Graphene mode locked fiber laser

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GRAPHENE MODE LOCKED FIBER LASERS

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Table of Contents

Acknowledgements	i
Table of Contents	ii
Summary	v
List of Figures	ix
List of Acronyms	xiv
Chapter 1 Introduction	1
1.1 Background	1
1.2 Motivation and Objectives	4
1.3 Major Contribution of the Thesis	9
1.4 Organization of the Thesis	12
Chapter 2 Theory of Vector Soliton Propagation in Fiber Lasers	15
2.1 Pulse Propagation in Optical Fibers	17
2.1.1 Dispersion and Birefringence in Optical Fibers.....	17
2.1.2 Nonlinear Effects in Optical Fibers	21
2.1.3 Nonlinear Schrödinger Equation and Soliton Solutions	22
2.2 Pulse Propagation in Erbium-doped Fiber	25
2.2.1 Gain Profile of Erbium-doped Fibers	25
2.2.2 Ginzburg-Landau Equation and Dissipative Soliton Solution.....	27
2.3 Fundamentals of Mode-locked Fiber Lasers	29
2.3.1 Principle of Mode Locking Technique	30
2.3.2 Active and Passive Mode-locking	31
2.3.3 Saturable Absorbers Mode Locking Fiber Lasers	33
2.3.4 Graphene Mode locker in Vector Soliton Fiber Lasers	38

2.4	Modelling of the Vector Soliton Fiber Laser	42
Chapter 3	Vector Soliton Operation in Graphene Mode-locked Fiber Lasers	45
3.1	Polarization Rotation and Locking of Multiple Vector Solitons.....	47
3.1.1	Experimental Results	48
3.1.2	Discussions	56
3.2	Multiple Vector Soliton Distribution	57
3.2.1	Bunches of Multiple Vector Solitons.....	60
3.2.2	Random Static Distribution of Vector Solitons	62
3.2.3	Restless Vector Solitons	63
3.2.4	Vector Soliton Rain.....	64
3.2.5	Giant Pulse State	68
3.2.6	Discussion	69
3.3	Summary	70
Chapter 4	Quasi-periodicity of Vector Solitons in Graphene Mode-locked Fiber Lasers	72
4.1	Experimental Results.....	75
4.2	Summary	85
Chapter 5	Noise-like Pulses and Rogue Waves in Graphene Mode-locked Fiber lasers	87
5.1	Noise-like Pulses	88
5.1.1	Experimental Results	89
5.1.2	Simulation Results	96
5.1.3	Discussions	100
5.2	Optical Rogue Wave	102
5.2.1	Experimental Results	104
5.2.2	Discussions	110
5.3	Summary	112

Chapter 6	Cavity Induced Modulation Instability of Fiber Lasers	114
6.1	Experimental Results.....	116
6.2	Discussion	123
6.3	Summary	124
Chapter 7	Dark Solitons in Fiber Lasers	126
7.1	Scalar Dark Solitons in All-normal Dispersion Fiber Lasers.....	128
7.1.1	Experimental Results	128
7.1.2	Numerical Results	135
7.1.3	Discussions and Summary	139
7.2	Dark Solitons in Dispersion-managed Fiber Lasers.....	141
7.2.1	Experimental Results	142
7.2.2	Numerical Results	148
7.2.3	Summary	149
Chapter 8	Dark-bright Solitons in Fiber Lasers.....	150
8.1	Dark-bright Solitons with Anomalous Cavity Dispersion	152
8.2	Dark-bright Solitons with Normal Cavity Dispersion.....	159
8.3	Dark-bright Solitons with Near-zero Cavity Dispersion.....	164
8.4	Summary	169
Chapter 9	Conclusions and Future Works.....	170
9.1	Conclusions	170
9.2	Recommendation for Future Works	173
Author's Publications		175
References.....		177

Summary

Study on the optical solitons formed in the erbium-doped fiber lasers is an attractive area of research. Solitons are stable localized wave packets that can propagate long distance in dispersive media without changing their shapes. They have been observed in different physical systems. The formation of optical solitons in the single mode fibers is a well-known phenomenon that has been extensively investigated. Either the bright solitons or dark solitons can be formed in a single mode optical fiber. Specifically, as shown by the nonlinear Schrödinger equation (NLSE), the bright solitons are formed in the anomalous dispersion single mode fibers (SMFs), and the dark solitons are formed in the normal dispersion single mode fibers. Furthermore, taking into account the fiber birefringence, various types of vector solitons, including the bright-bright vector solitons, dark-dark vector solitons and dark-bright vector solitons, could be formed in SMFs. The dynamics of the vector solitons are mathematically described by the coupled NLSEs.

Fiber laser is a perfect platform to generate and study the scalar and vector optical solitons. Different from the case of optical single mode fibers, the optical solitons formation in fiber lasers is a result of the mutual interaction among the cavity dispersion, fiber nonlinearity, laser gain saturation and gain bandwidth filtering. Strictly speaking, the solitons in fiber lasers are dissipative solitons which are governed by the Ginzburg-Landau Equation (GLE). However, NLSE solitons could also be obtained in fiber lasers under certain conditions. In the erbium-doped fiber lasers, bright solitons have been obtained

by passive mode locking of the lasers that could be achieved by inserting a saturable absorber into the laser cavity. Recently, graphene as a promising saturable absorber that has super broad operation bandwidth was proposed to use in the fiber lasers to generate vector solitons, which opened the study of bright-bright vector solitons in the mode-locked fiber lasers. Besides, scalar dark solitons and vector dark-bright solitons formed in the non-mode-locking fiber lasers is becoming another attracting field. In the current dissertation, the author presents a series of investigations on the bright-bright vector solitons formed in fiber lasers mode locked with graphene. Novel types of soliton generation in the non-mode-locking fiber lasers were also experimentally investigated and presented.

Firstly, vector soliton fiber lasers passively mode-locked by atomic layer graphene were set up. Graphene is a two dimensional atomic layer nanomaterial with unique optical properties. It possesses wavelength independent ultrafast saturable absorption. The author experimentally investigated the vector soliton operation of the graphene mode locked fiber lasers. It was shown that using graphene as a saturable absorber to mode lock the erbium-doped fiber lasers, vector solitons exhibit polarization rotation and polarization locked effects, as characterized by the new sets of sidebands in the optical spectrum. Moreover, various multiple vector soliton operations and interactions are experimentally investigated. In particular, the author firstly observed the vector soliton rain effect, which exhibits polarization rotating and polarization locked features in the graphene mode locked fiber lasers. The nonlinear dynamics of the vector solitons in graphene fiber laser was also investigated. Quasi-periodicity of single and multiple vector solitons in graphene mode-locked fiber laser was

experimentally verified as characterized by the appearance of the new extra sidebands in the optical spectrum.

Apart from the vector soliton operations of the graphene mode-locked fiber lasers, two distinct states of mode-locked pulses formed due to the soliton interactions were experimentally observed. Noise-like (NL) pulses in the graphene mode-locked fiber lasers were experimentally observed. It was shown that the formation of the NL pulses was due to the pulse-to-pulse attraction, which physically correlates with the ultrafast recovery feature of the graphene saturable absorber. Another interesting state of pulses observed in graphene mode-locked fiber lasers is the optical rogue wave generation. Rogue waves are the pulses with extreme high intensity, which were occasionally generated from the multiple pulse collisions, which is caused by the vector soliton interactions in the laser cavity. The character of the rogue wave was confirmed by the probability distribution of the peak pulse intensities.

In the non-mode-locking fiber lasers bright solitons can also be formed due to the modulation instability (MI) of light in the fiber laser cavity. The author experimentally investigated the MI of fiber lasers and verified that there are two kinds of MI, the conventional MI and the cavity induced modulation instability (CIMI). By carefully setting the cavity detuning, highly stable GHz level repetition rate pulse trains were generated in a ring cavity fiber laser. Study on the property of the generated pulse trains suggests that they were formed due to the cavity induced modulation instability lasing of the fiber laser. The author also observed the coexistence of the conventional modulation instability and the cavity induced modulation instability in the same fiber laser.

In the normal dispersion cavity fiber lasers without mode-locking, the evidence on the existence of the dark solitons was experimentally provided. Measured by a high speed system with a bandwidth of 33 GHz, dark solitons were unambiguously confirmed. Bunches of the dark solitons were also confirmed which was previously understood as dark pulses in a low speed system. Furthermore, dark solitons could not only be formed in the normal dispersion fiber lasers but also in the dispersion managed fiber lasers with normal cavity dispersion, suggesting that dark soliton is an intrinsic feature of the fiber lasers.

At last, using the same high speed measurement system, dark-bright solitons were experimentally investigated in the dispersion managed fiber lasers under different cavity dispersions. The author experimentally revealed the mechanism of the dark-bright soliton formation in the fiber laser. A technique for generating and detecting stable dark-bright soliton pulse trains in fiber lasers was proposed. It is envisaged that dark-bright solitons could be of potential applications in the future optics communications.

List of Figures

Figure 2.1 Pump scheme of erbium-doped fiber.	25
Figure 2.2 The absorption and emission spectra of the EDFs [1].....	27
Figure 2.3 Schematic diagram of NPR mode locking principle.	33
Figure 2.4 Schematic structure of the SESAM.....	35
Figure 2.5 Energy band structure of graphene.....	39
Figure 2.6 An experimental setup of the passively mode-locked fiber ring laser which consists of erbium-doped fiber, a wavelength-division multiplexer, a 1- input/2-output coupler, a polarization independent isolator, a polarization controller, and the graphene mode locker.	41
Figure 3.1 A schematic of the fiber laser setup. PC: Polarization controller; EDF: Erbium-doped fiber; SMF: Single-mode fiber; WDM: Wavelength- division multiplexer.	48
Figure 3.2 Vector soliton emission of the fiber laser. (a) Soliton spectra. Blue line: the total laser emission without passing through a polarizer; Red and Green lines: the two orthogonal polarization components resolved with an external cavity polarization beam splitter. (b) Oscilloscope trace of the solitons.	50
Figure 3.3 Oscilloscope traces of the pulse train in a phase locked vector soliton operation state. Blue line: the total laser emission without passing through a polarizer; Red and Green lines: the two orthogonal polarization components resolved with an external cavity polarization beam splitter.....	53
Figure 3.4 A polarization rotation vector soliton state of the laser. (a) The total and the polarization resolved soliton pulse trains with multiple solitons in cavity. (b) Optical spectra of the solitons. Blue line: the total laser emission without passing through a polarizer; Red and Green lines: the two orthogonal polarization components resolved with an external cavity polarization beam splitter.	54
Figure 3.5 A typical multiple vector soliton bunch state of the fiber laser. (a) and (b): Two states of the oscilloscope trace; (c) The details of the soliton bunch; (d) Polarization resolved optical spectra. Blue line: the total output; Green line: Output along the vertical axis; Red line: Output along the horizontal axis. (e) Autocorrelation trace of the pulse.	60

Figure 3.6 Random static vector soliton distribution at a low pump power.	62
Figure 3.7 Bunch of restless vector soliton bunch (video). (a) Soliton bunch moving in the cavity (Media 1); (b) Solitons oscillating in the bunch (Media 2).	63
Figure 3.8 Polarization locked vector soliton pulse trains: (a) A typical soliton rain at a pump power of 140 mW; (b) zoom-in of the soliton condense phase; (c) Soliton rain at a pump power of 200 mW; (d) Snapshot of the vector soliton rain (Media 3).	65
Figure 3.9 (a) Oscilloscope trace of the polarization locked vector soliton rain. Blue line: the total output; Green line: along the horizontal axis; Red line: along the vertical axis. (b) Oscilloscope trace of the polarization rotating vector soliton rain. Blue line: the total output; Green line: along the horizontal axis; Red line: along the vertical axis. (c) The optical spectrum of the polarization rotating vector soliton rain.	67
Figure 3.10 Giant pulse state when the soliton rain exploded at 200 mW pump power. (a) Oscilloscope trace; (b) 10 ns scale of the giant pulse; (c) Oscilloscope trace; (d) optical spectrum.	68
Figure 4.1 A schematic diagram of the graphene mode-locked fiber laser.	75
Figure 4.2 A state of the polarization locked vector soliton operation of the laser. (a) The total and the polarization resolved optical spectra of the vector soliton. The extra set of spectral sidebands is indicated by the arrows; (b) oscilloscope trace of the vector soliton pulse train. (c) The polarization resolved oscilloscope trace of the vector soliton pulse train.	77
Figure 4.3 Polarization locked vector soliton pulse trains with (a) two solitons and (b) four solitons in the cavity.	79
Figure 4.4 The total and the polarization resolved optical spectra of a polarization rotation vector soliton of the laser. The extra set of spectral sidebands is indicated by the arrows.	80
Figure 4.5 The total and the polarization resolved optical spectra of the vector solitons under quasi-periodic evolutions. The extra set of spectral sidebands is indicated by the arrows. (a) The case where the vector soliton has a large polarization ellipse; (b) The case where the strength of the two orthogonal polarization components of the vector soliton was comparable.	81

Figure 4.6 Oscilloscope traces of the vector soliton under quasi-periodic evolution. Blue line: the total vector soliton pulse train; Green line: the horizontal axis; Red line: the vertical axis.....	82
Figure 4.7 The quasi-periodic evolution of two vector solitons coexisting in cavity.....	85
Figure 5.1 A schematic of the experimental setup.....	89
Figure 5.2 Soliton operation of the fiber laser. (a) Optical spectrum measured. (b) The mode locked pulse train. (c) One “single pulse” of (b) detected by a high-speed oscilloscope.	90
Figure 5.3 (a) Multiple solitons state. (b) NL pulse operation of the fiber laser. (c) Optical spectrum. (d) Pulse profile measured with a high-speed oscilloscope. (e) Autocorrelation trace (pulse width corresponds to the sharp peak is about 1.2 ps).....	92
Figure 5.4 Multiple soliton pulses operation in a 30 m long cavity. (a) Optical spectrum measured. (b) Solitons within a bunch measured by a high-speed oscilloscope.....	93
Figure 5.5 Multiple soliton pulses operation in a 20 m long cavity. (a) Optical spectrum measured. (b) Solitons within a bunch in the cavity at a high pump power. (c) A twin pulse state at a low pump power.....	95
Figure 5.6 Numerically calculated pulse formation under different pumping strength. (a) The single pulse profile under gain=100 and (b) the corresponding optical spectrum of (a). (c) The NL pulse profile under gain= 6000 and (d) the corresponding time-averaged optical spectrum of (c). Please note that the optical pulse and spectrum of the NL pulse had been averaged over 1000 round trips.	96
Figure 5.7 Evolution with respect to cavity roundtrips of (a) the optical spectrum and (b) the optical pulse profile of the NL pulse. (c) The change of time-averaged optical spectra of the NL pulse under different laser gain.	98
Figure 5.8 Experimental setup of the fiber ring laser. PC polarization controller, EDF erbium doped fiber, SMF single-mode fiber, WDM wavelength-division multiplexer.	104
Figure 5.9 A typical mode locking state. (a) Optical spectrum and (b) oscilloscope trace.	105

Figure 5.10 A typical giant wave state. (a) Optical spectrum, (b) oscilloscope trace and (c) resolved trace in 20 GHz high speed oscilloscope.....	107
Figure 5.11 Optical rogue waves operation of the fiber laser. (a) Optical spectrum of the optical rogue wave; (b) Video record of a typical rogue wave emission measured by oscilloscope (Media 4).	108
Figure 5.12 Histogram (log scale) of the rogue wave state showing the distribution of the optical intensity maxima for 10^5 trace events.	109
Figure 5.13 Histogram (log scale) of the stable giant pulse state showing the distribution of the optical intensity maxima for 10^5 trace events.	109
Figure 6.1 Schematic illustration of the erbium doped fiber laser.	116
Figure 6.2 A state of 3 GHz pulse trains the fiber laser. (a) Oscilloscope trace; Inset: Trace in 3 ns scale; (b) Autocorrelation trace; (c) Optical spectrum.	118
Figure 6.3 A state of 32 GHz pulse trains the fiber laser. (a) Oscilloscope trace; (b) the autocorrelation trace; (c) Optical spectrum.	120
Figure 6.4 A state of high repetition rate pulses. The repetition rate is about 285 GHz. (a) Autocorrelation trace in 300 ps scale; Inset: trace in 180 ps scale. (b) Optical spectrum. Inset: Zoom-in of a spectral peak.	122
Figure 7.1 Schematic diagram of the all-normal dispersion fiber laser.	128
Figure 7.2 A train of the dark pulse emission of the fiber laser. Upper trace: laser emission measured with 33 GHz detection bandwidth; Lower trace: laser emission measured with 1 GHz detection bandwidth.....	130
Figure 7.3 Oscilloscope traces of the laser emission under different pump power. (a) ~68 mW (b) ~100 mW (c) ~400 mW (d) ~700 mW. The two curves in the trace represent two orthogonal polarization components.	130
Figure 7.4 Optical spectra of the laser under CW and dark pulse emissions...	131
Figure 7.5 Zoom-in of the laser emission. Upper trace: oscilloscope trace measured with 33 GHz bandwidth. Lower trace: oscilloscope traces measured with 5 GHz and 10 GHz bandwidth.....	134
Figure 7.6 A typical state of the dark soliton operation of the fiber laser numerically calculated. (a): The evolution of the dark soliton with the cavity round trips; (b) The soliton pulse profile; (c) The corresponding soliton spectrum.....	138

Figure 7.7 Schematic of the DM dark soliton fiber laser. WDM: wavelength division multiplexer. EDF: erbium doped fiber. SMF: single mode fiber. PCs: polarization controllers.	142
Figure 7.8 Dissipative soliton emission of the DM fiber laser. (a) Pulse train; (b) Spectrum; (c) Autocorrelator trace of the dissipative solitons.....	144
Figure 7.9 (a) Oscilloscope trace of dark solitons; (b) A single dark soliton; (c) Spectra of CW emission and dark soliton.....	144
Figure 7.10 Dark soliton emission of the laser. (a) Measured over one cavity roundtrip time; (b) and (c) Zoom-in at the positions that has one cavity roundtrip time separation.	147
Figure 7.11 Simulation of DM dark solitons in time domain with the net-cavity dispersion $\sim 0.092 \text{ ps}^2$	148
Figure 8.1 Schematic diagram of the dispersion managed fiber laser.	152
Figure 8.2 Dark-bright pulse formation process. (a)-(d):Four stages of the coherent coupling between the two polarization components.	155
Figure 8.3 A typical dark-bright pulse state. (a) Single dark bright pulse state; (b) autocorrelation trace of the bright pulse; (c) Multiple pulses; (d) Corresponding optical spectrum.	156
Figure 8.4 Evolution of dark-bright pulse pair under different pump power. (a) 33 dBm (2 W); (b) 34 dBm; (c) 35 dBm; (d) 36 dBm (4 W). Note: The soliton is randomly selected from a multiple solitons state.	157
Figure 8.5 Dark-bright pulse state in normal dispersion regime. (a) Oscilloscope trace; (b) Optical spectrum.....	160
Figure 8.6 Details of the dark-bright pulse in the normal dispersion regime. (a) Tens of pulses in 20 ns scale; (b) Single pulse in 2 ns scale.....	162
Figure 8.7 Coherent coupling optical spectrum of the two polarization components. The horizontal axis (green line) corresponds to the bright component and the vertical axis (red line) corresponds to the dark component.	164
Figure 8.8 Dark bright pulses in near zero dispersion. (a) Oscilloscope trace of 5 pulses; (b) Oscilloscope trace of a single pulse; (c) Autocorrelation trace of the bright pulse.....	165
Figure 8.9 Stable dark-bright vector soliton operation of the fiber laser.....	166
Figure 8.10 A state of multiple dark-bright solitons in the fiber laser.....	167

List of Acronyms

Abbreviations	Full Expressions
CIMI	Cavity Induced Modulation Instability
CNT	Carbon Nanotube
CVD	Chemical Vapour Deposition
CW	Continuous Wave
DCF	Dispersion-Compensating Fiber
DM	Dispersion-Managed
EDF	Erbium-Doped Fiber
FWHM	Full Width at Half Maximum
GHz	Gigahertz
GLE	Ginzburg-Landau Equation
GVD	Group Velocity Dispersion
MHz	Megahertz
MI	Modulation Instability
NL	Noise-Like
NLSE	Nonlinear Schrödinger Equation
NPR	Nonlinear Polarization Rotation
OSA	Optical Spectrum Analyzer
PBS	Polarization Beam Splitter
PC	Polarization Controller
SA	Saturable Absorber

SESAM	Semiconductor Saturable Absorber Mirror
SMF	Single Mode Fiber
SPM	Self Phase Modulation
SWCNT	Single-Wall Carbon Nanotube
WDM	Wavelength Division Multiplexer
XPM	Cross Phase Modulation

Chapter 1 Introduction

This introductory chapter is intended to provide a basic overview on the background theory of soliton fiber lasers and a brief introduction to the content of the dissertation. Section 1.1 presents the background of the theory of solitons and mode-locked fiber lasers. Section 1.2 presents the motivation and objectives of the research. Section 1.3 discusses the main contribution in this thesis. Section 1.4 provides an overview of the dissertation.

1.1 Background

Study of the solitons in the ultrafast optics is a rapidly growing field. It has become an active field of optics over the past two decades. The phenomenon of soliton was firstly described by John Scott Russell in a canal of Scotland in 1834. In his report, soliton was defined as a special kind of wave that can propagate over long distance without distortion [1]. The study of solitons covers a large area of nonlinear physics scenarios, including fluids and plasma physics and optics, biological and atmospheric systems, and nonlinear fiber optics and so on. In 1973, Hasegawa and Tappert firstly proposed the optical soliton propagation in the optical fibers, which opened the study of optical solitons in the field of nonlinear optics [2,3]. Optical soliton refers to an optical field with

a localized structure that does not change during propagation due to the balance between nonlinear and linear effects in the optical medium. Owing to its potential applications in the optical communications and optical signal processing [4-8], study of solitons in optical fibers have attracted intense interest in the last decades. The conventional temporal optical solitons can be basically classified as the bright soliton which is characterized by intensity peaks in the time domain and the dark solitons which is characterized by intensity dip in a continuous wave background [2,3]. The propagation of bright solitons and dark solitons in optical fibers is described by the nonlinear Schrödinger equation (NLSE), based on which optical bright solitons were supported when the group velocity dispersion (GVD) of the fiber is anomalous while optical dark solitons were supported when the GVD of the fiber is normal.

In the context of optical fibres, bright solitons are in the form of short pulse with ultrahigh peak intensities. To generate ultrafast pulses, fiber lasers are a suitable platform. In the mode-locked fiber lasers, an optical pulse can be automatically shaped into a soliton when the group velocity dispersion of the optical fiber is anomalous. Such a laser was named as soliton laser. The operation of soliton laser has been extensively investigated in the past decades [9-14]. Recently, the study of soliton fiber laser has been extended into the vector soliton fiber lasers [15]. Formation of vector soliton is attributed to the birefringent nature of the single mode optical fiber (SMF), i.e. SMF supports two orthogonal polarization modes with different refractive index in practice. The vector soliton propagation can be mathematically described by the coupled NLSE, in which bright-bright soliton, dark-bright soliton and dark-dark soliton have been theoretically predicted [8].

The bright-bright vector soliton can be generated in a vector soliton mode-locked fiber laser. A crucial condition for the vector soliton formation in fiber lasers is that no any polarization discrimination components should be used in the cavity. It has been demonstrated experimentally that even inserting a component with about 30% polarization discrimination could cause the nonlinear polarization rotation (NPR) mode locking of an all anomalous dispersion fiber laser [16]. Therefore, the main challenge for achieving vector soliton operation of a fiber laser is to find an appropriate saturable absorber (SA) that has polarization insensitive saturable absorption. So far, the recognized polarization insensitive SAs in mode-locked fiber laser include semiconductor saturable absorber mirror (SESAM), single wall carbon nanotube (SWCNT), graphene and so on [17-19]. As SESAM is the earliest proposed SA which has been mostly used for the generation of ultrashort pulses by passive mode locking of various types of lasers, the vector soliton properties in SESAM fiber lasers has been intensely investigated [20-24]. However, as the saturable absorption bandwidth of SESAM is narrow and fabrication of SESAM is rather complex and expensive, carbon based SAs with low cost and ultrafast recovery time was proposed. Recently, graphene as a novel wideband saturable absorber with low cost, high damage threshold value, large modulation depth and easy fabrication was proposed and widely studied in the mode-locking fiber lasers. Although the graphene as saturable absorber in mode locked fiber lasers has been verified, the performance of graphene in vector soliton fiber lasers is still not fully understood. It is meaningful to study the vector soliton formation and soliton dynamics in graphene mode-locked fiber lasers.

Apart from the bright solitons, the studies on other types of solitons, especially on dark-bright solitons and scalar dark solitons, still lack further experimental investigations. Recently, scalar dark pulses have been observed in CW fiber lasers [25], which provides a potential platform to observe the scalar dark solitons. Moreover, owing to the birefringence of the SMFs, vector dark-bright solitons and vector dark-dark solitons are also probably being experimentally observed under certain cavity conditions. The main challenge in experimental observation of these solitons is the limitation of the suitable measurement technique to detect narrow dark pulses. Thanks to the rapid advance of the ultrafast optoelectronics technology, we are now in the position to use ultrahigh-speed real-time electronic detection systems to directly measure narrow dark pulses. By using the high speed detection systems, the properties of the dark pulses can be revealed. Comparing to the bright solitons, dark solitons pulses show more stable behavior under perturbations, including amplifier noise, fiber losses and so on. The dark solitons are also less sensitive to the backgrounds and lower pulse shape distortion. Study of the scalar and vector dark solitons would be a promising field of the optical solitons and may find application in optical signal processing, optical computing and optical communications and so on.

1.2 Motivation and Objectives

Soliton formation in mode locked fiber lasers is a well-known phenomenon and has been extensively investigated. In the anomalous dispersion cavity fiber

lasers, soliton operation is obtained owing to the natural balance between the GVD and self-phase modulation (SPM) effect. However, due to the existence of gain and losses in a laser, strictly speaking, all the solitons formed in a fiber laser are essentially the dissipative solitons, whose dynamics is governed by the extended complex Ginzburg-Landau equation (CGLE) [26]. However, depending on the laser operation conditions, if the formed solitons have a spectral bandwidth that is far narrower than the effective gain bandwidth, the effect of the laser gain is purely to balance the cavity losses and it was experimentally found that the features of the formed solitons are more close to those of the nonlinear Schrodinger equation (NLSE) type of solitons. Theoretically speaking, the NLSE type of solitons is conservative in nature, while those of the CGLE type of solitons are dissipative in nature. They have very different features in soliton interaction. In a passively mode locked fiber laser, through finely controlling the laser operation conditions, one can actually fine tune the properties of the formed solitons, consequently, experimentally control the mutual soliton interaction.

