Two-stream mixing flow with streamwise vorticity

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TWO-STREAM MIXING FLOW WITH STREAMWISE VORTICITY

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Two-stream Mixing Flow with Streamwise Vorticity

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2005

Abstract

ABSTRACT

The aerodynamic characteristics of the single-lobe forced mixer have been extensively examined in the present dissertation. Nine single-lobe models with different parameters and geometry configurations were designed and fabricated. Penetration angles, lobe wavelengths, trailing edge configurations and modifications vary from case to case. Flat plate (to simulate plane free shear layer) and convoluted plate (to generate normal K-H vortex without streamwise vortex) have also been tested as baselines for comparison. Two velocity ratios, i.e., $r = 1:1$ and $0.4:1$ and Reynolds numbers (based on the wavelength of the basic single-lobe forced mixer) ranged from 9,200 to 40,000 have been chosen and tested.

Hot-wire anemometer (HWA) and laser Doppler anemometer (LDA) were employed to evaluate the two different types of vortices, namely Kelvin-Helmholtz (K-H in abbreviation) and streamwise vortices respectively, due to their different characteristics. The K-H vortices are shed periodically from the trailing edge of the model, while the streamwise vortices are spatially stable in the near wake. To characterize the periodical K-H vortices, fast Fourier transform (FFT) was performed on the velocity signals to extract its frequency information. Consecutively the other parameters of the K-H vortices, such as the mean wavelength and non-dimensional frequencies including Roshko number and Strouhal number could be examined at different flow conditions.

It was found that there were two series of K-H vortices shed after the trailing edge of the forced mixer, with both frequencies in proportion to the mean velocity of the oncoming streams. Both of these two frequencies are higher than that of the flat plate case. The Strouhal number increased with Reynolds number, but its increasing ratio decreased

Abstract

gradually. As a result, it will reach certain stable peak value at Reynolds number above 5×10^4 .

The streamwise vortices, as another contributor to the enhanced mixing performance, have been measured using a laser Doppler anemometer. Detailed 3-dimensional velocity fields at different cross-sections in the near wake after the trailing edge have been measured. It was found that at the same lobe wavelength, the initial mean streamwise vorticity was proportional to $\tan(\epsilon)$, where ϵ is the penetration angle. The model with lobe wavelength double the lobe height generated higher streamwise vorticity than the other models at different wavelengths. Semi-circular and rectangular models generated similar streamwise vorticity strength, and both are higher than the triangular model. The scalloping modification was beneficial to the generation of streamwise vorticity by forming additional streamwise vortices, whereas the scarfing modification on the present single-lobe forced mixer would suppress the generation of the streamwise vorticity.

The interaction of the streamwise vortices with the K-H vortices, namely 'pinched-off' effect, contributes to the mixing enhancement by stretching the K-H vortex tube and increasing its vorticity. On the other hand, the evolution of the streamwise vorticity with downstream distance is mainly dominated by the viscous dissipation and the deformation effects due to the K-H vorticity, except in some regions immediately after the mixer trailing edge where the stretching effects due to the streamwise mean flow acceleration taken place. Both of these two types of vortices are contributive to the mixing enhancement.

Key Words: lobed forced mixer, Kelvin-Helmholtz vortex, hot-wire anemometer, streamwise vortex, laser Doppler anemometer, interactions, mixing.

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NOMENCLATURE

Latin Symbols

A_{wake}	Wake Area of the test model, (m^2);
C_I	Normalized Streamwise Circulation, $C_I = \frac{\Gamma_I}{U \cdot \lambda}$;
E	Voltage Output from CTA (Constant Temperature Anemometer), (V);
E_i	Error of Item i (such as the pressure, temperature);
H_{dm}	Velocity Shape Factor of the Boundary Layer, $H_{dm} = \frac{\theta_d}{\theta_m}$;
L	Streamwise Length of the Lobed Forced Mixer, ($L=180$ mm);
L_m	Streamwise Length of the Lobe, ($L_m=74.3$ mm);
M	Averaged Mixedness, $M = \frac{1}{A} \int m \cdot dA$ or $M = \frac{1}{V} \int m \cdot dV$;
P	Static Pressure, (Pa);
P^*	Total Pressure, (Pa);
ΔP	Dynamic Pressure, $\Delta P = P^* - P$, (Pa);
R	Radius of the Lobe Trailing Edge Semi-circle;
Re	Reynolds number, $Re = \frac{\rho \bar{U} \lambda}{\mu} = \frac{\bar{U} \lambda}{\nu}$;
Ro	Roshko number, $Ro = \frac{f \cdot \lambda^2}{\nu}$;
R_0	Gas Constant, (for the air $R_0 = 287.06$ J / Kg · K);
S	Shape Factor, $S = \frac{1}{A_{wake}} \int_{wake} \left(\frac{u}{U}\right)^2 \cdot dA$;
Sc	Schmidt Number, $Sc = \nu / \kappa$;
St	Strouhal number, $St = \frac{f \cdot \lambda}{U} = \frac{Ro}{Re}$;
T	Temperature, (K);
U	Velocity, (m/s);
$\bar{U}, \bar{V}, \bar{W}$	Mean Velocities in the Streamwise, Spanwise and Traverse directions Respectively, (m/s);
u', v', w'	rms (root-mean-square) velocities in the Streamwise, Spanwise and Traverse directions Respectively, (m/s);
U_i	Uncertainty of Item i, (%);
X, Y, Z	Cartesian Coordinate System (streamwise, spanwise and cross-stream directions accordingly), (mm);
f	K-H Vortex Shedding Frequency, (Hz);
h	Height of the Lobe, (mm);

k	Turbulent Kinetic Energy, $k = \frac{1}{2}(u'^2 + v'^2 + w'^2)$, (m ² /s ²);
$\bar{k}$	Normalized Wake-Area Averaged Turbulent Kinetic Energy, $\bar{k} = \frac{1}{(\frac{1}{2}\bar{U}^2) \cdot A_{wake}} \int_{wake} k \cdot dA$;
m	Mixedness;
n	Number of the Lobes;
r	Velocity Ratio, $r = \frac{U_1}{U_2}$;
t	Thickness of the Lobed Forced Mixer, ($t=2\text{mm}$);
u	Local Velocity, (m/s);

Greek Symbols

Γ_I	Mean Streamwise Circulation of the Plane I, $\Gamma_I = \int_{AB} \vec{V} \cdot dy + \int_{BC} \vec{W} \cdot dz + \int_{CD} \vec{V} \cdot dy + \int_{DA} \vec{W} \cdot dz$;
ε	Penetration Angle (Half Divergence Angle) of the Lobed Forced Mixer;
θ_d	Displacement Thickness of the Boundary Layer, $\theta_d = \int_0^\infty (1 - \frac{u}{U}) dz$, (mm);
θ_m	Momentum Thickness of the Boundary Layer, $\theta_m = \int_0^\infty \frac{u}{U} (1 - \frac{u}{U}) dz$, (mm);
λ	Wavelength of the Lobe, (mm);
λ_{K-H}	Average K-H Vortex Wavelength, (mm);
μ	Dynamic Viscosity Coefficient, (for the air at 293K, $\mu = 1.811 \times 10^{-5} \text{ N} \cdot \text{s} / \text{m}^2$);
ν	Kinematic Viscosity Coefficient, $\nu = \mu / \rho$, (for the air at 293K, $\nu = 1.508 \times 10^{-5} \text{ m}^2 / \text{s}$);
ρ	Mass Density, (kg/m ³);
ϕ	Velocity Potential Function, $\vec{U} = \nabla \phi$;
ω	Vorticity, $\vec{\omega} = \nabla \times \vec{U}$, (s ⁻¹).
ω_x	Streamwise Vorticity, $\vec{\omega}_x = (\frac{\partial W}{\partial y} - \frac{\partial V}{\partial z}) \cdot \vec{i}$ or $\omega_x = \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z}$;

Subscripts

1	Upper Stream;
2	Lower Stream;
C	Point C of the Single-lobe Forced Mixer Trailing Edge or the Convuluted Plate (at the center of the semi-circle part of the trailing edge);
L	Point L of the Single-lobe Forced Mixer Trailing Edge or the Convuluted Plate (at the straight sidewall of the trailing edge).

Abbreviations

CP	Convoluted Plate;
FFT	Fast Fourier Transform;
FP	Flat Plate;
HWA	Hot wire Anemometer;
LDA	Laser Doppler Anemometer;
LM	Lobed Forced Mixer;
SNR	Signal to Noise Ratio;
TKE	Turbulent Kinetic Energy;
rms	Root Mean Square.

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Chapter 1. Introduction

1.1 K-H Vortices and Streamwise Vortices

Two-stream mixing is one of the most important and popular topics in the area of fluid mechanics. Dynamic instability exists in the mixing shear layer, and it will usually lead to the generation of spanwise vortices, namely the Kelvin-Helmholtz vortices after the two fluid dynamicists Lord Kelvin (1824~1907) and Hermann von Helmholtz (1821~1894). As shown in **Fig 1.1**, the K-H vortices are shed periodically from the trailing edge, which enhance mixing of the flow in the planes parallel to the direction of the basic flow. The alternating release of a surface vortex and trailing-edge vortex from the suction surface of the wing model in the separation vortex, leading-edge vortex and bluff-body effect regimes causes periodic instabilities in the wake, which are conventionally referred to as ‘vortex shedding’. The term ‘vortex shedding’ was firstly used by Tritton in 1988, but in most of the literature published thereafter, this term has sometimes been employed for all wake instabilities irrespective of how the vortical structures in the wake have evolved.

Another kind of vortices, namely the streamwise vortices, may appear together with the K-H vortices. It is a result of the energy transfer from the basic stream (primary stream) to the secondary stream through viscous mixing. Streamwise vortices mix the flow in the planes perpendicular to the direction of the basic flows. **Fig 1.2** shows typical streamwise vortices formed in the wake behind a triangular tab.

Streamwise vortices greatly enhance mixing performance. For example in the combustion process, they greatly enhance flame propagation and combustion efficiency (McVey 1988, McVey and Kennedy 1989).

To generate strong streamwise vortices, two methods are usually adopted – active control (perturbation method) and passive control (geometry method).

1.1.1 Active Control (Perturbation Method)

Active control is to generate streamwise vortices in a two-stream mixing situation, by perturbing the basic flows either mechanically or acoustically. The successful use of this method has been shown previously, for example, by Corcos (1979), Pierrehumbert and Widnall (1982), Corcos and Sherman (1984) and Corcos and Lin (1984).

Lasheras *et al* (1986) showed that the plane free shear layer was unstable to any small three-dimensional perturbations in the upstream conditions. This instability was found to result in the formation of a well-organized array of streamwise vortices on the braids between consecutive spanwise vortices.

Lasheras and Choi (1988) experimentally studied the three-dimensional instability of a plane free shear layer with small periodical sinusoidal perturbations. Using laser induced fluorescence and direct interface visualization, they observed the formation and evolution of the streamwise vortices. They also found that the characteristic time of growth for the two-dimensional shear instability is much shorter than that of the three-dimensional instability. During the formation of the streamwise vortex tubes, the spanwise vortices (or K-H vortices) maintain, to a great extent, their two-dimensionality, suggesting an almost uncoupled development of both instabilities; while later they will undergo nonlinear interactions with each other.

1.1.2 Passive Control (Geometry Method)

By designing the geometries of the splitter plates properly, there will be strong streamwise vortices generated in the mixing layer. This is because the geometries of the splitter plates will alter the direction of the basic flows. Compared with the active control, there are no moving parts in the passive control method.

Among different types of vortex generators, lobed forced mixers and tabs are two typical examples for the passive control devices to generate strong streamwise vorticity. **Fig 1.2** shows how a pair of counter-rotating streamwise vortices is formed in the wake of a triangular tab. In a continuing effort to increase mixing in free shear flows, vortex generators in the form of tabs have been investigated by many researchers, for example Rogers and Parekh (1994), Bohl and Foss (1996), Zaman (1996), Foss and Zaman (1999), Zaman (1999), Yu and Koh (2001), and Yu *et al* (2001).

Fig 1.3 shows how the streamwise vortex is formed in the wake of lobed forced mixer due to the twisting between streams 1 and 2, which will be explained in more details in Chapter 2. Although the combined effects of the two kinds of vortices, namely the K-H and streamwise vortices have been examined by many researchers, their individual characteristics are still yet to be determined in details, which will be the main focus of the present project.

1.2 Lobed Forced Mixer Flow

Lobed forced mixers are splitter plates with convoluted trailing edges designed to generate large-scale streamwise vorticity between two co-flowing streams. As typical passive vortex generators, their penetration regions are three dimensionally contoured surfaces. One typical example is shown in **Fig 1.4**.

Lobed forced mixers are employed so as to mix the two streams rapidly and efficiently. The parameters affecting the performance of the lobed forced mixers are: the configuration of the trailing edge, the penetration angle (it is linked with the height of the trailing edge), and the wavelength of the lobe. Section 2.2 describes lobed forced mixers in more details.

The trailing edge of the lobed forced mixer can be of different configurations, such as semi-circular, rectangular, triangular and sinusoidal, as shown in **Fig 1.5**.

As shown in existing literature, lobed mixers with parallel sidewalls have better mixing efficiency than others (for instance, Skebe *et al* 1987). In the present investigation, the semi-circular lobed forced mixer with parallel sidewalls would be studied extensively as the basic model for subsequent comparison with other models.

1.3 Mixing and Affecting Factors

1.3.1 Mixing and Mixedness

Mixing should be best defined at the molecular scale, which is much smaller than the scale on which the velocity and other parameters of the fluids are based. The measure of mixing is termed 'mixedness'. Qualitatively, molecular mixedness expresses the degree to which molecular scale mixing at certain point has advanced as a percentage of final molecular scale mixing. Supposing two fluids A and B are to be mixed with mass flow ratio r (in some cases $r=1$ but not always):

$$m_A : m_B = r, \quad (1-1)$$

so the final molecular scale mixing would also be r .

Imagine at certain point M in the flow field, the mass ratio between A and B is η , or

$$m_A : m_B = \eta. \quad (1-2)$$

The simplest scalar mixedness (denoted as m) is supposed to vary linearly with η . The relationship between m and η is shown in the solid lines of **Fig 1.6**. It can be expressed as:

$$m = \begin{cases} \frac{\eta}{r} & (\text{for } 0 \leq \eta \leq r) \\ \frac{\eta-1}{r-1} & (\text{for } r \leq \eta \leq 1) \end{cases}, \quad (1-3)$$

while $0 \leq m \leq 1$.

Here both η and m are the function of the coordinates.

$$\eta = \eta(x, y, z); \quad (1-4)$$

$$m = m(x, y, z). \quad (1-5)$$

The mixedness varies from 0 to 1 when η varies from 0 to r , and after that it returns to 0 when η changes from r to 1.

At any one cross-section, the averaged mixedness (M) is

$$M = \frac{1}{A} \int_A m \cdot dA; \quad (1-6)$$

and within one volume, the mixedness is:

$$M = \frac{1}{V} \int_V m \cdot dV. \quad (1-7)$$

So the problem left is to measure the mixedness at any point.

For the fluid dynamic researchers, the chemical reaction together with optical densitometry technique has widely been employed to measure the mixedness of fluids, mainly in the liquid media. The mixedness at one point is usually defined as the optical density (σ) of the chemical product normalized by the optical density of the fully mixed flow (σ_{final}), i.e.,

$$m = \frac{\sigma}{\sigma_{final}}. \quad (1-8)$$

The variation of m with σ is also schematically shown in dash-dotted line in **Fig 1.6**.

At the point where $\eta = r$, we have $\sigma = \sigma_{final}$, so m (at $\eta = r$) = 1;

At the point where $\eta = r'$ (r' depends on the concentrations of the chemical reactors in the fluids A and B), we have $\sigma = \sigma_{max} \geq \sigma_{final}$, so m (at $\eta = r'$) ≥ 1 at this point. This may be unreasonable for the mixedness definition. So to keep this definition meaningful, it is necessary to keep $\sigma_{max} = \sigma_{final}$, or $r = r'$. That is, the chemical reactors' concentrations should be optimized (usually stoichiometrical but not absolute) to have the maximum

optical density when the fluids A and B are fully mixed with mass ratio r . Thus the mixedness would be more reasonable as shown in dotted line in the figure.

The two examples shown above are two commonly used definitions on scalar mixing. However, the definitions of mixedness vary from case to case, depending on the flow parameters under investigation.

When the momentum is investigated, the mixedness at one point may be defined as (Yuan 1992):

$$m = 1 - \left(\frac{(U_1 - U) - (U - U_2)}{U_1 - U_2} \right)^2, \quad (1-9)$$

where U_1 and U_2 are the velocities of the unmixed streams; while U is the local streamwise velocity at that point.

For the case of a jet engine, the mixedness parameter (shape factor here) for thrust distribution was defined as (Bevilaqua 1974):

$$S = \left(\frac{U}{(U_1 + U_2)/2} \right)^2. \quad (1-10)$$

It should be noted that the shape factor S is either larger or equal to 1. It approaches to unity gradually while the mixing is going on. Actually the momentum distribution is the main interest in this definition. The shape factor will be employed in the present investigation for quantitative mixing evaluation.

1.3.2 Factors Affecting Mixing

It is well known that conduction, convection and radiation are the three elements for heat transfer process. The mixing is based on the mass transfer, which takes the form of diffusion and convection only. Radiation is not available for mass transfer. The governing

equations of heat conduction and mass diffusion are nearly the same, which is known as diffusion equation:

$$\frac{\partial \Psi}{\partial t} = \kappa \nabla^2 \Psi, \quad (1-11)$$

where Ψ is the distribution of temperature or mass concentration:

$$\Psi = \Psi(x, y, z, t), \quad (1-12)$$

and κ is thermal conductivity (heat transfer) or diffusion coefficient (mass transfer). For the gas, the diffusion coefficient is usually related to the temperature approximately as:

$$\kappa = A \cdot T^s, \quad (1-13)$$

in which A and s are parameterization constants.

The following analysis, originally applied to plane shear layer, makes no assumptions about the specific orientation of fluid motions. Therefore, it is assumed to be applicable to non-planar shear layers and flows with streamwise vorticity. The mixing process is characterized as a series of steps in a model due to Broadwell (1982). The first step is entrainment of pure irrotational fluid into the mixing layer, a Reynolds number independent process. The subsequent steps consist of the breakdown of eddies to successively smaller scales until the Kolomogorov microscale, the length scale at which viscous dissipation takes place, is reached. The dependence of molecular mixing on Reynolds number is determined by the ratio T : the characteristic time of diffusion across the microscale to the characteristic time of large scale entrainment. Broadwell (1982) estimated that this ratio varies with Reynolds number and Schmidt number as:

$$T \sim \frac{Sc}{\sqrt{Re_\delta}}. \quad (1-14)$$

Here the Schmidt number is proportional to {(kinematic viscosity) / (molecular diffusivity)} and is used in mass transfer in general and diffusion in flowing systems calculations in particular. It is generally defined in the following form:

$$Sc = \nu / \kappa, \quad (1-15)$$

where the diffusion coefficient is given in equation (1-13).

The Reynolds number here

$$Re_{\delta} = \frac{(U_2 - U_1) \cdot \delta}{\nu} \quad (1-16)$$

is based on the velocity difference across the layer ($U_2 - U_1$) and the entrainment length scale δ .

So in flows where large scale entrainment times are much larger than microscale diffusion times ($T \ll 1$), the mixing is said to be entrainment-limited: molecular mixing rates are dictated by entrainment rates and thus, are Reynolds number independent. Molecular mixing in this case becomes a good indicator of mixing at bulk scales.

While for $T \gg 1$, mixing is diffusion-limited. The microscale is reached before diffusion advances appreciably, causing molecular mixing to be sensitive to small scales, and thus, to be Reynolds number dependent.

In most aerospace applications, $Sc \sim 1$ (for air) and $Re_{\delta} \gg 1$ such that $T \ll 1$. As a result the molecular mixing is Reynolds number independent. It was therefore concluded that in order to ensure mixing results of practical interest, i.e., Reynolds number independent, it would be necessary to specifically monitor the dependence of molecular mixing on Reynolds number. This would determine the velocities necessary to produce Reynolds number independent results.

Although the wind tunnel employed in the present investigation is a low-speed tunnel, the Reynolds number defined by equation (1-16) is far above 1 and the ratio $T \ll 1$. This means the present mixing on forced mixer should be Reynolds number independent. The contribution due to molecular diffusion can be neglected, comparing with the contribution of the large scale entrainment. The large scale convection within turbulent mixing is mainly dominated by the vortex dynamics and other related turbulent structures. To examine the mixing performance of the present forced mixer, it may be reasonable to focus more on the characteristics of the different types of vortices and their interactions.

1.4 Objective

The objective of the present investigation is to examine the mixing characteristics of the lobed forced mixer flow with different flow conditions and geometric parameters. Some characteristics of the plane free shear layer have also been studied and used as a basis for comparison.

Since both the K-H vortices and the streamwise vortices play important roles in the mixing performances, it is necessary to examine their dynamic characteristics respectively in details.

1.4.1 Hot-wire Anemometer on K-H vortices

For the K-H vortices, instantaneous velocity measurements via a cross hot-wire probe together with spectral analysis method were used. The following information can be provided:

- (1) The existence of the K-H vortices and their breakdown location.

The existence of the K-H vortices can be detected through the peak(s) on energy spectrum. After a certain distance from the trailing edge, the K-H vortices will breakdown into smaller vortices, and at the same time the flow becomes turbulent. In the present investigation, the breakdown location of the K-H vortices is defined at the location where the peak on the spectrum is no longer visible.

- (2) The K-H vortex shedding frequency.

The frequency of the K-H vortices can be obtained from the dominant peak(s) in the spectrum. Sometimes there may be more than one peak in one spectrum, because of nonlinear effects.

(3) The average wavelength of the K-H vortices.

For the average streamwise wavelengths of the K-H vortices, they can be evaluated from their frequencies using the following equation (1-17).

$$\lambda_{K-H} = \bar{U} / f \quad (1-17)$$

(4) The parameters that affect the vortex shedding frequencies and their corresponding sizes;

The Reynolds number and velocity ratio are varied so as to determine the vortex shedding frequencies under different flow conditions.

(5) The relationships between non-dimensional frequencies (such as Roshko number and Strouhal number) and the Reynolds number:

$$Re = \frac{\bar{U} \cdot \lambda}{\nu} \quad (1-18)$$

Through the studies on the relationships between non-dimensional frequencies and the Reynolds number, the dynamic characteristics of the K-H vortices in the near wake of the lobed forced mixer flow can be determined.

1.4.2 Laser Doppler Anemometer on Streamwise Vortices

For the streamwise vortices, they are spatially stable and have no shedding frequency. LDA (laser Doppler anemometry) was used to measure the three dimensional velocity fields so as to study the streamwise vortices.

(1) Streamwise velocities were measured, so as to reveal the momentum exchange between the two streams. Furthermore the growth of the momentum thickness and the shape factor at downstream stations can be used to partially qualify the effectiveness of the mixing performance.

(2) Time-averaged velocity vectors of the secondary flow can be plotted to visualize how the streamwise vortices are formed and decayed.

(3) Streamwise vorticity contours can be calculated and plotted so as to quantify the magnitudes of the streamwise vortices and provide a quantitative understanding of the wake region.

(4) Mean streamwise vorticity, or circulation can be calculated and their trends can be studied.

(5) The Reynolds normal stresses, together with the turbulent kinetic energies can be evaluated from the LDA root-mean-square (rms) velocity results.

The LDA system mainly measures the time-averaged velocities. So the K-H vortices will not affect the LDA results on the streamwise vortices, because the K-H vortices are unstable and their time-averaged value is almost negligible. The Fourier transform of the HWA results cannot capture the spatially stable streamwise vortices. These make the HWA and LDA effective to evaluate the K-H and streamwise vortices respectively.

There are altogether eleven models tested in the present thesis. They are: 1. flat plate; 2. basic semi-circular single-lobe forced mixer (model a); 3,4. semi-circular single-lobe forced mixers at different height (models b & c); 5. convoluted plate (model d); 6,7. semi-circular single-lobe forced mixers at different wavelength (model e & f); 8. rectangular single-lobe forced mixer (model g); 9. triangular single-lobe forced mixer (model h); 10. scalloped single-lobe forced mixer (model i), and 11. scarfed single-lobe forced mixer (model j). They will be elaborated further in Section 3.2.

1.5 Layout of the Thesis

The thesis is organized as below:

Chapter 1 gives a brief introduction on the topic.

Chapter 2 provides a brief review on the previous investigations on the plane free shear layer and lobed forced mixers.

Chapter 3 presents the experimental facilities, test models and measurement techniques used in the present investigation. The uncertainties of the HWA and LDA measurements are evaluated afterwards.

Chapter 4 analyzes the effects of the penetration angle on the semi-circular single-lobe forced mixer. The flat plate, as one special case of zero penetration angle, will be tested briefly for one baseline; while the basic model will be evaluated in details in this chapter.

Chapter 5 presents the effects of the lobe wavelength on the semi-circular single-lobe forced mixer. Besides, the effects of the convoluted plate will also be analyzed in this chapter.

Chapter 6 explores the effects of the configuration and the modifications on the single-lobe forced mixer. The rectangular, triangular, scalloped and scarfed mixers are investigated.

Chapter 7 lists the conclusions from the experiments and puts forward some suggestions for improving the design of the lobed forced mixers.

Chapter 8 recommends some possible future work.

Figure 1.1 Kelvin-Helmholtz Vortices in Shear Layer

Figure 1.2 Contra-Rotating Streamwise Vortices Formed behind a Triangular Mixing Tab Inclined at 135° (placed in the on-coming stream)

Figure 1.3 Streamwise Vortices Generated Behind a Lobed Forced Mixer

Figure 1.4 Geometry of a Multi-Lobe Forced Mixer with Three Lobes (with parallel side-walls and semi-circular trailing-edge)

Figure 1.5 Different Trailing Edge Configurations for the Lobed Forced Mixer

Figure 1.6 Mixedness Definition at one Point

Chapter 2. Literature Review

In this chapter, previous investigations on plane free shear layer and lobed forced mixer flow will be briefly reviewed.

2.1 Plane Free Shear Layer

Instability is a property of the state of a system such that certain disturbance or perturbation introduced into the system will increase in magnitude. The maximum perturbation amplitude is always larger than the initial amplitude. For the instability of free shear layer, Helmholtz (1868) and Kelvin (1871) studied this problem as early as 1860s. In a plane free shear layer, large-scale spanwise vortices were observed over a wide range of Reynolds numbers. It is noticed that even when the Reynolds number is as high as 10^7 , the coherent structure in the free shear layer could still be observed clearly, as already shown in **Fig 1.1**. It is well known that the plane free shear layers are unstable. This kind of instability is commonly referred as ‘Kelvin-Helmholtz Shear Instability’ after the two fluid dynamicists who discovered it. The velocity and vorticity distributions for this shear flow are shown schematically in **Fig 2.1**.

The mechanism of instability is not yet fully understood, but there is certainly a connection with the nonlinear character of the equations of motion. The K-H instability is interesting, because it appears when there is an inflection point in the velocity profile. The mechanism for this kind of instability may be briefly described in the following manner. Let us consider a longitudinal strip of rotational fluid, separating the two irrotational regions as shown in **Fig 2.2(a)**. Supposing that this rotational zone is perturbed with a longitudinal wave. First, pressure differences between the two layers are

responsible for the growth of the perturbation amplitude (**Fig 2.2(b)**). Second, the crests of disturbance at the top of the interface and the troughs of disturbance at the bottom of the interface travel in opposite directions (**Fig 2.2(c)**). Velocity-induction will transform the sheet into a spiral as shown in **Fig 2.2(d)**. This spiral is usually referred to as Kelvin-Helmholtz cat-eye (also shown in **Fig 1.1**). Because the cat-eyes strain the interface of the two fluids, they are much contributive to the mixing enhancement.

2.1.1 Four Zones in the Plane Free Shear Layer

According to Sato (1959) and Freymuth (1966), four zones can be identified when a plane free shear layer becomes unstable (see **Fig 2.3**):

- I . The place of origin of the shear layer, that is, the location at which the boundary layer separates from the surface.
- II . The zone at which small disturbances of the shear layer grow (range of ‘linear’ growth). The disturbance can occur accidentally through environmental effects or it can be produced artificially by the sound from a loudspeaker or by a vibrating ribbon with a frequency f .
- III . The zone in which the growing disturbances cause the shear layer to roll up into discrete vortices. Through nonlinearity higher harmonics $2f$, $3f$, etc., with smaller amplitudes than the basic harmonic occur (Michalke 1972).
- IV . The zone where these vortices merge through ‘pairing’; in turn, these can be pairing of those vortices that have already ‘paired’. This successive pairing process continues until an apparent disintegration to turbulence sets in. Pairing is a process in which two adjacent vortices of the same rotary direction spin around

each other, come closer, and then merge. Such ‘coalescence’ results in the appearance of a subharmonic $f/2$ (Freymuth 1966 and Zaman and Hussain 1980).

2.1.2 Orr-Sommerfeld and Rayleigh Equations

For the linear stability analysis of 2-dimensional parallel shear flows such as the boundary layer, plane Poiseuille flow, Hagen-Poiseuille flow, plane mixing layer, wake and jet flow, the most commonly used governing equation is the Orr-Sommerfeld Equation.

For the velocity of basic parallel shear flow,

$$\vec{U} = U(z) \cdot i \quad (z_1 \leq z \leq z_2), \quad (2-1)$$

the Orr-Sommerfeld Equation is:

$$(i\alpha \text{Re})^{-1} (D^2 - \alpha^2)^2 \phi = (U - c)(D^2 - \alpha^2)\phi - U''\phi. \quad (2-2)$$

where

α – wave number, and it is a real number;

Re – Reynolds number of the basic flow;

$D = d/dz$; $D^2 = d^2/dz^2$;

ϕ -- flow function of the 2-D disturbance;

c – wave velocity, and it is a complex number, $c = c_r + ic_i$.

The criteria for stability are:

- If $\alpha \cdot c_i < 0$, flow is stable;
- If $\alpha \cdot c_i = 0$, flow is critically stable.
- If $\alpha \cdot c_i > 0$, flow is unstable;

For non-viscous presupposition, because the Re number will be infinite, the equation becomes Rayleigh Equation:

$$(U - c)(D^2 - \alpha^2)\phi - U''\phi = 0 \quad (2-3)$$

From this equation we can deduce two famous theories about the inviscid instability, namely the Rayleigh Inflection Point Theory and the Fjrtoft Theory, as described below.

For the inviscid parallel flow ($U=U_z$, $z_1 < z < z_2$), the inflection point is the prerequisite of instability, as Rayleigh Inflection Point Theory states.

$$U'' = 0 \quad (2-4)$$

Later Fjrtoft provided a more accurate prerequisite of instability, that is:

$$U''(U - U_s) < 0, \quad (z_1 < z < z_2) \quad (2-5)$$

in which U_s is the velocity of the inflection point,

$$U_s = U(z = z_s). \quad (2-6)$$

2.1.3 Instability of Plane Free Shear Layer

Helmholtz (1868) and Kelvin (1871) are the first two scientists who studied the instability of inviscid plane free shear layer. The linear Normal Mode has been adopted to analyze the evolution of the small disturbances by many scientists such as Orr (1907), Sommerfeld (1908), Heisenberg (1924), Tollmien (1929), Lin (1945, 1955) and Betchev & Criminale (1967). However, the disturbances would grow non-linearly in the third zone (see section 2.1.1) of the parallel shear flow. In the history, since Landau (1944) pointed out the non-linearity effects, Malkus & Veronis (1958), Stuart (1960), Watson (1960), Zhou H. (1982) all developed different modes in their analyses.

Here we consider two unidirectional stream mixing. The plane free shear layer is shown in **Fig 2.4**. The coordinates X, Y, Z denote the streamwise, spanwise and cross-stream (vertical here) directions respectively. The free shear layer is $X-Y$ ($x > 0$) half plane.

Under these flow conditions, supposing

$$U(z > 0) = U_1; \quad (2-7)$$

$$U(z < 0) = U_2. \quad (2-8)$$

The linear instability of non-viscous and incompressible flow can be theoretically analyzed as follows. Consider these two flows are of different density,

$$\rho(z > 0) = \rho_1; \quad (2-9)$$

$$\rho(z < 0) = \rho_2. \quad (2-10)$$

ζ is the interface of these two flows. The small disturbance is located on either side of ζ interface. Because of analyzing the instability here, the small disturbance can be assumed to be non-vortex. So the velocity potential functions ϕ can be expressed like these:

$$\phi(z > \zeta) = \phi_1; \quad (2-11)$$

$$\phi(z < \zeta) = \phi_2, \quad (2-12)$$

And because of incompressibility, they satisfy Laplace Equation:

$$\Delta\phi_1 = 0, \quad (z > \zeta); \quad (2-13)$$

$$\Delta\phi_2 = 0, \quad (z < \zeta). \quad (2-14)$$

(1) Boundary Conditions.

Because the disturbance is in limited district,

$$\nabla\phi(z \rightarrow +\infty) = U_1; \quad (2-15)$$

$$\nabla\phi \quad (z \rightarrow -\infty) = U_2. \quad (2-16)$$

(2) In another aspect, because the vortex plane ζ is the interface of the flows, suppose:

$$F(x, z, t) = z - \zeta(x, z, t) = 0. \quad (2-17)$$

We can get:

$$\frac{DF}{Dt} = 0. \quad (2-18)$$

in details, that is:

$$\frac{\partial\phi}{\partial z} = \frac{\partial\zeta}{\partial t} + \frac{\partial\phi}{\partial x} \frac{\partial\zeta}{\partial x} + \frac{\partial\phi}{\partial y} \frac{\partial\zeta}{\partial y}. \quad (2-19)$$

(3) Dynamics requirement.

Because of non-viscous condition, the surface tension can be neglected. So the pressures of the upper and lower side of the ζ plane would be the same:

$$\rho_1(c_1 - \frac{1}{2}(\nabla\phi_1)^2 - \frac{\partial\phi_1}{\partial t} - g\zeta) = \rho_2(c_2 - \frac{1}{2}(\nabla\phi_2)^2 - \frac{\partial\phi_2}{\partial t} - g\zeta). \quad (2-20)$$

where c_1 and c_2 are integration constants.

