Waveform and system design for colocated MIMO radar

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WAVEFORM AND SYSTEM DESIGN FOR COLOCATED MIMO RADAR

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Summary

This thesis analyzes and develops waveforms and signal processing techniques that are used to improve the performances of range, velocity, and direction-of-arrival (DOA) estimation for multiple-input-multiple-output (MIMO) radar systems with colocated transmit and receive antennas. The main contributions of this work consist of four parts, which are summarized as follows.

The starting point of this work is MIMO radar waveform design. The freedom to transmit non-coherent waveforms, i.e., the waveform diversity, enables many new features for MIMO radar to improve target detection and parameter estimation. However, it also introduces new challenges for the waveform design, e.g., minimizing the cross-ambiguity functions (CAF's). To tackle this problem, we consider the extension of two frequency-hopping waveforms from single-input-single-output (SISO) radar to MIMO radar, the Costas array coded (CAC) waveforms and the quadratic congruence coded (QCC) waveforms. Exhaustive search is carried out for suboptimal solutions to finding a set of codes that mutually exhibit least cross-ambiguity properties. The general problem of the code selection for global optimum is formulated as an open question. The suboptimal Costas and quadratic congruence codes are used throughout this thesis when other designs are considered.

The second contribution of this thesis is on receiver instrumental variable filter (IVF) design for joint range and Doppler sidelobe suppression. The commonly used matched filter (MF) maximizes the signal-to-noise ratio (SNR) of its output. However, it is inefficient in improving the signal-to-clutter ratio (SCR). Alternatively, an IVF enables a trade-off between the SNR and SCR at the receiver. We first analyze the performances of existing range domain only IVF designs using integrated sidelobe level (ISL) constraint and zero sidelobe (ZS) constraint, respectively. The results indicate the possibility of using phased-array radar for improved performance. Then a rectangular area centered at the origin on the range-Doppler plane is considered for joint clutter sidelobe suppression, where both MIMO and phased-array radar are considered. Closed-form solutions and simulation results

are provided. The conclusion is then drawn in a trade-off manner. Compromising the waveform diversity, the phased-array radar design turned out to be more efficient in terms of sidelobe suppression, SNR loss, and computational complexity.

Another advantage introduced by MIMO radar waveform diversity is the flexible design of transmit beampatterns, which is the topic of the third part of this thesis. Existing works have provided many solutions. However, little attention has been paid to several aspects of the performance such as the ripple within the energy focusing section, the sidelobe attenuation, and the transition bandwidth. Hence, we propose a feasibility problem (FP) formulation based on the nonlinear mapping from MIMO radar transmit beampattern design to multiple-input-single-output (MISO) finite impulse response (FIR) filter design. One of the advantages of the proposed method is that it takes quantitative controls of the above parameters in the design. Moreover, the minimum number of transmit antenna to achieve a given transmit beampattern is obtained. Via simulations, the transmit beampattern formula, which has a similar form as Kaiser's formula, is obtained for antenna selection given a set of design specifications. In addition, a robust design method is proposed. The robust design establishes a trade-off between the radio-frequency amplifier (RFA) requirements and the number of transmit antennas. It also leads to significant relaxation of waveform correlation requirements. As a result, the robust design uses a set of easily generated and widely used linear frequency modulated (LFM) waveforms to achieve the transmit beampattern identical to that designed under ideal orthogonality assumption.

The last contribution of this thesis is presenting a fairer comparison between MIMO and phased-array radar. Throughout this thesis, the performance of phased-array radar is considered in comparison with MIMO radar in various aspects. The most significant difference between the two systems is that phased-array radar transmits coherent waveforms whereas MIMO radar transmits non-coherent waveforms. Implicitly, it is the coherent gain versus the diversity gain. The two systems establish a trade-off for researchers and engineers to choose under different objectives, design environments, and given conditions. Via the comparisons on the basic system models, receiver identifiability, receiver IVF design, and transmit beampattern design, we conclude that MIMO radar outperforms phased-array radar when multiple targets exist whereas the latter outperforms the former when one is dealing with only a single target.

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Abbreviations

AAF	Auto-ambiguity function
ACF	Auto-correlation function
AF	Ambiguity function
ASNR	Array signal-to-noise ratio
BPSK	Binary phase-shift keying
CAC	Costas array coded
CAF	Cross-ambiguity function
CCF	Cross-correlation function
CS	Compressive sensing
DOA	Direction-of-arrival
DPSS	Discrete prolate spheroidal sequence
EVD	Eigenvalue decomposition
FIR	Finite-impulse-response
FP	Feasibility problem
GA	Genetic algorithm
ISL	Integrated sidelobe level
IVF	Instrumental variable filter
LFM	Linear frequency modulated
LS	Least-squares
MF	Matched filter
MIMO	Multiple-input multiple-output
MISO	Multiple-input single-output
MR	Matrix regularization
PN	Pseudo-noise
QCC	Quadratic congruence coded
RCS	Radar cross section

SA	Simulated annealing
SCR	Signal-to-clutter ratio
SDP	Semidefinite programming
SIMO	Single-input multiple-output
SISO	Single-input single-output
SNR	Signal-to-noise ratio
SVD	Singular value decomposition
TBP	Transmit beamspace processing
ULA	Uniform linear array
ZS	Zero sidelobe

Notations

$\mathbb{C}$	Complex number
$\mathbb{R}$	Real number
$\mathbb{Z}$	Integer
$\text{Re}\{\cdot\}$	Real part of a complex variable
$\text{Im}\{\cdot\}$	Imaginary part of a complex variable
$E\{\cdot\}$	Expectation
$\{\cdot\}^*$	Complex conjugation
$\{\cdot\}^T$	Matrix and vector transpose operator
$\{\cdot\}^H$	Matrix and vector transpose conjugate operator
$ \cdot $	Absolute value
$\ \cdot\ _F$	Frobenius norm
$\ \cdot\ _p$	ℓ_p -norm, $p \in \mathbb{Z}^+$
$\text{tr}\{\cdot\}$	Matrix trace
$\text{rank}\{\cdot\}$	Matrix rank
$\cdot\ \cdot$	Parallel vector
$\odot$	Hadamard (element-wise) product operator
$\otimes$	Kronecker product operator
$\text{diag}\{\cdot\}$	Diagonal matrix or vector of diagonal elements
$\text{vec}\{\cdot\}$	Matrix vectorization
mod	Modulo operator
$\succeq$	Positive semidefinite matrix operator
Variables	Lowercase or uppercase italic letters
Vectors	Lowercase boldface letters
Matrices	Uppercase boldface letters
$\mathbf{I}$	Identity matrix with appropriate dimension
$\mathbf{1}$	All-one vector with appropriate dimension

Chapter 1

Introduction

1.1 General Introduction

Radar has been invented and in use for more than 100 years. It is a system that transmits electromagnetic waveforms to detect objects such as aircrafts, missiles, ships, vehicles, etc., and determine range, velocity, and direction of the objects. A radar system physically consists of waveforms, a transmitter, and a receiver to fulfill the functionality of target detection and parameter estimation. The common applications of radar systems are in diverse areas including air traffic control, defense systems, ocean surveillance, and medical treatment, etc.

The simplest structure of a radar system is using one transmit antenna to transmit a single waveform, and one receive antenna to process the reflected echoes. It is also referred to as a single-input-single-output (SISO) radar. Alternatively, an array of antennas can be deployed at the receiver, which forms a single-input-multiple-output (SIMO) radar. If the transmitter also uses an array of antennas which transmit weighted versions of a single waveform, then the system becomes a phased-array radar. Sometimes phased-array radar uses single antenna at the transmitter, which is equivalent to a SIMO radar [1]. In this thesis, phased-array radar refers to the one using multiple antennas at both transmitter and receiver with a single waveform.

Recently, researchers have migrated the multiple-input multiple-output (MIMO) concept in communication systems to radar applications. As a result, a new paradigm, i.e., the MIMO radar system, is proposed in [2]. The transmit and receive antennas can be widely separated [3] or colocated [4], and the antenna array

can take various forms of geometry. Within the recent decade, MIMO radar systems have been intensively studied [5]. The key property of a MIMO radar is that it transmits multiple non-coherent waveforms, which is referred to as the waveform diversity. The uniqueness of MIMO radar systems is characterized by the waveform diversity. At the transmitter, the waveform diversity enables flexible designs of the transmit beampatterns which focus the signal energy within the spatial region(s) of interest. At the receiver, the echoes of the non-coherent waveforms carry different information of the target(s) such that various signal processing techniques can be developed for improved detection and parameter estimation. This thesis focus on the study of MIMO radar with colocated transmit and receive uniform linear array (ULA) antennas. For comparison, it is assumed that the phased-array radar uses an identical antennas structure. The rich literature of research on MIMO radar appears from four aspects, which is reviewed and summarized as follows.

(i) The design of MIMO radar waveforms with good auto- and cross-ambiguities [6–15]. The ambiguity function (AF) is one of the commonly used tools to characterize the waveform resolution properties of the time delay (range) and Doppler effects (velocity) [16]. It is seen in [17] that the extension from conventional AF to MIMO radar ambiguity function (MIMO AF) leads to the challenges of dealing with multiple auto-ambiguities and extra cross-ambiguities among the non-coherent waveforms. To simplify the problem, the Doppler effects are usually ignored, which results in waveform set design with satisfactory auto-correlations and minimized cross-correlations. Examples can be found in [6, 9–12]. The waveform designs without such simplification, i.e., waveform designs considering both delay and Doppler resolutions exist in [7, 8, 13]. Generally, MIMO radar waveform design is a difficult problem, especially when larger numbers of antennas are considered. This problem has been drawing much attention from researches.

(ii) The development of receiver signal processing technique for enhanced parameter estimation [18–23]. Conventional receivers of radar systems are based on least-squares (LS) solution which maximizes the signal-to-noise ratio (SNR) at the filter output. The LS filter is also called the matched filter (MF) [24]. However, the MF is inefficient in the presence of clutter. To balance the noise control and clutter control of a radar system, the instrumental variable filter (IVF) which consists of a matched and a mismatched filter, is proposed to be an alternative to the MF. The mismatched filtering technique has been used for sidelobe reduction applica-

tions [9], which achieves clutter suppression. The authors in [21] has extended the IVF design to MIMO radar case, and the designed IVF is able to suppress the sidelobes within the nearby range bins of the range bin of interest. Moreover, the IVF is able to compensate the imperfection of the orthogonal waveforms which is inevitable. As a result, a sharp peak and low sidelobes appear within a short range bins centered at the origin of the IVF filter output. A closed-form solution to the IVF design problem is proposed in [22], which furthers the investigation in [21]. An alternative design which considers both the transmitted waveforms and the IVF is seen in [20].

(iii) The design of the transmit beampattern and signaling strategy [25–38]. Conventionally, signal processing techniques are applied at the receiver side, whereas the transmission is simple and straightforward, e.g., omnidirectional transmission for SISO and standard MIMO radar, and phase shifting beamforming for phased-array radar. It is recently discovered [25] that the waveform diversity of MIMO radar allows flexible transmit beampattern design, which is also referred to as transmit beamforming [26] or transmit energy focusing [28]. Instead of omnidirectional transmission of a standard MIMO radar, the transmit beamforming focuses the energy of the transmitted signals within a predefined spatial section of interest, so that the direction-of-arrival (DOA) estimation at the receiver can be improved. The transmit beampattern design consists of two procedures which first optimizes the covariance matrix of the transmitted signals and then determines the signaling strategies that exhibit the obtained covariance matrix. For the first procedure, various optimization problems have been proposed, which are seen in [25–28, 30, 33, 36–38]. Generally, they can be categorized as the beampattern matching design and the maximum power design [27]. For the second procedure, researchers commonly carry out partially correlated waveform design to achieve the desired covariance matrix [26, 27, 29, 31, 35]. Alternatively, the transmit beamspace processing (TBP) technique is proposed as a more efficient and simpler method [28]. Instead of determining a set of waveforms, the TBP technique determines a weighting matrix which is imposed on a set of orthogonal waveforms. Hence the works [6–13] introduced in the above section (i) can be adopted here for the TBP based transmit beampattern design.

(iv) The comparison between MIMO and phased-array radar, and the development of MIMO radar architecture [1, 39–49]. Researchers have intensively com-

pared phased-array and MIMO radar, and demonstrated the fact that MIMO radar transmitting orthogonal waveforms enjoys the waveform diversity to extract useful information at receiver via the use of different waveforms. However, this comes with the loss of coherent processing gain which is preserved in phased-array radars. As a result, MIMO and phased-array radar yield the same overall beam pattern for DOA estimation [43]. Inspired by the comparison results, the authors in [43–45] introduced new array architectures referred to as the phased-MIMO radar, the transmit subaperturing radar, and the hybrid MIMO phased-array radar, respectively, to take both the advantages of waveform diversity and coherent processing gain. These proposed ideas have furthered the MIMO radar research potentials and offered more flexibility on the system design.

In addition to the above mentioned research topics, the investigations on MIMO radar system analysis [17, 50, 51], performance bounds [52–56], and signal processing techniques based on compressive sensing (CS) [57–59], have also received much research attention. Generally, MIMO radar signal processing is an attractive research topic that is receiving increasing attention of researchers.

1.2 Basic MIMO Radar Signal Model

MIMO radar systems can be categorized into different types according to the locations of the transmit and receive antennas and the use of waveforms [60]. Specifically, we consider MIMO radar systems with colocated ULA antennas in this thesis. The basic signal model of a MIMO radar with colocated ULA antennas is presented in this section. For the simplicity and consistency of this thesis, the assumptions on the waveforms and system are summarized as follows.

- The transmit and receive antennas are isotropic without mutual coupling.
- The transmit and receive antennas share the same target azimuth angles.
- The transmit and receive antenna arrays are calibrated.
- The waveforms are narrowband planewaves.
- The targets are point targets, whose shapes are neglected.
- The targets are located at far field, where the propagation is nondispersive.

- The far field is a 2 dimensional space, where the elevation angle is 0° .
- The channels are lossless time delay and phase delay channels.

A general architecture of a MIMO radar system consists of transmit signal processing unit and receiver signal processing unit respectively. The basic realizations of the two units are omnidirectional transmission at the transmitter and the MF bank and the receiver. Consider a MIMO radar system with N_T transmit antennas and N_R receive antennas. The N_T non-coherent baseband waveforms are denoted as $s_i(t)$, $i \in \{0, 1, \dots, N_T - 1\}$, whose sampled vector form is $\mathbf{s}_i$ with sample length N . For simplicity, we assume all the waveforms have unit norm, i.e., $\|\mathbf{s}_i\| = 1$. Let the transmit signal matrix be $\mathbf{S} \in \mathbb{C}^{N_T \times N}$, then for the omnidirectional transmission,

$$\mathbf{S} = [\mathbf{s}_0, \mathbf{s}_1, \dots, \mathbf{s}_{N_T-1}]^T. \quad (1.1)$$

Assume the transmit and receive ULA's have inter element spacing of half the carrier wavelength, and the counterclockwise angle 90° corresponds to the broadside, the transmit and receive steering vectors are then expressed as

$$\mathbf{e}_T(\theta) = [1, e^{-j\pi \cos \theta}, \dots, e^{-j\pi(N_T-1) \cos \theta}]^T, \quad (1.2)$$

$$\mathbf{e}_R(\theta) = [1, e^{-j\pi \cos \theta}, \dots, e^{-j\pi(N_R-1) \cos \theta}]^T, \quad (1.3)$$

where θ denotes the azimuth angle. Let K denote the number of targets that reflect the signals to the receiver, then for the k th target, $k \in \{0, 1, \dots, K-1\}$, we denote a_k as the complex amplitude of the target which is proportional to the radar cross section (RCS), θ_k as the azimuth angle of the target, τ_k as the round trip sample delay due to the range of the target, and v_k as the Doppler shift due to the velocity of the target. Denote an $N \times N$ sample shifting matrix as

$$\mathbf{J}_{\tau_k} = \begin{bmatrix} 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad (1.4)$$

whose $(i, i + \tau_k)$ th element is one, and other elements are zeros. Denote the Doppler shifting matrix as

$$\mathbf{V}_{v_k} = [\mathbf{v}_{v_k}, \mathbf{v}_{v_k}, \dots, \mathbf{v}_{v_k}]^T \in \mathbb{C}^{N_T \times N}, \quad (1.5)$$

where

$$\mathbf{v}_{v_k} = [1, e^{j2\pi v_k/f_s}, e^{j2\pi 2v_k/f_s}, \dots, e^{j2\pi(N-1)v_k/f_s}]^T. \quad (1.6)$$

Then the receive signal matrix $\mathbf{Y} \in \mathbb{C}^{N_R \times N}$ is given by

$$\mathbf{Y} = \sum_{k=0}^{K-1} a_k \mathbf{e}_R(\theta_k) \mathbf{e}_T^T(\theta_k) [(\mathbf{S}\mathbf{J}_{\tau_k}) \odot \mathbf{V}_{v_k}] + \mathbf{Z}, \quad (1.7)$$

where $\mathbf{Z}$ is an independent and identically distributed (i.i.d.) noise matrix whose rows have constant power σ^2 . The MF at the receiver is similar to the above expression but with estimated variables $\hat{\theta}_k$, $\hat{\tau}_k$, and $\hat{v}_k$, i.e.,

$$\mathbf{F}_{\text{MF}} = \{\mathbf{e}_R(\hat{\theta}_k) \mathbf{e}_T^T(\hat{\theta}_k) [(\mathbf{S}\mathbf{J}_{\hat{\tau}_k}) \odot \mathbf{V}_{\hat{v}_k}]\}^H, \quad (1.8)$$

and the MF output is then given by $\text{tr}\{\mathbf{Y} \cdot \mathbf{F}_{\text{MF}}\}$.

The MIMO AF has a similar form of $\text{tr}\{\mathbf{Y} \cdot \mathbf{F}_{\text{MF}}\}$. Assume $K = 1$, $a_0 = 1$, and $\mathbf{Z} = \mathbf{0}$, then the MF output becomes the matching of a set of waveforms with its delayed, Doppler shifted, and phase shifted version, which is equivalent to the MIMO AF derived in [17]. Hence the MIMO AF is given by

$$\chi_{\text{MIMO}}(\hat{\theta}, \hat{\tau}, \hat{v}) = \text{tr} \left\{ \mathbf{e}_R(\theta) \mathbf{e}_T^T(\theta) [(\mathbf{S}\mathbf{J}_{\tau}) \odot \mathbf{V}_v] \left(\mathbf{e}_R(\hat{\theta}) \mathbf{e}_T^T(\hat{\theta}) [(\mathbf{S}\mathbf{J}_{\hat{\tau}}) \odot \mathbf{V}_{\hat{v}}] \right)^H \right\}. \quad (1.9)$$

Note that the original definition of AF is a function of delay τ and Doppler frequency v without the consideration of azimuth angle θ . In [17] and (1.9), θ is considered to complete the general definition. However for waveform design based on the MIMO AF, θ is usually neglected, i.e., the waveform properties on the delay and Doppler domain are given more emphasis. Examples can be found in [7, 8, 13].

1.3 Research Aims and Thesis Structure

This thesis aims to develop novel and efficient signal processing techniques for MIMO radar systems with colocated antennas. First, we look into MIMO radar

waveforms which are the prerequisite for further investigations on the system performance analysis and signal processing unit design. Note that throughout the thesis we only consider single pulse waveforms, which is considered in most of the cited references in this thesis. Problems related to pulse train waveforms are not in the scope of the thesis. Then we aim to develop novel IVF's at the receiver for enhanced clutter suppression in both delay and Doppler domain. After that, we carry out MIMO radar transmit beampattern design, with the objectives of developing new methods and establishing new formulas to characterize the relationships among various design parameters. In addition, we also investigate the possibility of formulating a robust design against practical implementation problems. In this thesis, similar designs and performances of phased-array radar are provided when we design the receiver and transmitter for MIMO radar. Our last objective is to provide an over-all comparison between MIMO radar and phased-array radar within the scope of the research carried out in this thesis. Specifically, the contributions of this thesis are summarized as follows.

- We design MIMO radar waveforms with good auto- and cross-ambiguities using Costas and quadratic congruence arrays.
- Using the obtained waveforms, we design MIMO radar receiver IVF's for joint range and Doppler sidelobe suppression.
- We formulate a novel feasibility problem (FP) for MIMO radar transmit beampattern design, and empirically obtain transmit beampattern formula.
- Based on the transmit beamspace processing (TBP) technique and the FP problem formulation, we propose a robust design against waveform imperfections and hardware limitations.
- We compare MIMO and phased-array radar in terms of receiver clutter suppression, transmit beampattern design, and identifiability, and then draw our conclusions on the basis of performance trade-off.

The thesis is organized as follows. Chapter 2 focuses on MIMO radar waveform design. Specifically, we extend the existing coding methods using Costas and quadratic congruence arrays to the case of MIMO radar. For a SISO or phased-array radar, the objective is to find a single waveform whose auto-ambiguity func-

tion (AAF) has sharp peak and low sidelobes. However, for a MIMO radar, multiple waveforms are required, thus one need to consider not only multiple AAF's, but also multiple cross-ambiguity functions (CAF's). In this chapter, the MIMO AF is derived in continuous time domain with a more explicit expression. An exhaustive search is carried out on a subset of the array ensemble, and suboptimal solutions are obtained. Then the general optimization problem to find the global optimum frequency hopping codes for MIMO radar waveforms is formulated. However, finding the global optimum is challenging and the solution is not provided. Simulation results illustrate satisfactory performances of the suboptimal solutions, and indicate the potential advantages of the global optimal solutions.

In Chapter 3, we look into MIMO radar receiver design for clutter suppression. The waveforms obtained in Chapter 2 are used in this chapter for simulations. Conventionally, a MF is used at the radar receiver to maximize the SNR of the filter output. However, it has little control of the clutter. Hence the IVF's are proposed to trade-off the SNR and signal-to-clutter ratio (SCR). In this chapter, we first review and analyze the performances of existing IVF's, i.e., the designs based on the peak sidelobe level (PSL) constraint [21] and integrated sidelobe level (ISL) constraint [22]. Then we consider a more challenging and general design which jointly suppresses the clutter sidelobes in both time delay and Doppler domain. Similar designs using phased-array radar are also presented for comparison. We compare the performances using MIMO and phased-array radar in terms of sidelobe suppression level, SNR loss and, computational complexity. It is then illustrated that although the use of phased-array radar compromises the waveform diversity, better performances for the clutter suppression in range, and joint range and Doppler domain, can be achieved using a phased-array radar.

The transmit beampattern design for MIMO radar is intensively studied in Chapter 4 and 5. Chapter 4 develops novel formulation of the design which is achieved via the nonlinear mapping from the transmit beampattern design to the multiple-input single-output (MISO) finite impulse response (FIR) filter design. The proposed design formulates an FP, which is different from most of the existing formulations. The FP formulation takes quantitative controls of the ripples within the energy focusing section, the attenuation levels outside of the energy focusing section, and the transition bandwidth. Moreover, the minimum number of the required transmit antennas can be obtained using the proposed algorithm.

In addition, we empirically obtain the transmit beampattern formula for antenna selection, which has a similar form of the Kaiser's formula for filter length estimation. Based on the proposed design, Chapter 5 investigates the robust design against waveform imperfection and hardware implementation. Instead of using the weighting matrix optimized under ideal hardware and waveform correlation assumptions, we derive closed-form solution to the robust weighting matrix, and use a set of linear frequency modulated (LFM) waveforms to achieve the transmit beampatterns identical to those under ideal assumptions.

In Chapter 6, we compare the performances of MIMO and phased-array radar in terms of the basic system structure, receiver clutter suppression using IVF's, transmit beampattern design, and receiver identifiability. Since the IVF design using phased-array radar is discussed in Chapter 3, we put more emphasis on the remaining aspects. We first derive the basic signal model of phased-array radar systems using identical transmit and receive antennas as MIMO radar systems. Then the design of transmit beampattern using phased-array radar is considered and the performance is compared with that using MIMO radar. After that, we present several examples for the receiver identifiability of the two systems. Finally the conclusion of the comparison is drawn as a trade-off between the diversity gain (MIMO radar) and coherent gain (phased-array radar).

Chapter 2

MIMO Radar Waveform Coding Using Costas and Quadratic Congruence Arrays¹

2.1 Introduction

MULTIPLE-INPUT MULTIPLE-OUTPUT (MIMO) radar systems usually transmit orthogonal waveforms. For parameter estimation of an airborne target which is usually moving, a proper choice of the waveforms should satisfy that the orthogonality holds not only for time shifts, but also for frequency shifts. This is because the reflected echoes have a round-trip delay and a doppler shift. Therefore, the use of the ambiguity function (AF) which provides a more comprehensive way of analyzing signal ambiguity properties, is better than the use of only time domain auto- and cross-correlation functions, as a tool for waveform design.

In [6, 9], the authors proposed MIMO radar waveform design methods to reduce the auto-correlation sidelobe and aperiodic cross-correlation considering only time domain properties. In [61], an iterative algorithm for unimodular waveform design is proposed for improved detection performance. The estimation/detection accuracy and resolution may degrade drastically if the proposed waveforms happen to be highly correlated at certain range-doppler bins, resulting in high sidelobes appearing within the delay-doppler plane excluding the zero doppler cut in the

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plot of the cross-ambiguity function (CAF). To avoid this, we use auto-ambiguity function (AAF) and CAF instead of auto-correlation function (ACF) and cross-correlation function (CCF), to analyze the performance of the coded waveforms. Similar works can be found in [7, 8, 13]. In [7], the authors have provided some insights on discovering good frequency hopping codes to achieve desired MIMO AF's using simulate annealing (SA) algorithm for MIMO radar with separated antennas. In [8], the MIMO AF is extended to wideband case. In [13], waveform design method based on the inspiration from animal echolocation is presented, which results in enhanced delay-Doppler resolution. In this chapter, we look into the waveform design problem from a new perspective, i.e., the extension of well designed SISO radar waveforms, such as the Costas array coded (CAC) and quadratic congruence coded (QCC) waveforms, to MIMO radar systems.

The use of Costas array coded (CAC) and quadratic congruence coded (QCC) waveforms for radar systems dates back to several decades. These codes were commonly used in conventional single-input single-output (SISO) sonar and radar applications for their preferable auto-ambiguity and cross-ambiguity properties [62–65]. When only the delay and Doppler effect are considered (direction of arrival (DOA) excluded), a SISO radar can be considered as a special case of a phased-array radar. In this Chapter, the CAC and QCC waveforms are extended to MIMO scenario for the first time, and new optimum searching problems are formulated. One of the problems introduced by migrating SISO waveforms to MIMO radar is to deal with the CAF's. The CAF's can be considered as interference in the MIMO AF on the delay-Doppler plane. Hence, regardless of the diversity gain of MIMO radar systems [39], it is preferable that the CAF's are minimized and the MIMO AF approaches the SISO AF.

2.2 MIMO Radar Ambiguity Functions

The MIMO AF is derived and analyzed in [17], which can be equivalently expressed as a function of the azimuth angle θ , the round trip delay τ and the Doppler shift v as shown in (1.9). Here we simplify this expression by excluding θ , and derive the continuous domain MIMO AF using the AAF's and CAF's. In this way we put more emphasis on the delay Doppler properties of the waveforms. Following the notation in Section 1.2, the complex envelope of the baseband signal transmitted

Figure 2.1: Schematic Diagram of SISO (left) and MIMO (right) radar system.

by the i th transmit antenna is denoted as $s_i(t)$. Then at the m th receive antenna, $m \in \{0, 1, \dots, N_R - 1\}$, the received echo of $s_i(t)$, denoted as $s_{i,m}(t, \tau, v)$, is given by

$$s_{i,m}(t, \tau, v) = s_i(t - \tau) e^{j2\pi vt} e^{j\pi(i+m) \cos \theta}, \quad (2.1)$$

where the counterclockwise azimuth angle $\theta = 90^\circ$ for the broadside. τ and v represent respectively the delay and Doppler frequency. Hence the echoes of all the transmitted waveforms received at the m th receive antennas is then given by

$$y_m = \sum_{i=0}^{N_T-1} s_{i,m}(t, \tau, v) = \sum_{i=0}^{N_T-1} s_i(t - \tau) e^{j2\pi vt} e^{j\pi(i+m) \cos \theta}. \quad (2.2)$$

The schematic diagram of MIMO radar systems is depicted in Fig. 2.1. It can be seen that each receive antenna is fed with a mixture of the transmitted waveforms, and totally there are $N_T \times N_R$ versions of reflected echoes grouped by N_R receive antennas. For each receive antenna, MF's are used to extract the effect of each transmitted waveform respectively, thus there are totally $N_T \times N_R$ MF's and each MF output is a mixture of N_T components. Assume that the array processing at the receiver perfectly compensates the phase term, i.e., $\exp\{j\pi(i+m) \cos \theta\}$ in (2.1) and (2.2), then the AAF and CAF are given by

$$\chi_{\text{AAF}}^{i,i}(\tau, v) = \int s_i(t) s_{i,m}^*(t, \tau, v) dt, \quad (2.3)$$

$$\chi_{\text{CAF}}^{i,i'}(\tau, v) = \int s_i(t) s_{i',m}^*(t, \tau, v) dt, i \neq i'. \quad (2.4)$$

The MIMO AF can be obtained from the MF output under single target assumption [16], i.e.,

$$\chi_{\text{MIMO}}(\tau, v) = \left| \sum_{i=0}^{N_T-1} \sum_{m=0}^{N_R-1} \int s_i(t) y_m^*(t, \tau, v) dt \right|^2. \quad (2.5)$$

Using (2.1)-(2.4), (2.5) can be decomposed as

$$\chi_{\text{MIMO}}(\tau, v) = \left| \sum_{i=0}^{N_T-1} \sum_{m=0}^{N_R-1} \left(\chi_{\text{AAF}}^{i,i}(\tau, v) + \sum_{i'=0, i' \neq i}^{N_T-1} \chi_{\text{CAF}}^{i,i'}(\tau, v) \right) \right|^2. \quad (2.6)$$

For conventional radar waveform design, only one waveform is considered. Hence the problem is to obtain a SISO AF, i.e., $\chi_{\text{AAF}}^{0,0}(\tau, v)$ with sharp central peak and low sidelobes. However when a MIMO radar system is considered, one needs to optimize not only multiple AAF's, but also the CAF's. This is illustrated by (2.6). The CAF's are considered as interference in MIMO AF, and they are to be minimized. Suppose that each waveform has satisfactory AAF's and the CAF's are ideally zeroes, then take the system in Fig. 2.1 for example, the MIMO AF becomes

$$\chi_{\text{MIMO}} = \left| \sum_{i=0}^2 \sum_{m=0}^2 \chi_{\text{AAF}}^{i,i}(\tau, v) \right|^2 = 3 \left| \sum_{i=0}^2 \chi_{\text{AAF}}^{i,i}(\tau, v) \right|^2, \quad (2.7)$$

which is the summation of all the AAF's. Hence the objective of designing multiple MIMO radar waveforms is to minimize the CAF's while preserving the AAF property. As a result, we have a MIMO AF with a sharp peak at the origin and lowest sidelobes in the delay Doppler plane.

