Durability of woven composite structures

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DURABILITY OF WOVEN COMPOSITE STRUCTURES

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DURABILITY OF WOVEN COMPOSITE STRUCTURES

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ABSTRACT

The current study focuses on the durability of the composite structures, especially structures that are made of woven carbon-epoxy. The fatigue behaviour of woven composite structures is different from the fatigue behaviour of structures made of unidirectional (UD) and multidirectional (MD) composites. The fatigue behaviour of woven composites is much more complex than UD and MD composites because the interaction between warp and fill zone need to be taken into account. Thus, a thorough study on fatigue behaviour of woven composites is very important and much needed at this present time.

Material characterization has been done on L-930 flame retardant woven carbon-epoxy using accelerated testing methodology proposed by Miyano. The material characterization includes: determining the time-temperature shift factor, storage modulus master curve, constant strain rate (CSR) strength master curve, zero stress ratio fatigue strength master curve and fatigue strength at arbitrary load ratio, temperature and frequency for both tensile and shear properties. Several essential tests were performed to fully characterize a composite material: DMA test, constant strain rate (CSR) test at several different temperatures and zero stress ratio fatigue test at several different temperatures. There are evidences showing that accelerated testing methodology cannot be used to predict the shear fatigue strength at arbitrary load ratio but can be used to predict the shear fatigue strength at arbitrary temperature and zero load ratio. It was also found that modulus decay is linear over the log of number of cycles from the beginning until the end of fatigue life and the rate of modulus decay increases as the stress increases.

Finite element modelling and analysis were then carried out to predict the static strength and fatigue life of an I-beam structure using the material characterization parameters obtained earlier. A new "stiffness decay model" is introduced. The new model is based on maximum stress and modified Hashin criteria and was implemented in the USDFLD subroutines in conjunction with the finite element models. The static strength and fatigue life of an I-beam composite as well as stiffness decay were predicted and their results agreed very well with the experimental results.

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TABLE OF CONTENTS

ABSTRACT	2
ACKNOWLEDGEMENTS	3
TABLE OF CONTENTS	4
TABLE OF FIGURES	6
LIST OF TABLES	11
NOMENCLATURES	12
1 INTRODUCTION	14
1.1 Background	14
1.2 Objectives	15
1.3 Scope	15
1.4 Outline	16
2 LITERATURE REVIEW	18
2.1 Factors Affecting Fatigue Behaviour of Composite	18
2.2 Composite Damage Mechanism	23
2.3 Composite Failure Criteria	31
2.4 Composite Fatigue Modelling	39
2.5 Fatigue Strength Accelerated Testing Methodology	44
3 EXPERIMENTAL SET-UP	48
3.1 Specimen Preparation	48
3.2 Specimen Testing	52
4 MATERIAL CHARACTERIZATION	55
4.1 Tensile Properties Characterization	56
4.2 Shear Properties Characterization	68
4.3 Properties Characterization for Finite Element Modelling	75
5 FATIGUE MODELLING AND EXPERIMENTAL VALIDATION	83
5.1 Problem Definition	83
5.2 Finite Element Modelling	84
5.3 Finite Element Results and Experimental Validation	93

6	CONCLUSION AND FUTURE WORKS	102
6.1	Conclusion	102
6.2	Original Technical Contributions.....	103
6.3	Future Works	103
	LIST OF PUBLICATIONS	104
	REFERENCES	105
APPENDIX I	MAXIMUM STRESS PROGRESSIVE DAMAGE SUBROUTINE	111
APPENDIX II	MODIFIED HASHIN PROGRESSIVE DAMAGE SUBROUTINE.....	119

TABLE OF FIGURES

Chapter 2

Figure 2.1: Factors affecting the fatigue of fibre-reinforced composites	19
Figure 2.2: S-N fatigue data for unidirectional composite materials.....	20
Figure 2.3: Damage accumulation in laminated composites made of unidirectional layers during fatigue life.....	25
Figure 2.4: Damage development in a cross-ply laminate during tension-tension fatigue	26
Figure 2.5: Weave patterns of woven composites.	27
Figure 2.6: Micro-structural damages in woven composites: a) fibre breakage, b) matrix mico-cracking, c) fibre-matrix interfacial debonding.	27
Figure 2.7: Macro-structural damages in woven composites: a) transverse crack, b) shear failure, c) pure matrix region crack, d) delamination between fill and warp, e) delamination between adjacent layers and f) warp tensile failure.	28
Figure 2.8: Modulus decay and damage accumulation in woven-fabric composites during fatigue life.	29
Figure 2.9: Schematic of a unit cell.	29
Figure 2.10: Fatigue life comparison between unidirectional laminates (dashed line) and satin weaved fabric composites (solid line) under tension-compression fatigue loading.	30
Figure 2.11: 3D-fabrics category	31
Figure 2.12: Composite failure criteria classification.	32
Figure 2.13: Three dimensional state of stresses.	33
Figure 2.14: Evaluation of correlation coefficient C'_{12}	35
Figure 2.15: Composite fatigue modelling classification.	40
Figure 2.16: Tests needed in order to fully characterize a composite material.....	43
Figure 2.17: Flow chart of all the steps needed to fully characterized fatigue properties of a material based on Miyano's methodology.	45

Chapter 3

Figure 3.1: The arrangement of additional layers on prepreg for flat specimen before curing.....	49
Figure 3.2: Curing table.....	49
Figure 3.3: Autoclave	49
Figure 3.4: Curing process graph.	50
Figure 3.5: Woven CFRP specimen after curing, a) DMA specimen, b) shear CSR and fatigue specimen, c) tensile CSR and fatigue specimen.	50
Figure 3.6: a) I-beam prepreg stacking sequence, b) the arrangement of additional layers on prepreg for I-beam specimen before curing.....	51
Figure 3.7: I-beam specimen arrangement before curing process in autoclave.....	51
Figure 3.8: I-beam woven CFRP specimen after curing.....	51
Figure 3.9: a) DMA TA Q800, b) specimen installed in single cantilever clamp.	52
Figure 3.10: MTS810 systems with temperature chamber, a) for tension-tension fatigue testing, b) for shear and 4-point bending fatigue testing.	53
Figure 3.11: MTS 810 fixtures: a) tensile fixture with extensometer, b) shear fixture, c) 4-point bending fixture	54
Figure 3.12: Typical load input in MTS 810 fatigue test.	54

Chapter 4

Figure 4.1: L-930 flame retardant woven carbon-epoxy a) DMA specimen dimensions (mm) and b) DMA specimen picture.	56
Figure 4.2: a) Frequency sweep graphs at several temperatures for L-930-SC1 specimen, b) master curve of storage modulus for L-930-SC1 specimen. ...	57
Figure 4.3: Time-temperature shift factor of L-930 specimens ($T_g=109.375^{\circ}\text{C}$).....	58
Figure 4.4: a) CSR and fatigue specimen dimensions, b) L-930 CSR specimen ($a=0.45\text{mm}$, $b=1.35\text{mm}$).	59
Figure 4.5: CSR tensile stress vs strain graphs	60
Figure 4.6: Typical failure of L-930 specimen under tensile static loading.....	61

Figure 4.7: a) Tensile CSR results and b) master curve of tensile CSR at 25°C with curve fitting based on Christensen's theory ($\sigma_{s,0} = 1072.2056\text{MPa}$, $t'_1 = 1.2 \times 10^{20}$ and $n_r = 0.4$).....	61
Figure 4.8: The predicted tensile creep strength master curve.....	62
Figure 4.9: Typical normalized peak displacement against normalized number of cycles results	63
Figure 4.10: Tension-tension fatigue S-N curve at 25°C and zero stress ratio.	64
Figure 4.11: Tension-tension fatigue S-N curve at 100°C and zero stress ratio.	64
Figure 4.12: Tension-tension fatigue S-N curve at 175°C and zero stress ratio	64
Figure 4.13: Typical failed L-930 specimen under fatigue loading	65
Figure 4.14: Shifted zero stress ratio tension-tension fatigue strength vs log failure time.	65
Figure 4.15: Zero stress ratio tension-tension fatigue strength master curve	66
Figure 4.16: Predicted fatigue strength in master curve of zero stress ratio fatigue strength.	67
Figure 4.17: Predicted tension-tension fatigue strength vs log failure time curve at 4Hz, 80°C and stress ratio = 0.5.	67
Figure 4.18: Predicted tension-tension fatigue strength and validation results at 4Hz, 80°C and stress ratio = 0.5.	67
Figure 4.19: CSR Shear coupon dimensions based on ASTM D7078	68
Figure 4.20: CSR shear stress vs actuator displacement graphs.....	69
Figure 4.21: Typical failed specimen under shear CSR loading.....	70
Figure 4.22: a) Shear CSR results and b) master curve of shear CSR at 25°C with curve fitting based on Christensen's theory ($\sigma_{s,0} = 230\text{ MPa}$, $t'_1 = 1 \times 10^{-6}$ and $n_r = 0.056$).	70
Figure 4.23: The predicted shear creep strength master curve.	71
Figure 4.24: Typical shear fatigue peak displacement graph.....	71
Figure 4.25: Shear fatigue S-N curve at 25°C and zero stress ratio.	72
Figure 4.26: Shear fatigue S-N curve at 100°C and zero stress ratio.	72
Figure 4.27: Shear fatigue S-N curve at 175°C and zero stress ratio	73
Figure 4.28: Typical failed L-930 specimen under fatigue loading.	73

Figure 4.29: Shifted shear zero stress ratio vs log failure time.....	73
Figure 4.30: Shear zero stress ratio fatigue strength master curve.....	74
Figure 4.31: Predicted shear fatigue strength at 4Hz, 80°C and stress ratio = 0.5.....	75
Figure 4.32: Predicted shear fatigue strength and validation results at 4Hz, 80°C and stress ratio = 0.5.....	75
Figure 4.33: Predicted shear fatigue strength and validation results at 5Hz, 80°C and zero stress ratio.....	75
Figure 4.34: Tensile specimen picture (autoclave).	76
Figure 4.35: Tensile CSR stress vs strain graph at 25°C and 0.2mm/min (autoclave).	76
Figure 4.36: Typical failure of L-930 specimen under tensile static loading (autoclave).	77
Figure 4.37: Normalized tensile peak displacement vs normalized no of cycle to failure graphs at 25°C, zero stress ratio and maximum load: a) 60% and 65%, b) 70% and 75%, c) 80% and 90% of the static strength.....	77
Figure 4.38: S-N curve for L-930 specimen (autoclave) at 25°C and zero stress ratio	78
Figure 4.39: a) Shear specimen picture (autoclave), b) typical failure of L-930 specimen under shear static loading (autoclave) L-930 specimen at 25°C, maximum load = 11kN and zero stress ratio.	79
Figure 4.40: Shear CSR stress vs strain graph at 25°C and 1mm/min (autoclave).	79
Figure 4.41: Shear maximum displacement vs normalized no of cycle to failure graphs at 25°C and zero stress ratio	79
Figure 4.42: Shear S-N curve for L-930 specimen (autoclave) at 25°C and zero stress ratio	80
Figure 4.43: Tensile S-N curve comparison between L-930 woven carbon-epoxy made in curing table and autoclave.....	81
Figure 4.44: Shear S-N curve comparison between L-930 woven carbon-epoxy made in curing table and autoclave	82

Chapter 5

Figure 5.1: Specimen dimensions: a) isometric view, b) front view.....	83
Figure 5.2: Tensile coupon model.....	84
Figure 5.3: Convergence test results of shear coupon model	85

Figure 5.4: Shear coupon model	85
Figure 5.5: Symmetries in the I-beam model.	86
Figure 5.6: Convergence test results of 4-point bending I-beam model.....	86
Figure 5.7: I-beam model	87
Figure 5.8: Shear static stress vs strain of L-930 woven carbon-epoxy.....	89
Figure 5.9: a) Typical maximum displacement vs no of cycle result, b) typical modulus decay vs relative log no of cycle result	90
Figure 5.10: a) Normal modulus decay rate vs normal stress, b) Shear modulus decay rate vs shear stress.....	91
Figure 5.11: Flowchart of damage and failure in: a) maximum stress fatigue damage model, b) modified hashin fatigue damage model.....	92
Figure 5.12: a) Tensile static stress vs strain graph, b) tension-tension fatigue S-N curve.....	93
Figure 5.13: a) Shear static stress vs strain graph, b) failed elements in the static and fatigue shear coupon models (red).....	94
Figure 5.14: Shear coupon progressive fatigue results	95
Figure 5.15: I-beam static results.....	96
Figure 5.16: Failed elements in maximum stress and modified Hashin static damage models.....	96
Figure 5.17: Failed Failed I-beam woven CFRP in static test	97
Figure 5.18: Failed elements in maximum stress and modified Hashin fatigue damage models.....	98
Figure 5.19: Failed I-beam woven CFRP in fatigue test	99
Figure 5.20: I-beam S-N curve: a) 1st element failure, b) final failure	100
Figure 5.21: Relative stiffness against log number of cycles with maximum load = a) 1836N, b) 1887N, c) 1938N, d) 2040N.....	100

LIST OF TABLES

Chapter 4

Table 4.1:	L-930 woven carbon-epoxy DMA results.....	58
Table 4.2:	Average CSR results for L-930 specimen	59
Table 4.3:	Tension-tension fatigue results for L-930 specimen	63
Table 4.4:	Average shear CSR test results.	70
Table 4.5:	Shear fatigue results for L-930 specimen.	72
Table 4.6:	Tensile static properties in the warp and fill directions.	77
Table 4.7:	Zero stress ratio tension-tension fatigue test results table at 25°C (autoclave).	78
Table 4.8:	Zero stress ratio shear fatigue test results table at 25°C (autoclave).	80
Table 4.9:	L-930 flame retardant woven carbon-epoxy static properties.....	81
Table 4.10:	Burn test results.....	82

Chapter 5

Table 5.1:	Material properties relation with field variables (maximum normal stress criterion)	89
Table 5.2:	Material properties relation with field variables (modified Hashin criterion) .	89
Table 5.3:	I-beam 4-point bending static results.....	96
Table 5.4:	I-beam 4-point bending fatigue number of cycles to failure results: a) finite element models, b) experiments.....	99

NOMENCLATURES

σ_{11}	Normal stress in the fibre direction
σ_{22}	Normal stress in transverse to fibre direction
σ_{33}	Out of plane normal stress
τ_{12}	In plane shear stress
ε_{11}	Normal strain in the fibre direction
ε_{22}	Normal strain in the transverse direction
γ_{12}	In plane shear strain
E_{11}	Modulus elasticity of the ply in the direction of the fibres
E_{22}	Modulus elasticity of the ply in the transverse direction
ν_{12}	In plane Poisson's ratio
$\sigma_{11t}, \sigma_{22t}$	Normal tensile stress in the fibre and transverse direction
$\sigma_{11c}, \sigma_{22c}$	Normal compressive stress in the fibre and transverse direction
$\sigma_{11u}, \sigma_{22u}$	Ultimate normal stress in the fibre and transverse direction
$\sigma_{11tu}, \sigma_{22tu}$	Ultimate normal tensile stress in the fibre and transverse direction
τ_{12u}	Ultimate in plane shear stress
$\sigma_{11cu}, \sigma_{22cu}$	Ultimate normal compressive stress in the fibre and transverse direction
α	Shear stress contribution coefficient to the Hashin's fibre tensile initiation criterion
$\varepsilon_{11t}, \varepsilon_{22t}$	Normal tensile strain in the fibre and transverse direction
$\varepsilon_{11c}, \varepsilon_{22c}$	Normal compressive strain in the fibre and transverse direction
$\varepsilon_{11tu}, \varepsilon_{22tu}$	Ultimate normal tensile strain in the fibre and transverse direction
γ_{12u}	Ultimate in plane shear strain
$\varepsilon_{11cu}, \varepsilon_{22cu}$	Ultimate normal compressive strain in the fibre and transverse direction
τ_{23}, τ_{13}	Out of plane shear stress

τ_{23u}, τ_{13u}	Ultimate out of plane shear stress
E	Isotropic modulus elasticity
T	Temperature
T_0	Reference temperature
T_g	Glass transition temperature
$a_{T_0}(T)$	Time-temperature shift factor at particular temperature
ΔH	Activation energy
G	Gas constant
t_s	Static failure time
t_c	Creep failure time
t	Time
t'	Shifted time
f	Frequency
f'	Shifted frequency
t_f	Failure time
t'_f	Shifted failure time
N_f	Number of cycles to failure
σ_f	Fatigue strength
$\sigma_{f:1}$	Fatigue strength at stress ratio equal to one (creep strength)
$\sigma_{f:0}$	Zero stress ratio fatigue strength
SR	Stress ratio

Chapter One

INTRODUCTION

1.1 Background

The reasons for the increasing popularity of composites in weight critical application are their high specific stiffness and strength. Some examples of engineering applications where composites have become indispensable are in the area of sporting goods, aircraft and aerospace, automobile and marine. In these applications, cyclic and fluctuating loads are common and this type of loading condition will eventually cause fatigue in structures. The subject of fatigue and life prediction of materials and structures are usually intertwined. The current knowledge on fatigue and the prediction of life of composite structures is still at its infancy.

In the 60s and 70s, many engineers and researchers knew that metals suffered from fatigue and there was a misconception then that composites do not suffer from fatigue. There was however a handful of published literature on the fatigue behaviour of glass-fibre-composites. These composites exhibit a form of degradation in service that can be described as 'fatigue'. A simplistic description of this 'fatigue' phenomenon is that under cyclic loading condition, the load-bearing capacity of the materials falls with time and this results in failures at stress levels which are often well below the normal engineering strength. The mechanisms by which this deterioration occurs in composites are completely different from those which are responsible for the fatigue phenomena in metals. Not only are these mechanisms different, they are more complicated too. From the engineer's point of view, the challenge is to choose the appropriate material for a specific structural application so as to avoid either material or structural failure within the design life of a component or structure. Thus there is a need to understand the mechanisms of degradation in service and to be able to predict the life of a given composite under particular design condition.

Some of the earlier notable literature published in the area of fatigue response of fibre-reinforced composites were by Boller [1] in the early 70s, followed by Owen and

his collaborators [2]. While much of this early work on fatigue is focused on phenomenological studies, it quickly became apparent that an understanding of the micro-structural damage mechanisms responsible for failure under cyclic loading is a prerequisite for the development of new fatigue-resistant materials and also vital in the prediction of fatigue life. Researchers such as Reifsnider and Talreja [3] are associated with the key developments in the emerging field of damage mechanics. The build-up of fatigue damage is essentially a stochastic process and vital statistical interpretations of fatigue behaviour in conjunction with life prediction of composites were published by Hahn [4], Whitney [5] and Yang [6]. Almost all the composite fatigue life predictions mentioned earlier are empirical models which depend quite a lot on experimental results and only a few of them can be applied in the real structure. It leaves us with one concern: building a model which depends on fewer experiments. Miyano et al. [7] have made a considerable contribution of creating accelerated testing methodology in order to reduce the number of experiments needed to characterize composite material fatigue properties. Although a lot of research has been done in this area, more study is still needed in order to build a model which is simple, gives good failure prediction and depends less on experimental results of composite structures under fatigue loading.

1.2 Objectives

Under fatigue load, the behaviour of a composite structure depends on more variables than when it is under static load and incorporates different failure mechanisms as well. The first objective of this thesis is to fully characterize composite material properties using relevant experiments. The last objective of this thesis is to develop a new progressive damage model which gives information about stiffness degradation at every cycle and number of cycles to the failure as well.

1.3 Scope

The scope of work of the current research covers the following:

1. Comprehensive and extensive literature search and review.

A comprehensive and extensive literature search and review have been done in order to attain a better understanding and foundation to the research.

2. Static and fatigue properties characterization of composites.
The static and fatigue properties of a composite material were characterized using accelerated testing methodology developed by Miyano et al. [7]. The properties were characterized for arbitrary temperature, frequency and load ratio.
3. Static and fatigue damage modelling.
A new "stiffness decay model" that can predict the progressive static failure as well as stiffness degradation over the cycle and number of cycles to failure was introduced.
4. Experimental validation.
The new models developed for static and fatigue failure predictions are finally validated through experimental results of static and fatigue tests on woven CFRP I-beams.

