Indentation and impact of sandwich structures

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INDENTATION AND IMPACT OF SANDWICH STRUCTURES

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Abstract

High specific strength and stiffness dominate the choice of material selection in the fields of weight sensitive industries such as aerospace, automotive, wind turbine and ship building. To this end, sandwich constructions form a better choice compared to the conventional bare metal plates of same thickness. For practical load-bearing applications, it is often an important task to assess the sandwich structures for its safe working as well as its operational life under the given working conditions. In the present thesis, quasi-static indentation and low velocity impact loadings on the foam cored sandwich plates is investigated.

Initially, finite element (FE) models are developed for predicting the low velocity impact response of foam cored sandwich plates. The sandwich plates used for the present work have a core made of commercial aluminum alloy foam (Alporas) with faceplates made of either ductile aluminum (Al) or elastic carbon fiber reinforced plastic (CFRP). A spherical ended impactor of 2.65 kg mass with 6.7 m/s velocity is impacted on to a clamped sandwich plates. All the FE simulations are performed using 3D finite element models in the commercial FE code LS-DYNA. Selection of suitable constitutive models and erosion criterion for the failure analysis were investigated. Predicted load versus displacement curves were compared against experimental measurements.

Then, the relative performance of graded metal and polymeric foam cored sandwich plates is studied under low velocity impact loading. The metal foam sandwich plates are constructed using aluminum alloy foam (Alporas) core and polymeric foam sandwich plates are constructed using polyvinyl chloride (Divinycell H80 and H250) foam. A core of 40 mm thickness (with two layers of 20 mm each) and aluminum faceplates of 0.5 mm and 1.0 mm were used. Impact experiments were conducted with a hemispherical punch of mass 8.7 kg at a nominal velocity of 5.8 m/s. The effect of stepwise core grading on the maximum dynamic penetration force as well as energy absorption capacity is studied. To maximize the energy absorption or to minimize the mass of the sandwich plate for a given penetration force, alternatives to Alporas foam are chosen based on either equivalent density (viz., H250) or through-thickness compressive yield strength (viz., H80).

The second major contribution of this work is in the development of analytical models for

the indentation failure of composite sandwich plates under bending and also explores the failure mode map. The analytical models are developed for estimating the indentation behavior of circular composite sandwich plates with a rigid flat ended circular punch. Initially, core was treated as an elastic foundation to derive the design loads for the indentation failure. Additionally, analytical models are extended to elastic-perfectly plastic foundation. Conventional indentation analogy and radial compression analogy of top faceplate are used to derive the load displacement curves using small deformation theory. All the small deformation cases are derived by solving the differential equations exactly. Finally, large deformation of a plate subjected to indentation on rigid-perfectly plastic foundation is derived using Galerkin's weighted residual method using indentation analogy. Axisymmetric finite element (FE) models are used to validate the proposed analytical models.

Finally, competing failure modes are investigated for circular sandwich plates comprising quasi-isotropic E-glass/epoxy composite faceplates (with $[-60/0/60]_{ns}$ configuration) and Polyvinyl chloride (PVC) foam core under bending. Clamped sandwich plates were loaded using flat ended punch at the center of the plate. Three competing failure modes, viz., core indentation, core shear and face failure/microbuckling, are considered. Analytical estimates for the elastic response (stiffness) and initial failure load are proposed, and these are verified with experimental measurements and FE predictions. Analytical estimates for the failure modes are used to plot the failure mode map in non-dimensional plate radius versus faceplate thickness for a given material system. The failure mode map thus constructed is assessed by considering a few sandwich plate geometries. Normalized sandwich mass and failure load contours are superimposed onto the failure mode map to identify the locus of minimum weight design by numerical search. Effect of geometrical parameters of the sandwich plate on failure load and mode is also investigated.

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Nomenclature

$\bar{A}$	Ratio of length to width of the rectangular sandwich plate
a	Radius of the impactor/indenter
A_f^*	Equivalent stretching stiffness of the orthotropic faceplate
A_f	Stretching stiffness of the isotropic faceplate
A_{ij}	Components of the stretching stiffness matrix
ASTM	American society for testing and materials
$\bar{c}$	Nondimensional core thickness
c	Thickness of the core
CFRP	Carbon fiber reinforced polymer
COD	Crack opening displacement
CT	Computed tomography
D	Bending rigidity of the isotropic sandwich plate
D^*	Effective bending rigidity of the orthotropic sandwich plate
D_f^*	Effective bending rigidity of the orthotropic faceplate
D_f	Bending rigidity of the isotropic faceplate
D_{ij}	Components of the flexural rigidity matrix
DCB	Double cantilever beam
$\bar{E}$	Nondimensional modulus
$\bar{E}$	Plane strain value of Young's modulus

E_c	Young's modulus of the core
E_f	Young's modulus of the faceplates
E_s	Young's modulus of cell wall material core
E_{ijf}	Orthotropic Young's moduli of the faceplate
EPP	Elastic perfectly plastic foundation
f_n	Real part of n^{th} -order Hankel function of first kind
G_{IC}	Fracture energy release rate in mode I
g_n	Imaginary part n^{th} -order of Hankel function of second kind
GFPP	Glass fiber Polypropylene
GFRP	Glass fiber reinforced polymer
H	Total thickness of the sandwich plate
$H_n^{(1)}$	n^{th} -order Hankel functions of the first kind
$H_n^{(2)}$	n^{th} -order Hankel functions of the second kind
HSSA	Hybrid stainless steel assembly
i	Complex number
J_n	n^{th} -order Bessel function of first kind
k	Foundation stiffness
k_b	Bending stiffness
k_c	Contact stiffness
k_i	Localized stiffness
k_m	Membrane stiffness
k_s	Shears stiffness
K_x, K_y and K_z	Foundation moduli of the core material in Pasternak's foundation
k_{bs}	Equivalent stiffness from bending and shear

k_{eq}	Sandwich plate stiffness
K_{IC}	Stress intensity factor in mode I
K_{ss}	Steady state fracture toughness/stress intensity factor
$kei(r)$	Kernel function
λ	Plastic radius or radius of the crushed zone
L	Width of the rectangular plate
l	Cell size of foam or characteristic length in plate bending
M	Mass of the impactor in low velocity impact
M	Radial moment
m_f	Mass of the faceplate
m_{sw}	Equivalent mass of the sandwich plate
N	In plane stretching or compression
P	Reaction force/load on the impactor/indenter
P_{el}^i	Indentation failure load based on indentation analogy
P_{el}^{rc}	Indentation failure load based on radial compression analogy
PP	Polypropylene
PS	polystyrene
PU	Polyurethane
PVC	Polyvinyl chloride
q_r and q_z	Axisymmetric surface loads in r and z directions, respectively
q_x , q_y and q_z	Surface loading functions along x, y and z, respectively
$\bar{R}$	Nondimensional radius of the plate
R_c	Contact radius
R_e	Effective radius of a spherical indenter

RPP	Rigid perfectly plastic foundation
RVE	Representative volume element
S	Shear stiffness of the isotropic sandwich plate
S^*	Effective shear stiffness of the orthotropic sandwich plate
SENB	Single edge notched bend specimen
$\bar{t}$	Nondimensional faceplate thickness
t	Thickness of the faceplate
u, v and w	Displacement functions along x, y and z respectively
u_n	Real part of n^{th} -order Bessel function of first kind
v_0	Initial velocity of the impactor in low velocity impact
v_n	Imaginary part of n^{th} -order Bessel function of first kind
w_0	Indentation depth beneath the indenter
W_f	Work of fracture
δ	Localized indentation profile
δ_0	Localized indentation depth at the origin
δ_{el}	Downward displacement to cause plastic deformation in the foam
ϵ_c	Plastic yield strain of the core
$\bar{G}$	Nondimensional Shear modulus
κ	Shear correction factor
∇_r^2	Laplacian operator in polar co-ordinate system
ν_c	Poisson's ratio of the bulk material of core
ν_f	Poisson's ratio of faceplate
ν_p	plastic Poisson's ratio of foams
ν_{ijf}	Orthotropic Poisson's ratio of the faceplate

$\bar{\rho}$	Relative density of foam
ρ_f	Density of the faceplate
ρ_{sw}	Density of the sandwich
$\bar{\sigma}$	Nondimensional inplane strength of the faceplate
σ_f	Fracture strength or yield strength of faceplate
σ_s	Fracture strength or yield strength of bulk material of core
σ_{xx} , σ_{yy} and σ_{zz}	Normal stresses along cartesian coordinates x, y and z respectively
σ_c	Yield strength of core
$\bar{\tau}$	Nondimensional shear strength of the faceplate
τ	Time variable
τ_f	Transverse shear strength of the faceplate
τ_{rz}	Interface shear stress in polar coordinates
τ_{xy} , τ_{yz} and τ_{zx}	Shear stresses along xy, yz and zx planes respectively

Chapter 1

Introduction

This chapter discusses the advantages and various types of sandwich structures along with their constructional details. Finally, the objectives of the present research, scope of the work and thesis layout are presented.

1.1 Basics of the sandwich structure

A sandwich structure consists of two strong and stiff faceplates separated by a low density compliant core as shown in Figure 1.1. The faceplates are bonded to the core by a polymer resin. In general, engineering alloys such as steel, aluminum alloys or fiber reinforced polymer matrix composites are used as faceplates. Typical honeycombs, corrugated sheets or polymeric/metallic foams comprise the core as shown in Figure 1.2. The human skull is a typical natural sandwich construction. For a complete understanding of the sandwich structures, it is important to under-

Figure 1.1: Sandwich structure constructional details [1].

Homogeneous core materials:

Structured core materials:

Figure 1.2: Types of sandwich constructions [2].

stand the role of individual constituents and the load-carrying mechanism of the structure. The important functions of the sandwich constituents (viz. faceplates, adhesive and core) are listed here [3].

1.1.1 Functions of sandwich constituents

Functions of faceplates:

1. Faceplates carry the in-plane tensile/compressive loads along with bending moment.
2. Faceplates should be resistant to inter cell wall buckling in periodic cellular cores.
3. Good surface finish for easy integration with neighboring structural elements and should have resistance to humidity and temperature.

Functions of core:

1. The core must be stiff enough in the out-of-plane direction of the plate so as to ensure that the faceplates remain distant apart.
2. It must be stiff enough in the in-plane shear so as to ensure the faces not to slide with respect to each other during bending. Generally, honeycomb cores are susceptible to in-plane shear because of low plane shear stiffness. Polymer and metallic foam cores show a better performance in this respect.
3. It must be stiff enough to keep the faces nearly flat; to avoid local buckling/wrinkling of the faceplates.

4. It must be able to absorb energy under dynamic loading, through the process of crushing/densification, i.e. a constant plateau stress over a large strain.

Functions of Adhesive:

1. The role of the adhesive is to bond the core and faceplate, so as to maintain the sandwich effect for the operating life of the sandwich construction and to transfer the shear stress between the faceplate and core. Hence, the shear strength of the adhesive is an important property for its selection.

1.1.2 Sandwich structure and its similarity with I-section

One can think of an analogy between a sandwich construction and the conventional I-section. Most of the in-plane tensile/compressive stresses resulting from loading are taken up by the faceplates of the sandwich structure (which is similar to flanges in case of I-section) and the shear stresses are carried by the core (which is similar to web in case of I-section). Hence, correlating the faceplates with flanges and the core with web of I-beam is quite reasonable. These similarities are apparent in Figure 1.3.

Figure 1.3: Similarities of Sandwich construction and I-section [4].

1.2 Effectiveness of sandwich plates over monolithic plates

Vinson [5] compared a monolithic plate of thickness $2t$ with a sandwich plate, where each faceplate is of thickness t and the ratio of the faceplate to the core thickness is $1/20$. With this geometry, the flexural stiffness is 300 times that of the monolithic plate. For a given bending moment, the bending stress in sandwich faceplate is $1/30$ th of that of the stress developed in monolithic plate. In Figure 1.4, the flexural strength, stiffness and weight of a monolithic plate and the sandwich construction are compared for a given size [1]. It can be seen that the strength

and stiffness are consistently increasing with a small increase in the overall weight of the sandwich structure. Pflug and Verpost [6] compared the cost of a monolithic plate with a sandwich

Figure 1.4: Effectiveness of sandwich construction over monolithic plate [1].

plate for a given bending stiffness, as shown in Figure 1.5. Increasing the distance between the

Figure 1.5: Weight and material cost saving with sandwich construction [6].

faceplates increases the moment of inertia of the cross-section about the centroid. This increased

moment of the inertia leads to increased torsional as well as bending rigidity of the sandwich structure. However, thick core sandwich plates are prone to localized failure, due to its low modulus and hence impose a constraint on its maximum thickness. With this brief introduction highlighting the benefits of sandwich structures, the discussion is now focused on to their evolution and its commercial acceptability.

1.3 Background and motivation

The design of new material alternatives for weight-sensitive applications and extreme working environments is a difficult task for an engineer. Most of the structural components should bear the applied loads without failure over the designed life at minimum weight. Out of the available materials, sandwich plates have been shown to be one of the choices over monolithic plates in many aspects, such as high specific strength and stiffness. Additionally, any weight-sensitive structural element should satisfy, other flexibilities including directionality of material orientation (resulting from orthotropic faceplates and core materials), thermal and noise insulation, and fire resistance. Sandwich structures have a wide variety of applications ranging from interior decoration to spacecraft. Some typical applications include exterior and interior walls, roofs, ceilings, bridges, automotive, energy-absorbing bumpers, aerospace components, wind turbines, railway components and space craft parts.

During world war II (1939-1945), a plywood sandwich construction was used in the Mosquito night bomber [5]. GFRP/honeycomb and GFRP/balsa sandwich structures were used by the Wright Patterson Air Force Base for the Vultee BT-15 fuselage. In 1969, the Apollo Capsule (first successful landed space ship) demonstrated the sandwich structures applicability to extreme loading and environmental conditions. A schematic diagram of sandwich construction used for Apollo capsule is shown in Figure 1.6. In transport vehicles, reduced design weight

Figure 1.6: Schematic representation of sandwich construction used in Apollo capsule [7].

leads to improved fuel efficiency with reduced operational costs. Various sandwich structures

in A380 aircraft are shown in Figure 1.7. After installation of sandwich structures as structural

Figure 1.7: Applications of sandwich structures in A380 from [2]. In figure V/HTP : Vertical/Horizontal tail plane.

elements, loading can be more generic. Sandwich panels should be designed for these generic loading conditions. In the context of the aircraft and ship building industries, typical loadings are transient vibrations, bird impact, aerodynamic forces, water sloshing, impact of debris from the runway during landing and takeoff, and tool drops. Transient loadings are more general and common loading conditions that occur in most of the applications. To combat these issues, failure modes and the extent of the failure can be predicted from a series of experiments on structural elements such as beams and plates in laboratories.

The failure in sandwich construction could be either in the faceplates or core depending upon its geometry, mechanical properties for a given loading condition by considering perfect bonding between faceplate and the core. For a typical honeycomb based sandwich construction, failure modes such as skin compression, panel buckling, shear crimping, skin wrinkling, intra cell wall buckling and indentation are shown in Figure 1.8. Typical failure modes in foam-cored sandwich beams under uniformly-distributed load are shown in Figure 1.9 from [8]. Failure modes depend on the type of the material, loading and geometrical constraints. Hence, exhaustive studies are needed to capture all of the possible failure modes for a given sandwich construction. Under quasi-static loading, researchers [8] have established design maps to predict presence of possible failure modes for a given loading and boundary condition. These plots are termed as *failure*

Figure 1.8: Types of failure modes in a honeycomb core [4].

mode maps.

Figure 1.9: Failure modes in foam cored sandwich beams under distributed loading [8].

The preceding discussion gives an overview of typical failure modes under quasi-static loading conditions. Another type of loading of interest in the present study is low velocity impact, in which the possible failure modes are mostly either top faceplate failure or core local indentation, depending on the impact energy and type of sandwich. Hence failure modes under low velocity impact is not discussed, rather attention is given to development of reliable FE models.

In occupant safety, peak load and energy absorption plays a major role. In specific, high energy absorption with smaller peak loads are the essential objectives of the protective/sacrificial structures. Hence, in the current research, low velocity impact response of stepwise graded core

sandwich plates is investigated so as to minimize the peak force with high energy absorption or maximize the peak load with low energy absorption.

This background is a motivation to understand the behavior of sandwich plates under low velocity impact and quasi-static indentation loading conditions.

1.4 Objectives

The major objectives of the current work are

1. To establish reliable finite element (FE) models for predicting the impact response (viz., load versus displacement, energy absorption capacity and failure modes) of sandwich plates and to investigate the low velocity impact response of graded core sandwich plates.
2. To investigate the quasi-static indentation response (viz., stiffness, failure initiation load and load versus displacement) of sandwich plates and to construct failure mode maps and to find the minimum weight designs for a given load-bearing capacity.

1.5 Scope of the present work

The literature is reviewed to understand the extent of the research on the energy-absorption characteristics of sandwich structures with either metal or polymeric foam cored sandwich structures with either metallic or composite faceplates. The effect of low velocity impact and quasi-static indentation loading conditions on sandwich plate are thoroughly investigated in the current work.

Reliable FE models are developed (in commercial FE package LS-DYNA) to predicting the low velocity impact response of foam-cored sandwich plates with either metallic (ductile) or composite (brittle) faceplates. Effects of different constitutive models and hourglass control formulations have been studied to simulate the impact response. Predicted failure modes and impact response (viz., load versus displacement, energy absorption) are validated with experimental measurements.

The relative performance of the polymeric and metallic foams is studied (either for a given density or yield strength of the foam) under low velocity impact loading conditions. Additionally, the effect of step wise core grading on the peak load and energy absorption characteristics are studied. The failure modes are thoroughly examined to understand the energy absorption characteristics. Finite element predictions and analytical estimate compared against experimental measurements.

Analytical formulations are developed for the indentation behavior of sandwich plates. Flat ended circular punch is considered as indenter. Preliminary FE simulations revealed that punch

is making contact along its periphery during the indentation processes. Hence for analytical treatment total load is applied as an axisymmetric line load around the punch circumference. Initially the core is considered as an elastic foundation and bounding estimate is given for the indentation failure load. Later, the core is considered as an elastic perfectly plastic foundation, which is the typical response of the many commercially-available foams. All of these formulations are done using the assumption of small deformation theory. However, to estimate the indentation response at large indentation depths, large deformation is considered for the faceplate with core being rigid perfectly plastic foundation.

Failure mode maps and minimum weight designs are developed to investigate the effect of geometric and material parameters on the type of failure mode for a given material system and load bearing capacity. These failure mode maps are constructed by providing the bounding solutions for the indentation, core shear and microbuckling/faceplate failure modes. The indentation failure load is derived using the plate on an elastic foundation analogy. For estimating the core shear failure, bounding solution is defined on the displacement whereas faceplate failure load is estimated using the strength of materials approach by assuming symmetric bending about the sandwich plates neutral axis.

1.6 Report layout

Following the introduction chapter, the report layout is divided into following chapters.

Pertinent literature on sandwich structures under static and impact loading conditions is reviewed in chapter 2. The existing gaps in the behavior of sandwich structures are clearly identified. Chapter 3 provides details about the FE modeling of Alporas foam-cored sandwich plates subjected to low velocity impact using the commercial FE code LS-DYNA. Then the relative performance of polymeric and metal foam-cored sandwich plates under low velocity impact loading is studied in chapter 4. Additionally, the effect of stepwise core grading on the impact response is also investigated. Chapter 5 gives the analytical models for estimating the load-displacement relations and indentation failure initiation load of circular composite sandwich plates under quasi-static indentation. Simplified analytical models are used to represent a failure mode map of clamped circular composite plates under bending in chapter 6. Extensive finite element parametric studies are carried out to investigate the influence of various parameters on the failure modes, along with limited experimental studies. Finally, in chapter 7, the major conclusions are summarized and scope for further research is detailed.

Chapter 2

Literature review

With the evolution of sandwich structures as successful structural elements over the past decade, numerous studies have been performed and reported in the literature. Current literature review is confined to the low velocity impact and quasi-static indentation loading conditions on the sandwich structures. This chapter initially discusses the general manufacturing processes of metal and polymeric foams followed by their mechanical characterization, different constitutive models and numerical modeling strategies. Subsequently literature on sandwich structures subjected to low velocity impact and quasi-static indentation is discussed.

2.1 Introduction to foams

Of the different core materials available, honeycomb cores and foams have gained much importance, due to their manufacturability. A detailed description of manufacturing processes, the state-of-the-art of honeycomb sandwich structures and their real-time applications are covered by Bitzer [9]. Recently different innovative open cell periodic core materials (lattice structures) have been developed by Wadley et al. [10]. Foams manufactured from either metals or polymers have many engineering applications, such as sound and energy-absorbing components, automobiles parts and sporting goods, due to their advantageous physical, mechanical, thermal, electrical and acoustic properties over bulk metals or polymers. Later in this section, metal foams and polymeric foams will be discussed separately, because of the distinct manufacturing properties involved.

Foams can be either closed cell or open cell, based on whether the pores are inter-connected or closed. It is not within the scope of present chapter to cover all varieties of foams. So, the discussion will be confined to core materials which have been used in present study, namely closed cell aluminum foam with trade name Alporas[®] from Shinko Wire Company Ltd., Japan and closed cell Divinycell[®] PVC foams from Diab Inc.. Before going into the details of char-

acterization and constitutive modeling, it is worthwhile to understand the foams manufacturing processes.

2.1.1 Metal foams

Metals can be foamed using a range of methods. A wide range of issues related to metal foams, such as manufacturing, characterization methods, properties, constitutive models and commercial aspects are discussed in Ashby et al. [11]. Foamable metals include magnesium, lead, zinc, copper alloy, titanium, steel and gold, of which only aluminum and nickel have found commercial application. Nine methods, along with possible foamable metals, are listed below in square brackets along with their commercial names using the processes in parenthesis [11]:

1. Melt gas injection or air bubbling - [magnesium and aluminum] - (Cymat and Hydro)
2. Gas-releasing particle decomposition in the melt - [aluminum] - (Alporas)
3. Gas-releasing particle decomposition in semisolid - [lead, zinc, copper, titanium] - (Alu-light)
4. Casting using a polymer or a wax precursor as template- [aluminum, magnesium, nickel-chromium alloys, stainless steel, copper] - (Duocel)
5. Metal deposition on cellular preforms - [nickel and titanium] - (Incofoam)
6. Entrapped gas expansion - [titanium]
7. Hollow sphere structures - [nickel, cobalt and nickel—chromium alloys]
8. Co-compaction or casting of two materials with one leachable material - [aluminum]
9. Gas-metal eutectic solidification - [Copper, Nickel and Aluminum].

Out of the nine foaming processes listed above, the first five methods have gained commercial acceptability. With the above introduction to the existing manufacturing methods, the discussion below will be confined to the closed-cell aluminum alloy Alporas foam, which will be used in the present investigation.

2.1.1.1 Manufacturing of Alporas foam

From the preceding discussion, Alporas foam is manufactured using a gas-releasing particle decomposition process. This process is described by Miyoshi et al. [12] and Ashby et al. [13] see Figure 2.1.

Aluminum foams manufacturing involves melting the aluminum at 680°C followed by increasing the melt viscosity by adding 1.5% calcium; which oxidizes and forms CaO and CaAl_2O_4 particles. A foaming agent, TiH_2 is then added to the viscous melt to produce hydrogen gas bubbles. The molten material then cured for about 15 min, during which melt expands and fills up the mold. Then the foamed molten material is cooled in the mold.

The relative density, and hence the cell size (0.5 mm to 5 mm) of the Alporas foam, depends on the volume fraction of calcium and titanium hydride (0.02 to 0.7) added to the melt.

Figure 2.1: Alporas foam manufacturing process [12].

2.1.2 Polymeric foams

Polymers can be classified into thermoset's and thermoplastic's. Another classification is possible as rigid, semi-rigid and flexible depending on the physical characteristics. Polyvinyl chloride (PVC) foam was the first thermoplastic foam commercialized during World War II, followed by the Polyurethane (PU) and Polystyrene (PS) foams [14].

The production of most foamed polymers involves the formation of gas bubbles, followed by their growth and stabilization in the polymer melt. Gas bubbles are formed by dispersion of gas using blowing/foaming agents, which can be chemical or physical in nature. Chemical blowing agents (CBA) produce gas during process of manufacturing, whereas physical blowing agents (PBA) are inert gases, such as argon, is pumped and stirred.

Most thermoset foams are prepared by the simultaneous occurrence of polymer formation and gas generation, i.e. instead of thermoset polymer, monomers are used as raw materials [15]. A mixture of CBA and thermoplastic is fed to the forming tool to foam thermoplastics [16]. A wide variety of thermoplastic and thermosetting foamed polymers are documented by Landrock [15]. The PU foam attracted much attention of researchers as it is available in wide range of densities, cell structures and rigidities. Mills [17] compiled the micromechanical and FE modeling of PU, expanded polystyrene (EPS) and expanded polypropylene (EPP) foams with several case

studies. A detailed discussion on all the existing polymeric foams is not in the scope of in the present discussion. Hence manufacturing of PVC foams, which are used in the current studies are detailed here.

2.1.2.1 Manufacturing of Divinycell PVC foam

PVC foams can be made by the same traditional processes that are used for making unfoamed PVC products. Traditional manufacturing methods for PVC foams are a) Free-foaming and b) the Celuka/inward-foaming method [18, 19].

Divinycell grade H is a semi-rigid PVC foam. It is a mixture of a thermoplastic and a thermoset, expanded to low density. The manufacture of PVC, as described by DIAB, is summarized as follows [1]:

Divinycell PVC foam is manufactured in three phases. During the first phase, the PVC is gelatinized and the cell structure is created through decomposition of the blowing agents. In the second phase, the foam is expanded, mainly through reaction between Methylene diphenyl-diisocyanate (MDI) and water forming CO₂. In the third phase, the Divinycell is cured through completion of all chemical reactions. After molding skin removal an average plate has a dimension of 2440 x 1220 x 70 mm. This typical plate is usually cut into thinner sheets according to customer requirements.

2.2 Mechanical characterization of foams

Mechanical characterization of foams constitute evaluation of their cell morphology, density, chemical composition, uniaxial tension and compression, triaxial compression, fracture properties (fracture initiation energy, fracture propagation energy and corresponding toughness values) and plastic Poisson's ratio. Basic characterization of foams involves uniaxial compression test, due to its use as an energy absorbing structure many situations, through local collapse. Quasi-static tension, compression and shear test protocols are well established and data is available in number of publications. Hence attention is not given to such details in the present discussion. However, properties like plastic Poisson's ratio and fracture energy are more challenging to measure. Hence literature on these properties is listed here.

2.2.1 Metal foams

A comprehensive design guide with metal foams was given by Ashby et al. [11]. The effect of foam morphology and imperfections on their properties has also been studied [11, 20, 21].

2.2.1.1 Plastic Poisson's ratio

The Plastic Poisson's ratio plays a major role in defining the yield surface for the constitutive modeling of foam (which will be discussed in section 2.3.1). During the compression response, plasticity constitutes successive collapse of the cell walls (with negligible lateral deformation as shown in Figure 2.2) makes the plastic Poisson's ratio as almost zero. However, during the tension response of the foams, stretching and bending of the cell walls causes the plastic Poisson's ratio to be of same magnitude as elastic Poisson's ratio value.

Figure 2.2: Cell wise collapse of Alporas foam of 14% relative density [22].

Few measured values of plastic Poisson's ratio are available in literature are reproduced here. In compression, the value of plastic Poisson's ratio for Alporas foam (with a 8% relative density) was measured by Gioux [23] as 0.024. However, in tension, Motz and Pippan [24] reported the values of plastic Poisson's ratio for 250 kg/m³ (relative density: 9.25%) and 400 kg/m³ (relative density: 14.8%) as 0.332 and 0.440, respectively.

2.2.1.2 Fracture energy

Fracture energy is needed to understand the behavior of foam to fracture initiation and propagation. ASTM E813-89 and ASTM E1152-87 are the accepted standards for measuring the fracture energy. The fracture initiation energy, J_{IC} or G_{IC} , can be related to the stress intensity factor, K_{IC} , using Irwin's relation, $K_{IC} = \sqrt{\bar{E}J_{IC}}$, hence it is common practice to measure the J_{IC} or G_{IC} experimentally. where $\bar{E}(= E_s/(1 - \nu_s^2))$ is the plane strain value of Young's modulus

Gibson and Ashby [25] proposed the stress intensity factory, K_{IC} for brittle open cell foams as

$$K_{IC} = 0.65 \sigma_s \sqrt{\pi l} \bar{\rho}^{1.5} \quad (2.1)$$

where, l is the cell size, $\bar{\rho}$ is relative density and σ_s is the fracture strength or yield strength of the cell wall material.

Sugimura et al. [20] characterized the properties of Alporas foam (with a relative density 8%) using a notched beam specimen under four point bending and suggested using the unloading compliance as an indicator of crack extension. Fracture initiation energy J_{IC} is measured as 600 J/m² and corresponding stress intensity factor, K_{IC} , (calculated from Irwin relation with $\bar{E}$ = 1 GPa) as 0.8 MPa√m. The expression for steady state fracture toughness, K_{ss} (in terms of cell wall material Young's modulus E_s and strength σ_s) is given as

$$K_{ss} \approx \sqrt{E_s \sigma_s} l \bar{\rho}^{1.5} \quad (2.2)$$

Olurin et al. [26] conducted compact tension tests and double edge notched tension tests to measure the fracture initiation toughness J_{IC} and steady state fracture propagation toughness K_{ss} for Alporas foams. Equation for J_{ss} was found to be

$$K_{ss} \approx \sqrt{C_1 C_2} \sqrt{E_s \sigma_y} l \bar{\rho}^{1.75} \quad (2.3)$$

where, C_1 is ≈ 1 and $C_2 = 0.3$. The work of fracture W_f (the area under the traction separation plot) in the double edge notched tension test is approximately equal to the steady state fracture toughness J_{ss} from compact tension test (see Figure 2.3).

Motz and Pippan [27] measured fracture toughness values in terms of the critical J -integral, K_{IC} , J_{IC} , and the critical crack-tip opening displacement, COD using compact tension test specimens.

The tearing energy, γ is a representative property of the foam fracture response. Olurin et al. [28] measured γ from foam indentation tests using a circular flat-ended punch (with different diameters) is measured to be 7.45 kJ/m² for Alporas foams. **Ramamurthy and Kumaran [29]** extracted shear strength and tear energy from indentation experiments using conical indentors (with different cone angles). Experimental observations from two methods [28, 29] are within the experimental scatter. Apart from static characterization works on metal foams discussed, dynamic characterization and high strain-rate behavior of Alporas foam was measured [30–32].

Figure 2.3: Normalized steady-state energy release rate J_{ss} with respect to foam density [26].

2.2.2 PVC foams

List of experimental protocols and typical material properties are found in [33, 34]. Plastic Poisson's ratio in compression for polymeric foams was measured to be nearly zero [35, 36].

2.2.2.1 Fracture

The fracture initiation energy G_{IC} or J_{IC} of plastic materials is measured using the ASTM standard D5045-93. Viana and Carlsson[37] measured and reported the fracture initiation energy G_{IC} or J_{IC} for H30, H80, H100 and H200 foams as 112 J/m², 186 J/m², 309 J/m² and 625 J/m², respectively, and the corresponding K_{IC} as 0.064 MPa $\sqrt{m}$, 0.117 MPa $\sqrt{m}$, 0.169 MPa $\sqrt{m}$ and 0.370 MPa $\sqrt{m}$, respectively.

Kabir et al. [38] conducted dynamic fracture tests on PVC and rigid polyurethane (PUR) foams (using Instron Dynatup-8210) on single edge-notched bend specimens (SENB). The effect of different loading rates (0.00254 mm/s, 0.254 mm/s and 2.54 mm/s) on fracture toughness K_{IC} was also studied. The fracture toughness in the *flow direction* of Divinycell H130 foam was 0.22 ± 0.01 MPa $\sqrt{m}$, 0.23 ± 0.02 MPa $\sqrt{m}$ and 0.24 ± 0.01 MPa $\sqrt{m}$ for loading rates of 0.0254 mm/s, 0.254 mm/s and 2.54 mm/sec, respectively. Similarly in the *rise direction* of Divinycell H130 foam, fracture toughness was reported as 0.28 ± 0.01 MPa $\sqrt{m}$, 0.27 ± 0.02 MPa $\sqrt{m}$ and 0.28 ± 0.01 MPa $\sqrt{m}$ for loading rates of 0.0254 mm/s, 0.254 mm/s and 2.54 mm/sec, respectively.

Saenz et al. [39] conducted SENB and double cantilever beam (DCB) tests on H45, H60 and H100 to measure the critical fracture energy, G_{IC} . G_{IC} for H45 foam from SENB test was

reported as 0.11 ± 0.01 , whereas that from test was 0.24 ± 0.03 . Similarly for H60, SENB test G_{IC} was reported to be 0.24 ± 0.01 , whereas that from DCB test was 0.38 ± 0.04 . Similarly for H100 foam, G_{IC} was reported as 0.43 ± 0.04 from SENB test, where as that from DCB test was 0.89 ± 0.05 .

Readers are directed to [40–44] for dynamic properties and high strain rate behavior of Di-vinycell PVC foams. In the succeeding section, numerical modeling of the foams is discussed.

2.3 Multi-axial behavior of foams

Reliable FE models can predict and capture the stress and deformation states in intricate parts, which may not be an easy task using either real time experiments or analytical models. Modeling of foams involve developing constitutive models using theory of plasticity followed by their numerical implementation. Current section is devoted to review the constitutive models available in literature.

Gibson et al. [45] constructed the yield or failure surface of a cuboidal open-cell foam under hydrostatic loading. Failure surfaces were plotted for axi-symmetric and bi-axial loading in non-dimensional stress space. Under multi axial loading, the yield condition is given as

$$\pm \frac{\sigma_d}{\sigma_c} + 0.81 \bar{\rho} \left(\frac{\sigma_m}{\sigma_c} \right)^2 = 1 \quad (2.4)$$

where σ_d is the deviatoric stress, $\sigma_m (= \sigma_i/3)$ is the mean stress, σ_c is the core yield strength, $\bar{\rho}$ is the relative density of the foam. The yield function given by Eq. (2.4) can be applied to axisymmetric loading condition ($\sigma_2 = \sigma_3$) as well as biaxial loading ($\sigma_3 = 0$). Under biaxial loading, the yield function show an elliptic surface with a maximum principal stress cap in tension-tension and compression-compression quadrant. Estimated yield locus is compared with experimental measurements for PU, PE and aluminum foams by Triantafillou et al. [46]. Considered foams showed a spherical envelope in compression-compression stress quadrant against the buckling cap governed by maximum principal stress criterion.

No distinction is given made between metallic or polymeric foams. However several authors developed the constitutive models solely for either metallic or polymeric foams as discussed in preceding section.

2.3.1 Metallic foams

Miller [47] modified the Drucker-Prager yield function that depends on the plastic Poisson's ratio and a quadratic dependence on mean stress as:

$$f = \sigma_e - \gamma \sigma_m + \frac{\alpha}{d} \sigma_m^2 - d \quad (2.5)$$

where the new variables γ , α , d are

$$\begin{aligned} \gamma &= \frac{6\beta^2 - 12\beta + 6 + 9(\beta^2 - 1)/(1 + \nu_p)}{2(1 + \beta)^2} \\ \alpha &= \frac{45 + 24\gamma - 4\gamma^2 + 4\nu_p(2 + \nu_p)(-9 + 6\gamma - \gamma^2)}{16(1 + \nu_p^2)} \\ d &= \frac{\sigma_c^-}{2} \left[1 - \frac{\gamma}{3} + \sqrt{\left(1 - \frac{\gamma}{3}\right)^2 + \frac{4\alpha}{9}} \right] \\ \beta &= \frac{\sigma_c^-}{\sigma_c^+} \end{aligned} \quad (2.6)$$

where, σ_e is the effective stress, σ_m is the mean pressure, σ_c^+ and σ_c^- are the tensile and compressive strengths of the foam, respectively; d is the uniaxial strength, γ and α are the variables accounting for the yield function linear and quadratic dependence on pressure, respectively. Hence, the calibration of this model requires (i) the compressive stress versus strain response, (ii) the ratio of compressive to tensile yield strength, and (iii) the plastic Poisson's ratio, ν_p . Proposed yield function is compared against Drucker-Prager yield function using sandwich plate indentation simulations in commercial FE package Abaqus.

