Antenna array signal processing based methods for target detection and source estimation

Huang, Ling

2014

Huang, L. (2014). Antenna array signal processing based methods for target detection and source estimation. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/58904>

<https://doi.org/10.32657/10356/58904>

Antenna Array Signal Processing Based Methods for Target Detection and Source Estimation

HUANG LING

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirements for the degree of
Doctor of Philosophy

2014

ACKNOWLEDGEMENTS

I am sincerely grateful to all the individuals who have contributed to the successful completion of my graduate studies and this research thesis.

First and foremost, I wish to express my deepest gratitude to my supervisor, Professor LU Yilong, for his kindness, generosity, patience and consistent encouragement throughout my research project. His valuable suggestions, great insights and profound knowledge in the related research fields guide me through all the difficulties encountered during my research and lead me to the successful completion of my graduate study.

I would also extend my gratitude to Dr. GUO Xin, Dr. SUN Hongbo, and Dr. LIU Weixian who shared their knowledge and experience unselfishly. I would like to thank Dr. FENG Hongchuan for his help and careful explanations in related literatures of my project.

Besides, I would like to thank 2012 Final Year Project (FYP) student WEN Bihan and a group of students from a Design and Innovation Project (DIP) team 2011-E002 for their help in collecting experimental synthetic aperture radar data for random sparse array study in Chapter 4.

Especially, I am very grateful for the love and support of my parents and my dear husband Mr. HUANG Dezhao.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS.....	I
TABLE OF CONTENTS	II
SUMMARY.....	V
LIST OF FIGURES.....	VII
LIST OF TABLES.....	XIII
LIST OF ABBREVIATIONS.....	XIV
LIST OF SYMBOLS.....	XVI
CHAPTER 1 INTRODUCTION	1
1.1 BACKGROUND.....	1
1.2 MAJOR CONTRIBUTIONS	4
1.3 ORGANIZATION OF THE THESIS	6
CHAPTER 2 LITERATURE REVIEW	8
2.1 INTRODUCTION.....	8
2.2 REVIEW OF TARGET DETECTION	8
2.2.1 Wall Effects	10
2.2.2 Synthetic Aperture Radar (SAR) Imaging	12
2.2.3 Tomographic Imaging.....	15
2.2.4 Compressed Sensing (CS).....	18
2.3 REVIEW OF SOURCE ESTIMATION.....	20
2.3.1 Direction of Arrival (DOA) Estimation	22
2.3.2 Polarization Estimation	25
2.4 CONCLUDING REMARKS.....	26
CHAPTER 3 TARGET DETECTION BASED ON THROUGH-THE-WALL IMAGING	28
3.1 THROUGH-THE-WALL (TTW) TOMOGRAPHIC IMAGING.....	28
3.1.1 Multiple-View Through-The-Wall (TTW) Tomographic Imaging.....	29
3.2 COMPENSATION OF WALL EFFECTS	32
3.3 CHIRP SIGNAL	34
3.4 NUMERICAL EXAMPLES	37
3.5 CONCLUDING REMARKS.....	45
CHAPTER 4 TARGET DETECTION USING RANDOM SPARSE ARRAY	46
4.1 RANDOM SPARSE ARRAY (RSA).....	46
4.1.1 SAR Mode	47

4.1.2	Array Mode	48
4.2	COMPRESSED SENSING (CS) WITH RSA	49
4.2.1	Static Targets	50
4.2.2	Moving Targets	51
4.3	NUMERICAL EXAMPLES	55
4.3.1	Static Targets Detection	55
4.3.2	Moving Targets Detection	58
4.4	EXPERIMENTAL VALIDATION	63
4.4.1	Experimental Setup	63
4.4.2	Detection Results	66
4.5	DISCUSSIONS	78
4.5.1	Advantage	78
4.5.2	Limitation	78
4.6	CONCLUDING REMARKS	79
CHAPTER 5 DIRECTION OF ARRIVAL AND POLARIZATION ESTIMATIONS USING VECTOR CIRCULAR ARRAY		80
5.1	VECTOR CIRCULAR ARRAY (VCA)	80
5.1.1	Scalar Circular Array (SCA)	80
5.1.2	Proposed Vector Circular Array (VCA)	81
5.1.3	Mathematical Description of A VCA	83
5.2	DIRECTION OF ARRIVAL (DOA) AND POLARIZATION ESTIMATIONS	85
5.2.1	Signal Model	85
5.2.2	DOA Estimation Algorithm	89
5.2.3	Polarization Estimation Algorithm	92
5.3	NUMERICAL SIMULATIONS	93
5.3.1	Frequency Filtering	94
5.3.2	Parameter Estimations	95
5.3.3	Error Analysis	98
5.4	DOA AND POLARIZATION ESTIMATIONS IN A COLORED GAUSSIAN NOISE	105
5.4.1	Estimation Algorithm	105
5.4.2	Simulation Examples	107
5.5	DISCUSSIONS	110
5.5.1	Advantage	110
5.5.2	Limitation	110
5.6	CONCLUDING REMARKS	111
CHAPTER 6 DIRECTION OF ARRIVAL AMBIGUITY RESOLUTION OF ANTENNA ARRAY		112
6.1	AMBIGUITY PROBLEM IN A UNIFORM LINEAR ARRAY (ULA)	113

6.2	VECTOR CIRCULAR ARRAY (VCA) AS A COMPLETE ARRAY	116
6.2.1	Signal Model	116
6.2.2	Vector Arc Array (VAA)	119
6.3	PROPOSED COMPOSITE MULTIPLE RESOLUTION (CMR) APPROACH	122
6.3.1	Design Procedure	122
6.3.2	Ambiguity Resolution	124
6.3.3	Wideband DOA Estimation	127
6.4	NUMERICAL EXAMPLES	129
6.4.1	ULA to Improve Resolution of DOA.....	130
6.4.2	VAA to Resolve DOA Ambiguity	133
6.4.3	Resolve Ambiguity and Improve Resolution of DOA	133
6.4.4	Wideband DOA Estimation	137
6.4.5	Super Wideband DOA Estimation	142
6.5	DISCUSSIONS.....	147
6.6	CONCLUDING REMARKS.....	149
CHAPTER 7 CONCLUSIONS AND RECOMMENDATIONS.....		150
7.1	CONCLUSIONS	150
7.2	RECOMMENDATIONS FOR FUTURE WORK	151
BIBLIOGRAPHY.....		153
AUTHOR'S PUBLICATIONS.....		161
APPENDIX A: DERIVATION OF SOLUTION OF (5-17).....		163

SUMMARY

Antenna arrays provide significant contributions in areas such as wireless communications, radar and sonar. Antenna array signal processing has been gaining great interests in two major types of applications, target detection and source estimation. Target detection is important in areas such as fire rescuing, anti-terrorist and surveillance, in which through-the-wall (TTW) target detection and line-of-sight (LOS) target detection are equally demanded. Source estimation is important in areas such as vehicle tracking, patient safety, medical instrumentation monitoring and direction finding.

This thesis studies antenna array signal processing methods for radar target detection and source direction estimation. For radar target detection, both TTW and LOS target detections using radar imaging methods are studied. Since there are not many research works reported on tomographic imaging in TTW target detection, a schematic study on static target detection behind walls using single-view and multiple-view tomographic imaging methods are carried out. The findings show that multiple-view method gives relatively good performances both with and without walls. This study is only theoretical but it provides a potential direction of using the multiple-view method in TTW applications with loose time and space constraints. Besides, a random sparse array (RSA) and compressed sensing (CS) based target detection method in LOS condition is proposed since the approach of using a RSA and CS on target detection has not been investigated by other researchers. A RSA reduces hardware cost and simplifies system implementations and CS reduces the amount of data required. The RSA is characterized into two modes, the SAR mode and the array mode, for corresponding static and moving target detections. Both simulation and experiment results show that the proposed method can detect static and moving targets robustly over conventional methods with much less data and hardware cost. This proposed method can find its potential applications in imaging systems requiring large data storage and processing time.

For source direction estimation, direction of arrival (DOA) and polarization estimations are studied. Conventional uniform circular array (UCA) with vertically

polarized antenna elements is well known for DOA estimation of vertically polarized source signals from low grazing angles. However, recently there are increasing demands for DOA and polarization estimations of arbitrarily polarized signals from high grazing angles in military areas such as airborne applications. A UCA cannot meet the requirements, therefore a vector circular array (VCA) with identical elements polarized in radial directions on a circular grid is proposed along with a simple and intuitive estimation method. Extensive simulations and error analyses show that the VCA can estimate DOAs and polarizations of sources from high grazing angles with very small estimation errors.

DOA ambiguity problems due to large inter-element spacing of antenna arrays have been noticed since the first study of DOA estimation. Although some research works have been tried to solve the problem, no optimum solution has come out. In view of this and the property of a VCA, an ambiguity resolving approach based on a VCA, namely the composite-multiple-resolution (CMR) approach, is proposed. The basic idea is to make use of different portions of the array, namely the 2-element ULA and the vector arc array (VAA). Simulations and case studies are provided to investigate the performance of the approach. The results show that DOA ambiguity problem can be solved by using the approach and more interestingly, the operating bandwidth of the VCA can be greatly increased to more than 9:1 with the proposed approach. The VCA based approach may find its potential applications in high speed airborne applications.

LIST OF FIGURES

Fig. 2.1 Spatial model of a signal propagating through a wall (all reflected signals from the wall are not shown for simplicity).....	11
Fig. 2.2 Scenario of target detection behind walls using SAR [14] (reflected signals from the wall are not shown).	13
Fig. 2.3 System geometry of a monostatic radar tomography.....	16
Fig. 2.4 A block diagram of a beamformer.....	23
Fig. 2.5 Polarization decomposition.....	26
Fig. 3.1 Geometry of the multiple-view TTW tomographic imaging system (reflected signals from the wall are not shown).....	30
Fig. 3.2 Ray path of a signal propagating through a wall with thickness d_w and ray path of the signal propagating in the air (reflected signals from the wall are not shown for simplicity).	33
Fig. 3.3 Comparison of two baseband chirp pulses (up chirp) with DC = 0.4 and PRI = 1s, (a) the 1st chirp in the time domain, (b) spectrum of the 1st chirp ($B_1 = 20$ Hz), (c) the 2nd chirp in the time domain, and (d) spectrum of the 2nd chirp ($B_2 = 100$ Hz).	35
Fig. 3.4 Autocorrelation of the 2nd chirp pulse with $B = 100$ Hz, and cross-correlation between the two chirp pulses in Fig. 3.3.	37
Fig. 3.5 Autocorrelation of a stepped-frequency signal with $B = 100$ Hz, and cross-correlation between two stepped-frequency signals with same parameters as the two chirp pulses in Fig. 3.3.	37
Fig. 3.6 A flow chart of the numerical procedure in multiple-view tomographic imaging.	38
Fig. 3.7 Target detection result of single-view tomographic imaging using a chirp pulse with $[-15, 0]$ dB display threshold (a) when walls are absent, (b) when walls are present and wall effects are not compensated.	40
Fig. 3.8 Target detection result of multiple-view tomographic imaging using a chirp pulse with $[-15, 0]$ dB display threshold (a) when walls are absent, (b) when walls are present and wall effects are not compensated.	41
Fig. 3.9 Target detection result of multiple-view tomographic imaging using a stepped-frequency signal with $[-15, 0]$ dB display threshold (a) when walls are absent, (b) when walls are present and wall effects are not compensated.	42
Fig. 3.10 Target detection results of (a) single-view with a chirp pulse, (b) multiple-view with a chirp pulse when wall effects are compensated, (c) single-view with a stepped-frequency signal, and (d) multiple-view with a stepped-frequency signal when wall	

effects are compensated.	43
Fig. 3.11 Multiple targets detection result using a chirp pulse for (a) single-view and (b) multiple-view when wall effects are not compensated, (c) single-view and (d) multiple-view when wall effects are compensated.....	44
Fig. 4.1 Schematic diagram of a linear SAR mode RSA and its respective comparable geometry: uniform SAR.	47
Fig. 4.2 Schematic diagram of a linear array mode RSA and its respective comparable geometry: ULA.	48
Fig. 4.3 Flow chart of the static target detection process.	52
Fig. 4.4 Flow chart of moving target detection process.	54
Fig. 4.5 Static target detection results at SNR = 30 dB: (a) original target scene, (b) detection using full SAR data and back-projection, (c) detection using sparse RSA data and back-projection, (d) detection using sparse RSA data and compressed sensing.	56
Fig. 4.6 Detection resolutions at SNR = 0 dB: (a) back-projection with full SAR data, (b) compressed sensing with RSA data of the same size as full SAR data.	57
Fig. 4.7 Detection results using RSA data with compressed sensing at different SNR values: (a) 20 dB, (b) 10 dB, (c) 0 dB, (d) -10 dB.	58
Fig. 4.8 Original target scene (static target in blue circle and moving targets in red dots).	59
Fig. 4.9 Detection results at SNR = 30 dB: (a) moving target scene, (b) detection using full ULA data and beamforming, (c) detection using sparse RSA data and beamforming, (d) detection using sparse RSA data and compressed sensing.	60
Fig. 4.10 Moving target detection results using sparse RSA data and compressed sensing at different SNR values: (a) 20 dB, (b) 10 dB, (c) 0 dB, (d) -10 dB.	60
Fig. 4.11 Track reconstruction results at SNR = 30 dB: (a) moving target track, (b) reconstruction using full ULA data and beamforming, (c) reconstruction using sparse RSA data and beamforming, (d) reconstruction using sparse RSA data and compressed sensing.	62
Fig. 4.12 Reconstruction results of indistinct tracks at SNR = 30 dB: (a) actual moving target track, (b) reconstruction using full ULA data and beamforming, (c) reconstruction using sparse RSA data and beamforming, (d) reconstruction using sparse RSA data and compressed sensing.	62
Fig. 4.13 Equipment setup with colocated broadband transmitting and receiving antennas and an Agilent N3382A VNA as the transceiver.	63
Fig. 4.14 Ground truth of the target area with main targets: four palm trees, one street lamp post, and one trihedral corner reflector (circled in red).	65
Fig. 4.15 Imaging result using full SAR data and back-projection method with pixel size of	

0.01 m $\times$ 0.01 m	65
Fig. 4.16 Static target detection results with the same set of parameters as in Table 4.3 at SNR = -10 dB: (a) original target scene, (b) detection using full SAR data and back- projection, (c) detection using sparse RSA data and back-projection, (d) detection using sparse RSA data and compressed sensing.	66
Fig. 4.17 Detection result using back-projection with full SAR data in [-30, 0] dB scale with pixel size of 0.5 m $\times$ 0.5 m.	67
Fig. 4.18 Detection result using back-projection with sparse RSA data in [-30, 0] dB scale with pixel size of 0.5 m $\times$ 0.5 m.	67
Fig. 4.19 Detection result using compressed sensing with sparse RSA data in [-30, 0] dB scale with pixel size of 0.5 m $\times$ 0.5 m.	68
Fig. 4.20 Detection result using back-projection with sparse RSA data in [-10, 0] dB scale with pixel size of 0.5 m $\times$ 0.5 m.	69
Fig. 4.21 Detection result using compressed sensing with sparse RSA data in [-10, 0] dB scale with pixel size of 0.5 m $\times$ 0.5 m.	69
Fig. 4.22 The integrated errors of BP and CS results shown in merit order for four different percentages of data.	71
Fig. 4.23 The integrated errors of BP and CS results shown in histogram for four different percentages of data.	72
Fig. 4.24 Comparison of integrated errors for BP and CS with different percentages of data.	73
Fig. 4.25 Worst, Best, and Mean RSA-BP and RSA-CS images from 100 trials based on 20% random positions from the full SAR data.	74
Fig. 4.26 Worst, Best, and Mean RSA-BP and RSA-CS images from 100 trials based on 40% random positions from the full SAR data.	75
Fig. 4.27 Worst, Best, and Mean RSA-BP and RSA-CS images from 100 trials based on 60% random positions from the full SAR data.	76
Fig. 4.28 Worst, Best, and Mean RSA-BP and RSA-CS images from 100 trials based on 80% random positions from the full SAR data.	77
Fig. 5.1 A SCA is composed of uniformly spaced elements with identical amplitude and polarization direction (vertical polarization represented by bold red arrows).	81
Fig. 5.2 A VCA is composed of uniformly spaced elements with identical amplitude but different polarizations directions (radial polarizations in bold red arrows).	82
Fig. 5.3 Geometry of an N -element VCA.	83
Fig. 5.4 Illustration of the relations of a signal polarized in an arbitrary $\mathbf{E}$ direction to the spherical and Cartesian coordinates (θ is the elevation angle from the z -axis, ϕ is the azimuth angle from the x -axis, γ is the polarization angle, $\mathbf{k}$ is the wave vector in the	

signal propagation direction, $\mathbf{r}$ is the radiation vector, ρ is the projection of the source direction in x - y plane).....	86
Fig. 5.5 Array output signal powers and threshold level at u and v axes (u is axis 1 and v is axis 4 in this example): two peaks correspond to two source frequencies.	94
Fig. 5.6 MUSIC spectrums of two sources at the m th ULA (a) $m = 1$, (b) $m = 2$, (c) $m = 3$, and (d) $m = 4$ for indirect DOA (Φ_m^1, Φ_m^2) estimation (estimated values are labeled using vertical dotted lines).	95
Fig. 5.7 (a) Array geometry of the 8-element VCA and direction of source 1, (b) intersection of all half-cones of source 1, (c) array geometry of the 8-element VCA and direction of source 2, (d) intersection of all half-cones of source 2 (black dots represent antenna elements and black solid lines represent respective source directions in (a) and (c)).	97
Fig. 5.8 θ and ϕ estimation of source 1 over 100 trials at SNR equals to (a) 0 dB, (b) 5 dB, (c) 10 dB, (d) 15 dB (small gray dots are the individual estimations in different runs and large black circles are the mean estimations).	99
Fig. 5.9 θ and ϕ estimation of source 2 over 100 trials at SNR equals to (a) 0dB, (b) 5 dB, (c) 10 dB, (d) 15 dB (small gray dots are the individual estimations in different runs and large black circles are the mean estimations).	99
Fig. 5.10 RMS errors of (a) indirect DOA, (b) θ , (c) ϕ and (d) γ of sources with different elevation angles at different SNR values ranging from 0 dB to 30 dB.	101
Fig. 5.11 RMS errors of θ , ϕ and γ when θ is fixed, ϕ varies from 0° to 360° and γ varies from 0° to 90° : $\theta = 15^\circ$ in (a) – (c) and $\theta = 60^\circ$ in (d) – (f).	102
Fig. 5.12 RMS errors of θ , ϕ and γ when ϕ is fixed, θ varies from 5° to 85° and γ varies from 0° to 90° : $\phi = 90^\circ$ in (a) – (c) and $\phi = 180^\circ$ in (d) – (f).	102
Fig. 5.13 RMS errors of (a) indirect DOA, (b) θ , (c) ϕ and (d) γ at different SNR values from 0 dB to 30 dB when different number of sources is present.	104
Fig. 5.14 Individual (gray dots) and average (black circle) values of the 100 DOA estimations for signals with color noise at different SNR levels by conventional MUSIC and the adopted MFOC-MUSIC method.	108
Fig. 5.15 RMS errors of (a) indirect DOA, (b) θ , (c) ϕ and (d) γ at different SNR values from –10 dB to 20 dB in both white and colored Gaussian noise using MUSIC and MFOC-MUSIC.	109
Fig. 6.1 Flow chart of the progression in Chapter 6.....	113
Fig. 6.2 Illustration of phase difference between adjacent elements of a ULA.	113
Fig. 6.3 MUSIC spectrum at SNR = 20 dB when (a) $d = \lambda/2$, (b) $d = \lambda$, (c) $d = 5\lambda$, (d) $d = 10\lambda$	115

Fig. 6.4 Illustration of an 8-element VCA with 2 elements (gray dots) experiencing very large polarization mismatch (R_0 is the radius of the VCA, D is the diameter of the VCA and d is the inter-element spacing of the 2-element ULA).	120
Fig. 6.5 Flow chart of the CMR approach.	124
Fig. 6.6 Selection illustration of 3-element VAAs and 2-element ULAs in an 8-element VCA.	125
Fig. 6.7 Normalized signal powers at elements 1 to 4 in the 8-element VCA shown in Fig. 6.6 with 20 dB display dynamic range (DR) for each element.	130
Fig. 6.8 Results for $N = 8$, $D = d = 0.5\lambda$ and $d_{arc} = 0.19\lambda$ (no ambiguity occurs in both ULA and VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of full VCA, (b) MUSIC spectrum of 3-element VAA, (c) MUSIC spectrum of 1 st ULA (elements 1 & 5) and 2 nd ULA (elements 2 & 6) in Fig. 6.6, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.	132
Fig. 6.9 Results for $N = 8$, $D = d = \lambda$ and $d_{arc} = 0.38\lambda$ (ambiguity occurs in ULA but not in VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of full VCA, (b) MUSIC spectrum of 3-element VAA, (c) MUSIC spectrum of 1 st ULA (elements 1 & 5) and 2 nd ULA (elements 2 & 6) in Fig. 6.6, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.	134
Fig. 6.10 (a) Half-cones of possible real and ambiguous source directions, (b) the indicator matrix in Step 5 with 1° division in the θ axis and 5° division in the ϕ axis.	135
Fig. 6.11 Results for $N = 8$, $D = d = 3\lambda$ and $d_{arc} = 1.15\lambda$ (ambiguity occurs in both ULA and VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of 3-element VAA, (b) MUSIC spectrum of 1 st ULA (element 1 & 5) and 2 nd ULA (elements 2 & 6) in Fig. 6.6, (c) indicator matrix, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.	136
Fig. 6.12 Configuration of a 16-element VCA with diameter D , inter-element spacing d of a 2-element ULA and inter-element spacing d_{arc} of a VAA.	138
Fig. 6.13 Normalized signal powers at elements 1 to 8 in the 16-element VCA in Fig. 6.12 with 20 dB display dynamic range (DR) for each element.	138
Fig. 6.14 Selection illustration of 6-element VAAs and 2-element ULAs in a 16-element VCA.	139
Fig. 6.15 Results for $N = 16$, $D = d = 3\lambda$ and $d_{arc} = 0.58\lambda$ (ambiguity occurs in a ULA but not in a VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of 6-element VAA, (b) MUSIC spectrum of 1 st ULA (elements 1 &	

9) and 2 nd ULA (elements 2 & 10) in Fig. 6.14, (c) indicator matrix, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.	140
Fig. 6.16 Results for $N = 16$, $D = d = 6.2\lambda$ and $d_{arc} = 1.21\lambda$ (ambiguity occurs in a ULA but not in a VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of 6-element VAA, (b) MUSIC spectrum of 1 st ULA (elements 1 & 9) and 2 nd ULA (elements 2 & 10) in Fig. 6.14, (c) indicator matrix, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.	141
Fig. 6.17 Results for $N = 16$, $D = d = 12.5\lambda$ and $d_{arc} = 2.44\lambda$ with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.5\text{ GHz}$: (a) MUSIC spectrum of 6-element VAA, (b) MUSIC spectrum of 1 st ULA (elements 1 & 9) and 2 nd ULA (elements 2 & 10) in Fig. 6.14, (c) indicator matrix, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.	143
Fig. 6.18 (a) Final estimation result at $f = 1.5\text{ GHz}$ and source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ when 1 st ULA (elements 1 & 9) and 3 rd ULA (elements 3 & 11) are selected and one 6-element VAA is used, (b) final 3D DOA estimation result, (c) combination of results when different 2-element ULA pairs (1 st and 2 nd ULAs, 1 st and 3 rd ULAs in Fig. 6.14) are selected and one 6-element VAA is used, and (d) 3D combination result.	144
Fig. 6.19 Results for $N = 16$, $D = d = 25\lambda$ and $d_{arc} = 4.88\lambda$ with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 3.0\text{ GHz}$: (a) MUSIC spectrum of 6-element VAA, (b) MUSIC spectrum of 1 st ULA (elements 1 & 9) and 2 nd ULA (elements 2 & 10) in Fig. 6.14, (c) indicator matrix with 1° division in θ axis and 5° division in ϕ axis, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.	146
Fig. 6.20 (a) Final estimation result at $f = 3.0\text{ GHz}$ and source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ when 1 st ULA (elements 1 & 9) and 3 rd ULA (elements 3 & 11) are selected and one 6-element VAA is used (with 20 dB display threshold), (b) final 3D DOA estimation result, (c) combination of results when different 2-element ULA pairs (1 st & 2 nd ULAs and 1 st & 3 rd ULAs in Fig. 6.14) are selected and one 6-element VAA is used (with 20 dB display threshold), and (d) 3D combination result.	147
Fig. 6.21 Flow chart of the three scenarios using CMR approach.	148

LIST OF TABLES

Table 3.1: Transmitting Chirp Pulse Parameters	39
Table 4.1: Static Target Simulation Parameters	55
Table 4.2: Moving Target Simulation Parameters.....	59
Table 4.3: Experimental Parameters	64
Table 4.4: Four Types of Integrated Error – BP with different RSA data	72
Table 4.5: Four Types of Integrated Error – CS with different RSA data.....	72
Table 5.1: Polarization Angle Estimation at SNR = 30 dB	98
Table 5.2: Mean Errors of All Parameter Estimations of Source Two $(\theta_2, \phi_2, \gamma_2) = (30^\circ, 180^\circ, 45^\circ)$ at Different SNR Values	100

LIST OF ABBREVIATIONS

ADC	Analog-to-Digital Conversion
BP	Back-Projection
CBF	Conventional Beamforming
CMR	Composite-Multiple-Resolution
CS	Compressed Sensing
DBF	Digital Beamforming
DC	Duty Cycle
DOA	Direction of Arrival
DPD	Direct Position Determination
DSBF	Delay-and-Sum Beamforming
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Techniques
FBP	Filtered Back-Projection
FFT	Fast Fourier Transform
FOC	Fourth Order Cumulants
IMU	Inertial Measurement Units
LFM	Linear Frequency Modulation
LOS	Line-of-Sight
MF	Matched Filtering
MIMO	Multiple-Input-Multiple-Output
ML	Maximum Likelihood
MUSIC	Multiple Signal Classification
NLA	Non-uniform Linear Array
OMP	Orthogonal Matching Pursuit
PRI	Pulse Repetition Interval
RCS	Radar Cross Section
RF	Radio Frequency
RIP	Restricted Isometry Property
RMSE	Root Mean Square Error
RSA	Random Sparse Array

RX	Receiver
SAR	Synthetic Aperture Radar
SCA	Scalar Circular Array
SIMO	Single-Input-Single-Output
SNR	Signal-to-Noise Ratio
SVD	Singular Value Decomposition
TDOA	Time Difference of Arrival
TOA	Time of Arrival
TTW	Through-The-Wall
TX	Transmitter
UCA	Uniform Circular Array
ULA	Uniform Linear Array
URA	Uniform Rectangular Array
UWB	Ultra-Wideband
VAA	Vector Arc Array
VCA	Vector Circular Array
VNA	Vector Network Analyzer

LIST OF SYMBOLS

$\mathbf{A}^{arc}$	contains unit projections of all the source signals to all the antenna elements in a VAA
$\mathbf{a}_E$	unit vector in $\mathbf{E}$ direction
$\mathbf{a}_r$	unit direction vector of a source
$\mathbf{a}_\theta^q, \mathbf{a}_\phi^q$	unit vectors of E_θ^q and E_ϕ^q of the q th source
$\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$	unit vectors of the x , y and z axes in the Cartesian coordinate system
$\mathbf{b}(\Phi)$	virtual steering vector of the extended array aperture
$\mathbf{B}_S, \mathbf{B}_N$	signal and noise subspaces
$\mathbf{B}_N^{arc}$	noise subspace decomposed from $\mathbf{R}_z^{arc}$
c	light propagation velocity in the air
$\mathbf{C}_{4,s}$	FOC matrix of a source signal s
d	inter-element spacing of a 2-element ULA
$\mathbf{D}$	the indicator matrix
d_{arc}	inter-element spacing of a VAA
d_{in}	propagation path length inside a wall
d_w	wall thickness
$\delta\theta, \delta\phi$	searching steps of θ and ϕ
$\Delta\phi_{uv}$	azimuth angle difference between the u and v axes
$\Delta\psi_a$	phase difference at two adjacent ULA elements induced by a source from Φ_a
$\Delta x_m, \Delta y_m$	distance displacements from the $(m-1)$ th position to the m th position along x y axes
$\mathbf{E}$	incident electrical field of a source
$\mathbf{e}_m$	steering vector of the m th 2-element ULA in a VCA
E_m^q	projected amplitude of the q th source on the array elements in the m th 2-element ULA
E_θ, E_ϕ	components of $\mathbf{E}$ in the theta direction and in the phi direction
ϵ_r	relative permittivity

f_0	carrier frequency of a signal
f_L	low frequency limit governed by a 2-element ULA
f_U	upper frequency limit governed by a VAA
γ	linear polarization angle of a source
k	wavenumber
κ	chirp rate
λ	signal wavelength
$\mathbf{M}$	2D output DOA estimation result
μ_r	relative permeability
$\mathbf{N}_m$	noise matrix of the m th 2-element ULA
ω_q	the q th source angular frequency in radians
$\mathbf{P}_{arc}$	2D MUSIC spectrum of a VAA/the selection matrix
Φ	measurement matrix
Φ_a	ambiguity DOA
$\phi_{i1}, \phi_{(i+1)1}$	azimuth angles of the first elements in the i th and $(i+1)$ th ULAs
Φ_m	indirect DOA of a source with respect to the array axis of the m th 2-element ULA
ϕ_n	azimuth angle of the n th antenna element
R_1, R_2	propagation paths from TX to a wall and from a wall to a RX
R_1', R_2'	propagation paths from a wall to a target and from a target to a wall
$\mathbf{r}_0, \mathbf{r}_m$	the first and the m th azimuth position vector in SAR
$\mathbf{r}_n$	position vector of the n th antenna element
R_{yy}	covariance matrix of $\tilde{\mathbf{y}}_m^q$
$\mathbf{R}_z^{arc}$	covariance matrix of a VAA data
$\mathbf{S}^{arc}$	the steering vector matrix of a VAA with all steering vectors as diagonal elements
$\mathbf{s}, \mathbf{s}_{\Delta}^i$	static target scene and moving target scene at time instant t_i
σ	target reflectivity
$\sigma_g, \Delta\sigma_g$	reflectivity and difference reflectivity of the g th pixel
Σ	a diagonal matrix of size $N \times N$ containing eigenvalues of the signal and noise
Ψ	sparsifying matrix

$\psi_q(t)$	the time-varying phase of the q th source signal
T	pulse width of chirp signal in time
τ_w	time delay inside a wall
$\theta_{3dB}^{(H)}$	H -plane antenna beam width
θ, ϕ	elevation and azimuth angle of a source
$\mathbf{U}$	a unitary matrix containing eigenvectors of the signal and noise
u, v	axes of two 2-element ULAs
$\mathbf{V}$	another unitary matrix
v_w	propagation velocity inside a wall
v_{xi}, v_{yi}	moving target velocity component along x and y direction
$\mathbf{w}$	noise vector
$\mathbf{W}^{arc}$	noise matrix of a VAA
$\mathbf{Y}_m$	array output matrix of the m th 2-element ULA
$\mathbf{y}_m^q$	array output signal vector of the m th ULA induced by the q th source
$\tilde{\mathbf{y}}_m^q$	truncated signal of $\mathbf{y}_m^q$
$\mathbf{Z}^{arc}$	received signal matrix of a VAA

CHAPTER 1

INTRODUCTION

1.1 Background

Antenna arrays are indispensable tools in many applications ranging from wireless communications, radar, sonar and satellite [1-6]. They have advantages over a single antenna in that they possess higher angular resolution and broader field of view. Along with various antenna array signal processing methods, antenna arrays can provide high and robust performances in various applications in military and commercial areas. Different applications have different requirements in antenna types and signal processing methods. These requirements may keep changing as the time changes and the technology advances. Therefore, antenna array signal processing has always been an active research topic in areas such as radar signal processing, microwave imaging and source location. There are two most fundamental and important application types of array signal processing, target detection and source estimation. Target detection plays a vital role in areas such as fire rescuing, anti-terrorist and surveillance. Source estimation is very important in areas such as vehicle tracking, patient safety, medical instrumentation monitoring, and direction finding.

Through-the-wall (TTW) detection is one of the important research aspects of target detection. TTW applications are highly demanded in areas ranging from daily life activities to military usage [7]. Active target detection plays an extremely important role in TTW environment for a reason that seeing through buried rubble or concrete slabs in an earthquake with rapid detection and rescuing action save lives. Seeing through building walls with surveillance of moving terrorists enables rescuers to save victims quickly and accurately. Advanced knowledge of building structure, fire situations and the number of people trapped in the building in fire enables firefighters to locate tracked human and animals quickly and thus reduce injuries and deaths.

Traditional detection methods, including triangulation method [8] based on time of arrival (TOA), time difference of arrival (TDOA), direction of arrival (DOA), and joint methods between them, perform well in line-of-sight (LOS) environment. Weiss [9, 10] proposes a new method called direct position determination (DPD) which provides even higher accuracy. However, all of these methods may suffer great performance degradation when barriers block the LOS between sensors and targets and bring in path changes, multipath effects and other interferences. Since radio frequency (RF) electromagnetic wave radar with wavelength of a few centimeters has demonstrated its ability to penetrate through brick walls and detect targets behind them [11, 12], radar imaging methods are mostly favored in TTW applications.

