Numerical simulation of slider air bearings in head-disk interface system based on a mesh-free method

Li, Xu Hong

2006

Li, X. H. (2006). Numerical simulation of slider air bearings in head-disk interface system based on a mesh-free method. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/6078>

<https://doi.org/10.32657/10356/6078>

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Numerical Simulation of Slider Air Bearings in Head-Disk Interface System Based on A Mesh-free Method

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A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

2006

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Acknowledgments

I would like to express my deepest gratitude and thank to my supervisors A/P Du Hejun for his invaluable guidance, encouragement and patience throughout this study.

I am thankful for the opportunity provided by my co-supervisor Liu Bo, the manager of SMI Division in the Data Storage Institute (DSI).

I would also like to thank A/P Shu Chang in NUS and Prof. Qiu jinhao in Institute of Fluid Science, Tohoku University for having had the opportunity to share their values on education, knowledge. Appreciation also goes to Dr. Li Jianhua (Hitachi Japan), Dr. Hua Wei (DSI), and Dr. Ding Hang (NUS) for their great help to provide with an immense source of inspiration at an education level. Thanks also go to the friends and staffs of the Centre for mechanics of Micro-systems (CMMS) and Mechanics of Machine Lab (MOM) in NTU for their help and excellent service.

My gratitude also extends to my husband and my families for their support and encouragement in all the way.

Finally, I wish to thank the Nanyang Technological University (NTU) for providing me with the research scholarship, which makes this study possible.

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Summary

The magnetic disk drive has achieved extraordinary growth in areal density of data storage in the past few years. One of the key factors for this explosive growth in storage capacity is the substantially reduced and tiny spacing between the read/write head and recording media and the resulting more complex air bearing design. Research on alternative designs of air bearing has been constantly conducted with the objective to achieve consistently low flying height with high reliability and minimum sensitivities to altitude and contamination. To design and simulate such air bearing surfaces (ABS) with complex rail shapes and highly recessed regions, the generation of suitable meshes in current traditional numerical methods requires considerable time and manual effort in ABS modeling. Hence, numerical techniques that do not require meshes are attractive. In this thesis, a mesh-free methodology is developed for modeling and simulating air bearing sliders in hard disk drives. The developed CMMS Air Bearing Design Program implements mesh-free method to simulate complex air bearing sliders with shaped rails, multiple etch depths and highly recessed regions.

Recent decades have witnessed a research boom on mesh-free methods. It is well-known that mesh-free methods possess a few clear advantages over the methods requiring meshes, such as the requirement of node generation instead of mesh generation, and easy deletion/insertion of new nodes. Thus, it is not necessary to maintain detailed structural information about the computational meshes, comprising all nodal and element-based connectivity and hierarchical data. So far, there is no

research reported in the open literatures exploring the mesh-free method for solving ABS problems in hard disk drives. As a result, there are considerable interest and potential benefit in applying the mesh-free method to ABS simulation and design. Here introduced is the Least Square Finite Difference (LSFD) method, one of the mesh-free methods.

In this work, Least Square Finite Difference (LSFD) method was implemented to solve positive pressure sliders and negative pressure sliders based on the generalized lubrication theory. Good agreement is found between the numerical results obtained by the LSFD method and those reported in the literature, which clearly demonstrates the valid usage of this method in modern hard disk drives. To model air bearing sliders of complex geometry and shapes, the adaptive node generation scheme produced using ANSYS software in consideration of geometry and pressure profiles as well as the wall profile technique dealing with clearance discontinuities were applied. It was found that adding wall profile is a useful technique to deal with clearance discontinuities.

Study was also carried out to improve the computational efficiency of the LSFD method in solving the particular air bearing slider problems. It was found that the choice of various weight functions has significant influence on the efficiency of the LSFD method. A few original techniques to search supporting points in the LSFD method were also proposed and studied to improve the computational efficiency and reduce computational time. The effect of extrapolation factor on convergence speed is surveyed. In order to acquire faster convergence speed, two approximation methods for linearization of the equations are presented and compared.

Summary

In conclusion, the LSFD mesh-free method was found to be an effective method to efficiently and accurately solve the air bearing slider problems in head-disk interface system.

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List of Abbreviations and Symbols

Abbreviation	Meaning
AAB:	Advanced Air Bearing
ABS:	Air-bearing Surface
ADI:	Alternating Direction Implicit
AFC:	Antiferromagnetically Coupled
ASS:	Advanced Step Slider
DE:	Diffuse Element
DEM:	Diffuse Element Method
DFHT:	Dynamic Flying Height Tester
DSMC:	Direct Simulation Monte Carlo
EFGM:	Element-free Galerkin Method
FDM:	Finite Difference Method
FEM:	Finite Element Method
FIFD:	Factored Implicit Finite Difference
FTIR:	Frustrated Total Internal Reflection
FVM:	Finite Volume Method
GFD:	General Finite Difference
HDD:	Hard Disk Drive
HDI:	Head-Disk Interface
LSFD:	Least-Square Finite Difference
MLPG:	Meshless Local Petrov-Galkerkin Method
MQM:	Multiquadric Methods

List of Abbreviations and Symbols

MR:	Magnetoresistive
MRBFs:	Multiquadric Radial Basis Functions
MTBF:	Mean Time Before Failure
RKPM:	Reproducing Kernel Particle Methods
SNR:	Signal-to-Noise Ratio
SOR:	Successive-Over Relaxation
SPH:	Smoothed Particle Hydrodynamics
TPC:	Traverse Pressure Contour
TIRM:	Total Internal Reflection Microscopy

List of Abbreviations and Symbols**Symbols**

D	Inverse Knudsen number
f	Surface accommodation coefficient
h	Flying height
h_r	Wall profile height
h_0	Flying height of a reference point on the slider which assumed to have zero recess at the trailing edge center of the slider
H	Non-dimensional distance between air bearing surface and disk surface, normalized by h_0
HT	Taper height
K_n	Knudsen number
L	Reference length
P	Non-dimensional pressure, normalized by p_a
p_a	Ambient pressure
Q	Flow factor
U	Disk velocity component in the X direction
V	Disk velocity component in the Y direction
W_0	Normalized bearing load
X	Coordinate in slider length direction
$\bar{X}$	Coordinate of normalized load center along X-direction
X_0	Coordinate of normalized suspension load center along X-direction
XT	Taper length

List of Abbreviations and Symbols

Y	Coordinate in slider width direction
$\bar{Y}$	Coordinate of normalized load center along Y-direction
Y_0	Coordinate of normalized suspension load center along Y-direction
α	Pitch angle
β	Roll angle
γ	Taper angle
δ	Skew angle
λ_0	Molecular mean free path at atmospheric pressure
μ	Dynamic viscosity of the gas
Λ_x	Bearing number in the X direction $6\mu UL / P_a h_0^2$
Λ_y	Bearing number in the Y direction $6\mu VL / P_a h_0^2$
ω	Extrapolation factor

CHAPTER 1 Introduction

1.1 Background

Since IBM RAMAC 305's introduction in 1956, the hard disk storage has been by far the important member of the storage hierarchy in modern computers, as evidenced by the fraction of system cost devoted to that function. Magnetic hard disk provides reliable, high speed, high density and inexpensive storage media. In the late 1980s, disk drive specifications indicated a MTBF (mean time before failure) of 200 Khours while today's drives are specified at 1.2 million hours or longer. Areal density has further accelerated to an incredible 100% per year since 1997. This acceleration is the result of the introduction of MR (magnetoresistive) read heads in 1991, GMR (giant magnetoresistive) read heads in 1997 and AFC (antiferromagnetically coupled) media in 2001. An areal density of 100Gb/in² has already been achieved in today's hard disk drives (HDD). The hard disk drive industry's next goal is to increase the areal density to 1Tb/in². Recent trends in HDD price per megabyte show an even steeper reduction, 50% per year, making magnetic disk drives even more attractive as a storage technology. At this rate of improvement, the hard disk drive has become the most reliable, competitive potential means of storing data, and further improvements are expected to extend the life of magnetic hard disk drives well into the brilliant future.

Reliable operation of a modern magnetic disk drive depends critically on maintaining a controlled spacing between the magnetic recording head and the recording medium. Achieving higher areal densities requires the read/write heads to fly progressively closer to the disk surface, based on physical spacing laws. In a modern hard disk drive,

magnetic bits are written and stored on stacks of disks, which usually have a size from 1.0 inches to 3.5 inches in diameter and are mounted on a rotating spindle. The spindle usually rotates at a speed from a few thousands to tens of thousands revolutions per minute. The slider is mounted on the tip of a suspension arm, and the suspension arm is controlled by a position motor to locate the magnetic bits on the disk. The recording transducers are positioned at the end of a small ceramic 'slider' with a carefully designed air bearings that is self-pressurized by the air flow of the rapidly spinning disk (Fig. 1.1).

Fig. 1.1 The IBM Travelstar 25GB hard disk drive

When the disk spins at a high speed, the airflow between the slider and the disk creates an air bearing, which lifts the slider from the disk surface by a small distance, called the flying height. Careful design of air bearing and the head-disk interface

(HDI) is needed to reliably provide a physical spacing between the head's magnetic sensors and the disk down to less than 10nm in today's drives. This is equivalent to a stack of just about 30 average atoms, and all that at relative speeds up to 50 meters per second (over 100 miles/hour)!

Fig. 1.2 Travelstar 7K60 air bearing surface

The slider air bearing surface is designed with complex shapes to achieve the overall desired ABS performance characteristics. The femto ABS design for Travelstar 7K60 has three surfaces (Fig. 1.2). The shallow etch surface forms a step height inlet allowing airflow to pressurize the ABS surface. This results in the ABS surface generating positive pressure that lifts the slider away from the disk surface. The deep etch surface forms a negative pressure region that pulls the slider closer to the disk surface. These positive and negative pressure regions are designed to balance the downward force of the suspension gram load in a way that lifts the magnetic element to the desired fly height. The above extremely narrow spacing goal is achieved by

carefully designing the air bearing surface of the slider, which generally has a very complicated shape (Bogy, 1999 and Thomas, 1999).

1.2 Objective

Numerical simulation has been an indispensable tool for air bearing design. An efficient numerical program enables designers to evaluate many different designs without producing and testing each of them, thus shortening the design cycle time and saving cost. Much research has been conducted on air bearing simulation and design, however, due to geometrical complexity of air bearing surface, some methods of numerical simulation may be found unsuitable while some require extensive meshing capability so as to obtain accurate simulation results of air-bearing surface (ABS).

Mesh-free numerical methods are an efficient tool to solve partial differential equations. It is well-known that the mesh-free method has a few clear advantages over the mesh-based method such as the requirement of node generation instead of mesh generation and easy deletion/insertion of new nodes. They have been used in structural analysis and fluid mechanics, but no one has attempted to solve air-bearing slider problems in HDD using mesh-free methods from the open literatures. As a result, there is considerable interest in applying the mesh-free method that is capable of predicting the pressure distribution of sliders with complex geometry shapes and designing novel sliders. Consequently, the mesh-free-based methods for simulation of all kinds of sliders present rich, relatively unexplored area for research in computational fluid mechanics.

The primary goal of this thesis is to develop an efficient and powerful methodology that is capable of handling simulation and design of air bearing sliders with complex geometry shapes.

1.3 Overview

Modeling and computing of the air bearing force accurately and efficiently becomes the key issue in the design of sliders. Developing a mesh-free method for the air bearing problem involves two steps. The first step is to formulate and implement the mesh-free method for air bearing problems and verify the method using some simple examples with established analytic solutions or numerical solutions. The second step is to apply the mesh-free method developed to tackle the generalized Reynolds equations for complex positive sliders and negative sliders problems. The next six chapters are devoted to the detailed account of the numerical technique applied to solve the Reynolds equation for various slider models.

Chapter 2 gives a review on development of slider air bearings and a comprehensive introduction to the lubrication models for the gas layer with different thickness. Numerical techniques used previously in simulating air bearing surface are investigated and compared. Mesh-free methods are generally introduced in this chapter.

Chapter 3 presents a detailed description of the principle and some improvements of the least-square finite difference (LSFD) method. One-dimensional and two-dimensional formulae are shown. The resulting simultaneous nonlinear algebraic

equations are solved iteratively by Successive-Over Relaxation (SOR) method. The impact of different simplifying methods from simultaneous nonlinear equations to linear equations on convergence time is investigated and compared. Some new searching methods to acquire supporting points for the reference node are explored. Among these searching methods, no matter how sharply uneven the nodes distribute in the computational domain, supporting points are chosen from inner layer to outer layer around the reference node. It will largely expand the application of this method to models with complex geometry shapes and high variable gradient. The influence of the weighting functions on the results obtained is also studied. The best choice of the weighting functions is determined. These improvements remove the limitation of the LSFD method previously published and prolong its applications to most existing slider designs.

Chapter 4 deals with the one-dimensional and two-dimensional Reynolds equation using the LSFD mesh-free method. For the one-dimensional problem, both theoretical analysis and numerical simulation are carried out to show that the results obtained by the proposed mesh-free method are accurate for different models. Some simple two-dimensional sliders are simulated to show that the same is true. Both regular and irregular node distribution are implemented to provide greater flexibility for dealing with all kinds of potential complicated air bearing surfaces in modern magnetic drives. Moreover, the inverse problem, which determines the equilibrium-flying attitude, is solved using the Quasi-Newton method.

Chapter 5 demonstrates the applicability of the present schemes to positive sliders and discusses the treatment of special node issues in positive slider air bearings simulation

for hard disk drives. The piece-wise progression method with variable node spacing scheme based on geometric and pressure gradients is implemented for nodes generation. It gives the user the control to put more nodes where needed and provides smooth transition in different regions. Therefore, the LSFD mesh-free method is applied to compute effectively the pressure distribution of positive pressure sliders with rail shapes, such as single rail of IBM3380 taper flat slider, Global IBM3380 taper flat slider, Tri-pad slider and TPC2b slider.

In chapter 6, several negative slider designs are simulated using the mesh-free method. Besides adding wall profile to eliminate geometrical discontinuity, a variety of methods in node generation have been devised to minimize numerical error and overcome the potential instability problem of the LSFD method in solving negative pressure sliders. The wall profile technique and suitable node generation method are applied and verified to accurately acquire the ambient pressure in the recessed region of positive pressure sliders, such as two-rail straight slider, 1212 taper flat slider and the IBM3380 taper flat slider and pressure profiles in negative pressure sliders, such as negative pressure slider model 1 and advanced step slider (ASS).

The final chapter, Chapter 7, summarizes the research work and main conclusions so far, as well as future work.

CHAPTER 2 Literature Review

2.1 Introduction

A hard disk drive is basically a very compact, electronically controlled, mechanical device. It includes a spinning disk stack and a set of read/write magnetic recording heads positioned swiftly and accurately over magnetic disk surfaces by an electromagnetic actuator. Today, different kinds of hard disk drive play significant roles in diverse occasions. 3.5 inch form factor desktop drives can store more than 250 Gbytes per drive using 95 mm diameter disks with 10,000 RPM; 2.5 inch form factor designed for mobile, notebook can store more than 80 Gbytes on 65 mm disks rotating at 5400 PRM and a new form factor, 1 inch form factor microdrive designed for digital cameras, personal communications devices can store 5 Gbyte. Form factor evolutions to smaller sizes has been implemented without loss of capacity and often improved with higher performance and lower power dissipation.

2.2 Development Trend and Characteristics of Sliders

In modern hard disk drive industry, one of the most important objectives is to enhance the areal density of hard disk drive. Since 1991, the increasing rate of areal density has reached 60% per year with introduction of magnetoresistive (MR) read heads, and since 1997 the rate has further accelerated to an incredible 100% per year with giant magnetoresistive (GMR) heads and anti-ferromagnetically-coupled (AFC) media in 2001. Up to now, the industry has currently produced disk with an areal density of 100Gbits/in² and has demonstrated up to 150Gits/in² in the laboratory. An areal

density of 1 Tbits/in² is underway in research. To date few indications shows that non-magnetic technologies have such competitive potential to replace magnetic hard disk drives.

The hard disk drive evolution over the past half century has been led by miniaturization of every component in the system (Tab. 2.1). That trend evolved from a large aluminum externally pressurized hydrostatic slider in 1957 with spacing of 20μm and continued with the introduction of the ‘femto’ slider, which represents a key advancement in magnetic recording technology. Reduced slider size is motivated by several considerations such as economics, higher areal density, access time and data rate physical space requirements and new applications other than computer data storage.

Table 2.1 Evolutionary process of sliders with smaller form factors

Slider Size	Mini 100%	Micro 70%	Positive Pressure nano 62%	Negative Pressure nano 50%	Pico 30%	Femto 20%
Intro year	1980	1986	1991	1994	1997	2003
Length(mm)	4.00	2.80	2.50	2.00	1.25	0.85
Width(mm)	3.20	2.24	1.70	1.60	1.00	0.70
Height(mm)	0.86	0.60	0.43	0.43	0.30	0.23
Slider Mass(mg)	55.0	16.2	7.8	5.9	1.6	0.6

Higher recording density and signal resolution can be achieved if the read/write element which is attached to the slider can be maintained at a closer spacing over the magnetic disk rotating at a high speed. However if the slider gets too close, it can damage the disk and the data. If the slider is too far from the disk, the magnetic coupling between the disk media and the read write head is too weak to record and retrieve the data effectively. It is therefore important to control both lower and uniform distribution of the flying heights across the disk. Several parameters like slider shape (e.g. crown, camber and twist), air bearing surface tolerances (e.g. etch depths and rail shapes), suspension parameters (e.g. static attitude and gram load) and HDD tolerances (e.g. disk and suspension stacking and relative orientations of these stacks) affect the slider flying heights. Here, the study focuses on air bearing surface tolerances to get the expected flying characteristics.

Fig. 2.1 A sketch of the hard disk drive assembly

The slider has three degrees of freedom (Fig. 2.1). It can move up and down and rotate around the point where the suspension is attached in the pitch and roll directions, while the skew angle at each radial position is considered to be generally

fixed. The distance between the read-write element and the disk surface where magnetic information is stored and other parameters like pitch, skew and roll angles have a profound influence on the read back signals and consequently the performance of the hard disk drives. The motion of the slider reaches equilibrium when there is a balance of forces and moments generated by the extremely thin squeezed air layer under the slider and the opposing force and moments exerted by a pre-load of the suspension, which links the slider and the actuator. Other forces such as the intermolecular force become increasingly influential to the force/moment equilibrium if the spacing approaches a molecular distance. These pressure regions are carefully designed to create a high stiffness air bearing in a way that lifts the magnetic element to the desired flying height.

Since the first conception and application of the disk recording head element in 1956, the classical Reynolds equation has been found to predict well the performance of the air bearing operating with film thickness on the order of 8-10 μm . The sliders studied were either plane flat sliders or convex sliders with very large slenderness ratio (width to length ratio) of about 1.3. As the slider underwent design changes in order to increase the recording density, the result was a reduction in the bearing clearance from 8 to 1 μm . The single pad slider geometry also became more complex with a taper-flat surface and a slenderness ratio of 2.0. For example, the first self-acting air bearing appeared in the IBM 1301 storage system made of stainless steel with flying height of 6.4 μm in 1962. The first taper flat air bearing appeared in the IBM2305 made of ferrite with flying height 1.3 μm and 1.2Kgs perload force in 1971. To further increase the recording density, the bearing clearance was reduced from 1 μm to

0.5 μm . The slider geometry was also changed to the double pad arrangement with taper-flat surfaces and a slenderness ratio of 0.1 (Hsia, 1983). Such as, the first rail type ABS design was used in IBM 3340 Winchester drive. These were fabricated machined ferrite sliders in batches with 0.46 μm clearance and 20 grams preload force.

For the conventional sliders, the region between the rails is highly recessed so that the pressure there is generally assumed to be ambient due to high recessed height. However, by changing the rail design and reducing the recessed depth, sub-ambient pressure can be generated at some regions. It's well known that sub-ambient pressure slider exhibits some better characteristics than conventional sliders do. The sub-ambient pressure sliders have a higher stiffness, constant flying height, debris elimination effect, and take off at a lower velocity. These also are desirable for load-unload applications because of their self-loading capability. High stiffness (HS) type and fast take off (FTO) type negative sliders were investigated (Yoneoka, *et al.*, 1987). A well-known constant flying height slider, Traverse Pressure Contour (TPC) slider is reported (White, 1991). The two asymmetric TPC sections at each rail are designed to add extra lift to the slider when the slider has high skew angles. The TPC designs give near constant flying height under a wide range of conditions but are very sensitive to manufacturing tolerance. A constant flying height sub-ambient pressure slider - Guppy slider was designed, which generates a sub-ambient pressure at the region between the rails by means of its rail shapes (Yoneoka, *et al.*, 1991). "Parasol" slider was designed, which provides a constant flying height profile, stable low flying height, and debris elimination (Matsumoto, *et al.*, 1994). Seagate Advanced Air Bearing

(AAB) design was presented, which shows to control fly profile with shaped rails and partially stepped edges (Hardie, *et al.*, 1994).

2.3 Lubrication Equations

Because of the extremely narrow spacing between the slider and the disk, the gas in the spacing (microscopic level) is extremely rarefied and the gas molecules at a solid surface no longer experience the same velocity as the surface, i.e., there is slipping. The convectional lubrication theory (macroscopic level) developed for gas bearings cannot be applied to the type of bearings since the fluid inside the gas film cannot be treated strictly as a continuum.

Fig. 2.2 The Knudsen number limits on the mathematical models

The micro-flow is typically classified into four flow regimes according to their Knudsen number. The Knudsen number is defined as the ratio of the molecular mean free path (λ) to the minimum channel height (h) and is written as: $K_n = \lambda/h$. When the Knudsen number is less than 10^{-3} , it can be considered as a continuum, the Navier-

Stokes equations are still applicable; When the Knudsen number is between 10^{-3} and 10, they can neither be considered as continuum flows nor free molecular flows. A sub-classification is given in this regime, with slip flow defined as $10^{-3} < K_n < 0.1$ and transition flow for $0.1 < K_n < 10$. When the Knudsen number is greater than 10, the flow is in the free molecular region and the appropriate governing equations are governed by the collisionless Boltzmann equation (Fig. 2.2) (Sun, 2000). For all current hard disk drives, the Knudsen number is approximately in the region of 0.4 to 6.4.

Many attempts have been made to develop the slip correction models since Maxwell discovered the 'velocity slip' effect near a moving wall. Burgdorfer (1959) introduced a first-order correction to the continuum non-slip boundary condition used in the Reynolds equation. Hsia and Domoto (1983) considered a second-order slip-flow boundary condition for the velocity, and derived a modified Reynolds equation. Furthermore, Mitsuya (1993) introduced 1.5-order slip-flow boundary conditions. The three slip-flow models mentioned above are accurate only for a large inverse Knudsen number. Gans (1985), who first derived a lubrication equation using a linearized Boltzmann equation which is similar to Burgdorfer's first-order modified Reynolds equation, claimed that this equation is valid for all inverse Knudsen numbers. Fukui and Kaneko (1988) started from a linearized Boltzmann equation similar to Gans's with different solution method. Consequently, they derived the generalized Reynolds equation including thermal creep flow. Their results showed that the Burgdorfer's first-order slip-flow condition over-estimates the load-capacities, while the second-order slip-flow condition under-estimates it. To overcome the time-consuming work on the integro-differential equation for Posieuille flow velocity profiles, Fukui and

Kaneko (1990) proposed a polynomial fitting procedure to explicitly express Poseuille flow rate as a function of an inverse Knudsen number using polynomials. Therefore the Boltzmann-Reynolds equation is most suitable. Hwang's adjustable slip model (1996) is based on modified high-order slip-flow velocity distribution with three adjustable coefficients, which are corrected according to the Boltzman model. It is proposed to overcome the complicated and time-consuming difficulties of solving the Linearized Boltzmann equation.

The foregoing correction models can be expressed by the following non-dimensional steady state Reynolds equation

$$\frac{\partial}{\partial X} \left(QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial Y} \left(QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right) = 0 \quad (2.1)$$

Where P, H are non-dimensional parameters: pressure, distance between air bearing surface and disk surface, normalized by p_a, h_0 , respectively; p_a is the ambient pressure, h_0 is the flying height of a reference point on the slider which assumed to have zero recess at the trailing edge center of the slider. X, Y are coordinates in slider length direction and width direction. L is the length of the slider. The expression $\Lambda_x = 6\mu UL / P_a h_0^2$ and $\Lambda_y = 6\mu VL / P_a h_0^2$ are the bearing numbers in the X and Y directions respectively, which represent the relative importance between the convection effect and the diffusion effect. μ is the dynamic viscosity of the gas, U and V are the disk velocity components in the X and Y direction respectively. Q is the flow factor, which shows the difference between different rarefaction and slip models of the equation. Different Q value reflects different slipping order as follows

$$Q = 1 \quad \text{No-slip model}$$

$$Q = 1 + 6K_n / PH \quad \text{First-order slip model}$$

$$Q = 1 + 6K_n / PH + 6(K_n / PH)^2 \quad \text{Second-order slip model}$$

$$Q = 1 + 6\alpha K_n / PH + (8/3) * (K_n / PH)^2 \quad \text{1.5-order slip model}$$

$$Q = f(K_n / PH) \quad \text{Boltzmann-Reynolds model}$$

$$\alpha = (2 - f) / f$$

$$K_n = \lambda_0 / H_0 \quad (2.2)$$

α : Numerical constant

f : Surface accommodation coefficient

λ_0 : Molecular mean free path at atmospheric pressure

2.4 Methods Applied to ABS Simulation and Design

The lubrication equation is nonlinear, and hence, analytical solutions exist only for very simple bearing conditions (e.g., infinitely wide and steady). As a result, Gas film lubrication problem should be solved by direct method or some numerical methods, especially for complex bearing geometries. Direct Simulation Monte Carlo (DSMC) method is an example of the direct method. There are several numerical methods available in the literature for the solution of the generalized lubrication equation. These methods can be classified into the finite difference method (FDM), finite element method (FEM) and finite volume method (FVM).

2.4.1 Direct Simulation Monte Carlo (DSMC) Method

Direct Simulation Monte Carlo (DSMC) approach, introduced by Bird in the early 1970s, uses a stochastic algorithm to evaluate collision probabilities and scattering distribution according with kinetic theory. It was rigorously proved that DSMC is equivalent to a Monte Carlo solution of an equation “close” to the Boltzmann equation.

Despite the many advantages in using DSMC method for computing microscopic flows, particles methods remain costly. Due to computational expense, DSMC simulations are, at present, not competitive with slip-corrected Reynolds equations solvers as interactive design tool for slider air bearing (Huang and Bogy, 1998). However, DSMC computations remain useful for testing and validating continuum hydrodynamic calculations at high Knudsen numbers (up to 31.25).

2.4.2 Finite Difference Method

The finite difference method has been successfully used to solve the Reynolds equation, which requires relatively small memory, and short calculation times. The grid is easy to generate compared with the finite element method. In general, the finite difference method gives satisfactory solutions for the calculation of air bearing pressure and slider flying height.

Kogure, *et al.* (1983) reported investigations on static and dynamic characteristics of the negative pressure slider for magnetic recording disks. Based on the modified Reynolds equation, static and dynamic pressure distributions are calculated by

divergence formulation and a difference method. In order to minimize numerical instabilities, large numbers of grid points are required in the neighborhood of the trailing edge and the reverse step location, where large changes in pressure gradient exist. The solution and then process first divides the lubrication area by arbitrary grids for the discontinuity line, integrates lubrication equation over the meshing area and using the Gauss theorem.

However, in the pressure distribution calculation for a complex geometry slider, the finite difference method generally loses its advantages. When the geometry is complicated, usually a body-fitted mesh system that can smoothly follow the boundaries is generated by either solving a partial differential equation or simply using algebraic interpolation. However, the construction of body-fitting meshes and transformation of governing equations are not only tedious and problem-dependent, but also introduce additional geometrical error into the scheme and degrade the accuracy of the solution. Thus, in most cases, only Cartesian rectangular mesh systems have been fully explored in FDM simulation.

According to previous investigations, in addition to having enough mesh in high pressure gradient regions, an accurately specified wall profile is also necessary for getting accurate numerical results, which requires the clustering of very fine meshes in the narrow recessed wall regions. When the rail boundary is curved or not aligned with the coordinate axes, and a Cartesian rectangular mesh system is used, it is extremely difficult to achieve the above goals without refining the mesh in other regions where a fine mesh is not needed. This normally causes the mesh number to

become much too large for efficient numerical simulation, since the CPU time needed to get converged results increases very rapidly with the total grid number.

Therefore, the other methods are logical choices for flying height prediction in the complex geometry sliders.

2.4.3 Finite Element Method

The finite element method has gained popularity over the past decades in structural analysis, solid mechanics and fluid mechanics. The finite element technique has become an effective analysis tool for solving lubrication problems, which is insensitive to complex bearing film thickness and irregular bearing geometries. The pioneering FE works of Reddi (1969), Reddi and Chu (1970), Booker and Huebner (1972), Rohde and Oh (1975), and many others shed light and hope to the lubrication computational arena which, at the time, was dominated heavily by finite difference procedures. Mitsuya (1979) used the FE technique for investigating the molecular mean free path effects in gas lubricated slider bearings and found that problems with air bearing numbers as high as 10^4 could be studied without much difficulty. However, due to the presence of high-pressure gradients encountered in high bearing number problems, FE analysts must utilize extremely refined meshes in these high gradient areas to prevent numerical oscillations from appearing in the solution, thus raising the computational cost significantly. Efforts to reduce grid points without affecting the stability of the solution led to the introduction of upwind schemes.