2.3 CAC and QCC waveforms for MIMO Radar

Frequency hopping signals are commonly used in radar and sonar applications. However, in the family of frequency hopping signals, the well designed CAC and QCC waveforms, which have AAF's with sharp peak and low sidelobes, have been paid little attention in the context of MIMO radar waveform design. Next, we review the construction of the CAC and QCC signals, and address the optimization problems of the extension of CAC and QCC waveforms from SISO to MIMO radar. The conventional linear frequency modulated (LFM) waveform is considered as the basic waveform for the coding, which is illustrated in the following sections.

Figure 2.2: An example of (a) LFM, (b) CAC, and (c) QCC waveform representation, $M = 5$.

2.3.1 Costas and Quadratic Congruence Arrays

Let the coding sequence be $\mathbf{c}$ and we will use 2-D array plots to describe the LFM, CAC, and QCC signals. For a given bandwidth B , suppose the coding dimension is M , which means that the signal is equally divided into M segments and reordered along the time line, then the codes have a dimension of M , i.e., $\mathbf{c} \in \mathbf{Z}_+^M$. Figure 2.2 gives an example of the codes, which can be further simplified to 1-D arrays, such as $\mathbf{c}_a = \{1, 2, 3, 4, 5\}$, $\mathbf{c}_b = \{2, 1, 5, 3, 4\}$, and $\mathbf{c}_c = \{5, 1, 4, 4, 1\}$. There are several methods to generate Costas arrays, but it is still an open problem to find all of them. For illustration, we consider Welch's method as an example. More methods and in-depth details for Costas array construction can be found in [64, 65].

The codes for an $M \times M$ Welch-Costas array is given by

$$\mathbf{c}_{\text{CAC}}(n) = \alpha^n \bmod (M + 1), \quad n \in 1, M, \quad (2.8)$$

where α is a primitive root modulo prime $(M + 1)$. Under certain conditions, an $(M - 1) \times (M - 1)$ Costas array can be deduced from an $M \times M$ array [65]. Figure 2.2 (b) is generated with $M = 6$ and $\alpha = 3$ and dimension reduction of 1.

The codes for an $M \times M$ quadratic congruence array is given by [62, 63]

$$\mathbf{c}_{\text{QCC}}(n) = \beta(n - 1)^2 \bmod M, \quad n \in \{1, M\}, \quad (2.9)$$

where β is a positive integer and M is prime. Figure 2.2 (c) is generated with $M = 5$ and $\beta = 1$. It should be noticed that for Costas array, the appearance of all frequency levels within B is preserved, whereas for QCC some frequency levels are missing. This is seen in Figure 2.2 where the codes of Costas array occupy the entire bandwidth from code 1 to 5, but the codes of Quadratic Congruence array miss frequency bins 2 and 3.

Figure 2.3: A demonstration of sidelobe matrices. (a), auto-ambiguity of Figure 2.2 (b), cross-ambiguity between Figure 2.2 (b) and (c).

The ambiguity properties of the coded signals can be visualized using a sidelobe matrix. A sidelobe matrix is a 2-D convolution matrix of two coded arrays whose hits are replaced by 1. Figure 2.3 shows the auto-ambiguity sidelobe matrix of the CAC array and the cross-ambiguity sidelobe matrix between the CAC and QCC array in Figure 2.2. Note that practically CAC and QCC waveforms are not simultaneously used, here showing the example of the cross-ambiguity sidelobe matrix is for illustration purpose only. A sidelobe matrix agrees with the AAF or CAF of the corresponding coded waveforms, which is illustrated in Section 5.1 of [16]. Note that the sum of the coincidences is M^2 in a sidelobe matrix. The number of maximum coincidences in a cross-ambiguity sidelobe matrix is denoted as H , e.g., in Figure 2.3 (b), $H = 2$.

2.3.2 Multiple CAC and QCC Waveforms

The CAC and QCC signals have been used for conventional SISO sonar and radar applications with the objective of achieving sharp-peak-low-sidelobe AAF's. If we consider coding waveforms for MIMO radar systems, then N_T (instead of conventionally only one) CAC or QCC waveforms are needed. As the auto-ambiguity properties are well preserved by the nature of CAC and QCC waveforms [62–65], we mainly focus here on their cross-ambiguity properties. It has not been well studied that whether there exists N_T CAC waveforms or QCC waveforms which have desirable low cross-ambiguity values, given the coding dimension M . The objective of the coding can be explicitly stated as to find N_T waveforms such that the H of the cross-ambiguity sidelobe matrix between any waveform pair reaches the minimum. The general optimization problem for CAC and QCC waveforms

are presented as

$$\begin{aligned}
& \min_{\mathbf{c}_m} \max H \\
& \text{s.t. } m = 0, 1, \dots, N_T - 1, \\
& \mathbf{c}_m \in \text{Costas or quadratic congruence array.}
\end{aligned} \tag{2.10}$$

The above problem is difficult in terms of array generation and the computational complexity. Given a coding dimension M , the number of Costas arrays among all $M!$ frequency hopping codes is not easily determined. The results based on exhaustive search has proven that the number of Costas arrays increases monotonically with M when $M \leq 17$, and the trend reverses afterwards [16]. When $M = 17$, there exists 18276 Costas arrays, thus the total number of possible transmitted signal sets is $\binom{18276}{N_T}$, and for each set $\binom{N_T}{2}$ CAF's need to be evaluated in the exhaustive search. This is a computationally cumbersome procedure, especially when M and N_T are large. Usually, M is preferred to be large to reduce the sidelobe levels in the AAF's as the sidelobe peak reduction ratio is $1/M$ [16] (See Fig. 2.3 (a)).

For QCC waveforms, the number of quadratic congruence arrays is tractable. According to (2.9), we have

$$\begin{aligned}
\mathbf{c}_{\text{QCC}}(k) &= \beta(n-1)^2 \bmod M \\
&= (\beta + M)(n-1)^2 \bmod M,
\end{aligned} \tag{2.11}$$

which indicates that the number of quadratic congruence arrays is $(M-1)$, in which $\beta = 1, 2, \dots, M-1$. In an exhaustive search, the total number of possible transmitted signal sets is $\binom{M-1}{N_T}$, and $\binom{N_T}{2}$ CAF's need to be evaluated for each set. For a large M , this is still not computationally efficient.

Generally, we see from the above analysis that identifying multiple waveforms from the ensemble of frequency hopping codes, to exhibit a MIMO AF with sharp peak at the origin and low sidelobes in the Delay-Doppler plane, is difficult. In this Chapter, instead of looking for efficient solutions to this problem, we reveal the possibility of the extension of well designed SISO waveforms to MIMO radar, and provide suboptimal solutions for illustration.

Figure 2.4: The AAFs of CAC and QCC waveforms, $M = 5$. (a), $\mathbf{c}_{\text{CAC}} = \{2, 1, 5, 3, 4\}$. (b), $\mathbf{c}_{\text{QCC}} = \{5, 1, 4, 4, 1\}$.

2.4 Numerical Examples

In this section, we present examples of using CAC and QCC signals for MIMO radar systems where $N_T = 4$. A rectangular windowed LFM pulse (chirp) is used as the original signal for coding. The complex envelope of the LFM signal is given by

$$s(t) = \frac{1}{\sqrt{T}} \text{rect}\left(\frac{t}{T}\right) e^{j\pi\kappa t^2}, \quad (2.12)$$

where T is the LFM pulse duration, and κ is the chirp rate. The bandwidth of the pulse is $B = \kappa T$, and the bandwidth-time product is $BT = \kappa T^2$. In all the simulations in the following content, $N_T = 4$, $T = 1$ ms, and $k = 10^7$ s $^{-2}$, based on which the CAC and QCC signals are generated. Thus $B = 10$ kHz, $BT = 10$.

Figure 2.5: The AAFs of CAC and QCC waveforms, $M = 17$. (a), $\mathbf{c}_{\text{CAC}} = \{2, 1, 15, 7, 5, 11, 4, 16, 13, 17, 6, 14, 9, 12, 3, 8, 10\}$. (b), $\mathbf{c}_{\text{QCC}} = \{17, 1, 4, 9, 16, 8, 2, 15, 13, 13, 15, 2, 8, 16, 9, 4, 1\}$.

2.4.1 The AAF's

The AAF's of Both CAC and QCC signals exhibits desirable sharp peaks and low sidelobes. The sidelobes are caused by two factors: the sidelobes of *sinc* function and the hopping coincidences. The former is a property of LFM signals. The latter is due to auto-ambiguity sidelobe matrices, where there are coincidences within the Delay-Doppler plane. Figure 2.4 and 2.5 provide the plots of 4 AAF's of CAC and QCC waveforms with $M = 5, 17$ respectively. It can be seen that by increasing M , the auto-ambiguity properties are improved for both CAC and QCC cases. The sidelobes are lowered and spread into more areas.

Note that B and T are fixed for different coding dimensions, which indicates that the original LFM waveform is divided into 5 and 17 segments respectively

and then reordered to form the coded waveforms. The process of dividing and reordering of the LFM waveform introduces discontinuity of the phase between two consecutive segments. The discontinuity is more severe when M is larger. The sudden changes of the phases result in that the effective bandwidth of the coded waveforms is higher than B . Note that the range and Doppler resolution are inversely proportional to the bandwidth and time duration of the waveform [66]. Since the bandwidth is unchanged when increasing the coding dimension, the range resolution is also unchanged. However, the Doppler resolution is improved when increasing M introduces longer duration of signals. This is seen in Fig.2.4 (c) and (d). However here it is not considered as an improvement because the phase discontinuity is a more challenging than appropriately increasing B for hardware implementation. The phase discontinuity can be compensated via appropriate selections of B and T according to M . Such examples are not provided here. However in the later chapters, CAC and QCC waveforms with phase continuity are provided for other system designs.

2.4.2 The CAF's

This subsection addresses the new problem of waveform coding for MIMO radar systems, i.e., the evaluation of multiple CAF's. Neither to find all the Costas codes for large M nor to find N_T codes in the CAC or QCC pool with best cross-ambiguity properties is easy. In the simulation, we search in subsets of the ensemble and use several suboptimal solutions for demonstration. When $M = 5$, there are 40 Costas arrays and 4 quadratic congruence arrays. Since $N_T = 4$, the only choice for the QCC waveforms is to use all the existing codes. For CAC waveforms, we first calculate all the CAF's of a subset (10 out of 40) of the codes, and then search for 4 codes that on average exhibit lowest sidelobes. When $M = 17$, there are 16 quadratic congruence arrays but 18276 Costas arrays. In this case, we carry out global search for quadratic congruence codes, and suboptimal search for Costas codes. The Costas codes (27 out of 18276) are chosen from Golomb's and Welch's set [64], and the quadratic congruence codes are generated using (2.9). The codes

Figure 2.6: The CAF's of CAC and QCC waveforms. (a), CAC, $M = 5$. (b), QCC, $M = 5$. Each pair of subfigures are chosen from 6 CAF's among 4 waveforms. The left one corresponds to the CAF with maximum peak, and right one corresponds to the CAF of minimum peak).

selected after exhaustive searches are given below,

$$\begin{aligned}
 \mathbf{c}_{\text{CAC},5}^0 &= \{2, 1, 5, 3, 4\}, \quad \mathbf{c}_{\text{CAC},5}^1 = \{4, 3, 5, 1, 2\}; \\
 \mathbf{c}_{\text{CAC},5}^2 &= \{1, 3, 4, 2, 5\}, \quad \mathbf{c}_{\text{CAC},5}^3 = \{5, 2, 4, 3, 1\}; \\
 \mathbf{c}_{\text{QCC},5}^0 &= \{5, 1, 4, 4, 1\}, \quad \mathbf{c}_{\text{QCC},5}^1 = \{5, 2, 3, 3, 2\}; \\
 \mathbf{c}_{\text{QCC},5}^2 &= \{5, 3, 2, 2, 3\}, \quad \mathbf{c}_{\text{QCC},5}^3 = \{5, 4, 1, 1, 4\};
 \end{aligned}$$

(a) Left $\mathbf{c}_{\text{CAC}_17}^0, \mathbf{c}_{\text{CAC}_17}^1$. Right $\mathbf{c}_{\text{CAC}_17}^0, \mathbf{c}_{\text{CAC}_17}^2$.

(b) Left $\mathbf{c}_{\text{QCC}_17}^1, \mathbf{c}_{\text{QCC}_17}^2$. Right $\mathbf{c}_{\text{QCC}_17}^1, \mathbf{c}_{\text{QCC}_17}^3$.

Figure 2.7: The CAF's of CAC and QCC waveforms. (a), CAC, $M = 17$. (b), QCC, $M = 17$. Each pair of subfigures are chosen from 6 CAF's among 4 waveforms. The left one corresponds to the CAF with maximum peak, and right one corresponds to the CAF of minimum peak).

$$\begin{aligned}
 \mathbf{c}_{\text{CAC}_17}^0 &= \{2, 1, 15, 7, 5, 11, 4, 16, 13, 17, 6, 14, 9, 12, 3, 8, 10\}, \\
 \mathbf{c}_{\text{CAC}_17}^1 &= \{5, 17, 3, 1, 4, 14, 10, 7, 13, 12, 2, 11, 6, 8, 15, 16, 9\}, \\
 \mathbf{c}_{\text{CAC}_17}^2 &= \{13, 1, 15, 17, 14, 4, 8, 11, 5, 6, 16, 7, 12, 10, 3, 2, 9\}, \\
 \mathbf{c}_{\text{CAC}_17}^3 &= \{12, 16, 11, 3, 13, 10, 9, 15, 17, 5, 1, 6, 14, 4, 7, 8, 2\}; \\
 \mathbf{c}_{\text{QCC}_17}^0 &= \{17, 1, 4, 9, 16, 8, 2, 15, 13, 13, 15, 2, 8, 16, 9, 4, 1\}, \\
 \mathbf{c}_{\text{QCC}_17}^1 &= \{17, 4, 16, 2, 13, 15, 8, 9, 1, 1, 9, 8, 15, 13, 2, 16, 4\}, \\
 \mathbf{c}_{\text{QCC}_17}^2 &= \{17, 13, 1, 15, 4, 2, 9, 8, 16, 16, 8, 9, 2, 4, 15, 1, 13\}, \\
 \mathbf{c}_{\text{QCC}_17}^3 &= \{17, 14, 5, 7, 3, 10, 11, 6, 12, 12, 6, 11, 10, 3, 7, 5, 14\}.
 \end{aligned}$$

Figure 2.6 and 2.7 show the CAF's generated by the above 4 signal sets. For each set, there are 6 CAF's involved. To emphasize the cross-ambiguity properties, only the CAF's with maximum peak and minimum peak (2 out of 6) are presented for each set. Note that unlike the AAF's in Figure 2.4 and 2.5, the CAF's are not central symmetric. It is also indicated in this figure that increasing the coding dimension M results in reduction of the CAF sidelobes.

For the QCC waveforms when $M = 5$, we see from Fig. 2.6 (b) that the maximum sidelobe peak is 4.5 which is more likely to disturb the estimation of the true parameters. It is then seen in Fig. 2.7 (b) that by increasing M from 5 to 17, the maximum sidelobe peak is reduced to 0.2. Moreover, the sidelobes are distributed in more areas on the Delay-Doppler plane.

For the CAC signals, we see similar improvement by increasing the coding dimension M as shown in Fig. 2.6 (a) and 2.7 (a). We also see that given the same coding dimension, when M is small, the CAC waveforms performs better than the QCC waveforms for wider distributed sidelobes and their lower peaks. When M is increasing, The CAF's of the CAC waveforms have slightly higher peak values than those of QCC waveforms. However, this may not always be true. It should be noted that the construction of Costas arrays is much more complicated than that of quadratic congruence arrays. For $M = 17$, there is a total of 16 quadratic congruence arrays, while there are 18276 Costas arrays. The waveform sets used in the simulations are only chosen from a pool of 27 candidates which are easier generated. Thus the choice of the CAC waveform set is not globally optimized. There may be superior sets which cannot be easily found.

2.4.3 The MIMO AF's

Figure 2.8 shows the MIMO AF's of the CAC and QCC waveforms respectively, where $M = 17$. The plots in Figure 2.4 can be viewed as SISO AF's. The sidelobes are reduced, although not significantly, in MIMO AF's as compared to the corresponding SISO AF's. This is explained as follows. When all the AAF's and CAF's are summed to form the MIMO AF, the peaks of AAF's are coherently accumulated. However, the sidelobes in AAF's and peaks in CAF's occur in different delay-doppler areas, and they do not have coherent accumulations as the AAF peaks. Both the MIMO AF's of CAC and QCC waveforms have sidelobe peaks at around 0.1, and other sidelobes are more widely distributed in the delay-doppler

Figure 2.8: The MIMO AF's of (a) CAC and (b) QCC waveforms, $N_T = 4$, $M = 17$.

plane. This can be further improved by increasing the coding dimension M or number of antennas N_T .

2.5 Conclusions

In this chapter, the waveforms used for conventional SISO radar and sonar systems, i.e., the CAC and QCC waveforms, are for the first time extended to MIMO radar systems. The optimization problem for waveform coding that minimizes the CAF sidelobe levels are formulated. However, this optimization is difficult (especially for CAC waveforms) to solve in terms of the generation of Costas codes and the computational complexity. Instead of solving for global optimal solutions to this problem, we have presented several examples of suboptimal solutions via the

exhaustive search among subsets of the ensemble of the codes. Via the search of waveform sets ($N_T = 4$, $M = 5, 17$) with lower CAF peaks, we have obtained a suboptimal CAC waveform set in a pool of 27 candidates instead of the ensemble containing 18276 waveforms. We have also obtained an optimal QCC signal set via an exhaustive search from 16 candidates. Computer simulations show that on one hand, the extension of CAC and QCC waveforms from SISO to MIMO radar systems is feasible as the MIMO AF's exhibit satisfactory ambiguity properties; on the other hand, by increasing the coding dimension, the sidelobes in the MIMO AF's can be reduced. In addition, for the CAC waveforms, the ambiguity properties can be further improved if global optimum can be found.

In general, both the construction of Costas arrays and the search for N_T least correlated waveforms from the ensemble are challenging. The search also becomes tedious for QCC waveforms with large coding dimension M . Future research can focus on the design of efficient searching algorithms. The original waveform for coding can also be of other forms, not necessarily an LFM waveform. In the following chapters, the CAC and QCC waveforms are used in the simulations for performance analysis of MIMO radar systems designed to estimate various target parameters

Chapter 3

MIMO and Phased-Array Radar Receiver Design for Range and Doppler Sidelobe Suppression¹

3.1 Introduction

THE receiver [53, 54] instrumental variable filter (IVF) design for sidelobe suppression using multiple-input-multiple-output (MIMO) and phased-array radar is considered in this chapter. First, we analyze the performance of existing integrated sidelobe level (ISL) and the zero sidelobe (ZS) methods for range sidelobe suppression, and show the latter's superiority. The limitation of MIMO radar sidelobe suppression in a large number of range bins is discussed. It is shown that the phased-array radar avoids this limitation. Then we consider joint range and Doppler sidelobe suppression, and the solutions for both MIMO and phased-array radar are derived, classified, and analyzed. The conclusions are drawn through performance trade-off. The use of phased-array radar compromises the waveform diversity of MIMO radar (an excellent comparison work on the diversity is presented in [39]). However, with the objective of range, and joint range and Doppler sidelobe suppression, phased-array radar has better performance in terms of computational complexity, sidelobe suppression level and signal-to-noise ratio (SNR) loss. This will relieve the design workload for large scale radar systems. In addi-

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tion, we show via simulations that the bounds of clear area of matched filter output (ambiguity function) are inapplicable for the IVF design.

A MIMO radar system simultaneously transmits multiple waveforms and processes the reflected signals at the receiver with multiple antennas to detect targets and estimate target parameters such as the direction of arrival (DOA), Doppler frequency, range, and reflection coefficients. With the possibility of transmitting different waveforms, MIMO radar can achieve better system flexibility, spatial diversity and estimation resolution [5, 40]. This has been referred to as the MIMO radar waveform diversity.

As been addressed in Chapter 1, to maintain the functionality of MIMO radar in many applications, the transmitted waveforms are usually required to have very good auto- and cross-ambiguity properties. These properties are reduced to the auto- and cross-correlations when the Doppler effects are neglected. Consequently, the waveform design and optimization become the main focus in MIMO radar research [6–10, 14, 15]. However, waveforms with good ambiguity properties are difficult to find, especially when one needs to deal with a large number of transmit antennas and the Doppler effects cannot be neglected. To compensate the imperfection of the transmitted waveforms, it is possible to design matched and mismatched receiver filters [19, 22], or jointly design transmitted waveforms and receiver filters [20] and [21] to formulate a trade-off between signal-to-noise ratio (SNR) and signal-to-clutter ratio (SCR).

In [20], a cost function considering a joint design of transmitted waveforms and receiver filters to achieve the desired transmit beampattern together with the suppression of the auto- and cross-correlation sidelobes is proposed. However it is a computationally complex problem which involves optimizations of two matrices of large dimension. In [19, 21], the authors formulated the problem of designing an instrumental variable (IV) filter to suppress clutter from different range cells, and proposed the peak sidelobe level (PSL) method for the solution. In [22], the author proposed the integrated sidelobe level (ISL) method providing a closed-form solution with more computational efficiency, and briefly introduced the zero sidelobe (ZS) conditions. However, it has also been shown in [22] that the system would not be able to tolerate the Doppler effects.

In this chapter, we extend the IVF design problem to joint suppression of range and Doppler sidelobes, and illustrate the trade-off of choosing MIMO and phased-

array radar. First, the performances of ISL and ZS methods without considering the Doppler effects are analyzed. When the number of range cells for sidelobe suppression increases, MIMO radar fails to achieve the desired suppression level. In this case, it is shown that the phased-array radar can be used to fulfill the requirement. Second, the design for joint range and Doppler sidelobe suppression is considered. This is done with the inclusion of the Doppler shift matrix into the received signal model. We show that both MIMO and phased-array radar can be used for joint range-Doppler suppression of sidelobes. It is elaborated that phased-array radar offers better performance in terms of computational complexity, sidelobe suppression, and SNR loss. However, this is achieved at the price of losing the waveform diversity of MIMO radar.

3.2 System Model with Clutter

The MIMO radar system model considered in this chapter is similar to (1.7) but with the inclusion of the echoes from the clutter. Here we denote the shifting matrix $\mathbf{J}_{\tau_k}$ in (1.4) as $\mathbf{J}_p$, where p is the range bin index. We define a range section using P such that $p \in \{-P, \dots, -1, 1, \dots, P\}$. The existing clutter within the range bins from $-P$ to P are supposed to be suppressed. Denote the transmitted waveform matrix as in (1.1),

$$\mathbf{S} = [\mathbf{s}_0, \mathbf{s}_1, \dots, \mathbf{s}_{N_T-1}] \in \mathbb{C}^{N_T \times N}, \quad (3.1)$$

the covariance matrix of the transmitted waveforms with shift index p is then defined as

$$\mathbf{R}(p) = \lim_{N \rightarrow \infty} \mathbf{S} \mathbf{J}_p \mathbf{S}^H \triangleq \mathbf{S} \mathbf{J}_p \mathbf{S}^H, \quad (3.2)$$

where all the instantaneous values are used to replace the limit calculation. Suppose there are K_0 targets of interest at the nominal range bin where $p = 0$. Similarly, let K_p specify a set of clutter DOA's which are corresponding to non-zero range bins, then (1.7) is modified such that the received signal matrix consists of the target echoes, the clutter echoes, and the additive noise, i.e.,

$$\mathbf{Y} = \sum_{k=0}^{K_0-1} a_{k,0} \mathbf{e}_R(\theta_k) \mathbf{e}_T^T(\theta_k) \mathbf{S} + \sum_{p=-P, p \neq 0}^P \sum_{k=0}^{K_p-1} a_{k,p} \mathbf{e}_R(\theta_k) \mathbf{e}_T^T(\theta_k) \mathbf{S} \mathbf{J}_p + \mathbf{Z}, \quad (3.3)$$

Figure 3.1: Block Diagram of a MIMO Radar Receiver IVF.

where a_k in (1.7) is denoted as $a_{k,p}$ to represent the complex amplitudes of targets and clutter respectively. The middle term in the right hand side of (3.3) corresponds to the clutter responses, which is expected to be weakened via the filtering process at the receiver, and as a result to suppress the range sidelobes.

3.3 On IVF Range Sidelobe Suppression

A matched filter can maximize the attainable SNR at the output of the receiver [16]. However, its impact on clutter suppression near the peak of its output is insignificant. One way to improve clutter suppression is through the design of the transmitted waveforms. Several works have been done towards this direction [8–10]. Alternatively, an IVF can be used to replace the matched filter, and obtain a trade-off between SNR and SCR at the output of the filter. The schematic diagram of a MIMO radar IVF consisting of a matched filter and a mismatched filter is shown in Fig. 3.1. In this section, we first briefly restate the ISL and ZS methods [22] for the design of the IVF. Then we analyze their performances and draw two conclusions. (1) The ZS is superior to the ISL method in most situations. (2) The performance using MIMO radar has limitations when the number of range bins for suppression increases, but the use of phased-array radar can avoid the limitations.

The receive matrix of the matched filter under ideal waveform correlation properties is simply the Hermitian form of the transmit matrix, namely, $\mathbf{S}^H$. To main-

tain the mainlobe peak level, the IVF is formulated as

$$\mathbf{F}_{\text{IV}} = \mathbf{S}^H + \mathbf{\Delta X}, \quad (3.4)$$

where the columns in $\mathbf{\Delta} \in \mathbb{C}^{N \times (N-N_T)}$ is an orthonormal basis of the null space of $\mathbf{S}$, and $\mathbf{X} \in \mathbb{C}^{(N-N_T) \times N_T}$ is the matrix to be designed. Postmultiplying $\mathbf{Y}$ by $\mathbf{F}_{\text{IV}}$, the portion corresponding to clutter response is $\mathbf{S}\mathbf{J}_p(\mathbf{S}^H + \mathbf{\Delta X}) = \mathbf{R}(p) + \mathbf{S}\mathbf{J}_p\mathbf{\Delta X}$, then the ISL problem is formulated in [22] as

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{X}\|^2 \\ \text{s.t.} \quad & \sum_{p=-P, p \neq 0}^P \|\mathbf{R}(p) + \mathbf{S}\mathbf{J}_p\mathbf{\Delta X}\|^2 \leq 2P\zeta N_T, \end{aligned} \quad (3.5)$$

where ζ is the desired suppression level. On the other hand, the ZS method formulates the problem as a multidimensional least-squares problem, which aims to null the clutter response. The explicit expression is given by

$$\mathbf{A}_1 \mathbf{X} = \mathbf{B}, \quad (3.6)$$

where $\mathbf{A}_1 = [(\mathbf{S}\mathbf{J}_{-P}\mathbf{\Delta})^H, \dots, (\mathbf{S}\mathbf{J}_{-1}\mathbf{\Delta})^H, (\mathbf{S}\mathbf{J}_1\mathbf{\Delta})^H, \dots, (\mathbf{S}\mathbf{J}_P\mathbf{\Delta})^H]^H \in \mathbb{C}^{2PN_T \times (N-N_T)}$, and $\mathbf{B} = -[\mathbf{R}^H(-P), \dots, \mathbf{R}^H(-1), \mathbf{R}^H(1), \dots, \mathbf{R}^H(P)]^H \in \mathbb{C}^{2PN_T \times N_T}$.