LOS detection is another research aspect of target detection to be emphasized. Although LOS environment is less complicated than TTW environment, the requirement of detection algorithm performances becomes critical when both detection area and number of targets to be detected become large. Such condition may be mostly encountered in large area surveillance or maritime detection. Similarly as the TTW applications, radar imaging techniques are more preferred than traditional parameter-based methods such as TOA and DOA in these cases in LOS environment nowadays. The possible radar imaging techniques include synthetic aperture radar (SAR) imaging [13-15], beamforming [16-18], tomography [19, 20] and inverse scattering [21-23]. Antenna arrays have been proved to assist in the success of these imaging techniques. For example, a co-array configuration based wideband beamforming algorithm is able to produce high-resolution images [17].

Source DOA estimation using antenna arrays firstly appeared after the usage of antenna arrays in World-War II for radar applications. Various antenna array based DOA estimation algorithms have been proposed and investigated over the past several decades, including Capon [24, 25], multiple signal classification (MUSIC) [26, 27], estimation of signal parameters via rotational invariance techniques (ESPRIT) [28, 29], maximum likelihood (ML) [27, 30, 31], and the alternative methods [32]. Besides DOA information, polarization information is another significant aspect in source estimation. Although studied in some research works in

early years [33, 34], polarization estimation did not attract enough attention in source estimation. The major reason is that most of the source estimation applications are in ground surface wireless communications in which polarization information of signals is not required. However, there are increasing demands of passive source estimation, including both DOA and polarization estimations, for radars in marine and aerospace applications for homeland security nowadays.

In recent years, many research works have focused on joint estimation of DOA and polarization of signals, among which, diversely polarized antenna arrays with electromagnetic vector sensors are the major configuration for DOA and polarization estimation [35-38]. However, such array structures are difficult and troublesome to implement in practice and estimation algorithm is, for most of the time, complicated. To the best of the author's knowledge, so far no other simpler and intuitive methods have been reported for DOA and polarization estimations of source signals. Therefore, it is a challenging work to explore simple approaches in both antenna array deployment and parameter estimations.

There is always a typical problem coming along with the DOA estimation using conventional antenna arrays, including a uniform linear array (ULA), a uniform circular array (UCA) and a uniform rectangular array (URA). The problem is termed as the DOA ambiguity problem [39]. It is encountered as a result of symmetrical geometry of these arrays. Due to possible severe impacts of the ambiguity problem on DOA estimation result, this problem has attracted a lot of attention from many researchers. In general, there are two types of DOA ambiguity problem: one type is caused by the symmetrical property of the array manifold and is most severe in ULA; the other type is caused by large inter-element spacing of the uniform array (normally larger than half of source signal wavelength). The large inter-element spacing gives rise to grating lobes in the estimation result and is equivalently severe in all antenna array types. In order to obtain a good DOA estimation result, the ambiguity problem needs to be properly solved to mitigate large estimation errors caused by it.

Since the notice of the DOA ambiguity problem, a lot of research works have been done to study the ambiguities in the problem in the way of calculating

ambiguity sets of DOA angles [40-43]. Some works are carried out to resolve the ambiguity problems either by re-designing the array geometry [44, 45] or developing methods for resolving ambiguity issues in the context of signal processing [46, 47]. The former method is limited in practice since antenna array geometries cannot be changed once deployed. Therefore, the latter method based on signal processing is more attractive in real situations.

Over the past several decades, research activities with significant effects have taken place in the investigation of antenna arrays [35-38]. Their applications in military usage, commercial areas and surveillance applications function as a significant driver for the development of antenna array technologies. Although an antenna array is already a relatively mature theory, there is still much to be done in the areas of target detection and source estimation using antenna arrays.

1.2 Major Contributions

In this thesis, antenna array signal processing methods are studied with an emphasis on both target detection and source estimation.

First of all, a multiple-view tomographic imaging method is introduced to detect static targets in a TTW environment. It is a theoretical study of the imaging method based on antenna arrays. The multiple-view tomographic imaging method explores the full space diversity by employing an array of antenna elements around a room area and explores the frequency diversity by employing wideband signals. The performance of a single-view scenario is given along with the multiple-view scenario in numerical simulations. Both cases when walls are absent and present are also investigated for the two scenarios.

In order to investigate static and moving target detections using antenna arrays in LOS environment, a random sparse array (RSA) based imaging method is proposed. The method utilizes the compressed sensing (CS) technology, which greatly reduces the amount of measurement data required. To make the algorithm more specific for different target detections, the RSA is characterized into two modes, namely the SAR mode and the array mode. The RSA of SAR mode is for

static target detection and the RSA of array mode is for moving target detection. Performances of the proposed method in two modes are validated by comparing with conventional imaging algorithms such as back-projection (BP) and beamforming (BF). Numerical simulations are carried out to prove the effectiveness of the proposed method based on the RSA. Besides, performance of multiple static targets detection is validated through experimental results.

A new antenna array structure, namely the vector circular array (VCA), is proposed for estimation of sources with relatively high grazing angles and arbitrary polarizations. The reason is that a conventional scalar circular array (SCA) can only estimate low-grazing angle DOAs without the ability of estimating polarization angles. A simple and intuitive approach using the VCA is also proposed for both DOA and polarization estimations of sources from relatively high grazing angles. The VCA is divided into several 2-element ULAs. The estimation approach using subspace-based DOA estimation method (such as the MUSIC method) is based on these 2-element ULAs. Extensive simulations and error analyses demonstrate that the proposed approach can estimate those sources with very small estimation errors (mostly within one degree).

In practical situations, noise characteristics of antenna elements do not necessarily follow the white Gaussian noise. Additive white Gaussian noise is always assumed in the theoretical case, but colored Gaussian noise are mostly observed in practice. By adopting MUSIC-like method [48], the DOA estimation approach based on the VCA is extended to the DOA estimation in colored Gaussian noise environment. Superior performances of the approach in colored Gaussian noise are validated through numerical simulations.

DOA ambiguity problem due to large inter-element spacing can lead to severe consequences in critical applications, such as airborne applications, if not properly resolved. A new approach based on the previously proposed VCA is finally proposed for resolving DOA ambiguity problems in the 2-element ULAs of the VCA. The approach is called the composite-multiple-resolution (CMR) approach. In order to resolve the ambiguity problems without introducing new elements or structures into the array geometry, another subset of the VCA, namely the vector arc array (VAA),

is introduced to combine with the 2-element ULAs in the proposed approach. Extensive numerical simulations and case studies show the superior performance of the proposed approach. More interestingly, the approach is able to provide accurate DOA estimation of sources from a relatively wide frequency band by selecting multiple 2-element ULA pairs.

1.3 Organization of the Thesis

The remainder of the thesis is organized as follows.

In Chapter 2, we firstly review some basic theories and techniques for target detection in TTW environment. The most favorable techniques for TTW target detection are based on imaging. After that, we review some techniques for source estimation which is referred to as DOA estimation and polarization estimation. The review leads to the proposed research of the thesis.

In Chapter 3, we present a schematic study of static target detection in TTW environment. The method investigated is the tomographic imaging method. A multiple-view tomographic imaging is studied and investigated through numerical simulations.

Chapter 4 introduces two modes of the RSA concept for static and moving target detections, namely a SAR mode and an array mode. Based on these two modes, the performances of target detections using CS are investigated with numerical simulations. After that, an experimental example of static target detection using the SAR mode RSA is presented.

Chapter 5 introduces a new array structure named a VCA. The VCA is aiming for estimating relatively high-grazing angle DOAs and arbitrary polarization angles. An approach to DOA and polarization estimations for multiple sources is then proposed based on the subsets of the VCA, namely 2-element ULAs. The performance of the approach using the VCA is finally verified through extensive numerical simulations and error analyses. Moreover, the performance of the VCA in colored Gaussian noises is also studied.

Chapter 6 presents a new technique, the CMR approach, for resolving ambiguity problems in DOA estimation. It aims to solve the ambiguity problem caused by a large inter-element spacing of the 2-element ULAs in the proposed VCA. The design procedure of the approach is presented in detail and various numerical simulations are carried out to verify the performance of the approach. Wideband source DOA estimation using the approach is also verified through different case studies.

Finally, conclusions of the thesis and recommendations for future works are presented in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Target detection and source estimation have many applications in both civil and military sectors and have attracted a lot attention from researchers. Meanwhile, an antenna array has become increasingly important in areas such as radar, sonar and other civil or military applications as it can provide more information about targets and better performance than a single antenna. Thus we put emphasis on target detection and source estimation using an antenna array in this thesis.

2.2 Review of Target Detection

Nowadays, security and anti-terrorist applications become very important in both military and civilian areas. In these applications, target detection is especially important to achieve the desired purposes, such as TTW target detection in fire rescue and hostage rescue and LOS target detection in instrument tracking or surveillance and airborne precautions.

Due to the high complexity of the TTW environment, such as multiple path effects, multiple scattering, great power loss of the wall materials, conventional methods such as TOA and DOA are no longer suitable for detecting targets behind walls. Radar imaging methods have been gaining great attention in TTW target detection since they require no complicated calculation of TOA or DOA parameters. Reported works on imaging methods in TTW target detection are mainly SAR based imaging methods with appropriate modifications to account for walls effects. Cui *et al.* [13] introduce a BP algorithm to a TTW SAR imaging system and eliminate the wall effects by a time minimization method. Ahmad *et al.* [16] present a quadratically constraint capon beamformer to a TTW SAR imaging system with various wall effects incorporated. The results demonstrate that the proposed beamformer outperforms conventional delay-and-sum beamformer used in SAR imaging. In [14], a refocusing approach based on the idea of matched filtering is

introduced to the standard SAR imaging. The approach is to eliminate the strong wall effects due to ultra-wideband (UWB) frequency and wide incident angles required in standard SAR imaging system. An alternative TTW SAR imaging method is proposed in [18]. It synthesizes both a transmitting aperture and a receiving aperture and incorporates wall effects in a postdata acquisition beamformer. However, synthesizing two arrays are troublesome in practice. There are also reported works on imaging methods in TTW target detection based on antenna arrays. Wang and Amin [17] propose a TTW imaging approach based on a transmitter array and a receiver array with different standoff distances from the wall. The wall parameters are not exactly known. However, such approach requires the calculation of two or more target trajectories and target positions are the intersection points of these trajectories. All these SAR based TTW imaging methods only take into account the frequency diversity but do not explore full space diversity of the system.

In addition to the SAR based imaging, inverse scattering problem governed by wave equations is the other major category of TTW imaging. In [21], Zhang *et al.* propose a novel real time TTW imaging algorithm based on the spectrum Green's function. Complex scattering process due to walls is modeled by using the layered medium Green's function. They also study and adopt a time-reversal-MUSIC method for multiple targets localization behind walls in [23]. The Green's function is also involved in the method. A two-dimensional (2D) nonlinear inverse scattering technique is developed in [19] for imaging targets in a multilayered medium using four linear arrays. A tomography based method is described in [20] to reduce the TTW imaging to a subsurface reconstruction method. However, these methods all rely on incorporating the Green's function which is very complicated in calculation.

An important problem in preventing a clear target image in TTW environment is the incorrect compensation of wall effects [14, 49-53]. Since many TTW applications require only acceptable accuracy and resolution as long as target positions could be identified, robust and computationally-efficient algorithm is needed to produce images with acceptable resolution even when wall effects are not completely compensated. Conventional tomographic imaging employing beamforming method is simpler than that using the Green's function. Besides,

tomographic imaging can provide large space diversity and antenna arrays can help to improve the level of space diversity (such as MIMO array configuration). Since there have not been many research works on tomographic based TTW imaging methods using antenna arrays, we decide to study the performance of tomographic imaging based TTW target detection with antenna arrays.

LOS target detection is also attracting much attention from researchers especially in military areas. Imaging methods discussed for TTW imaging can be applied to LOS target detection. However, LOS target detection in military applications might require more accuracy and higher resolution than TTW target detection. Besides, easy deployment of detection system is another key requirement in military applications. In recent years, the concept of a RSA has been gaining great interest by researchers in applications other than array signal processing, especially military applications, due to its low level of power densities, diversity properties and improvement in survivability [54, 55]. It significantly increases resolution and decreases system cost, which is highly required in complex systems. With the emergence of an advanced technology, the CS technology, the RSA is being studied by several researchers in terms of its relations with CS [56, 57] in more recent years. However, the two new concepts have not been reported in target detections. Therefore, we decide to investigate the combination performance of the RSA and the CS in target detections in LOS environment.

The related methods in the studies of TTW and LOS target detections in this thesis as well as the wall effects in TTW environment are introduced in following subsections.

2.2.1 Wall Effects

Homogeneous walls are considered for simplicity. Phase shift and power attenuation are two major effects considered in the TTW target detection. Both of the two effects are related to signal propagation from the air to the wall and from the wall to the air. Since material changes at the boundary of the wall, a signal will change its direction at the air-wall interface due to refraction and change its direction again at the wall-air interface. Obviously, the refraction of the signal increases the total propagation path,

so do the time delay and power attenuation. Moreover, the signal propagation speed in the wall is lower than that for the signal in the air, so the total time delay will be further increased. A spatial model illustrating a signal penetrating through a wall is shown in Fig. 2.1.

Fig. 2.1 Spatial model of a signal propagating through a wall (all reflected signals from the wall are not shown for simplicity).

As illustrated, the actual propagation path of a signal through a wall is longer than the path when no wall is present. It has the expression as in (2-1). R_1 and R_2 represent paths on the side of sensors and R'_1 and R'_2 represent paths on the side of targets.

$$R_{tot} = R_1 + d_{in1} + R'_1 + R'_2 + d_{in2} + R_2 \quad (2-1)$$

where d_{in1} and d_{in2} are paths inside the wall. They are related to the wall thickness d_w and refraction angles as

$$\begin{aligned} d_{in1} &= d_w / \cos \theta_2 \\ d_{in2} &= d_w / \cos \theta_4 \end{aligned} \quad (2-2)$$

Wall effects can have non-ignorable influence, such as shifting in target positions, on the position information of a target behind the wall if the effects are accidentally ignored. However, accurate calculation of signal path is almost impossible for TTW applications, since wall effects depend highly on the wall materials as well as external environment factors, such as temperature and moisture contents of the wall. Therefore, it is normally assumed that wall properties are constant for simplicity so that wall effects can be approximated.

Different approaches for estimating signal paths or signal flight time have been tried in following research works. The conventional approximation method assumes constant velocity model [17], which results in larger position errors due to ignoring extra delay caused by a wall. Other methods with improved accuracy based on ray theory are presented in [50]. There are also methods to compute the true TOA inside a wall, such as the three-layer model in [18] and the two-layer model in [58, 59]. However, computational complexity must be accounted for or considered to achieve accurate wall parameter estimations because iterations and solutions of minimization problems are normally involved in the estimation process. A simple, fast and relatively precise approximation of the signal delay through a wall is presented in [60] by assuming an almost normal incidence of the signal to the air-wall interface.

2.2.2 Synthetic Aperture Radar (SAR) Imaging

SAR was originally invented for airborne applications [61]. SAR achieves a fine resolution image of a large target or area with a very large aperture synthesized by a single moving antenna. It has been proved to be a very useful technique for TTW imaging to acquire high resolution images by employing wideband signals [13-15, 62].

Resolution of the SAR images depend on two types of resolution: resolution in range direction and resolution in cross-range (azimuth) direction [63]. A wideband signal, such as a chirp signal or a stepped-frequency signal, has a compressed pulse duration τ (approximately equals to the signal bandwidth B) at the receiving antenna

after matched filtering (MF). Denoting the signal speed as c , the range resolution is then determined by

$$\Delta r = c/2B \quad (2-3)$$

The cross-range resolution is dependent on the beam width, θ_B , of the antenna. Assuming the wavelength of the transmitting signal is λ , the cross-range resolution becomes

$$\Delta x \approx r\theta_B = r\lambda/L \quad (2-4)$$

where L is the aperture length and r is the target range.

SAR imaging in ground applications has a different scenario from that in airborne applications. The aperture is synthesized by placing a transceiver (co-located transmitter and receiver) at different observation points along the scan line. The transceiver is composed of a transmitting antenna and a receiving antenna. In TTW imaging, the scan line is normally along a wall and it is divided into a number of observation positions with Δl distance in between. Fig. 2.2 shows a typical scenario of a SAR imaging system for target detection in TTW environment.

Fig. 2.2 Scenario of target detection behind walls using SAR [14] (reflected signals from the wall are not shown).

A transceiver is located y_a distance away from the wall. A scan line is divided into several number of observation points with a distance Δl and a total length L . The arrow represents the actual path of the signal transmitted by the transceiver, reflected back from the target and then received. Information obtained at all the observation points will be processed together to obtain an image using reconstruction methods, such as the well-known back-projection (BP) method [64, 65] which was firstly proposed for tomography in [66]; and its derivative, the filtered back-projection (FBP) method [67]. Beamforming method is recently presented as another efficient SAR imaging method by Ahmad *et al.* in [16, 18, 68], known as the synthetic aperture beamformer.

Assume the target is located at (x_t, y_t) , the transmitter (TX) position is (x_t, y_t) and the receiver (RX) position is (x_r, y_r) . The total path of the signal travelled from the TX to the target to the RX in Fig. 2.2 can be obtained similarly as in Section 2.2.1 and the signal TOA can then be calculated by

$$\tau_T = \frac{R_1 + R_1' + R_2' + R_2'}{c} + \frac{d_{in1} + d_{in2}}{c/\sqrt{\varepsilon_r}} \quad (2-5)$$

where $\varepsilon_r = \varepsilon_w/\varepsilon_0$ is the relative electrical constant of a wall and $c/\sqrt{\varepsilon_r}$ is the signal velocity inside the wall. For SAR imaging without walls, the target can be considered as locating on the paths of a constant TOA, which form several ellipses with the TX and the RX as two foci [69]. The target positions in the SAR imaging with walls can be obtained similarly, and the constant TOA becomes τ_T .

Divide the target scene into pixels of size $P \times Q$. Assume there are M observation points and K targets. The received signal at each observation point is a combination of the reflected energy from all the pixels. Consider a stepped-frequency signal with N steps. The received signal at the m th ($m = 1, 2, \dots, M$) observation point before image formation can be expressed in (2-6). For simplicity, we ignore power attenuation and denote the k th target reflectivity as σ_k .

$$y_m(n, t) = \sum_{k=1}^K \sigma_k \exp(-j2\pi f_n \tau_T(k, m)) \quad (2-6)$$

where f_n is the n th ($n = 1, 2, \dots, N$) frequency step and $\tau_T(k, m)$ is the TOA from the m th TX via the k th target to the m th RX. Consider using the BP method, the image is reconstructed by back-projecting the received signals along the original path [69] represented in (2-5). Projection can be denoted using the expression in (2-7).

$$p_m(\tau) = \sum_{n=1}^N y_m(n, t) \exp(j2\pi f_n \tau(m)) \quad (2-7)$$

Then the value of the (i, j) th ($i = 1, 2, \dots, P, j = 1, 2, \dots, Q$) pixel can be obtained by

$$I(i, j) = \frac{1}{M} \sum_{m=1}^M p_m(\tau_{ij}(m)) \quad (2-8)$$

where $\tau_{ij}(m)$ has the same expression as in (2-5) but is the time delay from the m th TX via the (i, j) th pixel to the m th RX.

2.2.3 Tomographic Imaging

The techniques of tomographic imaging were originally developed in the context of medical imaging, and have been used with a number of different kinds of radiation, including electromagnetic and acoustic waves [70].

The imaging system collects information from a full 360° view angle (which is different from SAR) and produces a cross-sectional image of the target. There are many types of tomographic imaging in regard to the geometries of the transmitter/receiver pairs, monostatic, bistatic and multistatic. Monostatic transmitter/receiver pair refers to a single antenna performing as both a TX and a RX. After obtaining information from the target at one position, the antenna is moved on to the next position to collect data of the target. After the data at all positions are collected, they are processed to form an image. Bistatic pair contains one RX and one transmitting antenna. The TX position is often fixed while the RX moves to different positions at different time instants. These geometries enable full space

diversity. Range resolution and azimuth resolution are quite similar to those of SAR imaging. Range resolution depends on the signal bandwidth and azimuth resolution depends on the area diameter as well as the wavelength of transmitting signal. Fig. 2.3 shows the system geometry of a monostatic radar tomography.

Fig. 2.3 System geometry of a monostatic radar tomography.

The BP method discussed in SAR imaging can also be used in tomographic imaging. Besides, beamforming algorithm is becoming an important reconstruction method in tomographic imaging [71]. It is not based on the projection theory and is suitable for both far-field and near-field conditions.

The basic idea of beamforming is to apply a phase weighting to the energies reflected by the target and coherently add them together. Similarly, assume there are M radar positions and K targets. Denote the reflectivity of the k th isotropic point target as σ_k . Neglecting the amplitude attenuation for a monostatic case, then the echo of the k th target $x_k(\theta)$ is expressed as

$$y_k(\theta) = \sigma_k \exp\left(-j \frac{4\pi R_k(\theta)}{\lambda}\right) \quad (2-9)$$

where θ is the rotation angle of the radar with respect to the x -axis and it is related to the radar position. $R_k(\theta)$ is the distance from the k th target to the radar position with rotation angle θ .

Dividing the target scene into $P \times Q$ pixels, then the reference function/phase weighting for the (i, j) th pixel has the form of

$$s_{ref_ij}(\theta) = \exp\left(j \frac{4\pi R_{ij}(\theta)}{\lambda}\right) \quad (2-10)$$

where $R_{ij}(\theta)$ is the distance from the (i, j) th pixel to the radar. Then the pixel can be reconstructed using the expression

$$I(i, j) = \int_0^{2\pi} \sum_{k=1}^K \sigma_k \exp\left\{-j \frac{4\pi}{\lambda} [R_k(\theta) - R_{ij}(\theta)]\right\} d\theta \quad (2-11)$$

The reconstruction of multistatic tomography using beamforming is similar to (2-11). Expressed in a discrete format, the multistatic tomography image reconstruction has the form of (2-12). Assume that the number of receiving radar position is M .

$$I_{multistatic}(i, j) = \sum_{m=1}^M \sum_{k=1}^K \sigma_k \exp\left\{-j \frac{4\pi}{\lambda} [R_k(m) - R_{ij}(m)]\right\} \quad (2-12)$$

where $R_k(m)$ and $R_{ij}(m)$ represent distances from the TX via the k th target and via the (i, j) th pixel, respectively to the m th receiving radar position. When the two distances coincide, energy will be accumulated at the target positions, otherwise energy will be dispersed.

Tomographic imaging has been studied for remote sensing, target detection and surveillance by some researchers. Michael *et al.* [72] investigate the geometric diversity of multistatic tomographic concept in locating moving targets and propose a matched filtering processing method for focusing the target. Sun *et al.* [73] investigate the tomographic imaging using single-tone continuous-wave (CW) signals via experimental results and good agreement is achieved between measured and simulated results. In [74], Song *et al.* make use of the tomographic imaging

concept to develop a two-dimensional nonlinear inverse scattering technique for imaging a multilayered medium simulating effects of building walls. Multi-frequency data with limited array view are able to achieve high-quality image reconstruction. However, there are not many other research works on TTW target detections base on tomographic imaging.

2.2.4 Compressed Sensing (CS)

Compressed sensing (CS) [75-77] is an emerging technology in recent few years and has gained great interest in many research areas. It challenges the Nyquist sampling theorem in signal processing. CS theory states that it is possible to perfectly recover sparse/compressible signals with highly incomplete information under proper transformation basis. This is very useful and attractive in areas requiring large amount of data processing and storage, such as image to image processing, medical imaging and radar imaging including SAR imaging and TTW radar imaging. It has been proved to be the most efficient technique especially in applications of radar imaging. In [78], Baraniuk and Steeghs introduce CS to radar imaging and successfully demonstrate its advantages in eliminating matched filtering and reducing receiver analog-to-digital conversion (ADC) bandwidth. In [79-81], CS is applied to TTW radar imaging to reduce the number of data samples to a very small amount while obtaining high-quality TTW radar images. In [82], the authors introduce CS concept in SAR and present an imaging modality with high-resolution map of the spatial distribution of targets and terrain using reduced number of transmitted/received signals. In [83], CS is applied to both simulation data and experimental SAR raw data to demonstrate the effectiveness on two separate one-dimensional (1-D) processing operations in range and azimuth dimensions. Recently in [84], the authors propose a random-frequency SAR imaging scheme based on CS. Limitations of stepped-frequency SAR system can be overcome by transmitting only a small number of frequencies. In the meantime, the available imaging range width is enlarged significantly when range and azimuth resolutions are maintained. However, to the best of the author's knowledge, there are not many works on CS based random array processing.

Suppose a signal $\mathbf{s}$ ($\mathbf{s} \in \mathbf{C}^N$) of length $N \times 1$ is called Q -sparse if the number of non-null elements in $\mathbf{s}$ is not larger than Q ($\|\mathbf{s}\|_0 \leq Q$). It can be the vector representing a sparse scene with Q targets. Suppose another signal $\mathbf{x}$ is sparse on a basis matrix Ψ which represents a linear transformation. Signal $\mathbf{x}$ of length $N \times 1$ is also Q -sparse if it can be written in the form of

$$\mathbf{x} = \Psi \mathbf{s} \quad (2-13)$$

where Ψ is called the sparsifying matrix of size $N \times N$. Signal $\mathbf{x}$ is usually measured by a measurement vector $\mathbf{y}$ in a system. The measurement vector $\mathbf{y}$ of length $M \times 1$ ($M < N$) is a linear projection of $\mathbf{x}$ on a random matrix Φ ,

$$\mathbf{y} = \Phi \mathbf{x} \quad (2-14)$$

where Φ is a $M \times N$ measurement matrix (where $M \ll N$). From above equations, we can find the relationships between $\mathbf{y}$ and $\mathbf{s}$. Obviously, $\mathbf{y}$ can be represented in terms of $\mathbf{s}$ via the following expression,

$$\begin{aligned} \mathbf{y} &= \mathbf{A} \mathbf{s} \\ \mathbf{A} &= \Phi \Psi \end{aligned} \quad (2-15)$$

To recover $\mathbf{s}$ from $\mathbf{y}$, one needs to solve a minimization problem expressed in (2-16).

$$\min_{\mathbf{s}} \|\mathbf{s}\|_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A} \mathbf{s} \quad (2-16)$$

And when noise is present, the problem becomes (2-17) and ξ is the uncertainty level.

$$\min_{\mathbf{s}} \|\mathbf{s}\|_1 \quad \text{s.t.} \quad \|\mathbf{y} - \mathbf{A} \mathbf{s}\|_2 \leq \xi \quad (2-17)$$

It has been proved in [75-77] that in order to exactly recover the desired sparse vector $\mathbf{s}$ using $\mathbf{y}$, the transformation matrix $\mathbf{A}$ must satisfy the *Restricted Isometry Property (RIP)*.

$$(1 - \delta) \|\mathbf{s}\|_2 \leq \|\mathbf{A} \mathbf{s}\|_2 \leq (1 + \delta) \|\mathbf{s}\|_2 \quad (2-18)$$

where δ is the smallest possible constant ($\delta > 0$). $\|\cdot\|_2$ represents the l_2 norm. The RIP

actually measures how well every set of Q columns of $\mathbf{A}$ forms approximately an orthonormal system. The smaller the positive constant δ is, the more possible to perfectly recover signals. The number of minimum measurements required is determined by the mutual coherence of matrix Φ and Ψ . Mutual coherence indicates the spreading extent of information associated with $\mathbf{s}$ among the entries of $\mathbf{y}$. The lower mutual coherence between Φ and Ψ is, the fewer number of measurements are needed to recover $\mathbf{s}$ from $\mathbf{y}$ [85]. The required number of measurements to recover $\mathbf{s}$ must satisfy the condition stated in [86],

$$M \geq C [\mu(\Phi, \Psi)]^2 NQ \log Q \quad (2-19)$$

where C is a constant and $\mu(\Phi, \Psi)$ is the mutual coherence between Φ and Ψ , of which the mathematical expression is,

$$\mu(\Phi, \Psi) = \max_{i \neq j} |\langle \Phi_i, \Psi_j \rangle| \quad (2-20)$$

where Φ_i and Ψ_j are columns of the respective matrix and this expression is only valid if these columns have unit norms.

Several approaches are used in CS to reconstruct the sparse signal, such as convex optimization [87, 88] and greedy algorithms [89-92]. Among these approaches, Orthogonal Matching Pursuit (OMP) [90] is a widely used version of greedy algorithms and $\mathbf{s}$ can be recovered by iteratively finding global minimum from locally optimal solution in each step.

2.3 Review of Source Estimation

DOA estimation has been an active research area for a long time [5, 6] and has many engineering applications, including wireless communications, radar, radio astronomy, navigation and tracking of various objects. Majority of traditional DOA estimation research activities were focused on signal sources with low grazing angles. There was less need for polarization estimations as for most ground-surface DOA estimations. The signal source grazing angles are very small and signal polarizations are vertical. In recent years, there are increasing demands from some

critical applications for both DOA and polarization estimations of signal sources with relatively high grazing angles and arbitrary polarizations, for examples, signal sources from space and airborne platforms.

Most of recent research activities for both DOA and polarization estimations exploit the electromagnetic property of antenna elements [35-38, 93, 94]. In [35] and [36], identical collocated six-component electromagnetic vector sensors are deployed for both direction and polarization estimations along with ESPRIT based algorithms. In [37], vector sensors with MUSIC based algorithm is extended in spatio-polarizational beamspace. In [38], arbitrarily collocated or oriented dipoles/loops/polarized antennas on an L-shaped array grid are made possible for multiple sources direction and polarization estimations with a root-MUSIC based algorithm. Wong [93] estimates multisource DOA and polarization by using three Cartesian components of electrical-field or magnetic-field. However, there are practical problems when vector sensors are used for DOA and polarization estimation such as the difficulty in implementing collocated elements due to the strong mutual coupling effects. A few works estimate source DOA and polarization by considering the polarization sensitivity of antenna arrays. In [94], a novel blind joint DOA and polarization estimation method is proposed for a polarization-sensitive uniform circular array. However, the method relies on the uniqueness of low-rank three way data decomposition, which is a little complicated. Therefore, simple approaches for both DOA and polarization estimations along with simple and easy-to-implement array structures are needed to be investigated.

The ambiguity problem in the DOA estimation using an antenna array has been arisen along with the research on DOA estimation [39]. Although classified into different ranks, the most severe ambiguity problem is the one associated with ULA due to large inter-element spacing (exceeds half-wavelength of a source signal). In practical applications, there may be signals with frequencies varying within a wide band. An antenna array is only able to accommodate all these signals by setting its inter-element spacing according to the highest frequency of the signals. However, very small inter-element spacing might results in very strong mutual coupling between antenna elements. Large inter-element spacing can help to reduce these

mutual coupling effects [95, 96] and at the meantime forms large array aperture which is very useful in many practical applications. Therefore DOA ambiguity problem associated with large inter-element spacing needs to be properly resolved before deploying antenna arrays.

Although not many, researchers have tried to investigate methods for resolving DOA ambiguity problem in a ULA due to large inter-element spacing. Gavish and Weiss [45] introduce a measuring method of similarity between array response vectors and give an array configuration selection scenario to avoid DOA ambiguities. However, this method is not feasible in practical situations. It is very difficult and troublesome to change the array geometry once antenna array are deployed. The classical method proposed in earlier times was termed the interferometry [46]. In the interferometry method, different baselines are used to resolve ambiguity from shorter baseline to longer baseline. In achieving this, at least three elements are required in the antenna array and they should be located along the same direction. However, ambiguity might not be resolved if long baseline is much larger than short baseline which corresponds to a large distinction in resolutions of the baselines. Besides, such method cannot sustain sources with different frequencies from a wide band. Only sources within a fixed frequency range can be estimated. There are also works resolving ambiguity problems based on signal processing method [47]. DOA estimation using non-uniform linear arrays (NLAs) are considered and two approaches for identifiable uncorrelated Gaussian sources are introduced. However, ambiguity problems in ULAs are not investigated by the work. Since ULAs have wider applications than NLAs due to its well-determined properties, there is a high demand to solve ambiguity problems in ULA.

The related information of DOA and polarization estimations is introduced in following sections.

2.3.1 Direction of Arrival (DOA) Estimation

There are many types of DOA estimation algorithms, including the traditional methods such as digital beamforming (DBF) [97], Capon, MUSIC, ESPRIT and ML. Among all these methods, DBF and MUSIC are the most popular methods and are discussed in detail in the following sections.

2.3.1.1 Digital Beamforming (DBF)

DBF enables an antenna array to capture signals in a particular direction without physically change the array pointing direction. It is also known as delay-and-sum beamforming (DSBF) or conventional beamforming (CBF) [97]. In DBF, each element is assigned with a complex weighting and the DOA is estimated by finding the peaks in the power spectrum of the array output.

We consider the beamforming process of an ULA. Assume a narrowband source signal $x(t)$ impinge on an N -element ULA. The output signal of the n th ($n = 1, 2, \dots, N$) element is denoted as $y_n(t)$. The n th element output is then multiplied by a complex weight w_n (consists of an amplitude weight and a phase shift) in a beamformer. A block diagram of the beamformer is shown in Fig. 2.4.

Fig. 2.4 A block diagram of a beamformer.