Garcia Suarez, *et al.* (1984) successfully solved gas bearing problems by implementing an upwind scheme together with a selective reduced integration procedure and a revision of boundary conditions. Although the implemented upwind scheme provides stable solutions and helps reduce a significant number of grid points, the resulting meshes still consist of a relatively large number of elements, and errors generated by the upwind scheme are difficult to assess.

Kubo (1988) adopted a method of solving the linearized Boltzmann equation directly by numerical iteration and developed a new finite element method for the rarefied gas lubrication equation.

Nguyen (1991) applied the p-type finite element method in solving the steady-state pressure under a Winchester-typed slider. The method used the different level shape function to enhance the computation accuracy. In this literature, only simple geometry sliders such as inclined plane slider, cylindrically crowned slider, and Winchester-typed slider are calculated.

Crone, *et al.* (1991) used finite element techniques in studying the effect of surface roughness on the flying height characteristics. Robert (1990) used finite element methods to determine the spacing profile that maximizes the stiffness in the normal, pitch and roll directions of a gas bearing. Yoneoka, *et al.* (1991) used finite element techniques in designing the Guppy slider.

Although the finite element methods offer a powerful tool to calculate the pressure distribution under arbitrary rail geometries, they are hampered by the large memory

requirements, much computational times and long model generation and re-generation times.

2.4.4 Finite Volume Method

The finite volume method is an integral method that can be implemented in the physical domain without the need for coordinate transformation. It allows unstructured meshes, which are much easier and more flexible to be generated. Due to their local conservative quality, the finite volume method is good at capturing sharp gradient, which are unavoidable in the air bearing simulation.

Based on finite volume technique, Cha, *et al.* (1995) simulated the steady-state flying conditions and dynamic response of sub-ambient pressure sliders with shaped rails. For the shaped rail sliders, a method of averaging the mass flow across the rail boundaries is implemented. Furthermore, the power-law scheme is implemented to calculate the mass flows. The resulting equation is solved using the alternating direction implicit (ADI) method.

Lu (1997) used a control volume formulation to discretize the lubrication equation. The hybrid scheme is used to treat the clearance discontinuities in arbitrarily shaped, multiple recessed level air bearings. An adaptive grid method based on the pressure gradient field is implemented which greatly facilitates grid generation. The implementation of a full approximation storage and full multi grid method dramatically improves the solver efficiency.

Wu (2000) adopted unstructured adaptive triangular mesh generation technique and vertex based on an upwind higher order finite volume scheme that suit slider air bearing simulation of hard disk drives.

Due to the unstructured nature of the mesh, additional information about connectivity between meshes must be stored, and a complicated data structure must be set up and used to locate the information needed. This indirect addressing results in increased computational time and memory.

2.5 Static Analysis of Flying Characteristics

In order to increase magnetic areal density, the head/disk spacing has been reduced tremendously. However, the reduction in the head/disk spacing increases the possibility of contact between the slider and disk, leading to an increasing chance for a catastrophic failure of the head-disk interface (HDI). Therefore, the static analysis of flying behavior of the head slider is important in evaluating the characteristics and reliability of a HDI under various operating conditions at the design stage.

The air bearing slider is supported by a suspension arm. The suspension provides stiffness in the vertical, roll, and pitch directions. The equilibrium position of the slider is determined by the suspension stiffness and the air bearing pressure distribution, where the pressure distribution is a function of the slider position. Therefore, the calculation of the slider flying height requires an iterative or simultaneous solution of the coupled equilibrium equation and Reynolds equation.

Computer programs for the static analysis of self-acting air bearings have been used for several decades in the research and development of air bearing sliders. As for the strategies to obtain the static flying characteristics of the slider, there have been several approaches.

White (1983) obtained static solutions by tracking the time-dependent second-order differential equation of slider motion asymptotically to steady state. Ruiz (1990) started with an initial attitude guessed and repeatedly updated it proportionally to the residuals of force and moments computed until the sums of force and moments acting on the slider vanish. The approach suffers from a lack of general applicability to sliders of any shape due to its empirical update scheme for the flying attitude.

Yamaura and Ono (1990) iteratively calculated slider flying height and attitude angles from time-independent Reynolds equation and three loading equations in translational, pitching and rolling directions by using the Newton-Raphson method, which converges quadratically. The scheme is not robust in the sense that divergence may occur if initial values of flying attitude parameters are not properly chosen. Crone, *et al.*, (1991) adopted an unconditionally stable and second order time accurate element-by-element scheme to study the slider transient response to disk roughness. Although the transient solution provides some useful information about the slider behavior, such an approach usually takes longer computational time than the steady state solution approach. As the steady state flying height is the primary concern, the transient solution becomes an unnecessary burden.

In addition to the asymptotic solution of the transient analysis, a few approaches to the steady state solution are available. Among them, the BFGS (named after Broyden, Fletcher, Goldfarb, Shanno) method and the Newton-Raphson method offer the most rapidly convergent processes.

Choi and Yoon (1994) implemented the BFGS to the flying attitude characteristics. This method is more efficient than the transient factored implicit finite difference (FIFD) approach and more generally applicable to the slider of any shape than an empirical approach. Peng (1995) applied the Newton-Raphson method to the equilibrium equation and the Galerkin formulation of the generalized Reynolds equation and a rapidly convergent computational scheme for the air bearing slider flying height attitude is achieved. The solution procedure starts with an initial guess of the equilibrium position, and the air bearing pressure. In every iteration, the increments are computed from finite element expressions, and the equilibrium position and pressure at the next iteration are updated. Convergence is achieved when the criteria are satisfied.

Cha (1995) used Quasi-Newton method to calculate the equilibrium flying attitude of Guppy slider. The Quasi-Newton method combines a globally convergent strategy with Newton's method, which has a fast local convergent property.

2.6 Optical Flying Height Testing Methodologies

When storage capacity in magnetic disk drives has largely increased, the accurate measurement of the spacing between slider and disk is very important. Although

magnetic disk storage is a relatively new technology, the principles of optical flying height testing were developed much earlier. Along the development in the HDDs, researches have been exploring different optical approaches to adapt to the decreasing in the flying height and all these approaches are based on the wave-motion nature of light. Many manufacturers are contemplating full production flying height testing for every slider as the flying height decreases, therefore, the repeatability, accuracy and reliability of traditional testing techniques has come under scrutiny. Since 1990's, great efforts have been put into improving the measurement sensitivity and accuracy. Many optical flying height testing methods have been reported in patents and papers. Generally, they will be classified into three families based on the angle of the incident light with respect to the normal. These three families are (Yei, 2005),

1. normal-incidence interferometry technique,
2. polarization interferometry technique, and
3. frustrated total internal reflection (FTIR) measurement technique.

In this section, the working principles, merits and demerits of the optical FH testing methods based on these three techniques will be explained respectively. The purpose of this review is to present a complete picture of the FH measurement technologies and the difficulties in the assurance of calibration accuracy, which leads to correct FH measurement.

2.6.1 Normal-incidence Interferometry Technique

The term normal-incidence interferometry technique is referred to the fact that the angle of incidence is zero. It is the case in which a wavefront is parallel to an interface and its ray path is perpendicular, or normal, to the interface as the wave impinges upon the interface as shown in Fig.2.3.

Fig. 2.3 A diagram illustrating the meaning of normal incidence, where the incident angle $\theta_i = 0^\circ$

There are some patents for FH testing which utilized this normal interferometry technique based on different calibration procedures. Some typical patents, which typically represent the evolution of the normal interferometry method used in FH testing will be introduced in this section.

2.6.1.1 Normal Interferometry with White -light Source

In this invention, broad spectrum light from a source such as a Xenon lamp directed substantially normal to the surface of the transparent disk, through the disk and onto

the slider, and the light reflected from the slider and from the surface of the disk close to the slider, is combined, and spectrally analyzed for constructive and destructive interference at each of a large number of wavelengths, to provide a plot of intensity vs. wavelength. The intensity vs. wavelength uses the ratio of the reflected light at each wavelength to the intensity of the source at each wavelength to give a 'normalized' plot of intensity vs. wavelength.

This method is easy to understand and by using this method, the flying height can be measured quite accurately when flying height is large. However, due to some limitations, the white light interferometry is no longer use for FH testing nowadays as the FH is lower than 10nm and keeps decreasing.

2.6.1.2 Normal Interferometry with Laser-scanning

In this invention, laser source is used to replace the white light source. The laser beam emitted from the laser light source is made to cast scanning the face of the magnetic head, which is opposite to the rotating quartz glass disk. When the face of the magnetic head is scanned like this by the laser beam, interference is caused in a space between the glass disk and the face of the magnetic head, which faces the glass disk, and interference fringes are formed on that face of the magnetic head, which is opposite to the glass disk.

The interference fringes are detected by a photo sensor and passed through a waveform memory section so that their strength distribution (as shown in Figure 2.4) can be measured by a computer section.

Fig. 2.4 A diagram showing light strengths of interference fringes

This invention is to provide a method and apparatus for enabling static behaviors such as pitch angle and balance of the magnetic head to be measured along with dynamic behaviors such as floating and frequency fluctuations, and also enabling the measurement to be made shorter in time and more accurate with a high SNR (signal-to-noise ratio). However, as common to the interferometry techniques, there is a drawback that the intensity of the interference pattern does not vary significantly with distance near the local maxima and minima and consequently the precision of techniques varies depending on the size of the gap to be measured. Moreover, to obtain the floating amount of the slider varies at least $1/4$ of the wavelength used. As the flying height lowers to less than 10nm and the slider size shrinks, this calibration method is no longer practical.

2.6.1.3 Three-wavelength Intensity Interferometry

The invention is the foundation of the well-known Dynamic Flying Height Tester (DFHT). In this invention, a mercury arc lamp light source is used to provide three

distinct wavelengths of light so that three separate interference fringes are generated. As shown in Fig.2.5, light from the mercury arc lamp is directed substantially normal to the surface of a transparent disk, through the disk and onto the slider on which a magnetic head is mounted.

Fig. 2.5 A diagrammatic view of the flying height testing assembly of the invention

The invention is widely used in the industry due to its capability of high speed self calibration of absolute interferometric intensity before each measurement, repeatable measurements for all slider materials, and same analysis algorithms for all flying heights down to contact. However, as the common drawback in the interferometry techniques, there is not enough sensitivity at contact if sliders were made of dielectric material or the extinction coefficient k is rather small for the slider material. Reflectivity of the slider is different from point to point, and if the calibration is only

done at one point, the measurement will carry a “substitution” error, which depends on the homogeneity of the slider material.

2.6.2 Polarization Interferometry Technique

2.6.2.1 Work Principle

The intensity-based interferometry has a serious drawback at extremely low FH, i.e., low measurement sensitivity. Polarization interferometry technique is a solution to this problem. In the polarization interferometry technology, laser is used as the light source due to the polarization required. Moreover, the light beam impinges upon the glass disk at an angle of incidence not equal to zero. Either an angle of incidence close to the Brewster angle or the Brewster angle is used.

Fig. 2.6 A diagram illustration the polarization interferometry, where the angle of incidence $\theta_i \neq 0^\circ$

As shown in Fig.2.6, a polarized light beam is directed to disk at an angle. A reflected beam passed a polarization beam splitter and is divided into the p-polarization and s-polarization, which are then detected by the photo detectors. The relative phase

difference between the two orthogonal polarization components and the reflected beam intensities are analyzed to find out the flying height. There are some patents for FH testing using this polarization interferometry technique. Most of these patents are from the same company, Zygo Corporation. Sliders are formed by composite materials, so the complex index of refraction on the slider may be different from different measurement spot. Predetermination for complex index of refraction causes error in FH.

2.6.2.2 Four-phase Polarization Interferometry

This invention takes the advantage of the interference phase information discarded by the normal incident interferometry. The setup of this invention is similar to that shown in Fig.2.6.

The beauty of the polarization interferometric technique is the phase channel. The measurement sensitivity can be guaranteed at low flying height even down to contact. However, the data acquisition and processing rate are very low, which is due in part to the method of phase modulation and the need to integrate over a full rotation of the disk. In this method, p-polarized light and s-polarized light experience different index of refraction n . If the two beams are in phase when they first enter the glass, they will become gradually phase shifted as they progress in their propagation. This phase shift is quite significant and in the case of polarization interferometry, gets directly mixed with the phase shift that is at the heart of the flying height measurement. It is not possible in practice to account for this phase shift in a clean manner. Moreover, there

is an overall drift in the interference phase due primarily to the presence of the high-speed phase modulator, resulting in an ambiguous phase offset.

2.6.3 Frustrated Total Internal Reflection (FTIR) Measurement Technique

2.6.3.1 Working Principle

Another approach to improve the measurement sensitivity for low FH is to use the evanescent wave. If an evanescent wave, such as that produced by total internal reflection, extends across a separating medium into a region occupied by a higher index of refraction material, energy may flow across the boundary (Fig.2.7). This phenomenon is known as frustrated total internal reflection, and is similar to quantum mechanical tunneling or barrier penetration.

Fig. 2.7 A drawing illustrates the frustrated total internal reflection

The expression for the total reflection coefficients for the p- and s-polarization state in this FTIR case is the same as those in the polarization interferometry except the

incident angle is greater than the critical angle. The frustrated total internal reflection technique has been used for gap measurement has a long history. In the US patent 'Apparatus and method for measuring linear nano-metric distance using evanescent radiation', Sides utilized this concept to measure the FH.

2.6.3.2 FH Testing Using Evanescent Radiation

The operation of this apparatus is based on the technique of Total Internal Reflection Microscopy (TIRM). The schematic diagram of this apparatus is shown in Fig.2.8. In this invention, a light source is used to direct incident electromagnetic radiation into a glass disk toward an interface with air at an angle equal to or exceeding the critical angle, θ_c , for TIR in the disk. A beveled edge is fabricated in the glass disk to achieve an incident angle exceeding the critical angle.

Fig. 2.8 A schematic view illustrating a preferred embodiment of the apparatus for measuring the distances between the first and second bodies

The beauty of this method is that the measurement sensitivity can be optimized to measure FH in different ranges by altering the penetration depth of the evanescent wave, e.g. shorter wavelength laser and/or utilize a disk with different refractive index. However, this FTIR technique is difficult to understand. Once the polarized light is

used, birefringence introduced in glass by high rotational speeds is a problem. Moreover, the light must strike the disk surface at an angle greater than critical angle complicates the use of this method at multiple radii on the same disk.

2.6.4 Summary

Comparing the merits and demerits of the three measurement techniques, namely normal interferometry, polarization interferometry and frustrated total internal reflection, we can conclude that the normal interferometry, which uses three wavelengths to estimate the flying height, is so far the best solution for the flying height testing.

CHAPTER 3 Mesh-Free Methods and the LSFD Method

3.1 Introduction

One common feature of traditional numerical methods is that they are all mesh-based methods that require structural meshes or non-structural meshes generated. For finite volume method (FVM), finite difference method (FDM) and finite element method (FEM), pre-processes are needed to establish a database containing nodal and elemental connectivity and hierarchical mesh information. In this case, the numerical results depend strongly on the mesh properties. Due to their good performance, these methods are widely used in all areas of engineering computation and design.

Despite of the popularity of traditional methods (such as FDM, FEM, and FVM) in the field of computational fluid mechanics simulations, a lot of new numerical schemes have been developed and explored in the past two decades. In general, one numerical method can only be applied to a narrow scope of flow problems more efficiently and successfully than the others. Many important complex problems still cannot be dealt with reliably and efficiently with standard numerical schemes. In addition to the bewildering physical complexities of fluid flows, flow problems in hard disk drive also involve complex geometries, for example, air bearing surface with complex shape rails and highly recessed regions. These present another major difficulty confronting the computational fluid dynamics, i.e., geometrical complexity.

To deal with the geometrical complexity, standard numerical schemes such as FDM, FEM and FVM employ various kinds of meshes.

To overcome the difficulties arising from the complex geometries, FV and FE methods use unstructured mesh to fit the shape of the physical domain. Usually, the unstructured mesh is a triangular mesh in two-dimension and a pyramidal mesh in three-dimension. However, unstructured mesh generation is not a trivial job. In many cases, mesh generation uses far more time and costs than the numerical solution itself. For example, the generation of a mesh for the simulation of airflow past an aircraft requires several months of work, while the solution computations may take only a few hours on a supercomputer. The generation of three-dimensional unstructured meshes for FE and FV methods, despite recent advances in the field, is still the bottleneck in many industrial computations.

Thus, one of the key advantages of mesh-free methods as compared to the standard methods is the saving of time and human-labor on the mesh construction when complex geometry is involved. Instead of mesh generation, mesh-free methods require node generation. From the point of view of computational efforts, node generation is seen as an easier and faster job. Another advantage of the mesh-free method is the ease of construction of high-order schemes. The construction of higher-order schemes on unstructured grids by standard schemes has encountered severe obstacles in the areas of stability and storage. Most programs are still based on linear elements, or, equivalently linear functional reconstruction. The use of mesh-free schemes can facilitate the construction of higher-order discretization. By making use

of mesh-free methods, we can also enjoy the computational ease of adding and subtracting nodes from the pre-existing nodes (Shu, *et al.*, 2004).

3.2 Concept of Mesh-Free Methods

In recent years, many new numerical schemes have been proposed to address the weakness of the standard numerical methods described previously, especially with regard to the geometrical complexity. Among the newly developed numerical schemes, a group of so-called mesh-less or mesh-free methods have especially attracted the attention of engineers, physicists and mathematicians. As its name implies, mesh-free methods are deliberately designed to eliminate the dependence on the mesh. The terms mesh-less and mesh-free refer to the ability of the method to construct functional approximation or interpolation entirely from the information at a set of nodes, without any pre-specified connectivity or relationships among the nodes. Some mesh-free methods do have a weak dependence on background meshes to carry out numerical quadrature calculation. Such methods are still regarded as mesh-free because there is no fixed connection among the nodes, but not “truly” mesh-free method due to the background meshes used (Shu, *et al.*, 2004).

3.3 Classification of Mesh-Free Methods

Among the first proposed mesh-free methods are the smoothed particle hydrodynamics (SPH) (Lucy, *et al.*, 1977) and generalized finite difference method (Liszka, 1984). After they were presented, more and more researchers showed their interests in the development of methods with mesh-free property. As a result, a

number of new mesh-free methods appeared in 1990s: the diffuse element method (DEM) (Nayroles, *et al.*, 1992), the element-free Galerkin method (EFGM) (Belytschko, *et al.*, 1994 1995 1996), multiquadric methods (MQM) (Kansa, 1990a,b), reproducing kernel particle methods (RKPM) (Liu *et al.*, 1995a,b; Liu *et al.*, 1996; Liu *et al.*, 1997), the partition of unity method (Melenk and Babuska 1996), HP-clouds methods (Duarte and Oden 1995, 1996) and meshless local Petrov-Galerkin method (MLPG) (1998 Atluri *et al.*, 1998, Zhu *et al.*, 1999). These mesh-free methods can be categorized by their characteristics.

From the point of view of the movement of nodes, the mesh-free methods may be classified into two groups: the particle method and the fixed node method. In the particle method, the nodes can be viewed as the flow particles with control mass and pass through the region of interest according to the flow. SPH method is among this type. Particle methods are highly appreciated in the cases of large deformation and extrusion, such as explosion under water, where the simulation of evolvement of the explosion is very important. However, in most cases of fluid flow, it is difficult and inefficient to trace particles in the flow field. It is more convenient to deal with the flow within a certain spatial region called control volumes. In the fixed node method, the positions of the nodes are fixed as its name implies. A node can be considered a very tiny region or control volume in the domain of interest, where the extensive properties of the passed flow particles are recorded and refreshed continuously.

If we take the form of governing equations into consideration, the fixed node method can be further subdivided into integral and non-integral types. The integral type employs numerical integration to solve a weak form of the governing partial

differential equations. Methods such as DEM, EFGM, HP-clouds, RKPM, MLPG fall into this category. The non-integral type is frequently, though not exclusively, related to some generalized or extended forms of the conventional FDM. The partial derivatives are discretized in terms of the nodal information. These are then substituted into a strong form of the governing partial differential equations to transform them into a system of linear or nonlinear algebraic equations. The FPM, GDF and MQM are in this category.

3.3.1 Integral Type Mesh-Free Methods

3.3.1.1 Smoothed Particle Hydrodynamics (SPH) Method

The idea of Smoothed Particle Hydrodynamics (SPH) method was initially proposed by Lucy (1977), and Monaghan (1988) improved it by providing a rationale based on so-called kernel approximation, which was generated from a local representation. Collocation was employed at the nodes to approximate the solution in the corresponding sub-domain. Therefore, SPH method does not require a background quadrature scheme, which makes SPH one of the most flexible mesh-free methods and very simple to implement. It should be noted that this simplicity and flexibility is at the cost of accuracy. As a result, a large number of nodes must be put into domain to achieve a reasonable accuracy in the practical applications. Duarte (1996) pointed out that the kernel estimation used in the SPH method is equivalent to the Shepard interpolant, which is well-known for its poor accuracy. Another well-known problem of the SPH method is the spatial instability of the method, often known as tensile

instability. Belytschko (1996) found that the instability phenomena usually occurred while a tensile stress and large derivative of the kernel were encountered.

3.3.1.2 Diffuse Element (DE) and Element-Free Galerkin (EFG) Methods

DEM, which was proposed by Nayroles, *et al.*, (1992), is the first one that uses moving least square technique to construct mesh-free approximations in a Galerkin method. After that, Belytschko, *et al.* (1994) refined the method and called his version EFGM. The EFGM and the DEM are similar in many ways except for two major differences: the EFGM includes certain terms in the derivatives of the interpolants which are omitted in the DEM and the EFGM employs Lagrange multipliers to enforce essential boundary conditions. One of the main drawbacks of these methods is the additional cost of building the moving least square functions. As compared with those of SPH, the computational effects are substantially more expensive. The need of a background quadrature scheme to evaluate the integrals, which appears in the weak forms used by the Galerkin method, also impairs the efficiency and applicability of the method to the practical applications.

3.3.1.3 Meshless Local Petrov-Galerkin (MLPG) Method

In the mesh-free methods of integral type mentioned above, they all need a background mesh to evaluate the integral of weak form of governing equations. Due to this reason, they bear the reputation of not being truly mesh-free methods. In order to develop a mesh-free method with more flexibility, Atluri, *et al.* (1998) presented a meshless local Petrov-Galerkin (MLPG) scheme to circumvent this requirement for a background mesh partition in numerical integration. An important contribution of this

method is that the concept of local weak form was firstly introduced into the field of mesh-free methods. This innovative idea allows the weak form of the governing equations to be satisfied on a local basis. Therefore, the evaluation of the local integral can be implemented in a regularly-shaped local domain, which is not related to the geometric shape of domain. Since there is no requirement for mesh either in the interpolation process or in the integration process, MLPG method is considered a “truly” mesh-free method. However, the price of this flexibility is the computational efforts. Because of the necessity of constructing the local integration at each node, much longer time is spent on the evaluation of local integral in the MLPG method than that of the other mesh-free methods of integral type, where the integrations are only carried out on a limited number of background meshes.

3.3.2 Non-Integral Type Mesh-Free Methods

3.3.2.1 Drawbacks of Integral Type Mesh-Free Methods in Flow Simulations

The integral type mesh-free methods enjoy great popularity in recent years. According to literature review, these methods except SPH are mainly applied to structure mechanics, and their applications to fluid dynamics are rarely found in the literatures. The first reason may lie with the flow problems itself. As discussed in the previous section, flow problems usually encounter both dynamic and geometric complexities. To effectively solve practical flow problems, a large number of nodes are required to capture the physical variation of flow parameters. The discretized equations of fluid mechanics are, after linearization, typically sparse. Iterative methods are usually

employed to solve them. The corresponding process is quite time-consuming. Comparatively, linear structure problems are simpler and require much less nodes. The second reason is that the MLS technique or its equivalent is employed in most of the integral mesh-free methods. As a result, the shape function at each node does not possess property of the Kronecker δ function. This drawback makes the discretization more complex. The third possible reason may be the most determinant one. The resultant algebraic equations by the above mesh-free methods such as MLPG are difficult to solve by iterative method. The tested iterative method is the Gaussian-Seidel method. In other words, one has to either find a suitable iterative algorithm to efficiently solve the system of algebraic equations or use the direct method. In general, the direct methods would be much more expensive than iterative methods when the number of nodes in the domain is larger than 1000. However, in most practical cases, 1000 nodes are not sufficient to catch the physical nature of the complex flow problems.

3.3.2.2 General Finite Difference (GFD) Method

Compared to the mesh-free methods of integral type, the mesh-free methods of non-integral type are more suitable for the fluid mechanic problems. General finite difference (GFD) method proposed by Liszka (1980) is considered one of the representatives for this group of mesh-free methods. Just as its name implies, GFD has the origin from the finite difference scheme, i.e., Taylor series expansion is used to approximate the derivatives of unknown functions or/and the unknown function. The main difference between FDM and GFDM lies in that GFDM applies least square technique to avoid the possible ill-conditioned configurations of the coefficient matrix

which arise from the arbitrary point distribution. Duarte (1995) pointed out that the underlying interpolation technique used in the GFD method has many similarities with the MLS method and is equivalent to EFGM and RKPM under special circumstances. The GFD method is more flexible than the EFGM since a collocation process at the nodes is used to construct the discrete set of equations governing the problem and therefore no other quadrature scheme is needed. However, the method also has the limitations similar to the other MLS based mesh-free methods, such as the lack of Kronecker δ property. Another drawback of the method is the low degree of regularity of the underlying basis functions. This implies that the solution of displacement in the structure mechanics may not be continuous. Up to date the applications of GFD method to fluid mechanics problem are not available in the literatures yet.

3.3.2.3 Multiquadric (MQ) Method

Multiquadric (MQ) method is another representative of non-integral mesh-free methods. In GFD method the use of least square technique make it achieve the mesh-free discretization, but in the MQ method the mesh-free property comes from its interpolation functions, i.e., Multiquadric Radial Basis Functions (MRBFs), which are “naturally” mesh-free. RBFs have been under intensive research as multi-dimensional interpolation tools. Their performance demonstrates that RBFs constitute a powerful framework for interpolating or approximating data on non-uniform grids. However, there are also some remarkable drawbacks in the RBFs based numerical schemes. One of them is that many radial basis functions have a constant number c , which is called shape parameter and determined by user. The accuracy of the final numerical solution

is indirectly influenced by the given shape parameter. Another drawback is that the generated coefficient matrix tends to be ill-conditioned (Kansa 1990 a) while the number of nodes in the domain increases. This drawback impairs its applicability to large and complex practical problems. The RBFs based numerical scheme is far from mature as compared with other mesh-free methods.

3.3.3 Desirable Mesh-Free Methods for Fluid Mechanics

Though much research has been done in the field of mesh-free methods, there are few mesh-free methods designed specially for the flow problems. The least square finite difference method is an ideal mesh-free method, which can deal with the complexity of flow problems. This mesh-free method possesses the following properties:

- Truly mesh-free (No need of background mesh)
- Accurate (h-p refinement)
- Efficient (at least iterative method can be easily implemented to solve the resultant algebraic equations)
- Kronecker δ shape function at the reference node
- Simple, easy to understand and program

Interest in mesh-free method and slider simulation was motivated by the desire to choose effective mesh-free schemes that are suitable in simulation and design of sliders in modern hard disk drives. Thus, after thorough literature review, the least square finite difference (LSFD) method is chosen and applied to the simulation of slider air bearings in magnetic disk drive in this thesis.

3.4 Least Square Finite Difference (LSFD) Method

To develop an efficient mesh-free method, a weighted least-square-based finite-difference scheme was proposed by Shu and Ding (2004). It can be viewed as a further development of the GFD method. The approximation of partial derivatives was formulated from the Taylor series expansions and the weighted least square technique. The method is simple to apply when compared to integral-based formulation, and much more flexible than conventional FD methods in practical applications.

3.4.1 1-D Taylor Series Formulation

Let us consider the one-dimensional grid points shown in Fig.3.1. For grid point 2, located midway between grid points 1 and 3 such that $\Delta x = x_2 - x_1 = x_3 - x_2$, the Taylor series expansions around point 2 give

$$\phi_1 = \phi_2 - \Delta x \left(\frac{d\phi}{dx} \right)_2 + \frac{1}{2} (\Delta x)^2 \left(\frac{d^2\phi}{dx^2} \right)_2 - \dots \quad (3.1)$$

$$\phi_3 = \phi_2 + \Delta x \left(\frac{d\phi}{dx} \right)_2 + \frac{1}{2} (\Delta x)^2 \left(\frac{d^2\phi}{dx^2} \right)_2 + \dots \quad (3.2)$$

Fig. 3.1 Configuration for One-dimensional FD scheme

Truncating the series just after the third term, and adding and subtracting the two equations, we obtain the following second-order finite-difference approximations for the first and second order derivatives:

$$\left(\frac{d\phi}{dx}\right)_2 = \frac{\phi_3 - \phi_1}{2\Delta x} \quad (3.3)$$

$$\left(\frac{d^2\phi}{dx^2}\right)_2 = \frac{\phi_1 - 2\phi_2 + \phi_3}{(\Delta x)^2} \quad (3.4)$$

The substitution of such expression into the differential equation leads to the finite difference equation.

FD scheme has good performance in the one-dimensional problems, but it confronts some difficulties when it is directly extended to multi-dimensional problems, especially those with complex geometries.

As shown above, one-dimensional (1D) Taylor series expansion is used in the development of the conventional FD schemes, where only the derivatives in one spatial direction are involved and considered as unknowns. To solve for these unknown derivatives, one needs to apply the 1D Taylor series expansion at some collocation points along the respective spatial direction. This procedure can be easily extended to the two-dimensional (2D) case, in which the 2D Taylor series expansion is applied. Since the formulation only requires the information about the relative positions of the supporting nodes to the reference node, the constructed scheme can be considered as a mesh-free approach.