This relationship also fits the basic flows,

$$\rho_1(c_1 - \frac{1}{2}U_1^2) = \rho_2(c_2 - \frac{1}{2}U_2^2). \quad (2-21)$$

As linear principle, the velocity potential function can be assumed to be composed of the basic velocity potential function together with the small disturbance potential:

$$\phi_1 = U_1 x + \phi_1' \quad (z > \zeta); \quad (2-22)$$

$$\phi_2 = U_2 x + \phi_2' \quad (z < \zeta), \quad (2-23)$$

The linear disturbance function and boundary conditions can be written as:

$$\Delta\phi_1' = 0; \quad (2-24)$$

$$\Delta\phi_2' = 0; \quad (2-25)$$

$$\nabla\phi_1'(z \rightarrow +\infty) = 0; \quad (2-26)$$

$$\nabla\phi_2'(z \rightarrow -\infty) = 0. \quad (2-27)$$

$$\frac{\partial\phi_1'}{\partial z} = \frac{\partial\zeta}{\partial t} + U_1 \frac{\partial\zeta}{\partial x} \quad (z = 0); \quad (2-28)$$

$$\frac{\partial\phi_2'}{\partial z} = \frac{\partial\zeta}{\partial t} + U_2 \frac{\partial\zeta}{\partial x} \quad (z = 0). \quad (2-29)$$

$$\rho_1(U_1 \frac{\partial\phi_1'}{\partial x} + \frac{\partial\phi_1'}{\partial t} + g\zeta) = \rho_2(U_2 \frac{\partial\phi_2'}{\partial x} + \frac{\partial\phi_2'}{\partial t} + g\zeta) \quad (z = 0). \quad (2-30)$$

Adopt normal mode solution, and suppose

$$(\zeta, \phi_1', \phi_2') = (\hat{\zeta}, \hat{\phi}_1', \hat{\phi}_2') \exp(i(\alpha x + \beta y - st)). \quad (2-31)$$

Substituting into (2-24),

$$\hat{\phi}_1' = A_1 \exp(-\tilde{k} z) + B_1 \exp(\tilde{k} z), \quad (2-32)$$

where A_1 and B_1 are arbitrary constants; and total wave number

$$\tilde{k} = (\alpha^2 + \beta^2)^{1/2}. \quad (2-33)$$

Substituting boundary condition (2-26),

$$B_1 = 0; \quad (2-34)$$

and

$$\hat{\phi}_1' = A_1 \exp(-\tilde{k} z). \quad (2-35)$$

Similarly

$$\hat{\phi}_2' = B_2 \exp(\tilde{k} z). \quad (2-36)$$

The eigenvalue 's' can be calculated. It is:

$$s = -i\tilde{k} \frac{\rho_1 U_1 + \rho_2 U_2}{\rho_1 + \rho_2} \pm \left(\frac{\tilde{k}^2 \rho_1 \rho_2 (U_1 - U_2)^2}{(\rho_1 + \rho_2)^2} - \frac{\tilde{k} g(\rho_1 - \rho_2)}{\rho_1 + \rho_2} \right)^{\frac{1}{2}}. \quad (2-37)$$

so:

$$\text{if } \frac{\tilde{k}^2 \rho_1 \rho_2 (U_1 - U_2)^2}{(\rho_1 + \rho_2)^2} - \frac{\tilde{k} g(\rho_1 - \rho_2)}{\rho_1 + \rho_2} \leq 0, \quad (2-38)$$

the flow is stable;

$$\text{and if } \frac{\tilde{k}^2 \rho_1 \rho_2 (U_1 - U_2)^2}{(\rho_1 + \rho_2)^2} - \frac{\tilde{k} g(\rho_1 - \rho_2)}{\rho_1 + \rho_2} > 0, \quad (2-39)$$

the flow is unstable.

That is,

$$\tilde{k} g(\rho_1^2 - \rho_2^2) > \tilde{k}^2 \rho_1 \rho_2 (U_1 - U_2)^2 \text{ -- Stable;} \quad (2-40)$$

$$\tilde{k} g(\rho_1^2 - \rho_2^2) = \tilde{k}^2 \rho_1 \rho_2 (U_1 - U_2)^2 \text{ -- Critically Stable;} \quad (2-41)$$

$$\tilde{k} g(\rho_1^2 - \rho_2^2) < \tilde{k}^2 \rho_1 \rho_2 (U_1 - U_2)^2 \text{ -- Unstable.} \quad (2-42)$$

It can be seen that for the basic flows U_1 and U_2 , if

$$U_1 \neq U_2, \quad (2-43)$$

the flow will be unstable, for a disturbance with a large wave number $\tilde{k}$ always can be found, and it is large enough to satisfy inequality (2-42).

From the analysis above it can be seen that the plane free shear layer with velocity difference between the upper and lower streams is always unstable.

By the way, for the case

$$U_1 = U_2, \quad (2-44)$$

the flow will be unstable for the case

$$\rho_1 > \rho_2. \quad (2-45)$$

This kind of instability is known as Gravity Instability, or Rayleigh-Taylor Instability.

In fact, besides the primary 2-D vortical structure, it has long been observed that the plane free shear layer also contains a well-organized array of streamwise vortices that superimpose onto the spanwise vortices, as shown in **Fig 2.5**. The presence of this secondary structure was observed by Konard (1976) and later by Breidenthal (1978, 1980 and 1981) and Jimenez (1983), among others. Bernal (1981) presented conclusive evidence that the streamwise streaks (previously observed in the plane views of the gas and liquid mixing layers of Konard and Breidenthal) were actually counter-rotating pairs of axial vortices superimposed onto the spanwise structure. Jimenez *et al* (1985) confirmed Bernal's findings through a three-dimensional reconstruction via digital image processing of motion pictures of a plane turbulent mixing layer. Their 3-D graphics reconstruction showed that after the transition to three-dimensionality, the mixing layer exhibited an array of counter-rotating pairs of streamwise vortices.

2.2 Lobed Forced Mixer

Lobed forced mixer is mainly used to enhance mixing between two unidirectional streams efficiently, especially on the aero-fan engine exhausts as an efficient ejector. An ejector is a fluid dynamic pump involving no moving parts. Energy is transferred from the primary stream to the secondary stream through viscous mixing. While ejectors have tremendous potential in aircraft application based on analytical predictions, the ability to implement them in effective system applications remains limited. One major reason is the slow mixing rate associated with conventional ejectors. Conventional ejectors rely on shear layer mixing, which requires long mixing ducts. This may increase the wall friction losses, duct weight and costs.

Since 1970's, many turbo-fan engines, such as RB211-524G/H (UK), CFM56-5c-2 (USA), F404-GE-F1D2 (USA), have adopted the lobed forced mixers onto their exhausts to mix the core and bypass flows together more effectively in the shortest possible distance with minimum pressure losses (Shumpert 1980). This kind of ejectors will:

- enhance the thrust of the engines by 5% (data from JT8D-17);
- reduce the length of the exhausters;
- reduce the fuel consumption by 2.5-3.0% (Kuchar and Chamberlin 1980);
- reduce the noise by 3.8dB (data from JT8D-17) (Crouch *et al* 1976);
- reduce the infrared radiation of the stealth planes by 95% (data from SA-365; MTR-390).

This application of lobed forced mixer in the turbo-fan engine ejectors have been studied in the past, as for example, by Paterson (1982 and 1984), Presz *et al* (1987 and 1988), Barber *et al* (1988), Koutomo and McGuirk (1989), Malecki *et al* (1990), Presz *et*

al (1994), Yu *et al* (1993, 1994, 1995, 1997 and 1999), Yu (1996), Yu and Xu (1997 and 1998), Yu and Yip (1997 a and b). However, in these investigations, mainly only the gross features of the flow field were investigated.

2.2.1 Mixing Mechanism of the Lobed Forced Mixer

It is clear that both the enhanced mixing areas and the two kinds of vorticity, Kelvin-Helmholtz vorticity (normal vorticity, or spanwise vorticity) and streamwise vorticity are all contributive to the enhanced mixing characteristics of lobed forced mixer.

2.2.1.1 Enhanced Mixing Interfacial Areas

As it is very clear, because of the convoluted trailing edge of the lobed forced mixer, the mixing area for the two coming flows will be enhanced. The extent for the enhanced area is closely related to the geometry of the trailing edge. For the four typical different trailing edges of the lobed forced mixer shown in **Fig 1.4**, the percentages of the enhanced mixing area (Δ) are listed here: (comparing with the flat splitter plate trailing edge)

- a. Semi-circular Trailing Edge, $\Delta = 4h/\lambda + (\pi/2 - 1)$;
- b. Rectangular Trailing Edge, $\Delta = 4h/\lambda + 1$;
- c. Triangular Trailing Edge, $\Delta = \csc(\beta/2) = \sqrt{(4h/\lambda)^2 + 1}$;
- d. Sinusoidal Trailing Edge, $\Delta = \int_0^\lambda (1 + \frac{4\pi^2 h^2}{\lambda^2} \cos^2(\frac{2\pi x}{\lambda}))^{1/2} dx$.

Here it should be emphasized that because the enhanced mixing area is not the only reason that leads to the enhanced mixing performance of the lobed forced mixer, the

geometry of the lobed forced mixer should not be judged only based on the enhanced mixing area (Δ). In fact it has been experimentally shown by Yu, Yeo and Teh (1995) that those with rectangular and semi-circular trailing edge works better for overall mixing performance.

2.2.1.2 Two Kinds of Vortices after the Trailing Edge

There are two kinds of vortices generated after the trailing edge of the lobed forced mixer, namely the Kelvin-Helmholtz vortices (or spanwise vortices, normal vortices) and the streamwise vortices, as shown in **Fig 2.6** schematically.

(a) Kelvin-Helmholtz Vortices (Spanwise, Normal Vortices)

K-H vortices are generated mainly because of the velocity difference between the upper and lower streams. This instability mechanism has been briefly described earlier in Section 2.1.2. For the same Reynolds number (or the mean velocity), the more the velocity difference (or the velocity ratio) the stronger the K-H vorticity would become.

The boundary layers and the geometry of the lobe may also affect the generation of the K-H vortices. The upper and lower boundary layers are critical initial conditions for the flow development inside the wake. The lobe geometry will change the local velocity, as shown in our experimental results later.

Due to the velocity difference between the upper and lower streams ($U_1 \neq U_2$), the K-H vortex tube shed from the trailing edge of the lobed forced mixer will become more and more inclined in the further downstream stations, as shown in **Fig 2.6 (a)**. This angle mainly depends on the relative magnitude of the two velocities. But for the velocity

matched case ($U_1=U_2$), the initial K-H vortex tube will not turn into the inclined situation throughout its evolution progress.

(b) Streamwise Vortices

Streamwise vortices are generated inviscidly, due to the unique geometry of the lobed forced mixer. As shown in **Fig 1.2** earlier, when the on-coming streams flowing through the lobed forced mixer, some streamlines, for example streams 1 and 2, will change their directions because of the blockage effects of the lobes. After the trailing edge, streams 1 and 2 will twist together to form a vortex in the streamwise direction.

(c) Interactions between the Two Kinds of Vortices

As a consequence of streamwise vortices, the K-H vortices at a small distance downstream of the trailing edge would be bulged at the lobe troughs and eventually pinched-off with adjacent vertical segments, as shown in **Fig 2.6(b)**. As a result of the skewing of the normal vortices at an angle relative to the lobed forced mixer exit plane, it is likely that the normal vortices would in turn provide similar pinch-off (squeezing) effects to the two rows of streamwise vortices with a lobe on the horizontal plane, as shown in **Fig 2.6(c)**. The consequence of this pinch-off effect would be the amalgamation and mutual annihilation of the streamwise vortices of alternate signs within one lobe (Yu and Yip 1997 a).

2.2.2 Previous Works on Lobed Forced Mixer

In fact, the rudiment of the lobed forced mixer was firstly put forward at the end of 1960's, for its mixing performance began to be considered in the application to aerofan-

engines. At that time, it was believed that the enhanced mixing area was the only reason for mixing enhancement.

In the end of 1970's, Bevilaqua (1978), Povinelli *et al* (1980) and Paterson (1982) found that there were strong secondary flows appeared in the wake of lobed forced mixer. The secondary flows generated played an important role in the mixing process. Since then the research on this kind of mixer began to flourish. Povinelli *et al* (1980) used different kinds of 3-D turbulent viscous models to calculate the flow field of a lobed forced mixer with 18 lobes. They found that the secondary flow was generated inviscidly, because the calculated results were found not sensitive to the different turbulent models used. Paterson (1982) used LDV to measure the 3-D velocity fields of the flow, and found that there were large-scale streamwise vorticity in the wake flow. He also concluded that both the lobe shape and its penetration angle were important parameters in determining the effectiveness of mixer performance. These results were further confirmed by the subsequent experimental investigations by Skebe *et al* (1988) and Barber *et al* (1988), and the computational studies by O'Sullivan *et al* (1996). Skebe *et al* (1988) considered how the shape of the lobe affects the strength of the secondary flow, and found that the parallel sidewalls were better than triangular ones for mixing and that the strength of the secondary flow was in proportion to the height and the penetration angle of the lobe. Similarly, Barber *et al* (1988) found that the strength of the streamwise vortices was higher when the lobe penetration region consisted of straight parallel sidewalls, and higher strength for the streamwise vorticity would result in a faster mixing rate downstream.

The water tunnel visualization tests of Werle *et al* (1987) and Manning (1991) have

shown that both the streamwise vorticity and the accompanying increase in initial interfacial area were significant contributors to the mixing enhancement of lobed forced mixers. The enhancement of mixing was also found to be stronger at higher velocity ratios. The experimental investigations by Eckerle *et al* (1992) and the computational studies by Tsui and Wu (1996) both supported the observation of Werle *et al* (1987) by examining the spatial average of the measured normal and shear stress components on successive transverse planes downstream of the mixer trailing edge. In particular, Werle *et al* (1987) further suggested that the structure of the flow behind the lobed forced mixer followed a three-step process by which the streamwise vortex cells were formed, intensified, and then broke down. Most intense mixing seemed to occur in the third region. Varying the velocity ratio across the lobe could cause a shift in the locations of these three regions. Similarly, Eckerle *et al* (1992) also suggested that the downstream mixing region of the forced mixer could be divided into three regions, for which the secondary motion would be generated in the first region, the counter-rotating streamwise vortices be formed in the second region, and the streamwise vortices break down in the third region, resulting in a significant improvement in turbulent mixing. From their experimental measurements of the lobed mixer with 15 full lobes, they also pointed out that the viscous effects must be included in analyses that predict the lobed mixer performance, which was confirmed by the following computational study of viscous effects on lobed mixer performance by O'sullivan *et al* (1996). From the numerical simulation of O'sullivan *et al* (1996), it was found that a thick inlet boundary layer led to filling of the lobe trough with low momentum fluid, resulting in a reduced lobe effective angle, which is known as boundary-layer blockage effect.

Velocity measurements by McCormick and Bennett (1994) using the triple-sensor hot film probe, in a lobed mixer similar to that of Eckerle *et al* (1992), however, concluded that the intense small-scale turbulence and mixing occurred at about two to four wavelengths downstream of the mixer trailing edge was due mainly to the deformation of the K-H (normal) vorticity into a pinched-off structure by the streamwise vorticity. Intense mixing was the direct result of the merging process between the horse-shoe vorticity at the lobe peaks and the vorticity generated by the lobe itself. No indication of break-down of the streamwise vorticity could be found within the range of measurement. However, only the results with laminar initial conditions had been reported by them. The role and the development of the Reynolds stresses had not been analyzed in details.

The measurement of Yu *et al* (1995) with initial turbulent boundary layers suggested that the appearance of the high turbulence region at around two to three wavelengths downstream of the trailing edge was a consequence of the positive production of Reynolds stresses. The flow became more homogeneous at the further downstream stations. Similar conclusions were drawn by Ukeiley *et al* (1992, 1993). Ukeiley *et al* (1992) performed a 2-D scalar version of the Proper Orthogonal Decomposition to examine the downstream flow field of the lobed mixer (same as that used by Eckerle *et al* (1992)) and concluded that the large-scale turbulent structures began to break down at about three wavelengths after the trailing edge and became more homogeneous at further downstream locations.

The mean flow and turbulence measurements using a two-component laser Doppler anemometer on the multi-lobe forced mixer with velocity ratio of 0.6 by Yu and Yip (1997 a) showed that the distance between two rows of neighboring streamwise vortices

of alternate signs within one lobe reduced with downstream distance, as shown in **Fig 2.7**. This reducing distance caused the neighboring streamwise vortices to interact with each other. They would amalgamate and annihilate each other after their formation. Besides, they found that the mixing process behind a lobed forced mixer was dictated initially by the streamwise vorticity, and gradually taken over by the increased turbulent mixing.

In short, previous studies focused on the same point – the streamwise vorticity in the wake of the lobe, for they are one of the two most important reasons for the enhanced mixing characteristics of the lobed forced mixer. The other important reason – the increased mixing interfacial area – is well known from the geometry of the lobe. It is still not clear as to how the individual effects of the K-H vortices and the streamwise vortices to the mixing characteristics of the lobed forced mixer.

Therefore, the consideration of the present project would be on how these two kinds of vortices affect the mixing characteristics individually. That is, what are the individual aerodynamic characteristics of the Kelvin-Helmholtz vortices and the streamwise vortices. Their interactions will also be considered. Spectral analysis would be adopted to study the characteristics of the Kelvin-Helmholtz vortices while LDA measurements would be used to study the streamwise vortices.

To eliminate any possible interactions between the neighboring streamwise vortices as already discovered by Yu and Yip (1997 a), single-lobe models would be adopted in the present investigation.

Figure 2.1 Velocity and Vorticity Distribution for the Plane Free Shear Layer

Figure 2.2 Mechanism of the K-H Instability
(taken from <http://joas.free.fr/kelvin/sommaire.htm>)

- zone I -- the origin of the free shear layer;
- zone II -- the linear growth zone;
- zone III -- the nonlinear growth zone;
- zone IV -- the vortices merge and pairing zone.

Figure 2.3 Four Zones of a Free Shear Layer (Sato 1959; Freymuth 1966)

Figure 2.4 Instability of the Plane Free Shear Layer

Figure 2.5 Streamwise Vortices Riding on Spanwise Vortices in Plane Free Shear Layer (Bernal 1981)

(a) Perspective View

(b) End View

(c) Additional Pinch-off Effect
(Yu and Yip 1997)

Figure 2.6 Two Kinds of Vortices after the Trailing Edge of a Lobed Forced Mixer

**Figure 2.7 Variation of the Vortex Spacing with Downstream Distance
(Taken from Yu and Yip, 1997 a)**

Chapter 3. Experimental Arrangements

The experiments were carried out in a low-speed, low-turbulence, open circuit suction type wind tunnel located in the Thermal and Fluids Research Laboratory, Nanyang Technological University (TFRL of NTU). **Fig 3.1** shows the schematic view of the experimental setup.

When the axial fan is switched on, the inlet air will flow through bellmouth, honeycomb, settling chamber, contraction cone and the test section. There is a splitter plate embedded on the center-line of the tunnel to separate the airflow into two parts. On either side of the splitter plate, there is the same area of flow. By incorporating different layers of screens and wire meshes on the upper and lower half of the bellmouth, the velocities of the upper (U_1) and lower (U_2) streams can be varied. Different test model is attached to the trailing edge of the splitter plate.

3.1 Wind Tunnel

The wind tunnel is open and suction type, which operates with the stable room temperature at about 25°C. The main parts of the wind tunnel (**Fig 3.1**) are as follows.

(1) Bellmouth

Bellmouth is the beginning of the wind tunnel. It is of square shape, and its dimensions at the leading and trailing edges are 735mm × 735mm and 550mm × 550mm respectively. The axial length of bellmouth is 83mm.

(2) Honeycomb

To straighten the airflow, a honeycomb is incorporated inside the wind tunnel.

The length of the honeycomb is 60mm.

(3) Settling chamber

A settling chamber is located between the honeycomb and the contraction cone to settle the airflow after the honeycomb and supply high quality airflow for contraction section. It is a square (550mm×550mm) shaped, and the axial length is 145mm.

(4) Contraction cone

A contraction cone is used for accelerating the inlet airflow. The entrance of the contraction cone is 550mm×550mm – the same as settling chamber size, and the exit is 183mm×183mm. So the contraction area ratio is 9:1. The inlet airflow will accelerate by about nine times after going through the contraction cone (for low speed flow). This large contraction ratio ensured a uniform core flow and a relative low turbulence level (less than 1% of the inlet bulk velocity) at entry to the test section.

The axial length of the contraction cone is 666mm.

(5) Splitter plate

A splitter plate is used here to divide the inlet airflow into two streams – upper and lower streams. It is a flat plate with 2mm thickness, embedded from the trailing edge of the bellmouth to almost the end of the contraction cone. The axial length of the splitter plate is 811mm.

The upper and lower streams have the same area at each cross-section. Making use of different sizes of grid and cloth at the bellmouth of the wind tunnel, the velocities of the two inlet airstreams can be varied.

(6) Test section

A test section is made of transparent Plexi-glass with thickness 10mm. It has a square cross section with the dimension 183mm×183mm, and a length of 585mm. The test

models are mounted at the end of the splitter plate of the wind tunnel, and on entrance to the test section at the central position.

(7) Safety screen

The safety screen is a sparse metal mesh, mounted between the test section and the diffuser. After the model, the airflow will go through the axial fan. It is necessary to mount the safety screen to prevent any flying object from entering the fan.

(8) Diffuser

The diffuser is used to join the test section and the axial fan. Its shape changes gradually along the axis, from square to circular. The inlet and outlet cross-section dimensions are 183mm×183mm and $\Phi 240$ mm respectively. Its axial length is 650mm.

(9) Axial fan and fan speed controller

The axial fan is driven by a 420W motor. By adjusting the rotating speed of the axial fan, the airflow speeds can be varied accordingly. The velocity range for both streams in the absence of the test model can be varied from 0 to about 10m/s.

3.2 Test Models

Eleven models have been tested in the present investigation, as will be described in details below.

The models used in our experiments have only one lobe, so as to eliminate any possible interactions between neighboring lobes as mentioned earlier in Chapter 2.

The models were made of 2mm thick fiberglass with round trailing edge. They were sprayed black to minimize the reflected laser light back into the LDA probe (refer to Fig 3.9) when performing LDA measurements, so as to reduce the level of background noise.

3.2.1 Flat Splitter Plate

A flat splitter plate model was tested as a baseline for comparison. The spanwise width of the flat plate is 183mm, which is the same as the width of the test section. Its streamwise length is 102mm, and the thickness is 2mm.

3.2.2 Basic Semi-circular Single-lobe Forced Mixer (model a)

The schematic diagram of the model a is shown in Fig 3.2. Its dimensions are as follows:

- $h = 30 \text{ mm}$ (height of the lobe);
- $\lambda = 60 \text{ mm}$ (wavelength of the lobe);
- $R = 15 \text{ mm}$ (radius of the trailing edge semi-circle);
- $\varepsilon = 22^\circ$ (penetration angle, or half divergence angle);
- $L_m = 74.3 \text{ mm}$ (axial length of the lobe);
- $t = 2 \text{ mm}$ (thickness of this model);

$L = 180 \text{ mm}$ (axial length of the lobed forced mixer);

$W = 180 \text{ mm}$ (width of the lobed forced mixer).

3.2.3 Semi-circular Single-Lobe Forced Mixers With Different Heights (models b and c)

To examine how the height of the lobed forced mixer affects its aerodynamic characteristics and mixing performance, mixers with different heights were designed and investigated. The schematic diagrams of models b and c are shown in **Fig 3.3**. The heights of models b and c are 15mm (or $\lambda/4$) and 22.5mm (or $3\lambda/8$) respectively. Their axial length of the penetration region are the same as the basic model a, i.e., $L_m=74.3\text{mm}$. So the penetration angles of model b and c are 11.4° and 16.8° respectively. The wavelengths for model a, b and c are the same.

3.2.4 Convoluted Plate (model d)

The convoluted plate would not be able to generate streamwise vortices (this will be explained in details later in Section 5.1). So it was designed and tested to compare with those models that can generate strong streamwise vorticity. However, the convoluted plate should be able to generate normal K-H vortices as well as the forced mixer.

The schematic diagram of the convoluted plate (model d) is shown in **Fig 3.4**. The trailing edge configuration of the convoluted plate is the same as that of the basic model a. The only difference is that the trailing edge of the convoluted plate is extended by 90mm in the streamwise direction.

3.2.5 Semi-circular Single-lobe Forced Mixers With Different Wavelengths (models e and f)

To examine how the wavelength of the lobed mixer affects its aerodynamic performances, models with different wavelengths were designed and tested. The schematic diagrams of models e and f are shown in **Fig 3.5**. The heights of models a, e and f are all the same, at $h=30\text{mm}$, but their lobe wavelengths are different. For basic model a, the wavelength is ($\lambda=2h=$) 60mm ; while for model e and f, their wavelengths are ($\lambda=h=$) 30mm and ($\lambda=4h=$) 120mm respectively.

3.2.6 Rectangular and Triangular Single-lobe Forced Mixers (models g and h)

To examine how the geometry of the forced mixer affects its mixing performance, models with rectangular and triangular configuration were designed and tested besides the basic mixer with semi-circular configuration. The schematic diagrams of models b and c are shown in **Fig 3.6**. Their lobe height and wavelength are the same as the basic model a.

3.2.7 Scalloped and Scarfed Single-lobe Forced Mixers (models i and j)

To check how the modification on the mixer affects its mixing characteristics, models with scalloping and scarfing effects were designed. The schematic diagrams of models i and j are shown in **Fig 3.7**. They are modified from the basic model a. Scalloping was achieved by eliminating up to 70% of the sidewall area at the penetration region; while the scarfing was to eliminate the mixer's penetration region by half angle, i.e., $\theta=(90^\circ-\varepsilon)/2=34^\circ$ from the side view, as shown in **Fig 3.7(2) (b)**.

3.3 Measurement Systems

The measurement systems consist of two main parts: the hot-wire anemometer and the laser Doppler anemometer.

3.3.1 Hot-wire Anemometer (HWA)

A hot-wire anemometer was used to detect the frequency of the vortex shedding from the trailing edge of the test models. Since the mixing performance has a close relationship with the vortex shedding frequency, the mixing characteristics of the lobed forced mixer owing to the K-H vortex could be examined by analyzing its vortex shedding frequency.

The hot-wire used in the present investigation was Pt 10%-Rh Wollaston coated with silver, and the diameter of wire after etching (away the silver) was $5\mu\text{m}$. Its length was about $1.2 \sim 1.5\text{mm}$, and has a resistance of about $11 \sim 14\ \Omega$. The overheat ratio is maintained at about 1.5. The wires were operated at CTA (Constant Temperature Anemometer) mode.

The instantaneous velocity should be sampled more than twice as fast as the waveform frequency of the K-H vortices themselves (Nyquist Frequency), which is up to 1500Hz in the present investigation. Generally speaking, for the $5\mu\text{m}$ hot-wire the range of its response frequency is from 0 to up to 30KHz or more. The hot-wire anemometer operated in CTA mode matches this requirement well.

3.3.1.1 Single Hot-wire Anemometer

The Dynamic Signal Analyzer (type: HP 35665A) was used together with the single hot-wire anemometer for fundamental spectrum analysis. This instrument is designed

specially for the spectrum analysis. The frequency range is from zero to 102.4kHz, which is wide enough for the present application. When it works, the measuring range can be varied, such as from 0 to 200Hz, 400Hz, 800Hz, 1.6KHz, or more. It measures power spectrum, so the vertical coordinate is power magnitude.

It has been mentioned above that the test rig was located in an air-conditioned room, and the temperature of the airflow was always maintained at 25°C. So no cold-wire (operated in CCA, or Constant Current Anemometer mode) is necessary for the temperature compensation when measuring the velocities.

For a single-wire probe to measure one-dimensional velocity, velocity calibration is necessary. For a cross-wire probe to measure two-dimensional velocity, both the velocity and pitch angle calibrations are necessary. For a hot-wire probe with three wires (or even more) to measure three-dimensional velocity, the complicated yaw calibration will be necessary.

One of the main reasons to use the single-wire together with the instrument Dynamic Signal Analyzer is that the velocity calibration could be ignored, because the output of one-dimensional voltage signals from the anemometer can be performed with FFT to achieve the frequency information directly.

However, the following two main reasons make it necessary for us to perform cross-wire measurements to achieve spectra with more accuracy:

- (1) Compared with the cross-wire measurements, only one-dimensional velocity could be measured by a single wire probe.
- (2) The K-H vortex is basically two-dimensional. So its velocity fluctuations can be reduced within two directions. The spectra obtained in these two directions could

be different. Though their dominant frequencies may probably be the same, their breakdown locations are not necessarily the same.

On the consideration of these two reasons, cross-wires were employed in the present investigation.

3.3.1.2 Cross Hot-wire Anemometer

The cross hot-wire was operated with Constant Temperature Anemometry (CTA) as well. It measures two-dimensional velocity at a point while ignoring the third dimensional velocity, i.e., perpendicular to the plane of the two wires. Refer to **Fig 3.8** for example, the cross-wire measures the velocity u and w in X and Z directions, while ignoring v in Y direction.

The output of CTA is voltage signal. From the modified King's Law, the relationship is:

$$E^2 = A + B \cdot U^n, \quad (3-1)$$

or polynomial approximation, especially when the voltage output is negative:

$$U = \sum_{i=0}^k \alpha_i \cdot E^i. \quad (3-2)$$

In our approximation, $k = 3$ is chosen. That is,

$$U = a + b \cdot E + c \cdot E^2 + d \cdot E^3. \quad (3-3)$$

(a) Velocity Calibration

Velocity calibration is used to determine the constants of a , b , c and d in equation (3-3) for each wire. The free stream of the wind-tunnel is of small velocity fluctuation and can be used to calibrate the hot-wire. By running the tunnel at different velocities,

different voltage outputs are documented correspondingly. Then a least square method was employed to calculate the constants for each wire. The velocity of the free stream was measured via a Pitot-static tube.

Pitot-static tube was used to measure the pressure difference. Consequently the velocity of the flow can be calculated from its corresponding pressure difference. For the wind tunnel used, because the total pressure (P^*) of the stream is the atmospheric pressure, ignoring the change of air density, Bernoulli's equation can be used:

$$\frac{P}{\rho} + \frac{U^2}{2} = \frac{P^*}{\rho}; \quad (3-4)$$

so

$$U = \sqrt{2(P^* - P)/\rho} = \sqrt{2\Delta P/\rho} \quad (3-5)$$

where

$$\rho = P^* / RT^*; \quad (3-6)$$

$$\Delta P = P^* - P, \quad (3-7)$$

and for air,

$$R = R_0 / M = 8314.4 / 28.96 = 287.06 \quad (J / Kg \cdot K). \quad (3-8)$$

For our experimental conditions,

$$P^* = 1.013 \times 10^5 Pa;$$

$$T^* = 25^\circ C = 298.15 K;$$

$$\rho = 1.18 Kg / m^3.$$

The manometer used in the present experiments was emplaced inclined, with a precision of 0.5Pa/unit.

(b) Angle Calibration

Angle calibration is to determine the inclination angles of the two wires to a known flow direction, for example, the X or streamwise direction in **Fig 3.8**. For one given free stream velocity U , when the cross-wire rotates in the X-Z plane around Y direction, the voltage outputs of the two wires will change, because the effective cooling velocity changes accordingly.

In the process of angle calibration, the cross-wire pitches at eleven angles, they are $\phi = +15^\circ, +12^\circ, +9^\circ, +6^\circ, +3^\circ, 0^\circ, -3^\circ, -6^\circ, -9^\circ, -12^\circ$ and -15° . Simultaneously two series of voltage signals were recorded for each pitch angle. (One pitch angle is increasing while another decreasing, for example $(\beta_1 + \phi)$ and $(\beta_2 - \phi)$.) Based on the velocity calibration via Pitot-static tube, the effective cooling velocities can be calculated from the known constants a , b , c and d . Then the least square method will be employed to determine the two pitch angles, β_1 and β_2 .

After velocity and angle calibrations, the cross-wire is ready to measure the unknown velocity fields with models installed in the wind tunnel. Signal drifting is the biggest disadvantage of the hot-wire anemometer. To ensure the accuracy of measurement, the velocity calibration should be repeated every one hour to avoid drifting of the hot-wire signals. The angle calibration needs not to be repeated, because the structure of the wire element will not change significantly once the hot-wire is fabricated.

Without considering the third velocity component, the uncertainty of the cross-wire velocity measurement is as low as 2.5% (see Section 3.4.1.1). However, this accuracy can be greatly reduced if the third component velocity is high enough to be comparable to the other two velocities. This disadvantage makes it improper for us to measure the

secondary flow of the lobed mixer using cross-wire. Increasing the wire numbers may incur many problems, such as the complicated fabrication of the hot-wire, the yaw calibration, and the mutual interactions among the wires. So to measure the velocity fields, laser Doppler anemometer will be employed due to its advantages of non-intrusiveness, high spatial and temporal resolution, and free from complicated and frequent calibrations.

3.3.3 Laser Doppler Anemometer (LDA)

The laser Doppler anemometer (LDA) is a useful tool for fluid dynamic investigations in both transparent liquids and gases, and has been used for more than three decades. It is a well-established technique that gives information about the flow velocity. Particular advantages of this method include: non-intrusive measurement, high spatial and temporal resolution, no need for calibration and the ability to measure reversed flows (Durst *et al* 1981).

The LDA data obtained in the present investigation may also be used for validation of different CFD methods and turbulence models.

3.3.3.1 Principles of One-dimensional LDA Measurement

One-dimensional LDA measurement is for one-dimensional velocity measurement. As shown in **Fig 3.9**, two laser beams of equal wavelength are focused at one point. This point is referred to as the measurement volume. The wave fronts of the laser beams are approximately plane, and as a result, the interference produces parallel fringes of light and darkness. The distance d_f between any two fringes depends on the wavelength (λ)

and the half angle (κ_A) between the incident beams, i.e.,

$$d_f = \frac{\lambda_f}{2 \sin \kappa_A}. \quad (3-9)$$

When a seeding particle moving with the fluid passes through the measurement volume, it will scatter light. The scattered light in the form of Doppler burst is collected by a receiving lens and focused onto a photo detector. The Doppler burst is a signal of fluctuation in intensity with a frequency (f_D) equal to the velocity (u) of the particle divided by the fringe spacing (d_f), i.e.,

$$f_D = u / d_f. \quad (3-10)$$

So, if f_D can be detected, the velocity of the particle can be calculated by

$$u = f_D \cdot d_f = \frac{f_D \cdot \lambda_f}{2 \sin \kappa_A}. \quad (3-11)$$

The Doppler bursts are filtered and amplified in the signal processor, which determines f_D for each particle by frequency analysis, using the Fast Fourier Transform (FFT) method. Up to this point, the particles have been assumed as passing through the measurement volume from top to bottom, scattering a Doppler burst signal with a frequency equals to the particle speed divided by the fringe distance. However, it must be noted that a particle traveling from bottom to top with the same speed will generate a signal having the same frequency, as shown in **Fig 3.10**. There is no information contained in the signal, which will enable the system to distinguish between upwards or downwards velocity. This problem is referred to as “velocity directional ambiguity”. Actually the single hot-wire anemometer also cannot discriminate reversed flow whereas the LDA system can overcome this problem by employing the technique of frequency shifting.

