New techniques for relay network beamforming on frequency-selective channels

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New Techniques for Relay Network Beamforming on Frequency-Selective Channels

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Abstract

User-cooperative techniques have recently gained tremendous interests in wireless communication society, for they are able to exploit the spatial diversity among distributed network nodes. In cooperative relay networks, data transmissions between sources and destinations can be assisted by relays which form a virtual antenna array.

In this thesis, we investigate relay network beamforming using the filter-and-forward (FF) relaying scheme on frequency-selective channels. The thesis makes original contributions by introducing two classes of new techniques for relay network beamforming design. One class involves designing the beamformers to achieve a distortionless response. The other class uses an alternative optimization criterion by minimizing the output power at the destination.

For the first class of technique for relay beamforming design, we consider a relay network on frequency-selective channels, in which the received signals are affected by the inter-symbol-interference (ISI), and the FF scheme is adopted to combat ISI. To adaptively select the optimal delay decision which crucially affects the performance, we formulate and solve the single-user relay beamforming problem using the frequency-domain approach. A new technique of distortionless design is introduced which aims at forcing the frequency response of equivalent channel from the source to the destination to be flat and linear phase. Hence the proposed approach eliminates the ISI completely, and it is shown to have lower computational complexity in selecting optimal decision delay according to channel realizations.

As the second class of technique for relay beamforming design, we develop a new criterion which amounts to minimize the destination output power also subject to the distortionless constraint and is described in frequency domain. An adjustable scalar amplifier at the destination side is introduced and its range of value and effects on the system performance are discussed. With the scalar amplifier simultaneously optimized with the relay beamformer coefficients, we demonstrate that the proposed new technique is equivalent to the distortionless SINR maximization criterion in terms of output SINR performance. In addition, the output power minimization scheme is extended to a multi-antenna destination scenario, where the destination also deploys a beamformer which is jointly designed with the relay beamformer.

By adopting the new technique of output power minimization, we propose a relay beamforming design without the distortionless constraint. We demonstrate that the proposed approach attains the optimal output SINR that is given by the SINR maximization design scheme. Moreover, although the same output SINR performance is achieved, when the individual relay transmit power constraint is taken into account, the proposed approach requires lower computational complexity than the SINR maximization scheme. In addition, a time-domain version of the distortionless constraint is proposed, and with this constraint, the efficient selection of decision delay is also considered with a time-domain approach.

Subsequently, we extend the output power minimization scheme to cater for multiple peer-to-peer FF relay beamforming. Two criteria are considered, in which one is to minimize the maximal output power among all destinations, and another is minimize the sum of the output power. The former one is proved to be equivalent to the worst SINR maximization scheme in terms of output SINR performance, and the latter one is shown to give the optimal average-output-SINR. Iterative algorithms based on second-order cone programming (SOCP) are proposed to solve these optimization problems.

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List of Abbreviations

AF	amplify-and-forward
BER	bit error rate
CSI	channel state information
DEF	distortion evaluation function
DF	decode-and-forward
DFT	discrete Fourier transform
DTFT	discrete-time Fourier transform
DOF	degree of freedom
FF	filter-and-forward
FIR	finite impulse response
FLOP	floating-point operations
ISI	inter-symbol interference
IUI	inter-user interference
MIMO	multiple input multiple output
MSE	mean square error
MMSE	minimum mean-square error
SDP	semidefinite programming
SDR	semidefinite relaxation
SINR	signal-to-interference-plus-noise ratio
SNR	signal-to-noise ratio
SOCP	second-order cone programming
TDD	time-division duplexing

List of Notations

$*$	Convolution
$\mathcal{F}\{\cdot\}$	Discrete-time Fourier transform of a sequence
$\Re\{\cdot\}$	Real-part of a complex number
$\otimes$	Kronecker product
$\oplus$	Direct sum of matrices
$(\cdot)^*$	Complex conjugation
$(\cdot)^T$	Transpose
$(\cdot)^H$	Hermitian transpose
$\text{diag}(\mathbf{a})$	Make a diagonal matrix whose diagonal entries are the elements of vector $\mathbf{a}$
$[A]_{i,j}$	Entry of matrix A in row i and column j
$ \cdot $	Absolute value of a complex number
$\ \cdot\ $	Euclidean norm of a vector
$\mathbf{I}_n$	$n \times n$ identity matrix
$\mathbf{0}_{m \times n}$	$m \times n$ all-zero matrix
$\mathbb{C}$	Field of complex numbers
$\mathbb{R}$	Field of real numbers
$f_i(m)$	Impulse response of the i th backward channel
$g_i(m)$	Impulse response of the i th forward channel
$h_i(m)$	Impulse response of the i th relay filter
$F_i(\omega)$	Frequency response of the i th backward channel
$G_i(\omega)$	Frequency response of the i th forward channel
$H_i(\omega)$	Frequency response of the i th relay filter
$h_{eqv}(m)$	Impulse response of the equivalent channel from the source to the input of the destination
$H_{eqv}(\omega)$	Frequency response of the equivalent channel from the source to the input of the destination
L_f	Length of backward channel
L_g	Length of forward channel
L_h	Length of relay filter
P_s	Source transmit power
P_0	Total relay transmit power budget
$P_{0,i}$	Individual relay transmit power budget

σ_n^2	Relay noise power
σ_v^2	Destination noise power
α	Destination scalar amplifier
γ	Desired power level of destination output signal
τ	Decision delay
R	Number of relays
σ_t	Delay spread factor
$\mathbf{w}$	Vector of relay beamforming coefficients
$\tilde{g}_{i,j}(m)$	Impulse response of the forward channel from the i th relay to the j th antenna of the multi-antenna destination
$\tilde{G}_{i,j}(\omega)$	Frequency response of the forward channel from the i th relay to the j th antenna of the multi-antenna destination
$\tilde{h}_{eqv}(m)$	Impulse response of the equivalent channel from the source to the input of the scalar amplifier of the multi-antenna destination
$\tilde{H}_{eqv}(\omega)$	Frequency response of the equivalent channel from the source to the input of the scalar amplifier of the multi-antenna destination
$t_j(m)$	Impulse response of the filter after the j th antenna of the multi-antenna destination
$T_j(\omega)$	Frequency response of the filter after the j th antenna of the multi-antenna destination
L_t	Length of filters at the multi-antenna destination
$\mathbf{z}$	Vector of beamforming coefficients of the multi-antenna destination
$f_{i,k}(m)$	Impulse response of the backward channel from the k th source to the i th relay
$g_{i,k}(m)$	Impulse response of the forward channel from the i th relay to the k th destination
$h_{eqv,k}(m)$	Impulse response of the equivalent channel from the k th source to the input of the k th destination
$h_{iui-j,k}(m)$	Impulse response of the IUI channel from the j th source to the input of the k th destination

Chapter 1

Introduction

1.1 Motivation

Within the realm of the wireless communications, both for academia and industry, there is endless pursuit for better capacity, reliability and coverage. Having been widely regarded as one of the successful techniques to achieve these goals, multi-antenna based multiple-input-multiple-output (MIMO) scheme has been intensively studied and adopted in standards such as WLAN [1], WiMAX [2], LTE and LTE-Advanced [3]. However, due to the physical size, battery life and processing power limitation of wireless terminal devices, it is still more practical to employ multiple antennas at the base station side.

In order to exploit the enormous merits introduced by MIMO techniques in networks with single-antenna devices, the concept of user-cooperation has recently gained tremendous interests in the wireless communication society [4–6], for they are able to exploit the spatial diversity among distributed network nodes. In cooperative relay networks, data transmissions between sources and destinations can be assisted by relays through certain relay protocols, among which the most popular ones are amplify-and-forward (AF) and decode-and-forward (DF) [6–9]. Without requiring decoding or re-encoding processing at the relay nodes, the AF scheme is commonly used due to its simplicity.

For the case where multiple relays exist in the network and they work cooperatively to assist the signal transmission from a source node to its desired destination node, the technique of relay beamforming, which is also known as network beamforming or distributed beamforming, has emerged recently [10–14]. The distributed relay nodes function cooperatively as a virtual multi-antenna system, and hence the cooperative diversity gain can be obtained. Problems arise as to coordinate all relays and make them work as a beamformer properly. There have been extensive work discussing the relay beamforming problem based on AF scheme [10–12, 15], which focus on networks in the frequency-flat channel environment and assume perfect knowledge of the channel state information (CSI).

Whereas, in order to deal with the frequency-selective channels, Chen *et al.* propose a filter-and-forward (FF) relaying scheme for distributed beamforming [16]. In this scheme, relays process the received signals with finite impulse response (FIR) filters and then re-transmit them towards the destination. Since the FF distributed beamforming functions as distributed channel equalization, along the same lines, Liang *et al.* introduce an additional decision feedback equalizer (DFE) at the destination side and take into account the direct link between the source and the destination [17].

Despite the great performance gain shown by FF relaying over the traditional AF scheme in dealing with frequency-selective channels, it is still of great interest to further investigate the FF relaying scheme. Rather than a simple extension of AF relaying scheme, FF scheme gives rise to some issues that specifically concern the filter design and frequency response characteristics of channels.

1.2 Objectives

The main objectives of this thesis are listed as follows.

- In FF relay beamforming, linear filters are introduced at the relay nodes to tackle the inter-symbol interference (ISI), which is caused by the frequency-selective channels. Then, the basic idea of eliminating ISI is to compensate for the frequency-selectivity of channels. In the literature on relay beamforming, considering and addressing the ISI problem directly from frequency domain have not been extensively studied. Therefore, the frequency-domain description and behavior of the relay network beamforming are to be characterized and investigated.
- It has been observed in [18] that the performance of the FF beamforming can be significantly affected by the slicer decision delay, and the decision delay is related to various system factors. However, for a given channel statistics and system parameters such as the filter length and the number of relays, there is no efficient design rule of selecting the optimal delay. Therefore, a new approach to optimally select the decision delay is to be studied.
- In the existing literature dealing with the relay beamforming design problems, the output SNR and the relay transmit power based criteria are most widely used. Besides, it is also of great interest to explore other design criteria which can have potential advantage or show new characteristics. Furthermore, exploring more criteria can also complete the framework of relay beamforming problems. Thus, new criteria for relay beamforming design are to be investigated.
- As a more meaningful network model which reflects more accurately the operation of practical systems, multi-user relay network beamforming problem is to be considered using new design criteria.

1.3 Major Contributions of the Thesis

The major contributions of this thesis are summarized as follows:

(1) Adaptive Decision Delay Selection Using Frequency-Domain Approach

We propose a new approach to relay beamforming in frequency-selective channels, where the FF relay scheme is adopted to combat the channel distortion. By formulating the relay beamforming problem in frequency domain, we impose a distortionless constraint to control the frequency response of the equivalent channel. It is proved that under a certain condition that is related to the length of the channel filters and the relay filters as well as the number of relay nodes, the equivalent channel is able to attain flat frequency response with linear phase through optimal relay beamforming. We provide a closed-form solution to the beamforming problem with distortionless constraint. In addition, the problem of adaptively selecting the optimal decision delay is also investigated, which achieves the highest output SINR for a channel realization. Simulation results demonstrate that the proposed approach achieved substantial performance improvement over the non-adaptive delay selection approaches.

(2) Output Power Minimization Design with Distortionless Constraint

We study the FF relay beamforming design problem with a new approach, which aims to minimize the total output power at the destination side subject to a distortionless constraint. By introducing an adjustable scalar amplifier at the destination side, we discuss the relay beamforming design scheme with minimizing output power in two cases. We first consider the scalar amplifier has a fixed value and a given signal gain level is required for the desired signal. We derive a range for the selection of signal gain in order to make it consistent with the relay transmit power constraint. We then consider the general design scheme where both the relay weight and the scalar amplifier are jointly

optimized to achieve the minimum output power also with a requirement for signal power gain. We show that the adaptive-scalar scheme always achieves the same output SINR performance as the scheme of SINR maximization with the distortionless constraint.

Furthermore, the output power minimization scheme is extended to the multi-antenna destination case, where each antenna at the destination node is followed by a linear FIR filter, which can be jointly designed and work collaboratively with the relay filters.

(3) Output Power Minimization for Optimal SINR Achievable Beamforming

Based on the idea of output power minimization, three relay beamforming approaches subject to different constraints are proposed. The first approach amounts to minimize the output power while keep the power of the desired signal at a constant level, and is subject to a total relay transmit power constraint. It is shown that, in terms of output SINR performance, this approach is equivalent to the SINR maximization formulation. The second approach also minimizes the output power with the desired signal power maintained at a constant level, but is subject to both total and individual relay transmit power constraint. This scheme is shown to be equivalent to SINR maximization. However, the proposed approach requires lower computational complexity in solving the problem. The third approach minimizes the output power under the constraint that the equivalent channel from the source to the destination is distortionless, *i.e.*, the frequency response is flat and with linear phase. Bearing this property, this approach can efficiently select the optimal decision delay.

(4) Relay Beamforming for Multi-User using Output Power Minimization

By extending the idea of output power minimization, we consider designing the FF relay beamforming for multiple peer-to-peer communications in frequency-selective channels. Two formulations are proposed. The first formulation aims to minimize the maximal

output power at the destination side among all users, under the constraints of both total and individual relay transmit power. We show that the proposed maximal output power minimization formulation is equivalent to the worst SINR maximization problem which leads to a sequence of semi-definite programming (SDP) feasibility problem. However, with the proposed formulation two iterative algorithms are provided, which are both based on second-order cone (SOCP) feasibility problem. The second formulation amounts to minimize the sum output power at all the destinations, and it is shown that this formulation renders the best performance in terms of mean output SINR among all users.

1.4 Organization of the Thesis

The organization of this thesis is as follows:

In Chapter 2, as a background of this thesis, the traditional MIMO beamforming and the progress of relay distributed beamforming are briefly reviewed. In Chapter 3, a relay beamforming scheme using frequency-domain method is proposed, which is able to handle the decision delay selection. In Chapter 4, the output power minimization scheme is studied, and its equivalence to the SINR maximization scheme is shown. In Chapter 5, the output power minimization scheme is further formulated in time-domain, and a non-distortionless design is proposed, which is shown to be equivalent to the usual SINR maximization scheme, and however, it shows advantage in computational complexity in the scenario of individual relay power constraint. In Chapter 6, the output power minimization scheme is extended to address the multi-user relay beamforming problem. We propose iterative algorithms to solve two multi-user beamforming formulations. Finally, Chapter 7 concludes this thesis and proposes future research directions.

Chapter 2

Background of Relay Network Beamforming

In this chapter, we briefly review the developing of MIMO communications techniques and the progress on the distributed relay beamforming. As we will depict in the sequel, the relay beamforming idea stems from and closely related to the more established MIMO techniques. Specifically, to achieve the merits of MIMO systems in terms of higher capacity, stronger reliability and better coverage abilities, and meanwhile to circumvent the limitation on amounting multiple antennas on wireless devices that are size-limited or power-limited, the distributed single-antenna terminals can be aggregated and cooperatively perform as a virtual multi-antenna system. Furthermore, within the discussion on relay distributed beamforming, we will provide a general view of different scenarios, formulation criteria, solution approaches, as well as the state-of-art developments and techniques.

2.1 MIMO Systems and Beamforming

The multiple-input-multiple-output (MIMO) technique have drawn considerable interests in both academia and industry, since deploying multiple antennas at both the transmitter and receiver side can significantly improve the communication performance over the

traditional single antenna systems [19–23].

The MIMO systems can provide array gain, diversity gain as well as multiplexing gain. By exploiting the multiplexing gain, the system capacity can be increased. Some well-known schemes include the diagonal Bell Labs layered space-times (D-BLAST) [24] and the vertical Bell Labs layered space-time (V-BLAST) [25].

While the spatial multiplexing techniques improve the system capacity, on the other hand, the spatial diversity techniques are utilized to improve the system reliability. The spatial diversity is achieved by sending multiple replicas of a signal from different transmit antennas, and the signal replicas arrive at the receiver from different paths and thus are assumed to undergo independent fading. Then by employing certain combining scheme at the receiver side, the effect of the fading on signal quality can be significantly reduced. This combining processing at the receiver side is the receive diversity scheme, in which the transmitter does not need to have the knowledge about the wireless channel. However, even if the channel information is unknown to the transmitter, the transmit diversity can also be exploited through the space-time coding techniques [26–31].

Moreover, if the transmitter also has the knowledge about the channel information, it has been shown that the system performance can be further improved. Specifically, in one aspect, with the channel state information, the transmitter is able to optimize its power allocation or input covariance matrix for multiple channels, thus benefitting the MIMO channel capacity [32, Chapter 10]. From another aspect, by utilizing the channel state information at the transmitter, transmit beamforming techniques can be adopted with additional array gain comparing to the space-time coding strategy [22]. An incisive one-sentence summary of the beamforming technique given in [14] is quoted here as:

“Beamforming is a versatile and powerful approach to receive, transmit, or relay signals of interest in a spatially selective way in the presence of interference and noise.”

The transmit beamforming approach is relatively new but draws significant interests among the communication society with the outburst of multi-antenna and MIMO techniques in communication systems. Therein the transmit beamforming can be used to exploit the spatial diversity gain as aforementioned, and also can be used for serving multiple users in the downlink channel by multi-user transmit beamforming [33–40]. Furthermore, from an information theoretic perspective, it is shown in [41] and [42] that, if the channel information is not known at the transmitter but the channel has a dominant mean or covariance direction, then beamforming is a capacity-achieving scheme.

2.2 Relay Network Beamforming

As has been shown previously, when the channel state information is known to the MIMO systems, the beamforming technique introduces further improvement in system performance. However, for the wireless devices that are limited by their physical sizes or the processing power, it is impractical for them to deploy multiple antennas. In this case, the relay network beamforming, or distributed relay beamforming [10, 13, 15, 43–45], realized by the cooperation of the distributed single-antenna network nodes can also attain the advantages of multi-antenna systems. Similar to the case of MIMO systems, distributed relay beamforming outperforms the distributed space-time coding schemes [46–48], which is the extension of conventional space-time coding.

In the sequel we briefly review different relay network models and scenarios for distributed relay beamforming problem, where we can see that various design criteria are utilized.

Single-User One-Way Relay Network Beamforming

The single-user one-way relay scenario is the basic and simplest topology for relay beamforming. It generally performs in a two-phase time-division duplexing (TDD) protocol.

In the first time phase, the source node broadcasts the signal to all the relay nodes as well as the destination nodes if the direct link exists. Then in the second time phase, the relay nodes relays the signal to the destination. In [15], the single-user one-way beamforming using AF relaying scheme is addressed. Therein, subject to a total relay transmit power constraint, the received SNR is optimized, and an analytical solution for the phase factor and the amplitude factor of the relaying coefficients is given. The similar problem is also discussed in [49]. Moreover, both AF and DF relaying schemes are considered in [50] for dealing with the relay beamforming, where the total relay transmit power constraint is also assumed. Taking into account the individual transmit power constraint for each relay node, in [10, 51, 52], the authors discussed the relay beamforming based on a criterion for maximizing the destination received SINR. They considered in [10] the relay networks with and without direct link, where an analytical solution is proposed for the case without direct link, and they showed that with the individual power constraint the optimal power transmitted by a relay is not necessary to attain its maximum allowable budget. The individual transmit power constraint is also considered in [53], where the SNR maximization problem is recast as a convex second-order cone programming (SOCP), which can be solved optimally and efficiently.

In the aforementioned work, the perfect knowledge of the CSI is assumed to be known. Therefore, as a more practical consideration, it draws much attention to cope with the non-ideal CSI. Relay beamforming using the quantized feedback of channel state information is addressed in [54], where a generalized Lloyd algorithm (GLA) [55, 56] is used to design the quantizer. In another way to handle the imperfect CSI, in [13, 57] the relay beamforming design is considered with the knowledge of second-order statistics of the CSI. Two beamforming design criteria are used, namely, *relay transmit power minimization* and *received SNR maximization*. And with the latter criterion, the individual

power constraint and the total power constraint are considered separately. In [58], the second-order statistics of CSI is also exploited, and the SNR maximization problem is more completely solved. Subsequently, within the second-order statistics CSI, authors in [59] propose a greedy method with linear complexity per iteration. Moreover, approaches that are based on the robust optimization techniques have been proposed in [53, 60–62] to address the robust relay beamforming problem, where the imperfect estimated CSI with bounded uncertainty region is used. Specifically, in [53] the SNR maximization problem with channel uncertainty is recast using S-Procedure [63, 64] and semidefinite relaxation techniques [65].

Furthermore, besides the criteria that aim to optimize the received SINR or the relay transmit power, there are also other criteria used for relay beamforming design appeared in the literature. In [66], an MMSE (minimum mean-square error) scheme is proposed for the distributed beamforming. With taking the direct link into account, [67] proposed a criterion of mutual information maximization.

Two-Way Relay Network Beamforming

Based on the concept of analog network coding [68], the one-way relay beamforming can be extended to two-way relay beamforming scheme, where the communications assisted by the relays are conducted bi-directionally. Depending on the time slots needed for one round of information exchange between the two transceivers, the two-way communication can be realized in several schemes [68–75], which are illustrated by Figure 2.1. In the literature that deal with the two-way relay beamforming problems, the two time-slot relaying scheme is most commonly employed due to its efficient use of the communication resource, which also attributes to the inherent ability of beamforming techniques for mitigating interference. In [76], the three time-slot scheme is studied and compared with

(a) Four time-slot two-way relaying scheme.

(b) Three time-slot two-way relaying scheme.

(c) Two time-slot two-way relaying scheme.

Figure 2.1: Two-way relaying schemes.

the two time-slot scheme. It is observed that the three time-slot scheme can outperform the two time-slot scheme under some conditions, such as direct link existing between two transceivers, provided that the direct link is strong enough.

In [77], the relay beamforming coefficients are jointly optimized with the transmit power of transceivers. Therein three approaches are studied, which are the approach of minimizing the sum transmit power of relays and two transceivers, the approach of minimizing merely the relay transmit power, and the SNR balancing approach. The SNR balancing approach, also known as max-min fair approach, aims at maximizing the lower received SNR of the two transceivers, which is solved by an iterative steepest descent algorithm. We note here that not only the relay powers, but also the transceiver powers are to be optimized.

To characterize the achievable rate region with the relay beamforming for a two-way multi-relay network, in [75] the authors have studied the problem of weighted sum-rate maximization, subject to the total relay power constraint and the individual relay power constraint. With the similar consideration, in [78] a beamforming design scheme is discussed, which aims at maximizing the sum rate under the total transceiver and relay power constraint. More interestingly, it is shown in [78] that the sum rate maximization approach renders the same beamforming weights as that given by the SNR balancing method.

In the context of frequency-selective channels environment, the two-way relay beamforming with FF relaying scheme is studied in [18] and [79] independently. Basically, the transmit power minimization and SNR balancing approaches are adopted in these work for designing the relay beamformer. In addition, in [18] an approach of minimization of the sum of MSEs is proposed.

Multi-User Relay Network Beamforming

In the above discussions, multiple relays cooperate and assist in the communications between two terminals, either single-directional or bi-directional, while the relay beamforming is also extended to the scenario of multiple pairs of users [12, 80–84]. Among these literature, in [12] the total relay transmit power is minimized under the constraint that the SINRs at destination nodes are above predefined thresholds. In this work, a semidefinite relaxation technique is exploited to recast the original problem as a semidefinite programming (SDP) which is convex and easy to handle but has relatively high computational loads. Subsequently [85] proposes an approximation technique and reformulates the power minimization problem to an SOCP convex problem. Due to the approximation, the SOCP based problem shows inferior performance to the SDP problem, and when the amount of user pairs increases, the performance loss becomes more severe. To tackle the problem raised by the SOCP approximation, an iterative method is proposed in [86, 87] where in each iteration the approximated SOCP problem is solved and an adaptation procedure is introduced. In this way, the performance can be successively improved. Moreover, the iterative method also avoids the problem of SDP based method of [12]. Specifically, the SDP based solution is a result of semidefinite relaxation (SDR) [88, 89], and due to the relaxation the solution obtained by SDP would be inaccurate, especially when the number of receivers significantly increases. Therefore, a concept of multi-group multicasting is also introduced in [86, 87], and in this scenario the iterative method shows performance advantage over the original SDP method.

In another way, the authors in [90] have considered an approach of maximizing the worst user SINR subject to relay power constraint, and this formulation leads to a SDP problem with semidefinite relaxation. Thus the similar disadvantage of the relaxation can also be encountered and gives rise to inaccurate or suboptimal solutions. To address

this problem, a method based on sequential convex programming is proposed in [91], where the semidefinite relaxation is circumvented.

The aforementioned work on multi-user relay beamforming suppose the transmit powers of all the source nodes are fixed while the relay transmit power are to be optimized. However, it is noted that with the source powers taken into consideration, the multi-user beamforming problem has more degrees-of-freedom for optimization, since the power allocation among users and relays then becomes controllable. Therefore, in [92], the transmit power allocation among all sources is jointly designed with the relay beamforming coefficients. Therein the beamforming design problem is formulated as minimizing the sum transmit power of all the source nodes and relays, subject to received SINR requirement at each destination node. This problem is further recast equivalently as a *difference of convex* (DC) programming [93] which is efficiently solved using a technique of *constrained concave convex procedure* (CCCP) [94].

Relay Network Beamforming on Frequency-Selective Channels

The AF based relay beamforming schemes discussed above which are aiming at frequency-flat channels cannot be directly adapted for frequency-selective channels. In order to deal with the frequency-selective channels, Chen *et al.* propose a filter-and-forward (FF) relaying scheme for relay beamforming [16]. With this new scheme, relays process the received signals using finite impulse response (FIR) filters and then re-transmit them towards the destination. Hence, the FF relaying scheme can be regarded as the extension of AF scheme, and the relay coefficient turns to a complex weighting vector rather than the simple complex scalar of the AF scheme. In [16], as usual in the frequency-flat scenario, the relay beamforming problem is formulated as relay power minimization problem and received SINR maximization problem. What makes a difference is that the ISI has to be taken into account.

Since it is noted that the FF relay beamforming in fact functions as distributed channel equalization to combat the ISI, then for extending this concept, in [17] Liang *et al.* introduce an additional decision feedback equalizer (DFE) at the destination side in order to further improve the equalization performance. Although they still use the maximizing received SINR as the optimization criterion, the SINR here is defined at the output of the destination DFE, which makes solving the problem much more challenging. What is more, Liang *et al.* further extend the FIR filtering for FF relaying to the IIR filtering, which provides a useful upper bound for the performance of FIR filter. Subsequently, Chen *et al.* and Liang *et al.* independently extend their previous work to cope with the case of two-way (bi-directional) transmissions [18, 79]. They extended the work in [77] that addresses the AF relay distributed beamforming for two-way transmission in frequency-flat channels.

In [95], also for tackling frequency-selective channels, a relay beamforming method is proposed by employing frequency-domain equalization at both the relay side and the destination side.

2.3 Implementation Issues

In this thesis, we focus on the one-way relay beamforming with centralized implementation, and we assume that all the wireless channel state information are perfectly known. Therefore, two aspects are concerned here, namely, how to estimate channels and how to calculate the beamforming coefficients.

Channel Estimation

We assume that during a training phase, the destination node estimates the all the relay-destination channels. On the other hand, each relay node estimates its own source-relay

channel, and forwards the channel information to the destination. Alternatively, using the relay retransmitted pilot signal that is originally from the source, the destination node can directly estimate the combined channel information of source-relay and relay-destination channels. Then the destination node can extract the source-relay channel information by employing deconvolution techniques.

Coefficients Calculation

Based on the channel information of all the source-relay and relay-destination channels, the destination nodes can calculate the relay beamforming coefficients and feed back them to the relays. Thus we need to assume in this thesis that all channels are slowly fading. Note that compared to the information data transmitted from the source to the destination via relays, the communication of calculated relay coefficients and channel information are of low data-rate and is performed once in a time block. In the multiple user pair relay beamforming scenario, any one destination node can be in charge of the coefficients calculation, or alternatively, an extra node may function as a dedicated processing center, who collects all the channel information, computes and feeds back the beamforming coefficients.

Chapter 3

Relay Beamforming using Distortionless Response Design in Frequency Domain with Adaptive Decision Delay Selection

3.1 Introduction

In this chapter, we consider a two-hop relay network on frequency-selective channels where multiple relays assist in the data transmission from a source to a destination. As has been introduced in Chapter 2, in order to deal with the frequency-selective channels, Chen *et al.* propose a filter-and-forward (FF) relaying scheme for distributed beamforming [16]. Furthermore, it is observed in [18] that the performance of the two-way FF beamforming scheme without equalization at the destination can be significantly affected by the slicer decision delay. Hence in [18], as a system parameter, a decision delay is chosen based on a given channel statistics. However, the fixed delay value is not guaranteed to be optimal for each channel realization. More importantly, there is no design rule for selecting the optimal delay that works well for a given channel statistics or system parameters such as the filter length and the number of relays. In fact for the one-way FF relaying [16, 17], the decision delay also affects the performance significantly, but the

optimal selection of the delay value for each channel realization and for different system parameters is not addressed therein.

In this chapter, we propose a new approach to relay beamforming on frequency-selective channels where the FF relay scheme is adopted at the relays. We formulate and solve the relay beamforming problem using the frequency domain approach since the frequency selectivity of the channels can be dealt with more directly and efficiently in this domain. The proposed beamforming design problem aims to optimize the signal-to-interference-plus-noise-ratio (SINR) at the destination subject to a total relay transmit power constraint plus a new set of constraints which achieve a distortionless equivalent channel response from the source to the destination. With the distortionless equivalent channel response approach, the decision delay parameter can be incorporated into the relay beamforming design which facilitates efficient selection of the optimal decision delay for different channel realization and statistics as well as different filter length at the relays. Furthermore, with the new approach, any inter-symbol interference can be eliminated at the destination. We provide a closed-form solution to the new constrained optimization problem and express the decision delay parameter as a scalar variable in the optimal weights. We further consider the problem of adaptively selecting the optimal decision delay for each channel realization which is shown to have low computational complexity as compared to the time-domain approach when adaptive decision delay is needed.

This chapter is organized as follows. In Section 3.2, the signal model is presented and the received SINR used for our beamforming design is defined. In Section 3.3, an evaluation for the channel distortion is introduced, and thus the distortionless constraint is presented. Section 3.4 formulates and solves the considered beamforming problem, and simulation results are provided in Section 3.5. Section 3.6 concludes this chapter.

3.2 Signal Model

3.2.1 Network Signal Model in Frequency Domain

As depicted in Figure 3.1, we consider a relay network with one source node, one destination node and R relay nodes, and all nodes are each equipped with a single antenna. We assume that the source and the destination are far away from each other, thus the direct link of source-destination is naturally ignored due to the severe attenuation. All the relay nodes work in the time-division duplexing (TDD) mode, and hence a signal transmission from the source to the destination consists of two phases as shown in Figure 3.1. That is, in the first phase, the source node broadcasts the signal of information symbols to the relays, and in the second phase, the signal received at each relay node is filtered and re-transmitted to the destination.

The wireless channels between the nodes are assumed to be frequency-selective, and are hence modeled as linear FIR filters. More specifically, the backward channel (a link from the source to a relay) corresponding to the i th relay is denoted by a column vector $\mathbf{f}_i$ of length L_f , which is represented as $\mathbf{f}_i = [f_i(0), \dots, f_i(L_f - 1)]^T$, $\forall i = 1, \dots, R$, where $f_i(m)$, $m = 0, \dots, (L_f - 1)$, is the filter coefficient. Similarly, the forward channel vector (a link from a relay to the destination) corresponding to the i th relay is denoted by $\mathbf{g}_i = [g_i(0), \dots, g_i(L_g - 1)]^T$, $\forall i = 1, \dots, R$. As in [17], $\mathbf{f}_i$ and $\mathbf{g}_i$ contain the combined effects of transmit pulse shaping filter, physical channel, receive filter and sampling. Throughout this chapter, we assume that the instantaneous CSI is perfectly known at the destination, as is also assumed in [16–18, 79].

The source node transmits an information-bearing sequence of symbols $s(m)$, with power $P_S = E\{|s(m)|^2\}$, where m is the symbol instant. Hence the sequence received at the i th relay is

$$r_i(m) = s(m) * f_i(m) + n_i(m), \quad (3.1)$$

Figure 3.1: Signal model of filter-and-forward relay network in frequency-selective channels.

where $*$ denotes the operation of convolution sum between sequences, and $n_i(m)$ denotes the additive white Gaussian noise with power $\sigma_n^2 = E\{|n_i(m)|^2\}$. The received signal $r_i(m)$ is first passed through the relay filter $\mathbf{h}_i = [h_i(0), \dots, h_i(L_h - 1)]^T$, after which the following sequence

$$t_i(m) = r_i(m) * h_i(m) = s(m) * f_i(m) * h_i(m) + n_i(m) * h_i(m), \quad (3.2)$$

is re-transmitted towards the destination. Then the sequence received at the destination is

$$\begin{aligned} y(m) &= \sum_{i=1}^R t_i(m) * g_i(m) + v(m) \\ &= s(m) * h_{eqv}(m) + \sum_{i=1}^R n_i(m) * h_i(m) * g_i(m) + v(m), \end{aligned} \quad (3.3)$$

where $v(m)$ is the additive white Gaussian noise with power $\sigma_v^2 = E\{|v(m)|^2\}$, and $h_{eqv}(m)$ denotes the impulse response of the equivalent channel from the source to the destination:

$$h_{eqv}(m) \triangleq \sum_{i=1}^R f_i(m) * h_i(m) * g_i(m). \quad (3.4)$$

Figure 3.2: Structure diagram of the equivalent channel $H_{eqv}(\omega)$ from the source to the destination.

Hence the frequency response of the equivalent channel from the source to the destination is given by

$$H_{eqv}(\omega) = \mathcal{F}\{h_{eqv}(m)\} = \sum_{i=1}^R F_i(\omega) \cdot H_i(\omega) \cdot G_i(\omega), \quad (3.5)$$

where $\omega \in [0, 2\pi]$ is the angular frequency in radians; $F_i(\omega)$, $H_i(\omega)$ and $G_i(\omega)$ are the frequency responses of $f_i(m)$, $h_i(m)$ and $g_i(m)$, respectively, *i.e.*, $F_i(\omega) = \mathcal{F}\{f_i(m)\}$, $H_i(\omega) = \mathcal{F}\{h_i(m)\}$ and $G_i(\omega) = \mathcal{F}\{g_i(m)\}$. For better illustration, the structure of the equivalent channel $H_{eqv}(\omega)$ is shown in Figure 3.2.

According to the theory of linear signals and systems, for a system $h(m)$ with a random signal $x(m)$ as its input, the power spectrum of the output signal $\tilde{x}(m) = x(m) * h(m)$ is given by $S_{\tilde{X}}(\omega) = S_X(\omega)|H(\omega)|^2$, where $S_X(\omega)$ is the power spectrum of $x(m)$. While in our signal model (3.3), the transmitted information symbols $s(m)$ are mutually independent, and thus the source signal has a constant power spectrum: $S_s(\omega) = P_S$. Hence, the frequency-domain expression of (3.3) is obtained as

$$S_Y(\omega) = P_S |H_{eqv}(\omega)|^2 + \sigma_n^2 \cdot \sum_{i=1}^R |H_i(\omega)G_i(\omega)|^2 + \sigma_v^2, \quad (3.6)$$

where $S_Y(\omega)$ is the power spectrum of $y(m)$.

3.2.2 SINR Expression in Frequency Domain

In this subsection, the received SINR at the destination is expressed in the frequency domain. Here, the *interference* refers to the inter-symbol-interference (ISI). For discussing the ISI in our signal model, here we temporarily ignore the noise terms in (3.3), and then the output of the equivalent channel $H_{eqv}(\omega)$ to the source signal $s(m)$ is $\tilde{s}(m) = \sum_k s(k)h_{eqv}(m-k)$. Hence, at the symbol instant $(m+\tau)$, the output signal can be expressed as

$$\tilde{s}(m+\tau) = s(m)h_{eqv}(\tau) + \sum_{k \neq m} s(k)h_{eqv}(m+\tau-k), \quad (3.7)$$

where τ denotes the decision delay. In (3.7) the desired symbol $s(m)$ is separated from the ISI caused by neighbouring symbols. Therefore, as an assessment of the signal quality for symbol-by-symbol detection, the SINR is defined at a detection instant $(m+\tau)$, as the ratio between the desired symbol power $E\{|s(m)h_{eqv}(\tau)|^2\}$ and the ISI power $E\{|\sum_{k \neq m} s(k)h_{eqv}(m-k)|^2\}$ plus noise power.