3.4.2 2-D Taylor Series Formulation

As shown in Fig.3.2, the functional value near reference node o can be approximated by the functional value and its derivatives at the reference node o by using the 2D Taylor series expansion within a local support area of the reference node:

Fig. 3.2 Supporting knots around a reference node.

$$\begin{aligned} \phi = & \phi_o + \Delta x \left(\frac{\partial \phi}{\partial x} \right)_o + \Delta y \left(\frac{\partial \phi}{\partial y} \right)_o + \frac{1}{2} (\Delta x)^2 \left(\frac{\partial^2 \phi}{\partial x^2} \right)_o + \frac{1}{2} (\Delta y)^2 \left(\frac{\partial^2 \phi}{\partial y^2} \right)_o + (\Delta x)(\Delta y) \left(\frac{\partial^2 \phi}{\partial x \partial y} \right)_o \\ & + \frac{1}{6} (\Delta x)^3 \left(\frac{\partial^3 \phi}{\partial x^3} \right)_o + \frac{1}{6} (\Delta y)^3 \left(\frac{\partial^3 \phi}{\partial y^3} \right)_o + \frac{1}{2} (\Delta x)^2 (\Delta y) \left(\frac{\partial^3 \phi}{\partial x^2 \partial y} \right)_o + \frac{1}{2} (\Delta x)(\Delta y)^2 \left(\frac{\partial^3 \phi}{\partial x \partial y^2} \right)_o + \dots \end{aligned} \quad (3.5)$$

Suppose that equation (3.5) is truncated to the third order derivatives. Then it has 9 unknowns. Among them, 2 are the first order derivatives, 3 are the second order derivatives, and 4 are the third order derivatives. Like the conventional FD scheme, 9 equations are needed to solve for these 9 unknowns. This can be achieved by applying equation (3.5) at 9 neighboring points. Suppose that all the 9 points are within a circular sub-domain D_o of radius d_o about node o . Application of equation (3.5) at 9 neighboring points gives

$$\phi_j - \phi_0 = s_j^T \cdot d\phi, \quad j = 1, 2, \dots, 9 \quad (3.6)$$

Where

$$s_j^T = \left[\Delta x_j, \Delta y_j, \frac{1}{2}(\Delta x_j)^2, \frac{1}{2}(\Delta y_j)^2, (\Delta x_j)(\Delta y_j), \frac{1}{6}(\Delta x_j)^3, \right. \\ \left. \frac{1}{6}(\Delta y_j)^3, \frac{1}{2}(\Delta x_j)^2(\Delta y_j), \frac{1}{2}(\Delta x_j)(\Delta y_j)^2 \right] \quad (3.7)$$

$$d\phi^T = \left[\left(\frac{\partial \phi}{\partial x} \right)_0, \left(\frac{\partial \phi}{\partial y} \right)_0, \left(\frac{\partial^2 \phi}{\partial x^2} \right)_0, \left(\frac{\partial^2 \phi}{\partial y^2} \right)_0, \left(\frac{\partial^2 \phi}{\partial x \partial y} \right)_0, \right. \\ \left. \left(\frac{\partial^3 \phi}{\partial x^3} \right)_0, \left(\frac{\partial^3 \phi}{\partial y^3} \right)_0, \left(\frac{\partial^3 \phi}{\partial x^2 \partial y} \right)_0, \left(\frac{\partial^3 \phi}{\partial x \partial y^2} \right)_0 \right] \quad (3.8)$$

In above equations, the subscript o denotes functional value at the reference node o ; subscript j ($j = 1, 2, \dots, 9$) denotes functional value at the supporting point j in D_o .

$\Delta x_j = x_j - x_o$ and $\Delta y_j = y_j - y_o$, where (x_j, y_j) denotes the Cartesian coordinates of node j . It is noted that in this development, Taylor series expansion was truncated after the third order derivatives. Thus, the second-order derivatives were approximated to second-order accuracy and the first-order derivatives to third-order accuracy. Furthermore, by defining

$$\Delta\phi^T = [\phi_1 - \phi_0, \dots, \phi_9 - \phi_0] \quad (3.9)$$

$$S^T = [s_1, \dots, s_9]_{9 \times 9} \quad (3.10)$$

Equation (3.6) is further assembled into the following succinct matrix form:

$$\Delta\phi = Sd\phi \quad (3.11)$$

The square matrix S contains all the geometric information about the distribution of the supporting nodes. If the matrix S is non-singular, the derivative vector $d\phi$ can be obtained as

$$d\phi = S^{-1} \Delta\phi \quad (3.12)$$

Application of equation (3.12) to discretize derivatives in a differential equation yields the requisite systems of finite-difference equations. Up to this point, the development is similar to that of the one-dimensional finite difference scheme. The only difference is that 9 supporting nodes surrounding the reference node are used in the present two-dimensional formulation.

When one or more of the supporting nodes are very close to the reference node, i.e. $(\Delta x_j, \Delta y_j) \approx 0$ for some j or some supporting nodes are very close to each other, the matrix may become ill-conditioned or singular.

To overcome the first difficulty, the radius d_o of the support domain is used to scale the local distance $(\Delta x_j, \Delta y_j)$,

$$\Delta \bar{x}_j = \frac{\Delta x_j}{d_o}, \quad \Delta \bar{y}_j = \frac{\Delta y_j}{d_o} \quad (3.13)$$

The condition number of resultant matrix $\bar{S}(\Delta \bar{x}, \Delta \bar{y})$ is greatly improved as compared with the matrix $S(\Delta x, \Delta y)$ and thus negative effect caused by small value of $(\Delta x, \Delta y)$ was alleviated. The derivative vector then leaves,

$$d\phi = D\bar{S}^{-1} \Delta\phi \quad (3.14)$$

$$\bar{S} = \begin{bmatrix} \frac{\Delta X_l}{d_0} & \frac{\Delta Y_l}{d_0} & \frac{1}{2} \left(\frac{\Delta X_l}{d_0} \right)^2 & \dots & \frac{1}{2} \left(\frac{\Delta X_l}{d_0} \right) \left(\frac{\Delta Y_l}{d_0} \right)^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\Delta X_{SN}}{d_0} & \frac{\Delta Y_{SN}}{d_0} & \frac{1}{2} \left(\frac{\Delta X_{SN}}{d_0} \right)^2 & \dots & \frac{1}{2} \left(\frac{\Delta X_{SN}}{d_0} \right) \left(\frac{\Delta Y_{SN}}{d_0} \right)^2 \end{bmatrix} \quad (3.15)$$

Where the scaling matrix **D** is the diagonal matrix,

$$D = \text{diag}(d_0^{-1}, d_0^{-1}, d_0^{-2}, d_0^{-2}, d_0^{-2}, d_0^{-3}, d_0^{-3}, d_0^{-3}, d_0^{-3}) \quad (3.16)$$

The second difficulty is solved by least-squares technique to optimize the approximation of the $d\phi$. The least-square technique allows an optimal approximation to be derived from an over-determined set of equations. It allows the use of a great number of nodes to bypass the problem of singularity.

Through the use of least-square technique to avoid the singularity of the coefficient matrix, it makes almost even error-distribution at the supporting points. This may not be the optimal distribution of approximation errors. For a given amount of total error, one would normally prefer the approximation error to be small in the crucial central region around the reference node, where the derivatives are evaluated, and be willingly to tolerate higher errors for points further out, since the latter is expected to have smaller influence on the desired derivatives. The redistribution of errors can be achieved by introducing a distance-related weighting function that assigns greater weightage to points near the reference node.

Five different weighting functions are examined in the present study:

$$W0_i = 1 \text{ (equivalent to no weighting)} \quad (3.17a)$$

$$W1_i = \sqrt{4/\pi}(1 - \bar{r}_i^2)^4 \quad (3.17b)$$

$$W2_i = 1/\bar{r}_i \quad (3.17c)$$

$$W3_i = 1 - 6\bar{r}_i^2 + 8\bar{r}_i^3 - 3\bar{r}_i^4 \quad (3.17d)$$

$$W4_i = 1/\bar{r}_i^4 \quad (3.17e)$$

where $\bar{r}_i = \frac{\sqrt{\Delta x_i^2 + \Delta y_i^2}}{d_o}$, the index i is the i th supporting point, and d_o denotes the radius of support domain, $0 \leq \bar{r}_i \leq 1$. The adoption of weighting function actually serves as the precondition procedure for derivative vector in such a way that,

$$d\phi_{9 \times 1} = D_{9 \times 9} \left(\bar{S}_{9 \times SN}^T W_{SN \times SN} \bar{S}_{SN \times 9} \right)^{-1} \bar{S}_{9 \times SN}^T W_{SN \times SN} \Delta\phi_{SN \times 1} \quad (3.18)$$

$$W = \begin{pmatrix} W_1 & & 0 \\ & \ddots & \\ 0 & & W_{SN} \end{pmatrix} \quad (3.19)$$

Where the matrix $\bar{S}$ and D are given in equations (3.15) and (3.16) respectively. Note that if only the first- and second-order derivative approximations are required, only the first five entries of $d\phi$ need to be considered. However, the inclusion of high-order terms can increase the accuracy of the method. It is also interesting to note that at each node the coefficient matrix remains unchanged for a fixed set of supporting points and its inverse matrix needs to be calculated only once. The increase in computational cost is acceptable as compared to the traditional FDM and the new

scheme gives the flexibility for complex problems. Furthermore, the derivatives given by equation (3.18) are independent of the governing equations, and can be used repeatedly for other problems with the same distribution of supporting points.

3.5 Successive Over Relaxation (SOR) Method

The generalized Reynolds equations for every inner point in computational domain are discretized to a nonlinear system of equations using the LSFD method. Successive over relaxation (SOR) method is employed to tackle these algebraic equations.

The SOR method is a method of solving a linear or nonlinear system of equations derived by extrapolating the Gauss-Seidel method. The extrapolation takes the form of a weighted average between the previous iteration and the computed Gauss-Seidel iterate successively for each component,

$$p_i^{(k)} = \omega \bar{p}_i^{(k)} + (1 - \omega) p_i^{(k-1)} \quad (3.20)$$

Where $\bar{p}$ denotes a Gauss-Seidel iterate and ω is the extrapolation factor. The idea is to choose a value for ω that will accelerate the convergence rate of the solution. A theorem shows that SOR fails to converge if ω is outside the interval (0,2).

We take the one-dimensional and two-dimensional generalized Reynolds equations as an example to demonstrate how to implement the SOR method to solve the nonlinear systems of equations. In order to apply SOR iteration method, the nonlinear algebraic equations should be simplified to linear algebraic equations. There are two

approximation methods to deal with the updating process, which are compared with each other according to the efficiency of the solution process.

3.5.1 The First Approximation Method for Linearization of the Equations

The one-dimensional generalized lubrication equation for plane sliders (refer to Fig.4.2 for the model in detail) can be expressed as

$$\begin{cases} \frac{d}{dX} \left[QPH^3 \frac{dP}{dX} - \Lambda_x PH \right] = 0 \\ Q = \frac{Q_p(D, \alpha)}{Q_{con}(D)} \\ H(X) = \frac{h_2 - h_1}{h_1} (1 - X) + 1 \\ \Lambda_x = 6\mu UL / (p_a h_0^2) \end{cases} \quad (3.21)$$

Where $Q_p(D, \alpha)$ is a Poiseuille flow rate coefficient and Q_{con} is the coefficient for a continuum flow. Quantities D and α are an inverse Knudsen number and an accommodation coefficient, respectively

$$D = D_0 PH = ph / (\mu / \sqrt{2RT}) \quad Q_{con} = D/6 \quad (3.22)$$

Power series expressions for flow rate coefficients, Q_p , are effective for flow rate calculations. First, for small inverse Knudsen numbers, D , where the slip flow assumption is reasonably valid, the asymptotic expression for $K_n \ll 1$ can be employed. If inverse Knudsen number D locates between 0.15 and 5, then Poiseuille flow rates Q can be obtained (Fukui, *et al.*, 1990)

$$Q = \frac{Q_p(D, \alpha)}{Q_{con}(D)} = 0.83112 + 7.50522/D + 0.93918/D^2 - 0.05814/D^3$$

$$= K_1 + K_2(PH)^{-1} + K_3(PH)^{-2} - K_4(PH)^{-3} \quad (3.23)$$

Where (for $(0.15 \leq D < 5)$)

$$K_1=0.83112, \quad K_2=7.50522D_0^{-1}, \quad K_3=0.93918D_0^{-2}, \quad K_4=0.05814D_0^{-3} \quad (3.24)$$

By means of $Q_p(D, \alpha)$ and $Q_{con}(D)$, the generalized lubrication equation based on the Boltzmann equation can then be expressed as

$$F = \frac{d}{dX} \left[(K_1 + K_2(PH)^{-1} + K_3(PH)^{-2} - K_4(PH)^{-3}) PH^3 \frac{dP}{dX} - \Lambda_x PH \right] = 0 \quad (3.25)$$

For the i th reference node, the generalized Reynolds equation for plane slider can be deduced to

$$F_i = \left(\frac{dP}{dX} \right)_i^2 [K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3}] +$$

$$\frac{dH}{dX} \bigg|_i \frac{dP}{dX} \bigg|_i [3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1}] +$$

$$\frac{d^2 P}{dX^2} \bigg|_i [K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2}] -$$

$$\Lambda_x H_i \frac{dP}{dX} \bigg|_i - \Lambda_x P_i \frac{dH}{dX} \bigg|_i \quad (3.26)$$

In one-dimensional LSFD method, the derivative of pressure can be expressed as

$$\frac{dP}{dX} \bigg|_i = \sum_j^{SN} a_{1j}^i P_j^i - (\sum_j^{SN} a_{1j}^i) P_i \quad \frac{d^2 P}{dX^2} \bigg|_i = \sum_j^{SN} a_{2j}^i P_j^i - (\sum_j^{SN} a_{2j}^i) P_i \quad (3.27)$$

In Eq.(3.26), only dP/dX , $d^2 P/dX^2$, P can be considered to provide pressure increment in the iteration process while other components hold constant values

obtained from previous iteration operation. Thus, the one-dimensional generalized Reynolds equation can be represented as

$$\begin{aligned}
 F_i = & \left(\frac{dP}{dX} \right)_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
 & \frac{dH}{dX} \left|_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \right. \\
 & \left. \left[\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right] \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \right. \\
 & \left. \Lambda_x H_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] - \Lambda_x P_i \frac{dH}{dX} \right|_i
 \end{aligned} \quad (3.28)$$

In the i th iteration, a pressure increment is applied in Eq.(3.28), we have

$$\begin{aligned}
 F_i + \Delta F_i = & \left(\frac{dP}{dX} \right)_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right] \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
 & \frac{dH}{dX} \left|_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right] \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \right. \\
 & \left. \left[\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) (P_i + \Delta P_i) \right] \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \right. \\
 & \left. \Lambda_x H_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right] - \Lambda_x (P_i + \Delta P_i) \frac{dH}{dX} \right|_i = 0
 \end{aligned} \quad (3.29)$$

Applying the Gauss-Seidel method, the pressure increment ΔP_i can be obtained by

$$\Delta P_i = \frac{-F_i}{deno} \quad (3.30)$$

$$\begin{aligned}
deno = & \left(\frac{dP}{dX} \right)_i \left(- \sum_j^{SN} a_{1j}^i \right) \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
& \left. \frac{dH}{dX} \right|_i \left(- \sum_j^{SN} a_{1j}^i \right) \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\
& \left(- \sum_j^{SN} a_{2j}^i \right) \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\
& \Lambda_x H_i \left(- \sum_j^{SN} a_{1j}^i \right) - \Lambda_x \left. \frac{dH}{dX} \right|_i
\end{aligned} \tag{3.31}$$

The i th pressure value will be refreshed in the i th iteration and thus is applied to the calculation of the $i+1$ th pressure value in the next iteration until the pressure value of total inner nodes in the computational domain is refreshed all over. The process will continue till convergence criterion as shown is satisfied.

$$\sum_{i=1}^{inode} F_i^2 \leq \varepsilon \tag{3.32}$$

Where $inode$ is the number of inner nodes in the computational domain, F_i is the residual of the i th algebraic equation. In the program, the convergence criterion ε is pre-set to 1.0e-05.

For the 2-D plane slider air bearings, the two-dimensional generalized Reynolds equation (Fig.4.15) is given by

$$\begin{cases} \frac{\partial}{\partial X} \left[QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right] + \frac{\partial}{\partial Y} \left[QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right] = 0 \\ Q = \frac{Q_p(D, \alpha)}{Q_{con}(D)} \\ \Lambda_x = \frac{6\mu_a UL}{h_0^2 P_a}, \quad \Lambda_y = \frac{6\mu_a VL}{h_0^2 P_a} \\ H(X) = 1 + \frac{L}{h_m} (1-X) \lg \alpha \\ \frac{\partial H}{\partial X} = -\frac{L}{h_m} \lg \alpha, \quad \frac{\partial H}{\partial Y} = 0 \end{cases} \quad (3.33)$$

Expanding Eq.(3.33) and using the LSFD method to discretize the differential equation yield

$$\begin{aligned} F_i = & \left[\left(\frac{\partial P}{\partial X} \right)_i^2 + \left(\frac{\partial P}{\partial Y} \right)_i^2 \right] [K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3}] + \\ & \left[\frac{\partial H}{\partial X} \frac{\partial P}{\partial X} + \frac{\partial H}{\partial Y} \frac{\partial P}{\partial Y} \right] [3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1}] + \\ & \left[\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2} \right] [K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2}] \\ & - H_i (\Lambda_x \frac{\partial P}{\partial X} + \Lambda_y \frac{\partial P}{\partial Y}) - P_i (\Lambda_x \frac{\partial H}{\partial X} + \Lambda_y \frac{\partial H}{\partial Y}) \end{aligned} \quad (3.34)$$

In the two-dimensional LSFD method, the derivatives of pressure can be expressed as

$$\begin{aligned} \frac{\partial P}{\partial X} \Big|_i &= \sum_j^{SN} a_{1j}^i P_j^i - (\sum_j^{SN} a_{1j}^i) P_i \quad \frac{\partial^2 P}{\partial X^2} \Big|_i = \sum_j^{SN} a_{3j}^i P_j^i - (\sum_j^{SN} a_{3j}^i) P_i \\ \frac{\partial P}{\partial Y} \Big|_i &= \sum_j^{SN} a_{2j}^i P_j^i - (\sum_j^{SN} a_{2j}^i) P_i \quad \frac{\partial^2 P}{\partial Y^2} \Big|_i = \sum_j^{SN} a_{4j}^i P_j^i - (\sum_j^{SN} a_{4j}^i) P_i \end{aligned} \quad (3.35)$$

Applying the principle of approximation discussed above for 1-D problem to simplify the nonlinear algebraic equation to linear equation,

$$\begin{aligned}
F_i + \Delta F_i = & \left[\frac{\partial P}{\partial X} \Big|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right) + \frac{\partial P}{\partial Y} \Big|_i \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) (P_i + \Delta P_i) \right) \right] \\
& \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
& \left(\frac{\partial H}{\partial X} \Big|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right) + \frac{\partial H}{\partial Y} \Big|_i \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) (P_i + \Delta P_i) \right) \right) \\
& \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\
& \left[\left(\sum_j^{SN} a_{3j}^i P_j^i - \left(\sum_j^{SN} a_{3j}^i \right) (P_i + \Delta P_i) \right) + \left(\sum_j^{SN} a_{4j}^i P_j^i - \left(\sum_j^{SN} a_{4j}^i \right) (P_i + \Delta P_i) \right) \right] \\
& \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\
& H_i \left[\Lambda_x \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right) + \Lambda_y \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) (P_i + \Delta P_i) \right) \right] \\
& - (P_i + \Delta P_i) \left(\Lambda_x \frac{\partial H}{\partial X} \Big|_i + \Lambda_y \frac{\partial H}{\partial Y} \Big|_i \right) = 0
\end{aligned} \tag{3.36}$$

Applying the Gauss-Seidel iteration method, the iterative step can be determined by

$$\Delta P_i = \frac{-F_i}{deno} \tag{3.37}$$

$$\begin{aligned}
deno = & \left[\left(\frac{\partial P}{\partial X} \right)_i \left(- \sum_j^{SN} a_{1j}^i \right) + \left(\frac{\partial P}{\partial Y} \right)_i \left(- \sum_j^{SN} a_{2j}^i \right) \right] \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
& \left[\frac{\partial H}{\partial X} \Big|_i \left(- \sum_j^{SN} a_{1j}^i \right) + \frac{\partial H}{\partial Y} \Big|_i \left(- \sum_j^{SN} a_{2j}^i \right) \right] \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\
& \left[\left(- \sum_j^{SN} a_{3j}^i \right) + \left(- \sum_j^{SN} a_{4j}^i \right) \right] \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\
& H_i \left(\Lambda_x \sum_j^{SN} a_{1j}^i + \Lambda_y \sum_j^{SN} a_{2j}^i \right) - \left(\Lambda_x \frac{\partial H}{\partial X} \Big|_i + \Lambda_y \frac{\partial H}{\partial Y} \Big|_i \right)
\end{aligned} \tag{3.38}$$

3.5.2 The Second Approximation Method for Linearization of the Equations

The second approximation method is to expand fully the discretized expression and neglect second order terms to acquire the pressure increment ΔP_i , instead of only keeping terms such as dP/dX , d^2P/dX^2 , P to acquire pressure increments.

For the one-dimensional Reynolds equation based on Fukui model, we substitute Eq.(3.27) into Eq.(3.26) and expand it

$$\begin{aligned}
 F_i + \Delta F_i = & \left[\sum_j^{SN} a'_{1j} P_j' - \left(\sum_j^{SN} a'_{1j} \right) (P_i + \Delta P_i) \right]^2 \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
 & \frac{dH}{dX} \Big|_i \left[\sum_j^{SN} a'_{1j} P_j' - \left(\sum_j^{SN} a'_{1j} \right) (P_i + \Delta P_i) \right] \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\
 & \left[\sum_j^{SN} a'_{2j} P_j' - \left(\sum_j^{SN} a'_{2j} \right) (P_i + \Delta P_i) \right] \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\
 & \Lambda_x H_i \left[\sum_j^{SN} a'_{1j} P_j' - \left(\sum_j^{SN} a'_{1j} \right) (P_i + \Delta P_i) \right] - \Lambda_x (P_i + \Delta P_i) \frac{dH}{dX} \Big|_i = 0
 \end{aligned} \tag{3.39}$$

Eliminating the higher order nonlinear terms, the pressure increment can be expressed as

$$\Delta P_i = \frac{-F_i}{deno} \tag{3.40}$$

$$\begin{aligned}
deno = & 2 \left[\left(\sum_j^{SN} a_{1j}^i \right)^2 P_i - \left(\sum_j^{SN} a_{1j}^i P_j^i \right) \left(\sum_j^{SN} a_{1j}^i \right) \right] \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
& \frac{dH}{dX} \Big|_i \left(- \sum_j^{SN} a_{1j}^i \right) \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\
& \left(- \sum_j^{SN} a_{2j}^i \right) \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\
& \Lambda_x H_i \left(- \sum_j^{SN} a_{1j}^i \right) - \Lambda_x \frac{dH}{dX} \Big|_i
\end{aligned} \tag{3.41}$$

For the two-dimensional Reynolds equation based on Fukui model, the principle is the same as above. Thus, the discretized two-dimensional Reynolds equation can be expanded and simplified into the succinct form to obtain the pressure increment from Eq.(3.34)

$$\begin{aligned}
F_i + \Delta F_i = & \left[\left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right)^2 + \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) (P_i + \Delta P_i) \right)^2 \right] \\
& \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
& \left(\frac{\partial H}{\partial X} \Big|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right) + \frac{\partial H}{\partial Y} \Big|_i \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) (P_i + \Delta P_i) \right) \right) \\
& \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\
& \left[\left(\sum_j^{SN} a_{3j}^i P_j^i - \left(\sum_j^{SN} a_{3j}^i \right) (P_i + \Delta P_i) \right) + \left(\sum_j^{SN} a_{4j}^i P_j^i - \left(\sum_j^{SN} a_{4j}^i \right) (P_i + \Delta P_i) \right) \right] \\
& \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\
& H_i \left[\Lambda_x \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right) + \Lambda_y \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) (P_i + \Delta P_i) \right) \right] \\
& - (P_i + \Delta P_i) \left(\Lambda_x \frac{\partial H}{\partial X} \Big|_i + \Lambda_y \frac{\partial H}{\partial Y} \Big|_i \right) = 0
\end{aligned} \tag{3.42}$$

$$\Delta P_i = \frac{-F_i}{deno} \tag{3.43}$$

$$\begin{aligned} deno = & 2 \left[\left(\frac{\partial P}{\partial X} \right)_i \left(- \sum_j^{SN} a_{1j}^i \right) + \left(\frac{\partial P}{\partial Y} \right)_i \left(- \sum_j^{SN} a_{2j}^i \right) \right] \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2 K_4 P_i^{-3} \right] + \\ & \left[\frac{\partial H}{\partial X} \right]_i \left(- \sum_j^{SN} a_{1j}^i \right) + \left[\frac{\partial H}{\partial Y} \right]_i \left(- \sum_j^{SN} a_{2j}^i \right) \left[3 K_1 P_i H_i^2 + 2 K_2 H_i + K_3 P_i^{-1} \right] + \\ & \left[\left(- \sum_j^{SN} a_{3j}^i \right) + \left(- \sum_j^{SN} a_{4j}^i \right) \right] \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\ & H_i \left(\Lambda_x \sum_j^{SN} a_{1j}^i + \Lambda_y \sum_j^{SN} a_{2j}^i \right) - \left(\Lambda_x \left[\frac{\partial H}{\partial X} \right]_i + \Lambda_y \left[\frac{\partial H}{\partial Y} \right]_i \right) \end{aligned} \tag{3.44}$$

The effects of the two approximation methods on computational time required are presented in Table 3.2 and Table 3.3 for one-dimensional and two-dimensional plane sliders, respectively. From Table 3.2, it can be seen clearly that the second method takes a much shorter computational time. It is about 21.3% faster. From Table 3.3, the same conclusion can be obtained. Therefore, for the generalized Reynolds equation, the second method will be a better choice requiring a shorter computational time yet with the same accuracy.

Table 3.1 Effect Of two approximation methods on computational time for solving 1-D plane slider

Approximation Method	Time	Results
1	45 sec	21.3% faster
2	35.42 sec	

Table 3.2 Effect Of two approximation methods on computational time for solving 2-D plane slider

Approximation Method	Time	Results
1	6 min 57 sec	12.1% faster
2	6 min 6 sec	

3.6 Influence of Weighting Function

The effect of five weighting functions as shown in equations (3.17a) ~ (3.17e) on the convergence and accuracy of the solution should be investigated. Although the use of least-square technique avoids singularity of the coefficient matrix, it has almost even error-distribution at the supporting points. The redistribution of errors can be achieved by introducing a distance-related weighting function that assigns greater weighting value to points near the reference node.

The pressure distributions obtained using different weighting functions for taper flat slider bearing (refer to chapter 5.2 for the model in detail) based on first-order slip model are shown in Fig.3.4 and Fig.3.5 for uneven and even node distributions, respectively. The scattering points distribution is generated in ANSYS software. When unevenly distributed nodes are used, Fig 3.4 shows that the weighting functions have significant influence on the results. The results in Fig 3.5 show that the pressure distribution is of little dependence on weighting functions when the nodes are distributed evenly with sufficiently dense grids.

Fig. 3.3 Pressure and uneven node distribution for different weighting functions (101*21)

Fig. 3.4 Pressure and even node distribution for different weighting functions (121*21)

For the case of unevenly distributed nodes, sharply denser grid nodes are assigned around the taper region and the edges, because these regions have high pressure-gradient. The weighting functions are found to have no significant influence on the convergence rate. To improve the accuracy of the numerical results, the distance-related weighting function is applied so that greater weightage is assigned to points closer to the reference node. In effect, the approximation of derivative at reference node is more closely related to the function values at those support points that are closer to the reference node comparing to support points that are further from the reference node. However, when the node number increases, the weighting functions are found to have little influence on the pressure distribution. It is found that the fifth weighting function is the best choice in terms of accuracy in simulating slider air bearing comparing with experiment and other numerical results, while it requires less number of nodes comparing to the other weighting functions.

3.7 Supporting Points Searching Methods

3.7.1 Global Searching Method

For each reference node, supporting points are selected in the whole domain according to the distance between the supporting points and the reference node in Fig. 3.6.

Fig. 3.5 Global searching method in plane slider

The closer the distance is, the more possible the knots in the whole domain are to be chosen as supporting points. If the distances among the reference node and neighboring knots are equivalent, the point with smaller X-coordinate value is firstly chosen and secondly the point with smaller Y-coordinate value. The distance-related searching method facilitates error distribution in arranging the supporting knots for the certain node. The points close to the reference node provide more contribution on discretizing the pressure derivative of the node compared with the points far away from it. This global searching method is very simple and convenient to implement for supporting points are acquired in the whole domain without considering the node distribution. Especially when the node is evenly scattered, the convergence speed is faster.

The plane slider is used to demonstrate performance of the global searching method. The length L of the plane slider is 1mm and its width is 0.825mm. The height at the trailing edge of the slider, $h_{\min}=50\text{nm}$, is used to determine the Knudsen number. The

slider is located at radial position of 18mm measured from the disk center with zero skew angle. The disk rotating speed is 5400RPM and the pitch angle is 0.1mrad. Fukui model is used to calculate the pressure distribution. Effects of different types of node layouts on number of supporting points, extrapolation factor and computational time are shown in Table 3.4. In ANSYS software, 50 is the number of element divisions per line. -2 indicates that the nodes distribute densely on the two sides and distribute sparsely in the center along a line with the ratio (2) of center division size to end division size. 0.2 being nominal ratio of last division size to first division size. In most cases, the node is distributed adaptively according to geometry and pressure gradients in simulating air bearing surface.

