Experimental and numerical investigations on cavitation bubble dynamics near a solid boundary

Yang, Yuanxiang

2014

Yang, Y. (2014). Experimental and numerical investigations on cavitation bubble dynamics near a solid boundary. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/59113>

<https://doi.org/10.32657/10356/59113>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**EXPERIMENTAL AND NUMERICAL
INVESTIGATIONS ON
CAVITATION BUBBLE DYNAMICS
NEAR A SOLID BOUNDARY**

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**SCHOOL OF CIVIL AND ENVIRONMENTAL
ENGINEERING**

2014

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INVESTIGATIONS ON
CAVITATION BUBBLE DYNAMICS
NEAR A SOLID BOUNDARY**

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2014

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to his supervisor, Associate Professor Tan Soon Keat, for his patient guidance, encouragement, constructive criticism and his strong commitment to supervise this study.

The author also wants to thank Dr. Wang Qian Xi (University of Birmingham) for introducing the author into this research area, which is full of novelties. His great work on the numerical simulations helps the author a lot.

The author gives many thanks to the fellows and technicians in the School of Civil and Environmental Engineering, Nanyang Environmental and Water Research Institute and Maritime Research Centre for their helps and suggestions.

The author is also grateful to the Nanyang Technological University for the opportunity to conduct the research work and the support in the form of research scholarship.

Finally, the author is deeply appreciative of his wife, his family and his friends for their patience, encouragement and support throughout the course of the Ph.D. study.

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ABSTRACT

Cavitation bubble dynamics near a solid boundary have been studied for quite a long time, primarily in the field of hydrodynamic science, to explain the cavitation corrosion and damage observed on ship propellers and hydraulic machines. Recently, however, there has been increasing interest in applying acoustic cavitation in certain other fields, like ultrasonic cleaning, sonochemistry, shockwave lithotripsy, and sonoporation. Although the occurrence areas of these cavitation bubbles are quite different (depending on the specific field), the physical fundamentals of these processes are more or less the same. This study attempts to elucidate the corrosion/cleaning mechanism of cavitation bubbles through investigations on a single cavitation bubble dynamics near a solid boundary.

The experiments on laser-induced cavitation bubble dynamics near a solid boundary were first presented. A single cavitation bubble was generated using a focused Q-switched Nd: YAG laser pulse. The bubble's maximum expanded volume was dominated by the energy of the incident laser pulse. The inception position of the single cavitation bubble had an important influence on the bubble dynamics, which was finely controlled using a micro-translation stage. Direct observations of the bubble evolutions using a high-speed video camera revealed detailed deformations of the cavitation bubble near the solid boundary: the non-spherical bubble generation; the nearly spherical shape during the first expansion phase; the formation of a liquid jet resulting from the asymmetry of the flow field; the counterjet only appearing in a certain range of stand-off distances; the various profiles after the jet impact, etc. Indirect detections of the acoustic signals released by the oscillating bubbles, using a hydrophone system, provided a much more accurate estimation of the bubble oscillation periods than those obtained from the bubble videos captured using the high-speed camera. The statistics of the non-dimensional bubble oscillation periods could be used to roughly indicate the bubble inception positions when the maximum expanded bubble radii were known. Temporal evolutions of the bubble size were measured in this study using a novel

method involving the application of AutoCAD. The energy deposited in the bubble was estimated using the maximum expanded volume after the jet impact.

Following this, numerical simulation works were conducted to show some detailed bubble dynamics which could not be observed using the experimental methods. A mathematical model based on the potential flow theory was used to describe the proposed problem. An approximate perturbation method was also developed using the method of matched asymptotic expansions to include the influence of the compressibility of the liquid. It was concluded that the velocity potential near the bubble surface satisfied Laplace's equation. A dimensionality reduction of the initial 3D potential problem to a final 1D solution was made in a cylindrical polar coordinate. Special treatments on the solid boundary and toroidal bubble were taken carefully. A numerical model based on the mixed Eulerian-Lagrangian (MEL) method and the boundary integral method (BIM) for bubble dynamics in a weakly compressible liquid near a solid boundary was created.

Finally, the numerical results calculated using the MEL-BIM model were verified using available analytical results and previous numerical results. Reliable comparisons were made between the numerical results and the experimental results obtained in this study, including those for the non-spherical bubble shape, the evolution of the equivalent bubble radius, and the movement of the bubble centroid. An important advantage of the numerical method was the ability to calculate the flow field dynamics near the bubble surface. The velocity vectors and pressure contours around the bubble surface could be simulated under a fixed grid across the liquid field. The jet impact dynamics and the induced hammer pressure that impinged on the solid boundary were calculated and proved using previous experimental measurements. The formation of the counterjet was also elucidated using the numerical simulations.

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LIST OF SYMBOLS

B_a	= incomplete Beta function
$\bar{b}$	= amplitude of incident harmonic standing wave
b	= non-dimensional amplitude of incident harmonic standing wave
$\bar{c}$	= speed of sound
c	= non-dimensional speed of sound
$\bar{c}_\infty$	= wave speed of the acoustic wave in the undisturbed liquid
C	= intersection curve of the bubble surface
ds	= initial distance between the bubble inception and the solid boundary
E	= complete elliptic integral of the second kind
E_{sur}	= surface energy stored in the bubble surface
E_{pot}	= potential energy acquired through the expansion of the bubble
f	= flow field
f_i, g_i	= arbitrary functions
f_i', g_i'	= first derivative of arbitrary functions with respect to time
F_1, G_1	= arbitrary functions having second-order derivatives
F_B	= Bjerknes force acting on the bubble
G	= free space Green's function
$\bar{h}$	= enthalpy

$\bar{h}_{\bar{x}}, \bar{h}_{\bar{y}}, \bar{h}_{\bar{z}}$	= enthalpy derivatives in three directions
h	= non-dimensional enthalpy
$\bar{k}$	= wave number of incident harmonic standing wave
k	= non-dimensional wave number of incident harmonic standing wave
K_1, K_2	= kernel functions
K	= complete elliptic integral of the first kind
k_1, k_2	= non-dimensional first and second bubble oscillation period
m_0, m_0', m_0''	= rate of change of the bubble volume, and its first and second derivatives with respect to time
M_1, M_2	= weight of the linear approximation
n	= number of the segments dividing the bubble surface curve
$\bar{p}$	= pressure
p	= non-dimensional pressure
$\bar{p}_{\bar{x}}, \bar{p}_{\bar{y}}, \bar{p}_{\bar{z}}$	= pressure derivatives in three directions
$\bar{p}_L$	= liquid pressure on the bubble surface
p_L	= non-dimensional liquid pressure on the bubble surface
$\bar{p}_v$	= vapor pressure inside the bubble
p_v	= non-dimensional vapor pressure inside the bubble
$\bar{p}_g$	= gas pressure inside the bubble
p_g	= non-dimensional gas pressure inside the bubble
p_{g0}	= non-dimensional initial gas pressure inside the bubble

$\bar{p}_\infty$	= hydrostatic liquid pressure
p_a	= local incident standing wave pressure
$\mathbf{p}, \mathbf{q}$	= points on the bubble surface
$\hat{\mathbf{q}}$	= image points of $\mathbf{q}$ about the solid boundary
P, Q, R, S	= tabulated polynomials
$\bar{R}_{max}$	= maximum expanded radius of the bubble
$\bar{R}_1, \bar{R}_2$	= principle radii of the curvature
$\bar{\mathbf{r}}$	= bubble surface position
$\mathbf{r}$	= non-dimensional bubble surface position
$\mathbf{r}_b$	= coordinate moving with a uniform stream at infinity
R_0	= non-dimensional initial bubble radius
r_{2*}	= non-dimensional second maximum expanded equivalent radius
S	= bubble surface
$\bar{t}$	= time
t	= non-dimensional time
t_0	= non-dimensional initial time
Δt	= non-dimensional time step
t_{c1}, t_{c2}	= first and second bubble oscillation period
t_{sd}	= first oscillation period of a spherical bubble in an infinite liquid
$\bar{\mathbf{u}}$	= velocity vectors
$\mathbf{u}$	= non-dimensional velocity vectors

$\bar{U}$	= typical liquid velocity on the bubble surface
$\bar{x}, \bar{y}, \bar{z}$	= three directional position near the bubble surface
$\tilde{x}, \tilde{y}, \tilde{z}$	= three directional position far from the bubble surface
x, y, z	= non-dimensional three directional position
$\bar{\rho}$	= density of the liquid
ρ	= non-dimensional density of the liquid
$\bar{\rho}_\infty$	= density of the liquid at infinity
$\bar{\sigma}$	= surface tension coefficient
σ	= non-dimensional surface tension
$\bar{\varphi}$	= velocity potential
φ	= non-dimensional velocity potential
ϕ	= velocity potential on the bubble surface
ϕ_0	= initial velocity potential on the bubble surface
Φ_{vr}	= velocity potential of the vortex ring
ψ	= normal velocity on the bubble surface
θ	= derivative of the velocity potential with respect to time
$\bar{\omega}$	= frequency of incident harmonic standing wave
ω	= non-dimensional frequency of incident harmonic standing wave
θ	= azimuthal angle of the bubble surface in a cylindrical polar coordinate
θ_0	= initial phase of incident standing wave

ε	= characteristic bubble-wall Mach number
λ	= ratio of the specific heats of the gas content inside the bubble
γ	= non-dimensional stand-off distance
ξ	= arc length variable
Γ	= velocity circulation of the vortex ring
τ	= tangential direction of the bubble surface
Δ	= small difference operator
∇	= partial differential operator

CHAPTER 1 INTRODUCTION

1.1 Background

A cavitation bubble occurs when the pressure in a volume of liquid drops below the corresponding saturated vapour pressure, causing a rapid phase transition and literally tearing the liquid apart. Cavitation bubbles are often observed in flow systems at control valves, pumps, propellers, impellers, and in the vascular tissues of plants. Cavitation bubbles may also be observed where there is local release of energy, such as the location of a focused laser pulse, or with an electrical discharge through a spark. Cavitation bubbles also occur in the presence of an acoustic field. Microscopic cavitation bubbles will be made to oscillate by the acoustic waves. If the acoustic intensity is sufficiently high, the bubbles will first grow in size and then rapidly collapse.

Based on the content inside the cavitation bubbles, the bubbles can be divided into two types: vapor bubbles and gas bubbles. As is known, vapor may evaporate into the cavitation bubbles from the surrounding liquid. Thus, the cavitation bubbles may not be a perfect vacuum, but have relatively low vapor pressure inside. The bubble thus formed is known as the pure vapor bubble. After reaching the maximum expanded volume, this low-pressure cavitation bubble begins to collapse due to the higher pressure of the surrounding liquid. As the bubble collapses, the pressure and temperature of the vapor within the bubble increases. The bubble eventually collapses into a minute fraction of its original size, and the vapor within the bubble dissipates into the surrounding liquid via a rather violent mechanism, which releases a significant amount of energy in the form of an acoustic shock wave and visible light.

For some very high pressure cavitation bubbles, such as those generated by a torpedo, an underwater mine, laser heating or spark discharge, a quantity of non-condensable contaminant gas is always present within the bubbles. The extreme high pressure gas inside the bubble pushes the bubble surface to expand outwards, and as a result the bubble may reach quite a large size. At the end of the subsequent

collapse, this type of bubble does not dissipate like the pure vapor bubble, but rebounds to a smaller volume than that at the first maximum expansion. This expansion and collapse process may repeat for several cycles. During these oscillation cycles, the energy deposited in the bubble is released into the ambient liquid until the bubble energy is finally exhausted.

The appearance of cavitation bubbles may cause many different results. Cavitation bubbles may sometimes be harmful in some engineering contexts, for example, they may cause cavitation corrosion on ship propellers and hydraulic machines, etc. However, cavitation bubbles are also sometimes employed in certain industrial and medical practices, like ultrasonic cleaning, chemical and bio-medical engineering, etc. Therefore, investigations of cavitation bubble dynamics are necessary and important.

1.2 Purpose and Scope

In the most common situation, when the cavitation bubbles are generated near a solid boundary, the bubble dynamics are quite different from those of bubbles generated in an infinite liquid. A liquid jet is always formed on the inner and top of the bubble, and this jet penetrates the opposite bubble wall and impinges on the solid boundary. The liquid jet is believed to be the main reason for the cavitation corrosion/cleaning. The purpose of this study is to present the detailed single cavitation bubble dynamics near a solid boundary.

To achieve its purpose, the study is divided into three main parts. The first part is a literature study, which consists of Chapter 2. The review of the motivation for this study reveals the significance of investigations of cavitation bubble dynamics. The review of experimental and numerical methods shows the development of study methods in this research area. The stated topic of focus – cavitation bubble dynamics near a solid boundary – indicates the scope of this study. Through the systemic review work, this study aims to contribute extensive knowledge of cavitation bubble dynamics to the existing literature.

The second part covers the experimental studies of the laser-induced cavitation bubble dynamics near a solid boundary. This part consists of Chapters 3 and 4.

Direct and indirect observations of the bubble evolutions are made to present various experimental results related to the bubble size and initial stand-off distances. Based on the statistics and analyses of these experimental results, this study attempts to provide some intuitive impressions of the cavitation bubble dynamics near a solid boundary.

The third part consists of Chapters 5 and 6, which presents the numerical investigations of the cavitation bubble dynamics near a solid boundary in a compressible liquid. Rigorous derivations of the mathematical and numerical models are presented. The numerical model is verified using some reasonable analytical and numerical results. Comparisons between the numerical and experimental results are also made in this study. The simulation results from the flow field dynamics are employed to reveal the interactions between the cavitation bubble and the solid boundary.

1.3 Thesis Organization

This thesis is divided into 7 chapters. The first chapter introduces the background and purpose of this study. In Chapter 2, a comprehensive literature review is presented, covering the motivations, research methods, and contents of this study. Chapter 3 introduces the experimental set-up in the lab and the data post-processing method. Chapter 4 shows the experimental results obtained using different measuring methods, and presents some statistics and analyses. Chapter 5 presents a developed MEL-BIM model for the simulations of the cavitation bubble dynamics near the solid boundary in a compressible liquid. In Chapter 6, the numerical model is first verified using both analytical and numerical results. The simulation results are then compared with the experimental results obtained in this study. Detailed dynamic features of the flow field near the bubble surface are also presented. Finally the general conclusions reached in this study are summarised in Chapter 7. Some recommendations are also made for future studies.

CHAPTER 2 LITERATURE REVIEW

2.1 Motivations for the Study of Cavitation

2.1.1 Preventions from cavitation damages

When a ship propeller rotates in water, low-pressure areas are formed behind the blades. The faster the blades move, the lower the pressure. As the liquid pressure at the blade reaches saturated vapor pressure, the fluid vaporizes and small cavitation bubbles are formed. These bubbles oscillate and collapse near the blades and cause pitting on the blade surface and produce noises at the same time. The pitting produces large wear and tear on various parts of the propeller and dramatically shortens the service life of the propeller (see Figure 2.1). For a submarine, the noises of propeller always betray its positions. Reports of extensive experiments on propellers of different designs, which include visual investigations of cavitations and measurements of noise, show that the dominant type of cavitation is in the form of tip vortex cavitation, accompanied by sheet cavitation on the leading edge and at the suction side of the blade. The characteristics of the noise produced depend on factors such as the advance coefficient, the cavitation number, and the propeller geometry. Of these factors, the advance coefficient is found to produce the most significant influence not only on cavitation noise but also on the inception of cavitation (Sharma et al. 1990).

In a hydraulic machine, cavitation inception frequently occurs just downstream from the point of minimum pressure, prior to the separation point, so that cavitation bubbles are swept up over the separation point as "separation bubble". Structural damage is then observed to occur near the reattachment point of the separated flow. Thus, cavitation should be a design consideration for a wide range of devices that are used to handle liquids. Cavitation bubbles can affect performance of many fluid machines, for example through increased drag of hydro-nautical vehicles, reduced the thrust produced by various propulsion systems, decreased power output and efficiency of turbines, and drop in head and efficiency produced by pumps (Arndt

1981b).

Figure 2.1 Cavitation damages on the impeller and propeller (<https://en.wikipedia.org/wiki/Cavitation>, 23-Jul-2013).

Military applications of torpedoes and depth bombs to destruct naval vessels and submarines demand detailed studies on underwater explosions. The same amount of explosive can cause much greater damages underwater than it would cause in air, because water is much less compressible than air. The physics of an underwater explosion are complex and involve a wide range of phenomena from combustion to shock waves. Immediately after detonation, a shock wave and an expanding high-pressure gas bubble will be formed. This shock wave moves at very high speed and generates a very high pressure. When it hits a ship or structure, the shock wave may inflict the first damage on the vessel/structure. The explosive product forms a high-pressure gas bubble, and the bubble expands and collapses for several cycles until it finally breaks up. The final stage of the collapse of the bubble is quite violent, which typically generates extreme high pressure and temperature, and is always accompanied by a high-speed jet directed towards the solid surface in its vicinity (Klaseboer et al. 2005).

2.1.2 General applications of cavitation bubble dynamics

Currently, the research trend is on the consideration to exploit the dynamic features of cavitation bubbles for certain applications, such as ultrasonic cleaning,

sonochemistry, shockwave lithotripsy, sonoporation etc. (Young 1989; Leighton 1994; Goldberg et al. 1994; Blake et al. 1999; Putterman & Weninger 2000; Reddy & Szeri 2002; Day 2005; Klaseboer et al. 2007; Calvisi, Iloreta & Szeri 2008).

One of the most widespread applications of ultrasound is surface cleaning. Ultrasonic cleaners are used to clean many different types of objects, including jewellery, lenses and other optical parts, dental and surgical instruments, industrial parts and electronic equipment (see Figure 2.2). This technique is commonly used in most chemistry laboratories. There are both merits and limitations in this application. Research works on the ultrasonic cleaning are always divided into two categories. The first involves enhancement of the mass transport via acoustic streaming and micro-streaming, which accelerates the dissolution of soluble contaminants. The second refers to the mechanical effects when cavitation occurs close to a boundary, and includes the shock waves, the collision of the bubble on the objective surface upon collapse and the micro-jetting (Maisonhaute et al. 2002).

Figure 2.2 Ultrasonic cleaning of glasses in an ultrasonic bath (http://en.wikipedia.org/wiki/Ultrasonic_cleaning, 23-Jul-2013).

Cavitation as a source of energy input for chemical processes is increasingly being investigated for its ability to generate high temperatures and pressures (hot spots) under nearly ambient conditions (Moholkar et al. 1999). Cavitation bubbles may be generated using ultrasound. The chemical effects of ultrasound do not come from a direct interaction of sound wave with the molecular species. Instead,

sonochemistry derives principally from acoustic cavitation bubbles: the formation, growth, and implosive collapse of bubbles. Cavitation bubble serves as a means of concentrating the diffuse energy of sound and is the underlying phenomenon responsible for sonochemistry and sonoluminescence. Many chemists and physicists are engaged in studies of acoustic cavitation, and achieved many interesting chemical and physical results (Suslick 1990; Margulis 1995; Crum 1994; Koda et al. 2004).

Another important application of the acoustic cavitations is the shockwave lithotripsy (Matula et al. 2002; Zabolotskaya et al. 2004; Calvisi et al. 2007, 2008). Shock wave lithotripsy is a common medical procedure for destroying kidney stones through the focusing of shock waves inside the human body (see Figure 2.3). These shock waves have an initial short compressive peak followed by a much longer rarefaction. Cavitation bubbles may be formed in this rarefaction area, which are believed to have significant contributions on the stone comminution (Pishchalnikov 2003).

Figure 2.3 Shockwave lithotripsy to kidney stones (http://en.wikipedia.org/wiki/Kidney_stone#Lithotripsy, 23-Jul-2013).

2.1.3 Roles of cavitation bubbles in membrane de-fouling

In the past decade, the volume of the reported words on ultrasonic cleaning of fouled membranes has increased substantially (Chai et al. 1999; Li et al. 2002;

Kobayashi et al. 2003; Chen et al. 2006). Membrane fouling affects both the quality and the quantity of product water (Zhu & Elimelech 1995) and ultimately shortens membrane life if the fouling is irreversible (Seidel and Elimelech, 2002). There are a number of approaches that could be used to reduce membrane fouling, and they include backwashing and chemical membrane cleaning (Gutman 1987). However, these methods are not ideal. Backwashing does not generally remove strongly adherent films or material trapped within the porous substructure of the membrane. In addition, degradation in flux capacity typically occurs after repeated backwashing. Furthermore, the use of chemical cleaning often does not solve the cleaning problem. Instead it may cause safety, waste treatment and disposal, as well as potential health issues.

Ultrasound emerges as an alternative technology for membrane fouling control. The compression cycles produced by the ultrasound can generate micro-bubbles. The bubbles release energies near the membrane, and results in the cleaning mechanics on the membrane surface. Ultrasonic cleaning has significant advantages over traditional methods to control membrane fouling. For example, chemicals would not be used in the membrane cleaning process. In addition, filtration is not interrupted during the cleaning process (Chen et al.2006). A pilot plant for sonication of submerged membranes to produce drinking water from surface water was constructed and placed at the Rhine Water Works in Biebesheim, Germany (Lauterborn & Urban, 2008), see Figure 2.4. Only 15 seconds of sonication after 30 minutes of filtration were sufficient to ensure the high performance of the membranes. No damage of the membranes was observed. Lauterborn & Urban suggested that more research work need to be carried out before sonication could be applied widely.

- = streamer-figure
- = bubble from aeration
- = part of fouled layer

Figure 2.4 Schematic diagram of the Lauterborn & Urban's (2008) experimental set-up.

Investigations of ultrasonic cleaning or control of membrane fouling are always associated with macroscopic ultrasonic dynamics (Chen et al. 2006). The cavitation bubbles generated in the acoustic field are quite small ($\mu\text{m}\sim\text{mm}$) and of extremely short lifetime ($\mu\text{s}\sim\text{ms}$). Investigation techniques of these cavitation processes are quite complicated. There are many reported works on the dynamics of a single cavitation bubble (Lauterborn & Bolle 1975, Lauterborn & Vogel 1984; Philipp & Lauterborn 1998; Ohl et al. 1999; Brujan et al. 2002; Lindau & Lauterborn 2003; Wolfrum et al. 2003), and the main objectives of these works are about damage caused by cavitations. These studies rarely focused on the mechanism of the cavitation cleaning. There are various controlling factors on the effects of cavitation cleaning, include the bubble size, the initial position from the object, and the characteristics of the foulant etc. In this study, the author tries to investigate single bubble dynamics near a solid boundary to reveal the cleaning mechanism of cavitation bubbles.

2.2 Experimental Studies on Cavitation Bubble Dynamics

2.2.1 Generation of a single cavitation bubble

There are several techniques that one could use to generate a single cavitation bubble. The kinetic-impulse technique (Benjamin & Ellis 1966) appears to be one that most closely models the natural inception process of bubble generation because the test liquid is momentarily put under tension. This technique suffers from the practical disadvantage that a small gaseous impurity or bubble must be positioned precisely in the test liquid prior to generation of the impulse. The problem of position and generation is overcome when a spark discharge is used (Gibson 1968, 1972b; Smith & Mesler 1972; Gibson & Blake 1980, 1982; Blake & Gibson 1981; Chahine 1977, 1982; Shima et al. 1981, 1983; Tomita et al. 1986; Kimoto et al. 1987; Cook et al. 1997; Buogo & Cannelli 2002; Gong et al. 2012), but the high-voltage electrodes that are placed in the liquid medium invariably disrupt the bubble dynamics in the last critical moments of collapse when the bubble has become highly deformed (see Figure 2.5). The laser generation technique has all the advantages of spark discharge without the disadvantages of electrodes (Lauterborn & Bolle 1975, Lauterborn 1982, Lauterborn & Vogel 1984, Lauterborn & Hentschel 1985; Vogel & Lauterborn 1988; Ward & Emmony 1991; Philipp & Lauterborn 1997, 1998; Ohl et al. 1999; Akhatov et al. 2001; Brujan et al. 2002; Lindau & Lauterborn 2003; Wolfrum et al. 2003; Yang et al. 2013). Technically the only fault of the laser technique is the mode of bubble generation, which, similar to spark discharge, involves intense local heating and vaporization of the liquid through application of a thermal impulse. Highest compliments should be accorded to Lauterborn W. for his pioneering work on the laser generation technique and continuous development works on this are on-going to this day.

Figure 2.5 A spark-generated bubble near a rubber beam (Gong et al. 2012).

2.2.2 Direct observation using high speed camera

Filming the evolutions of cavitation bubbles using a high-speed camera is the most popular method in the studies of cavitation bubble dynamics. The two important characteristics of a high-speed photography are the filming speed and resolution of the picture. The first paper that reported the use of high speed cinematographic photography was attributable to Knapp & Hollander for their work in 1948. A series of pictures depicting oscillating bubbles in the vicinity of an ogive nose in a high speed water tunnel were presented in the paper. These pictures were taken at a rate of 20,000 frames per second (fps). This frame rate was high enough to reveal the gross dynamic features of a cavitation bubble, which was not previously observable.

A great advancement came through the work of Ellis in 1956, who built a rotating mirror camera capable of taking series of pictures at a frame rate of up to a million fps. Benjamin & Ellis (1966) and Gibson (1968) provided high definition pictures on the liquid jet generation using various methods, and concluded that an average jet velocity of about 80 m/s could be observed under one atmospheric pressure. Hammitt and coworkers (1969-1972) presented a series of investigations into the dynamics of spark-generated bubbles in various configurations and confirmed jet formation directed towards solid boundaries as well as jet repulsion from flexible boundaries. Chahine and coworkers (1977-1982) also used the spark technique to investigate non-spherical bubble collapse near a free surface and a two-liquid interface, and between two solid walls. Their works showed the

obviously different bubble behaviors under different boundary conditions.

Lauterborn & Bolle (1975) presented their groundbreaking work on a laser-generated bubble that collapse in the neighborhood of a plane solid boundary. Although the frame rate was only 75,000 fps, but their work aroused the continuous investigations on laser-generated bubbles. Lauterborn & Vogel (1984) gave an example taken at 20,000 fps from the top view of a bubble, which showed the first second, and third collapses and rebounds of the bubble. They pointed out that the remarkable stability of the oscillating torus suggested that it should be interpreted as a bubble vortex ring, which was quite significant for the physical understanding of the toroidal bubble dynamics.

In the work of Ohl et al. (observed at 20 million fps, 1995), unprecedented details were captured and depicted the generic sequence of events leading to multiple shock waves and bubble shape deformations upon collapse. Philipp & Lauterborn (1998) recorded the characteristic effects of bubble dynamics, in particular the formation of a high-speed liquid jet and the emission of shock waves at the moment of collapse with high-speed photography at a frame rate of up to 1 million fps. The combination of high-speed photography of bubble dynamics from both side and bottom view with a shadowgraph illumination technique provided precise location of the emission of shock waves during collapse both in time and space (see Figure 2.6). A correlation with the erosion pattern caused by one single bubble was possible using this technique. Their works were recognized to be quite classic and cited by many other researchers (Brujan et al. 2001; Postema et al. 2004).

Figure 2.6 Laser-induced cavitation bubble dynamics recorded by Philipp & Lauterborn (1998) at a frame rate of 56,500 fps.

With the fast development of advanced digital technique, the speed and resolution of the camera continue to be enhanced, making it possible to obtain high resolution images at much higher frame rate. Using high-speed photography at a frame rate up to 5 million fps, Brujan et al. (2002) captured the temporal evolution of the liquid jet developed during bubble collapse, shock wave emission and the behavior of the “splash” effect. These detailed observations provided perfect corroboration for their simultaneous numerical simulations. The observations given by Lindau & Lauterborn (2003) using extremely high speed cinematography at a frame rate of up to 100 million fps elucidated the detailed shock wave scenario during bubble collapse, which had a spatial resolution in the order of micrometer. The bubble behaviors in the late stage of collapse and early stage of rebound were observed clearly at an angle of 45° from above the bubble. Kroninger et al (2010) combined the high speed cinematographic method and the particle tracking velocimetry (PTV) method to investigate the velocity field in the vicinity of a laser-generated cavitation bubble in water.

2.2.3 Indirect investigation through acoustic means

Beside the direct observation method of using a high speed camera, the detection of acoustic signals through the bubble life time is another important investigation method in the experimental studies of cavitation bubble dynamics. Three main devices are available to detect the pressure waves generated by the collapsing cavitations and they are: needle hydrophone, membrane hydrophone and fibre-optic hydrophone. The first two devices are made of Polyvinylidene fluoride (PVdF) sensitive elements, which can directly detect the pressure wave. The fibre-optic hydrophone detects the pressure waves based on the pertinence between the liquid pressure and optic reflectance of liquid.