Deshpande and Fleck [21] proposed two models namely a self-similar model and a differential hardening model of which self-similar yield surface model is widely used because of its simplicity. The self similar yield function is given as

$$f = \sqrt{\frac{1}{1 + (\alpha/3)^2}} (\sigma_e^2 + \alpha^2 \sigma_m^2) - \sigma_c \quad (2.7)$$

where the shape factor α is related to the plastic Poisson's ratio as

$$\alpha^2 = \frac{9}{2} \frac{1 - 2\nu_p}{1 + \nu_p} \quad (2.8)$$

Typical foams exhibit a plastic Poisson's ratio ν_p as zero, which gives $\alpha^2 = 9/2$. Unlike Miller [47], the self-similar yield surface model assumes uniaxial yield strength in tension and compression are equal. Hence, Miller [47] model is more generic in nature.

Gibson et al. [45], Miller [47] and Deshpande-Fleck [21] yield surfaces were compared with

the experimental data for Alporas by Gioux [23] (see Figure 2.4): Miller [47] and Deshpande-Fleck [21] yield surfaces fits well. Hanssen et al. [48] extended the Deshpande and Fleck [21] foam constitutive model to account for the statistical variation in the properties of the foam.

Figure 2.4: Comparison of various yield surfaces for an aluminum alloy Alporas foam [23].

2.3.2 Polymeric foams

Contributions towards the constitutive modeling of polymeric foams are discussed here.

Zhang et al. [49] developed a constitutive model for isotropic polymeric foam (on low-density polyurethane (PU), polystyrene (PS), and polypropylene (PP)) materials, which accounts for strain rate dependency and temperature effects. The yield locus is given as

$$f - \dot{f} = \frac{(\sigma_m - x_0)^2}{a} + \frac{\sigma_e^2}{b} - 1 = 0 \quad (2.9)$$

where f and $\dot{f}$ are the yield locus at quasi-static and high strain rate, x_0 , a and b are the material constants representing the origin, major and minor axis of the yield locus, which are functions of the plastic volumetric strain. These material constants are measured under uniaxial compression, hydrostatic compression and single lap-shear experiments. Strain rate effects are assumed to follow power law.

Deshpande and Fleck [35] have extended self-similar isotropic metallic foam model [21] to polymeric foams. It was found that, the yield function for Divinycell PVC foams is a quadratic of the form proposed by Deshpande and Fleck [21], capped by a maximum compressive principal stress criterion in the compression-compression quadrant as shown in Figure 2.5.

Figure 2.5: Yield surface for Divinycell PVC foams [35]. In figure solid line represents yield surface of Deshpande and Fleck [21] where as dashed line represents buckling cap (maximum compressive stress criterion).

Gdoutos et al. [36] conducted multi axial tests on H100 and H250 Divinycell PVC foams and compared their experimentally measured failure surface with that of Tsai-Wu 2D failure criterion. The comparison was well within the experimental scatter.

Gielen [50] developed a constitutive model for PVC foams that exhibit elastic behavior until failure under tension and elastic perfectly plastic behavior under compression. This attempt accounts for simultaneous tension and compression response.

2.4 Numerical modeling of foams

Most sandwich constructions have balsa wood, honeycombs, metallic or polymeric foams as cores, and strong and stiff metallic or composites as faceplates. Metallic honeycomb cores can be modeled on an individual cell basis using conventional J_2 metal plasticity constitutive models can effectively represent the honeycomb cell wall behavior [51]. Foams can be modelled at a microscopic scale by considering the individual cell walls and edges with cell wall material properties, whereas at a macroscopic level they are modelled as homogenous material with their respective properties. To give an overview of both modeling techniques, a few contributions are discussed here. In addition to constitutive models, failure/erosion criterion also plays a major role in numerical prediction of the failure mode. For macro/homogenized models, material erosion criterion is essential in predicting failure. In this section the discussion is devoted to different FE modeling strategies to numerically represent foam.

2.4.1 Micro-modeling of foams

A typical micro-modeling procedure consists of the capturing micro-structure of foams from micro-computed tomography (CT) scanner, filtering the images and reconstructing the images of the micro-CT to obtain the geometry for discretization in FE analysis as shown in Figure 2.6. Another method to micro-model foam involves a representative volume element (RVE) or statistically homogenous specimen. Numerical models of open-cell foams are created by modeling the cell edges with beam elements, whereas closed-cell foams can be modeled with shell and beam elements along the cell edges [52]. Microscopic models are difficult to construct, discretize and are computationally expensive.

Figure 2.6: Typical micro-models of foam.

2.4.2 Macro modeling of foams

Homogenized models were implemented and discussion is limited to homogenized material models available in commercial FE codes Abaqus and LS-DYNA.

ABAQUS implements the Deshpande and Fleck [21] constitutive model as `*CRUSHABLE_FOAM` model for polymeric/metallic foams with the choice of either isotropic or volumetric hardening. Hardening data can be incorporated using appropriate uniaxial or volumetric compression data. Additional user-subroutines are required to specify the damage initiation and propagation.

LS-DYNA [54] has MAT_HONEYCOMB (MAT_26), MAT_CRUSHABLE_FOAM (MAT_63), MAT_BILKHU/DUBIOS_FOAM (MAT_75) and MAT_MODIFIED_HONEYCOMB (MAT_126) as suitable material models for structural foams. Their yield functions and hardening laws were compared by Hanssen et al. [48]. Deshpande and Fleck [21] is available as MAT_DESHPANDE_FLECK_FOAM (MAT_154) foam model in LS-DYNA as programmed by Reyes et al. [55].

For a failure analysis it is necessary to have suitable failure/erosion criterion. In honeycomb cores, erosion can be incorporated using cell wall material fracture strain [51]. However, incorporating damage in homogenized foam models requires the selection of suitable erosion criterion. Reyes et al. [55] implemented different types of fracture criteria (viz., volumetric plastic strain criterion, and maximum principal stress criterion with accompanying Cockcroft and Latham's energy-based criterion to avoid premature element deletion) using the model proposed by Hanssen et al. [48] and concluded that the maximum principal stress criterion was able to predict the correct failure mode as compared to the maximum plastic volumetric strain criteria in quasi-static indentation simulations.

Lu et al. [56] performed indentation and impact simulations on Al alloy CYMAT foams resting on a rigid base using the commercial FE software ABAQUS by defining a cohesive layer in the foam to identify the damage. The mean diameter of the cohesive layer is equal to indenter diameter. Ivañez et al. [57, 58] simulated low and high velocity impact of PVC foam cored sandwich beams and plates using ABAQUS software. Failure of foam was modeled using user subroutine VUMAT with shear failure criterion. It can be observed from their work that the erosion surface of the foam is not smooth and elements along the erosion surface are distorted.

In summary, two methods (cohesive behavior [56] and material erosion [55, 58]) have been implemented in the literature to incorporate the failure of foam in sandwich plates. Under generic loading conditions, it is not possible to predict failure using cohesive zone modeling as the failure path is not known a priori to define the cohesive law. However, the material erosion criterion is the reliable choice.

2.5 Low velocity impact of sandwich plates

In this section, low velocity impact on sandwich structures is discussed with specific contributions on experimental as well as analytical modeling of the force-time and displacement-time response. A rigorous discussion is devoted to analytical modeling of low velocity impact.

2.5.1 Experimental studies

Two typical low speed impact apparatus are drop-weight (as shown in Figure 2.7a) and pendulum impact (Figure 2.7b); of which the drop-weight system is widely used. In the literature, researchers have conducted low velocity impact experiments on a wide variety of sandwich constructions including Graphite-epoxy/aluminum foam, Nomex honeycomb or Rohacell foam [59], woven GFRP/Coremat or honeycomb core [60], Graphite-epoxy/Nomex honeycomb [61], GFPP /Alporas [62], woven CFRP/Divinycell PVC foam [63], GFRP/aluminum honeycomb [64], hybrid stainless steel assembly (HSSA) plates [65], plain weave CFRP/Nomex honeycomb [66], aluminum 1100-H14/aluminum alloy 3003-H19 honeycomb [51], CFRP/Alporas foam [67], aluminum alloy AlMn1/Aluight AlSi10 foam [68], woven GFRP/PVC foam [69] and aluminum alloy AA5754-H32/aluminum alloy AA5052 honeycomb [70]. In all of these works, a spherical-ended impactor on either a square [59–63, 66] or a circular [51, 64, 65, 67–70] plate geometry is used. Typical failure modes involve delamination in the faceplates, fiber breakage, core crushing, debonding between core and faceplate and permanent core indentation. Typical damage initiation, propagation stages are shown in Figure 2.8.

Figure 2.7: Typical low velocity impact apparatus [61].

From the preceding discussion, it is evident that a wide variety of sandwich structures have been characterized for their low velocity impact response. However, it is important to investigate the relative performance of sandwich structures so as to minimize the weight of the sandwich panel for a given energy absorption capacity or to maximize the energy absorption for a given

Figure 2.8: stages of damage in low velocity impact of sandwich plates [66].

mass. A limited number of attempts were made in literature towards this objective. Relative performance of sandwich plates with Alporas[®] and Divinycell[®] PVC foams was studied by Compston et al. [71]. Faceplates for Alporas and PVC foam sandwich plates were constructed using lamina of 754 gsm (plain weave glass fiber/polypropylene prepreg) and 450 gsm (plain weave glass fiber/vinyl ester resin matrix using wet lay-up), respectively. However, the overall fiber content in each sandwich structure was maintained approximately consistent and the authors concluded that PVC foam sandwich plates have smaller overall deflection against Alporas sandwich plates. Relative performance of motorcycle helmet with aluminum alloy foam (Alu-light) shell against ABS polymeric shell was studied by Pinnoji et al. [72] and concluded that the resultant force on the head is lower in case of metal foam helmet compared to ABS polymeric helmet. Crupi et al. [68] investigated the relative performance of the GFRP/PVC foam

(75 kg/m³) sandwich plate against two types of aluminum alloy sandwich plates (from Schunk GmbH and Alulight GmbH) and concluded that PVC foam sandwich plates absorb higher energies compared to aluminum alloy sandwich panels. However, in the reported literature for relative performance evaluation; the sandwich panels have different types of faceplate materials. Hence, in current investigation, the relative performance of graded core sandwich structures with the same faceplates is investigated.

In contrast to the conventional sandwich construction with a single core layer, several layers of foams are often used in the recent literature to increase the energy absorption capacity. Zeng et al. [73] investigated the perforation of two core varieties (viz., monotonically increasing or decreasing density) in a sandwich plate using four different densities of polyepoxide hollow spheres layers. The authors concluded that increasing density configuration offers a higher energy absorptions with a low damage initiation force against decreasing density configuration. Yang and Qiao [74] have used two layers of aluminum honeycomb core sandwich structures to protect highway bridge girders under low velocity impact loading. Gardner et al. [75] used stepwise graded core with two, three, and four layers of foam core gradation to monotonically increase the acoustic wave impedance of sandwich plate in testing the blast resistance of sandwich structures using shock-tube apparatus. Experimental results showed that the monotonic increase in number of graded layers increases the blast resistance of the structure. To the best of author's knowledge, a comprehensive relative performance evaluation and failure modes comparison between PVC (Divinycell H grade) and aluminum alloy foam (Alporas) and effect of core grading have not been reported in literature. Hence, a detailed evaluation of the relative performance and effect of core grading of Alporas Al alloy and Divinycell PVC foams under low velocity impact is also the focus of the present work.

2.5.2 Analytical modeling of impact response

Expensive experimental methodologies can be replaced by approximate analytical models for design purposes. During the analytical treatment of impact behavior of sandwich structures, no distinction is made regarding the type of the core (viz., foam, honeycomb) due to the assumption of constant reactive pressure to represent the core deformation. An overview of the different analytical formulations related to sandwich beams and plates subjected to low velocity impact is given by Abrate [76]. Analytical estimations of the low velocity impact response consists of load-displacement relationships and the peak force. Peak force is estimated using the energy-balance model by [77]. In low velocity impact, strain rate and wave propagation effects are negligibly small. Hence quasi-static indentation laws are applicable to estimate the low velocity impact force [78]. Three different strategies to estimate the load-displacement relation are spring mass models, semi analytical models and higher order plate theories as discussed below:

2.5.2.1 Spring mass models

A two degree spring mass model for circular composite laminates was proposed by Shivakumar et al. [79] and to composite sandwich plate (supported at the edges) by Abrate [80] as shown in Figure 2.9. If the sandwich plate is resting on a rigid backing, then the global deflection of the sandwich plate w_0 becomes zero (due to absence of plate deflection) and hence system becomes a single degree of freedom system.

Figure 2.9: Two degree of freedom spring element model for a sandwich plate supported at the edges. In figure k_c : contact stiffness, k_b : bending stiffness, k_s : shear stiffness, k_m : membrane stiffness, m_{sw} : equivalent mass of the sandwich plate, M : mass of the impactor, w_0 : sandwich plate deflection and δ_0 : displacement of the impactor.

The bending stiffness k_b is estimated according to classical plate theory [81], the shear stiffness k_s is estimated using shear deformation plate theory [82, 83], the membrane stiffness k_m is estimated according to large deformation plate theories [84] and k_c is evaluated from the quasi-static indentation load-displacement relation. The Meyer's law provides the nonlinear load-displacement relation as $P = k_c \delta_0^{3/2}$. Assuming rigid impactor, contact stiffness is given by

$$k_c = \frac{4 E_f \sqrt{a}}{3 (1 - \nu_f^2)} \quad (2.10)$$

where E_f and ν_f are the Young's modulus and Poisson's ratio of the faceplate and a is the radius of the impactor. Abrate [80] attempted to provide the peak load estimates for a given impactor mass and impact energy using energy balance model. However explicit expressions for the contact stiffness terms k_b and k_s are not given in [80].

Hoo Fatt and Park [85] modeled the low velocity impact response of rectangular composite sandwich plates resting on rigid base and with different plate boundary conditions. The core

was considered as rigid perfectly plastic foundation (RPP) while the faceplates were considered either as plate (with only bending rigidity) or as membrane. Sandwich plate on rigid support can be represented as a spring mass system as shown in Figure 2.10a with arbitrary boundary conditions along the plate edges shown in Figure 2.10b.

Figure 2.10: Spring mass models for low velocity impact considered by Fatt and Park [85]. (a) single degree of freedom system and (b) two degree of freedom system. In figure a: is the impactor radius, k_{sw} : sandwich plate stiffness and σ_c is the core yield strength in compression.

The governing differential equations for the spring mass system shown in Figure 2.10a can be:

$$M \ddot{\delta}_0 + P + \pi a^2 \sigma_c = 0 \quad (2.11)$$

where the load P is given as

$$\text{for plate analogy :} \quad P = \frac{32}{15} \sqrt{2 D_f \sigma_c} \delta_0 \quad (2.12)$$

$$\text{for membrane analogy :} \quad P = \frac{8 \sqrt{A_f \sigma_c}}{3} \delta_0^{3/2} \quad (2.13)$$

where D_f and A_f are the bending rigidity and stretching stiffness of the faceplate, respectively. The governing differential equation Eq. (2.11) in conjunction with either Eq. (2.12) or Eq. (2.13) becomes sole function of variable δ_0 and hence solved numerically.

The spring mass system shown in Figure 2.10b has the following governing differential equa-

tions:

$$\begin{aligned}(M + m_{sw}) (\ddot{\delta}_0 + \ddot{w}_0) + P + \pi a^2 \sigma_c &= 0 \\ m_{sw} \ddot{w}_0 + k_{sw} w_0 - P - \pi a^2 \sigma_c &= 0\end{aligned}\quad (2.14)$$

where, $m_{sw} = 0.09 \rho_{sw} L^3 (c + 2t)$, ρ_{sw} is the density of the sandwich plate, L is the width of the sandwich plate and w_0 is the amplitude of the sandwich plate global deflection. The Eqs. (2.14) are coupled and can not be solved analytically. Hence, the nonlinear contact relations as given in Eqs. (2.12) and (2.13) are assumed to have a linear relation as $P = k_{1d} \delta_0$ to decouple the differential equations.

Olsson [86] gave analytical models for the large mass-low velocity impact on sandwich plates by considering the local core crushing and delamination. Spring mass system considered is the same as that shown in Figure 2.10b. For typical sandwich structures under indentation, Hertzian indentation of top faceplate is negligible leading to linear relation as $P = k_c \delta_0$ against $P = k_c \delta_0^{3/2}$. This is due to a simultaneous softening action due to core yielding and stiffening due to stretching of the faceplate. In large mass impact, the sandwich plate deforms in a similar fashion to that under quasi-static loading. If the mass of the sandwich plate is greater than two times the impactor mass, then the equivalent mass of the sandwich plate m_{sw} can be neglected. Load versus time, bottom faceplate displacement versus time relations are given as:

$$P(\tau) = v_0 \sqrt{M k_{eq}} \sin\left(\frac{\pi \tau}{\tau_i}\right); \quad w(\tau) = \frac{v_0 \sqrt{M k_{eq}}}{k_{bs}} \sin\left(\frac{\pi \tau}{\tau_i}\right) \quad (2.15)$$

where

$$\begin{aligned}\tau_i &= \pi \sqrt{\frac{M}{k_{eq}}}; & \frac{1}{k_{eq}} &= \frac{1}{k_c} + \frac{1}{k_b} + \frac{1}{k_s} \\ \frac{1}{k_{bs}} &= \frac{1}{k_b} + \frac{1}{k_s}; & k_c &= 8\sqrt{k D_f^*} \\ k_s &= S^* 1.467 \left(1 + \frac{\bar{A}}{10.12}\right) \\ k_b &= \frac{138 D^*}{4 L^2} \left(1 + \frac{0.29}{\bar{A}^5}\right) \text{ for a clamped plate} \\ &= \frac{59 D^*}{4 L^2} \left(1 + \frac{0.48}{\bar{A}^4}\right) \text{ for a simply supported plate}\end{aligned}\quad (2.16)$$

where $\bar{A}$ is the ratio of length to width of the sandwich plate, D^* and D_f^* are the effective bending rigidity of the orthotropic sandwich plate and orthotropic faceplate, respectively, S^* is the effective shear stiffness of the orthotropic sandwich plate, v_0 is the initial velocity of the

impactor, τ is the time and k is the foundation stiffness.

Zhou and Stronge [65] considered low velocity impact response of a hybrid stainless steel assembly, HSSA plates using a two degree of freedom spring mass model (as in Figure 2.9) to calculate the indentation law and hence the contact stiffness. The corresponding two DOF governing differential equations are

$$\begin{aligned} M \ddot{\delta}_0 + P &= 0 \\ m_{sw} \ddot{w}_0 + k_{bs} w_0 + k_m w_0^3 - P &= 0 \end{aligned} \quad (2.17)$$

where

$$\begin{aligned} P &= \chi \sqrt{(\delta_0 - w_0) \left[1 + \beta (\delta_0 - w_0)^2 \right]}; & m_{sw} &= \frac{\pi \rho_{sw} R^2 (7 v_f^2 + 56 v_f + 121)}{54 (v_f + 3)^2} \\ k_b &= \frac{16 \pi D_{eq}}{(1 - v_f) (3 + v_f) R^2}; & k_s &= \frac{4 \pi G_c (c + t)^2}{c [1 + 2 \ln(R/R_c)]} \\ \frac{1}{k_{bs}} &= \frac{1}{k_b} + \frac{1}{k_s}; & R_c &= \sqrt{\delta_{0m} a} \\ D_{eq} &= \frac{1}{2} E_f t (c + t)^2 + \frac{E_f t^3}{6}; & \chi &= \frac{16 \pi}{3} \sqrt{D_f \sigma_c} \\ \beta &= \frac{0.488}{t^2} & \delta &= \delta_0 \left(1 - \frac{r^2}{a^2} \right)^2 \\ k_m &= \frac{2 \pi E_f t}{(3 + v_f)^4} \left[\frac{191}{648} (1 + v_f)^4 + \frac{41}{27} (1 + v_f)^3 + \frac{32}{9} (1 + v_f)^2 + \frac{40}{9} (1 + v_f) + \frac{8}{3} \right] \end{aligned} \quad (2.18)$$

where R_c is the contact radius, ρ_{sw} is the planar density of the sandwich plate and is equal to $(2 \rho_f t + c \rho_c)$, ρ_f is the density of faceplates and ρ_c is the density of core. The equivalent mass of the sandwich plate m_{sw} is found close to one-fourth of the total panel mass for a v_f value of 0.3 with negligible shear contributions. The set of coupled differential equations Eq. (2.17) are solved numerically.

Malekzadeh et al. [87] modeled the impact behavior (load versus time and displacement versus time) using two different systems consisting of (a) spring-mass-damper (SMD) model for an elastic core and (b) a spring-mass-damper-dashpot (SMDD) model for a rigid-perfectly plastic core as shown in Figure 2.11. The faceplates are assumed to move relative to each other and hence the equivalent stiffness of the faceplate K_{face} is fixed to the datum. To estimate the displacements, strains and stresses in the core and faceplates an improved higher order sandwich plate theory (IHSAPT) (proposed by Malekzadeh et al. [88]) was used. IHSAPT is an improved version of the higher order sandwich plate theory (HSAPT) proposed by Frostig and Thom-

sen [89]. The improvements in IHSAPT includes first order shear deformation to the faceplates along with nonlinear transverse acceleration in the core.

Figure 2.11: Spring mass models used by Malekzadeh et al. [87]. (a) SMD model for an elastic core (b) a SMDD model for a rigid-perfectly plastic core. K_c : contact stiffness, K_{core} , K_{face} , and K_{sand} : core, faceplate, and sandwich plate stiffness, respectively; δ_0 , w_t , and w_s : displacement of the impactor, top faceplate and sandwich plate, respectively; and M_1 , M_f and M_{sand} : mass of the impactor, top faceplate and sandwich plate, respectively.

Foo et al. [90, 91] developed energy-balance models to estimate the load versus time and displacement versus time histories for the impact response of honeycomb core sandwich plates. Energy contributions from bending, shear and contact are considered. The governing equations were derived using law of conservation energy and momentum as

$$\begin{aligned}
 U_c + U_{bs} &= \frac{P^2}{2 k_{sw}} = \frac{1}{2} M [v_0^2 - v(\tau)^2] \\
 M [v_0 - v(\tau)] &= \int_0^\tau P d\tau
 \end{aligned} \tag{2.19}$$

where

$$\begin{aligned}
 U_c &= \frac{P^2}{2 k_c}; & U_{bs} &= \frac{P^2}{2 k_{sw}} \\
 k_{sw} &= \frac{k_c k_{bs}}{k_c + k_{bs}}; & k_{bs} &= \frac{k_b k_s}{k_b + k_s} \\
 k_c &= \left. \frac{dP}{d\delta_0} \right|_{\delta_0=0.1mm}; & k_b &= \frac{16 \pi D_{eq}}{(1 - \nu_f^2) a^2} \\
 k_s &= \frac{4\pi G_c (c+t)^2}{c [1 + 2\ln(a/R_c)]}; & D_{eq} &= \frac{1}{2} \frac{E_f t (c+t)^2}{2} + \frac{1}{6} E_f t^3
 \end{aligned} \tag{2.20}$$

where, U_c and U_{bs} are the strain energies from the contact and equivalent bending-shear energies of the sandwich plate. The contact radius of the indenter, R_c which depends on the impact force P , is needed to evaluate the shear stiffness k_s . It is known that R_c depends on impact force, P . However, the authors concluded that the shear stiffness k_s does not depend on R_c . Hence, an average value of 2.2 was used for the function $\ln(a/R_c)$ when R_c range from 0 to a .

2.5.2.2 Semi-empirical method

Anderson [92] analyzed the large mass-low energy impact case in a semi empirical manner. Spring mass system by considering core as elastic foundation (as shown in Figure 2.12a) is found to show stiff response compared to experimental measurements. This discrepancy is considered due to the degradation of material with impact. Hence, to incorporate material damage, dissipation elements, i.e. a damper is added into the spring mass system. A variety of spring mass systems viz., (a) velocity proportional damper (Voigt model) (b) displacement proportional damper (c) complex stiffness damper and (d) velocity proportional damper in series with spring (Maxwell model) as shown in Figures. 2.12b-2.12e, respectively, are considered.

Figure 2.12: Spring mass models considered by Anderson [92]. In figure (a) Nonlinear elastic, (b) velocity damping (Voigt model), (c) displacement damping (d) complex stiffness damping and (e) Maxwell model.

The overall equations of motion for the systems in Figure 2.12 are represented as:

$$\text{No damping :} \quad M \ddot{\delta}_0 + k_c \delta_0^n = 0 \quad (2.21a)$$

$$\text{Velocity proportional damping :} \quad M \ddot{\delta}_0 + \eta \dot{\delta}_0 + k_c \delta_0^n = 0 \quad (2.21b)$$

$$\text{Displacement proportional damping :} \quad M \ddot{\delta}_0 + d \operatorname{sgn}(\dot{\delta}_0) \delta_0 + k_c \delta_0^n = 0 \quad (2.21c)$$

$$\text{Complex stiffness damping :} \quad M \ddot{\delta}_0 + k^* (1 + \eta i) \delta_0^n = 0 \quad (2.21d)$$

$$\text{Maxwell's model :} \quad M \ddot{\delta}_0 = P; \quad \delta_0 - \frac{P}{c} - \frac{P}{n P} \left(\frac{P}{k_c} \right)^{1/n} \quad (2.21e)$$

where

$$\operatorname{sgn}(\dot{\delta}_0) = \begin{cases} -1 & \text{if } \dot{\delta}_0 < 0, \\ 0 & \text{if } \dot{\delta}_0 = 0, \\ 1 & \text{if } \dot{\delta}_0 > 0. \end{cases} \quad (2.22)$$

where c , d and η are the velocity, displacement and complex stiffness proportional damping parameters, respectively, k_c is the contact stiffness, δ_0 is the impactor displacement, M is the mass of the impactor, k^* is the complex stiffness value, $i = \sqrt{-1}$ and 'sgn' is the signum function. The parameters c , d , η and k_c were empirically evaluated from FE predictions. All the differential equations were integrated numerically using Runge-Kutta method.

Of all the proposed models, Maxwell's model (shown in Figure 2.12e) is found to yield physically-reasonable load-displacement histories. The deficiencies in the other models to incorporate degradation are as follows:

- Velocity proportional damping or Voigt model (Figure 2.12b): Having velocity dissipation in the spring mass system, initial velocity, v_0 to the impactor leads to a non-zero dissipation at time, $\tau = 0$. Since, the dissipation in the spring-mass system from the damper depends on the velocity. This non zero dissipation at $\tau = 0$ gives a non-zero reaction force at the start of the impact and is not apparent in the experimental observations.
- Displacement proportional damping (Figure 2.12c): Due to presence of the 'sgn' function, once the force reaches the maximum value, a change in sign of $\dot{w}_0$ leads physically unreasonable estimates for the impact force. Hence this model cannot predict the post-peak response.
- Complex stiffness model (Figure 2.12d): Having complex variable i , leads to difficulties in interpreting the real and complex portion of the impact force and hence is discouraged.

2.5.2.3 Higher order sandwich plate theories

Lee et al. [93] proposed a refined sandwich plate theory where, the faceplates are modeled as Mindlin's first order plates, while the core has a shear as well as a through-thickness stiffness. This allows one to predict the relative motion between the two faceplates and hence the transverse displacement function w is a function of through thickness co-ordinate. Typical displacement functions for the faceplates and the core are:

Faceplates:

$$\begin{aligned} u^{(i)}(x, y) &= u_0^{(i)}(x, y) - z^{(i)} \phi_x^{(i)}(x, y) \\ v^{(i)}(x, y) &= v_0^{(i)}(x, y) - z^{(i)} \phi_y^{(i)}(x, y) \\ w^{(i)}(x, y) &= w_0^{(i)}(x, y) \\ i &= 1, 2 \end{aligned} \quad (2.23)$$

Core:

$$\begin{aligned} u_c &= z_c \left[\frac{u_0^{(2)} - u_0^{(1)}}{c} + t \frac{\phi_x^{(2)} + \phi_x^{(1)}}{2c} \right] + \left[\frac{u_0^{(2)} + u_0^{(1)}}{2} + t \frac{\phi_x^{(2)} - \phi_x^{(1)}}{4} \right] \\ v_c &= z_c \left[\frac{v_0^{(2)} - v_0^{(1)}}{c} + t \frac{\phi_y^{(2)} + \phi_y^{(1)}}{2c} \right] + \left[\frac{v_0^{(2)} + v_0^{(1)}}{2} + t \frac{\phi_y^{(2)} - \phi_y^{(1)}}{4} \right] \\ w_c &= z_c \left[\frac{w_0^{(2)} - w_0^{(1)}}{c} + \frac{w_0^{(2)} + w_0^{(1)}}{2} \right] \end{aligned} \quad (2.24)$$

Using the principal of virtual work, a system of discretized finite element equations are developed for the assumed displacement field given by Eqs.(2.23) and (2.24). Newmark's integration method was used to solve the discretized finite element equations. However, the necessary indentation laws were empirically found from the quasi-static indentation experiments.

Yang and Qiao [94] investigated an orthotropic sandwich plate resting on a rigid base subjected to low velocity impact. Faceplates were considered as a first order shear deformable plates while the core is considered as a two parameter elastic foundation as shown in Figure 2.14.

The core behavior is governed by

$$\sigma_{zz}(x, y) = K_{zz} w(x, y); \quad \tau_{zx}(x, y) = K_{zx} v(x, y, -\frac{t}{2}); \quad \tau_{zy}(x, y) = K_{zy} u(x, y, -\frac{t}{2}) \quad (2.25)$$

$$K_{zz} = \frac{E_c}{c}; \quad K_{yz} = \frac{G_{yzc}}{c}; \quad K_{xz} = \frac{G_{xzc}}{c}; \quad (2.26)$$

Figure 2.13: Shear deformable sandwich plate under bending [93]

Figure 2.14: Sandwich plate with two parameter foundation [94].

while transverse deflection function is expressed as

$$w(x, y, z) = w(x, y) \frac{1}{\sinh(\gamma)} \sinh \left[\gamma \left(1 - \frac{z}{c} \right) \right] \quad (2.27)$$

where γ is an arbitrary constant that controls the through-thickness core displacement variations. Displacement functions for the shear deformable plate is similar to Eq. (2.23). Five simultaneous

governing differential equations are given to define the complete dynamic impact behavior. To solve five simultaneous differential equations, five unknown functions u_0 , v_0 , w , ϕ_x and ϕ_y are represented in terms of Fourier series as

$$u_0(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad (2.28a)$$

$$v_0(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn}(t) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \quad (2.28b)$$

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn}(t) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad (2.28c)$$

$$\phi_x(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{mn}(t) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad (2.28d)$$

$$\phi_y(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} E_{mn}(t) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \quad (2.28e)$$

Unknown coefficients A_{mn} , B_{mn} , C_{mn} , D_{mn} and E_{mn} are evaluated by substituting the expressions in the set of five differential equations. Assumed functions implicitly satisfy the simply supported boundary conditions. Having all the unknown functions in terms of Fourier series, the reaction force offered on to the punch also expressed in terms of Fourier series. Resulting system of equations are solved simultaneous for unknown coefficients.

Qiao and Yang [95] solved the impact of the sandwich plate resting on elastic half-space and hence the bottom faceplate is assumed to make a smooth contact with the elastic half space. The core is assumed to be an elastic anti-plane core. First order shear deformable plate theory was considered for faceplate and bending along with its stretching effects in the faceplates are accounted. Using the Ritz method. The governing equations of motion are written in matrix form and solved using the Lagrangian method. However, explicit expressions for the mass, stiffness matrices and force vectors are not provided.

2.6 Quasi-static indentation of sandwich structures

Indentation of sandwich plates is analytically modeled using a plate on elastic foundation. The literature is reviewed in this section with a focus on the analytical modeling of indentation behavior.

When a sandwich plate (either rectangular or circular geometry) with appropriate (simply

supported or clamped) boundary conditions subjected to localized loads, it undergoes localized deformation (deflection caused by core indentation) as well as global deformation (deflection caused from overall bending rigidity and shear stiffness). Without the loss of much accuracy, coupling between localized and global deformations can be neglected in analytical formulations. For modeling the local indentation, far field boundary conditions of sandwich plate has no effect. However, global deformation depends on the boundary conditions at the plate edges. Global deflection is obtained using the equivalent sandwich plate properties with the use of classical sandwich plate theories [83, 96, 97]. Herein analytical formulations for predicting the local indentation are discussed. Local indentation is solved as a thin faceplate resting on elastic foundation (either Winkler or Pasternak foundation). However, rigorousness of the formulation can be greatly improved by considering core as either elastic or rigid perfectly plastic foundation.

Thomsen [98] estimated the local indentation behavior of a simply-supported rectangular sandwich plate consisting of isotropic core and orthotropic faceplate. The faceplates are assumed to deform according to classical plate theory while the core is considered as a two parameter elastic foundation as shown in Figure 2.14. Core is represented by three stress components as given by Eq. (2.25) while the kinematic equations for the faceplate according to the classical plate theory are given as

$$u = u_0 - z \frac{dw}{dx}; \quad v = v_0 - z \frac{dw}{dy}; \quad w = w_0 \quad (2.29)$$

Stress components σ_{zz} , τ_{xz} and τ_{yz} are considered as primary unknown functions and expressed as double Fourier series as

$$\sigma_{zz}(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\left(\frac{m \pi x}{a}\right) \sin\left(\frac{n \pi y}{b}\right) \quad (2.30a)$$

$$\tau_{xz}(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \cos\left(\frac{m \pi x}{a}\right) \sin\left(\frac{n \pi y}{b}\right) \quad (2.30b)$$

$$\tau_{yz}(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin\left(\frac{m \pi x}{a}\right) \cos\left(\frac{n \pi y}{b}\right) \quad (2.30c)$$

Similarly, external surface loads q_x , q_y and q_z are also expressed as double Fourier series. By using differential element shown in Figure 2.15, the three equilibrium equations governing the unknown coefficients A_{mn} , B_{mn} and C_{mn} are developed. Having the expressions for unknown coefficients, the stress state in the core is estimated using Eq. 2.30c and displacements using Eq. 2.25. This estimates the local deformation. Overall global deformation is found using the

classical sandwich plate theory [96].

Figure 2.15: Rectangular plate under consideration by Thomsen [98].

Thomsen [99], derived analytical solution for indentation of circular sandwich plate with isotropic faceplate and isotropic core. Sandwich plate was considered as a plate resting on two-parameter elastic foundation model. Since, the face and core are isotropic, to make the solution axisymmetric, external surface loads ($q_r(r)$ and $q_z(r)$) are also considered to be axisymmetric. For an axisymmetric circular plate on elastic foundation, Thomsen [99] expressed interface normal stress σ_{zz} and interface shear stress τ_{rz} in terms of foundation stiffness (K_r and K_z) as

$$\sigma_{zz}(r) = K_z w(r); \quad \tau_{rz}(r) = K_r v(r, -\frac{t}{2}) \quad (2.31)$$

Expression for the foundation stiffness K_r and K_z are given by Vlasov and Leont'ev [100]. Considering the equilibrium of circular plate, two simultaneous differential equations were derived in terms of σ_{zz} and τ_{rz} . These differential equations were decoupled with the assumption that, in-plane displacements of mid surface of faceplate are negligibly small. These differential equations were converted into the form of two uncoupled Bessel differential equations through change of variables and solution was found in terms of Bessel functions. General solution for a circular sandwich plate under indentation load was given by

$$\sigma_{zz}(r) = C_1 u_0(r) + C_2 v_0(r) + C_3 f_0(r) + C_4 g_0(r) \quad (2.32)$$

Figure 2.16: Circular plate under consideration by Thomsen [99]. (a) Plate under consideration and (b) Free body diagram of the faceplate.

where

$$\begin{aligned} u_0(r) &= \text{Re}[J_0(\sqrt{i} \xi)]; \quad v_0(r) = \text{Im}[J_0(\sqrt{i} \xi)]; \\ f_0(r) &= \text{Re}[H_0^{(1)}(\sqrt{i} \xi)]; \quad g_0(r) = \text{Im}[H_0^{(2)}(\sqrt{i} \xi)]; \end{aligned} \quad (2.33)$$

where C_1 , C_2 , C_3 and C_4 are the arbitrary constants, l is the characteristic length equal to $\sqrt[4]{D_f/K_z}$, D_f is the bending rigidity of the faceplate, i is the complex number, J_0 is the zero-order Bessel function of first kind, $H_0^{(1)}$ and $H_0^{(2)}$ are zero-order Hankel functions of the first and second kind, respectively.