3.3.1 Performance Analysis of ISL and ZS Methods

The difference between the ISL and ZS methods can be noted from their formulations. The ISL method minimizes the SNR loss,

$$\text{SNRLoss} = \frac{\|\mathbf{S}^H + \mathbf{\Delta X}\|^2}{\|\mathbf{S}\|^2} = \frac{\|\mathbf{S}\|^2 + \|\mathbf{X}\|^2}{\|\mathbf{S}\|^2}, \quad (3.7)$$

with a desired sidelobe suppression level ζ , whereas the ZS method aims to suppress the sidelobes towards its minimum without considering the SNR loss. Hence it is reasonable to expect that the ISL solution yields less SNR loss, whereas the ZS solution yields the best suppression. However, we show that such trade-off is unnecessary because the ZS solution has the lowest sidelobe levels with only marginally higher SNR loss in most situations. The ZS solution also has much less

computational complexity.

The ISL constraint in (3.5) and the ZS condition in (3.6) can be equivalently expressed as

$$\|\mathbf{A}_1 \mathbf{X} - \mathbf{B}\|^2 \leq 2P\zeta N_T, \quad (3.8)$$

$$\|\mathbf{A}_1 \mathbf{X} - \mathbf{B}\|^2 = 0, \quad (3.9)$$

respectively, which explicitly show that they differ only from the sidelobe suppression levels, and when ζ approaches 0, the two equations become identical. Furthermore, if we include the objective function of minimizing $\|\mathbf{X}\|^2$ to the ZS condition, then the two methods are a quadratic constrained quadratic programming (QCQP) problem and a minimum norm problem, respectively. It has been stated in [22] that $\mathbf{A}_1$ has a high probability to be full rank, then the closed-form solution to ZS method can be obtained via the pseudo inverse, i.e.,

$$\hat{\mathbf{X}} = \begin{cases} \mathbf{A}_1^H (\mathbf{A}_1 \mathbf{A}_1^H)^{-1} \mathbf{B}, & 2PN_T \leq N - N_T, \\ (\mathbf{A}_1^H \mathbf{A}_1)^{-1} \mathbf{A}_1^H \mathbf{B}, & 2PN_T > N - N_T. \end{cases} \quad (3.10)$$

The closed-form solution of the ISL method is provided in (19) of [22], which is also a pseudo inverse based solution but uses matrix regularization. However as the matrix is full rank, the regularization would not be necessary, although tuning the regularization coefficient can reduce $\|\mathbf{X}\|^2$. Note that reducing the sidelobes (ZS), say -100 dB, is much better than suppressing them by a desired level (ISL), say $\zeta = -30$ dB, but in terms of power, these two values are quite trivial and such difference in sidelobe levels will not change $\|\mathbf{X}\|^2$ significantly.

Now we consider the computational complexity of ISL and ZS methods. The dominant computational load comes from matrix inversion. For ISL method, the Kronecker operation [22] increases matrix dimension, and its computational complexity is $O(1)(N - N_T)^3 N_T^3$. For the ZS method, it is $O(1) \min \{8P^3, (N - N_T)^3\}$. Because the design should follow $N \gg N_T$ and $P < N$, it then is easy to verify that the condition $\min \{8P^3, (N - N_T)^3\} < (N - N_T)^3 N_T^3$ is always satisfied. Hence the ZS method is much less computationally complex than the ISL method, e.g., if $N = 256$, $N_T = 5$, and $P = 20$, then the ISL solution requires about 3×10^4 times more computations than the ZS solution.

In conclusion, for MIMO radar, the ZS method outperforms the ISL method.

Firstly, it is always better to null the sidelobes, and the ZS method is aiming for this. Secondly, we have shown that the ZS method is a minimum norm problem, which also considers minimizing the SNR loss as in the ISL formulation. Lastly, the ZS method has substantially less computational complexity. The simulations in later section will provide clear evidence of these claims.

3.3.2 Limitation and Motivation

Limitation

The limitation of range sidelobe suppression using MIMO radar is that when P increases, the IVF loses the ability to suppress the sidelobes to the desired level. The constraint on P is found empirically in [22], which is expressed as

$$2P \leq \left\lfloor \frac{N - N_T}{N_T} \right\rfloor. \quad (3.11)$$

This is a stringent condition. If $N = 256$ and $N_T = 5$, then at most $P = 25$, which is merely one tenth of the waveform code length N .

We reveal the reason of this problem via the observation of matrix $\mathbf{A}_1$. The dimension of $\mathbf{A}_1$ is $2PN_T \times (N - N_T)$. If $2PN_T < (N - N_T)$, then $\mathbf{A}_1$ is surjective (meaning being full rank and having more columns than rows), and the problem is a standard minimum norm problem. In this situation, more than one solutions satisfies $\mathbf{A}_1 \mathbf{X} = \mathbf{B}$, and the minimum norm solution is selected as it minimizes the SNR loss in (3.7). If $2PN_T > (N - N_T)$, then $\mathbf{A}_1$ becomes injective (meaning being full rank and having more rows than columns), making the problem a standard least-squares problem. In this situation, $\mathbf{A}_1 \mathbf{X} = \mathbf{B}$ cannot be satisfied, the solution is obtained in the sense of least mean squared error, and it becomes obvious that the requirement on sidelobes cannot be achieved.

Motivation

Via the observation of $\mathbf{A}_1$, it can be seen that if one can preserve that $\mathbf{A}_1$ being surjective when P increases, then the limitation on P can be avoided. This is the motivation of using phased-array radar for this particular scenario. Note that the phased-array radar is a special case of MIMO radar which transmits coherent waveforms. We now show that matrix $\mathbf{A}_1$, now denoted as $\mathbf{A}_2$ for phased-array

radar, becomes more surjective, and thus more tolerant to the stringent condition when P is increased.

3.3.3 IVF Design Using Phased-Array Radar

Here we present ISL and ZS methods for phased-array radar. The transmitted waveform matrix of phased-array radar is

$$\tilde{\mathbf{S}} = [\mathbf{s}_0, \mathbf{s}_0, \dots, \mathbf{s}_0] \in \mathbb{C}^{N_T \times N}, \quad (3.12)$$

then the covariance matrix $\tilde{\mathbf{R}}(0)$ becomes an all-one matrix. Hence the norm term in the constraint of (3.5) becomes

$$\left\| \tilde{\mathbf{R}}(p) + \tilde{\mathbf{S}} \mathbf{J}_p \tilde{\mathbf{\Delta}} \tilde{\mathbf{X}} \right\|^2 = \left\| \mathbf{1}_{N_T} R_{0,0}(p) \mathbf{1}_{N_T}^T + \mathbf{1}_{N_T} \mathbf{s}_0^T \mathbf{J}_p \tilde{\mathbf{\Delta}} \tilde{\mathbf{X}} \right\|^2 \quad (3.13)$$

$$\triangleq \left\| \mathbf{1}_{N_T} \left[R_{0,0}(p) + \mathbf{s}_0^T \mathbf{J}_p \tilde{\mathbf{\Delta}} \mathbf{x} \right] \mathbf{1}_{N_T}^T \right\|^2 \quad (3.14)$$

$$= N_T^2 \left| R_{0,0}(p) + \mathbf{s}_0^T \mathbf{J}_p \tilde{\mathbf{\Delta}} \mathbf{x} \right|^2, \quad (3.15)$$

where $\mathbf{s}_0 = [s_0(0), s_0(1), \dots, s_0(N-1)]^T$, $\text{rank}\{\tilde{\mathbf{S}}\} = 1$, and $\tilde{\mathbf{\Delta}} \in \mathbb{C}^{N \times (N-1)}$ is the orthonormal null space basis matrix of $\tilde{\mathbf{S}}$. Note that (3.14) is obtained from (3.13) by setting

$$\tilde{\mathbf{X}} \triangleq \mathbf{x} \mathbf{1}_{N_T}^T. \quad (3.16)$$

Using (3.15), the formulations of ISL and ZS methods are written as

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{x}\|^2 \\ \text{s.t.} \quad & \|\mathbf{A}_2 \mathbf{x} - \mathbf{b}\|^2 \leq 2P\zeta, \quad (\text{ISL}) \end{aligned} \quad (3.17)$$

$$\text{or} \quad \|\mathbf{A}_2 \mathbf{x} - \mathbf{b}\|^2 = 0, \quad (\text{ZS}) \quad (3.18)$$

where $\mathbf{A}_2 = [(\mathbf{s}_0^T \mathbf{J}_{-P} \tilde{\mathbf{\Delta}})^H, \dots, (\mathbf{s}_0^T \mathbf{J}_{-1} \tilde{\mathbf{\Delta}})^H, (\mathbf{s}_0^T \mathbf{J}_1 \tilde{\mathbf{\Delta}})^H, \dots, (\mathbf{s}_0^T \mathbf{J}_P \tilde{\mathbf{\Delta}})^H]^H \in \mathbb{C}^{2P \times (N-1)}$, and $\mathbf{b} = [R_{0,0}(-P), \dots, R_{0,0}(-1), R_{0,0}(1), \dots, R_{0,0}(P)]^T \in \mathbb{C}^{2P \times 1}$. Note that $\mathbf{A}_2$, whose number of rows reduced from $2PN_T$ in $\mathbf{A}_1$ to $2P$, is more surjective. Hence, the phased-array solution would be N_T times more tolerant to P . $\mathbf{A}_2$ is shown in the appendix to be full rank, thus the solution is similarly obtained as for MIMO radar in (3.10). Additional advantage of phased-array radar can be observed in terms

of computational complexity. Because of dimension reduction of $\mathbf{A}_2$, the computational complexity of ISL and ZS methods using phased-array are $O(1)(N-1)^3$ and $O(1) \min \{8P^3, (N-1)^3\}$ respectively. The ZS solution has minimum computational complexity, and the phased-array solutions have less computational complexity than the MIMO radar solutions. Hence by compromising the waveform diversity of MIMO radar, improved performances in range sidelobe suppression can be achieved using phased-array radar.

3.4 Joint Range and Doppler Sidelobe Suppression

3.4.1 Relationship to Ambiguity Functions

When both range and the Doppler effects are taken into consideration, it is intuitive to relate the work to the ambiguity function which is in fact the matched filter output. Researchers have been investigating the properties of the ambiguity functions, in order to obtain a clear area surrounding the central peak as wide as possible in the range-Doppler plane. Several works can be found in [6–10, 14, 15, 17, 53, 54]. In [54] and [53], the authors derived theoretically the maximum achievable clear area for phased-array and MIMO radar, respectively, using matched filters. Such theoretical results are a performance bound for ambiguity function based design of MIMO radar waveforms. It can be seen from [6, 7, 9] and the discussion in Chapter 2 that such design is not easy, and the solutions are usually based on heuristic algorithms such as simulated annealing (SA) and genetic algorithm (GA).

However, the ambiguity function based waveform design (for matched filters) is not the only way to achieve the clear area in the range-Doppler plane. The alternative is via the receiver IVF design. In this case, as noted earlier, the receiver is not just a matched filter, but a parallel system consisting of a matched and a mismatched filter. Hence the bounds derived in [54] becomes inapplicable. It can be easily verified that the volume over the range-Doppler plane is altered because of the mismatched filter. It is possible to investigate the bounds of maximum achievable clear area for MIMO radar receivers with IVF's, but this is beyond the scope of this chapter. However, some properties of the IVF's are elaborated here. As shown in [54], the maximum clear area at the output of the matched filters for

phased-array radar is 4. Simulation results will be provided in the next section to show that by using an IVF, the clear area can exceed this bound, and hence a MIMO radar system can violate the bound in [53].

3.4.2 Joint Sidelobe Suppression

The joint range and Doppler sidelobe suppression aims to have a clear area surrounding the central peak on the range-Doppler plane, which is a design that considers two dimensions. Let the range step-size be n_0 . Note that when only the range bins are considered, $n_0 = 1$. Suppose the Doppler bins for suppression are q , $q \in \{-Q, \dots, 1, 1, \dots, Q\}$, then there are totally

$$U \triangleq (2P + 1)(2Q + 1) - 1 \quad (3.19)$$

range-Doppler bins for suppression. The Doppler shifting matrix defined in Chapter 1 is restated here for reference,

$$\mathbf{V}_q = [\mathbf{v}_q, \mathbf{v}_q, \dots, \mathbf{v}_q]^T \in \mathbb{C}^{N_T \times N}, \quad (3.20)$$

where

$$\mathbf{v}_q = [1, e^{j2\pi q v_0 / f_s}, e^{j2\pi q 2v_0 / f_s}, \dots, e^{j2\pi q (N-1)v_0 / f_s}]^T, \quad (3.21)$$

f_s is the sampling frequency, and v_0 is the Doppler step size. Hence the receive signal matrix in (3.3), $\mathbf{S}\mathbf{J}_p$, becomes $(\mathbf{S}\mathbf{J}_p) \odot \mathbf{V}_q$. In this situation, the covariance matrix $\mathbf{R}$ becomes a function of both p and q , i.e.,

$$\begin{aligned} \mathbf{R}(p, q) &= E \{ \mathbf{s}(n - pn_0) e^{j2\pi q v_0 n / f_s} \mathbf{s}^H(n) \} \\ &\triangleq [(\mathbf{S}\mathbf{J}_p) \odot \mathbf{V}_q] \mathbf{S}^H \\ &= \begin{bmatrix} R_{0,0}(p, q) & R_{0,1}(p, q) \cdots & R_{0,N_T-1}(p, q) \\ R_{1,0}(p, q) & R_{1,1}(p, q) \cdots & R_{1,N_T-1}(p, q) \\ \vdots & \cdots & \ddots & \vdots \\ R_{N_T-1,0}(p, q) & \cdots & \cdots & R_{N_T-1,N_T-1}(p, q) \end{bmatrix}. \end{aligned} \quad (3.22)$$

Stacking $[(\mathbf{S}\mathbf{J}_p) \odot \mathbf{V}_q] \mathbf{\Delta}$ on top of each other with ascending order of first p , and then q , respectively, we form a matrix $\mathbf{A}_3 \in \mathbb{C}^{UN_T \times (N - N_T)}$ for MIMO radar. Stacking

$[(\mathbf{s}_0^T \mathbf{J}_p) \odot \mathbf{v}_q^T] \mathbf{\Delta}_0$ on top of each other, we form a matrix $\mathbf{A}_4 \in \mathbb{C}^{U \times (N-1)}$ for phased-array radar. Similarly stacking $-\mathbf{R}(p, q)$ and $-R_{0,0}(p, q)$ on top of each other respectively, we obtain $\mathbf{C} \in \mathbb{C}^{U N_T \times N_T}$ and $\mathbf{c} \in \mathbb{C}^{U \times 1}$. Hence the ISL or ZS method for joint range and Doppler sidelobe suppression using MIMO and phased-array radar can be expressed as

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{X}\|^2 \\ \text{s.t.} \quad & \|\mathbf{A}_3 \mathbf{X} - \mathbf{C}\|^2 \leq 2P\zeta N_T^2, \quad (\text{ISL}) \end{aligned} \quad (3.23)$$

$$\text{or} \quad \|\mathbf{A}_3 \mathbf{X} - \mathbf{C}\|^2 = 0, \quad (\text{ZS}) \quad (3.24)$$

and

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{x}\|^2 \\ \text{s.t.} \quad & \|\mathbf{A}_4 \mathbf{x} - \mathbf{c}\|^2 \leq 2P\zeta, \quad (\text{ISL}) \end{aligned} \quad (3.25)$$

$$\text{or} \quad \|\mathbf{A}_4 \mathbf{x} - \mathbf{c}\|^2 = 0, \quad (\text{ZS}) \quad (3.26)$$

3.4.3 Doppler Resolution and Rank Deficiency

Under the narrowband assumption, it is assumed that the motion of the objects causes only a Doppler shift of the carrier frequency while the signal envelope is not affected [16]. In this way, the Doppler resolution is proportional to the dwell-time of the transmitted signals. Hence, the dwell-time, i.e., $T = N/f_s$ should be long enough to resolve the desired target Doppler frequencies. However, long dwell-time of single pulse transmission weakens the short range capability of the radar system as the reflected signals can be detected only after the transmission is completed. According to [16], if the desired Doppler resolution $v_0 = 1$ kHz, then the dwell-time should be approximately $1/1000 = 1$ ms, and the minimum range capability is approximately 300 km. Hence one needs to trade off the range capability and Doppler resolution.

The Doppler resolution of the IVF is reflected from the row rank of $\mathbf{A}_3$ and $\mathbf{A}_4$. Their dimension expansion as compared to $\mathbf{A}_1$ and $\mathbf{A}_2$ is due to the added Doppler bins. If the dwell-time is not long enough to resolve the desired Doppler frequencies, then $\mathbf{A}_3$ and $\mathbf{A}_4$ become row rank deficient. In other words, the rows in $\mathbf{A}_3$ or $\mathbf{A}_4$ only differ from an unresolvable Doppler step-size and become correlated.

Table 3.1: Illustration of rank deficiency.

N/f_s	10 ms	2 ms	1.7 ms	1 ms	0.1 ms
$\text{rank}\{\mathbf{A}_3\}$	200	200	175	127	30
$\text{rank}\{\mathbf{A}_4\}$	40	40	36	26	6
$N_T = 5, f_s = 4B, N = 1230, v_0 = 500 \text{ Hz}, \text{BT} = 410,$ $P = 0, \mathbf{A}_3 \in \mathbb{C}^{200 \times 1225}, \mathbf{A}_4 \in \mathbb{C}^{40 \times 1229}.$					

Table 3.1 illustrates the rank deficiency of the design problem using quadratic congruence coded (QCC) waveforms obtained in Chapter 1. Here $P = 0$, which allows to study the Doppler resolution without jointly considering time shift. The waveforms have equal bandwidth B and dwell-time T , and the coding dimension is 41. The bandwidth-time (BT) product is fixed to 410. It can be seen that 2 ms dwell-time preserves rank efficiency, but in this way the system cannot distinguish echoes within 600 km. Hence in the design, it is expected to relax the Doppler resolution requirement to reduce the dwell time. Generally, the design for joint suppression can be considered as a minimum norm or least-squares problem with rank deficiency, where matrix regularization technique can be used for closed-form solutions. The MIMO and phased-array radar ISL solutions are given as

$$\hat{\mathbf{X}} = \underbrace{\mathbf{A}_3^H (\mathbf{A}_3 \mathbf{A}_3^H + \gamma \mathbf{I})^{-1} \mathbf{C}}_{\mathbf{A}_3 \text{ surjective}} \text{ or } \underbrace{(\mathbf{A}_3^H \mathbf{A}_3 + \gamma \mathbf{I})^{-1} \mathbf{A}_3^H \mathbf{C}}_{\mathbf{A}_3 \text{ injective}}, \quad (3.27)$$

$$\hat{\mathbf{x}} = \underbrace{\mathbf{A}_4^H (\mathbf{A}_4 \mathbf{A}_4^H + \beta \mathbf{I})^{-1} \mathbf{c}}_{\mathbf{A}_4 \text{ surjective}} \text{ or } \underbrace{(\mathbf{A}_4^H \mathbf{A}_4 + \beta \mathbf{I})^{-1} \mathbf{A}_4^H \mathbf{c}}_{\mathbf{A}_4 \text{ injective}}, \quad (3.28)$$

where $\mathbf{I}$ is an identity matrix with appropriate dimension. If $\mathbf{A}_3$ and $\mathbf{A}_4$ are surjective and rank efficient, then the ZS solutions can be obtained by setting regularizing coefficients $\gamma = \beta = 0$. Recall that $\mathbf{A}_3 \in \mathbb{C}^{N_T \times (N - N_T)}$ and $\mathbf{A}_4 \in \mathbb{C}^{U \times (N - 1)}$, we can see that $\mathbf{A}_4$ is approximately N_T times more surjective than $\mathbf{A}_3$, and thus can have more clear area in the range-Doppler plane at the output of the IVF.

3.4.4 Performance Analysis of Joint Suppression

According to the previous analysis, it can be seen that on one hand, the time resolution is preserved due to $\mathbf{A}_1$ and $\mathbf{A}_2$ being full rank, which are obtained

when the Doppler effects are ignored ($Q=0$). On the other hand, the Doppler resolution can be preserved using the result in the previous subsection, i.e., $T \geq 2$ ms, which is obtained without time shift ($P = 0$). However, the joint range and Doppler design has extra range-Doppler bins where $pq \neq 0$, and the joint range-Doppler resolution may not be preserved. In this situation, $\mathbf{A}_3$ and $\mathbf{A}_4$ are still rank deficient. On the range-Doppler plane, the two dimensional design step-size is $(n_0/f_s, v_0)$. According to [66], the time resolution is approximately $1/(2B)$, and the Doppler resolution is approximately $1/T$. Hence the range-Doppler step-size has an area of approximately $1/(2BT)$. A strict necessary condition for $\mathbf{A}_3$ and $\mathbf{A}_4$ being full rank is

$$\begin{cases} v_0 > 1/T, \\ n_0/f_s > 1/2B. \end{cases} \quad (3.29)$$

In most situations, a relaxed condition that considers the area of the step-size, i.e.,

$$\frac{2BTv_0n_0}{f_s} > 1 \quad (3.30)$$

suffices to preserve the full rank of $\mathbf{A}_3$ and $\mathbf{A}_4$. Adding the Nyquist sampling condition, (3.30) is modified to

$$2B < f_s < 2BTv_0n_0 \Rightarrow Tv_0n_0 > 1. \quad (3.31)$$

Note that T , v_0 , and n_0 should be chosen relatively small to preserve the range capability and the design resolution. By considering the matrix dimensions, the solutions can be approximately classified as shown in Table 3.2, where the constraints of f_s are expressed in relation to the variables n_0 , v_0 , B , T , and U .

Next we illustrate that the full rank and surjective conditions for both MIMO and phased-array radar are difficult to achieve, because a large BT product is required. Consider the condition for MIMO radar for two situations, 1) $2B < (U+1)N_T/T$. In this situation, we have

$$\begin{cases} 2BT < (U+1)N_T, \\ (U+1)N_T/T < f_s < 2BTv_0n_0, \end{cases} \quad (3.32)$$

which indicates that

$$2BT < (U+1)N_T < 2BT^2v_0n_0, \quad (3.33)$$

Table 3.2: Joint Suppression Solution Classification

		Full Rank
MIMO	Surjective	$\max \{2B, (U+1)N_T/T\} \leq f_s \leq 2BTv_0n_0$
	Injective	$2B \leq f_s \leq \min \{2BTv_0n_0, (U+1)N_T/T\}$
Phased-Array	Surjective	$\max \{2B, (U+1)/T\} \leq f_s \leq 2BTv_0n_0$
	Injective	$2B \leq f_s \leq \min \{2BTv_0n_0, (U+1)/T\}$
		Low Rank
MIMO	Surjective	$f_s > \max \{2B, (U+1)N_T/T, 2BTv_0n_0\}$
	Injective	$\max \{2B, 2BTv_0n_0\} < f_s < (U+1)N_T/T$
Phased-Array	Surjective	$f_s > \max \{2B, (U+1)/T, 2BTv_0n_0\}$
	Injective	$\max \{2B, 2BTv_0n_0\} < f_s < (U+1)/T$

which highly limits the design flexibility, because $(U+1)N_T$ is bounded in a very narrow region as the value of Tv_0n_0 is usually close to (although larger than) 1. To avoid this, B should be increased, and thus we have the second situation. 2) $2B \geq (U+1)N_T/T$. In this situation, we have

$$\begin{cases} 2BT > (U+1)N_T, \\ 2B < f_s < 2BTv_0n_0, \end{cases} \quad (3.34)$$

which is easier to be satisfied, and there is no strict bound to limit suppression area U as long as the bandwidth of the transmitted signals is large enough. Note that T can also be appropriately increased to satisfy the above condition. Hence in general, the full rank and surjective condition is difficult to be satisfied unless the signals have large BT product. In our design, the BT product is fixed at 410, and the right inequality in (3.33) is not satisfied. Hence the full rank and surjective case is not shown in the examples.

The summary of the computational complexity are provided in Table 3.3 on top of this page. It can be observed that for joint range and Doppler suppression, if the $\mathbf{A}_3$ and $\mathbf{A}_4$ are injective, then the phased-array solution requires $[(N-1)/(N-N_T)]^3$ more times of computations than MIMO radar.

Table 3.3: Comparison of Computational Complexity

	Without Doppler Effects	
	ISL	ZS
MIMO	$O(1)(N - N_T)^3 N_T^3$ [22]	$O(1) \min \{8P^3 N_T^3, (N - N_T)^3\}$
Phased-Array	$O(1)(N - 1)^3$	$O(1) \min \{8P^3, (N - 1)^3\}$
	With Doppler Effects	
	ISL	ZS
MIMO	$O(1)(N - N_T)^3 N_T^3$ [22]	$O(1) \min \{U^3 N_T^3, (N - N_T)^3\}$
Phased-Array	$O(1) \min \{U^3, (N - 1)^3\}$	

3.5 Numerical Examples

In this section, we provide numerical examples to illustrate the designs and their performances. Firstly, the performances of ISL and ZS methods of MIMO and phased-array radar, for range sidelobe suppression are compared. Secondly, we show the simulation results for joint range and Doppler sidelobe suppression. The sidelobe suppression level, SNR loss, and computational complexity are the performance criteria. We also show that the theoretical bound derived in [53, 54] for matched filter clear area in the range-Doppler plane is inapplicable for receiver IVF design.

In [21] and [22], the authors used PSK codes to represent the transmitted waveforms and studied their correlation properties. Thus the range cell p is corresponding to the code shifts. When the codes are modulated to baseband signals, for example, 1 codeword corresponds to 10 samples, then p shifts in codes correspond to $10p$ shifts in the transmitted waveform samples. For comparison, we use BPSK codes without baseband modulation in the range sidelobe suppression examples. For the simulations considering Doppler effects, we use QCC waveforms with a coding dimension of 41 instead of BPSK waveforms. This is because in [22] it has been shown that the BPSK waveforms are sensitive to the Doppler effects. The QCC waveforms are more robust against the Doppler effects. As the study of Doppler effects needs to consider baseband modulation frequencies, we use baseband waveforms instead of the codes for simulations. The array signal-to-noise

(a) ISL, ASNR = 100 dB.

(b) ZS, ASNR = 100 dB.

(c) ISL, ASNR = 20 dB.

(d) ZS, ASNR = 20 dB.

Figure 3.2: MIMO radar uses the BPSK Hadamard codes scrambled by PN sequences, whereas phased-array radar uses the codes of the first waveform for MIMO radar. The mean curves are generated by taking the averages of 100 realizations. The standard deviations are too small, and omitted for clarity of the figures.

ratio (ASNR) is defined as

$$\text{ASNR} = \frac{N_R}{N_T} \cdot \frac{\text{tr}\{\mathbf{S}\mathbf{S}^H\}}{\text{tr}\{\mathbf{Z}\mathbf{Z}^H\}} = \frac{1}{\sigma^2}, \quad (3.35)$$

which is characterized by the noise power at the receiver. Since the IVF's are designed under noise free condition, the sidelobe levels will be affected by the ASNR. However note that in (3.7) the SNR loss is independent of ASNR. As this work mainly focuses on clutter suppression, high ASNR is assumed while the other parameters are investigated in the simulations.

3.5.1 Range Sidelobe Suppression Examples

To compare with the results in [21] and [22], the simulation parameters are set as follows: $N_T = 5$, $N = 256$, and $P = 20$. The transmitted codes are BPSK Hadamard codes scrambled by a pseudo-noise (PN) sequences. The comparison between MIMO and phased-array radar range sidelobe suppression is shown in Fig. 3.2, where the codes of the first waveform for MIMO radar are used for phased-array radar. Note that using matched filter will not result in the valley within $p \in [-20, 20]$. Also note that MIMO and phased-array radar have identical matched filter outputs [43]. From Fig. 3.2 (a) and (b), we observe that when the ASNR is high, nearly noise free, both MIMO and phased-array radar can satisfy the ISL constraints of suppressing the sidelobes within the predefined range cell region to -50 dB. The ZS solutions have flatter suppression levels as compared to the ISL solutions. Outside the predefined region, phased-array radar has lower sidelobes than MIMO radar. From Fig. 3.2 (c) and (d), we observe that when the ASNR becomes lower, the sidelobes outside the predefined region are not changed, but those within the predefined region degrade: the ISL constraints are not satisfied as the suppression levels rise up to around -30 dB for MIMO radar and -42 dB for phased-array radar. In addition, the sidelobe patterns obtained from ZS conditions approach those from ISL ones. Hence it is concluded that the noise at the receiver interferes the sidelobe suppression, but does not affect sidelobes outside of the predefined region, and phased-array radar solution is more tolerant to the noise at the receiver.

