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RELIABILITY ASSESSMENT OF THE COLLISION BETWEEN MARINE RISERS

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School of Civil and Environmental Engineering

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in fulfillment of the requirements for the degree of
Doctor of Philosophy**

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DECLARATION

This dissertation describes part of the research conducted at the School of Civil and Environmental Engineering, Nanyang Technological University between August 2008 and September 2012. It is the result of my own work and includes no collaboration work done by others, except where stated. In this dissertation, Chapter 3 is taken from my published journal paper (He and Low, 2012); Chapters 4 and 5 are taken from two submitted journal papers of mine respectively.

AUTHOR'S PUBLICATIONS

He, J.W. and Low, Y.M. (2012), An Approach for Estimating the Probability of Collision between Marine Risers, *Applied Ocean Research*, 35, 68-76.

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DEDICATION

This thesis is dedicated to those I love

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ABSTRACT

As offshore production shifts to deep water, riser systems become more vulnerable. Riser clashing is a critical issue needed to be addressed in the design stage. Two design principles are defined in the code and termed as *no collision allowed* and *collision allowed*. *Hydrodynamic interference* is found to be the main driver of riser collision. This thesis proposes *practical* approaches to evaluate the *probability of failure* due to riser collision. With this important parameter, designers can optimally determine the riser layout. The whole work is separated into three stages.

The first stage of work proposes an approach to predict the failure probability based on no collision allowed designs. The challenges are multi-faceted. Firstly, riser clashing is a rare event, thus collision sampling is impractical. Secondly, the nonlinearities induced by hydrodynamic interferences make the prediction much tougher. The approach relies on post-processing nodal positions obtained from time domain simulations and a properly defined process characterizing the response severity between two risers. A crossing rate analysis is conducted and the crossing rate is extrapolated to the failure threshold. The impact of hydrodynamic interferences on the failure probability is vast.

The second stage of work makes the above prediction more realistic by allowing considerations of multiple uncertainties, which are supposed to have huge impacts on the failure probability. Efforts are put on enhancing the efficiency and robustness of the approach. This approach involves the computation and extrapolation of crossing rate derivatives. The failure probability is achieved by approximating the classical reliability integral with an existing technique. The presence of multiple uncertainties increases the failure probability significantly.

In the third stage, a reliability assessment approach is proposed for collision allowed designs. The approach includes a local and a global stage. The bulk of effort is put on optimizing the global stage. The approach involves extrapolations of pre-collision statistical characteristics to the failure threshold. The post-collision behavior is

investigated particularly for this design principle. A case study evaluates the yield and fatigue limit states.

The above approaches are verified by Monte Carlo simulations.

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CHAPTER 1 INTRODUCTION

1.1 Background

1.1.1 Offshore systems and their components

Although the development of some innovative energy is speeded up in recent years, e.g. wind power and solar power, hydrocarbons are still dominating the energy consumption market worldwide in the form of heat and electricity energy. Oil and natural gas are the main products of hydrocarbons. In addition, hydrocarbons are the primary sources for chemical industry, e.g. one of the valuable products is the asphalt. The strong desire of exploiting the hydrocarbon resource tremendously boosts the development of offshore oil and gas production especially when the oil fields onshore are currently found to be depleting (Chakrabarti, 2005).

The birth year of offshore industry is commonly recognized to be 1947 when Kerr-McGee completed the first successful offshore well 4.6 m below the mean sea level in the Gulf of Mexico (Chakrabarti, 2005). It was the first oil production project operated in a place where the land is invisible. The drilling tools were mounted on an 11.6 m by 21.6 m wooden deck platform above the sea (Chakrabarti, 2005). Since the installation of this first platform, the offshore engineers have designed many innovative structures, e.g. fixed platforms, compliant towers and floating platforms. Moreover, the subsea structures are improved tremendously from fixed risers to compliant risers like flexibles, top-tensioned risers (TTRs), steel catenary risers (SCRs) and hybrid risers. Other subsea structures like mooring lines and pipelines are crucial components of an offshore system.

In fact, all these developments are tracing the oil field discoveries from shallow water to deepwater area, from benign ocean conditions to challenging and hostile environment, e.g. the Arctic region. By 1975, the oil production was successfully operated in a water depth of up to 144 m. Within the next three years, the maximum

operating depth dramatically increased by two times. The new record was set by the COGNAC platform which was made up of three separate structures in the water depth of 312 m (Sterling et al., 1979). Table 1.1 lists the completion years of some well-known platforms and their operation water depth from 1978 to 2010. It can be observed that the operation fields were moving from shallow water (<400 m) to deep water (400 m to 1500 m) and ultra-deep water (>1500 m). Besides, the deck size also increased dramatically. For example, the platform Deepwater Horizon completed in 2001 has a deck size of 112 m by 78 m, which is 35 times of that of the first platform completed by Kerr-McGee.

Table 1.1 Water depths in different projects

Year	Project	Water depth
1978	Cognoc, fixed platform	312 m (shallow)
1988	Bullwinkle, fixed platform	412 m (deep)
1993	Mars, top-tensioned platform	896 m (deep)
2003	Na Kika, semi-submersible	2300 m (ultra deep)
2010	Perdido, Spar	2450 m (ultra deep)

The exploitations in the ultra-deep water region are still in its early age, more efforts should be taken to make the operation reliable and safe. Nowadays, focuses are put on a non-traditional area, the Arctic region. It will doubtlessly become the next spotlight of offshore production since this region possesses the largest unexploited oil and gas reservoir (Robertson and Pierce, 2008). Some ongoing researches are conducted to improve the floating platforms and riser systems so as to adapt to the low-temperature environment and resist the ice collisions in the Arctic.

In the following context, offshore platforms as well as marine risers are introduced. Emphases are put on floating systems, in that they are currently the key players in industry. An offshore production system generally consists of three main components:

an offshore platform, a riser system and a mooring system. An offshore platform is a structure designed to locate in the sea under any weather condition and does not have access to the dry land. A riser is a pipe structure, which conveys the oil or gas under seabed to the platform above the mean sea level. Furthermore, a riser system is a number of risers designed to fulfill the missions of oil and gas production and sometimes drilling. A mooring system is commonly used to stabilize the floating platforms and control them to response within a safe domain. Unlike offshore buildings, offshore structures are manufactured onshore and transported by e.g. barge to oil fields after completion.

Offshore fixed structures are first used in offshore oil industry and initiate the upsurge of exploration and production of oil and gas towards the deeper sea at a rapid rate. For fixed platforms, the platform height must be greater than the water depth. If the platform is too large to be transported, it is usually pre-fabricated in parts to make the transportation available; then, all parts are assembled on site. It is not hard to envisage the high cost and difficulty of the installation in deepwater ocean. Therefore, this type of platforms is only suitable for shallow water.

(a)

(b)

Figure 1.1 Different types of offshore systems (a) Bottom supported and vertically moored structures (b) Floating production and subsea systems (Richardson et al., 2008)

In both deepwater and ultra-deep water environments, floating vessels seem to be the only feasible solution. Currently, the most advanced offshore floating platforms are able to operate in the water depth of about 2500 m (Perdido spar platform) and withstand any harsh environments. The popular types of floating platforms are Floating Production Storage and Offloading systems (FPSO), semi-submersible platforms, tension-leg platforms (TLPs) and Spar platforms. Every type of platform has its unique applicability to, e.g. water depth and sea states. Figure 1.1 summarizes different types of offshore systems operating in various levels of water depths. Generally, the tensioned platforms are not suitable for ultra-deep water, while semi-submersibles and FPSOs are appropriate in both deep and ultra-deep sea.

Marine riser is a critical component in an offshore system. Currently, there are four types of risers in common use. They are flexible risers, SCRs, TTRs and hybrid risers. A brief introduction is given below.

Figure 1.2 Schematic of a flexible riser system with a semi-submersible platform (Ghadimi, 1988)

Flexible riser

Flexible risers are widely applied in offshore industry, especially under benign environments. This type of riser has a relatively long development history. Current technologies enable it to serve under high pressure and temperature environment. The main characteristic of flexible risers is their low relative bending stiffness to axial stiffness. This feature makes it possible to withstand large vessel motions. The advantages of flexible risers include the availability of prefabrication and storage on reels, the lower costs on both transportation and installation, and the compatibility with compliant structures (Bai and Bai, 2005). However, flexible risers do have some disadvantages, e.g. they are technically limited to smaller sizes compared with other types. This limitation has negative impacts on the productivity. In addition, they are not suitable for application in ultra-deep water owing to a striking increase of fabrication cost with respect to water depth. Finally, the cost of flexible risers is relatively high due to the complex manufacturing processes and high cost of materials (Howells and Hatton, 1995; Hatton and Brownridge, 1995). Figure 1.2 shows a conventional layout of a flexible riser. A more detailed summary on conventional configurations is presented in Table 1.2.

Table 1.2 Standard flexible riser configurations (Brekke et. al., 2005)

Configuration Terms	Lazy Wave	Steep Wave	Free Hanging
Illustrations of Configurations
Configuration Terms	Steep S	Lazy S	Chinese Lantern
Illustrations of Configurations

Steel catenary riser

SCRs are fabricated with standard grade steel pipes. The pipes are welded and connected in the field and laid down to the ocean. SCRs can be hung freely in a longer length and are adopted under the conditions of higher pressure and temperature as well as larger diameter compared with flexible risers. Besides, SCRs have a low manufacturing cost (Howells, 1995). However, this type of risers is highly sensitive to fatigue and vessel change which inevitably increase the design costs. In addition, the requirement for a good welding quality and on-site fabrications also lead to a higher installation cost (Walters, 2003). Figure 1.3 presents the simplest layout of an SCR with a semi-submersible platform.

Figure 1.3 Schematic of a steel catenary riser connected to a semi-submersible (Walters, 2003)

Hybrid riser

Hybrid risers or a hybrid riser tower (HRT) makes use of a vertical bundle of steel risers to connect the seabed with a buoyancy can below the sea surface. The buoyancy cans are usually linked to the vessel with flexible line structures termed as jumpers. The objective of this layout is to use the relatively low cost steel risers to replace the major length of flexible risers so as to minimize the overall cost of a riser system. In addition, the component of flexible risers can accommodate the severe vessel motions (Hatton and Brownridge, 1995). If the system encounters a severe storm, the hybrid risers are allowed to be disconnected from the vessel. On the whole, this type of riser is insensitive to fatigue and available for pre-installation. Moreover, the installation cost is relatively low (McGrail and Lim, 2004). Unfortunately, hybrid risers have limitations on pressure and temperature resistances and manufacturing sizes in the section of flexible jumpers. Figure 1.4 is a typical schematic of a hybrid riser system.

Figure 1.4 Schematic of a hybrid riser system connected to a semi-submersible

Top tensioned riser

Unlike other compliant risers, TTRs are pre-tensioned and placed vertically; therefore the vessels can hardly move up and down and are limited to horizontal movements. Figure 1.5 illustrates all the features mentioned (the lines concentrating on the center of the deck are TTRs). TTRs are so versatile that they can perform the functions of production and injection, drilling and export. Because the risers are geometrically vertical lines extended from the sea surface to the bottom, the whole length of TTRs are minimized. One of the downsides of TTRs is that the required top tension has to be increased with the water depth owing to the increased weight of risers. Moreover, the vortex-induced vibration (VIV) effect is significant and the clashing probability is relatively high. It is overall costly to address these problems in a TTR system.

Figure 1.5 Schematic of a top tensioned riser system with a TLP

The compatibility between vessel types and riser types is summarized in Table 1.3. It is found that the first two types of the vessels are compatible with compliant risers such as flexible risers, SCRs and HRTs; the last two types are applicable to top tensioned risers like TTRs. Flexible risers and TTRs, in a general way, have higher chances of clashing. They are employed in the case studies in Chapters 3, 4 and 5. However, it does not mean that the other riser types are immune from the riser clashing without appropriate designs.

Table 1.3 The compatibility between vessel types and riser systems

Vessel types	Compatible riser systems
FPSOs	Flexible risers, SCRs and HRTs
Semi-submersibles	Flexible risers, SCRs, TTRs and HRTs
TLPs	TTRs
Spars platforms	TTRs

1.1.2 Riser clashing

As the emphasis of oil and gas industry moves towards greater water depths, deepwater marine risers play a vital role in the production of hydrocarbons. In a riser system, any damage or failure will inevitably cause a lower productivity and an ensuing increase in the overall cost. Furthermore, it may even lead to disastrous consequences such as ocean pollution, human casualties and severe damages on facilities. The oil leakage happened in Gulf of Mexico in 2010 is a painful factual example. Therefore, the riser system should be shielded in the design stage from various failure mechanisms.

Amongst other things, riser clashing is a topic of concern under the current circumstances. The consequence of riser clashing can be catastrophic. Both static contact and dynamic clashing can cause the risers to have fatigue or coating damage, and the latter may further lead to a loss of integrity (Rustad et al., 2008). A more detailed discussion on the consequence of riser clashing is provided in Section 2.3.5.3.

Before investigating the topic of riser clashing, some issues need to be clarified. For one thing, risers are commonly arranged in an array. When risers are placed adjacent to each other, hydrodynamic interaction will occur among them. A general term for this type of interaction is hydrodynamic interference. For another thing, riser deflection increases exponentially with water depth subjected to the same current magnitude (Huse, 1993). This fact makes riser clashing more risky in deep water. Hydrodynamic interference is generally considered to be the main driver of riser clashing (Sagatun, et al., 2002). It involves intricate physical processes while current understanding still falls short. Generally, hydrodynamic interferences can be categorized as vortex-induced vibration (VIV), wake shielding and wake-induced oscillation (WIO). Since VIV and WIO are recognized to be of importance for riser clashing, a substantial amount of research has been devoted to these aspects (Sagatun, et al., 2002; Huse and Kleiven, 2000; Blevins, et al., 2006; Huang, 2010). In industry,

software packages based on modal analysis are commonly used to predict VIV. Iwan and Blevins (Iwan and Blevins, 1974) established a wake oscillator model to simulate VIV response in the time domain. In terms of WIO, Blevins wake field model (Blevins, 2001) is a possible choice. Although Huse's model (Huse, 1993; Huse, 1994; Huse, 1996) is also popular, it does not account for the lift forces; hence it is impossible to simulate WIO (Det Norske Veritas, 2009). All in all, the available models are still not fully mature. For instance, the Blevins wake field model is invalid when clearance between two risers is smaller than about two diameters (Kavanagh, et al., 2000). A comprehensive literature review on hydrodynamic interference is done in Chapter 2.

In the actual riser design, the compliancy of marine risers implies that close riser spacing will increase the risk of mutual contact and collision (Elman and Alvim, 2008). The problem of riser clashing may be avoided by increasing the spacing between risers. Unfortunately, large riser spacing is prohibited in offshore design due to practical and economical reasons. For economical reason, designers generally prefer compact riser systems to minimize the deck size especially in deepwater fields where the number of connected risers tends to be large. For practical reason, larger riser spacing makes the riser system more difficult to be deployed. The two conflicting design requirements bring about the need for an optimized riser layout design which should have certain reliability against riser clashing.

However, in the literature, there is no sufficient guideline to address this concern. DNV code (Det Norske Veritas, 2009) only recommends a summation of two riser diameters as safe spacing to avoid VIV. However, a general approach to calculate the minimum allowable riser spacing on case-by-case basis is highly desired in industry. The efforts should be put on proposing approaches which are capable of computing probability of failure due to riser clashing.

Last but not least, it is necessary to discuss failure criteria of riser clashing. According to practical codes (Det Norske Veritas, 2009), two principles exist in the design strategy; they are *collision allowed* and *no collision allowed* riser designs.

No-collision-allowed riser design

For this design principle, the collision is normally not allowed even in accidental or extreme conditions. The probability of riser clashing should be controlled within a low level, e.g. less than 10^{-4} during field life, typically 20 years (Chakrabarti, 2005; Brekke et al., 2005). The probability 10^{-4} is quoted from Chapter 9 of Chakrabarti (2005). In this principle, a collision means failure of a riser system. Here, some studies contributing to the reduction of chance of riser clashing are introduced. These methods aimed at preventing the risers from mutual collisions under any circumstance. Rustad et al. (2008) introduced a PI-controller (proportional integral controller) to confine the horizontal displacement of the top tensioned risers in a TLP model. The main function of the PI-controller is to increase the speed of response and eliminate errors in a steady state of the riser system. Besides, Leira et al. (2002a) employed a new approach to improve the dynamic positioning (DP) systems in order to reduce the riser motions. These studies are capable of reducing the likelihood of riser clashing but fail to offer a quantitative conclusion on the reliability of riser system against collisions.

In a conventional riser design, the environmental loadings are random. The random source is commonly from irregular waves. In addition, the random sources could arise from many other sources such as current, hydrodynamic interaction, riser spacing, top-tension and discrepancy in dynamic properties of risers (Det Norske Veritas, 2009). In the literature, Leira et al. (2002b) and Huang (2010) analyzed the likelihood of riser clashing with respect to certain parameters. But their researches are merely based on static analysis. However, in an actual design, the versatile dynamic analysis is preferred.

Collision-allowed riser design

This principle states that a collision does not necessarily cause a failure in riser systems. Additional information about riser structural properties and factors characterizing the degree of collision severity are needed to assess the reliability against riser collisions. Obviously, additional efforts should be taken on assessing the consequence of a collision comparing with the previous design principle. However, this “less strict” failure criterion allows a more compact riser layout design. It is worth noticing that even though collisions are allowed, they should by no means be frequent. In fact, collisions should only take place under temporary, accidental and extreme conditions.

A complete collision-allowed assessment traditionally includes two separate analysis stages; namely, the global and local stage. A global analysis uses line elements to model a complete riser system, and involves all the environmental loadings. Certain collision parameters such as impact velocities and kissing angles are recorded as inputs for the local impact analyses. In a local analysis, a more detailed finite element (FE) model is built. An FE analysis is performed applying shell or solid elements. The stresses are extracted to evaluate the structural safety under the yield and fatigue criteria. The procedure for local impact analysis of risers is considered to be relatively well established, and there are many related studies (Wilmschurst et al., 2001; Alsos et al., 2010).

It is worth mentioning that, in collision-allowed design, the number of collisions is very small even though the collision rate dramatically increases. Researchers have already developed simulation tools that can predict riser collision and impact damage (Holmas et al., 2002). These simulation tools are realistic as necessary parameters are calibrated with laboratory tests and computational fluid dynamics. In these studies, the researchers only considered design parameters (e.g. top-tension factor and riser clearance) that yield high collision rates. However, for more commonly used parameters producing lower collision rates, a proper method is absent in the literature.

1.1.3 General considerations

Because of the significant nonlinearities in the problem (VIV, WIO, large deflections and drag forces), all the global riser dynamic analyses in this paper are conducted in time domain especially for critical design case. Static analysis and modal analysis are out of the scope of this work.

Besides, there are two differing schemes to do a dynamic analysis on risers. The first one is coupled analysis (Low and Langley, 2008), in which both the vessel and risers are treated as components of the global model subjected to various environmental loadings. The other one is a simplified scheme termed as decoupled analysis. In this scheme, vessel motions are prescribed and imposed on the top ends of risers. The latter scheme is much more efficient but less accurate. The essence of it is to isolate the analysis of riser systems. Comparing with coupled analysis, the loss of accuracy in decoupled analysis varies from case to case. In general, a smaller riser-to-platform weight ratio gives a more accurate result in decoupled analysis.

In actual design, a much larger number of sea states and model profiles should be considered in order to find out the “worst case”, which is a combination of, say, certain current profile, wave heading and vessel offset. However, it is not practical and distracting to consider all these in this thesis. Moreover, there are existing methods to identify the critical sea states and design parameters (Haver and Winterstein, 2009) which are independent of the work in the thesis. Therefore, all the case studies in this work consider only a limited number of sea states.

To be practical, the proposed methods in this thesis should make full use of simulation data that is already available as a consequence of other design requirements, such as ultimate riser tensions. However, some additional simulations are allowed if required.

1.2 Research objectives and scope

The ultimate goal of current work is to propose *practical* approaches to calculate the failure probability of a pair of risers against mutual collision. Tailor-made approaches are required to satisfy different design principles and consider multiple uncertainties.

Reliability assessment based on no-collision-allowed design

- Make use of the dynamic analysis data for a conventional riser design.
- Define an appropriate process to characterize the severity of motion between two risers.
- Expedite the production of the process to make the approach practical.
- Conduct a crossing rate analysis and extrapolate the crossing rate to the failure threshold with an appropriate extrapolation scheme. For simplicity, only randomness from waves is considered.

Prediction of riser clashing considering multiple uncertainties

- Shortlist a number of uncertain parameters which have significant impacts on failure probability.
- Make use of the dynamic analysis data for a conventional riser design. Additional simulations for revealing the impacts of uncertain parameters should be minimized.
- Employ an appropriate method to approximate the classical reliability integral. The response variables that are required in the method are achieved through extrapolations.
- Efforts are put on making the extrapolation scheme robust with limited data.

Reliability assessment based on collision-allowed design

- Make use of the dynamic analysis data for a conventional riser design.

- Efficiently establish an appropriate process to characterize the severity of motion between two risers.
- Conduct a local impact analysis to understand the induced stresses subjected to various initial conditions.
- Shortlist the crucial initial conditions for this problem, such as impact velocity and impact angle.
- Extrapolate the pre-collision statistical characteristics of the process to the failure threshold. Assign proper forms to statistically describe the initial conditions.
- Understand the statistical characteristics of the post-collision behavior.
- Deduce the form of failure probability corresponding to the collision allowed design.

1.3 Organization of thesis

In this thesis, Chapter 2 will document the state-of-the-art related to the topic of riser clashing in the literature. Since riser interference is crucial for riser clashing problems, researches on riser interference will be summarized in this chapter. Chapter 3 presents a proposed approach to predict the probability of riser clashing. Flexible risers are employed in the case studies. Chapter 4 aims to establish a scheme to efficiently predict riser clashing considering finite number of random parameters. Chapter 5 puts forward an approach to evaluate the probability of failure due to riser clashing with respect to collision allowed design. Top tension risers are applied in the case studies. A separate finite element analysis (FEA) is done to investigate the structural responses of the risers. Chapter 6 highlights the achievements made and the future work that is worth pursuing.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

A briefing was given in Chapter 1 on the current offshore industry including the types of floating platforms and risers. The marine riser is a critical component in an offshore system. Riser clashing is one of the potential problems that need to be addressed in the design stage. One difficulty is that hydrodynamic interferences including VIV, wake shielding and WIO make the prediction of riser clashing complicated. Moreover, significant hydrodynamic interference will result in high chance of riser clashing. It is universally accepted that WIO is the cause of most riser collisions and VIV contributes most to the clashing severity between risers (Sagatun et al., 2002). Hence, the understanding of hydrodynamic interference is a primary step for investigating this problem. A full literature review will be done on the characteristics of hydrodynamic interferences and corresponding numerical models in Section 2.3.3. Moreover, general approaches for investigating riser clashing are summarized in Section 2.4. Existing studies on both collision-allowed and no-collision-allowed riser systems are showcased in Section 2.5.

Besides, the factors that influence the degree of riser interferences and the designs aiming to minimize the interference are of interest. Those factors are showcased in Section 2.2.

2.2 Important factors on riser interferences

The degree of interferences between two adjacent risers depends on many factors, e.g. the environmental loadings and top tension. In fact, these factors are already summarized in the design code of riser systems including both compliant and top tension riser systems (Det Norske Veritas, 2009).

In the actual riser designs, some factors and potential issues have significant impacts on the degree of riser interferences. They should be carefully addressed at the design stage. These factors and potential issues are vessel motions, riser properties, environmental loadings, marine growths, riser configuration, accidental scenarios and riser spacing. Designers attempt to minimize the negative impacts of these factors on the riser systems. Some practical design details are exhibited herein. The purpose is to reveal the significant impact of hydrodynamic interference on the riser system design.

Vessel motions

When a vessel is subjected to environmental loadings, it will have responses in some translational directions and rotational directions. Generally, a vessel has six degrees of freedom, i.e. surge, sway, heave, roll, pitch and yaw which are visualized in Figure 2.1.

Figure 2.1 Degrees of freedom of a vessel

Harsh environmental loadings can lead to severe vessel motions to the extent that even the risers collide with each other. Some designs are introduced to mitigate the effects of vessel motions on the riser system. For example, the Single Line Offset Risers (SLORs) presented in Figure 2.2 are connected to the floating platform with flexible jumpers. Due to the compliancy of flexible jumpers, the impact of vessel motions on the production risers is dramatically reduced. In emergency situation, SLORs are allowed to disconnect from the vessel to avoid potential damage.

Figure 2.2 Four-riser arrangement Grouped SLORs (Natarajan and Hatton, 2009)

Some other designs, such as Spars and TLPs, aim to reduce the vessel motions. With the pre-tension exerted on the vessels, the motions are confined to be in the horizontal directions, namely surge, sway and yaw. These days, the dynamic positioning (DP) system (see Figure 2.3) is designed to stabilize the vessel motions. This system consists of position sensors, a thruster system, a DP control room and a power station. It makes use of the thrusters to resist the force-induced vessel motions and achieve the stability in a severe sea state. According to the report on offshore structures in the Gulf of Mexico (Richardson et al., 2008), quite a few floating platforms use DP systems to maintain a good stability.

