad (3.6)$$

where P_m is the motion pressure defined as

$$P_m = p - \frac{1}{2} \rho_0 \Omega^2 x^2 - \frac{1}{2} \rho_0 \Omega^2 y^2 + \rho_0 g x \sin(\Omega t) + \rho_0 g y \cos(\Omega t) \quad (3.7)$$

Hence,

$$-\frac{\partial P_m}{\partial x} = -\frac{\partial p}{\partial x} + \rho_0 \Omega^2 x - \rho_0 g \sin(\Omega t) \quad (3.8a)$$

$$-\frac{\partial P_m}{\partial y} = -\frac{\partial p}{\partial y} + \rho_0 \Omega^2 y - \rho_0 g \cos(\Omega t) \quad (3.8b)$$

$$-\frac{\partial P_m}{\partial z} = -\frac{\partial p}{\partial z} \quad (3.8c)$$

3.2.2 Non-dimensional governing equations

Using L , α/L , L^2/α , $\alpha^2 \rho/L^2$ and $q''L/k$ as scales for length, velocity, time, pressure and temperature respectively, and defining the dimensionless variables

$$X = \frac{x}{L} \quad Y = \frac{y}{L} \quad Z = \frac{z}{L} \quad (3.9a)$$

$$U = \frac{uL}{\alpha} \quad V = \frac{vL}{\alpha} \quad W = \frac{wL}{\alpha} \quad (3.9b)$$

$$\tau = \frac{t\alpha}{L^2} \quad P = \frac{P_m L^2}{\alpha^2 \rho} \quad \theta = \frac{T - T_c}{q''L/k} \quad (3.9c)$$

$$Ra_\omega = \frac{\beta \Omega^2 L^5 q''}{k \alpha \nu} \quad Ra^* = \frac{g \beta L^4 q''}{k \alpha \nu} \quad (3.9d)$$

$$Ta = \frac{4 \Omega^2 L^4}{\nu^2} \quad Pr = \frac{\nu}{\alpha} \quad (3.9e)$$

Ta , Ra_ω , Ra are widely used to reflect the effect of Coriolis force, centrifugal force and buoyancy force respectively. After coolant is determined, Prandtl number (Pr) is fixed and heat flux and rotation speed are independent variables, which are reflected by modified Rayleigh number (Ra^*) and Taylor number (Ta) respectively. Due to Rotation Rayleigh number (Ra_ω) not being an independent parameter but depending on Ra , Pr , and Ta , we will not discuss Ra_ω except to specify it explicitly. In some literature and experimental studies, Ra is used and it equals Ra^* divided by Nu . The Coriolis buoyancy force is neglected, due to $|\beta(T - T_c)| \ll 1$ in present studies. Then the dimensionless governing equations are shown below:

Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \quad (3.10)$$

Momentum equations:

$$\begin{aligned} \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} = & -\frac{\partial P}{\partial X} + Pr \left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right] \\ & + \underbrace{Ta^{0.5} Pr V}_{\text{Coriolis force term}} - \underbrace{Ra_{\omega} Pr X \theta}_{\text{Centrifugal force term}} \\ & + \underbrace{Ra * Pr \theta \sin(0.5 Ta^{0.5} Pr \tau)}_{\text{Buoyancy force term}} \end{aligned} \quad (3.11a)$$

$$\begin{aligned} \frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} = & -\frac{\partial P}{\partial Y} + Pr \left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right] \\ & - \underbrace{Ta^{0.5} Pr U}_{\text{Coriolis force term}} - \underbrace{Ra_{\omega} Pr Y \theta}_{\text{Centrifugal force term}} \\ & + \underbrace{Ra * Pr \theta \cos(0.5 Ta^{0.5} Pr \tau)}_{\text{Buoyancy force term}} \end{aligned} \quad (3.11b)$$

$$\frac{\partial W}{\partial \tau} + U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} = -\frac{\partial P}{\partial Z} + Pr \left[\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right] \quad (3.11c)$$

Energy equation:

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \quad (3.12)$$

For stationary case, Ω , Ta , Ra_{ω} , $\partial u / \partial t$, $\partial v / \partial t$, $\partial w / \partial t$, $\partial U / \partial \tau$, $\partial V / \partial \tau$,

$\partial W / \partial \tau$ are equal to zero and the general rotating model evolves into the stationary model.

By neglecting variables in Z direction, i.e., $w, \partial w / \partial z, \partial P_m / \partial z, \partial^2 u / \partial z^2, \partial^2 v / \partial z^2, \partial^2 w / \partial z^2, \partial^2 T / \partial z^2, W, \partial W / \partial Z, \partial P / \partial Z, \partial^2 U / \partial Z^2, \partial^2 V / \partial Z^2, \partial^2 W / \partial Z^2$, and $\partial^2 \theta / \partial Z^2$ are equal to zero, the general 3-D model is simplified as 2-D model. In stationary situation, steady 3-D model with 3×3 discrete heaters are developed. However, for the rotating case, 2-D model with 3×1 heaters are used to avoid computation difficulty caused by unsteady flow and temperature fields. The geometrical parameters and boundary conditions are changed according to different study objectives and described in each chapter later.

To examining the time evolution of the velocity and temperature fields, results for local, space-averaged and time-space-averaged Nusselt number ($Nu, \overline{Nu}, \overline{\overline{Nu}}$) can be evaluated from

$$Nu(Y) \Big|_{\phi=\phi} = \frac{I}{\theta(Y,Z)} \Big|_{\phi=\phi} = \frac{q'' L}{K \cdot [T(Y,Z) - T_c]} \Big|_{\phi=\phi} \quad (3.13a)$$

$$\overline{Nu}_i \Big|_{\phi=\phi} = \frac{q'' L}{\iint_{A_h} K \cdot [T(Y,Z) - T_c] dYdZ} \Big|_{\phi=\phi} \quad (i=1,2,3) \quad (3.13b)$$

$$\overline{\overline{Nu}} \Big|_{\phi=\phi} = \left(\sum_{i=1}^3 \frac{q'' L}{\iint_{A_h} K \cdot [T(Y,Z) - T_c] dYdZ} \Big|_{\phi=\phi} \right) / 3 \quad (3.13c)$$

$$\overline{\overline{\overline{Nu}}}_i = \frac{\int_0^{2n\pi} Nu_i d\phi}{2n\pi} \quad (3.13d)$$

$$\overline{\overline{\overline{Nu}}} = \left(\sum_{i=1}^3 \overline{\overline{\overline{Nu}}}_i \right) / 3 \quad (\text{averaged over one cycle}) \quad (3.13f)$$

3.3 Numerical methods

3.3.1 Discretized equations

In the present model, the flow is assumed incompressible and the fluid properties are constant. The Boussinesq approximation is also assumed to be applicable. It follows that the governing equation for a general variable transported by convection and diffusion in unsteady laminar flow and can be expressed in a general form known as the convection-diffusion equation:

$$\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho u\phi)}{\partial x} + \frac{\partial(\rho v\phi)}{\partial y} + \frac{\partial(\rho w\phi)}{\partial z} = \frac{\partial}{\partial x}(\Gamma_\phi \frac{\partial\phi}{\partial x}) + \frac{\partial}{\partial y}(\Gamma_\phi \frac{\partial\phi}{\partial y}) + \frac{\partial}{\partial z}(\Gamma_\phi \frac{\partial\phi}{\partial z}) + S_\phi \quad (3.14)$$

where the general variable ϕ stands for different dependent variables (u, v, w, T), while Γ_ϕ and S_ϕ represent the appropriate transport coefficients and source terms, respectively.

The three-dimensional form of Eq. (3.14) can be written as

$$\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = S_\phi \quad (3.15)$$

where J_x, J_y , and J_z are the total (convection plus diffusion) fluxes defined by

$$J_x = \rho u\phi - \Gamma_\phi \frac{\partial\phi}{\partial x} \quad (3.16a)$$

$$J_y = \rho v\phi - \Gamma_\phi \frac{\partial\phi}{\partial y} \quad (3.16b)$$

$$J_z = \rho w\phi - \Gamma_\phi \frac{\partial\phi}{\partial z} \quad (3.16c)$$

The integration of equation (3.10) over the control volume would give:

$$\frac{(\rho\phi)_P - (\rho\phi)_P^0}{\Delta t} \Delta x \Delta y \Delta z + J_e - J_w + J_n - J_s + J_f - J_b = S \Delta x \Delta y \Delta z = (S_c + S_p \phi_P) \Delta x \Delta y \Delta z \quad (3.17)$$

where the source term has been linearized in the usual manner, the quantities J_e, J_w, J_n, J_s, J_f , and J_b are the integrated total fluxes over the control-volume faces.

In a similar manner, we can integrate the continuity equation (3.10) over the control volume and obtain:

$$F_e - F_w + F_n - F_s + F_f - F_b = 0, \quad (3.18)$$

where F_e , F_w , F_n , F_s , F_f and F_b are the mass flow rates through the faces of the control volume. If pu at point e is taken to prevail over the whole interface e, we can write

$$F_e = (\rho u)_e \Delta y \Delta z, \quad (3.19a)$$

Similarly

$$F_w = (\rho u)_w \Delta y \Delta z, \quad (3.19b)$$

$$F_n = (\rho v)_n \Delta x \Delta z, \quad (3.19c)$$

$$F_s = (\rho v)_s \Delta x \Delta z, \quad (3.19d)$$

$$F_f = (\rho w)_f \Delta x \Delta y, \quad (3.19e)$$

$$F_b = (\rho w)_b \Delta x \Delta y. \quad (3.19f)$$

If we now multiply equation (3.18) by ϕ_p and subtract it from equation (3.17), we obtain

$$\begin{aligned} & \frac{\rho_p \phi_p - \rho_p \phi_p^0}{\Delta t} \Delta x \Delta y \Delta z - (J_e - F_e \phi_p) - (J_w - F_w \phi_p) + (J_n - F_n \phi_p) - (J_s - F_s \phi_p) \\ & + (J_f - F_f \phi_p) - (J_b - F_b \phi_p) = (S_C + S_p \phi_p) \Delta x \Delta y \Delta z \end{aligned} \quad (3.20)$$

The terms of $(J - F\phi)$ can be expressed as:

$$J_e - F_e \phi_p = a_E (\phi_p - \phi_E) \quad (3.21a)$$

$$J_w - F_w \phi_p = a_W (\phi_W - \phi_p) \quad (3.21b)$$

$$J_n - F_n \phi_p = a_N (\phi_p - \phi_N) \quad (3.21c)$$

$$J_s - F_s \phi_p = a_S (\phi_S - \phi_p) \quad (3.21d)$$

$$J_f - F_f \phi_p = a_F (\phi_F - \phi_p) \quad (3.21e)$$

$$J_b - F_b \phi_p = a_B (\phi_B - \phi_p) \quad (3.21f)$$

The three-dimensional discretization equation based on the general differential equation (3.14) can now be written as:

$$a_P \phi_P = a_E \phi_E + a_W \phi_W + a_N \phi_N + a_S \phi_S + a_F \phi_F + a_B \phi_B + b \quad (3.22)$$

where

$$a_E = D_e A(P_e) + \left[-F_e, 0 \right] \quad (3.23a)$$

$$a_W = D_w A(P_w) + \left[F_w, 0 \right], \quad (3.23b)$$

$$a_N = D_n A(P_n) + \left[-F_n, 0 \right], \quad (3.23c)$$

$$a_S = D_s A(P_s) + \left[F_s, 0 \right], \quad (3.23d)$$

$$a_F = D_f A(P_f) + \left[-F_f, 0 \right], \quad (3.23e)$$

$$a_B = D_b A(P_b) + \left[F_b, 0 \right], \quad (3.23f)$$

$$b = S_C \Delta x \Delta y \Delta z + a_p^0 \phi_p^0 \quad (3.23g)$$

$$a_p^0 = \frac{\rho_P \cdot \Delta v}{\Delta t} = \frac{\rho_P \Delta x \Delta y \Delta z}{\Delta t} \quad (3.23h)$$

$$a_P = a_E + a_W + a_N + a_S + a_F + a_B - S_P \Delta x \Delta y \Delta z + a_p^0 \quad (3.23i)$$

In the above, the corresponding conductances are defined by

$$\begin{aligned} D_e &= \frac{\Gamma_e \Delta y \Delta z}{(\delta x)_e}, \quad D_w = \frac{\Gamma_w \Delta y \Delta z}{(\delta x)_w}, \quad D_n = \frac{\Gamma_n \Delta x \Delta z}{(\delta y)_n}, \\ D_s &= \frac{\Gamma_s \Delta x \Delta z}{(\delta y)_s}, \quad D_f = \frac{\Gamma_f \Delta x \Delta y}{(\delta z)_f}, \quad D_b = \frac{\Gamma_b \Delta x \Delta y}{(\delta z)_b} \end{aligned} \quad (3.24)$$

and the Peclet number expresses the ratio of the strengths of convection to diffusion:

$$P_e = \frac{F_e}{D_e}, \quad P_w = \frac{F_w}{D_w}, \quad P_n = \frac{F_n}{D_n}, \quad P_s = \frac{F_s}{D_s}, \quad P_f = \frac{F_f}{D_f}, \quad P_b = \frac{F_b}{D_b} \quad (3.25)$$

In present problem, the sign of the source terms in the momentum equations is uncertain for each volume in different time stage. Therefore, the source term in Equ. 3.22 only has constant part and S_p equals to zero.

In general, there are five schemes, namely central difference, upwind, hybrid, exponential and power-law for formulation of convection diffusion. The power-law scheme is adopted in the present study as in the following:

$$A(|P|) = \begin{cases} 0, & (1 - 0.1|P|)^5 \end{cases} \quad (3.26)$$

3.3.2 The Staggered Grid and Momentum Equations

A staggered grid is defined as velocity components u , v , w , are stored in three different grid points. Pressure, temperature and the fluid properties are stored in the main grid point P (Fig. 3.2a). The velocity components are calculated for the points that lie on the surface of the main control volume. Thus the x -direction velocity component u is stored at the faces that are normal to the x -direction. Similarly, the y -direction velocity component v is stored at the faces that are normal to the y -direction, and the z -direction velocity component w is stored at the faces that are normal to the z -direction. The control volume for u is staggered on a half grid in the x -direction (Fig. 3.2b) and the control volume for v is staggered on a half grid in the y -direction (Fig. 3.1c), and the same for w in the z -direction.

If we focus our attention on the locations for u only, there is nothing unusual about this control volume. Its faces lie between the point e and the corresponding locations for the neighboring u velocities. The control volume is, however, staggered in relation to the normal control volume around the main grid point P . The staggering is in the x -direction only, such that the faces normal to that direction pass through the main grid points P and E . This layout realizes one of the main advantages of the staggered grid. The difference $p_P - p_E$ can be

used to calculate the pressure force acting on the control volume for the velocity u . The resulting discretization equation can be written as

(a) Main control volume

(b) Control volume for u

○ the main point
→ the grid point of u
↑ the grid point of v

(c) Control volume for v

Fig. 3.2: The staggered grid

$$a_e u_e = \sum a_{nb} u_{nb} + b + (p_P - p_E) A_e \quad (3.27)$$

For the three-dimensional situation, six neighbor u 's would be included outside the control volume. The neighbor coefficients a_{nb} account for the combined convection-diffusion influence at the control-volumes. The term b is defined in the same manner as in Eq. (3.23g), but the pressure gradient is not included in the source-term S_C and S_p . The term $(p_P - p_E) A_e$ is the pressure force acting on the u control volume, $A_e = \Delta y \Delta z$ is the area upon which the pressure difference acts.

The momentum equations for the other directions are treated in a similar manner. The discretization equations for v_n and w_f can be seen to be

$$a_n v_n = \sum a_{nb} v_{nb} + b + (p_P - p_N) A_n, \quad (3.28)$$

$$a_f w_f = \sum a_{nb} w_{nb} + b + (p_P - p_F) A_f \quad (3.29)$$

3.3.3 Pressure-Correction and Velocity-Correction

The momentum equations can be solved only when the pressure field is given or is somehow estimated. Unless the correct pressure field is employed, the resulting velocity field will not satisfy the continuity equation. Such an imperfect velocity field based on a guessed pressure field p^* will be denoted by u^* , v^* , w^* . This “starred” velocity field will result from the solution of the following discretization equations:

$$a_e u_e^* = \sum a_{nb} u_{nb}^* + b + (p_P^* - p_E^*) A_e \quad (3.30a)$$

$$a_n v_n^* = \sum a_{nb} v_{nb}^* + b + (p_P^* - p_N^*) A_n \quad (3.30b)$$

$$a_f w_f^* = \sum a_{nb} w_{nb}^* + b + (p_P^* - p_F^*) A_f \quad (3.30c)$$

In order to improve on the guessed pressure p^* such that the resulting starred velocity field will progressively get closer to satisfying the continuity equation, it is proposed that the correct pressure p is obtained from

$$p = p^* + p', \quad (3.31)$$

where p' will be called the pressure correction. The corresponding velocity corrections u' , v' , w' can be introduced in a similar manner:

$$u = u^* + u', \quad v = v^* + v', \quad w = w^* + w' \quad (3.32)$$

Subtracting Eq. (3.30a) from Eq. (3.27), one has

$$a_e u_e' = \sum_{nb} a_{nb} u_{nb}' + (p_P' - p_E') A_e \quad (3.33)$$

If the effect of velocity corrections of neighbor on the velocity correction u_e' are neglected, the result is

$$a_e u_e' = (p_P' - p_E') A_e \quad (3.34)$$

$$u_e' = \frac{A_e}{a_e} (p_P' - p_E') = d_e (p_P' - p_E') \quad (3.35)$$

Equation (3.35) is called the velocity-correction formula, which can also be written as

$$u_e = u_e^* + d_e (p_P' - p_E'), \quad (3.36)$$

This shows how the starred velocity u_e^* is to be corrected in response to the pressure corrections to produce u_e .