Consider that a snapshot of length K is obtained for each array element. Then the n th array element output signal can be denoted as $y_n(k)$, $k = 1, 2, \dots, K$. The k th sample of the beamformer output, $z(k)$ ($k = 1, 2, \dots, K$), is the weighted summation of all the k th samples of the N input signals to the beamformer and it can be expressed as

$$\begin{aligned} z(k) &= w_1^* y_1(k) + w_2^* y_2(k) + \dots + w_N^* y_N(k) \\ &= \mathbf{w}^H \mathbf{y}(k) \end{aligned} \quad (2-21)$$

where $(\cdot)^H$ is the Hermitian transpose operator. $\mathbf{w} = [w_1 \ w_2 \ \dots \ w_N]^T$ the weight vector, and $\mathbf{y}(k) = [y_1(k) \ y_2(k) \ \dots \ y_N(k)]^T$ the k th sample of the total array output. The power spectrum of the beamformer output as a function of direction can be computed as

$$P(\theta) = E\left\{\left|z(k)\right|^2\right\} = E\left\{\left|\mathbf{w}^H \mathbf{y}(k)\right|^2\right\} = \mathbf{w}^H \mathbf{R} \mathbf{w} \quad (2-22)$$

where $E\{\cdot\}$ is the statistical expectation operator, $\mathbf{R} = E\{\mathbf{y}(k)\mathbf{y}^H(k)\}$ is the data covariance matrix of $\mathbf{y}(k)$. By adjusting the weight vector, one can obtain a peak at the desired direction of interest.

For DBF, the weightings of all the elements are equal and phases can be selected to focus the array looking direction to the desired signal direction. The weight vector of a DBF is normally related to the array steering vector $\mathbf{e}(\theta)$ as

$$\mathbf{w} = \frac{1}{N} \mathbf{e}(\theta) \quad (2-23)$$

where $\mathbf{e}(\theta)$ denotes the steering vector of the antenna array for a signal coming from the direction θ . Thus the beamformer output power spectrum becomes

$$P_{DBF}(\theta) = \frac{\mathbf{e}^H(\theta) \mathbf{R} \mathbf{e}(\theta)}{N^2} \quad (2-24)$$

where $\mathbf{R}$ is the data covariance matrix of $\mathbf{y}(k)$ as defined in (2-22) and N is the number of antenna elements in the array. When the assumed direction is close to the real signal direction, there will be a peak in the spectrum. The resolution ability of DBF is determined by the bandwidth of the main lobe in the power spectrum.

2.3.1.2 Multiple Signal Classification (MUSIC)

MUSIC algorithm has higher resolution than DBF. It exploits the eigenstructure of the array output signal. The covariance matrix of the array output signal is formed and decomposed into two subspaces, namely the signal subspace $\mathbf{B}_S$ and the noise subspace $\mathbf{B}_N$. The two subspaces are orthogonal to each other. Due to the fact that the

array steering vector spans the same subspace as the signal subspace, the MUSIC power spectrum P_{MUSIC} has the form of a reciprocal of the product between the array steering vector and the noise subspace.

$$P_{MUSIC} = \frac{1}{\mathbf{e}^H(\theta) \mathbf{B}_N \mathbf{B}_N^H \mathbf{e}(\theta)} \quad (2-25)$$

where $\mathbf{e}(\theta)$ is the steering vector for a signal coming from the direction θ . Since the array steering vector is approximated to be orthogonal to the noise subspace, sharp peaks will appear in the power spectrum, which corresponds to the direction of the source signals.

2.3.2 Polarization Estimation

Polarization of an electromagnetic wave can be classified into three types: linear, circular and elliptical polarizations [98]. Linear and circular polarizations are the special types of elliptical polarization. A polarization is characterized by two parameters, the orientation angle/tilt angle ψ and the ellipticity ε (Fig. 2.5). The tilt angle ψ is the angle between the major semi-axis and the x -axis. The ellipticity is the major-to-minor-axis ratio and the corresponding ellipticity angle χ is equal to $\text{arccot}(\varepsilon)$. Circular polarization has an ellipticity of 1 and linear polarization has an ellipticity of 0 or infinity.

In traditional applications, such as wireless communications, the polarization information of a source signal is not required since all the antenna elements and source polarizations are kept to be vertical in most of the time. However, nowadays polarization information is gaining more and more interests and is highly demanded especially in some critical applications such as airborne applications. There are not as extensive research works of polarization estimation as that of DOA estimations. Most of the recent research works on polarization estimation focus on vector antenna array or vector sensors, for example, three crossed dipoles or monopoles [35, 36, 93].

Fig. 2.5 Polarization decomposition.

2.4 Concluding Remarks

In Chapter 2, we have reviewed related literatures and techniques on TTW target detection, LOS target detection, DOA and polarization estimations as well as resolving DOA ambiguity problems in antenna arrays.

Since existing research works have not investigated tomographic based TTW imaging using antenna arrays by exploiting full space diversity, we propose a schematic study on the tomographic TTW imaging based on four linear antenna arrays. A multiple-view tomographic imaging method is studied to detect static targets behind walls. The RSA concept and the CS technology are two emerging concepts. Research works on target detection have not considered the combination performance of a RSA and CS. Thus we propose a target imaging method based on a RSA and CS. The RSA is characterized into two modes to accommodate both static and moving target detection scenarios.

Although extensive research works are reported on DOA estimation of sources with mostly low grazing angles and vertical polarization, not many works are reported on both DOA and polarization estimations of sources with high grazing angles and arbitrary polarizations. In view of this condition, we propose a VCA for estimating both DOA and polarization information of sources with high grazing

angles and relatively arbitrary polarizations. This may be very useful in some critical applications such as airborne platform applications. Finally, due to the fact that ambiguity problems in DOA estimation require great attention and need to be solved in great request, we propose a composite-multiple-resolution (CMR) approach to solve the DOA ambiguity problems in ULA. This approach employs the properties of the VCA and fortunately is able to obtain unambiguous DOA estimation of sources from a wide frequency band.

CHAPTER 3

TARGET DETECTION BASED ON THROUGH-THE-WALL IMAGING

In this chapter we give a schematic study of the imaging method for TTW target detection based on tomographic imaging via simulation examples. A chirp signal is used in the imaging method. In Section 3.1, TTW tomographic imaging will be introduced and classified into two configurations: single-view and multiple-view. Algorithms for single-view and multiple-view TTW tomographic imaging will also be presented. In Section 3.2, a simple method for compensating wall effects is illustrated. In Section 3.3, chirp signal and its property is introduced. In Section 3.4, numerical examples are given to compare TTW target detection results from single-view and multiple-view tomographic imaging with chirp signal. Section 3.5 concludes the chapter.

3.1 Through-The-Wall (TTW) Tomographic Imaging

In TTW environment, investigation area is usually the inner space of a room. The room where targets are located is called the *target scene*. Since signals will experience greater loss when going through a wall than propagating in the air, a lot of information is needed to obtain an acceptable imaging result. Normally, tomographic imaging obtains a 360° view of the investigation area. In order to obtain intensive information of the TTW environment and give acceptable imaging results, a 360° of view is also required in the TTW tomographic imaging. However, since rotating radar sensor is not practical and time consuming to achieve the 360° view outside a room, a linear antenna array with distributed sensors becomes more implementable for investigating the room area with easier operation. By distributing sensors linearly along the wall of a room with uniform spacing, a TTW tomographic imaging system can be formed.

We classify the TTW tomographic imaging system with linear arrays to two configurations: single-view and multiple-view. Single-view refers to the case when

only one antenna sensor is performing as TX and RX and all other sensors are performing as RXs. This is effective when time is limited, in which case one image of the target scene can still be obtained after one transmission. Multiple-view refers to the case when most or all of the sensors located at different directions are used for both transmitting and receiving. Each transmission is from a different observation direction with respect to the room area. All the receiving sensors ensure a 360° view of the target scene, and the transmitting sensors provide additional views of the target scene. During each transmission and reception, target scene can be observed from a full 360° view and extensive information of target scene is obtained to form an image after all the observations.

3.1.1 Multiple-View Through-The-Wall (TTW) Tomographic Imaging

In this section, the multiple-view TTW tomographic imaging is introduced. It maximizes the space diversity of the system and achieves 360° of view not only at reception but also at transmission.

Obviously, the phase weighting in beamforming reconstruction will be more complicated when walls are present between sensors and targets than the phase weighting discussed in Section 2.2.3. Homogeneous walls are assumed and wall parameters are known a priori either by an on-site statistical measurement or an extraction from a pre-defined database. These parameters include relative permittivity, relative permeability and wall thickness, which are labeled as ϵ_r , μ_r and d_w , respectively. The target scene for this case is the inner area of a room enclosed by the homogeneous walls. Divide the area into $K = P \times Q$ pixels (P pixels along the x -axis, and Q pixels along the y -axis). N sensors are distributed uniformly around the room. Also assume that M targets are to be detected inside the room. For such TTW environment, as shown in Fig. 3.1, the round trip distance is composed of the distance of the signal travelled in the air and the distance of the signal travelled inside the wall. It is described in detail as follows.

Fig. 3.1 Geometry of the multiple-view TTW tomographic imaging system (reflected signals from the wall are not shown).

Denote the propagation path of a transmitting signal inside the wall as d_{in} and the propagation velocity inside the wall as v_w . The time delay introduced by the wall is calculated as

$$\tau_w = \frac{d_{in}}{v_w} = \frac{d_{in}\sqrt{\epsilon_r}}{c} \quad (3-1)$$

Due to the change in the propagation velocity of the signal inside the wall, an extra time delay is introduced each time when the signal propagates through the wall. The time delay has the form of

$$\Delta\tau_w = \frac{d_{in}}{v_w} - \frac{d_{in}}{c} = \frac{d_{in}(\sqrt{\epsilon_r} - 1)}{c} \quad (3-2)$$

Let the black pixel, $\mathbf{p}(x_i, y_j) = x_i\mathbf{a}_x + y_j\mathbf{a}_y$, represent the (i, j) th ($i = 1, 2, \dots, P, j = 1, 2, \dots, Q$) pixel position in the area and the gray pixel, $\mathbf{p}(x_m, y_m) = x_m\mathbf{a}_x + y_m\mathbf{a}_y$, represent the position of the m th target. According to the geometry of the multistatic

tomography, the actual round trip distance from the n th TX ($n = 1, 2, \dots, N$) via the (i, j) th pixel to the n' th RX ($n' = 1, 2, \dots, N$) can be given by

$$R_{ij}(n, n') = R_1 + d_{in1} + R'_1 + R'_2 + d_{in2} + R_2 \quad (3-3)$$

where d_{in1} represents the signal propagation path inside the wall at the forwarding direction and d_{in2} represents the signal propagation path inside the wall at the reflecting direction (as shown in Fig. 3.1). The first three terms on the right hand side represent the path of the transmitting signal travelled before arriving at the (i, j) th pixel (it is called the *transmission path*) and the rest three terms represent the path of the reflected signal from the pixel to the n' th RX (it is called the *reception path*).

For a single-view tomographic imaging system with N sensors, only one sensor is for transmitting and all the N sensors are for receiving. In the beamforming process, two steps are involved. They are matched filtering (MF) and phase weighting. The image at the (i, j) th ($i = 1, 2, \dots, P, j = 1, 2, \dots, Q$) pixel can then be reconstructed after the beamforming process

$$I_o^{\text{single}}(x_i, y_j) = \sum_{n'=1}^N \sum_{l=1}^L P_{MF}^{n'}(l) \exp\left(j2\pi \frac{R_{ij}(n')}{\lambda}\right) \quad (3-4)$$

where $P_{MF}^{n'}(l)$ is the phase found after MF at the n' th RX, l is the sample index and $l = 1, 2, \dots, L$, $R_{ij}(n')$ is the round-trip distance from the TX via the (i, j) th pixel to the n' th RX.

Multiple-view tomography with full transmission (all the sensors are for transmitting) extends the sensor geometry to N^2 and information is N times that of single-view tomography. Each transmission gives different information at the receiving end. The reconstructed image at the (i, j) th pixel can be written as

$$I_o^{\text{multiple}}(x_i, y_j) = \frac{1}{N} \sum_{n=1}^N \sum_{n'=1}^N \sum_{l=1}^L P_{MF}^{n'}(l) \exp\left(j2\pi \frac{R_{ij}(n, n')}{\lambda}\right) \quad (3-5)$$

where $P_{MF}^{n'}(l)$ is the same as that in (3-4) and $R_{ij}(n, n')$ is the round-trip distance from the n th TX via the (i, j) th pixel to the n' th RX. In multiple-view case, real target always contributes relatively high and constant energy to the corresponding pixel in each transmission. Energy at the real target pixel will accumulate after summing all the outputs of all the transmissions. However, ghost targets will appear at different pixels for different transmissions, summing the energies will not accumulate them. When averaging pixel output energies, the energy at the target pixel will be approximately equal to the reflected energy from the real target in one transmission while the energy at ghost target pixels will be attenuated. This approach enhances energies at the real target positions and disperses energies at the ghost target positions.

3.2 Compensation of Wall Effects

Phase weighting is very important to the beamforming result. Accurate phase weighting will enable all signals to focus on the real target position while inaccurate phase weighting will disperse the energy of the real target position and strengthen the energy of false target positions, forming ghost targets.

Wall effects are complicated and difficult to model in practical situations. However, a homogeneous wall with ideal reflection and refraction property suffices to investigate the TTW target imaging method. Therefore, it is usually assumed that walls are made of homogeneous material and only two optical phenomena, reflection and refraction, are taken into account. In TTW applications, estimation accuracy to only some extent is required for necessary actions to be taken by people outside the wall. It means that as long as targets can be detected and their rough positions can be obtained, necessary actions can be taken. Therefore, we adopt the wall compensation method assuming perpendicular signal to the air-wall interface in [60].

The estimated round-trip path is obtained by adding the extra path length caused by a wall to the direct path from a TX via a target to a RX when wall is not present. Consider the transmission path only (reception path can be obtained similarly). As shown in Fig. 3.2, the dashed straight line (R_d) is the direct path from

the TX to the target when wall is not present. The solid line represents the actual signal path from the TX to the target.

Fig. 3.2 Ray path of a signal propagating through a wall with thickness d_w and ray path of the signal propagating in the air (reflected signals from the wall are not shown for simplicity).

The extra delay caused by a wall to which the signal is assumed perpendicular is similar to that in (3-2) by substituting d_{in} by d_w . Then the estimated round-trip path can be calculated by.

$$R_{tot} = R_a + 2d_w(\sqrt{\epsilon_r} - 1) + R_a' \quad (3-6)$$

where R_a' is the direct path from the target to the RX when the wall is not present. Actually, we can also use other estimation methods for the actual distance travelled in the wall instead of using the wall thickness. But for simplicity, we consider that the distance inside the wall is the same as the wall thickness as in [60].

The phase weighting calculated from the estimated round-trip path in (3-6) is expressed in (3-7).

$$\text{phase weighting} = \exp\left(j \frac{2\pi R_{tot}}{\lambda}\right) \quad (3-7)$$

where λ is the signal wavelength. Compared with the real round-trip path (assuming homogeneous wall materials), the estimated round-trip path is underestimated. Therefore the wall effects cannot be completely compensated in the imaging process, resulting in slightly shifted target positions and possibly ghost targets around the real target positions. However, such rough position estimation is acceptable for TTW applications.

3.3 Chirp Signal

For TTW imaging, space diversity is very important for obtaining information from all the directions, and frequency diversity may be an extra advantage to achieve good results. The high penetration ability of wideband signals can prevent extensive signal power loss inside the wall, making wideband signals most preferable in TTW imaging. Therefore, a stepped-frequency signal can be used in TTW imaging. However, it might result in high sidelobes in the imaging results due to the frequency discontinuity of its frequency steps.

Chirp signal, a widely used tool in radar, recently becomes an attractive means for wireless communications and target localization [99-101]. It is a wideband signal with linear frequency modulation (LFM). Instantaneous frequency, f_t , of a chirp signal at time t is proportional to a constant, κ , called the chirp rate. Denoting bandwidth as B , the instantaneous frequency has the form of

$$f_t = f_0 + \kappa t \quad (3-8)$$

where f_0 is the carrier frequency of the signal, $\kappa = B/T$ (T is the pulse width in time). When $\kappa > 0$, the signal is called *up chirp* and when $\kappa < 0$, it is called *down chirp*. A chirp pulse with carrier frequency f_0 has the form of

$$s(t) = a(t) \exp(j2\pi f_0 t) \quad (3-9)$$

where $a(t) = \frac{1}{\sqrt{T}} \text{rect}\left(\frac{t-T/2}{T}\right) \exp(j\pi\kappa t^2)$ is the complex envelope of the chirp signal, $\text{rect}\left(\frac{t-T/2}{T}\right) = u(t) - u\left(\frac{t-T}{T}\right)$ is the rectangular function and $u(t)$ is the unit step function. T is the pulse width and the total signal duration can be denoted using the pulse repetition interval (PRI). Then the duty cycle (DC) defining the fraction of time that a transmitter is at on state can be defined as in (3-10).

$$\text{DC} = \frac{T}{\text{PRI}} \quad (3-10)$$

The following Fig. 3.3 shows a comparison of two chirp pulses with the same PRI and duty cycle but different bandwidths (or different chirp rates).

Fig. 3.3 Comparison of two baseband chirp pulses (up chirp) with $\text{DC} = 0.4$ and $\text{PRI} = 1$ s, (a) the 1st chirp in the time domain, (b) spectrum of the 1st chirp ($B_1 = 20$ Hz), (c) the 2nd chirp in the time domain, and (d) spectrum of the 2nd chirp ($B_2 = 100$ Hz).

When the bandwidth is relatively small, the spectrum of a chirp pulse is a spike-like curve. However, when the bandwidth increases, the spectrum is flattened. In fact, when bandwidth of the chirp pulse is further increased, its spectrum will be

further flattened. Therefore, the time-bandwidth product of a chirp pulse is much greater than one. It indicates the bandwidth efficiency.

The property that governs high robustness to disturbance is the correlation property of a chirp pulse. A matched filter at the receiving end speeds up the higher frequency at the trailing edge of the pulse relative to the lower frequency at the leading edge. Consequently the pulse is compressed to a width of $1/B$. The impulse response $h(t)$ of a matched filter can be written as

$$h(t) = s(T - t) \quad (3-11)$$

where T is the pulse width in time. The response is a time-reversed and time-shifted version of the chirp pulse. The autocorrelation function of a chirp pulse has the form of (3-12).

$$A(t) = \sqrt{BT} \frac{\sin \left[\pi BT \left(1 - \frac{|t-T|}{T} \right) \right]}{\pi BT} \exp[j2\pi f_0(t-T)], \quad t \in [0, 2T] \quad (3-12)$$

According to the expression of the autocorrelation function, we can see that autocorrelation gives a very sharp peak with the shape of a *sinc* function. It can be predicted that with severe noise disturbance, signal information can stand up from the noise floor. As an illustration, Fig. 3.4 shows the autocorrelation of the second chirp and the cross-correlation of the two chirps in Fig. 3.3 without any noise. Input signal to the correlator is delayed by an amount of $T/2$.

Compared to a chirp pulse, a stepped-frequency signal with the same bandwidth and duration can achieve a sharp peak similar to a chirp pulse but with relatively high level of sidelobes. The cross-correlation of two different stepped-frequency signals gives a low floor as chirp pulses. Fig. 3.5 shows the autocorrelation of a stepped-frequency signal with a bandwidth 100 Hz and the cross-correlation of two different stepped-frequency signals. The delay amount is the same as that of the chirp pulse in Fig. 3.4.

Fig. 3.4 Autocorrelation of the 2nd chirp pulse with $B = 100$ Hz, and cross-correlation between the two chirp pulses in Fig. 3.3.

Fig. 3.5 Autocorrelation of a stepped-frequency signal with $B = 100$ Hz, and cross-correlation between two stepped-frequency signals with same parameters as the two chirp pulses in Fig. 3.3.

3.4 Numerical Examples

Both a chirp pulse and a step-frequency signal are used in the TTW tomographic imaging for target detections in this section. The feasibility of a chirp pulse is demonstrated in detecting static targets via several numerical simulations as well as via the comparisons with a step-frequency signal.

Fig. 3.6 A flow chart of the numerical procedure in multiple-view tomographic imaging.

A MATLAB package is developed to simulate different transmission and reception scenarios in the numerical examples. Transmission consists of two coding steps, signal generation and phase shifts due to time delays. For simplicity, attenuation coefficients in all the round-trip propagations are assumed to be 1. Reflection process is modeled by multiplying received signals with the target reflectivity σ . Reception consists of matched filtering (MF) and beamforming reconstruction. In the generation of received signals, the time delay is based on an approximation that signal paths inside the walls are related to the wall thickness d_w by an averaged value (such as $\cos(45^\circ)$). When calculating phase weighting at the beamforming process, the estimated round-trip path is calculated according to (3-6) with the assumption that all signals are incident perpendicular to the air-wall interface. Fig. 3.6 shows a flow chart of the numerical procedure for multiple-view

tomographic imaging in the simulation. Single-view tomographic imaging is a subsection of the multiple-view case.

Without loss of generality, brick walls are assumed to be used in the simulations. The relative permittivity (ϵ_r) and permeability (μ_r) of brick walls are assumed to have the values of 4.0 and 1.0, respectively [102]. The investigation area is assumed to be a 3 m by 3 m room area with 10 cm wall thickness. Other elements, such as windows, doors, and furniture are not considered. It is an empty room area with only intended targets. The interested targets are point-like targets with size smaller than 10% of the sensor signal wavelength. All the multipath effects from the walls are ignored. Sensors only receive reflected signals from targets. Table 3.1 lists all the parameters of the transmitting chirp pulse in the simulations.

Table 3.1: Transmitting Chirp Pulse Parameters

Symbol	Quantity	Value
f_0	Frequency	1 GHz
λ	Wavelength	0.3 m
B	Bandwidth	500 MHz
f_s	Sampling frequency	50B
PRI	Pulse repetition interval	100 ns
DC	Duty cycle	0.4
τ_p	Uncompressed pulse width	40 ns
τ_c	Compressed pulse width	2 ns

Target reflectivity is referred to as the fraction of the intercepted power that is reradiated (scattered) by the target. In the simulation, the reflectivity of all the targets (σ) is set to be 1. The room area is divided into pixels of size 1 cm $\times$ 1 cm (about 3.33% of the signal wavelength). Target is assumed to be located inside one pixel. There are 16 sensors in total being distributed uniformly along the walls (4 sensors along each wall) with 50 cm distance from the wall. They are used for both transmission and reception. Each one of them transmits a signal when all others are turned to the receiving mode. The signal-to-noise ratio (SNR) is set to be 20dB for all the cases.

With the above multistatic geometry, tomographic imaging is able to achieve space diversity via a single observation from one sensor. In LOS condition, phase weighting is able to be determined accurately for each pixel in the room area by calculating the direct distance between sensors and the pixel. However, when walls are present, the phase weighting can only be estimated via the method in Section 3.2. Therefore, it is expected that a single observation is acceptable when walls are absent but may be unacceptable when walls are present. Fig. 3.7 shows the comparison between these two conditions with $[-15, 0]$ dB display threshold. The sensor located at the lower left corner of the room area is now in the transmitting mode.

Fig. 3.7 Target detection result of single-view tomographic imaging using a chirp pulse with $[-15, 0]$ dB display threshold (a) when walls are absent, (b) when walls are present and wall effects are not compensated.

When walls are present and the phase weighting in the beamforming reconstruction does not count the extra time delay induced by the walls, the imaging result shows a wide range of sidelobes around the actual target position compared to the result when walls are not present. Unlike the equal level of sidelobes around the target position in Fig. 3.7(a), it is found that most of the sidelobes in Fig. 3.7(b) are on the right side of the actual target position. This is raised by the inaccurate phase weighting. Compared to the relatively long propagation time from the target to the receiving sensors on the left side of the room area, the propagation time from the

target to the receiving sensors on the right side is short. The ratio of the extra time delay caused by walls over the left propagation time is then much smaller than the ratio of the extra wall delay over the right propagation time. Therefore, the inaccurate phase weighting causes much larger effects to the beamforming result of the right side sensors. The phase changes cannot be fully compensated for these sensors and the residual phase shifts may result in a focus on a ghost target position (a ghost target position appears on the right of the real target position as shown in Fig. 3.7(b)). It should be noted in this case that target shadowing of the chirp pulse when transmitting from the lower left corner to receiving sensors on the right may also occur. Such condition can also be considered as an insufficient observation of the target.

The multiple-view tomographic imaging enables sensors to obtain information of target scene from variant directions. By averaging the imaging results from all the views, the final imaging result can be obtained. Fig. 3.8 shows the multiple-view imaging result with $[-15, 0]$ dB display threshold.

Fig. 3.8 Target detection result of multiple-view tomographic imaging using a chirp pulse with $[-15, 0]$ dB display threshold (a) when walls are absent, (b) when walls are present and wall effects are not compensated.

In the above case when walls are present, the phase weighting does not include the extra phase change caused by walls, which means the phase change is not

correctly compensated. Fig. 3.8(b) shows clearly a relatively weak ghost target position or a relatively high level sidelobe. However, the real target position is well focused and no other obvious sidelobes around it. The result is much better than that in the single-view case.

We have also evaluated the performance of the multiple-view tomographic imaging using a stepped-frequency signal with the same bandwidth as that of the chirp pulse and 100 MHz frequency step. The results are similar to that using a chirp pulse. However, result using stepped-frequency signal gives a much higher sidelobe when walls are present and the phase shifts are not correctly compensated (see Fig. 3.9).

Fig. 3.9 Target detection result of multiple-view tomographic imaging using a stepped-frequency signal with $[-15, 0]$ dB display threshold (a) when walls are absent, (b) when walls are present and wall effects are not compensated.

When phase weighting counts the extra wall delay in the beamforming process, the wall effects can be well compensated in both single-view result and multiple-view result. Energies from the right side sensors refocus on the actual target position (Fig. 3.10(b) and Fig. 3.10(d)) in both conditions using a chirp pulse and a stepped-frequency signal. Single-view tomographic imaging results in both conditions still have high sidelobes around the real target position while multiple-view tomographic

imaging results give exactly the same results as that in LOS condition (conditions when walls are absent).

Fig. 3.10 Target detection results of (a) single-view with a chirp pulse, (b) multiple-view with a chirp pulse when wall effects are compensated, (c) single-view with a stepped-frequency signal, and (d) multiple-view with a stepped-frequency signal when wall effects are compensated.

With multiple-view tomographic imaging, target can be correctly focused if wall parameters are known. Even if the wall parameters are not exactly known, the detection result without compensation of wall effects is still acceptable. Besides well focusing targets, resolution of targets can also be improved using multiple-view

scenario. Fig. 3.11 gives a comparison of multiple targets detection result when wall effects are not compensated and multiple targets detection result when wall effects are compensated using a chirp pulse. Single-view result gives more than one ghost target position in both situations while multiple-view result gives clearly real target positions with low level sidelobes.

Fig. 3.11 Multiple targets detection result using a chirp pulse for (a) single-view and (b) multiple-view when wall effects are not compensated, (c) single-view and (d) multiple-view when wall effects are compensated.

As seen from the simulation results, the proposed multiple-view tomographic imaging system performs well and relatively meets the accuracy requirement of TTW target detection. However, it seems not feasible in military applications

requiring fast operation and high safety standards since the proposed system is time consuming and difficult to implement. Practical targets are always embedded in a complex environment, a large number of antenna sensors may be needed in order to obtain an acceptable image using the proposed system. Therefore, the system is hard to meet the demands of most of the military applications with high time efficiency and easy implementation. Military applications having low restrictions in time and implementation might be potential areas for the proposed system. Besides, some medical applications, such as breast cancer imaging, are also potential areas for the proposed system.

3.5 Concluding Remarks

In Chapter 3, we have presented a schematic study of target detections in TTW environment using tomographic imaging method in order to investigate the effects of antenna arrays on TTW tomographic imaging. Single-view and multiple-view tomographic imaging was introduced to represent two scenarios of antenna arrays, single-input-multiple-output (SIMO) and multiple-input-multiple-output (MIMO). By comparing the performances of single-view imaging and multiple-view imaging in two cases that walls are absent and present through several numerical simulations, it is found that the multiple-view tomographic imaging gives better imaging outcomes than single-view tomographic imaging especially when walls are present. The finding reveals that antenna arrays with MIMO scenario can greatly enhance the SNR of the system and somehow weaken the effects of the walls in TTW environment. However, time consuming is the major weakness of the multiple-view imaging system, which limits its feasibility in most practical military applications. It might be a potential consideration for applications requiring loose time constraint.

CHAPTER 4

TARGET DETECTION USING RANDOM SPARSE ARRAY

In this chapter, we present an approach based on a RSA and the CS technology for target detections in LOS environment without walls. In Section 4.1, the theory of a RSA will be introduced together with two modes of the RSA, namely the SAR mode and the array mode. In Section 4.2, algorithms of the RSA with CS for both static and moving target detections will be presented. In Sections 4.3 and 4.4, numerical examples and experimental validations will be given accordingly. In Section 4.5, the advantages and limitations of the proposed approach will be discussed. Section 4.6 concludes the chapter.

4.1 Random Sparse Array (RSA)

The concept of RSA was introduced and investigated in 1960's [103-105]. A RSA is defined as an array of randomly distributed elements with average inter-element spacing much larger than the signal wavelength [54, 106]. A RSA reveals its advantages in enlarging array aperture with relatively few antenna elements and distorting the symmetrical geometry of a conventional uniform array. The enlarged array aperture gives rise in the array resolution and the distorted array symmetry possibly mitigates periodic grating lobes caused by large inter-element spacing. In a RSA, the grating lobes are replaced by random sidelobes characteristics [106]. Therefore with a RSA, system cost can be greatly reduced.

The work of a RSA on imaging is firstly presented in [107] for forming building images from different angles. Ao *et al.* [108] apply the RSA concept to multi-channel TTW radar target imaging. Such a RSA is able to reduce the computational burden and operational complexity of an imaging system. However, there is not any research work on classifying a RSA. In this section, we present the detailed geometry of a RSA as well as its implementations. We characterize the RSA into two modes: one is the SAR mode and the other is the array mode.

4.1.1 SAR Mode

The SAR mode of a RSA, as its name indicates, is a synthetic aperture array with random sensor positions. It can be achieved by one moving sensor without implementing a number of sensors. Such configuration is of practical importance when only static targets or very slow moving targets are to be detected and the number of sensors is not enough to construct a long aperture array. Besides, when time is not a constraint for the imaging system, the RSA-SAR mode is a good choice. The linear RSA-SAR mode is similar to a conventional SAR case – using single sensor for transmitting and receiving at each azimuth position. But the difference is that the sensor is moved randomly to a position for transmitting and receiving in the RSA-SAR mode. The distance between adjacent azimuth positions is random, and these positions may be determined by inertial measurement units (IMU) with a ring laser gyro or a MEMS gyro [109, 110]. Fig. 4.1 shows a schematic diagram of a RSA-SAR mode of a linear configuration and its respective comparable geometry of a uniform SAR.

Fig. 4.1 Schematic diagram of a linear SAR mode RSA and its respective comparable geometry: uniform SAR.

4.1.2 Array Mode

The array mode of a RSA refers to an actual array composed of a set of randomly distributed elements. A small number of elements are located randomly forming a large aperture array. The element positions can also be determined using the method described in the RSA-SAR mode. For linear configuration, all the elements are located along one line. The RSA-array mode is able to reduce the complexity of moving target detection. The reason is that when using moving sensors, target images will be defocused if the relative speed between the moving sensors and the moving targets are not fully compensated. Besides, re-focusing algorithms bring new complexity to the system. However with static sensors, there is no need to compensate the smearing caused by Doppler effects and the moving targets can be better focused. Moreover, data collection time will be dramatically reduced in the RSA-array mode. The linear RSA-array mode is similar to a normal ULA. Fig. 4.2 shows an example of a RSA-array mode of linear configuration and its comparable geometry of ULA.

Fig. 4.2 Schematic diagram of a linear array mode RSA and its respective comparable geometry: ULA.

To make the comparison reasonable, the first and the last elements of a RSA are kept at the same positions as those of the respective SAR/ULA geometry, elements in between are randomly located. From the schematic geometries in Fig. 4.1 and Fig. 4.2, we can directly tell that a RSA needs much fewer elements and measurements than that for normal SAR and ULA; and it is much easier to implement.

Assume there are M random sparse azimuth positions or array elements in the system. The RSA can be denoted using a position matrix

$$\mathbf{P} = [\mathbf{r}_0 \ \mathbf{r}_1 \ \dots \ \mathbf{r}_m \ \dots \ \mathbf{r}_{M-1}]^T \quad (4-1)$$

where $\mathbf{r}_0$ is the first azimuth position in the RSA-SAR mode or the TX position in the RSA-array mode. The first position vector $\mathbf{r}_0$ is fixed and the m th position vector can be expressed in (4-2).

$$\mathbf{r}_m = \mathbf{r}_{m-1} + \Delta x_m \mathbf{a}_x + \Delta y_m \mathbf{a}_y, \quad m = 1, 2, \dots, M-1 \quad (4-2)$$

where $\mathbf{r}_m = x_m \mathbf{a}_x + y_m \mathbf{a}_y$, $\mathbf{a}_x$ and $\mathbf{a}_y$ are the unit vectors along x and y directions. Δx_m and Δy_m are the distance displacements from the $(m-1)$ th position along x y axes, respectively. Throughout this chapter, we consider the case when the displacement occurs at one direction (along the sensor moving direction in Fig. 4.2) only.

Although the two proposed schemes are different from each other, the underlying algorithms of them are the same when combined with CS. In the following section, we will derive the CS approach with the RSA for both static targets and moving targets, which is applicable for both RSA schemes.