Figure 2.3 General view of the DP system on a semi-submersible (Leira et al., 2002a)

Riser property

Here, riser property refers to the fundamental mechanical characteristics of a riser such as top tension. It is known that risers with high top tension can effectively reduce their deflections. Riser interference is therefore less significant and so does the probability of riser clashing. Moreover, DNV RP F203 (2009) stated that risers with different static and dynamic properties have high chance of riser clashing and the chance is minimized when their properties are the same. Last but not least, some designers enhance the resistance of risers against clashing by employing, for example, wear resistance and coating.

Environmental loading

Most of risers extend from the sea surface to seabed. They are exposed to various external forces along the depth of the ocean. Generally, the wave forces near the mean sea level and the current distributed along the water depth are the main external excitations. These excitations result in large deflections in risers and are likely to make them move closer to each other. Furthermore, current and wave are the stimulus of hydrodynamic interferences. Considering this, it is of help to explore methods to alleviate impacts of environmental forces on the riser systems. Do and Pan (2009) proposed a boundary controller for the riser system and found it effective to control the riser motions in severe sea states. Moreover, modal control is an alternative approach for lessening the riser motions induced by environmental loadings (Meirovitch, 1997; Gawronski, 1998).

Hydrodynamic interference

As mentioned, VIV, WIO and wake shielding are regarded as the main hydrodynamic interferences in this topic. Since they are the crucial factors for riser clashing, a comprehensive investigation is highly desired. The occurrence of VIV has something to do with the chaotic wake field around risers. Basically, in the industry, the

methodology is to make the wake field less chaotic so that the levels of VIV and WIO can be reduced. VIV suppression devices such as strakes, fairings and shrouds are the best choices (see Figure 2.4(a)-(b)). Blevins (2001) examined the performances of a number of VIV suppression devices. It was pointed out that the helical strakes and shrouds, for example, could reduce the VIV response by 70% to 90% compared with a plain cylinder. Streamlined fairings were expected to reduce VIV effects by 80%. The drawback of these devices is the cost burden on the riser systems. For wake shielding effects, the prediction is acceptable at current stage. It is usually examined at the preliminary design stage. Detailed literature reviews on hydrodynamic interference will be provided in Section 2.3.3.

Figure 2.4 VIV suppression: (a) riser strake (marktool.com); (b) riser fairing

Marine growth

Marine growth is commonly distributed in the offshore structures that are submerged in the ocean. The harmful aspects of marine growth on risers are basically the reduction of clearance between risers and the change of roughness on the riser surface. If marine growth is attached to the riser surface, the spacing between two risers may be significantly reduced in some sections. These sections will become a potential area for riser clashing. Moreover, the marine growth makes the surface of risers rougher.

Generally, the roughness will make the interference severer. Cleaning the marine growth is an effective way to get rid of marine growth, but it is costly and difficult to complete in deepwater area. Figure 2.5 presents the case that marine growth clings to the riser walls near the mean sea level.

Figure 2.5 Marine growths adhering to the riser surface (Stalenhoef and Raad, 1998)

Riser configuration

Figure 2.6 Riser configuration following separation principle (Rustad et al., 2009)

Riser configuration is determined at the design stage. There is certain restriction for different riser types in terms of configurations. A well designed riser configuration

can dramatically reduce the riser interference. For more compliant risers like flexible riser systems, a proper riser configuration plays an indispensable role in avoiding riser interferences. It is recommended that horizontal and vertical staggering of adjacent risers, which allows crossings of risers with different vertical positions, is practical and effective to avoid riser clashing (Rustad et al., 2009). Figure 2.6 illustrates a riser configuration following this principle, i.e. different hang-off angles at the top ends and different vertical positions at the buoyancy section.

Accidental load and scenarios

Accidental load and accidental scenarios are generally difficult to analyze, but these unexpected situations may lead to severe riser interference or even riser clashing. Some potential accidental scenarios should be analyzed beforehand, e.g. the dynamic analysis of the disconnection procedure of a flexible jumper and the assessment of clashing between the jumper and floater at the pontoon level (Brown et al., 1989). A possible model is shown in Figure 2.7. Moan (2009) proposed a systematic model to assess the reliability of an offshore system. Riser mutual collisions are treated as a potential accident in the model.

Figure 2.7 A flexible riser linking a fixed platform with a semi-submersible (Brown et al., 1989)

Riser spacing

Riser spacing exerts the most direct impact on the degree of riser interference. However, there is no generally accepted value for riser spacing. As is discussed in Section 1.1.2, there exists an optimized riser spacing for a given riser system. In the literature, some suggestions on riser spacing of specific riser systems are presented (see Table 2.1). By referring to the table, it can be concluded that there is no general criterion for an optimized riser spacing and the assessment is strongly case dependent. The different riser spacings recommended in Table 2.1 are empirical or from experiments. They may or may not consider the safety factors. In this table, D represents the diameter of a riser; D_1 and D_2 are diameters of two different risers. General approaches to assess proper riser spacing will be provided in Chapters 3, 4 and 5.

Table 2.1 Riser spacing in different studies

Authors	Riser spacing	Descriptions
DNV RP F203 (2009)	$D_1 + D_2$	Resist VIV effect without considering any safety factor (practical code)
Brekke et al. (2005)	$5D$	General practice for minimal spacing (empirical)
Fontaine et al. (2007)	$4D$	WIO is considered for SCRs, at least one clashing occurred in a short duration (experiment)
Niedzwecki and Duggal (1993)	$1.5D$	Collisions are observed between two cylinders in tandem in a short duration (experiment)
Ishii et al. (1991)	$3D$	Minimum spacing for flexible risers (empirical)

2.3 Hydrodynamic interference and analysis models*2.3.1 General*

Hydrodynamic interaction is the main driver of riser clashing. It is induced by ocean wave or current. It is of interest to have a deeper understanding of these environmental loadings which are summarized in Section 2.3.2. Generally,

hydrodynamic interference includes VIV, WIO and wake shielding. The mechanisms as well as the numerical models for hydrodynamic interferences are essential for current studies. Full literature reviews on these topics are shown in Section 2.3.3.

2.3.2 Wave and current

The influences of wave and current on offshore structures are inevitable in the ocean environment. These influences are generally treated as external forces to the structures. The forces cause severe motions of the structure and may lead to fatigue failure or yielding in some components. To address these potential failure modes, understanding of wave and current are necessary. Hence, the mechanisms and the models to describe them are introduced.

Wave

In the ocean, waves are mostly wind generated and commonly result from the wind blowing over a huge range of sea surface. Waves can travel up to several thousand kilometers or evolve to a destructive power over 10 meters height. If a sea state is directly affected by local winds, it is termed as a wind sea, in which waves are generated with a delay and hardly affected by the local wind at that time. In addition to wind generated wave, there are other types of wave like tsunamis which is a special type of wave caused by geological effects.

Ocean waves are essentially random. Due to the randomness, wave elevation at any location of the sea is unpredictable. Fortunately, the wave elevation can be described as a stochastic process. This process considers the physics determining the generation, growth, propagation and decay of waves.

In numerical analysis, there are some ideal wave models such as Airy wave model and second order wave model. Unfortunately, these models do not account for the

randomness in waves. To resolve this, a wave spectrum on frequency domain is required. JONSWAP and Pierson-Moskowitz wave spectrum are two typical options suitable for certain ocean regions. The resultant wave elevation is a Gaussian process.

Some laboratory experiments were conducted to examine the impacts of waves on the problem of riser clashing such as that by Duggal and Niedzwecki (1993). They proposed two sets of experiments: single flexible riser and a pair of flexible risers under either regular wave or random wave loadings. When wave passed through a riser, it induced transverse responses. It was found that the severity of transverse response was related to Keulegan-Carpenter (KC) number, namely $N_{kc} = UT/D$. Here, U is the amplitude of the flow velocity; T is the period of the transverse response or oscillation and D is a characteristic length scale of an object the flow is passing through say diameter of a cylinder. It was concluded that a small KC number gave relatively regular oscillation while a larger KC number produced instable and complicated responses.

Current

An ocean current is a nonstop movement of water in a constant direction. It is commonly generated by wind and tide. Furthermore, the current can be affected by factors such as shoreline configuration and interactions with an adjacent current. Ocean currents can flow for great distances and are considered as stable for a long period.

In the numerical models, current magnitude and distribution over water depth are always considered constant over a certain period. They may be uniformly distributed or proportionally decreased along the water depth. Moreover, current near the seabed is not uncommon in certain oceanic region.

In order to investigate the influence of current on riser clashing, Huse (1993; 1994; 1996) conducted some laboratory experiments on a riser array subjected to varied current magnitudes. The setup of the experiments is shown in Figure 2.8. The riser with larger diameter is the drilling riser while others are production risers. In the experiment, the magnitude of the current was increased gradually. As the current magnitude exceeded a certain value, the collision would be initiated and the frequency of collisions increased with the current velocity after the initialization. It would be even worse in deeper water, where strong currents are more common and the deflection of risers dramatically increases under the same current magnitude (Huang et al., 2002).

Figure 2.8 Setup of a top tensioned riser array (Huse, 1996)

2.3.3 Hydrodynamic interference

When hydrodynamic interference occurs among risers, there exist several mechanisms of riser motions. As mentioned, they are VIV, wake shielding and WIO.

2.3.3.1 VIV

When fluid flows around a cylinder, it will be separated into two parts and form two shear layers. The overlapping of these two layers generates the swirling vortices, which provide the external forces for VIV. As a result, VIV behaves as in-line and transverse oscillations of a riser subjected to a periodic shedding of vortices (API RP

2RD, 1998). It is universally agreed that VIV is severe when the shedding frequency approaches the natural frequency of the riser.

Figure 2.9 Displacement time history of a drilling riser (Huse, 1996)

Figure 2.9 shows the displacement time history of a drilling riser experiencing a lock-in vibration at its natural frequency. The vertical axis represents the number of riser diameter. The time history can be decomposed into a low-frequency and a high-frequency vibration (Huse, 1996). In Huse's experiment, it was observed that the high frequency VIV signal amplitude was about half of the riser diameter. However, Khalak and Williamson (1996) pointed out that the VIV amplitude could reach one diameter of a riser. The chaotic low-frequency signal was believed to be the result of WIO. The peak-to-peak amplitude could reach 40 diameters.

VIV increases drag coefficients so that drag forces on the riser are increased. A larger drag force results in greater deflection in a riser. It is unfavorable in a riser system especially when collisions are of concern. The amplified drag coefficient has something to do with the original drag coefficient. An empirical estimation is given by $C'_d = C_d \{1 + 1.043[2\sqrt{2}(A/D)]^{0.65}\}$, where C_d is the original drag coefficient; A is the VIV amplitude and D is the diameter of a riser (Vandiver, 1983). To alleviate the VIV effect, one needs to know the natural frequency of each riser in an array. This set of natural frequency should have an ample margin from that of environmental loadings (Huse, 1993; 1994). Another more costly way to resolve this

is to install VIV suppression devices on risers. Kavanagh et al. (2000) examined the performance of strakes (a type of suppression devices) in a riser.

Moreover, VIV has a self limiting nature. When the VIV amplitude reaches a relatively high level, e.g. one diameter, the regular vortex street will be disintegrated and the lift coefficient approaches zero. Consequently, there exist maximum VIV amplitudes of cylinders that are independent of the damping (Meneghini et al., 2005).

The topic of VIV phenomena is of concern for a long time in a wide range of industrial areas. The mass parameter, $m^* = m_{sys}/m_d$, which is defined as the ratio of system mass, m_{sys} , to displaced fluid mass, m_d , is a parameter relevant to characteristics of VIV responses (Feng, 1968). In the preliminary studies of VIV responses of line structures, cylinder models are widely employed in both experiments and numerical models. Some researches focus on high mass parameters while others on low mass parameters. Marine risers are structures with very low mass parameters, but significant work on this topic was absent until 1996. Khalak and Williamson (1996) carried out experiments for cylinders with a very low mass parameter, which was similar to that of marine risers. According to their experiments, the maximal response amplitude was about one diameter; and m^* significantly affected the characteristics of cylinder responses.

VIV models

To illustrate the VIV responses, a simplified VIV response model is provided herein. This model describes the interaction between vortex shedding and oscillations of a cylinder mounted on an elastic base (Meneghini et al., 2005). The vibrations are modeled in a single degree of freedom spring-mass-damper system (see Figure 2.10). Provided that the cylinder is free to move perpendicular to the flow direction, its motions can be described by an equation of motion (Meneghini et al., 2005)

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \frac{2}{\pi D m^*} U^2 C_l, \quad (2.1)$$

where ζ is the damping coefficient; ω_n is the natural frequency; m^* is the mass parameter; D is the diameter of cylinder; C_l is the lift coefficient; U is the incident current velocity and y is the displacement of the cylinder in the transverse direction.

Figure 2.10 A simple model for VIV analysis

Since the ratio of transverse displacement to diameter is of most concern, Eq. (2.1) is transformed to

$$\frac{D}{T_n^2} \ddot{Y} + 2\zeta\omega_n \frac{D}{T_n} \dot{Y} + \omega_n^2 Y = \frac{2}{\pi D m^*} U^2 C_l, \quad (2.2)$$

in which, the relationship between y and Y is given by

$$y = DY, \quad \dot{y} = \frac{D}{T_n} \dot{Y}, \quad \ddot{y} = \frac{D}{T_n^2} \ddot{Y}. \quad (2.3)$$

Here, T_n is the natural period of the cylinder.

A more advanced time domain VIV simulator was introduced by Iwan and Blevins (1974). This model considered the interaction between risers and fluids. Hence, this oscillator model consists of two components: a nonlinear and self-excited fluid

oscillator and a cylinder motion under the fluid forces. The fluid oscillator equation is provided by

$$\ddot{w} + K' \frac{u_t}{U} \frac{U}{D} w = (a'_1 - a'_4) \frac{U}{D} \dot{w} - a'_2 \frac{\dot{w}^3}{UD} + a'_3 \ddot{y} + a'_4 \frac{U}{D} \dot{y}, \quad (2.4)$$

where $K' = K/(a_0 + a_3)$ and $a'_i = a_i/(a_0 + a_3)$, $i=1,2,3,4$. The form of equation of motion depends on the boundary conditions. For illustration purpose, a simple boundary is prescribed herein, namely an elastically mounted cylinder. Under this boundary condition, the equation of motion has the form

$$\ddot{y} + 2\xi_T \omega_y \dot{y} + \omega_y^2 y = a''_3 \ddot{w} \frac{U}{D} + a''_4 w \frac{U}{D}, \quad (2.5)$$

in which $a''_i = \rho D^2 a_i / (m + a_3 \rho D^2)$ ($i=3,4$); $\omega_y = (k/m)^{1/2} / (1 + a_3 \rho D^2 / m)$; $\zeta_f = a_4 \rho D U / 2m \omega_y$; $\xi_T = [\zeta(k/m)^{1/2} / \omega_y + \zeta_f] / (1 + a_3 \rho D^2 / m)$. Here, ω_y is the circular natural frequency of the cylinder; k is the spring stiffness per unit length; ξ_T is a total effective damping coefficient composed of a component caused by structural viscous damping ζ and another one caused by viscous fluid damping ζ_f . w is a hidden flow variable first introduced in their work.

The left hand side of Eq. (2.4) describes the interaction between near wake and a boundary layer of the cylinder. The first term on the right hand side is a damping term while the second one limits the amplitude of fluid oscillation. The force placed on the fluid by the cylinder is represented by the last two terms. It is worth noticing that as the cylinder imposes forces on the fluid, opposed forces are induced. It results in an additional Eq. (2.5), which is a typical equation of motion with various external forces. These two equations feed back to each other. In other words, they are numerically “coupled”. As the frequency of the flow oscillations approaches the natural frequency of the cylinder, large-amplitude cylinder motion will be stimulated (Blevins, 2001).

A merit of this model is that hydrodynamic forces can be conveniently attained by utilizing a pre-established parameter database. In practice, these forces are prepared in computational fluid dynamics (CFD) tools. The disadvantages are from two aspects. First, it is inconvenient to apply this model on multiple risers. Second, it is necessary and expensive to build a large database that covers a wide range of Reynold numbers.

Internal flows

Internal flows have significant effects on VIV in risers. Amongst other things, Paidoussis (1998) devoted to the research on responses of slender structure with an axial flow. Monprapussorn et al. (2007) investigated the nonlinear vibrations of extensible risers in different internal flowing velocity profiles. Besides, Guo and Luo (2008) tested the VIV effects in the riser with internal flows.

2.3.3.2 Numerical models for wake

A wake is a region of disturbed flow caused by the fluid flowing around a solid body, e.g. a cylinder. This solid body can be termed as the upstream cylinder. Another cylinder placed in the wake will be termed as the downstream cylinder. Empirically, a cylinder is considered to be in the wake provided that the separation angle between the current and the center-to-center line of two cylinders is less than 20 degrees (Niedzwecki and Duggal, 1993).

There are two types of wake effects, i.e. wake shielding and WIO. The wake shielding effect reduces the drag forces on the downstream cylinder and makes the static position move upstream. These forces diminish with distance between the cylinders. Given that x is unchanged, the drag force is minimized when the downstream cylinder reaches the center line (see x coordinate in Figure 2.11). The greatest lift force occurs when the downstream cylinder is just outside the wake half width (b in Figure 2.11).

In terms of wake shielding, there are two popular analytical models. They are Huse's model and Blevins' model.

Figure 2.11 A wake profile behind a cylinder in stationary flow (Left) and the wake half width (Right) (Huse, 1993)

Wake formulation

By reference to Figure 2.11, a mathematical description of the wake field is given by Schlichting (1968). The wake half width b and the wake velocity U_0 at the center of upstream cylinder can be attained by Eqs. (2.6)-(2.8). With b and U_0 , the reduced velocity u at arbitrary downstream coordinates can be obtained from Eq. (2.9).

$$x_s = x + \frac{4D}{C_d}, \quad (2.6)$$

$$b = k_1 (C_d D x_s)^{0.5}, \quad (2.7)$$

$$U_0 = k_2 V (C_d D x_s)^{0.5}, \quad (2.8)$$

$$u = U_0 \exp(-0.693(\frac{y}{b})^2). \quad (2.9)$$

In the above equations, C_d is the drag coefficient; D is the diameter of the upstream cylinder. x is the distance in flow direction from the center of upstream cylinder while y is the transverse distance from the center line of the wake. V is the incident velocity for the upstream cylinder. k_1 and k_2 are empirical constant parameters. For a smooth cylinder, $k_1=0.25$ and $k_2=1.0$. To consider the effect caused by VIV, one can replace C_d with $C'_d = C_d \{1 + 1.043[2\sqrt{2}(A/D)]^{0.65}\}$ in Eqs. (2.6)-(2.8).

Figure 2.12 The wake velocity field behind a cylinder

Huse's model

Huse's wake model is a *parametric wake field model* which is based on Eqs.(2.6)-(2.9). Huse (1993) derived the forces on the downstream cylinders and wake velocities in the wake field by adopting the established wake formulation (Schlichting, 1968). Huse's model can be conveniently adopted on an array of cylinders. This model was verified by a series of experiments in the far wake (Huse, 1993; 1996).

Combining Eqs. (2.6)-(2.9), the reduced velocity is explicitly expressed as

$$u(x, y) = k_2 V \sqrt{\frac{C_d D}{x_s}} \exp\left(-0.693\left(\frac{y}{b}\right)^2\right). \quad (2.10)$$

In the above equation, all the quantities are illustrated in Figure 2.12. The wake velocity is obviously

$$U_0 = V - u. \quad (2.11)$$

The corresponding force on the downstream cylinder is conveniently given as

$$F = 0.5 \rho D C_d (V - u)^2, \quad (2.12)$$

where ρ is the fluid density. Similarly, the force on the cylinder placed in an array can be attained by simply replacing u with u_n in Eq. (2.12). Here, u_n is defined as the total reduced velocity, which is calculated by a root mean square summation of the reduced velocity of say n upstream cylinders at the position of a targeted downstream cylinder.

Kavanagh et al. (2000) performed experiments to verify Huse's model, and found it valid in the wake region greater than about $2D$ behind the upstream riser. The flow in the near wake region was not explicitly considered as it was a highly nonlinear phenomenon.

There are pros and cons of this model. First, the inflow velocity and drag forces can be computed in any location of the wake; second, this model is applicable to multiple cylinders. However, it is confined to static analysis. Lift forces are not accommodated in this model such that WIO responses cannot be described.

Blevins' model

Blevins' model (Blevins, 2001) is a *parametric mean force model*, in which the local drag and lift coefficients, $\overline{C}_d(x, y)$ and $\overline{C}_l(x, y)$, are attained beforehand numerically or experimentally. By placing another cylinder upstream, these coefficients are required to modify to $C_d(x, y)$ and $C_l(x, y)$ through

$$C_d = \overline{C}_d \frac{U_0^2}{V_0^2}, \quad (2.13)$$

$$C_l = \overline{C}_l \frac{U_0^2}{V_0^2}. \quad (2.14)$$

By referring to Figure 2.13, the forces imposed on the downstream risers are expressed as

$$F_x = 0.5 \rho V_{rel} D \frac{V^2}{U_0^2} [C_d(U_0 - \dot{x}) + C_l \dot{y}], \quad (2.15)$$

$$F_y = 0.5 \rho V_{rel} D \frac{V^2}{U_0^2} [-C_l(U_0 - \dot{x}) + C_d \dot{y}]. \quad (2.16)$$

In Figure 2.13, γ is termed as the attack angle. In Eqs. (2.15)-(2.16), V_{rel} is the relative velocity of the cylinder to the fluid. Both in Huse's model and Blevins's model, U_0 and V are assumed to be in the same direction. However, Mair and Maull (1971) suggested that the wake velocity did not act in the same direction as the free stream velocity, and they had a small separation angle.

Figure 2.13 Force diagram (DNV RP F203, 2009)

Finally, V_{rel} was approximated by Price and Piperni (1987) through drag coefficients, i.e.

$$V_{rel}(x, y) = V \sqrt{\frac{C_d(x, y)}{C_{gd}}}, \quad (2.17)$$

where C_{gd} is termed as the generic drag coefficient and it is measured outside the wake field.

A striking feature of Blevins' model is that force coefficients are given as functions of relative cylinder spacing. Commonly, the drag and lift coefficients are derived from model testing or two-dimensional CFD computation. Comparing with Huse's model, lift forces are considered in this model. Owing to this, WIO can be simulated in dynamic analysis.

Other models

In addition to the above two wake models, there are two models which are worthy of mentioning. The first one is an undisturbed fluid flow model which neglects the wake

field generated by a passing flow. The fluid is assumed to be ideal in the whole field. Hence, all the hydrodynamic interferences are not considered in the model. In practice, this model is used to identify the potential riser interference and is a good approximation if cylinders are placed far apart. The second model is a CFD model. This model depicts the viscous flow around a number of risers. They account for the hydrodynamic loadings and interaction effects between cylinders. Versatile as it may be, the high computational cost makes it unsuitable for design purpose.

2.3.3.3 Wake shielding and WIO

Wake shielding

API RP 2RD (1998) substantiated the shielding effect by experimental tests. If the diameters of both upstream and downstream cylinders are the same and the center-to-center spacing is $5D$, the drag force on the downstream cylinder is reduced by 45% due to the wake shielding. For the cases of $30D$ and $100D$, the reductions become 27% and 16% respectively. It is obvious that the wake shielding effect is non-negligible even in the far wake.

WIO and wake instability

WIO is defined as the low-frequency, large amplitude and chaotic motions of a cylinder in a wake. WIO induced by wake forces often follows an oval orbit. The generation of this orbit is explained herein. Firstly, the cylinder in the downstream is influenced by the large drag force and moves inside the wake; then, it moves towards the center line by lift force; after that, the reduced drag force will pull it to the upstream cylinder as well as the edge of wake, the cycle repeats in this way depicted as the dash line orbit in Figure 2.14. The oscillation orbit inputs energy into the cylinder and may result in the most harmful and unstable phase, i.e. *wake instability* in the cylinders (Blevins, 2001).

Figure 2.14 Illustration of WIO (Price and Piperni, 1987)

Figure 2.15 A critical curve for wake instability (Blevins et al., 2006)

Figure 2.16 Clearance between two jumpers along their arc length from the FPSO to the buoyancy tank (Wu et al., 2008)

Blevins et al. (2006) showed the region of potential wake instability in Figure 2.15. The wake instability is prone to occur within the critical curve, which is near the upstream cylinder, otherwise the wake induced forces will not be sufficient to encourage the oscillation.