For finding arbitrary constants in Eq. (2.32), author attempted to solve elastically supported circular plate under point load, and the results are compared with experimental measurements. Resulting constants for the problem under consideration are $C_1 = C_2 = C_4 = 0$ and

$$C_3 = \frac{P l^2}{4 D_f \sin[\arg(a_1 + i b_1)]} a_1 = \alpha'^2; \quad b_1 = \sqrt{1 - \alpha'^2}; \quad \alpha' = \frac{1}{8} \frac{t^2 l^2 K_r}{D_f}$$

where, P is the indentation load and K_r is the foundation shear modulus. The above analysis by Thomsen [99], the core was considered as an elastic body, however in general cores (eg. Alporas and Divinycell PVC foam core) exhibit elastic-perfectly plastic behavior. Additionally, small deformation is considered for the faceplates without any membrane stresses. Towards this end Olsson and McManus [101] solved the indentation problem using plate on elastic-perfectly plastic foundation analogy by accounting for large deformation of the faceplates.

Olsson and McManus [101], modeled axisymmetric isotropic circular faceplate plate resting on an isotropic elastic-perfectly plastic core/foundation under point load. The Equilibrium of the sandwich plate under consideration is shown in Figure 2.17. Shear deformations as well as large deformation of faceplate are considered.

Figure 2.17: Equilibrium of sandwich plate under consideration by Olsson and McManus [101].

Small deformation solution is extended to large deformation using

$$F_{NL}/F_L = 1 + k_m (w/t)^2 \quad (2.34)$$

where F_L and F_{NL} are the force estimates according to small and large deformation plate theory, respectively, t is the faceplate thickness and k_m is the membrane stiffness resulting from the large deformations of the faceplate. Due to limited solutions the large deformation of the plates, approximate interpolation method is used to find k_m . For orthotropic faceplates, equivalent isotropic bending rigidities are derived to extend the proposed isotropic solutions to orthotropic deformation profiles. Estimated load-displacement responses are in good agreement with the experimental measurements. However proposed methodology is mathematically challenging hence, Türk and Fatt [102] proposed an approximate solutions for predicting the indentation behavior.

Türk and Fatt [102] attempted to derive approximate solution for circular composite sandwich plate (with elastic faceplate and rigid-plastic core) indented by spherical indenter. Bending rigidity of the faceplates and the elastic strain energy of the core are neglected while the membrane stretching energy of the faceplates is considered. Indented displacement profile of circular plate was defined in Cartesian co-ordinates. Expression for load versus displacement are proposed by minimizing the total potential energy. Additionally, approximate solutions are proposed by considering the hemispherical indenter as a flat, with an effective radius (effective radius, $R_e = 0.4 \times$ radius of hemispherical nose). Assumed displacement profile of the faceplate in terms of equivalent radius was given in Eq. (2.35).

$$\delta(x, y) = \begin{cases} \delta_0 & 0 < x^2 + y^2 < R_e^2 \\ \delta_0 \left[1 - \frac{(x - R_e)}{(\lambda - R_e)} \right]^2 \left[1 - \frac{(y - R_e)}{(\lambda - R_e)} \right]^2 & R_e^2 < x^2 + y^2 < \lambda^2 \end{cases} \quad (2.35)$$

Contact law (load versus displacement) relation was given as

$$\delta_0 = \sqrt[3]{\frac{9 (P - \pi \sigma_c R_e^2)^2}{64 C_1 \sigma_c}} \quad (2.36)$$

where $C_1 = 8 \left[\frac{1}{45} (A_{11} + A_{22}) + \frac{1}{45} (2 A_{12} + 4 A_{66}) \right]$ and δ is the transverse displacement function, δ_0 is the indenter displacement, R_e is the effective radius, A_{ij} are the components of the stretching stiffness matrix of the faceplate, σ_c is the compressive strength of the core and λ is the plastic radius or radius of crushed zone. Absence of bending rigidity terms D_{ij} in expression for C_1 recalls the assumption of the present work that is, bending of faceplate was not considered.

Anderson and Madenci [103] provided the indentation load versus displacement relation between the spherical indenter and Graphite-epoxy/polymethacrylimide foam cored sandwich plate. All the laminate plies and the foam core are treated according to three dimensional elasticity. Contact area and the pressure distribution between the punch and sandwich plate are considered as variables and solved iteratively. Core is considered as elastic member hence the load versus displacement estimate is compared well with experimental measurements in the elastic region.

Sburlati [104] evaluated the indentation law for a sandwich plate loaded by either spherical ended punch (SEP) or flat ended punch (FEP). Contact pressure between the spherical indenter and the sandwich plate is assumed to have Hertzian pressure distribution where as the contact between flat punch and the sandwich plate as FEP on elastic half space. With the assumed pressure distribution, sandwich plate theory is used to obtain the load versus displacement relation.

The pressure distribution and estimated load versus displacement relations are

Pressure distribution in FEP:

$$p(r) = \frac{P}{2 \pi a \sqrt{a^2 - r^2}} \quad (2.37a)$$

Pressure distribution in SEP:

$$p(r) = \frac{4 G_f \sqrt{\bar{a}^2 - r^2}}{\pi (1 - \nu_f) a} \quad (2.37b)$$

Load-displacement relation:

$$w(r) = \frac{P}{2 \pi c G_c} \left[\ln \left(\frac{a^*}{R} \right) + \ln(2) - k^* \right] \quad (2.37c)$$

where

$$\bar{a} = \left(\frac{3 P a}{8} \frac{1 - \nu_f}{G_f} \right)^{1/3}$$

and a is the radius of the SEP or FEP, a^* is equal to a for FEP and $\bar{a}$ for SEP, G_f and ν_f are the shear modulus and Poisson's ratio of the faceplate, respectively, R is the plate radius, G_c is the core shear modulus, c is the core thickness and k^* is term depends on the indenter shape and plate boundary conditions. Author concluded that presence of G_c in the contact law shows that effect of shear modulus of the core plays a major role in the contact law.

Sburlati [105] extend previous work Sburlati [104] to FEP and SEP for square sandwich plates with either clamped or simply supported boundary conditions. Core and faceplate are considered as linear elastic and small deformation theory is assumed. Though the sandwich plate under consideration was square, author used circular plate in analytical formulations with the basis of approach used in Nemes and Simmonds [106] (in which authors proven that, low velocity impact/indentation on square sandwich plates can be modeled as a circular plates because of the fact that, impact/indentation is a localized deformation process). In Sburlati [104] works, author considered uniformly distributed circular patch pressure and Hertzian parabolic surface pressures as a surface loadings. Since faceplate and core are pure elastic, expressions were not reproduced because of its limited use.

Koissin et al. [107] attempted to derive analytical models for sandwich beams (plain strain) and circular plates (axisymmetric) subjected to point load. Core was considered as elastic-perfectly plastic and faceplates was considered as transversely isotropic. Small deformation as well as large deformations are considered for the faceplates. Free body diagram for the axisymmetric deformation is shown in Figure 2.18. Effect of faceplate stretching is brought out by deriving the solutions with and without stretching force, N . Governing differential equation for a circular plate with large deformation is

Figure 2.18: Equilibrium of sandwich plate under consideration by Koissin et al. [107].

$$\frac{d^3 w}{dr^3} + \frac{1}{r} \frac{d^2 w}{dr^2} - \left(\frac{N}{D_f} + \frac{1}{r^2} \right) \frac{dw}{dr} = \frac{1}{2 D_f} \left(\frac{P}{\pi r} - \sigma_c r \right) \quad (2.38)$$

General solution to the preceding equation has a form

$$w = P \alpha - \sigma_c \beta + \delta_{el} \quad (2.39)$$

where

$$\text{For linear solution, } N = 0 : \quad \alpha = \frac{\lambda^2}{16 \pi D_f} \frac{k+3+\nu_f}{k+1+\nu_f} \quad \beta = \frac{\lambda^2}{64 D_f} \frac{k+5+\nu_f}{k+1+\nu_f}$$

where $k = \lambda C D_f$, $C = 0.75 \sqrt{3} E^{1/3} D_f^{2/3}$, $E = 2 E_c / (1 + \nu_c) (3 - \nu_c)$, δ_0 is deflection of the indenter, δ_{el} is the downward displacement for elastic limit, P is the load on the indenter, D_f is the bending rigidity of the faceplate, E_c is the Young's modulus of the core, ν_c is the Poisson's ratio of the core, λ is the plastic radius, ϵ_c is the yield strain of core and c is the thickness of the core. Expressions for non-linear solution $N \neq 0$ is omitted for the sake of brevity.

To solve the large deformation of the faceplate leads to an unknown stretching force N , which depends on the indenter displacement δ_0 . Hence, an additional condition is proposed by accounting for the change in length of the faceplate. Explicit expressions for the load versus displacement response are not possible hence numerical approach is used to solve the resulting governing differential equations. Proposed analytical models were compared with experimental measurements and numerical simulations in ABAQUS and close agreement was observed. However, the estimates for the plastic radius, λ at a given indentation load, P , is not in good agreement with FE predictions.

Zhou and Stronge [65] proposed contact law for the indentation of elastic faceplates with rigid perfectly plastic core under finite strain conditions. Load versus indentation depth was

proposed by minimizing the total potential energy with respect to indentation depth. The displacement functions due to punch loading is assumed as (radial displacement, u and transverse displacement, δ)

$$\begin{aligned}\delta &= \delta_0 \left(1 - \frac{r^2}{\lambda^2}\right)^2 \\ u &= r(\lambda - 1)(A + B r)\end{aligned}\quad (2.40)$$

where constants 'A' and 'B' are defined as follows:

$$A = \frac{\delta_0 (-179 + 89 \nu_f)}{126 \lambda^3} \quad B = \frac{\delta_0^2 (13 \nu_f - 79)}{42 \lambda^4} \quad (2.41)$$

where λ is the plastic radius and ν_f is the Poisson's ratio of the faceplate material. Explicit expression for indentation law (load versus indentation displacement) was obtained as

$$P = \frac{16 \pi}{3} \sqrt{D_f \sigma_c \delta_0 \left[1 + \frac{(7505 + 4205 \nu_f - 2791 \nu_f^2) \delta_0^2}{17640 t^2}\right]} \quad (2.42)$$

where δ_0 is the deflection of the indenter, and D_f is the bending rigidity of the faceplate and σ_c is the compressive strength of the core.

In this section, analytical formulations for estimating the indentation load versus displacement curve are reviewed. However, the faceplates are considered as an elastic material while considering the core as either elastic or elastic perfectly plastic. However, in reality, the faceplates and the core have finite a strength. Hence estimating the strength of the sandwich plates under localized loads is essential for engineers. Towards this strength analysis of sandwich plates is discussed in the following section.

2.7 Analytical estimates for failure initiation load under localized loads

An axial rod under an applied compressive load can fail by either axial crushing or elastic buckling based on the length, aspect-ratio for a given material. Similarly, sandwich structure under an applied load can fail by different competing failure mechanisms. For safe design, it is essential to know the bounding loads for the initiation of various failure modes under localized loads.

Wen et al. [108] attempted to provide failure limit loads for indentation, core shear and bottom faceplate failure in sandwich beams under bending. Indentation failure is defined when

the top faceplate reaches its fiber shear strength τ_f . The limit loads for these failure modes are

Indentation failure:

$$P_i = 2 \pi a t \tau_f + \pi a^2 K_c \sigma_c \quad (2.43a)$$

Core shear failure:

$$P_{cs} = 2 \pi a c \tau_c K_c \quad (2.43b)$$

Bottom faceplate failure:

$$P_{bf} = \frac{2 \pi E_e \sigma_f (c + 2t)^2}{3 E_e F_1} \quad (2.43c)$$

where

$$E_e = E_c r'^3 + E_f (1 - r'^3); \quad F_1 = 2 \ln \left(\frac{2L}{\pi a} \right) + 0.0669; \quad r' = \frac{c}{c + 2t} \quad (2.44)$$

where K_c is the constraint factor for core crushing equals to 2.0, τ_f is the fiber shear strength of the faceplate material, a is the impactor/indenter radius, L is the width of the rectangular sandwich plate, c is the core thickness, σ_c is the core compressive strength, t is the faceplate thickness, τ_c is the shear strength of the core, σ_f is the faceplate failure strength, E_e is the effective Young's modulus of the sandwich plate, E_f is the Young's modulus of faceplate, E_c is the Young's modulus of the core.

Hoo Fatt and Park [109] proposed an analytical formulae for typical failure modes for sandwich plates under flat/spherical punch loading: (a) top faceplate tensile failure: occurs in thin faceplates undergoing large deformations (b) top faceplate shear failure: occurs in thick faceplates under small deformation with high transverse loads, (c) core shear failure: occurs in sandwich plates with thin faceplates and a weak core and (d) bottom faceplate failure: occurs when the core is strong enough to avoid both indentation and core shear failure as shown in Figure 2.19.

Shear failure of top faceplate:

$$P_{sf} = 2 \pi a t \tau_f + K_c \pi a^2 \sigma_c \quad (2.45a)$$

Tensile failure of top faceplate:

$$P_{tf} = \sqrt{2} A_{11} l_d \epsilon_f^{3/2} + \pi K_c \sigma_c R_e^2 \quad (2.45b)$$

Figure 2.19: Failure modes considered by Hoo Fatt and Park [109].

Core shear failure:

$$P_{cs} = \frac{32}{15} \sqrt{2 D_1 \sigma_c} \left(\frac{255 D_1 \gamma_c^4}{128 \sigma_c} \right)^{1/6} + \pi \sigma_c a^2 \quad (2.45c)$$

$$P_{cs} = \frac{8 \sqrt{C_1 \sigma_c}}{3} \left[\frac{9 C_1 \gamma_c^4}{\sigma_c} \right] + \pi \sigma_c a^2 \quad (2.45d)$$

Bottom faceplate failure:

$$P_{bf} = \frac{4 \pi \varepsilon_f (D_{11eq} + D_{12eq})}{\left(\frac{c}{2} + t \right) \left[2 \ln \left(\frac{2L}{\pi a} \right) + 0.669 \right]} \quad (2.45e)$$

where:

$$C_1 = 8 \left[\frac{A_{11} + A_{22}}{45} + \frac{2 A_{11} + 4 A_{22}}{49} \right] \quad (2.46)$$

$$D_1 = \frac{16384}{11025} (7D_{11} + 7D_{22} + 4D_{12} + 8D_{66})$$

where, l_d is the length of the damage is equal to $2 \pi R_e$ for ductile faceplates and $2 R_e$ for brittle faceplates, A_{ij} are the components of the stretching stiffness matrix of the faceplate, K_c is the constraint factor for core crushing equals to 2.0, R_e is the equivalent radius of the spherical indenter equals to $0.4 a$, a is the impactor radius, σ_c is the core compressive strength, t is the faceplate thickness, D_{ijeq} are the components of the bending stiffness matrix of the sandwich plate, D_{ij} are the elements of the bending stiffness matrix of faceplate, γ_c is the core shear failure

strain, ϵ_f is the faceplate tensile failure strain.

It is evident from the discussion that sandwich structures under localized loads fail by different failure modes viz., indentation, core shear and faceplate failure. It is of great interest to investigate the effect of geometry, material and loading parameters on the failure modes and loads and optimize the sandwich plate for weight.

2.8 Failure mode maps for sandwich structures

This section is devoted to the construction of failure mode maps for sandwich structures. As will be seen from the discussion, there has only one attempt to construct a failure mode map for a sandwich plate. Hence, the discussion is devoted to sandwich beam failure mode maps.

Concept of failure mode map is introduced by Triantafillou and Gibson [110] to investigate the effect of geometric parameters of the faceplate and core on the operative failure mode for a given material and loading parameters on aluminum/PU foam sandwich beams. To optimize the weight of the sandwich structure (subjected to either strength or stiffness as constraints) for a given load-bearing capacity, a minimum weight design concept is provided by Triantafillou and Gibson [111]. A detailed discussion on the concept of failure mode maps and minimum weight design maps is compiled in [11, 25]. Typical failure modes in the sandwich beams are shown in Figures 1.8 and 1.9. The operative failure mode depends on the behavior of the faceplate either ductile (viz., aluminum, steel) or brittle (viz., CFRP, GFRP). Hence the literature is classified according to the type of faceplate. Ductile faceplates are assumed to form plastic hinges, where as brittle faceplates are assumed to exhibit a linear elastic behavior until failure. Typical core-dominated failure modes in sandwich beams with either ductile or brittle faceplates are shown in Figure 2.20. Composites are weaker in compression, hence the dominated failure mode microbuckling occurs in top faceplate while ductile faceplates fail by tenacity and hence failure occur in bottom faceplate.

Limit load estimates for composite sandwich beams for failure modes shown in Figure 2.20 are

$$\text{Indentation: } P_{in} = b t \left(\frac{\pi^2 d E_f \sigma_c^2}{3 L} \right)^{1/3} \quad (2.47a)$$

$$\text{Core shear: } P_{cs} = 2 b d \tau_c \quad (2.47b)$$

$$\text{Microbuckling: } P_{mb} = \frac{4 b d t \sigma_f}{L} \quad (2.47c)$$

Figure 2.20: Failure modes in sandwich beams [112, 113].

Limit load estimates for meta sandwich beams for failure modes shown in Figure 2.20 are

$$\text{Indentation: } P_{in} = 4 b t \sqrt{\sigma_c \sigma_y} \quad (2.48a)$$

$$\text{Core shear: } P_{cs} = \frac{2 b t^2 \sigma_f}{l-s} + 2 b c \tau_c \left(1 + \frac{2 H}{l-s} \right) \text{ for mode A} \quad (2.48b)$$

$$P_{cs} = \frac{4 b t^2 \sigma_f}{l-s} + 2 b c \tau_c \quad \text{for mode B} \quad (2.48c)$$

$$\text{Face yield: } P_{fy} = \frac{4 b t (c+t) \sigma_f}{l-s} + \frac{b c^2 \sigma_c}{l-s} \quad (2.48d)$$

where b , d and L are the breadth, thickness and length of the sandwich beam, respectively, s is the distance between the loading rollers, t is the thickness of the faceplate, c is the thickness of the core, σ_c is the uniaxial strength of the core, σ_f is the faceplate yield stress, E_f is the modulus of the faceplate, τ_c is the shear strength of the core and H is the overhang length of the simply

supported beam. Sandwich bending can constitute indenter either of a spherical-ended punch or a flat-ended punch, while support being clamped or simply-supported subjected to either three or four point bending. All the contributions in the literature are listed in Table 2.1 with no emphasis on the type of the core (viz., honey comb, foam core).

Table 2.1: Review of literature on failure mode maps for sandwich beams.

Loading	Support	Composite faceplates		Metal faceplates	
		FEP	SEP	FEP	SEP
3-Point	Clamped		[114]	[115]	
	SS		[113, 114, 116]	[115, 117, 118]	
4-Point	Clamped				
	SS		[119]		[112]

FEP: flat ended punch, SEP: spherical ended punch and SS: simply support.

Of all the contributions in Table 2.1, the effect of beam support conditions (viz., simply support and clamped) on the failure map is investigated for woven GFRP/PVC foam core [114] and aluminum/aluminum alloy foam core [120] sandwich beams subjected to three-point bending by a flat ended circular punch. It is observed that failure mode map for clamped sandwich beams showed a shrink in size for microbuckling and core indentation failure regimes compared to that of simply supported beams.

Many attempts have been made in the literature to construct failure mode maps for sandwich beams, but very few attempts have been made to develop failure maps for sandwich plates with either a rectangular or a circular geometry. Towards this end, Sridhar [121] developed failure maps for simply-supported circular sandwich plates with aluminum faceplate and Alporas foam, subjected to circular flat-ended punch load at the center of the plate. A typical failure map for circular sandwich plate under bending is shown in Figure 2.21. For physically reasonable plates geometries face yield, core shear and indentation failure are found to be operative failure mechanisms.

2.9 Summary

The literature is reviewed with the objective of understanding the impact and indentation behavior of foam cored sandwich plates with either metallic or composite faceplates.

Fundamental aspects of the manufacturing processes of metallic and polymeric foams followed by their characterization for different properties viz., uniaxial tension, compression, shear, plastic Poisson's ratio, fracture initiation and propagation energy are reviewed. Additionally,

Figure 2.21: Typical failure mode maps for circular sandwich plates under bending in non-dimensional faceplate thickness t/c versus core thickness c/R space. [121].

constitutive modeling of foams and different FE modeling strategies (viz., micro and macro) are emphasized.

Subsequently, experimental investigations on the low velocity impact response of sandwich structures are briefly listed along with the different sandwich materials systems used by individual researchers. Rigorous discussion is devoted to analytical modeling of the low velocity impact and indentation response owing to their complexity.

Most of the analytical contributions for predicting the load-displacement response consider the faceplates as linear elastic with the core being either elastic or perfectly plastic with no failure. However the faceplates and core has a finite strength. Hence there is a need for estimating the bounding solutions for the failure load estimates. Towards this, upper bound estimates for the failure of sandwich structures is essential for designers and engineers. Hence the concept of failure mode map and minimum weight design concepts are reviewed.

From the literature review the following issues have been observed.

(a) Rigorous FE models are essential in predicting the low velocity impact behavior of foam cored sandwich structures are essential. Metallic honeycomb cores can be using cell wise basis hence well established J_2 plasticity can be used. However foams cell wise modeling is computationally intensive and expensive. Hence foam constitutive modeling and failure criterion plays a major role in FE modeling of foam cored structures.

(b) Single cored sandwich structures have gained popularity. However, recent FE investigations and limited experimental investigations have shown that graded core sandwich structures

have superior energy absorption characteristics over single cored sandwich structures. Hence, it is essential to investigate the effect of core grading under low velocity impact loading conditions on peak force and energy absorption.

Additionally, advances in manufacturing processes have lead to wide varieties of commercial feasible metallic and polymeric foams are available. However, no rigorous work is available to compare their relative performance under low velocity impact loading.

(c) All the indentation models available in literature are for spherical end indentors. No contributions are available for circular flat ended indenter with either elastic, rigid or elastic perfectly plastic behavior to the core while considering the bending, stretching behavior to the faceplates.

(d) Failure mode maps are well established for sandwich beams under bending and end compression. However, very limited attention on failure mode maps has been paid for sandwich plates under bending.

The rest of the work attempts to address above-mentioned issues.

Chapter 3

Impact modeling of foam cored sandwich plates with ductile or brittle faceplates

In this chapter, rigorous FE models were developed to predict the energy absorption characteristics of metal foam cored sandwich plates under low velocity impact loading. Faceplates of the sandwich plates were either elastic brittle composite laminates or ductile metals. The capabilities of different constitutive models and hourglass control methods on the prediction of failure modes and damage dimensions are investigated. Predicted impact response (viz., load versus time, energy versus time) are compared with experimental measurements and simple upper bound analytical estimates.

3.1 Peak load prediction for shear-plug failure mode

In this section, analytical models for failure load from Hoo Fatt and Park [109] are considered for comparison with numerical predictions and experimental measurements. The type of failure mode (viz., indentation or shear-plug) depends on the impact energy and the stiffness of the sandwich plate. Shear-plug failure mode was observed in the present experimental work as shown in Figure 4.1, which gives the details of faceplate stretching (fs), core crushing (cc) and core tear (ct) phenomena.

Figure 3.1: Failure by core shear-plug formation in ductile failure of isotropic faceplate.

Hoo-Fatt and Park [109] considered the faceplate stretching and core crushing as dominant mechanisms and derived the expression for sandwich plate (with linear elastic orthotropic faceplates and perfectly plastic core) failure initiation load (P_f) as:

$$P_f = \sqrt{2} A_{11f} l_{df} (\epsilon_{df})^{3/2} + \pi K_c \sigma_c R_e^2 \quad (3.1)$$

where

$$\begin{aligned} l_{df} &= 2 \pi R_e \text{ for ductile faceplates (Al),} \\ &= 2 R_e \text{ for brittle faceplates (CFRP),} \\ A_{11f} &= \frac{E_{11f} t}{(1 - \nu_{12f} \nu_{21f})}, \\ K_c &= 2.0, \\ R_e &= 0.4 a. \end{aligned}$$

where, l_{df} , A_{11f} , E_{11f} , t , ν_{ijf} and ϵ_{df} are the length of the damage, stretching stiffness, Young's modulus, thickness, Poisson's ratio and dynamic fracture strain of the faceplate material respectively, K_c is the constraint factor for core crushing, σ_c is the yield strength of the core, R_e is the effective radius of the impactor hemispherical nose and a is the impactor hemispherical nose radius. Hoo Fatt and Park [109] used an average value of 2.0 (from a range between 1.7 and 2.5) for the constraint factor, K_c based on the compression and indentation tests of Reddy et al. [108]. The effective radius, R_e is considered as the contact radius, which is again taken from the works of Hoo Fatt and Park [109].

In the case of ductile faceplates (Al), tensile failure occurs around the periphery of the effective radius of the impactor, so l_d is $2 \pi R_e$. In brittle faceplates (CFRP), radial fracture occurs underneath the impactor, so l_d is equal to $2 R_e$. It is assumed that the effect of strain-rate under the present loading conditions is negligibly small so the static tensile strain of faceplate is used

in place of the dynamic failure strain (ϵ_{df}). In the current work, the load required for the tearing of the foam in core shear failure around the circumference of impactor is also considered, so an additional term ($2 \pi R_e \gamma_c$) is added to Eq. (3.1). It is to be noted that, tear energy (γ_c) of the foams can be evaluated through instrumented indentation tests on foams [28]. Hence, the peak failure load for the shear plug failure is given by

$$P_f = P_{fs} + P_{cc} + P_{ct} \quad (3.2)$$

$$P_f = \sqrt{2} A_{11f} l_{df} (\epsilon_{df})^{3/2} + \pi K_c \sigma_c R_e^2 + 2 \pi R_e \gamma_c \quad (3.3)$$

It can be observed from Eq. (3.3) that, the failure load depends on the type and thickness of the faceplate and yield strength of the core, but independent of the core thickness.

3.2 Materials and their constitutive models

The sandwich plates considered in the present study consisted of aluminum alloy foam (Alporas supplied by GLEICH GmbH, Germany) with a 9% relative density as a core, and carbon fiber reinforced polymer matrix (CFRP) composite or aluminum as faceplates. Square sandwich plates of 100 mm x 100 mm size with a central circular diameter of 76 mm with cores of thickness 20 mm, 30 mm or 40 mm size and faceplates of thickness 0.5 mm or 1 mm are used in the sandwich plate construction. The core and faceplates are bonded using REDUX-332 adhesive film. Full details of material characterization and impact tests on Dynatup apparatus was explained by Mohan [122].

3.2.1 Core material

In this section, two constitutive models viz., Deshpande-Fleck (DF) model which is an isotropic constitutive model and the Homogenized-honeycomb (HC) model, which is an orthotropic constitutive model, are considered. In the orthotropic HC model, isotropic behavior is achieved by assigning the same material properties (such as elastic moduli and Poisson's ratio) in all the three directions. Before carrying out the numerical simulation of low velocity impact, it is necessary to calibrate the input properties of the material constitutive models by simulating the uniaxial compression test with a single element model of the test specimen.

Compression test simulation is performed on a cube modeled with a single eight node solid element employing one-point integration. The bottom four nodes of the cube are restrained in all the directions and a prescribed velocity is applied on the top four nodes such that the cube is compressed up to 80% engineering strain, similar to that in the experiments. To minimize inertia effects and to assure a quasi-static response, a prescribed velocity [55] is applied to the top nodes

of the cube as follows:

$$v(t) = \frac{\pi}{\pi - 2} \frac{d_{\max}}{T} \left[1 - \cos\left(\frac{\pi t}{T}\right) \right] \quad (3.4)$$

where $d_{\max}$ is the maximum applied displacement, T is the total duration of loading, t is the time variable and $v(t)$ is the velocity distribution function.

3.2.1.1 Calibration of DF constitutive model

A phenomenological homogenized constitutive model for the plastic behavior of metal foams is proposed by Deshpande and Fleck [21]. The yield surface is defined as a quadratic function of the mean stress and effective stress in stress space. The mean stress in the yield criterion shall account for the associated volume change in the porous foams. Yield criteria (Φ) for metal foams can be expressed as [21]:

$$\Phi = \hat{\sigma} - \sigma_y \quad (3.5)$$

where the equivalent stress $\hat{\sigma}$ is
$$\hat{\sigma}^2 = \frac{\sigma_e^2 + \alpha^2 \sigma_m^2}{1 + \left(\frac{\alpha}{3}\right)^2}$$

The yield surface shape factor, α is defined in terms of the plastic Poisson's ratio ν_p as

$$\alpha^2 = \frac{9}{2} \frac{1 - 2\nu_p}{1 + \nu_p} \quad (3.6)$$

The yield stress, σ_y as a function of effective strain, $\hat{\epsilon}$ is given by [54]

$$\sigma_y = \sigma_p + \gamma \frac{\hat{\epsilon}}{\epsilon_D} - \alpha_2 \ln \left[1 - \left(\frac{\hat{\epsilon}}{\epsilon_D} \right)^\beta \right] \quad (3.7)$$

where, σ_e is the von Mises effective stress, σ_m is the mean stress, σ_p is the plateau stress, ϵ_D is the densification strain, α_2 , γ and β are the curve-fit parameters as proposed by Hanssen et al. [48]. The densification strain is related to the foam relative density, $\bar{\rho}$ as $\epsilon_D = -\ln(\bar{\rho})$ based on Hallquist [54]. The parameters α_2 , γ and β need to be evaluated by customized curve-fitting of the true uniaxial compressive stress versus strain data based on Eq. (3.7). Effect of yield surface shape factor on the relative density for different foams has been reported by various researchers [11, 21, 123].

Yield surface shape factor α depends on the plastic Poisson's ratio of the foam according to Eq. (3.6). For Alporas foams with relative density range from 8% to 10%, the plastic Poisson's

ratio values lie in between 0.31 to 0.34 [11]. A value of plastic Poisson's ratio for Alporas with 8% relative density was measured by Gioux [23] as 0.024. However, Motz and Pippan [24] reported, values of plastic Poisson's ratio for Alporas foams with densities 250 kg/m^3 and 400 kg/m^3 as 0.332 and 0.440 respectively. These values are found to have wide differences from author to author. However, in the present work, a value of zero has been used according to the work of Reyes et al. [55].

True stress versus true strain is used as the data for curve fitting according to Eq. (3.7) using MATLAB software. Yield surface shape factor has been calculated based on zero plastic Poisson's ratio value from Eq. (3.6). Based on this procedure, the calculated input parameters of the DF constitutive model in numerical compression test are listed in Table 3.1. The corresponding engineering stress versus engineering strain curves from experiment, the true stress versus true strain used in the curve fitting, and the engineering stress versus engineering strain curve evaluated from a compression test simulation are shown in Figure 3.2a. The numerically-simulated engineering stress versus engineering strain curve agrees with the experimental response up to 60% engineering strain, subsequently there is a small deviation in the numerical engineering stress versus strain response from the experimental observations.

Table 3.1: Material properties of ALPORAS foam used in DF constitutive model

Property	Value
Mass density, ρ	250 kg/m^3
Young's modulus, E	1100 MPa
Yield stress, σ_y	1.53 MPa
Densification strain, ϵ_D	2.35
Poisson's ratio, ν_p	0
Yield surface shape factor, α	2.12
Curve fit parameter, γ	2.94 MPa
Curve fit parameter, α_2	576.3 MPa
Curve fit parameter, β	11.69
Volumetric fail constant [†] , CFAIL	0.035

[†] Appropriate failure mode is achieved using maximum principal stress erosion (MAXEPS = 0.5) criterion using MAT_ADD_EROSION option.

3.2.1.2 Calibration of HC constitutive model

To extend the functionality of the present numerical model to orthotropic foams, the orthotropic HC constitutive model is also considered. However, the required isotropic behavior is achieved by assigning the same properties in all directions. For a better understanding of the implementa-

(a)

(b)

Figure 3.2: Experimental and numerical uniaxial compression response of Alporas foam. (a) DF model and (b) HC model.

tion of constitutive model, the constitutive equations of the orthotropic HC model are discussed briefly and later the compression test simulation is performed. HC constitutive model is applicable for homogenized honeycombs and foams [54]. The individual nonlinear elastoplastic

uniaxial compressive behavior can be defined for normal stresses (σ_{ij}) and shear stresses (τ_{ij}). Coupling between these stress components is not considered. As compression progresses, the elastic properties of uncompressed material varies from their initial values to the fully compacted values at densification strain and linearly with the relative volume, V , as given (for one orthotropic direction) in the following equations.

$$\text{Current orthotropic Young's modulus, } E_{aa} = E_{AAU} + \beta (E - E_{AAU}) \quad (3.8)$$

$$\text{Current orthotropic shear modulus, } G_{ab} = G_{ABU} + \beta (G - G_{ABU}) \quad (3.9)$$

$$\text{Current normal stress, } \sigma_{aa}^{n+1^{trial}} = \sigma_{aa}^n + E_{aa} \Delta \epsilon_{aa} \quad (3.10)$$

$$\text{Current shear stress, } \tau_{ab}^{n+1^{trial}} = \tau_{ab}^n + 2 G_{ab} \Delta \epsilon_{ab} \quad (3.11)$$

where

$$\beta = \max \left[\min \left(\frac{1-V}{1-VF}, 1 \right), 0 \right],$$

VF is the relative volume at which the foam is fully compacted, E_{AAU} and G_{ABU} represent uncompressed orthotropic Young's and shear moduli, and E and G are the cell wall materials Young's and shear moduli respectively. It is to be noted that, VF is the relative volume at which densification starts. Relative volume is the ratio of current volume to initial volume of sample, so at the start of the compression test relative volume is equal to one. In uniaxial compression stress state, relative volume (V) can be related to volumetric strain $\epsilon_v = 1 - V$. This gives the relative volume at densification, $VF = 1 - \epsilon_D$.

In the literature, the densification strain is defined slightly differently by different authors. Gibson and Ashby [25] define the densification strain as the strain value after the densification process. This gives a densification value as 0.8 for the present foam and the corresponding VF is 0.2. However, Ashby et al. [11] define the densification strain as a strain value prior to densification process. This gives densification value as 0.6 for present foam and corresponding strain VF is 0.4. For this reason, a series of values i.e., 0.0, 0.2, 0.3 and 0.4 are considered for VF and so as to bring out the effect of VF clearly.

Material properties used in this simulation are shown in Table 3.2. Uniaxial compression engineering stress versus engineering strain curve used in the present work along with the input data curve used in HC constitutive model is shown in Figure 3.2b. To avoid a negative yield stress in the process of extrapolation, the input data curve is extended in negative abscissa (see Figure 3.2b). This extension is done based on the suggestions of Hallquist [54]. To understand the effect of VF on the uniaxial stress versus strain response of the foam, a quasi-static compression test simulation is performed in LS-DYNA explicit solver.

For the compression test simulation, a single element is compressed by the prescribing ve-

Table 3.2: Material properties of ALPORAS foam used in HC constitutive model

Property	Value/Response curve
Mass density, ρ	250 kg/m ³
Young's modulus, $E = E_{AAU} = E_{BBU} = E_{CCU}$	1100 MPa
Shear modulus, $G_{ABU} = G_{BCU} = G_{CAU}$	550 MPa
Plastic Poisson's ratio, ν_p	0.0
Yield stress, σ_y	1.8 MPa
Tensile strain at failure [†] , TSEF	0.35
Relative volume at compaction [‡] , VF	0.0, 0.2, 0.3, 0.4
Compression stress-strain curve, LCA = LCB = LCC = LCS	Figure 3.2b

[†] Failure strain in experiments is 3.5% as shown in Figure 3.4b.