Indeed, most of the previous experimental studies on the soliton operation of fiber lasers were focused on fiber lasers passively mode-locked with the nonlinear polarization rotation (NPR) technique. As to achieve the self-started mode locking of fiber lasers with the NPR technique, a polarizer or an equivalent component is inserted in the laser cavity, which fixes the polarization of the formed solitons everywhere in the cavity; the solitons formed in the fiber lasers thus have no polarization dynamics. Hence, from the dynamics point of view, they are equivalent to that of the scalar solitons. Multiple vector solitons, which are solitons with two orthogonally coupled

polarization components, can also be formed in a mode locked fiber laser if polarization sensitive components are removed out of the cavity [15]. The formation of vector solitons in a fiber laser is due to the vector nature of light propagation in the single mode fibers. Compared with the scalar solitons, vector solitons have more complicated dynamics, e.g. depending on the laser operation conditions people have experimentally observed group velocity locked vector solitons, the polarization locked vector solitons, and the polarization rotation vector solitons [24,27,28].

To achieve vector soliton operation of a fiber laser, it is essential to find an appropriate saturable absorber that has polarization insensitive saturable absorption. In the study of passively mode locked soliton fiber laser, graphene as a novel saturable absorber, is attracting great attention and is under intensive research nowadays. As graphene saturable absorbers are easy to make, and have superbroad bandwidth and variable modulation depth when different number of graphene layers is used, it is superior for generating vector solitons in fiber lasers. Various kinds of graphene have been reported that can be used as saturable absorbers in mode locked fiber lasers [29]. However, the property of the mode locking pulses by such a novel type of material was only experimentally studied recently, the vector soliton features of the graphene mode locked fiber lasers under different cavity dispersion are lacking. What is the property of the pulses in graphene mode locked fiber lasers? Is it a scalar soliton or vector soliton? Are there multiple soliton interactions and how do they interact with each other? Based on these questions, a systematic experimental study on the properties and dynamics of the vector solitons in graphene mode locked fiber lasers appears to be necessary and meaningful.

On the other hand, in the non-mode-locking regime of the soliton fiber lasers there are still various interesting features of soliton pulses that need to be investigated. Modulation instability (MI) of the fiber laser is an interesting topic which is related to the bright soliton formation. Conventional MI was considered only existing in the anomalous dispersion fiber and was utilized in fiber laser to generate scalar soliton pulses with high repetition rate as high as hundreds of GHz [30,31]. However, Haelterman *et al.* have theoretically shown that MI could still occur in a normal dispersion fiber cavity. The MI formation was attributed to the cavity boundary condition and this kind of MI aided by cavity detuning was also called the cavity induced modulation instability (CIMI). CIMI has been theoretical investigated, which has the similar properties as those of the conventional MI except that the cavity detuning could greatly alter the phase matching condition, and consequently the pulse repetition rate of the resulted laser intensity modulation. Haelterman *et al.* have also experimentally confirmed the existence of the CIMI in a passive ring fiber cavity [32]. However, the cavity induced MI lasing of fiber lasers and the exploitation of the effect for the generation of stable repetition rate variable GHz level pulses are so far not reported. It is interesting to study the CIMI in the fiber laser and its performance on the soliton formation in the laser cavity.

Dark soliton is another fascinating topic in the field of optical solitons. Although dark soliton theory has been studied for a long time, experimental observation of dark soliton is still difficult due to the limitation of the detection techniques for measuring narrow dark pulses. A type of dark pulse emission in an erbium-doped fiber (EDF) laser made of all-normal dispersion fibers was recently reported, whose dark soliton nature could not be verified owing to the

limitation of the experimental conditions. However, if the observed dark pulses were not the dark solitons formed in the fiber laser, then why the laser emission had a broad optical spectrum? What is the relation between the broad dark pulses and the broadened optical spectrum? Was there dark soliton formation in the fiber laser? All these are the questions that need to be answered.

Similarly, the existence of the dark-bright vector solitons also lacks experimental evidence. Is it possible to observe these solitons in the fiber laser? Under what condition these solitons can be generated? Study of dark soliton and dark-bright vector soliton in fiber laser will be a challenging but meaningful topic for the future optics communications.

The main objective of the thesis is through conducting extensive experimental investigations on the soliton fiber lasers working in the mode-locking regime and non-mode-locking regime to gain a comprehensive understanding on the vector soliton formation and interactions in the fiber lasers. In the mode-locking regime, a passively mode-locked fiber laser by graphene was established to investigate the formation and interactions of the vector solitons. In the non-mode-locking regime, scalar bright solitons formed by the cavity induced modulation instability in fiber laser was experimentally studied and compared with the conventional MI in a dispersion managed cavity. Furthermore, based on the previous experimental results, the existence of the scalar dark solitons was experimentally and numerically studied in the fiber laser with normal dispersion. In a dispersion managed fiber laser cavity with low birefringence, vector dark-bright solitons in fiber lasers are investigated under different cavity dispersion.

1.3 Major Contribution of the Thesis

First of all, fiber lasers passively mode-locked by atomic layer graphene have been built up and vector soliton nature of the formed solitons in the fiber laser have been confirmed. Vector soliton operations of the graphene mode-locked fiber laser have been achieved and comprehensively studied. In an all-anomalous dispersion cavity with weak birefringence, polarization rotation behaviour of multiple vector solitons was experimentally observed. Unlike the case of the polarization locked vector soliton formed in the laser, two extra sets of spectral sidebands always appear on the soliton spectrum of the polarization rotating vector solitons. The new sets of spectral sidebands were confirmed have the same formation mechanism as that of the Kelly sidebands.

In the same laser cavity mode locked with graphene, multiple vector soliton operation was also investigated. It is found that the vector multi-soliton state exhibited several characteristic modes, including the random static distribution of vector solitons, stable bunches of vector solitons, restless oscillations of vector solitons, rain of vector solitons, and emission of giant vector soliton pulses. Especially, the vector soliton rain was firstly experimentally observed and it has almost all the features of scalar soliton rain. Unlike the case of the scalar soliton, the vector soliton rains was verified by the polarization locking and rotating features, which also exhibits an extra set of sidebands in the optical spectrum. The formation mechanism of these vector soliton operations was

attributed to the multiple vector soliton formation and interactions in the anomalous dispersion laser cavity.

The nonlinear dynamics of the vector solitons is also investigated in the graphene mode-locked fiber laser. The quasi-periodicity of the vector solitons was successfully observed in an all anomalous dispersion cavity. The quasi periodicity was characterized by the soliton non-uniformity in the temporal pulse train and extra sideband appearance in optical spectrum. The physical mechanism for the quasi-periodic vector soliton evolution is the cavity induced soliton modulation instability.

Some specific states of the mode locking pulses, including the noise like pulses and rogue waves are also found in the graphene mode locked fiber lasers. Noises like pulses in graphene fiber lasers are experimentally observed and the results agree well with the simulation results. Based on the experimental and numerical studies, we found that the formation of the NL pulses is a combined consequence of the pulse breaking induced by the excessive nonlinearity of the cavity and the mutual attraction caused by the ultra-fast saturable absorption of the graphene. Rogue waves formed due to the soliton interactions are recorded in a multiple soliton fiber laser in the all anomalous dispersion cavity. We identify that the rogue wave generation is caused by the vector soliton interactions caused by the existence of unstable continuous wave (CW) emission in the laser cavity and its character is confirmed by the probability distribution of the pulse peak intensity.

In the non-mode-locking regime, where graphene is removed from the cavity, various kinds of solitons are also investigated. Variable repetition rate pulses

were generated in the fiber lasers. These pulse trains were formed due to the cavity induced modulation instability (CIMI) lasing of the fiber laser. Coexistence of the conventional modulation instability (MI) and the CIMI in the same fiber laser were also experimentally observed.

In the all normal dispersion cavity in which a polarizer is inserted, scalar dark pulse was observed. By using a high-speed detection system, the dark pulse was confirmed to be a dark soliton which is described by the NLSE. Scalar dark solitons were also observed in dispersion-managed cavity fiber lasers with normal net cavity dispersion.

Removing the polarizer from fiber laser and keeping low birefringence of the cavity, vector dark-bright pulses were firstly observed in the fiber laser under different cavity dispersions. The dark-bright pulses were obtained in the dispersion managed cavity under different cavity dispersions when the two orthogonal polarization components are coherently coupled. In the anomalous dispersion regime, the bright soliton is dominating and induces the dark pulses. While in the normal dispersion regime, the dark soliton is dominating and induces the bright pulses. In the near zero dispersion, dark and bright pulses are balanced and it was confirmed that dark-bright pulse was shaped into the dark-bright soliton. We believe that the study of dark soliton and vector dark-bright soliton pair will be a promising field and they can find applications in the future optics communications systems.

1.4 Organization of the Thesis

The thesis is organized as follows:

Chapter 1 serves as an introduction and provides a brief introduction to the research work of the thesis.

Chapter 2 presents the fundamental theory of the soliton propagation in optical fibers and fiber lasers, and the basic principle of the mode-locking technique in the fiber lasers. A mathematical model of the soliton propagation in mode-locked fiber lasers is established to study the soliton in fiber lasers by numerical simulations.

Chapter 3 presents the experimental investigation of the multiple vector soliton operation of the fiber lasers passively mode-locked with atomic layer graphene. We observed the polarization rotation and locking of multiple vector solitons in the fiber laser. We also present various interesting vector soliton patterns in the graphene mode-locked fiber laser with several video records. Specially, the existence of the polarization rotation and locking of the vector solitons rain are experimentally verified.

Chapter 4 addresses the experimental observation of the nonlinear dynamics of the vector solitons in the graphene mode locked fiber lasers. The quasi-periodicity of the vector soliton pulses was firstly observed in an all anomalous dispersion cavity fiber laser. The quasi-periodicity is formed by the soliton modulation instability in the fiber laser.

Chapter 5 focuses on the two distinct pulses formed in the graphene mode locked vector soliton fiber lasers. Firstly we present an investigation on the noise like pulses in the vector soliton fiber laser. The noise like pulse was both experimentally and numerically studied and it was confirmed as one of the intrinsic features of the graphene mode locked fiber lasers. In the second place, we presents the generation of the optical rogue waves in the graphene mode locked fiber lasers. The formation of the rogue wave was explained to be the results of the soliton collisions in the laser cavity.

Chapter 6 presents a GHz repetition rate pulses generation in a dispersion-managed fiber laser without mode locking. The high repetition rate pulse is formed by the cavity induced modulation instability of the laser cavity, which is different from the previous modulation instability occurred in the anomalous dispersion fibers. The two kinds of the modulation instability could simultaneously affect the soliton formation in the fiber laser.

Chapter 7 presents the experimental observation of the dark solitons in the fiber laser. The existence of the dark solitons was not only confirmed in the all normal dispersion cavity but also in the dispersion managed cavity with normal net cavity dispersion. We show that the dark soliton can be automatically formed in the normal dispersion fibers and is an intrinsic feature of the fiber laser.

Chapter 8 presents the experimental observation of the dark-bright pulses in the dispersion managed fiber lasers. The dark-bright soliton formation was systematically studied under different cavity dispersion, namely from the normal dispersion to anomalous dispersion. It was found that in the near zero

dispersion cavity with low birefringence, the dark bright pulses can be stable and shaped into the dark-bright solitons.

Chapter 9 concludes the dissertation and recommendations for the future works are also given.

Chapter 2 Theory of Vector Soliton

Propagation in Fiber Lasers

The soliton pulse propagation in optical fibers is simultaneously affected by the dispersion and nonlinear property of the optical fibers. The broadening of the light wave envelope is determined by the group velocity dispersion and the nonlinear property was mainly attributed to the Kerr effect. In mathematics the pulse propagation in optical fibers can be well described by the nonlinear Schrödinger equation (NLSE). While if the soliton pulse is propagating in gain fiber, for instance, an erbium doped fiber, the gain and gain dispersion introduced by erbium doped fiber and the saturable absorption effect need to be taken into account. Then the dominant equations are extended to the Ginzburg-Landau equations (GLEs) from the NLSE. Moreover, due to the existence of manufacturing imperfections, applied external stress and so on, the single mode fiber actually supports two orthogonal polarization modes with differing propagation constants. In other words, single mode optical fiber in practice is birefringent and there are two orthogonal polarization components of the light pulse propagating simultaneously. Owing to the soliton trapping effect, the two orthogonally polarization components of the vector solitons with different center wavelengths could couple together and propagate at the same group velocity. A vector state of soliton can be thus formed in birefringent SMFs and the fiber birefringence is an important factor that must be taken into account in

the study of vector soliton propagation in optical fibers. To describe the propagation of the vector soliton in fiber laser systems mathematically, the coupled NLSE and coupled GLE were thus introduced and the coupled GLEs are the basic equations in the modeling of the vector soliton fiber lasers.

Provided the dispersion is balanced by the nonlinearity, optical solitons can be shaped from the short pulses in a fiber laser. To generate the ultra-short pulses in the fiber lasers, the well-known mode locking technique is widely used. In a passively mode locking fiber laser, a train of equally spaced stable ultra-short pulses in temporal domain can be generated. Therefore, to understand the soliton fiber lasers, it is also necessary to introduce the theory of mode-locking technique and explain the vector soliton formation in the mode-locked fiber lasers.

The objective of this chapter is to present the basic theoretical background and main equations that govern the vector soliton pulse propagation in passively mode-locked fiber lasers. Section 2.1 gives a brief description of nonlinear pulse propagation in optical fibers. NLSE and the soliton solutions are introduced. Section 2.2 provides a mathematic description of nonlinear pulse propagation in a gain fiber. GLE and the dissipative soliton solutions are introduced. Section 2.3 presents the fundamental of mode-locking and introduces graphene as a saturable absorber in passively mode-locked fiber lasers. Section 2.4 presents the mathematic description of the vector soliton propagation. Based on the coupled GLE a numerical model of the vector soliton fiber lasers is introduced.

2.1 Pulse Propagation in Optical Fibers

When propagating in optical fibers, the properties of ultra-short pulses are undergoing complicated changes due to intramodal (i.e. chromatic) dispersion and intermodal delay effects [1]. Typical physical effects influencing pulses includes fiber dispersion, polarization mode dispersion and nonlinearity.

2.1.1 Dispersion and Birefringence in Optical Fibers

Dispersion is an intrinsic feature of the dispersive media. In the optical fibers, dispersion is defined as the dependence of the phase velocity in a medium on the optical frequency or the propagation mode. In the single mode optical fiber, there are mainly two types of dispersions need to be considered, namely the chromatic dispersion and the polarization mode dispersion.

The chromatic dispersion of an optical medium is the phenomenon that the phase velocity and group velocity of light wave propagating in a transparent medium depend on the optical frequency. Generally there are two sources of chromatic dispersions, namely material dispersion and waveguide dispersion. Material dispersion depends on the materials used to make optical fiber. For most fibers, material dispersion is the principle component of chromatic dispersion. The origin of chromatic dispersion is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons. In other words, the refractive index of the optical medium is a function of optical frequency of the light wave. Mathematically, the refractive index of an optical frequency ω was approximated by the Sellmeier equation as follows:

$$n^2(\omega) = 1 + \sum_{j=1}^m \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2} \quad (2.1.1)$$

where ω_j is the resonance frequency and B_j is the strength of j_{th} resonance. A three-term Sellmeier equation is typically used in silica, which is

$$n^2(\omega) = 1 + \frac{B_1 \omega_1^2}{\omega_1^2 - \omega^2} + \frac{B_2 \omega_2^2}{\omega_2^2 - \omega^2} + \frac{B_3 \omega_3^2}{\omega_3^2 - \omega^2} \quad (2.1.2)$$

The effect of fiber dispersion on the propagation of an optical pulse is mathematically defined via the Taylor expansion of mode-propagation constant β as a function of the pulse's carrier frequency ω :

$$\beta(\omega) = n(\omega) \frac{\omega}{c} = \beta_0 + (\omega - \omega_0) \beta_1 + \frac{1}{2} (\omega - \omega_0)^2 \beta_2 + \frac{1}{6} (\omega - \omega_0)^3 \beta_3 + \dots \quad (2.1.3)$$

$$\beta_m = \left(\frac{d^m \beta}{d\omega^m} \right)_{\omega=\omega_0} \quad (m=0,1,2,\dots) \quad (2.1.4)$$

where the zero-order term β_0 is the propagation constant at the carrier frequency, β_2 are related to the refractive index n and its derivatives through the following relations:

$$\beta_1 = \frac{1}{v_g} = \frac{n_g}{c} = \frac{1}{c} \left(n + \omega \frac{dn}{d\omega} \right) \quad (2.1.5)$$

$$\beta_2 = \frac{1}{c} \left(2 \frac{dn}{d\omega} + \omega \frac{d^2 n}{d\omega^2} \right) \quad (2.1.6)$$

where v_g is the group velocity of the pulse and n_g is the group index. The parameter β_2 represents the dispersion of the group velocity and is responsible for the optical pulse broadening when it propagates in the optical fiber. Thus β_2 is also known as the group velocity dispersion (GVD) parameter. The GVD

parameter can be positive or negative, corresponds to the anomalous dispersion region ($\beta_2 < 0$) and normal dispersion region ($\beta_2 > 0$) respectively. It is worth noting that the value of the β_2 varies with wavelength of the light wave and $\beta_2 = 0$ at $\lambda_D \approx 1.27 \mu\text{m}$. The fiber exhibits normal dispersion for wavelength $\lambda > \lambda_D$ and exhibit anomalous dispersion for wavelength $\lambda < \lambda_D$. In practice, as is more frequency encountered in engineering papers, the GVD parameter is also expressed by the dispersion parameter D which is given by

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2} \beta_2 = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} \quad (2.1.7)$$

The unit of parameter D is picoseconds of delay per nanometer of wavelength change per kilometre of fiber length (ps/nm/km). While the unit of β_2 is picoseconds squared per kilometre (ps²/km). For example, the dispersion of a typical standard single mode fiber is $D = 18 \text{ ps/nm/km}$, becomes $\beta_2 \approx -22.9 \text{ ps}^2/\text{km}$. The coefficient β_3 in Eq. (2.1.3) is the third order dispersion parameter. Third order dispersion could also affect the pulse propagation and its effect could be significant when $\beta_2 = 0$.

Apart from the chromatic dispersion, the pulses propagating in the single mode fiber also suffer from the polarization mode dispersion (birefringence). Polarization mode dispersion is due to the difference in the propagation characteristics of light waves with different polarization states which is caused by the intrinsic factors of the fiber, such as the geometric irregularities of the fiber core and internal stresses, or external factors such as fiber bending, twisting etc. A single mode fiber thus has two polarization eigenmodes and exhibits a certain birefringence. The strength of birefringence of a fiber is

defined as:

$$B = \delta\beta/k_0 = |n_x - n_y| \quad (2.1.8)$$

where $\delta\beta = |\beta_x - \beta_y|$ is the difference in the propagation constant, n_x and n_y are the effective indices of the two polarization modes of the fiber. If $n_x > n_y$, as the light polarized along the X -axis moves slower than the light polarization along the Y -axis. The X -axis is called the slow axis and the Y -axis is called the fast axis.

When the incident light is propagating a length of L , for a given value of B , the phase difference between the two polarization components of the light is as follows:

$$\Delta\phi = \frac{2\pi(n_x - n_y)L}{\lambda} = \frac{2\pi BL}{\lambda} = 2\pi \cdot \frac{L}{L_B} \quad (2.1.9)$$

where $L_B = \frac{\lambda}{n_x - n_y} = \frac{\lambda}{B}$ is called beat length. Physically, the beat length means

when the light propagates a length of L_B , phase difference between the two polarization components is 2π and the polarization of the light returns to its initial state. For standard single mode optical fibers, a typical value of beat length is around 1 meter. While for the polarization maintaining fibers, as a strong birefringence is deliberately built in the fibers, they could have a beat length of several millimeters.

2.1.2 Nonlinear Effects in Optical Fibers

Nonlinear effect in optical fibers is originated by the nonlinear response of polarization to strong optical field, which is another factor that must be taken into account in the study of pulse propagation in optical fibers. As the optical fiber is made of SiO₂, which has a symmetric molecular structure, the lowest-order nonlinear effects in optical fibers is the third order effect. In optical fibers there are various types of third-order nonlinear effects, such as optical Kerr effect, third-harmonic generation, four-wave mixing, stimulated Raman scattering and stimulated Brillouin scattering. The phase matching condition must be fulfilled for the occurrence of these third-order nonlinear effects. The optical Kerr effect is the most common nonlinear effect in optical fibers because the phase match conditions for optical Kerr effect are automatically fulfilled. Thus optical Kerr effect is always considered in the study of nonlinear pulse propagation in optical fibers. Optical Kerr effect originates from the intensity dependence of the refractive index which is given by

$$n(I) = n_0 + n_2 I \quad (2.1.10)$$

where I is the intensity of the light, n_0 is the refractive index of the fiber, and n_2 is the nonlinear-index coefficient which is determined by the third order nonlinear-property of the fiber. The optical Kerr effect demonstrates the intensity dependence of the refractive index, which leads to the formation of a large number of interesting nonlinear effects. The most widely studied effects are self-phase modulation (SPM) and cross-phase modulation (XPM). SPM is defined as the nonlinear phase modulation of a beam, which is responsible for the spectral broadening of pulses and the formation of the optical solitons in the

anomalous-dispersion fibers. The SPM effect can be described by the nonlinear phase shift of the pulses. Mathematically, the phase shift of pulses propagating in optical fibers with length L is described as

$$\phi = nk_0L = (n + n_2|E|^2)k_0L \quad (2.1.11)$$

where $k_0 = \frac{2\pi}{\lambda}$, n_2 is the second-order nonlinear refractive index of the optical fiber. The nonlinear phase shift induced by SPM is $\phi_{NL} = n_2|E|^2 k_0L$. XPM is a nonlinear effect where the optical intensity of one pulse influences the phase shift of another pulse by their interactions in optical fibers. For example, if there are two optical fields at frequency ω_1 and ω_2 , the total electrical field E can be described as

$$E = \frac{1}{2} \hat{x}[E_1 \exp(-i\omega_1 t) + E_2 \exp(-i\omega_2 t) + c.c] \quad (2.1.12)$$

The nonlinear phase shift introduced by the field E_2 to the field E_1 is

$$\phi_{NL} = n_2 k_0 L \cdot 2|E_2|^2 \quad (2.1.13)$$

Therefore, for the E_1 , adding the terms of SPM effect, the total nonlinear phase shift will be $\phi_{NL} = n_2 k_0 L(|E_1|^2 + 2|E_2|^2)$. In practice, when two or more optical pulses are propagating in the optical fibers, XPM always accompanies SPM. In the study of vector soliton transmission and soliton interactions, XPM is an important term when the two polarization components are coherent coupled.

2.1.3 Nonlinear Schrödinger Equation and Soliton Solutions

As discussed in 2.1.1 and 2.1.2, when we consider intense pulse propagation in optical fibers, dispersion and nonlinearity must be both considered. Derived from the well-known Maxwell equation with several approximations, optical

pulse propagation in single mode fibers is described by the nonlinear Schrodinger equation (NLSE) as follows:

$$\frac{\partial A}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i \gamma |A|^2 A \quad (2.1.14)$$

where A is the slow varying envelope of the optical pulse, γ is the nonlinear parameter, β_2 is the second order dispersion which corresponds to the GVD effect. It is noted high order dispersion effect is omitted. NLSE gives a simple description over the propagation of an optical pulse in optical fibers, which takes into account both the GVD and nonlinear effects. The nonlinear effects always generate a positive frequency chirp in the pulse and the GVD can introduce either positive or negative chirp, depending on the sign of GVD. If GVD introduces a negative chirp, i.e. the $\beta_2 < 0$, the frequency chirp can be eliminated by the balance between the nonlinear effects and the GVD. Under this condition the soliton solution can be obtained by solving the NLSE using the scattering method [1]. It is useful firstly to normalize the NLSE by introducing three dimensionless variables

$$U = \frac{A}{\sqrt{P_0}}, \xi = \frac{z}{L_D}, \tau = \frac{T}{T_0} \quad (2.1.15)$$

where $T = t_0 - z / v_g$ and Eq(2.1.14) can be written as

$$i \frac{\partial U}{\partial \xi} = \text{sgn}(\beta_2) \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} - N^2 |U|^2 U \quad (2.1.16)$$

where P_0 is the peak power, T_0 is the pulse width and $N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|}$. The

parameter N corresponds to the order of the soliton, $N = 1$ for the fundamental soliton, whose shape does not change on propagation. If $\text{sgn}(\beta_2) = -1$, which

means anomalous GVD regime is chosen and it corresponds to a bright soliton solution. The fundamental bright soliton solution is given by

$$u(\xi, \tau) = \eta \operatorname{sech}(\eta\tau) \exp(i\eta^2\xi/2) \quad (2.1.17)$$

where the parameter $u = NU$. The canonical form of the Eq(2.1.17) is by setting $\eta = 1$:

$$u(\xi, \tau) = \operatorname{sech}(\tau) \exp(i\xi/2) \quad (2.1.18)$$

The peak power P_0 required to support the fundamental soliton is

$$P_0 = \frac{|\beta_2|}{\gamma T_0^2} \approx \frac{3.11|\beta_2|}{\gamma T_{FWHM}^2} \quad (2.1.19)$$

where the FWHM of the soliton is defined using $T_{FWHM} \approx 1.76T_0$. The peak power P_0 is typically ~ 1 W near 1550 nm wavelength for $T_0 = 1$ ps, $\beta_2 = -1$ ps²/km and $\gamma = 3$ W⁻¹/km.

If $\operatorname{sgn}(\beta_2) = 1$, namely in the normal dispersion regime, by imposing a certain boundary condition, the dark soliton solutions can also be found by the inverse scattering method [1]. The general solution of a dark soliton is given by

$$u(\xi, \tau) = \eta[B \tanh(\zeta) - i\sqrt{1-B^2}] \exp(i\eta^2\xi) \quad (2.1.20)$$

Where $\zeta = \eta B(\tau - \tau_s - \eta B\sqrt{1-B^2})$. The parameter η represents the amplitude of the background, τ_s represents the location of the dip and B is the depth of the dip and governs the blackness of dark solitons. For dark solitons $|B| < 1$ are called gray solitons and $|B| = 1$ is called black soliton.

2.2 Pulse Propagation in Erbium-doped Fiber

2.2.1 Gain Profile of Erbium-doped Fibers

As the light propagation in rare-earth doped fibers can experience amplification process, the rare-earth doped gain fibers are widely used in optical amplifiers and optics communications, which is also one of the essential components in the fiber lasers. Many kinds of rare-earth ions can be used in the fiber laser with a wide range of operating wavelength from 400 nm to 4000 nm. Erbium-doped fibers became the intense focus since 1989 for its broad gain at 1.55 μm optical fiber communication window [33,34].