The most common method of resolving this ambiguity of the LDA system is to add a relatively small frequency shift to one of the beams as the particle crosses the measurement volume. A particle that is not moving will generate a signal of the shift frequency f_{shift} . A particle traveling downwards will generate a signal at the Doppler frequency plus f_{shift} , while a particle moving upwards will generate a frequency of f_{shift} minus the Doppler frequency. Frequency shifting is a powerful technique, which enables the instrument to sample reversing flows, and can also increase the range of measurable velocities. The principle of frequency shifting is also demonstrated in **Fig 3.10**. Before frequency shifting, the relationship between f_D and u is

$$f_D = \frac{u}{d_f} = \frac{2u \sin \kappa_A}{\lambda_f}, \quad (3-12)$$

which is a slant line passing through the origin. The velocity range that can be measured is from u_{min} to u_{max} , where u_{min} is often 0, and

$$u_{max} = \frac{\lambda_f}{2 \sin \kappa_A} f_{max}. \quad (3-13)$$

After shifting the frequency by an amount of f_{shift} , the new relationship will be expressed by the dashed line. Correspondingly, the velocity range will be changed to $u'_{min} \leq u \leq u'_{max}$, in which

$$u'_{min} = -\frac{\lambda_f}{2 \sin \kappa_A} f_{shift}, \quad (3-14)$$

$$u'_{max} = \frac{\lambda_f}{2 \sin \kappa_A} (f_{max} - f_{shift}). \quad (3-15)$$

The LDA system used in the present investigation provides the function of adjusting filtering frequency f_{min} and f_{max} , and the shifting frequency f_{shift} .

Fig 3.11 shows a typical Doppler burst obtained during the experiments.

3.3.3.2 Principles of Two-dimensional LDA Measurement

Based on one-dimensional LDA measurement, **Fig 3.12** shows the two-dimensional LDA measurement system. The two beams with green color measure the vertical velocity; while the other two beams with blue color measure the horizontal velocity. Both of these two measured velocities are in the plane perpendicular to the probe. Actually a 2-D system is the combination of two 1-D systems whose measurement volumes are overlapped as much as possible.

The Argon-ion laser (Coherent model Innova 70c, 3.0W) provided a coherent light source for the LDA system. The green (514.5nm) and blue (488nm) beams were separated, split and then transmitted through a 6m fiber-optic cable to a probe with a diameter of 83mm (TSI model 9800). A 350mm lens (TSI model 9253-350) with a 3.95° half-angle (κ_A) was placed in front of the probe to focus the four parallel beams to their common measurement volume.

A controller (TSI 95008) was used in conjunction with three step motors to automatically traverse the probe to the desired position. The controller could be either manually operated or remotely controlled by a PC (Pentium 266). The traversing mechanical system ensured the location of the probe volume to be within ± 0.01 mm in the three orthogonal directions. The PC was connected to the burst correlator. The sequence of the collecting data and subsequently moving the probe were managed with a software package (FIND 14 version) provided by TSI Incorporated.

Bragg shifting of frequency up to 40MHz (on each channel) was used to avoid

directional ambiguity. Fine water particles with sizes 5~10 micrometers generated by a commercial nebulizer were used to seed the airflow. Except in some regions immediately behind the trailing edge, data rates of 500 to 1000 Hz were normally obtainable. At each measuring point, the mean velocities together with the corresponding fluctuations (rms) were determined from populations of more than 4,000 samples together with a coincidence window of 1 μ s. Other coincidence window widths from 1 μ s to 100 μ s were also tested but no significant differences in the results were found. A careful appraisal of the errors associated with the LDA system has been conducted. The sources of error were mainly from velocity biasing, velocity gradient broadening (Durst *et al* 1981), the accuracy of the signal processor and the finite sampling size (Yanta and Smith 1973). Thus, the accuracy of the measured velocity components (normalized by the bulk mean velocity) can be expected to be about 1.2% and that of the normal stress to be about 2.2% (see Section 3.4.2.1).

In the present experiments, fine water droplets, sizes ranging from 2 to 10 μ m generated by a commercially available ultrasonic nebulizer (OMRON NE-U12) were used to seed the airflow. The total mass flux of the droplets is less than 0.2% of the main airflow.

The measurable velocity range for the LDA measurement system employed in the present experiments was about -200 m/s $\sim$ $+200$ m/s.

Three-orthogonal mean velocities ($\bar{U}$, $\bar{V}$ and $\bar{W}$) together with their corresponding rms (u' , v' and w') have been acquired at each cross-sectional plane. The measuring area was bounded by $-0.75 \leq y/\lambda \leq +0.75$, $-1 \leq z/\lambda \leq +1$ and at eight downstream locations where $x/\lambda = 0.5, 1, 1.5, 2, 3, 4, 5$ and 6 . At each cross-sectional plane, there were 475

measuring points (also see Section 4.3.2.1 for details).

3.4 Uncertainty Analysis

In this section, the uncertainties of both the hot-wire anemometer measurement and the laser Doppler anemometer measurement will be analyzed in details. Due to the complicity of flow measuring systems such as HWA and LDA employed in the present investigation, the random and systematic errors could arise from many sources.

3.4.1 Hot-wire Anemometer Measurements

The hot-wire anemometer measurement in the present investigation was to obtain two dimensional instantaneous velocity series. Based on the velocities, the vortex shedding frequency can be obtained via fast Fourier transformation. Consequently the non-dimensional frequencies, including the Roshko number and the Strouhal number can be obtained. So the uncertainty analysis will be performed on the velocity measurement, the frequency measurement and the non-dimensional frequencies' calculation.

3.4.1.1 Uncertainty of the Velocity Measurements

The random errors of hot-wire anemometry consist of errors from frequency response, spatial resolution, probe disturbance, curve-fit, signal analysis simplification, signal interpretation in high turbulence intensity flows, restricted sampling size, and noises from the circumstances. Most of these error sources have been discussed by Bruun (1995).

As have been discussed earlier, the K-H vortex shedding frequency in the present investigation is less than 1500Hz. It is only about 2% of the frequency response of the CTA employed (up to 30KHz), which is far beyond the Nyquist Frequency. Therefore,

the error due to the frequency response should be negligible.

The error arising from the spatial resolution in the present experiments is also negligible, because the heated filaments is less than 1.5mm, which is much smaller than the nominal length of the model and the momentum thickness (about 12 to 20 mm, see Figs 4.27 and 4.28) of the present shear layer.

The errors arising from probe disturbance have two aspects. One is the aerodynamic disturbance effect and the other is probe interference effect caused by vibration. Since the probe used in this study has a thick stem diameter (about 5mm), prong diameter (about 0.6mm), a short prong length (about 10mm), as well as a short prong spacing (about 2mm), the aerodynamic disturbance effects are very small. On the other hand, as the hot-wire filament is straight it should not have any significant vibration, especially in the low speed wind tunnel. Thus this error is not taken into consideration here.

The signals were acquired via A/D (analog to digital) conversion. With a gain of 20 and a proper offset setting, the hot-wire anemometer had an output voltage of $-10 \sim +10$ volts. The 16-bit SCB-68 A/D board had a resolution of 1.53×10^{-4} volts. So the error arising due to the A/D is negligible.

The sampling size of the hot-wire anemometer was up to 60,000. As a result, the error due to the restricted sampling size is also negligible.

So after ignoring the third component velocity magnitude ($w=0$), the uncertainty of the velocity measurement using hot-wire anemometer arises mainly from three types of errors: the error of the velocity measurement using Pitot-static tube ($U_{U,1}$), the error of the polynomial approximation ($U_{U,2}$) and the error of the angle calibration ($U_{U,3}$).

From equations (3-5) and (3-6), the velocities for calibration, which were measured

by Pitot-static tube are:

$$U = \sqrt{2 \frac{\Delta P}{P^*} R T^*}, \quad (3-16)$$

So the error of the velocity measurement using Pitot-static tube is due to the errors in measuring the atmospheric pressure, the temperature, and the pressure difference. From the present experimental conditions, the values of their errors and uncertainties are:

$$\begin{aligned} E_{P^*} &= 0.1 \text{ mmHg}; \\ E_{T^*} &= 0.5 \text{ K}; \\ E_{\Delta P} &= 0.5 \text{ Pa}; \\ U_{P^*} &= \frac{E_{P^*}}{2P^*} \approx 0.007\%; \\ U_{T^*} &= \frac{E_{T^*}}{2T^*} \approx 0.1\%; \\ U_{\Delta P} &= \frac{E_{\Delta P}}{2(\Delta P)} \approx 1.4\%. \end{aligned} \quad (3-17)$$

So the uncertainty of the velocity measured via Pitot-static tube ($U_{U,1}$) is:

$$\begin{aligned} U_{U,1} &= \frac{1}{U} \left(\left(\frac{\partial U}{\partial P^*} \cdot U_{P^*} \cdot P^* \right)^2 + \left(\frac{\partial U}{\partial T^*} \cdot U_{T^*} \cdot T^* \right)^2 + \left(\frac{\partial U}{\partial (\Delta P)} \cdot U_{\Delta P} \cdot \Delta P \right)^2 \right)^{\frac{1}{2}} \\ &\approx 0.7\%. \end{aligned} \quad (3-18)$$

On the other hand, from the polynomial fitted curve, for example, from **Fig 3.13**, the uncertainty due to the polynomial approximation is:

$$U_{U,2} \approx 2.2\%; \quad (3-19)$$

While for the angle calibration, it leads to uncertainty:

$$U_{U,3} \approx 1.0\%. \quad (3-20)$$

The overall uncertainty of the measured velocity components can be calculated by the Root-Sum-Square (RSS) method. RSS is a statistical method for calculating the

cumulative uncertainties when more than one component is involved, provided all these components are independent of each other. In the first step, each component is squared, then the sum of these squares is calculated, and finally the square root of the sum is evaluated. So the total uncertainty for the velocity measurement via hot-wire anemometer is:

$$U_U = \sqrt{(U_{U,1})^2 + (U_{U,2})^2 + (U_{U,3})^2} \approx 2.5 \%. \quad (3-21)$$

3.4.1.2 Uncertainty of the Vortex Shedding Frequency

From the repetition of the power spectrum measurement, the maximum uncertainty for the K-H vortex shedding frequency would be:

$$U_f = 5.6 \%. \quad (3-22)$$

3.4.1.3 Uncertainties of the Roshko and Strouhal Numbers

It can be seen from equations (4-13) and (4-18) that the uncertainty of the Roshko number arises due to the error of the frequency measurement; while the uncertainty of the Strouhal number should be attributed to the uncertainty of the velocity measurement besides the frequency uncertainty.

The uncertainty of the Roshko number is:

$$U_{Ro} = U_f = 5.6 \%; \quad (3-23)$$

And the Strouhal number uncertainty is:

$$\begin{aligned} U_{St} &= \frac{1}{St} \sqrt{\left(\frac{\partial(St)}{\partial f} \cdot f \cdot U_f\right)^2 + \left(\frac{\partial(St)}{\partial \bar{U}} \cdot \bar{U} \cdot U_{\bar{U}}\right)^2} \\ &= \sqrt{(U_f)^2 + (U_{\bar{U}})^2} \\ &= \sqrt{(5.6\%)^2 + (2.5\%)^2} \\ &= 6.1\%. \end{aligned} \tag{3-24}$$

All the uncertainties of the HWA measurement are listed in **Table 3-1**.

Item	Uncertainty
Total Pressure (P^*)	$U_{P^*} = 0.007\%$
Total Temperature (T^*)	$U_{T^*} = 0.1\%$
Gauge Pressure (ΔP)	$U_{\Delta P} = 1.4\%$
Velocity (U) (measured by Pitot-static tube)	$U_U = 0.7\%$
Velocity (U) (Measured by hot-wire anemometer)	$U_U = 2.5\%$
K-H Vortex Shedding Frequency (f)	$U_f = 5.6\%$
Roshko Number (Ro)	$U_{Ro} = 5.6\%$
Strouhal Number (St)	$U_{St} = 6.1\%$

Table 3-1 Uncertainties of the Hot-wire Anemometer Measurements

3.4.2 Laser Doppler Anemometer Measurements

The laser Doppler anemometer measurement in the present investigation was to obtain the 3-dimensional mean velocity fields ($\bar{U}, \bar{V}, \bar{W}$) together with their rms (root-mean-square) velocities ($\sqrt{u'^2}, \sqrt{v'^2}, \sqrt{w'^2}$), or normal stress ($\overline{u'^2}, \overline{v'^2}, \overline{w'^2}$). Then the vorticity distribution, the circulation, the momentum thickness and the shape factor are all calculated. Their respective uncertainties are analyzed as follows.

3.4.2.1 Uncertainties of the Mean Velocity and Normal Stress

Due to the complicity of the LDA system, the uncertainties of the mean velocity and normal stress using LDA arise from many aspects. Their total uncertainties can be decomposed into five parts: statistical, data filtering, accuracy of signal processor, velocity bias and finite sampling size.

(a) Statistical

For every measuring point, 3000 samples or instantaneous bursts have been collected as a compromise between statistical uncertainty and run time. Standard statistical formulas can be used to estimate uncertainty if the samples are independent and normally distributed. Considering the satisfactory data rates, the samples were reasonably assumed to be normally distributed (Lakao *et al* 1987). Using a 95% confidence interval, the statistical uncertainty in the mean velocity is about 0.5%. The uncertainty of the velocity variance is predicted by the distribution, which relates the uncertainty of the variance to its own magnitude (again assuming a normal distribution). The 95% confidence interval for each normal stress is about 2% of the value of the normal stress itself.

(b) Data Filtering

All samples outside three standard deviations from the mean were eliminated. Removing these points, however, will decrease the higher order statistics to some degree. The rationale for removing these points is that some of them are spurious, and can significantly distort the statistics. Therefore, removing the points generates slightly depressed, but much more reliable higher order statistics. Presumably, the truth lies somewhere in between the unfiltered and filtered sample sets, so all the turbulence statistics were calculated for both data sets. The difference between the two sets was defined as the data filtering uncertainty. This uncertainty is negligible in the mean values. However, the filtering process will give rise to about 0.5% uncertainty for the normal stress.

(c) Accuracy of Signal Processor

The accuracy of IFA 750 Digital Burst Correlator (manufactured by TSI Incorporated) has a maximum error of 0.05%, as stated in the manufacturer's specifications, even when the signal to noise ratio (SNR) is as low as -5dB.

(d) Velocity Bias Correction Transfer Function

The transfer function used to correct all the LDA data was

$$DR = 1 - 0.9 e^{-0.2|U|}, \quad (3-25)$$

where DR is the normalized data rate, and U is in m/s. The velocity bias correction can introduce uncertainty to the degree that the measured transfer function does not represent the true performance of the system. Minimum and maximum possible transfer functions

were defined to encompass all the data points. The functions were:

$$DR_{\min} = 1 - 0.98 e^{-0.2|U|}, \quad (3-26)$$

$$DR_{\max} = 1 - 0.8 e^{-0.2|U|}. \quad (3-27)$$

The differences in the means calculated with these transfer functions and Equation (3-25) were considered the uncertainty in the streamwise mean velocity due to the velocity bias correction.

(e) Finite Sampling Size

Micron-sized particles are required in the flow field when a LDA system is employed to measure velocity. Not only should the particles be able to follow the flow but also they are expected to produce continuous Doppler signals in time. This requires high particle concentration in the region of interest at all time. In the present investigation, the particle concentration could be low somewhere. A statistical analysis can be used to account for this kind of error. By assuming that the behavior of turbulence was Gaussian, the uncertainty for the rms quantities was found to be dependent on the number of data points instead of the flow conditions. From the statistical analysis, the error function was (based on 95% confidence limit):

$$error = \frac{1.96}{\sqrt{2N}} \approx \sqrt{\frac{2}{N}} \quad (3-28)$$

where N is the number of data samples per measuring point.

Table 3-2 lists all the estimated uncertainties arising from the sources of errors mentioned above.

	Statistical	Data Filtering	Accuracy of Signal Processor	Velocity bias	Finite Sampling Size
$\overline{U}, \overline{V}, \overline{W}$	0.5%	0	0.05%	1%	0
$\overline{u'^2}, \overline{v'^2}, \overline{w'^2}$	2%	0.5%	0.05%	0.5%	0.5%

Table 3-2 Uncertainties of LDA Generated from Respective Sources

So the total uncertainty of normalized mean velocity could be expected to be:

$$U_{\overline{U}, \overline{V}, \overline{W}} = \sqrt{(0.5\%)^2 + 0 + (0.05\%)^2 + (1\%)^2 + 0} = 1.2\% . \tag{3-29}$$

While the uncertainty of the normal stresses (normalized by U^2) is:

$$U_{\overline{u'^2}, \overline{v'^2}, \overline{w'^2}} = \sqrt{(2\%)^2 + (0.5\%)^2 + (0.05\%)^2 + (0.5\%)^2 + (0.5\%)^2} = 2.2\% . \tag{3-30}$$

3.4.2.2 Uncertainty of the Vorticity

The local vorticity is calculated via equation (4-34). Its uncertainty arises from two parts: one is the uncertainty of the mean velocity measurement using LDA; another is the uncertainty of the coordinate system. The error of the coordinate system is:

$$E_{\Delta} = 0.01 \text{ mm}, \tag{3-31}$$

so its uncertainty is

$$U_{\Delta} = \frac{1}{2} \frac{E_{\Delta}}{\Delta} = \frac{1}{2} \times \frac{0.01}{5} = 0.1\% . \tag{3-32}$$

So the uncertainty of the local vorticity is:

$$U_{\Omega} = \sqrt{8 \cdot (U_{\bar{U}, \bar{V}, \bar{W}})^2 + U_{\Delta}^2} = 3.4\%. \quad (3-33)$$

3.4.2.3 Uncertainty of the Circulation

Refer to equation (4-36), for the basic model a for example, the circulation is calculated from 36 mean velocities, in which 12 velocities are in traverse direction while other 24 velocities are in cross-stream direction. From the statistical analysis, for the sample larger than 30 ($N > 30$), the following formula can be used when analyzing the standard error of their mean value, based on 95% confidence:

$$U_{mean} = \frac{1.96}{\sqrt{N}} \cdot U \quad (3-34)$$

So the uncertainty of the circulation is:

$$U_{\Gamma} = \sqrt{\left(\frac{1.96}{\sqrt{36}} \times U_{\bar{U}, \bar{V}, \bar{W}}\right)^2 + U_{\Delta}^2} = 0.40\% \quad (3-35)$$

3.4.2.4 Uncertainty of the Momentum Thickness

The momentum thickness is calculated from equation (4-39). Due to the complicated integration, especially the complicity of the high-order-polynomial curve fitness, the uncertainty of the momentum thickness will be simplified by averaging $-u^2 + (U_1 + U_2)u - U_1U_2$ on its wake plane with about 300 measuring points ($N=300$). Suppose $U_1 + U_2 = 8$ m/s in the present LDA measurements, thus the uncertainty of the momentum thickness is:

$$U_{\theta} = \frac{1.96}{\sqrt{N}} \times ((2 \cdot U_{\bar{U}})^2 + (8 \cdot U_{\bar{U}})^2)^{\frac{1}{2}} = 0.93U_{\bar{U}} = 1.1\%. \quad (3-36)$$

3.4.2.5 Uncertainty of the Shape Factor

The shape factor employed in the present investigation for evaluation of mixedness is explained in equation (1-10) or (4-41). Its uncertainty arises mainly due to the error of the local time-averaged velocity measurement. Similar to the momentum thickness, the shape factor is averaged at every cross-section of the wake, and on average about 300 measuring points ($N=300$) would be employed for averaging. So the uncertainty of the shape factor would be:

$$U_s = \frac{1.96}{\sqrt{N}} \times (2 \cdot U_{\bar{U}}) = 0.23 U_{\bar{U}} = 0.28 \% . \tag{3-37}$$

All the uncertainties of the LDA measurements are listed in **Table 3-3**.

Item	Uncertainty
Mean Velocity ($\overline{U}, \overline{V}, \overline{W}$) (measured by LDA)	$U_{\overline{U}, \overline{V}, \overline{W}} = 1.2\%$
Reynolds Normal Stress ($\overline{u'^2}, \overline{v'^2}, \overline{w'^2}$)	$U_{\overline{u'^2}, \overline{v'^2}, \overline{w'^2}} = 2.2\%$
Local Streamwise Vorticity (Ω)	$U_{\Omega} = 3.4\%$
Circulation (Γ)	$U_{\Gamma} = 0.40\%$
Momentum Thickness (θ_m)	$U_{\theta} = 1.1\%$
Shape Factor (S)	$U_s = 0.28\%$

Table 3-3 Uncertainties of the LDA Measurements

Figure 3.1 Experimental Arrangements

Figure 3.2 Schematic Diagrams of the Basic Semi-circular Single-lobe Forced Mixer (model a) (dimensions in mm)

(1) Model b at Lobe Height $h=15\text{mm}$

(2) Model c at Lobe Height $h=22.5\text{ mm}$

Figure 3.3 Schematic Diagrams of the Semi-circular Single-lobe Forced Mixers at Different Lobe Heights (models b & c) (dimensions in mm)

Figure 3.4 Schematic Diagrams of the Convuluted Plate (model d) (dimensions in mm)

(1) Model e at Wavelength $\lambda = 30$ mm

(2) Model f at Wavelength $\lambda = 120$ mm

Figure 3.5 Schematic Diagrams of the Semi-circular Single-lobe Forced Mixers at Different Lobe Wavelengths (models e & f) (dimensions in mm)

(1) Rectangular single-lobe Forced Mixer (Model g)

(2) Triangular single-lobe Forced Mixer (Model h)

Figure 3.6 Schematic Diagrams of the Rectangular and Triangular Single-lobe Forced Mixers (models g & h) (dimensions in mm)

(1) Scalloped Single-lobe Forced Mixer (model i)

(2) Scarfed Single-lobe Forced Mixer (model j)

Figure 3.7 Schematic Diagrams of the Scalloped and Scarfed Single-lobe Forced Mixers (models i & j) (dimensions in mm)

Figure 3.8 Cross-wire Anemometer Calibration

Figure 3.9 Measurement Principle of Backscattered LDA System
(Taken from www.dantec.com)

Figure 3.10 Velocity Directional Ambiguity and Frequency Shifting

Figure 3.11 A Typical Doppler Burst Taken during the Experiment

Figure 3.12 Schematic of the 2-D LDA Measurement System
(Taken from LDA manual, TSI Incorporation, USA)

Figure 3.13 Hot-wire Anemometer Velocity Calibration Curve (polynomial fitted)

Chapter 4. Flow Characteristics of Single-lobe Forced Mixers at Different Penetration Angles

4.1 Introduction

The possible instabilities in two-stream mixing flows are one of the fundamental problems in classical fluid dynamics. It is well established that vorticity dynamics affect and control the mixing process to a great extent. Different velocity distributions in the shear layer usually lead to the generation of Kelvin-Helmholtz (K-H) vortices (namely normal vortices, or spanwise vortices, also see Chapter 2). The K-H vortices enhance mixing of the flow in the plane parallel to the direction of the primary flows. Another kind of vortices, namely the streamwise vortices, usually appears together with the K-H vortices. This results in additional energy transfer from the primary stream to the secondary stream. Consequently, the mixing in the planes perpendicular to the primary streams will be enhanced. Both types of vortices are significant contributors to the mixing performance.

It has long been observed that the plane free shear layer will lead to the formation of a well-organized array of streamwise vortices superimposed onto the K-H vortices. The presence of this secondary structure was initially observed by Bradshaw *et al* (1964) in an axisymmetric mixing layer; while the first systematic investigation of the streamwise vortices was performed by Konard (1976) and subsequently by Breidenthal (1981). However, streamwise vortices generated in the plane free shear layer normally are not significant, with smaller scale than the spanwise K-H vortices (Lasheras and Choi 1988); and some distances are also required for them to develop. Their appearances are also found to be very sensitive to initial conditions (Lasheras *et al* 1986; Bernal and Roshko 1986). To generate strong streamwise vortices so as to enhance the mixing performance,

two methods are commonly employed, i.e., the perturbation method and the geometry method. For the latter, by designing the geometries of the splitter plate properly without any moving parts, strong streamwise vortices could be generated in the mixing layer. Lobed forced mixer is one typical example. Lobed forced mixer has the characteristics of generating these two types of vortices – the small scale Kelvin-Helmholtz vortices (McCormick and Bennett, 1994) and the large scale streamwise vortices (Presz *et al* 1987; Yu and Yip 1997 a).

For the lobed forced mixer flow, according to the numerical simulation results of Barber (1988), the streamwise circulation (C_I) is in direct proportion to the tangent of the penetration angle (ε) (refer to **Fig 2.1**)

$$C_I \propto \tan(\varepsilon), \quad (4-1)$$

providing that the penetration angle is not larger than 22° . The main assumptions are that the inviscid fluid leaves the trailing edge of the mixer at an angle equal to the penetration angle of the mixer; and there is no separation at the troughs of the mixer.

In this chapter, three single-lobe forced mixers with different penetration angles ($\varepsilon_b = 11.4^\circ$, $\varepsilon_c = 16.8^\circ$ and $\varepsilon_a = 22.0^\circ$) have been examined so as to verify how the normal K-H vortices and streamwise vortices vary due to different penetration angles. The flat plate, which can be considered as one special case of the lobe forced mixer with penetration angle $\varepsilon_0 = 0^\circ$, was also tested as a basis for comparison.

Due to the different characteristics of the K-H and streamwise vortices, hot-wire anemometer (HWA) and laser Doppler anemometer (LDA) were employed to characterize these two types of vortices respectively. The information would be helpful to any further design of the lobed forced mixers with more efficient mixing performance.

The streamwise vortices are generally stable and have no distinct frequency peaks in the spectrum; correspondingly all the peaks in the spectrum should be attributed to the K-H vortices. So for the K-H vortices, spectrum analysis based on the HWA 2-dimensional velocity series can be used to evaluate this type of vortices by measuring their frequencies. Consequently the mean wavelengths (λ_{K-H}) of the K-H vortices could be calculated from their relationships with the corresponding dominant frequency:

$$\lambda_{K-H} \cdot f = \bar{U} , \quad (4-2)$$

in which the mean velocity is the average velocity of the two streams:

$$\bar{U} = \frac{U_1 + U_2}{2} . \quad (4-3)$$

Furthermore the frequently used non-dimensional vortex shedding frequencies, including the Roshko number and the Strouhal number, were examined and analyzed. For the streamwise vortices, laser Doppler anemometry (LDA) was employed to measure the 3-dimensional velocity fields. The contours of the streamwise mean velocities and vorticities, the corresponding secondary flow velocities, the circulation around a half-lobe, and the turbulent kinetic energies were evaluated and analyzed.

4.2 Hot-Wire Anemometer Measurements

4.2.1 Flat Plate

4.2.1.1 Initial Momentum Thickness

The initial momentum thickness at the trailing edge of the flat plate is formed due to the boundary layer, which has evolved from the bellmouth of the wind tunnel to the

Velocity $U_1=U_2$ (m/s)	3.40	4.42	5.66	6.86	8.13	9.25	10.36
Momentum thickness $\theta_{m,1}=\theta_{m,2}$ (mm)	0.55	0.46	0.42	0.39	0.36	0.34	0.33
Shape Factor $H_{dm}=\theta_d/\theta_m$	2.4	2.3	2.2	2.0	1.8	1.6	1.5

(a) Velocity Ratio $r = 1:1$

Velocity U_1 (m/s)	1.36	1.76	2.26	2.75	3.25	3.70	4.14
Momentum Thickness $\theta_{m,1}$ (mm)	0.99	0.71	0.64	0.60	0.56	0.53	0.49
Shape Factor $H_{dm}=\theta_d/\theta_m$	2.5	2.5	2.5	2.5	2.4	2.4	2.3

(b) Velocity Ratio $r = 0.4:1$
(U_2 and $\theta_{m,2}$ are the same as (a).)

Table 4-1 Initial Momentum Thickness for the Plane Free Shear Layer

trailing edge of the flat plate (see **Fig 3.1**) with the traveling distance of about 91 cm. For the two cases with different velocity ratios, the momentum thickness are tabulated in **Table 4-1(a)** and **(b)** respectively, which were measured at about 2 mm before the trailing edge of the flat plate using a single hot-wire anemometer.

The ratio of $\theta_{m,1}$ to $\theta_{m,2}$ is plotted in **Fig 4.1(a)**. With the increase in velocity, the shape factor of the boundary layer varies from about 2.5 to 1.5. So the experimental initial boundary layer condition covers both laminar and turbulent regimes. Though the turbulent initial condition is more practical, the laminar initial condition was also studied since turbulent initial conditions might mask the flow structure and bypass certain instabilities.

4.2.1.2 K-H Vortex Shedding Frequency

The K-H vortex was considered to shed at the frequency where the corresponding power magnitude reached its maximum value. **Fig 4.2** shows typical power spectra for the plane free shear layer (flat plate) case. For velocity ratio $r = 1:1$, due to their non-linear effects, super-harmonic peaks with smaller amplitudes were also found in the power spectrum (Michalke 1972).

As shown in **Fig 4.3**, the K-H vortex shedding frequency varies linearly with the mean velocity, which can be approximated as:

$$f_{FP} = 114.9 \bar{U} - 208.9 \quad (r = 1:1, 3.40 \text{ m/s} \leq \bar{U} \leq 10.36 \text{ m/s}); \quad (4-4)$$

$$f_{FP} = 100.1 \bar{U} - 147.7 \quad (r = 1:0.4, 2.39 \text{ m/s} \leq \bar{U} \leq 7.25 \text{ m/s}). \quad (4-5)$$

These results are consistent with the analytical results of Wu *et al* (1993), in which the vortex shedding frequency f is proportional to the mean velocity of the two streams:

$$f \propto \overline{U}. \quad (4-6)$$

Velocity ratio had little effects on the vortex shedding frequency. For the two velocity ratios involved in the present investigation, the K-H vortex shedding frequency measured were within 5%.

4.2.1.3 K-H Vortex Wavelength

Both the significant convection between the two mixing streams and the insignificant molecular diffusion take place simultaneously on the interface of the two streams. Consequently smaller K-H vortex wavelength, i.e., the interface between the two streams would be more strained, and the turbulence level together with the mixing performance would be enhanced correspondingly. The mean wavelength of the K-H vortices may be evaluated using Equation (4-2).

Fig 4.4 shows the K-H wavelength variation of the plane free shear layer. It is obvious that the K-H wavelengths decreased with the increase in the mean velocity until reaching certain stable values. The effects of the velocity ratio on the wavelength were as insignificant as on the frequency.

4.2.1.4 Strouhal Number

The dimensionless parameter for frequency is known as the Strouhal number after the Czech physicist Vincenz Strouhal (1850~1922) who, in 1878, first investigated the 'singing' of wires.

The Strouhal number of the plane free shear layer is defined as:

$$St_{FP} = \frac{f \cdot \overline{\theta}_m}{\overline{U}} = \frac{\overline{\theta}_m}{\lambda_{K-H}}, \quad (4-7)$$

in which

$$\overline{\theta}_m = \frac{1}{2}(\theta_{m,1} + \theta_{m,2}) \quad (4-8)$$

is the mean momentum thickness at the trailing edge.

The Strouhal number was found to be a constant value (0.032 ± 0.002) for the flat plate. This result agrees well with the analytical result of Monkewitz and Huerre (1982) as described in Chapter 2.

4.2.2 Basic Single-lobe Forced Mixer

4.2.2.1 Initial Momentum Thickness

Due to the convoluted penetration region of the mixer, the on-coming velocities of the two mixing streams will be affected. Consequently the initial momentum thickness at every point along the trailing edge will be different from the case of the plane free shear layer. As for example, the velocity profiles for the two sides of the apex (point C) are schematically shown in **Fig 4.5**.

As shown in this figure, the boundary layer at the upper side of point C (C_{upper}) would be ‘compressed’. Meanwhile, the airflow on the upper side was passing through convergent passage, therefore the local maximum velocity will be slightly higher than U_I . These two effects had caused the momentum thickness of the boundary layer here thinner than that at the initial point I as shown in **Fig 4.5**. **Table 4-2** lists the momentum thickness measured, denoted as $\theta_{m,CI}$. It can be seen from the Table that the momentum

thickness of the boundary layer at the upper side of point C decreased by about 20% due to the penetration region at the same Reynolds number.

On-coming velocity $U_1=U_2$ (m/s)	3.40	4.42	5.66	6.86	8.13	9.25	10.36
Momentum thickness (upper side) $\theta_{m,C1}$ (mm)	0.45	0.39	0.35	0.32	0.30	0.28	0.27
Momentum thickness (lower side) $\theta_{m,C2}$ (mm)	3.50 ($\pm 5\%$)						

Table 4-2 Initial Momentum Thickness at Point C of the Basic Single-lobe Forced Mixer

While at the lower side of point C (C_{lower}), the fluid would be decelerated, due to the divergent passage there. This phenomenon was named the ‘boundary layer blockage’ by O’Sullivan (1996). Strictly speaking, the momentum thickness measured at this location is not of the boundary layer but calculated from the velocity profile. So the 99% velocity line of the lower side was much further away from the mixer wall than that of the upper side. This had caused the momentum thickness for the lower side to become larger than that of the upper side one. As measured by the single hot-wire anemometer, the momentum thickness at this location was about 3.50mm, which was 10 times larger than that of the initial point *I*. Also its variation due to different on-coming velocity (U_2) in the present investigation was less than 5%. The smaller variation was due to the fact that the velocity profile, after non-dimensionalization, remained unchanged for different on-coming velocities. The ratio of $\theta_{m,C1}$ to $\theta_{m,C2}$ is plotted in **Fig 4.1(b)**.

Along the parallel sidewalls of the mixer, for example at point L shown in **Fig 4.6**, the initial boundary layers were very similar to the case of the flat plate. The momentum thickness measured at this point was within 5% of the flat plate case.

4.2.2.2 K-H Vortex Shedding Frequency

For the lobed mixer flow, the K-H vortex shedding was more complex. As measured in the experiments, there were always two main frequencies in the wake. This may be attributed to the fact that the initial conditions at points C and L were different. For the regions in the wake of the semi-circle part of the trailing edge, as shown in **Fig 4.6**, the dominant frequency was essentially the same in this region (denoted as f_C); while for the regions behind the parallel sidewalls of the trailing edge, the dominant frequency was also the same (denoted as f_L). **Fig 4.7** shows some typical spectra of the basic single-lobe forced mixer, both at points C and L. Similarly, the frequency may be approximated as:

$$f_{LM_C} = 134.5 \bar{U} - 191.5 \quad (r = 1:1, 3.40 \text{ m/s} \leq \bar{U} \leq 10.36 \text{ m/s}); \quad (4-9)$$

$$f_{LM_L} = 115.4 \bar{U} - 175.2 \quad (r = 1:1, 3.40 \text{ m/s} \leq \bar{U} \leq 10.36 \text{ m/s}). \quad (4-10)$$

$$f_{LM_C} = 218.8 \bar{U} - 330.0 \quad (r = 1:0.4, 2.39 \text{ m/s} \leq \bar{U} \leq 7.25 \text{ m/s}); \quad (4-11)$$

$$f_{LM_L} = 111.4 \bar{U} - 140.7 \quad (r = 1:0.4, 2.39 \text{ m/s} \leq \bar{U} \leq 7.25 \text{ m/s}). \quad (4-12)$$

Fig 4.8 shows the frequency variations both at points C and L of the basic single-lobe forced mixer. It is very clear that f_C was always higher than f_L for all the flow conditions investigated here. Actually f_L is rather close to the case of the plane free shear layer (flat plate). This is because the initial boundary layers and the corresponding momentum thickness at point L were very similar to the case of the plane free shear layer. Whereas

for point C, the initial conditions together with the momentum thickness were much different from the flat plate case as a result of the penetration region, especially on its lower side.

