Hysteresis models and fragility assessments of reinforced concrete structural components

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**HYSTERESIS MODELS AND FRAGILITY
ASSESSMENTS OF REINFORCED CONCRETE
STRUCTURAL COMPONENTS**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2014

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ASSESSMENTS OF REINFORCED CONCRETE
STRUCTURAL COMPONENTS**

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LIST OF PUBLICATIONS

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2. Zhiwei Huang, Li, B. and **Sengupta, P.**, “Reliability Assessment of Damaged RC Moment-Resisting Frame against Progressive Collapse under Static Loading Conditions,” *ASCE Journal of Engineering Mechanics*, Vol. 139 (1), 2013, pp. 1-17.
3. Zhiwei Huang, Li, B. and **Sengupta, P.**, “Reliability Assessment of Damaged RC Moment-Resisting Frame against Progressive Collapse under Dynamic Loading Conditions,” *Advances in Structural Engineering*, Vol. 17 (2), 2014, pp. 211-232.
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ABSTRACT

Earthquakes represent one of the most destructive natural disasters and the extent of casualties and damage in numerous incidences is well documented over the past decades. The devastating consequences of seismic events have emphasized the significance of the structural performance assessment. Fragility function, an essential component in seismic vulnerability estimation, is defined by the conditional probability of a particular structure exceeding a certain damage state when subjected to seismic excitations. Eventually, for development of fragility functions, characterization of the structural behavior and quantification of the structural damage states during seismic events are pertinent. Hence, the present research delineates the structural hysteretic behavior and different stages of damage experienced by reinforced concrete (RC) structural components for derivation of seismic fragility functions.

The basic requirement in modeling RC structural components under seismic loading is to define a constitutive load-deformation relationship capable of producing strength and stiffness degradation along with pinching at all displacement levels. This is a demanding task considering the numerous parameters contributing to the structural hysteretic behavior. The Bouc-Wen-Baber-Noori (BWBN) model, owing to its computational efficiency and mathematical tractability, is adopted as the basis of the current research and amended accordingly to simulate the hysteretic behavior of RC structural components. The Livermore Solver for ordinary Differential Equations (LSODE) is employed to solve the differential equations of the model. A database of RC beam-column joints and walls tested under quasi-static loading is compiled from the literature to determine the model parameters by a Genetic Algorithm (GA), a system identification technique and to successfully calibrate the analytical response with the experimental results. Subsequently, the sensitivity ranking of the model parameters is determined and the relationship between the model parameters and the structural features is derived by regression analysis using the existing database of the structural components. To facilitate structural analysis of RC buildings with beam-column joints and walls using the proposed analytical

approach, the hysteresis model is effectively implemented as a user element in ABAQUS.

For quantification of different stages of damage experienced by the structural components, the Park-Ang damage model is modified such that the range of the damage index (DI) is between zero and unity. The damage states are classified and quantified in the form of damage indices on the basis of the experimental response of the structural components. Drift ratio is selected as the engineering damage parameter (EDP) for defining the seismic demand in the structural components at any point of the loading history. On the basis of the maximum likelihood test results, a lognormal distribution is found acceptable as the theoretical distribution for fragility assessment of the structural components for all damage states. Thereafter, using incremental dynamic analysis approach, the structural models are subjected to increasing levels of scaled ground motion intensity suitable for Singapore until dynamic instability is reached. The minimum intensity of each ground motion at which the structural models exceed a certain damage state is ascertained to obtain the seismic fragility curves.

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LIST OF SYMBOLS

S_d	Spectral displacement
S_a	Spectral acceleration
V_y	Yield strength of structure
u_y	Yield level displacement
k_e	Elastic stiffness of the structure ($k_e = V_y / u_y$)
k_l	Post-yielding stiffness
r	Stiffness ratio ($r = k_l / k_e$)
$\bar{k}_p$	Degrading positive loading stiffness
$\bar{k}_n$	Degrading negative loading stiffness
P_{cr}	Cracking level load
D_{cr}	Cracking level displacement
P_y	Yield level Load in Takeda model
D_y	Yield level Displacement
P_l	Certain Load in a direction in Takeda model
k_r	Slope of the unloading segment from the curve in Takeda model
k_y	Slope of a line joining the yield point and the cracking point
D_{max}	Maximum curvature, rotation or deflection in the direction of loading
K	New Stiffness of system in DBL model
K_0	Primary Stiffness of system in DBL model

D_{yield}	Yield Deformation in DBL model
α	Constant in DBL model, $0.5 \leq \alpha \leq 0.6$
k_q	Unloading Stiffness at the inelastic segment of the Primary curve
U_y	Yield Deformation in Q-Hyst Model
U_m	Absolute value of the maximum deformation
U'_m	A point on Primary Curve symmetric to U_m with respect to the origin
S_I	Unloading slope from maximum deformation in Otani model
S_{OY}	Elastic slope for small amplitude deformation
D_{max}	Maximum deformation experienced
V_{cr}	Cracking load in Hysteresis Shear Model
V_y	Yield load in Hysteresis Shear Model
k_I	Unloading slope from a load lower than V_{cr}
k_2	Slope of the line connecting the Origin to the Cracking Point in opposite direction
k	Unloading slope from a load higher than V_{cr}
Δ_{cr}	Deflection at Cracking
Δ_y	Deflection at Yield
Δ	Deflection where Unloading starts
Δ_p	Previous Peak Displacement
V_p	Previous Peak Load
Δ_m	Maximum Displacement

V_m	Shear force on Primary Curve corresponding to maximum displacement
N	Axial Compressive Force
N_0	Nominal Concentric Axial Compressive capacity
n	A counter, tracing the successive number of cycles at maximum deflection
P_1, P_4	Primary Pivot Points
PP_2, PP_4	Pinching Pivot Points
β_1, β_2	Degree of pinching prior to Strength Degradation
d_{iMAX}	Maximum Displacement
d_{ti}	Strength Degradation Displacement
F_{iMAX}	Loadings associated with Maximum Displacement
F_{ti}	Loadings associated with Strength Degradation Displacement
K_0	Initial Elastic Region Stiffness
αK_0	Slope of Post-Yield Envelope Curve
K_u	Unloading Slope of the Post-Yield Envelop
K_r	Reloading from an Unloading Intercept to a Reloading Target
$E_{h,n}$	Dissipated Energy predicted by Low-Cycle Fatigue model at n th cycle
F_n	Reduced Strength predicted by Low-Cycle Fatigue model at n th cycle
$\bar{E}_{h,n}$	Normalized Energy Dissipation at the n th cycle
$(1 - \alpha)$	The Loss of Energy Dissipation Capacity at large values of n

β	Rate of Loss of Energy Dissipation
K_s	Post-Yielding Stiffness
K_c	Post-Capping (negative) Stiffness
F_r	Residual Strength
$\alpha_s, \alpha_c, \lambda$	Empirical Constants
F_n	Reduced Strength predicted by Low-Cycle Fatigue model at n th cycle
β_i	Cyclic Deterioration in Excursion i
E_i	Dissipated Hysteretic Energy in Excursion i
E_t	Hysteretic Energy Dissipation Capacity
$\beta_{s,i}$	Rate of Basic Strength Deterioration in Excursion i
$\beta_{c,i}$	Rate of Post-Capping Strength Deterioration in Excursion i
$\beta_{k,i}$	Rate of Unloading Stiffness Deterioration in Excursion i
$\beta_{a,i}$	Rate of Accelerated Reloading Stiffness Deterioration in Excursion i
FDR	Flexural Damage Ratio
k_0	Initial Stiffness
k_m	Reduced Secant Stiffness at the Maximum Displacement
k_f	Stiffness at Failure
δ_{max}	Peak Deformation
δ_u	Ultimate Deformation Capacity of the Structure
β	Calibration Parameter for Cyclic Damage

$E_c(\delta)$	Accumulated Hysteretic Energy per Loading Cycle
α	Empirical Constant
$\Sigma E_{si}^{\pm}$	Energy Dissipation in Primary Half Cycles for Positive (+) and Negative (-) Loading
$\Sigma E_i^{\pm}$	Energy Dissipation in Following Half Cycles respectively for Positive (+) and Negative (-) Loading
$E_U^{\pm}$	Ultimate Energy Capacity in Positive and Negative Monotonic Loading
DI_K	Stiffness-Based Damage Index
$\theta_p^{\pm} \Big _{currentPHC}$	Positive and Negative Peak Plastic Rotations in Primary Half Cycles (PHC)
$\theta_p^{\pm} \Big _{FHC}$	Positive and Negative Peak Plastic Rotations in Following Half Cycles (FHC)
D_T	Global Damage Index
D_i	Local Damage Index
w_i	Weight associated with D_i to obtain D_T
D_m	Softening Index corresponding to Maximum Softening
D_{pl}	Softening Index corresponding to Plastic Softening
D_F	Softening Index corresponding to Final Softening
T_m	Maximum Time Period of a Structure
T_{und}	Time Period of Undamaged Structure
T_{dam}	Time Period of Damaged Structure

I_D	Overall Damage Factor
t_E	Duration of the Seismic Event
$a_g(t)$	Ground Acceleration during Earthquake
m	System mass
u	Relative displacement of mass with respect to base displacement
$(\bullet)$	Derivatives with respect to time t
z	Hysteretic restoring force, a function of u
k	Stiffness
k_i	Initial tangent stiffness
k_f	Final tangent stiffness
α	Stiffness Ratio or Rigidity Ratio i.e. ratio of final and initial tangent stiffness
c	Viscous damping coefficient
t	Time
F_T	Total non damping restoring force
$F(t)$	Time dependent forcing function
ξ_0	Linear viscous damping ratio
ω_0	Pre-yield natural frequency of system
$f(t)$	Mass normalized forcing function
β	Hysteresis shape parameter
γ	Hysteresis shape parameter
n	Hysteresis shape parameter that controls curve smoothness
ν	Strength degradation parameter

η	Stiffness degradation parameter
$h(z)$	Pinching function
A	Parameter that regulates the tangent stiffness and ultimate hysteretic strength
ε	Dissipated hysteretic energy
δ_v	Constant that controls rate of strength degradation
δ_η	Constant that controls rate of stiffness degradation
z_u	Ultimate value of hysteretic strength, z i.e. z at $dz/du = 0$
q	Fraction of ultimate hysteretic strength z_u where pinching occurs
$sgn(\ )$	Signum function
ζ_1	Constant that controls magnitude of initial drop in slope, dz/du
ζ_2	Constant that controls rate of change of slope, dz/du
ζ_s	Measurement of total slip
p	Constant that controls rate of initial drop in slope, dz/du
ψ	Parameter that contributes to amount of pinching
δ_ψ	Parameter specified for desired rate of change of ζ_2 based on ε
λ	Parameter that controls rate of change of ζ_2 with change of ζ_1
f	Vector-valued function of t and y
y	Array of initial values with array-length neq
t	Initial value of the independent variable
S	Objective function
i	A measurement point
f	The mathematically modeled relation

F_i	Dependent variable
u_i	Independent variable
N	Number of data points measured
n	Number of unknown parameters
p_j	Parameters to be estimated
r_j	Possible parameter values
$\hat{R}$	Vector representing a possible solution set
u_j	Upper bound of r_j
l_j	Lower bound of r_j
d_j	Floating number corresponding to r_j within the limit of $[l_j, u_j]$
m_p, b_p	Constants computed for each population
$rawfitness_{mean}$	Average raw fitness of a population
$rawfitness_{max}$	Maximum raw fitness of a population
$scale\_fac$	User-specified scale factor
num_gen	Number of generations to be formed per population
gen	The generation count
$beginscale$	Percentage of generations before scaling starts $beginscale < 1.0$
E	Expected number of offspring (integer)
f_i	Fitness of organism i
f_{mean}	Average fitness of entire population
h_c	Cross-sectional Depths of columns
h_b	Cross-sectional Depths of beams

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b_j	Joint Width
b_c	Width of columns
b_b	Width of beams
f'_c	Concrete Compressive Strength in MPa
f_y	Yield Strength Of Reinforcing Steel in MPa
ρ_b	Beam Longitudinal Reinforcement Ratio in %
ρ_c	Column Longitudinal Reinforcement Ratio in %
ρ_j	Joint Core Transverse Reinforcement Ratio in %
n	Axial Load Ratio in %
l_w	Length of structural walls
h_w	Height of structural walls
t_w	Thickness of structural walls
ρ_h	Horizontal Reinforcement Ratio in %
ρ_v	Vertical Reinforcement Ratio in %
ρ_{be}	Boundary Reinforcement Ratio in %
e_{α_0}	Root Mean Square Error due to Variation of a parameter, say α_0
N	Number of data points for Input Displacement Function
$ e_{\alpha_0} $	Maximum Root Mean Square Error due to Variation α_0
a_1-a_{10}, b_1-b_{10}	Regression Coefficients
μ	Mean of the Lognormal Distribution
σ	Standard Deviation of the Lognormal Distribution

---

k	Shape Parameter of the Weibull and Gamma Distributions
λ	Scale Parameter of the Weibull and Gamma Distributions
α, β	Shape Parameters of the Beta Distribution
L	Maximum Likelihood Function
α	Significance Level
f	Degrees of freedom
m	Number of Intervals in χ^2 test
k	Number of Parameters in the Theoretical Distribution in χ^2 test
D_n	Maximum difference between the Theoretical and the Empirical CDFs
$F_X(x_i)$	Theoretical CDF
$S_X(x_i)$	Empirical CDF
$a_0 - a_5$	Regression Coefficients in Attenuation Relationship
Y	Ground Motion Amplitude
M_w	Moment Magnitude
R	Distance from the station to the centre of the corresponding fault plane, in km
ds_i	Damage State i
$\overline{PGA}, ds_i$	Median value of Peak Ground Acceleration at which the structure reaches the threshold levels of each damage state ds_i
β_{ds_i}	Standard Deviation of the Natural Logarithm of Peak Ground Acceleration each damage state ds_i
Φ	Standard Normal Cumulative Distribution Function

CHAPTER 1

INTRODUCTION

1.1 Research Background

Earthquakes are paramount among the natural hazards with the potential for massive casualties and socio-economic loss. Although the most detrimental impact of the seismic events is the human casualties, the huge economic loss due to the damage of various buildings and lifeline systems (e.g. transportation systems, water distribution systems etc.) is also of great concern. With growth of urbanization, more people tend to settle in favorable locations in terms of the opportunities available for flourishing their economy and the amenities accessible for livelihood. Thus in urban and metropolitan areas, with increasing population density and growth of infrastructures, more people and properties become potential targets of the impending earthquake event in that location. With the presence of numerous cities in the vicinity of seismic faults, the question of the inhabitants and the authorities is the amount of probable loss during future earthquakes.

Seismic performance assessment of Reinforced Concrete (RC) structures has gained increasing popularity among the researchers. Fragility functions play a crucial role in evaluating the potential severity of the consequences of a particular earthquake phenomenon i.e. the seismic vulnerability assessment of RC structures. Seismic fragility functions express the conditional probability of a particular structural component or a system exceeding a certain damage state i.e. a performance level when subjected to a given measure of ground motion intensity. Intensity Measure (IM) is a reference ground motion parameter versus which the probability of a structure exceeding a certain performance level is plotted. Intensity measure (IM) can be classified into two groups: Instrumental IM and Empirical IM. The intensity of ground motion when expressed as measured by an instrument or computed by processing recorded accelerograms, is termed instrumental IM. The most common instrumental IMs utilized by most researchers for structural seismic performance assessment are Peak Ground Acceleration (PGA), Peak Ground Velocity (PGV),

Spectral Displacement (S_d) and Spectral Acceleration (S_a). The empirical IMs are defined in terms of a discrete numerical seismic intensity scales derived from the qualitative assessments of damage. The empirical IMs like Mercalli-Cancani-Sieberg Intensity (MCS) scale, Modified Mercalli Intensity (MMI) scale have limited usage in seismic fragility assessments of structures. In seismic vulnerability assessment, the performance levels of any structure are derived on the basis of damage states representing the threshold levels of damage experienced by structures when subjected to ground shaking. The damage states are generally measured and classified on the basis of a numerical scale applicable for defining them. The basic methodologies involved in construction of seismic fragility curves can be classified into four categories: Empirical, Expert Opinion-based, Analytical and Hybrid (Papailia [P6]). The observed damage data from post-earthquake reconnaissance surveys provides the basis for development of Empirical fragility curves. In spite of being a realistic approach for constructing fragility curves, the quality of the results obtained in this method is affected by scarcity of the relevant data. Expert Opinion-based fragility curves are strictly dependent on the judgment of the consulted expert from his experience regarding seismic vulnerability evaluation of any structure. Analytical fragility curves are developed in an elaborative manner incorporating the characteristics of the hazard as well as the structures. In this method, the damage states are estimated by analyzing the structural models for increasing levels of earthquake intensity. Hybrid fragility curves are generated using various methods of damage prediction with a view to compensate the scarcity of the relevant observed data and the approximations in structural modeling.

Analytical evaluation of fragility curves being the most common approach in seismic performance assessment of structures emphasizes the importance of realistic modeling of the structural components. Analytical modeling of structural components can be conducted at three levels of refinement: Micro, Meso and Macro scale modeling methods. In micro scale modeling methods, an RC structural component is divided into a finite number of small steel and concrete elements. Despite being capable of modeling different loading patterns with reasonable accuracy, this method requires high numerical effort and much computational time

for large and complex structures. Meso scale models are the intermediate scale models which permit utilization of simplified kinematic hypotheses of the theory of beams with a consequent reduction of the size of the equation system, leading to faster analysis than the micro scale models. Truss models, hysteresis models, multi-spring models belong to the category of macro scale models which can represent the overall behavior of the structural component, such as deformation, energy dissipation capacity etc. The hysteresis models can be broadly classified into two groups, piecewise linear or polygonal hysteresis models (PHM) and smooth hysteresis models (SHM). In polygonal hysteresis models, stiffness changes generally occur at elastic, cracking, yielding, strength and stiffness degradation, crack and gap closing stages while in smooth hysteresis models continuous stiffness changes occur due to yielding and sharp changes take place during unloading and deterioration.

To develop a structural model capable of behaving like the actual reinforced concrete frame, an effort is undertaken to comprehend the behavior of RC structural components under seismic loading. Buildings constructed prior to the advent of the seismic provisions were mostly designed for gravity loading with little or no consideration of seismic resistance. Moreover, RC structures in the regions of low to moderate seismicity, like Singapore and Peninsular Malaysia are designed for gravity loading without following the modern seismic design codes. These gravity load-designed (GLD) or lightly-reinforced concrete (LRC) moment resisting frames have significant deficiencies like little or no joint core transverse reinforcement in the beam-column joint regions; widely spaced and poorly anchored transverse reinforcement in the columns and structural walls. Therefore, it is quite evident that performance of these structures may not be adequate to sustain earthquake-induced actions in the regions of low to moderate seismicity as observed in the Padang Earthquake (2009), damage from which is shown in **Fig. 1.1**.

Fig. 1.1: Structural Failure due to Insufficient Confinement in Beam-Column Joints in Padang Earthquake, 2009 (Photo Courtesy: Dr. Bing Li)

To avoid beam-column joint failure, current design codes require that the joints should be strong enough to withstand the yielding of connecting beams and columns. This can be achieved if failure of the beams occurs earlier than failure of the columns. The integrity of the joint must be ensured such that the strength and deformation capacity of the connected members can be developed. Strength degradation of the joint must be prevented to reduce further increment of lateral displacement. To fulfill all these requirements, the beam-column joint should be able to transfer the induced forces and achieve the desired performance. It is widely accepted that in a beam-column joint, adjacent member forces are transferred via two resisting mechanisms namely diagonal concrete compression strut mechanism and truss mechanism. In a LRC frame, beam-column joints which are not confined by the joint core transverse reinforcement cannot transfer induced loads by means of truss mechanism. Consequently, the joint shear forces are only resisted by the concrete compression strut. During earthquake, as soon as the diagonal concrete compression strength is reached, failure of beam-column joints cannot be avoided.

The structural walls are primary lateral load-resisting systems in numerous multistorey buildings and nuclear structures located in seismically active regions. In

low to moderate seismicity regions structural walls are detailed non-seismically hence they provide limited seismic resistance. The structural walls with less transverse reinforcement in the web and the boundary elements, when subjected to repeated cyclic deformation, possess poor energy dissipation characteristics, resulting in pinched hysteresis loops with significant strength degradation and possible sudden loss in lateral capacity (Hidalgo *et al.* [H4], Greifenhagen [G6], Kuang and Ho [K8]) as shown in **Fig. 1.2**.

Fig. 1.2: Shear Failure of Structural Walls during Chile Earthquake, 2010 (Photo Courtesy: AIR and RMS)

A constitutive model capable of capturing strength and stiffness degradation along with pinching at all displacement levels is a basic requirement in modeling and design of seismic-resistant structural components. The present research focuses on developing analytical hysteresis models for RC beam-column joints and structural walls and effectively implementing them in ABAQUS ([A1]). Thereafter, a damage model is proposed for quantification of various stages of damage using different ranges of damage index magnitudes corresponding to each damage state. Analytical fragility curves are constructed on the basis of the existing database of experimental results of RC structural walls and beam-column joints under quasi-static cyclic loading. Subsequently, seismic fragility curves of generic frames are developed by

conducting structural analysis for increasing levels of ground motion intensity representative of the Singapore region.

1.2 Objectives and Scope of the Research

The present research proposes of an analytical hysteresis model and a damage model for reinforced concrete (RC) structural components for evaluation of fragility curves of generic frames in Singapore when subjected to simulated earthquake loading. The primary objectives behind conducting this research are summarized hereunder:

- ❖ To develop the analytical hysteretic models for RC beam-column joints and structural walls under cyclic loading that accurately depict the original structural components.
- ❖ To implement the analytical hysteresis models of RC structural components in ABAQUS ([A1]) in the form of user elements to facilitate analysis of the structural systems and to obtain realistic structural response.
- ❖ To propose a damage model capable of defining different stages of structural damage in the form of a damage index so as to enable damage quantification of RC beam-column joints and structural walls under quasi-static cyclic loading.
- ❖ To construct analytical fragility functions for RC beam-column joints and structural walls using the proposed damage model for the existing experimental database of RC structural components.
- ❖ To derive seismic fragility curves for RC generic frames for the scaled ground motions of Singapore by conducting structural analysis at increasing levels of seismic intensity.

The scope of the present thesis is limited to development of hysteresis models of RC beam-column joints and structural walls. However, beam-column joints with transverse beams or slabs, retrofitted beam-column joints and structural walls are not covered in this research. Derivation of analytical models for other structural

components like columns, slab-column connections are not within the scope of the research. Effect of soil-structure interaction during evaluation of seismic fragility functions is ignored in this research. Moreover, influences of epistemic and aleatoric uncertainties resulting from structural modeling, ground motion data, material properties or structural analysis are kept out of the scope of the research due to time constraints.

1.3 Organization of Thesis

This thesis comprises eight chapters which are outlined hereunder:

Chapter 1 demonstrates the research background, primary objectives and scope of the research.

Chapter 2 consists of the state-of-the-art review of the analytical hysteresis models and damage models of RC structural components. A comprehensive survey of seismic fragility evaluation of RC structural components and buildings are also provided in this chapter.

Chapter 3 describes the original Bouc-Wen-Baber-Noori (BWBN) hysteresis model ([B10], [B2], [B3]) with the modifications proposed to it in this research. Basic information on the Livermore Solver for Ordinary Differential Equations (LSODE) and Genetic Algorithm (GA) employed respectively for solving the differential equations and estimating the analytical parameters involved in the proposed model are also included in this chapter.

Chapter 4 summarizes the experimental database of RC beam-column joints and structural walls under cyclic loading for estimation of the analytical model parameters using GA to calibrate the analytical results with the experimental response. On the basis of successful correlation between the experimental and model hysteresis loops of the structural components, the relationship between the model parameters and the structural characteristics is defined using regression analysis in this chapter.

Chapter 5 focuses on implementation of the proposed mathematical hysteresis model of RC beam-column joints and structural walls in the form of user elements in ABAQUS ([A1]). The hysteretic responses obtained from structural analysis are successfully calibrated against the experimental responses of beam-column joints, single and multi-storey structural walls and frame structures under quasi-static cyclic tests and shake table tests.

Chapter 6 illustrates the modification proposed to the Park-Ang damage model [P9]. The various stages of damage experienced by RC beam-column joints and structural walls are classified into four damage states as minor, low, moderate and severe on the basis of experimental results. The damage states are classified in the form of different magnitude ranges of damage indices using the existing experimental database and the proposed damage model. Inter-storey drift ratio is selected as Engineering Damage Parameter (EDP) for structural components.

Chapter 7 aims at establishing fragility functions of RC beam-column joints and structural walls using the existing experimental responses of the structural components. Generic frames with the beam-column joints and structural walls modeled as user elements are subjected to increasing levels of ground motion applicable for Singapore to evaluate the corresponding analytical seismic fragility curves.

Chapter 8 summarizes the conclusions drawn from the research conducted along with the recommendations for future research. Appendices A and B tabulate the experimental data for RC beam-column joints and structural walls utilized for calibrating the analytical model results and assessing the corresponding damage states and fragility functions respectively. Appendix C summarizes the scaled ground motions derived on the basis of attenuation relationships for Singapore.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

With occurrence of numerous seismic events across the globe, the concept of designing earthquake resistant structures has gained immense popularity among the engineers. With an intention of exploring the hysteretic behavior of reinforced concrete (RC) structures under quasi-static cyclic and seismic loading, quantifying different stages of structural damage and evaluating the seismic fragility curves for different structural performance levels, an extensive literature review is conducted as part of this research. Hence, a comprehensive synopsis of the present state-of-the-art along with the substantive findings is included in the following sections.

2.2 Research on Hysteresis Models

Extensive research is undertaken to date on prediction of hysteretic behavior of RC structural components. The Elasto-Plastic model introduced by Veletsos and Newmark ([V4]), as shown in **Fig. 2.1** is represented by an elastic curve indicating the cracked-section behavior with no incremental stiffness subsequent to yielding and unloading stiffness equal to the initial (elastic) one. V_y and u_y are defined as the yield strength and yield-level displacement respectively with k_e being the elastic stiffness ($k_e = V_y / u_y$) of the structure.