[‡] A VF value of 0.0 to 0.2 is suggested after studying the compression test simulation.

locity to the nodes on the top surface of the element according to Eq. (3.4). The front views of the simulation results at 80% compression strain for VF values 0.0, 0.2, 0.3 and 0.4 are shown in Figures 3.3a-3.3d. Engineering stress versus engineering strain responses are compared with that of experiments in Figure 3.3e.

Figure 3.3: (a)-(d) shows the front view of compression test simulation at 80 % compressive strain for different values of VF and (e) shows the stress versus strain response from experiments and from numerical simulation for different values of VF in HC model.

From Figure 3.3e, it can be observed that, as the value of VF increases, the lateral expansion of the foam increases. However, in experiments, the lateral expansion of foam is negligibly small. The effect of lateral expansion on the compression stress-strain response is also noticeable in Figure 3.3e. The sudden drop in compression stress value in Figure 3.3e represents the initiation

of lateral expansion of foam. It can be understood that, with a VF value close to zero (e.g., $VF = 0.2$), the HC model can reproduce the experimental behavior of foam up to 80% compressive strain. However, as the value of VF increases, the HC model is not able to reproduce the complete response of the foam behavior. Hence, for subsequent simulations, a value of 0.2 is assigned for VF .

Thus, for DF and HC constitutive models, the input material properties have been calibrated and confirmed for correctness. Tension tests are simulated on the dog-bone shaped foam specimens to find the failure/erosion strain. Dog-bone shaped specimens with reduced single point integration elements are used for the tension test simulation. One tab of dog-bone shape is constrained in all directions and velocity loading is applied to the other tab according to Eq. 3.4. Similar to the tension test experiments on foams, there is no necking phenomenon observed in the tension test simulation. Prescribed loading conditions along with boundary conditions and failure mode of the Alporas foam specimen is shown in Figure 3.4. Failure strain values viz., MAXEPS (in DF model) and TSSEF (in HC model) are calibrated to match the tensile stress-strain response predicted from the FE models with that of the experimental measurements. Calibrated failure strain values along with the predicted tensile stress-strain response are compared in Figure 3.4b. The input parameters for the foam constitutive models are based on the uniaxial compression test data hence the tension test simulation is predicting the stress-strain curve similar to uniaxial compression response.

During the low velocity impact loading, the DF model is able to account for the interaction between all the stress components and hence the failure strain value (MAXEPS) of 0.035 predict from the un-axial tension cannot be used to predict the load-displacement curve. Hence, for a given mesh, MAXEPS needs to be calibrated to match the low-velocity impact response (load-displacement) of the bare foam with that of experimental measurements. Using this methodology, MAXEPS value is calibrated and is found to be 0.5.

Unlike DF model, interaction between stress components is not considered in HC model and hence, using the same value of TSEF (0.35) is used to predict the impact load-displacement response without any additional calibration studies.

3.2.2 Faceplate materials

In the present work, to account for the effect of ductile and brittle faceplates under low velocity impact, half-hard aluminum alloy Al 1100 and unidirectional CFRP (Fiberdux-913C-HTA from Hexcel Composites, UK) are employed as the faceplate materials with 0.5 mm or 1 mm thickness.

Metal faceplates (Al 1100) are modeled using J_2 flow theory of plasticity using piece-wise linear plasticity material model (MAT_PIECEWISE_LINEAR_PLASTICITY, MAT_24). The half-hard cold worked Al sheets have nearly elastic-perfectly plastic stress-strain behavior and its

Figure 3.4: Finite element simulation of Alporas foam uniaxial tension test. (a) FE model (b) Comparison between FE predictions and experimental measurement.

properties are listed in Table 3.3. Failure strain values are calibrated to match with the experimental measurements and the same value is used in all the simulations. The unidirectional composite

Table 3.3: Material properties of the aluminum faceplate (Al 1100) used in piece-wise linear plasticity constitutive model

Property	Value
Mass density, ρ_{Al}	2700 kg/m ³
Young's modulus, E_{Al}	75 GPa
Yield stress, σ_{yAl}	116 MPa
Poisson's ratio, ν_{Al}	0.3
Plastic strain at failure [†] , FAIL	0.5

[†] Failure strain in experiments is 1.6%.

faceplates are modeled using a three dimensional composite failure material model (MAT_COMPOSITE_FAILURE_SOLID_MODEL, MAT_59). For the failure of composite faceplate, the maximum principal strain criterion is implemented using MAT_ADD_EROSION. The material

properties of CFRP material are provided in Table 3.4.

Table 3.4: Material properties of CFRP faceplate (Fiberdux-913C-HTA) used in three dimensional composite failure model

Property	Value
Mass density, ρ_{cfRP}	1100 kg/m ³
Young's modulus in 1 direction, E_{11cfRP}	150 GPa
Young's modulus in 2 and 3 directions, $E_{22cfRP} = E_{33cfRP}$	9.5 GPa
Shear modulus, $G_{12cfRP} = G_{13cfRP}$	5.43 GPa
Shear modulus, G_{22cfRP}	3.26 GPa
Poisson's ratio, $\nu_{21cfRP} = \nu_{31cfRP}$	0.16
Poisson's ratio, ν_{32cfRP}	0.45

Erosion is achieved based on maximum principal strain (MAXEPS = 0.035) criterion using M-AT_ADD_EROSION. Failure strain from experiments is 1.5%.

3.3 Finite element analysis

In this section, the low velocity impact behavior of bare foams and sandwich plates is discussed. The suitability of the two constitutive models under consideration (viz., DF and HC models) in low velocity impact is assessed by simulating bare foam impact. The effect of erosion criterion on the failure is also discussed. In all the impact simulations, plates (either sandwich or bare foam) of diameter 76 mm with steel hemispherical impactor of diameter 13 mm are used. An impactor of mass 2.65 kg and with a normal impact velocity of 6.7 m/s is impacted at the center of the clamped sandwich plate to achieve low velocity impact conditions. Same impactor mass and velocity are used for all the tests and simulations in the current chapter. The thickness of the Al alloy Alporas core is varied as 20 mm, 30 mm and 40 mm, where as faceplate thickness is varied as 0.5 mm and 1.0 mm. The point of impact is at the center of the plate. In DYNATUP 8250 apparatus, the plate under impact is fixed in-between the two square plates (one at the top and the other at the bottom of the sandwich plate) with central circular area of 76 mm diameter. This experimental setup works as a circular sandwich plate with clamped boundary conditions around the periphery. In addition to the impact response (viz., the load versus time and energy versus time results) appropriate failure mode predictions also play a major role in deciding the acceptance of numerical simulations. Hence, the discussion below will begin with the predicted failure modes and later focus on impact response.

Foam material properties used in the numerical simulations are listed in Tables 3.1 and 3.2. For the impactor, steel properties (viz., Young's modulus = 200 GPa, density = 7860 kg/m³ and

Poisson's ratio = 0.3) are assigned for the simulations. Meshing of the foam blanks is done such that, the mesh is refined in the region of impact to give an average aspect ratio of 1 to 2, with coarse mesh in the regions away from the impact region. Typical mesh details are shown in Figure 3.5. Both the facesheet and core are meshed with single integration point, eight node

Figure 3.5: Finite element model of sandwich plate used in present work. All the nodes around the circumference of the plate are clamped.

solid elements (viz., the constant stress solid element). To overcome the nonphysical hourglass behavior, hourglass control (viz., either stiffness based hourglass control or assumed strain co-rotational control) is used. It is found that, formation of a shear-plug is clearly seen using the stiffness based hourglass control and it is absent with the assumed strain co-rotational hourglass control. To avoid excessive stiffening, care is taken in post-processing stage to keep hourglass energy within the prescribed permissible limits (i.e., 10% of peak internal energy value).

Contact between core and rigid punch is achieved using erosion contact option. In bare foam impact simulations, soft-constraint formulation is used to avoid penetration of the slave surface (foam) into master surface (punch) resulting from huge difference in the material constants [54]. In the case of sandwich plate impact simulations, segment-based contact is used to define the impact between sandwich plate and rigid punch. Penalty scale factor is adjusted to overcome excessive penetration of the slave surface into master surface. To avoid negative volumes, interior contact is defined for the core region. To account for contact between eroded core material and parent core, single surface contact is defined. Debonding between the faceplate and the core is not observed in experiments, so the effect of adhesive layer is not considered in the numerical

simulation. Hence, tie constraints is applied between the faceplate and core material. All the contacts and sliding between the punch and sandwich component materials are assumed to be frictionless. The mass of the impactor is assigned using the point mass option in LS-DYNA. Noise in the reaction force is eliminated using Society of Automotive Engineers (SAE) filter, with a frequency of 2300 Hz.

All computations are carried out on a high-performance computing cluster facility using 4 nodes, each node consists of 8 CPUs (total of 32 CPUs) using MPP capabilities of LS-DYNA (version R4.2.1). Each node has 24 GB of shared memory.

3.3.1 Low velocity impact on bare foams

The effect of constitutive model and erosion criterion on the impact response and failure mode prediction can be effectively understood if the impact is performed on the bare foams. Hence, initially failure modes predicted by the numerical models are considered. To accomplish this task, impact on 20 mm thick Alporas foam block is studied. The failure modes predicted by the DF and HC models are shown in Figure 3.6. It is evident from Figure 3.6a that, DF model

Figure 3.6: Failure modes predicted by different constitutive models. (a) DF model with volumetric strain failure criterion. (b) and (c) DF model with maximum principal strain erosion criterion using assumed strain co-rotational stiffness hourglass control (type 6) and stiffness based hourglass control (type 4), respectively. (d) HC model with tensile strain failure criterion.

with volumetric strain failure criterion is not able to predict physically reasonable failure modes.

However, Figure 3.6b shows that the DF model with maximum principal strain erosion criterion and assumed strain based hourglass control (type 6) is able to predict pure core shear failure mode similar to that observed in experiments without shear-plug. Formation of shear-plug is observed with the use of DF model using maximum strain erosion criterion and stiffness based hourglass control (type 4) as shown in Figure 3.6c. Failure mode predicted by orthotropic HC model using tensile strain failure is shown in Figure 3.6d. It is evident that formation of shear-plug and failure mode are in good agreement with the experimental observations.

The accuracy of the simulations is checked using an energy balance. Towards this, the hourglass energy is maintained below 10% of peak internal energy value to make sure that the contributions from spurious energy modes is sufficiently low. Energy ratio is maintained approximately constant close to 1 to ensure overall equilibrium of the simulation. Negative sliding energy is maintained negligibly small to ensure the correctness of the contact definitions. A typical energy balance graph observed in the present work is shown in Figure 3.7.

The next aspect studied is the impact response (viz., load versus time and energy versus time)

Figure 3.7: Typical energy balance plots achieved in the numerical simulations.

of bare foams subjected to low velocity impact. The load versus time and energy versus time responses of 20 mm and 30 mm thick foams are compared with that of experimental measurements in Figure 3.8.

In Figure 3.8a, the load versus displacement response can be divided into three stages viz., (a) Elastic region (b) Plateau load region and (c) complete failure region. It can be observed that, for 20 mm and 30 mm thick cores, the plateau occurs approximately at same load value of

(a) Load versus time

(b) Energy versus time

Figure 3.8: Numerically predicted bare foam impact response comparison with experiments [122] for 20 mm and 30 mm thick cores.

approximately 0.9 kN independent of the core thickness. Effect of core thickness can be seen in the energy absorption plot Figure 3.8b. As the core thickness increased from 20 mm to 30 mm,

the energy absorption value increased from 12.5 J to 25 J (100 % increase).

3.3.2 Low velocity impact of sandwich plates

Table 3.5 lists various sandwich geometries simulated in the current work which corresponds to the experimental work of Mohan [122]. In the following discussion, the simulated impact response curves (viz., load versus time and energy versus time) are compared with that of experimental measurements.

Table 3.5: Numerical models solved in the present work.

Core thickness (mm)	Face plates			
	Thickness 0.5 mm		Thickness 1.0 mm	
	Aluminium	CFRP	Aluminium	CFRP
20	×	×	✓	✓
30	✓	✓	✓	✓
40	✓	✓	✓	✓

3.3.2.1 Impact of sandwich plates with 0.5 mm thick faceplate

Impact response of sandwich plate with 0.5 mm thick faceplate (Al/CFRP) and 30 mm or 40 mm core are shown in Figure 3.8. For comparison purpose, the responses with aluminum and CFRP faceplates are superimposed along with their respective experimental responses. It is to be noted from Figure 3.9a, that plateau load after initial peak is the same as that of the plateau load observed in bare foam impact in Figure 3.8. The duration of punch traversal by core tearing after complete failure of the faceplate is nearly the same for both faceplates.

In addition to the impact response, numerical models should also be able to predict correct failure modes as that observed in the experiments. The failure modes predicted by the present numerical models are compared with that of experiments in Figures 3.9a and 3.9b for aluminum sandwich plate. Ductile nature of the faceplate (Al), creates a hole (ductile fracture mode) whose diameter corresponds to the diameter of the punch (13 mm). In addition, the stretching of the faceplate leads to core crush region of 27 mm diameter surrounded by the hole. Figures 3.9c and 3.9d show the comparison of failure modes between the experimental observations and numerical simulations for CFRP sandwich plate. The damage zone is dominated along the fiber direction: damage zone dimensions of 35 mm along the fiber direction (longitudinal) and 20 mm along the fiber transverse direction, the same predicted by numerical simulation are 30 mm and 22 mm, respectively. Failure mode predictions confirm that the maximum principal strain based erosion criterion is reasonable for unidirectional composites under the present normal impact

(a) load versus time

(b) Absorbed energy versus time

Figure 3.9: Impact response of sandwich plate with 0.5 mm with 20 mm and 30 mm thick cores.

conditions.

Figure 3.8: Impact response of sandwich plate with 0.5 mm with 20 mm and 30 mm thick cores.

3.3.2.2 Impact of sandwich plates with 1.0 mm thick faceplates

Impact response (load versus time and energy versus time) for 1.0 mm thick faceplates (Al/CFRP) with 20 mm, 30 mm and 40 mm thick cores is shown in Figure 3.10. Peak loads in the Al and the CFRP sandwich plates are approximately 4.0 kN and 2.5 kN, respectively and is independent of the core thickness. However, core thickness has an effect on the amount of energy absorbed. For Al sandwich plate, the energy absorbed increased from 40 J to 55 J (an increase of 40%) as the thickness of the core changed from 20 mm to 40 mm. Similarly for CFRP sandwich plate the energy absorbed raised from 25 J to 40 J (an increase of 52%) as the thickness of the core increased from 20 mm to 40 mm. Absorbed energy curves in Figures 3.8, 3.8 and 3.10 show the cumulative energy with respect to time but not instantaneous energy absorption. Hence, the absorbed energy increases with time until it reaches a maximum value. The total absorbed energy can be read as the peak value of absorbed energy in the plot.

Impact is performed with 60 J of energy, however all the sandwich plates failed at 50 J by a shear-plug failure mode. So absorbed energy versus time plots did not reach peak value of impact energy (60 J).

3.3.3 Comparison of peak loads with analytical model

In this section, a comparison of numerical and analytical predictions with experimental results is listed in Table 3.6 so as to assess the accuracy of the proposed numerical modeling methodology.

Figure 3.9: Comparison of predicted failure modes with that of experiments. (a) and (c) the experimental failure modes for Al and CFRP sandwich plate, respectively. (b) and (d) the numerical failure modes for Al and CFRP sandwich plate, respectively.

Within experimental scatter, the peak loads are governed by faceplate thickness and material type but not by the thickness of the core, as observed from Eq. (3.3). So, for comparison purpose, the peak load values are taken from 30 mm thick core, though variation is negligible with other core thicknesses. In calculating the peak load values using Eq. (3.3), material properties from Tables 3.3 and 3.4 are used. The failure strain of the faceplate (ϵ_{crdf}) used in analytical investigation is taken from experiments rather than from the numerical simulations. Of the three mechanisms considered for estimating the failure load, contribution from faceplate stretching is dominant followed by core cursing and core tearing. Tear energy (γ_c) of the foam is taken as 6.7 kJ/m² [28]. The peak loads predicted by the analytical equation are in good agreement with that of the numerical simulations and experimental measurements.

Figure 3.10: Impact response of sandwich plate with 1.0 mm faceplate and 20 mm, 30 mm or 40 mm thick cores.

Table 3.6: Comparison of failure loads between experiments, analytical modeling and numerical predictions.

Faceplate Material	Faceplate Thickness (mm)	Failure Load (kN)			
		Experiments	Analytical	Numerical	
				DF model	HC model
Al	0.5	2.01	2.14	1.82	2.21
	1.0	3.73	4.08	4.07	4.08
CFRP	0.5	1.22	1.21	1.29	1.22
	1.0	2.41	2.25	2.12	2.33

3.4 Summary

Numerical models for the low velocity impact analysis of bare foams and sandwich plates have been developed using the LS-DYNA commercial finite element software. All the sandwich plates of Alporas foam core, with Al/CFRP faceplates failed by through-perforation failure, when impacted with a 60 J energy punch.

Two constitutive models, an isotropic DF constitutive model and an orthotropic HC constitutive model are considered for modeling the foam core. Though the foam core (Alporas) used in present work is an isotropic foam that can be modeled with DF model, an orthotropic HC constitutive model is also used to extend the present numerical model to orthotropic foams. Isotropic behavior is achieved by assigning the same properties in all directions. The foam constitutive models are calibrated using compression test simulation.

It is observed that, the peak load is governed by the type of faceplate but not by the thickness of the core. The energy absorbed depends on the type of the faceplate as well as the thickness of core.

The failure modes predicted by the DF constitutive model using volumetric strain failure criteria is found to be not in good agreement with that observed in experiments, whereas the maximum principal strain criteria is able to predict failure modes correctly. Hence, it is concluded that rather than the maximum stress criterion/maximum stress shear criterion used in literature the maximum principal strain criterion is able to predict the failure modes correctly with stiffness based hourglass control.

The simulated force versus time, and energy absorbed versus time responses along with the predicted failure modes and analytically-predicted peak loads are in good agreement with that of the experimental measurements.

Chapter 4

Relative performance of metallic and polymeric foam sandwich plates under low velocity impact

Chapter 3 provided FE modeling details of the sandwich plates subjected to low velocity impact and the importance of the constructive models and hourglass control and their effects in predicting the failure modes. In the present chapter, relative performance of sandwich plates consisting of Divinycell PVC and Alporas (ALP) foam core with aluminum faceplates is studied under low velocity impact. For a meaningful comparison, PVC H80 and H250 foams are selected with yield strength and density equal to that of Al alloy Alporas foam: H80 has same yield strength as that of Alporas foam and H250 has same mass density as that of Alporas. Effect of core grading on maximum dynamic penetration force and energy absorption capacity is also investigated by interchanging the layers of H80, H250 and Alporas foams. Using numerical models, failure modes and impact force-displacement response are predicted and compared with that of experiments. Upperbound analytical calculations were used to estimate the penetration force.

4.1 Methodology

4.1.1 Analytical modeling

In this section, analytical models for estimating the peak force (under low velocity impact) are summarized [124, 125].

4.1.1.1 Analytical modeling of bare foam impact

Flores-Johnson and Li [124, 125] investigated the indentation and low velocity impact response of polymethacrylimide (PMI) and polyetherimide (PEI) foams using different geometry of punches and proposed an analytical estimate for indentation load-displacement relation. These estimates will be used in present work.

When elastic-perfectly plastic foam experiences impact with a spherical punch, the impact force-displacement curve has a plateau region when the indentation depth is equal to the punch radius. An approximate analytical estimate for this plateau force (under spherical impactor) is expressed as [124]

$$P_{plateau} = P_{crushing} + P_{tearing} + P_{friction} \quad (4.1)$$

$$P_{plateau} = \pi R_i^2 \sigma_c + 2 \pi a \gamma_c + \frac{\pi^2}{2} a^2 \mu \sigma_c \quad (4.2)$$

where, a is the impactor hemispherical nose radius, σ_c is the yield strength of the core, γ_c is the tear energy of the core and μ is the coefficient of friction between the foam and the indenter. More details about the impact force-displacement curve characteristics will be discussed in section 4.2.2.1.

4.1.1.2 Analytical modeling of sandwich impact

Upper bound calculations for the penetration force estimates is summarized here from the work of Hoo-Fatt and Park [109]. Top faceplate damage initiation occurs by its tensile failure, involving membrane stretching. Schematic diagram of a typical shear plug failure mode is shown in Figure 4.1. Penetration force (P_f) for a sandwich plate with linear elastic perfectly plastic faceplate and perfectly plastic core can be expressed as follows:

$$P_f = 2\sqrt{2} \pi R_e \frac{E_f t}{(1 - \nu_f^2)} \epsilon_f^{3/2} + \pi K_c R_e^2 \sigma_c + 2 \pi a \gamma_c \quad (4.3)$$

where, t , E_f , ν_f and ϵ_f are the thickness, Young's modulus, Poisson's ratio and dynamic fracture strain of the faceplate material, respectively, R_e is the effective radius of the hemispherical impactor nose and K_c is the constraint factor for core crushing. In the work of Hoo-Fatt and Park [109], R_e is taken as 0.4 times the radius of the impactor, and K_c is chosen as 2.0 (an average value from a range between 1.7 and 2.5, based on the compression and indentation tests of Wen et al. [108]).

Figure 4.1: Failure by core shear-plug formation in ductile failure of isotropic faceplate.

4.1.2 Experimental investigation

This section discusses the details about materials and their characterization, specimen geometries and loading conditions used in the impact experiments.

4.1.2.1 Materials characterization

Alporas foams and H80 and H250 foams were used in present study. Alporas is a closed cell aluminum alloy foam manufactured by Shinko Wire Co., Japan. The relative density of Alporas used in present study has 8.5% with an average cell size of 4.7 mm. PVC foams of grades H80 and H250 are closed cell foams manufactured by Diab Inc. The numeric part in the nomenclature of the foams represents the nominal density (i.e. H80 and H250 have densities of 80 kg/m^3 and 250 kg/m^3 , respectively). However, the measured average densities of H80 and H250 foams were 68 kg/m^3 and 230 kg/m^3 with an average cell size of 0.4 mm and 0.15 mm, respectively. Half-hard cold worked aluminum sheets of thicknesses 0.5 mm and 1.0 mm were used as faceplate materials for sandwich plate construction.

Tensile tests were performed on dog-bone specimens of aluminum sheets and PVC foams prepared according to ASTM E8-04 and ISO 527, respectively. Alproas foam was also cut according to ASTM E8-04 with a minimum of seven cells in the gauge length to avoid cell size effects. The dogbone samples were gripped using wedge grips for Al faceplates, and mechanical side grips for PVC and Alporas foams. Clip-on extensometer (Instron 2630-100 series with 50 mm gauge length) was used to measure the strain during tensile testing. Uniaxial tension tests were performed using Instron 5500R universal testing machine with a displacement loading at 1 mm/min. Uniaxial tensile stress-strain response of aluminum sheets and foams is shown in Figures 4.2a and 4.2b, respectively. From the tensile tests of aluminum sheet material (in length,

width and 45° directions), it was confirmed to be nearly isotropic with elastic-perfectly plastic behavior. The effect of aluminum sheet thickness on Young's modulus value is found to be negligible. However, the yield strength increased up to 12 % with an increase in aluminum sheet thickness from 0.5 mm to 1.0 mm. The tensile modulus of Alporas foam is higher than that of PVC foams; however the failure stress is close to that of H80 within the experimental scatter and failure strain is lesser than that of PVC foams as shown in Figure 4.2b.

Compression tests on PVC foams were performed using 30 mm thick specimens with in-plane dimensions 50 mm x 50 mm, where as for Alporas foam a cuboid of 50 mm side length was used to avoid the size effects (minimum dimension of the specimen should be at least seven times the cell size) of the Alporas foam. Compression strain was calculated from the cross head displacement. Loading and unloading cycles were used in compression test to measure the corresponding loading and unloading modulus of the foams. Daniel and Cho [126] have shown that the through-thickness and in-plane compression properties differ and hence orthotropy needs to be considered for PVC foams. To account for orthotropy, compression tests were performed in through-thickness and in-plane (length and width) directions. The uniaxial compression test response of different foams tested is shown in Figure 4.3. Compression response of H250 and H80 shows that, the through-thickness strength is higher than in-plane strength (by a factor of 1.6 for H80 and 1.3 for H250). It was observed that the in-plane compressive strengths of H80 and H250 foams in length and width directions are close enough. However, the compression modulus of H250 foam in length direction is smaller in comparison to that in width direction, whereas H80 foam shows in-plane isotropy. The yield strengths of H80 and Alporas foams are almost the same (1.5 MPa for Alporas and 1.2 MPa for H80), however the rate of densification (change in stress with respect to change in strain) of Alporas foam is quite fast compared to that of H80 foam. Though the cell size of Alporas foam is larger than H80 foam, the fast rate of densification observed is due to cell wall material constitutive property.

4.1.2.2 Impact testing

Sandwich plates with in-plane dimensions 100 mm x 100 mm were fabricated by bonding aluminum faceplates to 40 mm thick foam layers. The faceplates on both sides of the core were either of thickness 0.5 mm or 1.0 mm. Two foam layers (viz., Alporas, H80 or H250) each of thickness 20 mm were bonded to give 40 mm thick core. PVC foam samples with in-plane dimensions 100 mm x 100 mm were cut from a foam block using circular saw and Alporas foam specimens were cut using electrical discharge machining (EDM) to minimize the damage to the cell walls.

Bonding between various interfaces was achieved using two-part methacrylate structural adhesive, MA310 (from ITW Plexus). Typical cured MA310 adhesive has an Young's modulus of

Figure 4.2: Uniaxial tension response of (a) aluminum face sheets and (b) PVC and aluminum alloy foams.

1.1 GPa, failure stress of 20 MPa, and failure strain of 10 % [127]. These properties ensure no premature failure of adhesive interface under low speed impact loading conditions. To maintain constant bondline thickness, a fiber glass mesh of 0.2 mm thickness was used at all the bonding

Figure 4.3: Uniaxial compression response of PVC and Alporas foams.

interfaces.

To understand the contributions of each constituents of the sandwich plate, impact tests were carried out on the aluminum sheets and foams of thickness 20 mm and 30 mm, and bare graded foams of thickness 40 mm in addition to the tests on sandwich plates. For effective comparison, H80 and H250 were selected against Alporas foam because H80 and Alporas foam have approximately the same yield strength whereas H250 and Alporas foam have approximately the same density. As an additional objective, the effect of core grading on impact response (peak force and energy absorption) is investigated.

All the impact tests were performed using Instron Dynatup 9250 apparatus with an impactor of 8.7 kg mass at 5.8 m/s velocity. Sandwich plate is clamped using two pneumatic plates (of size 100 mm x 100 mm with central circular hole of diameter 76 mm) on the top and bottom. Response from the impactor (tup) is recorded using the data acquisition system. Schematic view of the impact apparatus used in current experimental investigation is shown in Figure 4.4.

Three set of experiments were conducted on each type bare foam specimens (20 mm and 30

Figure 4.4: Schematic view of Dynatup 8250 apparatus [122].

mm) to ensure repeatability. The experimental scatter was found to be negligibly small. However owing to material availability constraint, only one set of experiments were conducted on 40 mm bare foams and sandwich plates. Possibility of experimental scatter and stochastic variations in the material types, adhesive amount and any experimental scatter are thus constrained by the sample size. In this experimental study, the impact response of nine 40 mm thick foams and eighteen sandwich plate designs were investigated.

4.1.3 Numerical modeling

Finite element (FE) models were developed in LS-Dyna to predict the impact behavior of the sandwich plates. Owing to the symmetry, quarter models were developed for numerical simulation and symmetric boundary conditions are applied on the planes of symmetry. A typical FE model of the sandwich plate with mesh details is shown in Figure 4.5. The clamped boundary conditions in the experiment were simulated by constraining all nodes around the curvilinear

boundary of the plate. The mesh was refined in the vicinity of the plate center so as to keep an element aspect ratio of 1-2 (with element dimensions of 0.5 mm) while coarse mesh was used in the regions away from the impact region. All the parts were meshed with constant stress solid

Figure 4.5: Quarter FE model of the graded sandwich plate. Nodes on the periphery of the sandwich plate are constrained in all directions and symmetry boundary conditions are imposed on the nodes in xz and yz plane.

element. Hourglass control was used to avoid spurious modes and care was taken to keep hourglass energy within 10% of peak internal energy value. Contact between plate and rigid punch was achieved using node to surface erosion contact option. Tie constraints were applied to simulate bonding between various interfaces. Selected adhesive ensures no interfaces de-bonding, hence tie constraint failure is not considered in the present FE model. Effect of friction between the impactor and the sandwich plate was neglected as the contact duration is very short. Excessive element distortions and smaller time steps (caused by thinning of element) were avoided by eroding elements if the time step decreases to 60% of its initial time step value.

Hemispherical impactor was modeled as a rigid body (using steel properties: Young's modulus of 200 GPa, density of 7860 kg/m³ and Poisson's ratio of 0.3) with lumped mass of 8.7 kg and an impact velocity of 5.8 m/s using PART INERTIA card. Noise in the reaction force is eliminated using Society of Automotive Engineers (SAE) filter, with a frequency of 2300 Hz. Aluminum faceplates were modeled using piece-wise linear plasticity material with J₂-flow the-

ory and isotropic hardening (MAT_PIECEWISE_LINEAR_PLASTICITY, MAT_24). Input material properties of aluminum faceplate are listed in Table 4.1. Core materials Alporas, H80 and H250 were modeled using homogenized honeycomb model (MAT_HONEYCOMB, MAT_26). In this constitutive model, the orthotropic foams can be modeled by defining different compression stress-strain curve in different directions. However, in homogenized honeycomb model coupling between the stress components is not considered. The material property values used for simulating Alporas, H80 and H250 foams are listed in Tables 4.2, 4.3 and 4.4, respectively. The loading modulus value measured from the present experiments is 300 MPa (from 50 mm thick specimens). However, Ramamurty and Paul [128] observed certain variability in the loading modulus of foams with respect to thickness of foams. Mean loading modulus values for 25 mm, 50 mm and 115 mm thick foams was reported as 180 MPa, 290 MPa and 460 MPa, respectively. In present work Alporas foams under investigation are of thickness 20 mm, hence Alporas foam was modeled using a Young's modulus of 180 MPa.

Table 4.1: Material properties of aluminum faceplate used in piece-wise linear plasticity constitutive model.

Property	Value
Mass density, ρ_{Al}	2700 kg/m ³
Young's modulus, E_{Al}	70 GPa
Yield stress, σ_{yAl} (0.5 mm)	116 MPa
(1.0 mm)	130 MPa
Poisson's ratio, ν_{Al}	0.3
Plastic strain at failure, EPSF (0.5 mm)	0.4
(1.0 mm)	0.5
Effective stress-strain curve	Figure 4.2a

Table 4.2: Material properties of Alporas foam used in honeycomb constitutive model.

Property	Value/Response curve
Mass density, ρ	250 kg/m ³
Young's modulus, $E = E_{AAU} = E_{BBU} = E_{CCU}$	180 MPa
Shear modulus, $G_{ABU} = G_{BCU} = G_{CAU}$	90 MPa
Plastic Poisson's ratio, ν_p	0.0
Yield stress, σ_y	1.5 MPa
Tensile strain at failure, TSEF	0.8
Relative volume at compaction, VF	0.0
Compression stress-strain curve, LCA = LCB = LCC = LCS	Curve (c) in Figure 4.3

For Alporas foam, always maximum strain failure criterion was used irrespective of its location in graded construction. However, for PVC foams H80 and H250, maximum strain failure

Table 4.3: Material properties of Divinycell H80 foam used in honeycomb constitutive model.

Property	Value/Response curve
Mass density, ρ	76 kg/m ³
Young's modulus, $E_{AAU} = E_{BBU}$	30 MPa
Young's modulus, $E = E_{CCU}$	49 MPa
Shear modulus, $G_{ABU} = G_{BCU} = G_{CAU}$	35 MPa
Plastic Poisson's ratio, ν_p	0.0
Yield stress, σ_y	1.2 MPa
Tensile strain at failure, TSEF	0.4
Volumetric strain at failure [†] , CFAIL	0.1
Relative volume at compaction, VF	0.0
Compression stress-strain curve, LCA = LCB	Curve (e) in Figure 4.3
Compression stress-strain curve, LCC = LCS	Curve (d) in Figure 4.3

[†] Volumetric fail strain is incorporated using MAT_ADD_EROSION option.

Table 4.4: Material properties of Divinycell H250 foam used in honeycomb constitutive model.

Property	Value/Response curve
Mass density, ρ	240 kg/m ³
Young's modulus, E_{AAU}, E_{BBU}	130 MPa, 146 MPa
Young's modulus, $E = E_{CCU}$	175 MPa
Shear modulus, $G_{ABU} = G_{BCU} = G_{CAU}$	116 MPa
Plastic Poisson's ratio, ν_p	0.0
Yield stress, σ_y	6.2 MPa
Tensile strain at failure, TSEF	0.6
Volumetric strain at failure [†] , CFAIL	0.125
Relative volume at compaction, VF	0.0
Compression stress-strain curve, LCA = LCB	Curve (b) in Figure 4.3
Compression stress-strain curve, LCC = LCS	Curve (a) in Figure 4.3

[†] Volumetric fail strain is incorporated using MAT_ADD_EROSION option.

criterion was used as long as there exists any backing layer. Otherwise, tensile volumetric strain ($\epsilon_{11} + \epsilon_{22} + \epsilon_{33}$) criterion [54] was used with MAT_ADD_EROSION option. A similar failure mode was simulated by Zhou et al. [69] using hydrostatic stress criterion. For example, in H80/H80 bare foam impact simulation, maximum strain criteria for the top layer of H80 and volumetric strain criterion for the bottom layer of H80 were used. These failure criterion were selected to simulate the observed failure modes. The damage parameters in the constitutive model are calibrated against experimental test data for the considered mesh size.

All computations were carried out on high performance computing cluster facility using 4 nodes, each node consists of 8 CPUs (total of 32 CPUs) using MPP capabilities of LS-DYNA

(version R5.1.1). Each node has 32 GB of shared memory.

4.2 Results and discussion

In this section, comparison between predicted as well as observed failure modes and impact force-displacement responses are discussed followed by a comparison of different graded bare foams and sandwich designs using bar charts.

4.2.1 Failure modes

4.2.1.1 Bare foams - one layer

Impact on bare Alporas foam blank leaves a straight hole through the thickness of the foam block. However, in PVC foams, the perforation path is straight up to 1/2 to 3/4th of foam blank thickness followed by conoid formation. Typical failure modes observed in experiments and that predicted using FE models, are shown in Figure 4.6. Brittle materials such as glass under localized indentation loads fail by pure conical mode of cracking [129]. Under impact loading, a shear-plug failure mode occurs in thicker plates while tensile failure with the formation of conoid is likely to occur in thinner plates. This is because comparatively bending becomes dominant in thinner plates and the conical portion of the failure surface always occurs on the lower side of the plates where tension is developed. A combination of shear plug and conical cracking failure mode is observed in concrete as reviewed by Li et al. [130]. The transition length from pure shear plug to cone cracking occurs when the penetration force in-front of the projectile is equal to the shear force in the remaining thickness of the concrete. Li et al. [130] also provided an empirical formula based on a large number of experiments for the shear-plug and conoid formation transition length in terms of compressive strength of the un-reinforced concrete and the ratio of target thickness (H) to projectile radius (R_i). Concrete is brittle under both tensile and compressive loading conditions, whereas cellularic PVC foams depict brittle behavior only under tensile loading due to the breaking of the cell-walls. Hence, the transition length in bear foams depends possibly on the tensile strength and H/R_i . Due to constraints on the different thicknesses of PVC foam samples and tearing energy data as a function of foam relative density, obtaining an empirical equation for the transition length for shear plug to conical mode of failure in these polymeric foams is not carried out in the current investigation. The base of the conoid (cone-shaped shear plug) observed in the experiments was neither circular nor elliptic but arbitrary in shape owing to orthotropy and inhomogeneity of the foams. Formation of the conoid is also observed in the works of Flores-Johnson and Li [124] during their indentation tests on the PEI closed-cell rigid foams (Airex R82.80) with hemispherical punch. Length of the conoid, L_c

Figure 4.6: Experimental and numerical failure modes and its dimensions of 20 mm thick foams. In (b), L_c and D_c are the length and diameter of the conoid, respectively.