As shown above, in order to define the SINR at a symbol instant, we can decompose mathematically the impulse response $h_{eqv}(m)$ into two parallel subsystems, where one subsystem has a flat frequency response. Specifically, from (3.4) we know that the equivalent channel $h_{eqv}(m)$ is a parallel combination of R single-relay links $(f_i(m)*h_i(m)*g_i(m))$, and each of them has the impulse response length of $L_c = (L_f + L_g + L_h - 2)$. Thus $h_{eqv}(m)$ can also be represented by an FIR filter with a length of L_c , whose impulse response vector is given by $\mathbf{h}_{eqv} = [h_{eqv}(0), \dots, h_{eqv}(L_c-1)]^T$. Therefore, if the decision delay τ is assumed to be an integer value from 0 to (L_c-1) , $\mathbf{h}_{eqv}$ can thus be decomposed as

$$\mathbf{h}_{eqv} = \mathbf{M}_\tau \mathbf{h}_{eqv} + \overline{\mathbf{M}}_\tau \mathbf{h}_{eqv}, \quad (3.8)$$

where

$$\mathbf{M}_\tau = \text{diag} \left\{ [\mathbf{0}_{1 \times \tau}, 1, \mathbf{0}_{1 \times (L_c - \tau - 1)}] \right\}, \tau = 0, \dots, L_c - 1$$

are a family of $L_c \times L_c$ matrices each with $[\mathbf{M}_\tau]_{(\tau+1),(\tau+1)} = 1$ being the only non-zero entry. Thus, the $(\tau+1)$ th tap is selected as the ISI-free part. Moreover, $\overline{\mathbf{M}}_\tau$ are a family of matrices that are defined as $\overline{\mathbf{M}}_\tau = \mathbf{I}_{L_c} - \mathbf{M}_\tau$ for each value of τ respectively.

However, in our signal model we are more concerned with representing this decomposition in the frequency domain. According to (3.5), the time domain and the frequency domain of the equivalent channel are linked by the Fourier transform. To express the operation of Fourier transform in a matrix form, we represent the result of the convolution sum $f_i(m) * h_i(m) * g_i(m)$ as a column vector $[\mathbf{f}_i * \mathbf{h}_i * \mathbf{g}_i] \in \mathbb{C}^{L_c \times 1}$. We also define the row vector $\mathbf{b}_{fgh}(\omega) = [1, e^{-j\omega \cdot 1}, \dots, e^{-j\omega \cdot (L_c-1)}] \in \mathbb{C}^{1 \times L_c}$ as a function of ω , where $\omega \in [0, 2\pi]$. Then $H_{eqv}(\omega)$ can be decomposed as

$$\begin{aligned} H_{eqv}(\omega) &= \sum_{i=1}^R \mathbf{b}_{fgh}(\omega) \cdot [\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i] \\ &= \sum_{i=1}^R \mathbf{b}_{fgh}(\omega) \cdot (\mathbf{M}_\tau + \overline{\mathbf{M}}_\tau) \cdot [\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i] \\ &= \mathbf{b}_{fgh}(\omega) \mathbf{M}_\tau \cdot \sum_{i=1}^R [\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i] + \mathbf{b}_{fgh}(\omega) \overline{\mathbf{M}}_\tau \cdot \sum_{i=1}^R [\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i]. \end{aligned} \quad (3.9)$$

Furthermore, to separate the relay filter coefficients $\mathbf{h}_i$, we can define $[\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i] = \boldsymbol{\Theta}_i \mathbf{h}_i$, where $\boldsymbol{\Theta}_i$ is a column-circulant matrix that is structured based on $f_i(m) * g_i(m)$. The detailed expression of $\boldsymbol{\Theta}_i$ is given in Appendix 3.7.2. Then inserting $[\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i] = \boldsymbol{\Theta}_i \mathbf{h}_i$ into (3.9), the squared norm of $H_{eqv}(\omega)$ is given by

$$|H_{eqv}(\omega)|^2 = |H_S(\omega)|^2 + |H_I(\omega)|^2 + H_c(\omega), \quad (3.10)$$

where

$$\begin{aligned} H_S(\omega) &\triangleq \mathbf{b}_{fgh}(\omega) \mathbf{M}_\tau \cdot \sum_{i=1}^R \boldsymbol{\Theta}_i \mathbf{h}_i, \\ H_I(\omega) &\triangleq \mathbf{b}_{fgh}(\omega) \overline{\mathbf{M}}_\tau \cdot \sum_{i=1}^R \boldsymbol{\Theta}_i \mathbf{h}_i, \end{aligned}$$

and

$$H_c(\omega) \triangleq 2 \cdot \Re \left\{ \left(\sum_{i=1}^R \Theta_i \mathbf{h}_i \right)^T \mathbf{M}_\tau \mathbf{b}_{fgh}^T(\omega) \cdot \mathbf{b}_{fgh}^*(\omega) \overline{\mathbf{M}_\tau} \left(\sum_{i=1}^R \Theta_i \mathbf{h}_i \right)^* \right\}$$

represents the cross term between $H_S(\omega)$ and $H_I(\omega)$.

With the above derivations, the received signal's power spectrum in (3.6) can be further written as

$$\begin{aligned} S_Y(\omega) = & \underbrace{P_S \cdot |H_S(\omega)|^2}_{\text{Desired signal}} + \underbrace{P_S \cdot |H_I(\omega)|^2 + P_S \cdot H_c(\omega)}_{\text{Inter-symbol interferece}} \\ & + \underbrace{\sigma_n^2 \cdot \sum_{i=1}^R |H_i(\omega) \cdot G_i(\omega)|^2 + \sigma_v^2}_{\text{Noise}}. \end{aligned} \quad (3.11)$$

Note that here we have decomposed the signal received at the destination into three components, namely, desired signal, ISI and noise. Using this decomposition, we can proceed to define the received SINR.

3.2.2.1 Received Signal Power Spectrum

For ease of exposition, let

$$\Psi = [\Theta_1, \dots, \Theta_R], \quad (3.12)$$

$$\mathbf{w} = [\mathbf{h}_1^H, \dots, \mathbf{h}_R^H]^T. \quad (3.13)$$

Then the term $\sum_{i=1}^R \Theta_i \mathbf{h}_i$, which appears in (3.10) and (3.11), can be rewritten as $\sum_{i=1}^R \Theta_i \mathbf{h}_i = \Psi \mathbf{w}^*$. Thus the power spectrum of the desired signal, which is expressed by the first component of (3.11), is given in a quadratic form:

$$P_S \cdot |H_S(\omega)|^2 = \mathbf{w}^H \tilde{\mathbf{Q}}_s^{(\omega, \tau)} \mathbf{w}, \quad (3.14)$$

where

$$\tilde{\mathbf{Q}}_s^{(\omega, \tau)} \triangleq P_S \cdot \Psi^T \mathbf{M}_\tau \mathbf{b}_{fgh}^T(\omega) \mathbf{b}_{fgh}^*(\omega) \mathbf{M}_\tau \Psi^*.$$

3.2.2.2 ISI Power Spectrum

Similarly, the power spectrum of the two interference terms in (3.11) can be derived as

$$P_S \cdot |H_I(\omega)|^2 = \mathbf{w}^H \tilde{\mathbf{Q}}_i^{(\omega, \tau)} \mathbf{w}, \quad (3.15)$$

$$P_S \cdot H_c(\omega) = \mathbf{w}^H \tilde{\mathbf{Q}}_{i2}^{(\omega, \tau)} \mathbf{w}, \quad (3.16)$$

respectively, where

$$\begin{aligned} \tilde{\mathbf{Q}}_i^{(\omega, \tau)} &\triangleq P_S \cdot \boldsymbol{\Psi}^T \bar{\mathbf{M}}_\tau \mathbf{b}_{fgh}^T(\omega) \mathbf{b}_{fgh}^*(\omega) \bar{\mathbf{M}}_\tau \boldsymbol{\Psi}^*, \\ \tilde{\mathbf{Q}}_{i2}^{(\omega, \tau)} &\triangleq P_S \cdot (\boldsymbol{\Psi}^T \mathbf{M}_\tau \mathbf{b}_{fgh}^T(\omega) \mathbf{b}_{fgh}^*(\omega) \bar{\mathbf{M}}_\tau \boldsymbol{\Psi}^* \\ &\quad + \boldsymbol{\Psi}^T \bar{\mathbf{M}}_\tau \mathbf{b}_{fgh}^T(\omega) \mathbf{b}_{fgh}^*(\omega) \mathbf{M}_\tau \boldsymbol{\Psi}^*) \end{aligned}$$

3.2.2.3 Noise Power Spectrum

We define the row vector $\mathbf{b}_h(\omega) = [1, e^{-j\omega \cdot 1}, \dots, e^{-j\omega \cdot (L_h - 1)}] \in \mathbb{C}^{1 \times L_h}$ as a function of ω , where $\omega \in [0, 2\pi]$. Consequently, the frequency response of each relay filter is further represented as $H_i(\omega) = \mathbf{b}_h(\omega) \mathbf{h}_i, \forall i = 1, \dots, R$. Given this notation, and also defining $\mathbf{D}_g^{(\omega)} \triangleq \text{diag}\{|G_1(\omega)|^2, \dots, |G_R(\omega)|^2\}$, after some derivations the noise term in (3.11) can be written as

$$\sigma_n^2 \cdot \sum_{i=1}^R |H_i(\omega) \cdot G_i(\omega)|^2 + \sigma_v^2 = \mathbf{w}^H \tilde{\mathbf{Q}}_n^{(\omega)} \mathbf{w} + \sigma_v^2, \quad (3.17)$$

where

$$\tilde{\mathbf{Q}}_n^{(\omega)} \triangleq \sigma_n^2 \cdot [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)]^H \cdot \mathbf{D}_g^{(\omega)} \cdot [\mathbf{I}_R \otimes \mathbf{b}_h(\omega)].$$

3.2.2.4 SINR Definition

From the above expressions in (3.14) to (3.17), $S_Y(\omega)$ in (3.11) can be rewritten as

$$S_Y(\omega) = \mathbf{w}^H \tilde{\mathbf{Q}}_s^{(\omega, \tau)} \mathbf{w} + \mathbf{w}^H (\tilde{\mathbf{Q}}_i^{(\omega, \tau)} + \tilde{\mathbf{Q}}_{i2}^{(\omega, \tau)}) \mathbf{w} + \mathbf{w}^H \tilde{\mathbf{Q}}_n^{(\omega)} \mathbf{w} + \sigma_v^2. \quad (3.18)$$

Here for convenience of explanation, again the noise term $(\mathbf{w}^H \tilde{\mathbf{Q}}_n^{(\omega)} \mathbf{w} + \sigma_v^2)$ is temporarily ignored and then we have $S_Y(\omega) = P_S |H_{eqv}(\omega)|^2$. Hence, the power of the received signal is $P_y = \frac{1}{2\pi} \int_{2\pi} S_Y(\omega) d\omega = \frac{P_S}{2\pi} \int_{2\pi} |H_{eqv}(\omega)|^2 d\omega$. Furthermore, according to the Parseval's Theorem of discrete-time Fourier transform (DTFT) and discrete Fourier transform (DFT) [96], we have $P_y = P_S \cdot \sum_{m=0}^{L_c-1} |h_{eqv}(m)|^2 = \frac{P_S}{2\pi} \int_{2\pi} |H_{eqv}(\omega)|^2 d\omega = \frac{P_S}{K} \sum_{k=1}^K |H_{eqv}(\omega_k)|^2$, where $K \geq L_c$ is the number of DFT frequency points and $\omega_k = 2\pi(k-1)/K$. That is, we can use the DFT to calculate the power, and hence throughout this chapter we assume $K \geq L_c$.

Similarly, taking the noise term into account, we can represent the powers of the signal, interference and noise components in (3.18) using DFT, respectively. Moreover, we prove in Appendix 3.7.1 that $\sum_{k=1}^K \tilde{\mathbf{Q}}_{i2}^{(\omega_k, \tau)} = \mathbf{0}$. Thus we can define the output SINR with respect to different τ as

$$SINR(\mathbf{w}, \tau) \triangleq \frac{\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w}}{\mathbf{w}^H (\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \mathbf{w} + \sigma_v^2}, \quad (3.19)$$

where

$$\begin{aligned} \mathbf{Q}_s^{(\tau)} &= \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{Q}}_s^{(\omega_k, \tau)}, \\ \mathbf{Q}_i^{(\tau)} &= \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{Q}}_i^{(\omega_k, \tau)}, \\ \mathbf{Q}_n &= \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{Q}}_n^{(\omega_k)}. \end{aligned}$$

Note that, matrices $\mathbf{Q}_s^{(\tau)}$ and $\mathbf{Q}_i^{(\tau)}$ are functions of τ .

3.3 Evaluation of the Channel Distortion

Having presented the network model and the SINR definition, we have constructed a framework for our relay beamforming design in the frequency domain. Bearing in mind

that our objective is to suppress the distortion effect on the information symbols caused by ISI, next we formulate an evaluation of the distortion across the frequency domain for the equivalent channel $H_{eqv}(\omega)$.

We start by defining an ISI evaluation that reveals the distortion level of the equivalent channel. Subsequently we investigate the circumstances under which the distortion level can be zero, *i.e.*, the ISI effect can be completely eliminated. Then, we come up with a *distortionless constraint* for the considered relay beamforming problem, which results in a zero ISI residual in the received signal at the destination.

3.3.1 Distortion Evaluation Function

The *distortion evaluation function* (DEF) for the equivalent channel $H_{eqv}(\omega)$ is defined in the frequency domain as the mean square error between $H_{eqv}(\omega)$ and the desired linear phase flat response, and it is expressed as

$$J(\mathbf{w}, \gamma, \tau) \triangleq \frac{1}{K} \sum_{k=1}^K |H_{eqv}(\omega_k) - \gamma e^{-j\omega_k \tau}|^2, \quad (3.20)$$

where

$$\begin{aligned} & |H_{eqv}(\omega) - \gamma e^{-j\omega \tau}|^2 \\ &= |H_{eqv}(\omega)|^2 - 2\gamma \cdot \Re \{ H_{eqv}(\omega) \cdot e^{j\omega \tau} \} + \gamma^2 \\ &= \left| \sum_{i=1}^R F_i(\omega) H_i(\omega) G_i(\omega) \right|^2 - 2\gamma \cdot \Re \left\{ e^{j\omega \tau} \sum_{i=1}^R F_i(\omega) H_i(\omega) G_i(\omega) \right\} + \gamma^2, \end{aligned} \quad (3.21)$$

where $\gamma > 0$ indicates the desired channel gain, and $\tau \in \{0, \dots, L_c - 1\}$ denotes the decision delay of the destination received sequence compared to the source transmitted sequence.

For convenience of mathematical representation, using the expression $H_i(\omega) = \mathbf{b}_h(\omega) \mathbf{h}_i$

as in (3.17), we can further rewrite (3.21) in matrix form as

$$\begin{aligned} & |H_{eqv}(\omega) - \gamma e^{-j\omega\tau}|^2 \\ &= \mathbf{u}^H \mathbf{d}_{fg}(\omega) \mathbf{d}_{fg}^H(\omega) \mathbf{u} - 2\gamma \cdot \Re \{ \mathbf{u}^H \mathbf{d}_{fg}(\omega) e^{j\omega\tau} \} + \gamma^2, \end{aligned} \quad (3.22)$$

where $\mathbf{d}_{fg}(\omega) \triangleq [F_1(\omega)G_1(\omega), \dots, F_R(\omega)G_R(\omega)]^T$ and $\mathbf{u} = [\mathbf{b}_h(\omega)\mathbf{h}_1, \dots, \mathbf{b}_h(\omega)\mathbf{h}_R]^H$.

With the definition of the overall relay coefficient vector $\mathbf{w}$ given by (3.13), after some derivations we can rewrite (3.22) as

$$\begin{aligned} & |H_{eqv}(\omega) - \gamma e^{-j\omega\tau}|^2 \\ &= \mathbf{w}^H \tilde{\mathbf{Q}}_0^{(\omega)} \mathbf{w} - 2\gamma \cdot \Re \{ \mathbf{w}^H \tilde{\mathbf{q}}_o^{(\omega)} e^{j\omega\tau} \} + \gamma^2, \end{aligned} \quad (3.23)$$

where

$$\begin{aligned} \tilde{\mathbf{q}}_o^{(\omega)} &\triangleq [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)]^H \cdot \mathbf{d}_{fg}(\omega), \\ \tilde{\mathbf{Q}}_0^{(\omega)} &\triangleq [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)]^H \cdot \mathbf{d}_{fg}(\omega) \mathbf{d}_{fg}^H(\omega) \cdot [\mathbf{I}_R \otimes \mathbf{b}_h(\omega)] \\ &= \tilde{\mathbf{q}}_o^{(\omega)} \cdot (\tilde{\mathbf{q}}_o^{(\omega)})^H. \end{aligned}$$

Thus, DEF $J(\mathbf{w}, \gamma, \tau)$ originally defined in (3.20) can be written in a quadratic form

as

$$\begin{aligned} & J(\mathbf{w}, \gamma, \tau) \\ &= \frac{1}{K} \sum_{k=1}^K \left(\mathbf{w}^H \tilde{\mathbf{Q}}_0^{(\omega_k)} \mathbf{w} - 2\gamma \cdot \Re \{ \mathbf{w}^H \tilde{\mathbf{q}}_o^{(\omega_k)} e^{j\omega_k\tau} \} + \gamma^2 \right) \\ &= \mathbf{w}^H \mathbf{Q}_0 \mathbf{w} - 2\gamma \cdot \Re \{ \mathbf{w}^H \mathbf{q}_o(\tau) \} + \gamma^2, \end{aligned} \quad (3.24)$$

where

$$\mathbf{q}_o(\tau) = \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{q}}_o^{(\omega_k)} e^{j\omega_k\tau}, \quad (3.25)$$

$$\mathbf{Q}_0 = \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{Q}}_0^{(\omega_k)} = \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{q}}_o^{(\omega_k)} (\tilde{\mathbf{q}}_o^{(\omega_k)})^H. \quad (3.26)$$

Note that the vector $\mathbf{q}_o(\tau)$ is a function of the decision delay τ . This property facilitates the subsequent discussion on the relay beamforming design with adaptive decision delay.

3.3.2 Minimization of DEF

For a given set of $(\mathbf{w}, \gamma, \tau)$ values, DEF indicates how close the equivalent channel $H_{eqv}(\omega)$ is able to approach the desired linear phase flat frequency response, and thus it embodies the level of ISI residual in the signal received at the destination. On the other hand, given parameters (γ, τ) , we are interested in the lowest achievable value of DEF with varying relay coefficients $\mathbf{w}$. This leads to an unconstrained optimization problem:

$$J_{\min} = \min_{\mathbf{w}} J(\mathbf{w}, \gamma, \tau). \quad (3.27)$$

Taking the derivative of $J(\mathbf{w}, \gamma, \tau)$ with respect to $\mathbf{w}^*$ and equating it to 0, we obtain

$$\frac{\partial J(\mathbf{w}, \gamma, \tau)}{\partial \mathbf{w}^*} = \mathbf{Q}_0 \mathbf{w} - \gamma \mathbf{q}_o(\tau) = 0. \quad (3.28)$$

The solution to the linear equation (3.28) gives the optimal $\mathbf{w}$ for problem (3.27). Depending on whether the matrix $\mathbf{Q}_0$ is of full rank, we have two cases for the possible results.

3.3.2.1 When $\mathbf{Q}_0$ is full rank

In this case, the solution to the linear equation in (3.28) is unique and is given by

$$\check{\mathbf{w}} = \gamma \mathbf{Q}_0^{-1} \mathbf{q}_o(\tau). \quad (3.29)$$

Hence we can obtain the optimal value of $J(\mathbf{w}, \gamma, \tau)$ as

$$J_{\min} = J(\check{\mathbf{w}}, \gamma, \tau) = \gamma^2 (1 - \mathbf{q}_o^H(\tau) \mathbf{Q}_0^{-1} \mathbf{q}_o(\tau)). \quad (3.30)$$

Since the matrix $\mathbf{Q}_0^{-1}$ is positive semi-definite, it is for sure that $\mathbf{q}_o^H \mathbf{Q}_0^{-1} \mathbf{q}_o$ is non-negative, hence we have $\gamma^2 (1 - \mathbf{q}_o^H \mathbf{Q}_0^{-1} \mathbf{q}_o) \leq \gamma^2$. On the other hand, according to the definition of DEF in (3.20), $J(\mathbf{w}, \gamma, \tau)$ is also non-negative. As a result, $0 \leq J_{\min} \leq \gamma^2$.

3.3.2.2 When $\mathbf{Q}_0$ is rank deficient

In this case, the linear equation (3.28) is under-determined and has infinitely many solutions, and so does the optimization problem (3.27). To further simplify the linear equation, we perform the eigen-decomposition to matrix $\mathbf{Q}_0$ as $\mathbf{Q}_0 = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{U}$ is an orthogonal matrix and $\mathbf{\Lambda}$ is a diagonal matrix with $\mathbf{Q}_0$'s eigenvalues on the diagonal in a descending order. Then substituting the decomposition of $\mathbf{Q}_0$ into (3.28) yields

$$\mathbf{U}\mathbf{\Lambda}\mathbf{U}^H\mathbf{w} = \gamma\mathbf{q}_o(\tau). \quad (3.31)$$

Assuming the rank of $\mathbf{Q}_0$ is r_0 , partition the matrix $\mathbf{U}$ into two blocks as $\mathbf{U} = [\mathbf{U}_1, \mathbf{U}_2]$, where $\mathbf{U}_1 \in C^{(RL_h) \times r_0}$ and $\mathbf{U}_2 \in C^{(RL_h) \times (RL_h - r_0)}$. Then we have

$$\mathbf{U}\mathbf{\Lambda}\mathbf{U}^H\mathbf{w} = [\mathbf{U}_1, \mathbf{U}_2] \begin{bmatrix} \mathbf{\Lambda}_1 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}_1^H \\ \mathbf{U}_2^H \end{bmatrix} \mathbf{w} = \gamma\mathbf{q}_o(\tau), \quad (3.32)$$

where $\mathbf{\Lambda}_1$ is an $(r_0 \times r_0)$ diagonal matrix with the r_0 non-zero eigenvalues of $\mathbf{Q}_0$ on its diagonal. From (3.32), it is clear that $\mathbf{U}_1\mathbf{\Lambda}_1\mathbf{U}_1^H\mathbf{w} = \gamma\mathbf{q}_o(\tau)$, and then we can obtain

$$\mathbf{\Lambda}_1\mathbf{U}_1^H\mathbf{w} = \gamma\mathbf{U}_1^H\mathbf{q}_o(\tau), \quad (3.33)$$

which is equivalent to (3.28). Hence, the solution space of the linear equation (3.33) gives a region of $\mathbf{w}$ within which DEF achieves its minimum value $J_{\min}$. Next we investigate the lowest possible value of $J_{\min}$ and prove that $J_{\min}$ can be zero.

3.3.2.3 Condition for $\min_{\mathbf{w}} J(\mathbf{w}) = 0$

In the above discussions, we have transformed the original problem of minimizing DEF (3.27) into a problem of finding the solution space of a linear equation (3.33). However, it is not clear yet how small the value of $J_{\min}$ can be. Now we prove that the minimum value of DEF can be zero, as long as some conditions are met. Hence it means that the

frequency response $H_{eqv}(\omega)$ is able to be completely flat with linear phase. For simplicity of notation, we define a matrix $\mathbf{B}$ as

$$\mathbf{B} \triangleq [\tilde{\mathbf{q}}_o^{(\omega_1)}, \dots, \tilde{\mathbf{q}}_o^{(\omega_K)}]. \quad (3.34)$$

Consequently, the vector $\mathbf{q}_o(\tau)$ in (3.25) and the matrix $\mathbf{Q}_0$ in (3.26) can be rewritten as

$$\mathbf{q}_o(\tau) = \frac{1}{K} \mathbf{B} \cdot \vec{\varphi}(\tau) \quad (3.35)$$

and

$$\mathbf{Q}_0 = \frac{1}{K} \mathbf{B} \mathbf{B}^H, \quad (3.36)$$

respectively, where $\vec{\varphi}(\tau) = [e^{j\omega_1 \cdot \tau}, \dots, e^{j\omega_K \cdot \tau}]^T$. Before proceeding with the proof of $\min_{\mathbf{w}} J(\mathbf{w}) = 0$, we prove two lemmas first.

Lemma 1: Given $L_c = (L_f + L_g + L_h - 2)$, the matrix $\mathbf{B}$ can be decomposed as $\mathbf{B} = \mathbf{\Psi}^T \mathbf{\Phi}_{fgh}^T$, where $\mathbf{\Psi} \in \mathbb{C}^{L_c \times (RL_h)}$ and $\mathbf{\Phi}_{fgh} \in \mathbb{C}^{K \times L_c}$. Moreover, if $RL_h \geq L_c$, the rank of matrix $\mathbf{B}$ is L_c .

Proof: See Appendix 3.7.2

Lemma 1 indicates the condition under which the matrix $\mathbf{Q}_0$ can be rank deficient. Since according to (3.36), we know that $\mathbf{Q}_0$ has the same rank as $\mathbf{B}$, which equals L_c when $RL_h \geq L_c$. Moreover, according to (3.26) we know that the dimension of $\mathbf{Q}_0$ is $(RL_h \times RL_h)$. Therefore, as long as $RL_h > L_c$ the matrix $\mathbf{Q}_0$ is rank deficient. Otherwise, $\mathbf{Q}_0$ is of full rank.

Lemma 2: If $RL_h \geq L_c$, the vectors $\vec{\varphi}(\tau), \tau = 0, \dots, (L_c - 1)$, which are defined after (3.36), lie in the column-space of the matrix $\mathbf{B}^H$.

Proof: See Appendix 3.7.3

Next, according to the above lemmas, we can finally show the condition for $\min_{\mathbf{w}} J(\mathbf{w}, \gamma, \tau) = 0$.

Theorem 1: If L_h , L_c and the relay number R satisfy the condition that $RL_h \geq L_c$, the minimum value of DEF $J(\mathbf{w}, \gamma, \tau)$ is zero.

Proof: See Appendix 3.7.4

Theorem 1 states that, when certain condition is met, the minimum value of DEF can be zero. Hence, the equivalent channel $H_{eqv}(\omega)$ is able to achieve flat frequency response with linear phase.

3.3.3 The Distortionless Constraint

The above discussion on “minimization of DEF” provides a criterion for relay beamforming design which attempts to minimize the difference between $H_{eqv}(\omega)$ and a desired distortionless linear phase channel. The solution to the minimizing DEF criterion depends on the relationship between RL_h and L_c . If $RL_h < L_c$ is assumed, $\mathbf{Q}_0$ has full rank (RL_h) and thus we have a unique solution for the relay beamforming weights $\mathbf{w}$, but J_{min} cannot be 0, which means the distortionless $H_{eqv}(\omega)$ is not achievable. If $RL_h = L_c$, $\mathbf{Q}_0$ has full rank (L_c) and we also have a unique solution for $\mathbf{w}$, and J_{min} can be 0. If $RL_h > L_c$ is supposed, $\mathbf{Q}_0$ is rank deficient and there exist multiple solutions for $\mathbf{w}$ that lie in the solution space of the under-determined linear equation (3.33). All of these solutions lead to $J_{min} = 0$, *i.e.*, distortionless $H_{eqv}(\omega)$.

However, if DEF is merely minimized, although the distortionless can be guaranteed by requiring $RL_h \geq L_c$, the noise effects (at both the relay side and the destination side) are totally ignored. Especially in the case of $RL_h > L_c$ where multiple solutions exist, we do not have a quality evaluation for different $\mathbf{w}$ given by the solution space. Moreover, variables γ and τ in DEF cannot be optimized within the minimizing DEF criterion. Therefore, if $RL_h > L_c$ is assumed where extra degree-of-freedom (DOF) is available due to the rank deficiency of $\mathbf{Q}_0$, DEF is better to be used as a constraint with additional

criteria such as SINR maximization. Following the above discussion, assuming $RL_h > L_c$, since therein $\min_{\mathbf{w}} J(\mathbf{w}) = 0$ is proven, based on (3.24) we formulate a constraint:

$$\mathbf{w}^H \mathbf{Q}_0 \mathbf{w} - 2\gamma \cdot \Re \{ \mathbf{w}^H \mathbf{q}_0(\tau) \} + \gamma^2 = 0. \quad (3.37)$$

We term this equation as the *distortionless constraint*, which indicates that under this constraint, the magnitude response $|H_{eqv}(\omega)|$ is forced to be flat and the phase response $\angle H_{eqv}(\omega)$ is forced to be linear. Thus, the equivalent channel imposes no distortion on the transmitted information sequences. Furthermore, according to the discussion in the previous subsection, the distortionless constraint (3.37) is equivalent to the linear constraint given by (3.33).

Next, with the distortionless constraint taken into account, we formulate an optimization problem and solve it for the relay beamforming problem.

3.4 Maximizing SINR with Distortionless Constraint

In this section, we present a relay beamforming design approach based on the channel distortionless criterion, where the received SINR is maximized with the distortionless constraint as well as a power constraint.

3.4.1 Problem Formulation

To incorporate the distortionless constraint, we assume $RL_h > L_c$ throughout this section and the subsequent section. Under this condition, the problem is formulated as

$$\begin{cases} \max_{\mathbf{w}, \gamma} & SINR(\mathbf{w}) \\ s.t. & J(\mathbf{w}, \gamma, \tau) = 0 \\ & P_R(\mathbf{w}) \leq P_0, \end{cases} \quad (3.38)$$

where $P_R(\mathbf{w})$ denotes the total relay transmit power. Moreover, the frequency-domain relationship corresponding to the total relay transmit signal (3.2) is

$$S_{t,i}(\omega) = P_S \cdot |F_i(\omega) \cdot H_i(\omega)|^2 + \sigma_n^2 |H_i(\omega)|^2, \quad (3.39)$$

where $S_{t,i}(\omega)$ represents the power spectrum of the transmitted signal at the i th relay. Along with the expression $H_i(\omega) = \mathbf{b}_h(\omega)\mathbf{h}_i$ as in (3.17), the total relay transmit power is given by

$$\begin{aligned} P_R &= \sum_{i=1}^R \left(\frac{1}{K} \sum_{k=1}^K S_{t,i}(\omega_k) \right) \\ &= \frac{1}{K} \sum_{k=1}^K \sum_{i=1}^R (P_S |F_i(\omega_k) \mathbf{b}_h(\omega_k) \mathbf{h}_i|^2 + \sigma_n^2 |\mathbf{b}_h(\omega_k) \mathbf{h}_i|^2) \\ &= \frac{1}{K} \sum_{k=1}^K \mathbf{w}^H \left([\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega_k)]^H \mathbf{D}_f^{(\omega_k)} [\mathbf{I}_R \otimes \mathbf{b}_h(\omega_k)] \right) \mathbf{w}, \end{aligned} \quad (3.40)$$

where $\mathbf{D}_f^{(\omega)} = P_S \cdot \text{diag}\{|F_1(\omega)|^2, \dots, |F_R(\omega)|^2\} + \sigma_n^2 \cdot \mathbf{I}_R$. Moreover, by denoting

$$\mathbf{D} = \frac{1}{K} \sum_{k=1}^K \left([\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega_k)]^H \mathbf{D}_f^{(\omega_k)} [\mathbf{I}_R \otimes \mathbf{b}_h(\omega_k)] \right),$$

the total relay transmit power (3.40) is finally represented as

$$P_R(\mathbf{w}) = \mathbf{w}^H \mathbf{D} \mathbf{w}. \quad (3.41)$$

Thus, according to the definition given by (3.19), (3.24) and (3.41), this optimization problem can be explicitly expressed as

$$\begin{aligned} \max_{\mathbf{w}, \gamma} \quad & \frac{\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w}}{\mathbf{w}^H (\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \mathbf{w} + \sigma_v^2} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{Q}_0 \mathbf{w} - 2\gamma \cdot \Re \{ \mathbf{w}^H \mathbf{q}_o(\tau) \} + \gamma^2 = 0 \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \end{aligned} \quad (3.42)$$

3.4.2 A Closed-Form Solution

Next we derive a closed-form solution to the above optimization. First of all, we can draw the following conclusion.

Lemma 3: Suppose that a vector $\mathbf{w}_1$ satisfies the distortionless constraint (3.37) for a given γ and τ . Then the equations hold: $\mathbf{w}_1^H \mathbf{Q}_s^{(\tau)} \mathbf{w}_1 = \gamma^2 P_S$ and $\mathbf{w}_1^H \mathbf{Q}_i^{(\tau)} \mathbf{w}_1 = 0$.

Proof: See Appendix 3.7.5.

Moreover, as discussed in Section III, when the matrix $\mathbf{Q}_0$ is rank deficient, the distortionless constraint can be converted to a linear constraint as shown in (3.33). Therefore, the problem (3.42) is equivalent to

$$\begin{aligned} \max_{\mathbf{w}, \gamma} \quad & \frac{\gamma^2 P_s}{\mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_v^2} \\ \text{s.t.} \quad & \mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{w} = \gamma \mathbf{U}_1^H \mathbf{q}_o(\tau) \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \end{aligned} \quad (3.43)$$

Since $\gamma \neq 0$, by defining $\hat{\mathbf{w}} = \mathbf{w}/\gamma$, we can obtain

$$\begin{aligned} \max_{\hat{\mathbf{w}}, \gamma} \quad & \frac{P_s}{\hat{\mathbf{w}}^H \mathbf{Q}_n \hat{\mathbf{w}} + \sigma_v^2 / \gamma^2} \\ \text{s.t.} \quad & \mathbf{\Lambda}_1 \mathbf{U}_1^H \hat{\mathbf{w}} = \mathbf{U}_1^H \mathbf{q}_o(\tau) \\ & \gamma^2 \hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} \leq P_0. \end{aligned} \quad (3.44)$$

The numerator of the objective function is a constant term, and thus the maximization problem can be turned into a minimization problem:

$$\begin{aligned} \min_{\hat{\mathbf{w}}, \gamma} \quad & \hat{\mathbf{w}}^H \mathbf{Q}_n \hat{\mathbf{w}} + \frac{\sigma_v^2}{\gamma^2} \\ \text{s.t.} \quad & \mathbf{\Lambda}_1 \mathbf{U}_1^H \hat{\mathbf{w}} = \mathbf{U}_1^H \mathbf{q}_o(\tau) \\ & \gamma^2 \hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} \leq P_0. \end{aligned} \quad (3.45)$$

It can be easily shown by contradiction that the second constraint is always active, *i.e.*, $\hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} = P_0/\gamma^2$. Specifically, assume that a vector $\hat{\mathbf{w}}_1$ and a scalar γ_1 have been found as a set of optimal solution to problem (3.45) and they make the power constraint inactive, *i.e.*, $\gamma_1^2 \hat{\mathbf{w}}_1^H \mathbf{D} \hat{\mathbf{w}}_1 < P_0$. In this case, we can always find a $\gamma_2 > \gamma_1$ such that $\gamma_2^2 \hat{\mathbf{w}}_1^H \mathbf{D} \hat{\mathbf{w}}_1 = P_0$, and meanwhile makes the objective function $(\hat{\mathbf{w}}_1^H \mathbf{Q}_n \hat{\mathbf{w}}_1 + \sigma_v^2/\gamma_2^2) < (\hat{\mathbf{w}}_1^H \mathbf{Q}_n \hat{\mathbf{w}}_1 + \sigma_v^2/\gamma_1^2)$, which, however, contradicts the optimality of $\{\hat{\mathbf{w}}_1, \gamma_1\}$. Therefore, it can be concluded

that the equality of the second constraint in (3.45) always holds, and then the problem can be further equivalent to

$$\begin{aligned}
 \min_{\hat{\mathbf{w}}, \gamma} \quad & \hat{\mathbf{w}}^H \mathbf{Q}_n \hat{\mathbf{w}} + \frac{\sigma_v^2}{\gamma^2} \\
 s.t. \quad & \mathbf{\Lambda}_1 \mathbf{U}_1^H \hat{\mathbf{w}} = \mathbf{U}_1^H \mathbf{q}_o(\tau) \\
 & \hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} = \frac{P_0}{\gamma^2}.
 \end{aligned} \tag{3.46}$$

To solve this optimization problem, we seek the Lagrangian multiplier method. For the details on solving it please refer to Appendix 3.7.6, and the optimal solution is given by

$$\begin{aligned}
 \hat{\mathbf{w}}_0(\tau) &= \mathbf{v}(\tau) \\
 \gamma_0(\tau) &= \sqrt{\frac{P_0}{\mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau)}},
 \end{aligned} \tag{3.47}$$

where

$$\begin{aligned}
 \mathbf{v}(\tau) &= \left(\left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right) \\
 &\quad \times \left(\mathbf{\Lambda}_1 \mathbf{U}_1^H \left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau).
 \end{aligned}$$

Given the solution in (3.47) for $\hat{\mathbf{w}}$ and γ , and along with the definition of $\hat{\mathbf{w}} = \mathbf{w}/\gamma$, the optimal solution to the original problem (3.42) is given by

$$\mathbf{w}_0(\tau) = \gamma_0(\tau) \cdot \hat{\mathbf{w}}_0(\tau) = \sqrt{\frac{P_0}{\mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau)}} \cdot \mathbf{v}(\tau). \tag{3.48}$$

Note that here the optimal weight $\mathbf{w}_0$ is a function of a scalar τ , and this fact gives rise to the subsequent discussions about the optimal selection of the decision delay τ .