The correction formulas for the velocity components in the other directions can be written similarly:

$$v_n = v_n^* + d_n (p_P' - p_N') \quad (3.37)$$

$$w_f = w_f^* + d_f (p_P' - p_F') \quad (3.38)$$

Thus, all the preparation needed for obtaining a discretization equation for p' is complete.

The integrated form of the continuity equation (3-1) becomes

$$\begin{aligned} & [(\rho u)_e - (\rho u)_w] \Delta y \Delta z + [(\rho v)_n - (\rho v)_s] \Delta x \Delta z \\ & + [(\rho w)_f - (\rho w)_b] \Delta x \Delta y = 0. \end{aligned} \quad (3.39)$$

By substituting all the velocity components by the velocity-correction formulas, we obtain the following discretization equation for p' :

$$a_P p'_P = a_E p'_E + a_W p'_W + a_N p'_N + a_S p'_S + a_F p'_F + a_B p'_B + b \quad (3.40)$$

where

$$a_E = \rho_e d_e \Delta y \Delta z \quad (3.41a)$$

$$a_W = \rho_w d_w \Delta y \Delta z \quad (3.41b)$$

$$a_N = \rho_n d_n \Delta x \Delta z \quad (3.41c)$$

$$a_S = \rho_s d_s \Delta x \Delta z \quad (3.41d)$$

$$a_F = \rho_f d_f \Delta x \Delta y \quad (3.41e)$$

$$a_B = \rho_b d_b \Delta x \Delta y \quad (3.41f)$$

$$a_P = a_E + a_W + a_N + a_S + a_F + a_B \quad (3.41g)$$

$$b = [(\rho u^*)_w - (\rho u^*)_e] \Delta y \Delta z + [(\rho v^*)_s - (\rho v^*)_n] \Delta x \Delta z + [(\rho w^*)_f - (\rho w^*)_b] \Delta x \Delta y \quad (3.41h)$$

Since the values of the density ρ will normally be available only at the main grid points, the interface densities such as ρ_e may be calculated by any convenient interpolation. Whatever the method of interpolation, the value of ρ_e must be consistently used for the two control volumes to which the interface belongs to.

The term b represents a "mass source". If b is zero, it means that the starred velocities do satisfy the continuity equation, and no pressure correction is needed. This condition is employed to judge if the simulation result is converging. The above formulation completes all the needed equations for obtaining the velocity components and pressure.

The above set of discretization equations for each variable is solved by the line-by-line procedure, which is the combination of the Tri-Diagonal Matrix Algorithm (TDMA) and the Gauss-Seidal iteration technique with additional block-corrected method for fast convergence.

The SIMPLE algorithm consists of solving the pressure equation to obtain the pressure field and solving the pressure-correction equation only to correct the velocities. The sequence of operations is as follows:

1. Guess the pressure field p^* or use the previous convergent result.
2. Solve the momentum equations, such as Eqs. (3.30a,b,c), to obtain u^*, v^*, w^* .
3. Solve the p' equation.
4. Calculate p from Eq. (3.31) by adding p' to p^* .
5. Calculate u, v, w from their starred values using the velocity-correction formulas (3.36) to (3.38).
6. Solve the discretization equation for other ϕ' 's (such as temperature equation) because it influences the flow field through fluid properties, source terms, etc. (eg. for natural and **mixed** convection).
7. Treat the corrected pressure p as a new guessed pressure p^* , return to step 2, and repeat the whole procedure until a converged solution is obtained.

3.3.4 Under-relaxation

Under-relaxation is a very useful device for nonlinear problems. It is employed to avoid divergence in the iterative solution of a strongly nonlinear phenomena. In this respect, a weighted average of the newly calculated value and the previous value are taken at each point. $\tilde{\phi}^{n+1}$ represents the newly calculated value of ϕ at the $n+1$ iteration. It is obtained from

$$a_P^\phi \tilde{\phi}_P^{n+1} = \sum_{nb} a_{nb}^\phi \phi_{nb}^n + S_\phi$$

where $\sum$ = sum over N, S, E, W, F,

The weighted average value is

$$\phi_p^{n+1} = \alpha \tilde{\phi}_p^{n+1} + (1 - \alpha) \phi_p^n \quad (3.43)$$

where $\tilde{\phi}_p^{n+1}$ represents the under-relaxed value of ϕ_p at the $n+1$ iteration and α ($0 < \alpha \leq 1$) is the under-relaxation parameter. The under-relaxed value is calculated from:

$$\left(\frac{a_p^\phi}{\alpha} \right) \phi_p^{n+1} = \sum_{nb} a_{nb} \phi_{nb}^n + \left[S_c^\phi + (1 - \alpha) \frac{a_p^\phi}{\alpha} \phi_p^n \right] \quad (3.44)$$

The effect of under-relaxation factor on convergence rate is considerable. Unacceptably slow convergence or divergence of the solution is obtained if the factors are too low or too high, respectively. To solve the momentum equation, it is suggested to under-relax u^*, v^*, w^* with a relaxation factor α about 0.3-0.5 (depending on the computation cases). The other under-relaxation factors for the pressure and temperature are set to 0.5 and 0.4-0.6, respectively.

3.3.5 Solution Procedures

A numerical simulation program is developed with Compaq Fortran 6.5, as shown in Fig. 3.3. The main program is responsible to set up grids, geometrical parameters and initial boundary conditions, read data from file or write data to file, assign the value of iteration, calculate time and call subroutine, TIMITR, to get a convergent result in each time step. Integer variables, ISTEP and ITER, represent the time iteration number and maximum iteration number in each time step respectively. Stationary cases are treated as special cases whose rotation speed equals to zero. The time step, DT, is assigned to large value, such as 10^{20} , and only one Timitr iteration is required. For conjugate heat transfer, the geometrical domain is consisted of solid zones and fluid zone. Numerical solutions are obtained in both regions simultaneously by solving the continuity, momentum, and energy equations in the entire computational domain. By specifying the appropriate thermophysical properties in each region, the governing equations are reduced to the proper forms for each of the regions. A

degenerate form of the momentum equations, obtained by assign very large viscosities, is solved in the solid region (Patankar 1978, Heindel *et al.* 1995c,d).

The parameters of geometry are given or calculated, and the initial values of u, v, w, p, p', b, a_p are set in SETUP1. The coefficients of the equations are established in SETUP2, where three routines, SOLVE, DIFLOW, and GAMSOR are called. The SOLVE routine is called upon to solve the equations. In the SOLVE block, the equations are solved by using of TDMA. In DIFLOW, the formulation of the convection diffusion $A(|P|)$ is calculated. The transport coefficient Γ_ϕ and source term S_ϕ are set for u, v, w, ϕ in GAMSOR. Uniformed grid is generated in UGRID, and results are printed in OUTPUT1. The process of calculation can be shown in the screen by OUTPUT. The grid points and surface of control volume are generated in GRID. The initial conditions are set up for the unsteady problem, whereas the initial values are assumed for a steady problem in BEGIN. The density of fluid is set in DENSE while the boundary conditions are written in BOUND. UPDATE will refresh the data with the new computation results. GTIME can change the time step when necessary. The flow chart is shown in Figure 3.3 illustrating the over-all computational scheme.

Fig. 3.3 Flow chart of the computer program

3.3.6 Convergence Criterion

Convergence of the computational scheme is declared when the following convergence criterion is satisfied for each steady-state result:

$$S_{\max} < 10^{-8} \quad \text{for velocity } u, v, w \quad (3.45)$$

$$\frac{|\phi_{ijk}^{n+1} - \phi_{ijk}^n|_{\max}}{|\phi_{ijk}^{n+1}|_{\max}} \leq 10^{-4} \quad \text{for temperature} \quad (3.46)$$

where SMAX is the largest absolute value of the “mass source” used in the motion pressure correction equations and ϕ represents any dependent variable, and n referred to the value of ϕ at the n th iteration level. However, the use of the maximum changes of the variables as convergence criteria has to be used with caution. Pantankar (1980) pointed out that if under-relaxation is being employed it may not be sufficient by monitoring the changes in dependent variables from iteration to iteration. To reduce the likelihood of false convergence related to small but persistent changes between successive iteration due to under-relaxation, an examination scheme is also implemented between iteration Z and $1-20$. Although this comparison method does not eliminate the possibility of false convergence completely, the method can nevertheless reduce it. In the present study, strong under-relaxation is required for the velocities, particularly on the W-component because of the strong source term resulted from the buoyancy. On the other hand, temperature required only minor under-relaxation. As a result, the speed of the velocities convergence much slower than the convergent speed of the temperature. Typically, the convergence criterion of temperature is the first to be satisfied, and is followed by the W-component being the last to be satisfied. For each Ra^* , the converged solution for a lower Ra^* is used as the initial guess for the next around of computation to reduce the computational times.

At higher Ra^* , It was found that severe under-relaxation was required to obtain convergence. To determine whether the converged solutions corresponded to physically steady solutions, an unsteady version of the code has been used to check the results at the highest Ra^* , 10^7 and 10^8 . The unsteady code is fully implicit in time, and used the same iteration scheme as the steady version of the code to obtain convergence within a **given** time step. Convergence for a given time step is assumed when the changes in temperature and velocities between successive iterations, for all grid points, is less than **0.001x** the maximum value of the dependent variables at that time step. The convergence criterion for the steady

state is that the changes in all dependent variables between successive time steps, for all grid points, is less than $0.0001 \times$ the maximum value of the dependent variables.

3.3.7 Model validation

The numerical results from Heindel *et al.* (1995a) are used for the first validation for $Ra^* = 10^5 - 10^8$, $Pr = 5, 25$. By employing a similar (but not identical) $55 \times 31 \times 91$ non-uniform grid, and using the same symmetry assumption, the results of Nu are within 4.5%. The row-averaged Nu at different Ra^* are given in Table 3.1.

Table 3.1: Comparisons of the row-averaged Nusselt numbers with those of Heindel *et al.* (1995a) for $Pr = 5.0$ (water)

Nu	$Ra^* = 10^5$			$Ra^* = 10^7$			$Ra^* = 10^8$		
Rows	3	2	1	3	2	1	3	2	1
Present computation	4.337	5.236	5.819	11.07	12.19	14.87	16.37	19.59	21.57
Heindel <i>et al.</i>	4.298	4.948	6.018	10.80	12.43	15.12	17.11	19.70	23.96
Percent	0.91	0.58	3.3	2.5	1.93	1.65	4.32	0.56	2.54

Considering the complexities of the physical and small scale structure involved in R-B convection, RNG k- ϵ turbulent simulation provide by FLUENT 5.5 is used to check the flow model ($Ra^*=4.6 \times 10^8$). The maximum error for temperature field between laminar model and turbulent model is less than 2%. Study of Fernando and Smith (2001) shows that constant flux boundary condition makes cell patterns much flatter than constant temperature boundary conditions and results that flow appears to become turbulent as high as $Ra > 10^8$. Together with comparison with experimental flow visuality of Poleniti *et al.* (1993), the laminar model is proper in present study fields ($Ra^* \leq 1 \times 10^9$). On the other hand, rotation increase flow

stability. Therefore, the rotating studied fields ($10^3 \leq Ra \leq 10^7$, $0 \leq Ta \leq 10^4$, $Pr = 0.7$) are confined within laminar flow.

The further validation for rotation case will be shown in the later chapters. Compared with forced convection, present velocity gradient for air is very low because the maximum velocity magnitude is about 10^{-2} m/s and the flow is not boundary layer flow. However, the temperature gradient is relatively high and the rotating periodic temperature fields are easier to measured and available in the paper of Hamady *et al.* (1994). Therefore, validation of the program and most discussion are based on temperature results.

3.4 Method to sort out the effects of Coriolis and centrifugal forces

Theoretical analysis to sort out the effects of various body forces is an important tool as shown in Greenspan (1968), Vanyo (1993). Motion pressure, shown in Equ. 3.7, is not only useful to simplify the governing equations but also beneficial to sort out the effect of centrifugal force. When the density is constant or its change is small and rotation speed is slow enough [$(\omega^2 r/g) \ll 1$, Greenspan, 1968], there is no difference for flow field and temperature field whether there is centrifugal force term in the governing equations, as shown in the work of Hamady *et al.* (1994).

In many studies, Ra , Ra , Ta , E_k are widely used to reflect effects of various body forces (Hamady *et al.*, 1994, Zhang, 2003). A worthwhile observation concerns the very meaning of dimensionless number such as Ra , Ta , E_k . For example, we are often told that the Gr can be interpreted as the parameter describing the ratio of buoyancy to viscous forces in the natural convection boundary layer. To see error in this interpretation, consider the natural convection of water ($Pr=5$) along the cold vertical wall in the enclosure, where $Gr \sim 10^8$ - 10^9 ($Ra \sim 2 \times 10^7$ - 2×10^8): according to the above interpretation, the viscous forces must be neglected in comparison with the body force because the Gr is enormous visa unity. This is

certainly not true. For example, in the case of water, there exists a balance between friction and buoyancy: without a balance of forces, the wall jet cannot exist in the steady state. By themselves, dimensionless numbers have no meaning (Bejan, 1995). On the other hand, among the three independent dimensionless parameters (Ra, Pr, Tu) in present problem, Ta is the only parameter related to rotation speed Ω and can be used to gauge the effect of body forces that are related to rotation. Unfortunately, there are three body forces, inertial, Coriolis and centrifugal forces, that are all related to rotation.

Therefore, it is not proper to study the effects of the body forces by studying the above dimensionless numbers. However, this can be resolved by means of a straightforward numerical method, shown in Table 3.2. To study the effect of centrifugal force and Coriolis force, respective terms in the momentum equations are neglected in numerical simulations. In the non-inertial frame, buoyancy force, rotating in the enclosure, is the driving force that must always be included in simulations, with Ra^* reflecting the effect of buoyancy force.

Table 3.2: Cases for rotation simulations

Case	Rotation	Terms neglected	Purpose
1	No		Reference case.
2	Yes	None	All effects included. Real case.
3	Yes	Centrifugal force and Coriolis force	To study the effects of inertial force.
4	Yes	Centrifugal force	To study the effects of Coriolis force.
5	Yes	Coriolis force	To study the effects of centrifugal force.

Chapter 4

Experimental Studies

The experimental setup and data post-process methodology are described in this chapter. Due to rotation, some special instruments and methodology have been used. The most difficult challenge is to keep thermocouple work perfectly at the same time because there are more than 70 solder joints in 12 thermocouples in the rotation situation. Experimental results are used to validate the numerical simulations.

4.1 Experimental setup

Figure 4.1 shows the experimental setup, consisting of a test cell, a mounting apparatus and associated instruments for system control and collecting data. The main components required in present experimental study are listed in Table 4.1.

Fig. 4.1: Schematic of the test facility

Table 4.1 List of the main experimental equipment

1	Test cell 1	Customized	1 set	3×1 heaters
2	Test cell 2	Customized	1 set	3×3 heaters
3	Test bed	Customized	1 set	
4	Slip rings	6118-111-8	3 sets	Lebow Associates, Inc.
5	Servo motor	NPSA-Z	1 set	Nikki Denso Pte. Ltd.
5	Servo driver	P064036	1 set	Nikki Denso Pte. Ltd.
6	Data Acquisition/Switch Unit	HP 34970A	1 set	Hewlett Packard Co. Ltd.
7	Variable Voltage Transfer	B-1 300VA	3 sets	Voltac Co. Ltd.
8	Fan	109S025	1 set	Sanyo Denki Co. Ltd.
9	Voltmeter	J&D 1000	3 sets	Dakin Engineering Pte. Ltd.
10	Ammeter	J&D 1000	3 sets	Dakin Engineering Pte. Ltd.
11	Tachometer	TM-4010	1 set	Line Seiki Co. Ltd.
12	Thermocouple	K	12 sets	
13	Thermocouple Calibration	18B	1 set	ThermaCal Inc. Pte. Ltd.
14	Computer	300PL	1 set	IBM Co. Ltd.
15	Data collection Software	HP Benchlink Data logger	1 set	Hewlett Packard Co. Ltd.
16	Data postprocessing software	Customized		

4.1.1 Test cell

In present study, two test cells with a 3 x 1 (counter-clockwise rotation) or 3 x 3 (inclined orientations) heater array, are manufactured and installed to compare with 2-D analysis and 3-D analysis respectively. The coordinate system and the definition of inclined angle are shown in Fig. 4.2.