Rayleigh (1917) might be the first to theoretically propound the emission of high-pressure pulses by a collapsing bubble. The pressure wave was generated from a strong compression of the bubble contents in an imploding spherical cavitation bubble. The first experimental evidence of this pressure pulse was given by Harrison (1952) using acoustic measurements. Guth (1954) appeared to be the first to observe the shock wave with a schlieren technique. Later measurements of single bubbles confirmed that even for collapse that comes into contact with a solid boundary, a shock wave was always generated. The pressure amplitudes were estimated to reach as high as 1 GPa (Jones & Edwards 1960).

Hentschel & Lauterborn (1982) described their attempts at establishing the relationship between the single bubble dynamics and the sonic/shock waves radiated using the pressure-time curves picked up through a microphone. Vogel & Lauterborn (1988) developed a technique to determine the pressure of short acoustic pulses generated at the inception of cavitation bubble and subsequent collapse by using a sensitive pressure transducer with a frequency bandwidth smaller than that of the acoustic pulses. The pressure wave emission caused by the bubble collapse in the vicinity of a solid boundary was especially important for understanding the mechanism of cavitation corrosion.

When a bubble collapse in the vicinity of a solid boundary, the oscillation frequency of the bubble was reduced (reported in the work given by Strasberg in 1953) and therefore, a prolongation of the collapse time should be investigated. This

prolongation factor, which was dependent on the distance between the inception position of the bubble and the solid boundary, was measured experimentally by Vogel & Lauterborn (1988), see Figure 2.7.

Figure 2.7 Prolongation factors measured by Vogel & Lauterborn (1988).

2.3 Analytical Solutions for Spherical Bubble Dynamics

Studies on the spherical bubble dynamics was first considered in connection with underwater explosions (Herring 1941; Cole 1948; Trilling 1952; Keller & Kolodner 1956). The experimental results on the spherical bubble dynamics were not precise in the early works (Keller & Kolodner 1956). Along with the enhancement of camera speed and development of new bubble generation techniques (Board & Kimpton 1974), the experimental investigations on the spherical bubble dynamics became more and more convenient and accurate. The

experimental results on the spherical bubble dynamics were always employed to validate the proposed numerical model (Lee et al. 2007).

Rayleigh (1917) presented a classical analytical solution for the spherical bubble dynamics in an infinite, incompressible and inviscid liquid. The work is still widely cited today. Herring refined this analytical solution in 1941, and studied the motion of explosion bubbles with arbitrary pressure variation inside the bubble, including a first order correction for compressibility of the liquid. Besides these early works, many different analytical studies on the same problem were also reported in later years (Hickling & Plesset 1964; Jahsman 1968; Epstein & Keller 1972; Flynn 1975; Lastman & Wentzell 1979, 1981; Keller & Miksis 1980). In 1986 and 1987, Prosperetti & Lezzi studied the radial dynamics of a spherical bubble in a compressible fluid using a simplified singular-perturbation method to the first and second order in the bubble-wall Mach number. A one-parameter family of approximate equations was adopted to describe the spherical bubble dynamics. They pointed out that the first-order correction to the incompressible results captured a large extent of the effect of liquid compressibility, and the second-order term had only a minor influence. In the later reported numerical works (Dassie & Reali 1996; Popinet & Zaleski 2002; Lee et al. 2007; Wang & Blake 2011, 2012), this analytical method was always employed to validate the proposed numerical models.

2.4 Numerical Simulations and Numerical Methods

2.4.1 Boundary Integral Method (BIM)

Numerical simulation of an oscillating bubble is complex due to the fact that the discretization of the governing equations must follow the evolving bubble surface within the calculation domain. BIM is particularly suitable for this kind of problems as it involves discretization of the bubble surface only. Canot & Achard (1991) reviewed the general information on BIM for potential flow problems. BIM has been particularly favored for investigation of bubble formation and oscillation near solid boundaries or free surfaces (Oguz & Prosperetti 1990a, 1990b;

Boulton-Stone 1993a) due to the non-spherical deformed shapes.

BIM has been well developed for simulating the evolution of bubbles under the assumption that the liquid is inviscid and irrotational (Blake, Taib & Doherty 1986; Chahine & Perdue 1988; Boulton-Stone 1993; Pearson, Blake & Otto 2004). BIM was also successfully extended to simulate a toroidal bubble dynamics (doubly-connected geometry, the potential solution was non-unique) through introducing a dynamic cut in the flow domain (Best & Kucera 1992). Another approach was reported by Lundgren & Mansour (1991) to deal with the non-unique solution of doubly-connected problem. A vortex ring inside the bubble was introduced to render the potential unique of a toroidal bubble.

Brujan et al. (2002) employed BIM based on an incompressible liquid impact model to simulate the bubble evolutions during the final stage of the collapse of a cavitation bubble near a solid boundary. The pressure contours and the velocity vectors in the liquid surrounding the bubble were also presented. The comparisons between their experimental and numerical data were favorable with regard to the bubble shapes (see Figure 2.8). Lee et al. (2007) also successfully used BIM to study the formation of a toroidal bubble at the end of the bubble collapse near a solid boundary. They extended the simulation into the rebound of the toroidal bubble by considering the loss of system energy. The energy loss was incorporated into the mathematical model by a discontinuous jump in the potential energy at the minimum volume during the short collapse-rebound period, which was accompanied by shock wave emission.

Figure 2.8 Comparisons between the experimental observations and the numerical simulations using the BIM at the final stage of the collapse of a cavitation bubble near a solid boundary (Brujan et al. 2002).

2.4.2 Finite Difference Method (FDM)

Conceptually, FDM is the ideal numerical simulation method for the bubble dynamic problem of solving the non-approximated Navier-Stokes equations. With appropriate boundary conditions, the flow field dynamics in the entire domain can be obtained. The key difficulty in accomplishing this task arises from the changing geometry of the bubble surface in time, which oscillates in volume and shape. The precise location of the bubble surface must be known in the computational domain so that boundary conditions may be applied. A finite difference method for solving the steady free-boundary problems was specifically developed by Ryskin & Leal (1983, 1984). Boundary-fitted orthogonal coordinates were taken to map the physical domain to a square computational grid. The numerical generation of the boundary-fitted coordinates assured that the bubble surface is coincident with a coordinate line, which eliminated the difficulties associated with tracking the evolving bubble surface. Kang & Leal (1987, 1989) generalized this method to deal with time dependent bubble deformation without volume change. This method of

simulation was found to exhibit some artificial revises, but still captured most of the interesting dynamical features of the oscillating bubble.

Popinet & Zaleski (2002) presented a numerical method based on a finite volume formulation using both a fixed grid (Blanco & Magnaudet 1995; Legendre 1996) and a front-tracking approach (Harlow & Welch 1965; Hirt & Nichols 1981) to solve the axisymmetric Navier-Stokes equations (see Figure 2.9). In this method, the bubble surface was tracked using surface points connected with cubic splines, which allowed the precise treatment of the surface integral terms appearing in the finite volume formulation.

Figure 2.9 Fixed grid and front-tracking bubble surface in the numerical simulation presented by Popinet & Zaleski (2002).

2.4.3 MEL-BIM model

Most of the previous simulation results of cavitation bubble dynamics near a solid boundary presented good agreements with the experimental results only up to the end of the first collapse (Blake et al. 1986; Zhang et al. 1994; Popinet & Zaleski 2002). After the jet impact, the rebounding of the toroidal bubble was quite difficult to simulate well (see Figure 2.10). In fact, the effects of liquid compressibility, the energy loss due to the emission of acoustic and shock waves into the liquid, chemical reactions, molecular relaxation in the gas, heat and mass transfer at the bubble surface should all be taken into account. Some of these effects could be primary, some are secondary, and some are negligible (Geers & Hunter 2002).

Figure 2.10 Temporal evolution of the equivalent radius of the bubble as given by experiments (Lauterborn & Ohl 1997) and numerical simulations given by (Popient & Zaleski 2002), which showed the numerical divergence after the jet impact.

In this study, the method of matched asymptotic expansions (Wang & Blake 2010, 2011) was used to incorporate the influence of the compressibility of the liquid. Due to the existence of a rigid boundary, a jet forms on the top of the bubble and impacts the opposite wall of the bubble, which necessitate the implementation of the vortex ring model (Wang et al. 1996b). The numerical model based on the mixed Eulerian-Lagrangian (MEL) method and the direct boundary integral method (BIM) for bubble dynamics in a weakly compressible liquid was established by Wang et al. (2010).

One of the principal advantages of this MEL-BIM model is that the boundary mesh of the model follows the transient bubble surface, and thus the bubble surface and the unknowns on the surface are direct solutions, unlike those obtained by interpolations in domain approaches. This MEL-BIM model was found to successfully march the simulations to the second and further cycles of the bubble oscillations (Wang & Blake 2010, 2011).

2.5 Cavitation Bubble Dynamics near a Solid Boundary

The cavitation bubble dynamics near a solid boundary had been investigated for a long time. However some basic mechanisms of the bubbles, such as the relations between the bubble oscillation periods and the stand-off distances, the formation of counterjet, the liquid flow inside the jet tunnel etc., are still not described very well. In this study, the author tried to use both experimental and numerical methods to elucidate these basic mechanisms.

Benjamin and Ellis (1966) observed the non-spherical collapse of an initially spherical bubble near a solid boundary towards which a high speed liquid jet was directed. This high speed liquid jet impacting against the solid boundary was thought to be the main cause of the cavitation damage (see Figure 2.11). Earlier research works carried out by Naude & Ellis (1961); Shutler & Mesler (1965) also showed similar phenomenon. After that, a large number of experimental works on the bubble dynamics near a solid boundary were reported (Gibson 1972b; Lauterborn & Bolle 1975; Gibson & Blake 1980, 1982; Tomita et al. 1986; Vogel & Lauterborn 1988; Ward & Emmony 1991; Philipp & Lauterborn 1997, 1998; Ohl et al. 1999; Lindau & Lauterborn 2003; Yang et al. 2013). These experimental results presented many detailed phenomena that took place during the evolutions of the bubble, such as the non-spherical deformations, the formations of the liquid jet, and the appearances of the counterjet etc.

Figure 2.11 Damage on aluminium specimens caused by 100 cavitation bubbles with a same maximum expanded radius but at different stand-off distances (Philipp & Lauterborn 1998).

Besides these exhaustive experimental investigations, numerical simulations also played an important role in the investigation of the non-spherical bubble dynamics near a solid boundary. Plesset & Chapman (1971) numerically examined the deformations of bubble shapes near a solid boundary and evaluated the velocity of the liquid jet, whose simulated bubble dynamics and liquid jet velocity were in good agreement with the experimental results of Lauterborn & Bolle (1975). Because the available analytical analysis was limited to asymptotic studies in which the deformation of the bubble was confined to a small perturbation range (Chahine 1982), numerical simulation had become more and more popular for investigating the detailed phenomena of bubble dynamics near a solid boundary. Blake, Taib & Doherty (1986) employed the BIM to study the growth and collapse of transient vapor cavitation bubbles near a solid boundary in the presence of buoyancy forces and an incident stagnation-point flow. Best & Kucera (1992) examined precisely the cavitation bubble dynamics with non-condensable gas inside the bubble. They found that the presence of non-condensable bubble contents could arrest the liquid jet formation. Best (1993) extended the BIM to examine the toroidal phase of an

underwater explosion bubble near a solid boundary which was developed after the liquid jet impact. This was achieved via the addition of a cut in the fluid domain to retain the singly connected geometry. Klaseboer et al. (2005) presented the numerical simulations of the explosion bubble interacting with a submerged rigid boundary using a three-dimensional bubble dynamic code in conjunction with a structural code. The bubble code is based on the boundary-element method (BEM) and had been coupled to a structural finite-element code (PAM-CRASHTM).

CHAPTER 3 EXPERIMENTAL METHOD

3.1 Experimental Design

Figure 3.1 shows a schematic diagram of the experimental set-up. A single cavitation bubble was generated by using a focused laser pulse, and a high-speed camera system was used to film the bubble evolutions. In order to clearly photograph the bubble shapes, a back lighting was required. A hydrophone system was employed to detect the acoustic signals generated during the bubble oscillations. The output electric signals detected by the hydrophone system were recorded by using a digital oscilloscope. A suitable cuvette was needed to contain the water for generating bubbles inside. The cuvette needed to be positioned very stably and accurately, so a micro-translation stage was employed. Then all of these apparatuses needed to be mounted on a plane and stable working deck.

Figure 3.1 Schematic diagram of the experimental set-up.

3.2 Apparatus

3.2.1 Laser system

A Q-switched Nd: YAG laser system (LOTIS TII LS-2134UTF) was used to generate a single laser pulse (see Figure 3.2).

Figure 3.2 Laser system including laser emitter, cooling system, power supply and remote control.

The laser beam generated from the laser emitter was stable and repeatable, which was mainly attributed by the Q-switched system. In this study, the single shot mode was used to generate a single laser pulse and produce a single cavitation bubble in the cuvette. The energy of the laser pulse was controlled by changing the pump lamp energy. Some general characteristics of the laser pulse were shown in Table 3.1.

Table 3.1 Laser pulse specifications.

Wavelength (nm)	Pulse duration (ns)	Beam divergence (mrad)	Jitter ($\pm$ ns)	Beam diameter (mm)
532	6-7	≤ 0.8	1.0	≤ 6.0

3.2.2 Optical path

The laser pulse coming out from laser emitter should be transferred and focused into the cuvette through a series of optical components (see Figure 3.3). A 532nm filter was employed to filter the noise optical waves and maintain the purity of the 532nm optical waves. A 45° mirror was used to change the transfer direction of the laser beam. Then the laser beam was focused into the cuvette through a convex lens. The cuvette walls were made of microscope slides (SAIL BRAND), and the bottom was made of PVC. The dimensions of the cuvette were 25mm in length, 25mm in width, and 75mm in height. The cuvette was positioned on a micro-translation stage (STANDA), which had a travel range of 15mm, and sensitivity of 0.01mm. The base mounts and accessories used in this study were all opto-mechanical products from STANDA.

Figure 3.3 Optical path operated in the study.

3.2.3 High-speed video camera

A high-speed video camera (*i-SPEED 3*) provided by the Olympus was

operated in this study (see Figure 3.4). The *i-SPEED 3* was the latest addition to the Olympus *i-SPEED* range, and had been designed to an advanced specification providing high resolution, extreme low light sensitivity and up to 150,000 fps recording.

Figure 3.4 High-speed video camera from the Olympus (*i-SPEED 3*).

The Olympus CDU (Controller Display Unit) incorporated a large 8.4" high resolution LCD monitor and no extra monitor or PC was required for the operation. The 'clip select editor' allowed the operator to quickly identify and save relevant image frames through the cavitation bubble evolutions. In this study, the camera was operated in a frame rate of 100,000 fps, and the resolution was 108×80 pixels (Quality with Speed mode). The *i-SPEED* Software Suite was used to post-process the videos recorded during the experiments.

3.2.4 Hydrophone system

The hydrophone system was manufactured by Precision Acoustics (PA) LTD (see Figure 3.5). The PA high performance hydrophone system comprised a 100MHz wide-band preamplifier, which was phantom powered through the DC coupler, and a plug-in interchangeable PVdF probe. The PA submersible integrated preamplifier provided immediate signal buffering and acted as a precision 50 Ohm source. The PA DC coupler completed with its own power supply providing dc

power to the submersible preamplifier whilst acting as an acoustic signal coupler between the preamplifier and the user's measurement system. In this study, a 1 mm diameter PVdF probe with a 28 micron PVdF element was employed.

Figure 3.5 Hydrophone system manufactured by PA.

The acoustic signals detected by the hydrophone system were recorded by using a digital oscilloscope (see Figure 3.6). This oscilloscope had a maximum sample rate of up to 1GS/s, which was adequate for this study. The recorded waveform data could be either analysed directly using the oscilloscope or transferred to the control computer using a Flash Memory.

Figure 3.6 Digital oscilloscope (Tektronix DPO 2012) used to record the acoustic signals detected by the hydrophone system.

3.2.5 Computer controller

A computer was used as the control centre of the experimental set-up (see Figure 3.7). The videos captured using the high-speed camera and acoustic waveforms recorded using the hydrophone system were all transferred into the computer for post-processing. The softwares installed in the computer for data processing included *i-SPEED suite* (Olympus), *OPENCHOICE DESKTOP APPLICATION* (Tektronix), AutoCAD (AUTODESK), MATLAB (MathWorks) etc.

Figure 3.7 Control computer from Acer inc.

3.2.6 Optical set-up

Due to the usage of the high-power laser, which was of high risk, the installation of the optical components must be very accurate and stable in this study. Then the experimental apparatuses were all installed on an optical table (DAEIL SYSTEMS, vibration isolation systems), which provided a solid workstation (see Figure 3.8). The cable connections of the apparatuses followed the overall design given in Figure 3.1.

Figure 3.8 Assembly of the experimental apparatuses, which included the bubble generation system, the high-speed observation station and the acoustic detection system.

3.3 Experimental Procedure

The experimental procedure mainly included the following steps:

1. Debugging of experimental apparatuses: the output of the laser pulse was normal and stable; the high-speed camera ran well and stores good; the hydrophone could detect acoustic signals and the records of the oscilloscope were continuous and clear.

2. Adjusting of optical components: the laser pulse was focused into the cuvette and the focus point should locate nearly at the center of the water volume; the camera lens was adjusted to capture the bubble shape clearly, and the captured bubble should be located at the center of the CDU screen.

3. Determination of reference scale: a photograph of the transparent referential ruler was captured using the high-speed camera at the frame rate of 100,000 fps, and then the size of the bubble captured by the high-speed camera at the same operating conditions was judged using this reference scale (see Figure 3.9).

Figure 3.9 From left to right: transparent referential ruler, reference scale captured using the high-speed camera and bubble picture captured at the same operating conditions.

4. Fixing of hydrophone probe: the tip of the PA probe should be positioned at a sufficient distance away from the laser-induced cavitation bubble to reduce the disturbance on the bubble dynamics. The probe was held by an L shaped mount as shown in Figure 3.10.

Figure 3.10 Fixing of the hydrophone probe.

5. Triggering and synchronisation: an edge type trigger was set up in the oscilloscope, only when the voltage signals transferred from the hydrophone system reached this trigger point, the recording of the waveform would start.

6. Initialization of bubble position: at the beginning of the experiments,

the laser pulse should be focused on the surface of the PVC bottom centre through the side wall to mark the zero reference position. Then the travel distance of the micro-translation stage was defined as the stand-off distance ds between the bubble inception and the solid boundary, which could be read directly from the scale of the micro-stage.

7. High-speed video: the laser system was aroused to generate a single shot (see Figure 3.11), and simultaneously the trigger switch provided in the high-speed camera system was pressed down to mark the time point of the bubble generation. The 'clip select editor' was operated to quickly identify and save relevant image frames through the bubble evolutions to the memory card.

Figure 3.11 Bubble generation instant. The 532nm laser pulse is green.

8. Acoustic signal: the acoustic waveforms recorded by the oscilloscope were transferred into the Flash Memory for further analysis.

9. Repetitive operation: step 7 and 8 were repeated to obtain experimental results for different cases through changing the bubble initial positions and altering the energy of the laser pulse.

3.4 Data Processing

3.4.1 High-speed video

The videos of the bubble evolutions captured by using the high-speed camera were transferred from the memory card to the control computer, and then opened using the *i-speed suite* software (see Figure 3.12). Then the videos were saved as pictures for the following usage in this report. After calibration using the reference scale, the spherical bubble radius and centroid position could be measured manually using the software as shown of the red dots. Due to the manual work, there was about ± 1 pixel's measurement error for determining the bubble radius.

Figure 3.12 Window interface of the *i-speed suite* software.

3.4.2 Non-spherical bubble size

In this study, the non-spherical bubble size was determined using the powerful AutoCAD software. This approach was firstly proposed by Yang et al. (2013), which was found to be much more efficient and convenient, and with high level of accuracy as compared to other reported approaches (Baghdassarian et al. 1999; Gonzalez-Avila et al. 2011).

Figure 3.13 Schematic diagram of the approach adopted to determine the volume of an axisymmetric non-spherical bubble. The horizontal dash-line indicated the solid boundary, and the vertical dash-line was the axis of symmetry. The bubble surface was traced by the solid white line.

The volume of axisymmetric non-spherical bubbles could be estimated using the volume of revolution about the axis of symmetry as follows,

$$V = S \times 2\pi D_X = \frac{4}{3}\pi R_{eq}^3 \quad (3.1)$$

where S was the area of the rotational region bounded by the bubble surface curve and the axis of symmetry ; D_X was the distance between the mass centre of the rotational region and the axis of symmetry as illustrated in Figure 3.13. After tracing the bubble surface, the mass centre D and area S of the rotational region could be calculated using the AutoCAD software tools (command: massprop). The tracing work of the bubble surface was artificial, which may include some small errors. The jet flow inside the bubble was not distinct and this part of the volume was not subtracted from the total bubble volume. This omission might cause certain measurement error but it was believed being negligible. The term R_{eq} in Equation (3.1) was the equivalent radius of a spherical bubble having the same volume as the non-spherical bubble. In follow sections, the equivalent radius was used to indicate

the bubble size.

3.4.3 Acoustic signals

When a bubble was generated or jet impact took place, a pressure wave would be released and the impulse so formed could be detected through acoustic means (Vogel & Lauterborn 1988; Wolfrum et al. 2003). A hydrophone system associated with an oscilloscope could capture these transient pressure impulses. The wave peak appeared in the recorded acoustic waveform indicated the pressure impulse, and the timing measurements could be directly taken using cursors on the oscilloscope (Vertical Line ④ and ⑥ in Figure 3.14). The acoustic waveforms were also saved as data files into the control computer, which could be replotted using MATLAB.

Figure 3.14 A screenshot of the oscilloscope, which shows the measurement of the bubble periods.

3.5 Summary

This chapter generally introduced the experimental set-up in the study. As a quite complex system, the set-up included the overall design, the selection of experimental apparatuses and purchase, the assembly, testing and operation. The post-processing of experimental data was another important work, which needed many professional softwares to deal with the original data captured using the high-speed camera and hydrophone system.

CHAPTER 4

LASER-INDUCED CAVITATION BUBBLE DYNAMICS NEAR A SOLID BOUNDARY

4.1 Non-spherical Deformation

It can be observed that a cavitation bubble is formed when the incident laser pulse has provided sufficient energy to tear the liquid apart. The bubble expands to a large size, and the maximum expanded size depends on the incident laser energy. The difference between the interior gas pressure and the exterior liquid pressure causes the bubble to expand and collapse. When a bubble is generated near a solid boundary, the radial liquid flow in the direction towards the boundary is retarded, which causes the unequal pressure distribution across the bubble surface. This pressure gradient produces different accelerations of the upper and lower bubble wall, and leads to the formation of a liquid jet on the inner and upper part of the bubble. After the jet has penetrated the opposite bubble wall, the bubble becomes toroidal and starts to rebound in this form. At a certain time, the toroidal bubble rejoins and continues to expand and collapse in singly-connected form. This process may be repeated for several cycles until the bubble energy dissipates into the ambient liquid.

Various examples of bubble evolutions during the first two oscillations are shown herein. The bubble has a maximum expanded radius of $\bar{R}_{max} = 1.2$ mm and different non-dimensional stand-off distances ($\gamma = ds/\bar{R}_{max}$, where ds is the distance between the solid boundary and the bubble position at inception). The stand-off distance ranges from 3.0 to 0.08. The frame that recorded the first bright light flash caused by the laser-induced plasma is accounted as the inception time of the bubble. These pictures were captured using the high-speed camera with a frame rate of 100,000 fps. The rigid boundary in these pictures located at the interface between the upper white part and the lower dark green part, which was indicated in

Figure 3.13. The width of each picture that appears in this study is approximately 4.6 mm, unless specified otherwise.

4.1.1 Stand-off distance $2.2 < \gamma \leq 3.0$

Figure 4.1 shows the bubble evolutions near a solid boundary when γ was greater than 2.2. Upon careful comparison of the various test cases, it was noticed that the bubble deformations for these cases were nearly identical for the first two oscillations. During the first oscillation, the retarding effect of the solid boundary on the liquid flow was weak, and the bubbles were nearly spherical. The small and late appearance of the elongation in the direction normal to the solid boundary lasts for a very short time before the first collapse (e.g. Figure 4.1a, Row 3, Frame 3). A small amount of liquid in the region above the bubble was accelerated more strongly than the other parts near the bubble surface during the first collapse phase. Therefore, a thin jet developed, approved by the thin tail appearing at the bottom of the bubble (e.g. Figure 4.1a, Row 3, Frame 7-10). After the jet penetration, the bubble became like a funnel. There was no counterjet appearing for these cases. After the second maximum expanded volume, the tail gradually disappeared (e.g. Figure 4.1a, Row 4, Frame 2 - 4). The only slight difference between these cases is that the lower part of the bubble during the second oscillation became sharper when the distance was reduced (comparing from Figure 4.1a to Figure 4.1d).

(a) $\gamma = 3.0$

(b) $\gamma = 2.7$

(c) $\gamma = 2.5$

(d) $\gamma = 2.3$

Figure 4.1 Non-spherical deformations of a bubble with $\bar{R}_{max} = 1.2$ mm, 100,000 fps ($\Delta t = 10$ μ s), frame width 4.6 mm at (a) $\gamma = 3.0$; (b) $\gamma = 2.7$; (c) $\gamma = 2.5$; (d) $\gamma = 2.3$.

4.1.2 Stand-off distance $2.0 \leq \gamma \leq 2.2$

When the stand-off distance was smaller than 2.2, the influence of the solid boundary on the bubble deformation became obvious. The elongation of the bubble in the direction normal to the solid boundary appeared a little earlier but was more pronounced than before the first collapse. During the second oscillation, the counterjet appeared at the top of the bubble from the beginning of the second expansion, and became clearer with smaller stand-off distance (comparing Figure 4.2a and Figure 4.2b). The lower part of the bubble didn't touch the solid boundary all along. After the second maximum expanded volume, the counterjet gradually disappeared.

(a) $\gamma = 2.2$

(b) $\gamma = 2.0$

Figure 4.2 Non-spherical deformations of a bubble with $\bar{R}_{max} = 1.2$ mm, 100,000 fps ($\Delta t = 10$ μ s), frame width 4.6 mm at (a) $\gamma = 2.2$; (b) $\gamma = 2.0$.

4.1.3 Stand-off distance $1.2 < \gamma < 2.0$

In this range, the rebounding bubble had the tendency to form a shape like the inverse spade (e.g. Figure 4.3a, Row 3, Frame 7). For the first two cases, $\gamma=1.7$ and 1.5, the upper part of the bubble oscillated while retaining the inverse spade shape, and the lower part of the bubble touched the solid boundary and became flattened (e.g. Figure 4.3a, b). The counterjet grew much higher than the previous cases of $\gamma > 2.0$, and finally separated from the main bubble after the second maximum expanded volume. At $\gamma = 1.3$, after the jet impact, the lower part of the rebounding bubble touched the solid boundary before forming the inverse spade shape (e.g. Figure 4.3c, Row 3, Frame 8). The top part of the bubble sank down towards the solid boundary, and the lower part of the bubble spread outwards on the surface of the solid boundary. The bubble finally formed an isosceles trapezoid shape (e.g. Figure 4.3c, Row 4, Frame 3).

(a) $\gamma = 1.7$

(b) $\gamma = 1.5$

(c) $\gamma = 1.3$

Figure 4.3 Non-spherical deformations of bubble with $\bar{R}_{max} = 1.2$ mm, 100,000 fps ($\Delta t = 10$ μ s), frame width 4.6 mm at (a) $\gamma = 1.7$; (b) $\gamma = 1.5$; (c) $\gamma = 1.3$.

4.1.4 Stand-off distance $1.0 < \gamma \leq 1.2$

For this unique range, the bubble was just touching the solid boundary at the moment of jet impact (e.g. Figure 4.4a, Row 3, Frame 8). Before the jet impact, the bubble oscillated nearly spherically and the lower part of the bubble became a little flattened in the late stage of the first collapse phase. After the jet impact, the bubble expanded and collapsed like a small mound on the solid boundary (e.g. Figure 4.4a, b, Row 3). There was still a counterjet appearing at the top of the bubble.