As mentioned, Huse and Blevins made significant contributions to the wake instability analysis, hence, both models are popularly used in the industry. A comparison between these models was done by Wu et al., (2008). According to Figure 2.16, Blevins' model is always more conservative than Huse's model for riser clashing predictions. In the far wake, Huse's and Blevins's model gave similar results. However, in the near wake, where velocity gradients and lift forces were large, Blevins' wake model predicted observable collisions while Huse's model did not. This consequence is owing to the fact that Blevins' model is able to account for lift forces while Huse's model fails to do so.

Besides Huse and Blevins, Huang et al. (2002) accomplished plenty of experiments for wake interference under different experimental setups, e.g. varied ways of cylinder mountings; two cylinders with different diameters; differing positions of a cylinder in the wake. The main finding was that the unstable area of the downstream cylinder was enlarged with the increase of reduced velocity. An extension work was conducted on analyzing two top tensioned risers (Huang, 2010).

2.4 General approaches for riser clashing analysis

Riser interference describes the severity of relative motions between two risers. The extreme scenarios of riser interference are riser collisions. Therefore, riser interference analysis is crucial for riser clashing analysis. This section emphasizes the general practice for analyzing riser clashing.

It is conventional to implement a global riser system analysis to reveal potential issues needed to be improved. One of these issues is the interference between risers. The global analysis includes static and dynamic analyses of a riser system exposed to various environmental loadings. The resulting outputs are commonly riser clearances, collision event records and the like. As discussed in Section 1.1.3, time domain dynamic analysis is applied in the approaches in Chapters 3, 4 and 5. Besides dynamic analysis, other options are static analysis, modal analysis, wake instability analysis and local impact analysis.

2.4.1 Static and modal analysis

A static analysis inspects the static state of a riser system involving all the available static loadings like floater offsets, current loadings and wake shielding. Current and offsets may be applied in any possible direction to ensure that the designed riser system is sustainable in all possible ocean environments. The magnitudes of these forces are usually compared to reveal the most critical case.

Basically, the static analysis is treated as a preliminary study on riser interference. If the closest distance between two risers is less than $2D$, a further dynamic analysis is highly recommended (Kavanagh et al., 2000). The outcomes of static analysis are mainly the initial positions and stresses which are respectively the mean positions and mean stresses for the subsequent dynamic analysis. A comprehensive summary of static analyses on both top tensioned riser and compliant risers (flexible risers and SCRs) is documented in DNV (2001).

Another method that is applied in the preliminary design is an eigen value analysis approach, which is used to determine the eigen frequencies and eigen modes of riser systems (Det Norske Veritas, 2009). This analysis examines dynamic properties of a riser system, which aims at avoiding destructive resonance effects.

2.4.2 Wake interference analysis

If wake interference is demonstrated to be a possible scenario in dynamic analysis e.g. a significant portion of risers undergoes severe interference, a wake interference analysis is recommended to carry out. The most unfavorable situation of wake interference is the so called wake instability in which large riser motions will occur and the clashing is almost unavoidable. Moreover, VIV is also considered in the analysis.

Commercial software

In practice, to account for the hydrodynamic interferences, some established numerical tools are applied. Orcaflex (Orcina, 2012) is one of a few reputable software specialized on riser analysis. Two numerical models are available for time domain simulations of wake interferences between two risers. They are Huse's model and Blevins' model (see Section 2.3.3.2)

Time domain VIV is simulated using either Milan wake oscillator or Iwan and Blevins's wake oscillator (Orcina, 2012). Both wake oscillator models account for the lift force but say nothing on the VIV effects on the drag forces. Moreover, the application of these models is limited to steady state conditions. Besides these oscillator models, SHEAR7 (Vandiver and Li, 2005) and VIVANA (Marintek, 2012) are popularly employed in the industry and academy.

CFD

Among numerical tools, CFD models have excellent performance in wake instability analysis. Sagantun et al. (2002) presented a simulator based on CFD to assess the risk of collision between two risers. The impact velocity and collision frequency were estimated in the simulation. This program was so versatile that the VIV and WIO effects can be isolated and reflected in the output. It gave insights to the contributions

of the WIO on the riser responses. The simulation results generally agreed well with some experimental data. The limitation of this work is the computational cost and the inflexibility on model set-up.

Figure 2.17 Comparison between the wake measurements behind a stationary cylinder and the classical wake theory for $Re=35000$ (Saint-Marcoux et al., 2009)

Other methods

It was shown in some experiments that VIV in the upstream riser makes the wake further expand and change the forces imposed on the downstream riser. However, the classical wake field model did not consider this effect (Saint-Marcoux et al., 2009). The discrepancies were found to be large in the near wake and remained non-negligible in the far wake (see Figure 2.17). Blevins and Wu (2007) put forward the superposition of two wakes, which had a much better performance than the classical one. The new model was found to be effective in describing a more realistic wake field when the upstream riser was under VIV. This new model requires further verifications by more experiments and is not available in the numerical tools currently.

Analytical methods

The analytical approaches are available in the literature but limited to modal analysis. Wu et al. (2002) proposed a new approach, which is based on the Huse wake model and eigen value approach, to analyze the stability of a downstream cylinder in the wake. The wake was first divided into stable, unstable and critical regions. The stability conditions were derived from the characteristic equation and further converted to four inequalities according to Routh's criteria. The inequalities could be violated in two conditions, i.e. Hopf bifurcation and stationary bifurcation. Wu et al. (2002) mentioned that, for marine risers, stationary bifurcation was more likely to occur. Huang (2010) continued the above-mentioned work on two top tensioned risers. The idea of stable and unstable regions was adopted. He suggested that if a riser fell into the stable regime, it would tend to move to an equilibrium point within certain range of current velocity. However, provided that it went into the instable regime, it would easily fall into the stable region under small disturbances and converged to an equilibrium position. When the current velocity increased to a critical level, the stable and unstable regions would disappear and the wake instability motions would take place in the downstream riser. This critical current velocity was relevant to initial spacing, top tension factor and ratio of the lateral fluid force. The relation between critical velocity and these three parameters were shown in Huang (2010). However, his work was limited to uniform currents and modal analysis.

2.4.3 Local impact analysis

Local impact analysis is necessary for collision-allowed riser designs. In general practice, global riser models are built and some simulations are processed. The resulting clashing events and the corresponding data, e.g. locations and impact velocities, are captured in a crude global riser model; after that, a detailed local riser finite element (FE) model is established to attain realistic impacts between risers.

The most important results are peak stresses and stress ranges. To obtain these, an FE model must be applied and a dynamic analysis is highly recommended. To establish a local riser FE model, some advices are given by DNV RP F203 (2009). They are summarized as followed: (1) the length of risers should be lengthy enough to eliminate boundary effects on the collision locations; (2) The element should either be shell or solid volume, the mesh density should be sufficiently refined to capture the local dynamics and stress; (3) contact modeling should be less sensitive to the peak stress. A Lagrange multiplier technique (Papadopoulos and Solberg, 1998) and the penalty stiffness method (Hunek, 1993) may be applicable; (4) Participating mass (Sagatun et al., 1999) should be determined before defining the riser mass; (5) Time steps should be small enough to capture the impact process; (6) A regular meshing is of help for the convergence. The yielding and fatigue damage are evaluated after FE analysis.

Figure 2.18 Relative impact velocity (Kalleklev et al., 2003)

Usually, the analyses are performed for a range of different relative impact velocities U_n and impact angles θ , which are visualized in Figure 2.18 (Kalleklev et al., 2003). The effects on the risers depend on many factors like riser wall thickness,

participating mass (Li and Morrison, 2000), coating conditions and contact angle. For example, the riser with coating and strake can significantly reduce the effect of impact.

2.5 Literature review relevant to two design principles

This section summarizes the researches on collision-allowed and no-collision-allowed riser designs as well as riser failure detection and consequences.

2.5.1 No-collision-allowed design

Generic guidelines

Regarding this category, some guidelines are found in the literature. Seyed and Patel (1991) carried out comprehensive analysis on flexible riser design. A full review was done on the selection of design parameters and analysis methods. Ishii et al. (1991) conducted a well designed experiment to assess the proper gap for flexible risers. They pointed out that a narrow riser gap was beneficial for installation, accommodation as well as the reduction in bundle torsion. However, the gap should also be large enough to avoid possible riser interference and collision events. A 3D clearance between two adjacent risers without considering safety factor was recommended. To diminish the unfavorable interference due to close proximity, spacers were introduced. Moreover, the high bending stress should be accommodated in the locations of spacers. The design and analysis for TLP risers was carried out by Lim and Hatton (1991). In the riser clearance analysis, they mentioned that the clearance criteria of risers during and after installation were different. For installed risers, the dynamic analysis was especially crucial for those risers having different dynamic characteristics. The up-to-date design approaches for both compliant and tensioned riser systems are available in many latest codes, e.g. DNV, API and ABS (2008). Among these codes, DNV RP F203 (2009) has a relatively systematic recommended practice on preventing risers from mutual collisions.

Methods to reduce riser dynamics

Some methods are introduced to alleviate riser dynamics so that the riser clashing becomes almost impossible. In terms of motion restriction of risers, modal control and boundary control are two major methods. The modal control is mature owing to the long developing history in mechanical engineering and is a conventional way to analyze riser dynamics (Meirovitch, 1997; Gawronski, 1998). However, boundary control seems to be more practical and efficient on riser dynamics control. Do and Pan (2008) proposed a hydraulic system at the top ends of risers to reduce the transverse motions of flexible risers (Figure 2.19). This boundary controller was improved to apply on a three-dimensional riser model (Do and Pan, 2009), which is shown in Figure 2.20. The controller is designed to resist disturbances induced by wave, wind and currents in the ocean environment. Further experimental data are required to verify the performance of this boundary controller.

Figure 2.19 Riser coordinates and the hydraulic system (Do and Pan, 2008)

Figure 2.20 Riser coordinates and the improved hydraulic system (Do and Pan, 2009)

For top tensioned risers, Rustad et al., (2008) introduced dynamical controllers to achieve equal effective riser lengths. The method is practical but requires further demonstration by experimental data.

2.5.2 Collision-allowed design

Leira et al. (2002a; 2002c) proposed an approach to assess a collision allowed riser system as is presented in Figure 2.21. The approach was based on an integrated program HYBER (Holmas et al., 2002). In their work, the hydrodynamic forces were pre-established by CFD program and the results were stored to apply in the subsequent response simulation. Then a contact search scheme was used to detect the collision events in the simulation result and the desired pipe stresses were collected to compute the cumulative fatigue damage and fulfill other tasks.

Figure 2.21 The model for a case study (Leira et al., 2002b)

Figure 2.22 An overview of the set-up for the dual riser clashing tests (Herfjord and Holmas, 2005)

Leira's work was verified by a riser interference experiment (Herfjord and Holmas, 2005), whose setup is exhibited in Figure 2.22. Comparing with the data in experiment, the numerical results slightly over-predicted the collision velocity and frequency. The clinging tendency of risers with strakes was found to be predictable in the simulation.

Niedzwecki and Duggal (1993) did meaningful work on riser clashing analysis. Their experimental set-up is similar to that in Figure 2.22 but with clearance equal to $1.5D$.

The time history of relative displacement between two flexible cylinders was extracted. Since the signal was non-Gaussian, it was transformed to a Gaussian process applying Hermite transformation technique. A crossing rate analysis was conducted to predict the riser clashing. It resulted in an accurate to conservative prediction by comparing with experimental observations. A local impact analysis was absent in their work. A detailed guideline for FE model set-up for local impact analysis is summarized in Chapter 5.

2.5.3 Failure detection and consequences

Figure 2.23 presents the possible failure mechanisms of flexible risers. As can be seen, the external sheath damage, aged internal sheath and PVDF sheath pull out account for 60% of all the failure cases. Such statistics are derived from a total of 106 flexible risers reported by United Kingdom Continnence Society (UKCS) operators (UKOOA, 2001). Generally, there are five failure modes for risers: fatigue, corrosion, erosion, pipe blockage or flow restriction and accidental damage (Bai and Bai, 2005). Figure 2.24 captured external sheath damages in the process of installation.

Figure 2.23 Flexible riser system failure mechanisms (UKOOA, 2001)

Figure 2.24 An example of external sheath damage on a riser surface (Bai and Bai, 2005)

Riser collisions can lead to most of these failure modes. For example, the collisions may damage the coating or strakes of a riser and make it vulnerable to the acid ocean environment. Elman and Alvim (2008) proposed that most damage cases of flexible risers were due to external sheath damage, corrosion and fatigue induced damage to the tensile armours. Frequent collisions between risers will initiate small cracks on the surface of the riser and end up with fatigue failure. In addition, a severe riser clashing may even results in immediate yielding. A costly replacement or repairing is needed after clashing event. Even worse, the clashing may end up with leakage and flooded problems. All these consequences will lead to an increase of overall cost and the loss of production. Moreover, the leakage and flooded problems will pollute the environment and endanger the safety of the personnel and platform (Bai and Bai, 2005). In short, the riser system should be shielded in the design stage from failure due to riser collisions.

The possible cracks in a flexible riser need to be detected by detection devices. In Figure 2.25, Elman and Alvim (2008) set up a failure detection system for identifying

the occurrence of armour wire failure. The system was prepared for long term offshore field trials.

Figure 2.25 Sketch of flexible riser monitoring system (Elman and Alvim, 2008)

2.6 Chapter summary

This chapter reported actual designs aiming to avoid riser clashing in current offshore industry. Besides, the characteristics and numerical models of hydrodynamic interferences that occur in a riser system were comprehensively studied. The practical approaches for riser clashing analysis were summarized. Emphasis was put on studies relevant to both no-collision-allowed and collision-allowed designs. It is found that, for no-collision-allowed design, there is no available approach to assess the probability of clashing in a given riser system. For collision-allowed design, the existing approaches are not practical to implement. In view of the gaps in the literature, Chapters 3, 4 and 5 will propose practical approaches for assessing the probability of failure due to riser clashing. Tailor-made approaches are required to satisfy e.g. different design principles.

The main role of this chapter is to provide literature reviews for subsequent chapters. Section 2.2 introduces some possible random variables which are critical for riser clashing problem and are later referred to in Section 4.2.2. The wave, current, VIV

and WIO models introduced in Section 2.3 are applied in all the case studies in Chapters 3-5. Similarly, the wake interference analysis described in Section 2.4.2 is employed in all the case studies in the subsequent chapters. For Section 2.4.3, the conventional local analysis approach is directly applied in the approach proposed in Chapter 4. The important concepts, no-collision-allowed design and collision-allowed design, are introduced in Sections 2.5.1-2.5.2. They are the basics for the proposal of approaches in Chapters 3-5.

CHAPTER 3 PREDICTING THE PROBABILITY OF COLLISION – UNCERTAINTIES FROM RANDOM WAVES

3.1 Introduction

It is highlighted in Chapter 1 that there are two different design principles in riser designs. Chapter 2 reviewed the studies relevant to both principles. No collision allowed design strictly prohibits any collision during the service life. In practice, a probability of clashing lower than 10^{-4} per annum should be guaranteed (Chakrabarti, 2005). If the service life is 20 years and the probability of clashing is 10^{-4} per annum, the probability of failure P_f will be about 2×10^{-3} . This small probability implies that the productivity of simulating collisions is extremely low. In practice, only a limited duration of nonlinear dynamic analyses are adopted. As a result, the great difficulty is to predict the rare event with limited data at hand. Moreover, there is no available approach for predicting the probability of collision.

This chapter strives to develop an approach for calculating the probability of riser collision. It is assumed that the numerical models for VIV and wake field (see Section 2.3.3) are properly calibrated. The approach is based on simulation data available for a conventional riser design. Case studies, involving two flexible risers, are conducted to verify various hypotheses made in the approach. The prediction is compared with the Monte Carlo simulation.

3.2 Extreme value prediction

Riser clashing is essentially an extreme value problem. Hence, existing techniques for extreme value analysis are explored in this section.

3.2.1 Overview

The current problem involves the prediction of extreme values from existing data, which are commonly the riser responses obtained from time-domain dynamic

analyses. As mentioned, the main obstacle comes from the fact that the extremes are too expensive to be obtained from the available simulation data due to excessive computational time. Consequently, some techniques specialized in solving extreme-value problems are used. They commonly rely on the assumption that the extreme values or exceedances above a certain threshold follow a certain distribution provided that the sample size is adequate.

It is easy to name a few commonly used extreme value distributions e.g. the Weibull distribution and the generalized extreme value (GEV) distribution. The idea of peaks-over-threshold (POT) is promising because it makes full use of existing data. Moreover, the POT method can be improved by techniques such as de Hann estimators and the bootstrapping approach (Naess and Clausen, 2001; Naess and Gaidai, 2009). However, this method performed unfavorably in some reliability analyses of offshore systems (Naess et al., 2009)

Recently, Naess's research team developed a technique for crossing rate extrapolation. This technique (hereinafter referred to as Naess method) is found to be robust and suitable for reliability analysis of offshore systems. The validity of Naess method for the riser clashing problem will be investigated, and the method is summarized in Section 3.2.4.

3.2.2 Random sea state

As mentioned, riser response is crucial for this problem. It can be deemed as a stochastic process. In reality, the uncertainties contributing to this process come from many sources e.g. riser property and drag coefficient. In this chapter, it is assumed that the randomness in the system is contributed merely by random waves. To consider multiple sources of uncertainties, another new approach is proposed in Chapter 4.

In a short-term design approach, it is presumed that the most critical "load case" happens during a designed storm. This storm follows a given return period say 100

years. The sea state in the storm is assumed to be stationary for a short duration of typically 3 hours. The features of the random waves in that sea state are characterized by a wave spectrum $S_{\eta\eta}(\omega)$, where ω is the frequency.

A fundamental assumption which is generally valid in deep water is that the wave elevation $\eta(t)$ is Gaussian (Chakrabarti, 2005). A commonly used technique for producing the process $\eta(t)$ is random phase scheme (Elgar et al., 1985), where $\eta(t)$ is expressed as a summation of N wave components, that is

$$\eta(t) = \sum_{n=1}^N A_n \cos(\omega_n t + \phi_n), \quad (3.1)$$

where $A_n = \sqrt{2S_{\eta\eta}(\omega_n)\Delta\omega}$, $\Delta\omega$ is the frequency interval and ϕ_n is a random phase that is uniformly distributed in the interval $[0, 2\pi)$. As $N \rightarrow \infty$, $\eta(t)$ approaches a Gaussian distribution but the convergence rate is slow with respect to N . Moreover, for a given N , there always exists an upper bound in the distribution which makes the extremes underestimated.

Another approach is termed as random coefficient scheme (Shinozuka, 1972), in which $\eta(t)$ is represented as

$$\eta(t) = \sum_{n=1}^N [a_n \cos(\omega_n t) + b_n \sin(\omega_n t)], \quad (3.2)$$

where a_n and b_n are Gaussian distributed random variables with zero mean and variance of $S_{\eta\eta}(\omega_n)\Delta\omega$. In this form, $\eta(t)$ is always Gaussian, as a result this scheme is applied in the case studies.

3.2.3 Monte Carlo simulation and the Poisson approximation

Monte Carlo simulation is a robust scheme for attaining failure probability for any defined stochastic process. It is basically a sampling technique conducted in a defined random parameter domain. In this case, each sample is a time domain simulation of 3-hour duration. A failure is defined by one or more collisions within this duration. The probability of failure can be estimated as

$$P_f \approx \frac{\hat{n}}{\hat{N}}, \quad (3.3)$$

in which $\hat{N}$ is the total number of realizations, and $\hat{n}$ is the number of realizations that exceed the failure threshold. The accuracy of Eq. (3.3) should be guaranteed by a relatively large $\hat{n}$ if P_f is small.

The unfavorable aspect of the Monte Carlo simulation method is the extremely low efficiency especially when the probability is small. Suppose that the probability of riser clashing is $P_f = 10^{-4}$, which is slightly smaller than the allowable $P_f = 2 \times 10^{-3}$, at least 10^5 samples are needed for an estimate of P_f with a coefficient of variation (CoV) of 0.3. Moreover, the computational cost is very high for producing each sample due to the complicated nonlinear dynamic analysis. Hence, the employment of Eq. (3.3) must be regarded as unfeasible herein.

To solve extreme value problems, the Poisson approximation method is commonly used. To apply this method, consider a stochastic process $X(t)$, which represents the degree of severity of riser interference. The actual definition of $X(t)$ is rather complicated and will be postponed to the later section. In addition, an upcrossing rate v_z^+ is introduced to denote the likelihood of surpassing a given threshold z in $X(t)$. v_z^+ is defined as the mean amount of upcrossings per second with respect to a threshold z and a duration T .

The computation of v_z^+ can be accomplished by Monte Carlo simulations by assuming ergodicity. First, the total number of upcrossings, denoted as n_c , are counted in the simulation with a duration of T_0 which may comprise a number of independent time domain simulations (i.e. $T_0 = mT$). The counting process is illustrated in Figure 3.1. Then, this number is divided by the duration T_0 to produce v_z^+ , i.e. n_c/mT . Here,

v_z^+ associates with duration T . The m at the denominator should be sufficiently large to ensure an accurate estimate of the actual mean.

Figure 3.1 Illustration of up-crossing counting

Following Poisson approximation, the probability of failure can be expressed as (Lin, 1967)

$$P_f = 1 - \exp(-v_z^+ T). \quad (3.4)$$

In Eq. (3.4), it is assumed that successive upcrossings are independent. The P_f produced by this method is well known to be conservative and it becomes exact when $z \rightarrow \infty$, or correspondingly, $P_f \rightarrow 0$ (Lin, 1967). Furthermore, this approximation is usually good provided that $X(t)$ is not a typical narrowband process. Since riser clashing is a relatively rare incident, and riser deflections are unlikely to be narrowband (owing to hydrodynamic damping), the Poisson approximation can be regarded to be acceptable for the present problem.

It is worth mentioning that, to accumulate sufficient crossings at failure threshold, the computational cost is as intensive as that in Eq. (3.3). However, the concept of

crossing rate is worth pursuing because observations at less critical z are much cheaper to obtain. It seems practical to extrapolate v_z^+ to the failure threshold based on observations at relatively low levels of z . Two techniques for the extrapolation will be considered. The details are provided as follows.

3.2.4 Extrapolations of crossing rates

The first extrapolation technique to be introduced is commonly used. It is based on the GEV distribution that follows from classical extreme value theory. Suppose that $X_{\max}$ denotes the extreme value of $X(t)$ within a duration T . When the threshold z is large, the cumulative density function (cdf) for $X_{\max}$ asymptotes to the GEV distribution which has the form

$$F_{X_{\max}}(z) = \exp \left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\}. \quad (3.5)$$

Here, ξ , σ and μ are parameters of the distribution. The desired threshold z is always out of the range of available data. Since $P_f = 1 - F_{X_{\max}}(z)$, Eq. (3.4) can be transformed to the form of probability of survival $F_{X_{\max}}(z)$. The combination of this and Eq. (3.5) produces

$$\ln(v_z^+ T) = \frac{-1}{\xi} \ln \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]. \quad (3.6)$$

By employing the existing v_z^+ , $\ln(v_z^+ T)$ is plotted against z . The least squares method is adopted to fit $\ln(v_z^+ T)$ to the form written in Eq. (3.6). Then the parameters are attained from the fitting. The P_f can be achieved by putting the parameters and the desired threshold z back to Eq. (3.5). This GEV method is notorious for its high sensitivity to the tail region of the available data, thus the choice of the range to be utilized is critical. However, a robust rule for this decision seems to be absent, hence the extrapolation quality is difficult to guarantee.

In view of this, Naess method (Naess and Gaidai, 2009) is introduced here. This method makes the extreme value prediction more robust. Naess and his co-researchers found that in many applications of offshore engineering, the crossing rates at high threshold can be approximated by an empirical expression

$$v_z^+ = q(z) \exp[-a(z-b)^c], \quad (3.7)$$

where a , b and c are constant parameters, and $q(z)$ can be treated as a constant for large z . The value q is recommended to be sampled from the tail region and taken as the mean of $v_z^+ / f_X(z)$, where $f_X(z)$ is the probability density of $X(t)$. To achieve the parameters, Eq. (3.7) is rearranged to

$$\ln \left[-\ln \left(\frac{v_z^+}{q} \right) \right] = \ln(a) + c[\ln(z-b)]. \quad (3.8)$$

Let $y_1 = \ln[-\ln(v_z^+/q)]$ and $x_1 = \ln(z-b)$, the parameter q should be selected such that y_1 and x_1 follow an approximated linear relationship. b should be as small as possible while maintaining the linearity. Then the remained parameters are achieved by a linear fitting adopting maximum likelihood method on Eq. (3.8).

If the total simulation duration is long enough say 30 h, a 95% confidence interval for the estimated v_z^+ can be attained from $v_z^+ \pm 1.96\hat{s}(z)/\sqrt{m}$, where $\hat{s}(z)$ is the sample standard deviation. The fluctuation of the crossing rate at the tail is generally large owing to the fewer available sampling points. Thus, it is recommended (Naess and Gaidai, 2009) to exclude data points whose confidence band violates a criterion, i.e. $2 \times 1.96\hat{s}(z)/v_z^+ \sqrt{m} \leq 1$.

