Modeling and analysis based techniques for fault diagnosis of planetary gearboxes

Hong, Liu

2014

Hong, L. (2014). Modeling and analysis based techniques for fault diagnosis of planetary gearboxes. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/61679>

<https://doi.org/10.32657/10356/61679>

NANYANG
TECHNOLOGICAL
UNIVERSITY

**MODELING AND ANALYSIS BASED TECHNIQUES
FOR FAULT DIAGNOSIS OF PLANETARY
GEARBOXES**

HONG LIU

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

2014

MODELING AND ANALYSIS BASED TECHNIQUES FOR FAULT DIAGNOSIS OF PLANETARY GEARBOXES

NANYANG
TECHNOLOGICAL
UNIVERSITY

HONG LIU
B. ENG.

School of Mechanical and Aerospace Engineering
Nanyang Technological University, Singapore

A thesis submitted to Nanyang Technological University in partial
fulfillment of the requirement for the degree of Doctor of
Philosophy

2014

Acknowledgement

Foremost, I would like to express my sincere gratitude to my supervisor, Assistant Professor Jaspreet Singh Dhupia, for his kind and patient guidance, keen insights, encouragements and friendship throughout my research. He was always around whenever I needed him, and helped me focus in the right direction. His motivation and persistence helped me professionally. From him, I have not only learned the method to conduct research but also gained an ability to identify problems in research. It is of a great benefit and an honor to work with him.

I owe my deep gratitude to Professor Wang Danwei for his kind instructions and advices. His professional knowledge and experience help me greatly in my research.

I would like to thank the National Renewable Energy Laboratory (NREL) at Golden, Colorado, US to share their valuable experimental data with us.

Gratitude is expressed to the director of the Mechantronics Laboratory and all the lab technicians in the laboratory for their self-giving help with my experiments.

I am also grateful to Nanyang Technological University for providing me with the financial support and excellent research facilities to accomplish my PhD research.

Finally, I would like to express my gratitude to my family and my friends for their unwavering patience and great support through the entire journey.

Table of Contents

ACKNOWLEDGEMENT.....	I
TABLE OF CONTENTS.....	II
SUMMARY.....	V
LIST OF TABLES.....	VII
LIST OF FIGURES.....	VIII
LIST OF SYMBOLS.....	XIII
CHAPTER 1 INTRODUCTION.....	1
1.1 MOTIVATION.....	1
1.2 PROBLEM STATEMENT.....	3
1.3 OBJECTIVES	5
1.4 KEY CONTRIBUTIONS OF THE THESIS.....	5
1.5 ORGANIZATION OF THE THESIS.....	6
CHAPTER 2 LITERATURE REVIEW.....	9
2.1 FAILURE MODES IN GEARBOXES.....	10
2.1.1 Gear failures.....	10
2.1.2 Bearing failures	11
2.1.3 Shaft failures.....	12
2.1.4 Other sources of failures in gearboxes.....	13
2.2 MEASUREMENTS IN GEARBOX MONITORING	14
2.2.1 Vibration based monitoring.....	15
2.2.2 Acoustic emission based monitoring	16
2.2.3 Electrical current signal analysis.....	18
2.2.4 Wear debris analysis	18
2.2.5 Temperature based monitoring	19
2.3 GENERAL STRUCTURE OF THE VIBRATION BASED DIAGNOSTIC SYSTEM.....	20
2.4 DYNAMIC AND ANALYTICAL VIBRATION MODELING FOR FAULT DIAGNOSIS OF GEARBOXES	21
2.4.1 Lumped-parameter dynamic model of a single stage gear-set.....	22
2.4.2 Mathematical amplitude-frequency modulated vibration model of gear-set.....	26
2.5 RECENT ADVANCES IN SIGNAL ANALYSIS METHODS FOR GEARBOX FAULT DIAGNOSIS.....	28
2.5.1 Time-domain signal analysis.....	29
2.5.2 Spectral analysis.....	33
2.5.3 Time-frequency signal analysis	34

2.6	PLANETARY GEAR SYSTEM.....	38
2.6.1	<i>Geometry and configurations of planetary gearbox.....</i>	<i>39</i>
2.6.2	<i>Gear mesh frequency of planetary gearbox</i>	<i>41</i>
2.6.3	<i>Modulation effect due to the rotating carrier with respect to the mounted sensor.....</i>	<i>43</i>
CHAPTER 3	DYNAMIC MODELING AND EXPERIMENTAL INVESTIGATION OF PLANETARY GEARBOX.....	45
3.1	TWO-DIMENSIONAL MECHANICAL MODEL OF PLANETARY GEARBOX	45
3.1.1	<i>Dynamic model of planetary gear-set</i>	<i>46</i>
3.1.2	<i>Effect of a gear defect on gear mesh stiffness in a planetary gear system.....</i>	<i>50</i>
3.1.3	<i>Model application – an integrated electro-mechanical drivetrain</i>	<i>54</i>
3.2	VIBRATION MEASUREMENTS OF PLANETARY TEST RIGS AND AMPLITUDE MODULATION PHENOMENON IN THE MEASUREMENTS	55
3.2.1	<i>Laboratory planetary gearbox test rig</i>	<i>56</i>
3.2.2	<i>Planetary gearbox test rig at the National Renewable Energy Laboratory.....</i>	<i>56</i>
CHAPTER 4	GEOMETRY BASED VIBRATION MODELS FOR FAULT DIAGNOSIS OF PLANETARY GEAR-SET.....	59
4.1	VIBRATION SIGNAL MODELS AND MONITORING TECHNIQUES FOR PLANETARY GEARBOX	60
4.2	FOURIER SERIES ANALYSIS OF HEALTHY PLANETARY GEAR SYSTEM	63
4.3	FOURIER SERIES ANALYSIS OF PLANETARY GEAR WITH LOCAL FAULTS	69
4.3.1	<i>Local fault on the annulus gear.....</i>	<i>70</i>
4.3.2	<i>Local fault on the sun gear.....</i>	<i>72</i>
4.3.3	<i>Local fault on a planet gear</i>	<i>74</i>
4.4	EXPERIMENTAL SIGNAL ANALYSIS.....	75
4.5	REMARKS ON DEVELOPED GEOMETRY BASED MODEL	78
CHAPTER 5	DIAGNOSTIC SIGNAL ANALYSIS FOR MONITORING PLANETARY GEARBOX.....	80
5.1	INTRODUCTION TO SIGNAL ANALYSIS TECHNIQUES FOR DIAGNOSIS OF GEARBOX.....	81
5.2	PROPOSED ALGORITHM FOR GEARBOX FAULT DIAGNOSIS IN TIME-DOMAIN	84
5.2.1	<i>Dynamic time warping.....</i>	<i>85</i>
5.2.2	<i>Fast dynamic time warping</i>	<i>87</i>
5.2.3	<i>Correlated kurtosis.....</i>	<i>89</i>
5.2.4	<i>Proposed fault diagnosis algorithm</i>	<i>91</i>

5.3	SIMULATION BASED INVESTIGATION OF PROPOSED ALGORITHM'S PERFORMANCE	97
5.3.1	<i>Detection and location of a gear defect in single stage fixed axis gearbox</i>	<i>98</i>
5.3.2	<i>Dynamic model of planetary gear transmission.....</i>	<i>101</i>
5.3.3	<i>Detection and location of a gear defect in planetary gearbox.....</i>	<i>104</i>
5.4	EXPERIMENTAL RESULTS	107
5.5	REMARKS ON PROPOSED DIAGNOSTIC ANALYSIS METHOD	111
CHAPTER 6	CONCLUSION AND FUTURE WORKS.....	113
6.1	CONCLUSIONS	113
6.2	FUTURE WORKS	116
REFERENCES.....		118

Summary

This thesis proposes analytical models and monitoring methods based on these models to carry out fault diagnosis of planetary gearboxes and assist in developing robust fault alarm framework for early detection of gear failures. The proposed techniques can be applied not only to planetary wind turbine gearboxes but also to other general planetary gearboxes in varied engineering applications.

The equally-spaced planetary gearbox is an important power-train component employed in many electro-mechanical engineering systems whose failures can result in significant capital losses and pose safety concerns. For example, a modern wind turbine is usually designed for a life span of over 20 years. Failure of its drive train components, such as the planetary gearbox and the generator, can leave the wind turbine out of service for long periods and therefore contribute to majority of the downtime and incurred maintenance costs over the entire lifetime of the wind turbine. Vibration based diagnostic techniques to detect gear tooth damage in gearboxes haven been widely studied. However, the complex nature of measured vibration spectra, resulting from rotating planet pinions with respect to the stationary sensor mounted on the gearbox housing, makes an effective implementation of these techniques as a challenge.

The research presented in this thesis addresses the challenges of vibration based monitoring of planetary gearbox by (1) understanding the dynamic behavior of planetary gears and the amplitude modulation (AM) phenomena due to the relative motions of the planet pinions with respect to the stationary sensor through a lumped parameter modeling based approach, (2) identifying

the location of the additional frequency components in measured vibration spectra resulting due to general gear faults, which are shown to depend on the geometry of the planetary gear-set and the gear carrying the local fault and (3) developing a fault alarm algorithm to diagnose incipient gear failure for industrial applications. The results are validated through dynamic simulations and experimental data from two planetary gearbox test rigs: a 4KW laboratory planetary gearbox test rig with the artificial seeded gear failure and a 750 kW tested gearbox for wind turbine application installed in a 2.5 MW dynamometer test facility at the National Renewable Energy Laboratory (NREL) at Golden, Colorado, US. The second facility is capable of providing static, highly accelerated life and model-in-the-loop tests. Both experiments collected vibration data through stationary accelerometers attached to the housing of the gearbox.

The original contribution of the thesis include: (1) A mathematical AM-FM (amplitude modulated/frequency modulated) signal model developed to simulate the distinct spectrum of planetary gearboxes operating under healthy and faulty conditions; (2) Fourier series analysis based on this mathematical signal model that identifies the additional frequency components appearing due to local faults and describe the relationship between the location of these frequency components and the geometry of the planetary gear-set and (3) a new time domain approach that combines dynamic time warping and correlated kurtosis to detect gear faults and identify their location, thus assisting in the development of a robust fault alarm system.

Keywords: Planetary gear system; Modulation; Gear-set fault; Diagnosis

List of Tables

TABLE 2.1: SUMMARY OF DIAGNOSIS TECHNIQUES USED FOR GEARBOX FAILURES	
.....	15
TABLE 2.2: ASSEMBLY CONFIGURATION OF PLANETARY GEAR SYSTEM.....	39
TABLE 5.1: SYSTEM PARAMETERS FOR THE EXAMPLE PLANETARY GEAR SYSTEM	
.....	104

List of Figures

FIGURE 1.1. EXAMPLES OF PLANETARY GEARBOX APPLICATIONS (A) WIND TURBINE PLANETARY GEARBOX AND (B) HELICOPTER PLANETARY TRANSMISSION GEARBOX.....	1
FIGURE 1.2. CATASTROPHE CAUSED BY THE GEAR FAILURE IN A WIND TURBINE..	2
FIGURE 1.3. OUTLINE OF THE THESIS	6
FIGURE 2.1. TYPICAL SCHEME FOR CONDITION MONITORING SYSTEM FOR GEARBOX DIAGNOSIS.	9
FIGURE 2.2. TYPICAL GEAR TOOTH DEFECT (A) SEEDED CRACKED TOOTH ON A SPUR GEAR [5] AND (B) NATURALLY WORN TEETH ON AN INDUSTRIAL HELICAL GEAR [16].....	11
FIGURE 2.3. AN ILLUSTRATION OF THE COMPONENTS IN A TYPICAL ROLLING ELEMENT BEARING.....	12
FIGURE 2.4. EXAMPLES OF BEARING FAILURES.	12
FIGURE 2.5. AN ILLUSTRATION OF THE SHAFT MISALIGNMENT.	13
FIGURE 2.6. A VIEW OF A WIRELESS ACCELEROMETER MOUNTED ON THE GEAR HOUSING.	16
FIGURE 2.7. ACOUSTIC EMISSION SENSOR LOCATED CLOSE TO THE MONITORING COMPONENTS [46].	17
FIGURE 2.8. AN ILLUSTRATION OF THE NON-INTRUSIVE CURRENT SENSING SYSTEM.....	18
FIGURE 2.9. MANUAL SAMPLING FROM THE LOCATION OF OIL SAMPLING PORT [52].....	19
FIGURE 2.10. A THERMOMETER TO MONITOR THE CONDITION OF THE MACHINE.	19
FIGURE 2.11. DATA FLOW IN THE VIBRATION BASED DIAGNOSTIC SYSTEM.	20
FIGURE 2.12. THE LUMPED PARAMETER MODEL OF A SINGLE STAGE GEAR PAIR.	22
FIGURE 2.13. GEAR TOOTH MESH STIFFNESS UNDER HEALTHY AND FAULTY CASE.	24
FIGURE 2.14. SIMULATED VIBRATION SPECTRUM OF A SPUR GEAR-PAIR.	25
FIGURE 2.15. SIDEBANDS AROUND GEAR MESH HARMONICS IN A PRESENCE OF PINION TOOTH DEFECT.	26

FIGURE 2.16. FREQUENCY RESPONSE OF TIME SYNCHRONOUS AVERAGE AS A SPECIAL CASE OF COMB FILTERS (A) COMB LIKE MAGNITUDE RESPONSE AND (B) LINEAR PHASE RESPONSE.....	30
FIGURE 2.17. THE VIBRATION SPECTRUM BEFORE (A) AND AFTER (B) THE TIME SYNCHRONOUS AVERAGE PROCESSING.	31
FIGURE 2.18. DYNAMIC TIME WARPING OF TIME SERIES (A) BEFORE (B) AFTER PROCESSING.	32
FIGURE 2.19. SHORT TIME FOURIER TRANSFORM SPECTROGRAM OF $x(t)$	35
FIGURE 2.20. THE WAVELET TRANSFORM OF $x(t)$	36
FIGURE 2.21. THE WIGNER-VILLE DISTRIBUTION OF $x(t)$	38
FIGURE 2.22. EXAMPLE FOR AN INDUSTRIAL PLANETARY GEAR SYSTEM WITH BEARINGS.	38
FIGURE 2.23. AN ILLUSTRATION OF PLANETARY GEAR SYSTEM WITH THREE PLANETS.	42
FIGURE 2.24. (A) INFLUENCE OF PLANET i ON THE VIBRATION RESPONSE MEASURED BY THE SENSOR, (B) PLANET GEAR LOCATION AT THE INSTANT OF MAXIMUM INFLUENCE ON THE VIBRATION RESPONSE AT THE SENSOR.	43
FIGURE 3.1. TWO DIMENSIONAL LUMPED PARAMETER MODEL OF A PLANETARY GEAR-SET.....	47
FIGURE 3.2. (A) ANNULUS GEAR MESHES WITH PLANETS IN THE ROTARY REFERENCE FRAME (B) SUN GEAR MESHES WITH PLANETS IN THE ROTARY REFERENCE FRAME.	52
FIGURE 3.3. PERIODIC FLUCTUATION OF THE GEAR MESH STIFFNESS BETWEEN THE SUN AND PLANET GEARS WITH LOCAL GEAR TOOTH FAULT ON THE SUN.	53
FIGURE 3.4. THE INTEGRATED ELECTRO-MECHANICAL DRIVETRAIN MODEL IN MATLAB/SIMULINK.	54
FIGURE 3.5. (A) SCHEMATIC OF THE TEST RIG, (B) PLANETARY GEARBOX TEST RIG AND (C) SEEDED GEAR FAULT ON THE INTERNAL RING GEAR OF THE PLANETARY GEAR-SET.	55
FIGURE 3.6. (A) NREL 2.5 MW DYNAMOMETER TEST FACILITY (CREDIT: LEE JAY FINGERSH/NREL #16913), (B) SCHEMATIC OF THE TEST FACILITY (CREDIT: NREL) AND (C) VIBRATION SENSOR MOUNTED ON THE ANNULUS GEAR RADIAL 6 AND 12 O'CLOCK.	57

FIGURE 4.1. A PLANETARY GEAR-SET WITH 4 PLANETS AND A SENSOR MOUNTED ON THE FIXED ANNULUS GEAR.	63
FIGURE 4.2. SIMULATED VIBRATION SPECTRUM AROUND GEAR MESHING FREQUENCY FOR A HEALTHY EQUALLY SPACED 4-PLANETS GEAR-SET AS PERCEIVED BY A FIXED SENSOR ON THE ANNULUS.	68
FIGURE 4.3. SIMULATED VIBRATION SPECTRUM AROUND GEAR MESHING FREQUENCY DUE TO LOCAL FAULT ON THE ANNULUS GEAR FOR AN EQUALLY SPACED 4-PLANETS GEAR-SET, $N_A = 99$, $N_S = 29$ AND $N_P = 35$	71
FIGURE 4.4. SIMULATED VIBRATION SPECTRUM AROUND GEAR MESHING FREQUENCY DUE TO LOCAL FAULT ON THE SUN GEAR FOR AN EQUALLY SPACED 4-PLANETS GEAR-SET, $N_A = 99$, $N_S = 29$ AND $N_P = 35$	73
FIGURE 4.5. SIMULATED VIBRATION SPECTRUM AROUND GEAR MESHING FREQUENCY DUE TO LOCAL FAULT ON A PLANET GEAR FOR AN EQUALLY SPACED 4-PLANETS GEAR-SET, $N_A = 99$, $N_S = 29$ AND $N_P = 35$ (A) VIBRATION SPECTRUM AND (B) POWER SPECTRUM.	74
FIGURE 4.6. MEASURED ACCELERATIONS FROM AN3 AND AN4 AROUND MESH FREQUENCY UNDER 25% OF RATED POWER AND 1200 RPM NOMINAL HIGH SPEED SHAFT ROTATIONAL SPEED.	75
FIGURE 4.7. (A) FRETTING CORROSION OCCURRED ALONG A LINE OF ACTION AT THE SUN PINION (CREDIT: GEARTECH, NREL #19750) AND (B) POLISHING WEAR AT THE ANNULUS GEAR (CREDIT: GEARTECH, NREL #19749).	76
FIGURE 4.8. (A) PREDICTED SPECTRUM FOR FAULTY ANNULUS GEAR, (B) PREDICTED SPECTRUM FOR FAULTY SUN GEAR AND (C) MEASURED VIBRATION SPECTRUM AROUND GEAR MESHING FREQUENCY FOR AN EQUALLY SPACED 3-PLANETS GEAR-SET, $N_A = 99$, $N_S = 21$ AND $N_P = 39$	77
FIGURE 4.9. SPECTRUM WHEN THE GEARBOX OPERATED AT (A) AT 25% OF RATED POWER AND 1800 RPM AND (B) 50% OF RATED POWER AND 1800 RPM. ...	78
FIGURE 5.1. DYNAMIC TIME WARPING OF TWO TIME SERIES (A) BEFORE AND (B) AFTER PROCESSING.	85
FIGURE 5.2. OPTIMAL WARPING PATH SEARCHED BY DYNAMIC PROGRAMMING.	86
FIGURE 5.3. WINDOW AND OPTIMAL PATH EVALUATED DURING DIFFERENT LEVEL/RESOLUTIONS WHEN RUNNING RECURSIVE FAST DTW ALGORITHM WITH PRE-DEFINED RADIUS OF 4 CELLS.	88

FIGURE 5.4. FAST DYNAMIC TIME WARPING OF THE TWO EXAMPLED TIME SERIES (A) BEFORE, AND (B) AFTER PROCESSING.	89
FIGURE 5.5. CK VERSUS KURTOSIS FOR DIFFERENT SIGNALS CONTAINING DISTINCT PEAKS.	90
FIGURE 5.6. FLOWCHART OF THE PROPOSED TIME DOMAIN FAULT DETECTION METHOD.	91
FIGURE 5.7. SCHEME OF DETERMINING THE PRE-DEFINED WINDOW FUNCTION. ..	94
FIGURE 5.8. REFERENCE SIGNAL ESTIMATION FOR PLANETARY GEAR-SET.	94
FIGURE 5.9. THE TIME RESAMPLING ALGORITHM.	96
FIGURE 5.10. PERFORMANCE OF TIME RESAMPLING ALGORITHM: (A) TEST AND REFERENCE SIGNAL, (B) TEST AND REFERENCE SIGNAL AFTER FAST DTW, (C) THE RAW RESIDUAL SIGNAL, AND (D) THE RESIDUAL SIGNAL AFTER THE PROPOSED TIME RESAMPLE ALGORITHM.	97
FIGURE 5.11. (A) SIMULATED SIGNALS OF THE GEARBOX UNDER HEALTHY AND FAULTY PINION CONDITION $x_H(T)$ AND $x_{PF}(T)$; (B) SPECTRA OF $x_H(T)$ AND $x_{PF}(T)$ AFTER BAND-PASS FILTER; (C) $x_H(T)$ AND REFERENCE SIGNAL $y(T)$ AFTER INITIAL PHASE ESTIMATION BUT BEFORE FAST DTW; (D) $x_H(T)$ AND REFERENCE SIGNAL $y(T)$ AFTER FAST DTW; (E) $x_{PF}(T)$ AND REFERENCE SIGNAL $y(T)$ AFTER INITIAL PHASE ESTIMATION BUT BEFORE FAST DTW AND (F) $x_{PF}(T)$ AND REFERENCE SIGNAL $y(T)$ AFTER FAST DTW.	98
FIGURE 5.12. RESIDUAL SIGNALS AFTER FAST DTW (A) THE RAW RESIDUAL SIGNALS RZ AFTER FAST DTW, AND (B) THE RESIDUAL SIGNALS Z AFTER THE PROPOSED TIME RESAMPLE.	100
FIGURE 5.13. (A) ERROR FUNCTION AND (B) GEAR MESHING STIFFNESS UNDER HEALTHY AND FAULTY CASE.	102
FIGURE 5.14. SIMULATED RESIDUAL VIBRATION SIGNALS FROM THE PLANETARY GEAR-SET UNDER HEALTHY AND LOCAL SUN GEAR FAULTED CASES AFTER APPLICATION OF THE PROPOSED DIAGNOSIS APPROACH.	104
FIGURE 5.15. RESIDUAL SIGNAL UNDER SUN GEAR FAULTY CASE.	107
FIGURE 5.16. THE PRE-DEFINED WINDOW FUNCTION FOR THE PLANETARY GEARBOX ON THE TEST RIG AS A SUM OF SINUSOIDAL SIGNALS THAT ARE PERIODIC AT CARRIER ROTATION HARMONICS (A) DISCRETE MAGNITUDE SPECTRUM OF W_j AND (B) DISCRETE PHASE SPECTRUM OF ψ_j	108