Velocity ratio can much more significantly be seen to affect the frequency at point C. For the basic single-lobe forced mixer, when the velocity ratio decreases from 1 to 0.4, the frequency at point C increased by 100%. While at point L, the velocity ratio had little effects on the K-H vortex shedding frequency, which is similar to the flat plate case.

The higher K-H vortex shedding frequencies of the forced mixer may explain in part, why its mixing performance is more effective than the plane free shear layer.

4.2.2.3 K-H Vortex Wavelength

Fig 4.9 shows the wavelengths of the basic single-lobe forced mixer, both at points C and L. It is obvious that the K-H wavelengths decreased with the increase in the mean velocity until reaching certain stable values. Whereas for the two velocity ratios investigated, the wavelength after point C of the forced mixer was found to be lower than that after point L. For the forced mixer at velocity ratio 1:1, the wavelengths after point C are about 80% of the flat plate values. At velocity ratio 0.4:1, they are about 50% only. Varying the velocity ratios had more effects on the wavelength at point C than at point L. At point C, the wavelength decreased by about 30~40% when the velocity ratio changed from 1:1 to 0.4:1; while at point L there was no significant differences for the two velocity ratios.

4.2.2.4 Roshko Number

To correlate periodic wakes, the Roshko number was usually adopted (Roshko 1955; Berger and Wille 1972; Simmons 1977; Levi 1983). It is a non-dimensional frequency. The frequency (f) can be correlated by the Roshko number (Ro) and the Reynolds number (Re), based on the wavelength of the lobe (λ) and the kinematic viscosity of the air (ν), i.e.,

$$Ro = \frac{f \cdot \lambda^2}{\nu} \quad (4-13)$$

From the relationship between the K-H vortex shedding frequency and the mean velocity, it is easy to deduce that the Roshko number increases linearly with the increase of the Reynolds number at both points C and L, with the following regression equations:

$$Ro_{LM_C} = 8.07Re - 44300 \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (4-14)$$

$$Ro_{LM_L} = 6.92Re - 40500 \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (4-15)$$

$$Ro_{LM_C} = 13.1Re - 76300 \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (4-16)$$

$$Ro_{LM_L} = 6.68Re - 32500 \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900). \quad (4-17)$$

Although the increases of the Roshko number to the Reynolds number are linear at different points (Points C and L) and at different velocity ratios, their magnitudes of the increasing rates are different. These increasing rates are rather important for evaluating the mixing performance of the lobed mixer, which will be explained in the subsequent paragraphs in more details.

4.2.2.5 Strouhal Number

For the forced mixers, the Strouhal number is defined as the ratio of the nominal length (λ) to the mean wavelength of the K-H vortices as follows:

$$St = \frac{f \cdot \lambda}{U} = \frac{\lambda}{\lambda_{K-H}}. \quad (4-18)$$

From the above equation it can be seen that the physical meaning of Strouhal number is actually how many roll-up, or K-H cat-eyes within each nominal length (lobe wavelength here) in the streamwise direction. While the relationship among the Reynolds number, Roshko number and Strouhal number is:

$$St = \frac{Ro}{Re}. \quad (4-19)$$

According to the regression equations of the Roshko number, i.e. equations (4-14) to (4-17), the Strouhal number for the basic mixer can be fitted as follows:

$$St_{LM_C} = 8.07 - \frac{44300}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (4-20)$$

$$St_{LM_L} = 6.92 - \frac{40500}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (4-21)$$

$$St_{LM_C} = 13.1 - \frac{76300}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (4-22)$$

$$St_{LM_L} = 6.68 - \frac{32500}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900). \quad (4-23)$$

These results are also shown in **Fig 4.10**. It can be easily deducted that the Strouhal number increases with increasing Reynolds number while the increasing rate decreases:

$$\text{Rate of increase} = \frac{d(St)}{d(Re)} \propto \frac{1}{(Re)^2}. \quad (4-24)$$

The Strouhal number increases rapidly when Reynolds number changes from 9,200 to about 20,000; whereas for Reynolds number larger than 20,000, the Strouhal number increases relatively slowly. Stable Strouhal number values are expected to be achieved at Reynolds number above 5×10^5 . This is because the viscous force and inertial force play different roles at different Reynolds number ranges, as will be elucidated in section 4.2.4 later together with other test models carried out by other researchers. From the definition of the Strouhal number it can be seen that the higher the Strouhal number, the smaller the wavelength of the K-H vortex. The lobed forced mixer is therefore expected to achieve better mixing at high Strouhal number. The relationship between the Strouhal number and the Reynolds number provides some interesting information. From the viewpoint of the K-H vortices, when the Reynolds number is less than 5×10^5 , the mixing performance of the forced mixer increases with the Reynolds number; when the Reynolds number is larger than 5×10^5 , the mixing performance reaches its maximum and becomes stable thereafter. The velocity ratio affects Strouhal number at point C more significantly than at point L.

4.2.3 Comparison at Different Penetration Angles

4.2.3.1 Initial Momentum Thickness

Tables 4-3 and 4-4 list out the momentum thickness measured at two sides of point C for the other two models tested in this chapter with different penetration angles, 11.4° and 16.8° respectively.

On-coming velocity $U \text{ (m/s)}$	3.40	4.42	5.66	6.86	8.13	9.25	10.36
Momentum thickness $\theta_{m,C1} \text{ (mm)}$	0.50	0.42	0.38	0.35	0.33	0.31	0.30
Momentum thickness $\theta_{m,C2} \text{ (mm)}$	2.20($\pm 5\%$)						

Table 4-3. Initial Momentum Thickness at Point C of the Single-lobe Forced Mixer b ($\epsilon=11.4^\circ$)

On-coming velocity $U \text{ (m/s)}$	3.40	4.42	5.66	6.86	8.13	9.25	10.36
Momentum thickness $\theta_{m,C1} \text{ (mm)}$	0.48	0.41	0.36	0.33	0.32	0.30	0.28
Momentum thickness $\theta_{m,C2} \text{ (mm)}$	2.80($\pm 5\%$)						

Table 4-4. Initial Momentum Thickness at Point C of the Single-lobe Forced Mixer c ($\epsilon=16.8^\circ$)

Comparing **Tables 4-3** and **4-4**, the momentum thickness of the boundary layer formed at the upper side of point C changed with the penetration angle. Generally speaking, when the penetration angle increased from 11.4° , 16.8° to 22.0° , the upper-side momentum thickness decreased from about 90%, 85% to 80% of the value of the flat plate case. But at the lower-side of point C, the momentum thickness changed more, with magnitudes of 2.20mm, 2.80mm to 3.50mm respectively. This is the main difference for the initial conditions of the three cases.

For point L, the boundary layers on either side were similar to the flat plate case. This is in line with the fact that the flow conditions at point L were much similar to the flat plate case.

4.2.3.2 K-H Vortex Wavelength

Figs 4.11 and 4.12 show the K-H wavelength variations for the two models at different penetration angle (or lobe height), i.e., model b (Penetration Angle $\varepsilon=11.4^\circ$) and c (Penetration Angle $\varepsilon=16.8^\circ$) respectively. Together with Fig 4.3 (the wavelength variation for the plane free shear layer) and Fig 4.9 (the wavelength variation for the basic single-lobe forced mixer with penetration angle $\varepsilon=22.0^\circ$), it is obvious that when the lobe penetration angle ε increases from 0 to 22.0° , the K-H wavelength decreases, and this phenomenon is more obvious at point C. Consequently the expected mixing performance due to the contribution of the K-H vortices would be enhanced.

4.2.3.3 Strouhal Number

Figs 4.13 and 4.14 show the Strouhal number variations for the two models with different penetration angle (or lobe height), i.e., model b (Penetration Angle $\varepsilon=11.4^\circ$) and c (Penetration Angle $\varepsilon=16.8^\circ$) respectively. Comparing with the basic mixer model with penetration angle $\varepsilon=22.0^\circ$, the Strouhal number variation for the three models are quite similar. Nevertheless, their maximum Strouhal numbers were rather different for the three models with different penetration angles, as shown in Fig 4.15. From this figure it can be seen that for point C, the peak Strouhal numbers increase with the lobe penetration angle, especially at velocity ratio $r=0.4:1$. However, the increments of the Strouhal

numbers at point L are not that significant. This difference should be attributed to the different initial conditions at points C and L, as shown in section 4.2.3.1. At point C, due to the dramatically changed velocity profiles and the corresponding momentum thickness, it is obvious that the measured Strouhal number depends greatly on the penetration angle. While at point L, the initial velocity profiles, or boundary layers, did not change as much comparing to the flat plate case, and as a result the Strouhal number would not change too much. With the increase of the penetration angle, the maximum Strouhal number increases also, and the corresponding mixing performance would also be improved.

4.2.4 Comparison with other Models

4.2.4.1 Circular Cylinder

It is well known that the periodical Karman Vortex Street is shed in the wake of circular cylinder, for Reynolds number larger than a certain value say about 50. Roshko (1954) experimentally obtained two well-known equations for the non-dimensional frequency St_d for the wake of a circular cylinder, as shown in **Fig 4.16**:

$$St_d = 0.212 - 4.5 / Re_d; \quad (50 < Re_d < 150) \quad (4-25)$$

$$St_d = 0.212 - 2.7 / Re_d. \quad (300 < Re_d < 2000) \quad (4-26)$$

Although the curve only indicates that vortex streets exist up to $Re=1000$, they actually occur into turbulent range of $Re=10^7$ and higher. Linhard (1966) experimentally showed that at sufficiently large Reynolds numbers about 3×10^5 , a constant Strouhal value of about 0.212 was traditionally taken.

4.2.4.2 Wing Models

Huang *et al* (2001) have studied and evaluated similar experiments in the water tunnel on a wing model (NACA 0012), using the particle tracking flow visualization method (PTFD) and particle image velocimetry (PIV). Their main results are shown in Fig 4.17. Similarly the relationship between the Reynolds number and Strouhal number, both of which based on the physical width of the wing model, are:

$$St = 0.159 - \frac{28.604}{Re}. \quad (4-27)$$

From the above equation they have concluded that at large Reynolds numbers, the Strouhal number of vortex shedding of the NACA 0012 wing would attain a constant value of 0.159, which is slightly larger than the maximum Strouhal number 0.15 for a vertical flat plate and is lower than the 0.212 of a circular cylinder.

One critical question here is why the Strouhal numbers for all the three models, i.e., the circular cylinder, the wing model, and the lobed forced mixers tested in the present investigation, increase gradually with the Reynolds number and finally reach certain stable maximum values at high Reynolds number. As the Reynolds number increases, the test models shift from viscous-dominated regime to inertial-dominated regime gradually. At high Reynolds number, it can be assumed that they are flying in an inviscid fluid. In this case, it seems that the wavelength of the unstable K-H vortex is no longer dependent on the Reynolds number but is only dependent on the structure of the flying body itself. Consequently the Strouhal number, which represents the ratio between the nominal size of the flying body and the K-H wavelength, as one of the most important non-dimensional frequencies, would keep constant too.

Although the Strouhal number variation tendencies of different test models listed here are much similar, their relative magnitudes are much different. This is mainly because the test media, the geometry of the test models and thus their nominal length scales are different. Thus the corresponding Reynolds number ranges, in which the Strouhal number behaves alike, are much different for the three bluff bodies, i.e., the circular cylinder, the wing model and the present forced mixer.

4.3 Laser Doppler Anemometer Measurements

As mentioned in section 4.1, the three-dimensional mean velocity fields and their corresponding fluctuations in the near wake of the two-stream mixing flow have been measured using LDA. The streamwise velocity, the mean velocity vectors of the secondary flow, the streamwise vorticity including its mean and peak values, the momentum thickness, the mean Reynolds normal stresses together with the turbulent kinetic energies have been non-dimensionalized and are presented from **Fig 4.18** to **4.31**.

4.3.1 Streamwise Velocity Contours and Mean Secondary Velocity Vectors

Streamwise velocity has been non-dimensionalized by the nominal velocity, or mean velocity of the two streams. Due to the symmetry of the models and the flow fields, only half lobe (left side here) will be shown in the following paragraphs.

The mean secondary velocity vector is:

$$\vec{V}_s = \bar{V} \cdot \vec{j} + \bar{W} \cdot \vec{k} . \quad (4-28)$$

in which V and W are both non-dimensionalized by the nominal velocity. Similarly only the right half of the lobe will be shown in the diagrams. The arrow base of the secondary velocity vector is at the measuring point.

4.3.1.1 Velocity Ratio $r = 1:1$

For the basic forced mixer flow, although the velocities of the upper and lower on-coming air-flows (U_1 and U_2) are the same, the streamwise velocity fields after the trailing edge will deviate from the mean value by up to 30%, due to the effects of the penetration region.

For the basic single-lobe forced mixer and at cross-sectional plane of $x/\lambda = 0.5$ (**Fig 4.18 b(1)**), accumulation of the low momentum fluids could be found near the upper trough regions, while the fluids near the lower trough region accelerated. It should be noted that this difference was the direct result from the mixer geometry with only one lobe. The upper side airflow went through a divergent passage, while the lower side airflow went through a convergent one. No flow separations were detected at both troughs. The asymmetrical geometry of the mixer led to the asymmetrical flow fields. This also led to the streamwise vorticity asymmetry – the streamwise vortices generated here were different from those generated after the multi-lobed forced mixer, as typically measured by Eckerle *et al* (1992) and Yu and Yip (1997 a). The maximum value of the secondary flow was at about 60% of the mean streamwise velocity at this station.

At the subsequent station $x/\lambda = 1$, the streamwise velocity contours began to diffuse, as shown in **Fig 4.18 b(2)**. The secondary velocity vectors were the strongest at this station with maximum value at about $0.75 \bar{U}$.

At further downstream stations at $x/\lambda = 2$ and 3 , the streamwise velocity contours continued to diffuse with the velocity gradients diminished gradually, as shown in **Fig 4.18 b(3)** and **(4)**. The magnitude of the secondary flow also decreased, with maximum values at about $0.5 \bar{U}$ and $0.4 \bar{U}$ respectively.

At station $x/\lambda = 4$ and shown in **Fig 4.18 b(5)**, the streamwise velocity gradients reduced further. This was accompanied by a rapid decrease in the strength of the streamwise vorticity. Maximum secondary flow was about $0.20\sim 0.25 \bar{U}$ only.

At the final measured station $x/\lambda = 6$, the streamwise velocity distribution across the entire wake appears to be fairly uniform as shown in **Fig 4.18 b(6)**.

For the other two single-lobe forced mixers with different height (or penetration angle), as shown in **Figs 4.19b** and **4.20b** respectively, the development for the streamwise velocity contours and the secondary flows were similar to the case of the basic single-lobe forced mixer. However, their magnitudes of the secondary flow were about 50% (for model b) and 30% (for model c) smaller respectively than the basic mixer, as might have been expected due to their smaller penetration angles. It should be noted here that the secondary flow scales of **Figs 4.18**, **4.19** and **4.20** were different, as shown in **Figs 4.18a**, **4.19a** and **4.20a** respectively.

4.3.1.2 Velocity Ratio $r = 0.4:1$

For this case, the velocity of the upper-side on-coming airflow was 40% of the lower side. The normalized streamwise velocity contours are shown in **Figs 4.18c**, **4.19c**, **4.20c** and **4.21** for the basic single-lobe forced mixer, the other two forced mixers with different lobe height and the flat plate respectively.

For the basic single-lobe forced mixer and at $x/\lambda = 0.5$, as shown in **Fig 4.18c(1)**, the streamwise velocity contours were clustered near the trailing edge, and the velocity gradients there were much higher than those for the case of velocity ratio $r = 1:1$ (see **Fig 4.18b**). This explains why the K-H vortices for this case were much stronger than the former case with the velocity of the two on-coming streams matched. The maximum value of the secondary flow was at about $0.7\bar{U}$.

At $x/\lambda = 1$, the streamwise velocity contours began to diffuse. The secondary velocity vectors peaked at this station with maximum value at about $0.8\bar{U}$, as shown in **Fig 4.18c(2)**. Rapid exchange of momentum between these two streams happened here, and

this contributed much to the mixing enhancement.

At $x/\lambda = 2, 3, 4$ and **Figs 4.18c(3), (4), (5)**, the streamwise velocity contours continued to diffuse with the velocity gradients decreased. The magnitudes of the secondary flows kept decreasing, with maximum values at about $0.55\bar{U}$, $0.40\bar{U}$ and $0.20\bar{U}$ respectively.

At the final station of the near field, i.e. $x/\lambda = 6$ and **Fig 4.18c(6)**, the streamwise velocity contours continued to diffuse. The mixing was yet to complete, and the momentum exchange between these two streams would continue into the far field. Meanwhile the magnitudes of the secondary flow velocity vectors decreased further.

While for the other two single-lobe forced mixers with different height (or penetration angle), both the streamwise velocity contours and the secondary flows were much similar to the case of the basic single-lobe forced mixer. However, their secondary flow magnitudes were weaker than the basic mixer, due to their smaller penetration angles.

For the flat plate, only the cross-section at $x = 120$ mm is shown here. It can be seen clearly that the streamwise velocity contours are essentially parallel to the trailing edge geometry, as shown in **Fig 4.21**.

4.3.2 Streamwise Vorticity

Similar to the Kelvin-Helmholtz vorticity, the streamwise vorticity is another important factor that contributes to the mixing performance of the lobed mixer. In the following paragraphs the streamwise vorticity, including its contours and peak streamwise vorticity, and the circulation (or mean streamwise vorticity) will be analyzed.

4.3.2.1 Streamwise Vorticity Contours

The vorticity is obtained by taking the curl of the velocity vector, that is:

$$\vec{\Omega} = \nabla \times \vec{V} \quad (4-29)$$

For the Cartesian coordinate system, it is

$$\vec{\Omega} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}, \quad (4-30)$$

in which u , v and w are the instantaneous velocities in the x , y and z directions.

Correspondingly the streamwise vorticity is

$$\vec{\Omega}_x = \Omega_x \cdot \vec{i} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \cdot \vec{i}. \quad (4-31)$$

It can be approximated to be

$$\Omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \approx \frac{\Delta w}{\Delta y} - \frac{\Delta v}{\Delta z}. \quad (4-32)$$

The interval distances of the measuring points are

$$\Delta y = \Delta z = 5 \text{ mm} \quad (4-33)$$

in the present investigation as shown in **Fig 4.22**. And the whole measuring plane is one big rectangle including $19 \times 25 = 475$ measuring points, with the size of $(19-1)\Delta y \times (25-1)$

$\Delta z = 90 \text{ mm} \times 120 \text{ mm}$.

Due to the stable characteristics of the streamwise vorticity, the mean velocity $\bar{V}$ and $\bar{W}$ could be used. That is,

$$\Omega_x \approx \frac{\Delta w}{\Delta y} - \frac{\Delta v}{\Delta z} = \frac{\Delta \bar{W}}{\Delta y} - \frac{\Delta \bar{V}}{\Delta z} = \bar{\Omega}_x. \quad (4-34)$$

Refer to **Fig 4-22**, for the four neighboring points, denoted as (m, n) , $(m, n+1)$, $(m+1, n)$ and $(m+1, n+1)$, the streamwise vorticity is calculated as follows:

$$\begin{aligned} \bar{\Omega}_x = \frac{\Delta \bar{W}}{\Delta y} - \frac{\Delta \bar{V}}{\Delta z} = \frac{1}{2} & \left(\frac{\bar{W}_{(m,n+1)} - \bar{W}_{(m,n)}}{\Delta y} + \frac{\bar{W}_{(m+1,n+1)} - \bar{W}_{(m+1,n)}}{\Delta y} \right) \\ & - \frac{1}{2} \left(\frac{\bar{V}_{(m+1,n)} - \bar{V}_{(m,n)}}{\Delta z} + \frac{\bar{V}_{(m+1,n+1)} - \bar{V}_{(m,n+1)}}{\Delta z} \right). \end{aligned} \quad (4-35)$$

So actually the streamwise vorticity measured is the mean vorticity of this small rectangle in grey color with size of $\Delta y \times \Delta z = 5\text{mm} \times 5\text{mm}$ shown in **Fig 4.22**. Based on the former results obtained by Yu and Yip (1997 a), this arrangement should be accurate enough to provide a global picture of the streamwise vorticity distribution.

The contours of the non-dimensional streamwise vorticity for the basic single-lobe forced mixer (model a) are shown in **Fig 4.23**. To non-dimensionalize the streamwise vorticity, it was divided by the factor $(\bar{U}/\lambda)$, in which $\bar{U}$ is the mean streamwise velocity while λ is the wavelength of the lobed mixer.

(a) Velocity Ratio $r=1:1$

As usual, results at six consecutive cross-sections are shown in **Fig 4.23a**. Generally speaking the streamwise vorticity is positive on the left half lobe, while negative on the right half lobe.

At cross-section $x/\lambda=0.5$ and **Fig 4.23a(1)**, one pair of counter-rotating streamwise vortices was generated immediately behind the trailing edge of each lobe. It was rather clear that the left vortex was rotating in the anti-clockwise direction, or in $+X$ direction; while the right one was rotating in clockwise direction or in $-X$ direction. Their symmetry was not perfect because the vorticity was calculated from 8 corresponding

velocities, as expressed in equation (4-32). So the uncertainty of the vorticity was about three times higher than the velocity's. However the magnitudes of these two streamwise vortices were nearly the same. The maximum non-dimensional vorticity is about ± 1.40 .

At the subsequent stations $x/\lambda=1$ and 2 shown in **Figs 4.23a(2)** and (3), the streamwise vorticity contours were similar to those at the cross-section $x/\lambda=0.5$. There was one pair of vortices located in the center of the measurement plane, while at the edge of the measurement plane the vorticity magnitude was almost zero. The maximum vorticity is about ± 1.40 .

At stations $x/\lambda=3$ and 4, the cores of the streamwise vorticity seemed to be shifting. As shown in **Figs 4.23a(4)** and (5), the right streamwise vortex core had moved in the cross-stream direction until the location $z = -12$ mm. However, the vortex shifting in spanwise direction (or Y direction) was not obvious. These phenomena were different from the multi-lobe forced mixer measured by Yu & Yip (1997 a), in which the streamwise vortices were found to obviously shift in the spanwise direction. This difference can only be attributed to the fact that the present research model has only one lobe, as a result the effect from the neighboring lobes would not be presented.

At the last measured station in the near field $x/\lambda=6$ and **Fig 4.23a(6)**, the streamwise vorticity had mostly decayed to a negligible level. Actually even the maximum magnitude was only about ± 0.40 , and at most locations it was almost zero.

(b) Velocity Ratio $r=0.4:1$

Because the velocities of the upper and lower streams are different, the vorticity fields for this case were more complicated than that for the case where the velocities of the two

on-coming streams matched. The contours are shown in **Fig 4.23b**.

At the first cross-section measured $x/\lambda=0.5$, the streamwise vortices were mainly concentrated on the upper troughs and lower peaks. The vorticity reached its maximum value there at about ± 1.05 . At the upper peak and lower trough, the vorticity is of smaller magnitude at about ± 0.50 .

At the following two stations $x/\lambda=1$ and 2 , the streamwise vortices seemed to be intensified. From the contours it could be seen that they are away from the parallel sidewalls.

At stations $x/\lambda=3$ and 5 , the streamwise vortices began to breakdown. The maximum magnitudes at these two cross-sections are ± 0.95 and ± 0.55 respectively. No obvious shifting of the vortex core could be detected.

At $x/\lambda=6$, the streamwise vortices continued to breakdown to a magnitude less than ± 0.20 .

4.3.2.2 Peak Streamwise Vorticity

The peak streamwise vorticity for the basic single-lobe forced mixer has been plotted in **Fig 4.24**. For both the two velocity ratios involved in the present investigation, the variations for the peak streamwise vorticity were almost the same. Very little changed within the first three wavelengths distance after the trailing edge, but decayed much faster in the subsequent three wavelengths.

From the comparison between the mean (see **Fig 4.24**) and peak streamwise vorticity of the basic single-lobe forced mixer (model a), it could be seen that the peak vorticity diffused slower than the mean value. Actually the mean streamwise vorticity began to

diffuse immediately after the trailing edge of the mixer; while the peak vorticity only started to diffuse three wavelengths after the trailing edge. This was mainly because the vortex core, where the peak vorticity could be reached usually, would be affected later than its periphery when the two adjacent streamwise vortices merged with each other.

4.3.2.3 Mean Streamwise Vorticity (Circulation)

The strength of the mean streamwise vorticity ($\overline{\omega_x}$), which may be considered as one important parameter to quantify the effectiveness of mixer performance, could be evaluated using the following expression:

$$C_I = \frac{\Gamma_I}{U \cdot \lambda}, \quad (4-36)$$

in which Γ_I is the streamwise circulation of the plane **I**, where $\partial \mathbf{I}$ encompasses half lobe:

$$\Gamma_I = \iint_I \omega_x \cdot dA = \overline{\omega_x} \cdot A = \oint_{\partial I} \vec{U}_s \cdot d\vec{l} = \int_{AB} \vec{V} \cdot d\vec{y} + \int_{BC} \vec{W} \cdot d\vec{z} + \int_{CD} \vec{V} \cdot d\vec{y} + \int_{DA} \vec{W} \cdot d\vec{z} \quad (4-37)$$

The results of the normalized streamwise circulation for the three mixers with different lobe heights are plotted in **Fig 4.25**. It is obvious that the circulation decreased almost exponentially with x/λ for all cases. For every model, the magnitudes for the two velocity ratios were similar to each other. However, for different models with different lobe height, the circulations were different. The initial values are listed in **Table 4-5**. According to the analysis of Skebe *et al* (1988) and numerical simulation results of Barber (1988), the streamwise circulation is in direct proportion to the tangent of the penetration angle,

$$C_I \propto \tan(\varepsilon) \tag{4-38}$$

provided the penetration angle is not larger than 22°. From **Table 4-5** it can be seen that the above scaling law agrees well for a single-lobe mixer.

	Model b <i>h</i> = 15 mm	Model c <i>h</i> = 22.5 mm	Model a (Basic) <i>h</i> = 30 mm
Penetration Angle (ε)	$\varepsilon_b = 11.4^\circ$	$\varepsilon_c = 16.8^\circ$	$\varepsilon_a = 22.0^\circ$
$\tan(\varepsilon)$	0.202	0.303	0.404
$C_I(r=1)$	0.324	0.478	0.628
$C_I(r=0.4)$	0.262	0.400	0.557
$\tan(\varepsilon_b) : \tan(\varepsilon_c) : \tan(\varepsilon_a) = 2 : 3 : 4 = C_{I,b} : C_{I,c} : C_{I,a}$			

Table 4-5 Relationship Between the Initial Normalized Streamwise Circulation and $\tan(\varepsilon)$

4.3.3 Momentum Thickness

The shear layer entrainment momentum thickness in the mixing layer is defined as:

$$\theta_m = \int_{-\infty}^{+\infty} \frac{(U_1 - u) \cdot (u - U_2)}{(U_1 - U_2)^2} \cdot dZ \tag{4-39}$$

in which U_1 and U_2 are the velocities of the on-coming upper and lower streams, while u is the local streamwise velocity. Z is the cross-stream direction (see **Fig 4.21**).

4.3.3.1 Plane Free Shear Layer

The momentum thickness of the plane free shear layer has been measured by many researchers, such as Bell and Mehta (1992) and Salman *et al* (2003). However, the results

were not same, mainly because the initial conditions varied from case to case. For example, the initial boundary layers of the two streams measured by Bell and Mehta (1992) were much closer to laminar, with a shape factor of 2.52 and 2.24 for high and lower speed streams respectively, and a velocity ratio of 0.6. Salman *et al* (2003) measured turbulent boundary layers with a shape factor of 1.524 and 1.438 respectively, and a velocity ratio of 0.57. As shown in **Fig 4.26**, the momentum thickness with the initial laminar boundary layers seemed to increase much faster than that with initial turbulent boundary layers.

For the present investigation, only one case is shown here in solid line. The initial boundary layers were transitional, with shape factor of 1.91 and 1.64 respectively. The momentum thickness of the present investigation was closer to the result of Salman *et al* (2003). This was in line with the fact that the initial boundary layers of the present investigation were more inclined to be turbulent but not yet fully developed.

4.3.3.2 Basic Single-lobe Forced Mixer

Fig 4.27 shows the momentum thickness growth of the basic single-lobe forced mixer (model a). It is clear that the momentum thickness grew very fast within the initial two wavelengths after the trailing edge; but slowed down at $x/\lambda \geq 3$. This tendency was the same as the multi-lobe forced mixer measured by Yu and Yip (1997 a). However, the magnitudes of the momentum thickness for these two cases were different, mainly due to the different lobe height of these two cases. From the comparison it seemed that the momentum thickness was proportional to the lobe height. Both Yu & Yip (1997) and the present investigation used the lobe forced mixers with 22 degree of penetration angle.

4.3.3.3 Single-lobe Forced Mixers at Different Heights

The momentum thickness of the three models, a, b and c are shown in Fig 4.28. Although the variation trends for the three models were almost the same, their respective magnitudes were different. This difference could be attributed to the fact that the K-H vortices in their mixing layers were different. For model a, due to its highest penetration angle, the K-H vortices generated in its mixing layers were the strongest as shown before. As a result, the momentum exchange between the two streams was the most significant, and this had led to its highest momentum thickness magnitude.

4.3.4 Turbulent Kinetic Energy

The turbulent kinetic energy is another important parameter relating to the mixing performance evaluation. In the present investigation the wake-area averaged turbulent kinetic energy of the basic single-lobe forced mixer had been examined and presented in Fig 4.29. Here the wake area boundary was defined by the region bounded by $-0.5 \leq y/\lambda \leq +0.5$ and $u/\bar{U} \leq 0.95$ for the velocity matched case ($r = 1:1$). While for the velocity unmatched case, it was defined by the region bounded by $-0.5 \leq y/\lambda \leq +0.5$, and at a location where the influence of the secondary flow to the mean flow was insignificant, i.e., less than 5% of the bulk mean velocity of the two streams.

$$\bar{k}(x/\lambda) = \frac{1}{\left(\frac{1}{2}\bar{U}^2\right) \cdot A_{wake}} \int_{wake} k \cdot dA = \frac{1}{\left(\frac{1}{2}\bar{U}^2\right) \cdot A_{wake}} \int_{wake} \frac{1}{2}(u'^2 + v'^2 + w'^2) \cdot dA \quad (4-40)$$

From the results it could be seen that for the basic single-lobe forced mixer, the streamwise components of the Reynolds Normal Stresses were of the largest magnitude among the three components. For the two velocity ratios, all the three components

together with the total turbulent kinetic energy began to increase after the trailing edge, reaching their respective maximum values at about $x/\lambda = 4$ for the case $r = 0.4:1$, or at about $x/\lambda = 5$ for the case $r = 1:1$. The details of the turbulent kinetic energy distributions at the downstream location of $x/\lambda = 0.5$ are plotted in **Fig 4.30** together with the trailing edge configuration. For the case $r = 1:1$ shown in **Fig 4.30(a)**, the magnitude of turbulent kinetic energy at point L was higher than that of point C. However, their magnitudes for the case $r = 0.4:1$ were similar, as shown in **Fig 4.30(b)**.

Most importantly was that the relative magnitude of these two cases with different velocity ratios. From the comparison it was obvious that the turbulent kinetic energy for the case $r = 0.4:1$ was almost six times the values of the case $r = 1:1$. This was far beyond the difference between their mean streamwise vorticity magnitudes (about 15% only, as will be shown in the next paragraph). In fact, similar results were found by Eckerle *et al* (1992), whose LDA measurements showed that the average root-mean-square (*rms*) velocity for the case $r = 2:1$ were approximately two to three times of the values measured at the corresponding planes for $r = 1:1$. It was likely that this significant difference may be attributed to the K-H vortex, which was much stronger for the case $r = 0.4:1$ than the case $r = 1:1$, as shown in the results of K-H vortices earlier. Comments that can be drawn from here are that the K-H vortices contributed more to the turbulent kinetic energy than the streamwise vortices. The magnitude of the turbulent kinetic energy is largely responsible for the mixing performance of the lobed forced mixer. In this aspect, careful consideration should be taken into the formation and the dynamic characteristics of the K-H vortices. For the streamwise vorticity, though it does not contribute significantly to the turbulent kinetic energy as the K-H vorticity, it strains the

interface of the two mixing streams, and thereby increases the molecular diffusion between the two streams. The mixing between these two streams would therefore be greatly enhanced. Actually the well-organized streamwise vortices were spatially stationary, as similarly pointed out by Plesniak *et al* (1994), which was generated in the curved two-stream turbulent mixing layers. So the streamwise vorticity contributed less to the velocity fluctuations (or *rms* velocities) and the corresponding turbulent kinetic energy.

Fig 4.31 provides the turbulent kinetic energy of the two other lobed mixers with different lobe heights. From the comparison among the three single-lobe forced mixers with height 15mm (model c), 22.5mm (model d) and 30mm (basic model a), it is obvious that the wake-averaged turbulent kinetic energy increased with the lobe height, or penetration angle. The results on the Strouhal number increments for the three models shown earlier explained this well. With the increase of the Strouhal numbers, the wavelength of the K-H vortex decreased, consequently the interface between the upper and lower streams would be more strained. As a result, the fluctuations together with the turbulent kinetic energy would increase.

4.3.5 Shape Factor

The mixing of the two flows within the wake region in terms of momentum distribution is of interest. Therefore it may be sufficient to define mixedness parameter for the two streams in terms of the shape factor of the streamwise mean velocity distribution, i.e.,

$$S = \frac{1}{A_{wake}} \int_{wake} \left(\frac{u}{U} \right)^2 \cdot dA \quad (4-41)$$

The definition used here is similar to that used by Bevilaqua (1974) to evaluate the thrust distribution (also see equation (1-10)). Ideally, at the location where the two streams at the wake region with different velocities were to be completely mixed, and spatial uniformity was achieved, the momentum distribution would be uniform and the shape factor should be equal to unity. The variation of the shape factor as a function of downstream distance should provide a useful indication of the extent of the spatial uniformity that the two streams could achieve. Unfortunately for the velocity-matched case ($r = 1:1$), the shape factor variation was so small that its uncertainty (magnitude at about $\sim 0.28\%$) would conceal its variation. Actually the maximum deviation of its variation was only about $0.5\sim 0.8\%$ from the ideal mixed condition for this case, which was too small to quantify its actual difference. So only for the case at velocity ratio $0.4:1$ would be shown in the present investigation. For comparison, the shape factor of the flat plate would also be shown as a baseline.

Fig 4.32 shows the shape factor variations for the flat plate and the three lobed mixers with different lobe heights. It can be seen that the shape factor of the flat plate was the highest, followed by the lobed mixer b at lobe height 15 mm, and then the mixer c at lobe height 22.5 mm. The shape factor of the basic model a was the lowest, which implied the best mixing performance achieved among the four models.

For the flat plate, it seemed that its shape factor varied relatively linearly with x/λ , which was similar to its momentum thickness variation. Although there were also streamwise vortices imposing on the normal vortices, they were relatively weak. Mainly only the normal K-H vortices contributed to its mixing performance in this case. So its mixing performance would be much inferior to the lobed mixer.