(as shown in Figure 4.6) of H80 and H250 foams was found to be 6 mm and 10 mm, respectively. Impact tests on 20 mm and 30 mm thickness samples (viz., H80 and H250) revealed that L_c does not depend on the thickness of the foam but on the foam density under impact. Diameter of the conoid, D_c (chord length of an arbitrary conoid) of H80 was found to vary from 20 mm to 25 mm and that of H250 was found to vary from 25 mm to 40 mm due to in-plane orthotropy of the PVC foams. Accurate prediction of arbitrary conoid shapes is not possible owing to the fact that unavailability of constitutive model for orthotropic foams accounting for the interaction between stress components. The estimated angle of the conoid of H80 was found to vary from 30° to 54° and that of H250 was found to vary from 30° to 53° .

The numerically simulated failure zone diameter for the Alporas foam is in fairly good agreement with that of the experimental measurements. The diameter of the conoid, D_c of H80 foam

in the experiments was measured to be varying from 20 mm to 24 mm (from three repeated experiments) against 22 mm and 26 mm (in two orthotropic directions x and y) predicted by the FE model. The D_c for H250 foam was measured to be varying from 35 mm to 40 mm (from three repeated experiments) against the FE prediction of 29 mm and 33 mm (in two orthotropic directions x and y). Experimentally observed length of the shear-plug of Alporas foam is very small because of the collapse or densification due to large cell size. In PVC foams, length of the conoid is of the order of foam blank thickness due to low densification of the foam resulting from small cell size as shown in Figure 4.6. The numerically-predicted length of the shear plugs, L_c were not in agreement with that of the experiments because in the numerical simulation the elements get compressed and eroded based on stable time increment (to avoid very small time step in the computations).

4.2.1.2 Bare graded foams - two layers

The cut section views of bare graded foams in Figure 4.7 shows that the failure modes observed in test specimens. Alporas foam always fails by shear plug formation irrespective of its location in the graded construction. The failure mode in the PVC foam layer depends on its location in the graded construction. If a PVC foam layer is followed by stronger backing layer, then the front PVC foam layer fails by pure shear plug failure mode and the conoidal fracture is either completely absent or small. In H80/H80, H250/H250, H80/ALP and H80/H250 specimens, top layers fail by pure shear plug formation because of the presence of a backing layer of equal or higher strength. In H250/ALP specimen, the rear layer (Alporas) is low in strength, so the conoidal fracture is present in front layer (H250) while the failure in Alporas is pure shear plug with diameter equal to the conoid diameter in H250 core (see Figure 4.7e). Typical shear plugs and conoid fragments are shown in Figure 4.8.

4.2.1.3 Sandwich plates

All sandwich plates tested failed by through-hole shearing except those with 0.5 mm faceplate and H250 foam as a second layer (viz., H250/H250, H80/H250 and ALP/H250). Since aluminum faceplates with 0.5 mm thickness is not sufficient enough to act as a stiff backing for H250 foam, conoid formation has occurred. Experimental failure modes for H250/H250 sandwich plate with 1.0 mm and 0.5 mm faceplates are shown in Figure 4.9. In numerical simulations, a layer of elements between bottom faceplate and core failed without any visible failure in the bottom faceplate. Sizes of the conoid in 0.5 mm and 1.0 mm sandwich plates were not accurately predicted by the present numerical models. Prediction of accurate conoid size is possible by careful calibration of erosion parameters, which in turn depend on the mesh size. A small conoid formation is observed in 1.0 mm faceplates experiments (as shown in Figure 4.9b) is possibly due

Figure 4.7: Experimental failure modes of 40 mm thick bare foams.

Figure 4.8: Typical shear plugs and conoids observed in bare graded foams. In figure (a) ALP/ALP (b) H80/H80 (c) H250/H250 (d) H250/ALP and (e) H80/H250.

to foam orthotropy.

Figure 4.9: Failure modes in H250/H250 sandwich plate.

4.2.2 Impact response: Force versus displacement

4.2.2.1 Bare foams of 20 mm thickness

Comparisons between numerically-predicted and experimentally-measured impact force-displacement curves are shown in Figure 4.10 along with summary of mass of the foam blanks, dynamic penetration force and energy absorption values. It can be seen that, impact event (force-displacement) consists of three stages; (a) linear region: gradual increase in force as the penetration depth increases from zero value (b) plateau region: impact force reaches a plateau value as penetration depth is equal to punch radius and force remains constant (c) failure: formation of shear plug (in Alporas) or sudden conoid formation (in H80 or H250) occurs. When a cellular structure

Figure 4.10: Force-displacement response of bare foams of thicknesses 20 mm and 30 mm.

experiences contact with a impactor, the cell walls ahead of the impactor buckles followed by a layer-wise collapse of thin section and their propagation leading to its densification. As impact progresses, contributions of tearing of the cell walls around the loading impactor and densification effect of foam decides either increase or constant plateau force or stress.

Analytical estimation of plateau force according to Eq. (4.2) need tear energy of the core, γ_c . According to Olurin et al. [28], tear energy of Alporas foam is 6.7 kJ/m^2 . Using material properties of Alporas from Table 4.2, the plateau force for Alporas foam can be calculated as 569 N (which is in close agreement with experimental measurement). Analytical plateau forces of H80 and H250 were not verified due to a lack of a tear energy data. From Eq. (4.2) it is evident that, plateau force is independent of the thickness of the foam. The same observation can be seen from the experimental data in Figure 4.10, as the thickness of the H80 foams increases from 20 mm to 30 mm, there is no change in plateau force. Similar observations were noted for Alporas foams [131]. This behavior in soft foams is due to localized damage at the core surface (localization effects) dominates the thickness effects (increase in force with respect to increase in thickness of the foam) of the core. However, thickness effect is clearly seen in H250 (strong) foam as shown in Figure 4.10 against soft foams (viz., H80 and Alporas).

For a given yield strength of 1.5 MPa, selecting H80 foam (of mass 13 g) in place of Alporas (of mass 45 g) leads to a decrease in maximum impact force from 0.8 kN to 0.4 kN (-50%) and a decrease in energy absorption from 9.8 J to 4.2 J (-57%). This is because the Young's modulus of the Alporas foam is higher than that of H80 foam. For a given density of 250 kg/m^3 , choosing H250 in place of Alporas foam results an increase in maximum impact force from 0.8 kN to 2.6 kN (+225%) and an increase in energy absorption from 9.8 J to 26.8 J (+173%).

Correlation between the failure mode and impact force-displacement curve is evident. For Alporas foam, failure is a through-thickness perforation, so there is no sudden drop in the force-displacement curve. Due to the formation of conoid in PVC foams, there is a sudden drop in the force-displacement curve. This sudden drop in force occurs approximately at the 1/2 to 3/4th of foam blank thickness. Although the FE models of H80 foam are able to predict the formation of conoid, the sudden drop in force-displacement curve is delayed by 4 mm as shown in Figure 4.10. This delay is possibly due to the approximate nature in calibration of erosion strain value used for element deletion or due to the inability of constitutive model to account for the interaction between stress components.

4.2.2.2 Bare foams of thickness 40 mm

The experimentally measured impact response of bare foams of thickness 40 mm is shown in Figure 4.11. Damage initiation force of 40 mm thick graded foam is governed by the top foam layer and is equal to that of front layer alone (20 mm). Effect of adhesive layer is clearly seen

if both foam layers (viz., ALP/ALP, H80/H80, ALP/H80 and H80/ALP) are soft. The elastic stiffness is highest for H250/ALP panel out of all the tested panels because of the combined effect of two high Young's modulus foams (viz., Alporas and H250) considered. This ensures highest elastic energy absorption capacity and damage initiation force.

4.2.2.3 Sandwich plates

Sandwich plates were constructed with aluminum faceplates and Alporas foam and/or PVC foams as a core for evaluating the relative performance of Alporas foam and PVC foams. Comparison between experimental measurements and numerical predictions of force-displacement response of 0.5 mm aluminum sandwich plate is shown in Figure 4.12. For a given yield strength of 1.5 MPa, replacing Alporas foam with H80 results an increase in maximum dynamic force from 1.3 kN to 1.7 kN (+30%) and correspondingly a decrease in energy absorption from 37.1 J to 32.4 J (-12.7%), a decrease in mass from 135 g to 70 g (-48%) and an increase in specific energy absorption from 0.278 kJ/kg to 0.463 kJ/kg (+66.5%). Similarly, replacing Alporas aluminum foam with H250 for a given density of 250 kg/m³, the maximum dynamic force increases from 1.3 kN to 4.0 kN (+207%), the energy absorption increases from 37.1 J to 98.55 J(+165%) and the specific energy absorption increases from 0.278 kJ/kg to 0.725 kJ/kg (+160%). Similar trends were observed for sandwich plates with 1.0 mm faceplates with respect to maximum dynamic force and energy absorption performance values.

From the force-displacement measurements, it is obvious that the maximum dynamic force is governed by the faceplate thickness. Though the yield strength and Young's modulus of Alporas sandwich are higher than that of H80 foam, the maximum dynamic force is smaller in Alporas sandwich plate than H80 sandwich plate. This observation is consistent with sandwich plates having either 0.5 mm or 1.0 mm thick faceplates. Possible reason is due to large cell size (4.7 mm) triggering cell size effects in Alporas foam sandwich against small cell size of H80 foam (0.4 mm). However, these size effects in Alporas foam get attenuated as the faceplate thickness increases.

A comparison between numerical predictions, analytical estimates and experimental measurements is shown in Figure 4.12 so as to assess the accuracy of the implemented numerical modeling methodology. For the estimation of the maximum dynamic force values using Eq. (4.3), material properties from Tables 4.1-4.4 were used. The failure strain of the faceplate (ϵ_f) used in analytical investigation is taken from 0.5 mm thick specimen tension test data. It is evident from Figure 4.8 that the shear plugs of H250 are higher (9 mm to 10 mm) in diameter compared to Alporas and H80 foams (5 mm to 6 mm). This explains that, the effective radius, R_e varies with respect to the material under impact and is higher for stronger foams. To verify this numerically, the finite element simulation output is captured (just before the maximum force)

Figure 4.11: Experimental force-displacement response of bare foams: (a) low yield strength foams and (b) presence dense foam.

and shown in Figure 4.13 for Alporas, H80 and H250 sandwich plates with 0.5 mm faceplate. It is evident that contact radius is higher for a sandwich plate with stronger foam for a given load. Hence, for the maximum dynamic force estimation of Alporas and H80 (soft foam based) sand-

Figure 4.12: Comparison between experimental, numerical and analytical responses of sandwich plates.

sandwich plates R_e was taken as $0.4 R_i$ as proposed by Hoo Fatt and Park [109]. However, for H250 (strong foam based) sandwich plates R_e was estimated by geometric scaling of $0.4 R_i$ (based on

core shear plug diameter) and a good approximate was found to be $0.75 R_i$. Analytically predicted maximum penetration forces for H80 and Alporas sandwich plates are close enough as shown in Figure 4.12.

Figure 4.13: Effect of sandwich type on the contact radius (just before the face failure or maximum dynamic penetration force for 0.5 mm sandwich plate).

4.2.3 Comparison charts

For effective comparison between different sandwich constituents, bare graded foams and sandwich designs, bar charts are plotted in Figures 4.14 and 4.15 showing the maximum dynamic force, energy absorption and panel weight. Comparison chart for the sandwich constituents is shown in Figure 4.14a. Thickness of the foam has no effect on the maximum dynamic force in H80 and Alporas foams because of the failure localization in the foams. However, in H250 foam, the maximum dynamic force increases with the increase in foam thickness.

Bar chart in Figure 4.14b summarizes and compares the impact responses of 40 mm thick graded foams. Damage initiation force and maximum dynamic force are different in bare graded foams impact. The damage initiation force depends on type of the foam on the impact side. Soft and compliant foam on the impact side of the sandwich plate will have lower damage initiation force, with no effect on overall energy absorption (viz., H80/H250). Maximum dynamic force depends on the location of the stronger foam in the graded configuration. For a given weight; maximum dynamic force or energy absorption capacity can be improved by using H250 foam in place of Alporas foam (viz., H80/H250 instead of H80/ALP) and improved performance is obtained when the stronger foam is on the impact side (viz., H250/H80 instead of H80/H250).

Bar charts in Figure 4.15 summarizes and compares the impact response of the graded sandwich plates with 0.5 mm and 1.0 mm thickness faceplates. Effect of grading sequence of soft foams is nullified in sandwich construction, i.e. ALP/H80 and H80/ALP sandwich plates have same maximum dynamic force and energy absorption capacities. Increase in faceplate thickness nullifies the effects of variation in cell size of Alporas foam and difference in moduli of soft foams (H80 and ALP) on the maximum dynamic force and energy absorption. Using stronger

Figure 4.14: Comparison chart of maximum dynamic force, energy absorption and mass of (a) sandwich constituents and (b) bare graded foam.

foams as the front layer in graded construction increases the overall performance of the sandwich plate substantially.

4.3 Summary

The compressive yield strength of Alporas and H80 foams was close to 1.5 MPa. However, the densification process of Alporas foam is rapid (due to cell wall material constitutive property) in comparison to that of H80. The H80 and H250 foams are found to have higher strength in through-thickness direction compared to in-plane directions. For the same weight, choosing H250 gives 300% (based on compression strength of 6.2 MPa against 1.5 MPa of Alporas) stronger structure when compared to Alporas foam structure.

Alporas foams always fail by shear plug failure mode irrespective of its location in the graded configuration. However, in PVC foams (H80 and H250), the failure mode depends on their location in the graded configuration. If a PVC foam layer is followed by an equally strong or stiff backing layer, then PVC foam fails by pure core shear plug otherwise by a combination of shear plug and conoid (conical shear plug) failure. Accurate numerical prediction of the shear

Figure 4.15: Comparison chart of maximum dynamic force, energy absorption and mass of (a) 0.5 mm and (b) 1.0 mm.

plug and conoid dimensions demands a constitutive model, which considers the foam orthotropy and coupling between all the stress components.

The maximum dynamic force does not depend on the thickness of soft foams (viz., Alporas or H80) due to localized failure in soft foams. However, in dense foams (H250), the maximum dynamic force increases with an increase in thickness of the foam due to absence of localization effects. In the impact of graded bare foams, damage initiation force is governed by the top foam layer and is equal to that of front layer alone. However, the maximum dynamic force in the graded bare foam depends on the location of the strong foam (H250) in the graded configuration.

In graded core sandwich plates, the effect of core grading can be seen only if the yield strength of the cores is different. Using cores of the same yield strength in a graded core has no effect on the sandwich plate performance in terms of the maximum dynamic force or energy absorption. With the use of graded core sandwich plates with thin faceplate, one can maximize the energy absorption with small maximum penetration force by using soft foam on the impact side followed by a strong foam backing. Similarly, one can maximize the maximum dynamic force by decreasing the energy absorption value by using stronger foam on the impact side followed by softer foam. Presence of softer foams as a front layer in graded sandwich plate leads to an early

triggering of indentation phenomena, so the damage initiation force is smaller in comparison to sandwich plates with stronger foams as a front layer. An increase in faceplate thickness from 0.5 mm to 1.0 mm nullifies the cell size effects of the underlying foam (viz., Alporas). Sandwich plates with H250 as core outperform (all the other designs considered) in terms of damage initiation force and energy absorption for a given mass. The maximum penetration force predicted from numerical models and their analytical estimates are in close agreement with that of experimental measurements.

Though all the conclusions drawn on sandwich plates were based on one set of experiments, the sample size is expected to have minimal effect on repeatability, so that overall conclusions remains valid.

Chapter 5

Indentation models for circular composite sandwich plates

In the preceding chapter, relative performance of the graded core sandwich plates is studied under low velocity impact loading. However, experiments are always expensive and time consuming. Hence, it is important for the engineers to have a design guide lines to predict the peak force and load-displacement response. This design values are possible using the analytical models. Negligible wave effects in low velocity impact conditions lead to model it as quasi-static loading condition. Hence, in the current chapter, an attempt is made to estimate the indentation failure load, load-displacement relations and local deformation profiles of the sandwich plates.

Most of the existing studies focused on the indentation of sandwiched beam or plates by line or point load. In this paper, analytical formulations are developed for the indentation of composite sandwich circular plates by a rigid, circular and flat-ended indenter, where the loading is considered on the outer circumference of the indenter. In all the formulations, the faceplate is considered as an elastic material. Firstly, the core is treated as an elastic foundation and a bounding condition is obtained to estimate the indentation failure load. Secondly, the core is treated as an EPP foundation and indentation load versus displacement relation is proposed. Both formulations are solved using indentation as well as radial compression analogy within the context of small deformation theory. To estimate the deformation behavior for indentation depths of order of thickness, large large deformations of the faceplate needs to be accounted for. Towards this, faceplates in-plane membrane stretching is included in the governing differential equations, and are solved using Galerkin's weighted residual method. The analytical predictions are compared with finite element calculations.

5.1 Assumptions

The following assumptions are made in the analytical modelling of the indentation problem analysis:

1. The deformation of the faceplates are small when compared to its thickness.
2. The sandwich plate is of infinite radius and through thickness in axisymmetric domain
3. Foam core undergoes yielding according to maximum normal stress criterion.
4. Flat punch is assumed to impose an axisymmetric line load (around the punch circumference) on the faceplate.

Based on the afore mentioned assumptions, two analogies (viz., indentation analogy and radial compression analogy) were used to understand the response of the sandwich plates subjected to localized loads.

5.2 Sandwich plate on a rigid base: indentation analogy

In this section, we present the solution for the indentation failure initiation load and explicit expression of the load versus displacement for a sandwich plate on a rigid backing are provided.

Consider a sandwich circular plate of radius R , consisting of a core of thickness c , sandwiched between two identical elastic faceplates each of thickness t . The sandwich plate rests on a rigid backing as shown in Figure 5.1a, and subjected to indentation load via a rigid circular indenter of diameter $2a$. The Young's modulus and Poisson's ratio of the core are E_c and ν_c , respectively; and σ_c is the compressive strength of the core material. The faceplates are made from a material with Young's modulus E_f and Poisson's ratio ν_f . Let P be the centrally applied load and δ the corresponding displacement of the punch during the indentation.

When the flat circular punch comes in contact with the sandwich plate, due to the relatively high modulus of the faceplate in comparison with that of the core, the deformed shape of the faceplate does not conform to shape of the punch; hence the faceplate experiences contact with the punch only along the punch circumference as shown in Figure 5.1b. In the following analysis, we assume the applied indentation load P is distributed over the circumference of a circle of radius a as an axisymmetric line load, $p = P/(2\pi a)$. Consequently, the contact interaction between the faceplate and the indenter in the region $0 \leq r \leq a$ is not considered in this analytical modeling.

Two cases are considered: (i) an elastic core and (ii) an elastic-perfectly plastic (EPP) core. The bending of a plate on an elastic foundation and subject to an axisymmetric line load has been

presented by Panc [132]. We begin by providing a summary of this analysis, for completeness. This is then used as a basis for the formulation for the indentation of a sandwiched plate with an elastic-perfectly plastic (EPP) core. The purpose of the analysis is three-fold to determine (i) the indentation failure load, (ii) the load versus displacement response of the indenter and (iii) the local deformed profile of the top faceplate when subjected to an arbitrary indentation load $P = 2\pi a p$.

Figure 5.1: Sandwich plate indentation analogy.

5.2.1 Plate on an elastic foundation

For the analytical treatment of the problem, an infinitely long top faceplate ($R \gg a$) is considered to be resting on an infinitely thick ($c \gg t$) elastic foundation with foundation stiffness k as shown in Figure 5.1b. Two coordinate systems are used in the analysis: coordinate system $\underline{r}-\underline{z}$ is centered at the intersection of the longitudinal axis of the punch and the mid surface of the sandwich plate while coordinate system $r-z$ is centered at the intersection of the longitudinal axis of the punch and the mid surface of the top faceplate (see Figure 5.1b).

The governing differential equation for a plate on an elastic foundation and subject to axisymmetric line load $p = P/(2\pi a)$ at a radial distance $r = a$ is given by [132]

$$D_f \nabla_r^2 \nabla_r^2 w + k w = 0 \quad (5.1)$$

where $w = w(r)$ is the transverse displacement at a radial distance r , D_f is the bending rigidity

of the faceplate and ∇_r^2 is the Laplacian operator in polar coordinates. D_f and ∇_r^2 are defined as

$$D_f = \frac{E_f t^3}{12 (1 - \nu_f^2)}; \quad \nabla_r^2 = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} = \frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right). \quad (5.2)$$

In Eq. (5.1), the foundation modulus k is the reaction pressure per unit displacement (N/mm²/mm) defined conventionally as $k = E_c/c$. As the core thickness $c \rightarrow \infty$, the foundation modulus $k \rightarrow 0$, which leads to a physically unreasonable value for k . Hence, to limit the value of the foundation modulus k for thick core, following limits are set [101]:

$$k = \frac{E_c}{c^* (1 - \nu_c^2)} \quad (5.3a)$$

$$c^* = \min(c, c^{3D}); \quad c^{3D} = \frac{128}{27} \left[\frac{2 D_f (1 - \nu_c^2)}{E_c} \right]^{1/3}. \quad (5.3b)$$

The general solution to the governing differential equation Eq. (5.1) is given by [132]

$$w(r) = A_1 J_0 \left(\frac{r}{l} e^{i\psi} \right) + A_2 J_0 \left(\frac{r}{l} e^{-i\psi} \right) + A_3 H_0^{(1)} \left(\frac{r}{l} e^{i\psi} \right) + A_4 H_0^{(2)} \left(\frac{r}{l} e^{-i\psi} \right) \quad (5.4)$$

where $\psi = \pi/4$, $l (= \sqrt[4]{D_f/k})$ is a characteristic length, J_n is the Bessel function of first kind of n^{th} index, $H_n^{(1)}$, $H_n^{(2)}$ are the first and second Hankel functions of n^{th} index, respectively and A_j ($j = 1$ to 4) are complex valued constants of integration.

By dividing the plate into two regions: an inner region $0 \leq r \leq a$ and an outer region $a \leq r \leq \infty$, the transverse displacement is expressed in the following form:

$$\text{for } 0 \leq r \leq a: \quad w(r) = w_i(r) = A_{1i} u_0(r) + A_{2i} v_0(r) + A_{3i} f_0(r) + A_{4i} g_0(r) \quad (5.5a)$$

$$\text{for } a \leq r \leq \infty: \quad w(r) = w_o(r) = A_{1o} u_0(r) + A_{2o} v_0(r) + A_{3o} f_0(r) + A_{4o} g_0(r) \quad (5.5b)$$

The expressions for f_n , g_n , u_n and v_n are given in Appendix A.

The conditions of finite deflection and vanishing slope at $r = 0$ and vanishing deflection at the far field ($r \rightarrow \infty$) give: $A_{4i} = A_{3i} = A_{1o} = A_{2o} = 0$.

The remaining integration constants, A_{1i} , A_{2i} , A_{3o} and A_{4o} are evaluated from the continuity of transverse displacement, slope, bending moment and shear force at $r = a$.

With the above stated boundary conditions, the deflection profile of a circular sandwich plate with elastic faceplates and elastic core, and subject to an arbitrary indentation load P via a rigid

circular punch of radius a is obtained as [132]

$$\text{for } 0 \leq r \leq a : \quad w_i(r) = \frac{P l^2}{4 D_f} [f_0(a) u_0(r) - g_0(a) v_0(r)] \quad (5.6a)$$

$$\text{for } a \leq r \leq \infty : \quad w_o(r) = \frac{P l^2}{4 D_f} [u_0(a) f_0(r) - v_0(a) g_0(r)] \quad (5.6b)$$

Equation (5.6) give the load-deflection response and the deformed shape of the top faceplate. The punch displacement δ at an applied load P is obtained from $\delta = w_i(r = a)$.

Guided by the results of FE simulations (as will be elaborated in Sec. 5.6.2), indentation failure is defined to occur when the core material beneath the punch has yielded. Hence, at the instant of indentation failure, the reactive pressure applied by the foundation on the faceplate is equal to the compressive yield strength of the core, σ_c . Thus, the punch deflection at failure δ_{el} , is given by

$$w(a) = \delta_{el} = \frac{\sigma_c}{k} \quad (5.7)$$

where k is the stiffness of the foundation given by Eq. (5.3). The corresponding indentation load at failure is obtained by substituting Eq. (6.8) in (5.6):

$$P_{el}^i = \frac{4 D_f \sigma_c}{k l^2} \frac{1}{u_0(a) f_0(a) - v_0(a) g_0(a)} \quad (5.8)$$

5.2.2 Plate on an elastic-perfectly plastic foundation

Solution for the indentation of a sandwiched plate consisting of elastic faceplates and an elastic-perfectly plastic (EPP) core is developed, following the method presented above for an elastic core. The indentation of a plate on an EPP foundation is shown in Figure 5.2. As before, the load P , from the punch is modelled as an axisymmetric line load of magnitude $p = P/(2\pi a)$ where a is the punch radius. Within the plastic zone $0 \leq r \leq \lambda$, the core has been compressed to its yield strength and thus exerts a uniform pressure of magnitude σ_c on the top faceplate. However, in the region $\lambda \leq r \leq \infty$, the deformation of the core is still elastic and the core exerts a load of magnitude $k w$ on the top faceplate, where w is the transverse displacement of the faceplate.

When the sandwich plate is subject to an increasing magnitude of indentation load, the deformation is initially elastic; at this stage the transverse displacement is as given in Eq. (5.6). When the applied load $P = P_{el}^i$, Eq. (5.8), the core will start to yield. The plastic radius λ at the onset of plastic yield in the core is assumed to be $\lambda = a$. For an indentation load $P > P_{el}^i$, the plastic zone will propagate outward with $\lambda > a$

For simplicity, we divide the deformation region into three zones: (i) the plastic zone in the

Figure 5.2: Indentation of an elastic plate on elastic perfectly plastic foundation.

interval $0 \leq r \leq a$ and transverse displacement w_{pi} , (ii) the plastic zone in the $a \leq r \leq \lambda$ with transverse displacement designated as w_{po} , and (iii) the elastic zone in the interval $\lambda \leq r \leq \infty$ with a transverse displacement denoted by w_e . The governing differential equations for the three zones are given by

$$\text{for } 0 \leq r \leq a: \quad D_f \nabla_r^2 \nabla_r^2 w_{pi} = -\sigma_c \quad (5.9a)$$

$$\text{for } a \leq r \leq \lambda: \quad D_f \nabla_r^2 \nabla_r^2 w_{po} = -\sigma_c \quad (5.9b)$$

$$\text{for } \lambda \leq r \leq \infty: \quad D_f \nabla_r^2 \nabla_r^2 w_e + k w_e = 0 \quad (5.9c)$$

Adopting a solution method similar to that for the elastic foundation, and by ensuring finite deflection and vanishing slope at $r = 0$ and vanishing deflection at $r = \infty$, the general solutions to the differential equations take the form

$$\text{for } 0 \leq r \leq a: \quad w_{pi}(r) = B_1 r^2 + B_2 - \frac{\sigma_c r^4}{64 D_f} \quad (5.10a)$$

$$\text{for } a \leq r \leq \lambda: \quad w_{po}(r) = B_3 r^2 \ln(r) + B_4 \ln(r) + B_5 r^2 + B_6 - \frac{\sigma_c r^4}{64 D_f} \quad (5.10b)$$

$$\text{for } \lambda \leq r \leq \infty: \quad w_e(r) = B_7 f_0(r) + B_8 g_0(r) \quad (5.10c)$$

where $f_0(r)$ and $g_0(r)$ are as defined in Appendix A with $\psi = \pi/4$, and B_j ($j=1$ to 8) are integration constants. The integration constants are determined from continuity of transverse displacement, slope, bending moment and shear force at both $r = a$ and $r = \lambda$, and the expressions for the constants are given in Appendix A.

An additional unknown is the relationship between the indentation load P and the plastic radius λ . For any arbitrary indentation load $P > P_{el}^i$, the extend of the plastic radius λ is found

using the condition at $r = \lambda$:

$$w_{po}(\lambda) = \frac{\sigma_c}{k} \quad (5.11)$$

Substituting Eq. (5.10b) in Eq. (5.11) leads to an explicit relation between indentation load P versus plastic radius λ as

$$P = \frac{\pi}{2 \eta_{12}} \left[8 \sqrt{2} l^2 \eta_7 (\sigma_c \lambda^2 - 2 \eta_{11}) - \sqrt{2} \eta_8 \left[\sigma_c \lambda^4 - 2 \frac{P_{el}^i}{\pi} (\lambda^2 - a^2) \right] + 16 \lambda l (\eta_6 \eta_{11} - 2 \sigma_c l^2 \eta_5) \right] \quad (5.12)$$

where expressions for η_j ($j = 1$ to 13) are given in Appendix A.

The implementation of the above solution is as follows. Consider a sandwich plate with an elastic-perfectly plastic core that rests on a rigid backing, and with given geometrical dimensions and material properties for the core and faceplates. When subject to an increasing indentation load P by a flat circular indenter of radius a , the deformation is initially elastic and the transverse displacement of the top faceplates at this stage can be determined from Eq. (5.6). When $P = P_{el}^i$, indentation failure occurs and the plastic zone $\lambda = a$; here the indentation failure load P_{el}^i , is given by Eq. (5.8). With further increase in the load beyond the indentation failure load, the plastic zone spreads, and the plastic zone at any given load $P > P_{el}^i$ is obtained by solving for λ in Eq. (5.12) and the corresponding deformation profile of the top faceplate is then obtained from Eq. (5.10).

The prediction from the analytical models (i.e. elastic foundation and elastic-perfectly plastic foundation) for a sandwich plate resting on a rigid backing will later be compared with finite element simulation results, in section 5.6. In the following section, the indentation response of a circular composite sandwich plate with clamped boundary is analyzed.

5.3 Clamped sandwich plate: radial compression analogy

Now we consider a circular sandwiched plate of radius R , that is fully clamped along the circumference and subject to a load P via a rigid circular punch of radius a . We assume $R \gg a$ and that the faceplates are elastic. The geometrical and material properties are denoted in a similar manner as for the sandwich plate resting on a rigid foundation.

During the bending of the sandwich plate due to the indentation loading, the top faceplate is under compression while bottom faceplate is under tension as shown in Figure 5.3a. The deformation of the sandwich plate is made up of two components: the global displacement of the sandwich plate due to bending and shear, and the displacement of the top faceplate due to localized indentation by the punch. First, we focus on the local deformation in order to determine the indentation load at failure. Later on we determine the total displacement of the indenter at a

given applied load.

For analytical modeling of the indentation behavior of the sandwich plate, an infinite long top faceplate is considered to undergo buckling on an infinitely thick foundation [113] as shown in Figure 5.3b. The punch load P induces a radial compressive force N in the top faceplate and a net radial shear force of intensity $p = P/(2\pi a)$ at $r = a$ as shown in Figure 5.3b. The magnitude of the faceplate in-plane compressive force, N depends on the extent of indentation depth and hence on radial bending moment, M . Consequently, the relationship between the radial moment M and radial compressive force N is required in order to determine the displacement profile.

Figure 5.3: Radial compression analysis of a circular clamped plate. (a) Symmetric bending of a sandwich plate and (b) Radial compression analogy of indentation failure.

5.3.1 Relationship between M and N

During the axisymmetric bending of an elastic circular sandwich plate, radial ϵ_r and circumferential ϵ_θ strains develop in the plate as given by

$$\epsilon_r = \epsilon_r^{(0)} + z \epsilon_r^{(1)}; \quad \epsilon_\theta = \epsilon_\theta^{(0)} + z \epsilon_\theta^{(1)} \quad (5.13a)$$

where

$$\epsilon_r^{(0)} = \frac{du}{dr} + \frac{1}{2} \left(\frac{dw}{dr} \right)^2; \quad \epsilon_r^{(1)} = -\frac{d^2w}{dr^2}; \quad \epsilon_\theta^{(0)} = \frac{u}{r}; \quad \epsilon_\theta^{(1)} = -\frac{1}{r} \frac{dw}{dr} \quad (5.13b)$$

In Eq. (5.13), $\epsilon_r^{(0)}$ and $\epsilon_\theta^{(0)}$ are the mid-surface strains, u is the in-plane radial displacement, w is the transverse displacement, and z is a transverse coordinate centered on the mid-surface of the sandwich plate, see Figure 5.3a. Using elastic stress-strain relation for plane stress, the resulting in-plane radial resultant force, N and radial moment M can easily be shown to be

$$N = \int_0^{c/2+t} \sigma_r d\bar{z} = \int_{c/2}^{c/2+t} \sigma_r d\bar{z} = \frac{E_f}{1-\nu_f^2} t \left[\epsilon_r^{(0)} + \frac{c+t}{2} \left(\epsilon_{rr}^{(1)} + \nu \epsilon_\theta^{(1)} \right) \right] \quad (5.14)$$

and

$$\begin{aligned}
 M &= \int_{-(c/2+t)}^{c/2+t} \sigma_r z dz = \int_{-(c/2+t)}^{-c/2} \sigma_r z dz + \int_{c/2}^{c/2+t} \sigma_r z dz \\
 &= \frac{t E_f}{1 - \nu_f^2} \left[\frac{(c+t)^2}{2} + \frac{t^2}{6} \right] \left(\epsilon_{rr}^{(1)} + \nu \epsilon_{\theta\theta}^{(1)} \right) \\
 &\approx \frac{E_f}{1 - \nu_f^2} t \frac{(c+t)^2}{2} \left(\epsilon_{rr}^{(1)} + \nu \epsilon_{\theta\theta}^{(1)} \right)
 \end{aligned} \tag{5.15}$$

where we have assumed zero in-plane displacement ($u = 0$) and that the magnitude of the radial stress and radial force in the core is much smaller than the corresponding value in the faceplate.