3.4.3 Adaptive Selection of Decision Delay τ

Recalling (3.35), $\mathbf{q}_o(\tau)$ can be represented in terms of matrix $\mathbf{B}$. Then substituting this $\mathbf{q}_o(\tau)$ expression into (3.48) yields

$$\mathbf{w}_0(\tau) = \gamma_0(\tau) \cdot \mathbf{Q}_w \vec{\varphi}(\tau), \tag{3.49}$$

where

$$\begin{aligned}\gamma_0(\tau) &= \sqrt{\frac{P_0}{\vec{\varphi}^H(\tau) (\mathbf{Q}_w^H \mathbf{D} \mathbf{Q}_w) \vec{\varphi}(\tau)}}, \\ \mathbf{Q}_w &= \frac{1}{K} \left(\left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right) \\ &\quad \times \left(\mathbf{\Lambda}_1 \mathbf{U}_1^H \left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right)^{-1} \mathbf{U}_1^H \mathbf{B}.\end{aligned}$$

Substituting $\mathbf{w}_0$ and $\gamma_0(\tau)$ expressed by (3.49) into the objective function of (3.43), we have the optimal SINR as a function of the decision delay τ :

$$\begin{aligned}\text{SINR}_{opt}(\tau) &= \frac{\gamma_0(\tau)^2 \cdot P_s}{\mathbf{w}_0^H(\tau) \mathbf{Q}_n \mathbf{w}_0(\tau) + \sigma_v^2} \\ &= \frac{P_s \cdot P_0}{\vec{\varphi}(\tau)^H \mathbf{Q}_w^H (P_0 \cdot \mathbf{Q}_n + \sigma_v^2 \cdot \mathbf{D}) \mathbf{Q}_w \vec{\varphi}(\tau)}.\end{aligned}\tag{3.50}$$

Since the scalar τ takes integer values from 0 to $(L_c - 1)$, then for a specific channel realization, we can choose a best decision delay τ_0 that leads to the optimal output SINR:

$$\tau_0 = \arg \max_{\tau \in \{0, \dots, (L_c - 1)\}} \text{SINR}_{opt}(\tau).\tag{3.51}$$

Thus, by substituting τ_0 into (3.49), the optimal weight vector corresponding to the optimal delay can be given by

$$\mathbf{w}_{opt} = \sqrt{\frac{P_0}{\vec{\varphi}^H(\tau_0) (\mathbf{Q}_w^H \mathbf{D} \mathbf{Q}_w) \vec{\varphi}(\tau_0)}} \cdot \mathbf{Q}_w \vec{\varphi}(\tau_0).\tag{3.52}$$

Remark: In the above discussion, we compare the SINR achieved by different decision delay values, and then select the optimal one which renders the highest SINR. It is noted that, the SINR maximization approach in [16] is also able to incorporate this process, *i.e.*, calculating the SINR for each value of delay and searching for the optimal delay. However, in that case, the approach of [16] needs to reconstruct the relevant matrices ($\mathbf{Q}_s$ and $\mathbf{Q}_i$) corresponding to different delays and solve the SINR maximization problem

for each delay value. In contrast, the proposed approach solves the problem (3.38) only once, and obtains the SINR as a function of τ as shown in (3.50). Then we just need to compare the SINR with respect to different values of τ , and hence the best delay can be selected for a channel realization.

3.4.4 Computational Complexity

Now we analyze the computational complexity of the proposed approach and compare it with the approach of [16]. We use the number of *floating-point operations* (FLOPs) to indicate the computational complexity, and here a FLOP represents either a complex addition or a complex multiplication. For both approaches, we discuss the computational complexity in two stages, where one is the matrices construction stage and the other is the problem solving stage.

3.4.4.1 Matrices Constructing Stage

According to (3.43), solving the proposed formulation requires matrices $\mathbf{D}$, $\mathbf{Q}_n$ as well as $\mathbf{Q}_0$ that gives the eigen-decomposition $\mathbf{\Lambda}_1$ and $\mathbf{U}_1$. On the other hand, for the time-domain approach of [16] the corresponding matrices $\mathbf{D}_T$, $\mathbf{Q}_{n,T}$, $\mathbf{Q}_{s,T}$ and $\mathbf{Q}_{i,T}$ are required (here the matrices' subscripts are modified for avoiding ambiguity). The matrices $\mathbf{Q}_{s,T}$ and $\mathbf{Q}_{i,T}$ are dependent on the decision delay, and thus if the approach of [16] is required to adapt the delay to channel realizations, both $\mathbf{Q}_{s,T}$ and $\mathbf{Q}_{i,T}$ have to be calculated for each possible delay value to find the optimal delay. Therefore, we focus on comparing the complexity of $\mathbf{Q}_0$ with the aggregation complexity of $\mathbf{Q}_{s,T}$ and $\mathbf{Q}_{i,T}$.

According the definition in (3.26) and assuming $K = L_c$, the computational complexity (FLOPs) for constructing $\mathbf{Q}_0$ is given by

$$C_{F.1} = 2R^2L_h^2(L_f + L_g + L_h - \frac{5}{2}) + (RL_h + R)(L_f + L_g + L_h - 2). \quad (3.53)$$

On the other hand, by exploiting the structures of matrices $\mathbf{Q}_{s,T}$ and $\mathbf{Q}_{i,T}$ and counting the complex arithmetic operations for obtaining both matrices, with respect to each delay the aggregation computational complexity (FLOPs) for constructing $\mathbf{Q}_{s,T}$ and $\mathbf{Q}_{i,T}$ can be obtained as

$$C_{T,1} = \begin{cases} 2R^2L_h^2(L_f + L_g - \frac{L_h}{3} - \frac{3}{2}) + 2RL_hL_fL_g \\ \quad + \frac{2R^2L_h}{3} - RL_h(L_f + L_g - 1), & L_f + L_g \geq L_h \\ 2R^2L_h^2(L_h - \frac{L_f}{3} - \frac{L_g}{3} - \frac{1}{6}) + 2RL_hL_fL_g \\ \quad + (\frac{2}{3}R^2 - RL_h)(L_f + L_g - 1), & L_f + L_g < L_h \end{cases} \quad (3.54)$$

It is noted that $C_{F,1}$ and $C_{T,1}$ have the same highest order of scale. Furthermore, to calculate $\mathbf{Q}_{s,T}$ and $\mathbf{Q}_{i,T}$ for every delay value $\tau \in 0, \dots, (L_c - 1)$, the total computational complexity is $L_c \cdot C_{T,1}$ FLOPs. In contrast, for the proposed approach $\mathbf{Q}_0$ is independent of delay and needs to be calculated only once. Figure 3.3 illustrates an example for the comparison between $C_{F,1}$ and $L_c \cdot C_{T,1}$.

3.4.4.2 Problem Solving Stage

Here, for an illustrative purpose, only the major operations are counted. For solving the proposed problem, we need to eigen-decompose an $RL_h \times RL_h$ matrix $\mathbf{Q}_0$, and calculate the inverse of an $RL_h \times RL_h$ matrix $(\mathbf{Q}_n + (\sigma_v^2/P_0)\mathbf{D})$ and an $L_c \times L_c$ matrix $(\mathbf{\Lambda}_1\mathbf{U}_1^H(\mathbf{Q}_n + (\sigma_v^2/P_0)\mathbf{D})^{-1}\mathbf{U}_1\mathbf{\Lambda}_1^H)$. The computational complexity of these operations are $O((RL_h)^3)$, $O((RL_h)^3)$ and $O(L_c^3)$ FLOPs, respectively. Furthermore, according to (5.49) and assuming $K = L_c$, searching for the optimal delay requires $O(L_c^3)$ FLOPs.

While solving the maximizing SINR problem of [16], the inverse square root of an $RL_h \times RL_h$ matrix $\mathbf{D}_T$ is calculated and requires $O((RL_h)^3)$ FLOPs. For each delay value an $RL_h \times RL_h$ matrix $(\mathbf{D}_T^{-1/2}(\mathbf{Q}_{i,T} + \mathbf{Q}_{n,T})\mathbf{D}_T^{-1/2} + (\sigma_v^2/P_0)\mathbf{I})$ needs to be inversed

(a)

(b)

Figure 3.3: Comparison of Computational complexity for constructing matrices. $L_c \cdot C_{T-1}$ denotes the FLOPs for constructing $\mathbf{Q}_{s,T}$ and $\mathbf{Q}_{i,T}$ in [16] when adaptive delay is needed, and C_{F-1} denotes the FLOPs for constructing $\mathbf{Q}_0$. (a) With varying number of relays, and $L_f = L_g = L_h = 5$. (b) With varying relay filter length, and $L_f = L_g = 5$, $R = 20$.

with a complexity of $O((RL_h)^3)$ FLOPs. However, to search for the optimal delay from

$\tau = 0, \dots, L_c$, the matrix inversion has to be performed L_c times to different matrices.

Therefore, we conclude that for the approach of [16] to adapt to a optimal delay, it generally requires a higher computational load than the proposed approach.

3.5 Numerical Simulation

In the simulation, the performance of the proposed adaptive delay approach is investigated, where we consider a network with $R = 20$ (unless otherwise specified) relays. Throughout this section, the relay noise power and destination noise power are assumed to be $\sigma_n^2 = \sigma_v^2 = 1$, and the source power P_s is 10dB higher than the noise power. The coefficients of channel impulse responses are modeled as independent quasi-static Rayleigh fading, and are hence generated as zero-mean complex Gaussian random variables with the exponential power delay profile [97]

$$p(m) = \frac{1}{\sigma_t} \sum_{l=0}^{L_x-1} e^{-m/\sigma_t} \delta(m-l), \quad (3.55)$$

where σ_t characterizes the delay spread, and $L_x \in \{L_f, L_g\}$ denotes the length of the backward channels or the forward channels. Throughout the simulations, $L_f = L_g = 5$, and $L_h = 5$ unless otherwise specified. All the channels are assumed to be quasi-static and perfectly known by the algorithm, which means that the channels are randomly generated but remain static over a frame time, during which multiple successive symbols are transmitted. The algorithm calculates the relay weights at the beginning of each frame based on the known CSI (a channel realization), and the same set of weights are used during this frame period.

Through the simulations, we compare the performance of the proposed adaptive delay approach with the SINR maximization approach of [16] by Chen *et al.* We note that in [16] the decision delay is not considered, which means therein τ is set to zero. For comparison in our simulations, by choosing different element of vector $\mathbf{s}$ in (16) of [16], the approach of Chen *et al.* can be modified to work on different decision delays. However, for the approach of [16] the decision delay needs to be pre-selected prior to solving the beamforming problem. As we have discussed, if the delay is not pre-selected, the approach

Figure 3.4: SINR performance versus total relay transmit power P_0 . Delay spread factor $\sigma_t = 2$. The SINR maximization approach of [16] is shown with different pre-selected decision delays τ_d .

of [16] can also carry out the solving procedure with respect to each decision delay and choose one of the solution that renders the highest SINR, but this process requires higher computational load than the proposed approach.

3.5.1 Impacts of Decision Delay

In the first simulation, Figure 3.4 illustrates the SINR performance versus the total relay transmit power constraint. Herein, for the approach of [16], different delay values τ_d are determined in advance and kept fixed during the simulation with multiple trials. Each of the SINRs is obtained by averaging over 5000 channel realizations (i.e., 5000 trials). It is observed that, the approach of [16] is outperformed by our approach when it does not pre-select an appropriate decision delay τ_d . In contrast, our proposed method determines the optimal delay which varies with respect to different channel realizations.

Note that in Figure 3.4, we only show the values of pre-selected delay $\tau_d = 0, \dots, 4$.

Figure 3.5: SINR performance versus non-adaptive decision delay τ_d , with different total relay transmit power budget P_0 . Delay spread factor $\sigma_t = 2$. The proposed approach is independent of τ_d .

For better illustration of the performance varying with different values of τ_d for the non-adaptive approach [16], Figure 3.5 shows the SINR performance versus the non-adaptive decision delay. Since actually the proposed approach is independent of the non-adaptive decision delay τ_d , here plotting the proposed approach versus τ_d is just for the purpose of comparison with the non-adaptive delay selection approach of Chen *et al.* It is observed that the non-adaptive approach takes peak SINR value at $\tau_d = 4$, where the peak value is close to the proposed approach. The reason why the peak SINR of [16] is close to the proposed approach can be explained as follows. Comparing the proposed distortionless approach with the approach of [16], it is noted that the approach of [16] is formulated as maximizing SINR with power constraint, and the proposed approach is formulated as maximizing SINR with power constraint and distortionless constraint. That is, the proposed approach has the same optimization objective as the approach of [16], and moreover has an additional constraint. Therefore, some degrees-of-freedom

Figure 3.6: Histogram of the distribution of optimal delay selected by the proposed approach for 5000 realizations. Power budget $P_0 = 10\text{dB}$. Delay spread factor $\sigma_t = 2$. $L_f = L_g = 5$.

(DOF) is consumed for satisfying the distortionless constraint, and the remaining DOFs for optimizing SINR are fewer than the approach of [16]. Thus, if the same delay is used for both approaches, the proposed approach is suboptimal to the approach of [16]; and if the same delay is used, while there are sufficient DOFs the performance of the two approaches will be very close.

From Figure 3.4 and Figure 3.5 it seems that for the approach of [16], a fixed delay $\tau_d = 4$ is optimal for different channel realizations. To explain this observation, we use Figure 3.6 to illustrate the histogram of optimal delay distribution selected by the proposed approach for 5000 realizations. As Figure 3.6 (a) shows, in this scenario the most probable optimal delay is $\tau_d = 4$. However, in the scenario of Figure 3.6 (b), both $\tau_d = 2$ and $\tau_d = 3$ have high probability to appear, and thus neither one is necessarily optimal for different realizations. In the subsequent simulation we will see that the

optimal delay is not only affected by the channel realization, but also affected by other system parameters.

3.5.2 Effects of Relay Filter Length

In the next simulation, the effects of the relay filter length L_h on the performance are shown by Figure 3.7 and Figure 3.8. Therein, Figure 3.7 shows the SINR performance versus the non-adaptive decision delay, where the proposed approach is independent of τ_d . We see that, the non-adaptive approach [16] suffers from poor SINR when the pre-selected decision delay is not chosen appropriately. We also observe that, the optimal decision delay for the non-adaptive approach is affected by the filter length. Although there seems to exist relation between L_c and the optimal delay, this observation is restricted to the scenario that all the relay filters have the same length. However, if not all the relay nodes have the same filter length, there would be no obvious relation between the various lengths of relay filters and the optimal delay, and thus it is more difficult to pre-select the optimal decision delay for the non-adaptive approach. Whereas the proposed approach determines the decision delay according to channel realizations and achieves the SINR that is close to the optimal performance of the approach of [16]. In Figure 3.8 the SINR performance is drawn with respect to the relay filter length. We can see that for the non-adaptive method, if the decision delay is not appropriately chosen, the SINR performance stays at a relatively low level and cannot be significantly improved with the increase of relay filter length. Moreover, comparing to the non-adaptive approach with $\tau_d = 5$, the proposed approach shows advantage over the range of shorter relay filter length ($L_h < 5$), although performances of the two approaches are close while longer filter length is used.

Figure 3.7: SINR performance versus non-adaptive decision delay τ_d , with different relay filter lengths L_h . Delay spread factor $\sigma_t = 2$. The proposed approach is independent of τ_d . The rectangles indicate the peak values of the SINR maximization approach [16].

Figure 3.8: SINR performance versus relay filter length L_h . Delay spread factor $\sigma_t = 2$. The SINR maximization approach of [16] is shown with different pre-selected decision delays τ_d .

Figure 3.9: SINR performance versus non-adaptive decision delay τ_d , with different delay spread factor σ_t . The proposed approach is independent of τ_d . The rectangles indicate the peak values of the SINR maximization approach [16].

3.5.3 Effects of Delay Spread Factor

Next, Figure 3.9 and Figure 3.10 illustrate the effects of the delay spread factor σ_t . It is observed from Figure 3.9 that for different values of the delay spread factor σ_t , the non-adaptive approach [16] takes peak values at different positions, even when the relay filters length are fixed. Thus, although we note from Figure 3.7 that the optimal delay is related to the relay filter length, σ_t results in additional difficulty for the non-adaptive approach to pre-select the optimal delay. Figure 3.10 illustrates that the performances of both approaches could be significantly affected with the increase in σ_t . If the delay is not pre-selected appropriately, the SINR of the non-adaptive approach rolls off quickly compared with the proposed approach.

Figure 3.10: SINR performance versus delay spread factor σ_t . The SINR maximization approach of [16] is shown with different pre-selected decision delays τ_d .

3.5.4 Effects of Number of Relays

The effect of the number of relays is also illustrated in Figure 3.11. We note that here $L_c = L_f + L_g + L_h - 2 = 13$, and the starting point of the horizontal axis is $R = 4$. Therefore, we have $RL_h = 20 > L_c = 13$, which indicates the distortionless condition is satisfied at $R = 4$ and at all the values of $R > 4$. It is seen that, when there are sufficient relays (*e.g.*, $R \geq 15$), the performance of the proposed approach is close to the non-adaptive approach [16] with optimal pre-selected decision delay, while the approach of [16] can be superior for a small number of relays. This is explained as follows. As we have mentioned previously that the proposed approach has the same optimization objective (SINR maximization) as the approach of [16], and moreover has an additional constraint. Hence, if the same delay is used for both approaches, the proposed approach is suboptimal to the approach of [16] since some of the DOFs are consumed by the distortionless constraint. For example, at the point $R = 4$ in Figure 3.11, 13 DOFs

Figure 3.11: SINR performance versus number of relays R . Delay spread factor $\sigma_t = 2$. The SINR maximization approach of [16] is shown with different pre-selected decision delays τ_d .

are used for distortionless constraint and the dimension of the optimization variable $\mathbf{w}$ is $RL_h = 20$, while the approach of [16] has more DOFs for maximizing SINR thus it performs better even with non-optimal delay. However, with the increase of R the dimension of $\mathbf{w}$ also increases, but the DOFs required for distortionless constraint are still 13, and then there are abundant DOFs for optimizing SINR thus the performance of the proposed approach gets close to that of [16] with optimal decision delay.

3.5.5 BER Performance

In the last simulation, the bit error rate (BER) performance versus total relay transmit power is shown in Figure 3.12. Here, the simulated data are generated with QPSK modulation scheme. It can be seen that, with a higher relay transmit power, the proposed approach achieves a lower BER. Comparing the performance differences due to the selection of decision delay, the BER performance reflects a trend that is consistent with the

Figure 3.12: BER performance versus the total relay transmit power P_0 . Delay spread factor $\sigma_t = 2$. The SINR maximization approach of [16] is shown with different pre-selected decision delays τ_d .

SINR performance in Figure 3.4.

3.6 Conclusion

In this chapter, we proposed a new approach to relay beamforming in frequency-selective channels, where the FF relay scheme was adopted to combat the channel distortion. By formulating the relay beamforming problem in frequency domain, we imposed a distortionless constraint to control the frequency response of the equivalent channel. It is proved that under a certain condition that is related to the length of the channel filters and the relay filters as well as the number of relay nodes, the equivalent channel is able to achieve exact flatness through optimal relay beamforming. We provided a closed-form solution to the beamforming problem with distortionless constraint. In addition, the problem of adaptively selecting the optimal decision delay was also investigated, which

achieves the highest output SINR for a channel realization. Simulation results demonstrated that the proposed approach achieved substantial performance improvement over the non-adaptive delay selection approaches.

3.7 Appendices

3.7.1 Proof of $\sum_{k=1}^K \tilde{\mathbf{Q}}_{i2}^{(\omega_k, \tau)} = \mathbf{0}$

From the definition of $\tilde{\mathbf{Q}}_{i2}^{(\omega, \tau)}$ after (3.16) we can obtain

$$\sum_{k=1}^K \tilde{\mathbf{Q}}_{i2}^{(\omega_k, \tau)} = P_S \cdot \mathbf{\Psi}^T (\mathbf{M}_\tau \mathbf{A} \bar{\mathbf{M}}_\tau + \bar{\mathbf{M}}_\tau \mathbf{A} \mathbf{M}_\tau) \mathbf{\Psi}^*, \quad (3.56)$$

where $\mathbf{A} = \sum_{k=1}^K \mathbf{b}_{fgh}^T(\omega_k) \cdot \mathbf{b}_{fgh}^*(\omega_k) \in \mathbb{C}^{L_c \times L_c}$. Based on the definition of $\mathbf{b}_{fgh}(\omega)$, the (l, m) th entry of matrix $\mathbf{A}$ can be expressed as

$$[\mathbf{A}]_{l,m} = \sum_{k=1}^K e^{j \frac{2\pi(k-1)}{K} (l-m)}. \quad (3.57)$$

Along with $K \geq L_c$, we can further obtain

$$\sum_{k=1}^K e^{j \frac{2\pi(k-1)}{K} (l-m)} = \begin{cases} \sum_{k=1}^K e^{j0} = K, & l = m \\ \frac{1 - e^{j2\pi(l-m)}}{1 - e^{j \frac{2\pi(l-m)}{K}}} = 0, & l \neq m \end{cases} \quad (3.58)$$

Therefore, $\mathbf{A}$ is a diagonal matrix, and hence it is obvious that $\mathbf{M}_\tau \mathbf{A} \bar{\mathbf{M}}_\tau = \mathbf{0}$ and $\bar{\mathbf{M}}_\tau \mathbf{A} \mathbf{M}_\tau = \mathbf{0}$. Then from (3.56) we conclude that $\sum_{k=1}^K \tilde{\mathbf{Q}}_{i2}^{(\omega_k, \tau)} = \mathbf{0}$. ■

3.7.2 Proof of Lemma 1

Comparing (3.21) and (3.23), it is obtained that

$$H_{eqv}(\omega) = \mathbf{w}^H \tilde{\mathbf{q}}_o^{(\omega)}. \quad (3.59)$$

On the other hand, from the first equation of (3.9), the frequency response of the equivalent channel $H_{eqv}(\omega)$ can be written as

$$H_{eqv}(\omega) = \sum_{i=1}^R \mathbf{b}_{fgh}(\omega) \cdot [\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i]. \quad (3.60)$$

Furthermore, by representing the result of the convolution $f_i(m) * g_i(m)$ as a column vector $\mathbf{b}_i = \mathbf{f}_i * \mathbf{g}_i = [b_{i,1}, \dots, b_{i,L_b}]^T$, where $L_b = (L_f + L_g - 1)$, we obtain that $[\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i] = \mathbf{\Theta}_i \mathbf{h}_i$, where

$$\mathbf{\Theta}_i = \begin{bmatrix} b_{i,1} & 0 & & 0 \\ \vdots & b_{i,1} & \ddots & \vdots \\ b_{i,L_b} & \vdots & & \vdots \\ 0 & b_{i,L_b} & \ddots & 0 \\ \vdots & 0 & & b_{i,1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & b_{i,L_b} \end{bmatrix} \in \mathbb{C}^{L_c \times L_h}$$

is a column-circulant matrix. Substituting $[\mathbf{f}_i * \mathbf{g}_i * \mathbf{h}_i] = \mathbf{\Theta}_i \mathbf{h}_i$ into (3.60), and using (3.12) and (3.13), we have

$$H_{eqv}(\omega) = \mathbf{b}_{fgh}(\omega) \cdot \mathbf{\Psi} \begin{bmatrix} \mathbf{h}_i \\ \vdots \\ \mathbf{h}_R \end{bmatrix} = \mathbf{w}^H \mathbf{\Psi}^T \mathbf{b}_{fgh}^T(\omega). \quad (3.61)$$

Comparing (3.61) and (3.59) it can be obtain that

$$\tilde{\mathbf{q}}_o^{(\omega)} = \mathbf{\Psi}^T \mathbf{b}_{fgh}^T(\omega). \quad (3.62)$$

Moreover, according to the definition of matrix $\mathbf{B}$ in (3.34), we can rewrite $\mathbf{B}$ using the result of (3.62) as

$$\mathbf{B} = [\mathbf{\Psi}^T \mathbf{b}_{fgh}^T(\omega_1), \dots, \mathbf{\Psi}^T \mathbf{b}_{fgh}^T(\omega_K)] \triangleq \mathbf{\Psi}^T \mathbf{\Phi}_{fgh}^T, \quad (3.63)$$

where $\mathbf{\Phi}_{fgh} \triangleq [\mathbf{b}_{fgh}^T(\omega_1), \dots, \mathbf{b}_{fgh}^T(\omega_K)]^T \in \mathbb{C}^{K \times L_c}$, and $\mathbf{\Psi} \in \mathbb{C}^{L_c \times (RL_h)}$. Note that $\mathbf{\Phi}_{fgh}$ has full column rank L_c since we have assumed $K \geq L_c$, and $\mathbf{\Psi}$ has full row rank L_c if $RL_h \geq L_c$. Thus, when $RL_h \geq L_c$ is satisfied, the rank of $\mathbf{B}$ is also L_c . $\blacksquare$

3.7.3 Proof of Lemma 2

According to the conclusion of **Lemma 1**, we have

$$\mathbf{B}^H = \mathbf{\Phi}_{fgh}^* \mathbf{\Psi}^*. \quad (3.64)$$

Based on (3.63), the structure of Φ_{fgh}^* is shown as follows:

$$\begin{aligned}
 \Phi_{fgh}^* &= [\mathbf{b}_{fgh}^T(\omega_1), \mathbf{b}_{fgh}^T(\omega_2), \dots, \mathbf{b}_{fgh}^T(\omega_K)]^H \\
 &= \begin{bmatrix} e^{j\omega_1 \cdot 0} & e^{j\omega_1 \cdot 1} & \dots & e^{j\omega_1 \cdot (L_c-1)} \\ e^{j\omega_2 \cdot 0} & e^{j\omega_2 \cdot 1} & \dots & e^{j\omega_2 \cdot (L_c-1)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\omega_K \cdot 0} & e^{j\omega_K \cdot 1} & \dots & e^{j\omega_K \cdot (L_c-1)} \end{bmatrix} \\
 &= [\vec{\varphi}(0), \vec{\varphi}(1), \dots, \vec{\varphi}(L_c-1)], \tag{3.65}
 \end{aligned}$$

where $\vec{\varphi}(\tau)$ is defined after (3.36).

To investigate the column space of the matrix $\mathbf{B}^H$, we define the i th column of Ψ^* as $\tilde{\mathbf{p}}_i \in \mathbb{C}^{L_c \times 1}$, ($i = 1, \dots, RL_h$). Thus, according to (3.64) we obtain

$$\begin{aligned}
 \mathbf{B}^H &= \Phi_{fgh}^* [\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \dots, \tilde{\mathbf{p}}_{RL_h}] \\
 &= [\Phi_{fgh}^* \tilde{\mathbf{p}}_1, \Phi_{fgh}^* \tilde{\mathbf{p}}_2, \dots, \Phi_{fgh}^* \tilde{\mathbf{p}}_{RL_h}]. \tag{3.66}
 \end{aligned}$$

Moreover, along with the structure of Φ_{fgh}^* shown in (3.65), we further obtain

$$\mathbf{B}^H = \left[\sum_{\tau=0}^{L_c-1} \vec{\varphi}(\tau) \cdot [\tilde{\mathbf{p}}_1]_{\tau}, \dots, \sum_{\tau=0}^{L_c-1} \vec{\varphi}(\tau) \cdot [\tilde{\mathbf{p}}_{RL_h}]_{\tau} \right], \tag{3.67}$$

where $[\tilde{\mathbf{p}}_i]_{\tau}$ denotes the τ th element of vector $\tilde{\mathbf{p}}_i$. Note that each column of $\mathbf{B}^H$ lies in the space spanned by $\vec{\varphi}(\tau), \tau = 0, \dots, (L_c - 1)$, where the dimension of the space is L_c . Then it is obvious that when $RL_h \geq L_c$ the columns of $\mathbf{B}^H$ span the same space as the vectors $\vec{\varphi}(\tau)$ span, *i.e.*, $\vec{\varphi}(\tau)$ lies in the column-space of matrix $\mathbf{B}^H$. ■

3.7.4 Proof of Theorem 1

Here we rewrite DEF $J(\mathbf{w}, \gamma, \tau)$ (3.24) using matrix $\mathbf{B}$ defined in (3.34), and the following derivation shows the procedure of *completion of square*:

$$\begin{aligned} J(\mathbf{w}, \gamma, \tau) &= \mathbf{w}^H \mathbf{Q}_0 \mathbf{w} - \gamma \mathbf{w}^H \mathbf{q}_o(\tau) - \gamma \mathbf{q}_o^H(\tau) \mathbf{w} + \gamma^2 \\ &= \frac{1}{K} \mathbf{w}^H (\mathbf{B} \mathbf{B}^H) \mathbf{w} - \frac{\gamma}{K} \mathbf{w}^H \mathbf{B} \vec{\varphi}(\tau) - \frac{\gamma}{K} \vec{\varphi}^H(\tau) \mathbf{B}^H \mathbf{w} + \gamma^2 \\ &= \frac{1}{K} \mathbf{w}^H \mathbf{B} (\mathbf{B}^H \mathbf{w} - \gamma \vec{\varphi}(\tau)) - \frac{\gamma}{K} \vec{\varphi}^H(\tau) (\mathbf{B}^H \mathbf{w} - \gamma \vec{\varphi}(\tau)) \\ &\quad + \gamma^2 - \frac{\gamma^2}{K} \vec{\varphi}^H(\tau) \cdot \vec{\varphi}(\tau), \end{aligned}$$

where $\vec{\varphi}(\tau) = [e^{j\omega_1 \tau}, \dots, e^{j\omega_K \tau}]^T$, hence $\vec{\varphi}^H(\tau) \cdot \vec{\varphi}(\tau) = K$. Then we can continue to derive:

$$\begin{aligned} J(\mathbf{w}, \gamma, \tau) &= \frac{1}{K} (\mathbf{w}^H \mathbf{B} - \gamma \vec{\varphi}^H(\tau)) (\mathbf{B}^H \mathbf{w} - \gamma \vec{\varphi}(\tau)) + 0 \\ &= \frac{1}{K} \|\mathbf{B}^H \mathbf{w} - \gamma \vec{\varphi}(\tau)\|_2^2. \end{aligned} \tag{3.68}$$

According to the conclusion of **Lemma 2** that the column vector $\vec{\varphi}(\tau)$ lies in the column space of matrix $\mathbf{B}^H$, thus the term in the l_2 -norm in (3.68)

$$\mathbf{B}^H \mathbf{w} - \gamma \vec{\varphi}(\tau) = 0 \tag{3.69}$$

has solutions. Hence, now we complete the proof that the minimum value of the distortion evaluation function $J(\mathbf{w}, \gamma, \tau)$ is zero. ■

3.7.5 Proof of Lemma 3

According to (3.20), the distortionless constraint $J(\mathbf{w}_1, \gamma, \tau) = 0$ indicates that $H_{eqv}(\omega_k) = \gamma e^{-j\omega_k \tau}$ for each $\omega_k, k = 1, \dots, K$. Recalling (3.9), (3.12) and (3.13), we can write $H_{eqv}(\omega)$ as $\mathbf{b}_{fgh}(\omega) \cdot \Psi \mathbf{w}_1^*$, and hence the distortionless constraint can be expressed as

$$\begin{bmatrix} \mathbf{b}_{fgh}(\omega_1) \Psi \mathbf{w}_1^* \\ \vdots \\ \mathbf{b}_{fgh}(\omega_K) \Psi \mathbf{w}_1^* \end{bmatrix} = \gamma \begin{bmatrix} e^{-j\omega_1 \tau} \\ \vdots \\ e^{-j\omega_K \tau} \end{bmatrix},$$

which can be further written as $\Phi_{fgh} \Psi \mathbf{w}_1^* = \gamma \cdot \vec{\varphi}^*(\tau)$ according to (3.64). Note that the $(K \times L_c)$ matrix Φ_{fgh} has the property $\Phi_{fgh}^H \Phi_{fgh} = K \cdot \mathbf{I}_{L_c}$, and then we can obtain

$$\Psi \mathbf{w}_1^* = \frac{\gamma}{K} \cdot \Phi_{fgh}^H \vec{\varphi}^*(\tau). \quad (3.70)$$

On the other hand, using (3.14) and (3.19) we have

$$\mathbf{w}_1^H \mathbf{Q}_s^{(\tau)} \mathbf{w}_1 = \frac{P_S}{K} \sum_{k=1}^K |\mathbf{b}_{fgh}(\omega_k) \mathbf{M}_\tau \Psi \mathbf{w}_1^*|^2. \quad (3.71)$$

Inserting (3.70) into (3.71) yields

$$\begin{aligned} \mathbf{w}_1^H \mathbf{Q}_s^{(\tau)} \mathbf{w}_1 &= \frac{\gamma^2 P_S}{K^3} \sum_{k=1}^K |\mathbf{b}_{fgh}(\omega_k) \mathbf{M}_\tau \Phi_{fgh}^H \vec{\varphi}^*(\tau)|^2 \\ &= \frac{\gamma^2 P_S}{K^3} \sum_{k=1}^K |e^{-j\omega_k \tau} \sum_{j=1}^K e^{j\omega_j \tau} e^{-j\omega_j \tau}|^2 = \gamma^2 P_S. \end{aligned} \quad (3.72)$$

Similarly, together with (3.15) and (3.72), we can obtain

$$\begin{aligned} \mathbf{w}_1^H \mathbf{Q}_i^{(\tau)} \mathbf{w}_1 &= \frac{P_S}{K} \sum_{k=1}^K |\mathbf{b}_{fgh}(\omega_k) (\mathbf{I}_{L_c} - \mathbf{M}_\tau) \Psi \mathbf{w}_1^*|^2 \\ &= \frac{P_S}{K} \sum_{k=1}^K |\mathbf{b}_{fgh}(\omega_k) \Psi \mathbf{w}_1^* - \gamma e^{-j\omega_k \tau}|^2 \\ &= \frac{P_S}{K} \sum_{k=1}^K |\gamma e^{-j\omega_k \tau} - \gamma e^{-j\omega_k \tau}|^2 = 0. \end{aligned} \quad (3.73)$$

The proof is completed. ■

3.7.6 Solving Problem (3.46)

The Lagrangian associated with the problem (3.46) is

$$\begin{aligned} \mathcal{L}(\hat{\mathbf{w}}, \gamma, \vec{\lambda}, \mu) &= \hat{\mathbf{w}}^H \mathbf{Q}_n \hat{\mathbf{w}} + \frac{\sigma_v^2}{\gamma^2} + \vec{\lambda}^T \Re\{\mathbf{\Lambda}_1 \mathbf{U}_1^H \hat{\mathbf{w}} - \mathbf{U}_1^H \mathbf{q}_o(\tau)\} \\ &\quad + \mu(\hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} - \frac{P_0}{\gamma^2}). \end{aligned} \quad (3.74)$$

Then we can obtain the following conditions:

$$\frac{\partial \mathcal{L}(\hat{\mathbf{w}}, \gamma, \vec{\lambda}, \mu)}{\partial \hat{\mathbf{w}}^*} = (\mathbf{Q}_n + \mu \mathbf{D}) \hat{\mathbf{w}} + \mathbf{U}_1 \mathbf{\Lambda}_1^H \vec{\lambda} = 0, \quad (3.75)$$

$$\frac{\partial \mathcal{L}(\hat{\mathbf{w}}, \gamma, \vec{\lambda}, \mu)}{\partial (\gamma^2)} = \frac{-\sigma_v^2 + \mu P_0}{\gamma^4} = 0, \quad (3.76)$$

$$\mathbf{\Lambda}_1 \mathbf{U}_1^H \hat{\mathbf{w}} - \mathbf{U}_1^H \mathbf{q}_o(\tau) = 0, \quad (3.77)$$

$$\hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} - \frac{P_0}{\gamma^2} = 0. \quad (3.78)$$

To solve these equations, first, from (3.76) we obtain that

$$\mu = \frac{\sigma_v^2}{P_0}. \quad (3.79)$$

Substituting (3.79) into (3.75) and solving for $\hat{\mathbf{w}}$ yield

$$\hat{\mathbf{w}} = -(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D})^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \vec{\lambda}. \quad (3.80)$$

Then substituting (3.80) into (3.77) and solving for $\vec{\lambda}$ yield

$$\vec{\lambda} = - \left(\mathbf{\Lambda}_1 \mathbf{U}_1^H (\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D})^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau). \quad (3.81)$$

Substituting (3.81) back into (3.80), we can obtain the optimal $\hat{\mathbf{w}}$ that is a function of parameter τ as

$$\hat{\mathbf{w}}_0(\tau) = \mathbf{v}(\tau), \quad (3.82)$$

where for avoiding ambiguity of notation, we introduce a vector

$$\begin{aligned} \mathbf{v}(\tau) = & \left((\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D})^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right) \\ & \times \left(\mathbf{\Lambda}_1 \mathbf{U}_1^H (\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D})^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau). \end{aligned}$$

Then, using (3.78) and (3.82) we can solve for the optimal γ that is also a function of τ as

$$\gamma_0(\tau) = \sqrt{P_0 / \mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau)}. \quad (3.83)$$

Therefore, the solution of problem (3.46) is given by (3.82) together with (3.83).