Bilinear Degrading Stiffness model by Clough and Johnston ([C10]) operates on a bilinear primary curve with ascending post yielding branches i.e. strain hardening and stiffness degradation during load reversals as observed in **Fig. 2.2**. The bilinear degrading stiffness model is categorized by stiffness regimes namely elastic stiffness k_e ; unloading stiffness k_e ; positive degrading stiffness $\overline{k_p}$; negative degrading stiffness $\overline{k_n}$ and post-yielding stiffness k_1 defined in terms of k_e and stiffness ratio r (with magnitude ranging between $\pm 5\%$) as $k_1 = rk_e$.

Fig. 2.1: Load-Deflection Diagram of Simple Elasto-plastic Model
(Veletsos and Newmark [V4])

Fig. 2.2: Load-Deflection Diagram of Bilinear Degrading Stiffness Model
(Clough [C10])

On the basis of experimental observations, Takeda model ([T1]) was introduced where the trilinear primary curve represents the uncracked, cracked and post-yielding stages as shown in **Fig. 2.3**. The cracking level load P_{cr} and displacement D_{cr} form the coordinates of the first break in the load-deflection diagram and nonlinear deformation initiates after section crack. The yield load P_y is calculated assuming a parabolic compressive stress-strain curve for concrete whereas the yield deflection D_y is measured to be the sum of deflections caused by curvature based

on cracked section, rebar-slippage, test platform deformation and shear. Depending on different magnitude ranges of load P_I several rules are prescribed to define the hysteresis response under load reversals. Here D_I represents the displacement corresponding to load P_I . When yield load P_y is exceeded in one direction, the slope of the unloading curve k_r is calculated by the following expression in terms of maximum deflection in the loading direction D_{max} ; yield deflection D_y and slope of the line joining yield point in one direction and cracking point in the other direction k_y .

$$k_r = k_y (D_y / D_{max})^{0.4} \quad (2.1)$$

Fig. 2.3: Load-Deflection Diagram of Trilinear Degrading Stiffness Model

(Takeda *et al.* [T1])

The load deflection relationship of the simple peak-oriented degrading bilinear (DBL) model proposed by Imbeault and Nielsen ([I2]) is shown in **Fig. 2.4**. The DBL system changes its stiffness only when the prior maximum is exceeded in any direction according to the following mathematical expression:

$$K = K_0 (D_y / D_{max})^\alpha \quad (2.2)$$

where K is the new stiffness of system; K_o is the primary stiffness of system; D_y is the yield deformation; D_{max} is the maximum deformation in any direction beyond yielding and α is a constant ranging between 0.5 to 0.6.

Fig. 2.4: Load-Deflection Diagram of Degrading Bilinear Model
(Imbeault and Nielsen [I2])

The Q-hysteresis model developed by Saidi and Sozen ([S3]) has a bilinear primary curve with ascending post-yielding branches with stiffness degradation at unloading and load reversal as shown in **Fig. 2.5**. The unloading stiffness at the inelastic segment of the primary curve k_q is denoted by the following expression.

$$k_q = k(U_y/U_m)^\alpha \quad (2.3)$$

Here k is the elastic stiffness of the primary curve, U_y is the yield deformation and U_m is the absolute maximum deformation, α is a constant with magnitude 0.4 for single degree of freedom system. The reloading stiffness is determined as the slope of the line $X_oU'_m$ with U'_m being the point on the primary curve symmetric to U_m with respect to the origin.

Fig. 2.5: Load-Deflection Diagram of Q-Hysteresis Model (Saidi and Sozen [S3])

The hysteresis model introduced by Otani ([O1]) has a bilinear primary curve with break point at yielding of the section as shown in **Fig. 2.6**. Unloading from maximum deformation maintains slope S_1 as mentioned in the expression hereunder.

$$S_1 = S_{OY} (U_y / U_m)^\alpha \quad (2.4)$$

where U_y is the yield level displacement; U_m is the maximum deformation experienced; S_{OY} is the slope of the elastic curve and α is an empirical constant.

Fig. 2.6: Load-Deflection Diagram of Hysteresis Model (Otani [O1])

The load-deformation relationship of Hysteresis Shear model introduced by Ozcebe and Saatcioglu ([O3]) is derived based on statistical analysis of the experimental data (**Fig. 2.7**). Unloading from a load higher than the cracking load V_{cr} but lower than the yield load V_y follows slope k as defined hereunder:

$$k = k_1 - \frac{k_1 - k_2}{\Delta_y - \Delta_{cr}} (\Delta - \Delta_{cr}) \quad (2.5)$$

where k_1 is the cracking stiffness; k_2 is slope of the line joining the cracking point and the yield point; Δ_{cr} , Δ_y and Δ are the cracking level, yield level and unloading deflections respectively. When V_y is exceeded in any loading direction, unloading stiffnesses from loads above and below V_{cr} are obtained as follows.

Fig. 2.7: Load-Deflection Diagram of Hysteresis Shear Model
(Ozcebe and Saatcioglu [O3])

$$k = k_2 \left\langle 1 - 0.05 \frac{\Delta}{\Delta_y} \right\rangle \quad (2.6)$$

$$k = 0.6k_2 \left\langle 1 - 0.07 \frac{\Delta}{\Delta_y} \right\rangle \quad (2.7)$$

Reloading follows the primary curve until the load is reversed at a load higher than V_{cr} . When V_{cr} is exceeded in the direction of loading, reloading up to V_{cr} follows a straight line passing through point (Δ_p, V'_p) . Reloading beyond V_{cr} follows a straight line up to the primary curve passing through point (Δ_m, V'_m) and beyond the intersection the reloading branch follows the primary curve. When unloading ceases before reaching the zero load axes, reloading traces a straight line aiming at the immediately preceding peak point.

$$V'_p = V_p e^{\alpha(\Delta_p/\Delta_y)} \quad (2.8)$$

$$\alpha = 0.82(N/N_0) - 0.14 < 0.0 \quad (2.9)$$

$$V'_m = V_m e[\beta n + \gamma(\Delta_m/\Delta_y)] \quad (2.10)$$

$$\beta = -0.014\sqrt{\Delta_m/\Delta_y} \quad (2.11)$$

$$\gamma = -0.010\sqrt{n} \quad (2.12)$$

Here Δ_p is the shear displacement corresponding to previous peak shear force V_p ; V_m is the shear force corresponding to maximum displacement Δ_m ; V'_p and V'_m are the respective shear forces aimed at previous peak displacement and maximum displacement during unloading; N is the axial compressive force; N_0 is the nominal axial load capacity under concentric loading; n is a counter tracing number of cycles at constant displacement; α , β , γ are mathematical coefficients.

In Pivot Hysteresis model developed by Dowell, Seible and Wilson ([D4]), four branches of the strength envelope of the monotonically increasing positive and negative loading as displayed in **Fig. 2.8** represent the elastic stiffness, strain-hardening stiffness, strength degradation from shear failure, confinement failure or bar pull-out and the linearly decreasing residual strength. The four quadrants of the

model (Q_1 , Q_2 , Q_3 and Q_4) are defined by the horizontal axis and the elastic loading lines (not the vertical axis).

Fig. 2.8: Strength Envelope and Quadrant Definition in Pivot Hysteresis Model
(Dowell *et al.* [D4])

The primary pivot points (P_1 , P_4) and the pinching pivot points (PP_2 , PP_4) as defined in **Fig. 2.9** determine the amount of softening with increasing displacement and the degree of pinching following load reversal respectively. Post exceedance of the yield displacement in either direction, the modified strength envelope is defined by lines joining PP_4 and PP_2 to S_1 and S_2 (previous maximum displacements in both directions) respectively.

Fig. 2.9: Pivot Point Designation in Pivot Hysteresis Model (Dowell *et al.* [D4])

The position of pinching pivot points changes when maximum displacement $d_{i\max}$ is greater than the strength degradation displacement d_{ti} as denoted hereunder.

$$\beta_i^* = \frac{F_{i\max}}{F_{ti}} \beta_i \quad (2.13)$$

Here β_i defines the degree of pinching for a ductile flexural response prior to strength degradation; $F_{i\max}$ and F_{ti} are the loadings corresponding to $d_{i\max}$ and d_{ti} respectively. In quadrants 1 and 3, loading follows lines away from P_1 and P_3 respectively while unloading moves along lines towards P_1 and P_3 correspondingly. In quadrants 2 and 4, loading takes place along lines towards PP_2 and PP_4 respectively while unloading follows lines away from P_2 and P_4 .

An energy-based hysteresis model was developed by Sucuoglu and Erberik ([S20]) for deteriorating single degree of freedom systems. The model operates on a bilinear curve with elastic stiffness K_0 ; post-yielding stiffness αK_0 ; unloading stiffness K_u and reloading stiffness K_r as shown in **Fig. 2.10**.

Fig. 2.10: Load-Deflection Diagram of Energy-based Hysteresis Model
(Sucuoglu and Erberik [S20])

Unloading from maximum displacement maintains elastic stiffness while unloading from an intermediate displacement point follows the slope of the line between the reloading target at current maximum post-elastic displacement and its unloading

intercept. Reloading slope K_r depends on the strength corresponding to the target point at the current maximum displacement in the respective direction.

In order to relate the loss in the energy dissipation capacity in the displacement cycle with the associated reduced strength level, let a deteriorating system with yield strength F_y and yield displacement u_y be subjected to a displacement pattern with amplitude u_m as shown in **Fig. 2.11**.

Fig. 2.11: Relationship between the reduced strength and the dissipated energy of Energy-based Hysteresis Model (Sucuoglu and Erberik [S20])

The energy dissipated in the first cycle is:

$$E_{h,1} = 2.5F_y(u_m - u_y) \quad (2.14)$$

$$E_{h,n} = 2F_n(u_m - u_y) \quad (2.15)$$

$$E_{h,n}/E_{h,1} = 0.8(F_n/F_y) \quad (2.16)$$

$E_{h,n}$ and F_n are the dissipated energy predicted by low-cycle fatigue model and the associated reduced strength at n th cycle, respectively. The normalized dissipated energy $\bar{E}_{h,n}$ at the equivalent n th cycle number is

$$\bar{E}_{h,n} = \alpha + (1-\alpha)e^{\beta(1-n)} \quad (2.17)$$

Here $(1-\alpha)$ is the loss of energy dissipation capacity for larger values of n and β is the rate of loss of dissipated energy.

Pinching hysteresis model by Ibarra, Medina and Krawinkler ([I1]) is defined by a piece-wise linear backbone curve with three control points representing yield strength F_y , peak strength F_c and residual strength F_r as shown in **Fig. 2.12**. Here, K_e is the elastic stiffness; δ_y is the yield displacement; K_s is the post-yielding stiffness with $K_s = \alpha_s K_e$; δ_c is the displacement at peak strength; K_c is the post-capping (negative) stiffness with $K_c = \alpha_c K_e$; δ_r is the displacement corresponding to residual strength $F_r = \lambda F_y$ and α_s , α_c , λ are empirical constants.

Fig. 2.12: Backbone Curve of Pinching Hysteresis Model
(Ibarra, Medina and Krawinkler [I1])

Fig. 2.13 presents the load-deformation relationship of the pinching hysteresis model in absence of deterioration where the unloading path follows the elastic stiffness K_e . The reloading path is divided into two segments with stiffnesses $K_{rel,a}$ and $K_{rel,b}$ through a break point, defined by κ_f , the maximum pinched strength and the corresponding displacement κ_d .

Fig. 2.13: Load-Deflection Diagram of Pinching Hysteresis Model in absence of deterioration (Ibarra, Medina and Krawinkler [11])

The cyclic deterioration β_i in excursion i is defined by:

$$\beta_i = \left\langle E_i / (E_t - \sum_{j=1}^i E_j) \right\rangle^c \quad (2.18)$$

Here E_i is the dissipated hysteretic energy in excursion i ; $\sum E_j$ is the hysteretic energy dissipated in all previous excursions during both positive and negative loading; E_t is the hysteretic energy dissipation capacity, $E_t = \gamma F_y \delta_y$; γ is a parameter representing E_t as a function of twice the elastic strain energy at yielding ($F_y \delta_y$) and c is an exponent defining the rate of deterioration with magnitude ranging between 1.0 to 2.0. Throughout the loading history, magnitude of β_i must be within the range $0 < \beta_i \leq 1$ otherwise hysteretic energy capacity is assumed to be exhausted with collapse being imminent. The cyclic deterioration comprises four individual deterioration modes involving degradation of basic strength, post-capping strength, unloading stiffness and accelerated reloading stiffness as shown in **Fig. 2.14**. Basic strength deterioration is associated with reduction in yield strength F_y and post-yielding stiffness K_s during cyclic loading. Post-capping strength deterioration is related to translation of post-capping branch towards origin.

Fig. 2.14: Four Deterioration modes of Pinching Hysteresis Model:

- a) Basic Strength Deterioration; b) Post-Capping Strength Deterioration;
- c) Unloading Stiffness Deterioration; d) Accelerated Reloading Stiffness

Deterioration (Ibarra, Medina and Krawinkler [I1])

Unloading stiffness deterioration is associated with reduction of unloading stiffness under repeated loading. Accelerated reloading stiffness is related to increase in the absolute maximum positive or negative displacement of past loading cycles. The mathematical expressions of the four deterioration modes are as follows:

$$F_{y,i}^{\pm} = (1 - \beta_{s,i}) F_{y,i-1}^{\pm} \quad (2.19)$$

$$K_{s,i}^{\pm} = (1 - \beta_{s,i}) K_{s,i-1}^{\pm} \quad (2.20)$$

$$F_{ref,i}^{\pm} = (1 - \beta_{c,i}) F_{ref,i-1}^{\pm} \quad (2.21)$$

$$K_{u,i} = (1 - \beta_{k,i}) K_{u,i-1} \quad (2.22)$$

$$\delta_{t,i}^{\pm} = (1 + \beta_{a,i}) \delta_{t,i-1}^{\pm} \quad (2.23)$$

Here $F_{y,i}^{\pm}$, $K_{s,i}^{\pm}$ and $K_{u,i}$ are the deteriorated yield strength, post-yielding stiffness and unloading stiffness after excursion i respectively; $F_{ref,i}^{\pm}$ is the intersection of the vertical axis with the projection of the post-capping branch for displacement cycle i in positive and negative loading directions; $\delta_{t,i}^{\pm}$ is the incremented absolute maximum positive or negative displacement after excursion i ; $\beta_{s,i}$, $\beta_{c,i}$, $\beta_{k,i}$ and $\beta_{a,i}$ are the rates of basic strength deterioration, post-capping strength deterioration, unloading stiffness deterioration and accelerated reloading stiffness deterioration in excursion i , respectively.

2.3 Study on Multi-spring Models

Several multi-spring models for RC beam-column joints and structural walls are proposed to date to obtain realistic hysteretic behavior. Alath and Kunnath model ([A3]) simulates the joint shear deformation using a rotational spring model with degrading hysteresis as shown in **Fig. 2.15**. The shear stress-strain envelope is determined empirically while the cyclic hysteresis response is obtained on the basis of experimental cyclic response.

Fig. 2.15: Alath and Kunnath Joint Model ([A3])

Biddah and Ghobarah model ([B8]) comprises separate rotational springs for the joint shear and bond-slip deformation as observed in **Fig. 2.16**. The shear stress-strain relationship of the joint was evaluated using a tri-linear idealization based on the softening truss model proposed by Hsu ([H9]). The cyclic response of the joint shear spring and the bond-slip spring respectively follow a hysteresis relationship with no pinching effect and a bilinear hysteresis model with pinching effect.

Fig. 2.16: Biddah and Ghobarah Joint Model ([B8])

Youssef and Ghobarah ([Y1]) proposed a joint element which contains two diagonal translational springs connecting the opposite corners of the panel zone and twelve translational springs located at the panel interface zone to simulate the joint shear deformations and other modes of inelastic behavior like bond-slip, concrete crushing etc. respectively as shown in **Fig. 2.17**.

Fig. 2.17: Youssef and Ghobarah Joint Model ([Y1])

Lowes and Altoontash proposed a 4-node joint element with twelve degrees of freedom (L10)], as shown in **Fig. 2.18**, which explicitly represents three types of inelastic mechanisms of beam-column joints under reversed cyclic loading. Eight zero-length translational springs simulate the bond-slip response of beam and column longitudinal reinforcement while four zero-length shear springs define the interface shear deformations. A panel zone zero-length rotational spring simulates the shear deformation of the joint. The bar stress versus slip relationship is idealized from experimental response of the beam-column joint specimens. The cyclic response of the panel zone is obtained from a highly pinched hysteresis relationship and the interface shear is simulated using a relatively stiff elastic load-deformation response deduced from experimental data.

Fig. 2.18: Lowes and Altoontash Joint Model ([L10])

A simplification to the Lowes and Altoontash Joint Model is introduced by Altoontash ([A5]) as presented in **Fig. 2.19**. This simplified model consists of four zero-length rotational springs located at the beam-column interfaces to simulate the member-end rotations due to bond-slip behavior. The panel zone rotational spring is utilized to simulate the shear deformation of the joint. The constitutive relationship for the panel zone remains identical to the Lowes and Altoontash (L10) model, enabling the calculation of constitutive parameters based on structural properties and experimental responses.

Fig. 2.19: Altoontash Simplified Joint Model ([A5])

Shin and LaFave ([S10]) modeled the joint by rigid elements located along the edges of the panel zone and a rotational spring embedded in one of the four hinges linking the adjacent rigid elements, as observed in **Fig. 2.20**. Two rotational springs located at the beam-joint interfaces simulate the member-end rotations due to inelastic behavior of the beam longitudinal reinforcements and plastic hinge rotations due to inelastic behavior of the beam separately.

Fig. 2.20: Shin and LaFave Joint Model ([S10])

In order to analyze the pseudo-dynamic earthquake response of a full-scale seven storey RC wall-frame test structure, Kabeyasawa *et al.* ([K1]) developed a three vertical line elements model (TVLEM) with infinitely rigid beams at the top and

bottom floor levels as shown in **Fig. 2.21**. Two boundary elements represent the axial stiffness of the outside columns located in the test structure. The central one-component vertical element represents the wall panel which consists of a vertical spring, a horizontal spring and a rotational spring located at the base.

Fig. 2.21: Kabeyasawa *et al.* Wall Model ([K1])

Vulcano and Bertero ([V5]) attempted to modify the outer vertical spring of TVLEM model using a spring assembly as found in **Fig. 2.22**, to simulate the physical behavior of cracking and yielding. The single topmost spring represents the uncracked concrete while the two parallel springs denote the cracked concrete and steel respectively. The steel spring follows a bilinear curve and the cracked concrete spring either seizes to act representing the cracked stage or takes up action indicating the closing of cracks.

Fig. 2.22: Vulcano and Bertero Spring Assembly ([V5])

Vulcano *et al.* ([V6]) further replaced the rotational spring by additional vertical springs as observed in **Fig. 2.23** to simulate the axial behavior of the web and the gradual yielding of the vertical reinforcement more smoothly.

Fig. 2.23: Vulcano *et al.* Multiple Vertical Spring Model ([V6])

Linde and Bachmann ([L8]) developed a macro element to represent the inelastic seismic behavior of shear walls controlled by flexure, with modest influence of shear cracking in the hysteretic response. This model comprises three vertical springs and one vertical spring connected to the rigid beams to simulate the axial and flexural behavior and shear behavior, respectively.

Fig. 2.24: Linde and Bachmann Wall Model ([L8])

The macro wall element developed by Youssef and Ghobarah ([Y1]) contains four steel and concrete springs to represent the behavior of steel reinforcement and concrete strut and to define the plastic hinge region. Among the four springs, two exterior springs indicate the wall boundary region while the two interior springs represent the remaining wall section. Two diagonal springs represent the shear behavior of the wall.

Fig. 2.25: Youssef and Ghobarah Wall Model ([Y1])

2.4 Investigation on Damage Models

The concept of defining different stages of structural damage in the form of damage indices is effective owing to its simplicity in application. Typically damage indices are dimensionless parameters intended to range between 0 for undamaged structure and 1 for collapsed structure with intermediate parameters indicating certain degree of structural damage. However, the expression for damage index needs to be generalized such that it can incorporate different structural aspects. With this intention in mind, several damage models defining the expressions for damage indices are proposed by researchers to date. Damage models can be broadly classified into two categories: Local Damage models and Global Damage models. In local damage models, the degree of damage experienced by a structural component is measured using the specific damage index while the global damage models represent the overall damage state of a structure. Thus the global damage of any structure is dependent on the severity and distribution of the local damage of the structural components.

2.4.1. Local Damage Models

The local Damage index proposed by most researchers is a quantity where excess deformation, energy dissipation or the combination of both is taken into account. Sozen ([S18]) suggested inter-storey drift as the damage measure.

$$\% \text{ of damage} = 50 \times \text{maximum interstorey drift}(\%) - 25 \quad (2.24)$$

On the basis of analysis results of structural components and small-scale structures, it was concluded that inter-storey drift magnitude smaller than 1% leads to damage of non-structural components while inter-storey drift magnitude larger than 4% may result in irreparable structural damage. Structural collapse is observed to occur when inter-storey drift magnitude surpasses 6%.

Banon *et al.* ([B4]) proposed the structural damage to be dependent on stiffness degradation and defined flexural damage ratio (FDR) as the ratio of initial stiffness

(k_0) to the reduced secant stiffness at the maximum displacement (k_m) as shown hereunder. The definition of stiffness degradation is explained in **Fig. 2.26**.

$$FDR = k_0 / k_m \quad (2.25)$$

Roufaiel and Meyer ([R6]) redefined FDR as the rate of stiffness change from maximum deformation (k_m) to the initial condition (k_0) divided by the same from the instant of failure (k_f) to the initial condition (k_0) as follows:

$$FDR = \frac{k_f}{k_m} \cdot \frac{(k_m - k_0)}{(k_f - k_0)} \quad (2.26)$$

Fig. 2.26: Definition of Stiffness Degradation ([R6])

Toussi and Yao ([T3]) and Stephens and Yao ([S19]) introduced inter-storey drift and permanent drift as damage measures since they are closely related to the plastic deformations of a structure. According to this model structural damage is classified into four groups: a) Safe with storey drift ratio not exceeding 1%; b) Lightly damaged with permanent drift ratio 0.5%; c) Damaged with permanent drift ratio 1% and d) Critically damaged with top storey displacement showing some

aperiodicity at the end of the record with poor correlation between base shear and top level displacement.

The Park-Ang damage model ([P9]) considers a linear combination of damage caused by excessive deformation and repeated cyclic loading. In this model, damage index (DI) is calculated based on the following expression:

$$DI = \left\langle \frac{\delta_{max}}{\delta_u} + \beta \left| \frac{\delta}{\delta_u} \right|^\alpha \frac{dE}{E_c(\delta)} \right\rangle \quad (2.27)$$

where δ_{max} is the peak deformation, δ_u is the ultimate deformation capacity of the structure, β is a cyclic deterioration parameter, α being an empirical constant, $E_c(\delta)$ is the accumulated hysteretic energy per loading cycle. Kratzig ([K7]) suggested the dissipated energy to be the damage measure as follows.

$$DI^\pm = \frac{\Sigma E_{si}^\pm + \Sigma E_i^\pm}{E_U^\pm + \Sigma E_i^\pm} \quad (2.28)$$

where $\Sigma E_{si}^\pm$ and $\Sigma E_i^\pm$ are the energy dissipations in primary and following half cycles respectively for positive (+) and negative (-) loading. $E_U^\pm$ is the ultimate energy capacity in positive and negative monotonic loading respectively.

The expression for stiffness-based damage index (DI_K) proposed by Ghobarah *et al.* ([G3]) based on the static push over analysis is presented hereunder:

$$DI_K = 1 - (K_{final} / K_{initial}) \quad (2.29)$$

Here $K_{initial}$ and K_{final} are the initial slopes of the base shear vs. top deflection relationship resulting from the push-over analysis and the earthquake time-history analysis respectively. The plastic rotation based damage index by Mehanny and Deierlein ([M5]) is expressed below.

$$DI^{\pm} = \frac{\theta_p^{\pm} \Big|_{currentPHC}^{\alpha} + \sum_{i=1}^{n^{\pm}} \theta_p^{\pm} \Big|_{FHC,i}^{\beta}}{\theta_{pu}^{\pm \alpha} + \sum_{i=1}^{n^{\pm}} \theta_p^{\pm} \Big|_{FHC,i}^{\beta}} \quad (2.30)$$

$$DI = \sqrt[\gamma]{DI^{+\gamma} + DI^{-\gamma}} \leq 1.0 \quad (2.31)$$

Here $\theta_p^{\pm} \Big|_{currentPHC}$ and $\theta_p^{\pm} \Big|_{FHC}$ are the positive and negative peak plastic rotations in primary half cycles (PHC) and following half cycles (FHC) respectively and α , β and γ are the calibration parameters. For DI calculation, $\theta_p^{\pm} \Big|_{FHC}$ of the previous cycle is considered to be $\theta_p^{\pm} \Big|_{currentPHC}$ for the next cycle in the model.

2.4.2. Global Damage Models

Global damage indices are usually computed by combining the local damage indices across the structure or by using the overall structural characteristics, like the modal parameters. Global damage indices derived by local damage indices generally employ some weighing systems to define the distribution and severity of individual local indices. Park, Ang and Wen ([P9]) defined the global damage index D_T of any structural system as the sum of the local damage indices of the structural components D_i weighted by the energy absorbing contribution factor λ_i as follows.

$$D_T = \sum_i \lambda_i D_i \quad (2.32)$$

where $\lambda_i = E_i / \sum E_i$ and E_i is the total absorbed hysteretic energy by the i th structural component.

Bracci ([B11]) generalized the expression for global damage index D_T in terms of weight w_i and local damage index D_i of the i^{th} structural component.

$$D_T = \frac{\sum_i w_i D_i^{(b+1)}}{\sum_i w_i D_i^b} \quad (2.33)$$

Here higher magnitude of exponent b emphasizes the influence of more severely damaged elements.

Gunturi and Shah ([G9]) proposed cost weighting for calculation of global damage index D_T .

$$D_T = \frac{\sum_{i=1}^n w_i D_i}{\sum_{i=1}^n w_i} \quad (2.34)$$

where w_i is the weighting factor i.e. replacement cost of i th component; D_i is the dollar damage index of i th component and n is the number of structural components. Dollar damage index is a ratio of the repair cost to the replacement cost. It is assumed that Dollar damage index is equal to the Response damage index i.e. ratio of the maximum response experienced by a structural component to the response capacity of the component.