FIGURE 5.17. MEASURED VIBRATION SPECTRUM AROUND GEAR MESH FREQUENCY.
.....109

FIGURE 5.18. THE RESIDUAL SIGNAL BETWEEN THE MEASURED VIBRATION
SIGNAL AND REFERENCE SIGNAL (A) AFTER THE PROPOSED DIAGNOSIS
APPROACH PROCESSING (B) ONLY AFTER INITIAL PHASE MATCHING.110

List of Symbols

Chapter 2

Pd	Pitch diameter of a rolling bearing
AE	Acoustic emission
x_p	The pinion translational displacement in x direction
y_p	The pinion translational displacement in y direction
θ_p	The pinion angular displacement
x_g	The gear translational displacement in x direction
y_g	The gear translational displacement in y direction
θ_g	The gear angular displacement
t	Time
Q	The degrees of freedom vector
δ	The deflection along the line of action
α	Pressure angle
r_p	Base radius of the pinion
r_g	Base radius of the gear
F_m	The dynamic gear mesh force
c	Gear mesh damping
k	Gear mesh stiffness
m	Mass
I	Mass moment of inertia
T_{in}	Input torque
T_{out}	Load torque
M	Mass matrix

C	Damping matrix
K	Stiffness matrix
F	Force matrix
f_m	Gear mesh frequency
f_s	Shaft rotational frequency
n	Integer
m	Integer
M	Maximum number of the m th gear mesh harmonic
X_m	Amplitude coefficient of the m th gear mesh harmonic
θ_m	Phase coefficient of the m th gear mesh harmonic
N_g	Number of teeth of the gear
$x(t)$	Any vibration signal
$a_m(t)$	Amplitude modulation function of the m th gear mesh harmonic
$b_m(t)$	Phase modulation function of the m th gear mesh harmonic
A_{mn}	Amplitude coefficient of the n th harmonic of $a_m(t)$
B_{mn}	Amplitude coefficient of the n th harmonic of $b_m(t)$
N	Maximum number of the n th harmonic of $a_m(t)$ and $b_m(t)$
α_{mn}	Phase coefficient of the n th harmonic of $a_m(t)$
β_{mn}	Phase coefficient of the n th harmonic of $b_m(t)$
$y(t)$	Any vibration signal
$z(t)$	Any vibration signal
TSA	Time synchronous average

Δt	Time interval
sf	Sampling frequency
R	Number of Records
L	Number of data per sample
k	Integer
r	The r th Records
$\mathbf{D}$	Cumulative distance matrix
i	Integer
j	Integer
c_{ij}	Euclidean distance
x_i	The i th element of the signal x
y_j	The j th element of the signal y
N_x	The length of the signal x
N_y	The length of the signal y
$w(t)$	Window function
t	Time instant
j	Imaginary part
f	Frequency
STFT_x	Short-time Fourier transform of the signal x
WT_x	Wavelet transform of the signal x
a	Scale parameter of the wavelet transform
$\psi(t)$	Mother wavelet
WVD_x	Wigner-Ville transform of the signal x
IMF	Intrinsic model function of Hilbert-Huang transform
$r(t)$	Residual signal of Hilbert-Huang transform

$h_{ij}(t)$	A signal used in the j th iteration to find the i th IMF
$m_{ij}(t)$	Envelop signal of the j th iteration to find the i th IMF
$c_i(t)$	The i th IMF
N_a	The number of teeth of the annulus gear
N_s	The number of teeth of the sun gear
N_p	The number of teeth of the planet pinion
λ	The least mesh angle
ψ_i	The i th planet located angle
l	Integer
q	Integer
ω_m	Gear mesh frequency
ω_p	Rotational frequency of the planet pinion
ω_s	Rotational frequency of the sun gear
ω_c	Rotational frequency of the carrier

Chapter 3

x_s	Translational displacements of the sun gear in x coordinate
y_s	Translational displacements of the sun gear in y coordinate
θ_s	Rotational angle of the sun gear
x_c	Translational displacements of the carrier in x coordinate
y_c	Translational displacements of the carrier in y coordinate

θ_c	Rotational angle of the carrier
φ_i	Initial located angle of planet i
ψ_i	The planet located angle of planet i
R_c	Radius of the carrier
x_i	Absolute translational displacements of the i th planet with respect to the origin in x coordinate
y_i	Absolute translational displacements of the i th planet with respect to the origin in y coordinate
x_{pi}	Relative translational displacements of the i th planet with respect to the pin-hole on the carrier in x coordinate
y_{pi}	Relative translational displacements of the i th planet with respect to the pin-hole on the carrier in y coordinate
θ_i	Rotational angle of the planet i
R_s	Base radius of the sun gear
R_p	Base radius of the planet pinion
δ_{spi}	Deflection between the sun gear and the planet i
δ_{api}	Deflection between the annulus gear and the planet i
T	Kinetic energy
V	Potential energy
m_s	Mass of the sun gear
I_s	Mass moment of inertia of the sun gear
m_c	Mass of the carrier
I_c	Mass moment of inertia of the carrier

m_p	Mass of the planet pinion
I_p	Mass moment of inertia of the planet pinion
k_s	Bearing supporting stiffness of the sun gear
k_c	Bearing supporting stiffness of the carrier
k_{pi}	Bearing supporting stiffness of the planet i
k_{spi}	Mesh stiffness between the sun gear and the planet i
k_{api}	Mesh stiffness between the annulus gear and the planet i
t	Time
$\mathbf{Q}(t)$	Degrees of freedom vector
P	Number of the planet pinions
$\mathbf{M}$	Mass matrix
$\mathbf{C}$	Damping matrix based on the Rayleigh damping ratio
$\mathbf{C}_b$	Bearing supporting damping
$\mathbf{K}_b$	Bearing supporting stiffness
$\mathbf{K}(t)$	Time-varying gear mesh stiffness
$\mathbf{F}(t)$	Externally applied torques vector
γ_{si}	Relative phase between the i th sun-planet mesh and the first sun-planet mesh
γ_{ai}	Relative phase between the i th annulus-planet mesh and the first annulus-planet mesh
γ_{as}^i	Relative phase between the i th annulus-planet mesh and sun-planet mesh
ω_s	Rotational frequency of the sun gear

ω_c	Rotational frequency of the carrier
ω_p	Rotational frequency of the planet pinion
ω_m	Gear mesh frequency
$k_{api}(t)$	Mesh stiffness between the annulus gear and the planet i
l	Integer
a_l	Amplitude coefficient of the l th harmonic
b_l	Amplitude coefficient of the l th harmonic
N_s	Number of teeth of the sun gear
N_p	Number of teeth of the planet pinion
N_a	Number of teeth of the annulus gear
G	Gear ratio of a gear-set
T_c	Rotational period of the carrier
$w(t)$	Window function
$u(t)$	Step function
$U_i(t)$	Function in terms of $u(t)$ defined in Eq. (3.17)

Chapter 4

MF	Gear mesh frequency
ψ_i	The initial angular location of planet i
P	Number of planets
ω_s	Rotational frequency of the sun gear
ω_c	Rotational frequency of the carrier
ω_p	Rotational frequency of the planet pinion
ω_m	Gear mesh frequency

N_s	Number of teeth of the sun gear
N_p	Number of teeth of the planet pinion
N_a	Number of teeth of the annulus gear
θ_j	Initial phase angle of the j^{th} harmonic component
X_{ij}	Fourier coefficient of j^{th} harmonic of the vibration contribution from the annulus and i^{th} planet mesh
t	Time
$x(t)$	Vibration response from planets meshing with the annulus gear
j	Integer
j	Imaginary part
$y(t)$	Vibration response from planets meshing with the sun gear
δ_j	Initial phase angle of the j^{th} harmonic component
Y_{ij}	Fourier coefficient of j^{th} harmonic of the vibration contribution from the sun and i^{th} planet mesh
$z(t)$	Vibration response perceived by the sensor
$w(t)$	Window function
W_k	Fourier series coefficients of the k th harmonic
X_m	Fourier series coefficients of the m th harmonic
k	Integer
m	Integer
q	Integer
$v(t)$	Residual signal of a vibration response
$a_m(t)$	Amplitude modulation function of the m th harmonic

$b_m(t)$	Phase modulation function of the m th harmonic
ω_{af}	Characteristic fault frequency of the annulus gear
ω_{sf}	Characteristic fault frequency of the sun gear
ω_{pf}	Characteristic fault frequency of the planet pinion

Chapter 5

Fast DTW	Fast dynamic time warping
CK	Correlated kurtosis
X	Any time series
Y	Any time series
N	Length of a time series
M	Length of a time series
W	A warp path
K	Length of the warp path
i	Index
j	Index
k	Index
D	Cost matrix
t	Time
$x(t)$	Any time signal
$y(t)$	Any time signal
β	Phase
f	Frequency
dt	Time step
T	Period of interest

CK_M	Correlated kurtosis of M -shift
δ	Impulse signal
f_m	Gear mesh frequency
θ	Initial phase
A	Amplitude coefficient
$w(t)$	Window function
φ	Initial phase
W_j	Fourier series coefficient of j th harmonic
ψ_j	Phase of j th harmonic
f_c	Rotational frequency of the carrier
RZ	Raw residual signal
Z	Residual signal after resample
f_{sh}	Rotational frequency of the shaft
N_p	Number of pinion teeth
N_g	Number of gear teeth
f_{ns}	Nominal shaft rotational frequency
$a(t)$	Amplitude modulation function
$b(t)$	Phase modulation function
$n(t)$	Noise function
SNR	Signal to noise ratio
M	Mass matrix
C	Damping matrix based on the Rayleigh damping ratio
K(t)	Time-varying gear mesh stiffness
C_b	Bearing supporting damping

$\mathbf{K}_b$	Bearing supporting stiffness
$\mathbf{F}(t)$	Externally applied torques vector
$\mathbf{Q}(t)$	Degrees of freedom vector
$K_{\max}$	Maximum values of gear meshing stiffness
$K_{\min}$	Minimum values of gear meshing stiffness
ΔK_{loss}	Stiffness loss due to the gear tooth failure
N_s	Number of the sun gear teeth
N_a	Number of the annulus gear teeth
N_p	Number of the planet pinion teeth
f_s	Rotational frequency of the sun gear
f_p	Rotational frequency of the planet pinion
RMS	Root mean square
G	Gear ratio

Chapter 1 Introduction

1.1 Motivation

Planetary gear-sets are widely employed in powertrains of energy, aerospace and maritime engineering. Their applications include bucket wheel excavator used in lignite mines, heavy-duty wind turbine gearbox and transmission gearbox of helicopters (Fig. 1.1). Generally, planetary gear-sets employ P number of identical planet pinions, which allow the net load to be shared through multiple gear meshes between the planets and the mating sun, as well as, annulus gear. This enables a larger torque-to-weight ratio, quieter operation and a more robust system than normal fixed-axis/parallel-axis gear pairs. However, any failures with these heavy-duty planetary gearboxes usually result in huge loss of capital, expensive repair and overhaul and large unscheduled downtimes. For example, wind turbine planetary gearboxes have been referred to as “Achilles’ heel” of wind turbines [1] because of their high probability of failure in nearly all currently operational wind turbine models. Such failures in

Figure 1.1. Examples of planetary gearbox applications (a) Wind turbine planetary gearbox and (b) helicopter planetary transmission gearbox.

Figure 1.2. Catastrophe caused by the gear failure in a wind turbine.

gearboxes can further be accompanied by a catastrophic damage of the entire wind turbine system, as shown in Figure 1.2. Similarly, fatigue cracks were found in the planet carriers of two US Army UH-60A Black Hawk helicopter main transmissions had the potential to cause severe flight safety incidents [2]. Therefore, a practical and robust planetary gearbox monitoring system is critically needed to be able to provide the warning of eminent damages and help in scheduling the maintenance to avoid system failures.

In general, such monitoring systems are called as “condition health monitoring systems”. These condition monitoring equipment consist of the diagnostic and prognostic sub-system for varied engineering machines, where key parameters of the machines are monitored to provide measurements. Any significant change in one or more of these measurements may be indicative of a developing fault within the system. The use of condition monitoring can assist in scheduling maintenance to minimize the unscheduled downtime for machines and taking other preemptive actions to prevent accidents, while improving the reliability of the system by assuring its operation within safe limits. Currently, the industrial practice to carry out gearbox monitoring does not consider the

knowledge of the fault source. However, as a planetary gearbox contains several gears, identifying a faulty gear by a condition monitoring method prior to disassembling the gearbox can help reduce the time to physically find the faulty gear. More importantly, with such information a replacement gear could be ordered beforehand, which will reduce the downtimes caused by dismantling the gearbox and leaving it idle while waiting for the replacement part.

1.2 Problem statement

Vibration, acoustic signal and oil debris monitoring are the three most common fault diagnostic techniques of condition monitoring systems among them. Vibration based diagnosis is gradually becoming the most popular monitoring technique because of ease of measurement which allows for no interference with the normal operation of machines [3]. A detailed vibration spectral analysis can yield accurate results regarding specific condition of the system such as fault source, remaining life for the system, fault severity and etc. However, for planetary gear systems, such detailed vibration analysis is a challenge due to the complexities in the measured vibration signal, which is caused by the multiple simultaneous meshing of different planet pinions and the revolution of these planet pinions with respect to the sun gear [4, 5]. For a healthy fixed-axis gear, the vibration spectrum only consists of significant frequency components located at the gear meshing frequency (MF) and its harmonics. Any gear fault or manufacturing error introduces sidebands in the vibration spectrum around MF harmonics. However, in planetary gear-sets, dominant sidebands around MF harmonics can be observed even when no fault or no severe manufacturing error exists. Moreover, some researchers have reported that the dominant frequency components may not even occur at the

gear meshing harmonics for specific gear-set geometries [6, 7]. These have been recognized to be caused by the varying phase angles and motions of the planet pinions with respect to the mounted sensor.

The success of fault diagnosis and prognosis in the planetary gearboxes using vibration measurement has been demonstrated in [2, 5, 8-10]. While Ref. [8] shows classical statistical analysis of vibration signals is still sensitive for the detection of localized gear faults in the planetary gearbox, it is unable to identify the localized fault to isolate the fault location in gearbox. Although Ref. [5] proposes a technology to separate out the data for each planet gear by using window functions, this technology is difficult to implement in practice as it requires multiple sensors and the knowledge of the time instant when the planets pass through the sensor location. Model based monitoring methods for planetary gearboxes are also discussed in [2, 9, 10]. However, these methodologies limit themselves to the carrier plate crack fault as no explicit model of the planetary gear-sets was further explored to identify the locations of the additional frequency components introduced by the damaged tooth. So far no proper mathematical or analytical analysis has been provided to explain why and where additional sidebands appear at certain frequency locations when the local gear fault or manufacturing error arises in planetary gear-sets. Moreover, in practice, gearboxes often operate under some fluctuation around nominal load/speed conditions during their normal service. Load fluctuations could introduce additional modulation sidebands that make the vibration spectrum more complex to analyze. Speed fluctuations result in a variation of both the modulations and their carrier frequencies (gear mesh harmonics) that blurs the sideband components in the spectra of the vibration measurement,

often making it difficult to be recognized. Such smearing effect can be abated by the time synchronous average/order tracking technique that acquires the measurements at identical angle increment from the rotary encoder [11]. Unfortunately, such equipment are usually absent in many industrial applications. Therefore, besides a better understanding of the vibration spectrum obtained from planetary gearbox under healthy and faulty condition, more intelligent methods for extracting fault information for industrial drive systems are required as well.

1.3 Objectives

The objective of the research is to develop an intelligent fault diagnostic algorithm, which integrates models, simulation, and experimental data, to diagnose gear failure in the planetary gear system with an application to the wind turbine gearbox. The presented architecture can be also applied to a variety of practical engineering systems containing planetary gearboxes. Thus, this research works on: (1) investigation of the sidebands characteristics of the vibration spectrum for planetary gearboxes measured by the stationary sensor mounted on gearbox housing under healthy and faulty conditions; (2) study of the fault detection method from measured vibration spectrum from realistic power transmission operating under speed/load fluctuation condition; (3) Experimental validation from planetary gearbox test rigs.

1.4 Key contributions of the thesis

The original contributions of this thesis include:

- 1) A mathematical AM-FM (amplitude modulated/frequency modulated) signal model is developed to simulate the distinct vibration spectrum of planetary gearboxes operating under healthy and faulty conditions;
- 2) Fourier series analysis based on this mathematical signal model explains the measured vibration pattern to identify additional frequency components that depend on the geometry of the planetary gear-set and the location of the gear faults [12];
- 3) A new time domain approach that combines dynamic time warping and correlated kurtosis is proposed to detect gear faults and identify their location for assisting in development of a robust fault alarm system [13].

1.5 Organization of the thesis

The outline of the thesis is illustrated in the Fig. 1.3 and organized as follows.

Figure 1.3. Outline of the thesis

This Chapter has described the motivation, the problem statement and the objectives to develop the condition monitoring system for planetary gearboxes. The key originality of this research is also highlighted in this Chapter.

Chapter 2 covers the historical aspects of gearbox diagnosis. First, the gearbox failures are classified, different sensors/measurements for gearbox condition monitoring reported in literature are compared and their advantages and the disadvantages of each measurement method are summarized. The vibration models of normal fixed-axis/parallel-axis gear pair are reviewed, based on which, the dynamic response of a single stage spur gear due to the gear tooth fault and its corresponding vibration signatures is discussed in details. Afterwards, the applications of the various time domain, frequency domain and time-frequency domain techniques in fault diagnosis are discussed. At the end of this chapter, the planetary gear system is introduced. Unique geometries and assemble conditions of planetary gear system are given for a deeper understanding of the planetary gear-set.

Chapter 3 proposes a two-dimensional time-varying lumped parameter model to study the dynamics of planetary gear sets operating under varying dynamic conditions. This discrete dynamic model considers the periodically time-varying gear mesh stiffness and non-linearity associated with tooth separations. Amplitude modulations and planets phasing relationship are also taken in account in the model, which is due to the rotation of the carrier with respect to the sensor at a fixed position on the planetary transmission housing. The simulations are compared with experimental results from the planetary gearbox test rigs that are also illustrated at the end of this Chapter.

Chapter 4 develops a mathematical vibration response model of the planetary gear-set to consider the effect of local faults that may occur at the annulus gear, any of planet gears, or the sun gear. Additional frequency components resulting from these local faults have been identified by the Fourier series analysis, which are proven to be located at frequencies that depend on the geometry of the planetary gear-set and the gear carrying the local fault. The formulation presented can assist in developing robust feature extraction algorithms for early detection of planetary gearbox failures. This formulation has been published in *Mechanism and Machine Theory*.

Chapter 5 presents a new time-domain diagnostic algorithm combining the fast dynamic time warping (Fast DTW) as well as the correlated kurtosis (CK) techniques to characterize the gear malfunctions by identifying the corresponding damaged gear and its position. The presented diagnosis approach is useful for developing automatic fault alarm system in both fixed axis as well as epicyclic gearboxes used in practical industrial drive systems. The possibility to detect mechanical gear faults through electrical signatures in electro-mechanical drive-trains system such as wind turbines is also explored in the middle of this chapter. The novel time-domain diagnosis algorithm has been published in *Journal of Sound and Vibration*.

Finally, Chapter 6 concludes the overall thesis and summarizes the key contribution. Possible future work in implementing the proposed model-based diagnosis techniques and prognosis of the planetary gear system are also proposed at the end of the chapter.

Chapter 2 Literature Review

Gearboxes, the most critical components of drive-trains, are widely applied in both mechanical and electro-mechanical hybrid engineering systems. The function of a gearbox is to transmit rotational motion from a driving prime mover to a driven machine, which allows machines to operate within their efficient speed conditions. Section 2.1 reviews the gearbox fault sources as well as their failure modes. Any unexpected failures in a gearbox result in long unscheduled down-time, significant capital losses and can even cause severe injury or catastrophe [1]. Therefore, it is important to diagnose gearbox faults in order to schedule predictive maintenance while these faults are still in their early stages. In the last three decades, extensive experimental and theoretical investigations are attracted to study the gearbox damage monitoring techniques. Figure 2.1 illustrates the scheme of a typical gearbox monitoring system. There are different types of measurements/approaches to the detection of faults in geared systems, such as acoustic signal analysis, debris monitoring and vibration analysis. They are briefly reviewed in the Section 2.2. The next three sections mainly reviews previous works in the fields of the vibration-based

Figure 2.1. Typical scheme for condition monitoring system for gearbox diagnosis.

diagnosis for gear defects, which are relevant to the research presented in this thesis. Firstly, the general structure of vibration-based diagnostic system is described in the Section 2.3. Then, Section 2.4 reviews the dynamic and analytical vibration models in order to study the vibration response in presence of gear defects. Finally, a brief review of some recent advances in signal analysis methods, which are used to extract the gear fault signatures/features of gear defects from vibration signal, is given in the Section 2.5. The last section, Section 2.6 covers the basic knowledge of planetary gear system as this research work carries out fault diagnosis of planetary gearboxes.

2.1 Failure modes in gearboxes

Gearbox diagnosis techniques, fault detection and isolation, can be classified based on various criteria, such as the fault sources, failure modes, measured signals and analysis algorithms. For the purpose of this overall review, gearbox defects are introduced according to the source of their failures in the first section and then the later sections provides a discussion on measured signal and their respective analyzed technique that can detect and isolate the gear failures. The different fault sources in gearboxes are as follows:

2.1.1 Gear failures

A gear is a rotatory machine part having cut teeth, which meshes with another toothed part in order to transmit torque from one shaft to the other at a constant angular velocity. The gear tooth profile currently in use are almost universally involute profiles [14]. During its operation, each gear tooth alternatively meshes and detaches with teeth of neighboring pinion. Hence, the load of these gear teeth are continuously changing, make them susceptible to breakage [15]. The typical modes of gear tooth failure include cracked tooth, spalled tooth and

Figure 2.2. Typical gear tooth defect (a) seeded cracked tooth on a spur gear [5] and (b) naturally worn teeth on an industrial helical gear [16].

worn tooth [5, 16-19] as shown in Fig. 2.2. Such gear defects in gearboxes are among the most serious failures in a gearbox [20, 21], which have potential to cause catastrophic failure of the whole system. Moreover, the gear manufacturing and assembling errors, such as tooth profile deviations, pitch errors and base circle eccentricities (also known as run-out), also reduce the gearbox performance [22]. Thus, the early detection of these gear faults is very crucial for the safe and efficient operation of the whole engineering system.

2.1.2 Bearing failures

Bearings allow the constrained relative motion between two or more parts in mechanical systems. The typical rolling element bearing is made up of outer race, inner-race, cage and rolling elements [23] as illustrated in Fig 2.3 (Pd represents the pitch diameter, i.e., the distance between the center of the two opposite rolling elements). The life of a rolling element bearing is determined by load, temperature, lubrication and installation. Although bearings are designed to work well in non-ideal conditions, issues such as poor lubrication, overload and over-speed still often account for them to fail quickly earlier than the designed life [24]. Bearing failures can be classified as inner-race defects, outer-race defects and rolling element defects, some examples are shown in Fig.