(a) $\gamma = 1.2$

(b) $\gamma = 1.1$

Figure 4.4 Non-spherical deformations of a bubble with $\bar{R}_{max} = 1.2$ mm, 100,000 fps ($\Delta t = 10$ μ s), frame width 4.6 mm at (a) $\gamma = 1.2$; (b) $\gamma = 1.1$.

4.1.5 Stand-off distance $0.6 < \gamma \leq 1.0$

In this range, the lower part of the bubble touched the solid boundary during the late stage of the first expansion phase (e.g. Figure 4.5a, Row 2, Frame 2 and Figure 4.5b, Row 1, Frame 8). At the first maximum expanded volume, the lower part of the bubble spreads smaller than the bubble maximum radius (e.g. Figure 4.5a, b, Row 2, Frame 4). The bubble became a little elongated in the direction normal to the solid boundary during the following collapse phase. During the subsequent oscillation, the bubble evolved like a volcano (e.g. Figure 4.5a, Row 4, Frame 3 and Figure 4.5b, Row 4, Frame 6).

(a) $\gamma = 1.0$

(b) $\gamma = 0.8$

Figure 4.5 Non-spherical deformations of a bubble with $\bar{R}_{max} = 1.2$ mm, 100,000 fps ($\Delta t = 10$ μ s), frame width 4.6 mm at (a) $\gamma = 1.0$; (b) $\gamma = 0.8$.

4.1.6 Stand-off distance $\gamma \leq 0.6$

When the stand-off distance γ was smaller than 0.6, the lower part of the bubble touched the solid boundary during the early stage of the first expansion phase (e.g. Figure 4.6a, Row 1, Frame 3; Figure 4.6b, Row 1, Frame 2; Figure 4.6c, Row 1, Frame 2; Figure 4.6d, Row 1, Frame 2). At the first maximum expanded volume, the lower part of the bubble spread larger than the bubble's maximum radius (e.g. Figure 4.6a, Row 2, Frame 4; Figure 4.6b, Row 2, Frame 4; Figure 4.6c, Row 2, Frame 3; Figure 4.6d, Row 2, Frame 3). The elongation of the bubble in the direction normal to the solid boundary during the late stage of the first collapse phase was relatively small, and the bubble was curved convexly (e.g. Figure 4.6a, Row 2, Frame 9; Figure 4.6b, Row 2, Frame 10). After the first collapse, the bubble rebounded slightly, and a splash (Tong et al. 1999) appeared at the top of the bubble (e.g. Figure 4.6a, Row 4, Frame 1 and Figure 4.6b, Row 4, Frame 1). When the stand-off distance reduced to smaller than 0.4, the bubble appeared more convex and the splash during the second oscillation was not very obvious.

(a) $\gamma = 0.6$

(b) $\gamma = 0.4$

(c) $\gamma = 0.25$

(d) $\gamma = 0.08$

Figure 4.6 Non-spherical deformations of a bubble with $\bar{R}_{max} = 1.2$ mm, 100,000 fps ($\Delta t = 10$ μ s), frame width 4.6 mm at (a) $\gamma = 1.0$; (b) $\gamma = 0.8$; (c) $\gamma = 0.25$; (d) $\gamma = 0.08$.

4.2 Analogous Bubble Shape

After a large number of bubble evolutions were observed, it was found that for the bubbles having different maximum radii but equivalent non-dimensional stand-off distances, the deformations of the bubbles were quite analogous. The differences appeared only on the bubble sizes and oscillation periods.

4.2.1 First oscillation

The first frame of each row in Figure 4.7 shows the bubble shape at about 10 μ s after its generation (the frame immediately after the appearance of the bright light flash frame). The fifth frame of each row shows the bubble shape at its first maximum expanded volume, and the difference between the bubble sizes is clear. The last frame of each row shows the end of the first bubble collapse. The exact time each photo was taken is indicated at the bottom of each frame.

For the case $\gamma = 2.0$, during the expansion phase, the bubble grew almost

spherically. In the late stage of the collapse phase, the bubble lost spherical symmetry by becoming elongated in the direction normal to the solid boundary. As the collapse progressed, the upper part of the bubble became concave and formed a liquid jet towards the solid boundary, which is quite obvious in the last frame of each row.

For the case $\gamma = 1.5$, the liquid jet formed a little earlier than the last case due to the retarding effects of the solid boundary becoming stronger. From the last frame of each row, it can be discerned that after jet impact, a distinct counterjet appeared at the top of the collapsed bubble.

For the case $\gamma = 1.0$, the bubble initially grew spherically. As the bubble migrated towards the solid boundary, the lower part of the bubble became flattened. During the collapse phase, the bubble became elongated in the direction normal to the solid boundary, and a liquid jet formed on the inner and upper part of the bubble towards the solid boundary. At the end of the first collapse, a faint counterjet can be observed in the last frame of each row.

For the case $\gamma = 0.6$, the lower part of the bubble touched the solid boundary in the early stage of the expansion phase and remained in contact with the boundary. After the first maximum expanded volume, the upper part of the bubble became highly concave, and resembled a volcano. At the end of the first collapse, the bubble became a thin layer attached to the solid boundary.

(a) $\gamma = 2.0$

(b) $\gamma = 1.5$

(c) $\gamma = 1.0$

(d) $\gamma = 0.6$

Figure 4.7 Analogous first oscillations of a bubble with different maximum bubble radii generated at (a) $\gamma = 2.0$; (b) $\gamma = 1.5$; (c) $\gamma = 1.0$; (d) $\gamma = 0.6$.

4.2.2 Second oscillation

The first frame of each row in Figure 4.8 shows the bubble shape at the beginning of the second oscillation. The fifth frame of each row shows the bubble shape at the second maximum expanded volume. The last frame of each row shows the end of the second collapse. The exact time each photo was taken is indicated at the bottom of each frame.

For the case $\gamma = 2.0$, after the jet impact, a small counterjet appeared at the top of the bubble. The bubble then expanded to an inverse spade shape, and the tail of the bubble slightly touched the solid boundary. After the second maximum expanded volume, the counterjet gradually disappeared, and the tail became more and more slim. At the end of the second collapse, the bubble became a dot in the liquid.

For the case $\gamma = 1.5$, the counterjet was much more obvious. The bubble developed to the inverse spade shape, while the lower part of the bubble touched the solid boundary in the early stage of the second expansion phase. As the expansion progressed, the upper part of the bubble remained in the spade shape, but the lower

part spread outward on the surface of the solid boundary. After the second maximum expanded volume, the counterjet became weak and the upper part of the bubble fell towards the solid boundary.

For the case $\gamma = 1.0$, the bubble remained in contact with the solid boundary and looked like a small mound. The only difference at the different volumes was the size of the mound. At the second maximum expanded volume, the lower part of the bubble spreads quite largely. The counterjet became invisible, which means that $\gamma = 1.0$ is a critical value of non-dimensional stand-off distance for the appearance of the counterjet.

For the case $\gamma = 0.6$, the bubble developed into an erupted volcano shape during the second oscillation. The bottom of the bubble became much wider on the solid boundary, and a splash appeared on the upper part of the bubble. No counterjet could be observed for this case. At the end of the collapse, only a fairly thin bubble sheet remained.

(a) $\gamma = 2.0$

(b) $\gamma = 1.5$

(c) $\gamma = 1.0$

Figure 4.8 Analogous second oscillations of a bubble with different maximum bubble radii generated at (a) $\gamma = 2.0$; (b) $\gamma = 1.5$; (c) $\gamma = 1.0$; (d) $\gamma = 0.6$.

4.3 Bubble Generation Form

When a short laser pulse (6 ns) is focused into the liquid volume, a very high light intensity, associated with a considerable strength in the electric field, is achieved. There, the laser pulse causes heating of impurities in the liquid, and/or dielectric breakdown with avalanche ionization, and creates a plasma spot. The plasma then expands to form a cavitation bubble. The duration of optical breakdown (bubble generation) is quite short, lasting only hundreds of nanoseconds (Ohl, et al. 1999), i.e. $O(10^{-7})$ s. Such a short generation duration means that the high speed camera employed in this study is not fast enough to capture the entire status of bubble generation. A combination of several tests under identical operation conditions is presented herein to show the bubble generation form (see Figure 4.9). The bright light emission in the first frame was caused by the laser-produced plasma. The forming bubble is visible as a dark border around the central bright spot, as shown in the second frame. It can be discerned that initially the bubble was generated non-spherically, and looked more like a water-drop. However, it quickly became spherical due to the isotropic ambient liquid pressure and the surface

tension.

Figure 4.9 Bubble generation induced by a focused Nd: YAG laser pulse in the liquid volume.

4.4 Liquid Jet Formation

When a bubble is generated near a solid boundary, a liquid jet always forms on the inner and upper part of the bubble due to the retarding effect of the solid boundary. The intensity of the retarding effect depends on the stand-off distance, which determines the duration of the liquid jet formation. In this study, the formation of the jet is estimated through the concave-down of the upper part of the bubble. Figure 4.10 shows the jet formation time of a bubble with a maximum radius $\bar{R}_{max} = 1.5 \text{ mm}$ at different stand-off distances. The first frame of each row shows the bubble shape just before the jet formation, and the last frame of each row shows the jet impact instant. It can be discerned that the liquid jet formed later when the stand-off distance became larger. When the stand-off distance was smaller than 0.5, the liquid jet formed at about $50\mu\text{s}$ before the jet impact. When the stand-off distance was larger than 2.5, the jet formed at less than $10\mu\text{s}$ before the jet impact.

Figure 4.10 Jet formations for a bubble with $\bar{R}_{max} = 1.5$ mm at different stand-off distances.

4.5 Counterjet Formation

A counterjet forms only for a certain range of stand-off distances. Vogel et al. (1989) pointed out that a counterjet developed only for $\gamma > 1.0$. In this study, the counterjet could be found only when $1.0 < \gamma \leq 2.2$, which is slightly different from the values given by Lindau & Lauterborn (2003, $1.0 < \gamma < 3.0$). Lindau & Lauterborn pointed out that the duration of visibility of the counterjet became shorter when γ was increased to larger than 2.0. In this study, the counterjet was sometimes not visible when the frame rate of the camera was inadequate. The formation of the counterjet was always coupled with the jet impact. For γ larger than 2.2, the liquid jet impacted on the opposite bubble wall quite weakly, or did not happen at all, and therefore no counterjet arose. When the stand-off distance was smaller than 1.0, no counterjet could be observed because the lower part of the bubble touched the solid boundary during the first oscillation, and the liquid jet directly impinged on the solid boundary.

Figure 4.11 shows the counterjet formations of a bubble with a maximum radius of 1.5mm at different stand-off distances ranging from 1.2 to 2.2. The frames of each row show the counterjet development after jet impact and the interframe time is $10\mu s$. It can be observed that the size of the counterjet depends on the stand-off distance. The largest counterjet appeared in the case $\gamma = 1.4$. For the case $\gamma = 1.2$, the bubble was just touching the solid boundary at the moment of jet impact, and the counterjet did not develop completely. When γ was larger than 1.2, the counterjet developed completely, and the size decreased as γ increased.

Lindau & Lauterborn (2003) explained the formation mechanism of the counterjet as the intersection of the shock wave. The high pressure at the intersection points was presumably followed by a tension wave which gave rise to cavitation inception and so formed the counterjet. In this study, a hypothesis is proposed that the counterjet may form due to the back-flow of the liquid through the jet tunnel. After the liquid jet penetrates the opposite bubble wall, a thin tunnel may form inside the bubble, which allows the liquid to flow inside. This hypothesis may be also supported by the phenomenon of the counterjet disappearing during the second expansion phase, which is believed to be caused by the rejoining of the bubble.

Figure 4.11 Counterjet formations for a bubble with $\bar{R}_{max} = 1.5$ mm at different stand-off distances after the jet impact. The frame size is 4.6×1.7 mm.

4.6 Bubble Oscillation Periods

By observing the bubble evolutions given in Section 4.1, it can be discerned that the bubble oscillation periods vary with the stand-off distances for a bubble with an equivalent maximum radius. A rough estimation of the bubble oscillation periods can be made using the time signatures on the film strip captured with the high-speed camera. But this estimation includes an inherent measurement error of $\pm 1/fps$ owing to the limitation of the camera speed. Therefore, a more accurate method is demanded to obtain the bubble oscillation periods.

4.6.1 Accuracy of periods recorded with the hydrophone system

As is known, when a bubble is generated or jet impact takes place, a pressure impulse will be released, and can be detected through acoustic means. These acoustic signals captured using a hydrophone system can be used to determine the bubble oscillation periods. Figure 4.12 shows several typical acoustic waveforms recorded using the hydrophone system, and the corresponding bubble shapes captured with the high-speed camera during the first two bubble oscillations. The bubble oscillation periods detected using the hydrophone system are defined as the time intervals between the pressure impulses. The interval t_{c1} between the first and second pulse in the acoustic waveform is called the first oscillation period, the interval t_{c2} is called the second oscillation period, and so on. It can be discerned that the oscillation periods recorded using the hydrophone system agree very well with the time signatures on the pictures captured with the high-speed camera. Due to the existence of inherent measurement error when the high-speed camera is used, the bubble periods measured using the hydrophone system should be much more accurate than those based on the film strip.

(a) $\gamma = 0.6$

(b) $\gamma = 1.0$

(c) $\gamma = 1.4$

(d) $\gamma = 2.0$

(e) $\gamma = 2.4$

(f) $\gamma = 3.0$

Figure 4.12 Comparisons of oscillation periods for a bubble with $\bar{R}_{max} = 1.5 \text{ mm}$ at (a) $\gamma = 0.6$; (b) $\gamma = 1.0$; (c) $\gamma = 1.4$; (d) $\gamma = 2.0$; (e) $\gamma = 2.4$;

(f) $\gamma = 3.0$ measured using the high-speed camera at 100,000 fps and the hydrophone system.

4.6.2 Non-linear variation of the bubble oscillation periods

Figure 4.13 shows the first two bubble oscillation periods acquired using the hydrophone system for a bubble with $\bar{R}_{max} = 1.5$ mm at different stand-off distances. The peak value of the first oscillation period appeared at $\gamma \approx 0.6$, and $t_{c1} \approx 335 \mu s$. When the stand-off distance was smaller than 0.6, the first oscillation period decreased rapidly to about $300 \mu s$ at $\gamma \approx 0.1$. When the stand-off distance was larger than 0.6, the value of t_{c1} gradually reached about $280 \mu s$ at $\gamma \approx 3.0$. For the second oscillation period, the peak value appeared at $\gamma \approx 1.2$, and $t_{c2} \approx 250 \mu s$. The second oscillation period for $\gamma < 0.5$ could not be detected by the hydrophone due to the dissipation of the bubble. In the range of $0.5 < \gamma < 1.2$, the second oscillation period decreased quite steeply, from about $250 \mu s$ to about $140 \mu s$. When $\gamma > 1.2$, the decreasing slope of t_{c2} became slightly gentle, at about $150 \mu s$ at $\gamma \approx 3.0$.

(a)

(b)

Figure 4.13 Bubble oscillation periods acquired using the hydrophone system for a bubble with $\bar{R}_{max} = 1.5$ mm at various stand-off distances: (a) first oscillation period t_{c1} ; (b) second oscillation period t_{c2} .

4.6.3 Non-dimensional statistics on the bubble oscillation periods

In order to generate comparable statistics on the bubble oscillation periods for the cases of different maximum radii, a non-dimensionalization of the oscillation periods is required. The first oscillation periods of spherical bubbles with different maximum radii in an infinite fluid are used to create this non-dimensionalization (Rayleigh 1917).

$$t_{sd} = 2 \times 0.915 \bar{R}_{max} \sqrt{\frac{\bar{\rho}_{\infty}}{\bar{p}_{\infty} - \bar{p}_v}} \quad (4.1)$$

where $\bar{p}_{\infty} = 98.07$ kPa is the ambient pressure, $\bar{p}_v = 2.98$ kPa is the vapour pressure of water at 20 °C, and $\bar{\rho}_{\infty} = 1000$ kg/m³ is the density of water.

Figure 4.14 below shows the non-dimensional bubble oscillation periods, obtained using the hydrophone system. The maximum expanded radii of the bubbles ranged from 1.0 mm to 1.6 mm. The bubbles were generated at different

stand-off distances ($\gamma \leq 3.0$) from the solid boundary. The non-dimensional first oscillation period, $k_1 = t_{c1}/t_{sd}$, reached the peak value at $\gamma \approx 0.6$ for all bubble sizes. The peak value decreased from about 1.25 to 1.15 when $\bar{R}_{max}$ increased from 1.0 mm to 1.6 mm, as can be discerned from the distribution of the data bands shown in Figure 4.14a. For $\gamma < 0.6$, k_1 finally decreased to about 1.10. It should be noted that this trend is different from the results reported by Lindau & Lauterborn (2003) for a bubble with $\bar{R}_{max} = 1.5$ mm, which is shown as the dash line in Figure 4.14a. In Lindau & Lauterborn's results, k_1 continued to increase and finally approached about 1.35 at $\gamma = 0.1$. It is plausible that the difference arises from the fact that Lindau & Lauterborn extrapolated the value of k_1 for $\gamma < 0.5$ based on their experimental results for $\gamma > 0.5$, which may be not reliable. For $\gamma > 0.6$, k_1 decreased with γ and finally approached 1.0.

The non-dimensional second oscillation period, $k_2 = t_{c2}/t_{sd}$, varied with γ for different $\bar{R}_{max}$ and formed a nearly identical curve. The value of k_2 rose rapidly for small stand-off distances, and reached a peak value of around 0.9 in the vicinity of $\gamma = 1.2$. It then decreased gradually with γ , and approached 0.55 asymptotically.

(a)

(b)

Figure 4.14 (a) The non-dimensional first oscillation period k_1 and (b) the non-dimensional second oscillation period k_2 versus γ for bubbles with different maximum radii of $\bar{R}_{max} = 1.0, 1.2, 1.4, 1.6$ mm.

4.7 Acoustic Pressure Waves

4.7.1 Profile of the acoustic waveform

Figure 4.15a shows a typical acoustic waveform detected using the hydrophone system for a bubble with $\bar{R}_{max} = 1.5$ mm at $\gamma = 2.0$. Figure 4.15b shows the magnification of the waveform at the optical breakdown of the bubble. It can be discerned that the voltage signal had a rather fast rising to about 24V within a time interval of about 50 ns after triggering, which indicates the quite short duration of the bubble generation. Due to the existence of the solid boundary, a reflected wave was generated, which can be discerned from the small wave peak after the bubble generation. The reflected acoustic wave had a fairly similar profile to that of the optical breakdown wave.

Figure 4.15 Acoustic waveform detected using the hydrophone system for a bubble with $\bar{R}_{max} = 1.5$ mm at $\gamma = 2.0$: (a) low time resolution; (b) high time resolution.

4.7.2 Acoustic wave speed

The acoustic wave speed through the water is an important characteristic in the study of hydroacoustics. As is known, the velocity of sound in the water is about 1500 m/s, which can be verified in this study. Figure 4.16 shows the high time resolution of acoustic waves detected after the optical breakdowns of bubbles with $\bar{R}_{max} = 1.5$ mm at $\gamma = 1.0$, 2.0 and 3.0 . The time differences between the optical breakdown waves and the reflected acoustic waves for these three cases are about 2.0 , 4.0 , and $6.0 \mu\text{s}$. The travelling distances of these acoustic waves should be 3.0 ,

6.0 and 9.0 mm respectively, which are diploid dimensional stand-off distances. Dividing the travelling distances by the time differences, it can be found that the acoustic wave speed is, appropriately and as expected, 1500 m/s.

Figure 4.16 High time resolution of the acoustic waves detected after the optical breakdown of a bubble with $\bar{R}_{max} = 1.5$ mm at (a) $\gamma = 1.0$, black solid line; (b) $\gamma = 2.0$, red dash line; (c) $\gamma = 3.0$, blue dot line.

4.8 Bubble Sizes

4.8.1 Temporal evolution of the equivalent bubble radius

Figure 4.17 shows the time histories of the equivalent bubble radius for a bubble with $\bar{R}_{max} = 1.5$ mm at $\gamma = 0.6, 1.2, 2.0$ using the data processing methods given in Equation (3.1). It can be observed that the time histories of the equivalent bubble radius for these three cases were nearly identical during the first expansion phase, and that the radius approached the equivalent maximum expanded radius of 1.5 mm. After that, the reductions of the equivalent bubble radius started to diverge due to the retarding effects of the solid boundary on the bubble evolutions, particularly for the case $\gamma = 0.6$. The second maximum expanded equivalent radius was significantly influenced by the stand-off distance, about 1.05 mm for $\gamma = 0.6$ and 2.0, which is significantly smaller than that for the intermediate case of $\gamma = 1.2$, about 1.15 mm.

Figure 4.17 Time histories of the equivalent radius of a bubble with $\bar{R}_{max} = 1.5 \text{ mm}$ at $\gamma = 0.6, 1.2, 2.0$.

4.8.2 Energy dissipation

A direct manifestation of the energy deposited in the bubble is the maximum expanded volume during each bubble oscillation cycle. The first maximum expanded volume is always taken as the benchmarked volume that indicates the incident laser energy in the bubble generation, while the second maximum expanded volume indicates the residual bubble energy after the jet impact.

Figure 4.18 presents the non-dimensional second maximum expanded equivalent radius $r_{2*} = \bar{R}_{2max}/\bar{R}_{max}$ for bubbles with $\bar{R}_{max} = 1.0, 1.2, 1.4, 1.6 \text{ mm}$ at $0.5 \leq \gamma \leq 3.0$. The profiles of r_{2*} versus γ are similar for different maximum radii. The value of r_{2*} is around 0.55 at $\gamma = 0.5$, and increases rapidly to about 0.8 in the vicinity of $\gamma = 0.8$. When $\gamma > 0.8$, r_{2*} decreases with γ monotonically and approaches 0.6 near $\gamma = 3.0$.

Figure 4.18 The second maximum expanded equivalent bubble radii for a bubble with $\bar{R}_{max} = 1.0, 1.2, 1.4, 1.6$ mm at $0.5 \leq \gamma \leq 3.0$.

Assuming that the bubble at the second maximum expanded volume is at thermodynamic equilibrium with its ambient liquid, the energy deposited in the bubble must then consist of two parts: the surface energy E_{sur} and the potential energy E_{pot} . Through the definition of surface tension, σ (energy per unit surface area, always taken to be 0.0728 J/m^2 for 20°C water), the total surface energy stored in the bubble surface is $\sigma \cdot 4\pi R_{2max}^2$. The potential energy acquired through the expansion of the bubble implies the work done by the liquid system on the bubble system. As the ambient liquid has to be displaced outward during the second expansion, the work done is given as the volume of the second maximum expanded bubble multiplied by the ambient static liquid pressure, i.e. $p_\infty \cdot 4\pi R_{2max}^3/3$. Thus, a general expression (Brennen 1995) for the net energy deposited in the bubble at the second maximum expanded volume is:

$$\begin{aligned} E_{dep} &= E_{sur} + E_{pot} = 4\pi\sigma R_{2max}^2 + \frac{4}{3}\pi p_\infty R_{2max}^3 \\ &= \frac{4}{3}\pi R_{2max}^2 (3\sigma + p_\infty R_{2max}) \cong \frac{4}{3}\pi p_\infty R_{2max}^3 \end{aligned} \quad (4.2)$$

The surface energy term in Equation (4.2) may be ignored as $3\sigma \ll p_\infty R_{2max}$. It

can be deduced from (4.2) that the residual bubble energy at the second maximum expanded volume is proportional to the cube of the second maximum expanded equivalent radius. A 70% residual radius at the second maximum expanded volume as observed indicates that about two-thirds of the bubble energy has dissipated.

4.9 Summary

This chapter presents detailed features of bubble evolutions near a solid boundary. The bubble always generates non-spherically, and then expands to a large size. The maximum expanded bubble size depends on the incident laser energy. When the bubble is generated near a solid boundary, the bubble deforms non-spherically and a liquid jet forms on the inner and upper part of the bubble. For a bubble generated at initial stand-off distances greater than the maximum bubble radius, the bubble remains nearly spherical during the first oscillation cycle, and only slight elongation appears in the direction normal to the solid boundary. The formation time of the liquid jet depends on the stand-off distance. After the liquid jet has penetrated the opposite bubble wall, the bubble always forms a funnel-like shape. When the stand-off distance is between 1.0 and 2.2, a counterjet appears at the top of the rebounding bubble after the jet impact, and gradually disappears during the second collapse phase. Similarly, the size of the counterjet depends on the stand-off distance. When the stand-off distance continues to decrease, the lower part of the bubble touches the solid boundary during the first oscillation cycle, and the bubble resembles a volcano attached to the solid boundary during the second oscillation.

The pertinent global features of the bubble evolutions at different stand-off distances are also presented. The non-linear varied bubble oscillation periods are related to both the bubble's maximum radii and the stand-off distances. The measurements of the oscillation periods using the hydrophone system appear to be more convenient and accurate than those obtained using the high-speed camera. The non-dimensional oscillation periods k_1 and k_2 for various maximum bubble radii and stand-off distances are investigated. The distinctive feature of the results, which differs from the observations reported in other literature, is the decay of k_1 for the

stand-off distances $\gamma < 0.5$. The acoustic pressure waveforms recorded using the hydrophone system can be used to verify the sound speed in the water. The statistics of the second maximum expanded volume indirectly show the residual bubble energy after the jet impact. In addition, about 30% reduction of the maximum expanded bubble radius corresponds to about two-thirds of the bubble energy dissipation.

CHAPTER 5

MEL-BIM MODEL FOR NUMERICAL SIMULATION

5.1 Potential Flow Theory

Study of dynamics of a cavitation bubble typically considers a bubble with radius of $O(\mu m - mm)$ subjected to an acoustic wave with high frequency ultrasound $O(10^0 - 10^3)$ kHz and at high intensity $O(10^1 - 10^3)$ w/cm^2 (Young 1989; Leighton 1994; Blake et al. 1999). Acoustic bubble dynamics are typically dominated by inertial effects as indicated by a high Reynolds number, $O(10^2 - 10^4)$. The time scales of the bubble life are quite short, typically $O(\mu s - ms)$, thus limiting the dispersion of vorticity in a very thin layer adjacent to the bubble surface. Bubble dynamics can thus be modeled approximately based on the potential flow theory. The buoyancy force is ignored in this study because of the fairly small bubble size $O(\mu m - mm)$.

Significant liquid compressibility due to high-speed motions usually occurs when thermal effects on the liquid are unimportant (Plesset & Prosperetti 1977). In this study, it is assumed that thermal effects on the liquid are insignificant. The liquid state is thus completely defined by a single thermodynamic variable. The sound speed $\bar{c}$ and the enthalpy $\bar{h}$ of the liquid can be defined as follows (the bar over the symbols indicates the dimensional quantities):

$$\bar{c}^2 = \frac{d\bar{p}}{d\bar{\rho}}, \quad \bar{h} = \int_{\bar{p}_\infty}^{\bar{p}} \frac{d\bar{p}}{\bar{\rho}} \quad (5.1a, b)$$

where the reference pressure $\bar{p}_\infty$ is the pressure in the undisturbed liquid (hydrostatic pressure).

The equation of mass conservation is

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} + \bar{\nabla} \cdot (\bar{\rho} \bar{\mathbf{u}}) = 0 \quad (5.2)$$

By assuming that the flow is irrotational, a velocity potential $\bar{\varphi}$, such that $\bar{\mathbf{u}} = \bar{\nabla}\bar{\varphi}$, is introduced. The equation of mass conservation can then be written as

$$\bar{\nabla}^2 \bar{\varphi} + \frac{1}{\bar{\rho}} \frac{D\bar{\rho}}{D\bar{t}} = 0 \quad (5.3)$$

where $D/D\bar{t} = \partial/\partial\bar{t} + \bar{\mathbf{u}} \cdot \bar{\nabla}$.