In Table 3.4, when the node distribution is uneven, the number of supporting points will increase from 15 to 50, which leads to less accuracy of the results. On the other hand, the extrapolation factor required by the SOR method will decrease accordingly from 0.5 to 0.1 compared with the even node distribution, which results in the increased computational time from 4 seconds to 15 seconds, even 2100 seconds.

Table 3.3 Influence of node distribution on number of supporting points, extrapolation factor, time and ratio in global searching method

Node Distribution(x*y)	No. of Supporting Points	Extrapolation Factor	Computational Time(sec)	The Minimum Ratio of Supporting Points along X to Y Direction	The Maximum Ratio of Supporting Points along X to Y Direction
50*50	15	0.5	4	3/4=0.75	4/3=1.3
50(0.2)*50(-2)	50	0.1	15	4/11=0.36	13/3=4.3
97(0.2)*97(-2)	50	0.3	35*60=2100	4/15=0.27	17/3=5.67

Thus, the drawback of the global searching method is that it requires a great number of supporting points especially for sharply uneven node distribution to make the matrix in Eq.(3.18) inverse. In some extreme cases, even a large number of supporting points does not guarantee the matrix inverse or the convergence of the results. The reason is that, if the node distribution along the x-direction is denser, supporting points in the axis are in the majority of them chosen by the space. The inverse condition requires even supporting point distribution along X-direction and Y-direction respectively. Therefore, more neighboring points are included in the whole domain to satisfy the inverse need. It will extend the searching time and reduce the calculation accuracy.

3.7.2 Local Searching Method

Actually, it is observed that valid supporting points for the reference node are located within a local support area, instead of the whole domain. Thus, a sub-domain centered around the node is built to restrict the searching range in Fig.3.7, which effectively saves searching time. When the node distribution is not sharply uneven in the whole computational domain, the length d_0 of the square sub-domain is considered to be a constant value during the searching procedure. For sharply uneven node distribution, the value d_0 varies according to the node density.

Fig. 3.6 Local searching method in plane slider

Although the local searching method is timesaving, the distance-based searching methods become inefficient, even fail in the cases of increasingly large total number of nodes or sharply uneven node layout, which generally occur in simulation of sliders with complex rail shapes and highly recessed regions.

3.7.3 Layer Searching Methods

A new searching method choosing supporting points from inner layer to outer layer around the reference node is proposed (Fig.3.8). The layer searching method can accommodate various node layouts, no matter how sharply uneven the nodes distribute in the computational domain.

Fig. 3.7 Layer searching method in plane slider

Six different layer searching methods are designed to determine supporting knots in Fig.3.9-Fig.3.13. 8 points in the first layer among all layer searching methods is common. The actual differences among them lie in points' choice of other layers. In the first layer method (Fig.3.9), points along Y-direction are chosen with priority to smaller X coordinate value.

Fig. 3.8 The first layer searching method

In the second method (Fig.3.10), points along X-direction are chosen with priority to smaller Y coordinate value.

Fig. 3.9 The second layer searching method

In Fig.3.11, the third method acquires points along X-direction by increasing Y coordinate difference between the reference node and supporting points with priority to smaller Y coordinate.

Fig. 3.10 The third layer searching method

In Fig.3.12, the fourth method arranges points along Y-direction by increasing X coordinate and Y coordinate difference with priority to smaller y coordinate.

Fig. 3.11 The fourth layer searching method

The fifth searching method determines points along Y-axis by increasing X coordinate value with priority to smaller Y coordinate value in Fig.3.13. The sixth method chooses points along X-direction with increasing X coordinate and Y coordinate simultaneously in Fig.3.14.

Fig. 3.12 The fifth layer searching method

Fig. 3.13 The sixth layer searching method

Plane slider is considered as an example to compare the performance of all six searching methods. In Table 3.5, influence of different searching methods on the number of supporting points required, extrapolation factor, computational time and ratios are compared. It is observed that all the layer searching methods require fewer number of supporting knots than the distance-based methods, i.e. local method and global method. Meanwhile, the computational times required by the layer methods are much shorter than those required by the distance-base methods. In conclusion, the fourth layer searching method is the best choice to acquire stable solutions in shortest time among the layer methods.

Table 3.4 Comparison of influence of different searching methods on number of supporting points, extrapolation factor, time and ratio

Searching methods	No. of supporting points	Extrapolation factor	Computational time(sec)	The minimum ratio of supporting points along x to y direction	The maximum ratio of supporting points along x to y direction
1(Layer method)	15	0.7	32	$3/3=1$	$4/2=2$
2(Layer method)	15	1.0	23	$3/4=0.75$	$3/2=1.5$
3(Layer method)	15	0.8	34	$2/4=0.5$	$3/3=1.0$
4(Layer method)	15	1.2	16	$2/3=0.66$	$4/3=1.33$
5(Layer method)	15	0.7	45	$2/3=0.6$	$4/2=2.0$
6(Layer method)	15	1.0	24	$2/4=0.5$	$3/2=1.5$
Local method	50	0.2	6*60	$4/15=0.27$	$17/3=5.6$
Global method	50	0.3	35*60	$4/15=0.27$	$17/3=5.67$

3.8 Influence of Extrapolation Factor in SOR Iteration Method

In general, it is not possible to compute in advance the value of extrapolation factor ω that will maximize the convergence rate of the SOR iteration method. Thus, the value should be predetermined by experience and trial process.

The plane slider with even and uneven node distributions are calculated with different extrapolation factors, which is used to investigate influence of the factors on computational time and accuracy. The air bearing pressure is calculated by solving the

lubrication equation with the Fukui model. For even node distribution in computational domain, three sets of extrapolation factors are chosen to verify the result and time. As shown in Table 3.6, the computational time will increase with the decreasing extrapolation factor. Once extrapolation factor increases, the pressure values in the iteration will be updated with larger pressure increments, which thus reduce the iteration time substantially in convergence process. Although computational time required in uneven node case is higher as shown in Table 3.6, actually it doesn't need so many numbers of nodes to achieve the result of same accuracy as that in the even case. In conclusion, to accelerate convergence speed, the extrapolation factor should be chosen as large as possible on condition that the iteration process is convergent.

Table 3.5 Influence of extrapolation factor on computational time

Node Distribution	Extrapolation Factor	Computational Time(sec)
97*97	1.2	6
	0.5	31
	0.1	198
97(0.2)*97(-2)	1.2	16
	0.5	69
	0.1	441

3.9 Concluding Remarks

The least square finite difference method is very simple and easy to be understood and implemented, especially as compared with other mesh-free methods. In the LSFD

method, the number of supporting points in the distance-based searching method largely increases in the cases of sharply uneven node layouts, which reduces the accuracy of numerical results. Therefore, the layer method searching the supporting points for the reference node is proposed. It improves the accuracy and reduces the computational time compared with the distance-based searching methods. Moreover, in order to acquire faster convergence speed, two approximation methods for linearization of the equations are presented and compared. In addition, the influence of weight functions on calculated result is investigated. The fifth weighting function is verified to be the best choice among all weighting functions. Moreover, the effect of extrapolation factor on convergence speed is surveyed. The value of extrapolation factor predetermined by experience and trial process should be chosen as large as possible on the condition that the result is convergent.

CHAPTER 4 Implementation and Verification of the LSFD Method for Slider Air Bearings

4.1 Introduction

The complex shape and operating requirements of sliders emphasize the need to thoroughly understand the theory of lubrication and the characteristics of various film configurations for the designer.

For infinitely long bearings ($L \rightarrow \infty$) (Fig.4.1), one-dimensional Reynolds equation can be used to calculate the pressure profile and other parameters such as load and pressure center. Infinite length films and simple two-dimensional sliders are easier to analyze, and the fundamental feature of the film geometry, i.e., the mechanism by which pressure is developed, is more easily understood. Hence these simple models provide a practical basis for understanding the fundamental characteristics and physical differences among gas-film bearings. Another purpose of simulating simple one-dimensional and two-dimensional slider air bearing problems in this chapter is to verify the efficiency and accuracy of the LSFD mesh-free method introduced in Chapter 3.

One can use these fundamental principles to obtain new design concepts and to establish performance characteristics for comparison purposes. To evaluate the effects of geometry parameter on pressure distribution and load-carrying capability for gas

films, the influence of different design parameters on the performance of air bearing can be easily studied through the usage and analysis of numerical methods.

In this chapter, the LSFD method is implemented to solve some simple 1-D and 2-D slider air bearing problems to demonstrate the applicability and validity of the LSFD method.

Fig. 4.1 Slider stationary coordinate system

4.2 Mathematical Model of Plane-Wedge Slider Bearing

The study of lubricating films begins appropriately with the plane-wedge film (see Fig.4.2). It is the simplest shape to analyze, and its study provides qualitative

understanding of the properties of more complex film shapes. Since the films treated are infinitely long, side flow and derivatives with respect to Y are zero. The dimension is selectively normalized with respect to the trailing (here the minimum) film thickness $h_1 = h_m$ and bearing breadth B so that

$$H(X) = \frac{h_2 - h_1}{h_1} (1 - X) + 1 \quad (4.1)$$

Where H is the normalized film-thickness h/h_1 and $X=x/B$.

Fig. 4.2 The infinite long inclined slider bearing

As shown in Fig.4.2, the slider width is divided into a limited number of nodes. Except for the two end nodes (boundary node), the residual nodes are internal nodes. The pressure values of supporting points near a reference node can be approximated by the functional value and derivatives at the reference node by using the 1-D Taylor series expansion

$$P_j^i = P_i + \Delta X_{ij} \left(\frac{dP}{dX} \right)_i + \frac{1}{2} \Delta X_{ij}^2 \left(\frac{d^2 P}{dX^2} \right)_i + \frac{1}{3!} \Delta X_{ij}^3 \left(\frac{d^3 P}{dX^3} \right)_i + \frac{1}{4!} \Delta X_{ij}^4 \left(\frac{d^4 P}{dX^4} \right)_i \quad (4.2)$$

In general, the functional value p_j^i represents the pressure of the j th supporting point around the i th reference node where $j=1,\dots,SN$ (number of support points), ΔX_{ij} is the distance between the i th reference node and the j th support point.

Re-arranging above equations and writing them in matrix form, one has

$$\begin{Bmatrix} P_1^i - P_i \\ \vdots \\ P_j^i - P_i \\ \vdots \\ P_{SN}^i - P_i \end{Bmatrix} = \begin{bmatrix} \Delta X_{i1} & \frac{1}{2}\Delta X_{i1}^2 & \frac{1}{3!}\Delta X_{i1}^3 & \frac{1}{4!}\Delta X_{i1}^4 \\ \vdots & \vdots & \vdots & \vdots \\ \Delta X_{ij} & \frac{1}{2}\Delta X_{ij}^2 & \frac{1}{3!}\Delta X_{ij}^3 & \frac{1}{4!}\Delta X_{ij}^4 \\ \vdots & \vdots & \vdots & \vdots \\ \Delta X_{iSN} & \frac{1}{2}\Delta X_{iSN}^2 & \frac{1}{3!}\Delta X_{iSN}^3 & \frac{1}{4!}\Delta X_{iSN}^4 \end{bmatrix} \begin{Bmatrix} \left. \frac{dP}{dX} \right|_i \\ \left. \frac{d^2P}{dX^2} \right|_i \\ \left. \frac{d^3P}{dX^3} \right|_i \\ \left. \frac{d^4P}{dX^4} \right|_i \end{Bmatrix} \quad (4.3)$$

The derivatives at the reference node are determined by the functional values at this node and the support points of the node as

$$dP_{4 \times 1} = D_{4 \times 4} \left(\bar{S}_{4 \times SN}^T W_{SN \times SN} \bar{S}_{SN \times 4} \right)^{-1} \bar{S}_{4 \times SN}^T W_{SN \times SN} \Delta P_{SN \times 1} \quad (4.4)$$

Where the scaling matrix D is a $order \times order$ diagonal matrix ($order = 4$), and W is an $SN \times SN$ diagonal matrix

$$D = \begin{bmatrix} d_0^{-1} & & & \\ & d_0^{-2} & & \\ & & d_0^{-3} & \\ & & & d_0^{-4} \end{bmatrix}_{4 \times 4} \quad \mathbf{W} = \begin{bmatrix} W_1 & & 0 \\ & \ddots & \\ 0 & & W_{SN} \end{bmatrix}_{SN \times SN}$$

$$\bar{S}^T = \begin{bmatrix} \frac{\Delta X_{i1}}{d_0} & \dots & \frac{\Delta X_{ij}}{d_0} & \dots & \frac{\Delta X_{iSN}}{d_0} \\ \frac{\Delta X_{i1}^2}{2d_0^2} & \dots & \frac{\Delta X_{ij}^2}{2d_0^2} & \dots & \frac{\Delta X_{iSN}^2}{2d_0^2} \\ \frac{\Delta X_{i1}^3}{3!d_0^3} & \dots & \frac{\Delta X_{ij}^3}{3!d_0^3} & \dots & \frac{\Delta X_{iSN}^3}{3!d_0^3} \\ \frac{\Delta X_{i1}^4}{4!d_0^4} & \dots & \frac{\Delta X_{ij}^4}{4!d_0^4} & \dots & \frac{\Delta X_{iSN}^4}{4!d_0^4} \end{bmatrix}_{4 \times SN}$$

$$dP = \left\{ \begin{array}{c} \frac{dP}{dX} \Big|_i \\ \frac{d^2P}{dX^2} \Big|_i \\ \frac{d^3P}{dX^3} \Big|_i \\ \frac{d^4P}{dX^4} \Big|_i \end{array} \right\} \quad \Delta P = \left\{ \begin{array}{c} P_1^i - P_i \\ \vdots \\ P_j^i - P_i \\ \vdots \\ P_{SN}^i - P_i \end{array} \right\} \quad (4.5)$$

$$dP_{4 \times 1} = C_{4 \times SN} \Delta P_{SN \times 1} \quad (4.6)$$

d_0 : Maximal distance among the i th reference node and the j th support points.

Therefore, the first-, second-order derivative approximations are obtained by considering the first five entries of dP in Eq.(4.2). The inclusion of high-order terms can increase the accuracy of the LSFD method. It is observed that the coefficient matrix $C_{4 \times SN}$ at each node remains unchanged for a fixed set of supporting points and its inverse matrix needs to be calculated only once. Thus, the LSFD method gives the flexibility for complex problems. Moreover, the derivatives given in Eq.(4.4) are independent of the lubrication equations with different flow models, and can be used repeatedly for different models with the same distribution of node layout.

4.2.1 Continuum Flow

The one-dimensional Reynolds equation for gas bearing based on continuum flow is (Gross, 1977)

$$\begin{cases} \frac{d}{dX} \left[PH^3 \frac{dP}{dX} - \Lambda_x PH \right] = 0 \\ H(X) = \frac{h_2 - h_1}{h_1} (1 - X) + 1 \\ \Lambda_x = 6\mu UL / (p_a h_0^2) \end{cases} \quad (4.7)$$

The pressures at boundary nodes are equal to ambient pressure. For the i th inner node in the computation domain; the left hand side of the Reynolds equation in Eq.(4.7) can be expressed as

$$F_i = \left(\frac{dP}{dX} \right)_i^2 H_i^3 + 3P_i H_i^2 \frac{dH}{dX} \bigg|_i \frac{dP}{dX} \bigg|_i + \frac{d^2 P}{dX^2} \bigg|_i P_i H_i^3 - \Lambda_x H_i \frac{dP}{dX} \bigg|_i - \Lambda_x P_i \frac{dH}{dX} \bigg|_i = 0 \quad (4.8)$$

Where $i = 1, \dots, \text{Inode}$ (Inode: the total number of inner nodes)

Based on the LSFD mesh-free method, the pressure derivatives at the i th reference node can be discretized as

$$\begin{Bmatrix} \frac{dP}{dX} \bigg|_i \\ \frac{d^2 P}{dX^2} \bigg|_i \\ \frac{d^3 P}{dX^3} \bigg|_i \\ \frac{d^4 P}{dX^4} \bigg|_i \end{Bmatrix} = \begin{bmatrix} a_{11}^i & \dots & a_{1j}^i & \dots & a_{1SN}^i \\ a_{21}^i & \dots & a_{2j}^i & \dots & a_{2SN}^i \\ a_{31}^i & \dots & a_{3j}^i & \dots & a_{3SN}^i \\ a_{41}^i & \dots & a_{4j}^i & \dots & a_{4SN}^i \end{bmatrix} \begin{Bmatrix} P_1^i - P_i \\ \vdots \\ P_j^i - P_i \\ \vdots \\ P_{SN}^i - P_i \end{Bmatrix} \quad (4.9)$$

The derivatives of pressure at the i th reference node can be expressed as the weighting summation of pressure difference between the reference node P_i and support points P_j^i

$$\left. \frac{dP}{dX} \right|_i = \sum_j^{SN} a_{1j}^i (P_j^i - P_i) \quad \left. \frac{d^2 P}{dX^2} \right|_i = \sum_j^{SN} a_{2j}^i (P_j^i - P_i) \quad (4.10)$$

This can be re-arranged as

$$\left. \frac{dP}{dX} \right|_i = \sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \quad \left. \frac{d^2 P}{dX^2} \right|_i = \sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \quad (4.11)$$

Substitution of Eq.(4.11) into Eq.(4.8) gives

$$\begin{aligned} F_i = & \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right)^2 H_i^3 + 3P_i H_i^2 \left. \frac{dH}{dX} \right|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) \\ & + \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right) P_i H_i^3 - \Lambda H_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) - \Lambda P_i \left. \frac{dH}{dX} \right|_i = 0 \end{aligned} \quad (4.12)$$

Now, the Reynolds equation is discretized into a nonlinear system of equations, which are solved subsequently by the Successive Over-Relaxation (SOR) method to acquire pressure solutions. The idea of SOR method is to choose a value for extrapolation factor ω that will accelerate the rate of convergence of the iterations to the solution. In the iteration method, the pressure P_i^k of the i th reference node in Eq.(4.12) is updated at the k th iteration to obtain new pressure value P_i^{k+1} , which is the weighting summation of previous pressure and a pressure increment (Eq.(4.13)).

The whole procedure progresses continually till the convergence criterion is satisfied in Eq.(4.14)

$$P_i^{k+1} = P_i^k + \omega \Delta P_i^k \quad (4.13)$$

$$\sum_{i=1}^{inode} F_i^2 \leq \varepsilon \quad (4.14)$$

Since the discretized algebraic equations Eq.(4.12) are nonlinear, expanding Eq.(4.12) and neglecting higher order nonlinear items, pressure increment ΔP_i at this iteration can be solved

$$\begin{aligned} 0 = F_i + \Delta F_i = \frac{dP}{dX} \bigg|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right) H_i^3 + \\ 3P_i H_i^2 \frac{dH}{dX} \bigg|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right) + \\ \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) (P_i + \Delta P_i) \right) P_i H_i^3 - \\ \Lambda_x H_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) (P_i + \Delta P_i) \right) - \Lambda_x (P_i + \Delta P_i) \frac{dH}{dX} \bigg|_i \end{aligned} \quad (4.15)$$

Simplifying further the above equation yields

$$\begin{aligned} 0 = F_i + \left\{ \frac{dP}{dX} \bigg|_i \left(- \sum_j^{SN} a_{1j}^i \right) H_i^3 + 3P_i H_i^2 \frac{dH}{dX} \bigg|_i \left(- \sum_j^{SN} a_{1j}^i \right) + \right. \\ \left. \left(- \sum_j^{SN} a_{2j}^i \right) P_i H_i^3 - \Lambda_x H_i \left(- \sum_j^{SN} a_{1j}^i \right) - \Lambda_x \frac{dH}{dX} \bigg|_i \right\} \Delta P_i \end{aligned} \quad (4.16)$$

Consequently, achieving the pressure increment ΔP_i

$$\Delta P_i = \frac{-F_i}{deno} \quad (4.17)$$

$$\begin{aligned} deno = \left\{ \frac{dP}{dX} \bigg|_i \left(- \sum_j^{SN} a_{1j}^i \right) H_i^3 + 3P_i H_i^2 \frac{dH}{dX} \bigg|_i \left(- \sum_j^{SN} a_{1j}^i \right) + \right. \\ \left. \left(- \sum_j^{SN} a_{2j}^i \right) P_i H_i^3 - \Lambda_x H_i \left(- \sum_j^{SN} a_{1j}^i \right) - \Lambda_x \frac{dH}{dX} \bigg|_i \right\} \end{aligned} \quad (4.18)$$

4.2.2 First-Order Model

The one-dimensional Reynolds equation based on first-order model is expressed as follows (Burgdorfer, 1959)

$$\begin{cases} \frac{d}{dX} \left[QPH^3 \frac{dP}{dX} - \Lambda_x PH \right] = 0 \\ Q = 1 + 6 \frac{K_n}{PH} \\ H(X) = \frac{h_2 - h_1}{h_1} (1 - X) + 1 \\ \Lambda_x = 6\mu UL / (p_a h_0^2) \end{cases} \quad (4.19)$$

The first term on the left hand side of Eq.(4.19) is a component of poiseuille flow induced by a pressure gradient. The second term is a component of Couette flow induced by the relative slider motion between two bearing surfaces. According to the principle of the LSFD method, for the i th inner node, the first-order Reynolds equation can be expressed as

$$\begin{aligned} F_i = & \left(\frac{dP}{dX} \right)_i^2 H_i^3 + 3P_i H_i^2 \frac{dH}{dX} \bigg|_i \frac{dP}{dX} \bigg|_i + P_i H_i^3 \frac{d^2 P}{dX^2} \bigg|_i + 12K_n H_i \frac{dH}{dX} \bigg|_i \frac{dP}{dX} \bigg|_i \\ & + 6K_n H_i^2 \frac{d^2 P}{dX^2} \bigg|_i - \Lambda_x H_i \frac{dP}{dX} \bigg|_i - \Lambda_x P_i \frac{dH}{dX} \bigg|_i = 0 \end{aligned} \quad (4.20)$$

Substitution of Eq.(4.11) into Eq.(4.20) gives

$$\begin{aligned} F_i = & \left(\frac{dP}{dX} \right)_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] H_i^3 + 3P_i H_i^2 \frac{dH}{dX} \bigg|_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] + \\ & P_i H_i^3 \left[\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right] + 12K_n H_i \frac{dH}{dX} \bigg|_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] + \\ & 6K_n H_i^2 \left[\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right] - \Lambda_x H_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] - \Lambda_x P_i \frac{dH}{dX} \bigg|_i \end{aligned}$$

(4.21)

Applying the SOR method, the iterative step can be determined by

$$\Delta P_i = \frac{-F_i}{deno} \quad (4.22)$$

$$\begin{aligned} deno = & \left\{ \frac{dP}{dX} \Big|_i \left(-\sum_j^{SN} a_{1j}^i \right) H_i^3 + 3P_i H_i^2 \frac{dH}{dX} \Big|_i \left(-\sum_j^{SN} a_{1j}^i \right) + \right. \\ & \left(-\sum_j^{SN} a_{2j}^i \right) P_i H_i^3 + 12K_n H_i \frac{dH}{dX} \Big|_i \left(-\sum_j^{SN} a_{1j}^i \right) + \\ & \left. 6K_n H_i^2 \left(-\sum_j^{SN} a_{2j}^i \right) - \Lambda_x H_i \left(-\sum_j^{SN} a_{1j}^i \right) - \Lambda_x \frac{dH}{dX} \Big|_i \right\} \end{aligned} \quad (4.23)$$

4.2.3 Second-Order Model

Hsia, *et al.* (1983) proposed a higher order approximation equation for larger Knudsen numbers than the range for Burgdorfer's equation by considering both the first- and second-order slip flows. The main effect of the second order slip term is to increase the slip effects by permitting more side leakage flow, resulting in a decrease in the load carrying capacity for a fixed clearance height. The one-dimensional Reynolds equation based on second-order model is

$$\begin{cases} \frac{d}{dX} \left[QPH^3 \frac{dP}{dX} - \Lambda PH \right] = 0 \\ Q = 1 + 6 \frac{K_n}{PH} + 6 \left[\frac{K_n}{PH} \right]^2 \\ H(X) = \frac{h_2 - h_1}{h_1} (1 - X) + 1 \\ \Lambda_x = 6\mu UL / (p_a h_0^2) \end{cases} \quad (4.24)$$

Expanding Eq.(4.24) and rewriting the Reynolds equation based on second-order model at the i th reference node as

$$F_i = \left(\frac{dP}{dX} \right)_i^2 \left[H_i^3 - \frac{6K_n^2 H_i}{P_i^2} \right] + \frac{dH}{dX} \bigg|_i \frac{dP}{dX} \bigg|_i \left[3P_i H_i^2 + 12K_n H_i + \frac{6K_n^2}{P_i} \right] + \frac{d^2 P}{dX^2} \bigg|_i \left[P_i H_i^3 + 6K_n H_i^2 + \frac{6K_n^2 H_i}{P_i} \right] - \Lambda_x H_i \frac{dP}{dX} \bigg|_i - \Lambda_x P_i \frac{dH}{dX} \bigg|_i = 0 \quad (4.25)$$

According to the pressure derivatives given in Eq.(4.11), the second-order Reynolds equation at the i th reference node can be expressed as

$$F_i = \left(\frac{dP}{dX} \right)_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] \left[H_i^3 - \frac{6K_n^2 H_i}{P_i^2} \right] + \frac{dH}{dX} \bigg|_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] \left[3P_i H_i^2 + 12K_n H_i + \frac{6K_n^2}{P_i} \right] + \left[\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right] \left[P_i H_i^3 + 6K_n H_i^2 + \frac{6K_n^2 H_i}{P_i} \right] - \Lambda_x H_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] - \Lambda_x P_i \frac{dH}{dX} \bigg|_i \quad (4.26)$$

Applying the SOR method, the pressure increment ΔP_i can be obtained by

$$\Delta P_i = \frac{-F_i}{deno} \quad (4.27)$$

$$deno = \left\{ \frac{dP}{dX} \bigg|_i \left(- \sum_j^{SN} a_{1j}^i \right) \right] \left[H_i^3 - \frac{6K_n^2 H_i}{P_i^2} \right] + \frac{dH}{dX} \bigg|_i \left(- \sum_j^{SN} a_{1j}^i \right) \left[3P_i H_i^2 + 12K_n H_i + \frac{6K_n^2}{P_i} \right] + \left(- \sum_j^{SN} a_{2j}^i \right) \left[P_i H_i^3 + 6K_n H_i^2 + \frac{6K_n^2 H_i}{P_i} \right] - \Lambda_x H_i \left(- \sum_j^{SN} a_{1j}^i \right) - \Lambda_x \frac{dH}{dX} \bigg|_i \right\} \quad (4.28)$$

4.2.4 Fukui Model

Fukui and Kaneko (1988) started from a linearized Boltzmann equation based on the BGK model. The basic equation is decomposed so as to describe the fundamental flows that depend on the gradients of temperature, pressure and velocity. By considering mass conservation, these flows are incorporated later into a generalized Reynolds equation.

For slider bearings with finite width, the generalized Reynolds equation is as follows

$$\begin{cases} \frac{d}{dX} \left[QPH^3 \frac{dP}{dX} - \Lambda_x PH \right] = 0 \\ Q = \frac{Q_p(D, \alpha)}{Q_{con}(D)} \\ H(X) = \frac{h_2 - h_1}{h_1} (1 - X) + 1 \\ \Lambda_x = 6\mu UL / (p_a h_0^2) \end{cases} \quad (4.29)$$

Where $Q_p(D, \alpha)$ is a Poiseuille flow rate coefficient and Q_{con} is the coefficient for a continuum flow. Quantities D and α are an inverse Knudsen number and an accommodation coefficient, respectively

$$D = D_0 PH = ph / (\mu / \sqrt{2RT}) \quad Q_{con} = D/6 \quad (4.30)$$

Power series expressions for flow rate coefficients, Q_p , are effective for flow rate calculations. First, for small inverse Knudsen numbers, D , where the slip flow assumption is reasonably valid, the asymptotic expression for $K_n \ll 1$ can be employed. Power series representations for $0.15 \leq D \leq 5$ and $0.01 \leq D < 0.15$ were calculated using least squares method. The results are as follows (Fukui and Kaneko, 1990):

$$\begin{cases} Q_p = D/6 + 1.0162 + 1.0653/D - 2.1354/D^2 & (5 \leq D) \\ Q_p = 0.13852D + 1.25087 + 0.15653/D - 0.00969/D^2 & (0.15 \leq D < 5) \\ Q_p = -2.22919D + 2.10673 + 0.01653/D - 0.0000694/D^2 & (0.01 \leq D < 0.15) \end{cases} \quad (4.31)$$

If inverse Knudsen number D falls between 0.15 and 5, then Poiseuille flow rates Q can be obtained by using Eq.(4.30) and the second equation of Eq.(4.31)

$$\begin{aligned} Q &= \frac{Q_p(D, \alpha)}{Q_{con}(D)} = 0.83112 + 7.50522/D + 0.93918/D^2 - 0.05814/D^3 \\ &= K_1 + K_2(PH)^{-1} + K_3(PH)^{-2} - K_4(PH)^{-3} \end{aligned} \quad (4.32)$$

Where $((0.15 \leq D < 5))$

$$K_1=0.83112 \quad K_2=7.50522D_0^{-1} \quad K_3=0.93918D_0^{-2} \quad K_4=0.05814D_0^{-3}$$

Thus, similarly, the corresponding coefficients for other inverse Knudsen numbers are shown in Table 4.1.