Figure 2.3. An illustration of the components in a typical rolling element bearing.

(a) Inner-race severe spalling defect [25] (b) Outer-race moderate spalling defect [25]

Figure 2.4. Examples of bearing failures.

2.4. Common bearing failure modes include flaking, spalling and pitting defects produced by friction or wear processes [25-28].

2.1.3 Shaft failures

The failure patterns arising in gearbox shafts are similar to those arising in any other rotary machine. Therefore, conventional shaft fault diagnosis techniques can also be applied to detect gearbox shaft defects. Misalignment, imbalance are the two most common shaft fault modes. Shaft misalignment is the

Figure 2.5. An illustration of the shaft misalignment.

condition in which two adjacent shafts are not adequately aligned to each other [29]. Misalignment of shafts can be either parallel, angular or the combination of the two misalignment. The parallel misalignment means that the centerlines of both shafts are parallel but they have certain offset errors. While the angular misalignment means the shafts are at certain angle error to each other. Shaft imbalance is a condition wherein the shaft's center of mass and rotational axis do not coincide. Shaft misalignment and imbalance can cause increased vibration and dynamic load at some components of the gearbox, resulting in quicker damage and reduced the lifetime of the whole drive-trains (Fig. 2.5) [30, 31].

2.1.4 Other sources of failures in gearboxes

To connect the driving and driven equipment to the gearbox, input and output shaft couplings are used. Shaft couplings connect the end of two shafts, and allow the smooth transfer of power between them. Improper assembly of coupling can also result in shaft misalignments. Coupling failure often happens

in the heavy-duty power transmission systems such as in energy or maritime applications. Normally, proper maintenance is sufficient to keep couplings to work smoothly and to enable them to reach their full service life. However, improper installation, poor coupling selection and operation beyond design capabilities can result in higher failure rated coupling [32, 33]. As gearboxes are usually used in industrial powertrains, they often work under severe environment conditions. Therefore, the debris such as dust, rubble and chips arising from their own wear may get trapped within gearboxes. This debris also results in a reduced lubrication quality, which shortens the life of gearboxes. Therefore, proper cleaning and change of lubrication is recommended for gearboxes especially when working under adverse conditions [34].

2.2 Measurements in gearbox monitoring

The signal analysis based algorithms for condition monitoring of gearbox usually depend on sensors employed in the data acquisition. Researchers have proposed several different monitoring methods for gearbox fault detection and identification [18-23]. These monitoring methods can be classified into vibration measurement, acoustic emission and noise measurements, electrical current measurement, wear debris analysis and temperature measurement. For a complete review, these monitoring methods are introduced briefly in this Section. Table 2.1 presents a summary of the monitoring methods that have been studied for condition monitoring of gearbox in the published literature. The vibration based diagnosis has been the most popular monitoring technique because of ease of measurement and it is applied to monitor the planetary gearbox in this research work.

Table 2.1: Summary of diagnosis techniques used for gearbox failures

	Vibration signal	Acoustic emission and noise signal	Electrical current signal	Wear Debris
Gear defects	[3], [5], [8-9], [12-15], [17- 18], [33-35], [57], [64], [76]	[48-50]	[53-55]	[56-57]
Bearing defects	[24-26], [41], [43]	[46], [51- 52]	[53]	N/A
Shaft misalignment and coupling failures	[29]	[49]	[30]	N/A

2.2.1 Vibration based monitoring

From Table 2.1, it can be observed that the vibration measurement is the most widely studied method for condition monitoring of rotary machineries. The typical frequency in this application ranges from 1 Hz to 10K Hz. A healthy gearbox exhibits certain level of vibration response because of the gear tooth stiffness, manufacturing errors, tooth backlash and so on [4, 35-38]. However, in a presence of gear defects, bearing defects or shaft misalignment, the vibration response of gearboxes become more severe [3, 39-41]. The shaft rotational frequency and its harmonics are located in the low frequencies region, which contains the fault signature of the shaft fault as shaft misalignments [42].

Figure 2.6. A view of a wireless accelerometer mounted on the gear housing.

The gear mesh frequency of gearboxes and its first five harmonics are usually captured for diagnosis of gear failure [3]. These gear mesh harmonics are from several hundred Hz to a few kHz (<5 kHz). It has been recognized that the signals generated by bearing faults are primarily to be found in the high frequency region of the natural frequencies of bearing rings and rolling elements, at such high frequencies in the tens of kHz [43]. Therefore, by measuring and analyzing the vibration signal of gearboxes, it is possible to determine both the fault source and the severity of the defect [44, 45]. Different sensors can be applied to measure the vibration signal of gearboxes, such as accelerometer, laser displacement sensor and proximity sensor [31]. Especially, accelerometers can be easily mounted on a gear housing to measure the vibration of the gearbox externally, which would not interfere with the normal operation of the gearbox (Fig 2.6). Thus, the vibration-based diagnosis has been a preferred monitoring technique [3] because of its ease of measurement.

2.2.2 Acoustic emission based monitoring

Figure 2.7. Acoustic emission sensor located close to the monitoring components [46].

Acoustic emission (AE) signals are defined as transient elastic waves generated from a rapid release of strain energy caused by a deformation or damage on the surface of a material [46]. The application of AE to non-destructive testing of material typically takes place between 100 kHz and 1 MHz range of frequency [47]. Sources of emitting AE signal in gearbox include cracks, deformations, frictions and so on. For instance, the interaction of surface asperities and impingement of the bearing rollers over a defect on an outer or inner races will result in the generation of acoustic emission [48]. AE sensors are also capable for detecting these transient elastic waves generated by gear defects [49, 50]. Moreover, machine deterioration and malfunction can be detected using AE at earlier stages compared to vibration [51]. However, the main drawback with the application of the AE technique is the attenuation of the signal and as such the AE sensor has to be close to its source as illustrated in Fig. 2.7. However, such locations are often inaccessible in practical operations of the gearbox and the sensor can only be placed on the gearbox housing. Therefore, the AE signal originating from the defective component will suffer

Figure 2.8. An illustration of the non-intrusive current sensing system.

severe attenuation before reaching the sensor [52]. Because of this disadvantage, the application of AE is not as popular as vibration method.

2.2.3 Electrical current signal analysis

Electro-mechanical systems such as geared transmissions driven by induction machines are widely used in a number of industrial applications because of their power-to-weight ratio, reduced cost and high reliability. Consequently, electrical current signature analysis has been the most recent addition as a non-intrusive and easy to measure condition monitoring technique (Fig 2.8) [53]. Electrical current signal analysis is an efficient and new method to detect the fluctuation of the gear load caused by the gearbox failure because the faulted components introduce sidebands across line frequency. Therefore, it is straightforward to use electrical current signature to diagnose gearbox faults [54]. A few number of research works [30, 55] have been published to investigate the capability and efficiency of current signature analysis in condition monitoring of gear defects.

2.2.4 Wear debris analysis

Figure 2.9. Manual sampling from the location of oil sampling port [52].

Wear debris analysis is based on oil investigation for assessing machine condition. There are six main types of wear particles generated from common machinery, corresponding to rubbing, spherical, laminar, cutting, fatigue chunk, or severe sliding particles. Each of these wear particles have their own distinctive characteristics [56]. Therefore, a representative sample of the lubrication fluid of gearboxes can provide information on the wear modes, wear sources, and wear phases present in the gearbox. Researchers [57] have proven its efficiency in condition monitoring of gearboxes. Although wear debris analysis is a powerful health monitoring method, it is time consuming, with manual sampling, requiring expensive laboratory set ups, and human expert inspection of the wear debris samples (Fig. 2.9). These disadvantages hinder wear debris analysis, and therefore it has not been popular in industry.

2.2.5 Temperature based monitoring

Figure 2.10. A thermometer to monitor the condition of the machine.

If severe defects exist in the gearbox, the temperature of lubrication oil will be significantly different from the normal case during the operation [58]. Temperature monitoring needs a thermometer (Fig. 2.10) to monitor the temperature change in the gearbox. Although it is simple and robust to detect advanced faults in gearboxes, it is insensitive to early stage defects and usually cannot identify the source of the failures in machines.

2.3 General structure of the vibration based diagnostic system

This thesis concentrates in the vibration monitoring of gear failures. Vibration based diagnostic system allows the track of the development of the gear degradation, which in turn allows for implementing preventive activities in order to avoid unpredictable downtimes and serious failures. Figure 2.11 summarizes the data flow of a typical vibration based diagnostic system. There are internal and external sources of vibration excitations for the gear set. The main source of the internal one is induced by the time varying mesh stiffness [59]. Moreover, the degradation of a gear as a fault progresses results in a degradation of the gear mesh stiffness over the life of a gearbox [60, 61]. Thus, the common local gear faults are modelled in dynamic simulation study by assuming a local drop a general squared wave form gear meshing stiffness function, which is reviewed in the Section 2.4. The external sources of

Figure 2.11. Data flow in the vibration based diagnostic system.

excitation are mainly induced by the fluctuation of the input velocity and torque, which complicates the measured signal. Therefore, analysis of machine vibration signals is very important to extract the fault signature/features in the complex measured signal for condition monitoring of the gearboxes. Some typical signal analysis for the vibration based diagnostic system is introduced in the section 2.5. These extracted fault signatures/features contain the necessary discriminative information for the fault classifier to have the chance of classification. The support vector machine, artificial neural networks, genetic algorithm has been widely used as the classification techniques of gear diagnosis and prognosis [10, 62, 63]. These data-driven approaches have the potential to discriminate the failure modes, assess the damage level and predict the remaining lifetime based on the extracted fault signatures/features. However, due to the need of large sets of the training data to implement the classifiers, they will be studied in the future. In the current stage, this research work mainly contributes to the extraction of the fault signatures/features in planetary gear systems.

2.4 Dynamic and analytical vibration modeling for fault diagnosis of gearboxes

As the main part of the power transmission system, the gearboxes are employed in many engineering machines. Fault diagnosis system can guarantee that this critical component maintained in good condition. When the gears are defective, they will introduce impacts to the normal meshing vibration. Thus, it is crucial to study the influence of gear failure on the dynamic behavior of a geared system to understand the nature of the fault signature in the measured vibration signals [64].

Figure 2.12. The lumped parameter model of a single stage gear pair.

2.4.1 Lumped-parameter dynamic model of a single stage gear-set

A single stage spur gear transmission model with six degrees of freedom is proposed in [59] (Fig. 2.12). The driving pinion p and driven gear g are assumed as rigid bodies connected to each other along the line of action through the corresponding gear mesh stiffness $k(t)$ and viscous damping c [65]. These gears are held by bearings, which allow them to translate in x and y directions and freely rotate about their centers in the x - y transverse plane of gear. Thus, the motion of each gear can be defined using translational coordinate x and y , and the angular coordinate θ . $\mathbf{Q}(t)$ is the degrees of freedom vector that contains two coordinates for translational vibration and a coordinate for torsional motion for each gear in the plane as:

$$\mathbf{Q}(t) = [x_p \ y_p \ \theta_p \ x_g \ y_g \ \theta_g]^T$$

where x_p is defined as the pinion translational displacement in x direction; y_p is the pinion translational displacement in y direction; θ_p is the pinion angular displacement; x_g is the gear translational displacement in x direction; y_g is the

gear translational displacement in y direction; θ_g is the gear angular displacement.

The deflection along the line of action is expressed as:

$$\delta(t) = (x_p - x_g) \sin \alpha + (y_p - y_g) \cos \alpha + \theta_p r_p - \theta_g r_g \quad (2.1)$$

where, r_p and r_g are the radius of the base circle of the pinion and gear, respectively. α is the pressure angle of the gear pair.

Further, defining the dynamic gear mesh forces of the gear pair $F_m(t)$ as:

$$F_m(t) = c \dot{\delta}(t) + k(t) \delta(t) \quad (2.2)$$

In which, c is the damping ratio and $k(t)$ represents the time varying gear mesh stiffness.

The resulting equations of motion describing the transverse and torsional model are then:

$$\left. \begin{aligned} m_p \ddot{x}_p + F_m(t) \cos \alpha &= 0 \\ m_p \ddot{y}_p + F_m(t) \sin \alpha &= 0 \\ I_p \ddot{\theta}_p + F_m(t) r_p &= T_{in} \\ m_g \ddot{x}_g - F_m(t) \cos \alpha &= 0 \\ m_g \ddot{y}_g - F_m(t) \sin \alpha &= 0 \\ I_g \ddot{\theta}_g - F_m(t) r_g &= T_{out} \end{aligned} \right\} \quad (2.3)$$

where, m and I are the mass and mass moment of inertia of the pinion p and gear g denoted by the subscript p and g respectively. The external torque T_{in} and T_{out} represent the driving torque and load torque. Assembling all these equations (2.1-2.3) results in a global equation of motion for the fixed axis gearbox that can be expressed in matrix form as:

$$\mathbf{M}\ddot{\mathbf{Q}}(t) + \mathbf{C}\dot{\mathbf{Q}}(t) + \mathbf{K}(t)\mathbf{Q}(t) = \mathbf{F}(t) \quad (2.4)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}(t)$ is the time-varying gear meshing stiffness matrix, $\mathbf{F}(t)$ is the externally applied torques vector. The mesh stiffness between an engaged geared pair consists of two parts: one associated with the local Hertzian deformation and the other associated with the tooth bending deflection. Finite element analysis and empirical equations have been approximated for the calculation of mesh stiffness [66, 67]. The detailed variation in the gear mesh stiffness due to the gear tooth faults is studied in [16] and [17], wherein it has been concluded that such faults are always accompanied by a stiffness reduction. The failure characteristics of impulse response and the sideband frequency in fixed-axis gear-set have been shown to be similar (i.e., the fault signature appeared as specific frequency locations) except their magnitude. The square waveform was shown to be a good

Figure 2.13. Gear tooth mesh stiffness under healthy and faulty case.

Figure 2.14. Simulated vibration spectrum of a spur gear-pair.

approximation for meshing stiffness function under healthy operating conditions in spur gear pairs [59]. Thus, it is a reasonable assumption that a gear defect can be simulated by assuming a local drop in a squared waves form meshing stiffness function as illustrated in Fig. 2.13 [68].

The vibration dynamic response of the single stage gear-set is solved by numerical integration of Eq. 2.4 implemented in MATLAB. The healthy case (Fig. 2.14) is characterized by the dominance of the gear mesh frequency f_m and its harmonics. Figure 2.15 represents the simulated vibration spectrum of the pinion in the case of a broken pinion tooth. As a consequence, new frequency components appear at $nf_m \pm mf_s$ (n, m are integers) as sidebands, where f_s represents the rotational frequency of the defected pinion.

The validation of these simulation results can be found in many published research literature and experimental work [3, 35, 39], where the authors observed that the gear mesh harmonics usually dominates the energy of the measured gear vibration signal. The fault information is generally and mostly contained in the modulated signal. Thus, when the vibration signals of gearbox are used in the early fault detection, it is common to analyze the modulation

sidebands in the vibration signals get commonly used. Further, by analyzing the sidebands location, the various characteristic frequencies can be used to determine gear fault sources [44].

2.4.2 Mathematical amplitude-frequency modulated vibration model of gear-set

As lumped parameter models require appropriate choices of equivalent masses connected by appropriate stiffness and damping elements. A detailed system parameter identification of planetary gear-sets, such as gear mass moments of inertia, the gear mesh stiffness, and the damping and stiffness of the bearings, is required for accurate results from this approach. The end-users and maintenance engineers of gearbox often lack such system parameter information and its estimation requires a high level of skill and effort. Therefore, researchers have also developed the mathematical amplitude-frequency modulated vibration model of gear-sets as the analytical model to describe the vibration patterns of gearboxes for ease of the application and understanding [35]. In general, vibration signals for healthy gears are dominated by gear meshing vibration. Considering a pair of fixed-axis gears that mesh under a constant load and speed, and assuming that initially all teeth on the gears are

Figure 2.15. Sidebands around gear mesh harmonics in a presence of pinion tooth defect.

identical and equally spaced. Since the contact stiffness varies periodically with the number of teeth in contact, and as the contacting point on tooth surface varies, the resulting vibration will be excited at the meshing frequency. If one of the gear has N teeth, with its shaft carrying this gear has rotation frequency f_s , the meshing vibration $x(t)$ can be expressed as [3]:

$$x(t) = \sum_{m=1}^M X_m \cos(2\pi m f_m t + \theta_m) \quad (2.5)$$

where m ($0, 1, \dots, M$) is the mesh harmonic (MF) number, X_m , θ_m are the amplitude and initial phase at the m -th harmonic frequency f_m (i.e., $f_m = m \times N_g \times f_s$, where N_g represents the tooth number of the gear and f_s is the shaft rotation frequency of the respective gear).

Now considering that this gear has a local defect such as a fatigue crack which affects the stiffness of the neighboring teeth such that it changes the vibration behavior as the affected teeth mesh with the other gear. Such changes in vibration behavior can be defined by the amplitude and phase modulation functions, $a(t)$ and $b(t)$, respectively. Moreover, manufacturing errors such as pitch errors, base circle eccentricities (also known as run-out) and misalignments can also similarly modulate the vibration signal and introduce non-zero frequency components at gear rotational frequency around the mesh frequencies [22, 36]. As the modulation is periodic with the gear shaft rotation frequency f_s , these functions can be represented as a Fourier series as [69]:

$$a_m(t) = \sum_{n=0}^N A_{mn} \cos(2\pi n f_s t + \alpha_{mn}). \quad (2.6)$$

$$b_m(t) = \sum_{n=0}^N B_{mn} \cos(2\pi n f_s t + \beta_{mn}). \quad (2.7)$$

Without loss of generality, the initial phase θ , α and β can be set to 0 for simplicity. Therefore, the modulated gear meshing vibration $y(t)$ due to the local gear fault is

$$y(t) = \sum_{m=0}^M X_m (1 + a_m(t)) \cos(2\pi m N_g f_s t + b_m(t)). \quad (2.8)$$

While, different type of local defect and their severity may affect the amplitude and phase of the vibration response in varied manner, however, it does not change the frequency locations in the response since the periodicity is conserved.

After filtering out the MF and its harmonics, the residual signal can be written as [7]:

$$z(t) = \sum_{m=0}^M X_m a_m(t) \cos(2\pi m N_g f_s t + b_m(t)). \quad (2.9)$$

2.5 Recent advances in signal analysis methods for gearbox fault diagnosis

The intrinsic dynamic nature of gearbox is reviewed in the last section. Such vibration signals measured from these systems contain rich information about machinery health conditions. Therefore, important fault features, such as those presented in the sidebands components, can be extracted from these signals for fault detection and diagnosis, if proper analysis methods are applied. An overview of the related studies is given in this section. This review is structured as follows. The time-domain methods are first revisited in Section 2.5.1. Some

of these techniques, correlated kurtosis and dynamic time warping, are further explored in the Chapter 5 to analyze the practical vibration signals with the speed fluctuations around the nominal operating conditions. Then in Section 2.5.2, the frequency-domain techniques are briefly reviewed. In planetary gear-sets, the vibration spectrum is quite complex. Dominant sidebands around gear mesh harmonics can be observed even when no fault or no severe manufacturing error exists. Therefore, the details of the vibration spectrum of the planetary gear system is studied in the Chapter 4, which is useful in diagnosing the source of faults and developing intelligent techniques for condition health monitoring of the planetary gear systems. The time-frequency analysis techniques are powerful tool to analyze the non-stationary signal. However, these methods usually require expert knowledge to implement and analyze their result. Therefore, they haven't been well accepted by the maintenance engineers. For a complete review, typical time-frequency algorithms are introduced in Section 2.5.3.

2.5.1 Time-domain signal analysis

Statistical fault diagnosis

These methods can provide analyses in terms of the statistical average in the time domain [31]. The classical statistical indicators are sensitive to the gear faults by comparing the statistical value in the healthy and faulty case, such as root mean square (RMS), crest factor and kurtosis [8]. This kind of statistical analysis is usually unable to provide more detailed diagnostic information like the identification of the localized fault to isolate the fault location in gearbox components. Moreover, correlated kurtosis (CK) has been introduced in the vibration based diagnostic system recently [70] as CK takes the advantages of

the periodicity of the gear fault impact. The detailed evaluation of the correlated kurtosis is given in the Chapter 5 and it is further combined with the dynamic time warping method to diagnose the planetary gearbox. However, these signal analysis methods is usually based on the stationary/quasi-stationary assumption.

Time synchronous average

Time synchronous average (TSA) is usually applied as a pre-processing technique to remove noise from gearbox vibration signal. It can remove not only the background noise but also periodic events that are not exactly synchronous with the shaft period while the gear is being monitored [3]. In general, TSA is a signal averaging process over a large number of cycles, synchronous with the running speed of a specific shaft in the gearbox measured by the rotary encoder. It resamples the measured noisy signal from the time-domain into the angular domain. When the measured vibration signal is synchronously averaged with the rotating of the gear of interest, the resulting signal is an estimate of the average meshing vibration of that particular gear over a complete revolution. For a sampling frequency sf , the time interval between two samples is $\Delta t = 1 / sf$. If the obtained signal $x(t)$ consists of R records (revolutions), and each record has L data samples, then the TSA signal

Figure 2.16. Frequency response of time synchronous average as a special case of comb filters (a) comb like magnitude response and (b) linear phase response.

can be obtained by [71]:

$$y(t) = \frac{1}{R} \sum_{r=0}^{R-1} y(t + rL\Delta t) \quad (2.10)$$

where $t = k\Delta t$, for $k = 0$ to $(L-1)$. In addition, if the number of samples per record is not exactly equal from one record to another as the motor speed is more or less not the same for every rotation; an interpolation process should be taken in each record to ensure each record has L data samples. TSA can also be understood as a linear comb filter illustrated by Figure 2.16. Advanced gear tooth damage can often be identified readily by the direct inspection of the TSA trace. Figure 2.17 represents the spectrum of a raw measured vibration data from a gearbox and the data with higher signal to noise ratio after implementing the time synchronous average trace. An extension to the TSA algorithm, order

Figure 2.17. The vibration spectrum before (a) and after (b) the time synchronous average processing.

tracking method, was presented in [11]. The order tracking method to resample data with respect to the angular position requires the rotary encoder to remove the operational speed fluctuations.