DiPasquale and Cakmak ([D2], [D3]) developed a damage model on the basis of evolution of the natural period of a time-varying linear system equivalent to the actual non-linear system for a series of non-overlapping time windows. In this regard, softening indices are utilized to relate the changes in the natural periods of a structure to the intensity of structural damage. This global damage index takes into consideration the combined effects of stiffness degradation and plastic deformation. The variation in fundamental period during a seismic event is shown in **Fig. 2.27**. The expressions for various softening indices in terms of three different periods of structures are summarized below.

$$D_m = 1 - \frac{T_{und}}{T_m} \quad (2.35)$$

$$D_{pl} = 1 - \frac{T_{dam}^2}{T_m^2} \quad (2.36)$$

$$D_F = 1 - \frac{T_{und}^2}{T_{dam}^2} \quad (2.37)$$

Here, D_m , D_{pl} and D_F represent the softening indices corresponding to maximum softening, plastic softening and final softening respectively; T_m is the maximum period of a structure; T_{und} and T_{dam} are the periods of undamaged and damaged structure.

Fig. 2.27: Variation of Fundamental Period during a Seismic Event
(Williams *et al.* [W3])

Cosenza and Manfredi ([C14]) suggested an overall damage factor I_D in terms of the energy content of a seismic event as follows:

$$I_D = \frac{2g}{\pi} \frac{I_A}{PGA \cdot PGV} \quad (2.38)$$

Here I_A is the Arias intensity, a measure of earthquake destructiveness based on the root mean square acceleration (RMSA) as follows:

$$I_A = \frac{\pi}{2g} \cdot t_E \cdot RMSA^2 = \frac{\pi}{2g} \int_0^{t_E} a_g(t)^2 dt \quad (2.39)$$

where t_E is the duration of the seismic event and $a_g(t)$ is the ground acceleration during earthquake.

2.5 Study on Seismic Fragility Functions of RC Structures

Fragility functions play a significant role in evaluating the potential severity of the consequences of a particular earthquake phenomenon. Seismic fragility functions represent the conditional probability of a particular structural component or a system exceeding a certain damage state i.e. a performance level when subjected to a given measure of ground motion intensity. Over the past few decades, extensive research is devoted to the seismic vulnerability assessment of RC structures.

Aslani and Miranda ([A8]) proposed fragility functions of slab-column connections in the existing RC non-seismically detailed buildings with respect to the peak inter-storey drift ratio by interpreting the experimental results of 82 specimens from the literature. The fragility functions were evaluated corresponding to four damage states i.e. light cracking, severe cracking, punching shear failure and loss of lateral load carrying capacity.

Pagni and Lowes ([P2]) and Brown *et al.* ([B12]) identified inter-storey drift, number of load cycles, drift in combination with the number of loading cycles, joint shear strain and joint shear strain in combination with the number of loading cycles as the five potential engineering demand parameters for obtaining fragility functions of older and modern beam-column joints respectively on the basis of previous experimental results. The fragility functions corresponding to five methods of repair i.e. cosmetic repair, epoxy injection, patching, concrete replacement and joint replacement were derived in the research.

Gulec *et al.* ([G7]) evaluated the fragility functions of low aspect ratio walls of rectangular, flanged and barbell cross-sections in terms of the methods of repair associated with the damage states obtained from the past experimental results. Damage states were characterized using the direct indicators of damage such as cracking and crushing of concrete, residual displacement due to sliding and

reinforcement buckling and fracture. The associated methods of repair of walls are cosmetic repair, epoxy injection, partial wall replacement and complete wall replacement.

Singhal and Kiremidjian ([S12]) developed seismic vulnerability curves and damage probability matrices for low, mid and high-rise RC frame structures by conducting nonlinear dynamic analysis. Spectral acceleration and root mean square acceleration were considered to be the ground motion intensity measure and the Park-Ang damage model was adopted for damage quantification in this study.

Mosalam *et al.* ([M8]) modeled the gravity load-designed two-storey two-bay RC frames with and without infill in the finite element system DIANA and performed push over analysis using the dynamic plastic hinge method. The seismic fragility curves for gravity load-designed low-rise bare and infilled RC frames in the area of Memphis, Tennessee, United States were assessed using the trilinear capacity curves.

Dumova-Jovanoska ([D5]) evaluated the earthquake intensity-damage relations in the form of fragility curves and damage probability matrices for RC frame-wall structures in Skopje (Macedonia) region. The damage probability matrices were determined on the basis of the defined damage states and the probability distributions of the occurrence of damage by earthquakes of given intensity. The fragility curves of 6-storey RC frame and 16-storey RC wall-frame structures with seismic design were developed by conducting nonlinear dynamic analyses for 240 synthetic time histories using the damage probability matrices.

Masi ([M2]) derived the seismic fragility functions of bare frames, regularly infilled frames and open ground storey frames representative of post-1970 gravity load-designed RC existing buildings in Italy. The frame structures were modeled using DRAIN-2D software to conduct the non-linear time-history analyses with the artificial and natural accelerograms. Takeda hysteresis modeling rules ([T1]) were applied to beams and columns of the frames while each masonry panel was modeled by using plane elements having elasto-plastic load-deformation principles ([V4]).

Kappos *et al.* ([K4]) employed the capacity spectrum method to evaluate the seismic fragility curves of RC buildings with and without infills for different levels of seismic design. The uncertainty in the damage states and the variability of the structural capacity were obtained from HAZUS ([F3]).

Erberik and Elnashai ([E3]) established the fragility curves for a 5-storey schematic flat slab building with masonry infill designed according to US seismic codes. 10 recorded accelerograms compatible with the code spectrum were selected for obtaining the damage states of the building in terms of the inter-storey drift ratio. Uncertainties resulting from the material strength of RC structures were taken into account in this study. The seismic fragility curves derived from the inelastic response history analyses of RC flat slab structures were compared with the same of the moment-resisting RC frame structures. The comparison study concluded that the earthquake losses for flat slab structures are in the same range as for the moment-resisting frame structures.

Vacareanu *et al.* ([V1]) performed the seismic vulnerability assessment of representative residential RC buildings in Bucharest, Romania designed with low and medium level seismic codes using HAZUS ([F3]) and ATC-40 ([A9]) methodology. Monte Carlo simulation method was adopted to estimate the random variables required for push-over analysis to generate the family of capacity curves. On the basis of the expected probability of collapse, ranking of different building types in Romania was proposed so that the ranking of buildings can be regarded as a decision tool for seismic retrofitting.

Akkar *et al.* ([A2]) developed seismic fragility curves for 32 low and mid rise existing RC buildings in Düzce, Turkey by conducting pushover analysis and nonlinear dynamic analysis for 82 recorded accelerograms. A displacement controlled nonlinear static procedure was employed with an inverted triangular lateral load distribution for obtaining the capacity curves of each building. Peak ground velocity (PGV) was considered as a measure of strong motion intensity in the fragility functions developed for the buildings since the inelastic dynamic response displacements were observed to be significantly better correlated with

PGV than peak ground accelerations (PGA) across the structural period ranging from 0.2 to 1.0 seconds.

Rossetto and Elnashai ([R5]) generated seismic fragility curves for low-rise infilled RC frames designed with the old Italian seismic code by interpreting the responses of adaptive pushover analysis and nonlinear dynamic analysis of the equivalent SDOF structure for ten accelerograms. In this study the vulnerability curves were derived using the analytical damage statistics by incorporating the uncertainties resulting from the ground motion data and damage state identification for any structural type and seismo-tectonic environment.

Kappos *et al.* ([K5]) developed seismic fragility curves for RC frame and wall-frame buildings in terms of PGA and spectral displacement using hybrid methodology by combining the statistical data with the results from non-linear dynamic or static analyses. A wide variety of buildings with Low, mid and high-rise, three different configurations; explicitly bare, regularly infilled and soft ground storey building, four classes of seismic design specifically no code, low code, moderate code and high code were modeled and analyzed in this study for seismic vulnerability study. For all buildings inelastic static and dynamic analyses were carried out using SAP2000N and DRAIN2000 respectively. RC members were modeled using lumped plasticity beam-column elements and infill walls were modeled using the diagonal strut elements for the inelastic static analyses and the shear panel isoparametric elements for the inelastic dynamic analyses.

Ramamoorthy ([R2]) performed seismic fragility estimates of RC buildings of 1 to 10 storeys tall on the basis of simulated response obtained from nonlinear time history analysis for a suite of synthetic ground motion developed for Memphis, Tennessee. Since the seismic response of buildings is sensitive to the frequency content of the earthquake and the fundamental building period, bivariate fragility estimates defined as the conditional probability of attaining or exceeding a specified performance level for given values of spectral acceleration and fundamental building period, were developed for generic buildings. Approximate confidence

bounds on the fragility assessments were derived to reflect the inherent epistemic uncertainty in the predicted values.

Ellingwood *et al.* ([E1]) assessed several low to mid rise steel and RC buildings representative of design and construction practice in Central and Eastern United States. A nonlinear static push-over analysis of each frame was performed to identify the general characteristics of the structural behavior and define the capacity curve. A nonlinear time history analysis was conducted for each frame in Opensees ([M6]) using different sets of synthetic ground motions to assess the seismic demand on the frames. The seismic demands on frames were observed to be strongly dependent on the ground motion model indicating that the impact of epistemic uncertainty in ground motion modeling and performance assessment is substantial.

Erberik ([E2]) derived analytical fragility curves of RC frame buildings constructed between 1973 and 1999 by performing nonlinear dynamic analysis for a set of 100 recorded accelerograms. The energy-based hysteresis model ([S20]) was employed in this study to explore the influence of deterioration in structural properties under repeated loading cycles for the existing low rise and mid rise RC frame buildings. The bilinear capacity curves for buildings were obtained by pushover analysis. The analytical fragility curves were in good agreement with the observed damage after the Düzce earthquake. The degrading characteristics of structures were observed to have significant influence on the fragility curves in this study. Considering the high sensitivity of the fragility curves to limit state definitions, incorporation of uncertainty in the capacity by quantifying the variability in the limit states was found to be quite important.

Özer and Erberik ([O4]) generated seismic fragility curves in terms of in terms of maximum inter-storey drift ratio for 3, 5, 7 and 9 storey RC moment-resisting frame structures in Turkey with poor, medium and good seismic design considerations for four damage states by performing non-linear time-history analysis. In this study, the main parameters affecting the structural fragility were considered as number of storeys, structural deficiencies quantified as superior, typical or poor subclass and

ground motion intensity level. Structural damage was observed to increase with decreasing structural sub-class quality and increasing number of storeys for buildings. Probabilistic limit states were found essential in fragility studies as the deterministic limit states cannot be obtained with much confidence and quantification of limit states directly affects the resulting fragility.

2.6 Conclusions

On the basis of the extensive literature review, the conclusions drawn are summarized hereunder:

- i) Most of the analytical hysteresis models developed to date does not reflect realistic response for RC beam-column joints and structural walls under cyclic and seismic loading. Elasto-plastic ([V4]) and Degrading Bilinear Model ([I1]) are unable to represent the hysteretic behavior for structural components. Clough Model ([C10]) does not incorporate the pinching effect. Q-Hysteresis ([S3]) and Otani Models ([O1]) consider the concrete section to be elastic up to yield with initiation of nonlinearity in concrete after the appearance of cracks. The number of rules required being quite large, performances of Takeda Model ([T1]), Pivot Hysteresis Model ([D4]) and Hysteresis Shear Model ([O3]) are relatively poor comparing to their complexity. The analytical and observed force-displacement path of Energy-based hysteresis model ([S20]) is not quite satisfactory at larger cycle numbers because pinching due to anchorage slip is not accounted for in the model.
- ii) The macro scale models developed for RC beam-column joints and structural walls contain multiple springs to predict their overall hysteretic response. While formulating the macro scale models, it is assumed in most of the research that plane cross-sections remain plane for rigid elements which may affect the accuracy of the overall response of the structural components. With increase in the number of springs for presenting each behavioral aspect of the structural component separately, the complexity associated becomes a serious concern. On the other hand, available simplified models are not capable of producing

- required response. Moreover, the accuracy of the hysteretic response of each spring is significant in obtaining realistic response from the macro models.
- iii) Damage indices provide the means for quantifying structural damage experienced under varying loading conditions. The most widely used damage indicator, inter-storey drift, fails to incorporate the effects of repeated cyclic loading. Similarly, Flexural damage ratio (FDR) does not take the effects of cumulative damage caused by repeated cyclic loading into account. The maximum permanent drift when used as a damage measure provides significant discrepancies due to the variation of structural configuration and loading types. The main problem in using the Park-Ang damage model ([P9]) lies in the determination of the ultimate displacement capacity δ_u and degradation parameter β . The regression equations proposed by most researchers yield much lower values for β making the contribution of the energy term in the damage index negligible. The softening damage indices do not explicitly incorporate the effects of dissipated hysteretic energy and strength degradation of a structure. Moreover, the most common approach of obtaining global damage index by combining the local damage indices with a weighting scheme does not consider the difference between various structural components leading to erroneous response.
- iv) The extensive research conducted for seismic fragility assessment of RC frame structures reveals the importance of selection of the structural characteristics (year of construction, number of storeys, configuration of structure i.e. bare or infill frame etc.) representative of a region. Then, it is necessary to model the representative structure adequately so as to obtain the generalized and realistic response for the capacity and performance evaluation. However, this is often the less prioritized step in seismic vulnerability assessment. The ground motion datasets also must be scaled to become compatible with the region for which the vulnerability assessment is being carried out. To date no research is conducted on seismic fragility assessment of RC frame structures located in Singapore. Thus, the present research is aimed at defining hysteretic models of RC

structural components and quantifying the structural damage under seismic loading to assess the seismic vulnerability of RC frame structures in Singapore.

CHAPTER 3

HYSTERESIS MODELS FOR RC STRUCTURAL COMPONENTS

3.1 Introduction

Analytical prediction of hysteretic behavior of Reinforced Concrete (RC) structural components requires a load-deformation relationship capable of producing requisite strength and stiffness degradation along with pinching at all displacement levels. Moreover, the hysteresis model must be generic, computationally efficient and mathematically tractable such that it can be applicable to random input functions. Thus, it can be clearly perceived that development of hysteresis models satisfying all above requirements becomes stringent considering the numerous parameters contributing to the structural behavior. Hence, an utmost effort is undertaken to predict the hysteretic behavior of RC structural components on the basis of the Bouc-Wen-Baber-Noori smooth hysteresis model (SHM).

3.2 Description of Hysteresis Model

The Bouc-Wen-Baber-Noori (BWBN) hysteresis model selected as the basis of the present research is amended accordingly to simulate the hysteretic behavior of RC structural components. Bouc ([B10]) suggested a versatile, smoothly varying hysteresis model for a single-degree-of-freedom (SDOF) system under forced vibration. Baber and Wen ([B2]) extended the model with inclusion of stiffness and/or strength degradation as a function of hysteretic energy dissipation. Baber and Noori ([B3]) further incorporated pinching effects in the model.

The equation of motion for a single-degree-of-freedom system as shown in **Fig. 3.1** can be presented hereunder:

$$m\ddot{u} + c\dot{u} + F_T[u(t), z(t), t] = F(t) \quad (3.1)$$

where u is the relative displacement of mass m with respect to ground motion; c is the linear viscous damping coefficient; $F_T[u(t), z(t), t]$ is the non-damping restoring force consisting of the linear restoring force αku and the hysteretic restoring force $(1-\alpha)kz$; α is stiffness ratio i.e. the ratio of final asymptote tangent stiffness k_f to initial stiffness k_i with magnitudes 1 for a linear system and 0 for a nonlinear system and $F(t)$ is the time-dependent forcing function.

Fig. 3.1: Schematic Model of a Single-Degree of Freedom Hysteretic System
(Foliente *et al.* [F4])

Dividing both sides of Eq. (3.1) by m , the following standard expression is obtained:

$$\ddot{u} + 2\xi_0\omega_0\dot{u} + \alpha\omega_0^2u + (1-\alpha)\omega_0^2z = f(t) \quad (3.2)$$

where ξ_0 is the linear damping ratio, $c/2\sqrt{k_im}$; ω_0 is the pre-yield system natural frequency, $\sqrt{k_i/m}$ and $f(t)$ is the mass-normalized forcing function.

Hysteretic restoring force is a function of hysteretic displacement z and thus the relationship between z and u is as follows:

$$\dot{z} = h(z) \frac{A\dot{u} - \nu(\beta|\dot{u}|z|^{n-1}z + \gamma\dot{u}|z|^n)}{\eta} \quad (3.3)$$

where A designates the tangent stiffness; β , γ and n are the hysteretic shape parameters; ν and η are the strength and stiffness degradation parameters respectively and $h(z)$ is the pinching function.

For a non-pinching and non-degrading system, hysteresis is defined by a continuous function and hysteretic stiffness is always zero at local maximum or minimum i.e. the point on the load-deformation curve where velocity changes its sign. So, at an infinitesimal distance dz away from z_{max} where velocity is close to but not equal to zero and $\dot{z}_{max} \approx \dot{z}$,

$$\dot{z}_{max} \approx 0 = A\dot{u} - \nu(\beta|\dot{u}|z|^{n-1}z + \gamma\dot{u}|z|^n)$$

$$\text{Or } z_{max} = \pm \{A/\nu(\beta + \gamma)\}^{1/n} \quad (3.4)$$

Inclusion of variation of A can contribute to the versatility of the model. However, the parameter A is somewhat redundant as both hysteretic stiffness and hysteretic force can be varied by the stiffness ratio and the hysteresis shape parameters. Thus, for simplicity the magnitude of A is set to unity.

3.2.1 Stiffness Ratio or Rigidity Ratio

Stiffness ratio or Rigidity ratio α is defined as the ratio of the final asymptote tangent stiffness k_f to the initial stiffness k_i . The magnitude of α lies between zero for a complete nonlinear system to unity for a linear system. In the original BWBN model, α was considered to be of constant magnitude. However, based on experimental results of RC structural components under repeated cyclic loading, it can be well recognized that their stiffness decreases after attaining a certain

displacement and thus, considering stiffness ratio of constant magnitude is impractical. Therefore, in the present research, the expression of α is made to be a function of D_{max} as follows:

$$\alpha = \alpha_0 e^{(-0.1D_{max})} \quad (3.5)$$

where α_0 is the magnitude of α at the initial stage and D_{max} is the absolute maximum displacement. Thus D_{max} is defined as the maximum positive displacement for $u > 0$ and the absolute maximum negative displacement for $u < 0$. A comparison of experimental load-deformation curve of RC beam-column joint specimen P5 tested by Pessiki *et al.* ([P12]) with model load-displacement plots using constant and displacement-based α is shown in **Fig. 3.2**.

Fig. 3.2: Experimental and Model (with Constant α and Displacement-based α) Load Deformation Plot of RC Beam-Column Joint Specimen 5 (Pessiki *et al.* [P12])

Similarly, the hysteresis loops of RC wall specimen 22 tested by Hidalgo *et al.* ([H4]) obtained from experimental results and model responses with constant and displacement-based α is shown in **Fig. 3.3**. From **Figs. 3.2** and **3.3**, it is quite

evident that use of displacement-based α is more accurate in determination of hysteretic behavior of RC structural components.

Fig. 3.3: Experimental and Model (with Constant α and Displacement-based α) Load Deformation Plot of RC Wall Specimen 22 (Hidalgo *et al.* [H4])

3.2.2 Hysteresis Shape Parameters

The hysteresis shape parameters β , γ and n and their interaction determines the basic hysteresis shape. The absolute values of β and γ inversely influence the hysteretic stiffness and strength, as well as the smoothness of the hysteresis loop. The combination of magnitudes of β and γ influences the hysteresis loops to maintain a hardening or softening load-deformation relationship. During loading, parameter n controls the sharpness of the transition from initial to asymptotic slope. For $n = 1$, the relationships between the combined magnitudes of β and γ and their impact on the hysteresis loops are presented in **Fig. 3.4** and described hereunder.

$$i) \left. \begin{array}{l} \beta + \gamma > 0 \\ \gamma - \beta < 0 \end{array} \right\} \text{Weak Softening}$$

- ii) $\left. \begin{array}{l} \beta + \gamma > 0 \\ \gamma - \beta = 0 \end{array} \right\}$ Weak Softening on loading, mostly linear unloading
- iii) $\left. \begin{array}{l} \beta + \gamma > \gamma - \beta \\ \gamma - \beta > 0 \end{array} \right\}$ Strong Softening on loading and unloading, narrow loop
- iv) $\left. \begin{array}{l} \beta + \gamma = 0 \\ \gamma - \beta < 0 \end{array} \right\}$ Weak Hardening
- v) $\left. \begin{array}{l} 0 > \beta + \gamma \\ \beta + \gamma > \gamma - \beta \end{array} \right\}$ Strong Hardening

Fig. 3.4: Possible Shapes of Hysteresis loops under different combinations of β and γ for $n=1$ (Baber *et al.* [B1])

3.2.3 Hysteretic Energy

Hysteretic energy ε is defined as the irrecoverable strain energy absorbed by the hysteretic element. Hysteretic energy is utilized to estimate the strength and

stiffness degradation of RC structural components approximately. The total strain energy consisting of the irrecoverable hysteretic energy and the recoverable strain energy, is expressed as continuous integral of hysteretic force, $f(t)$ over total displacement u . Since the recoverable strain energy tends to be quite small compared to the irrecoverable hysteretic energy, the hysteretic energy is approximately considered equal to the total energy. Thus, the expression for hysteretic energy is presented as follows.

$$\varepsilon(t) = \int_{u(0)}^{u(t)} f \cdot du = (1 - \alpha) \cdot \omega_0^2 \int_{u(0)}^{u(t)} z(u, t) \cdot dt \cdot \frac{du}{dt} = (1 - \alpha) \cdot \omega_0^2 \int_0^t \ddot{x}(u, t) \cdot \dot{u}(t) \cdot dt \quad (3.6)$$

3.2.4 Strength and Stiffness Degradation

Gradual decrease in the strength of a structural component when it is cyclically loaded at the incremental displacement level is termed as strength degradation. Progressive loss of lateral stiffness of a structural component in each loading cycle is defined as stiffness degradation. Strength and stiffness degradation parameters ν and η respectively, are the functions of total hysteretic energy ε (see eqn. 3.6) as shown in the following expressions:

$$\nu(\varepsilon) = 1 + \delta_\nu \varepsilon \quad (3.7)$$

$$\eta(\varepsilon) = 1 + \delta_\eta \varepsilon \quad (3.8)$$

where δ_ν and δ_η are the designated rates of strength and stiffness degradation respectively at different displacement levels. The effect of increasing magnitudes of δ_ν on hysteresis load-deformation loops is presented in **Fig. 3.5** while the influence of increasing magnitudes of δ_ν is shown in **Fig. 3.6**. When the magnitudes of δ_ν and δ_η are zero, the structure does not degrade its strength and stiffness as shown in **Figs. 3.5** and **3.6** respectively. Due to increase in δ_η , both

hysteretic force and hysteretic stiffness degrade whereas increase in δ_v reduces the hysteretic force without changing the hysteretic stiffness.

Fig. 3.5: Strength Degradation with Changing δ_v , when all other parameters and input function are same

Fig. 3.6: Stiffness Degradation with Changing δ_η , when all other parameters and input function are same

3.2.5 Pinching Function

Due to the presence of high shear forces, opening and closing of cracks and rebar slippage, the pinching phenomenon takes place. Pinching function $h(z)$ is a function of multiple parameters as stated in the following expression.

$$h(z) = 1 - \zeta_1 e^{-\left(\frac{z \operatorname{sgn}(\dot{u}) - qz_{\max}}{\zeta_2} \right)^2} \quad (3.9)$$

where ζ_1 determines the severity of pinching or the magnitude of initial drop in slope dz/du having a magnitude varying from 0 to 1. ζ_2 causes the pinching region to spread and q sets pinching level as a fraction of z_{max} . Both ζ_1 and ζ_2 vary with hysteretic energy ε (see eqn. 3.6) as follows:

$$\zeta_1(\varepsilon) = \zeta_s \left\langle 1 - e^{-p\varepsilon} \right\rangle \quad (3.10)$$

$$\zeta_2(\varepsilon) = (\psi + \delta_\psi)(\lambda + \zeta_1) \quad (3.11)$$

where p is the rate of initial drop in slope; ζ_s is the total slip; ψ controls the amount of pinching; δ_ψ is the rate of pinching spread and λ regulates the rate of change of ζ_2 with change of ζ_1 . The continuous nature of pinching function produces smooth hysteresis loops with a gradual transition from almost zero stiffness to maximum hysteretic stiffness. Effect of pinching due to change in magnitudes of ζ_s on hysteretic behavior is shown in **Fig. 3.7**.

Fig. 3.7: Effect of Pinching on Hysteresis loops

3.3 Solving Procedure for Analytical Hysteresis Model

The complete hysteresis model can be represented in its analytical form as follows:

$$\ddot{u} + 2\xi_0\omega_0\dot{u} + \alpha\omega_0^2u + (1-\alpha)\omega_0^2z = f(t) \quad (3.12)$$

$$\dot{z} = \left\langle 1 - \zeta_s (1 - e^{-p\varepsilon}) e^{-\left(\frac{z \operatorname{sgn}(\dot{u}) - q \{1 + \delta_v \varepsilon (\beta + \gamma)\}^{1/n}}{\psi + \delta_\psi \varepsilon} \right)^2} [\lambda + \zeta_s (1 - e^{-p\varepsilon})]^2} \right\rangle$$

$$\times \left\langle \frac{\dot{u} - (1 + \delta_v \varepsilon) (\beta |\dot{u}| |z|^{n-1} z + \gamma |\dot{u}| |z|^n)}{1 + \delta_\eta \varepsilon} \right\rangle \quad (3.13)$$

$$\varepsilon(t) = (1 - \alpha) \omega_0^2 \int_0^t \ddot{x}(u, t) \dot{u}(t) dt \quad (3.14)$$

Since in Eq. (3.12) - Eq. (3.14), all derivatives appear in the first power and the variables vary with time at different rates, the hysteresis model consists of a stiff set of ordinary differential equations (ODE), which can be solved numerically by using Gear's backward differential formulae ([G1]). In the present research, the Livermore Solver for Ordinary Differential Equations (LSODE) is selected for solving the ODEs involved in the proposed analytical model ([H6], [R1]). The input displacement function required for computation, may not necessarily be continuous, even discrete data points from an external file can serve the purpose. LSODE requires the user to convert the system of ODEs into an array of first order ODEs.

$$\frac{dy}{dt} = f(t, y) \quad (3.15)$$

where y is a vector containing the set of ODEs and f is a vector-valued function of t and y . Subsequently, it can be written as:

$$\begin{pmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{pmatrix} = \begin{pmatrix} u(t) \\ \dot{u}(t) \\ z(t) \\ \varepsilon(t) \end{pmatrix} \quad (3.16)$$

The hysteresis model Eqs. (3.12)- (3.14) can be rewritten based on Eq. (3.16) as follows:

$$\dot{y}_1 = y_2 \quad (3.17)$$

$$\dot{y}_2 = -2\xi_0 \omega_0 y_2 - \alpha \omega_0^2 y_1 - (1 - \alpha) \omega_0^2 y_3 + f(t) \quad (3.18)$$

$$\dot{y}_3 = 1 - \zeta_s \left(1 - e^{-py_4} \right) e^{-\left(y_3 \operatorname{sgn}(y_2) - q \left\{ 1 / (1 + \delta_\nu y_4) (\beta + \gamma) \right\}^{1/n} \right)^2 / (\psi + \delta_\psi y_4)^2 \left[1 + \zeta_s \left(1 - e^{-py_4} \right) \right]^2} \\ \times \left\langle \frac{y_2 - (1 + \delta_\nu y_4) \beta |y_2| |y_3|^{n-1} y_3 + \gamma y_2 |y_3|^n}{1 + \delta_\eta y_4} \right\rangle \quad (3.19)$$

$$\dot{y}_4 = (1 - \alpha) \omega_0^2 y_2 y_3 \quad (3.20)$$

Computing time to solve the model and numeric noise can be greatly reduced if both force and displacement are of the same order of magnitude. Therefore, on the basis of experimental results of RC structural components, kilonewton (kN) and millimeter (mm) as units of load and displacement respectively are found to be preferable. With known displacement function and suitably estimated analytical parameters, LSODE solver can solve these equations without much problem.