Table 4.1 Coefficients for various inverse Knudsen numbers in Fukui model

Inverse Knudsen Number (D)	K1	K2	K3	K4
$(0.15 \leq D < 5)$	0.83112	$7.50522D_0^{-1}$	$0.93918D_0^{-2}$	$0.05814D_0^{-3}$
$(5 \leq D)$	1	$6.0972D_0^{-1}$	$6.3918D_0^{-2}$	$12.8124D_0^{-3}$
$(0.01 \leq D < 0.15)$	-13.37514	$12.64038D_0^{-1}$	$0.09918D_0^{-2}$	$0.0004164D_0^{-3}$

By means of $Q_p(D, \alpha)$ and $Q_{con}(D)$, the generalized lubrication equation based on the Boltzmann equation can then be expressed as

$$F = \frac{d}{dX} \left[(K_1 + K_2(PH)^{-1} + K_3(PH)^{-2} - K_4(PH)^{-3})PH^3 \frac{dP}{dX} - \Lambda_x PH \right] = 0 \quad (4.33)$$

For the i th reference node, the generalized Reynolds equation can be deduced to

$$\begin{aligned} F_i = & \left(\frac{dP}{dX} \right)_i^2 \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\ & \left. \frac{dH}{dX} \right|_i \left. \frac{dP}{dX} \right|_i \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\ & \left. \frac{d^2 P}{dX^2} \right|_i \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\ & \Lambda_x H_i \left. \frac{dP}{dX} \right|_i - \Lambda_x P_i \left. \frac{dH}{dX} \right|_i \end{aligned} \quad (4.34)$$

Substitution of equation (4.11) into equation (4.34) gives

$$\begin{aligned} F_i = & \left(\frac{dP}{dX} \right)_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\ & \left. \frac{dH}{dX} \right|_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\ & \left[\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right] \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\ & \Lambda_x H_i \left[\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right] - \Lambda_x P_i \left. \frac{dH}{dX} \right|_i \end{aligned} \quad (4.35)$$

Applying the SOR method, the pressure increment ΔP_i can be obtained by

$$\Delta P_i = \frac{-F_i}{deno} \quad (4.36)$$

$$\begin{aligned}
deno = & \left(\frac{dP}{dX} \right)_i \left(- \sum_j^{SN} a_{1j}^i \right) \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
& \frac{dH}{dX} \left| \left(- \sum_j^{SN} a_{1j}^i \right) \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \right. \\
& \left. \left(- \sum_j^{SN} a_{2j}^i \right) \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \right. \\
& \left. \Lambda_x H_i \left(- \sum_j^{SN} a_{1j}^i \right) - \Lambda_x \frac{dH}{dX} \right|
\end{aligned} \tag{4.37}$$

Fukui proposed interpolating a flow rate database to rapidly calculate ultra-thin lubrication problems. The database is created through numerical calculation based on the linearized Boltzmann equation. The interpolation method reduces the calculation time required, while the accuracy remains high, thereby it is a practical tool for ultra-thin flying head sliders.

4.3 Theoretical and Numerical Solutions of 1-D Reynolds equation

According to the principle and equations depicted above, the 1-D fluid lubrication problems based on various flow models were solved numerically for micro-flow and nano-flow in the space between the slider and the disk. The boundary condition is that pressures of boundary points are equal to atmospheric pressure. For the purpose of verification, the calculated pressure distributions, load carrying capacities, load center for air bearing will be compared with those obtained from analytical methods or other numerical methods.

4.3.1 Continuum Flow

In order to compare the present results with analytical solutions reported earlier (Gross, 1977), film-thickness ratio $h_1/h_2=1.5, 2.0$ are studied with air bearing number of $\Lambda_x=10$. The predicted pressure profile by the LSFD method is shown in Fig.4.3 together with the analytical results. The results of the LSFD simulation and the analytical results are in good agreement. The maximum value of relative error is 1.1369%. Moreover, the effects of film-thickness ratio on pressure distribution are also shown in Fig.4.3.

Fig. 4.3 Comparison of pressure profiles for the infinitely long plane-wedge gas film having film-thickness ratio $h_1/h_2=1.5, 2$, $\Lambda=10$

The effects of bearing number on pressure distribution are shown in Fig.4.4. The pressure values increase as the film-thickness ratio and air bearing number increase for the continuum flow.

Fig. 4.4 Effect of air bearing number on pressure profile for the infinitely long plane-wedge gas film

The load capacities of this system, obtained from the numerical and analytic method, as a function of air bearing number are shown in Fig 4.5, when the thin film ratio are 1.5, 2. The load capacity integrated from pressure distribution, which is calculated using the Reynolds equation based on continuum flow by the LSFD method, is also in good agreement with that using analytical method for pressure distribution.

The effects of air bearing number and film-thickness ratio on load are shown in Figs.4.6 and 4.7. Note that the asymptotic load for high Λ is delayed as H_1 increases as shown in Fig 4.6. This is reasonable, since Λ is based on the trailing-edge thickness. The value of Λ is inversely proportional to the square of this dimension. It follows that the influence of Λ based on local film thickness is bigger than that of the film thickness for air bearing load capacities. The predicted load capacity profiles for all

five air bearing numbers are included in Fig 4.7. The maximum value of load capacity increases gradually with the air bearing number.

Fig. 4.5 Comparison of bearing load for the infinitely long plane-wedge gas film having film-thickness ratio $h_1/h_2=1.5, 2$

Fig. 4.6 Effect of film-thickness ratio and bearing number on bearing load for the infinitely long plane-wedge gas films

Fig. 4.7 Effect of film-thickness ratio on load for the infinitely long plane-wedge gas films for various bearing numbers

It is apparent that, both the load and the moment of the pressure distribution are transcendental functions of the film-thickness ratio H_1 and of air bearing number Λ_x . Because of the dependence on Λ_x , the position of the center of pressure is not determined completely by the film-thickness ratio. Fig.4.8 further compares the center of pressure calculated using the LSFD method with that using analytical method when film-thickness ratio are 1.5, 2 respectively. Similarly, the result differences of two methods are very little as shown in Fig.4.8.

Fig.4.9 shows that the effects of bearing number and film-thickness ratio upon center of pressure. In general, the film-thickness ratio of a gas-lubricated pivoted plane-surface bearing will not remain constant under load changes, since $\bar{x} = \bar{x}(H_1, \Lambda_x)$.

The point of maximum pressure moves to the trailing edge, when the film-thickness ratio increases.

Fig. 4.8 Comparison of center of pressure for the infinitely long plane-wedge gas film having film-thickness ratio $h_1/h_2=1.5, 2$

Fig. 4.9 Effect of bearing number on center of pressure for the infinitely long plane-wedge gas films

4.3.2 Numerical Results of First-order Slip Model Flow

Fig.4.10 shows a comparison between the pressures distributions obtained from analytical (Gross, 1977) and the LSFD methods for the Reynolds equation based on first-order slip model for certain Knudsen number. This figure shows that the maximum value of relative error is 1.315%.

Fig. 4.10 Comparison of pressure distribution based on first-order slip model along the straight slider bearing

4.3.3 Comparison of Numerical Results for Various Flow Models

To solve the Reynolds equation numerically, the derivatives were discretized with their Least Square Finite Difference approximations. The resulting set of nonlinear equations is solved iteratively using Successive Over-Relaxation method. For the

Fukui-Kaneko correction, the power series method is used to acquire flow rate coefficients, which is effective for flow rate calculations.

For comparison, we consider a system of length $L = 5\mu\text{m}$ with outlet height $h_2=50\text{nm}$, and inlet height $h_1=100\text{nm}$. The ambient pressure p_a is atmospheric pressure, and the ambient temperature $T_0=273\text{K}$. The mean free path is 62nm , and the viscosity of the gas is $\mu=2.08\text{e-}05\text{Ns/m}^2$. The accommodation coefficient is unity ($\alpha=1$), so the walls are fully accommodating. The Knudsen number at the exit is $K_n=1.25$, the air bearing number is $\Lambda=61.6$.

When molecules strike a fully accommodating thermal wall ($\alpha=1$), molecules emitting from the boundary obey the local Maxwell distribution function whose temperature and macroscopic velocities are those at the boundary. If the accommodation coefficient α is less than 1, then $(1-\alpha)$ of the molecules arriving at the boundary reflect specularly and α of them reflect diffusely. If the accommodation coefficient α equals zero, molecules arriving at the boundary reflect specularly without change of speed. The main effect of the surface accommodation coefficient is to increase the degree of slip and decrease the load carrying capacity by permitting more side leakage flow. Generally, accommodation coefficient is set to 0.89 or 1.0 for most gas-surface interactions.

The solutions of the Reynolds equations based on various models using the LSFD method are presented in Fig.4.11. The normalized load capacity and pressure center values are summarized in Table 4.2. As in Fig.4.11, the pressure profile from the

continuum (no-slip) Reynolds equation is too high. The pressure profile given by the first order slip-correction, as compared with the Fukui prediction, over-estimates the pressure and hence the load capacity, whereas the second-order slip model under-estimates them.

Fig. 4.11 Comparison of pressure distribution with the generalized lubrication equation solution of Fukui and Kaneko, and the Reynolds equations results with first- and second-order slip, continuum models ($K_n=1.25, \Lambda=61.6$)

Table 4.2 Load capacities and pressure centers ($\Lambda=61.6, K_n=1.25, \alpha=1.0$)

	Continuum	First-order	Second-order	Fukui
Load Capacity	0.36042435	0.22471682	0.15386772	0.174340804
Pressure Center	0.68680831	0.61657915	0.58523999	0.597125271

4.3.4 Step Slider Air Bearing

In the one-dimensional case, first-order Burgdorfer lubrication equation can be integrated numerically for two-phase slider-bearing configurations. One of these is the Rayleigh step bearing.

As the theory of thick-film lubrication became generally accepted, a natural desire arose to use the theory to deduce the optimum load-carrying shape. It is not surprising that Lord Rayleigh (1912) first performed this investigation (Gross, 1977). Rayleigh applied the calculus of variations to determine the optimum geometry for the incompressible film. Now the corresponding bearing is called the Rayleigh step bearing (Fig.4.12). In calculating the step slider pressure distribution, because the film thickness is not continuous, so a slope is used to approximate it (Fig.4.13), where $B_1=0.4B$, $B_2=0.1B$, $B_3=0.5B$.

Fig. 4.12 Step slider bearing with length ratio $\lambda=B_2/B_1$

Fig. 4.13 Modified step slider bearing

Fig.4.14 illustrates the pressure profiles for an infinitely long bearing having $H_1=h_1/h_2=2.33$, $\Lambda=26.6$, and $\lambda=B_2/B_1=1$. The maximum pressure occurs at the conjunction of two steps. The maximum relative error between the analytical and numerical solution is 5.38%. The larger error is caused by the approximation in the numerical model, which uses a slope instead of pure step.

According to analytic calculation results (Gross, 1977), the load carried by the Rayleigh step bearing is less than that carried by the optimum plane slider bearing. This conclusion is based on infinitely long constant-viscosity films. For finite long bearings, more side flow is expected from the step bearing than that from the plane slider bearing because the maximum pressure of the step bearing occurs at the step where the film thickness is maximal, a condition that encourages side flow. Recognizing the characteristics, Archibald (Gross, 1977) suggested curving the step in the plane of the bearing to contain the region of large film thickness, thus inhibiting

side flow. The maximum pressure in a plane slider bearing occurs where the film thickness is nearly minimal. Therefore we expect the plane slider bearing to take comparatively more load when the slenderness ratio, L/B is small.

Fig. 4.14 Pressure profiles for the infinitely long gas step bearings

4.4 Theoretical and Numerical Solutions of 2-D Reynolds Equation

4.4.1 Introduction

In the head/disk interface system, the rotation of the disk surface causes the motion of air under the slider that creates the pressure and provides the lifting force that causes the head to float above the disk. The prediction of the vertical force on the head is a crucial design calculation since the head will not accurately read or write if it flies too

high or low. Since this height is typically smaller than the mean free path of molecules in the air, micro and nano-scopic effects are important.

Traditionally, macroscopic hydrodynamic equations (e.g., Navier-Stokes, Reynolds) have been used to model slider air bearing problems. Thus far, three kinds of modified Reynolds equations have been presented: those based on the first-order slip-flow model (Burgdorfer, 1959) (herein named the first-order modified Reynolds equation), those based on the second-order slip-flow model (Hsia, *et al.*, 1983) (the second-order modified Reynolds equation), and those based on the Boltzmann equation (Fukui, *et al.*, 1990) (generalized Reynolds equation). The former two models hold for a small K_n value, while the latter one holds no matter how large the K_n value is. Therefore, the generalized equation is most suitable. However, the first- and second-order modified Reynolds equations are still widely in practical use. Therefore, it is necessary to calculate the pressure distribution and the resultant force using the LSFD method for different models. The objective is to investigate the validity of the three-dimensional slider air bearing simulation using the LSFD mesh-free method compared with other numerical methods.

4.4.2 Mathematical Model of Flat Slider Bearing

Fig.4.15 shows a typical three-dimensional (3-D) flat slider gas bearing configuration. The flying height is normalized with respect to the trailing (here the minimum) film thickness h_m .

$$H(X) = 1 + \frac{L}{h_m} (1 - X) \lg \alpha \quad (4.38)$$

Where H is normalized the film-thickness $H = h/h_m$, $X = x/L$ and $Y = y/L$. Here, only pitch angle α is considered.

To calculate the pressure distribution of the above 3-D slider air bearing, the 2-D computation domain of slider air bearing is divided into a number of nodes, which include inner nodes and boundary nodes. The pressure of a reference node can be approximated by using the 2-D Taylor series expansion

Fig. 4.15 The inclined slider air bearing

$$\begin{aligned}
 P_j^i = & P_i + \Delta X_{ij} \left(\frac{\partial P}{\partial X} \right)_i + \Delta Y_{ij} \left(\frac{\partial P}{\partial Y} \right)_i + \frac{1}{2} (\Delta X_{ij})^2 \left(\frac{\partial^2 P}{\partial X^2} \right)_i + \frac{1}{2} (\Delta Y_{ij})^2 \left(\frac{\partial^2 P}{\partial Y^2} \right)_i + \\
 & (\Delta X_{ij}) (\Delta Y_{ij}) \left(\frac{\partial^2 P}{\partial X \partial Y} \right)_i + \frac{1}{6} (\Delta X_{ij})^3 \left(\frac{\partial^3 P}{\partial X^3} \right)_i + \frac{1}{6} (\Delta Y_{ij})^3 \left(\frac{\partial^3 P}{\partial Y^3} \right)_i + \quad (4.39) \\
 & \frac{1}{2} (\Delta X_{ij})^2 (\Delta Y_{ij}) \left(\frac{\partial^3 P}{\partial X^2 \partial Y} \right)_i + \frac{1}{2} (\Delta X_{ij}) (\Delta Y_{ij})^2 \left(\frac{\partial^3 P}{\partial X \partial Y^2} \right)_i
 \end{aligned}$$

Where P_j^i ($j = 1, \dots, SN$) is the pressure of the j th supporting point around the i th reference node, P_i is the pressure of the reference node. SN: the number of supporting points, $i = 1, Inode$ (the number of inner nodes)

In Eq.(4.39), the number of supporting points should be equal to or larger than the total number of the expanded terms in Taylor expansion to guarantee the matrix inverse.

Re-arrange the above equation to matrix form

$$\begin{Bmatrix} P_1^i - P_i \\ \vdots \\ P_j^i - P_i \\ \vdots \\ P_{SN}^i - P_i \end{Bmatrix} = \begin{bmatrix} \Delta X_{i1} & \Delta Y_{i1} & \frac{1}{2}(\Delta X_{i1})^2 & \frac{1}{2}(\Delta Y_{i1})^2 & \Delta X_{i1}\Delta Y_{i1} & \frac{1}{6}(\Delta Y_{i1})^3 \\ & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta X_{ij} & \Delta Y_{ij} & \frac{1}{2}(\Delta X_{ij})^2 & \frac{1}{2}(\Delta Y_{ij})^2 & \Delta X_{ij}\Delta Y_{ij} & \frac{1}{6}(\Delta X_{ij})^3 \\ & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta X_{iSN} & \Delta Y_{iSN} & \frac{1}{2}(\Delta X_{iSN})^2 & \frac{1}{2}(\Delta Y_{iSN})^2 & \Delta X_{iSN}\Delta Y_{iSN} & \frac{1}{6}(\Delta X_{iSN})^3 \end{bmatrix} \begin{bmatrix} \frac{1}{6}(\Delta Y_{i1})^3 & \frac{1}{2}(\Delta X_{i1})^2\Delta Y_{i1} & \frac{1}{2}\Delta X_{i1}(\Delta Y_{i1})^2 \\ \vdots & \vdots & \vdots \\ \frac{1}{2}(\Delta X_{ij})^2\Delta Y_{ij} & \frac{1}{2}(\Delta X_{ij})^2\Delta Y_{i1} & \frac{1}{2}\Delta X_{ij}(\Delta Y_{ij})^2 \\ \vdots & \vdots & \vdots \\ \frac{1}{6}(\Delta Y_{iSN})^3 & \frac{1}{2}(\Delta X_{iSN})^2\Delta Y_{iSN} & \frac{1}{2}\Delta X_{iSN}(\Delta Y_{iSN})^2 \end{bmatrix} \begin{bmatrix} \frac{\partial P}{\partial X} \Big|_i & \frac{\partial P}{\partial Y} \Big|_i & \frac{\partial^2 P}{\partial X^2} \Big|_i & \frac{\partial^2 P}{\partial Y^2} \Big|_i & \frac{\partial^2 P}{\partial X \partial Y} \Big|_i & \frac{\partial^3 P}{\partial X^3} \Big|_i & \frac{\partial^3 P}{\partial Y^3} \Big|_i & \frac{\partial^3 P}{\partial X^2 \partial Y} \Big|_i & \frac{\partial^3 P}{\partial Y^2 \partial X} \Big|_i \end{bmatrix}^T \quad (4.40)$$

Based on the theory introduced in Chapter 3, approximate derivatives follow

$$dP_{9 \times 1} = D_{9 \times 9} \left(\bar{S}^T_{9 \times SN} W_{SN \times SN} \bar{S}_{SN \times 9} \right)^{-1} \bar{S}^T_{9 \times 9} W_{SN \times SN} \Delta P_{SN \times 1} \quad (4.41)$$

Where the scaling matrix D

$$D = \text{diag}(d_0^{-1}, d_0^{-1}, d_0^{-2}, d_0^{-2}, d_0^{-2}, d_0^{-3}, d_0^{-3}, d_0^{-3}, d_0^{-3})$$

$$dP = \left[\frac{\partial P}{\partial X} \Big|_i, \frac{\partial P}{\partial Y} \Big|_i, \frac{\partial^2 P}{\partial X^2} \Big|_i, \frac{\partial^2 P}{\partial Y^2} \Big|_i, \frac{\partial^2 P}{\partial X \partial Y} \Big|_i, \frac{\partial^3 P}{\partial X^3} \Big|_i, \frac{\partial^3 P}{\partial Y^3} \Big|_i, \frac{\partial^3 P}{\partial X^2 \partial Y} \Big|_i, \frac{\partial^3 P}{\partial Y^2 \partial X} \Big|_i \right]^T$$

$$\bar{S}^T = \begin{bmatrix} \frac{\Delta X_{il}}{d_0} & \dots & \frac{\Delta X_{ij}}{d_0} & \dots & \frac{\Delta X_{isN}}{d_0} \\ \frac{\Delta Y_{il}}{d_0} & \dots & \frac{\Delta Y_{ij}}{d_0} & \dots & \frac{\Delta Y_{isN}}{d_0} \\ \frac{\Delta X_{il}^2}{2d_0^2} & \dots & \frac{\Delta X_{ij}^2}{2d_0^2} & \dots & \frac{\Delta X_{isN}^2}{2d_0^2} \\ \frac{\Delta Y_{il}^2}{2d_0^2} & \dots & \frac{\Delta Y_{ij}^2}{2d_0^2} & \dots & \frac{\Delta Y_{isN}^2}{2d_0^2} \\ \frac{\Delta X_{il} \Delta Y_{il}}{d_0^2} & \dots & \frac{\Delta X_{ij} \Delta Y_{ij}}{d_0^2} & \dots & \frac{\Delta X_{isN} \Delta Y_{isN}}{d_0^2} \\ \frac{\Delta X_{il}^3}{6d_0^3} & \dots & \frac{\Delta X_{ij}^3}{6d_0^3} & \dots & \frac{\Delta X_{isN}^3}{6d_0^3} \\ \frac{\Delta Y_{il}^3}{6d_0^3} & \dots & \frac{\Delta Y_{ij}^3}{6d_0^3} & \dots & \frac{\Delta Y_{isN}^3}{6d_0^3} \\ \frac{\Delta X_{il}^2 \Delta Y_{il}}{2d_0^3} & \dots & \frac{\Delta X_{ij}^2 \Delta Y_{ij}}{2d_0^3} & \dots & \frac{\Delta X_{isN}^2 \Delta Y_{isN}}{2d_0^3} \\ \frac{\Delta Y_{il}^2 \Delta X_{il}}{2d_0^3} & \dots & \frac{\Delta Y_{ij}^2 \Delta X_{ij}}{2d_0^3} & \dots & \frac{\Delta Y_{isN}^2 \Delta X_{isN}}{2d_0^3} \end{bmatrix}_{9 \times SN}$$

$$\mathbf{W} = \begin{pmatrix} W_1 & & 0 \\ & \ddots & \\ 0 & & W_{SN} \end{pmatrix} \quad \Delta P = \begin{Bmatrix} P_1^i - P_i \\ \vdots \\ P_j^i - P_i \\ \vdots \\ P_{SN}^i - P_i \end{Bmatrix} \quad (4.42)$$

In computing the pressure distribution of slider air bearing, the number of supporting points is constant. Thus, d_0 is the maximum distance among the reference node and supporting knots. The pressure derivative of different orders at the reference node is approximated by the information at a set of supporting nodes, without any pre-specified connectivity among the nodes. Moreover, it is not necessary to maintain detailed structural information about the computational mesh, comprising all nodal and element-based connectivity and hierarchical data.

4.4.3 Continuum Flow

The generalized slider air bearing shape is shown in Fig 4.15. The origin of the X, Y and Z coordinate system is chosen to be in the plane of the driving surface at the center of the bearing leading edge. The film thickness is in general a function of both X and Y.

The analysis of finite, steady films with plane-wedge slider air bearing starts with two-dimensional continuum Reynolds equation

$$\begin{cases} \frac{\partial}{\partial X} \left[PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right] + \frac{\partial}{\partial Y} \left[PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right] = 0 \\ \Lambda_x = \frac{6\mu_a UL}{h_0^2 P_a}, \quad \Lambda_y = \frac{6\mu_a VL}{h_0^2 P_a} \\ H(X) = 1 + \frac{L}{h_m} (1-X) \tan \alpha \\ \frac{\partial H}{\partial X} = -\frac{L}{h_m} \tan \alpha, \quad \frac{\partial H}{\partial Y} = 0 \end{cases} \quad (4.43)$$

The non-dimensional pressures of boundary nodes are equal to ambient pressure. For every inner node, the left hand side of the Reynolds equation in Eq.(4.43) can be written as

$$\begin{aligned} F_i &= \left(\frac{\partial P}{\partial X} \right)_i^2 H_i^3 + 3P_i H_i^2 \frac{\partial H}{\partial X} \bigg|_i \frac{\partial P}{\partial X} \bigg|_i + \frac{\partial^2 P}{\partial X^2} \bigg|_i P_i H_i^3 - \Lambda_x H_i \frac{\partial P}{\partial X} \bigg|_i \\ &\quad - \Lambda_y P_i \frac{\partial H}{\partial X} \bigg|_i + \left(\frac{\partial P}{\partial Y} \right)_i^2 H_i^3 + 3P_i H_i^2 \frac{\partial H}{\partial Y} \bigg|_i \frac{\partial P}{\partial Y} \bigg|_i + \frac{\partial^2 P}{\partial Y^2} \bigg|_i P_i H_i^3 = 0 \end{aligned} \quad (4.44)$$

Where $i = 1, \dots, \text{Inode}$ (Inode: the number of inner nodes)

Based on the LSFD method, the derivatives of pressure at the reference node can be expressed as the weighting function of pressure differences between the node and supporting points,

$$\begin{Bmatrix} \frac{\partial P}{\partial X} \bigg|_i \\ \frac{\partial P}{\partial Y} \bigg|_i \\ \frac{\partial^2 P}{\partial X^2} \bigg|_i \\ \frac{\partial^2 P}{\partial Y^2} \bigg|_i \end{Bmatrix} = \begin{bmatrix} a_{11}^i & \dots & a_{1j}^i & \dots & a_{1SN}^i \\ a_{21}^i & \dots & a_{2j}^i & \dots & a_{2SN}^i \\ a_{31}^i & \dots & a_{3j}^i & \dots & a_{3SN}^i \\ a_{41}^i & \dots & a_{4j}^i & \dots & a_{4SN}^i \end{bmatrix} \begin{Bmatrix} P_1^i - P_i \\ \vdots \\ P_j^i - P_i \\ \vdots \\ P_{SN}^i - P_i \end{Bmatrix} \quad (4.45)$$

Eq. (4.45) can be expressed in the following form:

$$\begin{aligned}
\left. \frac{\partial P}{\partial X} \right|_i &= \sum_j^{SN} a_{1j}^i (P_j^i - P_i) & \left. \frac{\partial^2 P}{\partial X^2} \right|_i &= \sum_j^{SN} a_{3j}^i (P_j^i - P_i) \\
\left. \frac{\partial P}{\partial Y} \right|_i &= \sum_j^{SN} a_{2j}^i (P_j^i - P_i) & \left. \frac{\partial^2 P}{\partial Y^2} \right|_i &= \sum_j^{SN} a_{4j}^i (P_j^i - P_i)
\end{aligned} \quad (4.46)$$

Or, in more detail

$$\begin{aligned}
\left. \frac{\partial P}{\partial X} \right|_i &= \sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i & \left. \frac{\partial^2 P}{\partial X^2} \right|_i &= \sum_j^{SN} a_{3j}^i P_j^i - \left(\sum_j^{SN} a_{3j}^i \right) P_i \\
\left. \frac{\partial P}{\partial Y} \right|_i &= \sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i & \left. \frac{\partial^2 P}{\partial Y^2} \right|_i &= \sum_j^{SN} a_{4j}^i P_j^i - \left(\sum_j^{SN} a_{4j}^i \right) P_i
\end{aligned} \quad (4.47)$$

Where $a_{1j}^i, a_{2j}^i, a_{3j}^i, a_{4j}^i$ are weighting coefficients of different order derivatives. These are determined by the coordinates of the reference node and supporting points.

Substitution of Eq.(4.17) into Eq.(4.44) gives

$$\begin{aligned}
F_i &= \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right)^2 H_i^3 + 3P_i H_i^2 \left. \frac{\partial H}{\partial X} \right|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) + \\
&\quad \left(\sum_j^{SN} a_{3j}^i P_j^i - \left(\sum_j^{SN} a_{3j}^i \right) P_i \right) P_i H_i^3 - \Lambda_s H_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) - \Lambda P_i \left. \frac{\partial H}{\partial X} \right|_i + \\
&\quad \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right)^2 H_i^3 + 3P_i H_i^2 \left. \frac{\partial H}{\partial Y} \right|_i \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right) + \\
&\quad \left(\sum_j^{SN} a_{4j}^i P_j^i - \left(\sum_j^{SN} a_{4j}^i \right) P_i \right) P_i H_i^3
\end{aligned} \quad (4.48)$$

Applying the SOR method, the iterative step can be determined

$$\begin{aligned}
deno = & \frac{\partial P}{\partial X} \Big|_i \left(- \sum_j^{SN} a_{1j}^i \right) H_i^3 + 3P_i H_i^2 \frac{\partial H}{\partial X} \Big|_i \left(- \sum_j^{SN} a_{1j}^i \right) + \\
& \left(- \sum_j^{SN} a_{3j}^i \right) P_i H_i^3 - \Lambda_x H_i \left(- \sum_j^{SN} a_{1j}^i \right) - \Lambda_x \frac{\partial H}{\partial X} \Big|_i + \\
& \frac{\partial P}{\partial Y} \Big|_i \left(- \sum_j^{SN} a_{2j}^i \right) H_i^3 + 3P_i H_i^2 \frac{\partial H}{\partial Y} \Big|_i \left(- \sum_j^{SN} a_{2j}^i \right) + \left(- \sum_j^{SN} a_{4j}^i \right) P_i H_i^3
\end{aligned} \tag{4.49}$$

The iteration procedure is the same as that for the solution of one-dimensional Reynolds equation presented in previous section.