Dynamic time warping

Dynamic time warping (DTW) technique finds the optimal alignment between two time series by allowing one of the time series to be “warped” non-linearly by stretching or shrinking along its time axis. Thus, DTW can be used to determine the similarity between the two time series. Fig. 2.18 illustrates the alignment of two time series processed by DTW. DTW algorithm has been shown to be effective in recognition, data mining and signal processing [72], recently it was successfully used to detect and quantify common faults in a reciprocating compressor through current signals of the driving motor [73]. DTW can be used to suppress the supply frequency component and highlight the sideband components and shown promising results where the aligned signals by DTW do not lose information and is suitable for fault diagnosis. The

Figure 2.18. Dynamic Time Warping of time series (a) before (b) after processing.

optimal warping path is selected based on the cumulated matrix. Once the cumulated distance matrix $\mathbf{D}$ is built, the alignment warping path can be found by a simple back tracking which runs through the low-cost areas “valleys” on the cumulative distance matrix $\mathbf{D}$. However, the quadratic $O(N^2)$ time and space complexity of this algorithm limits its usefulness only to short time series containing at most a few thousand data points.

2.5.2 Spectral analysis

Spectral analysis is the most popular and common diagnosis technique in the field of machine monitoring. Most industrial condition monitoring systems apply Fourier transform based method to detect the gearbox failure [74]. Due to the rotating nature of the gearbox, the signature of localized faults of the gear teeth (e.g., crack, spalling and breakage) and bearing components (e.g., outer-race, inner-race, and rolling elements) generally manifests as periodic transient impulses. The corresponding fault signature is a modulation phenomenon of gear mesh frequency (or structure resonance frequency) being modulated by a certain fault characteristic frequency [75]. This amplitude modulated–frequency modulated (AM–FM) vibration signal is composed of gear mesh harmonics and sideband components, and thus the gearbox fault may be identified via detecting the presence of the sidebands. However, the existence of background noise corrupts transient impulses and masks the sidebands in practice, and thus increases the difficulty to identify specific faults. Therefore, the noise suppression and fault signature enhancement techniques are crucial for the success of implementing spectral analysis. For example, time synchronous averaging is usually used as a pre-processing technique to extract the periodic sideband components by suppressing the asynchronous noises in a vibration

signal. Afterwards, the user-defined band-pass filtering and envelop detection methods are used to remove irrelevant frequency components and then extract the demodulating signal. The resulting amplitude and phase modulated signals are further analyzed as “envelop spectra” to determine whether the faults originate from bearings or gear teeth. Moreover, these extracted sidebands can be used to determine the severity of the fault and estimate the remaining useful life of the gearbox. However, no consistent guidance exists to aid the maintenance engineers choose proper parameters of the band-pass filter in a practical monitoring case. To address this problem, spectral kurtosis technique has been developed recently to locate the highest signal to noise ratio region in the frequency domain [76].

2.5.3 Time-frequency signal analysis

Joint time–frequency analysis is an effective approach to addressing non-stationary issues. With this method, signals are presented in a time–frequency–amplitude/energy density 3D space. Hence, both the constituent frequency components and their time variation features can be revealed [44]. To date, various time–frequency analysis methods have been proposed. The typical methods include linear time–frequency representations such as short time Fourier transform and wavelet transform [77], bilinear/quadratic time–frequency distributions based on Wigner–Ville distribution [78].

Short-time Fourier transform and wavelet transform

The short-time Fourier transform adds a time variable to the traditional Fourier spectrum, thus allowing it to investigate the time-varying nature of a signal. With this method, a short time window moves along time to slice a signal. The short time Fourier transform is obtained as follows. For any signal $x(t)$, suppose

Figure 2.19. Short time Fourier transform spectrogram of $x(t)$.

$w(\tau - t)$ is a window function centered at time t (where t is a time variable). Then the observed or segmented signal through this window is $x(t) w(\tau - t)$. Sliding the window through the time span of interest and applying the Fourier transform to each segment leads to the short time Fourier transform:

$$\text{STFT}_x(t, f) = \int_{-\infty}^{+\infty} x(\tau) w(\tau - t) \exp(-j2\pi f \tau) d\tau \quad (2.11)$$

Once the window function and its length are chosen, the time–frequency resolution of the short-time Fourier transform is fixed. In applications, a shorter time window should be used to localize the time instant of higher frequency components. A longer time window is preferred to pinpoint the location of lower frequency components.

To illustrate, a simulated non-stationary signal $x(t)$ is generated with a Gaussian white noise $n(t)$ at a signal-to-noise ratio of 3 dB:

$$x(t) = \sin(2\pi f_1 t) + 2 \cos(2\pi f_2 t + 150 \cos(2\pi f_m t)) + n(t) \quad (2.12)$$

Figure 2.20. The wavelet transform of $x(t)$.

where $f_1 = f_2 = 500$ Hz and $f_m = 2$ Hz. The sampling frequency is 2000 Hz, and $x(t)$ has a length of 1024 data points. Figure 2.19 shows the short-time Fourier transform spectrogram of $x(t)$ obtained using a Hanning window of length 128.

The wavelet transform employs wavelets as a basis, instead of sinusoidal functions. It adds a scale variable in addition to the time variable in the inner product transform. Hence, it is effective for time–frequency localization, and is suited to transient signal analysis [79]. For any energy limited signal $x(t) \in L^2(\mathbb{R})$, the wavelet transform can be defined as:

$$\text{WT}_x(t, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(\tau) \psi\left(\frac{\tau - t}{a}\right) d\tau \quad (2.13)$$

where $\psi(t)$ is the mother wavelet/wavelet basis, a is the scale parameter and t is the time shift.

Wavelet transform iteratively decomposes the approximation signals of lower frequency, but does not further work on the signals of higher frequencies. For higher frequency components, wavelet transform has a better time localization but a lower frequency resolution. For lower frequency components, the

frequency resolution is higher whereas the time localization is worse. Figure 2.20 shows the continuous wavelet transform spectrogram of $x(t)$ by using complex Morlet 32 as the mother wavelet. Although, wavelet transform is effective in analyzing self-similar signal with the wavelet basis. While various types of wavelet basis have been proposed, there is still a lack of a well-accepted effective method to guide choosing a suitable wavelet basis to match the signal structure [79].

Wigner-Ville distribution

For any signal $x(t)$, the Wigner–Ville distribution is defined as:

$$\text{WVD}_x(t, f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) \exp(-j2\pi f \tau) d\tau \quad (2.14)$$

As the Wigner–Ville distribution is not linear, the Wigner–Ville distribution of a sum of multiple signal components is not equal to the sum of the Wigner–Ville distributions of these signal components. For example, the Wigner–Ville distribution of signal $x(t)=x_1(t) + x_2(t)$ is:

$$\text{WVD}_x(t, f) = \text{WVD}_{x_1}(t, f) + \text{WVD}_{x_2}(t, f) + 2 \text{Re}[\text{WVD}_{x_1, x_2}(t, f)] \quad (2.15)$$

where the cross-term results from the interference between time and frequency due to the nonlinear quadratic transform:

$$\text{WVD}_{x_1, x_2}(t, f) = \int_{-\infty}^{\infty} x_1\left(t + \frac{\tau}{2}\right) x_2^*\left(t - \frac{\tau}{2}\right) \exp(-j2\pi f \tau) d\tau \quad (2.16)$$

The Wigner–Ville distribution does not involve any window function and thus is free from the interference between time localization and frequency resolution. The product of time interval and frequency bandwidth reaches the lower bound

Figure 2.21. The Wigner-Ville distribution of $x(t)$.

of the Heisenberg uncertainty principle. Consequently it has the highest time–frequency resolution among all the time–frequency distributions. However, for multi-component signals in mechanical systems, its applications are inevitably hindered by the cross-term interferences as shown in Fig. 2.21. Therefore, the pseudo-Wigner–Ville distribution is developed to attenuate the effect of cross-term interferences [78].

2.6 Planetary gear system

Planetary gear system also referred as epicyclic gear system usually consists of an annulus gear (also called ring gear), several planet pinions, a central or sun

Figure 2.22. Example for an industrial planetary gear system with bearings.

Table 2.2: Assembly configuration of planetary gear system

Input	Output	Stationary
Sun Gear	Planet Carrier	Annulus Gear
Planet Carrier	Annulus Gear	Sun Gear
Sun Gear	Annulus Gear	Planet Carrier

gear and a carrier connecting all the planets. This structure allows for a large torque-to-weight ratio, quiet operation and a more robust system than normal fixed-axis gear pairs. Therefore, planetary gear systems are usually employed in heavy duty systems. The different geometry and configurations of the planetary gear system is firstly introduced in this section, in which the equally-spaced planetary gear-sets are widely used in industrial. Then, it is described the calculation methods of the gear mesh frequency in planetary gear-sets. Finally, the modulation phenomena of the measured vibration spectrum in planetary gear-sets are reviewed at the end of this section.

2.6.1 Geometry and configurations of planetary gearbox

Any given of planetary gear set has three main components: the sun gear, the planet gears with the planet gears' carrier and the ring or annulus gear. Each of these three components can be the input, the output or can be held stationary. Table 2.2 shows different assembly conditions. In this thesis, the investigation are illustrated using the configuration with the stationary annulus gear, which is the most common configuration applied in the industrial engineering systems.

A planetary gear system is compact in space, but has a complex structure. It needs a high-quality manufacturing process. When designing the planetary gear system, it must be recognized that not every combination of gear teeth and located angle can be assembled. In order to determine the relationship among

the numbers of teeth of the sun gear s , N_s , the planet gears p , N_p and the annulus gear a , N_a and the number of planet gears, P ; the parameters must satisfy the following three conditions:

Condition 1: $N_a = N_s + 2N_p$. For standard gear system, this is the condition necessary for the center distances of the gears to match.

Condition 2: Setting a certain planet gear located at 0 degree without any loss of generality, other planets can only be assembled in any set of discrete angles as long as they are integer multiple of the least mesh angle which can be defined

as $\lambda = \frac{2\pi}{N_s + N_a}$.

Condition 3: $m(N_p + 2) < m(N_s + N_p) \sin \frac{\pi}{P}$. Here, m denotes module of the gear. Satisfying this condition insures that adjacent planet gears can operate without tip interfering with each other.

If all P planets of a planetary gear set can be assembled at equal angular distances with respect to the axis of the sun gear (Without loss of generality, the planets can be assumed to be located at $\psi_i = 2\pi(i-1)/P$ where $i = 1, 2, 3, \dots, P$), the planetary gear set thus assembled is called an equally spaced gear set. Otherwise, the planetary gear set is called an unequally spaced gear set.

For either type of planetary gear-set, there are three possible phasing relationships:

(i) *In-phase gear meshes*

Planetary gear-sets are said to have in-phase gear meshes when all the planets are in a similar stage of gear tooth mesh with both the sun and annulus gears. This occurs if $N_s\psi_i = 2\pi n$ and $N_a\psi_i = 2\pi l$ ($n, l \in \mathbb{Z}$, and $\mathbb{Z}$ refers to the integer space). This condition is true if and only if both the number of teeth on the sun, N_s , and the annulus, N_a , are integer multiples of P , i.e., N_s/P and N_a/P are integers. Under this condition, the load is divided equally among the multiple sun-planet and annulus-planet tooth meshes.

(ii) *Sequentially phased gear meshes*

Planetary gear-sets are said to have sequentially phased gear meshes if different planets are in dissimilar stages of gear tooth mesh with both the sun and annulus gears. This occurs if $N_s\psi_i \neq 2\pi m$, $N_a\psi_i \neq 2\pi l$ but $\sum_{i=1}^P N_s\psi_i = q\pi$,

$\sum_{i=1}^P N_a\psi_i = n\pi$ ($m, l, q, n \in \mathbb{Z}$). This phase condition has significant benefits in reducing vibration and noise levels of the system. Therefore, it is the more common configuration used in industrial applications. It can be proven that in-phase and sequentially phased gear meshes are the only possible phasing condition for equally spaced planetary gear systems.

(iii) *Arbitrarily phased gear meshes*

The arbitrary phase relationship is when none of the conditions for in-phase or sequentially phase gear sets are met. This gear mesh condition can occur only if the planets are positioned unequally.

2.6.2 Gear mesh frequency of planetary gearbox

Figure 2.23. An illustration of planetary gear system with three planets.

In fixed-axis gear system, gear meshing frequency or gear tooth meshing frequency is the potential vibration frequency for any machine that contains gears; and can be evaluated by multiplying the number of teeth by the rotational frequency of the gear. However, calculating gear meshing frequency for the planetary gear-set requires somewhat more information regarding its design.

Figure 2.23 illustrates a typical planetary gear-set configuration with P planets whose angular rotation frequencies are $-\omega_p$, wherein the sun gear s and the planet carrier c act as the input and output respectively, their angular rotation frequency are ω_s and ω_c . The annulus gear a is stationary, which means $\omega_a = 0$. There are different methods to calculate gear meshing frequency of the planetary gear system. One of these methods is to calculate the relationship of peripheral speed. As the points are meshing they must have the same tangential velocities; therefore ($r = mN$, pitch radius of the gear, m represents as the gear module in this case), $\omega_s r_s = \omega_p r_p + \omega_c (r_s + r_p)$; $\omega_a r_a = \omega_p r_p - \omega_c (r_s + r_p)$. The other method is to consider the planetary gear-set in a rotary reference frame, in which the reference frame has the opposite rotary velocity with respect to the

Figure 2.24. (a) Influence of planet i on the vibration response measured by the sensor, (b) Planet gear location at the instant of maximum influence on the vibration response at the sensor.

carrier. Then both of them can be derived that the gear tooth meshing frequency ω_m of the planetary gear-set is:

$$\omega_m = \omega_c N_a = (\omega_s - \omega_c) N_s = (\omega_p + \omega_c) N_p \quad (2.17)$$

2.6.3 Modulation effect due to the rotating carrier with respect to the mounted sensor

In the most practical industrial systems, the vibration sensors for monitoring the planetary gearbox are mounted on the stationary annulus gear/gear housing. These vibration response measured by the sensors fixed to the annulus gear of planetary gear-sets can provide valuable diagnostic information while providing no interference to normal gearbox operation. However, such vibration response of planetary gear-sets has been reported to contain distinct sideband patterns in the measured vibration spectrum. This is because when a planet moves toward the fixed vibration sensor, the level of vibration measured increases, reaches a peak when the planet is closest to the sensor, and then decreases as the planet recedes. Therefore, the vibration response due to periodic gear meshes as measured by a fixed sensor is modulated by carrier rotation, which gives rise to the sideband patterns observed in the measured vibration spectrum. Ref. [80]

defined a Hanning window $w(t)$ to describe this amplitude modulation (AM). It was assumed that each planet influences the vibration signal at the fixed sensor for a duration of T_c/P . As Fig. 2.24 illustrates, when a planet i approaches the sensor, its influence will increase for the first $T_c/2P$ duration, reaching a maximum when the planet is at the sensor location and then gradually diminishing to zero at the end of the next $T_c/2P$ duration. After this, the next planet $i+1$ will dominate the response at the sensor for the next T_c/P duration.

A Hanning function, $w(t)$, is used to represent this phenomenon and can be expressed as

$$w(t) = \frac{1}{2} - \frac{1}{2} \cos\left(\frac{2\pi P t}{T_c}\right) = \frac{1}{2} - \frac{1}{2} \cos(P\omega_c t) \quad (2.18)$$

Thus, for a planet i positioned at angle ψ_i , the i^{th} window function can be defined as

$$w_i(t) = w\left(t - \frac{\psi_i}{\omega_c}\right) U_i(t) \quad (2.19)$$

where, $U_i(t)$ can be expressed in terms of step function $u(t)$ as

$$U_i(t) = \sum_{k=1}^{\infty} \left\{ u\left[t - \left(\frac{(k-1)P + i - 1}{P}\right)T_c\right] - u\left[t - \left(\frac{(k-1)P + i}{P}\right)T_c\right] \right\} \quad (2.20)$$

Chapter 3 Dynamic Modeling and Experimental Investigation of Planetary Gearbox

Planetary gears are considered to be one of the most important drive-train components and are widely used in various heavy-duty machinery applications such as automobiles, helicopters and maritime ships transmissions. In spite of the critical functions of planetary gears, vibration and noise issues continue to cause major troubles within their applications. The vibration and noise from planetary gears not only deteriorate the working environment, but also reduce the durability and reliability of the engineering systems [81, 82]. As the vibration and noise problems are closely related to the dynamic behavior of planetary gears caused mainly by gear mesh deformation, the main objective of this chapter is to investigate (1) a time-varying model of a planetary gear set to study the dynamics of planetary gear-set, (2) the formulations of the planetary gear mesh stiffness and (3) the amplitude modulation (AM) phenomena due to the relative motions of the planet pinions with respect to the mounted sensor on the stationary annulus gear. Thus, the modeling tasks employed in this chapter consists of two major steps. The first step is to study the dynamic behaviors of planetary gear-sets by using a lumped parameter dynamic model that includes the time-varying mesh stiffness. The second step in this chapter is to understand the amplitude modulation phenomenon in the measured signals perceived by the fixed vibration sensor.

3.1 Two-dimensional mechanical model of planetary gearbox

Dynamic analyses of planetary gear systems based on the lumped parameter method have been reported in literature. Recent publications regarding the

dynamics of planetary gears are summarized at the beginning at this section. Lin and Parker [83] have developed a dynamic model of planetary gears focusing on free vibration characteristics to predict their natural frequencies and the vibration modes. Later, Parker [84] investigate the distinctive vibration mode properties of planetary gears with hybrid finite element-lumped parameter model, in which the elastic continuum ring gear is modeled by finite and lumped parameter method is used to model other parts of planetary gears. Parker [85, 86] also examines the dynamic behavior of spur planetary gears using two models: (i) a two-dimensional lumped-parameter model, and (ii) a finite element model. It is found that responses from the dynamic analysis using lumped parameter and finite element models are successfully compared qualitatively and quantitatively. Kahraman [87] and Velez [88] have investigated the manufacturing errors in the planetary gears, such as eccentricities, run-out errors and planet position errors, and simulates their contribution to the quasi-static and dynamic load sharing amongst the planets. To the authors' knowledge, most of these vibration models define the degrees-of-freedom in the vicinity of a rigid body motion as the state of reference, which rotates with the constant carrier angular rotational speed Ω_c . Then, small vibrations of gears are further superimposed on this nominal rigid gear body rotational motion. Therefore, this study focuses primarily to develop equations of motion for the planetary gear-sets in the inertial reference frame.

3.1.1 Dynamic model of planetary gear-set

The dynamic model of planetary gear system used in this study is given in Fig. 3.1. The influences of unequal planet load sharing as well as elastic deformations of gear bodies will not be included in this study and the

flexibilities will be kept limited to the gear meshes and radial bearings. In this two-dimensional (2D) lumped-parameter model, the gears are assumed as rigid bodies connected to each other along the line of action through the corresponding gear mesh stiffness and viscous damping. The gears are represented as lumped inertias, the gear meshes are modeled as nonlinear springs with tooth contact loss and periodically varying stiffness due to changing tooth contact conditions. These gears are held by bearings, which allow them to translate in x and y directions and freely rotate about their centers in the x - y transverse plane of gear. Thus, the motion of each gear can be defined using translational coordinate x and y , and the angular coordinate θ [89]. Correspondingly, the supporting bearings are simplified as linear springs. Thus, the motion of the sun gear can be described with the translational coordinates x_s and y_s , and the angular coordinate θ_s . Similarly, the motion of the carrier is defined by the translational displacements x_c and y_c , and the rotational angle θ_c . Since the shaft of the planet gear is mounted on the carrier, the motion of the planet gears is the superposition of the carrier motion and the planet motion relative to the carrier. φ_i is the initial located angle of planet i . ψ_i is the planet located angle of planet i and $\psi_i = \varphi_i + \theta_c$.

Figure 3.1. Two dimensional lumped parameter model of a planetary gear-set.

$$x_i = x_c + R_c \cos \psi_i + x_{pi} \quad (3.1)$$

$$y_i = y_c + R_c \sin \psi_i + y_{pi} \quad (3.2)$$

In which, x_i and y_i are the absolute translational displacements of the i th planet with respect to the origin. x_{pi} and y_{pi} are the relative translational displacements of the i th planet with respect to the pin hole on the carrier. The i th planet pinion has rotation θ_i .

Thus, the gear mesh deformation between the sun gear and the i th planet gear can be defined by the following (R is the radius of the base circle):

$$\begin{aligned} \delta_{spi} &= (x_s - x_i + R_c \cos \psi_i) \sin(\alpha - \psi_i) + (y_s - y_i + R_c \sin \psi_i) \cos(\alpha - \psi_i) \\ &\quad + (\theta_s - \theta_c) R_s - [- (\theta_p - \theta_c)] R_p \\ &= (x_s - x_c - x_{pi}) \sin(\alpha - \varphi_i - \theta_c) + (y_s - y_c - y_{pi}) \cos(\alpha - \varphi_i - \theta_c) \\ &\quad + \theta_s R_s + \theta_i R_p - \theta_c (R_s + R_p) \end{aligned} \quad (3.3)$$

Similarly, the gear mesh deformation between the annulus gear and the i th planet gear can be written as:

$$\begin{aligned} \delta_{api} &= (x_i - R_c \cos \psi_i) \sin(\alpha + \psi_i) - (y_i - R_c \sin \psi_i) \cos(\alpha + \psi_i) \\ &\quad - (\theta_i - \theta_c) R_p - [- (0 - \theta_c)] R_a \\ &= (x_c + x_{pi}) \sin(\alpha + \varphi_i + \theta_c) - (y_c + y_{pi}) \cos(\alpha + \varphi_i + \theta_c) \\ &\quad - \theta_i R_p - \theta_c (R_a - R_p) \end{aligned} \quad (3.4)$$

Lagrange's equation is used to in order to derive the equations of motion [90]. The kinetic and potential energy of the planetary gear needs to be expressed in the generalized coordinates of the inertia reference frame. The kinetic energy is the sum of the translational kinetic energy and the rotational kinetic energy. The total kinetic energy of the planetary gear-set equals to the summing all the

kinetic energy for the sun gear, the carrier, and the planet pinions (assume there is P number of planets in the analyzed planetary gear-set):

$$T = \frac{1}{2} \left[m_s (\dot{x}_s^2 + \dot{y}_s^2) + I_s \dot{\theta}_s^2 \right] + \frac{1}{2} \left[m_c (\dot{x}_c^2 + \dot{y}_c^2) + I_c \dot{\theta}_c^2 \right] + \sum_{i=1}^P \frac{1}{2} \left[m_p (\dot{x}_i^2 + \dot{y}_i^2) + I_p \dot{\theta}_i^2 \right] \quad (3.5)$$

where, $\dot{x}_i = \dot{x}_c + \dot{x}_{pi} - R_c \sin \theta_c \dot{\theta}_c$ and $\dot{y}_i = \dot{y}_c + \dot{y}_{pi} + R_c \cos \theta_c \dot{\theta}_c$.