3.4 Estimation of Hysteresis Model Parameters

A problem where output response for a given input is known, but the parameters involved in the process equations are not known, is termed a system identification problem or inverse parameter estimation problem (Sage [S1]). Similarly, the present problem of estimating the magnitude of the parameters involved in the hysteresis model is an inverse parameter estimation problem. However, the performance of the hysteresis model is not only dependent on the individual magnitude of the parameters, but also on the interaction between them. Thus, it is almost impossible to accurately identify the parameters without a systematic search. The objective of suitable parameter estimation is to minimize the difference between the experimental results and the model output for a given input function as shown in **Fig. 3.8** so that the hysteresis model can be practical and applicable to a wide range of similar problems.

Fig. 3.8: System Identification Problem

3.4.1 Optimization Methods

The methodology of minimizing the difference between the observed and modeled dataset by adjusting the parameter magnitudes turns parameter estimation into an optimization problem. Plenty of literature is devoted to optimization problems as reported by Collins et al. ([C12]), Sage ([S1]), Lin ([L7]), Goldberg ([G5]), Ghanem and Shinozuka ([G2]), Lybanon and Messa ([L11]), Heine ([H3]), Xu ([X1]) and many others. On the basis of the existing literature, some widely used optimization methods are schematically presented in **Fig. 3.9**. The optimization methods are generally classified into discrete and continuous methods. Discrete optimization is generally concerned with integers and used to solve scheduling problems, network optimization problems etc. Continuous optimization methods can be essentially categorized into local and global optimizers. Local optimizers generally opt for calculus based techniques like extrema and gradient methods. Thus, local optimization method requires quite less efficiency to generate good quality results. Global optimizers basically attempt to identify the best set of parameters such that the results can represent the observed dataset. Since Genetic Algorithms are one of the most popular global optimizers, they are adopted for the present parameter estimation problem.

3.4.2 Genetic Algorithm (GA)

Numerous investigations are carried out to elucidate the theoretical basis and mechanism of Genetic Algorithm (GA). In brief, the foundation of GA lies in the theory of natural evolution, the ultimate optimizer. During the continuous struggle for survival in a given environment, an organism develops some favorable variation to enhance its chances for survival. Thus, after mutation the organism is more likely to survive, reproduce and transfer the favorable variation to its descendants. In due course, the optimally adapted organisms will be the inhabitants of the given environment resulting in global optimization. Similarly, GA employs probabilistic rules to produce and select parameters. Then, with the help of mutation and crossover on a random basis, GA identifies the best set of parameters to reach the global optimum.

Fig. 3.9: Optimization Methods (Sage [S1])

The primary reason for selecting GA as the system identification tool is its versatility and user friendliness. GA permits the user to incorporate the calculus based approaches, if required, in the original subroutine to enable hybridization for achieving better quality of results. Even the level of accuracy of the result can be regulated in a problem-specific manner according to the choice of the user. Since the experimental results generally include significant error, local optimization techniques are not preferable for handling those data. GA can cope with the noisy data well without affecting its performance. The objective function can be discontinuous, non-differentiable, linear or nonlinear, with or without constraints. Moreover, GA does not require the initial point estimates of the parameters, a range of magnitudes within which the best set of parameters is expected to lie can serve the purpose. Although interval selection of the parameter set does not affect the outcome of GA, it can make significant difference in the computational time required to converge to the ultimate solution. However, quick insight can be gained about the problem at hand and a trend can be recognized promptly regarding the parameter intervals. Hence, to identify the magnitudes of the hysteresis model parameters, a Genetic Algorithm (GA) subroutine is written in Visual Fortran based on the existing literature (Goldberg [G5], Carroll [C3], Lybanon and Messa [L11], Heine [H3]).

The program organization is characterized by four nested loops. The innermost loop (Loop 4) is the actual GA that generates a population, checks the solver (LSODE) calculation as well as selects and mates pairs to crossover and mutate. Since the parameters are generated on random basis, solver computation is ensured after each run to prevent GA from falsely recognizing the erroneous sums of squares as better fit and reaching local optima or even non-optima. Loop 3 executes the GA a user-specified number of times; each time with a different randomly chosen initial population and Loop 2 progressively decreases or shrinks the parameter interval. GA is an adaptive algorithm with an ability to discover the initial erroneous ranges for the input parameters. If an incorrect interval is provided and the optimal parameter lies outside the interval, the results tend to be clustered near the interval that should be readjusted. Loop 1 subsequently shifts the interval in the direction of the clustering and starts over. Although improper interval selection for the

parameters does not create hurdle in the performance of GA, it can cause considerable difference in the computational time required to reach the ultimate solution.

3.4.3 Objective and Fitness Function

The objective function can be defined by any of the two common stochastic estimators, Maximum Likelihood Estimation (MLE) and Ordinary Least Squares (OLS). Since Maximum likelihood estimation method requires the information about the variance of the error, which is unknown in the present problem, this approach cannot be utilized. On the contrary, Ordinary Least Squares method works without prior knowledge of estimated parameters and variances of measurement errors. In this method, it is assumed that the error is independent and normally distributed with expected value zero. Thus the objective function becomes

$$S = \sum_{i=1}^N \left[f(p_1, p_2, \dots, p_j, \dots, p_n; u_i) - F_i \right]^2 \quad (3.21)$$

where f represents the analytical model estimating the dependent variable F_i based on the measurements of independent variable u_i , i denotes a measurement point, p_j signifies the parameters to be estimated, n is the number of unknown parameters and N is the number of measured data points. Adjustment of the parameters to minimize the objective function S is the fitting process. The present problem is over-determined with number of data points being larger than the number of unknown parameters ($N > n$) (Lybanon and Messa [L11]). Global optimum can be reached by minimizing Eq. 3.21.

The solution vector $\hat{R}$ can be represented in terms of r_j , the possible magnitudes of the parameters as follows.

$$\hat{R} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_j \\ \vdots \\ r_n \end{bmatrix} \quad (3.22)$$

If u_j and l_j designate the upper and lower bounds of r_j , the expression for r_j is:

$$r_j = d_j \times (u_j - l_j) + l_j \quad (3.23)$$

Here d_j signifies a floating number corresponding to r_j within the limits of $[l_j, u_j]$ with $0.0 \leq d_j \leq 1.0$. Thus, the solution vector $\hat{R}$ can be represented as:

$$\hat{R} = \hat{D} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_j \\ \vdots \\ d_n \end{bmatrix} \quad (3.24)$$

With d_j varying from 0 to 1, crossover and mutation schemes can be applied using random number generator functions.

The *prefitness* is the initial fitness of a population and can be obtained by the following equation:

$$Prefitness(\hat{R}) = \sum_{i=1}^N \left[f(p_1, p_2, \dots, p_j, \dots, p_n; u_i) - F_i \right]^2 \quad (3.25)$$

In order to solve Eq. 3.21 *rawfitness* is computed using the following expression:

$$rawfitness = Max_{prefitness} - Prefitness(\hat{R}) \quad (3.26)$$

Here $Max_{prefitness}$ represents the maximum *prefitness* of a population i.e. the worst fit for the parameters. The best fit for the parameters can be obtained when

rawfitness of a population is closest to $Max_{prefitness}$. Higher magnitude of fitness, produces better quality of solution and thus the expression in Eq. 3.26 turns the minimization problem (Eq. 3.21) into a maximization problem.

3.4.4 Scaling

Scaling of *rawfitness* is conducted by GA to obtain *finalfitness* of a population used for selection and reproduction purposes. In order to facilitate convergence at later stages, linear scaling is employed. Few specific parameter set tends to control the initial selection process resulting in unsuitable competition and with less diverse population in the final stage; the simulation fails to maintain its focus (Goldberg [G5]). So, negative and positive scaling of fitness is employed in earlier and final stages respectively.

For $gen > beginscale \times num\_gen$

$$fitness(\hat{R})_{gen} = m_p \times rawfitness(\hat{R}) + b_p \quad (3.27)$$

The expressions for constants m_p , b_p are presented below.

$$m_p = \frac{(scale\_fac - 1) \times rawfitness_{mean}}{rawfitness_{max} - rawfitness_{mean}} \quad (3.28)$$

$$b_p = \frac{rawfitness_{mean} \times (rawfitness_{max} - scale\_fac \times rawfitness_{mean})}{rawfitness_{max} - rawfitness_{mean}} \quad (3.29)$$

For $gen < 0.1 \times num\_gen$

$$scale\_fac = nscale \quad (3.30)$$

Where gen is the generation count, $beginscale$ is the percentage of generations before initiation of scaling ($beginscale < 1.0$), num_gen is the number of generations per population, *fitness* is the fitness computed for each population and

generation, $rawfitness_{mean}$ and $rawfitness_{max}$ are the average and maximum $rawfitness$ of a population respectively, $scale\_fac$ is a user-specified scale factor, $nscale$ is the negative scaling with magnitude 0.1 employed at the initial stage of the simulation. With increase in the number of generations, higher computational efficiency and computing time are required. Although large scale factors increase the accuracy of the solution, the chances of reaching premature convergence amplify accordingly. Usually good quality results can be obtained with $scale\_fac = 1000$, $beginscale = 0.5$ and $num\_gen = 10$.

3.4.5 Selection Strategy

Stochastic sampling without replacement is the basis of selection (Lybanon and Messa [L11], Goldberg [G5]). If vector $\hat{D}$ represents a contender solution with fitness f_i , the expected number of offsprings generated (E) is as follows.

$$E = \frac{f_i}{f_{mean}} \quad (3.31)$$

Here f_{mean} is the average fitness of the entire population. If an offspring is selected E number of times randomly, the organism is abandoned and no longer obtainable for selection. Additionally, the best organism of each population is identified and placed unaltered into the next generation to avoid early loss of information through cross over or mutation (De Jong [D1]).

3.4.6 Crossover, Mutation, Seeding and Shrinkage

GA permits the user to select any of the four crossover strategies: Arithmetic Crossover, One-point Crossover, Combined One-point and Arithmetic Crossover and Combined Arithmetic and Two-point Crossover. The fourth strategy i.e. Combined Arithmetic and Two-point Crossover being capable of performing effectively for the problems with multiple parameters is selected for the hysteresis model parameter estimation. The crossover rate designated by the user has a

magnitude of 0.9 with 90% of the pairs crossed over. Let $\hat{D}_1$ and $\hat{D}_2$ be the aspiring solutions selected and paired for crossover.

$$\hat{D}_1 = \begin{bmatrix} d_{1,1} \\ d_{2,1} \\ \vdots \\ d_{j,1} \\ \vdots \\ d_{n,1} \end{bmatrix} \text{ and } \hat{D}_2 = \begin{bmatrix} d_{1,2} \\ d_{2,2} \\ \vdots \\ d_{j,2} \\ \vdots \\ d_{n,2} \end{bmatrix} \quad (3.32)$$

The crossed over offspring is

$$\hat{D}_{12} = \begin{bmatrix} d_{1,1} \\ d_{2,1} \\ \vdots \\ d_{i-1,1} \\ t \\ d_{i+1,2} \\ d_{i+2,2} \\ \vdots \\ d_{n,2} \end{bmatrix} \quad (3.33)$$

$$\text{with } t = p \times d_{i,1} + (1-p) \times d_{i,2} \quad (3.34)$$

Here i is a random number between 1 and n and p is a random floating point between 0.0 and 1.0.

Various mutation strategies applicable to GA are Uniform Mutation, Boundary Mutation and Non-uniform Mutation with and without decay. In uniform mutation, magnitude of d_j is randomly varied to a floating point number between 0.0 and 1.0.

In boundary mutation the magnitude of d_j is changed at random to either 0.0 or 1.0.

In non-uniform mutation, a small magnitude is added to or subtracted from d_j such that it does not surpass 1.0 or drop below 0.0. The effect of decay at higher generation counts can be included or excluded from this mutation strategy. The user-specified variables pr_u , pr_b and pr_n denote the probabilities of

uniform, boundary and non-uniform mutations respectively with the corresponding magnitudes of 0.1, 0.01 and 0.1.

Seeding is applied to enhance the efficiency of GA by using the information of offspring generation from a previous run to the next run. Thus, with the help of seeding, the solution vectors are not created on a random basis, but are produced on the basis of best vectors obtained from the previous run. Shrinkage is useful in narrowing down the initially defined parameter intervals and resolving the convergence criterion. The extent of shrinkage can be determined by the user and the magnitude of shrinkage is considered to be 0.5 in the present problem.

3.5 Summary

This chapter illustrates the original Bouc-Wen-Baber-Noori hysteresis model along with the modification proposed in this research to assimilate the hysteretic behavior of RC beam-column joints and structural walls. The model derived from the equation of motion for a single degree of freedom system comprises various parameters to exhibit different structural response characteristics like strength and stiffness degradation, pinching, hardening and softening etc. The parameter stiffness ratio α specified to be of constant magnitude in the original model is amended to be displacement-based in the present research. The hysteresis model with modified α can perform better than the same with constant α with respect to the experimental results of RC structural components.

The differential equations involved in the mathematical model are solved by employing Livermore Solver for Ordinary Differential Equations (LSODE). When hysteretic force and displacement are of similar order of magnitude, the solving time can be significantly reduced. Thus, reviewing the existing literature containing the experimental results of RC structural components, kilonewton (kN) and millimeter (mm) as units of load and displacement respectively are perceived to be preferable.

The structural hysteretic behavior is dependent on the individual parameter magnitudes and their interaction. Hence, Genetic Algorithm (GA) is utilized to

systematically identify the parameter magnitudes. GA can perform satisfactorily with discontinuous objective functions and erroneous initial estimates of the parameters. However, in spite of being versatile and convenient in nature, GA requires significant computational time if improper intervals of parameters are prescribed in the subroutine. But with completion of each run, the user can gain quick insight about the problem at hand and can assist the program to enhance its performance.

CHAPTER 4

CALIBRATION OF HYSTERESIS MODELS USING EXPERIMENTAL RESULTS

4.1 Introduction

With the hysteresis model being illustrated in the previous chapter, it is essential to ensure the effectiveness of the model and accuracy of the solver and the system identification methodology. In order to examine the performance of the proposed hysteresis model, a database containing experimental results of reinforced concrete (RC) beam-column joints and structural walls under quasi-static cyclic loading is compiled from the existing literature. Thus, the past experimental results of structural components act as a benchmark on the basis of which the reliability of the proposed hysteresis model is successfully judged.

4.2 Model Sensitivity to Parameter Variations

Parameter sensitivity analysis is essential when dealing with the system identification techniques. A sensitive parameter when deviated from its sought-after magnitude will show significant error. Thus, by changing the magnitude of sensitive parameters, better correlation can be achieved. On the contrary, a less sensitive parameter, even when it fluctuates from its sought-after magnitude, can produce a reasonable response since its contribution to the final response is relatively less. Therefore, providing narrower ranges for less sensitive parameters can increase simplicity in the procedure without affecting the quality of the results.

Let $[Y]$ be the hysteretic force for a given input displacement function and the magnitude of the analytical parameters are estimated accordingly. Then, the magnitude of each parameter is varied from -50% to +50% of its original value keeping the remaining parameters constant. If due to the variation of a parameter, say α_0 , the hysteretic force becomes $[Y']$, then the root mean square error e_{α_0} will be as follows.

$$e_{\alpha_0} = \left\langle \sum_{i=1}^N (Y - Y')^2 \right\rangle^{1/2} \quad (4.1)$$

Here N is the number of data points for input displacement function. The maximum error related to the variation of α_0 , termed as $|e_{\alpha_0}|$ can be obtained by the following expression.

$$|e_{\alpha_0}| = \text{maximum}(e_{\alpha_0}) \quad (4.2)$$

Thus, the maximum root mean square (RMS) error associated with each parameter variation can be obtained. The parameter with the highest magnitude of maximum root mean square error is ranked as 1 based on its sensitivity and the parameter with the lowest magnitude of maximum RMS error is considered to be the least sensitive.

4.2.1 Parameter Sensitivity Study for Beam-Column Joints

In pursuance of judging the sensitivity of the hysteresis model to the variation of associated analytical parameters, a sample set of model parameters is deduced from beam-column joint specimens as follows.

$$\alpha_0 = 0.045, \omega_0 = 2.45, \xi_0 = 0.02, \beta = 0.06, \gamma = -0.009, n = 1.0, \delta_v = 0.00005, \\ \delta_\eta = 0.0004, \zeta_s = 0.93, q = 0.04, p = 0.06, \psi = 0.8, \delta_\psi = 0.128, \lambda = 0.1.$$

Varying magnitude of each parameter from -50% to +50%, the maximum root mean square (RMS) error associated with each parameter variation is calculated and the sensitivity ranking of the model parameters is determined on the basis of maximum RMS error. The maximum RMS error and the corresponding sensitivity ranking of the model parameters are summarized in **Table 4.1**. By plotting the root mean square error for each model parameter within the range of its variation, a Spider diagram is obtained as shown in **Fig. 4.1**.

Table 4.1: Parameter Sensitivity Ranking for Beam-Column Joints

Parameter	Maximum Root mean square error	Rank
α_0	9.45	7
ω_0	80.57	1
ξ_0	0.45	14
β	34.44	4
γ	3.04	9
n	34.71	3
δ_v	6.50	8
δ_η	21.15	5
ζ_s	64.32	2
ψ	15.73	6
δ_ψ	2.80	10
p	0.70	13
q	2.03	11
λ	0.83	12

Fig. 4.1: Spider Diagram of Root Mean Square Error versus Percentage Variation of Each Parameter for Beam-Column Joints

4.2.2 Parameter Sensitivity Study for Structural Walls

With the aim of judging the sensitivity of the hysteresis model to the variation of associated analytical parameters for structural walls, a sample set of their magnitude is considered as follows.

$$\alpha_0 = 0.012, \quad \omega_0 = 10, \quad \xi_0 = 0.02, \quad \beta = 0.4, \quad \gamma = -0.01, \quad n = 1.0, \quad \delta_v = 0.00003, \\ \delta_\eta = 0.0006, \quad \zeta_s = 0.75, \quad q = 0.08, \quad p = 0.05, \quad \psi = 0.75, \quad \delta_\psi = 0.2, \quad \lambda = 0.1.$$

The maximum RMS errors associated with variation of each parameter magnitude from -50% to +50% are gathered in **Table 4.2**. The corresponding sensitivity ranking of the model parameters are determined accordingly and also summarized in **Table 4.2**. A Spider diagram is obtained by plotting the RMS errors for the model parameters versus the range of their variations, as shown in **Fig. 4.2**.

Table 4.2: Parameter Sensitivity Ranking for Walls

Parameter	Maximum Root mean square error	Rank
α_0	14.36	9
ω_0	163.66	2
ξ_0	13.18	10
β	87.46	3
γ	16.94	8
n	32.72	6
δ_v	13.01	11
δ_η	63.75	4
ζ_s	225.33	1
ψ	53.98	5
δ_ψ	13.18	10
p	2.97	13
q	11.40	12
λ	19.23	7

Fig. 4.2: Spider Diagram of Root Mean Square Error versus Percentage Variation of Each Parameter for Walls

In general, variation of magnitude of analytical parameters displays influences in peak load, stiffness at maximum load, strength deterioration, stiffness degradation and pinching function. The analytical parameters, stiffness ratio α_0 and system natural frequency ω_0 with their changing magnitudes, affect the peak load and stiffness of the hysteresis loop. Hysteresis shape parameters β , γ and n can individually influence the shapes of the hysteresis loops along with the hysteresis load and stiffness. With increase or decrease of degradation parameters δ_v and δ_η , structural wall experiences more or less degradations respectively. Due to increase in δ_η , both hysteretic force and hysteretic stiffness degrade whereas increase in δ_v reduces the hysteretic force without changing the hysteretic stiffness. ζ_s controls the amount of total slip in the hysteresis loop as no slip can be observed when this parameter is of zero magnitude whereas with an increase in its magnitude, greater slip is found in the loop. As q sets the pinching level as a fraction of $z_{\max}$, the

influence of its varying magnitude can significantly affect pinching. Variation of p contributes to the rate of drop in the slope. Moreover, with an increase or decrease in ψ , the amount of pinching behaves proportionately and due to increase in the magnitude of δ_ψ , the pinching region spreads. A change in λ also affects the amount and spread of pinching in the hysteresis loop. In brief, system properties $(\alpha_0, \omega_0, \xi_0)$ and hysteresis shape parameters (β, γ, n) control the skeleton of hysteresis loops; degradation parameters (δ_v, δ_η) determine strength and stiffness deteriorations and pinching parameters $(\zeta_s, \psi, \delta_\psi, p, q, \lambda)$ govern the magnitude of slip, the amount and spread of pinching. However, the primary purpose of conducting sensitivity analysis is to facilitate estimation of parameters using Genetic Algorithm (GA). GA being an adaptive algorithm can perform satisfactorily even if wrong lower and upper bounds are provided for the model parameters, but at the cost of computational time. Thus, sensitivity ranking of parameters for beam-column joints and structural walls shown in **Tables 4.1** and **4.2** can be utilized to identify the most sensitive and the least parameters. By providing broader ranges of magnitudes for sensitive parameters in the GA input files, better correlation can be achieved. Similarly narrower ranges can be provided for less sensitive parameters to reduce computational time without affecting the quality of the results obtained from GA. in order to reduce computational time.

4.3 Calibration with Experimental Results of RC Structural Components

The reliability and accuracy of the proposed hysteresis model has to be confirmed by calibrating the model output against the experimental results. Hence, the experimental results of RC beam-column joints and structural walls under quasi-static cyclic loading are assembled from the existing literature to serve the purpose. Considering the common practice of constructing lightly reinforced concrete structures in Singapore, the interior and exterior beam-column joint specimens and RC wall specimens with non-seismic detailing are selected for calibration. The tested specimens are selected in such a way that a wide variation in terms of

structural geometry, material properties and axial loading levels can be covered for both structural components. After selecting the tested specimens from the literature, their load-deformation data are retrieved from the graphical experimental results. Then, the experimental dataset is utilized to estimate the magnitudes of the hysteresis model parameters for the input displacement function using a Genetic Algorithm (GA). Thus, on the basis of the estimated model parameters, the analytical load-deformation results of the structural components can be obtained using Livermore Solver for Differential Equations (LSODE). The entire process is summarized in the form of a flowchart as presented in **Fig. 4.3**. Thus, the comparison between the experimental and analytical hysteresis load-displacement plots of RC structural components can define the accuracy of the hysteresis model, solver and system identification methodology.

4.3.1 Interior Beam-Column Joints

The structural characteristics of the interior beam-column joint specimens with non-seismic details selected for calibration are summarized in Appendix A. Here h_c and h_b represent the corresponding cross-sectional depths of columns and beams in mm; b_j is the joint width in mm considered as the smaller of $\{b_c, b_b + h_c\}$; b_c and b_b are the width of columns and beams respectively in mm; f'_c is the concrete compressive strength in MPa; f_y is the yield strength of reinforcing steel in MPa; ρ_b and ρ_c are the longitudinal reinforcement ratio in % respectively for beam and column sections; ρ_j is the joint core transverse reinforcement ratio in % and n is the axial load ratio in %. RC non-seismically detailed interior beam-column joint specimens include Unit O1 by Hakuto *et al.* ([H1]), Units 1 and 2 tested by Liu *et al.* ([L9]), Units PEER-1450, PEER-2250, CD15-1450, CD30-1450, CD30-2250, PADH-14 and PADH-22 tested by Lehman *et al.* ([A4], [W1]), P2, P4, P5, P7, P8 and P9 tested by Pessiki *et al.* ([P12]) and JXO-B1, JXO-B2 and JXO-B8 by Joh *et al.* ([J1], [J2]). The comparison between the experimental and analytical hysteresis load-deformation plots for selected interior beam-column joints are shown in **Fig. 4.4**.

Fig. 4.3: Flow Chart of the Process of Resolving Hysteretic behavior of RC Structural Components

Fig. 4.4: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Interior Beam-Column Joints with Non-seismic Detailing (continued)

Fig. 4.4: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Interior Beam-Column Joints with Non-seismic Detailing

4.3.2 Exterior Beam-Column Joints

The structural characteristics of the exterior beam-column joint specimens with non-seismic details selected for calibrating the hysteresis model are summarized in Appendix A. Here h_c and h_b represent the corresponding cross-sectional depth of columns and beams in mm; b_j is the joint width in mm considered as the smaller of $\{b_c, b_b + h_c\}$; b_c and b_b are the width of columns and beams respectively in mm; f'_c is the concrete compressive strength in MPa; f_y is the yield strength of reinforcing steel in MPa; ρ_b and ρ_c are the longitudinal reinforcement ratio in % respectively for beam and column sections; ρ_j is the joint core transverse reinforcement ratio in % and n is the axial load ratio in %. RC non-seismically detailed exterior beam-column joint specimens comprise Units O6 and O7 tested by Hakuto *et al.* ([H1]), Units EJ1, EJ2, EJ3 and EJ4 tested by Liu *et al.* ([L9]), Units 1, 2, 3, 4, 5 and 6 tested by Pantelides *et al.* ([P5]) and Test# 2, Test# 4, Test# 5 and Test# 6 by Clyde *et al.* ([C11]). The comparison between the experimental and analytical hysteresis load-deformation plots for selected exterior beam-column joints are shown in **Fig. 4.5**.