Substituting Equations (5.1 a, b) into Equation (5.3) obtains

$$\bar{\nabla}^2 \bar{\varphi} + \frac{1}{\bar{c}^2} \left(\frac{\partial \bar{h}}{\partial \bar{t}} + \bar{\nabla} \bar{\varphi} \cdot \bar{\nabla} \bar{h} \right) = 0 \quad (5.4)$$

The equation for momentum conservation (N.S. equation) is

$$\frac{\partial \bar{\mathbf{u}}}{\partial \bar{t}} + \bar{\mathbf{u}} \cdot \bar{\nabla} \bar{\mathbf{u}} = -\frac{1}{\bar{\rho}} \bar{\nabla} \bar{p} \quad (5.5)$$

From Equation (5.1b), it can be calculated that

$$d\bar{h} = \frac{d\bar{p}}{\bar{\rho}} \quad (5.6)$$

which is equal to

$$\bar{h}_{\bar{x}} d\bar{x} + \bar{h}_{\bar{y}} d\bar{y} + \bar{h}_{\bar{z}} d\bar{z} = \frac{\bar{p}_{\bar{x}} d\bar{x} + \bar{p}_{\bar{y}} d\bar{y} + \bar{p}_{\bar{z}} d\bar{z}}{\bar{\rho}} \quad (5.7)$$

Equalizing each term of Equation (5.7),

$$\bar{\nabla} \bar{h} = \frac{1}{\bar{\rho}} \bar{\nabla} \bar{p} \quad (5.8)$$

The equation of momentum conservation (5.5) then becomes

$$\frac{\partial}{\partial \bar{t}} \bar{\nabla} \bar{\varphi} + \frac{1}{2} \bar{\nabla} (\bar{\nabla} \bar{\varphi} \cdot \bar{\nabla} \bar{\varphi}) = -\bar{\nabla} \bar{h} \quad (5.9)$$

Integrating this equation once obtains the Bernoulli equation

$$\frac{\partial \bar{\varphi}}{\partial \bar{t}} + \frac{1}{2} |\bar{\nabla} \bar{\varphi}|^2 + \bar{h} = 0 \quad (5.10)$$

In order to find the expressions for $\bar{c}$ and $\bar{h}$, an equation of state of the liquid is required. The Tait model relating pressure to density of the fluid is employed, which indicates the compressibility property:

$$\frac{\bar{p} + \bar{B}}{\bar{p}_{\infty} + \bar{B}} = \left(\frac{\bar{\rho}}{\bar{\rho}_{\infty}} \right)^n \quad (5.11)$$

The values $\bar{B} = 3049.13 \text{ bars}$ and $n = 7.15$ give an excellent fit to the experimental pressure-density relation for water up to 10^5 bars (Fujikawa &

Akamatsu 1980), and $\bar{\rho}_\infty$ is the density of the liquid at infinity.

Substituting the relation (5.11) into (5.1a, b), yields

$$\bar{c}^2 = \frac{n(\bar{p} + \bar{B})}{\bar{\rho}} = \frac{n}{\bar{\rho}_\infty} (\bar{p}_\infty + \bar{B})^{\frac{1}{n}} (\bar{p} + \bar{B})^{\frac{n-1}{n}} \quad (5.12a)$$

$$\begin{aligned} \bar{h} &= \frac{\bar{c}^2 - \bar{c}_\infty^2}{n-1} = \frac{\bar{c}_\infty^2}{n-1} \left(\frac{\bar{c}^2}{\bar{c}_\infty^2} - 1 \right) = \frac{\bar{c}_\infty^2}{n-1} \left(\frac{\bar{p} + \bar{B}}{\bar{p}_\infty + \bar{B}} \frac{\bar{\rho}_\infty}{\bar{\rho}} - 1 \right) \\ &= \frac{\bar{c}_\infty^2}{n-1} \left(\left(\frac{\bar{p} + \bar{B}}{\bar{p}_\infty + \bar{B}} \right)^{(n-1)/n} - 1 \right) \end{aligned} \quad (5.12b)$$

where $\bar{c}_\infty$ is the wave speed of the acoustic wave in the undisturbed liquid, and

$$\bar{c}_\infty^2 = \frac{n(\bar{p}_\infty + \bar{B})}{\bar{\rho}_\infty} \quad (5.13)$$

The enthalpy $\bar{h}$ around the equilibrium pressure $\bar{p}_\infty$ can be expanded using a Taylor series expansion as follows:

$$\begin{aligned} \bar{h} &= \frac{\bar{c}_\infty^2}{n-1} \left(\left(\frac{\bar{p} + \bar{B}}{\bar{p}_\infty + \bar{B}} \right)^{\frac{n-1}{n}} - 1 \right) = \frac{\bar{c}_\infty^2}{n-1} \left(\left(1 + \frac{\bar{p} - \bar{p}_\infty}{\bar{p}_\infty + \bar{B}} \right)^{\frac{n-1}{n}} - 1 \right) \\ &= \frac{\bar{c}_\infty^2}{n-1} \left(\left(1 + \frac{n-1}{n} \frac{\bar{p} - \bar{p}_\infty}{\bar{p}_\infty + \bar{B}} + \frac{\left(\frac{n-1}{n} \right) \left(\frac{n-1}{n} - 1 \right)}{2} \left(\frac{\bar{p} - \bar{p}_\infty}{\bar{p}_\infty + \bar{B}} \right)^2 + \dots \right) - 1 \right) \\ &= \frac{\bar{c}_\infty^2}{n} \frac{\bar{p} - \bar{p}_\infty}{\bar{p}_\infty + \bar{B}} - \frac{\bar{c}_\infty^2}{2} \frac{1}{n^2} \left(\frac{\bar{p} - \bar{p}_\infty}{\bar{p}_\infty + \bar{B}} \right)^2 + \dots \\ &= \frac{\bar{p} - \bar{p}_\infty}{\bar{\rho}_\infty} - \frac{1}{2\bar{c}_\infty^2} \left(\frac{\bar{p} - \bar{p}_\infty}{\bar{\rho}_\infty} \right)^2 + \dots \end{aligned} \quad (5.14)$$

To complete the mathematical formulations for the solutions of this potential problem, knowledge of the boundary conditions on the bubble surface is necessary. On the bubble surface, the liquid pressure is given as

$$\bar{p}_L = \bar{p}_v + \bar{p}_g - \bar{\sigma} \left(\frac{1}{\bar{R}_1} + \frac{1}{\bar{R}_2} \right) \quad (5.15)$$

where $\bar{p}_v$ is the vapor pressure inside the bubble, which is a function of the temperature of the bubble wall only. This pressure is assumed to be small for most cases of investigation ($\bar{p}_v \sim 2 \text{ kPa}$ at 20°C) and is often negligible (Brennen 1995). $\bar{p}_g$ is the non-condensable gas pressure inside the bubble. $\bar{R}_1$ and $\bar{R}_2$ are the

principle radii of the curvature of the bubble surface. $\bar{\sigma}$ is the surface tension coefficient, which should always be 0.0728 N/m (for 20°C water). Equation (5.15) requires the normal stresses on the bubble surface to be continuous.

The kinematic boundary condition on the bubble surface is given as

$$\frac{d\bar{\mathbf{r}}}{d\bar{t}} = \bar{\nabla}\bar{\varphi} \quad \text{for } \bar{\mathbf{r}} \text{ on } S \quad (5.16)$$

The enthalpy of the liquid on the bubble surface is obtained from (5.12b):

$$\bar{h}_L = \frac{\bar{c}_\infty^2}{n-1} \left(\left(\frac{\bar{p}_L + \bar{B}}{\bar{p}_\infty + \bar{B}} \right)^{(n-1)/n} - 1 \right) \quad (5.17)$$

Assuming that the bubble is located at a harmonic standing wave along the z -axis, the boundary condition at infinity is given as

$$\bar{\varphi}|_{\bar{r} \rightarrow \infty} = \frac{\bar{b}}{2} \cos(\bar{\omega}\bar{t} + \bar{k}\bar{z} + \theta_0) + \frac{\bar{b}}{2} \cos(\bar{\omega}\bar{t} - \bar{k}\bar{z} - \theta_0) \quad (5.18)$$

where $\bar{b}$, $\bar{k}$, $\bar{\omega}$ and θ_0 are the amplitude, wave number, frequency, and initial phase, respectively.

5.2 Non-dimensional Normalization

The non-dimensionalization approach given by Prosperetti & Lezzi (1986) is adopted in this study: all lengths are scaled with respect to $\bar{R}_{max}$ (the maximum radius of the bubble); the time is scaled by $\bar{R}_{max}/\bar{U}$, where $\bar{U}$ is the typical liquid velocity on the bubble surface, which is given by $\bar{U} = (\bar{p}_\infty/\bar{\rho}_\infty)^{1/2}$; the enthalpy is scaled by $\bar{U}^2$. The detailed non-dimensionalization is shown as follows:

$$\mathbf{r} = \frac{\bar{\mathbf{r}}}{\bar{R}_{max}}, \quad t = \frac{\bar{U}}{\bar{R}_{max}} \bar{t}, \quad \varphi = \frac{\bar{\varphi}}{\bar{R}_{max}\bar{U}} \quad (5.19a,b,c)$$

$$h = \frac{\bar{h}}{\bar{U}^2}, \quad c = \frac{\bar{c}}{\bar{c}_\infty}, \quad p = \frac{\bar{p} - \bar{p}_\infty}{\bar{p}_\infty} \quad (5.20a,b,c)$$

where the dimensional symbols are indicated with a overbar. The non-dimensional ones are presented without any indication, unless specified otherwise.

Substituting these non-dimensional symbols into Equation (5.4) and Equation (5.10) yields

$$\nabla^2 \varphi + \frac{\varepsilon^2}{c^2} \left(\frac{\partial h}{\partial t} + \nabla \varphi \cdot \nabla h \right) = 0 \quad (5.21)$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2} |\nabla \varphi|^2 + h = 0 \quad (5.22)$$

where ∇ is defined in non-dimensional terms, which means $\nabla = \bar{R}_{max} \bar{\nabla}$. And

$$\varepsilon = \frac{\bar{U}}{\bar{c}_\infty} \quad (5.23)$$

is the characteristic bubble-wall Mach number, which is assumed to be small in this study.

After a transformation of the Equation (5.12b), and substituting Equations (5.20a, b) and (5.23) into it, the non-dimensional sound speed c becomes,

$$c^2 = 1 + \varepsilon^2(n-1)h \quad (5.24)$$

The non-dimensional expansion form of the enthalpy yielded from Equation (5.14) is

$$h = p - \frac{1}{2} \varepsilon^2 p^2 + O(\varepsilon^3) \quad (5.26)$$

In this study, only the first order of Mach number is considered, thus

$$h = p + O(\varepsilon^2) \quad (5.27)$$

Substituting Equation (5.27) into Equation (5.21) and Equation (5.22) yields

$$\nabla^2 \varphi + \frac{\varepsilon^2}{c^2} \left(\frac{\partial p}{\partial t} + \nabla \varphi \cdot \nabla p \right) = O(\varepsilon^4) \quad (5.28)$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2} |\nabla \varphi|^2 + p = O(\varepsilon^2) \quad (5.29)$$

The boundary condition on the bubble surface becomes

$$\frac{d\mathbf{r}}{dt} = \nabla \varphi \quad \text{for } \mathbf{r} \text{ on } S \quad (5.30)$$

The liquid pressure on the bubble surface can be obtained using Equation (5.20c)

$$p_L = \frac{\bar{p}_L - \bar{p}_\infty}{\bar{p}_\infty} = -1 + \left(p_v + p_g - \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right) \quad (5.32)$$

where

$$p_v = \frac{\bar{p}_v}{\bar{p}_\infty}, \quad p_g = \frac{\bar{p}_g}{\bar{p}_\infty}, \quad \sigma = \frac{\bar{\sigma}}{\bar{R}_{max} \bar{p}_\infty} \quad (5.33a,b,c)$$

$$R_1 = \frac{\bar{R}_1}{\bar{R}_{max}}, \quad R_2 = \frac{\bar{R}_2}{\bar{R}_{max}} \quad (5.34d,e)$$

The boundary condition at infinity becomes

$$\varphi|_{r \rightarrow \infty} = \frac{b}{2} \cos(\omega t + kz + \theta_0) + \frac{b}{2} \cos(\omega t - kz - \theta_0) \quad (5.35)$$

where

$$b = \frac{\bar{b}}{\bar{R}_{max}\bar{U}}, \quad k = \bar{R}_{max}\bar{k}, \quad \omega = \frac{\bar{R}_{max}}{\bar{U}}\bar{\omega} \quad (5.36a,b,c)$$

and it is found that

$$\frac{k}{\omega} = \frac{\bar{R}_{max}\bar{k}}{\bar{R}_{max}\bar{\omega}/\bar{U}} = \frac{\bar{U}}{\bar{\omega}/\bar{k}} = \frac{\bar{U}}{\bar{c}_\infty} = \varepsilon \quad (5.37)$$

5.3 Computational Model: Second-order Theory

An approximate perturbation theory using the method of matched asymptotic expansions (Wang & Blake, 2010) is employed in this study to include the influence of the compressibility of the liquid (see Appendix A1). The investigation in this study mainly focuses on the fluid dynamics near the bubble surface $(\bar{x}, \bar{y}, \bar{z}) = O(\bar{R}_{max})$, where the velocity potential satisfies Laplace's equation:

$$\nabla^2 \varphi = O(\varepsilon^2) \quad (5.38)$$

The kinematic boundary condition on the bubble surface is

$$\frac{dr}{dt} = \nabla \varphi + O(\varepsilon^2) \quad \text{on } S \quad (5.39)$$

The dynamic boundary condition on the bubble surface is

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2} |\nabla \varphi|^2 + p_L = O(\varepsilon^2) \quad \text{on } S \quad (5.40)$$

And the far-field boundary condition is

$$\varphi|_{r \rightarrow \infty} = (\varphi)^o = f_0(t) + \varepsilon f_1(t) + \varepsilon g_1(t)z + \frac{m_0(t)}{r} + O(\varepsilon^2) \quad (5.41)$$

where

$$f_0(t) = b \cos(\omega t) \cos(\theta_0), \quad f_1(t) = -m_0'(t) \quad (5.42a,b)$$

$$g_1(t) = -b\omega \cos(\omega t) \sin(\theta_0) \quad (5.42c)$$

$$m_0(t) = \int_S \frac{\partial \varphi_0}{\partial n} dS \quad (5.42d)$$

Examining these equations, the following conclusion can be reached: to the first-order in ε , the potential flow problem can be reduced to solving Laplace's

equation with the compressible effects of the liquid appearing only in the far-field boundary condition (5.41). Based on the derivation of Equation (5.41), the flow velocity in the far-field at the initial time can be obtained as

$$\nabla\varphi|_{r\rightarrow\infty,t=t_0} = -\varepsilon b\omega \cos(\omega t)\sin(\theta_0)\mathbf{k} \quad (5.43)$$

where $\mathbf{k}$ is the unit vector along the z-axis.

The equations from (5.38) to (5.43) are based on an absolute coordinate system fixed to the solid boundary. As is known, a bubble subjected to an acoustic wave always simultaneously oscillates radially and moves in the z-direction. To simplify the problem, a coordinate $\mathbf{r}_b$ moving with a (time-dependent) uniform stream of the acoustic wave at infinity, in which the flow velocity vanishes at infinity, is adopted. This problem is then reduced to only considering the local bubble oscillations, and not the relative movement in the z-direction. The transforms of the coordinates are shown as follows:

$$t_b = t \quad (5.44a)$$

$$\begin{aligned} \mathbf{r}_b &= \mathbf{r} + \varepsilon(\phi_a^{in} - \phi_a^{re})\mathbf{k} \\ &= \mathbf{r} - \varepsilon b \sin(\omega t_b) \sin(kz_b + \theta_0) \mathbf{k} \end{aligned} \quad (5.44b)$$

In this moving coordinate, the bubble centroid is at the $z_b = 0$ position, therefore

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_b} + \varepsilon b\omega \cos(\omega t_b) \sin(\theta_0) \frac{\partial}{\partial z_b}, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial x_b} \quad (5.45a,b)$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y_b}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z_b} \quad (5.45c,d)$$

Substituting Equation (5.45) into Equations (5.38)-(5.43) yields

$$\nabla_b^2 \varphi = O(\varepsilon^2) \quad (5.46)$$

$$\frac{d\mathbf{r}_b}{dt_b} = \varepsilon b\omega \cos(\omega t_b) \sin(\theta_0)\mathbf{k} + \nabla_b \varphi + O(\varepsilon^2) \text{ on } S \quad (5.47)$$

$$\frac{\partial \varphi}{\partial t_b} + \varepsilon b\omega \cos(\omega t_b) \sin(\theta_0) \frac{\partial \varphi}{\partial z_b} + \frac{1}{2} |\nabla_b \varphi|^2 + p_L = O(\varepsilon^2) \text{ on } S \quad (5.48)$$

$$\varphi|_{r_b \rightarrow \infty} = f_0(t_b) + \varepsilon f_1(t_b) + \varepsilon g_1(t_b) z_b + \frac{m_0(t_b)}{r_b} + O(\varepsilon^2) \quad (5.49)$$

$$\nabla_b \varphi|_{r_b \rightarrow \infty, t_b = t_0} = \varepsilon g_1(t_0) \mathbf{k} \quad (5.50)$$

where the operator ∇_b is in terms of $\mathbf{r}_b$, which makes the estimation $\mathbf{r}_b = \mathbf{r} + O(\varepsilon)$ in (5.49).

The following decomposition of φ is then made:

$$\varphi = f_0(t_b) + \varepsilon f_1(t_b) + \varepsilon g_1(t_b)z_b + \phi \quad (5.51)$$

Substituting Equation (5.51) into Equation (5.46) to Equation (5.50) yields

$$\nabla_b^2 \phi = O(\varepsilon^2) \quad (5.52)$$

$$\frac{d\mathbf{r}_b}{dt_b} = \nabla_b \phi + O(\varepsilon^2) \quad \text{on } S \quad (5.53)$$

$$\begin{aligned} \frac{d\phi}{dt_b} &= \frac{1}{2} |\nabla_b \phi|^2 - (p_L + f_0'(t_b) + \varepsilon f_1'(t_b) + \varepsilon g_1'(t_b)z_b) \\ &\quad + O(\varepsilon^2) \quad \text{on } S \end{aligned} \quad (5.54)$$

$$\phi|_{r_b \rightarrow \infty} = \frac{m_0(t_b)}{r_b} + O(\varepsilon^2) \quad (5.55)$$

$$\nabla_b \phi|_{r_b \rightarrow \infty, t_b = t_0} = 0 \quad (5.56)$$

Also, substituting Equation (5.51) into the definition of $m_0(t_b)$ in Equation (5.42d) results in

$$\begin{aligned} m_0(t_b) &= \int_S \frac{\partial \varphi_0}{\partial n} dS = \int_S \frac{\partial \varphi}{\partial n} dS + O(\varepsilon) \\ &= \int_S \nabla_b (f_0(t_b) + \varepsilon f_1(t_b) + \varepsilon g_1(t_b)z_b + \phi) \cdot \mathbf{n} + O(\varepsilon) \\ &= \int_S \frac{\partial \phi}{\partial n} dS + O(\varepsilon) \end{aligned} \quad (5.57)$$

Substituting Equation (5.57) and Equation (5.42) into Equation (5.54) yields

$$\begin{aligned} \frac{d\phi}{dt_b} &= 1 + \frac{1}{2} |\nabla_b \phi|^2 - \left(p_v + p_g - \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right) \\ &\quad + b \sin(\omega t_b) \cos(\theta_0) + \varepsilon m_0''(t_b) \\ &\quad - \varepsilon b \omega^2 \sin(\omega t_b) \sin(\theta_0) z_b + O(\varepsilon^2) \end{aligned} \quad (5.58a)$$

$$\begin{aligned} \frac{d}{dt_b} (\phi - \varepsilon m_0'(t_b)) &= 1 + \frac{1}{2} |\nabla_b \phi|^2 - \left(p_v + p_g - \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right) \\ &\quad + b \sin(\omega t_b) \cos(\theta_0) - \varepsilon b \omega^2 \sin(\omega t_b) \sin(\theta_0) z_b + O(\varepsilon^2) \end{aligned} \quad (5.58b)$$

The original term $\varepsilon m_0''(t_b)$ in (5.58a) is moved to the left-hand side as

shown in Equation (5.58b) to avoid the calculation of the second-order derivative of $m_0(t_b)$ with respect to time. Compared with the incompressible flow modeling, three additional terms appear in (5.58a) for the dynamic condition on the bubble surface:

The first term, $b\sin(\omega t_b)\cos(\theta_0)$, is the local first-order spherical wave pressure at the bubble centre.

The second term, $\varepsilon m_0''(t_b)$, is associated with the second-order outgoing wave due to the oscillations of the bubble. The first two terms yield only spherical wave field effects.

The third term, $-\varepsilon b\omega^2 \sin(\omega t_b)\sin(\theta_0)z_b$, is associated with the well known Bjerknes force acting on the bubble. The Bjerknes force is the resultant force due to the pressure gradient acting over the bubble, which is always believed to break the spherical symmetry of the bubble. As the bubble size is much smaller than the acoustic wavelength, the local pressure in the bubble center can be used to replace the pressure on the bubble surface. The Bjerknes force acting on the bubble to the first-order is given as,

$$\begin{aligned}
F_B &= \oint_S p_a \mathbf{n} dS \\
&= \oint_S \left(\frac{b}{2} \omega \sin(\omega t_b + kz_b + \theta_0) + \frac{b}{2} \omega \sin(\omega t_b - kz_b - \theta_0) \right) \mathbf{n} dS \\
&= b\omega \oint_S \left(\sin(\omega t_b)\cos(\theta_0) - \omega \varepsilon z_b \sin(\omega t_b) \sin(\theta_0) + O(\varepsilon^2) \right) \mathbf{n} dS
\end{aligned} \tag{5.59}$$

Using the Gauss divergence theorem, $\iiint_V (\nabla \cdot \mathbf{F}) dV = \oint_S \mathbf{F} \cdot \mathbf{n} dS$, the Bjerknes force becomes:

$$\begin{aligned}
F_B &= b\omega \int_V \nabla \left(\sin(\omega t_b)\cos(\theta_0) - \omega \varepsilon z_b \sin(\omega t_b) \sin(\theta_0) + O(\varepsilon^2) \right) dV
\end{aligned}$$

$$\begin{aligned}
&= b\omega \int_V (-\omega\varepsilon \sin(\omega t_b) \sin(\theta_0)) \mathbf{k} dV \\
&= -V \varepsilon b \omega^2 \sin(\omega t_b) \sin(\theta_0) \mathbf{k}
\end{aligned} \tag{5.60}$$

which is associated with the third additional term in (5.58a).

5.4 Numerical Model Description

In this study, a MEL-BIM model is employed to simulate the bubble dynamics near a solid boundary. Due to the axisymmetric character of this potential problem, the dimensionality for the solution is reduced from original 3-D to integral 2-D through the rotation of the semi-cross-section in a cylindrical polar coordinate. Furthermore, the problem is reduced further to 1-D using the Boundary Integral Method (BIM) by integrating through the polar angle, as shown in Figure 5.1.

(a) Initial 3-D problem

(b) Final 1-D solution

Figure 5.1 Reduction of dimensionalities of the axisymmetric potential problem.

5.5 Boundary Integral Method

As previously presented, the velocity potential ϕ of the liquid near the bubble surface satisfies Laplace's Equation (5.52). When the bubble has a piecewise smooth surface S , a solution may be represented in terms of a boundary integral

equation, which is developed from the Green's second identity (Blake 1985):

$$c(\mathbf{p})\phi(\mathbf{p}) + \int_S \phi(\mathbf{q}) \frac{\partial}{\partial n} \left(\frac{1}{|\mathbf{p}-\mathbf{q}|} \right) dS = \int_S \frac{\partial \phi(\mathbf{q})}{\partial n} \frac{1}{|\mathbf{p}-\mathbf{q}|} dS \quad (5.61)$$

where $\mathbf{p} \in \Omega + S$, $\mathbf{q} \in S$, $\frac{\partial}{\partial n}$ is the normal derivative outward from S , and $c(\mathbf{p}) = \begin{cases} 4\pi, & \text{if } \mathbf{p} \in \Omega \\ 2\pi, & \text{if } \mathbf{p} \in S \end{cases}$.

$\mathbf{p}$ on S is chosen to yield an equation for either ϕ or $\frac{\partial \phi}{\partial n}$ on S if the other is specified. Once both are known on S , Equation (5.61) can be used to generate ϕ at any point in the flow field. In axisymmetric problems, ϕ and $\frac{\partial \phi}{\partial n}$ are independent of the rotational angle, and the integrals in Equation (5.61) can be performed analytically in the azimuthal angle θ . A one-dimensional boundary integral equation on the intersection curve C of the bubble surface at the plane $\theta = 0$ is represented as

$$2\pi\phi(\mathbf{p}) = \int_C \left(\frac{\partial \phi(\mathbf{q})}{\partial n} K_1 - \phi(\mathbf{q}) K_2 \right) dl(\xi) \quad (5.62)$$

where the intersection curve C is discretized as n linear elements using $n + 1$ nodes, which is taken as $N = 60$ in this calculation. These elements are parameterized by arc length ξ . Then a set of general linear algebraic equations are derived, which relate the potential ϕ and its normal derivative $\frac{\partial \phi}{\partial n}$ on the bubble surface through two kernel functions K_1 and K_2 . The kernel functions K_1 and K_2 are given as follows, and their detailed derivations are presented in Appendix A2.

$$K_1 = \frac{4r(\xi)K(k)J(\xi)}{D(\xi)} \quad (5.63)$$

$$K_2 = \frac{-4r(\xi)}{D(\xi)^3} \left\{ \left[\frac{dz(\xi)}{d\xi} (r(\xi) + r_0) - \frac{dr(\xi)}{d\xi} (z(\xi) - z_0) - \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \right] \frac{E(k)}{1 - k^2(\xi)} + \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 K(k) \right\} \quad (5.64)$$

where

$$J(\xi) = \left[\left(\frac{dz(\xi)}{d\xi} \right)^2 + \left(\frac{dr(\xi)}{d\xi} \right)^2 \right]^{\frac{1}{2}} \quad (5.65a)$$

$$D(\xi) = [(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2]^{\frac{1}{2}} \quad (5.65b)$$

$K(k), E(k)$ are the complete elliptic integrals of the first and second kind, which can be obtained from the approximations in Hastings (1955):

$$K(k) = P(x) - Q(x) \ln(x) \quad (5.66a)$$

$$E(k) = R(x) - S(x) \ln(x) \quad (5.66b)$$

where

$$x = 1 - k^2(\xi) \quad (5.67a)$$

$$k^2(\xi) = \frac{4r(\xi)r_0}{(r(\xi)+r_0)^2+(z(\xi)-z_0)^2} \quad (5.67b)$$

and P, Q, R and S are tabulated polynomials.

5.6 Initial Condition

5.6.1 Vapor bubble

This type of bubble is filled with only saturated vapor ($p_g = 0$ and p_v is always negligible), and starts with a very small radius of R_0 . The velocity potential on the bubble surface is given by Blake and Gibson (1981) as

$$\phi_0(R_0, t_0) = -R_0 \left\{ \frac{2}{3} \left[\left(\frac{1}{R_0} \right)^3 - 1 \right] \right\}^{\frac{1}{2}} \quad (5.68)$$

Here, the initial time t_0 is the time it takes from inception (a tiny point) to the start state with radius R_0 , which can be expressed in terms of an incomplete Beta function:

$$t_0 = 3 \left(\frac{3}{2} \right)^{\frac{1}{2}} B_a \left(\frac{5}{6}, \frac{3}{2} \right), \quad a = (R_0)^3 \quad (5.69)$$

5.6.2 Gas bubble

When the bubble contains not only the saturated vapor, but also some quantity of contaminant gas, the thermodynamics of the bubble is quite different from that of the pure vapor bubble. If there is no appreciable mass transfer of gas to or from the liquid, the gas pressure inside the bubble follows this formula:

$$p_g = p_{g0} \left(\frac{R_0}{R} \right)^{3\lambda} \quad (5.70)$$

where λ is the ratio of the specific heats of the gas content. Clearly $\lambda = 1$ implies an isothermal behavior, and $\lambda > 1$ models adiabatic behaviors. It should be understood that the accurate evaluation of the behavior of the gas inside the bubble requires the solution of the mass, momentum, and energy equations, which is quite complicated. In this study, only a simple thermodynamic correlation is considered as shown in Equation (5.70). For the initial condition on the gas bubble surface, the generalized Rayleigh-Plesset equation is dominant:

$$R_0 \ddot{R}_0 + \frac{3}{2} \dot{R}_0^2 = -1 + p_{v0} + p_{g0} - \frac{2\sigma}{R_0} \quad (5.71)$$

Once the initial gas pressure inside the bubble is given, the initial bubble radius R_0 can be calculated using Equation (5.71). The initial velocity potential on the bubble surface is then given as:

$$\phi_0(R_0, t_0) = -R_0 \dot{R}_0 \quad (5.72)$$

5.7 Influence of Solid Boundary

To simulate the expansion and collapse of a cavitation bubble near a solid boundary, the fluid is modeled as inviscid and irrotational. A series of equations from Equation (5.52) to Equation (5.58) for the bubble dynamics in a compressible fluid is introduced in the previous sections. In addition to the boundary conditions on the bubble surface and at infinity, another boundary condition caused by the solid boundary is applied, which requires

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = 0 \quad (5.73)$$

The expansion phase is started from a very small spherical bubble of radius R_0 , with the velocity potential on the bubble surface given by

$$\phi_0(R_0, t_0) = \phi_0^R(R_0, t_0) \left(1 + \frac{R_0}{[(z+\gamma)^2 + r^2]^{\frac{1}{2}}} \right) \quad (5.74)$$

where $\phi_0^R(R_0, t_0)$ is the initial velocity potential on the bubble surface without solid boundary, and r and z are the coordinates of the bubble surface. It should be

noted that the coordinate system here is different from the moving one, as the $z = 0$ axis does not pass the bubble centroid, but is now fixed again on the solid boundary. An inverse transform of coordinates should be taken. The non-dimensional variable γ is defined as

$$\gamma = \frac{ds}{R_{max}} \quad (5.75)$$

where ds is the initial distance between the bubble inception and the solid boundary. The second term in the parentheses of (5.74) represents the correction to the initial velocity potential on the bubble surface due to the existence of the solid boundary.