This method yields accurate results if the extrapolation of v_z^+ is limited to no more than three orders of magnitude. This extrapolation technique gives the best performance for systems that resemble a Gumbel extreme value distribution (Naess and Gaidai, 2009). This is the first time to apply Naess method to the riser clashing problem, thus it is of interest to investigate its performance.

3.3 Description of proposed approach

3.3.1 General considerations

Marine risers are usually modeled as line elements using numerical approaches such as the finite element method, lumped mass approach or finite difference formulation. The time history of the nodal positions is extracted directly from the simulation results. In this work, the commercial software used is based on the lumped mass approach; however, the procedure introduced in this section is applicable to any other numerical model provided that all the nodal positions are available at all time steps.

Before proceeding further, it is necessary to reveal the difficulty of developing an appropriate method. First of all, even though Section 3.2 has introduced the relevant techniques for extreme value analysis based on existing data, a proper definition of $X(t)$ is still absent. In fact the definition of $X(t)$ is not straightforward, and it will be revealed that the common way of using the instantaneous minimum gap (i.e. clearance) between a pair of risers is inappropriate. Second, even if the clearance between two risers is promising for $X(t)$, a problem of high computation cost remains, as the computation of this quantity is extremely expensive. Thus, the main bulk of the proposed method is to resolve these difficulties in a satisfactory manner.

3.3.2 Calculation of the riser clearance

Although the clearance is inappropriate to define $X(t)$, it is crucial for the final definition. For this reason, a method for calculating $X(t)$ will be outlined below. Basically, a riser is modeled as a number of nodes connected by straight segments. The possibility of shape functions between nodes, as can be obtained through the finite element method, will not be considered as it increases the problem complexity unnecessarily.

Consider two elements j and k , whose nodal coordinates are denoted as j_1, j_2 and k_1, k_2 respectively. According to fundamental geometry, the minimum distance between the vectors (i.e. infinitely long lines) is

$$d_{j,k} = \left| \frac{(\mathbf{j}_1 - \mathbf{k}_1) \cdot (\mathbf{j}_2 - \mathbf{j}_1) \times (\mathbf{k}_2 - \mathbf{k}_1)}{(\mathbf{j}_2 - \mathbf{j}_1) \times (\mathbf{k}_2 - \mathbf{k}_1)} \right|. \quad (3.9a)$$

From now on, the minimum distance $d_{j,k}$ is termed as distance. In reality, Eq. (3.9a) may not be valid for all scenarios, in that the elements are of finite length. Under this condition, the relative position between two elements is of importance. In order to discuss comprehensively on all possible scenarios, five additional equations are provided below:

$$d_{j,k} = \min_{t_1, t_2=1,2} \left(\left| \mathbf{j}_{t_1} - \mathbf{k}_{t_2} \right| \right), \quad (3.9b)$$

$$d_{j,k} = \left| \left(\mathbf{k}_1 - \mathbf{j}_1 \right) - \frac{(\mathbf{k}_1 - \mathbf{j}_1) \cdot (\mathbf{k}_2 - \mathbf{k}_1)(\mathbf{k}_2 - \mathbf{k}_1)}{\left| (\mathbf{k}_2 - \mathbf{k}_1) \right|^2} \right|, \quad (3.9c)$$

$$d_{j,k} = \left| \left(\mathbf{j}_1 - \mathbf{k}_1 \right) - \frac{(\mathbf{j}_1 - \mathbf{k}_1) \cdot (\mathbf{j}_2 - \mathbf{j}_1)(\mathbf{j}_2 - \mathbf{j}_1)}{\left| (\mathbf{j}_2 - \mathbf{j}_1) \right|^2} \right|, \quad (3.9d)$$

$$d_{j,k} = \left| \left(\mathbf{j}_2 - \mathbf{k}_2 \right) - \frac{(\mathbf{j}_2 - \mathbf{k}_2) \cdot (\mathbf{j}_2 - \mathbf{j}_1)(\mathbf{j}_2 - \mathbf{j}_1)}{\left| (\mathbf{j}_2 - \mathbf{j}_1) \right|^2} \right|, \quad (3.9e)$$

$$d_{j,k} = \left| \left(\mathbf{k}_2 - \mathbf{j}_2 \right) - \frac{(\mathbf{k}_2 - \mathbf{j}_2) \cdot (\mathbf{k}_2 - \mathbf{k}_1)(\mathbf{k}_2 - \mathbf{k}_1)}{\left| (\mathbf{k}_2 - \mathbf{k}_1) \right|^2} \right|. \quad (3.9f)$$

Figure 3.2 Different relative positions of two elements; shortest distance governed by (a) element-to-element, (b) node-to-node and (c) node-to-element.

The selection of a proper equation requires some explanations. In the subsequent discussion, let $\mathbf{p}_j$ and $\mathbf{p}_k$ represent the position vectors of the closest points of the two vectors, but they may not be located within the elements themselves. By reference to Figure 3.2(a), consider a scenario in which both $\mathbf{p}_j$ and $\mathbf{p}_k$ are located within their corresponding elements. Under this condition, it is obvious that Eq. (3.9a) applies. Figure 3.2(b) describes the second scenario such that the distance is governed by two nodes on the respective elements. This situation is related to Eq. (3.9b), and since there are totally four possible node-to-node combinations, the “min” sign in Eq. (3.9b) produces the critical case. In the final scenarios, the distance is defined as one element’s node to its projection on another element, as illustrated in Figure 3.2(c). There are altogether four such combinations. They correspond to Eq. (3.9c)-(3.9f). The condition when the elements are parallel requires special consideration. Since there exist infinite pairs of $\mathbf{p}_j$ and $\mathbf{p}_k$, Eq. (3.9a) becomes undefined, and equations pertaining to the second or third scenario (Figure 3.2(b)-(c)) should be applied.

Having calculated the distance for all element combinations, the clearance between two risers can be approximated as

$$C(t) = \min_{\substack{j=1:n \\ k=1:n}} [d_{j,k}(t)] - D, \quad (3.10)$$

where n is the number of elements in one riser considered in the collision analysis, and D is the sum of the radii of the two risers. In the previous analysis, there are no simplifications made in computing the individual element distances; thus, this procedure will be referred to as the “exact algorithm”.

The exact algorithm is computationally demanding primarily because of the need to analyze the relative positions of the elements in order to identify the scenario and select the appropriate equation. A computationally efficient procedure termed the “simple algorithm” is introduced here, in which the distance is estimated from the mid-nodes of the elements, i.e.

$$d_{j,k}(t) = \left| \frac{\mathbf{j}_1(t) + \mathbf{j}_2(t)}{2} - \frac{\mathbf{k}_1(t) + \mathbf{k}_2(t)}{2} \right|. \quad (3.11)$$

The riser clearance corresponding to the simple algorithm is obtained by combining Eq. (3.10) and Eq. (3.11).

In addition to the simple algorithm, there is another approximation algorithm for calculating the riser distance. In this algorithm, $d_{j,k}$ is defined as the shortest distance between the mid-node of element j and the entire element k . Compared to the simple algorithm, the efficiency of this algorithm is compromised due to the extra procedure of identifying the relative position between two elements. Moreover, the value of $d_{j,k}$ differs if the distance is associated with the mid-node of element k and the element j . This discrepancy may become problematic in application. Owing to these two reasons, this algorithm will not be applied.

3.3.3 Definition of the process $X(t)$

As mentioned earlier, the clearance $C(t)$ is not a suitable choice for $X(t)$, and this statement will be demonstrated here. The theory introduced in Section 3.2 presumes that the failure threshold, z , is positive. This is the classical formulation of the single-barrier problem with positive threshold. On the other hand, it is clear that $C(t)$ is always positive, and failure occurs when $C(t) = 0$. It is convenient but not imperative to transform the present downcrossing problem to an upcrossing one by a simple transformation

$$X(t) = -\frac{C(t)}{I}, \quad (3.12)$$

where I is the initial clearance, corresponding to the static riser configuration. In this way, $X(t) \leq 0$, with $X(t) = 0$ denoting failure.

Now, suppose that there is a typical system comprising two flexible risers hung in the lazy wave configuration. Owing to a separation angle, the risers are closer at the top

end. The details of this system and the loading condition are secondary to the current discussion, and they will be presented afterwards. Figure 3.3 shows a typical time history of $X(t)$ that is generated by Eq. (3.12). It can be observed that $X(t)$ is usually -1 or close to -1 with a few peaks. Another more in-depth analysis shows that most of the data points are contributed by the pair of elements directly attached to the platform.

Figure 3.3 Time history of process $X(t)$ generated by post-normalization approach

In Figure 3.3, take a typical peak that is labeled as Scenario 1 as an example. This peak is produced when the two elements are approaching each other. When these two elements move apart, the clearance will be governed by the gap at the connection point, hence $X(t) = -1$, i.e. Scenario 2. Owing to boundary conditions, it is impossible for these elements to meet, despite that their initial clearance is small. However, they are chosen to define $X(t)$. This example reveals the illogical scenario in which the behavior of an element pair that will never collide is extrapolated to predict the probability of clashing.

To illustrate how a suitable definition of $X(t)$ can be attained, consider another example comprising two pairs of elements. The first pair is initially separated by a relatively small initial clearance, say 1 m. In a dynamic analysis, the clearance can reach a minimum of 0.95 m (5% reduction). The second pair has a higher initial clearance of 4 m; however the response is more severe and the minimum clearance is 2 m (50% reduction). Following the criteria of Eq. (3.12), the first pair will be selected for $X(t)$ as the minimum clearance is smaller (0.95 m versus 2 m). However, the second pair is more likely to collide based on engineering judgment.

The inspiring aspect of the above example is that the percentage of relative movement (in this case 50% versus 5% for the second and first pair respectively) is more representative for the likelihood of collision. Following this principle, $C_{j,k}(t)$ is first normalized with respect to its initial value $I_{j,k}$, resulting in a new parameter

$$N_{j,k}(t) = \frac{C_{j,k}(t)}{I_{j,k}}. \quad (3.13)$$

Here, $N_{j,k}$ is termed as normalized clearance. It is expected to characterize the severity with regards to clashing for the element pair j and k , at any given time, with a smaller $N_{j,k}(t)$ indicating higher severity. At the static configuration, $N_{j,k} = 1$ for all j and k . Next, $X(t)$ is taken as the largest value of $-N_{j,k}(t)$, i.e.

$$X(t) = \max[-N_{j,k}(t); j, k \in \{1, 2, \dots, n\}]. \quad (3.14)$$

In Eq. (3.14), the negative sign is to guarantee that this is an upcrossing rate problem. As mentioned, $X(t) \leq 0$ with equality representing failure. When $X(t) < -1$, all element pairs are moving apart. The above procedure is referred to as a pre-normalized approach, because normalization is carried out prior to extracting the minimum value. Similarly, Eq. (3.12) is a post-normalized approach.

3.3.4 Concepts of critical pair

In this subsection, a concept named as critical pairs is introduced. This concept is derived from the notion that only a small number of element combinations, known as

the critical pairs, are responsible for characterizing the essential feature of $X(t)$, i.e. the peaks that govern the extremes. In contrast, some element combinations such as those near the top ends of risers (see Figure 3.4) are very unlikely to contribute to $X(t)$. If the critical pairs can be identified beforehand, a substitute of $X(t)$ is written as

$$\hat{X}(t) = \max_{(j,k) \in P} [-N_{j,k}(t)] \quad (3.15)$$

where P is the set of critical pairs. This new process $\hat{X}(t)$ can be generated quickly as P is supposed to be a small set.

The next issue needed to be addressed is the method to sieve out critical pairs in an efficient and accurate manner. The method is based on two ideas. First, even a simulation is short; it may include enough information for selecting the critical pairs. Second, the highly efficient simple algorithm (Eq. (3.11)), may be helpful to order the criticality among element combinations although it is insufficiently rigorous for computation of actual clearances.

Figure 3.4 Model of the riser system

Now, assume that the available information is a short time series of $N_{j,k}(t)$, $j = 1 : n$, $k = 1 : n$. A simple criterion needs to be established for comparing the criticality of the various time series, and two possible measures are

$$M_{j,k} = \max_{t=0:\tau} [-N_{j,k}(t)], \quad (3.16)$$

$$M_{j,k} = \text{var}_{t=0:\tau} [-N_{j,k}(t)], \quad (3.17)$$

where τ is the maximum duration for consideration. Eq. (3.16) extracts the maxima over the time series, while Eq. (3.17) takes the variance. Both are reasonable measures of the criticality, with a higher $M_{j,k}$ corresponding to greater criticality. It is found that Eq. (3.16) is superior. One reason is that Eq. (3.16) tends to provide information at the extremes, which is desirable. However, Eq. (3.17) puts no emphasis on the tails. Moreover, Eq. (3.16) can be computed more quickly. Thus, the element pairs associated with larger $M_{j,k}$ are selected as the critical pairs according to Eq. (3.16).

Unfortunately, the direct implementation of the above scheme fails, because the simple algorithm is unable to provide the accurate order of criticality among all the element pairs. This may be ascribed to the over-simplifications made in the algorithm. But this simple algorithm is found to be suitable to shortlist a larger group of *candidate pairs* that are potentially critical.

In view of this, an efficient two-stage sieving scheme is proposed. In the first stage, the simple algorithm is applied on a very short duration T_1 to select a number, p_1 of candidate pairs from the original set of element combinations. In the second stage, the exact algorithm is applied to ensure that the criticality from the pool of candidatures is in the correct order. In this stage, the simulation duration and number of critical pairs is T_2 and p_2 respectively. It is recommended that $T_2 > T_1$, for the selection of critical pairs is more crucial.

Table 3.1 Recommended parameters in proposed approach

Parameters	Recommended Values
n	Configuration dependent (see Section 3.3.4)
T_1, T_2	600 s, 1200 s
p_1, p_2	$n, 0.25n$

Table 3.1 presents the recommended values of T_1 and T_2 , which the author considers appropriate for most situations. The variable n (see for example Eq. (3.10)) represents the minimum element number that is included in the analysis. The prescription of a minimum element number is to alleviate the computational burden, and it is motivated by the hypothesis that collision is more likely to occur at the upper portion of a riser. When hydrodynamic interference is neglected, the recommended n should satisfy $d_{n,n} > 11D$ in the static state. This criterion is based on several case studies performed by the authors. Moreover, the validity of the recommended n can be reassured by comparison with the locations of the critical pairs. With hydrodynamic interference, the recommended criterion is $d_{n,n} > 25D$, since the interference between two cylinders is negligible provided that their separation in the direction of the current exceeds $25D$ (Wu et al., 2003). Finally, the recommend p_1 and p_2 are governed by n (see Table 3.1).

3.3.5 Summary of the proposed approach

The components of the proposed approach have been introduced in the previous sections. In this connection, it is appropriate to summarize the whole approach for predicting riser clashing. Moreover, some of the important parameters are provided here. The procedure of proposed approach is illustrated in Figure 3.5.

The procedure can be divided into three main portions, namely the identification of the candidate pairs, the identification of critical pairs, and finally the extreme value analysis. In the first part, the nodal positions are collected from the time domain simulation for a very short duration, for example $T_1 = 600$ s. The simple algorithm is applied to calculate the individual element clearances, which are subsequently

pre-normalized. After that, a sieving technique is used to extract a pool of candidate pairs up to say $p_1 = 100$.

Figure 3.5 Flowchart of the proposed approach

The second part involves the finalization of the critical pairs. The procedures are quite similar to the first. The main differences are that the exact algorithm is employed, the duration is longer, say $T_2 = 1200$ s, and the number of critical pairs should be quite small, such as $p_2 = 40$.

In the last part, the nodal positions are collected again from a dynamic analysis, but this time for the full duration (i.e. 10×3 h storm), and only for nodes associated with the critical pairs. After pre-normalization, $\hat{X}(t)$ is then obtained and used in place of

$X(t)$ for extreme value analysis. The upcrossings for various thresholds z are counted in order to determine the crossing rate ν_z^+ . Next, the crossing rates are extrapolated based on Naess method to the failure threshold $z = 0$. Finally, the probability of failure P_f is attained based on Poisson approximation.

3.4 Case studies

3.4.1 Description of system

Time domain simulations are carried out using the commercial software Orcaflex (Orcina Ltd, 2012). By reference to Figure 3.4, the model includes two flexible risers installed in the lazy wave configuration in a water depth of 605 m. The two ends of a riser are pinned to an FPSO unit and the seabed respectively. The gap at the topside is $5.5D$, and the separation angle is 7 deg. All the other details of the riser system are summarized in Tables 3.2 and 3.3.

Table 3.2 Description of the various riser sections

Riser Section Types	Section Length (m)	Element Number	Included in Analysis
Flexible (upper section)	605	95	Yes
Flexible with buoyancy	80	8	No
Flexible (lower section)	341	17	No

The JONSWAP spectrum is chosen to characterize the sea state of the 3-h storm. A 100-year return period is selected which followed with a significant waveheight equal to 15.7 m and an average period equal to 13.5 s. In order to induce more severe response, both the incident wave and current are in the direction of the y -axis, which is perpendicular to the symmetrical plane of the risers. The wave spectrum is divided into 250 components of equal frequency interval from 0.2 rad/s to 1.2 rad/s. The surface velocity of the current is 1.2 m/s, and the current decreases linearly to 0.36 m/s at the seabed.

Table 3.3 Essential properties of flexible risers

	Riser Section Types	
	Flexible	Flexible with Buoyancy
Outside diameter (mm)	478.4	873.0
Wall thickness (mm)	133.1	330.4
Dry mass (kg/m)	200.0	431.1
Area (m ²)	0.1444	0.5632
Flexural rigidity EI (kNm ²)	153	153
Drag coefficient C_d	1.200	0.979
Hang-off angle (deg)	10	N.A.
Content density (kg/m ³)	500	500

Morison equation is adopted to generate wave loadings on the nodes of a riser. The vessel motions in six degrees-of-freedom are specified through Response Amplitude Operators (RAOs). The RAOs define the first-order motions, which are critical for riser clashing problem (Chakrabarti, 2005). In this work, the slow drift response due to second-order wave loads is not simulated. In fact, it can be done in a more efficient way, i.e. to perform the clashing analysis at several representative vessel positions so as to incorporate the effects of slow drift and mean offset.

The sea state is assumed to be stationary for 3 h. Ten three-hour sea states are analyzed, so that $T = 3$ h, $m = 10$ and $T_0 = 30$ h, from which the crossing rates are extrapolated to estimate P_f . This total duration $T_0=30$ h is typical but not fixed for the final design. Orcaflex is so versatile that it includes VIV and WIO based on theoretical models mentioned in Section 3.1. However, DNV (Det Norske Veritas, 2009) does not recommend including VIV in the riser clearance assessment, perhaps because the interaction between VIV and WIO is not fully understood yet. As an alternative, it is recommended (Det Norske Veritas, 2009) that the VIV effect can be accounted for by increasing the collision criteria from zero to $2D$. This criterion can be easily incorporated in the proposed approach by replacing D with $3D$ in Eq. (3.10).

In the following contents, two case studies with and without hydrodynamic interference will be performed to verify the proposed approach.

3.4.2 Case study 1 – with hydrodynamic interference

One novel aspect of the proposed approach is the two-stage sieving strategy, which pertains to the concept of critical pairs. Hence, the feasibility of critical pairs should be first examined. To this end, the process $\hat{X}(t)$, which includes 40 critical pairs ($p_2 = 40$), is generated for a short duration of 250 s. Here, $X(t)$ plays a crucial role as a reference solution, and it is produced for the complete 95×95 element combinations using the exact algorithm. The time taken to evaluate $X(t)$ is 250 times that of $\hat{X}(t)$; this huge difference highlights the computational advantage granted by the sieving method compared to a direct approach.

Figure 3.6 Time history of comparison between $X(t)$ and $\hat{X}(t)$

The comparison is shown in Figure 3.6. It is apparent that $\hat{X}(t)$ correctly reproduces all the peaks of $X(t)$, in spite of inconsistencies at lower levels. Although the upcrossings for all positive thresholds are counted, only the crossings at a suitably high level, e.g. $z > -0.85$, are employed for extrapolation. Thus, the major concern is the ability to capture the peaks, which represent the essential information at the extreme levels. In this regard, the sieving strategy for procuring $\hat{X}(t)$ is demonstrated to be effective. There is a huge discrepancy in the lower region, and although it would not affect the extremes, it is interesting to discuss the underlying reason. A value below -1 implies that all associated element pairs have moved apart relative to their initial positions. This condition is difficult to achieve for $X(t)$ because it involves all the element combinations. In contrast, $\hat{X}(t)$ only involves a few participants such that it has high chance to attain $\hat{X}(t) < -1$.

Figure 3.7 Effect of number of critical pairs on the upcrossing rate

The effect of p_2 (number of critical pairs) on the crossing rates is investigated. Figure 3.7 plots the crossing rates of $\hat{X}(t)$ for $p_2 = 10, 40$, and 95×95 (all possible combinations). The number of candidate pairs, p_1 , is kept constant at 100. It can be

deduced that the choice of $p_2 = 40$ is adequate since there is no observable improvement when p_2 increases to 95×95 . When $p_2 = 10$, the crossing rates are still consistent at high levels, even if there are disparities at lower levels. This implies that the tail is governed by a handful of critical pairs, and a good prediction can be envisaged even when p_2 is quite small. The influence of p_1 on the computational burden is negligible, as the candidate pairs are calculated by the simple algorithm. The choice of n also affects the performance. The maximum element number n is taken as 95, representing the upper section of a riser (Table 3.2). Table 3.4 lists the locations of the critical pairs, which are concentrated in the upper portions of the risers. The largest element number (j and k) is 11. Since $n = 95$, which is much larger than 11, the chosen n is deemed to be sufficient.

Table 3.4 Critical pair locations (j, k) in case study 1

Order	Critical Pairs							
1-8	(3,4)	(4,3)	(4,5)	(4,4)	(5,4)	(5,5)	(3,3)	(3,2)
9-16	(5,6)	(2,2)	(2,3)	(6,5)	(6,6)	(6,7)	(7,6)	(7,7)
17-24	(2,1)	(1,1)	(7,8)	(1,2)	(8,7)	(8,8)	(8,9)	(3,1)
25-32	(9,9)	(9,8)	(4,2)	(5,3)	(9,10)	(6,4)	(10,10)	(7,5)
33-40	(10,9)	(5,7)	(8,6)	(6,8)	(7,9)	(9,7)	(10,11)	(8,10)

Figure 3.8 Upcrossing rate extrapolation by GEV and Naess method

Extrapolation of the crossing rates plays an indispensable role for a reliable prediction of P_f . The crossing rate for a 30-h duration is computed, and extrapolated to failure using two techniques, namely the GEV and Naess methods. The use of tail-end data is of concern. There are pros and cons of applying these data. On one hand, these data are most representative of the extremes, but on the other hand, they are subjected to higher sampling errors. For Naess method, there exists a practical guide to select the usable range of data points; the upper bound of the data range should satisfy a confidence band criterion, whereas the lower bound is chosen to ensure that the data are as linear and extensive as possible (see Section 3.2.4). A similarly simple rule is unavailable for the GEV extrapolation. However, to allow a consistent comparison, the GEV extrapolation will be based on the same data as Naess method. Two data sets are considered; the first is based on the abovementioned criterion, while the latter includes the first set and employs extra data points at the tails. Figure 3.8 compares the extrapolation for the two techniques. The GEV extrapolation is found to be sensitive to the different data ranges, whereas Naess method has a more stable performance. This implies that when adopting the GEV extrapolation, a very different P_f can ensue by including or excluding certain data points.

Figure 3.9 Upcrossing rate (30 h) extrapolation by Naess method ($q = 0.08$, $b = 0$, $a = 12.63$, $c = 1.79$; the range between two dash lines is the extrapolation range)

Figure 3.9 displays the crossing rate extracted from a 30-h simulation. The crossing rate is extrapolated from $v_z^+ = 1.0 \times 10^{-4}$ at $z = -0.3$, down to $v_z^+ = 3.0 \times 10^{-7}$ at the desired threshold $z = 0$, corresponding to $P_f = 3.2 \times 10^{-3}$. Since the extrapolation is about three orders of magnitude for v_z^+ , it is more convincing to verify this result through additional simulations. To this end, 750 three-hour sea states are simulated, amounting to $T_0 = 2250$ h, and the extrapolation is shown in Figure 3.10. The inclusion of data reduces the confidence band in the tail region, hence the crossing rate can be extrapolated from $z = -0.225$. The extrapolation is within two orders of magnitude for v_z^+ . There is good agreement between the results for 30 h and 2250 h. However, from Figure 3.10, it is found that the tail deviates downwards from the fitted line. Since the amount of data here is adequate to eliminate sampling errors, the tail is likely to reflect the actual feature of the riser response. Hence, the current prediction errs on the conservative side.