From radial Eq. (5.14) and Eq. (5.15), the resultant force N is therefore related to the radial bending moment M according to

$$\frac{N}{M} = \frac{1}{(c+t)} + \frac{2}{(c+t)^2} \frac{\epsilon_{rr}^{(0)}}{\left(\epsilon_{rr}^{(1)} + \nu \epsilon_{\theta\theta}^{(1)} \right)} \approx \frac{1}{c+t} \tag{5.16}$$

The relation between M and N during axisymmetric deformation of an elastic sandwich plate given in Eq. (5.16) can be shown to also be valid for beams within the context of small deformation. Having the relation between N and M , the radial compression analogy of a plate on elastic and EPP foundation will now be discussed

5.3.2 Radial compression of elastic plate on elastic foundation

The analysis of the radial compression of a plate on elastic foundation (Figure 5.3b) is presented here for the case of small deformation. The governing differential equation for a plate resting on an elastic foundation can be written as

$$D_f \nabla_r^2 \nabla_r^2 w + N \nabla_r^2 + k w = 0 \tag{5.17}$$

In obtaining the solution to the governing Eq. (5.17), we have as before divided the deformed plate into two domains: an inner domain $0 \leq r \leq a$ with the transverse displacement $w_i(r)$ and an outer domain region $a \leq r \leq \infty$ with transverse displacement $w_o(r)$. By defining a non-dimensional radius, $\rho = r/l$, where $l = \sqrt[4]{D_f/k}$ and making use of the relationship between radial force N and radial bending moment M from Eq. (5.16), the general solution to Eq. (5.17)

can be shown to be

$$\text{for } 0 \leq r \leq a: \quad w = w_i(r) = C_{1i} u_0(r) + C_{2i} v_0(r) + C_{3i} f_0(r) + C_{4i} g_0(r) \quad (5.18a)$$

$$\text{for } a \leq r \leq \infty: \quad w = w_o(r) = C_{1o} u_0(r) + C_{2o} v_0(r) + C_{3o} f_0(r) + C_{4o} g_0(r) \quad (5.18b)$$

where u_0 , v_0 , f_0 and g_0 are as defined in Appendix (A), but with ψ given by

$$\psi = \frac{1}{2} \left(\pi - \tan^{-1} \frac{\sqrt{1-\chi^2}}{\chi} \right) \quad \text{where} \quad \chi = \frac{P(1+\nu_f)}{16\pi(c+t)\sqrt{kD_f}} \left[2 \ln \left(\frac{R}{a} \right) + \frac{a^2}{R^2} - 1 \right] \quad (5.19)$$

The conditions of finite deflection and zero slope at $r = 0$ and zero deflection as $r \rightarrow \infty$ lead to $C_{3i} = C_{4i} = C_{1o} = C_{2o} = 0$. The displacement functions thus reduce to

$$\text{for } 0 \leq r \leq a: \quad w_i(r) = C_{1i} u_0(r) + C_{2i} v_0(r) \quad (5.20a)$$

$$\text{for } a \leq r \leq \infty: \quad w_o(r) = C_{3o} f_0(r) + C_{4o} g_0(r) \quad (5.20b)$$

The remaining integration constants C_{1i} , C_{2i} , C_{3o} and C_{4o} are evaluated using continuity conditions for displacement, slope, moment and shear force at $r = a$. The expressions for these constants of integration are given in Appendix C. Equation (5.20) enable the determination of the deformed shape of the top faceplate at any applied indentation load $P (= 2\pi ap)$; this does not include the global deformation of the sandwich plate due to the overall bending and shear.

The crushing of the core and hence the indentation failure of the sandwich is determined by the local deformation profile of the top faceplate and not the global deformation of the sandwich. Consequently the load at the onset of indentation failure is determined from the condition $w_i(r = a) = \sigma_c/k$; Using this condition in Eq. (5.20a) this leads to an implicit expression that can be solved for the limit load P_{el}^{rc} based on the radial compression analogy:

$$[C_{1i} u_0(a) + C_{2i} v_0(a)] - \frac{\sigma_c}{k} = 0 \quad (5.21)$$

The load versus displacement response of the punch includes the global displacement associated with bending and shear. The total displacement δ of the punch at a given applied indentation load P is therefore given by

$$\delta = \frac{P}{k_b} + \frac{P}{k_s} + w_i(r = a) \quad (5.22)$$

where the local displacement w_i is given by Eq. (5.20), and k_b and k_s are the bending and

shear stiffness of the sandwich plate, respectively, and are given by

$$\begin{aligned}
 k_b &= \frac{16 \pi D_{sw}}{a^2} \left[\frac{R^2}{a^2} - \frac{a^2}{R^2} - 4 \ln \left(\frac{R}{a} \right) \right]^{-1} \\
 k_s &= \frac{2 \pi S_{sw}}{\ln(R/a)} \\
 D_{sw} &= \frac{E_f c^2 t}{2 (1 - \nu_f^2)} \left(1 + 2 \frac{t}{c} + \frac{4}{3} \frac{t^2}{c^2} + \frac{E_c c}{6 E_f t} \frac{1 - \nu_f^2}{1 - \nu_c^2} \right) \\
 S_{sw} &= \kappa (c G_c + 2 t G_f)
 \end{aligned} \tag{5.23}$$

where D_{sw} and S_{sw} are the equivalent bending rigidity and shear stiffness of the sandwich plate respectively, R is the radius of the plate, a is the punch radius, E_f is the Young's modulus of the faceplate, G_c and G_f are the shear modulus of the core and the faceplates respectively, and κ is the shear correction factor estimated using Madabhushi-Raman and Davalos [133] formulation. Having the overall stiffness of the sandwich plate, the displacement δ of the punch at a given load P can be determined by Eq. (5.22).

5.3.3 Radial compression of an elastic plate on an elastic-perfectly plastic foundation

In this section, the load versus displacement, plastic radius for a given load and deformation profile is estimated using the analogy of radial compression of the top faceplate on an elastic-perfectly plastic (EPP) foundation, for clamped sandwich plate subjected to an axisymmetric line load $p = P/(2\pi a)$

A typical instance of radial compression of the top faceplate on EPP foundation is shown in Figure 5.4. Similar to the approach used in Sec. 5.2.2, we assume that the core is already crushed in the region $0 \leq r \leq a$ with a load of P_{el}^{rc} . Hence, the minimum value for the plastic radius, λ is a and the corresponding indentation load is P_{el}^{rc} , which is calculated using Eq. (5.21). Any arbitrary value of indentation load, $P > P_{el}^{rc}$ leads to an increase in the plastic radius, λ .

Figure 5.4: Radial compression of plate on elastic perfectly plastic foundation.

The problem is divided into three regions and the governing differential equation of the three regions is written as:

$$\text{for } 0 \leq r \leq a: \quad D_f \nabla_r^2 \nabla_r^2 w_{pi} + N \nabla_r^2 w_{pi} = -\sigma_c \quad (5.24a)$$

$$\text{for } a \leq r \leq \lambda: \quad D_f \nabla_r^2 \nabla_r^2 w_{po} + N \nabla_r^2 w_{po} = -\sigma_c \quad (5.24b)$$

$$\text{for } \lambda \leq r \leq \infty: \quad D_f \nabla_r^2 \nabla_r^2 w_e + N \nabla_r^2 w_e + k w_e = 0 \quad (5.24c)$$

The general solution to the differential equations is

$$\text{for } 0 \leq r \leq a: \quad w_{pi}(r) = F_{1i} + F_{2i} \ln(r) + F_{3i} J_0(\beta r) + F_{4i} Y_0(\beta r) - \frac{\sigma_c r^2}{4N} \quad (5.25a)$$

$$\text{for } a \leq r \leq \lambda: \quad w_{po}(r) = F_{1o} + F_{2o} \ln(r) + F_{3o} J_0(\beta r) + F_{4o} Y_0(\beta r) - \frac{\sigma_c r^2}{4N} \quad (5.25b)$$

$$\text{for } \lambda \leq r \leq \infty: \quad w_e(r) = F_{1e} u_0(r) + F_{2e} v_0(r) + F_{3e} f_0(r) + F_{4e} g_0(r) \quad (5.25c)$$

where, $f_0(r)$, $g_0(r)$, $u_0(r)$ and $v_0(r)$ are defined in Eq. (A.1) with ψ value according to Eq. (5.19), Y_0 is the Bessel function of second kind expressed in Appendix A, $\beta = \sqrt{N/D_f}$, and N is the radial compressive force in the faceplate. To have a finite deflection at $r = 0$ and zero deflection at $r = \infty$; $F_{2i} = F_{4i} = F_{1e} = F_{2e} = 0$. Hence (5.25) reduce to

$$\text{for } 0 \leq r \leq a: \quad w_{pi}(r) = F_1 + F_2 J_0(\beta r) - \frac{\sigma_c r^2}{4N} \quad (5.26a)$$

$$\text{for } a \leq r \leq \lambda: \quad w_{po}(r) = F_3 + F_4 \ln(r) + F_5 J_0(\beta r) + F_6 Y_0(\beta r) - \frac{\sigma_c r^2}{4N} \quad (5.26b)$$

$$\text{for } \lambda \leq r \leq \infty: \quad w_e(r) = F_7 f_0(r) + F_8 g_0(r) \quad (5.26c)$$

As before, the eight unknown constants F_j ($j = 1$ to 8) are found using the continuity conditions at $r = a$ and $r = \lambda$ and are given in Appendix D.

Additionally, to find the relation between the applied load, P and the plastic radius λ , we use moment continuity at $r = \lambda$ and the displacement boundary condition Eq. (5.11) to obtain the implicit relation as

$$\frac{F_7}{l^2} \left[\frac{l}{\lambda} (1 - \nu_f) \Upsilon_1 - \Upsilon_3 \right] + \frac{F_8}{l^2} \left[\frac{l}{\lambda} (1 - \nu_f) \Upsilon_2 - \Upsilon_4 \right] = -\frac{M}{D_f} \Big|_{r=\lambda} \quad (5.27)$$

where the radial moment, M is

$$M = -D_f \left[\frac{\beta}{r} (1 - \nu_f) (F_6 Y_1(\beta r) - F_5 J_1(\beta r)) + \beta^2 (F_6 Y_0(\beta r) - F_5 J_0(\beta r)) - \frac{\sigma_c}{2N} (1 + \nu_f) - \frac{F_4}{r^2} (1 - \nu_f) \right] \quad (5.28)$$

All the constants in Eq. (5.27) and Eq. (5.28) are defined in Appendix D.

The punch displacement δ at a given load P includes the displacement of the sandwich due to bending and shear, as for the elastic foundation, is given by

$$\delta = \frac{P}{k_b} + \frac{P}{k_s} + w_{pi}(r = a) \quad (5.29)$$

where the bending stiffness k_b and shear stiffness k_s of the sandwich plate are given by Eq. (5.23).

5.4 Indentation analogy: RPP foundation (large deformation theory)

The analysis presented above is based on the assumption of small deformation, which is only valid for punch (or top faceplate) displacement of about $t/2$ where t is the faceplate thickness. To predict the indentation behavior of the sandwich plate subjected to large deformation, the in-plane displacement, u needs to be considered, and the in-plane radial stretching force, N_r (same as N in the preceding sections) and circumferential stretching force N_θ in the faceplate become predominantly important. In this section, large deformation of the top faceplate is considered and resulting set of differential equations are solved by using Galerkin's weighted residual method. Towards this, a thin elastic plate is considered to be resting on RPP foundation as shown in Figure 5.5.

Figure 5.5: Indentation of a plate on RPP foundation.

Governing differential equations [84] for a plate resting on RPP foundation considering large deformation effect is shown to be

$$D_f r \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) \right] = r Q + r N_r \frac{dw}{dr} \quad (5.30a)$$

$$r \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (r^2 N_r) \right] = -\frac{E_f t}{2} \left(\frac{dw}{dr} \right)^2 \quad (5.30b)$$

where Q is shear force at a radial distance r . The circumferential stretching force, N_θ is related to the radial stretching force N_r as follows:

$$N_\theta = \frac{d}{dr} \left(r \frac{dN_r}{dr} \right) \quad (5.31)$$

Additionally, in-plane displacement can be related to radial stretching force as

$$u_r = \frac{1}{E_f t} \left[r \frac{dN_r}{dr} + (1 - \nu_f) N_r \right] \quad (5.32)$$

For a plate with axisymmetric line load at a radius $r = a$, the discontinuous distribution in shear force can be written as a single function using Heaviside step function, $\mathcal{H}$ as

$$r Q = \frac{P}{2\pi} \mathcal{H}(r - a) - \frac{\sigma_c r^2}{2} \quad (5.33)$$

where

$$\mathcal{H}(r - a) = \begin{cases} 0 & \text{if } r < a, \\ 1 & \text{if } r \geq a. \end{cases} \quad (5.34)$$

where $P = 2\pi a p$ is the punch load on the RPP foundation.

To start with the method of Galerkin weighted residual method, a transverse displacement function, $w(r)$, is assumed to be represented as a single polynomial from the region $0 \leq r \leq \lambda$ (against different functions for regions $0 \leq r \leq a$ and $a \leq r \leq \lambda$ as assumed in preceding sections) and is represent as

$$\text{for } 0 \leq r \leq \lambda: w(r) = w_0 \left(1 - \frac{r^2}{\lambda^2} \right)^3 = w_0 \left[1 - 3 \frac{r^2}{\lambda^2} + 3 \frac{r^4}{\lambda^4} - \frac{r^6}{\lambda^6} \right] \quad (5.35)$$

which satisfies the following boundary conditions

$$\begin{aligned} \text{at } r = 0: & \quad w = w_0; \quad w' = 0 \\ \text{at } r = \lambda: & \quad w = 0; \quad w' = 0; \quad M = 0 \end{aligned} \quad (5.36)$$

where $w_0 = w(r = 0)$ is the displacement at the center of the plate. Substituting Eq. (5.35)

in Eq. (5.30b) followed by successive integration gives

$$N = -w_0^2 \frac{E_f t}{2} \left[\frac{3}{10} \frac{r^{10}}{\lambda^{12}} - \frac{9}{5} \frac{r^8}{\lambda^{10}} + \frac{9}{2} \frac{r^6}{\lambda^8} - 6 \frac{r^4}{\lambda^6} + \frac{9}{2} \frac{r^2}{\lambda^4} \right] + \frac{\overline{C_1}}{2} + \frac{\overline{C_2}}{r^2} \quad (5.37)$$

within the region $0 \leq r \leq \lambda$.

To have a finite value of N at $r = 0$, $\overline{C_2}$ must be zero. Using the boundary condition $N = 0$ at $r = \lambda$ gives

$$\text{for } 0 \leq r \leq \lambda : N = -w_0^2 \frac{E_f t}{4} \left[\frac{3}{10} \frac{r^{10}}{\lambda^{12}} - \frac{9}{5} \frac{r^8}{\lambda^{10}} + \frac{9}{2} \frac{r^6}{\lambda^8} - 6 \frac{r^4}{\lambda^6} + \frac{9}{2} \frac{r^2}{\lambda^4} - 3 \frac{1}{\lambda^2} \right] \quad (5.38)$$

Having the expression for deflection w from Eq. (5.35) and in-plane compressive force N from Eq. (5.38), one can perform successive integration of Eq. (5.30a) to get the expression for central deflection w_0 . However, for the sake of simplicity, Eq. (5.30a) is solved using Galerkin's weighted residual method. Substituting Eq. (5.38) in Eq. (5.30a) and solving the differential equation by minimizing the weighted residue gives

$$\frac{3}{112} E_f t w_0^3 + \frac{6}{5} D_f w_0 + \frac{1}{80} \lambda^4 \sigma_c - \frac{P \lambda^2}{16 \pi} \left(1 - \frac{a^2}{\lambda^2} \right)^4 = 0 \quad (5.39)$$

Rearranging Eq. (5.39) leads to

$$w_0^3 + \vartheta_1 w_0 + \vartheta_2 = 0 \quad (5.40)$$

where

$$\vartheta_1 = \frac{56 t^2}{15 (1 - \nu_f^2)}; \quad \vartheta_2 = \frac{7}{15} \frac{\lambda^4 \sigma_c}{E_f t} - \frac{7 P \lambda^2}{3 \pi E_f t} \left(1 - \frac{a^2}{\lambda^2} \right)^4$$

Positive root to Eq. (5.40) can be written as

$$w_0 = \varpi_1 + \varpi_2 \quad (5.41)$$

where

$$\begin{aligned} \varpi_{1,2} &= \left(-\frac{\vartheta_2}{2} \pm \sqrt{\vartheta_3} \right)^{1/3} \\ \vartheta_3 &= \left(\frac{\vartheta_1}{3} \right)^3 + \left(\frac{\vartheta_2}{2} \right)^2 \end{aligned} \quad (5.42)$$

The load displacement relation is found using the shear force boundary condition at $r = \lambda$

(assuming $\lambda > a$ in Eq. (5.33))

$$2 \pi \lambda Q(r = \lambda) = P - \pi \lambda^2 \sigma_c \quad (5.43)$$

Evaluating shear force according to the assumed displacement function, Eq. (5.35) gives

$$Q(r) = -D_f \frac{d^2}{d} \left(\frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right) = \frac{48 D_f w_0 r}{\lambda^6} (3 r^2 - 2 \lambda^2) \quad (5.44a)$$

$$Q(r = \lambda) = \frac{96 w_0 D_f}{\lambda^3} \quad (5.44b)$$

This shear force, Q at $r = \lambda$ should act downwards on the plate to have a contact with the RPP foundation. Hence, in the calculations, a negative sign needs to be considered. Substituting Eq. (5.44b) in Eq. (5.43) gives an implicit expression between two variables λ and P as follows

$$P + \frac{64 \pi w_0 D_f}{\lambda^2} - \pi \lambda^2 \sigma_c = 0 \quad (5.45)$$

For any given value of λ (that is $> a$), the load indentation load, P is calculated from the preceding expression since w_0 is a function of λ and P . Hence, using Eq.(5.41), displacement of the top faceplate at the origin is evaluated. Displacement of the punch is found using Eq. (5.35) at $r = a$. Hence the load-displacement relation is established.

5.5 Finite element modeling

The accuracy of the analytical models for the indentation behavior of sandwich plates is verified by comparison with finite element (FE) predictions. To predict the indentation response of the sandwich plates, axisymmetric FE models were developed in ABAQUS CAE version 6.11. The computational geometry along with the loading and boundary conditions are shown in Figure 5.6. Faceplates and core were meshed using CAX4R (4-node bilinear axisymmetric reduced integration) elements while the flat punch (FP) was modeled as an analytical rigid body with a fillet radius of 0.5 mm along the punch periphery. To understand the effect of the analytical analogy implemented, indentation simulations are also conducted using ring punch (RP). The core was meshed with 60 elements in the thickness direction and faceplate is modeled using 4 elements in thickness direction, while maintaining 200 elements in the radial direction for both faceplate and core. Stiffness hourglass control was used to avoid spurious energy modes resulting from reduced integration. Smooth displacement loading was applied to the punch to simulate the quasi-static loading condition.

Faceplates were modeled as a linear elastic, isotropic material with Young's modulus of

25.3 GPa and a Poisson's ratio of 0.24. These are representative properties of quasi-isotropic $[-60/0/60]_{ns}$ E-glass/epoxy laminate.

The material properties of the core used in the simulation were obtained by the experiment. Closed cell PVC foams of grades H35, H45, H80 and H100 (supplied by Diab Inc.) were tested in uniaxial compression. The specimens have a 50 mm by 50 mm square cross section, and a height of 25 mm for H35, 20 mm for H45 and H80, and 17 mm for H100. All the tests were conducted on Instron 5567 at a nominal speed of 1 mm/min. The measured uniaxial compression response of the foams together with the assumed ideal elastic-perfectly plastic curves are shown in Figure 5.7 while the measured properties are given in Table 5.1.

Core was modeled using two methodologies: (a) continuum foam model in Fig. 5.6a and (b) interaction model in Fig. 5.6b. In the continuum foam model, elastic behavior is governed by the Young's modulus and Poisson's ratio values and an elastic-perfectly plastic (EPP) foam was modeled using the crushable foam material model of Deshpande and Fleck [21] with isotropic hardening and a compression yield stress ratio of 1.1. The faceplates are tied to the foam core to avoid interfacial adhesive failure. The compressive stress-strain responses of all the foam densities are shown in Fig. 5.7.

Figure 5.6: FE modelling strategies of the sandwich plate. Where, $a = 10$ mm, $c = 50$ mm, $t = 0.7$ mm and $R = 8a$ are used.

In interaction model, the core and bottom faceplate are not modelled explicitly rather the reaction force (from the foam core/foundation) is applied on the faceplate bottom surface (usually termed as a slave surface in interaction terminology). This is achieved using Abaqus user interaction subroutine UINTER with input variables as k and σ_c . To define the interaction, a dummy

rigid surface (termed as master surface) is modeled to interact with a deformable slave surface. All the degrees of freedom of the rigid master surface are constrained. Elastic foundation is simulated by applying the “ $k w$ ” pressure on the slave surface. In simulating the EPP foundation, if “ $k w$ ” is greater than σ_c then a reaction force of σ_c is applied on the top faceplate bottom. Hence, this is considered as a maximum (through-thickness) normal stress yield criterion, similar to the yield criterion used in the analytical modeling.

The UINTER methodology can not be used to simulate the radial compression analogy as both top as well as bottom faceplate undergo deflection. Hence, only CF model with *Crushable_foam option is used for radial compression analogy.

Figure 5.7: Uniaxial compressive stress-strain response of PVC foam for various densities as marked.

Table 5.1: Properties of foams

Property	Foams*			
	H35	H45	H80	H100
Density (kg/m ³)	35	45	80	100
Young's Modulus (MPa)	22	35	49	52
Yield strength (MPa)	0.5	0.65	1.2	2
Yield strain (%)	2.273	1.857	2.449	3.846

* For all the foams an elastic Poisson's ratio of 0.0, plastic Poisson's ratio of 0.0 are used.

To be consistent with the analytical formulations which were based on small deformation

theory, the simulation was carried out with the non-linear geometry option in Abaqus, i.e. NL-GEOM, switched off.

To simulate the indentation analogy (CF model), the bottom faceplate was constrained in all degrees of freedom; whereas to simulate the radial compression analogy (CF model), the sandwich plate circumference was constrained in all degrees of freedom.

5.6 Results and discussion

In this section, analytical (stiffness, load-displacement relation, punch displacement-plastic radius and deflection profiles of top faceplate) estimates for the plates on EF and EPP foundation (using indentation and radial compression analogy) are compared against the FE predictions. Initially, the elastic solution for the indentation analogy (taken from [132]) is used as a benchmark solution to assess the stiffness prediction from different FE models under consideration.

Later, the load-displacement and punch displacement versus plastic radius estimates (for the EPP foundation) are compared against the FE predictions. Finally, the deflection profiles (for EF and EPP foundations) are compared for the indentation analogy. After conforming that the small deformation solution is in agreement with FE predictions, large deformation solution (RPP foundation) is compared with the FE prediction.

5.6.1 Elastic foundation: small deformation theory

Stiffness estimates from proposed FE methodologies viz., CF and UINTER model are compared with the analytical estimates from Panc [132]. In FE models, the stiffness is predicted using one step solution. A comparison of stiffness estimates among different strategies are listed in Table 5.2.

Table 5.2: Comparison of stiffness prediction from different methodologies using indentation analogy.

		Methodology									
		FE									
Foam	Anal.	Continuum foam					UINTER				
		FP	†	RP	‡	§	FP	†	RP	‡	§
H35	957.7	964.7	-0.7	985.1	-2.9	-2.1	966.6	-0.9	991.4	-3.5	-2.6
H45	1587.7	1472.6	7.2	1498.0	5.6	-1.7	1604.3	-1.0	1629.9	-2.7	-1.6
H80	2275.9	2004.7	11.9	2030.4	10.8	-1.3	2329.3	-2.3	2318.2	-1.9	0.5
H100	2423.4	2117.4	12.6	2142.4	11.6	-1.2	2489.0	-2.7	2465.0	-1.7	1.0
† $\frac{\text{Anal} - \text{FP}}{\text{Anal}} \times 100;$		‡ $\frac{\text{Anal} - \text{RP}}{\text{Anal}} \times 100;$				§ $\frac{\text{FP} - \text{RP}}{\text{FP}} \times 100$					

The percentage difference between the flat punch (FP) and ring punch (RP) (in both CF and UINTER models) reveals that the maximum percentage difference in stiffness is 2.6%, which decreases with increasing foam density.

Using continuum foam model, the percentage error between the FP and analytical estimates increases with respect to increase in foam density. Similar trends are observed in RP stiffness predictions. This due to the differences in representing the foam yield behavior in numerical (*Crushable_foam) and analytical model.

The stiffness estimates from UINTER model reveal that the percentage error in FP predictions with respect to analytical estimates increases with respect to foam density. This is possible due to the difference in loading condition i.e. in analytical modeling the load is considered to be axisymmetric line load. However, the predictions from RP model is shows a decrease in percentage error w.r.t increase in foam density.

Hence, it is evident that the analytical models are better validated using either FP/RP using UINTER models.

The load-displacement values are compared between the CF model using FP and the analytical estimates in the Section 5.6.3, for the radial compression analogy.

5.6.2 Indentation limit load

In this section, the efficacy of the proposed bounding condition in predicting the indentation failure load is verified. In the FE simulation, indentation failure initiation is defined as the instant of change in slope in the load versus displacement curve (to be discussed later). As shown in Figure 5.8a, the predicted indentation failure loads by both the indentation and radial compression analogy are in good agreement with the finite element prediction for all the foam densities considered. We note that the failure load increases with increasing foam density, and there is insignificant difference between the estimated failure load for a sandwich plate resting on a rigid backing (i.e. indentation analogy) and a circular plate with clamped edge (radial compression analogy). This ensures the core indentation failure is dictated by the local deformation characteristics of the sandwich plate.

(a) Indentation analogy

(b) Radial compression analogy

Figure 5.8: Effect of foam density on indentation failure load. Analytical estimates for P_{el}^i and P_{el}^{rc} are calculated using Eq. (5.8) and Eq. (5.21), respectively.

The indentation failure load shown in Figure 5.8 can be fitted to the relation

$$\frac{P_{el}^i}{\sigma_s a^2} = \alpha \left(\frac{\rho_f}{\rho_s} \right)^\beta \quad (5.46)$$

where ρ_f is the foam density, σ_s and ρ_s are the yield strength and density of foam solid cell wall material. Solid PVC has a typical density ρ_c of 1300 kg/m³ and yield strength σ_c of 35.4 MPa. The results of the current analysis are described by Eq. (5.46) with $\alpha = 4.12$ and $\beta = 1.1$.

5.6.3 Load versus displacement response: small deformation theory

In the preceding section, the limit load corresponds to the load to create the plastic radius, $\lambda = a$. This section provides the load versus displacement relationship when the plastic radius is $\lambda > a$. The load versus displacement response for a sandwich plate with elastic-perfectly plastic core predicted using the indentation analogy Eq. (5.12) and faceplate radial compression analogy Eq. (5.29) is shown in Figure 5.9 and Figure 5.10, respectively: it is a typical bi-linear response with two distinct linear portions with varying slopes. The analytical load-displacement response is compared with the corresponding FE predictions in same figure. The predicted initial elastic stiffness and the load-displacement response for $P \leq P_{el}$ are in good agreement with the corresponding FE solution for both the indentation and radial compression analogies.

As the deformation progresses, beyond the initial failure, the FE predictions (from UINTER model) are in good agreement in the indentation analogy. However, the discrepancy between the analytical estimates and the FE (CF model) predictions is quite high in both indentation analogy and faceplate radial compression analogy. This is due to the difference in the yield criterion implemented in the CF model (Deshpande and Fleck [21] criterion) and analytical formulations (maximum normal stress criterion). Since FE and analytical models are based on small deformation theory, both are compared for $w(a)/t$ of 4.0.

5.6.4 Evolution of the plastic radius: small deformation theory

When the applied punch displacement equals the indentation failure initiation load, the analytical model predicts a plastic radius, $\lambda = a$. The through thickness stress contours at the instant of indentation failure is shown as inserts in Figure 5.8a: the extent of the plastic zone, λ at the onset of indentation failure equal to punch radius a .

When the applied punch displacement δ is greater than the elastic limit δ_{el} , the plastic zone spreads towards the outer boundary of the sandwich plate. In the FE simulation, the plastic zone was quantified by determining the location when the equivalent plastic strain reached the yield strain value at the surface of the core in contact with the top faceplate. The evolution of the plastic radius λ with the punch displacement is shown in Figure 5.11a and 5.11b for indentation analogy and radial compression analogy, respectively. For an applied punch displacement, $\delta > \delta_{el}$ there

Figure 5.9: Comparison of load-displacement responses by different formulations using indentation analogy.

is reasonable agreement between the FE and the analytical values of plastic radius. A small discrepancy between the FE (CF models) prediction and the analytical estimate is due to the difference in the yield criterion.

5.6.5 Deformed shape of the top faceplate: small deformation theory

In this section, the deformation profile estimates for the indentation analogy is compared against the FE predictions (for EF and EPP foundations).

The deformation profiles for elastic foundation (indentation analogy) is shown in Figure 5.12. For elastic foundation with low density foam (viz., H35) reveal that the punch is in contact with

Figure 5.10: Indentation load P versus punch displacement $\delta = w(a)$ for sandwich plates with EPP core.

the top faceplate around the circumference. However, as the foam density increases, the faceplate conforms to the punch shape. For H100 foam, the deformation profile in the domain $0 \leq r \leq a$, reveal that analytical deflection profiles are in reasonable agreement with the FE (UINTER-RP) prediction as UINTER-RP model is also simulating the axisymmetric line load. The FE models with FP are conforming to the punch shape as predicted by CF and Uinter models.

However from the EPP foundation deformation profiles (Figure 5.13) reveal that the punch is always experiencing contact only at its circumference irrespective of the foam density. Hence, it is reasonable to consider the flat punch loading as an axisymmetric line-load along its circumference. This is clearly observed from the deformation profiles in the domain $0 \leq r \leq a$.

Irrespective of the foundation type (viz., EF and EPP) and punch geometry (viz., FP, RP), the analytical deflection profile estimates in the region $r \geq a$ are in good agreement with FE predictions using UINTER model. However, the FE predictions using CF model are deviating far from the analytical estimates with no wave characteristic response as also observed by Steeves and Fleck [113]. This deviation is solely due to the differences in the foam yield criterion.

In summary, there is a good agreement (of deflection profiles) between the analytical estimates and FE prediction using UINTER model. However the FE predictions from CF model are showing a discrepancy due the differences in the yield criterion.

(a) Indentation analogy

(b) Radial compression analogy

Figure 5.11: Punch displacement versus plastic radius. Plastic radius for radial compression analogy is estimated using Eq. (5.27).

5.6.6 Indentation on RPP foundation: large deformation theory

In this section, the efficacy of the proposed analytical model for the large deformation of plate is compared against the FE predictions (consisting EPP foam core). Estimated load versus displacement relation is compared with the FE predictions in Figure 5.14a i.e. for rigid foundation

Figure 5.12: Comparison of deformation profiles by different formulations using indentation analogy. In figure deformation profile for elastic foundation is estimated using Eq. (5.6) at $\delta = \delta_{el}$.

formulation assumption there is no deformation until the loads attain a critical value. At zero indentation depth, punch experiences a preload to equilibrate the reaction from RPP foundation. As the indentation progresses, good agreement (of load-displacement response) is observed between FE predictions and analytical estimates. Having infinite foundation modulus ($k = \infty$) for RPP foundation, the failure initiation loads based on RPP foundation analogy is higher in comparison to that for a foundation with finite modulus (elastic foundation) as shown in Figure 5.14b. As the RPP foundation can not be simulated the deformation profiles (as shown in Figure 5.15a) and load versus plastic radius (as shown in Figure 5.15b) are not compared against the FE predictions.

Faceplate deflection profiles from analytical and FE (UINTER) predictions (as shown in

Figure 5.13: Comparison of deformation profiles by different formulations using indentation analogy. In figure deformation profile for elastic foundation is estimated using Eq. (5.10) at $\delta = 3 \times \delta_{el}$.

Figures 5.12 and 5.13) reveal that, the faceplate deflection profile on an elastic as well as EPP foundation shows a damped harmonic oscillation characteristic feature leads to a tensile stress in the foundation at far field as observed in [113, 134]. However, this tensile stresses in the foundation are negligible in the FE (CF model) predictions. Additionally, in the RPP foundation deformation profiles reveals the absence of tensile stresses in the foundation as shown in 5.15a.

Figure 5.14: Indentation analogy comparison between FE prediction and analytical estimate. In (a) P_r^i is estimated from Eq. 5.45. In (b) RPP foundation estimates corresponds to zero punch displacement.

Figure 5.15: Deformation profile and plastic radius evolution in large deformation of plates. (a) Analytical deformation profile, Eq. (5.35) of a plate (with large deformation) on RPP foundation using indentation analogy at $\lambda = 1.5 a$. (b) Plastic radius versus applied load, Eq. (5.45) on RPP foundation using indentation analogy.

5.7 Summary

Analytical formulations are proposed to estimate the indentation response of composite sandwich plates subjected to quasi-static indentation by a rigid circular flat ended punch. The faceplate was

assumed to have linear elastic behavior with axisymmetric deformation and the indentation response has been modeled using conventional indentation analogy and recently proposed faceplate radial compression analogy.

A bounding condition is proposed to estimate the indentation failure load. Since typical polymer and metallic foams show EPP response, the elastic foundation analogy is extended to EPP foundation. These formulations are solved assuming small deformations of the faceplate.

The analytical model predicts the indentation failure initiation loads accurately well. Although the radial compression analogy is able to predict the failure initiation load, the load versus displacement response and load versus plastic radius response deviate from the FE predictions. Reasonable agreement is observed between FE predictions and analytical estimates for the load versus indentation displacement, plastic radius and deflection profile of the top faceplate at applied loads that satisfy small deformation theory conditions.

In order to accurately predict the load versus displacement response for indentation depths that are much greater than the faceplate thickness, large deformations are considered in the faceplates by considering core as RPP foundation. Good agreement (in terms of stiffness, load-displacement response, punch displacement-plastic radius and faceplate deflection profiles) is observed between analytical estimates and experimental predictions.

Chapter 6

Simplified failure analysis of circular composite sandwich plates under bending

In the preceding chapter, indentation of composite sandwich plates subjected to quasi-static indentation loading is studied by considering the faceplates as elastic with core being either elastic or elastic perfectly plastic. In real world, sandwich structures under localized loading conditions undergo not only core indentation failure but also core shear and faceplate failure depending on geometrical, material and loading parameters. The effect of these parameters on the operative failure mechanism is studied in this chapter.

6.1 Bending response of composite sandwich plates

In this section, analytical formulae for the elastic stiffness and failure initiation load of a clamped circular sandwich plate are presented.

6.1.1 Stiffness response

Consider a clamped circular sandwich plate with symmetric faceplates perfectly bonded to foam core, which is loaded by a rigid circular flat-ended punch at the center of the plate as shown in Fig. 6.1. Let R be the plate radius, a the punch radius, c the core thickness and t the faceplate thickness. The Young's and shear moduli of core are E_c and G_c , respectively; σ_c and τ_c are the strengths of the core material in uniaxial compression and shear loading conditions, respectively. The faceplates of the sandwich has Young's modulus E_f , Poisson's ratio ν_f and strength σ_f . Let P be the centrally applied load and δ the corresponding displacement of the punch during the bending of sandwich plate.

For mathematical analysis, following non-dimensional geometrical (viz., $\bar{c}$, $\bar{t}$, $\bar{R}$), and mate-

Figure 6.1: Terminology of clamped circular sandwich plate under circular flat-ended punch.

rial parameters (viz., $\bar{E}$, $\bar{G}$, $\bar{\rho}$, $\bar{\sigma}$, $\bar{\tau}$) are defined as follows:

$$\begin{aligned} \bar{c} &= \frac{c}{R}; & \bar{t} &= \frac{t}{c}; & \bar{R} &= \frac{R}{a}; & \bar{\sigma} &= \frac{\sigma_c}{\sigma_f} \\ \bar{\tau} &= \frac{\tau_c}{\sigma_f}; & \bar{E} &= \frac{E_c}{E_f}; & \bar{G} &= \frac{E_c}{G_f}; & \bar{\rho} &= \frac{\rho_c}{\rho_f} \end{aligned} \quad (6.1)$$

When a flat circular punch comes in contact with a sandwich plate, owing to the high modulus of the faceplate, the deformed shape of faceplate does not conform to punch shape and hence the faceplate experiences contact with the punch only along a circle with radius equal to the punch radius a . Hence, the load is transferred from punch to the sandwich plate as a line load along the circumference of the punch with radius a . For analytical treatment, the total load P is considered to be distributed over a circle of radius a as an axisymmetric line load.