Table 4.2: Two test cells and their objectives

	Heater block of the test cell		Coolant	Study fields		Purpose
	Number	Length×width×height		Stationary	Rotation	
1.	3×1	76.2×12.7×9 mm ³	air	0 rpm	2 ~ 30 rpm	2-D analysis
2.	3×3	12.7×12.7×9 mm ³	water	0 rpm		3-D analysis

Table 4.3: Testing conditions and measurement variables

No.	Heaters	Coolant	Testing boundary conditions	Rotation /Inclined	Measurement variables				
					Input power		Thermocouples		
					Voltmeter	Ammeter	Heaters	Substrate	Cold wall
1	3×1	Air	Conjugate	Inclined angle	1	1	3	6	3
2	3×3	Water	Constant heat flux	Rotation speed	9	9	9	3	3

Fig. 4.2: Coordinate system and inclined angle ϕ

(1) The test cell with a 3x1 heater array

This test cell is used for both stationary and rotation experimental study with air. As shown schematically in Figs. 4.3 and 4.4, it consists of a substrate plate (Heaters wall), an opposing cold plate (Isothermal wall), a side wall, two drive walls compressed between two clamping plates by 6 bolts passing through each corner. The material of the substrate plate is Teflon for strength at the experimental temperature and low thermal conductivity ($k=0.35 \text{ W/mK}$) acted as insulate substrate plate. The side wall is made of Lexan ($k=0.2 \text{ W/mK}$), a semi-transparent material with low thermal conductivity.

Fig.4.3: Schematic of the test cell with a 3x1 heater array

The substrate plate provides a support for holding a 3×1 array of $76.2 \times 12.7 \times 9 \text{ mm}^3$ oxygen-free copper blocks spaced at 3.2mm apart. From the engineering view-port, the most important characteristic of the flow is the rate of heat transfer across each heater. As shown in the Fig.4.4, each back is instrumented with one K-type thermocouple located in holes drilled from the front surface of the back. Thermal grease is used to reduce the effect of thermal contact resistance in the interfaces. To provide an iso-flux heat input, three thin plate resistive heaters ($76 \times 10 \times 2 \text{ mm}^3$) are soldered onto the back of each copper block. The heaters are carefully selected such that the variations in their resistance are less than 2 percent. At last, except the cold plate, the test cell is bounded with sponge, an insulation cover ($k=0.023 \text{ W/mK}$) with 20 mm width, to further minimize heat loss. The cold plate opposite to the heaters is made of copper, which due to its large thermal conductivity ($k=398 \text{ W/m.K}$) provided a near-uniform plate temperature. A fan is used to increase the heat transfer between the copper plate and environment, thus keeping a constant plate temperature.

Fig.4.4: Schematic of the thermocouple arrangement in the test cell

with a 3 x 1 heater array

The side wall locates between the heater plate and the cold plate. The frame-shaped lexan wall is used as the side wall of the enclosure and also controls the spacing between the heater plate and cold plate. At the both sides of test cell, there are two drive walls, used to connect the test cell and the shaft. Following the motion of the shaft, the test cell is allowed for variation of the test cell orientation.

The thermocouple arrangement is shown in Fig.4.4, indicated by a circle and a number nearby. In order to obtain accurate and reliable results, thermocouples are calibrated and tested. The maximum error associated with each thermocouples reading is estimated to be less than 0.3°C . Thermocouples no. 1–3 are embedded in the copper blocks to monitor the surface temperature of each copper block. Thermocouples no. 4–9 are used to monitor the temperature distribution in the substrate. The other three thermocouples are used to monitor

the surface temperature distribution of the isothermal wall. The temperature variation in the experimental tests is found to be less than 0.3°C.

Experimental and numerical studies of Heindel *et al.* (1995a,c) show that when the aspect ratio of heater (A_{thr}) is larger than 3, the difference of row-averaged Nu between 2-D analysis and 3-D analysis are less than 5% in stationary orientation. Because the value of present A_{thr} equals to 6.0 and the effect of rotation in present study ranges to heat transfer is small, it's reasonable to compare the results of experiment and 2-D analysis.

(2) The test cell with a 3x3 heater array

This test cell is used for stationary experimental study with water and comparing with 3-D analysis. The substrate plate provides a support for holding a 3 x 3 array of 12.7 x 12.7 x 9 mm³ oxygen-free copper blocks spaced at 3.2mm apart. For experiments using water, de-ionized water was vigorously boiled in a breaker to remove any air bubbles. The water was then cooled for several minutes and poured slowly into the test cell, and care was taken to prevent air entrainment. The side wall of the test cell is made of Lexan, a transparent material. No air bubble is observed in the experimental studies. However, considering the volume expansion of water, a special structure has been designed as shown in Figure 4.6. The annular clearance (0.1mm in radius direction) allows the volumetric expansion of water by compressing the air in the upper annular section.

Fig.4.5: Schematic of the thermocouple arrangement in the test cell
with a **3x3** heater array

Figure 4.6: Special structure to allow for volumetric expansion of liquid

4.1.2 Mounting apparatus servo motor section and slip rings

A servo motor, three slip rings and the test cell are assembled together by three aluminum shafts via one coupling and two drive walls. A steel test bed with orientation holes for each component is used to keep all of them in the same axis.

The servo motor is controlled by the servo driver (NIKKI DENSO), which has three control models: (1) position deviation; (2) motor speed; (3) output torque. Here, the first control model is used. But before this, the bubble equipment should be used to give the original reference orientation then see the parameter of the servo driver to reach the desired inclined orientation.

The special slip ring brush assembly is used to transfer such inputs and outputs between stationary and rotary components of the experiment equipment. The brushes are made of silver-plated brass spring stock to provide the required pressure for a good connection between the rotational parts and stationary parts. Three slip rings provide 28 terminals and are assembled onto the aluminum shaft.

Fig. 4.7: Servo motor and other attached equipment

4.1.3 Thermocouples and calibration

Due to rotation, each thermocouple is separated to four sections: one connects the ports between the slip ring and Data Acquisition Unit; one is installed in the stationary part of the slip ring; one is rotated with the shaft and has an output port as rotation part of the slip ring; the last one connects this port and the temperature measured place. To carry out the experiment, 78 joints should be monitored to ensure perfect contact, as shown in Fig. 4.8, and it is a great challenge. In addition, thermocouple readings are taken with and without rotation to examine the frictional heating effects between the slip springs and the brushes. It must be ensured that in all cases the thermocouple readings vary less than $\pm 0.05^\circ\text{C}$.

One result of thermocouple calibration is shown in Fig. 4.9 and the others are shown in Table 4.4.

Fig. 4.8: Joints in the slip rings

4.1.4 Software

The running of data acquisition program and post-processing program to deal with variable fluid properties are shown in Figs. 4.10 and 4.11 respectively.

Fig. 4.9: Calibration results for a four-joint thermocouple

Table 4.4: Calibration results ($Y=A+BX$)

101	-2.2071	0.9910	0.994
102	-2.4432	1.0066	0.995
103	-2.158	1.0032	0.991
104	-2.0256	1.0024	0.996
105	-1.9786	1.0138	0.997
106	-2.1035	0.9957	0.993
107	-2.3478	0.9896	0.997
108	-1.9874	1.0029	0.995
109	-1.8798	0.9987	0.998
110	-2.1084	1.0005	0.995
111	-1.9874	0.9978	0.993
112	-1.9987	1.0102	0.995

Fig. 4.10: The running of data acquisition program in one rotating situation

Fig. 4.11: The running of post-processing program

4.1.5 Additional instruments

The power supplied to each heater is calculated from the product of the voltage drop and current measured by a voltmeter and ammeter, respectively. The error associated with the

measurement is estimated to be 1.3%. The temperatures are sampled and stored by using a HP 3054 DAU (Data Acquisition Switch Unit). The output of DAU is connected to the PC for storing and analyzing of data.

4.2 Experimental procedures

The experiments are conducted with air and water for stationary cases and air for rotation cases. After each filling procedure, the test cell is clamped to the mounting apparatus and carefully leveled using a bubble balance. The test enclosure is either oriented vertically or inclined at a prescribed angle using a universal protractor and balance. Transformers are used for regulating the power supply to heaters. Voltmeters and ammeters are employed for power consumption measurements. The power supplied to the heater array is then varied accordingly. Rotation speed is changed by AC Sever Motor.

Temperatures and power inputs are recorded when steady state is reached. Steady condition usually deems to have been achieved when the temperature difference between any element and the cold plate doesn't vary by more than 0.5°C over 10 minute interval, as shown in Fig. 4.10. At steady state, temperatures are scanned 300 times during 10 minute interval and averaged. A computer is employed for storing and analyzing data.

4.3 Data reduction

Different researchers use different material to carry out experiment studies. When the effect, such as conjugate heat transfer, is small enough that it could not affect the basic trends, most researchers used correction factors to filter this effect and compare each other, as shown in Poleniti *et al.* (1993), Heindel *et al.* (1995a,b), Zhang (2003). Otherwise or the effect is the focus of the study, more complex numerical model should be developed to more accurately reflect the experimental conditions (Hamady *et al.* 1989, 1994, Heindel *et al.* 1995c,d).

4.3.1 Effect of conjugate heat transfer

When water is used as coolant, correction factors are used to consider conjugate heat transfer. Fig. 4.12 shows the geometrical and physical parameters to estimate those effects for water. The contact thermal resistances between each surface are neglected. The heat transfer coefficient (h) is assumed $10 \text{ W/m}^2\cdot\text{K}$ for the boundary condition between sponges and environment. In fact, the results are not sensitive when h varies from $1 \text{ W/m}^2\cdot\text{K}$ to $10 \text{ W/m}^2\cdot\text{K}$.

Fig. 4.12 Geometrical and physical parameters for the test cell of water (Unit: mm)

Fig. 4.13 Heat dissipation distribution for the experimental study of water

In Fig. 4.13, the calculations illustrate the heat-dissipation distribution between each surface in one vertical orientation ($\phi=90^\circ$) and one horizontal orientation ($\phi=180^\circ$). The values of some heat transfer paths are not given because they are less than 0.5%. In general, heat loss from the sponges, bottom wall and top wall is small (about 2%). Based on these, the heat loss factors and correction factors are shown in Table 4.5.

Table 4.5: Correction factors for water ($q'''=1 \times 10^6 - 1 \times 10^8 \text{ W/m}^3$, $T_c=20^\circ\text{C}$)

ϕ	Heat loss factor η	Correction factor		
		K_{C1}	K_{C2}	K_{C3}
45°	0.979	0.889~0.895	0.897~0.950	0.886~0.897
90°	0.988	0.863~0.887	0.891~0.924	0.902~0.924
135°	0.981	0.896~0.905	0.899~0.950	0.886~0.897
180°	0.987	0.861~0.897	0.895~0.934	0.868~0.896

However, for natural convection with air whose heat transfer capability is less than one tenth of water, the effect of heat transfer in the substrate even exceeds that of pure convection heat transfer. Therefore, experimental results are compared with numerical simulations considering conjugate effects directly. The geometrical and physical parameters are same to that of water shown in Fig. 4.12 except the width of the sponge increases from 38.7 mm to 50 mm to ensure small heat loss.

4.3.2 Effect of radiation

In the experimental study for air ($10^\circ\text{C} \leq \Delta T \leq 40^\circ\text{C}$), radiation heat transfer has been considered in stationary cases. Discrete transfer radiation model (DTRM), provided by FLUENT 6.0, is adopted due to Planck number (Pl) and optical thickness (aD) are equal to 0.139 and 2.54×10^{-4} and they are less than unity greatly. Radiation factors, varying from 0.92–0.97, are used and applied to rotation cases because the effect of rotation to heat transfer

is small. The trend is close to that of Sanjeev and Bahgat (2000) in the temperature increase range of $10^{\circ}\text{C} \leq \Delta\bar{T} \leq 60^{\circ}\text{C}$.

4.3.3 Data reduction

The tests are conducted under conditions of equal power dissipation for each of the heaters powered. All fluid properties are evaluated at a film temperature defined by

$$T_f = \frac{T_h + T_c}{2} \quad (4.1)$$

where T_c is the average of the three temperatures measured on the cold plate surface and T_h is the local surface temperature of a given heater.

For experimental study for water, the data were reduced in terms of Ra and Nu as defined in the equations (3.9d) and (3.13b).

In particular, the local Nusselt number is calculated from:

$$Nu(i, j) = \frac{hL_z}{k} = \frac{\kappa Q_i L_z}{kA_h(T_h - T_c)} \quad (4.2)$$

$$Q_i = \eta Q_{i0} \quad (4.3)$$

where A_h is the wetted surface area of the flush-mounted heater. Q_i is the total power dissipated by heater i , as measured from voltage and current readings. η , κ are heat loss factor and correction factor respectively.

4.4 Uncertainty Analysis

The uncertainty in the quantities computed from experimental measurements (Nu , Ra , Ta , q , and T) is computed using the procedure outlined by Poleniti *et al.* (1993).

The overall uncertainty in the Nu and Ra varies with the power input. Smaller power input results in a larger uncertainty primarily because of a lower temperature difference across the

enclosure and a higher fraction of energy lost by conduction. As the power input is increased, the uncertainty in the experimental data decreases.

The average Nu and δNu for each heater are calculated using the following expressions:

$$\overline{Nu(i, j)} = \frac{\bar{h}L_z}{k} = \frac{Q_i(i, j) L_z}{kA_h(\bar{T}_h - T_c)} \quad (4.4)$$

$$\delta \overline{Nu(i, j)} = \sqrt{\delta^2 Q + \delta^2 L_z + \delta^2 A_h + \delta^2 (\bar{T}_h - T_c)} \quad (4.5)$$

The maximum uncertainties in L , A_h and $(T_h - T_c)$ were calculated to be $\delta L = 0.8\%$, $\delta A_h = 2\%$, $\delta L = 1.6\%$, and $\delta(T_h - T_c) = 2\%$, respectively. The maximum uncertainty in Q_c , including the uncertain of radiation and heat loss caused by different boundary conditions ($h = 1 - 10 \text{ W/m}^2 \cdot \text{K}$) was estimated to 19.1%. Using above equation, the maximum uncertainty in $\overline{Nu}$ was estimated to be 19.3%.

The Rayleigh number, Ra_L , and δRa_L are calculated using the following expression:

$$Ra = \frac{g\beta(T_h - T_c)L_z^3}{\nu\alpha} \quad (4.6)$$

$$\delta Ra = \sqrt{\delta^2 (\bar{T}_h - T_c) + 9\delta^2 L_z} \quad (4.7)$$

As mentioned before, the maximum uncertainties in L and $(T_h - T_c)$ were calculated to be $\delta L = 0.8\%$, and $\delta(T_h - T_c) = 2\%$, respectively. Using equation, the maximum uncertainty in Ra was estimated to be 3.2%.

When the rotation speed approaches to as slow as 2.975 rpm., tachometer cannot be used and we must count the rotation number in 20 minutes. The resolution of eye for rotation is assumed to be half of a cycle. Then the maximum uncertain of measured rotation speed is shown below:

$$\delta_{\omega} = \frac{0.5}{2.975 \times 20} = 0.84\%$$

It should be noted that it was assumed that there was no uncertainty associated with fluid properties. On the other hand, the uncertain in the inclined angle and the manufacture precision of the enclosure will also affect the results directly which could not be reflected in present analysis. Thus, uncertainties may be somewhat higher than indicated.

4.5 Experimental results

A series of stationary experiment are performed with the following conditions: $\phi = 0^\circ$, 45° , 90° , 135° , 180° . The heater input power changes from 0.113W to 1.45W for air and 0.4W to 8W for water.

Procedures for rotational experiments are similar to that in the stationary mode. Power to the AC motor is controlled to provide different rotational speeds. The servo system of AC motor is configured to rotate at 2.975 rpm, 5.85 rpm, 14.525 rpm, 20.83rpm and 27.175 rpm respectively.

Experimental results will be presented in the later chapters and compared with the numerical simulation results.

Chapter 5

Effects of Inclination on Natural Convection Heat Transfer Characteristics from an Array of Discrete Heat Sources

The work in this chapter is to investigate the complex interactions of thermal and internal fluid flow fields in an inclined shallow enclosure for different coolants. An interesting flow pattern evolution with different characters has been studied to give new insights and greater understanding of the flow and heat transfer behavior. Correlations for practical use are given and compared with experiments as well as data from literature. Optimization based on geometrical location of discrete heat sources is provided.

5.1 Introduction

Convection heat transfer in stationary orientations can be treated as a special case of rotation heat transfer whose rotation speed is infinitesimally small. However, the complex interactions of thermal and internal fluid flow fields in a stationary finite-size, inclined fluid system confined by surrounding walls are still a challenging subject faced by professional engineer in some trades. General study is focused on the rectangular with typically a single hot wall and a cold wall (Jaluria 1980, Bejan 1995, Incropera et al. 1996). In fact, even the simplest case of R-B flow can reveal a wealth of the hydrodynamic regimes and phenomena, resulting great challenge to numerical investigations. With the advent of electronics cooling, natural convection with an array of discrete heat sources represents a much younger and contemporary important sub-field. The problem of discrete heat sources attached to an

inclined or horizontal surface of an electronic package is widely encountered in the industry. However, literature review shows that studies in these fields are considerably limited.