Fig. 4.5: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Exterior Beam-Column Joints with Non-seismic Detailing (continued)

Fig. 4.5: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Exterior Beam-Column Joints with Non-seismic Detailing (continued)

Fig. 4.5: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Exterior Beam-Column Joints with Non-seismic Detailing

4.3.3 Walls with Rectangular Cross-sections

A database of lightly reinforced concrete wall specimens with rectangular cross-sections is compiled from the literature to calibrate the analytical response with the experimental results. A summary of the specimen characteristics is provided in Appendix B. In the table, l_w , h_w and t_w are the length, height and thickness of structural walls in mm correspondingly; f'_c is the concrete compressive strength in MPa; f_y is the reinforcing steel yield strength in MPa; ρ_h , ρ_v and ρ_{be} are the horizontal, vertical and boundary reinforcement ratio in % respectively and n is the

axial load ratio in %. RC wall specimens with rectangular cross-sections include Specimens 72, 73, 165, 170 and 175 by Hirosawa *et al.* ([H7]), Specimens 1-4, 6-16, 21-26, 28-30 and 32 tested by Hidalgo *et al.* ([H4]), Specimens M1, M2, M3 and M4 tested by Greifenhagen *et al.* ([G6]), wp111-9, wp111-10, wp1105-7, wp1105-8, wp110-5, wp110-6 by Massone *et al.* ([M3]), U1.0, U1.5, C1.0, C1.5, U1.0-BC, U1.5-BC by Kuang *et al.* ([K8]). The comparison between the experimental and analytical hysteresis load-deformation plots for selected RC wall specimens with rectangular cross-sections are shown in **Fig. 4.6**.

Fig. 4.6: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Rectangular Cross-sections (continued)

Fig. 4.6: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Rectangular Cross-sections (continued)

Fig. 4.6: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Rectangular Cross-sections

4.3.4 Walls with Flanged Cross-sections

RC wall specimens with flanged cross-sections are selected from the literature for calibrating the analytical response against experimental results. The summary of the specimen characteristics is enlisted in Appendix B where l_w , h_w and t_w are the length, height and thickness of structural walls in mm correspondingly; f'_c is the concrete compressive strength in MPa; f_y is the reinforcing steel yield strength in MPa; ρ_h , ρ_v and ρ_{be} are the horizontal, vertical and boundary reinforcement ratio in % respectively and n is the axial load ratio in %. RC wall specimens with flanged cross-sections include S5 and S7 by Maier *et al.* ([M1]), 36M830 and 36M850 by Sato *et al.* ([S6]) and LW1, LW2, LW3, LW4, LW5, MW1, MW2 and MW3 by Li *et al.* ([L5]). The comparison between the experimental and analytical hysteresis load-deformation plots for selected RC wall specimens with flanged cross-sections are shown in **Fig. 4.7**.

Fig. 4.7: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Flanged Cross-sections (continued)

Fig. 4.7: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Flanged Cross-sections

The comparison between the hysteretic energy calculated based on the experimental and analytical load-deformation curves of the beam-column joint and structural wall specimens obtained from the literature are shown in **Figs. 4.8** and **4.9**.

Fig. 4.8: Comparison between Experimental and Analytical Hysteretic Energy of Reinforced Concrete Beam-Column Joints

Fig. 4.9: Comparison between Experimental and Analytical Hysteretic Energy of Reinforced Concrete Structural Walls

The correlation between the experimental and analytical hysteretic energy proves that the analytical model can perform satisfactorily for lightly reinforced concrete beam-column joints and structural walls.

4.4 Correlation between the Model Parameters and Structural Features

On the basis of effective correlation between the experimental and model hysteresis loops for the wide range of RC beam-column joint and wall specimens, it can be perceived that the analytical hysteresis force for any structural components can be obtained using the proposed approach when the displacement history and the model parameters are known. Since the model parameters are estimated based on experimental results using GA, they may not be identified properly for any specimen when the experimental outcome is not available. Therefore, in order to resolve this difficulty, approximate magnitudes of the analytical parameters are proposed in this research in terms of the physical parameters of the structural component. The structural characteristics selected for interior and exterior beam-column joints include h_b , h_c , b_j , f'_c , f_y , ρ_j , ρ_b , ρ_c and n where h_c and h_b represent the corresponding cross-sectional depths of columns and beams in mm; b_j is the joint width in mm considered as the smaller of $\{b_c, b_b + h_c\}$; b_c and b_b are the width of columns and beams respectively; f'_c is the concrete compressive strength in MPa; f_y is the yield strength of reinforcing steel in MPa; ρ_b and ρ_c are the longitudinal reinforcement ratio in % respectively for beam and column sections; ρ_j is the joint core transverse reinforcement ratio in % and n is the axial load ratio in %. The structural features for structural walls include l_w , h_w , t_w , f'_c , f_y , ρ_h , ρ_v , ρ_{be} and n where l_w , h_w and t_w are the length, height and thickness of structural walls in mm correspondingly; f'_c is the concrete compressive strength in MPa; f_y is the reinforcing steel yield strength in MPa; ρ_h , ρ_v and ρ_{be} are the horizontal, vertical and boundary reinforcement ratio in % respectively and n is the axial load ratio in %. The structural characteristics of RC beam-column joint and

wall specimens from the available literature are summarized in Appendices A and B respectively. The magnitudes of the corresponding model parameters at the level of 95% correlation are estimated accordingly. Thereafter, a series of nonlinear multivariate regression analyses are performed to obtain the expressions for hysteresis model parameters in terms of the physical characteristics of the structural components.

In the estimation of the model parameters, n and λ are found to be very close to 1 and .0001 respectively for all structural components and thus no regression analysis is carried out for this parameter. Moreover, the magnitude of β is kept constant at 0.06 for beam-column joint specimens as the magnitudes of β obtained for the available specimens are quite close to 0.06. For each of the remaining parameters, two sets of regression analysis are required to be performed for RC interior and exterior beam-column joints as reasonable differences in magnitudes of the model parameters can be observed for them. Similarly, for estimation of the remaining parameters for RC structural walls with rectangular and flanged cross-sections, two sets of regression analysis are required to be performed owing to the reasonable differences in the magnitudes of the model parameters for the wall specimens with these two different cross-sections.

In order to obtain simplified expressions for the model parameters, the physical characteristics of the structural components obtained from the literature are normalized. Thereafter, the regression analyses are carried out to obtain the best suited expressions for the model parameters in terms of the normalized structural physical characteristics. From the regression analysis, exponential functions of linear variables multiplied by various coefficients are found as best fitted with the model parameters obtained from GA. The expressions of the model parameters α_0 , ω_0 , ξ_0 , β , δ_v , δ_η , ζ_s , q , p , ψ , δ_ψ for beam-column joints, denoted as P_1 , in terms of ten variables and seven coefficients a_1 - a_7 are presented as follows. Similarly, the generalized expression for the model parameter γ (denoted as P_2) is included hereunder.

$$P_1 = e^{a_1 \times (h_b/h_c) \times (b_j/h_c) + a_2 \times (f'_c/f_y) + a_3 \times \rho_j + a_4 \times \rho_b + a_5 \times \rho_c + a_6 \times n + a_7} \quad (4.5)$$

$$P_2 = -e^{a_1 \times (h_b/h_c) \times (b_j/h_c) + a_2 \times (f'_c/f_y) + a_3 \times \rho_j + a_4 \times \rho_b + a_5 \times \rho_c + a_6 \times n + a_7} \quad (4.6)$$

The magnitudes of a_1 – a_7 for the model parameters for RC interior and exterior beam-column joints are summarized in **Tables 4.3** and **4.4** respectively. Comparison between the parameter magnitudes obtained by GA and calculated by nonlinear regression analysis is shown in **Figs. 4.10** and **4.11** for RC interior and exterior beam-column joints, respectively.

In the same way, the expressions for the model parameters α_0 , ω_0 , ξ_0 , β , δ_v , δ_η , ζ_s , q , p , ψ , δ_ψ , λ of structural walls denoted as P_3 , in terms of ten variables and seven coefficients b_1 – b_7 are presented as follows.. The generalized expression for the model parameter γ (denoted as P_4) is presented below.

$$P_3 = e^{b_1 \times (l_w/h_w) \times (l_w/t_w) + b_2 \times (f'_c/f_y) + b_3 \times \rho_h + b_4 \times \rho_v + b_5 \times \rho_{be} + b_6 \times n + b_7} \quad (4.7)$$

$$P_4 = -e^{b_1 \times (l_w/h_w) \times (l_w/t_w) + b_2 \times (f'_c/f_y) + b_3 \times \rho_h + b_4 \times \rho_v + b_5 \times \rho_{be} + b_6 \times n + b_7} \quad (4.8)$$

The magnitudes of b_1 – b_7 for the model parameters for RC structural walls with rectangular and flanged cross-sections are summarized in **Tables 4.5** and **4.6** respectively. Comparison between the parameter magnitudes obtained by GA and calculated by nonlinear regression analysis is presented in **Figs. 4.12** and **4.13** for RC structural walls with rectangular and flanged cross-sections respectively.

Table 4.3: Summary of Magnitudes of the Coefficients for Each Parameter for Interior Beam-Column Joints

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
α_0	1.8520	-36.7570	1.7170	1.0000	0.0660	0.0410	-5.7560
ω_0	-0.4830	5.2790	-0.8430	0.1280	0.1270	-0.0090	1.1630
ζ_s	0.0045	-0.3120	-0.1590	0.0010	-0.0060	-0.0010	-0.0010
ψ	0.0560	0.3970	0.0840	-0.0800	0.0280	0.0020	-0.2560
δ_ψ	-0.0350	2.2650	-0.0840	-0.0720	0.0250	0.0002	-2.1610
δ_ν	0.6980	0.7530	3.5030	-0.2080	0.3840	0.0080	-12.4350
ξ_0	0.2390	-8.5200	-3.4100	-0.7540	0.0830	-0.0170	-2.0810
γ	-0.0314	0.5860	-0.1650	-0.0644	-0.0801	-0.0006	-4.4744
δ_η	1.2010	-7.2090	0.6480	0.5090	-0.3370	0.0205	-10.1150
p	-0.0822	2.1507	-0.8796	-0.5972	0.5498	0.0049	-3.2021
q	0.1848	-9.8510	2.0165	-0.3706	-0.0333	-0.0250	-1.7180

Table 4.4: Summary of Magnitudes of the Coefficients for Each Parameter for Exterior Beam-Column Joints

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
α_0	0.4700	-36.3430	1.9230	1.8640	-2.5160	-0.0830	-3.1410
ω_0	-0.2040	4.3430	3.8300	0.1850	0.1210	0.0050	0.2330
ζ_s	0.0025	-0.9270	-4.1480	-0.0110	0.0260	0.0010	-0.0010
ψ	0.0360	-3.5560	2.3780	0.0380	-0.0930	-0.0002	0.0457
δ_ψ	0.0153	-7.5440	-1.2120	0.1390	-0.2830	0.0020	-1.4020
δ_ν	0.3080	-3.1390	-3.1650	0.2760	-0.2000	0.0170	-10.5210
ξ_0	0.0937	-12.9720	3.9040	0.3124	-0.2827	0.0055	-3.0050
γ	-0.0687	-0.4345	1.7630	-0.0186	-0.0632	0.0010	-4.3930
δ_η	0.0996	-2.9696	2.0570	-0.7835	0.3482	-0.0150	-6.3633
p	0.1430	-32.3720	-15.7270	-0.0230	0.0212	0.0074	0.0970
q	0.1421	-1.5651	2.2256	-0.0706	-0.0235	-0.0403	-2.6730

Fig. 4.10: Comparative study between the Analytical Parameters Estimated by GA and by Regression Analysis for Interior Beam-Column Joints (continued)

Fig. 4.10: Comparative study between the Analytical Parameters Estimated by GA and by Regression Analysis for Interior Beam-Column Joints

Fig. 4.11: Comparative study between the Analytical Parameters Estimated by GA and by Regression Analysis for Exterior Beam-Column Joints (continued)

Fig. 4.11: Comparative study between the Analytical Parameters Estimated by GA and by Regression Analysis for Exterior Beam-Column Joints

Table 4.5: Summary of Magnitudes of the Coefficients for Each Parameter for RC walls with Rectangular Cross-sections

	b_1	b_2	b_3	b_4	b_5	b_6	b_7
α_0	0.0190	40.5330	0.1390	-0.3100	-0.0186	-0.8320	-9.4060
β	0.0195	-1.8000	-0.0020	-0.0570	0.0380	-0.0290	2.3970
ω_0	-0.0030	3.3300	0.7720	-0.3620	0.0060	0.0330	3.8720
ζ_s	-0.0004	-0.1650	0.0630	0.0110	0.0014	-0.0016	-0.0370
ψ	-0.0050	6.4690	0.4320	-0.0800	0.0170	0.0370	-1.2880
δ_ψ	0.0035	19.6410	0.5830	0.3680	0.1030	0.0720	-3.5780
δ_v	0.0430	21.1210	-0.1520	0.1700	0.0760	-0.0330	-11.3420
ξ_0	0.0012	-6.9420	-1.2390	0.6830	-0.0020	-0.0120	-3.1990
γ	-0.0120	-16.2790	-0.9000	0.4870	0.0340	-0.1120	2.6440
δ_η	0.0070	1.4250	-1.2760	0.8220	-0.0720	-0.0220	-7.6250
p	0.0012	0.6250	-0.2950	0.0070	0.0043	-0.0150	-1.4020
q	0.0087	-12.1310	-0.4100	-0.7690	-0.0760	-0.0360	-0.9050

Table 4.6: Summary of Magnitudes of the Coefficients for Each Parameter for RC Walls with Flanged Cross-sections

	b_1	b_2	b_3	b_4	b_5	b_6	b_7
α_0	0.0290	23.9860	-1.2120	0.6640	-0.0160	-0.7450	-5.2570
β	0.0230	3.8960	-2.7640	1.9920	-0.0490	0.0490	2.2670
ω_0	-0.0050	3.0300	0.7320	-0.3820	0.0030	0.0110	3.9390
ζ_s	-0.0006	-0.1950	0.0580	0.0080	-0.0026	0.0012	-0.0470
ψ	-0.0030	1.9010	1.2500	-0.9000	0.0060	0.0520	-1.1240
δ_ψ	0.0060	6.7500	-3.8930	2.7060	0.0300	0.0630	-1.5770
δ_v	0.0560	14.3860	1.5560	-0.4490	0.0700	0.1620	-13.2710
ξ_0	0.0012	-6.9420	-1.2390	0.6830	-0.0120	-0.0020	-3.1990
γ	0.0190	-21.1180	6.6700	-5.7440	1.1270	0.0290	0.5420
δ_η	0.0070	1.4250	-1.2760	0.8220	-0.0220	-0.0720	-7.6250
p	-0.0030	0.2720	0.2220	-0.3860	0.0030	0.0190	-1.4050
q	0.0225	-10.5330	0.5120	-1.1420	-0.0860	-1.2380	-0.0600

Fig. 4.12: Comparative study between the Analytical Parameters Estimated by GA and by Regression Analysis for RC Walls with Rectangular Cross-Sections
(continued)

Fig. 4.12: Comparative study between the Analytical Parameters Estimated by GA and by Regression Analysis for RC Walls with Rectangular Cross-Sections

Fig. 4.13: Comparative study between the Analytical Parameters Estimated by GA and by Regression Analysis for RC Walls with Flanged Cross-sections (continued)

Fig. 4.13: Comparative study between the Analytical Parameters Estimated by GA and by Regression Analysis for RC Walls with Flanged Cross-sections

Based on the regression equations, the mathematical parameters associated with the analytical model can be defined in terms of the structural characteristics of RC walls. However, the main motive behind this correlation study is to make the model user-friendly by trimming the efforts needed to run the Genetic Algorithm extensively. As soon as the user gains a quick impression about the approximate parameter sets for a specific reinforced concrete wall, its hysteresis response under a specific displacement history can be attained using any suitable solver. Thus, the hysteretic behavior of any structural component can be comprehended based on its structural features under a certain displacement history.

4.5 Numerical Examples

Four numerical examples are presented in this section to select the magnitudes of the analytical parameters of RC beam-column joints and structural walls based on the regression equations using Eqns. 4.5-4.8 and **Tables 4.3 - 4.6**.

RC interior and exterior beam-column joint specimens Unit P2 by Pessiki *et al.* ([P12]) and Unit 2 by Pantelides *et al.* ([P5]) are chosen for the numerical example to calibrate the model response using parameters obtained by Regression analysis with the experimental results obtained from literature. Similarly, RC structural wall specimens with rectangular and flanged cross-sections, Specimen 1 by Hidalgo *et al.* ([H4]) and LW3 by Li *et al.* ([L5]) are chosen for the numerical example to calibrate the model response using parameters obtained by regression analysis with the experimental results obtained from the literature.

1) The structural characteristics of RC interior beam-column joint specimen Unit P2 by Pessiki *et al.* are as follows.

- ◆ Joint aspect ratio = 0.67

(Beam depth (h_b) = 400 mm and Column cross-sectional depth (h_c) = 600 mm)

- ◆ Joint width (b_j) = 400 mm

- ◆ Concrete compressive cylinder strength of the specimen (f'_c) = 34.5 MPa

- ◆ Average yield strength of steel reinforcement bars (f_y) = 456 MPa
- ◆ Joint core transverse reinforcement ratio (ρ_j) = 0.0%
- ◆ Column longitudinal reinforcement ratio (ρ_c) = 0.27%
- ◆ Beam longitudinal reinforcement ratio (ρ_b) = 1.8%
- ◆ Column axial load ratio (n) = 27%

The parameter magnitudes for the Interior beam-column joint obtained from regression analysis equations (Eqns. 4.5-4.6) and **Table 4.3** are as follows.

$$\alpha_0 = 0.0092, \quad \omega_0 = 4.8932, \quad \xi_0 = 0.0141, \quad \beta = 0.06, \quad \gamma = -0.0088, \quad n = 1.00, \\ \delta_v = 0.00001, \quad \delta_\eta = 0.00009, \quad \zeta_s = 0.9419, \quad q = 0.0227, \quad p = 0.0543, \quad \psi = 0.7911, \\ \delta_\psi = 0.1251, \quad \lambda = 0.0001.$$

The experimental and analytical hysteresis loops for Unit P2 are shown in **Fig. 4.14** where both of the hysteresis loops are in well agreement with each other.

Fig. 4.14: Experimental and Analytical Load- Deflection Plots of RC Interior Beam-Column Joint Specimen from Numerical Example

2) The structural characteristics of RC exterior beam-column joint specimen Unit 2 by Pantelides *et al.* are as follows.

- ◆ Joint aspect ratio = 1.0
(Beam depth (h_b) = 400 mm and column cross-sectional depth (h_c) = 400 mm)
- ◆ Joint width (b_j) = 400 mm
- ◆ Concrete compressive cylinder strength of the specimen (f'_c) = 33 MPa
- ◆ Average yield strength of steel reinforcement bars (f_y) = 469 MPa
- ◆ Joint core transverse reinforcement ratio (ρ_j) = 0.0%
- ◆ Column longitudinal reinforcement ratio (ρ_c) = 2.47%
- ◆ Beam longitudinal reinforcement ratio (ρ_b) = 3.12%
- ◆ Column axial load ratio (n) = 25%

According to regression analysis equations ((Eqns. 4.5-4.6)) and **Table 4.4**, the final parameter magnitudes of Unit 2 are as follows.

$$\alpha_0 = 0.0005, \quad \omega_0 = 3.9509, \quad \xi_0 = 0.0344, \quad \beta = 0.06, \quad \gamma = -0.0094, \quad n = 1.00, \\ \delta_v = 0.00006, \quad \delta_\eta = 0.00022, \quad \zeta_s = 0.9509, \quad q = 0.0201, \quad p = 0.1314, \quad \psi = 0.7704, \\ \delta_\psi = 0.1171, \quad \lambda = 0.0001.$$

The experimental and analytical hysteresis loops for Unit 2 are shown in **Fig. 4.15** which shows good correlation between both the hysteresis loops.

Fig. 4.15: Experimental and Analytical Load- Deflection Plots of RC Exterior Beam-Column Joint Specimen from Numerical Example

3) The structural characteristics of RC rectangular wall Specimen 1 by Hidalgo *et al.* are recorded as follows.

- ◆ Wall length (l_w) = 1000 mm
- ◆ Wall height (h_w) = 2000 mm
- ◆ Wall thickness (t_w) = 120 mm
- ◆ Concrete compressive cylinder strength of the specimen (f'_c) = 19.4 MPa
- ◆ Average Yield strength of steel for reinforcement bars (f_y) = 392 MPa
- ◆ Transverse reinforcement ratio (ρ_h) = 0.131%
- ◆ Longitudinal reinforcement ratio (ρ_v) = 0.251%
- ◆ Boundary reinforcement ratio (ρ_{be}) = 8.50%
- ◆ Axial Load ratio (n) = 0.0 %

The magnitude of the model parameters of the wall depending on regression analysis equations (Eqns. 4.7-4.8) can be obtained from **Table 4.5** as follows.

$\alpha_0 = 0.0005$, $\omega_0 = 59.476$, $\xi_0 = 0.0289$, $\beta = 14.844$, $\gamma = -8.018$, $n = 1.00$,
 $\delta_v = 0.00008$, $\delta_\eta = 0.00030$, $\zeta_s = 0.9764$, $q = 0.072$, $p = 0.255$, $\psi = 0.446$, $\delta_\psi = 0.213$, $\lambda = 0.0001$.

The experimental and analytical hysteresis loops for Specimen 1 are shown in **Fig. 4.16** which illustrates the accuracy of the estimated parameters.

Fig. 4.16: Experimental and Analytical Load- Deflection Plots of RC Structural Wall Specimens with Rectangular Cross-sections from Numerical Example

4) The structural characteristics of RC flanged wall LW3 are noted hereunder.

- ◆ Wall length (l_w)= 2000 mm
- ◆ Wall height (h_w)= 2000 mm
- ◆ Wall thickness (t_w)= 120 mm
- ◆ Concrete compressive cylinder strength of the specimen (f'_c)= 34.8 MPa
- ◆ Average Yield strength of steel for reinforcement bars (f_y)= 382 MPa
- ◆ Transverse reinforcement ratio (ρ_h)= 0.5%
- ◆ Longitudinal reinforcement ratio (ρ_v)= 0.5%

- ◆ Boundary reinforcement ratio (ρ_{be})= 1.4%
- ◆ Axial Load ratio (n)= 5%

The magnitude of the model parameters of the flanged wall depending on regression analysis equations (Eqns. 4.7-4.8) can be calculated from **Table 4.6** as follows.

$\alpha_0 = 0.0013$, $\omega_0 = 78.720$, $\xi_0 = 0.0163$, $\beta = 16.37$, $\gamma = -3.0668$, $n = 1.00$, $\delta_v = 0.00007$, $\delta_\eta = 0.00034$, $\zeta_s = 0.961$, $q = 0.044$, $p = 0.243$, $\psi = 0.573$, $\delta_\psi = 0.333$, $\lambda = 0.0001$.

The analytical hysteresis loops for Specimen LW3 are calibrated against the respective experimental results and as shown in **Fig. 4.17**, they are in good agreement with each other.

Fig. 4.17: Experimental and Analytical Load- Deflection Plots of RC Structural Wall Specimens with Flanged Cross-sections from Numerical Example

4.6 Summary

Calibration of the hysteresis model against experimental results of the RC beam-column joint and structural wall specimens obtained from the existing literature proves the effectiveness of the proposed hysteresis model. Comparison between the

experimental and analytical hysteresis load-deformation plots shows the accuracy and precision of the solver and system identification algorithm. The tested specimens are selected in such a way such that a wide variation in terms of structural geometry, material properties, reinforcement detailing and axial loading levels can be covered for both structural components.

Parameter sensitivity analysis is essential when dealing with the system identification techniques. A sensitive parameter when deviated from its sought-after magnitude will show significant error. Thus, by changing the magnitude of sensitive parameters, better correlation can be achieved. On the contrary, a less sensitive parameter, even when it is fluctuated from its sought-after magnitude, can produce a reasonable result as its contribution to the final response is relatively less. Therefore, providing narrower ranges for less sensitive parameters can increase simplicity in the procedure without affecting the quality of the results.

Since the model parameters are estimated based on experimental results using GA, they may not be identified properly for any specimen when the experimental outcome is not available for that particular specimen. Therefore, in order to resolve this difficulty, approximate magnitudes of the analytical parameters are proposed in this research in terms of the physical parameters of the structural components. Based on the regression equations, the mathematical parameters associated with the analytical model can be defined in terms of the structural characteristics of tested specimens. However, the main motive behind this correlation study is to make the model user-friendly by trimming the efforts needed to run the Genetic Algorithm extensively. As soon as the user gains a quick impression about the approximate parameter sets for a specific structural component, its hysteresis response under a specific displacement history can be attained using any suitable solver. Thus, the hysteretic behavior of any structural component can be comprehended based on its structural features under a definite displacement history.

CHAPTER 5

IMPLEMENTATION OF HYSTERESIS MODELS FOR RC STRUCTURAL COMPONENTS

5.1 Introduction

The proposed analytical model with the suggested mathematical parameters can be employed to deduce the hysteresis load-deformation plots for simple Reinforced Concrete (RC) beam-column joints and structural walls. However, in order to perform structural analysis of a complex structural system and to determine its hysteretic behavior, the proposed approach may be tedious to carry out. For this purpose, a simulation platform is required where the proposed model can be incorporated adequately to determine the hysteretic behavior of the structural system using similar approach. Hence the proposed analytical model is implemented in ABAQUS (version 6.9-1) ([A1]) to enhance its user-friendliness.