For the basic mixer model a, its enhanced mixing interfacial areas, the presence of strong K-H vortices and streamwise vortices all contributed to its mixing enhancement. There was a fast decrease within the first two wavelengths downstream of its trailing edge, which indicated a rapid variation of the momentum distribution within the wake region, as a consequence of streamwise vorticity shed by its lobe. The slopes of the variation curve were also found to decrease, which implied the momentum mixing was almost completed.

For the lobed mixers b and c, their shape factors were between the flat plate and the basic model a, which were in accordance with their lobe heights. The intensity of the normal K-H vortices and streamwise vortices, together with their interfacial areas, were all weaker or less than those of the basic model a but stronger or larger than those of the flat plate case. It was also found that the shape factor leveling off location would shift further downstream when the lobe height was reduced, indicating a slower mixing rate.

4.4 Further Discussion

The results in the present chapter show the relative contributions of the K-H vortices and the streamwise vortices in the mixing process downstream of lobed forced mixers at different heights. It was well established from previous investigations (see section 2.2.2) that the height of the mixer largely determined the strength of the secondary flow. Strong streamwise vorticity would lead to more efficient mixing and this in turn depends on the geometry of the mixer. In general, mixers with parallel sidewalls are superior to the other geometries (Skebe *et al* 1988; Yu *et al* 1995), in terms of generating strong secondary flow. The subsequent decay however depends on the initial flow conditions, for example upstream boundary layer thickness and velocity ratios. Both thick boundary layers and higher velocity ratios would lead to a faster decay of the streamwise vorticity (from about 5-6 wavelengths at the velocity ratio 1:1 to about 3 wavelengths at velocity ratio 1:3). The corresponding mixing distance, i.e., at the location where the two streams are mixed well (at least in the mean sense) would also be shortened at high velocity ratios (Yu *et al* 1998). The thickening of the boundary layers would not suppress the generation of the secondary flow as long as no flow separation appeared at the troughs.

It is obvious from the above that some general features on the lobed mixer flow have been established. The present investigation intends to establish some links between the K-H vortices and the streamwise vortices — how do they actually interact with each other? The present investigation is by no means conclusive but should be able to provide some additional insight on the interactions of the two vortices. In the following discussion, our attention will focus more on the velocity ratio $r = 0.4:1$. References will only be made to case $r = 1:1$ whenever it is necessary. At velocity ratio $r = 0.4:1$, as may

be expected, the strength of the streamwise vorticity near the trailing edge was similar to that found in $r=1:1$ (Yu *et al* 1998). The subsequent decay length is also similar for both cases indicating that in the absence of the neighboring lobes, the decay length should also be independent of the velocity ratios.

Based on the measurements presented in section 4.3.4, the generation of TKE responsible for mixing enhancement is due largely to the breakdown of the K-H vortices. The geometry of the lobe modified firstly the trailing edge conditions for the formation of K-H vortices. The subsequent generation of the streamwise vorticity modified the mixing length structure further causing the K-H vortex to break down earlier (relative to a flat plate). The spectrum measurements at points C and L actually show they breakdown at different locations. The former appears earlier than the latter by about half wavelength. If one considers a normal vortex tube shed immediately from the trailing edge as shown in **Fig 3.2**, the shape of which should follow closely to the shape of the lobe's trailing edge profile. By considering the breakdown locations of the K-H vortices at points C and L respectively, it is obvious that the normal vortex tube would be tilted after it was being shed from the trailing edge. The same observation was also found in the film visualization experiments of McCormick and Bennett (1994), in which they estimated the tilted angle to be the same as the lobe penetration.

At this point if the normal vortex tube is not affected by the streamwise vorticity, they should maintain the same shape until they finally breakdown as in the case of convoluted plate (see Chapter 5). It should be pointed out that the tube does not breakdown simultaneously at every locations but rather point C would break down at a location slightly earlier than point L. The time averaged measurement of the downstream wake

revealed a similar trend that the high level of TKE firstly appears at point C and was followed by point L. In the presence of streamwise vorticity, the normal vortex line will be stretched to form the ‘pinched off’ structure as described by McCormick and Bennett (1994) and Yu and Yip (1997 a), as schematically shown in **Fig 2.6(b)**. However, it appeared that the high TKE (as shown in the time-averaged measurements) in our present investigation did not appear at the pinched off location. It appears firstly at the lobe peak regions where the normal vortex line was stretched most. As the vortex line was stretched, the diameter of the tube would be reduced. This may explain why the wavelength of the K-H vortices became smaller causing an earlier breakdown of the K-H vortices at the corresponding location. This was followed by breakdown of the K-H vortices at the other locations. In fact, similar observation was also found in the experiment of a coaxial lobed nozzle (Yu and Xu 1998).

It is necessary to analyze the ‘pinched off’ phenomenon in more details. The basic principle has been explained in the literature review on the lobed forced mixer in Section 2.2.2 and **Fig 2.6(b)**. However, the earlier analysis was focused on the structure of the K-H vortex tube only. Actually the interactions of the streamwise vortices with the K-H vortices are more than structural deformation. As shown in **Fig 4.33**, when the K-H vortex tube is deformed from **(a)** to be ‘pinched-off’ **(b)**, it will be stretched, especially at point C. As it is well known from the conservation of angular momentum, i.e., the Bio-Savart Law in the fluid dynamics,

$$\vec{v} = \frac{\Gamma}{4\pi} \int \frac{d\vec{l} \times (\vec{r} - \vec{r}_0)}{\|\vec{r} - \vec{r}_0\|^3}, \quad (4-42)$$

with the stretching of the vortex tube and decreasing of its diameter, its rotating speed

will increase. For example, consider one vortex tube shown in (c) with diameter R , length L , mass density ρ , total mass M , angular velocity ω and total angular momentum I . We have:

$$M = \rho \pi R^2 L \quad (4-43)$$

and

$$I = L \cdot \int_0^R \rho \omega r \cdot 2\pi r \cdot dr. \quad (4-44)$$

So

$$\omega = \frac{3\sqrt{\pi}}{4} I \cdot M^{-\frac{3}{2}} \cdot \rho^{\frac{1}{2}} \cdot L^{\frac{1}{2}} \quad (4-45)$$

Suppose that the mass density, total mass and total angular momentum are all constants for this vortex tube, it can be clearly seen from equation (4-45) that the angular velocity of the fluid is proportional to the root of its length scale. If the vortex tube is stretched to get doubled in length, its angular velocity will increase about 40% simultaneously. Consequently its vorticity will increase 40% as well, because the vorticity is double of the angular velocity of the fluids,

$$\vec{\Omega} = 2\vec{\omega} \quad (4-46)$$

or

$$\Omega = 2\omega. \quad (4-47)$$

Although the K-H vortex tube generated in the wake of lobed mixer is much more complicated than this example schematically shown in **Fig 4.33(c)**, there is no doubt that with the ‘pinched-off’ effects going on, its rotating speed will increase. Consequently the turbulence level together with the TKE at point C will increase correspondingly, which

was what have been detected in the LDA measurements. The increments definitely enhance the mixing in the following downstream wakes. So from this point of view, the interaction of the streamwise vortices with the K-H vortices is beneficial to the mixing performance of the lobed mixer.

Of another particular note in the present investigation is that at high Reynolds number, the Strouhal numbers along the trailing edge of a lobed forced mixer would become almost constant. At high Reynolds number ($Re \geq 5 \times 10^5$), the inertial force becomes dominant while the viscous forced is negligible. As a result, the normal Kelvin-Helmholtz vortices are shed inviscidly. Their mean wavelength is found to be dependent only on the nominal size of the model, but not on the Reynolds number. Correspondingly the ratio between the nominal size of the model and the mean K-H wavelength, i.e., the Strouhal number, will remain constant at high Reynolds numbers. However, due to the complicated model configuration of the lobed forced mixer, there will be two maximum Strouhal numbers for one model at one velocity ratio. A small range of the Strouhal number can be easily identified along the trailing edge. For example in the present geometry under investigation, the Strouhal number at velocity ratio 0.4:1 lies within 6.68 to 13.1 at high Reynolds number for the basic single-lobe forced mixer. In addition, the trailing edge circulation is almost constant regardless of the initial conditions and in the absence of the neighboring lobes, the decay length was found to be within five wavelengths.

Finally, it must also be pointed out that the present investigation did not include the interactions of the neighboring vortices. As shown in the earlier measurements, at high velocity ratios, the streamwise vortices tend to decay faster. It may be possible to

speculate based on the present results. For the case of $r = 0.4:1$ at Reynolds number 1.5×10^4 , the K-H vortex firstly broke down at about 1.2 wavelengths from the trailing edge for point L. The time-averaged measurements for the mean velocity also showed that the distance between any two contour levels increased significantly. The rapid diffusion for the streamwise mean velocities would facilitate the interactions between two neighboring vortices. This may explain why the streamwise vortices would decay even faster at higher velocity ratios.

4.5 Conclusions

Hot-wire anemometer and Laser Doppler Anemometer have been employed in the present investigation to measure and evaluate the respective characteristics of the Kelvin-Helmholtz vortices and the streamwise vortices generated in the wake of a single-lobe forced mixer, at two velocity ratios, namely 1:1 and 0.4:1. The Kelvin-Helmholtz vortices generated in the plane free shear layer (flat plate) have been compared. From the experiments the following conclusions can be drawn.

(1) The streamwise vorticity increases with the increase in the penetration angle, provided that the angle is less than 22 degree. Similar to the multi-lobe forced mixer, the single-lobe forced mixer follows the scaling law of the initial mean streamwise vorticity well.

(2) The Kelvin-Helmholtz vortex shedding frequency is proportional to the mean velocity for all the models tested. For the forced mixer flow, there are two dominant K-H vortex shedding frequencies in the wake: one is behind the semi-circle part of the trailing edge (f_C), the other is behind the parallel side-walls (f_L); and f_C is always higher than f_L , especially for the forced mixer flow. The frequencies of the flow structures in the forced mixer are some 30% (at point L) to 110% (at point C) higher than those of the flat plate case.

(3) The mean K-H vortex wavelengths for a flat plate are larger than those of the lobed forced mixer. Varying the velocity ratios had more significant effect on the wavelength at point C than at point L.

(4) The Strouhal number of the plane free shear layer is about 0.031, based on the mean momentum thickness. For the lobed forced mixer flow, the Strouhal number

increases with the Reynolds number while the increasing ratio decreases gradually until it reaches certain stable values both for point C and L. Varying the velocity ratios had more significant effect on the Strouhal number at point C than at point L. When the velocity ratio was changed from 1:1 to 0.4:1, the Strouhal number at point C increased by almost 100%. Furthermore it may be expected that higher peak Strouhal number could be achieved when the velocity ratio approaches 0:1.

(5) The interaction of the streamwise vortices with the K-H vortices makes the latter structure 'pinched-off' and rotate faster with higher vorticity, resulting in more mixing enhancement. The generation of turbulent kinetic energy is mainly due to the Kelvin-Helmholtz vortices while the presence of the streamwise vortices enhance its generation further. Both of them contribute to the mixing enhancement performance.

(6) The shape factor, as an indicator of mixing performance, decreases when the penetration angle increases from 0 to 22 degree, which implies that better mixing performance could be achieved at larger penetration angle.

Figure 4.1 Ratio of Momentum Thickness for Two Sides of (a) Flat Plate and (b) Basic Single-lobe Forced Mixer (model a) at Point C

Figure 4.2 Typical Power Spectra of the Plane Free Shear Layer (Flat Plate)

Figure 4.3 K-H Vortex Shedding Frequencies after the Trailing Edge of the Flat Plate (FP)

Figure 4.4 K-H Wavelength Variations for Plane Free Shear Layer (FP)

Figure 4.5 Velocity Profiles and Momentum Thickness of the Upper and Lower Sides of Point C (C is the Apex of the Penetration Region of the Forced Mixer)

Figure 4.6 The Measuring Locations of K-H Vortices for the Single-lobe Forced Mixer

Figure 4.7 Typical Power Spectra of the Basic Single-lobe Forced Mixer (model a)

Figure 4.8 The K-H Vortex Shedding Frequencies after the Trailing Edge of the Basic Single-lobe Forced Mixer (model a)

Figure 4.9 K-H Wavelength Variations for Basic Single-lobe Forced Mixer (model a)

Figure 4.10 The Strouhal Number Variation with the Reynolds Number for Basic Single-lobe Forced Mixer (model a)

Figure 4.11 K-H Vortex Wavelength Variation for Single-lobe Forced Mixer at Penetration Angle $\varepsilon=11.4^\circ$ (model b)

Figure 4.12 K-H Vortex Wavelength Variation for Single-lobe Forced Mixer at Penetration Angle $\varepsilon=16.8^\circ$ (model c)

Figure 4.13 The Strouhal Number Variation with the Reynolds Number for Single-lobe Forced Mixer at Penetration Angle $\varepsilon=11.4^\circ$ (model b)

Figure 4.14 The Strouhal Number Variation with the Reynolds Number for Single-lobe Forced Mixer at Penetration Angle $\varepsilon=16.8^\circ$ (model c)

Figure 4.15 Maximum Strouhal Number Range for the Models at Different Penetration Angles at $\varepsilon=22.0^\circ$ or $h=30$ mm (Basic Model a); $\varepsilon=16.8^\circ$ or $h=22.5$ mm (Model c); and $\varepsilon=11.4^\circ$ or $h=15$ mm (Model b)

(a) Circular Cylinder in Uniform Cross-flow

(b) Strouhal Number as the Function of Reynolds Number for the Wake of Circular Cylinder

Figure 4.16 Dynamical Characteristics for the Wake of the Circular Cylinder (Roshko 1953)

(a) Sketch of the Uniform Flow Passing Over Wing Model (NACA 0012)

(b) Functional Relationship between Roshko Number and Reynolds Number for the Wing Model

(c) Functional Relationship between Strouhal Number and Reynolds Number for the Wing Model

Figure 4.17 Dynamic Characteristics for the Wake of Wing Model (Huang *et al* 2000)

(a) Coordinate System and Vector Magnitude Scale

(1) $x/\lambda = 0.5$

(2) $x/\lambda = 1.0$

(3) $x/\lambda = 2.0$

(4) $x/\lambda = 3.0$

(5) $x/\lambda = 4.0$

(6) $x/\lambda = 6.0$

(b) Velocity Ratio $r=1:1$

(1) $x/\lambda = 0.5$

(2) $x/\lambda = 1.0$

(3) $x/\lambda = 2.0$

(4) $x/\lambda = 3.0$

(5) $x/\lambda = 4.0$

(6) $x/\lambda = 6.0$

(c) Velocity Ratio $r=0.4:1$

Figure 4.18 Contours of the Normalized Streamwise Velocity $(U/\bar{U})$ and the Corresponding Secondary Flow Velocity Vectors $(\bar{V}_s/\bar{U})$ at Successive Downstream Stations after the Trailing Edge for the Basic Single-lobe Forced Mixer at (b) Velocity Ratio $r=1:1$ and (c) Velocity Ratio $r=0.4:1$

(a) Coordinate System and Vector Magnitude Scale

(4) $x/\lambda = 4.0$

(5) $x/\lambda = 6.0$

(b) Velocity Ratio $r = 1:1$

(1) $x/\lambda = 0.5$

(2) $x/\lambda = 1.0$

(3) $x/\lambda = 2.0$

(4) $x/\lambda = 4.0$

(5) $x/\lambda = 6.0$

(c) Velocity Ratio $r=0.4:1$

Figure 4.19 Contours of the Normalized Streamwise Velocity $(U/\bar{U})$ and the Corresponding Secondary Flow Velocity Vectors $(\vec{V}_s/\bar{U})$ at Successive Downstream Stations after the Trailing Edge for the Single-lobe Forced Mixer (Model b) at (b) Velocity Ratio $r=1:1$ and (c) Velocity Ratio $r=0.4:1$

(a) Coordinate System and Vector Magnitude Scale

(2) $x/\lambda = 1.0$ (3) $x/\lambda = 2.0$

(4) $x/\lambda = 4.0$

(5) $x/\lambda = 6.0$

(b) Velocity Ratio $r = 1:1$

(1) $x/\lambda = 0.5$

(2) $x/\lambda = 1.0$

(3) $x/\lambda = 2.0$

(4) $x/\lambda = 4.0$

(5) $x/\lambda = 6.0$

(c) Velocity Ratio $r=0.4:1$

Figure 4.20 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) and the Corresponding Secondary Flow Velocity Vectors ($\vec{V}_s/\bar{U}$) at Successive Downstream Stations after the Trailing Edge for the Single-lobe Forced Mixer (Model c) at (b) Velocity Ratio $r=1:1$ and (c) Velocity Ratio $r=0.4:1$

Figure 4.21 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) at Station of $x = 120$ mm after the Trailing Edge of Flat Plate for Velocity Ratio $r = 0.4:1$

Figure 4.22 Schematic Diagram for the Streamwise Vorticity Measurements, $\Delta y = \Delta z = 5$ mm

(1) $x/\lambda=0.5$

(2) $x/\lambda=1.0$

(3) $x/\lambda=2.0$

(4) $x/\lambda=3.0$

(5) $x/\lambda=4.0$

(6) $x/\lambda=6.0$

a. Velocity Ratio $r=1:1$

(1) $x/\lambda=0.5$

(2) $x/\lambda=1.0$

(3) $x/\lambda=2.0$

(4) $x/\lambda=3.0$

(5) $x/\lambda=4.0$

(6) $x/\lambda=6.0$

b. Velocity Ratio $r=0.4:1$

Figure 4.23 Contours of the Normalized Streamwise Vorticity at Downstream Stations after the Trailing Edge of Basic Single-lobe Forced Mixer (Model a) for (a) Velocity Ratio $r=1:1$ and (b) Velocity Ratio $r=0.4:1$

Figure 4.24 Variation of Peak Streamwise Vorticity with Downstream Distance for Basic Single-lobe Forced Mixer

Figure 4.25 Variation of the Normalized Streamwise Circulation with Downstream Distance of Single-lobe Forced Mixers at Different Heights

Figure 4.26 Momentum Thickness Growth in the Mixing Layer of the Plane Free Shear Layer

Figure 4.27 Momentum Thickness Growth in the Mixing Layer of the Lobe Forced Mixer

Figure 4.28 Momentum Thickness Growth in the Mixing Layer of the Single-Lobe Forced Mixer with Different Heights at $r=0.4:1$

Figure 4.29 Normalized Wake-Area Averaged Turbulent Kinetic Energy ($\bar{k}$) and Reynolds Normal Stresses in the Wake of Basic Single-lobe Forced Mixer

Figure 4.30 Distributions of the Normalized Turbulent Kinetic Energy ($\bar{k}$) after the Trailing Edge of the Basic Single-lobe Forced Mixer, $x/\lambda=0.5$

Figure 4.31 Normalized Wake-Area Averaged Turbulent Kinetic Energy ($\bar{k}$) of Single-lobe Forced Mixer with Different Heights, $h=15\text{ mm}$ (Model b, in Real Lines) and $h=22.5\text{ mm}$ (Model c, in Broken Lines)

Figure 4.32 Variation of Shape Factor for the Flat Plate and Single-lobe Forced Mixer at Different Lobe Heights, Velocity Ratio $r = 0.4:1$

(a) Initial K-H Vortex Tube Shed after Trailing Edge of Lobed Forced Mixer

(b) Pinched-off K-H Vortex Tube in the Downstream Stations after Trailing Edge with higher Rotating Speed ($\omega_2 > \omega_1$)

(c) Schematic Diagram for Vortex Tube Stretching

Figure 4.33 Interactions of Streamwise Vortices with the K-H Vortices

Chapter 5. Flow Characteristics of the Convolute Plate and Single-lobe Forced Mixers at Different Wavelengths

5.1 Introduction

The purpose of the present investigation is to provide a better understanding of the vortical and turbulent structures in the lobed forced mixer shear layer and to examine the mixing effectiveness of lobed forced mixer under different flow conditions and geometries. The Kelvin-Helmholtz vortex shedding frequency, Strouhal number, the development of streamwise vorticity and the growth of momentum thickness were examined. In the previous chapter, the height of the lobe had been examined in details. The effects of another important parameter of the mixer, the lobe wavelength, will be studied in this chapter.

The characteristics of the convolute plate that can only generate the normal K-H vortices will first be studied with the objective to reveal the respective contributions of the two different types of vortices in relations to the basic lobed mixer in Chapter 4.

The interfacial area for the two on-coming streams will be increased as a result of the convolute trailing edge surfaces for the lobed forced mixer. The convolute plate (model d), which is designed to have the same trailing edge configuration as the basic single-lobe forced mixer (model a in Chapter 4), is expected to have better mixing performance than the flat plate, while it should be less than the lobed forced mixer. This is because the airflow would be re-straightened to a great extent before reaching the trailing edge as shown in **Fig 5.1**. As a result, the secondary flow generated in its shear layer would be weaker than that of the lobed forced mixer flow and the streamwise vortices should be insignificant or likely be negligible. However, the increased interfacial areas between the two mixing streams and the normal K-H vortices, to some extent,

would enhance the mixing performance of the convoluted plate.

Similar investigation had been carried out previously by Manning (1991) with a multi-lobe forced mixer. By examining chemical reaction between the two on-coming streams (added with hydroxide ions and phenolphthalein respectively) via optical densitometry technique in a water tunnel, the molecular mixing of three models, namely flat plate, convoluted plate and lobed forced mixer, had been investigated. One of the major conclusions was that the convoluted plate provided about half the mixing augmentation of the lobed mixer. However, the measurements in optical density had some shortcomings in absolute accuracy even after the complicated calibration procedures, and that the free shear layers might also be influenced by the duct walls. Of great importance in Manning's investigation was that the normal K-H vortices shed after the convoluted plate and the forced mixer were considered to be of the same intensity and significance. Besides Manning (1991), Yu and Yip (1997 a) had measured the streamwise vorticity generated after the convoluted plate, normal lobed mixer and scalloped mixer using a two-component LDA. It was found that the shear layer growth for the convoluted plate, which had little influence on the streamwise vorticity, depended solely on the roll-up of the normal vorticity and was therefore much lower than the cases of normal and scalloped mixers with streamwise vorticity. However, detailed measurements, especially the intensity of the normal K-H vortices, were yet to be obtained, which would be carried out in the present investigation.

As far as the lobe's wavelength effects on mixing are concerned, little work has been carried out in the previous studies, though the wavelength is deemed as another important parameter besides the lobe height. The wavelength affects the spanwise length-scale of

the streamwise vortices. As shown in **Fig 5.2**, the spanwise and the vertical length-scales of the streamwise vortices are (Wu, 1993):

$$L_{span} = \lambda / 2; \quad (5-1)$$

$$L_{vert} = h. \quad (5-2)$$

To optimize the streamwise vortex, these two scales should be matched so that the streamwise vortex could rotate in a circular form, that is:

$$\lambda / 2 = h. \quad (5-3)$$

It should be noted that the basic single-lobe forced mixer (model a) is designed according to this principle.

As has already been shown in Chapter 4, in order to generate strong streamwise vortices, the height of the lobe (h) or the penetration angle (ε) should not be too small. Three single-lobe forced mixers have already been examined at different heights at $h=15$ mm, 22.5 mm and 30 mm respectively, with the same lobe wavelength at 60 mm. The results show that the basic model with height of 30 mm (or penetration angle at $\varepsilon = 22^\circ$) generates the strongest streamwise vorticity, which is also directly proportional to $\tan(\varepsilon)$. Test models at different wavelengths but at the same height ($h = 30\text{mm}$) will be examined. As shown in **Fig 3.2**, the wavelength of model e is 30 mm; while model f is 120 mm. The ratio of half wavelength to height for model e and f is 0.5 and 2.0 respectively. For model a, this ratio is optimized to be at 1.0. As a result, the spanwise length scale of the streamwise vortices would be modified. It would be compressed for model e, while it would be extended for model f. The intensity of the streamwise vortices generated within the wake of these three models at different lobe wavelengths would be different in someways. They will also be examined and compared in this chapter.

Besides the streamwise vorticity, the normal K-H vortex shed after the convoluted plate, models e and f, are expected to be different, because the initial conditions and the boundary conditions for different types of test models are different. Similar to the former models at different lobe heights (models a, b and c), the K-H vortex shedding frequencies, their mean wavelength, and the non-dimensional frequency or Strouhal number would be investigated in this chapter for the convoluted plate (model d) and the other two single-lobe forced mixers at different wavelengths (models e and f).

5.2 Hot-Wire Anemometer Measurements

5.2.1 Convolute Plate

5.2.1.1 K-H Vortex Shedding Frequency

Similar to the lobed mixer flow, there were always two dominant frequencies in the convolute plate's wake flow. It is because the initial flow conditions at points C and L were different. For the regions in the wake of the semi-circle part of the trailing edge, the dominant frequency was essentially the same in this region (denoted as f_C); and for the regions behind the parallel side-walls of the trailing edge, the dominant frequency was also the same (denoted as f_L). It should be noted that f_C was always higher than f_L under any flow condition. **Fig 5.3** shows typical spectra for the convolute plate at point C. The relationships between the frequency and the velocity for the convolute plate may be approximated as (see **Fig 5.4**):

$$f_{CP_C} = 80.9 \bar{U} - 120 \quad (r = 1:1, 3.40 \text{ m/s} \leq \bar{U} \leq 10.36 \text{ m/s}) ; \quad (5-4)$$

$$f_{CP_L} = 79.2 \bar{U} - 81.6 \quad (r = 1:1, 3.40 \text{ m/s} \leq \bar{U} \leq 10.36 \text{ m/s}); \quad (5-5)$$

$$f_{CP_C} = 116.0 \bar{U} - 75.5 \quad (r = 0.4:1, 2.39 \text{ m/s} \leq \bar{U} \leq 7.25 \text{ m/s}); \quad (5-6)$$

$$f_{CP_L} = 83.8 \bar{U} - 66.3 \quad (r = 0.4:1, 2.39 \text{ m/s} \leq \bar{U} \leq 7.25 \text{ m/s}); \quad (5-7)$$

Velocity ratio can be much more significantly seen to affect the frequency at point C than at point L. When the velocity ratio decreased from 1:1 to 0.4:1, the frequency at point L changed little while at point C it increased by 60%. The K-H vortex shedding frequency at point L of the convolute plate was very similar to the flat plate case (see **Fig 4.3**). For a velocity ratio equals to unity ($r = 1:1$), both frequencies measured at points C and L were similar to the flat plate case. It can be expected that if the extension part of

the convoluted plate were long enough, the two frequencies at points C and L may become the same as the flat plate case: the velocities before the trailing edge of the fully extended convoluted plate would be thoroughly re-aligned within the streamwise direction.

5.2.1.2 K-H Vortex Wavelength

Fig 5.5 shows the wavelength variation of the convoluted plate, both at points C and L. It is clear that the K-H wavelength decreased with the increase in the mean velocity until reaching certain stable values. Varying the velocity ratios had more effects on the wavelength after point C than after point L.

Comparing with the basic lobed forced mixer, the wavelength of the convoluted plate is of larger magnitude both at points C and L. The extension part of the convoluted plate had weakened the intensity of the K-H vortices. Furthermore, there would be no vortex stretching effects in the absence of the streamwise vortices. This may be another reason that had weakened the strength of the K-H vortices in the wake of the convoluted plate.

5.2.1.3 Strouhal Number

Similar to that of the forced mixer, the Strouhal number for the convoluted plate is defined as the ratio of the nominal length (λ) to the mean wavelength of the K-H vortices:

$$St = \frac{f \cdot \lambda}{U} = \frac{\lambda}{\lambda_{K-H}}. \quad (5-8)$$

From equations (5-4) to (5-7), the relationships between the Reynolds number and the Strouhal number can be approximated as follows:

$$St_{CP_C} = 4.85 - \frac{28650}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (5-9)$$

$$St_{CP_L} = 4.75 - \frac{19500}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (5-10)$$

$$St_{CP_C} = 6.96 - \frac{18050}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (5-11)$$

$$St_{CP_L} = 5.03 - \frac{15850}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (5-12)$$

As shown in **Fig 5.6**, the variation of the Strouhal number for the convoluted plate was very similar to the basic single-lobe forced mixer (see **Fig 4.10**): it increased with increasing Reynolds number and decreasing velocity ratio. For Reynolds number less than 20,000, the Strouhal number increased rapidly; while for Reynolds number larger than 20,000 it increased relatively slowly. Stable Strouhal number values were found at Reynolds number above 5×10^5 . The velocity ratio affects Strouhal number behind point C more significantly than behind point L.

The stable values of the Strouhal number at high Reynolds number for the convoluted plate were lower than the values of the basic single-lobe forced mixer, see **Figs 5.6** and **4.10**. This can only be imputed to the extension part of the convoluted plate comparing to the basic single-lobe forced mixer. By extending the trailing edge of the basic model a, the convoluted plate re-straightens the on-coming airflows. As a result, the upper and lower airflows would become similar to the flat plate case, to some extent. Therefore for the convoluted plate case, the Strouhal number, as an indicator of the significance of the K-H vortices, would be lower than the basic single-lobe forced mixer. It can also be noted that with the increase of the extension part of the convoluted plate from 0 to infinite, the significance of the K-H vortices would also decrease and eventually matched

that of the flat plate case. However, due to its corrugated trailing edge configuration, the interfacial areas between the two mixing streams would be larger than that of the flat plate case. Actually the interfacial areas of the convoluted plate were about 2.07 times of the flat plate case. As a result, the mixing performance of the convoluted plate would be better than the flat plate.

5.2.2 Single-lobe Forced Mixers at Different Wavelengths

5.2.2.1 K-H Vortex Shedding Frequency

The K-H vortex shedding frequencies for the models e and f are shown in Figs 5.7 and 5.8 respectively, for the two different points C and L at two velocity ratios $r=1:1$ and $0.4:1$. Similar to the case of the basic model a, they all increased linearly with the mean velocity. The linear functions for model e ($\lambda=30\text{mm}$) can be concluded as follows:

$$f_{LMe_C} = 110.0 \bar{U} - 98.6 \quad (r=1:1, 3.40\text{m/s} \leq \bar{U} \leq 10.36\text{m/s}) ; \quad (5-13)$$

$$f_{LMe_L} = 85.2 \bar{U} - 78.9 \quad (r=1:1, 3.40\text{m/s} \leq \bar{U} \leq 10.36\text{m/s}) ; \quad (5-14)$$

$$f_{LMe_C} = 183.1 \bar{U} - 219.5 \quad (r=0.4:1, 2.39\text{m/s} \leq \bar{U} \leq 7.25\text{m/s}) ; \quad (5-15)$$

$$f_{LMe_L} = 96.8 \bar{U} - 120.4 \quad (r=0.4:1, 2.39\text{m/s} \leq \bar{U} \leq 7.25\text{m/s}) ; \quad (5-16)$$

While for model f ($\lambda=120\text{mm}$), the K-H vortex shedding frequencies can be fitted as:

$$f_{LMf_C} = 96.6 \bar{U} - 120.3 \quad (r=1:1, 3.40\text{m/s} \leq \bar{U} \leq 10.36\text{m/s}) ; \quad (5-17)$$

$$f_{LMf_L} = 81.8 \bar{U} - 117.6 \quad (r=1:1, 3.40\text{m/s} \leq \bar{U} \leq 10.36\text{m/s}) ; \quad (5-18)$$

$$f_{LMf_C} = 163.3 \bar{U} - 210.2 \quad (r=0.4:1, 2.39\text{m/s} \leq \bar{U} \leq 7.25\text{m/s}) ; \quad (5-19)$$

$$f_{LMf_L} = 105.1 \bar{U} - 141.8 \quad (r = 0.4:1, \quad 2.39 \text{ m/s} \leq \bar{U} \leq 7.25 \text{ m/s}); \quad (5-20)$$

From the comparison with the basic model a with a lobe wavelength of 60 mm, it can be seen that the K-H vortex shedding frequency for both model e and f were lower than the basic model a. The lobe of model e at wavelength $\lambda=30$ mm was too thin to alter the on-coming flow fields. While the lobe for model f at wavelength $\lambda=120$ mm was so wide that it had over modified the initial flow conditions. The LDA measurements would give more detailed velocity fields information later, from which it could be seen that flow separations had happened near point C. This would suppress the intensity of the normal K-H vortices greatly.

5.2.2.2 K-H Vortex Wavelength

The variations of the K-H vortex wavelength are plotted in **Figs 5.9** and **5.10** for models e and f respectively.

Similar to the basic model a, the K-H mean wavelength shed in the shear layers of models e and f decreased gradually with the increasing Reynolds number. Finally they reach certain stable values at Reynolds number larger than 5×10^5 , i.e., the inertia-dominated regime where the viscous effects became negligible. The stable K-H wavelength magnitudes of the models a, e and f are tabulated in **Table 5-1**.

It can also be seen that the mean wavelength of the K-H vortex at point C was always smaller than that at point L, and depended more on the velocity ratio than that at point L. This was mainly due to the fact that the initial flow conditions at point L were much similar to the flat plate case, which did not depend on the velocity ratio greatly. Whereas

the initial flow conditions at point C were greatly modified, which depended on the velocity ratio to a great extent.

Same Lobe Height ($h=30\text{mm}$)	Different Lobe Wavelength λ (mm)	K-H Wavelength λ_{K-H} (mm)			
		Point C		Point L	
		$r=1:1$	$r=0.4:1$	$r=1:1$	$r=0.4:1$
Model a	60	7.43	4.58	8.67	8.98
Model e	30	9.09	5.46	11.74	10.33
Model f	120	10.34	6.12	12.22	9.51

Table 5-1 Stable K-H Wavelengths for Models at Different Lobe Wavelengths

Due to their lower vortex shedding frequencies, the K-H wavelengths for both models e and f are higher than the basic model a, both at points C and L. This means that the K-H vortices' contribution to the mixing enhancement for models e and f would not be as significant as the basic model a.

5.2.2.3 Strouhal Number

The variations of the Strouhal number are shown in Figs 5.11 and 5.12 for models e and f respectively. For model e with lobe wavelength at ($\lambda=$) 30 mm, the Strouhal number variation can be approximated as:

$$St_{LM\ e_C} = 6.60 - \frac{23650}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \tag{5-21}$$

$$St_{LM\ e_L} = 5.11 - \frac{18950}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \tag{5-22}$$

$$St_{LM\ e_C} = 10.98 - \frac{52400}{Re} \quad (r = 0.4 : 1, \quad 9,200 \leq Re \leq 27,900); \quad (5-23)$$

$$St_{LM\ e_L} = 5.81 - \frac{28750}{Re} \quad (r = 0.4 : 1, \quad 9,200 \leq Re \leq 27,900); \quad (5-24)$$

While model f has lobe wavelength at ($\lambda=$) 120 mm, its Strouhal number variation can be approximated as:

$$St_{LM\ f_C} = 5.80 - \frac{28700}{Re} \quad (r = 1 : 1, \quad 13,100 \leq Re \leq 40,000); \quad (5-25)$$

$$St_{LM\ f_L} = 4.91 - \frac{28100}{Re} \quad (r = 1 : 1, \quad 13,100 \leq Re \leq 40,000); \quad (5-26)$$

$$St_{LM\ f_C} = 9.80 - \frac{50200}{Re} \quad (r = 0.4 : 1, \quad 9,200 \leq Re \leq 27,900); \quad (5-27)$$

$$St_{LM\ f_L} = 6.31 - \frac{33850}{Re} \quad (r = 0.4 : 1, \quad 9,200 \leq Re \leq 27,900); \quad (5-28)$$

Comparing with the basic single-lobe forced mixer (modal a), it was obvious that the maximum Strouhal numbers were different for models with different wavelengths. As shown in **Fig 5.13**, the basic model a had the highest maximum Strouhal numbers. For the single-lobe forced mixer with the same lobe height (models a, e and f), when the wavelength increased from 0 to the critical value ($\lambda=2h=60$ mm), the maximum Strouhal numbers increased correspondingly. This was because the lobe with smaller wavelength could not modify the flow fields or the initial conditions dramatically so that the significance of the K-H vortices were still weak, or only slightly stronger than the flat plate case. Beyond the critical value, the maximum Strouhal numbers began to decrease, especially at point C. This was mainly because the penetration region of the lobe over modified the flow fields near point C. Consequently the significance of the K-H vortices

would decrease at point C, as well as the corresponding maximum Strouhal number.