On the other hand, the potential energy of the planetary gear-set is expressed as:

$$V = \frac{1}{2} k_s (x_s^2 + y_s^2) + \frac{1}{2} k_c (x_c^2 + y_c^2) + \sum_{i=1}^P \frac{1}{2} \left[k_{pi} (x_{pi}^2 + y_{pi}^2) + k_{spi} \delta_{spi}^2 + k_{api} \delta_{api}^2 \right] \quad (3.6)$$

where, k_s , k_c and k_{pi} are the stiffness of the supporting. k_{spi} and k_{api} are the gear mesh stiffness of sun-planet i and annulus-planet i , respectively. Besides when δ_{spi} and $\delta_{api} < 0$, their values are compulsorily set to 0 as the teeth lose contact and the resulting spring force will be equal to zero. The back collisions of the teeth are usually not taken into account in planetary gear-set models, because the gear backlash values of a planetary gear-set are considerably larger than those of fixed-axis gear pair in order to ensure easy assembly and prevent contact on both flanks of the gear teeth [81].

$\mathbf{Q}(t)$ is defined as the degrees of freedom vector that contains two coordinates for translational vibration and a coordinate for torsional motion for each gear in the plane containing the gear.

$$\mathbf{Q}(t) = [x_s \ y_s \ \theta_s \ x_c \ y_c \ \theta_c \ x_{p1} \ y_{p2} \ \theta_1 \ \cdots \ x_{pP} \ y_{pP} \ \theta_P]^T \quad (3.7)$$

The equations of motion for the planetary gear are then derived by using Lagrange's equation:

$$\frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{Q}_q} \right) - \frac{\partial T}{\partial Q_q} + \frac{\partial V}{\partial Q_q} = T_q \quad (3.8)$$

$$\ddot{x}_i = \ddot{x}_c + \ddot{x}_{pi} - R_c \sin \theta_c \ddot{\theta}_c - R_c \cos \theta_c \dot{\theta}_c^2 \quad (3.9)$$

$$\ddot{y}_i = \ddot{y}_c + \ddot{y}_{pi} + R_c \cos \theta_c \ddot{\theta}_c - R_c \sin \theta_c \dot{\theta}_c^2 \quad (3.10)$$

where, T_q represent the externally applied torque. Substitution of Eqs (3.1-3.7) and Eqs (3.9-3.10) into Eq. (3.8). leads to the equation of motion. Assembling all the individual equation results in a global equation of motion for the gearbox that can be expressed in matrix form as:

$$\mathbf{M}\ddot{\mathbf{Q}}(t) + [\mathbf{C} + \mathbf{C}_b]\dot{\mathbf{Q}}(t) + [\mathbf{K}(t) + \mathbf{K}_b]\mathbf{Q}(t) = \mathbf{F}(t) \quad (3.11)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix based on the Rayleigh damping ratio, $\mathbf{K}(t)$ is the time-varying gear meshing stiffness matrix, $\mathbf{C}_b$ and $\mathbf{K}_b$ are the bearing damping and stiffness matrices, $\mathbf{F}(t)$ is the externally applied torques vector. The dynamic equations of the lumped parameter of gear-sets can be numerically integrated in MATLAB/Simulink environment by using either the Newmark time integration method or the ODE solver of the MATLAB.

3.1.2 Effect of a gear defect on gear mesh stiffness in a planetary gear system

As several identical planets are employed and multiple teeth mesh in a planetary gear-set, the mesh phasing properties of phasing between the various sun-planet meshes and phasing between the various annulus-planet meshes for general planetary gear-sets must be carefully considered to avoid analysis and

simulation results' errors [91]. This is because that the stiffness of these meshes varies as the gears rotate because of the changed number of teeth in contact. While using this model for dynamic analysis, it is essential to correctly define the relative phase of these multiple tooth meshes as the gears rotate. The equally spaced planetary gear-set with 4-planet ($P = 4$) shown is used as an example to illustrate how to correctly define the relative phase relationship of these multiple tooth mesh stiffness in this section. The i th planet in this gear-set is located at circumferential angle $\psi_i = \frac{2\pi(i-1)}{P}$. The phasing relationships of mesh stiffness that should be taken into consideration can be classified as: (i) γ_{si} is defined as the relative phase between the i th sun-planet mesh and the first sun-planet mesh (γ_{s1} is set to 0 for the simplicity and without loss of generality), (ii) γ_{ai} is defined as the relative phase between the i th annulus-planet mesh and the first annulus-planet mesh and (iii) γ_{as}^i is defined as the relative phase between the i th annulus-planet mesh and sun-planet mesh. Figure 3.2 illustrates a scenario that considers the planetary gear-set in a rotary reference frame, which has been fixed to the carrier. In this reference frame, the carrier becomes stationary, annulus rotates anti-clockwise with $-\omega_c$ and the sun gear rotates clockwise with $\omega_s - \omega_c$. Therefore, it can be observed that there exists a time lead $\frac{\psi_i}{\omega_c}$ between the first annulus-planet and the i th annulus-planet mesh. Vice versa, there is a time lag $\frac{\psi_i}{\omega_s - \omega_c}$ between the first sun-planet and the i th sun-planet mesh. The periodicity of the mesh stiffness of the annulus-planet 1 k_{ap1} ensures it has a Fourier series representation:

Figure 3.2. (a) Annulus gear meshes with planets in the rotary reference frame
(b) Sun gear meshes with planets in the rotary reference frame.

$$k_{apl}(t) = \sum_{l=0}^{\infty} [a_l \sin l\omega_m t + b_l \cos l\omega_m t] \quad (3.12)$$

where, ω_m is the gear mesh frequency of the planetary gear set. Thus, it can yield:

$$k_{api}(t) = \sum_{l=0}^{\infty} \left[a_l \sin l\omega_m \left(t - \frac{\psi_i}{\omega_c} \right) + b_l \cos l\omega_m \left(t - \frac{\psi_i}{\omega_c} \right) \right] \quad (3.13)$$

Recalling that:

$$\omega_m = N_a(\omega_a + \omega_c) = N_s(\omega_s - \omega_c) = N_p(\omega_p + \omega_c). \quad (3.14)$$

where N denotes the number of teeth on corresponding gear as indicated by the subscript.

Thus, it can derive that $\gamma_{ai} = -N_r \psi_i$. Similarly, $\gamma_{si} = N_s \psi_i$. Ref. [91] has shown that the relative phase between the i th annulus-planet mesh and sun-planet mesh γ_{as}^i is based on the gear design parameters and gear tooth profiles.

However, it also proves that $\gamma_{as}^i = \gamma_{as}^1 = \gamma$ implying that the phasing between all the sun-planet meshes and planet-annulus meshes is a constant. Therefore,

the omission or arbitrary choosing of this initial phase γ usually does not change any of the vibration characteristics.

In the simulated work, the gear mesh stiffness is modeled as a time-varying function with square waveform to simulate the effect of variation in the gear meshing stiffness as shown in Fig. 3.3 after the proper evaluation of the phasing relationship described in this section. The detailed variation in the gear meshing stiffness due to the gear tooth defects is studied in [16] and [17], wherein it has been concluded that such faults are always accompanied by a stiffness reduction as the contact between the teeth can be partly lost. The failure characteristics of impulse response and the sideband frequency in fixed-axis gear-set have been shown to be similar (i.e., the fault signature appeared as specific frequency locations) except their magnitude. The square waveform was shown to be a good approximation for meshing stiffness function under healthy operating conditions in spur gears [59]. Therefore the reduction in the mesh stiffness is modeled as a local drop in general squared wave form meshing stiffness function as shown in Fig. 3.3.

Figure 3.3. Periodic fluctuation of the gear mesh stiffness between the sun and planet gears with local gear tooth fault on the sun.

3.1.3 Model application – an integrated electro-mechanical drivetrain

An electro-mechanical drive-train consisting of a planetary gearbox driving a permanent magnetic synchronous generator is developed and implemented in MATLAB/ Simulink block diagram and shown in Fig. 3.4. The integrated hybrid model is used to illustrate the feasibility and the interest of tooth fault detection. The analysis of the vibration and electric signals will be presented in the later chapters based on the model. A constant input torque of 100 Nm is assumed to be applied at the input shaft connected to the sun gear of the planetary gearbox in this model. The planetary gearbox is modeled using a lumped parameter approach described in the Section 3.1.1 and 3.1.2. A three phase permanent magnetic synchronous generator (PMSM) and the coupling mechanical shafts are modeled by using the pre-set model 12 in MATLAB/Simulink PMSM library and Mechanical Shaft block in SimPower Systems library. The models of the PMSM, and the lumped parameter of planetary gearbox are coupled together in MATLAB/Simulink environment which is then numerically integrated by the Runge-Kutta 45 solver (ode 45). This electro-mechanical model that couples planetary gear-sets with an AC

Figure 3.4. The integrated electro-mechanical drivetrain model in MATLAB/Simulink.

Figure 3.5. (a) Schematic of the test rig, (b) Planetary gearbox test rig and (c) Seeded gear fault on the internal ring gear of the planetary gear-set.

generator programmed in MATLAB/Simulink enables the simulation of the dynamic response of the coupled system and allows an investigation of the interactions between the mechanical vibrations generated by gears under healthy conditions and in presence of gear tooth faults and the electric currents on driven generator.

3.2 Vibration measurements of planetary test rigs and amplitude modulation phenomenon in the measurements

3.2.1 Laboratory planetary gearbox test rig

A laboratory planetary gearbox test rig is developed and shown in Fig. 3.5. The input shaft of the gearbox is connected to a 3-phase AC motor (4 kW 4 pole) controlled using a standard industrial drive. A generator, with a resistive load bank, is driven by the output shaft of the gearbox. The two tested gearboxes have back-to-back planetary gear-sets, each having four planets in an equally spaced configuration with number of ring gear teeth $N_r = 84$, number of sun gear teeth $N_s = 28$, and number of planet gear teeth $N_p = 28$. As a result of the back-to-back scheme, the overall gear ratio of the gearbox equals to $G \times 1/G = 4 \times 1/4 = 1$, where G represents the gear ratio of a single planetary gear-set. The vibration signals are measured at a sampling rate of 20K Hz using an accelerometer attached to the gearbox housing outside the ring gear. The accelerometer is used with the Kistler coupler and is then fed to a Pentium 4 computer through a PCI dSPACE real time control card (DS 1104) as the data acquisition system. An artificial seeded gear tooth fault is introduced on one of the ring gear teeth (Fig. 3.5 (c)). This manual seeded gear damage is created along the line of action by using electro-discharge machine (EDM), which removes around 10% percentage of metal material with respect to the gear tooth circular thickness along the line of action on the flank of one gear tooth.

3.2.2 Planetary gearbox test rig at the National Renewable Energy Laboratory

The vibration measurements from a 750 kW test gearbox on a 2.5 MW dynamometer test facility at the National Renewable Energy Laboratory (NREL) are also analyzed in the later chapter. This facility is capable of providing static, highly accelerated life and model-in-the-loop tests. The tested gearbox used in

this work consists of an equally spaced 3-planets gear-set ($N_a = 99$, $N_s = 21$, $N_p = 39$) and two parallel gear stages that was damaged during its operation on a turbine at the Ponnequin wind farm, CO, USA. It was installed and put into unattended operation on 14 September 2009. However, the turbine was stopped 5 October 2009 after the bearing temperature exceeded the threshold and reports of oil loss from the gearbox that resulted in damage to its internal bearings and gears. The high speed stage gear teeth showed signs of significant

Figure 3.6. (a) NREL 2.5 MW dynamometer test facility (Credit: Lee Jay Fingersh/NREL #16913), (b) Schematic of the test facility (Credit: NREL) and (c) Vibration sensor mounted on the annulus gear radial 6 and 12 o'clock.

overheating. Subsequently the gearbox was removed from the turbine and shipped back to NREL to avoid a potential catastrophic failure. Since the damage was not catastrophic, the gearbox was retested in the NREL's 2.5 MW dynamometer before it was disassembled. Measurements obtained from two accelerometers, AN3 and AN4 mounted on the annulus gear (Fig. 3.6), were used for analysis. After the dynamometer test, the gearbox was sent to a rebuild shop, where it was disassembled and a detailed failure analysis was conducted. This detailed failure analysis indicated damage to the annulus gear and the sun gear of the planetary gear-set. Severe fretting corruptions were found on the sun pinion and moderate gear scuffing and polishing wear on the annulus gear. The photographs of the damaged sun pinion and annulus gear are presented in the next chapter with the vibration based diagnostic results.

Chapter 4 Geometry based Vibration Models for Fault

Diagnosis of Planetary Gear-set

Equally-spaced planetary gearboxes are important power-train components for varied engineering systems. Their failures can result in significant capital losses and pose safety concerns. The vibration measurements perceived by a sensor mounted on the gearbox housing can provide valuable diagnostic information while providing no interference to the normal gearbox operation. However, such vibration based monitoring techniques are difficult to implement in planetary gearboxes due to the complex nature of measured vibration spectrum that is a result of planets revolving with respect to the stationary sensors mounted on the gearbox housing. Previous research carrying out simulation and experiments using such measurements have reported distinct sideband patterns in the resulting vibration spectra, which differ significantly from the spectra of a normal fixed-axis/parallel gear pair system. In this chapter, Fourier series analysis is used to explain these distinct sidebands patterns that contain rich diagnostic information. The results obtained are useful to understand the cause of the observed vibration behavior in both healthy and faulty planetary gearboxes, and identify the locations of these additional frequency components introduced by the damaged gear in an otherwise complex measured vibration spectrum. Thus, the formulation presented in this chapter can assist in developing robust feature extraction algorithms for early detection of planetary gearbox failures. The theoretical derivations presented in this chapter are validated by both dynamic simulations, and experiments on a dynamometer test bed using a 750 kW gearbox damaged during its operation while installed in a

wind turbine. The predicted fault frequencies for observed faults in the annulus and sun gears of the gearbox are vividly presented in the experimentally measured frequency spectrum.

4.1 Vibration signal models and monitoring techniques for planetary gearbox

Planetary gearboxes are commonly employed in heavy-duty powertrains of energy, maritime and aerospace engineering. Some of the typical applications of their use include wind turbine gearboxes, planetary reducers for ships and transmission gearboxes of helicopters. Any failures with these planetary systems have a potential to cause expensive repair and overhaul, large unscheduled down-time and safety concerns. The perceived vibration response of sensors fixed to the annulus gear of planetary gear systems can provide useful diagnostic information pertaining to the health of such systems [92-94]. Therefore, it is essential to understand the vibration behavior of healthy as well as faulty planetary gearboxes for development of robust condition health monitoring techniques that can detect faults at earlier stages.

For an error-free fixed-axis/parallel gear pair, the vibration spectrum contains only dominant components at the gear meshing frequency (MF) and its harmonics. If there is any gear damage, e.g., a tooth on one gear develops a spall or a crack; it results in modulations of the amplitude and frequency in the original vibration measurements that is periodic at the shaft rotation frequency and its harmonics [3, 16, 35]. The analytical formulation describing this phenomenon is reviewed as Eqs. (2.5-2.8) in the Chapter Two. Since amplitude and frequency modulations appear as sidebands around the dominant gear MF and its harmonics in the vibration spectrum, this behavior suggests that a

frequency-domain early-detection algorithm of damaged tooth should look for changes in the amplitudes of the sidebands around MF and its harmonics that are integer multiples of its shaft rotation frequency, associated with the gear meshing harmonics [36, 44].

However, as planetary gear-sets employ P number of identical planet pinions that revolve around the sun gear, the vibration signals picked up by a fixed sensor attached to planetary-gearbox housing differ significantly from that of fixed-axis/parallel gear systems [3, 16, 35]. Fourier series analysis has been used to explain the asymmetrical sidebands of annulus-planets gear meshes observed in the vibration spectrum in a healthy equally-spaced planetary gear system [6, 80]. According to their formulation, the total vibration spectrum, i.e., sum of the vibration response contributed from each of the individual planet gear measured by a fixed sensor, will have dominant components at frequencies that are integer multiple of the number of planets times carrier order or at frequencies whose difference from MF is an integer multiple of number of planets times the carrier order.

The success of fault detection and prognosis in the planetary gearboxes using vibration measurement has been demonstrated in [2, 9, 95]. However, the methodologies presented in these works limited themselves to the carrier plate crack fault as no explicit model of the planetary gear-sets was further explored to identify the locations of the additional frequency components introduced by the damaged tooth. The lumped-parameter dynamic model of a planetary gearbox was recently used to predict sideband behaviors of the planetary gear-set due to the local manufacturing errors [87]. However, such models require appropriate choices of equivalent masses connected by appropriate stiffness and

damping elements which result in governing differential equation of motion associated with a large number of degrees of freedom. The end-users and maintenance engineers of gearbox often lack such system parameter information and its numerical integration is computationally expensive. Moreover, it does not provide adequate fundamental understanding of the reason for additional dominant frequency components that arise due to presence of faults in the measured vibration spectrum and their dependence on the geometry of the planetary gear-sets the way analytical solutions do [96]. Thus, many researchers usually prefer to study the condition-based maintenance techniques based on analytical/mathematical vibration signal model of gearbox as it is easy to implement [3, 9, 35, 95]. So far no proper mathematical or analytical analysis has been provided to explain why and where additional sidebands appear at certain frequency locations when local gear fault or manufacturing error arises in planetary gear-sets.

This work extends the formulation presented in [6, 80] that considers the Fourier series analysis of planets meshing with annulus gear under healthy condition by further exploring the vibration signals from sun-planets gear meshes that were ignored previously. This more complete model for the vibration response from a planetary gear-set enables further investigation and succinct mathematical explanation of vibration spectrum, presented herein, under the influence of general gear tooth damage; such as cracked tooth, spalled tooth or gear eccentricity, which may occur at any of the gears in an equally-spaced planetary gear-set. The effect of local faults on the sidebands in the measured vibration spectrum picked by a fixed sensor mounted on the annulus gear reveals important frequency features, which can be useful for early

Figure 4.1. A planetary gear-set with 4 planets and a sensor mounted on the fixed annulus gear.

detection algorithm of damaged tooth and diagnosing the source of faults in a planetary gear system. The derived conclusions are validated by both dynamic simulations and experiments on a 750 kW gearbox similar to those installed in wind turbines. The gearbox used for experimental validation was observed to have faults in annulus and sun gear, which could be correlated to the dominant vibration response predicted at frequency components determined by Fourier series analysis presented in this work.

4.2 Fourier series analysis of healthy planetary gear system

Figure 4.1 illustrates a typical equally spaced planetary gear-set configuration with 4 planets wherein the sun gear s and the planet carrier c act as the input and output respectively. The i th planet in this gear-set is located at angular position $\psi_i = \frac{2\pi(i-1)}{P}$, where P = number of planets. If P planets rotate counter-clockwise with angular rotation frequencies of ω_p , the sun gear s and

the carrier c will rotate clockwise with an angular rotation frequency of ω_s and ω_c respectively. The annulus gear a is stationary, which implies $\omega_a = 0$. It can be derived that the gear MF ω_m of the planetary gear-set is:

$$\omega_m = N_a(\omega_a + \omega_c) = N_s(\omega_s - \omega_c) = N_p(\omega_p + \omega_c), \quad (4.1)$$

where, N denotes the number of teeth on corresponding gear as indicated by the subscript.

When planetary gear systems are manufactured precisely (i.e., they are error free) and each planet-annulus gear pair transmits the same load, the vibration contribution (which is proportional to the gear meshing force [97]) from the annulus gear and i^{th} planet mesh is periodic at the gear mesh frequency ω_m . The vibration waveform resulting from different annulus-planet gear mesh will be similar but shifted in time [6] because of the variation in the phase at the mesh. Thus, the vibration signal from planet i meshing with the annulus gear $x_i(t)$ can be written as [80]:

$$\begin{aligned} x_i(t) &= x\left(t - \frac{\psi_i}{\omega_c}\right) = \sum_{j=1}^J X_{ij} \cos\left(jN_a\omega_c\left(t - \frac{\psi_i}{\omega_c}\right) + \theta_j\right) \\ &= \sum_{j=1}^J X_{ij} \cos(jN_a\omega_c t - jN_a\psi_i + \theta_j) \end{aligned} \quad \dots (4.2)$$

where, X_{ij} is the Fourier coefficient of j^{th} harmonic of the vibration contribution from the annulus and i^{th} planet mesh, θ_j is the initial phase angle of the j^{th} harmonic component and $-N_a\psi_i$ is the phase angle between the annulus gear meshes with planet i and planet 1.

Similarly, the vibration signal from the sun gear and i^{th} planet mesh is also periodic at the gear mesh frequency ω_m and can be written as $y_i(t)$:

$$\begin{aligned} y_i(t) &= y\left(t + \frac{\psi_i}{\omega_s - \omega_c}\right) = \sum_{j=1}^J Y_{ij} \cos\left(j\left(\omega_m\left(t + \frac{\psi_i}{\omega_s - \omega_c}\right) + \gamma\right) + \delta_j\right) \\ &= \sum_{j=1}^J Y_{ij} \cos\left(jN_s(\omega_s - \omega_c)t + jN_s\psi_i + j\gamma + \delta_j\right) \end{aligned} \quad (4.3)$$

where, Y_{ij} is the Fourier coefficient of j^{th} harmonic of the vibration contribution from the sun and i^{th} planet mesh, δ_j is the initial phase angle of the j^{th} harmonic component and $N_s\psi_i$ is the phase angle between the sun gear meshes with the planet i and planet 1. The phase angle γ represents the phase difference between sun-planets meshes and annulus-planets meshes [91]. Under ideal conditions, $X_{ij} = X_j$ and $Y_{ij} = Y_j$ as planets share the load evenly. Also, without any loss of generality, θ_j and δ_j can be set to 0 for simplicity in this work.

As a planet moves toward the vibration sensor, the level of the measured vibration signal increases, reaches a peak when the planet is closest to the sensor, and then decreases as the planet gear recedes. Therefore, the influence of planet i on vibration response at the sensor location will be periodic with the carrier rotation frequency. An example of assumed function representation to describe this influence can be found in [7] and the details of such a carrier modulation function are presented in Eqs. (2.18-3.20) of the literature reviewed chapter. Thus, the sensor perceived vibration signal $z(t)$ due to annulus-planets and sun-planets meshes can be modeled as the super-position of each individual ring-planet/sun-planet mesh:

$$z(t) = \sum_{i=1}^P w_i(t)[x_i(t) + y_i(t)] \quad (4.4)$$

where, $w_i(t) = w\left(t - \frac{\psi_i}{\omega_c}\right)$ is the window function corresponding to i th planet.