The minimum achievable range sidelobe average values versus the ASNR is shown in Fig. 3.3. In one iteration, the average value of sidelobes within the suppression region under different ASNR is evaluated. This produces the “1 ite” curves. Then 100 realizations are simulated and averaged to produce the mean curves. We see that the minimum achievable sidelobe levels are inversely proportional to the ASNR, and phased-array radar can always achieve better sidelobe suppression along the ASNR axis than MIMO radar. Further advantages of phased-array radar is illustrated in Fig. 3.4. The SNR loss caused by MIMO radar is roughly 7 times of that caused by phased-array radar when $P = 20$. The SNR loss of phased-array radar is always less than 1 dB, as its value for ZS method is 0.9925 dB. When $\zeta < -40$ dB, the SNR loss of both curves are very close to the ZS condition results. This means that the ZS conditions null the sidelobes within

Figure 3.3: Minimum achievable range sidelobe averages versus ASNR for MIMO and phased-array radar, $N_T = 5$, $P = 20$ and $N = 256$.

Figure 3.4: SNR loss versus ζ for ISL constraints, and SNR loss of ZS conditions, $N_T = 5$, $N = 256$, and $P = 20$.

the predefined range cells to the lowest possible levels while the SNR loss increases only marginally compared to that of the ISL method. Hence the analysis in Section 3.3.1 is verified. According to Table 3.3, the ZS solution is also more computationally efficient than the ISL for phased-array radar as long as $2P \leq (N - 1)$, which would be practically true for most cases.

The SNR loss versus P and the mean values of the minimum achievable sidelobe levels versus P for MIMO and phased-array radar are shown in Fig. 3.5 and 3.6,

Figure 3.5: SNR loss versus P for ISL and ZS constraints, $N_T = 5$, $N = 256$, and $\zeta = -50$ dB.

Figure 3.6: Minimum achievable range sidelobe averages versus P for MIMO and phased-array radar, $N_T = 5$, $N = 256$, and $\text{ASNR} = 90$ dB.

respectively. These two figures belong to the full rank categories in Table II when $Q = 0$. The values of P corresponding to the peaks in Fig. 3.5 and the sharp transition points in Fig. 3.6 are in fact corresponding to the region that $\mathbf{A}_1$ and $\mathbf{A}_2$ turn from surjective to injective. The discussion in Section 3.3.2 and 3.3.3 is supported by these two figures. MIMO radar has less SNR loss when P increases, but $\mathbf{A}_1$ turns injective rather quickly, making the design a least-squares problem, and the sidelobes can be only suppressed to -20 dB, which is close to the matched filter as shown in Fig. 3.2. On the contrary, phased-array radar has increasing SNR

loss, but is able to be surjective (preserve being a minimum norm problem) and can suppress the sidelobes to -100 dB for about 100 more range bins as compared to MIMO radar.

3.5.2 Joint Range and Doppler Sidelobe Suppression Examples

In this section, the QCC baseband modulated waveforms are used for simulations. The carrier frequency is set to 1 GHz, and the Doppler region for suppression is $[-10, 10]$ kHz. This region corresponds to velocities from -500 to 500 m/s. The Doppler step size is $v_0 = 500$ Hz, which corresponds to 25 m/s, i.e., the Doppler region is divided into 40 cells. Using simulations, we analyze the performances of both MIMO and phased-array radar, in relation to the average sidelobe level on the $U = (2P+1)(2Q+1) - 1$ range-Doppler locations and the SNR loss. For simplicity and emphasis on the key points, we provide surjective low rank, injective full rank and injective low rank examples for both MIMO and phased-array radar.

Fig. 3.7 shows the joint range and Doppler sidelobe suppression results for MIMO and phased-array radar, respectively. It can be seen that the rectangular area of $U = 1680$ range-Doppler bins are cleared in all the subfigures. The suppression level of MIMO radar is above -30 dB, whereas that of phased-array radar can reach lower than -40 dB. Fig. 3.7 (a) and (b) illustrate the surjective low rank cases, where $\mathbf{A}_3 \in \mathbb{C}^{8400 \times 9020}$ and $\mathbf{A}_4 \in \mathbb{C}^{1680 \times 9020}$, $\gamma = 0.1$ and $\beta = 0.01$. The SNR loss is more than 15 and 18 dB respectively. Fig. 3.7 (c) and (d), where $\mathbf{A}_3 \in \mathbb{C}^{8400 \times 1230}$ and $\mathbf{A}_4 \in \mathbb{C}^{1680 \times 1230}$, illustrate the injective full rank case, which are typical least-squares problems. We observe that MIMO radar has grating lobes uniformly located at different range cuts beyond U . Phased-array radar does not have such grating lobes, but the sidelobes beyond the region for suppression is very high. It can also be seen that without matrix regularization, the SNR loss reaches 20 and 34 dB respectively, which is not acceptable. Note that Fig. 3.7 (a) and (b) have range suppression of $40/9020 = 0.0044$ of the dwell-time while Fig. 3.7 (c) and (d) have range suppression $40/1230 = 0.0326$ of the dwell-time. Hence it can be expected that there are grating lobes at different range cuts in Fig. 3.7 (a) and (b) at $p > 60$ and $p < -60$.

As mentioned earlier, for extended range capability of the radar systems, the

(a) Surjective low rank, suppression -22.3618 dB, SNR loss 15.4318 dB. (b) Surjective low rank, suppression -51.6470 dB, SNR loss 18.5252 dB.

(c) Injective full rank, suppression -23.9289 dB, SNR loss 20.8893 dB. (d) Injective full rank, suppression -50.6290 dB, SNR loss 34.7518 dB.

(e) Injective low rank, suppression -30.2858 dB, SNR loss 9.1763 dB. (f) Injective low rank, suppression -41.4738 dB, SNR loss 10.0235 dB.

Figure 3.7: Joint Range and Doppler sidelobe suppression using MIMO and phased-array radar. QCC baseband waveforms with coding dimension 41 are used, $P = 20$, $Q = 20$, $v_0 = 500$ Hz, $n_0 = 2$ samples, $BT = 410$, and $ASNR = 90$ dB. (a), (c), and (e) are for MIMO radar where the waveforms have identical bandwidth and dwell-time. (b), (d), and (f) are for phased-array radar. (a) (b) $B = 205$ kHz, $T = 2$ ms, $f_s = 22B$, $\gamma = 0.1$, and $\beta = 0.01$. (c) (d) $B = 205$ kHz, $T = 2$ ms, $f_s = 4B$, and $\gamma = \beta = 0$. (e) (f) $B = 4.1$ MHz, $T = 0.1$ ms, $f_s = 4B$, $\gamma = 0.5$, and $\beta = 0.2$.

(a) Zero range and Doppler cuts of Fig. 3.7 (e) (f). (b) Sidelobe suppression vs. SNR loss.

(c) Grating lobe peaks vs. γ and β . (d) Injective low rank joint suppression of an area of 8.

Figure 3.8: Performances of injective low rank formulation using MIMO and phased-array radar, where $P = 20$, $Q = 20$, $N_T = 5$, $\text{ASNR} = 90$ dB, $B = 4.1$ MHz, $T = 0.1$ ms, and $f_s = 3B$. (a) Zero range and Doppler cuts of Fig. 3.7 (e) (f). (b) Minimum achievable sidelobe averages versus SNR loss, where $\gamma, \beta \in \{1.9, 1.7, 1.5, 1.3, 1.1, 0.9, 0.7, 0.5, 0.3, 0.2, 0.1, 0.09\}$. (c) Grating lobe peaks versus γ and β . (d) Phased-array joint sidelobe suppression of an area of 8, where $P = 123$, $Q = 200$, $B = 4.1$ MHz, $f_s = 4B$, $v_0 = 500$ Hz, and time shift step-size is 2 samples. The sidelobe average value is -40.7422 dB and SNR loss is 19.8016 dB.

dwel-time should not be too long. A long dwell-time also increases the dimensions of matrices $\mathbf{X}$ and $\mathbf{x}$ and thus increases the workload for optimization. In addition, we see from Fig. 3.7 (c)-(d) that although the full rank formulations have desirable Doppler resolution at the origin, they suffer from high SNR loss. Hence a moderate compromise on the Doppler resolution, i.e., reducing the dwell-time is desirable, which is illustrated in 3.7 (e) and (f). It can be seen that by appropriately choosing γ and β , the SNR loss for both MIMO and phased-array radar is reduced to about 10 dB and the sidelobe suppression for MIMO radar improves. Moreover, the grating lobes only appear along the zero range cut.

The performances of injective low rank case is illustrated in Fig. 3.8. The zero range ($p = 0$) and zero Doppler ($q = 0$) cuts are shown in Fig. 3.8 (a), where for MIMO radar the grating lobe reaches -5.032 dB and for phased-array radar it reaches -5.801 dB. The Doppler cell located at -3 dB level of the main lobes corresponds to $q = \pm 7$, hence the resolution of Doppler effects along the zero range line is 3500 Hz, corresponding to velocity of 175 m/s. The averages of the joint suppression levels versus the SNR loss are shown in Fig. 3.8 (b). We observe from the monotonicity of the curves and the values of γ and β that, when γ or β decreases, the mean sidelobe level decreases at the expense of increasing the SNR loss. For a given SNR loss, the phased-array radar can have around 13 dB more sidelobe suppression than MIMO radar. The figure also indicates that there exists a critical region for either γ or β bounded by $[0.1, 2]$, within which both the SNR loss and the minimum achievable sidelobes satisfy the designing criteria. If γ and β are smaller, then the SNR loss exceeds 10 dB, which may be unacceptable; if they are larger, then for MIMO radar the minimum achievable sidelobes become higher than -23 dB, which defeats the main objective. The sound choices of γ or β are found based on empirical results. Note that the SNR loss is independent of range and Doppler values, which can be observed from (3.7). Fig. 3.8 (c) shows the grating lobe peaks versus the choice of γ and β in $[0.1, 2]$. It is observed that when $\beta < 0.2$, the grating lobe of phased-array radar is more sensitive towards the change of the parameter, which increases rapidly as β decreases, and it tends to exceed the mainlobe at the origin. However when $\beta \geq 0.2$, the phased-array radar has lower grating lobe peaks than those of MIMO radar. Combining the results found in Fig. 3.8 (a) and (b), we can find a preferable value, e.g., $\beta = 0.2$, for phased-array radar, as it gives very low grating lobe peaks, less than 10 dB SNR loss, and sidelobe suppression level at about -40 dB. Similarly, we can find a suitable value of γ for MIMO radar, but with relatively degraded performance.

3.5.3 The Maximum Clear Area

For simplicity, we use the simulation results of phased-array radar IVF for determining the clear area. A clear area of 8 surrounding the origin on the range-Doppler plane is seen in Fig. 3.8 (d). The area chosen obeys the conditions in [54], which are (1) convex, and (2) symmetric. The derivation in [54] assumes an impulse at the origin, which is impractical. Hence in our simulation, the mainlobe of the cen-

tral peak is neglected. The peak sidelobe level is -11.53 dB, i.e., 0.0703 in height. Substituting $\eta = 0.0703V_0$ into (16) of [54], we have

$$C \leq V_0 (0.25V_0 - 0.0703V_0)^{-1} = 5.5648. \quad (3.36)$$

However, as shown in the figure, the “ η clear” area $C = 200 \text{ kHz} \times 40 \text{ } \mu\text{s} = 8 > 5.5648$, which contradicts the above condition. This example illustrates that the bounds derived based on matched filter ambiguity function for the clear area are inapplicable for an IVF. Hence it is argued that the similar bounds derived in [53] for MIMO radar are also inapplicable for IVF, and the bounds for the clear area of IVF outputs need further investigation.

3.6 Conclusions

In this Chapter, we have addressed the receiver IVF design problem for clutter suppression using both MIMO and phased-array radar. We compared the existing ISL and ZS methods, and show the latter’s advantages under no consideration of the Doppler effects. We revealed the limitation of ISL and ZS methods using MIMO radar under the stringent condition of having large number of range bins for suppression, and formulated the problem using phased-array coherent transmission to avoid this limitation. The Design of joint range and Doppler sidelobe suppression is also considered. The Doppler resolution and rank deficiency are discussed and 8 cases of solution are classified. Simulation results show that full rank (long dwell-time) formulations yield high SNR loss and grating lobes. The injective low rank formulation moderately compromises the Doppler resolution, but with a significant reduction of SNR loss and lower suppression levels for both MIMO and phased-array radar. The computational complexity of all scenarios is also provided. It can be concluded that, the use of phased-array radar loses the waveform diversity, but can improve the performances of the receiver IVF for range, or joint range and Doppler sidelobe suppression in terms of computational complexity, suppression level, and SNR loss. Finally, we present the simulation results of phased-array radar, which illustrate that the clear area bounds derived in [54] for matched filter (ambiguity function) is inapplicable when using an IVF at the receiver. Hence it is worth of investigating the existence of such a bound for the receiver IVF.

Appendix: On the Rank of $\mathbf{A}_2$

Assume there exists a set of scalars α_p , not all zero, such that the linear combination of rows in $\mathbf{A}_2$ is zero vector, i.e.,

$$\sum_{p=-P, p \neq 0}^P (\alpha_p \mathbf{s}_0^T \mathbf{J}_p) \tilde{\Delta} = \mathbf{0}, \quad (3.37)$$

then the equality holds if and only if either of the two conditions below is satisfied:

1. $\mathbf{T} = [\mathbf{J}_{-P}^T \mathbf{s}_0, \dots, \mathbf{J}_{-1}^T \mathbf{s}_0, \mathbf{J}_1^T \mathbf{s}_0, \dots, \mathbf{J}_P^T \mathbf{s}_0]^T$ is row rank deficient, where $\mathbf{T}$ is a $2P \times N$ banded Toeplitz matrix constructed by stacking shifted versions of $\mathbf{s}_0^T$ on top of each other;
2. $\exists p \in \{-P, -P+1, \dots, -1\} \cup \{1, 2, \dots, P\}$, such that $\mathbf{s}_0^T \mathbf{J}_p$ lies in the left null space of $\tilde{\Delta}$.

To verify the first condition, we make $\mathbf{T}$ a square matrix by appropriately choosing P and N such that $2P = N$ (if N is odd, then $2P = N - 1$, and we add $\mathbf{s}_0^T \mathbf{J}_{\pm P \pm 1}$ at the bottom or on top of the matrix). $\mathbf{T}$ can be considered as the snapshots matrix of a passive sensor array, which is usually proven to be nonsingular under the condition of enough number of snapshots and existence of noise (see [67, 68] and references therein). Alternatively, we see from [69] that the probability of $\mathbf{T}$ being singular without noise is close to zero. And the explicit expression of $\det \{\mathbf{T}\}$ has been derived in [70], which indicates that the waveforms we generated can hardly yield 0 determinant of $\mathbf{T}$. Hence, we can at most have N linearly independent rows in $\mathbf{T}$, i.e., $\text{rank} \{\mathbf{T}\} = \min \{2P, N\}$.

Now consider the second condition. As $\tilde{\Delta}$ consists of $N - 1$ orthonormal columns and N rows, its left nullity is 1. Hence the basis of the left null space of $\tilde{\Delta}$ is of dimension $1 \times N$. We also have $\mathbf{s}_0^T \tilde{\Delta} = \mathbf{0}$, which means that $\mathbf{s}_0^T$ is in the left null space of $\tilde{\Delta}$. Hence, the second condition can be restated as $\exists p \in \{-P, -P+1, \dots, -1\} \cup \{1, 2, \dots, P\}$, such that

$$\mathbf{s}_0^T \mathbf{J}_p || \mathbf{s}_0^T \Rightarrow \mathbf{s}_0^T \mathbf{J}_p = \rho \mathbf{s}_0^T, \quad (3.38)$$

where ρ is a real constant. When $p > 0$, we have

$$\rho \mathbf{s}_0^T = [\rho s_0(0), \rho s_0(1), \dots, \rho s_0(N-1)], \quad (3.39)$$

$$\mathbf{s}_0^T \mathbf{J}_p = [\underbrace{0, 0, \dots, 0}_p, s_0(0), s_0(1), \dots, s_0(N-p-1)], \quad (3.40)$$

where $s_0(i)$ is the i th element of $\mathbf{s}_0^T$, then (3.38) becomes

$$\begin{aligned} \rho s_0(i) &= \begin{cases} 0, & i = 0, 1, \dots, p, \\ s_0(i-p), & i = p+1, \dots, N-1, \end{cases} \\ &= 0, \quad i = 0, 1, \dots, N-1, \end{aligned} \quad (3.41)$$

which means $\mathbf{s}_0^T = \mathbf{0}$, an unrealistic solution. Combining the results of both conditions, (3.37) does not hold for $2P \leq N-1$, thus $\text{rank}\{\mathbf{A}_2\} = 2P$; when $2P > N-1$, the first condition can be easily satisfied and $\mathbf{T}$ becomes full column rank, thus we have

$$\text{rank}\{\mathbf{A}_2\} = \text{rank}\{\mathbf{T}\tilde{\Delta}\} = \text{rank}\{\tilde{\Delta}\} = N-1, \quad (3.42)$$

i.e., $\text{rank}\{\mathbf{A}_2\} = \min\{2P, N-1\}$. ■

Chapter 4

MIMO Radar Transmit Beampattern Design With Ripple and Transition Band Control¹

4.1 Introduction

THE direction-of-arrival (DOA) estimation is a classical problem in array and radar systems. Transmit beamforming is able to focus the energy of the transmitted signal within a predefined spatial region so that the signal-to-noise ratio at receiver can be effectively improved. In this way the DOA estimation can be more accurate and noise robust. The waveform diversity of multiple-input-multiple-output (MIMO) radar systems offers many advantages in comparison to the phased-array counterpart. One of the advantages is that it allows one to design MIMO radar with flexible transmit beampatterns, which can effectively improve the signal energy at the receiver side for enhanced target angle estimation. Many researchers have proposed solutions to this problem in the recent decade. However, these designs pay little attention to certain aspects of the performance such as the ripples within the energy focusing section, the attenuation of the sidelobes, the width of the transition band, the angle step-size, and the required number of transmit antennas. In this Chapter, we first propose several methods which can

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indirectly or directly control the ripple levels within the energy focusing section and the transition bandwidth. These methods are based on existing problem formulations. More importantly, we reformulate the design as a feasibility problem (FP). Such a formulation enables a more flexible and efficient design which achieves the most preferable beampatterns with the least system cost. Using this formulation, an empirical MIMO radar beampattern formula is obtained. This MIMO radar beampattern formula is similar to Kaisers formula in conventional finite-impulse-response (FIR) filter design. The performances of the proposed methods and formulations are evaluated via numerical examples.

Compared to the phased-array counterpart, MIMO radar enjoys the advantage of waveform diversity but has drawbacks in terms of signal-to-noise ratio (SNR) loss [41]. To preserve the waveform diversity of MIMO radar while taking the advantage of the coherent processing gain of phased-array radar, new configurations of radar architecture such as the phased-MIMO radar and hybrid MIMO phased-array radar have been proposed [43–45]. The signaling strategies discussed in these works use multiple beams (from subarrays), each of which is generated using a different waveform. Besides these array partitioning based techniques, the MIMO radar transmit beampattern synthesis (or transmit energy focusing) based on signal correlations has also been receiving much attention recently [20, 25–32, 34, 36–38, 71–75]. This approach makes full use of the waveform diversity introduced by MIMO radar to enhance the DOA estimation. The method focuses the energy of transmitted waveforms within one or several predefined angle sections where multiple targets are likely to exist. With an appropriate design, the performance can be improved compared to conventional phased-array radar or MIMO radar without beampattern synthesis. In addition, the designed system does not need to sweep the field at transmitter as in conventional phased-array radar.

MIMO radar transmit beampattern is characterized by the covariance matrix of the transmitted waveforms, thus the objective is to design several transmitted waveforms to achieve the desired covariance matrix. In this situation, the desired covariance matrix usually lies between a scaled identity matrix (orthogonal waveforms) and a scaled all-one matrix (coherent waveforms), thus the common mode of operation that MIMO radar transmits orthogonal waveforms is altered.

There are usually two procedures to achieve the design. In Procedure 1, the optimal covariance matrix is obtained according to the desired beampattern. Then

in Procedure 2, the waveforms are optimized according to the obtained covariance matrix. The beampattern synthesis problem for MIMO radar was first formulated in [25] as a minimization of a cost function, with the solution based on a gradient search. In [36] and [37], a closed-form least-squares solution and a low-complexity method based on the modification of the cost function were proposed, respectively. In [26] (the full version of [25]), the authors proposed new optimization problems in which the positive semidefinite matrix constraint was parameterized by a set of coefficients, to achieve iterative solutions. In [27] and [29], the authors provided a more comprehensive analysis on transmit beampattern synthesis, in which several designing scenarios, such as maximum power design, beampattern matching design, and minimum sidelobe design, were investigated. In [30] and [31], the covariance matrix was parameterized using the coordinates of a hypersphere, resulting in an iterative solution as well as a closed-form solution. More recently, the design has been extended to wideband applications [34].

It was noted in [26,29] and [31] that the second design step, i.e., the design of the waveforms according to the optimized covariance matrix, is not easy to achieve. In [71–73], and their full version [28], a more efficient technique called transmit beamspace processing (TBP) was proposed. The TBP technique introduces a weighting matrix to several orthogonal waveforms used in the standard MIMO radar. In this way, the covariance matrix of the transmitted waveforms becomes the quadratic form of the weighting matrix, and once the covariance matrix is obtained, the weighting matrix can be easily derived using eigenvalue decomposition.

Although many solutions and discussions have been made upon this problem, very little attention has been paid to several aspects of the performance such as the ripple within the energy focusing section, the sidelobe attenuation, the transition bandwidth, and the system efficiency in terms of the choice of design step-size of the angle and the number of transmit antennas. In this chapter, a comprehensive discussion of transmit beampattern design for MIMO radar is provided. Specifically, a weighted beampattern matching design method based on an ℓ_1 -norm cost function is proposed, which can indirectly control the ripples within the energy focusing section and the transition bandwidth. Two closed-form solutions to the ℓ_2 -norm cost function are proposed. They are based on matrix regularization (MR) and singular value decomposition (SVD) respectively. The former is able to indirectly control the ripples and the transition bandwidth. More importantly, we

reformulate the transmit beampattern design as a feasibility problem (FP). This formulation is superior to minimization based optimization problems because it has a full control of the ripple levels, the sidelobe levels, the desired power levels, the transition bandwidth, and the number of transmit antennas. Subsequently, A MIMO radar beampattern formula for antennas selection is obtained empirically based on the proposed FP. This formula provides an approximation of the number of transmit antennas required for different design specifications.

4.2 Signal Model and Transmit Beamspace Processing

Recall the transmit signal model in Section 1.2. Consider a MIMO radar system with N_T transmit antennas and N_T orthogonal waveforms of sample length N . The i th waveform sequence is denoted as $\mathbf{s}_i$, where $i \in \{0, 1, \dots, N_T - 1\}$. Let the transmit signal matrix be $\mathbf{S} \in \mathbb{C}^{N_T \times N}$, then for the omnidirectional transmission as shown in (1.1),

$$\mathbf{S} = [\mathbf{s}_0, \mathbf{s}_1, \dots, \mathbf{s}_{N_T-1}]^T, \quad (4.1)$$

where $\|\mathbf{s}_i\|^2 = 1$ is the elemental power constraint. Since in this chapter we do not consider the receiver side of the radar system, the transmit steering vector (1.2) is simply denoted as

$$\mathbf{e}(\theta) = [1, e^{-j\pi \cos \theta}, \dots, e^{-j\pi(N_T-1) \cos \theta}]^T, \quad (4.2)$$

instead, where θ denotes the azimuth angle. The transmit beampattern is defined as the power distribution along different steering directions, i.e.,

$$P(\theta) = \mathbf{e}^T(\theta) \mathbf{R} \mathbf{e}^*(\theta), \quad (4.3)$$

where

$$\mathbf{R} = \lim_{N \rightarrow \infty} \mathbf{S} \mathbf{S}^H \quad (4.4)$$

is the covariance matrix of the transmitted waveforms. The covariance matrix is approximated using a finite number of samples, and it is assumed that $\mathbf{R} \triangleq \mathbf{S} \mathbf{S}^H \approx \mathbf{I}$.

The original design of MIMO radar transmit beampattern consists of two steps which first optimizes the covariance matrix $\mathbf{R}$ followed by the corresponding waveform design. In this case, the design of N_T partially correlated waveforms is needed to achieve the optimized $\mathbf{R}$, which can be found in [26,29] and [31]. Instead of transmitting N_T partially correlated waveforms, the TBP technique [28] introduces a weighting matrix $\mathbf{W} \in \mathbb{C}^{N_T \times K}$, $K \leq N_T$ at the transmitter. In this way, only K orthogonal waveforms are needed for N_T antennas. Denote the matrix stacking the K orthogonal waveforms as

$$\mathbf{S}_{\text{TBP}} = [\mathbf{s}_0, \mathbf{s}_1, \dots, \mathbf{s}_{K-1}]^T \in \mathbb{C}^{K \times N}, \quad (4.5)$$

then the transmit signal matrix (1.1) modifies to $\mathbf{S} = \mathbf{W}\mathbf{S}_{\text{TBP}}$, and the transmit beampattern becomes

$$\begin{aligned} P(\theta) &= \mathbf{e}^T(\theta) \mathbf{R} \mathbf{e}^*(\theta) \\ &= \mathbf{e}^T(\theta) \mathbf{W} \mathbf{S}_{\text{TBP}} \mathbf{S}_{\text{TBP}}^H \mathbf{W}^H \mathbf{e}^*(\theta) \\ &\approx \mathbf{e}^T(\theta) \mathbf{W} \mathbf{W}^H \mathbf{e}^*(\theta), \end{aligned} \quad (4.6)$$

where $\mathbf{S}_{\text{TBP}} \mathbf{S}_{\text{TBP}}^H \approx \mathbf{I}$, and the covariance matrix of the transmitted waveforms becomes $\mathbf{R} = \mathbf{W} \mathbf{W}^H$. It can be seen from (4.6) that by using TBP, it becomes unnecessary to design partially correlated waveforms. The weighting matrix can be easily obtained via simple matrix manipulations such as eigenvalue decomposition. Another advantage of the TBP technique is that the number of orthogonal waveforms required can be less than that of the transmit antennas, i.e.,

$$K_{\min} = \text{rank}\{\mathbf{R}\}. \quad (4.7)$$

Hence the TBP technique simplifies the transmit beampattern design by avoiding complicated design of partially correlated waveforms as well as reducing the number of required waveforms. Note that the TBP technique is irrelevant to the first design step, i.e., optimizing the covariance matrix $\mathbf{R}$. Here we introduce the TBP technique to emphasize its efficiency in the second design step. In addition, the use of TBP technique reveals the possibility that the transmit beampattern design for MIMO radar can be mapped to conventional FIR filter design, which will be illustrated in Section 4.4, and through this way an FP formulation is proposed. In

the following content, we focus on the design of $\mathbf{R}$. We now express the transmit beampattern in matrix form. According to the properties of trace and Kronecker product, (4.3) can be rewritten in an inner product form:

$$\begin{aligned}
 P(\theta) &= \mathbf{e}^T(\theta) \mathbf{R} \mathbf{e}^*(\theta) \\
 &= \text{tr}\{\mathbf{e}^*(\theta) \mathbf{e}^T(\theta) \mathbf{R}\} \\
 &= [\text{vec}\{\mathbf{e}^*(\theta) \mathbf{e}^T(\theta)\}]^H \text{vec}\{\mathbf{R}\} \\
 &= [\mathbf{e}^H(\theta) \otimes \mathbf{e}^T(\theta)] \text{vec}\{\mathbf{R}\}.
 \end{aligned} \tag{4.8}$$

Let $\Delta\theta$ be an appropriately chosen step-size for the angle resolution in the design, then more explicitly we have

$$\begin{bmatrix} P(0^\circ) \\ P(\Delta\theta) \\ P(2\Delta\theta) \\ \vdots \\ P(180^\circ) \end{bmatrix} = \begin{bmatrix} \mathbf{e}^T(0^\circ) \otimes \mathbf{e}^H(0^\circ) \\ \mathbf{e}^T(\Delta\theta) \otimes \mathbf{e}^H(\Delta\theta) \\ \mathbf{e}^T(2\Delta\theta) \otimes \mathbf{e}^H(2\Delta\theta) \\ \vdots \\ \mathbf{e}^T(180^\circ) \otimes \mathbf{e}^H(180^\circ) \end{bmatrix} \text{vec}\{\mathbf{R}\}, \tag{4.9}$$

which represents a set of linear equations. Denote (4.9) as

$$\mathbf{p} = \mathbf{A} \text{vec}\{\mathbf{R}\}, \tag{4.10}$$

where $\mathbf{A} \in \mathbb{C}^{[(180/\Delta\theta)+1] \times N_T^2}$, then the linear equations corresponding to the angle section(s) where the transmit energy is supposed to be focused can be extracted to form $\mathbf{p}_{\text{in}} = \mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\}$, and those left is $\mathbf{p}_{\text{out}} = \mathbf{A}_{\text{out}} \text{vec}\{\mathbf{R}\}$. Note that $\Delta\theta$ can be appropriately chosen such that $180/\Delta\theta$ is an integer. There are various other formulations to the design problem as noted in [27, 28, 30], and the references therein. However, all these formulations can be equivalently represented using (4.9), because (4.9) reflects the ultimate objective of the design, namely, the control of the spatial energy level w.r.t. each angle.