It can be seen that the basic model a had the most effective design in wavelength to generate the strongest K-H vortices so that mixing performance could be enhanced.

5.3 Laser Doppler Anemometer Measurements

5.3.1 Streamwise Mean Velocity Contours and Secondary Mean Velocity Vectors

5.3.1.1 Convolved Plate

The coordinate system used for the LDA measurement of the convoluted plate (shown in **Fig 5.14a**) was the same as that for the basic single-lobe forced mixer. The extent of the cross sectional planes were chosen to be the same as the lobed forced mixer also, that is, $+0.75\lambda \geq y \geq -0.75\lambda$ and $+\lambda \geq z \geq -\lambda$. Due to the extension part of the convoluted plate model, which was 1.5 times of the model wavelength, only four downstream stations after the trailing edge of the convoluted plate in the near field, i.e., $x/\lambda = 0.5, 1, 2$ and 4 , were measured and at two velocity ratios 1:1 and 0.4:1. As expected, the velocity fields measured were symmetrical about X-Z plane to within 7% (based on mass flux evaluation), only the results on the left half side are shown here.

(a) Velocity Ratio $r = 1:1$

Similar to the lobed forced mixer, the streamwise velocity fields after the trailing edge will deviate from the mean value by about 20% due to the penetration region.

As shown in **Fig 5.14b** (1) to (4) for the four downstream stations, the velocity difference disappeared gradually via diffusion. At the first station $x/\lambda = 0.5$ and **Fig 5.14b** (1), the streamwise velocity at $z = +3/4 \lambda$ was about 20% higher than the mean streamwise velocity; while it was about 20% lower at around $z = -3/4 \lambda$. At further downstream stations, the streamwise velocity would become more uniform. For example, at the subsequent two downstream stations $x/\lambda = 1$ and 2 shown in **Figs 5.14b** (2) and (3), the maximum deviations were about 10% of the mean streamwise velocity. By the final

measured station at $x/\lambda = 4$ and **Fig 5.14b** (4), the velocity distribution had become quite uniform in the streamwise direction (X) with maximum deviation at about 5% of the mean streamwise velocity.

(b) Velocity Ratio $r = 0.4:1$

For this case, the velocity of the upper-side on-coming airflow was 40% of the lower-side. The normalized streamwise velocity contours for the four downstream stations are shown in **Fig 5.14c** (1) to (4) respectively. At $x/\lambda = 0.5$ (**Fig 5.14c** (1)), the streamwise velocity contours followed closely the shape of the convolutioned plate's trailing edge. With the momentum exchange between the two streams going on, the gradients of the streamwise velocity also decreased gradually. The streamwise velocity contours were seen to diffuse consequently at the further downstream stations. Nevertheless, the velocity difference within the measured region would be maintained until the last measured station at $x/\lambda = 4$ shown in **Fig 5.14c** (4). The mixing would continue into the far field.

The secondary flows in the wake of the convolutioned plate were very weak. The maximum magnitude of the secondary flow did not exceed 8% of the nominal velocity, even in the very near wake, i.e., at the station $x/\lambda = 0.5$. As it had been pointed out earlier, although its penetration region had altered the directions of the two on-coming flows, its extension part re-aligned the altered flow directions back to the streamwise direction. As a result, secondary flows almost disappeared in the wake. As shown, the convolutioned plate was able to suppress the generation of strong streamwise vorticity.

5.3.1.2 Single-lobe Forced Mixer at Wavelength $\lambda=30$ mm

The coordinate system used for the LDA measurement of model e is shown in **Fig 5.15a**. The height of the present model is the same as models a and f. The extent of the cross-sectional planes being measured is also the same as the basic lobed forced mixer, that is, $+1.5h \geq y \geq -1.5h$ and $+2h \geq z \geq -2h$. Five downstream stations after the trailing edge of model e in the near field, i.e., $x/(2h) = 0.5, 1, 2, 4$ and 6 were measured at two velocity ratios 1:1 and 0.4:1.

Generally speaking, the flow pattern of this model was similar to that of the basic model a. However, due to the fact that the wavelength of model e (shown in **Fig 5.16**) is only half of basic model a (shown in **Fig 5.2**), the secondary flow that had passed through the lower trough (flow 1) would have less mass flux to match with that had passed through the upper trough (flow 2). The spanwise length scale of the streamwise vortices (L_{span}) was equal to the wavelength of the present model e, or half of the basic model a. Comparing **Fig 5.16** with **Fig 5.2**, it was clear that the expected circular shape of the streamwise vortex would be compressed (at least in size) in the spanwise direction. As a result, the shape of the streamwise vortex generated by the model e would be deformed to become somewhat elliptical. As a result, the streamwise vortices in the present model e would entrain less mass flux comparing to the basic model a. This had led to its mixing performance not as good as the basic model a.

5.3.1.3 Single-lobe Forced Mixer at Wavelength $\lambda=120$ mm

The results for this model (model f) are plotted in **Fig 5.17**. Because the wavelength of the present model is 120mm (or double that of the basic model a), there is essentially

no parallel sidewall for this model. This caused the directions of secondary flows to diverge, especially for the flow that had passed through the lower trough (flow 1) for the case of velocity ratio 0.4:1 (which is circled within the ellipses shown in **Figs 5.17c** (1) and (2)). To generate streamwise vortices, flow 1 (**Fig 5.16**) was expected to travel in the positive cross-stream direction (+Z); while another secondary flow (flow 2), which had passed through the upper trough, was expected to travel in the negative cross-stream direction (-Z). Flow with non-uniform directions consumes kinetic energies but not effective for the streamwise vortices generation. So the streamwise vorticity for this model was not as strong as that for the models with parallel sidewalls. This was in agreement with the conclusions made by Skebe *et al* (1988) who deduced that the mixing performance of the model with parallel sidewalls is better than that without the parallel sidewalls.

Another observation was that with the increase in wavelength, the flow separation at the trough was more likely to appear. For the velocity ratio 1:1 and at cross-section $x/(2h)=0.5$, it was found that one small region near the lower trough (as circled in ellipse in **Fig 5.17b** (1)) whose velocity in the cross-stream direction (Z direction) was mainly negative. This region with negative velocity showed the existence of flow separation. It was clear that once the flow separation happens, pressure loss together with the drag and noise generation would increase dramatically. So from this point of view, the double wavelength designed model may not be effective for mixing enhancement.

5.3.2 Mean Streamwise Vorticity

As pointed out earlier, the strength of the mean streamwise vorticity ($\overline{\omega_x}$) was one

important indicator to quantify the effectiveness of mixer performance, which may be evaluated using the equation (4-35). Γ_I , which is defined in equation (4-37), is the streamwise circulation of the plane **I** whereas ∂I encompasses half lobe.

The results of the normalized streamwise circulation for the three mixers at different lobe wavelengths, that is model a, e and f, are plotted in **Fig 5.18** and **Fig 5.19** for the two velocity ratios. Results from the model e are shown in dotted lines. It should be pointed out that the rectangular cross-section plane **I** (or rectangle ABCD, see **Fig 4.25**) on which the circulation had been calculated differs from case to case. Actually the plane of basic model a was two times larger than that of model e, while it was only half of that of the model f. The normalizations were performed based on their respective lobe wavelengths.

It is obvious that for every model, the variation of the normalized streamwise circulation for the two velocity ratios were very similar to each other. However, for different models with different lobe wavelength, the magnitudes of circulation after normalization were some different. As may have been expected, the streamwise circulation for models e and f were both smaller than that of the basic model a.

For model e, the initial streamwise circulation at the first station measured, i.e., $x/(2h)=0.5$ was similar to that formed in the basic model a. This showed the initial strength of the streamwise vorticity for models a and e was similar in magnitudes. However, the decay rate for model e was much faster than that of model a. As it is pointed out earlier, it was because the secondary flow 1 with high positive cross-stream velocity (+w) entrained less mass flux to match the secondary flow 2 that had passed through the upper trough with high negative cross-stream velocity (-w). So the streamwise vortices generated in the wake of model e would entrain less mass flux

comparing to those in the wake of the basic model a. The smaller-scale streamwise vortex entraining less momentum would also decay faster, which had been shown in dashed lines in **Fig 5.18** and **Fig 5.19**. This was in line with the turbulent energy cascades theory that during the vortex breakdown, it broke down into smaller scale vortices all the way until it reaches the Kolmogorov scale. So the small-scale vortex was expected to breakdown earlier and faster than the large-scale vortex.

While for model f, due to the flow separation at the lower trough and the non-uniform secondary velocity fields corresponding to non-parallel sidewalls, its streamwise vorticity was initially about 30% weaker than that of the basic model a. However, their variation trends were similar to each other, as shown in **Figs 5.18** and **5.19**.

5.3.3 Momentum Thickness

The momentum thickness in the mixing layer is defined in equation (4-39). It is one important indicator for mixing performance, especially for the mixing between two streams with different velocities. **Fig 5.20** shows the momentum thickness growth of the convoluted plate (model d) and single-lobe forced mixers at different wavelengths (models a, e and f). Here the momentum thickness was normalized by their same lobe height ($h=30$ mm).

For the convoluted plate, the momentum thickness varied almost linearly with x/λ , which was very similar to the flat plate case. This was in line with the fact that neither the flat plate nor the convoluted plate could generate strong streamwise vortices. Correspondingly only the normal K-H vortices were mainly responsible for the momentum exchange between the two on-coming streams. Besides, there would be no

non-linear interaction effects between the streamwise vortices and the K-H vortices, which would usually enhance the momentum exchange and the mixing performance.

While for the lobed mixer models a, e and f, which were of the same lobe height but different wavelengths, it appeared that the lobe wavelength does not affect the momentum thickness significantly. However, their momentum growth tendencies were much different to those of the flat plate and convoluted plate. From **Fig 5.20** it could be seen that the increasing rate was very rapid from the trailing edge to $x/\lambda=2.0$ for all the three lobed mixers. The increasing rate of the convoluted plate was obviously lower than the other three cases initially. The results were expected, because in the absence of the streamwise vorticity, the entrainment for the convoluted plate case would rely solely on the 2-dimensional roll-up of the normal K-H vorticity that was obviously lower than the cases with streamwise vorticity. The comparison highlights the fact that the presence of the streamwise vorticity greatly enhanced the initial entrainment growth rate. However, the growth rates for the mixers reduced and became lower than that of the convoluted plate case beyond $x/\lambda=2.0$. It was not surprising that with the streamwise vorticity decaying gradually, the momentum exchange would be restrained and the corresponding entrainment growth rate became slower than that of the convoluted plate case, which had an undisturbed roll-up of the K-H vorticity from the trailing edge.

5.3.4 Shape Factor

The variation of the momentum shape factor with the downstream distance for the velocity matched case (velocity ratio 1:1) was very small compared to the unmatched velocity case. The calculated results for velocity ratio 1:1 were too small to quantify the

actual difference between different test models since its uncertainty was at about 0.28 %. Similar to the results of the momentum thickness, only the velocity ratio 0.4:1 was shown in **Fig 5.21** for the convoluted plate and the single-lobe forced mixers at different wavelengths. The results of the flat plate are also shown for comparison.

From the figure it can be seen that the shape factor of the convoluted plate was always lower than the flat plate. This was due to its increased interfacial areas and the stronger K-H vortices (compared with the K-H vortices of the flat plate) that enhanced the momentum mixing between the two on-coming streams. However, the shape factor for the convoluted plate was found to be much larger than the mixer at different wavelengths. This was mainly due to the facts that there was no streamwise vortices generated in the mixing layer, and the normal K-H vortices shed within its wake were also weaker than those of the lobed mixer. Furthermore in the absence of the streamwise vorticity, there would be no interactions between the normal and streamwise vortices.

The shape factor varied almost linearly with x/λ both for the flat plate and the convoluted plate, which seems to be in accordance with the fact that there was no significant streamwise vortex in the mixing layer of these two cases.

Comparing with the flat plate and the convoluted plate, the shape factors of the lobed mixers were of smaller magnitudes, which implied better mixing performance could be achieved by the mixer. Their decrease in the first two wavelengths downstream of the trailing edge indicated a rapid variation of the momentum distribution within the wake region, as a consequence of the streamwise vortices shed by the mixer. At further downstream stations, the variation slopes were found to decrease gradually. This was mainly due to the decay of the streamwise vortices. As expected, the momentum

exchange between the two streams would decrease as the streamwise vortices decayed gradually.

The shape factor of the basic model a was found to be lower than the other two mixers at lobe wavelength 30 mm and 120 mm respectively. This was due to the fact that the intensities of the normal K-H and streamwise vortices were stronger for the basic model a than those for the other two mixers with different lobe wavelengths, although the mixing interfacial areas of the three cases were the same.

5.4 Further Discussion

The relative contributions of K-H vortices and streamwise vortices in the mixing process downstream of the lobed forced mixers with different wavelengths have been examined in the present chapter. It can be shown from the present investigations that similar to the lobe height, the lobe wavelength also determines the strength of these two types of vortices significantly. Strong vorticity would lead to more efficient mixing and this in turn depends on the lobe wavelength as well as the lobe height.

For the convoluted plate, the initial K-H vortices shed from its trailing edge are weaker than the basic forced mixer, although stronger than those of the plane free shear layer. On the other hand, it has been revealed in section 4.4 that streamwise vortices stretched the K-H vortex tube (see **Fig 4.32**). This will be beneficial to the strengthening of the K-H vortices. But for the convoluted plate, this strengthening is not available due to the absence of the streamwise vorticity. The intensity of the K-H vortices within the wake of the convoluted plate would also be weaker than those in the wake of the basic forced mixer. As a result, the mixing contributions of the K-H vortices generated in the wake of the convoluted plate and the basic forced mixer are very different. So the conclusion made by Manning (1991) that the relative contributions of the K-H vortices (together with the increased interfacial areas) and the streamwise vortices were almost equal for the mixing augmentation may be worthy of reconsideration. From this point of view, the relative contributions of the K-H vortices are even more likely to be superior to the streamwise vortices.

It has been analyzed in the last chapter the effects of the pinched-off phenomenon together with the interactions of the streamwise vortices with the K-H vortices. In this

chapter, the interactions of the K-H vortices with the streamwise vortices will be further analyzed.

It is well known that the Navier-Stokes equation takes the following form:

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \vec{v} + \frac{\nu}{3} \nabla(\nabla \cdot \vec{v}) + \vec{f}, \quad (5-29)$$

in which $\vec{f}$ is the fluid body force.

Take curl ($\nabla \times$) onto this equation, the vorticity dynamics equation will be like this:

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega} \cdot \nabla) \vec{v} - \vec{\Omega}(\nabla \cdot \vec{v}) + \frac{1}{\rho^2} (\nabla \rho) \times (\nabla P) + \nu \nabla^2 \vec{\Omega} + \nabla \times \vec{f}, \quad (5-30)$$

in which

$$\vec{\Omega} = \nabla \times \vec{v}. \quad (5-31)$$

For the present investigation, some terms of the right hand in this equation can be simplified. For instance, for the second term $-\vec{\Omega}(\nabla \cdot \vec{v})$, because of the incompressibility characteristics of the low speed airflow, we can have

$$\nabla \cdot \vec{v} = 0, \quad (5-32)$$

or

$$-\vec{\Omega}(\nabla \cdot \vec{v}) = 0 \quad (5-33)$$

as well.

For the third term $\frac{1}{\rho^2} (\nabla \rho) \times (\nabla P)$, because the temperature is kept constant throughout the experiments, the mass density ρ is only dependent on the local pressure P . As a result, the directions of $\nabla \rho$ and ∇P are always the same. That is, $(\nabla \rho) \times (\nabla P) = 0$, or

$$\frac{1}{\rho^2}(\nabla\rho)\times(\nabla P)=0. \quad (5-34)$$

And for the last term $\nabla\times\vec{f}$, the only volume force onto the fluids is its gravity force,

$$\vec{f}=-g\cdot\vec{z}, \quad (5-35)$$

in which g is the gravity acceleration, and z is the coordinate component in the cross-stream direction. So

$$\nabla\times\vec{f}=0. \quad (5-36)$$

So the vorticity dynamics equation in the present investigation can be simplified to:

$$\frac{D\vec{\Omega}}{Dt}=(\vec{\Omega}\cdot\nabla)\vec{v}+\nu\nabla^2\vec{\Omega}. \quad (5-37)$$

This means only the vortex stretching and distortion term $(\vec{\Omega}\cdot\nabla)\vec{v}$ and viscous dissipation term $\nu\nabla^2\vec{\Omega}$ are contributive to the vorticity evolution. For the streamwise vorticity in the Cartesian coordinate system, equation (5-37) can be expressed in details like the following:

$$\frac{D\Omega_x}{Dt}=\Omega_x\cdot\frac{\partial u}{\partial x}+\Omega_y\cdot\frac{\partial u}{\partial y}+\Omega_z\cdot\frac{\partial u}{\partial z}+\nu\nabla^2\Omega_x. \quad (5-38)$$

From this equation it is very clear that the K-H vorticity (Ω_y and Ω_z) has an effect on the streamwise vorticity's evolution as well, which is similar to the wake flow of a delta-tab investigated by Yu *et al* (2001).

On the other hand, because

$$\frac{D\Omega_x}{Dt}=\frac{\partial\Omega_x}{\partial t}+(\vec{v}\cdot\nabla)\Omega_x=\frac{\partial\Omega_x}{\partial t}+u\frac{\partial\Omega_x}{\partial x}+v\frac{\partial\Omega_x}{\partial y}+w\frac{\partial\Omega_x}{\partial z}, \quad (5-39)$$

and the streamwise vorticity is almost stationary, or time-independent:

$$\frac{\partial \Omega_x}{\partial t} = 0, \quad (5-40)$$

together with the fact that the secondary flow velocities (v and w) are mostly smaller than the streamwise velocity (u), which means the transportation of the streamwise vorticity is mainly oriented in the streamwise direction, we can simplify the equation further to

$$\frac{D\Omega_x}{Dt} \approx u \frac{\partial \Omega_x}{\partial x}. \quad (5-41)$$

Here it can be seen that the time evolution of the streamwise vorticity is mainly linked to the decay (or growth) of the streamwise vorticity with downstream distance. Furthermore combine equations (5-38) and (5-41), we have:

$$u \frac{\partial \Omega_x}{\partial x} \approx \Omega_x \cdot \frac{\partial u}{\partial x} + \Omega_y \cdot \frac{\partial u}{\partial y} + \Omega_z \cdot \frac{\partial u}{\partial z} + \nu \nabla^2 \Omega_x. \quad (5-42)$$

This means that the decay of the streamwise vorticity with downstream distance ($\frac{\partial \Omega_x}{\partial x}$) is related to the stretching effect ($\Omega_x \cdot \frac{\partial u}{\partial x}$) from itself, distortion effects from K-H vorticity ($\Omega_y \cdot \frac{\partial u}{\partial y} + \Omega_z \cdot \frac{\partial u}{\partial z}$), and the fluidic viscous effect ($\nu \nabla^2 \Omega_x$).

It should be noted that the above discussion is focused on the local instantaneous streamwise vorticity. Of special importance is the streamwise mean vorticity transport equation, which may be written as (Bradshaw *et al* 1981):

$$\begin{aligned} \frac{D\Omega_x}{Dt} = & \Omega_x \cdot \frac{\partial U}{\partial x} + \Omega_y \cdot \frac{\partial U}{\partial y} + \Omega_z \cdot \frac{\partial U}{\partial z} + \frac{\partial}{\partial x} \left(\frac{\partial \overline{u'v'}}{\partial z} - \frac{\partial \overline{u'w'}}{\partial y} \right) \\ & + \frac{\partial^2}{\partial y \partial z} (\overline{v'^2} - \overline{w'^2}) + \left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial y^2} \right) (\overline{v'w'}) + \nu \nabla^2 \Omega_x. \end{aligned} \quad (5-43)$$

Suppose:

$$P_1 = \Omega_x \cdot \frac{\partial U}{\partial x}; \quad (5-44)$$

$$P_2 = \Omega_y \cdot \frac{\partial U}{\partial y} + \Omega_z \cdot \frac{\partial U}{\partial z}; \quad (5-45)$$

$$P_3 = \frac{\partial}{\partial x} \left(\frac{\partial \overline{u'v'}}{\partial z} - \frac{\partial \overline{u'w'}}{\partial y} \right); \quad (5-46)$$

$$P_4 = \frac{\partial^2}{\partial y \partial z} (v'^2 - w'^2); \quad (5-47)$$

$$P_5 = \left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial y^2} \right) (\overline{v'w'}); \quad (5-48)$$

$$P_6 = \nu \nabla^2 \Omega_x. \quad (5-49)$$

Apart from P_6 , i.e., the viscous term, P_1 , P_2 , P_3 , P_4 and P_5 are essentially the production terms for the streamwise vorticity. They can be broadly divided into two groups: P_1 and P_2 describe secondary flow production of the first kind (pressure-driven), while P_3 , P_4 and P_5 are concerned with secondary flow production of the second kind (Reynolds stress-driven). The focus of the present discussion will be on the effects of each production term on the evolution of streamwise vorticity with downstream distance.

P_1 represents the vortex stretching effect due to the streamwise mean flow acceleration. The effect of this term is difficult to quantify by examining the streamwise mean velocity distribution on a particular cross-sectional plane alone. However, qualitatively, some regions, for example the region that appears immediately behind the forced mixer can be examined. This region is initially occupied by low-momentum fluids, where the boundary layer is shedding from the trailing edge. As the flow progresses further downstream, this velocity defect region will rapidly be filled by the higher-momentum fluids from the surroundings as a result of the streamwise vorticity generated. Hence, the U is likely to increase in the streamwise vorticity direction. This provides a

positive production to the streamwise vorticity at the corresponding region. It should be noted that the flow becomes more uniformly distributed in the far wake region, so the stretching effects are confined to the region immediately after the forced mixer, where the boundary layer is shed from the trailing edge of the mixer. Whereas on one whole cross-sectional plane, this term is approximated to be zero due to the continuity characteristics:

$$\iint_{\text{Cross-section}} \frac{\partial U}{\partial x} \cdot dydz = \frac{\partial}{\partial x} \iint_{\text{Cross-section}} U \cdot dydz = \frac{\partial q}{\partial x} = 0, \quad (5-50)$$

in which

$$q = \iint_{\text{Cross-section}} U \cdot dydz \quad (5-51)$$

is the total volume flux of the flow.

P_3 and P_5 are production terms due to the shear stress components, including $\overline{u'v'}$, $\overline{u'w'}$ and $\overline{v'w'}$. From the cross hot-wire measurements, the magnitudes of these shear stresses are of the magnitude about 0.01, after non-dimensionalized by $\overline{U}^2$. For example at point C ($x=10$ mm, $y=0$, $z=30$ mm) in the high turbulence region of basic model a, the detailed datum is $\frac{\overline{u'v'}}{\overline{U}^2} = 0.015$. More extensive measurements using LDA in the wake of

a single inverted delta-tab have been carried out by Yu *et al* (2001), in which the measured shear stresses were of the same magnitudes as the present investigation. So the production terms of P_3 and P_5 can be concluded to be so small that they can be ignored.

P_4 is the production term due to the anisotropy of the wake flow. Refer to **Fig 4.29** in the last chapter it can be seen that the difference between v'^2 and w'^2 are of the magnitude about 0.002 throughout the near wake region. So this production term is

ignorable as well as P_3 and P_5 . As a result, only the term P_2 is left besides P_1 and P_6 . This is also true for the single inverted tab that has been measured by Yu *et al* (2001), in which P_2 is measured with magnitude about 0.2.

P_2 represents the vortex distortion effect due to the non-uniformity of the streamwise velocity in both spanwise (y) and cross-stream (z) directions. Actually this term shows how the normal K-H vorticity (Ω_y, Ω_z) affects the streamwise vorticity (Ω_x). So this term is of our main interests. Because this term relates both the K-H vorticity and the non-uniformity of the streamwise velocity, it is difficult to quantify this term point by point. Refer to **Fig 5.22**, points M and N are two symmetric points in the wake region of the forced mixer. The K-H vorticity at these two points are denoted as Ω_{K-H} and Ω'_{K-H} respectively. They can be decomposed into two parts, which are

$$\vec{\Omega}_{K-H} = \Omega_y \cdot \vec{j} + \Omega_z \cdot \vec{k} \quad (5-52)$$

$$\begin{aligned} \vec{\Omega}'_{K-H} &= \Omega'_y \cdot \vec{j} + \Omega'_z \cdot \vec{k} \\ &= \Omega_y \cdot \vec{j} - \Omega_z \cdot \vec{k} \end{aligned} \quad (5-53)$$

in which $\vec{j}$ and $\vec{k}$ are unit vectors of Y and Z coordinates, and

$$\Omega'_y = \Omega_y \quad (5-54)$$

while

$$\Omega'_z = -\Omega_z \quad (5-55)$$

On the other hand, the streamwise velocity derivatives satisfy the following relationships at these two points, also due to the fact that the flow fields are symmetrical about X - Z plane:

$$\left(\frac{\partial U}{\partial y}\right)_M = \left(\frac{\partial U}{\partial(-y)}\right)_N = -\left(\frac{\partial U}{\partial y}\right)_N ; \quad (5-56)$$

$$\left(\frac{\partial U}{\partial z}\right)_M = \left(\frac{\partial U}{\partial z}\right)_N, \quad (5-57)$$

So the term P_2 at points M and N are:

$$P_{2,M} = \Omega_y \cdot \left(\frac{\partial U}{\partial y}\right)_M + \Omega_z \cdot \left(\frac{\partial U}{\partial z}\right)_M \quad (5-58)$$

$$\begin{aligned} P_{2,N} &= \Omega_y' \cdot \left(\frac{\partial U}{\partial y}\right)_N + \Omega_z' \cdot \left(\frac{\partial U}{\partial z}\right)_N \\ &= \Omega_y \cdot \left(-\left(\frac{\partial U}{\partial y}\right)_M\right) + (-\Omega_z) \cdot \left(\frac{\partial U}{\partial z}\right)_M \\ &= -P_{2,M} \end{aligned} \quad (5-59)$$

It can be seen from the above equation that the distortion effects of the K-H vorticity on the streamwise vorticity at points M and N are of the same magnitude but negative effects. On the other hand, the initial streamwise vorticity (at $x=0$) should be symmetrical (same magnitude but with different sign) about X-Z plane too. So the evolution of the streamwise vorticity would be essentially symmetrical about X-Z plane. Meanwhile, the neighboring streamwise vortices would be pinched towards the X-Z plane. Furthermore this would lead to their merging with each other. According to the vortex merging results of Carton *et al* (2002) and Waugh (1992), for the mergering with opposite signed strains, the overall vorticity after merging is almost the algebra summation of the vorticity before merging. So this merging process is actually the decaying process, because the vorticity before merging is of the same magnitude while opposite signs. So the deformation of the K-H vortex tube would most likely to hasten the decaying of the streamwise vorticity. However, the effect of the term P_2 is difficult to quantify point by point because it varies according to the coordinates.

In summary, the evolution of the streamwise vorticity is mainly dominated by the normal K-H vorticity's distortion effects and the essential fluidic viscous effects. Only in the regions immediately behind the trailing edge does the self-stretching effects from streamwise vorticity itself exist. While the other production terms (P_3 , P_4 and P_5) may have some localized effects on the flow field, they should not have any significant effect on the overall trend of the streamwise vorticity development.

5.5 Conclusions

Normal Kelvin-Helmholtz vortex and streamwise vortex have been examined in this chapter for the models of convoluted plate (model d) and two other single-lobe forced mixers at different wavelengths (models e and f), using both the hot-wire anemometry and the laser-Doppler anemometry. Their results have been compared with the basic single-lobe forced mixer (model a). From this chapter the following conclusions can be drawn.

(1) The normal Kelvin-Helmholtz vortices generated in the wake of the convoluted plate are weaker than those of the basic single-lobe forced mixer, but stronger than those of the flat plate. No obvious streamwise vorticity can be generated in the wake of the convoluted plate.

(2) For single-lobe forced mixers with the same lobe height $h=30\text{mm}$, the basic model a with lobe wavelength $\lambda(=2h)=60\text{ mm}$ has stronger K-H vortex shedding than the other two models e and f with lobe wavelength $\lambda(=h)=30\text{ mm}$ and $\lambda(=4h)=120\text{ mm}$ respectively. Model e with half wavelength design of basic model a cannot change the on-coming flows so much; while model f with double wavelength design over modifies the on-coming flows.

(3) For single-lobe forced mixers with the same lobe height $h=30\text{ mm}$, the basic model a at lobe wavelength $\lambda(=2h)=60\text{ mm}$ has stronger streamwise vortex generation than the other two models e and f with lobe wavelength $\lambda(=h)=30\text{ mm}$ and $\lambda(=4h)=120\text{ mm}$ respectively. For model e, the streamwise vortices decay the fastest. While for model f, due to flow separation and non-uniform secondary flow directions, the mean streamwise vorticity is the weakest among the three models.

(4) The momentum thickness variation of the convoluted plate decrease almost linearly with x/λ , which is very similar to the flat plate case. Whereas for the single-lobe forced mixers at different wavelengths, the momentum thickness varies in a similar manner: initially it increases rapidly within two lobe wavelengths, and the increase slows down at further downstream stations due to the decaying of the streamwise vorticity.

(5) The convoluted plate can give a mixing performance better than the flat plate, but worse than that of the lobed mixer. The basic lobed mixer has superior mixing enhancement to the other two lobed mixers with different lobe wavelengths, due to its strongest normal K-H and streamwise vortices.

(6) For the mixer flow, both the distortion due to the normal K-H vorticity and the intrinsic viscous dissipation would affect the evolution of the mean streamwise vorticity. The self-stretching effect from streamwise vorticity itself only exists in the regions immediately after the trailing edge of the mixer.

Figure 5.1 Schematic Diagram of the Air-flow Passing through the Convolved Plate

Figure 5.2 Spanwise and Vertical Length Scales of the Streamwise Vortices in the Wake of Lobed Forced Mixer

Power Magnitude

(1) $U_1 = U_2 = 5.66 \text{ m/s}$, $f = 340 \text{ Hz}$.
Measuring Point C(30, 0, 30)mm.

Power Magnitude

(2) $U_1 = 5.66 \text{ m/s}$, $U_2 = 2.26 \text{ m/s}$, $r = 0.4:1$, $f = 320 \text{ Hz}$.
Measuring Point C(30, 0, 30)mm.

Figure 5.3 Typical Power Spectra of the Convoluted Plate under Different Flow Conditions

Figure 5.4 The K-H Vortex Shedding Frequencies after the Trailing Edge of the Convoluted Plate (model d)

Figure 5.5 The Wavelength of the K-H vortices Shed after the Trailing Edge of the Convolved Plate (model d)

Figure 5.6 The Strouhal Number Variation with the Reynolds Number for the Convolved Plate (model d)

Figure 5.7 The K-H Vortex Shedding Frequencies after the Trailing Edge of the Single-lobe Forced Mixer at Wavelength $\lambda=30\text{mm}$ (model e)

Figure 5.8 The K-H Vortex Shedding Frequencies after the Trailing Edge of the Single-lobe Forced Mixer at Wavelength $\lambda=120\text{mm}$ (model f)

Figure 5.9 The K-H Wavelength Variation with the Reynolds Number for Single-lobe Forced Mixer at Wavelength $\lambda=30mm$ (model e)

Figure 5.10 The K-H Wavelength Variation with the Reynolds Number for Single-lobe Forced Mixer at Wavelength $\lambda=120mm$ (model f)

Figure 5.11 The Strouhal Number Variation with the Reynolds Number for Single-lobe Forced Mixer at Wavelength $\lambda=30\text{mm}$ (model e)

Figure 5.12 The Strouhal Number Variation with the Reynolds Number for Single-lobe Forced Mixer at Wavelength $\lambda=120\text{mm}$ (model f)

Figure 5.13 Variations of the Maximum Strouhal Number for the Models at Different Wavelengths: $\lambda=30$ mm (Model e), $\lambda=60$ mm (Basic Model a) and $\lambda=120$ mm (Model f)

a. Coordinate System

b. Velocity Ratio $r=1:1$

c. Velocity Ratio $r=0.4:1$

Figure 5.14 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) at Downstream Stations after the Trailing Edge of Convolved Plate for (b) Velocity Ratio $r=1:1$ and (c) Velocity Ratio $r=0.4:1$

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(a) Coordinate System and Vector Magnitude Scale

(1) $x/(2h) = 0.5$

(2) $x/(2h) = 1.0$

(3) $x/(2h) = 2.0$

(4) $x/(2h) = 4.0$

(5) $x/(2h) = 6.0$

(b) Velocity Ratio $r = 1:1$

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(1) $x/(2h) = 0.5$

(2) $x/(2h) = 1.0$

(3) $x/(2h) = 2.0$

(4) $x/(2h) = 4.0$

(5) $x/(2h) = 6.0$

(c) Velocity Ratio $r = 0.4:1$

Figure 5.15 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) and the Corresponding Secondary Flow Velocity Vectors ($\vec{V}_s/\bar{U}$) at Successive Downstream Stations after the Trailing Edge of Single-lobe Forced Mixer at Wavelength $\lambda=30\text{mm}$ (model e) for (b) Velocity Ratio $r=1:1$ and (c) Velocity Ratio $r=0.4:1$

Figure 5.16 Schematic of the Streamwise Vortices Generated in the Wake of Single-lobe Forced Mixer with Half Wavelength $\lambda=30\text{ mm}$ (model e)

(a) Coordinate System and Vector Magnitude Scale

Flow Separation Region.