The overall deflection of the sandwich structure is composed of bending deflection (δ_b) and shear deflection (δ_s) [96]. Using Kirchhoff's classical plate theory (CPT), the bending component of deflection, δ_b , in terms of equivalent bending rigidity, D_{eq} , is given [135] as

$$\delta_b^K = \frac{P a^2 T_1}{16 \pi D_{eq}} \quad (6.2)$$

where

$$\begin{aligned} D_{eq} &= E_f c^2 t T_2 \approx \frac{1}{2} \frac{E_f c^2 t}{(1 - \nu_f^2)} \\ T_1 &= \left[\bar{R}^2 - \frac{1}{\bar{R}^2} - 4 \ln(\bar{R}) \right] \\ T_2 &= \frac{1}{2(1 - \nu_f^2)} \left(1 + 2\bar{t} + \frac{4}{3}\bar{t}^2 + \frac{\bar{E}}{6\bar{t}} \frac{1 - \nu_f^2}{1 - \nu_c^2} \right) \end{aligned}$$

Mindlin's first order shear deformation theory (FSDT) gives the shear component of the deflection, δ_s [83] as

$$\delta_s^M = \frac{P \ln(\bar{R})}{2 \pi A_{44eq}} \quad (6.3)$$

where the equivalent shear stiffness, A_{44eq} of the sandwich plate is given by

$$A_{44eq} = \kappa (G_c c + 2 G_f t)$$

In the above equation, shear correction factor, κ is estimated according to Madabhushi-Raman and Davalos [133] as

$$\begin{aligned} \kappa &= \left[A_{44} - \frac{A_{45}^2}{A_{55}} \right]^{-1} \left/ \left[\sum_{n=1}^N \left(\bar{Q}_{44}^n - \frac{\bar{Q}_{44}^{n2}}{\bar{Q}_{55}^n} \right)^{-1} \right. \right. \\ &\quad \left. \left\{ P_n (z_{n+1} - z_n) + \frac{R_n}{2} (z_{n+1}^2 - z_n^2) + \frac{V_n}{3} (z_{n+1}^3 - z_n^3) \right. \right. \\ &\quad \left. \left. + \frac{W_n}{4} (z_{n+1}^4 - z_n^4) + \frac{X_n}{5} (z_{n+1}^5 - z_n^5) \right\} \right] \end{aligned} \quad (6.4)$$

where

$$\begin{aligned} P_n &= M_n^2 + H_n^2 Z_n^2 - 2 M_n H_n z_n + U_n^2 + \frac{J_n^2 z_n^2}{4} \\ &\quad - U_n J_n z_n^2 + 2 M_n U_n - M_n J_n z_n^2 - 2 H_n U_n z_n + H_n J_n z_n^3 \\ R_n &= 2 M_n H_n - 2 H_n^2 z_n + 2 H_n U_n - H_n J_n z_n^2 \\ V_n &= H_n^2 - \frac{J_n^2 z_n^2}{2} + U_n J_n + M_n J_n - z_n H_n J_n; \end{aligned}$$

$$\begin{aligned}
 W_n &= H_n J_n; \quad X_n = \frac{J_n^2}{4} \\
 M_n &= \sum_{m=1}^{n-1} H_m (z_{m+1} - z_m) \\
 U_n &= \sum_{m=1}^{n-1} J_m (z_{m+1}^2 - z_m^2) \\
 H_n &= \bar{Q}_{1i}^n \psi_{1i}; \quad J_n = \bar{Q}_{1i}^n \phi_{1i} \quad i = 1, 2, 6
 \end{aligned}$$

The matrix elements ψ_{ij} and ϕ_{ij} are calculated from in-plane stretching $[A]$, out of plane coupling $[B]$ and bending $[D]$ stiffness submatrices as

$$\psi = -D^{-1}B[A - BD^{-1}B]^{-1}; \quad \delta = [D - BA^{-1}B]^{-1} \quad (6.5)$$

where N is the total number of layers (equal to 3 in the present work) in the laminate, n is any arbitrary layer under consideration, $\bar{Q}_{ij}$ are the reduced coefficients, z_n and z_{n+1} are the top and bottom surface distances of the n^{th} layer from the mid-section of the sandwich plate.

For sandwich plates with thin faceplate ($\bar{t} < 0.1$), the assumption of negligible transverse shear stiffness contribution from the faceplate approximation is reasonable. However, for sandwich plates with thick faceplates ($\bar{t} > 0.1$), the transverse shear stiffness contribution from faceplates is not negligible and hence is considered in estimating the $A_{44_{eq}}$ according to Eq. (6.3). FSDT stiffness estimate for thick sandwich plates is erroneous (as will be elaborated in Sec. 6.4) due to its assumption of constant transverse stress distribution.

An accurate estimate for sandwich plate stiffness is possible with layer-wise third order shear deformation theory, which is beyond the scope of the present study. Hence, (an equivalent single layer) third order plate theory (TPT) is used to estimate the shear component of the deflection (δ_s). TPT solution is derived from CPT solution according to the procedure given by Reddy and Wang [82] as

$$\delta_s^R = P \left[\frac{C_4}{D_{eq}} + \frac{C_5 \bar{D}_{eq}}{D_{eq} \bar{A}_{44_{eq}}} \frac{I_0(\theta_1)}{\sqrt{\xi_1}} \right] \quad (6.6)$$

where,

$$\begin{aligned}
 C_4 &= \frac{1}{2\pi} \frac{\xi_2}{\xi_1} \frac{\bar{D}_{eq}}{\bar{A}_{44_{eq}}} \left[\ln(\bar{R}) + \frac{I_0(\theta_1) - I_0(\theta_2)}{\theta_2 I_1(\theta_2)} \right] \\
 C_5 &= \frac{1}{2\pi} \frac{\xi_2}{\sqrt{\xi_1}} \left[\frac{1}{\theta_2 I_1(\theta_2)} - \frac{I_0(\theta_1) K_1(\theta_2)}{I_1(\theta_2)} - K_0(\theta_1) \right] \\
 F_{eq} &= E_f c^5 T_3; \quad H_{eq} = E_f c^7 T_4
 \end{aligned}$$

$$\begin{aligned}
A_{44_{eq}} &= G_c c T_5; \quad D_{44_{eq}} = G_c c^3 T_6; \quad F_{44_{eq}} = G_c c^5 T_7 \\
\alpha &= \frac{4}{3 c^2 T_8}; \quad \beta = 3 \alpha \\
\bar{D}_{eq} &= D_{eq} - \alpha F_{eq} = E_f c^2 t T_9 \\
\bar{A}_{44_{eq}} &= A_{44_{eq}} - \beta D_{44_{eq}} = G_c c T_{10} \\
\bar{D}_{44_{eq}} &= D_{44_{eq}} - \beta F_{44_{eq}} = G_c c^3 T_{11} \\
\hat{A}_{44_{eq}} &= \bar{A}_{44_{eq}} - \beta D_{44_{eq}} = G_c c T_{12} \\
(D_{eq} H_{eq} - F_{eq}^2) &= E_f^2 c^9 t T_{13} \\
\xi_1 &= \frac{D_{eq} \hat{A}_{44_{eq}}}{\alpha^2 (D_{eq} H_{eq} - F_{eq}^2)} = \frac{T_{14}}{c^2} \\
\xi_2 &= \frac{\bar{D}_{eq} \bar{A}_{44_{eq}}}{\alpha^2 (D_{eq} H_{eq} - F_{eq}^2)} = \frac{T_{15}}{c^2} \\
\theta_1 &= \sqrt{\xi_1} a = \frac{\sqrt{T_{14}}}{\bar{c} \bar{R}}; \quad \theta_2 = \sqrt{\xi_1} R = \frac{\sqrt{T_{14}}}{\bar{c}} \\
T_3 &= \frac{2}{5 (1 - \nu_f^2)} \left[\left(\frac{1}{2} + \bar{t} \right)^5 + \frac{1}{32} \left(\bar{E} \frac{1 - \nu_f^2}{1 - \nu_c^2} - 1 \right) \right] \\
T_4 &= \frac{2}{7 (1 - \nu_f^2)} \left[\left(\frac{1}{2} + \bar{t} \right)^7 + \frac{1}{128} \left(\bar{E} \frac{1 - \nu_f^2}{1 - \nu_c^2} - 1 \right) \right] \\
T_5 &= \left(4 \kappa' \bar{t} \frac{1 + \nu_c}{\bar{G}} + 1 \right) \\
T_6 &= \frac{1}{12} + \kappa' \frac{1 + \nu_c}{\bar{G}} \left(\bar{t} + 2 \bar{t}^2 + \frac{4}{3} \bar{t}^3 \right) \\
T_7 &= \frac{1}{80} + \kappa' \frac{1 + \nu_c}{\bar{G}} \left(\frac{\bar{t}}{4} + \bar{t}^2 + 2 \bar{t}^3 + 2 \bar{t}^4 + \frac{4}{5} \bar{t}^5 \right) \\
T_8 &= (1 + 2 \bar{t})^2 \\
T_9 &= T_2 - \frac{4 T_3}{3 \bar{t} T_8} \\
T_{10} &= T_5 - \frac{4 T_6}{T_8} \\
T_{11} &= T_6 - \frac{4 T_7}{T_8} \\
T_{12} &= T_{10} - \frac{4 T_{11}}{T_8} \\
T_{13} &= T_2 T_4 - \frac{T_3^2}{\bar{t}} \\
T_{14} &= \frac{9 \bar{E} T_2 T_8^2 T_{12}}{32 (1 + \nu_c) T_{13}}
\end{aligned}$$

$$T_{15} = \frac{9 \bar{E} T_8^2 T_9 T_{10}}{32 (1 + \nu_c) T_{13}}$$

where F_{eq} , H_{eq} , D_{44eq} and F_{44eq} are higher order stiffness terms, κ' is the faceplate contribution factor, and I_n and K_n are the n^{th} order modified bessel functions of first and second kind, respectively. κ' controls the contribution of the faceplate transverse shear stiffness to the total shear stiffness of the sandwich plate. This factor can found empirically by matching the analytical stiffness from Eq. (6.6) with either experimentally measured stiffness or from FE prediction for any thick sandwich geometry ($\bar{t} > 0.1$ and $\bar{c} < 0.1$).

From the load-displacement relations, sandwich plate stiffness k_{sw} can be expressed as

$$k_{sw} = \frac{k_b k_s}{k_b + k_s} \quad (6.7)$$

where $k_b = P/\delta_b^K$ is calculated using Eq. (6.2), $k_s = P/\delta_s^M$ is calculated from Eq. (6.3) for first order shear stiffness estimate or $k_s = P/\delta_s^R$ can be calculated from Eq. (6.6) for third order shear stiffness estimate.

In addition to stiffness prediction, it is essential to estimate the failure initiation load for estimating the sandwich plate behavior and is discussed in the following section.

6.1.2 Failure initiation load

Three possible failure modes, viz., localized core indentation, core shear and face failure/microbuckling as shown in Fig. 6.2 are considered as the competing failure modes for sandwich plates under bending. Presence of similar failure modes in sandwich beams with metallic and composite face plates is documented in [11, 136, 137].

Figure 6.2: Schematic (axisymmetric section) view of failure modes in circular composite sandwich plates.

The faceplates of composite sandwich beams remain elastic while it undergoes core indentation and core shear failure [113] and the same is applicable for composite sandwich plates.

Hence, in the current analytical treatment, the faceplates are considered to have a linear elastic behavior and the PVC foam an elastic behavior under bending. The composite faceplates considered in the current work are of quasi-isotropic configuration, $[-60/0/60]_{ns}$. Homogenized properties of the laminate such as in-plane Young's modulus, shear modulus and Poisson's ratio are estimated analytically [138, 139] and the same are used in analytical estimates.

6.1.2.1 Indentation

Indentation failure is considered to occur when the through-thickness stress in the core beneath the punch reaches its compressive yield strength, σ_c , as shown in Fig. 6.2a. The total indentation region is divided into two parts: plastic indentation region λ_p with the reaction force from the core is equal to core compressive strength and elastic indentation region λ_e with the reaction force equal to $k w$.

Indentation analysis of composite sandwich plates has been attempted by several authors. Olsson and McManus [101] gave an analytical estimate for the indentation load of circular sandwich plates subjected to indentation by spherical punch by considering the effect of stretching and large deformation. Koissin et al. [107] studied the indentation behavior of plate resting on elastic-plastic foundation loaded by a spherical punch with large deformations.

Lee and Tsotsis [140] investigated the indentation of sandwich plate under flat-ended circular punch by considering plate on elastic foundation. Giunta et al. [141] derived analytical indentation models for composite sandwich plate subjected to flat-ended square punch and compared the results of CPT, FSDT and TPT theories. In the available indentation models for sandwich plates under flat-ended punch loading [140, 141], authors assumed that the faceplate conforms to punch shape. In reality, due to high stiffness of the faceplate, as the indentation process progresses, the punch experiences contact along the periphery of the punch as shown in Fig. 6.2a. Hence, in current work, circular punch is considered to impose an axisymmetric line load at a radius a .

Guided by FE simulations (as will be elaborated in Sec. 6.4), indentation failure is defined to occur when the core under the punch has yielded, i.e. λ_p equals to punch radius a . Hence, at the instant of indentation failure, the reactive pressure applied by the core/foundation is equal to compressive yield strength of the core as given by

$$k w(a) = \sigma_c \quad (6.8)$$

In the above equation, the deflection of the faceplate $w(r)$ was found using the analogy of a plate on elastic foundation with a circumferential load along the radius a and solution is given by Panc [132].

The limit load for indentation is estimated using Eq. (6.8) as a bounding condition and is

expressed as

$$P_{in} = \frac{4 D_f \sigma_c}{k l^2} \frac{1}{u_0 f_0 - v_0 g_0} \quad (6.9)$$

where

$$\begin{aligned} D_f &= \frac{E_f t^3}{12 (1 - \nu_f^2)}, \\ l^2 &= \sqrt{\frac{D_f}{k}}, \\ u_0 &= \operatorname{Re} J_0(\theta_3), \\ v_0 &= \operatorname{Im} J_0(\theta_3), \\ f_0 &= \operatorname{Re} H_0^{(1)}(\theta_3), \\ g_0 &= \operatorname{Im} H_0^{(1)}(\theta_3), \\ \theta_3 &= \frac{a(i+1)}{\sqrt{2}l} = \frac{i+1}{\sqrt{2}} \left[\frac{12 \bar{E} (1 - \nu_f^2)}{\bar{c}^4 \bar{R}^4 \bar{t}^3} \right]^{1/4}, \end{aligned}$$

D_f is the bending rigidity of the faceplate, i is the imaginary number $\sqrt{-1}$, l is the characteristic length, k is the foundation stiffness defined as the ratio of core compressive modulus to the thickness (E_c/c), and $H_n^{(1)}$ and J_n are the n^{th} order Hankel and Bessel functions of first kind, respectively.

Though the present work deals with quasi-isotropic laminate, Eq. (6.9) is still applicable for any general symmetric laminate configurations (with orthotropic deformation profiles) using the concept of length scaling [101].

6.1.2.2 Core shear

Core shear failure is assumed to occur when the radial shear strain in the core exceeds the failure strain of the core. In early attempts for estimating the core shear failure of metallic sandwich beams, contribution from faceplates was not considered [96, 142]. However, in recent attempts, contribution from faceplates is considered by accounting for the work from circumferential plastic hinges [11]. In the context of composite sandwich beams [113, 116, 137], contribution from thin faceplates is considered to be negligible. However for thick faceplates, contribution from faceplates needs to be considered. Hence, in the current work, the faceplate contribution is considered by giving bounding estimate on the displacement rather than the load. Schematic view of core shear failure mode is shown in Fig. 6.2b. At the instant of core shear, the faceplate remains elastic. However, there is a stiffness contribution from faceplates to the failure load. Hence, load at the instant of core shear is estimated from the stiffness by defining the displacement for core

shear as follows:

$$P_{cs} = k_{sw} \delta_{cs} \quad (6.10)$$

where

$$\delta_{cs} = \frac{\tau_c}{G_c} \frac{R-a}{2} \quad (6.11)$$

k_{sw} is the sandwich plate stiffness, δ_{cs} is the critical displacement for the core shear failure and G_c is the shear modulus of the core. It is evident from Diab data sheet [143] that the shear modulus of the core, G_c , can be related to the Young's modulus by $E_c/[2(1 + \nu_c)]$ and hence in the current investigation, E_c is considered to be the primary variable rather than G_c .

6.1.2.3 Face failure or microbuckling

Failure of the faceplate in tension is termed its failure (which usually occurs in bottom faceplate) and that in compression is termed as microbuckling (which usually occurs in top faceplate). Most laminates are weak in compression (due to fibre kinking) rather than in tension, hence microbuckling is the dominant failure mode. However, for some unbalanced laminate configuration (such as the laminate considered in present study, $[-60/0/60]_{ns}$), the tensile strength is lower than the compressive strength. Analytical treatment for both failure modes is the same due to assumption of symmetric bending of sandwich plate about the neutral axis as shown in Fig. 6.2c.

During symmetric bending of a sandwich plate, the radial stress, σ_{rr} (at an arbitrary distance, z , from the neutral axis) in the faceplates is related to the radial moment, M_{rr} , as

$$\sigma_{rr} = \frac{M_{rr}}{2I} z \quad (6.12)$$

where

$$I = \int_{-\frac{c}{2}}^{\frac{c}{2} + t} z^2 dz = t^3 T_{16}$$

$$T_{16} = \left(\frac{1}{3} + \frac{1}{2\bar{t}} + \frac{1}{4\bar{t}^2} \right)$$

At the limiting case of faceplate failure or microbuckling, equating radial stress in faceplates to limiting strength and radial moment to limiting moment, Eq. (6.12) becomes

$$\sigma_f = \frac{M_L^f}{2I} \left(\frac{c}{2} + t \right) \quad (6.13)$$

According to Kirchhoff's plate theory limiting bending moment, M_L^f is

$$M_L^f = \frac{P(1 + \nu_f)}{8\pi} T_{17} \quad (6.14)$$

where

$$T_{17} = 2 \ln(\bar{R}) + \frac{1}{\bar{R}^2} - 1$$

Substituting Eq. 6.13 into Eq. 6.12 gives the load required for faceplate failure or microbuckling as

$$P_{ff/mb} = \frac{16\pi\sigma_f t^2 T_{16}}{(1 + \nu_f) T_{17}} \left(1 + \frac{1}{2\bar{t}}\right)^{-1} \quad (6.15)$$

6.2 Experimental method

This section describes the constituent materials of the sandwich construction and their mechanical characterization followed by the bending tests on sandwich plates. All tests (unless otherwise mentioned) were performed on Instron 5567 (with 50 kN load cell) under displacement control at a nominal speed of 1 mm/min. Sandwich constituents were characterized using three to five samples to account for repeatability.

6.2.1 Materials and measured properties

Closed cell PVC foams of grades H35 and H250 (supplied by Diab Inc., Bangkok, Thailand) were used as the core material for sandwich plate construction. The measured average densities of H35 and H250 foams were 35 kg/m³ and 230 kg/m³ with an average cell size of 0.65 mm and 0.15 mm, respectively.

Through-thickness compression tests on PVC foams were performed using foams of thickness 25 mm (for H35) and 20 mm (for H250) with in-plane dimensions of 50 mm x 50 mm. In-plane compression tests were performed on a cuboid of 25 mm (for H35) and 20 mm (for H250) foams. Compression strain was calculated from cross head displacement. The uniaxial compression response of H35 and H250 foams is shown in Fig. 6.3a. Compression response of H35 and H250 shows that the through-thickness strength is higher than in-plane strength by a factor of 1.5 for H35 and 1.44 for H250.

Tensile tests were performed on dog-bone specimens of PVC foams machined according to ISO 527 from 25 mm (H35) and 20 mm (H250) thick foam blanks. The dogbone samples were gripped using mechanical side grips in the testing machine. A clip-on extensometer (Instron 2630-100 series) with 50 mm gauge length was used to measure the strain during tensile testing.

Typical tensile response of the foams is shown in Fig. 6.3b. Variation in tensile response of foams in length and width direction was found to be negligible. Tensile stress-strain behavior can be idealized as a bilinear response curve with small hardening exponent, which increases with an increase in the density of the foam. Measured compressive and tensile properties of PVC foams are listed in Table 6.1.

Table 6.1: Properties of Divinycell PVC foams

Property	Foam type	
	H35	H250
Density (kg/m ³)	38	230
Compressive modulus (MPa)		
TT, IP	22, 14	232, 135
Compressive strength (MPa)		
TT, IP	0.5, 0.3	5.6, 4
Tensile modulus (MPa)-IP	25	151
Tensile failure strain-IP	0.07	0.2
Shear strength [†]	0.3	3.9

[†] Lower bound shear strength values are taken from Diab data sheet [143].

Tensile, compression and shear tests were performed (in 1 and 2 direction) on the uni-directional laminate for characterization. Tension test specimens (in 1 and 2 direction) were of 1.5 mm thick, 25 mm wide and 150 mm length. Aluminum end tabs of 40 mm length were abraded, degreased and bonded to tension specimen using Araldite[®] epoxy adhesive. A strain gage rosette (0° - 90°) of 350 Ω with 10 mm gauge length was bonded to tension specimens for measuring strain. All the tension tests were conducted using MTS 810 material test system. In-plane shear test was performed according to ASTM standard D 7078 (V-notched rail shear method). Double-V notch specimen was cut from 2 mm thick specimen in two orthotropic directions. Strain gage rosette (same type as that used in tension test) was bonded to the specimen at $\pm 45^\circ$ configuration to the loading axis. During the shear test across the fibre direction (S12), the fibres were not sheared; hence the shear stress-strain curve was reported up to 5% elongation, as per the ASTM standard. Nominal tensile and shear stress-strain response is shown in Fig. 6.4. The measured GFRP properties are listed in Table 6.2.

Compression tests were performed (in 1 and 2 directions) on 1.5 mm thick and 20 mm wide specimens using IITRI fixture with 10 mm gauge length. The strain gauges were not mounted onto the specimens, as only the compression failure strength values are of interest. Specimens with visible bending deformation were discarded for the strength estimation.

(a)

(b)

Figure 6.3: Nominal stress-strain curves of Divinycell PVC foams in uniaxial (a) compression (b) tension.

Figure 6.4: Nominal tensile and in-plane shear stress-strain curves of unidirectional GFRP laminate.

Table 6.2: Properties of E-glass/epoxy composite

Property	Value
Density (kg/m^3)	2200
Tensile moduli (GPa) E_{11} , E_{22}	46, 13
Tensile strength (MPa) X_t , Y_t	1330, 50
Poisson's ratio ν_{12} , ν_{31} , ν_{23} (assumed)	0.26, 0.074, 0.44
Compressive strength (MPa) X_c , Y_c	537, 130
Shear moduli (GPa) G_{12} , G_{31} , G_{23} (estimated)	7, 6.5, 4.48
Shear strength (GPa) S_{12} , S_{21}	65, 40
QI-laminate homogenized properties [†]	
Tensile moduli (GPa) E_{xx} , E_{yy} , E_{zz}	25.3, 25.3, 13
Poisson's ratio ν_{yx} , ν_{zx}	0.24, 0.175
Shear moduli (GPa) G_{xy} , G_{yz}	10, 5.46
Ultimate tensile strength (MPa) $X_t^{\ddagger}$, Y_t	407, 240
Ultimate compressive strength (MPa) X_c , Y_c	275, 250
Ultimate shear strength (MPa) S_{xy}	76

[†] Elastic properties of the laminate was found using [139] and strength values were estimated using Hashin damage criterion [138].

[‡] Experimental strength value was measured to be 395 MPa.

6.2.2 Sandwich plate construction and test method

Faceplate material, GFRP laminates (with quasi-isotropic configuration $[-60/0/60]_{\text{ns}}$) were cured from unidirectional (UD) E-glass fibre/epoxy prepreg in an autoclave. Sandwich plates were

constructed by bonding the GFRP laminates to PVC foam. The GFRP laminates used were with $[-60/0/60]_{1s}$ and $[-60/0/60]_{2s}$ configuration of nominal thicknesses 0.7 mm and 1.5 mm, respectively. The H35 PVC foam cores used were either of 5 mm or 25 mm thickness. Composite laminates were bonded using MA310 (two-part methacrylate structural adhesive obtained from ITW Plexus). Typical cured MA310 adhesive has an Young's modulus of 1.1 GPa, tensile failure strength of 20 MPa, shear failure strength of 22 MPa and tensile failure strain of 10 % [127]. These adhesive properties ensure no premature failure of bonded interface. The adhesive was cured at room temperature under a pressure of 1 kPa for one day.

Four sandwich designs were constructed for the experimental investigation and the load-displacement measurements were used to assess the proposed analytical models. Geometric details of the sandwich designs are listed in Table 6.3. Design #(1) and #(2) were expected to fail by core indentation mode, and designs #(3) and #(4) were expected to fail by core shear failure. In-plane dimensions of the fabricated sandwich plates were 200 by 200 mm for indentation designs and 250 mm by 250 mm for core shear designs.

The sandwich plate was clamped between two rectangular steel plates which has a central hole of radius R as shown in Fig. 6.1 and flat-ended circular punch was used to load the sandwich plate at the center of the plate. The punch loading was measured from crosshead movement. Each indentation experiment was repeated twice to ensure repeatability.

6.3 Finite element modeling

Three dimensional quarter symmetric FE models are developed (in LS-Dyna explicit solver) to predict the load versus displacement behavior. Accuracy of the analytical estimates are verified with FE predictions. Faceplates and core are modeled using single integration point solid elements. Punch is modeled as a rigid body with steel properties (viz., Young's modulus = 200 GPa, density = 7860 kg/m^3 and Poisson's ratio = 0.3). Smooth velocity loading is applied to the punch to simulate quasi-static loading condition (kinetic energy is kept as low as 10% of peak internal energy value). Tie constraints were applied between the core and the faceplates as there was no debonding failure.

Composite faceplates were modeled as homogenized solid with its elastic behavior governed by classical laminate plate theory and the failure criterion of Chang and Chang [144] as incorporated in the MAT_59 model of LS-DYNA. The composite failure model simulates the 3-dimensional behavior of an orthotropic composite structure. This constitutive model predicts four major failure modes, namely tensile failure, transverse shear failure, compressive failure and delamination. In the current work, failure of the faceplate is avoided by assigning very large values to the strength parameters in core shear and core indentation modes. For the nu-

Table 6.3: Comparison of stiffness and failure load values among experiments, analytical and FE predictions.

	Sandwich plate design			
	#(1)	#(2)	#(3)	#(4)
c (mm)	25	25	5	5
R (mm)	80	80	80	110
t (mm)	0.7	1.5	1.5	1.5
$\bar{c}$	0.3125	0.3125	0.0625	0.0455
$\bar{t}$	0.028	0.060	0.300	0.300
Stiffness (kN/m)				
Experiment	638	875	534	412
Analytical				
TPT-Eq. (6.6)	848	1037	474	443
FSDT-Eq. (6.3)	671	722	218	213
FE	533	830	476	368
Fail load (kN)				
Experiment [Failure mode]	0.39 [I]	0.702 [I]	0.638 [CS]	0.545 [CS]
Analytical				
Indentation (I)-Eq. (6.9)	0.433	0.744	0.502	0.748
Core shear (CS)-Eq. (6.10)	1.053	1.286	0.588	0.755
Face failure (FF)-Eq. (6.12)	26.525	57.392	12.342	12.802
FE [Failure mode]	0.4 [I]	0.782 [I]	0.692 [I/CS]	0.736 [CS]
Displacement at failure (mm)				
Analytical	0.65 [†]	1.03 [†]	1.24 [§]	1.70 [§]
Experiment	0.6	0.84	1.10	1.60
FE	0.8	0.8	1.36	1.79

[†] Calculated from analytical indentation load/analytical stiffness from FSDT.

[§] Calculated using Eq. (6.10).

merically verified faceplate microbuckling failure mode, the stresses σ_{xx} and σ_{yy} are compared with strength values X_c and Y_c , respectively (in the post processing phase) to predict the failure. Homogenized properties of the quasi-isotropic laminate are listed in Table 6.2.

PVC foam core was modeled using homogenized honeycomb model (MAT_26 in LS-Dyna), without the interaction among stress components [54, 131]. The uniaxial compression responses in three orthogonal directions and the shear stress-strain responses will determine MAT_26 behavior. Deformations involved in foam are less than 40% at the instant of failure, and hence viscoelastic effects are negligible [35].

6.4 Assessment of analytical models

In this section, the stiffness and failure initiation load estimates from the proposed analytical models are compared with the experimental measurements and FE predictions. The details of the sandwich plates considered and their dimensions are listed in Table 6.3. Designs #(1) and #(2) have thin faceplates ($\bar{t} < 0.1$) whereas designs #(3) and #(4) have thick faceplates ($\bar{t} > 0.1$). Experimentally measured load-displacement curves of these designs are shown in Figs. 6.5 and 6.6, respectively. All of them predominantly show typical bi-linear response without any peak load for the duration of punch displacement. The initial slope of the (linear portion of load-displacement curve) is a measure of the sandwich plate stiffness. The load at which the nonlinearity sets in determines the initial failure load.

Figure 6.5: Assessment of analytical equations for indentation failure mode.

The stiffness of the sandwich plate is analytically estimated using the FSDT and TPT using Eq. (6.3) and Eq. (6.6), respectively. For thin ($\bar{t} < 0.1$) sandwich designs #(1) and #(2), FSDT estimates (671 kN/m and 722 kN/m) are in agreement with ($671/638 = 1.05$ and $722/875 = 0.83$) and FE predictions ($671/533 = 1.25$ and $722/830 = 0.87$). However, for thick ($\bar{t} > 0.1$) sandwich designs #(3) and #(4), FSDT estimates (218 kN/m and 213 kN/m) are found to differ significantly from experimental measurements ($218/534 = 0.41$ and $213/412 = 0.52$) and FE predictions

Figure 6.6: Assessment of analytical equations for core shear failure mode. Contour scale for (b) is same as that of shown in (a).

($218/476 = 0.46$ and $213/368 = 0.58$).

For equivalent single layer TPT theory, the face contribution factor κ' of 0.2 is found by matching analytical stiffness estimate with experimental stiffness measurement of design #(3) thus analytical estimates for designs #(3) is in agreement with experimental measurements and FE predictions. The same face contribution factor, κ' is valid for all sandwich plate designs.

From Table 6.3, it is evident that, TPT estimate for design #(4) using κ' of 0.2 is in agreement with experimental measurements and FE predictions. Using equivalent single layer TPT for sandwich designs #(1) and #(2), the stiffness estimates (848 kN/m and 1037 kN/m) are found to be in agreement with that of experimental measurements ($848/638 = 1.32$ and $1037/878 = 1.18$).

Collapse loads are calculated using Eqs. (6.9), (6.10) and (6.12) for estimating the minimum load of the three considered failure modes. From Table 6.3, it is evident that designs #(1) and #(2) are dominated by core indentation failure mode, as it has the lowest value when compared with core shear and faceplate failure estimates. Analytically estimated load-displacement response is compared with that of experimental measurement as well as the FE prediction in Fig. 6.5, and good agreement is seen among them. Contours of through-thickness stresses σ_{zz} stress in Fig. 6.5 validates the yield condition in Eq. (6.8) and is confirmed to be a good approximation for estimating the bounding load for indentation failure mode. At the instant of core indentation failure, the faceplates remain elastic (as verified through FE simulations).

For design #(3) and #(4), the failure load estimates by indentation and core shear modes are close to one another as listed in Table 6.3. To experimentally judge this failure either as a pure core shear or indentation is not possible as there was no instrumentation to measure core radial shear strain. Hence, the FE simulation results along with the load-displacement curves are shown in Fig. 6.6. Contours of through-thickness stresses σ_{zz} and transverse shear stress τ_{yz} reveal that design #(3) failed by indentation and core shear simultaneously whereas design #(4) failed by pure core shear. In FE simulation, core shear failure is considered at the instant when the transverse shear stress reaches the shear strength (0.3 MPa for H35 foam) of the foam. Bounding estimates of the punch displacement for core shear estimate is in agreement with that of experiments and FE simulations. For all sandwich designs #(1) through #(4), the numerically predicted load-displacement responses deviate from the experimental measurements after the collapse load. This is due to the absence (presence) of perfectly clamped boundary condition in the experiment (simulation).

Faceplate failure is absent in sandwich structures with weak foam (H35) as a core and the same has been observed by Steeves and Fleck [113]. Hence, to assess the face failure estimate given by Eq. (6.15), stronger H250 foam is considered for the core and validated against the FE simulation. A sandwich plate design with $t = 0.7$ mm, $c = 7$ mm, $a = 31.875$ mm and $\bar{R} = 8$ analyzed for the faceplate failure. The analytical estimate for face failure is 5.5 kN against the FE prediction 7.1 kN. This confirms that the analytical estimate serves as a good design estimate ($5.5/7.1 = 0.77$). Possible discrepancy between analytical estimate and FE prediction is due to absence of large deformations in the current analytical formulation. Since the current faceplate under consideration, $[-60/0/60]_{ns}$ is weak in tension (Y-direction), contour plot of σ_{yy} is shown in Fig. 6.7 to predict the failure. At the instant of faceplate failure, the compressive and shear

stresses in the core are much smaller than the failure strengths, which conforms the faceplate failure mode.

Figure 6.7: Faceplate failure of a sandwich plate with H250 foam as core.

6.5 Failure mode map for sandwich plates

The active failure mode for a sandwich plate is the one with the lowest possible failure load of all the failure modes considered, viz., core indentation, core shear and micro buckling. Failure mode map (which gives the safe design parameters and minimum mass of the sandwich plate) is constructed by plotting the contour lines of normalized failure initiation load and mass of sandwich plate on $\bar{l}$ versus $\bar{c}$ space for a given non-dimensional ratios of strength (viz., $\bar{\sigma}$, $\bar{\tau}$), stiffness (viz., $\bar{E}$, and $\bar{G}$) and loading ($\bar{R}$) parameters. This method was originally applied to sandwich beams by Triantafillou and Gibson [110] and a detailed discussion is given by Gibson and Ashby [25]. The non-dimensional structural load index $\hat{P}$ is expressed as

$$\hat{P} = \frac{P}{\pi a^2 \sigma_f} \quad (6.16)$$

In order to construct the mass contours and to find the minimum mass trajectory, non-dimensional mass of the sandwich plate, $\hat{M}$, is defined as

$$\hat{M} = \frac{M}{\pi R^3 \rho_f} = \bar{c} \bar{\rho} + 2 \bar{t} \bar{c} \quad (6.17)$$

Using Eqs.(6.9), (6.10), (6.15) and (6.16), non-dimensional failure loads for indentation, core shear and faceplate failure/microbuckling failure modes are written as

$$\hat{P}_{in} = \frac{2}{\sqrt{3}} \pi \frac{\bar{\sigma} \bar{c}^2 \bar{R}^2 \bar{t}^{3/2}}{\sqrt{\bar{E}} (1 - \nu_f^2)} \frac{1}{u_0 f_0 - v_0 g_0} \quad (6.18)$$

$$\hat{P}_{cs} = \frac{(1 + \nu_c) \bar{c} \bar{R} (\bar{R} - 1) \bar{t}}{\bar{E}} \times \left[\frac{T_1}{16 \bar{c}^2 \bar{R}^2 \bar{t} T_2} + \frac{\pi T_{18}}{T_2} + \frac{2 \pi (1 + \nu_c) \hat{C}_5 T_9}{\bar{E} T_2 T_{10}} I_0(\theta_1) \right]^{-1} \quad (6.19)$$

where

$$T_{18} = \frac{(1 + \nu_c) T_9 T_{15}}{\pi \bar{E} T_{10} T_{14}} \left\{ \ln(\bar{R}) + \frac{\bar{c}}{\sqrt{T_{14}} I_1(\theta_2)} [I_0(\theta_1) - I_0(\theta_2)] \right\}$$

$$\hat{C}_5 = \frac{1}{2 \pi} \frac{T_{15}}{T_{14}} \left[\frac{\bar{c}}{\sqrt{T_{14}} I_1(\theta_2)} - I_0(\theta_1) \frac{K_1(\theta_2)}{I_1(\theta_2)} - K_0(\theta_1) \right]$$

$$\hat{P}_{ff/mb} = \frac{16 \bar{c}^2 \bar{t}^2 \bar{R}^2 T_1}{(1 + \nu_f) T_{17}} \left(1 + \frac{1}{2 \bar{t}} \right)^{-1} \quad (6.20)$$

An example of failure mode map (together with force and mass contours) is shown in Fig. 6.8 for a sandwich plate with E-glass/epoxy faceplate and H35 foam core, which gives the non-dimensional ratios of strength (viz., $\bar{\sigma} = 0.0021$, $\bar{t} = 0.00125$), stiffness (viz., $\bar{E} = 0.00087$, and $\bar{G} = 0.004$), loading ($\bar{R} = 8$) and mass ($\bar{\rho} = 0.0173$) parameters. For plotting current failure mode map, the tensile strength in Y-direction, Y_t is taken as σ_f . It is evident from Fig. 6.8 that the faceplate failure is absent (for physically reasonable geometric parameters, $\bar{c}$, $\bar{t}$) in the failure mode map since the H35 foam is too soft to trigger GFRP faceplate failure. Minimum mass trajectory is plotted by a numerical search for $\bar{c}$ and $\bar{t}$ coordinates at which the mass of sandwich plate $\hat{M}$ is minimum for a selected finite number of force contours, $\hat{P}$. Due to the implicit nature of failure load equations, explicit expressions for the failure regime boundaries and the relation between $\hat{M}$ and $\hat{P}$ are not possible. Sandwich designs (#(1) through #(4)) discussed in section 6.4 are marked in the failure mode map. As seen from the map, designs #(1) and #(2) fall in the indentation failure regime. However, design #(3) remains in the indentation regime although it

Figure 6.8: Failure mode map for sandwich plate with PVC foam core (H35) and glass/epoxy laminate (with quasi-isotropic lay-up). This is plotted for non-dimensional variables $\bar{R} = 8$, $\bar{\sigma} = 0.0021$, $\bar{\tau} = 0.00125$, $\bar{E} = 0.00087$ and $\bar{G} = 0.004$.

failed by a combination of core shear and indentation as shown in Fig. 6.6a. This is expected along the failure regime boundaries due to approximate nature of bounding estimates although the variation in failure load by core shear and indentation mode for design #3 is small. Design #4 is in core shear regime as estimated by analytical model and FE prediction.