Figure 2.1 Pump scheme of erbium-doped fiber.

Amplification of the erbium-doped fiber (EDF) is realized through stimulated emission after the pumping Er^{3+} ion from their ground state to the excited states. In an erbium-doped fiber laser, the pumping scheme of the Er^{3+} ion can be considered as a three-level system, as shown in Figure 2.1. There are two

mainly efficient pumping for the erbium-doped fiber: 980 nm and 1480 nm wavelengths. In this thesis, 1480 nm pump was chosen as the pump source. In the process of amplifications, the broadened gain of the EDF is homogeneous and the gain coefficient of EDF is given by

$$g(\omega) = \frac{G}{1 + (\omega - \omega_a)^2 T_2^2 + P/P_s} \quad (2.2.1)$$

where G is the small signal gain, ω is the frequency of the incident light, ω_a is the resonant frequency of the dopant energy levels, T_2 is the dipole relaxation time, P is the power of the incident light, P_s is the saturation power of the erbium-doped fiber. In practice, when considering the amplification of a single pulse, the term $P/P_s \ll 1$ for most rare-earth doped fiber and thus can be neglected. Then the gain coefficient is approximated as

$$g(\omega) = \frac{G}{1 + (\omega - \omega_a)^2 T_2^2} \approx G \left[1 - \frac{(\omega - \omega_a)^2}{\Omega_g^2} \right] \quad (2.2.2)$$

where Ω_g is the FWHM of the gain spectrum, namely the gain bandwidth of the fiber. Eq. (2.2.2) shows that the gain is governed by a Lorentzian profile. For a Lorentzian profile, g is inversely related to the dipole relaxation time T_2 of the dopants. A typical absorption and emission spectrum of erbium-doped is shown in Figure 2.2 for example. It is worth noting that a double peak structure was found in Figure 2.2, which is due to the erbium ion doped into the silica fibers, in which there are interaction of the ions with the silica host and other co-dopants such as germania and alumina within the fiber core. The energy levels of erbium ions are thus not discrete but consist of several close space levels instead. In an erbium doped fiber laser, the bandwidth of the gain can be as large as 40 nm and correspondingly support ultrashort pulses as short as ~60 fs.

Figure 2.2 The absorption and emission spectra of the EDFs [1].

2.2.2 Ginzburg-Landau Equation and Dissipative Soliton Solution

Generally, the interaction of the doped ions with the light is governed by the Maxwell-Bloch equations [1]. However, in the case of that the pulse width is larger comparing with the dipole relaxation time T_2 of the Er^{3+} ions ($T_2 < 0.1$ ps), the rate equation approximation can be made and consequently the gain term can be simply added into the NLSE.

As the gain spectrum of the erbium-doped fiber is limited, the gain dispersion is to reduce the gain for spectral components away from the gain peak. The gain is thus frequency dependent which is approximated by:

$$g(\omega) = g(\omega_0) + \left. \frac{\partial g}{\partial \omega} \right|_{\omega=\omega_0} (\omega - \omega_0) + \left. \frac{1}{2} \frac{\partial^2 g}{\partial \omega^2} \right|_{\omega=\omega_0} (\omega - \omega_0)^2 + \dots \quad (2.2.3)$$

Without loss of the generality, assuming that the pulse carrier frequency coincides with the central gain frequency, the equation describing optical pulse propagation in the doped fibers becomes:

$$\frac{\partial A}{\partial z} = -\beta_1 \frac{\partial A}{\partial t} - \frac{i}{2} (\beta_2 + igT_2^2) \frac{\partial^2 A}{\partial T^2} + i\gamma |A|^2 A + \frac{g}{2} A \quad (2.2.4)$$

Here g is the gain coefficient, T_2 is the life time of the excited states, α_2 is the two-photon absorption coefficient. On account of the gain saturation, the peak gain coefficient g is time dependent and its variation with time is governed by [1]

$$\frac{\partial g}{\partial t} = \frac{G - g}{T_1} - \frac{g |A|^2}{E_s} \quad (2.2.5)$$

where G is the small signal gain, and E_s is the saturation energy. T_1 is the population decay time, which is around 10 ms for erbium doped ions. Normally, the pulse width is much narrower than 10 ms and the T_1 term is therefore negligible during pulse amplification, and $g(t)$ can be approximated as

$$g(t) = G \exp \left[-\frac{1}{E_s} \int_{-\infty}^t |A|^2 dt \right] \quad (2.2.6)$$

The saturation energy is typically 10 μ J and the pulse energy is much smaller than it. Therefore the gain saturation can be neglected if a single pulse is considered. However, the saturation of the gain must be considered if there are thousands of pulses passing through the gain fiber as their average power will saturate the gain. Through transforming T_2 into the frequency domain, Eq. (2.2.4) is rewritten as

$$\frac{\partial A}{\partial z} = -\beta_1 \frac{\partial A}{\partial t} - \frac{i}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + i\gamma |A|^2 A + \frac{g - \alpha}{2} A + \frac{g}{2\Omega_g^2} \frac{\partial^2 A}{\partial T^2} \quad (2.2.7)$$

where g represents the gain and α represents the loss. Eq. (2.2.7) is known as

the Ginzburg-Landau equation (GLE). If $g = 0$ and $\alpha = 0$, the equation return backs to the NLSE. The GLE is nonintegrable and no exact solitary wave solutions can be obtained. The general solution of GLE in an anomalous dispersion regime is a chirped hyper-secant pulse which is given by

$$\mu(\xi, \tau) = N_s \sec h(p\tau) \exp[iK_s \xi - iq \ln(\cosh p\tau)] \quad (2.2.8)$$

where $\xi = \frac{z|\beta_2|}{T_0^2}$, $\tau = \frac{T}{T_0}$, $u = \left(\frac{\gamma T_0^2}{|\beta_2|} \right)^{1/2}$. N_s , p , q and K_s are constants depending

on the parameter $d = \frac{gT_0^2}{|\beta_2|}$. Eq. (2.2.8) is a dissipative soliton solution. Unlike

the NLSE soliton solution, these solitons are frequency chirped which is influenced by the fiber gain. The peak power required to form a soliton is

$$P_0 = \frac{|\beta_2|}{\gamma T_0^2} \approx \frac{3.11|\beta_2|}{\gamma T_{FWHM}^2} \quad (2.2.9)$$

2.3 Fundamentals of Mode-locked Fiber Lasers

To study the pulse propagation in fiber lasers, the optical pulses must be generated first. In the fiber lasers, basically there are two techniques used in the generation of short optical pulses, including Q-switching and mode-locking. Q-switching could generate pulses with the width of only several nanoseconds while the mode-locked pulses can be even shorter. In the ultrafast fiber lasers, mode-locking technique is always necessary for the generation of ultrashort pulses. With the rapid development of optical communications in the last three decades, mode-locked fiber lasers have been capable of producing pulses with

widths in a very wide range of from tens of femtoseconds to nanosecond and a wide repetition rate range.

Mode locking is an important technique for the generation of soliton pulses in fiber lasers. In this section, we present the physics of mode locking in fiber lasers and introduce various kinds of mode-locking technique in fiber lasers.

2.3.1 Principle of Mode Locking Technique

In a fiber laser, there are a large number of longitudinal modes falling in the gain bandwidth. The principle of the mode-locking technique is to lock the phase of longitudinal modes in a laser. The frequency spacing between the longitudinal modes is given by $\Delta\nu = c/L$, where L is the optical length during one round trip in the cavity. The total optical field can be written as $E(t) = \sum_{m=-M}^M E_m \exp(i\phi_m - i\omega_m t)$, where E_m , ϕ_m , and ω_m are respectively the amplitude, phase and the frequency of a specific mode among $(2M + 1)$ modes.

In a multimode CW laser, normally all the modes operate independently of each other with no definite phase relationship among them. Thus the interference term in the total intensity averages out to zero. Mode locking occurs if the phases of longitudinal modes are synchronized, in other words, the phase difference between two neighboring longitudinal modes is locked to a constant value ϕ . Assuming all the modes have the same amplitude E_0 , the sum of all the modes is given as [1]

$$|E(t)|^2 = \frac{\sin^2[(2M + 1)\pi\Delta\nu T + \phi/2]}{\sin^2(\pi\Delta\nu T + \phi/2)} E_0^2 \quad (2.3.1)$$

where $|E(t)|^2$ is a period function of time with period $\tau_r = 1/\Delta\nu$, which equals to the round trip time of the laser cavity. The pulse width τ_p can also be estimated from Eq. (2.3.1) which is given by

$$\tau_p = [(2M + 1)\Delta\nu]^{-1} \quad (2.3.2)$$

The pulse width is inversely related to the spectral bandwidth over which phases of various modes can be synchronized. In practice, the exact relationship between the pulse width and the gain bandwidth depends on the nature of the gain broadening.

2.3.2 Active and Passive Mode-locking

Classified by the mode locking methods, mode locking technique can be classified as active mode locking and passive mode locking.

- **Active mode locking**

Active mode-locking of fiber laser is usually obtained by inserting an active modulator into the cavity to modulate either the amplitude or the phase of the intra-cavity optical field at a frequency that equals to integer multiples of the cavity longitude mode spacing [35-37]. When the modulation frequency is the integral times of the laser cavity frequency, the laser can be active mode-locked and output pulse train is synchronized to the external modulation signal. The actively mode-locked fiber laser can be realized using different kinds of modulation methods by employing an acousto-optic modulator (amplitude modulation), an electro-optic modulator, a Mach-Zehnder integrated-optic modulator, or a semiconductor electro absorption modulator. Active mode

locking is a preferred choice for producing ultralow-jitter pulses at high repetition rates as high as tens of GHz with pulse duration in the picosecond range. However, as there are too many pulses in the lasers, which share the energy of the intra cavity, the energy of each pulse is weak and the pulse width is correspondingly broad. Moreover, a broad pulse width also means low peak power. Due to the low peak power of the active mode-locking pulses, nonlinear SPM is so weak that soliton shaping is almost impossible in actively mode locked lasers. Besides, owing to the gain competition between the pulses, drop-out problem also exists in the actively mode-locked fiber lasers [38,39]. To overcome these shortcomings of active mode locking, passive mode locking lasers were therefore proposed which may solve these problems.

- **Passive mode locking**

Passively mode-locked is a widely used mode-locking technique which does not require any externally modulated media or devices [40-42], but only employs a saturable absorber (SA) in the fiber laser cavity to realize the mode-locking. In many cases, the saturable absorber can also start the mode-locking process. The pulse generation process sometimes could begin automatically after switching on the laser, which is known as self-starting mode locking. Compared with active mode locking, passively mode locking is capable of producing ultrashort optical pulses as it makes use of a nonlinear effect of the saturable absorber (SA), which is able to modulate the resonator losses much faster than any electronic modulator. The SA refers to an optical device or medium that exhibits an intensity-dependent transmission [43], which selectively absorbs low intensity light and transmits light with sufficiently high

intensity. Saturable absorbers in passively mode-locked lasers could be divided into two categories: artificial saturable absorber based on nonlinear light interference and real saturable absorber based on material's nonlinear optical absorption property. The mainly recognized passive mode lockers currently include nonlinear polarization rotation (NPR), semiconductor saturable absorber mirror (SESAM), carbon nano-tube and graphene based saturable absorbers and so on. Passive mode locking of these saturable absorbers is briefly introduced as follows.

2.3.3 Saturable Absorbers Mode Locking Fiber Lasers

As discussed above, saturable absorbers (SAs) play an important role in the passively mode-locked fiber lasers, which have been used for passive mode-locking since 1970s. In the passively mode locked SA fiber laser, the pulse formation process is actually quite complex. In this section, a brief introduction of several kinds of saturable absorber mode locking is given.

- **NPR mode locking**

Figure 2.3 Schematic diagram of NPR mode locking principle.

NPR mode locking is based on the so-called artificial SAs which rely on SPM and XPM induced nonlinear phase shift. The physics of the mode locking

process can be illustrated in Figure 2.3. When the light with an arbitrary polarization is incident into the setup, the polarization of the light will be transferred to be linear because of the polarizer before the fiber. As the incident light is propagating in the fiber, it splits into two components along the two polarization axes of the fiber, respectively. Due to the existence of the birefringence, the polarization of the light will become elliptically polarized after propagating in a certain length. The polarization of the light will rotate in each beat length as the fiber length increases on the condition that the intensity of the light is weak. However, if the intensity of the light is strong, under the optical Kerr effect, namely the SPM and XPM of the fiber, a nonlinear phase shift is introduced, and the polarization of the light is thus changed. The transmission of the light through the analyzer will also depend on the light intensity. It can be expected that the stronger the light, the larger is the light intensity transmission through the analyzer if the orientation of the polarization controller and the polarizer are properly set. In this situation an artificial saturable absorption is generated. As the nonlinear absorption is based on the optical Kerr effect, the recovery time of NPR mode locking is in the order of several femtoseconds, in which ultra-short mode-locked pulses can be generated. It is worth noting that in an NPR mode locked fiber laser, a polarization dependent component must be inserted into the laser cavity. The existence of a polarization discrimination component in cavity will fix the polarization of light at the cavity position. Under effect of the cavity boundary condition the polarization of light is then fixed everywhere in the cavity and no vector soliton dynamics could be obtained. Therefore, solitons formed in a NPR mode locked fiber laser mainly display the scalar soliton features.

- **Semiconductor saturable absorber mirror**

SESAM is intensity and wavelength dependent mirror, which was widely used as a saturable absorber in commercial mode locked lasers. Typically, a SESAM contains a semiconductor Bragg mirror and (near the surface) a single quantum well absorber layer. A typical structure of SESAM is shown in Figure 2.4. The wavelength dependent response is caused by the Bragg mirror, which is GaAs/AlAs on the GaAs substrate. On top of the mirror is the quantum well, which is made of InGaAs. The InGaAs quantum well is the saturable absorber material which is responsible for the intensity-dependent reflection. The mode locking technique of SESAM is based on the mutual interactions of light with the laser gain medium and the saturable absorber.

Figure 2.4 Schematic structure of the SESAM

The saturable absorption property of the SESAM can be described by several parameters, including the recovery time, the modulation depth, the bandwidth, the saturation intensity and the non-saturable losses. To obtain ultrashort pulses, a short recovery time is preferred in the mode-locking lasers. SESAM has a bi-temporal recovery time and the shortest time can be several picoseconds or sub-picoseconds. The longer recovery time is suitable for the start of the mode locking while the shorter recovery time could enable ultrafast pulse generation.

However, to achieve a ~ps recovery time, extra complicated techniques such as a suboptimal low-temperature growth environment or high-energy heavy-ion implantation are required. The modulation depth of the SESAM depends on the design of the Bragg stack layer. SESAM with resonant Bragg stacks can have a large modulation depth but with a limited bandwidth height. SESAM based on anti-resonant Bragg stacks can have a large bandwidth but a limited modulation depth. In this case, to increase the modulation depth, a high non-saturable loss must be introduced.

SESAM can work at various wavelengths by changing the parameters in the design, however, the bandwidth of a specific SESAM is normally narrow, which restricts the SESAM as a wideband mode locker. Moreover, the damage threshold of the SESAM is generally low and the fabrication system of SESAM is expensive and complex. As a result of these shortcomings of the SESAM, recently, new materials with wide operation bandwidth and lower cost were being sought to replace the SESAM in the mode-locked fiber laser.

- **Carbon nanotube saturable absorbers**

Recently, carbon based saturable absorbers have attracted a lot of interests in the study of ultrafast fiber laser [29]. Carbon based nano-materials exhibit several often unforeseen properties, which can be valuable in various fields of study, such as nanotechnology, electronics, and optics and so on. Among the various carbon based materials, carbon nanotubes (CNTs), especially single-walled carbon nanotubes (SWCNTs), were firstly found to exhibit saturable absorption property in 2003 [44].

CNT is allotropes of carbon with a cylindrical nanostructure. It is a seamless cylinder rolled by a hexagonal network of carbon atoms. The diameter of the CNT is normally several nanometres and the length of CNT is typically a few microns. In terms of its chirality, a SWCNT can be classified into semiconducting, metallic or semi-metallic one [45]. As current synthesis methods can not accurately control the desired chirality or diameter of the tube. Normally, SWCNTs are the hybrid of both semiconductor and metallic tubes. In semiconducting nanotubes the excitonic absorption has an ultrafast recovery time which plays a key role in the saturable absorption property of SWCNT. While the metallic nanotubes play the role of the recombination centers, which are responsible for the ultrafast recovery time of SWCNTs. As a direct band-gap material, SWCNTs' bandgap is determined by their nanotube structures. Therefore, for specific SWCNTs with a certain diameter and chiral angle, only the light with a particular wavelength can be absorbed [46]. A broadband saturable absorption mode locker may be realized by mixing SWCNTs with large number of different diameters and chiral angles.

Comparing to the SESAM, SWCNT as a mode locker has an intrinsically ultrafast recovery time (sub picosecond), a wider operating spectral range, and is easier for fabrication with a lower cost. A lot of achievements of SWCNT as saturable absorber in fiber laser have been indeed gained in this area. By using a stretched pulse design, the shortest pulse achieved thus far from SWCNT mode locked fiber lasers is 74 fs [47]. The maximum reported average power is as large as ~250 mW. Pulses generated by SWCNT fiber laser have also been used in nonlinear compression [48], amplification [49-51] and super continuum generation [49,52,53].

However, the shortcomings of the SWCNTs mode locker are still obvious: As the operating wavelength of the SWCNT depends on its structure and SWCNTs tend to form bundles that finish up as scattering sites, the coexistence of SWCNTs with different diameters will introduces extra linear losses, which makes the mode locking in the fiber laser difficult to be achieved. Moreover, these effects also limit the SWCNT as a wideband saturable absorber. To avoid the intrinsic large losses associated with the various tube sizes in SWCNTs, graphene was proposed as novel saturable absorbers. The details of the graphene will be discussed in the next section.

2.3.4 Graphene Mode locker in Vector Soliton Fiber Lasers

Recently, another kind of carbon based material, a two dimension allotrope of carbon which is called graphene, has attracted tremendous attentions for its unique properties as saturable absorbers in the next-generation of the passively mode locked fiber lasers. Graphene is a one-atom-thick planar sheet of sp²-bonded carbon atoms that are densely packed in a honeycomb crystal lattice. The most substantial feature of the graphene is that the bandgap of the graphene is different from the conventional semiconductor saturable absorbers: Graphene has zero band-gap and a linear dispersion relation. A schematic diagram of the energy band structure of graphene is shown in Figure 2.5. Due to this unique energy band structure and combined with the Pauli Exclusion Principle, the optical interband transitions of graphene are frequency independent and thus could absorb infrared light without bandwidth limitation. Moreover, comparing to traditional semiconductor materials, graphene has a large absorption modulation depth with low saturation intensity. Considering these unique

features of graphene, it is ambiguously potential used as a wide band saturable absorber to replace the SESAM in the mode locked fiber lasers and its performance has been verified by experiments.

Figure 2.5 Energy band structure of graphene.

There have been various methods to fabricate the graphene mode locker, such as liquid phase exfoliation [54-56], chemical vapor deposition (CVD)[17,57-62], carbon segregation [63], graphene oxide [64], reduced graphene oxide [65] and micro-mechanical cleavage [66,67]. In this thesis, we focus on the graphene made from the CVD method. In the semiconductor industry, CVD method has been a mature technology and it is the most useful technology for deposition and material preparation. Using the CVD method, a cheap and common way is to grow the graphene on nickel substrate, on which the graphene films can be monolayer or few-layers [61,62]. Copper foil is also a perfect substrate to grow the graphene, on which the graphene films are monolayer [68]. CVD-deposited graphene is highly crystalline and the thickness of the graphene film is

controllable. These films are continuous and could be lifted off and transferred to an optical fiber pigtail end easily, which is particularly suitable for the experimental study of fiber laser.

So far, the CVD graphene has been claimed that it possesses wavelength-insensitive ultrafast saturable absorption as a “full-band” mode locker [17]. Dissipative solitons with continuous wavelength tuning as large as 30 nm (1570–1600 nm) has been obtained in an erbium-doped dissipative soliton fiber laser mode locked with atomic layer graphene [59].

Similar to SESAMs and SWCNTs, atomic layer graphene as a saturable absorber is polarization independent. Therefore, the graphene mode locked fiber laser can support vector soliton formation. Furthermore, as graphene saturable absorbers are easy to make, and have broad bandwidth and variable modulation depth when different number of graphene layers is used, graphene mode-locked fiber lasers could be a perfect test bed for the study of vector soliton mode locked fiber lasers.

Figure 2.6 An experimental setup of the passively mode-locked fiber ring laser which consists of erbium-doped fiber, a wavelength-division multiplexer, a 1-input/2-output coupler, a polarization independent isolator, a polarization controller, and the graphene mode locker.

In this dissertation, experimental study of the graphene mode locked fiber laser was designed as a ring cavity connected by single mode fiber with necessary fiber coupled components. Figure 2.6 shows a typical experimental setup of the graphene mode-locked fiber laser. The components connected in the cavity including erbium-doped fiber, a wavelength-division multiplexer (WDM), a 1-input/2-output coupler (10:90), a polarization independent isolator, a polarization controller and the graphene mode locker.

2.4 Modelling of the Vector Soliton Fiber Laser

As mentioned in section 2.1, scalar soliton propagation in optical fiber is described by NLSE and scalar pulse propagation in fiber lasers can be described by GLE. However, the NLSE above only describes the propagation in one spatial dimension. Considering that the single mode fiber is birefringent, strictly speaking, the soliton propagation along two orthogonal polarizations is different. The coupling between the two polarization components must be taken into account and these factors are described by the coupled version of NLSE as follows [1]

$$\begin{aligned}\frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + i\gamma \left(|u|^2 + \frac{2}{3}|v|^2 \right) u + i\frac{\gamma}{3} v^2 u^* \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 v}{\partial t^2} + i\gamma \left(|v|^2 + \frac{2}{3}|u|^2 \right) v + i\frac{\gamma}{3} u^2 v^*\end{aligned}\quad (2.4.1)$$

where u and v are the two normalized slowly varying pulse envelopes along the slow and the fast axes, u^* and v^* represent their conjugates, $2\beta = 2\Delta n/\lambda$ is the wave-number difference and $2\delta = 2\beta\lambda/2\pi c$ is the group velocity difference. The additional terms v of the first term on the right hand side refers to the XPM effect. The second term of the right hand sides refers to the effect of four wave mixing. The coupled NLSE can be used to describe the vector soliton pulse propagation in birefringent single mode optical fibers.

Furthermore, when we consider the pulse propagation in fibre lasers, gain and losses must be also taken into account, and all the solitons formed in the fiber lasers are essentially the dissipative solitons. To describe the vector soliton pulse propagation in fiber laser, the NLSE can be further developed into the coupled GLEs as follows

$$\begin{aligned}
 \frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + \frac{\beta_3}{6} \frac{\partial^3 u}{\partial t^3} + i\gamma \left(|u|^2 + \frac{2}{3} |v|^2 \right) u + i \frac{\gamma}{3} v^2 u^* + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2} \\
 \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 v}{\partial t^2} + \frac{\beta_3}{6} \frac{\partial^3 v}{\partial t^3} + i\gamma \left(|v|^2 + \frac{2}{3} |u|^2 \right) v + i \frac{\gamma}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2}
 \end{aligned}
 \tag{2.4.2}$$

The coupled GLEs describe the vector pulse propagation within the whole cavity including the amplification effect of the erbium doped fiber gain. The coupled GLEs are traditionally used to setup a mathematic modelling of the vector soliton fiber laser. However, the soliton pulses circulating in a laser cavity are also subjected to the actions of the cavity components and feedback. These effects are not considered in the coupled GLEs.

To describe the soliton operation of a fiber laser more accurately, in this dissertation we use the pulse tracing method so that the laser cavity effects can be included in the numerical model. A basic idea of the pulse tracing method is that in the simulation process, one starts the calculation with an arbitrary initial weak pulse and let the pulse circulate in the laser cavity with specific conditions. Combing with the pulse tracing method we can solve the coupled GLE in a numerical approach. As there is no explicit analysis of the nonlinear differential equations, the standard split-step Fourier method [1] is applied to numerically solve the coupled GLE.

Before solving the equations, a proper step size needs to be carefully selected so that the results can be stable. In the process of solving the equations, it is worth mentioning that the impact of graphene as a SA in the cavity must also be considered. Mathematically, the effect of the SAs on the pulses is accounted by multiplying its Jones matrix to the optical field of the pulse. In this dissertation, we only consider the effect of graphene which is a real SA with a finite

recovery time. The transmission property of graphene can be directly modelled by

$$\frac{\partial l_s}{\partial t} = \frac{l_s - l_0}{T_{rec}} - \frac{P(t)}{E_{sat}^{SA}} l_s \quad (2.4.3)$$

where T_{rec} is the absorption recovery time, l_s is the saturation absorption of the absorber, l_0 is the initial absorption of the absorber, $P(t)$ is the temporal power of the pulse and E_{sat}^{SA} is saturation energy of the SA. As graphene is a fast saturable absorber, whose recovery time is usually much faster than the pulse duration, the loss l_s can be modelled as [69]:

$$l_s = \frac{l_0}{1 + P(t) / P_{sat}} \quad (2.4.4)$$

where P_{sat} is the saturation power of the graphene.

Based on the above mentioned cavity transmission function of a fiber laser mode-locked by the graphene, a result will be obtained after calculation of one round trip time and it will be the initial pulse of the next round of calculation. The calculation repeats until a steady result is obtained, which will be the soliton solution of the designed fiber laser. By using this method we can not only simulate the soliton formation process of the fiber laser mode locked with graphene but also scalar and vector dark soliton formation in the non-mode-locked fiber laser cavity. All the simulation results mentioned in this thesis would be based on the pulse tracing method.

Chapter 3 Vector Soliton Operation in

Graphene Mode-locked Fiber Lasers

Vector soliton formation in birefringent single mode fibers (SMFs) was theoretically extensively investigated in the past. C. R. Menyuk first theoretically studied optical pulse propagation in birefringent optical fibers [70]. He found that above a certain pulse intensity level two orthogonally polarized solitons with different centre wavelengths could couple together and propagate at the same group velocity. The entity of the coupled solitons was later known as a group velocity locked vector soliton. V. V. Afanasjev theoretically investigated the pulse propagation in weakly birefringent SMFs and found an approximate analytic solution for the polarization rotation vector soliton [28]. Christodoulides and Joseph [71], and Akhmediev *et al.* [72] theoretically predicted different forms of polarization locked vector solitons in weakly birefringent optical fibers. In contrast to the theoretical studies, except that the group velocity locked vector soliton was experimentally confirmed [27], experimental observations on the other types of the theoretically predicted vector solitons were hampered by the stringent requirement on the birefringence property of the SMFs. To form these types of vector solitons the SMFs need to have stable weak birefringence over a long distance, while in practice it is difficult to make such SMFs. However, it was recently found that the requirement could be leveraged if the optical pulse is propagating in a fiber

cavity, where as far as the average birefringence over one cavity roundtrip is close to zero, the phase locked vector solitons could be formed.