4.2 Compressed Sensing (CS) with RSA

It has recently been proved that the property of a RSA can satisfy the requirement of CS [56, 57]. However, the approach of a RSA combined with CS for target detection has not been investigated. Therefore, a RSA imaging scheme combined with CS in the target imaging system is presented with the aim of reducing the operation complexity as well as the data collection time and storage. We present the approach

in both static and moving target detection cases.

4.2.1 Static Targets

In this section, the approach is based on the first scheme — the RSA-SAR mode, in which only one sensor is used. Assume that a RSA consists of M random sparse azimuth positions. Spacing between adjacent positions is random. In the imaging system, a stepped-frequency signal composed of N frequency bins is considered. The sensor transmits the stepped-frequency signal and receives the echo at one RSA position and moves to the next position for both transmitting and receiving.

It is assumed that the target scene is sparse (reasonable as long as the number of intended targets is much less than the number of pixels). The investigated area is divided into a number of pixels of size $d_x \times d_y$, where $x = 1, 2, \dots, K$, and $y = 1, 2, \dots, L$. Pixel in which target locates can be denoted using the target reflectivity. The target reflectivity (σ) is defined as the fraction of the intercepted power that is reradiated (scattered) by the target. It has a value between 0 and 1, 0 means no reflection and 1 means 100% reflection. Reflectivity of other pixels has similar definition as that of the target reflectivity. The reflectivity at the (x, y) th pixel is denoted using σ_{xy} . Representing the target scene using the reflectivity of its pixels as a column vector [81], the expression is shown in (4-3).

$$\mathbf{s} = [\sigma_{11}, \sigma_{21}, \dots, \sigma_{K1}, \sigma_{12}, \sigma_{22}, \dots, \sigma_{xy}, \dots, \sigma_{KL}]^T \quad (4-3)$$

where $\sigma_{xy} = 0$ when no target is located in the (x, y) th pixel.

A signal experiences an amplitude modulation and a phase shift before received by the RSA sensor. The received echo from one pixel is related to the scene reflectivity as in (4-4). Representing an index g using the pixel index x and y , $g = x + (y-1)K$ [111] where g ranges from 1 to KL , then the received signal of the n th frequency bin at the m th RSA sensor position is expressed as (4-4) for noise free case.

$$\mathbf{x}[m, n] = \sum_{g=1}^{KL} \sigma_g \exp(-j2\pi f_n \tau_{mg}) \quad (4-4)$$

where $\tau_{mg} = 2R_{mg}/c$. R_{mg} represents the path length from the m th sensor position to the g th pixel position, c is the light propagation velocity, σ_g is the reflectivity of the g th pixel and f_n is the n th frequency bin.

Measurement data of one RSA sensor can be written as the summation of received signals from all the pixels to the RSA sensor position. Expressing the total measurement data in matrix format, including measurement data at all the sensor positions in the RSA, and selecting R frequency bins for processing, we get a matrix expression in (4-5).

$$\mathbf{y} = \Phi \mathbf{x} + \mathbf{w} = \Phi \Psi \mathbf{s} + \mathbf{w} \quad (4-5)$$

where

$$\begin{aligned} \Psi &= [\Psi_1^T \ \Psi_2^T \ \cdots \ \Psi_N^T]^T_{MN \times KL} \\ [\Psi_n]_{mg} &= \exp(-j2\pi f_n \tau_{mg}) \\ \tau_{mg} &= \frac{2|\mathbf{r}_m - \mathbf{r}_g'|}{c} \\ n &= 1, 2, \dots, N, \quad g = 1, 2, \dots, KL \end{aligned}$$

In these expressions, $\mathbf{x}$ represents the received vector of size $MN \times 1$ and Φ is the $MR \times MN$ measurement matrix for selecting frequency bins randomly, $\mathbf{w}$ is the noise vector of size $MR \times 1$, $\mathbf{r}_m$ and $\mathbf{r}_g' = x_g \mathbf{a}_x + y_g \mathbf{a}_y$ are the position vectors of the m th RSA sensor and the g th pixel, respectively. Each row in Φ can be chosen to be a vector containing all zeros except one and only one “1” randomly located inside the vector [112]. In this way, Φ and Ψ can have minimum mutual coherence. The target scene can then be recovered using the OMP reconstruction method. Fig. 4.3 summarizes the static target detection process as well as the steps to reconstruct static target scene in a flow chart.

4.2.2 Moving Targets

Change detection is a commonly used technique in moving target detections. In change detection, the moving target scene can be obtained by differencing the observed scenes at different time instants. By using this approach, static backgrounds

will be cancelled and moving targets will be remained. In this section, the RSA-array mode is used for deriving the approach of moving target detection.

Fig. 4.3 Flow chart of the static target detection process.

Assume the RSA consists of M randomly located antenna elements along the x -axis. The center antenna element performs as a TX and RX. The rest elements perform as RXs only. It is usually assumed that the velocity is high enough for a target to move to distinct pixels at different observation time instants. The purpose is to ensure a clear image of the moving targets.

We divide the target scene into $K \times L$ pixels. According to [113], the investigated moving target scene can be considered sparse. Target scene at time instant t_i is the same as that in (4-3) and can be denoted in (4-6) with an additional index i indicating the time instant at which the scene is observed.

$$\mathbf{s}^i = [\sigma_{11}^i, \sigma_{21}^i, \dots, \sigma_{K1}^i, \sigma_{12}^i, \sigma_{22}^i, \dots, \sigma_{xy}^i, \dots, \sigma_{KL}^i]^T \quad (4-6)$$

where σ_{xy}^i represents the reflectivity of the (x, y) th pixel at time instant t_i . $\sigma_{xy}^i = 0$ if no target is located inside the (x, y) th pixel at t_i . By differencing the two scenes

between time instant t_i and t_{i-1} , we can obtain the moving target scene at time instant t_i [111] as shown in (4-7).

$$\begin{aligned}\mathbf{s}_{\Delta}^i &= \mathbf{s}^i - \mathbf{s}^{i-1} \\ &= [\Delta\sigma_{11}, \Delta\sigma_{21}, \dots, \Delta\sigma_{K1}, \Delta\sigma_{12}, \Delta\sigma_{22}, \dots, \Delta\sigma_{KL}]^T\end{aligned}\quad (4-7)$$

$$\Delta\sigma_{xy} = \sigma_{xy}^i - \sigma_{xy}^{i-1} = \begin{cases} -\sigma_q, & \text{target move out} \\ 0, & \text{no target} \\ \sigma_q, & \text{target move in} \end{cases}\quad (4-8)$$

The difference signal at the m th RX can be written as the summation of difference signals from all the KL pixel positions instead of the summation of signals from all the target positions. The reason is that the reflection from a non-target pixel can be ignored due to its zero reflectivity. There are two components in the difference signals, the noise-only terms from non-target pixels and the signal-plus-noise terms from target pixels. By employing the index g in Section 4.2.1, the difference signal from the g th pixel position in the scene can be expressed by

$$x_{\Delta g}^i[m, n] = \Delta\sigma_g \exp(-j2\pi f_n \tau_{Tgm}) \quad (4-9)$$

where $\Delta\sigma_g = \Delta\sigma_{xy}$ is the difference reflectivity of the g th pixel, τ_{Tgm} is the round trip time delay from the center TX via the g th pixel to the m th RX, f_n is the n th frequency bin. Combine the difference signals from all the pixels and represent the combination $\mathbf{x}_{\Delta}^i[m, n]$ in (4-10),

$$\mathbf{x}_{\Delta}^i[m, n] = \sum_{g=1}^{KL} \Delta\sigma_g \exp(-j2\pi f_n \tau_{Tgm}) + \mathbf{w}_{\Delta}[m, n] \quad (4-10)$$

where $\Delta\sigma_g, f_n, \tau_{Tgm}$ have the same definitions as in (4-9) and $\mathbf{w}_{\Delta}[m, n]$ represents the difference noise between time t_i and t_{i-1} on the n th frequency signal bin at the m th RX.

When R frequency bins are selected, the entire measurement data vector can be written as in (4-11).

$$\mathbf{y}_\Delta^i = \Phi \mathbf{x}_\Delta^i + \mathbf{w}_\Delta^i = \Phi \Psi \mathbf{s}_\Delta^i + \mathbf{w}_\Delta^i \quad (4-11)$$

where Φ and Ψ have the same definitions as that in (4-5), but the formats of individual elements in Ψ are different. The element in Ψ in (4-11) is defined as $[\Psi_n]_{gm} = \exp(-j2\pi f_n \tau_{Tgm})$, where $\tau_{Tgm} = (|\mathbf{r}_g - \mathbf{r}_T'| + |\mathbf{r}_m' - \mathbf{r}_g|)/c$. The fixed TX position vector in the RSA is denoted using $\mathbf{r}_T$. The difference signal vector is denoted using $\mathbf{x}_\Delta^i$ of size $MN \times 1$ and the difference noise vector is denoted using $\mathbf{w}_\Delta^i$.

Fig. 4.4 Flow chart of moving target detection process.

We can also easily extend the moving target detection at two time instants case to multiple time instants case. Then the moving target scene at time t_i has a slightly different expression from (4-7).

$$\mathbf{s}_\Delta = \mathbf{s}^i - \mathbf{s}^{i-1} + \mathbf{s}^{i-2} - \mathbf{s}^{i-3} + \cdots + \mathbf{s}^1 - \mathbf{s}^0. \quad (4-12)$$

where $\mathbf{s}^0$ represents the vector containing reflectivity of the initial target scene. It is noted that the number of time instants $i+1$ should be set to an even number so that moving targets at multiple time instants can be observed while the static background can be cancelled. Information of moving targets at multiple time instants might be

very useful when the target track is the only information needed in the application and the target moving direction is not required. A flow chart showing the moving target detection process is presented in Fig. 4.4.

4.3 Numerical Examples

Several numerical experiments are carried out on detection of both static and moving targets. Through the imaging process, target positions can be obtained and targets can be detected. Different imaging results are compared in each simulation example, including results using CS method in the RSA for both static and moving target detections, results using the time-domain BP method in SAR for static target detection and results using the CBF method in ULA for moving target detection.

4.3.1 Static Targets Detection

Four point targets are assumed in the system model. A SAR mode RSA is located along the x -axis. The investigation area where the four point targets are located is of size $5 \text{ m} \times 5 \text{ m}$, ranging from 0 m to 5 m in x -axis and 15 m to 20 m in y axis. The pixel size of the area is set to be $0.1 \text{ m} \times 0.1 \text{ m}$. Assume that all targets' reflectivity have the same value ($\sigma = 1$), and reflectivity of pixels without targets is set to be zero ($\sigma = 0$). The four targets are located at (1.5, 17.9) m, (1.3, 17.5) m, (2.7, 17.5) m, and (4.1, 17.5) m. Table 4.1 lists the parameters used in the simulation examples and Fig. 4.5(a) shows the static target scene. To give a fair comparison, the aperture lengths of normal SAR and the RSA-SAR mode are both set to be around 5 m. Therefore normal SAR requires much larger number of azimuth positions than that

Table 4.1: Static Target Simulation Parameters

Symbol	Quantity	Value
f_{st}	Starting frequency	2 GHz
f_{ed}	Ending frequency	3 GHz
N	Number of frequency bins	201
N_s	Number of azimuth positions in SAR	101
d_s	Azimuth sampling spacing of SAR	5 cm
M_{RSA}	Number of azimuth positions in RSA	5
r	Percentage of M_{RSA}/N_s	5% of N_s
R	Selected frequency bins	20% of N

Fig. 4.5 Static target detection results at $\text{SNR} = 30$ dB: (a) original target scene, (b) detection using full SAR data and back-projection, (c) detection using sparse RSA data and back-projection, (d) detection using sparse RSA data and compressed sensing.

the RSA requires. Results of BP and CS using normal SAR data and RSA data are compared. All results are shown in $[-30, 0]$ dB scale.

In order to meet the measurement requirement of CS and keep the data amount as small as possible, we choose an amount around $5\% \times 20\% = 1\%$ of the full SAR data as the sparse RSA data size in the simulation. The 5% refers to the percentage of the number of the RSA azimuth positions to the full SAR positions and 20% refers to the percentage of the selected frequency bins. Choose a very good signal quality, $\text{SNR} = 30$ dB, to show the performances of all the methods at conditions very close to noise free case. In subsequent parts, more practical SNR values will be considered. The simulation results using full SAR data and sparse RSA data are shown in Fig. 4.5. The conventional BP method and CS method using the same amount of sparse data are compared.

It is easy to see from Fig. 4.5(c) and Fig. 4.5(d) that a RSA with CS can perform much better than a RSA with BP algorithm when using the same amount of data in imaging the target scene. Many sidelobes appear around the real target positions even though all the targets can be identified in Fig. 4.5(b). There is no sidelobe appear when using a RSA and CS in Fig. 4.5(d). It is concluded that by using a RSA and CS, much less amount of data can generate much clearer detection results than normal SAR imaging method.

In addition to the improvement in sidelobe levels, the detection resolution can also be improved by using a RSA and CS. The data size of the RSA is increased to the same amount of full SAR data in order to compare the resolutions. In this case, same number of azimuth positions and frequency bins are used in both the RSA and the conventional SAR, but the aperture of the RSA becomes much larger than that of SAR. We choose a relatively week quality signal with an SNR = 0 dB in this example. The comparison of resolutions of the SAR and the RSA is shown in Fig. 4.6. It is observed that the resolution of target detection using a RSA with CS is greatly improved than that using a SAR with the BP method.

Fig. 4.6 Detection resolutions at SNR = 0 dB: (a) back-projection with full SAR data, (b) compressed sensing with RSA data of the same size as full SAR data.

However, the reconstructed images using a RSA and CS are degraded when the noise level goes up (SNR value is decreased). Sidelobes appear and noises arise in the images as shown in Fig. 4.7. In spite of this phenomenon, one can still see the target positions easily when the SNR value is not lower than 0 dB.

Fig. 4.7 Detection results using RSA data with compressed sensing at different SNR values: (a) 20 dB, (b) 10 dB, (c) 0 dB, (d) -10 dB.

4.3.2 Moving Targets Detection

In the moving target scene reconstruction, the RSA-array mode is used. Initial moving target scene is shown in Fig. 4.8. Three point targets are located at (1.0, 16.8) m, (1.2, 17.4) m, (2.8, 17.6) m. It is assumed that the first one is a static target with a strong reflectivity ($\sigma = 1$) and the other two targets are moving with a relatively weak reflectivity ($\sigma = 0.1$). For simplicity, only velocity in the x - y plane is considered for moving targets. The velocities are defined as two dimensional vectors: $v_i = (v_{xi}, v_{yi})$, ($i = 1, 2$), where v_{xi} represents the velocity component along x direction and v_{yi} represents the velocity component along y direction. The pixel size is again set to be $0.1 \text{ m} \times 0.1 \text{ m}$. Performance of a RSA with CS is compared with that of a ULA of the same aperture with CBF method. Five antenna elements are assumed in the array (about 5% of the ULA elements with the same aperture). We select 20% of frequency bins. Then the RSA data is about 1% of the ULA data. Table 4.2 lists the simulation parameters in this example.

Fig. 4.8 Original target scene (static target in blue circle and moving targets in red dots).

Table 4.2: Moving Target Simulation Parameters

Symbol	Quantity	Value
f_{st}	starting frequency	2 GHz
f_{ed}	ending frequency	3 GHz
N	number of frequency bins	201
N_a	number of antenna elements in ULA	101
d_a	array element spacing of ULA	5 cm
M_{RSA}	number of antenna elements in RSA	5
r	percentage of M_{RSA}/N_a	5% of N_a
R	selected frequency bins	20% of N
PRI	pulse repetition interval of TX	0.1 s

Case 1: Velocities of the two moving targets are assumed to be $v_1 = (-2, -1)$ m/s and $v_2 = (2, -1)$ m/s. In this case, we aim to detect moving targets from two different observation time instants. After a PRI, new positions of the two moving targets become (1.0, 17.3) m, and (3.0, 17.5) m. This means that the two targets both move to distinct pixels at the two observation time instants.

Fig. 4.9 Detection results at SNR = 30 dB: (a) moving target scene, (b) detection using full ULA data and beamforming, (c) detection using sparse RSA data and beamforming, (d) detection using sparse RSA data and compressed sensing.

Fig. 4.10 Moving target detection results using sparse RSA data and compressed sensing at different SNR values: (a) 20 dB, (b) 10 dB, (c) 0 dB, (d) -10 dB.

From the simulation results in Fig. 4.9, one can see that the RSA imaging with CS can recover moving target scene perfectly with very few measurements. CBF algorithm nearly fails when using the sparse RSA measurements. The detection results using CBF and the full ULA data of the same aperture as the RSA are still not good as the results of using the RSA data with CS. However, the performances of the RSA with CS at low SNR values are necessary to be investigated. Fig. 4.10 shows the moving target detection results at SNR values from 20 dB down to -10 dB. As shown in Fig. 4.10(d), moving targets cannot be fully detected when $\text{SNR} = -10$ dB.

Case 2: Velocities of the two moving targets are still assumed to be $v_1 = (-2, -1)$ m/s and $v_2 = (2, -1)$ m/s. However, target track information is assumed to be the most important thing in this case. We observe several time instants ($i > 2$) and obtain a change detection result using data from all the time instants. According to (4-12), the number of time instants to be observed should be even number. We take 4 time instants for example ($i = 3$) since two time instants have already been considered in Case 1. It is expected that target tracks formed by 4 positions will be obtained in the track reconstruction. The following Fig. 4.11 shows the track reconstruction results using different geometries and methods. It is found that using the RSA data with CS can completely reconstruct the moving target tracks. However, the RSA data with CBF method completely misses the tracks.

Case 3: Velocities of the first moving target is kept the same as previous cases but the velocity of the second target is reduced to $v_2 = (0.5, -1)$ m/s. After one PRI, the second target will move 0.05 m in x direction and 0.1 m in y direction. Therefore, it can only move to a distinct pixel in y direction but indistinct pixel in x direction after two observations. We test the performance of using the RSA data with CS on reconstructing the target tracks in this case. Since target migration occurs in this condition, we can expect that the performance of using the RSA with CS degrades. Fig. 4.12 shows the results at $\text{SNR} = 30$ dB.

Fig. 4.11 Track reconstruction results at SNR = 30 dB: (a) moving target track, (b) reconstruction using full ULA data and beamforming, (c) reconstruction using sparse RSA data and beamforming, (d) reconstruction using sparse RSA data and compressed sensing.

Fig. 4.12 Reconstruction results of indistinct tracks at SNR = 30 dB: (a) actual moving target track, (b) reconstruction using full ULA data and beamforming, (c) reconstruction using sparse RSA data and beamforming, (d) reconstruction using sparse RSA data and compressed sensing.

4.4 Experimental Validation

An experiment on static targets detection is carried out with the help of a Final Year Project (FYP) student and a group of students from a Design and Innovation Project (DIP) team in collecting full SAR data. The experiment is to better verify the performance of the RSA and CS in practical environment. Three scenarios are compared: reconstruction using full SAR data with BP method, reconstruction using sparse RSA data with BP method and reconstruction of sparse RSA data with CS. To give a fair comparison, the synthetic apertures of the SAR and the RSA are constructed to be the same.

4.4.1 Experimental Setup

An experimental test is carried out outside a laboratory area to verify the performance of the proposed scheme in the static target detection case. A simple SAR sensor setup is shown in Fig. 4.13. It consists of two colocated broadband ridged horn antennas for transmitting and receiving, an Agilent N3382A Vector Network Analyzer (VNA) for signal generation and measurements, and a laptop with LabView software for automated measurement control. The two antennas are broadband ridged horn antennas with frequency specification from 0.8 to 8.0 GHz.

Fig. 4.13 Equipment setup with colocated broadband transmitting and receiving antennas and an Agilent N3382A VNA as the transceiver.

The target scene of interest outside the laboratory area is about 20 m long and 20 m wide. An aperture of 13 m is synthesized for both full SAR and the RSA. The number of azimuth positions is 372 and the spacing between adjacent azimuth positions is 35 mm in the full SAR. The spacing between azimuth positions of the RSA is random and the number of azimuth positions in the RSA is a portion of the number in the full SAR. A wideband signal ranging from 4 to 6 GHz is synthesized by a number of frequency steps. The frequency step is set to be 1.25 MHz and there are 1601 frequency steps in total. The H -plane beamwidth of the antenna from its test report is listed in Table 4.3 along with all other parameters. The RSA azimuth position number is 40% of that in the full SAR and we choose a larger percentage (80% for instance) of selected frequency bins in the experiment for the RSA to tolerant unexpected interferences or noises in the experimental case.

Table 4.3: Experimental Parameters

Symbol	Quantity	Value
f_{st}	Starting frequency	4 GHz
f_{ed}	Ending frequency	6 GHz
N	Number of frequency bins	1601
N_s	Number of azimuth positions in SAR	372
d_s	Azimuth sampling spacing	35 mm
M_{RSA}	Number of azimuth positions in RSA	148
r	Percentage of M_{RSA}/N_s	40% of N_s
R	Selected frequency bins	80% of N
$\theta_{3dB}^{(H)}$	H -plane antenna beam width	$36^\circ \pm 3^\circ$
R_0	Distance between SAR to the scene	30 m

A photo of the ground truth investigated is shown in Fig. 4.14. There are six targets in total. The four palm trees are considered as main static targets. One trihedral corner reflector is placed at a position between two palm trees at the center, but a little further than them in range direction. It is an intended introduced interference. Near the second tree from the left, there is a street lamp post made of metal and is another strong interference in the scene. Fig. 4.15 gives the two-dimensional SAR image with pixel size of $0.01 \text{ m} \times 0.01 \text{ m}$. The azimuth and range resolution of the image can be estimated by (4-13) and (4-14).

$$\Delta X = \frac{\lambda R}{L} = \frac{0.06 \times 30}{(372-1) \times 0.035} = 0.138 \text{ m} \quad (4-13)$$

$$\Delta Y = \Delta R = \frac{c}{2B} = \frac{3 \times 10^8}{2 \times 2 \times 10^9} = 0.075 \text{ m} \quad (4-14)$$

Fig. 4.14 Ground truth of the target area with main targets: four palm trees, one street lamp post, and one trihedral corner reflector (circled in red).

Fig. 4.15 Imaging result using full SAR data and back-projection method with pixel size of $0.01 \text{ m} \times 0.01 \text{ m}$.

4.4.2 Detection Results

Before presenting the experimental results, it is necessary to give the respective simulation results using the same parameter set as in the experiment. As seen from Fig. 4.15, the six targets are located approximately at (6.6, 35) m, (9.5, 32) m, (12.8, 29.5) m, (8.5, 29.5) m, (4.5, 29.5) m and (0, 29.5) m in the sequence from further to nearer and from left to right. Fig. 4.16 shows the target scene and detection results using BP method and CS through simulations. To reduce the processing time, the pixel size is set to be $0.5 \text{ m} \times 0.5 \text{ m}$ in the simulation.

Fig. 4.16 Static target detection results with the same set of parameters as in Table 4.3 at $\text{SNR} = -10 \text{ dB}$: (a) original target scene, (b) detection using full SAR data and back-projection, (c) detection using sparse RSA data and back-projection, (d) detection using sparse RSA data and compressed sensing.

In the experimental results, we set the pixel size to the same value ($0.5 \text{ m} \times 0.5 \text{ m}$) as that in Fig. 4.16 with the purpose of reducing processing time and giving a fair comparison. OMP algorithm is applied in CS reconstruction with the RSA data. Result using full SAR data and BP method is shown in Fig. 4.17 and compared with the image in Fig. 4.15, the corner reflector becomes difficult to identify in this figure after the pixel size is increased. Fig. 4.18 shows the detection result using sparse

RSA data and BP algorithm and Fig. 4.19 shows the detection result using sparse RSA data and CS approach.

Fig. 4.17 Detection result using back-projection with full SAR data in $[-30, 0]$ dB scale with pixel size of $0.5 \text{ m} \times 0.5 \text{ m}$.

Fig. 4.18 Detection result using back-projection with sparse RSA data in $[-30, 0]$ dB scale with pixel size of $0.5 \text{ m} \times 0.5 \text{ m}$.

Fig. 4.19 Detection result using compressed sensing with sparse RSA data in $[-30, 0]$ dB scale with pixel size of $0.5 \text{ m} \times 0.5 \text{ m}$.

To see the results more clearly, we decrease the image display scale of Fig. 4.18 and Fig. 4.19 to $[-10, 0]$ dB and show the resulted images in Fig. 4.20 and Fig. 4.21 respectively. In addition, we also circled necessary places in both images to emphasize the real target positions. By comparing these two figures, we can see that CS with sparse RSA data produces better result in Fig. 4.21 than BP with the same sparse data in Fig. 4.20. In Fig. 4.21, the lamp is recognizable. Four palm trees can still be identified in the image except the one nearest to the lamp. This is possibly due to the strong reflections of the lamp which submerges the reflection of the second palm tree. The corner reflector is totally not recognizable due to weak reflection. Targets in Fig. 4.20 are not recognizable at all. When comparing the result in Fig. 4.21 with that in Fig. 4.17, we can find that detection result of CS with sparse RSA data is not as good as that using BP method with full SAR data. In this case, in the viewpoint of perfect target detection, full SAR method gives much better result than a RSA and CS. It is concluded that in practical experiments, target detection using CS and a RSA may not be ideal, but it decreases data storage and processing time with acceptable performance. Targets may not be fully and correctly detected when the environment consists of many unknown sources, possibly include the

undesired reflections and possible multiple scattering from the static background. When data storage and processing time is not the concern, BP method with more SAR data may have better detection result than CS method with less RSA data.

Fig. 4.20 Detection result using back-projection with sparse RSA data in $[-10, 0]$ dB scale with pixel size of $0.5 \text{ m} \times 0.5 \text{ m}$.

Fig. 4.21 Detection result using compressed sensing with sparse RSA data in $[-10, 0]$ dB scale with pixel size of $0.5 \text{ m} \times 0.5 \text{ m}$.

A quantitative error analysis is carried out on the experimental data to further compare the detection performance of BP with RSA data with that of CS with RSA data. The RSA data is selected in such a way that the first and last azimuth positions of the full SAR data are always selected and data at the positions in between are randomly selected. Since the RSA data is randomly selected, 100 trials are performed to obtain a statistical comparison of both detection results. We obtain results when different sizes of data are selected as the RSA data, including 20%, 40%, 60% and 80% of full SAR positions.

Full SAR detection is considered as the reference detection result since it has the best performance among the three results. In each trial, the difference between CS with RSA detection and full SAR detection and the difference between BP with RSA detection and full SAR detection are obtained. The integrated errors Δ_{err} over all the pixels of these two results are calculated for all the 100 trials.

$$\Delta_{err} = \frac{1}{KL} \sum |\mathbf{I}_{det} - \mathbf{I}_{ori}| \quad (4-15)$$

where $\mathbf{I}_{det}$ refers to the normalized detection result of CS with RSA or BP with RSA. $\mathbf{I}_{ori}$ is the normalized reference target image which is the full SAR image in this case. KL represents the total number of pixels in the image. Therefore, the integrated error has a value greater than 0 but smaller than 1.

The integrated errors (in dB) of both BP and CS with four different percentages of RSA data are shown in Fig. 4.22. The errors are shown in merit order. As found from the figure, CS with RSA always gives smaller integrated errors than BP with RSA from low data percentages to high data percentages. This indicates that CS with RSA maintain a better performance than BP with RSA throughout the 100 trials. The integrated errors for four different data percentages are also shown in histograms in Fig. 4.23. It shows the distribution of integrated errors over the 100 trials. The error of CS with RSA centralizes on a value much smaller than that of BP with RSA centralizes on. According to Fig. 4.22 and Fig. 4.23, the detailed values of the worst errors, the best errors, the median errors and the mean errors of the four cases can be obtained and they are listed in Table 4.4 for BP with RSA data and in Table 4.5 for

CS with RSA data. As the data amount is increased, the integrated errors for both BP and CS decrease. However, the error of CS with RSA is much smaller than that of BP with RSA in all the 100 trials. The worst error of CS with RSA is smaller than the best error of BP with RSA. Therefore, the performance of CS with RSA is better than that of BP with RSA according to these results.

Fig. 4.22 The integrated errors of BP and CS results shown in merit order for four different percentages of data.

Fig. 4.23 The integrated errors of BP and CS results shown in histogram for four different percentages of data.

Table 4.4: Four Types of Integrated Error – BP with different RSA data

Percentage of positions	RSA-BP			
	Worst (dB)	Best (dB)	Median (dB)	Mean (dB)
20%	-10.9	-20.8	-14.5	-14.4
40%	-11.1	-23.3	-17.4	-17.5
60%	-13.2	-24.6	-18.6	-18.8
80%	-14.3	-26.9	-20.8	-20.9

Table 4.5: Four Types of Integrated Error – CS with different RSA data

Percentage of positions	RSA-CS			
	Worst (dB)	Best (dB)	Median (dB)	Mean (dB)
20%	-24.9	-29.4	-27.2	-27.2
40%	-26.7	-30.0	-28.1	-28.2
60%	-26.5	-29.9	-28.7	-28.7
80%	-27.6	-30.5	-29.1	-29.1

(a) Comparison of integrated errors (RSA-BP)

(b) Comparison of integrated errors (RSA-CS)

Fig. 4.24 Comparison of integrated errors for BP and CS with different percentages of data.

A clear comparison of the integrated errors when four different percentages of RSA data are used with BP and CS are shown in Fig. 4.24(a) and Fig. 4.24(b) respectively. This comparison shows the difference between the integrated errors at different cases and CS with RSA has better performance than that of BP with RSA.

Fig. 4.25 Worst, Best, and Mean RSA-BP and RSA-CS images from 100 trials based on 20% random positions from the full SAR data.

Fig. 4.26 Worst, Best, and Mean RSA-BP and RSA-CS images from 100 trials based on 40% random positions from the full SAR data.

Fig. 4.27 Worst, Best, and Mean RSA-BP and RSA-CS images from 100 trials based on 60% random positions from the full SAR data.

Fig. 4.28 Worst, Best, and Mean RSA-BP and RSA-CS images from 100 trials based on 80% random positions from the full SAR data.

According to the integrated errors, we can find out in which trial the worst and the best detection results occur. The worst results of BP with RSA and CS with RSA may not happen in the same trial; and so do the best results. The worst and the best detection results among 100 trials as well as the average detection results of 100

trials for the four different data percentages are shown from Fig. 4.25 to Fig. 4.28. Take 20% data for example, the worst result of BP with RSA occurs in the 31st trial and the best result occurs in the 15th trial; the worst result of CS with RSA occurs in the 31st trial and the best result occurs in the 43rd trial. Since the azimuth resolution of the target image is almost two times the range resolution and the pixel sizes of the resulted image are the same in range and azimuth directions, we use a rectangular box with smaller length in range and larger width in azimuth direction to represent the possible target position area. Any point inside the box area can be considered as a target. According to the worst and best results from individual trails, we can see that CS with RSA can detect some targets but BP with RSA cannot detect targets. The average results illustrate that the overall performance of CS with RSA is better than that of BP with RSA.

4.5 Discussions

4.5.1 Advantage

The target detection approach using imaging method based on a RSA and CS presented in this chapter can achieve higher resolution and lower sidelobe levels than traditional array configurations of the same aperture with conventional imaging methods in both static and moving target detections. A RSA gives a large aperture with very small number of sensors or very small number of data collection positions. System cost is therefore dramatically reduced. The utilization of the CS method enables the approach with a RSA being able to greatly increase the resolution and mitigate sidelobes. Besides, reduction in processing time and space for data storage leads to decreased system complexity.

4.5.2 Limitation

The approach based on a RSA and CS also has its own limitations besides the aforementioned advantages. The performance of a RSA with CS is not very good in real situations since practical environment is much complicated and huge number of unexpected interferences may come from the vertiginous background. More robust

detection methods may need to be investigated for real complicated environment in the future.

4.6 Concluding Remarks

In Chapter 4, we have presented an approach by combining a RSA with CS for target detections in open space environment. We have introduced two modes of a RSA and the performance has been evaluated through simulation examples of static and moving target detections and experimental validations of static target detections. Results show that a RSA with CS has some advantages but also has limitations. When the environment is composed of many unexpected interferences or multiple scatterings, a RSA with CS has degraded performances. Besides, when the target scene is too large and too many elements for a RSA are required, the RSA is not rational to be chosen. The method using a RSA and CS can be used for TTW detections however the performance may not be rational. It is not very appropriate for most TTW detections in its present form. The approach of a RSA combined with CS can be considered as a basis for TTW detection algorithms since the method requires less data storage and lower hardware cost. However, new and advanced algorithms need to be developed to take into account wall effects and signal penetration losses in TTW environment and to optimize the approach based on a RSA and CS.

CHAPTER 5

DIRECTION OF ARRIVAL AND POLARIZATION ESTIMATIONS USING VECTOR CIRCULAR ARRAY

In this chapter, we will propose a simple and intuitive approach based on a new array structure, named VCA, for DOA and polarization estimations. In Section 5.1, the geometry and mathematical theory of the proposed VCA will be introduced along with its comparisons with the conventional SCA. In Section 5.2, estimation algorithms including DOA and polarization parameters estimation will be presented. In Section 5.3, several numerical experiments of source estimation using VCA will be carried out. In Section 5.4, the DOA and polarization estimation approach will be extended to sources in colored Gaussian noise. Its performance will be compared with that in white Gaussian noise. In Section 5.5, the advantages and limitations of the proposed approach will be discussed. Section 5.6 concludes this chapter.