5.2 Macro Model for RC Beam-Column Joints

Since the proposed model has already established its ability to determine the hysteretic behavior of RC beam-column joints, the only task to be performed at this stage is to establish the compatibility between the model and the simulation platform. As the output in the proposed model is the overall hysteretic behavior of RC beam-column joints, a single component macro-model is suitable for embedding the mathematical equations in it. The macro model of beam-column joints prescribed in this research is a four node two dimensional idealization of an interior beam-column joint. The model consists of four truss elements forming the boundary of the joint panel, eight connector springs at the joint-beam and joint-column interfaces and two diagonal springs connecting the opposite corners of the joint boundaries as depicted in **Fig. 5.1**. The connector spring elements are considered to be elastic springs with concrete mechanical properties. Four rigid elements are attached to the two beam elements and two column elements to ease the connectivity of the joint-beam and joint-column regions. Thus, the connector

springs work as adjoining elements to link each node of the joint boundary with the corresponding node of the rigid element to ensure connectivity between the joint region (comprising of joint panel and diagonal springs) and beam or column elements.. As shown in **Fig. 5.1**, each joint node is connected to one vertical spring and one horizontal spring.

Fig. 5.1: Schematic diagram of the Beam-Column Joint Macro Model

In this model, mass elements are considered to be lumped at the joint nodes. The diagonal spring elements being the user-elements include the spring orientations, spring inclinations and the analytical model parameters to deliver the hysteretic

response of the beam-column joints. If the diagonal spring is connected from the left bottom node of the joint panel to the right top node, the orientation magnitude is specified as 1. On the other hand, when the diagonal spring is connected from the right bottom node of the joint panel to the left top node, the orientation magnitude is designated as 2. A user subroutine is written in Visual Fortran which acts as a link between Livermore Solver for Ordinary Differential Equations (LSODE) necessary for solving the equations and user element properties of the diagonal springs. When the incorporation of the model equations into the subroutine is complete, ABAQUS ([A1]) can run the input file successfully and develop the hysteresis force for the input displacement function utilizing the spring inclinations (obtained from joint dimensions), spring orientations and the hysteresis model parameters. However, in order to obtain the desired structural response, the hysteresis parameters are required to be estimated separately using Genetic Algorithm (GA) or the regression analysis equations. The identified parameter magnitudes are then inserted in the user-element properties thereafter.

5.3 Calibration with Experimental Results of RC Beam-Column Joints

The performance of the macro model for RC interior and exterior beam-column joints with non-seismic details is verified on the basis of the available experimental results. As mentioned earlier, the structural characteristics of the relevant interior and exterior beam-column joint specimens are summarized in Appendix A.

5.3.1 Interior Beam-Column Joint Specimens

The database containing experimental details of RC non-seismically detailed interior beam-column joint specimens include Unit O1 by Hakuto *et al.* ([H1]), Units 1 and 2 tested by Liu *et al.* ([L9]), Units PEER-1450, PEER-2250, CD15-1450, CD30-1450, CD30-2250, PADH-14 and PADH-22 tested by Lehman *et al.* ([A4], [W1]), P2, P4, P5, P7, P8 and P9 tested by Pessiki *et al.* ([P12]) and JXO-B1, JXO-B2 and JXO-B8 by Joh *et al.* ([J1], [J2]). The wide variation in terms of structural features, material properties and loading characteristics covered in the

selection process of tested beam-column joints specimens increase the applicability of the proposed macro model. The comparison between the experimental and analytical hysteresis load-deformation plots for selected interior beam-column joint specimens are shown in **Fig. 5.2**. From this figure, it is quite evident that the implementation of interior beam-column joints is successful.

Fig. 5.2: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Interior Beam-Column Joints with Non-seismic Detailing (continued)

Fig. 5.2: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Interior Beam-Column Joints with Non-seismic Detailing

5.3.2 Exterior Beam-Column Joint Specimens

The database containing experimental details of RC non-seismically detailed exterior beam-column joint specimens comprise Units O6 and O7 tested by Hakuto *et al.* ([H1]), Units EJ1, EJ2, EJ3 and EJ4 tested by Liu *et al.* ([L9]), Units 1, 2, 3, 4, 5 and 6 tested by Pantelides *et al.* ([P5]) and Test# 2, Test# 4, Test# 5 and Test# 6 by Clyde *et al.* ([C11]). The wide variation in terms of structural features, material properties and loading characteristics covered in the selection process of tested beam-column joints specimens increase the applicability of the proposed

macro model. The comparison between the experimental and analytical hysteresis load-deformation plots for selected exterior beam-column joint specimens are shown in **Fig. 5.3**. From this figure, it is quite evident that the implementation of exterior beam-column joints is successful.

Fig. 5.3: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Exterior Beam-Column Joints with Non-seismic Detailing (continued)

Fig. 5.3: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Exterior Beam-Column Joints with Non-seismic Detailing

5.4 Macro Model for RC Structural Walls

Since the proposed model has already established its good performance in determining the hysteretic behavior of RC walls, the only task to be conducted at this stage is to establish the compatibility between the model and the simulation platform. As the output in the proposed model is the overall hysteretic behavior of RC walls, a single component macro-model is suitable for embedding the mathematical equations in it. The wall macro model prescribed in this research consists of two horizontal truss elements, two vertical flexure springs with concrete

material properties and two diagonal springs connecting the opposite corners of the boundaries of the wall panel as depicted in **Fig. 5.4**. The total mass of the wall is assumed to be lumped at the nodes of the wall panel.

Fig. 5.4: Schematic diagram of the Structural Wall Macro Model

The diagonal spring elements being the user-element include the spring orientations, spring inclinations and the analytical model parameters to deliver hysteretic response of the wall. If the spring is connected from left bottom to right top, the orientation magnitude is specified as 1 and if it is connected from right bottom to left top, then orientation magnitude is specified as 2. A user subroutine is written in Visual Fortran which acts as a link between the LSODE solver necessary for solving the equations and user element properties of the diagonal springs within the wall boundary. When the incorporation of the model equations into the subroutine is complete, ABAQUS ([A1]) can run the input file successfully and

develop the hysteresis force for a given displacement function utilizing the spring inclinations (obtained from wall dimensions), spring orientations and the hysteresis model parameters. However, in order to obtain the desirable structural response, the hysteresis parameters are estimated separately and then the identified parameter magnitudes are inserted in the user-element properties.

5.5 Calibration with Experimental Results of RC Structural Walls

The sole purpose of calibrating the analytical model responses with the wide range of experimental results of RC walls obtained from the literature is to ensure the adequacy and accuracy of the proposed macro model. As mentioned earlier, the structural characteristics of the relevant structural wall specimens is summarized in Appendix B. Thus, the appropriateness of the implementation of the mathematical hysteresis model proposed in previous chapters, is judged for single and multiple storey wall specimens by comparing the model response with the experimental results.

5.5.1 Single Storey RC Structural Walls

RC wall specimens with rectangular cross-sections include Specimens 72, 73, 165, 170 and 175 by Hirosawa *et al.* ([H7]), Specimens 1-4, 6-16, 21-26, 28-30 and 32 tested by Hidalgo *et al.* ([H4]), Specimens M1, M2, M3 and M4 tested by Greifenhagen *et al.* ([G6]), wp111-9, wp111-10, wp1105-7, wp1105-8, wp110-5, wp110-6 by Massone *et al.* ([M3]), U1.0, U1.5, C1.0, C1.5, U1.0-BC, U1.5-BC by Kuang *et al.* ([K8]). RC wall specimens with flanged cross-sections include S5 and S7 by Maier *et al.* ([M1]), 36M830 and 36M850 by Sato *et al.* ([S6]) and LW1, LW2, LW3, LW4, LW5, MW1, MW2 and MW3 by Li *et al.* ([L5]). The comparison between the experimental and analytical hysteresis load-deformation plots for selected RC wall specimens with rectangular cross-sections are shown in **Fig. 5.5**.

Fig. 5.5: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Rectangular Cross-sections (continued)

Fig. 5.5: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Rectangular Cross-sections

Similarly, the comparison between the experimental and analytical hysteresis load-deformation plots for selected RC wall specimens with flanged cross-sections are shown in **Fig. 5.6**. From these figures, it is quite evident that the implementation of RC walls is successful.

Fig. 5.6: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Flanged Cross-sections (continued)

Fig. 5.6: Experimental and Analytical Load Deformation Plots of Reinforced Concrete Structural Walls with Flanged Cross-sections

5.5.2 Multiple Storey RC Structural Walls

Reinforced Concrete multi-storey wall specimens WDH4a, WDH4b, WDH5a, WDH5b by Belmouden and Lestuzzi ([B6]) are modeled in ABAQUS ([A1]) based on the schematic diagram on **Fig. 5.4**. The test walls have rectangular cross-sections and identical dimensions with the length of wall (l_w)= 900 mm, height of each storey of the wall (h_w)=1360 mm with the total height being 4080 mm, thickness of wall (t_w)=100 mm as shown in **Fig. 5.7**. The concrete compressive strength (f'_c) of WDH4 and WDH5 series specimens are 36.3 and 36.5 MPa respectively while the average yield strength (f_y) of reinforcing steel are 481.4 and 553.9 MPa correspondingly. Total vertical reinforcement ratio ρ_t , web vertical reinforcement ratio ρ_v and boundary vertical reinforcement ratio ρ_{be} for WDH4 and WDH5 series specimens are 0.47% and 0.60%; 0.73% and 1.73% and 0.42% and 0.42% respectively. The web horizontal reinforcement ratio ρ_h for WDH4 and WDH5 series specimens are 0.04% and 0.03% and respectively. The shake table tests were conducted on the wall specimens for two synthetic ground motions. The wall footing was rigidly connected to the shake table, which operates in one horizontal direction and can move up to 125 mm in each direction. The axial load applied (N) is 3% of the resistance of the gross area for both types of walls ($0.03f_cA_g$) and thus axial load ratio (n) is equal to 3%.

In order to model the three storey wall specimens in ABAQUS ([A1]), the hysteresis model parameters are calculated on the basis of the structural features of the walls using regression equations obtained in Chapter 4. Thereafter the boundary conditions and the loading history of the model wall are kept identical to the test wall. The reliability of the analytical approach for predicting the hysteresis load-deformation relationship of multi storey structural wall is proved by comparing the numerical results of a three storey RC wall with the experimental outcome obtained from the literature. The comparisons between the simulation responses of the wall macro model with the experimental load-deformation plot are presented in **Fig. 5.8** which proves the appropriateness of the implementation.

Fig. 5.7 Structural Geometry of Reinforced Concrete Three Storey Test Walls
([B6])

Fig. 5.8 Experimental and Analytical Hysteresis Load Deformation Plots of Reinforced Concrete Three Storey Walls

5.6 Comparison with Experimental Results of RC Frame Structures

The proposed macro model for beam-column joints shown in **Fig. 5.1** is implemented in a three-storey three-bay gravity load designed frame tested in University of Pavia by Calvi *et al.* ([C1]). The frame was designed according to the Engineering practice followed in 1970 and hence no transverse reinforcement was placed in the joint region and smooth steel bars with mechanical properties similar to those typically used in older periods, were adopted for both the longitudinal and

transverse reinforcement. The structural geometry of the test frame is illustrated in **Fig. 5.9**. The columns have a uniform cross-section, 200 mm $\times$ 200 mm with the longitudinal and transverse reinforcement ratio being 0.75% and 0.12% respectively. The beams of the test frame also have a uniform cross-section 200 mm $\times$ 330 mm with the longitudinal and transverse reinforcement ratio ranging between 0.82% to 0.99% and 0.04% to 0.08% respectively.

Fig. 5.9 Structural Geometry of Reinforced Concrete Three Storey Three Bay Test Frame ([C1])

The frame system was subjected to quasi-static loading at increasing levels of top displacement, applied to the structure using three electro-mechanical actuators connected to the closest beam through a steel lever arm. The loading history consisted of a series of three cycles at increasing level of top drift ($\pm 0.2\% \pm 0.6\% \pm 1.2\%$) with one final cycle at $\pm 1.6\%$.

The ABAQUS ([A1]) model of the test frame is constructed on the basis of the proposed macro model shown in **Fig. 5.1**. The test frame structure consists of eight exterior beam-column joints and four interior beam-column joints. The hysteresis model parameters of the beam-column joint elements are calculated on the basis of their structural features using regression equations obtained in Chapter 4. Thereafter the boundary conditions and the loading history of the model frame are kept identical to the test structure. The comparison between the experimental and the simulation results for the 1st, 2nd and 3rd storey shear versus the corresponding inter-storey displacements and the base shear versus the top storey displacement are shown in **Fig. 5.10**. The correlation between the experimental and analytical hysteresis responses of RC frame structure justifies the reliability of implementation of the proposed analytical model.

Fig. 5.10: Experimental and Analytical Hysteresis Load Deformation Plots of Reinforced Concrete Frame (continued)

Fig. 5.10: Experimental and Analytical Hysteresis Load Deformation Plots of Reinforced Concrete Frame

5.7 Summary

The proposed analytical hysteresis model with the suggested mathematical parameters can be employed to deduce the hysteresis load-deformation plots for RC beam-column joints and structural walls. However, in order to elucidate the hysteretic behavior of a structural system containing multiple beam-column joints and walls, the proposed model is implemented in ABAQUS ([A1]) (version 6.9-1) in the form of macro elements for beam-column joints and structural walls to enhance its user-friendliness. Two diagonal springs connecting the opposite corners of the boundaries of joint and wall panels contain the user-element properties, i.e. the spring orientations, spring inclinations and the analytical model parameters to deliver the required hysteretic response of the structural components. A user subroutine is written in Visual Fortran which acts as a medium between the LSODE solver and the user element properties of the diagonal springs. When the hysteresis model parameters are estimated adequately and the model equations are incorporated into the subroutine, ABAQUS ([A1]) can deliver the hysteresis force for a given displacement function utilizing the spring inclinations, spring orientation and the model parameters. The correlation between the simulation and experimental responses of RC beam-column joints, single and multi-storey

structural walls and frame structures justify the exactness and efficiency of the macro models and the proposed methodology.

CHAPTER 6

DAMAGE QUANTIFICATION OF RC STRUCTURAL COMPONENTS

6.1 Introduction

Quantitative damage evaluation plays a pivotal role in seismic performance assessment of structures. The concept of defining different stages of structural damage in the form of damage indices is effective owing to its simplicity in application. However, the expression for damage index needs to be generalized such that it can incorporate different structural aspects well. In the present research, a damage model inspired from the Park-Ang damage model ([P9]) is proposed for damage quantification of Reinforced Concrete (RC) beam-column joints and structural walls on the basis of the relevant experimental results from the existing literature.

6.2 Proposed Damage Model

The damage index proposed by most researchers is a quantity where inelastic deformation, energy dissipation or the combination of both is taken into account. To date, the Park-Ang damage model ([P9]) is one of the most widely used damage models among the researchers. However, a significant problem in this model is that the damage index magnitude can exceed unity when the structural failure is assumed to occur. In order to resolve this deficiency, a damage model based on the Park-Ang damage model ([P9]) is suggested in this research where the expression for damage index (DI) is presented hereunder.

$$DI = \left\langle (1 - \beta) \frac{\delta_{max}}{\delta_u} + \beta \int \frac{\delta}{\delta_u} \frac{dE}{E_c} \right\rangle \quad (6.1)$$

Here δ_{max} is the peak deformation, δ_u is the ultimate deformation capacity of the structural component, β is the cyclic deterioration parameter, dE is the

incremental dissipated energy and E_c is the cumulative hysteretic energy per loading cycle obtained by the area under the force-displacement curve.

In order to estimate an expression for β , it is assumed that at the ultimate limit state of structure, damage index (DI) magnitude is 1. Hence,

$$\left\langle (1 - \beta) \frac{\delta_{max}}{\delta_u} + \beta \int \frac{\delta_{max}}{\delta_u} \frac{dE}{E_c(\delta)} \right\rangle = 1$$

$$\beta = \frac{\frac{\delta_{max}}{\delta_u} - 1}{\frac{\delta_{max}}{\delta_u} \left\langle 1 - \int \frac{dE}{E_c} \right\rangle} \quad (6.2)$$

The databases of past experimental results of RC interior and exterior beam-column joints and structural walls with rectangular and flanged cross-sections are summarized in Appendices A and B respectively. This database of RC beam-column joints and structural walls is utilized to calibrate the parameter (δ_u) involved in the proposed damage model. Once δ_u is identified on the basis of past experimental results, the magnitude of β for a structural component can be obtained based on Eqn. 6.2 when the hysteretic load-deformation response is known.

In order to obtain simplified expressions for δ_u in terms of the physical characteristics of the structural components i.e. the structural geometry, material properties, reinforcement ratio and axial load ratio, they are normalized. Thereafter, the multivariate regression analyses are carried out to obtain the best suited expressions for the model parameters in terms of the normalized structural physical characteristics. From the regression analysis, exponential functions of linear variables multiplied by various coefficients are found as best fitted with the δ_u obtained from the experimental results. The expressions of δ_u for interior and exterior beam-column joints and structural walls with rectangular and flanged

cross-sections in terms of ten variables and seven constant coefficients are presented as follows.

For RC interior beam-column joints,

$$\delta_u/h(\%) = e^{-0.33 \times (h_b/h_c) \times (b_j/h_c) + 7.7 \times (f'_c/f_y) + 0.94 \times \rho_j + 0.1 \times \rho_b + 0.03 \times \rho_c + 0.01 \times n + 1.24} \quad (6.3)$$

For RC exterior beam-column joints,

$$\delta_u/h(\%) = e^{-0.13 \times (h_b/h_c) \times (b_j/h_c) + 5.7 \times (f'_c/f_y) + 0.94 \times \rho_j + 0.1 \times \rho_b + 0.01 \times \rho_c + 0.03 \times n + 1.12} \quad (6.4)$$

For RC walls with rectangular cross section,

$$\delta_u/h_w(\%) = e^{0.02 \times (l_w/h_w) \times (l_w/t_w) - 13.3 \times (f'_c/f_y) + 3.0 \times \rho_h - 0.2 \times \rho_v - 0.14 \times \rho_{be} - 0.14 \times n + 1.25} \quad (6.5)$$

For RC walls with flanged cross section,

$$\delta_u/h_w(\%) = e^{-0.003 \times (l_w/h_w) \times (l_w/t_w) + 18 \times (f'_c/f_y) - 10.5 \times \rho_h + 9.6 \times \rho_v - 1.7 \times \rho_{be} - 0.03 \times n + 1.22} \quad (6.6)$$

Here h_c and h_b represent the corresponding cross-sectional depths of columns and beams in mm; b_j is the joint width in mm considered as the smaller of $\{b_c, b_b + h_c\}$; b_c and b_b are the width of columns and beams respectively in mm; l_w , h_w and t_w are the length, height and thickness of structural walls in mm respectively; f'_c is the concrete compressive strength in MPa; f_y is the yield strength of reinforcing steel in MPa; ρ_b and ρ_c are the longitudinal reinforcement ratio in % respectively for beam and column sections; ρ_j is the joint core transverse reinforcement ratio in %; ρ_h , ρ_v and ρ_{be} are the horizontal, vertical and boundary reinforcement ratio in % respectively for wall sections; n is the axial load ratio in %; h is the storey height for beam-column joints. Since the selected interior and exterior beam-column joint and structural wall specimens are lightly

reinforced, the simplified expressions of δ_u are also applicable to lightly reinforced concrete structural components only. The correlation between the experimental and empirical magnitudes of ultimate drift ratio for RC beam-column joints and structural walls is shown in **Fig. 6.1** and **Fig. 6.2** respectively.

Fig. 6.1 Comparison between the Predicted and Observed Ultimate Drift Capacity (%) of RC Interior and Exterior Beam-Column Joints

Fig. 6.2 Comparison between the Predicted and Observed Ultimate Drift Capacity (%) of RC Walls with Rectangular and Flanged Cross-sections

The damage indices for the beam-column joints and structural walls can be calculated based on Eqs. 6.1 -6.6. In order to facilitate visualization of the damage evolution in the beam-column joint and structural wall specimens from the existing literature, the calculated damage indices versus drift for two beam-column joint

specimens and two wall specimens are plotted in **Figs. 6.3** and **6.4** respectively. Due to page constraint, the calculated damage indices versus drift for all beam-column joint and wall specimens cannot be included here.

Fig. 6.3 Relationship between the Calculated Damage Indices and Observed Drift Ratio (%) of RC Interior and Exterior Beam-Column Joints

Fig. 6.3 Relationship between the Calculated Damage Indices and Observed Drift Ratio (%) of RC Interior and Exterior Beam-Column Joints

6.3 Damage States and Method of Repair

In the present research, damage states represent the threshold levels of damages experienced by the structural components under a given loading environment. Generally damage states are characterized by different stages sustained by the structure when subjected to external loading and the indicators of different damage states in structural components are initiation of cracking, measurement of the crack width, extent of concrete crushing, sliding shear displacement, reinforcement bar yielding, buckling and fracture etc. However, the method of repair to restore the structural components to its pre-damaged condition provides a basic measure of the economical impact of the seismic loading.

On the basis of the existing literature, the method of repair associated with each damage state of RC structural components is explored and the range of the damage index magnitudes for each damage state is derived from the database. When the beam-column joints due to seismic loading exhibit initial hair line cracks at the beam-column interfaces and within the joint area with the maximum crack width within the joint being less than 0.5 mm, repairing the surface finish can adequately improve the aesthetic appearance of the joints. Similarly, when the damage in RC structural walls due to seismic loading is limited to initiation of flexural and shear cracks with the crack width less than 0.5 mm, restoration of the strength and stiffness are not required while refurbishing the surface finish can adequately improve the aesthetic appearance of the structural walls. This damage state can be designated as minor damage and the method of repair opted for the first damage state is termed as Cosmetic Repair.

If the beam-column joints face yielding of beam longitudinal reinforcement, with the maximum crack width within the joint being greater than 0.5 mm but less than 2 mm, the damage state is assumed as light. At this damage state, the structural walls experience yielding of horizontal, vertical and boundary reinforcements and significant opening of flexural and shears cracks. The widths of the flexural and shear cracks are greater than 0.5 mm but less than 3 mm. In order to restore strength and stiffness of the structural components and to prevent further water infiltration,

corrosion and fire damage, injection of cracks with epoxy resin is opted for the second damage state.

When concrete spalling occurs in the beam-column joints at the extent of 10-30% and the cracks extend into the beams and/ or columns, it cannot be repaired by epoxy injection. Similarly, when the structural walls exhibit buckling of longitudinal reinforcement at the boundary region, concrete crushing at the compression toes and flexural cracks of width exceeding 3 mm, it cannot be repaired by means of epoxy injection. Thus the damage sustained by the structural components at this stage can be considered to be moderate and to restore the structural components to its pre-earthquake condition the damaged concrete and rebars need to be replaced.

If the beam-column joints experience major spalling of surface concrete and crushing of concrete extending to the joint core and the structural wall incurs sliding at the interface between the wall web and the foundation, widespread crushing of concrete and fracture of reinforcement bars that cannot be repaired by replacing the damaged concrete and rebars, wide diagonal cracks which cannot be restored by epoxy injections and shear cracks of width exceeding 3 mm, the damage state is regarded as severe. At this damage stage, the complete structural component needs to be replaced. In view of the above-mentioned damage conditions, the damage index magnitudes for the structural components from literature are identified and grouped together to provide the ranges of damage index magnitudes for the four damage states in **Table 6.1**.

Table 6.1: Damage Indices corresponding to Damage States and Method of Repair

Damage State	Method of Repair	Range of Damage Index (DI)
1. Minor	Cosmetic Repair	$DI < 0.10$
2. Low	Epoxy Injection	$0.10 < DI < 0.40$
3. Moderate	Partial Replacement	$0.40 < DI < 0.75$
4. Severe	Complete Replacement	$DI > 0.75$

6.4 Engineering Demand Parameter

The experimental results of Reinforced Concrete (RC) interior and exterior beam-column joints and structural walls with rectangular and flanged cross-sections summarized in Appendices A and B provide a basis for development of each damage state. However, to utilize this database, engineering damage parameters are required to be identified such that they can predict the observed damage adequately. An engineering demand parameter (EDP) is a scalar or a functional quantity that can define the seismic demand of a structural element at any point of the loading history.

Pagni and Lowes ([P2]) and Brown and Lowes ([B12]) proposed a variant on dissipated hysteretic energy for assessment of RC beam-column joints, namely, a demand parameter whose functional form included maximum story drift and number of load (or displacement) cycles as variables. Unfortunately, many sources did not report the relationship between damage and number of loading cycles and so the Lowes demand parameter was not pursued. To date, drift ratio is the most reported engineering demand parameter for most structural elements. Drift ratio is defined as the displacement caused in a structural element due to a specific loading history divided by the height of the structural component. In the present research, drift ratio is considered to be the measure of seismic demand for RC beam-column joints and structural walls.

6.5 Method of Data Combination

With an objective of developing the fragility functions for RC beam-column joints and structural walls, relationships between the EDP and the methods of repair required to restore the structural components for each damage state were evaluated based on the experimental results obtained from literature. Since a range of damage index magnitudes are specified for each method of repair, the damage data must be combined accordingly to obtain the desirable results. However, there are two approaches for combining the damage data as follows.

Method 1: For each specimen, the EDP magnitudes corresponding to each damage state associated with a specific method of repair are used. This approach delivers maximum number of data points for each method of repair at the cost of more dispersion in the data range. Moreover, the method shows bias towards higher EDP levels.

Method 2: In this method the EDP magnitudes related to the lowest damage state for each specimen associated with a specific method of repair are adopted. This methodology gives lesser number of data points for each method of repair in comparison with Method 1. However, this method introduces relatively less bias towards higher EDP levels.

The damage data is combined using both methods and the mean and standard deviation of the dataset are presented in **Table 6.2**. This table shows that the mean EDP is quite close for both methods while the standard deviation is relatively high for data set combined using Method 1. Thus, among the two methods of data combination, Method 2 is observed to be more appropriate for data combination and selected for generating fragility functions. **Fig. 6.5** shows the compiled damage states versus damage indices for RC structural walls with rectangular and flanged cross-sections. **Fig. 6.6** shows the compiled damage states versus damage indices for RC interior and exterior beam-column joints. **Fig. 6.7** shows the compiled damage states versus drift ratio data for RC structural walls with rectangular and flanged cross-sections. **Fig. 6.8** shows the compiled damage states versus drift ratio data for RC interior and exterior beam-column joints.