4.4.4 First-Order Model

The two-dimensional Reynolds equation based on first-order model is (Burgdorfer, 1959)

$$\begin{cases}
\frac{\partial}{\partial X} \left[QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right] + \frac{\partial}{\partial Y} \left[QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right] = 0 \\
Q = 1 + 6 \frac{K_a}{PH} \\
\Lambda_x = \frac{6\mu_a UL}{h_0^2 P_a}, \quad \Lambda_y = \frac{6\mu_a VL}{h_0^2 P_a} \\
H(X) = 1 + \frac{L}{h_m} (1-X) \tan \alpha \\
\frac{\partial H}{\partial X} = -\frac{L}{h_m} \tan \alpha, \quad \frac{\partial H}{\partial Y} = 0
\end{cases} \tag{4.50}$$

Similarly, again for the i th inner node, the Reynolds equation can be expressed as

$$\begin{aligned}
F_i = & \left(\frac{\partial P}{\partial X} \right)_i^2 H_i^3 + 3P_i H_i^2 \frac{\partial H}{\partial X} \bigg|_i \frac{\partial P}{\partial X} \bigg|_i + \frac{\partial^2 P}{\partial X^2} \bigg|_i P_i H_i^3 + 12K_n H_i \frac{\partial P}{\partial X} \bigg|_i \frac{\partial H}{\partial X} \bigg|_i \\
& + 6K_n H_i^2 \frac{\partial^2 P}{\partial X^2} \bigg|_i - \Lambda_x H_i \frac{\partial P}{\partial X} \bigg|_i - \Lambda_x P_i \frac{\partial H}{\partial X} \bigg|_i + \left(\frac{\partial P}{\partial Y} \right)_i^2 H_i^3 \\
& + 3P_i H_i^2 \frac{\partial H}{\partial Y} \bigg|_i \frac{\partial P}{\partial Y} \bigg|_i + \frac{\partial^2 P}{\partial Y^2} \bigg|_i P_i H_i^3 + 12K_n H_i \frac{\partial P}{\partial Y} \bigg|_i \frac{\partial H}{\partial Y} \bigg|_i + 6K_n H_i^2 \frac{\partial^2 P}{\partial Y^2} \bigg|_i = 0
\end{aligned} \tag{4.51}$$

Using the LSFD method to discretize the derivative terms

$$\begin{aligned}
F_i = & \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right)^2 H_i^3 + 3P_i H_i^2 \frac{\partial H}{\partial X} \bigg|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) + \\
& \left(\sum_j^{SN} a_{3j}^i P_j^i - \left(\sum_j^{SN} a_{3j}^i \right) P_i \right) P_i H_i^3 + 12K_n H_i \frac{\partial H}{\partial X} \bigg|_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) + \\
& 6K_n H_i^2 \left(\sum_j^{SN} a_{3j}^i P_j^i - \left(\sum_j^{SN} a_{3j}^i \right) P_i \right) - \Lambda_x H_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) - \Lambda_x P_i \frac{\partial H}{\partial X} \bigg|_i + \\
& \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right)^2 H_i^3 + 3P_i H_i^2 \frac{\partial H}{\partial Y} \bigg|_i \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right) + \\
& \left(\sum_j^{SN} a_{4j}^i P_j^i - \left(\sum_j^{SN} a_{4j}^i \right) P_i \right) P_i H_i^3 + 12K_n H_i \frac{\partial H}{\partial Y} \bigg|_i \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right) + \\
& 6K_n H_i^2 \left(\sum_j^{SN} a_{4j}^i P_j^i - \left(\sum_j^{SN} a_{4j}^i \right) P_i \right)
\end{aligned} \tag{4.52}$$

Applying the SOR method, the iteration step can be obtained by

$$\Delta P_i = \frac{-F_i}{deno}$$

$$\begin{aligned}
deno = & \frac{\partial P}{\partial X} \Big|_i \left(- \sum_j^{SN} a_{1j}^i \right) H_i^3 + 3P_i H_i^2 \frac{\partial H}{\partial X} \Big|_i \left(- \sum_j^{SN} a_{1j}^i \right) + \left(- \sum_j^{SN} a_{3j}^i \right) P_i H_i^3 + \\
& 12K_n H_i \frac{\partial H}{\partial X} \Big|_i \left(- \sum_j^{SN} a_{1j}^i \right) + 6K_n H_i^2 \left(- \sum_j^{SN} a_{3j}^i \right) + \frac{\partial P}{\partial Y} \Big|_i \left(- \sum_j^{SN} a_{2j}^i \right) H_i^3 + \\
& 3P_i H_i^2 \frac{\partial H}{\partial Y} \Big|_i \left(- \sum_j^{SN} a_{2j}^i \right) + \left(- \sum_j^{SN} a_{4j}^i \right) P_i H_i^3 + 12K_n H_i \frac{\partial H}{\partial Y} \Big|_i \left(- \sum_j^{SN} a_{2j}^i \right) + \\
& 6K_n H_i^2 \left(- \sum_j^{SN} a_{4j}^i \right) - \Lambda_x H_i \left(- \sum_j^{SN} a_{1j}^i \right) - \Lambda_x \frac{\partial H}{\partial X} \Big|_i
\end{aligned} \quad (4.53)$$

4.4.5 Second-Order Model

The two-dimensional Reynolds equation based on second-order model is (Hsia, *et al.*, 1983)

$$\begin{cases}
\frac{\partial}{\partial X} \left[QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right] + \frac{\partial}{\partial Y} \left[QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right] = 0 \\
Q = 1 + 6 \frac{K_n}{PH} + 6 \left[\frac{K_n}{PH} \right]^2 \\
\Lambda_x = \frac{6\mu_a UL}{h_0^2 P_a}, \quad \Lambda_y = \frac{6\mu_a VL}{h_0^2 P_a} \\
H(X) = 1 + \frac{L}{h_m} (1-X) \tan \alpha \\
\frac{\partial H}{\partial X} = -\frac{L}{h_m} \tan \alpha, \quad \frac{\partial H}{\partial Y} = 0
\end{cases} \quad (4.54)$$

Similarly, again for every inner node, one has

$$\begin{aligned}
F_i = & \left[\left(\frac{\partial P}{\partial X} \right)_i^2 + \left(\frac{\partial P}{\partial Y} \right)_i^2 \right] \left[H_i^3 - \frac{6K_n^2 H_i}{P_i^2} \right] + \left(\frac{\partial H}{\partial X} \Big|_i \frac{\partial P}{\partial X} \Big|_i + \frac{\partial H}{\partial Y} \Big|_i \frac{\partial P}{\partial Y} \Big|_i \right) \\
& \left[3P_i H_i^2 + 12K_n H_i + \frac{6K_n^2}{P_i} \right] + \left(\frac{\partial^2 P}{\partial X^2} \Big|_i + \frac{\partial^2 P}{\partial Y^2} \Big|_i \right) \\
& \left[P_i H_i^3 + 6K_n H_i^2 + \frac{6K_n^2 H_i}{P_i} \right] - \Lambda_x H_i \frac{\partial P}{\partial X} \Big|_i - \Lambda_x P_i \frac{\partial H}{\partial X} \Big|_i = 0
\end{aligned} \quad (4.55)$$

Using the LSFD method to discretize the differential equation

$$\begin{aligned}
 F_i = & \left[\left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right)^2 + \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right)^2 \right] \\
 & \left[H_i^3 - \frac{6K_n^2 H_i}{P_i^2} \right] + \left(\frac{\partial H}{\partial X} \right)_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) + \frac{\partial H}{\partial Y} \left(\sum_j^{SN} a_{2j}^i P_j^i - \left(\sum_j^{SN} a_{2j}^i \right) P_i \right) \\
 & \left[3P_i H_i^2 + 12K_n H_i + \frac{6K_n^2}{P_i} \right] + \\
 & \left[\left(\sum_j^{SN} a_{3j}^i P_j^i - \left(\sum_j^{SN} a_{3j}^i \right) P_i \right) + \left(\sum_j^{SN} a_{4j}^i P_j^i - \left(\sum_j^{SN} a_{4j}^i \right) P_i \right) \right] \\
 & \left[P_i H_i^3 + 6K_n H_i^2 + \frac{6K_n^2 H_i}{P_i} \right] - \Lambda_x H_i \left(\sum_j^{SN} a_{1j}^i P_j^i - \left(\sum_j^{SN} a_{1j}^i \right) P_i \right) - \Lambda_x P_i \frac{\partial H}{\partial X} \Big|_i = 0
 \end{aligned} \tag{4.56}$$

Applying the Gauss-Seidel iteration method, the iterative step can be determined by

$$\begin{aligned}
 \Delta P_i = & \frac{-F_i}{deno} \\
 deno = & \left(\frac{\partial P}{\partial X} \right)_i \left(- \left(\sum_j^{SN} a_{1j}^i \right) \right) + \frac{\partial P}{\partial Y} \left(- \left(\sum_j^{SN} a_{2j}^i \right) \right) \left[H_i^3 - \frac{6K_n^2 H_i}{P_i^2} \right] + \\
 & \left(\frac{\partial H}{\partial X} \right)_i \left(- \left(\sum_j^{SN} a_{1j}^i \right) \right) + \frac{\partial H}{\partial Y} \left(- \left(\sum_j^{SN} a_{2j}^i \right) \right) \left[3P_i H_i^2 + 12K_n H_i + \frac{6K_n^2}{P_i} \right] + \\
 & \left[\left(- \left(\sum_j^{SN} a_{3j}^i \right) \right) + \left(- \left(\sum_j^{SN} a_{4j}^i \right) \right) \right] \left[P_i H_i^3 + 6K_n H_i^2 + \frac{6K_n^2 H_i}{P_i} \right] - \\
 & \Lambda_x H_i \left(- \left(\sum_j^{SN} a_{1j}^i \right) \right) - \Lambda_x \frac{\partial H}{\partial X} \Big|_i
 \end{aligned} \tag{4.57}$$

4.4.6 Fukui Model

The two-dimensional Reynolds equation based on Fukui model is (Fukui, *et al.*, 1988)

$$\begin{cases}
\frac{\partial}{\partial X} \left[QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right] + \frac{\partial}{\partial Y} \left[QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right] = 0 \\
Q = \frac{Q_p(D, \alpha)}{Q_{con}(D)} \\
\Lambda_x = \frac{6\mu_a UL}{h_0^2 P_a}, \quad \Lambda_y = \frac{6\mu_a VL}{h_0^2 P_a} \\
H(X) = 1 + \frac{L}{h_m} (1 - X) \tan \alpha \\
\frac{\partial H}{\partial X} = -\frac{L}{h_m} \tan \alpha, \quad \frac{\partial H}{\partial Y} = 0
\end{cases} \quad (4.58)$$

Thus, the Reynolds equation based on Fukui-model can be expressed as

$$\begin{aligned}
F &= \frac{\partial}{\partial X} \left[(K_1 + K_2(PH)^{-1} + K_3(PH)^{-2} - K_4(PH)^{-3}) PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right] + \\
&\frac{\partial}{\partial Y} \left[(K_1 + K_2(PH)^{-1} + K_3(PH)^{-2} - K_4(PH)^{-3}) PH^3 \frac{\partial P}{\partial Y} \right] = 0
\end{aligned} \quad (4.59)$$

The coefficients K_1 , K_2 , K_3 , and K_4 , are given by Table 4.1.

Using the LSFD method to discretize the differential equation yields

$$\begin{aligned}
F_i &= \left[\left(\frac{\partial P}{\partial X} \right)_i^2 + \left(\frac{\partial P}{\partial Y} \right)_i^2 \right] [K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3}] + \\
&\left[\frac{\partial H}{\partial X} \frac{\partial P}{\partial X} + \frac{\partial H}{\partial Y} \frac{\partial P}{\partial Y} \right] [3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1}] + \\
&\left[\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2} \right] [K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2}] - \\
&- H_i (\Lambda_x \frac{\partial P}{\partial X} + \Lambda_y \frac{\partial P}{\partial Y}) - P_i (\Lambda_x \frac{\partial H}{\partial X} + \Lambda_y \frac{\partial H}{\partial Y})
\end{aligned} \quad (4.60)$$

Applying the SOR iteration method, the iterative step can be determined by

$$\Delta P_i = \frac{-F_i}{deno}$$

$$\begin{aligned}
deno = & \left[\left(\frac{\partial P}{\partial X} \right)_i \left(-\sum_j^{SN} a_{1j}^i \right) + \left(\frac{\partial P}{\partial Y} \right)_i \left(-\sum_j^{SN} a_{2j}^i \right) \right] \left[K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3} \right] + \\
& \left[\frac{\partial H}{\partial X} \Big|_i \left(-\sum_j^{SN} a_{1j}^i \right) + \frac{\partial H}{\partial Y} \Big|_i \left(-\sum_j^{SN} a_{2j}^i \right) \right] \left[3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1} \right] + \\
& \left[\left(-\sum_j^{SN} a_{3j}^i \right) + \left(-\sum_j^{SN} a_{4j}^i \right) \right] \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\
& H_i \left(\Lambda_x \sum_j^{SN} a_{1j}^i + \Lambda_y \sum_j^{SN} a_{2j}^i \right) - \left(\Lambda_x \frac{\partial H}{\partial X} \Big|_i + \Lambda_y \frac{\partial H}{\partial Y} \Big|_i \right)
\end{aligned} \tag{4.61}$$

4.5 Node Distribution for 2-D Plane Slider Air Bearing

4.5.1 Regular Node Distribution

Fig.4.15 shows a typical three-dimensional (3-D) plane slider bearing. Under the steady-state operation, the plate slider will float above the spinning disk with an averaged pitch angle, subjected to a small dynamic variation during the operation. The length L of the slider is $4\mu\text{m}$ and its width is $3.3\mu\text{m}$. The height at the trailing edge of the slider, $h_{\min}=25\text{nm}$, is used to determine the Knudsen number. The pitch angle is 0.01rad . The disk speed U at the slider is 25m/s . The gas considered here is argon with temperature $T_0=0^\circ\text{C}$, density $\rho=1.78\text{kg/m}^3$, the viscosity of the gas is $\mu=2.08\text{e-}05\text{Ns/m}^2$. Argon is chosen in place of air because a single element gas is simpler in direct simulation Monte Carlo (DSMC) method. So, the air bearing number is equal to 197. The Knudsen number of outlet is equal to 2.5. Fukui model is used to calculate the pressure distribution.

Fig 4.16 shows pressure plots from the DSMC solutions (Huang, *et al.*, 1997) while Fig.4.17 shows the corresponding results obtained from the generalized Reynolds equation using the LSFD method for the case $h_{\min}=25\text{nm}$ ($K_n=2.5$). The top two plots in Fig.4.16 show the pressure profile viewed from the front and side using the DSMC method, while Fig.4.16 left-down plot depicts the contours of the pressure profile. Similar plots are presented in Fig.4.17 for $h_{\min} = 5\text{nm}$.

Comparing Fig. 4.16 with Fig. 4.17, it was found that very small relative error is detected between the results from DSMC method and the LSFD mesh-free method. Thus, lower flying height of the slider (without changing anything else) is drawn to 5nm, as shown in Fig. 4.18. The computation domain is discretized into 50×50 node distribution, as shown in Fig.4.19. The area with larger pressure gradient is divided into node layout with smaller space. The computation time required is about 14 second on P4 (2.2G CPU) workstation for the calculation of pressure.

Fig. 4.16 The 3-D pressure profile using DSMC method for $K_n=2.5$ ($h_{\min}=25\text{nm}$)

Fig. 4.17 The 3-D pressure profile using the LSFD method for $K_n=2.5$ ($h_{\min}=25\text{nm}$)

Fig. 4.18 The 3-D pressure profile using the LSFD method for $K_n=12.5$ ($h_{min}=5nm$)

Fig. 4.19 Node distribution for the two-dimensional plane slider

When the minimum spacing $h_{\min}$ becomes smaller (Fig.4.18), it can be observed from these pressure profiles that the peak position moves toward the trailing edge of the slider and the peak value increases. The pressure gradient near the trailing edge is also found to be higher.

It was found that the numerical solution using the LSFD mesh-free method agrees well with the result of the DSMC method (Huang, 1997). Moreover, it was shown that the numerical scheme can deal with the fluid problem in air bearing sliders with nano-level flying height.

In Table 4.3, the air bearing force for different Knudsen numbers K_n and bearing number Λ are listed. The bearing force is defined as

$$W = \frac{1}{g} \int_A (P - P_a) dA \quad (4.62)$$

Where the g is the gravitational acceleration.

In Table 4.3, the relative difference increases with the Knudsen number until K_n reaches about 12.5 and then it drops quickly to 1.067% when K_n is about 20.83. The reason for the relative difference drop is that the influence of the Knudsen number on the load carrying capacity decreases as the bearing number increases. From the modified Reynolds equation, the bearing number is the ratio of the Couette flow rate to the Poiseuille flow rate. Hence, as Λ increases, the Couette flow in the lubrication flow becomes dominant over the Poiseuille flow. Since the flow rate of the Couette

flow is independent of the Kundsens number K_n , the influence of the Knudsen number on the load capacity decreases as Λ increases.

A graph of pressure profiles as a function of Knudsen number is plotted in Fig.4.20. It shows that the bearing force increases with the increase of Knudsen number.

Table 4.3 Air bearing forces of the plane slider with different K_n

h_{min}	K_n	Λ	$W(\times 10^{-3}mg)$		Relative Difference
			DSMC[22]	LSFD	
25	2.5	197	1.571	1.596	1.579%
15	4.17	548	2.753	2.826	2.633%
10	6.25	1232	4.028	4.167	3.45%
5	12.5	4928	6.95	7.198	3.57%
3	20.83	13689	9.835	9.940	1.067%
2	31.25	30800	12.355	12.52	1.33%

Fig. 4.20 Pressure profiles of the plane slider with different K_n

The predicted pressure profiles for all the four attitudes obtained using Fukui model are presented in Fig.4.21. The maximum pressure value decreases gradually on reducing the inlet height h_1 (or pitch angle) as anticipated.

From the viewpoint of the load capacity, the bearing force is proportional to the value of pitch angle, as shown in Fig.4.21. This means that by decreasing the pitch angle, one can decrease the bearing force. The bearing force increases with the increase of Knudsen number. When the Knudsen number is larger, the large bearing force makes it difficult to hold the read/write head lower, for high-density HDD designs. One of the practical methods of solving the problem is to reduce the pitch angle appropriately.

Fig. 4.21 Pressure distribution for different pitch angles ($\Lambda=197, K_n=2.5, h_{min}=25$)

4.5.2 Irregular Node Distribution

Current sliders in hard disk drives are characterized by complex rail shapes and highly recessed air bearing regions between the rails. The rails are joined to the fully recessed regions by abruptly changing wall profiles determined by the etching process.

In order to deal with sliders with complex geometry, such as the rail boundary that is curved or not aligned with the coordinate axes, a reasonable node layout should be developed. If a Cartesian rectangular node system is used, it is extremely difficult to achieve the above goals without refining the local nodes space in other regions where a fine system of node distribution is not needed. This normally causes the node number to become too large for efficient numerical simulation, since the CPU time needed to get converged results increases very rapidly with the total node number.

Because of its geometric flexibility in constructing node layout around complex configurations and relative convenience in incorporating an adaptive methodology, the triangular grid distribution - an irregular node layout, has become popular in recent years in many applications and is conveniently produced by ANSYS software. The advantage of using the irregular node distribution with the LSFD method lies in its robustness and efficiency of the application, which may not be fully demonstrated in rectangular node distribution. Compared with regular node distribution, this method can apply powerfully in all kinds of sliders with complex shapes due to its ability to generate optimal distribution of the existing node points. When the number of nodes is large, the searching time to get supporting points increases accordingly due to the distance-based searching method compared with layer searching method in the case of

regular node layout. However, the searching time is not the dominative part of the total computational time. Instead, the total computational time will decrease since the LSFD method based on irregular node layout is more efficient and robust.

In ANSYS software, triangular grid generation is divided into two modes. One is free mode shown in Fig.4.22 and the other one is map mode shown in Fig.4.23. The pressure distributions are shown in Fig.4.24 and Fig.4.25. It was found that very small relative differences are detected between the results produced by regular and irregular node layout.

Fig. 4.22 Irregular node distribution (free mode) of the plane slider

Fig. 4.23 Irregular node distribution (map node) of the plane slider

Fig. 4.24 3-D pressure profile of the plane slider with irregular free node distribution
for $K_n=1.27$

Fig. 4.25 3-D pressure profile of the plane slider with irregular map node distribution
for $K_n=1.27$

4.6 Results for 2-D Taper-flat Slider Air Bearing

Under heavy loadings it may be desirable to conduct away a large amount of heat through the bearing housing. A flat slider is unlikely to be satisfactory under these conditions, and the use of a fixed taper-flat or a step bearing may be required.

The film in a taper-flat slider bearing (Fig.4.26) consists of an inlet wedge followed by a parallel section. The mathematical complications necessary to match boundary conditions at the interior boundary are such that no analytical solutions are available for the finite taper-flat bearing, even with an incompressible film.

Fig.4.27 shows a comparison among the results obtained for a taper flat slider air bearing with film-thickness ratio $H_1=h_1/h_2=2$ using the LSFD mesh-free method and the FD method based on continuum model (Gross, 1977). The results generally agree well with each other and the maximum of relative error among those results is about 3.8%.

Fig. 4.26 The taper flat slider bearing

Fig. 4.27 Comparison of load for the taper flat slider bearings having film-thickness ratio $h_1/h_2=2$

Another taper flat slider gas bearing, whose dimensions are shown in Fig.4.28, is also considered. Slider bearings are employed in magnetic recording disk systems, where the slider flying spacing is reduced to micro-value. So, a film pressure distribution is normally calculated using the modified Reynolds equation introduced by Burgdorfer (1959), which considers first-order slip flow effects. The discrete nonlinear algebraic equation is solved by the SOR method.

Fig. 4.28 Dimension of the taper flat slider bearing

The pressure distribution for high $\Lambda=1000$ in the tapered flat slider bearing is shown in Fig.4.29. The lubrication equation is solved using node distribution with variable size space, in which node distribution with small space is assigned at the tapered flat region and at the trailing edge where large pressure gradients are formed. One rail was divided into regular rectangular node layout which consists of 2121 nodes (101 nodes in the X direction, 11 nodes in the Y) in Fig.4.30. The nodes were concentrated

around the region of maximum pressure gradient. A smooth and reasonable pressure distribution without any numerical instability is obtained.

Fig. 4.29 Pressure distribution along the centerline of the taper flat slider air bearing

Fig. 4.30 Node distribution in the taper flat slider

In the taper flat slider bearing, the pressure in the converging section will rise parabolically and peak just ahead of the junction between sections. At the junction point the pressure will be greater than ambient pressure by an amount ΔP_i . This high pressure at the leading edge occurs without film-thickness discontinuity. Thus the side flow of a finite taper-flat bearing is less than that of a step bearing with the same film thickness, slenderness, and length ratios.

The influence of compressibility is to elevate the value of the mean pressure in the trailing section to approach that of the peak film pressure. For the step slider bearing, the peak pressure occurs at the internal boundary owing to the flow-resistance discontinuity there. For the taper flat slider bearing, the peak pressure occurs just upstream of the internal boundary. In the leading region of the step slider bearing, compressibility effects tend to reduce the load component sustained there. However, the leading region of the taper flat slider bearing does not, in general, suffer this disadvantage.

4.7 Solutions for Steady State Flying Height

In most air bearing simulations, the air bearing attitude is the desired result for a given suspension load. This is called the inverse problem.

The solution starts at a guessed slider attitude. The pressure distribution is solved, and then the resulting bearing force as well as the location of the force is compared with the suspension preload and its location respectively. When the difference among them is greater than the specified number, the Quasi-Newton iteration method is employed to search for the new slider attitude (Ellis *et. al*, 1995).

The normalized bearing load and location can be expressed as

$$W = \int_0^b \int_0^1 (P - 1) dX dY$$

$$\bar{X} = \frac{1}{W} \int_0^b \int_0^1 X(P - 1) dX dY$$

$$\bar{Y} = \frac{1}{W} \int_0^b \int_0^l Y(P-1) dXdY \quad (4.63)$$

Where b is the slider's width non-dimensionalized by the slider's length. Let W_0 , (X_0, Y_0) be the suspension load and the location of load point non-dimensionalized by $p_a L^2$ and L respectively;. Then the normalized force imbalance is defined as

$$R_1 = \int_0^b \int_0^l (P-1) dXdY - W_0 \quad (4.64)$$

$$R_2 = \frac{\int_0^b \int_0^l X(P-1) dXdY}{\int_0^b \int_0^l (P-1) dXdY} - X_0 \quad (4.65)$$

$$R_3 = \frac{\int_0^b \int_0^l Y(P-1) dXdY}{\int_0^b \int_0^l (P-1) dXdY} - Y_0 \quad (4.66)$$

R_1, R_2, R_3 are used as the convergence criterion for the Quasi-Newton search of the steady state slider flying attitude.

To find the equilibrium flying attitudes, the flying height, pitch and roll angle of the slider are varied iteratively until the force and location are balanced. In this section, the application of the Quasi-Newton algorithm to the inverse problem of air-bearing analysis is presented (Garret, 1984), where the equilibrium flying attitudes, $Z^* = (H^*, \theta^*, \varphi^*)$ corresponds $R = [R_1, R_2, R_3]^T = 0$.

The first step of the Quasi-Newton algorithm is to apply Newton's method. That is, starting at some initial guess, Z_0 , the next iteration is obtained using

$$Z_+ = Z_0 - J^{-1}(Z_0)R(Z_0) \quad (4.67)$$

Where J is the Jacobian matrix of R at Z_0 . The reason for using Newton's method first is that in case the initial guess is relatively close to the equilibrium flying attitude, Z^* , the fast local convergence property of Newton's method is fully utilized. In fact, Newton's method converges quadratically if the initial guess is sufficiently close to the roots; thus, it is said to be a second order method. However, if the initial guess is not close to Z^* , then the Newton method is either very slow in convergence or, in some cases, the method leads the solution to diverge away from Z^* .

The next step in the Quasi-Newton algorithm is to decide whether or not the Newton step is satisfactory or not, which is defined as

$$s_n = -J^{-1}(Z_n)R(Z_n) \quad (4.68)$$

In fact, for the inverse problem in the air-bearing analysis, the norm of Jacobian matrix is very small. This results in a large magnitude for the Newton step, and unless the solution is very close to Z^* , the solution oscillates as it tries to correct for the large steps. Consequently, until the solution is near Z^* , one rejects the Newton step and proceeds to the next iteration using a step size determined through a line-search algorithm.

The line-search algorithms are, in general, used to redirect the solution when the Newton step steers it outside the convergence region of Newton's method. The algorithm is mainly applied to minimization problems, and the idea of a line-search algorithm is that given a descent direction, q_n (which is a direction in which the objective function decreases), find a scalar, r_n , such that the next iteration will be

satisfactory. In applying the line-search algorithm, one focuses on the corresponding minimization problem for the inverse problem:

$$\min r(h, \theta, \varphi) = \frac{1}{2} R^T(Z) R(Z) \quad (4.69)$$

Where $\frac{1}{2}$ is added for algebraic convenience. Note that every solution to $R=0$ is a solution to Eq.(4.67). Although, in general, Eq.(4.67) might contain a local minimum that is not a solution to $R=0$, the functions, R_1, R_2 and R_3 do not contain such local minima; therefore, every solution to Eq.(4.67) is also a solution to $R=0$.

To find the flying attitude that minimizes the objective function, r the backtracking line-search algorithm is used. Starting with $r_n = 1$, one uses a line-search algorithm to reduce r_n until one obtains

$$r(Z_n + r_n q_n) < r(Z_n) + \varepsilon r_n \nabla r(Z_n)^T q_n \quad (4.70)$$

Where

$$\nabla r(Z_n) = J^T R(Z) \quad (4.71)$$

Since only a small decrease in the function value is desired, the value of ε is set at 10^{-4} .

The Newton direction given by, s_n , is a descent direction since

$$\nabla r^T s_n = -R^T J J^{-1} R = -R^T R < 0 \quad (4.72)$$

Thus, the Newton direction is used in the backtracking algorithm. The next iteration flying attitude updates as $Z_{m+1} = Z_n + r_n s_n$ until the solution is within some tolerance of Z^* .

In the above algorithm, it is necessary to calculate the Jacobian matrix of R . However, since the elements of R are found by integrating the generalized lubrication equation, an analytical formulation of the Jacobian matrix cannot be found. Instead, the elements of the Jacobian matrix are obtained numerically using a difference method, e.g.,

$$\frac{\partial R_1}{\partial h} = \frac{R_1(h + \Delta h) - R_1(h)}{\Delta h} \quad (4.73)$$

To calculate the Jacobian matrix numerically at each iteration takes large computational effort. Thus, instead of calculating a new Jacobian matrix at each iteration, one approximates the Jacobian matrix using the secant method by Broyden [79]. Given the Jacobian matrix at some iteration step, J_n the Jacobian at the next step is approximated using

$$J_{n+1} = J_n + \frac{(y_n - J_n s_n) s_n^T}{s_n^T s_n} \quad (4.71)$$

where

$$y_n = R(Z_{n+1} - Z_n)$$

$$Z_{n+1} = Z_n + s_n$$

Consequently, the Jacobian matrix is calculated only once initially, and then updated for the subsequent steps.

In summary, the Quasi-Newton method is obtained by combining the backtracking line-search algorithm with Newton's method, which is a very robust technique for calculating the equilibrium flying attitude of a slider.

A typical three-dimensional flat slider gas bearing is employed to determine the flying attitude subject to pre-applied suspension force and moment (Fig.4.15). The dimension of the slider is the same as the computation model in Fig.4.16 (Huang, 1997). The suspension force is equal to 1.596×10^{-2} mg, and the location of the force is $1.797 \mu\text{m}$ and $1.384 \mu\text{m}$ along the x-direction and y-direction, respectively. At first, an initial flying attitude (h_m^0, α^0) is assumed. The initial guesses for minimum flying height and pitch angle are 20nm, and 0.015rad respectively. Given the values of flying attitude parameters, the pressure distribution over the slider surface can be calculated by solving the generalized Reynolds equation using the LSFD method in the pressure calculation module. Integrating the pressure over the computation domain, the force and its location are obtained. Using the Quasi-Newton method, after several iterations, the result converges to the balanced flying attitude: $h_m=25\text{nm}$, $\alpha=0.01\text{rad}$. Thus, it is feasible to use the coupled procedure of the LSFD method and Quasi-Newton method to determine the accurate flying attitude.