As the window function corresponding to i th planet $w_i(t)$ is periodic with a fundamental frequency of the carrier rotation frequency ω_c , it can be expanded as a Fourier series as $\sum_{k=-K}^K W_k e^{jk\omega_c t} e^{-jk\omega_c \frac{\psi_i}{\omega_c}}$. Here j is the imaginary part, the first exponent term refers to the k th harmonic of frequency ω_c , W_k represent the Fourier series coefficients of the k th harmonic of frequency ω_c and the second exponential term has a constant power corresponding to the phase shift between different planets. Similarly, vibration signal from the i th planet- annulus gear mesh $x_i(t)$ is periodic with a fundamental frequency of the gear MF ω_m , and can be expanded as a Fourier series as $\sum_{m=-M}^M X_m e^{jmN_a \omega_c t} e^{-jmN_a \omega_c \frac{\psi_i}{\omega_c}}$. Here the first exponent term refers to the m th harmonic of frequency ω_m and the second exponential term has a constant power corresponding to the phase shift between different planets. The Fourier coefficients W_k and X_m can be experimentally determined from spectral analysis using envelope analysis/ demodulation for a given healthy planetary gearbox under a certain operating conditions and stored as pre-defined values. Further, by Eqs. (4.2) and (4.4), the vibration signal $w_i(t)x_i(t)$ from planet i due to annulus-planet mesh perceived by a fixed sensor can be further expanded as a complex Fourier series (j is the imaginary part):

$$\begin{aligned}
w_i(t)x_i(t) &= w\left(t - \frac{\psi_i}{\omega_c}\right)x\left(t - \frac{\psi_i}{\omega_c}\right) \\
&= \left(\sum_{k=-K}^K W_k e^{jk\omega_c t} e^{-jk\omega_c \frac{\psi_i}{\omega_c}} \right) \left(\sum_{m=-M}^M X_m e^{jmN_a\omega_c t} e^{-jmN_a\omega_c \frac{\psi_i}{\omega_c}} \right) \\
&= \sum_{k=-K}^K \sum_{m=-M}^M W_k X_m e^{j(mN_a+k)\omega_c t} e^{-j(mN_a+k)\psi_i}
\end{aligned} \tag{4.5}$$

Thus, the overall sensor perceived vibration signal $x(t)$ due to the vibration contribution from all the annulus-planet gear meshes can be expressed by summing $w_i(t)x_i(t)$ as:

$$x(t) = \sum_{i=1}^P w_i(t)x_i(t) = \sum_{k=-K}^K \sum_{m=-M}^M W_k X_m e^{j(mN_a+k)\omega_c t} \left(\sum_{i=1}^P e^{-j(mN_a+k)\psi_i} \right) \tag{4.6}$$

Since, for equally-spaced planetary gear-set, $\psi_i = \frac{2\pi(i-1)}{P}$

$$\sum_{i=1}^P e^{-j(mN_a+k)\frac{2\pi(i-1)}{P}} = \begin{cases} P & \text{when } k + mN_a = \text{integer multiple of } P \\ 0 & \text{otherwise} \end{cases} \tag{4.7}$$

Therefore, fixed-sensor responses $x(t)$ from healthy annulus-planets gear meshes can be expected in frequency regions in the immediate neighborhoods of the gear meshing frequency and only at planet carrier rotational harmonics that are integer multiples of the number of equi-spaced planets. This phase cancellation phenomenon from individual planets has also been studied and verified in [6, 7, 80].

From Eqs. (4.3) and (4.4) and following the same procedure, the sensor perceived vibration signal $y(t)$ due to sun-planets meshes can be expanded as:

Figure 4.2. Simulated vibration spectrum around gear meshing frequency for a healthy equally spaced 4-planets gear-set as perceived by a fixed sensor on the annulus.

$$\begin{aligned}
 y(t) &= \sum_{i=1}^P w_i(t) y_i(t) = \sum_{i=1}^P w \left(t - \frac{\psi_i}{\omega_c} \right) y \left(t + \frac{\psi_i}{\omega_s - \omega_c} \right) \\
 &= \sum_{i=1}^P \left(\sum_{k=-K}^K W_k e^{jk\omega_c t} e^{-jk\omega_c \frac{\psi_i}{\omega_c}} \right) \left(\sum_{m=-M}^M Y_m e^{j(m(N_s(\omega_s - \omega_c)t + \gamma))} e^{jmN_s(\omega_s - \omega_c) \frac{\psi_i}{\omega_s - \omega_c}} \right) \\
 &= \sum_{k=-K}^K \sum_{m=-M}^M W_k Y_m e^{j(mN_a + k)\omega_c t} e^{jm\gamma} \left(\sum_{i=1}^P e^{j(mN_s - k)\psi_i} \right)
 \end{aligned} \tag{4.8}$$

Similar with Eq. (4.7),

$$\sum_{i=1}^P e^{j(mN_s - k) \frac{2\pi(i-1)}{P}} = \begin{cases} P & \text{when } mN_s - k = \text{integer multiple of } P \\ 0 & \text{otherwise} \end{cases} \tag{4.9}$$

For equally-spaced planetary gear-set, it can be derived that $N_s + N_a = \text{integer multiple of } P$. Therefore, if $mN_a + k = \text{integer multiple of } P$, only then $mN_s - k = \text{integer multiple of } P$. This implies that fixed-sensor response $y(t)$ from healthy planet-sun gear meshes can also be expected only at planet carrier rotational harmonics that are integer multiples of the number of equi-spaced planets, which is the same as in the case of planet-annulus gear meshes. Figure 4.2 gives the simulated spectrum of the overall vibration signal for a healthy

equally-spaced 4-planets gear-set picked by a fixed sensor, which has $N_a = 99$, $N_s = 29$ and $N_p = 35$. The details of simulation which is based on a lumped parameter dynamic model of planetary gearbox can be found in Eqs. (3.1-3.11) of the Chapter Three. It can be observed that the sideband behaviors from the overall vibration response only appear at locations that are integer multiple of the number of planets times the carrier order (92^{th} , 96^{th} , 100^{th} , 104^{th} carrier order) and MF (99^{th} carrier order) is canceled in this sequential mesh design of the gearbox.

4.3 Fourier series analysis of planetary gear with local faults

In this section, Fourier series analysis of equally-spaced planetary gear-set $\left(\psi_i = \frac{2\pi(i-1)}{P}\right)$ is extended to account for additional frequency components in the vibration spectrum that appear in presence of a local fault, such as a damaged gear tooth on annulus, sun or planet gears. The variations observed in the amplitude and phase at these additional frequency components enable the detection of a damaged planetary gearbox.

In case of a fixed-axis gear pair, a general local fault on one of the gears results in localized variations in amplitude and phase in the vibration response which is dominated by the MF and its harmonics. These variations can be defined by the amplitude and phase modulation functions, $a_f(t)$ and $b_f(t)$ respectively. The residual signal (i.e., after the removal of meshing harmonics from the original signal) of modulated gear meshing vibration $v(t)$ can be expressed as [69]:

$$v(t) = \sum_{j=1}^J X_j a_j(t) \cos(j\omega_m t + b_j(t)) \quad (4.10)$$

When a local gear fault exists in the annulus-planets/sun-planets meshes, it would also result in amplitude and phase/frequency modulation effect as described by Eq. (4.10). Thus, the amplitude and phase modulated responses for the vibration signal from an individual planet i meshing with annulus and sun gears can be represented as $x'_i(t)$ and $y'_i(t)$ respectively:

$$x'_i(t) = \sum_{m=1}^M X_m \left(1 + a_m \left(t - \frac{\psi_i}{\omega_c} \right) \right) \cos \left(m N_a \omega_c \left(t - \frac{\psi_i}{\omega_c} \right) + b_m \left(t - \frac{\psi_i}{\omega_c} \right) \right) \quad (4.11)$$

$$y'_i(t) = \sum_{m=1}^M Y_m \left(1 + a_m \left(t + \frac{\psi_i}{\omega_s - \omega_c} \right) \right) \cos \left(m \left(N_s (\omega_s - \omega_c) \left(t + \frac{\psi_i}{\omega_s - \omega_c} \right) + \gamma \right) + b_m \left(t + \frac{\psi_i}{\omega_s - \omega_c} \right) \right) \quad \dots (4.12)$$

where

$$\begin{cases} a_m(t) = \sum_{l=1}^L A_{ml} \cos(l \times \omega_{\text{characteristic}} t) \\ b_m(t) = \sum_{l=1}^L B_{ml} \cos(l \times \omega_{\text{characteristic}} t) \end{cases} \quad (4.13)$$

The characteristic fault frequency of annulus gear is $\omega_{af} = \frac{\omega_m}{N_a} = \omega_c$, the

characteristic fault frequency of sun gear is $\omega_{sf} = \frac{\omega_m}{N_s} = \omega_s - \omega_c$ and the

characteristic fault frequency of planet is $\omega_{pf} = \frac{\omega_m}{N_p} = \omega_p + \omega_c$.

4.3.1 Local fault on the annulus gear

Figure 4.3. Simulated vibration spectrum around gear meshing frequency due to local fault on the annulus gear for an equally spaced 4-planets gear-set, $N_a = 99$, $N_s = 29$ and $N_p = 35$.

From Eqs. (4.10) and (4.11), if the damaged gear tooth is on the annulus gear, the residual signal $w_i(t)x_i'(t)$ (after the removal of meshing harmonics from the original signal) from planet i and annulus-gear mesh can be written as:

$$w_i(t)x_i'(t) = w\left(t - \frac{\psi_i}{\omega_c}\right) a\left(t - \frac{\psi_i}{\omega_c}\right) x\left(t - \frac{\psi_i}{\omega_c} + b\left(t - \frac{\psi_i}{\omega_c}\right)\right) \quad (4.14)$$

As characteristic fault frequency of ring gear is $\omega_{df} = \omega_c$, Eq. (4.14) can be further extended as a complex Fourier series as:

$$\begin{aligned} w_i(t)x_i'(t) &= \left(\sum_{k=-K}^K W_k e^{jk\omega_c t} e^{-jk\omega_c \frac{\psi_i}{\omega_c}} \right) \left(\sum_{n=-N}^N A_n e^{jn\omega_c t} e^{-jn\omega_c \frac{\psi_i}{\omega_c}} \right) \left(\sum_{m=-M}^M \sum_{l=-L}^L X_m B_l e^{j(mN_a+l)\omega_c t} e^{-j(mN_a+l\omega_c) \frac{\psi_i}{\omega_c}} \right) \\ &= \sum_{k=-K}^K \sum_{n=-N}^N \sum_{l=-L}^L \sum_{m=-M}^M W_k A_n B_l X_m e^{j(mN_a+k+n+l)\omega_c t} e^{-j(mN_a+k+n+l)\psi_i} \\ &= \sum_{q=-Q}^Q \sum_{m=-M}^M V_q X_m e^{j(mN_a+q)\omega_c t} e^{-j(mN_a+q)\psi_i} \\ &= \sum_{q=-Q}^Q \sum_{m=-M}^M V_q X_m e^{j(mN_a+q)\omega_c t} e^{-j(mN_a+q)\psi_i} \\ &\dots \end{aligned} \quad (4.15)$$

where, $\sum_{q=-Q}^Q V_q e^{j(q\omega_c t)} e^{j(q\psi_i)}$ is defined to be equal to

$$\sum_{k=-K}^K \sum_{n=-N}^N \sum_{l=-L}^L W_k A_n B_l e^{j(k+n+l)\omega_c t} e^{j(k+n+l)\psi_i} . \text{ Comparing Eq. (4.15) with Eq. (4.6), it can}$$

be seen that the additional dominant frequency components measured by a fixed sensor for an equally-spaced planetary gear-set having a damaged tooth on the ring gear will appear at $m\omega_m + q\omega_c$ (m, q are integer number) but only at an integer multiple of the number of planets (P) times the carrier rotational frequency ω_m , which is the same as in the case of healthy gear-set. However, such local fault on the ring gear will change the magnitude of these sidebands as compared to the healthy case. Statistical indicators can be used to detect such variations in the individual amplitudes caused by the damage. Thus, in Fig. 4.3, it can be observed that the magnitude of 92th, 96th, 100th and 104th carrier orders has changed due to the local annulus fault and new frequency components appear at such as 88th, 108th and 120th carrier order. All these frequency components are at integer multiple of the number of planets (=4).

4.3.2 Local fault on the sun gear

Using the same analogy, if the sun gear has a local fault, such as a damaged tooth, the amplitude and phase modulation functions, $a_m(t)$ and $b_m(t)$ in Eqs. (4.12) and (4.13), are periodic with the characteristic frequency

$$\omega_{sf} = \frac{\omega_m}{N_s} = \omega_s - \omega_c . \text{ Thus, the sensor perceived residual vibration signal}$$

$$\sum_{i=1}^P w_i(t) y_i'(t) \text{ due to the sun-planets meshes can be expanded as Fourier series:}$$

$$\begin{aligned}
\sum_{i=1}^P w_i(t) y_i'(t) &= \sum_{k=-K}^K \sum_{q=-Q}^Q \sum_{m=-M}^M W_k V_q Y_m e^{j((mN_a+k)\omega_c + q(\omega_s - \omega_c))t} e^{jm\gamma} \left(\sum_{i=1}^P e^{j(mN_s+q-k)\nu_i} \right) \\
&= \sum_{k=-K}^K \sum_{q=-Q}^Q \sum_{m=-M}^M W_k V_q Y_m e^{j\left(mN_a\omega_c + k\omega_c + q\frac{N_a}{N_s}\omega_c\right)t} e^{jm\gamma} \left(\sum_{i=1}^P e^{j(mN_s+q-k)\frac{2\pi(i-1)}{P}} \right)
\end{aligned} \tag{4.16}$$

Similar with the ring gear fault case, it can be deduced that the local sun gear fault will introduce additional non-zero frequency components in the measured vibration spectrum at frequency locations corresponding to $mN_s + q - k =$ integer multiple of P . Thus, a local fault on the sun gear can be indicated by frequency components at $\left(mN_a + q\frac{N_a}{N_s} + k\right)\omega_c$ only when $mN_s + q - k =$ integer multiple of P . Such frequency locations are referred as the fault signature, which can be used for feature extraction to detect local sun gear fault in a planetary gear system. Fig. 4.4 shows the simulated spectrum of vibration signal due to a local fault on the sun gear for a four equally-spaced planetary gear-set. It can be observed that additional frequency components appear at

Figure 4.4. Simulated vibration spectrum around gear meshing frequency due to local fault on the sun gear for an equally spaced 4-planets gear-set, $N_a = 99$, $N_s = 29$ and $N_p = 35$.

$\left(mN_a + q \frac{N_a}{N_s} + k \right) \omega_c$ due to the sun gear fault only when $mN_s + q - k = \text{integer}$

multiple of P ($=4$), such as the components at carrier orders of

$$81.93 \left(= 99 - 5 \times \frac{99}{29} + 0 \right) \quad . \quad 91.17 \left(= 99 - 2 \times \frac{99}{29} - 1 \right) \quad \text{and}$$

$$109.2 \left(= 99 + 3 \times \frac{99}{29} + 0 \right).$$

4.3.3 Local fault on a planet gear

If a planet gear, i_0 , has a damaged gear tooth, it will affect the vibration response generated because of its meshing with the annulus gear, as well as the sun gear once every rotation of that planet. However, as other planets are

Figure 4.5. Simulated vibration spectrum around gear meshing frequency due to local fault on a planet gear for an equally spaced 4-planets gear-set, $N_a = 99$, $N_s = 29$ and $N_p = 35$ (a) vibration spectrum and (b) Power spectrum.

assumed to be healthy, the vibration response will not be cancelled completely as indicated by Eq. (4.6) and (4.9) due to additional contributions resulting from the local fault on planet i_0 . Since the planet rotation frequency relative to the carrier is $\omega_c + \omega_p$, the amplitude and phase modulation functions corresponding to planet i_0 fault are periodic with fundamental frequency $\omega_c + \omega_p$. Therefore, the additional frequency components in vibration response spectrum due to damaged tooth on planet i_0 will be located at $m\omega_m + q(\omega_c + \omega_p) + k\omega_c = \left(mN_a + q\frac{N_a}{N_p} + k\right)\omega_c$, where m , q and k can be any integer numbers, which is

the case for the components observed at $84.86\left(=99 - 5 \times \frac{99}{35} + 0\right)$,

$101.8\left(=99 + \frac{99}{35} + 0\right)$ and $107.5\left(=99 + 3 \times \frac{99}{35} + 0\right)$ in Fig. 4.5. Because no

phase cancellation occurs in this case, the spectrum contains more frequency

components. Thus, the variation in amplitude and phase at $\left(mN_a + q\frac{N_a}{N_p} + k\right)\omega_c$

can suggest a local fault at a planet.

4.4 Experimental signal analysis

Figure 4.6. Measured accelerations from AN3 and AN4 around mesh frequency under 25% of rated power and 1200 RPM nominal high speed shaft rotational speed.

The formulation presented till now was experimentally validated based on measurements from a 750 kW test gearbox on a 2.5 MW dynamometer test facility at the National Renewable Energy Laboratory (NREL). This facility is capable of providing static, highly accelerated life and model-in-the-loop tests. The tested gearbox used in this work consists of an equally spaced 3-planets gear-set ($N_a = 99$, $N_s = 21$, $N_p = 39$) and two parallel gear stages that was damaged during its operation on a turbine at the Ponnequin wind farm, CO, USA. Measurements obtained from two accelerometers, AN3 and AN4 mounted on the annulus gear were used for analysis as illustrated in the Section 3.2.2. Figure 4.6 presents the measured acceleration spectrum from this gearbox during the test at 25% of rated power and 1200 RPM nominal high speed shaft rotational speed. After the dynamometer test, the gearbox was sent to a rebuild shop, where it was disassembled and a detailed failure analysis [98, 99] was conducted. This detailed failure analysis indicated damage to the annulus gear and the sun gear of the planetary gear-set. Severe fretting corrosions were found on the sun pinion and moderate gear scuffing and polishing wear on the annulus gear (Fig. 4.7). The details of measured vibration spectrum of AN4 around

Figure 4.7. (a) Fretting corrosion occurred along a line of action at the sun pinion (Credit: GEARTECH, NREL #19750) and (b) polishing wear at the annulus gear (Credit: GEARTECH, NREL #19749).

planetary gear mesh frequency are shown in Fig. 4.8 and compared with predicted results by Eqs. (4.11-4.13). A set of sidebands implying sun gear fault (highlighted in red) can be observed at frequency locations such as $87.57 \left(= 99 - 2 \times \frac{99}{21} - 2 \right)$, $104.7 \left(= 99 + \frac{99}{21} + 1 \right)$ and $110.4 \left(= 99 + 2 \times \frac{99}{21} + 2 \right)$ times the carrier frequency. Frequency sidebands that are integer multiple of the

Figure 4.8. (a) Predicted spectrum for faulty annulus gear, (b) Predicted spectrum for faulty sun gear and (c) measured vibration spectrum around gear meshing frequency for an equally spaced 3-planets gear-set, $N_a = 99$, $N_s = 21$ and $N_p = 39$.

Figure 4.9. Spectrum when the gearbox operated at (a) at 25% of rated power and 1800 RPM and (b) 50% of rated power and 1800 RPM.

number of planets ($=3$) and far from planetary gear meshing frequency of 99^{th} carrier order imply the annulus gear fault (highlighted in blue), such as 87^{th} , 90^{th} , 114^{th} , 117^{th} and 120^{th} carrier orders, which is consistent with the visual inspection shown in Fig. 4.8. Further, this wind turbine planetary gearbox is then tested in the other two operational conditions, which are 25% of rated power and 1800 RPM nominal high speed shaft rotational speed and 50% of rated power and 1800 RPM nominal high speed shaft rotational speed, respectively. The experimental spectra of these two other operational conditions are also given in Fig. 4.9, which exhibit similar vibration patterns around the gear mesh frequency.

4.5 Remarks on developed geometry based model

The vibration response measured by a sensor fixed relative to the annulus gear in an equally-spaced planetary gear system has been investigated in this chapter. A Fourier series analysis explaining the vibration pattern of healthy equally-spaced planetary gear-sets considering both the gear meshes between the planets and the annulus, as well as, gear meshes between the planets and the sun gear is considered. The vibration response model is further explored to consider the effect of local faults that may occur at the annulus gear, any of planet gears, or the sun gear. Additional frequency components resulting due to local faults have been identified, which are located at frequencies that depend on the geometry of the planetary gear-set and the location of the local fault. This result is especially useful to locate frequency components, also referred as a fault signature, that can provide rich fault information for a given planetary gear system. Finally, the theoretical conclusions drawn in this paper are validated on a dynamometer test facility using a 750kW gearbox that was damaged during its operation while installed on a wind turbine. The dynamic simulation results presented herein and the experimental validation of planetary gearbox indicate that the knowledge of fault signatures can be used with such a measurement to assist in feature extraction algorithm for early detection of gearbox failures from a measured vibration spectrum, which may be contaminated with noise and disturbance from other processes.

Chapter 5 Diagnostic Signal Analysis for Monitoring Planetary Gearbox

Spectral analysis techniques to process vibration measurements have been widely studied to characterize the state of gearboxes. However, in practice, the modulated sidebands resulting from the local gear fault are often difficult to extract accurately from an ambiguous/blurred measured vibration spectrum due to the limited frequency resolution and small fluctuations in the operating speed of the machine that often occurs in an industrial environment. To address this issue, a new time-domain diagnostic algorithm is developed and presented herein for monitoring of gear faults, which shows an improved fault extraction capability from such measured vibration signals. This new time-domain fault detection method combines the fast dynamic time warping (Fast DTW) as well as the correlated kurtosis (CK) techniques to characterize the local gear fault, and identify the corresponding faulty gear and its position. Fast DTW is employed to extract the periodic impulse excitations caused from the faulty gear tooth using an estimated reference signal that has the same frequency as the nominal gear mesh harmonic and is built using vibration characteristics of the gearbox operation under presumed healthy conditions. This technique is beneficial in practical analysis to highlight sideband patterns in situations where data is often contaminated by process/measurement noises and small fluctuations in operating speeds that occur even at otherwise presumed steady-state conditions. The extracted signal is then resampled for subsequent diagnostic analysis using CK technique. CK takes advantages of the periodicity of the geared faults; it is used to identify the position of the local gear fault in

the gearbox. Based on simulated gear vibration signals, the Fast DTW and CK based approach is shown to be useful for condition monitoring in both fixed axis as well as epicyclic gearboxes. Finally the effectiveness of the proposed method in fault detection of gears is validated using experimental signals from a planetary gearbox test rig. For fault detection in planetary gear-sets, a window function is introduced to account for the planet motion with respect to the fixed sensor, which is experimentally determined and is later employed for the estimation of reference signal used in Fast DTW algorithm.