Nowadays, vector solitons fiber laser has captured great attentions for its potential applications in optics communications. To generate vector solitons, passively mode locked fiber laser without polarization sensitive components is an essential condition. Recently, vector soliton operation of a fiber laser mode-locked with atomic layer graphene was reported. Graphene has been shown to have not only extraordinary electronic properties but also novel optical properties. An example of graphene's novel optical properties is its broadband saturable absorption, which has been exploited for passively mode locking lasers of different wavelengths [17,59,65,73,74]. The two dimensional structure of graphene determines that its saturable absorption is independent of the light polarization when the light is incident perpendicularly to its plane. Like a semiconductor saturable absorption mirror (SESAM), this polarization insensitive saturable absorption of graphene could be used to passively mode lock a laser and introduce no polarization discrimination. Moreover, as graphene saturable absorbers are easy to make, and have superbroad bandwidth and variable modulation depth when different number of graphene layers is used, it is superior for generating vector solitons in fiber lasers.

In this chapter firstly we confirms the vector nature of the solitons formed in the graphene mode-locked erbium-doped fiber lasers and then we focus on the further study of the vector soliton formation and multiple vector soliton interactions in the graphene mode-locked erbium-doped fiber lasers. In Section 3.1, we introduce the polarization rotation features of multiple vector solitons formed in an all anomalous dispersion fiber laser mode locked with atomic

layer graphene. In Section 3.2, in the same graphene mode-locked fiber laser cavity various vector multi-soliton operations and their mutual interactions are experimentally investigated. Section 3.3 summarizes the results and shows the conclusion.

3.1 Polarization Rotation and Locking of Multiple Vector Solitons

Vector solitons was first experimentally observed by S. T. Cundiff *et al.* in a passively mode locked fiber laser [75]. In their experiment both the polarization rotating and the polarization locked vector solitons were obtained. Vector soliton formation in passively mode locked fiber lasers were also experimentally confirmed by several other groups [22,76-78]. However, their fiber lasers were mode locked with the SESAM and the polarization rotation and locking properties have not been experimentally verified in graphene fiber laser.

In this section we investigate the polarization rotating and polarization locked vector solitons in a fiber laser mode locked by graphene. It was found that different from the case of the polarization locked vector soliton formed in the laser, two extra sets of spectral sidebands always appear on the soliton spectrum of the polarization rotating vector solitons. We confirm that the new sets of spectral sidebands have the same formation mechanism as that of the Kelly sidebands. The extra spectral sidebands can be considered as a characteristic of the vector soliton formation in the fiber laser.

3.1.1 Experimental Results

Figure 3.1 A schematic of the fiber laser setup. PC: Polarization controller; EDF: Erbium-doped fiber; SMF: Single-mode fiber; WDM: Wavelength-division multiplexer.

The graphene mode-locked fiber laser is schematically shown in Figure 3.1. The laser cavity is a ring that consists of 0.7 m erbium-doped fiber (EDF Er80-8/125 from Liekki) with GVD of $-22.9 \text{ ps}^2/\text{km}$ and 22 m standard SMF. In addition, a 10% fiber output coupler was used to output the signal, a polarization independent isolator was used to force the unidirectional operation of the ring cavity, and an intra-cavity polarization controller (PC) was used to fine tune the linear cavity birefringence. All these passive components are made of the SMF or pigtailed with SMFs. The laser was pumped by a high power Raman fiber laser (KPS-BT2-RFL-1480-60-FA) of wavelength 1480 nm. The

graphene was fabricated with the chemical vapour deposition method [17]. The CVD deposited graphene is a free standing atomic layer graphene film, which is high crystalline and continuous. It can be readily lifted off from the deionized water and transferred onto a fiber pigtail. The number of the layers of the graphene can be identified from the Raman spectra and optical contrast spectroscopy. As more layers of the graphene corresponds to a high absorption, by simply measuring the saturable absorption strength of the graphene we can estimate the number of the layers of the graphene. It is estimated that the graphene used have 3~5 atomic layers with a saturable absorption modulation depth of ~23%. The soliton spectra of the laser were measured with an optical spectrum analyzer (ANDO AQ6317). The soliton pulse train emitted by the laser was monitored by a 1 GHz oscilloscope (Lecroy Waverunner 6100) together with three 1.2 GHz photo-detectors (Thorlab DET01CFC).

Figure 3.2 Vector soliton emission of the fiber laser. (a) Soliton spectra. Blue line: the total laser emission without passing through a polarizer; Red and Green lines: the two orthogonal polarization components resolved with an external cavity polarization beam splitter. (b) Oscilloscope trace of the solitons.

Self-started mode locking was always obtained in the laser when the pump power is increased to ~ 90 mW. Immediately after mode locking, multiple pulses were normally obtained in the laser cavity. Occasionally the pulses were

observed moving randomly in the cavity. However, through carefully adjusting the intracavity polarization controller and/or changing the pump power, a stable state where the multiple pulses stay stationary with respect to each other in the cavity, could be obtained. Such a state was shown in Figure 3.2 for example. Figure 3.2(a) is the measured optical spectrum. Kelly sidebands are obvious on the spectrum [79,80], confirming that the pulses are optical solitons. The central wavelength of the solitons is at 1562.40 nm. Figure 3.2(b) is the measured oscilloscope trace of the soliton pulses. Several uniform soliton pulses trains can be clearly observed, indicating that the solitons possess the soliton energy quantization property. We note that different from the soliton spectra obtained from a NPR mode locked fiber laser, apart from the Kelly sidebands, there are also another set of spectral sidebands (indicated by the arrows) on the soliton spectrum shown in Figure 3.2(a). At first sight one may confuse that all the sharp spectral peaks on the soliton spectrum are the Kelly sidebands. However, as one carefully adjusts the intra cavity PC, it will be observed that one set of the spectral sidebands shift their positions remarkably on the soliton spectrum, while the other set of spectral sidebands have almost no position change. The latter set of spectral sidebands is the Kelly sidebands. We point out that the appearance of the other set of spectral sidebands is a result of the vector soliton formation in the fiber laser. To clearly identify the difference of the spectral sidebands, it is better to measure the polarization resolved spectra of the vector solitons. As in our laser there is no any polarization discrimination component in the cavity, the net cavity birefringence could be tuned to a very small value. Consequently, the formed solitons are all vector soliton and consist of two orthogonal polarization components. To simultaneously measure the

polarization resolved spectra of the vector solitons, experimentally we used a fiber pigtailed external cavity polarization beam splitter to separate the two orthogonal polarization components. In order to balance the fiber pigtail induced linear polarization rotation, we inserted a polarization controller between the laser output port and the polarization beam splitter. The polarization resolved spectra of the solitons measured are shown in Figure 3.2(a). On the polarization resolved spectra while the Kelly sidebands always exhibit as spectral peaks, the other set of spectral sidebands displays either as a spectral peak or a spectral dip, between the two orthogonal polarization components there is always a peak-dip relationship, indicating the existence of coherent energy exchange between the two soliton components. In a previous work it has been shown that such spectral sidebands are formed due to the coherent coupling between the two polarization components of the vector solitons in a laser [23]. Here it is worth noting that four-wave mixing between the two orthogonal polarization components of light in weakly birefringent SMFs were experimentally observed by Trillo *et al.* [81] and numerically investigated by Blow *et al.* [82]. It was shown that this effect could cause polarization instability. However, different from the case of light propagation in SMFs, which can be treated as a conservative system, light propagation in a laser cavity also experiences gain and losses. The formation of a stable soliton in a laser needs to fulfill the gain-loss balance. Probably it is because of this difference no soliton polarization instability was observed in our experiment. Moreover, we note that a soliton circulating in a cavity also has to satisfy the cavity boundary condition. Haelterman *et al.* have theoretically shown that the cavity detuning could alter the phase matching condition of the coherent wave

coupling in a dispersive cavity [83]. Indeed, in our experiments it was observed that the positions of the extra spectral sidebands vary remarkably with the adjustment of the paddles of the intra-cavity PC.

Figure 3.3 Oscilloscope traces of the pulse train in a phase locked vector soliton operation state. Blue line: the total laser emission without passing through a polarizer; Red and Green lines: the two orthogonal polarization components resolved with an external cavity polarization beam splitter.

Like the appearance of Kelly sidebands is a characteristic of the soliton formation in a laser, we found that the appearance of the above extra spectral sidebands is a characteristic of the vector soliton formation in our fiber laser. Experimentally, we further noticed that depending on the laser operation conditions, two types of vector solitons could be formed: The polarization locked vector solitons and polarization rotation vector solitons. Figure 3.3 shows the oscilloscope traces of a state with multiple polarization locked vector solitons in cavity. In this case all vector solitons in the cavity have exactly the same polarization and their polarization features remain unchanged as they

propagate in the cavity. Therefore, when measured after an external cavity polarizer, after every cavity roundtrip all the solitons in the cavity remain the same pulse intensity. Optical spectra of all the polarization locked vector solitons are similar to those as shown in Figure 3.2(a), except that the positions of the extra spectral sidebands vary.

Figure 3.4 A polarization rotation vector soliton state of the laser. (a) The total and the polarization resolved soliton pulse trains with multiple solitons in cavity. (b) Optical spectra of the solitons. Blue line: the total laser emission without passing through a polarizer; Red and Green lines: the two orthogonal

polarization components resolved with an external cavity polarization beam splitter.

Figure 3.4 shows the oscilloscope traces and spectra of a polarization rotation vector soliton state. Without passing through an external cavity polarizer, the oscilloscope trace shows no difference to that of a polarization locked state, as shown in Figure 3.3. However, from the polarization resolved traces, one can clearly identify the soliton pulse intensity variations. Carefully checking the pulse intensity variations, it can be identified that the polarization of the vector solitons shown in Figure 3.4 is rotating in the cavity. After every three cavity-roundtrips their polarization rotated back. We found experimentally that whenever the polarization of the vector solitons was rotating in the cavity, another two sets of weak spectral sidebands (indicated with arrows in Figure 3.4(b) further appeared on the soliton spectrum. Comparing Figure 3.4(b) with Figure 3.2(a) it is easy to see that apart from the Kelly sidebands, there are another three sets of spectral sidebands visible on the soliton spectrum now. Based on the polarization resolved spectra, one set of the sidebands is due to the coherent energy exchange between the two orthogonal polarization components, as described above. Each of the other two sets of the weak new sidebands (indicated with arrows in Figure 3.4(b)) appears only on spectrum of one of the two orthogonal polarization components, respectively. Moreover, no matter how the two orthogonal polarization components are split, the weak new sidebands always exhibit as spectral peaks while no spectral dip could be obtained. The feature of the weak new sidebands is similar to that of the Kelly sidebands.

3.1.2 Discussions

Obviously, the new sets of weak spectral sidebands are formed due to the polarization rotation of the vector solitons. The periodic polarization rotation of the vector solitons introduces an extra periodic soliton parameter variation, therefore, leading to the formation of another set of spectral sidebands. As the soliton polarization rotation has a different period than that of the soliton circulation in the cavity, the new spectral sidebands have different positions to those of the conventional Kelly sidebands. We point out that similar to the formation of the Kelly sidebands, the formation of the new weak spectral sidebands is a linear effect. They are caused by the constructive interference between the dispersive waves emitted by the solitons as their polarizations periodically rotate in the cavity. For a vector soliton it has two orthogonal polarization components, in a state of polarization rotation each soliton polarization component would have different phases. It could be due to the subtle difference between the two polarization components that each soliton component formed its own new spectral sidebands under polarization rotation of the vector solitons. Finally, we point out that although the polarization rotation spectral sidebands have the same formation mechanism as those of the Kelly sidebands, their positions varied remarkably with the laser operation conditions, because the rotation speed of the vector solitons varied with the experimental conditions.

3.2 Multiple Vector Soliton Distribution

Multiple soliton formation in mode locked fiber lasers is a well-known phenomenon and has been extensively investigated in the past [84,85]. It has been shown that various mechanisms could lead to the formation of multiple solitons. These mechanisms include the wave-breaking effect, the effective spectral filter effect, the soliton peak clamping effect, and the soliton shaping of dispersive waves. Indeed, in previous experimental studies on the multiple solitons formed in the fiber lasers, people have observed various modes of multiple soliton operation, such as soliton bunches, soliton collisions [86], vibration of soliton pairs [87], restless solitons [21], bound state of solitons [88-92] and so on. Some of these effects can be traced back as a result of the direct soliton interaction of the dissipative solitons, or the dispersive waves mediated NLSE type of soliton interaction. Therefore, based on the different features of the multiple soliton operation of a fiber laser one can get an insight into the properties of the formed solitons. Recently, a novel form of multiple soliton operation named as “soliton rain” was observed by S. Chouli *et al.* [93,94]. It was shown that the multiple soliton formation in a fiber laser could even manifest the process of the rain-drop formation in the nature, or in another word, the multiple soliton interaction in a fiber laser follows the universal statistics of the many body systems. However, most of the previous experimental studies on the multiple soliton operation of fiber lasers were focused on fiber lasers passively mode-locked with the nonlinear polarization rotation (NPR) technique, in which the scalar solitons are always formed.

To study the multiple vector soliton formation, the semiconductor saturable absorber mirror (SESAM) was initially employed in the fiber laser cavity [21,95]. In an experiment with a SESAM as the passive mode locker Zhao *et al.* have observed a state of so-called bunched restless vector solitons [21]. Vector soliton bunching controlled by SESAMs with different recovery times was also experimentally investigated by R. Gumenyuk *et al.* [95]. Recently, the mode locking of fiber lasers with atomic-layer graphene based saturable absorbers has attracted considerable attention of research [17,96,97]. Compared with the SESAMs, graphene has a number of novel characteristics, e.g. their saturable absorption have super broad bandwidth and ultrafast recovery time, which render that the formed multiple vector solitons could have many new dynamic features.

In this section we report on an experimental study of the multiple vector soliton operation of an erbium doped fiber laser mode-locked with the atomic-layer graphene. We show experimentally that depending on the laser cavity design and the concrete laser operation conditions, five different characteristic modes, bunches of multiple vector solitons, random static distribution of vector soliton, vector soliton rain, restless vector soliton and giant pulse of the multiple soliton operation were obtained in the vector soliton fiber lasers. Especially worth mentioning is that vector soliton rains with either the polarization locked or the polarization rotating vector solitons were revealed.

The schematic diagram of graphene mode-locked fiber ring laser is the same as that shown in Figure 3.1. The 0.7 m highly erbium doped fiber (Liekki Er80-8/125) with anomalous dispersion was still used as the gain medium. Other

fibers used were the standard single mode fiber (SMF28) with a GVD parameter of 18 (ps/nm)/km. The cavity length is slightly different which is about 22.6 m. The atomic layer graphene used was also synthesized with the CVD method [17]. The graphene sheets were carefully deposited on the end facet of an optical fiber, which was inserted in a FC/APC connector. It is noted that the graphene sheets used here is same as that in Section 3.1. The graphene have 3~5 atomic layers with a saturable absorption modulation depth of ~23%. In order to measure more details of the pulse train in the time domain, a 33 GHz high-speed oscilloscope (Agilent DSA-X 93204A) together with two 45 GHz photo-detectors (New focus 1014) was used.

3.2.1 Bunches of Multiple Vector Solitons

Figure 3.5 A typical multiple vector soliton bunch state of the fiber laser. (a) and (b): Two states of the oscilloscope trace; (c) The details of the soliton bunch; (d) Polarization resolved optical spectra. Blue line: the total output; Green line: Output along the vertical axis; Red line: Output along the horizontal axis. (e) Autocorrelation trace of the pulse.

Multiple solitons were always formed initially in the laser cavity when the mode locking was achieved at a pump power of ~ 80 mW. A typical state of the multiple solitons immediately after the mode locking is shown in Figure 3.5(a). The solitons were randomly distributed in the cavity but they were not static. The solitons moved slowly in the cavity and eventually came together, forming

a soliton bunch in the cavity, as shown in Figure 3.5(b). After the soliton bunch was formed, the laser then operated in a stable state, and the same soliton bunch repeated with the cavity repetition rate on the oscilloscope trace. Figure 3.5(c) shows a zoom-in structure of the stable soliton bunch. In the particular case there were 11 solitons in the bunch. The separation between the solitons was not constant and the average distance was ~ 1 ns. Figure 3.5(d) shows the corresponding optical spectra of the laser emission. The central wavelength of the solitons was at 1563.6 nm, and the 3 dB bandwidth was ~ 2.3 nm. Two sets of spectral sidebands appeared on the spectra. The one indicated by arrows was the Kelly sidebands. It confirmed that the mode locked pulses were solitons [79,80]. Apart from the Kelly sidebands, there was another set of sidebands. The set of sidebands exhibits peak-dip alteration in the polarization resolved spectra, whose appearance was a result of the coherent energy exchange between the two polarization components of the vector solitons [23]. Therefore, based on the characteristic optical spectra (particularly the additional peak-dip spectral sidebands) one can easily determine if the vector solitons are formed. It is noted that these extra sidebands always appeared on the spectra of the mode locked pulses, suggesting that the as-formed solitons were always vector solitons. Except the spectral sidebands, the optical spectra profile was smooth. The state of vector soliton bunch was very stable. It could last several hours if no laser operation conditions were changed. Figure 3.5(e) shows the measured autocorrelation trace of the solitons. The measured pulse width was 1.8 ps. If a sech^2 pulse profile is assumed, the actual pulse width was about 1.17 ps. The time bandwidth product of the pulses is 0.335, indicating that the pulses are transform-limited. A state of single vector soliton in cavity was obtained

through carefully reducing the pump power to ~ 30 mW. The energy of a single vector soliton of the laser is estimated to be ~ 8 pJ.

3.2.2 Random Static Distribution of Vector Solitons

Figure 3.6 Random static vector soliton distribution at a low pump power.

By tuning the paddles of the PC but without introducing any unstable CW components, a vector soliton bunch could explode. Subsequently a state of steady random vector soliton distribution over the whole cavity was obtained. Figure 3.6 shows a typical example. There were 7 solitons coexisting in the cavity and they are far apart from each other. The state shown was obtained under a pump power of ~ 50 mW. If the pump power was then increased, a CW spectral component appeared on the soliton spectrum, indicating that a CW background would be introduced into the cavity. In this case the solitons started to move in the cavity due to the CW mediated soliton-soliton long distance interaction force. Once the CW component was suppressed through carefully reducing the pumping, another state of randomly distributed multiple vector

soliton could be obtained. Similar phenomena were also observed previously on the multiple scalar soliton fiber lasers [98]. It shows that the unstable CW beam provides a global soliton interaction mechanism among the vector solitons formed in a laser.

3.2.3 Restless Vector Solitons

Figure 3.7 Bunch of restless vector soliton bunch (video). (a) Soliton bunch moving in the cavity (Media 1); (b) Solitons oscillating in the bunch (Media 2).

At a relatively stronger pump power (~ 130 mW), as a result of appearance of unstable CW component in the cavity an initially stable state of randomly distributed multiple solitons was destroyed and the solitons started to move in the cavity and eventually formed a soliton bunch. However, the vector solitons in the bunch were not static, but moved constantly with respect to each other, forming a so-called restless vector soliton bunch. A similar restless vector soliton bunch was also observed in a SESAM mode locked fiber laser by Zhao *et al.* [21]. As the saturable absorption properties of SESAM differ from those of the graphene only in that they have different recovery time and saturable absorption strength, it is not surprising why similar multiple vector soliton operation could be obtained in the graphene mode locked fiber lasers. In our

experiments two different types of the restless vector soliton evolutions were observed. Figure 3.7 (see the Media 1 and Media 2 in the attachments) shows these two types of the restless vector soliton evolutions, respectively. Figure 3.7(a) shows a stroboscopic record of vector soliton bunch movement. The soliton bunch moved as a unit in the cavity, while some solitons within the bunch moved at different velocities. Therefore, vector soliton collisions occurred as indicated by the appearance of the high intensity peaks in the pulse train. Despite of the collisions among the vector solitons within the bunch, the solitons could not escape from the bunch. The collision among the solitons became more frequent with the increase of the pump power. Figure 3.7(b) shows a stroboscopic record of another restless vector soliton evolution, where no soliton collisions occurred. However, the relative soliton positions within the bunch still varied.

3.2.4 Vector Soliton Rain

If the pump power was further increased, the number of solitons formed in the cavity continuously increased. At an appropriate PC setting, an interesting dynamic pattern as shown in Figure 3.8(a) could be obtained. In the scalar soliton lasers such a state was called “soliton rain” [93]. The state was called as “soliton rain” because when measured with a low speed detection system, the soliton pulse train in one cavity length could be phenomenally considered as composed of two parts: a soliton flow part (the left hand side of the screen) and a soliton condensed part (the right hand side of the screen), and the solitons are constantly moving from the soliton flow part to the soliton condensed part, which is analogous to the process of rain droplet formation (evaporation from

the noisy background/falling to the condensed part) in the nature. As can be seen in Figure 3.8(b), when measured with our high-speed detection system the soliton condensed phase is actually a bunch of solitons close to each other. It was experimentally observed that stronger pump power could induce higher density of the solitons within the bunch. Figure 3.8(c) shows a soliton rain state obtained at the pump power of 200 mW. Not only more solitons were formed in the train but also a large pulse was formed at the end of the flow. The large pulse was a result of the soliton collision within the train. It is worth noting that all the solitons in the train are constantly moving. Figure 3.8(d) (see Media 3 in the attachments) shows a record of the state.

Figure 3.8 Polarization locked vector soliton pulse trains: (a) A typical soliton rain at a pump power of 140 mW; (b) zoom-in of the soliton condense phase; (c) Soliton rain at a pump power of 200 mW; (d) Snapshot of the vector soliton rain (Media 3).

The main difference between the vector solitons and scalar solitons is that a vector soliton has two coupled orthogonal polarization components. Depending on their coupling the polarization of the vector soliton could be either fixed or rotating [28,75]. Therefore, different from the scalar soliton rain, by using the polarization resolved measurement technique we are able to identify two different types of vector soliton rains, the polarization locked and rotation vector soliton rain. Figure 3.9(a) shows the case of the polarization locked vector soliton rain. It can be clearly seen that all the solitons in the flow are uniform both in the total output and the polarization resolved outputs, which is a feature of the polarization locked vector solitons. Figure 3.9(b) was obtained by slightly changing the setting of the polarization controllers from that of a polarization locked vector soliton rain state. In this way a polarization rotation vector soliton rain state could be obtained. Comparing with Figure 3.9(a), it can be seen while all the solitons have the same pulse heights in the total output, the soliton pulse height varied periodically in the polarization resolved outputs, indicating that the solitons are polarization rotating vector solitons. In particular, the polarization of the vector solitons restore the original polarization every triple of the cavity round-trip time. Polarization rotation of the vector solitons could also be identified from their soliton spectra. Figure 3.9(c) shows the corresponding optical spectrum of the solitons shown in Figure 3.9(b). On the spectrum two extra sets of spectral sidebands could be observed, which were caused by the polarization rotation of the vector solitons in the laser cavity as mentioned in Section 3.1.

Figure 3.9 (a) Oscilloscope trace of the polarization locked vector soliton rain. Blue line: the total output; Green line: along the horizontal axis; Red line: along the vertical axis. (b) Oscilloscope trace of the polarization rotating vector soliton rain. Blue line: the total output; Green line: along the horizontal axis; Red line: along the vertical axis. (c) The optical spectrum of the polarization rotating vector soliton rain.

3.2.5 Giant Pulse State

Figure 3.10 Giant pulse state when the soliton rain exploded at 200 mW pump power. (a) Oscilloscope trace; (b) 10 ns scale of the giant pulse; (c) Oscilloscope trace; (d) optical spectrum.

When the pump power is high enough (~ 200 mW), under an appropriate PC setting another multiple vector soliton operation state as shown in Figure 3.10 could be observed. Figure 3.10(a) shows a typical oscilloscope trace of the state. It displayed as a stable pulse train, and each pulse could have very large pulse energy. In our experiment pulse energy as large as 60 nJ had been obtained. Figure 3.10(b) shows the single pulse profile measured with our high-speed detection system. As the pump power increased the pulse width also increased, but the pulse peak power only increased slightly. Figure 3.10(c) shows the autocorrelation trace of the “giant pulses”. It has an obvious pedestal. On top of

the pedestal there is a pulse peak with pulse duration of about 1.29 ps, which is similar to that measured in the multiple vector soliton state. Figure 3.10(d) shows the optical spectrum of the giant pulses. It has a triangular shape with blurred spectral sidebands. Based on the measured autocorrelation trace and the pulse spectrum, we conclude that that giant pulses are made of many tightly packed solitons with random soliton separations.

3.2.6 Discussion

It is to note that similar multiple soliton patterns have also been observed in the scalar soliton fiber lasers, which shows that the formation of these multiple soliton patterns is a general feature of the soliton lasers, which is independent on whether they are scalar or vector solitons. In a previous work on the scalar soliton interaction in the passively mode locked fiber lasers it have been shown that various modes of the multiple scalar soliton operation could be traced back to the specific features of the soliton interaction in the cavity [98]. Experimentally, we found that the same is valid for the vector solitons. Despite of the fact that a vector soliton consists of two orthogonally coupled polarization components, and therefore has more complicated dynamics than that of a scalar soliton, the interaction between the vector solitons in a fiber laser cavity is still mainly determined by the same types of interactions mechanisms: the unstable CW caused global type of soliton interaction, the dispersive wave mediated long range soliton interaction, and the direct soliton interaction, in the fiber lasers that are mode locked by the saturable absorbers like the SESAM or graphene, there is also a absorber caused attraction force, whose strength and action range depends on the saturable absorption strength

and the recovery time of the saturable absorbers. The various modes of the vector soliton operation are actually the combined consequence of the mutual actions of these soliton interaction mechanisms on the multiple solitons formed in the laser cavity, e.g. under the effects of a weak unstable CW in the cavity and the saturable absorber effect, the multiple solitons would always have the tendency of soliton bunching. Depending on the strength of the dispersive waves generated by the solitons, one would expect that the solitons in a bunch could be either static or moving restlessly. A soliton rain state could be considered as a special case of the soliton bunch formed under strong pumping, where a lot of solitons are formed in the cavity. In this case due to the fast saturable absorption of the absorber, the soliton distribution displays an increasing density with the time. A state of bunched solitons is formed due to the direct soliton interaction [98]. So far in our experiments no stable states of the closely bunched vector solitons have been observed. Nevertheless, we believe that the giant pulse emission states could be formed as a result of the direct soliton interaction in the laser. However, in the current case many solitons are bunched together with random soliton separations, leading to the observed features of the multiple vector soliton laser emission.