4.3 The Design of the Covariance Matrix

A standard phased-array radar transmits coherent waveforms. With the elemental power constraint, the covariance matrix is an all-one matrix, thus the transmit beampattern is equivalent to the delay-and-sum response. For a standard MIMO radar transmitting orthogonal waveforms, the covariance matrix is an identity matrix, resulting in a uniform spatial response. The waveform diversity of MIMO radar enables flexible control over the transmit beampattern by generating a covariance matrix between the two extremes.

4.3.1 Existing Design Formulations

Generally, the design of transmit beampattern for MIMO radar can be considered as a constrained optimization problem. In the literature, two common design categories can be identified, which are the beampattern matching design and the maximum power design. The commonly used cost functions and constraints are noted in the following.

- In beampattern matching design, a desired transmit beampattern, $P_d(\theta)$, is specified. The desired power level within the focusing angle section is denoted as P_0 , and that outside this region is 0. Usually, $P_d(\theta)$ is one or several rectangular functions. Similar to $\mathbf{p}_{\text{in}}$ and $\mathbf{p}_{\text{out}}$, we have $\mathbf{p}_{\text{d,in}}$ and $\mathbf{p}_{\text{d,out}}$, respectively. The commonly used cost functions in [25–27, 30, 36, 37], which have been argued to be better than others, are

$$J_1(\mathbf{R}) = \|\mathbf{p}_d - \mathbf{A} \text{vec}\{\mathbf{R}\}\|_2, \quad (4.11)$$

$$J_2(\mathbf{R}) = \|\mathbf{p}_d - \mathbf{A} \text{vec}\{\mathbf{R}\}\|_1, \quad (4.12)$$

where $J_2(\mathbf{R})$ is the ℓ_1 -norm version of $J_1(\mathbf{R})$. Note that in some of the literature, there is a cross term added to $J_1(\mathbf{R})$, e.g. (19) in [27] and (4) in [30]. This term is irrelevant to the covariance matrix design. Instead, it helps in the second design step to reduce the cross-correlations of the partially correlated waveforms. As we adopt the TBP technique for the design, which uses orthogonal waveforms, the use of the cross term is unnecessary.

- The maximum power design appears in [27] and [28]. The former maximizes

the minimum power within the energy focusing section, whereas the later maximizes the ratio of the focused power and the total power. Note that there is no desired focusing power level involved in the maximum power design technique. Hence the cost functions can be written as:

$$J_3(\mathbf{R}) = -\min \{ \mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\} \}, \quad (4.13)$$

$$J_4(\mathbf{R}) = -\frac{\| \mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\} \|_1}{\| \mathbf{A} \text{vec}\{\mathbf{R}\} \|_1}. \quad (4.14)$$

On one hand $\mathbf{R}$ needs to be a covariance matrix, thus $\mathbf{R}$ is positive semidefinite. On the other hand, to improve the transmission efficiency, the elemental power constraint needs to be satisfied. These two constraints are expressed as

$$C_1 : \quad \mathbf{R} \succeq \mathbf{0}, \quad (4.15)$$

$$C_2 : \quad \text{diag}\{\mathbf{R}\} = \mathbf{1}. \quad (4.16)$$

It should be noted that in the beampattern matching design, C_2 may lead to failure in achieving P_0 within the focusing section of the resulting transmit beampattern. The power levels of the transmit beampattern are implicitly governed by the power assigned to the waveforms (e.g., C_2 or other constraints on the power of the waveforms). For a given N_T with C_2 satisfying that each waveform has unit power, it is intuitive to expect P_0 has an upper limit. If P_0 is chosen above this limit, then the desired power focusing level would be beyond that can be achieved by the given waveforms. In this chapter, such conflict is illustrated in Section 4.5, but a quantitative analysis, which is worth of further research attention, is not provided. Hence, if C_2 incurs problems in the design, it could be relaxed to

$$C_3 : \quad \text{tr}\{\mathbf{R}\} = N_T \quad (4.17)$$

to weaken the conditions on $\mathbf{R}$. If the solution still fails to achieve P_0 , then C_3 can be further relaxed in the design, e.g., a larger value could be assigned to $\text{tr}\{\mathbf{R}\}$.

4.3.2 Indirect Control of Ripples

The Weighted Beampattern Matching Design

It has been illustrated in [26] and [30] that the use of $J_2(\mathbf{R})$ outperforms $J_1(\mathbf{R})$ in terms of the ripple levels within the energy focusing section. Hence we propose a weighted cost function based only on $J_2(\mathbf{R})$, which is expressed as

$$J_5(\mathbf{R}, \mu) = \mu \|\mathbf{p}_{d,\text{in}} - \mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\}\|_1 + (2 - \mu) \|\mathbf{p}_{d,\text{out}} - \mathbf{A}_{\text{out}} \text{vec}\{\mathbf{R}\}\|_1, \quad (4.18)$$

where $0 \leq \mu \leq 2$. The value 2 is chosen to normalize the weight to each term in (4.18). It can be verified that $J_5(\mathbf{R}, 1) = J_2(\mathbf{R})$, thus $J_2(\mathbf{R})$ is a special case of $J_5(\mathbf{R}, \mu)$. The value 2 can be replaced by any arbitrary positive value without altering the cost function. Note that $J_5(\mathbf{R}, \mu)$ enables the trade-off between the designing error within and outside the energy focusing section(s). Hence the design becomes more flexible as it can emphasize more on either the energy focusing or the sidelobe reduction. If $\mu \rightarrow 2$, then the ripples within the focusing section would be small. If $\mu \rightarrow 0$, then the sidelobe levels would become small. Simulations in Section 4.5 will illustrate these points.

Closed-Form Solutions to $J_1(\mathbf{R})$

It has been noted in [36,37], and [30] that $J_1(\mathbf{R})$ can be considered as a least-squares problem with the following linear equation representation

$$\mathbf{A} \text{vec}\{\mathbf{R}\} = \mathbf{p}_d. \quad (4.19)$$

The rows in $\mathbf{A}$ have N_T^2 elements with $(2N_T - 1)$ distinct values, which are

$$\{e^{j\pi \cos[(1-N_T)\theta]}, e^{j\pi \cos[(2-N_T)\theta]}, \dots, e^{j\pi \cos[(N_T-1)\theta]}\}. \quad (4.20)$$

Hence if

$$\frac{180}{\Delta\theta} + 1 \geq 2N_T - 1, \quad (4.21)$$

which is usually satisfied, then $\text{rank}\{\mathbf{A}\} = 2N_T - 1$. In general $\mathbf{A}$ is rank deficient. The closed-form solution in [36] suffers from inversion of a rank deficient matrix.

In [30] the authors has considered the positive semidefinite constraint and the elemental power constraint into the formulation, which complicates the formulation and hence the solutions.

We propose two closed-form solutions based on matrix regularization (MR) and singular value decomposition (SVD), respectively, without any modification of (4.19). The positive semidefinite constraint C_1 and elemental power constraint C_2 are relaxed. The two solutions are given by

$$\text{vec}\{\tilde{\mathbf{R}}_{\text{MR}}\} = (\mathbf{A}^H \mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{A}^H \mathbf{p}_d, \quad (4.22)$$

$$\text{vec}\{\tilde{\mathbf{R}}_{\text{SVD}}\} = \mathbf{V}_1 \boldsymbol{\Sigma}_1^{-1} \mathbf{U}_1^H \mathbf{p}_d + \mathbf{V}_2 \mathbf{x}, \quad (4.23)$$

where λ is a regularization parameter, and $\mathbf{x}$ is an arbitrary vector. $\mathbf{U}_1$, $\boldsymbol{\Sigma}_1$, $\mathbf{V}_1$, and $\mathbf{V}_2$ are from the SVD of $\mathbf{A}$:

$$\mathbf{A} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^H \quad (4.24)$$

$$= \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_2 \end{bmatrix}^H, \quad (4.25)$$

where $\boldsymbol{\Sigma}_1 \in \mathbb{R}_+^{(2N_T-1) \times (2N_T-1)}$, and $\mathbf{x}$ is an arbitrary vector. We first investigate the similarity and difference between the two solutions, then we discuss how the solutions can be modified to satisfy C_1 .

The SVD based solution (4.23) is the general solution using pseudo inverse for least-squares problems. In a general problem, one would use a minimum norm constraint on the variable, resulting in $\mathbf{x} = \mathbf{0}$. The MR based solution (4.22) adds a perturbation parameter λ to the pseudo inverse minimum norm solution, and in this way the ripples can be controlled. To illustrate this point, substitute (4.23) and (4.25) into the left-hand side of (4.19), the designed transmit beampattern using (4.23) is then given by

$$\begin{aligned} \mathbf{A} \text{vec}\{\tilde{\mathbf{R}}_{\text{SVD}}\} &= \mathbf{U}_1 \boldsymbol{\Sigma}_1 \mathbf{V}_1^H \mathbf{V}_1 \boldsymbol{\Sigma}_1^{-1} \mathbf{U}_1^H \mathbf{p}_d + \mathbf{U}_1 \boldsymbol{\Sigma}_1 \mathbf{V}_1^H \mathbf{V}_2 \mathbf{x} \\ &= \mathbf{U}_1 \mathbf{U}_1^H \mathbf{p}_d, \end{aligned} \quad (4.26)$$

where $\mathbf{V}_1^H \mathbf{V}_2 = \mathbf{0}$ because $\mathbf{V}$ is a unitary matrix. Hence, $\mathbf{x}$ does not alter the designed beampattern. For the MR based solution (4.22), it can be verified using

(4.25) that if $\lambda \rightarrow 0$, then designed beampattern approaches (4.26), i.e., $\mathbf{U}_1 \mathbf{U}_1^H \mathbf{p}_d$, and if $\lambda \rightarrow \infty$, then the designed beampattern becomes $\mathbf{A} \mathbf{A}^H \mathbf{p}_d$. This is how λ indirectly controls the ripples.

Although the SVD based solution is not able to control the ripples, we present this solution here to show an advantage of the SVD based solution, especially under the unavoidable constraint C_1 . Note that the solutions (4.22) and (4.23) are obtained without considering the constraint C_1 (being positive semidefinite), which must be satisfied. It is shown in the appendix that $\tilde{\mathbf{R}}_{\text{MR}}$ and $\tilde{\mathbf{R}}_{\text{SVD}}$ are Hermitian. However, they may have negative eigenvalues. For the MR based solution (4.22), the only way to deal with the negative eigenvalues is to discard negative eigenvalues of $\tilde{\mathbf{R}}_{\text{MR}}$, which may cause severe degradation of performance especially when the negative eigenvalues are large. However for the SVD based solution (4.23), it may be possible to take the advantage of the variable $\mathbf{x}$ to force the eigenvalues of $\tilde{\mathbf{R}}_{\text{SVD}}$ being nonnegative. Let $\mathbf{R}_H$ be an arbitrary Hermitian matrix, then the problem of determining $\mathbf{x}$ after obtaining (4.23) is stated as

$$\begin{aligned} \text{find } & \mathbf{x}, \mathbf{R}_H \\ \text{s.t. } & \mathbf{R}_H = \mathbf{R}_H^H, \\ & \text{vec} \{ \mathbf{R}_H \} = \mathbf{V}_2 \mathbf{x}, \\ & \tilde{\mathbf{R}}_{\text{SVD}} \succeq \mathbf{0}. \end{aligned} \tag{4.27}$$

As the work of this chapter mainly focuses on the ripple and transition band control in the transmit beampattern design, the solution to the above problem is not discussed here. The problem (4.27) is noted here for the future research as well as for the completeness of the solution (4.23).

In this subsection, two methods to indirectly control the ripples have been discussed. The control is indirect because there are no explicit constraints or objective functions to determine the ripple levels. Although the ripples can be affected by tuning the newly introduced parameters μ or λ , quantitative assessment of ripple levels is not known before the solution is obtained. Although the SVD based closed-form solution is unable to indirectly control the ripples, its relation to the MR solution and the existence of the arbitrary parameter $\mathbf{x}$ is interesting and worth of further investigation.

4.3.3 Direct Control of Ripples

Additional constraints can be imposed to explicitly control the ripple levels within the energy focusing section. The formulations using such constraints are referred to as direct control. Let the maximum tolerable ripple level be δ , then the additional constraints can be formulated as follows. For beampattern matching design, the ripple levels depend on P_0 , thus we set

$$\begin{aligned} C_4 : \quad & \max \{ \mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\} \} \leq P_0 + 0.5\delta, \\ & \min \{ \mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\} \} \geq P_0 - 0.5\delta. \end{aligned} \quad (4.28)$$

For maximum power design, the peak level is independent of P_0 , thus the ripple constraint can be set as

$$C_5 : \quad \max \{ \mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\} \} - \min \{ \mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\} \} \leq \delta. \quad (4.29)$$

Note that C_4 and C_5 can be used with any cost function. Also note that $\mathbf{p}_d$ is a function of P_0 , where the letter is the energy level within the energy focusing section of the former. For $\mathbf{p}_d$ -dependent cost functions, C_4 and C_5 are identical. For $\mathbf{p}_d$ -independent cost functions, the use of C_4 will modify the design to a $\mathbf{p}_d$ -dependent case. The relationship between the ripple level and the transition bandwidth can be explained using correlation properties. It can be seen from (4.9) that the adjacent rows in $\mathbf{A}$ are correlated because of the small variation $\Delta\theta$. If δ is small, then $\text{vec}\{\mathbf{R}\}$ has nearly identical Hermitian angles [76] with the rows corresponding to the passband. This will make the Hermitian angles between $\text{vec}\{\mathbf{R}\}$ and the adjacent rows near the passband quite close to each other, resulting in a wide transition band. On the contrary, if δ is increased, the Hermitian angles between $\text{vec}\{\mathbf{R}\}$ and passband rows in $\mathbf{A}$ can vary, which will provide more degree of freedom to let the Hermitian angles between $\text{vec}\{\mathbf{R}\}$ and stopband rows increase, so that the transition bandwidth would be shortened.

4.3.4 Convexity

In the previous subsections, 5 cost functions and 5 constraints have been reviewed or proposed. Different combinations of the cost functions and constraints form dif-

ferent constrained optimization problems. To ensure that the problems formulated are efficiently solvable, the cost functions and constraints need to satisfy convexity. Here the convexity is briefly discussed and details can be found in [77]. Let $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote the real and imaginary parts of a complex variable, then it can be shown that

$$J_1^2(\mathbf{R}) = \left\| \text{Re}\{\mathbf{p}_d\} - \begin{bmatrix} \text{Re}\{\mathbf{A}\} \\ -\text{Im}\{\mathbf{A}\} \end{bmatrix}^T \begin{bmatrix} \text{Re}\{\text{vec}\{\mathbf{R}\}\} \\ \text{Im}\{\text{vec}\{\mathbf{R}\}\} \end{bmatrix} \right\|_2^2 + \left\| \text{Im}\{\mathbf{p}_d\} - \begin{bmatrix} \text{Im}\{\mathbf{A}\} \\ \text{Re}\{\mathbf{A}\} \end{bmatrix}^T \begin{bmatrix} \text{Re}\{\text{vec}\{\mathbf{R}\}\} \\ \text{Im}\{\text{vec}\{\mathbf{R}\}\} \end{bmatrix} \right\|_2^2, \quad (4.30)$$

which is a convex function. Similarly, $J_2(\mathbf{R})$ and $J_5(\mathbf{R}, \mu)$ are convex. For $J_3(\mathbf{R})$, C_4 , and C_5 , the portion $\mathbf{A}_{\text{in}} \text{vec}\{\mathbf{R}\}$ can be verified to be an affine function by substituting real and imaginary parts into $\mathbf{A}_{\text{in}}$ and $\text{vec}\{\mathbf{R}\}$. Hence $J_3(\mathbf{R})$, C_4 , and C_5 are convex. C_1 , C_2 , and C_3 are well-formed constraints that can be used with any of the convex cost functions. Hence combinations of the above cost functions and constraints can be efficiently solved using the tools (e.g., the solver *SeDuMi* or *SDPT3*) provided in [78]. Note that $J_4(\mathbf{R})$ is not convex, but an efficient solution via eigenvalue decomposition for $J_4(\mathbf{R})$ with C_1 and C_2 are provided in [28]. In this chapter, we mainly use the tools in [78] to solve the optimization problems. We therefore emphasize more on appropriate convex problem formulations and hence the computational complexity analysis for the solutions is not provided.

4.4 Covariance Matrix Design as a Feasibility Problem

4.4.1 Motivation

The design of the transmit beampattern for phased-array radar with ULA transmitter can be related to the design of digital finite-impulse-response (FIR) filters, where the tapped delay line is replaced by spatial delay, and the filter length is corresponding to the number of transmit antennas [74, 75]. The MIMO radar transmit beampattern design based on TBP can then be considered as a generalization

Figure 4.1: The block diagram of MIMO radar transmit beampattern design based on TBP, shown as a MISO FIR filter design.

of conventional filter design, which is a multiple-input-single-output (MISO) filter. This is depicted in Fig. 4.1, where $\tau = e^{j\pi \cos \theta}$, and the filter coefficient $w_{m,k}$ is the $\{m, k\}$ th element of the weighting matrix $\mathbf{W}$. According to (4.6), the transmit beampattern is in fact the energy density spectrum resulting from the filter coefficients. Another observation is that the positive semidefinite constraint and the power constraint are required in MIMO radar design, but they are unnecessary in conventional FIR filter design.

It is well known that in a typical FIR filter design problem, the passband and stopband ripples and the transition bandwidth are key parameters to characterize the performance of the filter. Reducing the ripples and the transition bandwidth results in increasing the filter length. For a set of given specifications, an empirical formula (Kaiser's formula) provides an approximation of the filter length [79]. As noted earlier in the context of MIMO radar transmit beampattern design, the ripple control has seldom been considered. This motivates us to study the ripple control performance and its relationship with the transition bandwidth in MIMO radar transmit beampattern design. The ripples exist due to the truncation of the ideal impulse response, which is referred to as the Gibbs phenomenon [79]. For a radar system, the filter length, i.e., the number of transmit antennas is limited, thus it is intuitive to expect that if the number of transmit antennas is fixed, then reducing the ripples will result in larger transition bandwidth. If the number of transmit

antennas can be adjusted, then there exists a trade-off among the ripple levels, transition bandwidth, and the number of transmit antennas. In the following sections, such a trade-off will be quantitatively investigated using an expression similar to Kaiser's formula.

4.4.2 Design Reformulated as a Feasibility Problem

In the previous sections, $\mathbf{A}$ is partitioned into $\mathbf{A}_{\text{in}}$ and $\mathbf{A}_{\text{out}}$, indicating a design with zero transition bandwidth. In this section, a transition band ΔB is introduced. For simplicity, the desired beampatterns are chosen symmetrical w.r.t. 90° . Let the passband edges be θ_P ($90^\circ < \theta_P < 180^\circ$) and $180^\circ - \theta_P$, thus the stopband edges are $\theta_S \triangleq \theta_P + \Delta B$ and $180^\circ - \theta_S$. The rows in $\mathbf{A}$ which correspond to the passband and stopband are now denoted as $\mathbf{A}_P(\theta_P)$ and $\mathbf{A}_S(\theta_S)$ respectively. Because $\theta_S \triangleq \theta_P + \Delta B$, $\mathbf{A}_S(\theta_S)$ is a function of both θ_P and ΔB , and therefore denoted as $\mathbf{A}_S(\theta_P, \Delta B)$. Let the stopband attenuation be ε , then the FP is formulated as

$$\begin{aligned}
 &\text{find } \mathbf{R} \\
 &\text{s.t. } \mathbf{R} \succeq \mathbf{0}, \\
 &\quad \text{diag}\{\mathbf{R}\} = \mathbf{1} \text{ or } \text{tr}\{\mathbf{R}\} = N_T, \\
 &\quad \max \{\mathbf{A}_P(\theta_P) \text{vec}\{\mathbf{R}\}\} \leq P_0 + 0.5\delta, \\
 &\quad \min \{\mathbf{A}_P(\theta_P) \text{vec}\{\mathbf{R}\}\} \geq P_0 - 0.5\delta, \\
 &\quad \max \{\mathbf{A}_S(\theta_P, \Delta B) \text{vec}\{\mathbf{R}\}\} \leq \varepsilon.
 \end{aligned} \tag{4.31}$$

Let N_{bot} denote the initial estimate of the minimum number of transmit antennas, and N_{top} denote the maximum number of transmit antennas allowed in the radar system, then the algorithm to find the number of transmit antennas is summarized as follows:

Step 0: Set designing parameters: θ_P , ΔB , δ , P_0 , and ε .

Step 1: Start at initial value $N_T \leftarrow N_{\text{bot}}$.

Step 2: Obtain the solution to (4.31) using the tools in [78].

Step 3: If the problem is not feasible, then $N_T \leftarrow N_T + 1$, and go to **Step 4**. Else if the problem is feasible, then the problem is solved, and the $\mathbf{R}$ obtained is the optimal solution.

Step 4: If $N_T \leq N_{\text{top}}$, go to **Step 2**. Else if $N_T > N_{\text{top}}$, then the problem

Figure 4.2: Mapping from MIMO radar transmit beampattern to the frequency response of FIR filter.

cannot be solved, N_{top} should be increased.

Note that the FP formulation depends on P_0 . If P_0 is chosen inappropriately, then the elemental power constraint or trace constraint will conflict with P_0 , and the design will fail. An alternative formulation independent of P_0 can be easily obtained by subtracting the 3rd and 4th constraints of (4.31) if necessary, i.e.,

$$\max \{ \mathbf{A}_P(\theta_P) \text{vec}\{\mathbf{R}\} \} - \min \{ \mathbf{A}_P(\theta_P) \text{vec}\{\mathbf{R}\} \} \leq \delta. \quad (4.32)$$

4.4.3 The MIMO Radar Beampattern Formula

In conventional FIR filter designs, the empirical formula obtained by Kaiser [79] provides an efficient way to determine the approximate filter length required to achieve the ripples and transition bandwidth requirements. As shown in Fig. 4.1, the design of transmit beampattern for MIMO radar is a generalized MISO FIR filter design problem with the tap delay nonlinearly mapped into propagation delay. In this case, the estimation of the number of transmit antennas (filter length) is a more complicated problem which cannot be easily solved. In this section, we establish a relationship between the estimated number of transmit antennas and other system parameters, to which we refer as the MIMO radar transmit beampattern formula. The parameters involved in the design are N_T , θ_P , ΔB , δ , ϵ , P_0 , and $\Delta\theta$.

The Nonlinear Mapping

According to Fig. 4.2, the relationship between the conventional FIR filter and MIMO radar transmit beampattern is that $e^{-j\pi 2f}$ is mapped to $e^{j\pi \cos \theta}$. Because such mapping is nonlinear, the locations of θ_P and θ_S affect the design, while in Kaiser's formula, the design is independent of the locations of passband and stopband edges. The parameters mapped to an equivalent FIR filter are as follows: passband edge: $0.5 |\cos \theta_P|$, stopband edge: $0.5 |\cos \theta_S|$, transition bandwidth: $0.5 (|\cos \theta_S| - |\cos \theta_P|)$. The estimated number of transmit antennas is denoted as $\hat{N}_T(\theta_P, \Delta B, \delta, \varepsilon, P_0)$. Note that $\theta_S = \theta_P + \Delta B$. In Kaiser's formula the passband and stopband edges do not affect the estimation of the filter length. Instead, the transition bandwidth does, which appears in its denominator [79]. However, due to the nonlinear mapping from filter tap delay to antenna propagation delay, the locations of the passband and stopband edges become nontrivial. To eliminate the effect of the locations of passband and stopband edges in the beampattern formula, we set $\theta_P = 120^\circ$ in the initial design. In fact, any other value of $\theta_P \in (90^\circ, 180^\circ)$ can be chosen for the initial design. Using the nonlinear mapping, it can be shown that $\hat{N}_T(120^\circ, \Delta B, \delta, \varepsilon, P_0)$ is equal to the estimation of the number of transmit antennas for any arbitrary location $\tilde{\theta}_P, \Delta \tilde{B}$, with an approximately chosen ΔB , i.e.,

$$\hat{N}_T(\tilde{\theta}_P, \Delta \tilde{B}, \delta, \varepsilon, P_0) = \hat{N}_T(120^\circ, \Delta B, \delta, \varepsilon, P_0). \quad (4.33)$$

Equating the equivalent transition bandwidth by letting $|\cos(\tilde{\theta}_P + \Delta \tilde{B})| - |\cos \tilde{\theta}_P| = |\cos(120 + \Delta B)| - |\cos 120|$, we have

$$\Delta B = \arccos \left\{ \cos(\tilde{\theta}_P + \Delta \tilde{B}) - \cos \tilde{\theta}_P + \cos 120^\circ \right\} - 120^\circ. \quad (4.34)$$

In other words, if the design is for passband edge at $\tilde{\theta}_P$ and transition bandwidth of $\Delta \tilde{B}$, then the estimated number of antennas is equivalent to the estimation using $\theta_P = 120^\circ$ and ΔB evaluated via (4.34).

Kaiser's Formula Like Expression

The properties observed in Kaiser's formula [79] are also applicable for the beampattern formula, i.e.,

$$\hat{N}_T \propto \log_{10}(\delta \varepsilon), \quad (4.35)$$

and

$$\hat{N}_T \propto \frac{1}{\Delta B}. \quad (4.36)$$

The step-size $\Delta\theta$ for the angles in the design corresponds to the spectral resolution of an FIR filter frequency response. If it is chosen too large, then the beampattern may not be able to preserve the energy between two consecutive design angles. If it is chosen small, then the complexity of the design increases as the dimension of $\mathbf{A}$ increases. For simplicity, we set $\Delta\theta = 1^\circ$ in the design. The estimated number of transmitted antennas can then be expressed as

$$\hat{N}_T(120^\circ, \Delta B, \delta, \varepsilon, P_0) \approx \frac{af(P_0)\log_{10}(\delta\varepsilon) + b}{c\Delta B}, \quad (4.37)$$

which indicates that the MIMO radar transmit beampattern formula has the form of the Kaiser's formula, with the inclusion of an additional variable P_0 . It will be shown in Section 4.5.4 that the function f and the coefficients a , b , and c can be empirically obtained via simulations.

4.5 Numerical Examples

In this section, the advantages of the proposed cost function $J_5(\mathbf{R}, \mu)$ is first illustrated. Then the performances of the closed-form solutions presented in Section III are compared and analyzed. After that, the performances considering the direct ripple control are investigated by comparing different combinations of cost functions and constraints. All the optimization problems without closed-form solutions are solved using public domain optimization tools [78]. The parameters for simulations are set as follows: $N_T = 10$, $\Delta\theta = 1^\circ$, and the angle section of energy focusing is $\Theta_1 = [60^\circ, 120^\circ]$. Note that in the literature, the design of multiple beams has been considered. However, this can also be achieved via the individual design of each beam and then appropriately adding them together. Hence it is sufficient to discuss only the single beam design. C_1 is considered as a global constraint hence omitted. The simulations to obtain a , b , c and f for the beampattern formula are provided at the end of this section.

Figure 4.3: Optimized beampatterns with indirect ripple control, where $N_T = 10$, $P_0 = 20$, and C_3 included. (a) Beampattern matching design, $P_0 = 20$, $\mu = 0.5, 1.5$. (b) The dB view of (a).

Figure 4.4: Optimized beampatterns with indirect ripple control where $N_T = 10$, and $P_0 = 30$. (a) Failure of beampattern matching design due to the conflict between $P_0 = 30$ and C_3 . (b) The same design as in (a) but with C_3 excluded. $\text{tr}\{\mathbf{R}\}$ is 15.6424, 15.3885, 13.9783, and 16.1968, respectively.

4.5.1 Performances of Indirect Ripple Control

The simulation results of the designs without the ripple control constraints are provided in Fig. 4.3–4.8. Fig. 4.3 compares the proposed method using $J_5(\mathbf{R}, \mu)$, with two popular designs using $J_1(\mathbf{R})$ and $J_2(\mathbf{R})$. It is indicated that different choices of μ put different weights on the error within and outside the energy focusing section. If $\mu < 1$, then minimizing the sidelobes is more important than minimizing the error within the focusing section, and vice versa if $\mu > 1$. The

Figure 4.5: Optimized beampatterns using the proposed $J_5(\mathbf{R}, \mu)$ versus different choices of μ , where $N_T = 10$, C_3 excluded, and $P_0 = 30$.

beampattern designed using $J_5(\mathbf{R}, 1.5)$ appears to be the best solution as it has minimum passband ripples and satisfactory sidelobes observed from Fig. 4.3(a) and (b). Note that although the solution using $J_5(\mathbf{R}, 0.5)$ results in large ripples, it has minimum sidelobes as shown in Fig. 4.3(b).