5.1 Introduction to signal analysis techniques for diagnosis of gearbox

Early detection of local gear faults in practical industrial environments is crucial to optimize the maintenance schedule and reduce the financial cost of gearbox damage [54, 100]. Vibration based diagnosis using a sensor fixed to a gearbox housing is the most preferred monitoring technique because of the ease of measurement and no interference with the normal operation of the system [3, 98]. If there is any local gear fault, it results in amplitude and frequency modulations in the original vibration measurements whose sidebands are periodic at the shaft rotation frequency and its harmonics [16, 35]. However, gearboxes often operate under some small fluctuation around nominal load/speed conditions during their normal service [42, 101]. These fluctuations result in a variation of both the modulations and their carrier frequencies (gear mesh harmonics) that blurs the sideband components in the spectra of the vibration measurement, often making it difficult to be recognized [44]. Such smearing effect can be abated by the order tracking technique or the time synchronous averaging (TSA) that acquires the measurements synchronized at identical angle increment instead of the identical sampling period [11, 102].

Although TSA is a well-established technique for analyzing gearbox vibration signal [71, 103], its commercial implementation is limited because of the requirement for additional shaft mounted encoders to provide a measure of shaft angular position and sophisticated interpolation algorithms to resample the vibration data. Since such equipment and resources lead to increased cost to applications, they are usually absent in most industrial applications [99]. In such cases, the conventional method is to extract the measurement over a shorter time duration using a sliding window during which the gearbox is presumed to operate under stationary condition. However, these shorter length vibration signals are usually analyzed using Fourier transforms that has limitations such as the limited frequency resolution and spectral leakage, while the small operational speed oscillations continue to exist.

To avoid the extra cost incurred in implementation of TSA and shortcomings of the Fourier transform based analysis, a time domain method that uses dynamic time warping (DTW) was recently employed to detect common faults in a reciprocating compressor through current signals of the driving motor in [73], though no identification or diagnosis of fault signals was demonstrated. In most industrial gearboxes, which contain several gear pairs, the fault identification and diagnosis is of interest as replacement gears can be ordered before the actual disassembly and physical inspection of gears, which in turn reduces the machine downtime [44]. Further, even though DTW algorithm was indicated to be effective in recognition, data mining and signal processing [72], it has an $O(N^2)$ time and space complexity that limits its usefulness only to small time series containing at most a few thousand data points [104]. This makes the DTW algorithm hard to be applied to vibration signal monitoring for

applications such as gearboxes, where data is usually measured for several seconds at a sampling rate which is of the order of thousands of Hertz to capture the characteristic vibration phenomena occurring around the meshing frequency and its harmonics

To address the limitations of DTW time and space complexity, as well as, characterize the local gear fault by identifying the corresponding faulted gear and its position, a new time-domain diagnostic algorithm combining the fast dynamic time warping (Fast DTW) as well as the correlated kurtosis (CK) techniques is proposed herein. Fast DTW runs in linear $O(N)$ time and space complexity [104], has been applied to fault diagnostic application in geared transmissions for the first time in this work to highlight the sideband patterns resulting from the local gear fault. The fault diagnosis algorithm introduces an estimated reference signal that has the same frequency as the nominal gear mesh frequency and is built using vibration characteristics of the gearbox when operating under presumed healthy conditions. The effect of shifting and distortion that usually occurs between a measured signal and an estimated reference signal due to errors in estimation as well as small speed fluctuations is minimized by Fast DTW as it eliminates these distortion effects by allowing an “elastic” stretching or compressing within the two time series to determine the best fit between the estimated reference and measured signal. Thus, it enables an improved time-domain fault diagnosis algorithm wherein the residual signal is more sensitive to faults through filtering out of normal process disturbances. After processing the vibration signal using FAST DTW, the resulting residual signal is resampled for subsequent identification of damaged gear and its position using correlated kurtosis. This step is necessary as the correlated

kurtosis analysis takes advantages of the rotating periodicity of the local gear faults to identify the position of the damaged gear tooth, which is restored during the resampling. The applicability of the technique has been demonstrated through both simulations and experiments. Simulations for monitoring of fixed-axis and epicyclic gear-sets show that the vibration signals processed using the proposed technique have residual signal containing rich fault information which is more sensitive to the gear faults. The effectiveness of this approach is also demonstrated experimentally through measured vibration signals from a 4kW planetary gearbox test rig.

The rest of the paper is organized as follows. In Section 5.2, the proposed time domain approach for gear fault diagnosis based on the Fast DTW and correlated kurtosis using vibration signal is described. Section 5.3 investigates the effectiveness of the proposed method using MATLAB/Simulink simulation studies. Experimental validation using a controlled planetary gearbox test rig is described in Section 5.4. Finally, Section 5.5 concludes the work.

5.2 Proposed algorithm for gearbox fault diagnosis in time-domain

Dynamic time warping has been applied to process data from motor currents to detect mechanical faults in a reciprocating compressor [73]. But the quadratic time and space complexity of this algorithm limits DTW's application. To address this limitation, an improved time domain fault diagnosis method, which combines Fast DTW and correlated kurtosis, based on vibration measurement from a rotary machine is proposed in this work. This approach demonstrates good performance for extracting fault signature from measured vibration data, and is able to identify the local gear fault, i.e., the position of the damaged gear

Figure 5.1. Dynamic time warping of two time series (a) before and (b) after processing.

in the gearbox. The background of dynamic time warping, fast dynamic time warping and correlated kurtosis techniques are summarized in the first part of this section for the ease of the readers. Afterwards, a brief description of the proposed algorithm is presented.

5.2.1 Dynamic time warping

Dynamic time warping technique finds the optimal alignment between two time series by allowing the given time series to be “warped” non-linearly by stretching or shrinking along its time axis. Thus, DTW can be used to determine the similarity between the two time series. Figure 5.1 illustrates the alignment of two time series processed by DTW. Note that, DTW algorithm is able to achieve alignment by non-uniform “warping”, i.e., while the time series Y is shrunk in the start, it is stretched later to align with the time series X .

The dynamic time warping problem can be stated formally as follows: Given two time series, X and Y , of length N and M , respectively.

$$X = x_1, x_2, \dots, x_i, \dots, x_N \quad (5.1)$$

$$Y = y_1, y_2, \dots, y_j, \dots, y_M \quad (5.2)$$

Construct a warp path W :

Figure 5.2. Optimal warping path searched by dynamic programming.

$$W = w_1, w_2, \dots, w_k, \dots, w_K$$

where K is the length of the warp path and the k th element of the warp path is

$$w_k = (i, j) \quad (5.3)$$

such that i is an index from time series X , and j is an index from time series Y .

The warp path must satisfy the following conditions:

- a) $w_1 = (1,1)$ which implies that the warp path must start at the beginning of each time series,
- b) $w_K = (N,M)$ which implies that the wrap path must finish at the end of both time series, and,
- c) if $w_k = (i, j)$, and $w_{k+1} = (i', j')$; then $i' \in (i, i+1)$, $j' \in (j, j+1)$, which implies that between the start and end of the wrap path every index in both of the given time series must be utilized.

Dynamic time warping finds the optimal warp path that minimizes the accumulative distance (usually Euclidean distance) between the two time series by typically using a dynamic programming approach. For this approach, a two dimensional cost matrix $\mathbf{D}$ (also referred as the accumulative distance matrix) of dimension $N \times M$ is constructed. Figure 5.2 shows an example to determine optimal warp path based on the cost matrix for the alignment of two time series signals shown in Fig. 5.1. The detailed dynamic programming approach is described in [72]. This algorithm is quadratic in both time and space complexity, as each cell in cost matrix $\mathbf{D}$ is required to be filled exactly once.

5.2.2 Fast dynamic time warping

As a result of quadratic time and space complexity of DTW algorithm, its implementation to monitoring data containing only around 10K measurement points may require a gigabyte range of memory space. Therefore, speeding up computational time and reducing memory requirement is crucial for successful implementation of DTW algorithm to most vibration based fault diagnosis problems. A multi-level approach, called Fast DTW algorithm, has been developed that runs in linear $O(N)$ time and space complexity [104]. Fast DTW algorithm can be implemented using a three step recursive approach. First, a lower level/resolution time series is created that has half as many points as the input time series. Next, an optimal path in this low level/resolution is found, which is projected to the original higher level/resolution input time series. Finally, the projected path is expanded by a pre-defined radius to form a search window that is passed to the DTW algorithm. Thus, the fast implementation the DTW algorithm only evaluates the cells in the search window rather than the complete cost matrix of $O(N^2)$ dimension. Figure 5.3 shows this three step

Figure 5.3. Window and optimal path evaluated during different level/resolutions when running recursive Fast DTW algorithm with pre-defined radius of 4 cells.

approach of Fast DTW algorithm pictorially using the same time series as given in Fig. 5.1. Shaded cells represent the search window, in which red shaded cells represent the projected path and green shaded cells represent the pre-defined radius.

The performance of the programmed Fast DTW code to remove small fluctuations that can be found in the measured signals can be further evaluated using a simulated case. Consider a test signal $x(t)$ and reference signal $y(t)$:

$$x(t) = \begin{cases} A \cos(2\pi f(t + 0.02t^2)) & 0 \leq t \leq 1, \\ A \cos(2\pi f(t - 0.01t^2) + \beta) & 1 < t \leq 2, \end{cases} \quad \beta = \pi - 2\pi f(1.02) \quad (5.4)$$

$$y(t) = A \cos(2\pi f t) \quad (5.5)$$

where $t \in [0, 2]$, amplitude $A = 1$ and rotational frequency $f = 10$ Hz. For this test, a simulation time step dt of 1×10^{-4} s and pre-defined radius of 20 cells is chosen. Figure 5.4 shows the test signal and reference signal before and after application of Fast DTW algorithm. From Fig. 5.1 and Fig. 5.4, it can be concluded that DTW/Fast DTW can align two similar signals that contain small

Figure 5.4. Fast dynamic time warping of the two exemplified time series (a) before, and (b) after processing.

phase difference or small speed oscillation that is often the case in practical applications. This is an important property which is responsible for the improved sensitivity of residual signal obtained from the fault detection algorithm, which is described in detail in Section 5.2.4.

5.2.3 Correlated kurtosis

A fault in rotating machine such as a local gear fault introduces periodic impacts that appear as impulse-like peaks in the measurements [70]. Kurtosis has been recommended for detection of such peaks in measurement signal [39]. Although, a high kurtosis value for a given data set indicates a presence of a distinct peak, however, kurtosis value decreases when the measured data set contains periodic impulses repeating at the period of a fault. Correlated kurtosis takes advantage of the periodicity of the faults, and therefore, is used in this work to detect periodic impulses introduced by a faulted gear tooth. Correlated kurtosis of M -shift for measured data set X is defined as [70]:

$$CK_M(T) = \frac{\sum_{n=1}^N \left(\prod_{m=0}^M x_{n-mT} \right)^2}{\left(\sum_{n=1}^N x_n^2 \right)^{M+1}} \quad (5.6)$$

Kurtosis = 998, $CK_1(100) = 0$, $CK_2(100) = 0$, $CK_M(T \neq \text{integer multiple of } 100) = 0$

Kurtosis = 98.01, $CK_1(100) = 0.09$, $CK_2(100) = 0.008$,
 $CK_M(T \neq \text{integer multiple of } 100) = 0$

Kurtosis = 98.01, $CK_1(100) = 0.09$, $CK_2(100) = 0.008$,
 $CK_M(T \neq \text{integer multiple of } 100) = 0$

Figure 5.5. CK versus Kurtosis for different signals containing distinct peaks.

where T is the period of interest (the period for fault signature that needs to be detected). Correlated kurtosis of first and second-shift, $M = 1$ or 2 , is studied in this paper as the higher shift CK may lose their numerical precisions in the practical application [70]. From (5.6), it can be seen that the CK value approaches a maximum only when the period of interest T matches with the period of the impulses. Figure 5.5 illustrates the CK_M versus Kurtosis for several simple signals. Fig. 5.5(a) shows a signal with distinct peak, while Fig. 5.5(b) and (c) are defined as $0.3 \sum_{k=0}^{\lfloor n/100 \rfloor} \delta_{n-k100}$ and $0.1 \sum_{k=0}^{\lfloor n/100 \rfloor} \delta_{n-k100}$, respectively. It can be seen that while Kurtosis value decreases as the peaks in

data set become periodic, the CK value increases about the specified period. However, comparing Fig. 5.5(b) and (c), both Kurtosis and CK values are not sensitive to the amplitude of the peaks in the data set. Therefore, to investigate the variations in amplitude of data sets, which indicates the severity of the fault, the RMS value is also calculated for the presented simulations and experimental results in the later sections.

5.2.4 Proposed fault diagnosis algorithm

The key steps of the proposed diagnosis approach in this study are band-pass filtering, reference signal estimation, Fast DTW implementation and residual signal analysis of processed vibration measurement to detect gear faults in gear-sets, which are presented as a flowchart in Fig. 5.6. The details of this approach are given as follows:

Step 1: Band-pass filtering. The measured vibration signal is pre-processed by band-pass filtering around the dominant gear mesh frequency harmonic f_m to remove other gear mesh harmonics as well as remaining non-fault related frequency components resulting from high frequency noise and other process disturbances that are commonly injected in an industrial drives. The pre-

Figure 5.6. Flowchart of the proposed time domain fault detection method.

processed vibration signal is denoted as $x(t)$.

Step 2: Reference signal estimation. An estimated signal is built using vibration characteristics of the gear-set operation under presumed healthy conditions, which has the same frequency as the nominal gear mesh harmonic f_m after the band-pass filtering. This reference signal is used as an input in Step 3. The procedure for estimating the reference signal for both fixed axis as well as the planetary gear-set is described separately. This is because the planetary gear-set vibration signals have more spectral components than a fixed axis gear systems and therefore, the estimation technique is also more involved.

- (1) *Fixed axis gear-set:* Considering an error-free fixed axis gear pair meshing under a constant load and speed, its mesh vibration $y(t)$ around a certain gear mesh frequency harmonic f_m can be expressed as a sinusoidal signal [35]:

$$y(t) = A \cos(2\pi f_m t + \theta) \quad (7)$$

which can be used as the reference signal $y(t)$ in the subsequent Fast DTW algorithm. The amplitude of reference signal is estimated by finding the root mean square (RMS) value for the raw measured vibration signal after band-pass filtering stage, $x(t)$ that has a length of N as:

$$A = \sqrt{2}x_{\text{rms}} = \sqrt{\frac{2}{N} \sum_N x_n^2} \quad (8)$$

The nominal gear mesh frequency f_m can be calculated from the nominal operational speed of the system. Its time axis t is chosen to have the same length N and time step as $x(t)$ in this study. The initial phase θ of the reference signal is estimated by sweeping the phase angle of the reference

signal from 0 to 2π in small increments and finding the phase angle that yields the least mean square error between $x(t)$ and $y(t)$.

- (2) *Planetary gear-set*: When a planet pinion moves with the carrier toward the vibration sensor mounted on the stationary gearbox housing, the level of the measured vibration signal increases, reaches a peak (≤ 1) when the planet is closest to the sensor, and then decreases as the planet gear recedes (≥ 0). The window function $w(t) \in [0, 1]$, models the effect of this amplitude modulation (AM) phenomenon that is periodic with the frequency of carrier rotation [80]. Therefore, the mesh vibration signal $y(t)$ around a certain gear mesh frequency harmonic f_m of a healthy planetary gear-set can be expressed as sinusoidal signal with AM effect [105]:

$$y(t) = w(t + \varphi) (A \cos(2\pi f_m t + \theta)) \quad (5.9)$$

Further, $w(t)$ can be expanded as a Fourier series consisting of a sum of sinusoidal functions containing the carrier rotational frequency and its harmonics as:

$$w(t + \varphi) = \sum_{j=0}^J W_j \cos(2\pi j f_c (t + \varphi) + \psi_j) \quad (5.10)$$

where, f_c is the nominal speed of the carrier that can be calculated from the nominal operational speed of the gear-set. Since the actual pattern of the window function $w(t)$ relates to the structure of the gearbox and the position of the sensor only, the value of amplitude W_j and initial phase ψ_j of j th harmonic in Eq. (5.10) should be independent from carrier rotational frequency f_c . The value of amplitude W_j and initial phase ψ_j of j th harmonic in Eq. (5.10) can be found and stored as pre-defined parameters to describe the window function for a given planetary gearbox. These

values, W_j and ψ_j , are determined using the steps described in Fig. 5.7. First, the measured vibration signal under healthy condition is first passed through a band pass filter around the mesh harmonic of interest. Then, the envelope of the measured vibration signal can be extracted by hardware/software envelope detector/demodulation algorithm. Afterwards, this envelope signal is normalized within the $[0, 1]$ range to form the window function $w(t)$. Further, the order tracking and time synchronous averaging (TSA) techniques can be applied to synchronize the window signal based on the carrier rotational angle to attenuate aperiodic noise. Finally, the discrete Fourier series transform is employed to determine the discrete magnitude spectrum of W_j and discrete phase spectrum of ψ_j . The overall scheme of reference signal estimation for planetary gear-set using these evaluated parameters for window function, W_j and ψ_j , is illustrated in

Figure 5.7. Scheme of determining the pre-defined window function.

Figure 5.8. Reference signal estimation for planetary gear-set.

Fig. 5.8. The initial phase φ of the window function can be found by sweeping φ from 0 to 2π in small increments to find the value of φ that yields the least mean square error between the envelop of $x(t)$ and $w(t + \varphi)$. Further, the amplitude A and the initial phase θ of Eq. (5.9) can be also estimated by the same methods presented in fixed-axis case. Afterwards, Eqs. (5.9) and (5.10) are used to generate the reference signal $y(t)$ as shown in Fig. 5.8 for the planetary gear-set, which is employed in the subsequent Fast DTW algorithm.

Step 3: Fast DTW implementation and residual signal analysis. The two signals, pre-processed measured signal $x(t)$ and estimated reference signal $y(t)$, are matched in time domain using the Fast DTW algorithm described in Section 5.2.2. The aim of applying Fast DTW is to reveal the difference between the two signals $x(t)$ and $y(t)$, which is highlighted by evaluating the residual signal. The raw residual signal is defined as $|x_{warped} - y_{warped}|$, where x_{warped} and y_{warped} are obtained from signals x and y respectively, after transforming with the wrap path obtained from Fast DTW. If the gearbox is operating under ideal healthy condition, the measured vibration signal after band-pass filtering should be similar to the estimated reference signal. Small phase/rotational frequency differences that may occur between x and y because of the typical machine operation characteristics can be removed by warping them along the time axis (similar to the illustrations in Fig. 5.1 and Fig. 5.4 using Fast DTW algorithm). Thus, the residual signal under healthy condition is smoothed out and has lower RMS value when processed by Fast DTW. If a damaged gear exists, it introduces periodic impact/impulse-like response in the measured signal at its characteristic fault period T . Thus, the residual signal under faulty condition

contains periodic peak values with high amplitude. The RMS value of the residual signal is employed to detect variation in the amplitude of the residual. A higher RMS value indicates a larger difference between the measured signal and the reference signal and hence indicates the severity of the fault. Moreover, damaged gear teeth on different gears have different characteristic fault period T related to rotational frequency of the shaft carrying the gear. Thus, by evaluating the $CK_M(T)$ for all possible characteristic fault frequencies arising from possible tooth damage at different gears from the residual signal can identify the position of the fault. The challenge to evaluate the $CK_M(T)$ on the raw residual signal obtained after the application of Fast DTW algorithm is that the length of residual signal K is usually different from the length N of the original signals. The reason for this difference in length of data can be deduced from Fig. 5.2 and constrain (c) on the wrap path wherein it can be observed that the index j' may equal to j in the warping path function $w_k=(i, j)$, $w_{k+1}=(i', j')$.

```

Input:  RZ: Raw Residual Signal —  $\mathbf{RZ} = rz_1, rz_2, \dots, rz_k, \dots, rz_K$ 
        W: Warping path —  $\mathbf{W} = w_1, w_2, \dots, w_k, \dots, w_K$ ;  $w_k = (i, j)$ ,  $w_{k+1} = (i', j')$ 
        —  $i$ : index of vibration signal  $x$ 
        —  $j$ : index of reference signal  $y$  with constant nominal speed
Output: Z: Residual signal after resample —  $\mathbf{Z} = z_1, z_2, \dots, z_n, \dots, z_N$ 

FOR  $k=1:K$ 
{
    IF  $j' = j$ 
    {
         $z_n = \text{Max}(rz_k, rz_{k+1})$ 
    }
    ELSE
    {
         $z_n = rz_k$ 
         $n = n+1$ 
    }
}

```

Figure 5.9. The time resampling algorithm.

Figure 5.10. Performance of time resampling algorithm: (a) test and reference signal, (b) test and reference signal after Fast DTW, (c) the raw residual signal, and (d) the residual signal after the proposed time resample algorithm.

Therefore, a resampling algorithm is required to restore the length of the raw residual signal back to the original measured signal before employing $CK_M(T)$ to identify the fault position. This time resampling algorithm is presented in Fig. 5.9. The capability of the time resample algorithm is illustrated in Fig. 5.10. A periodic reference signal with period of 275 data points and a quasi-periodic test signal with a period fluctuating around the nominal period of 275 data points is shown before and after application of Fast DTW in Fig. 5.10(a) and (b) respectively. The peaks of the raw residual in Fig. 5.10(c) are also a quasi-periodic signal due to the quasi-periodic characteristics of the test signal and the warp path generated by the Fast DTW algorithm. Figure 5.10(d) gives the residual signal that is resampled using the index j of the periodic reference signal along the original time axis of the reference. The period of this residual signal is restored back to 275 data points, which is the same as the reference signal.

5.3 Simulation based investigation of proposed algorithm's performance

The performance of the proposed approach is investigated using both simulations and experiments. In this section, the performance of algorithm is tested by developing an analytical model for a single stage of fixed axis gearbox and a dynamic model for an equally-spaced planetary gearbox.