1.4 Outline

This report is organized in the following structure:

- **Chapter 1 – Introduction**

Chapter one explains about the background, objective of the research, the scope of the thesis and the outline of this report.

- **Chapter 2 – Literature Review**

Literature review covers all the theories and finding of published work related to the current study.

- **Chapter 3 – Experimental Set-Up**

The set-up of the experiments and devices that are used in the research are presented.

- **Chapter 4 – Material Characterization**

The step by step procedures of composite material static and fatigue properties characterization are explained in this chapter.

- **Chapter 5 – Fatigue Modelling and Experimental Validation**

The procedures of developing the new "stiffness decay model" to predict the static and fatigue performance of a woven CFRP structure as well as experimental validation results are discussed in this chapter.

- **Chapter 6 – Conclusion and Future Works**

Conclusions deduced from the results presented in Chapter 4 and 5, as well as the future works that can be done are discussed in this chapter.

Chapter Two

LITERATURE REVIEW

It is well known that composites have high strength to weight ratio, which makes the use of composites popular nowadays especially where light weight structures are of utmost importance. Sporting goods, aircraft, automobile and shipbuilding are just a few examples where composites are normally used. Despite these advantages, the knowledge about composites is not complete yet and further research is still needed in several areas. One of those is composites under fatigue loading. The behaviour of composites under fatigue loading is completely different from that of metal. Fatigue in metal occurs by the initiation of a single crack, which propagates until catastrophic failure occurs. In contrast to metal, damage build up in composites is in a global fashion rather than in a localized fashion. Composites have several damage accumulation mechanisms, fibre-matrix debonding, matrix cracking, delamination and fibre fracture. These damage mechanisms can occur independently or interactively depending on material properties and testing conditions [8].

The purpose of this literature review is to gain a better understanding of composite materials under fatigue loading. Several things will be discussed here: factors affecting fatigue behaviour, damage mechanism, failure criteria, fatigue modelling and accelerated testing methodology.

2.1 Factors Affecting Fatigue Behaviour of Composite

Several factors of an inherent and external nature that affect the fatigue behaviour of fibre-reinforced composites are compiled in Figure 2.1. Each of these factors will be discussed in some detail in subsequent sections.

– Types of fibre

As the main load carrier in composites, the type of fibres used affect the composites fatigue behaviour as the fibres carry most of the load. Figure 2.2 shows typical plots of peak tensile stress versus log cycles to failure for three common fibre-reinforced composites. The S-N curve of glass fibre-reinforced plastic (GFRP) shows a

more drastic drop in its fatigue strength than that of carbon fibre-reinforced plastic (CFRP). The use of very stiff carbon fibre limits the strain in the composite and thus prevents large deformation in the matrix which can lead to premature initiation of damage. On the other hand, the use of less stiff glass fibre allows for large deformation in the matrix giving rise to fatigue failure. The fatigue performance of kevlar fibre-reinforced plastic (KFRP) is more complicated than that of CFRP and GFRP due to the fact that kevlar fibre is fatigue sensitive [9]. The task of compiling a fatigue database for composites can be rather daunting, considering that there are many different types of fibre materials available in the market. Some typical glass fibres that are found in the market are E-glass, ECR-glass, C-glass and S-glass fibres. S-glass, for example, has higher stiffness and strength in comparison with the other glass fibres. Each of these glass fibres react differently under a corrosive environment as well [10]. The effect of corrosion on fatigue of composites will be discussed later. There are several different types of carbon fibres such as T300, AS4 and IM6, among these, IM6 has smaller fibre diameter but higher stiffness and strength [11, 12].

Figure 2.1: Factors affecting the fatigue of fibre-reinforced composites

Figure 2.2: S-N fatigue data for unidirectional composite materials [9].

– Types of matrix

Several researchers have shown that the fatigue strength of glass fibre-reinforced composites is significantly dependent on the properties of the resin [1, 2]. Fatigue damage in the form of crack initiation usually starts in the matrix region. There are researches that show the advantage of thermoplastic resin over thermoset resin in terms of ductility and toughness [13-15]. These resulted in considerably longer fatigue life of thermoplastics resin [15]. The other advantage of a tougher resin is its higher interlaminar fracture toughness which will result in increased fatigue resistance against delamination [16]. The fracture toughness of fibre-reinforced composite is affected by the interface between matrix and fibre as well. Weaker interface tends to improve the fracture toughness by resisting crack propagation through the matrix, but reduces the effectiveness of the stress transfer [17]. In order to get the desirable performance, interfacial adhesion can be controlled by surface treatment, such as plasma treatment, fibre sizing and coating, electro-discharge, dry and wet oxidation [18].

Resistance to crack propagation in the matrix material can also be increased by adding rubber particles in the resin [19]. Adding nano-particles in the matrix material can increase the composite tensile strength, impact strength and fatigue life quite significantly [20]. The addition of nano-particles such as carbon nanotubes in polymer matrices will also allow damage sensing via electrical signals. Micro-scale damage such as inter-fibre failure and matrix micro-cracking and macroscopic damage such as delamination and rupture of fibre bundles can be detected using this damage sensing method [21, 22]. Besides being able to sense damage, its piezo-resistive properties also makes it possible to measure strain-rate in carbon nanotube-polymer composites [21, 23].

- Stacking sequence and type of reinforcement

The fatigue damage mechanism of composites depends on their stacking sequence and reinforcement type, thus defining the unique fatigue properties of stacked composites. The effect of stacking sequence and reinforcement type on fatigue damage mechanism of composite materials will be discussed later in the section on composite damage mechanism.

- Loading conditions

The fatigue damage and failure response of composites depend largely on the loading conditions [9, 24-26]. For instance, the fatigue performances of a particular composite under tension-tension fatigue will be different from those under tension-compression or compression-compression fatigue [24]. Fibre failure is the main failure mode in unidirectional composites under tension-tension fatigue [9]. However, during tension-compression and purely compressive cycling, cracks propagate through the spreading of fibre buckling failure zones [24-26]. In various combinations of axial tension-compression cycling, the fatigue resistance of unidirectional composites decreases as the compressive stress increases [24]. Due to the poor compression response of aramid fibre composites, the compression stress will be likely to be even more damaging to composite containing aramid fibre than those of carbon fibre and glass fibre [27].

– Environmental conditions

The effect of three environmental conditions, i.e. temperature, moisture and corrosion, on fibre-reinforced composites properties have been studied and will be discussed in the following.

○ Temperature

One of the earliest and perhaps most researched of environmental conditions that affect the fatigue properties of composites is temperature. Temperature is known to degrade and age the mechanical properties of the resin material. The type of resin used will therefore affect the fatigue performance of composites at high temperature [28]. Miyano et al. [7, 29-31] used the strong relationship between time and temperature in composite fatigue performance to build an accelerated testing methodology which will be discussed later in this chapter.

○ Moisture

Moisture is known to affect the properties of the resin but not those of the fibre. Moisture affects the thermo-mechanical properties of the resin through plasticization or hydrolytic or chemical degradation of the resin network. This will in turn reduce the composites life and maximum service temperature (moisture lowers the glass transition temperature of the resin). One important parameter in defining the effect of moisture is the moisture diffusion rate. Several factors that affect the moisture diffusion rate are [32]:

- The polarity of the molecular structure
- The degree of cross-linking
- The degree of crystallinity in the case of a thermoplastic matrix
- The presence of residuals in the material

○ Corrosion

Fibre-reinforced composites are well known for their high corrosion resistance compared to metals. Thus fibre-reinforced composites are found in many structural applications of a corrosive nature such as components like pipe,

scrubber, beam, etc [10]. Recent research showed that fibre-reinforced composites are susceptible to acidic corrosive environments [10, 33]. It was discovered that a polymer matrix degrades faster in the acidic environment [33] and fibre, especially E-glass fibre, failed at a much lower load than the design load due to environmental stress corrosion cracking (ESCC) [10]. The problem of ESCC escalates at higher acid concentration coupled with temperature and load. There are several glass fibres such as ECR, C and S-glass fibres that give better resistance against ESCC [10]. ECR-glass essentially is boron and fluorine free E-glass. With the removal of boron and fluorine, the chemical resistance - especially acid resistance - of ECR-glass fibre is vastly improved [34]. C-glass fibre was developed specifically to resist chemical attack and S-glass fibre is a high strength glass fibre that gives stability under extreme corrosive environments as well [10].

2.2 Composite Damage Mechanism

The damage mechanism of composites in a fatigue environment will be reviewed and discussed in this section. The particular case of fibre-reinforced composites subjected to cyclic tensile-tensile loading will be emphasized. The two main micro-structural damage mechanisms commonly observed in composites under cyclic loading are fibre failure and matrix failure [3, 35].

– Fibre failure

Fibre failure in composites regardless of static or fatigue failure is classified into two modes of failure : tensile and compressive fibre failure [35]. The typical tensile fibre failure modes are fibre pull-out, fibre fracture and debonding. Fibre pull-out failure occurs where both fibre and matrix are brittle [36].

For a composite in tension, local fibre fracture occurs in the early loading stage and stress redistribution follows. After which, debonding of fibre from matrix occurs and this is followed by fibre breakage that leads to the final failure [35]. Compressive fibre failure is however less dependent on fibre strength and depends more on fibre stability such as fibre micro-buckling and kinking. Free edge and area in the vicinity of voids are the places where fibre micro-buckling usually initiates [35]. Compressive fibre failure is also affected by fibre misalignment. It has been reported that a 0.25°

fibre misalignment can reduce the compressive strength of a unidirectional composite up to 70% of its initial value [35].

- Matrix failure

Matrix failure can be distinguished into two: matrix failure in a ply (inter-fibre fracture) and matrix failure in between plies (delamination) [37]. Inter-fibre fracture most likely starts at fibre-resin interface then propagates to the resin. On the other hand, delamination is caused by interlaminar stress, which is a direct effect of microcracks in the resin [35]. Free edges of multi-directional laminates generally produce interlaminar shear stress singularities that initiate microcracks. In general, the severity of the free-edge effect depends on the ply orientation of two adjacent plies.

The failure process as a result of these various damage mechanisms depends on the type of reinforcement in the composites; unidirectional, multi-directional, woven and 3D reinforcement, which will be explained one by one in the following.

- Unidirectional composites

The fatigue response of unidirectional composites under tension-tension fatigue load is typically a function of the fibre properties of the composites and the alignment of the fibre from the loading axis. In the early stage of the fatigue response of composites with fibres aligned in the loading direction, matrix cracks will initiate in the direction along the fibres [38, 39]. As the cyclic load continues, the cracks grow and accumulate into several stress concentration spots. When the maximum cyclic load reaches the residual strength, fibre matrix breakage will cause total failure. The damage mechanism of unidirectional composites aligned at smaller ply angle (less than 20°) will be essentially the same as the damage mechanism of unidirectional composites aligned at 0° [40]. However, the final failure of unidirectional composites aligned at larger ply angle (20° or more) will most likely to be dominated by matrix failure [40].

- Multidirectional composites

Two types of multidirectional composites will be discussed here, namely the cross-ply laminate and angle-ply laminate. Figure 2.3 illustrates the damage accumulation versus life for a cross-ply laminate under tension-tension fatigue loading. The damage in the Y axis is quantified from the stiffness degradation, with damage value equal to 1 means that the laminate has failed. The progression of damage that cause the damage accumulation in Figure 2.3 is shown in Figure 2.4 [41]. At the initial stage of fatigue life, damage accumulates very rapidly. The initiation of fatigue damage is the appearance of matrix cracks perpendicular to loading direction, followed by matrix cracks along the fibre in the transverse plies. As the laminate is further stressed (middle stage of fatigue life), the crack density increases and this is followed by crack coupling and fibre/matrix debonding (debonding here depends on the interfacial strength between fibre and matrix). The next stage of damage mechanism is in the form of delamination. Delamination will initiate near the free edges because of high edge interlaminar stresses. As the loading continues, the size of delamination will grow. The last stage of damage mechanism is breakage of the fibres aligned in the loading direction. This description of damage mechanism and their progression is also applicable to the unidirectional laminated composites.

Figure 2.3: Damage accumulation in cross-ply laminate during fatigue life [41].

Figure 2.4: Damage development in a cross-ply laminate during tension-tension fatigue[41].

Fatigue damage mechanism in angle-ply laminates is however very dependent on its ply orientation with matrix failure dominating for larger ply angles. The final failure in the form of fibre breakage is more common in smaller ply angles in angle-ply laminates. More intensive and detailed study in this area is much needed, especially in understanding the relation between ply orientation and fatigue properties of practical laminates.

– Woven composites

Woven composites have many advantages compared to unidirectional and multi-directional composites. Some of these advantages are better impact resistance, damage tolerance, dimensional stability over a large range of temperature and ease of manufacturing. They have however lower overall in-plane properties than unidirectional composites [41]. The structural behaviour of woven composites is affected by the fibre material, matrix material, weave pattern, fabric geometry, fibre volume fraction and laminate configuration [42-46].

Woven composites come in several different weave patterns such as plain, twill and satin weaves as illustrated in Figure 2.5. Plain weave is a symmetrical weave as shown in Figure 2.5(a). It offers good stability and reasonable porosity but very difficult to be formed to a complex shape. Twill weave (Figure 2.5(b)) on the other hand, is very easy to be formed to a complex shape, have higher mechanical properties and smoother surface but it offers slight reduction in stability. Satin weave is basically twill weave with fewer intersections of warp and fill as shown in Figure 2.5(c). This type of weave is very flat, have good mechanical properties and

can be formed easily to complex shape, but the stability is low and has asymmetric effect [47]. In addition, the number of fibres per tow of these woven composites is also varied. There are four standard amounts: 1,000 fibres/tow (1k), 3,000 fibres/tow (3k), 6,000 fibres/tow (6k) and 12,000 fibres/tow (12k).

Figure 2.5: Weave patterns of woven composites [47].

The micro-structural damages that occur in woven composites under fatigue loading are normally in the form of matrix micro-cracking, fibre breakage, crack coupling and fibre-matrix interfacial debonding as shown in Figure 2.6. Transverse crack (fill direction of the weave), shear failure (warp direction of the weave), pure-matrix region cracks, delamination between fill and warp, delamination between adjacent layers and warp tensile failure are the usual macroscopic damage mechanisms as shown in Figure 2.7.

Figure 2.6: Micro-structural damages in woven composites: a) fibre breakage, b) matrix micro-cracking, c) fibre-matrix interfacial debonding [48].

Figure 2.7: Macro-structural damages in woven composites: a) transverse crack, b) shear failure, c) pure matrix region crack, d) delamination between fill and warp, e) delamination between adjacent layers and f) warp tensile failure [48].

Figure 2.8 shows the modulus decay and damage accumulation in woven-fabric composites during fatigue life. Modulus decay is basically the ratio between the modulus at a particular percentage of life and the initial modulus. It also shows that fatigue life of woven composites is typically divided into three stages; the initial stage, middle stage and final stage. Micro-structural damages and transverse cracks in the fill direction of the weave as shown in Figure 2.7 are formed during the initial stage of the fatigue life. There is also rapid decay of modulus during this stage which is mainly caused by strain and stress concentrations in the geometrically repeating unit cell of Figure 2.9. In the middle stage of the fatigue life, the main damage mechanisms with reference to Figure 2.7 are shear failure of the warp (b), matrix cracks (c) and, delamination between fill and warp (d) as well as between adjacent layers (e). In the final stage of the fatigue life, all the damage modes will grow rapidly. At the stress concentration locations, fibres will break resulting in the final failure of the composite.

Figure 2.8: Modulus decay and damage accumulation in woven-fabric composites during fatigue life [41].

Figure 2.9: Schematic of a unit cell [49].

Curtis [50] compared the behaviour of laminated woven composites with that of equivalent non-woven composite laminates under reversed axial cyclic loading. Three different stacking sequences were reported : Lay-up A – $[90^\circ, 0^\circ, 0^\circ, 90^\circ]_s$, lay-up B – $[+45^\circ, -45^\circ, 0^\circ, 90^\circ]_s$ and lay-up C – $[0^\circ, 90^\circ, +45^\circ, -45^\circ]_s$. The S-N diagrams of these test specimens are presented in Figure 2.10. Transverse cracks were observed to develop in the early stage of the fatigue response of non-woven coupon with lay-up A. Longitudinal interlaminar cracks and delamination between 0° and 90° layers then started to appear, and finally the specimen failed with evidence of fibre breakage. Similar failure mechanism and process can be observed in the woven coupon with lay-up A configuration, but damage and failure were confined to individual tow of fibres [50]. Figure 2.10(a) shows that the fatigue curve of woven coupon is lower than that of the non-woven coupon for specimens with lay-up A. One reason for this is that woven coupon has greater fibre instability meaning the

buckling of 0° fibres will create high shear stress at the resin and interface region of the buckled 0° fibres [50].

(a) Lay-up A

(b) Lay-up B

(c) Lay-up C

Figure 2.10: Fatigue life comparison between unidirectional laminates (dashed line) and satin weaved fabric composites (solid line) under tension-compression fatigue loading[50].

In the case of the non-woven coupons with lay-up B configuration, damage initiates in the $+45^\circ$ and -45° layers leading to delamination between the interface between the $+45^\circ$ and 0° layers. For the woven coupons with lay-up B, initial damage in the form of delamination occurs between 0° and 90° layers followed by transverse cracks in 90° tows and cracks in the resin rich regions between tows [50]. The damage and failure mechanisms of both woven and non-woven specimens with lay-up C configuration are similar to those of specimens of lay-up B. Figures 2.10(b) and 2.10(c) show similar fatigue response for both non-woven and woven specimens.

- Three dimensional-woven fabric composites

Some recent composites developed for structural application are the three dimensional (3D) woven fabric composites. In comparison to two dimensional (2D) woven fabric composites, 3D-fabrics generally are known to have higher delamination toughness and impact damage resistance [51-53]. The introduction of z-binders in 3D-fabrics will however induce a resin rich region around the z-binders, and this will give rise to microstructural damage in the form of local in-plane distortion, fibre breakage and crimping [51]. Under in-plane fatigue loading, these initial micro-damages will grow into macroscopic damage before final failure. As a result, the in plane fatigue properties of 3D-fabrics are considerably less than those of 2D-fabrics [51].

Based on the weaving process, 3D-fabrics can be classified into three types [54-56]: 2D-weaving 3D-fabrics, 3D-weaving 3D-fabrics and non-woven 3D-fabrics (produced by non-woven noobing process) as shown in Figure 2.11. The 2D-weaving process uses one set of yarns in the length-wise direction (warp) and another set of yarns in the transverse direction (fill). The process to produce 2D-fabrics and 3D-fabrics are the same, the only difference is just the number of sheet (single sheet for 2D-fabrics and multiple sheets for 3D-fabrics). The 3D-weaving process, on the other hand, uses one set of warp yarns and two perpendicular sets of fill yarns (horizontal fill and vertical fill) [56].

Figure 2.11: 3D-fabrics category[56].

2.3 Composite Failure Criteria

Composites can fail through several different individual damage mechanisms as described in earlier sections. To complicate matters, one damage mechanism can interact with another damage mechanism. Therefore to predict the failure of composites under loading, a suite of failure criteria may be needed. Many researchers have contributed towards setting up a database of composite failure criteria [57-60]. The extensively published failure criteria can be classified into two major research groups [61-63] as illustrated in Figure 2.12. The "Mode-Independent" criteria refer to final failure occurring with no interaction among the various modes of failure. Whereas "Mode-Dependent" criteria are criteria based on the interaction among the various modes of failure.