5.1 Vector Circular Array (VCA)

5.1.1 Scalar Circular Array (SCA)

A conventional UCA can be defined as a scalar array. It is referred to as a SCA in this chapter. A SCA is composed of a set of identical elements uniformly distributed along a circle with the same polarization direction (mostly vertical to the array plane). Fig. 5.1 shows the configuration of a SCA with 8 identical elements. Element polarization directions at the corresponding positions are represented by bold red arrows. As Fig. 5.1 shows, all the elements in a SCA have identical vertical polarization.

Fig. 5.1 A SCA is composed of uniformly spaced elements with identical amplitude and polarization direction (vertical polarization represented by bold red arrows).

Limited by the configuration, a SCA can only be deployed using monopole/dipole-like antenna elements. According to the properties of such type of antennas, a SCA is mainly used for DOA estimation of sources from relatively low grazing angles with vertical polarization and is less effective in estimating sources from relatively high grazing angles with arbitrary polarization. Therefore, a SCA mainly finds its applications in areas involving only vertical polarization such as most earth surface wireless communications.

5.1.2 Proposed Vector Circular Array (VCA)

Inspired by the fact that sources from relatively high grazing angles are gaining great and important interest in some special applications and polarization information of sources is also becoming more and more important, we propose a VCA with the aim of estimating sources from relatively high grazing angles and with possibly arbitrary polarization directions.

A VCA has a similar definition as that of a SCA. It is an array composed of uniformly distributed elements with same amplitude but different orientations along a circle. Fig. 5.2 shows a configuration of the proposed VCA with 8 elements. All the elements have the same amplitude but are radially polarized in different directions. Radial polarization, as its name indicates, is the polarization with electrical field

direction pointing outwards from the center of the array or pointing towards to the center of the array radially. The bold red arrows in Fig. 5.2 represent the radial polarizations at the corresponding element positions. The proposed VCA is actually a radially polarized circular array. It is not a real vector array. We call it vector due to the different polarization directions of all the elements in horizontal direction and to distinguish it from the conventional vertical polarized circular array (the scalar circular array). Since we target sources from high-grazing angles, we mainly focus on horizontal polarization and ignore vertical polarization in the proposed VCA. The vertical polarization components cannot be resolved by the proposed VCA.

Fig. 5.2 A VCA is composed of uniformly spaced elements with identical amplitude but different polarizations directions (radial polarizations in bold red arrows).

Unlike a SCA, a VCA is able to achieve more flexibility in antenna elements design according to its requirement of antenna element polarizations. It is expected that possible choice of antenna element types in a VCA can be dipole/monopole with reflector, cavity antenna, patch antenna, horn antenna and many other types. Benefited from its geometry, a VCA can estimate direction of source signals with relatively high grazing angles and arbitrary polarization, which refers to linearly, circularly and elliptically polarized signals. However, it might have limitations on estimating the tilt angles and senses in elliptical polarization in general, but it is still

able to determine whether the polarization is elliptical. For sources with low grazing angles, the VCA is limited to source signals with horizontal polarization or polarizations close to horizontal which have little polarization mismatch with the antenna elements (for vertical polarization or polarizations close to vertical polarization, there are very large polarization losses, thus source directions cannot be correctly estimated).

5.1.3 Mathematical Description of A VCA

Assume that a VCA consists of N elements with uniform spacing. In order to be consistent with our estimation approach derived later, only even number of elements is considered, that is $N = 2M$. A VCA can be seen as a circular array constituted by M number of 2-element ULAs with their array axes crossed at the center of the VCA. All the 2-element ULAs have different orientations from each other. Each 2-element ULA can be used to perform DOA estimation.

Fig. 5.3 Geometry of an N -element VCA.

Fig. 5.3 shows the geometry of an N -element VCA. The VCA is placed in the Cartesian coordinate system in such a way that its phase center coincides with the origin. There is a corresponding array axis associated with each ULA pointing to

different directions. The line connecting the two elements' positions in an ULA forms the array axis vector. Without loss of generality, let's consider the elements are counted anti-clockwise with increasing numbers from the x -axis. The angular separation between consecutive elements is uniform and has the expression of $\Delta\phi = 2\pi/N$. Then the first element has a zero azimuth angle and the n th element has an azimuth angle ϕ_n of the form in (5-1),

$$\phi_n = \Delta\phi(n-1) \quad (5-1)$$

R_0 represents the radius of the VCA. It is related to the inter-element spacing d of the 2-element ULA by the equation $d = 2R_0$. With the expression of the azimuth angle and the radius of the array, the position vector of the n th element can be represented using the unit vectors of the axes in the Cartesian coordinate system along with its azimuth angle ϕ_n .

$$\mathbf{r}_n = R_0 (\mathbf{a}_x \cos \phi_n + \mathbf{a}_y \sin \phi_n) \quad (5-2)$$

where $\mathbf{a}_x$ and $\mathbf{a}_y$ are the unit vectors of the x and y axes in the Cartesian coordinate system.

Without loss of generality, we can consider unit position vectors of the first M elements starting counter-clockwise from the x -axis are the defined array axes of all the M 2-element ULAs, for example the first three ULA axes are the axes pointing to the directions of element orientations labeled as 1, 2, 3 in Fig. 5.3. Therefore, the azimuth angle of the m th array axis is the same as that of the m th element ϕ_m . The m th array axis can be represented as a unit vector in terms of the unit vectors of the coordinate system. The m th axis has an orientation vector which can be written as (5-3).

$$\mathbf{a}_m = \mathbf{a}_x \cos \phi_m + \mathbf{a}_y \sin \phi_m \quad (5-3)$$

where $\phi_m = \phi_n$ ($m, n = 1, 2, \dots, M$). According to the antenna array theory, there is a steering vector corresponds to each 2-element ULA. All the M steering vectors have the same format and the m th steering vector can be written as the expression in (5-4),

$$\mathbf{e}_m = \left[\exp\left(jk\left(-\frac{d}{2}\right)\cos\Phi_m\right) \quad \exp\left(jk\frac{d}{2}\cos\Phi_m\right) \right]^T \quad (5-4)$$

where $k = 2\pi/\lambda$ is the wavenumber, λ is the source signal wavelength, d is the inter-element spacing of 2-element ULAs and is equal to half wavelength of the source in order to prevent any direction ambiguity due to grating lobes. The angle Φ_m refers to the DOA angle of the source signal with respect to the m th array axis.

5.2 Direction of Arrival (DOA) and Polarization Estimations

5.2.1 Signal Model

In the remainder of this chapter, we consider only linearly polarized plane wave sources from far-field. Assume that there are Q source signals impinging on the N -element VCA from the far field. All of them are narrowband plane wave sources polarized linearly in the space. The definition of a source with elevation angle (θ) and azimuth angle (ϕ) in relation to the Cartesian and spherical coordinate systems is shown in Fig. 5.4. The electrical field $\mathbf{E}$ of the source can be decomposed into two orthogonal components in the electrical field plane (the gray plane in Fig. 5.4), one is in the theta direction (E_θ) and the other is in the phi direction (E_ϕ). Then the polarization angle (γ) of the source is defined as the angle between the theta component (E_θ) and the total electrical field $\mathbf{E}$. The left two-dimensional (2D) gray plane in Fig. 5.4 clearly shows the coordinate of the source polarization and the small cross at the center indicates that the signal propagation direction is going into the gray plane. It is obvious from Fig. 5.4 that θ ranges from 0° to 90° in the vertical plane, ϕ ranges from 0° to 360° in the x - y plane and γ ranges from 0° to 90° in the electrical field plane.

We represent the direction and polarization parameters of the q th source using $(\theta_q, \phi_q, \gamma_q)$, then the direction vector of the q th source can be expressed as (6-5) in terms of unit vectors of the Cartesian coordinate system.

$$\mathbf{a}_r^q = \mathbf{a}_x \sin\theta_q \cos\phi_q + \mathbf{a}_y \sin\theta_q \sin\phi_q + \mathbf{a}_z \cos\theta_q \quad (5-5)$$

Fig. 5.4 Illustration of the relations of a signal polarized in an arbitrary $\mathbf{E}$ direction to the spherical and Cartesian coordinates (θ is the elevation angle from the z -axis, ϕ is the azimuth angle from the x -axis, γ is the polarization angle, $\mathbf{k}$ is the wave vector in the signal propagation direction, $\mathbf{r}$ is the radiation vector, ρ is the projection of the source direction in x - y plane).

Due to the fact that the polarization of a signal will cause different amplitudes in different directions in space, array elements with different polarization orientations will receive different powers from the same signal. Moreover, signals from different DOAs will excite array elements with different phase shifts. Therefore, the m th 2-element ULA receives a phase-shifted and amplitude-modulated version of the q th source signal when it hits on the array elements. Decompose the incident electrical field $\mathbf{E}$ into two orthogonal components as in (5-6) and (5-7).

$$\mathbf{E} = \mathbf{a}_\theta^q E_\theta^q + \mathbf{a}_\phi^q E_\phi^q \quad (5-6)$$

where

$$\begin{aligned}
\mathbf{E} &= E\mathbf{a}_E \\
\mathbf{a}_E &= \mathbf{a}_\theta^q \cos \gamma_q + \mathbf{a}_\phi^q \sin \gamma_q \\
E_\theta^q &= E \cos \gamma_q \\
E_\phi^q &= E \sin \gamma_q
\end{aligned} \tag{5-7}$$

where $\mathbf{a}_\theta^q$ and $\mathbf{a}_\phi^q$ are the respective unit vectors of E_θ^q and E_ϕ^q , $\mathbf{a}_E$ is the unit vector in $\mathbf{E}$ direction, and E is the magnitude of $\mathbf{E}$. We can easily find the relationships between $\mathbf{a}_\theta^q$, $\mathbf{a}_\phi^q$ and the x, y, z unit vectors of the coordinate system. They are shown in (5-8).

$$\begin{aligned}
\mathbf{a}_\theta^q &= \mathbf{a}_x \cos \theta_q \cos \phi_q + \mathbf{a}_y \cos \theta_q \sin \phi_q - \mathbf{a}_z \sin \theta_q \\
\mathbf{a}_\phi^q &= -\mathbf{a}_x \sin \phi_q + \mathbf{a}_y \cos \phi_q
\end{aligned} \tag{5-8}$$

To obtain the projection of the total electrical field on the m th array, we carry out a dot product between $\mathbf{E}$ and the orientation vector $\mathbf{a}_m$ in (5-3). The product is shown in (5-9). It gives the amplitude modulation of the source signal projected on the array elements in the m th ULA.

$$\begin{aligned}
E_m^q &= \mathbf{E} \cdot \mathbf{a}_m \\
&= \cos \theta_q \cos(\phi_m - \phi_q) E_\theta^q + \sin(\phi_m - \phi_q) E_\phi^q
\end{aligned} \tag{5-9}$$

Consider Q sources with Q different frequencies. Assume the q th source signal having a frequency f_q and its signal expression is

$$x_q(t) = \exp(j2\pi f_q t) \tag{5-10}$$

In this case, the wave number k of the source signal is equal to $2\pi/\lambda_q = 2\pi f_q/c$. Then the steering vector of the m th 2-element ULA excited by the q th source can be written as

$$\begin{aligned}
\mathbf{e}_m(\theta_q, \phi_q) &= \begin{bmatrix} \exp(jk\mathbf{r}_m^1 \cdot \mathbf{a}_r^q) \\ \exp(jk\mathbf{r}_m^2 \cdot \mathbf{a}_r^q) \end{bmatrix} \\
&= \begin{bmatrix} \exp(-jkR_0 \sin \theta_q \cos(\phi_q - \phi_m)) \\ \exp(jkR_0 \sin \theta_q \cos(\phi_q - \phi_m)) \end{bmatrix}
\end{aligned} \tag{5-11}$$

where $\mathbf{r}_m^1$ and $\mathbf{r}_m^2$ are the position vectors of the respective elements in the m th ULA numbered as 1 and 2. Without loss of generality, we consider the element with smaller index as the first element in the ULA. Then the ULA array output caused by the q th source after sampling is expressed in (5-12).

$$\mathbf{y}_m^q = \mathbf{I}E_m^q \mathbf{e}_m(\theta_q, \phi_q) \mathbf{x}_q + \mathbf{n}_m^q \tag{5-12}$$

where $\mathbf{x}_q$ is the q th signal vector of length $1 \times N_s$ (N_s is the number of snapshots), $\mathbf{I}$ is a 2×2 identity matrix, $\mathbf{y}_m^q$ is the $2 \times N_s$ array output signal vector induced by the q th source and $\mathbf{n}_m^q$ is the respective noise vector at the m th ULA. An additive white Gaussian noise with zero mean and variance σ^2 is assumed and is independent with the sources. Besides, all the sources are also independent with each other. Therefore, the array output of the m th ULA is a linear summation of outputs from all the source signals. Expressed in matrix, the array output has the form in (5-13).

$$\mathbf{Y}_m = \mathbf{A}_m \mathbf{S}_m(\theta_q, \phi_q) \mathbf{X} + \mathbf{N}_m \tag{5-13}$$

where

$$\begin{aligned}
\mathbf{A}_m &= [\mathbf{I}E_m^1 \ \mathbf{I}E_m^2 \ \cdots \ \mathbf{I}E_m^q \ \cdots \ \mathbf{I}E_m^Q] \\
\mathbf{S}_m(\theta_q, \phi_q) &= \begin{bmatrix} \mathbf{e}_m^1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \mathbf{e}_m^2 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & & & \vdots \\ 0 & 0 & 0 & \cdots & \mathbf{e}_m^q & \cdots & 0 \\ \vdots & \vdots & & \ddots & & & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & & \mathbf{e}_m^Q \end{bmatrix} \\
\mathbf{X} &= [\mathbf{x}_1 \ \mathbf{x}_2 \ \cdots \ \mathbf{x}_q \ \cdots \ \mathbf{x}_Q]^T
\end{aligned}$$

In the above expressions, $\mathbf{e}_m(\theta_q, \phi_q)$ is denoted using $\mathbf{e}_m^q$ for short, ignoring the bracketed terms. $\mathbf{Y}_m$ and $\mathbf{N}_m$ are the respective array output matrix and noise matrix of size $2 \times N_s$ of the m th ULA.

Inspired by the fact that frequency filtering can be applied to the received signals at the receiver side to separate narrowband source signals with different frequencies, we implement a frequency filtering before the estimation process to extract the correct signals of different sources. Fast Fourier Transform (FFT) is performed to obtain a frequency domain received signal first followed by a windowing process before parameter estimation. A number of samples, N_t , is picked out from each frequency peak as well as adjacent regions of the peak by the windowing process. After that, a $1 \times N_t$ truncated version of $\mathbf{y}_m^q$ can be formed by the selected samples, denoting as $\tilde{\mathbf{y}}_m^q$ in (5-14).

$$\tilde{\mathbf{y}}_m^q = \mathbf{I}E_m^q \mathbf{e}_m(\theta_q, \phi_q) \tilde{\mathbf{x}}_q + \tilde{\mathbf{n}}_m^q \quad (5-14)$$

where $\tilde{\mathbf{x}}_q$ represents the truncated source signal of length N_t and $\tilde{\mathbf{n}}_m^q$ is the respective truncated noise vector. The respective truncated signal can be used for estimating DOA and polarization parameters of the corresponding source.

5.2.2 DOA Estimation Algorithm

For our DOA estimation approach based on a VCA, most traditional super-resolution estimation methods can be used, such as Capon [24, 25, 114], MUSIC [26, 27], maximum likelihood (ML) [27, 30, 31, 115], and the derivations [32]. We simply consider MUSIC, the most popular DOA estimation method, in our approach.

We can obtain a covariance matrix of the truncated output signal of the m th ULA induced by the q th source. The covariance matrix is shown in (5-15).

$$\begin{aligned}
R_{yy} &= \tilde{\mathbf{y}}_m^q \cdot (\tilde{\mathbf{y}}_m^q)^T \\
&= (E_m^q)^2 \left(\mathbf{e}_m(\theta_q, \phi_q) \tilde{\mathbf{x}}_q(t) \tilde{\mathbf{x}}_q^T(t) \mathbf{e}_m^T(\theta_q, \phi_q) + \frac{\tilde{\mathbf{n}}_m(t) \tilde{\mathbf{n}}_m^T(t)}{(E_m^q)^2} \right) \\
&= (E_m^q)^2 R_{y'y'},
\end{aligned} \tag{5-15}$$

where $R_{y'y'}$ is the covariance matrix of the truncated signal with a positive scale on the noise covariance matrix in the case when the source is not polarized and equal power is received on all the ULA elements. In other words, R_{yy} is equal to $R_{y'y'}$ when $(E_m^q)^2 = 1$. However, when polarization effect is considered, a positive amplitude modulation on the covariance matrix of the truncated signal is always found without any phase changes. When carrying out singular value decomposition (SVD) on the covariance matrix, the subspace decomposition will not be changed compared to non-polarized signals. The only term changed is the eigenvalue since it is related to the power of signals and noise. In view of this, we can conclude that DOA estimation using 2-element ULAs is able to give undistorted DOA spectrum by employing conventional MUSIC algorithm.

For the m th 2-element ULA, a MUSIC power spectrum can be constructed using steering vector $\mathbf{e}_m(\Phi)$ of the ULA and the noise subspace $\mathbf{B}_N$ decomposed from the truncated signal covariance matrix R_{yy} . The spectrum is shown in (5-16).

$$P_m(\Phi) = \frac{1}{\mathbf{e}_m^H(\Phi) \mathbf{B}_N \mathbf{B}_N^H \mathbf{e}_m(\Phi)} \tag{5-16}$$

where Φ represents the DOA angle from 0° to 180° with respect to the m th array axis. Since the inter-element spacing of 2-element ULAs is always kept half-wavelength of the source signal, there will be no grating lobes in the DOA estimation. Therefore, only one peak can be found from each power spectrum which corresponds to the DOA of the source signal with respect to the m th ULA axis. In order to distinguish this DOA value from the actual 2D DOA values (θ, ϕ) of the source, we named Φ the “*indirect DOA*” in this chapter.

There will be M indirect DOA values obtained from MUSIC spectrums in total for the q th source. Consider upper-half space of the x - y plane only. Then according to the symmetrical property of a ULA, one DOA value will result in a half-cone ambiguity area above the VCA plane (x - y plane) for a single source. The half-cone is symmetrical with respect to the array axis and centered at the phase center of the 2-element ULA (the origin of the coordinate system). By combining all the M half-cones, a total of C_M^2 number of intersection lines will appear. If the system is noise free, all the half-cones will intersect together forming one and only one intersection line. This line represents the actual source direction in the space. However if noise is present, all the lines may not coincide together. In this case, we are able to determine the source direction by averaging the estimated directions from individual intersection lines. In doing this, we need to investigate the 2-element ULAs pair by pair.

We select two ULAs each time and assign their array axes as u and v axes. Without loss of generality, u is assigned to the ULA axis with smaller element index and v is assigned to the ULA with larger element index than the u -axis. The angle that u -axis lags v -axis is equal to the azimuth angle difference between the first element in them. The direct 2D DOA (θ_q, ϕ_q) is related to the indirect DOA values Φ_u^q and Φ_v^q of the two ULAs as in (5-17).

$$\begin{aligned}\cos \Phi_u^q &= \mathbf{a}_r^q \cdot \mathbf{a}_u = \sin \theta_q \cos(\phi_q - \phi_u) \\ \cos \Phi_v^q &= \mathbf{a}_r^q \cdot \mathbf{a}_v = \sin \theta_q \cos(\phi_q - \phi_v)\end{aligned}\tag{5-17}$$

where Φ_u^q and Φ_v^q are the indirect DOAs with respect to the u and v axes. The estimated values of the indirect DOAs, $(\hat{\Phi}_u^q, \hat{\Phi}_v^q)$, can be obtained from the MUSIC spectrums. Thus there are two unknowns, θ_q and ϕ_q , in the two equations of (5-17). It is possible to work out the unique solutions of (5-17) as listed in (5-18), (5-19) and (5-20).

$$\hat{\theta}_q = \arcsin \sqrt{\left(\cos \hat{\Phi}_u^q\right)^2 + \left(\frac{\cos \hat{\Phi}_v^q - \cos \hat{\Phi}_u^q \cos(\Delta \phi_{uv})}{\sin(\Delta \phi_{uv})}\right)^2}\tag{5-18}$$

$$\hat{\phi}_q = \begin{cases} \phi_u + \arccos(a_1) \in [\phi_u, \phi_u + \pi], & \text{if } a_2 \geq 0 \\ \phi_u - \arccos(a_1) \in [\phi_u - \pi, \phi_u], & \text{if } a_2 < 0 \end{cases} \quad (5-19)$$

where

$$\begin{aligned} a_1 &= \frac{\cos \hat{\Phi}_u^q}{\sqrt{(\cos \hat{\Phi}_u^q)^2 + \left(\frac{\cos \hat{\Phi}_v^q - \cos \hat{\Phi}_u^q \cos(\Delta\phi_{uv})}{\sin(\Delta\phi_{uv})} \right)^2}} \\ a_2 &= \frac{\cos \hat{\Phi}_v^q - \cos \hat{\Phi}_u^q \cos(\Delta\phi_{uv})}{\sin(\Delta\phi_{uv}) \sqrt{(\cos \hat{\Phi}_u^q)^2 + \left(\frac{\cos \hat{\Phi}_v^q - \cos \hat{\Phi}_u^q \cos(\Delta\phi_{uv})}{\sin(\Delta\phi_{uv})} \right)^2}} \\ \Delta\phi_{uv} &= \phi_v - \phi_u \end{aligned} \quad (5-20)$$

where $\Delta\phi_{uv}$ is the azimuth angle difference between the u and v axes. The detailed procedure to solve (5-17) can be found in Appendix A. Denote the total number of combinations of ULA pairs as $P = C_M^2$, the final estimation of 2D source DOA can be obtained by averaging the estimated directions $(\hat{\theta}_q^p, \hat{\phi}_q^p)$ from all the individual combinations as shown in (5-21).

$$\bar{\theta}_q = \frac{1}{C_M^2} \sum_{p=1}^{C_M^2} \hat{\theta}_q^p, \quad \bar{\phi}_q = \frac{1}{C_M^2} \sum_{p=1}^{C_M^2} \hat{\phi}_q^p \quad (5-21)$$

It is expected from (5-21) that the more combinations of ULA pairs, the smaller error of the final estimated directions.

5.2.3 Polarization Estimation Algorithm

Consider two ULAs with u and v axes. Projections of the incident field $\mathbf{E}$ on both array axes can be obtained from the equation (5-9), in which only E_θ^q and E_ϕ^q are the unknown terms. The obtained equations can be written in matrix form in (5-22).

$$\begin{bmatrix} E_u^q \\ E_v^q \end{bmatrix} = \begin{bmatrix} \cos \hat{\theta}_q \cos \hat{\phi}_q & -\sin \hat{\phi}_q \\ \cos \hat{\theta}_q \sin \hat{\phi}_q & \cos \hat{\phi}_q \end{bmatrix} \begin{bmatrix} E_\theta^q \\ E_\phi^q \end{bmatrix} \quad (5-22)$$

For two unknowns in two equations, we can estimate the unknowns easily with unique solutions. By solving (5-22), the estimation of (E_θ^q, E_ϕ^q) can be obtained along with the estimated polarization angle γ_q as shown in (5-23) and (5-24).

$$\begin{bmatrix} \hat{E}_\theta^q \\ \hat{E}_\phi^q \end{bmatrix} = \begin{bmatrix} \cos \hat{\theta}_q \cos \hat{\phi}_q & -\sin \hat{\phi}_q \\ \cos \hat{\theta}_q \sin \hat{\phi}_q & \cos \hat{\phi}_q \end{bmatrix}^{-1} \begin{bmatrix} E_u^q \\ E_v^q \end{bmatrix} \quad (5-23)$$

$$\hat{\gamma}_q = \arctan \left(\frac{\hat{E}_\phi^q}{\hat{E}_\theta^q} \right) \quad (5-24)$$

Therefore similarly as Section 5.2.2 for DOA estimation, the estimated polarization angles of all the individual combinations can be averaged to give the final solution of γ_q as shown in (5-25).

$$\bar{\gamma}_q = \frac{1}{C_M^2} \sum_{p=1}^{C_M^2} \hat{\gamma}_q^p \quad (5-25)$$

where C_M^2 denotes the total number of combinations of ULA pairs or the total number of estimated polarization angles.

5.3 Numerical Simulations

Several numerical simulations are carried out to verify the performance of our proposed approach using the VCA. Multiple source estimations are considered in the simulations.

Two linearly polarized narrowband sources ($Q = 2$) are considered impinging on a VCA of 8 elements. Therefore, N is equal to 8 and there are four 2-element ULAs ($M = 4$). The source signals are plane waves with different frequencies: $f_1 = 1.2$ GHz and $f_2 = 1.3$ GHz. Values of their DOA and polarization directions are $(\theta_1, \phi_1, \gamma_1) = (60^\circ, 45^\circ, 10^\circ)$ and $(\theta_2, \phi_2, \gamma_2) = (30^\circ, 180^\circ, 45^\circ)$. In order to prevent any ambiguous indirect DOA estimations for all the sources, the inter-element spacing of the 2-element ULAs should be kept no larger than half-wavelength of the signal with

maximum possible frequency in range (f_2 in this example). Therefore, we set the inter-element spacing to half of the minimum wavelength among all the sources.

5.3.1 Frequency Filtering

Frequency filtering is carried out in the frequency domain. Fig. 5.5 shows the frequency domain array output signal powers of two ULAs with u and v axes. Since output signals at two elements of one ULA produce the same power distribution, the signal power at one element in the ULA is enough for us to implement frequency filtering.

Fig. 5.5 Array output signal powers and threshold level at u and v axes (u is axis 1 and v is axis 4 in this example): two peaks correspond to two source frequencies.

When a source is present, the power spectrum of the output signal will give a peak with high power at the corresponding frequency. At other frequencies, the powers will be relatively low because most of the powers come from the noise. In order to filter out correct samples of different signals, a more general method is taken to separate peaks with high powers from that with low powers. We calculate the mean power of all the output signals as well as the mean noise power across the entire array and pick out sample indexes with powers higher than a threshold level

from the mean signal power. The threshold power is around 30 dB above the mean noise power. Truncated signal of a ULA can be obtained by selecting samples with the filtered indexes from the individual ULA signal.

5.3.2 Parameter Estimations

Indirect DOA Φ and direct 2D DOA angles (θ, ϕ) estimations are shown in following figures. Frequencies of the two source signals are also estimated along with the indirect DOA values. Fig. 5.6 shows MUSIC power spectrums of both sources at the four ULA pairs.

Fig. 5.6 MUSIC spectrums of two sources at the m th ULA (a) $m = 1$, (b) $m = 2$, (c) $m = 3$, and (d) $m = 4$ for indirect DOA (Φ_m^1, Φ_m^2) estimation (estimated values are labeled using vertical dotted lines).

Fig. 5.7(a) and (c) show the real directions of the two sources with respect to the VCA in the Cartesian coordinate system and Fig. 5.7(b) and (d) show the intersections of all the half-cones of the two sources generated from the four 2-

element ULAs. It can be found that the intersection lines of any two half-cones nearly converge to a single line in the space. This line represents the estimated source direction. We can observe clearly that source 1 is coming from the first quadrant of the x - y plane with relatively low grazing angle and source 2 is from the $-x$ direction with relatively high grazing angle.

(a)

(b)

(c)

(d)

Fig. 5.7 (a) Array geometry of the 8-element VCA and direction of source 1, (b) intersection of all half-cones of source 1, (c) array geometry of the 8-element VCA and direction of source 2, (d) intersection of all half-cones of source 2 (black dots represent antenna elements and black solid lines represent respective source directions in (a) and (c)).

Polarization angle of each source can then be estimated using the above estimated elevation and azimuth angles $(\hat{\theta}, \hat{\phi})$ according to (5-25). We list the estimated polarization angles as well as their estimation errors of the two sources using different ULA combinations in Table 5.1 at SNR = 30 dB. In this case, the estimation errors of both sources are less than one degree which is very small.

Table 5.1: Polarization Angle Estimation at SNR = 30 dB

Combination (u, v)	Source 1 ($\gamma = 10^\circ$)		Source 2 ($\gamma = 45^\circ$)	
	estimate ($^\circ$)	error ($^\circ$)	estimate ($^\circ$)	error ($^\circ$)
(1, 2)	9.87	-0.19	44.66	-0.34
(1, 3)	10.04	0.04	44.96	-0.04
(1, 4)	9.73	-0.27	43.27	-1.73
(2, 3)	10.03	0.03	44.85	-0.15
(2, 4)	9.82	-0.18	44.10	-0.90
(3, 4)	10.09	0.09	45.36	0.36

5.3.3 Error Analysis

An error analysis is performed in order to better understand the performance of the proposed approach using the VCA. We carry out $L = 100$ trials for the estimation of each parameter. Fig. 5.8 shows the values of the estimated elevation (θ) and azimuth angles (ϕ) in all the 100 trials for source 1 at different SNR values. The mean values of both θ and ϕ are also calculated and plotted in the figure. Besides, the actual source angles are labeled using dotted lines. Fig. 5.9 shows the individual estimations as well as the mean estimation for source 2. We can find that the individual estimations have a large spread around the actual angle at low SNR values for both sources. As SNR increases, the spreading is narrowed. However, the mean value of the estimations does not vary much from low SNR values to high SNR values and the error between the mean value and the actual direction is less than one degree for all the cases.

Fig. 5.8 θ and ϕ estimation of source 1 over 100 trials at SNR equals to (a) 0 dB, (b) 5 dB, (c) 10 dB, (d) 15 dB (small gray dots are the individual estimations in different runs and large black circles are the mean estimations).

Fig. 5.9 θ and ϕ estimation of source 2 over 100 trials at SNR equals to (a) 0dB, (b) 5 dB, (c) 10 dB, (d) 15 dB (small gray dots are the individual estimations in different runs and large black circles are the mean estimations).

We now list the mean errors of all the estimated parameters, including indirect DOA (Φ), elevation (θ), azimuth (ϕ) and polarization (γ) angles, to give a clear view of the VCA performance at different SNR values. Take source 2 for example, the mean errors are all shown in Table 5.2. The mean errors in all the cases are all within one degree.

Table 5.2: Mean Errors of All Parameter Estimations of Source Two
($\theta_2, \phi_2, \gamma_2$) = (30°, 180°, 45°) at Different SNR Values

SNR (dB)	Parameters			
	Indirect DOA (°)	θ (°)	ϕ (°)	γ (°)
0	0.9720	-0.1264	0.5998	-0.4611
10	0.7960	-0.0984	0.4618	-0.3924
20	0.8420	-0.1100	0.4937	-0.4348
30	0.9030	-0.1359	0.5336	-0.4617

Root mean square error (RMSE) of a parameter ω is defined in (5-26),

$$\text{RMSE} = \sqrt{\frac{1}{L} \sum_{l=1}^L (\hat{\omega}_l - \omega)^2} \quad (5-26)$$

where $\hat{\omega}_l$ can be the indirect DOA, direct $\bar{\theta}$, $\bar{\phi}$, and $\bar{\gamma}$ in our simulations. RMS errors of indirect DOA, elevation, azimuth and polarization angles when single source is present are shown in Fig. 5.10. These sources have exactly the same azimuth angle ($\phi = 180^\circ$) and polarization angle ($\gamma = 45^\circ$), but different elevation angles (θ) ranging from low grazing angle to high grazing angle. When SNR increases, RMS error decreases for all the parameters. At all SNR values, RMS errors are very small for sources with high grazing angles, but relatively large values for sources with low grazing angles. Especially for very low grazing angle ($\theta = 85^\circ$), the estimation of polarization angle fails, which is consistent with the statement of our VCA in Section 5.1.2.

Fig. 5.10 RMS errors of (a) indirect DOA, (b) θ , (c) ϕ and (d) γ of sources with different elevation angles at different SNR values ranging from 0 dB to 30 dB.

Fig. 5.11 shows RMS error variations of the direct 2D DOA and polarization angles when θ is fixed at 15° or 60° and both ϕ and γ are varying. As the focus is on the estimation sensitivity to the signal directions, here we assume a high quality signal of SNR = 20 dB for error analysis shown in both Fig. 5.11 and Fig. 5.12. Fig. 5.12 shows RMS error variations when ϕ is fixed at 90° or 180° and both θ and γ are varying. The varying ranges of θ , ϕ , and γ are from 5° to 85° , 0° to 360° , and 0° to 90° respectively. Since the proposed VCA is mainly for sources with relatively high grazing angles, we here only investigate sources with elevation angles smaller than 85° . For 0° or very small elevation angles, the sources come from the main lobe of the antenna elements, thus the DOA estimation of these sources is more robust. Therefore, we here only consider error variations for elevation angle starting from 5° . The estimation error of ϕ when θ is fixed at a small elevation angle in Fig. 5.11(b) is larger than that when θ is fixed at a large elevation angle in Fig. 5.11 (e).

Fig. 5.11 RMS errors of θ , ϕ and γ when θ is fixed, ϕ varies from 0° to 360° and γ varies from 0° to 90° : $\theta = 15^\circ$ in (a) – (c) and $\theta = 60^\circ$ in (d) – (f).

Fig. 5.12 RMS errors of θ , ϕ and γ when ϕ is fixed, θ varies from 5° to 85° and γ varies from 0° to 90° : $\phi = 90^\circ$ in (a) – (c) and $\phi = 180^\circ$ in (d) – (f).