5.2 Methodology and Analysis

5.2.1 Model Geometry and Parameters

Fig. 5.1 : Physical configuration, boundary conditions and the coordinate system

An electronic package consisting of a 3×3 array of discrete heat sources mounted to one wall of a rectangular enclosure is shown in Fig. 5.1. A uniform flux condition is imposed at the back of each heater source, while the back of the substrate is assumed to be adiabatic. The opposite wall is maintained isothermal, and the remaining walls are assumed adiabatic. Because symmetry is observed in previous fully 3-D calculations (Heindel *et al.* 1995, Tou and Zhang 2003) and also proved by experimental study of Poleniti *et al.* (1993), a plane of symmetry at $x=D/2$ is used to reduce the computational domain by a factor of two. The

definition of inclined angle ϕ is shown in Fig. 5.1 (c). The geometrical parameters are shown in table 5.1.

Table 5.1: Summary on geometric parameters

(a) Fixed geometric parameters $H=95.3$ mm, $L=57.2$ mm, $L_z=12.7$ mm, $W=12.7$ mm									
H/L_z	L/L_z	L_x/L_z	P_x/L_z	P_z/L_z	H_1/L_z	H_3/L_z	W_1/L_z	W_3/L_z	
7.5	4.5	1	1.25	1.25	2.0	2.0	0.5	0.5	
(b) Variable geometric parameter									
ϕ	$= 0^\circ$ to 180° , at 5° intervals (2-D model)								
ϕ	$= 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 135^\circ, 150^\circ, 180^\circ$ (3-D model)								

5.2.2 Governing Equations and Boundary Conditions

The governing equations are shown below and can be treated as a special case of Eqs.

3.10-3.12 without rotation force term and unsteady term:

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0. \quad (5.1)$$

Momentum equation

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} = -\frac{\partial P}{\partial X} + Pr \left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right]. \quad (5.2a)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} = -\frac{\partial P}{\partial Y} + Pr \left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right] + Ra^* Pr \theta \sin \phi. \quad (5.2b)$$

$$U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} = -\frac{\partial P}{\partial Z} + Pr \left[\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right] + Ra^* Pr \theta \cos \phi. \quad (5.2c)$$

Energy equation

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2}. \quad (5.3)$$

In the above, P is the dimensionless pressure defined as $P = P_m L_z^2 / (\alpha^2 \rho)$ and P_m is the motion pressure and defined as

$$P_m = p + \rho g y \sin \phi + \rho g z \cos \phi. \quad (5.4)$$

Thus,

$$\left. \begin{aligned} -\frac{\partial P_m}{\partial x} &= -\frac{\partial p}{\partial x}, \\ -\frac{\partial P_m}{\partial y} &= -\frac{\partial p}{\partial y} - \rho g \sin \phi, \\ -\frac{\partial P_m}{\partial z} &= -\frac{\partial p}{\partial z} - \rho g \cos \phi. \end{aligned} \right\} \quad (5.5)$$

Boundary conditions:

$$\left. \begin{aligned} \text{At } X=0: \quad & U = \frac{\partial V}{\partial X} = \frac{\partial W}{\partial X} = 0, \quad \frac{\partial \theta}{\partial X} = 0. \\ \text{At } X=2.25: \quad & U = V = W = 0, \quad \frac{\partial \theta}{\partial X} = 0. \\ \text{At } Y=0: \quad & U = V = W = 0 \\ & \frac{\partial \theta}{\partial Y} = -1, \quad \text{for the iso-flux region (heaters)} \\ & \frac{\partial \theta}{\partial Y} = 0, \quad \text{for the adiabatic region (the rest).} \\ \text{At } Y=1: \quad & U = V = W = 0, \quad \theta = 0. \\ \text{At } Z=0 \text{ and } Z=7.5: \quad & U = V = W = 0, \quad \frac{\partial \theta}{\partial Z} = 0. \end{aligned} \right\} \quad (5.6)$$

The numerical procedures are the same as described in the chapter 3. However, finer grid $60 \times 30 \times 120$ is necessary due to the complex flow fields for inclined angle ϕ changing from 135° to 180° while grid $40 \times 20 \times 80$ is used for ϕ varying from 0° to 135° . Grid independence checks are carried out in three orientations ($\phi = 0^\circ, 90^\circ, 180^\circ$) with 2 times finer grid, requiring more than one week of computational time in the SGI ORIGIN 2000 work station. In the early study, FLUENT is used as a tool to do validation and numerical investigation.

5.3 Results and Analyse

5.3.1 Experimental results and numerical validation

Figure 5.2 shows the comparison between numerical simulations ($1 \times 10^2 \leq Ra^* \leq 1 \times 10^8$, $Pr = 5, 9$) and experimental data ($8 \times 10^5 \leq Ra^* \leq 1 \times 10^8$, $Pr \approx 5$). In the experiments, the heaters in the same row have about the same temperature, probably due to the weak velocity in the horizontal direction in different inclined positions. Since column-to-column variations are insignificant, row averaged $\overline{Nu}$ can be used without any loss of generality. (This phenomena is also proved by present numerical simulations and reported by experimental study of Polentini *et al.* (1994) in the angular position of $90^\circ, 120^\circ, 135^\circ, 150^\circ$ and 180° for water and FC-77.) Better agreement is obtained for the middle row of heaters and the maximum differences between numerical simulations and experiment data are about 9%. In fact, the maximum differences take place in the bottom row of heaters. The temperature increases for the bottom row of heaters are smallest and are more sensitive to the conjugate effect and heat loss that is difficult to evaluate accurately due to the complex geometrical structure and thermal contact resistances. The correlations for $\overline{Nu}$ vs. Ra^* are presented in Table 5.2.

5.3.2 Effects of inclination on flow pattern evolution

For all cases of numerical results, $\overline{Nu}$ generally reduces to about 1.5, as Ra^* decreases to its low limit of 100, which is less than the critical value of 1708. Theoretically, $\overline{Nu}$ assumes 1 for pure conduction with Ra^* approaches zero. In this conduction regime, $\overline{Nu}$ remains essentially unaltered until Ra^* approaches the lower critical value of 1708. As Ra^* passes the critical value there is clearly a sharp increase in heat transfer rate. The slopes of the curves change from zero to positive values. The increase in heat transfer rate is accomplished with the onset of natural convection.

(a) $\phi = 45^\circ$ (b) $\phi = 90^\circ$

(c) $\phi = 135^\circ$

(d) $\phi = 180^\circ$

Fig. 5.2: The effects of inclination on heat transfer for water

Table 5.2 Summary of correlations of $\overline{Nu}$ vs. Ra at various inclination angles

$\phi = 45^\circ$ (Heaters face up) Row 1: Row 2: Row 3:	$\overline{Nu} = 0.387Ra^{0.270}$ $\overline{Nu} = 0.284Ra^{0.265}$ $\overline{Nu} = 0.240Ra^{0.261}$
$\phi = 90^\circ$ (Vertical orientation) Row 1: Row 2: Row 3:	$\overline{Nu} = 0.274Ra^{0.263}$ $\overline{Nu} = 0.341Ra^{0.262}$ $\overline{Nu} = 0.531Ra^{0.256}$
$\phi = 135^\circ$ (Heaters face down) Row 1: Row 2: Row 3:	$\overline{Nu} = 0.549Ra^{0.251}$ $\overline{Nu} = 0.317Ra^{0.266}$ $\overline{Nu} = 0.250Ra^{0.270}$
$\phi = 180^\circ$ (Heated from below) Row 1 & Row 3: Row 2:	$\overline{Nu} = 0.482Ra^{0.237}$ $\overline{Nu} = 0.239Ra^{0.284}$

In general, it is observed that there is a flow pattern evolution with inclined angle and Ra^* , as shown in Table 5.3. When the hot wall is on the top of the cold wall ($\phi = 0^\circ$), even though the flow field looks complex and has the multi-cellular form, the velocity magnitude is so small (-1×10^{-3} m/s) that it could not affect the temperature field. As a result, conduction heat transfer in the liquid coolant dominates, as shown in Fig. 5.3(a). It is known that two flow patterns, two-dimensional uni-cellular and three-dimensional multi-cellular flow patterns exist, when the inclined angle increases from the vertical orientation ($\phi = 90^\circ$) to the horizontal orientation ($\phi = 180^\circ$), as shown in Fig. 5.3. However, in the horizontal orientation, there is a complex flow pattern evolution which, to the authors' knowledge, has not been reported before.

(When the inclined angle is in the range of $0^\circ < \phi < 180^\circ$, it is natural that row 3 is called top row. When study the continuous evolution process in the horizontal orientation

($\phi = 180^\circ$), there is no top row from the geometrical opinion. However, the name is still used to maintain consistent from the previous discussions).

Table 5.3: Flow pattern evolution for water ($Pr \approx 5$)

Ra^* ϕ	Flow pattern evolution		
	$Ra^* = 4.6 \times 10^5 \longrightarrow 4.5 \times 10^6 \longrightarrow 2.5 \times 10^7$		
0° ↓	Multi-cellular flow pattern, conduction heat transfer (Fig. 5.3(a))		
90° ↓	Uni-cellular boundary layer convection (Fig. 5.3(b))		
150° ↓	Multi-cellular convection (Fig. 5.3(c))		
180°	Toroidal convection → Bimodal convection → R-B convection (Fig. 5.4) (Fig. 5.5) (Fig. 5.6)		

In the low Ra^* , toroidal convection takes places, as shown in Fig. 5.4 and Fig. 5.7(a). The hot fluid rises in the central region as a result of buoyancy forces, until it reaches near the top cold wall where it turns radially outward, towards the sidewalls while it is cooled. Then it turns downward near those walls. Finally, the restriction imposed by the bottom wall (substrate wall) forces the fluid to turn radially inward, receiving heat when it passes above the discrete heat sources. The flow path is completed as the colder fluid is entrained to the ascending flow at the center of the enclosure. The maximum velocity appears in the upper section of the center heater. Compared with other flow patterns, Toroidal convection is deleterious to heat transfer due to the high peak temperature in the center heater. However, in these Ra^* ranges for water, even though there is obvious difference in Nu as shown in Fig. 5.2(d), the difference in temperature for each heater is small ($\Delta T < 1^\circ\text{C}$). The flow field is so weak that it could not affect the temperature field greatly. As a result, it is found that the all nine heaters almost keep to the same average temperature in the experiment. Sezai and

Mohamad (2000) studied the flow pattern with a single heat source for air and discussed only the toroidal convection.

With increasing Ra^* , bimodal convection occurs, as shown in Fig. 5.5 and Fig. 5.7(b). In general, the flow pattern consists of two rolls rotating clockwise at the short side of the enclosure, with the warm up-streams of the two rolls mixing above the middle row of heaters. Near the lateral wall, velocity in the X-direction is obvious, just as the case of toroidal convection. But in the central region, there is a 2-D bimodal convection dominating the flow reflecting a continuing evolution process. Bimodal convection is also reported by Mukutmoni and Yang (1993) when studying the constant wall temperature problem in a shallow enclosure.

When Ra^* is high enough, R-B convection occurs, and takes the form of four-roll pattern, as seen in Fig. 5.6 and Fig. 5.7(c) (for $Ra^* = 2.5 \times 10^7$). In general, the paths of particles in Fig. 5.6 are parallel, reflecting weak movement in X-direction and the flow is close to 2-D, resulting in almost the same temperature in the same row of heaters. In Fig. 5.7(c), the two side rolls, due to transition effects, can be called developing R-B rolls here. With further increase in Ra^* , convection with more rolls are expected. According to Cole and Fernando (1998), constant heat flux boundary condition makes cell patterns much faster than constant temperature boundary conditions. For constant wall temperature of the bottom boundary, the flow appears to become turbulent for $Ra > 1.6 \times 10^4 Pr^{1/4}$ while $Ra^* > 10^8$ for constant heat flux bottom boundary. Experimental studies of Polentini *et al.* (1993) showed laminar flow even in $Ra^* = 4.6 \times 10^8$, which supports our laminar flow assumption.

Fig. 5.3: Flow and temperature fields on Y-Z planes at X=0 for water at different angular positions

Fig. 5.4: Flow field for water showing toroidal convection ($Ra^*=4.6 \times 10^5$, $\phi = 180^\circ$)

Fig. 5.5: Flow field for water showing bimodal convection ($Ra^*=4.5 \times 10^6$, $\phi = 180^\circ$)

Fig. 5.6: Flow field for water showing R-B convection ($Ra^*=2.5 \times 10^7$, $\phi = 180^\circ$)

5.3.3 Effects of inclination on distortion of temperature field

When the inclined angle approaches the horizontal orientation, distortion in temperature field begins to appear, and intensify at the top rows of heaters as shown in Fig. 5.8b ($\phi = 135^\circ$). This greatly enhances heat transfer for the top row of heaters. From the vector plot in Figs. 5.9 and 5.10, the distorted temperature fields could be explained as follows. In the vertical orientation, from the fringe to the heater, there is a sharp velocity increase. But at the heater, the velocity magnitude remains constant. From mass continuity, some fluid is pushed from the fringe to the heater. There is thus span-wise motion even though it is weak. But at the heater, no such span-wise motion exists, due to more uniform velocity magnitude. However, in the inclined orientation, even at the heaters the velocity profile shows two peaks. Correspondingly, there are two local high temperature regions at the heaters, resulting in temperature field distortion (Fig. 5.8(b)). This trend is intensified with increasing Ra^* or with greater deviation from the vertical orientation to the horizontal orientation, resulted from a stronger x-velocity, with transition from 2-D flow pattern to a 3-D one.

(a) Toroidal convection, $Ra^* = 4.6 \times 10^5$ (b) Bimodal convection, $Ra^* = 4.5 \times 10^6$ (c) R-B convection, $Ra^* = 2.5 \times 10^7$

Fig. 5.7: Flow and temperature field for Y-Z planes at $X=0$ for water in the horizontal orientation ($\Phi = 180^\circ$)

Fig. 5.8: Temperature distortion for water at different inclined angles ($Ra^*=2.5 \times 10^7$)

Fig. 5.9: Vector plot for water at selected $Z=1, 2, 2.5, 3.75, 5, 6.5$ planes
($Ra^*=2.5 \times 10^7$, $\phi = 90^\circ$)

Fig. 5.10: Vector plot for water at selected $Z=1, 2, 2.5, 3, 3.75, 5, 6.5$ planes
($Ra^*=2.5 \times 10^7$, $\phi = 120^\circ$)

5.3.4 Effects of inclination on local heat transfer rate

Figure 5.11 shows the local heat transfer rate at $\phi=0^\circ$. In this orientation, even though the flow field is still complex, the velocity magnitude is very small and it has no capability to affect the temperature field. The conduction heat transfer plays the main role. Edge effect for each heater can be seen clearly. But the jump of Nu at the edge of heaters is smooth. The Nu distributions of all nine heaters are almost the same. It demonstrates that the convection has little affect to the temperature field.

As shown in Fig. 5.12 for $\phi=90^\circ$, there is a decrease in Nu for heaters from row 1 to row 3. At the effect of lateral advection, there are local sharp increase in the leading edge of heaters and relative smooth increase in the trailing edge of each heater. The Nu difference for the same row is very small, as the result of small horizontal velocity in the enclosure.

When ϕ increases to 150° , the heat transfer for the bottom row of heaters is improved, as shown in Fig. 5.13. However, heat transfer of the top row of heaters, which has the maximum temperature of the enclosure and determines the heat transfer capability at last, become severe. Also inside the heater, there is distortion of local distribution for Nu which can not be seen in vertical orientation.

The distribution of Nu in the substrate wall for different flow pattern at $\phi=180^\circ$ is shown in figure 5.14. Generally, it is symmetrical to center of X and Z axis and there is a mild inward reduction in Nu from the side heater to the center heater, as the result of flow field. The difference for different Ra^* is very small. That means, the lowest Nusselt number is located at the center of the center heater, even though there is no great difference for the local lowest Nu for each heater. However, inside each heater, there is an obviously reduction for Nu from the edge to the center caused by edge effect.

Fig. 5.11: Nu distribution for water ($Ra^* = 2.1 \times 10^7$, $\phi = 0^\circ$) Fig. 5.12: Nu distribution for water ($Ra^* = 2.1 \times 10^7$, $\phi = 90^\circ$)

Fig. 5.13: Nu distribution for water ($Ra^*=2.1 \times 10^7$, $\phi=150^\circ$)

(a) Toroidal convection

(b) Bimodal convection

(c) R-B convection

Fig. 5.14: Nu distribution for water ($\phi=180^\circ$)

5.3.5 Assessment of two- and three- dimensional effects

According to study of Wroblewski and Joshi (1994) for a square heat source, two-dimensional calculations yielded a maximum heater temperature 2.8 times higher than that predicted from the three-dimensional model that could capture edge effects. Heindel *et al.* (1995a,b) carried out two and three-dimensional calculations for a 3x3 array of discrete heaters and the difference between 2-D and 3-D models are 15%.