Conventional notations are used in the subsequent discussion on the failure criteria. Three normal stresses of σ_{ij} and six components of shear stresses τ_{ij} (where $i, j = 1, 2, 3$) shown in Figure 2.13 represent the general state of stresses at a material point. The corresponding normal and shear strains are defined by ε_{ij} and γ_{ij} respectively, and the Poisson's ratios are denoted by ν_{ij} . Subscripts c and t in the stress or strain component indicate compressive and tensile respectively, and subscript u indicates the ultimate stress or strain component.

Figure 2.12: Composite failure criteria classification.

Figure 2.13: Three dimensional state of stresses.

– Mode-independent failure criteria

Mode-independent failure criteria are used to predict the damage and failure of the material without directly identifying the various modes of failure. A mode-independent failure criterion is usually defined with only one equation and this makes the criterion easy to apply. Unfortunately, these criteria do not reveal information about the nature of the damage [61]. This group of failure criteria can be further sub-divided into two groups: polynomial and parametric criteria [63].

○ Polynomial criteria

One of the earliest and most popular quadratic criterion is the Tsai-Hill or Azzi-Tsai criterion [59]. The earlier yield criterion for isotropic material was proposed by Hill [64], this was later modified by Azzi and Tsai [62], and Tsai [65] for predicting failure of fibre-reinforced composites. This criterion states that there is material failure in the composite if the following inequality is violated.

$$\left(\frac{\sigma_{11}}{\sigma_{11u}}\right)^2 - \left(\frac{\sigma_{11}}{\sigma_{11u}}\right)\left(\frac{\sigma_{22}}{\sigma_{22u}}\right) + \left(\frac{\sigma_{22}}{\sigma_{22u}}\right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}}\right)^2 < 1 \quad (2.1)$$

The other popular quadratic criterion is the Tsai-Wu criterion [58] with the same condition of the violation of the inequality of the equation means material failure :

$$F_1\sigma_{11} + F_{11}\sigma_{11}^2 + F_2\sigma_{22} + F_{22}\sigma_{22}^2 + 2F_{12}\sigma_{11}\sigma_{22} + F_{66}\tau_{12}^2 < 1 \quad (2.2),$$

where

$$\begin{aligned}
 F_1 &= \frac{1}{\sigma_{11tu}} + \frac{1}{\sigma_{11cu}} & F_2 &= \frac{1}{\sigma_{22tu}} + \frac{1}{\sigma_{22cu}} & F_{11} &= -\frac{1}{\sigma_{11tu}\sigma_{11cu}} \\
 F_{22} &= -\frac{1}{\sigma_{22tu}\sigma_{22cu}} & F_{66} &= \frac{1}{\sigma_{12u}^2} & F_{12} &= -\frac{1}{2\sigma_{11tu}\sigma_{11cu}}
 \end{aligned} \tag{2.3}$$

Hoffman [66] derived a more general form of the quadratic equation of the Tsai-Hill [59] equation. The Hoffman criterion caters for different tensile and compressive strength of the composites. The Hoffman criterion is similar to the Tsai-Wu criterion [58] except for:

$$F_{12} = -\frac{1}{2\sqrt{\sigma_{11tu}\sigma_{11cu}\sigma_{22tu}\sigma_{22cu}}} \tag{2.4}$$

Chamis [67] also attempted to account for the differences in the tensile and compressive strength by introducing two compensation constants to the Tsai-Hill failure criterion.

$$\left(\frac{\sigma_{11}}{\sigma_{11u}}\right)^2 + \left(\frac{\sigma_{22}}{\sigma_{22u}}\right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}}\right)^2 - C_{12}C'_{12}\left(\frac{\sigma_{11}}{\sigma_{11u}}\right)\left(\frac{\sigma_{22}}{\sigma_{22u}}\right) < 1 \tag{2.5}$$

$$C_{12} = \frac{(1 + 4\nu_{12} - 3\nu_{13})E_{22} + (1 - \nu_{23})E_{11}}{\sqrt{E_{11}E_{22}(2 + \nu_{12} + \nu_{13})(2 + \nu_{21} + \nu_{23})}} \tag{2.6}$$

where E_{11} and E_{22} are the longitudinal and transverse modulus elasticity associated with the 1 and 2 directions respectively. The constant C_{12} depends only on the fundamental material properties while the constant C'_{12} has different value for each quadrant in the failure locus as shown in Figure 2.14.

Figure 2.14: Evaluation of correlation coefficient C'_{12} [67].

Franklin and Marin [68] included the biaxial stress condition in order to achieve better accuracy in composite failure prediction especially when dealing with complex stresses. Based on this criterion, material failure occurs if the following inequality is violated [68].

$$\frac{\sigma_{11}^2 - C\sigma_{11}\sigma_{22}}{\sigma_{11tu}\sigma_{11cu}} + \frac{\sigma_{22}^2}{\sigma_{22tu}\sigma_{22cu}} + \left(\frac{\tau_{12}}{\tau_{12u}}\right)^2 + \sigma_{11}\frac{\sigma_{11c} - \sigma_{11t}}{\sigma_{11tu}\sigma_{11cu}} + \sigma_{22}\frac{\sigma_{22c} - \sigma_{22t}}{\sigma_{22tu}\sigma_{22cu}} < 1 \quad (2.7)$$

where C is a floating constant that depends on the biaxial stress condition chosen.

Basically, the accuracy of the prediction of failure increases correspondingly with the increase of the polynomial order. With the higher order polynomial criteria, the solution gets more complex and laborious. For example, there are some successful cubic criteria [69, 70]. A cubic criterion is more flexible in application compared to a quadratic criterion. There are more interaction parameters involved which makes the criterion more complicated [63].

All the above mentioned failure criteria share a common weakness. These failure criteria use Cartesian stress components referred to each loading type

which make them not suitable for large deformation. Hilton [71] attempted to solve this problem by creating invariant deterministic failure criterion:

$$C \left(\frac{J_1}{\gamma_1} \right)^{b_1} \left(\frac{J_2}{\gamma_2} \right)^{b_2} \left(\frac{J_3}{\gamma_3} \right)^{b_3} = 1 \quad (2.8)$$

where parameters C , b_1 , b_2 and b_3 are obtained from experiments. J_k and γ_k are invariants of the second order tensor τ_{ij} (stress) and τ_{ijU} (failure strength) which can be written as

$$\begin{aligned} J_1 &= \tau_{ii} \\ J_2 &= \tau_{ij} \tau_{ji} \\ J_3 &= \tau_{ij} \tau_{jk} \tau_{ki} \end{aligned} \quad (2.9)$$

and

$$\begin{aligned} \gamma_1 &= \tau_{iiU} \\ \gamma_2 &= \tau_{ijU} \tau_{jiU} \\ \gamma_3 &= \tau_{ijU} \tau_{jkU} \tau_{kiU} \end{aligned} \quad (2.10)$$

○ Parametric criteria

Parametric criteria utilize a series (usually of trigonometric function) other than a polynomial. Some developed the criterion using the Fourier expansion [72, 73] while others used the sine series [74]. The accuracy of parametric criteria depends significantly on the number of terms used in the series [63].

There are two major drawbacks of mode-independent failure criteria in comparison with mode-dependent failure criteria. The mode-independent criteria can predict the damage and failure of composite materials, but they do not reveal the nature of the damage or failure modes. Also the accuracy of the mode-independent criteria in one region of the failure envelopes cannot be improved without affecting the accuracy in the other region of the failure envelopes [63].

– Mode-dependent failure criteria

Mode-dependent failure criteria are sets of criteria that are used to predict the damage and failure of material corresponding to each individual mode of failure. These criteria normally come in a set of equations, with each equation for each particular failure mode. Two of the earliest known mode-dependent criteria are the maximum stress and the maximum strain criteria [75]. Based on the maximum stress criterion, a composite will not fail until the one of the following strain inequalities is violated.

$$\begin{aligned} \sigma_{11t} < \sigma_{11tu} \quad \sigma_{22t} < \sigma_{22tu} \quad \tau_{12} < \tau_{12u} \\ \sigma_{11c} < \sigma_{11cu} \quad \sigma_{22c} < \sigma_{22cu} \end{aligned} \quad (2.11)$$

Similarly, in maximum strain criterion, failure in the composite will not occur until one of the following inequalities is violated.

$$\begin{aligned} \varepsilon_{11t} < \varepsilon_{11tu} \quad \varepsilon_{22t} < \varepsilon_{22tu} \quad \gamma_{12} < \gamma_{12u} \\ \varepsilon_{11c} < \varepsilon_{11cu} \quad \varepsilon_{22c} < \varepsilon_{22cu} \end{aligned} \quad (2.12)$$

A more sophisticated mode-dependent failure criterion was developed by Hashin [57, 76]. The Hashin failure criterion consisted of four different failure modes as defined in the following equations.

Fibre tension ($\sigma_{11} \geq 0$)

$$\left(\frac{\sigma_{11}}{\sigma_{11tu}} \right)^2 + \alpha \left(\frac{\tau_{12}}{\tau_{12u}} \right)^2 \leq 1 \quad (2.13)$$

Fibre compression ($\sigma_{11} \leq 0$)

$$\left(\frac{\sigma_{11}}{\sigma_{11cu}} \right)^2 \leq 1 \quad (2.14)$$

Matrix tension ($\sigma_{22} \geq 0$)

$$\left(\frac{\sigma_{22}}{\sigma_{22tu}} \right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}} \right)^2 \leq 1 \quad (2.15)$$

Matrix compression ($\sigma_{22} \leq 0$)

$$\left(\frac{\sigma_{22}}{2\tau_{23u}} \right)^2 + \left[\left(\frac{\sigma_{22}}{2\tau_{23u}} \right)^2 - 1 \right] \frac{\sigma_{22}}{\sigma_{22cu}} + \left(\frac{\tau_{12}}{\tau_{12u}} \right)^2 \leq 1 \quad (2.16)$$

Based on the Hashin criterion, failure in particular mode occurs if inequality for that particular mode is violated. Several researchers have developed new mode-dependent criteria based on the extension and modification of the Hashin criterion. Puck [77] extended Mohr's hypothesis to composite materials. After which Kroll-Hufenbach [78] merged the Hashin criterion with the Puck criterion. More recently, Davila and Camanho [79] have successfully developed a mode dependent failure criterion by combining several failure criteria for each failure modes. This criterion states that failure in a particular mode occurs if any one of the following inequality in equations (2.17) to (2.26) is violated.

Matrix compression ($\sigma_{22} \leq 0$)

$$\left(\frac{\tau_{eff}^T}{\tau_{12u}^T} \right)^2 + \left(\frac{\tau_{eff}^L}{\tau_{12u}^L} \right)^2 \leq 1 \quad (2.17)$$

where

$$\tau_{eff}^T = \left(|\tau^T| + \eta^T \sigma_n \right) \quad \tau_{eff}^L = \left(|\tau^L| + \eta^L \sigma_n \right) \quad (2.18)$$

and σ_n , τ^T , τ^L are the normal, transverse shear and longitudinal shear stresses that act on the fracture plane. The symbols η^T and η^L are the corresponding internal material frictions to be determined experimentally.

Matrix tension ($\sigma_{22} \geq 0$)

$$(1-g) \frac{\sigma_{22}}{\sigma_{22u}} + g \left(\frac{\sigma_{22}}{\sigma_{22u}} \right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}} \right)^2 \leq 1 \quad (2.19)$$

$$g = \frac{G_{Ic}}{G_{IIc}} \quad (2.20)$$

where G_{Ic} and G_{IIc} are the critical energy release rates in the mode I and II loading respectively.

Fibre tension ($\sigma_{11} \geq 0$)

$$\frac{\varepsilon_{11}}{\varepsilon_{11u}} \leq 1 \quad (2.21)$$

Fibre compression ($\sigma_{11} \leq 0$ & $\sigma_{22} \leq 0$)

$$\frac{|\tau_{12}| + \eta^L \sigma_{22}}{\tau_{12u}} \leq 1 \quad (2.22)$$

Fibre compression ($\sigma_{11} \leq 0$ & $\sigma_{22} \geq 0$)

$$(1-g) \frac{\sigma_{22}}{\sigma_{22u}} + g \left(\frac{\sigma_{22}}{\sigma_{22u}} \right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}} \right)^2 \leq 1 \quad (2.23)$$

Matrix damage in biaxial compression

$$\left(\frac{\tau_{eff}^{mT}}{\tau_{12u}^T} \right)^2 + \left(\frac{\tau_{eff}^{mL}}{\tau_{12u}^L} \right)^2 \leq 1 \quad (2.24)$$

$$\tau_{eff}^{mT} = \left(-\sigma_{22} \cos \alpha (\sin \alpha - \eta^T \cos \alpha) \right) \quad (2.25)$$

$$\tau_{eff}^{mL} = \left(\cos \alpha (|\tau_{12}| + \eta^T \sigma_{22} \cos \alpha) \right) \quad (2.26)$$

where τ_{eff}^{mT} and τ_{eff}^{mL} are the effective transverse and longitudinal stresses in the misalignment frame. The parameter α is the fracture angle which is to be determined in an iterative manner.

Although mode-dependent criteria have the advantage over mode-independent criteria in terms of their capability in providing failure mode information, it does not mean that mode-dependent criteria are always more accurate than mode-independent criteria. The accuracy here is relative. One particular failure criterion might give better estimation in one part of the failure envelope but inaccurate estimation in the other part. Thus, choosing the appropriate criterion for a particular case is required [62] and using a suite of failure criteria is currently the best option.

2.4 Composite Fatigue Modelling

In order to reduce the number of tests for predicting composite fatigue failure, composite fatigue modelling is needed. There are currently three main groups of composite fatigue models: fatigue life model, phenomenological model and progressive damage model. Figure 2.15 illustrates the three main groups and their associated

composite fatigue models. Each model will be looked at and discussed with some details in the subsequent sections.

Figure 2.15: Composite fatigue modelling classification.

– Fatigue life models

Current fatigue life models normally utilize one of the failure criteria as the base and an empirical S-N curve as an input. Such fatigue life models can be used to predict the number of cycles to failure but they do not account for the damage accumulation [80]. There are currently several fatigue life models available in the literature, Jen [81, 82] and Philippidis [83] developed a deterministic fatigue life model that is basically a modification of the Tsai-Hill criterion, as shown below :

$$\left(\frac{\sigma_{11}}{\sigma_{11f}}\right)^2 + \left(\frac{\sigma_{22}}{\sigma_{22f}}\right)^2 + \left(\frac{\tau_{12}}{\tau_{12f}}\right)^2 - \left(\frac{\sigma_{11}}{\sigma_{11f}}\right)\left(\frac{\sigma_{22}}{\sigma_{22f}}\right) < 1 \quad (2.27)$$

where σ_{11f} , σ_{22f} and τ_{12f} are the fatigue failure stresses in the S-N curves. This criterion can be used to model fatigue life for any stress ratio and frequency as long as the S-N curves are available for the corresponding stress ratio and frequency [83]. Reifsnider [84] presented a fatigue life model based on the micro-structural level. Fawaz [85] developed a model that is able to predict the S-N curve of a unidirectional (UD) laminate with arbitrary ply orientation based on the S-N curve of a laminate with fibres aligned in one orientation. Paramonov [86] successfully developed a statistical fatigue life model which can predict the minimum and maximum number of cycles to failure of a composite structure.

- Phenomenological models

Phenomenological models include the description of the damage in composites during fatigue loading by modelling the degradation of one particular property of composites. There are two common phenomenological models available: residual stiffness model and residual strength model.

- Residual strength models

This phenomenological model used experimental observation to describe the strength loss of composites. This model can be sub-divided into two models: sudden death model and wear-out model. The residual strength in the sudden death model is kept constant over a certain number of cycles and is then suddenly degraded drastically when it reaches the critical number of cycles to failure. On the other hand, the residual strength in the wear-out model is continually decreasing over the number of cycles following a certain predetermined equation. Several researchers have developed and successfully applied this model for use in glass fibre-reinforced composites [87, 88], and Diao and Mai [89] have presented a statistical model of residual strength to predict the fatigue life of composite laminates.

- Residual stiffness models

This model describes the stiffness loss of composite laminates based on experimental observation. One major advantage of the residual stiffness model over the residual strength model is that only the stiffness of composites is needed for material characterization. Several notable residual stiffness models have been developed and published in the literatures [87, 90, 91]. One such model is by Whitworth [91] :

$$E_{(n)} = E_{(0)} \left(\frac{S}{c_1 S_u} \right)^{\frac{1}{c_2}} \left[-h \ln(n+1) + \left(c_1 \frac{S_u}{S} \right)^{\frac{m}{c_2}} \right]^{\frac{1}{m}} \quad (2.28)$$

where $E_{(0)}$, $E_{(n)}$, S_u and S respectively are initial stiffness, stiffness at n cycles, ultimate strength and applied stress. The parameters c_1 , c_2 , h and m are obtained from experiments.

Phenomenological models have one common weakness. It can only predict the fatigue behaviour of composite laminates under mono-axial fatigue loading and cannot account for the complex stress state in the real structure. To get the correct parameters for this fatigue model, laboratory tests must simulate the same complex stress state as the real structure in order to fully characterize the material.

– Progressive damage models

This model is currently the most advanced model compared to the earlier models that were discussed. Progressive damage model is able to predict not only the number of cycles to failure but also the degradation of the properties in the composite structures via the use of fracture criteria. This model can be divided into two groups : model predicting damage growth and model predicting residual mechanical properties [80] .

○ Models predicting damage growth.

Since the late 80s, there were models that can be used to predict damage growth [92-95]. Some models were developed to predict damage growth from either notch [92] or holes [93]. Bergmann [94] developed an empirical delamination propagation model which combines all the modes (mode I tension, mode II shear and mode III shear) in one equation. The governing equation of the Bergmann model is:

$$\frac{dA}{dN} = c_1 (f(G_t))^n = c_2 \varepsilon^n A^m \quad (2.29)$$

where G_t is the total of mode I, II and III energy release rates, A is the delaminated area and N is the respective number of cycles. The parameters c_1 , c_2 , n and m are determined from experiments. By assuming constant width and a_0 as the initial crack length, Bergmann model can be written in the form of:

$$N = \frac{a^{(1-m)} - a_0^{(1-m)}}{(1-m)c\sigma^n} \quad (2.30)$$

Dahlen [95] has successfully built an empirical delamination propagation model that includes the effect of shear reversal in mode II delamination growth.

Shear reversal takes place when the surfaces bounding the delamination are moving in both positive and negative directions [95].

- Models predicting residual mechanical properties.

This model requires the relationships of the residual mechanical properties of composites with their damage variables. Shokrieh [96-99] has constructed a model which is able to predict the fatigue damage progression of complicated composite structures using a modified Hashin failure criterion provided that the properties of the composite materials are fully characterized. In order to fully characterize a composite material, experimental results based on the three loading conditions of tension, compression and shear on fibres and resins are needed. For clarity, an illustration of the required tests is shown in Figure 2.16. In order to fully characterize a composite material, for each combination of load and fibre or matrix testing, two different set of tests are needed [96]:

- Fatigue test until certain number of cycles continued by static test in order to get the residual stiffness and strength.
- Fatigue test to get the S-N curve.

Figure 2.16: Tests needed in order to fully characterize a composite material [96].

In order to make the model less expensive, Paepegem [100-108] implemented a cycle jump. The interval between two successive cycles where the fatigue damage law is evaluated, is small in the beginning but increases as

the cycle advances [108]. Hochard [109] developed a similar progressive fatigue model specially catered for woven composites.

A progressive damage model has more advantages when compared to the other models but it can be very complex and expensive in terms of computational solution as well as in terms of the number of experiments needed to fully characterize the material properties. The fatigue life model is rather straight forward and affordable from the computational solution and experimental point of view. One has to bear in mind that fatigue life model can only predict the number of cycles to first element failure (initial failure), which in practice may not be the final failure.