The condition of no flow through the solid boundary is incorporated into the calculation using an image bubble about the solid boundary (see Figure 5.2).

$$\begin{aligned} c(\mathbf{p})\phi(\mathbf{p}) + \int_S \phi(\mathbf{q}) \frac{\partial}{\partial n} \left(\frac{1}{|\mathbf{p}-\mathbf{q}|} + \frac{1}{|\mathbf{p}-\hat{\mathbf{q}}|} \right) dS \\ = \int_S \frac{\partial}{\partial n} (\phi(\mathbf{q})) \left(\frac{1}{|\mathbf{p}-\mathbf{q}|} + \frac{1}{|\mathbf{p}-\hat{\mathbf{q}}|} \right) dS \end{aligned} \quad (5.76)$$

where $\hat{\mathbf{q}}$ is the image point of $\mathbf{q}$ about the plane $z = 0$.

Figure 5.2 Geometry for the simulation of a cavitation bubble dynamics near a solid boundary.

5.8 Vortex Ring Model

In an asymmetric environment, a high-speed liquid-jet often forms during the bubble collapse, which subsequently impacts upon the opposite bubble wall. Once jet impact has penetrated the opposite bubble wall, the liquid domain is transformed from a singly-connected to a doubly-connected form. A circulation is then generated around the toroidal bubble, since a velocity potential jump occurs at the impact point. The velocity potential function ϕ in a doubly-connected flow field is not unique. In order to render the velocity potential ϕ unique, a vortex ring model (Wang et al.1996) is employed.

The vortex ring can be placed anywhere inside the bubble ring. In this study, it is placed at the location (a, c) of the cross section of the bubble (Figure 5.3b):

$$a = \frac{x_{c1} + x_{c2}}{2}, \quad c = \frac{z_{max} + z_{min}}{2} \quad (5.77a,b)$$

where z_{max} and z_{min} are the maximum and minimum values of the z-coordinates of the cross section, and x_{c1} and x_{c2} are the x-coordinates of the two intersections of the horizontal line $z = c$ and the cross section.

Figure 5.3 Transition of a bubble from singly-connected form to doubly-connected form: (a) Immediately before impact, (b) Immediately after impact.

The strength of the vortex ring is called the velocity circulation Γ , which is defined by the velocity integration along any closed curve C that threads through the torus. The velocity circulation is equal to the jump of the velocity potential ϕ across the contact points at the instant of impact.

$$\Gamma = \oint_C \nabla \phi \cdot d\mathbf{l} = \phi_{n+1} - \phi_1 \quad (5.78)$$

where ϕ_{n+1} and ϕ_1 are the velocity potentials at the impact point, and n is the number of elements. For the incompressible potential flow, the circulation Γ is invariable in time.

The velocity potential is now decomposed into two parts: the velocity potential of the vortex ring Φ_{vr} and a remnant velocity potential Φ :

$$\phi = f_0(t) + \varepsilon f_1(t) + \varepsilon g_1(t)r + \Phi + \Phi_{vr} \quad (5.79)$$

and

$$\Phi_{vr} = \Gamma \Phi_0(x, z, a, c) \quad (5.80)$$

Φ is continuous in the flow field. $\Phi_0(x, z, a, c)$ is the velocity potential due to a vortex ring of unit circulation strength inside the bubble at the cross section in the $x - z$ plane. The corresponding analytical solution of the velocity field $\mathbf{v}_0(x, z, a)$ for the unit vortex ring, which has a radius of a , can be obtained using the Biot-Savart law (Wang et al. 1996b). The velocity potential $\Phi_0(x, z, a, c)$ must vanish at infinity. Its value at a point s_j on the bubble surface is obtained as follows:

$$\Phi_0(x_j, z_j, a, c) = \Phi_0(x_1, z_1, a, c) + \int_0^{s_j} \mathbf{v}_0(x, z - c, a) d\mathbf{s} \quad (5.81)$$

where $\Phi_0(x_1, z_1, a, c)$ is the velocity potential at node 1, which can be obtained by the integration of $\mathbf{v}_0(x, z - c, a)$ along the z -axis from infinity to node 1, with the following result:

$$\Phi_0(x_1, z_1, a, c) = \int_{-\infty}^{z_1} \mathbf{v}_0(0, z - c, a) d\mathbf{s} = \frac{1}{2} \left(\frac{z_1 - c}{\sqrt{(z_1 - c)^2 + a^2}} + 1 \right) \quad (5.82)$$

The velocity of the vortex ring is given as $\mathbf{v}_{vr}(x, z, a) = \Gamma \mathbf{v}_0(x, z - c, a)$. Thus the velocity potential of the vortex ring satisfies Laplace's equation in the flow field and vanishes at infinity. Substituting Equation (5.79) into Equation (5.52), Equation (5.53), Equation (5.55) and Equation (5.58b) yields the boundary value problem for the toroidal bubble, and setting $\phi = \Phi + \Phi_{vr}$ yields

$$\nabla^2(\phi) = O(\varepsilon^2) \quad (5.83)$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}_{vr} + \nabla \Phi + O(\varepsilon^2) \quad \text{on } S \quad (5.84)$$

$$\frac{d}{dt}(\phi - \varepsilon m_0'(t)) = 1 + \frac{1}{2}|\mathbf{v}_{vr} + \nabla\phi|^2$$

$$\begin{aligned} & -\left(p_v + p_g - \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right) + b\sin(\omega t)\cos(\theta_0) \\ & -\varepsilon b\omega^2 \sin(\omega t)\sin(\theta_0)z + O(\varepsilon^2) \end{aligned} \quad (5.85)$$

$$\Phi|_{r \rightarrow \infty} = \frac{m_0(t)}{r} + O(\varepsilon^2) \quad (5.86)$$

where $\mathbf{v}_{vr} = \nabla\Phi_{vr}$ is the induced velocity due to the vortex ring.

However, as the bubble evolves and translates, the bubble surface may be too close to the vortex ring, resulting in the numerical instability. To avoid this instability, the vortex ring is relocated using Equation (5.77) when its minimum distance from the bubble surface is less than $0.02R_{eq}$, where R_{eq} is the equivalent bubble radius $R_{eq} = \frac{3}{4\pi}V^{1/3}$. When the relocation is performed, Equation (5.83) above is first used to find the total velocity potential, after which the following is obtained:

$$\phi = \Phi^{old} + \Phi_{vr}^{old} \quad (5.87)$$

where Φ_{vr}^{old} and Φ^{old} are the velocity potential of the vortex ring and the remnant velocity potential before relocation of the vortex ring. Since the total velocity potential remains unchanged after the relocation of the vortex ring, the following equation can be used to find the remnant velocity potential Φ^{new} after the relocation

$$\phi = \Phi^{new} + \Phi_{vr}^{new} \quad (5.88)$$

where Φ_{vr}^{new} is the velocity potential of the vortex ring after the relocation of the vortex ring using Equation (5.77).

5.9 Solution Procedures of MEL-BIM Model

A numerical model based on the mixed Eulerian-Lagrangian (MEL) method and the Boundary Integral Method (BIM) was constructed by Wang et al. (2010) for simulating bubble dynamics in a weakly compressible liquid. One of the principal

advantages of this MEL-BIM model is that the boundary mesh of the model follows the transient bubble surface. Thus, the bubble surface and the unknowns on the surface are direct solutions, unlike those obtained using interpolations in domain approaches.

5.9.1 Singly-connected phase

The velocity potential ϕ and the normal velocity $\partial\phi/\partial n$ on the bubble surface are related to each other as shown in Equation (5.61). The section curve C is then discretized to n linear segments using $n_1 = n + 1$ nodes, as shown in Figure 5.1(b). The velocity potential ϕ and the normal velocities $\partial\phi/\partial n$ are both assumed to be distributed linearly in each segment. The flow chart of the MEL-BIM model is shown in Figure 5.4.

Figure 5.4 Flow chart of the MEL-BIM model.

In each new time step, the new bubble surface S and the velocity potential ϕ on the surface are calculated from the solutions at the previous time step as follows:

$$\mathbf{r}(t + \Delta t) = \mathbf{r}(t) + \Delta t \cdot \nabla \phi + O(\Delta t^2) \quad (5.89)$$

and

$$\phi(\mathbf{r}(t + \Delta t), t + \Delta t) = \phi(\mathbf{r}(t), t) + \Delta t \cdot \frac{D\phi}{Dt} + O(\Delta t^2) \quad (5.90)$$

After the new bubble surface and velocity potential on the bubble surface are obtained, re-gridding of the bubble surface is carried out to keep the equalization of segment length for the stability of the solutions. A cubic spline interpolation method (Appendix A3) is then used to calculate the value at each new node.

The normal velocity $\partial\phi/\partial n$ on the bubble surface is obtained using the BIM, and the tangential velocity on the bubble surface can be obtained using the following difference approximation:

$$\frac{\partial\phi_j}{\partial\tau} = \frac{d_{j-1}^2\phi_{j+1} - [d_{j-1}^2 - d_j^2]\phi_j - d_j^2\phi_{j-1}}{d_j d_{j-1} (d_j + d_{j-1})}, j = 2, 3, \dots, n \quad (5.91)$$

This equation is obtained by fitting a quadratic through the three points

$$(-d_{j-1}, \phi_{j-1}), \quad (0, \phi_j), \quad (d_j, \phi_{j+1})$$

and taking the slope of the quadratic at the point j ($\partial\phi/\partial\tau = 0$ at the bottom and top points of the bubble (node 1 and node n)).

$$d_j = [(z_{j+1} - z_j)^2 + (r_{j+1} - r_j)^2]^{\frac{1}{2}} \quad (5.92)$$

is the length of each segment. Once the normal and tangential velocities on the bubble surface are obtained, the new position of each node can be obtained using Equation (5.89).

$D\phi/Dt$ in Equation (5.90) can be calculated using Equation (5.58b). The function $m_0(t)$ is obtained by the integration of the known normal velocity distribution $\partial\phi/\partial n$ over the bubble surface as shown in Equation (5.57). Its derivative to time $m_0'(t)$ needed in Equation (5.58b) is calculated using a least-square method for stability. The velocity potential on the new bubble surface can then be obtained using Equation (5.90).

The choice of a suitable time step Δt in Equation (5.89) and Equation (5.90) is quite important. If the time step is too long, the updates of the solutions may not be accurate enough due to the non-linear character of the problem. However, if the time step is too short, the calculation time may be prodigious. A relationship given by Gibson and Blake (1982) is

$$\Delta t = \frac{\Delta\phi}{\max_S (|D\phi/Dt|)} \quad (5.93)$$

where $\Delta\phi$ is taken as some constant, and the maximum value defined in the denominator is obtained over all nodes on the bubble surface.

For the first time step, values for ϕ are obtained using the initial conditions given by Equation (5.68) or Equation (5.72). After successive iteration of the procedures given above, the bubble shape and velocity potential on the bubble surface can be calculated at any selected time point. In this study, due to the existence of the solid boundary, the initial condition given in Equation (5.74) and Green's integral formula given in Equation (5.76) are employed.

As is known, the bubble surface may lose smoothness in the MEL-BIM simulations, in which small-scale numerical errors develop rapidly into a sawtooth-like instability (Longuet-Higgins & Cokelet 1976; Blake et al. 1986, 1987). The instability profile depends on meshing and is thus a numerical instability. A five-point smoothing formula reported by Longuet-Higgins & Cokelet (1976) is the most well-known technique to remedy this problem. This smoothing is carried out about once every 5 time steps. The numerical error due to the smoothing is of the second order of small quantities in terms of the mesh size, and has only a very small effect on the solution.

5.9.2 Doubly-connected phase

The numerical transformation of a singly-connected bubble to a toroidal one is carried out by removing nodes 1 and $n + 1$, corresponding to the jet impact point. This transformation is performed when the distance between these two nodes is less than about 0.02 times the equivalent bubble radius. The bubble surface and the remnant velocity potential are re-interpolated into its doubly-connected shape and re-discretized using cubic splines (see Figure 5.3). After the jet impact, the governing equations of the problem should use Equation (5.83) to Equation (5.86), following which the numerical procedures will be quite similar to those of the singly-connected case.

As the bubble evolves in the toroidal form, the liquid jet may become very narrow and the bubble has a tendency to rejoin. Best (1994) and Lee et al. (2007) modelled the rejoining of an axisymmetric bubble, which took place on the axis of symmetry. In this study, the rejoining is performed when the distance between the

nearest node on the bubble surface and the axis of symmetry is less than 0.02 times the equivalent bubble radius. In the rejoining process, the nearest node and one of its neighbours, either the node above or below it (whichever is closer to the axis), are placed on the axis of symmetry (see Figure 5.5). The total velocity potential remains constant at these two nodes for such small displacements.

Figure 5.5 Transition from toroidal phase to singly-connected phase: (a) Immediately before rejoining; (b) Immediately after rejoining.

After the rejoining, the vortex ring is taken away and its velocity potential is added to the total velocity potential. A doubly-connected bubble is now transformed into a singly-connected one.

5.10 Flow Field Modeling

In order to describe the flow field dynamics adjacent to the bubble surface, the velocity potential, velocity and pressure in the flow field should also be calculated. A fixed grid is employed to fill the research region, including the bubble and sufficient ambient flow field (see Figure 5.6). The physical quantities at these grid points can be calculated using the difference method and BIM method. The grid points inside the bubble or exactly on the bubble surface are not taken into calculation. In addition, the grid points outside but quite near the bubble surface should be treated specially due to the numerical instability.

Figure 5.6 The calculation region filled with fixed grid points.

5.10.1 Difference method

As the potential flow near the bubble surface satisfies Laplace's Equation (5.52), the velocity potential in the flow field can be calculated using Green's integral formula as shown in Equation (5.61), in which $c(\mathbf{r}, t) = 4\pi$ is used

$$\phi(\mathbf{r}, t) = \frac{1}{4\pi} \int_S \left(\frac{\partial \phi(\mathbf{q}, t)}{\partial n} G(\mathbf{r}, \mathbf{q}) - \phi(\mathbf{q}, t) \frac{\partial G(\mathbf{r}, \mathbf{q})}{\partial n} \right) dS(\mathbf{q}) \quad (5.94)$$

$\mathbf{r}$ and $\mathbf{q}$ are the field point and source point respectively. The velocity in the flow field can then be calculated through the difference equations:

$$\mathbf{v}(\mathbf{r}, t) = \frac{\phi(\mathbf{r} + \Delta \mathbf{r}, t) - \phi(\mathbf{r} - \Delta \mathbf{r}, t)}{2\Delta \mathbf{r}} \quad (5.95)$$

The partial derivative of the velocity potential with respect to time, $\partial \phi / \partial t$, in the flow field is given as

$$\frac{\partial \phi}{\partial t} = \frac{\phi(\mathbf{r}, t) - \phi(\mathbf{r}, t - \Delta t)}{\Delta t} \quad (5.96)$$

Based on Equation (5.95) and Equation (5.96) above, the pressure in the flow field can be obtained using the Bernoulli Equation (5.29):

$$p = 1 - \frac{\partial \phi}{\partial t} - \frac{1}{2} |\mathbf{v}|^2 - p_a + O(\varepsilon^2) \quad (5.97)$$

5.10.2 BIM method

However, as is known, the difference method for the solution of non-linear problems is not always very accurate. A BIM method is therefore presented here for the solution of the non-linear flow field dynamics. The velocity potential near the bubble surface satisfies Laplace's equation:

$$\nabla^2 \phi = O(\varepsilon^2) \quad (5.98)$$

After making a partial derivative with respect to the time, it becomes

$$\frac{\partial}{\partial t} (\nabla^2 \phi) = \nabla^2 \frac{\partial \phi}{\partial t} = O(\varepsilon^2) \quad (5.99)$$

Setting $\theta = \frac{\partial \phi}{\partial t}$, yields

$$\nabla^2 \theta = O(\varepsilon^2) \quad (5.100)$$

The partial derivative of the velocity potential with respect to the time on the bubble surface can be calculated using the Bernoulli Equation (5.29):

$$\theta = -p_L - \frac{1}{2} |\mathbf{v}|^2 - p_a + O(\varepsilon^2) \quad (5.101)$$

Its normal derivative can be calculated using the BIM:

$$2\pi\theta(\mathbf{r}, t) = \int_S \left(\frac{\partial \theta(\mathbf{q}, t)}{\partial n} G(\mathbf{r}, \mathbf{q}) - \theta(\mathbf{q}, t) \frac{\partial G(\mathbf{r}, \mathbf{q})}{\partial n} \right) dS(\mathbf{q}) \quad (5.102)$$

The partial derivative of the velocity potential with respect to the time in the flow field can then be obtained using

$$\theta(\mathbf{r}, t) = \frac{1}{4\pi} \int_S \left(\frac{\partial \theta(\mathbf{q}, t)}{\partial n} G(\mathbf{r}, \mathbf{q}) - \theta(\mathbf{q}, t) \frac{\partial G(\mathbf{r}, \mathbf{q})}{\partial n} \right) dS(\mathbf{q}) \quad (5.103)$$

Finally, the pressure in the flow field can be obtained again using Bernoulli Equation (5.29):

$$p = 1 - \theta - \frac{1}{2} |\mathbf{v}|^2 - p_a + O(\varepsilon^2) \quad (5.104)$$

where $\mathbf{v}$ is obtained using the difference method in Equation (5.95).

5.11 Summary

In this chapter, a systematic derivation of the mathematical formulations for describing the cavitation bubble dynamics near a solid boundary is presented. Based on the potential flow theory, the governing equations are developed. After the non-dimensionalization, the method of matched asymptotic expansions is employed to incorporate the compressibility of the liquid. The velocity potential near the bubble surface satisfies Laplace's equation, which can be solved using the BIM. Due to the existence of a solid boundary, an image bubble is introduced to deal with the asymmetry of the flow field. The vortex ring model is adopted for the calculation of the toroidal bubble dynamics after the jet impact. Two methods are presented for the calculation of the flow field dynamics near the bubble surface using a fixed grid. These methods are: (1) the difference method, and (2) the BIM method.

CHAPTER 6

SIMULATION RESULTS: VERIFICATION AND COMPARISON WITH EXPERIMENTAL RESULTS

6.1 Comparison with Analytical Solution

6.1.1 Rayleigh-Plesset equation

The Rayleigh bubble is a single spherical bubble expanding and collapsing in an infinite, incompressible, inviscid and irrotational liquid. The generalized Rayleigh-Plesset equation (Rayleigh 1917) for the spherical bubble dynamics is:

$$\frac{\bar{p}_L - \bar{p}_\infty}{\bar{\rho}_\infty} = \bar{R}(t) \frac{d^2 \bar{R}(t)}{d\bar{t}^2} + \frac{3}{2} \left(\frac{d\bar{R}(t)}{d\bar{t}} \right)^2 \quad (6.1)$$

After non-dimensionalization, it becomes

$$p_L = R(t) \frac{d^2 R(t)}{dt^2} + \frac{3}{2} \left(\frac{dR(t)}{dt} \right)^2 \quad (6.2)$$

For a pure vapor bubble, the liquid pressure on the bubble surface is given as $p_L = -1 + \left(p_v - \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right)$. If the vapor pressure p_v and surface tension σ are neglected, $p_L = -1$. Substituting this into Equation (6.61) yields

$$-1 = R(t) \frac{d^2 R(t)}{dt^2} + \frac{3}{2} \left(\frac{dR(t)}{dt} \right)^2 \quad (6.3)$$

An analytical solution for this equation is

$$\left(\frac{dR(t)}{dt} \right)^2 = \frac{2}{3} \left(\frac{1}{R(t)} \right)^3 - \frac{2}{3} \quad (6.4)$$

For a pure vapour bubble growing from a non-dimensional initial radius of $R_0 = 0.1$ at $t_0 = 0.001553$, calculated using Equation (5.69), the initial velocity potential on the bubble surface can be obtained using Equation (5.68), i.e.

$$\phi_0 = 2.5806796.$$

Following this, the numerical simulation results obtained using the MEL-BIM model are compared with the analytical solutions of the Rayleigh-Plesset equation for the relations between the bubble radial velocity and bubble radius, during both the expansion and collapse phase, as shown in Figure 6.1. It can be observed that they agree quite well.

Figure 6.1 Comparison between the numerical results obtained using the MEL-BIM model and the analytical solutions of the Rayleigh-Plesset equation for the radial velocity versus radius during both the expansion and collapse phase for a spherical pure vapor bubble.

In the numerical simulation using the MEL-BIM model, the bubble reaches the maximum radius at the non-dimensional time 0.918710, which is in good agreement with the result of Taib (1985), i.e. 0.919478. Instability in the radial velocity starts to occur during the collapse phase at the non-dimensional time 1.826251 in the simulation using the MEL-BIM model, and the exact life time of a Rayleigh bubble is 1.82936.

To obtain the bubble radial velocity and bubble radius versus the time, the second-order ordinary differential Equation (6.2) can be integrated using a variable

step Runge-Kutta Method. The comparisons between the numerical simulations using the MEL-BIM model and the solutions of the Raylei-Plesset equation are shown in Figure 6.2. It can be observed that the results from using these two different approaches agree quite well.

(a)

(b)

Figure 6.2 Comparisons between the numerical results obtained using the

MEL-BIM model and the solutions of the Rayleigh-Plesset equation for a spherical pure vapor bubble dynamics: (a) bubble radius, (b) bubble radial velocity.

6.1.2 Keller-Herring equation

Prosperetti and Lezzi (1986) presented the radial dynamics of a spherical bubble in a compressible liquid by means of a simplified singular-perturbation method to the first order in the bubble-wall Mach number. They gave a general Keller-Herring equation (KHE) formulation, as follows:

$$\begin{aligned} \left(1 - (1 + \kappa)\varepsilon \frac{dR(t)}{dt}\right) R(t) \frac{d^2 R(t)}{dt^2} + \frac{3}{2} \left(1 - \left(\frac{1}{3} + \kappa\right) \varepsilon \frac{dR(t)}{dt}\right) \left(\frac{dR(t)}{dt}\right)^2 \\ = \left(1 + (1 - \kappa)\varepsilon \frac{dR(t)}{dt}\right) h_L + \varepsilon R \frac{dh_L}{dt} \end{aligned} \quad (6.5)$$

where κ is an arbitrary parameter. The results of the KHE are insensitive to the parameter κ . In this study, κ is set at $\kappa = 0$ (the Keller form). If setting $\varepsilon \rightarrow 0$, Equation (6.5) will be reduced to the Rayleigh-Plesset equation [(6.1) above].

Considering that a bubble contains not only the saturated vapor, but also some quantity of non-condensable contaminant gas, the liquid pressure on the bubble surface is given as

$$h_L = p_L = -1 + \left(p_v + p_{g0} \left(\frac{R_0}{R}\right)^{3\lambda} - \frac{2\sigma}{R}\right) \quad (6.6)$$

A second-order ordinary differential equation for the calculation of a spherical gas bubble dynamics in a compressible liquid can then be obtained:

$$\begin{aligned} \left(1 - \varepsilon \frac{dR(t)}{dt}\right) R(t) \frac{d^2 R(t)}{dt^2} + \frac{3}{2} \left(1 - \frac{1}{3} \varepsilon \frac{dR(t)}{dt}\right) \left(\frac{dR(t)}{dt}\right)^2 \\ = \left(1 + \varepsilon \frac{dR(t)}{dt}\right) \left(-1 + p_{g0} \left(\frac{R_0}{R(t)}\right)^{3\lambda} - \frac{2\sigma}{R(t)}\right) \\ + \varepsilon R(t) \left(-\frac{3\lambda}{R(t)} p_{g0} \left(\frac{R_0}{R(t)}\right)^{3\lambda} \frac{dR(t)}{dt} + \frac{2\sigma}{R(t)^2} \frac{dR(t)}{dt}\right) \end{aligned} \quad (6.7)$$

The surface tension coefficient σ is taken as $\sigma = \frac{\bar{\sigma}}{\bar{R}_{max}\bar{p}_{\infty}}$ and $\bar{\sigma} = 0.0728\text{N/m}$ (for 20 °C water). The liquid surrounding the bubble is initially at rest. The

maximum expanded radius $\bar{R}_{max}$ is 1.0mm. The bubble expands from a starting radius of $R_0 = 0.1$, and a fairly high gas pressure inside the bubble of $p_{g0} = 425.0$. The vapour pressure p_v is neglected. The whole process of bubble evolution is assumed to be adiabatic and the ratio of the specific heats of the gas content is assumed to be $\lambda = 1.4$. A fourth-order Runge-Kutta method is used to integrate this equation accurately.

Figure 6.3 shows the evolutions of the gas bubble radius and the radial velocity versus time, using different methods for both the compressible and incompressible cases. It can be observed that the first maximum expanded bubble radius in the compressible liquid cannot reach 1.0, only about 0.94. Furthermore, during the following oscillation cycles, the maximum expanded radii become smaller and smaller. This phenomenon is caused by the dissipation of the bubble energy that results from the compressibility of the liquid. The differences between the results obtained using the MEL-BIM model and the KHE model become more and more obvious after the second oscillation. A possible reason is that the introduction of the compressibility parameter aggravates the numerical errors using the MEL-BIM model.

(a)

(b)

Figure 6.3 Comparisons between numerical results obtained using the MEL-BIM model and the KHE model for a spherical gas bubble dynamics in a compressible liquid; and the Rayleigh-Plesset equation is solved for the incompressible case: (a) bubble radius; (b) bubble radial velocity.

In a coordinate with the origin located at the bubble centre, the flow field dynamics at a fixed point (1.0, 1.0) are calculated as shown in Figure 6.4. The numerical results obtained using different methods are compared, and it is found that the coincidence is quite good between the results obtained using the MEL-BIM model and the KHE model for the compressible case.

(a) The velocity potential: $\phi = -\frac{R^2 \dot{R}}{r}$ (6.8)

(b) The radial velocity: $\phi_r = \frac{R^2 \dot{R}}{r^2}$ (6.9)

(c) The partial derivative of the velocity potential with respect to the time:

$$\phi_t = -\frac{2R\dot{R}^2 + R^2\ddot{R}}{r} \quad (6.10)$$

(d) The liquid pressure: $p = 1 - \phi_t - \frac{1}{2}|\phi_r|^2$ (6.11)

Figure 6.4 Comparisons between the numerical results obtained using the MEL-BIM model and the KHE model for the flow field dynamics at the point $r = (1.0, 1.0)$ near a spherical gas bubble in a compressible liquid; and the Rayleigh-Plesset equation solved for the incompressible case: (a) the velocity potential; (b) the radial velocity; (c) the partial derivative of the velocity potential with respect to the time; (d) the liquid pressure.

6.2 Comparison with Previous Non-spherical Bubble Simulation

In this section, comparisons are made between the simulation results obtained using the MEL-BIM model and those reported by Taib (1985) on the expansion and collapse of a vapor bubble in an incompressible liquid near a solid boundary. The bubble shape, movement of the bubble centroid, and jet velocity as a function of time for different values of γ are compared.