Figure 3.10 Upcrossing rate (2250 h) extrapolation by Naess method ($q = 0.10$, $b = 0$, $a = 12.78$, $c = 1.73$; the range between two dash lines is the extrapolation range)

An important point needs to be clarified. The “reference solution” $P_f = 3.0 \times 10^{-3}$ (Figure 3.10) is not based on direct Monte Carlo simulations. The correctness of this result resides on the assumption that the linearity of the data, when written in the form of Eq. (3.8), remains valid up to the failure threshold. Although the variable trend of the crossing rate curve may be of concern, in order to strictly verify this, about 3.6×10^3 three-hour simulations are required. In view of this, a slight compromise in verification is tolerated. If the actual P_f is exceedingly small, such as 10^{-7} , then the prediction may be highly inaccurate due to the lengthy extrapolation. Notwithstanding, for design purposes, the possibly high error is not a major concern under this circumstance, because the result has already supplied the desired information that the riser design is over conservative, and the following task is to optimize the existing design e.g. reduce the riser gap.

Figure 3.11 Comparison of upcrossing rates (30 h) based on pre- and post-normalization

Another novelty of the approach is the definition of $X(t)$ based on pre-normalization (Eqs. (3.13) and (3.14)). This is compared with the more obvious choice of using the

clearance, which is post-normalized for the purpose of comparison (Eq. (3.12)). In Section 3.3.3, it has been argued that post-normalization will produce unreasonable results. In this connection, Figure 3.11 compares the crossing rates of $X(t)$ based on pre- and post-normalization for a 30-h simulation. It can be seen that the trends differ significantly. Furthermore, upon extrapolation, the post-normalization produces a P_f of 3.2×10^{-4} , which is an order of magnitude more conservative than the reference solution (see Figure 3.10). The conservativeness may be due to the fact that the post-normalization fails to capture the more critical instant clearances produced by the element pairs outside the range of top-ends.

The computational expense of the proposed algorithm is of practical concern. For this case study, it requires about 90 h to complete the 30-h simulation using a single 2.50 GHz processor. On the same machine, the entire procedure to estimate P_f , as outlined in Figure 3.5, needs only 24 min. Dynamic analyses are performed in any case for other design requirements such as ultimate riser tensions. At a relatively small cost (0.4% computational overhead), an engineer would also be able to estimate the probability of riser clashing.

3.4.3 Case study 2 – without hydrodynamic interference

Another case study with 3000 h simulations, without considering the hydrodynamic interference and current, is performed. The rigorous Monte Carlo simulations are not available due to the unaffordable computational time. To permit a fair comparison, the target P_f is designed to be similar to that of the first case study. The layout of this model is unchanged (see Figure 3.4), except that the gap at the top is prescribed as $0.6D$.

As shown in Figure 3.12, the crossing rate is extrapolated from $\nu_z^+ = 2.7 \times 10^{-5}$ at $z = -0.32$, down to $\nu_z^+ = 1 \times 10^{-7}$ at the failure threshold $z = 0$. The corresponding P_f is

1.4×10^{-3} , which is close to the result for case study 1. Since $0.6D$ is very small compared with the criteria applied in actual design ($>5D$), this confirms the well known fact that flow induced vibration is a major driver of riser clashing. Omitting this phenomenon will produce an unrealistic design.

Figure 3.12 Upcrossing rate (3000 h) extrapolation by Naess method for case study 2 ($q = 0.36$, $b = -0.05$, $a = 14.05$, $c = 1.09$; the range between two dash lines is the extrapolation range)

Comparing Figures 3.10 and 3.12, it is obvious that the latter exhibits a higher degree of linearity. The nonlinear characteristic in Figure 3.10, especially at the tail region, can be attributed to geometric and hydrodynamic nonlinearities. Geometric nonlinearity results from an increase in the lateral stiffness due to large deflection, while the hydrodynamic nonlinearity is caused by an increase of the wake interference effect when two risers are in close proximity. The two nonlinearities cause the crossing rate behavior to be less well behaved.

The overall favorable performance of the proposed approach can be attributed to various reasons. First, the underlying hypotheses such as the critical pair concept have

been demonstrated to be valid. Second, because ocean waves are Gaussian, the Gumbel extreme value distribution is the domain of attraction for most offshore engineering problems in spite of nonlinearities; consequently, Naess extrapolation method is suitable. Third, the robustness of the extrapolation technique also plays an important part.

3.5 Chapter summary

This chapter outlines a practical approach for estimating P_f from limited simulation data, based on existing theory for extreme response prediction. The approach has two main novel aspects. First, a process $X(t)$ is defined by pre-normalizing the clearance between each element pair. This definition is crucial for the proposed approach, for an inappropriate definition of $X(t)$ produces incorrect results. Second, as the computation cost for $X(t)$ is formidable, a two-stage sieving procedure is proposed to identify critical element pairs. It tremendously expedites the production of crossing rates without compromising accuracy.

There are two case studies. The first one considering hydrodynamic interferences was used to validate the proposed approach. The second one without hydrodynamic interferences was conducted to compare with the first one so as to study the impact of the interference. The P_f is calculated by extrapolating the crossing rates based on 30-h simulations. By increasing the simulation duration considerably, it is found that the extrapolation is consistent, generally erring on the conservative side. Further, it is shown that the presence of hydrodynamic interference significantly increases the likelihood of collision. Finally, although the approach is tested on flexible risers, it should in principle be also applicable to other riser types, such as top tensioned risers.

The approach in this chapter only considers randomness from irregular waves. However, in actual case, uncertainty comes from many other sources such as riser properties and current. To investigate the impacts of uncertain parameters and

compute the failure probability considering multiple sources of uncertainties, a new approach will be proposed in Chapter 4.

CHAPTER 4 PREDICTING THE PROBABILITY OF COLLISION – MULTIPLE UNCERTAINTIES

4.1 Introduction

Chapter 3 proposed an approach for predicting the probability of failure of a no collision allowed riser system. The method is valid only when the randomness mainly comes from the irregular wave. However, this assumption may result in an unrealistic prediction provided that other uncertainties are significant such as current, hydrodynamic interaction, riser spacing, top-tension and discrepancy of riser properties (Det Norske Veritas, 2009). In fact, the abovementioned uncertain parameters have striking impacts on the probability of failure, which will be demonstrated in this chapter.

In view of this, the goal of this work is to establish an analogous approach to Chapter 3 while considering multiple sources of uncertainties. As will be shown later, the main difficulties arise from the high computational intensity and the significant error in crossing rate extrapolations. The proposed approach should be able to address these issues in addition to those mentioned in Chapter 3. Monte Carlo simulations are provided as a benchmark to the performance of the approach. For illustration purpose, only a few key random parameters will be shortlisted in the analysis. The proposed approach is, in principle, prepared for all types of riser systems, but only top tension riser system is used in the case studies.

4.2 Probability of collision under multiple random parameters

4.2.1 General considerations

This work requires the extrapolations of available crossing rates to the threshold of interest. This task is fulfilled by the approach proposed in Chapter 3. The procedures

are summarized in Section 3.3.5. For compatibility, the definition and symbol of both variables and parameters used in Chapter 3 still apply to this chapter.

4.2.2 Choice and description of the random variables

The probability of riser clashing depends on many parameters. Some of these parameters are controllable while others are not. The uncertainties obviously come from the uncontrollable parameters. It is worth mentioning that significant uncertainties (i.e. large variations) of parameters do not necessarily lead to large impacts on the likelihood of riser clashing. In a strict sense, a complete sensitivity studies should be done to sieve out a few of critical parameters. In this work, four random variables are shortlisted based on Det Norske Veritas (2009), which is summarized in Table 4.1. The variables are denoted by $\mathbf{Y} = [Y_1 \ Y_2 \ Y_3 \ Y_4]^T$, and are described below.

Table 4.1 Summary of the random variables

Variable	Description	Distribution type	Median	CoV, lower bound	CoV, upper bound
Y_1	Current	Lognormal	1.0 m/s	0.05	0.10
Y_2	Drag coefficient	Lognormal	1.1	0.15	0.20
Y_3	RAO factor	Lognormal	1.0	0.05	0.10
Y_4	Riser mass	Normal	190 kg/m	$0.05\sqrt{2}$	$0.1\sqrt{2}$

(1) The current magnitude significantly affects the deflection of a riser and is accordingly relevant to the chance of mutual contacts. Besides, it influences the riser responses through VIV. In reality, ocean current varies in magnitude on a monthly basis from 0.1 to 1.5 m/s (Anawat et al., 2010). In the designs, the current magnitude may be prescribed to be linearly decreased or uniformly distributed along the water depth. Some cases may involve a linearly decreased section(s) and a uniform

distributed section(s). In this work, a uniform current is prescribed and Y_1 represents the current velocity magnitude.

(2) The empirical parameter Y_2 (drag coefficient) generally suffers from poor accuracy, as it is obtained from a scaled model tests under unrealistic circumstances. Due to this limitation, significant errors in Y_2 are expected. Besides, the errors may be ascribed to scale effects, different flow characteristics and so on.

(3) As mentioned in Section 1.1.3, the RAO (response amplitude operator) governs the vessel motions in a decoupled analysis. The vessel motions in turn have large impacts on riser interferences (see Section 2.2). The uncertainties in the RAO are caused by fluctuations in the vessel mass and damping, as well as errors induced in the computation of the hydrodynamic loads. Here, Y_3 is a parameter to represent the randomness in the RAO with the form $Y_3 R(\omega)$, in which $R(\omega)$ is the RAO at the median value of Y_3 corresponding to frequency ω .

(4) The riser mass plays an important role in the characteristics of riser responses, in that it determines the natural periods of the riser. The riser mass may change due to e.g. variation in the conveyed fluids. For riser clashing problem, the difference of mass between two risers is a more representative parameter (Det Norske Veritas, 2009). Let the mass of the risers M_1 and M_2 be independent and normally distributed, the difference $\Delta = M_1 - M_2$ also follows a normal distribution, and has a variance $\sigma_\Delta^2 = \sigma_{M_1}^2 + \sigma_{M_2}^2$. In this work, Δ is prescribed by keeping the mass of the upstream riser M_1 constant, while varying the mass of the downstream one to account for the uncertainty, i.e. $Y_4 = \overline{M}_1 + \Delta$ (the bar represents the mean).

By reference to Det Norske Veritas (2010), Y_1 , Y_2 and Y_3 follow lognormal distributions and Y_4 a normal distribution. Owing to the fact that these random variables pertain to differing mechanisms, it is reasonable to assume them to be

independent. Table 4.1 lists the median values and the ranges of coefficient of variation (CoV) of the four random parameters. The CoV is defined as $\sigma_{Y_j} / \bar{Y}_j$, where σ_{Y_j} and $\bar{Y}_j$ are the mean and standard deviation respectively, and the ranges are recommended in (Det Norske Veritas, 2010). In this study, two typical sets of CoVs are investigated. They correspond to the upper bounds and the lower bounds of the CoVs respectively. It is also convenient to term these two cases as the *large uncertainty* case and the *small uncertainty* case. It is necessary to clarify that, for the riser mass, the recommended CoV range of 0.05 ~ 0.10 refers to a single riser, i.e. M_1 or M_2 . The CoV for Y_4 is increased by a factor of $\sqrt{2}$, as Y_4 models the discrepancy between M_1 and M_2 .

In this case, the medians are adopted for defining the baseline case parameters rather than the means, which are more commonly used. The reason is that the parameters will be later transformed to standard normal space, and the medians are kept unchanged in the monotonic transformation. Moreover, the median is generally more robust as it is not sensitive to sampling errors by much. Once the medians and the CoVs are known, the parameters of a lognormal distribution can be obtained explicitly.

4.2.3 Monte Carlo simulation

The probability of failure has a form

$$P_f = \int p_Y(\mathbf{y}) f_Y(\mathbf{y}) d\mathbf{y}, \quad (4.1)$$

where $p_Y(\mathbf{y})$ represents the probability of failure on the condition that $\mathbf{Y} = \mathbf{y}$; $f_Y(\mathbf{y})$ denotes the joint probability density function of $\mathbf{Y}$. Hence, Eq. (4.1) is a multiple integral of dimension m , where m is the total number of random variables considered, i.e. four in this case.

In order to benchmark the proposed reliability method introduced in Section 4.4.4, Monte Carlo simulation is applied in this work, which is a robust and accurate method. Under these circumstances, the P_f based on Monte Carlo simulation has the form

$$P_f = E[p_Y] \approx \frac{1}{N} \sum_{n=1}^N p_Y(\mathbf{y}^{(n)}), n = 1, 2, 3, \dots, N; \mathbf{y}^{(n)} \text{ sampled from } f_Y(\cdot) \quad (4.2)$$

where $\mathbf{y}^{(n)}$ is the n th random variate sampled from $f_Y(\cdot)$. To make use of Eq. (4.2), a 30-h simulation based on $\mathbf{y}^{(n)}$ is performed and the crossing rate is extrapolated to the failure threshold to achieve ν_0^+ . Finally, the $p_Y(\mathbf{y}^{(n)})$ is evaluated by $1 - \exp(-\nu_0^+ T)$. Here, it can be assumed that the extrapolation based on 30-h simulation is sufficiently accurate, as this prerequisite has been verified in He and Low (2012). Moreover, the Naess extrapolation method has been demonstrated to be reliable for numerous offshore engineering applications (Naess and Gaidai, 2007; Naess et al., 2007).

In the Monte Carlo simulation, the accuracy of the mean prediction is usually evaluated by the means of the CoV of the estimator, which is presented as

$$\delta \approx \frac{s}{\sqrt{N E[p_Y]}}, \quad (4.3)$$

where s is the sample standard deviation of $\{p_Y(\mathbf{y}^{(1)}), p_Y(\mathbf{y}^{(2)}), \dots, p_Y(\mathbf{y}^{(N)})\}$.

As mentioned earlier, two sets of CoVs will be considered. The large uncertainty case is of more interest since it exposes the riser system to the “worse case”. Thus, it will be examined first. To save the efforts on performing another intensive Monte Carlo simulation for the small uncertainty case, the importance sampling technique is employed. This technique is commonly used for improving the efficiency of Monte Carlo simulation, but now it is applied simply for permitting the variates to be sampled from another density $h_Y(\cdot)$ instead of the target density $f_Y(\cdot)$. The importance sampling estimate for P_f is written as

$$P_f \approx \frac{1}{N} \sum_{n=1}^N \frac{p_Y(\mathbf{y}^{(n)}) f_Y(\mathbf{y}^{(n)})}{h_Y(\mathbf{y}^{(n)})}, n = 1, 2, 3, \dots, N; \mathbf{y}^{(n)} \text{ sampled from } h_Y(\cdot) \quad (4.4)$$

In this small uncertainty case, $h_Y(\cdot)$ refers to the density from the large uncertainty case, while $f_Y(\cdot)$ is the actual density with the smaller CoV. The sampling variability can again be estimated using Eq. (4.3), but here s denotes the standard deviation of $\{S^{(1)}, S^{(2)}, \dots, S^{(N)}\}$, where $S^{(n)} = p_Y(\mathbf{y}^{(n)})f_Y(\mathbf{y}^{(n)})/h_Y(\mathbf{y}^{(n)})$.

4.2.4 Proposed reliability method

In the literature, there are numerous methods to approximate the probability integral in Eq. (4.1). Direct numerical integration and Monte Carlo simulation are two robust methods to achieve the P_f but suffer from formidable computational cost to ensure acceptable accuracy. Approximate approaches, such as asymptotic expansion (Papadimitriou et al., 1997), univariate dimension reduction (Rahman and Xu, 2004) and point estimates (Hong, 1996), seem more practical in application. Unfortunately, the improved efficiency is hardly satisfactory for the designers. For instance, the asymptotic approximation involves intensive processes of searching for the maxima points. In this case, at least 30-h simulation is required to accurately evaluate a single $p_Y(\mathbf{y})$, not to mention the large number of total evaluation points. In view of this, these methods are deemed as impractical in the present context.

The proposed reliability method is established on the perturbation method (Papadimitriou et al., 1997), which substantially expedite the asymptotic expansion approach. The success of this method stems from the Taylor series expansion of the response function $p_Y(\cdot)$ about μ_Y (μ denotes the mean). By applying the second-order perturbation method on Eq. (4.1), one obtains

$$P_f = E[p_Y] = p_Y(\mu_Y) + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \frac{\partial^2 p_Y(\mu_Y)}{\partial Y_i \partial Y_j} \text{cov}(Y_i, Y_j). \quad (4.5)$$

In this work, the perturbation method is implemented slightly different from a common practice. Firstly, the non-Gaussian correlated variables $\mathbf{Y}$ is transformed to independent standard normal variables $\mathbf{U}$. Commonly, this task may be fulfilled by the Rosenblatt transformation (Melchers, 1999), written as $\mathbf{U} = T(\mathbf{Y})$. But in this work,

since the random variables are already independent, the transformation is simply by letting

$$\mathbf{U} = \frac{\mathbf{V} - \mu_{\mathbf{V}}}{\sigma_{\mathbf{V}}}, \quad \mathbf{V} = [\ln Y_1 \quad \ln Y_2 \quad \ln Y_3 \quad Y_4]^T. \quad (4.6)$$

Then, Eq. (4.1) becomes $P_f = \int p_{\mathbf{U}}(\mathbf{u}) f_{\mathbf{U}}(\mathbf{u}) d\mathbf{u}$. By applying the second-order perturbation method, Eq. (4.1) is transformed to

$$P_f = E[p_{\mathbf{U}}] = p_{\mathbf{U}}(\mathbf{0}) + \frac{1}{2} \sum_{j=1}^m \frac{\partial^2 p_{\mathbf{U}}(\mathbf{0})}{\partial U_j^2}. \quad (4.7)$$

The expression of Eq. (4.7) is significantly simplified owing to the fact that $\mu_{\mathbf{U}} = \mathbf{0}$, $\text{cov}(U_i, U_i) = 1$, and $\text{cov}(U_i, U_j) = 0$. Besides, the transformation has some other benefits in this context. First, since the cross terms are zero, the amount of computation is reduced. Second, Eq. (4.5) is accurate to second-order, however, Eq. (4.7) is effectively accurate up to third-order. This statement necessitates simple justification. It is useful to point out some facts in Eq. (4.7), i.e. $E[U_j] = 0$, $E[U_i^3] = 0$ and U_i and U_j are independent. Following these, in Eq. (4.7), the third-order terms, all of which involve $E[U_i^3]$ and $E[U_i^2 U_j] = E[U_i^2] E[U_j]$, disappear in the $\mathbf{U}$ -space. In contrast, the third-order terms, $E[(Y_i - \bar{Y}_i)^3]$ and $E[(Y_i - \bar{Y}_i)^2 (Y_j - \bar{Y}_j)]$, are non-zero in $\mathbf{Y}$ -space. Being exactly what is mentioned, the transformation allows the third moments of $f_{\mathbf{Y}}(\cdot)$ to be accounted for at no additional cost. Finally, the expansion point $\mathbf{U} = \mu_{\mathbf{U}} = \mathbf{0}$ here pertains to the median of $\mathbf{Y}$. The advantage of median over mean comes from the fact that the former statistical quantity is generally invariant under a monotonic transformation. This property is desired in the actual design. Moreover, the median is a better measure of central tendency than the mean. This in principle renders it a more ideal expansion point than the mean.

To apply Eq. (4.7), $\partial^2 p_{\mathbf{U}}(\mathbf{0})/\partial U_j^2$ firstly needs to be evaluated numerically. Here, the central difference formula is adopted to obtain this second-derivative, i.e.

$$\frac{\partial^2 p_{\mathbf{U}}(\mathbf{0})}{\partial U_j^2} = \frac{p_{U_j}(\Delta u_j) + p_{U_j}(-\Delta u_j) - 2p_{U_j}(0)}{(\Delta u_j)^2}, \quad (4.8)$$

where Δu_j is a step size between two adjacent sampling points; $p_{U_j}(u)$ is the probability of failure under the condition that one dimension (i.e. U_j) of $\mathbf{U}$ is given a value of u while the remained entries are zero. Obviously, there are totally $2m$ evaluation points for $p_{\mathbf{U}}(\mathbf{u})$ (excluding $p_{\mathbf{U}}(\mathbf{0})$ which is the baseline case). For each point, a 30-h simulation is required. Subsequently, crossing rates are extrapolated from the simulation data. The computation effort is unaffordable in design, not to mention that the extrapolation is sensitive to several factors such as the range of data chosen. Although $p_{\mathbf{U}}(\mathbf{u})$ itself may not be very sensitive to the extrapolation process, which was demonstrated in He and Low (2012), the emphasis here should be put on $\partial^2 p_{\mathbf{U}}(\mathbf{0})/\partial U_j^2$ but not $p_{\mathbf{U}}(\mathbf{u})$ by referring to Eq. (4.7).

Owing to the limitation of the above simple scheme, it is proposed to perform the extrapolation on $\partial^2 p_{\mathbf{U}}(\mathbf{0})/\partial U_j^2$ directly. First of all, $p_{\mathbf{U},z}(\mathbf{0})$ is defined as the probability that $X(t)$ exceeds the threshold z , which is presented as

$$p_{\mathbf{U},z}(\mathbf{0}) = 1 - \exp[-v_{\mathbf{U},z}^+(\mathbf{0})T]. \quad (4.9)$$

Here, $v_{\mathbf{U},z}^+(\mathbf{u})$ is the crossing rate when $\mathbf{U}=\mathbf{u}$, e.g. $\mathbf{U}=\mathbf{u}=\mathbf{0}$ in Eq. (4.9). Moreover, $p_{\mathbf{U},0}(\mathbf{0}) = p_{\mathbf{U}}(\mathbf{0})$. The first attempt is obviously to calculate $\partial^2 p_{\mathbf{U},z}(\mathbf{0})/\partial U_j^2$ for various threshold z with Eq. (4.8). Then, $\partial^2 p_{\mathbf{U},z}(\mathbf{0})/\partial U_j^2$ is extrapolated to $z=0$. Unfortunately, to the best knowledge of the author, there is no available extrapolation technique, including Naess method, to fulfill this task. However, by differentiating Eq. (4.9) with respect to U_j twice and applying the chain rule gives

$$\frac{\partial^2 p_{\mathbf{U},z}(\mathbf{0})}{\partial U_j^2} = V_{j,z}'' T \exp[-v_{\mathbf{U},z}^+(\mathbf{0})T] - (V_{j,z}')^2 T^2 \exp[-v_{\mathbf{U},z}^+(\mathbf{0})T], \quad (4.10)$$

where

$$V'_{j,z} = \frac{\partial v_{U,z}^+(\mathbf{0})}{\partial U_j}, \quad V''_{j,z} = \frac{\partial^2 v_{U,z}^+(\mathbf{0})}{\partial U_j^2}, \quad (4.11)$$

are the partial derivatives of $v_{U,z}^+$ with respect to U_j at $\mathbf{U} = \mathbf{0}$. Here, in Eq. (4.11),

$V'_{j,z}$ and $V''_{j,z}$ can be approximated using central difference formulae, i.e.

$$V'_{j,z} = \frac{v_{U_j,z}^+(\Delta u_j) - v_{U_j,z}^+(-\Delta u_j)}{2\Delta u_j}, \quad (4.12)$$

$$V''_{j,z} = \frac{v_{U_j,z}^+(\Delta u_j) + v_{U_j,z}^+(-\Delta u_j) - 2v_{U_j,z}^+(0)}{(\Delta u_j)^2}. \quad (4.13)$$

By reference to Eqs. (4.12) and (4.13), $V'_{j,z}$ and $V''_{j,z}$ are found to be akin to the crossing rates. Hence, the task of extrapolations may be accomplished by Naess method, which is a robust technique for crossing rate extrapolation.

Here, similar to the extrapolation formula for $v_{U,z}^+(\mathbf{0})$ (Eq. (3.7)), it is proposed to use the Naess form for the derivatives, so that

$$V'_{j,z} = q' \exp[-a'(z - b')^{c'}], \quad (4.14)$$

$$V''_{j,z} = q'' \exp[-a''(z - b'')^{c''}], \quad (4.15)$$

where the parameters a' , b' , etc are fitted in the same way as that in Section 3.2.4.

The validity of this extrapolation strategy will be examined in the case study, but there are several supportive points for this strategy. The Naess form of Eq. (3.7) is rather flexible, allowing linear ($c = 1$), concave ($c > 1$) or convex ($c < 1$) behaviors on a semi-log graph. Further, $V'_{j,z}$ and $V''_{j,z}$ are linear functions of $v_{U_j,z}^+(\cdot)$. Taking $V''_{j,z}$ as an example, its components $v_{U_j,z}^+(\Delta u_j)$, $v_{U_j,z}^+(-\Delta u_j)$ and $v_{U_j,z}^+(0)$ are each approximated by the Naess form. Thus, it seems reasonable that $V''_{j,z}$ can be

described likewise, especially if one considers that either $v_{U_j,z}^+(\Delta u_j)$ or $v_{U_j,z}^+(-\Delta u_j)$ is likely to be dominating.

In the computation of Eqs. (4.12) and (4.13), it is recommended that the step size Δu_j should not be small. The reason is that the crossing rates are sampled from the number of upcrossings n , where n is an integer. Hence, the discrepancy of n between sample points must be sufficient. For the large uncertainty case, Δu_j is prescribed to be one standard deviation, i.e. $\Delta u_j = 1$. To avoid replicate simulations, the evaluation points are reused for the small uncertainty case, and Δu_j is reduced accordingly.