2.5 Fatigue Strength Accelerated Testing Methodology

In order to fully characterize a composite material under arbitrary temperatures, frequencies and load ratios, a large number of tests need to be done. Thus, accelerated testing methodology is very important in order to reduce the number of test needed. Miyano et al. [7, 29-31] have successfully developed a method to predict fatigue strength of composite materials under arbitrary temperatures, frequencies and load ratios. Several types of composite under different types of load have been tested [110-117]. There are 4 hypotheses used in Miyano's method [7, 112]:

1. Same failure mechanism for static, creep and fatigue failure
2. Same time-temperature superposition principle for all strengths
3. The linear cumulative damage law for monotonic (constant strain rate) loading
4. Linear dependence of fatigue strength upon stress ratio

When these hypotheses are met, the fatigue strength under arbitrary combination of frequency, stress ratio and temperature can be determined based on:

1. Master curve of constant strain rate (CSR) strength
2. Master curve of fatigue strength for zero stress ratio

The flow chart of all the steps needed to fully characterize the fatigue properties of a composite material based on Miyano's method can be seen in Figure 2.17, followed by the explanation of each steps.

Figure 2.17: Flow chart of all the steps needed to fully characterize fatigue properties of a material based on Miyano's methodology.

1. Master curve of storage modulus

Storage modulus is the ratio of the stress to strain under vibratory conditions and measures the energy stored. In order to get the time-temperature shift factor, master curves of storage modulus need to be constructed from the dynamic mechanical analysis (DMA) at several different frequencies and temperatures. Then, the storage modulus at each temperature needs to be shifted so they will overlap with each other. By doing so, the time-temperature shift factor can be found using:

$$a_{T_0}(T) = \frac{t}{t'} \quad (2.31)$$

Then, the activation energy (ΔH) can be calculated by using the following equation.

$$\log a_{T_0}(T) = \frac{\Delta H}{2.303G} \left(\frac{1}{T} - \frac{1}{T_0} \right) \quad (2.32),$$

where G is the gas constant, $8.314 \times 10^{-3} \text{ kJ} / (\text{K.mol})$.

2. Master curves of constant strain rate (CSR) strength

To get master curves of CSR strength, CSR tests at several different temperatures and strain rates need to be carried out. Then, by using the same way as that used in the master curve of storage modulus, master curves of CSR strength can be constructed. The

time-temperature shift factor of CSR strength master curve is assumed to be the same as time-temperature shift factor determined earlier from the storage modulus master curve [114].

3. Prediction of creep strength master curve

Miyano [7, 112] proposes the prediction method of creep strength master curve from master curve of static strength based on Christensen's theory [118, 119]. This theory is based on a linear cumulative damage (LCD) law which states:

$$\int_0^{t^*} \frac{dt}{t_c [\sigma(t)]} = 1 \quad (2.33)$$

In order to predict the creep strength master curve, the CSR strength master curve needs to be curve fitted using equation (2.34) [7, 118, 119].

$$\log \sigma_s = \log \sigma_{s,0}(t'_0, T_0) - \log \left[1 + \left(\frac{t'}{t'_1} \right)^{n_r} \right] \quad (2.34)$$

As the results the values of $\sigma_{s,0}$, t'_1 and n_r are obtained. Subsequently, these values are substituted into equation (2.35) [7, 118, 119], so the master curve of creep strength can be achieved.

$$\log \sigma_c = \log \sigma_{s,0}(t'_0, T_0) - \log \left[1 + \left(\frac{1}{n_r} + 1 \right)^{n_r} \left(\frac{t'}{t'_1} \right)^{n_r} \right] \quad (2.35)$$

4. Master curves of zero stress ratio fatigue strength

Fatigue test needs to be performed to get S-N curves at base frequency and zero stress ratio for several temperatures. Stress ratio is defined as the ratio between minimum and maximum stress. Zero stress ratio is the condition arises when minimum stress is zero with any predetermined maximum stress. Then, the S-N curves for each temperature need to be shifted in order to get master curve of fatigue strength at zero stress ratio using:

$$f' = f \cdot a_{T_0}(T) \quad (2.36)$$

and

$$t'_f = \frac{t_f}{a_{T_0}(T)} = \frac{N_f}{f'} \quad (2.37)$$

5. Prediction of fatigue strength for arbitrary frequency, stress ratio and temperature

At the end, fatigue strength can be predicted for arbitrary frequency, stress ratio and temperature (f, R, T) by implementing linear dependence of fatigue strength upon stress ratio as:

$$\sigma_f(t_f; f, R, T) = \sigma_{f;1}(t_f; f, T)R + \sigma_{f;0}(t_f; f, T)(1 - R) \quad (2.38)$$

One note of caution here is that Miyano's methodology has two known limitations. The first is that the methodology uses a linear cumulative damage (LCD) law which is generally unsatisfactory except for constant strain rate to failure [120]. The other is that Miyano's methodology cannot be used when there are hysteretic heating [121], and hysteretic heating normally occurs at high frequency fatigue test (10Hz or more).

Chapter Three

EXPERIMENTAL SET-UP

In order to characterize a composite material and validate the failure models, several types of test need to be carried out. In this chapter, the tests preparation and set-up are presented and discussed in details.

3.1 Specimen Preparation

In order to create a specimen, two steps of preparation need to be conducted, they are: prepreg processing and curing process.

3.1.1 Prepreg processing

Woven CFRP prepreg is used as the raw material to fabricate woven CFRP specimen. Prepreg needs to be cut and stacked based on the required shape and dimension prior to curing process.

3.1.2 Curing process

There are 3 different types of specimen that were fabricated: tensile, shear and I-beam specimens. The details of curing process preparation of all the specimens will be discussed in the following.

- DMA, tensile and shear specimens

The curing process of DMA, tensile and shear specimens are relatively the same, as all of them are categorized as flat specimens. Before going through the curing process, prepreg for the flat specimens must be placed on top of an aluminium plate that is already wrapped by non-porous teflon. Subsequently, layers of porous teflon, breather and non-porous teflon need to be added on top of the prepreg (Figure 3.1).

The curing process can be carried out either using a curing table (Figure 3.2) or an autoclave (Figure 3.3). There are several differences between the curing process in the curing table and autoclave. The curing table can only cure flat specimen. On the other hand, an autoclave can cure specimens with 3 dimensional (3D) shape as well as flat specimens. The pressure in the autoclave is 6.9 bar in comparison to ambient pressure of 1 bar in the curing table. The typical specimen fabricated using autoclave tends to

have higher modulus and strength in comparison with the same specimen fabricated using a curing table.

Figure 3.1: Arrangement of additional layers on prepreg for flat specimen before curing.

Figure 3.2: Curing table.

Figure 3.3: Autoclave.

The curing process in curing table and autoclave take place for about three hours. The whole curing process can be divided into three major steps: temperature ramping, temperature dwelling, and cooling (Figure 3.4). Figures 3.5(a), (b) and (c) show the final DMA, shear and tensile specimens respectively. The details about all specimen dimensions will be discussed in Chapter 4.

Figure 3.4: Curing process graph.

Figure 3.5: Woven CFRP specimen after curing, a) DMA specimen, b) shear CSR and fatigue specimen, c) tensile CSR and fatigue specimen.

– I-beam specimen

As an I-beam specimen has 3D shape, it is not possible to cure it properly on a curing table. To produce the I-beam specimen in the autoclave, 2 metal beams with thickness equal to the width of the I-beam web and 2 other beams with width at least the same as the width of the I-beam flange are needed. These metal beams are then

wrapped with non-porous teflon, breather and porous teflon respectively. The I-beam web and flanges are 6 layers thick and to fabricate this, prepregs are stacked as shown in Figure 3.6(a). The arrangement of the prepreg, metal beams and additional layers for curing process is shown in Figure 3.6(b) and subsequently need to be wrapped by vacuum bag before the curing process (Figure 3.7). Figure 3.8 shows the picture of I-beam woven CFRP after the curing process.

Figure 3.6: a) I-beam prepreg stacking sequence, b) the arrangement of additional layers on prepreg for I-beam specimen before curing.

Figure 3.7: I-beam specimen arrangement before curing process in autoclave.

Figure 3.8: I-beam woven CFRP specimen after curing.

3.2 Specimen Testing

The three types of test performed here in the characterization of a composite material are DMA (dynamic mechanical analysis), constant strain rate (CSR) and fatigue tests. CSR and fatigue tests are carried out with the MTS 810 and DMA is performed with the TA Q800. All the tests set-up will be discussed in the following.

3.2.1 TA Q800

TA Q800 (Figure 3.9(a)) is a very accurate pneumatic dynamic mechanical analysis (DMA) instrument with the following specifications:

- Maximum force : 18 N
- Minimum force : 0.0001 N
- Force resolution : 0.00001 N
- Strain resolution : 1 nanometer
- Frequency range : 0.01 to 200 Hz
- Temperature range : - 150 to 600 °C
- Heating rate : 0.1 to 20 °C/min

In order to maintain the accuracy of DMA TA Q800, three different calibrations are first performed: mass, zero and compliance calibrations. Subsequently, the specimen is installed in the clamp, in this case single cantilever clamp as shown in Figure 3.9(b). Before starting the test, several variables such as the frequency range and the temperature range are set. The details about all the variables that are used in the test will be discussed later in Chapter 4.

Figure 3.9: a) DMA TA Q800, b) specimen installed in single cantilever clamp.

3.2.2 MTS 810

MTS 810 (Figure 3.10) is a hydraulic testing machine that can deliver very high actuator speed which is suitable for fatigue test. The specifications of MTS 810 are:

- Maximum force : 100 kN
- Type : Hydraulic
- Maximum testing range : 150 mm
- Stiffness : 2.6×10^8 N/m
- Weight : 540 kg
- Standards : DIN and ISO

Figure 3.10: MTS810 systems with temperature chamber, a) for tension-tension fatigue testing, b) for shear and 4-point bending fatigue testing.

There are several devices need to be activated in conjunction with MTS 810: hydraulic pump, cooling water pump and cooling tower. Moreover, 3 different fixtures were used: tensile (Figure 3.11(a)), shear (Figure 3.11(b)) and 4-point bending (Figure 3.11(c)) fixtures. In the case of the CSR test, displacement rates need to be determined before starting the test. As for fatigue test, several parameters need to be determined in the set-up: test frequency, target set-point and amplitude. The target set-point in the MTS 810 set-up is equivalent to the mean load (F_{mean}) and the amplitude is half of the difference between the maximum and the minimum loads ($\Delta F / 2$). Figure 3.12 shows the typical sinusoidal load input in the MTS 810 fatigue test.

Figure 3.11: MTS 810 fixtures: a) tensile fixture with extensometer, b) shear fixture, c) 4-point bending fixture.

Figure 3.12: Typical load input in MTS 810 fatigue test.

Chapter Four

Material Characterization

The basic material for the composite structure is first characterized prior to the fatigue tests and subsequently fatigue modelling. Take note that in some figures a non-zero axis is used to give clarity to the plots. In order to fully characterized fatigue properties of a material for arbitrary frequency, load ratio and temperature, an infinite number of tests need to be performed. Accelerated testing methodology as explained in Chapters 2 and 3 requires 3 types of tests that need to be done in order to fully characterize a material:

1. Dynamic mechanical analysis (DMA)

DMA tests were performed at certain frequency ranges for several different temperatures in order to construct a master curve of storage modulus and to get the time–temperature shift factors.

2. Constant strain rate (CSR) / static test

In order to build the master curve of CSR, CSR/static tests at several different temperatures and displacement rates were done. The creep strength master curve can also be predicted after the master curve of CSR is constructed.

3. Fatigue test at zero stress ratio

Master curve of fatigue strength at zero stress ratio can be constructed using master curve of CSR and S-N curves at a particular frequency, zero stress ratio and several different temperatures. In order to get the S-N curves, fatigue tests at several different load magnitudes need to be completed.

The material that was characterized is L-930 flame retardant woven carbon-epoxy prepreg (L-930 woven carbon-epoxy). L-930 woven carbon-epoxy is a balanced plain weave carbon-epoxy with 12,000 fibres/tow (12k) and 1.2 thread/cm construction and area weight = 193gr/m². It consists of flat GT-700 carbon fibres and toughened flame retardant epoxy as the matrix. The recommended curing cycle is at 120°C and 100psi (6.9bar) for 60 minutes. Based on the data sheet, the tensile modulus and tensile

strength of L-930 woven carbon-epoxy are 62.5GPa and 1225MPa respectively. The material characterization for L-930 woven carbon-epoxy will next be described in detail with particular attention to tensile and shear properties.

4.1 Tensile Properties Characterization

Tensile properties characterization of L-930 woven carbon-epoxy can be divided into several steps: storage modulus master curve, constant strain rate (CSR) strength master curve, creep strength master curve, zero stress ratio fatigue strength master curve and fatigue strength prediction and validation at arbitrary temperature, frequency and load ratio. Detailed discussions of all these steps are in the following.

4.1.1 Storage Modulus Master Curve

DMA tests were done to L-930 flame retardant woven carbon-epoxy with dimensions as can be seen in Figure 4.1. These tests capture the storage modulus at frequencies from 0.1 Hz to 10 Hz (frequency sweep) and temperatures from 30°C to 240°C (with 5°C intervals). The lower limit of the frequency range was chosen to be 0.1 Hz because the test will take much longer time if a lower frequency is chosen. On the other hand, the higher limit cannot be more than 10 Hz because it will cause resonance, which will make the result inaccurate. An interval of 5°C was picked so as to have sufficient data points for overlapping in the forming of the master curve.

Figure 4.1: L-930 flame retardant woven carbon-epoxy a) DMA specimen dimensions (mm) and b) DMA specimen picture.

The test was repeated 4 times in order to ensure the validity of the results (L-930-Single Cantilever: SC1, SC2, SC3 and SC4). The test result of SC1 specimen test is shown in Figure 4.2(a). The single temperature frequency sweep results are then shifted to

overlap with the other temperature results in order to construct one smooth master curve of storage modulus graph as can be seen in Figure 4.2(b). Using equation (4.1), the time-temperature shift factor, $a_{T_0}(T)$, can be analysed as shown in Figure 4.3.

$$a_{T_0}(T) = \frac{t}{t'} \quad (4.1)$$

Then, in order to get the activation energies (ΔH_1 and ΔH_2) and glass transition temperature (T_g), the time-temperature shift factor results in Figure 4.3 need to be curve fitted using equation (4.2).

$$\log a_{T_0}(T) = \frac{\Delta H_1}{2.303G} \left(\frac{1}{T} - \frac{1}{T_0} \right) H(T_g - T) + \left[\frac{\Delta H_1}{2.303G} \left(\frac{1}{T_g} - \frac{1}{T_0} \right) + \frac{\Delta H_2}{2.303G} \left(\frac{1}{T} - \frac{1}{T_g} \right) \right] (1 - H(T_g - T)) \quad (4.2)$$

Table 4.1 lists out all the ΔH_1 , ΔH_2 and T_g results obtained from the tests.

Figure 4.2: a) Frequency sweep graphs at several temperatures for L-930-SC1 specimen, b) master curve of storage modulus for L-930-SC1 specimen.

Figure 4.3: Time-temperature shift factor of L-930 specimens ($T_g=109.375^\circ\text{C}$).

Table 4.1: L-930 woven carbon-epoxy DMA results.

Test No	ΔH_1 (kJ/mol)	ΔH_2 (kJ/mol)	T_g ($^\circ\text{C}$)
1	143.93	633.22	110.00
2	126.47	629.25	116.00
3	122.91	644.07	105.50
4	115.50	631.36	106.00
Avg	127.20	634.47	109.38

4.1.2 Tensile Constant Strain Rate Strength Master Curve

In order to construct constant strain rate (CSR) master curve, CSR tensile tests were performed at 5 different temperatures: 25°C , 60°C , 100°C , 140°C and 175°C and at each temperature, 2 different displacement rates were used (360mm/min and 3.6mm/min). These test temperatures are chosen such that responses below and above the T_g can be studied. The same goes for the displacement rates, these displacement rates were chosen to get the good data spreading while considering the limitations of the MTS810 fatigue machine. The dimensions of L-930 woven carbon-epoxy CSR specimen are shown in Figure 4.4. The results of all the tests are shown in Figure 4.5. Table 4.2 presents the average CSR strength and modulus against log failure time for all test specimens. A typical mode of failure regardless of temperature is shown in Figure 4.6.

Figure 4.4: a) CSR and fatigue specimen dimensions, b) L-930 CSR specimen.

Table 4.2: Average CSR results for L-930 specimen.

Test Descriptions	Time	Log Time	Load	Strength	E_{11}
	s	Log(min)	N	MPa	MPa
25°C and 360mm/min	0.53	-2.05	11853.50	1053.64	59794.09
60°C and 360mm/min	0.52	-2.06	12174.10	1082.14	63969.25
100°C and 360mm/min	0.52	-2.06	11874.46	1055.51	65847.62
140°C and 360mm/min	0.60	-2.00	11781.91	1047.28	65690.32
175°C and 360mm/min	0.52	-2.06	11780.82	1047.18	66543.15
25°C and 3.6mm/min	56.93	-0.02	11967.89	1063.81	59386.05
60°C and 3.6mm/min	57.65	-0.02	12670.82	1126.30	63843.94
100°C and 3.6mm/min	57.39	-0.02	12430.54	1104.94	62564.74
140°C and 3.6mm/min	53.41	-0.05	11952.05	1062.40	66984.27
175°C and 3.6mm/min	46.23	-0.11	10138.95	901.24	62254.54

Figure 4.5: CSR tensile stress vs strain graphs.

Figure 4.6: Typical failure of L-930 specimen under tensile static loading.

By rearranging equation (4.1), the shifted time is:

$$t' = \frac{t}{a_{T_0}(T)} \quad (4.3).$$

Using the results of $a_{T_0}(T)$ in Figure 4.3, a master curve of tensile CSR is obtained as shown in Figure 4.7. The master curve of tensile CSR then is curve fitted using Christensen's equation [7, 118, 119] :

$$\log \sigma_s = \log \sigma_{s,0}(t'_0, T_0) - \log \left[1 + \left(\frac{t'}{t'_1} \right)^{n_r} \right] \quad (4.4)$$

Figure 4.7: a) Tensile CSR results and b) master curve of tensile CSR at 25°C with curve fitting based on Christensen's theory ($\sigma_{s,0} = 1072.2\text{MPa}$, $t'_1 = 1.2 \times 10^{20}$ and $n_r = 0.4$).

4.1.3 Tensile Creep Strength Master Curve

As explained previously in Chapter 2, the creep strength master curve is predicted from the CSR strength master curve using linear cumulative damage (LCD) law. Based on Christensen's theory [7, 118, 119], creep strength master curve can be predicted using the following equation.

$$\log \sigma_c = \log \sigma_{s,0}(t'_0, T_0) - \log \left[1 + \left(\frac{1}{n_r} + 1 \right)^{n_r} \left(\frac{t'}{t'_1} \right)^{n_r} \right] \quad (4.5)$$

Where, the values of $\sigma_{s,0}$, t'_1 and n_r were obtained from the tensile CSR strength master curve. The predicted tensile creep strength master curve together with the CSR master curve are shown in Figure 4.8.

Figure 4.8: The predicted tensile creep strength master curve.

4.1.4 Zero Stress Ratio Tension-Tension Fatigue Strength Master Curve

Zero stress ratio S-N curves at several different temperatures are needed to build zero stress ratio fatigue strength master curve. The specimen dimensions and materials used were identical to those used in the tensile CSR tests. The test frequency for all of the fatigue tests is kept at 5Hz. Fatigue tests at 3 different temperatures of 25°C, 100°C and 175°C at several different load magnitudes were carried out. The typical plot of peak displacement normalized by the failure displacement is shown in Figure 4.9. The number of cycles to failure results of all the tests are tabulated in Table 4.3. The results of the tests at 25°C, 100°C and 175°C are compiled and plotted as S-N curves and the average results are extracted as shown in Figures 4.10, 4.11 and 4.12 respectively. A typical failed

specimen due to tension-tension fatigue load regardless of the temperature is shown in Figure 4.13.

Table 4.3: Tension-tension fatigue results for L-930 specimen.