(1) $x/(2h)=0.5$

(2) $x/(2h)=1.0$

(3) $x/(2h) = 2.0$

(4) $x/(2h) = 4.0$

(5) $x/(2h) = 6.0$

(b) Velocity Ratio $r = 1:1$

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Non-uniform Secondary Flow Directions

(1) $x/(2h) = 0.5$

(2) $x/(2h) = 1.0$

(3) $x/(2h) = 2.0$

(4) $x/(2h) = 4.0$

(5) $x/(2h) = 6.0$

(c) Velocity Ratio $r = 0.4:1$

Figure 5.17 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) and the Corresponding Secondary Flow Velocity Vectors ($\vec{V}_s/\bar{U}$) at Successive Downstream Stations after the Trailing Edge of Single-lobe Forced Mixer at Wavelength $\lambda=120\text{mm}$ (model f) for (b) Velocity Ratio $r=1:1$ and (c) Velocity Ratio $r=0.4:1$

Figure 5.18 Variation of the Normalized Streamwise Circulation with Downstream Distance for the Single-lobe Forced Mixers at Different Wavelengths, Velocity Ratio $r = 1:1$

Figure 5.19 Variation of the Normalized Streamwise Circulation with Downstream Distance for the Single-lobe Forced Mixers at Different Wavelengths, Velocity Ratio $r = 0.4:1$

Figure 5.20 Momentum Thickness Growth in the Mixing Layer of the Convoluted Plate and Single-Lobe Forced Mixer at Different Wavelengths, Velocity Ratio $r=0.4:1$

Figure 5.21 Variation of Shape Factor for the Flat Plate, Convoluted Plate and Single-lobe Forced Mixer at Different Wavelengths, Velocity Ratio $r=0.4:1$

Figure 5.22 K-H Vorticity Distributions at Two Symmetrical Points M and N

Chapter 6. Flow Characteristics of Single-lobe Forced Mixer with Different Trailing Edge Configurations and Modifications

6.1 Introduction

As it has been pointed out earlier in the review of lobed mixer flow, both the lobe shape (or configuration) and its parameters are important in determining the effectiveness of mixer performances (Paterson 1982). In Chapters 4 and 5, the lobe parameters especially its height and wavelength, have been examined in details. In this chapter, the lobe configurations will be further discussed. Single-lobe forced mixers with rectangular (model g) and triangular (model h) trailing edge configurations are examined. Both of them have the same geometric parameters as the basic semi-circular single-lobe forced mixer model a, including the penetration angle 22 degree, lobe height at 30 mm and wavelength at 60 mm. Models g and h had already been described in Section 3.2.6 and shown in **Fig 3.6**. For the rectangular model g, it has straight parallel sidewalls similar to the basic semi-circular model a. However, the straight parallel sidewalls, which are usually helpful for the generation of streamwise vorticity (Barber *et al* 1988), are not available for the triangular model h. The upstream conditions for the three mixers with different configurations are the same. Comparison will be made afterwards.

The scalloping effect is achieved by eliminating up to 70% of the sidewall area at the penetration region of the lobe. The investigations of Presz *et al* (1994) suggested that using the scalloped lobes in the lobed mixer of aggressive penetration angles could reduce the likelihood of boundary-layer separation, which usually happens to the normal lobed mixer when the penetration angle is larger than 22 degree. The investigation of Presz *et al* (1994) suggested that some additional streamwise vortices would be formed as a result of scalloping effect, and thereby enhanced mixing performance could be

achieved. This was further confirmed in the experiments conducted by Yu *et al* (1997), that the streamwise-circulation strength of the scalloped mixer was found to be about 30% higher than the normal mixer. Unfortunately, it was also observed by Yu *et al* (1997) that when the penetration angle of the multi-lobe scalloped mixer was increased from 22° to 35° , the magnitude of the streamwise circulation, however, did not increase significantly. This may be due largely to a shorter effective length in the penetration region that may hamper the growth of the secondary flow. Meanwhile, all the above conclusions are based on the experiments with the multi-lobe forced mixer. While for the single-lobe forced mixer, detailed investigations are yet to be carried out. To elucidate the flow characteristics of the single-lobe mixer with scalloping modification, and to compare the possible benefits of scalloping effect on multi-lobe and single-lobe mixer, the scalloped single-lobe forced mixer (model i, also see **Fig 3.7a**), will be examined in the present investigation. It was modified based on the basic model a, so that comparison could be directly made between models a and i afterwards.

Apart from the scalloped mixer, another kind of modification on basic model a, namely scarfed mixer will also be examined and compared. It has been tested on the multi-lobe forced mixer that the scarfing effect could enhance the strength of the trailing-edge streamwise circulation by up to 25~30% more than the normal unmodified mixer, as measured by Koutmos and McGuirk (1995) using LDA. The scarfed single-lobe forced mixer j is shown schematically in **Fig 3.7b**. Different from the scalloping effect that removing the sidewalls, the scarfing effect is to remove the mixer's penetration region from the side view by half angle, i.e., $\theta = (90^\circ - \varepsilon)/2 = 34^\circ$, in which the penetration angle $\varepsilon = 22^\circ$. Also this is different from the scarfing multi-lobe mixer, which is typically achieved

by alternately extending and cutting back the lobes at the trailing edge.

Hot-wire anemometry and laser Doppler anemometry would be employed to characterize the aerodynamic performance of the K-H vortex and streamwise vortex respectively. It should be emphasized that till now the experimental methods are still dominant in investigating the performance of the lobed forced mixer. This is mainly because the shear layer of the forced mixer is a highly complex three-dimensional convoluted shear layer, especially the K-H instability is difficult to capture in the simulations. As a result, there are still great inconsistencies between simulations and experiments. Actually computational studies of the lobed forced mixers to date have employed a hierarchy of numerical methods. These range from reduced-order point vortex models, as in Strickland *et al* (1998), to the solution of the Reynolds-averaged form of the Navier-Stokes equations as in Koutmos and McGuirk (1989). The most commonly used turbulence model for lobed forced mixer simulation has been the standard linear $k-\varepsilon$ model of Launder and Spalding (1974), which has been firstly employed by Koutmos and McGuirk (1989) and then followed by other researchers such as Malecki and Lord (1990), Abolfadl and Sehra (1991), Tsui and Wu (1996), O'Sullivan *et al* (1996), and Salman *et al* (2003). Of these, the studies of Tsui and Wu (1996), O'Sullivan *et al* (1996), and Salman *et al* (2003) contain thorough validations of the computations against experimental data. For example, the most important factor causing discrepancies between the experiments and the simulation in the investigation of Tsui and Wu (1996) was the 'swirling flow', which could not be modeled in their calculations. Actually the swirling flow they mentioned is the K-H vortices. While the recent prediction of lobed mixer vortical structures with a $k-\varepsilon$ turbulent model by Salman *et al*

(2003) showed that there was a lag of 1.75 lobe heights in the shear-layer development with respect to the measured data; while for the global parameters such as momentum thickness and streamwise circulation generally showed an under-prediction of 20% and an over-prediction of 65% with respect to the measured data, respectively.

Since the combined effects of scalloping and scarfing on a multi-lobe mixer cannot promote any significant streamwise-circulation enhancement (Yu *et al* 2000), a single-lobe forced mixer combined with both scalloping and scarfing modifications would not be examined in the present experiment furthermore.

6.2 Hot-Wire Anemometer Measurements

6.2.1 Rectangular Single-lobe Forced Mixer (model g)

6.2.1.1 K-H Vortex Shedding Frequency

Similar to the basic semi-circle single-lobe forced mixer (model a) flow, there were always two dominant frequencies in the rectangular single-lobe forced mixer's wake flow, because the initial flow conditions at point C and L were largely different. **Fig 6.1** shows some typical spectra of this model at point C. The relationships between the frequency and the velocity may be approximated as (also see **Fig 6.2**):

$$f_{LM\ g_C} = 83.7 \bar{U} - 52.4 \quad (r = 1:1, 3.40\text{ m/s} \leq \bar{U} \leq 10.36\text{ m/s}) ; \quad (6-1)$$

$$f_{LM\ g_L} = 78.5 \bar{U} - 87.0 \quad (r = 1:1, 3.40\text{ m/s} \leq \bar{U} \leq 10.36\text{ m/s}) ; \quad (6-2)$$

$$f_{LM\ g_C} = 184.4 \bar{U} - 265.8 \quad (r = 0.4:1, 2.39\text{ m/s} \leq \bar{U} \leq 7.25\text{ m/s}) ; \quad (6-3)$$

$$f_{LM\ g_L} = 80.5 \bar{U} - 87.2 \quad (r = 0.4:1, 2.39\text{ m/s} \leq \bar{U} \leq 7.25\text{ m/s}) ; \quad (6-4)$$

Same as the basic model a, the velocity ratio can be much more significantly seen to affect the frequency at point C than at point L for model g. When the velocity ratio decreases from 1:1 to 0.4:1, the frequency at point C increased by 60%; whereas at point L it changed very little. The K-H vortex shedding frequency at point L for model g was similar to the flat plate case (see Chapter 4). The velocity ratio had little effects on its vortex shedding frequency because the initial conditions at point L of model g were similar to the flat plate case.

6.2.1.2 K-H Vortex Wavelength

Fig 6.3 shows the wavelength variation of the model g, both at point C and L. It is

clear that the K-H wavelengths decreased with the increase in mean velocity until reaching certain stable values. The wavelength of the K-H vortices shed after point C was smaller than after point L. For velocity ratio 1:1, the wavelength after point C was about 85~90% of that after point L; while for velocity ratio 0.4:1, the former was only half of the latter. Varying the velocity ratio had more effects on the wavelength at point C. At point L, the K-H wavelength is almost not affected by the velocity ratio. This was mainly due to the fact that the upstream conditions at point L were much similar to the flat plate case. Whereas at point C, due to great changes of the upstream conditions, the wavelength at this point was much different from the flat plate case.

6.2.1.3 Strouhal Number

From equations (6-1) to (6-4), the relationships between the Reynolds Number and the Strouhal Number for rectangular single-lobe forced mixer model g can be approximated as follows:

$$St_{LM\ g_C} = 5.02 - \frac{12500}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (6-5)$$

$$St_{LM\ g_L} = 4.71 - \frac{20800}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (6-6)$$

$$St_{LM\ g_C} = 11.06 - \frac{63450}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (6-7)$$

$$St_{LM\ g_L} = 4.83 - \frac{20850}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (6-8)$$

As shown in **Fig 6.4**, the variation of the Strouhal number for the present model is very similar to that of the basic single-lobe forced mixer: it increased with increasing

Reynolds number while the increase ratio decreases gradually. For Reynolds number less than 20,000, the Strouhal number increased rapidly; while for Reynolds number larger than 20,000, the Strouhal number increases relatively slowly. Stable Strouhal number values could be achieved at Reynolds number larger than 5×10^5 . The velocity ratio affects Strouhal Number behind point C more significantly than behind point L, which was mainly due to the different upstream conditions at these two locations. The maximum Strouhal numbers at these two locations were different too. For velocity ratio 1:1, the maximum values at point L and C were 4.71 and 5.02 respectively; while for velocity ratio 0.4:1, their maximum values were 4.83 and 11.06 respectively.

6.2.2 Triangular Single-lobe Forced Mixer (model h)

6.2.2.1 K-H Vortex Shedding Frequency

Because point C for the triangular single-lobe forced mixer was a singular point, there were no obvious peaks found in the spectral results. All the K-H vortices were shed from the linear sidewall of the model. **Fig 6.5** shows some typical spectra of this model at point L. The K-H vortex shedding frequencies are shown in **Fig 6.6**. They all increased linearly with the mean velocity. The linear functions can be approximated as follows:

$$f_{LM\ h_L} = 99.1\bar{U} - 75.9 \quad (r = 1:1, 3.40\text{ m/s} \leq \bar{U} \leq 10.36\text{ m/s}); \quad (6-9)$$

$$f_{LM\ h_L} = 108.4\bar{U} - 62.6 \quad (r = 0.4:1, 2.39\text{ m/s} \leq \bar{U} \leq 7.25\text{ m/s}); \quad (6-10)$$

Comparing with the basic model a, the K-H vortex shedding frequency of the present model h was a little lower than that at point L for the model a.

6.2.2.2 K-H Vortex Wavelength

The variations of the K-H vortex wavelength are plotted in **Fig 6.7** for point L of the triangular single-lobe forced mixer model h. With the increase in Reynolds number, their mean wavelength would decrease until it reached certain stable values at high Reynolds number where the flow was dominated by inertial forces only.

As it had been pointed out earlier, smaller K-H vortex wavelength means the interface between the two streams would be more strained. As a result, the mixing performance would be improved correspondingly. For the two velocity ratios involved, the wavelength decreased about 15% when the velocity ratio changed from 1:1 to 0.4:1.

Comparing with the basic model a, the K-H vortex wavelength of the present model h was at about 10% larger than that at point L of the basic model a.

6.2.2.3 Strouhal Number

The variations of the Strouhal number with the Reynolds number for the triangular lobed mixer model h had been plotted in **Fig 6.8**, which can also be approximated as follows:

$$St_{LM\ h_L} = 5.95 - \frac{17550}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (6-11)$$

$$St_{LM\ h_L} = 6.50 - \frac{14500}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (6-12)$$

It can be seen that the Strouhal number varied in the same way as the other models. Initially it increased fast but the rate of increase decreased gradually, until it reached certain stable values, i.e., 5.95 and 6.50 for the case with velocity ratio 1:1 and 0.4:1 respectively. The increment between the two velocity ratios was not as high as point C of

the semi-circular (basic model a) or rectangular (model g) models, but some higher than point L of the two models (a and g).

6.2.3 Scalloped Single-lobe Forced Mixer (model i)

6.2.3.1 K-H Vortex Shedding Frequency

The K-H vortex shedding for the scalloped forced mixer, due to its scalloping effects, is more complicated than the basic forced mixer model a. As shown schematically in **Fig 6.9**, some additional streamwise vortices would be formed near the vicinity of the scalloped sidewalls, which are of smaller length scale than the main streamwise vortices. These additional streamwise vortices were formed due to the pressure difference between the two streams near the removed sidewalls. They rotate in the Y-Z plane while going along with the main flow in the X direction. Similar to the main streamwise vortices, it is difficult for us to document these vortices using hot-wire anemometer because they are spatially stationary. These additional streamwise vortices would merge with the main streamwise vortices after the trailing edge, because they are of the same signs. These amalgamations strengthen the overall streamwise vorticity, as will be further examined in the LDA results.

Figs 6.10 (1) and (2) show typical power spectra at point C and L respectively under the same flow conditions. It's rather clear that significant peaks can only be seen at point C, while no significant peaks can be detected at point L.

Fig 6.11 shows the relationships between the K-H vortex shedding frequency and the mean velocity of the coming flows at point C of the scalloped single-lobe forced mixer model i. As usual the frequency varied linearly with the mean velocity for both velocity

ratios involved. The linear relationships can be approximated as follows:

$$f_{LMi_C} = 135.4\bar{U} - 198.3 \quad (r = 1:1, 3.40\text{ m/s} \leq \bar{U} \leq 10.36\text{ m/s}); \quad (6-13)$$

$$f_{LMi_C} = 220.2\bar{U} - 334.4 \quad (r = 0.4:1, 2.39\text{ m/s} \leq \bar{U} \leq 7.25\text{ m/s}). \quad (6-14)$$

Actually the K-H vortex shedding at point C of the scalloped model was almost the same as the basic model a. This was because the initial flow conditions of these two models at point C were almost the same (differences less than 3%), although they were different at point L.

6.2.3.2 K-H Vortex Wavelength

The variations of the K-H vortex wavelength are plotted in **Fig 6.12** (for model i at point C). Point L is not shown because the K-H vortices were replaced by some smaller streamwise vortices near the sidewall.

It can be seen that the mean K-H vortex wavelength at point C decreased with the increase of the Reynolds number before reaching certain stable values when the inertial force dominated the flow regime. It decreased as well when the velocity ratio changed from 1:1 to 0.4:1 by about 40%. As shown earlier, decreasing the K-H wavelength would result in achieving better mixing contributed from the K-H vortices. The wavelength magnitudes at point C for this model were almost the same as the basic model a, because the initial flow conditions at point C of these two models were very similar to each other.

6.2.3.3 Strouhal Number

Fig 6.13 shows the variations of the Strouhal number at point C with the Reynolds number. The Strouhal number variation can be approximated as follows:

$$St_{LMi_C} = 8.12 - \frac{45800}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (6-15)$$

$$St_{LMi_C} = 13.2 - \frac{77300}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (6-16)$$

Here it is rather clear that the Strouhal number variation at point C of the present model is the same (the difference is within the error) as the basic model a. As pointed out earlier, this is because the flow conditions at point C of these two models are almost the same, though significantly different at point L.

6.2.4 Scarfed Single-lobe Forced Mixer (model j)

6.2.4.1 K-H Vortex Shedding Frequency

Typical power spectra for point C of the scarfed single-lobe forced mixer model j are shown in **Figs 6.14 (1) and (2)** for the two velocity ratios $r = 1:1$ and $0.4:1$ respectively. The measuring point C is 10mm away from the trailing edge of the mixer. For (1) $U_1 = U_2 = 4.42\text{m/s}$, the K-H vortex shedding frequency at point C is around 390Hz; whereas for case (2) $U_1 = 4.42\text{m/s}$ and $U_2 = 1.77\text{m/s}$, the K-H vortex shedding frequency is about 290Hz. For all the other velocities measured under these two velocity ratios, the linear functions of the vortex shedding frequency shown in **Fig 6.15** are as follows:

$$f_{LMj_C} = 119.1\bar{U} - 136.4 \quad (r = 1:1, \quad 3.40\text{m/s} \leq \bar{U} \leq 10.36\text{m/s}); \quad (6-17)$$

$$f_{LMj_C} = 194.0\bar{U} - 317.8 \quad (r = 0.4:1, \quad 2.39\text{m/s} \leq \bar{U} \leq 7.25\text{m/s}); \quad (6-18)$$

Whereas for point L, the K-H vortex shedding frequency (not listed here) is very close to the basic model a, which have been discussed in Chapter 4 already, due to the same initial flow conditions at this measuring point.

6.2.4.2 K-H Vortex Wavelength

The variations of the K-H vortex wavelength shed after point C of the present model are plotted in **Fig 6.16**. The wavelength variations at point L were the same as those of the basic model a (see **Fig 4.9**), so they are not shown here.

While at point C, the wavelength of the present scarfed model j was slightly larger than that at point C of the basic model a due to the present model's scarfing effect, which shortened the axial length of the penetration region of the mixer slightly. This implies that the K-H vortex mixing contribution of the present model j was slightly less effective than that of the basic model a.

6.2.4.3 Strouhal Number

Fig 6.17 shows the variations of the Strouhal number at point C with the Reynolds number, which can be approximated as follows:

$$St_{LMj_C} = 7.14 - \frac{31500}{Re} \quad (r = 1:1, \quad 13,100 \leq Re \leq 40,000); \quad (6-19)$$

$$St_{LMj_C} = 11.64 - \frac{73500}{Re} \quad (r = 0.4:1, \quad 9,200 \leq Re \leq 27,900); \quad (6-20)$$

The Strouhal number increased rapidly for Reynolds number less than 20,000. It would reach a stable maximum value at Reynolds number about 5×10^5 . From the results it can be seen that for the case at the velocity ratio 1:1, the Strouhal number reached stable values earlier than the case at the velocity ratio 0.4:1. This might be due to the inertial dominated regions for these two cases are being different in someway.

6.2.5 Comparison among Different Configurations

The K-H vortices shed within the wakes of four single-lobe forced mixers with different configurations or modifications have been analyzed above. Together with the basic model a, some comparisons can be made consequently. The interfacial contact areas, or the initial K-H vortex tube length, is larger for the rectangular and scalloped models than the basic model a, while it is smaller for the triangular and scarfed models than the basic model a. For the present investigation, the interfacial contact areas of the basic semi-circular model a is about 15% larger than the triangular model h.

The Strouhal numbers, as one of the most representative non-dimensional vortex shedding frequency, have been listed above for different geometry configurations. Accordingly the maximum Strouhal numbers for different models are listed in **Table 6-1**.

Maximum Strouhal number	C ($r=1:1$)	L ($r=1:1$)	C ($r=0.4:1$)	L ($r=0.4:1$)
Basic Model a	8.07	6.92	13.1	6.68
Rectangular model g	5.02	4.71	11.06	4.83
Triangular model h	/	5.95	/	6.50
Scalloped Model i	8.12	/	13.2	/
Scarfed model j	7.14	6.87	11.64	6.65

**Table 6-1 Maximum Strouhal Numbers for Models
with Different Geometry Configurations**

From the table it is clear that for a lobed mixer with rectangular or triangular configurations, their maximum Strouhal numbers are smaller than the basic semi-circular model a. This implies that the K-H vortices shed from the rectangular or triangular

trailing edges are weaker than the semi-circular trailing edge. The latter has smaller K-H mean wavelength than the former. Furthermore, there is no point C for the triangular mixer. As it has been shown, the point C can usually shed stronger K-H vortices. So it can be seen that the normal K-H vortices shed after the trailing edge of the triangular mixer are especially weak. Consequently from the viewpoint of the K-H vortices, better mixing performance can be provided by the mixer with semi-circular trailing edge, or the basic model a.

Whereas for the two modifications, namely scalloped and scarfed models, the intensity differences of their normal K-H vortices compared with the basic model a are not very significant. Although it is unlikely that strong normal K-H vortices would be shed after point L for scalloped mixer, the additional streamwise vortices there are contributive to the mixing performance as well. The vortex shedding at point C would not be seriously affected by the scalloping effect, which takes place at the parallel sidewalls only. This is also true for the point C of the scarfed mixer. Although the scarfing effect removed some sidewalls at point L, the initial flow conditions there were changed very slightly, which were still very close to the flat plate case. So it seemed that the modification of either scalloping or scarfing would not change the strength of the normal K-H vortices significantly.

6.3 Laser Doppler Anemometer Measurements

6.3.1 Streamwise Velocity Contours and Mean Secondary Velocity Vectors

The measuring planes are the same as those for the basic lobed mixer (model a), that is, $+0.75\lambda \geq y \geq -0.75\lambda$ and $+\lambda \geq z \geq -\lambda$, in which wavelength of the lobe trailing edge $\lambda=60\text{mm}$. Five downstream stations after the trailing edge of the model in the near field, i.e., $x/\lambda = 0.5, 1, 2, 4$ and 6 have been measured at the two velocity ratios $1:1$ and $0.4:1$.

Only the streamwise velocity contours on the left half side and the secondary velocity vectors on the right half side will be shown together at three representative stations $x/\lambda = 0.5, 2$ and 4 .

6.3.1.1 Rectangular Single-lobe Forced Mixer (model g)

The coordinate system used for the LDA measurement of the rectangular single-lobe forced mixer is shown in **Fig 6.18a**.

(a) Velocity Ratio $r = 1:1$

The streamwise velocity of the on-coming flow decelerates at both troughs up to 20% of the mean velocity. As shown in **Fig 6.18b** (1) to (3) respectively for the three representative downstream stations, the velocity contours diffused gradually with downstream distance. By the final measured station at $x/\lambda = 6$, the velocity distribution had become almost uniform in the streamwise direction.

The secondary flows were mainly restricted within the wake of the troughs. Strongest secondary flow at the first station examined was about 70% of the mean streamwise velocity scale, as shown in **Fig 6.18a**. Then it would become smaller and smaller at the

further down-stream stations.

(b) Velocity Ratio $r = 0.4:1$

For this case, the velocity of the upper side on-coming airflow was 40% of the lower side. As shown in **Fig 6.18c** (1) to (3), the streamwise velocity contours had the same shape as the lobe trailing edge initially, and they diffused subsequently at downstream distance. It should be mentioned here that the contours were plotted from $y/\lambda = -0.75$ to $+0.25$, to show the flow fields in the vicinity of the mixer sidewall. However, the velocity gradients were maintained until the last measured station at $x/\lambda = 6$. The mixing would continue into the far wake.

The secondary flows in the wake of the present mixer are shown at the right side, from $y/\lambda = 0$ to $+0.75$. Strong secondary flows could be found in the wake of the lobe troughs, with maximum value at about 65% of the mean streamwise velocity at the station $x/\lambda = 0.5$. With measuring plane moving downstream, the secondary flow became weaker, with maximum value at about 45% of the mean streamwise velocity at the station $x/\lambda = 2$ or 32% at $x/\lambda = 4$.

6.3.1.2 Triangular Single-lobe Forced Mixer (model h)

The coordinate system used for the LDA measurement of model h and the length scale of the mean streamwise velocity are shown in **Fig 6.19a**.

(a) Velocity Ratio $r = 1:1$

As shown in **Fig 6.19b** (1), the streamwise velocity decelerated at the upper troughs,

whereas accelerated at the lower trough. Both deviations were up to 20% of the mean streamwise velocity. With the flow going along X direction, the streamwise velocity became more and more uniform.

The secondary flow fields were similar to the basic semi-circular and rectangular lobed mixers. However, the secondary flows of the triangular lobed mixer were the weakest among the three models. As shown in **Figs 6.19b** (1) (2) and (3), the strongest secondary flows at the three stations were 55%, 40% and 30% of the mean streamwise velocity respectively. This was mainly due to the significant boundary blockage effects (O'sullivan *et al* 1996) of this model. A thick inlet boundary layer led to filling of the lobe trough with low-momentum fluid, resulting in a reduced lobe effective angle, and hence, reduced streamwise circulation, compared to a configuration with a thin inlet boundary layer. The sidewalls of the triangular model are not parallel but about 53° inclined to each other, as shown in **Fig 3.6**. It is expected that the boundary layers attached on these two sidewalls would strengthen each other, especially at the trough region. As a result, the reduction of its effective penetration angle would be larger than other models with parallel sidewalls. Consequently the secondary flows would be somewhat suppressed, with smaller magnitudes than those of the semi-circular and rectangular lobed mixers.

(b) Velocity Ratio $r = 0.4:1$

The results for this case are shown in **Fig 6.19c**. At the first station measured, the streamwise velocity contours were in the vicinity of the linear sidewall of the mixer trailing edge. Steep velocity gradients could be seen clearly due to the different velocities

of the upper and lower streams. With the flow going down and the mixing going on, the contours would disperse and deviate from the linear sidewall. The streamwise velocity gradients became smaller and smaller.

Strong secondary flows could be found in the wake of lower trough. Due to the lower velocity of the upper on-coming stream, secondary flows in the wake of upper troughs were weak. Both of them became weaker and weaker with the flow going downstream due to viscous effects and momentum diffusion or mixing.

6.3.1.3 Scalloped Single-lobe Forced Mixer (model i)

The streamwise mean velocity contours and the mean secondary velocity vectors are shown in **Fig 6.20** for the scalloped single-lobe forced mixer. It could be seen that the streamwise velocity gradients became less steep at further downstream stations. Also the streamwise velocity gradients were steeper for the case with velocity ratio 0.4:1 than the case 1:1, especially near the interfacial areas between these two mixing streams. For the velocity ratio 1:1, the streamwise velocity fields would deviate from their mean value by about 30%, which was owing to the penetration region. Accumulation of the low momentum fluids could be found near the upper trough region initially. However, they would diffuse to the whole flow field gradually. For the velocity ratio 0.4:1 case, the maximum secondary velocities were about 70%, 62% and 35% at stations $x/\lambda = 0.5, 2.0$ and 4.0 respectively. Whereas for velocity ratio 1:1, they were about 80%, 70% and 38% at the three stations respectively. No flow separations were detected throughout the whole flow fields.

Actually the results were very similar to the basic model a, which were plotted in **Fig**

4.18 already. However, small differences between these two models were that the initial secondary velocity vectors for the scalloped model were slightly stronger than those for the basic model a, while the former case decayed slightly faster. These differences can be explained as follows. Owing to the scalloping effect, some additional streamwise vortices would be generated near the notches, as shown earlier in **Fig 6.9**. They were rotating in the same directions as the main streamwise vortices. As a result, the secondary velocity vectors together with the total streamwise vorticity would be slightly stronger than those of basic model a. However, the consequent merging process between the additional and main streamwise vortices would weaken the total streamwise vorticity. As far as we know, the merger with like signed strain is less efficient due to the final deformation and filamentation of the resulting vortex (Waugh 1992). Later the total circulation on the half plane **I**, or mean streamwise vorticity would be quantitatively analyzed and compared to the basic model a.

6.3.1.4 Scarfed Single-lobe Forced Mixer (model j)

Fig 6.21 shows the fundamental LDA results for this model. From the comparison of this model with the basic model a, it could be seen that the streamwise velocity gradient of the present model is not as great as that of the basic model a. This was in accordance with the former results of the K-H vortex. Owing to the scarfing effect, the penetration region could not change the on-coming flows as significantly as that of the basic model a.

However, the secondary flow types of these two models were nearly the same, except that the secondary flow of the present model was somewhat weaker than that of the basic model. As expected, the scarfing effect shortens the axial length of the penetration region

although the penetration angle remains unchanged; consequently the secondary flow generated in its wake would be weakened. This effect was especially apparent for the lower stream, because the penetration region was cut off mainly in the lower side.

6.3.2 Mean Streamwise Vorticity

Figs 6.22 and 6.23 show the variations of the normalized streamwise circulation, or mean streamwise vorticity with downstream distance of single-lobe forced mixers with different geometry configurations, including the basic model a with semi-circular trailing edge, model g with rectangular trailing edge and model h with triangular trailing edge for velocity ratio $r=1:1$ and $0.4:1$ respectively. From the figures it could be observed that the rectangular model g had the highest mean streamwise vorticity, which was followed by the basic semi-circular model a. This was because that the penetration regions of the rectangular model g and the semi-circular basic model a were different. Actually the end view area of the upper half penetration region for the rectangular model g is ($\lambda = 2h$):

$$A_{PR,g} = \frac{\lambda}{2} \cdot h = h^2, \quad (6-21)$$

whereas the area for the penetration region of semi-circular model a is

$$A_{PR,a} = \frac{\lambda}{4} \cdot h + \frac{1}{2} \cdot \pi \cdot \left(\frac{h}{2}\right)^2 = 0.89 h^2. \quad (6-22)$$

It is clear that $A_{PR,g}$ is about 12.4% larger than $A_{PR,a}$. This means that the penetration region for rectangular model is 12.4% larger than the semi-circular model a. As a result, the circulation of rectangular model was about 5% higher than semi-circular model, as measured in the experiments. However, the drag on the penetration region for rectangular model should be larger than the semi-circular model due to the larger blockage areas.

Most importantly, there is stress concentration problem for the rectangular model design; and it is heavier in weight. All these shortcomings reduce its application, although it can generate a slightly stronger streamwise vorticity.

The mean streamwise vorticity magnitude in the wake of the triangular model h was the lowest among the three models. One reason was that there was no parallel sidewall for the triangular model, which led to serious boundary blockage effects. The parallel sidewalls were very important for the streamwise generation. It is also because the penetration region for this model was the smallest on the upper side with blockage areas

$$A_{PR,h} = \frac{1}{2} \cdot \frac{\lambda}{2} \cdot h = 0.5 h^2. \quad (6-23)$$

From the two figures, the circulation of the triangular model was about 60% of the basic semi-circular model for both the two velocity ratios involved.

Figs 6.24 and 6.25 show the variation of the normalized streamwise circulation with downstream distance of single-lobe forced mixers with different modified configurations, i.e., the scalloped model i and scarfed model j, for velocity ratios $r = 1:1$ and $0.4:1$ respectively. Results of the unmodified basic model a was also shown here for comparison. It could be observed that the relative magnitudes and variation trends for the three models were almost the same for both velocity ratios. All of them decayed exponentially with downstream distance, and reached negligible magnitude within the near field of the wake region at $x/\lambda=6$.

However, the initial circulations and the decay rates were somewhat different. For the scalloped model, its initial circulation was of the highest magnitude, which were about 12% higher than the basic model a. The reason had been explained earlier, which was mainly due to additional streamwise vortices formed near the notches. These additional

vortices strengthened the magnitude of the total streamwise vortices. Meanwhile, it could be seen from the figure that the streamwise vorticity would breakdown faster for the scalloped mixer than the basic model a. Similar phenomenon has been observed in the serial experiments of the multi-lobe forced mixers by Yu *et al* (1996). The faster streamwise vorticity-decaying rate of the scalloped mixer should be imputed to the vortex merging among the main streamwise vortex and the other two additional streamwise vortices. As far as it is known, the vortex merging between like signed vortices would weaken the vorticity of the resulting vortex due to its deformation and filamentation (Waugh 1992).

While for the scarfed model j, the streamwise circulation generated in its wake was about 75% of the basic unmodified model a only because of its shortened axial length of the penetration region. To shorten the axial length means to reduce the lobe height as well. The lobe height at its lower side is about 16.9 mm only as shown in **Fig 3.7(2)**. It has already been proved in Chapter 4 that the streamwise vorticity is in direct proportion to the lobe height. So it is not strange that the streamwise vorticity in the wake of scarfed model would be inferior to the basic model a.

6.3.3 Momentum Thickness

Fig 6.26 shows the momentum thickness growth rate in the mixing layer of the single-lobe forced mixers with different configurations at velocity ratio 0.4:1. The basic model a is also shown in dotted line for comparison. It is very clear that the rectangular model g had the highest momentum thickness, which was mainly due to the fact that the mean streamwise vorticity for this geometry configuration is the highest.

Among them the momentum thickness in the wake of triangular model h was especially low. This result was in accordance with the former conclusions on the K-H and streamwise vortices. For the triangular model h, it had non-parallel sidewalls only, and the K-H vortices were shed with lower Strouhal number. Also the mean streamwise vorticity for this model was the lowest among the three models. Together with the fact that it had smallest mixing contact areas for the two on-coming streams, the mixing performance of the triangular model h was the lowest.

Fig 6.27 shows the momentum thickness growth in the mixing layer of the single-lobe forced mixers with modified configurations together with the unmodified model a at velocity ratio 0.4:1. The scalloped model i had the highest initial momentum thickness, and grew rapidly from the station $x/\lambda = 0.5$ to 1.0. Both of these two characteristics should be attributed to the scalloping effect, which lead to the additional streamwise vortices that strengthen the main streamwise vorticity and enhance the rapid mixing performance. The scarfed model j had the lowest momentum thickness throughout the near field, which was in line with the facts that both the K-H vortices and mean streamwise vorticity were of the least magnitudes among the three models.

However, it should be noted that the momentum evolution tendencies for the four models were all similar to the basic model a. It increased rapidly at the first three stations but slowed down at further downstream stations in the near wake field. This was in line with the fact that both the normal K-H vortices and the streamwise vortices would breakdown or decay in the near wake field. The normal K-H vortices usually breakdown within a distance of about two lobe-wavelengths after the trailing edge. It could be seen from **Figs 6.23** and **6.25** that at velocity ratio 0.4:1, the mean streamwise vorticity

decayed so fast that it almost reached negligible values at the station $x/\lambda = 5$.

6.3.4 Shape Factor

The shape factors variation for different trailing edge configurations and modifications, which were employed to evaluate the mixing performance of different models, are plotted in **Figs 6.28** and **6.29**. Similar to the momentum variation, only the cases at velocity ratio 0.4:1 are shown here. The basic model a is also shown as a baseline for comparison.

It could be observed that the shape factor variation for the rectangular single-lobe forced mixer was very close to that of the basic model a. This was in accordance with the magnitudes of their K-H and streamwise vortices. From the former results, it has been shown that the intensity of the K-H vortices for the rectangular mixer was weaker than those of the basic semi-circular mixer. On the other hand, however, the streamwise vorticity of the rectangular mixer was about 5% stronger than the basic model a. As a compromise, it was possible that the mixing performances of these two mixers are very similar, since the K-H and streamwise vortices are both contributive to the mixing enhancement. While for the triangular mixer, it has been shown that neither its normal K-H vortices nor streamwise vortices was stronger than those of semi-circular model. As expected, its mixing performance would be worse than the basic model a, which can be seen from its higher shape factors.