6.2.1 Bubble shape

The bubble shapes at selected non-dimensional time points obtained using the MEL-BIM model are shown as follows for $\gamma = 2.0$, 1.5 and 1.0 respectively. The selected time points in this report are not very consistent with Taib's as the time step chosen in this MEL-BIM model is much smaller than Taib's. Nevertheless, the evolutions of the bubble shapes are quite similar in these two results.

i, The case $\gamma = 2.0$

During the expansion phase, the bubble expands almost spherically. During the collapse phase, it becomes slightly elongated in the direction normal to the solid boundary. As the collapse progresses, the upper part of the bubble becomes flattened and a liquid jet forms (see Figure 6.5). In the simulation using the MEL-BIM model, the end time of the expansion phase is 0.996881 (0.998769 in Taib's), and the impact time of the liquid jet to the opposite bubble wall is about 2.020418 (2.034596 in Taib's). It can be observed that the results agree quite well.

(a) Expansion phase (MEL-BIM model) (b) Collapse phase (MEL-BIM model)

(c) Expansion phase (Taib's)

(d) Collapse phase (Taib's)

Figure 6.5 Bubble shapes for $\gamma = 2.0$ at non-dimensional time points: (a) A, 0.001568 B, 0.010659 C, 0.038616 D, 0.152887 E, 0.318887 F, 0.982882; (b) A, 0.986882 B, 1.728872 C, 1.959423 D, 1.988294 E, 2.000056 F, 2.008774 G, 2.014940 H, 2.020356; (c) A, 0.001553 B, 0.018112 C, 0.051295 D, 0.135380 E, 0.321236 F, 0.998769; (d) A, 0.998769 B, 1.725801 C, 1.882079 D, 1.951385 E, 1.993804 F, 2.010326 G, 2.204469 H, 2.034596.

ii, The case $\gamma = 1.5$

Similar to that of the previous case, the bubble expands almost spherically, and becomes slightly elongated in the direction normal to the solid boundary. Late in the collapse phase, the upper part of the bubble becomes flattened and a liquid jet forms. The liquid jet in this case is obviously narrower than that in the previous case (see Figure 6.6). The end time of the expansion phase obtained using the MEL-BIM model is 1.023227 (1.033903 in Taib's), and the impact time of this case is 2.080700 (2.096912 in Taib's). Hence the results are slightly different from Taib's.

(a) Expansion phase (MEL-BIM model) (b) Collapse phase (MEL-BIM model)

(c) Expansion phase (Taib's) (d) Collapse phase (Taib's)

Figure 6.6 Bubble shapes for $\gamma = 1.5$ at non-dimensional time points: (a) A, 0.001568 B, 0.010996 C, 0.041521 D, 0.163233 E, 0.501234 F, 1.009227; (b) A, 1.013227 B, 1.697219 C, 1.962858 D, 2.010814 E, 2.030537 F, 2.045956 G, 2.060229 H, 2.072150 I, 2.080308; (c) A, 0.001553 B, 0.024625 C, 0.085576 D, 0.209916 E, 0.474799 F, 1.033903; (d) A, 1.033903 B, 1.724544 C, 1.880317 D, 2.015098 E, 2.038946 F, 2.058315 G, 2.076982 H, 2.096912.

iii, The case $\gamma = 1.0$

The bubble expands spherically at the beginning, but as it migrates towards the solid boundary, the lower part of the bubble becomes flattened. During the collapse phase, the bubble also becomes elongated in the direction normal to the solid boundary. As the collapse progresses, a sharp liquid jet forms on the inner and upper part of the bubble (see Figure 6.6). It can be discerned that there is an intersection of the lower part of the bubble shape during the collapse phase, which appears in both the MEL-BIM results and Taib's results. The end time of the expansion phase obtained using the MEL-BIM model is 1.067394 (1.074744 in Taib's), and the impact time of this case is 2.155505 (2.164147 in Taib's).

(a) Expansion phase (MEL-BIM model) (b) Collapse phase (MEL-BIM model)

Figure 6.7 Bubble shape for $\gamma = 1.0$ at non-dimensional time points: (a) A, 0.001568 B, 0.011961 C, 0.047285 D, 0.183400 E, 0.357401 F, 1.053394; (b) A, 1.057394 B, 1.697386 C, 1.969265 D, 2.032540 E, 2.061008 F, 2.084526 G, 2.107443 H, 2.130362 I, 2.153747; (c) A, 0.001553 B, 0.009124 C, 0.050120 D, 0.142422 E, 0.319365 F, 1.074744; (d) A, 1.074744 B, 1.856425 C, 2.027375 D, 2.052816 E, 2.102190 F, 2.126404 G, 2.148998 H, 2.164146.

6.2.2 Centroid movement

Figure 6.8 shows the movement of the bubble centroid for the three different values of γ . There is a slight divergence from the solid boundary at the beginning of the expansion phase for all cases. After this, however, the bubble migrates towards the solid boundary, and more rapidly during the collapse phase. It can be observed that the results obtained using the MEL-BIM model match those given by Taib quite well.

Figure 6.8 Comparisons of the movements of the bubble centroid initiated at three different stand-off distances.

6.2.3 Jet velocity

Figure 6.9 shows the velocity of the upper part of the bubble at the axis of symmetry for the three different values of γ . It can be observed that the velocity decreases quite rapidly during the expansion phase. In the early collapse phase, the velocity increases slowly. After the liquid jet forms, the velocity increases rapidly. However, during the later stage of the collapse, the velocity remains almost uniform, which is quite clear for $\gamma = 1.0$. The jet velocity reaches 15.922405 (16.0 in Taib's) for $\gamma = 2.0$, 10.970025 (10.9 in Taib's) for $\gamma = 1.5$, and 8.137387 (8.5 in Taib's) for $\gamma = 2.0$ at the jet impact time. It can be discerned that the agreements between these results are quite good.

Figure 6.9 Comparisons of the velocities of the upper part of the bubble at the axis of symmetry initiated at three different stand-off distances.

6.3 Initial Conditions Set-up for Comparison with Experimental Results

In order to assess the applicability of the developed MEL-BIM model to real cases, the numerical simulation results should be compared with the measured experimental results. The most difficult problem before the comparisons are made is the set-up of initial conditions for the numerical simulations. In fact, the initial bubble radius, initial gas pressure inside the bubble, initial radial velocity of the bubble surface, and the ratio of the specific heats of the gas content are all desired. A possible approach to obtain these quantities is searching the existing literature for similar cases, but this may not always be successful. Another possible way to derive these quantities is using the analytical solutions of the spherical bubble dynamics in a compressible liquid. As is known, the initial phenomena of bubbles near and far away from the solid boundary are quite similar, so the KHE model can be solved to

obtain the initial conditions. The comparisons are made for a bubble with $\bar{R}_{max} = 1.5\text{mm}$. The surface tension coefficient for an air-water interface is $\bar{\sigma} = 0.072\text{kg/s}^2$. The density of the water is $\bar{\rho} = 1000\text{kg/m}^3$. The hydrostatic pressure at infinity is $\bar{p}_{\infty} = 98070\text{Pa}$, and the vapor pressure is $\bar{p}_v = 2980\text{Pa}$. The process of the bubble evolution is assumed to be adiabatic, and the derivations of the initial conditions using the KHE model are shown as follows:

(1) The ratio of the specific heats of the gas content λ depends on the degrees of the freedom (f) of the gas molecule, as $\lambda = 1 + \frac{2}{f}$. The gas content inside the bubble is presumed to be mostly made of oxygen and hydrogen which are initially contained or ionized by the laser pulse. These gases are all diatomic with 5 degrees of freedom (at room temperature: three translational and two rotational degrees of freedom). λ is taken to be 1.4.

(2) When the bubble is initially at the first maximum expanded radius $R_0 = 1.0$, the radial velocity of the bubble surface is presumed to be zero, and the gas pressure inside the bubble decides the second maximum rebounding radius. Figure 6.10 below shows the influence of the initial gas pressure inside the bubble on the evolution of the spherical bubble radius using the KHE model. From the experimental results previously shown in Figure 4.18, it can be discerned that the second maximum expanded equivalent bubble radii are mostly about 0.7 times that of the first maximum expanded radii. Therefore, the initial gas pressure inside the bubble is chosen as $p_{g0} = 0.008$ to cause the second maximum expanded radius to reach 0.7.

Figure 6.10 Comparisons of the evolutions of a spherical bubble radius with different initial gas pressures inside the bubble, solved using the KHE model. The bubble is initiated with the maximum expanded radius and zero radial velocity in a compressible liquid.

(3) The real bubble starts from a tiny point and the gas pressure inside the bubble is quite large. The initial radius is set to be 0.1 in this simulation. Based on the adiabatic process, the initial gas pressure inside the bubble can be calculated using $p_g = p_{g0} \left(\frac{R_0}{R} \right)^{3\lambda} = 0.08 \times \left(\frac{1.0}{0.1} \right)^{3 \times 1.4} = 126.8$. This gas pressure is substituted into the KHE model, and the initial radius is changed to 0.1. The last quantity needed is the initial radial velocity of the bubble surface, which should be estimated to keep the first maximum radius at 1.0 and the second maximum radius at 0.7. The initial radial velocity for a spherical vapor bubble with an initial radius of 0.1 can be calculated using Equation (6.4), which makes it 25.8. The radial velocity for the KHE model should then be slightly larger than this value due to the compressibility of the liquid, and it is found to be about 32.5.

Figure 6.11 Comparisons of the evolutions of a spherical bubble radius with different initial radial velocities, solved using the KHE model. The bubble is initiated with a radius of 0.1 and an initial gas pressure inside the bubble of 126.8 in a compressible liquid.

(4) The initial conditions ($R_0 = 0.1, p_{g0} = 126.8, V_{R0} = 32.5$ and $\lambda = 1.4$) obtained using the KHE model can now be substituted into the MEL-BIM model for the simulations of non-spherical bubble dynamics near a solid boundary. Due to the fairly complicated boundary conditions and instability of the numerical simulations, the initial conditions require constant refinements.

6.4 Simulated Bubble Shapes

6.4.1 Bubble evolutions until the end of second collapse

i, The case $\gamma = 2.1$

Figure 6.12 below shows the simulated bubble evolutions for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 2.1$. The simulation results are presented in a non-dimensional

coordinate. During the expansion phase, the bubble expands almost spherically. During the collapse phase, it becomes slightly elongated in the direction normal to the solid boundary. As the collapse progresses, the upper part of the bubble becomes flattened and a liquid jet forms. The evolution of the bubble shapes during the first oscillation is quite similar to the case previously presented in Figure 6.5, the only difference being the time period. Due to the existence of contaminant gas inside the bubble, the bubble continues to rebound in a toroidal form. During the second expansion phase, the bubble forms a shape like a pair of ears. The bubble then rejoins at some time point and continues to expand. After reaching the second maximum expanded volume, the bubble starts to collapse. The shoulders of the upper part of the bubble collapse faster than the central part and a projecture forms in the center. The central part of the upper part of the bubble then collapses quicker than the shoulders and finally impacts on the lower bubble wall. The lower part of the bubble starts to dent from the beginning of the second collapse phase. After the impact of the upper and lower bubble wall, the bubble continues to collapse in the doubly-connected form.

Figure 6.12 Bubble evolutions for a bubble with $\bar{R}_{max}= 1.5\text{mm}$ at $\gamma = 2.1$ during (a) first expansion phase at non-dimensional time points A, 0.00000 B, 0.02825 C, 0.09488 D, 0.18705 E, 0.27989 F, 0.46979 G, 0.93241; (b) first collapse phase at A, 1.15280 B, 1.47575 C, 1.68559 D, 1.89271 E, 1.96635 F, 1.99807; (c) second expansion phase in toroidal form at A, 2.00059 B, 2.00702 C, 2.02202 D, 2.04344 E,

2.06495 F, 2.10776 G, 2.15034 H, 2.21447; (d) continued expansion phase after rejoining at A, 2.21469 B, 2.23171 C, 2.27283 D, 2.38786 E, 2.78110; (e) second collapse phase in singly-connected form at A, 2.96106 B, 3.20410 C, 3.37131 D, 3.53567 E, 3.59385 F, 3.62743; (f) continued collapse phase in doubly-connected form at A, 3.62939 B, 3.64565 C, 3.65767 D, 3.66971 (amplified).

ii, The case $\gamma = 1.5$

Figure 6.13 below shows the simulated bubble evolutions for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 1.5$. The simulation results are presented in a non-dimensional coordinate. The bubble expands almost spherically, and becomes slightly elongated in the direction normal to the solid boundary. Late in the collapse phase, the upper part of the bubble becomes flattened and a tongue-like liquid jet forms. The time period of the first oscillation for this gas bubble is different from that of the vapor bubble case (see Figure 6.6). After the jet penetrates the opposite bubble wall, the bubble continues to collapse in the toroidal form and reaches a fairly small volume. The bubble then rebounds and forms a shape like a pair of ears. Due to the existence of the solid boundary, the lower part of the bubble touches the boundary during the second expansion phase. The bubble rejoins at some time point and continues to expand. During the second collapse phase, the shoulders of the upper part of the bubble collapse faster than the central part.

(a)

(b)

(c)

(d)

Figure 6.13 Bubble evolutions for a bubble with $\bar{R}_{max}= 1.5\text{mm}$ at $\gamma = 1.5$ during (a) first expansion phase at non-dimensional time points A, 0.00000 B, 0.02947 C, 0.09681 D, 0.19481 E, 0.29531 F, 0.48825 G, 0.96718; (b) first collapse phase at A, 1.19469 B, 1.52011 C, 1.73664 D, 1.95278 E, 2.02863 F, 2.06118; (c) continued collapse after jet impact at A, 2.06123 B, 2.06493 C, 2.06986 D, 2.07355 (amplified); (d) second expansion phase in toroidal form at A, 2.07356 B, 2.08550 C, 2.11332 D, 2.15317 E, 2.19280 F, 2.27217 G, 2.35143 H, 2.47034; (e) continued expansion phase after rejoining at A, 2.47073 B, 2.56905 C, 2.75536 D, 2.92667 E, 3.09557; (f) second collapse phase in singly-connected form at A, 3.10052 B, 3.41744 C, 3.62763 D, 3.83867 E, 4.04853 F, 4.15383.

iii, The case $\gamma = 1.0$

Figure 6.14 below shows the simulated bubble evolutions for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 1.0$. The simulation results are presented in a non-dimensional coordinate. The bubble expands spherically at the beginning, but as the bubble migrates towards the solid boundary, the lower part of the bubble becomes flattened. During the first collapse phase, the bubble becomes elongated in the direction normal to the solid boundary. As the collapse progresses, a sharp liquid jet forms on

the inner and upper part of the bubble. After the liquid jet penetrates the opposite bubble wall, the bubble collapses in a toroidal form and keeps this form throughout the second oscillation.

(e)

Figure 6.14 Bubble evolutions for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 1.0$ during (a) first expansion phase at non-dimensional time points A, 0.00000 B, 0.03025 C, 0.10215 D, 0.20309 E, 0.30497 F, 0.50594 G, 1.00255; (b) first collapse phase at A, 1.23504 B, 1.56830 C, 1.78930 D, 2.01085 E, 2.08853 F, 2.12185; (c) continued collapse after jet impact at A, 2.12206 B, 2.14569 C, 2.16125 D, 2.17693 E, 2.19259 F, 2.20043; (d) second expansion phase in toroidal form at A, 2.22897 B, 2.29568 C, 2.39070 D, 2.48575 E, 2.67752 F, 3.15127; (e) second collapse phase in doubly-connected form at A, 3.18332 B, 3.81061 C, 4.12738 D, 4.17781.

iv, The case $\gamma = 0.5$

Figure 6.15 below shows the simulated bubble evolutions for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 0.5$. The simulation results are presented in a non-dimensional coordinate. The bubble expands spherically at the beginning, but the lower part of the bubble becomes flattened during the early stage of the first expansion phase and touches the solid boundary before reaching the maximum expanded volume. After the liquid jet penetrates the opposite bubble wall and impacts on the solid boundary, the bubble continues to collapse in a toroidal form, and the space between the inner bubble surface becomes more and more wide. The bubble keeps the doubly-connected form during the expansion and collapse phase of the second oscillation.

Figure 6.15 Bubble evolutions for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 0.5$ during (a) first expansion phase at non-dimensional time points A, 0.00000 B, 0.03154 C, 0.10655 D, 0.21139 E, 0.31872 F, 0.52847 G,

1.04867; (b) first collapse phase at A, 1.27776 B, 1.60223 C, 1.82113 D, 2.03830 E, 2.11462 F, 2.14699; (c) continued collapse after jet impact at A, 2.14777 B, 2.17384 C, 2.21302 D, 2.23863 E, 2.26458 F, 2.27749; (d) second expansion phase in toroidal form at A, 2.30928 B, 2.36429 C, 2.52027 D, 2.67446 E, 3.06172; (e) second collapse phase in doubly-connected form at A, 3.08611 B, 3.56425 C, 3.77564 D, 3.85549 E, 3.88331.

6.4.2 Comparison with experimental results

Figure 6.16 below shows the comparisons between the numerical simulation results obtained using the MEL-BIM model and the experimental results captured using the high-speed camera for bubble shape evolutions near a solid boundary. The bubble has a maximum expanded radius of 1.5mm. In the case $\gamma = 2.4$, the bubble shapes agree quite well during the first oscillation period. After the jet impact, the bubble first rebounds like a pair of ears. Due to the invisibility of the liquid jet inside the bubble, only the outlines of the bubble shapes can be compared. After rejoining, a tail appears in the lower part of the bubble, which can be observed in both the experimental and numerical results. In the late stage of the second collapse phase, the tail disappears and a small liquid jet forms again on the inner and upper part of the bubble, and penetrates the opposite bubble wall for the second time. Once again, the bubble rebounds like a pair of ears, but the volume is much smaller than that during the second expansion phase.

For $\gamma = 1.6$, the results obtained by the experiment and the simulation also have a good agreement for the first oscillation. After the liquid jet penetrates the opposite bubble wall, the toroidal bubble first rebounds like a pair of ears, but due to the existence of the solid boundary, the lower part of the bubble touches the solid boundary during the expansion phase, and then the bubble rejoins and continues to expand. The counterjet appearing during the second expansion phase in the experimental results cannot be reflected in the numerical simulations. The counterjet is believed to be the result of back-flow of the liquid through the jet

tunnel inside the bubble, and not part of the bubble body. After the toroidal bubble rejoins, the jet tunnel disappears in the simulation results, and the counterjet simultaneously disappears in the experimental results. Although the rebound of the bubble during the second expansion phase is much larger in the simulation than that observed in the experiment, the bubble profiles are quite similar.

In the last case $\gamma = 0.6$, the difference between the experimental and numerical results is negligible for the first oscillation. During the second oscillation, the splash appearing on the top of the experimental bubble cannot be simulated using the numerical model. The volcano-like shape is obvious in both results.

(a) $\gamma = 2.4$

(b) $\gamma = 1.6$

(c) $\gamma = 0.6$

Figure 6.16 Comparisons of the shape evolutions between the experimental and simulation results for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at (a) $\gamma = 2.4$; (b) $\gamma = 1.6$; (c) $\gamma = 0.6$.

6.5 Equivalent Bubble Radius

Figure 6.17 below shows the histories of the equivalent bubble radius as obtained from the experiments and simulations for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 2.0$. The spherical gas bubble evolution calculated using the KHE model is employed for the reference. It can be observed that the numerical results agree quite well with the experimental results for the first bubble oscillation. However, during the second oscillation, the differences between the numerical and experimental results are quite obvious, for both the size and time period. For the difference between the experimental and numerical results after the jet impact, a probable explanation is proposed here. When the liquid jet impacts on the opposite bubble wall, a shock wave may be emitted (e.g. as reported by Lindau & Lauterborn 2003), which can be detected using the hydrophone system. The emission of the shock wave clearly dissipates some amount of bubble energy, which is not considered in this study. This explanation was also supported by Popinet & Zaleski (2002), who pointed out that the emission of shock waves needed to be introduced into the simulation.

Figure 6.17 Temporal evolution of the equivalent radius of a bubble with

$\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 2.0$, as obtained using the experimental measurement, the numerical simulation and the KHE model.

Figure 6.18 below shows the temporal evolutions of the equivalent radius of a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at different initial stand-off distances. It can be observed that when the stand-off distance decreases from 3.0 to 1.0, the second maximum expanded equivalent radius increases from about 1.0 mm to 1.4 mm. However, when the stand-off distance continues to decrease to 0.5, the value of the second maximum expanded equivalent radius decreases to about 1.2 mm, and the simulated bubble loses stability shortly after the second maximum expanded volume. The simulation results of the second maximum expanded equivalent radius have a similar trend to that of the experimental results given previously in Figure 4.18, which indicates that the peak value of the second maximum expanded equivalent radius appears to be around $\gamma = 0.8$.

Figure 6.18 Temporal evolutions of the equivalent radius of a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at different initial stand-off distances.

6.6 Bubble Centroid Position

Figure 6.19 below shows another important comparison between the experimental and simulation results: the movement of the bubble centroid near the solid boundary. The symbols represent the experimental results while the solid lines represent the numerical simulation results. Although there are some deviations between these two results, the general trends of the bubble centroid movement are quite similar. It can be observed that when a bubble is generated closer to the solid boundary, the bubble moves further away from the solid boundary in the early stage of the first expansion phase; this can be observed in both the experimental and numerical results. In contrast, the bubble migrates rapidly towards the solid boundary in the late stage of the first collapse phase. The rebound of the bubble position during the second oscillation becomes more obvious when the initial stand-off distance decreases.

Figure 6.19 Comparisons of the movements of the bubble centroid for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at different stand-off distances. The symbols are the experimental results, and the solid lines show the numerical simulation results.

6.7 Flow Field Dynamics

6.7.1 Velocity vectors and pressure contours

A distinct advantage of the numerical simulations is that the whole flow fluid dynamics can be captured at any time point. Figure 6.20 below shows the fluid dynamics around the bubble surface for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 0.6$, simulated using the MEL-BIM model together with the pictures captured during the experiment at the corresponding time point and frame size. The left column shows the velocity vector distributions. The upper left shows the first expansion phase, which can be readily deduced from the directions of the velocity vectors. The lower left shows the first collapse phase, and the velocity vectors point inwards. The magnitudes of the velocities near the symmetric axis are larger than the rest, which indicate the imminent formation of the liquid jet. The right column shows the corresponding pressure contours near the bubble surface. The dash lines show the bubble shapes calculated using the MEL-BIM model, which agree well with the experimental results on the left.

Figure 6.20 Flow field dynamics simulated using the MEL-BIM model for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 0.6$. The left column shows the

velocity field and right column shows the pressure contours. The dash lines are the bubble shapes obtained using the numerical simulations. The bubble pictures captured in the experiments are at: (a) the first expansion phase, about 110 μ s after bubble generation; (b) the first collapse phase, about 250 μ s after bubble generation.

6.7.2 Jet impact dynamics

The liquid jet impact is always believed to be the main mechanism of the bubble corrosion on the solid boundary. However, due to the fairly short duration of the jet impact, the measurements using the experimental methods are not always accurate enough. The numerical simulation provides a helpful understanding of the liquid jet impact dynamics. Figure 6.21 below shows the velocity fields around a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 1.0$ from the jet formation to the jet impact. It can be discerned that the simulation results agree quite well with the experimental results on the bubble shapes. The maximum jet velocity at the impact instant obtained using the numerical model is about 81.1 m/s, which is just in the range of the experimental results: 76.8~113.9 m/s, given by Philipp & Lauterborn (1998).

Figure 6.21 The velocity vectors in the flow field near a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 1.0$ from the jet formation to the jet impact. The width of each frame is 2.3 mm.

6.7.3 Counterjet formation

A hypothesis is proposed in Section 4.5, that the counterjet may be formed due to the back-flow of the liquid through the jet tunnel, and this now can be proved using the numerical simulations. Figure 6.22(a) below shows the velocity vectors around a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 2.0$ immediately before and after jet impact. It can be observed that before the jet impact, the velocity vectors along the symmetric axis above the bubble direct downwards. However, after the jet impact, there are two different directions for the velocity vectors inside the jet tunnel. The velocity flow directed upwards is believed to be the cause of the counterjet, and the velocity flow directed downwards causes the formation of a liquid tail in the lower part of the bubble. This explanation of the counterjet formation is also supported by the fact that when the toroidal bubble reaches the second maximum expanded volume, the bubble rejoins and the jet tunnel vanishes, and the counterjet starts to disappear. Figure 6.22(b) shows the velocity vectors immediately before and after the rejoining of a toroidal bubble. The upward and downward liquid flow inside the jet tunnel before the rejoining maintains the counterjet and bubble tail. After the rejoining, there is no upward liquid flow, so the counterjet starts to disappear. However, the downward flow is maintained for a long time, and the bubble tail becomes larger.

(a)

(b)

Figure 6.22 The velocity vectors in the flow field near a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 2.0$ (a) counterjet formation after jet impact, the width of each frame is 2.3 mm ; (b) counterjet disappearance after rejoining of the toroidal bubble, the width of each frame is 4.6 mm.

6.7.4 Jet-induced hammer pressure impact on the solid boundary

The impact of a liquid jet on a solid boundary generates a great water hammer pressure, and this pressure is linearly dependent on the jet velocity at the point of impact on the solid boundary. Figure 6.23 below shows the hammer pressure distribution on the solid boundary at the instant of jet impact for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 1.6$. The hammer pressure caused by the jet impact is about

26.3MPa as calculated in the numerical simulation, which is close to the experimental result of 34MPa given by Philipp & Lauterborn (1998). Such a great hammer pressure may cause significant corrosion, or a cleaning effect, on the solid boundary.

Figure 6.23 The hammer pressure distribution on the solid boundary at the jet impact instant for a bubble with $\bar{R}_{max} = 1.5\text{mm}$ at $\gamma = 1.6$.

6.8 Summary

The numerical simulation results obtained using the MEL-BIM model are first compared with the analytical Rayleigh-Plesset equation for a spherical bubble dynamics in an infinite and incompressible liquid. When the compressibility of the liquid is incorporated, the KHE model is employed for the comparisons. The good agreements between the numerical simulation results and the analytical solutions strongly verify the credibility of the proposed MEL-BIM model.

The simulation results concerning non-spherical vapor bubble dynamics near a solid boundary for the first oscillation, as reported by Taib (1985), are employed to test the ability of the MEL-BIM model to deal with non-spherical bubble dynamics. It is found that the numerical results obtained using the MEL-BIM model match Taib's results quite well, including those for the non-spherical bubble shape, bubble centroid movement, and liquid jet velocity.

For the comparisons with the experimental results, the most important thing at

the beginning of the simulations is the set-up of the initial conditions. The KHE model is employed to approach the initial conditions for the non-spherical bubble, as the model is believed to be quite efficient and reliable. Various numerical simulation results are used to make comparisons with the experimental results, including those for the bubble shapes, the equivalent bubble radii and the bubble positions. Although there are some differences between the numerical and experimental results, which may be caused by some mechanisms not able to be considered in this study, such as the emission of shock waves and the viscosity of the liquid, the numerical simulations can still represent the bubble dynamics near the solid boundary very well. Due to the limitation of the experimental measurements, the flow field dynamics near the deformable bubbles cannot always be shown clearly. The numerical simulations can compensate for these shortcomings of the experimental methods, as they can present the velocity vectors and pressure contours around the non-spherical bubble at any time point. The jet impact dynamics and formations of the counterjet can also be revealed using the simulation methods.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The occurrences of cavitations in many natural, industrial and bio-medical processes may sometimes be beneficial or sometimes harmful, but the physical fundamentals of these processes are quite similar. This study presented detailed investigations of the non-spherical cavitation bubble dynamics near a solid boundary. The investigations were made using two different methods: experiments and numerical simulations, which were always complementary in providing an in-depth understanding of the bubble dynamics.

In the experiments, the single cavitation bubble was generated by a focused laser pulse, and the bubble's maximum expanded radius depended on the incident laser energy. Evolutions of the bubble shapes were captured using a high-speed camera with a frame rate of 100,000 fps. A hydrophone system was employed to monitor the acoustic signals generated by the transient pressure impulses released by the bubble. The initial stand-off distances between the bubble inception and the solid boundary were controlled using the micro-translation stage.