25°C		100°C		175°C	
Test Description	Nf	Test Description	Nf	Test Description	Nf
	Cycles		Cycles		Cycles
977.78 MPa 1 st	10947	1013.33 MPa 1 st	264	844.44 MPa 1 st	73
977.78 MPa 2 nd	110916	1013.33 MPa 2 nd	454	844.44 MPa 2 nd	6
888.89 MPa 1 st	210203	960.00 MPa 1 st	1926	844.44 MPa 3 rd	21
888.89 MPa 2 nd	25851	960.00 MPa 2 nd	263	800.00 MPa 1 st	1579
888.89 MPa 3 rd	284700	960.00 MPa 3 rd	3814	800.00 MPa 2 nd	726
888.89 MPa 4 th	46616	906.67 MPa 1 st	6760	800.00 MPa 3 rd	502
844.44 MPa 1 st	12529	906.67 MPa 2 nd	5244	755.56 MPa 1 st	3000
844.44 MPa 2 nd	22123	906.67 MPa 3 rd	1514	755.56 MPa 2 nd	10
844.44 MPa 3 rd	193104	853.33 MPa 1 st	185680	755.56 MPa 3 rd	7753
844.44 MPa 4 th	250024	853.33 MPa 2 nd	25643	711.11 MPa 1 st	12582
800.00 MPa 1 st	149252	853.33 MPa 3 rd	117301	711.11 MPa 2 nd	12251
800.00 MPa 2 nd	33524	853.33 MPa 4 th	4018	711.11 MPa 3 rd	9720
800.00 MPa 3 rd	158414	800.00 MPa 1 st	294430	666.67 MPa 1 st	14389
800.00 MPa 4 th	820640	800.00 MPa 2 nd	234947	666.67 MPa 2 nd	15034
800.00 MPa 5 th	644258	800.00 MPa 3 rd	10242	666.67 MPa 3 rd	12079
711.11 MPa 1 st	278143	800.00 MPa 4 th	4540	622.22 MPa 1 st	26016
711.11 MPa 2 nd	589146	800.00 MPa 5 th	102806	622.22 MPa 2 nd	18783
711.11 MPa 3 rd	1399892

Figure 4.9: Typical normalized peak displacement against normalized number of cycles results.

(a) Graphical plots

Max Stress	Nf
MPa	Cycles
977.78	60931.5
888.89	141842.5
844.44	119445
800.00	361217.6
711.11	755413

(b) Extracted average data

Figure 4.10: Tension-tension fatigue S-N curve at 25°C and zero stress ratio.

(a) Graphical plots

Max Stress	Nf
MPa	Cycles
1013.33	328
960	1201
906.67	4506
853.33	83161
800	129393

(b) Extracted average data

Figure 4.11: Tension-tension fatigue S-N curve at 100°C and zero stress ratio.

(a) Graphical plots

Max Stress	Nf
MPa	Cycles
844.44	33
800	936
755.56	3588
711.11	11518
666.67	13834
622.22	22400

(b) Extracted average data

Figure 4.12: Tension-tension fatigue S-N curve at 175°C and zero stress ratio.

Figure 4.13: Typical failed L-930 specimen under fatigue loading.

Using the same method as that used for the generation of the master curve of CSR strength, $a_{T_0}(T)$ values for each temperatures are obtained from Figure 4.3. The shifted time (t') is calculated using equation (4.3). As a result, zero stress ratio fatigue strength in the shifted time domain is obtained as shown in Figure 4.14. Finally the master curve of zero stress ratio fatigue strength can be constructed by joining all the point that have the same number of cycles to failure as shown in Figure 4.15.

Figure 4.14: Shifted zero stress ratio tension-tension fatigue strength vs log failure time.

Figure 4.15: Zero stress ratio tension-tension fatigue strength master curve.

4.1.5 Tension-Tension Fatigue Strength Prediction and Validation

The master curve of zero stress ratio fatigue strength and the master curve of creep strength have successfully been constructed, thus the fatigue strength of L-930 woven carbon-epoxy at arbitrary stress ratio, frequency, temperature and load magnitude can be predicted. The validity of the selected prediction was at 80°C, 4Hz and 0.5 load ratio. The shift factor ($a_{T_0}(T)$) value is obtained from Figure 4.3 and the shifted time (t') is calculated using equation (4.3) for several number of cycles. Subsequently, the fatigue strength is found by matching the calculated shifted time (t') for particular number of cycles in the master curve of zero stress ratio fatigue strength as shown in Figure 4.16. The creep strength for the particular shifted time (t') range is found directly from the creep strength master curve (Figure 4.8). Subsequently, using the following equation

$$\sigma_f(t_f, f, R, T) = \sigma_f(t_f, f, R = 1, T)R + \sigma_f(t_f, f, R = 0, T)(1 - R) \quad (4.6)$$

the S-N curve at the desired load ratio can be predicted as shown in Figure 4.17. Experimental validation was done in order to verify the prediction as shown in Figure 4.18. The experimental validation results agree very well with the predicted results.

Figure 4.16: Predicted fatigue strength in master curve of zero stress ratio fatigue strength.

Figure 4.17: Predicted tension-tension fatigue strength vs log failure time curve at 4Hz, 80°C and stress ratio of 0.5, also SR of 0 and 1.

Figure 4.18: Predicted tension-tension fatigue strength and validation results at 4Hz, 80°C and stress ratio of 0.5.

4.2 Shear Properties Characterization

The same set of steps that was used to characterize the tensile properties is repeated here to characterize shear properties. As for storage modulus master curve, the shift factor in Figure 4.3 is also applicable to the shear properties as the time-temperature shift factor is independent of the loading mode. Figure 4.19 shows the shear coupon dimensions based on ASTM D7078. The complete step by step shear properties characterization of L-930 woven carbon-epoxy will be discussed in details in the following.

Figure 4.19: Shear coupon dimensions based on ASTM D7078.

4.2.1 Shear Constant Strain Rate Strength Master Curve

CSR shear tests are done at 5 different temperatures (25°C, 60°C, 100°C, 140°C and 175°C) in order to build master curve of CSR shear strength. Figure 4.20 shows the shear stress against actuator displacement graphs of all CSR shear tests. The average CSR shear test results at all temperatures are tabulated in Table 4.4.

Figure 4.20: CSR shear stress vs actuator displacement graphs.

A typical failed specimen under shear CSR is shown in Figure 4.21. The shear strength results in log time domain from all the temperatures and the master curve of shear CSR are plotted in Figure 4.22(a) and (b) respectively.

Table 4.4: Average shear CSR test results.

Test Description	Time	Log Time	Max Load	Shear Strength
	s	Log (min)	N	MPa
25°C and 100mm/min	2.07	-1.46	5582.75	82.30
100°C and 100mm/min	2.18	-1.44	3296.68	49.17
175°C and 100mm/min	2.70	-1.35	936.62	14.65
25°C and 1mm/min	193.00	0.51	4875.33	72.10
60°C and 1mm/min	270.40	0.65	3587.00	57.32
100°C and 1mm/min	311.30	0.72	2215.14	36.70
140°C and 1mm/min	360.00	0.78	1431.78	23.32
175°C and 1mm/min	195.90	0.51	602.59	10.21

Figure 4.21: Typical failed specimen under shear CSR loading.

Figure 4.22: a) Shear CSR results and b) master curve of shear CSR at 25°C with curve fitting based on Christensen's theory ($\sigma_{s,0} = 230$ MPa, $t'_1 = 1 \times 10^{-6}$ and $n_r = 0.056$).

4.2.2 Shear Creep Strength Master Curve

The shear creep strength master curve is predicted via the use of equation (4.5) with $\sigma_{s,0}$, t'_1 and n_r values were attained from shear CSR master curve. The predicted shear creep strength master curve together with the shear CSR master curve are presented in Figure 4.23.

Figure 4.23: The predicted shear creep strength master curve.

4.2.3 Shear Zero Stress Ratio Fatigue Strength Master Curve

Shear fatigue tests were performed at 5Hz, zero stress ratio and for 3 different temperatures (25°C, 100°C and 175°C) in order to get the S-N curve at suggested temperatures and ultimately to build the shear zero stress ratio fatigue strength master curve. The fatigue specimen is identical to the CSR shear specimen. Figure 4.24 and Table 4.5 show the typical plot of peak displacement against normalized number of cycles and the tabulated results of the number of cycles to failure of all the tests. The results of the tests at 25°C, 100°C and 175°C are compiled and plotted as S-N curves and the average results are extracted as shown in Figures 4.25, 4.26 and 4.27 respectively. Figure 4.28 shows a photograph of a typical failed specimen due to shear fatigue regardless of the temperature.

Figure 4.24: Typical shear fatigue peak displacement graph.

Table 4.5: Shear fatigue results for L-930 specimen.

25°C		100°C		175°C	
Test Description	Nf (Cycles)	Test Description	Nf (Cycles)	Test Description	Nf (Cycles)
90% 1 st	513	80% 1 st	99	70% 1 st	166
90% 2 nd	137	80% 2 nd	54	70% 2 nd	499
90% 3 rd	275	70% 1 st	147	60% 1 st	1857
90% 4 th	413	70% 2 nd	303	55% 1 st	7792
80% 1 st	380	60% 1 st	522	50% 1 st	942848
80% 2 nd	338	50% 1 st	11690	50% 2 nd	1350144
80% 3 rd	1247	45% 1 st	22464		
80% 4 th	9380	45% 2 nd	16068		
70% 1 st	3037	36% 1 st	1201585		
70% 2 nd	13863				
70% 3 rd	57236				
65% 1 st	167041				
60% 1 st	1813118

% Load	Nf (Cycles)
90%	335
80%	2836
70%	24712
65%	167041
60%	1813118

(a) Graphical plots

(b) Extracted average data

Figure 4.25: Shear fatigue S-N curve at 25°C and zero stress ratio.

% Load	Nf (Cycles)
80%	77
70%	225
60%	522
50%	11690
45%	19266
36%	1201585

(a) Graphical plots

(b) Extracted average data

Figure 4.26: Shear fatigue S-N curve at 100°C and zero stress ratio.

Figure 4.27: Shear fatigue S-N curve at 175°C and zero stress ratio.

Figure 4.28: Typical failed L-930 specimen under fatigue loading.

Figure 4.29 shows the shifted shear zero stress ratio fatigue strength at all temperatures. Connecting all the data of the same number of cycles to failure, the master curve of zero stress ratio shear fatigue strength is constructed as shown in Figure 4.30.

Figure 4.29: Shifted shear zero stress ratio vs log failure time.

Figure 4.30: Shear zero stress ratio fatigue strength master curve.

4.2.4 Shear Fatigue Strength Prediction and Validation

Figure 4.31 shows the predicted shear fatigue strength obtained for condition of 4Hz, 80°C and stress ratio of 0.5. The experimental results were however far off from the prediction as shown in Figure 4.32. This is because the Christensen's equations (equations (4.4) and (4.5)) used in the prediction of the creep strength master curve from constant strain rate (CSR) strength master curve are only valid for constant strain rate (CSR) case that have linear stress-strain relation (for example the tensile CSR test result of Figure 4.5). The shear CSR test result, on the other hand, has a relatively large non-linear stress-strain relationship. This could have been the likely cause for the deviation between the experimental and predicted fatigue strength at 4Hz, 80°C and stress ratio of 0.5.

Another set of experiments was performed at 5Hz, 80°C and zero stress ratio and the results agree very well with the predicted results as shown in Figure 4.33. It can be deduced that the shear fatigue strength prediction at zero stress ratio is still valid. Since the creep strength master curve was not utilized in predicting the shear fatigue strength at zero stress ratio, hence it can be deduced that the discrepancies mentioned above is caused by non-linear shear stress-strain relationship which was not accounted for in the creep strength master curve.

Figure 4.31 : Predicted shear fatigue strength at 4Hz, 80°C and stress ratio = 0.5.

Figure 4.32 : Predicted shear fatigue strength and validation results at 4Hz, 80°C and stress ratio = 0.5.

Figure 4.33 : Predicted shear fatigue strength and validation results at 5Hz, 80°C and zero stress ratio.

4.3 Properties Characterization for Finite Element Modelling

The material characterizations that have been done up until this point were using coupon specimens fabricated in the curing table. The woven CFRP I-beam used in the experimental validations of the structural fatigue described in Chapter 5 was

manufactured using the autoclave, hence a new set of tests was conducted to characterize coupons fabricated using the autoclave. The details of the new characterization will be discussed in the following.

4.3.1 Tensile Properties Characterization

The tensile specimen is 250mm long with gauge length of 150mm, 16mm wide and 4 layers thick (Figure 4.34). The end tabs were made of the same raw material. Static tensile tests were done in both warp and fill direction and the results are presented in Figure 4.35. The failure mode of warp and fill specimens are typically the same as shown in Figure 4.36. It is also similar to the failure mode of the specimen fabricated in the curing table (Figure 4.6). The tensile static properties in the warp and fill directions of the specimen fabricated in the autoclave are tabulated in Table 4.6. It can be seen that the modulus and strength in the warp and fill directions are very close. Thus the tensile fatigue characterization will only be done in the warp direction and the fatigue properties in the fill direction are assumed to be the same as the fatigue properties in the warp direction.

Figure 4.37 shows the normalized peak displacement against normalized number of cycles results from all the fatigue tests in the warp direction. Table 4.7 and Figure 4.38 show the tabulated number of cycles results and S-N curve respectively.

Figure 4.34: Tensile specimen picture (autoclave).

Figure 4.35: Tensile CSR stress vs strain graph at 25°C and 0.2mm/min (autoclave).

Figure 4.36: Typical failure of L-930 specimen under tensile static loading (autoclave).

Table 4.6: Tensile static properties in the warp and fill directions.

Warp	E11	σ_{11U}	Peak Load
	Mpa	Mpa	N
1	66997.8	1216.3	13537.9
2	66945.6	1215.2	13525.1
Avg	66971.7	1215.8	13531.5

Fill	E11	σ_{22U}	Peak Load
	Mpa	Mpa	N
1	67109.1	1188.8	13398.0
2	66723.4	1191.8	13765.7
Avg	66916.3	1190.3	13581.9

(a)

(b)

(c)

Figure 4.37: Normalized tensile peak displacement vs normalized no of cycle to failure graphs at 25°C, zero stress ratio and maximum load: a) 60% and 65%, b) 70% and 75%, c) 80% and 90% of the static strength.

Table 4.7: Zero stress ratio tension-tension fatigue test results table at 25°C (autoclave).

Test Description	No of Cycles to Failure (Cycles)
90% 1 st	33
90% 2 nd	27
80% 1 st	248
80% 2 nd	387
75% 1 st	502
75% 2 nd	4521
70% 1 st	7482
70% 2 nd	5308
65% 1 st	158656
65% 2 nd	91440
60% 1 st	360911
60% 2 nd	683417

Figure 4.38: S-N curve for L-930 specimen (autoclave) at 25°C and zero stress ratio.

4.3.2 Shear Properties Characterization

The dimensions of the shear specimen are exactly the same as the dimensions of the shear specimen for previous characterization (Figure 4.19). Figure 4.39(a) shows the shear specimen fabricated using the autoclave. The shear static test results are presented in Figure 4.40 and the typical failed specimen is shown in Figure 4.39(b). The failure mode is similar to the failure mode of the shear specimen fabricated in the curing table (Figure 4.21). The normalized peak displacement against normalized number of cycles results are shown in Figure 4.41. The number of cycles results are tabulated in Table 4.8 and the S-N curve is presented in Figure 4.42.

Figure 4.39: a) Shear specimen picture (autoclave), b) typical failure of L-930 specimen under shear loading (autoclave).

Figure 4.40: Shear CSR stress vs strain graph at 25°C and 1mm/min (autoclave).

Figure 4.41: Shear maximum displacement vs normalized no of cycle to failure graphs at 25°C and zero stress ratio.

Table 4.8: Zero stress ratio shear fatigue test results table at 25°C (autoclave).

Test Description	No of Cycle to Failure (Cycles)
95% 1 st	50
90% 1 st	63
80% 1 st	165
75% 1 st	269
70% 1 st	457
67% 1 st	12009
62% 1 st	7749
62% 2 nd	1346370
60% 1 st	356265
60% 2 nd	119909
57% 1 st	1634903

Figure 4.42: Shear S-N curve for L-930 specimen (autoclave) at 25°C and zero stress ratio.

The characterization of the coupon fabricated using the autoclave is now complete. Tables 4.9(a) and (b) show the tabulated static properties of L-930 woven carbon-epoxy fabricated using curing table and autoclave respectively. It can be seen that the autoclave L-930 woven carbon-epoxy has higher normal and shear modulus and static strength. In the case of fatigue properties, the tension-tension fatigue strength for the autoclaved material has less scatter as shown in Figure 4.43. The shear fatigue strength of the specimens fabricated in the autoclave is considerably higher than the specimens fabricated in the curing table (Figure 4.44). The difference in the tensile and shear static and fatigue properties between specimen fabricated using autoclave and curing table is

likely caused by the difference in the fibre volume fraction. Table 4.10 shows the fibre volume fraction obtained from the burn test of specimen fabricated in curing table and autoclave. It can be seen that the specimen fabricated in the autoclave has higher fibre volume fraction because the additional pressure in the fabrication process (6.9 bar in the autoclave against 1 bar in the curing table). It means the specimen fabricated in the autoclave have more volume of fibres within the same total volume, which gives this specimen higher strength and modulus in comparison to the specimen fabricated in curing table (as fibres carry more load than matrices).

Table 4.9 : L-930 flame retardant woven carbon-epoxy static properties.

(a) Curing Table		(b) Autoclave	
$E_{11t} = E_{22t}$	59.40 GPa	$E_{11t} = E_{22t}$	66.97 GPa
$\sigma_{11tu} = \sigma_{22tu}$	1063.80 MPa	$\sigma_{11tu} = \sigma_{22tu}$	1215.80 MPa
G_{12}	3.86 GPa	G_{12}	4.56 GPa
τ_{12u}	83.50 MPa	τ_{12u}	92.28 MPa
ν_{12}	0.20	ν_{12}	0.05

Figure 4.43: Tensile S-N curve comparison between L-930 woven carbon-epoxy made in curing table and autoclave.

Figure 4.44: Shear S-N curve comparison between L-930 woven carbon-epoxy made in curing table and autoclave.

Table 4.10 : Burn test results.

Autoclave	Fibre volume fraction (%)	Curing table	Fibre volume fraction (%)
1	59.54	1	58.36
2	61.55	2	57.87
3	61.80	3	58.81
Average	60.96	Average	58.35

Chapter Five

Fatigue Modelling and Experimental Validation

The characterization of the woven CFRP test coupons under static and fatigue loading were studied thoroughly in Chapter 4. In this chapter, empirical damage models are developed based on the experimental data of the coupon tests with the aim of predicting the static and fatigue response of a real structure. A beam with an I-shaped cross section is chosen as this structure has 3-dimensional shape, relatively complex stress state and very common in real life. The beam is subjected to four-point bending and the predicted structural response is compared with the experimental result. Take note that some figures are plotted with a non-zero axis to accentuate the results.

5.1 Problem Definition

The I-beam structure is made of L-930 woven carbon-epoxy and has the dimensions as shown in Figure 5.1. The fabrication process of the I-beam was explained in details in Chapter 3. This structure will undergo 4-point bending static and fatigue loading, with 80mm loading span and 270mm supporting span, at room temperature (25°C). For the fatigue test, the loading will be in sinusoidal form with frequency of 5 Hz and zero load ratio (R) as shown in Figure 3.12.

Figure 5.1: Specimen dimensions (mm): a) isometric view, b) front view.

5.2 Finite Element Modelling

For the prediction of the fatigue behaviour of the I-beam, two models were constructed: static damage and fatigue damage models. Abaqus v6.10 is used as the tool to build and run the models. The process of building the models is discussed in the following.

5.2.1 Finite Element Models

Three finite element models were constructed: tensile coupon, shear coupon and 4-point bending I-beam models. The tensile and shear coupon models were used to validate the static and fatigue damage models by comparing the finite element results with the tensile and shear coupon test results. The I-beam model was used to predict the static and fatigue behaviour of the composite I-beam subjected to 4-point bending load.