Chapter 4

A New Frequency-Domain Approach using Power Minimization with Distortionless Response Constraint

4.1 Introduction

In the previous chapter, we have discussed the relay beamforming problem based on a commonly used criterion, which aims at minimizing the received SINR at the destination side. However, with this design criterion, the output power level at the destination is not directly controlled, and in practice the SINR is not as easily measurable as the signal power. We note that in the literature on relay beamforming designs, criteria based on optimizing transmit power of the source or relays are also frequently used. By employing these criteria, the major concern lies in the power control aspects, such as the battery life of terminal equipments and interference control in the network.

In this chapter, we consider the filter-and-forward (FF) relay beamforming design with an alternative approach, which aims to minimize the output power at the destination side and meanwhile subject to a distortionless response constraint. By introducing an adjustable scalar amplifier at the destination side, we discuss the relay beamforming design scheme with minimizing output power in two cases. We first consider the scalar

to be fixed and a given signal gain level is required for the desired signal. We derive a range for the selection of signal gain in order to make it consistent with the relay transmit power constraint. We then consider the general design scheme where both the relay weight and the scalar amplifier are jointly optimized, which aims at achieving the minimum output power and fulfilling a signal power gain requirement. We show that the adaptive-scalar scheme always attains the same output SINR performance as that given by the SINR maximization with the distortionless constraint.

Furthermore, the scheme of output power minimization is extended to the multi-antenna destination case, where each antenna at the destination node is followed by a linear FIR filter, which can be jointly designed and work collaboratively with the relay filters. In this way, the burden for equalizing the frequency-selective channels from the source to the destination can be allocated between the relay nodes and the destination node, thus leading to further improvement of the system performance, since more system resource and degrees-of-freedom are available

The organization of this chapter is as follows. In Section 4.2, the signal model is described. In Section 4.3, an evaluation for the channel distortion is introduced, and the problem formulation is presented. Section 4.4 discusses the beamforming problem with a fixed scalar, and Section 4.5 discusses the beamforming problem with the adaptive scalar. In Section 4.6, the output power minimization scheme is generalized to a scenario where the destination node is equipped with multiple antennas. Simulation results are provided in Section 4.7. In Section 4.8, conclusions are drawn.

4.2 Signal Model

As depicted in Figure 4.1, we consider a relay network with one source node, one destination node and R relay nodes, and all nodes are each equipped with a single antenna.

Figure 4.1: Signal model of filter-and-forward relay network in frequency-selective channels.

It is noted that the only difference between this network model and Figure 3.2 which is discussed in the previous chapter lies in the scalar amplifier at the destination node. We will discuss in sequel jointly designing this scalar factor with the relay coefficients.

The wireless channels between the nodes are assumed to be frequency-selective, and are hence modeled as linear FIR filters. As defined in Chapter 3, the backward channel and forward channel are represented as $\mathbf{f}_i = [f_i(0), \dots, f_i(L_f - 1)]^T$ and $\mathbf{g}_i = [g_i(0), \dots, g_i(L_g - 1)]^T$, respectively. These channels are assumed to be perfectly known.

The source node transmits an information-bearing sequence of symbols $s(m)$, with power $P_S = E\{|s(m)|^2\}$, and then the sequence received at the destination is

$$y(m) = \sum_{i=1}^R t_i(m) * g_i(m) + v(m), \quad (4.1)$$

where $v(m)$ is the additive white Gaussian noise with power $\sigma_v^2 = E\{|v(m)|^2\}$. At the destination side, the received signal is amplified by a scalar amplifier $\alpha \in \mathbb{R}$ and output the sequence

$$z(m) = \alpha \cdot y(m) = \alpha \cdot s(m) * h_{eqv}(m) + \alpha \cdot n_{pro}(m) + \alpha \cdot v(m), \quad (4.2)$$

where

$$h_{eqv}(m) \triangleq \sum_{i=1}^R f_i(m) * h_i(m) * g_i(m) \quad (4.3)$$

denotes the impulse response of the equivalent channel from the source to the input of the destination node, and

$$n_{pro}(m) \triangleq \sum_{i=1}^R n_i(m) * h_i(m) * g_i(m) \quad (4.4)$$

is the propagating noise from the relay nodes. Moreover, (4.2) can be decomposed as

$$\begin{aligned} & z(m + \tau) \\ &= \alpha \cdot \sum_k h_{eqv}(m + \tau - k)s(k) + \alpha \cdot (n_{pro}(m + \tau) + v(m + \tau)) \\ &= \underbrace{\alpha \cdot h_{eqv}(\tau)s(m)}_{\text{Desired signal}} + \underbrace{\alpha \cdot \sum_{k \neq m} h_{eqv}(m + \tau - k)s(k)}_{\text{ISI}} \\ &\quad + \underbrace{\alpha \cdot (n_{pro}(m + \tau) + v(m + \tau))}_{\text{Noise}}. \end{aligned} \quad (4.5)$$

It is seen that the desired symbol $s(m)$ is separated from the ISI introduced by neighbouring symbols.

4.3 Output Power Minimization with Distortionless Constraint

4.3.1 Frequency-Domain Representation

In our signal model (4.5), the transmitted information symbols $s(m)$ are supposed to be mutually independent, and hence the source data sequence has a constant power spectrum: $S_s(\omega) = P_S$. Hence, according to (4.5) and following the derivations in Chapter 3,

the frequency-domain expression corresponding to (4.5) is obtained as

$$\begin{aligned}
 S_Z(\omega) = & \underbrace{\alpha^2 P_S \cdot |H_S(\omega)|^2}_{\text{Desired signal}} + \underbrace{\alpha^2 P_S \cdot |H_I(\omega)|^2 + \alpha^2 P_S \cdot H_c(\omega)}_{\text{Inter-symbol interferece}} \\
 & + \underbrace{\alpha^2 \sigma_n^2 \cdot \sum_{i=1}^R |H_i(\omega) \cdot G_i(\omega)|^2 + \alpha^2 \sigma_v^2}_{\text{Noise}}.
 \end{aligned} \tag{4.6}$$

where $S_Z(\omega)$ is the power spectrum of $z(m)$, and the notations involved are summarized as follows:

$$\begin{aligned}
 H_S(\omega) &\triangleq \mathbf{b}_{fgh}(\omega) \mathbf{M}_\tau \cdot \sum_{i=1}^R \boldsymbol{\Theta}_i \mathbf{h}_i, \\
 H_I(\omega) &\triangleq \mathbf{b}_{fgh}(\omega) \overline{\mathbf{M}}_\tau \cdot \sum_{i=1}^R \boldsymbol{\Theta}_i \mathbf{h}_i, \\
 H_c(\omega) &\triangleq 2 \cdot \Re e \left\{ \left(\sum_{i=1}^R \boldsymbol{\Theta}_i \mathbf{h}_i \right)^T \mathbf{M}_\tau \mathbf{b}_{fgh}^T(\omega) \mathbf{b}_{fgh}^*(\omega) \overline{\mathbf{M}}_\tau \left(\sum_{i=1}^R \boldsymbol{\Theta}_i \mathbf{h}_i \right)^* \right\}, \\
 \mathbf{b}_{fgh}(\omega) &\triangleq [1, e^{-j\omega \cdot 1}, \dots, e^{-j\omega \cdot (L_c-1)}] \in \mathbb{C}^{1 \times L_c}, \\
 \mathbf{M}_\tau &\triangleq \text{diag} \{ [\mathbf{0}_{1 \times \tau}, 1, \mathbf{0}_{1 \times (L_c-\tau-1)}] \}, \tau = 0, \dots, L_c - 1, \\
 \overline{\mathbf{M}}_\tau &\triangleq \mathbf{I}_{L_c} - \mathbf{M}_\tau,
 \end{aligned}$$

and $\boldsymbol{\Theta}_i$ is an $L_c \times L_h$ column-circulant matrix with $[b_{i,1}, \dots, b_{i,L_b}, \mathbf{0}_{1 \times (L_c-L_b)}]^T$ in the first column, whose structure can explicitly be shown in Section 3.7.2.

Since we have obtained the power spectra of the desired signal, ISI and noise components, then the average power of each component can be given by

$$P_{sig} = \mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w}, \quad P_{ISI} = \mathbf{w}^H \mathbf{Q}_i^{(\tau)} \mathbf{w}, \quad P_{noi} = \mathbf{w}^H \mathbf{Q}_n \mathbf{w},$$

where the notations have been defined in Chapter 3 are rewritten as follows:

$$\begin{aligned}
 \mathbf{Q}_s^{(\tau)} &\triangleq \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{Q}}_s^{(\omega_k, \tau)}, \\
 \mathbf{Q}_i^{(\tau)} &\triangleq \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{Q}}_i^{(\omega_k, \tau)}, \\
 \mathbf{Q}_n &\triangleq \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{Q}}_n^{(\omega_k)}, \\
 \tilde{\mathbf{Q}}_s^{(\omega, \tau)} &\triangleq P_S \cdot \boldsymbol{\Psi}^T \mathbf{M}_\tau \mathbf{b}_{fgh}^T(\omega) \mathbf{b}_{fgh}^*(\omega) \mathbf{M}_\tau \boldsymbol{\Psi}^*, \\
 \tilde{\mathbf{Q}}_i^{(\omega, \tau)} &\triangleq P_S \cdot \boldsymbol{\Psi}^T \overline{\mathbf{M}}_\tau \mathbf{b}_{fgh}^T(\omega) \mathbf{b}_{fgh}^*(\omega) \overline{\mathbf{M}}_\tau \boldsymbol{\Psi}^*, \\
 \tilde{\mathbf{Q}}_n^{(\omega)} &\triangleq \sigma_n^2 \cdot [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)]^H \cdot \mathbf{D}_g^{(\omega)} \cdot [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)],
 \end{aligned}$$

and $K \geq L_c$ is the number of DFT frequency points and $\omega_k = 2\pi(k-1)/K$.

4.3.2 Distortionless Constraint

The *distortion evaluation function* (DEF) for the equivalent channel $H_{eqv}(\omega)$ is defined in the frequency domain as the mean square error between $H_{eqv}(\omega)$ and the desired linear phase flat response, and it is expressed as

$$J(\mathbf{w}, \gamma, \tau, \alpha) \triangleq \frac{1}{K} \sum_{k=1}^K |\alpha H_{eqv}(\omega_k) - \gamma e^{-j\omega_k \tau}|^2, \quad (4.7)$$

where $\gamma > 0$ indicates the desired channel gain, and $\tau \in \{0, \dots, L_c - 1\}$ denotes the decision delay of the destination received sequence comparing to the source transmitted sequence. Also note that the scalar amplifier α is involved here.

Also following the derivations in Chapter 3, the DEF $J(\mathbf{w}, \gamma, \tau, \alpha)$ originally defined in (4.7) can be written in a quadratic form as

$$\begin{aligned}
 J(\mathbf{w}, \gamma, \tau, \alpha) &= \frac{1}{K} \sum_{k=1}^K \left(\alpha^2 \mathbf{w}^H \tilde{\mathbf{Q}}_0^{(\omega_k)} \mathbf{w} - 2\alpha\gamma \cdot \Re \{ \mathbf{w}^H \tilde{\mathbf{q}}_o^{(\omega_k)} e^{j\omega_k \tau} \} + \gamma^2 \right) \\
 &= \alpha^2 \mathbf{w}^H \mathbf{Q}_0 \mathbf{w} - 2\alpha\gamma \cdot \Re \{ \mathbf{w}^H \mathbf{q}_o(\tau) \} + \gamma^2,
 \end{aligned} \quad (4.8)$$

where

$$\begin{aligned}\mathbf{q}_o(\tau) &\triangleq \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{q}}_o^{(\omega_k)} e^{j\omega_k \cdot \tau}, \\ \mathbf{Q}_0 &\triangleq \frac{1}{K} \sum_{k=1}^K \tilde{\mathbf{q}}_o^{(\omega_k)} (\tilde{\mathbf{q}}_o^{(\omega_k)})^H, \\ \tilde{\mathbf{q}}_o^{(\omega)} &\triangleq [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)]^H \cdot \mathbf{d}_{fg}(\omega), \\ \mathbf{d}_{fg}(\omega) &\triangleq [F_1(\omega)G_1(\omega), \dots, F_R(\omega)G_R(\omega)]^T.\end{aligned}$$

are following the definition in Chapter 3.

Furthermore, as the previous definition, the distortionless constraint is formulated as

$$J(\mathbf{w}, \gamma, \tau, \alpha) = \alpha^2 \mathbf{w}^H \mathbf{Q}_0 \mathbf{w} - 2\alpha\gamma \cdot \Re \{ \mathbf{w}^H \mathbf{q}_o(\tau) \} + \gamma^2 = 0. \quad (4.9)$$

According to the discussion in Section 3.3.3, the distortionless constraint can be linearized as

$$\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{w} = \frac{\gamma}{\alpha} \mathbf{U}_1^H \mathbf{q}_o(\tau), \quad (4.10)$$

which is under the condition that $RL_h \geq L_c$.

4.3.3 Problem Formulation

Now we can proceed to present the problem formulation using the idea of output power minimization. First, the total received power at the input of the destination side is

$$P_{\text{Rx}} = \mathbf{w}^H (\mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \mathbf{w} + \sigma_v^2. \quad (4.11)$$

Thus the output power after the scalar amplifier is $\alpha^2 \cdot P_{\text{Rx}}$. Then, we can formulate a relay beamforming problem that aims at “minimizing the output power subject to distortionless constraint and power constraint”:

$$\begin{aligned}\min_{\mathbf{w}, \alpha} \quad & \alpha^2 \cdot \left(\mathbf{w}^H (\mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \mathbf{w} + \sigma_v^2 \right) \\ \text{s.t.} \quad & \mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{w} = \frac{\gamma}{\alpha} \mathbf{U}_1^H \mathbf{q}_o(\tau) \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.\end{aligned} \quad (4.12)$$

where γ is a system parameter that is pre-defined. Here, γ specifies the output signal power level.

Next, we will discuss the solution of this optimization problem.

4.4 Beamforming without Adaptive Scalar

As a special case, here we consider that the scalar at the destination is fixed, and without loss of generality, it is supposed to be $\alpha = 1$. Then α is no longer an optimization variable as in the original problem (4.12). Thus, the problem formulation turns to

$$\begin{aligned}
 \min_{\mathbf{w}} \quad & \mathbf{w}^H (\mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \mathbf{w} + \sigma_v^2 \\
 s.t. \quad & \Lambda_1 \mathbf{U}_1^H \mathbf{w} = \gamma \cdot \mathbf{U}_1^H \mathbf{q}_o(\tau) \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{4.13}$$

Furthermore, we know that the distortionless constraint gives rise to $\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} = (\gamma/\alpha)^2 P_s$ and $\mathbf{w}^H \mathbf{Q}_i^{(\tau)} \mathbf{w} = 0$, and therefore with these relation the above problem is rewritten as

$$\begin{aligned}
 \min_{\mathbf{w}} \quad & \gamma^2 P_s + \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_v^2 \\
 s.t. \quad & \Lambda_1 \mathbf{U}_1^H \mathbf{w} = \gamma \cdot \mathbf{U}_1^H \mathbf{q}_o(\tau) \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{4.14}$$

Note that the output signal power term $\gamma^2 P_s$ and receiver noise power σ_v^2 in the objective function are irrelevant to the optimization variable $\mathbf{w}$ and thus they can be omitted:

$$\begin{aligned}
 \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{Q}_n \mathbf{w} \\
 s.t. \quad & \Lambda_1 \mathbf{U}_1^H \mathbf{w} = \gamma \cdot \mathbf{U}_1^H \mathbf{q}_o(\tau) \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{4.15}$$

The interpretation for this formulation is: keeping a given constant gain (specified by γ) for the desired signal, and with the ISI being completely eliminated (due to the distortionless constraint), the output noise power is suppressed, which therefore leads to a way of obtaining good signal quality and thus good SNR.

It can be seen that as long as there is a proper value of parameter γ that simultaneously satisfies the distortionless constraint and the power constraint, the problem (4.15) is feasible. Hence, a problem arises that how to specify a “proper” γ , or whether there exists an optimal γ . Next we will address this issue on the possible value range of γ .

4.4.1 Feasible Region of the Constraints

Here we consider the constraints set:

$$\begin{cases} \mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{w} = \gamma \cdot \mathbf{U}_1^H \mathbf{q}_o(\tau) \\ \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0 \end{cases} \quad (4.16)$$

which consists of the distortionless constraint and the relay transmit power constraint. The problem now is to determine the possible value of γ that makes the set of constraints feasible. By performing a variable substitution $\hat{\mathbf{w}} = \mathbf{D}^{\frac{1}{2}} \mathbf{w}$, we obtain:

$$\begin{cases} (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{D}^{-\frac{1}{2}}) \hat{\mathbf{w}} = \gamma \cdot \mathbf{U}_1^H \mathbf{q}_o(\tau) \\ \|\hat{\mathbf{w}}\|_2^2 \leq P_0 \end{cases} \quad (4.17)$$

We start from the linear constraint, which is an under-determined linear equation as long as the condition $RL_h > L_c$ is satisfied. For simplicity of notation, define an $L_c \times RL_h$ matrix $\mathbf{A} = \mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{D}^{-\frac{1}{2}}$ that has full row-rank, and a vector $\mathbf{b} = \mathbf{U}_1^H \mathbf{q}_o(\tau)$. Then the linear constraint is written as

$$\mathbf{A} \hat{\mathbf{w}} = \gamma \cdot \mathbf{b}. \quad (4.18)$$

The pseudo-inverse of the full row-rank matrix $\mathbf{A}$ is calculated as

$$\mathbf{B} = \text{pinv}(\mathbf{A}) = \mathbf{A}^H (\mathbf{A} \mathbf{A}^H)^{-1}. \quad (4.19)$$

Therefore, the general solution to the under-determined linear equation $\mathbf{A}\hat{\mathbf{w}} = \gamma\mathbf{b}$ is expressed as

$$\hat{\mathbf{w}} = \gamma \cdot \mathbf{B}\mathbf{b} + (\mathbf{I}_{RL_h} - \mathbf{B}\mathbf{A})\mathbf{u}, \quad \forall \mathbf{u} \in \mathbb{C}^{RL_h}. \quad (4.20)$$

Now, take into account the second constraint in (4.17). The ℓ_2 -norm of $\hat{\mathbf{w}}$ can be given by

$$\begin{aligned} \|\hat{\mathbf{w}}\|_2^2 &= \gamma^2 \|\mathbf{B}\mathbf{b}\|_2^2 + \|(\mathbf{I} - \mathbf{B}\mathbf{A})\mathbf{u}\|_2^2 \\ &\quad + \gamma \cdot \mathbf{b}^H \mathbf{B}^H (\mathbf{I} - \mathbf{B}\mathbf{A})\mathbf{u} + \gamma \cdot \mathbf{u}^H (\mathbf{I} - \mathbf{A}^H \mathbf{B}^H) \mathbf{B}\mathbf{b}, \end{aligned} \quad (4.21)$$

By using (4.19), we can further obtain that $\mathbf{b}^H \mathbf{B}^H (\mathbf{I} - \mathbf{B}\mathbf{A})\mathbf{u} = 0$ and $\mathbf{u}^H (\mathbf{I} - \mathbf{A}^H \mathbf{B}^H) \mathbf{B}\mathbf{b} = 0$. Therefore, the ℓ_2 -norm of $\hat{\mathbf{w}}$ can be expressed as

$$\|\hat{\mathbf{w}}\|_2^2 = \gamma^2 \|\mathbf{B}\mathbf{b}\|_2^2 + \|(\mathbf{I} - \mathbf{B}\mathbf{A})\mathbf{u}\|_2^2, \quad \forall \mathbf{u} \in \mathbb{C}^{RL_h}. \quad (4.22)$$

Next we substitute the expression (4.22) for the norm of $\hat{\mathbf{w}}$ into the quadratic constraint in (4.17), and then we have

$$\gamma^2 \|\mathbf{B}\mathbf{b}\|_2^2 + \|(\mathbf{I} - \mathbf{B}\mathbf{A})\mathbf{u}\|_2^2 \leq P_0, \quad (4.23)$$

from which it is further obtained that

$$\gamma^2 \leq \frac{P_0 - \|(\mathbf{I} - \mathbf{B}\mathbf{A})\mathbf{u}\|_2^2}{\|\mathbf{B}\mathbf{b}\|_2^2}, \quad \forall \mathbf{u} \in \mathbb{C}^{RL_h}. \quad (4.24)$$

For an arbitrary $\mathbf{u} \in \mathbb{C}^{RL_h}$, its ℓ_2 -norm is obviously unbounded: $0 \leq \|\mathbf{u}\|_2^2 < +\infty$. Consequently, the norm term in the numerator of (4.24) can also be shown to be unbounded:

$$0 \leq \|(\mathbf{I} - \mathbf{B}\mathbf{A})\mathbf{u}\|_2^2 \leq \|\mathbf{I} - \mathbf{B}\mathbf{A}\|_2^2 \|\mathbf{u}\|_2^2 < +\infty. \quad (4.25)$$

Therefore, combining (4.24) and (4.25) as well as the fact that γ^2 being non-negative, we can obtain the possible range of γ :

$$0 \leq \gamma^2 \leq \frac{P_0}{\|\mathbf{B}\mathbf{b}\|_2^2}. \quad (4.26)$$

Hence, if the value of the system parameter γ satisfies the condition given by (4.26), the constraint set (4.16) is always feasible. That is, the non-negative parameter γ is shown to be upper-bounded, and the upper bound is related to the CSI of all the channels.

4.4.2 γ and Active Power Constraint

Having obtaining the upper-bound for non-negative parameter γ , we are further concerned with whether a lower-bound exists. In fact we find that the concern about lower-bound is related to whether the power constraint is active. To begin with, assuming that the value of γ is within the range given by (4.26), we have the Lagrangian associated with the problem (4.15) as follows:

$$\begin{aligned} \mathcal{L}(\mathbf{w}, \lambda, \mu) \\ = \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \lambda^T (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{w} - \gamma_0 \mathbf{U}_1^H \mathbf{q}_o(\tau)) + \mu (\mathbf{w}^H \mathbf{D} \mathbf{w} - P_0). \end{aligned} \quad (4.27)$$

Hence, the Karush-Kuhn-Tucker (KKT) conditions [98] are shown as

$$(\mathbf{Q}_n + \mu \mathbf{D}) \mathbf{w} + \mathbf{U}_1 \mathbf{\Lambda}_1^H \lambda = 0, \quad (4.28)$$

$$\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{w} = \gamma \mathbf{U}_1^H \mathbf{q}_o(\tau), \quad (4.29)$$

$$\mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0, \quad (4.30)$$

$$\mu \geq 0, \quad (4.31)$$

$$\mu (\mathbf{w}^H \mathbf{D} \mathbf{w} - P_0) = 0. \quad (4.32)$$

From (4.28) and (4.29) we can obtain that

$$\begin{aligned} \mathbf{w}(\tau, \mu) &= \gamma (\mathbf{Q}_n + \mu \mathbf{D})^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \\ &\times (\mathbf{\Lambda}_1 \mathbf{U}_1^H (\mathbf{Q}_n + \mu \mathbf{D})^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau). \end{aligned} \quad (4.33)$$

For simplicity of notation, we define a scalar-value function of μ

$$f(\mu) = \mathbf{w}^H(\tau, \mu) \mathbf{D} \mathbf{w}(\tau, \mu), \quad \mu \geq 0, \quad (4.34)$$

to denote the total relay transmit power, or the left-hand side of the power constraint in (4.15).

4.4.2.1 Power Constraint is Inactive

According to the complementary slackness condition (4.32), if the equality $\mu = 0$ holds it is required that $\mathbf{w}^H \mathbf{D} \mathbf{w} < P_0$, which means that the power constraint is not active. Hence,

$$f(0) = \mathbf{w}^H(\tau, 0) \mathbf{D} \mathbf{w}(\tau, 0) = \gamma^2 \cdot b(\tau) < P_0, \quad (4.35)$$

where

$$\begin{aligned} b(\tau) &= \mathbf{q}_o^H(\tau) \mathbf{U}_1 (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{Q}_n^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{Q}_n^{-1} \mathbf{D} \\ &\quad \times \mathbf{Q}_n^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{Q}_n^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau). \end{aligned}$$

According to (4.35), if the power constraint is inactive, we have $\gamma^2 < P_0/b(\tau)$. Therefore, on the contrary, a necessary condition for the power constraint to be active is

$$\gamma^2 \geq \frac{P_0}{b(\tau)}. \quad (4.36)$$

That is, if we want the power constraint to be active at the optimal solution, the parameter γ should be lower-bounded. This conclusion is discussed in more detail subsequently.

4.4.2.2 Power Constraint is Active

On the other hand, again from the complementary slackness condition (4.32), if $\mu > 0$ we have $\mathbf{w}^H \mathbf{D} \mathbf{w} = P_0$, which means the power constraint is active. To solve the optimization (4.15) with the active power constraint, now the problem is to substitute (4.33) into the power constraint to solve for multiplier μ . That is, we aim at solving for μ from

$$f(\mu) = \mathbf{w}^H(\tau, \mu) \mathbf{D} \mathbf{w}(\tau, \mu) = P_0. \quad (4.37)$$

Figure 4.2: Illustration of function $f(\mu)$.

Firstly, as a illustrative purpose, for a channel realization we plot the function $f(\mu)$ with $\mu \geq 0$ as in Figure 4.2.

There are two observations about the figure:

- (1) $f(\mu)$ is a monotonically decreasing function of μ when $\mu \geq 0$;
- (2) $f(\mu)$ is lower bounded.

Therefore, when $\mu \geq 0$, the maximum value of μ is

$$f_{upper} = f(0) = \mathbf{w}^H(\tau, 0) \mathbf{D} \mathbf{w}(\tau, 0) = \gamma^2 \cdot b(\tau), \quad (4.38)$$

where $b(\tau)$ is defined in (4.35). On the other hand, the asymptotic lower bound of $f(\mu)$ is

$$f_{lower} = f(\mu \rightarrow \infty) = \mathbf{w}^H(\tau, \mu \rightarrow \infty) \mathbf{D} \mathbf{w}(\tau, \mu \rightarrow \infty). \quad (4.39)$$

More specifically, we note that when $\mu \rightarrow \infty$, it can be obtained

$$(\mathbf{Q}_n + \mu \mathbf{D})^{-1} \approx \frac{1}{\mu} \mathbf{D}^{-1}. \quad (4.40)$$

With this result, and from (4.33) we can obtain

$$\mathbf{w}(\tau, \mu \rightarrow \infty) \approx \gamma \mathbf{D}^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{D}^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau). \quad (4.41)$$

Therefore, from (4.39) and (4.41) we have

$$f_{lower} \approx \gamma^2 \cdot a(\tau), \quad (4.42)$$

where $a(\tau) = \mathbf{q}_o^H(\tau) \mathbf{U}_1 (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{D}^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau)$.

Since we have obtained the upper and lower bound of $f(\mu)$ with $\mu \geq 0$, to ensure equation (4.37), $f(\mu) = P_0$, to have solution, it is required that

$$\gamma^2 \cdot a(\tau) = f_{lower} < P_0 \leq f_{upper} = \gamma^2 \cdot b(\tau), \quad (4.43)$$

or equivalently:

$$\frac{P_0}{b(\tau)} \leq \gamma^2 \leq \frac{P_0}{a(\tau)}. \quad (4.44)$$

Moreover, note in (4.26) that

$$\begin{aligned} & \|\mathbf{Bb}\|_2^2 \\ &= \left\| (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{D}^{-\frac{1}{2}})^H (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{D}^{-\frac{1}{2}} \mathbf{D}^{-\frac{1}{2}} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau) \right\|_2^2 \\ &= \mathbf{q}_o^H(\tau) \mathbf{U}_1 (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{D}^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau) = a(\tau). \end{aligned} \quad (4.45)$$

Thus, the upper bound for γ^2 given by (4.44) is consistent with the upper bound given by (4.26). On the other hand, the lower bound γ^2 given by (4.44) is consistent with (4.36).

4.4.3 Solving the Non-Adaptive Scalar Problem

Here we summarize the solution of problem (4.15). As discussed in the above subsections, the solution of optimal problem is given by (4.33), where the Lagrangian multiplier μ needs to be determined depending on whether the power constraint is active or not.

4.4.3.1 Power Constraint is Inactive

If the given value of parameter γ satisfies $0 < \gamma^2 < P_0/b(\tau)$, the power constraint is inactive, and then the Lagrangian multiplier in (4.33) is $\mu = 0$. Therefore, in this case the optimal solution is given by

$$\mathbf{w}_1 = \gamma \cdot \mathbf{Q}_n^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{Q}_n^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau). \quad (4.46)$$

Then the optimal value is

$$\mathbf{w}_1^H \mathbf{Q}_n \mathbf{w}_1 = \gamma^2 \cdot \mathbf{q}_o^H(\tau) \mathbf{U}_1 (\mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{Q}_n^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau). \quad (4.47)$$

We observe that the optimal value is increasing w.r.t. γ .

4.4.3.2 Power Constraint is Active

On the other hand, if γ satisfies $P_0/b(\tau) \leq \gamma^2 \leq P_0/a(\tau)$, the power constraint is active. Then we need to solve (4.37) for the Lagrangian multiplier μ . Since we have observed that $f(\mu)$ is a monotonically decreasing function w.r.t. μ , a simple bisection method can be used to search for the μ that holds the equation. The optimal solution is written as

$$\mathbf{w}_2 = \gamma \cdot \mathbf{v}(\tau, \mu(\gamma)) \quad (4.48)$$

where

$$\begin{aligned} \mathbf{v}(\tau, \mu(\gamma)) &= (\mathbf{Q}_n + \mu(\gamma) \mathbf{D})^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \\ &\times (\mathbf{\Lambda}_1 \mathbf{U}_1^H (\mathbf{Q}_n + \mu(\gamma) \mathbf{D})^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau), \end{aligned}$$

where the solved Lagrangian multiplier $\mu(\gamma)$ indicates that it is a function of parameter γ . The optimal value is

$$\mathbf{w}_2^H \mathbf{Q}_n \mathbf{w}_2 = \gamma^2 \cdot \mathbf{v}^H(\tau, \mu(\gamma)) \mathbf{Q}_n \mathbf{v}(\tau, \mu(\gamma)). \quad (4.49)$$

To demonstrate the optimal value of problem (4.15) with varying γ values, again for ease of illustration, we define another a function of μ :

$$g(\mu) = \mathbf{v}^H(\tau, \mu) \mathbf{Q}_n \mathbf{v}(\tau, \mu), \quad (4.50)$$

and we plot $g(\mu)$ with $\mu \geq 0$ as in Fig 4.3. It is seen that $g(\mu)$ is an increasing function of μ . Moreover, by observing (4.37) and Figure 4.2 we conclude that, for $P_0/b(\tau) \leq \gamma^2 \leq P_0/a(\tau)$, $\mu(\gamma)$ is also an increasing function of γ . Hence, we conclude that the compound function $g(\mu(\gamma))$ is an increasing function of γ . That is, similar to the inactive power constraint case, the optimal value $\mathbf{w}_2^H \mathbf{Q}_n \mathbf{w}_2 = \gamma^2 \cdot g(\mu(\gamma))$ is increasing w.r.t. γ .

Figure 4.3: Illustration of function $g(\mu)$.

Now we have solved problem (4.15) with a given value of parameter γ . However, a questions naturally arises is whether there exists an “optimal” value of γ in terms of certain quality of service (QoS) measurement. The next section will address this question.

4.5 Beamforming with Adaptive Scalar

Next, we consider directly solving the original problem formulation (4.12), where the relay beamforming weight $\mathbf{w}$ and the equalization scalar α are jointly optimized. We will provide a closed-form solution to it.

Again, due to the distortionless constraint the relations hold: $\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} = (\gamma/\alpha)^2 P_s$ and $\mathbf{w}^H \mathbf{Q}_i^{(\tau)} \mathbf{w} = 0$, and therefore the problem (4.12) is rewritten as

$$\begin{aligned} \min_{\mathbf{w}, \alpha} \quad & \gamma^2 \cdot P_s + \alpha^2 \cdot (\mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_v^2) \\ \text{s.t.} \quad & \mathbf{\Lambda}_1 \mathbf{U}_1^H \mathbf{w} = \frac{\gamma}{\alpha} \mathbf{U}_1^H \mathbf{q}_o(\tau) \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \end{aligned} \quad (4.51)$$

Since the signal power $\gamma^2 P_s$ is a constant term in the objective function, thus it can be omitted. Let $\hat{\mathbf{w}} = \alpha \cdot \mathbf{w}$, then the above optimization is equivalent to

$$\begin{aligned} \min_{\hat{\mathbf{w}}, \alpha} \quad & \hat{\mathbf{w}}^H \mathbf{Q}_n \hat{\mathbf{w}} + \alpha^2 \cdot \sigma_v^2 \\ \text{s.t.} \quad & \mathbf{\Lambda}_1 \mathbf{U}_1^H \hat{\mathbf{w}} = \gamma \cdot \mathbf{U}_1^H \mathbf{q}_o(\tau) \\ & \hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} \leq \alpha^2 \cdot P_0. \end{aligned} \quad (4.52)$$

This problem is quite similar to problem (3.45) in Chapter 3. Thus following the derivation procedure as previously discussed, we can obtain the optimal solution herein:

$$\begin{aligned} \hat{\mathbf{w}}_0(\gamma, \tau) &= \gamma \cdot \mathbf{v}(\tau) \\ \alpha_0(\gamma, \tau) &= \gamma \cdot \sqrt{\frac{\mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau)}{P_0}}, \end{aligned} \quad (4.53)$$

where

$$\begin{aligned} \mathbf{v}(\tau) &= \left(\left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right) \\ &\quad \times \left(\mathbf{\Lambda}_1 \mathbf{U}_1^H \left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau). \end{aligned}$$

Given the solution in (4.53) for $\hat{\mathbf{w}}$ and α as well as the definition of $\hat{\mathbf{w}} = \alpha \cdot \mathbf{w}$, the optimal weight of the original problem (4.51) is given by

$$\mathbf{w}_0(\tau) = \frac{\hat{\mathbf{w}}_0(\gamma, \tau)}{\alpha_0(\gamma, \tau)} = \sqrt{\frac{P_0}{\mathbf{v}^H(\tau)\mathbf{D}\mathbf{v}(\tau)}} \cdot \mathbf{v}(\tau). \quad (4.54)$$

Note that here the optimal weight $\mathbf{w}_0$ is irrelevant of parameter γ . That is to say, when the relay weight vector and the scalar amplifier are jointly optimized, the parameter γ does not affect the weight but controls the scalar at the destination side.

Moreover, the objective function of (4.51) with optimal solution can be given by

$$\begin{aligned} & \gamma^2 \cdot P_s + \alpha_0^2 \cdot (\mathbf{w}_0^H \mathbf{Q}_n \mathbf{w}_0 + \sigma_v^2) \\ &= \gamma^2 \cdot \left(\mathbf{v}^H(\tau) \left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right) \mathbf{v}(\tau) + P_s \right) \\ &= \gamma^2 \cdot \mathbf{q}_o^H(\tau) \mathbf{U}_1 \left(\mathbf{\Lambda}_1 \mathbf{U}_1^H \left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_1 \mathbf{\Lambda}_1^H \right)^{-1} \mathbf{U}_1^H \mathbf{q}_o(\tau) + \gamma^2 P_s. \end{aligned} \quad (4.55)$$

Note that it monotonically increases w.r.t. γ . On the other hand, the power of received signal before the scalar equalizer is obtained as

$$\begin{aligned} & \frac{\gamma^2}{\alpha^2} P_s + \mathbf{w}_0^H \mathbf{Q}_n \mathbf{w}_0 + \sigma_v^2 \\ &= \frac{\mathbf{v}^H(\tau) \left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right) \mathbf{v}(\tau) + P_s}{\mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau) / P_0}, \end{aligned} \quad (4.56)$$

which is irrelevant of γ . That is to say, the parameter γ does not affect the received signal power, but it can be used to control the scalar equalizer α (refer to (4.53)) and hence control the power level for the subsequent detection process, especially if the subsequent processors have requirement for an input dynamic range.