Due to the existence of the solid boundary, the flow field became asymmetric around the bubble surface. The bubble started to oscillate non-spherically after generation, and the deformation patterns depended on the stand-off distances. For the bubbles with different maximum expanded radii, the deformation patterns of the bubbles were quite similar for the equivalent non-dimensional stand-off distances. The only differences concerned the bubble oscillation periods and the bubble sizes. A representative bubble with $\bar{R}_{max} = 1.2$ mm at different initial stand-off distances was selected to reveal the non-spherical deformations of the bubble. For the cases with small stand-off distances, i.e. $\gamma \leq 1.0$, the lower part of the bubble touched the solid boundary before the liquid jet impact. During the second oscillation, the bubble evolved like a volcano, and a splash appeared on the top of the bubble. For

the cases with stand-off distance $\gamma > 1.0$, the liquid jet penetrated the opposite bubble wall before the lower part of bubble touched the solid boundary, and the bubble transformed into a toroidal form. When $1.0 < \gamma \leq 2.0$, the rebounding toroidal bubble touched the solid boundary during the second expansion phase, and formed different shapes, such as inverse spade, small mound, isosceles trapezoid etc. For $2.0 < \gamma \leq 3.0$, the bubble was observed to maintain a nearly spherical shape during the first oscillation, and to form a funnel-like shape during the second oscillation. A counterjet was observed on the top of the toroidal bubble only in the range of $1.0 < \gamma < 2.2$, and the counterjet gradually disappeared after the toroidal bubble had reached the second maximum expanded volume. The size of the counterjet was also dependent on the initial stand-off distances.

Another important phenomenon characterising the influence of the solid boundary on the bubble dynamics was the variation of bubble oscillation periods. A fairly accurate method using the hydrophone system to estimate the bubble oscillation periods was employed in this study. The non-linear performance of the bubble oscillation periods provided the researchers with an indirect approach to determine the stand-off distances. The non-dimensional oscillation periods k_1 and k_2 , for different maximum bubble radii and different stand-off distances, were investigated statistically. The distinct results, which differed from those reported in the previous literatures, were those concerning the decay of the first oscillation periods after the initial stand-off distances were smaller than 0.5. The non-dimensional first oscillation periods reached the maximum values at $\gamma \approx 0.6$ for all bubble sizes. The peak value of k_1 decreased when the maximum bubble radius increased. For the non-dimensional second oscillation period k_2 , a nearly identical curve was obtained for different maximum bubble radii, and the maximum value appeared at $\gamma \approx 1.2$. The equivalent radii of the bubbles over their whole life spans were obtained in this study using AutoCAD software. This method is believed to be quite convenient and of high accuracy. The statistics of the second maximum expanded radii indirectly showed the residual bubble energies after the liquid jet impact. An approximate 30% reduction of the maximum expanded bubble radii corresponded to an approximate two-thirds energy loss in the cavitation bubble using the potential energy expression.

In order to mathematically describe the cavitation bubble dynamics near a solid boundary in a compressible liquid, a systematic derivation of the mathematical model was made in this study. The bubble dynamics was modeled approximately based on the potential flow theory. An approximate perturbation theory using the method of matched asymptotic expansions was employed to include the influence of the compressibility of the liquid in the mathematical model. In the perturbation method, the fluid domain was finely divided into two regions: the inner region near the bubble and the outer region far away from the bubble. The matching of asymptotic expansions that developed the velocity potential near the bubble surface satisfied Laplace's equation, which could be solved using the BIM. Due to the existence of a solid boundary, an image bubble was introduced to deal with the asymmetry of the flow field. The vortex ring model was adopted for the calculation of the toroidal bubble dynamics. Two different methods were used in the calculation of the flow field dynamics near the bubble surface under a fixed grid.

A MEL-BIM model was then created to model the bubble dynamics near a solid boundary. Due to the axisymmetric character of this potential problem, the dimensionality for the solution was reduced from original 3-D to integral 2-D. As the velocity potential near the bubble surface satisfied Laplace's equation, the problem was reduced further to a 1-D solution using the BIM. Two kernel functions were derived to relate the velocity potential and the normal velocity on the bubble surface, and, finally, a set of linear algebraic equations was obtained. The velocity potential and bubble surface location at each new time step were derived from the values of the last time step using the MEL-BIM model.

In order to prove the reliability and accuracy of the MEL-BIM model, the numerical simulation results obtained using the MEL-BIM model were compared with the analytical solutions for a spherical bubble dynamics in the incompressible (Rayleigh-Plesset equation) and compressible (Keller-Herring equation) liquid. For a pure vapor bubble, the results agreed quite well. However, for a gas bubble, some numerical errors appeared due to the instability of the MEL-BIM model after several oscillation cycles. The flow field dynamics at a fixed point were also calculated for the comparisons. Some previous numerical results for the non-spherical vapor bubble dynamics near a solid boundary were used to make a

comparison to verify the ability of the MEL-BIM model to deal with this type of problem.

The numerical simulations were conducted with the objective of compensating for the shortcomings of the experimental studies. The initial conditions derived using the KHE models were found to be quite useful. A large number of numerical simulation results were presented for comparisons with the experimental results, including those concerning the bubble shapes, the bubble equivalent radii and the bubble centroid positions. Although some numerical errors occurred, which may have been caused by some mechanisms not involved in this study (emission of the shock waves, viscosity of the liquid etc.), the numerical simulations could still represent the bubble dynamics near the solid boundary quite well. The numerical simulations also complemented the experiments, which could reveal the velocity vectors and pressure distributions near the bubble surface at any time point. The formations of the counterjet and the hammer pressure impact on the solid boundary at the instant of jet impact were also investigated in detail through the numerical simulations in this study.

7.2 Recommendations

Although detailed investigations have been made in this study for the non-spherical bubble dynamics near a solid boundary, some recommendations for further investigation into this topic may still be made. From the experimental perspective, the bubble pictures taken in this study are not sufficiently adequate for the studies of some rapid dynamic phases, such as the bubble generation and the liquid jet impact, which means that a camera with a higher speed is required. The liquid jet tongue inside the bubble is also not visible in this study, and this may be corrected through some optical means. As it is an important phenomenon, the hammer pressure impact on the solid boundary may be detected using the membrane hydrophone in place of the solid boundary. Some improvements to the numerical model may also be feasible. The inclusion of viscosity in similar simulations may introduce some unknown phenomena. The shock waves that occur during the jet impact may also be introduced to restrain the over-rebounding of the

toroidal bubble through some energy dissipation method.

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APPENDICES

A1 Matched Asymptotic Expansions Method

In order to develop the matched asymptotic expansions, the fluid domain is divided into two regions: the inner region near the bubble where $(\bar{x}, \bar{y}, \bar{z}) = O(\bar{R}_S)$ and the outer region far away from the bubble where $(\bar{x}, \bar{y}, \bar{z}) = O(c_\infty T)$, here T is the period of the oscillating bubble (see Figure A1.1).

(a) Inner region

(b) Outer region

Figure A1.1 Schematic diagram of the two divided fluid regions.

A1.1 Inner expansion

In the non-dimensional inner region $\mathbf{r} = (x, y, z) = O(1)$, the expansions of the potential φ and pressure p are defined as follows:

$$\varphi(\mathbf{r}, t) = \varphi_0(\mathbf{r}, t) + \varepsilon \varphi_1(\mathbf{r}, t) + \dots \quad (\text{A1.1})$$

$$p(\mathbf{r}, t) = p_0(\mathbf{r}, t) + \varepsilon p_1(\mathbf{r}, t) + \dots \quad (\text{A1.2})$$

It was noted that the first-order correction in terms of the bubble-wall Mach number to the incompressible results captured a large extent the effect of compressibility, with the higher order having only a minor influence. In this study, only the first-order accuracy in terms of Mach number ε was considered. Substituting these expansions into the continuity Equation (5.28) and Bernoulli Equation (5.29) to yield the inner field equation for $i = 0, 1$

$$\nabla^2 \varphi_0 = 0 \quad (\text{A1.3})$$

$$\nabla^2 \varphi_1 = 0 \quad (\text{A1.4})$$

and

$$\frac{\partial \varphi_0}{\partial t} + \frac{1}{2} |\nabla \varphi_0|^2 + p_0 = 0 \quad (\text{A1.5})$$

$$\frac{\partial \varphi_1}{\partial t} + \nabla \varphi_0 \cdot \nabla \varphi_1 + p_1 = 0 \quad (\text{A1.6})$$

φ_0 and φ_1 satisfy the Laplace's equation, and their general solutions can be written as follows, using the second Gauss identity:

$$\begin{aligned} \varphi_i(\mathbf{r}, t) &= f_i(t) + g_i(t)z \\ &+ \int_S \left(\frac{\partial \varphi_i(\mathbf{q}, t)}{\partial n} G(\mathbf{r}, \mathbf{q}) - \varphi_i(\mathbf{q}, t) \frac{\partial G(\mathbf{r}, \mathbf{q})}{\partial n} \right) dS(\mathbf{q}), \\ i &= 0, 1 \end{aligned} \quad (\text{A1.7})$$

where S is the bubble surface, $\mathbf{n}$ is the unit outward normal on the surface, $\mathbf{q}$ is the integration variable on the surface, and the free space Green's function is

$$G(\mathbf{r}, \mathbf{q}) = \frac{1}{|\mathbf{r} - \mathbf{q}|} \quad (\text{A1.8})$$

The unknown functions $f_i(t)$ and $g_i(t)$ for $i = 0, 1$ are arbitrary functions of time, and to be determined by the matching between the inner and outer expansions later.

To prepare for the matching between the inner and outer expansions, the outer limit of the inner expansion is needed. In the outer region, where $(\bar{x}, \bar{y}, \bar{z}) = O(c_\infty T)$, the outer variable is introduced as

$$\begin{aligned} \tilde{\mathbf{r}} &= (\tilde{x}, \tilde{y}, \tilde{z}) = \frac{1}{c_\infty T} (\bar{x}, \bar{y}, \bar{z}) = \frac{R_s}{c_\infty T} (x, y, z) \\ &= \varepsilon(x, y, z) = \varepsilon \mathbf{r} = O(1) \end{aligned} \quad (\text{A1.9})$$

where

$$\frac{R_{max}}{c_\infty T} = \frac{O(UT)}{c_\infty T} = O\left(\frac{U}{c_\infty}\right) = O(\varepsilon) \quad (\text{A1.10})$$

The following estimation is made for the Green's function in the outer limit of the inner region:

$$G(\mathbf{r}, \mathbf{q}) = \frac{1}{|\mathbf{r} - \mathbf{q}|} = \frac{1}{\left|\frac{\tilde{\mathbf{r}}}{\varepsilon} - \mathbf{q}\right|} = \frac{\varepsilon}{\tilde{r}} + O(\varepsilon^2) \quad (\text{A1.11})$$

where $\tilde{r} = |\tilde{\mathbf{r}}|$, and

$$O\left(\frac{\tilde{r}}{\varepsilon}\right) = O\left(\frac{1}{\varepsilon}\right) \quad \text{and} \quad O(\mathbf{q}) = O(1) \quad (\text{A1.12a,b})$$

As $1/\varepsilon$ is very large, $\mathbf{q}$ is eliminated in the denominator. Similarly,

$$\frac{\partial G(\mathbf{r}, \mathbf{q})}{\partial n} = \mathbf{n} \cdot \nabla_{\mathbf{q}} G(\mathbf{r}, \mathbf{q}) = \mathbf{n} \cdot \frac{\mathbf{r} - \mathbf{q}}{|\mathbf{r} - \mathbf{q}|^3} = \mathbf{n} \cdot \frac{\varepsilon^2 \tilde{\mathbf{r}} - \varepsilon^3 \mathbf{q}}{|\tilde{\mathbf{r}} - \varepsilon \mathbf{q}|^3} = O(\varepsilon^2) \quad (\text{A1.13})$$

where $\nabla_{\mathbf{q}}$ is in terms of $\mathbf{q}$.

Substituting these two estimations Equation (A1.11) and Equation (A1.13) into Equation (A1.7) to yield the outer limits of the inner expansions,

$$\begin{aligned} (\varphi_i)^o &= f_i(t) + g_i(t) \frac{\tilde{z}}{\varepsilon} + \int_S \left(\frac{\partial \varphi_i(\mathbf{q}, t)}{\partial n} \frac{\varepsilon}{\tilde{r}} \right) dS(\mathbf{q}) + O(\varepsilon^2) \\ &= f_i(t) + g_i(t) \frac{\tilde{z}}{\varepsilon} + \varepsilon \frac{m_i(t)}{\tilde{r}} + O(\varepsilon^2) \quad i = 0, 1 \end{aligned} \quad (\text{A1.14})$$

where

$$m_i(t) = \int_S \frac{\partial \varphi_i(\mathbf{q}, t)}{\partial n} dS(\mathbf{q}) \quad (\text{A1.15})$$

Then the outer limit of the inner expansion to the first order in terms of ε is

$$\begin{aligned} (\varphi)^o &= (\varphi_0)^o + \varepsilon (\varphi_1)^o \\ &= f_0(t) + g_0(t) \frac{\tilde{z}}{\varepsilon} + \varepsilon \frac{m_0(t)}{\tilde{r}} + \varepsilon f_1(t) + \varepsilon g_1(t) \frac{\tilde{z}}{\varepsilon} + O(\varepsilon^2) \\ &= f_0(t) + g_0(t) z + \frac{m_0(t)}{r} + \varepsilon f_1(t) + \varepsilon g_1(t) z + O(\varepsilon^2) \end{aligned} \quad (\text{A1.16})$$

where $r = |\mathbf{r}|$.

A1.2 Outer expansion

Using the outer variable $(\tilde{x}, \tilde{y}, \tilde{z})$ defined in Equation (A1.9), the continuity Equation (5.28) and Bernoulli Equation (5.29) become

$$\tilde{\nabla}^2 \varphi + \frac{1}{c^2} \left(\frac{\partial p}{\partial t} + \varepsilon^2 \tilde{\nabla} \varphi \cdot \tilde{\nabla} p \right) = O(\varepsilon^2) \quad (\text{A1.17})$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2} \varepsilon^2 |\tilde{\nabla} \varphi|^2 + p = O(\varepsilon^2) \quad (\text{A1.18})$$

where the operator $\tilde{\nabla}$ is defined in terms of $\tilde{\mathbf{r}}$, $\tilde{\nabla} = (1/\varepsilon) \nabla$.

The outer expansions of the potential φ and pressure p are defined as follows:

$$\varphi(\tilde{\mathbf{r}}, t) = \phi_0(\tilde{\mathbf{r}}, t) + \varepsilon \phi_1(\tilde{\mathbf{r}}, t) + \dots \quad (\text{A1.19})$$

$$p(\tilde{\mathbf{r}}, t) = P_0(\tilde{\mathbf{r}}, t) + \varepsilon P_1(\tilde{\mathbf{r}}, t) + \dots \quad (\text{A1.20})$$

Substituting them into Equations (A1.19) and Equation (A1.20) to yield the outer field equation:

$$\tilde{\nabla}^2 \phi_0 + \frac{\partial P_0}{\partial t} = 0 \quad (\text{A1.21})$$

$$\tilde{\nabla}^2 \phi_1 + \frac{\partial P_1}{\partial t} = 0 \quad (\text{A1.22})$$

and

$$\frac{\partial \phi_0}{\partial t} + P_0 = 0 \quad (\text{A1.23})$$

$$\frac{\partial \phi_1}{\partial t} + P_1 = 0 \quad (\text{A1.24})$$

Combining Equations (A1.21) and (A1.23), (A1.22) and (A1.24), yields

$$\tilde{\nabla}^2 \phi_i - \frac{\partial^2 \phi_i}{\partial t^2} = 0 \quad i = 0, 1 \quad (\text{A1.25})$$

Thus, far from the bubble, the first two order outer potential solutions ϕ_0 and ϕ_1 satisfy the linear wave equation.

As the bubble is much smaller than the wavelength and the far field is not affected by the existence of the bubble, the leading outer solution ϕ_0 is thus the acoustic standing wave Equation (5.35). Together with the relation given by (5.37), one can get

$$\begin{aligned} \phi_0|_{r \rightarrow \infty} &= \phi_a = \phi_a^{in} + \phi_a^{re} \\ &= \frac{b}{2} \cos(\omega t + \omega \varepsilon z + \theta_0) + \frac{b}{2} \cos(\omega t - \omega \varepsilon z - \theta_0) \end{aligned} \quad (\text{A1.26})$$

The general solution ϕ_1 is the well known d'Alembert solution of the wave equation

$$\phi_1 = \frac{F_1(t - \tilde{r}) + G_1(t + \tilde{r})}{\tilde{r}} \quad (\text{A1.27})$$

where F_1 and G_1 are two arbitrary functions having second-order derivatives. According to the Sommerfeld radiation condition, ϕ_1 cannot contain incoming

waves, but only outgoing, thus

$$\phi_1 = \frac{F_1(t-\tilde{r})}{\tilde{r}} \quad (\text{A1.28})$$

The inner limits of the outer expansions can be obtained by taking a Taylor series expansion as follows:

$$\begin{aligned} (\phi_0)^i &= \frac{b}{2} \cos((\omega t + \theta_0) + \omega \varepsilon z) + \frac{b}{2} \cos((\omega t - \theta_0) - \omega \varepsilon z) \\ &= \frac{b}{2} \cos(\omega t + \theta_0) + \frac{b}{2} \omega \varepsilon z (\cos(\omega t + \theta_0))' \\ &\quad + \frac{b}{2} \cos(\omega t - \theta_0) - \frac{b}{2} \omega \varepsilon z (\cos(\omega t - \theta_0))' + O(\varepsilon^2) \\ &= \frac{b}{2} (\cos(\omega t + \theta_0) + \cos(\omega t - \theta_0)) \\ &\quad + \frac{b}{2} \omega \varepsilon z (-\sin(\omega t + \theta_0) + \sin(\omega t - \theta_0)) + O(\varepsilon^2) \\ &= b \cos(\omega t) \cos(\theta_0) - \varepsilon b \omega z \cos(\omega t) \sin(\theta_0) + O(\varepsilon^2) \end{aligned} \quad (\text{A1.29})$$

$$(\phi_1)^i = \frac{F_1(t-\tilde{r})}{\tilde{r}} = \frac{F_1(t-\varepsilon r)}{\varepsilon r} = \frac{F_1(t)}{\varepsilon r} - F_1'(t) + O(\varepsilon) \quad (\text{A1.30})$$

Combining the above results yields the inner limit of outer expansion to the first order in terms of ε as

$$\begin{aligned} (\phi)^i &= b \cos(\omega t) \cos(\theta_0) - \varepsilon b \omega z \cos(\omega t) \sin(\theta_0) \\ &\quad + \frac{F_1(t)}{r} - \varepsilon F_1'(t) + O(\varepsilon^2) \end{aligned} \quad (\text{A1.31})$$

A1.3 Matching

Then the author employed the Van Dyke's matching principle (Van Dyke 1975) to equate the outer limit of the inner expansion Equation (A1.16) and the inner limit of the outer expansion Equation (A1.31), yields

$$f_0(t) = b \cos(\omega t) \cos(\theta_0), \quad g_0(t) = 0, \quad F_1(t) = m_0(t) \quad (\text{A1.32a,b,c})$$

$$f_1(t) = -F_1'(t) = -m_0'(t), \quad g_1(t) = -b \omega \cos(\omega t) \sin(\theta_0) \quad (\text{A1.33d,e})$$

With F_1 defined above, one can obtain the outer solution ϕ_1 as

$$\phi_1 = \frac{m_0(t-\tilde{r})}{\tilde{r}} \quad (\text{A1.34})$$

Then the first two outer solution terms are

$$\begin{aligned} \varphi(\tilde{\mathbf{r}}, t) = & \frac{b}{2} \cos(\omega t + \omega \tilde{z} + \theta_0) + \frac{b}{2} \cos(\omega t - \omega \tilde{z} - \theta_0) \\ & + \varepsilon \frac{m_0(t-\tilde{r})}{\tilde{r}} + O(\varepsilon^2) \end{aligned} \quad (\text{A1.35})$$

The first term of the solution is the incident acoustic wave, and the second term is due to a point source whose strength is equal to the rate of change of the bubble volume.

A2 Boundary Integral Method (BIM)

A2.1 Formulations

For any sufficient smooth function ϕ which satisfies Laplace's equation in a domain Ω having piecewise smooth surface S , Green's integral formula can be written as (Jaswon & Symm 1977; Brebbia 1978):

$$c(\mathbf{p})\phi(\mathbf{p}) + \int_S \phi(\mathbf{q}) \frac{\partial}{\partial n} \left(\frac{1}{|\mathbf{p}-\mathbf{q}|} \right) dS = \int_S \frac{\partial}{\partial n} (\phi(\mathbf{q})) \frac{1}{|\mathbf{p}-\mathbf{q}|} dS \quad (\text{A2.1})$$

where $\mathbf{p} \in \Omega + S$, $\mathbf{q} \in S$, $\frac{\partial}{\partial n}$ is the normal derivative outward from S , and

$$c(\mathbf{p}) = \begin{cases} 4\pi, & \text{if } \mathbf{p} \in \Omega \\ 2\pi, & \text{if } \mathbf{p} \in S \end{cases} \quad (\text{A2.2})$$

And set,

$$\frac{\partial}{\partial n} (\phi(\mathbf{q})) = \psi(\mathbf{q}) \quad (\text{A2.3})$$

$$G(\mathbf{p}, \mathbf{q}) = \frac{1}{|\mathbf{p}-\mathbf{q}|} \quad (\text{A2.4})$$

where $\mathbf{q}$ is the integration variable on the surface, $\mathbf{p}$ is the unknown points, $\psi(\mathbf{q})$ is the normal velocity at point $\mathbf{q}$, and $G(\mathbf{p}, \mathbf{q})$ is Green's function due to a unit source in an infinite fluid domain.

Using cylindrical polar coordinates with $\mathbf{p} = (r_0, 0, z_0)$ and $\mathbf{q} = (r, \theta, z)$

respectively. If the surface S is parameterized by arc length variable ξ , then yields

$$\mathbf{q} = (r(\xi), \theta, z(\xi)) \quad (\text{A2.5})$$

Then, the integration of the Green function becomes

$$\begin{aligned} I_1 &= \int_S G(\mathbf{p}, \mathbf{q}) dS = \int_S G(r_0, z_0, r(\xi), z(\xi), \theta) dS \\ &= \iint G(r_0, z_0, r(\xi), z(\xi), \theta) r(\xi) \left[\left(\frac{dz(\xi)}{d\xi} \right)^2 + \left(\frac{dr(\xi)}{d\xi} \right)^2 \right]^{\frac{1}{2}} d\xi d\theta \\ &= \int_0^1 d\xi r(\xi) \left[\left(\frac{dz(\xi)}{d\xi} \right)^2 + \left(\frac{dr(\xi)}{d\xi} \right)^2 \right]^{\frac{1}{2}} \\ &\quad \int_0^{2\pi} G(r_0, z_0, r(\xi), z(\xi), \theta) d\theta \end{aligned} \quad (\text{A2.6})$$

and,

$$\begin{aligned} G(r_0, z_0, r(\xi), z(\xi), \theta) &= \frac{1}{[(r(\xi)\cos\theta - r_0)^2 + (r(\xi)\sin\theta)^2 + (z(\xi) - z_0)^2]^{\frac{1}{2}}} \\ &= \frac{1}{[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2 - 4r(\xi)r_0\cos^2\frac{\theta}{2}]^{\frac{1}{2}}} \end{aligned} \quad (\text{A2.7})$$

Then

$$\begin{aligned} I_1 &= \int_0^1 d\xi r(\xi) \left[\left(\frac{dz(\xi)}{d\xi} \right)^2 + \left(\frac{dr(\xi)}{d\xi} \right)^2 \right]^{\frac{1}{2}} \\ &\quad \int_0^{2\pi} \frac{d\theta}{[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2 - 4r(\xi)r_0\cos^2\frac{\theta}{2}]^{\frac{1}{2}}} \\ &= \int_0^1 \frac{d\xi r(\xi) \left[\left(\frac{dz(\xi)}{d\xi} \right)^2 + \left(\frac{dr(\xi)}{d\xi} \right)^2 \right]^{\frac{1}{2}}}{[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2]^{\frac{1}{2}}} \\ &\quad \int_0^{2\pi} \frac{d\theta}{\left[1 - \frac{4r(\xi)r_0\cos^2\frac{\theta}{2}}{(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2} \right]^{\frac{1}{2}}} \end{aligned} \quad (\text{A2.8})$$

Set

$$k^2(\xi) = \frac{4r(\xi)r_0}{(r(\xi)+r_0)^2+(z(\xi)-z_0)^2} \quad (\text{A2.9})$$

The θ integral becomes

$$\begin{aligned} \int_0^{2\pi} \frac{d\theta}{\left[1 - \frac{4r(\xi)r_0 \cos^2 \frac{\theta}{2}}{(r(\xi)+r_0)^2+(z(\xi)-z_0)^2}\right]^{\frac{1}{2}}} &= \int_0^{2\pi} \frac{d\theta}{\left[1 - k^2(\xi) \cos^2 \frac{\theta}{2}\right]^{\frac{1}{2}}} \\ &= 4 \int_0^{\frac{\pi}{2}} \frac{d\alpha}{[1 - k^2(\xi) \cos^2 \alpha]^{\frac{1}{2}}} = 4 \int_0^{\frac{\pi}{2}} \frac{d\alpha}{[1 - k^2(\xi) \sin^2 \alpha]^{\frac{1}{2}}} = 4K(k) \end{aligned} \quad (\text{A2.10})$$

where $K(k)$ is the complete elliptic integral of the first kind, which can be get from the approximations in Hastings (1955).