Tensile coupon model

Only a quarter of the tensile coupon was modelled in order to reduce the computational time. The convergence test that has been done shows that the accuracy of the model is independent of the mesh size. The final tensile coupon model with its constraint is shown in Figure 5.2. A linear shell element is used with a mesh size of 0.5mm by 0.5mm which gives a total of 2400 elements.

Figure 5.2: Tensile coupon model.

Shear coupon model

In order to validate the damage model especially for the shear damage model, a shear coupon model was built. A convergence test was done by running shear static test simulations on several different mesh sizes and the results in the form of % change of shear strength against number of element are shown in Figure 5.3. The model uses a linear shell element and the mesh size is varied from 1.15mm by 1.15mm to 0.5mm by 0.5mm at the stress concentration area, which gives a total of 5079 elements. Figure 5.4 shows the final shear coupon model with its constraints.

Figure 5.3: Convergence test results of shear coupon model.

Figure 5.4: Shear coupon model.

4-point bending I-beam model

Figure 5.5 shows the symmetries of a 4-point bending loading case of an I-beam. For a more computationally efficient model, a quarter of the I-beam was modelled. A convergence test was done by running 4-point bending static test simulations on several different mesh densities and the results in the change of failure load against number of element are shown in Figure 5.6. The final I-beam model used a linear shell element with a mesh size of 1mm by 1mm with a refinement of 0.1mm by 0.1mm at the stress concentration area which gives a total of 15,737 elements. Constraints are imposed at the X-Symmetry and Z-Symmetry and contacts are modelled at loading and supporting points as shown in Figure 5.7. The local 1-direction (the direction of σ_{11}) is aligned to the global z-direction for the whole model. The local 2-direction (the direction of σ_{22}) is aligned to the global x-direction for the flanges and to the global y-direction for the web as shown in Figure 5.7.

Figure 5.5: Symmetries in the I-beam model.

Figure 5.6: Convergence test results of 4-point bending I-beam model.

Figure 5.7: I-beam model.

5.2.2 Static damage models

Two static damage models were developed. The first static damage model used the maximum stress criterion, which stated that failure will occur if any one of the following inequalities is violated

$$\sigma_{11} < \sigma_{11u} \quad \sigma_{22} < \sigma_{22u} \quad \tau_{12} < \tau_{12u} \quad (5.1)$$

where σ_{ij_u} are the ultimate strengths.

The other static damage model employed the modified Hashin failure criterion. The original Hashin failure criterion was developed for unidirectional (UD) laminae and consists of four failure modes: fibre tension and compression (1st direction), matrix tension and compression (2nd direction) as can be seen in equations (2.10) to (2.13). In the case of L-930 flame retardant woven carbon-epoxy which is balanced woven CFRP, the properties in the 1st and 2nd directions are assumed to be the same and as fibres aligned in both directions, fibre tension and compression equations are also applied to both 1st and 2nd directions. The final form of modified Hashin failure criterion which is used in the static damage model is defined as:

Fibre tension ($\sigma_{11} \geq 0$)

$$\left(\frac{\sigma_{11}}{\sigma_{11u}} \right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}} \right)^2 \leq 1 \quad (5.2)$$

Fibre compression ($\sigma_{11} \leq 0$)

$$\left(\frac{\sigma_{11}}{\sigma_{11cu}} \right)^2 \leq 1 \quad (5.3)$$

Fibre tension ($\sigma_{22} \geq 0$)

$$\left(\frac{\sigma_{22}}{\sigma_{22tu}} \right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}} \right)^2 \leq 1 \quad (5.4)$$

Fibre compression ($\sigma_{22} \leq 0$)

$$\left(\frac{\sigma_{22}}{\sigma_{22cu}} \right)^2 \leq 1 \quad (5.5)$$

Based on the modified Hashin failure criterion, failure in a particular mode occurs if inequality for that particular mode is violated.

The tensile stress remains linear over the tensile strain until failure as found earlier in Figure 4.35. Thus, E_{11} and E_{22} are modelled constant until failure in the static damage model. The relationship of shear stress to shear strain is however non-linear. The L-930 woven carbon-epoxy undergoes shear plastic deformation as can be seen in Figure 5.8. Thus, in modelling shear properties, 6 material points are used to model the non-linear effect as shown in Figure 5.8.

USDFLD is used as the subroutine for static damage model. By using this subroutine, user-defined field variables can be connected to the material elastic properties. In this case, three user-defined field variables were used: FV1, FV2 and FV3. The connections between these user-defined variables with the material elastic properties of maximum stress and modified Hashin models can be seen in Tables 5.1 and 5.2 respectively. The subroutines program of the maximum stress and modified Hashin static damage models can be found in the Appendix I and II respectively. By using these subroutines in conjunction with the quarter I-beam finite element model, the progressive static failure of I-beam can be simulated.

Figure 5.8: Shear static stress vs strain of L-930 woven carbon-epoxy.

Table 5.1 : Material properties relation with field variables (maximum stress criterion).

Condition	FV1	FV2	FV3	E_{11}	E_{22}	ν_{21}	G_{12}
No failure	0	0	0	E_{11}	E_{22}	ν_{21}	G_{12}
1 st direction normal failure	1	0	0	0	E_{22}	0	G_{12}
2 nd direction normal failure	0	1	0	E_{11}	0	0	G_{12}
1 st and 2 nd directions normal failure	1	1	0	0	0	0	0
Shear failure	0	0	1	E_{11}	E_{22}	0	0
1 st direction normal and shear failure	1	0	1	0	E_{22}	0	0
2 nd direction normal and shear failure	0	1	1	E_{11}	0	0	0
1 st and 2 nd directions normal and shear failure	1	1	1	0	0	0	0

Table 5.2 : Material properties relation with field variables (modified Hashin criterion).

Condition	FV1	FV2	FV3	E_{11}	E_{22}	ν_{21}	G_{12}
No failure	0	0	0	E_{11}	E_{22}	ν_{21}	G_{12}
1 st direction failure	1	0	1	0	E_{22}	0	0
2 nd direction failure	0	1	1	E_{11}	0	0	0
1 st and 2 nd directions failure	1	1	1	0	0	0	0

5.2.3 Fatigue damage model

A fatigue damage model was developed to predict the stiffness degradation as well as the number of cycles to failure. Two fatigue damage models were developed: maximum stress fatigue damage and modified Hashin fatigue damage models. The only difference between both models is the way the models determine the final failure. The

derivation of the damage equation as well as the way to determine failure for both models will be discussed in the following.

The fatigue life of woven composites is typically divided into three stages: the initial, middle and final stages. Figure 5.9(a) shows a typical experimental result of maximum displacement against number of cycles in the tension-tension fatigue test. This result can be converted to modulus decay against relative log number of cycles as shown in Figure 5.9(b). It is interesting to note that the modulus decay is linear over the log number of cycles from the beginning until near to the end of fatigue life of the test coupon. This phenomenon is observed in all the coupon test results.

Figure 5.9: a) Typical maximum displacement vs number of cycle result, b) typical modulus decay vs relative log no of cycle result.

Another interesting phenomenon that is observed in the test results is the rate of modulus decay increases as the stress increases as shown in Figures 5.10(a) and (b). This phenomenon together with the fact that modulus decay is linear over the log number of cycles then were used to calculate the new modulus (E^{new}) at the end of every increment as defined in equations (5.6) and (5.7). This is the newly developed "stiffness decay model".

$$\frac{\Delta E}{E^{initial}} = \left(\frac{d\left(\frac{E}{E^{initial}}\right)}{d(\log N)} \right) \Delta(\log N) \quad (5.6)$$

$$E^{new} = E^{old} + \Delta E \quad (5.7)$$

where $\Delta(\log N)$ is the difference between log number of cycles of current and previous increments. $E^{initial}$, E^{old} and ΔE are the static modulus, the modulus at the end of the previous increment and the difference between the modulus at the end of the current and previous increments respectively. The sign in equation (5.7) is plus because the modulus decay rate (Figures 5.10(a) and (b)) is already negative.

The last thing that needs to be addressed is the failure determination. The way to determine failure in both maximum stress and modified Hashin fatigue damage models are different and will be discussed separately in the following.

Figure 5.10: a) Normal modulus decay rate vs normal stress, b) Shear modulus decay rate vs shear stress.

Maximum stress fatigue damage model

The proposed maximum stress fatigue damage model is also developed based on the phenomenon of linear modulus decay over log number of cycles that is observed earlier. The newly proposed damage model is:

$$D^{new} = \frac{\log(10^{(D^{old} \cdot \log(N_f))} + \Delta N)}{\log(N_f)} \quad (5.8)$$

where $\log(N_f)$ is the log number of cycles to failure that is calculated from the S-N curve at a particular state of stress (Figures 4.38 and 4.42), D^{new} and D^{old} are the damage parameters at the current and previous increments respectively, and ΔN is the difference between the number of cycles of the current and previous increments. This damage equation is calculated separately for each stress component at every increment.

The failure in the maximum stress fatigue damage model will occur when the damage parameter in equation (5.8) equals to one. The flowchart and the subroutine of the maximum stress fatigue damage model are shown in Figure 5.11(a) and Appendix I respectively.

Figure 5.11: Flowchart of damage and failure in: a) maximum stress fatigue damage model, b) modified Hashin fatigue damage model.

Modified Hashin fatigue damage model

Besides the modulus degradation of equations (5.6) and (5.7), the modified Hashin fatigue damage model also incorporates strength degradation. The proposed strength degradation model is:

$$\sigma_f^{new} = \sigma_f^{old} - \left((\sigma_s - \sigma) \left(\frac{\Delta N}{Nf} \right) \right) \quad (5.9)$$

Equation (5.9) is calculated separately for each stress component at every increment. The degraded fatigue strengths (σ_f^{new}) replace the ultimate strengths (σ_u) in equations (5.2) to (5.5), and this allows one to determine failure at a particular element. Figure 5.11(b) shows the flowchart of the modified Hashin fatigue damage model, where H is

the left side of equations (5.2) to (5.5). The subroutine of the modified Hashin fatigue damage model can be found in Appendix II.

5.3 Finite Element Results and Experimental Validation

5.3.1 Damage Model Validation

Before using the model to predict the static and fatigue performance of a woven CFRP I-beam structure, the damage models are validated by comparing the finite element results of the coupon models with the coupon experimental results.

Tensile damage model validation

Figure 5.12(a) shows the predicted tension stress and strain response of maximum stress and modified Hashin static damage models in comparison with the experimental results, which shows that the models give very good prediction in the tension static case. Figure 5.12(b) shows the S-N curve comparison between experimental results and fatigue damage models results. Both maximum stress and modified Hashin fatigue damage models give very good prediction in the tension-tension fatigue case as shown.

Figure 5.12: a) Tensile static stress vs strain graph, b) tension-tension fatigue S-N curve.

Shear damage model validation

The predicted failure in static and fatigue shear coupon for both maximum stress and modified Hashin damage models is shown in Figure 5.13(b). The models failed in the mid-section where the cross section area is minimum. The shear stress against strain graph in Figure 5.13(a) shows that the maximum stress and modified Hashin static

damage models give very good prediction in the case of shear static. Figure 5.14 shows the S-N curves of fatigue damage models in comparison with experimental results, which shows that maximum stress and modified Hashin fatigue damage models give relatively good prediction in the shear fatigue case as well.

Figure 5.13: a) Shear static stress vs strain graph, b) failed elements (shown in red) in the static and fatigue shear coupon models.

Figure 5.14: Shear coupon progressive fatigue results.

The coupons validation of the damage models that have been developed gave very promising results and this gives the required confidence in using the damage models to predict structural response with a much more complicated stress state.

5.3.2 Static Four-Point Bending Finite Element Results and Experimental Validation

In order to validate the progressive damage models that have been developed, beside validation using coupon models, a detailed experimental investigation was conducted in parallel to the numerical study. The test is a standard four-point bending and the structure is an I-beam made of L-930 woven carbon-epoxy. Under static four-point loading, the woven CFRP I-beam gives a structural response as shown in Figure 5.15. The predicted results of maximum stress and modified Hashin static damage models are plotted for comparison as well. It can be seen that the predicted results generally agreed with the experimental results (Table 5.3). The proposed models predict the maximum load and the flexural stiffness of the woven CFRP I-beam quite well and also give accurate prediction of the failure location. The post-mortem of the failure shows that a crack initiated at the top flange that is in contact with the roller and is possibly caused by compression in the length-wise direction of the I-beam. It then propagated to the web and this possibly due to low shear resistance in the web. The failure location (failed element) of the maximum stress and modified Hashin models are relatively the same as shown in Figures 5.16. A photograph of the failure region of the tested I-beam is shown in Figure 5.17.

Figure 5.15: I-beam static results.

Table 5.3 : I-beam 4-point bending static results.

	Failure Load (N)			
	Max Stress	Modified Hashin	Experiment 1	Experiment 2
1 st Failure	2439.54	2439.54	NA	NA
Final Failure	2486.40	2460.82	2459.47	2336.50

Figure 5.16 : Failed elements (shown in red) in maximum stress and modified Hashin static damage models.

Figure 5.17: Failed I-beam woven CFRP in static test.

5.3.3 Fatigue Four-Point Bending Finite Element Results and Experimental Validation

The test rig for four-point bending fatigue is the same standard four-point bending rig used in the static test and the structure is an I-beam made of woven CFRP of identical dimensions. The failure mode and location of the maximum stress and modified Hashin fatigue damage models are relatively the same as the static model results as shown in Figure 5.18. Figure 5.19 shows a photograph of the fatigue failure region of the tested I-beam. As can be seen in the S-N curves in Figures 5.20, the difference between maximum stress and modified Hashin fatigue damage models is very small. Both models give an accurate number of cycles to failure prediction. Table 5.4 shows the number of cycles to failure results of fatigue damage models in comparison with the experimental results. Plots of the relative stiffness against log number of cycles response for maximum load of 1836N, 1887N, 1938N and 2040N are presented in Figures 5.21(a), (b), (c) and (d) respectively, where relative stiffness is defined as fatigue stiffness divided by initial stiffness. The finite element models predicted the relative stiffness response over the

number of cycles reasonably well at lower loads. At higher loads, the finite element models predicted the response rather well at the beginning and deviate slightly as the cycle progresses. One of the main possible reason for this discrepancy may be due to the imperfection of the specimens. It must be mentioned that the fabrication process of the I-beam specimen is more complicated compared to the fabrication process of the tensile and shear specimens, and geometric imperfections of the I-beam specimens are ultimately unavoidable. As a result, more data scatter is found in the I-beam specimen experimental results. The finite element model on the other hand, is modelled with an assumption that the I-beam is perfect. The other possible reason for the deviation between the experimental and predicted results is that the current model assumes the same tensile and compressive static and fatigue properties. The finite element results are slightly stiffer than the experimental results for the static case. The stiffness difference becomes more obvious in the case of fatigue, especially at the end of fatigue life. It is possible that the compression-compression fatigue properties differ from the tension-tension fatigue properties. It is recommended that one do the compression-compression fatigue material characterization in the future to improve the current finite element model.

Figure 5.18 : Failed elements (shown in red) in maximum stress and modified Hashin fatigue damage models.

Figure 5.19: Failed I-beam woven CFRP in fatigue test.

Table 5.4 : I-beam 4-point bending fatigue number of cycles to failure results: a) finite element models, b) experiments.

(a)					(b)	
Load (N)	Max Stress		Modified Hashin		Test Description	Nf (Cycles)
	1 st Element Failure	Final Failure	1 st Element Failure	Final Failure		
2040	560	699	560	699	2040N 1 st	160
1938	3000	3499	3000	3476	2040N 2 nd	522
1887	6600	7486	6600	7444	1938N 1 st	2676
1836	14000	14232	14500	14512	1938N 2 nd	312
					1887N 1 st	1555
					1887N 2 nd	3688
					1836N 1 st	280596
					1836N 2 nd	159452
					1836N 3 rd	6558

Figure 5.20: I-beam S-N curve: a) 1st element failure, b) final failure.

Figure 5.21: Relative stiffness against log number of cycles with maximum load = a) 1836N, b) 1887N, c) 1938N, d) 2040N.

The new models that have been developed and implemented to a structure in 4-point bending, successfully predict the static and fatigue behaviour of the woven CFRP I-beam. The prediction in general correlated well in some situations and in other situations they showed the deficiency in the current damage models developed.

Chapter Six

Conclusion and Future Works

6.1 Conclusion

In summary, significant conclusions that can be deduced from the results presented in Chapters 4 and 5 are:

1. The tensile fatigue properties of L-930 woven carbon-epoxy for arbitrary frequency, temperature and load ratio were fully characterized and successfully validated using experimental results.
2. The shear fatigue properties of L-930 woven carbon-epoxy for arbitrary frequency, temperature and load ratio were fully characterized but failed in the validation process at 4Hz, 80°C and stress ratio of 0.5. In this case they were far off from the experimental validation results. The main reason for this failure is that the Christensen's equation can only be used on linear stress-strain CSR responses; however the shear CSR stress-strain responses are very non-linear.
3. The shear fatigue properties of L-930 woven carbon-epoxy for arbitrary temperature, zero load ratio and frequency of 5Hz were successfully validated as the creep strength master curve was not utilized in the prediction. This further supports the limitation of Christensen's equation mentioned in the above point 2.
4. The static strength of an I-beam structure made of L-930 woven carbon-epoxy was modelled and analyzed using finite element software. The analysis results from the maximum stress and modified Hashin static damage models agree very well with the experimental validations.
5. The fatigue life of an I-beam structure made of L-930 woven carbon-epoxy was modelled and analyzed. The maximum stress and modified Hashin fatigue damage models give considerably good prediction on fatigue life as well as the stiffness decay as confirmed by the experimental validation results.

6.2 Original Technical Contributions

The original technical contributions from this current research are:

1. Found evidences showing that accelerated testing methodology cannot be used to predict the shear fatigue strength of L-930 woven carbon-epoxy at arbitrary load ratio but can be used to predict the shear fatigue strength of L-930 woven carbon-epoxy at arbitrary temperature and zero load ratio.
2. It was found that modulus decay is linear over the log of number of cycles from the beginning until the end of fatigue life and the rate of modulus decay increases as the stress increases for L-930 woven carbon-epoxy material.
3. A new "stiffness decay model" is presented to predict static and fatigue progressive failure in composites.

6.3 Future Works

The future researches should focus on the following:

1. Building shear creep strength master curve for L-930 woven carbon-epoxy by doing creep test at several different temperatures and load magnitudes.
2. Fully characterize shear fatigue properties of L-930 woven carbon-epoxy for arbitrary frequency, temperature and load ratio utilizing creep strength master curve discussed above.
3. Compression material characterization (static and fatigue).
4. Validation using other types of structures such as n-beam and sandwich.
5. Improving the current damage models to include the compression data and the effect of temperature.
6. To account for the non-linear shear CSR in predicting fatigue failure for non-zero load ratio.

LIST OF PUBLICATIONS

Journals Publications

1. Satrio Wicaksono and Chai, Gin Boay. (2012). A review of advances in fatigue and life prediction of fiber-reinforced composites. Proceedings of the Institution of Mechanical Engineers - Part L: Journal of Materials: Design and Applications, , 1-17.
2. Satrio Wicaksono and Chai, Gin Boay. (2013). The Response of Woven CFRP under Static and Fatigue Loading. Advanced Materials Research, Vol 651, 221-226.

Conferences Publications

1. Satrio Wicaksono and Chai G.B., 2012, The Response of Woven CFRP under Static and Fatigue Loading, 2012 International Conference on Engineering Materials, Singapore, Singapore, 30-31 December 2012.
2. Satrio Wicaksono and Chai G.B., 2013, Failure of Laminated Panels with Initial Delamination in Tension, The 5th Regional Conference on Mechanical and Aerospace Technology, Bangkok, Thailand, 12-13 February 2013.
3. Satrio Wicaksono and Chai G.B., 2013, Fatigue Life Prediction of Woven CFRP Structure, 17 International Conference on Composite Structures, Porto, Portugal, 17-21 June 2013.