The failure of the design is illustrated in Fig. 4.4(a), where $P_0 (=30)$ is inappropriately chosen under the constraint C_3 . As mentioned at the end of Section 4.3.1, the value of P_0 should not be chosen beyond the capability of the transmitter where the total transmit energy is constrained by $N_T = 10$. It can be seen that the solutions of $J_1(\mathbf{R})$ and $J_2(\mathbf{R})$ can only have power level around 20. A few points in the beampatterns designed using $J_5(\mathbf{R}, \mu)$ are reaching 30. Generally, all the methods presented in Fig. 4.4(a) fail the design. This problem can be solved via the relaxation of C_3 , which is shown in Fig. 4.4(b). It can also be solved by increasing the total transmit energy, which is straightforward and not presented via simulations. A further observation with $J_5(\mathbf{R}, \mu)$ is shown in Fig. 4.5, where it is seen that the ripple is small when μ approaches 2, while the sidelobes are relatively high. The ripple is large when μ approaches 0, while the sidelobes become relatively low. However, such a trade-off is empirical and the ripple control is not considered in the design formulations.

The optimized transmit beampatterns based on maximum power design are shown in Fig. 4.6, where the solution to $J_3(\mathbf{R})$ is obtained using [78], and the solution to $J_4(\mathbf{R})$ is based on the eigenvalue decomposition of $\sum_{\theta} \mathbf{e}(\theta) \mathbf{e}^H(\theta)$, where

Figure 4.6: Maximum power design examples, where $N_T = 10$, C_3 included, and P_0 irrelevant.

Figure 4.7: Optimized beampatterns with indirect ripple control using closed-form solutions to $J_1(\mathbf{R})$, $P_0 = 20$, $\mathbf{x} = \mathbf{0}$. All the traces are normalized to N_T . The solution of $\{J_2, C_3\}$ is provided for comparison. (a) Comparison between iterative solution and closed-form solutions. (b) The dB view of (a).

$\mathbf{e}(\theta)$ is the steering vector as defined in (4.2). Note that in maximum power design, the peak level of the power is independent of P_0 . Comparing Fig. 4.6 with Fig. 4.3(a), it can be seen that the solution to $J_4(\mathbf{R})$ has a similar performance with the solution to $J_5(\mathbf{R}, 0.5)$, although the former is not tunable. The solution to $J_3(\mathbf{R})$ has a wider mainbeam with lower energy focusing levels. In addition, the two methods have similar sidelobe levels.

The performances of the closed-form solutions to $J_1(\mathbf{R})$ are shown in Fig. 4.7 and 4.8 respectively. The iterative solution to $J_2(\mathbf{R})$ is also provided for compar-

Figure 4.8: Optimized beampatterns with indirect ripple control using closed-form solutions to $J_1(\mathbf{R})$, where $N_T = 10$, $P_0 = 20$, and $\mathbf{x} = \mathbf{0}$.

ison. It can be seen from Fig. 4.7 that both solutions have certain degradation. The SVD based solution has larger ripples and higher sidelobes, whereas the MR based solution has slightly less focused power compared to the $J_2(\mathbf{R})$ solution. In general, both solutions are quite satisfactory. Fig. 4.8 shows the beampatterns versus different values of λ . According to (4.23) and (4.44), if $\lambda = 0$ and $\mathbf{x} = \mathbf{0}$, then $\tilde{\mathbf{R}}_{\text{MR}} = \tilde{\mathbf{R}}_{\text{SVD}}$. Hence the advantage of $\tilde{\mathbf{R}}_{\text{MR}}$ is that it can control the ripples via λ , albeit indirectly. The advantage of $\tilde{\mathbf{R}}_{\text{SVD}}$ is that the variable $\mathbf{x}$ may be chosen as noted in (4.27) to force $\tilde{\mathbf{R}}_{\text{SVD}}$ being positive semidefinite to avoid the degradation caused by discarding negative eigenvalues.

4.5.2 Performances of Direct Ripple Control

It is shown using Fig. 4.3–4.8 that none of the existing methods is able to control the passband ripples, whereas the proposed method using cost function $J_5(\mathbf{R}, \mu)$, and the closed-form solution $\tilde{\mathbf{R}}_{\text{MR}}$ allow ripple control. However, the ripple control is indirect, i.e., we empirically change the values of μ and λ without knowing the effect on the ripple levels of the resultant beampatterns. It should be noted that the solution to $J_2(\mathbf{R})$ appears to have very small ripples, but if Θ_1 and P_0 are changed, the resultant ripples are not guaranteed to be small. Hence the use of constraints C_4 and C_5 are necessary, so that a certain level of ripple tolerance can be allowed in the design.

Figure 4.9: Optimized beampatterns with direct ripple control where $N_T = 10$, C_3 excluded, and C_5 included. (a) Beampattern matching design using $J_2(\mathbf{R})$ and Θ_1 under different values of P_0 , $\delta = 0.1$. (b) The same as (a) except Θ_1 is modified to $[68^\circ, 112^\circ]$.

Figure 4.10: Maximum power design using $J_3(\mathbf{R})$ under different values of δ , where Θ_1 is modified to $[65^\circ, 115^\circ]$, C_3 and C_5 included.

Fig. 4.9–4.10 provide a few examples of the transmit beampattern design using ripple control constraints C_4 and C_5 . C_4 is used for beampattern matching design and C_5 is used for maximum power design. It can be seen from Fig. 4.9(a) that the ripple levels within the passband are perfectly constrained within δ . However, this is achieved by compromising the transition bandwidth. Since there can be some tolerance within the passband, we can reduce the passband width accordingly to adjust the beampattern. This is shown in Fig. 4.9(b). Note that although Θ_1 is modified to range $[68^\circ, 112^\circ]$ for the solution, the desired passband is still $[60^\circ, 120^\circ]$.

The optimized beampatterns from the maximum power design using $J_3(\mathbf{R})$ with the trace constraint C_3 are shown in Fig. 4.10. We observe that the trade-off between the passband ripples and transition bandwidth can be achieved when the total energy constraint is imposed.

In general, the proposed cost function, closed-form solutions, as well as the ripple level constraints enable flexible design of the transmit beampattern. The ripple levels can be controlled indirectly (via tuning μ or λ) or directly (via tuning δ), and as a result one can freely choose the constraints and parameters for various design criteria such as minimum sidelobes, minimum passband ripples, narrowest transition bandwidth, etc. The design can be easily extended to multiple-passband beampatterns.

4.5.3 Feasibility Problem Solution Examples

The simulation results using the proposed FP formulation are shown in Fig. 4.11. The design parameters are set as follows: $\theta_p = 120^\circ$, $\Delta B = 10^\circ$, $\delta = \varepsilon = 0.1$, and $\Delta\theta = 1^\circ$. To compare the results with the methods reviewed and proposed in Section 4.3, we first solve the FP formulation to obtain the minimum number of antennas N_T . Using N_T , we then perform the optimization formulated by J_2 under two situations: 1) The simulation result using J_2 and C_4 where $P_0 = 25$ without C_3 is shown in Fig. 4.11 (a). The resultant trace of the covariance matrix is 14.3068, and $N_T = 21$. 2) The simulation result using J_2 , C_5 , and C_3 , without P_0 , is shown in Fig. 4.11 (b), where the maximum power focusing level is 40.33, and $N_T = 23$. These figures also indicate the possibility of improving the design using FP formulation upon the cost function based optimizations. It can be seen that the FP solutions have extra sidelobe suppression than the direct control l_1 -norm cost function based solutions. Another observation is that the sidelobes of the FP solution do not decay away from the mainlobe, but the sidelobes using cost function $J_2(\mathbf{R})$ do. This is because the sidelobe attenuation constraint in the FP formulation (4.31) only aims to suppress the sidelobes below ε , while the cost function formulation $J_2(\mathbf{R})$ minimizes the errors, resulting in the sidelobes approaching the desired value, i.e., 0, along the angle axis away from the mainlobe. The sidelobes for FP solution are strictly bounded under -25 dB, which is more preferable for rejection of echoes from targets existing outside the energy focusing region at the receiver. Moreover, the FP solution method is more flexible, because

(a) $P_0 = 25$, C_3 excluded. (b) P_0 excluded, C_3 included.

Figure 4.11: Optimized beampatterns using FP formulation, where $\theta_P = 120^\circ$, $\Delta B = 10^\circ$, and $\delta = \varepsilon = 0.1$. The minimal N_T is obtained via FP solution, then it is used for the solution using J_2 . (a) P_0 dependent design, where $P_0 = 25$, N_T is found to be 21, and $\text{tr}\{\mathbf{R}\} = 14.3068$. (b) P_0 independent design, where $N_T = \text{tr}\{\mathbf{R}\} = 23$, and the maximal focusing level is 40.33.

ΔB and ε can be tuned in the design, which is not possible in any other method discussed in Section 4.3. Hence it is indicated that the FP method yields the optimal solution which satisfies all the design constraints in a single iteration of the algorithm.

Figure 4.12: MIMO radar beampattern formula simulation results and approximations, where $\theta_P = 120^\circ$. (a) N_T vs. P_0 , where $\Delta B = 10^\circ$, and $\delta = \varepsilon = 0.1$. (b) N_T vs. ΔB , where $P_0 = 30$, and $\delta = \varepsilon = 0.1$. (c) N_T vs. $\log_{10}(\delta\varepsilon)$, where $\Delta B = 10^\circ$, and $P_0 = 30$.

4.5.4 Beampattern Formula Verification

To obtain the beampattern formula, we use P_0 -dependent constraints and exclude the constraints on the signal power, i.e., C_2 and C_3 , in the FP formulation (4.31). This is because the normalized ripple level depends on the desired peak level P_0 in the energy focusing section, which should be known a priori. The exclusion of C_2 and C_3 also avoids their conflict with P_0 . Note that the values of P_0 and ε together determine the sidelobe attenuation in dB. Fig. 4.12 shows the simulation results and their approximations to obtain the MIMO radar beampattern formula. From Fig. 4.12 (a) we observe the logarithmic relationship between N_T and P_0 , thus

(4.37) can be modified as

$$\hat{N}_T(120^\circ, \Delta B, \delta, \varepsilon, P_0) \approx \frac{a (\log_{10}(\delta\varepsilon)) (\log_{10} P_0) + b}{c\Delta B}. \quad (4.38)$$

Except for the new parameter P_0 , we observe from Fig. 4.12 (b) and (c) that the properties of Kaiser's formula are preserved in MIMO radar scenario, where ΔB is inversely proportional to N_T , and $\log_{10}(\delta\varepsilon)$ is linear related to N_T . According to the figures, a , b , and c can be related using the following approximations, i.e.,

$$\begin{cases} \frac{-2a \log_{10} P_0 + b}{10c} \approx 8 \log_{10} P_0 + 11, \\ \frac{-2a \log_{10} 30 + b}{c\Delta B} \approx \frac{220}{\Delta B}, \\ \frac{a (\log_{10} 30) (\log_{10}(\delta\varepsilon)) + b}{10c} \approx -4.2 \log_{10}(\delta\varepsilon) + 14. \end{cases} \quad (4.39)$$

The above approximated equations are solved empirically. Specifically, we search within possible numerical values of a and b into (4.39) and select the corresponding approximation curve that best matches the discrete experimental results by visual observation. Note that (4.39) can also be solved using data fitting techniques. The coefficients are thus approximated as

$$\begin{cases} a \approx -34.22c, \\ b \approx 128.7c. \end{cases} \quad (4.40)$$

Substituting (4.40) into (4.38), the beampattern formula is expressed as

$$\hat{N}_T(120^\circ, \Delta B, \delta, \varepsilon, P_0) \approx \frac{-34.22 (\log_{10}(\delta\varepsilon)) (\log_{10} P_0) + 128.7}{\Delta B}. \quad (4.41)$$

The experiment results verifying this formula are shown in Table 4.1, where $P_0 = 50$. The first two rows show that changing δ and ε will not affect the filter length as long as $\delta\varepsilon$ is constant. The 3rd and 4th rows compare the results using the nonlinear mapping equation (4.34) when $\tilde{\theta}_P = 100^\circ \neq 120^\circ$. The last two rows show the design with more strict conditions, where $\Delta B = 8^\circ$ and $\varepsilon = 0.001$. The nonlinear mapping is again verified by these two examples. We observe slight mismatch between the estimated value and the true value, but this is acceptable as Kaiser's formula also has such mismatch. Generally, it can be seen from the last

Table 4.1: MIMO Radar Beampattern Formula Verifications

$\tilde{\theta}_P$	$\Delta\tilde{B}$	ΔB	δ	ε	$\hat{N}_T$	N_T
120°	10°	10°	0.1	0.01	30.3116	29
120°	10°	10°	0.01	0.1	30.3116	29
100°	10°	11.9415°	0.1	0.1	20.5148	21
120°	12°	12°	0.1	0.1	20.4148	21
133°	10°	8.0710°	0.1	0.001	44.7496	40
120°	8°	8°	0.1	0.001	45.1569	40

two columns that the estimated number of transmit antennas is quite close to the true values.

4.6 Conclusions

The transmit beampattern design for MIMO radar has been receiving much attention in the recent decade. However, the solutions proposed in the literature have paid little attention to several aspects of the performance such as the ripple control, transition bandwidth issue, the sidelobe attenuation, the angle step-size of the design, and the minimum number of transmit antennas, etc. This chapter first proposed several methods for indirect and direct control of the passband ripples. More importantly, it is then shown that the MIMO radar transmit beampattern design is similar to, but more complicated than traditional low-pass FIR filter design problem. Hence we reformulated this problem as an FP problem, which is able to design the beampattern with quantitative specifications of the passband ripple δ , stopband attenuation ε , and transition bandwidth ΔB . Moreover, with the FP formulation, we then empirically obtained the MIMO radar beampattern formula, which is similar to Kaiser's formula for FIR filter length estimation. The design angle step-size is set to $\Delta\theta = 1^\circ$ in the simulations. In fact, if the design criteria are not so strict, then $\Delta\theta$ can be increased to reduce the dimension of $\mathbf{A}$. On the other hand, if the design requires strict ripple control, substantial sidelobe attenuation and very narrow transition bandwidth, then $\Delta\theta$ needs to be further reduced. Hence, the MIMO radar beampattern formula as a function of $\Delta\theta$ is worth of further investigation. In addition, there are several other issues that are worth of further investigation, i.e., the quantitative assessment of the conflict between the constraint C_2 and power level P_0 , the performance analysis of the free variable

$\mathbf{x}$ in (4.23), and the computational complexity analysis for all the optimization problems involved in this chapter.

Meanwhile, it should be noted that the design discussed in this chapter only takes a single passband centered at the broadside for example. In fact, more flexible desired beampatterns can be used. The reason that we exclude such examples is because they have already been considered in several existing works such as [5] and the references therein. Nulling certain directions or assigning multiple passbands are possible and can be immediately modified by reordering $\mathbf{A}_P$ and $\mathbf{A}_S$, or adding extra constraints of similar form. Instead of including examples of such cases, we intensively discuss the primary single passband design to emphasize our original contributions.

It should be noted that all the designs and derivations in this chapter are based on the assumption of ideal orthogonal waveforms, and the efficiency of the radio-frequency amplifier (RFA) for the implementation of the transmit power constraints are not considered. As such, the work of this chapter can be considered as a theoretical investigation of the transmit beampattern design. In the next chapter, we look into practical issues for the implementation of the designed transmit beampattern, and propose a robust design which considers the efficiency of RFA's, and uses easily generated, widely used linear frequency modulated (LFM) waveforms rather than using the more complicated nearly orthogonal waveforms.

Appendix: Proof of $\tilde{\mathbf{R}}_{MR}$ and $\tilde{\mathbf{R}}_{SVD}$ Being Hermitian

Proof: According to (4.9), the rows in $\mathbf{A}$, i.e., $\mathbf{e}^T(\theta) \otimes \mathbf{e}^H(\theta)$, $\theta \in \{0^\circ, \Delta\theta, \dots, 180^\circ\}$, can be represented as

$$\mathbf{e}^T(\theta) \otimes \mathbf{e}^H(\theta) = \{\text{vec}\{\mathbf{e}^*(\theta)\mathbf{e}^T(\theta)\}\}^T, \quad (4.42)$$

where, $\mathbf{e}^*(\theta)\mathbf{e}^T(\theta)$ is a Hermitian matrix. Hence each column in

$$\mathbf{A}^H = \mathbf{V}\mathbf{\Sigma}^H\mathbf{U}^H \quad (4.43)$$

is the vectorization of a Hermitian matrix. Substituting (4.24) into (4.22), we have

$$\begin{aligned}
 \text{vec}\{\tilde{\mathbf{R}}_{\text{MR}}\} &= (\mathbf{A}^H \mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{A}^H \mathbf{p}_d \\
 &= (\mathbf{V} \Sigma^H \mathbf{U}^H \mathbf{U} \Sigma \mathbf{V}^H + \lambda \mathbf{I})^{-1} \mathbf{V} \Sigma^H \mathbf{U}^H \mathbf{p}_d \\
 &= [\mathbf{V} (\Sigma^H \Sigma + \lambda \mathbf{I}) \mathbf{V}^H]^{-1} \mathbf{V} \Sigma^H \mathbf{U}^H \mathbf{p}_d \\
 &= \mathbf{V} (\Sigma^H \Sigma + \lambda \mathbf{I})^{-1} \Sigma^H \mathbf{U}^H \mathbf{p}_d.
 \end{aligned} \tag{4.44}$$

Note that (4.43) can be considered as a set of linear operations $(\Sigma^H \mathbf{U}^H)$ on the columns of $\mathbf{V}$, yielding the columns in $\mathbf{A}^H$ to be the vectorization of Hermitian matrices. In (4.44), $\Sigma^H \mathbf{U}^H$ performs the same column operations as in (4.43) on $\mathbf{V} (\Sigma^H \Sigma + \lambda \mathbf{I})^{-1}$ instead of $\mathbf{V}$. Note that $(\Sigma^H \Sigma + \lambda \mathbf{I})^{-1}$ is a diagonal matrix which only scales the columns of $\mathbf{V}$. Hence all the columns in $\mathbf{V} (\Sigma^H \Sigma + \lambda \mathbf{I})^{-1} \Sigma^H \mathbf{U}^H$ are the vectorization of Hermitian matrices, and any linear combinations (any $\mathbf{p}_d$) of them will result in a vector that can be reordered as a Hermitian matrix, i.e., $\tilde{\mathbf{R}}_{\text{MR}}$ is Hermitian. $\tilde{\mathbf{R}}_{\text{SVD}}$ can be easily verified to be Hermitian in the same way when $\mathbf{x} = \mathbf{0}$. ■

Chapter 5

Robust Design of Transmit Beampattern Using LFM Waveforms¹

5.1 Introduction

A robust design of the transmit beampatterns for colocated MIMO radar is considered in this chapter. The work is based on the approach proposed in Chapter 4. Two practical issues are considered as an extension to that work. In the Procedure 1 discussed in Chapter 4 which determines optimal the covariance matrix, neither the elemental power constraint [27] nor the total power constraint is efficient for hardware implementation using radio-frequency amplifiers (RFA's). Instead, we propose a relaxed power constraint which lies between the two constraints. This has not yet been discussed or reported in the literature. More importantly in Procedure 2 of Chapter 4 that determines the signaling strategy to achieve the covariance matrix obtained in Procedure 1, we relax the requirement of orthogonal waveforms for the TBP technique, and formulate an optimization problem to obtain the weighting matrix and a set of linear frequency modulated (LFM) waveforms to achieve the covariance matrix that was obtained under ideal orthogonality assumptions. The set of LFM waveforms have identical chirp rate and time duration but different initial frequencies with uniform step-size. Although

¹Contents of this chapter have been submitted to Signal Processing, June 2013.

many nearly orthogonal waveforms have been discovered in literature [6, 9, 11–13] (and Chapter 2 of this thesis), the implementation of these waveform is relatively complicated. On the contrary, the set of LFM waveforms exhibit stronger correlations, but they are easy to generate and are widely used waveforms in practical radar systems. The proposed robust design in this chapter generally exhibits two advantages. i) It is more suitable for hardware implementation because of more flexible choices among the robust waveforms, antennas and the RFA's. ii) It uses easily generated and practically used LFM waveforms to achieve beampatterns identical to those using perfectly orthogonal waveforms. The robust design is not limited to LFM waveforms, and can also be used with other radar waveforms.

5.2 Signal Model

The TBP based signal model (Section 4.2) is reviewed here for reference. The TBP technique uses K ($K \leq N_T$) waveforms and a weighting matrix $\mathbf{W} \in \mathbb{C}^{N_T \times K}$ to construct the transmit signal matrix $\mathbf{S} \in \mathbb{C}^{N_T \times N}$, where N is the length of the probing signal sequence $\mathbf{s}_i$, $i \in \{0, 1, \dots, K-1\}$. The waveforms are assumed to be unit norm, i.e., $\|\mathbf{s}_i\|^2 = 1$. Let the waveform matrix for TBP be

$$\mathbf{S}_{\text{TBP}} = [\mathbf{s}_0, \mathbf{s}_1, \dots, \mathbf{s}_{K-1}]^T, \quad (5.1)$$

then

$$\mathbf{S} = \mathbf{W}\mathbf{S}_{\text{TBP}}. \quad (5.2)$$

Suppose the transmit array is a uniform linear array (ULA) with inter element spacing of half the carrier wavelength, and the counterclockwise angle 90° corresponds to the broadside, the transmit steering vector is then expressed as $\mathbf{e}(\theta) = [1, e^{-j\pi \cos \theta}, \dots, e^{-j\pi(N_T-1) \cos \theta}]^T$, where θ denotes the azimuth angle. The transmit beampattern is defined as the power distribution along different steering directions, i.e.,

$$P(\theta) = \mathbf{e}^T(\theta) \mathbf{R} \mathbf{e}^*(\theta), \quad (5.3)$$

and

$$\mathbf{R} = \lim_{N \rightarrow \infty} \mathbf{S} \mathbf{S}^H \quad (5.4)$$

is the transmit covariance matrix. The covariance matrix is approximated using a finite number of samples, and it is assumed that $\mathbf{R} \triangleq \mathbf{S}\mathbf{S}^H$.

Let $\Delta\theta$ be an approximately chosen step-size for the angle resolution in the design. Then (4.3) can be written in the following matrix form

$$\begin{bmatrix} P(0^\circ) \\ P(\Delta\theta) \\ P(2\Delta\theta) \\ \vdots \\ P(180^\circ) \end{bmatrix} = \begin{bmatrix} \mathbf{e}^T(0^\circ) \otimes \mathbf{e}^H(0^\circ) \\ \mathbf{e}^T(\Delta\theta) \otimes \mathbf{e}^H(\Delta\theta) \\ \mathbf{e}^T(2\Delta\theta) \otimes \mathbf{e}^H(2\Delta\theta) \\ \vdots \\ \mathbf{e}^T(180^\circ) \otimes \mathbf{e}^H(180^\circ) \end{bmatrix} \text{vec}\{\mathbf{R}\}, \quad (5.5)$$

which represents a set of linear equations. $\text{vec}\{\cdot\}$ is the matrix vectorization operator, and $\otimes$ denotes the Kronecker product. Denote (5.5) as

$$\mathbf{p} = \mathbf{A} \text{vec}\{\mathbf{R}\}, \quad (5.6)$$

where $\mathbf{A} \in \mathbb{C}^{[(180/\Delta\theta)+1] \times N_T^2}$. As shown in Chapter 4, the design of the covariance matrix is nonlinearly mapped to the design of a multiple-input-single-output (MISO) finite impulse response (FIR) filter. In this way, a desired single beam can be characterized by passband, stopband, and transition band. For simplicity, the desired beampattern is chosen symmetrical w.r.t. 90° . Let the passband edges be θ_P ($90^\circ < \theta_P < 180^\circ$) and $180^\circ - \theta_P$, thus the stopband edges are $\theta_S \triangleq \theta_P + \Delta B$ and $180^\circ - \theta_S$. The rows of $\mathbf{A}$ which correspond to the passband and stopband are extracted to form $\mathbf{A}_P(\theta_P)$ and $\mathbf{A}_S(\theta_S)$ respectively. Because $\theta_S \triangleq \theta_P + \Delta B$, $\mathbf{A}_S(\theta_S)$ is a function of both θ_P and ΔB , and therefore denoted as $\mathbf{A}_S(\theta_P, \Delta B)$.

5.3 Relaxation of the Power Constraint

The Procedure 1 of the transmit beampattern design optimizes $\mathbf{R}$. The original power constraint for MIMO radar transmit beampattern design is noted in [26] and [27], which is the elemental power constraint

$$C_1 : \quad \text{diag}\{\mathbf{R}\} = \mathbf{1}, \quad (5.7)$$

where $\mathbf{1}$ is an all-one vector with appropriate dimensions. It is argued in [30] that (5.7) is required for the system to have maximum power efficiency. However, such an argument ignores the RFA *gain flatness*, and *gain linearity*. The *gain flatness* is defined as the deviation of the gain at the output of the RFA. The *gain linearity* exhibits the property that within a certain range of power levels in the *dynamic range*, the gain is linear. This means that the nonlinearity of the RFA is a case-by-case problem and should not be treated generally. More details on practical realization of RFA's can be found in [80]. Besides the practical issues, also note that (5.7) is a mathematically a stringent constraint, and the use of this constraint may result in degradation of the designed beampattern or even failure. Such examples can be found in [27] and Chapter 4 Fig. 4.4. In Fig. 4.4, the total power constraint

$$C_2: \quad \text{tr}\{\mathbf{R}\} \leq N_T \quad (5.8)$$

is used as an alternative to (5.7), where $\text{tr}\{\cdot\}$ is the matrix trace operator. In this way, although the theoretical solution can be obtained, the power dynamic ranges of the RFA's differ significantly, which is inefficient for hardware implementation.

To avoid the above two extremes and make use of the linearity of the RFA's, the relaxation of the constraint on the transmit power can be used for more efficient designs. The efficiency is in terms of the establishment of the trade-off between the RFA linearity property and the number of transmit antennas. Let the relaxation parameter for the transmit power be η , then the relaxed power constraint is given by

$$C_3: \quad (1 - \eta)\mathbf{1} \leq \text{diag}\{\mathbf{R}\} \leq (1 + \eta)\mathbf{1}, \quad (5.9)$$

where the vector inequality " $\leq$ " means the elements of the left-hand side vector is not greater than the corresponding elements of the right-hand side vector. The value of η can be flexibly chosen according to the RFA properties.

Let maximum tolerable passband ripple level be δ and the stopband attenuation be ε . Based on the FP formulation in Section 4.4.2, the transmit power relaxed

FP is formulated as

$$\text{find } \mathbf{R} \quad (5.10)$$

$$\text{s.t. } \mathbf{R} \succeq \mathbf{0}, \quad (5.11)$$

$$(1 - \eta)\mathbf{1} \leq \text{diag}\{\mathbf{R}\} \leq (1 + \eta)\mathbf{1}, \quad (5.12)$$

$$\max \{\mathbf{A}_S(\theta_P, \Delta B) \text{vec}\{\mathbf{R}\}\} \leq \varepsilon, \quad (5.13)$$

$$\max \{\mathbf{A}_P(\theta_P) \text{vec}\{\mathbf{R}\}\} - \min \{\mathbf{A}_P(\theta_P) \text{vec}\{\mathbf{R}\}\} \leq \delta, \quad (5.14)$$

where (5.11) means $\mathbf{R}$ being positive semidefinite, (5.13) ensures the stopband ripples are upper bounded by ε , and (5.14) constrains the passband ripples within δ .

Let N_{bot} denote the initial estimate of the minimum number of transmit antennas, and N_{top} denote the maximum number of transmit antennas allowed in the radar system, then the algorithm to find the number of transmit antennas is summarized as follows:

Step 0: Set designing parameters: θ_P , ΔB , δ , and ε .

Step 1: Start at initial value $N_T \leftarrow N_{\text{bot}}$.

Step 2: Solve (5.10)-(5.14) using the tools in [78].

Step 3: If the problem is not feasible, then $N_T \leftarrow N_T + 1$, and go to **Step 4**. Else if the problem is feasible, then the problem is solved, and the $\mathbf{R}$ obtained is the optimal solution.

Step 4: If $N_T \leq N_{\text{top}}$, go to **Step 2**. Else if $N_T > N_{\text{top}}$, then the problem cannot be solved, N_{top} should be increased.

5.4 Robust Signaling Strategy

In this section, we propose a robust solution to Procedure 2, which determines the signaling strategy to exhibits the optimized $\mathbf{R}$. The optimized $\mathbf{R}$ is denoted as $\hat{\mathbf{R}}$. We first summarize the existing strategies and then discuss the proposed robust design.