4.5.1 Equivalence to SINR Maximization Scheme

The adaptive-scalar scheme achieves the SINR performance of the Distortionless Maximization SINR scheme that is presented in Chapter 3, while further provides the ability to control the output signal power level.

In fact, by comparing the optimal solution (3.48) obtained by SINR maximization and the optimal solution here (4.54), we note that they are identical. However, by solving the output power minimization problem, a additional factor α is obtained which indicates the output signal power level.

4.5.2 Relation to Fixed Scalar Problem

As we have mentioned previously, the non-adaptive scalar problem that is covered in Section 4.4 is a special case of the adaptive scalar scheme with a fixed value of α .

Therefore, with a given value of α , using the relation between α and γ shown by (4.53), the “optimal” γ can be obtained as follows, which leads to the same SINR performance as given by the SINR maximization scheme under the distortionless constraint:

$$\gamma = \alpha \cdot \sqrt{\frac{P_0}{\mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau)}}. \quad (4.57)$$

Nevertheless, if the output signal power is required and the α is fixed, the analysis in Section 4.4 is still applicable.

4.6 Extension to Multi-Antenna Destination

As has been discussed in the previous chapter and so far in this chapter, in a relay network, the frequency-selective equivalent channel from the source to the destination can be equalized through the distributed beamforming. Thus the processing can also be regarded as distributed equalization. Recall that the additional equalizer at the destination side was considered in [17, 18], where the performance gain over the scheme without destination equalizer is demonstrated by the numerical simulation. However, therein single antenna is assumed at each receiver. In order to further exploit spatial diversity gain, destination receiver with multiple antennas is considered in [99] for frequency-flat

channels, where the received signals from each destination antenna are combined by a receive beamformer whose coefficients are jointly optimized with the distributed relay beamformer.

In this section, for relay beamforming design on the frequency-selective channels, we extend the output power minimization scheme to the case that multiple antennas are employed on the destination node. Here all the wireless channels between nodes of the relay network are supposed to be frequency-selective, and linear filtering processing is performed at both the relay side and the destination side. Therefore, following Chapter 3, we present a signal model that is more conveniently expressed in frequency domain. Based on the frequency-domain signal model, the beamforming processing at the filter-and-forward relays and at the destination node are jointly designed, which aims at optimizing the output SINR at the destination side. An iterative alternate optimization technique is utilized to calculate the beamforming coefficients of both the relay side and the destination side, and several different initialization schemes are discussed. Due to the additional system resource is employed, the performance gain over the single-antenna destination scenario is obtained and is shown by numerical simulation.

4.6.1 Multi-Antenna Destination with Receiving Filters

As depicted in Figure 4.4, we consider a relay network with one single-antenna source node, R single-antenna relay nodes and one multi-antenna destination node. As assumed in Chapter 3, the direct link from the source to the destination does not exist. The relay nodes also work in the TDD mode, and a complete signal transmission consists of two phases, namely, the broadcasting phase and the joint relay and destination beamforming phase. The backward channels and the forward channels are assumed to be frequency-selective and are modeled as FIR filters. Specifically, the backward channel corresponding to the i th relay is expressed as $\mathbf{f}_i = [f_i(0), \dots, f_i(L_f - 1)]^T$, where

Figure 4.4: Signal model of filter-and-forward relay network with multi-antenna destination.

$f_i(m)$'s, $m = 1, \dots, (L_f - 1)$, are the filter coefficients; the forward channel from the i th relay to the j th antenna of the destination is denoted as $\tilde{\mathbf{g}}_{ij} = [\tilde{g}_{ij}(0), \dots, \tilde{g}_{ij}(L_g - 1)]^T$, $i = 1, \dots, R, j = 1, \dots, N$. Throughout this chapter, we assume that the instantaneous CSI is perfectly known.

Similar to the single-antenna case, the received signal at the j th antenna of the destination node is given as

$$\begin{aligned} y_j(m) &= \sum_{i=1}^R t_i(m) * \tilde{g}_{i,j}(m) + v_j(m) \\ &= s(m) * \sum_{i=1}^R f_i(m) * h_i(m) * \tilde{g}_{i,j}(m) + \sum_{i=1}^R n_i(m) * h_i(m) * \tilde{g}_{i,j}(m) + v_j(m), \end{aligned} \quad (4.58)$$

where $t_i(m)$ is the signal transmitted by the i th relay, $n_i(m)$ is the additive white Gaussian noise at the i th relay with power $\sigma_n^2 = E\{|n_i(m)|^2\}$, and $v_j(m)$ is the additive white Gaussian noise at the j th receive antenna with power $\sigma_v^2 = E\{|v_j(m)|^2\}$. Subsequently, the received signal $y_j(m)$ at each of the destination antenna is processed by an FIR filter $\mathbf{a}_j = [a_j(0), \dots, a_j(L_a - 1)]^T$, respectively, and then they are added up and multiplied

by a scalar amplifier α , to produce the output signal:

$$\begin{aligned}\tilde{y}(m) &= \alpha \cdot \sum_{j=1}^N y_j(m) * a_j(m) \\ &= \alpha \cdot s(m) * \tilde{h}_{eqv}(m) + \alpha \cdot \sum_{i=1}^R n_i(m) * c_i(m) + \alpha \cdot \sum_{j=1}^N v_j(m) * a_j(m),\end{aligned}\quad (4.59)$$

where

$$\tilde{h}_{eqv}(m) \triangleq \sum_{i=1}^R f_i(m) * h_i(m) * \sum_{j=1}^N \tilde{g}_{ij}(m) * a_j(m)$$

denotes the impulse response of the equivalent channel from the source to the input of the destination amplifier α , and

$$c_i(m) \triangleq h_i(m) * \sum_{j=1}^N \tilde{g}_{ij}(m) * a_j(m),$$

is the equivalent channel from the input of the j th relay to the destination output. According to the definition of $\tilde{h}_{eqv}(m)$ and $c_i(m)$, it is obvious that $\tilde{h}_{eqv}(m) = \sum_{i=1}^R (f_i(m) * c_i(m))$.

Furthermore, the frequency response of $c_i(m)$ is given as

$$C_i(\omega) = \mathcal{F}\{c_i(m)\} = H_i(\omega) \sum_{j=1}^N \tilde{G}_{ij}(\omega) A_j(\omega), \quad (4.60)$$

where $\omega \in [0, 2\pi]$ is the angular frequency; $H_i(\omega) = \mathcal{F}\{h_i(m)\}$, $\tilde{G}_{ij}(\omega) = \mathcal{F}\{\tilde{g}_{ij}(m)\}$ and $A_j(\omega) = \mathcal{F}\{a_j(m)\}$. Similarly, we can obtain the frequency response of $\tilde{h}_{eqv}(m)$

$$\tilde{H}_{eqv}(\omega) = \mathcal{F}\{\tilde{h}_{eqv}(m)\} = \sum_{i=1}^R F_i(\omega) C_i(\omega) \quad (4.61)$$

where $F_i(\omega) = \mathcal{F}\{f_i(m)\}$. Therefore, the frequency-domain expression of (4.59) is obtained as

$$S_{\tilde{Y}}(\omega) = \alpha^2 P_S \left| \tilde{H}_{eqv}(\omega) \right|^2 + \alpha^2 \sigma_n^2 \sum_{i=1}^R |C_i(\omega)|^2 + \alpha^2 \sigma_v^2 \sum_{j=1}^N |A_j(\omega)|^2, \quad (4.62)$$

where $S_{\tilde{Y}}(\omega)$ is the power spectrum of $\tilde{y}(m)$.

4.6.2 Jointly Beamforming Design Problem

4.6.2.1 Distortion Evaluation Function

Here, comparing with the previous signal model for single-antenna destination, a scalar amplifier α is involved and the equivalent channel $\tilde{H}_{eqv}(\omega)$ has a new definition. Thus, with the notations

$$\mathbf{w} = [\mathbf{h}_1^H, \dots, \mathbf{h}_R^H]^T, \quad (4.63)$$

$$\mathbf{z} = [\mathbf{a}_1^H, \dots, \mathbf{a}_N^H]^T, \quad (4.64)$$

the definition of the *distortion evaluation function* (DEF) is modified as follows

$$J(\mathbf{w}, \mathbf{z}, \gamma, \tau, \alpha) \triangleq \frac{1}{K} \sum_{k=1}^K \left| \alpha \cdot \tilde{H}_{eqv}(\omega_k) - \gamma e^{-j\omega_k \tau} \right|^2, \quad (4.65)$$

where γ indicates the desired channel gain, and τ denotes the decision delay of the destination output sequence comparing to the source transmitted sequence.

To derive an expression for the DEF explicitly as a function of the relay coefficients $\mathbf{w}$ and destination coefficients $\mathbf{z}$, we can rewrite $\tilde{H}_{eqv}(\omega)$ as

$$\begin{aligned} \tilde{H}_{eqv}(\omega) &= \sum_{i=1}^R F_i(\omega) (\mathbf{b}_h(\omega) \mathbf{h}_i) \sum_{j=1}^N \tilde{G}_{ij}(\omega) (\mathbf{b}_a(\omega) \mathbf{a}_j) \\ &= \sum_{i=1}^R \left\{ F_i(\omega) \sum_{j=1}^N \tilde{G}_{ij}(\omega) (\mathbf{b}_a(\omega) \mathbf{a}_j) \right\} \mathbf{b}_h(\omega) \mathbf{h}_i. \end{aligned} \quad (4.66)$$

where $\mathbf{b}_a(\omega) = [1, e^{-j\omega \cdot 1}, \dots, e^{-j\omega \cdot (L_a - 1)}]$. We further define $\mathbf{d}_g^{(i)}(\omega) \triangleq [\tilde{G}_{i1}(\omega), \dots, \tilde{G}_{iN}(\omega)]^T$ and $\mathbf{u} \triangleq [\mathbf{b}_a(\omega) \mathbf{a}_1, \dots, \mathbf{b}_a(\omega) \mathbf{a}_N]^H$, and thus (4.66) can be further written as

$$\tilde{H}_{eqv}(\omega) = \sum_{i=1}^R \{ F_i(\omega) \mathbf{u}^H \mathbf{d}_g^{(i)}(\omega) \} \mathbf{b}_h(\omega) \mathbf{h}_i. \quad (4.67)$$

We also define $\mathbf{k} = [\mathbf{b}_h(\omega)\mathbf{h}_1, \dots, \mathbf{b}_h(\omega)\mathbf{h}_R]^H$ and $\mathbf{d}_f(\omega) \triangleq [F_1(\omega), \dots, F_R(\omega)]^T$, and then (4.67) can be further expressed as

$$\begin{aligned}\tilde{H}_{eqv}(\omega) &= \mathbf{k}^H \begin{bmatrix} F_1(\omega)\mathbf{u}^H \mathbf{d}_g^{(1)}(\omega) \\ \vdots \\ F_R(\omega)\mathbf{u}^H \mathbf{d}_g^{(R)}(\omega) \end{bmatrix} \\ &= \mathbf{k}^H \cdot \text{diag} \{ \mathbf{d}_f(\omega) \} \cdot [\mathbf{d}_g^{(1)}(\omega), \dots, \mathbf{d}_g^{(R)}(\omega)]^T \mathbf{u}^*. \end{aligned} \quad (4.68)$$

Note that, using *Kronecker product*, the vectors $\mathbf{u}$ and $\mathbf{k}$ can be written as $\mathbf{u} = [\mathbf{I}_N \otimes \mathbf{b}_a^*(\omega)] \mathbf{z}$ and $\mathbf{k} = [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)] \mathbf{w}$, respectively. Thus (4.68) can be finally written as

$$\tilde{H}_{eqv}(\omega) = \mathbf{w}^H \mathbf{D}_{fg}^{(\omega)} \mathbf{z}^*, \quad (4.69)$$

where

$$\mathbf{D}_{fg}^{(\omega)} = [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)]^H \cdot \text{diag} \{ \mathbf{d}_f(\omega) \} \cdot [\mathbf{d}_g^{(1)}(\omega), \dots, \mathbf{d}_g^{(R)}(\omega)]^T [\mathbf{I}_N \otimes \mathbf{b}_a^*(\omega)]^*.$$

Substitute (4.69) into (4.65), and the DEF can be written in a quadratic form of variable $\mathbf{w}$ as

$$J(\mathbf{w}, \mathbf{z}, \gamma, \tau, \alpha) = \alpha^2 \cdot \mathbf{w}^H \mathbf{Q}_h \mathbf{w} - 2\alpha\gamma \text{Re} \{ \mathbf{w}^H \mathbf{q}_h(\tau) \} + \gamma^2, \quad (4.70)$$

where

$$\begin{aligned}\mathbf{Q}_h &= \frac{1}{K} \sum_{k=1}^K \mathbf{D}_{fg}^{(\omega_k)} \mathbf{z}^* \mathbf{z}^T (\mathbf{D}_{fg}^{(\omega_k)})^H, \\ \mathbf{q}_h(\tau) &= \frac{1}{K} \sum_{k=1}^K e^{-j\omega_k \tau} \mathbf{D}_{fg}^{(\omega_k)} \mathbf{z}^*.\end{aligned}$$

For a given set of channel property parameters (γ, τ) , we are interested in the smallest achievable value of DEF with varying relay coefficients $\mathbf{w}$ and $\mathbf{z}$. Since in the multi-antenna destination case, the equivalent channel $\tilde{H}_{eqv}(\omega)$ is constituted by the backward and forward channels, the relay filters as well as the destination FIR filters, then the length of the equivalent channel can be shown to be $(L_f + L_g + L_h + L_a - 3)$. Therefore,

similar to the previous single-antenna scenario, we have the following conclusion about the minimum of DEF:

Given L_f , L_g , L_h and L_a as the filter lengths of $\mathbf{f}_i$, $\tilde{\mathbf{g}}_{i,j}$, $\mathbf{h}_i$ and $\mathbf{a}_j$, respectively, the minimum value of DEF is zero if the following two conditions hold: $RL_h \geq (L_f + L_g + L_h + L_a - 3)$ and $NL_t \geq (L_f + L_g + L_h + L_a - 3)$.

4.6.2.2 Expression of Output Power $P_{\text{out}}(\mathbf{w}, \mathbf{z})$

To formulate the output power minimization scheme in the multi-antenna destination scenario, first we will define the output power. Therefore, when the distortionless constraint holds, according to (4.62) and (4.65), the output power is defined as

$$\begin{aligned} P_{\text{out}}(\mathbf{w}, \mathbf{z}) &= \frac{\alpha^2}{K} \sum_{k=1}^K P_S \left| \frac{\gamma}{\alpha} e^{-j\omega_k \tau} \right|^2 + \frac{\alpha^2}{K} \sum_{k=1}^K \sigma_n^2 \sum_{i=1}^R |C_i(\omega_k)|^2 + \frac{\alpha^2}{K} \sum_{k=1}^K \sigma_v^2 \sum_{j=1}^N |A_j(\omega_k)|^2 \\ &= \gamma^2 P_S + \frac{\alpha^2 \sigma_n^2}{K} \sum_{k=1}^K \sum_{i=1}^R |C_i(\omega_k)|^2 + \frac{\alpha^2 \sigma_v^2}{K} \sum_{k=1}^K \sum_{j=1}^N |A_j(\omega_k)|^2 \\ &= \gamma^2 P_S + \alpha^2 \sigma_n^2 \mathbf{w}^H \mathbf{Q}_{n,h} \mathbf{w} + \alpha^2 \sigma_v^2 \mathbf{z}^H \mathbf{z}, \end{aligned} \quad (4.71)$$

where

$$\begin{aligned} \mathbf{Q}_{n,h} &= \frac{1}{K} \sum_{k=1}^K (\mathbf{Z}^{(\omega_k)})^H \mathbf{D}_g^{(\omega_k)} \mathbf{Z}^{(\omega_k)}, \\ \mathbf{D}_g^{(\omega)} &= \text{blkdiag} \left\{ \mathbf{d}_g^{(1)}(\omega) (\mathbf{d}_g^{(1)}(\omega))^H, \dots, \mathbf{d}_g^{(R)}(\omega) (\mathbf{d}_g^{(R)}(\omega))^H \right\}, \\ \mathbf{Z}^{(\omega)} &= [\mathbf{I}_R \otimes ([\mathbf{I}_N \otimes \mathbf{b}_a^*(\omega)] \mathbf{z})] [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega)]. \end{aligned}$$

4.6.2.3 Expression of Relay Transmit Power $P_R(\mathbf{w})$

Now we derive the expression of the total relay transmit power. To express P_R in terms of $\mathbf{w}$, we note that the power spectrum of the transmit signal by the i th relay is

$$S_{t,i}(\omega) = P_S \cdot |F_i(\omega) \cdot H_i(\omega)|^2 + \sigma_n^2 |H_i(\omega)|^2. \quad (4.72)$$

Along with the expression $H_i(\omega) = \mathbf{b}_h(\omega)\mathbf{h}_i$ defined previously, the total relay transmit power can be derived as:

$$\begin{aligned}
 P_R &= \sum_{i=1}^R \left(\frac{1}{K} \sum_{k=1}^K S_{t,i}(\omega_k) \right) \\
 &= \frac{1}{K} \sum_{k=1}^K \sum_{i=1}^R (P_s |F_i(\omega_k) \mathbf{b}_h(\omega_k) \mathbf{h}_i|^2 + \sigma_n^2 |\mathbf{b}_h(\omega_k) \mathbf{h}_i|^2) \\
 &= \frac{1}{K} \sum_{k=1}^K \mathbf{w}^H \left([\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega_k)]^H \mathbf{D}_f^{(\omega_k)} [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega_k)] \right) \mathbf{w}, \tag{4.73}
 \end{aligned}$$

where, $\mathbf{D}_f^{(\omega)} = P_s \cdot \text{diag}\{|F_1(\omega)|^2, \dots, |F_R(\omega)|^2\} + \sigma_n^2 \cdot \mathbf{I}_R$. Further denoting

$$\mathbf{D} = \frac{1}{K} \sum_{k=1}^K \left([\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega_k)]^H \mathbf{D}_f^{(\omega_k)} [\mathbf{I}_R \otimes \mathbf{b}_h^*(\omega_k)] \right),$$

then (4.73) can be finally expressed as a quadratic form of variable $\mathbf{w}$:

$$P_R(\mathbf{w}) = \mathbf{w}^H \mathbf{D} \mathbf{w}. \tag{4.74}$$

Now we are ready to formulate the relay beamforming problem.

4.6.2.4 Output Power Minimization with Distortionless Constraint

Similar to the single-antenna destination case, we propose a relay beamforming approach, where the output power at the destination side is minimized with a distortionless constraint as well as a total relay transmit power constraint:

$$\begin{cases} \min_{\mathbf{w}, \mathbf{z}, \gamma, \alpha} & \alpha^2 \cdot P_{\text{out}}(\mathbf{w}, \mathbf{z}) \\ \text{s.t.} & J(\mathbf{w}, \mathbf{z}, \gamma, \tau, \alpha) = 0 \\ & P_R(\mathbf{w}) \leq P_0. \end{cases} \tag{4.75}$$

Then, according to the definition give by (4.70), (4.71) and (4.74), this optimization problem (4.75) can be explicitly expressed as:

$$\begin{aligned}
 \min_{\mathbf{w}, \mathbf{z}, \gamma, \alpha} \quad & \gamma^2 P_s + \alpha^2 \sigma_n^2 \mathbf{w}^H \mathbf{Q}_{n,h} \mathbf{w} + \alpha^2 \sigma_v^2 \mathbf{z}^H \mathbf{z} \\
 \text{s.t.} \quad & \alpha^2 \mathbf{w}^H \mathbf{Q}_h \mathbf{w} - 2\alpha \gamma \text{Re} \{ \mathbf{w}^H \mathbf{q}_h(\tau) \} + \gamma^2 = 0 \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \tag{4.76}
 \end{aligned}$$

4.6.3 Alternate Algorithm

In this section, we use an alternate algorithm to give a suboptimal solution for the relay beamforming coefficients $\mathbf{w}$ and destination beamforming coefficients $\mathbf{z}$. It is shown that with either $\mathbf{w}$ or $\mathbf{z}$ being given, we can find a closed-form solution for the other variable. Therefore, $\mathbf{w}$ and $\mathbf{z}$ can be calculated alternately with an initial value.

4.6.3.1 Updating $\mathbf{w}$ and α given $\mathbf{z}$

Now we present a closed-form solution to the optimization problem with an initial value of $\mathbf{z}$. Firstly, similar to the treatment with the single-antenna destination case, here with given $\mathbf{z}$ the distortionless constraint in (4.76) can be recast as a linear equation

$$\mathbf{\Lambda}_{h1} \mathbf{U}_{h1}^H \mathbf{w} = \frac{\gamma}{\alpha} \mathbf{U}_{h1}^H \mathbf{q}_h(\tau), \quad (4.77)$$

where $\mathbf{\Lambda}_{h1}$ is a diagonal matrix with all the non-zero eigenvalues of $\mathbf{Q}_{n,h}$ on the diagonal in a descending order, and $\mathbf{U}_{h1}$ is the matrix containing the corresponding eigenvectors.

Therefore, the distortionless constraint in (4.76) can be replaced by the linear constraint shown in (4.77). Now proceed with the solution for $\mathbf{w}$ and α . Since $\gamma \neq 0$, we can define $\hat{\mathbf{w}} = \mathbf{w}/\gamma$. The solving procedure is similar to the single-antenna case, and the solution is given by

$$\begin{aligned} \hat{\mathbf{w}}_0 &= \gamma \cdot \mathbf{v} \\ \alpha_0 &= \gamma \cdot \sqrt{\frac{\mathbf{v}^H \mathbf{D} \mathbf{v}}{P_0}}, \end{aligned} \quad (4.78)$$

where

$$\begin{aligned} \mathbf{v} &= \left(\left(\sigma_n^2 \mathbf{Q}_{n,h} + \frac{\sigma_v^2 \mathbf{z}^H \mathbf{z}}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_{h1} \mathbf{\Lambda}_{h1}^H \right) \\ &\quad \times \left(\mathbf{\Lambda}_{h1} \mathbf{U}_{h1}^H \left(\sigma_n^2 \mathbf{Q}_{n,h} + \frac{\sigma_v^2 \mathbf{z}^H \mathbf{z}}{P_0} \mathbf{D} \right)^{-1} \mathbf{U}_{h1} \mathbf{\Lambda}_{h1}^H \right)^{-1} \mathbf{U}_{h1}^H \mathbf{q}_h(\tau). \end{aligned}$$

Therefore, with the definition of $\hat{\mathbf{w}} = \mathbf{w}/\gamma$, the optimal solution of the original problem (4.76) is given by

$$\mathbf{w}_0 = \gamma_0 \cdot \hat{\mathbf{w}}_0 = \sqrt{\frac{P_0}{\mathbf{v}^H \mathbf{D} \mathbf{v}}} \cdot \mathbf{v}. \quad (4.79)$$

4.6.3.2 Updating $\mathbf{z}$ given $\mathbf{w}$ and α

According to (4.70) we have

$$\mathbf{w}^H \mathbf{D}_{fg}^{(\omega)} \mathbf{z}^* = \mathbf{z}^H (\mathbf{D}_{fg}^{(\omega)})^T \mathbf{w}^*.$$

Then DEF in (4.70) can be rewritten in a quadratic form of variable $\mathbf{z}$ as

$$J(\mathbf{w}, \mathbf{z}, \gamma, \tau, \alpha) = \alpha^2 \mathbf{z}^H \mathbf{Q}_a \mathbf{z} - 2\alpha\gamma \text{Re} \{ \mathbf{z}^H \mathbf{q}_a(\tau) \} + \gamma^2, \quad (4.80)$$

where

$$\begin{aligned} \mathbf{Q}_a &= \frac{1}{K} \sum_{k=1}^K (\mathbf{D}_{fg}^{(\omega_k)})^T \mathbf{w}^* \mathbf{w}^T (\mathbf{D}_{fg}^{(\omega_k)})^*, \\ \mathbf{q}_a(\tau) &= \frac{1}{K} \sum_{k=1}^K e^{-j\omega_k \tau} (\mathbf{D}_{fg}^{(\omega_k)})^T \mathbf{w}^*. \end{aligned}$$

On the other hand, according to (4.71) we can also obtain the quadratic form of variable $\mathbf{z}$ as follows:

$$\begin{aligned} & \frac{\alpha^2 \sigma_n^2}{K} \sum_{k=1}^K \sum_{i=1}^R |C_i(\omega_k)|^2 + \frac{\alpha^2 \sigma_v^2}{K} \sum_{k=1}^K \sum_{j=1}^N |A_j(\omega_k)|^2 \\ &= \alpha^2 \cdot \mathbf{z}^H (\sigma_n^2 \mathbf{Q}_{n,a} + \sigma_v^2 \mathbf{I}_N) \mathbf{z}, \end{aligned} \quad (4.81)$$

where

$$\begin{aligned} \mathbf{Q}_{n,a} &= \frac{1}{K} \sum_{k=1}^K \mathbf{A}^H \{ \mathbf{I}_R \otimes [\mathbf{b}_h^T(\omega_k) \mathbf{b}_h^*(\omega_k)] \} \mathbf{A}, \\ \mathbf{A} &= \tilde{\mathbf{H}}^H (\hat{\mathbf{D}}_g^{(\omega)})^H [\mathbf{I}_N \otimes \mathbf{b}_a^*(\omega)], \\ \tilde{\mathbf{H}} &= \text{blkdiag} \{ \mathbf{h}_1, \dots, \mathbf{h}_R \}, \\ \hat{\mathbf{D}}_g^{(\omega)} &= [\mathbf{d}_g^{(1)}(\omega), \dots, \mathbf{d}_g^{(R)}(\omega)]. \end{aligned}$$

Therefore, with given $\mathbf{w}$ and α , the problem (4.76) can be expressed as a optimization with respect to $\mathbf{z}$:

$$\begin{aligned} \min_{\mathbf{z}} \quad & \alpha^2 \cdot \mathbf{z}^H (\sigma_n^2 \mathbf{Q}_{n-a} + \sigma_v^2 \mathbf{I}_N) \mathbf{z} \\ \text{s.t.} \quad & \alpha^2 \mathbf{z}^H \mathbf{Q}_a \mathbf{z} - 2\alpha \gamma \text{Re} \{ \mathbf{z}^H \mathbf{q}_a(\tau) \} + \gamma^2 = 0. \end{aligned} \quad (4.82)$$

where α is a constant factor in the objective function and thus can be omitted. Similar to the result of (4.77), the constraint in (4.82) is equivalent to a linear equation $\mathbf{\Lambda}_{a1} \mathbf{U}_{a1}^H \mathbf{z} = (\gamma/\alpha) \mathbf{U}_{a1}^H \mathbf{q}_a(\tau)$, where $\mathbf{\Lambda}_{a1}$ is a diagonal matrix with all the non-zero eigenvalues of $\mathbf{Q}_{n-a}$ on its diagonal, and $\mathbf{U}_{a1}$ is the matrix containing the corresponding eigenvectors. Thus (4.82) can be rewritten as

$$\begin{aligned} \min_{\mathbf{z}} \quad & \mathbf{z}^H (\sigma_n^2 \mathbf{Q}_{n-a} + \sigma_v^2 \mathbf{I}_N) \mathbf{z} \\ \text{s.t.} \quad & \mathbf{\Lambda}_{a1} \mathbf{U}_{a1}^H \mathbf{z} = \frac{\gamma}{\alpha} \mathbf{U}_{a1}^H \mathbf{q}_a(\tau). \end{aligned} \quad (4.83)$$

The optimal solution is obtained by using Lagrangian multiplier method as

$$\begin{aligned} \mathbf{z}_0 = \frac{\gamma}{\alpha} \bigg((\sigma_n^2 \mathbf{Q}_{n-a} + \sigma_v^2 \mathbf{I}_N)^{-1} \mathbf{U}_{a1} \mathbf{\Lambda}_{a1}^H \bigg) \\ \times \left(\mathbf{\Lambda}_{a1} \mathbf{U}_{a1}^H (\sigma_n^2 \mathbf{Q}_{n-a} + \sigma_v^2 \mathbf{I}_N)^{-1} \mathbf{U}_{a1} \mathbf{\Lambda}_{a1}^H \right)^{-1} \mathbf{U}_{a1}^H \mathbf{q}_a(\tau). \end{aligned}$$

Now that the updating of $\mathbf{z}$ and $\mathbf{w}$ can be proceeded iteratively until the change between two successive iterations is smaller than a error tolerance.

4.7 Numerical Simulation

In the simulation, the performance of the proposed adaptive delay approach is investigated. We consider a network where the number of relays is $R = 10$ (unless otherwise specified) relays. Throughout this section, the relay noise power and destination noise power are assumed to be $\sigma_n^2 = \sigma_v^2 = 1$, and the source power P_s is 10dB higher than the

noise power. The coefficients of channel impulse responses are modeled as independent quasi-static Rayleigh fading, and are hence generated as zero-mean complex Gaussian random variables with the exponential power delay profile [97]

$$p(m) = \frac{1}{\sigma_t} \sum_{l=0}^{L_x-1} e^{-m/\sigma_t} \delta(m-l), \quad (4.84)$$

where σ_t characterizes the delay spread and here $\sigma_t = 2$ is assumed, and $L_x \in \{L_f, L_g\}$ denotes the length of the backward channels or the forward channels. Throughout the simulations, $L_f = L_g = 5$, and $L_h = 5$ unless otherwise specified. All the channels are assumed to be quasi-static and perfectly known by the algorithm, which means that the channels are randomly generated but remain static over a frame time, during which multiple successive symbols are transmitted. The algorithm calculates the relay weights at the beginning of each frame based on the known CSI (a channel realization), and the same set of weights are used during this frame period.

4.7.1 Fixed α scheme

In Fig. 4.5, Fig. 4.6 and Fig. 4.7, we display the performance of output power minimization scheme with fixed scalar α . Since we have provided the value range of parameter γ with fixed α , the experiment results are illustrated with γ being the lower bound value and the upper bound value. Fig. 4.5 shows the output SINR performance, from which we can observe that, using either upper bound γ or lower bound γ does not lead to consistently better performance than that obtained by the other. It is also illustrated that the decision delay largely affects the performance.

The output power performance given by the proposed beamforming scheme with fixed scalar is illustrated by Fig. 4.6 and Fig. 4.7, and are with respect to different decision delays. We note that for delay values $\tau = 0$ and $\tau = 4$, the output noise power is not

Figure 4.5: Performance of beamforming scheme with fixed scalar: output SINR versus total relay power constraint P_0 . $\alpha = 1$.

significantly affected but the output signal power is influenced. This observation accounts for the different SINR performance with different delays.

4.7.2 Adaptive α scheme

We have shown that the output power minimization scheme with adaptive scalar renders the same output SINR performance as the SINR maximization scheme considered in Chapter 3. Thus the experiments here focus on the output power behavior. Fig. 4.8 displays the output noise power with γ fixed to be 1. Since γ is fixed, then the output signal power is fixed to be $\gamma^2 P_s$. Thus the varying noise power gives rise to the varying output SINR.

4.7.3 Adaptive γ scheme with α fixed

In (4.57) we have shown that, given a fixed value of α , we can also calculate a corresponding γ which gives rise to the same output SINR as given by the SINR maximization

Figure 4.6: Performance of beamforming scheme with fixed scalar: output signal/noise power versus total relay power constraint P_0 . $\alpha = 1$, decision delay $\tau = 0$.

Figure 4.7: Performance of beamforming scheme with fixed scalar: output signal/noise power versus total power constraint P_0 . $\alpha = 1$, decision delay $\tau = 4$.

Figure 4.8: Performance of beamforming scheme with adaptive scalar: output noise power versus total relay power constraint P_0 , with different decision delays τ . $\gamma = 1$.

scheme with distortionless design. Hence, this can be regarded as adaptive γ scheme with α fixed. Fig. 4.9, Fig. 4.10 and Fig. 4.11 display the performance of beamforming with and without adaptive γ . In Fig. 4.9, the output SINR performance with respect to total power constraint is illustrated. With the scalar α fixed to be 1, the effects of different pre-given γ value is shown for the fixed scalar scheme. The value of γ is chosen as either the lower or upper bound that is given by (4.44). We can see that, the output SINR performance of the adaptive γ scheme always outperforms the fixed γ case. More specifically, as discussed in Section 4.5, the output SINR performance of the adaptive γ scheme does not affected by the value of α . However, when the power budget P_0 is small, the output SINR performance of fixed γ scheme for given upper bound γ is close to the performance of the adaptive γ scheme. On the other hand, when the power budget is relatively large (here, $P_0 > 20dB$), the performance of fixed γ scheme for given lower bound γ is close to that of the adaptive γ scheme.

Figure 4.9: For fixed $\alpha = 1$, performance comparison of beamforming scheme with adaptive γ and fixed γ : output SINR versus total relay power constraint P_0 . Decision delay $\tau = 4$.

Figure 4.10: Illustration of upper and lower bound γ of the fixed scalar scheme, averaged by multiple channel realizations, and compared with the “optimal” γ obtained by adaptive γ scheme. Scalar amplifier $\alpha = 1$.

Figure 4.11: For fixed $\alpha = 1$, performance comparison of beamforming scheme with adaptive γ and fixed γ : output noise power versus total relay power constraint P_0 . Decision delay $\tau = 4$.

In Figure 4.10, with $\alpha = 1$, by averaging over multiple channel realizations, the averaged upper and lower bound γ for fixed γ scheme and the “optimal” γ given by the adaptive γ scheme (calculated using (4.57) with fixed $\alpha = 1$) are illustrated. From this experiment result we can see that, the “optimal” value of γ which renders the best output SINR performance in deed lies in the upper and lower bound we have derived and given in (4.44). The same trend as in the previous experiment is also observed: when the power budget P_0 is small, the upper bound γ is close to the “optimal” γ given by the adaptive γ scheme. When the power budget is relatively large (here, $P_0 > 20dB$), the lower bound γ is close to the “optimal” γ .

From problem (4.14) and (4.52), it is noted that for a given value of γ , the output noise power reflects the optimal value of the original output power minimization problem. Specifically, the output interference power is eliminated due to the distortionless constraint and the output desired signal power maintained at a constant level, and there-

Figure 4.12: Relay beamforming with multi-antenna destination: output SINR performance versus total relay transmit power budget P_0 ; $L_f = L_g = L_t = 5$, $L_h = 2$, $N = 4$.

fore, the output noise power is the crucial component of the total output power. Thus, the output noise power is displayed in Figure 4.11 for both the fixed γ scheme and the adaptive γ scheme. With α fixed to be 1, γ of the adaptive γ scheme is determined by (4.57).

4.7.4 Multi-Antenna at Destination

In Figure 4.12 and Figure 4.13, the performance of multi-antenna destination joint beamforming is shown, where different setting of antenna number is considered. It is observed that, by employing multi-antenna as well as FIR filters at the destination side, the system performance is superior to the traditional single-antenna destination case. Moreover, for the above two signal model scenarios, the corresponding performances with respect to the delay decision τ are shown by Figure 4.14 and Figure 4.15. It is observed that the multi-antenna destination scenario performs more robust to the different decision delays

Figure 4.13: Relay beamforming with multi-antenna destination: output SINR performance versus total relay transmit power budget P_0 ; $L_f = L_g = L_h = L_a = 5$, $N = 8$.

than the single-antenna case. This is because, with the introducing of the destination FIR filters, the total filter length of controllable filters are increased, comparing to the single-antenna destination case where only the relay nodes are deployed with controllable FIR filters.

4.8 Conclusion

In this chapter, we discussed designing the filter-and-forward (FF) relay beamforming with an alternative approach, which aims to minimize the destination output power subject to a distortionless response constraint. By introducing a scalar equalizer at the destination side, we considered the relay beamforming problem using the new scheme in two cases. We first considered the scalar is fixed and a given signal gain level is required for the desired signal. We derived a range for the selection of signal gain in order to make it consistent with the relay transmit power constraint. We then considered the general

Figure 4.14: Relay beamforming with multi-antenna destination: output SINR performance versus decision delay γ ; $L_f = L_g = L_a = 5, L_h = 2, N = 4$.