Then yields,

$$I_1 = \int_0^1 K_1(r_0, z_0, r(\xi), z(\xi)) d\xi \quad (\text{A2.11})$$

where K_1 is the first kernel function, and

$$K_1 = \frac{4r(\xi)K(k)J(\xi)}{D(\xi)} \quad (\text{A2.12})$$

$$J(\xi) = \left[\left(\frac{dz(\xi)}{d\xi} \right)^2 + \left(\frac{dr(\xi)}{d\xi} \right)^2 \right]^{\frac{1}{2}} \quad (\text{A2.13})$$

$$D(\xi) = [(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2]^{\frac{1}{2}} \quad (\text{A2.14})$$

The integration of the normal derivative of the Green function can be deduced using the similar procedures. The normal direction of the surface can be expressed as

$$\hat{\mathbf{n}} = \frac{1}{\left[\left(\frac{dz(\xi)}{d\xi} \right)^2 + \left(\frac{dr(\xi)}{d\xi} \right)^2 \right]^{\frac{1}{2}}} \left(\frac{dz(\xi)}{d\xi} \cos \theta, \frac{dz(\xi)}{d\xi} \sin \theta, -\frac{dr(\xi)}{d\xi} \right) \quad (\text{A2.15})$$

and,

$$\nabla G(r_0, z_0, r(\xi), z(\xi), \theta) = \frac{-(r(\xi) \cos \theta - r_0, r(\xi) \sin \theta, z(\xi) - z_0)}{\left[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2 - 4r(\xi)r_0 \cos^2 \frac{\theta}{2} \right]^{\frac{3}{2}}} \quad (\text{A2.16})$$

The normal derivative of the Green function can be achieved by

$$\frac{\partial G(r_0, z_0, r(\xi), z(\xi), \theta)}{\partial n} = \hat{\mathbf{n}} \cdot \nabla G(r_0, z_0, r(\xi), z(\xi), \theta)$$

$$\begin{aligned}
&= \frac{-\left[(r(\xi)\cos\theta - r_0)\frac{dz(\xi)}{d\xi}\cos\theta + r(\xi)\sin\theta\frac{dz(\xi)}{d\xi}\sin\theta - (z(\xi) - z_0)\frac{dr(\xi)}{d\xi}\right]}{\left[\left(\frac{dz(\xi)}{d\xi}\right)^2 + \left(\frac{dr(\xi)}{d\xi}\right)^2\right]^{\frac{1}{2}} \cdot \left[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2 - 4r(\xi)r_0\cos^2\frac{\theta}{2}\right]^{\frac{3}{2}}} \\
&= \frac{-\left[-r_0\frac{dz(\xi)}{d\xi}\cos\theta + r(\xi)\frac{dz(\xi)}{d\xi} - (z(\xi) - z_0)\frac{dr(\xi)}{d\xi}\right]}{\left[\left(\frac{dz(\xi)}{d\xi}\right)^2 + \left(\frac{dr(\xi)}{d\xi}\right)^2\right]^{\frac{1}{2}} \cdot \left[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2 - 4r(\xi)r_0\cos^2\frac{\theta}{2}\right]^{\frac{3}{2}}}
\end{aligned} \tag{A2.17}$$

Then, the integration of the normal derivative of the Green function is given as,

$$\begin{aligned}
I_2 &= \int_S \frac{\partial G(r_0, z_0, r(\xi), z(\xi), \theta)}{\partial n} dS \\
&= \iint \frac{\partial G(r_0, z_0, r(\xi), z(\xi), \theta)}{\partial n} r(\xi) \left[\left(\frac{dz(\xi)}{d\xi}\right)^2 + \left(\frac{dr(\xi)}{d\xi}\right)^2\right]^{\frac{1}{2}} d\xi d\theta \\
&= \\
&\quad - \left\{ \int_0^1 d\xi r(\xi) \left[r(\xi) \frac{dz(\xi)}{d\xi} - \right. \right. \\
&\quad \left. \left. (z(\xi) - z_0) \frac{dr(\xi)}{d\xi} \right] \int_0^{2\pi} \frac{d\theta}{\left[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2 - 4r(\xi)r_0\cos^2\frac{\theta}{2}\right]^{\frac{3}{2}}} - \right. \\
&\quad \left. \int_0^1 d\xi r(\xi) \frac{dz(\xi)}{d\xi} \int_0^{2\pi} d\theta \frac{r_0\cos\theta}{\left[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2 - 4r(\xi)r_0\cos^2\frac{\theta}{2}\right]^{\frac{3}{2}}} \right\}
\end{aligned} \tag{A2.18}$$

Making the same substitution as (A2.9), yields

$$\begin{aligned}
I_2 &= - \left\{ \int_0^1 \frac{d\xi r(\xi) \left[r(\xi) \frac{dz(\xi)}{d\xi} - (z(\xi) - z_0) \frac{dr(\xi)}{d\xi} \right]}{\left[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2\right]^{\frac{3}{2}}} \int_0^{2\pi} \frac{d\theta}{\left[1 - k^2(\xi)\cos^2\frac{\theta}{2}\right]^{\frac{3}{2}}} - \right. \\
&\quad \left. \int_0^1 d\xi r(\xi) \frac{dz(\xi)}{d\xi} \frac{r_0}{\left[(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2\right]^{\frac{3}{2}}} \int_0^{2\pi} d\theta \frac{\cos\theta}{\left[1 - k^2(\xi)\cos^2\frac{\theta}{2}\right]^{\frac{3}{2}}} \right\}
\end{aligned} \tag{A2.19}$$

The first θ integral can be written as

$$\begin{aligned} \int_0^{2\pi} \frac{d\theta}{[1-k^2(\xi)\cos^2\frac{\theta}{2}]^{\frac{3}{2}}} &= 4 \int_0^{\frac{\pi}{2}} \frac{d\alpha}{[1-k^2(\xi)\cos^2\alpha]^{\frac{3}{2}}} \\ &= 4 \int_0^{\pi/2} \frac{d\alpha}{[1-k^2(\xi)\sin^2\alpha]^{\frac{3}{2}}} = 4\Pi(k, k) \quad (\text{A2.20}) \end{aligned}$$

and the second θ integral is

$$\begin{aligned} \int_0^{2\pi} d\theta \frac{\cos\theta}{[1-k^2(\xi)\cos^2\frac{\theta}{2}]^{\frac{3}{2}}} &= 4 \int_0^{\frac{\pi}{2}} \frac{(2\cos^2\eta-1)d\eta}{[1-k^2(\xi)\cos^2\eta]^{\frac{3}{2}}} \\ &= \frac{8}{k^2(\xi)} \int_0^{\frac{\pi}{2}} \frac{(k^2(\xi)\cos^2\eta-1)d\eta}{[1-k^2(\xi)\cos^2\eta]^{\frac{3}{2}}} + \left(\frac{8}{k^2(\xi)} - 4\right) \int_0^{\frac{\pi}{2}} \frac{d\eta}{[1-k^2(\xi)\cos^2\eta]^{\frac{3}{2}}} \\ &= \left(\frac{8}{k^2(\xi)} - 4\right) \int_0^{\frac{\pi}{2}} \frac{d\eta}{[1-k^2(\xi)\cos^2\eta]^{\frac{3}{2}}} - \frac{8}{k^2(\xi)} \int_0^{\frac{\pi}{2}} \frac{d\eta}{[1-k^2(\xi)\cos^2\eta]^{\frac{1}{2}}} \\ &= \left(\frac{8}{k^2(\xi)} - 4\right) \Pi(k, k) - \frac{8}{k^2(\xi)} K(k) \quad (\text{A2.21}) \end{aligned}$$

where $\Pi(k, k)$ is the complete elliptic integral of the third kind, which can be written in term of the complete elliptic integral of the second kind $E(k)$ as,

$$\Pi(k, k) = \frac{E(k)}{1-k^2} \quad (\text{A2.22})$$

hence,

$$\begin{aligned} I_2 &= \int_S \frac{\partial G(r_0, z_0, r(\xi), z(\xi), \theta)}{\partial n} dS \\ &= -4 \int_0^1 \frac{d\xi r(\xi)}{[(r(\xi)+r_0)^2 + (z(\xi)-z_0)^2]^{\frac{3}{2}}} \\ &\quad \left\{ \left[\frac{dz(\xi)}{d\xi} (r(\xi)+r_0) - \frac{dr(\xi)}{d\xi} (z(\xi) - z_0) - \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \right] \frac{E(k)}{1-k^2(\xi)} + \right. \\ &\quad \left. \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 K(k) \right\} \\ &= \int_0^1 K_2(r_0, z_0, r(\xi), z(\xi)) d\xi \quad (\text{A2.23}) \end{aligned}$$

where K_2 is the second kernel function, and

$$K_2 = \frac{-4r(\xi)}{D(\xi)^3} \left\{ \left[\frac{dz(\xi)}{d\xi} (r(\xi) + r_0) - \frac{dr(\xi)}{d\xi} (z(\xi) - z_0) - \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \right] \frac{E(k)}{1-k^2(\xi)} + \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 K(k) \right\} \quad (A2.24)$$

where $D(\xi)$ is given by (A2.14), and $K(k), E(k)$ is the complete elliptic integral of the first and second kind, which can be get from the approximations in Hastings (1955),

$$K(k) = P(x) - Q(x) \ln(x) \quad (A2.25a)$$

$$E(k) = R(x) - S(x) \ln(x) \quad (A2.25b)$$

where

$$x = 1 - k^2(\xi) \quad (A2.26)$$

and P, Q, R and S are tabulated polynomials as follows.

Table A2.1 The coefficients of the complete elliptic integral.

Equation	$f(x)=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4$			
Values	$P(x)$	$Q(x)$	$R(x)$	$S(x)$
a_0	1.38629436112	0.5	1.0	0
a_1	0.09666344259	0.12498593597	0.44325141463	0.24998368310
a_2	0.03590092383	0.06880248576	0.06260601220	0.09200180037
a_3	0.03742563713	0.03328355346	0.04757383546	0.04069697526
a_4	0.01451196212	0.00441787012	0.01736506451	0.00526449639

A2.2 Approximation method

In the computation, a linear surface – linear functions (L-L) approximation for the bubble surface shape and the potential and its normal derivative on the bubble surface is provided. ϕ_j and ψ_j are assumed to be single valued at the end points of the linear segments which approximate the surface, see Figure 5.1b. If the segment is parameterized by ξ in the range (0,1), make a definition as

$$\begin{cases} M_1(\xi) = 1 - \xi \\ M_2(\xi) = \xi \end{cases} \quad (\text{A2.27})$$

On segment S_j ,

$$\begin{cases} r(\xi) = r_j M_1(\xi) + r_{j+1} M_2(\xi) \\ z(\xi) = z_j M_1(\xi) + z_{j+1} M_2(\xi) \\ \phi(\xi) = \phi_j M_1(\xi) + \phi_{j+1} M_2(\xi) \\ \psi(\xi) = \psi_j M_1(\xi) + \psi_{j+1} M_2(\xi) \end{cases} \quad (\text{A2.28})$$

The collocation points are move to the end points of each segment, yielding $n + 1$ equations with $n + 1$ unknowns, where n is the number of the segments. The integrals on each segment can be written

$$\int_{S_j} dS \frac{\partial}{\partial n} (\phi(q)) \frac{1}{|p_i - q_j|} = b_{1ij} \psi_j + b_{2ij} \psi_{j+1} \quad (\text{A2.29})$$

where

$$\begin{aligned} b_{kij} &= S_j \int_0^1 d\xi M_k(\xi) \int_0^{2\pi} d\theta \frac{1}{|p_i - q(\xi, \theta)|} \\ &= \int_0^1 M_k(\xi) K_1(r_0, z_0, r(\xi), z(\xi)) d\xi \end{aligned} \quad (\text{A2.30})$$

K_1 is the first kernel function. Similarly,

$$\int_{S_j} dS \phi(q) \frac{\partial}{\partial n} \left(\frac{1}{|p_i - q_j|} \right) = a_{1ij} \phi_j + a_{2ij} \phi_{j+1} \quad (\text{A2.31})$$

where

$$\begin{aligned} a_{kij} &= S_j \int_0^1 d\xi M_k(\xi) \int_0^{2\pi} d\theta \frac{\partial}{\partial n} \left(\frac{1}{|p_i - q(\xi, \theta)|} \right) \\ &= \int_0^1 M_k(\xi) K_2(r_0, z_0, r(\xi), z(\xi)) d\xi \end{aligned} \quad (\text{A2.32})$$

and K_2 is the second kernel function.

a_{kij} and b_{kij} can be calculated directly using standard Gauss-Legendre quadrature (see Appendix A4.1). But when the collocation point p_i is in the same segment with q_j , the integrand is singular and must be treated specially. The singular integrals are performed by subtracting the logarithmic term to remove the singularity, then using a quadrature scheme incorporating the logarithm to complete the integration. Also when the p_i is on the axis of symmetry, the integrations can be treated specially.

i, p_i and q_j in the same segment, $i = j$, singularity at $\xi = 0$

This singularity occurs when p_i is the point $(r_0, z_0) = (r(0), z(0))$. The Equations (A2.30) and (A2.32) become

$$\begin{aligned} b_{kij} &= S_j \int_0^1 d\xi M_k(\xi) \int_0^{2\pi} d\theta \frac{1}{|p_i - q(\xi, \theta)|} \\ &= \int_0^1 M_k(\xi) \frac{4r(\xi)K(k)J(\xi)}{D(\xi)} d\xi \end{aligned} \quad (\text{A2.33})$$

$$\begin{aligned} a_{kij} &= S_j \int_0^1 d\xi M_k(\xi) \int_0^{2\pi} d\theta \frac{\partial}{\partial n} \left(\frac{1}{|p_i - q(\xi, \theta)|} \right) \\ &= \int_0^1 M_k(\xi) \frac{-4r(\xi)}{D(\xi)^3} \left\{ \left[\frac{dz(\xi)}{d\xi} (r(\xi) + r_0) - \frac{dr(\xi)}{d\xi} (z(\xi) - z_0) - \right. \right. \\ &\quad \left. \left. \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \right] \frac{E(k)}{1 - k^2(\xi)} + \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 K(k) \right\} d\xi \end{aligned} \quad (\text{A2.34})$$

where

$$J(\xi) = \left[\left(\frac{dz(\xi)}{d\xi} \right)^2 + \left(\frac{dr(\xi)}{d\xi} \right)^2 \right]^{\frac{1}{2}} \quad (\text{A2.35})$$

and

$$D(\xi) = [(r(\xi) + r_0)^2 + (z(\xi) - z_0)^2]^{\frac{1}{2}} \quad (\text{A2.36})$$

By writing

$$\ln(x) = \ln\left(\frac{x}{\xi^2}\right) + 2 \ln(\xi) \quad (\text{A2.37})$$

And using Hastings' approximations, the logarithmic singularity is separated out as follows

$$\begin{aligned} b_{kij} &= S_j \int_0^1 d\xi M_k(\xi) \int_0^{2\pi} d\theta \frac{1}{|p_i - q(\xi, \theta)|} \\ &= 4 \left\{ \int_0^1 M_k(\xi) \frac{r(\xi)J(\xi)}{D(\xi)} \left[P(x(\xi)) - Q(x(\xi)) \ln\left(\frac{x(\xi)}{\xi^2}\right) \right] d\xi + \right. \end{aligned}$$

$$2 \int_0^1 M_k(\xi) \frac{r(\xi)J(\xi)}{D(\xi)} Q(x(\xi)) \ln\left(\frac{1}{\xi}\right) d\xi \} \quad (\text{A2.38})$$

$$\begin{aligned} a_{kij} &= S_j \int_0^1 d\xi M_k(\xi) \int_0^{2\pi} d\theta \frac{\partial}{\partial n} \left(\frac{1}{|\mathbf{p}_i - \mathbf{q}(\xi, \theta)|} \right) \\ &= \int_0^1 M_k(\xi) \frac{-4r(\xi)}{D(\xi)^3} \left\{ \frac{R(x(\xi)) - S(x(\xi)) \ln\left(\frac{x(\xi)}{\xi^2}\right)}{1 - k^2(\xi)} \left[\frac{dz(\xi)}{d\xi} (r(\xi) + r_0) - \right. \right. \\ &\quad \left. \frac{dr(\xi)}{d\xi} (z(\xi) - z_0) - \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \right] + \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \left[P(x(\xi)) - \right. \\ &\quad \left. Q(x(\xi)) \ln\left(\frac{x(\xi)}{\xi^2}\right) \right] \Big\} d\xi - \\ &8 \int_0^1 M_k(\xi) \frac{r(\xi)}{D(\xi)^3} \left\{ \frac{S(x(\xi))}{1 - k^2(\xi)} \left[\frac{dz(\xi)}{d\xi} (r(\xi) + r_0) - \frac{dr(\xi)}{d\xi} (z(\xi) - z_0) - \right. \right. \\ &\quad \left. \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \right] + \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 Q(x(\xi)) \Big\} \ln\left(\frac{1}{\xi}\right) d\xi \end{aligned} \quad (\text{A2.39})$$

The first integral in these two equations contains no singularity, and can be integrated by standard Gauss-Legendre quadrature. The other integrals contain an explicit singularity of log type which can be integrated using the quadrature scheme tabulated by Stroud and Secrest (1966) for the integral (see Appendix A4.2).

$$\int_0^1 f(x) \ln\left(\frac{1}{x}\right) dx \quad (\text{A2.40})$$

ii, $\mathbf{p}_i$ and $\mathbf{q}_j$ in the same segment, $i = j+1$, singularity at $\xi = 1$

This is the case when $\mathbf{p}_i$ is the point $(r_0, z_0) = (r(1), z(1))$. Similarly, by writing

$$\ln(x) = \ln\left(\frac{x}{(1-\xi)^2}\right) + 2 \ln(1 - \xi) \quad (\text{A2.41})$$

Obtain,

$$\begin{aligned} b_{kij} &= S_j \int_0^1 d\xi M_k(\xi) \int_0^{2\pi} d\theta \frac{1}{|\mathbf{p}_i - \mathbf{q}(\xi, \theta)|} \\ &= 4 \left\{ \int_0^1 M_k(\xi) \frac{r(\xi)J(\xi)}{D(\xi)} \left[P(x(\xi)) - Q(x(\xi)) \ln\left(\frac{x(\xi)}{(1-\xi)^2}\right) \right] d\xi + \right. \end{aligned}$$

$$2 \int_0^1 M_k(\xi) \frac{r(\xi)J(\xi)}{D(\xi)} Q(x(\xi)) \ln\left(\frac{1}{1-\xi}\right) d\xi \} \quad (\text{A2.42})$$

Substituting $\eta = 1 - \xi$ into the second integral, yields

$$\begin{aligned} b_{kij} = & 4 \left\{ \int_0^1 M_k(\xi) \frac{r(\xi)J(\xi)}{D(\xi)} \left[P(x(\xi)) - Q(x(\xi)) \ln\left(\frac{x(\xi)}{(1-\xi)^2}\right) \right] d\xi + \right. \\ & \left. 2 \int_0^1 M_k(1-\eta) \frac{r(1-\eta)J(1-\eta)}{D(1-\eta)} Q(x(1-\eta)) \ln\left(\frac{1}{\eta}\right) d\xi \right\} \end{aligned} \quad (\text{A2.43})$$

Similarly,

$$\begin{aligned} a_{kij} = & S_j \int_0^1 d\xi M_k(\xi) \int_0^{2\pi} d\theta \frac{\partial}{\partial n} \left(\frac{1}{|\mathbf{p}_i - \mathbf{q}(\xi, \theta)|} \right) \\ = & \int_0^1 M_k(\xi) \frac{-4r(\xi)}{D(\xi)^3} \left\{ \frac{R(x(\xi)) - S(x(\xi)) \ln\left(\frac{x(\xi)}{(1-\xi)^2}\right)}{1-k^2(\xi)} \left[\frac{dz(\xi)}{d\xi} (r(\xi) + r_0) - \right. \right. \\ & \left. \frac{dr(\xi)}{d\xi} (z(\xi) - z_0) - \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \right] + \frac{2}{k^2(\xi)} \frac{dz(\xi)}{d\xi} r_0 \left[P(x(\xi)) - \right. \\ & \left. \left. Q(x(\xi)) \ln\left(\frac{x(\xi)}{(1-\xi)^2}\right) \right] \right\} d\xi - \\ & 8 \int_0^1 M_k(1-\eta) \frac{r(1-\eta)}{D(1-\eta)^3} \left\{ \frac{S(x(1-\eta))}{1-k^2(1-\eta)} \left[\frac{dz(1-\eta)}{d(1-\eta)} (r(1-\eta) + r_0) - \right. \right. \\ & \left. \frac{dr(1-\eta)}{d(1-\eta)} (z(1-\eta) - z_0) - \frac{2}{k^2(1-\eta)} \frac{dz(1-\eta)}{d(1-\eta)} r_0 \right] + \\ & \left. \frac{2}{k^2(1-\eta)} \frac{dz(1-\eta)}{d(1-\eta)} r_0 Q(x(1-\eta)) \right\} \ln\left(\frac{1}{\eta}\right) d\eta \end{aligned} \quad (\text{A2.44})$$

and the treatments to the integrals are similar to the last case.

iii, $\mathbf{p}_i$ on the axis of symmetry

When the point $\mathbf{p}_i$ is on the axis of symmetry, that is $(r_0, z_0) = (0, z_0)$, the integrations can be simplified as follows

$$b_{kij} = \int_0^{2\pi} d\theta \int_0^1 M_k(\xi) \frac{r(\xi)J(\xi)}{[r^2(\xi) + (z(\xi) - z_0)^2]^{\frac{1}{2}}} d\xi$$

$$= 2\pi \int_0^1 M_k(\xi) \frac{r(\xi)J(\xi)}{[r^2(\xi)+(z(\xi)-z_0)^2]^{\frac{1}{2}}} d\xi \quad (\text{A2.45})$$

and

$$a_{kij} = -2\pi \int_0^1 M_k(\xi) \frac{r(\xi)\left(r(\xi)\frac{dz(\xi)}{d\xi} - (z(\xi)-z_0)\frac{dr(\xi)}{d\xi}\right)}{[r^2(\xi)+(z(\xi)-z_0)^2]^{\frac{3}{2}}} d\xi \quad (\text{A2.46})$$

A2.3 Linear algebraic equations

Based on the above work, one can get a set of equations as follows

$$\begin{aligned} 2\pi\phi_i + \sum_{j=1}^{n1-1} (a_{1ij}\phi_j + a_{2ij}\phi_{j+1}) \\ = \sum_{j=1}^{n1-1} (b_{1ij}\psi_j + b_{2ij}\psi_{j+1}) \quad i = 1, 2, \dots, n1 \end{aligned} \quad (\text{A2.47})$$

where $n1$ is the number of the nodes, and $n1 = n + 1$, n is the number of segments, and after some combinations, the following formulas are obtained,

$$2\pi\phi_i + \sum_{j=1}^{n1-1} \hat{H}_{ij}\phi_j = \sum_{j=1}^{n1-1} G_{ij}\psi_j \quad i = 1, 2, \dots, n1 \quad (\text{A2.48})$$

where

$$\hat{H}_{ij} = a_{1ij} + a_{2i(j-1)} \quad \text{for } j = 2, 3, \dots, n1 - 1 \quad (\text{A2.49})$$

and

$$G_{ij} = b_{1ij} + b_{2i(j-1)} \quad \text{for } j = 2, 3, \dots, n1 - 1 \quad (\text{A2.50})$$

For $j = 1$ and $j = n1$,

$$\hat{H}_{i1} = a_{1i1}, \quad G_{i1} = b_{1i1} \quad (\text{A2.51a,b})$$

$$\hat{H}_{in1} = a_{2i(n1-1)}, \quad G_{in1} = b_{2i(n1-1)} \quad (\text{A2.51c,d})$$

Defining $H_{ij} = \hat{H}_{ij} + 2\pi\delta_{ij}$, where $\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$, a more general formula of

the equations is given:

$$H\phi = G\psi \quad (\text{A2.52})$$

which can be solved using the Gauss elimination with partial pivoting. An additional device which proved helpful in increasing the accuracy of the computation has been to replace the diagonal elements of the matrix H_{ij} by

$$H_{ii} = 4\pi - \sum_{j \neq i} H_{ij} \quad (\text{A2.53})$$

This formula is applicable for a closed surface like a bubble. It cannot be applied for an opened surface like a free surface. This relation is deduced from the fact that the solution of an interior problem of the Laplace equation with a constant boundary condition is a constant.

A3 Cubic Spline Interpolation Method

The position of the bubble surfaces, and the velocity potential distribution on the surfaces, are only known explicitly at the calculation nodes. After re-gridding during each time step of the developed MEL-BIM model, the location and velocity potential at the new nodes are required, and must therefore be interpolated. Cubic splines are the lowest order interpolates which provide the second derivatives, which vary linearly between adjacent nodes.

In this calculation, the cubic splines are taken under the first boundary conditions, which means that the values of a function $y = f(x)$ at the knots $x_1 < x_2 < \dots < x_{n-1} < x_n$ is given as $y_1, y_2, \dots, y_{n-1}, y_n$, and the first derivatives at the endpoints are also known as $y'(x_1)$ and $y'(x_n)$. The calculations of the first derivatives $y'(x_j)$ ($j = 2, 3, \dots, n-1$) at the rest $n-2$ knots are given as:

$$a_1 = 0, b_1 = y'(x_1) \quad (\text{A3.1})$$

$$h_j = x_{j+1} - x_j, j = 1, 2, \dots, n-1 \quad (\text{A3.2})$$

$$\alpha_j = \frac{h_{j-1}}{h_{j-1} + h_j}, j = 2, 3, \dots, n-1 \quad (\text{A3.3})$$

$$\beta_j = 3 \left[\frac{(1-\alpha_j)(y_j - y_{j-1})}{h_{j-1}} + \frac{\alpha_j(y_{j+1} - y_j)}{h_j} \right], j = 2, 3, \dots, n-1 \quad (\text{A3.4})$$

$$a_j = -\frac{\alpha_j}{2 + (1-\alpha_j)\alpha_{j-1}}, j = 2, 3, \dots, n-1 \quad (\text{A3.5})$$

$$b_j = \frac{\beta_j - (1-\alpha_j)b_{j-1}}{2 + (1-\alpha_j)\alpha_{j-1}}, j = 2, 3, \dots, n-1 \quad (\text{A3.6})$$

$$y'(x_j) = a_j y'(x_{j+1}) + b_j, j = n-1, n-2, \dots, 2 \quad (\text{A3.7})$$

The second derivatives at all knots are given as:

$$y''(x_j) = \frac{6(y_{j+1}-y_j)}{h_j^2} - \frac{2(2y'(x_j)+y'(x_{j-1}))}{h_j}, j = 1, 2, \dots, n-1 \quad (\text{A3.8})$$

$$y''(x_n) = \frac{6(y_{n-1}-y_n)}{h_{n-1}^2} - \frac{2(2y'(x_n)+y'(x_{n-1}))}{h_{n-1}} \quad (\text{A3.9})$$

The integral of the function between $[x_1, x_n]$ is given as

$$\begin{aligned} T &= \int_{x_1}^{x_n} y(x) dx \\ &= \frac{1}{2} \sum_{i=1}^{n-1} (x_{i+1} - x_i) (y_i + y_{i+1}) \\ &\quad - \frac{1}{24} \sum_{i=1}^{n-1} (x_{i+1} - x_i)^3 (y_i'' + y_{i+1}'') \end{aligned} \quad (\text{A3.10})$$

The values and derivatives at the interpolant points s can be calculated using the following formulas:

$$\begin{aligned} y(s) &= \left[\frac{3}{h_i^2} (x_{i+1} - s)^2 - \frac{2}{h_i^3} (x_{i+1} - s)^3 \right] y_i \\ &\quad + \left[\frac{3}{h_i^2} (s - x_i)^2 - \frac{2}{h_i^3} (s - x_i)^3 \right] y_{i+1} \\ &\quad + h_i \left[\frac{1}{h_i^2} (x_{i+1} - s)^2 - \frac{1}{h_i^3} (x_{i+1} - s)^3 \right] y'(x_i) \\ &\quad - h_i \left[\frac{1}{h_i^2} (s - x_i)^2 - \frac{1}{h_i^3} (s - x_i)^3 \right] y'(x_{i+1}) \end{aligned} \quad (\text{A3.11})$$

$$\begin{aligned} y'(s) &= \frac{6}{h_i} \left[\frac{1}{h_i^2} (x_{i+1} - s)^2 - \frac{1}{h_i} (x_{i+1} - s) \right] y_i \\ &\quad - \frac{6}{h_i} \left[\frac{1}{h_i^2} (s - x_i)^2 - \frac{1}{h_i} (s - x_i) \right] y_{i+1} \\ &\quad + \left[\frac{3}{h_i^2} (x_{i+1} - s)^2 - \frac{2}{h_i} (x_{i+1} - s) \right] y'(x_i) \\ &\quad + \left[\frac{3}{h_i^2} (s - x_i)^2 - \frac{2}{h_i} (s - x_i) \right] y'(x_{i+1}) \end{aligned} \quad (\text{A3.12})$$

$$y''(s) = \frac{1}{h_i^2} \left[6 - \frac{12}{h_i} (x_{i+1} - s) \right] y_i$$

$$\begin{aligned}
& + \frac{1}{h_i^2} \left[6 - \frac{12}{h_i} (s - x_i) \right] y_{i+1} \\
& + \frac{1}{h_i} \left[2 - \frac{6}{h_i} (x_{i+1} - s) \right] y'(x_i) \\
& - \frac{1}{h_i} \left[2 - \frac{6}{h_i} (s - x_i) \right] y'(x_{i+1})
\end{aligned} \tag{A3.13}$$

where $s \in [x_i, x_{i+1}]$ and $i = 1, 2, \dots, n$.

A4 Quadrature Method

A4.1 Standard Gauss-Legendre quadrature

In numerical analysis, a quadrature rule is an approximation of the definite integral of a function, usually stated as a weighted sum of function values at specified points within the domain of integration. A 6-point Gaussian-Legendre quadrature rule is employed in this study. The domain of integration for such a rule is conventionally taken as $[-1, 1]$, so the rule is stated as

$$\int_{-1}^1 f(x) dx \approx \sum_{i=1}^n w_i f(x_i) \tag{A4.1}$$

Table A4.1 Weights and abscissas for n=6 in Gaussian-Legendre quadrature.

i	weight - w_i	abscissa - x_i
1	0.3607615730481386	-0.6612093864662645
2	0.3607615730481386	0.6612093864662645
3	0.4679139345726910	-0.2386191860831969
4	0.4679139345726910	0.2386191860831969
5	0.1713244923791704	-0.9324695142031521
6	0.1713244923791704	0.9324695142031521

A4.2 Stroud-Secrest quadrature

The integrals which contain an explicit singularity of log type can be integrated using the quadrature scheme tabulated by Stroud and Secrest (1966). A 6-point Stroud-Secrest quadrature rule is employed in this calculation.

$$\int_0^1 f(x) \ln\left(\frac{1}{x}\right) dx \approx \sum_{i=1}^n w_i f(x_i) \ln\left(\frac{1}{x_i}\right) \quad (\text{A4.2})$$

Table A4.2 Weights and abscissas for n=6 in Stroud-Secrest quadrature.

i	weight - w_i	abscissa - x_i
1	0.23876366257854756972	0.021634005844116948995
2	0.30828657327394679296	0.12958339115495079613
3	0.24531742656321038598	0.31402044991476550879
4	0.14200875656647668542	0.53865721735180214454
5	0.055454622324886290015	0.75691533737740285216
6	0.010168958692932275886	0.92266885137212023733