3.3 Summary

In summary, we have experimentally investigated the vector soliton operation of a fiber laser mode locked with the atomic layer graphene. Both the polarization locked and polarization rotation vector solitons were observed in

our laser. It was shown that the cavity effect always induces resonant energy exchange between the two orthogonal polarization components of the vector solitons. However, no polarization instability of the solitons was observed. For the polarization rotation vector solitons, it was found that two new sets of spectral sidebands would appear on the vector soliton spectrum, whose formation is due to the periodic polarization rotation of the vector solitons.

We have also experimentally studied the multiple vector soliton operation and observed various characteristic operation modes of the multiple vector soliton operation of the laser, these include the vector soliton bunches, random static vector soliton distribution, restless vector soliton bunches, giant pulse emission, and vector soliton rains. Different from the case of scalar soliton rain, either the polarization-locked vector soliton rain or the polarization rotation vector soliton rain was further identified. Based on results of the experimental studies, we point out that the appearance of the different modes of the multiple vector soliton operation could be well understood by the soliton interaction caused by the following four types of main mechanisms, the unstable CW related global type of soliton interaction, the dispersive wave mediated long range soliton interaction, the direct soliton interaction, and the saturable absorber induced soliton-soliton attraction. Our experimental results demonstrate that in terms of the soliton interaction in the laser cavity both the vector solitons and the scalar solitons show similar features.

Chapter 4 Quasi-periodicity of Vector Solitons

in Graphene Mode-locked Fiber Lasers

Passively mode-locked fiber lasers have been intensively investigated both due to their practical applications as a cost-effective ultrashort pulse source and rich dissipative soliton dynamics, which can be well described by the complex Ginzburg-Landau equation (CGLE) [26]. Theoretical studies on the dissipative solitons have revealed that apart from their stability against perturbations, they can also exhibit different types of complicate dynamics [26,99]. These include the soliton pulsations, soliton quasi-periodicity, soliton period-doubling bifurcations and route to chaos. Indeed, all of these features of the dissipative solitons have been experimentally confirmed, e. g. the soliton period doubling and tripling effect was first reported by K. Tamura *et al.* [100], the soliton period doubling and quasi periodicity was observed by G. Sucha *et al.* in an additive pulse mode locked laser [101], the quasi-periodic route to chaos was reported by F. Sanchez *et al.* in an erbium-doped fiber laser [102], and by S. Bolton and M. Acton in a Kerr-lens mode locked Ti:sapphire laser [103]. L. Zhao *et al.* obtained both experimentally and numerically soliton period-doubling route to chaos in a fiber laser [104]. Moreover, detailed experimental studies have shown that associate with the occurrence of the soliton dynamics, extra spectral sidebands always appear on the soliton spectra, which was later turned out to be a result of the soliton modulation instability (MI) in the lasers [105]. The soliton MI leads to coherent wave coupling between the solitons and

dispersive waves. Under strong coherent wave mixing a dissipative soliton could display various types of dynamic features.

However, the majority of the previous experimental studies on the dissipative soliton dynamics were focused on the mode locked solid state lasers with a polarization selective element in cavity, or fiber lasers mode locked with the nonlinear polarization rotation (NPR) technique, where in order to achieve the self-started passive mode locking a polarizer was inserted into the cavity. The polarization selective element or polarizer defines the polarization of light at the cavity position. Under the effect of cavity boundary condition it further fixes the polarization of light everywhere in the cavity. Solitons obtained in the lasers are essentially scalar solitons. In a mode locked fiber laser if there is no any polarization sensitive components in cavity, the vector nature of light needs to be considered, consequently the vector solitons, which are solitons with two coupled orthogonal polarization components, would be formed [24]. Compared with the scalar solitons, a vector soliton could exhibit more complicated dynamics.

Several groups have experimentally demonstrated vector soliton operation in SESAM fiber lasers. Using a semiconductor saturable absorber mirror (SESAM) as the mode locker S. Cundiff *et al.* have first experimentally obtained the polarization locked and polarization rotation vector solitons in a linear cavity fiber laser [75]. L. Zhao *et al.* have experimentally investigated features of the vector solitons formed in a ring fiber laser, and observed period-doubling bifurcation of the vector solitons [106]. Nevertheless, experimental study on features of the vector solitons is still rare and there is still no study on this topic based on the graphene fiber laser, to the best of our knowledge.

In this chapter, we present results of experimental studies on the vector soliton formation and dynamics in an erbium-doped fiber laser passively mode-locked with atomic-layer graphene. We show experimentally that apart from the periodic vector soliton evolution in the laser cavity, under certain conditions the vector solitons formed in the fiber laser can exhibit quasi-periodic evolution, where both the pulse energy and the polarization of the vector solitons vary quasi-periodically with the cavity roundtrips. Moreover, like the quasi-periodic evolution of the scalar solitons obtained in fiber lasers, associated with the occurrence of the quasi-periodic dynamics of the vector solitons, an extra set of spectral sidebands appeared on the optical spectrum of the solitons. The extra spectral sideband formation was found to be a threshold effect, which is a characteristic of the MI of the solitons formed in a laser. Hence, the experimental result suggests that it is the cavity induced MI that has caused the quasi-periodic dynamics of the vector solitons. Quasi-periodicity of multiple vector solitons was also observed, where the vector solitons exhibit uncorrelated quasi-periodic evolutions in the cavity. In the quasi-periodic state the vector solitons no longer possess the soliton energy quantization feature. Section 4.1 presents the experimental results. Section 4.2 discusses the results. Conclusion is given in Section 4.3.

4.1 Experimental Results

Figure 4.1 A schematic diagram of the graphene mode-locked fiber laser.

The cavity configuration of the vector soliton fiber laser is schematically shown in Figure 4.1. The laser was pumped by a Raman fiber laser of wavelength 1480 nm. The ring cavity has a length of 22.6 m. A piece of 0.7 m erbium doped fiber (EDF Er80-8/125 from Liekki) with GVD of -18 ps/nm/km was used as the gain medium. The rest fibers used in the cavity were all standard single mode fibers. A 10% fiber output coupler was employed to output the signal. A polarization independent isolator was used in the cavity to force the unidirectional operation of the ring cavity, and a polarization controller (PC) was inserted in the cavity to fine tune the linear cavity birefringence. A few-layer graphene film, grown with the CVD was used to mode lock the fiber laser. The graphene was transferred to an end of a fiber pigtail, which was then connected to another fiber through a fiber connector. It is noted that the graphene film we used is the same as that of Section 3.1 and the properties of the graphene will not be further discussed. An optical spectrum analyzer (ANDO AQ6317) was used to monitor the optical spectrum of the solitons, and

a 1 GHz oscilloscope (Lecroy Waverunner 6100) together with three 2 GHz photo-detectors (Thorlab DET01CFC) was used to record the soliton pulse trains. To make sure that vector solitons are formed, we have checked the polarization loss of each individual component used in our laser. To experimentally resolve the features of the two orthogonally polarized components of the vector solitons, a fiber pigtailed polarization beam splitter was connected to the output port of the fiber laser with a PC inserted in between. The function of the external cavity PC is to balance the effect of weak linear birefringence of the output fiber.

Figure 4.2 A state of the polarization locked vector soliton operation of the laser. (a) The total and the polarization resolved optical spectra of the vector soliton. The extra set of spectral sidebands is indicated by the arrows; (b) oscilloscope trace of the vector soliton pulse train. (c) The polarization resolved oscilloscope trace of the vector soliton pulse train.

Mode locking of the laser was achieved at a pump power of ~ 90 mW. Initially, multiple mode locked pulses were obtained. However, through carefully adjusting the pump power, a single pulse operation state was then achieved. Under the single pulse operation the pump power became as low as 39 mW. Figure 4.2 shows for example a state of the single soliton operation of the fiber laser. The optical spectra of the pulse are shown in Figure 4.2(a). The existence of the Kelly sidebands on the spectra confirms that the mode locked pulse is a soliton. To prove that the observed soliton is a vector soliton, we note that apart from the Kelly sidebands, there is another set of spectral sidebands (indicated by arrows) on the soliton spectra. The extra set of spectral sidebands differs from the Kelly sidebands because their positions vary sensitively with the change of the linear cavity birefringence. In addition, on the polarization resolved spectra the extra set of sidebands exhibits clear difference from the

Kelly sidebands: the Kelly sidebands always display as spectral peaks, while the new set of sidebands show a dip-peak relation between the two orthogonal polarization components, indicating the existence of coherent energy exchange between them [23]. Figure 4.2(b) is the corresponding pulse train of the soliton. The time interval between the pulses is 113 ns, which coincides with the roundtrip time of the cavity. Based on the polarization resolved pulse trains shown in Figure 4.2(c), it is to conclude that the vector soliton is a polarization locked vector soliton as no polarization rotation exists. We note that despite of the existence of coherent energy exchange between the two polarization components, no linearly polarized soliton was obtained in our experiment. The observed vector soliton is elliptically polarized as no matter how the external cavity PC was adjusted, none of the two polarization resolved components could be turned to zero. In our experiments once the laser was mode locked, the obtained mode locked pulses were vector solitons, indicating that the average cavity birefringence of our laser was small. Depending on the pump strength, multiple phase locked vector solitons coexisting in cavity were also obtained, as shown in Figure 4.3(a) and (b). In the steady states all the vector solitons have exactly the same soliton parameters, indicating that under the polarization locked vector soliton operation the soliton energy is still quantized [107].

Figure 4.3 Polarization locked vector soliton pulse trains with (a) two solitons and (b) four solitons in the cavity.

Figure 4.4 The total and the polarization resolved optical spectra of a polarization rotation vector soliton of the laser. The extra set of spectral sidebands is indicated by the arrows.

Starting from a polarization locked vector soliton state, when pump power of the laser or the orientation of the paddlers of the intra cavity PC was carefully changed, the polarization of the vector solitons then became unlocked. Consequently, the polarization of the vector solitons rotated in the cavity. Associated with the soliton polarization rotation, two extra sets of spectral sidebands were found to appear on the spectrum of the vector soliton, as shown in Figure 4.4 (indicated by arrows). A similar phenomenon was also reported by Cundiff *et al.*[75]. However, the appearance of the new spectral sidebands did not affect the pulse height of the vector solitons as on the oscilloscope trace the vector soliton pulses were still uniform. In the Chapter 3 we have already experimentally investigated the formation mechanism of the two sets of extra spectral sidebands [108]. They were found to be the Kelly sidebands caused by the periodic polarization rotations of the vector solitons in the cavity. The periodic polarization rotation of the vector solitons introduces another periodicity in the laser, which causes extra constructive interference between

the dispersive waves radiated by the solitons as they circulate in the cavity. As a polarization rotating vector soliton has two orthogonal polarization components with unlocked phases, each polarization component would form its own Kelly sidebands. Therefore, two extra sets of spectral sidebands appeared on the vector soliton spectrum, while on the polarization resolved spectra of the vector soliton, only one set of the extra Kelly sidebands would be observable, as also shown in Figure 4.4. We point out that in the case where multiple vector solitons coexist in the cavity, although each of the vector solitons rotated at the same speed of rotation, the polarization orientations of the vector solitons were different.

Figure 4.5 The total and the polarization resolved optical spectra of the vector solitons under quasi-periodic evolutions. The extra set of spectral sidebands is

indicated by the arrows. (a) The case where the vector soliton has a large polarization ellipse; (b) The case where the strength of the two orthogonal polarization components of the vector soliton was comparable.

Figure 4.6 Oscilloscope traces of the vector soliton under quasi-periodic evolution. Blue line: the total vector soliton pulse train; Green line: the horizontal axis; Red line: the vertical axis.

Both the polarization locked and polarization rotation vector solitons could be easily obtained in our laser. Because the solitons are dissipative in nature, the energy of the vector solitons increases as the pump power increases. Experimentally we noticed that as the energy of the vector solitons increased to a certain level, an extra set of spectral sidebands further appeared on the vector soliton spectrum, as shown in Figure 4.5 (indicated by arrows). As the new extra set of spectral sidebands became clearly visible, the pulse height of the vector solitons on the oscilloscope trace also gradually became non-uniform, as shown in Figure 4.6. Based on the polarization resolved soliton pulse traces, it is easy to see that the polarization rotation of the vector soliton also became quasi-periodic with the cavity roundtrips.

Two different situations were revealed through the polarization resolved measurement. Figure 4.5(a) shows a case where the vector soliton has a large polarization ellipse. In this case the new extra spectral sidebands only appeared on the strong polarization component of the vector soliton. On the spectrum of the weak polarization component there is no sign of the new extra spectral sidebands. Figure 4.5(b) shows another case where the strength of the two orthogonal polarization components of the vector soliton was comparable. In this case the new extra spectral sidebands appeared at the same position on both polarization components. This experimental result suggests that the appearance of the new extra spectral sidebands is a threshold effect. Its appearance requires that the pulse intensity of the soliton is sufficiently strong. In addition, comparing with the results shown in Figure 4.5(a) and (b), which were taken under two different intracavity PC settings but the same pump power, the positions of the new extra spectral sidebands also depend on the laser cavity detuning.

In the past the formation mechanisms of various spectral sidebands of the scalar solitons formed in fiber lasers were both experimentally and numerically investigated and it was shown that coherent wave mixing between the solitons and dispersive waves could occur in a laser, which was previously named as the soliton modulation instability [105]. Under the effect of the soliton modulation instability the energy of the soliton and dispersive waves are coherently coupled. Under strong coherent energy coupling the energy of the soliton could become varying as it propagates in the cavity. A soliton circulating in a laser cavity also has to satisfy the cavity boundary condition, which modifies the phase matching condition of the coherent wave mixing. As shown by Haelterman *et al.*, the

cavity feedback effect is reflected as that the resonant frequency now depends on the cavity detuning as well [83]. The features of the observed new extra spectral sidebands well match those formed due to the soliton MI reported for the scalar solitons.

The soliton MI is a threshold effect. Like the case of scalar solitons, in our experiment once the intensity of the vector solitons became beyond the MI threshold, a new set of spectral sidebands appeared at the positions where the phase matching condition between the soliton and dispersive waves is fulfilled. However, different from the scalar solitons, a vector soliton has two coupled orthogonal polarization components. Further depending on the strength of them, either both of the two polarization components or one of them could become unstable. Nevertheless, in either case the evolution of the vector soliton would become quasi-periodic. Here it is worth noting that because the coherent wave mixing is between the soliton and dispersive waves, no spectral dip-peak but only the spectral peaks were observed on the polarization resolved spectra. Furthermore, changing the cavity detuning also shifted the positions of the spectral sidebands on the soliton spectrum. From a quasi-periodic vector soliton evolution state, if the pump power was further increased, instead of that the laser emission evolved to the chaotic state, a new soliton would be generated, eventually the vector soliton evolution became periodic or quasi-periodic with less soliton energy variation. Further increasing the pump power, the same process as described above repeated.

Figure 4.7 The quasi-periodic evolution of two vector solitons coexisting in cavity.

Quasi-periodic vector soliton intensity variations of multiple vector soliton in cavity were also observed. Figure 4.7 shows for example a case of two vector solitons coexisting in the cavity. Each of the vector solitons exhibits the quasi-periodic energy variation, as clearly shown by their oscilloscope traces. In the state the solitons in the cavity have uncorrelated soliton intensity variations. No soliton energy quantization exists in the case. With too many vector solitons in cavity, the interaction among the solitons could become very complicated.

4.2 Summary

In conclusion, we have experimentally studied the vector soliton operation of an erbium-doped fiber laser passively mode locked with atomic layer graphene. It was found that under certain operation conditions the vector solitons of the laser could exhibit quasi-periodic evolution. Similar to the case of quasi-periodic evolution of the scalar solitons observed in a fiber laser, in the state a new set of spectral sidebands appeared on the spectra of the vector solitons, and the

appearance of the new spectral sidebands was a threshold effect. Based on the features of the vector solitons, we pointed out that the appearance of the extra spectral sidebands is due to the cavity induced soliton modulation instability, and the quasi-periodic vector soliton evolution is a result of the strong resonant energy coupling between the vector soliton and the dispersive waves. However, in comparison with the MI of the scalar solitons, our experimental results show that depending on the strength of each of the two orthogonal polarization components of the vector soliton, the MI could occur on one of the polarization components or both of them. Quasi-periodicity and quasi-periodic route to chaos are intrinsic features of the nonlinear dynamic systems. Our experimental results further confirm that the passively mode locked soliton fiber laser is a nonlinear dynamic system. No matter if scalar or vector solitons are formed in the laser, its dynamics follows the universal rules of the nonlinear dynamics.

Chapter 5 Noise-like Pulses and Rogue Waves

in Graphene Mode-locked Fiber lasers

In a passively mode locked fiber laser, soliton pulse formation in the cavity is a complicated dissipative process, in which except the impact of the saturable absorber, the balanced interaction among the laser gain, loss, cavity dispersion and the nonlinear Kerr effect also plays an important role, and directly affects the formed soliton pulse parameters. If the cavity parameters are not suitably controlled, the soliton fiber laser may operate in the regime of some distinct pulses such as noise like pulses, extreme pulses. Noise like pulses have been a well-documented phenomenon which has been investigated in the scalar soliton fiber lasers [109-111]. Rogue wave is a novel state which was reported recently [112,113]. However, both of them have not been observed in the vector soliton fiber lasers. It is interesting and meaningful to investigate the formation and properties of the noise like pulses and rogue waves in a vector soliton fiber laser mode locked with graphene.

In this chapter, we report on the noise like pulses and rogue waves formed in the graphene mode locked vector soliton fiber lasers. In Section 5.1, we report on the experimental observation of noise-like (NL) pulses in a fiber laser passively mode locked by atomic layer graphene. Based on the experimental and numerical studies, we found that the formation of the NL pulses is a combined consequence of the pulse breaking induced by the excessive

nonlinearity of the cavity and the mutual attraction of the soliton pulses caused by the ultra-fast saturable absorption of the graphene. The large energy and broadband spectrum of the NL pulses may find applications in super-continuum generation and metrology where low coherence is required. The experiment also suggests a new research direction for the graphene-based ultrafast laser photonics. In Section 5.2, optical rogue waves are experimentally observed in an all anomalous dispersion erbium-doped fiber laser mode-locked with atomic layer graphene. We identify that pulses with extreme high intensity were occasionally generated from the multiple pulse collisions, which is caused by the vector soliton interactions under existence of unstable continuous wave (CW) emission in the laser cavity. The probability distribution of pulse peak amplitudes confirms the rogue wave character of the observed giant pulse.

5.1 Noise-like Pulses

The noise-like pulse formation in fiber lasers passively mode locked by the nonlinear polarization rotation (NPR) technique has been widely studied and its properties and formation mechanism are already clear. It might be of importance to study the features of the NL pulses formed in other fiber lasers that are mode locked by different methods. In this paper, we experimentally and numerically show that the noise like pulses can also be formed in a graphene mode locked fiber laser. Moreover, extensive experimental and the numerical studies have further shown that the pulse-to-pulse attraction, which physically correlates with the ultrafast recovery feature of the graphene saturable absorber,

could have played a role on the formation of the NL pulses in the fiber laser. Based on the observed experimental results we point out that the graphene mode locked NL pulse emission fiber lasers could be a promising large energy pulse source with the advantages of compactness, small size, all-fiber-format and low cost.

5.1.1 Experimental Results

Figure 5.1 A schematic of the experimental setup.

The experimental setup of the graphene mode locked fiber laser is schematically shown in Figure 5.1. The laser was pumped by a high power fiber Raman Laser source (KPS-BT2-RFL-1480-60-FA) with a central wavelength of 1480 nm, and a maximum pump power up to 5 W. The total cavity length was about ~70 m, comprising of 3 m erbium doped fiber (EDF) with group velocity

dispersion (GVD) of -48 (ps/nm)/km and ~67 m standard single mode fiber (SMF) with GVD of 18 (ps/nm)/km. The laser cavity was dispersion managed. It had a total cavity dispersion of about -8.50 ps². A 10% output coupler was used in the cavity to output the signal. A polarization independent isolator was employed to force the unidirectional operation of the ring cavity. An intra-cavity polarization controller (PC) was used to fine tune the linear cavity birefringence. All the passive components were made of the SMF. The graphene was fabricated by the chemical vapor deposition method. Optical Raman spectroscopy technique (Witec Alpha 300R) was used to identify the layer number of the graphene, and it was found that the graphene used had 3~4 layers.

Figure 5.2 Soliton operation of the fiber laser. (a) Optical spectrum measured. (b) The mode locked pulse train. (c) One “single pulse” of (b) detected by a high-speed oscilloscope.

The ring cavity fiber laser has a self-starting mode locking threshold of about 200 mW due to intra-cavity loss. Figure 5.2(a) shows a typical optical spectrum of the mode locked laser. Clear Kelly sidebands are visible on the spectrum, which is a characteristic of the soliton formation in the anomalous dispersion cavity fiber lasers. In addition, a continuous wave spectral component is also on the spectrum, indicating that an unstable CW coexists with the soliton pulses in the cavity. The central wavelength of the solitons is located at 1573.5 nm and the 3-dB bandwidth is ~ 1.7 nm. Figure 5.2(b) shows an oscilloscope trace of the state measured with a low speed (500 MHz) detection system. The roundtrip time of the pulses is about 350 ns, which matches with the total cavity length of 70 m. Under the low speed detection where the internal structures of the mode locked pulses cannot be clearly resolved, the pulses on the oscilloscope trace look like a single pulse. However, measured with a 20 GHz high-speed oscilloscope (Agilent 86100A), the “single pulse” is visualized as shown in Figure 5.2(c). The “single pulse” in Figure 5.2(b) turns out to be a soliton bunch. Within the soliton bunch the relative positions of the solitons are unstable. The solitons are moving restlessly. By increasing the pump power while keeping all the other laser parameters unchanged, more soliton pulses emerged. In other words, the increase of pump power does not boost the energy of each individual solitons but increase the soliton number.

Figure 5.3 (a) Multiple solitons state. (b) NL pulse operation of the fiber laser. (c) Optical spectrum. (d) Pulse profile measured with a high-speed oscilloscope. (e) Autocorrelation trace (pulse width corresponds to the sharp peak is about 1.2 ps).

By adjusting the orientation of the intra-cavity PC, NL pulses could be generated from a multi-soliton state. A state thus obtained is shown in Figure 5.3. Figure 5.3(a) shows the multi-soliton state of the laser just before the NL pulse was formed. After changing the orientation of the intra-cavity PC, a NL pulse state as shown in Figure 5.3(b) was formed. The optical spectrum of the

Figure 5.3(a) state is the same as shown in Figure 5.2(a). Figure 5.3(c) shows the optical spectrum of Figure 5.3(b) state. Different from the multi-soliton operation state where the optical spectrum of the laser exhibits clear Kelly sidebands (see Figure 5.2(a)), the optical spectrum of the NL pulse state shown in Figure 5.3(b) is blurred and has no spectral sidebands. Figure 5.3(d) shows the high resolution oscilloscope trace of Figure 5.3(b). It is easy to see that within the mode-locked pulse there are a large number of solitons inside. The solitons travel together as one entity in the cavity. The NL pulse state shown in Figure 5.3(b) is very stable, which can last several hours until people change it. Figure 5.3(e) shows the autocorrelation trace of the NL pulses. It shows a narrow peak atop a broad pedestal, which is a typical characteristic of the NL pulse states. If a sech^2 pulse shape is assumed, the average soliton pulse width in a NL pulse is estimated to be 1.2 ps.

Figure 5.4 Multiple soliton pulses operation in a 30 m long cavity. (a) Optical spectrum measured. (b) Solitons within a bunch measured by a high-speed oscilloscope.

To study the formation mechanism of the NL pulses, we used another laser that has a shorter cavity length of 30 m. The mode locking operation can still be achieved but with a higher threshold (250 mW). The optical spectrum of the multi-soliton state of the laser is shown in Figure 5.4(a), in which the Kelly sidebands are observable, indicating that the total cavity dispersion is still anomalous. The central wavelength is now shifted to 1575 nm and the 3-dB bandwidth is ~3.5 nm. As shown in Figure 5.4(b), the multiple solitons tend to form a bunch in a shorter laser cavity. If the pump power is decreased, the number of solitons in the bunch reduces until the mode locking state is lost. It is noted that in this process no single soliton could be obtained. We attribute the phenomenon as due to the relative high loss of the cavity induced by the graphene. When the quality of the graphene used is not high, we found that the cavity losses could be large. In this case only multiple solitons or the noise like pulse could be easily obtained under strong pump. It also explains why the mode-locking threshold of the current laser is as high as 250 mW.

Figure 5.5 Multiple soliton pulses operation in a 20 m long cavity. (a) Optical spectrum measured. (b) Solitons within a bunch in the cavity at a high pump power. (c) A twin pulse state at a low pump power.

Further shortening the cavity length by reducing the SMF, the optical bandwidth of the soliton spectrum further broadens. For example, with a length of 13 m SMF (cavity length is 20 m), the 3-dB bandwidth increased to as large as ~ 4 nm, as shown in Figure 5.5(a). When the cavity length is reduced to 20 m, a stable soliton bunch is always obtained as shown in Figure 5.5(b). Compared with the state shown in Figure 5.4(b), no soliton can escape out of the bunch. When the pump power is decreased to the mode locking threshold value, a stable twin-pulse soliton state is generated, as shown in Figure 5.5(c). Interestingly, the twin-pulse soliton state cannot be obtained in a longer laser cavity.

Based on the above experimental results, we can conclude that in a long cavity the new solitons formed could move away from the others, eventually become freely moving solitons. While in a short cavity, the formed solitons are trapped by others and tend to form a soliton bunch. This phenomenon indicates that the pulse breaking takes place in the single mode fiber and it can be deduced that more individual soliton pulses can be formed if the length of SMF is correspondingly increased. As the length of SMF in the cavity increases (see Figure 5.2), soliton bunch will become more unstable and finally all the solitons will come out of the bunch and move freely in the cavity.

5.1.2 Simulation Results

Figure 5.6 Numerically calculated pulse formation under different pumping strength. (a) The single pulse profile under gain= 100 and (b) the corresponding optical spectrum of (a). (c) The NL pulse profile under gain= 6000 and (d) the corresponding time-averaged optical spectrum of (c). Please note that the optical pulse and spectrum of the NL pulse had been averaged over 1000 round trips.