To evaluate insensitivity of the approach to the initial values of flying attitude parameters, six initial guesses are applied to flat slider with zero skew angles as listed in.

Table 4.4 Six initial guesses tested

Case Number	h_m (nm)	α (rad)	Iteration Number
1	20	0.015	6
2	15	0.015	6
3	20	0.010	4
4	15	0.010	8
5	10	0.015	8

4.8 Summary

The LSFD mesh-free method is implemented to solve the Reynolds equation based on various flow models in simple one-dimensional and two-dimensional slider air bearings. It is found that there are very small relative discrepancies between the result from the LSFD mesh-free method and other analytic and numerical results. The LSFD mesh-free method is both mathematically simple and computationally accurate due to the stability of the algorithm in solving simple slider problems. Moreover, both regular and irregular node distribution are implemented to provide greater flexibility for dealing with all kinds of potential complicated air bearing surfaces in modern magnetic drives. In addition, the inverse problem, which determines the equilibrium-flying attitude, was also solved using the Quasi-Newton method to further validate the model. On the other hand, the principle of node generation for a simple one rail in taper flat slider and plane slider can also be applied to complex sliders with high gradients of pressure and geometry. It is proved that the LSFD mesh-free method is an efficient and robust computational method for solving simple slider air bearing problems in HDD.

CHAPTER 5 Node Generation and Numerical Simulation of Positive Sliders

5.1 Node Generation Principle

To further demonstrate the feasibility and effectiveness of the LSFD mesh-free method for solving air bearing surface problems, more complex positive sliders are modeled and solved using the LSFD scheme in this chapter.

A successful numerical solution depends as much on a system of adequate nodes as on the numerical algorithm used. Due to the special geometry of the slider air bearing surface, the solution of the generalized Reynolds equation entails additional node requirements. For example, the discontinuities along the rail edges and the front taper have to be treated carefully to ensure meaningful solutions. The principle of node generation is also described in this chapter.

In the mesh-free methods, instead of using mesh generation, one actually employs nodes generation. Finite number of points can be distributed on a computational domain evenly or unevenly without concern of structural connectivity. Unlike mesh generation in conventional FD/FE methods, there is no pre-requirement for the distribution of points in the present mesh-free method. Therefore, points or nodes generation in the present method is consequently easier in complex domains as compared with mesh generation.

According to the specific characteristic of practical problems, there are many ways to generate points. For example, for applications in the regular domain, where the solution is sufficiently smooth, the points in the domain can be generated in the same manner as the rectangular grid in conventional finite-difference schemes. The boundary points can simply be the intersections of the grid with boundary, irrespective of its shape. For problems in irregular domain, we can get reasonable nodes distribution in the domain either by random point generation or by using algebraic function. Furthermore, for many applications, the solution may need different resolutions for different regions; high resolution being typically required for regions near boundaries and high pressure gradients. That is to say, when we use either mesh-based or point-based methods, the density of mesh/point distribution should reflect that need. Therefore, in such circumstances, the distribution of the nodes or points in the computational domain must be generated either adaptively or by using known information about the specific physical problem. Both of them can be implemented easily in the present method as we can freely add or delete nodes instead of re-meshing.

The current code developed implements a piece-wise geometric progression to produce rectangular node layout. Although the node generation in some critical areas can significantly affect solution accuracy due to the special nature of the air bearing geometry (clearance discontinuity, change of slope at the taper end), the piece-wise geometric progression method is very flexible and adaptive to complicated slider geometry. The variable node spacing scheme based on geometric progression has been popular in finite difference solutions; because it gives the user the control to put

more nodes where needed and provides smooth transition from a coarse node spacing region to a fine node spacing region. In this method, the computational domain is divided into several intervals in both directions, and a geometric progression is used in each interval. It is important that the node distances at the junction of two intervals be comparable in order to minimize numerical error.

In the present method, the computational domain is firstly divided into intervals in both the X and Y directions. The end points of intervals are called the control points. The coordinates of these control points as well as their node indices are given. Therefore, the number of nodes in each interval is known. For each interval, a spacing ratio is given, which is the ratio of successive node spacing. In ANSYS software, if the ratio is positive and larger than unity (Fig.5.1), the spacing increases with the ratio of last division size to first division size. While the ratio is positive and smaller than unity (Fig.5.2), the spacing decreases with the same ratio. If it is negative, it means nominal ratio of center division size to end division size (Fig.5.3). Ratio defaults to 1.0 (uniform spacing). Then a geometric progression is used to generate the node distribution in each interval.

Fig. 5.1 Division and ratio 30(5) in ANSYS software

Fig. 5.2 Division and ratio 30(0.1) in ANSYS software

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Fig. 5.3 Division and ratio 30(-2) in ANSYS software

The rule of thumb for a sound node system is that all important features should be covered by sufficient number of nodes. There are two factors to significantly influence node distribution in simulation of air bearing surface. One is geometry shape and the other is pressure profile.

In single rail of IBM 3380 taper flat slider, the height gradient on the taper part is more than 190 times larger than that on the flat part, which brings numerical difficulty in dealing with the severe gradient discontinuity. Thus, the single rail of IBM 3380 taper flat slider is considered as a valuable example to demonstrate the principle mentioned of numerical simulation and node generation.

5.2 Single Rail of IBM 3380 Taper Flat Slider and Node Generation

5.2.1 Pressure Solution for a Given Slider Attitude

The static pressure distribution of an IBM 3380 taper flat slider is calculated using the numerical scheme developed in this thesis to compare against that obtained from the finite volume method.

The pressure over the recessed center region in the IBM 3380 taper flat slider is considered to be the atmosphere pressure as boundary condition, since the heights of the rails are much larger than that of the center region compared with the minimum

flying height in the trailing edge. Thus, only the two rails are divided into finite nodes and calculated by the LSFD method with the generalized Reynolds equation to acquire the pressure profile.

Fig.5.4 and Fig.5.5 show the rail shape and dimension of the IBM3380 taper flat slider. Table 5.1 and Table 5.2 show the geometrical parameters and the flying height attitude parameter of the IBM3380 taper flat slider, respectively. The inner rail and outer rail have the same width value Y_L . Here, we define the roll angle β as positive when the outer rail is higher than the inner rail. Besides, the whole slider is assumed to have the same velocity at a fixed position of track.

Fig. 5.4 IBM3380 taper flat slider

Table 5.1 Geometry parameter of the IBM3380 taper flat slider

Taper Length (XT)	0.368E-03m
Center of Preload (XF0)	0.2162E-02m
Total Length (TXL)	0.405E-02m
Rail Width (YL)	0.50E-03m
Total Width (TYL)	0.30E-02m
Taper Height (HT)	0.599E-05m

Fig. 5.5 Dimension of the IBM 3380 taper flat slider

Table 5.2 Flying attitude parameter of single rail of the IBM3380 taper flat slider

Minimum Flying Height (h_0)	243nm
Pitch Angle (α)	80 μ rad
Roll Angle (β)	4 μ rad
Skew Angle (δ)	0 μ rad
Taper Angle (γ)	15.189mrad
Velocity of Disk	22m/s

Table 5.3 shows the numerical parameters used in simulating the slider, such as weighing function w and extrapolation factor ω . These parameters are chosen to

ensure faster convergence speed and more accurate result. For example, the Taylor order chosen to assure the accuracy of the LSFD method, by which the first-order derivatives were approximated to third-order accuracy, and the second derivatives were approximated to second-order accuracy. The extrapolation factor ω chosen is able to lead to result convergence in the shortest computational time. The layer searching method can cope with the sharply uneven node distribution.

Table 5.3 Numerical parameter of single rail of the IBM3380 taper flat slider

Taylor Order (Order)	9
Number of Supporting Point (Supnode)	15
Extrapolation Factor (ω)	1.2
Weighting Function (w)	5
Searching Method	Layer Searching Method
Computational Time	16 second

The two-dimensional Reynolds equation based on Fukui model is (Fukui, *et al.*, 1988)

$$\frac{\partial}{\partial X} \left[QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right] + \frac{\partial}{\partial Y} \left[QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right] = 0 \tag{5.1}$$

At the i th reference node, the generalized Reynolds equation can be expanded as

$$\begin{aligned}
F_i = & \left[\left(\frac{\partial P}{\partial X} \right)_i^2 + \left(\frac{\partial P}{\partial Y} \right)_i^2 \right] [K_1 H_i^3 - K_3 P_i^{-2} H + 2K_4 P_i^{-3}] + \\
& \left[\frac{\partial H}{\partial X} \left| \frac{\partial P}{\partial X} \right|_i + \frac{\partial H}{\partial Y} \left| \frac{\partial P}{\partial Y} \right|_i \right] [3K_1 P_i H_i^2 + 2K_2 H_i + K_3 P_i^{-1}] + \\
& \left[\frac{\partial^2 P}{\partial X^2} \right|_i + \frac{\partial^2 P}{\partial Y^2} \right|_i \left[K_1 P_i H_i^3 + K_2 H_i^2 + K_3 P_i^{-1} H_i - K_4 P_i^{-2} \right] - \\
& - H_i \left(\Lambda_x \frac{\partial P}{\partial X} \right|_i + \Lambda_y \frac{\partial P}{\partial Y} \right|_i) - P_i \left(\Lambda_x \frac{\partial H}{\partial X} \right|_i + \Lambda_y \frac{\partial H}{\partial Y} \right|_i) = 0
\end{aligned} \tag{5.2}$$

The rail configurations of the slider depend on the magnitude of the height at the discretization nodes, since the slider region is divided into finite nodes. Evaluation of the height at the discrete points in the computational domain can be carried out based on their location on the slider configuration, which is determined by their coordinates.

In Fig.5.5, the height H_i and height gradient $\partial H / \partial X|_i$ in the taper part are very different from those in the flat part. For the taper flat, flying height and height gradient can be expressed as ($X \leq XT$)

$$\begin{aligned}
H(X) &= \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{XT}{TXL} - X \right)tg\gamma \right] + 1 \\
\frac{\partial H}{\partial X} &= -\frac{TXL}{h_m} (tg\gamma + tg\alpha) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} tg\beta
\end{aligned} \tag{5.3}$$

Flying height and height gradient of the flat part in the IBM3380 slider can be represented as ($X > XT$)

$$\begin{aligned}
H(X) &= \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right] + 1 \\
\frac{\partial H}{\partial X} &= -\frac{TXL}{h_m} tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} tg\beta
\end{aligned} \tag{5.4}$$

Fig.5.6 shows the comparison between the results from the LSFD method and those from the finite volume method (FVM) based on Fukui model for the IBM3380 air bearing slider with taper angle γ (Cha, *et al.*, 1995). The figure describes the pressure distribution obtained at the middle of the inner rail. The results generally agree well with each other and the maximum relative error among those results is about 2.8%. Fig.5.7 shows the 3-D pressure profile of a single rail of the IBM3380 slider air bearing.

Fig. 5.6 Comparison of pressure profiles of the IBM3380 taper flat slider using the LSFD method and the finite volume method (Cha, *et al.*, 1995)

Fig. 5.7 The 3-D pressure profile of single rail in the IBM 3380 slider using the LSFD method

5.2.2 Node Generation for Single Rail of the IBM 3380 Taper Flat Slider

The IBM3380 taper flat slider is used as an example of node generation for the piece-wise geometric progression. The parameters in Table 5.4 are used to generate the nodes plotted in Fig. 5.8. The node distribution used is 166×21 . The number of node ratios is one more than the number of control points, because n control points divide the domain into $n + 1$ intervals. In the geometry aspect, height is continuous (Fig.5.9), while height gradient is discontinuous at the junction of the taper and the flat sections. The height gradient $\partial H / \partial X$ on the taper part is more than 190 times larger than that on the flat part as shown in Fig.5.10.

Table 5.4 Node specification of single rail in the IBM3380 taper flat slider

X Coordinate (mm)	[0,0.268] [0.268,4.05]
X control Coordinate (mm)	0.268
X Control Point Indices	20, 145
X Node Ratios	-2, -5
Y Coordinate (mm)	[0,0.5]
Y control Coordinate (mm)	No
Y Control Point Indices	20
Y Node Ratios	-2

Fig. 5.8 Node distribution for single rail of the IBM3380 taper flat slider

Fig. 5.9 Height profile for single rail of the IBM3380 taper flat slider

Fig. 5.10 Height gradient $\partial H/\partial X$ in single rail of the IBM3380 taper flat slider

On the aspect of pressure distribution, predicted pressure gradient sharply increases in regions near the line between the taper part and the flat part as well as around the trailing edge. Therefore, the numerical difficulty in dealing with the gradient discontinuity is tackled by choosing denser nodes close to the joint line among these areas. The control point is designed to 0.268mm in Table 5.4 to prevent having nodes on the joint line. It is observed that locations of left node and right node along X-direction are 0.359281047542mm and 0.371882098628mm respectively, so that the lines with X coordinate of 0.368mm is avoided successfully. The choice of the two nodes depends on the slider design.

The difference of node coordinate along X-direction is shown in Fig.5.11. The nodes located in the regions of taper end and trailing edge should be denser than other regions due to high geometry and pressure gradients. The solution with the joint point as a node is compared with the one with no node at the joint point in Fig.5.12. It can

be seen that the computed air bearing pressure experiences large fluctuations (upper curve) when the original node ($XT=0.368e-03$) is kept unchanged, while the result with no node at the joint point gives much smoother pressure variation at the taper edge.

Fig. 5.11 Coordinate difference (ΔX) in single rail of the IBM3380 taper flat slider

Fig. 5.12 Effect of node avoiding at the taper end. (o): node at joint point; (Δ): no node at joint point

5.3 Positive Pressure Sliders

The above section only describes pressure distribution of single rail in the IBM3380 taper flat slider, which provides the foundation work to implement the LSFD mesh-free method into other positive pressure sliders with complex rail shapes. Thus, in this section, the whole slider air bearing is attempted and calculated to obtain the pressure profile. Several positive sliders are chosen as models, such as IBM3380 taper flat slider, Tripad slider, TPC slider to verify the LSFD mesh-free method. For this kind of sliders, the height difference between rails and recessed region is so large that the pressure of recessed region can be considered as the ambient pressure.

5.3.1 Global IBM3380 Taper Flat Slider

5.3.1.1 Pressure Solution for a Given Slider Attitude

The IBM 3380 slider is used as a typical example of positive slider (Fig.5.13) to prove the feasible application of the LSFD method in simulating positive slider air bearings. The center recessed region is considered to be of the ambient pressure as boundary condition. From the left plot in Fig.5.13, the rail configurations of the slider depend on the magnitude of the height at the discretization nodes.

If $X \leq X_T$ and $Y < Y_L$ and $Y_L < Y < (T_{YL} - Y_L)$ (Fig. 5.5), the flying height is obtained by

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{T_{YL}}{TXL} - Y \right)tg\beta + \left(\frac{X_T}{TXL} - X \right)tg\gamma \right] + 1 \quad (5.5)$$

If $X > X_T$ and $Y < Y_L$ and $Y_L < Y < (T_{YL} - Y_L)$ (Fig. 5.5), the flying height is represented as

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right) tg\beta \right] + 1 \quad (5.6)$$

If $(TYL-YL) > Y > YL$ (Fig. 5.5), the flying height is equal to

$$H(X) = \text{Constant value} \quad (5.7)$$

Therefore, the pressure profile can be obtained by using the LSFD mesh-free method based on the flying height and other parameters such as velocity (Fig. 5.13). Comparing the pressure distribution obtained here (Fig.5.13) with that of single rail of the slider presented in Fig.5.7, it is observed that they agree exactly with each other, which means the mesh-free method is successfully implemented to solve the global IBM3380 taper flat slider.

Fig. 5.13 Geometry and pressure profile of the IBM3380 taper flat slider

5.3.1.2 Node Generation of Global IBM3380 Taper Flat Slider

The node layout has some difference between the single rail of the IBM3380 slider and the global IBM3380 slider. The whole air bearing surface under the slider, instead

of single rail, is simulated to acquire the pressure profile for the global IBM3380 slider. Since the center recess region is considered to be of the ambient pressure as boundary condition in the global IBM3380 taper flat slider, nodes in the region should be sparsely distributed. Table 5.5 shows that 10 control points are distributed in the center recessed region ([0.5mm,2.5mm]), while 20 control points are placed in the rail regions ([0,0.5mm], [2.5mm,3.0mm]). Therefore, the computational solver with the distance-based searching method results in the same pressure result (Fig.5.13) as that of the single rail, which shows the validity of the LSFD method in solving the global taper flat slider problem with suitable node layout presented in Fig.5.14.

Table 5.5 Node specification of the global IBM3380 taper flat slider without wall profile

X Coordinate (mm)	[0,0.268] [0.268,4.05]
X control Coordinate (mm)	0.268
X Control Point Indices	20, 145
X Node Ratios	-2, -5
Y Coordinate (mm)	[0,0.5] [0.5,2.5] [2.5,3.0]
Y control Coordinate (mm)	0.5, 2.5
Y Control Point Indices	20, 10, 20
Y Node Ratios	-2, 1, -2

Fig. 5.14 Node distribution of the IBM 3380 taper flat slider (166*51)

5.3.2 Tri-pad Slider

Tri-pad slider is a modified two straight rails taper flat design with two shortened rails and a small third pad located at the center of the trailing edge that carries the read/write element, shown in Figure 5.15 (Michael, *et al.*, 1996). The two front pads are close to the outer edges of the slider to maximize roll stiffness and are typically terminated beyond the middle of the slider for adequate pitch stiffness. The rear center pad minimizes roll effects on the magnetic signal. The tri-pad slider operates at a high pitch angle which reduces front corner contacts and lowers the spacing at the rear pad. The pressure of the tri-pad slider is calculated with 300 μ rad pitch angle and 40nm flying height, shown in Figure 5.15. The node layout is 180 $\times$ 130. The node distribution is denser on the rear pad than that on other regions.

Fig. 5.15 Geometry and pressure profile of the tri-pad slider

5.3.3 TPC2b Slider

A well-known constant flying height slider is the Traverse Pressure Contour (TPC) slider. The rail shape is plotted in Figure 5.16 (Cha, *et al.*, 1995). The constant space between the slider and the disk is achieved by the different role played by the TPC sections that are cut along the entire length of the rails. The two asymmetric TPC sections at each rail are designed to add extra lift to the slider when the slider has high skew angles. By compressing and expanding the incoming flow at a skew angle using the partially recessed steps, the TPC slider provides constant fly height across the entire disk radius. More nodes are concentrated at the trailing edge to avoid spike. The 3-D pressure distribution is plotted in Figure 5.16.

Fig. 5.16 Geometry and pressure profile of the TPC2b slider

5.4 Summary

The piece-wise progression method with variable node spacing scheme based on geometric and pressure gradients is implemented for nodes generation. It gives the user the control to put more nodes where needed and provides smooth transition in different regions. The LSFD mesh-free method is applied to compute effectively the pressure distribution of positive pressure sliders with rail shapes, such as IBM3380 taper flat slider, Tri-pad slider and TPC2b slider. It is demonstrated that the LSFD method with suitable node layout is an effective and alternative method to model and simulate the air bearing problems of various positive sliders.

CHAPTER 6 Node Generation and Numerical Simulation of Negative Sliders

6.1 Introduction

The increasing of the bit density on a data track, and hence the storage capacity results in head-to-disk spacing (flying height) to be below 10nm in modern hard disk drives. Besides the extremely low spacing, it is also desirable to have a constant flying height over the entire disk, a low take-off/landing velocity and a stiff air bearing. In order to meet these strict performance requirements, air bearings of significant complexity have been proposed (White, 1983; Hardie, *et al.*, 1994; White, 1997). New features in air bearing designs include shaped rails, multiple etch depths and recessed regions. It is extremely important to accurately simulate those regions characterized by sharp pressure gradient and rapidly changing elevations in numerical models.

Since height discontinuity exists between regions of different heights in negative pressure sliders, which results in divergence of the nonlinear systems of equations discretized from the lubrication equation, a new technique named adding wall profile is implemented to eliminate elevation discontinuity in the numerical computational process. Therefore, this approach will not only prevent the potential numerical divergence problems caused by height discontinuity, but also enable the application of the LSFD mesh-free method to all possible complex slider designs.

6.2 Wall Profile Technique

This wall profile technique can eliminate numerical difficulties caused by clearance discontinuity in negative pressure sliders. It builds a ramp between regions with different heights. Since the height discontinuity also exists in positive pressure sliders, the ambient pressure can be obtained by the technique in the recessed region when the whole air bearing surface is implemented to simulate. Several positive pressure sliders such as two-rail slider, 1212 taper flat slider are tackled to demonstrate the wall profile technique in the mesh-free method prior to simulating negative pressure sliders.

6.2.1 Two-Rail Slider with Discontinuous Height and Node Generation

6.2.1.1 Pressure Solution for a Given Slider Attitude

Two-rail slider, as a conventional positive pressure slider, whose dimension is shown in Fig.6.1, has two rails and recessed region. Generally, the recessed region is considered to have the ambient pressure due to significant height difference between recessed region and rails. Here, however, it represents a clearance discontinuity with abrupt change in geometry. Taking the two-rail slider for example, the entire region under the slider is calculated to simulate the pressure profile with a given flying spacing $1\mu\text{m}$.

In order to suppress numerical difficulties caused by clearance discontinuities presented in the sliders, the wall profile should be designed and the clustering of very fine nodes is distributed in the narrow recessed wall region. The width of the wall

profile is 0.02mm from the rail edge and the height of it is 40μm as shown in Table 6.1. The flying attitude parameter and numerical parameter are represented in Table 6.2 and Table 6.3, respectively.

Fig. 6.1 The two-rail slider

Table 6.1 Geometrical parameter of the two-rail slider with wall profile

Slider Length (TXL)	1.0E-03m
Slider Width (TYL)	0.6E-03m
Rail Width (W1)	0.1E-03m
Wall Profile Width (W2-W1)	0.02E-03m
Wall Profile Height (hc)	4.0E-05m

Table 6.2 Flying attitude parameter of the two-rail slider with wall profile

Minimum Flying Height (h_0)	1.0E-06m
Pitch Angle (α)	1.25mrad
Roll Angle (β)	0 μ rad
Skew Angle (δ)	0 μ rad
Radial Position	18mm
RPM	5400

Table 6.3 Numerical parameter of the two-rail slider with wall profile

Taylor Order (Order)	9
Number of Supporting Point (Supnode)	15
Extrapolation Factor (ω)	0.8
Weighting Function (w)	5
Searching Method	Layer Searching Method
Computational Time	36 minute

Fig. 6.2 Dimension of the two-rail slider

In Fig.6.2, if $Y < W1$ or $W4 < Y < TYL$ the height and height gradient of the rail part is obtained by

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right] + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha, \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.1)$$

If $W2 > Y > W1$ (Fig.6.2), the height and height gradient of the wall profile in outer rail is represented as

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(Y - \frac{W1}{TXL} \right)tg\delta \right] + 1$$

$$tg\delta = \frac{h_c}{W2 - W1} \quad \frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\beta - tg\delta) \quad (6.2)$$

If $W4 > Y > W3$ (Fig.6.2), the height and height gradient of the wall profile in inner rail is represented as

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{W4}{TXL} - Y \right)tg\delta \right] + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\beta + tg\delta) \quad (6.3)$$

If $W4 > Y > W3$ (Fig.6.2), the height and height gradient of the cavity in center region is determined by

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right] + \frac{h_c}{h_m}$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha, \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.4)$$

Fig.6.3 shows the 3-D pressure plots from the generalized Reynolds equation solution using the LSFD mesh-free method for the case of $h_{min}=1\mu m$ ($K_n=0.064$). It can be seen from Fig.6.4 and Fig.6.5 that the pressures of the recessed region approach the ambient pressure and agree exactly with the result obtained from the finite volume method (CMLAir32). Fig.6.6 describes the pressure distribution of different recessed heights without the wall profile, which indicates that the pressure of the cavity region approaches the ambient pressure when height of the cavity is larger. Therefore, it is observed that adding wall profile is important to accurately and effectively simulate negative slider air bearings.

Fig. 6.3 The 3-D pressure profile for the two-rail slider with wall profile

Fig. 6.4 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the two-rail slider along $y= 0.475\text{E-}04\text{m}$

Fig. 6.5 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the two-rail slider along $x=0.1022E-03m$

Fig. 6.6 Pressure profiles of the two-rail slider with different recessed heights along $x=0.475E-04m$

6.2.1.2 Node Generation of the Two-Rail Slider

The steep clearance change along the rail boundaries may also cause significant numerical errors if care is not taken in node generation. Thus, the nodes should be denser among the regions with large height and pressure gradients and become sparser with gradual transitions to other regions.

Along X-direction, node spacing in the leading edge and trailing edge is smaller than other regions (Fig.6.7), since the predicted pressure gradients $\partial P/\partial X$ are much larger in these areas than those in other regions and the height gradients $\partial H/\partial X$ are a constant value (Fig.6.8). Along Y-direction, on the one hand, the height gradient $\partial H/\partial Y$ (Fig.6.9) and predicted pressure gradient $\partial P/\partial Y$ change sharply in the areas of the wall profile. On the other hand, rail boundaries should be avoided to be chosen as nodes due to sensitive property in the lines between rails and recessed regions (Fig.6.10). Based on the principle described, the computational domain is divided into 91×109 nodes as shown in Fig.6.11 and detailed division and ratio are depicted in Table 6.4.

Fig. 6.7 Coordinate difference (ΔX) in the two-rail slider with wall profile

Table 6.4 Node specification of the two-rail slider with wall profile

X Coordinate (mm)	[0,0.2] [0.2,0.72] [0.72,1.0]
X Control Coordinate (mm)	0.2,0.72
X Control Point Indices	20, 35, 35
X Node Ratios	2, 1, 0.3125
Y Coordinate (mm)	[0, 0.095] [0.095, 0.105] [0.105, 0.1525] [0.1525, 0.4475] [0.4475, 0.495] [0.495, 0.505] [0.505, 0.6]
Y Control Coordinate (mm)	0.095, 0.105, 0.1525, 0.4475, 0.495, 0.505
Y control Point Indices	26, 5, 13, 20, 13, 5, 26
Y Node Ratios	-2, 1, 0.5, -2, 2, 1, -2

Fig. 6.8 Height gradient $\partial H / \partial X$ in the two-rail slider with wall profile

Fig. 6.9 Height gradient $\partial H / \partial Y$ in the two-rail slider with wall profile

Fig. 6.10 Coordinate difference (ΔY) in the two-rail slider with wall profile

Fig. 6.11 Node distribution of the two-rail slider (91*109) with wall profile

6.2.2 1212 Taper Flat Slider with Discontinuous Height

The 1212 taper flat slider is taken as a global slider example with discontinuous heights to acquire the pressure distribution using the LSFD method with adding wall profile technique, in which normalized heights of the three boundary points (at $x=0$, 0.2mm, 1.0mm) are 1, 2, and 12 as shown in Fig.6.12. The slider possesses clearance discontinuities near rail boundaries and height gradients discontinuities between the taper and flat regions. The whole air bearing surface is simulated to acquire the pressure profile, where geometry parameter, attitude parameter and numerical parameter are shown in Table 6.5-Table 6.7.