Figure 5.15 shows Nu distributions along different lines at the heater plate. To distinguish each other, the edge effect for $X=\pm 1.75$ is called remote edge effect while the edge effect for $X=\pm 0.5$ is called the inner edge effect. In Fig. 5.15(a), for conduction heat transfer at $\phi=0^\circ$, edge effects are mild compared with convection heat transfer. The difference between different rows of heaters is small. In Fig. 5.15(b), at vertical orientation, the bottom row of heaters has the highest Nu and the edge effect clearly increases heat transfer. This explains why the results of 2-D model with neglected edge effects are 20% lower compared to that of 3-D model (Tou and Zhang, 2003). In this flow pattern, remote edge effect is near to the inner edge effect, due to the weak velocity in the X-direction.

Similar to previous discussion, in Fig. 5.15(c), the value of Nu reduces from the top row of heaters to the bottom row. Compared with Fig. 5.15 (b), heat transfer for the bottom row of heaters in Fig. 5.15(c) turns worse. However, heat transfer of the top row, that has the maximum temperature of the enclosure and determines the heat transfer capability, is improved. It is more obvious in the horizontal orientation, as shown in Fig. 5.15(d), where, because the lowest $\overline{Nu}$ among the three rows, is higher than those of other orientations. Thus, the horizontal orientation is best for heat transfer.

(a) $\phi = 0^\circ$

(b) $\phi = 90^\circ$

(c) $\phi = 150^\circ$

(d) $\phi = 180^\circ$

Fig. 5.15: Local Nusselt number distribution for water at different inclined angles

($Ra^* = 1 \times 10^5, Pr = 5$)

From Fig. 5.15, we find that there is a clear difference between remote edge effect and inner edge effect for 3-D results. However, there are only small differences in the local Nu distribution along the center lines of the three column of heaters. As seen in Fig. 5.15, they are also close to the 2-D results for all inclined angular positions. Furthermore, when the heater aspect ratio is greater than 3 as shown in Fig. 5.16, three-dimensional predictions approached those of the two-dimensional model, with row averaged Nu agreeing to within 5%. This observation agrees with the studies of Wroblewski and Joshi (1994) and Heindel *et al.* (1995a,b). It was proved by Heindel *et al.* (1995c,d) that 2-D conjugate model is well suited for predicting general trends associated with the effect of real substrate. To avoid computation difficulty of 3-D conjugate model, 2-D conjugate model was used and the key trends are still captured by this simplified model. In the present experimental study, heater aspect ratio equals to 6. Therefore, 2-D model can be used in the later discussion without losing generality.

Fig. 5.16: Effect of heater aspect ratio ($Ra^*=1 \times 10^5$, $Pr=5$, $\phi=90^\circ$)

5.3.6 Effects of Prandtl number

Present results of 3-D numerical simulations in vertical orientation are shown in Fig. 5.17. In addition, computations are carried out for various fluids ($Pr=0.7, 5, 20, 120$) in all inclined angles ($\phi=0^\circ\sim 180^\circ$) for 2-D modeling, as shown in Fig. 15.18. It is found that, with other parameters unaltered, the effects of Pr are (i) For most liquids (except metal liquids), whose Pr is greater than 1, the effects of Pr are negligible, and (ii) For most gases whose Pr are about 0.7, the effects of Pr are small and the differences between liquids and gases in $\overline{Nu}$ and $\overline{Nu}$ are small.

Fig. 5.17: The effects of Pr in vertical orientation from 3-D numerical simulations

In fact, the experimental study of Polentini *et al.* (1994) showed that water ($Pr\approx 5$) and FC-77 ($Pr\approx 25$) obey the same empirical correlations from the vertical orientation ($\phi=90^\circ$) to the horizontal orientation ($\phi=180^\circ$). The effect of Pr in the vertical orientation and horizontal orientation (heated from below) for non-discrete heat sources has been studied by others. The

following equations are given by Berkovsky and Polevikov (from Bejan 1995), Bejan (1995), Globe and Dropkin (1959) respectively to describe the effect of Pr .

$$g(Pr) = \left(\frac{Pr}{Pr + 0.2} \right)^{0.28} \quad (Pr < 10^5, Ra < 10^{13}, \phi = 90^\circ) \quad (5.7)$$

$$g(Pr) = \begin{cases} Pr^{0.25} & (Pr < 1) \\ 1 & (Pr \geq 1) \end{cases} \quad (\text{For boundary layer flow and } \phi = 90^\circ) \quad (5.8)$$

$$g(Pr) = Pr^{0.074} \quad (0.02 \leq Pr \leq 8750, 1.51 \times 10^5 \leq Ra \leq 6.76 \times 10^8, \phi = 180^\circ) \quad (5.9)$$

where g is the function of Pr .

Present studies are comparable with the above equations as they are all weak functions of Pr . In particular, Eq. (5.8) is derived from scale analysis based on the assumption that the buoyancy force is balanced by friction force (for $Pr \gg 1$) or inertial force (for $Pr \ll 1$) in vertical orientation. Because this assumption can be applied to all inclined orientations for natural convection, perhaps this equation may be applicable to all orientations to give an estimate for heat transfer that may be useful for new coolants.

(a) At different Rayleigh number

(b) At different rows

Fig. 5.18: The effects of Prandtl number in different orientations from 2-D numerical simulations

5.3.7 Comparison between experimental and numerical results for air

In the literature, experimental results with discrete heat sources for air are not available and the data for liquid coolants cannot be used and compared. The conjugate flow and temperature fields of air are shown in Fig. 5.19. In general, the flow and temperature profiles in the fluid zone are close to the prediction without real substrate. However, conjugate simulations predict nearly isothermal conditions at the heater/fluid interface because of the high thermal conductivity of copper blocks. In addition, the temperature difference between different rows of heaters is reduced greatly, especially for two top rows, due to low thermal resistance of copper block and small distance between them. At last, due to thermal spread in the substrate, it is found that fluid circulation encompasses the entire fluid enclosure when we compared Fig. 5.3b with Fig. 5.19c. Even though the inclined orientation has noticeable effects to the fluid zone, the effect to heat transfer in the solid zones is small.

Figure 5.20 shows the comparison between numerical simulations considering complex geometrical structures and experimental data. The maximum deviation between them is 15.6%. In the real experimental condition, detail geometrical structure, thermal contact resistance and the boundary conditions of control zones can never be described perfectly in the numerical simulations. Another important factor is the thermal property uncertainty of materials, including coolant and solids. For small heat generation and low temperature increase, the effect of heat loss is relatively important and difficult to be calculated accurately. In the high heat generation and high temperature rise, Bossinesq approximation and radiation heat transfer will produce more uncertainty. Therefore, in these regions, more errors are expected. Considering the above uncertain and other data shown in the literature (Ho and Chang, 1994), the comparison is encouraging.

(a) $\phi = 0^\circ$

(b) $\phi = 45^\circ$

(c) $\phi = 90^\circ$

Fig. 5.19: Flow and temperature fields in the experimental conditions
($Ra^* = 5.12 \times 10^4$)

At last, after taking average of all discrete heaters, the present results are ready to compare with those available in the literature. For an arbitrary angle of inclination, Bejan and others proposed the following empirical formula for an enclosure with uniform hot and cold surfaces opposite to each other (Bejan 1995, Incropera *et al.* 1996):

$$0^\circ < \phi < 90^\circ, \quad \overline{Nu}(\phi) = 1 + [\overline{Nu}(90^\circ) - 1] \sin \phi \quad (5.10)$$

$$90 < \phi < \phi^*, \quad \overline{Nu}(\phi) = \overline{Nu}(90^\circ) (\sin \phi)^{1/4} \quad (5.11)$$

$$\phi^* < \phi < 180^\circ, \quad H/L < 10, \quad \overline{Nu} = \left[\frac{\overline{Nu}(90^\circ)}{\overline{Nu}(180^\circ)} (\sin \phi^*)^{1/4} \right]^{\phi/\phi^*} \overline{Nu}(180^\circ) \quad (5.12)$$

(a) $\phi = 45^\circ$

(b) $\phi = 90^\circ$

(c) $\phi = 135^\circ$ (d) $\phi = 180^\circ$

Fig. 5.20: The effects of inclination on heat transfer for air

In the above, there is a local minimum Nu at a special tilt angle ϕ^* , up to the aspect ratio, Ra and Pr . However, Eqn.5.10 cannot be used directly because Nu is not equal to unity for the pure conduction heat transfer with discrete heat sources. It should be replaced with $\overline{Nu}(0^\circ)$. Fig. 5.21 shows a good agreement between these empirical correlations and the present study. There is a slight oscillation in Nusselt number as inclination angle changes from 30° to 150° . It is close to the observation of Tou and Zhang. (2003) .

Fig. 5.21: The effect of inclination on heat transfer

5.3.8 Optimization based on geometrical location of discrete heat sources

A design process requires the evaluation of a number of available options to determine the most appropriate case that meets the primary design requirements, e.g., maximum mean heat transfer, minimum peak temperature in the system. For natural convection, constraints come from not only computation challenge, but also design and manufacture requirements. In many actual application for natural convection, conjugate and radiation effects cannot be

neglected. However, due to complexity, these effects are neglected and the model is often simplified from the real PCBs and packages to the blocks and heat sources with uniform heat flux to capture the main character. It is also widely used by thermal design engineers in the beginning of layout design. Detailed zoom-in models are time and computational resource consuming and only used for the important situation that needs high reliability. Present work tries to derive general instructions based on the simplified models.

There are two optimization criteria: (1) maximum global performance or maximization of $\overline{Nu}$; (2) minimization of the hot-spot temperatures or maximization of $\overline{Nu}_i$. The geometry has three degrees of freedom, locations of three heat sources. The optimization method is performed by enumerating all possible locations of heaters in the enclosure and recording the case with the best heat transfer performance. The optimal results based on previous criteria are shown in Figs. 5.22, 5.23 respectively, together with the clockwise flow patterns.

When conduction heat transfer dominates ($Ra^*=10^3$) as shown in Figs 5.22a, 5.23a, the optimal distributions are uniform. After the onset of convection, the increasing Ra^* value has the effect of pushing the heat sources towards the starting cold current of the clockwise circulation. The trailing heat source is least sensitive to this influence. This phenomena agrees well with the observation of Silva *et al.* (2004) in a square enclosure.

In Fig. 5.2b or 5.12, it's easy to find that the heat transfer performances of three heat sources are not uniform and the performances of heaters row 1 and row 2 are better than heater row 3. The optimal results based on the maximization of $\overline{Nu}_i$ shown that two beginning heat sources are in contact with each other. It is an important consideration in practice: Improving the performance of the weakest component.

Fig. 5.22: Optimal location of discrete heat sources based on maximization of $\overline{Nu}$

Fig 5.23: Optimal location of discrete heat sources based on maximization of $\overline{Nu}_i$

5.4 Conclusions

The following conclusions can be concluded from present studies:

- (1) An experimental study is carried out to study natural convection in an enclosure with 3x3 discrete heat sources for water at various orientations. In general, heaters in the same row have almost the same temperature, due to weak velocity in the horizontal direction.
- (2) In-depth 2-D and 3-D numerical simulations for laminar flow are implemented and compared with experimental results. Complex flow pattern evolution, flow field, and temperature fields are observed due to interactions of buoyancy forces tangential and normal to the heater surfaces.
- (3) In the horizontal orientation for water, with increasing Ra^* , the flow pattern evolutions are complex and three main characteristics can be identified: toroidal convection, bimodal convection and R-B convection. For increase in heat transfer, the horizontal orientation is best because the lowest $\overline{Nu}$ among the three rows is higher than those in other orientations.
- (4) On inclining the enclosure towards the horizontal orientation (heated from below) with liquid coolants, the flow and temperature fields become complex and distorted. Three-dimensional effects, such as fluid flow in horizontal direction and distorted temperature field begins to appear and intensify, which greatly enhance the heat transfer for the top row of heaters.
- (5) Edge effect increases heat transfer greatly. In the situation that the effects of edge can be negligible or small, the results of 2-D model are close to those of 3-D model.
- (6) For most liquids (except metal liquids), whose Pr is greater than 1, the effects of Pr are negligible. For most gases whose Pr are about 0.7, the effects of Pr are small and the differences between liquids and gases in $\overline{Nu}$ and $\overline{\overline{Nu}}$ are small. According to scale

analysis, the fluid motion is balanced between buoyancy force and friction force. It can be used to develop and test new coolants.

- (7) Optimal location of discrete heat sources in the vertical orientation shows that the spacing between heaters is not uniform after the onset of convection. Increasing Ra^* value has the effect of pushing the heat sources towards the starting cold current of the clockwise circulation. It is beneficial for thermal design engineers in the layout of electronic components.

Chapter 6

Rotating Heat Transfer in an Enclosure of Square Cross-section

In the current work, an in-house two-dimensional program for natural convection on an air-filled differentially-heated rotating enclosure of square cross-section is developed and validated with the published experiment results of Mach-Zehnder interferometer. The effects of Coriolis force and centrifugal force are sorted out clearly from others for the first time by the numerical simulations. Study in flow pattern evolution and heat transfer characteristics in this chapter can be applied to all similar rotation situations.

6.1 Introduction

In the physical world, all natural objects are rotating which must be taken into account at sufficiently large rotation rates. It so happened that the convection in rotating fluids up to quite recently was a neglected subject, but which may have potentially important implications and applications. The monographs on the theory of rotating fluids were prepared by Greenspan (1968), Vanyo (1993) and Boubnov & Golitsyn (1995), containing most of what is necessary for a basic understanding of this theory. From literature review in this field, experimental investigations and theoretical analysis are the two main methods and many non-dimensional numbers, such as Strouhal number, Rossby number, Ekman number, and Taylor number, rotation Rayleigh number, are widely used to analysis data and sort out the effects of various body forces coupled in the rotating fluids. Numerical investigation is a great challenge, especially for present unsteady problem.

6.2 Model Formulation

6.2.1 Governing equations and boundary conditions

Fig: 6.1: Physical problem and coordinate system

A two-dimensional enclosure of side H filled with air executes a steady uniform counterclockwise angular velocity about its longitudinal axis, as shown in Fig. 6.1, with the geometrical layout and the Cartesian coordinates (x, y) rotating with the cavity. The surface at $y=-H/2$ and $y=H/2$ are at uniform cold and hot temperatures (T_c, T_h) , respectively. The temperatures along the lateral wall are assumed linear distribution between T_h and T_c to consider conjugate heat transfer in the lateral wall of the experiments. The definition of angular position ϕ is shown in Fig. 6.1(b). For natural convection, the Bossinesq approximation is used (Buhler and Oertel, 1982, Lee and Lin, 1996). The terms representing the thermal and rotational buoyancies and Coriolis force are, respectively, equal to $\rho_0 \bar{g} \beta (T - T_c)$, $-\rho_0 \beta (T - T_c) \bar{\Omega} \times (\bar{\Omega} \times \bar{r})$ and $-2\rho_0 [1 - \beta (T - T_c)] \cdot \bar{\Omega} \times \bar{V}$. The resulting lamina- flow development can be described as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6.1)$$

Momentum equations:

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = & -\frac{1}{\rho_0} \frac{\partial P_m}{\partial x} + \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + 2\Omega v - 2\Omega v \beta (T - T_c) \\ & - \Omega^2 x \beta (T - T_c) + g \beta (T - T_c) \sin(\Omega t) \end{aligned} \quad (6.2a)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = & -\frac{1}{\rho_0} \frac{\partial P_m}{\partial y} + \nu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - 2\Omega u + 2\Omega u \beta (T - T_c) \\ & - \Omega^2 y \beta (T - T_c) + g \beta (T - T_c) \cos(\Omega t) \end{aligned} \quad (6.2b)$$

Energy equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \quad (6.3)$$

where P_m is the motion pressure.

Using H , αH , H^2/α , $\alpha^2 \rho / H^2$ and $T_h - T_c$ as scales for length, velocity, time, pressure and temperature respectively, and the following dimensionless variables are defined:

$$\left. \begin{aligned} X &= \frac{x}{H} & Y &= \frac{y}{H} & U &= \frac{u H}{\alpha} & V &= \frac{v H}{\alpha} \\ \tau &= \frac{t \alpha}{H^2} & P &= \frac{p_m H^2}{\alpha^2 \rho} & \theta &= \frac{T - T_c}{T_h - T_c} \\ Ra_\omega &= \frac{\beta \Omega^2 H^4 (T_h - T_c)}{\alpha \nu} & Ra &= \frac{g \beta H^3 (T_h - T_c)}{\alpha \nu} \\ Ta &= \frac{4 \Omega^2 H^4}{\nu^2} & Pr &= \frac{\nu}{\alpha} \end{aligned} \right\} \quad (6.4)$$

The Coriolis buoyancy force is neglected, due to $|\beta(T - T_c)| \ll 1$ in present studies.