In order to gain an insight on the formation mechanism of the NL pulses, we performed numerical simulations with a model based on the coupled complex Ginzburg-Landau equations (GLEs) similar to that subscribed in reference [23], but the mode-locking component was replaced by the graphene saturable absorber. To make the simulation possibly close to the experimental results, the following simulation parameters were adopted: nonlinear fiber coefficient $\gamma = 3 \text{ W}^{-1} \text{ km}^{-1}$; erbium fiber gain bandwidth $\Omega_g = 16 \text{ nm}$; fiber dispersions $D''_{EDF} = -32 \text{ (ps/nm) /km}$, $D''_{SMF} = 18 \text{ (ps/nm) /km}$ and $D''' = 0.1 \text{ (ps}^2\text{/nm)/km}$; cavity length $L = 3.0 \text{ m}_{EDF} + 65 \text{ m}_{SMF} = 68 \text{ m}$; cavity birefringence $L/L_b = 100$ and the nonlinear gain saturation energy $P_{sat} = 50 \text{ pJ}$. The nonlinear response of the graphene saturable absorber is governed by the rate equation as follows:

$$\frac{\partial l_s}{\partial t} = -\frac{l_s - l_0}{T_{rec}} - \frac{|u|^2 + |v|^2}{E_{sat}} l_s \quad (5.1.1)$$

Where T_{rec} is the absorption recovery time of graphene, l_0 is the initial absorption of the absorber, and E_{sat} is the absorber saturation energy. In view that graphene is an ultra-fast saturable absorber with a large modulation depth, we used the following saturable absorption parameters for simulation, $E_{sat} = 1 \text{ nJ}$, $l_0 = 0.3$ and $T_{rec} = 100 \text{ fs}$.

Figure 5.7 Evolution with respect to cavity roundtrips of (a) the optical spectrum and (b) the optical pulse profile of the NL pulse. (c) The change of time-averaged optical spectra of the NL pulse under different laser gain.

Once the laser gain coefficient G exceeds 60 km^{-1} , which corresponds to the threshold pump power required to balance the cavity loss, a stable single pulse could be numerically obtained. Figure 5.6(a) shows a typical state numerically calculated under the laser gain coefficient $G = 100 \text{ km}^{-1}$. It is well-known that within a fiber laser mode locked by NPR with net cavity group velocity dispersion approaching zero, the combined actions of two types of soliton-shaping mechanisms: the balance between the fiber nonlinear Kerr effect and the linear cavity dispersion, and the other caused by the laser gain saturation and dispersion, results in the generation of asymmetric optical sidebands [114]. Herein, from the optical spectrum shown in Figure 5.6(b) where the Kelly spectral sidebands are asymmetric, one can conclude that the formation of asymmetric spectral sidebands is a feature of the soliton lasers with net near zero dispersion, which is independent on the concrete mode locking component.

By further increasing the laser gain, the soliton pulse intensity continuously increases and its pulse width decreases. Eventually the pulse breakups at a pump power of $G = 400 \text{ km}^{-1}$. The generation of multi-soliton pulses is correlated with the excessive nonlinearity of the optical pulse propagation along a long segment of the anomalous dispersion optical fiber, as reported by Zhao *et al.* previously [115]. In the following, the multi-soliton pulses encounter the nonlinear shaping enabled by the nonlinear response of graphene. The transient nonlinear changes of transmission of graphene show two characteristic time constants: a fast component (100 fs) attributed mainly to the carrier-carrier intraband scattering and a slow component (1 ps) caused by the carrier-phonon intraband scattering and electron-hole inter-band recombination processes. The fast component of the saturable absorption of graphene exerts an attraction force among the multi-soliton pulses, making them unable to separate in time domain. Consequently, the multiple soliton pulses are tightly bound, resulting in the formation of the NL pulse state, as shown in Figure 5.6(c) and Figure 5.6(d). To further uncover the impact of the absorber recovery time, we deliberately employed a slower recovery time (500 fs) for simulation whereas other parameters are kept unchanged. The state of multi-soliton propagation can be numerically obtained, but they are well separated. In the case the NL pulse state is hardly formed, indicating that a faster saturable absorber, which exhibits stronger mutual attraction force, helps the generation of the NL pulse state. To probe the internal structure, the roundtrip-to-roundtrip simulation is also carried out. Different from the single soliton where the optical spectra are almost identical and indistinguishable from one round to the other, the NL pulse state exhibits significant fluctuations in either the roundtrip-to-roundtrip optical

spectra or pulse profiles, as shown in Figure 5.7. Each of the individual spectra or pulse profiles is highly structured and randomly varied with the evolution of the cavity roundtrips. Upon averaging for 1000 rounds a time-averaged optical spectral and pulse intensity profile as shown in Figure 5.6(c) and Figure 5.6(d) are obtained, which are comparable with the corresponding OSA and high speed oscilloscope trace measurements in experiments (see Figure 5.3). The change of optical spectra under different pump power is also summarized in Figure 5.7(c), from which the optical spectral sidebands are hardly observable. We find that the absence of spectral sidebands is a direct consequence of the NL pulse state. We know that the formation of the Kelly sidebands is resulted by the constructive interference between the intra-cavity dispersive wave and soliton wave, which co-circulate within the laser cavity. However, the restless movement and collision among the multi-soliton pulses in a NL pulse can make the dispersive wave and soliton wave unable to fulfil the mutual phase matching condition, which therefore suppresses the generation of optical spectral sidebands.

5.1.3 Discussions

Based on the experimental and numerical results discussed above, we can summarize the features of the current graphene mode locked dispersion-managed cavity fiber lasers as the following. There exist two mode locking operation regimes, the multi-soliton regime and the NL pulse regime. In which regime a laser operating can be easily distinguished from the optical spectrum of the laser emission. Usually, the multi-soliton (resp. NL pulse) has a smooth (resp. blurred) optical spectrum with (resp. without) optical spectral sidebands,

and requires relatively lower (resp. higher) pump power. By controlling the intra-cavity birefringence (through the polarization controllers), the laser operation can be switched between the multi-soliton and NL pulse state. Moreover, the cavity length and net cavity dispersion also have an impact on the laser operation. In a short fiber laser cavity, the multi-soliton bunch state is readily obtained. With the increase of the SMF length, which enhances the accumulated optical nonlinearity and shifts the total cavity dispersion towards anomalous dispersion regime, more individual solitons will move out of the bunch, and finally the soliton bunch collapses.

The unique nonlinear optical property of graphene also contributes to the formation of the NL pulse in the experiment. Unlike the soliton bunch observed in SESAM fiber laser [21], where all the solitons are almost uniform which can be considered as an effect caused by the bandwidth limitation of SESAM. Graphene has a wider operation wavelength. Therefore, no saturable bandwidth limiting effect exists. Like the NPR mode locking, the fast response of graphene saturable absorption imposes an extra soliton attraction force to the solitons. Under the effect the solitons tend to form a bunch in the cavity. Moreover, as the intensities of the pulses are unequal in the current laser cavity, the soliton bunch cannot be as stable as that in SESAM mode-locked fiber lasers. Once the pump power is sufficiently high, too many solitons will be formed and interact with each other simultaneously and NL pulses will be correspondingly obtained. The energy of the NL pulse could be large because all the solitons in the cavity are in a “bunch”. A NL pulse usually looks like a “single pulse” in the cavity. However, once the length of SMF is increased, in which pulse breaking effect take places [115], where soliton bunches could

collapse and finally the solitons move away from each other, a multiple soliton operation state would be formed in the cavity. Thus we can conclude that formation of both the multi-soliton and NL pulse state is an intrinsic feature of the graphene mode locked dispersion-managed fiber lasers.

5.2 Optical Rogue Wave

Rogue waves, or extreme waves, refer to the strong and brief events emerged unexpectedly in the nonlinear systems. Investigation of the rogue waves was originated from the freak wave appeared spontaneously in the ocean, and it has been extended to other scientific field with nonlinear dynamics. The study on the rogue waves in optics started from the Solli *et al.* who firstly observed the optical rogue waves in photonic crystal fibers with super-continuum generation [112]. Comparing to the ocean systems, an unambiguous advantage of the nonlinear optics system is that the experimental observation and record of the rogue wave events are much easier. Thus far, there have been various experimental studies on the rogue waves in optical dissipative systems, including Raman fiber amplifier and lasers, semiconductor lasers, nonlinear driven cavities, pump-modulated fiber lasers, Ti: Sapphire lasers and so on [116-122]. In these works the formation mechanism of the rogue waves are also different, including modulation and nonlinear spectral instability, multistability, multi-soliton collision and so on. The optical rogue wave formation could be of various reasons under different conditions.

A mode locked fiber laser, as a dissipative system, which takes into account

dispersion, Kerr nonlinearity, cavity birefringence, pumping and so on, is another perfect platform for optical rogue wave generation [14]. A. Zaviyalov *et al.* numerically studied the rogue wave generation in mode-locked fiber lasers with normal dispersion, which provides a theoretical potential for the experimental generation of the rogue waves [113]. Moreover, J. M. Soto-Crespo *et al.* numerically showed that the probability of the optical rogue waves in mode-locked fiber lasers with saturable absorber is higher than those in other systems [123]. Recently, based on the soliton bunching in the cavity, the optical rogue waves were firstly experimentally generated in an erbium-doped passively mode-locked fiber lasers by nonlinear polarization rotation technique [124]. However, as the solitons in the fiber lasers mode locked by the NPR technique are scalar solitons, optical rogue waves in vector soliton fiber lasers with other polarization independent saturable absorbers have never been studied before.

In this section, we demonstrate an experimental observation of optical rogue waves in a vector soliton fiber laser. Unlike the previous studies, the optical rogue wave generation is based on the multiple vector soliton formation and interaction in the fiber laser. To the best of our knowledge, this is also the first report on the rogue wave generation in graphene mode locked fiber lasers.

5.2.1 Experimental Results

Figure 5.8 Experimental setup of the fiber ring laser. PC polarization controller, EDF erbium doped fiber, SMF single-mode fiber, WDM wavelength-division multiplexer.

The fiber ring laser is schematically shown in Figure 5.8. The laser was pumped by a high power fiber Raman Laser source (KPS-BT2-RFL-1480-60-FA) of wavelength 1480 nm through a wavelength division multiplexer (WDM) coupler. A 0.7 m long erbium doped fiber (EDF Er80-8/125 from Liekki) with group velocity dispersion (GVD) of 18 ps/nm/km was used as the gain medium and 21.9 m single mode fiber (SMF) with GVD 18 ps/nm/km was employed to connect the components in the cavity. A 10% output coupler was used to output the signal. All the passive components are made of the SMF. The graphene which plays the role of nonlinear saturable absorber in the cavity was synthesized by the CVD method, with 2-3 layers of graphene sheets deposited into the fiber patch cord. The laser operation is monitored by a 1 GHz

oscilloscope (Lecroy Waverunner 6100) together with a 1.2 GHz photo-detector (Thorlab DET01CFC) and an optical spectrum analyzer (ANDO AQ6317B).

Figure 5.9 A typical mode locking state. (a) Optical spectrum and (b) oscilloscope trace.

At a pump power of ~ 90 mW, mode locking of the laser is self-started. Figure 5.9 is a typical vector soliton operation of the fiber laser. The optical spectra of the pulses are shown in Figure 5.9(a). The existence of the Kelly sidebands confirms the soliton features of the pulses. It is noted that apart from the Kelly sidebands, an extra set of sidebands was observed. In the polarization resolved spectra, the extra sideband exhibits dip and peak in the vertical axis and

horizontal axis, respectively, which suggests that they are caused by the coherent energy exchange between the two polarization components. This set of sidebands is formed by the four wave mixing between the two polarization components [23]. Figure 5.9(b) is the oscilloscope trace of the laser emission. The solitons are forming a bunch and the separation between each bunch is ~ 113 ns, which is the round trip time of the cavity, indicating the pulses are formed due to the mode locking.

Figure 5.10 A typical giant wave state. (a) Optical spectrum, (b) oscilloscope trace and (c) resolved trace in 20 GHz high speed oscilloscope.

By increasing the pump power, more solitons come out in the cavity and several bifurcations can be achieved. As the pump power increases to as large as ~200 mW, by tuning the polarization controller carefully, a pulse train with much higher intensity was observed. As shown in Figure 5.10(a), the optical spectrum of the pulses is blurred, indicating the high intensity pulses are not comprised by a single soliton but a bunch of solitons close to each other. Measured by a high speed system, as shown in Figure 5.10(c), it was found that each pulse in one round trip time actually is a “giant pulse” which consists of plenty of solitons moving as a unit in a distinct region. This giant pulse has been reported in Section 3.2.

Figure 5.11 Optical rogue waves operation of the fiber laser. (a) Optical spectrum of the optical rogue wave; (b) Video record of a typical rogue wave emission measured by oscilloscope (Media 4).

By carefully tuning the paddles of the polarization controller and the pump power, the relative position of the solitons in the giant pulses is no longer stationary. The giant pulses became unstable and collapsed into plenty of solitons moving in the whole laser cavity. A typical optical spectrum of the laser emission in this regime is shown in Figure 5.11(a). Unlike the optical spectrum in Figure 5.9(a) and Figure 5.10(a), there is one unstable continuous wave and two more sets of sidebands observed in the spectrum, which indicates the modulation instability and polarization rotating effect induced into the cavity [105,125]. In the video of Figure 5.11(b) (see Media 4 in the attachment),

the movement of the pulse train is found to be unpredictable and there is pulse with extreme high intensity come out in several seconds. Although the emergence of the extreme pulse are unpredictable, the position of the extreme pulses are deterministic. In other words, the extreme pulses always come out at the same position in the cavity.

Figure 5.12 Histogram (log scale) of the rogue wave state showing the distribution of the optical intensity maxima for 10⁵ trace events.

Figure 5.13 Histogram (log scale) of the stable giant pulse state showing the distribution of the optical intensity maxima for 10⁵ trace events.

To further study the formation of the extreme pulses. We measure the trend of the peak intensity of the laser output in a specific time. The peak intensity in each round trip time was recognized as one sample and the oscilloscope automatically record $\sim 1.1 \times 10^5$ samples in about one hour. By using these data, the histogram (in log scale) of the intensity of the vector soliton pulses was plotted, as shown in Figure 5.12. The histogram displays as an L-shape with a long tail, suggesting that the extreme pulses formed in the cavity are rogue waves. It is worth mentioning that there is a small peak around amplitude of ~ 1.2 , which is a typical amplitude value of giant pulse. In other words, although it was an unstable state, the pulse collision occurs still more frequently, tending to form a giant pulse but unstable. In this view, the formation of the rogue waves could be considered as an intermediate bifurcation between the stable giant pulses and soliton bunches. For comparison, we also plot the histogram of the giant pulse state, as shown in Figure 5.13. It can be clearly seen that almost all the values of the peak intensity range from 1.1 to 1.3 and no tails can be found from the histogram, suggesting it is a stable state which is significantly different from that of the rogue wave state. It is worth noting that intensity of the giant pulses is much higher than soliton bunches. Therefore, in the histogram the intensity is stable at a high level, comparing to the rogue wave state. However, the maximum intensity achieved in the stable giant pulse state is smaller than that of rogue wave state.

5.2.2 Discussions

To further confirm the extreme pulses are rogue waves, we claim that the extreme pulses have the three key features of the rogue waves [112]. Firstly, the

pulse we observed is a rare event with extremely high amplitude. It can be confirmed by the video in the experiment and probability density function. Secondly, the pulse propagation could be traced over long distance. Thirdly, the events of the rogue waves could arise more frequently by tuning the pump power and polarization controllers. Therefore, the extreme pulse formed in our cavity is a controllable and deterministic optical rogue waves.

Unlike the rogue waves reported in the previous work, we believe that formation of the rogue waves in our fiber laser is formed by the vector soliton interactions and polarization rotation of the vector solitons. Soliton interactions in fiber laser have been investigated by our previous works and three types of soliton interaction were reported [98]. In the current cavity, the optical rogue waves are always formed when the unstable continuous wave come out in the cavity. It can be deduced that the global type of soliton interactions, which is caused by the existence of continuous wave plays an important role in the formation of the laser cavity. Furthermore, as our fiber laser is mode-locked by graphene, in which vector solitons can be achieved. It can be expected that vector soliton interaction must be more complicated.

Another factor that may contribute to the formation of rogue waves is the polarization rotating effect of the vector solitons. It is worth noting that both in the giant pulses and soliton bunches states, the polarization of the vector solitons are locked, which could be observed from the optical spectrum and the pulse trains. When the polarization is unlocked, two more Kelly sidebands appear in the spectrum of the vector solitons. It can be expected that the polarization instability increases the complexity of the vector soliton

interactions. As the polarization rotation has a certain period, it is plausible that at a certain point phases of the polarization rotating vector solitons are coherent which cause an extreme pulse formed. The time interval between adjacent two extreme pulses depends on the period of the polarization rotation and the period of the soliton movement.

5.3 Summary

In summary, we have experimentally and numerically investigated the formation mechanism of the NL pulses in an erbium-doped dispersion-managed fiber laser passively mode locked by a few-layer graphene saturable absorber. The NL pulses can be regarded as another mode locking regime enabled by graphene saturable absorber, and characterized by sideband-free and broad optical spectrum and narrow peak atop a broad pedestal in its auto-correlation trace. Despite of its noise-like property, this state can lead to large energy operation, making graphene saturable absorber applicable for high power laser photonics. We have shown that the formation of the NL pulse is related to the ultrafast saturable absorption of the CVD graphene.

We have also experimentally observed optical rogue waves in a vector soliton fiber laser mode locked with atomic-layer graphene. Extreme pulses are generated by the vector soliton interaction in the soliton bunches and combining with the polarization rotating effect in the cavity. The formation of the rogue waves highly depends on the parameters of the cavity could be considered as an intermediate bifurcation between the stable giant pulses and soliton bunches.

These works may be also useful for a better understanding on the operation of the mode locked fiber lasers.

Chapter 6 Cavity Induced Modulation

Instability of Fiber Lasers

Modulation instability (MI) refers to the physical process that the intensity of an initially stable intense carrier wave automatically becomes periodic pulsing [126]. In the field of nonlinear optics, MI was observed in the case of intense light propagation in the anomalous dispersion fibers where the nonlinear light propagation is described by the nonlinear Schrödinger Equation. Physically, MI is a four wave mixing process where the strong carrier wave at a frequency ω is automatically phase matched with two weak sidebands at frequencies $\omega \pm \Omega$. The coherent wave coupling among them causes the intensity of the intense carrier wave change periodically.

It is of particular interest to the MI effect in a fiber laser as it could lead to the ultra-high repetition rate pulse emission of a laser [30,31,127,128]. Masataka *et al.* [30,127,128] proposed a MI soliton fiber laser that produced soliton pulse trains with 115 GHz repetition rate. Gong *et al.* [31] reported the generation of 660 GHz soliton pulse trains in an erbium doped fiber laser based on the MI method. However, determined by the gain feature of the MI process and the threshold conditions of the laser oscillation, the repetition rate of the pulse trains generated by the MI in a fiber laser is normally higher than ~100 GHz. Stable pulse trains with several GHz or tens of GHz repetition rate could not be generated with the technique.

There is no MI for light propagation in the normal dispersion fibers. However, Haelterman *et al.* have theoretically shown that MI could still occur in a normal dispersion fiber cavity. It was found that the cavity boundary condition could introduce phase matching between the strong laser beam that circulates in the cavity and weak perturbation sidebands, leading to the appearance of MI of the laser beam [83,129]. The cavity detuning aided MI was also called the cavity induced modulation instability. Theoretical studies have shown that the CIMI has similar properties as those of the conventional MI except that the cavity detuning could greatly alter the phase matching condition, and consequently the pulse repetition rate of the resulted laser intensity modulation. Through carefully controlling the laser beam intensity and the cavity detuning, stable MI pulse trains with GHz to tens of GHz level, variable repetition rate could be generated.

The appearance of the CIMI is independent of the cavity dispersion. It exists both in the normal and the anomalous dispersion cavity. The existence of the CIMI in a passive ring fiber cavity have been experimentally verified [32]. However, the cavity induced MI lasing of fiber lasers and the exploitation of the effect for the generation of stable repetition rate variable GHz level pulses are so far not reported. In this paper, we provide the experimental evidence on the cavity induced MI lasing of an erbium-doped fiber ring laser. We show experimentally that by exploiting the effect in a fiber ring laser stable GHz level repetition rate variable pulse trains can be generated. In our fiber laser the CIMI has a lower pump threshold than that of the conventional MI lasing. Under even stronger pump power, coexistence of the CIMI and the conventional MI in the

fiber laser was also experimentally observed. The experimental result clearly shows that they are two different processes in the fiber laser.

In this Chapter, we report experimental observation of variable high repetition rate pulses in modulation instability (MI) dispersion-managed fiber lasers. Owing to the modulation instability induced by the cavity boundary condition, an ultra-stable 3 GHz pulse train was observed with a short pulse width of ~340 fs. We also successfully observed a modulated 285 GHz pulse train which was formed due to the MI in anomalous dispersion fiber. Both the conventional MI and the MI induced by cavity boundary condition coexisted simultaneously. The experiment results are shown in Section 6.1. In Section 6.2 the results are discussed. The results are summarized in Section 6.3.

6.1 Experimental Results

Figure 6.1 Schematic illustration of the erbium doped fiber laser.

Figure 6.1 shows a schematic of the fiber laser. The laser had a ring cavity with

a length of ~ 13.5 m. Among the cavity fibers a piece of 3 m highly doped EDF with a GVD parameter of -48 ps/nm/km was used as the gain medium. The other fibers used were the standard single mode fiber (SMF-28) with a GVD parameter of 18 ps/nm/km. The cavity is dispersion-managed and has net anomalous cavity dispersion. A polarization independent isolator was employed in the cavity to force the unidirectional operation of the ring cavity, and an intra-cavity polarization controller (PC) was used to fine-tune the linear cavity birefringence. The laser was pumped by a 1480 nm high power fiber Raman Laser source whose maximum output pump power is as high as 5 W. A 10% output coupler was used to output the laser emission.

Figure 6.2 A state of 3 GHz pulse trains the fiber laser. (a) Oscilloscope trace; Inset: Trace in 3 ns scale; (b) Autocorrelation trace; (c) Optical spectrum.

A critical condition for obtaining the modulation instability in the fiber laser is that the cavity loss must be sufficiently small, so that high efficient lasing can be achieved. The threshold of continuous wave (CW) emission in our fiber laser was ~ 30 mW. 1.5 mW CW output was obtained as the pump power increased to 100 mW. As the pump power increased to 2 W, the output could be as high as ~ 65 mW. As there are neither polarization dependent components nor saturable absorber in the fiber laser cavity, initially the laser always operated in the CW regime. However, it was experimentally observed that as the pump power was increased to ~ 200 mW, through carefully tuning the paddles of the intra-cavity PC, stable pulsed emission of the laser could be obtained. The pulse width and repetition rate of the pulsed oscillation varied with the pump power and the cavity detuning. Generally, the higher the pump power the narrower the pulse width. While at the same pump power as the cavity detuning was changed, various pulse repetition rates in the range from GHz to tens of GHz could be obtained. Figure 6.2 shows for example a typical stable pulsed oscillation state of the laser obtained at a pump power of 800 mW. Figure 6.2(a) is the

oscilloscope trace of the laser emission. The pulses have a repetition rate of ~ 3 GHz, which is about 200 times of the fundamental cavity repetition rate. The FWHM of the pulses was measured with a commercial autocorrelator whose scan range is ~ 180 ps. The measured autocorrelation trace is shown in Figure 6.2(b). The pulse width is estimated to be ~ 340 fs if a sech^2 pulse shape is assumed. The optical spectrum of the pulses is shown in Figure 6.2(c). The 3-dB bandwidth of the spectrum is about 11.5 nm. So the time-bandwidth product of the pulses is about 0.469, which indicates that the pulses are not transform-limited but chirped.

The state shown in Figure 6.2 was very stable. Although the laser was put in a noise environment without any protection, the state could last several hours until the operation was switched off. Despite of the fact that there is no any polarization sensitive element in the cavity, experimentally we found that the pulses were linearly polarized. We believe that it was caused by the polarization instability effect of the lasers. Carefully examining the oscilloscope trace it is easy to see that there was very weak CW component associated with the pulses. However, the weak CW and the high repetition rate pulses have together formed a cooperative state that is almost invariant with time.

Figure 6.3 A state of 32 GHz pulse trains the fiber laser. (a) Oscilloscope trace; (b) the autocorrelation trace; (c) Optical spectrum.

With the pump power fixed and carefully varying the cavity detuning by changing the orientation of the paddles of the intra-cavity PC, other repetition rate pulse trains were also obtained. Figure 6.3 shows for example a state with a repetition rate of ~ 32 GHz. As the oscilloscope used only has a bandwidth of 33 GHz, the pulse repetition rate is close to its bandwidth limit. Therefore, the measured oscilloscope trace is strongly distorted. Nevertheless, one can still determine its pulse repetition rate as ~ 32 GHz. The pulses were further measured with a commercial autocorrelator, as shown in Figure 6.3(b). The adjacent pulses have an interval of 30.4 ps. The optical spectrum of the pulses is

shown in Figure 6.3(c). The 3-dB bandwidth of the spectrum is about 9.4 nm. Some spectral spikes are observed on the top of the spectrum. Zoomed in the spectrum, the spectral spikes have an interval of ~ 0.26 nm, which coincides with the pulse repetition rate of 32 GHz. The appearance of the spectral spikes is a clear indication that the formation mechanism of the high repetition rate stable pulses is a result of the MI of the laser. Based on the properties of the pulse that they have much lower pulse repetition rate than the conventional MI lasing of the fiber lasers, and their pulse repetition rate was dependent on the cavity detuning, we conclude that they are caused by the CIMI. We note that no such spectral spikes were observed in Figure 6.2(c). It is because the pulse repetition rate in that case was too low. The corresponding interval of the spectral peaks is too small to be resolved by the optical spectrum analyzer used. It is worth mentioning that, in our laser cavity, the repetition rates of the pulses shown in Figure 6.2 and Figure 6.3 are variable by tuning the PC. We also obtained ~ 2 GHz, ~ 12 GHz, and ~ 20 GHz pulse trains and so on.

Figure 6.4 A state of high repetition rate pulses. The repetition rate is about 285 GHz. (a) Autocorrelation trace in 300 ps scale; Inset: trace in 180 ps scale. (b) Optical spectrum. Inset: Zoom-in of a spectral peak.

Increasing the pump power to 2 W, emission of the laser with an even higher pulse repetition rate was further obtained. A typical state is shown in Figure 6.4. As the pulse repetition rate was far beyond the maximum bandwidth of our electronic detection system (33 GHz), no pulse trains could be detected on the oscilloscope trace. However, the high repetition rate pulse train could still be recognized by the autocorrelation measurement, as shown in Figure 6.4(a). The autocorrelation trace shows that the pulses had an interval of ~ 3.5 ps, corresponding to a pulse repetition rate of ~ 285 GHz. The autocorrelation trace also shows that the pulses were not strictly uniform and there was pulse intensity modulation. Figure 6.4(b) is the corresponding optical spectrum. There are strong spectral peaks on the spectrum. The interval of the spectral peaks is about 2.5 nm, indicating a pulse repetition rate of about ~ 300 GHz, which quite well matches with the pulse repetition rate measured with the autocorrelation trace. Zoomed in the spectral peaks, it is found that there are further small

spectral spikes on the peaks. The spikes have an interval of ~ 0.1 nm, as shown in the inset of Figure 6.4(b). The small spectral peaks indicate the existence of another modulation with a frequency of about 10 GHz. It was experimentally identified that the interval of the small spectral spikes varied with the cavity detuning. Changing the orientation of the paddles of the intra-cavity polarization controller the small modulation could be strengthened or weakened.