Table 6.2: Comparative Study for the EDP versus Damage States Dataset
Combination Methods

Interior Beam-Column Joints								
Method of Data Combination	Mean				Standard Deviation			
	Damage State 1	Damage State 2	Damage State 3	Damage State 4	Damage State 1	Damage State 2	Damage State 3	Damage State 4
1	0.807	2.560	4.607	5.605	0.264	0.609	1.122	1.335
2	0.775	2.566	4.729	5.735	0.219	0.557	1.063	1.278
Exterior Beam-Column Joints								
Method of Data Combination	Mean				Standard Deviation			
	Damage State 1	Damage State 2	Damage State 3	Damage State 4	Damage State 1	Damage State 2	Damage State 3	Damage State 4
1	0.684	2.288	4.163	5.187	0.372	1.219	2.207	2.743
2	0.635	2.328	4.160	5.187	0.305	1.147	2.142	2.688
Walls with Rectangular Cross-sections								
Method of Data Combination	Mean				Standard Deviation			
	Damage State 1	Damage State 2	Damage State 3	Damage State 4	Damage State 1	Damage State 2	Damage State 3	Damage State 4
1	0.150	0.407	0.657	1.382	0.108	0.246	0.336	0.712
2	0.133	0.333	0.533	1.334	0.071	0.177	0.283	0.709
Walls with Flanged Cross-sections								
Method of Data Combination	Mean				Standard Deviation			
	Damage State 1	Damage State 2	Damage State 3	Damage State 4	Damage State 1	Damage State 2	Damage State 3	Damage State 4
1	0.154	0.536	0.896	1.241	0.165	0.242	0.499	0.684
2	0.124	0.311	0.497	1.243	0.031	0.077	0.123	0.307

Fig. 6.5 Damage States versus Damage Indices plot using Method 2 for Data Combination for RC Interior and Exterior Beam-Column Joints

Fig. 6.6 Damage States versus Damage Indices plot using Method 2 for Data Combination for RC Walls with Rectangular and Flanged cross-sections

Fig. 6.7 Damage States versus Drift Ratio (%) plot using Method 2 for Data Combination for RC Interior and Exterior Beam-Column Joints

Fig. 6.8 Damage States versus Drift Ratio (%) plot using Method 2 for Data Combination for RC Walls with Rectangular and Flanged cross-sections

6.6 Summary

Damage quantification plays an important role in seismic performance assessment of structures. To date, the Park-Ang damage model is one of the most widely used damage models among the researchers. However, a significant problem in this model is that the damage index magnitude can exceed unity when the structural failure is assumed to occur. In order to overcome this deficiency, a modified Park-Ang Damage model is proposed in this research to quantify and classify the damage states of RC beam-column joints and structural walls in terms of damage index. Drift ratio is selected as the engineering damage parameter (EDP) i.e. is a scalar capable of defining the seismic demand of a structural element at any point of the loading history. An experimental database of rectangular and flanged RC wall specimens and interior and exterior beam-column joints under simulated seismic loading is compiled from literature to obtain the EDP versus damage states data for RC beam-column joints and structural walls.

CHAPTER 7

FRAGILITY ASSESSMENT OF RC STRUCTURAL COMPONENTS

7.1 Introduction

Fragility functions play an essential role in seismic performance assessment of structures. To date, fragility functions are considered to be an effective tool in the quantification of seismic vulnerability of structures. Fragility functions represent the conditional probability of a particular structure exceeding a certain damage state when subjected to seismic excitations. On the basis of the selected Engineering Demand parameter (EDP) i.e. drift ratio and the four damage states data, seismic fragility functions for Reinforced Concrete (RC) beam-column joints and structural walls are evaluated here. Thereafter, the seismic fragility curves for three, six and nine storey RC structural walls and bare frames are developed for the scaled ground motions representative of Singapore.

7.2 Standard Probability Distributions

The Engineering Damage parameter (EDP) and damage states data are utilized to create the families of empirical cumulative distribution functions (CDF) which can define the probability of exceeding each damage state for a specific magnitude of EDP by using four widely used probability distributions, Lognormal distribution, Weibull distribution, Beta distribution and Gamma distribution. The lognormal distribution is the probability distribution of a random variable whose logarithm is normally distributed. The probability distribution functions (PDF) for the lognormal distribution is as follows.

$$f_x(x) = \begin{cases} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right], & \text{for } x \geq 0 \\ 0, & \text{elsewhere} \end{cases} \quad (7.1)$$

Here μ and σ are the mean and standard deviation of the natural logarithm of the demand parameter. As observed in Eq. 7.1, this distribution can only consider positive magnitudes of the demand parameter x . The lognormal distribution is widely used for evaluation of fragility functions. The probability density function (PDF) for the Weibull distribution is dependent on two parameters i.e. k , the shape of the distribution and λ , the scale parameter.

$$f_x(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda} \right)^{k-1} e^{-\left(\frac{x}{\lambda} \right)^k}, \text{ for } k, \lambda > 0, x \geq 0 \quad (7.2)$$

The beta distribution is defined within the interval $[0, 1]$. The probability density function (PDF) for the beta distribution utilizes two positive valued shape parameters α and β as described below.

$$f_x(x) = \begin{cases} \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}, & \text{for } 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases} \quad (7.3)$$

Here $B(\alpha, \beta)$ is the beta function as follows.

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)} \quad (7.4)$$

Similar to the lognormal distribution, the gamma distribution is also one-sided i.e. it also works for positive magnitudes of the demand parameter. The probability density function (PDF) for the gamma distribution is mentioned hereunder.

$$f_x(x) = \begin{cases} \frac{\lambda^k}{\Gamma(k)} x^{k-1} e^{-\lambda x}, & \text{for } x \geq 0 \\ 0, & \text{elsewhere} \end{cases} \quad (7.5)$$

Here k is the shape of the distribution, λ is the scale parameter and $\Gamma(k)$ is the gamma function as follows.

$$\Gamma(k) = \int_0^{\infty} y^{k-1} e^{-y} dy \quad (7.6)$$

7.3 Method of Maximum Likelihood

Generally, two methods are adopted to determine the parameters associated with the cumulative distribution functions of four standard probability distributions so as to calibrate them with the empirical datasets: Method of Moments and Method of Maximum Likelihood. In Method of Moments, the CDF parameters are computed from the mean and variance of the empirical datasets which can be quite different from the mean and variance of the population resulting in introduction of some error in the calibration process. Hence, Method of Maximum Likelihood is used to determine the distribution parameters required to define the CDFs for the empirical datasets. This method uses the sample likelihood function which is defined as:

$$L = \prod_{i=1}^n f_x(x_i, p) \quad (7.7)$$

where f_x is the derivative of the CDF with respect to the random variable, x_i is the individual data point and p is the vector of distribution parameters. The maximum likelihood estimate (MLE) of p is the value that maximizes the likelihood function.

The four standard probability distributions are tested using goodness-of-fit tests to identify the standard CDF best fitted to the empirical CDF. Evaluation of the goodness-of-fit of the fragility functions using hypothesis testing was introduced by Shinozuka *et al* ([S11]). In the present research, three standard goodness-of-fit tests namely, Chi-square (χ^2) test, Kolmogorov-Smirnov (K-S) test and Lilliefors test are selected to quantify the goodness of the four selected standard CDFs with respect to the empirical CDF.

In order to perform a χ^2 test, the empirical and the theoretical datasets corresponding to each damage state are subdivided into m intervals. When the

numbers of observations in each interval are n_i for the observed data and e_i for the theoretical data, it can be shown that ([H8])

For $n \rightarrow \infty$

The quantity $\sum_{i=1}^m \frac{(n_i - e_i)^2}{e_i}$ approaches the χ^2 distribution with $f = m - 1 - k$ degrees of freedom.

Here k is the number of parameters in the theoretical distribution. At the significance level α , the theoretical distribution is acceptable if

$$\sum_{i=1}^m \frac{(n_i - e_i)^2}{e_i} < c_{1-\alpha, f} \quad (7.8)$$

where $c_{1-\alpha, f}$ is the value of the χ^2 distribution with f degrees of freedom at a CDF value of $(1-\alpha)$. To obtain satisfactory results from χ^2 test, the number of intervals m and the number of observations in each interval must be at least 5 ([H2], [B7]) and the total number of data points must exceed 50 ([K6]). In the present study, m is chosen as 5 and α is selected to be 5%, with $c_{1-\alpha, f} = 5.992$ for all cases. However, due to relatively less number of data points for RC non-seismically detailed beam-column joints and RC walls with flanged cross-sections, the results of the χ^2 tests become less reliable.

The maximum difference between the theoretical and the empirical CDFs, D_n is ([P2]):

$$D_n = \max |F_X(x_i) - S_X(x_i)| \quad (7.9)$$

where $F_X(x_i)$ is the theoretical CDF and $S_X(x_i)$ is the empirical CDF for i th observation. D_n is a random variable with a distribution dependent on the sample size n and the CDF of D_n is related to the significance level α as

$$P(D_n \leq D_n^\alpha) = 1 - \alpha \quad (7.10)$$

On the basis of K-S test, if D_n is less than or equal to the tabulated value of D_n^α , ([B7]) the theoretical distribution is acceptable at the significance level of α (here, 5%). Although K-S test is appropriate for any size of the dataset, its accuracy is questionable when the distribution parameters are estimated from the data ([B7], [K6], [L6]), as in the present case.

The Lilliefors test is a variant of the K-S test to account for the evaluation of the distribution parameters from the sample data ([L6]). This test evaluates the normality of a given data and is appropriate for the small sample sizes. It is used to assess the acceptability of the lognormal distribution by testing the logarithm of the available data at 5% significance level.

7.4 Seismic Fragility Functions for RC Beam-Column Joints

For evaluation of seismic fragility functions of RC beam-column joints, the distribution parameters for the lognormal, Weibull, beta and gamma distribution are assessed on the basis of the method of maximum likelihood. The distribution parameters of the four standard probability distributions for RC interior and exterior beam-column joints are summarized in **Table 7.1**. The χ^2 and K-S tests are conducted for all four standard probability distributions. The results of χ^2 and K-S tests for all beam-column joint datasets are summarized in **Tables 7.2** and **7.3**. From these tables it is quite evident that the most commonly employed lognormal distribution is acceptable as the theoretical distribution for all damage states of RC interior and exterior beam-column joints. In addition, the lognormal distribution can pass Lilliefors test for all but one damage states for both types of beam-column joints, as shown in **Table 7.4**. Thus the lognormal distribution is finalized for assessing the theoretical fragility functions for beam-column joint specimens. The empirical and theoretical (using Lognormal Distribution) fragility functions for RC interior and exterior beam-column joints are plotted in **Figs. 7.1** and **7.2** respectively.

Table 7.1: Distribution Parameters for Lognormal, Weibull, Beta and Gamma Distributions

Interior Beam-Column Joints								
Damage States	Lognormal		Weibull		Beta		Gamma	
	μ	σ	k	λ	α	β	k	λ
1	-0.298	0.308	3.360	0.848	0.324	0.181	12.504	0.062
2	0.916	0.242	4.064	2.777	0.499	0.233	21.261	0.121
3	1.525	0.252	3.919	5.129	0.402	0.224	19.782	0.239
4	1.720	0.245	6.213	3.982	0.465	0.226	20.194	0.284
Exterior Beam-Column Joints								
Damage States	Lognormal		Weibull		Beta		Gamma	
	μ	σ	k	λ	α	β	k	λ
1	-0.569	0.491	2.068	0.693	0.713	0.547	4.334	0.147
2	0.725	0.497	2.094	2.510	0.364	0.319	4.118	0.565
3	1.297	0.512	1.968	4.528	0.383	0.298	3.772	1.103
4	1.513	0.524	1.928	5.660	0.391	0.289	3.724	1.393

Table 7.2: χ^2 test result for Lognormal, Weibull, Beta and Gamma Distributions

Interior beam-column joints									
Damage States	$c_{I-\alpha, f}$	Lognormal		Weibull		Beta		Gamma	
		Eq. 7.8	Correct CDF	Eq. 7.8	Correct CDF	Eq. 7.8	Correct CDF	Eq. 7.8	Correct CDF
1	5.992	2.33	True	2.33	True	23.75	False	3.83	True
2	5.992	5.50	True	5.08	True	34.25	False	5.58	True
3	5.992	2.83	True	5.83	True	23.75	False	4.33	True
4	5.992	2.83	True	62.33	False	26.25	False	3.83	True
Exterior beam-column joints									
Damage States	$c_{I-\alpha, f}$	Lognormal		Weibull		Beta		Gamma	
		Eq. 7.8	Correct CDF	Eq. 7.8	Correct CDF	Eq. 7.8	Correct CDF	Eq. 7.8	Correct CDF
1	5.992	4.33	True	4.33	True	7.92	False	4.33	True
2	5.992	5.00	True	1.92	True	11.25	False	2.58	True
3	5.992	3.25	True	3.25	True	13.92	False	3.25	True
4	5.992	3.25	True	5.25	True	11.92	False	3.25	True

Table 7.3: K-S tests Results for Lognormal, Weibull, Beta and Gamma Distributions

	Interior beam-column joints				Exterior beam-column joints			
	Damage State 1	Damage State 2	Damage State 3	Damage State 4	Damage State 1	Damage State 2	Damage State 3	Damage State 4
D_n^α	0.300	0.300	0.300	0.300	0.330	0.330	0.330	0.330
Lognormal Distribution								
D	0.158	0.211	0.211	0.263	0.188	0.188	0.188	0.188
p	0.956	0.742	0.742	0.462	0.912	0.912	0.912	0.912
Correct CDF	True	True	True	True	True	True	True	True
Weibull Distribution								
D	0.211	0.211	0.211	0.895	0.188	0.125	0.125	0.125
p	0.742	0.742	0.742	0.000	0.912	0.999	0.999	0.999
Correct CDF	True	True	True	False	True	True	True	True
Beta Distribution								
D	0.421	0.421	0.421	0.421	0.250	0.313	0.250	0.250
p	0.049	0.049	0.049	0.049	0.633	0.348	0.633	0.633
Correct CDF	False	False	False	False	True	True	True	True
Gamma Distribution								
D	0.158	0.211	0.158	0.211	0.188	0.188	0.188	0.188
p	0.956	0.742	0.956	0.742	0.912	0.912	0.912	0.912
Correct CDF	True	True	True	True	True	True	True	True

Table 7.4: Results of Lilliefors tests for Lognormal Distribution

Damage States	Interior beam-column joints				Exterior beam-column joints			
	Lognormal Distribution				Lognormal Distribution			
	D_n^α	D	p	Correct CDF	D_n^α	D	p	Correct CDF
1	0.195	0.127	0.500	True	0.213	0.222	0.033	False
2	0.195	0.154	0.271	True	0.213	0.169	0.247	True
3	0.195	0.129	0.500	True	0.213	0.183	0.156	True
4	0.195	0.161	0.213	True	0.213	0.175	0.207	True

Fig. 7.1 Empirical and Theoretical (Lognormal) Fragility Curves for RC Interior Beam-column Joints

Fig. 7.2 Empirical and Theoretical (Lognormal) Fragility Curves for RC Exterior Beam-column Joints

7.5 Seismic Fragility Functions for RC Structural Walls

For evaluation of seismic fragility functions of RC structural walls, the distribution parameters for the lognormal, Weibull, beta and gamma distribution are assessed on the basis of the method of maximum likelihood. The distribution parameters of the four standard probability distributions for RC structural walls with rectangular and flanged cross-sections are summarized in **Table 7.5**. The results of χ^2 and K-S tests for the datasets are summarized in **Table 7.6** and **Table 7.7**. From these tables it is quite evident that the most commonly employed lognormal distribution is acceptable as the theoretical distribution for all damage states of RC walls with rectangular and flanged cross-sections. In addition, the lognormal distribution can pass Lilliefors tests for all damage states of both types of RC walls, as shown in **Table 7.8**.

Table 7.5: Distribution Parameters for Lognormal, Weibull, Beta and Gamma Distributions

Walls with Rectangular Cross-sections								
Damage States	Lognormal		Weibull		Beta		Gamma	
	μ	σ	k	λ	α	β	k	λ
1	-1.789	0.419	2.846	0.198	2.674	3.911	5.186	0.034
2	-0.646	0.417	2.854	0.638	2.686	3.936	5.202	0.11
3	0.114	0.407	2.935	1.360	2.963	4.666	5.479	0.222
4	0.359	0.500	2.293	1.819	1.808	1.923	4.448	0.361
Walls with Flanged Cross-sections								
1	-2.253	0.526	1.882	0.137	0.400	3.081	7.997	0.014
2	-0.784	0.297	3.977	0.512	1.304	1.429	11.038	0.043
3	-0.224	0.271	4.231	0.892	1.347	0.278	13.328	0.062
4	0.124	0.294	3.786	1.275	1.158	0.177	11.854	0.100

Table 7.6: χ^2 test results for Lognormal, Weibull, Beta and Gamma Distributions

Walls with Rectangular cross-sections									
Damage States	$c_{I-\alpha,f}$	Lognormal		Weibull		Beta		Gamma	
		Eq.	Correct	Eq.	Correct	Eq.	Correct	Eq.	Correct
		7.8	CDF	7.8	CDF	7.8	CDF	7.8	CDF
1	5.992	2.25	True	4.25	True	5.63	True	3.63	True
2	5.992	5.38	True	4.75	True	6.00	False	3.13	True
3	5.992	0.88	True	4.75	True	5.63	True	1.38	True
4	5.992	1.50	True	1.88	True	9.20	False	1.63	True
Walls with Flanged cross-sections									
Damage States	$c_{I-\alpha,f}$	Lognormal		Weibull		Beta		Gamma	
		Eq.	Correct	Eq.	Correct	Eq.	Correct	Eq.	Correct
		7.8	CDF	7.8	CDF	7.8	CDF	7.8	CDF
1	5.992	2.91	True	5.09	True	60.18	False	0.18	True
2	5.992	0.18	True	0.18	True	16.55	False	0.18	True
3	5.992	1.25	True	3.75	True	46.25	False	1.25	True
4	5.992	0.40	True	0.40	True	47.20	False	0.40	True

Table 7.7: K-S tests Results for Lognormal, Weibull, Beta and Gamma Distributions

	Walls with Rectangular cross-sections				Walls with Flanged cross-sections			
	Damage State 1	Damage State 2	Damage State 3	Damage State 4	Damage State 1	Damage State 2	Damage State 3	Damage State 4
D_n^α	0.152	0.152	0.152	0.256	0.231	0.231	0.234	0.256
Lognormal Distribution								
D	0.100	0.088	0.063	0.120	0.182	0.091	0.063	0.080
p	0.798	0.907	0.997	0.990	0.602	0.999	1.000	1.000
Correct CDF	True	True	True	True	True	True	True	True
Weibull Distribution								
D	0.150	0.150	0.125	0.200	0.182	0.061	0.094	0.080
p	0.304	0.304	0.532	0.649	0.602	1.000	0.998	1.000
Correct CDF	True	True	True	True	True	True	True	True
Beta Distribution								
D	0.125	0.125	0.113	0.200	0.818	0.273	0.563	0.680
p	0.532	0.532	0.666	0.649	0.000	0.143	0.000	0.000
Correct CDF	True	True	True	True	False	False	False	False
Gamma Distribution								
D	0.100	0.100	0.075	0.160	0.091	0.061	0.094	0.080
p	0.798	0.798	0.973	0.877	0.999	1.000	0.998	1.000
Correct CDF	True	True	True	True	True	True	True	True

Table 7.8: Results of Lilliefors tests for Lognormal Distribution

Damage States	Walls with Rectangular cross-sections				Walls with Flanged cross-sections			
	Lognormal Distribution				Lognormal Distribution			
	D_n^α	D	p	Correct CDF	D_n^α	D	p	Correct CDF
1	0.152	0.100	0.798	True	0.154	0.097	0.500	True
2	0.152	0.088	0.907	True	0.154	0.110	0.376	True
3	0.152	0.063	0.997	True	0.157	0.112	0.376	True
4	0.256	0.120	0.990	True	0.180	0.132	0.301	True

Thus the lognormal distribution is finalized for assessing the theoretical fragility functions for RC wall specimens. The empirical and theoretical (using Lognormal Distribution) fragility functions for RC walls with rectangular and flanged cross-sections are plotted in **Figs. 7.3** and **7.4** respectively.

Fig. 7.3: Empirical and Theoretical (Lognormal) Fragility Curves for RC walls with Rectangular cross-sections

Fig. 7.4: Empirical and Theoretical (Lognormal) Fragility Curves for RC walls with Flanged cross-sections

7.6 Seismic Fragility Assessment of RC Structures

In order to evaluate the seismic fragility functions of three, six and nine storey Reinforced Concrete (RC) structural walls and bare frames, scaled ground motions representative of Singapore are utilized in this research. Non-seismically detailed beam-column joints and walls, typically used in gravity load designed frame and wall structures are selected for this purpose. The ground motion selection, structural characteristics and fragility assessments are illustrated in the following sections.

7.6.1 Ground Motion Selection

Since the seismic behavior of RC structural systems are sensitive to the characteristics of the earthquake excitation (e.g., intensity, duration and frequency content), selection of representative seismic actions for a region is of paramount importance. Singapore is a modern city with a population of about 3.7 million living in an area of about 600 km². Due to land shortage, more than 80% of the population in Singapore lives in high-rise residential buildings. Although Singapore is located in a low seismicity region, the country is exposed to long-distance earthquake originating from Sumatra. The long-period ground motions may pose certain threat to the high-rise buildings in Singapore which may have natural periods close to the predominant period of the ground motion.

Megawati *et al.* ([M7]) developed an attenuation relationship having earthquake magnitude and source-station distance, which is able to represent the complicated and time-consuming ground-motion simulations. The functional form adopted for the estimation of the horizontal ground-motion parameters follows the basic principles of wave propagation in elastic media as described below:

$$\ln(Y) = a_0 + a_1(M_w - 6) + a_2(M_w - 6)^2 + a_3 \ln(R) + (a_4 + a_5 M_w) + \varepsilon \ln(Y) \quad (7.11)$$

Here Y is the geometric mean of the horizontal peak ground acceleration, peak ground velocity or response spectral acceleration values at 5% damping ratio for

various natural periods. The unit for the acceleration values is cm/s^2 and that for velocity is cm/s . M_w is the moment magnitude and R is the distance from the station to the centre of the corresponding fault plane, in km. The quadratic term of $a_2(M_w - 6)^2$ is used to account for the fact that the corner period of earthquake source spectrum increases with the earthquake magnitude and the source area, and the rate of increase in ground-motion amplitude Y becomes slower for larger value of M_w . Therefore, the regression coefficient a_2 generally has negative values. Coefficient a_3 represents the geometrical attenuation rate, whereas a_4 and a_5 account for the inelastic attenuation. The term $\varepsilon \ln(Y)$ accounts for the variation in the ground motion parameter and has a mean value of 0.0 and a standard deviation value $\sigma \ln(Y)$ representing the standard deviation of the model due to the randomness in the source process. The regression coefficients $a_0 - a_5$ were obtained to best fit the simulated data using a least-squares procedure ([M7]). For a potential earthquake M_w of 9 in the Mentawai region in the Sumatran megathrust at 600 km distance from the source of seismic excitation to the centre of the corresponding fault plane (R), the response spectrum at 5% damping ratio is obtained on the basis of Eq. (7.11). The response spectral acceleration as shown in **Fig. 7.5**, is shown by a set of three lines where the central solid line represents the mean spectrum (μ) and the two enclosing dashed lines denote the mean $\pm$ one standard deviation spectra ($\mu \pm \sigma$).

30 scaled ground motion records are extracted from the Next Generation Attenuation (NGA) database ([P1]) for Singapore to evaluate the fragility functions for three, six and nine storey RC walls and frames without infills such that the squared residuals between the generated acceleration response spectrum for Singapore and the scaled ground motion records become minimized. The target acceleration response spectrum and the acceleration response spectra for the selected scaled ground motions are shown in **Fig. 7.6**. The details of the scaled ground motions are summarized in Appendix C.

Fig. 7.5 Pseudo Acceleration Response Spectra at 5% Damping Ratio in Singapore for a Potential Earthquake of M_w 9 in the Mentawai Region

Fig. 7.6 Target Acceleration Response Spectra along with the Acceleration Response Spectra for Selected Scaled Ground Motions at 5% Damping Ratio in Singapore

7.6.2 Structural Model

After selection of representative ground motions for Singapore, the structural walls and frame structures are modeled such that they can reflect the construction practice of the region. According to the present practice, the structural walls have less or no boundary confinement and the beam-column joints of the frame structure contains limited or no transverse reinforcement in the joint core as found in the existing literature ([P4], [L3]). The schematic diagrams of the adopted three storey structural wall and frame are presented in **Figs. 7.7** and **7.8** respectively.

Fig. 7.7 Structural Geometry of Reinforced Concrete Three Storey Generic Wall
([P4])

The RC wall has rectangular cross-sections with the length of wall (l_w) = 2650 mm, height of each storey of the wall (h_w) = 3600 mm and thickness of wall (t_w) = 200

mm. The concrete compressive strength (f'_c) and the average yield strength (f_y) of reinforcing steel are 30 MPa and 460 MPa respectively. Total vertical reinforcement ratio ρ_v and horizontal reinforcement ratio ρ_h are 2.0% and 0.20% respectively. No boundary reinforcement is provided in the generic wall. The axial load applied (N) is 20% of the resistance of the gross area for all three types of walls ($0.2f_cA_g$) and thus axial load ratio (n) is equal to 20%.

Fig. 7.8 Structural Geometry of Reinforced Concrete Three Storey Three Bay Generic Frame ([P4], [L3])

The columns of the RC frame have a uniform cross-section, 300 mm $\times$ 900 mm with the longitudinal and transverse reinforcement ratio being 2.54% and 0.13% respectively for six and nine storey RC frames. The columns of the three-storey RC frame has a uniform cross-section, 300 mm $\times$ 600 mm with the longitudinal and transverse reinforcement ratio being 2.54% and 0.13% respectively. The beams of

the frame also have a uniform cross-section $300 \text{ mm} \times 600 \text{ mm}$ with the longitudinal and transverse reinforcement ratio being 0.89% and 0.26% respectively. No joint core transverse reinforcement is provided in the beam-column joints of the test frame. The axial load applied (N) is 20% of the resistance of the gross area for all three types of frames ($0.2f_c A_g$) and thus axial load ratio (n) is equal to 20%. The structural characteristics, reinforcement ratio and axial loading history of the six and nine storey generic walls and frames are mostly kept unchanged. The structural walls and frames are modeled in ABAQUS ([A1]) on the basis of the macro model shown in **Figs. 5.1** and **5.4**. The model parameters are calculated on the basis of their structural characteristics in order to obtain the desired response.