Then the governing equations and boundary conditions become

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6.5)$$

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6.1)$$

Momentum equations:

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = & -\frac{1}{\rho_0} \frac{\partial P_m}{\partial x} + \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + 2 \Omega v - 2 \Omega v \beta (T - T_c) \\ & - \Omega^2 x \beta (T - T_c) + g \beta (T - T_c) \sin(\Omega t) \end{aligned} \quad (6.2a)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = & -\frac{1}{\rho_0} \frac{\partial P_m}{\partial z} + \nu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - 2 \Omega u + 2 \Omega u \beta (T - T_c) \\ & - \Omega^2 y \beta (T - T_c) + g \beta (T - T_c) \cos(\Omega t) \end{aligned} \quad (6.2b)$$

Energy equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \quad (6.3)$$

where P_m is the motion pressure.

Using H , α/H , H^2/α , $\alpha^2 \rho/H^2$ and $T_h - T_c$ as scales for length, velocity, time, pressure and temperature respectively, and the following dimensionless variables are defined:

$$\left. \begin{aligned} X &= \frac{x}{H} & Y &= \frac{y}{H} & U &= \frac{u H}{\alpha} & V &= \frac{v H}{\alpha} \\ \tau &= \frac{t \alpha}{H^2} & P &= \frac{P_m H^2}{\alpha^2 \rho} & \theta &= \frac{T - T_c}{T_h - T_c} \\ Ra_\omega &= \frac{\beta \Omega^2 H^4 (T_h - T_c)}{\alpha \nu} & Ra &= \frac{g \beta H^3 (T_h - T_c)}{\alpha \nu} \\ Ta &= \frac{4 \Omega^2 H^4}{\nu^2} & Pr &= \frac{\nu}{\alpha} \end{aligned} \right\} \quad (6.4)$$

The Coriolis buoyancy force is neglected, due to $|\beta(T - T_c)| \ll 1$ in present studies.

Then the governing equations and boundary conditions become

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6.5)$$

The governing equations contain three independent dimensionless governing parameters, namely, Prandtl number (Pr), Rayleigh number (Ra), and Taylor number (Ta). In addition, rotational Rayleigh number (Ra_ω) reflects the effect of centrifugal buoyancy force, but it depends on the other dimensionless governing parameters.

6.2.2 Solution and numerical validation

The basic equations for the present unsteady rotating cavity flow are solved numerically, using our own program. The steady-state results are obtained by two convergence criteria: (1) $S_{MAX} \leq 1 \times 10^{-8}$, where S_{MAX} is the largest absolute value of the “mass source” used in the motion pressure correction equations, and (2) $MAX((\theta_{i,j}^{n+1} - \theta_{i,j}^n) / \theta_{i,j}^{n+1}) \leq 1 \times 10^{-5}$. Typical computation process for the present periodic oscillation problem is shown in Fig. 6.2, where after about six rotations ($\tau = 6\tau_0$), the final periodic oscillation results are obtained. To ensure correct results, additional computation work was carried out beyond 6 rotations. The numerical oscillations required to reach the final state varies from 5 to 100 rotations. The $\Delta\tau$ is the time step in numerical simulations and varies from $1/72 \tau_0$ to $1/360 \tau_0$. Although five types of solution (stable two-cell solution, stable steady multi-cell solution, periodic oscillation, chaotic oscillation and symmetry-breaking oscillation led by sub-harmonic bifurcation) have been reported by Yang and Wang (2003), in our study, periodic oscillation and chaotic oscillation results have been obtained, and only the former is studied in detail.

Compared with forced convection, velocity gradient of natural convection is very low because normally the maximum velocity magnitude is about $10^{-2} - 10^{-1}$ m/s and present flow is not boundary layer flow. However, the temperature gradient is high and easier to be measured successfully than flow field, shown in the work of Hamady *et al.* (1994). Therefore, validation of the program and most discussion are based on temperature results.

Fig: 6.2: Typical computation process for present periodic oscillation problem

In view of the complex flow to be simulated, program tests for both non-rotating ($\Omega = 0$ rpm) and rotating cases ($\Omega = 15, 17.5$ rpm) are conducted and compared with six available experimental results of Hamady *et al.* (1994) by means of Mach-Zehnder interferometer, as shown in Fig. 6.3, and found to have close agreement. Figure 6.3 also shows that the present 2-D model can provide a useful tool to analyse a real situation; length of the cavity in the experiment is ten times the side (H). In the present situation, the buoyancy, centrifugal, and Coriolis forces all lie in the x - y plane and are not neglected in the 2-D model. For the inclined angle approaching 180° , the cavity heated from below gives rise to the well-known R-B convection, which is unstable and no convergent result exists (Fig. 6.3(c)). Finally, a grid-independence test was conducted. The differences in whole $\overline{Nu}$ evolution process predicted from the 36×36 and 80×80 grids are within 1% during the entire periodic oscillation. In addition, evolution of temperature from 10 grids has been checked and to

ensure the small variation ($< 2\%$). Different time steps are checked to ensure convergence when Ta is higher than 10^6 , and some simulations may require up to one week of computational time with a Pentium IV PC. For example, if we use 80×80 mesh grids, it may take half minute to get one convergence result for steady problem. However, for present unsteady problem, 72 time steps are used in each circulation and 250 circulations are necessary to make sure whether the final periodic has been reached as shown in Fig. 6.10. Then the total computation time is estimated to $0.5 \times 72 \times 250 = 9000$ minutes = 6.25 days. If 360 time steps are used in each circulation that is necessary in high Taylor number, it will take more than one month. Therefore, 2-D numerical simulations are adopted and the research work is confined in the periodic field.

6.3. Results and discussions

6.3.1 Flow and temperature fields without rotation

We first present the flow and temperature fields without rotation as shown in Fig. 6.4. In Fig. 6.4(a), the flow is extremely slow because of the gravitationally stable condition. In this orientation, pure conduction dominates heat transfer. Fig. 6.4(a)-(c) show that the convective flow grows in strength as the angular position ϕ increases. A clockwise circulation induced by the buoyancy force dominates the motion of fluid when the hot wall is in the left side of the enclosure. This motion persists to 180° , after which buoyancy force reverses flow direction and induces a counterclockwise circulation when the hot wall is in the right side of the enclosure, as illustrated for 270° in Fig. 6.4(d). The flow and temperature fields from 180° to 360° are symmetric with respect to 180° (heated from below). For stationary cases, 180° will be the transition angle in which clockwise circulation evolves to counterclockwise circulation, and at 180° , the circulation is uncertain because of the unstable of R-B convection.

Fig: 6.3: Comparison of computed isotherms with experiment results

(upper figures in each of (a), (b), etc.) of Hamady *et al.* (1994) ($Ra = 3 \times 10^5$, $Pr = 0.7$)

(a) $\phi = 0^\circ$, $\Omega = 0$ rpm

(b) $\phi = 45^\circ$, $\Omega = 0$ rpm

(c) $\phi = 90^\circ$, $\Omega = 0$ rpm

Fig: 6.4: Flow and temperature fields in a stationary cavity

$$(Ra=1.2 \times 10^5, Ta=0, Pr=0.7)$$

6.3.2 Flow and temperature fields with rotation

The velocity vector plot and temperature fields at $Ra=1.2 \times 10^5$, $Ta=8.9 \times 10^4$ and $Ra_w=496$ are shown in Fig. 6.5. As in the stationary case, there are two basic flow patterns: clockwise and counterclockwise circulations, reflecting that buoyancy force still plays an important role. However, in the orientation of 180° , clockwise circulation dominates the flow. The transition angle position is around 270° that is caused by the inertial force. Even though

beyond 180° buoyancy force tends to induce a counterclockwise circulation, the flow inertia from the clockwise circulation still persists and put off the transition angle for flow reversal. The Coriolis force has an aiding effect on the clockwise motion. With the accumulation of gravitational buoyancy force, four counterclockwise cells first appear at the four corners while in the core clockwise flow still persists, as shown in Fig. 6.5(f), (g). With increasing angle, the gravitational force increases so that the size of the corner cells grows bigger. At 315° , in Fig. 6.5(h), the effect of the buoyancy force almost balances the effect of the inertial force and the four corner cells combine into two, while the weakened clockwise circulation in the core at 270° now disappears completely. The transition from clockwise to counterclockwise motion is completed. It should be noted that the Coriolis force is now directed outward from the core and tends to stabilize the motion, thus reducing the heat transfer to the wall. Accordingly, at 0° , shown in Fig. 6.5(a), there is a transition from counterclockwise to clockwise circulation. In the whole cycle, no unstable angular position exists, since R-B convection does not occur. Background rotation increases flow stability, as mentioned by others (Vanyo 1993, Boubnov and Golitsyn 1995, Fernando and Smith 2001).

(a) $\phi = 0^\circ$, $\Omega = 8.5$ rpm

(b) $\phi=45^\circ$, $\Omega=8.5$ rpm

(c) $\phi=90^\circ$, $\Omega=8.5$ rpm

(d) $\phi=135^\circ$, $\Omega=8.5$ rpm

(e) $\phi=180^\circ$, $\Omega=8.5$ rpm

(f) $\phi=225^\circ$, $\Omega=8.5$ rpm

(g) $\phi=270^\circ$, $\Omega=8.5$ rpm

Fig: 6.5: Flow and temperature fields in a rotating cavity

$$(Ra=1.2 \times 10^5, Pr=0.7, Ta=8.9 \times 10^4)$$

6.3.3 The effects of Coriolis force and centrifugal force

The complex change in the flow and temperature fields can be explained by studying the effects of various body forces that dominate the flow. In Fig. 6.6, the dashed lines are the 'special' results considering the effects of buoyancy and inertial forces and neglecting the effects of Coriolis and centrifugal forces. The continuous lines are the 'real' results considering all forces. In general, the dash lines have the same trend and relatively small

difference in mean Nusselt number with the continuous lines, indicating that heat transfer is dominated by inertial and buoyancy forces. The difference between the two lines is attributed to the centrifugal force or Coriolis force or both. After the centrifugal force is added, the results (indicated by circles) have almost no change compared with 'special' results. If the Coriolis force is considered, the results (indicated by asterisks) are very close to the 'real' results. This shows that the difference is mainly caused by the Coriolis force even though it is small, and that the effect of centrifugal force can be neglected.

(a) $Ra=1.2 \times 10^5$, $Pr=0.7$, $Ta=8.9 \times 10^4$, $Ra_w=496$

(b) $Ra=1.2 \times 10^5$, $Pr=0.7$, $Ta=1 \times 10^6$, $Ra_\omega=5.57 \times 10^3$

(c) $Ra=1 \times 10^6$, $Pr=0.7$, $Ta=1 \times 10^6$, $Ra_\omega=6.63 \times 10^4$

Fig: 6.6: Effect of Coriolis and centrifugal forces to mean Nusselt number
in one periodic oscillation

For a vertical rotation cylinder, the effects of centrifugal force can be neglected when $(\omega^2 r/g) \ll 1$ (Greenspan 1968). If the density of fluid is constant, the centrifugal force will disappear in normal computation because it only affects the pressure distribution (Vanyo 1993). Considering the change of density, and using the Boussinesq approximation, we introduce the rotational Rayleigh number Ra_ω to reflect the effect of centrifugal buoyancy force. However, it is not an independent governing parameter. From governing equations, the effect of Coriolis force is proportional to rotation speed and the effect of centrifugal force is proportional to the square of rotation speed. So it is expected that under high rotation speed, centrifugal force will dominate the flow. However, due to inertial force, the motion may have already evolved into solid body rotation that will be discussed later. Lee and Lin (1996) discussed the case where $Ra=10^2$, $Ta=10^2$, $Pr=0.7$ and $Ra_\omega=10^6$ and concluded that in this condition the centrifugal (buoyancy) force has significant influences on the flow. However, at these low Ra and Ta , the estimate Ra , is actually about 6.6×10^{-4} , hence the flow dominated by the centrifugal force does not occur in the present situation.

The effect of Coriolis force is complex. As mentioned before, when Coriolis force tends to promote the flow circulation, it increases the mean heat transfer. When the flow motion is in the reverse direction of Coriolis force, a reversed effect on the mean heat transfer occurs. The effect of Coriolis force is more obvious with increasing rotation speed and low buoyancy force, as seen in comparing the differences in Fig. 6.6 (a)-(c).

Finally, effects of various body forces on the local Nusselt number at the angular position of 270° are shown in Fig. 6.7. Under the assigned boundary conditions, Coriolis force has no effect on local Nusselt number at the two edge points. But between the edges, its effect is small but obvious.

(a) $Ta=8.9 \times 10^4$, $Ra_w=496$ (b) $Ta=1 \times 10^6$, $Ra_w=5.57 \times 10^3$

Fig. 6.7: Effect of Coriolis and centrifugal forces to local Nusselt number

at the angular position of 270° ($Ra=1.2 \times 10^5$, $Pr=0.7$)

Present results based on this numerical method are some different to work of Buhler and Oertel (1982), Hamady *et al.* (1994), Zhang (2003) whose results are based on analyse of dimensionless numbers, such as Ta , Ek . According to scale analysis of Bejan (1995), these numbers have no meaning by themselves. This straightforward numerical method provides a direct and powerful tool to sort out effects of various body forces.

6.3.4 The effects of inertial force

From previous discussions, the differences in the flow and temperature fields between rotation and stationary are mainly attributable to the inertial force. In the stationary case, due to weak velocity and small acceleration, the effect of inertial force is relative small, but with increasing of rotation speed, inertial force increases rapidly. The difficulty of studying the effect of inertial force is that there is no independent dimensionless governing parameter to describe it and inertial force is coupled with other body forces that are also related to rotation. In present range of studies, the effect of Coriolis and centrifugal force is small, and Taylor number then reflects the effect of inertial force.

The effect of inertial force to overall Nusselt number of the cold plate can be seen in Fig. 6.8. For the stationary case ($Ta=0$), as shown in Fig. 6.8(a), two local maximum values of Nu correspond to two local maximum strengths of clockwise and counterclockwise circulations. Here, the first transition angular position, $\phi_1=180^\circ$, corresponds to the evolution from clockwise to counterclockwise circulation. The second transition angular position, $\phi_2=0^\circ$ or 360° , corresponds to the reverse evolution. These two transition angular positions, ϕ_1 and ϕ_2 correspond to two local minimum values of Nu . With increasing rotation, the inertial force extends the angular position range of clockwise circulation and the counterclockwise circulation becomes weakened. As a result, the two transition angular positions increase and

the increase in ϕ_1 is larger than that of ϕ_2 . Finally, ϕ_2 and the second local maximum of Nu disappears when Ra is close to Ta , as shown in Fig. 6.8(d). Obviously, the inertial force plays an essential role in maintaining the clockwise motion.

The periodic oscillation of Nu is caused by the buoyancy force. The inertial force tries to reduce the oscillation, and it is expected that when inertial force is high enough to outweigh the thermal oscillation caused by the buoyancy force, the oscillation becomes smaller and leading finally to solid body rotation. This trend is clearly reflected in Fig. 6.8(e), (f).

In the study of vertical rotation box for high Prandtl number and fixed high Rayleigh number, Rossby found that solid body rotation would occur if $[Ta^{\frac{1}{2}} / Ra^{\frac{2}{5}}] \gg Ra^{\frac{4}{15}}$. In solid body rotation, Nu approaches unity due to pure conduction heat transfer (Boubnov and Golitsyn 1995). Sometimes the value of Nu for the solid body rotation may be slightly higher than unity, determined by the definition of Nu . A similar phenomenon can be seen in Tang (1975) and Kim *et al.* (2000).

From the previous analysis, the solid body rotation is determined by the relative strength of inertial force and buoyancy force. If the inertial force is kept constant and the buoyancy force is increased greatly, it is expected that the oscillation of Nu will increase again, as shown in Fig. 6.9(c), where Ra increases to 1×10^6 . However, higher Rayleigh number brings about difficulties in numerical simulations due to the instability of fluid motion. It is difficult to get convergent results for low rotation speeds and low Taylor numbers ($Ta < 1 \times 10^4$) due to the R-B convection. On the other hand, even though we can obtain convergent results in high Taylor number ($Ta = 1 \times 10^7$) and can expect the solid body rotation becoming obvious (referring to the mean value Nu in Fig. 6.10), the periodic oscillation evolves into chaotic oscillation. Continuing increase in rotation speed will turn the flow turbulent that is outside the present scope.

Fig: 6.8: Increasing effect of inertial force with increasing rotation speed
at lower Rayleigh Number ($Ra=1.2 \times 10^5$, $Pr=0.7$)

Fig. 6.9: Increasing effect of inertial force with increasing rotation speed
at higher Rayleigh Number ($Ra=1 \times 10^6$, $Pr=0.7$)

Fig: 6.10: Chaotic periodic oscillation in high Rayleigh number ($Ta/Ra = 10$)

6.3.5 The effects of buoyancy force

For fixed rotation speed and Taylor number ($Ta=1 \times 10^5$), the mean Nusselt number at different Rayleigh number is showed in Fig. 6.11. Clearly, increasing buoyancy force causes higher oscillation of Nu . The whole evolution process with increasing Ra and fixed Tu is a reverse process compared to the previously described process with the increasing Ta and fixed Ra . Since the difference in results between $Ta=1 \times 10^3$ and $Ta=1 \times 10^4$ is small, the result of $Ta=5 \times 10^4$ is shown and it is close to the solid body rotation with the characteristics of small oscillation and low Nu .