Figure 4.15: Relay beamforming with multi-antenna destination: output SINR performance versus decision delay γ ; $L_f = L_g = L_h = L_a = 5, N = 8$.

case where both the relay weight and the scalar amplifier are jointly optimized to achieve the minimum output power, and also with a signal gain requirement. We showed that the adaptive scalar scheme always achieves the SINR performance that is given by SINR maximization scheme with distortionless response design.

Chapter 5

An Alternative Time-Domain Approach using Power Minimization for Attaining Optimal SINR

5.1 Introduction

In the previous chapter, we present the output minimization scheme for relay beamforming. Therein the problem is modeled, formulated and solved using the frequency-domain method. What is more, the idea of *distortionless evaluation function* (DEF) and the subsequent *distortionless constraint* are represented in the frequency domain. We also demonstrate that the distortionless output power minimization scheme is equivalent to the distortionless SINR maximization scheme in terms of achieving the same output SINR performance.

Motivated by the above mentioned results obtained by the frequency-domain description, in this chapter, we generalize the output power minimization scheme to the time domain. It is found the time-domain counterpart of network model description and relay beamforming design are equivalent to that in the frequency domain, and more over, gives rise to easier expression. Additionally, inspired by the equivalence between the output power minimization and SINR maximization schemes subject to the distortionless con-

straint, we formulate a new output power minimization scheme which is shown to be equivalent to the SINR maximization scheme without the distortionless constraint.

Specifically, in this chapter, based on the output power minimization scheme, three relay beamforming design problems are considered, which are with different scenarios of relay transmit power constraint. The first scheme minimizes the output power while keeping the power of the desired signal at a constant level, and subject to a total relay transmit power constraint. It is shown that, in terms of output SINR, this scheme is equivalent to the SINR maximization formulation. The second scheme also minimizes the output power with the desired signal power maintains a constant level, but subject to both total and individual power constraint. It is shown that this scheme is equivalent to the SINR maximization. However, the proposed approach requires lower computational complexity in solving the problem. The third scheme minimizes the output power under the constraint that the equivalent channel from the source to the destination is flat and linear phase. Note that this constraint is a time-domain counterpart of the distortionless constraint that has been discussed within the frequency domain by the previous chapters. Baring this property, we can efficiently select the optimal decision delay.

The organization of this chapter is as follows. In Section 5.2, the signal model is described and the inter-symbol interference is defined. In Section 5.3, FF Beamforming with Total Relay Power Constraint is discussed. Section 5.4 discusses FF Beamforming with Individual Relay Power Constraint. Section 5.5 discusses FF Beamforming with Adaptive Decision Delay. Simulation results are provided in Section 5.6, and Section 5.7 concludes this chapter.

Figure 5.1: Signal model of filter-and-forward relay network in frequency-selective channels.

5.2 Signal Model

As depicted in Figure 5.1, we follow the signal model as in Section 4.2 of Chapter 4, and here only the single-antenna destination scenario is considered. As defined in the previous chapters, the backward channel and forward channel that are corresponding to the i th relay are denoted by $\mathbf{f}_i = [f_i(0), \dots, f_i(L_f - 1)]^T$ and $\mathbf{g}_i = [g_i(0), \dots, g_i(L_g - 1)]^T$, respectively. All the instantaneous CSI are supposed to be perfectly known. The i th relay filter is denoted as $\mathbf{h}_i = [h_i(0), \dots, h_i(L_h - 1)]^T$.

Therefore, following the expression in Section 4.2 of Chapter 4, at the destination side the output signal that has passed the scalar amplifier $\alpha \in \mathbb{R}$ is expressed as

$$\begin{aligned} z(m) &= \alpha \cdot y(m) \\ &= \alpha \cdot s(m) * h_{eqv}(m) + \alpha \cdot n_{pro}(m) + \alpha \cdot v(m), \end{aligned} \quad (5.1)$$

where

$$h_{eqv}(m) \triangleq \sum_{i=1}^R f_i(m) * h_i(m) * g_i(m) \quad (5.2)$$

denotes the impulse response of the equivalent channel from the source to the input of the destination node, and

$$n_{pro}(m) \triangleq \sum_{i=1}^R n_i(m) * h_i(m) * g_i(m) \quad (5.3)$$

is the propagating noise from the relay nodes.

Next, for better illustration and facilitating the subsequent problem formulation, we rewrite the overall time-domain signal model (5.1) in matrix form.

To start with, by represent the result of the convolution $f_i(m) * g_i(m)$ as a column vector $\mathbf{b}_i = \mathbf{f}_i * \mathbf{g}_i = [b_{i,1}, \dots, b_{i,L_b}]^T$, where $L_b = (L_f + L_g - 1)$. The equivalent channel $h_{eqv}(m)$ can be further written in matrix form as

$$\mathbf{h}_{eqv} = \sum_{i=1}^R \Theta_i \mathbf{h}_i = \Psi \mathbf{w}^*, \quad (5.4)$$

where $\Psi = [\Theta_1, \dots, \Theta_R]$, $\mathbf{w} = [\mathbf{h}_1^H, \dots, \mathbf{h}_R^H]^T$, and

$$\Theta_i = \begin{bmatrix} b_{i,1} & 0 & & 0 \\ \vdots & b_{i,1} & \ddots & \vdots \\ b_{i,L_b} & \vdots & & \vdots \\ 0 & b_{i,L_b} & \ddots & 0 \\ \vdots & 0 & & b_{i,1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & b_{i,L_b} \end{bmatrix} \quad (5.5)$$

is a column-circulant matrix with dimension $\mathbb{C}^{L_c \times L_h}$. Therefore, the first term in (5.1) can be rewritten as

$$s(m) * h_{eqv}(m) = \mathbf{h}_{eqv}^T \mathbf{s}(m) = \mathbf{w}^H \Psi^T \mathbf{s}(m), \quad (5.6)$$

where $\mathbf{s}(m) = [s(m), s(m-1), \dots, s(m-L_c+1)]^T$.

The propagation noise defined in (5.3) can be expressed in matrix form as

$$n_{pro}(m) = \sum_{i=1}^R (\mathbf{G}_i \mathbf{h}_i)^T \mathbf{n}_i(m), \quad (5.7)$$

where the $(L_g + L_h - 1) \times L_h$ column-circulant matrix is defined as

$$\mathbf{G}_i = \begin{bmatrix} g_i(0) & 0 & & 0 \\ \vdots & g_i(0) & \ddots & \vdots \\ g_i(L_g - 1) & \vdots & & \vdots \\ 0 & g_i(L_g - 1) & \ddots & 0 \\ \vdots & 0 & & g_i(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & g_i(L_g - 1) \end{bmatrix}, \quad (5.8)$$

and $\mathbf{n}_i = [n_i(m), n_i(m - 1), \dots, n_i(m - L_g - L_h + 2)]^T$.

Then, from (5.6) and (5.7), we have

$$z(m) = \alpha \cdot \mathbf{w}^H \mathbf{\Psi}^T \mathbf{s}(m) + \alpha \cdot \sum_{i=1}^R (\mathbf{G}_i \mathbf{h}_i)^T \mathbf{n}_i(m) + \alpha \cdot v(m). \quad (5.9)$$

As seen in the previous Chapters, the ISI can also be represented. Suppose that at the time instance (m) , we want to estimate the transmitted symbol $s(m - \tau)$ from $z(m)$, and τ denotes the decision delay. By separating the $(\tau + 1)$ th column from the matrix $\mathbf{\Psi}^T$, we can further decompose (5.9) as:

$$\begin{aligned} z(m) = & \underbrace{\alpha \cdot \mathbf{w}^H \vec{\psi}_\tau^T s(m - \tau)}_{\text{Desired signal}} + \underbrace{\alpha \cdot \mathbf{w}^H \overline{\mathbf{\Psi}}_\tau^T \bar{\mathbf{s}}_\tau(m)}_{\text{ISI}} \\ & + \underbrace{\alpha \cdot \sum_{i=1}^R (\mathbf{G}_i \mathbf{h}_i)^T \mathbf{n}_i(m)}_{\text{Noise}} + \alpha \cdot v(m), \end{aligned} \quad (5.10)$$

where $\vec{\psi}_\tau$ is the $(\tau + 1)$ th row of $\mathbf{\Psi}$, and hence $\overline{\mathbf{\Psi}}_\tau$ is the submatrix of $\mathbf{\Psi}$ with removing the $(\tau + 1)$ th row, and $\bar{\mathbf{s}}_\tau(m) = [s(m), \dots, s(m - \tau + 1), s(m - \tau - 1), \dots, s(m - L_c + 1)]^T$. Therefore, we see that the desired symbol $s(m)$ is separated from the ISI introduced by neighbouring symbols.

5.3 FF Beamforming with Total Relay Power Constraint

5.3.1 Problem Formulation

In this section, we will present the formulation of output power minimization with constraint on the total relay transmit power. Then firstly, we derive the expression for the transmit power of each relay node. According to the network signal model, the transmit signal from the i th relay is expressed as

$$t_i(m) = s(m) * f_i(m) * h_i(m) + n_i(m) * h_i(m). \quad (5.11)$$

It can be further expressed in matrix form as:

$$t_i(m) = (\mathbf{F}_i \mathbf{h}_i)^T \tilde{\mathbf{s}}(m) + \mathbf{h}_i^T \tilde{\mathbf{n}}_i(m), \quad (5.12)$$

where

$$\mathbf{F}_i = \begin{bmatrix} f_i(0) & 0 & & 0 \\ \vdots & f_i(0) & \ddots & \vdots \\ f_i(L_f - 1) & \vdots & & \vdots \\ 0 & f_i(L_f - 1) & \ddots & 0 \\ \vdots & 0 & & f_i(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & f_i(L_f - 1) \end{bmatrix}, \quad (5.13)$$

is an $(L_f + L_h - 1) \times L_h$ column-circulant matrix, $\tilde{\mathbf{s}}(m) = [s(m), s(m-1), \dots, s(m-L_f-L_h+2)]^T$, and $\tilde{\mathbf{n}}_i(m) = [n_i(m), n_i(m-1), \dots, n_i(m-L_h+1)]^T$. Therefore, the total relay transmit power is given by

$$\begin{aligned} P_{Tot} &= \sum_{i=1}^R E|t_i(m)|^2 \\ &= \sum_{i=1}^R \mathbf{h}_i^T (P_s \cdot \mathbf{F}_i^T \mathbf{F}_i^* + \sigma_n^2 \cdot \mathbf{I}_{L_h}) \mathbf{h}_i^* \\ &= [\mathbf{h}_1^T, \dots, \mathbf{h}_R^T] \cdot (\tilde{\mathbf{F}} + \sigma_n^2 \cdot \mathbf{I}_{RL_h}) \cdot [\mathbf{h}_1^T, \dots, \mathbf{h}_R^T]^H \\ &= \mathbf{w}^H \mathbf{D} \mathbf{w} \end{aligned} \quad (5.14)$$

where $\tilde{\mathbf{F}} = P_s \cdot \text{blkdiag}\{\mathbf{F}_1^T \mathbf{F}_1^*, \dots, \mathbf{F}_R^T \mathbf{F}_R^*\}$, and $\mathbf{D} = \tilde{\mathbf{F}} + \sigma_n^2 \cdot \mathbf{I}_{RL_h}$.

On the other hand, according to the expression for output signal decomposition (5.10), the output power at the destination side is given by

$$E|z(m)|^2 = \alpha^2 \cdot \mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} + \alpha^2 \cdot \mathbf{w}^H \mathbf{Q}_i^{(\tau)} \mathbf{w} + \alpha^2 \cdot \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \alpha^2 \sigma_v^2, \quad (5.15)$$

where

$$\begin{aligned} \mathbf{Q}_s^{(\tau)} &= P_s \cdot \vec{\psi}_{\tau+1}^T \vec{\psi}_{\tau+1}^*, \\ \mathbf{Q}_i^{(\tau)} &= P_s \cdot \bar{\Psi}_{\tau+1}^T \bar{\Psi}_{\tau+1}^*, \\ \mathbf{Q}_n &= \sigma_n^2 \cdot \text{blkdiag}\{\mathbf{G}_1^T \mathbf{G}_1^*, \dots, \mathbf{G}_R^T \mathbf{G}_R^*\}. \end{aligned} \quad (5.16)$$

Hence, the output power minimization problem is formulated as

$$\begin{aligned} \min_{\mathbf{w}, \alpha} \quad & \alpha^2 \cdot \mathbf{w}^H \left(\mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n \right) \mathbf{w} + \alpha^2 \sigma_v^2 \\ \text{s.t.} \quad & \alpha \cdot \mathbf{w}^H \vec{\psi}_{\tau+1}^T = \gamma \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0, \end{aligned} \quad (5.17)$$

where γ is a pre-defined system parameter, which indicates the power level of the output signal. Thus, without loss of generality, γ can be set as a real number. This problem formulation is inspired by the distortionless output power minimization discussed in Chapter 4. Here we note that, the distortionless constraint does not appear, and instead, another linear constraint is imposed. Then the equivalent channel is not forced to be flat and linear phase, but we will subsequently show that this formulation gives rise to a relay beamformer that achieves the optimal output SINR, although the equivalent channel is not distortionless.

5.3.2 Problem Solution

Now we provide a closed-form solution of problem (5.17). From the linear constraint $\alpha \cdot \mathbf{w}^H \vec{\psi}_{\tau+1}^T = \gamma$ and the definition of $\mathbf{Q}_s$ in (5.15), we can obtain

$$\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} = P_s \cdot \mathbf{w}^H \vec{\psi}_{\tau+1}^T \vec{\psi}_{\tau+1}^* \mathbf{w} = P_s \cdot \left| \mathbf{w}^H \vec{\psi}_{\tau+1}^T \right|^2 = \frac{\gamma^2 P_s}{\alpha^2}. \quad (5.18)$$

Therefore, substitute this result into (5.17) and also perform a variable substitution $\hat{\mathbf{w}} = \alpha \cdot \mathbf{w}$. Then the optimization can be rewritten as

$$\begin{aligned} \min_{\mathbf{w}, \alpha} \quad & \hat{\mathbf{w}}^H \left(\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n \right) \hat{\mathbf{w}} + \alpha^2 \sigma_v^2 \\ \text{s.t.} \quad & \hat{\mathbf{w}}^H \vec{\psi}_{\tau+1}^T = \gamma \\ & \hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} \leq \alpha^2 P_0. \end{aligned} \quad (5.19)$$

Now it is noted that this problem is similar to problem (4.52) in Chapter 4, where the linear constraint (linearized version of distortionless constraint) is slightly different but the solving procedure is quite similar. Therefore, following the same solving method we directly give the optimal solution as

$$\begin{aligned} \hat{\mathbf{w}}_0(\gamma, \tau) &= \gamma \cdot \mathbf{v}(\tau) \\ \alpha_0(\gamma, \tau) &= \gamma \cdot \sqrt{\mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau) / P_0}, \end{aligned} \quad (5.20)$$

where

$$\mathbf{v}(\tau) = \frac{(\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n + (\sigma_v^2 / P_0) \mathbf{D})^{-1} \vec{\psi}_{\tau+1}^T}{\vec{\psi}_{\tau+1}^* (\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n + (\sigma_v^2 / P_0) \mathbf{D})^{-1} \vec{\psi}_{\tau+1}^T}.$$

Given the solution in (5.20) for $\hat{\mathbf{w}}$ and α as well as the definition of $\hat{\mathbf{w}} = \alpha \cdot \mathbf{w}$, the optimal weight of the problem (5.17) is given by

$$\mathbf{w}_0(\tau) = \frac{\hat{\mathbf{w}}_0(\gamma, \tau)}{\alpha_0(\gamma, \tau)} = \sqrt{\frac{P_0}{\mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau)}} \cdot \mathbf{v}(\tau). \quad (5.21)$$

Note that here the optimal weight $\mathbf{w}_0$ is irrelevant of parameter γ . That is to say, when the relay weight vector and the scalar equalizer are jointly optimized, the parameter γ does not affect the weight but controls the scalar amplifier α and thus controls the output signal power level at the destination side.

5.3.3 Relation to the SINR Maximization Scheme

As demonstrated by (5.21), the optimal solution for the relay coefficient using the output power minimization scheme does not depend on the system parameter γ . Then a question arises is what relation exists between the output power based scheme and the SINR based scheme which also does not require the parameter γ . Here we show that the proposed minimizing output power scheme is in fact equivalent to the scheme of “SINR maximization under total relay power constraint” of [16], in term of achieving the optimal output SINR performance.

According to the linear constraint of the problem formulation (5.17), we can express α using γ and $\mathbf{w}$ as follows:

$$\alpha = \frac{\gamma}{\mathbf{w}^H \vec{\psi}_{\tau+1}^T}. \quad (5.22)$$

Then substitute the above equation into the objective function of (5.17) and again use the fact that $\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} = P_s \cdot \mathbf{w}^H \vec{\psi}_{\tau+1}^T \vec{\psi}_{\tau+1}^* \mathbf{w}$, and they yield the following problem, in which the optimization variable α is eliminated:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \gamma^2 P_s + \frac{\gamma^2 P_s \cdot \left(\mathbf{w}^H \left(\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n \right) \mathbf{w} + \sigma_v^2 \right)}{\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w}} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \end{aligned} \quad (5.23)$$

Note that $\gamma^2 P_s$ is a constant term in the objective function and can be omitted without affecting the optimal solution for variable $\mathbf{w}$. Thus this optimization can be further

equivalent to a maximization problem:

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w}}{\mathbf{w}^H \left(\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n \right) \mathbf{w} + \sigma_v^2} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \end{aligned} \quad (5.24)$$

However, this is just the “SINR maximization under total relay power constraint” formulation in [16] (although the expression of $\mathbf{w}$, $\mathbf{Q}_s^{(\tau)}$, $\mathbf{Q}_i^{(\tau)}$ and $\mathbf{Q}_n$ are totally different). Here we also note that the optimal solution for $\mathbf{w}$ is irrelevant of parameter γ , which is consistent with the observation obtained from (5.20). That is to say, the parameter γ does not affect the optimal SINR, but it does control the output power.

5.4 FF Beamforming with Individual Relay Power Constraint

In this section, we further discuss the output power minimization scheme subject to both the individual relay transmit power constraint and the total relay transmit power constraint. The individual power constraint is a more practical concern, which naturally arises since the distributed relay nodes have their own power budget due to physical limits such as the battery lifetime. The transmit power management is also a consideration in terms of controlling interference, either to other nodes or to other networks that share the same spectrum resources.

5.4.1 Problem Formulation

According to the matrix-form expression of the transmit signal from the i th relay (5.12), we can obtain the individual relay transmit power as

$$P_i = E|t_i(m)|^2 = \mathbf{h}_i^T (P_s \cdot \mathbf{F}_i^T \mathbf{F}_i^* + \sigma_n^2 \cdot \mathbf{I}_{L_h}) \mathbf{h}_i^*. \quad (5.25)$$

Also define notation

$$\mathbf{D}_i = \text{diag}\{\mathbf{m}_i\} \otimes (P_s \cdot \mathbf{F}_i^T \mathbf{F}_i^* + \sigma_n^2 \cdot \mathbf{I}_{L_h}), \quad (5.26)$$

where $\mathbf{m}_i = [\mathbf{0}_{1 \times (i-1)}, 1, \mathbf{0}_{1 \times (R-i)}]$ is a $1 \times R$ vector with the i th element being the only non-zero element, then (5.25) can be further rewritten as

$$P_i = \mathbf{w}^H \mathbf{D}_i \mathbf{w}. \quad (5.27)$$

Now we have given the expression for individual relay transmit power. Then using the same idea and being parallel to problem (5.17), the output power minimization problem with individual and total relay transmit power constraint is formulated as

$$\begin{aligned} \min_{\mathbf{w}, \alpha} \quad & \alpha^2 \cdot \mathbf{w}^H \left(\mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n \right) \mathbf{w} + \alpha^2 \cdot \sigma_v^2 \\ \text{s.t.} \quad & \alpha \cdot \mathbf{w}^H \vec{\psi}_{\tau+1}^T = \gamma \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0 \\ & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R, \end{aligned} \quad (5.28)$$

where $P_{0,i}$ is the individual power budget for the i th relay.

Following the same procedure of Section 5.3.3, it can be proved that the proposed scheme is equivalent to the ‘‘SINR maximization under individual relay power’’ formulation of [16]. Despite the equivalence between the two problem formulations, we will subsequently illustrate that our proposed formulation requires a lower computational loads for solving the problem.

5.4.2 Problem Solution

Unlike the problem (5.17) solved in the previous section, due to the newly included individual power constraints, the problem (5.28) does not have a closed-form solution.

Hence we will have to retreat to solve it using numerical methods. In order to adopt efficient numerical methods, here we transform the original problem (5.28) to a second-order cone convex programming.

To begin with, again according to $\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} = P_s \cdot \mathbf{w}^H \vec{\psi}_{\tau+1}^T \vec{\psi}_{\tau+1}^* \mathbf{w}$, and using the variable substitution $\hat{\mathbf{w}} = \alpha \cdot \mathbf{w}$, problem (5.28) can be equivalent to

$$\begin{aligned} \min_{\hat{\mathbf{w}}, \alpha} \quad & \hat{\mathbf{w}}^H \left(\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n \right) \hat{\mathbf{w}} + \alpha^2 \cdot \sigma_v^2 \\ \text{s.t.} \quad & \hat{\mathbf{w}}^H \vec{\psi}_{\tau+1}^T = \gamma \\ & \hat{\mathbf{w}}^H \mathbf{D} \hat{\mathbf{w}} \leq \alpha^2 P_0 \\ & \hat{\mathbf{w}}^H \mathbf{D}_i \hat{\mathbf{w}} \leq \alpha^2 P_i, \quad i = 1, \dots, R. \end{aligned} \quad (5.29)$$

In order to transform the above problem into a second-order cone programming, here the following notations are defined:

$$\begin{aligned} \mathbf{b} &= [\vec{\psi}_{\tau+1}, 0]^T, \\ \mathbf{u} &= [\hat{\mathbf{w}}^T, \alpha]^T, \\ \tilde{\mathbf{Q}}^{(\tau)} &= \begin{bmatrix} \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n & \mathbf{0}_{RL_h \times 1} \\ \mathbf{0}_{1 \times RL_h} & \sigma_v^2 \end{bmatrix}. \end{aligned} \quad (5.30)$$

Then the objective function and the linear constraint can be rewritten respectively as

$$J(\mathbf{u}) = \mathbf{u}^H \tilde{\mathbf{Q}}^{(\tau)} \mathbf{u} \quad (5.31)$$

and

$$\mathbf{u}^H \mathbf{b} = \gamma. \quad (5.32)$$

To deal with the set of power constraints, the following notations are defined:

$$\mathbf{d} = [\mathbf{0}_{1 \times RL_h}, \sqrt{P_0}], \quad \tilde{\mathbf{D}} = \begin{bmatrix} \mathbf{D}^{1/2} & \mathbf{0}_{RL_h \times 1} \\ \mathbf{0}_{1 \times RL_h} & 0 \end{bmatrix}, \quad (5.33)$$

and

$$\mathbf{d}_i = [\mathbf{0}_{1 \times RL_h}, \sqrt{P_{0.i}}], \quad \tilde{\mathbf{D}}_i = \begin{bmatrix} \mathbf{D}_i^{1/2} & \mathbf{0}_{RL_h \times 1} \\ \mathbf{0}_{1 \times RL_h} & 0 \end{bmatrix},$$

$$i = 1, \dots, R \quad (5.34)$$

where $\mathbf{D}^{1/2}$ and $\mathbf{D}_i^{1/2}$ are the principal square roots of matrices $\mathbf{D}$ and $\mathbf{D}_i$, respectively.

Then, the power constraints can be rewritten as

$$\begin{aligned} \|\mathbf{u}^H \tilde{\mathbf{D}}\|_2 &\leq \mathbf{u}^H \mathbf{d}, \\ \|\mathbf{u}^H \tilde{\mathbf{D}}_i\|_2 &\leq \mathbf{u}^H \mathbf{d}_i, \end{aligned} \quad (5.35)$$

which are a set of second-order cone constraints of variable $\mathbf{u}$.

Therefore, according to the rewritten version of objective function and constraints that are given above, we can recast the problem (5.29) as

$$\begin{aligned} \min_{\mathbf{u}} \quad & \mathbf{u}^H \tilde{\mathbf{Q}}^{(\tau)} \mathbf{u} \\ \text{s.t.} \quad & \mathbf{u}^H \mathbf{b} = \gamma \\ & \|\mathbf{u}^H \tilde{\mathbf{D}}\|_2 \leq \mathbf{u}^H \mathbf{d} \\ & \|\mathbf{u}^H \tilde{\mathbf{D}}_i\|_2 \leq \mathbf{u}^H \mathbf{d}_i, \quad i = 1, \dots, R. \end{aligned} \quad (5.36)$$

This problem is a convex optimization and can be efficiently solved using interior point methods. Furthermore, by introducing an auxiliary variable t , the above convex problem can be further rewritten as a standard second-order cone programming:

$$\begin{aligned} \min_{\mathbf{u}, t} \quad & t \\ \text{s.t.} \quad & \|\mathbf{u}^H \tilde{\mathbf{Q}}^{1/2}\|_2 \leq t \\ & \mathbf{u}^H \mathbf{b} = \gamma \\ & \|\mathbf{u}^H \tilde{\mathbf{D}}\|_2 \leq \mathbf{u}^H \mathbf{d} \\ & \|\mathbf{u}^H \tilde{\mathbf{D}}_i\|_2 \leq \mathbf{u}^H \mathbf{d}_i, \quad i = 1, \dots, R. \end{aligned} \quad (5.37)$$

Remark: Note that in [16], the scheme of “SINR maximization subject to individual relay power constraints” is solved as a feasibility problem using the bisection search technique. That is, the SOCP feasibility problem has to be solved multiple times for finding the optimal solution, and the times of solving SOCP depends on the initial value and the termination condition. On the contrary, we have shown that the proposed scheme is equivalent to the SINR maximization scheme, but the proposed approach solves the convex programming (5.37) only once using the efficient interior point methods. The worst-case complexity of solving (5.37) using interior point method is comparable to that of solving one iteration of SINR maximization, since in terms of problem dimension problem (5.37) has only one more variable than the SINR maximization problem of [16]. Hence, the proposed consumes lower computational resources to achieve the optimal SINR performance, and when the number of relays is large or when the power budget is abundant (thus the initial interval for bisection search is large), the advantage in computational complexity of the proposed approach is more obvious.

5.5 FF Beamforming with Adaptive Decision Delay

Recalling the solution (5.21) for the problem of total relay power constraint, it is seen that to calculate the optimal weight $\mathbf{w}_0(\tau)$ for different value of delay decision τ , we have to form the matrix $\mathbf{Q}_i^{(\tau)}$ and solve the problem for each τ . The output SINR yielded by different τ value is further compared to identify an “optimal” decision delay. The procedure is not efficient, and in this section we aim to find the proper decision delay in a suboptimal but efficient way without solving the same problem multiple times.

Recall the frequency-domain beamforming approach with distortionless constraint that is covered in Chapter 3, where the decision delay can be efficiently determined due to the flat and linear phase equivalent channel resulted in by the distortionless constraint.

Here, in order to exploit the merit of distortionless design for facilitating the decision delay adapting, we will present a time-domain counterpart of the distortionless constraint.

5.5.1 Distortionless Constraint in Time-Domain

Inspired by the original frequency-domain distortionless constraint defined in Section 3.3.3 of Chapter 3 and also in Section 4.3.2 of Chapter 4, our objective here for providing a time-domain counterpart can be described as making the equivalent channel $\mathbf{h}_{eqv}$ perform as a single-tap FIR filter. That is, $\mathbf{h}_{eqv}$ has a flat frequency response with linear phase, which resembles the effect of the frequency-domain distortionless constraint. More specifically, we require the impulse response of the equivalent channel to be $\mathbf{h}_{eqv} = (\gamma/\alpha) \cdot \mathbf{1}_\tau$, $\tau = 0, \dots, L_c - 1$, where $\mathbf{1}_\tau = [\mathbf{0}_{1 \times \tau}, 1, \mathbf{0}_{1 \times (L_c - \tau - 1)}]^T$ is a vector with only the $(\tau + 1)$ th element being non-zero.

In order to incorporating a parameter τ to denote the decision delay in the definition of the distortionless constraint, here we introduce an $L_c \times L_c$ Fourier matrix

$$\mathbf{\Phi} = [\vec{\varphi}(0), \vec{\varphi}(1), \dots, \vec{\varphi}(L_c - 1)] \quad (5.38)$$

where $\vec{\varphi}(\tau) = [1, e^{-j\omega_1 \cdot \tau}, \dots, e^{-j\omega_{L_c} \cdot \tau}]^T$. Then we can obtain the frequency response of a single-tap FIR filter:

$$\vec{\varphi}(\tau) = \mathbf{\Phi} \cdot \mathbf{1}_\tau. \quad (5.39)$$

Moreover, noting that $\mathbf{\Phi}^H \mathbf{\Phi} = L_c \cdot \mathbf{I}_{L_c}$ which is due to the orthogonal property of the Fourier matrix, we can further obtain

$$\mathbf{1}_\tau = \frac{1}{L_c} \mathbf{\Phi}^H \vec{\varphi}(\tau). \quad (5.40)$$

Hence, together with (5.4), the distortionless requirement $\mathbf{h}_{eqv} = (\gamma/\alpha) \cdot \mathbf{1}_\tau$ can be rewritten as

$$\mathbf{h}_{eqv} - \frac{\gamma}{\alpha} \cdot \mathbf{1}_\tau = \mathbf{\Psi} \mathbf{w}^* - \frac{\gamma}{\alpha L_c} \cdot \mathbf{\Phi}^H \vec{\varphi}(\tau) = 0. \quad (5.41)$$

The matrix $\Psi \in \mathbb{C}^{L_c \times RL_h}$ is defined in (5.4), and it is obviously that as long as $RL_h \geq L_c$, Ψ has full row rank and hence there exists a weight vector $\mathbf{w}$ that holds the equation (5.41). Then we always assume $RL_h \geq L_c$ in the adaptive decision delay beamforming scheme using distortionless design.

5.5.2 Problem Formulation and Solution

Based on the above analysis and definitions, the problem of time-domain output power minimization with distortionless constraint is formulated as

$$\begin{aligned} \min_{\mathbf{w}, \alpha} \quad & \alpha^2 \cdot \mathbf{w}^H \left(\mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n \right) \mathbf{w} + \alpha^2 \sigma_v^2 \\ \text{s.t.} \quad & \Psi^* \mathbf{w} = \frac{\gamma}{\alpha L_c} \cdot \Phi^T \vec{\varphi}^*(\tau) \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \end{aligned} \tag{5.42}$$

We note that here the distortionless constraint is naturally a linear constraint. This is unlike the frequency-domain distortionless design, where the distortionless constraint is originally formulated as a quadratic form and then transformed to a simple linear constraint.

What is more, comparing this formulation with the total power constraint formulation (5.17), it is obviously seen that these two formulation differs in the linear constraint. The linear constraint in (5.17) uses only one degree-of-freedom, however, the linear distortionless constraint of (5.42) consumes more degree-of-freedom. With this observation and noting that the objective and the other constraint are identical in these two formulations, we conclude that the distortionless formulation is sub-optimal to the non-distortionless formulation.

Upon realizing the similarity to the previous problems, the solution procedure is straightforward. From the definition of matrices $\mathbf{Q}_s^{(\tau)}$ and $\mathbf{Q}_i^{(\tau)}$ in (5.16), it is obtained

that

$$\begin{aligned}
 \mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)} &= P_s \cdot \left(\vec{\psi}_{\tau+1}^T \vec{\psi}_{\tau+1}^* + \bar{\Psi}_{\tau+1}^T \bar{\Psi}_{\tau+1}^* \right) \\
 &= P_s \cdot \left(\vec{\psi}_{\tau+1}^T \vec{\psi}_{\tau+1}^* + \sum_{\substack{k=1, \\ k \neq (\tau+1)}}^{L_c} \vec{\psi}_k^T \vec{\psi}_k^* \right) \\
 &= P_s \cdot \Psi^T \Psi^*.
 \end{aligned} \tag{5.43}$$

Then, taking into account the linear constraint in (5.42), we have

$$\begin{aligned}
 \mathbf{w}^H (\mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)}) \mathbf{w} &= P_s \cdot \mathbf{w}^H \Psi^T \Psi^* \mathbf{w} = P_s \cdot \|\Psi^* \mathbf{w}\|_2^2 \\
 &= \frac{\gamma^2 P_s}{\alpha^2 L_c^2} \cdot \|\Phi^T \vec{\varphi}^*(\tau)\|_2^2 = \frac{\gamma^2 P_s}{\alpha^2}.
 \end{aligned} \tag{5.44}$$

Therefore, substituting this result into the objective function, the problem formulation (5.42) can be rewritten as

$$\begin{aligned}
 \min_{\mathbf{w}, \alpha} \quad & \gamma^2 P_s + \alpha^2 \cdot \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \alpha^2 \sigma_v^2 \\
 s.t. \quad & \Psi^* \mathbf{w} = \frac{\gamma}{\alpha L_c} \cdot \Phi^T \vec{\varphi}^*(\tau) \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{5.45}$$

Similar to the problem (5.17), and by using the variable substitution $\hat{\mathbf{w}} = \alpha \cdot \mathbf{w}$, we can obtain the optimal solution as

$$\begin{aligned}
 \hat{\mathbf{w}}_0(\gamma, \tau) &= \frac{\gamma}{L_c} \cdot \mathbf{v}(\tau), \\
 \alpha_0(\gamma, \tau) &= \frac{\gamma}{L_c} \cdot \sqrt{\mathbf{v}(\tau)^H \mathbf{D} \mathbf{v}(\tau) / P_0},
 \end{aligned} \tag{5.46}$$

where

$$\begin{aligned}
 \mathbf{v}(\tau) &= \left(\left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \Psi^T \right) \\
 &\quad \times \left(\Psi^* \left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \Psi^T \right)^{-1} \Phi^T \vec{\varphi}^*(\tau).
 \end{aligned}$$

Given the solution in (5.46) for $\hat{\mathbf{w}}$ and α as well as the definition of $\hat{\mathbf{w}} = \alpha \cdot \mathbf{w}$, the optimal weight of the problem (5.45) is given by

$$\mathbf{w}_0(\tau) = \frac{\hat{\mathbf{w}}_0(\gamma, \tau)}{\alpha_0(\gamma, \tau)} = \sqrt{\frac{P_0}{\mathbf{v}^H(\tau) \mathbf{D} \mathbf{v}(\tau)}} \cdot \mathbf{v}(\tau). \quad (5.47)$$

Again, note that here the optimal weight $\mathbf{w}_0$ is irrelevant of γ and L_c , and it is a function of delay decision τ .

Next we discuss the adaptive selection of delay decision. Substituting the optimal solution $\mathbf{w}_0(\tau)$ and $\alpha_0(\gamma, \tau)$ into the objective function of (5.45) yields

$$\begin{aligned} J(\hat{\mathbf{w}}_0, \alpha_0) &= \gamma^2 P_s + \alpha_0^2 \cdot \mathbf{w}_0^H \mathbf{Q}_n \mathbf{w}_0 + \alpha_0^2 \sigma_v^2 \\ &= \gamma^2 P_s + \vec{\varphi}^T(\tau) \mathbf{B} \vec{\varphi}^*(\tau), \end{aligned} \quad (5.48)$$

where

$$\mathbf{B} = \Phi^* \left(\Psi^* \left(\mathbf{Q}_n + \frac{\sigma_v^2}{P_0} \mathbf{D} \right)^{-1} \Psi^T \right)^{-1} \Phi^T.$$

Since the scalar τ takes integer values from 0 to $(L_c - 1)$, then for a specific channel realization, we can choose a best decision delay τ_0 that leads to the minimal output power, and hence achieves the maximal output SINR:

$$\tau_0 = \arg \min_{\tau \in \{0, \dots, (L_c - 1)\}} J(\hat{\mathbf{w}}_0(\gamma, \tau), \alpha_0(\gamma, \tau)). \quad (5.49)$$

Thus, by substituting τ_0 into (5.47), the optimal weight vector corresponding to the optimal delay can be given by

$$\mathbf{w}_{opt} = \mathbf{w}_0(\tau_0) = \sqrt{\frac{P_0}{\mathbf{v}^H(\tau_0) \mathbf{D} \mathbf{v}(\tau_0)}} \cdot \mathbf{v}(\tau_0). \quad (5.50)$$

Note that for a given parameter γ , we merely need to calculate the matrix $\mathbf{B}$ and the optimal delay τ can be efficiently found. Then the optimal weight $\mathbf{w}_{opt}$ can also be determined efficiently, since all the complicated matrix calculations for computing (5.50) have already been worked out when the matrix $\mathbf{B}$ is calculated.