Publications in Progress

1. Satrio Wicaksono and Chai G.B. (2013). Life Prediction of Woven CFRP Structure Subject to Static and Fatigue Loading. Composite Structures.
2. Satrio Wicaksono and Chai G.B. (2013). Static and Fatigue Properties of L-930 Woven Carbon-Epoxy in Tension, Shear and Bending. Composite Science and Technology.

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APPENDIX I

MAXIMUM STRESS PROGRESSIVE DAMAGE STATIC & FATIGUE SUBROUTINE

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SUBROUTINE USDFLD(FIELD,STATEV,PNEWDT,DIRECT,T,CELENT,TIME,DTIME,
1 CMNAME,ORNAME,NFIELD,NSTATV,NOEL,NPT,LAYER,KSPT,KSTEP,KINC,
2 NDI,nshr,coord,jmac,jmtyp,matlayo,laccflg)
C
  INCLUDE 'ABA_PARAM.INC'
C PROGRESSIVE FATIGUE SUBROUTINE
C MATERIAL AND STRENGTH PARAMETERS
C X means 1st direction Y means 2nd direction
C T means tensile C means compression
C XS is the shear strength
C XTM is the meeting point of the high and low tensile s-n curve
C XTMI is the minimum tensile stress that will affect the fatigue
C properties
C XSF is the end of linear fatigue s-n curve and the starting
C XSM is the meeting point of the high and low shear s-n curve
C XSMI is the minimum shear stress that will affect the fatigue
C point of quadratic fatigue s-n curve
C properties
C all in MPa
  PARAMETER(XT=1215.77D0,YT=1215.77D0)
  PARAMETER(XC=-1215.77D0,YC=-1215.77D0,XS=92.28D0)
  PARAMETER(XTM=720.652D0,XSF=87.78178D0,XSM=55.10507D0)
  PARAMETER(XTMI=197.349D0,XSMI=11.513D0)
  PARAMETER(RXE=0.872419D0,RXS=0.320541D0)
C
C FATIGUE STRENGTH CONSTANTS
C CONDITION: TEMP=25DEGC FREQ=5HZ STRESS RATIO=0
C THE CONSTANT VALUES ARE FOR S-N CURVE IN LOG Nf DOMAIN
C
C THE SHEAR STRESS AND PLASTIC SHEAR STRAIN (6 LINES)
C
C GI = INITIAL G12
C
C THE LAST ELASTIC REGION STRAIN ; BEGINNING OF PLASTIC REGION
C ALS,BLS,AHS,BHS ARE THE LINE CONSTANTS OF
C THE  $\Delta(G12/G12_{initial})/\Delta \log N_f$  graph
C ALT,BLT,AHT,BHT ARE THE LINE CONSTANTS OF
C THE  $\Delta(E11/E11_{initial})/\Delta \log N_f$  graph
  PARAMETER(CATH=4.741D0,CBTH=-112.054D0,CCTH=1223.361D0)
  PARAMETER(CATL=2.6519D0,CBTL=-106.43D0,CCTL=1265.2D0)
  PARAMETER(CASH=1.2675D0,CBSH=-16.159D0,CCSH=106.48D0)
  PARAMETER(CASL=0.067D0,CBSL=-4.2298D0,CCSL=78.271D0)
  PARAMETER(CDS=-3.4949D0,CES=92.28D0)
  PARAMETER(TS1=20.D0,TS2=40.D0,TS3=50.D0)
  PARAMETER(TS4=60.D0,TS5=70.D0,TS6=92.356D0)
  PARAMETER(ES1=0.D0,ES2=8300.721D-6,ES3=17159.31D-6)
  PARAMETER(ES4=36792.89D-6,ES5=67202.33D-6,ES6=144416.2D-6)

```

```

PARAMETER(GI=4558.D0,ES0=4387.737D-6)
PARAMETER(ALT=-2.777D-5,BLT=-3.190D-4)
PARAMETER(AHT=-6.843D-5,BHT=2.979D-2)
PARAMETER(ALS=-1.598D-3,BLS=2.993D-4)
PARAMETER(AHS=-2.030D-2,BHS=1.001D0)
C
  CHARACTER*80 CMNAME,ORNAME
  CHARACTER*3  FLGRAY(15)
  DIMENSION FIELD(NFIELD),STATEV(NSTATV),DIRECT(3,3),T(3,3),TIME(2),
    * coord(*),jmac(*),jmtyp(*)
  DIMENSION ARRAY(15),JARRAY(15)
C
C INITIALIZE FAILURE FLAGS FROM STATEV.
C ZF, ZM AND ZS ARE THE MODULUS DEGRADATION VARIABLES
C ZFF, ZMF AND ZSF ARE THE DAMAGE VARIABLES
C TT IS THE TOTAL NUMBER OF CYCLE
C ES IS THE THE SHEAR MODULUS DEGRADATION FROM STATIC PART
  ZF  = STATEV(1)
  ZM  = STATEV(2)
  ZS  = STATEV(3)
  TT  = STATEV(4)
  ES  = STATEV(5)
  ZFF = STATEV(6)
  ZMF = STATEV(7)
  ZSF = STATEV(8)
  ZEF = STATEV(9)
  ZEM = STATEV(10)
  ZES = STATEV(11)
C
C GET STRESSES FROM PREVIOUS INCREMENT
  CALL GETVRM('S',ARRAY,JARRAY,FLGRAY,jrcd,
    $  jmac, jmtyp, matlayo, laccflg)
  S11 = ARRAY(1)
  S22 = ARRAY(2)
  S12 = ARRAY(4)
C THE ABSOLUTE STRESS
  AS11 = ABS(S11)
  AS22 = ABS(S22)
  AS12 = ABS(S12)
C NEW TOTAL NUMBER OF CYCLES = FTT
C TT=THE OLD TOTAL NUMBER OF CYCLES
C DTIME=DELTA NUMBER OF CYCLES (ADDITIONAL NUMBER OF CYCLES IN THAT
C PARTICULAR INCREMENT)
  FTT = TT + DTIME
C
C FATIGUE STRENGTH EQUATION
  GTT = log10(TT+(10.D0**(-20.D0)))
  GFTT = log10(FTT+(10.D0**(-20.D0)))
C
C TO PUT THE NEW TOTAL TIME IN THE STATEV
  STATEV(4) = FTT
C
C FIBER 2ND DIRECTION TENSILE/COMPRESSIVE FAILURE
C GXT1 is the log failure Nf at current state of stress
C EF and EEF are the measure of residual modulus
C FOR FTT < 1 - IT WILL DO STATIC ANALYSIS
C FOR FTT > 1 - IT WILL DO FATIGUE ANALYSIS
  IF (FTT .LT. 1.D0) THEN
    IF (ZFF .LT. 1.D0) THEN
      ZFF = AS11/XT

```

```

IF (ZFF .LT. 1.D0) THEN
  FIELD(1) = 0.D0
  STATEV(1) = 0.D0
  STATEV(6) = 0.D0
  STATEV(9) = 1.D0
ELSE
  FIELD(1) = 1.D0
  STATEV(6) = ZFF
  STATEV(9) = 0.D0
ENDIF
ELSE
  FIELD(1) = 1.D0
  STATEV(9) = 0.D0
ENDIF
ELSE
  IF (ZFF .LT. 1.D0) THEN
    IF (AS11 .LT. XT) THEN
      IF (AS11 .GT. XTMI) THEN
        IF (AS11 .GT. XTM) THEN
          CAT1=CATH
          CBT1=CBTH
          CCT1=CCTH
          GXT1F = ((-CBT1)-SQRT((CBT1**2.D0)
* -(4.D0*CAT1*(CCT1-AS11)))) / (2.D0*CAT1)
          ZZFF = (log10((10.D0**(ZFF*GXT1F)
* +DTIME)))/GXT1F
          STATEV(6) = ZZFF
          AT1=AHT
          BT1=BHT
        ELSE
          CAT1=CATL
          CBT1=CBTL
          CCT1=CCTL
          AT1=ALT
          BT1=BLT
        ENDIF
          GXT1 = ((-CBT1)-SQRT((CBT1**2.D0)
* -(4.D0*CAT1*(CCT1-AS11)))) / (2.D0*CAT1)
          ZZF = (log10((10.D0**(ZF*GXT1)
* +DTIME)))/GXT1
          STATEV(1) = ZZF
          DEDNF=(AT1*AS11)+BT1
          DNF=(log10((10.D0**(ZF*GXT1))+DTIME))
* -(ZF*GXT1)
          DEF=DEDNF*DNF
          ZZEF=ZEF+DEF
          IF (ZZEF .GT. ZEF) THEN
            ZZEF=ZEF
          ENDIF
          IF (ZZEF .GT. 1.D0) THEN
            STATEV(6) = 1.D0
          ENDIF
          IF (ZZEF .LT. 0.D0) THEN
            ZZEF = 0.D0
          ENDIF
          EEf=1.D0-ZZEF
          IF (STATEV(6) .LT. 1.D0) THEN
            IF (EEF .LT. 1.D0) THEN
              FIELD(1) = EEF
              STATEV(9) = ZZEF
            
```

```

ELSE
  FIELD(1) = 1.D0
  STATEV(9) = 0.D0
ENDIF
ELSE
  FIELD(1) = 1.D0
  STATEV(9) = 0.D0
ENDIF
ELSE
  FIELD(1) = 1.D0-ZEF
ENDIF
ELSE
  STATEV(6) = 1.D0
  FIELD(1) = 1.D0
  STATEV(9) = 0.D0
ENDIF
ELSE
  FIELD(1) = 1.D0
ENDIF
END IF
C
C FIBRE 2ND DIRECTION TENSILE/COMPRESSIVE FAILURE
IF (FTT .LT. 1.D0) THEN
  IF (ZMF .LT. 1.D0) THEN
    ZMF = AS22/YT
    IF (ZMF .LT. 1.D0) THEN
      FIELD(2) = 0.D0
      STATEV(2) = 0.D0
      STATEV(7) = 0.D0
      STATEV(10) = 1.D0
    ELSE
      FIELD(2) = 1.D0
      STATEV(7) = ZMF
      STATEV(10) = 0.D0
    ENDIF
  ELSE
    FIELD(2) = 1.D0
    STATEV(10) = 0.D0
  ENDIF
ELSE
  IF (ZMF .LT. 1.D0) THEN
    IF (AS22 .LT. YT) THEN
      IF (AS22 .GT. XTMI) THEN
        IF (AS22 .GT. XTM) THEN
          CAT=CATH
          CBT=CBTH
          CCT=CCTH
          GXT2F = ((-CBT)-SQRT((CBT**2.D0)
* -(4.D0*CAT*(CCT-AS22)))) / (2.D0*CAT)
          ZZMF = (log10((10.D0**((ZMF*GXT2F)
* +DIME)))/GXT2F
          STATEV(7) = ZZMF
          AT2=AHT
          BT2=BHT
        ELSE
          CAT=CATL
          CBT=CBTL
          CCT=CCTL
          AT2=ALT
          BT2=BLT

```

```

ENDIF
GXT2 = ((-CBT)-SQRT((CBT**2.D0)
* -(4.D0*CAT*(CCT-AS22)))) / (2.D0*CAT)
ZZM = (log10((10.D0**(ZM*GXT2)
* +DTIME))/GXT2
STATEV(2) = ZZM
DEDNM=(AT2*AS22)+BT2
DNM=(log10((10.D0**(ZM*GXT2))+DTIME))
* -(ZM*GXT2)
DEM=DEDNM*DNM
ZZEM=ZEM+DEM
IF (ZZEM .GT. ZEM) THEN
ZZEM=ZEM
ENDIF
IF (ZZEM .GT. 1.D0) THEN
STATEV(7) = 1.D0
ENDIF
IF (ZZEM .LT. 0.D0) THEN
ZZEM = 0.D0
ENDIF
EEM=1.D0-ZZEM
IF (STATEV(7) .LT. 1.D0) THEN
IF (EEM .LT. 1.D0) THEN
FIELD(2) = EEM
STATEV(10) = ZZEM
ELSE
FIELD(2) = 1.D0
STATEV(10) = 0.D0
ENDIF
ELSE
FIELD(2) = 1.D0
STATEV(10) = 0.D0
ENDIF
ELSE
FIELD(2) = 1.D0-ZEM
ENDIF
ELSE
STATEV(7) = 1.D0
FIELD(2) = 1.D0
STATEV(10) = 0.D0
ENDIF
ELSE
FIELD(2) = 1.D0
ENDIF
ENDIF
C
C SHEAR FAILURE
C CONDITION: TEMP=25DEGC FREQ=5HZ STRESS RATIO=0
C
IF (FTT .LT. 1.D0) THEN
IF (ZSF .LT. 1.D0) THEN
IF (AS12 .GE. TS1) THEN
IF (AS12 .LT. TS2) THEN
ESN=ES0+ES1+((AS12-TS1)*(ES2-ES1)/(TS2-TS1))
EES=1-((AS12/ESN)/GI)
ELSE
IF (AS12 .LT. TS3) THEN
ESN=ES0+ES2+((AS12-TS2)*(ES3-ES2)/(TS3-TS2))
EES=1-((AS12/ESN)/GI)
ELSE

```

```

        IF (AS12 .LT. TS4) THEN
            ESN=ES0+ES3+((AS12-TS3)*(ES4-ES3)/(TS4-TS3))
            EES=1-((AS12/ESN)/GI)
        ELSE
            IF (AS12 .LT. TS5) THEN
                ESN=ES0+ES4+((AS12-TS4)*(ES5-ES4)/(TS5-TS4))
                EES=1-((AS12/ESN)/GI)
            ELSE
                ESN=ES0+ES5+((AS12-TS5)*(ES6-ES5)/(TS6-TS5))
                EES=1-((AS12/ESN)/GI)
            ENDIF
        ENDIF
    ENDIF
ENDIF
ENDIF
ENDIF
IF (AS12 .LT. XS) THEN
    IF (EES .GT. ES) THEN
        FIELD(3) = EES
        STATEV(5) = EES
    ELSE
        FIELD(3) = ES
        STATEV(5) = ES
    ENDIF
    STATEV(11) = 1.D0
ELSE
    FIELD(3) = 1.D0
    STATEV(8) = 1.D0
    STATEV(5) = 1.D0
    STATEV(11) = 0.D0
ENDIF
ELSE
    FIELD(3) = 1.D0
    STATEV(11) = 0.D0
ENDIF
ELSE
    IF (ZSF .LT. 1.D0) THEN
        IF (AS12 .LT. XS) THEN
            IF (AS12 .GT. XSMI) THEN
                IF (AS12 .GT. XSF) THEN
                    GX12 = (AS12-CES)/CDS
                    GX12F = GX12
                    ZZSF = (log10((10.D0**((ZSF*GX12F))
* +DTIME))/GX12F
                    STATEV(8) = ZZSF
                    AS=AHS
                    BS=BHS
                ELSE
                    IF (AS12 .GT. XSM) THEN
                        CAS=CASH
                        CBS=CBSH
                        CCS=CCSH
                        GX12F=(((-CBS)-SQRT((CBS**2.D0)
* -(4.D0*CAS*(CCS-AS12)))))/(2.D0*CAS)
                        ZZSF = (log10((10.D0**((ZSF*GX12F))
* +DTIME))/GX12F
                        STATEV(8) = ZZSF
                        AS=AHS
                        BS=BHS
                    ELSE
                        CAS=CASL

```

```

        CBS=CBSL
        CCS=CCSL
        AS=ALS
        BS=BLS
    ENDIF
    GX12=((-CBS)-SQRT((CBS**2.D0)
*   -(4.D0*CAS*(CCS-AS12))))/(2.D0*CAS)
    ENDIF
    ZZS = (log10((10.D0** (ZS*GX12)
*   +DTIME))/GX12
    STATEV(3) = ZZS
    DEDNS=(AS*AS12)+BS
    DNS=(log10((10.D0** (ZS*GX12))+DTIME))
*   -(ZS*GX12)
    DES=DEDNS*DNS
    ZZES=ZES+DES
    IF (ZZES .GT. ZES) THEN
        ZZES=ZES
    ENDIF
    IF (ZZES .GT. 1.D0) THEN
        STATEV(8) = 1.D0
    ENDIF
    IF (ZZES .LT. 0.D0) THEN
        ZZES = 0.D0
    ENDIF
    E12S=1.D0-(ZZES*(1.D0-ES))
    IF (STATEV(8) .LT. 1.D0) THEN
        IF (E12S .LT. 1.D0) THEN
            FIELD(3)=E12S
            STATEV(11)=ZZES
        ELSE
            FIELD(3)=1.D0
            STATEV(11)=0.D0
        ENDIF
    ELSE
        FIELD(3) = 1.D0
        STATEV(3) = 1.D0
        STATEV(11) = 0.D0
    ENDIF
    ELSE
        FIELD(3)=1.D0-(ZES*(1.D0-ES))
    ENDIF
    ELSE
        STATEV(8) = 1.D0
        FIELD(3) = 1.D0
        STATEV(11) = 0.D0
    ENDIF
    ELSE
        FIELD(3) = 1.D0
    ENDIF
ENDIF
C
C
C STATE TRANSITION DIAGRAM
C
C FV1: MATRIX COMPR/TENS FAILURE
C FV2: FIBER/MATRIX SHEAR FAILURE
C FV3: MATERIAL DAMAGE (SHEAR NONLINEARITY)
C         FV1 FV2 FV3    E1  E2  NU12  G12
C

```

```

C (0) NO FAILURE      0  0  0  --> E1  E2  NU12  G12
C (1) FIBER (COMPR/TENS)  1  0  0  --> 0  E2  0  G12
C (2) MATRIX (COMPR/TENS)  0  1  0  --> E1  0  0  G12
C (3) FIBER & MATRIX    1  1  0  --> 0  0  0  0
C (4) SHEAR FAILURE     0  0  1  --> E1  E2  0  0
C (5) FIBER & SHEAR     1  0  1  --> 0  E2  0  0
C (6) MATRIX & SHEAR    0  1  1  --> E1  0  0  0
C (7) FIBER, MATRIX & SHEAR 1  1  1  --> 0  0  0  0
C
C UPDATE FIELD VARIABLES
C
C
C THIS SUBROUTINE WEAKNESS
C NO ELEMENT DELETION WHICH MEANS ELEMENT PROPERTIES REVIVAL IS POSSIBLE
C THIS PART NEED TO BE MODIFIED SO ELEMENT REVIVAL IS NOT POSSIBLE
C
  RETURN
  END

```


APPENDIX II

MODIFIED HASHIN PROGRESSIVE DAMAGE STATIC & FATIGUE SUBROUTINE