6.2 Discussion

Based on the observed experimental results, we concluded that the pulse trains shown in Figure 6.2 and Figure 6.3 were generated due to the CIMI lasing. To further support our conclusion, we note that the pulses were not mode locked pulses as no any mode locking technique was applied in the laser. Secondly, the pulses were not formed through the conventional MI process. The appearance of the conventional MI in a fiber has normally a high threshold. Determined by the gain feature of the MI process, as theoretically shown, the gain at low frequencies is much smaller than those at high frequencies [1]. Therefore, the MI pulses trains usually have an ultrahigh repetition rate. As demonstrated by previous experimental results on the MI in fibers or fiber lasers, the repetition rates of the MI pulses in fiber laser were usually >100 GHz. In our laser cavity, the repetition rates of the pulses shown in Figure 6.2 and Figure 6.3 are variable, ranging from ~ 1 GHz to ~ 40 GHz, which is much smaller than previously reported ones. For the cavity induced MI, the gain spectrum is highly dependent on the cavity detuning. Under a certain cavity detuning, gain value could be

sufficient high at low frequency [129]. In other words, one could observe a low frequency modulated pulses induced by CIMI. Furthermore, the modulation frequency of the CIMI is not dependent on the pump power but the cavity detuning, which matches the experimental observations. These analyses are also verified by the results in Figure 6.4. The 285 GHz pulse train shown in Figure 6.4 was induced by the conventional MI as the pump power was sufficiently high and the net cavity dispersion is anomalous. The high repetition rate didn't change too much as the cavity detuning was changed, but the low frequency modulation displayed as the small spectral spikes did change with the cavity detuning. Therefore, they are evidences of the CIMI, and the results shown in Figure 6.4 provide an evidence of the coexistence of the conventional MI and the CIMI in the same fiber laser.

6.3 Summary

In conclusion, we have experimentally demonstrated stable variable GHz level repetition rate pulse train generation in an erbium-doped fiber laser. It is shown that at fixed pump strength the pulse repetition rate can be varied by changing the cavity detuning, showing that they are caused by the cavity induced modulation instability as theoretically predicted by Haelterman *et al.* Unlike the conventional modulation instability lasing of a fiber laser, where the repetition rate of the MI pulses is uniquely determined by the fiber dispersion and the laser beam intensity, the pulse train of the cavity induced modulation instability is further varied by the cavity detuning. Therefore, through appropriately setting the cavity detuning and the pump power, stable GHz repetition rate pulse trains

can be generated in a fiber laser. To the best of our knowledge, this is the first experimental demonstration on the cavity induced MI lasing of the fiber lasers. At high pump power, coexistence of the cavity induced MI and the conventional MI lasing was also experimentally observed. In this case the ultrahigh repetition rate pulses caused by the conventional MI is intensity modulated by the cavity induced MI. Nevertheless, the conventional MI can be avoided by operating the laser in the net normal cavity dispersion regime.

Chapter 7 Dark Solitons in Fiber Lasers

In the previous chapter 3 to chapter 6, the solitons or pulses discussed can be simply classified as bright soliton or bright pulses. Based on the NLSE, bright solitons are formed in the anomalous dispersion fibers. Formation of bright solitons has been found to be an intrinsic feature of the mode locked fiber lasers with an anomalous dispersion cavity [85]. While in the normal dispersion fibers, another type of solitons called dark solitons, namely a localized intensity dip on a continuous wave (CW) background, are formed. In the non-mode-locking regime, one would naturally expect that dark solitons could be observed in the normal dispersion cavity fiber lasers.

Indeed, a novel type of dark pulse emission in an erbium-doped fiber (EDF) laser made of all-normal dispersion fibers was observed recently [25]. However, due to the limitation of the experimental conditions its dark soliton nature could not be verified. Later in a comment on the paper by Coen and Sylvestre [130], it was pointed out that the experimentally observed dark pulses could not be the dark solitons formed in the fiber laser. Because in the experiment an electronic detection system with a bandwidth of 1 GHz was used, if the formed dark solitons had only several picosecond pulse width, they would have not been detectable by the detection system. Hence the experimentally observed dark pulses with a near 100% darkness should have a much broader pulse width. Coen and Sylvestre's comment is obviously correct. The key to solve the

problem is to find a way to measure in real time the narrow dark pulses formed in the laser. However, different from the bright solitons, whose existence in a laser could be easily identified even with a moderate speed photo-detector and oscilloscope, it is unfortunately not so straightforward to detect a dark soliton formed in a laser. This probably is the main reason why so far no experimental observation on the individual dark soliton in fiber lasers has been reported.

The objective of this chapter is to present the experimental evidences of the dark solitons in the fiber laser and show that the dark soliton formation is an intrinsic feature of the fiber lasers. Using an ultrahigh speed detection system with a temporal resolution of 30 ps we further experimentally investigated the dark pulse emission of our fiber laser. In the Section 6.1 we show clear experimental evidence on the existence of dark solitons in the all-normal dispersion cavity fiber lasers. Moreover, we identified in a subsequent experiment that the observed dark pulses were in fact bunches of the dark solitons formed in the fiber laser. In the Section 6.2 we show experimental observation of dark solitons in an erbium-doped dispersion-managed cavity fiber laser with net normal cavity dispersion. Despite of the existence of a piece of anomalous dispersion fiber which does not support dark solitons, our experimental results show that as far as the average cavity dispersion is in the normal regime, dark solitons could still be formed in the fiber laser. Both of these results are confirmed by the numerical simulations and the features of the formed dark solitons could be well explained based on the NLSE.

7.1 Scalar Dark Solitons in All-normal Dispersion Fiber Lasers

7.1.1 Experimental Results

Figure 7.1 Schematic diagram of the all-normal dispersion fiber laser.

Based on the theoretical prediction on the dark soliton, an all-normal dispersion cavity erbium-doped fiber laser was constructed. The schematic diagram of the cavity is shown in Figure 7.1. We used a piece of 3 m erbium-doped fiber (EDF) that has a erbium-doping concentration of 2880 ppm and a value of group velocity dispersion (GVD) of -32 (ps/nm)/km as the gain medium of the laser, and made the total cavity an arbitrary length of 10.5 m. The rest of the cavity fibers used were the dispersion compensation fiber (DCF) that has GVD of -4 (ps/nm)/km. Again a high power Fiber Raman Laser (KPS-BT2-RFL-1480-60-FA) at wavelength 1480 nm was used as the pump source. The pump light was

coupled to the fiber cavity through a wavelength division multiplexer (WDM), and a 10% fiber coupler was used to output the laser emission. All the intra-cavity components (WDM, output coupler, and isolator) were either specially made with the DCF or pigtailed with it. The total cavity dispersion was estimated as about $\sim 0.167 \text{ ps}^2$. An optical spectrum analyzer (Ando AQ-6315B) and a high-speed oscilloscope (DSA-x 93204A 33 GHz, Agilent Technologies) together with two 45 GHz photo-detectors (Newfocus Model 1014) were used to monitor the laser output.

A major improvement of the current experiment from that reported previously is that an ultrahigh speed detection system was used to monitor the laser emission in the real-time. The detection system (photodetector and oscilloscope) has a temporal resolution as short as $\sim 30 \text{ ps}$. Tested with a femtosecond mode-locked fiber laser source, we also experimentally confirmed that the rising and falling time of the detection system is $\sim 12 \text{ ps}$, respectively. The new detection system allows us to easily distinguish a picosecond-level dark pulse from the background noise. An external cavity fiber-pigtailed polarization beam splitter (PBS) together with an external cavity polarization controller was used to resolve the two orthogonal polarization components of the laser emission. Therefore, based on the measured results we could also determine if the dark pulses obtained are linearly polarized or not.

Figure 7.2 A train of the dark pulse emission of the fiber laser. Upper trace: laser emission measured with 33 GHz detection bandwidth; Lower trace: laser emission measured with 1 GHz detection bandwidth.

Figure 7.3 Oscilloscope traces of the laser emission under different pump power. (a) ~68 mW (b) ~100 mW (c) ~400 mW (d) ~700 mW. The two curves in the trace represent two orthogonal polarization components.

We first repeated the previous experiment and confirmed that all the dark pulse emission features reported were reproducible in the current laser. Figure 7.2 shows for example the single dark pulse emission of the laser measured with a 1 GHz bandwidth detection system and the high speed detection system, respectively. Measured with a 1 GHz detection bandwidth a nice stable dark pulse train with the fundamental cavity repetition rate was obtained. When measured with the high-speed detection system the dark pulse train was still visible on the oscilloscope trace, however, the CW background then displayed large amplitude high frequency noise-like intensity modulations. Confirmed by the high-speed measurement the dark pulses indeed have sub-nanosecond pulse width.

Figure 7.4 Optical spectra of the laser under CW and dark pulse emissions.

To determine the formation mechanism of the observed broad dark pulses, we then experimentally investigated the variation of laser emission with the pump power increase. A generic evolution observed is shown in Figure 7.3. At a relatively low pump power (~ 68 mW), the fiber laser emitted weak continuous wave (CW), as shown in Figure 7.3(a). In this state the measured oscilloscope

trace is stable. We note that in each of the plots in Figure 7.3 the two oscilloscope traces corresponding to each of the two orthogonal polarization components of the laser emission are displayed. As the emission of the fiber laser was actually linearly polarized, one trace has therefore always zero intensity. The trace serves here as a reference to indicate the actual noise level of the detection system. Figure 7.4 shows the optical spectra of the laser emissions. The center wavelength of the laser emission was at 1577.4 nm. In the stable CW laser emission the optical spectrum was a narrow line with a 3 dB bandwidth of ~ 0.07 nm. As the pump power was increased to ~ 100 mW, the oscilloscope trace then gradually became unstable. Narrow dark pulses also appeared obviously on the oscilloscope trace, as shown in Figure 7.3(b). As the pump power was further increased, more and more dark pulses appeared on the oscilloscope trace, as shown in Figure 7.3(c). Associated with the appearance of the dark pulses, the optical spectrum of the laser emission also became broader. A typical spectrum measured at 400 mW pumping power is shown in Figure 7.4, whose 3 dB bandwidth is ~ 0.2 nm. To highlight the features of the dark pulses, in Figure 7.5(a) we have zoomed in a segment of the oscilloscope trace. Within a time interval of ~ 600 ps, there were 5 dark pulses. The durations of the dark pulses on the oscilloscope trace were ~ 30 ps, which is the limitation of the detection system. Therefore, the real dark pulse width could be even narrower than it. Based on the 3 dB spectral bandwidth of the laser emission and assuming that the dark pulses have a transform-limited hyperbolic-tangent profile, we estimated that the width of the dark pulses should be ~ 13 ps. At the pump power of 400 mW the laser output power was ~ 9.6 mW. Considering that the average cavity dispersion of β_2 is ~ 15.9 ps²/km, assuming that a black

soliton was formed and the nonlinear fiber coefficient is $\gamma = 3 \text{ W}^{-1}\text{km}^{-1}$, based on the soliton formula $P_0 = \frac{\beta_2}{\gamma\tau^2}$ the estimated soliton pulse width would be 13.1 ps. Therefore, it is to believe that the dark pulses on the oscilloscope trace should be the dark solitons formed in the fiber laser.

We had also experimentally studied the filter effects of the detection system on the measured results. As displayed in Figure 7.5(b), by changing the bandwidth of the detection system to 5 GHz, 10 GHz, the measured intensity dips would become broader and much shallower. Hence, despite of the existence of the dark solitons in the fiber lasers, if measured with a narrow-bandwidth detection system the dark solitons would not be detectable.

Figure 7.5 Zoom-in of the laser emission. Upper trace: oscilloscope trace measured with 33 GHz bandwidth. Lower trace: oscilloscope traces measured with 5 GHz and 10 GHz bandwidth.

As shown in Figure 7.3(b) and (c), the dark solitons are randomly distributed in the whole cavity, and the stronger the pump strength, the denser the dark solitons. However, an interesting effect was observed experimentally that above a certain pump level, in our experiment which was about ~ 700 mW, the dark solitons always automatically bunched together, as shown in Figure 7.3(d). As a result of the dark soliton bunching a broad dark pulse was then formed in the

cavity, e.g. on the oscilloscope trace shown in Figure 7.3(d) a broad dark pulse with a pulse width of 450 ps was formed, and immediately after the broad dark pulse there were many dark solitons approaching it. We had experimentally measured the state with a 1 GHz bandwidth detection system and confirmed that the soliton bunching was displayed as a nanosecond-level dark pulse. The experimental result clearly explained how the broad dark pulses reported previously were formed. We note that formation of bound states of solitons and soliton bunching is a well-known feature of the bright solitons formed in fiber lasers [85,89-92,94]. Bound states of dark solitons were theoretically predicted by V. V. Afanasjev *et al.*[131]. Our experimental result shows that, like the bright solitons, the dark solitons formed in a fiber laser also have the tendency of bunching.

7.1.2 Numerical Results

To confirm that dark solitons could form in our fiber laser, in particular, the formed dark solitons have comparable soliton parameters to those observed experimentally, we further numerically simulated the laser operation under the experimental conditions. We used exactly the same model as reported in [85] to simulate the operation of the fiber laser. It has been shown previously that the model could faithfully reproduce the various features of the bright solitons experimentally observed in the passively mode locked fiber lasers with the nonlinear polarization rotation technique [85]. Although under the dark soliton emission the laser is no longer operating in the mode locking regime, we note that the basic properties of the nonlinear light propagation in the cavity are still

the same. Hence, the same model can also be used to simulate the dark soliton operation of the fiber laser.

We used the following coupled extended Ginzburg-Landau equations to describe the nonlinear light propagation in the cavity fibers:

$$\begin{cases} \frac{\partial u}{\partial z} = i\beta u - \delta \frac{\partial u}{\partial t} - \frac{ik''}{2} \frac{\partial^2 u}{\partial t^2} + \frac{k'''}{6} \frac{\partial^3 u}{\partial t^3} + i\gamma(|u|^2 + \frac{2}{3}|v|^2)u + \frac{i\gamma}{3} v^2 u^* + \frac{G}{2} u + \frac{G}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial v}{\partial z} = -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{ik''}{2} \frac{\partial^2 v}{\partial t^2} + \frac{k'''}{6} \frac{\partial^3 v}{\partial t^3} + i\gamma(|v|^2 + \frac{2}{3}|u|^2)v + \frac{i\gamma}{3} u^2 v^* + \frac{G}{2} v + \frac{G}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2} \end{cases} \quad (7.1.1)$$

where u and v are the normalized envelopes of the light along the two orthogonal polarization axes of the fiber. $2\beta = 2\pi\Delta n/\lambda$ is the wave-number difference between the two polarization modes. $2\delta = 2\beta\lambda/2\pi$ is the inverse group velocity difference, β_2 is the second order dispersion coefficient, k''' is the third order dispersion coefficient, and γ represents the nonlinearity of the fiber. G is the saturable gain coefficient of the fiber. For the undoped fiber $G = 0$, and for the erbium-doped fiber we further considered the gain saturation as

$$G = G_0 \exp\left[-\frac{\int (|u|^2 + |v|^2) dt}{E_{\text{sat}}}\right] \quad (7.1.2)$$

where G_0 is the small signal gain coefficient, and E_{sat} is the normalized saturation energy.

To also take into account the actions of the various cavity components such as the polarizer and output coupler on the light, we used the so-called pulse tracing method for the simulation. Briefly, we monitored a small segment of the light field within the laser cavity. Starting the simulation with a weak CW light that

has a small dip on top of it, we let the light circulate in the cavity. When the light encounters a cavity component, we multiply the Jones matrix of the component to the light field. After one cavity roundtrip we then used the result of the previous round calculation as the input of the next round calculation. The process was repeated until a stable field was obtained. With the technique not only the nonlinear light propagation effects in the cavity fiber but also the cavity feedback and cavity boundary condition were automatically considered.

Figure 7.6 A typical state of the dark soliton operation of the fiber laser numerically calculated. (a): The evolution of the dark soliton with the cavity round trips; (b) The soliton pulse profile; (c) The corresponding soliton spectrum.

To make the simulations as close as possible to the experimental situation, we used the following parameters for our simulations: The nonlinear fiber coefficient $\gamma = 3 \text{ W}^{-1}\text{km}^{-1}$; Erbium fiber gain bandwidth $\Omega_g = 16 \text{ nm}$; Fiber dispersions $k''_{\text{EDF}} = 40.7 \text{ ps}^2/\text{km}$, $k''_{\text{DCF}} = 5.1 \text{ ps}^2/\text{km}$ and $k''' = 0.01 \text{ ps}^3/\text{km}$; Cavity length $L = 3.0 \text{ m EDF} + 6.5 \text{ m DCF} = 9.5 \text{ m}$; Cavity birefringence $L/L_b = 9.5/100$ and the nonlinear gain saturation energy $E_{\text{sat}} = 480 \text{ pJ}$. Like in the experiment we used the laser gain coefficient and the cavity linear phase delay bias as the control parameters.

Independent of the concrete cavity parameters, as far as the linear cavity phase delay bias was set in the no-mode-locking regime, stable dark solitons could always be obtained in our simulations. With a cavity linear phase delay bias setting of 0.9π and the gain coefficient of $G_0 = 1100 \text{ km}^{-1}$, a typical stable dark

soliton numerically calculated is shown in Figure 7.6. Figure 7.6(a) shows that with the cavity round trips, the dark pulse is stable, suggesting that it is a dark soliton of the laser. Figure 7.6(b) shows the calculated pulse intensity profile. By fitting the pulse intensity profile with that of a standard dark soliton, the dark pulse has duration of about 11.5 ps, which is comparable with that estimated from the experimentally measured soliton spectrum. Figure 7.6(c) is the corresponding calculated dark soliton spectrum. The calculated soliton spectrum also has a sharp narrow peak, which is similar to the experimental result. No spectral sidebands are observable on the soliton spectrum as the formed dark soliton has a broad pulse width and the cavity has a short length of 9.5 m. Depending on the laser gain and gain saturation energy settings, numerically gray solitons with different darkness were also obtained. Associated with the different darkness of the gray solitons the pulse width also varied. The numerical result explains why no energy quantization was observed for the dark solitons in the experiments.

7.1.3 Discussions and Summary

Different from the bright solitons formed in the mode locked fiber lasers, where one can effectively control the number of solitons in the cavity through carefully controlling the pump strength, it was found experimentally that it is almost impossible to control the number of the dark solitons. To understand the feature of the dark solitons we note that Kivisher *et al.* have theoretically shown that different from the NLSE type of bright solitons, whose formation requires a certain fixed pulse intensity threshold, the formation of the NLSE type of dark solitons has no threshold [132]. Any small initial intensity dip could be evolved into dark solitons. Our experimental result well confirmed the theoretical

prediction. All the dark solitons were automatically formed in the cavity once the intracavity laser intensity was sufficiently strong, which is different from the bright soliton formation in a fiber laser, where in order to form the bright solitons a mode locked pulse needs to be formed in prior. As the dark solitons can be easily formed, and in a fiber ring laser there are plenty sources, e.g. the environmental noise and the mode beating, that could cause weak intensity dips, many dark solitons were simultaneously formed in the laser cavity. The dark solitons are randomly embedded in the laser emission. However, due to their narrow pulse width they are undetectable with even a moderate (<15 GHz) speed detection system. Only under sufficiently strong pumping where the dark solitons have bunched together and formed a giant dark pulse, their existence would then become detectable with even 1 GHz bandwidth detection systems as reported previously [25].

In summary, we have experimentally investigated the dark pulse emission of an all-normal dispersion fiber laser by using an ultrahigh speed detection system. Based on the real-time monitoring on the laser emission, we confirmed the NLSE type dark soliton formation in the fiber laser. Especially our experimental results have shown that the dark soliton formation is an intrinsic feature of the fiber lasers. As the dark soliton formation has no threshold, many dark solitons are always formed in a laser. However, their existence could not be easily detected if a moderate bandwidth detection system is used except that the dark solitons are bunched together to form a giant dark pulse.

7.2 Dark Solitons in Dispersion-managed Fiber Lasers

A fascinating feature of the fiber lasers is that their cavities can be dispersion-managed (DM), meaning that the cavity can be made of fibers with opposite sign of group velocity dispersion. Light circulation in a DM cavity fiber laser is analog to the light propagation in an endless DM fiber transmission line [133]. It has been theoretically shown that such a DM fiber transmission line supports both bright and dark DM solitons [134-136]. Experimentally, the DM bright solitons have been observed in the DM cavity fiber lasers [137]. However, to the best of our knowledge, no experimental observation on the DM dark solitons has been reported. To better understand the dark soliton formation in fiber lasers and the dark soliton features, we further studied the DM dark solitons in a net normal dispersion cavity EDF laser. We found experimentally that dark solitons could also be formed in a DM cavity fiber laser as far as the net cavity dispersion is positive. Moreover, the formed dark solitons have similar features as those observed in the all-normal dispersion fiber lasers. Numerical simulations also confirmed the dark soliton formation in DM fiber lasers.

7.2.1 Experimental Results

Figure 7.7 Schematic of the DM dark soliton fiber laser. WDM: wavelength division multiplexer. EDF: erbium doped fiber. SMF: single mode fiber. PCs: polarization controllers.

An erbium-doped dispersion-managed cavity fiber laser as shown in Figure 7.7 was constructed. The ring cavity consisted of a piece of ~ 3.0 m, 2880 ppm EDF with a group velocity dispersion (GVD) parameter of -32 (ps/nm)/km and ~ 3 m single mode fiber (SMF) whose GVD parameter is 18 (ps/nm)/km. The rest of the fibers used was the dispersion compensation fiber (DCF) with a GVD parameter of -4 (ps/nm)/km. The total cavity length was ~ 13 m. A polarization dependent isolator was used to force the unidirectional operation of the cavity, and an in-line polarization controller (PC) was inserted in the cavity to fine-tune the birefringence of the cavity. A 10% fiber coupler was used to output the signal. The laser was pumped by a high power Fiber Raman Laser (KPS-BT2-RFL-1480-60-FA) of wavelength 1480 nm. An optical spectrum analyzer (Ando AQ-6315B) and an ultrahigh speed oscilloscope (DSA-x 93204A 33

GHz, Agilent Technologies) together with two 45 GHz photo-detectors (NewFocus Model 1014) were used to monitor the spectral and temporal characteristics of the laser output simultaneously.

The total dispersion of the cavity is estimated $\sim 0.092 \text{ ps}^2$. We note that the laser has exactly the same cavity configuration as a passively mode locked fiber laser with the nonlinear polarization rotation (NPR) technique [138]. Indeed, depending on the linear cavity phase delay bias setting, self-started mode locking of the laser could be obtained, and consequently the laser was operating in a dissipative soliton emission state. A typical state of the dissipative soliton operation of the laser is shown in Figure 7.8. The optical spectrum of the soliton (see Figure 7.8(b)) has the characteristic sharp spectral edges with the central wavelength at 1568 nm, and a 3 dB bandwidth of $\sim 13.7 \text{ nm}$. The FWHM pulse width of the dissipative solitons was directly measured with a commercial autocorrelator (FR-103XL). It was $\sim 5.8 \text{ ps}$ (see Figure 7.8(c)). The dissipative solitons have a time-bandwidth product of ~ 9.7 .

Figure 7.8 Dissipative soliton emission of the DM fiber laser. (a) Pulse train; (b) Spectrum; (c) Autocorrelator trace of the dissipative solitons.

Figure 7.9 (a) Oscilloscope trace of dark solitons; (b) A single dark soliton; (c) Spectra of CW emission and dark soliton.

Like the case of the dark soliton operation of the all-normal dispersion cavity fiber laser the dark soliton operation of the laser was obtained by setting the laser in the non-mode-locking linear cavity phase delay bias regime [25,85].

Experimentally this was achieved by fixing all the other laser parameters but continuously tuned the orientation of the PC so that the laser moved away from a bright dissipative soliton operation state. It was observed that the laser emission changed from a bright soliton emission to a dark soliton emission state. A typical oscilloscope trace of the dark soliton emission of the fiber laser under a pump power of ~500 mW is shown in Figure 7.9(a). For comparison we have also shown the detection system noise level as a reference signal in the figure. On the oscilloscope trace shown in Figure 7.9(a), the dark solitons are displayed as narrow intensity dips on a strong CW background. To highlight we also zoomed in a segment of the oscilloscope trace and displayed it in Figure 7.9(b). The dark pulse has a duration of ~23 ps, which is limited by the bandwidth of the oscilloscope. Comparing the dark pulse with the system background noise signal, one can unambiguously identify the existence of the dark solitons in the laser. Under the dark soliton emission, the spectrum of the laser became obviously broader than that of the CW emission, as shown in Figure 7.9(c). Based on the measured soliton spectral bandwidth of 0.25 nm, if a transform-limited hyperbolic-tangent pulse profile is assumed, we estimate that the dark soliton pulse width is ~ 10.5 ps. Corresponding to the state shown in Figure 7.9 the laser output power was 6.8 mW. Assuming that the formed solitons are black, then following Matera *et al.* [139] the soliton pulse width τ and the average power of the CW background is related through $P_0 = \beta/\gamma\tau^2$, where γ is the Kerr coefficient of the fiber and β is the average GVD of the fiber laser. Using $\gamma = 3 \text{ W}^{-1}\text{km}^{-1}$ and $\beta = 7.08 \text{ ps}^2/\text{km}$, we estimate that the pulse duration would be ~10.9 ps, which well matches the experimental result. We note that it is experimentally challenge to detect dark solitons formed in a laser

[130]. The dark intensity dips shown in Figure 7.9(a) would be undetectable even if a moderate speed (15 GHz) detection system would be used. Given that the dark solitons are always randomly formed in the laser cavity (see Figure 7.9(a)) and have narrow pulse width, in the meantime, the coexistence of other nonlinear dynamic effects also unavoidably complicates the laser emission [140], it seems understandable that dark soliton emission of fiber lasers has been so far less addressed.