For the other two different modified mixers, the normal K-H vortices were almost of the same intensity to the basic model a. However, their streamwise vorticities were different. For the scalloped mixer, because higher streamwise circulation would be

formed in its wake region, its mixing performance was expected to be better than that of the basic model a. Whereas for the scarfed mixer, its mixing performance was likely to be inferior to the basic model a due to its weaker streamwise vorticity. These had been revealed in the results of their shape factor variations. It could be seen that the shape factor of the scalloped mixer was a little bit lower than that of the basic model a, implying that the mixing shear layer of the scalloped mixer was more uniform than that of the basic model a. However, the difference between these two mixers was not very significant. While for the scarfed mixer, its shape factor was considerably higher than that of the basic model a. It seemed that to achieve the same uniformity level for the mixing streams as the basic model a or scalloped mixer, the scarfed single-lobe mixer requires more mixing distance. Its mixing performance was inferior to that of the basic model a or scalloped mixer.

6.4 Further Discussion

The relative contributions of K-H vortices and streamwise vortices in the mixing process downstream of lobed forced mixers with different configurations or modifications have been revealed in the present chapter. It is well established from the present investigations that the lobe configuration determines the intensity of these two types of vortices significantly as well as lobe parameters such as lobe height and wavelength. Strong vorticity, both the normal K-H vorticity and the streamwise vorticity, would lead to more efficient mixing and this in turn depends on the lobe geometry as well as the lobe parameters.

The single-lobe rectangular model can generate slightly higher streamwise vorticity, which is in line with the results obtained from multi-lobe forced mixer by Yu *et al* (1995) at three velocity ratios 1:1, 1:2 and 1:3. However, its K-H vortex shedding is found to be inferior to the basic model a. The reason why the normal K-H vortices shed after the rectangular mixer were weaker than that of the basic semi-circular mixer may be explained as follows. As the rectangular model had larger penetration region (comparing from their end views as shown in **Figs 3.2(a)** and **3.6(1)a**, also see equations (6-21) and (6-22)), the secondary flows of the rectangular mixer would be stronger than the basic model a, which was further testified by the LDA measurements. Unfortunately, the secondary flows only contribute to the generation of the streamwise vortices but not to the normal K-H vortices, especially at point L, because to generate K-H vortices it is preferable that the on-coming fluids go in the streamwise direction (X direction). With the increase of the secondary velocities, the primary velocity (u) would decrease accordingly. This decrease would weaken the intensity of the K-H vortices. Together with the higher

drag coefficient penalty, stress and weight problems, it appears that the shortcomings of the rectangular model exceed its merit.

For the triangular model, due to its smallest interfacial contact areas and there is no point C after which strong K-H vortices are usually shed, the mixing contribution due to the normal K-H vortices is expected to be much less than that of the basic model a. Also it suppresses the streamwise vorticity greatly, due to its non-parallel sidewalls and the boundary layer blockage effects, which reduce the effective penetration angle.

For the scalloped and scarfed models, the K-H vortex shedding are similar to the basic model a. However, the streamwise vorticity strength was found to be different from that of the single-lobe forced mixer.

For the normal lobed forced mixer, it has been proved experimentally that the penetration angle should not exceed 22° for the prevention of flow separation problem. Similar numerical conclusion by O'sullivan (1996) has been predicted but with over prediction that the penetration angle could be up to 30 degree. However, by employing the scalloped mixer, the penetration angle may be increased up to 35° without serious flow separation. This has firstly been observed by Presz *et al* (1994). The reason why the aggressive scalloped mixer can suppress the flow separation may be explained briefly like this. It is well known that flow separation happens where there is a negative pressure gradient when the flow is passing through a divergent passage, or

$$\frac{\partial P}{\partial l} \leq 0. \quad (6-24)$$

For illustration, **Fig 6.30** shows the flow separation happening at the lower trough region point P (P is within the boundary layer usually), when the penetration angle of the normal lobed mixer is larger than 22° . However, in the case of the scalloped model, i.e., the

parallel sidewalls are mostly absent, the pressure near the lower trough region will be increased due to the entrainment of the airflow from the upper stream to the lower stream as shown in **Fig 6.9(a)** via the additional streamwise vortices a and b. As a result, the separation condition (6-24) would unlikely to be satisfied any more, and again the flow would be driven in positive pressure gradient. However when the penetration angle is too large (larger than the critical angle 35° , as measured by Presz *et al*), the flow separation would appear again due to the excessively divergent flow passage at the lower trough region.

In the present investigation, the scalloped model is designed at the same penetration angle as the basic model a for direct comparison. It has been shown by Yu *et al* (1997) that the scalloping effects could enhance the streamwise circulation up to 30% for multi-lobe mixer, compared with the streamwise circulation of a normal unmodified multi-lobe mixer with the same penetration angle and trailing edge configuration. However, in the present investigation with single-lobe forced mixer at penetration angle 22° , the streamwise circulation for the scalloped model was found to increase about 12% comparing to that for the basic model a. It should be pointed out that this smaller increment should not be imputed to the unchanged penetration angle but to the difference between multi-lobe and single-lobe models, because actually there would be no significant increase in the streamwise circulation when the penetration angle of the scalloped mixer increases from 22° to aggressively scalloped angle 35° (Yu *et al* 1997). On the other hand, from the smaller increment in the streamwise circulation of the present single-lobe mixer, it can be deducted that there were indeed significant interactions between the neighboring streamwise vortices for the multi-lobe mixer.

Actually their transverse interactions have also been observed in the flow visualization results by Yu *et al* (1997) already. From this point of view, the interactions between the neighboring streamwise vortices are contributive to the streamwise circulation; and the scalloped mixer with multi-lobe is superior to that with single-lobe for the generation of streamwise vorticity.

For the scarfing effects, the LDA measurements of Koutmos and McGuick had shown that a scarfed multi-lobe forced mixer can generate streamwise circulation 25~30% stronger than that by the normal lobed mixer. Unfortunately, the scarfing effects on the multi-lobe forced mixer, which is achieved by alternately extending and cutting back the lobes at the trailing edge cannot be similarly performed on the present single-lobe forced mixer. For the present mixer it has only one lobe, so the scarfing effect can only be accomplished by cutting this whole lobe. This procedure shortens the axial length of the mixer's penetration region and reduces its lobe height meanwhile. So the scarfing of the single-lobe mixer is trying to achieve high mixing performance within less distance. Unfortunately by reducing its lobe height, the streamwise vorticity magnitude would be greatly suppressed to about 75% of that of the basic model a. So the scarfing effect on the single-lobe forced mixer was not effective.

Noting that the present experiments were carried out at low speeds. However, some detailed differences are expected at higher speeds. As shown in the experimental studies by Tew *et al* (1995), the mixing rate would be reduced at high convective Mach numbers (mean speed of the two streams). For example, the mixing rate was found to decrease about 10% when the convective Mach number increased from 0.43 to 0.58. The lobed forced mixers were also found to be less effective in generating streamwise vorticity in

subsonic-supersonic flows than in subsonic-subsonic or supersonic-supersonic flows. However, it can be expected that the gross mixing features would be very similar at both low and high speed flows.

6.5 Conclusions

In this chapter, the single-lobe forced mixers with two other different geometry configurations, i.e., rectangular and triangular trailing edges, and two modified configurations, i.e., the scalloped and scarfed configurations, have been examined using hot-wire anemometer and laser Doppler anemometer on their K-H vortices and streamwise vortices respectively. From the experiments the following conclusions can be drawn.

(1) The K-H vortices shed after the rectangular and triangular lobed mixers are weaker than the basic semi-circular lobed mixer.

(2) Both the scalloping and the scarfing effects cannot change the strength of the K-H vortices significantly.

(3) The rectangular model can generate 5% higher circulation, or mean streamwise vorticity than the basic semi-circular model; while the triangular model can only generate 60% circulation of the basic semi-circular model.

(4) The single-lobe scalloped mixer generates 12% higher initial mean streamwise vorticity, which decays faster than the basic unmodified model; while the scarfed single-lobe mixer generates about 75% mean streamwise vorticity of the basic model only. Also it seems that the interactions between the neighboring streamwise vortices of the multi-lobe mixer strengthen the total streamwise vorticity.

(5) Both the rectangular and the scalloped models have thicker momentum thickness than the basic model a; whereas both the triangular and the scarfed models have thinner momentum thickness than the basic model a.

(6) The mixing performance of the basic mixer a is similar to that of the rectangular

mixer, and both are better than that of the triangular mixer. The scalloping on single-lobe forced mixer improves its mixing performance though not as effective as that on the multi-lobe forced mixer, while the scarfing on single-lobe forced mixer worsens its mixing performance.

Power Magnitude

Power Magnitude

Figure 6.1 Typical Power Spectra of Rectangular Single-lobe Forced Mixer (model g) under Different Flow Conditions

Figure 6.2 The K-H Vortex Shedding Frequencies after the Trailing Edge of Rectangular Single-lobe Forced Mixer (model g)

Figure 6.3 The Wavelength of the K-H vortices Shed after the Trailing Edge of Rectangular Single-lobe Forced Mixer (model g)

Figure 6.4 The Strouhal Number Variation with the Reynolds Number for Rectangular Single-lobe Forced Mixer (model g)

Power Magnitude

Power Magnitude

Figure 6.5 Typical Power Spectra for Point L of Triangular Single-lobe Forced Mixer (model h) under Different Flow Conditions

Figure 6.6 The K-H Vortex Shedding Frequencies after the Trailing Edge of Triangular Single-lobe Forced Mixer (model h)

Figure 6.7 The Wavelength of the K-H vortices Shed after the Trailing Edge of Triangular Single-lobe Forced Mixer (model h)

Figure 6.8 The Strouhal Number Variation with the Reynolds Number for Triangular Single-lobe Forced Mixer (model h)

Figure 6.9 Schematic of the Vortices in the Vicinity of the Scalloped Single-lobe Forced Mixer (model i) Trailing Edge

Power Magnitude

(1) $U_1 = U_2 = 5.66 \text{ m/s}$, $f = 560 \text{ Hz}$.
Measuring Point C(10, 0, 30)mm.

Power Magnitude

(2) $U_1 = U_2 = 5.66 \text{ m/s}$, No Significant Peaks.
Measuring Point L(10, 15, 0)mm.

Figure 6.10 Typical Power Spectra of Scalped Single-lobe Forced Mixer (model i) at Point C and L

Figure 6.11 The K-H Vortex Shedding Frequencies after the Trailing Edge of Scalped Single-lobe Forced Mixer (model i)

Figure 6.12 The Wavelength of the K-H vortices Shed after the Trailing Edge of Scalloped Single-lobe Forced Mixer (model i)

Figure 6.13 The Strouhal Number Variation with the Reynolds Number for Scalloped Single-lobe Forced Mixer (model i)

Figure 6.14 Typical Power Spectra of Scarfed Single-lobe Forced Mixer (model j) under Different Flow Conditions

Figure 6.15 The K-H Vortex Shedding Frequencies after the Trailing Edge of Scarfed Single-lobe Forced Mixer (model j)

Figure 6.16 The Wavelength of the K-H vortices Shed after the Trailing Edge of Scarfed Single-lobe Forced Mixer (model j)

Figure 6.17 The Strouhal Number Variation with the Reynolds Number for Scarfed Single-lobe Forced Mixer (model j)

(a) Coordinate System and Vector Magnitude Scale

(b) Velocity Ratio $r=1:1$

Figure 6.18 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) and the Corresponding Secondary Flow Velocity Vectors ($\vec{V}_s/\bar{U}$) at Successive Downstream Stations after the Trailing Edge of Rectangular Single-lobe Forced Mixer (model g) for (b) Velocity Ratio $r = 1:1$ and (c) Velocity Ratio $r = 0.4:1$

(a) Coordinate System and Vector Magnitude Scale

(1) $x/\lambda = 0.5$

(b) Velocity Ratio $r = 1:1$

(1) $x/\lambda = 0.5$

(2) $x/\lambda = 2.0$

(3) $x/\lambda = 4.0$

(c) Velocity Ratio $r = 0.4:1$

Figure 6.19 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) and the Corresponding Secondary Flow Velocity Vectors ($\vec{V}_s/\bar{U}$) at Successive Downstream Stations after the Trailing Edge of Triangular Single-lobe Forced Mixer (model h) for (b) Velocity Ratio $r = 1:1$ and (c) Velocity Ratio $r = 0.4:1$

(a) Coordinate System and Vector Magnitude Scale

(b) Velocity Ratio $r=1:1$

(1) $x/\lambda = 0.5$

(2) $x/\lambda = 2.0$

(3) $x/\lambda = 4.0$

(c) Velocity Ratio $r = 0.4:1$

Figure 6.20 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) and the Corresponding Secondary Flow Velocity Vectors ($\vec{V}_s/\bar{U}$) at Successive Downstream Stations after the Trailing Edge of Scalped Single-lobe Forced Mixer (model i) for (b) Velocity Ratio $r = 1:1$ and (c) Velocity Ratio $r = 0.4:1$

(a) Coordinate System and Vector Magnitude Scale

(b) Velocity Ratio $r = 1:1$

(1) $x/\lambda = 0.5$

(2) $x/\lambda = 2.0$

(3) $x/\lambda = 4.0$

(c) Velocity Ratio $r = 0.4:1$

Figure 6.21 Contours of the Normalized Streamwise Velocity ($U/\bar{U}$) and the Corresponding Secondary Flow Velocity Vectors ($\vec{V}_s/\bar{U}$) at Successive Downstream Stations after the Trailing Edge of Scarfed Single-lobe Forced Mixer (model j) for (b) Velocity Ratio $r = 1:1$ and (c) Velocity Ratio $r = 0.4:1$

Figure 6.22 Variation of the Normalized Streamwise Circulation with Downstream Distance for Single-lobe Forced Mixers with Different Configurations, Velocity Ratio $r = 1:1$

Figure 6.23 Variation of the Normalized Streamwise Circulation with Downstream Distance for Single-lobe Forced Mixers with Different Configurations, Velocity Ratio $r = 0.4:1$

Figure 6.24 Variation of the Normalized Streamwise Circulation with Downstream Distance of Single-lobe Forced Mixers with Different Modifications, Velocity Ratio $r = 1:1$

Figure 6.25 Variation of the Normalized Streamwise Circulation with Downstream Distance of Single-lobe Forced Mixers with Different Modifications, Velocity Ratio $r = 0.4:1$

Figure 6.26 Momentum Thickness Growth in the Mixing Layer of the Single-Lobe Forced Mixers with Different Configurations, Velocity Ratio $r = 0.4:1$

Figure 6.27 Momentum Thickness Growth in the Mixing Layer of the Single-Lobe Forced Mixers with Different Modifications, Velocity Ratio $r = 0.4:1$

Figure 6.28 Variation of Shape Factor for the Single-lobe Forced Mixer with Different Configurations, Velocity Ratio $r=0.4:1$

Figure 6.29 Variation of Shape Factor for the Single-lobe Forced Mixer with Different Modifications, Velocity Ratio $r=0.4:1$

Figure 6.30 Flow Separation at the Lobe Trough of the Forced Mixer, $\epsilon > 22^\circ$

Chapter 7. Conclusions

The two-stream mixing with streamwise vorticity, which is one of the most important topics in the area of fluid dynamics, has been investigated via a single-lobe forced mixer. The mixing indicators, including the Strouhal number for the Kelvin-Helmholtz vortex, the local and mean vorticity for the streamwise vortex, and the shape factor have been examined to reveal the mixing performance of the single-lobe forced mixer. For comparison, lobed mixers at different penetration angles (including the flat plate), different wavelength design, the convoluted plate, mixers with different geometric configurations and modifications have been extensively investigated. Based on the measurements using HWA and LDA, some understandings on the complicated mixing characteristics, such as the vortex shedding frequency of the Kelvin-Helmholtz vortices and their corresponding mean wavelengths, the non-dimensional Strouhal number, the generation and decay of the streamwise vortices, the local and mean streamwise vorticity, the Reynolds normal stresses together with the turbulent kinetic energy, the momentum thickness and shape factor of the mixing layer, and the interactions between the normal K-H vortices and the streamwise vortices, are examined and subsequently presented. The following remarks can be summarized from the preceding chapters.

1. There are two series of K-H vortices shed after the trailing edge of the basic lobed mixer, with both frequencies increase linearly with the mean velocity of the on-coming streams. Both of these two frequencies are higher than that of the plane free shear layer, or flat plate case. The Strouhal number increases with the Reynolds number while the rate of increase decreases gradually with increasing Reynolds number. As a result, it will reach certain stable value at Reynolds number above 5×10^5 . Varying the velocity ratio

affects the Strouhal number at point C more than at point L (C is at the center of the semi-circle part of the lobe trailing edge, while L is at the straight sidewall of the lobe trailing edge). Furthermore, higher maximum Strouhal number might be expected to be achieved when the velocity ratio approaching 0:1. The K-H vortices appear to be more important than the streamwise vortices for the generation of the turbulent kinetic energy, though these two types of vortices are both responsible for the enhanced mixing performance of the lobed mixer.

2. For the single-lobe forced mixers at different penetration angles ($\leq 22^\circ$), the streamwise vorticity measured near the trailing edge follows the scaling law as the multi-lobe forced mixer. That is, the initial mean streamwise vorticity is directly proportional to the tangent of the penetration angle ($\tan\epsilon$). The momentum thickness increases with the penetration angle too, while the shape factor decreases, which implies that better mixing performance could be achieved at larger penetration angle ($\leq 22^\circ$).

3. The convoluted plate cannot generate strong streamwise vortices. The strength of its K-H vortices is also weaker than that of the lobed mixer while stronger than that of the flat plate case. Its lower shape factor implies better mixing performance than the plane free shear layer. For the single-lobe forced mixer of different wavelengths, the basic model with wavelength twice the lobe height ($\lambda=2h$) has the strongest K-H vortices and streamwise vortices. The effects of the wavelength on the momentum thickness are not as significant as the penetration angle. The mixing performance of the basic mixer is better than that of other two mixers at half or double lobe wavelength design, while all of them are better than that of the convoluted plate and flat plate cases.

4. For different geometry configurations, the K-H vortices shed after the basic semi-

circular lobed mixer are stronger than that of the rectangular and triangular lobed mixers; while the scalloping and the scarfing effects cannot change the strength of the K-H vortices significantly. The rectangular model can generate 5% higher mean streamwise vorticity than the basic semi-circular model, whereas the triangular model can only generate 60% circulation of the basic semi-circular model. The scalloping modification has 12% higher initial mean streamwise vorticity than the basic model with faster decaying. The scarfed model can only generate 75% mean streamwise vorticity of the basic model. After the trailing edge, the basic model has a momentum thickness thinner than those in the rectangular and the scalloped models but thicker than those in the triangular and the scarfed models. The mixing performance of the basic mixer is almost the same as that of the rectangular mixer, and both are better than the triangular mixer. Scalloping on single-lobe forced mixer improves its mixing performance though not as effective as on multi-lobe forced mixer, while the scarfing on single-lobe forced mixer did not show any improvements in its mixing performance.

5. The interaction between the K-H vortices and streamwise vortices makes the structure of the K-H vortex tube to deform like ‘pinched-off’. The pinched-off effect is contributive to the mixing enhancement of the lobed mixer. The evolution of the streamwise vorticity with downstream distance is mainly dominated by the viscosity and distortion effects from the K-H vorticity, except in the region immediately after the mixer where the self-stretching effects due to the streamwise mean flow acceleration take place.

For a particular lobed forced mixer, the strength of the streamwise vorticity is not strongly dependent on the initial flow conditions, but mainly on the mixer’s configuration. This has already been proved in multi-lobe experiments by Yu *et al*

(1995). But for the K-H vortices, they are dependent on both the initial flow conditions (mainly velocity ratio and Reynolds number) and the geometry of the mixer. So to achieve the best and fast mixing performance between two unidirectional streams using lobed forced mixer, the design of the mixer's configuration is very important. From the present investigation it is promising to employ those with semi-circular trailing edge configuration with penetration angle at about 22 degree. Also the mixer is expected to work at the flow regimes in which maximum Strouhal numbers could be achieved. The lobe wavelength should also be designed to be double the lobe height, which is usually dependent on the other structural conditions. The scalloping modification is also a good choice if possible.

Chapter 8. Recommendations

This dissertation has focused on the experimental investigations of the mixing performance of the lobed forced mixer. Subsonic flow regime has been examined. Actually the lobed mixer is presently utilized not only in the subsonic regime but also in the transonic and supersonic regimes. It is believed that the flow characteristics at different flow regimes will be somewhat different. So the mixing performance of the lobed mixer at transonic and supersonic regimes is worthy of more investigations.

On the other hand, the present work is mainly focused on the momentum exchange between the two streams. But heat transfer is of equal importance in its utilization. For example, for the lobed forced mixer used in aero-fan engines, there are dramatic temperature differences between the two mixing streams. Till now little of the heat exchange problem has been carried out on the lobed mixer flow. The heat exchange problem will be one further target.

With the development of the computer science in the recent three decades, CFD, or computational fluid dynamics has been one powerful tool to solve very complicated fluid problems. There is a great deal of interest in simulating lobed forced mixer flows, because to design an optimum mixer on experimental evidence alone is expensive and nontrivial due to the wide range of parameters that can be varied. Actually computational studies of the lobed forced mixers to date have employed a hierarchy of numerical methods. However, great inconsistencies and discrepancies have arisen from the comparisons due to the complicated three-dimensional wake fields. The present results would be useful for further development of CFD in producing the complicated flow fields, especially the non-linear interactions between the K-H and streamwise vortices,

which maybe still unrevealed throughout the present investigation.

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Appendix A. Code for Angle Calibration of the Cross Hot-wire Anemometer

```

%open the files collected from the NI labview
%ioput just corresponds to the angle of the files
files='0 -15 -12 -9 -6 -3 3 6 9 12 15';
k=0; num=11;ioput=zeros(11,3)
ioput(:,3)=[0;-15;-12;-9;-6;-3;3;6;9;12;15]*3.1415926/180

%set initial value as zero

SY1=0;SY2=0;SX1=0;SX2=0;SDX1=0;SDX2=0;
S1=0;
S2=0;

while isempty(files)==0
    k=k+1;
    [filesig,files]=strtok(files);
    infile=[filesig '.txt'];
    [dd]=eff(infile,k);
    ioput(k,1)=dd(k,1);
    ioput(k,2)=dd(k,2);

%ioput =cos(0~15)-U/Uoc !!!!!!!A3+(B3*e3bar)+C3*(e3bar^2)+D3*(e3bar^3)!!!!!!

    ioput2(k,1)=cos(ioput(k,3))-
(3.0559+0.4524*ioput(k,1)+0.0652*ioput(k,1)^2+0.0057*ioput(k,1)^3)/(3.0559+0.4524*
ioput(1,1)+0.0652*ioput(1,1)^2+0.0057*ioput(1,1)^3);
    ioput2(k,2)=cos(ioput(k,3))-
(2.7786+0.3931*ioput(k,2)+0.0428*ioput(k,2)^2+0.0030*ioput(k,2)^3)/(2.7786+0.3931*
ioput(1,2)+0.0428*ioput(1,2)^2+0.0030*ioput(1,2)^3);
    ioput2(k,3)=sin(ioput(k,3));
    %X2=X1;

% DX1=X1^2;DX2=DX1;
%XY1=X1*Y1;
%XY2=X2*Y2;
%SY1=SY1+Y1;SY2=SY2+Y2;
%SX1=SX1+X1;SX2=SX2+X2;
%SDX1=SDX1+DX1;SDX2=SDX1;
%S1=S1+XY1;
%S2=S2+XY2;

```

```

end
[a1,b1]=linecalibrate(ioput2(:,1),ioput2(:,3),11)
[a2,b2]=linecalibrate(ioput2(:,2),ioput2(:,3),11)

fid=fopen('vm.txt','a');
fprintf(fid,'%g ',ioput);
fclose(fid)
%EANG1=(atan((11*S1-SX1*SY1)/(11*SDX1-SX1*SX1)))*180/3.1415926
EANG1=atan(b1)*180/3.1415926
EANG2=atan(b2)*180/3.1415926

%EANG2=(atan((11*S2-SX2*SY2)/(11*SDX2-SX2*SX2)))*180/3.1415926

p(2,1)=EANG1;p(2,2)=EANG2;p
fid=fopen('para.txt','a');
fprintf(fid,'%8.4f %8.4f/n',p);

```

```

%eff function
%caculate the mean of the E1 E2

```

```

function [iop]=eff(infile,m)
data=csvread(infile);
E1=data(:,1);
E2=data(:,2);

E1bar=mean(E1);
E2bar=mean(E2);
iop(m,1)=E1bar;

iop(m,2)=E2bar;
%%%%%%%%
return

```

Appendix B. The Fourier Transform

1. Introduction

Linear transforms, especially Fourier and Laplace transforms, are widely used in solving problems in science and engineering. The Fourier transform is used in linear systems analysis, antenna studies, optics, random process modeling, probability theory, quantum physics, and boundary-value problems (Brigham 1988) and has been very successfully applied to restoration of astronomical data (Brault and White 1971). The Fourier transform, a pervasive and versatile tool, is used in many fields of science as a mathematical or physical tool to alter a problem into one that can be more easily solved. Some scientists understand Fourier theory as a physical phenomenon, not simply as a mathematical tool. In some branches of science, the Fourier transform of one function may yield another physical function (Bracewell 1965).

The Fourier transform, named for Jean Baptiste Joseph Fourier (1768~1830), is an integral transform that re-expresses a function in terms of sinusoidal basis functions, i.e. as a sum or integral of sinusoidal functions multiplied by some coefficients ("amplitudes"). In signal processing and related fields, the Fourier transform is typically thought of as decomposing a signal into its component frequencies and their amplitudes. The wide applicability stems from several useful properties of the transforms:

- The transforms are linear operators and, with proper normalization, are unitary as well (a property known as Parseval's theorem or, more generally, as the Plancherel theorem, and most generally via Pontryagin duality).
- The transforms are invertible, and in fact the inverse transform has almost the same form as the forward transform.

- The sinusoidal basis functions are eigenfunctions of differentiation, which means that this representation transforms linear differential equations with constant coefficients into ordinary algebraic ones. (For example, in a linear time-invariant physical system, frequency is a conserved quantity, so the behavior at each frequency can be solved independently.)
- By the convolution theorem, Fourier transforms turn the complicated convolution operation into simple multiplication, which means that they provide an efficient way to compute convolution-based operations such as polynomial multiplication and multiplying large numbers.
- Fast algorithms, based on the fast Fourier transform (FFT), exist to evaluate Fourier transforms on computers.

In terms of signal processing, the Fourier transform takes a time series representation of a signal function and maps it into a frequency spectrum, where ω is angular frequency. That is, it takes a function in the time domain into the frequency domain; it is a decomposition of a function into harmonics of different frequencies.

When the function f is a function of time and represents a physical signal, the transform has a standard interpretation as the spectrum of the signal. The magnitude of the resulting complex-valued function F represents the amplitudes of the respective frequencies (ω), while the phase shifts are given by $\arctan$ (imaginary parts/real parts).

However, it is important to realize that Fourier transforms are not limited to functions of time, and temporal frequencies. They can equally be applied to analyze spatial frequencies, and indeed for nearly any function domain.

2. (Continuous) Fourier Transform (CFM/FM)

Most often, the unqualified term "Fourier transform" refers to the continuous Fourier transform.

The Fourier transform, in essence, decomposes or separates a waveform or function into sinusoids of different frequency which sum to the original waveform. It identifies or distinguishes the different frequency sinusoids and their respective amplitudes (Brigham, 4). The Fourier transform of $f(x)$ is defined as:

$$F(s) = \int_{-\infty}^{+\infty} f(x) \cdot \exp(-i \cdot 2\pi xs) \cdot dx. \quad (\text{B-1})$$

Applying the same transform to $F(s)$ gives

$$f(\omega) = \int_{-\infty}^{+\infty} F(s) \cdot \exp(-i \cdot 2\pi \omega s) \cdot ds. \quad (\text{B-2})$$

If $f(x)$ is an even function of x , that is $f(x) = f(-x)$, then $f(\omega) = f(x)$. If $f(x)$ is an odd function of x , that is $f(x) = -f(-x)$, then $f(\omega) = f(-x)$. When $f(x)$ is neither even nor odd, it can often be split into even or odd parts.

To avoid confusion, it is customary to write the Fourier transform and its inverse so that they exhibit reversibility:

$$F(s) = \int_{-\infty}^{+\infty} f(x) \cdot \exp(-i \cdot 2\pi xs) \cdot dx, \quad (\text{B-3})$$

$$f(x) = \int_{-\infty}^{+\infty} F(s) \cdot \exp(i \cdot 2\pi xs) \cdot ds, \quad (\text{B-4})$$

so that

$$f(x) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} f(x) \cdot \exp(-i \cdot 2\pi xs) \cdot dx \right] \cdot \exp(i \cdot 2\pi xs) \cdot ds. \quad (\text{B-5})$$

As long as the integral exists and any discontinuities, usually represented by multiple integrals of the form $\frac{1}{2}[f(x+) + f(x-)]$, are finite. The transform quantity $F(s)$ is often represented as $\tilde{f}(s)$ and the Fourier transform is often represented by the operator $\mathfrak{R}$ (Bracewell 1965).

There are functions for which the Fourier transform does not exist; however, most physical functions have a Fourier transform, especially if the transform represents a physical quantity. Other functions can be treated with Fourier theory as limiting cases. Many of the common theoretical functions are actually limiting cases in Fourier theory.

Usually functions or waveforms can be split into even and odd parts as follows

$$f(x) = E(x) + O(x), \quad (\text{B-6})$$

where

$$E(x) = \frac{1}{2} [f(x) + f(-x)], \quad (\text{B-7})$$

$$O(x) = \frac{1}{2} [f(x) - f(-x)], \quad (\text{B-8})$$

and $E(x)$, $O(x)$ are, in general, complex. In this representation, the Fourier transform of $f(x)$ reduces to:

$$2 \int_0^{+\infty} E(x) \cdot \cos(2\pi xs) dx - 2i \int_0^{+\infty} O(x) \cdot \sin(2\pi xs) dx. \quad (\text{B-9})$$

It follows then that an even function has an even transform and that an odd function has an odd transform. Additional symmetry properties are shown in **Table 1** (Bracewell 1965).

An important case from **Table 1** is that of a Hermitian function, one in which the real part is even and the imaginary part is odd, *i.e.*, $f(x) = f^*(-x)$. The Fourier transform of a

Hermitian function is even. In addition, the Fourier transform of the complex conjugate of a function $f(x)$ is $F^*(-s)$, the reflection of the conjugate of the transform.

Function	Transform
real and even	real and even
real and odd	imaginary and odd
imaginary and even	imaginary and even
complex and even	complex and even
complex and odd	complex and odd
real and asymmetrical	complex and asymmetrical
imaginary and asymmetrical	complex and asymmetrical
real even plus imaginary odd	real
real odd plus imaginary even	imaginary
Even	even
odd	odd

Table 1. Symmetry Properties of the Fourier Transform

Since the Fourier transform $F(s)$ is a frequency domain representation of a function $f(x)$, the s characterizes the frequency of the decomposed cosinusoids and sinusoids and is equal to the number of cycles per unit of x (Bracewell 1965). If a function or waveform is not periodic, then the Fourier transform of the function will be a continuous function of frequency (Brigham 1988).

3. Discrete Fourier Transform (DFT)

Because a digital computer works only with discrete data, numerical computation of the Fourier transform of $f(t)$ requires discrete sample values of $f(t)$, which we will call f_k . In addition, a computer can compute the transform $F(s)$ only at discrete values of s , that is, it can only provide discrete samples of the transform, F_r . If $f(kT)$ and $F(rs_0)$ are the k th and r th samples of $f(t)$ and $F(s)$, respectively, and N_0 is the number of samples in the signal in one period T_0 , then

$$f_k = T f(kT) = T_0 N_0^{-1} f(kT) \quad (\text{B-10})$$

and

$$F_r = F(rs_0), \quad (\text{B-11})$$

where

$$s_0 = 2 \pi T_0^{-1}. \quad (\text{B-12})$$

The discrete Fourier transform (DFT) is defined as

$$F_r = \sum_{k=0}^{N_0-1} f_k \cdot \exp(-i \cdot r \Omega_0 k), \quad (\text{B-13})$$

where $\Omega_0 = 2\pi N_0^{-1}$. Its inverse is

$$f_k = N_0^{-1} \sum_{r=0}^{N_0-1} F_r \cdot \exp(i \cdot r \Omega_0 k). \quad (\text{B-14})$$

These equations can be used to compute transforms and inverse transforms of appropriately sampled data. Proofs of these relationships are in Lathi (1992).

4. Fast Fourier Transform (FFT)

The Fast Fourier Transform (FFT) is a DFT algorithm developed by Tukey and Cooley in 1965, which reduces the number of computations from something on the order

of N_0^2 to $N_0 \log_2 N_0$. There are basically two types of Tukey-Cooley FFT algorithms in use: decimation-in-time and decimation-in-frequency. The algorithm is simplified if N_0 is chosen to be a power of 2, but it is not a requirement.

5. Accuracy and Approximations

Even the "exact" FFT algorithms have errors when finite-precision floating-point arithmetic is used, but these errors are typically quite small; most FFT algorithms, e.g. Cooley-Tukey, have excellent numerical properties. The upper bound on the relative error for the Cooley-Tukey algorithm is $O(e \log n)$, compared to $O(e n^{3/2})$ for the naive DFT formula (Gentleman and Sande 1966), where e is the machine floating-point relative precision.

6. References

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Appendix C. Memoir

I was born in Yixing, Jiangsu Province, People's Republic of China in 1974.

From year of 1993 to 1997 I joined Department of Propulsion and Power, Beijing University of Aeronautics and Astronautics (Beijing, China) for undergraduate study with major of Aero-engine. The final year project is 'Stress and Strain Analyses of Monolithic Aircraft Turbine Plate'. By designing the turbine plate and its blades together, the stress with largest magnitude was found to be reduced by about 15% to 20%. I was awarded Bachelor of Engineering after four years' curricula.

My Master of Science was achieved from the Institute of Mechanics, Chinese Academy of Sciences (Beijing, China) from 1997 to 2000. The project 'Generation of Supersonic Carrier Gas with High Enthalpy' was carried out, to generate steam gas with high temperature and high speed via the combustion of hydrogen and oxygen under different pressure conditions. Some critical problems, such as the mixing of the hydrogen and oxygen, the ignition problem, the control of mass flow by the critical nozzles, the pressure adjustment by the second throat, and the stability of the combustion have been discussed briefly.

The present thesis is my Ph.D. dissertation. I joined the School of Mechanical and Aerospace Engineering, Nanyang Technological University (Singapore) for the degree of Ph.D. in 2001. The research title is 'two-stream mixing flow with streamwise vorticity'. The degree will be awarded in July 2005.

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Mao Ronghai

May 12, 2005

有志者事竟成！