```

SUBROUTINE USDFLD(FIELD,STATEV,PNEWDT,DIRECT,T,CELENT,TIME,DTIME,
1 CMNAME,ORNAME,NFIELD,NSTATV,NOEL,NPT,LAYER,KSPT,KSTEP,KINC,
2 NDI,nshr,coord,jmac,jmtyp,matlayo,laccflg)
C
  INCLUDE 'ABA_PARAM.INC'
C SUBROUTINE FOR HASHIN PROGRESSIVE FATIGUE WITH 6 INPUT SHEAR LINES
C FAILURE IS DETERMINED BY STRESS (HASHIN) ONLY
C
C PROGRESSIVE FATIGUE SUBROUTINE
C MATERIAL AND STRENGTH PARAMETERS
C X means 1st direction Y means 2nd direction
C T means tensile C means compression
C XS is the shear strength
C XTM is the meeting point of the high and low tensile s-n curve
C XTMI is the minimum tensile stress that will affect the fatigue
C properties
C XSF is the end of linear fatigue s-n curve and the starting
C XSM is the meeting point of the high and low shear s-n curve
C XSMI is the minimum shear stress that will affect the fatigue
C point of quadratic fatigue s-n curve
C properties
C all in MPa
C RXE and RXS are the relative stiffness at failure for
C tensile and shear stiffness respectively
  PARAMETER(XT=1215.77D0,YT=1215.77D0)
  PARAMETER(XC=-1215.77D0,YC=-1215.77D0,XS=92.28D0)
  PARAMETER(XTM=720.652D0,XSF=87.78178D0,XSM=55.10507D0)
  PARAMETER(XTMI=197.349D0,XSMI=11.513D0)
  PARAMETER(RXE=0.872419D0,RXS=0.320541D0)
C
C FATIGUE STRENGTH CONSTANTS
C GI = INITIAL G12
C
C THE LAST ELASTIC REGION STRAIN ; BEGINNING OF PLASTIC REGION
C ALS,BLS,AHS,BHS ARE THE LINE CONSTANTS OF
C THE  $\Delta(G12/G12_{initial})/\Delta \log N_f$  graph
C ALT,BLT,AHT,BHT ARE THE LINE CONSTANTS OF
C THE  $\Delta(E11/E11_{initial})/\Delta \log N_f$  graph
  PARAMETER(CATH=4.741D0,CBTH=-112.054D0,CCTH=1223.361D0)
  PARAMETER(CATL=2.6519D0,CBTL=-106.43D0,CCTL=1265.2D0)
  PARAMETER(CASH=1.2675D0,CBSH=-16.159D0,CCSH=106.48D0)
  PARAMETER(CASL=0.067D0,CBSL=-4.2298D0,CCSL=78.271D0)
  PARAMETER(CDS=-3.4949D0,CES=92.28D0)
  PARAMETER(TS1=20.D0,TS2=40.D0,TS3=50.D0)
  PARAMETER(TS4=60.D0,TS5=70.D0,TS6=92.356D0)
  PARAMETER(ES1=0.D0,ES2=8300.721D-6,ES3=17159.31D-6)
  PARAMETER(ES4=36792.89D-6,ES5=67202.33D-6,ES6=144416.2D-6)

```

```

PARAMETER(GI=4558.D0,ES0=4387.737D-6)
PARAMETER(ALT=-2.777D-5,BLT=-3.190D-4)
PARAMETER(AHT=-6.843D-5,BHT=2.979D-2)
PARAMETER(ALS=-1.598D-3,BLS=2.993D-4)
PARAMETER(AHS=-2.030D-2,BHS=1.001D0)
C
CHARACTER*80 CMNAME,ORNAME
CHARACTER*3 FLGRAY(15)
DIMENSION FIELD(NFIELD),STATEV(NSTATV),DIRECT(3,3),T(3,3),TIME(2),
* coord(*),jmac(*),jmtyp(*)
DIMENSION ARRAY(15),JARRAY(15)
C
C INITIALIZE FAILURE FLAGS FROM STATEV.
C ZF, ZM AND ZS ARE THE MODULUS DEGRADATION VARIABLES
C ZFF, ZMF AND ZSF ARE THE DAMAGE VARIABLES
C BFF AND BMF ARE THE FAILURE MEASURE FOR HASHIN CRITERION
C 1=FAILED, 0=OK
C TT IS THE TOTAL NUMBER OF CYCLES
C ES IS THE THE SHEAR MODULUS DEGRADATION FROM STATIC PART
ZF = STATEV(1)
ZM = STATEV(2)
ZS = STATEV(3)
TT = STATEV(4)
ES = STATEV(5)
SX = STATEV(6)
SY = STATEV(7)
SS = STATEV(8)
ZEF = STATEV(9)
ZEM = STATEV(10)
ZES = STATEV(11)
BFF = STATEV(12)
BMF = STATEV(13)
ZSF = STATEV(14)
ZFF = STATEV(15)
ZMF = STATEV(16)
C
C GET STRESSES FROM PREVIOUS INCREMENT
CALL GETVRM('S',ARRAY,JARRAY,FLGRAY,jrcd,
$ jmac, jmtyp, matlayo, laccflg)
S11 = ARRAY(1)
S22 = ARRAY(2)
S12 = ARRAY(4)
C THE ABSOLUTE STRESS
AS11 = ABS(S11)
AS22 = ABS(S22)
AS12 = ABS(S12)
C NEW TOTAL NUMBER OF CYCLES = FTT
C TT=THE OLD TOTAL NUMBER OF CYCLES
C DTIME=DELTA NUMBER OF CYCLES (ADDITIONAL NUMBER OF CYCLES IN THAT
C PARTICULAR INCREMENT)
FTT = TT + DTIME
C
C FATIGUE STRENGTH EQUATION
GTT = log10(TT+(10.D0**-20.D0))
GFTT = log10(FTT+(10.D0**-20.D0))
C
C TO PUT THE NEW NUMBER OF CYCLES IN THE STATEV
STATEV(4) = FTT
C
C SHEAR DAMAGE CONSTANT

```

C CONDITION: TEMP=25DEGC FREQ=5HZ STRESS RATIO=0
C

```

IF (FTT .LT. 1.D0) THEN
  STATEV(8)=XS
  IF (STATEV(12) .LT. 1.D0) THEN
    IF (STATEV(13) .LT. 1.D0) THEN
      IF (AS12 .GE. TS1) THEN
        IF (AS12 .LT. TS2) THEN
          ESN=ES0+ES1+((AS12-TS1)*(ES2-ES1)/(TS2-TS1))
          EES=1-((AS12/ESN)/GI)
        ELSE
          IF (AS12 .LT. TS3) THEN
            ESN=ES0+ES2+((AS12-TS2)*(ES3-ES2)/(TS3-TS2))
            EES=1-((AS12/ESN)/GI)
          ELSE
            IF (AS12 .LT. TS4) THEN
              ESN=ES0+ES3+((AS12-TS3)*(ES4-ES3)/(TS4-TS3))
              EES=1-((AS12/ESN)/GI)
            ELSE
              IF (AS12 .LT. TS5) THEN
                ESN=ES0+ES4+((AS12-TS4)*(ES5-ES4)/(TS5-TS4))
                EES=1-((AS12/ESN)/GI)
              ELSE
                ESN=ES0+ES5+((AS12-TS5)*(ES6-ES5)/(TS6-TS5))
                EES=1-((AS12/ESN)/GI)
              ENDIF
            ENDIF
          ENDIF
        ENDIF
      ENDIF
    ENDIF
  ENDIF
  IF (EES .GT. ES) THEN
    FIELD(3) = EES
    STATEV(5) = EES
  ELSE
    FIELD(3) = ES
    STATEV(5) = ES
  ENDIF
  STATEV(11) = 1.D0
  STATEV(14) = 0.D0
  ENDIF
ENDIF
ELSE
  IF (STATEV(12) .LT. 1.D0) THEN
    IF (STATEV(13) .LT. 1.D0) THEN
      IF (AS12 .LT. XS) THEN
        IF (AS12 .GT. XSMI) THEN
          IF (AS12 .GT. XSF) THEN
            GX12 = (AS12-CES)/CDS
            GX12F = GX12
            ZZSF = (log10((10.D0**(ZSF*GX12F))
* +DTIME))/GX12F
            STATEV(14) = ZZSF
            AS=AHS
            BS=BHS
            X12F = 10.D0**(GX12F)
            IF (X12F .GT. 0.D0) THEN
              SSS = SS - ((XS-AS12)*(DTIME/X12F))
              IF (SSS .LT. 0.1D0) THEN
                STATEV(8) = 0.1D0
                SS = 0.1D0
              ENDIF
            ENDIF
          ENDIF
        ENDIF
      ENDIF
    ENDIF
  ENDIF

```

```

ELSE
STATEV(8) = SSS
SS = SSS
ENDIF
ENDIF
ELSE
IF (AS12 .GT. XSM) THEN
CAS=CASH
CBS=CBSH
CCS=CCSH
GX12F=(((-CBS)-SQRT((CBS**2.D0)
* -(4.D0*CAS*(CCS-AS12)))))/(2.D0*CAS)
ZZSF = (log10((10.D0** (ZSF*GX12F))
* +DTIME))/GX12F
STATEV(14) = ZZSF
AS=AHS
BS=BHS
X12F = 10.D0** (GX12F)
IF (X12F .GT. 0.D0) THEN
SSS = SS - ((XS-AS12)*(DTIME/X12F))
IF (SSS .LT. 0.1D0) THEN
STATEV(8) = 0.1D0
SS = 0.1D0
ELSE
STATEV(8) = SSS
SS = SSS
ENDIF
ENDIF
ELSE
CAS=CASL
CBS=CBSL
CCS=CCSL
AS=ALS
BS=BLS
ENDIF
GX12=(((-CBS)-SQRT((CBS**2.D0)
* -(4.D0*CAS*(CCS-AS12)))))/(2.D0*CAS)
ENDIF
ZZS = (log10((10.D0** (ZS*GX12))
* +DTIME))/GX12
STATEV(3) = ZZS
DEDNS=(AS*AS12)+BS
DNS=(log10((10.D0** (ZS*GX12))+DTIME))
* -(ZS*GX12)
DES=DEDNS*DNS
ZZES=ZES+DES
IF (ZZES .GT. ZES) THEN
ZZES=ZES
ENDIF
IF (ZZES .GT. 1.D0) THEN
ZZES = 0.D0
ENDIF
IF (ZZES .LT. 0.D0) THEN
ZZES = 0.D0
ENDIF
E12S=1.D0-(ZZES*(1.D0-ES))
IF (STATEV(12) .LT. 1.D0) THEN
IF (STATEV(13) .LT. 1.D0) THEN
IF (E12S .LT. 1.D0) THEN
FIELD(3)=E12S

```

```

        STATEV(11)=ZZES
    ELSE
        FIELD(3)=1.D0
        STATEV(11)=0.D0
    ENDIF
ENDIF
ENDIF
ELSE
    FIELD(3)=1.D0-(ZES*(1.D0-ES))
ENDIF
ENDIF
ENDIF
ENDIF
ENDIF
ENDIF
C
C
C FIBER 1ST DIRECTION TENSILE/COMPRESSIVE FAILURE
C GXT1 is the log failure Nf at current state of stress
C EF and EEF are the measure of residual modulus
C FOR FTT < 1 - IT WILL DO STATIC ANALYSIS
C FOR FTT > 1 - IT WILL DO FATIGUE ANALYSIS
    IF (FTT .LT. 1.D0) THEN
        STATEV(6)=XT
        IF (STATEV(12) .LT. 1.D0) THEN
            IF (S11 .GE. 0.D0) THEN
                ZF1=((S11/XT)**2.D0)+((S12/XS)**2.D0)
            ELSE
                ZF1=(S11/XC)**2.D0
            ENDIF
            IF (ZF1 .LT. 1.D0) THEN
                FIELD(1) = 0.D0
                STATEV(1) = 0.D0
                STATEV(12) = 0.D0
                STATEV(9) = 1.D0
            ELSE
                FIELD(1) = 1.D0
                STATEV(1) = 1.D0
                STATEV(12) = 1.D0
                STATEV(9) = 0.D0
                FIELD(3) = 1.D0
                STATEV(14) = 1.D0
                STATEV(5) = 1.D0
                STATEV(11) = 0.D0
                STATEV(15) = 1.D0
                ZS = 1.D0
            ENDIF
        ELSE
            FIELD(1) = 1.D0
            FIELD(3) = 1.D0
            STATEV(14) = 1.D0
            STATEV(11) = 0.D0
            STATEV(15) = 1.D0
            ZS = 1.D0
        ENDIF
    ELSE
        IF (STATEV(12) .LT. 1.D0) THEN
            IF (AS11 .LT. XT) THEN
                IF (AS11 .GT. XTMI) THEN
                    IF (AS11 .GT. XTM) THEN
                        CAT1=CATH
                    
```

```

CBT1=CBTH
CCT1=CCTH
GXT1F = ((-CBT1)-SQRT((CBT1**2.D0)
*  -(4.D0*CAT1*(CCT1-AS11)))) / (2.D0*CAT1)
XT1F = 10.D0**(GXT1F)
IF (XT1F .GT. 0.D0) THEN
  SSX = SX - ((XT-AS11)*(DIME/XT1F))
  IF (SSX .LT. 1.D0) THEN
    STATEV(6) = 1.D0
    SX = 1.D0
  ELSE
    STATEV(6) = SSX
    SX = SSX
  ENDIF
ENDIF
ZZFF = (log10((10.D0**(ZFF*GXT1F)
*  +DIME))/GXT1F)
STATEV(15) = ZZFF
AT1=AHT
BT1=BHT
ELSE
CAT1=CATL
CBT1=CBTL
CCT1=CCTL
AT1=ALT
BT1=BLT
ENDIF
GXT1 = ((-CBT1)-SQRT((CBT1**2.D0)
*  -(4.D0*CAT1*(CCT1-AS11)))) / (2.D0*CAT1)
ZZF = (log10((10.D0**(ZF*GXT1)
*  +DIME))/GXT1)
STATEV(1) = ZZF
DEDNF=(AT1*AS11)+BT1
DNF=(log10((10.D0**(ZF*GXT1))+DIME))
*  -(ZF*GXT1)
DEF=DEDNF*DNF
ZZEF=ZEF+DEF
IF (ZZEF .GT. ZEF) THEN
  ZZEF=ZEF
ENDIF
IF (ZZEF .GT. 1.D0) THEN
  ZZEF = 0.D0
ENDIF
IF (ZZEF .LT. 0.D0) THEN
  ZZEF = 0.D0
ENDIF
EEF=1.D0-ZZEF
IF (STATEV(12) .LT. 1.D0) THEN
  IF (EEF .LT. 1.D0) THEN
    FIELD(1) = EEF
    STATEV(9) = ZZEF
  ELSE
    FIELD(1) = 1.D0
    STATEV(9) = 0.D0
  ENDIF
ELSE
  FIELD(1) = 1.D0
  STATEV(1) = 1.D0
  STATEV(9) = 0.D0
  FIELD(3) = 1.D0

```

```

STATEV(14) = 1.D0
STATEV(11) = 0.D0
STATEV(15) = 1.D0
ZS = 1.D0
ENDIF
ELSE
FIELD(1) = 1.D0-ZEF
ENDIF
ENDIF
IF (S11 .GE. 0.D0) THEN
ZF1=((S11/SX)**2.D0)+((AS12/SS)**2.D0)
ELSE
ZF1=(S11/(-SX))**2.D0
ENDIF
IF (ZF1 .GE. 1.D0) THEN
FIELD(1) = 1.D0
STATEV(1) = 1.D0
STATEV(12) = 1.D0
STATEV(9) = 0.D0
FIELD(3) = 1.D0
STATEV(14) = 1.D0
STATEV(11) = 0.D0
STATEV(15) = 1.D0
ZS = 1.D0
ENDIF
ELSE
FIELD(1) = 1.D0
STATEV(1) = 1.D0
STATEV(9) = 0.D0
FIELD(3) = 1.D0
STATEV(14) = 1.D0
STATEV(11) = 0.D0
STATEV(15) = 1.D0
ZS = 1.D0
ENDIF
ENDIF
C
C FIBRE 2ND DIRECTION TENSILE/COMPRESSIVE FAILURE
IF (FTT .LT. 1.D0) THEN
STATEV(7)=YT
IF (STATEV(13) .LT. 1.D0) THEN
IF (S22 .GE. 0.D0) THEN
ZF2=((S22/YT)**2.D0)+((S12/XS)**2.D0)
ELSE
ZF2=(S22/YC)**2.D0
ENDIF
IF (ZF2 .LT. 1.D0) THEN
FIELD(2) = 0.D0
STATEV(2) = 0.D0
STATEV(13) = 0.D0
STATEV(10) = 1.D0
ELSE
FIELD(2) = 1.D0
STATEV(2) = 1.D0
STATEV(13) = 1.D0
STATEV(10) = 0.D0
FIELD(3) = 1.D0
STATEV(14) = 1.D0
STATEV(5) = 1.D0
STATEV(11) = 0.D0

```

```

STATEV(16) = 1.D0
ZS = 1.D0
ENDIF
ELSE
FIELD(2) = 1.D0
FIELD(3) = 1.D0
STATEV(14) = 1.D0
STATEV(11) = 0.D0
STATEV(16) = 1.D0
ZS = 1.D0
ENDIF
ELSE
IF (STATEV(13) .LT. 1.D0) THEN
IF (AS22 .LT. YT) THEN
IF (AS22 .GT. XTMI) THEN
IF (AS22 .GT. XTM) THEN
CAT=CATH
CBT=CBTH
CCT=CCTH
GXT2F = ((-CBT)-SQRT((CBT**2.D0)
* -(4.D0*CAT*(CCT-AS22)))) / (2.D0*CAT)
XT2F = 10.D0**(GXT2F)
IF (XT2F .GT. 0.D0) THEN
SSY = SY - ((YT-AS22)*(DTIME/XT2F))
IF (SSY .LT. 1.D0) THEN
STATEV(7) = 1.D0
SY = 1.D0
ELSE
STATEV(7) = SSY
SY = SSY
ENDIF
ENDIF
ZZMF = (log10((10.D0**(ZMF*GXT2F))
* +DTIME))/GXT2F
STATEV(16) = ZZMF
AT2=AHT
BT2=BHT
ELSE
CAT=CATL
CBT=CBTL
CCT=CCTL
AT2=ALT
BT2=BLT
ENDIF
GXT2 = ((-CBT)-SQRT((CBT**2.D0)
* -(4.D0*CAT*(CCT-AS22)))) / (2.D0*CAT)
ZZM = (log10((10.D0**(ZM*GXT2))
* +DTIME))/GXT2
STATEV(2) = ZZM
DEDNM=(AT2*AS22)+BT2
DNM=(log10((10.D0**(ZM*GXT2))+DTIME))
* -(ZM*GXT2)
DEM=DEDNM*DNM
ZZEM=ZEM+DEM
IF (ZZEM .GT. ZEM) THEN
ZZEM=ZEM
ENDIF
IF (ZZEM .GT. 1.D0) THEN
ZZEM = 0.D0
ENDIF

```



```

IF (ZZEM .LT. 0.D0) THEN
  ZZEM = 0.D0
ENDIF
EEM=1.D0-ZZEM
IF (STATEV(13) .LT. 1.D0) THEN
  IF (EEM .LT. 1.D0) THEN
    FIELD(2) = EEM
    STATEV(10) = ZZEM
  ELSE
    FIELD(2) = 1.D0
    STATEV(10) = 0.D0
  ENDIF
ELSE
  FIELD(2) = 1.D0
  STATEV(2) = 1.D0
  STATEV(10) = 0.D0
  FIELD(3) = 1.D0
  STATEV(14) = 1.D0
  STATEV(11) = 0.D0
  STATEV(16) = 1.D0
  ZS = 1.D0
ENDIF
ELSE
  FIELD(2) = 1.D0-ZEM
ENDIF
ENDIF
IF (S22 .GE. 0.D0) THEN
  ZF2=((S22/SY)**2.D0)+((AS12/SS)**2.D0)
ELSE
  ZF2=(S22/(-SY))**2.D0
ENDIF
IF (ZF2 .GE. 1.D0) THEN
  FIELD(2) = 1.D0
  STATEV(2) = 1.D0
  STATEV(13) = 1.D0
  STATEV(10) = 0.D0
  FIELD(3) = 1.D0
  STATEV(14) = 1.D0
  STATEV(11) = 0.D0
  STATEV(16) = 1.D0
  ZS = 1.D0
ENDIF
ELSE
  FIELD(2) = 1.D0
  STATEV(2) = 1.D0
  STATEV(10) = 0.D0
  FIELD(3) = 1.D0
  STATEV(14) = 1.D0
  STATEV(11) = 0.D0
  STATEV(16) = 1.D0
  ZS = 1.D0
ENDIF
ENDIF
C
C FV1: MATRIX COMPR/TENS FAILURE
C FV2: FIBER/MATRIX SHEAR FAILURE
C FV3: MATERIAL DAMAGE (SHEAR NONLINEARITY)
C
C UPDATE FIELD VARIABLES
C

```

```
C  
C  
  RETURN  
  END
```