When the pump power was further increased, the density of dark solitons in the cavity increased. Experimentally it was observed that beyond a certain pump level (800 mW in the laser), the generated dark solitons always spontaneously bunch together to form a large dark pulse, whose pulse duration could vary from ~500 ps to ~3 ns depending on the pump power level and cavity conditions. The same effect was also observed in the all-normal dispersion cavity fiber lasers [25]. The experimental result shows that the large “dark pulse” is actually a bunch of many dark solitons. Under steady state operation, the dark soliton bunches were also stable in the cavity, which could be detected even with a 1 GHz detection system. These features of the dark solitons are exactly the same as those observed in the all-normal dispersion cavity fiber lasers [25]. We also experimentally investigated the effect of the anomalous dispersion SMF on the DM dark solitons by deliberately changing the SMF length. Operation of the fiber lasers with net cavity dispersion of 0.161 ps^2 , 0.092 ps^2 , 0.023 ps^2 and -0.046 ps^2 , was experimentally investigated respectively. It was found that dark solitons could always be obtained as far as the net cavity dispersion was normal, while no dark solitons were detected in the net anomalous dispersion cavity fiber laser.

Figure 7.10 Dark soliton emission of the laser. (a) Measured over one cavity roundtrip time; (b) and (c) Zoom-in at the positions that has one cavity roundtrip time separation.

We note that different from the bright soliton formation, which requires that the pulse intensity is above a certain threshold, the formation of dark solitons has no threshold. Any small intensity dip in a strong CW background could evolve into a dark soliton [1]. Due to this feature of the dark solitons it is experimentally difficult to control the number of dark solitons formed in a laser. Generally, many dark solitons were simultaneously formed in a laser. Figure 7.10 shows a typical case of the dark solitons formed in the whole laser cavity. We emphasize that except some small local changes caused by the dark soliton interaction, the overall soliton patterns repeated with the cavity period, confirming that the dark pulses are dark solitons rather than laser intensity fluctuations. In our experiment no dark soliton energy quantization was observed.

7.2.2 Numerical Results

Figure 7.11 Simulation of DM dark solitons in time domain with the net-cavity dispersion $\sim 0.092 \text{ ps}^2$.

To confirm the experimental observations, we have further numerically simulated the dark soliton formation in DM fiber lasers. We used a model as described in [85]. To make the simulations comparable with our experiment, we used the following parameters: the orientation of the intra-cavity polarizer to the fiber fast birefringent axis $\Phi = 0.125\pi$; the nonlinear fiber coefficient $\gamma = 3 \text{ W}^{-1}\text{km}^{-1}$; the erbium fiber gain bandwidth $\Omega_g = 16 \text{ nm}$; fiber dispersions $k''_{EDF} = 40.7 \text{ ps}^2/\text{km}$, $k''_{SMF} = -22.9 \text{ ps}^2/\text{km}$, $k''_{DCF} = 2.54 \text{ ps}^2/\text{km}$ and $k''' = 0.1 \text{ ps}^3/\text{km}$; cavity linear birefringence $L/L_b = 0.1$ and the gain saturation energy $P_{sat} = 50 \text{ pJ}$. $L_{EDF} = 3 \text{ m}$; $L_{DCF} = 7 \text{ m}$; and the length of the SMF was changed from 0 to 9 m. In our simulations an arbitrary weak dip input was used as the initial condition. It was found that when the linear cavity phase delay bias was biased in the non-mode-locking regime, stable dark soliton could always be obtained as far as the net cavity dispersion is in the normal regime. Figure 7.11 shows a dark soliton

numerically obtained in the case of 3.5 m SMF in the cavity. It shows that stable dark soliton can be formed in a DM cavity fiber laser.

7.2.3 Summary

In summary, we have first experimentally demonstrated dark soliton formation in a dispersion-managed cavity fiber laser with net normal cavity dispersion. We found that the formed dark solitons have similar properties as those observed in the all-normal dispersion cavity fiber lasers. Numerical simulation has also confirmed our experimental observation of dark soliton formation in the DM fiber lasers. The result shows again that the dark soliton formation is an intrinsic feature of the normal dispersion cavity fiber lasers.

Chapter 8 Dark-bright Solitons in Fiber Lasers

Vector temporal soliton formation in the single mode optical fibers was extensively investigated in the past. The dynamics of the vector soliton formation can be well governed by the coupled NLSEs which take into account the birefringence in the SMFs. With a weak birefringence assumption, various kinds of vector solitons have been deduced from the coupled NLSEs, such as bright-bright soliton, dark-bright soliton and dark-dark soliton [71,141-144]. In the previous chapters, bright-bright vector solitons have been investigated in vector soliton mode-locked fiber lasers. However, the properties of the other vector solitons have not been discussed. In this chapter we turn to the study of dark-bright vector solitons in the fiber lasers.

Christodoulides *et al.* firstly theoretically predicted the existence of dark-bright solitons in the single mode fiber [141]. Based on the NLSE, they pointed out that under a weak birefringence, the dark-bright vector solitons can exist in different dispersion regimes with different properties. In normal dispersion regime, the dark soliton plays a dominant role and in anomalous dispersion regime the bright soliton dominates. Their predictions provide a guideline for the experimental study on the dark-bright solitons in fiber lasers.

According to the theoretical prediction of Christodoulides *et al.*, there are two crucial conditions for the dark-bright soliton formation in fiber lasers. Firstly, the birefringence of the fiber cavity must be sufficiently weak so that the two polarization components of the light can be strongly coupled, and secondly, the energy of the intra cavity should be sufficiently high to support the formation of dark solitons. Similar to the case of scalar dark soliton, the main challenge in the dark-bright soliton experiment study is how to detect the existence of the dark solitons as they are difficult to be observed in the time domain. Although there are several experimental observations in fiber lasers in which they claim that the “dark-bright pulse” was obtained, strictly speaking, these pulses cannot be considered as dark-bright solitons [145-147]. There are still no systematic experimental studies on the formation of the dark-bright solitons in fiber lasers, to the best of the author’s knowledge.

In this chapter, we present experimental studies on dark-bright pulses in an erbium-doped fiber laser. Stable dark-bright soliton pulse train was formed in a dispersion-managed fiber laser with near zero cavity dispersion. Through the experimental studies, we illustrate the formation process of the dark-bright solitons step by step and provide the technique to generate the dark-bright solitons in the fiber lasers.

To generate the dark-bright solitons, both dark pulse and bright pulse must be automatically obtained in the cavity. Therefore proper cavity dispersion must be chosen. Based on the previous experiments, it was found that in all anomalous dispersion cavities, dark soliton was difficult to be achieved, while bright pulses cannot be shaped into solitons without mode locking in all normal dispersion

cavities. A dispersion-managed cavity is thus necessary in order to achieve the dark-bright soliton operation of the fiber lasers. Our experimental results are all based on the dispersion managed fiber lasers under different cavity dispersion. Section 8.1 presents the dark-bright pulse formation in the anomalous cavity dispersion laser. In anomalous dispersion, bright pulse is formed first and shaped into a bright soliton, the cross polarization coupling then leads to the formation of the dark pulses. Section 8.2 presents the case of normal cavity dispersion. In normal dispersion, dark pulse is shaped into the dark soliton, and cross polarization coupling then leads to the formation of the bright pulses. Section 8.3 presents the case of near zero cavity dispersion, in which both the dark and bright components can be simultaneously shaped into a stable dark-bright soliton in the laser cavity. Section 8.4 summarizes the results and concludes the chapter.

8.1 Dark-bright Solitons with Anomalous Cavity Dispersion

Figure 8.1 Schematic diagram of the dispersion managed fiber laser.

The dispersion managed fiber laser is schematic shown in Figure 8.1. The 3 m long EDF is of normal dispersion with GVD -48 ps/nm/km and 13.5 m long SMF is of anomalous dispersion with GVD 18 ps/nm/km. In addition, a 10% fiber output coupler was used to output the signal, a polarization independent isolator was used to force the unidirectional operation of the ring cavity, and an intra-cavity polarization controller was used to fine tune the linear cavity birefringence. All these passive components are made of the SMF or pigtailed with SMFs. The laser was pumped by a high power Raman fiber laser (KPS-BT2-RFL-1480-60-FA) of wavelength 1480 nm with maximum pump power as high as 5 W. The output optical spectra of the fiber laser were measured by an optical spectrum analyser (ANDO AQ6317). The soliton pulse train emitted by the fiber laser was monitored by a 33 GHz oscilloscope (Agilent DSA-X 93204A) together with two 45 GHz photo-detectors (New Focus 1014).

As there is no polarization components inserted into the cavity, the laser is initially working at continuous wave regime. When the pump power was increased to ~ 400 mW, CIMI induced soliton pulse train (see Chapter 6) was achieved. By using the polarization beam splitter, the two orthogonal polarization components of the laser emission can be observed simultaneously. The CIMI induced solitons were found to be scalar solitons and in the other polarization component the intensity of the light is almost zero. By tuning the paddles of the PC, the intensity of the other polarization is strengthened and the number of the CIMI solitons was thus reduced because the energy was transferred to the other polarization. If the birefringence can be sufficiently small at this state and by increasing the pump power to 2 W, the coupling between the two polarizations could be observed, as shown in Figure 8.2.

Figure 8.2(a)-(d) illustrate four states of the coupling between the two polarization components. The blue curve indicates the CMI induced bright soliton pulses and the green curve indicates the CW of the other polarization that can generate dark pulses. Initially, as shown in Figure 8.2(a), the bright component is much stronger and the CW level of the dark component is almost zero. The background level of the dark component can be increased by carefully tuning the PC. Correspondingly, the width of the dark pulse is decreased. Figure 8.2(d) shows the state that the CW background of the dark pulse reaches to its maximum level when the pump power is fixed to be 2 W.

Figure 8.2 Dark-bright pulse formation process. (a)-(d): Four stages of the coherent coupling between the two polarization components.

Figure 8.3 A typical dark-bright pulse state. (a) Single dark bright pulse state; (b) autocorrelation trace of the bright pulse; (c) Multiple pulses; (d) Corresponding optical spectrum.

Figure 8.4 Evolution of dark-bright pulse pair under different pump power. (a) 33 dBm (2 W); (b) 34 dBm; (c) 35 dBm; (d) 36 dBm (4 W). Note: The soliton is randomly selected from a multiple solitons state.

Through the process shown in Figure 8.2, a typical dark-bright pulse was achieved. Figure 8.3 (a) shows an oscilloscope trace of a single dark-bright pulse. Measured by the oscilloscope, the FWHMs of the dark pulse and the bright pulse are comparable which are around ~ 30 ps. As the bandwidth of the oscilloscope is 33 GHz, the pulse width of the dark pulse should be < 30 ps. The width of the bright pulse was measured by a commercial autocorrelator (FR-103XL) which is ~ 1 picosecond, as shown in Figure 8.3(b). Figure 8.3(c) shows a state of multiple dark-bright pulses in the cavity. It can be seen that the width of the dark pulse is not always as narrow but variable. Actually, the width of the dark pulse depends on the coupling and the pump power. Figure 8.3(d) shows the corresponding optical spectrum. The optical spectrum reveals that the two polarization components are coherently coupled. Figure 8.4 shows the shape of

the dark-bright pulse under different pump power. It is clear that the width of the dark pulse decreased as the pump increases. It can be expected that under sufficiently high pump power dark pulse can be shaped into the dark soliton.

From the experimental results it can be summarized as three key features of the dark-bright pulse pair in anomalous dispersion cavity. First, the dark pulse is always generated with the bright soliton and formed a dark-bright pulse pair, indicating that the formation of the dark-bright pulse is attributed to the coherent coupling between the two polarization components. Second, the bright pulse has been shaped into the soliton while the shape of the dark pulse is still variable, which indicates that the dark pulse is not a soliton of the NLSE type and it was a bright soliton and a dark pulse pair. Third, the dark pulse could be shaped into the dark soliton under sufficiently high pump power (~ 4 W), which suggests that the dark-bright soliton is still possible in the anomalous dispersion cavity. It can be concluded from these features that the cross polarization coupling and high pump power are two essential conditions to generate the dark-bright pulses. However, in practice the pump power usually cannot be too high to avoid damage on the components in the laser cavity. Moreover, if the pump power is too high, the dark pulse in anomalous dispersion will not be stable. Thus, it is difficult to achieve a stable dark-bright soliton pair in the anomalous dispersion fiber laser. A proper method to solve these problems is to add some dispersion compensating fiber into the cavity with normal dispersion which could stabilize the dark pulse. Besides, reducing the cavity loss is a method to wear off the dependence on the high pump power.

8.2 Dark-bright Solitons with Normal Cavity Dispersion

Based on the experimental results in Section 8.1, the length of the anomalous dispersion fiber in the cavity seems to be too long and dark pulse was not stable. Therefore, keeping the same cavity configuration, we reduce the length of SMF to 10 m so the total cavity length now is ~13 m, and the cavity is in the normal dispersion regime.

Unlike the case of anomalous dispersion, the fiber laser always works at CW regime and the CIMI pulse is not achieved. As the pump power increases to 2 W, a dark pulse train could be achieved. By properly further tuning the PC carefully, dark-bright pulse train was found and a typical state of dark-bright pulses is shown in Figure 8.5. Figure 8.5(a) shows an oscilloscope trace in one and half round trip time. It is found that the dark pulse train is not uniform and CW background is a little noisy, which is obviously different from the bright pulses in the anomalous dispersion. This result is similar to the cases of the scalar dark pulses discussed in chapter 7, indicating that the dark pulse is automatically generated in the normal dispersion fibers. Figure 8.5(b) shows the corresponding optical spectrum. The red curve corresponds to the bright component which is with weaker intensity and the green curve corresponds to the dark component which is with stronger intensity. It can be seen that the coupling occurs at ~1580 nm.

Figure 8.5 Dark-bright pulse state in normal dispersion regime. (a) Oscilloscope trace; (b) Optical spectrum.

To study the details of the dark-bright pulses, a 20 ns scale picture of oscilloscope trace was captured which is shown in Figure 8.6(a). It is clear that dark pulses are not uniform and only a few dark pulses are correspondingly coupled with a bright pulse. In fact, because there is no threshold value for the formation of a certain dark pulse, dark pulses could be generated all the time. Due to the existence of the gain competition, not all the automatically generated dark pulses in the cavity are shaped in the dark solitons. Some of the dark pulses are not with sufficient energy due to the limitation of the intra cavity energy. Even there is coupling between the two polarization components, the energy is still not enough to support the formation of the dark pulse or even the bright solitons. Figure 8.6(a) actually shows various different stages of the formation of the dark-bright pulse. These pulses can be considered as various dark-bright pulses at different stages of the formation with different energies. It can be expected that if the energy are high enough, some of them could be shaped into the dark-bright solitons. Figure 8.6(b) show a single dark-bright pulse which could be considered almost shaped into the dark-bright solitons. The FWHM of the dark components and the bright components are both approaching the bandwidth limit of the oscilloscope. Unlike the case in anomalous dispersion, the width of the bright components is not as narrow as several picoseconds and it cannot be measured by the autocorrelator, which supports our prediction that the dark soliton leading to the formation of the bright soliton and their FWHM should be comparable which are both ~ 20 ps.

Figure 8.6 Details of the dark-bright pulse in the normal dispersion regime. (a) Tens of pulses in 20 ns scale; (b) Single pulse in 2 ns scale.

Based on the above experiments results, the formation of the dark-bright soliton in the normal dispersion regime could be summarized as follows: Firstly, the dark pulses with different intensities automatically emerged in the fiber laser due to the intrinsic property of the dark soliton formation in optical fibers. When the two orthogonal polarizations components are coherently coupled, the

bright pulse can be induced from the dark pulse. The shape of the bright pulse depends on the strength of the coupling and the total energy of the vector pulse pair. Dark-bright soliton could be formed when both of the polarization components are with sufficient power. However, in most cases the dark pulse could be shaped into the solitons but the bright pulse was not.

Comparing with the case of anomalous dispersion, the dark pulse can be shaped into the soliton more easily and 2 W of pump power is sufficient for dark soliton formation. However, the formation of the bright pulse is uncontrollable in the laser cavity. Moreover, due to the intrinsic non-uniform property of the dark soliton, plenty of pulses are always formed simultaneously in the cavity which shares the energy of the cavity and the pulses are thus not so stable.

8.3 Dark-bright Solitons with Near-zero Cavity Dispersion

Figure 8.7 Coherent coupling optical spectrum of the two polarization components. The horizontal axis (green line) corresponds to the bright component and the vertical axis (red line) corresponds to the dark component.

Considering the situations encountered in the normal dispersion regime and the anomalous dispersion regime. A more stable dark-bright pulse train could be expected in the near zero cavity dispersion regime. To this end a dispersion managed fiber laser working at the near zero cavity dispersion was constructed by adding segments of single mode fiber and dispersion compensating fiber into the cavity.

Figure 8.8 Dark bright pulses in near zero dispersion. (a) Oscilloscope trace of 5 pulses; (b) Oscilloscope trace of a single pulse; (c) Autocorrelation trace of the bright pulse.

In this case, coupled dark and bright pulse pairs are always obtained in the cavity when the two orthogonal polarization components are coupled. Figure 8.7 shows the optical spectrum when the two polarization components are coupled with each other. The horizontal axis (green line) corresponds to the bright component in the temporal domain and the vertical axis (red line) corresponds to the dark component in the temporal domain. As long as the coupling state is obtained, dark-bright pulse pair could be easily found by slightly tuning the PC. Figure 8.8 shows the oscilloscope trace of dark-bright pulses. Figure 8.8(a) shows that there are five pulses in 500 ns interval and there is only one dark-bright pulse pair in each cavity round trip time. Figure 8.8(b) shows the details of the dark-bright pulse pair. Measured by the autocorrelator, the FWHM of the bright pulse is ~ 1 ps, as shown in Figure 8.8(c). The FWHM of the dark pulse displayed on the oscilloscope trace is ~ 20 ps.

Figure 8.9 Stable dark-bright vector soliton operation of the fiber laser.

Figure 8.10 A state of multiple dark-bright solitons in the fiber laser.

To show the stability of the formed dark-bright pulse pair, a dark-bright pulse train measured in 2 μs scale is shown in Figure 8.9. It can be seen that the fluctuation of the pulse intensities is quite small. The dark-bright pulse emission state was quite stable and could be kept for tens of minutes. Multiple pulses were also achievable in the cavity. The number of the pulses in each round trip time can be controlled by controlling the PC. A multiple pulse state is shown in Figure 8.10. However, the multiple dark-bright pulses in the cavity are not as stable as single pulse state and the intensities of the pulses are not uniform either.

From the results obtained, we can conclude that the formation of the dark-bright soliton in near zero dispersion regimes mainly depends on the coherent coupling. Because we did not obtain any pulses before the dark-bright pulse emerged. The only condition for the pulse formation is the two orthogonal polarization components are coupled coherently. Moreover, the two polarization components in the zero dispersion regime is more balancing, in

other words, the dark components and the bright components are comparable and plays the equivalent roles in the soliton formation.

The stability of the dark-bright pulses in this regime is significantly improved which could be attributed to the suitable dispersion management. As mentioned in Section 8.1, the dark-bright pulses in anomalous dispersion regime could be more stable by adding certain length dispersion compensating fiber. Now it has been verified that when the cavity dispersion is compensated to zero, the pulses could be more stable in the cavity. Besides, as the power can be allocated properly into the two polarization components, the soliton shaping in the zero dispersion regimes is also better than other regimes. The dark components and bright components could be both shaped into the solitons without too high pump power (~ 2.5 W is sufficient). The dark-bright pulses can be expected to be more stable after the solitons have been formed.

8.4 Summary

In summary, we presented the experimental study on the dark-bright soliton generation in an erbium-doped fiber laser. Through systematic investigations on the dark-bright pulse pair formation in different cavity dispersion regimes, we found that stable dark-bright solitons could be formed in the fiber laser when the net cavity dispersion is set near the net zero point. Technically, this was achieved with a dispersion-managed laser cavity. Another essential condition for dark-bright soliton formation is low birefringence of the cavity so that the coherent coupling could occur between the two polarizations of the light in the fiber. A near zero dispersion regime is preferred to keep the balance between the dark component and the bright component. Fulfilling the above conditions, a stable dark-bright soliton pulse train can be generated.

Through the above experimental studies, the mechanism of the dark-soliton generation is also revealed. The dark-bright soliton formation is due to the strong cross polarization coupling of light in the fiber laser, which coincides with the theoretical prediction of Christodoulides *et al.*[141]. The generation of stable dark-bright soliton pulses provides a potential on the study of the vector dark-bright soliton and other vector solitons for optics communications.

Chapter 9 Conclusions and Future Works

In this dissertation, a comprehensive study on the vector solitons formed in the graphene mode locked fiber lasers and the dark solitons formed in non-mode-locking erbium-doped fiber lasers were presented. This chapter summarizes the accomplishments of the research and provides some suggestions for the future works.

9.1 Conclusions

- **Mode-locking Regime**

In the mode locking regime, we have experimentally investigated the vector soliton operation of fiber lasers mode locked with the atomic layer graphene. Similar to the case of SESAM mode locked fiber lasers, both the polarization locked and polarization rotation vector solitons were experimentally obtained in the graphene mode locked fiber lasers. The cavity effect always induces resonant energy exchange between the two orthogonal polarizations components of the vector solitons, and for the polarization rotation vector solitons, two new sets of spectral sidebands appeared on the vector soliton spectrum.

Under the multiple vector soliton operation of a fiber laser, various multiple vector soliton patterns were experimentally observed, including the vector soliton bunches, random static vector soliton distribution, restless vector soliton bunches, giant pulse emission, and vector soliton rains. In particular, the polarization-locked vector soliton rain and the polarization rotation vector soliton rain were experimentally identified. Based on results of the experimental studies, the appearance of the different modes of the multiple vector soliton operation could be well understood by the soliton interaction caused by the following four types of main mechanisms, the unstable CW related global type of soliton interaction, the dispersive wave mediated long range soliton interaction, the direct soliton interaction, and the saturable absorber induced soliton-soliton attraction.

The nonlinear dynamics of the vector solitons in graphene fiber lasers was also investigated. Quasi-periodicity of the vector solitons was experimentally observed. The cavity induced soliton modulation instability was the origin of the quasi-periodicity. The vector solitons in the laser could be emitted with both quasi-periodic pulse energy variation and polarization rotation, which can be characterized by the respectively extra sidebands appearance in the optical spectrum.

- **Non-Mode-locking Regime**

In the non-mode-locked fiber lasers where no graphene was inserted in the cavity, stable various GHz level repetition rate pulse trains were generated. At fixed pump strength the pulse repetition rate can be varied by changing the cavity detuning, showing that they were caused by the cavity induced

modulation instability. CIMI is different from the conventional modulation instability lasing of a fiber laser. The repetition rate of the conventional MI pulses is uniquely determined by the fiber dispersion and the laser beam intensity, while the pulse train of the cavity induced modulation instability is further related to the cavity detuning. At high pump power, coexistence of the cavity induced MI and the conventional MI lasing was also experimentally observed. In this case the ultrahigh repetition rate pulses caused by the conventional MI is intensity modulated by the cavity induced MI. Nevertheless, the conventional MI can be avoided by operating the laser in the net normal cavity dispersion regime.

Changing the cavity to all normal dispersion, the dark pulse emission was experimentally obtained. The existence of the NLSE dark solitons is experimentally verified with a high speed detection system. The formation mechanism of the dark pulse in the laser was revealed which is due to the normal cavity dispersion of the fiber laser. As the dark soliton formation has no threshold, many dark solitons are always formed in a laser. The dark solitons were also observed in the dispersion managed cavity fiber lasers as long as the cavity dispersion is normal. The experimental results suggest that dark soliton formation is an intrinsic feature of the net normal dispersion cavity fiber lasers.

Operating a dispersion-managed fiber laser in the near zero-dispersion regime, a novel type of vector solitons called dark-bright soliton was generated experimentally. Based on the previous reported theoretical predictions and through systematically studying the features of the dark-bright pulse pair under different cavity dispersions, it is shown that the dark-bright soliton formation is

due to the coherent coupling between the two polarizations components of the light in the fiber laser. The generation of stable dark-bright soliton pulses provides a technique on the stable dark soliton generation in fiber lasers. It is expected that the generated stable dark-bright solitons pairs and the dark solitons could find applications in the future optics communications systems.

9.2 Recommendation for Future Works

The studies presented in this thesis also provide various interesting directions for the study in the future. In this section, a brief discussion on the possible future works was presented.

Recently, different methods of fabricating graphene have been proposed. In this dissertation we have investigated the vector soliton operation of atomic layer graphene mode locked fiber lasers where the graphene was made with the CVD method. It would be interesting to study the vector soliton operation of fiber lasers mode locked with other kinds of graphene, such as the oxide graphene, reduced oxide graphene and so on. Moreover, some other two-dimensional nanomaterials with similar atomic structure as graphene were also produced, including the topological insulators, molybdenum disulphide. These two-dimensional nanomaterials may also have saturable absorption. It would be interesting to study the soliton properties of the fiber lasers mode locked with these two-dimensional nanomaterials.

Another interesting area of the study is the graphene mode locking fiber lasers working at other wavelengths, for instance, at 2 micrometer. Comparing to the

erbium-doped fiber lasers working at 1.5 μm , Tm-doped fiber lasers have attracted intense attentions. However, the study of the mode locked thulium-doped fiber lasers is at an early stage and there are still plenty of works can be done. It is expected that the thulium-doped fiber lasers working at 2 μm would be the focus of the mode locked fiber laser studies in the near future.

Study on the scalar dark solitons and vector dark solitons is of fundamental and practical importance. Thus far, experimental study on the dark soliton is at its early stage. Thanks to the development of the high speed optoelectronic measurement systems, one can now observe the dark solitons directly on the high speed oscilloscope. However, there are still plenty of questions on the dark solitons that need to be answered. What is the FWHM of a standard fundamental dark soliton? Are there bound states of dark solitons? With even higher speed detection systems, these questions could be answered. Similar questions could also be asked on the dark-bright vector solitons. To satisfactorily clarify these questions would be important tasks of the future studies. Moreover, studies on the possible applications of the scalar and vector dark solitons could be other interesting research topics in the future.

Author's Publications

1. **Y. F. Song**, H. Zhang, D. Y. Tang, D. Y. Shen, "Polarization rotation vector solitons in a graphene mode-locked fiber laser," *Optics Express*, **20** (24), 27283-27289 (2012).
2. **Y. F. Song**, H. Zhang, D. Y. Tang, D. Y. Shen, "Vector multi-soliton operation and interaction in a graphene mode-locked fiber laser," *Optics Express*, **21** (8), 10010-10018 (2013).
3. **Y. F. Song**, L. Li, D. Y. Tang, and D. Y. Shen, "Quasi-periodicity of vector solitons in a graphene mode-locked fiber laser," *Laser Physics Letters*, **10** (12), 125103 (2013).
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6. L. Li, **Y. F. Song**, H. Zhang, and D. Y. Shen, and D. Y. Tang, "Dark soliton operation fiber lasers," *Lasers and Electro-Optics Pacific Rim (CLEO-PR)*, Kyoto, Japan, 2013.
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8. **Y. F. Song**, J. Guo, L. M. Zhao, D. Y. Shen and D. Y. Tang, “280 GHz dark soliton fiber laser,” *Optics Letters*, accepted.
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