5.4.1 Existing Strategies and Drawbacks

The Procedure 2 of the design is generally expressed as to find the transmit signal matrix $\mathbf{S}$ such that

$$\mathbf{S}\mathbf{S}^H = \hat{\mathbf{R}}. \quad (5.15)$$

Substituting (5.2) into (5.15), we have

$$\mathbf{W}\mathbf{S}_{\text{TBP}}\mathbf{S}_{\text{TBP}}^H\mathbf{W}^H = \mathbf{W}\mathbf{W}^H = \hat{\mathbf{R}}, \quad (5.16)$$

where ideal orthogonality property is assumed. In this way, as noted in Chapter 4, the problem becomes the design of the weighting matrix $\mathbf{W}$. An intuitive solution of $\mathbf{W}$ is via the eigenvalue decomposition of $\hat{\mathbf{R}}$. The value of K is then determined by the rank of $\hat{\mathbf{R}}$, i.e., $K = \text{rank}\{\hat{\mathbf{R}}\}$. A rank deficient $\hat{\mathbf{R}}$ has its eigenvalue decomposition using non-square eigenvector matrix, i.e., $\hat{\mathbf{R}} = \hat{\mathbf{Q}}\hat{\mathbf{\Lambda}}\hat{\mathbf{Q}}^H$, where $\hat{\mathbf{Q}} \in \mathbb{C}^{N_T \times K}$, and $\hat{\mathbf{\Lambda}} \in \mathbb{C}^{K \times K}$, then a simple solution can be obtained as

$$\hat{\mathbf{W}} = \hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U}_0, \quad (5.17)$$

where $\mathbf{U}_0 \in \mathbb{C}^{K \times K}$ is an arbitrary unitary matrix. The TBP based solution (5.17) is superior to partially correlated waveforms design (e.g., [26], [29], and [31]) because of its simplicity and efficiency. However, we concern two practical issues about this solution. i) although many works have been carried out to discover waveforms with good auto- and cross-correlations, as noted in Chapter 2, it is still a generally difficult problem, especially for large number of transmit antennas. ii) The waveforms discovered only exhibit an approximation of ideal correlations, which means (5.16) is implemented with an approximation, i.e.,

$$\mathbf{W}\mathbf{S}_{\text{TBP}}\mathbf{S}_{\text{TBP}}^H\mathbf{W}^H \approx \hat{\mathbf{R}}. \quad (5.18)$$

Hence certain degradations of the designed transmit beampattern are inevitable. Instead of using the above assumption and approximation, we propose the design based on (5.2) but with waveforms without good correlation properties, e.g., the easy-to-generate LFM waveforms.

5.4.2 The Robust Design

As seen in the existing work [28] and also in Chapter 4, conventionally, the impact of $\mathbf{W}$ only lies in that with the use of perfectly orthogonal waveforms, it exhibits the optimized covariance matrix. However, as a weighting matrix that can be flexibly tuned, $\mathbf{W}$ can be constrained such that it can compensate the non-ideality of waveform correlations. This motivates us to consider a robust design. We denote the waveform matrix for robust design as $\mathbf{S}_{\text{RTBP}} \in \mathbb{C}^{K \times N}$. Note that instead of being mutually (nearly) orthogonal, the rows of $\mathbf{S}_{\text{RTBP}}$ can be correlated. Then the robust design is expressed as

$$\mathbf{W} \mathbf{S}_{\text{RTBP}} \mathbf{S}_{\text{RTBP}}^H \mathbf{W}^H = \hat{\mathbf{R}}. \quad (5.19)$$

Let $\tilde{\mathbf{R}} \triangleq \mathbf{S}_{\text{RTBP}} \mathbf{S}_{\text{RTBP}}^H$, then the following derivations provide the solution to the robust design.

$$\mathbf{W} \tilde{\mathbf{R}} \mathbf{W}^H = \hat{\mathbf{R}} \quad (5.20)$$

$$\iff \mathbf{W} \tilde{\mathbf{Q}} \tilde{\mathbf{\Lambda}} \tilde{\mathbf{Q}}^H \mathbf{W}^H = \hat{\mathbf{Q}} \hat{\mathbf{\Lambda}} \hat{\mathbf{Q}}^H \quad (5.21)$$

$$\iff \mathbf{W} \tilde{\mathbf{Q}} \sqrt{\tilde{\mathbf{\Lambda}}} = \hat{\mathbf{Q}} \sqrt{\hat{\mathbf{\Lambda}}} \mathbf{U} \quad (5.22)$$

$$\iff \hat{\mathbf{W}} = \hat{\mathbf{Q}} \sqrt{\hat{\mathbf{\Lambda}}} \mathbf{U} \left(\sqrt{\tilde{\mathbf{\Lambda}}} \right)^{-1} \tilde{\mathbf{Q}}^H, \quad (5.23)$$

where $\tilde{\mathbf{Q}}, \tilde{\mathbf{\Lambda}} \in \mathbb{C}^{K \times K}$ are from the eigenvalue decomposition of $\tilde{\mathbf{R}}$, and $\mathbf{U} \in \mathbb{C}^{K \times K}$ is an arbitrary unitary matrix. It can be seen that (5.17) is a special case of (5.23) when $\tilde{\mathbf{\Lambda}} = \mathbf{I}$, i.e., the waveforms are perfectly orthogonal. It is also noted that the existence of the robust solution only requires $\tilde{\mathbf{\Lambda}}$ to be invertible, i.e., $\mathbf{S}_{\text{RTBP}}$ is full row rank. This is a significant relaxation of the waveform requirement. We also observe that in (5.23), $\hat{\mathbf{Q}} \sqrt{\hat{\mathbf{\Lambda}}}$ is fixed because it is from the obtained $\hat{\mathbf{R}}$, whereas $\mathbf{U}$ and $\tilde{\mathbf{\Lambda}}$ are tunable. Further discussions on tuning $\mathbf{U}$ and $\tilde{\mathbf{\Lambda}}$ are provided in the following subsections.

5.4.3 Quantization Error and the Unitary Matrix $\mathbf{U}$

It is indicated in chapter 4 that the design of the transmit beampattern is equivalent to the design of a MISO FIR filter. We note here that the finite-word-length effects are inherent in digital filters when one is considering the practical implementation

[81]. Let the word rounding step-size of the waveforms and the multiplier output be Q_1 and Q_2 respectively. Let the quantization noise of the waveforms be $e_S^k(n)$, $k \in \{0, 1, \dots, K-1\}$. Let the quantization noise of the multiplier output be $e_M^{l,k}(n)$, $l \in \{0, 1, \dots, N_T-1\}$. The quantization noises are i.i.d. random variables with uniform distribution over a small quantization step-size, i.e., $e_S^k(n) \sim \mathcal{U}(-0.5Q_1, 0.5Q_1)$, $e_M^{l,k}(n) \sim \mathcal{U}(-0.5Q_2, 0.5Q_2)$. It then follows that the corresponding means and variances are given as $\eta_S = \eta_M = 0$, $\sigma_S^2 = Q_1^2/12$, and $\sigma_M^2 = Q_2^2/12$. Denote

$$\hat{W}_k(\theta) = \sum_{l=0}^{N_T-1} w_{l,k} e^{-j\pi l \cos \theta}, \quad (5.24)$$

which is the spatial domain response of the k th branch of the MISO filter. The overall output noise power σ_{MISO}^2 due to the finite-word-length effects is then given by

$$\begin{aligned} \sigma_{\text{MISO}}^2 &= \underbrace{\sum_{k=0}^{K-1} \sigma_S^2 \int_{\theta} |\hat{W}_k(\theta)|^2 d\theta}_{\text{Waveform Quantization}} + \underbrace{\sum_{k=0}^{K-1} \sum_{l=0}^{N_T-1} \sigma_M^2}_{\text{Multiplier Output Quantization}} \\ &= \frac{K}{12} \left(Q_1^2 \|\hat{\mathbf{W}}\|^2 + N_T Q_2^2 \right), \end{aligned} \quad (5.25)$$

where the Parseval's theorem [81] is used to evaluate the integral. It is seen from (5.25) that the output quantization noise power is proportional to $\|\hat{\mathbf{W}}\|^2$. Hence the optimization problem to obtain a matrix $\mathbf{U}$ that minimizes the output noise power σ_{MISO}^2 is expressed as

$$\begin{aligned} \min_{\mathbf{U}} \quad & \|\hat{\mathbf{W}}\|^2 \\ \text{s.t.} \quad & \mathbf{U}\mathbf{U}^H = \mathbf{I}. \end{aligned} \quad (5.26)$$

According to (5.23), we have

$$\begin{aligned}
\|\hat{\mathbf{W}}\|^2 &= \text{tr}\{\hat{\mathbf{W}}\hat{\mathbf{W}}^H\} \\
&= \text{tr}\left\{\hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U}\sqrt{\tilde{\mathbf{\Lambda}}^{-1}}\tilde{\mathbf{Q}}^H\tilde{\mathbf{Q}}\sqrt{\tilde{\mathbf{\Lambda}}^{-1}}\mathbf{U}^H\sqrt{\hat{\mathbf{\Lambda}}}\hat{\mathbf{Q}}^H\right\} \\
&= \text{tr}\left\{\hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{U}^H\sqrt{\hat{\mathbf{\Lambda}}}\hat{\mathbf{Q}}^H\right\} \\
&= \text{tr}\left\{\sqrt{\hat{\mathbf{\Lambda}}}\hat{\mathbf{Q}}^H\hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{U}^H\right\} \\
&= \text{tr}\{\hat{\mathbf{\Lambda}}\mathbf{U}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{U}^H\}.
\end{aligned} \tag{5.27}$$

Hence (5.26) is equivalent to

$$\begin{aligned}
&\min_{\mathbf{U}} \quad \text{tr}\{\hat{\mathbf{\Lambda}}\mathbf{U}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{U}^H\} \\
&\text{s.t.} \quad \mathbf{U}\mathbf{U}^H = \mathbf{I}.
\end{aligned} \tag{5.28}$$

Note that the solution (5.17) under ideal correlation assumption has fixed norm of the weighting matrix, and thus the unitary matrix $\mathbf{U}_0$ is trivial. The discussions on the solutions to (5.28) can be found in [82] and the references therein, which is beyond the scope of this chapter. Next, we investigate the eigenvalue spread of $\tilde{\mathbf{R}}$, and formulate the overall optimization problem for Procedure 2 with the consideration of (5.28).

5.4.4 The Eigenvalue Spread of $\tilde{\mathbf{R}}$

The eigenvalue spread of $\tilde{\mathbf{R}}$ (equivalently $\tilde{\mathbf{\Lambda}}$) is determined by the correlation property of the waveforms. Let the diagonal elements of $\tilde{\mathbf{\Lambda}}$ be in descending order, i.e., $\{\tilde{\lambda}_{\max}, \dots, \tilde{\lambda}_{\min}\}$, then the eigenvalue spread, denoted as ρ , is defined as

$$\rho = \frac{\tilde{\lambda}_{\max}}{\tilde{\lambda}_{\min}}. \tag{5.29}$$

It can be seen from (5.20)-(5.23) that proposed robust design is able to use the waveforms without good orthogonality to achieve the transmit beampattern identical to that designed under ideal orthogonality assumption. Hence we take the most popular and commonly used radar waveforms-the linear frequency modulated (LFM) waveforms (chirps) for example to study the eigenvalue spread of $\tilde{\mathbf{R}}$.

The consideration of using a set of LFM waveforms for MIMO radar system can be found in [6]. Here, we use a similar way as in [6] to generate a set of LFM waveforms and quantitatively study the relationship between the eigenvalue spread of $\tilde{\mathbf{R}}$ and the waveform parameters. Note that because of using multiple waveforms, we do not consider both rising and falling chirps. Within a given time and bandwidth, rising and falling chirps inevitably have multiple intersections. Also, the time-frequency positions of multiple rising and falling chirps are not easy to be defined. Hence we consider parallel chirps instead. Let the bandwidth of the baseband waveforms be B . Let the duration of the single pulse transmission be T . Let the initial frequency step-size be f_0 . Let the chirp rate be κ . The set of K non-orthogonal LFM waveforms are then expressed as

$$s_k(t) = \frac{1}{\sqrt{T}} \exp \left\{ j2\pi \left(kf_0t + \frac{1}{2}\kappa t^2 \right) \right\}, \quad (5.30)$$

where $k \in \{0, 1, \dots, K-1\}$. K is determined by examine the rank of $\hat{\mathbf{R}}$ as indicated by (4.7). The parameters f_0 and κ are confined by B , T , and K : the instantaneous frequency of $s_{K-1}(t)$ at time T should be no greater than B , i.e.,

$$\begin{aligned} \left. \frac{d \left((K-1)f_0t + \frac{1}{2}\kappa t^2 \right)}{dt} \right|_{t=T} &\leq B \\ \iff (K-1)f_0 + \kappa T &\leq B \\ \iff \kappa &\leq \frac{(B - (K-1)f_0)}{T}. \end{aligned} \quad (5.31)$$

Substituting (5.31) into (5.30), we obtain the expressions of K LFM waveforms which are straightforward and hence omitted. The correlation between $s_k(t)$ and $s_{k+\Delta k}(t)$, $\Delta k \in \{0, 1, \dots, K-1\}$, at zero lag, which is the $(k, k + \Delta k)$ th element

of $\tilde{\mathbf{R}}$, is given by

$$\begin{aligned}
|R_{k,k+\Delta k}(f_0)| &= \left| \int_0^T s_k(t) s_{k+\Delta k}^*(t) dt \right| \\
&= \frac{1}{T} \left| \int_0^T e^{j2\pi \{k f_0 t + \frac{1}{2} \kappa t^2 - (k+\Delta k) f_0 t - \frac{1}{2} \kappa t^2\}} dt \right| \\
&= \frac{1}{T} \left| \int_0^T \exp \{-j2\pi \Delta k f_0 t\} dt \right| \\
&= |\text{sinc} \{\pi f_0 \Delta k T\}|.
\end{aligned} \tag{5.32}$$

It is observed that if $\Delta k = 0$, then (5.32) becomes the auto-correlation of $s_k(t)$ at zero lag, which gives unity.

An intuitive solution to f_0 in (5.32) is by setting $f_0 T = 1$, thus $f_0 = 1/T$. Then $\forall \Delta k, |R_{k,k+\Delta k}(f_0)| = 0$. Therefore $\tilde{\lambda}_{\max} = \tilde{\lambda}_{\min}$. Due to the sampling effect, small values exist in off-diagonal elements of $\tilde{\mathbf{R}}$. Choosing $f_0 T = 1$ is the best way to de-correlate the LFM waveforms. However, T is usually chosen very small to preserve the range ability of the radar system, which is indicated in Section 3.4.3. In this situation, f_0 is a large initial frequency for the LFM waveforms, which reduces the bandwidth efficiency. Since the robust design allow waveforms without good correlation properties, it is possible to select $f_0 < 1/T$. If f_0 is chosen too small such that the LFM waveforms are highly correlated, then large values will appear in the diagonal elements of $\tilde{\mathbf{\Lambda}}^{-1}$, which results in a large value of $\|\hat{\mathbf{W}}\|^2$ and amplifies the quantization error. Hence, with the consideration of (5.28), the overall optimization problem for the second step of MIMO radar transmit beampattern design based on the TBP technique is formulated as

$$\begin{aligned}
&\min_{\mathbf{U}, f_0} |R_{k,k+\Delta k}(f_0)| + |f_0| \\
&\text{s.t.} \quad \left| \text{tr}\{\hat{\mathbf{\Lambda}} \mathbf{U} \tilde{\mathbf{\Lambda}}^{-1} \mathbf{U}^H\} - N_T \right| \leq \zeta, \\
&\quad \mathbf{U} \mathbf{U}^H = \mathbf{I},
\end{aligned} \tag{5.33}$$

where ζ is a small positive real number. The first constraint ensures that $\|\hat{\mathbf{W}}\|^2$ resulting from the robust design is close to, if no less than, that resulting from the design under ideal assumptions. The solution to (5.33) identifies $\mathbf{U}$ and $\tilde{\mathbf{\Lambda}}$, which are then substituted into (5.23) to obtain the weighting matrix. Efficient

solutions to (5.33) is not within the scope of this chapter. Instead, we provide several empirical solutions to illustrate the advantages of the robust design in the next section.

5.5 Numerical Examples

In this section, we provide several examples of the robust design. First, we illustrate the trade-off between the number of transmit antennas and the relaxation of the transmit power. Then we focus on the empirical solutions to (5.33). Some examples selecting f_0 and $\mathbf{U}$ are presented.

The trade-off between the number of transmit antennas and the relaxation of the transmit power is illustrated in Tables 5.1 and 5.2, where a symmetric transmit beampattern is considered and $\theta_P = 100^\circ$. In Table 5.1, the estimated number of transmit antennas for various specifications of the design parameters are provided. It can be seen that C_1 , the elemental power constraint, is a mathematically stringent condition such that several designs fail to satisfy it. On the contrary, the use of C_2 , the total power constraint, yields the best solutions in terms of the required number of antennas. However, the price paid by using C_2 is shown in Table 5.2, where the transmit power ranges (max – min) are the largest. It then can be noted that the use of C_3 establishes the trade-off among η , $\hat{N}_T$, and the power range. The 4th and 10th rows indicate that when the specifications are stringent, C_1 fails the design, C_2 results in minimum number of antennas with the most difficult hardware requirement, whereas C_3 eases the linearity requirement of the RFA by almost 50% of the requirement using C_2 , with nearly the same number of antennas required. Hence for implementation, C_3 provides more flexible solutions according to the hardware and system conditions.

The robust designs of the transmit beampatterns are shown in Fig. 5.1, where the beampatterns of omnidirectional transmission, and the standard TBP designs using the quadratic congruence coded (QCC) waveforms proposed in Chapter 2 and the LFM waveforms are provided for comparison. The bandwidth and time duration of the LFM waveforms are set as $B = 100$ kHz, and $T = 1$ ms. A non-symmetric desired beampattern is used, where the passband edges are 70° and 120° , $\Delta B = 20^\circ$, and $\varepsilon = \delta = 0.1$. Because (5.20)-(5.23) indicate that the transmit beampattern is independent of $\mathbf{U}$, we set $\mathbf{U} = \mathbf{U}_0 = \mathbf{I}$. Note that $\mathbf{U}$ only affects

(a) $f_0 = 800 \text{ Hz}$, $\|\hat{\mathbf{W}}\|^2 = 12.9635$

(b) $f_0 = 900 \text{ Hz}$, $\|\hat{\mathbf{W}}\|^2 = 11.9692$

(c) $f_0 = 1 \text{ kHz}$, $\|\hat{\mathbf{W}}\|^2 = 11.1096$.

Figure 5.1: The designed transmit beampatterns using standard and robust TBP techniques with different f_0 , where $\hat{N}_T = 11$, and $\mathbf{U} = \mathbf{U}_0 = \mathbf{I}$.

Table 5.1: Comparison of Different Power Constraints, $\theta_P = 100^\circ$.

	ΔB	δ	ε	$\hat{N}_T$			
				C_1	C_2	(η)	C_3
1	15°	1	1	9	8	(20%)	9
2	15°	0.1	1	11	11	(20%)	11
3	15°	1	0.35	N/A	10	(20%)	16
4	15°	1	0.35	N/A	10	(30%)	11
5	15°	0.5	0.5	N/A	10	(20%)	10
6	10°	1	1	15	13	(20%)	14
7	10°	0.1	1	18	17	(20%)	17
8	10°	0.1	0.8	21	17	(20%)	18
9	10°	0.1	0.5	N/A	18	(20%)	25
10	10°	0.1	0.5	N/A	18	(30%)	18

Table 5.2: Resultant RFA Minimum and Maximum Power from Table 5.1.

	C_1		C_2		C_3	
	min	max	min	max	min	max
1	1.00	1.00	0.54	1.26	0.82	1.13
2	1.00	1.00	0.41	1.29	0.82	1.15
3	1.00	1.00	0.27	1.39	0.80	1.20
4	1.00	1.00	0.27	1.39	0.70	1.29
5	1.00	1.00	0.26	1.40	0.80	1.19
6	1.00	1.00	0.37	1.30	0.81	1.17
7	1.00	1.00	0.32	1.25	0.81	1.17
8	1.00	1.00	0.40	1.23	0.81	1.17
9	1.00	1.00	0.21	1.32	0.80	1.20
10	1.00	1.00	0.21	1.32	0.70	1.30

$\|\hat{\mathbf{W}}\|^2$ which is indicated in (5.27). It is seen from Fig. 5.1 the robust design can compensate the imperfection of the waveform correlations and recovers the designed beampatterns to the one resulting from the theoretical result $\hat{\mathbf{R}}$. It is also seen that reducing the correlations among the LFM waveforms results in that the standard TBP based design approaches the robust TBP design. However, note that even at the least correlation point $f_0 = 1$ kHz, there still exist mismatches between

Figure 5.2: $1/\rho$ versus f_0 , where $B = 100$ kHz, and $T = 1$ ms.

the two. The advantage of the robust design is therefore illustrated. Also note that when $f_0 = 800$ Hz and $\mathbf{U} = \mathbf{I}$, $\|\hat{\mathbf{W}}\|^2 = 42.9635$, which is unsatisfactory. Next, we illustrate the tuning of f_0 and $\mathbf{U}$ to trade-off waveform correlations, bandwidth usage, and to reduce $\|\hat{\mathbf{W}}\|^2$.

The relationship between the eigenvalue spread of $\tilde{\mathbf{R}}$ and f_0 is shown in Fig. 5.2. We observe that periodically the correlations are minimized when $f_0 T$ is a positive integer, which agrees with (5.32). For sufficient bandwidth usage, it is preferable that $f_0 < 1/T$. However, reducing f_0 increases the correlations of the LFM waveforms, which results in large values appearing in $\tilde{\mathbf{A}}^{-1}$ and $\|\hat{\mathbf{W}}\|^2$ becomes large. In this situation, $\mathbf{U}$ plays a role to bound $\|\hat{\mathbf{W}}\|^2$. Empirical results to study the impact of $\mathbf{U}$ on $\|\hat{\mathbf{W}}\|^2$ is provided in Fig. 5.3, where the specifications of the desired beampattern is the same as in Fig. 5.1, $\hat{N}_T = 11$, and f_0 is chosen along the slope within $0 < f_0 < 1$ kHz. Among 1000 independent realizations of $\mathbf{U}$ for 3 different values of f_0 respectively, there exist $\mathbf{U}$'s that minimize $\|\hat{\mathbf{W}}\|^2$. The minimum values of $\|\hat{\mathbf{W}}\|^2$ are 14.9353, 12.1387, and 10.6882 for $f_0 = 800$, 850, and 1 kHz, respectively. Hence when $f_0 = 800$ Hz, a better choice of $\mathbf{U}$ is able to reduce $\|\hat{\mathbf{W}}\|^2$ from 42.9635 in Fig. 5.1 (a) to 14.9353, which is a significant improvement.

Figure 5.3: $\|\hat{\mathbf{W}}\|^2$ versus random realizations of $\mathbf{U}$ for different f_0 , where $\hat{N}_T = 11$, and the minimum values of $\|\hat{\mathbf{W}}\|^2$ are 14.9353, 12.1387, and 10.6882 for $f_0 = 800$, 850, and 1 kHz, respectively.

5.6 Conclusions

This chapter has presented a robust design of the transmit beampatterns based on the TBP technique for colocated MIMO radar. The proposed design uses a relaxed power constraint with the consideration of the required number of antennas and the *gain flatness* and *gain linearity* of the RFA's. In this way a trade-off between the number of antennas and the RFA's is established and the hardware implementation become more efficient and flexible. Instead of imposing ideal correlations of the waveforms, or using waveforms with very good correlation properties, we use a set of non-orthogonal LFM waveforms to achieve the design. The designed beampatterns are identical to those designed under ideal waveform correlation assumption. An optimization problem is formulated to select i) an initial frequency step-size for the LFM waveforms, and ii) a unitary matrix $\mathbf{U}$ to minimize the effects of quantization error. Empirical solutions are provided and illustrated via numerical examples. Several issues can be noted for further investigation. The waveforms for the robust design do not limit to LFM waveforms, other waveforms can also be used. Moreover, the theoretical solution to (5.33) is still open, the techniques introduced in [82] and the references therein are worth of exploring for efficient solutions.

Chapter 6

Comparison Between MIMO and Phased-Array Radar¹

6.1 Introduction

SINCE the idea of multiple-input multiple-output (MIMO) radar was first proposed in 2004 [2], the comparison between MIMO radar and its counterpart, phased-array radar, has been drawing much research attention [1,39–49]. Generally, it comes to a common agreement that MIMO radar systems enjoy the waveform diversity that enables flexible waveform and system designs. However, phased-array radar systems enjoy the coherent processing gain that usually results in higher signal-to-noise ratio (SNR). According to [39], the difference between the two systems has been described as the diversity gain versus the coherent gain. As a result, the advantages of MIMO radar over phased-array radar is rather unclear. The work of [41] provides a good discussion on this.

In this chapter, we first compare the basic signal models of MIMO and phased-array radar. It is then proven that MIMO radar with omnidirectional transmission of orthogonal waveforms has identical matched filter (MF) response as phased-array radar with steered transmission of coherent waveforms. Therefore, the virtual array effect, which has been considered as the advantage of MIMO radar, is in fact also

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a property of phased-array radar. After that, we focus on the comparison of the two systems on the topics that have been discussed in previous chapters. For receiver instrumental variable filter (IVF) design in Chapter 3, it can be seen that phased-array radar has already been considered and investigated in detail by us, and thus more efforts are given on the comparison of transmit beampattern design. In addition, we study the identifiability property of the two systems. Finally, we draw conclusions on the comparison between the two systems within the scope of the design and performance aspects involved in the previous chapters of this thesis. The subscriptions $\{\cdot\}_{\text{MM}}$ and $\{\cdot\}_{\text{PH}}$ in this chapter are used to distinguish similar parameters of MIMO and phased-array radar respectively.