This is due to the fact that when signals come from high grazing angles, the projections of the electrical field in the x - y plane are smaller than the projections when signals come from low grazing angles. Then the polarization mismatches between the projections and the antenna elements in the former case become larger than the mismatches in the latter case. Noise tolerant ability of the former case is thus lower than that of the latter case. From Fig. 5.11(c) and Fig. 5.11(f), we can see that when the signal comes from different elevation angles, the RMS error of the polarization angle has different dependences on azimuth angles, but the variations are very small for both cases. According to our proposed approach using the VCA, estimation error of polarization angle depends on the estimation errors of both elevation angle and azimuth angle. This is the reason why the dependences on azimuth angles are different in the two cases. From Fig. 5.12, we can see that when ϕ is fixed, all the RMS errors are very small and the variations of RMS error for all parameters with different elevation angles are almost the same. This result reveals that the dependence of polarization angle estimation on azimuth angle is larger than the dependence of polarization angle estimation on elevation angle. From Fig. 5.12(b)-(c) and Fig. 5.12(e)-(f), we can see that for both ϕ and γ , when elevation angle is very small, the estimation error is slightly larger than that of large elevation angles. This may also be caused by the difference in polarization mismatches between the signal projection on the x - y plane and the antenna elements. Polarization estimation errors are very small for elevation angles up to 75° and become larger after that except for $\gamma = 0^\circ$ and 90° ; in fact, our simulation studies reveal that the RMS errors are less than one degree when elevation angle is less than 75° . When elevation angle is larger than 75° , RMS errors increase for both θ and γ . The phenomenon is possibly caused by the variation of the antenna element radiation pattern at different elevation angles. The antenna radiation pattern in the VCA is pointing towards the 0° elevation angle which has the largest antenna gain, and the antenna gain degrades when elevation angle increases to a larger value. Therefore, when elevation angle is larger than 75° , the antenna gain may be very small for the received signals, which leads to a large RMS error. The RMS error of γ has different variations for different polarization angles. It shows that when γ is 0° or 90° , the error is quite small but very large otherwise. This is because of that at these two critical polarization angles (0° or 90°) and relatively large elevation angles (85°), the

projections of the electrical field on the x - y plane is relatively large and there are at least one or two 2-element ULAs experiencing very small polarization mismatch with the projections. Thus estimation error becomes small at these two angles. In fact, when elevation angle is greater than 85° and near to 90° , the approach may not be robust enough and the RMS error will be very large when γ is 0° , and relatively smaller when γ is 90° due to the small polarization mismatch between signals and antenna elements.

In general, antenna radiation properties and polarization properties of both source signals and antenna elements have non-ignorable influences on the performance of the proposed approach based on the VCA. However, all the RMS errors are quite flat and small in different situations; thus the DOA and polarization estimations should be robust and accurate in all relatively high grazing angle directions (elevation angle up to 75°).

Fig. 5.13 RMS errors of (a) indirect DOA, (b) θ , (c) ϕ and (d) γ at different SNR values from 0 dB to 30 dB when different number of sources is present.

To further investigate the performance of the approach using the VCA, we compared the RMS errors when different number of sources is present. Fig. 5.13 shows the comparison result when number of sources varying from one to seven. It is found that when number of sources increases, the estimation error increases at different SNR values. The core reason may be mainly due to the errors in the estimation of covariance matrix of the array output signal. The ideal covariance matrix can be achieved when the signal snapshots have infinite length and the decomposed noise subspace as well as the estimated DOA values from MUSIC spectrums will be the most accurate ones. However, in practice, infinite snapshot length cannot be achieved, thus estimation errors will increase. For our approach, the more sources are present, the shorter the length of the truncated signals will be. Since the truncated signals are used for covariance matrix estimation of each 2-element ULA in our approach, error in both covariance matrix and decomposed noise subspace will increase and thus errors in DOA estimation will also increase. This is a limitation of our approach. However, the estimation error is still not very large for all the cases.

5.4 DOA and Polarization Estimations in a Colored Gaussian Noise

Since colored noise is mostly observed in practical antenna arrays, it is necessary to investigate the performance of our VCA in colored Gaussian noise.

5.4.1 Estimation Algorithm

It has been demonstrated that fourth order cumulants (FOC) has the ability of noise suppression and array aperture extension. It is asymptotically blind to Gaussian noise. Therefore, DOA estimation in colored Gaussian noise employing FOC has been gaining great interest since 1990s', such as FOC-MUSIC [116, 117]. A more generalized method, the $2q$ -MUSIC method [118], was proposed later to give general performance study of such algorithms and recently another method, MFOC-MUSIC [48], is proposed for less computational complexity and higher computation speed.

For estimating single far-field narrowband plane wave source using an N -element ULA, the received snapshots can be written in a matrix form as in (5-27).

$$\mathbf{X} = \mathbf{e}(\Phi)\mathbf{s} + \mathbf{W} \quad (5-27)$$

where $\mathbf{X} = [x_1, x_2, \dots, x_N]^T$ is of size $N \times N_s$, x_n is the signal vector of size $N_s \times 1$ received at the n th element, $\mathbf{e}(\Phi)$ is the steering vector of size $N \times 1$, Φ is the indirect DOA angle, $\mathbf{s}$ is the source signal of length $1 \times N_s$, and $\mathbf{W} = [w_1, w_2, \dots, w_N]^T$ is the noise matrix of size $N \times N_s$.

For zero-mean complex stationary measurements, the definition of FOC can be expressed by (5-28) according to [117, 119].

$$\begin{aligned} cum\{x_i, x_j^*, x_k, x_l^*\} &= E\{x_i x_j^* x_k x_l^*\} \\ &\quad - E\{x_i x_j^*\} E\{x_k x_l^*\} \\ &\quad - E\{x_i x_k\} E\{x_j^* x_l^*\} \\ &\quad - E\{x_i x_l^*\} E\{x_j^* x_k\} \end{aligned} \quad (5-28)$$

where $(i, j, k, l) \in [1, 2, \dots, N]$ and $()^*$ denotes the complex conjugate. Each “ x ” represents snapshot received at one of the array elements. A matrix $\mathbf{C}_{4,x}$ of size $N^2 \times N^2$ [118] with the cumulants in (5-28) as the $[(i-1)N+j, (k-1)N+l]$ th element can be obtained in (5-29) instead of the $N \times N$ covariance matrix in MUSIC.

$$\mathbf{C}_{4,x} = \mathbf{b}(\Phi)\mathbf{C}_{4,s}\mathbf{b}(\Phi)^H \quad (5-29)$$

where $\mathbf{b}(\Phi) = \mathbf{e}(\Phi) \otimes \mathbf{e}(\Phi)$ is the virtual steering vector ($N^2 \times 1$) of the extended array aperture, and $\mathbf{C}_{4,s}$ represents the corresponding FOC matrix of source signals. The matrix $\mathbf{C}_{4,x}$ can also be eigen-decomposed into signal subspace $\mathbf{B}_s$ of size $N^2 \times 1$ and noise subspace $\mathbf{B}_N$ of size $N^2 \times (N^2 - 1)$ as the matrix in MUSIC. The signal subspace coincides with the virtual steering vector $\mathbf{b}(\Phi)$. Therefore, the power spectrum of MFOC-MUSIC can be written as

$$P_{MFOC-MUSIC}(\Phi) = \frac{1}{\mathbf{b}(\Phi)^H \mathbf{B}_N \mathbf{B}_N^H \mathbf{b}(\Phi)} \quad (5-30)$$

where Φ is the indirect DOA value varying from 0° to 180° and $()^H$ represents the complex conjugate transpose.

In our VCA, the FOC is calculated for each 2-element ULA and the size of matrix $\mathbf{C}_{4,x}$ becomes 4×4 .

5.4.2 Simulation Examples

We carry out a simulation to verify the performance of a VCA in source estimation in colored Gaussian noise using MFOC-MUSIC method and compare the results with that in white Gaussian noise using MUSIC. Both DOA and polarization estimations are considered.

The total antenna element in the VCA is selected as 4 just for simple verification in this section, which means the VCA is a 4-element VCA and total two 2-element ULAs are used. First element locates on the x -axis. A single narrowband plane wave source coming from directions $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 45^\circ)$ is assumed in the simulation. Both white and colored Gaussian noises are assumed independent with the source signal. The number of snapshots is set to be 2000 for estimating both matrices in MUSIC and MFOC-MUSIC. In total, 100 runs are carried out to obtain all the results.

Fig. 5.14 shows a comparison of the estimated directions at all the runs using MUSIC and MFOC-MUSIC in colored noise. Mean value of all the runs is indicated in each sub-figure. It shows that at low SNR values, the VCA can perform quite well in colored Gaussian noise by employing MFOC-MUSIC.

(a) SNR = 10 dB

(b) SNR = 0 dB

(c) SNR = -10 dB

Fig. 5.14 Individual (gray dots) and average (black circle) values of the 100 DOA estimations for signals with color noise at different SNR levels by conventional MUSIC and the adopted MFOC-MUSIC method.

Fig. 5.15 RMS errors of (a) indirect DOA, (b) θ , (c) ϕ and (d) γ at different SNR values from -10 dB to 20 dB in both white and colored Gaussian noise using MUSIC and MFOC-MUSIC.

Fig. 5.15 shows the error analysis of the four source parameters, indirect DOA, elevation angle, azimuth angle and polarization angle. MUSIC and MFOC-MUSIC in both with and colored Gaussian noise are compared at different SNR values varying from -10 dB to 20 dB. From the error analysis, we can tell that MUSIC and MFOC-MUSIC have similar performance in both white and colored Gaussian noise at high SNR values. However as the SNR value decreases, their performances diverge. At this condition, MFOC-MUSIC is able to give much better performance in colored Gaussian noise than its performance in white Gaussian noise as well as the MUSIC performance in both white and colored noise.

5.5 Discussions

5.5.1 Advantage

The proposed DOA and polarization estimation approach using a VCA is simple and intuitive. It can achieve good performance in estimating sources with both white and colored Gaussian noise by simply employing MUSIC/MUSIC-like method. The approach makes use of two or more 2-element ULAs to derive the direct elevation, azimuth and polarization angles of multiple sources with relatively high grazing angles and arbitrary linear polarizations. Estimation error analysis shows that the proposed approach is accurate, robust, and generally speaking, insensitive to signal directions.

Compared with the classical polar-MUSIC [33, 34] method, our approach based on the VCA is a completely different approach. In fact, it is relatively more robust when different polarization mismatches occur between signals and antenna elements. Besides, it can work well even when elements are not uniform, as long as there are a group of 2-element ULAs. In practical implementations, a 2-element ULA can be grouped by selecting the received data from the two respective channels at the receiver side and processing together as a complete array data set. The rest 2-element ULAs can also be grouped through selecting respective data sets from the receiver channels. By selecting the correct data sets from the receiver side, the signals can be separately correctly. This way also enables the possible implementation of non-uniform VCA.

5.5.2 Limitation

Of course, the approach based on a VCA also has some limitations in estimating elliptically polarized signals. It cannot completely estimate the tilt angles and senses of elliptical polarizations. Besides, in some critical low grazing angles, our VCA is not able to give very good performance due to large polarization losses between signals and antenna elements. Moreover, the approach is not a general solution of DOA and polarization estimation using a VCA as a complete array. Therefore, further work, such as derivation of algorithms when considering all the 2-element ULAs as a whole or when dividing the VCA into other sub-arrays instead of ULAs,

needs to be done to investigate the VCA as a complete array or sub-complete array, which is more practical.

5.6 Concluding Remarks

In Chapter 5, we have presented a VCA for multiple sources DOA and polarization estimations with a simple and intuitive approach. Since conventional SCA can only estimate sources with low-grazing angles and vertical polarization, the VCA is proposed for estimating signals with relatively high grazing angles and arbitrary polarization angles in most cases. High grazing angle and arbitrary polarization angle estimations are more desirable in some applications (for example airborne applications) than a conventional SCA. By investigating the performance of a VCA in estimating sources with different directions through extensive simulations and error analysis, it is found that the VCA can perform quite well with very small estimation errors in estimating sources with high grazing angle and especially arbitrary linear polarization angles at different SNR values. Besides, the VCA also performs well in colored Gaussian noise by employing MUSIC-like algorithm (MFOC-MUSIC) for DOA estimations. However, the VCA has difficulties in estimating sources with elliptical polarization since the phase shifts of different 2-element ULAs depends on the polarization information and cannot be compensated in the estimation process. When estimating multiple sources, the proposed approach is rational for sources with different frequencies. Thus approaches that can estimate correlated sources using the VCA need to be investigated in the future.

CHAPTER 6

DIRECTION OF ARRIVAL AMBIGUITY

RESOLUTION OF ANTENNA ARRAY

In the preceding chapter, we have introduced a VCA with its inter-element spacing properly set at half-wavelength of a source signal. The only ambiguity problem in this case leads to infinite ambiguity directions forming a half-cone across the 2-element ULA in the upper-half space of the x - y plane. The half-cone is a possible area where the real source DOA is located in. Such ambiguity problem can be resolved by intersecting at least two different half-cones. However, if inter-element spacing is increased to a value larger than half-wavelength of the source signal, grating lobes might appear in the DOA spectrum obtained by subspace-based estimation methods (e.g. MUSIC). Without resolving the ambiguity problem in the 2-element ULA, the proposed VCA will completely fail in estimating source directions and polarizations. In viewing that 2D antenna array has good ambiguity resolving ability than 1D array (for example 2D circular array does not suffer ambiguity problems of 1D linear array in the upper-half space of the x - y plane) and to minimize system cost without adding new antenna elements, we bring the subset of the VCA, namely the vector arc array (VAA), into our consideration. A VAA is expected to provide an unambiguous DOA estimation with relatively low resolution while the 2-element ULA can provide an ambiguous DOA estimation with relatively high resolution. By combining these two estimations, an unambiguous estimation with high resolution is expected to be obtained. Therefore in this chapter, a composite multiple-resolution (CMR) approach by using the estimation results of a VAA to resolve the ambiguity problems in the estimation results of 2-element ULAs is proposed. It can resolve the ambiguity problem caused by large inter-element spacing in the VCA and successively achieve wideband DOA estimation.

In Section 6.1, we introduce the DOA ambiguity problem in a 2-element ULA. In Section 6.2, the theory of a VCA as a complete array is presented and the VAA is introduced. In Section 6.3, the proposed CMR approach is given in detail. In Section

6.4, respective numerical examples are carried out to verify the proposed approach. In Section 6.5, the advantage and limitation of the proposed approach are discussed. Section 6.6 concludes this chapter. A flow chart in shows more clearly the above progression of this chapter.

Fig. 6.1 Flow chart of the progression in Chapter 6.

6.1 Ambiguity Problem in a Uniform Linear Array (ULA)

Consider a simple 2-element ULA with inter-element spacing d for DOA estimation as shown in Fig. 6.2. Assume that a single narrowband far-field plane wave source with a frequency f and a wavelength λ impinges on the array.

Fig. 6.2 Illustration of phase difference between adjacent elements of a ULA.

As illustrated in Fig. 6.2, phase difference caused by the signal path difference between the two adjacent elements can be expressed by (6-1).

$$\Delta\psi = \frac{2\pi}{\lambda} \Delta R = \frac{2\pi}{\lambda} d \cos \Phi \quad (6-1)$$

where $\Phi \in [0^\circ, 180^\circ]$ represents source DOA angle with respect to array axis (the x -axis in Fig. 6.2). The Φ used here is to be consistent with the notation of the indirect DOA in Chapter 5. For a test DOA Φ_a , the ULA gives a phase difference at the two adjacent elements, denoting as $\Delta\psi_a$. Difference between the phase difference of the test direction Φ_a and the phase difference of a real DOA Φ can be denoted in (6-2).

$$\Delta\beta = \Delta\psi_a - \Delta\psi = \frac{2\pi}{\lambda} d (\cos \Phi_a - \cos \Phi) \quad (6-2)$$

According to [96, 120], when d is greater than half-wavelength of the source signal, DOA ambiguity occurs if $\Delta\beta$ is integer multiple of 2π which is expressed in (6-3).

$$\frac{2\pi}{\lambda} d (\cos \Phi_a - \cos \Phi) = \pm 2n\pi \quad (6-3)$$

where n is any positive integer starting from 1. With this relation, the array steering vector corresponding to direction Φ_a can be written as a scalar multiple of the steering vector corresponding to the real DOA Φ . The set of Φ_a solved from (6-3) are ambiguous directions and will appear in the DOA spectrum obtained using methods such as beamforming, MUSIC or ESPRIT, as grating lobes.

For example, consider a narrowband far-field plane wave source signal impinging on a 2-element ULA lying along x -axis. The signal forms a DOA angle $\Phi = 120^\circ$ with respect to the ULA. When d is no larger than $\lambda/2$, we can obtain one unambiguous direction in MUSIC spectrum as shown in Fig. 6.3(a). However, when d is larger than $\lambda/2$, ambiguous direction will appear in the MUSIC spectrum. Assume d is equal to one wavelength λ , the ambiguous direction can be calculated according to (6-3). The expression is shown in (6-4).

$$\frac{2\pi}{\lambda} \cdot \lambda (\cos \Phi_a - \cos 120^\circ) = \pm 2n\pi \quad (6-4)$$

Only $n = 1$ and a positive right hand side value can give a valid ambiguous direction $\Phi_a = 60^\circ$. This ambiguous direction can be correctly observed along with the real direction in the MUSIC spectrum in Fig. 6.3(b). When d is increased to a much larger value, there exist many solutions of (6-3) and plenty of ambiguity directions appear in the spectrum. Fig. 6.3(c) and Fig. 6.3(d) show respectively the MUSIC spectrum when d is equal to 5λ and 10λ .

Fig. 6.3 MUSIC spectrum at SNR = 20 dB when (a) $d = \lambda/2$, (b) $d = \lambda$, (c) $d = 5\lambda$, (d) $d = 10\lambda$.

In the proposed DOA estimation approach using a VCA in Chapter 5, MUSIC is applied to each 2-element ULA output to obtain the corresponding indirect DOA estimation. The estimated indirect DOA values are the inputs to the subsequent estimations of source parameters including elevation, azimuth and polarization angles. However, when inter-element spacing is larger than half-wavelength of source signal, ambiguous directions appear in the indirect DOA spectrum. This prevents successful estimation of all the source parameters. Therefore, resolving ambiguity in MUSIC spectrum is greatly required in our VCA.

Due to the fact that it is difficult or relatively impossible to change the array geometry once deployed in practice, it is better for us to investigate an effective ambiguity resolution method based on the existing array configuration of the VCA.

6.2 Vector Circular Array (VCA) as a Complete Array

In Chapter 5, the VCA is divided into 2-element ULAs and DOA estimation approach is developed using these ULAs. It is now necessary to obtain a general description of our VCA as a complete array.

6.2.1 Signal Model

Similarly as in Section 5.2.1, we assume Q far-field narrowband plane wave sources with different frequencies impinge on an N -element VCA. All sources are linearly polarized. The q th source signal with elevation angle θ_q , azimuth angle ϕ_q , and polarization angle γ_q can be represented by a more general expression in (6-5) than that in (5-10).

$$x_q(t) = a_q(t) \exp \left\{ j \left[\omega_q t + \psi_q(t) \right] \right\} \quad (6-5)$$

where $a_q(t)$ is the amplitude coefficient, ω_q is the q th source angular frequency in radians, $\psi_q(t)$ is the time-varying phase of the q th source signal. The electrical field $\mathbf{E}$ of the q th source can be written as

$$\mathbf{E}_q = \mathbf{a}_{\theta q} E_{\theta q} + \mathbf{a}_{\phi q} E_{\phi q} \quad (6-6)$$

where

$$\begin{aligned} E_{\theta q} &= x_q(t) \cos \gamma_q \\ E_{\phi q} &= x_q(t) \sin \gamma_q \\ \mathbf{a}_{\theta q} &= \mathbf{a}_x \cos \theta_q \cos \phi_q + \mathbf{a}_y \cos \theta_q \sin \phi_q - \mathbf{a}_z \sin \theta_q \\ \mathbf{a}_{\phi q} &= -\mathbf{a}_x \sin \phi_q + \mathbf{a}_y \cos \phi_q \end{aligned}$$

We now consider antenna elements in the VCA separately instead of pairing them. Then, the unit polarization direction vector of the n th antenna element

coincides with its unit position vector and can be expressed in terms of unit vectors of the Cartesian coordinate system by (6-7).

$$\mathbf{a}_n = \mathbf{a}_x \cos \phi_n + \mathbf{a}_y \sin \phi_n \quad (6-7)$$

where ϕ_n is the azimuth angle of the n th element with respect to the x -axis. It can be expressed in (6-8).

$$\phi_n = \frac{2\pi}{N}(n-1), \quad n=1, 2, \dots, N \quad (6-8)$$

Denote radius of the circular array as R_0 , then the steering vector of the N -element VCA corresponding to the q th source has the form of (6-9).

$$\mathbf{e}(\theta_q, \phi_q) = \left[\left\{ \exp(jk_q R_0 \sin \theta_q \cos(\phi_q - \phi_n)) \right\}_{n=1}^N \right]^T \quad (6-9)$$

where $k_q = 2\pi/\lambda_q$ is the wavenumber of the q th source.

The projection of $\mathbf{E}_q$ on the n th antenna element can be obtained by the dot product of $\mathbf{E}_q$ and $\mathbf{a}_n$ in (6-10).

$$\mathbf{E}_q \cdot \mathbf{a}_n = x_q(t) \left[\cos \theta_q \cos(\phi_n - \phi_q) \cos \gamma_q + \sin(\phi_n - \phi_q) \sin \gamma_q \right] \quad (6-10)$$

Denote signal length as N_s , the m th sample of the array output signal can be expressed using $\mathbf{z}(m)$ in (6-11).

$$\mathbf{z}(m) = \sum_{q=1}^Q \mathbf{A}^q \mathbf{e}(\theta_q, \phi_q) x_q(m) + \mathbf{w}(m) \quad (6-11)$$

where

$$\begin{aligned} \mathbf{A}^q &= \text{diag} \left\{ a_n^q \right\}_{n=1}^N \\ a_n^q &= \cos \theta_q \cos(\phi_q - \phi_n) \cos \gamma_q + \sin(\phi_q - \phi_n) \sin \gamma_q \end{aligned}$$

In (6-11), $\mathbf{z}(m)$ and $\mathbf{w}(m)$ are the m th array output sample and the m th noise sample respectively, and have the same size of $N \times 1$, $x_q(m)$ is the m th discrete sample of the

q th source signal. $\mathbf{A}^q$ contains unit projections of the q th source signal to all the antenna elements. Denote in matrix form, the total array output of size $N \times N_s$ can be expressed by (6-12).

$$\mathbf{Z} = \mathbf{A}\mathbf{S}\mathbf{X} + \mathbf{W} \quad (6-12)$$

where

$$\begin{aligned} \mathbf{A} &= [\mathbf{A}^1 \ \mathbf{A}^2 \ \dots \ \mathbf{A}^q \ \dots \ \mathbf{A}^Q] \\ \mathbf{S} &= \text{diag}\{\mathbf{e}^q\}_{q=1}^Q \\ \mathbf{X} &= [\mathbf{x}_1^T \ \mathbf{x}_2^T \ \dots \ \mathbf{x}_q^T \ \dots \ \mathbf{x}_Q^T]^T \\ \mathbf{W} &= [\mathbf{w}_1^T \ \mathbf{w}_2^T \ \dots \ \mathbf{w}_n^T \ \dots \ \mathbf{w}_N^T]^T \end{aligned}$$

$\mathbf{A}$ is the real-valued projection matrix of size $N \times NQ$, $\mathbf{S}$ is the steering vector matrix of size $NQ \times Q$, $\mathbf{X}$ is the source signal matrix of size $Q \times N_s$, $\mathbf{W}$ is the noise vector matrix of size $N \times N_s$, $\mathbf{e}^q$ is the short form of $\mathbf{e}(\theta_q, \phi_q)$, $\mathbf{x}_q$ is the q th source signal in discrete form of size $1 \times N_s$ and $\mathbf{w}_n$ is the noise vector at the n th antenna element of size $1 \times N_s$.

Assume white Gaussian noise is considered and independence exists among all the signals and noise. It is easy to obtain the covariance matrix of the VCA array output as shown in (6-13).

$$\begin{aligned} \mathbf{R}_z &= E[\mathbf{Z}\mathbf{Z}^H] \\ &= E[(\mathbf{A}\mathbf{S}\mathbf{X} + \mathbf{W})(\mathbf{A}\mathbf{S}\mathbf{X} + \mathbf{W})^H] \\ &= E[\mathbf{A}\mathbf{S}\mathbf{X}\mathbf{X}^H\mathbf{S}^H\mathbf{A}^H] + E[\mathbf{W}\mathbf{W}^H] \\ &= \mathbf{A}\mathbf{R}_z'\mathbf{A}^H + \sigma^2\mathbf{I} \end{aligned} \quad (6-13)$$

where $\mathbf{I}$ is the identity matrix of size $N \times N$ and σ^2 is the noise power. Obviously, $\mathbf{R}_z'$ is associated with the array output covariance matrix when signal is not polarized, we denote the matrix as $\mathbf{R}_z''$. Its expression in terms of $\mathbf{R}_z'$ and singular value decomposition (SVD) can be expressed by (6-14).

$$\begin{aligned}\mathbf{R}_z'' &= \mathbf{\Lambda} \mathbf{R}_z' \mathbf{\Lambda}^H \\ \mathbf{R}_z'' &= \mathbf{U} \mathbf{\Sigma} \mathbf{V}^*\end{aligned}\tag{6-14}$$

where $\mathbf{\Lambda}$ is of size $N \times NQ$ and has the same format as $\mathbf{A}$ but with all the sub-matrix $\mathbf{A}^q$ replaced by an identity matrix of size $N \times N$, $\mathbf{U}$ is a unitary matrix of size $N \times N$ containing eigenvectors of the signal and noise, $\mathbf{\Sigma}$ is a diagonal matrix of size $N \times N$ containing eigenvalues of the signal and noise and $\mathbf{V}$ is another unitary matrix of size $N \times N$.

For a polarized signal, different projections will be generated on antenna elements with different polarization directions. Hence, a_n^q in $\mathbf{A}$ will vary from element to element and so as the signal powers received at different elements. For two elements with 180° azimuth angle difference, two projections with same magnitude but opposite signs will be obtained. Multiplication of $\mathbf{R}_z'$ with $\mathbf{A}$ and $\mathbf{A}^H$ will change both amplitude and phase of the values inside $\mathbf{R}_z'$. Therefore, SVD on $\mathbf{R}_z$ will give different (in both amplitude and phase) eigenvectors and different (in amplitude only) eigenvalues. The subspaces spanned by both signal and noise eigenvectors will be changed and noise subspace will not be orthogonal to the array steering vector at the real signal direction. In observing this phenomenon, it is concluded that conventional subspace-based DOA estimation algorithms, such as MUSIC and ESPRIT, cannot be directly applied on the complete data covariance matrix $\mathbf{R}_z$ of the VCA. The estimated DOA using $\mathbf{R}_z$ will be incorrect.

6.2.2 Vector Arc Array (VAA)

It has been found from Chapter 5 that for any polarized signals, our VCA is able to obtain relatively high power at least on one 2-element ULA, reduced power at the neighboring ULAs and very low power at the other ULAs. Such power difference is caused by different polarization mismatches between signals and ULAs. When the variation of the received signal powers at different elements is very large, the VCA is becoming a non-uniform circular array with elements of high power working properly while elements of low power working improperly. Fig. 6.4 gives an illustration of the situation when two elements in an 8-element VCA experience very large polarization loss. R_0 represents the radius of the VCA. Denote the diameter of

the VCA as D , the inter-element spacing of the 2-element ULA as d , then $D = 2R_0$ and $d = 2R_0$.

Fig. 6.4 Illustration of an 8-element VCA with 2 elements (gray dots) experiencing very large polarization mismatch (R_0 is the radius of the VCA, D is the diameter of the VCA and d is the inter-element spacing of the 2-element ULA).

Inspired by the fact that two elements with 180° phase difference in the VCA will experience same polarization mismatch, we can always consider a VCA to be composed of two arc arrays. For example, each arc array in Fig. 6.4 consists of three elements (connected with black solid arc lines). Since the polarization directions of all the elements inside one arc array are different, we name the arc array as a vector arc array (VAA).

Elements in one VAA are receiving relatively similar signal powers and the projections on elements in the same arc array have the same sign. The received signal of VAA has a similar expression as that of the received signal of the complete VCA. The expression is shown in (6-15).

$$\mathbf{Z}^{arc} = \mathbf{A}^{arc} \mathbf{S}^{arc} \mathbf{X} + \mathbf{W}^{arc} \quad (6-15)$$

where

$$\begin{aligned}
\mathbf{A}^{arc} &= \begin{bmatrix} \mathbf{A}_{arc}^1 & \mathbf{A}_{arc}^2 & \dots & \mathbf{A}_{arc}^q & \dots & \mathbf{A}_{arc}^Q \end{bmatrix} \\
\mathbf{S}^{arc} &= \text{diag} \left\{ \mathbf{e}_{arc}^q \right\}_{q=1}^Q \\
\mathbf{X} &= \begin{bmatrix} \mathbf{x}_1^T & \mathbf{x}_2^T & \dots & \mathbf{x}_q^T & \dots & \mathbf{x}_Q^T \end{bmatrix}^T \\
\mathbf{W}^{arc} &= \begin{bmatrix} \mathbf{w}_{arc1}^T & \mathbf{w}_{arc2}^T & \dots & \mathbf{w}_{arcg}^T & \dots & \mathbf{w}_{arcG}^T \end{bmatrix}^T
\end{aligned}$$

G denotes number of elements in one VAA and $g = 1, 2, \dots, G$ is the arc element index,

$\mathbf{A}_{arc}^q = \text{diag} \left\{ a_{arcg}^q \right\}_{g=1}^G$ is of size $G \times G$, a_{arcg}^q has the same expression as that in (6-11)

but with n replaced by $arcg$, $\mathbf{e}_{arc}^q = \left[\left\{ \exp \left(jk_q R_0 \sin \theta_q \cos (\phi_q - \phi_{arcg}) \right) \right\}_{g=1}^G \right]^T$, $\mathbf{S}^{arc}$ is of

size $GQ \times Q$, and $\mathbf{W}^{arc}$ is of size $G \times N_s$. The covariance matrix of the VAA data is similar to that in (6-13) and can be obtained in (6-16),

$$\begin{aligned}
\mathbf{R}_z^{arc} &= E \left[\mathbf{Z}^{arc} (\mathbf{Z}^{arc})^H \right] \\
&= E \left[\mathbf{A}^{arc} \mathbf{S}^{arc} \mathbf{X} \mathbf{X}^H (\mathbf{S}^{arc})^H (\mathbf{A}^{arc})^H \right] + E \left[\mathbf{W}^{arc} (\mathbf{W}^{arc})^H \right] \quad (6-16) \\
&= \mathbf{A}^{arc} \mathbf{R}_z^{arc'} (\mathbf{A}^{arc})^H + \sigma^2 \mathbf{I}^{arc}
\end{aligned}$$

where $\mathbf{R}_z^{arc'}$ is a portion of $\mathbf{R}_z'$ in (6-13) and has a size of $GQ \times GQ$ and $\mathbf{I}^{arc}$ is the identity matrix of size $G \times G$. In this case, matrix $\mathbf{A}^{arc}$ contains projections with the same sign, which brings only power changes to $\mathbf{R}_z^{arc'}$. Singular value decomposition of $\mathbf{R}_z^{arc}$ in (6-17) will result in same eigenvectors as but different eigenvalues from those decomposed from the covariance matrix associated with $\mathbf{R}_z^{arc'}$ when signal is not polarized, we denote the matrix as $\mathbf{R}_z^{arc''}$.

$$\mathbf{R}_z^{arc} = \mathbf{U}^{arc} \mathbf{\Sigma}^{arc} (\mathbf{V}^{arc})^* \quad (6-17)$$

where $\mathbf{U}^{arc}$ is a unitary matrix of size $G \times G$ containing eigenvectors of the signal and noise in a G -element VAA, $\mathbf{\Sigma}^{arc}$ is a diagonal matrix of size $G \times G$ containing eigenvalues of the signal and noise and $\mathbf{V}^{arc}$ is another unitary matrix of size $G \times G$.

The signal and noise subspaces obtained from (6-17) are the same as that spanned by the eigenvectors decomposed from $\mathbf{R}_z^{arc}$. Thus the noise subspace is orthogonal to the arc array steering vectors. Conventional MUSIC can then be applied in DOA estimation using the VAA. However since the overall signal quality degrades across the arc array, it is expected that a reduced resolution will be obtained in the MUSIC spectrum. The MUSIC power spectrum of a VAA can be expressed by (6-18).

$$\mathbf{P}_{arc}(\theta, \phi) = \frac{1}{(\mathbf{e}_{arc}^q)^H \mathbf{B}_N^{arc} (\mathbf{B}_N^{arc})^H \mathbf{e}_{arc}^q} \quad (6-18)$$

where $\mathbf{B}_N^{arc}$ is the noise subspace obtained from (6-17). The elevation angle θ varies from 0° to 90° and azimuth angle ϕ varies from 0° to 360° . Assume that searching steps of θ and ϕ are $\delta\theta$ and $\delta\phi$ and their angle axes are of length K and L respectively. Then $\delta\theta = 90^\circ/(K-1)$ and $\delta\phi = 360^\circ/(L-1)$. The size of $\mathbf{P}_{arc}$ is $K \times L$.

6.3 Proposed Composite Multiple Resolution (CMR) Approach

In order to resolve the ambiguity problem of 2-element ULAs when inter-element spacing is larger than half of the signal wavelength without adding additional antenna elements in the VCA, it is expected to make use of the data of a VAA.