5.6 Numerical Simulation

In the simulation, the performance of the proposed adaptive delay approach is investigated. Throughout this section, the relay noise power and destination noise power are assumed to be $\sigma_n^2 = \sigma_v^2 = 1$, and the source power P_s is 10dB higher than the noise power. The coefficients of channel impulse responses are modeled as independent quasi-static Rayleigh fading, and are hence generated as zero-mean complex Gaussian random variables with the exponential power delay profile [97]

$$p(m) = \frac{1}{\sigma_t} \sum_{l=0}^{L_x-1} e^{-m/\sigma_t} \delta(m-l), \quad (5.51)$$

where σ_t characterizes the delay spread, and $L_x \in \{L_f, L_g\}$ denotes the length of the backward channels or the forward channels. Throughout the simulations, $L_f = L_g = 5$, and $L_h = 5$ unless otherwise specified. All the channels are assumed to be quasi-static and perfectly known by the algorithm, which means that the channels are randomly generated but remain static over a frame time, during which multiple successive symbols are transmitted. The algorithm calculates the relay weights at the beginning of each frame based on the known CSI (a channel realization), and the same set of weights are used during this frame period.

Figure 5.2 and Figure 5.3, respectively for relays number $R = 10$ and $R = 20$, illustrate the SINR performance versus total relay transmit power P_0 . The distortionless minimizing power scheme can adaptively select the optimal decision delay. We note that for the non-distortionless power minimization scheme, if the decision delay is not properly selected, the performance loss would be rather severe. On the other hand, if the delay is optimally chosen, the performance of the non-distortionless power minimization would outperform its distortionless counterpart, and when the relay number is sufficiently large, the two performances are quite close. Due to the equivalence between the power

Figure 5.2: SINR performance versus total relay transmit power P_0 . The Non-distortionless Min Power scheme is shown with different pre-selected decision delays τ_d . $R = 20$.

Figure 5.3: SINR performance versus total relay transmit power P_0 . The Non-distortionless Min Power scheme is shown with different pre-selected decision delays τ_d . $R = 10$.

Figure 5.4: SINR performance versus relay filter length L_h . The Non-distortionless Min Power scheme is shown with different pre-selected decision delays τ_d . $R = 20$.

minimization and SINR maximization we have discussed, the results of Figure 5.2 is the same as that of Figure 3.4.

Figure 5.4 and Figure 5.5 illustrate the SINR performance versus relay filter length L_h , with the total relay power constraint. We can see that there exists a upper bound of SINR performance for a certain L_h value. Moreover, as can be observed, for the non-distortionless approach, when the decision delay is relatively large, *e.g.*, here $\tau_d = 4$ and $\tau_d = 5$, the relay filter length affects the performance significantly. On the other hand, we can see that, if the decision delay is not properly set, *e.g.*, here $\tau_d = 1$, then by increasing L_h the performance cannot be significantly improved.

To show the effects of number of relays more clearly, Figure 5.6 displays the SINR performance versus R , with the total relay power constraint. It is seen that, if there are not sufficient number of relay nodes, the distortionless design would be inferior to the non-distortionless design with even non-optimal decision delay. This is because some

Figure 5.5: SINR performance versus relay filter length L_h . The Non-distortionless Min Power scheme is shown with different pre-selected decision delays τ_d . $R = 10$.

Figure 5.6: SINR performance versus number of relays R . The Non-distortionless Min Power scheme is shown with different pre-selected decision delays τ_d .

Figure 5.7: SINR performance versus individual relay transmit power $P_{0,i}$ for different pre-selected decision delays τ_d . $R = 10$.

Figure 5.8: Average number of iterations versus individual relay transmit power budget $P_{0,i}$ for different decision delays τ .

degrees-of-freedom are employed to attain the distortionless response that is required by the distortionless constraint.

In Figure 5.7, we compare the proposed Min Power with individual constraints scheme to Chen's Max SINR scheme [16]. As discussed previously, the two schemes should give the same results. The slight difference is due to the numerical tolerate error setting. Specifically, since for Chen's approach, a bisection search method is used to solve a sequence of feasibility problem, then the termination tolerate error affects the accuracy of the solution. In this experiment we set the tolerate $\epsilon = 0.1$. For the proposed approach, we do not involve the bisection method and thus the tolerate error issue as that in the bisection search is not encountered. In Figure 5.8, we show the average number of bisection iterations, corresponding to result of Figure 5.7. As we have mentioned, the worst-case complexity of solving (5.37) using interior point method is comparable to that of solving one iteration of SINR maximization, since in terms of problem dimension problem (5.37) has only one more variable than the SINR maximization problem of [16]. Then comparing the number of iterations embodies the comparison of computational loads. We note in Figure 5.8 that for the SINR maximization approach, the iteration number increases with the increase of output SINR. While the proposed approach is not iteratively solved, and thus there is always one iteration only.

5.7 Conclusion

In this chapter, we considered designing the filter-and-forward (FF) relay beamforming in frequency-selective channels. An alternative approach is proposed which aims to minimize the output power at the destination side. Three relay beamforming schemes based on different constraints are discussed. The first scheme minimizes the output power while keeping the power of the desired signal at a constant level, and subject to a total relay transmit power constraint. It is shown that, in terms of output SINR, this scheme is equivalent to the SINR maximization formulation. The second scheme also minimizes the

output power with the desired signal power maintained at a constant level, and subject to both total and individual power constraint. We have shown that this scheme is equivalent to the SINR maximization, however, the proposed approach shows lower computational complexity in solving the problem. The third scheme minimizes the output power subject to the constraint that the equivalent channel response from the source to the destination is distortionless, *i.e.*, flat and linear phase. Baring this property, we can efficiently select the optimal decision delay that would significantly affect the system performance.

Chapter 6

Multi-User Relay Beamforming Using Power Minimization Scheme

6.1 Introduction

In the previous chapter, for merely a single source and a single destination, we have formulated the output power minimization beamforming scheme in time domain, and we have shown that the power based formulation is equivalent to the SINR maximization scheme in terms of the output SINR performance. In this chapter, we consider extending this beamforming scheme to cater for the scenario of multiple source-destination pair communications.

To address the multiple peer-to-peer relay beamforming problem on frequency-selective channels, a worst-SINR maximization criterion based on filter-and-forward relaying scheme is adopted in [90]. Therein, to solve the optimization problem, semidefinite relaxation (SDR) technique is used to relax the problem as a convex semidefinite programming (SDP). However, due to the relaxation the feasible optimal solution for the original problem may not be found. The same problem of relaxation is also encountered by the scheme proposed in [12] for relay beamforming in flat channels, where the total relay transmit power is to be minimized. Then, to address this relaxation problem of [12], a second-order cone programming (SOCP) based iterative relay transmit power minimization methods

is proposed by [86, 87], where the performance is illustrated to outperform the SDP relaxation method, and it is also superior to a more conservative SOCP based method [85]. In another way to address the SDR problem encountered by the worst SINR minimization scheme, a sequential convex programming algorithm is proposed in [91].

In this chapter, based on the idea of output power minimization that is considered in the previous chapters, we propose two formulations for the multi-user relay beamforming. The first formulation aims to minimize the maximal output power at the destination side among all the users, under the constraints of both total and individual relay transmit power. We demonstrate that the proposed maximal output power minimization formulation is equivalent to the worst SINR maximization problem which leads to a sequence of SDP feasibility problem. With the proposed formulation, two algorithms for solving it are provided, which are both based on SOCP feasibility problem. The second formulation amounts to minimize the sum output power at all the destinations, and it is shown that this formulation renders the best performance in terms of average output SINR among user.

The organization of this chapter is as follows. In Section 6.2, the signal model is given. In Section 6.3, the problem formulation and solution for the maximal output power minimization criterion is presented and its relation to SINR maximization is discussed. Section 6.4 the problem formulation and solution for the sum output power minimization criterion is presented. Simulation results are provided in Section 6.5. Section 6.6 concludes this chapter.

6.2 Signal Model

As depicted in Figure 6.1, we consider a relay network with K source-destination pair nodes and R relay nodes, and all nodes are each equipped with a single antenna. We

Figure 6.1: Signal model of multi-user filter-and-forward relay network in frequency-selective channels.

assume that the source nodes and their corresponding destination nodes are far away from each other, thus the direct link of source-destination is ignored due to the severe attenuation. All the relay nodes work in the TDD mode, and hence a signal transmission from all the sources to their destinations consists of two phases. Specifically, in the first phase, all the source nodes broadcast the signals of information symbols to the relays, and in the second phase, the signals received at each relay nodes is filtered and re-transmitted to the destination nodes.

As with the single-user case, the frequency-selective channels among the nodes can be similarly represented. Also note that there are channels between sources and their non-counterpart destinations, and in that case the received signal from the non-counterpart sources are regarded as inter-user interference (IUI). More specifically, the backward channel from the k th source to the i th relay is denoted by column vector $\mathbf{f}_{i,k}$ of length L_f , which is represented as $\mathbf{f}_{i,k} = [f_{i,k}(0), \dots, f_{i,k}(L_f-1)]^T, \forall i = 1, \dots, R, k = 1, \dots, K$,

where $f_{i,k}(m)$, $m = 0, \dots, (L_f - 1)$, is the filter coefficient. Similarly, the forward channel vector from the i th relay to the k th destination is denoted by $\mathbf{g}_{i,k} = [g_{i,k}(0), \dots, g_{i,k}(L_g - 1)]^T$, $\forall i = 1, \dots, R$, $k = 1, \dots, K$. As supposed in [17], $\mathbf{f}_{i,k}$ and $\mathbf{g}_{i,k}$ contain the combined effects of transmit pulse shaping filter, physical channel, receive filter and sampling. The instantaneous CSI is also assumed to be perfectly known.

Each source node transmits an information-bearing sequence of symbols $s_k(m)$, with a common transmit power $P_S = E\{|s_k(m)|^2\}$, where m is the symbol instant. Hence the sequence received at the i th relay is

$$r_i(m) = \sum_{k=1}^K s_k(m) * f_{i,k}(m) + n_i(m), \quad (6.1)$$

where $n_i(m)$ denotes the additive white Gaussian noise with power $\sigma_n^2 = E\{|n_i(m)|^2\}$. The received signal $r_i(m)$ is first passed through the relay filter $\mathbf{h}_i = [h_i(0), \dots, h_i(L_h - 1)]^T$, after which the following sequence

$$t_i(m) = r_i(m) * h_i(m), \quad (6.2)$$

is re-transmitted towards the destination. Then the sequence received at the k th destination is

$$y_k(m) = \sum_{i=1}^R t_i(m) * g_{i,k}(m) + v_k(m), \quad (6.3)$$

where $v_k(m)$ is the additive white Gaussian noise with power $\sigma_v^2 = E\{|v_k(m)|^2\}$. At the destination side, the received signal is multiplied by a complex number α_k and the output sequence is given by

$$\begin{aligned} z_k(m) &= \alpha_k \cdot y_k(m) \\ &= \alpha_k \cdot s_k(m) * h_{eqv,k}(m) + \alpha_k \cdot \sum_{\substack{j=1, \\ j \neq k}}^K s_j(m) * h_{iui-j,k}(m) \\ &\quad + \alpha_k \cdot n_{pro,k}(m) + \alpha_k \cdot v_k(m), \end{aligned} \quad (6.4)$$

where

$$h_{eqv,k}(m) \triangleq \sum_{i=1}^R f_{i,k}(m) * h_i(m) * g_{i,k}(m) \quad (6.5)$$

denotes the impulse response of the equivalent channel from the k th source to the input of the k th destination node, and

$$h_{iui-j,k}(m) \triangleq \sum_{i=1}^R f_{i,j}(m) * h_i(m) * g_{i,k}(m), \quad j \neq k \quad (6.6)$$

denotes the impulse response of the *inter-user interference* (IUI) channel from the j th source to the input of the k th destination, and

$$n_{pro,k}(m) \triangleq \sum_{i=1}^R n_i(m) * h_i(m) * g_{i,k}(m) \quad (6.7)$$

is the propagating noise that propagates from the relay nodes to the k th destination nodes.

6.2.1 ISI and IUI in Output Signal

Next we will explicitly express the ISI and IUI for each user in this multi-user network. To start with, we first rewrite the overall signal model (6.4) in matrix form. Recalling the definition (6.6) for the j th IUI channel that interferes user k , we represent the result of the convolution $f_{i,j}(m) * g_{i,k}(m)$, ($j, k \in \{1, \dots, K\}, i \in \{1, \dots, R\}$), as a column vector $\mathbf{b}_{i,j,k} = \mathbf{f}_{i,j} * \mathbf{g}_{i,k} = [b_{i,j,k}(0), \dots, b_{i,j,k}(L_b - 1)]^T$, where $L_b = (L_f + L_g - 1)$. Then $h_{iui-j,k}(m)$ can be further written in matrix form as:

$$\mathbf{h}_{iui-j,k} = \sum_{i=1}^R \Theta_{i,j,k} \mathbf{h}_i = \Psi_{j,k} \mathbf{w}^*, \quad (6.8)$$

where $\mathbf{\Psi}_{j,k} = [\mathbf{\Theta}_{1,j,k}, \dots, \mathbf{\Theta}_{R,j,k}]$, $\mathbf{w} = [\mathbf{h}_1^H, \dots, \mathbf{h}_R^H]^T$, and

$$\mathbf{\Theta}_{i,j,k} = \begin{bmatrix} b_{i,j,k}(0) & 0 & & 0 \\ \vdots & b_{i,j,k}(0) & \ddots & \vdots \\ b_{i,j,k}(L_b - 1) & \vdots & & \vdots \\ 0 & b_{i,j,k}(L_b - 1) & \ddots & 0 \\ \vdots & 0 & & b_{i,j,k}(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & b_{i,j,k}(L_b - 1) \end{bmatrix} \quad (6.9)$$

has the dimension of $\mathbb{C}^{L_c \times L_h}$. Therefore, the IUI term in (6.4) is expressed as

$$\begin{aligned} & \alpha_k \cdot \sum_{\substack{j=1 \\ j \neq k}}^K s_j(m) * h_{iui-j,k}(m) \\ &= \alpha_k \cdot \sum_{\substack{j=1 \\ j \neq k}}^K \mathbf{h}_{iui-j,k}^T \mathbf{s}_j(m) = \alpha_k \cdot \mathbf{w}^H \sum_{\substack{j=1 \\ j \neq k}}^K \mathbf{\Psi}_{j,k}^T \mathbf{s}_j(m), \end{aligned} \quad (6.10)$$

where $\mathbf{s}_j(m) = [s_j(m), s_j(m-1), \dots, s_j(m-L_c+1)]^T$.

Similarly, the equivalent channel of the k th user-pair (6.5) can also be rewritten in matrix form as

$$\mathbf{h}_{eqv,k} = \sum_{i=1}^R \mathbf{\Theta}_{i,k,k} \mathbf{h}_i = \mathbf{\Psi}_{k,k} \mathbf{w}^*, \quad (6.11)$$

and the first term in (6.4) is expressed as

$$\alpha_k \cdot s_k(m) * h_{eqv,k}(m) = \alpha_k \cdot \mathbf{h}_{eqv,k}^T \mathbf{s}_k(m) = \alpha_k \cdot \mathbf{w}^H \mathbf{\Psi}_{k,k}^T \mathbf{s}_k(m). \quad (6.12)$$

Moreover, the propagation noise defined in (6.7) can be expressed in matrix form as

$$n_{pro,k}(m) = \sum_{i=1}^R (\mathbf{G}_{i,k} \mathbf{h}_i)^T \mathbf{n}_i(m), \quad (6.13)$$

where $\mathbf{n}_i = [n_i(m), n_i(m-1), \dots, n_i(m-L_g-L_h+2)]^T$, and $\mathbf{G}_{i,k}$ is an $(L_g+L_h-1) \times L_h$

column-circulant matrix with $[g_{i,k}(0), \dots, g_{i,k}(L_g - 1), \mathbf{0}_{1 \times (L_h - 1)}]^T$ as the first column:

$$\mathbf{G}_{i,k} = \begin{bmatrix} g_{i,k}(0) & 0 & & 0 \\ \vdots & g_{i,k}(0) & \ddots & \vdots \\ g_{i,k}(L_g - 1) & \vdots & & \vdots \\ 0 & g_{i,k}(L_g - 1) & \ddots & 0 \\ \vdots & 0 & & g_{i,k}(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & g_{i,k}(L_g - 1) \end{bmatrix}. \quad (6.14)$$

Therefore, from (6.10), (6.12) and (6.13), the destination output signal (6.4) can be written as

$$\begin{aligned} z_k(m) &= \alpha_k \cdot \mathbf{w}^H \mathbf{\Psi}_{k,k}^T \mathbf{s}_k(m) + \alpha_k \cdot \mathbf{w}^H \sum_{\substack{j=1 \\ j \neq k}}^K \mathbf{\Psi}_{j,k}^T \mathbf{s}_j(m) \\ &\quad + \alpha_k \cdot \sum_{i=1}^R (\mathbf{G}_{i,k} \mathbf{h}_i)^T \mathbf{n}_i(m) + \alpha_k \cdot v_k(m). \end{aligned} \quad (6.15)$$

Moreover, suppose that at the time instance (m) , at each destination node the corresponding transmitted symbol $s_k(m - \tau)$ is to be estimated from $z_k(m)$, where τ denotes the decision delay. By separating the $(\tau + 1)$ th column from the matrix $\mathbf{\Psi}_{k,k}^T$, we can further decompose (6.15) as

$$\begin{aligned} z_k(m) &= \underbrace{\alpha_k \cdot \mathbf{w}^H (\bar{\psi}_{k,k}^{(\tau)})^T s_k(m - \tau)}_{\text{Desired signal}} \\ &\quad + \underbrace{\alpha_k \cdot \mathbf{w}^H (\bar{\mathbf{\Psi}}_{k,k}^{(\tau)})^T \bar{\mathbf{s}}_k^{(\tau)}(m)}_{\text{ISI}} + \underbrace{\alpha_k \cdot \mathbf{w}^H \sum_{\substack{j=1 \\ j \neq k}}^K \mathbf{\Psi}_{j,k}^T \mathbf{s}_j(m)}_{\text{IUI}} \\ &\quad + \underbrace{\alpha_k \cdot \sum_{i=1}^R (\mathbf{G}_{i,k} \mathbf{h}_i)^T \mathbf{n}_i(m)}_{\text{Noise}} + \alpha_k \cdot v_k(m), \end{aligned} \quad (6.16)$$

where $\vec{\psi}_{k,k}^{(\tau)}$ is the $(\tau+1)$ th row of $\mathbf{\Psi}_{k,k}$, and hence $\overline{\mathbf{\Psi}}_{k,k}$ is the submatrix of $\mathbf{\Psi}_{k,k}$ with removing the $(\tau+1)$ th row, and $\tilde{\mathbf{s}}_k^{(\tau)}(m) = [s_k(m), \dots, s_k(m-\tau+1), s_k(m-\tau-1), \dots, s_k(m-L_c+1)]^T$, which is the residual vector of $\mathbf{s}_k(m)$ with the $(\tau+1)$ th element removed. Therefore, we see that the desired symbol $s_k(m)$ is separated from the ISI introduced by neighbouring symbols.

6.3 Minimization of Maximal Output Power

6.3.1 Problem Formulation

From (6.2), we know that the transmit signal from the i th relay is expressed as:

$$t_i(m) = \sum_{k=1}^K s_k(m) * f_{i,k}(m) * h_i(m) + n_i(m) * h_i(m). \quad (6.17)$$

It can be further represented in matrix form as

$$t_i(m) = \sum_{k=1}^K (\mathbf{F}_{i,k} \mathbf{h}_i)^T \tilde{\mathbf{s}}_k(m) + \mathbf{h}_i^T \tilde{\mathbf{n}}_i(m), \quad (6.18)$$

where $\tilde{\mathbf{n}}_i(m) = [n_i(m), n_i(m-1), \dots, n_i(m-L_h+1)]^T$, $\tilde{\mathbf{s}}_k(m) = [s_k(m), s_k(m-1), \dots, s_k(m-L_f-L_h+2)]^T$. $\mathbf{F}_{i,k}$ is an $(L_f+L_h-1) \times L_h$ column-circulant matrix with $[f_{i,k}(0), \dots, f_{i,k}(L_f-1), \mathbf{0}_{1 \times (L_h-1)}]^T$ as the first column:

$$\mathbf{F}_{i,k} = \begin{bmatrix} f_{i,k}(0) & 0 & 0 \\ \vdots & f_{i,k}(0) & \ddots & \vdots \\ f_{i,k}(L_f-1) & \vdots & & \vdots \\ 0 & f_{i,k}(L_f-1) & \ddots & 0 \\ \vdots & 0 & & f_{i,k}(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & f_{i,k}(L_f-1) \end{bmatrix}. \quad (6.19)$$

Note that the transmitted signal $s_k(m)$ from different users are supposed to be uncorrelated with each other. Therefore, the individual relay transmit power is given by

$$P_i = E|t_i(m)|^2 = \mathbf{h}_i^T (P_s \cdot \sum_{k=1}^K \mathbf{F}_{i,k}^T \mathbf{F}_{i,k}^* + \sigma_n^2 \cdot \mathbf{I}_{L_h}) \mathbf{h}_i^*. \quad (6.20)$$

Define a notation

$$\mathbf{D}_i = \text{diag}\{\mathbf{m}_R^i\} \otimes (P_s \cdot \sum_{k=1}^K \mathbf{F}_{i,k}^T \mathbf{F}_{i,k}^* + \sigma_n^2 \cdot \mathbf{I}_{L_h}), \quad (6.21)$$

where $\mathbf{m}_R^i = [\mathbf{0}_{1 \times (i-1)}, 1, \mathbf{0}_{1 \times (R-i)}]$ is a $1 \times R$ vector with the i th element being the only non-zero element. Then (6.20) can be further rewritten as

$$P_i = [\mathbf{h}_1^T, \dots, \mathbf{h}_R^T] \cdot \mathbf{D}_i \cdot [\mathbf{h}_1^T, \dots, \mathbf{h}_R^T]^H = \mathbf{w}^H \mathbf{D}_i \mathbf{w}. \quad (6.22)$$

Consequently, the total relay transmit power can be obtained as

$$P_{Tot} = \sum_{i=1}^R P_i = \mathbf{w}^H \mathbf{D} \mathbf{w}, \quad (6.23)$$

where $\mathbf{D} = \sum_{i=1}^R \mathbf{D}_i = \tilde{\mathbf{F}} + \sigma_n^2 \cdot \mathbf{I}_{RL_h}$, and $\tilde{\mathbf{F}} = P_s \cdot \text{blkdiag}\{\sum_{k=1}^K \mathbf{F}_{1,k}^T \mathbf{F}_{1,k}^*, \dots, \sum_{k=1}^K \mathbf{F}_{R,k}^T \mathbf{F}_{R,k}^*\}$.

On the other hand, according to (6.16), the output power at the k th destination node is given by

$$\begin{aligned} E|z_k(m)|^2 &= |\alpha_k|^2 \cdot \left(\mathbf{w}^H \mathbf{Q}_{s,k}^{(\tau)} \mathbf{w} + \mathbf{w}^H \mathbf{Q}_{isi,k}^{(\tau)} \mathbf{w} \right. \\ &\quad \left. + \mathbf{w}^H \mathbf{Q}_{iui,k} \mathbf{w} + \mathbf{w}^H \mathbf{Q}_{n,k} \mathbf{w} + \sigma_v^2 \right), \end{aligned} \quad (6.24)$$

where the following notations are used:

$$\begin{aligned} \mathbf{Q}_{s,k}^{(\tau)} &= P_s \cdot (\vec{\psi}_{k,k}^{(\tau)})^T (\vec{\psi}_{k,k}^{(\tau)})^*, \\ \mathbf{Q}_{isi,k}^{(\tau)} &= P_s \cdot (\overline{\Psi}_{k,k}^{(\tau)})^T (\overline{\Psi}_{k,k}^{(\tau)})^*, \\ \mathbf{Q}_{iui,k} &= P_s \cdot \sum_{\substack{j=1, \\ j \neq k}}^K (\overline{\Psi}_{j,k}^{(\tau)})^T (\overline{\Psi}_{j,k}^{(\tau)})^*, \\ \mathbf{Q}_{n,k} &= \sigma_n^2 \cdot \bigoplus_{i=1}^R \mathbf{G}_{i,k}^T \mathbf{G}_{i,k}^*. \end{aligned} \quad (6.25)$$

where $\bigoplus_{i=1}^R \mathbf{G}_{i,k}^T \mathbf{G}_{i,k}^* = \mathbf{G}_{1,k}^T \mathbf{G}_{1,k}^* \oplus \mathbf{G}_{2,k}^T \mathbf{G}_{2,k}^* \oplus \dots \oplus \mathbf{G}_{R,k}^T \mathbf{G}_{R,k}^*$ is the direct sum of R matrices.

The maximal output power minimization problem is formulated as

$$\begin{aligned}
 \min_{\mathbf{w}, \vec{\alpha}} \max_k \quad & |\alpha_k|^2 \cdot \mathbf{w}^H \mathbf{Q}_k \mathbf{w} + |\alpha_k|^2 \cdot \sigma_v^2 \\
 s.t. \quad & \alpha_k \cdot \mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T = \gamma_k, \quad k = 1, \dots, K \\
 & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0,
 \end{aligned} \tag{6.26}$$

where $\mathbf{Q}_k = \mathbf{Q}_{s,k}^{(\tau)} + \mathbf{Q}_{si,k}^{(\tau)} + \mathbf{Q}_{iwi,k} + \mathbf{Q}_{n,k}$, $\vec{\alpha} = [\alpha_1, \dots, \alpha_K]^T$, and $\gamma_k = \gamma \cdot e^{j\phi_k}$ with γ as a pre-defined system parameter that is common to every user; P_0 is the total relay power budget, and $P_{0,i}$ is the individual relay power budget. The phase ϕ_k will be discussed in the problem solution section.

6.3.2 Relation to the SINR Maximization Scheme

Here we show that the proposed minimizing maximal output power scheme is equivalent to the scheme of worst SINR maximization of [90].

According to the linear constraint of the problem formulation (6.26), it can be obtained

$$\alpha_k = \frac{\gamma_k}{\mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T}, \quad k = 1, \dots, K. \tag{6.27}$$

Substitute the above equation into the objective function of (6.26) and use the facts that $\mathbf{w}^H \mathbf{Q}_{s,k}^{(\tau)} \mathbf{w} = P_s \cdot \mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T (\vec{\psi}_{k,k}^{(\tau)})^* \mathbf{w}$ as well as $|\gamma_k|^2 = \gamma^2$, and they yield the following problem, in which the variables α_k are eliminated:

$$\begin{aligned}
 \min_{\mathbf{w}} \max_k \quad & \gamma^2 P_s + \frac{\gamma^2 \left(\mathbf{w}^H \mathbf{Q}_{ipn,k}^{(\tau)} \mathbf{w} + \sigma_v^2 \right)}{\mathbf{w}^H \mathbf{Q}_{s,k}^{(\tau)} \mathbf{w}} \\
 s.t. \quad & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0,
 \end{aligned} \tag{6.28}$$

where $\mathbf{Q}_{ipn.k}^{(\tau)} = \mathbf{Q}_{isi.k}^{(\tau)} + \mathbf{Q}_{iui.k} + \mathbf{Q}_{n.k}$ represents the covariance matrix of interferences (both inter-symbol and inter-user) and noise.

Note that in the objective function, $\gamma^2 P_s$ and γ^2 are constant terms that are irrelevant to the optimization variables and thus can be omitted. This optimization can be further expressed as

$$\begin{aligned} \max_{\mathbf{w}} \min_k & \quad \frac{\mathbf{w}^H \mathbf{Q}_{s.k}^{(\tau)} \mathbf{w}}{\mathbf{w}^H \mathbf{Q}_{ipn.k}^{(\tau)} \mathbf{w} + \sigma_v^2} \\ \text{s.t.} & \quad \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0.i}, \quad i = 1, \dots, R \\ & \quad \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \end{aligned} \tag{6.29}$$

However, this is just the “worst SINR maximization” formulation in [90] (although the expression of $\mathbf{w}$, $\mathbf{Q}_s^{(\tau)}$, $\mathbf{Q}_i^{(\tau)}$ and $\mathbf{Q}_n$ are totally different). Here we also note that the optimal solution for $\mathbf{w}$ is irrelevant of parameter γ_k as long as the module $|\gamma_k|$, $k = 1, \dots, K$, equals to each other.

6.3.3 Problem Solution

Starting from the problem formulation (6.26) and introducing in an auxiliary variable t , we can reformulate the problem into its epigraph form:

$$\begin{aligned} \min_{\mathbf{w}, \tilde{\alpha}, t} & \quad t \\ \text{s.t.} & \quad |\alpha_k|^2 \cdot \mathbf{w}^H \mathbf{Q}_k \mathbf{w} + |\alpha_k|^2 \cdot \sigma_v^2 \leq t, \quad k = 1, \dots, K \\ & \quad \alpha_k \cdot \mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T = \gamma_k, \quad k = 1, \dots, K \\ & \quad \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0.i}, \quad i = 1, \dots, R \\ & \quad \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0. \end{aligned} \tag{6.30}$$

According to (6.27), the optimization variable $\vec{\alpha}$ can be eliminated and we can obtain an equivalent problem:

$$\begin{aligned}
 \min_{\mathbf{w}, t} \quad & t \\
 \text{s.t.} \quad & \mathbf{w}^H \mathbf{Q}_k \mathbf{w} + \sigma_v^2 \leq t \cdot \left| \frac{\mathbf{w}^H (\bar{\psi}_{k,k}^{(\tau)})^T}{\gamma_k} \right|^2, k = 1, \dots, K \\
 & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{6.31}$$

Moreover, in order to transform the quadratics in the constraints to a more convenient Euclidean-norm form, we define the following notations:

$$\begin{aligned}
 \mathbf{u} &= [\mathbf{w}^T, 1]^T, \\
 \hat{\mathbf{Q}}_k &= \mathbf{Q}_k \oplus \sigma_v^2, \\
 \hat{\mathbf{D}} &= \mathbf{D} \oplus 0, \\
 \hat{\mathbf{D}}_i &= \mathbf{D}_i \oplus 0, \\
 \mathbf{d}_k &= [\bar{\psi}_{k,k}^{(\tau)}, 0]^T,
 \end{aligned} \tag{6.32}$$

where “ $\oplus$ ” denotes the direct sum of matrices. Then (6.31) can be further written as

$$\begin{aligned}
 \min_{\mathbf{u}, t} \quad & t \\
 \text{s.t.} \quad & \left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u} \right\| \leq \sqrt{t} \cdot \left| \frac{\mathbf{u}^H \mathbf{d}_k}{\gamma_k} \right|, k = 1, \dots, K \\
 & \left\| \hat{\mathbf{D}}_i^{1/2} \mathbf{u} \right\| \leq \sqrt{P_{0,i}}, i = 1, \dots, R \\
 & \left\| \hat{\mathbf{D}}^{1/2} \mathbf{u} \right\| \leq \sqrt{P_0} \\
 & \mathbf{u}_{\text{end}} = 1,
 \end{aligned} \tag{6.33}$$

where $\hat{\mathbf{Q}}_k^{1/2}$, $\hat{\mathbf{D}}_i^{1/2}$ and $\hat{\mathbf{D}}^{1/2}$ are the matrix square roots of $\hat{\mathbf{Q}}_k$, $\hat{\mathbf{D}}_i$ and $\hat{\mathbf{D}}$, respectively, and $\mathbf{u}_{\text{end}}$ denotes the last element of vector $\mathbf{u}$.

Assuming the optimal value of problem (6.33) is known as t^* , for any given $t \geq t^*$ all the constraints can be satisfied; while on the other hand, the constraints are infeasible for a given $t < t^*$, since otherwise, this $t < t^*$ is a solution in the feasible region and thus contradicts the optimality of t^* . Therefore, we can solve (6.34) through solving a sequence of feasibility problem:

$$\begin{aligned}
 & \text{find } \mathbf{u} \\
 & s.t. \quad \left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u} \right\| \leq \sqrt{t} \cdot \left| \frac{\mathbf{u}^H \mathbf{d}_k}{\gamma_k} \right|, \quad k = 1, \dots, K \\
 & \quad \left\| \hat{\mathbf{D}}_i^{1/2} \mathbf{u} \right\| \leq \sqrt{P_{0,i}}, \quad i = 1, \dots, R \\
 & \quad \left\| \hat{\mathbf{D}}^{1/2} \mathbf{u} \right\| \leq \sqrt{P_0} \\
 & \quad \mathbf{u}_{\text{end}} = 1.
 \end{aligned} \tag{6.34}$$

This problem is not a convex programming even with a given value of t , since the first constraint is non-convex. However, a relation is noted that $\text{Re} \{ \mathbf{u}^H \mathbf{d}_k / \gamma_k \} \leq | \mathbf{u}^H \mathbf{d}_k / \gamma_k |$, where the equality holds when $\mathbf{u}^H \mathbf{d}_k / \gamma_k$ is real-valued. Consequently, similar to the treatment performed by [85, 87], the first constraint in (6.33) can be strengthened as

$$\left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u} \right\| \leq \sqrt{t} \cdot \text{Re} \left\{ \frac{\mathbf{u}^H \mathbf{d}_k}{\gamma_k} \right\}, \tag{6.35}$$

which, with a given value of t , is a second-order cone and thus is a convex set of the variable $\mathbf{u}$. Therefore, by replacing the first constraint of (6.34) with (6.35), the problem (6.34) is approximated by a strengthened problem:

$$\begin{aligned}
 & \text{find } \mathbf{u} \\
 & s.t. \quad \left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u} \right\| \leq \sqrt{t} \cdot \text{Re} \left\{ \frac{\mathbf{u}^H \mathbf{d}_k}{\gamma_k} \right\}, \quad k = 1, \dots, K \\
 & \quad \left\| \hat{\mathbf{D}}_i^{1/2} \mathbf{u} \right\| \leq \sqrt{P_{0,i}}, \quad i = 1, \dots, R \\
 & \quad \left\| \hat{\mathbf{D}}^{1/2} \mathbf{u} \right\| \leq \sqrt{P_0} \\
 & \quad \mathbf{u}_{\text{end}} = 1.
 \end{aligned} \tag{6.36}$$

Note that for a given t , (6.36) is an SOCP and can be efficiently solved using interior-point methods [98, Chapter 11].

Traditionally, to find the optimal value t^* through solving a sequence of feasibility problem (6.36), a bisection search method can be adopted as shown by [98, Algorithm 4.1]. However, since here we use the relation $\text{Re}\{\mathbf{u}^H \mathbf{d}_k / \gamma_k\} \leq |\mathbf{u}^H \mathbf{d}_k / \gamma_k|$ to replace the original constraint $\|\hat{\mathbf{Q}}_k^{1/2} \mathbf{u}\| \leq \sqrt{t} \cdot |\mathbf{u}^H \mathbf{d}_k / \gamma_k|$ with a convex constraint (6.35) for each $k \in \{1, \dots, K\}$, the approximated constraint is strengthened over the original one. More specifically, the new constraint renders a smaller feasible region that is a subset of the feasible region given by the original constraint. Therefore, the optimal solution obtained by solving the approximated problem (6.36) is not necessarily optimal to the original problem (6.34), unless the equalities hold for each k : $\text{Re}\{\mathbf{u}^H \mathbf{d}_k / \gamma_k\} = |\mathbf{u}^H \mathbf{d}_k / \gamma_k|$.

However, note that for the equality in relation $\text{Re}\{\mathbf{u}^H \mathbf{d}_k / \gamma_k\} \leq |\mathbf{u}^H \mathbf{d}_k / \gamma_k|$ to hold, it is required that $(\mathbf{u}^H \mathbf{d}_k / \gamma_k)$ is real-valued. According to the definition of $\mathbf{u}$ and $\mathbf{d}_k$ in (6.32), it is required that $(\mathbf{w}^H \vec{\psi}_{k,k}^{(\tau)} / \gamma_k)$ is real-valued for $k = 1, \dots, K$. Here, $\mathbf{w}$ and $\vec{\psi}_{k,k}^{(\tau)}$ are in general complex-valued, and γ_k is also a complex number but only its modulus $|\gamma_k|$ is given as a system parameter. Therefore, in order to make the equality $\text{Re}\{\mathbf{u}^H \mathbf{d}_k / \gamma_k\} = |\mathbf{u}^H \mathbf{d}_k / \gamma_k|$ hold, the phase of γ_k should be the same as that of $\mathbf{w}^H \vec{\psi}_{k,k}^{(\tau)}$. Given this observation, the approximation can be refined iteratively together with the iterative feasibility search. Specifically, at the m th iteration of the feasibility search, if problem (6.36) is feasible and thus a solution to the variable $\mathbf{u}^{(m)}$ is obtained for the current value of $t = t^{(m)}$, then the phase of γ_k can be adjusted as

$$\gamma_k^{(m+1)} = \left| \gamma_k^{(m)} \right| e^{j\phi_k^m}, \quad (6.37)$$

where $\phi_k^{(m)} = \angle(\mathbf{u}^{(m)H} \mathbf{d}_k) = \angle(\mathbf{w}^{(m)H} \vec{\psi}_{k,k}^{(\tau)})$ is the phase of $\mathbf{w}^{(m)H} \vec{\psi}_{k,k}^{(\tau)}$. The bisection search combined with adjusting phase of γ_k is summarized by Algorithm 1.