6.2 On Matched Filter and Virtual Array

For simplicity and fair comparison, we assume that the two systems employ identical uniform linear antenna arrays and share the same target azimuth angle θ . The physical difference between the two systems is that MIMO radar transmits omnidirectional non-coherent waveforms whereas phased-array radar transmits steered coherent waveforms. The receive signal matrix of a MIMO radar system (1.7) is rewritten as

$$\mathbf{Y}_{\text{MM}} = \sum_{k=0}^{K-1} a_k \mathbf{e}_{\text{R}}(\theta_k) \mathbf{e}_{\text{T}}^T(\theta_k) [\mathbf{S}_{\text{MM}} \mathbf{J}_{\tau_k} \odot \mathbf{V}_{v_k}] + \mathbf{Z}. \quad (6.1)$$

Let the estimated variables (angle, delay, Doppler) at the receiver be $\hat{\theta}_k$, $\hat{\tau}_k$, and $\hat{v}_k$, then the MF output is given by

$$\begin{aligned} \psi_{\text{MM}}(\hat{\theta}_k, \hat{\tau}_k, \hat{v}_k) &= \text{tr} \{ \mathbf{Y}_{\text{MM}} \cdot \mathbf{F}_{\text{MM}} \} \\ &= \text{tr} \left\{ \sum_{k=0}^{K-1} a_k \mathbf{e}_{\text{R}}(\theta_k) \mathbf{e}_{\text{T}}^T(\theta_k) [\mathbf{S}_{\text{MM}} \mathbf{J}_{\tau_k} \odot \mathbf{V}_{v_k}] \right. \\ &\quad \times [\mathbf{S} \mathbf{J}_{\hat{\tau}_k} \odot \mathbf{V}_{\hat{v}_k}]^H \mathbf{e}_{\text{T}}^*(\hat{\theta}_k) \mathbf{e}_{\text{R}}^H(\hat{\theta}_k) \left. \right\} + \text{tr} \{ \mathbf{Z} \mathbf{F}_{\text{MM}} \}. \end{aligned} \quad (6.2)$$

To illustrate the virtual array effect, the round-trip delay and Doppler effects are assumed to be perfectly compensated (similar assumptions are used in [71]). Under

the assumption that the waveforms are orthogonal, (6.2) is simplified to

$$\begin{aligned}\psi_{\text{MM}}(\hat{\theta}_k) &= \text{tr} \left\{ \sum_{k=0}^{K-1} a_k \mathbf{e}_{\text{R}}(\theta_k) \mathbf{e}_{\text{T}}^T(\theta_k) \mathbf{e}_{\text{T}}^*(\hat{\theta}_k) \mathbf{e}_{\text{R}}^H(\hat{\theta}_k) \right\} + \text{tr} \{ \mathbf{Z} \mathbf{F}_{\text{MM}} \} \\ &= \sum_{k=0}^{K-1} a_k \mathbf{e}_{\text{R}}^H(\hat{\theta}_k) \mathbf{e}_{\text{R}}(\theta_k) \mathbf{e}_{\text{T}}^T(\theta_k) \mathbf{e}_{\text{T}}^*(\hat{\theta}_k) + \text{tr} \{ \mathbf{Z} \mathbf{F}_{\text{MM}} \}.\end{aligned}\quad (6.3)$$

For phased-array radar systems, the azimuth angle to be estimated is used at the transmitter such that the transmitted coherent waveforms are steered to $\hat{\theta}_k$. Denote the coherent waveform used here as $\mathbf{s}_0$, the receive signal matrix and the MF output are then given by

$$\mathbf{Y}_{\text{PH}} = \sum_{k=0}^{K-1} a_k \mathbf{e}_{\text{R}}(\theta_k) \mathbf{e}_{\text{T}}^T(\theta_k) \left[\left(\mathbf{e}_{\text{T}}^*(\hat{\theta}_k) \mathbf{s}_0 \mathbf{J}_{\tau_k} \right) \odot \mathbf{V}_{v_k} \right] + \mathbf{Z}, \quad (6.4)$$

$$\mathbf{F}_{\text{PH}} = \left\{ \mathbf{e}_{\text{R}}(\hat{\theta}_k) \mathbf{e}_{\text{T}}^T(\hat{\theta}_k) \left[\left(\mathbf{e}_{\text{T}}^*(\hat{\theta}_k) \mathbf{s}_0 \mathbf{J}_{\hat{\tau}_k} \right) \odot \mathbf{V}_{\hat{v}_k} \right] \right\}^H. \quad (6.5)$$

With the same assumption of the compensation of delay and Doppler variables, the MF output of phased-array radar is then simplified as

$$\begin{aligned}\psi_{\text{PH}}(\hat{\theta}_k) &= \text{tr} \{ \mathbf{Y}_{\text{PH}} \cdot \mathbf{F}_{\text{PH}} \} \\ &= \text{tr} \left\{ \sum_{k=0}^{K-1} a_k \mathbf{e}_{\text{R}}(\theta_k) \mathbf{e}_{\text{T}}^T(\theta_k) \mathbf{e}_{\text{T}}^*(\hat{\theta}_k) \mathbf{s}_0 \mathbf{s}_0^H \mathbf{e}_{\text{T}}^T(\hat{\theta}_k) \mathbf{e}_{\text{T}}^*(\hat{\theta}_k) \mathbf{e}_{\text{R}}^H(\hat{\theta}_k) \right\} + \text{tr} \{ \mathbf{Z} \mathbf{F}_{\text{PH}} \} \\ &= N_{\text{T}} \text{tr} \left\{ \sum_{k=0}^{K-1} a_k \mathbf{e}_{\text{R}}(\theta_k) \mathbf{e}_{\text{T}}^T(\theta_k) \mathbf{e}_{\text{T}}^*(\hat{\theta}_k) \mathbf{e}_{\text{R}}^H(\hat{\theta}_k) \right\} + \text{tr} \{ \mathbf{Z} \mathbf{F}_{\text{PH}} \} \\ &= N_{\text{T}} \sum_{k=0}^{K-1} a_k \mathbf{e}_{\text{R}}^H(\hat{\theta}_k) \mathbf{e}_{\text{R}}(\theta_k) \mathbf{e}_{\text{T}}^T(\theta_k) \mathbf{e}_{\text{T}}^*(\hat{\theta}_k) + \text{tr} \{ \mathbf{Z} \mathbf{F}_{\text{PH}} \}.\end{aligned}\quad (6.6)$$

Note that apart from the noise terms, (6.2) and (6.6) only differ from a scalar N_{T} , which indicate identical overall beampatterns of the two systems. It is further indicated that the virtual array effect, which has been addressed in many existing works [5, 40, 42, 46] is also a property of phased-array radar. The virtual array takes the advantage of multiple transmit and receive antennas. The overall beampattern is equivalent to that obtained using an antenna array resulting from the convolution of the transmit and receive antennas. At either transmit or receive side, the antenna

element spacing can be increased to allow certain grating lobes with narrower beamwidth, then at the alternative side, half wavelength element spacing is used such that the grating lobes would lie in the sidelobe sector. As a result, the overall beampattern has improved angle resolution and sidelobe reduction. Note that adjusting the antenna element spacing alters the values of transmit and receive steering vectors, and the changes of the values are identical in (6.2) and (6.6). Hence it is easy to verify that both systems enjoy the performance improvement resulting from virtual array. Other examples of virtual array effects that can be obtained are discussed in [5, 40, 42, 46], and these are not presented in this thesis.

6.3 On Transmit Beampattern Design

The design of the transmit beampattern of a radar system aims to focus the energy of the transmitted signals to the desired spatial section(s) in order to enhance the direction of arrival (DOA) estimation at the receiver. In this section, we consider this problem for phased-array radar from the perspective of finite impulse response (FIR) filter design. For MIMO radar, we have formulated the design as a feasibility problem (FP) in Chapter 4 and 5, and demonstrated its advantages over the existing methods. In [27], the authors provided transmit beampattern design examples using phased-array radar for comparison, and showed the inability in achieving the desired beampattern. This design is based on a minor modification to the MIMO radar problem formulation, i.e., adding a rank constraint. However, such a problem formulation may not be suitable for phased-array radar. Alternatively, we use the spectral factorization technique [83] which has been considered to be better than conventional filter design methods. It would be shown that the two systems can achieve similar transmit beampatterns for identical number of transmit antennas and waveform energy. This is contradictory to the existing argument that MIMO radar can achieve more flexible transmit beampattern design over its phased-array counterpart. Simulation results are provided and discussed.

Recall that in Section 4.2, the transmit beampattern of MIMO radar was written as

$$\begin{aligned} P_{\text{MM}}(\theta) &= \mathbf{e}^T(\theta) \mathbf{R}_{\text{MM}} \mathbf{e}^*(\theta) \\ &= \mathbf{e}^T(\theta) \mathbf{W} \mathbf{W}^H \mathbf{e}^*(\theta). \end{aligned} \tag{6.7}$$

For phased-array radar, the weighting matrix simplifies to a column vector, i.e.,

$$\mathbf{w} = [w(0), w(1), \dots, w(N_T - 1)]^T, \quad (6.8)$$

then the corresponding transmit beampattern is given by

$$\begin{aligned} P_{\text{PH}}(\theta) &= \mathbf{e}^T(\theta) \mathbf{R}_{\text{PH}} \mathbf{e}^*(\theta) \\ &= \mathbf{e}^T(\theta) \mathbf{w} \mathbf{s}_0^T \mathbf{s}_0^* \mathbf{w}^H \mathbf{e}^*(\theta) \\ &= \mathbf{e}^T(\theta) \mathbf{w} \mathbf{w}^H \mathbf{e}^*(\theta), \end{aligned} \quad (6.9)$$

where $\mathbf{R}_{\text{PH}}$ denotes the covariance matrix of $\mathbf{w} \mathbf{s}_0^T$.

The FP algorithm for MIMO radar is described in Section 4.4.2 and hence not reproduced here. The phased-array transmit beampattern design can be considered as the design of a single branch of the MISO filter, which is a conventional single-input single-output (SISO) FIR filter. This is shown in Fig. 4.1. The spectral factorization technique discussed in [83] is an efficient technique for the design of the FIR filter coefficients, i.e., the weight vector for transmitted waveform. The authors in [27] convert their MIMO radar formulation to a phased-array radar formulation by adding the constraint $\text{rank}\{\mathbf{R}_{\text{MM}}\} = 1$. This constraint is nonconvex and complicates the solution. Moreover, the iterative solution fails to achieve the desired beampattern. In fact, such a formulation may not be suitable for phased-array radar. Alternatively, let the autocorrelation coefficients associated with $\mathbf{w}$ be expressed as

$$r(k) = \sum_{i=-N_T+1}^{N_T-1} w(i)w(i+k), \quad (6.10)$$

where $r(k) = r(-k)$, and the vector form of $r(k)$ be

$$\mathbf{r} = [r(0), r(1), \dots, r(N_T - 1)]^T. \quad (6.11)$$

Instead of directly solving for $\mathbf{w}$, [83] uses $\mathbf{r}$ as the optimization variable. The magnitude squared frequency response of the filter can be represented by the Fourier

transform of $\mathbf{r}$, i.e.,

$$R(f) = \sum_{k=-N_T+1}^{N_T-1} r(k) e^{-j2\pi f k} \quad (6.12)$$

$$\begin{aligned} &= \left(\sum_{k=0}^{N_T-1} w(k) e^{-j2\pi f k} \right) \left(\sum_{k=0}^{N_T-1} w(k) e^{-j2\pi f k} \right)^* \\ &= |W(f)|^2 \quad (\text{squared frequency response}) \end{aligned} \quad (6.13)$$

$$= 1 + 2 \begin{bmatrix} \cos(2\pi f) \\ \cos(2\pi 2f) \\ \vdots \\ \cos(2\pi(N_T-1)f) \end{bmatrix}^T \mathbf{r}, \quad (6.14)$$

where $f \in [-0.5, 0.5]$ is the normalized frequency. Replacing $-2\pi f$ by $\cos \theta$, the FP for phased-array radar transmit beampattern design is formulated as:

$$\begin{aligned} &\text{find } \mathbf{r} \\ &\text{s.t. } [1, 0, 0, \dots, 0] \mathbf{r} = N_T, \\ &\quad \max \{ \mathbf{B}_P(\theta_P) \mathbf{r} \} - \min \{ \mathbf{B}_P(\theta_P) \mathbf{r} \} \leq \delta, \\ &\quad \max \{ \mathbf{B}_S(\theta_P, \Delta B) \mathbf{r} \} \leq \varepsilon. \end{aligned} \quad (6.15)$$

According to (6.14), a matrix $\mathbf{B}$ is formed as

$$\begin{bmatrix} 1 & 2 \cos(\cos 0^\circ) & \cdots & 2 \cos((N_T-1) \cos 0^\circ) \\ 1 & 2 \cos(\cos \Delta\theta) & \cdots & 2 \cos((N_T-1) \cos \Delta\theta) \\ 1 & 2 \cos(\cos 2\Delta\theta) & \cdots & 2 \cos((N_T-1) \cos 2\Delta\theta) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2 \cos(\cos 180^\circ) & \cdots & 2 \cos((N_T-1) \cos 180^\circ) \end{bmatrix},$$

and $\mathbf{B}_P$ and $\mathbf{B}_S$ are extracted from $\mathbf{B}$ in the same way as $\mathbf{A}_P$ and $\mathbf{A}_S$ are extracted from matrix $\mathbf{A}$. Once $\mathbf{r}$ is optimally selected, the spectral factorization techniques can be used to obtain $\mathbf{w}$, which are summarized in the appendix of [83].

Fig. 6.1 provides a comparison between the transmit beampatterns designed for

Figure 6.1: Simulation results of the transmit beampatterns. FP formulation for MIMO radar is compared with FP formulation and spectral factorization for phased-array radar. $N_T = 29$, $\theta_P = 115^\circ$, $\Delta B = 8^\circ$, $\delta = \varepsilon = 0.1$.

MIMO and phased-array radar. The two solutions have nearly identical mainlobes and sidelobe peaks. This indicates that with appropriate designs, both MIMO and phased-array radar result in transmit beampatterns having similar passband, stopband, transition band, and passband/stopband ripples. The designs use same number of transmit antennas and total waveform energy.

6.4 On Parameter Identifiability

The identifiability of MIMO and phased-array radar systems has been theoretically discussed in Chapter 1 of [5]. In this section, we look into this issue via several simulation examples which are grouped into two cases.

- **Case 1**, identifiability using traditional MF's.
- **Case 2**, identifiability using classical eigenstructure based method, i.e., the MUSIC spectrum.

The transmit beamforming discussed in Chapter 4, 5, and the previous section is considered for both systems in both cases. For simplicity, we only provide the expression of MUSIC spectrum for MIMO radar. The expression for phased-array

radar can be obtained by replacing $\mathbf{W}$ with $\mathbf{w}$, and the $\{\cdot\}_{\text{MM}}$ terms with the corresponding $\{\cdot\}_{\text{PH}}$ terms. Neglecting the round trip delay and Doppler variable and considering the robust transmit beamforming weighting matrix $\hat{\mathbf{W}}$ in (5.23), (6.1) can be written in the following matrix form

$$\mathbf{Y}_{\text{MM}} = \mathbf{E}_{\text{R}} \mathbf{D} \mathbf{E}_{\text{T}}^T \hat{\mathbf{W}} \mathbf{S}_{\text{MM}} + \mathbf{Z}, \quad (6.16)$$

where $\mathbf{E}_{\text{R}} = [\mathbf{e}_{\text{R}}(\theta_0), \mathbf{e}_{\text{R}}(\theta_0), \dots, \mathbf{e}_{\text{R}}(\theta_{K-1})] \in \mathbb{C}^{N_{\text{T}} \times K}$, and $\mathbf{E}_{\text{T}}$ is constructed in the same way. $\mathbf{D} \in K \times K$ is a diagonal matrix whose diagonal elements are the reflection coefficients a_k . According to (5.19) and (5.23), the covariance matrix of the received data matrix is then given by

$$\begin{aligned} \mathbf{R}_{\text{MM}}^{\mathbf{Y}} &= E \{ \mathbf{Y}_{\text{MM}} \cdot \mathbf{Y}_{\text{MM}}^{\text{H}} \} \\ &= E \left\{ \mathbf{E}_{\text{R}} \mathbf{D} \mathbf{E}_{\text{T}}^T \hat{\mathbf{R}} \mathbf{E}_{\text{T}}^* \mathbf{D}^H \mathbf{E}_{\text{R}}^H \right\} + \sigma^2 \mathbf{I}. \end{aligned} \quad (6.17)$$

The expectation in (6.17) takes the instantaneous values as an approximation, which has the following form

$$\hat{\mathbf{R}}_{\text{MM}}^{\mathbf{Y}} = \mathbf{E}_{\text{R}} \mathbf{D} \mathbf{E}_{\text{T}}^T \hat{\mathbf{R}} \mathbf{E}_{\text{T}}^* \mathbf{D}^H \mathbf{E}_{\text{R}}^H + \sigma^2 \mathbf{I}. \quad (6.18)$$

The MUSIC spectrum is then given by [84, 85]

$$P_{\text{M}}(\theta) = \frac{1}{\mathbf{e}_{\text{R}}^H(\theta) \mathbf{E}_{\text{n}} \mathbf{E}_{\text{n}}^H \mathbf{e}_{\text{R}}(\theta)}, \quad (6.19)$$

where the columns of $\mathbf{E}_{\text{n}}$ are extracted from the eigenvectors of $\hat{\mathbf{R}}_{\text{MM}}^{\mathbf{Y}}$ which correspond to the $N_{\text{R}} - K$ smallest eigenvalues.

The comparisons are conducted for receiver using MF's (**Case 1**) or MUSIC estimation (**Case 2**). For each case, single and multiple target situations are considered. For the MF case, the conventional transmission, i.e., omnidirectional transmission for MIMO radar and steered transmission for phased-array radar, are also considered to illustrate Section 6.2. In the simulations, the transmit beam pattern specifications are identical to the example presented Fig. 6.1, i.e., $\theta_{\text{P}} = 115^\circ$, $\Delta B = 8^\circ$, $\delta = \varepsilon = 0.1$, and $N_{\text{T}} = 29$. The quadratic congruence coded (QCC) waveforms are used for the simulations. According to Chapter 2, we set the coding dimension $M = 41$ and have totally 40 quadratic congruence arrays (see (2.11)).

(a)

(b)

Figure 6.2: The MF based overall beampatterns for single target. (a) Omnidirectional transmission for MIMO radar, and Steered transmission for phased-array radar. (b) TBP used for MIMO radar, and weights obtained via spectral factorization used for phased-array radar.

29 sets of the ensemble are empirically selected to exhibit least cross-ambiguity functions. For single target ($K = 1$), the target angle is assumed to be 85° . For multiple targets, the number of target are $K = 4$, whose azimuth angles are 70° , 72° , 85° , and 100° , respectively.

Figure 6.3: The MF based overall beampatterns for multiple targets. (a) Omnidirectional transmission for MIMO radar, and Steered transmission for phased-array radar. (b) TBP used for MIMO radar, and weights obtained via spectral factorization used for phased-array radar.

The MF outputs for single target is presented in Fig. 6.2, where a noise free environment is assumed. It is seen from Fig. 6.2 (a) that the overall beampatterns for MIMO radar with omnidirectional transmission and phased-array radar with steered transmission are nearly identical. The minor mismatch is caused by the

(a)

(b)

Figure 6.4: MUSIC spectrum for single and multiple targets, where $\text{ASNR} = -10$ dB.

imperfection of the QCC waveforms. Hence the theoretical equivalence between the overall beampatterns of MIMO and phased-array radar is illustrated. We see from Fig. 6.2 (b) that the use of TBP technique reduces the sidelobes of the overall beampattern for MIMO radar. However, the designed weights for transmit beamforming causes higher sidelobes as compared to the steered transmission in

Fig. 6.2 (a). Therefore it is indicated that although as seen in Fig. 6.1 that phased-array radar is able to generate nearly identical transmit beampatterns as MIMO radar, it is not able to achieve similar overall beampattern improvement using conventional MF's.

The overall beampatterns in the presence of multiple targets are shown in Fig. 6.3. We observe from Fig. 6.3 (a) that the theoretical equivalence between the overall beampatterns of the two systems is preserved for multiple targets. In Fig. 6.3 (b) similar properties as in Fig. 6.2 (b) are noted. The MIMO radar TBP technique is able to reduce the sidelobes, while phased-array radar does not enjoy this advantage. Moreover, phased-array radar becomes unable to resolve the targets at 70° and 72° . Hence we can conclude based on Fig. 6.2 and 6.3 that for direction-of-arrival (DOA) estimation using MF's, transmit beamforming is more effective for MIMO radar than phased-array radar for sidelobe reduction without degradation of angular resolution. The overall beampattern (**Case 1**) is usually considered as a basic criterion to analyze the performance of DOA estimation. In the following content, we consider the popular eigenstructure based MUSIC algorithm for improved angle resolution.

The simulation results of **Case 2**, i.e., the DOA estimation using the MUSIC algorithm are shown in Fig. 6.4, where a single realization of the estimation is presented. As we have many number of antennas $N_T = N_R = 29$, a lower array signal-to-noise ratio (ASNR) is assumed for more clear comparison. The ASNR is defined in (3.35) of Section 3.5. We rewrite (3.35) in dB here for reference,

$$\text{ASNR} = 10 \lg \left\{ \frac{N_R}{N_T} \cdot \frac{\text{tr}\{\mathbf{S}\mathbf{S}^H\}}{\text{tr}\{\mathbf{Z}\mathbf{Z}^H\}} \right\} = -10 \lg \sigma^2. \quad (6.20)$$

In Fig. 6.4, $\text{ASNR} = -10$ dB, i.e., $\sigma^2 = 10$. It is seen from Fig. 6.4 (a) that MIMO radar with TBP technique does not exhibit an obvious advantage compared to its omnidirectional transmission and phased-array radar. On the contrary, phased-array radar usually generates best estimation results when a single target is involved because of its coherent processing gain. When it comes to multiple target, the advantage of MIMO radar with TBP technique is revealed by Fig. 6.4 (b). Phased-array radar fails the DOA estimation because of the waveform coherence. For MIMO radar with omnidirectional transmission and TBP transmission, the later exhibits better performance in terms of sidelobe level and resolution. It is seen

Figure 6.5: Multiple targets DOA estimation using MF, MF with transmit beamforming, and MUSIC algorithm, respectively with clutter existing at 30° and 160° , where $\text{ASNR} = -10$ dB. (a) MF with conventional transmission. (b) MF with transmit beamforming. (c) MUSIC Spectrum with transmit beamforming

from the zoomed-in subfigure in Fig. 6.4 (b) that the later is able to resolve the targets at 70° and 72° whereas the former is not.

An example of multiple targets DOA estimation with the existence of clutter is provided in Fig. 6.5. It is assumed that the angle section of interest are from 65° to 115° . We observe from this Fig. 6.5 (a) that conventional method estimates every reflected signal and thus has peaks corresponding to clutter. In Fig. 6.5 (b), the advantage of transmit beamforming is verified in terms of clutter suppression. Meanwhile the properties of sidelobe reduction and resolution using MIMO radar are preserved as they are in Fig. 6.3 (b). In Fig. 6.5 (c), it is seen that phased-array radar fails due to waveform coherence, MIMO radar with omnidirectional transmission estimates the DOA's of both targets and clutter without being

able to resolve the targets at 70° and 72° . However, MIMO radar with transmit beamforming successfully suppresses the clutter and resolve these closely located targets.

6.5 Conclusions

In this chapter, we have compared MIMO and phased-array radar systems in terms of several performance aspects, i.e., the overall beam pattern, transmit beam pattern design, and the parameter identifiability. For MF based overall beam pattern, phased-array radar enjoys coherent gain from the steered coherent transmission, while at the receiver, it does not have diversity gain. MIMO radar does not preserve the coherent transmission gain at transmitter, but it has diversity gain at receiver. As a result, the two systems have the same overall beam pattern. It is also proven that MIMO and phased-array radar can form nearly identical transmit beam pattern with identical transmit and receive antennas. Further analysis on receiver identifiability has revealed that for single target issue, phased-array radar is sufficient for DOA estimation. However for multiple targets, MIMO radar, especially MIMO radar with TBP technique exhibits better performance in terms of angle resolution as well as sidelobe reduction. Note that in Chapter 3 in the receiver IVF design, phased-array radar has turned out to be better than MIMO radar in terms of sidelobe suppression level, computational complexity, and SNR loss. This is also a single target scenario. Hence our conclusions on the comparison is stated as follows. Within the scope of topics discussed in this thesis, phased-array radar is more suitable for single target estimation and analysis because of its coherent gains, whereas MIMO radar is more suitable for a system involving multiple targets because its waveform diversity enables simultaneous estimation of the targets. Hence, the comparison of the two systems can be considered as a trade-off for different situations and system requirements.

However, it should be noted that the spatial smoothing technique has been established for several decades for multiple coherent target identification [86–89]. This means that phased-array radar is able to achieve DOA estimation for multiple targets simultaneously if techniques like spatial smoothing is used. Therefore, a fairer quantitative comparison on parameter identifiability of MIMO and phased-array radar is worth of further investigation.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

This thesis has investigated the waveform and system design of multiple-input multiple-output (MIMO) radar systems with colocated transmit and receive antennas. Specifically, the waveform coding using Costas and quadratic congruence arrays, the receiver instrumental variable filter (IVF) design for clutter sidelobe suppression, the transmit beampattern design, and the comparison between MIMO and phased-array radar on several performance aspects, have been discussed, and new methods and designs are proposed.

The waveform diversity of MIMO radar systems calls for waveforms with good auto- and cross-ambiguity properties, which evokes the selection of multiple waveforms in the procedures of waveform design. In Chapter 2, we have extended two popular single-input single-output (SISO) radar coding sequences, i.e., the Costas and the quadratic congruence arrays, into MIMO radar systems. Via suboptimal exhaustive search, we have obtained several Costas array coded (CAC) and quadratic congruence array coded (QCC) waveforms with satisfactory ambiguity properties. Such coding methods are used in later chapters for the simulations for the performances of other designs, e.g., the receiver IVF performance analysis in Chapter 3, the robust transmit beampattern design in Chapter 5, and the receiver identifiability test in Chapter 6.

In Chapter 3, we have looked into the IVF design at MIMO radar receiver for clutter suppression. Based on the existing results for only range sidelobe suppression [21, 22], we have presented the performance comparison of the integrated

sidelobe level (ISL) constraint and the zero sidelobe (ZS) constraint, and illustrated the later's advantages. We have also extended the IVF design for joint range and Doppler sidelobe suppression. Closed-form solutions are derived and categorized for different system specifications. We have also shown via simulations that the bounds of the volumes on the delay-Doppler plane in the IVF's output cannot be similarly characterized by the similar bounds derived for matched filter (MF) [53, 54]. In addition, the design and performance of phased-array counterpart has also been provided for comparison in terms of sidelobe suppression level, computational complexity, and signal-to-noise ratio (SNR) loss.

Chapter 4 and 5 focus on the transmit beampattern design employing the waveform diversity of MIMO radar systems. We first provided a comprehensive review of existing methods for the transmit beampattern design. Different from all the existing methods, we mapped MIMO radar transmit beampattern design to a multiple-input single-output (MISO) finite impulse response (FIR) filter design, and proposed a design using a feasibility problem formulation, which allows quantitative controls of ripples and transition band. Moreover, using the proposed FP design, we have obtained an empirical formula, i.e., the transmit beampattern formula, to estimate the required number of transmit antennas to achieve the desired transmit beampattern. The transmit beampattern formula is similar to Kaiser's formula to estimate the FIR filter length given a desired transfer function. A robust transmit beampattern design was investigated in Chapter 5. We have considered two practical issues. i) the efficiency of the radio frequency amplifiers (RFA's), and ii) the imperfection of the waveform correlations. As a result, the robust design is achieved by relaxing the elemental transmit power constraint, and the use of a set of nonorthogonal linear frequency modulated (LFM) waveforms. We have shown theoretically that the robust design using LFM waveforms is able to achieve identical transmit beampattern designed using ideal orthogonal waveforms. In addition, the robust design does not limit to the use of LFM waveforms.

In Chapter 6, we have compared MIMO and phased-array radar within the scope of this thesis. The two basic transmit models, i.e., omnidirectional transmission of orthogonal waveforms for MIMO radar and steered transmission of coherent waveforms for phased-array radar, has yield nearly identical MF overall beampatterns which only differ from a scalar N_T . For transmit beampattern design, it is also proven that phased-array radar can achieve nearly identical transmit beam-

patterns as MIMO radar with identical transmit and receive antennas. To form a flexible transmit beampattern for phased-array radar, the steering vector was replaced by a weighting vector obtained using spectral factorization method. Further investigation and comparison have been carried out on receiver parameter identifiability using eigenstructure based estimation method, i.e., the MUSIC algorithm. Based on the results in Chapter 3, and the theoretical comparison and support of simulation results in this chapter, we have drawn the following conclusion in a trade-off manner. Phased-array radar is more efficient to deal with single target because its coherent gain enhances the estimation in terms of resolution and sidelobe reduction. On the contrary, MIMO radar is more efficient to deal with multiple targets because its waveform diversity is able to simultaneously identify parameters of multiple target echoes. In addition, it is important to note that although phased-array radar can achieve nearly identical transmit beampatterns as MIMO radar, it is not able to meet the constant power constraint. By using multiple waveforms, MIMO radar is able to meet the power constraint by regularizing the diagonal elements of the covariance matrix to be constant.

7.2 Future Work

There are several issues that are worth of further research attention. First, several optimization problems in this thesis have not been efficiently solved. In Chapter 2, (2.10) was proposed without a solution. Instead, we carried out suboptimal exhaustive search. In Chapter 4, (4.27) was formulated to identify the $\mathbf{x}$ in the closed-form solution. Instead of solving this problem, we provided some references to find efficient solutions. In Chapter 5, we have formulated an overall robust design of the transmit beampattern using LFM waveforms (5.33). Empirical solutions are provided to illustrate the significance of the proposed design, but theoretical solutions were not discussed. The above mentioned optimization problems are worth of investigation. In addition to the optimization problems, several problems are discussed below for furthering the knowledge and understanding of the corresponding topics.

For the receiver IVF design, we have proven in Chapter 3 that the bounds derived for MF ambiguity functions are inapplicable for IVF outputs. Hence further research can be conducted towards the direction of finding similar bounds. For the

comparison of MIMO and phased-array radar, more efficient methods for phased-array radar are usually neglected as seen in many existing works. Therefore, the identifiability of the two systems should be further analyzed where the spatial smoothing method [86] should be considered for phased-array radar.

Author's Publications

Journal Papers

1. G. Hua and S. S. Abeysekera, "Receiver Design for Range and Doppler Side-lobe Suppression Using MIMO and Phased-Array Radar," *IEEE Trans. Signal Process.*, vol. 61, no. 6, pp. 1315-1326, March 2013.
2. G. Hua and S. S. Abeysekera, "MIMO Radar Transmit Beampattern Design With Ripple and Transition Band Control," *IEEE Trans. Signal Process.*, vol. 61, no. 11, pp. 2963-2974, June 2013.
3. G. Hua and S. S. Abeysekera, "Efficient Design of Transmit Beampattern for Colocated MIMO Radar," *Signal Processing* submitted, June 2013.

Conference Proceedings

1. G. Hua and S. S. Abeysekera, "Collocated MIMO Radar Waveforms Coding using Costas and Quadratic Congruence Arrays," in *Proc. International Conference on Information, Communications, and Signal Processing (ICICS)*, IEEE, Singapore, 2011.
2. G. Hua and S. S. Abeysekera, "Collocated MIMO Radar Transmit Beamforming using Orthogonal Waveforms," in *Proc. International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, IEEE, Kyoto, 2012.
3. G. Hua and S. S. Abeysekera, "On the Transmit Beampattern Design Using MIMO and Phased-Array Radar," in *Proc. International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, IEEE, Vancouver, 2013.
4. G. Hua and S. S. Abeysekera, "A Comparison on DOA Parameter Identifiability for MIMO and Phased-Array Radar," in *Proc. International Conference on Information, Communications, and Signal Processing (ICICS)*, IEEE, Tainan, 2013.

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