Further, it is emphasized that the same irregular wave train must be used for the evaluation points to minimize sampling errors. Finally, the simulation duration for $v_{U_j,z}^+(\Delta u_j)$ and $v_{U_j,z}^+(-\Delta u_j)$ need not be as long as 30 h. The reason is that for derivatives, it suffers less from the irregular wave because of the cancellation effect; hence the trends of $V'_{j,z}$ and $V''_{j,z}$ tend to be more regular. In this work, a duration of 6 h is selected. The total number of evaluation is $2m$. For $m = 4$, the additional simulation duration to account for the various uncertainties is $2 \times 4 \times 6 = 48$ h, which is only 1.6 times the original 30-h simulation for $\mathbf{U} = \mathbf{0}$. This makes the proposed reliability approach an affordable one that can be implemented in practice.

4.3 Case studies

4.3.1 Details of numerical model

As top-tensioned risers in deep water are susceptible to mutual collisions, this type of riser system is selected as the model for this case study. The properties pertaining to the riser dynamic simulations are provided in Table 4.2. The global riser model is described in Figure 4.1. A tensioned leg platform is located in a water depth of 500 m. In this context, uncoupled scheme is applied to accomplish the dynamic analysis (see Section 1.1.3). Thus, the riser motions are independent of the vessel motions and the

number of risers does not affect the overall dynamic characteristics. For simplicity, two risers are modeled with a center-to-center space of $11D$ (clearance equals to $10D$). The total riser length is 520 m, with 20 m in the air. A commercial package, Orcaflex (Orcina, 2012), is used to perform the time domain simulations. The riser is modeled as a line structure with $M = 104$ equal-length elements. The software allows the modeling of VIV and WIO, and both effects are enabled during the simulations.

Table 4.2 Essential riser properties

Parameter	Value
Young's modulus	$2.05 \times 10^{11} \text{ N/m}^2$
Outer diameter	0.268 m
Wall thickness	14 mm
Added mass coefficient	1.0
Water density	1025 kg/m^3

Figure 4.1 Global riser model

The JONSWAP spectrum is chosen to characterize the sea state. To stimulate extreme responses between two risers, significant wave height H_S and average period T_z are

assigned to be 15.7 m and 13.5 s respectively. The wave spectrum $S_{\eta\eta}(\omega)$ is divided into 250 components of equal frequency interval from $\omega = 0.2 \sim 1.2$ rad/s. Then, the wave train is generated by assigning random phases to the resulting 250 sinusoidal wave components. Both current and incoming wave are in the x direction (see Figure 4.1). Here, the platform motions are defined using RAOs which are random (see Section 4.2.2). Although RAOs only account for the first-order motion, this component is most responsible for riser collision while the second-order responses are not critical in this context (Det Norske Veritas, 2009).

4.3.2 Baseline case

Figure 4.2 Crossing rate extrapolations of baseline case (6 h and 30 h simulation data)

A so-called baseline case is defined by $\mathbf{U} = \mathbf{0}$, or equivalently, when Y_j are at their median values. A 30-h simulation is performed to establish this case, which is supposed to be ready for direct use in the actual design. Figure 4.2 plots out the crossing rate $\nu_{\mathbf{U},z}^+(\mathbf{0})$ and the fitted curve up to $z = 0$ generated by Naess method. As indicated in Figure 4.2, $\nu_{\mathbf{U},0}^+(\mathbf{0}) = 1.02 \times 10^{-7} \text{ s}^{-1}$ and correspondingly $p_{\mathbf{U},0}(\mathbf{0}) = 1.10 \times 10^{-3}$,

which is the probability of failure without accounting for the extra four uncertain parameters.

In this work, the extrapolation from 30-h data is regarded as sufficiently accurate to be the benchmark result. The supportive argument is that the extrapolation is within the recommended three orders of magnitude, similar to that considered in Chapter 3. In addition to the 30-h simulation, another 6-h data are provided herein to examine the sensitivity of extrapolation caused by differing amount of data. As such, it is obvious from Figure 4.2 that the extrapolation of the 6-h data is compromised due to insufficient amount of data.

4.3.3 Large uncertainty case

In the large uncertainty case, the upper bounds of the CoV of the variables (see Table 4.1) are taken. The proposed method requires 6-h simulations for each evaluation points. To show the favorable performances of only 6-h data, 30-h simulations are also considered as benchmarks. As such, 30-h simulations are performed to evaluate $v_{U_j,z}^+(\Delta u_j)$ and $v_{U_j,z}^+(-\Delta u_j)$ for each variable U_j , where $\Delta u_j = 1$. The trends are plotted in Figure 4.3(a)-(d) for the four variables: U_1 = current; U_2 = drag coefficient; U_3 = RAO factor; U_4 = riser mass. The crossing rates are generated based on the values listed in Table 4.3.

Table 4.3 The values of variables corresponding to $k\Delta u_j$

Variable j	$k\Delta u_j$		
	$k = -1$	$k = 0$	$k = 1$
1	0.90	1.0	1.10
2	0.88	1.1	1.32
3	0.90	1.0	1.10
4	163	190	217

Figure 4.3 Crossing rates of (a) U_1 , $U_1 \pm \Delta u_1$; (b) U_2 , $U_2 \pm \Delta u_2$; (c) U_3 , $U_3 \pm \Delta u_3$; (d) U_4 , $U_4 \pm \Delta u_4$

It is of interest to discuss the results of this sensitivity study. First, the crossing rate increases with current due to the fact that a stronger current generates more severe VIV responses (Figure 4.3(a)). The drag coefficient C_D has the most meaningful impacts on the response owing to crucial role of C_D in both wave and VIV. Since the wave and VIV responses are increased monotonically with C_D , the crossing rate also follows such trend (Figure 4.3(b)). A larger RAO factor leads to more severe top-end motions of the risers. As expected, the crossing rate increases with the RAO factor but is less sensitive compared to the former variables (Fig. 3(c)). In spite of this, its effects are still non-negligible. The riser mass variable has some unique characteristics compared to the foregoing variables, e.g. the variation of the crossing

rate against the riser mass is not monotonic. By reference to Figure 4.3(d), both positive and negative changes of U_4 lead to an increase of the crossing rate. The reason is that when the two risers have the same mass, their deflections are the most synchronized due to similar dynamic characteristics. When there is a discrepancy of mass between two risers, their responses will become less synchronized, which results in higher likelihood of collision. It is observed from Figure 4.3(d) that the amount of change of the crossing rate is asymmetrical, in which a negative change produces a larger effect by comparing with that of a positive change. This asymmetrical behavior is very complicated, but it may be relevant to the asymmetry in the position of the risers with respect to the current, namely one in the upstream and the other in the downstream experiencing WIO effects.

Figure 4.4 Extrapolation of (a) $V'_{1,z}$; (b) $V'_{2,z}$; (c) $V'_{3,z}$; (d) $V'_{4,z}$

Figure 4.5 Extrapolation of $V''_{j,z}$: (a) $j = 1$; (b) $j = 2$; (c) $j = 3$; (d) $j = 4$

A conclusion of the above sensitivity study is that all four random variables significantly affect the crossing rate. To compute $\partial^2 p_{U,z}(\mathbf{0}) / \partial U_j^2$, the derivatives $V'_{j,z}$ and $V''_{j,z}$ are evaluated by Eqs. (4.12) and (4.13) and plotted in Figures 4.4(a)-(d) and 4.5(a)-(d). Once again, comparisons will be made between 6-h and 30-h data. To conduct the extrapolations, Eqs. (4.14) and (4.15) are rearranged to a form similar to Eq. (3.8). The linearity of this form needs to be tested here. To this end, Figure 4.6(a) plots the trend of $\ln[-\ln(V''_{2,z}/q'')]$ against $\ln(z-b'')$, while Figure 4.6(b) shows $\ln[-\ln(V'_{2,z}/q')]$ against $\ln(z-b')$. For any other variable, the same quality of linearity is observed when written in the form of Eq. (3.8). To save

the space, they are not plotted out. The linearity found in Figure 4.6 justifies the use of Naess form to carry out the extrapolation. Besides, the extrapolation curves relevant to 6-h and 30-h data are indistinguishable.

Figure 4.6 Linear fitting by Naess method for (a) $V''_{2,z}$ and (b) $V'_{2,z}$

Figures 4.4(a)-(d) and 4.5(a)-(d) illustrate the extrapolations of $V'_{j,z}$ and $V''_{j,z}$ respectively for all four variables. By comparing these two figures with Figure 4.2, it is expected that the discrepancies between the 6-h and 30-h extrapolations for $V'_{j,z}$ and $V''_{j,z}$ are remarkably smaller than that for $v_{U,z}^+(0)$. The derivatives of the crossing rate are less sensitive to the amount of available data, because the randomness due to the stochastic waves is somewhat mitigated during a finite difference calculation. In this approach, all the crossing rates are generated following the same wave time history. Therefore, the subtraction or addition will not increase the variation. The reduction of randomness is probably due to the offset of noise in the crossing rates during the finite difference calculation. By substituting $z = 0$ into Eqs. (4.14) and (4.15), $V'_{j,0}$ and $V''_{j,0}$ are obtained and presented in Table 4.4. Then, Eq. (4.10) (with $z = 0$) is used to determine $\partial^2 p_U(\mathbf{0})/\partial U_j^2$ from $V'_{j,0}$ and $V''_{j,0}$. Table 4.4 lists the values for $\frac{1}{2}\partial^2 p_U(\mathbf{0})/\partial U_j^2$, which represents the alteration of the failure

probability due to the uncertainty of the j th variable according to the rationale behind the perturbation method. It is quantitatively shown in Table 4.4 that all four variables contribute considerably to the increase of the probability of failure.

Table 4.4 Results of the proposed approach for the large uncertainty case

Variable j	30-h data			6-h data		
	$V_{j,0}''$	$V_{j,0}'$	$\frac{1}{2} \frac{\partial^2 p_U(\mathbf{0})}{\partial U_j^2}$	$V_{j,0}''$	$V_{j,0}'$	$\frac{1}{2} \frac{\partial^2 p_U(\mathbf{0})}{\partial U_j^2}$
1	3.90×10^{-7}	1.57×10^{-7}	2.10×10^{-3}	5.10×10^{-7}	1.06×10^{-7}	2.75×10^{-3}
2	7.58×10^{-7}	8.02×10^{-7}	4.06×10^{-3}	8.22×10^{-7}	6.32×10^{-7}	4.42×10^{-3}
3	5.66×10^{-7}	2.02×10^{-7}	3.06×10^{-3}	4.08×10^{-7}	2.45×10^{-7}	2.20×10^{-3}
4	1.88×10^{-7}	-2.88×10^{-7}	1.01×10^{-3}	2.13×10^{-7}	-2.20×10^{-7}	1.15×10^{-3}

From Eq. (4.10), $\partial^2 p_U(\mathbf{0})/\partial U_j^2$ is given by the sum of $V_{j,0}'' T \exp[-v_{U,0}^+(\mathbf{0})T]$ and $-(V_{j,0}')^2 T^2 \exp[-v_{U,0}^+(\mathbf{0})T]$. The two constant components $T \exp[-v_{U,0}^+(\mathbf{0})T]$ and $-T^2 \exp[-v_{U,0}^+(\mathbf{0})T]$ are calculated and found to be 1.08×10^4 and -1.17×10^8 . By putting $V_{j,0}''$ and $V_{j,0}'$ into the two terms, one can conclude that from Table 4.4 $V_{j,0}'' T \exp[-v_{U,0}^+(\mathbf{0})T]$ is a dominant term compared with $-(V_{j,0}')^2 T^2 \exp[-v_{U,0}^+(\mathbf{0})T]$. It can be explained as follows. For small probabilities, $p_U(\mathbf{0}) \approx v_{U,0}^+(\mathbf{0})T$, namely $p_U(\mathbf{0})$ is linearly related to $v_{U,0}^+(\mathbf{0})$. It then follows that $\partial^2 p_U(\mathbf{0})/\partial U_j^2$ and $V_{j,0}''$ are also linear and $\partial^2 p_U(\mathbf{0})/\partial U_j^2$ depends only on $V_{j,0}''$. Here, $-(V_{j,0}')^2 T^2 \exp[-v_{U,0}^+(\mathbf{0})T]$ is a non-linear correction term. When the probabilities are large to the extent that $1 - \exp(-\theta) \approx \theta$ is no longer valid, $-(V_{j,0}')^2 T^2 \exp[-v_{U,0}^+(\mathbf{0})T]$ will become non-negligible.

The P_f obtained by the different methods and cases are summarized in Table 4.5. There are two important findings from this table. First, the presence of uncertainties

increases the probability of failure by an order of magnitude. Hence, the consideration of various uncertainties during a riser interference reliability assessment is meaningful. Second, the P_f derived from the 6-h and 30-h simulation data are very close. This promising result justifies the robustness of the proposed approach.

Table 4.5 Comparison of probability of failure obtained by different methods

	Large uncertainty case	Small uncertainty case
Monte Carlo simulation	1.36×10^{-2}	5.75×10^{-3}
Proposed method, 6 h data	1.16×10^{-2}	5.12×10^{-3}
Proposed method, 30 h data	1.13×10^{-2}	4.94×10^{-3}
Simple scheme, 6 h data	8.09×10^{-3}	3.87×10^{-3}
Simple scheme, 30 h data	9.89×10^{-3}	4.37×10^{-3}

Monte Carlo simulations are performed to benchmark the accuracy of the proposed method. Here, each sample involves a 30-h simulation, which requires 16 h of computer processing time, running on a machine with four 3.4 GHz parallel processors. Owing to time constraint, only 130 samples were considered. Referring to Table 4.5, the Monte Carlo simulation for P_f has a disparity of 17% from the proposed method. The inaccuracies in the proposed method can be attributed to the finite difference estimates for the derivatives $V'_{j,z}$ and $V''_{j,z}$, and the truncated Taylor expansion in the perturbation method. However, given the high sensitivity nature of failure prediction, this level of accuracy by the proposed method is satisfactory.

The Monte Carlo simulation also has relatively high sampling variability. The CoV of the estimator is determined from Eq. (4.3) and found to be 0.132. To further investigate the data, Monte Carlo simulations are plotted out as a probability density function in Figure 4.7. Most of the samples are below 0.01 while others are distributed from 0.01 to about 0.12. On the whole, this distribution has a large CoV and is highly skewed. Due to the large CoV, the process of convergence is usually slow. Moreover, the significant positive skewness results in a relatively high chance of sampling large conditional failure probabilities. When a large value is sampled, the sample mean will be greatly distorted if the sample size is inadequate. However, such

large-value sample is required as it reflects the actual statistical characteristics. As such, the direct sampling method relies on a sizable amount of samples to maintain an acceptable accuracy of the mean. An importance sampling technique may be of help to expedite the Monte Carlo simulation, which is an interesting topic for the future work. It is also of interest to plot out the samples of the conditional failure probabilities (see Figure 4.8). The spiky pattern of these samples, characterized by the sharp peaks, reflects the high skewness of the variable.

Figure 4.7 Probability density function of conditional failure probabilities from Monte Carlo simulation

Figure 4.8 Samples of conditional failure probabilities from Monte Carlo simulations

Table 4.6 Results of the simple scheme for the large uncertainty case

Variable j	30-h data				6-h data		
	$p_{U_j}(\Delta u_j)$	$p_{U_j}(-\Delta u_j)$	$\frac{1}{2} \frac{\partial^2 p_U(0)}{\partial U_j^2}$		$p_{U_j}(\Delta u_j)$	$p_{U_j}(-\Delta u_j)$	$\frac{1}{2} \frac{\partial^2 p_U(0)}{\partial U_j^2}$
1	4.83×10^{-3}	9.72×10^{-4}	1.80×10^{-3}		2.70×10^{-3}	7.77×10^{-4}	6.39×10^{-4}
2	8.15×10^{-3}	8.64×10^{-4}	3.41×10^{-3}		8.02×10^{-3}	7.13×10^{-4}	3.27×10^{-3}
3	3.59×10^{-3}	5.12×10^{-4}	9.51×10^{-4}		2.91×10^{-3}	8.42×10^{-4}	7.76×10^{-4}
4	2.13×10^{-3}	5.33×10^{-3}	2.63×10^{-3}		3.02×10^{-3}	3.77×10^{-3}	2.30×10^{-3}

It is of interest to examine the performance of the simple scheme (refer to the description following Eq. (4.8)). The conditional probabilities of failure corresponding to $p_{U_j}(\Delta u_j)$ and $p_{U_j}(-\Delta u_j)$ are calculated by extrapolating 6-h and 30-h data (see Table 4.6), while the resulting P_f can be found in Table 4.5. The simple scheme with 6-h simulations shows unsatisfactory accuracy. It can be ascribed to two deficiencies. First, the simple scheme is based on extrapolating the crossing rate instead of its derivatives. It has already been shown in Figure 4.2 that the accuracy of extrapolating crossing rates is compromised due to limited data. Second, the discrepancy is produced in the process of the finite difference formulation. In Eq. (4.8), several terms involving certain level of errors are summed up. In contrast, the finite difference calculation is performed prior to the extrapolation containing any errors in the proposed method, hence it does not face this problem. Although the simple scheme with 30-h data produces reasonable results, this performance is achieved at a much higher computational expense.

It is worth discussing the appropriateness of the perturbation method and Naess method for calculating the probability of riser clashing under other circumstances. The perturbation method (see Eq. (4.5)) is a general method based on approximating the integral (see Eq. (4.1)) by Taylor series expansion, typically up to second order. It is difficult to foresee the accuracy of the method from general principles. The reason is that, firstly, the performance of this method depends on the integral behavior which

makes it problem dependent; secondly, there is no precedent work to verify that perturbation method is in general suitable for riser clashing prediction. However, the current study shows that the perturbation method is appropriate for the present problem based on comparison with Monte Carlo simulations. This suggests that the perturbation method is promising for riser systems similar to top-tension risers and with similar random variables. For riser systems with dramatically different response behavior, say flexible risers, Monte Carlo simulations are required to verify the validity of applying this method.

Naess extrapolation method is known to be appropriate for the crossing rate of structural response. Unfortunately, there is no study in the literature to investigate the possibility of applying Naess method to predict the first and second derivatives of riser structural responses. However, the Naess method has potential to be appropriate for predicting derivatives of riser structural responses. Firstly, Naess method is generally suitable for extrapolating crossing rate with linearity or mild nonlinearity after logarithm which makes it applicable to a wide range of problems (see Section 4.2.4). Secondly, the derivatives of crossing rate always have similar trends to the dominating crossing rate in Eqs. (4.12)-(4.13), which makes the extrapolation of derivatives has almost the same quality as that of the crossing rate itself.

4.3.4 Small uncertainty case

For the small uncertainty case, the CoV of the variables are assigned the lower bounds in Table 4.1. No new simulations are required here. In the perturbation method, the same evaluation points are employed, but the step sizes now correspond to $\Delta u_1 = 2$, $\Delta u_2 = 1.33$, $\Delta u_3 = 2$, and $\Delta u_4 = 2$. For Monte Carlo simulation, the importance sampling technique is applied to calculate P_f by exploiting the simulation results for the large uncertainty case that are already available (see the discussion below Eq. (4.4)).

Table 4.7 Results of the proposed approach for the small uncertainty case

Variable j	30-h data				6-h data		
	$V_{j,0}''$	$V_{j,0}'$	$\frac{1}{2} \frac{\partial^2 p_v(\mathbf{0})}{\partial U_j^2}$		$V_{j,0}''$	$V_{j,0}'$	$\frac{1}{2} \frac{\partial^2 p_v(\mathbf{0})}{\partial U_j^2}$
1	9.75×10^{-8}	7.85×10^{-8}	5.26×10^{-4}		1.28×10^{-7}	5.30×10^{-8}	6.88×10^{-4}
2	4.26×10^{-7}	6.02×10^{-7}	2.29×10^{-3}		4.62×10^{-7}	4.74×10^{-7}	2.49×10^{-3}
3	1.42×10^{-7}	1.01×10^{-7}	7.65×10^{-4}		1.02×10^{-7}	1.23×10^{-7}	5.50×10^{-4}
4	4.70×10^{-8}	-1.44×10^{-7}	2.53×10^{-4}		5.33×10^{-8}	-1.10×10^{-7}	2.88×10^{-4}

The intermediate results from the proposed method and the simple scheme are produced in Tables 4.7 and 4.8 respectively, while the P_f results are reported in Table 4.5. The prediction by the proposed method is again reasonably close to the Monte Carlo result. As expected, with the reduced uncertainty in the random variables, the P_f is smaller than the previous case. The CoV of the importance sampling estimator is 0.138. It is not necessary to discuss these results in detail since the trends are similar to the previous case.

Table 4.8 Results of the simple scheme for the small uncertainty case

Variable j	30-h data				6-h data		
	$p_{U_j}(\Delta u_j)$	$p_{U_j}(-\Delta u_j)$	$\frac{1}{2} \frac{\partial^2 p_v(\mathbf{0})}{\partial U_j^2}$		$p_{U_j}(\Delta u_j)$	$p_{U_j}(-\Delta u_j)$	$\frac{1}{2} \frac{\partial^2 p_v(\mathbf{0})}{\partial U_j^2}$
1	4.83×10^{-3}	9.72×10^{-4}	4.50×10^{-4}		2.70×10^{-3}	7.77×10^{-4}	1.60×10^{-4}
2	8.15×10^{-3}	8.64×10^{-4}	1.92×10^{-3}		8.02×10^{-3}	7.13×10^{-4}	1.84×10^{-3}
3	3.59×10^{-3}	5.12×10^{-4}	2.38×10^{-4}		2.91×10^{-3}	8.42×10^{-4}	1.94×10^{-4}
4	2.13×10^{-3}	5.33×10^{-3}	6.58×10^{-4}		3.02×10^{-3}	3.77×10^{-3}	5.74×10^{-4}

4.4 Chapter summary

In addition to random waves, there are numerous uncertain parameters that can affect the probability of collision. As shown in Section 4.3, the effect of extra uncertain parameters is an increase of one order of magnitude in P_f . Hence, the risk of riser

collision can be substantially underestimated without considering these parameters. The proposed approach in this chapter extends the work in Chapter 3 by allowing some parameters in the model to be random. These variables are current, drag coefficient, RAO factor and riser mass. The innovative aspects of this work come from: first, the transformation of variables to standard normal space; second, the new extrapolation process. As a result, the primary advantage of the method is its computational affordability. A top-tensioned riser system is selected for the case study. The predicted result agrees well with that from Monte Carlo simulations.

CHAPTER 5 RELIABILITY ANALYSIS BASED ON COLLISION ALLOWED APPROACH

5.1 Introduction

The last two chapters developed approaches on assessing the probability of failure based on no-collision-allowed design. In this chapter, emphasis will be put on collision allowed design. With reference to this design principle, it is stated that a collision does not necessarily cause a failure in riser systems. Additional information about riser structural properties and parameters characterizing the degree of collision severity are needed to assess the reliability against riser collisions. The parameters are termed *collision parameters* henceforth. In this research area, at least three aspects have to be investigated so as to ensure satisfactory reliability. They are prediction of riser collisions, riser local impact analysis and assessment of collision factors.

The prediction of riser collisions is well addressed in Chapters 3 and 4. Efforts should be put on riser local impact analysis and assessment of collision parameters. Generally, in a riser clearance assessment, the local impact analysis belongs to local stage; the assessment of collision parameters is relevant to the global stage. For completeness, both of them will be performed in this chapter although local impact analysis is relatively mature in industry. This work strives to develop a probability-based approach, which is analogous to that in Chapters 3 and 4, for the collision allowed riser design. Finally, probability of yielding failure and the resultant fatigue damage will be computed.

Compared to the no-collision-allowed riser design, the current design has a much higher chance of clashing. However, the number of collision events observed is very limited. Hence, the assessment of collision parameters should not be based on Monte Carlo simulations. The main challenge of this work is to deduce the collision

parameters from the pre-collision riser responses. Moreover, the nonlinearities involved make this task more difficult to achieve.

5.2 Local impact analysis

5.2.1 General considerations

(a)

(b)

Figure 5.1 Two risers in contact: (a) cross-sectional view; (b) three-dimensional view

Usually, the local impact analysis is undertaken using the collision parameters obtained from a global analysis for each collision observed. Alternatively, as in this work, the local analysis is conducted in advance for prescribed ranges of collision parameters. For convenience in application, the results are interpolated or fitted with curve/surface.

First of all, some terms need to be defined. Figure 5.1(a) illustrates the cross-sections of two risers in contact. Here, $\mathbf{V}_1$ and $\mathbf{V}_2$ are the absolute velocities of Risers 1 and 2. The relative velocity $\mathbf{U}_{\text{rel}}$ is decomposed into normal and tangential components U_n and U_t . The kissing angle Θ is the angle between $\mathbf{U}_{\text{rel}}$ and U_n . Figure 5.1(b) is a three-dimensional model of two risers. The contact angle α is defined as the included angle between two contacted riser sections.