7.6.3 Seismic Fragility Functions

In order to obtain the fragility curves, incremental dynamic analysis method ([V1]) is adopted in this research to determine the minimum intensity of each ground motion at which the structure exceeds each damage states. Thus non-linear dynamic analyses are performed for 30 scaled ground motions for the structural models in ABAQUS ([A1]) at increasing levels of ground motion intensity by using constant increments until the dynamic instability of the structure is reached. The modified Park-Ang damage model proposed in this research is utilized for component level damage quantification. The global damage index of the structural system is considered as the sum of the local damage indices of the structural components weighted by the total absorbed hysteretic energy for each structural component as shown in Eq. 2.32. Hence, the peak ground acceleration records for all four damage states for three types of structural walls and frames are compiled using method 2 of data combination as described in Chapter 6.

Lognormal distribution being the most widely used probability distribution for seismic fragility evaluation is utilized in this research. Thus, the conditional probability of exceeding a particular damage state ds_i , for a certain peak ground acceleration (PGA) is defined by the following relationship.

$$P[ds \geq ds_i / PGA] = \Phi \left\langle \frac{1}{\beta_{ds_i}} \ln \frac{PGA}{\overline{PGA, ds_i}} \right\rangle \quad (7.12)$$

Here $\overline{PGA, ds_i}$ is the median value of peak ground acceleration at which the structure reaches the threshold levels of each damage state ds_i and β_{ds_i} is the standard deviation of the natural logarithm of peak ground acceleration each damage state ds_i . Φ is the standard normal cumulative distribution function. The magnitudes of $\overline{PGA, ds_i}$ (in terms of g) and β_{ds_i} for the four damage states for three, six and nine storey walls and bare frames are summarized in **Table 7.9**.

Table 7.9: Estimated PGA based Fragility Curve Parameters for Lognormal Distribution

Structure Type	Damage State 1		Damage State 2		Damage State 3		Damage State 4	
	$\overline{PGA, ds_1}$ (g)	β_{ds_1}	$\overline{PGA, ds_2}$ (g)	β_{ds_2}	$\overline{PGA, ds_3}$ (g)	β_{ds_3}	$\overline{PGA, ds_4}$ (g)	β_{ds_4}
Three Storey Wall	0.037	1.045	0.071	1.198	0.133	0.936	0.214	1.092
Six Storey Wall	0.032	1.045	0.069	1.198	0.127	0.936	0.182	1.092
Nine Storey Wall	0.028	1.045	0.068	1.198	0.120	0.936	0.160	1.092
Three Storey Frame	0.024	1.045	0.071	1.198	0.133	0.936	0.150	1.092
Six Storey Frame	0.019	1.045	0.053	1.196	0.103	0.936	0.120	1.092
Nine Storey Frame	0.014	1.045	0.043	1.198	0.081	0.936	0.094	1.092

The PGA -based seismic fragility curves of the three, six and nine storey walls and frames are shown in **Figs. 7.9 - 7.14** respectively.

Fig. 7.9 Seismic Fragility Curves for Three Storey RC Wall for the four Damage States

Fig. 7.10 Seismic Fragility Curves for Six Storey RC Wall for the four Damage States

Fig. 7.11 Seismic Fragility Curves for Nine Storey RC Walls for the four Damage States

Fig. 7.12 Seismic Fragility Curves for Three Storey RC Frames for the four Damage States

Fig. 7.13 Seismic Fragility Curves for Six Storey RC Frames for the four Damage States

Fig. 7.14 Seismic Fragility Curves for Nine Storey RC Frames for the four Damage States

7.7 Summary

Development of fragility functions is a pertinent stage in seismic performance assessment of reinforced concrete (RC) structures. Fragility functions demonstrate the probability of exceeding different damage states in structures under seismic loading. Drift ratio is selected as the engineering damage parameter (EDP) i.e. is a scalar capable of defining the seismic demand of a structural element at any point of the loading history. An experimental database of RC interior and exterior beam-column joints and wall specimens with rectangular and flanged cross-sections under simulated seismic loading is compiled from literature to obtain the drift-based fragility curves of the structural components. On the basis of goodness-of-fit tests, lognormal distribution is chosen as the best fit with the empirical datasets for generation of fragility functions.

As a consequence of the successful correlation between the experimental and analytical results, three, six and nine storey walls and frames without infills are modeled in ABAQUS ([A1]) using the proposed hysteresis model utilizing the macro models described in Chapter 5. The structural characteristics of the generic wall and frame structures i.e. the structural geometry, reinforcement detailing; are defined in accordance with common practice in Singapore. The ground motion database is obtained on the basis of the pseudo-acceleration response spectrum for Singapore calculated from the attenuation relationships developed by Megawati *et al.* ([M7]). Non-linear dynamic analysis is conducted using 30 scaled ground motion data extracted from the Next Generation Attenuation (NGA) database. Thereafter, using the incremental dynamic analysis approach, the generic mid-rise structures are subjected to increasing levels of ground motion intensity until the dynamic instability of the structure is reached. Thus the threshold levels of ground motion intensities at which the structural system exceeds each damage state are ascertained to obtain the PGA-based seismic fragility functions. Lognormal distribution is chosen as the best fit for generation of fragility curves for three, six and nine storey structural walls and frame structures without infills.

CHAPTER 8

CONCLUSIONS AND FUTURE WORK

8.1 Conclusions

Analytical evaluation of fragility curves being the most common approach in seismic performance assessment of structures emphasizes the importance of realistic modeling of the structural components. Thus, a constitutive model capable of capturing strength and stiffness degradation along with pinching at all displacement levels is a basic requirement in modeling and design of earthquake-resistant structural components. An utmost effort is undertaken in this research to derive an analytical hysteresis model and a damage model for reinforced concrete (RC) structural components for evaluation of fragility curves of generic frames in Singapore when subjected to simulated earthquake loading. The brief summary on the research conducted and the substantive findings of this research are summarized hereunder.

This research illustrates the proposed modification of the Bouc-Wen-Baber-Noori hysteresis model to simulate the hysteretic behavior of RC beam-column joints and structural walls. The model derived from the equation of motion for a single degree of freedom system comprises various parameters to exhibit different structural response characteristics like strength and stiffness degradation, pinching, hardening and softening etc. The parameter stiffness ratio α specified to be of constant magnitude in the original model is amended to be displacement-based in the present research. The hysteresis model with modified α can perform better than the same with constant α with respect to the experimental results of RC structural components.

The differential equations involved in the mathematical model are solved by employing Livermore Solver for Ordinary Differential Equations (LSODE). When hysteretic force and displacement are of similar order of magnitude, the solving time can be significantly reduced. Thus, reviewing the existing literature containing

the experimental results of RC structural components, kilonewton (kN) and millimeter (mm) as units of load and displacement respectively are perceived to be preferable.

The structural hysteretic behavior is dependent on the individual parameter magnitudes and their interaction. Hence, Genetic Algorithm (GA) is utilized to systematically identify the parameter magnitudes. GA can perform satisfactorily with discontinuous objective functions and erroneous initial estimates of the parameters. However, in spite of being versatile and convenient in nature, GA requires significant computational time if improper intervals of parameters are prescribed in the subroutine. But with completion of each run user can gain quick insight about the problem at hand and can assist the program to enhance its performance.

Calibration of the hysteresis model with experimental results of the RC beam-column joint and structural wall specimens obtained from existing literature proves the effectiveness of the proposed approach. Comparison between the experimental and analytical hysteresis load-deformation plots show the exactness and precision of the applied methodology. Parameter sensitivity analysis is essential when dealing with the system identification techniques. A sensitive parameter when deviated from its sought-after magnitude will show reasonable error. Thus, by changing the magnitude of sensitive parameters, better correlation can be achieved. On the contrary, a less sensitive parameter, even when it is fluctuated from its estimated value, can produce a reasonable response as its contribution to the final response is relatively less. Therefore, providing narrower ranges for less sensitive parameters can increase simplicity in the procedure without affecting the quality of the results. On the basis of effective correlation between the experimental and model hysteresis loops for the wide range of RC structural components, it can be perceived that the analytical hysteresis force for any structural component can be obtained using the proposed approach when the displacement history and the model parameters are known.

Since the model parameters are estimated based on experimental results using GA, they may not be identified properly for any specimen when the experimental outcome is not available for that particular specimen. Therefore, in order to overcome this difficulty, approximate magnitudes of the analytical parameters are proposed in this research in terms of the physical parameters of the structural components. Based on the regression equations, the mathematical parameters associated with the analytical model can be recognized in terms of the structural characteristics of tested specimens. However, the main motive behind this correlation study is to make the model user-friendly by reducing the effort needed to run the Genetic Algorithm extensively. As soon as the user gains a quick impression about the approximate parameter sets for a specific structural component, its hysteresis response under a specific displacement history can be attained using any suitable solver. Thus, the hysteretic behavior of any structural component can be comprehended based on its structural features under a definite displacement history.

The proposed analytical model with the suggested mathematical parameters can be utilized to derive the hysteresis load-deformation plots for simple RC beam-column joints and structural walls. However, in order to determine the hysteretic behavior of a complex structural system containing multiple beam-column joints and structural walls and to perform structural analysis, the proposed approach may be tedious to carry out. For this purpose, a simulation platform is required where the proposed model can be incorporated adequately to determine hysteretic behavior of the complex structural wall system using similar approach. Hence, the proposed analytical model is implemented in ABAQUS ([A1]) in the form of a macro element to further enhance its user-friendliness. The correlation between the simulation and experimental responses of RC beam-column joints and structural walls justifies the accuracy and efficiency of the method.

Quantitative damage evaluation plays a pivotal role in seismic performance assessment of structures. To date, the Park-Ang damage model is one of the most widely used damage models among the researchers. However, a significant problem in this model is that the damage index magnitude can exceed unity when the

structural failure is assumed to occur. In order to resolve this deficiency, a modified Park-Ang Damage model is proposed in this research to quantify and classify the damage states of RC beam-column joints and structural walls in terms of damage index. Drift ratio is selected as the engineering damage parameter (EDP) i.e. is a scalar capable of defining the seismic demand of a structural element at any point of the loading history. An experimental database of rectangular and flanged RC wall specimens and interior and exterior beam-column joints under simulated seismic loading is compiled from the literature to comprehend the methods of repair associated with each damage state for the structural components. Thereafter, the EDP magnitudes related to the lowest damage states for each specimen associated with a specific method of repair is adopted to obtain the EDP versus damage states data for RC beam-column joints and structural walls.

Development of fragility functions is a pertinent stage in seismic performance assessment of RC structures. Fragility functions demonstrate the probability of exceeding different damage states in structures under seismic loading. Drift ratio is selected as the engineering demand parameter (EDP) i.e. a scalar capable of defining the seismic demand of a structural element at any point of the loading history. An experimental database of RC wall specimens with rectangular and flanged cross-sections and interior and exterior beam-column joints under simulated seismic loading is compiled from the literature to obtain the EDP versus damage states data for assessment of the fragility functions. On the basis of goodness-of-fit tests, lognormal distribution is chosen as the best fit with the empirical datasets for generation of fragility functions. Since performance of the damage model and accuracy of fragility evaluation for structural walls when subjected to ground motions are strongly sensitive to the hysteretic behavior of RC structural components, the proposed hysteresis model is implemented in the form of a macro element in ABAQUS ([A1]). As a consequence of the successful correlation between the experimental and analytical results, three, six and nine storey structural walls and frame structures without infills are modeled using the proposed approach and non-linear dynamic analysis is performed using 30 scaled ground motion records for Singapore. Thereafter, incremental dynamic analysis approach is incorporated in this research and the structural components are subjected to

increasing levels of ground motion intensity by using constant increments in intensity until dynamic instability of the structure is reached to determine the minimum intensity of each ground motion at which the structural system exceeds each damage state and to obtain their seismic fragility curves on the basis of peak ground accelerations.

8.2 Future Work

The Present research is undertaken to derive an analytical hysteresis model and a damage model for RC beam-column joints and walls for evaluation of fragility curves of generic frame structures in Singapore when subjected to simulated earthquake loading. However, there are some areas related to this research which cannot be covered due to time constraints. Hence, future research can be carried out to enlighten the following subjects.

The proposed hysteresis model is developed to represent the behavior of RC beam-column joints and walls. RC beam-column joints with transverse beams or slabs are not within the scope of the research. Thus, hysteresis models for RC beam-column joints with transverse beams or slabs, RC columns, slab-column connections, coupling beams can be an interesting area for future research.

In this research the structural performance assessment under seismic loading is limited to RC frames without infills. Further research can be conducted on evaluation of seismic performance of RC frames with infill and RC wall-frame structures.

Primarily low to mid-rise RC bare frames are the subject of seismic fragility assessment in this research. Future research can be directed to seismic performance assessment of high rise RC buildings, industrial buildings, bridges, storage vessels etc.

Moreover, the effect of soil-structure interaction is not taken into account in the present research. Consideration of soil-structure interaction in seismic vulnerability assessment of RC structures can be an exciting area of future research.

The effect of epistemic and aleatoric uncertainties resulting from structural modeling, ground motion data, material properties or structural analysis can be taken into account in future for seismic performance assessment.

The available experimental investigations conducted by past reviewers on quasi-static cyclic loading tests of lightly reinforced concrete structural components are quite limited. More experimental work in this field can be conducted to increase the size of the database and enable more realistic analytical modeling.

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Appendix A: Summary of RC Beam-Column Joint Specimens from Literature

A.1: Synopsis of RC Interior Beam-Column Joint Specimens

Researchers	Specimen ID	h_b (mm)	h_c (mm)	b_j (mm)	f_c (MPa)	f_y (MPa)	ρ_j (%)	ρ_b (%)	ρ_c (%)	n (%)
Hakuto <i>et al.</i> [H1]	O1	500	300	460	41	325	0	1.81	1.97	0
Liu <i>et al.</i> [L9]	Unit 1	500	300	460	43.8	320	0	1.81	1.97	0
	Unit 2	500	300	460	48.9	320	0	1.81	1.97	12
Lehman <i>et al.</i> ([A4], [W1])	PEER14	500	450	400	31.8	420	0	1.14	1.45	11
	PEER22	500	450	400	38.4	517	0	1.88	2.78	9
	CD1514	500	450	400	29.8	420	0	1.14	1.45	12
	CD3014	500	450	400	42.5	420	0	1.14	1.45	8
	CD3022	500	450	400	38.1	476	0	1.88	2.78	9
	PADH14	500	450	400	42.9	420	0	1.14	1.45	8
	PADH22	500	450	400	36.3	517	0	1.88	2.78	10
Pessiki <i>et al.</i> [P12]	P2	400	600	400	34.5	456	0	1.79	2.0	27
	P4	400	600	400	27.6	518	0	1.79	1.9	34
	P5	400	600	400	27.6	427	0.23	1.79	1.9	34
	P7	400	600	400	20.7	460	0	1.20	2.0	46
	P8	400	600	400	20.7	460	0	0.99	2.0	46
	P9	400	600	400	27.6	461	0	1.20	2.0	10

Researchers	Specimen ID	h_b (mm)	h_c (mm)	b_j (mm)	f_c (MPa)	f_y (MPa)	ρ_j (%)	ρ_b (%)	ρ_c (%)	n (%)
Joh <i>et al.</i> ([J1], [J2])	JXO-B1	350	300	300	21.3	371	0.27	1.52	1.18	17
	JXO-B2	350	300	300	20.8	371	0.27	0.81	1.18	17
	JXO-B8	350	300	300	25.6	404	0.27	1.14	2.07	15

A.2: Synopsis of RC Exterior Beam-Column Joint Specimens

Researchers	Specimen ID	h_b (mm)	h_c (mm)	b_j (mm)	f_c (MPa)	f_y (MPa)	ρ_j (%)	ρ_b (%)	ρ_c (%)	n (%)
Hakuto <i>et al.</i> [H1]	O6	500	460	460	34	353	0	1.5	0.85	0
	O7	500	300	460	31	353	0	1.5	0.85	0
Liu <i>et al.</i> [L9]	EJ1	500	300	460	33.7	320	0	1.5	0.85	0
	EJ2	500	300	460	29.2	320	0	1.5	0.85	0
	EJ3	500	300	460	34	320	0	1.5	0.85	25
	EJ4	500	300	460	36.5	320	0	1.5	0.85	23
Pantelides <i>et al.</i> [P5]	Unit 1	400	400	400	33	469	0	3.12	2.47	10
	Unit 2	400	400	400	33	469	0	3.12	2.47	25
	Unit 3	400	400	400	34	469	0	3.12	2.47	10
	Unit 4	400	400	400	34	469	0	3.12	2.47	25
	Unit 5	400	400	400	31.7	469	0	3.12	2.47	10
	Unit 6	400	400	400	31.7	469	0	3.12	2.47	25
Clyde <i>et al.</i> [C11]	Test#2	400	450	300	46.2	462	0	4.17	2.22	10
	Test#4	400	450	300	41	462	0	4.17	2.22	25
	Test#5	400	450	300	37	462	0	4.17	2.22	25
	Test#6	400	450	300	40.1	462	0	4.17	2.22	10

Appendix B: Summary of RC Wall Specimens from Literature

B.1: Synopsis of RC Wall Specimens with Rectangular Cross-sections

Researchers	Specimen ID	l_w (mm)	h_w (mm)	t_w (mm)	f_c (MPa)	f_y (MPa)	ρ_h (%)	ρ_v (%)	ρ_{be} (%)	n (%)
Hirosawa <i>et al.</i> [H7]	72	1700	1600	160	17.3	410	0.26	0.51	5.68	11.4
	73	1700	1600	160	20.8	410	0.26	0.51	5.68	9.4
	165	800	800	60	23.5	454	0.23	0.22	5.29	0
	170	1200	800	60	24.5	454	0.23	0.24	3.53	0
	175	1600	800	60	25.5	454	0.23	0.22	2.65	0
Greifenhagen <i>et al.</i> [G6]	M1	1000	610	100	50.69	504	0.37	0.34	0.00	2.2
	M2	1000	610	100	50.98	504	0	0.34	0.00	2.2
	M3	900	610	80	20	624	0.26	0.39	0.00	9.5
	M4	900	610	80	24.4	624	0.26	0.39	0.00	5
Hidalgo <i>et al.</i> [H4]	1	1000	2000	120	19.4	392	0.131	0.251	8.50	0
	2	1000	2000	120	19.6	402	0.246	0.251	8.50	0
	4	1000	2000	120	19.5	402	0.381	0.251	10.58	0
	6	1300	1800	120	17.6	314	0.131	0.259	6.54	0
	7	1300	1800	120	18.1	471	0.246	0.125	6.54	0
	8	1300	1800	120	15.7	471	0.246	0.259	6.54	0
	9	1300	1800	100	17.6	366	0.255	0.255	7.00	0
	10	1300	1800	80	16.4	367	0.25	0.25	7.31	0
	11	1400	1400	100	16.3	362	0.127	0.255	5.71	0

Researchers	Specimen ID	l_w (mm)	h_w (mm)	t_w (mm)	f_c (MPa)	f_y (MPa)	ρ_h (%)	ρ_v (%)	ρ_{be} (%)	n (%)
Hidalgo <i>et al.</i> [H4]	12	1400	1400	100	17	366	0.255	0.127	5.71	0
	13	1400	1400	100	18.1	370	0.255	0.255	5.71	0
	14	1700	1200	80	17.1	366	0.125	0.25	4.41	0
	15	1700	1200	80	19	366	0.25	0.125	4.41	0
	16	1700	1200	80	18.8	366	0.25	0.25	4.41	0
	21	1300	1800	100	24.2	258	0	0	4.62	0
	22	1300	1800	100	17.2	222	0	0	4.62	0
	23	1300	1800	100	24.2	333	0.25	0	8.54	0
	24	1300	1800	100	23.9	323	0	0.25	4.62	0
	25	1400	1400	100	23.9	352	0	0	4.29	0
	26	1400	1400	100	17.7	262	0	0	4.29	0
	28	1400	1400	100	23.3	258	0	0.25	4.29	0
	29	1500	1050	80	23.2	400	0	0	5.00	0
	30	1500	1050	80	17.9	356	0	0	5.00	0
	32	1500	1050	80	23.3	344	0	0.25	5.00	0
Massone <i>et al.</i> [M3]	wp111-10	1372	1219	152	31.4	424	0.27	0.25	0.87	10
	wp1105-8	1372	1219	152	31.9	424	0.27	0.25	0.87	5
	wp1105-7	1372	1219	152	32	424	0.27	0.25	0.87	5
	wp110-5	1372	1219	152	29.9	424	0.27	0.25	0.87	0
	wp110-6	1372	1219	152	31	424	0.27	0.25	0.87	0
Kuang <i>et al.</i> [K8]	U1.0	1200	1200	100	30.4	520	1.05	0.92	0	0
	U1.5	1200	1800	100	34.9	520	1.05	0.92	0	0

Researchers	Specimen ID	l_w (mm)	h_w (mm)	t_w (mm)	f_c (MPa)	f_y (MPa)	ρ_h (%)	ρ_v (%)	ρ_{be} (%)	n (%)
Kuang <i>et al.</i> [K8]	C1.0	1200	1200	100	35.2	520	1.05	1.05	0	0
	C1.5	1200	1800	100	34.2	520	1.05	1.05	0	0
	U1.0-BC	1200	1200	100	31.2	520	1.05	0.92	0	0
	U1.5-BC	1200	1800	100	33.8	520	1.05	0.92	0	0

B.2: Synopsis of RC Wall Specimens with Flanged Cross-sections

Researchers	Specimen ID	l_w (mm)	h_w (mm)	t_w (mm)	f_c (MPa)	f_y (MPa)	ρ_h (%)	ρ_v (%)	ρ_{be} (%)	n (%)
Maier <i>et al.</i> [M1]	S5	1180	1200	100	37.3	574	1.01	1.13	1.13	6.3
	S7	1180	1200	100	34.1	555	1.01	1.13	1.13	27.3
Sato <i>et al.</i> [S6]	36M8-30	2150	1400	150	39.3	296	1.16	1.16	1.16	5.1
	36M8-50	2150	1400	150	37.5	528	0.72	0.72	0.72	5.3
Li <i>et al.</i> [L5]	LW1	2000	2000	120	40.2	382	0.5	0.5	1.4	0
	LW2	2000	2000	120	41.6	382	0.5	0.5	1.4	5
	LW3	2000	2000	120	34.8	382	0.5	0.5	1.4	5
	LW4	2000	2000	120	39.8	382	0.5	0.5	1.4	0
	LW5	2000	2000	120	35.6	382	0.5	0.5	1.4	5
	MW1	2000	3000	120	41.2	382	0.5	0.5	1.4	5
	MW2	2000	3000	120	39.6	382	0.5	0.5	1.4	5
	MW3	2000	3000	120	40.3	382	0.5	0.5	1.4	5

Appendix C: Scaled Ground Motion Database

Ground Motion ID	NGA#	Scale Factor	Event	Station	Magnitude	V_{s30} (m/s)	Mean Squared Error
GM1	2459	0.1168	Chi-Chi- Taiwan-03, 1999	CHY026	6.2	226	0.0275
GM2	1779	0.2823	Hector Mine, 1999	El Centro Array #10	7.13	202.8	0.0547
GM3	2568	0.3347	Chi-Chi- Taiwan-03, 1999	KAU010	6.2	201	0.0711
GM4	1803	0.2446	Hector Mine, 1999	LB - City Hall	7.13	381.2	0.0734
GM5	2503	0.1854	Chi-Chi- Taiwan-03, 1999	CHY094	6.2	221.9	0.0762
GM6	1195	0.0627	Chi-Chi- Taiwan, 1999	CHY026	7.62	226	0.0765
GM7	2512	0.3009	Chi-Chi- Taiwan-03, 1999	CHY115	6.2	228.6	0.0792
GM8	2511	0.2693	Chi-Chi- Taiwan-03, 1999	CHY114	6.2	228.6	0.0921
GM9	1538	0.0591	Chi-Chi- Taiwan, 1999	TCU112	7.62	201	0.094
GM10	1553	0.0523	Chi-Chi- Taiwan, 1999	TCU141	7.62	209.2	0.0982
GM11	1610	0.2232	Duzce- Turkey, 1999	Kutahya	7.14	274.5	0.0991
GM12	2497	0.1715	Chi-Chi- Taiwan-03, 1999	CHY082	6.2	193.7	0.1052
GM13	1542	0.0366	Chi-Chi- Taiwan, 1999	TCU117	7.62	198.6	0.1143
GM14	1398	0.1371	Chi-Chi- Taiwan, 1999	KAU087	7.62	276.1	0.1144
GM15	1540	0.048	Chi-Chi- Taiwan, 1999	TCU115	7.62	215.3	0.1153
GM16	1238	0.0459	Chi-Chi- Taiwan, 1999	CHY092	7.62	253.7	0.1164
GM17	2741	0.256	Chi-Chi- Taiwan-04, 1999	CHY082	6.2	193.7	0.1175
GM18	2505	0.2103	Chi-Chi- Taiwan-03, 1999	CHY099	6.2	228.8	0.1217
GM19	2701	0.1866	Chi-Chi- Taiwan-04, 1999	CHY026	6.2	226	0.1233
GM20	2583	0.568	Chi-Chi- Taiwan-03, 1999	KAU088	6.2	201	0.1293

Ground Motion ID	NGA#	Scale Factor	Event	Station	Magnitude	V_{s30} (m/s)	Mean Squared Error
GM21	2471	0.2098	Chi-Chi- Taiwan-03, 1999	CHY044	6.2	194.5	0.1324
GM22	1790	0.2316	Hector Mine, 1999	Huntington Beach - Lake St	7.13	370.8	0.134
GM23	1356	0.1323	Chi-Chi- Taiwan, 1999	KAU010	7.62	201	0.1457
GM24	2088	0.4583	Nenana Mountain- Alaska, 2002	Anchorage International Airport	6.7	274.5	0.1479
GM25	2492	0.1799	Chi-Chi- Taiwan-03, 1999	CHY076	6.2	169.8	0.1482
GM26	2064	0.5433	Nenana Mountain- Alaska, 2002	Anchorage - K2-04	6.7	279.4	0.1483
GM27	2463	0.126	Chi-Chi- Taiwan-03, 1999	CHY032	6.2	192.7	0.1531
GM28	2486	0.2553	Chi-Chi- Taiwan-03, 1999	CHY066	6.2	212	0.1565
GM29	1180	0.0475	Chi-Chi- Taiwan, 1999	CHY002	7.62	235.1	0.1584
GM30	1539	0.0821	Chi-Chi- Taiwan, 1999	TCU113	7.62	230.3	0.1599