Fig. 6.12 The 1212 taper flat slider with wall profile

Table 6.5 Geometrical parameter of the 1212 taper flat slider with wall profile

Slider Length (TXL)	1.0E-03m
Slider Width (TYL)	0.6E-03m
Taper Length (XT)	0.2E-03m
Rail Width (W1)	0.1E-03m
Wall Profile Width (W2-W1)	0.02E-03m
Wall Profile Height (h _c)	4.0E-05m

Table 6.6 Flying attitude parameter of the 1212 taper flat slider with wall profile

Minimum Flying Height (h ₀)	1.0E-06m
Pitch Angle (α)	1.25mrad
Roll Angle (β)	0μrad
Skew Angle (δ)	0μrad
Radial Position	18mm
RPM	5400

Table 6.7 Numerical parameter of the 1212 taper flat slider with wall profile

Taylor Order (Order)	9
Number of Supporting Point (Supnode)	15
Extrapolation Factor (ω)	0.8
Weighting Function (w)	5
Searching Method	Layer Searching Method
Computational Time	60 minutes

Fig. 6.13 Dimension of the 1212 taper flat slider with wall profile

In Fig.6.13, height and height gradient of the taper parts ($X < X_T$ and $Y < W$ or $X < X_T$ and $W_4 < Y < (TYL - W_4)$) are obtained by (area 1)

$$H(X) = (1 - X)48.75 - 38.0 + (1 - X)tg\alpha + \left(\frac{TYL}{TXL} - Y\right)tg\beta$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\alpha + 48.75), \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.5)$$

Height and height gradient of the flat parts ($(TXL - X_T) > X > X_T$ and $Y < W_1$ or $(TXL - X_T) > X > X_T$ and $W_4 < Y < (TYL - W_4)$) in Fig.6.13 are represented as (area 2)

$$H(X) = \frac{TXL}{h_m} \left[(1 - X)tg\alpha + \left(\frac{TYL}{TXL} - Y\right)tg\beta \right] + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} \operatorname{tg} \alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} \operatorname{tg} \beta \quad (6.6)$$

Height and height gradient of the wall profile ($X < X_T$ and $Y > W_1$ and $0.921e-03 + 0.195 * X - 8.0 * Y \geq 0$) in Fig.6.13 are expressed as (area 3 down)

$$H(X) = \frac{TXL}{h_m} \left[-0.04875 * X + 2.0 * Y - \frac{0.18925E-03}{TXL} + (1-X) \operatorname{tg} \alpha + \left(\frac{TYL}{TXL} - Y \right) \operatorname{tg} \beta \right]$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} (\operatorname{tg} \alpha + 0.04875) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} (\operatorname{tg} \beta - 2.0) \quad (6.7)$$

Height and height gradient of the wall profile ($X < X_T$ and $Y < W_4$ and $0.003879 - 0.195 * X - 8.0 * Y \leq 0.0$) in Fig.6.13 are denoted as (area 3 upper)

$$H(X) = \frac{TXL}{h_m} \left[-0.04875 * X - 2.0 * Y + \frac{0.101075E-02}{TXL} + (1-X) \operatorname{tg} \alpha + \left(\frac{TYL}{TXL} - Y \right) \operatorname{tg} \beta \right]$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} (\operatorname{tg} \alpha + 0.04875) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} (\operatorname{tg} \beta + 2.0) \quad (6.8)$$

Height and height gradient of the wall profile ($TXL - X_T > X > X_T$ and $Y < W_1$) in Fig.6.13 are denoted as (area 4 down)

$$H(X) = \frac{TXL}{h_m} \left[(1-X) \operatorname{tg} \alpha + \left(\frac{TYL}{TXL} - Y \right) \operatorname{tg} \beta + \left(Y - \frac{W_1}{TXL} \right) \operatorname{tg} \delta \right]$$

$$\operatorname{tg} \delta = \frac{h_c}{W_2 - W_1}, \quad \frac{\partial H}{\partial X} = -\frac{TXL}{h_m} \operatorname{tg} \alpha, \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} (\operatorname{tg} \beta - \operatorname{tg} \delta) \quad (6.9)$$

Height and height gradient of the wall profile ($(TXL - X_T) > X > X_T$ and $W_3 < Y < W_4$) in Fig.6.13 are obtained by (area 4 upper)

$$H(X) = \frac{TXL}{h_m} \left[(1-X) \operatorname{tg} \alpha + \left(\frac{TYL}{TXL} - Y \right) \operatorname{tg} \beta + \left(\frac{W_4}{TXL} - Y \right) \operatorname{tg} \delta \right]$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} \operatorname{tg} \alpha, \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} (\operatorname{tg} \beta + \operatorname{tg} \delta) \quad (6.10)$$

Height and height gradient of the recessed region in Fig.6.13 are represented as (area 5)

$$H(X) = \frac{TXL}{h_m} \left[(1-X) \operatorname{tg} \alpha + \left(\frac{TYL}{TXL} - Y \right) \operatorname{tg} \beta \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} \operatorname{tg} \alpha, \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} \operatorname{tg} \beta \quad (6.11)$$

Fig.6.14 shows the 3-D pressure plots of the 1212 taper flat slider with the wall profile using the LSFD method for the case of $h_{\min} = 1 \mu\text{m}$ ($K_n = 0.064$). From Fig.6.15 and Fig.6.16, the pressures of the recessed region approach the ambient pressure and the results agree well with those from the finite volume method (CMLAir32).

Fig. 6.14 The 3-D pressure profile for the 1212 taper flat slider

Fig. 6.15 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the 1212 taper flat slider along $x = 0.2009E-03m$

Fig. 6.16 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the 1212 taper flat slider along $x = 0.510320601E-04m$

6.2.3 IBM3380 Taper Flat Slider with Discontinuous Height and Node Generation

6.2.3.1 Pressure Solution for a Given Slider Attitude

The IBM3380 taper flat slider (Fig.6.17) has the similar shape to 1212 taper flat slider. However, the pressure and height gradients of IBM3380 taper flat slider are much larger than those of 1212 taper flat slider, especially in the region of the wall profile and the area between the taper and flat parts. The geometry parameter, flying attitude and numerical parameter are described in Table6.8-Table6.10. Height profiles in different areas of the IBM3380 slider will affect the pressure result significantly.

Fig. 6.17 The IBM3380 taper flat slider with wall profile

Table 6.8 Geometrical parameter of the IBM3380 taper flat slider with wall profile

Slider Length (TXL)	0.405E-02m
Slider Width (TYL)	0.3E-02m
Taper Length (XT)	0.368E-03m
Rail Width (W1)	0.50E-03m
Wall Profile Width (W2-W1)	0.01E-03m
Wall Profile Height (h _c)	0.2236E-04m
Taper Height (HT)	0.559E-05m

Table 6.9 Flying attitude parameter of the IBM3380 taper flat slider with wall profile

Minimum Flying Height (h ₀)	243nm
Pitch Angle (α)	80μrad
Roll Angle (β)	0μrad
Skew Angle (δ)	0μrad
Taper Angle (γ)	15.189mrad
Radial Position	18mm
RPM	5400

Table 6.10 Numerical parameter of the IBM3380 taper flat slider with wall profile

Taylor Order (Order)	9
Number of Supporting Point (Supnode)	15
Extrapolation Factor (ω)	0.8
Weighting Function (w)	5
Searching Method	Layer searching method
Computational Time	60 mintue

In Fig. 6.13, height and height gradient of the taper parts are represented as (area 1)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{XT}{TXL} - X \right)tg\gamma \right] + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\alpha + tg\gamma), \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.12)$$

Height and height gradient of the flat parts in Fig. 6.13 are represented as (area 2)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right] + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.13)$$

Height and height gradient of the wall profile in Fig. 6.13 are expressed as (area 3 down)

$$H(X) = \frac{TXL}{h_m} \left[\frac{-0.01519021739 * X + 0.2236 * Y - 0.105967E-03}{TXL} + (1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right]$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\alpha + 0.01519021739) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\beta - 0.2236) \quad (6.14)$$

Height and height gradient of the wall profile in Fig. 6.13 are denoted as (area 3 upper)

$$H(X) = \frac{TXL}{h_m} \left[\frac{-0.01519021739 * X - 0.2236 * Y + 0.564833E-03}{TXL} + (1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right]$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\alpha + 0.01519021739) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\beta + 0.2236) \quad (6.15)$$

Height and height gradient of areas 2, 4, 5 in Fig. 6.13 have the same equations as Eq.(6.6) and Eq. (6.9)~ Eq. (6.11).

The obtained 3-D pressure profile of the IBM3380 taper flat slider with the wall profile is plotted in Fig.6.18 using the LSFD method for the case of $h_{\min}=243\text{nm}$ ($K_n=0.261$). At the same time, the pressures of the recessed region are found to approach the ambient pressure and the calculated results agree well with those obtained by the finite volume method (Cha, 1995) as shown from Fig.6.19 and Fig.6.20.

Fig. 6.18 The 3-D pressure profile in the IBM3380 slider with wall profile for $K_n=0.261$ ($h_{\min}=243\text{nm}$)

Fig. 6.19 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the IBM3380 slider along $y = 0.25E-03m$

Fig. 6.20 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the IBM3380 slider along $x = 0.358E-03m$

6.2.3.2 Node Generation of the IBM3380 Taper Flat Slider with Wall profile

We take the IBM3380 taper flat slider as an example to describe how to deal with node distribution in the case of the wall profile. The wall profile is designed to have $10\mu\text{m}$ width and $40\mu\text{m}$ height. Node layout is generated according to height gradients shown in Fig.6.21 and Fig.6.22 and the predicted pressure gradient.

In X-direction (Table 6.11), the former two control points designed to avoid the junction point ($X_T=0.368\text{mm}$) eliminate geometrical gradient discontinuity. Moreover, since pressure gradients on both sides of the junction point and trailing edge are larger, suitable divisions and ratios are designated to satisfy with the requirements of high resolution on high gradients and smooth transition from sparse region to dense region.

In Y-direction (Table 6.11), sensitive points ($y=0.50\text{mm}$, 2.5mm) on rails are avoided to be chosen as nodes by using neighboring points ($y=0.495\text{mm}$, 0.51mm) close to them. The third and fourth control points with proportional divisions and ratios ensure comparative node spacing on both sides of the sensitive points in the slider, which is very important in successfully obtaining numerical results. The consequential coordinate differences along X, Y directions in the IBM3380 taper flat slider with the wall profile are represented in Fig.6.23 and Fig.6.24. The resultant node distribution is shown in Fig.6.16 with total nodes number of $102*125$.

Fig. 6.21 Height gradient $\partial H/\partial X$ in the IBM3380 taper flat slider with wall profile

Fig. 6.22 Height gradient $\partial H/\partial Y$ in the IBM3380 taper flat slider with wall profile

Table 6.11 Node specification of the IBM3380 taper flat slider with wall profile

X Coordinate (mm)	[0, 0.358] [0.358, 0.378] [0.378, 0.557] [0.557, 3.692] [3.692,4.05]
X Control Coordinate (mm)	0.358, 0.378, 0.557, 3.692
X Control Point Indices	20, 1 ,10, 50, 20
X Node Ratios	-2, 1, 2, -5, -2
Y Coordinate (mm)	[0, 0.495] [0.495, 0.51] [0.51, 0.7575] [0.7575, 2.2475] [2.2475, 2.495] [2.495, 2.51] [2.51, 3.0]
Y Control Coordinate (mm)	0.495, 0.51, 0.7575, 2.2475, 2.495, 2.51
Y Control Point Indices	28, 1, 14, 10, 14, 1, 28
Y Node Ratios	-2, 1, 0.5, 1, 2, 1, -2

Fig. 6.23 Coordinate difference (ΔX) in the IBM3380 taper flat slider with wall profile

Fig. 6.24 Coordinate difference (ΔY) in the IBM3380 taper flat slider with wall profile

Fig. 6.25 Node distribution of the IBM3380 taper flat slider (102*125) with wall profile

6.3 Negative Pressure Sliders and Node Generation

6.3.1 Negative Pressure Slider Model 1

In order to simulate the negative pressure slider, according to the principle of fluid mechanics for air bearing surface, the negative pressure slider model 1 (Fig.6.26) modified from 1212 taper flat slider is calculated to obtain negative pressure profile and verify the LSFD mesh-free method for application to simulation of this kind of slider. Generally, the negative pressure slider has higher pressure gradient than the positive pressure slider in some regions, in which pressures change sharply from positive value to negative value. It brings a great challenge to develop the mesh-free method for applications to simulation of more modern and complex head sliders.

In the negative pressure slider model 1, the wall profile is designed to eliminate discontinuous height gradients, which has 0.05mm width and 0.39 μ m height as shown in Table 6.12. The flying attitude parameter and numerical parameter are represented in Table 6.13 and Table 6.14, respectively.

Again, the height profiles of different regions will influence significantly the pressure distribution in the air bearing surface.

Fig. 6.26 The negative pressure slider model 1

Table 6.12 Geometrical parameter of the negative pressure slider model 1

Slider Length (TXL)	0.14E-02m
Slider Width (TYL)	0.1E-03m
Taper Length (XT)	0.2E-03m
Rail Length (L2)	0.1E-02m
Wall Profile Width (L3-L2)	0.05E-03m
Wall Profile Height (h _c)	0.39E-04m
Taper Height (HT)	0.975E-05m

Table 6.13 Flying attitude parameter of the negative pressure slider model 1

Minimum Flying Height (h_0)	1.0E-06m
Pitch Angle (α)	833.3 μ rad
Roll Angle (β)	0 μ rad
Skew Angle (δ)	0 μ rad
Taper Angle (γ)	48.7114mrad
Radial Position	18mm
RPM	5400

Table 6.14 Numerical parameter of the negative pressure slider model 1

Taylor Order (Order)	9
Number of Supporting Point (Supnode)	15
Extrapolation Factor (ω)	1.2
Weighting Function (w)	5
Searching Method	Layer Searching Method
Computational Time	60 minute 46 second
Computational Node	161*49

Fig. 6.27 Dimension of the negative pressure slider model 1

In Fig.6.27, height and height gradient of the taper part are represented as (area 1)

$$H(X) = (1-X)68.25 - 57.5.0 + (1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y\right)tg\beta$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\alpha + 68.25), \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.16)$$

Height and height gradient of the flat part are obtained by (area 2)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y\right)tg\beta \right] + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.17)$$

Height and height gradient of the wall profile region are obtained by (area 3)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y\right)tg\beta + \left(X - \frac{L2}{TXL}\right)tg\gamma \right] + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\alpha - tg\gamma) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.18)$$

Height and height gradient of the extension region are expressed as (area 4)

$$H(X) = \frac{TXL}{h_m} \left[(1-X) \operatorname{tg} \alpha + \left(\frac{TYL}{TXL} - Y \right) \operatorname{tg} \beta \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} \operatorname{tg} \alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} \operatorname{tg} \beta \quad (6.19)$$

The height gradient $\partial H / \partial X$ in the wall profile is much larger than that in other regions (Fig.6.28), which represents abrupt pressure change, while height gradient $\partial H / \partial Y$ is assumed to be uniform in the whole domain (Fig.6.29). Therefore, denser nodes are clustered along the wall profile and the taper part, while nodes are distributed sparsely in other regions.

Fig. 6.28 Height gradient $\partial H / \partial X$ in the negative pressure slider model 1

Fig. 6.29 Height gradient $\partial H/\partial Y$ in the negative pressure slider model 1

The 3-D pressure profile for the negative pressure slider model 1 is obtained as described in Fig.6.30, which agrees well with predicted pressure trend according to fluid mechanics principle in node generation. Fig.6.31 and Fig.6.32 indicate that the pressure results agree well with those from the finite volume method (CMLAir32). It further proves the feasibility of the LSFD method for solving the negative pressure head slider problems.

Fig. 6.30 The 3-D pressure profile for the negative pressure slider model 1

Fig. 6.31 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the negative pressure slider model 1 along $y=0.68672862E-04m$

Fig. 6.32 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the negative slider 1 along $x = 0.7279919624E-03m$

6.3.2 Advanced Step Slider (ASS) and Node Generation

6.3.2.1 Pressure Solution for a Given Slider Attitude

An advanced sub-ambient pressure step slider, which consists of a step bearing with sub-micron depth and a cavity for generating sub-ambient pressure force to reduce variation of flying height caused by changing attitudes and disk velocities, is studied here. The slider is illustrated in Fig.6.33. The dimension of the slider is 1.25mm long, 1.0mm wide, and 0.3mm high, which is a so-called pico slider. The ASS has a step bearing 0.2 μm at the leading edge, a cross rail, a cavity for generating sub-ambient pressure force and a center pad. The flying height at the trailing edge of the center pad is fixed at 30nm and the pitch attitude at 80 μrad . The sub-ambient pressure step slider is placed on 16 mm diameter disk drives with 7200RPM. The air bearing load of the

slider is in equilibrium with the sum of the suspension load and sub-ambient pressure force when the slider flies over the disk.

Step depth of the step slider is crucial to decrease the sensitivity of flying height to disk velocity. Basically, the advanced step slider, a sub-ambient pressure step slider, shows the uniform flying height across the disk surface and small decrease of flying height at high altitude.

Fig. 6.33 The advanced step slider

Attaching two straight rails to either side of air bearing surface of the advanced step slider, the pressure distribution will change accordingly. It is found that the roll stiffness will be improved by balancing attitude along roll direction through the two rails. However, these side rails also introduce clearance discontinuities among rails and cavity regions, which bring much more challenge to get convergence results. It

can be tackled by adding wall profile and suitable node distribution to acquire the pressure profile.

Fig. 6.34 Dimension of the advanced step slider

In Fig.6.34, height and height gradient of the step bearing are determined by (area 1)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right] + \frac{h_s}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.20)$$

Height and height gradient of the cross rail and the center pad can be represented as (area 2, 3)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right] + \frac{h_s}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}tg\beta \quad (6.21)$$

Height and height gradient of the wall profile in area 4 are denoted as (area 4)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{W1}{TXL} - Y \right)tg\gamma1 \right] + \frac{h_c}{h_m} + 1$$

$$tg(\gamma1) = \frac{h_s - h_c}{WT} \quad \frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\gamma1 + tg\beta) \quad (6.22)$$

Height and height gradient of the wall profile in area 5 are expressed as (area 5)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{W1}{TXL} - Y \right)tg\gamma1 \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(-tg\gamma1 + tg\beta) \quad (6.23)$$

Height and height gradient of the wall profile in area 6 are obtained (area 6)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta - 0.065X - 0.075Y + \frac{0.2575E-04}{TXL} \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\alpha + 0.065) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\beta + 0.075) \quad (6.24)$$

Height and height gradient of the wall profile in area 7 are represented as (area 7)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta - 0.065X + 0.075Y + \frac{0.5075E-04}{TXL} \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\alpha + 0.065) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\beta - 0.075) \quad (6.25)$$

Height and height gradient of the wall profile in area 8 are obtained (area 8)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{W1}{TXL} - Y \right)tg\gamma_2 \right] + \frac{h_c}{h_m} + 1$$

$$tg(\gamma_2) = \frac{h_c}{WT} \quad \frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\gamma_2 + tg\beta) \quad (6.26)$$

Height and height gradient of the wall profile in area 9 are represented as (area 9)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(Y - \frac{W8}{TXL} \right)tg\gamma_2 \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m}tg\alpha \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(-tg\gamma_2 + tg\beta) \quad (6.27)$$

Height and height gradient of the wall profile in area 10 are expressed as (area 10)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{L1}{TXL} - X \right)tg\gamma_3 \right] + \frac{h_s}{h_m} + 1$$

$$tg(\gamma_3) = \frac{h_s}{WT} \quad \frac{\partial H}{\partial X} = -\frac{TXL}{h_m}(tg\gamma_3 + tg\alpha) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m}(tg\beta) \quad (6.28)$$

Height and height gradient of the wall profile in area 11 are acquired by (area 11)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(X - \frac{L4}{TXL} \right)tg\gamma \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} (tg\alpha - tg\gamma) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} (tg\beta) \quad (6.29)$$

Height and height gradient of the wall profile in area 12 are obtained by (area 12)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{L5}{TXL} - X \right)tg\gamma \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} (tg\alpha + tg\gamma) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} (tg\beta) \quad (6.30)$$

Height and height gradient of the wall profile in area 13 are expressed as (area 13)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(\frac{W3}{TXL} - Y \right)tg\gamma \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} (tg\alpha) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} (tg\beta + tg\gamma) \quad (6.31)$$

Height and height gradient of the wall profile in area 14 are represented as (area 14)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta + \left(Y - \frac{W6}{TXL} \right)tg\gamma \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} (tg\alpha) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} (tg\beta - tg\gamma) \quad (6.32)$$

Height and height gradient of the cavity region in area 15 are expressed as (area 15)

$$H(X) = \frac{TXL}{h_m} \left[(1-X)tg\alpha + \left(\frac{TYL}{TXL} - Y \right)tg\beta \right] + \frac{h_c}{h_m} + 1$$

$$\frac{\partial H}{\partial X} = -\frac{TXL}{h_m} (tg\alpha) \quad \frac{\partial H}{\partial Y} = -\frac{TXL}{h_m} tg\beta \quad (6.33)$$

Tables 6.15~ 6.17 show the geometrical parameter, flying height attitude parameter and numerical parameter of the advanced step slider. Fig.6.35 shows the normalized sub-ambient pressure in advanced step slider with 30nm of minimum flying height. Fig.6.36 and Fig.6.37 compare the pressure results obtained by the LSFD method with those from the finite volume method (CMLAir32). The results are found to agree well with each other.

Table 6.15 Geometrical parameter of the advanced step slider

Slider Length (TXL)	1.25E-03m
Slider Width (TYL)	1.0E-03m
Step Length (XT)	0.2E-03m
Cross Rail Length (L4-L2)	0.22E-03m
Wall Profile Width WT(L3-L2)	0.02E-03m
Cavity Height (h_c)	1.5E-06m
Step Height (h_s)	0.2E-06m

Table 6.16 Flying attitude parameter of the advanced step slider

Minimum Flying Height (h_0)	30.0E-09m
Pitch Angle (α)	80 μ rad
Roll Angle (β)	0 μ rad
Skew Angle (δ)	0 μ rad
Radial Position	16mm
RPM	7200

Table 6.17 Numerical parameter of the advanced step slider

Taylor Order (Order)	9
Number of Supporting Point (Supnode)	15
Extrapolation Factor (ω)	0.6
Weighting Function (w)	5
Searching Method	Layer Searching Method
Computational Time	6 hours
Computational Node	170*141

Fig. 6.35 The 3-D pressure profile in the advanced step slider for $K_n=2.116$

Fig. 6.36 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the advanced step slider along $x = 0.157779277053E-03m$

Fig. 6.37 Comparison of pressure profiles using the LSFD method and the finite volume method (CMLAir32) in the advanced step slider along $y = 0.50d-03m$

6.3.2.2 Node Generation of the Advanced Step Slider with Wall profile

The advanced step slider is used to illustrate how to generate the nodes layout in negative sliders. It can be anticipated that the pressure peak on the positive region is very high, and yet the negative pressure is very low on the neighboring region. Hence, it is valuable to discuss how to achieve reasonable node distribution in the advanced step slider.

It is particularly important to have enough node resolution in junction areas among the recessed region and surrounding regions, in which pressures drop rapidly. When the node is prescribed manually using the piecewise geometric progression method, it is of course possible to concentrate the node sufficiently at the recessed region. Since step bearing has different height from the cross rail, the wall profile is positioned to eliminate the height discontinuity, leaving denser nodes distributed in this ramp region. In general, the wall profiles always lie in transition regions to connect those regions with different heights, such as cavity region, cross rail region and center pad region.

The same principle of node generation is used in practice along X, Y-directions in the negative pressure slider problems. Thus, node generation should be based on anticipated pressure and height gradients (Fig.6.38 and Fig.6.39) with the aid of the wall profile (Table 6.18). The resultant differences of nodes coordinates along both directions are shown in Fig.6.40 and Fig.6.41. Total node distribution for the advanced step slider is presented in Fig.6.42.

Fig. 6.38 Height gradient $\partial H/\partial X$ in the advanced step slider

Fig. 6.39 Height gradient $\partial H/\partial Y$ in the advanced step slider

Table 6.18 Node specification of the advanced step slider

X Coordinate (mm)	[0,0.16] [0.16, 0.199] [0.199, 0.201] [0.201, 0.418] [0.418, 0.439] [0.439, 0.441] [0.441, 0.462] [0.462, 0.726] [0.726, 0.99] [0.99, 1.028] [1.028, 1.032] [1.032, 1.141] [1.141, 1.25]
X Control Coordinate (mm)	0.16, 0.199, 0.201, 0.418, 0.439, 0.441, 0.462, 0.726, 0.99, 1.028, 1.032, 1.141
X Control Point Indices	18, 20, 1, 30, 11, 1, 11, 25, 20, 11, 1, 10, 10
X Node Ratios	0.1, 1, 1, -8, 1, 1, 1, 10, 0.125, 1, 1, 5, 0.1
Y Coordinate (mm)	[0,0.16] [0.16, 0.188] [0.188, 0.192] [0.192, 0.22][0.22, 0.35] [0.35, 0.378] [0.378, 0.382] [0.382, 0.41] [0.41, 0.59] [0.59, 0.618] [0.618 ,0.622] [0.622, 0.65] [0.65, 0.80] [0.80, 0.828] [0.828, 0.832] [0.832, 0.86] [0.86, 1.0]
Y Control Coordinate (mm)	0.16, 0.188, 0.192, 0.22, 0.35, 0.378, 0.382, 0.41, 0.59, 0.618 ,0.622, 0.65, 0.80, 0.828, 0.832, 0.86
Y Control Point Indices	7, 1, 7, 15, 8, 1, 8, 14
Y Node Ratios	1, 1, 1, -5, 1, 1, 1, -5

Fig. 6.40 Coordinate difference (ΔX) in the advanced step slider

Fig. 6.41 Coordinate difference (ΔY) in the advanced step slider

Fig. 6.42 Node distribution of the advanced step slider (170&141)

6.4 Summary

The negative pressure sliders bring great challenge to numerical simulation of air bearing sliders due to larger pressure gradient in some regions compared with positive pressure sliders. Besides adding the wall profile to eliminate geometrical discontinuity, a variety of methods in node generation have been devised to minimize numerical error and overcome the potential instability problem of the LSFD method in solving negative pressure sliders. Firstly, it is found that node spacing around the area of the largest pressure gradient is an important basic value for node generation in the negative pressure sliders. Next, the slider air bearing has a geometrical peculiarity, namely, the inherent clearance discontinuity problem. This requires nodes layout based on height change and height gradient. The proposed approach is to choose points positioned on both sides of sensitivity point to intentionally avoid those significant sensitive points. Finally, the anticipated pressure gradient should be a valuable factor to be considered in nodes generation. It is proved that adding the wall profile and adaptive nodes distribution based on pressure and height gradients would enable the LSFD method to successfully simulate negative pressure sliders.

CHAPTER 7 Conclusions and Future Work

7.1 Conclusions

The main purpose of the research is to develop a powerful, general purpose mesh-free based air bearing design software that can model and simulate the complex modern air bearing sliders for hard disk drives (HDD). The model should be able to incorporate all main air bearing characteristics that are being used in hard disk drives, including shaped rails, arbitrary wall profiles and multiple etch depths. To achieve the goal, a simple, yet efficient mesh-free method – the least square finite difference (LSFD) method for solving the rarefied gas lubrication equation, is introduced and applied to solve the air bearing slider problems. The key point is the exploration of the mesh-free method for ABS modeling, so that it will facilitate the simulation of all types of sliders with complex geometry and highly recessed region.

Firstly, to enhance the computational efficiency of the mesh-free LSFD method, some new techniques to search supporting points for the reference node were proposed and explored. This would greatly expand the application of the mesh-free method to slider models with complicated geometry and high variable height gradient. In addition, the influence of the weighting functions on the results obtained was also studied and the best choice of the weighting functions was determined. Moreover, the extrapolation factor was predetermined through numerical trial process to provide the fastest convergence. Two approximation methods for the linearization of the discretized equations are also presented and compared to achieve optimal convergence speed. These improvements eliminated some potential limitation of the LSFD method

previously published and further facilitated its employment in most existing slider models.

Secondly, the one-dimensional and two-dimensional Reynolds equations with various slip models based on the mesh-free method were solved to verify the methodology by comparing with the theoretical and numerical results previously published. The results were found to agree well with each other. Furthermore, both regular and irregular node distribution were implemented to provide greater flexibility for dealing with all types of potential complicated air bearing surfaces in modern magnetic drives. The inverse problem, which determines the equilibrium-flying attitude, was also solved using the Quasi-Newton method to further validate the model.

Thirdly, the full potential of the mesh-free LSFD method in dealing with the fluid problems in head disk interface has been demonstrated by its applications to different types of positive sliders, such as Tri-pad slider, TPC slider and IBM 3380 taper flat slider. The piece-wise progression method with variable node spacing scheme based on geometric and pressure gradients is implemented for nodes generation. Positive sliders models based on the generalized lubrication were tackled. Good agreement is found between the simulated results and results obtained from other numerical methods.

Finally, a few low flying negative pressure sliders are attempted and presented. The technique of adding wall profile is proposed and designed to eliminate geometrical discontinuity in height. It is found that adding wall profiles, piece-wise geometric

progression approach as well as a powerful adaptive strategy for node generation improve the solution accuracy and convergence.

The mesh-free LSFD method, together with the proposed node generation strategy, was proved to be an efficient way for solving the generalized Reynolds equation for air bearing sliders with complex rail systems.

7.2 Future Work

Continued growth of storage densities in the industry of hard disk drives requires new innovations of air bearing surface design, which presents great challenges in the design in general. The mesh-free method and numerical model developed in this thesis for ABS simulation is a significant and successful first step in establishing a general and versatile tool for designing novel sliders. Further improvement and work are needed to improve this methodology, so that it will be able to more efficiently and robustly deal with the modeling and simulation of more complex sliders required for even higher recording densities and ever lower flying height.

Firstly, to further improve the convergence rate and the efficiency for solving the highly nonlinear problems, the multi-grid method should be explored and implemented in future. Present nonlinear algebraic equations discretized from the generalized Reynolds equation by the LSFD method are tackled by the Successive-Over Relaxation (SOR). In the quest to speed up and improve the convergence process, the SOR method based on single grid level shows inefficiency, since it is observed that the convergence of the solution slows down as the iteration process

continues. The process of obtaining the final solution iteratively from an initial guess is called “smoothing”. Brandt (1977) demonstrated that only those error components with wavelength comparable to the mesh size are smoothed efficiently, and that error components with longer wavelengths are smoothed at progressively slower rates. However, the long wavelength (smooth) error can be smoothed more efficiently if the solution is moved to a coarser grid. As a result, by moving the solution back and forth between a set of different grid levels, both long and short wavelength error components can be smoothed efficiently. Moreover, the efficiency of the multi-grid method is optimum compared to single grid iterative method in the sense that the computation effort is linearly proportional to the number of degrees of freedom. The multi-grid method becomes even more superior when the system of equations becomes larger. Thus, the existing solver based on the LSFD method with multi-grid method will speed up the convergence process, which largely improves the solver's efficiency.

Secondly, the advantage of the LSFD mesh-free method lies in easy node generation without element connectivity. Present node distribution is generated in consideration of predicted pressure and known height gradients. Thus, automatic node generation, refinement and adaptation techniques should be explored and implemented to generate several levels of good quality node layout that may be required in multi-grid method over sliders with complex rail shapes, so that the advantage of mesh-free method over standard numerical methods will be fully realized.

Finally, current design of air bearing surfaces is mainly based on engineers experience and trial-and-error approach as well as basic parametric optimization through design

parameters change. Those parametric optimizations may be only able to produce some incremental performance improvement rather than a global optimum solution. To overcome this limitation restricted by the present modeling and designing technology, topological optimization should ultimately be studied and employed in the computational modeling tool. The topology optimization will be able to produce some novel air bearing designs through an iterative optimization process starting from an initial slider of a general rectangle geometry and uniform thickness and meeting the requirements of a relatively constant flying height over the entire disk, a low take-off/landing velocity as well as low sensitivity to manufacturing tolerances.

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