6.3.1 Design Procedure

In observing that when inter-element spacing is larger than half-wavelength of the source signal, 2-element ULA gives ambiguous direction estimation with high resolution; while VAA gives unambiguous direction estimation with relatively low resolution, we design an ambiguity resolution approach in following steps.

Step 1) Determine valid elements in each VAA according to the received signal powers on all the elements. The valid elements are the elements with relatively high powers while elements with very small powers are considered as invalid elements.

Step 2) Divide the complete VCA into 2-element ULAs and obtain the corresponding received signals for each 2-element ULA.

Step 3) Select any two ULAs from the valid elements and obtain two power spectrums of the indirect DOA estimation using MUSIC method on the two sets of ULA data. Then find out all the directions of the indirect DOA in the spectrum, including both ambiguous and real directions. Denote the two sets of indirect DOA as Φ_1 and Φ_2 .

Step 4) Choose two indirect DOAs, each from the two MUSIC spectra, at one time and calculate the corresponding 2D source direction (elevation and azimuth angles). A valid solution corresponds to a “1” and an invalid solution corresponds to a “0”. The binary number is stored in an empty matrix of size $K \times L$ (same size as the 2D MUSIC spectrum matrix in Section 6.2.2). Its rows correspond to the elevation angle θ index and its columns correspond to the azimuth angle ϕ index. Continue choosing and calculating until all the combinations of two indirect DOAs in the two spectrums are processed. The matrix containing “1” and “0” indicators is referred to as the *indicator matrix*.

Step 5) Obtain a 2D MUSIC spectrum using the output data of a VAA and find out the possible 2D source directions from the spectrum. This 2D spectrum acts as a *selection matrix*.

Step 6) Multiply the *selection matrix* in Step 5 with the *indicator matrix* obtained in Step 4, the final estimated source directions can be obtained from the resulting matrix.

A clear flow of the approach is shown in Fig. 6.5. Since the approach is associated with two types of antenna arrays with different sizes and different resolutions, we name it as the composite multiple-resolution (CMR) approach.

Fig. 6.5 Flow chart of the CMR approach.

6.3.2 Ambiguity Resolution

Data is collected using the complete VCA as described in Section 6.2.1 and can be obtained as in (6-11). Consider the same VCA as that in Fig. 6.4, $N = 8$ and elements 4 and 8 are invalid elements (filled in gray in Fig. 6.6). Elements connected in red belong to one arc array and elements connected in blue belong to the other arc array. Assume selecting the i th and $(i+1)$ th (for example, $i = 1$ means the first and second ULAs are chosen as pair u and pair v respectively in Fig. 6.6) 2-element ULAs from the valid elements (elements 1 to 3 and elements 5 to 7 in Fig. 6.6 are valid elements).

Fig. 6.6 Selection illustration of 3-element VAAs and 2-element ULAs in an 8-element VCA.

The steering vector of the i th ULA can be expressed as in (6-19). Φ_i is the indirect DOA with respect to the i th ULA axis, which can be obtained from the MUSIC spectrum.

$$\mathbf{e}_i^{ULA}(\Phi_i) = [\exp(jkR_0 \cos \Phi_i) \exp(-jkR_0 \cos \Phi_i)]^T \quad (6-19)$$

For the q th source signal, the array output signal at the i th ULA can be selected from the output signals of the VCA and is written in the form of (6-20) and (6-21), m denotes the sample index of the output signal.

$$\mathbf{z}_i(m) = \sum_{q=1}^Q \mathbf{A}_i^q \mathbf{e}_i(\theta_q, \phi_q) x_q(m) + \mathbf{w}_i(m) \quad (6-20)$$

where

$$\mathbf{A}_i^q = \begin{bmatrix} a_{i1}^q \\ a_{i2}^q \end{bmatrix}, \quad a_{i1}^q = -a_{i2}^q$$

$$\mathbf{e}_i(\theta_q, \phi_q) = \begin{bmatrix} \exp(jkR_0 \sin \theta_q \cos(\phi_q - \phi_{i1})) \\ \exp(-jkR_0 \sin \theta_q \cos(\phi_q - \phi_{i1})) \end{bmatrix}^T \quad (6-21)$$

The steering vector $\mathbf{e}_i(\theta_q, \phi_q)$ in (6-21) is equivalent to that in (6-9) with only two elements selected. However, as discussed in Section 6.2.1, projections with different signs will result in different subspace decompositions. In order to apply the DOA estimation approach described in Chapter 5, we multiply the selected signals by a matrix $\mathbf{h}$ and obtain another set of data as in (6-22).

$$\mathbf{z}_i'(m) = \mathbf{h} \mathbf{z}_i(m) = \sum_{q=1}^Q \mathbf{h} \mathbf{A}_i^q \mathbf{e}_i(\theta_q, \phi_q) x_q(m) + \mathbf{h} \mathbf{w}_i(m) \quad (6-22)$$

where

$$\mathbf{h} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

MUSIC method is then applied on the i th and $(i+1)$ th 2-element ULA data to estimate the respective indirect DOAs Φ_i and Φ_{i+1} .

When the inter-element spacing d is larger than half-wavelength of the source signal, several peaks will be generated for both Φ_i and Φ_{i+1} . We denote the ambiguous value of Φ_i and Φ_{i+1} as Φ_i^a and Φ_{i+1}^a respectively. Then each combination of these two DOAs will give two equations as in (6-23).

$$\begin{aligned} \cos \Phi_i^a &= \sin \theta_q \cos(\phi_q - \phi_{i1}) \\ \cos \Phi_{i+1}^a &= \sin \theta_q \cos(\phi_q - \phi_{(i+1)1}) \end{aligned} \quad (6-23)$$

where ϕ_{i1} and $\phi_{(i+1)1}$ are the azimuth angles of the i th and $(i+1)$ th ULA respectively. The small number 1 indicates that the azimuth angle of the corresponding ULA coincides with the azimuth angle of the first element in that ULA. Solve all the

possible combinations of Φ_i^a and Φ_{i+1}^a and store the valid solutions $(\tilde{\theta}_q, \tilde{\phi}_q)$ using a binary indicator 0 or 1 at the corresponding positions in an empty matrix $\mathbf{D}$ of size $K \times L$. The row of $\mathbf{D}$ corresponds to the azimuth angle searching axis of length $1 \times L$ as defined in Section 6.2.2 and the column of $\mathbf{D}$ corresponds to the elevation angle searching axis of length $1 \times K$.

2D MUSIC spectrum of the VAA (three elements in Fig. 6.6) is able to give a low resolution in source DOA estimation. It can be obtained as in (6-18). The power spectrum is the selection matrix of size $K \times L$. Then the final estimated DOA $(\hat{\theta}_q, \hat{\phi}_q)$ can be obtained by selecting the indicator from $\mathbf{D}$ using the selection matrix $\mathbf{P}_{arc}$. The position of the selected indicator in $\mathbf{D}$ gives the corresponding source direction as expressed in (6-24).

$$\begin{aligned} \mathbf{M}(k, l) &= \mathbf{D}(k, l) \cdot \mathbf{P}_{arc}(k, l) \\ (\hat{\theta}_q, \hat{\phi}_q) &= \arg \underset{\mathbf{M}(k, l) > 0}{[\mathbf{M}]} \end{aligned} \quad (6-24)$$

where $k = 1, 2, \dots, K$ and $l = 1, 2, \dots, L$.

It is expected that when the resolution of the DOA estimation using a VAA is narrow enough to select the correct source directions, the ambiguity will be easily resolved. When the resolution of the VAA is too wide, ambiguous directions might also be selected. However, proved by Ding and Xiao in [44] that circular array has no ambiguity when the number of elements is no smaller than 8, it is expected that with more elements, the VAA may have sharp enough resolution to resolve the ambiguity.

6.3.3 Wideband DOA Estimation

According to the geometry of the VCA in Fig. 6.4, the inter-element spacing d_{arc} of a VAA is related with the inter-element spacing d of a 2-element ULA by the expression in (6-25).

$$\begin{aligned} d_{arc} &= 2R_0 \sin(\pi/N) \\ &= d \sin(\pi/N) \end{aligned} \quad (6-25)$$

where N is the number of elements in the VCA.

We have an interesting finding that a ULA and a VAA can give different frequency limits, beyond which ambiguity occurs due to different inter-element spacings of the two arrays. That is to say, theoretically, the inter-element spacing of a ULA gives the lower frequency limit (f_L) that causes no ambiguity in indirect DOA estimation; the inter-element spacing of a VAA gives the upper frequency limit (f_U) that causes no ambiguity in direct 2D DOA estimation. Take the ratio of the two frequency limits and represent it using the inter-element spacings of a ULA and a VAA, we can get an expression as in (6-26).

$$\frac{f_U}{f_L} = \frac{\lambda_{ULA}}{\lambda_{arc}} = \frac{d}{d_{arc}} = \frac{1}{\sin(\pi/N)} \quad (6-26)$$

For an 8-element VCA, the ratio is around 2.6. When the number of elements increases to 16, the ratio increases to a value around 5.1.

The lower frequency limit can be reduced further since frequencies smaller than this limit give larger wavelengths and no ambiguity. Then we have a question to raise, will the upper limit be further increased without any ambiguity?

Benefited from the ambiguity resolution approach, we are able to observe that the proposed approach is suitable for three scenarios. Firstly, for lower frequency band (smaller than the low frequency limit), both a ULA and a VAA have no ambiguity. The DOA and polarization estimation method using a VCA in Chapter 5 can be used to obtain the source directions. Secondly, for frequencies within the low frequency limit and the high frequency limit, a ULA will give ambiguous direction estimation but a VAA is unambiguous. Source DOA estimation can be achieved directly by a VAA; but estimation using a ULA can be used to provide resolution improvement for the arc array estimation. Thirdly, for frequencies beyond the upper frequency limit, number of ambiguities in a ULA spectrum is further increased. A VAA with relatively more number of elements has no ambiguity in 2D DOA estimation, it can be used to resolve the ambiguity in a ULA on one hand; on the other hand, the ULA estimation result can be used to improve the estimation resolution of the VAA.

Fortunately, the proposed approach is able to provide our VCA the ability to sustain wide frequency bands according to the three scenarios. Signals with frequencies within this band can be estimated with fixed array geometry and without any DOA ambiguity. After the DOA estimation, estimation of signal polarizations is also possible by using this approach.

6.4 Numerical Examples

To provide a better verification on the proposed ambiguity resolution approach, we carry out several simulation experiments in this section. The determination of the number of elements in the VCA is also presented.

Since the aim is to resolve ambiguity in DOA estimation, it is enough to consider single source estimation in this section. We assume that a narrowband far-field plane wave source is impinging on an 8-element VCA ($N = 8$) as in Fig. 6.4. We consider a relatively good signal quality with $\text{SNR} = 10$ dB for all the simulations. Additive white Gaussian noise is considered and independences exist among all the signals and noise in all the simulations. The signal parameters are set to be $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and signal frequency is $f = 1.2$ GHz. The wavelength of the source signal is thus $\lambda = c/f = 0.25$ m. Illuminated by this source, the VCA can be divided into two arc arrays same as that in Fig. 6.6 (elements 1, 2 and 3 form the first VAA and elements 5, 6 and 7 form the second VAA). The division of the arc arrays can be obtained by comparing signal powers at different elements in the VCA. A frequency spectrum of signals at half of the VCA elements (4 elements in this case) is shown in Fig. 6.7. All the powers are normalized to the maximum element power and only 20 dB dynamic range (shaded area in gray) of the normalized power is shown.

It is observed from Fig. 6.7 that signal powers at elements 1 to 3 have small variations from each other while signal power at element 4 is much lower than them. It is easy to conclude that signal power at element 8 in the other half of the VCA is also very small compared to elements 5 to 7 due to the symmetrical property of a circular array. Therefore, according to the procedure of our proposed approach, element 4 and element 8 will not be used and two arc arrays are formed (elements 1

to 3 form the first arc array and elements 5 to 7 form the second arc array). This division coincides with the illustration in Fig. 6.6.

Fig. 6.7 Normalized signal powers at elements 1 to 4 in the 8-element VCA shown in Fig. 6.6 with 20 dB display dynamic range (DR) for each element.

In Sections 6.4.1 to 6.4.4, the inter-element spacing d of a 2-element ULA in the VCA is varied to achieve different illustration purposes. In Section 6.4.5, d is fixed and f is varied with the purpose of investigating the achievable wideband frequency.

6.4.1 ULA to Improve Resolution of DOA

Keep the source frequency (1.2 GHz) unchanged. Assume inter-element spacing of the ULA, d , is equal to $\lambda/2$, then the diameter D of the VCA is $\lambda/2$ and the inter-element spacing d_{arc} of the VAA is $\lambda \sin(\pi/N)/2$ (about 0.19λ when $N = 8$) which is less than $\lambda/2$. In other words, both a 2-element ULA and a VAA do not have any ambiguous directions in the DOA spectrum. The estimation results of an 8-element

VCA with $D = d = 0.5\lambda$ and $d_{arc} = 0.19\lambda$ are shown in Fig. 6.8. For the full VCA, DOA estimation using MUSIC completely misses the true DOA as shown in Fig. 6.8(a) (the cross point of the two dotted lines represent the position of the true source DOA). However, MUSIC spectrum of VAA can give unambiguous DOA estimation although with a peak widely spread in the spectrum as indicated by the white area in Fig. 6.8(b) (the white area coincides with the cross point). This spectrum acts as a selection matrix in latter estimation steps. As in Fig. 6.6, elements 1 and 5 are selected as the 1st ULA and elements 2 and 6 are selected as the 2nd ULA. The MUSIC spectrums of the two selected 2-element ULAs are shown in Fig. 6.8(c). Two real indirect DOAs of these two ULAs, 120° and 110.7° , are labeled in the figure. We can see that there is no ambiguity direction in both spectrums. The DOA estimation approach based on a VCA presented in Chapter 5 can be applied to obtain the estimated DOA value and form the indicator matrix in Step 4 in Section 6.3.1. Fig. 6.8(d) shows the final estimation result after the selection matrix with 20 dB display threshold. In this example, the indicator matrix contains only one “1” corresponding to the position of the final DOA estimation. The 2D DOA parameters are correctly obtained and resolution is greatly improved in Fig. 6.8(d) compared with that in Fig. 6.8(b).

Of course, the two selected ULAs can also be the first and the third (elements 3 and 7) ULA or the second and the third ULA in Fig. 6.6. The estimation result is similar as that in Fig. 6.8. Result in Fig. 6.8 clearly demonstrates that a VAA gives a wide peak in the MUSIC spectrum; however, the DOA resolution can be improved with the help of a 2-element ULA. On the other hand, when a ULA produces ambiguous DOA estimation with high resolution, a VAA can be used to select the correct DOA from all the estimations as long as the peak of the VAA spectrum covers the correct DOA only. We demonstrate the ambiguity resolving ability of a VAA in the next sub-section.

Fig. 6.8 Results for $N=8$, $D=d=0.5\lambda$ and $d_{arc}=0.19\lambda$ (no mabiguity occurs in both ULA and VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f=1.2\text{GHz}$: (a) MUSIC spectrum of full VCA, (b) MUSIC spectrum of 3-element VAA, (c) MUSIC spectrum of 1st ULA (elements 1 & 5) and 2nd ULA (elements 2 & 6) in Fig. 6.6, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.

6.4.2 VAA to Resolve DOA Ambiguity

Increase inter-element spacing, d , of the ULA to a value larger than $\lambda/2$ but smaller than $\lambda/[2\sin(\pi/N)]$ (about 1.3λ for the 8-element VCA). In this case, there will be no ambiguity in the DOA estimation using the VAA. We set d to be λ and Fig. 6.9 shows the estimation results of the 8-element VCA with $D = d = \lambda$ and $d_{arc} = \lambda\sin(\pi/N)$ (about 0.38λ when $N = 8$). The source direction is kept to be the same as previous section, $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$.

The MUSIC spectrum of the full VCA gives two false peaks (Fig. 6.9(a)) and the spectrum of the VAA gives one correct peak (Fig. 6.9(b)). In this case, the estimation result using the VAA is still unambiguous but both ULAs give one ambiguous direction along with the true indirect DOAs (Fig. 6.9(c)). Due to this reason, the DOA estimation approach using a VCA described in Chapter 5 cannot be applied to obtain the final 2D source DOA since one cannot distinguish the true indirect DOA from the ambiguous ones. Four half-cones will be generated and there will be two valid intersection lines among these half-cones (as shown in Fig. 6.10(a)). According to the proposed ambiguity resolution approach in this chapter, the intersection lines correspond to two possible solutions as shown in Fig. 6.10(b). The two solutions form the indicator matrix. Each pixel in the indicator matrix has a division of 1° in the θ axis and a division of 5° in the ϕ axis. The final DOA estimation result in Fig. 6.9(d) is then obtained as the selection result from Fig. 6.10(b) using the selection matrix in Fig. 6.9(b).

From the simulation example, it is easy to tell that the proposed approach is able to successfully resolve ambiguity when ambiguous direction appear in the ULA spectrums but not appear in the VAA spectrum.

6.4.3 Resolve Ambiguity and Improve Resolution of DOA

When further increase d to a value larger than $\lambda/[2\sin(\pi/N)]$ (1.3λ when $N = 8$), the inter-element spacing d_{arc} of the VAA reaches a value beyond $\lambda/2$, ambiguity will occur in both a ULA and a VAA. We set d to be 3λ and Fig. 6.11 shows the estimation results of the 8-element VCA with $D = d = 3\lambda$ and $d_{arc} = 3\lambda\sin(\pi/N)$ (about 1.15λ when $N = 8$).

Fig. 6.9 Results for $N = 8$, $D = d = \lambda$ and $d_{arc} = 0.38\lambda$ (ambiguity occurs in ULA but not in VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of full VCA, (b) MUSIC spectrum of 3-element VAA, (c) MUSIC spectrum of 1st ULA (elements 1 & 5) and 2nd ULA (elements 2 & 6) in Fig. 6.6, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.

(a)

(b)

Fig. 6.10 (a) Half-cones of possible real and ambiguous source directions, (b) the indicator matrix in Step 5 with 1° division in the θ axis and 5° division in the ϕ axis.

Fig. 6.11 Results for $N = 8$, $D = d = 3\lambda$ and $d_{arc} = 1.15\lambda$ (ambiguity occurs in both ULA and VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of 3-element VAA, (b) MUSIC spectrum of 1st ULA (element 1 & 5) and 2nd ULA (elements 2 & 6) in Fig. 6.6, (c) indicator matrix, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.

Number of ambiguous directions increases extensively in ULA spectrums when d is increased to a large value and so as directions in the VAA spectrum. In this case, the VAA is not able to solve the ambiguity problem in the ULA as indicated in Fig. 6.11(d). There is an ambiguous direction with relatively the same power as that of the real direction (gray spikes in Fig. 6.11(e)). Therefore, one cannot distinguish the real direction from this ambiguous direction. In Fig. 6.11 and following figures, the indicator matrix has a 1° division in the θ axis and 5° division in the ϕ axis.

6.4.4 Wideband DOA Estimation

Inspired by the work in [44], we intend to increase the number of elements in a VCA in order to achieve an unambiguous DOA estimation using the VAA when source frequency or D/λ reaches a relatively large value (for example, the value 6 of D/λ in Fig. 6.11). By achieving unambiguous estimation in a VAA, ambiguity in a ULA can be resolved and wideband estimation can be obtained. It is obvious that resolution of a VAA can be improved when number of elements in the VAA is increased. A configuration of a 16-element VCA with labeled diameter D , inter-element spacing d of a ULA and inter-element spacing d_{arc} of a VAA is shown in Fig. 6.12. Since the inter-element spacing limit of a VAA without ambiguous direction estimation is $d_{arc} = \lambda/2$, the corresponding limit of the inter-element spacing d of a ULA is increased to about 2.56λ when N is increased to 16 (much larger than 1.3λ when $N = 8$). Keep the VCA diameter D the same value as that in Section 6.4.3 and change only the number of elements to 16, then $D = 3\lambda$, $d = 3\lambda$ and $d_{arc} = 0.58\lambda$. Illuminated by the same source, the output signal powers at half elements of the 16-element VCA are shown in Fig. 6.13. All the powers are normalized to the maximum element power and only 20 dB dynamic range (shaded in gray) of the normalized power is shown.

It is observed from Fig. 6.13 that signal powers at the 8 elements appear to be relatively symmetrical with respect to the center axis between elements 2 and 3. Powers at elements 2 and 3 have relatively the same value, so as elements 1 and 4, elements 5 and 8 and elements 6 and 7, among which elements 6 and 7 have the smallest powers.

Fig. 6.12 Configuration of a 16-element VCA with diameter D , inter-element spacing d of a 2-element ULA and inter-element spacing d_{arc} of a VAA.

Fig. 6.13 Normalized signal powers at elements 1 to 8 in the 16-element VCA in Fig. 6.12 with 20 dB display dynamic range (DR) for each element.

According to the proposed approach and symmetrical property, elements 6 and 7 as well as elements 14 and 15 are not used. Therefore, two 6-element arc arrays will be formed as shown in Fig. 6.14. Similarly as denoted in Fig. 6.6, elements connected in red belong to one arc array and elements connected in blue belong to the other arc array. Any two 2-element ULAs (for example, two ULAs labeled with u and v axes in Fig. 6.14) can be selected to generate the indicator matrix. The corresponding estimation results for $D = d = 3\lambda$ and $d_{arc} = 0.58\lambda$ are shown in Fig. 6.15.

Fig. 6.14 Selection illustration of 6-element VAAs and 2-element ULAs in a 16-element VCA.

As Fig. 6.15 shows, the ambiguity of the ULA can be resolved using the VAA and the resolution of the VAA can be improved by the ULA although there are extensive ambiguous directions in the ULA spectrums.

When the diameter D of the VCA is further increased, a ULA has even more number of ambiguous directions. When d_{arc} reaches a similar relation with λ as that in Fig. 6.11, say $d_{arc} = 1.2\lambda$, the inter-element spacing d of the ULA is then equal to $d_{arc}/\sin(\pi/N) = 6.15\lambda$, and D becomes 6.15λ . Fig. 6.16 shows the estimation results for $D = d = 6.2\lambda$ and $d_{arc} = 1.21\lambda$.

Fig. 6.15 Results for $N = 16$, $D = d = 3\lambda$ and $d_{arc} = 0.58\lambda$ (ambiguity occurs in a ULA but not in a VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of 6-element VAA, (b) MUSIC spectrum of 1st ULA (elements 1 & 9) and 2nd ULA (elements 2 & 10) in Fig. 6.14, (c) indicator matrix, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.

Fig. 6.16 Results for $N = 16$, $D = d = 6.2\lambda$ and $d_{arc} = 1.21\lambda$ (ambiguity occurs in a ULA but not in a VAA) with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.2\text{GHz}$: (a) MUSIC spectrum of 6-element VAA, (b) MUSIC spectrum of 1st ULA (elements 1 & 9) and 2nd ULA (elements 2 & 10) in Fig. 6.14, (c) indicator matrix, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.

Even though diameter of the VCA is increased to a very large value, the source direction can be correctly and unambiguously estimated using a 16-element VCA (as shown in Fig. 6.16(d) and Fig. 6.16(e)). However in a similar case, an 8-element VCA cannot give unambiguous estimation. Therefore, accurate DOA estimation of wider band sources can be achieved by using a VCA with number of elements no less than 16.

6.4.5 Super Wideband DOA Estimation

The frequency of signals that can be estimated by a normal array is limited by the inter-element spacing of the array. Therefore, estimation of sources within a wideband cannot be completely achieved by a single fixed array configuration. From Section 6.3.3, we have obtained the estimated bandwidth ratio of an 8-element and a 16-element VCA. We choose a 16-element VCA due to its larger bandwidth ratio and investigate its ability to estimate super wideband DOAs.

In this sub-section, we fix the diameter D of our VCA to be a constant value, for example 2.5 m, just for illustration. Then the inter-element spacing of both a ULA and a VAA are fixed to be $d = 2.5$ m and $d_{arc} = 0.488$ m. The unambiguous frequency limited by a ULA is 60 MHz and the unambiguous frequency limited by a VAA is around 306 MHz. Sources of frequencies within this range can be estimated without any ambiguity. The ratio of these two frequencies is more than 5:1 which is the estimated bandwidth ratio. When frequency increases to a larger value than 306 MHz, we can obtain an additional bandwidth ratio. Two cases with high source frequencies are studied and compared from Fig. 6.17 to Fig. 6.20.

Case 1: Set source frequency to be 1.5 GHz, which is much higher than the upper frequency limit 306 MHz. We obtain estimation results for diameter $D = 12.5 \lambda$, inter-element spacing of the ULA $d = 12.5 \lambda$, and inter-element spacing of the VAA $d_{arc} = 2.44 \lambda$ in Fig. 6.17.

Fig. 6.17 Results for $N = 16$, $D = d = 12.5\lambda$ and $d_{arc} = 2.44\lambda$ with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 1.5$ GHz: (a) MUSIC spectrum of 6-element VAA, (b) MUSIC spectrum of 1st ULA (elements 1 & 9) and 2nd ULA (elements 2 & 10) in Fig. 6.14, (c) indicator matrix, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.

Fig. 6.18 (a) Final estimation result at $f = 1.5$ GHz and source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ when 1st ULA (elements 1 & 9) and 3rd ULA (elements 3 & 11) are selected and one 6-element VAA is used, (b) final 3D DOA estimation result, (c) combination of results when different 2-element ULA pairs (1st and 2nd ULAs, 1st and 3rd ULAs in Fig. 6.14) are selected and one 6-element VAA is used, and (d) 3D combination result.

The real DOA with relatively high power is observed along with an ambiguous DOA with relatively low power in Fig. 6.17(e). The power difference between these two DOAs is around 4 dB. For single source estimation, one can simply choose the one with higher power to be the estimated DOA. However, a more robust and

accurate estimation result can be obtained by employing multiple selections of 2-element ULAs and combining their results. For example, when the first and the third ULAs (elements 1 & 9 and elements 3 & 11 for the respective ULAs in Fig. 6.14) are chosen, the estimation results at $f = 1.5$ GHz (shown in Fig. 6.18(a) and Fig. 6.18(b)) are different from that in Fig. 6.17(d) and Fig. 6.17(e) except the real DOA. Fig. 6.17(d) and Fig. 6.18(a) are multiplied and normalized to obtain the 2D combination result in Fig. 6.18(c). The respective 3D combination result is shown in Fig. 6.18(d).

Case 2: Increase the source frequency to 3.0 GHz and the estimation results for diameter $D = 25\lambda$, inter-element spacing of the ULA $d = 25\lambda$ and inter-element spacing of the VAA $d_{arc} = 4.88\lambda$ are shown in Fig. 6.19.

From Fig. 6.19(d), we can find that there exist other DOA indicators along with the real DOA in the final estimation result. However, the powers of other DOAs are smaller (typically around 10 dB lower in Fig. 6.19(d)) than that of the real DOA and one can identify the real DOA from the figure for single source estimation. The estimation result when a different selection of 2-element ULAs is made (1st ULA with elements 1, 9 and 3rd ULA with elements 3, 11 are chosen in the selection) is shown in Fig. 6.20(a) and Fig. 6.20(b). The two matrices in Fig. 6.19(d) and Fig. 6.20(a) are multiplied and normalized to obtain the 2D combination result in Fig. 6.20(c). The respective 3D combination result is shown in Fig. 6.20(d).

According to the results in Fig. 6.18(d) and Fig. 6.20(d), it is quite obvious that powers of false DOAs are dispersed while the power of real DOA is not when combining results obtained from multiple selections of 2-element ULAs. The false DOAs can be successfully eliminated in the estimation result by employing this approach. The additional bandwidth achieved up to frequency 3.0 GHz is about $3.0 \text{ GHz} / 306 \text{ MHz} = 9.8$. We may still increase the frequency further and it is expected that correct DOA estimation might be achieved with more number of 2-element ULA selections at even higher source frequency. After these wideband DOA estimation approaches, the successive approaches for estimating polarization of signals proposed in Chapter 5 can be continued without any difficulty.

Fig. 6.19 Results for $N = 16$, $D = d = 25\lambda$ and $d_{arc} = 4.88\lambda$ with source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ and $f = 3.0$ GHz: (a) MUSIC spectrum of 6-element VAA, (b) MUSIC spectrum of 1st ULA (elements 1 & 9) and 2nd ULA (elements 2 & 10) in Fig. 6.14, (c) indicator matrix with 1° division in θ axis and 5° division in ϕ axis, (d) final DOA estimation result from Step 6 (with 20 dB display threshold), and (e) final 3D DOA estimation result.

Fig. 6.20 (a) Final estimation result at $f = 3.0$ GHz and source direction $(\theta, \phi, \gamma) = (30^\circ, 180^\circ, 30^\circ)$ when 1st ULA (elements 1 & 9) and 3rd ULA (elements 3 & 11) are selected and one 6-element VAA is used (with 20 dB display threshold), (b) final 3D DOA estimation result, (c) combination of results when different 2-element ULA pairs (1st & 2nd ULAs and 1st & 3rd ULAs in Fig. 6.14) are selected and one 6-element VAA is used (with 20 dB display threshold), and (d) 3D combination result.

6.5 Discussions

The key point of the proposed CMR approach in this chapter is the combination of two different sub-arrays, the VAA and the 2-element ULA in a VCA. A ULA governs the practical lower frequency limit of source signals and a VAA governs the

practical upper frequency limit according to the restrictions on their inter-element spacing. The proposed approach is suitable for three scenarios. When source with very low frequency is to be estimated, the VAA is not necessary to be used and DOA estimation can be based on 2-element ULAs. When source frequency increases, DOA estimation based on ULAs becomes ambiguous and the VAA can be used to resolve the ambiguity problem. When source frequency increases further, both ULAs and the VAA become ambiguous but the problem can be resolved by employing multiple selections of ULA pairs. The three scenarios suitable for CMR approach are summarized in Fig. 6.21.

Fig. 6.21 Flow chart of the three scenarios using CMR approach.

Nevertheless, if number of elements in a VCA is small, a VAA cannot give unambiguous estimation for the case when the array inter-element spacing of the VAA is larger than half-wavelength of the source signal. In this case, a VAA cannot help to resolve ambiguity problem in a ULA. However, when increasing the number of elements to 16, unambiguous estimation can be achieved and can even be extended for sources with frequencies within a much wider band. Therefore, the proposed approach is relatively robust for a VCA with number of elements no smaller than 16.

6.6 Concluding Remarks

In Chapter 6, we have presented a CMR approach for resolving DOA ambiguity problem using a VCA. Since resolving ambiguity problems in arrays with larger inter-element spacing is very important and changing antenna array geometry is difficult in some critical applications, we propose the estimation approach so that the array can have large inter-element spacing and its geometry needs not change when estimating sources with different frequencies. The proposed approach includes the use of a VAA and 2-element ULAs divided from the VCA. The VAA can provide unambiguous DOA estimation with relatively low resolution and the ULA can provide ambiguous DOA estimation with high resolution. By combining these two estimations, an unambiguous DOA estimation with high resolution can be obtained. Through various simulations and case studies, it is found that the approach is able to provide completely unambiguous estimation when the number of elements in the circular array is 16 or more. Besides, by employing one pair of 2-element ULAs to estimate sources with high frequencies and employing more than one pair of 2-element ULAs to estimate sources with even higher frequencies, the approach brings the ability to the VCA to estimate sources within a wide frequency band when array configuration is fixed.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

In this thesis, a TTW target detection method based on multiple-view tomographic imaging has been investigated. The multiple-view tomographic imaging explores full space diversity using an array with 360 degree of view angle and employs a chirp signal to achieve high range resolution before image formation. The method gives a robust detection of targets even when wall effects are not completely compensated.

To further investigate the antenna array applications in target detection, we have proposed a LOS target detection approach using the combination of a RSA and the CS technology. Two schemes of the RSA are studied for static and moving targets detections, respectively. The corresponding CS based approaches of these two schemes are derived and numerical simulations are provided to verify their performances. To validate the approach based on the RSA and CS, an experiment is carried out to detect sparse static targets in an outdoor area. Both simulation and experimental results have shown that a RSA combined with CS gives better detection results than that for conventional imaging methods.

In addition to target detection, we also investigate the antenna array applications in source estimation. We propose a VCA and a simple approach for both DOA and polarization estimations using the proposed array. Various simulations and error analysis have been performed to investigate the performances of the proposed approach in both white and colored Gaussian noise scenarios. The approach based on the VCA is able to give accurate estimation of sources with relatively high grazing angles and arbitrary linear polarizations as well as sources with relatively low grazing angles and some special linear polarization angles. It is also able to estimate sources with circular and elliptical polarizations but cannot determine the tilt and senses of elliptical polarizations in general. In addition, with MUSIC-like method

(the MFOC-MUSIC), the approach using the VCA gives better estimations in colored Gaussian noise than in white Gaussian noise.

To solve the DOA ambiguity problem caused by large inter-element spacing of the ULA, we propose a CMR approach based on our VCA. The VCA is divided into VAAs, which is able to provide a coarse DOA estimation. When the inter-element spacing of a 2-element ULA exceeds half-wavelength of the source signal, the ULA will give a fine but ambiguous DOA estimation. The ambiguity problem in the ULA can be solved by the VAA. With this approach, we then apply the VCA to wideband DOA estimation and design the bandwidth of sources that can be estimated by the VCA without any ambiguity. Simulations are performed to verify the performance of the proposed ambiguity resolution approach. The results reveal that the approach is indeed effective and the idea may be extended to other types of antenna arrays to give unambiguous DOA estimation. The VCA based approach is expected to find wide applications in areas where conventional circular arrays and rectangular arrays cannot be used.