The rest of the designs viz., #5 through #15 are selected to further assess the failure mode map in the following section. However, only numerical models were used as a bench mark to assess the analytical models as well as failure modes for designs #5 through #15. This is due to the constraint on the minimum faceplate thicknesses with $[-60/0/60]_{ns}$ quasi-isotropic configuration. Dimensions of the sandwich plate designs #5 through #15 shown in Fig. 6.8 are listed in Table 6.4.

Table 6.4: Geometric parameters of the sandwich designs #(5)–#(15)

Design	c (mm)	t (mm)	R (mm)	$\bar{c}$	$\bar{t}$
#(5)	25	3.75	80	0.3125	0.150
#(6)	25	7	80	0.3125	0.280
#(7)	10	2.8	80	0.1250	0.280
#(8)	5	2.1	62.5	0.0800	0.420
#(9)	10	4	80	0.1250	0.400
#(10)	5	1.4	62.5	0.0800	0.280
#(11)	10	0.28	80	0.1250	0.028
#(12)	5	0.7	255	0.0196	0.140
#(13)	5	1.4	255	0.0196	0.280
#(14)	10	0.7	255	0.0392	0.070
# (15)	10	0.28	255	0.0392	0.028

6.5.1 Assessment of failure mode map

In this section, the constructed failure mode map is thoroughly verified for its accuracy in estimating the failure initiation load for the identified failure mode. Sandwich designs #(5) through #(15) are discussed here starting with indentation failure dominated designs followed by core shear dominated designs.

6.5.1.1 Indentation

The bounding estimate for the indentation failure is derived from equilibrium solution when the applied load is sufficient to cause the plastic collapse of core underneath the punch area locally. Sandwich plates with thin faceplates and thick core show a change in slope (degradation in stiffness as shown in Fig. 6.5 for designs #(1) and #(2)). Designs with either thick faceplates and thick core (viz., designs #(5), #(6) and #(9)) or thin faceplate and thin core (viz., designs #(7), #(8), #(10) and #(11)), a change in slope in the load-displacement is observed when $\lambda_p \gg a$ as shown in Fig. 6.9.

Figure 6.9: Indentation failure in sandwich designs #5 through #11). Contour scale for σ_{zz} is same as that shown in Fig. 6.5.

Change in slope can be either a degradation in stiffness (core failure) or an increase in stiffness (which represents the initiation of stretching phenomenon). For designs #(5), #(6) and #(9), a degradation in stiffness is observed while for designs #(7), #(8), #(10) and #(11), an increase in stiffness is observed as shown in Fig. 6.9. Although all geometries failed by core indentation, the characteristic difference in load-displacement curve is due to the variations in $\bar{c}$ and $\bar{l}$. As seen from Fig. 6.9b, contours of σ_{zz} for designs #(10) and #(11) confirms to the bounding condition $\lambda_p = a$ (at the change in slope in load-displacement curve). However the analytical estimate for failure load is far away from the FE prediction. This discrepancy is due to the fact that the sandwich plate under goes overall global deflection before it experiences the localized core indentation failure. However, in the bounding estimate, the overall bending of the sandwich plate prior to core indentation failure is not considered. Hence, these geometries can be modeled accurately using higher order sandwich plate theories such as the one proposed by Santiuste et al. [145] in estimating the indentation response. However, existing sandwich plate theories treat the core as a linear elastic material.

6.5.1.2 Core shear

Contours of transverse shear stress τ_{yz} for designs #(12) through #(15) are shown in Fig. 6.10, which confirms the failure by core shear. Comparison between analytical estimates and FE predictions in terms of stiffness, failure load and displacement are also shown in Fig. 6.10. Since $(R - a)$ is the same for the designs #(12)-#(15), analytical estimate for the punch displacement to produce core shear failure (according to Eq. (6.11)) is same. Sandwich designs with thin core (#(12) and #(13)) are found to show no significant difference in punch displacement both for the core shear initiation as well as complete shear of the core thickness. However, for sandwich designs with thick core (#(14) and #(15)), when maximum value of τ_{yz} reaches the shear strength, τ_c , the corresponding displacement is taken as core shear failure. No degradation in stiffness (in load-displacement) has been predicted by FE models due the fact that the stretching of faceplates dominates the core shear failure. Analytical estimates for stiffness, failure load and displacement are in agreement with that of numerical predictions.

Figure 6.10: Fringe scale for τ_{yz} is same as that shown in Fig. 6.6. Analytical stiffness, k estimate is based on TPT.

6.6 Effect of geometric parameters on the failure loads and modes

Failure mode map is constructed in terms of two free variables, $\bar{c}$ and $\bar{t}$ for given values of remaining non-dimensional variables in Eq. (6.1). In this section, the effect of each variable (either $\bar{c}$ or $\bar{t}$) on failure mode and load is studied. Five paths have been defined in the failure mode map. Path A is the line connecting designs #(1)–#(2)–#(5)–#(6), which represents the line along which the faceplate thickness (t) is being increased (for a given $\bar{c}$), and illustrates the effect faceplate thickness on the indentation failure load. Paths B and C are the lines connecting designs #(12)–#(13) and #(15)–#(14), respectively, and these lines illustrate the effect of face plate thickness on the core shear failure load. Paths D and E are the lines that connect designs #(6)–#(7)–#(10)–#(13) and #(1)–#(11)–#(15), respectively. Paths D and E bring out the effect of increase in radius of the plate for a given faceplate thickness.

The analytical estimate of indentation load along path A (for $\bar{c} = 0.3125$) is plotted in Fig. 6.11a. Analytical estimates are compared with numerical predictions and experimental measurements. It is evident that as the faceplate thickness increases, the discrepancy between bounding load estimate ($\lambda_p = a$) and load at the change in slope of the load-displacement curve increases. However, for a physically reasonable sandwich panels, faceplates are usually thin, so the proposed bounding estimate gives a good design estimate.

(a) Path A

(b) Path B and C

Figure 6.11: Effect of faceplate thickness on the (a) indentation and (b) core shear failure mode.

The effect of faceplate thickness on the core shear failure load is shown in Fig. 6.11b for $\bar{c} = 0.0186$ and 0.0392 . It is evident that analytical estimates show a trend similar to that of FE predictions.

Path D is plotted in Fig. 6.12a for investigating the effect of sandwich plate radius (for a given $\bar{t} = 0.28$) on the failure mode. It can be observed that the analytical estimate (for failure load) is in agreement with FE predictions. For design #(7), a high discrepancy between the bounding condition $\lambda_p = a$ and the load at the change in slope is observed. This is because design #(7) has a thin face and thin core, and hence after the indentation failure, no degradation in the stiffness (as shown in Fig 6.9b) is observed. Rather, an increase in stiffness is evident due to stretching of the faceplates.

Figure 6.12: Effect of sandwich plate radius on the failure modes for (a) $\bar{t} = 0.28$ and (b) $\bar{t} = 0.028$.

(b) Path E

Figure 6.12: Effect of sandwich plate radius on the failure modes for (a) $\bar{t} = 0.28$ and (b) $\bar{t} = 0.028$.

The effect of plate radius, R (for $\bar{t} = 0.028$) on the indentation failure load is shown in Fig. 6.13b. It is evident that the analytical estimate is able to predict the trend that is in agreement with FE predictions. However, the FE prediction of design #11 deviates from the analytical estimate. Recalling from section 6.5.1.1, this discrepancy between FE prediction and analytical estimate is due to the fact the sandwich plate undergoes overall global bending before the local indentation. Design #15 is at the intersection of indentation and core shear failure mode. From FE simulations, design #15 is confirmed to fail by core shear in Fig. 6.10. However, the failure mode map suggests design #15 will fail by indentation failure. Though the failure modes predictions are different, the failure loads offered by the two failure mode estimates is close enough to the analytical estimate. Such deviations are rather expected in bounding solution.

6.7 Summary

Bending response of clamped circular composite sandwich plate with E-glass/epoxy faceplates and PVC foam core has been studied.

Stiffness estimates for sandwich plates using equivalent single layer FSDT does not provide satisfactory estimates for a range of sandwich geometries considered in present work. A generic expression for the stiffness of the sandwich plate for all the geometries is possible only with layer wise laminate theories, which is beyond the scope of the present work. Hence, a semi-analytical method is proposed (based on equivalent single layer TPT) to estimate the stiffness of the sandwich plate.

Localized core indentation, core shear and face failure are considered as the dominant failure modes of clamped circular sandwich plates. Bounding estimates for limit loads have been developed for each failure mode. The proposed core shear failure load estimate considers the contribution from the faceplates and core. The accuracy of analytical estimates were checked against the (limited) experimental measurements and FE predictions. The limit load for face failure/micro buckling failure mode was verified using FE simulations only, as the geometry of the plate is beyond the capacity of the existing test jig for the considered sandwich plate material system. Core indentation dominated designs with thin face and thin core can be better modeled using higher order sandwich plate theories as these geometries undergo overall global deflection before it undergoes core indentation failure. Bounding analytical estimates were found to be in good agreement ($\pm 20\%$) with experimental measurements as well as FE predictions for the indentation failure mode.

Analytical bounding estimates for failure loads and the mass of sandwich plate were normalized, and a failure mode (or design) map is constructed in normalised core thickness and faceplate thickness, $\bar{c}$ versus $\bar{t}$ plane to understand the effect of geometric parameters (for a given material and loading parameters) on the sandwich plate failure response. This failure mode map for circular composite sandwich plates is first of its kind in literature.

Chapter 7

Conclusions and Future Work

This chapter summarizes the major contributions and conclusions of the present study followed by recommendations for further investigation.

7.1 Conclusions

Main conclusions from the current investigation are listed as follows:

- Foams cannot be modeled on cell wise basis (as modeled in honeycomb cores) to have computationally feasible analysis. Hence, constitutive models play a major role in representing the foam behavior in numerical simulations. Two constitutive models are for foams considered: (a) Deshpande Fleck (DF) and (b) Homogenized honeycomb (HC) model.

DF model accounts for deviatoric and distortional components of energy with isotropic hardening behavior and considers hardening as a function of plastic strain. To implement this model either hydrostatic tension or compression response of the foam is needed. However the hydrostatic loading apparatus is not common. FE simulations reveal that, default failure criterion viz., volumetric strain criterion is not able to predict physically reasonable failure modes (as shown in Figure 7.1c). After thorough investigation, maximum principal strain criterion with stiffness hourglass control is found to predict accurate failure modes (as can be seen in Figure 7.1d), damage zone dimensions and load-displacement histories.

HC model accounts for all six components of the stress tensor (with no interaction). Hardening is included according to the six stress components and hence yield surface is a function of engineering volumetric strain. Uniaxial compression and/or shear test data is required to implement HC constitutive model. This makes the HC model easy for implementation. Through investigation revealed that foam is accurately represented by HC model even without interaction between all the components of the stress tensor. Default

tensile strain failure criterion is good enough to predict the failure modes (as shown in Figure 7.1e) observed in experiments.

Figure 7.1: Failure modes predicted by different constitutive models. (a) DF model with volumetric strain failure criterion. (b) DF model with stiffness based hourglass control (d) HC model with tensile strain failure criterion.

- Recent advances in manufacturing processes led to the production of metallic and polymeric foams. Hence it is essential to investigate the relative performance of wide variety of foams. In the present work, relative performance of Divinycell PVC foams and Alporas foams is studied, under low velocity impact loading. Either for a given foams density (as shown in Figure 7.2a) or compressive yield strength (as shown in Figure 7.2b), Divinycell (PVC) polymeric foams is found to perform better than Alporas foams with characteristics of high peak load and/or energy absorption capacity. Similar trends are observed in sandwich plates impact response (as shown in Figures 7.3a-7.3b).

Figure 7.2: Relative performance of Divinycell PVC and Al alloy Alporas foams. Comparison charts based on impact tests on bare foams.

Figure 7.3: Relative performance of Divinycell PVC and Al alloy Alporas foams. Comparison charts based on impact tests on sandwich plates.

Literature revealed that, step wise core grading is found to show better impact response than single core layer sandwich construction under wave controlled high velocity impact and shock loading conditions. Effect of step wise core grading on impact response (viz., load-displacement response, peak failure load and energy absorption capacity) is also investigated. Against conventional sandwich structures with one foam layer as core, recent trends suggest use of two or more core layers (step wise core grading).

Bare graded foam designs with stronger foams on the impact side (followed by high density foams) showed a higher energy absorption capacity compared to designs with softer foams on the impact side as shown in Figure 7.4a. It is also evident that the damage initiation load different from peak load in the designs with softer foams on the impact side.

Similarly, in graded core sandwich plates with softer foam layer on the impact side followed by a stronger foam has a higher energy absorption (as shown in Figure 7.4b) against the sandwich plate with high density foam on the impact side followed by low density foam with little effect on the energy absorption capacity. Effect of faceplate thickness on the peak load and specific energy absorption is also investigated for conventional (as shown in Figure 7.5a) as well as graded core sandwich plates (as shown in Figure 7.5b). It is evident that, as the faceplate thickness increases, there is an increase performance in terms of peak load and specific energy absorption.

Figure 7.4: Comparison among the (a) bare graded foams and (b) graded sandwich plates.

Figure 7.5: Effect of faceplate thickness on (a) conventional and (b) graded core sandwich plates.

- Analytical indentation failure loads for sandwich plates either resting on rigid base or clamped around the periphery are derived. Indentor with flat ended punch is considered in the present study. In analytical modeling, flat punch is assumed to impose an axisymmetric line load on the faceplate. Conventional indentation analogy is used to model indentation of sandwich plate resting on rigid base whereas top-faceplate-radial-compression analogy is used to model the sandwich plate clamped around the periphery. Indentation failure is defined to occur when the core under the punch reaches its compressive strength.

Core is considered as elastic, elastic perfectly plastic with small deformation to the faceplate. Differential equations of equilibrium are solved exactly.

In large deformation analysis, core is considered as rigid perfectly plastic foundation using classical indentation analogy. As differential equations resulting from large deformation theory cannot be solved exactly, Galerkin's weighted residual method is used.

Good agreement is achieved between FE predictions and analytical estimates in terms of indentation failure load and load-displacement response.

- Failure mode maps are constructed for circular composites sandwich plates to investigate the effect of geometric parameters (viz., plate radius, faceplates and core thickness) on the dominant failure modes under bending. Bending of the sandwich plates is achieved using a circular flat ended indentor at the center of the plate. Flat indentor is assumed to impose an axisymmetric line on the sandwich plate, around its circumference. Core indentation, core shear and top faceplate microbuckling or bottom faceplate tensile failure are the competing failure modes.

Bending component of deformation of the sandwich plates is estimated using Kirchhoff's classical plates theory. Shear deflection is derived using Reddy's (equivalent single layer) third order shear deformation theory.

Indentation and faceplate failure loads are derived by imposing a bounding solution on the failure load. However, core shear failure is proposed by defining the bounding solution on the indentor displacement. Contribution from the faceplates is considered in estimating the core shear failure load.

For a physically reasonable geometrical parameters, core indentation and core shear are found to be dominant failure modes. Using the failure mode map, trajectory of minimum weight designs are plotted using numerical search as shown in Figure 7.6. Constructed failure mode map is assessed rigorously using FE simulations. It is observed that for very thick faceplates (of 6 mm) indentation failure loads are deviate from FE predictions.

Figure 7.6: Failure mode map for sandwich plate with PVC foam core (H35) and glass/epoxy laminate (with quasi-isotropic lay-up). This is plotted for non-dimensional variables $\bar{R} = 8$, $\bar{\sigma} = 0.0021$, $\bar{\tau} = 0.00125$, $\bar{E} = 0.00087$ and $\bar{G} = 0.004$. Arrows shows the minimum mass trajectory.

7.2 Contributions

- Reliable FE models of low velocity impact of foam cored sandwich structures is developed.
- Relative performance of metallic and polymeric foams is investigated under low velocity impact response. Additionally, effect of step wise core grading is investigated.
- Analytical indentation models are developed for circular sandwich plates that are either resting on rigid base or clamped around the circumference. Small as well as large deformations to the faceplate is considered.
- Failure mode map is constructed for clamped circular composite sandwich plate under

bending.

7.3 Scope for future work

Further investigations related to the current scope of the work are as follows:

- Having reliable three dimensional FE models to predict the low velocity impact response and failure modes, it would be informative to conduct parametric study to understand the effect of core thickness, faceplate thickness and geometry of the sandwich panel on the energy absorption characteristics.
- Viscoplastic properties of polymeric foams play a dominant role when the compaction strain is greater than 40%. In the current investigations, PVC foams are characterized for uniaxial tension, compression and shear loading conditions. However, it is essential to investigate the viscoelastic and viscoplastic properties of the PVC foams to understand the time dependant recovery during unloading stage.
- Current investigation deals with the indentation of sandwich plates subjected to small deformation theory. Large deformation theory is used to understand the effect of faceplate stretching on load-displacement response by considering core as rigid perfectly plastic foundation using indentation analogy. However it would be interesting to investigate the large deformation by considering core as elastic perfectly plastic foundation. Similar formulations can be attempted using faceplate-radial-compression analogy.
- Current investigation considers the foam core as either elastic or elastic perfectly plastic foundation. Though few typical foams show an elastic perfectly plastic response, they do exhibit a slight increase in strength during the plateau region. Hence, current analysis can be further extended by considering core reaction according to as bilinear or an even arbitrary power law.
- Low velocity impact response can be estimated using the quasi-static indentation law. Hence, proposed indentation models can be extended to predict the low velocity impact response of composite sandwich plates either resting on rigid base or clamped plates under bending.
- Failure mode map has been proposed for the clamped circular sandwich plates subjected to bending by circular flat ended punch. Similar investigation needs to be extended for simply supported boundary conditions to investigate the effect of boundary conditions on the failure mode regimes. Additional considerations include spherical end to the indenter and plates with rectangular geometry.

Appendix A

Indentation of a plate on an elastic foundation

In evaluating the transverse displacement, the functions f_n , g_n , u_n and v_n are defined as

$$\begin{aligned} u_n(r) &= \operatorname{Re} J_n \left(\frac{r}{l} e^{i\psi} \right); & v_n(r) &= \operatorname{Im} J_n \left(\frac{r}{l} e^{i\psi} \right) \\ f_n(r) &= \operatorname{Re} H_n^{(1)} \left(\frac{r}{l} e^{i\psi} \right); & g_n(r) &= \operatorname{Im} H_n^{(1)} \left(\frac{r}{l} e^{i\psi} \right) \end{aligned} \quad (\text{A.1})$$

where J_n and Y_n is the Bessel function of first and second kind of n^{th} index, respectively; $H_n^{(1)}$ and $H_n^{(2)}$ are the first and second Hankel functions of n^{th} index, respectively; and $n=0$ and 1. Y_0 is introduced in the radial compression of plate on EPP foundation. These functions are

$$\begin{aligned} J_0(r\varsigma) &= 1 - \frac{r^2\varsigma^2}{4} + \frac{r^4\varsigma^4}{64} + \dots \\ J_1(r\varsigma) &= \frac{r\varsigma}{2} - \frac{r^3\varsigma^3}{16} + \frac{r^5\varsigma^5}{384} + \dots \\ Y_0(r\varsigma) &= \frac{2}{\pi} \ln \left(\frac{r\varsigma}{2} \right) + \frac{2\gamma}{\pi} + \frac{r^2\varsigma^2}{2\pi} \left[1 - \gamma - \ln \left(\frac{r\varsigma}{2} \right) \right] + \dots \\ Y_1(r\varsigma) &= -\frac{2}{\pi r\varsigma} + \varsigma \left[\frac{a}{\pi} \ln \left(\frac{r\varsigma}{2} \right) - \frac{\varsigma}{2\pi} (1 - 2\gamma) \right] + \dots \\ H_0^{(1)}(r\varsigma) &= \frac{1}{\pi} \left[\pi + 2i \ln \left(\frac{r\varsigma}{2} \right) + 2i\gamma \right] - \frac{1}{4\pi} r^2 \varsigma^2 \left[\pi + 2i \ln \left(\frac{r\varsigma}{2} \right) - 2i + 2i\gamma \right] \dots \\ H_1^{(1)}(r\varsigma) &= -\frac{2i}{\pi r\varsigma} + \frac{r\varsigma}{2\pi} \left[2i \ln \left(\frac{r\varsigma}{2} \right) + 2i\gamma - i + \pi \right] + \dots \end{aligned} \quad (\text{A.2})$$

where $\varsigma = e^{i\psi}/l$, $\gamma = 0.57721$ is Euler's constant, and $i = \sqrt{-1}$.

Appendix B

Indentation of plate on EPP foundation

The coefficients in the displacement function Eq. (5.10) for a plate on elastic perfectly plastic foundation based on indentation analogy are defined as follows.

$$B_1 = B_5 + \frac{P}{8 \pi D_f} [\ln(a) + 1] \quad (\text{B.1})$$

$$B_2 = B_6 + \frac{P a^2}{8 \pi D_f} [\ln(a) - 1] \quad (\text{B.2})$$

$$B_3 = \frac{P}{8 \pi D_f} \quad (\text{B.3})$$

$$B_4 = a^2 B_3 \quad (\text{B.4})$$

$$B_5 = \frac{\sigma_c}{4 D_f \eta_{12}} \left\{ 8 \lambda l^3 \eta_5 [\ln(\lambda) + 1] + \frac{\lambda l \eta_6}{2} [4 \lambda^2 \ln(\lambda) + \lambda^2 + 2 a^2] \right. \\ \left. - \sqrt{2} l^2 \eta_7 [4 \lambda^2 \ln(\lambda) + 2 \lambda^2 + a^2] + \frac{\eta_8}{2 \sqrt{2}} [\lambda^4 \ln(\lambda) + 16 l^4 + a^2 \lambda^2] \right\} \quad (\text{B.5})$$

$$B_6 = \frac{\sigma_c}{64 D_f \eta_{12}} \left\{ 128 \lambda l^3 \eta_5 [a^2 \ln(\lambda) - \lambda^2] \right. \\ + 4 \lambda l \eta_6 [8 a^2 \lambda^2 \ln(\lambda) - 3 \lambda^4 - 4 a^2 \lambda^2 - 64 l^4] \\ - \sqrt{2} l^2 \eta_7 [64 a^2 \lambda^2 \ln(\lambda) - 36 \lambda^4 - 16 a^2 \lambda^2 - 256 l^4] \\ \left. - \sqrt{2} \eta_8 [a^2 \lambda^4 (3 - 4 \ln(\lambda)) + \lambda^6 + 128 l^4 \lambda^2 - 64 a^2 l^4] \right\} \quad (\text{B.6})$$

$$B_7 = \frac{\sigma_c l^3}{2 D_f \eta_{13}} \left\{ 2\sqrt{2} l [4\eta_2 l^2 - \eta_1 (\lambda^2 - a^2)] - 8\lambda l^2 f_0(\lambda) + \lambda g_0(\lambda) (2a^2 - \lambda^2) \right\} \quad (\text{B.7})$$

$$B_8 = \frac{-\sigma_c l^3}{2 D_f \eta_{12}} \left\{ 2\sqrt{2} l [4\eta_1 l^2 + \eta_2 (\lambda^2 - a^2)] + 8\lambda l^2 g_0(\lambda) + \lambda f_0(\lambda) (2a^2 - \lambda^2) \right\} \quad (\text{B.8})$$

where

$$\begin{aligned}\eta_1 &= f_1(\lambda) - g_1(\lambda); & \eta_2 &= f_1(\lambda) + g_1(\lambda) \\ \eta_3 &= f_0(\lambda) - g_0(\lambda); & \eta_4 &= f_0(\lambda) + g_0(\lambda) \\ \eta_5 &= [f_1(\lambda)]^2 + [g_1(\lambda)]^2; & \eta_6 &= [f_0(\lambda)]^2 + [g_0(\lambda)]^2 \\ \eta_7 &= \eta_2 f_0(\lambda) - \eta_1 g_0(\lambda); & \eta_8 &= \eta_1 f_0(\lambda) + \eta_2 g_0(\lambda) \\ \eta_9 &= \eta_3 g_1(\lambda) - \eta_4 f_1(\lambda); & \eta_{10} &= \eta_3 f_1(\lambda) + \eta_4 g_1(\lambda) \\ \eta_{11} &= \frac{P_{el}^i}{2\pi} - \frac{\sigma_c \lambda^2}{2} \\ \eta_{12} &= \sqrt{2} [4 \eta_7 l^2 - \eta_8 (\lambda^2 - a^2)] - 4 l \lambda \eta_6 \\ \eta_{13} &= \sqrt{2} [4 \eta_{10} l^2 + \eta_9 (\lambda^2 - a^2)] - 4 l \lambda \eta_6\end{aligned}\tag{B.9}$$

The indentation failure load P_{el}^i is given by Eq. (5.8), and f_n , g_n , u_n and v_n are given in Appendix A.

Appendix C

Radial compression of plate on elastic foundation

The coefficients in the displacement function Eq. (5.20) for a plate on elastic foundation based on radial compression analogy are given as follows.

$$C_{1i} = -\frac{P l^3}{2 \pi a D_f \Delta} \{ \Lambda_2 [f_0 \Lambda_8 - g_0 \Lambda_7] + \Lambda_3 [-v_0 \Lambda_8 + g_0 \Lambda_6] - \Lambda_4 [f_0 \Lambda_6 - v_0 \Lambda_7] \} \quad (C.1)$$

$$C_{2i} = \frac{P l^3}{2 \pi a D_f \Delta} \{ \Lambda_1 [f_0 \Lambda_8 - g_0 \Lambda_7] + \Lambda_3 [-u_0 \Lambda_8 + g_0 \Lambda_5] - \Lambda_4 [-u_0 \Lambda_7 + f_0 \Lambda_5] \} \quad (C.2)$$

$$C_{3o} = -\frac{P l^3}{2 \pi a D_f \Delta} \{ \Lambda_1 [-v_0 \Lambda_8 + g_0 \Lambda_6] + \Lambda_2 [u_0 \Lambda_8 - g_0 \Lambda_5] - \Lambda_4 [u_0 \Lambda_6 - v_0 \Lambda_5] \} \quad (C.3)$$

$$C_{4o} = \frac{P l^3}{2 \pi a D_f \Delta} \{ \Lambda_1 [f_0 \Lambda_6 - v_0 \Lambda_7] + \Lambda_2 [u_0 \Lambda_7 - f_0 \Lambda_5] - \Lambda_3 [u_0 \Lambda_6 - v_0 \Lambda_5] \} \quad (C.4)$$

where

$$\begin{aligned} \Delta = & \Lambda_1 \{ \Lambda_6 [f_0 \Lambda_{12} - g_0 \Lambda_{11}] + \Lambda_7 [-v_0 \Lambda_{12} + g_0 \Lambda_{10}] - \Lambda_8 [f_0 \Lambda_{10} - v_0 \Lambda_{11}] \} \\ & + \Lambda_2 \{ \Lambda_5 [-f_0 \Lambda_{12} + g_0 \Lambda_{11}] + \Lambda_7 [-g_0 \Lambda_9 + u_0 \Lambda_{12}] + \Lambda_8 [f_0 \Lambda_9 - u_0 \Lambda_{11}] \} \\ & + \Lambda_3 \{ \Lambda_5 [v_0 \Lambda_{12} - g_0 \Lambda_{10}] + \Lambda_8 [u_0 \Lambda_{10} - v_0 \Lambda_9] + \Lambda_6 [g_0 \Lambda_9 - u_0 \Lambda_{12}] \} \\ & + \Lambda_4 \{ \Lambda_5 [f_0 \Lambda_{10} - v_0 \Lambda_{11}] + \Lambda_6 [-f_0 \Lambda_9 + u_0 \Lambda_{11}] - \Lambda_7 [u_0 \Lambda_{10} - v_0 \Lambda_9] \} \end{aligned} \quad (C.5)$$

and the functions Λ_n are defined as follows:

$$\begin{aligned} \Lambda_1(r) &= u_1(r) \cos(\psi) - v_1(r) \sin(\psi); & \Lambda_2(r) &= u_1(r) \sin(\psi) + v_1(r) \cos(\psi) \\ \Lambda_3(r) &= f_1(r) \cos(\psi) - g_1(r) \sin(\psi); & \Lambda_4(r) &= f_1(r) \sin(\psi) + g_1(r) \cos(\psi) \\ \Lambda_5(r) &= u_0(r) \cos(2\psi) - v_0(r) \sin(2\psi); & \Lambda_6(r) &= u_0(r) \sin(2\psi) + v_0(r) \cos(2\psi) \\ \Lambda_7(r) &= f_0(r) \cos(2\psi) - g_0(r) \sin(2\psi); & \Lambda_8(r) &= f_0(r) \sin(2\psi) + g_0(r) \cos(2\psi) \\ \Lambda_9(r) &= u_1(r) \cos(3\psi) - v_1(r) \sin(3\psi); & \Lambda_{10}(r) &= u_1(r) \sin(3\psi) + v_1(r) \cos(3\psi) \\ \Lambda_{11}(r) &= f_1(r) \cos(3\psi) - g_1(r) \sin(3\psi); & \Lambda_{12}(r) &= f_1(r) \sin(3\psi) + g_1(r) \cos(3\psi) \end{aligned} \quad (C.6)$$

In Eq. (C.1), functions $f_n = f_n(r)$, $g_n = g_n(r)$, $u_n = u_n(r)$, $v_n = v_n(r)$ and $\Lambda_n(r)$ are evaluated at $r = a$ using the expressions in Appendix A.

Appendix D

Radial compression of plate on EPP foundation

Using continuity conditions at $r = a$ and $r = \lambda$ leads to the deflection profile given by Eq. (5.26) for a plate on an EPP foundation based on radial compression analogy and the corresponding integration constants F_j are given by

$$F_1 = \frac{P}{2\pi N} \ln(a) + F_3 \quad (D.1)$$

$$F_2 = \frac{P \lambda l \beta \Upsilon_9 \Upsilon_{11} - 4 l \Upsilon_{10} \Upsilon_8 + 4 N \lambda \delta_{el} (\Upsilon_6 \Upsilon_1 - \Upsilon_2 \Upsilon_5)}{4 \lambda N \Upsilon_9 \beta l J_1(\beta \lambda)} \quad (D.2)$$

$$F_3 = \frac{1}{N} \left[\frac{\sigma_c \lambda^2}{4} + N \delta_{el} - \frac{P}{2\pi} \ln(\lambda) \right] + \frac{P}{2\pi \lambda N \beta} \frac{J_0(\beta a)}{J_1(\beta \lambda)} + \frac{1}{\lambda N \beta l \Upsilon_9} \frac{J_0(\beta \lambda)}{J_1(\beta \lambda)} [l \Upsilon_{10} \Upsilon_8 - N \lambda \delta_{el} (\Upsilon_6 \Upsilon_1 - \Upsilon_2 \Upsilon_5)] \quad (D.3)$$

$$F_4 = \frac{P}{2\pi N} \quad (D.4)$$

$$F_5 = \frac{P}{4N} Y_0(\beta a) + F_2 \quad (D.5)$$

$$F_6 = -\frac{P}{4N} J_0(\beta a) + F_2 \quad (D.6)$$

$$F_7 = -\frac{1}{\lambda N \Upsilon_9} [\beta^2 l^2 (l g_0(\lambda) \Upsilon_{10} + \lambda N \delta_{el} \Upsilon_2) - N \lambda \delta_{el} \Upsilon_6] \quad (D.7)$$

$$F_8 = \frac{1}{\lambda N \Upsilon_9} [\beta^2 l^2 (l f_0(\lambda) \Upsilon_{10} + \lambda N \delta_{el} \Upsilon_1) - N \lambda \delta_{el} \Upsilon_5] \quad (D.8)$$

where

$$\begin{aligned} \Upsilon_1 &= f_1(\lambda) \cos(\psi) - g_1(\lambda) \sin(\psi); & \Upsilon_2 &= f_1(\lambda) \sin(\psi) + g_1(\lambda) \cos(\psi) \\ \Upsilon_3 &= f_0(\lambda) \cos(2\psi) - g_0(\lambda) \sin(2\psi); & \Upsilon_4 &= f_0(\lambda) \sin(2\psi) + g_0(\lambda) \cos(2\psi) \\ \Upsilon_5 &= f_1(\lambda) \cos(3\psi) - g_1(\lambda) \sin(3\psi); & \Upsilon_6 &= f_1(\lambda) \sin(3\psi) + g_1(\lambda) \cos(3\psi) \\ \Upsilon_7 &= g_0(\lambda) \Upsilon_1 - f_0(\lambda) \Upsilon_2; & \Upsilon_8 &= g_0(\lambda) \Upsilon_5 - f_0(\lambda) \Upsilon_6 \\ \Upsilon_9 &= \beta^2 l^2 \Upsilon_7 - \Upsilon_8; & \Upsilon_{10} &= \frac{P}{2\pi} - \frac{\sigma_c \lambda^2}{2} \\ \Upsilon_{11} &= J_0(\beta a) Y_1(\beta \lambda) - Y_0(\beta a) J_1(\beta \lambda) \end{aligned} \tag{D.9}$$

where the Bessel functions J_n , Y_n are defined in [Appendix A](#).

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Journal publications

1. A. Rajaneesh, I. Sridhar, and S. Rajendran. Impact modeling of foam cored sandwich plates with ductile or brittle faceplates. *Composite Structures*, 94(5):17451-1754, 2012.
2. A. Rajaneesh, I. Sridhar, and S. Rajendran. Relative performance of metal and polymeric foam sandwich plates under low velocity impact. *International Journal of Impact Engineering*, 65:126-136, 2014.
3. A. Rajaneesh, I. Sridhar, and S. Rajendran. Simplified failure analysis of circular composite sandwich plates under bending. *International Journal of Mechanical Sciences*, under minor revision.
4. A. Rajaneesh, I. Sridhar, and A. R. Akisanya. Indentation models for composite sandwich plates. *Composite Part B: Engineering*, Manuscript in preparation.

International Conferences

1. A. Rajaneesh, I. Sridhar, and S. Rajendran. Numerical modeling of low velocity impact response on metal foam cored sandwich panels: Effect of various facesheet materials. 18th *International Conference on Composite Materials*, 21 - 26 August 2011, Korea.
2. A. Rajaneesh, I. Sridhar, and S. Rajendran. Relative performance of polymeric and metal foams. 10th *International Conference on Sandwich Structures*, 27 - 29 August 2012, France.
3. A. Rajaneesh, I. Sridhar, and S. Rajendran. Failure mode maps for clamped circular composite sandwich plates under bending. 2nd *International conference on materials for advanced technologies*, 30 June - 5 July 2013, Singapore.