Fig: 6.11: Effect of Buoyancy force to Mean Nusselt number
in one periodic oscillation ($Ta=1 \times 10^5$, $Pr=0.7$)

The onset of solid body rotation is shown in Fig. 6.12(a), and present results agree with the experimental results of Tang (1975) when $Ra \approx 1 \times 10^6$ and $Pr \approx 7$. It should be pointed out that solid body rotation occurs at low rotation speed ($\Omega \approx 90$ rpm). In stationary cases, conduction will take place when driving force, buoyancy force, is small. In this study, conduction heat transfer in stationary ($\Omega = 0$ rpm) can be treated as a special case of solid body rotation with $\Omega \rightarrow 0$ rpm. Rotation delays the onset of convection (Fernando and Smith 2001). In Fig.12(b), the critical Rayleigh numbers for the onset of convection are shown below:

(a) fixed Rayleigh number

(b) fixed Taylor number ($Pr=0.7$)

Fig. 6.12: Solid body rotation and onset of convection

$$1 \times 10^2 < Ra_c < 5 \times 10^3 \quad (Ta = 0) \quad (6.10a)$$

$$5 \times 10^3 < Ra_c < 1 \times 10^4 \quad (Ta = 1 \times 10^5) \quad (6.10b)$$

where the value of Nu for solid body rotation limit is slightly higher than unity. The phenomena that rotation delays the onset of convection is indicated in the Fig. 6.12(b). However, the conduction in the case of $Ra=5 \times 10^3$ and $Ta=10^5$ ($\Omega \approx 9$ rpm) is solid body rotation also. Obviously, rotation may inhibit heat transfer and result in solid body rotation or increase heat transfer at high Rayleigh number, depending on the ratio of the driving buoyancy force and the driven inertial force.

6.4. Conclusions

After validation with the experimental results of Hamady et al. (1994), a comprehensive numerical simulations are carried out in the range of $Ta \leq 1 \times 10^6$, $Ra \leq 1 \times 10^6$, and $Pr=0.7$. The evolution process of flow and temperature fields is studied and explained by deeper and clearer understanding of various body forces. The following conclusions can be drawn:

- (1) The physically realizable periodic oscillation solutions are identified numerically and validated with the published unsteady experimental results perfectly.
- (2) In low rotation speed ranges, due to buoyancy force, there are clockwise and counterclockwise circulations, and correspondingly two local peak Nusselt numbers in each periodic oscillation. In transition stages, more vortices are predicted.
- (3) With increasing rotation speed and increasing effect of inertial force, clockwise circulation is enlarged and tends to overcome the counterclockwise circulation. The second local peak Nusselt number becomes weakened and disappeared when $(Ta/Ra) \sim 1$. Inertial force also reduces the oscillation of Nusselt number in general.

- (4) But after the rotation convection enters the solid-body rotation mode, heat transfer is reduced. Hence it is noteworthy that relative effect of driving buoyancy and driven inertial forces can lead to increase or decrease in heat transfer.
- (5) The effect of Coriolis force is small and differentiated clearly from those of inertial and centrifugal forces. The centrifugal effect to heat transfer is found to be negligible in the present studied ranges.
- (6) In the present study range, R-B convection does not happen in rotating convection. Therefore, rotation increases flow stability. But it delays the onset of cohvection, another expression of solid body rotation.

Chapter 7

Effects of Rotation on Natural Convection Heat Transfer Characteristics from an Array of Discrete Heat Sources

Effects of rotation on the periodic convection heat transfer characteristics from an array of discrete heat sources are investigated comprehensively in this chapter. The study covers oscillation phenomena from periodic to chaotic, flow and temperature evolution from stationary situation to the slow rotation, effects of various body forces, correlation equations, validation by experiment results after considering conjugate effects and optimal distribution of discrete heat sources.

7.1 Introduction

The rotating flow is one of the most challenging research fields of fluid mechanics. The effects of multiple, interacting body forces found in rotating systems will be the subject of extensive research effort in the future as the applications in the rotary machines, the guided missiles and space-based manufacturing process increase in importance. However, the studies for electronics cooling under the background rotation are scarce so far. Thermal-rotation coupling has a profound effect on flow and heat transfer, the focus of the current work. A numerical simulation and experimental investigations are carried out to explore this challenge subject on the unconventional natural convection phenomena due to rotation.

7.2 Model Formulation

7.2.1 Governing equations

The physical and geometry conditions are showed in Fig. 7.1. The enclosure, filled with air, executes a steady uniform counterclockwise angular velocity about a horizontal axis. Three discrete heat sources are flushed-mounted onto one of the sidewalls, while the opposite wall acts as a uniform cold surface. The remaining walls are insulated from the surroundings.

Fig. 7.1 : Physical and geometrical arrangement

For natural convection, Bossinesq approximation is used and the non-inertial reference frame XOY is shown in Fig. 7.1. Using L , αL , L^2/α , $\alpha^2 \rho / L^2$ and $q'' L/k$ as scales for length, velocity, time, pressure and temperature, respectively, the dimensionless governing equations and boundary conditions are shown below:

Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7.1)$$

Momentum equations:

$$\begin{aligned} \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = & -\frac{\partial P}{\partial X} + Pr \left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right] + \underbrace{Ta^{0.5} Pr V}_{\text{Coriolis force term}} \\ & - \underbrace{Ra_{\omega} Pr X \theta}_{\text{Centrifugal force term}} + \underbrace{Ra * Pr \theta \sin(0.5 Ta^{0.5} Pr \tau)}_{\text{Buoyancy force term}} \end{aligned} \quad (7.2a)$$

$$\begin{aligned} \frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = & -\frac{\partial P}{\partial Y} + Pr \left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right] - \underbrace{Ta^{0.5} Pr U}_{\text{Coriolis force term}} \\ & - \underbrace{Ra_{\omega} Pr Y \theta}_{\text{Centrifugal force term}} + \underbrace{Ra * Pr \theta \cos(0.5 Ta^{0.5} Pr \tau)}_{\text{Buoyancy force term}} \end{aligned} \quad (7.2b)$$

Energy equation:

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \quad (7.3)$$

Boundary conditions:

$$\left. \begin{aligned} \text{At } X = -A_H/2: U=V=0, \frac{\partial \theta}{\partial X} &= 0. \\ \text{At } X = A_H/2: U=V=0, \frac{\partial \theta}{\partial X} &= 0. \\ \text{At } Y = -A_W/2, U=V=0, \theta &= 0. \\ \text{At } Y = A_W/2: U=V=0, \\ \frac{\partial \theta}{\partial Y} &= -1 \quad \text{for the iso-flux regions (heaters),} \\ \frac{\partial \theta}{\partial Y} &= 0 \quad \text{for the insulated regions.} \end{aligned} \right\} \quad (7.4)$$

To examining the time evolution of the velocity and temperature fields, results for local, space-averaged and time-space-averaged Nusselt number (Nu , $\overline{Nu}$, $\overline{\overline{Nu}}$) can be evaluated from

$$\left. \begin{aligned}
 Nu(Y)|_{\phi=\phi} &= \frac{1}{\theta(Y)|_{\phi=\phi}} = \frac{q'' L}{K \cdot [T(Y) - T_c]} \Big|_{\phi=\phi} \\
 \overline{Nu}_i|_{\phi=\phi} &= \frac{q'' L}{\int_L K \cdot [T(Y) - T_c] dX} \Big|_{\phi=\phi} \quad i=1,2,3 \\
 \overline{\overline{Nu}}|_{\phi=\phi} &= \left[\sum_{i=1}^3 \frac{q'' L}{\int_L K \cdot [T(Y) - T_c] dX} \right] / 3 \\
 \overline{\overline{Nu}}_i &= \frac{\int_0^{2\pi} Nu_i d\phi}{2\pi} \quad \overline{\overline{Nu}} = \left(\sum_{i=1}^3 \overline{\overline{Nu}}_i \right) / 3 \quad (\text{averaged over one cycle})
 \end{aligned} \right\} \quad (7.5)$$

7.2.2 Solution and numerical validations

The basic equations for the present unsteady rotating enclosure flow are solved numerically, using the program described before. The steady-state results are obtained by two convergence criteria: (1) $S_{MAX} \leq 1 \times 10^{-10}$, where S_{MAX} is the largest absolute value of the “mass source” used in the motion pressure correction equations, and (2) $MAX((\theta_{i,j}^{n+1} - \theta_{i,j}^n) / \theta_{i,j}^{n+1}) \leq 1 \times 10^{-5}$. The numerical oscillations required to reach the final state varies from 5 to 200 rotations. The $\Delta\tau$, τ_0 are the dimensionless time step in numerical simulations and dimensionless time required for one rotation respectively. In present study, $\Delta\tau$ equals to $1/72 \tau_0$. Some simulations may require up to one week of computational time with a Pentium IV PC.

Grid-independence tests were conducted. The differences in $\overline{Nu}$, $\overline{Nu}$ predicted from the proposed grid (20x90) and finer grid (40x 120) are within 1% during the entire periodic oscillation. The difference of the last temperature fields is kept within 2% also. Different time steps $\Delta\tau$ ($1/72 \tau_0 \sim 1/360 \tau_0$) are checked to ensure convergence results when Ta is higher than 10^3 . Through these tests, the proposed numerical method is considered to be suitable for the present problem.

7.3 Results and Discussion

The study fields are showed in Table 7.1.

Table 7.1: Summary of model parameters

Ra^*	Ta	Pr	A_w	A_H
$10^3 \sim 10^7$	$0 \sim 10^4$	0.7	1, 2	7.5
Notes: Character length, $L=12.7$ mm				

7.3.1 Oscillation phenomena

Under a non-inertial frame, the driving force rotates in the enclosure and results in oscillation phenomena. When the time for one cycle equals the time for finishing one rotation, it is called uni-rotation periodic oscillation ($n=1$). Otherwise, it is called multi-rotation oscillation ($1 < n < \infty$).

Typical result for mi-rotation periodic oscillation problem is shown in Fig. 7.2, where after about five rotations ($\tau = 5\tau_0$), the final periodic oscillation pattern is obtained. To ensure correct result, additional computation work was carried out beyond 5 rotations. Fig. 7.3 shows the Nu evolution for multi-rotating periodic oscillation. With the same Ra^* and increased Ta , chaotic oscillation occurs as shown in Fig. 7.4 where even after 175 rotations,

there is still no sign of periodic oscillation. At the special values of flow parameters, the flow may have different features, i.e. periodic oscillating or chaotic oscillating flows, close to observation of Yang and Wang (2003) who studied the bifurcation and stability of combined free and forced convection in rotating curved ducts.

(a) Mean Nusselt Number

(b) Mean Nusselt Number for each heater

Fig. 7.2: Uni-rotation periodic oscillation ($n=1$)

(a) Mean Nusselt Number

(b) Mean Nusselt Number for each heater

Fig. 7.3: Multi-rotating periodic oscillation ($n=2$) Fig. 7.4: Chaotic oscillation ($n=\infty$)

7.3.2 Flow and temperature fields without rotation

Due to close relationship between stationary and low rotation case, the discussion will start from stationary case firstly. In the vertical orientation, $\phi=90^\circ$ (270°), the enclosure is filled with strong uni-cellular clockwise (counterclockwise) circulation, as shown in Fig. 7.5c. In the orientations that ϕ are close to 0° or 180° , there are different multi-cellular flow patterns shown in table 5.3. In other words, there are clockwise vortex and counterclockwise vortex at the same time. Especially, in the orientation that ϕ equals 0° or 180° , the strength of two circulations equals each other and the flow pattern and temperature fields are symmetric, as shown in Fig. 7.5a, and 7.5e. In the whole cycle ($\phi=0^\circ\sim 360^\circ$), the flow and temperature fields are symmetric with respect to 180° .

7.3.3 Flow and temperature fields with rotation

When the velocity caused by thermal buoyancy force is so weak that conduction heat transfer dominated in the stationary situations, the temperature fields have a hierarchical structure, as shown in Fig. 5.3a, 6.3a, 6.4a. There are two anisomalous vortex in Fig. 7.6(a) as the result of transition stage for motion from counterclockwise to clockwise circulation. However, clockwise motion is stronger. And in temperature field, the convection heat transfer resulted from clockwise motion can be seen clearly and hierarchical structure doesn't appear. Therefore, it is expected a higher Nu compared to stationary case where conduction heat transfer dominates. In Fig. 7.6(b) ($\phi=45^\circ$), even though the clockwise motion dominates the flow, the temperature field shows hierarchical structure and results in an expected local minimum Nu . However, it is different to the conduction heat transfer whose Nu is close to unity. From Fig. 7.6(c) to Fig. 7.6(e), clockwise motion dominates the fluid. But when ϕ equals to 225° , the flow enters the transition stage again. In the core region, counterclockwise circulation appears and becomes stronger and stronger. The clockwise circulation falls back

on the corner regions and become weaken and weaken until disappear at last. In this orientation or near by, hierarchical structure appears again and bring about the second local minimum Nusselt number. In transition stage, there are more vortex in the enclosure which caused much difficult in numerical simulations. After transition stage, the counterclockwise motion dominates the flow as shown in Fig. 7.6(h). However, the counterclockwise motion is not as strong as clockwise. The flow and temperature fields for the phase angle of 360° are completely same to those for the phase angle of 0° . It proves a periodic process exists in this rotation situation.

Level	T
10	0.536
9	0.482
8	0.428
7	0.375
6	0.321
5	0.268
4	0.214
3	0.161
2	0.107
1	0.054

(a) $\phi = 0^\circ$, $\Omega = 0$ rpm (b) $\phi = 45^\circ$, $\Omega = 0$ rpm

Fig.7.5: Flow and temperature fields without rotation ($Ra^*=10^5$, $Ta=0$, $Pr=0.7$, $A_w=1$)

Level	T
10	0.355
9	0.320
8	0.284
7	0.249
6	0.213
5	0.178
4	0.142
3	0.107
2	0.071
1	0.036

(a) $\phi=0^\circ$, $\Omega \approx 1$ rpm

Level	T
10	0.371
9	0.334
8	0.297
7	0.260
6	0.223
5	0.186
4	0.149
3	0.111
2	0.074
1	0.037

(b) $\phi=45^\circ$, $\Omega \approx 1$ rpm

Level	T
10	0.301
9	0.271
8	0.240
7	0.210
6	0.180
5	0.150
4	0.120
3	0.090
2	0.060
1	0.030

(c) $\phi=90^\circ$, $\Omega \approx 1$ rpm

Fig.7.6: Flow and temperature fields with rotation

$$(Ra^*=1 \times 10^5, Ta=1 \times 10^3, Pr=0.7, A_W=1)$$

7.3.4 Effects of Coriolis and centrifugal forces

The results are showed in Fig 7.7. In general, the effect of Coriolis force and centrifugal force is small in present study ranges. As shown in Fig. 7.7a, these two forces can be neglected. For fixed Ra^* , the increase of Ta brings about the increase effect of Coriolis and centrifugal forces. As shown in Fig. 7.7b, the effect of these two forces increases obviously but still small. For fixed Ta , the increase of Ra^* means the increasing effect of buoyancy

force. As shown in Fig. 7.7c, the dashed line is perfect close to continuous line. It shows that the flow is dominated by thermal buoyancy force and inertial force. Since then, Ta is used to represent the effect of inertial force.

(a) $Ra^*=1 \times 10^5$, $Ta=1 \times 10^3$, $Pr=0.7$, $A_W=1$.

(b) $Ra^*=1 \times 10^5$, $Ta=1 \times 10^4$, $Pr=0.7$, $A_W=1$.

(c) $Ra^*=1 \times 10^6$, $Ta=1 \times 10^4$, $Pr=0.7$, $A_W=1$

Fig.7.7: The effect of Coriolis and centrifugal forces in different situations

7.3.5 Effects of inertial force on heat transfer

For the stationary case, as shown by zero Ta in Fig. 7.8 and Fig. 7.9a, three phenomenon can be seen: (1) There are three local peak Nu in one period. The two sided peak points are responding to the strongest points of clockwise and counterclockwise motions while the middle value is the result of R-B convection. (2) Corresponding to three local peak Nu , there are three inflexion points and the angles at the inflexion points are shown in Table 7.2 as ϕ_1 , ϕ_2 , ϕ_3 . (3) The minimum Nu of 1.76 is close to unity due to conduction heat transfer. The effects due to rotation is summarized in Table 7.2:

Table 7.2 Effects of rotation ($Ra^*=1 \times 10^5$, $Pr=0.7$)

Ta	No. of peak Nu	Nu_{max}	Nu_{min}	ΔNu ($Nu_{max} - Nu_{min}$)	ϕ_1	ϕ_2	ϕ_3
0	3	4.520	1.760	2.76	$\sim 0^\circ$	$\sim 160^\circ$	$\sim 200^\circ$
10^3	2	4.139	2.660	1.479	$\sim 30^\circ$	$\sim 225^\circ$	
10^4	2	3.586	2.549	1.037	$\sim 85^\circ$	$\sim 305^\circ$	
10^5	1	3.577	3.065	0.512	$\sim 135^\circ$

Fig. 7.8: Effect of Taylor number ($Ra^*=1 \times 10^5$, $Pr=0.7$, $A_w=1$)

(a) $Ta=0$ (b) $Ta=1 \times 10^3$

Fig. 7.9: Mean Nusselt number for each row of heater at different Taylor number

$$(Ra^*=1 \times 10^5, Pr=0.7, A_w=1)$$

For the stationary and low rotation cases, thermal buoyancy force causes two or more local peaks in Nu . And the flow consists of clockwise circulation, counterclockwise circulation and two transition stages between them. With the increase in rotation speed, the clockwise circulation is enlarged and tends to overcome other circulations. As a result, except for the first local peak Nu , the others become weaken and disappeared when the rotation speed is high enough. From the table 7.2, it can be seen that with increasing rotation and Ta , the swing of Nu is reduced and the minimum Nu is improved gradually. Especially, conduction heat transfer ($Nu \approx 1$) disappears.