7.2 Recommendations for Future Work

Based on the studies and findings of the proposed techniques in this thesis, several suggestions on future work are listed as follows.

In this thesis, only one type of TTW target imaging techniques is discussed. It is not the optimum method in practical situations; different situations might require different techniques or combinations of several techniques. Experiments may be carried out to validate the methods and the effects of walls. Models of wall effects need to be studied further in order to give more accurate target detection. The approach based on a RSA and CS for moving target detection has not been validated via experimental data due to the constraint in time and hardware. In the future, an experiment may be carried out to test the performance of the proposed approach. In addition, a RSA might not be limited to be used with CS method only; it can be used in combination with other algorithms. More works need to be done to investigate the processing algorithms for a RSA. Most importantly and interestingly, applications of a RSA in TTW target detections need be investigated by incorporating wall effects in

newly developed algorithms so that the work is more practical and feasible for future military applications.

For the proposed DOA and polarization estimation approach based on a VCA, some work can be tried in the future, including estimation of tilt angles and senses of elliptically polarized sources because elliptic polarization is more common in practice. Moreover, the performance of the proposed approach based on a VCA needs to be compared with that of the polar-MUSIC method, which is a more generalized method for diversely polarized arrays. There are some areas on resolving ambiguity problems in a ULA using the approach based on a VCA that need to be investigated and improved in the future. One idea might be that more sub-sets of the VCA or the VAA can be divided and used for DOA estimation. For example, when 4-element sub-VAA with maximum signal power is used, the estimation result might be more robust to noise due to the overall increased signal quality. Therefore, sub-VAA with less number of elements but higher overall signal quality can be investigated in DOA estimation in the future. Moreover, two arc arrays in the VCA might have different phase information due to their different position information. It is interesting to find the relationships between these two arc arrays and investigate their possible benefit on increasing the robustness of estimation results. Besides, another possible research aspect is that by dividing the VCA into several clusters, the upper frequency limit might be changed and the possible band of the source frequency might be varied.

BIBLIOGRAPHY

- [1] D. H. Johnson and D. E. Dudgeon, *Array Signal Processing*. Englewood Cliffs: NJ: Prentice Hall, 1993.
- [2] R. A. Monzingo and T. W. Miller, *Introduction to Adaptive Arrays*. New York: John Wiley and Sons, 1980.
- [3] H. L. Van Trees, *Optimum Array Processing, Part IV of Detection, Estimation and Modulation Theory*. New York: John Wiley and Sons, 2002.
- [4] B. D. Van Veen and K. M. Buckley, "Beamforming: a versatile approach to spatial filtering," *IEEE ASSP Mag.*, vol. 5, pp. 4-24, 1988.
- [5] L. C. Godara, "Applications of antenna arrays to mobile communications. I. Performance improvement, feasibility, and system considerations," *Proc. IEEE*, vol. 85, pp. 1031-1060, 1997.
- [6] L. C. Godara, "Application of antenna arrays to mobile communications. II. Beam-forming and direction-of-arrival considerations," *Proc. IEEE*, vol. 85, pp. 1195-1245, 1997.
- [7] E. J. Baranoski, "Through wall imaging: Historical perspective and future directions," in *IEEE Int. Conf. Acoust., Speech, Signal Process.*, 2008, pp. 5173-5176.
- [8] L. Hui, H. Darabi, P. Banerjee, and L. Jing, "Survey of Wireless Indoor Positioning Techniques and Systems," *IEEE Trans. Systems, Man, and Cybernetics-Part C: Applications and Reviews*, vol. 37, pp. 1067-1080, 2007.
- [9] A. J. Weiss, "Direct position determination of narrowband radio frequency transmitters," *IEEE Signal Process. Lett.*, vol. 11, pp. 513-516, 2004.
- [10] A. Amar and A. J. Weiss, "Advances in direct position determination," in *Proc. Sensor Array and Multichannel Signal Processing Workshop*, 2004, pp. 584-588.
- [11] L. M. Frazier, "Surveillance through walls and other opaque materials," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 11, pp. 6-9, 1996.
- [12] J. A. McVay, K. M. Yemelyanov, A. Hoorfar, and N. Engheta, "Through-the-wall imaging and sensing: an electromagnetic perspective," in *IEEE Workshop on Signal Processing Applications for Public Security and Forensics*, 2007, pp. 1-4.
- [13] G. L. Cui, L. J. Kong, and J. Y. Yang, "A Back-projection algorithm to stepped-frequency synthetic aperture through-the-wall radar imaging," in *1st Asian and Pacific Conference on Synthetic Aperture Radar*, 2007, pp. 123-126.
- [14] M. Dehmollaian and K. Sarabandi, "Refocusing through building walls using synthetic aperture radar," *IEEE Trans. Geosci. Remote Sens.*, vol. 46, pp. 1589-1599, 2008.

- [15] X. X. Liu and H. Leung, "Through the wall imaging using chaotic modulated ultra wideband synthetic aperture radar," in *IEEE Int. Conf. Acoust., Speech and Signal Process.*, 2007, pp. III-1257-III-1260.
- [16] F. Ahmad, M. G. Amin, and S. A. Kassam, "A beamforming approach to stepped-frequency synthetic aperture through-the-wall radar imaging," in *2005 1st IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing*, 2005, pp. 24-27.
- [17] G. Wang and M. G. Amin, "Imaging through unknown walls using different standoff distances," *IEEE Trans. Signal Process.*, vol. 54, pp. 4015-4025, 2006.
- [18] F. Ahmad, M. G. Amin, and S. A. Kassam, "Synthetic aperture beamformer for imaging through a dielectric wall," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, pp. 271-283, 2005.
- [19] L.-P. Song, C. Yu, and Q.-H. Liu, "Through-wall imaging (TWI) by radar: 2-D tomographic results and analyses," *IEEE Trans. Geosci. Remote Sens.*, vol. 43, pp. 2793-2798, 2005.
- [20] A. Vertiy and S. Gavrilov, "Subsurface tomography application for through-wall imaging," in *Proc. 9th International Conference on Electromagnetics in Advanced Applications (ICEAA-05) and 11th European Electromagnetic Structures Conference (EESC-05). September, 2005*, pp. 12-16.
- [21] W. Zhang, L. Li, and F. Li, "Autofocusing imaging through the unknown building wall with EM inverse scattering algorithm," *Microwave and Optical Technology Letters*, vol. 52, pp. 732-736, 2010.
- [22] E. A. Marengo and F. K. Gruber, "Subspace-based localization and inverse scattering of multiply scattering point targets," *EURASIP J. Appl. Signal Process.*, vol. 2007, pp. 192-192, 2007.
- [23] W. J. Zhang, A. Hoorfar, and L. Li, "Through-the-wall target localization with time reversal MUSIC method," *Progress In Electromagnetics Research*, vol. 106, pp. 75-89, 2010.
- [24] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57, pp. 1408-1418, 1969.
- [25] P. Handel, P. Stoica, and T. Soderstrom, "Capon method for doa estimation: accuracy and robustness aspects," in *IEEE Winter Workshop on Nonlinear Digital Signal Processing*, 1993, pp. P_7.1-P_7.5.
- [26] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. 34, pp. 276-280, 1986.
- [27] P. Stoica and A. Nehorai, "MUSIC, maximum likelihood, and Cramer-Rao bound," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, pp. 720-741, 1989.
- [28] R. Roy, A. Paulraj, and T. Kailath, "Estimation of Signal Parameters via Rotational Invariance Techniques - ESPRIT," in *IEEE Military Communications Conf. - Communications-Computers: Teamed for the 90's*, 1986, pp. 41.6.1-41.6.5.

- [29] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech and Signal Process.*, vol. 37, pp. 984-995, 1989.
- [30] P. Stoica and A. B. Gershman, "Maximum-likelihood DOA estimation by data-supported grid search," *IEEE Signal Process. Lett.*, vol. 6, pp. 273-275, 1999.
- [31] M. H. Li and Y. L. Lu, "Maximum likelihood DOA estimation in unknown colored noise fields," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, pp. 1079-1090, 2008.
- [32] M.-H. Li and Y.-L. Lu, "Improving the performance of GA-ML DOA estimator with a resampling scheme," *Signal Process.*, vol. 84, pp. 1813-1822, 2004.
- [33] B. Friedlander and A. J. Weiss, "A direction finding algorithm for diversely polarized arrays," *Digital Signal Processing-A Review Journal*, vol. 2, pp. 123-134, 1992.
- [34] A. L. Swindlehurst and M. Viberg, "Subspace fitting with diversely polarized antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 41, pp. 1687-1694, 1993.
- [35] K. T. Wong and M. D. Zoltowski, "Closed-form direction finding and polarization estimation with arbitrarily spaced electromagnetic vector-sensors at unknown locations," *IEEE Trans. Antennas Propag.*, vol. 48, pp. 671-681, 2000.
- [36] M. D. Zoltowski and K. T. Wong, "ESPRIT-based 2-D direction finding with a sparse uniform array of electromagnetic vector sensors," *IEEE Trans. Signal Process.*, vol. 48, pp. 2195-2204, 2000.
- [37] K. T. Wong and M. D. Zoltowski, "Self-initiating MUSIC-based direction finding and polarization estimation in spatio-polarizational beam space," *IEEE Trans. Antennas Propag.*, vol. 48, pp. 1235-1245, 2000.
- [38] K. T. Wong, L. Linshan, and M. D. Zoltowski, "Root-MUSIC-based direction-finding and polarization estimation using diversely polarized possibly collocated antennas," *IEEE Antennas Wireless Propag. Lett.*, vol. 3, pp. 129-132, 2004.
- [39] R. O. Schmidt, "A signal subspace approach to multiple emitters location and spectral estimation," Ph.D. dissertation, Stanford Univ., Palo Alto, CA, 1981.
- [40] C. Proukakis and A. Manikas, "Study of ambiguities of linear arrays," in *IEEE Int. Conf. Acoust., Speech, Signal Process.*, 1994, pp. IV/549-IV/552 vol.4.
- [41] A. Flieller, P. Larzabal, and H. Clergeot, "Study of ambiguities in array manifold: a general framework," in *Proc. 8th IEEE Signal Processing Workshop on Statistical Signal and Array Processing*, 1996, pp. 574-577.
- [42] A. Manikas and C. Proukakis, "Modeling and estimation of ambiguities in linear arrays," *IEEE Trans. Signal Process.*, vol. 46, pp. 2166-2179, 1998.

- [43] T. Liu and L. Xiao, "About the Ambiguities in DOA Estimation," *Chinese Journal of Electronics*, vol. 19, pp. 747-751, 2010.
- [44] Q. Ding and X. Xiao, "DOA ambiguity vs. array configuration for subspace-based DF methods," in *Proc. CIE Int. Conf. Radar*, 1996, pp. 488-492.
- [45] M. Gavish and A. J. Weiss, "Array geometry for ambiguity resolution in direction finding," *IEEE Trans. Antennas Propag.*, vol. 44, pp. 889-895, 1996.
- [46] E. Jacobs and E. W. Ralston, "Ambiguity resolution in interferometry," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-17, pp. 766-780, 1981.
- [47] Y. I. Abramovich, N. K. Spencer, and A. Y. Gorokhov, "Resolving manifold ambiguities in direction-of-arrival estimation for nonuniform linear antenna arrays," *IEEE Trans. Signal Process.*, vol. 47, pp. 2629-2643, 1999.
- [48] J. H. Tang, X. C. Si, and P. Chu, "Improved MUSIC algorithm based on fourth-order cumulants," *Systems Engineering and Electronics*, vol. 32, pp. 256-259, 2010.
- [49] J. Chen, G. Y. Fang, and F. Li, "Image correction for UWB through-wall imaging radar," in *7th International Symposium on Antennas, Propagation & EM Theory*, 2006, pp. 1-4.
- [50] S. S. Gauthier and W. Chamma, "Surveillance through concrete walls," *Proc. SPIE 5403*, pp. 597-608, 2004.
- [51] P. J. Shargo and J. W. Melody, "Model-based correction of through-wall SAR imagery via raytracing," in *IEEE Radar Conf.*, 2007, pp. 706-711.
- [52] Y.-S. Yoon and M. G. Amin, "High-resolution through-the-wall radar imaging using beamspace MUSIC," *IEEE Trans. Antennas Propag.*, vol. 56, pp. 1763-1774, 2008.
- [53] L. Chen and O. Shan, "A time-domain beamformer for UWB through-wall imaging," in *TENCON 2007*, 2007, pp. 1-4.
- [54] D. Yavuz, "Frequency and focal region properties of random sparse arrays," *IEEE Trans. Antennas Propag.*, vol. 32, pp. 456-465, 1984.
- [55] H. Steyskal and J. K. Schindler, "Beamforming and signal processing with sparse, random arrays for space based radar," in *Proc. IEEE Aerosp. Conf.*, 2004, pp. 887-896 Vol.2.
- [56] L. Carin, "On the relationship between compressive sensing and random sensor arrays," *IEEE Antennas Propag. Mag.*, vol. 51, pp. 72-81, 2009.
- [57] L. Carin, L. Dehong, and G. Bin, "Coherence, compressive sensing, and random sensor arrays," *IEEE Antennas Propag. Mag.*, vol. 53, pp. 28-39, 2011.
- [58] R. Tanaka, "Report on SAR imaging," *Internal Report*, 2003.
- [59] M. Aftanas, J. Rovnakova, M. Drutarovsky, and D. Kocur, "Efficient method of TOA estimation for through wall imaging by UWB radar," in *IEEE Int. Conf. Ultra-Wideband*, 2008, pp. 101-104.

- [60] L. Chen and O. Shan, "Through-wall surveillance using ultra-wideband short pulse radar: numerical simulation," in *2nd IEEE Conf. Industrial Electronics and Applications*, 2007, pp. 1551-1554.
- [61] C. A. Wiley, "Synthetic aperture radars: a paradigm for technology evolution," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 21, pp. 440-443, 1985.
- [62] M. Dehmollaian, M. Thiel, and K. Sarabandi, "Through-the-wall imaging using differential SAR," *IEEE Trans. Geosci. Remote Sens.*, vol. 47, pp. 1289-1296, 2009.
- [63] M. I. Skolnik, *RADAR Handbook*, 2nd ed. New York: McGraw-Hill, 1990.
- [64] S. Xiao, D. C. Munson, Jr., S. Basu, and Y. Bresler, "An $N^2/\log N$ back-projection algorithm for SAR image formation," in *Conference Record of the Thirty-Fourth Asilomar Conference on Signals, Systems and Computers*, 2000, pp. 3-7 vol.1.
- [65] A. F. Yegulalp, "Fast backprojection algorithm for synthetic aperture radar," in *The Record of the 1999 IEEE Radar Conference*, 1999, pp. 60-65.
- [66] A. Boag, Y. Bresler, and E. Michielssen, "A multilevel domain decomposition algorithm for fast $O(N^2/\log N)$ reprojection of tomographic images," *IEEE Trans. Img. Process.*, vol. 9, pp. 1573-1582, 2000.
- [67] C. E. Yarman, B. Yazc, and M. Cheney, "Bistatic synthetic aperture radar imaging for arbitrary flight trajectories," *Trans. Img. Proc.*, vol. 17, pp. 84-93, 2008.
- [68] F. Ahmad, M. G. Amin, and S. A. Kassam, "Through-the-wall wideband synthetic aperture beamformer," in *IEEE Antennas and Propagation Society International Symposium*, 2004, pp. 3059-3062 Vol.3.
- [69] Y. Ding and D. C. Munson, Jr., "A fast back-projection algorithm for bistatic SAR imaging," in *Int. Conf. Image Process.*, 2002, pp. II-449-II-452 vol.2.
- [70] H. D. Griffiths and C. J. Baker, "Fundamentals of Tomography and RADAR," *Advances in Sensing with Security Applications*, vol. 2, pp. 171-187, 2006.
- [71] F. Simonetti and L. Huang, "From beamforming to diffraction tomography," *Journal of Applied Physics*, vol. 103, pp. 103110-103110-7, 2008.
- [72] M. C. Wicks, B. Himed, H. Bascom, and J. Clancy, "Tomography of Moving Targets (TMT) for Security and Surveillance," *Advances in Sensing with Security Applications*, vol. 2, pp. 323-339, 2006.
- [73] H. B. Sun, H. C. Feng, and Y. L. Lu, "High resolution radar tomographic imaging using single-tone CW signals," in *Proc. IEEE int. Radar Conf.*, Washington DC, USA, 2010, pp. 975-980.
- [74] L. P. Song, C. Yu, and Q. H. Liu, "Through-wall imaging (TWI) by radar: 2-D tomographic results and analyses," *IEEE Trans. Geosci. Remote Sens.*, vol. 43, pp. 2793-2798, 2005.

- [75] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, pp. 489-509, 2006.
- [76] E. J. Candès, J. K. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Comm. Pure and Appl. Math.*, vol. 59, pp. 1207-1223, 2006.
- [77] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, pp. 1289-1306, 2006.
- [78] R. Baraniuk and P. Steeghs, "Compressive radar imaging," in *Proc. IEEE Radar Conf.*, 2007, pp. 128-133.
- [79] Y.-S. Yoon and M. G. Amin, "Imaging of behind the wall targets using wideband beamforming with compressive sensing," in *Proc. IEEE/SP 15th Workshop on Statistical Signal Process.*, 2009, pp. 93-96.
- [80] M. Leigsnering, C. Debes, and A. M. Zoubir, "Compressive sensing in through-the-wall radar imaging," in *Proc. IEEE Int. Conf. Acous., Speech, Signal Process.*, 2011, pp. 4008-4011.
- [81] Q. Huang, L. Qu, B. H. Wu, and G. Y. Fang, "UWB Through-Wall Imaging Based on Compressive Sensing," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, pp. 1408-1415, 2010.
- [82] V. M. Patel, G. R. Easley, D. M. Healy, Jr., and R. Chellappa, "Compressed synthetic aperture radar," *IEEE J. Sel. Topics Signal Process.*, vol. 4, pp. 244-254, 2010.
- [83] M. T. Alonso, P. Lopez-Dekker, and J. J. Mallorqui, "A novel strategy for radar imaging based on compressive sensing," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, pp. 4285-4295, 2010.
- [84] J. G. Yang, J. Thompson, X. T. Huang, T. Jin, and Z. M. Zhou, "Random-frequency SAR imaging based on compressed sensing," *IEEE Trans. Geosci. Remote Sens.*, vol. 51, pp. 983-994, 2013.
- [85] D. L. Donoho, M. Elad, and V. N. Temlyakov, "Stable recovery of sparse overcomplete representations in the presence of noise," *IEEE Trans. Inf. Theory*, vol. 52, pp. 6-18, 2006.
- [86] E. Candes and J. Romberg, "Sparsity and incoherence in compressed sampling," *Inverse Problems*, vol. 23, 2006.
- [87] T. Goldstein and S. Osher, "The split bregman method for l1-regularized problems," *SIAM Journal on Imaging Sciences*, vol. 2, pp. 285-332, 2009.
- [88] J. Romberg. L1-magic, Aug. 2008 [Online]. Available: <http://www.acm.caltech.edu/l1magic/>
- [89] S. Mallat, *A Wavelet Tour of Signal Processing*: Academic Press, 1999.
- [90] D. Needell and R. Vershynin, "Uniform uncertainty principle and signal recovery via regularized orthogonal matching pursuit," *J. Found. Comput. Math.*, vol. 9, pp. 317-334, 2009.

- [91] D. Needell and R. Vershynin, "Greedy signal recovery and uncertainty principles," in *Proc. SPIE*, Bellingham, WA, 2008, p. 68140J.
- [92] D. L. Donoho, Y. Tsaig, I. Drori, and J. L. Starck, "Sparse solution of underdetermined systems of linear equations by stagewise orthogonal matching pursuit," *IEEE Trans. Inf. Theory*, vol. 58, pp. 1094-1121, 2012.
- [93] K. T. Wong, "Direction finding/polarization estimation-dipole and/or loop triad(s)," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 37, pp. 679-684, 2001.
- [94] X. Zhang, Y. Shi, and D. Xu, "Novel blind joint direction of arrival and polarization estimation for polarization-sensitive uniform circular array," *Progress In Electromagnetics Research*, vol. 86, pp. 19-37, 2008.
- [95] M. J. L. Alves and M. S. de Alencar, "Mutual coupling effects in a linear adaptive antenna array with random spacing," in *IEEE 18th International Symposium on Personal, Indoor and Mobile Radio Communications*, 2007, pp. 1-4.
- [96] C. W. Kim, B. Luo, Y. C. Liang, and M. Y. W. Chia, "MIMO-OFDM radar array configuration for resolving DOA ambiguity," in *IEEE International Conference on Communication Systems*, 2012, pp. 85-89.
- [97] M. Bartlett, *An Introduction to Stochastic Process*. New York: Cambridge Univ. Press, 1956.
- [98] Wikipedia. Polarization (waves) [Online]. Available: [http://en.wikipedia.org/wiki/Polarization_\(waves\)](http://en.wikipedia.org/wiki/Polarization_(waves))
- [99] A. Springer, W. Gugler, M. Huemer, L. Reindl, C. C. W. Ruppel, and R. Weigel, "Spread spectrum communications using chirp signals," in *IEEE/AFCEA EUROCOMM Information Systems for Enhanced Public Safety and Security*, 2000, pp. 166-170.
- [100] ----, "nanoNET Chirp Based Wireless Networks," in *White Paper Nanotron Technologies GmbH*, 2007.
- [101] X. W. Wang, M. R. Fei, and X. Li, "Performance of chirp spread spectrum in wireless communication systems," in *11th IEEE Singapore Int. Conf. Comm. Syst.*, 2008, pp. 466-469.
- [102] L. M. Frazier, "Radar surveillance through solid materials," *Proc. SPIE*, pp. 139-146, 1997.
- [103] H. Unz, "Linear Arrays with arbitrarily distributed elements," *IRE Trans. Antennas Propagat.*, vol. 8, pp. 222-223, 1960.
- [104] A. L. Maffett, "Array factors with nonuniform spacing parameter," *IRE Trans. Antennas Propagat.*, vol. 10, pp. 131-136, 1962.
- [105] Y. T. Lo, "A mathematical theory of antenna arrays with randomly spaced elements," *IEEE Trans. Antennas Propagat.*, vol. 12, pp. 257-268, 1964.
- [106] B. D. Steinberg, *Principles of aperture and array system design: including random and adaptive arrays*. New York: Wiley Press, 1976.
- [107] R. Innocenti, "Sparse array of RF sensors for sensing through the wall," *Proc. SPIE*, pp. 65470B-65470B, 2007.

- [108] Z. J. Ao, L. J. Kong, Y. Jia, and J. Y. Yang, "Multi-channel target cooperative detection in through-the-wall-radar imaging based on random sparse array," in *2011 IEEE CIE Int. Conf. Radar 2011*, pp. 176-178.
- [109] G. Dissanayake, S. Sukkarieh, E. Nebot, and H. Durrant-Whyte, "The aiding of a low-cost strapdown inertial measurement unit using vehicle model constraints for land vehicle applications," *IEEE Trans. Robot. Automat.*, vol. 17, pp. 731-747, 2001.
- [110] F. X. Cao, D. K. Yang, A. G. Xu, J. Ma, W. D. Xiao, C. L. Law, *et al.*, "Low cost SINS/GPS integration for land vehicle navigation," in *Proc. the IEEE 5th International Conference on Intelligent Transportation Systems*, 2002, pp. 910-913.
- [111] L. Huang and Y. L. Lu, "Radar imaging with compressed sensing for detecting moving targets behind walls," in *IET Int. Conf. Radar Systems (Radar 2012)*, 2012, pp. 1-5.
- [112] Y.-S. Yoon and M. G. Amin, "Compressed sensing technique for high-resolution radar imaging," *Proc. SPIE*, pp. 69681A-69681A, 2008.
- [113] M. Amin, F. Ahmad, and W. J. Zhang, "A compressive sensing approach to moving target indication for urban sensing," in *2011 IEEE Radar Conf. (RADAR)*, 2011, pp. 509-512.
- [114] R. J. Weber and H. Yikun, "Analysis for Capon and MUSIC DOA estimation algorithms," in *IEEE Antennas and Propagation Society International Symposium (APSURSI '09)* 2009, pp. 1-4.
- [115] T. T. Zhang, Y. L. Lu, and H. T. Hui, "Compensation for the mutual coupling effect in uniform circular arrays for 2D DOA estimations employing the maximum likelihood technique," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, pp. 1215-1221, 2008.
- [116] B. Porat and B. Friedlander, "Direction finding algorithms based on high-order statistics," in *Int. Conf. Acoust., Speech, and Signal Process.*, 1990, pp. 2675-2678.
- [117] B. Porat and B. Friedlander, "Direction finding algorithms based on high-order statistics," *IEEE Trans. Signal Process.*, vol. 39, pp. 2016-2024, 1991.
- [118] P. Chevalier, A. Ferreol, and L. Albera, "High-resolution direction finding from higher order statistics: the 2q-MUSIC algorithm," *IEEE Trans. Signal Process.*, vol. 54, pp. 2986-2997, 2006.
- [119] M. C. Dogan and J. M. Mendel, "Applications of cumulants to array processing .I. Aperture extension and array calibration," *IEEE Trans. Signal Process.*, vol. 43, pp. 1200-1216, 1995.
- [120] C. M. Tan, S. E. Foo, M. A. Beach, and A. R. Nix, "Ambiguity in MUSIC and ESPRIT for direction of arrival estimation," *Electronics Letters*, vol. 38, pp. 1598-1600, 2002.

AUTHOR'S PUBLICATIONS

JOURNAL PAPERS

- [1] L. Huang, Y. L. Lu, "Performance analysis of direct position determination for emitter source positioning," *American Journal of Signal Processing*, vol. 2, pp. 41-45, Jul 2012.
- [2] L. Huang, Y. L. Lu, "Target detection from microwave imaging based on random sparse array and compressed sensing," *Progress In Electromagnetic Research B*, vol. 53, pp.333-354, 2013.
- [3] L. Huang, Y. L. Lu, "Vector circular array based DOA and polarization estimations," submitted to *IEEE Trans. on Antennas and Propagation*.
- [4] L. Huang, Y. L. Lu, "A composite multiple resolution method for resolving DOA ambiguities based on vector circular array", to be submitted to *IEEE Trans. on Antennas and Propagation*.

CONFERENCE PAPERS

- [1] L. Huang and Y. L. Lu, "Performance analysis of direct position determination for emitter source positioning," *Proc. 2010 IEEE Antennas and Propagation Society International Symposium*, Toronto, Canada, 11-17 Jul. 2010.
- [2] L. Huang, Y. L. Lu, and W. Liu, "Using chirp signal for accurate RFID positioning," *Proc. 2010 International Conference on Communications, Circuits and Systems*, Chengdu, China, 28-30 Jul. 2010.
- [3] L. Huang and Y. L. Lu, "Through-the-wall tomographic imaging using chirp signals," *Proc. 2011 IEEE Antennas and Propagation Society International Symposium*, Spokane, Washington, USA, 3-8 Jul. 2011.
- [4] L. Huang and Y. L. Lu, "Through-the-wall moving target detection with compressed sensing," *Proc. 2012 IEEE Antennas and Propagation Society International Symposium*, Chicago, IL, USA, Jul. 2012.
- [5] L. Huang and Y. L. Lu, "Radar imaging with compressed sensing for detecting moving targets behind walls," *Proc. 2012 IET International Conference on Radar Systems*, Glasgow, UK, 22-25 Oct. 2012.
- [6] L. Huang and Y. L. Lu, "Multiple sources DOA and polarization estimation using vector circular array," *Proc. 2013 IET International Radar Conference*, Xian, China, 14-16 Apr. 2013.

- [7] L. Huang and Y. L. Lu, "DOA estimation in colored Gaussian noise using vector circular array," *Proc. 2013 IEEE Antennas and Propagation Society International Symposium*, Orlando, Florida, USA, 7-13 Jul. 2013.
- [8] L. Huang, Y. L. Lu, S. W. Yang and H. B. Zhang, "FOC-MUSIC method for signal parameter estimations using vector circular array," *Proc. 5th IEEE International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications (MAPE 2013)*, Chengdu, China, 29-31 Oct. 2013. (Received **Best Paper Award**).

Appendix A: Derivation of Solution of (5-17)

Denote the azimuth angle difference between u and v axes as $\Delta\phi_{uv}$,

$$\Delta\phi_{uv} = \phi_v - \phi_u \quad (\text{A-1})$$

Substituting (A-1) into the second equation of (5-17), we can represent $\cos\Phi_v^q$ using $\cos\Phi_u^q$.

$$\begin{aligned} \cos\Phi_v^q &= \sin\theta_q \cos(\phi_q - (\phi_u + \Delta\phi_{uv})) \\ &= \sin\theta_q \left[\cos(\phi_q - \phi_u) \cos(\Delta\phi_{uv}) + \sin(\phi_q - \phi_u) \sin(\Delta\phi_{uv}) \right] \quad (\text{A-2}) \\ &= \cos\Phi_u^q \cos(\Delta\phi_{uv}) + \sin\theta_q \sin(\phi_q - \phi_u) \sin(\Delta\phi_{uv}) \end{aligned}$$

By rearranging the known terms, we can get the following equation.

$$\frac{\cos\Phi_v^q - \cos\Phi_u^q \cos(\Delta\phi_{uv})}{\sin(\Delta\phi_{uv})} = \sin\theta_q \sin(\phi_q - \phi_u) \quad (\text{A-3})$$

Therefore, by squaring and summing up the first equation in (5-17) and (A-3), we can obtain a solution of θ_q . Since the elevation angle is within $[0^\circ, 90^\circ]$ of which the sine function is non-negative, it can be written as (A-4).

$$\theta_q = \arcsin \sqrt{(\cos\Phi_u^q)^2 + \left(\frac{\cos\Phi_v^q - \cos\Phi_u^q \cos(\Delta\phi_{uv})}{\sin(\Delta\phi_{uv})} \right)^2} \quad (\text{A-4})$$

Substituting (A-4) back into the first equation in (5-17) and (A-3), we can obtain another two equations by rearranging known and unknown terms.

$$\begin{aligned} \frac{\cos\Phi_u^q}{\sqrt{(\cos\Phi_u^q)^2 + \left(\frac{\cos\Phi_v^q - \cos\Phi_u^q \cos(\Delta\phi_{uv})}{\sin(\Delta\phi_{uv})} \right)^2}} &= \cos(\phi_q - \phi_u) \\ \frac{\cos\Phi_v^q - \cos\Phi_u^q \cos(\Delta\phi_{uv})}{\sin(\Delta\phi_{uv}) \sqrt{(\cos\Phi_u^q)^2 + \left(\frac{\cos\Phi_v^q - \cos\Phi_u^q \cos(\Delta\phi_{uv})}{\sin(\Delta\phi_{uv})} \right)^2}} &= \sin(\phi_q - \phi_u) \end{aligned} \quad (\text{A-5})$$

We simplify (A-5) to (A-6) using the substitutions in (A-7).

$$\begin{aligned} a_1 &= \cos \beta_q \\ a_2 &= \sin \beta_q \end{aligned} \tag{A-6}$$

where

$$\begin{aligned} a_1 &= \frac{\cos \Phi_u^q}{\sqrt{(\cos \Phi_u^q)^2 + \left(\frac{\cos \Phi_v^q - \cos \Phi_u^q \cos(\Delta \phi_{uv})}{\sin(\Delta \phi_{uv})} \right)}} \\ a_2 &= \frac{\cos \Phi_v^q - \cos \Phi_u^q \cos(\Delta \phi_{uv})}{\sin(\Delta \phi_{uv}) \sqrt{(\cos \Phi_u^q)^2 + \left(\frac{\cos \Phi_v^q - \cos \Phi_u^q \cos(\Delta \phi_{uv})}{\sin(\Delta \phi_{uv})} \right)}} \\ \beta_q &= \phi_q - \phi_u \end{aligned} \tag{A-7}$$

To solve (A-6), we need to carry out a condition check on the sign of a_1 and a_2 .

Condition 1: when $a_1 > 0, a_2 \geq 0$, we obtain a solution of β_q .

$$\beta_q = \arccos(a_1), \quad \in \left[0, \frac{\pi}{2} \right] \tag{A-8}$$

Condition 2: when $a_1 < 0, a_2 \geq 0$,

$$\beta_q = \arccos(a_1), \quad \in \left[\frac{\pi}{2}, \pi \right] \tag{A-9}$$

Condition 3: when $a_1 > 0, a_2 < 0$,

$$\beta_q = 2\pi - \arccos(a_1), \quad \in \left[\frac{3\pi}{2}, 2\pi \right] \tag{A-10}$$

Condition 4: when $a_1 < 0, a_2 < 0$,

$$\beta_q = 2\pi - \arccos(a_1), \quad \in \left[\pi, \frac{3\pi}{2} \right] \tag{A-11}$$

By combining the solutions from all the conditions and using a_1 to represent the solutions, we obtain the common solution of β_q .

$$\begin{aligned} \text{If } a_2 \geq 0, \quad \beta_q &= \arccos(a_1) && \in [0, \pi] \\ \text{If } a_2 < 0, \quad \beta_q &= -\arccos(a_1) && \in [-\pi, 0] \end{aligned} \quad (\text{A-12})$$

By substituting $\beta_q = \phi_q - \phi_u$ into (A-12), we can obtain the final solution of ϕ_q as in (5-19).