There are two failure mechanisms for riser collisions, i.e. material yielding and fatigue. For yield criterion, the maximum von Mises stress S_v is extracted from the stress time series. According to the work done by Leira et al. (2002b), S_v depends primarily on U_n and α . For TTRs, α is usually very small, and global simulations in this work also reveal that $\alpha < 1^\circ$. Since S_v increases with α , it is conservatively assumed that $\alpha = 1^\circ$, in which case S_v reduces to a function of U_n alone.

Fatigue damage based on S - N curves usually relies on the stress ranges of first principal stress. An impact generally contains a number of stress cycles of different stress ranges, but it is convenient to adopt the largest stress range ΔS_{max} for fatigue calculations. Similarly, according to the codes (Det Norske Veritas, 2009), ΔS_{max} is presumed to be dependent on both U_n and Θ .

5.2.2 Finite element analysis

The FE analysis is carried out using a commercial software LS-DYNA (LSTC, 2010). In this FE analysis, the steel grade is L80. Table 5.1 lists the material properties and

relevant information of risers. Figure 5.2 shows the finite element model, which comprises SHELL163 shell elements. It represents a scenario of two riser sections prior to contact. The target riser is kept stationary and has a longer length so that the fixed boundary conditions have negligible effect on the impact region (Chan et al., 2000). The contact riser section is a free body with a prescribed initial velocity, while the length should be determined so that the correct mass participating in a collision is well represented. Methods to calculate the participating mass exist in the literature (Sagatun et al., 1999), but for simplicity, the riser length is taken to be 3 m. This length is suggested by Wilmshurst et al. (2001), who conducted local riser impact analyses similar to this work. The stress results are extracted from the target riser section.

Table 5.1 Riser properties

Parameter	Value
Yield stress	$550 \times 10^8 \text{ N/m}^2$
Young's modulus	$2.05 \times 10^{11} \text{ N/m}^2$
Poisson ratio	0.3
Outer diameter	0.268 m
Inner diameter	0.240 m
Added mass coefficient	1.0
Drag coefficient	1.1
Wall thickness	14 mm
Cross-sectional area	0.012 m^2
Steel density	7850 kg/m^3
Content	800 kg/m^3
Water density	1025 kg/m^3
Effective density in model (steel+content+added mass)	16265 kg/m^3

Figure 5.2 A finite element model for local analysis

Figure 5.3 Von mises stress S_v as a function of U_n

Figure 5.4 Surface plot of maximum stress range $\Delta S_{\max}(U_n, \Theta)$

Figure 5.3 plots out S_v varied with U_n , and a quadratic equation is fitted to the data points. Figure 5.4 displays a surface plot of $\Delta S_{\max}(U_n, \Theta)$. It is known that for a given Θ , $\Delta S_{\max}$ varies linearly with U_n . If U_n is kept constant, the relationship between $\Delta S_{\max}$ and Θ follows approximately a second-order polynomial (Kalleklev et al., 2003). Followed by these two rules, the surface is fitted to the form

$$\Delta S_{\max}(U_n, \Theta) = (b_0 + b_1\Theta + b_2\Theta^2)U_n, \quad (5.1)$$

where $b_0 = 50.43$, $b_1 = 0.239$, $b_2 = 0.00603$.

5.3 Global analysis for prediction of collision parameters

5.3.1 General considerations

In the global dynamic analysis, risers are modeled as a set of lumped mass connected by spring with structural damping. The hydrodynamic forces are computed by Morison's equation. VIV and WIO are simulated with the models summarized in Chapter 2, i.e. Iwan and Blevins wake oscillator model and Blevins wake field model

(Iwan and Blevins, 1974; Blevins, 2001). The riser system can be either coupled or uncoupled from the floating platform. For an uncoupled analysis, the platform motions are computed in advance and applied as boundary conditions with the use of RAOs.

In offshore engineering, a design sea state is assumed to be statistically stationary for a short duration, typically three hours. For the yield criterion, the goal is to calculate the probability of failure P_f for a given sea state. The proposed method is supposed to be applicable to both short term and long term design. In a short term design, a typical sea state is prescribed to represent the most critical one within the whole service life. P_f is calculated based on this sea state. Certainly, this sea state should be well chosen and some recommendations could be found from practical codes (Chakrabarti, 2005). For the fatigue criterion, the aim is to compute the partial fatigue damage arising from the given sea state. The total damage accumulates from a number of sea states that characterizes the long-term environmental condition.

In reality, there are many uncertainties in the riser interference problem. Some relevant work concerning multiple sources of uncertainties is accomplished by the author in Chapter 4. In this work, for simplicity, the uncertainty is assumed to be from irregular waves alone. However, it is in principle valid to apply the scheme summarized in Chapter 4 in this work so that multiple uncertainties can be accounted for. Let $\eta(t)$ represent the incident wave elevation, which is a stochastic process. $\eta(t)$ is generated from a random wave spectrum $S_{\eta\eta}(\omega)$ through a summation of linear wave components with random phases. As a result, the riser response is also stochastic. In addition to the local impact behavior elaborated in Section 5.2, the probabilistic assessment requires at least two more pieces of information. These are the rate of collision and the probabilistic descriptions of the collision parameters. They will be discussed as follows.

5.3.2 Estimating the collision rate

Riser clashing is essentially a rare event problem. It is infeasible to achieve the probability of collisions through direct sampling. Hence, a crossing rate analysis is established based on a process $X(t)$ characterizing the severity of collision. Finally, the probability of collision is attained by extrapolating the crossing rate to failure threshold.

This subsection aims to introduce two methods of estimating the collision rate (collision events per unit time). The first method to be presented has been established in Section 3.2.4 and is termed as Naess method (see Eqs. (3.7) and (3.8)). The process $X(t)$ is produced by Eqs. (3.13) and (3.14). It is worth noticing that the normalization of $C_{jk}(t)$ with respect to its initial value (see Eq. (3.13)) is crucial if the initial clearances are different among different element pairs, such as flexible risers studied in Chapter 3. However, normalization seems not very crucial for TTRs.

Another classical technique for the estimation of crossing rates for non-Gaussian processes is the Hermite transformation introduced by Winterstein (1988). Duggal and Niedzwecki (1994) applied this technique to the collision analysis between two flexible cylinders. A briefing of the application procedure of this technique is summarized herein. First, a stochastic process $X(t)$ is approximated by another non-Gaussian process $Y(t)$. $Y(t)$ is a monotonic function with respect to a standard Gaussian process $U(t)$, i.e. $Y(t) = g(U(t))$. The first few statistical moments of $X(t)$ are used to establish the function $g(\cdot)$. In practice, $g(\cdot)$ has an analytical form if the number of moments are four or less. It is common to employ the first four moments, namely the mean μ_X , standard deviation σ_X , skewness γ_X , and kurtosis κ_X . Then, for a softening response ($\kappa_X > 3$), $g(\cdot)$ has the form

$$g(U) = \mu_X + \kappa_X \sigma_X \left[U + c_3 (U^2 - 1) + c_4 (U^3 - 3U) \right], \quad (5.2)$$

where $c_3 = \gamma_x / [4 + 2\sqrt{1 + 1.5(\kappa_x - 3)}]$, $c_4 = [\sqrt{1 + 1.5(\kappa_x - 3)} - 1]/18$ and $\kappa = 1/\sqrt{1 + 2c_3^2 + 6c_4^2}$.

Next, the crossing rate for the threshold z can be approximated as

$$v_x^+(z) \approx v_y^+(z) = v_u^+(0) \exp \left\{ -\frac{[g^{-1}(z)]^p}{2} \right\}, \quad (5.3)$$

where $v_u^+(0) = v_x^+(\mu_x)$, and the inverse function $g^{-1}(\cdot)$ exists, and has an analytical expression (Winterstein, 1988).

5.3.3 Understanding the collision mechanisms

The problem of riser clashing involves many nonlinear phenomena such as WIO, VIV, large deflections and drag forces. These nonlinear features make the problem very complicated. Before proceeding further, it is imperative to understand the characteristics of the current problem. To this end, extensive Monte Carlo simulations of the global model are performed on an illustrative TTR system, details of which will be described in Section 5.5. The Monte Carlo simulation includes 604 collision events.

The process $X(t)$ is exemplified in Figure 5.5 to visualize a collision event, $X(t)=0$ denotes the occurrence of a collision event. It is observed that a collision is always followed by a rapid succession of several impacts. Hence, a collision is defined as a series of successive impacts which should have a high level of dependency between each other. However, adjacent events can be assumed to be independent provided that the interval is adequate. Referring to Figure 5.5, an initial impact (Scenario 1) is followed by additional impacts (Scenario 2), but they are not restricted to the same riser location. The subsequent impacts are believed to be generated by VIV and geometric nonlinearity (Nygard et al., 2001). It is obvious that the collision rate mentioned in Section 5.3.2 refers to events.

Figure 5.5 Plot of a sample time history of $X(t)$ for illustrating a collision event

Consider a collision event, let V_0 , V_1 , V_2 , etc denote the relative normal velocities of the impacts, where V_0 corresponds to the initial impact. These velocities can be conveniently transformed to V_0 , $V_0 + \delta_1$, $V_0 + \delta_2$, etc, by letting $\delta_i = V_i - V_0$. For the yield criterion, the largest velocity within an event is used to define U_n for the local impact analysis, i.e.

$$U_n = \max\{V_0, V_0 + \delta_1, V_0 + \delta_2, \dots\} = V_0 + V_\Delta, \quad (5.4)$$

in which $V_\Delta = \max\{0, \delta_1, \delta_2, \dots\}$ is the difference between subsequent impact velocities and the initial one. It can be interpreted as the additional velocity due to the presence of subsequent impacts.

Among these impact velocities, V_0 is in principle easiest to predict, for it is unaffected by the prior impacts. To make this conjecture useful, V_0 and V_Δ should be independent. A scatter diagram is constructed from the Monte Carlo results shown in Figure 5.6. The correlation coefficient is found to be -0.069, which implies that V_0 and V_Δ are essentially uncorrelated.

The correlation coefficient presented is for further information. The uncorrelation between the two variables does not guarantee that they are independent. However, the

pattern of the scatter diagram (see Fig. 5.6) suggests that there is no obvious dependency between V_0 and V_Δ . Therefore, it seems reasonable to postulate that V_0 and V_Δ are statistically independent. This is a critical assumption made in the proposed approach. The performance of the approach strongly relies on the independence of these two velocity components.

The independence casts light on the prediction of impact velocity; provided that the probability density functions (pdfs) of V_0 and V_Δ can be estimated, then the pdf of U_n can be acquired from the convolution integral

$$f_{U_n}(u) = \int_0^u f_{V_\Delta}(u-v)f_{V_0}(v)dv, \quad (5.5)$$

where the limits of the integration account for the fact that $f_{V_\Delta}()$ and $f_{V_0}()$ are zero for negative arguments.

Figure 5.6 Scatter diagram of V_0 and V_Δ (correlation coefficient = -0.069; sample size=604)

The physical meaning of independence is explained as follows. The wave-frequency riser response and the WIO govern the “gross motions”. In other words, they are the responsible drivers for initial impact velocity V_0 . Upon impact and consequential rebounds, the lock-in VIV phenomenon, which is typically of a much higher

frequency, causes the risers to collide again, although not necessarily at the same location. However, the momentum that is responsible for V_0 is not entirely lost. It is reserved for the subsequent impact velocities V_1 , V_2 , etc, with additional velocity components δ_i governed by other mechanisms. Thus, V_0 (gross motions) and δ_i (VIV and geometric nonlinearity) are independent. So are V_0 and V_Δ .

The mean, standard deviation and skewness (standardized third moment) of V_0 are 0.26 m/s, 0.11 m/s and 0.40 respectively, while those of V_Δ are 0.17 m/s, 0.17 m/s and 1.5. These statistics indicate that both V_0 and V_Δ are significant, and that both variables have skewed distributions. The prediction of the statistics of V_0 and V_Δ will be presented in the following subsections. The kissing angle Θ is relevant only to fatigue, and will be discussed in Section 5.4.2.

5.3.4 Statistics of the initial impact velocity

From vector analysis, it is found that $V_0 = \dot{X}(t)$ at the moment of contact; the details are provided in Appendix A. V_0 is strictly positive, while $\dot{X}(t)$ can be either positive or negative depending on upcrossing (impact) or downcrossing (rebound) the zero level. For convenience, let the random variable $\dot{X}_c$ represent $\dot{X}(t)$ under the condition that $X = 0$ (either impact or rebound). Since only positive $\dot{X}_c$ is of interest, let $\dot{X}_c^+$ be the positive $\dot{X}_c$. The next endeavor is to estimate the pdf of $\dot{X}_c$ so that $f_{\dot{X}_c^+}()$ can be attained by retrieving only the positive part of $f_{\dot{X}_c}()$ and renormalizing. Finally, $f_{V_0}() = f_{\dot{X}_c^+}()$.

Now, consider the case in which $X(t)$ is approximated by the Hermite transformation process $Y(t)$ (see Section 5.3.2). Then, if $f_{\dot{Y}}()$ is achievable, it may be used to approximate $f_{\dot{X}}()$. First of all, $Y(t) = g[U(t)]$, in which $g()$ is furnished by Eq. (5.2). Differentiating this equation with respect to t and applying the chain rule, one gets

$$\dot{Y}(t) = \kappa \sigma_X [3c_4 U^2(t) + 2c_3 U(t) + 1 - 3c_4] \dot{U}(t). \quad (5.6)$$

Here, $U(t)$ is a stationary Gaussian process, so $\dot{U} \sim N(0, \sigma_U^2)$, where σ_U can be deduced from $v_U^+(0) = \sigma_U / 2\pi\sigma_U$, and $N(\bullet, \bullet)$ is the usual denotation for a normal distribution. According to Eq. (5.6), $\dot{Y} \sim N(0, \sigma_Y^2)$, but σ_Y depends on $U(t)$. By taking $\sigma_Y(U(t))$ and putting $U(t) = g^{-1}[Y(t)]$ into it, one obtains

$$\sigma_Y(Y) = \kappa \sigma_X \{3c_4 [g^{-1}(Y)]^2 + 2c_3 [g^{-1}(Y)] + 1 - 3c_4\} \sigma_U. \quad (5.7)$$

The pdf of $\dot{X}_c$ can be estimated as $N(0, \sigma_Y^2(0))$.

There are some limitations of the Hermite model. First, the approximated process $\dot{Y}(t)$ is Gaussian, but $\dot{X}(t)$ can be non-Gaussian (as will be seen in the case studies). In addition, $\dot{X}(t)$ may be non-stationary and depends on the riser clearances due to WIO effects. Although Eq. (5.7) does allow σ_Y to vary with Y , the relationship is prescribed in the Hermite model, and may not represent the true behavior. The source of these limitations is explained as follows. $Y(t)$ can be mapped to a Gaussian process through $U(t) = g^{-1}[Y(t)]$. Similarly, $W(t) = g^{-1}[X(t)]$ but $W(t)$ may not be a Gaussian process although the pdf of $W(t)$ is (almost) Gaussian for fixed t . The reason is that $W(t_1)$ and $W(t_2)$ are generally not jointly Gaussian, which violate the definition of a Gaussian process. In short, the Hermite model reproduces the non-Gaussian pdf of $X(t)$, but the complexity of non-Gaussian processes means that it cannot simultaneously satisfy $\dot{X}(t)$.

To avoid the limitations of the Hermite model, and noting that $f_{\dot{X}}(\cdot)$ is dependent on $X(t)$, it is proposed to derive the pdf at $X(t)=0$ by extrapolating the behavior at pre-collision thresholds for a range of X . Because it is untenable to extrapolate the probability densities directly, extrapolation of statistical moments is used instead.

Moreover, since the fourth moment (kurtosis) is often subjected to large sampling error due to lack of data, it is wise to focus on the first three moments alone, which also allows one to construct a skewed pdf (non-Gaussian).

Let $r_k(x) = E[\dot{X}^k | X = x]$ represent the k th raw moment conditional on X . A simple and straightforward scheme for estimating $r_k(x)$ is to build a sample of $\dot{X}^k$, whose corresponding x resides within a preset uniform window $x \pm \Delta x$. The problem is that the error is large if the window is too wide (i.e. large Δx). On the other hand, if Δx is small, the sample size will be insufficient which will also lead to large error. To resolve this difficulty, a one-dimensional Gaussian smoothing technique (Davies, 2005) is adopted for current application. The raw moments are estimated as

$$r_k(x) = \frac{\sum_{j=1} \dot{X}^k(t_j) \psi_j}{\sum_{j=1} \psi_j}, \quad (5.8)$$

where

$$\psi_j = \exp\left(-\frac{[X - X(t_j)]^2}{2\Omega\sigma_X}\right) \quad (5.9)$$

is a bell-shaped weighting function and Ω is a scale parameter. A smaller Ω makes the effective sampling concentrate on local data (i.e. around x), thus alleviating the smoothing effect. If Ω is too small, say 0.1, the result becomes indistinguishable with that of the uniform window. In contrast, a large Ω makes the result less realistic by excessively sampling the region far away from x . Hence, the optimized Ω is the smallest value that produces a smooth plot. In this study, $\Omega = 0.5$ is empirically recommended to be the first trial. Finally, the mean, variance, and skewness are calculated respectively from $\mu_{\dot{X}}(x) = r_1(x)$, $\sigma_{\dot{X}}^2(x) = r_2(x) - r_1^2(x)$ and $\gamma_{\dot{X}}(x) = [r_3(x) - 3\mu_{\dot{X}}(x)\sigma_{\dot{X}}^2(x) - \mu_{\dot{X}}^3(x)]/\sigma_{\dot{X}}^3(x)$. It is more convenient to discuss the procedure of the extrapolation with the aid of case studies. Therefore, it will be presented in Section 5.3

The moments of $\dot{X}$ at $X = 0$ are simply $\mu_{\dot{X}_c} = \mu_{\dot{X}}(0)$, $\sigma_{\dot{X}_c}^2 = \sigma_{\dot{X}}^2(0)$, and $\gamma_{\dot{X}_c} = \gamma_{\dot{X}}(0)$. The subsequent task is to construct $f_{\dot{X}_c}(\cdot)$ from the three moments. There are many available methods for this density estimation problem; see Low (2012) for a recent review and discussion. It is proposed to use the shifted lognormal distribution (SLD), which has a right-skewed density of the form (Yuan, 1933)

$$f_{\dot{X}_c}(\xi) = \frac{1}{(\xi - \hat{\lambda})\sqrt{2\pi}\hat{\sigma}} \exp\left\{-\frac{1}{2\hat{\sigma}^2}[\ln(\xi - \hat{\lambda}) - \hat{\mu}]^2\right\}, \quad \xi > \hat{\lambda} \quad (5.10)$$

where $\hat{\mu}$, $\hat{\sigma}$, and $\hat{\lambda}$ are the parameters of the distribution. These parameters can be determined explicitly from the three moments as follows (Low, 2012)

$$\gamma_{\dot{X}_c} = \left[\exp(\hat{\sigma}^2) + 2\right]\sqrt{\exp(\hat{\sigma}^2) - 1}, \quad \therefore \hat{\sigma}^2 = \ln\left(1 + \left[2\sinh\left(\frac{1}{3}\sinh^{-1}\left(\frac{-\gamma_{\dot{X}_c}}{2}\right)\right)\right]^2\right), \quad (5.11a)$$

$$\hat{\mu} = \frac{1}{2}\ln\left(\frac{\sigma_{\dot{X}_c}^2}{\exp(\hat{\sigma}^2) - 1}\right) - \frac{1}{2}\hat{\sigma}^2, \quad (5.11b)$$

$$\hat{\lambda} = \mu_{\dot{X}_c} - \exp\left(\hat{\mu} + \frac{\hat{\sigma}^2}{2}\right). \quad (5.11c)$$

In the SLD, $\mu_{\dot{X}_c}$ and $\sigma_{\dot{X}_c}^2$ are allowed to use any value, while $0 < \gamma_{\dot{X}_c} < \infty$. For the case where $\gamma_{\dot{X}_c}$ is negative, one can simply adopt $|\gamma_{\dot{X}_c}|$, and mirror the pdf about $\mu_{\dot{X}_c}$. This operation reverses the sign of $\gamma_{\dot{X}_c}$, but preserves $\mu_{\dot{X}_c}$ and $\sigma_{\dot{X}_c}^2$. The mirroring is implemented by replacing ξ by $2\mu_{\dot{X}_c} - \xi$ on the right-hand side of Eq. (5.10). One benefit of the SLD is that it approaches the normal distribution in the limit $|\gamma_{\dot{X}_c}| \rightarrow 0$.

5.3.5 Statistics of the differential impact velocity

The next step is to predict the statistics of V_{Δ} . This task is envisaged to be challenging owing to the numerous complicated factors involved. In this work, attempts to develop a semi-analytical model based on understanding VIV characteristics have

been unsuccessful. It seems that $f_{V_\Delta}()$ can be estimated only by actual simulations. Since the collision events are rare, the endeavors should be devoted to increasing the collision rate through artificial means while preserving the statistical characteristics of V_Δ .

The proposed strategy is to estimate $f_{V_\Delta}()$ from an additional simulation with only a fraction of the original simulation duration. The original riser model is slightly changed to provoke a very large collision rate. More specifically, the riser spacing for two risers is reduced. The basic notion of this strategy is that a small variation should dramatically increase the collision rate without modifying the structural nonlinearity and hydrodynamics by much. Going by this, the statistics of V_Δ is supposed to be almost unaltered. Moreover, the reduced spacing will affect V_0 , but not necessarily V_Δ , since V_0 and V_Δ are independent. In short, the basic idea is that provided the risers collide, the post-collision behavior should be similar. This hypothesis will be verified in Section 5.5. With a sizeable database of V_Δ obtained from the supplementary simulation, a distribution can be fitted to the empirical data.

Some practical considerations related to the implementation of the proposed scheme deserve mentioning. In an uncoupled analysis, changing the position of one (or both) riser(s) has no influence on the platform motions. However, this may not be true for a coupled analysis. In this case, an uncoupled analysis should be employed for the supplementary simulation, using the platform motions obtained from the main simulation cases.

5.4 Limit state evaluation

5.4.1 Yield criterion

An appropriate stress S_c allowed for collision should be well chosen. One concern is that S_c should constitute only a portion of the yield stress, in that reserve capacity is

needed to resist other types of loadings. In fact, adoption of the collision allowed design approach may further optimize the riser layout compared to no collision allowed design, but the extra costs need to be paid for. In the case study, the effect of different S_c will be investigated. For a given S_c , the probability of failure due to single collision event is written as

$$P_1 = \int_{u^*}^{\infty} f_{U_n}(u) du, \quad (5.12)$$

where u^* is the value corresponding to failure, i.e. $S_v(u^*) = S_c$.

The failure assessment for a no collision allowed design is a first passage problem, in which the occurrence of collision means failure. For this classical problem, it is commonly assumed that successive upcrossings of the failure threshold ($z=0$) are independent. This assumption is usually valid for rare events. Then, the probability of failure can be estimated as (Lin, 1967)

$$P_f = 1 - \exp(-N), \quad (5.13)$$

where $N = v_x^+(0)T$ is the average number of times $X(t)$ upcrosses the failure threshold in a duration T , while the procedure for estimating $v_x^+(0)$ has been explained in Section 5.3.2.

The procedure to obtain P_f slightly differs for a collision-allowed design. The main discrepancy is that on average only a fraction P_1 of all collisions result in failure. Hence, the mean number of failures in a duration T becomes NP_1 . Similarly, by assuming that the intervals between adjacent collision events follow a Poisson distribution, the failure probability is stated as

$$P_f = 1 - \exp(-NP_1). \quad (5.14)$$

Compared to Eq. (5.13), the variable N in Eq. (5.14) is allowed to be larger given that P_f remains unchanged, as P_1 is supposed to be much smaller than one, say 0.1. Since the greater N can be yielded by reducing the riser spacing, it is equivalent to state that,

compared to the other design principle, the collision allowed design tolerates a more compact riser layout given the same target P_f . The riser spacing may be further reduced by assigning a smaller P_1 in Eq. (5.14), which can be realized by strengthening the riser material. However, one should notice that an excessively large N is prohibited even in the current design principle and there is a lower bound in P_1 due to technical constraints.

5.4.2 Fatigue criterion

The fatigue assessment method adopted in this work is based on the S - N curve, in which the relationship between a constant stress range S and the number of cycles to failure N_f is prescribed for varied types of materials. In practice, the S - N is given explicitly by $N_f(S) = KS^{-m}$, where K and m are parameters determined from laboratory cyclic tests. For the case involving irregular stress cycles, the fatigue damage d is assumed to accumulate linearly (Miner's rule), such that

$$d = \sum_j \frac{n_j}{N_f(S_j)} = \frac{1}{K} \sum_j n_j S_j^m, \quad (5.15)$$

where n_j is the number of cycles corresponding to S_j . Generally, the structure is deemed to fail due to fatigue damage provided that d exceeds unity divided by a design safety factor (Det Norske Veritas, 2011). In fatigue design, it is normal to evaluate the mean damage $E[d]$, and apply a safety factor to account for uncertainties.