The figure 7.9 shows that in the rotation situations, heat transfer behavior of row 1 and row 3 becomes asymmetrical gradually with the increase of rotation speed and Taylor number. It is resulted from the evolution process in the flow and temperature fields.

In Fig.7.10, the multi-rotation periodic oscillation is shown. It should be pointed out that it has the same Ra^* and Tu , as shown in Fig. 6c and the only difference is A_w , the aspect ratio. With increasing rotation speed, chaotic oscillation as shown in Fig. 7.4 will take place.

Fig. 7.10: Mean Nusselt number for each row of heater showing multi-rotation oscillation

$$(Ra^*=1 \times 10^5, Ta=1 \times 10^4, Pr=0.7, A_w=2)$$

7.3.6 Effects of thermal buoyancy force

The effects of heat flux on heat transfer can be obtained with a fixed Tu and different Ra^* , as shown in Figs. 7.11 and 7.12. When Ra^* is as low as 10^3 , conduction heat transfer with characteristic of low and flat Nu is seen. When Ra^* is increased to 10^4 , the fluctuation of Nu indicates the onset of convection and there is only one peak Nu . Further increase in Ra^* shows more oscillations in Nu and two or three local peak Nu appears in Fig. 7.11. These evolution phenomena seem a reverse process discussed in Fig. 7.8 and Table 7.2, showing that the evolution process is affected by the combined effect of rotation and heat flux.

Fig. 7.11: Effect of modified Rayleigh number ($Ta=1 \times 10^4$, $Pr=0.7$, $A_W=1$)

(a) Row 1

(b) Row 2

(c) Row 3

Fig. 7.12: Mean Nusselt number for each row of heater at different modified Rayleigh number

$$(Ta = 1 \times 10^4, Pr = 0.7, A_w = 1)$$

7.3.7 Effects of rotation on time-and-space averaged heat transfer

The Fig. 7.13 shows that rotation delays the onset of convection, pointed out by Fernando and Smith (2001) also. This is especially clear when Ta is as high as 10^4 . As a result of continuity, rotation reduces heat transfer performance in the low heat flux range and increases it in the relatively high heat flux ranges. The summary of correlations of $\overline{Nu}$ as a function of Ra^* , at different Ta is shown in Table 7.3.

Table 3: Summary of correlations of $\overline{Nu}$ at different Ta

$A_w=1$		
$Ta=0$	$\overline{Nu} = 0.3691 Ra^{*0.2028}$	$10^3 \leq Ra^* \leq 10^7$
$Ta=10^2$	$\overline{Nu} = 0.3367 Ra^{*0.2083}$	$10^3 \leq Ra^* \leq 10^7$
$Ta=10^3$	$\overline{Nu} = 0.2996 Ra^{*0.2139}$	$10^3 \leq Ra^* \leq 10^7$
$Ta=10^4$	$\overline{Nu} = 0.1649 Ra^{*0.2505}$	$10^4 \leq Ra^* \leq 10^7$
$A_w=2$		
$Ta=0$	$\overline{Nu} = 0.3983 Ra^{*0.1930}$	$10^2 \leq Ra^* \leq 10^6$
$Ta=10^2$	$\overline{Nu} = 0.3184 Ra^{*0.2098}$	$10^2 \leq Ra^* \leq 10^7$
$Ta=10^3$	$\overline{Nu} = 0.2281 Ra^{*0.2341}$	$10^3 \leq Ra^* \leq 10^7$
$Ta=10^4$	$\overline{Nu} = 0.1694 Ra^{*0.2506}$	$10^3 \leq Ra^* \leq 10^7$

(a) $A_w = 1$ (b) $A_w = 2$

Fig. 7.13: Effects of Ra^* and Ta on the overall averaged Nusselt number
in different aspect ratio

7.3.8 Comparison between experimental and numerical results and conjugate effects

In experimental studies, the measured temperatures do not show periodical variation even in the low rotation speed, as shown in Fig. 4.8 where the temperature of any element (heater) does not vary by more than 0.1°C over 10 minute interval. The reason is explained from parameters in Table 7.4.

Table 7.4: Parameter for copper, Teflon, Lexan and air

	Air	Copper	Teflon	Lexan
$\rho C_p (J/m^3 \cdot K)$	1.15×10^3	3.42×10^6	2.31×10^6	1.56×10^6
$k (W/m \cdot K)$	0.026	387.6	0.35	0.2

If there is heat transfer between a unit volume of solid zone and neighbor fluid zone, for example, between copper block and air, the temperature decrease of copper is about 1/1000 that of air because the value of ρC_p for copper is about 1/1000 than that of air. At the same time, due to the higher thermal conductivity of copper, this temperature change will be weakened by the neighbor copper elements. Therefore, rotation will result in a steady temperature field in the solid zones due to high thermal inertia of the solid compared with air. In fact, it has been used in the last chapter, boundary condition shown in **Eqs. 6.8**.

Flow and temperature evolution process in one rotation is shown in Fig. 7.14. In Fig. 7.14a, a counterclockwise circulation in the fluid zone can be seen clearly. Correspondingly, there is a clockwise circulation from Fig. 7.14c to 7.14f. Fig. 7.14b shows a transition stage from counterclockwise to clockwise circulation and Fig. 7.14f,g show an opposite transition process. During transition, more vortices arise in the flow field. The temperature fields are also shown to be a periodic phenomenon resulting from flow fields, close to the previous discussion. However, temperature fields in the solid zones seem almost constant in the whole rotation, supporting the explanation of thermal inertia.

(a) $\phi = 0^\circ$

(b) $\phi = 45^\circ$

(c) $\phi = 90^\circ$

(d) $\phi = 135^\circ$

(e) $\phi = 180^\circ$

(f) $\phi = 225^\circ$

Fig 7.14: Flow and temperature fields in the experimental conditions

$$(Ra=1 \times 10^4, Ta=3 \times 10^3, Pr=0.7, Aw=2)$$

The experimental results are compared with numerical simulation results in Fig. 7.15 and Fig. 7.16. The mean error between them is less than 15%. Considering complex geometrical structure, thermal contact resistance, uncertainties of boundary conditions, the results are considered encouraging. An interesting phenomenon in Fig. 7.15 is that the temperature increases for three heaters are close together. The obvious difference of heat transfer performance for three row of heaters discussed in references of Heindel *et al.* (1995) without considering the substrate effect is not valid for air. In addition, it can be seen from Fig. 7.16 that the effect of rotation to mean heat transfer is small in present study range.

(a) $Ta = 3.72 \times 10^1$ (b) 3.1×10^3

Fig. 7.15: Comparison between numerical and experimental results

Fig. 7.16: Effects of rotation on heat transfer

7.3.9 Optimal distribution of discrete heat sources

Because of complexity, the conjugate and radiation effects are neglected and the geometrical configuration has three freedoms: locations of three heaters. However, computation work for these unsteady problems considering all possible combinations is still prohibitive. Partial optimal work has been carried out to identify the general trends that affect the performance and give some design advices from the opinion of thermal management.

Firstly, calculations are carried out when the middle row of heater is fixed at the center of heater wall and the location of the two side heaters is changed symmetrically. From results shown in Fig. 7.17, value of Nu for heater 1 is higher than others greatly. The reason is pointed out in the study of flow pattern evolutions. There are clockwise and counterclockwise circulations in each periodic. However, the clockwise circulation is so strong that it lasts most time of periodic. Therefore, in most time, heater 1 is located near the starting cold current of the clockwise circulation and it results in a better heat transfer performance. From Fig. 7.17,

greater distance between the heaters will result in better performance, except the side heaters are too close to the corner where inanimate fluid is deleterious to heat transfer. From the opinion of thermal management, the optimal location is 2 ± 0.083 even though minimization is the development trend of electronic package. Unfortunately, from Fig. 7.18, the optimal location will change with Ra^* and Ta .

To improve the performance of the weakest component, middle heater, more calculations are carried out when the side heaters are fixed and the middle heater changes freely along the heater wall. Final results in Fig. 7.19 show that the optimal distributions are not uniform and they are pushed toward the starting cold current of the dominated clockwise circulation, close to the description by Silva *et al.* (2004) in the stationary situation where the enclosure is dominated by clockwise motions. With rotation, the optimal distribution is affected by balancing of these two motions. The optimal distribution of discrete heat sources in rotating fluid is close to results in the stationary situation if their circulations are same.

Fig. 9.17: Optimal location of side heaters ($Ra^*=1 \times 10^5$, $Ta=1 \times 10^4$, $Pr=0.7$, $A_w=1$)

(a) Effect of Ra^* (b) Effects of Ta Fig. 7.18 Effects of Ra^* and Tu to optimal distribution

Fig. 7.19: Optimal location of center heater ($Ra^*=1 \times 10^5$, $Ta=1 \times 10^4$, $Pr=0.7$, $A_W=1$)

7.4. Conclusions

The following conclusions can be drawn from the present study:

- (1) Three physically realizable phenomena, uni-rotation periodic oscillation, multi-rotation periodic oscillation and chaotic oscillation are identified.
- (2) For the stationary and low rotation cases, thermal buoyancy force causes two or more local peaks in Nu . And the flow consists of clockwise circulation, counterclockwise circulation and two transition stages between them. With the increase of rotation speed, the clockwise circulation is enlarged and tends to overcome other circulations. As a result, except for the first local peak Nu , the others become weakened and disappeared when the rotation speed is high enough. In the rotation situations, heat transfer behavior for the two side rows of heaters is not symmetrical in each cycle due to asymmetrical behavior of clockwise and counterclockwise motions.

- (3) In general, rotation increases flow stability, results in imbalance of clockwise and counterclockwise circulations, improves heat transfer in the weak stages and reduces the oscillation of Nusselt number. It may increase or reduce average heat transfer performance. These phenomena are determined by the combined effect of thermal buoyancy force and inertial force. The effects of Coriolis force and centrifugal force to heat transfer are small in present study ranges. The correlations between Ra^* , Ta , and Nu are shown in Table 7.3.
- (4) Rotation results in a steady temperature field for solid zones and periodic flow and temperature fields for fluid zones even in the low rotation speed range.
- (5) The optimal distribution of discrete heat sources in rotating fluid is close to results in the stationary situation if their circulations are same.

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

2-D and 3-D numerical simulations are developed and carried out to study flow pattern evolution and heat transfer characteristics in an inclined/rotating enclosure with discrete heat sources. All present simulation results are consistent with existing reported work (Tang 1975, Catton et al. 1978, Hamady et al. 1989, 1994, Heindel et al. 1995, Tou et al. 1999, Tou and Zhang 2003), as well as the present experimental results. This gives confidence to all the new simulation results.

In stationary cases, the following conclusions can be drawn:

- (1) Complex flow pattern evolution, flow field, and temperature fields are observed due to interactions of buoyancy forces tangential and normal to the heater surfaces.
- (2) In the horizontal orientation for water, with increasing Ra^* , the flow pattern evolutions are complex and three main characteristics can be identified: toroidal convection, bimodal convection and R-B convection. For increase in heat transfer, the horizontal orientation is best because the lowest $\overline{Nu}$ among the three rows is higher than those in other orientations.
- (3) On inclining the enclosure towards the horizontal orientation (heated from below) for liquid coolants, the flow and temperature fields become complex and distorted. Three-dimensional effects, such as fluid flow in horizontal direction and distorted

temperature field begin to appear and intensify, which greatly enhance the heat transfer for the top row of heaters.

- (4) Edge effect increases heat transfer greatly. In the situation that the effects of edge can be negligible or small, the results of 2-D model are close to those of 3-D model.
- (5) For most liquids (except metal liquids), whose Pr is greater than 1, the effects of Pr are negligible. For most gases whose Pr are about 0.7, the effects of Pr are small and the differences between liquids and gases in $\overline{Nu}$ and $\overline{\overline{Nu}}$ are small. According to scale analysis, the fluid motion is balanced between thermal buoyancy force and friction force. It can be used to develop and test new coolants.
- (6) Optimal location of discrete heat sources in the vertical orientation shows that the spacing between heaters is not uniform after the onset of convection. Increasing Ra^* value has the effect of pushing the heat sources towards the starting cold current of the clockwise circulation. It is beneficial for thermal design engineers in the layout of electronic components.

In rotating cases, the main conclusions are shown below:

- (1) Three physically realizable phenomena, uni-rotation periodic oscillation, multi-rotation periodic oscillation and chaotic oscillation can be identified and perfect agreement to the published periodic experimental results.
- (2) In low rotation speed ranges, due to buoyancy force, there are clockwise and counterclockwise circulations, and correspondingly two or more local peak Nusselt numbers in each periodic oscillation. In transition stages, more vortices are predicted.
- (3) With increasing rotation speed and increasing effect of inertial force, clockwise circulation is enlarged and tends to overcome the counterclockwise circulation. The

second local peak Nusselt number becomes weakened and disappeared. Inertial force also reduces the oscillation of Nusselt number in general.

- (4) But after the rotation convection enters the solid-body rotation mode, heat transfer is reduced. Hence it is noteworthy that relative effect of driving buoyancy and driven inertial forces can lead to increase or decrease in heat transfer. It is the important finding of present study and should be paid attention in the applications.
- (5) The effect of Coriolis and Centrifugal force is small and differentiated clearly from those of other forces by a straightforward numerical method. This is one of the important contributions of the present study.
- (6) In the present study range, R-B convection does not happen in rotating convection. Therefore, rotation increases flow stability. But it delays the onset of convection, another expression of solid body rotation.
- (7) Rotation reduces oscillation in Nusselt number, improves heat transfer in the weak stages but may reduce or increase mean heat transfer performance. In the rotation situations, heat transfer behavior for the two side **rows** of heaters is not symmetrical in each cycle due to asymmetrical behavior of clockwise and counterclockwise motions. The correlations between Ra^* , Ta , and Nu are given.
- (8) Rotation results in a steady temperature field for solid zones and periodic flow and temperature fields for fluid zones even in the low rotation speed range.
- (9) The most important conclusion for thermal design engineers is that the optimal distribution of discrete heat sources in rotating fluid is close to results in the stationary situation if their circulations are same.
- (10) Numerical methods have a distinct advantage over the experimental study in generating database and sorting out the effects of Coriolis force and centrifugal force to others. Model parameters and body forces can be changed at will with ease while

it is very difficult or impossible to do with an experimental study. Numerical model is capable of revealing complex flow and temperature field structure and is versatile in parametric study. However, computer computation ability is a great limitation.

8.2 Recommendations

Rotation has obvious effect to flow structure, flow pattern evolution, local heat transfer for natural convection in the present closed rotating system. However, it has relatively small effect to the mean heat transfer. The recommended work in the future are shown below:

- (1) For periodic rotating heat transfer, inertial force plays an important role. It means the Prandtl number must be taken into consideration to enrich the present work. In addition, different aspect ratios should be investigated in detail.
- (2) The study of complete transition stage from uni-rotation periodic flow to chaotic flow or body rotation flow or turbulent flow is difficult to do by numerical simulations. Experimental studies with Mach-Zehnder interferometer and laser sheet flow visualization used in the rotation situations to measure the temperature and flow fields are suggested to explore these challenges.

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Appendix: List of journal publications

1. C.P Tso, L.F. Jin, S.K.W Tou and X.F. Zhang, 2004, Flow pattern evolution in natural convection cooling from an array of discrete heat sources in a rectangular cavity at various orientations, *International Journal of Heat and Mass Transfer*, Vol. 47, pp. 4061-4073.
2. C.P Tso, L.F. Jin and S.K.W Tou, Flow pattern evolution, natural convection characteristics and effects of various body forces in a rotating enclosure of square cross-section, submitted to an *International Journal*.
3. L.F. Jin, C.P Tso and S.K.W Tou, Experimental and numerical studies on a rotating cavity with the discrete heat sources and conjugate effects, submitted to an *International Journal*.
4. L.F. Jin, S.K.W Tou, and C.P Tso, Effects of rotation on natural convection cooling from an array of discrete heat sources in a rectangular cavity, submitted to an *International Journal*.