Algorithm 1 Bisection search combined with adjusting phase of γ_k .

```

1: Initialization: Given  $t_l$  and  $t_u$ , and the optimal value  $t^*$  is assumed to meet  $t_l \leq t^* \leq t_u$ . And Given error tolerance  $\epsilon$ . Iteration number  $m = 0$ .
2: repeat
3:    $m := m + 1$ .
4:    $t^{(m)} := (t_l + t_u)/2$ .
5:   Solve the convex SOC feasibility problem (6.36).
6:   if Problem (6.36) is feasible then
7:      $t_u := t^{(m)}$ .
8:      $\gamma_k^{(m+1)} = |\gamma_k^{(m)}|e^{j\phi_k^{(m)}}$ , where  $\phi_k^{(m)} = \angle(\mathbf{u}^{(m)H}\mathbf{d}_k)$ .
9:   else
10:     $t_l := t^{(m)}$ .
11:   end if
12: until  $(t_l - t_u) \leq \epsilon$ 
    
```

The motivation of Algorithm 1 is to iteratively make $(\mathbf{u}^H \mathbf{d}_k / \gamma_k)$ close to being real-valued. However, a critical problem arises in the procedure of bisection search iteration. Specifically, recall that at an iteration when $t = t^{(m)}$, the strengthened constraint (6.35) makes the feasible region of (6.36) smaller than that of the original problem (6.34). Hence, if the current value $t = t^{(m)}$ makes the problem (6.36) infeasible, this $t^{(m)}$, however, is possible to render a feasible solution to the original problem (6.34), *i.e.*, $t^{(m)} > t^*$. In that case, according to the bisection search procedure shown in Algorithm 1, the lower bound of the updated searching interval for t is $t_l = t^{(m)}$. Consequently, the values of t that is smaller than the current value $t^{(m)}$ but somehow contains the optimal value t^* of the original problem will be discarded, and the subsequent iterations cannot get closer to t^* . That is, there is a performance gap between the approximated and original problem, and the gap is not able to be reduced through iterations.

On the contrary, also due to the strengthened constraint (6.35), it is noted that if a $t^{(m)}$ leads to a feasible solution to the approximated problem (6.36), then this $t^{(m)}$ also makes the original problem (6.34) feasible. This is easily demonstrated: for a given $t^{(m)}$ and a given γ_k if a $\mathbf{u}$ is found to meet $\|\hat{\mathbf{Q}}_k^{1/2} \mathbf{u}\| \leq \sqrt{t^{(m)}} \cdot \text{Re}\{\mathbf{u}^H \mathbf{d}_k / \gamma_k\}$, and then

we can obtain $\|\hat{\mathbf{Q}}_k^{1/2} \mathbf{u}\| \leq \sqrt{t^{(m)}} \cdot \text{Re}\{\mathbf{u}^H \mathbf{d}_k / \gamma_k\} \leq \sqrt{t^{(m)}} \cdot |\mathbf{u}^H \mathbf{d}_k / \gamma_k|$ which satisfies the original constraint. Therefore, if we construct a searching path for t that is feasible for each iteration, we can avoid the possible discarding of the interval of t that contains the optimal t^* . To design such a search path with this property, first we note that if the approximated constraint is satisfied: $\|\hat{\mathbf{Q}}_k^{1/2} \mathbf{u}^{(m)}\| \leq \sqrt{t^{(m)}} \cdot \text{Re}\{\mathbf{u}^{(m)H} \mathbf{d}_k / \gamma_k^{(m)}\}$, we have

$$\left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u}^{(m)} \right\| \leq \sqrt{t^{(m)}} \cdot \left| \mathbf{u}^{(m)H} \mathbf{d}_k / \gamma_k^{(m)} \right| = \sqrt{t^{(m)}} \cdot \left| \mathbf{u}^{(m)H} \mathbf{d}_k / \gamma_k^{(m+1)} \right|, \quad (6.38)$$

where $\gamma_k^{(m+1)}$ is given by (6.37). Therefore, we can scale down $t^{(m)}$ to make the equality hold. More specifically, assume a scaling factor β_k , ($0 < \beta_k \leq 1$), that gives rise to $\|\hat{\mathbf{Q}}_k^{1/2} \mathbf{u}^{(m)}\| = \sqrt{\beta_k t^{(m)}} \cdot |\mathbf{u}^{(m)H} \mathbf{d}_k / \gamma_k^{(m+1)}|$, and thus β_k can be calculated as

$$\beta_k = \frac{\left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u}^{(m)} \right\|^2}{t^{(m)} \cdot \left| \mathbf{u}^{(m)H} \mathbf{d}_k / \gamma_k^{(m+1)} \right|^2}. \quad (6.39)$$

Since for each constraint indexed by k there is a common t , to guarantee that the updated t still makes the constraints feasible for each k , then the value of t can be updated as

$$t^{(m+1)} = (\max_k \beta_k) \cdot t^{(m)}. \quad (6.40)$$

The problem solving procedure discussed above is summarized in Algorithm 2.

Discussion: The advantage of the proposed SOCP based algorithm over the SDP based method adopted by [90]: Although as shown in Section 5.3.3 that the proposed scheme based on the output power minimization is equivalent to the SINR-based formulation considered by [90], the solving algorithm, solution and system performance are different. The relay beamforming problem is also solved via iterative feasibility search in [90], but in each iteration a semidefinite relaxed optimization is to be solved, thus can lead to inaccurate solution. While the proposed SOCP based approach successively modifies the approximated constraint through iteration and meanwhile searches for the optimal t value (optimal value for the original problem formulation).

Algorithm 2 ‘Feasible path’ search combined with adjusting phase of γ_k .

- 1: **Initialization:** Given $t^{(0)}$, and the optimal value t^* is assumed to meet $t^* \leq t^{(0)}$.
And Given error tolerance ϵ . Iteration number $m = 0$.
 - 2: **repeat**
 - 3: $m := m + 1$.
 - 4: $t^{(m)} = t^{(m-1)}$.
 - 5: Solve the SOCP feasibility problem (6.36) for $\mathbf{u}^{(m)}$.
 - 6: $\gamma_k^{(m+1)} = |\gamma_k^{(m)}| e^{j\phi_k^{(m)}}$, where $\phi_k^{(m)} = \angle(\mathbf{u}^{(m)H} \mathbf{d}_k)$.
 - 7: $\beta_k = \|\hat{\mathbf{Q}}_k^{1/2} \mathbf{u}^{(m)}\|^2 / (t^{(m)} \cdot |\mathbf{u}^{(m)H} \mathbf{d}_k / \gamma_k^{(m+1)}|^2)$.
 - 8: $t^{(m+1)} = (\max_k \beta_k) \cdot t^{(m)}$.
 - 9: **until** $(t^{(m)} - t^{(m+1)}) \leq \epsilon$
-

6.4 Minimization of Sum of Output Power

6.4.1 Problem Formulation

In this section, we present another multiple user beamforming scheme. The problem is formulated as

$$\begin{aligned}
 \min_{\mathbf{w}, \vec{\alpha}} \quad & \sum_{i=1}^K (|\alpha_k|^2 \cdot \mathbf{w}^H \tilde{\mathbf{Q}}_k \mathbf{w} + |\alpha_k|^2 \cdot \sigma_v^2) \\
 s.t. \quad & \alpha_k \cdot \mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T = \gamma_k, \quad k = 1, \dots, K \\
 & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0-i}, \quad i = 1, \dots, R \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{6.41}$$

This problem formulation amounts to suppress the output power as an aggregation, and thus it does not focus on balancing the performances among different users, which is achieved by the previous formulation.

By introducing a vector auxiliary variable $\vec{t} = [t_1, \dots, t_K]^T$, we can rewrite the above

problem as

$$\begin{aligned}
 & \min_{\mathbf{w}, \vec{\alpha}, \vec{t}} \sum_{k=1}^K t_k \\
 & s.t. \quad \mathbf{w}^H \tilde{\mathbf{Q}}_k \mathbf{w} + \sigma_v^2 \leq \frac{t_k}{|\alpha_k|^2}, \quad k = 1, \dots, K \\
 & \quad \alpha_k \cdot \mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T = \gamma_k, \quad k = 1, \dots, K \\
 & \quad \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R \\
 & \quad \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{6.42}$$

From the linear constraint of (6.42), we can obtain that $\alpha_k = \gamma_k / (\mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T)$, and substituting it into the first constraint of (6.42) yields

$$\begin{aligned}
 & \min_{\mathbf{w}, \vec{t}} \sum_{k=1}^K t_k \\
 & s.t. \quad \mathbf{w}^H \tilde{\mathbf{Q}}_k \mathbf{w} + \sigma_v^2 \leq t_k \cdot \left| \frac{\mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T}{\gamma_k} \right|^2, \quad k = 1, \dots, K \\
 & \quad \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R \\
 & \quad \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{6.43}$$

Now this optimization problem resembles (6.31) in structure, but the newly introduced auxiliary variable is not a scalar, and thus the one-dimensional bisection method cannot be used. However, the multi-dimensional bisection search is of high computational complexity, and the computational load increases exponentially with respect to the dimension of $\vec{t}$, *i.e.*, the number of user pairs. Nevertheless, we can still use an iterative algorithm that is similar to Algorithm 1 to solve this problem, which will be presented subsequently.

6.4.2 Problem Solution

By using the notations defined in (6.32), the above problem can be approximated as

$$\begin{aligned}
 \min_{\mathbf{u}, \vec{t}} \quad & \sum_{k=1}^K t_k \\
 s.t. \quad & \left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u} \right\| \leq \sqrt{t_k} \cdot \left| \frac{\mathbf{u}^H \mathbf{d}_k}{\gamma_k} \right|, \quad k = 1, \dots, K \\
 & \left\| \mathbf{D}_i^{1/2} \mathbf{u} \right\| \leq \sqrt{P_{0,i}}, \quad i = 1, \dots, R \\
 & \left\| \mathbf{D}^{1/2} \mathbf{u} \right\| \leq \sqrt{P_0} \\
 & \mathbf{u}_{\text{end}} = 1.
 \end{aligned} \tag{6.44}$$

Since there are two sets of optimization variables, $\mathbf{u}$ and $\vec{t}$, we can address this problem in an alternate manner. Specifically, first supposing $\mathbf{u}$ is given and fixed, from (6.44) we have

$$\begin{aligned}
 \min_{\vec{t}} \quad & \sum_{k=1}^K t_k \\
 s.t. \quad & \left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u} \right\| \leq \sqrt{t_k} \cdot \left| \frac{\mathbf{u}^H \mathbf{d}_k}{\gamma_k} \right|, \quad k = 1, \dots, K.
 \end{aligned} \tag{6.45}$$

Note that the t_k 's are independent among all the constraint. Thus it is easily concluded that for the objective to be minimized, the t_k in each constraint should be minimized independently. That is, the t_k 's should make every constraint active. Therefore, give $\mathbf{u}$ the optimal t_k is obtained as

$$t_k = \frac{\left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u}^{(m)} \right\|^2}{\left| \mathbf{u}^{(m)H} \mathbf{d}_k / \gamma_k^{(m+1)} \right|^2}. \tag{6.46}$$

On the other hand, supposing $\vec{t}$ is given and fixed, from (6.44) we have a feasibility problem:

$$\begin{aligned}
 & \text{find } \mathbf{u} \\
 & s.t. \quad \left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{u} \right\| \leq \sqrt{t_k} \cdot \left| \frac{\mathbf{u}^H \mathbf{d}_k}{\gamma_k} \right|, \quad k = 1, \dots, K \\
 & \quad \left\| \mathbf{D}_i^{1/2} \mathbf{u} \right\| \leq \sqrt{P_{0,i}}, \quad i = 1, \dots, R \\
 & \quad \left\| \mathbf{D}^{1/2} \mathbf{u} \right\| \leq \sqrt{P_0} \\
 & \quad \mathbf{u}_{\text{end}} = 1.
 \end{aligned} \tag{6.47}$$

In order to solve this non-convex problem efficiently, again we replace the first set of constraints with the approximated second-order cone constraints as given in (6.35). Then problem (6.47) is turned to a convex SOCP feasibility problem and can be solved efficiently with interior points methods. Furthermore, since the variables $\mathbf{u}$ and $\vec{t}$ are alternately optimized iteratively, then at each iteration after calculating $\mathbf{u}$, the approximated SOCP solution can be refined by updating the phase of γ_k as shown by (6.37).

The procedure discussed above for solving this problem is summarized in Algorithm 3.

Algorithm 3 ‘Feasible path’ search combined with adjusting phase of γ_k .

- 1: **Initialization:** Given $t_k^{(0)}, k = 1, \dots, K$, and the optimal value t^* is assumed to meet $t^* \leq \sum_{k=1}^K t_k^{(0)}$. And Given error tolerance ϵ . Iteration number $m = 0$.
 - 2: **repeat**
 - 3: $m := m + 1$.
 - 4: $t_k^{(m)} = t_k^{(m-1)}$.
 - 5: Solve the SOCP feasibility problem (6.36) for $\mathbf{u}^{(m)}$.
 - 6: $\gamma_k^{(m+1)} = |\gamma_k^{(m)}| e^{j\phi_k^{(m)}}$, where $\phi_k^{(m)} = \angle(\mathbf{u}^{(m)H} \mathbf{d}_k)$.
 - 7: $t_k^{(m+1)} = \|\hat{\mathbf{Q}}_k^{1/2} \mathbf{u}^{(m)}\|^2 / |\mathbf{u}^{(m)H} \mathbf{d}_k / \gamma_k^{(m+1)}|^2$.
 - 8: **until** $\|\vec{t}^{(m)} - \vec{t}^{(m+1)}\| \leq \epsilon$
-

6.5 A Feasible Initialization

In the previously discussed iterative algorithms, we always need feasible starting points, *i.e.*, t_l and t_u for Algorithm 1, $t^{(0)}$ for Algorithm 2, and $t_k^{(0)}$ for Algorithm 3. To provide a feasible initialization, we can, of course, simply adopt a feasibility search method to find a qualified initial point. However, the feasibility search is inefficient since it has to calculate the optimization problem multiple times merely for find a initial value. In this section, we present efficient initialization schemes that can be used for the three proposed algorithms.

6.5.1 Initialization for Algorithms 1 and 2

Recalling the problem (6.30), the optimal α_k 's are in general different from each other, so are γ_k 's. If we assume that all α_k 's are equal and each γ_k is given the value of their common modulus $|\gamma_k| = \hat{\gamma}$, the optimal value of (6.30) will be upperbounded by the optimal value of the following relaxed problem:

$$\begin{aligned}
 \min_{\mathbf{w}, \alpha, \hat{t}} \quad & \hat{t} \\
 s.t. \quad & |\alpha|^2 \cdot \mathbf{w}^H \mathbf{Q}_k \mathbf{w} + |\alpha|^2 \cdot \sigma_v^2 \leq \hat{t}, \quad k = 1, \dots, K \\
 & \alpha \cdot \mathbf{w}^H (\vec{\psi}_{k,k}^{(\tau)})^T = \hat{\gamma}, \quad k = 1, \dots, K \\
 & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R \\
 & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0.
 \end{aligned} \tag{6.48}$$

Therefore, the optimal solution of this problem, $\hat{t}^*$, can be used as an initial value of Algorithms 1 and 2 for solving (6.30). To solve (6.48), following the definition in (6.32)

and further defining notations:

$$\begin{aligned}
 \hat{\mathbf{w}} &= \alpha \cdot \mathbf{w}, \quad \mathbf{v} = [\hat{\mathbf{w}}^T, \alpha]^T, \\
 \mathbf{b} &= [\mathbf{0}_{RL_h \times 1}, \sqrt{P_0}], \\
 \mathbf{b}_i &= [\mathbf{0}_{RL_h \times 1}, \sqrt{P_{0.i}}], \quad i = 1, \dots, R,
 \end{aligned} \tag{6.49}$$

then (6.48) can be rewritten as

$$\begin{aligned}
 \min_{\mathbf{v}, \hat{t}} \quad & \hat{t} \\
 s.t. \quad & \left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{v} \right\| \leq \hat{t}, k = 1, \dots, K \\
 & \mathbf{d}_k^H \mathbf{v} = \hat{\gamma}, k = 1, \dots, K \\
 & \left\| \mathbf{D}_i^{1/2} \mathbf{v} \right\| \leq \mathbf{b}_i^H \mathbf{v}, i = 1, \dots, R \\
 & \left\| \mathbf{D}^{1/2} \mathbf{v} \right\| \leq \mathbf{b}^H \mathbf{v}.
 \end{aligned} \tag{6.50}$$

This is an SOCP convex problem, which can be efficiently solved using interior point methods. Therefore, the initial upper bound t_u in Algorithm 1 and the initial t in Algorithm 2 can be both set as $\hat{t}^*$.

On the other hand, as for the lower bound t_l in Algorithm 1, according to original formulation (6.26) as well as (6.28), a practical initialization can be

$$t_l = \gamma^2 P_s \leq \max_k (|\alpha_k|^2 \cdot \mathbf{w}^H \mathbf{Q}_k \mathbf{w} + |\alpha_k|^2 \cdot \sigma_v^2).$$

6.5.2 Initialization for Algorithm 3

We present two schemes for the initialization of Algorithm 3.

Scheme 1: Just using the initialization obtained previously for Algorithm 1 and 3, a simple initialization is given by

$$\tilde{t}_k = \tilde{t}^* \cdot \mathbf{1}_{K \times 1}.$$

Scheme 2: Similar to the previous subsection, the problem (6.42) can be relaxed to

$$\begin{aligned}
 \min_{\mathbf{v}, \vec{t}} \quad & \sum_{k=1}^K \hat{t}_k \\
 \text{s.t.} \quad & \left\| \hat{\mathbf{Q}}_k^{1/2} \mathbf{v} \right\| \leq \hat{t}_k, k = 1, \dots, K \\
 & \mathbf{d}_k^H \mathbf{v} = \hat{\gamma}, k = 1, \dots, K \\
 & \left\| \mathbf{D}_i^{1/2} \mathbf{v} \right\| \leq \mathbf{b}_i^H \mathbf{v}, i = 1, \dots, R \\
 & \left\| \mathbf{D}^{1/2} \mathbf{v} \right\| \leq \mathbf{b}^H \mathbf{v}.
 \end{aligned} \tag{6.51}$$

This is also an SOCP convex problem, and therefore, the initial t_k in Algorithm 3 can be set as $\hat{t}_k^*$ for $k = 1, \dots, K$.

6.6 Numerical Simulation

In the simulation, the performance of the proposed adaptive delay approach is investigated. Here we consider a network where the number of relays is $R = 10$ (unless otherwise specified) relays. Throughout this section, the relay noise power and destination noise power are assumed to be $\sigma_n^2 = \sigma_v^2 = 1$, and the source power P_s is 10dB higher than the noise power. The coefficients of channel impulse responses are modeled as independent quasi-static Rayleigh fading, and are hence generated as zero-mean complex Gaussian random variables with the exponential power delay profile [97]

$$p(m) = \frac{1}{\sigma_t} \sum_{l=0}^{L_x-1} e^{-m/\sigma_t} \delta(m-l), \tag{6.52}$$

where σ_t characterizes the delay spread, and $L_x \in \{L_f, L_g\}$ denotes the length of the backward channels or the forward channels. All the channels are assumed to be quasi-static and perfectly known by the algorithm, which means that the channels are randomly generated but remain static over a frame time, during which multiple successive symbols

are transmitted. The algorithm calculates the relay weights at the beginning of each frame based on the known CSI (a channel realization), and the same set of weights are used during this frame period.

The SOCP convex problems involved in each iteration of all the three proposed algorithms are solved using the CVX software [100] with MOSEK [101] as the solver.

6.6.1 Output SINR Versus Power Constraint

Figure 6.2 illustrates the performance of the worst SINR among users with respect to individual power constraint. The individual power budget is P_0 and the total power budget is set to be $5P_0$. For comparison, the SINR given by the semidefinite relaxation (SDR) is also shown. As discussed previously in this chapter, due to the relaxation of the original problem, the result given by SDR can be regarded as an upper bound of the worst SINR. As is observed from the figure, the proposed Algorithm 2 attains the upper bound. As expected, Algorithm 2 performs worse than Algorithm 1, since as discussed in Section 6.3.3, the feasibility search strategy of Algorithm 1 is likely to discard the interval that contains the optimal solution. Algorithm 3, which solves the sum output power minimization problem, shows a performance that is better than Algorithm 1, although it does not aim to optimize the worst output SINR.

Viewing the performance comparison from another perspective, Figure 6.3 displays the mean of output SINR among all the users. It is seen that now Algorithm 3 renders a slightly better performance than that of Algorithm 2. This result demonstrates that Algorithm 2, *i.e.*, the maximal output power minimization criterion, gives rise to a more balanced SINR performance among all the users. However, by using Algorithm 3, the output SINR of each user can differ from each other more significantly. It is also observed that the Algorithm 1 has the worst performance in terms of mean output SINR.

Figure 6.2: Worst SINR among users versus individual transmit power P_0 . Number of user pairs $K = 3$, number of relays $R = 10$, filter length $L_h = 3$, decision delay $\tau = 3$.

Figure 6.3: Mean SINR among users versus individual transmit power P_0 . Number of user pairs $K = 3$, number of relays $R = 10$, filter length $L_h = 3$, decision delay $\tau = 3$.

6.6.2 Effects of Decision Delay

Recalling the single-user FF relay beamforming discussed in the previous chapters, therein the decision delay parameter plays an important role since it affects the output SINR significantly. Similarly, we will see that the decision delay is also an important factor with the multi-user case. In Figure 6.4, we display the performance of the worst SINR among users with respect to different decision delays. The individual power budget is P_0 and the total power budget is set to be $5P_0$. From this figure we note that the output SINR varies with different delays. Specifically, starting from $\tau = 0$, the output SINR increases monotonically until it attains a peak at certain value of τ . Then with value of τ increasing after the peak, the output SINR decreases monotonically. That is, there exists an optimal delay, and moreover we note that in this example where the relay filter length $L_h = 3$, the optimal decision delay is $\tau = 2$ or $\tau = 3$. Furthermore, it can also be observed that with different delays the three algorithms maintain the relative performance relation.

As another example, in Figure 6.5 the mean of output SINR is also compared with varying decision delays. We can still see that Algorithm 3 renders a slightly better mean output SINR performance than Algorithm 2. Moreover, to illustrate the computational loads of the proposed algorithms, we show in Figure 6.6 the number of feasibility search iterations. The iteration number can be used as a reasonable measurement since for the three proposed algorithms, the core computation is basically the same, which is to solve (6.36) in each feasibility search iteration. We note that, the iteration number of Algorithm 1 shows a inverse-U shape trend, which is opposite to SINR performance as shown in Figure 6.4 and Figure 6.5. This can be explained as follows. Since Algorithm 1 is a bisection search method, the iteration numbers is completely determined by the initial lower and upper bound of t . Therefore, when the value of decision delay τ increases, as

Figure 6.4: Worst SINR among users versus decision delay τ . Number of user pairs $K = 3$, number of relays $R = 10$, filter length $L_h = 3$.

shown by Figure 6.4 and Figure 6.5, the SINR performances first increase to a peak and then decrease, which means that the output power first decreases to a minimum and then increases, thus making the initial upper bound of bisection method first decrease and then increase. Therefore, the number of iterations needed for bisection method also first decreases and then increases correspondingly.

6.6.3 Effects of Number of User Pairs

In Figure 6.7, Figure 6.8 and Figure 6.9, we display the performance of the worst SINR among users with respect to different numbers of user pairs. We can see that with the increase of user pairs, the performances given by Algorithm 1 and 3 deviate more from that of Algorithm 1. More interestingly, with the increase of user pairs, the worst SINR performance of Algorithm 1 surpasses Algorithm 3. However, as shown in Figure 6.9, now the computational load of Algorithm 1 is constantly lower than that of the other

Figure 6.5: Mean SINR among users versus decision delay τ . Number of user pairs $K = 3$, number of relays $R = 10$, filter length $L_h = 3$.

Figure 6.6: Average number of iterations versus decision delay τ . Number of user pairs $K = 3$, number of relays $R = 10$, filter length $L_h = 3$.

Figure 6.7: Worst SINR among users versus number of user pairs K . Number of relays $R = 10$, filter length $L_h = 3$, decision delay $\tau = 3$.

two algorithms, while in the experiments with the same scenario the SINR performance differences among the three algorithms is always less than 2. Therefore, in this case Algorithm 1 can be used as a efficient suboptimal approximation of Algorithm 2 and 3.

6.6.4 Effects of Number of Relays and Relay Filter Length

In Figure 6.10, Figure 6.11 and Figure 6.12, we display the performance comparison with respect to different numbers of number of relays. As shown in Figure 6.10, when the number of relays is not sufficient, the worst SINR performance of Algorithm 3 is close to Algorithm 1. As the number of relays increases, the worst SINR performance of Algorithm 3 deviates from Algorithm 1 and gets closer to Algorithm 2. More interestingly, as shown by Figure 6.12, with the increasing of relays number, the iterations needed for both Algorithm 2 and Algorithm 3 increase accordingly; however, the iteration number of Algorithm 1 decreases monotonically. This can be explained as follows. Since Algorithm 1

Figure 6.8: Mean SINR among users versus number of user pairs K . Number of relays $R = 10$, filter length $L_h = 3$, decision delay $\tau = 3$.

Figure 6.9: Average number of iterations versus number of user pairs K . Number of relays $R = 10$, filter length $L_h = 3$, decision delay $\tau = 3$.

Figure 6.10: Worst SINR among users versus number of relays R . Number of user pairs $K = 3$, filter length $L_h = 3$, decision delay $\tau = 3$.

is a bisection search method, the iteration numbers is completely determined by the initial lower and upper bound of t . Therefore, when the relay number increases, as shown by Figure 6.10 and Figure 6.11, the SINR performances increase, which means that the output power decreases, thus leading to the decrease of initial upper bound of bisection method. Therefore, the number of iterations needed for bisection method decreases.

In Figure 6.13 and Figure 6.14, we display the performance comparison with respect to different relay filter lengths. The similar trend to the previous experiment can be observed here. The SINR performances increase with respect to the relay filter length, and also a upper bound for the SINR performance for different decision delay is observed. With respect to the computational complexity, we can see from Figure 6.14 that the number of iterations needed for Algorithm 1 also decrease with the increasing of the relay filter length. The explanation is similar to the previous experiment, *i.e.*, the SINR performance increases while the output power decreases.

Figure 6.11: Mean SINR among users versus number of relays R . Number of user pairs $K = 3$, filter length $L_h = 3$, decision delay $\tau = 3$.

Figure 6.12: Average number of iterations versus number of relays R . Number of user pairs $K = 3$, filter length $L_h = 3$, decision delay $\tau = 3$.

Figure 6.13: Worst SINR among users versus relay filter length L_h . Number of user pairs $K = 3$, number of relays $R = 10$, decision delay $\tau = 3$.

Figure 6.14: Average number of iterations versus relay filter length L_h . Number of user pairs $K = 3$, number of relays $R = 10$, decision delay $\tau = 3$.

Figure 6.15: Worst output SINR performance comparison of Algorithm 2 with SDR randomization, for the multi-group multicasting case. Number of sources $K = 3$, number of destinations in each group: 3, number of relays $R = 10$, decision delay $\tau = 3$.

6.6.5 Comparison with SDR Randomization

From the previous experiments, we have seen that the proposed Algorithm 2 always renders the worst SINR performance that is most close to the SDR upper bound. Therefore, in Figure 6.15 the SINR performance comparison of Algorithm 2 with SDR randomization method is illustrated, where for each SDR-based solution 1000 randomization candidate vectors are generated. The experiment scenario is set as the multi-group multicasting, which is considered in [87]. Since there are much more destination nodes, the SDP-based solution is likely to have a rank larger than 1, and then the original problem may not be accurately solved. However, the proposed approach circumvents the SDR and iteratively refined the approximated constraint, and hence the proposed approach is possible to give rise to superior performance.

6.7 Conclusion

In this chapter, we consider designing the FF relay beamforming for multiple peer-to-peer communication in frequency-selective channels. Two approaches are proposed. The former approach aims to minimize the maximal output power at the destination side among all the users, under the constraints of both total and individual relay transmit power. We have demonstrated that the proposed maximal output power minimization formulation is equivalent to the worst SINR maximization problem which leads to a sequence of SDP feasibility problem. While the latter approach aims to minimize the sum of the output power of all the destinations. For solving the proposed formulation problems, three iterative algorithms are provided, which are all based on SOCP feasibility problem with a successive modification of the approximated constraint. Moreover, the proposed algorithms does not suffer the problem encountered by SDP when the number of user pairs is relatively large.

Chapter 7

Conclusion and Future Work

7.1 Conclusion

This thesis has focused on relay beamforming on frequency-selective channels using filter-and-forward relaying scheme. In the following, we briefly review the major results obtained by each chapter of this thesis.

While the filter-and-forward relaying scheme has been introduced in literature to tackle the relay beamforming design on frequency-selective channels, a problem encountered by this beamforming scheme is that the system performance is crucially affected by the decision delay. Hence in Chapter 3, we have addressed the problem of efficiently determining the optimal decision delay. Specifically, by expressing the relay beamforming problem in the frequency domain, we have introduced a distortionless design into the SINR maximization formulation, which has the effect of forcing the equivalent channel from the source to the destination to be frequency-flat and linear phase. Therefore, the parameter of decision delay is conveniently incorporated into the design problem and thus is efficiently optimized. Although additional constraint (distortionless) is imposed, simulation results demonstrated that the performance of the proposed approach is still close to the non-distortionless approach as long as there are a sufficient amount of relays.

To further study the proposed concept of distortionless design, and inspired by the traditional minimum variance distortionless response (MVDR) technique in array signal processing, in Chapter 4 we have proposed an output power minimization scheme for single-user relay beamforming design. Realizing the inherent difference between traditional antenna array beamforming and relay distributed beamforming, we have introduced an adjustable scalar amplifier at the destination node. Then we discuss the relay beamforming design with minimizing output power in two cases, namely, fixed scalar amplifier and adaptive scalar amplifier. For the former case, we derive a range for the parameter of signal gain, within which the signal gain is consistent with the relay transmit power constraint. For the latter case, we consider the general design scheme where both the relay coefficients and the scalar amplifier are jointly optimized to achieve the minimum output power under a signal gain requirement. We have shown that the adaptive-scalar scheme always achieves the same SINR performance as that given by the SINR maximization with distortionless constraint. Furthermore, the scheme of output power minimization is generalized to the multi-antenna destination case, where each antenna at the destination node is followed by a linear FIR filter, which can be jointly designed and work collaboratively with the relay filters.

As stated above, under the distortionless constraint, we have proposed a new relay beamforming design scheme using output power minimization, and have demonstrated its equivalence to the SINR maximization scheme. In Chapter 5, we have extended the new design scheme to cater for the non-distortionless case. Similarly, we have shown that the non-distortionless renders the optimal SINR which is given by the SINR maximization design. However, when dealing with individual relay power constraint, the proposed output power based approach requires significantly lower computational complexity in attaining the optimal SINR performance than the SINR maximization scheme. In the

same chapter we have also presented a time-domain counterpart of the previously defined distortionless constraint, which offers an alternative approach to solve the problem with distortionless design. By using the distortionless constraint, we are able to determine the optimal decision delay within the output power minimization design scheme.

Finally, in Chapter 6, as a natural extension, we have considered the FF relay beamforming for multiple peer-to-peer communication using the idea of output power minimization. For the multi-user design problem, we have proposed two formulations, namely, to minimize the maximal output power among all the users and to minimize the sum output power at all the destinations. We have demonstrated that the former formulation is equivalent to the worst SINR maximization problem. However, using the SDR technique, the worst SINR maximization problem leads to a sequence of SDP feasibility problem, while the proposed formulation is solved by iterative algorithms which are based on SOCP feasibility problem. We have shown that the proposed approach renders better performance than that of the SDR with randomization. On the other hand, the second proposed formulation is shown to give rise to the best performance in terms of average output SINR among users.

7.2 Future Work

Based on the studies in this thesis, several suggestions for the future research work are proposed as follows.

Robustness issues of FF relay beamforming: In this thesis, we have always assumed that the channel state information is completely known. However, in practice the channel information is estimated and inevitably contains estimation errors. Although within the relay beamforming design for frequency-flat channels the robustness issue against channel uncertainty has been discussed in literature, the robustness issues on

channel uncertainty in frequency-selective channels are not extensively studied. Therefore, the robustness related issues such as modeling the frequency-selective relay channels uncertainty, and the robust relay beamforming design that aims at handling the channel error or channel uncertainty, are worth further study.

Implementing adaptive algorithms: We have stated in this thesis that the relay beamforming design on frequency-selective channels can be regarded as a distributed equalization scheme. On the other hand, we note that the conventional equalization techniques in point-to-point wireless communications can be realized in an adaptive way, in order to tackle the conditions such as fast-changing channels and high computational complexity. Thus, a question arises that how we can implement adaptive algorithms to the relay distributed beamforming/equalization design problem.

Adaptive relay filter length: In this thesis, it is always assumed that the filters at all the relay nodes have the same fixed length. However, due to the different qualities of the instant channel realizations that are corresponding to different relays, the relay filter lengths are not necessary to be identical. Therefore, the effects of varying relay filter length on the system performance are worth to be studied, and the design problem for adaptive selection of filter length needs to be addressed correspondingly.

Further study in multi-user case: In this thesis, to deal with the multi-user relay beamforming design problem, we have employed the output SINRs as the core performance evaluation. However, there still exist many other measurements for the global system performance considering the tradeoff among the multiple data streams corresponding to multiple user pairs, which will lead to different design criteria, such as the data rate based, BER based and MSE based criteria. Moreover, based on the consideration of computational loads encountered by the relay networks dealing with multiple users, more efficient algorithms for multi-user relay beamforming are also worth to be explored.

Decision delays in multi-user case: As has been discussed in Chapter 6, the decision delays for all the user pairs are fixed and equal. Due to different channel conditions among user pairs, the decision delay for each user is not necessary to be the same as that of other users, and it should be adaptively selected according to the channel realization just as discussed in Chapter 3 for the single-user case. Therefore, a naturally arises problem is to optimize each user-pair's decision delay in the multi-user relay beamforming design.

Author's Publications

Journal Papers and Manuscripts

- (1) **Tao Wang**, Boon Poh Ng, and Meng Hwa Er, "Frequency-domain approach to relay beamforming with adaptive decision delay for frequency-selective channels," *IEEE Trans. Signal Processing*, vol. 61, no. 22, pp. 5563-5577, Nov. 2013.
- (2) **Tao Wang**, Meng Hwa Er, and Boon Poh Ng, "Multi-user filter-and-forward relay beamforming for frequency-selective channels based on output power minimization," submitted for publication.
- (3) **Tao Wang**, Meng Hwa Er, and Boon Poh Ng, "Filter-and-forward relay beamforming with adaptive decision delay using output power minimization," submitted for publication.
- (4) Wei Chen, Joni Polili Lie, Boon Poh Ng, **Tao Wang**, and Meng Hwa Er, "Joint gain/phase and mutual coupling array calibration technique with single calibrating source", *International Journal of Antennas and Propagation*, Vol. 2012 (2012).

Conference Papers

- (1) **Tao Wang**, Meng Hwa Er, and Boon Poh Ng, "Filter-and-forward relay beamforming using output power minimization," *International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Florence, Italy, May 2014.

- (2) **Tao Wang**, Boon Poh Ng, and Meng Hwa Er, "DOA estimation of amplitude modulated signals with less array sensors than sources," *International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Kyoto, Japan, March 2012.
- (3) **Tao Wang**, Boon Poh Ng, Ying Zhang, and Meng Hwa Er, "Multiple peer-to-peer communications for two-way relay networks," *International Conference on Information, Communications and Signal Processing (ICICS)*, Singapore, Dec. 2011.

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