Two approximations are made to simplify the analysis. First, the collisions are conservatively assumed to occur at the same location, so that damages from all collision are valid to be summed up. Second, the stress ranges for each event are assumed to be equal to $\Delta S_{\max}$, which is a function of U_n and Θ (see Section 5.2.2). Some information that is useful for understanding the expression of $E[d]$ will be provided later. A sea state contains a number of collision events, while each event has several impacts. For each impact, a number of stress cycles ensue. Let the expected duration of a collision event be denoted as $E[T_c]$ and the average period of each stress

cycle T_s , which is determined from the local analysis. Then, the total number of stress cycles for the entire collision event is approximated as $N_c = E[T_c]/T_s$. Hence, the expected fatigue damage can be estimated as

$$\begin{aligned} E[d] &= N_c N \frac{1}{K} E[\Delta S_{\max}^m(\Theta, U_n)] \\ &= N_c N \frac{1}{K} \int_0^\infty \int_0^{\pi/2} \Delta S_{\max}^m(\theta, u_n) f_{\Theta U_n}(\theta, u_n) d\theta du_n \quad (5.16) \end{aligned}$$

where $N = v_x^+(0)T$. By using Eq. (5.1) and assuming that Θ and U_n are independent, the above double integral may be separated into single integrals, i.e.

$$E[d] = N_c N \frac{1}{K} \left[\int_0^\infty (b_0 + b_1\theta + b_2\theta^2)^m f_\Theta(\theta) d\theta \right] \left[\int_0^\infty u_n^m f_{U_n}(u_n) du_n \right]. \quad (5.17)$$

The remaining tasks are to determine N_c and $f_\Theta(\cdot)$.

First, consider the parameter $N_c = E[T_c]/T_s$. A collision event starts from the initial contact and ends when the risers move apart (i.e. $X(t) < 0$), such that T_c is approximated as the interval between the upcrossing and subsequent downcrossing of the threshold $z = 0$ (see Figure 5.5). Let $T_c^*(z)$ denote the upcrossing and downcrossing interval for an arbitrary threshold z . Now, plot out $E[T_c^*(z)]$ and extrapolate it to $z=0$, one can estimate $E[T_c]$ by letting $E[T_c] = E[T_c^*(0)]$. In fact, this is an approximation. Suppose that there are no contact forces during dynamic simulation. In this special case, the two risers will pass through each other after “contact”, and the predicted T_c represents the actual collision duration. In reality, the existence of contact force causes resisting forces between two risers, and reduces the collision duration. Thus, the predicted T_c is conservative.

The next step is to discuss the distribution of Θ . An extrapolation scheme similar to that for producing $f_{U_n}(\cdot)$ is established. But this time, only the first two moments (μ_Θ and σ_Θ) are extrapolated, so that $f_\Theta(\cdot)$ is a normal distribution. The reason for this

change is based on the fact that $\Delta S_{\max}(U_n, \Theta)$ depends less strongly on Θ , so that $f_{\Theta}()$ may not need to be that accurate.

Finally, it is worth mentioning that the damage computed by the above procedure constitutes only a fraction of that contributed by riser collision for a particular sea state. To carry out the complete fatigue assessment, the fatigue damage from other sources (e.g. dynamic tensions and bending moments) should be added in. However, the approach for the complete fatigue damage is well established and is out of the scope of this work. In practice, several sea states should be analyzed to reflect the long-term environmental condition. Low and Cheung (2012) presented a variety of methods for performing long-term fatigue analysis.

5.5 Case study

5.5.1 Global model

The local model has been presented in Section 5.2. A global model comprising two TTRs attached to a tensioned leg platform is used in the case study. A commercial software, Orcaflex (Orcina Ltd., 2012), is used to build the riser model and perform the tasks of time domain simulations. The contact effects are enabled between risers. The two risers are $7.7D$ apart ($D=0.268$) and have the same length of 520 m (20 m in the air and the rest immersed in a 500 m deep ocean). A single riser has 104 elements with equal length. Both wake interference and VIV are enabled in the model. Other properties of the riser model are presented in Table 5.1.

Commonly, fatigue damage assessment contains the analysis of several sea states, but for illustrative purposes, only one sea state is considered. The JONSWAP spectrum is chosen to characterize the sea state. To simulate extreme responses between two TTRs, a significant wave height of 15.7 m and an average period of 13.5 s are designed in the spectrum model. The wave spectrum is divided into 250 components of equal frequency interval from 0.2 rad/s to 1.2 rad/s. The current magnitude is 1 m/s

and uniformly distributed along the water depth. The incident wave and current are collinear, aligned in a direction such that the two risers are in tandem relative to the environmental loads. This sea state represents the most critical load case duration the service life.

The wave forces acting on the risers are represented by Morison's equation. The Iwan and Blevins wave oscillator model (Iwan and Blevins, 1974) is used to simulate VIV responses, while WIO is attended by the Blevins wake field model (Blevins, 2001). A decoupled analysis scheme is used. RAOs for a six degree-of-freedom rigid body motions are calculated in advance. For simplicity, the second-order slow drift responses are neglected, as they are generally not critical to the riser clashing problem (Det Norske Veritas, 2009).

The total simulation duration considered for the proposed approach is 12 h ($4 \times 3h$). About 6 collision events are captured within the 12-h simulation, but this quantity is far from sufficient for a direct probabilistic analysis. This necessitates the adaptation of the proposed method, which does not rely on the number of collision events. For verification, Monte Carlo simulations (1104 h) are performed, and consequently 604 collision events are captured. The summary of this database has been furnished in Section 5.3.1 to verify that V_0 and V_Δ are uncorrelated. Moreover, other results are presented in separate subsection as follows.

5.5.2 Collision rate

Figure 5.7 plots the crossing rate $v_X^+(z)$ calculated based on 12 h simulation. Naess method and the Hermite model are used to extrapolate $v_X^+(z)$ to the collision threshold (see Section 5.3.2). The collision rates obtained by Naess method is $1.48 \times 10^{-4} \text{ s}^{-1}$, which is close to the Monte Carlo result of $1.51 \times 10^{-4} \text{ s}^{-1}$. However, the Hermite prediction, which is $6 \times 10^{-5} \text{ s}^{-1}$ is too non-conservative. According to the

theory of random vibrations (Lin, 1967), the crossing rate depends on the joint probability density $f_{\dot{x}\ddot{x}}()$. As explained in Section 5.3.2, the Hermite model may be able to represent $f_{\dot{x}}()$ accurately, but the striking nonlinear effects in this problem make it difficult to predict $f_{\dot{x}}()$, not to mention $f_{\dot{x}\ddot{x}}()$. The resulting errors may explain the relatively poor performance of the Hermite model at least in this problem.

Figure 5.7 Extrapolation of crossing rate $v_X^+(z)$

5.5.3 Initial impact velocity

The process $X(t)$ is sampled at 0.01 s interval and $\dot{X}(t)$ is attained by numerical differentiation. Using the procedure described in Section 5.3.4, the statistical moments of $\dot{X}(t)$ against different x are evaluated. $\mu_{\dot{X}}(x)$ is found to be almost zero for all x . Figure 5.8(a) plots $\sigma_{\dot{X}}(x)$ against x ; the U-shape trend is believed to be contributed by two conflicting factors. One factor is that the lateral stiffness of a TTR increases with displacement, so that $\sigma_{\dot{X}}(x)$ reduces with x due to increasing structural resistance. On the other hand, wake interference does encourage vibration as the risers

get closer (larger x). When the two risers are sufficiently close, the second factor dominates the overall riser response, thus directing the trend upwards. Figure 5.8(b) plots the trend of $\gamma_{\dot{X}}(x)$. The negative skewness is ascribed to the geometric nonlinearity, which skews the response towards lower structural resistance (smaller x), and it finally leads to a left-skewed response.

Figure 5.8 Extrapolation of the moments of $\dot{X}$: (a) standard deviation; (b) skewness

In view of the substantial nonlinearity in both $\sigma_{\dot{X}}(x)$ and $\gamma_{\dot{X}}(x)$ (see Figure 5.8(a)-(b)), they are fitted to a third-order polynomial and extrapolated to the collision threshold. The extrapolation range indicated in Figure 5.8(a)-(b) requires to be taken care of. Amongst other things, the data range used for extrapolation should correspond to a smooth and well behaved curve. Thus, it is sensible to neglect data that are erratic at the tail region due to insufficient samples. Moreover, $\gamma_{\dot{X}}(0) = -1.83$, implies that the skewness is significant. It is of interest to show the prediction of $\sigma_{\dot{X}}(x)$ based on the Hermite model, hence, it is included in Figure 5.8(a). There is obvious discrepancy between the actual result and that produced by the Hermite model. Moreover, the Hermite model always predicts zero skewness for $\dot{X}$, which is wrong in most cases such as the present one.

Figure 5.9 Pdf of V_0

Figure 5.9 compares the pdfs of V_0 constructed by various methods mentioned. The proposed approach uses three moments and the SLD to approximate $\dot{X}_c$, and as a

result, $f_{V_0}()$ is procured (see Section 5.3.4). The Hermite model estimates $\dot{X}_c$ following the procedure presented in Section 5.3.4. Monte Carlo simulations are treated as the benchmark; here the pdf (in the form of a scaled histogram) is constructed from the actual impact velocities. It is found that the proposed approach agrees well with Monte Carlo simulations. In contrast, the Hermite model produces a much fatter tail, which is an unrealistic prediction. To show the critical tail region of the pdfs more clearly, the probability of exceedance, $P_e() = 1 - F_{V_0}()$, is plotted out in Figure 5.10. Here, $F_{V_0}()$ denotes the cumulative distribution function. The tail performance of the proposed approach is found to be favorable.

Figure 5.10 Probability of exceedances of V_0

5.5.4 Differential impact velocity

The supplementary simulation (see Section 3.5) for estimating $f_{V_A}()$ is set at 4 h, which is 33% of the main simulation duration of 12 h. Generally, 1D or 2D reduction in riser spacing is sufficient to dramatically increase the number of collisions. But bear in mind that more collisions may not always pay off, as the independence

between collision events may become questionable (small time interval between adjacent events). In this work, a riser spacing of $5D$ is used, which results in 330 collision events.

Figure 5.11 Comparison of Monte Carlo simulation and the supplementary simulation: (a) pdf; (b) probability of exceedance

Next, $f_{V_{\Delta}}()$ is achieved from the samples of V_{Δ} . A first attempt is naturally to fit the data to a theoretical distribution, unfortunately, in the present case, none of the commonly known distributions (such as exponential, Weibull, lognormal) are found to be suitable. To avoid unnecessary errors made due to inappropriate fitting, and in the spirit of simplicity, the straightforward histogram method is employed.

Figure 5.11(a) compares the pdf of V_{Δ} estimated from the $5D$ riser spacing with the Monte Carlo result ($7.7D$ spacing). For a better understanding of the influence of the riser spacing on $f_{V_{\Delta}}()$, another set of 9-h simulation is conducted with $6D$ riser spacing, which generates 230 collision events. The approximate agreement between the three curves suggests that $f_{V_{\Delta}}()$ is not very sensitive to the riser spacing, which demonstrates the hypothesis made in Section 5.3.5. Figure 5.11(b) plots the probability of exceedance. The supplementary simulations with $5D$ and $6D$ spacing seem to be conservative. Taking an exceedance probability of 0.1 as an example, the $5D$ spacing simulation overestimates V_{Δ} by 37%. To sum up, the accuracy of the proposed supplementary simulation method can be regarded to be satisfactory considering the complexity of the problem. Moreover, this simple method generates a sizable amount of data at a modest computational cost.

5.5.5 Yield assessment

Following the procedure presented in Section 4.1, the probability of failure for the yield criterion can be calculated for a given S_c . Figure 5.12 displays the variation of P_f with different varied S_c , which is computed by different approaches. The disparity between the proposed approach and Monte Carlo simulations is acceptable in engineering sense. Although the approach is conservative, there is no obvious alternative approach that is equally efficient.

Figure 5.12 Plot of P_f against S_c

Remember that the Hermite model significantly underestimates the collision rate. In order to isolate this effect, the P_f corresponding to the Hermite model is calculated using the correct collision rate. Observed from Figure 5.12, the Hermite model is found to be more conservative than the proposed approach. However, the disparity between these two methods is almost unperceivable as they follow the same procedure for constructing $f_{V_\Delta}()$, in which the most error rests. The advantage of the proposed approach may not be obvious in this case, but it is in fact rewarding due to the accurate prediction of $f_{V_0}()$. Considering other cases where V_0 is dominant due to, e.g. different riser configurations, the superiority of the proposed approach will be more significant.

The design P_f can be reduced by decreasing either P_1 or the collision rate (see Eq. (5.16)). In practice, the collision rate can be controlled through various ways, for instance adjusting the pretension or the riser spacing. However, in this work, it is unaffordable to target at lower collision rates, due to practical concerns on Monte

Carlo simulations. In the current circumstance, the Monte Carlo simulations used a total CPU time of 28 days, running on a computer with four 3.4 GHz processors. The computational cost would be even higher for cases with lower collision rates. The alternative of lowering P_1 requires a more accurate estimate of $f_{U_n}()$, which implies a longer supplementary simulation for securing a more accurate tail in $f_{V_\Delta}()$.

5.5.6 Fatigue assessment

With reference to Eq. (5.15), the following values of the $S-N$ parameters are adopted: $m = 4.7$, $K = 10^{17.446}$, corresponding to high strength steel (Det Norske Veritas, 2011). First of all, the number of cycles per second is obtained from the local analysis, i.e. $T_s = 1/60$ s. The remaining information required to calculate the fatigue damage are μ_Θ , σ_Θ , $E[T_c]$, and $E[U_n^m]$. Table 5.2 presents the results obtained by the proposed approach and Monte Carlo simulations. For the proposed approach, μ_Θ , σ_Θ and $E[T_c]$ are acquired from extrapolation, as shown in Figures 5.13 and 5.14. In the Monte Carlo simulation, the kissing angles Θ are collected from the initial impact of each collision event, while T_c is the duration between the first and final impact of each collision event.

It is observed that μ_Θ and σ_Θ are in good agreement, whereas the proposed approach slightly overestimates T_c and $E[U_n^m]$. The overestimation of $E[U_n^m]$ is due to the error in $f_{V_\Delta}()$, while the reason for the conservative prediction of T_c has been discussed in Section 5.4.2. The fatigue damage is over predicted by 22%, which should be acceptable for engineering purposes, in consideration of the huge uncertainty of fatigue in general. In this case, the fatigue damage is rather small, the reason being it only pertains to a single 3-h sea-state, and the effects from riser tensions, etc have not been considered. A statement is also valid that fatigue is not the governing failure mechanism for the riser system at least in this work.

(a)

(b)

Figure 5.13 Extrapolation of the moments of Θ : (a) standard deviation; (b) skewness

Figure 5.14 Extrapolation of the expected collision duration

Table 5.2 Comparison of parameters related to fatigue assessment

	Proposed approach	Monte Carlo simulation
μ_θ (deg)	57	53
σ_θ (deg)	28	26
$E[T_c]$ (s)	1.95	1.75
$E[U_n^m]$	0.0978	0.0841
Expected fatigue damage	9.1×10^{-8}	8.2×10^{-8}

5.6 Chapter summary

This chapter presented a new approach for probabilistic assessment of collision allowed riser design. For comprehensive understanding, both local and global stages were presented. The local finite element analysis simulated the impact between two riser sections. The useful outcomes were the stresses pertaining to different normal velocities U_n and kissing angles θ . For the global analysis, environmental forces including the hydrodynamic effects were considered. The statistics of collision

parameters such as U_n and Θ were predicted. Particularly, for better predicting U_n , a short supplementary simulation was elaborated.

The proposed approach was tested on a TTR system as it is susceptible to mutual riser clashing in deep water, which is currently a great concern in offshore industry. The global and local analyses were combined to evaluate the yield and fatigue limit states. The results obtained by the proposed approach were found to be in good agreement with the expensive Monte Carlo simulations.

CHAPTER 6 CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

Prior to riser clearance assessment, a determination of failure criterion is necessitated. In this regard, two options are furnished in the practical codes, namely no-collision-allowed design and collision-allowed design. This thesis presented practical approaches on assessing the reliability of riser systems against riser-to-riser collisions. The contributions are mainly on two aspects, i.e. the probabilistic modeling of the riser clashing problems and the enhancement of efficiency in the proposed approaches. The main findings are categorized into three themes.

Reliability assessment based on no-collision-allowed design

In a no-collision-allowed riser design, the engineers need to guarantee that the probability of failure due to riser clashing is very small. Thus, an approach that caters to assessing the probability of riser clashing in a given model is desired. To this end, a practical approach was proposed for evaluating P_f from limited simulation data, assisted by a well established extrapolation technique for extreme responses. The main findings are summarized as follows:

- By comparing with the post-normalized clearance, the pre-normalized clearance, $X(t)$, is found to be more robust and appropriate to represent the severity of motion with respect to clashing events.
- The concept of critical pairs is critical for expediting the production of a surrogate process $\hat{X}(t)$, which has the same peaks as that of $X(t)$. A two-stage sieving procedure is demonstrated to be capable of identifying the critical pairs.
- The proposed approach based on 30-h simulation data gives favorable prediction on P_f comparing with the Monte Carlo simulations but slightly errs on the conservative side.

- The presence of hydrodynamic interference significantly increases the likelihood of riser collisions.
- Naess method is robust for the crossing rate extrapolation in riser clashing problem while the GEV method is sensitive to the amount of data adopted.

Prediction of riser clashing considering multiple sources of uncertainties

One deficiency of the above work is that the randomness is considered only from the irregular wave train. However, in reality, uncertainties also come from other sources. It is found that the selected uncertain parameters have significant impacts on the probability of riser clashing. In view of these, the approach introduced in Chapter 3 is improved by allowing considerations of multiple uncertainties. The reliability assessment in this work relies on the classical reliability integral. An approximation scheme termed as perturbation method is employed to solve this integral. The main findings are summarized as follows:

- The prediction by the proposed approach agrees well with that by the Monte Carlo simulations.
- The case study investigates four random parameters, namely current, drag coefficient, RAO factor and riser mass. Each one has meaningful impacts on the P_f . On the whole, the presence of additional uncertainties increases the P_f by an order of magnitude.
- All the four random variables are transformed to a standard normal space, by which an improvement of accuracy is yielded.
- The new extrapolation process involves the first and second order derivatives of crossing rates. Compared to crossing rates, the extrapolation of their derivatives renders the approach more robust. Naess method is demonstrated to be suitable for the extrapolations.
- The second order perturbation method is appropriate for evaluating P_f with respect to riser clashing.
- Only a small amount of additional simulations (48 h) is required to implement the reliability analysis considering four random parameters.

Reliability assessment based on collision-allowed design

In collision-allowed riser design, the main problem lies in the fact that the existing approaches are not practical. In view of this, a general approach is proposed to assess the riser interference following the collision allowed criterion. The approach comprises a local impact analysis and a global riser simulation. The local stage aims at attaining the collision-induced stresses pertaining to different impact velocities U_n and kissing angles θ . The global stage considers all the effects crucial for riser clashing. The goal is to predict the statistics of collision parameters, e.g. U_n and θ with a limited amount of simulation data. The main findings are summarized as follows:

- Impact velocity is found to be the most critical parameter determining the severity of a collision. Kissing angle, contact angle and participating mass have meaningful impacts on the collisions. Collision duration, stress cycle and kissing angle are responsible for determining the fatigue damage due to riser clashing.
- A Gaussian smoothing technique is shown to be effective to reveal the trend of statistics of pre-collision responses.
- The impact velocity is divided into two components, V_0 and V_Δ , pertaining to the pre-collision response and the post-collision response respectively. V_0 and V_Δ are shown to be independent. V_0 is successfully obtained by extrapolating the pre-collision responses to the clashing threshold. V_Δ is approximated through supplementary simulations. The approximation is found to be good by comparing with the actual post-collision behavior.
- The shifted lognormal distribution is appropriate to describe the impact velocity.
- This approach is efficient because only a short duration supplementary simulation is required. In this case, the simulation duration is 4-h.
- The proposed approach is found to yield good prediction by comparing with Monte Carlo simulations.

- The collision-allowed riser design results in a more compact riser design comparing with no collision allowed design.

6.2 Recommendations for future work

In the literature, a substantial amount of work is devoted to the topics relevant to riser clashing. For example, the establishment of numerical tools on wake interference and VIV; riser local impact analysis using FE method; the control of riser motions in order to avoid mutual collisions; prediction of wake instability which is likely to cause riser collisions. The work in this thesis has applied some of the state-of-the-art in the literature. The main contributions are on developing practical approaches for assessing the reliability of riser clashing against mutual collisions. The recommendations for possible future work are summarized as follows:

Figure 6.1 Upper section of the four-riser model (static state)

Reliability assessment in a riser system (multiple riser pairs)

- All the proposed approaches in this thesis are based on only two risers. However, in practice, a riser system comprises more than a pair of risers (see Figure 6.1). Thus, it is useful to develop approaches to evaluate the probability of failure when more than two risers are included. To be practical, a necessary step is to sieve out a handful of critical riser pairs, such that the probability of

riser clashing in the riser system can be deduced from that in the critical riser pairs. A sieving method is required to select the critical riser pairs. The approach should also allow the consideration of multiple sources of uncertainties in the model.

Uncertainties in hydrodynamic interference models

- In time domain simulations, the numerical models for hydrodynamic interferences are assumed to be properly calibrated. In fact, only the recommended parameters were applied in the models. Thus, it is recommended to calibrate the numerical models through CFD analysis. Furthermore, hydrodynamic interference is the most decisive driver of riser clashing. Since the uncertainties have striking impacts on the failure probability, a comprehensive sensitivity study should be conducted in the VIV and WIO models with respect to their parameters. An approach incorporating the uncertainties in VIV and WIO models as well as those considered in Chapter 4 is of great value in the actual design.

Fatigue life considering effects of riser clashing

- In a collision allowed design, riser clashing is not an impossible scenario. A severe riser clashing may initiate a crack on the riser surface. This crack is likely to dramatically reduce the fatigue life of the riser system. Thus, it is of great significance to reevaluate the fatigue life of a riser system within the service life by considering the residual effects of riser collisions. Under this circumstance, the fatigue crack growth analysis should be applied.

Reliability assessment of clashing between flexible jumpers

- Flexible jumpers in a hybrid riser system (see Figure 1.4) should be treated in a different manner from common risers. For one thing, they are connected between a platform and a buoyancy can below the sea surface instead of the seabed. For another thing, both ends of the jumpers are movable while one end

of a riser is fixed to the seabed. Besides, the distribution of external forces, including waves, surface current and vessel motions, along the jumpers significantly differs from that of risers. In design, flexible jumpers are arranged in bundles, thus the probability of mutual clashing is high due to the small spacing. In short, a reliability analysis approach for flexible jumpers against mutual collisions is desired in offshore industry.

Independence of V_0 and V_Δ

- In Section 5.3.3, an assumption that V_0 and V_Δ are independent has been made. This is a critical assumption for the proposed approach in Chapter 5. Therefore, it is worth investigating more advanced methods in a future work to test the independence between V_0 and V_Δ . Some existing techniques can be found in the literature such as Blum et al. (1961), Fogle et al. (2008) and Skaug and Tjostheim (1993).

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APPENDIX A

Figure A.1 depicts the cross-sectional view of two riser elements j and k , initially in contact. Since it is the relative position that is of interest, assume that Riser 1 is stationary, and Riser 2 is moving away with a normal velocity U_n and a tangential velocity U_t . The clearance $C_{jk}(t)$ is defined as the distance between center-to-center, minus D . After Δt , $C_{jk}(t)$ can be expressed as

$$\begin{aligned} C_{jk}(t) &= \sqrt{(D + U_n \Delta t)^2 + (U_t \Delta t)^2} - D \\ &= \sqrt{D^2 + 2U_n D \Delta t + (U_n^2 + U_t^2)(\Delta t)^2} - D \end{aligned} \quad (\text{A.1})$$

Since Δt is small, $(\Delta t)^2$ is negligible and Eq. (A.1) can be simplified to

$$C_{jk}(t) = D \left(1 + \frac{2U_n \Delta t}{D} \right)^{1/2} - D \approx U_n \Delta t. \quad (\text{A.2})$$

Hence, at the time of contact, $\dot{C}_{jk}(t) = U_n$. For clarity, it is assumed that the risers are moving apart, but the same principle applies when Riser 2 is approaching Riser 1. Hence, by reference to Eq. (3.14), it is apparent that $\dot{X}(t)$ is the normal velocity during contact. The assumption is that the closest element pair (j, k) is invariant within a small time step Δt . In practice, even if (j, k) does change (to an adjacent pair, for example), the consequence is small.

Figure A.1 Cross-sectional view of two riser elements