5.3.1 Detection and location of a gear defect in single stage fixed axis gearbox

A simulated case is presented herein: a single stage fixed axis gear-set with a pinion having 10 teeth and a gear having 13 teeth. Let mesh harmonic order $m = 1$, its amplitude $A_1 = 1$, number of pinion teeth $N_p = 10$, number of gear teeth $N_g = 13$ and rotational frequency of pinion is defined as

Figure 5.11. (a) Simulated signals of the gearbox under healthy and faulty pinion condition $x_h(t)$ and $x_{pf}(t)$; (b) spectra of $x_h(t)$ and $x_{pf}(t)$ after band-pass filter; (c) $x_h(t)$ and reference signal $y(t)$ after initial phase estimation but before Fast DTW; (d) $x_h(t)$ and reference signal $y(t)$ after Fast DTW; (e) $x_{pf}(t)$ and reference signal $y(t)$ after initial phase estimation but before Fast DTW and (f) $x_{pf}(t)$ and reference signal $y(t)$ after Fast DTW.

$$f_s = \begin{cases} 0.08t + 10, t \in [0, 1] \\ -0.04t + 10, t \in [1, 2] \end{cases} \text{ to simulate small fluctuations around the nominal}$$

rotational speed $f_{ns} = 10$ Hz. The amplitude and phase modulation (AM & FM) functions due to a local fault on the pinion are defined as $a(t) = 0.3\cos(2\pi f_s t)$ and $b(t) = 0.1\cos(2\pi f_s t)$. The simulated vibration signals of the gearbox under healthy and pinion faulty condition are generated as $x_h(t)$ and $x_{pf}(t)$, respectively:

$$x_h(t) = A_1 \cos(2\pi N_p f_s t) + n(t) \quad (5.11)$$

$$x_{pf}(t) = A_1(1 + a(t))\cos(2\pi N_p f_s t + b(t)) + n(t) \quad (5.12)$$

where, $n(t)$ is the Gaussian white noise whose SNR=0 dB. This mathematical signal characteristics (Eqs. (5.11 and 5.12)) for healthy and faulty cases respectively has been widely used for simulation studies of gear diagnosis by researchers [3, 35, 44]. Figure 5.11(a) plots the time domain waveform of simulated healthy signal $x_h(t)$ and faulty signal $x_{pf}(t)$, which indicates that the AM-FM features are mostly concealed by the noise. Afterwards, the proposed time domain fault diagnosis approach described in last section is applied to both $x_h(t)$ and $x_{pf}(t)$. Due to the fluctuations in the operational speed, the fault feature cannot be clearly identified even from the spectra obtained after the band-pass filtering of raw signals to remove the high frequency noise (as seen from Fig. 5.11(b)). Figure 5.11(c) and (d) present the simulated vibration signal $x_h(t)$ and the reference signal $y(t)$ before and after application of the proposed Fast DTW process, which is similar as the alignment of the simulated faulty vibration signal $x_{pf}(t)$ and the reference signal $y(t)$ in Fig. 5.11(e) and (f). It can be seen that the simulated and reference signals are aligned together as a result of Fast DTW application. For calculation of $CK_M(T)$, T corresponding to local fault

period of pinion is $T_p = (1 / f_{ns}) / dt = 1000$ while T corresponding to the local fault period for the gear is $T_g = (1 / (N_p f_{ns} / N_g)) / dt = 1300$. Figure 5.12(a) and (b) gives the raw residual signals r_z (just after the application of Fast DTW) and the residual signals z (after subsequent application of the proposed resampling algorithm). $CK_M(T)$ is calculated for both residual signals for healthy as well as faulty case. It can be observed from Fig. 5.12 that the proposed resampling algorithm for the raw residual signal after Fast DTW is necessary to employ $CK_M(T)$ to identify the fault position. For example, the r_z based $CK_1(1000)$

Healthy condition: $RMS=0.06$, $CK_1(T_p)=0.6 \times 10^{-5}$, $CK_1(T_g)=0.7 \times 10^{-5}$; $CK_2(T_p)=0.03 \times 10^{-9}$, $CK_2(T_g)=0.3 \times 10^{-9}$.
 Local pinion fault: $RMS=0.12$, $CK_1(T_p)=1.6 \times 10^{-5}$, $CK_1(T_g)=7.5 \times 10^{-5}$; $CK_2(T_p)=0.4 \times 10^{-9}$, $CK_2(T_g)=7.3 \times 10^{-9}$.

Healthy condition: $RMS=0.06$, $CK_1(T_p)=1.3 \times 10^{-5}$, $CK_1(T_g)=1.9 \times 10^{-5}$; $CK_2(T_p)=0.7 \times 10^{-9}$, $CK_2(T_g)=0.5 \times 10^{-9}$.
 Local pinion fault: $RMS=0.13$, $CK_1(T_p)=23.3 \times 10^{-5}$, $CK_1(T_g)=2.7 \times 10^{-5}$; $CK_2(T_p)=63.9 \times 10^{-9}$, $CK_2(T_g)=1.1 \times 10^{-9}$.

Figure 5.12. Residual signals after Fast DTW (a) the raw residual signals r_z after Fast DTW, and (b) the residual signals z after the proposed time resample.

under healthy and local pinion faulty condition remains at small values 0.6×10^{-5} and 1.6×10^{-5} , respectively. This variation in CK_1 value is insensitive to the fault at the pinion location when compared to the z based $CK_1(1000)$ under healthy and faulty conditions with values of 1.3×10^{-5} and 23.3×10^{-5} respectively. Further, the z based $CK_1(1300)$ is insensitive to pinion fault and remains at smaller value of 2.7×10^{-5} , which indicates that the $CK_1(T)$ value is sensitive only for values of T corresponding to the time period of the characteristic fault frequency and its multiples. Similar trend can be also observed in the CK_2 values in Fig. 5.12.

5.3.2 Dynamic model of planetary gear transmission

The dynamic model of planetary gear system used in this study is given in the Section 3.1.1. The gears are assumed as rigid bodies connected to each other along the line of action through the corresponding gear mesh stiffness and viscous damping [59]. These gears are held by bearings, which allow them to translate in x and y directions and freely rotate about their centers in the x - y transverse plane of gear. Such model has been widely applied to study the dynamics of industrial planetary gearbox. The motion of the sun gear is defined with the translational displacement x_s and y_s , and the angular coordinate θ_s . Similarly, the motion of the carrier is defined by x_c , y_c , and θ_c . α is the pressure angle and ψ_i is the initial angle location for planet i . An error function e_{spi} and e_{rpi} with 5um amplitude error and a profile similar to saw-tooth [106] are used to describe the gear uncertain imperfections such as deviations in the dimensions and shape of the gears due to the manufacturing error as shown in Fig. 5.13(a). Thus, the gear mesh deformation along the line of action between

Figure 5.13. (a) Error function and (b) gear meshing stiffness under healthy and faulty case.

the sun gear and the i th planet gear can be defined as (R is the radius of the base circle):

$$\begin{aligned} \delta_{spi} = & (x_s - x_i + R_c \cos \psi_i) \sin(\alpha - \psi_i) + (y_s - y_i + R_c \sin \psi_i) \cos(\alpha - \psi_i) \\ & + (\theta_s - \theta_c) R_s - [-(\theta_p - \theta_c)] R_p + e_{spi} \end{aligned} \quad (5.13)$$

Similarly, the gear mesh deformation between the ring gear and the i th planet gear can be written as:

$$\begin{aligned} \delta_{api} = & (x_i - R_c \cos \psi_i) \sin(\alpha + \psi_i) - (y_i - R_c \sin \psi_i) \cos(\alpha + \psi_i) \\ & - (\theta_i - \theta_c) R_p - [-(0 - \theta_c)] R_r + e_{api} \end{aligned} \quad (5.14)$$

where, the i th planet pinion has rotation θ_i . x_i and y_i are the translational displacements of the i th planet. If δ_{spi} and $\delta_{api} < 0$, their values are compulsorily set to 0 as the teeth lose contact and the resulting spring force will be equal to zero. The back collisions of the teeth are usually not taken into account in planetary gear-set models, because the gear backlash values of a planetary gear-set are considerably larger than those of fixed-axis gear pair in order to ensure easy assembly and prevent contact on both flanks of the gear teeth [81]. The

global equation of motion for the gearbox that can be expressed in matrix form as:

$$\mathbf{M}\ddot{\mathbf{Q}}(t) + [\mathbf{C} + \mathbf{C}_b]\dot{\mathbf{Q}}(t) + [\mathbf{K}(t) + \mathbf{K}_b]\mathbf{Q}(t) = \mathbf{F}(t) \quad (5.15)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}(t)$ is the time-varying gear meshing stiffness matrix, $\mathbf{C}_b$ and $\mathbf{K}_b$ are the bearing damping and stiffness matrices, $\mathbf{F}(t)$ is the externally applied torques vector, and $\mathbf{Q}(t)$ is the degrees of freedom vector that contains two coordinates for translational vibration and a coordinate for torsional motion for each gear in the plane containing the gear. Figure 15(b) illustrates a square waveform used to describe the time-varying gear meshing stiffness between a gear pair that form the corresponding element in matrix $\mathbf{K}(t)$. The degradation of a gearbox as a fault progresses results in a degradation of the gear mesh stiffness over the life of a gearbox [60, 61]. In this work, the common local gear faults are modelled in dynamic simulation study by assuming a local drop a general squared wave form gear meshing stiffness function (Fig. 5.13(b)). The rationale of this approach is based on the investigation on the variation in the gear meshing stiffness studied in [16], wherein it was concluded that such faults are always accompanied by a local reduction in the gear meshing stiffness. In this paper, the maximum and minimum values of gear meshing stiffness are assumed to be $K_{\max} = 5 \times 10^8$ N/m, $K_{\min} = 3 \times 10^8$ N/m, respectively. The 25% and 50% local stiffness loss ΔK_{loss} is used to simulate the moderate and severe local sun gear faulted cases respectively. The dynamic equations of the lumped parameter of gear-sets are numerically integrated in MATLAB/Simulink environment. A constant input

Table 5.1: System parameters for the example planetary gear system

	Sun	Planet	Ring	Carrier
Number of teeth	28	36	100	
Module (mm)	2	2	2	
Pressure angle (deg)		20	20	
Contact ratio		1.5	1.5	
Mass (kg)	0.41	0.79		5.43
Inertia moment (kg.m ²)	2.53×10^{-4}	6.52×10^{-4}		2.66×10^{-4}
Bearing stiffness (N/m)	100	10^8		10^8
Mean mesh stiffness (N/m)		4×10^8	4×10^8	

torque 300 Nm is exerted on the input side and $k\dot{\theta}_{\text{output}}^2$ is used as a load torque.

The parameter details of the investigated gear-set are provided in the Table 5.1.

5.3.3 Detection and location of a gear defect in planetary gearbox

Vibration signal analysis

In this simulation study, a gear-set having four planets in equally spaced configuration, with sun gear and planets' carrier acting as input and output

Healthy case: $\text{RMS} = 3 \times 10^{-7}$, $\text{CK}_1(T_{\text{sunf}}) = 1.5 \times 10^{-5}$, $\text{CK}_1(T_{\text{ringf}}) = 1.5 \times 10^{-5}$, $\text{CK}_1(T_{\text{planetf}}) = 1.3 \times 10^{-5}$;

Moderate fault: $\text{RMS} = 4 \times 10^{-7}$, $\text{CK}_1(T_{\text{sunf}}) = 2.3 \times 10^{-5}$, $\text{CK}_1(T_{\text{ringf}}) = 1.3 \times 10^{-5}$, $\text{CK}_1(T_{\text{planetf}}) = 1.3 \times 10^{-5}$;

Severe fault: $\text{RMS} = 7 \times 10^{-7}$, $\text{CK}_1(T_{\text{sunf}}) = 2.5 \times 10^{-5}$, $\text{CK}_1(T_{\text{ringf}}) = 1.5 \times 10^{-5}$, $\text{CK}_1(T_{\text{planetf}}) = 1.6 \times 10^{-5}$;

White noise: $\text{CK}_1(T_{\text{sunf}}) = 1.1 \times 10^{-5}$, $\text{CK}_1(T_{\text{ringf}}) = 1.1 \times 10^{-5}$, $\text{CK}_1(T_{\text{planetf}}) = 1.1 \times 10^{-5}$.

Figure 5.14. Simulated residual vibration signals from the planetary gear-set under healthy and local sun gear faulted cases after application of the proposed diagnosis approach.

respectively is investigated. Number of ring gear teeth $N_r = 100$, number of sun gear teeth $N_s = 28$, and number of planet gear teeth $N_p = 36$. The dynamic response under healthy conditions and a local gear tooth defect in the sun gear of the planetary gear-set is simulated using the previously described lumped parameter model. A 15 dB wide band white noise is added to the dynamic responses to simulate the practical measured signal. The nominal rotational frequency for sun f_s , carrier f_c and planets f_p are 69.56 Hz, 15.21 Hz and 27.05 Hz under steady operating conditions, respectively. The simulated signal is then pre-processed using a band-pass filter with the central frequency around the gear mesh frequency $f_m = f_c N_r = 1521$ Hz. For this planetary gear-set at the given simulated operating conditions, the local sun gear fault frequency can be evaluated as $f_{sunf} = f_m / N_s = (f_s - f_c) = 54.35$ Hz. Therefore, for evaluation of correlated kurtosis, T corresponding to local sun gear fault is $T_{sunf} = (1 / f_{sunf}) / dt$. Similarly, T corresponding to local ring gear fault is $T_{ringf} = (1 / f_{ringf}) / dt$, and T value corresponding to local planet gear fault is $T_{planetf} = (1 / f_{planetf}) / dt$, respectively. Figure 5.14 presents the simulated residual vibration signals under healthy and local sun gear tooth faulted conditions. The evaluated CK_1 with $T = T_{sunf}$, T_{ringf} , and $T_{planetf}$, and RMS values of the residual signals are shown for different fault severity level. Additionally, the CK values of a white noise are also given as a reference in Fig. 5.14. It can be observed that the RMS values of the residual signals increase when the fault severity increases. $CK_1(T_{sunf})$ is found to have significant increase in presence of faults that indicates the gear fault happens at the sun gear. However, $CK_1(T_{ringf})$ and $CK_1(T_{planetf})$ are still remaining at smaller values comparable to the white noise case, which increase or decrease somewhat under the local sun gear tooth fault conditions.

Comparing these CK_1 values in Fig. 5.14, it implies that strong periodic components resulting from local gear fault happen and only happen at the local sun gear fault frequency in the residual signal after the proposed fault diagnosis algorithm. Consequently, a selection of appropriate thresholds on the RMS and CK_1 value can allow for detection of gear faults and its position.

Thus, from simulations of fixed axis as well as planetary gear-sets, it can be seen that a selection of appropriate thresholds on the RMS and CK_M values can enable detection of gear faults and its position using the proposed algorithm.

Electrical current signal analysis

Electrical current signal analysis provides an alternative and non-intrusive way to detect mechanical faults through electrical signatures and have attracted researchers' interest for gearbox diagnostics. Thus, the integrated electro-mechanical model is used to investigate the capability of the proposed diagnosis algorithm on processing electrical signal, which couples a planetary gear-set with an AC generator programmed in MATLAB/Simulink as developed in the Section 3.1.3. Figure 5.15 gives the residual signal under healthy condition and local sun gear faulty condition. Periodic peaks can be clearly observed in the residual signal in presence of local sun gear fault. Under healthy condition, the RMS value and $CK_1(T_{sunf})$ for this residual signal equals to $1.5e-4$ and $8.4e-5$, respectively. $CK_1(T_{ringf})$ and $CK_1(T_{planetf})$ equal to $7.9e-5$ and $7.6e-5$. When local sun gear fault is introduced in the simulation, the RMS value and $CK_1(T_{sunf})$ of the corresponding residual signal is found to increase dramatically to $2.1e-4$ and $23.7e-5$, respectively. However, $CK_1(T_{ringf})$ and $CK_1(T_{planetf})$ decreases or remain at nearly the same value of $8.1e-5$ and $9.9e-5$ respectively. Additionally, the CK values of a white noise are also given in Fig. 5.15. These CK_1 values

Health: $CK_1(T_{sunf}) = 8.4 \times 10^{-5}$, $CK_1(T_{ringf}) = 7.9 \times 10^{-5}$ and $CK_1(T_{planetf}) = 7.6 \times 10^{-5}$
 Sun gear fault: $CK_1(T_{sunf}) = 23.7 \times 10^{-5}$, $CK_1(T_{ringf}) = 8.1 \times 10^{-5}$, $CK_1(T_{planetf}) = 9.9 \times 10^{-5}$
 White noise: $CK_1(T_{sunf}) = 7.2 \times 10^{-5}$, $CK_1(T_{ringf}) = 7 \times 10^{-5}$ and $CK_1(T_{planetf}) = 6.9 \times 10^{-5}$

Figure 5.15. Residual signal under sun gear faulty case.

implies that strong periodic components resulting from tooth damaged gear happen and only happen at the local sun gear fault frequency in the residual signal after the proposed fault diagnosis algorithm. Thus, a selection of appropriate thresholds on the RMS and CK_1 value can allow for detection of gear faults and its position. Simulation results show that the proposed diagnosis algorithm also provides an effective and easy implementation of time domain approach to detect faults and identify its location in planetary gear based on the electrical stator current signal [107].

5.4 Experimental results

The proposed Fast DTW algorithm along with fault identification using correlated kurtosis was applied on experimentally measured vibration signals from a planetary gearbox test rig. The proposed Fast DTW algorithm along with fault identification using correlated kurtosis was applied on experimentally

Figure 5.16. The pre-defined window function for the planetary gearbox on the test rig as a sum of sinusoidal signals that are periodic at carrier rotation harmonics (a) discrete magnitude spectrum of W_j and (b) discrete phase spectrum of ψ_j .

measured vibration signals from the 4KW laboratory planetary gearbox test rig.

The details of the test rig are introduced in the Section 3.2.1.

The vibration transmission function from the gear mesh to sensor location described in Step 2 in Section 2.4 can be experimentally obtained through demodulation of the measured vibration signal under healthy condition with controlled constant speed to determine the amplitude modulation (AM) function as illustrated in Fig. 5.7. The magnitude of this AM function is then normalized from 0 to 1. Figure 5.16 shows the pre-defined AM function of the tested planetary gearboxes that is estimated from the envelope signal of the measured vibration of the test rig under presumed healthy condition. Since according to Eq. (10) the j th component of window function is located at j th harmonic of carrier frequency, therefore, the carrier order instead of frequency is shown on the x-axis of Fig. 5.16. Further note that, this pre-defined AM function contains the vibration contribution from the manufacturing errors that may exist under the presumed healthy conditions at the beginning of the test.

Figure 5.17. Measured vibration spectrum around gear mesh frequency.

The vibration signal is measured with motor operating under open-loop speed control mode with nominal speed of 1400 RPM. However, small deviations and fluctuations of the operational speed could be observed as evident from the measured spectra shown in Fig. 5.17. Figure 5.17 illustrates the measured vibration spectrum around the nominal gear mesh frequency $f_m = 1400/60/(GN_r) = 490$ Hz for both healthy as well as ring gear fault conditions. It can be observed that the measured peak of gear mesh frequency deviates from its nominal value and the spectra are blurred due to the inherent speed fluctuations during the test. Afterwards the proposed time domain fault diagnosis approach described in Section 5.2.4 is applied to the pre-processed data filtered with a FIR band-pass filter with the central frequency 490 Hz and bandwidth 70 Hz used to extract the signal around the gear mesh frequency. The characteristic local ring gear fault frequency is $f_{ringf} = f_c = f_m / N_r = 5.83$ Hz, a corresponding T_{ringf} equals to $(1 / f_{ringf}) / dt$ is chosen to evaluate the CK_M value. As N_s equals to N_p in this planetary gear-set, T_{sunf} also equals to $T_{planetf}$. The residual signals obtained after application of Fast DTW are presented in Fig. 5.18(a). This residual signal clearly highlights the differences between

Figure 5.18. The residual signal between the measured vibration signal and reference signal (a) after the proposed diagnosis approach processing (b) only after initial phase matching.

the measured vibration signal and the reference signal obtained during the healthy and ring gear fault conditions. The RMS, increase from 1.2×10^{-4} under the healthy condition to 4.9×10^{-4} under the ring gear faulty condition. $CK_1(T_{ringf})$ and $CK_2(T_{ringf})$ also increases from 7.9×10^{-5} and 6.8×10^{-9} under the healthy condition to compared large values 19.7×10^{-5} and 47.7×10^{-9} , which imply that a local pinion gear fault exists. On the other hand, while $CK_1(T_{sunf}) = CK_1(T_{planetf})$ and $CK_2(T_{sunf}) = CK_2(T_{planetf})$ also increase to 8.2×10^{-5} and 8.9×10^{-9} , they still remains at comparatively small values. The effectiveness of proposed algorithm for fault detection from practical measured signals is further demonstrated in Fig. 5.18(b), which presents the residual

signal obtained from measured vibration signal and reference signal without transforming either signal using the warp path obtained using Fast DTW. It can be observed that the residual signal obtained for healthy and a ring gear fault condition does not contain any statistical differences, and it is difficult to detect gear faults without application of Fast DTW. Thus, it can be concluded that Fast DTW processing optimally matches the measured vibration signal to reference signal that results in more accurate feature extraction using the proposed approach.

5.5 Remarks on proposed diagnostic analysis method

Industrial gearboxes usually exhibit small fluctuations in speed and load around nominal operating conditions. Under these real industrial conditions, the usual Fourier transform based approach has limitations as the fault signature often appears to be blurred and is difficult to identify. To address the problem of real fast varying speed fluctuations within small range, a new time domain approach to detect the local gear faults and identify their location from measured vibration signal is presented in this paper. This new time-domain fault detection method combines the fast dynamic time warping (Fast DTW) as well as the correlated kurtosis (CK) techniques. The performance and applicability of the proposed approach is investigated using analytical and dynamic simulation of fixed axis and planetary gear systems, as well as measured vibration signal from a planetary gearbox. The mathematical modeling, computer simulation and experimental results indicate, (a) the proposed approach is able to detect gear faults, identify their location and have potential to assess the degradation level of the gear mesh stiffness (b) only the correlated kurtosis corresponding to particular time interval that is related to the

fault signature is sensitive and shows a significant increase in its value in presence of gear tooth fault, (c) the residual signal obtained after application of Fast DTW to experimentally measured vibration signals highlights fault signals more clearly as compared to evaluated residual signal without the application of Fast DTW. Thus, the presented diagnosis approach in this paper is useful for developing automatic diagnostic process in complex industrial machinery systems include both practical fixed axis as well as epicyclic gearboxes.

Chapter 6 Conclusion and Future Works

6.1 Conclusions

A geometry-based analytical vibration models for describing the vibration spectrum obtained from planetary gearboxes in presence of faults has been presented in this thesis. This developed amplitude modulated/frequency modulated (AM-FM) signal model has been successfully tested and validated by diagnosing faults in a 750 kW planetary gearbox that as damaged during its installation in a wind turbine. This gearbox was tested within the 2.5 MW dynamometer test facility at the National Renewable Energy Laboratory (NREL). Compared with other published gearbox vibration signal models, the geometry-based analytical vibration model presented in this thesis provides a quick and efficient prediction of dominant frequency components observed in the vibration spectrum measured by a fixed sensor mounted on the stationary ring gear in case of a gear damage. Further, this analytical model also provides a succinct mathematical understanding of the reason for occurrence of additional frequency components in presence of gear faults and its relationship to the geometry of the planetary gear set. This contribution of research work has been published in *Mechanism and Machine Theory*. The advantages of the developed analytical model are summarized as follows:

- (1) The model presented in this thesis requires only the basic knowledge of the gearbox geometry, i.e., the number of teeth on annulus, sun and planet gears, number of planets and locations of planets, and is easily implemented by the end-users and maintenance engineers; unlike other complex and involved dynamic modeling task requiring much

consideration and careful adaptations to enable their practical implementation.

- (2) Previous research work usually used Hanning windows to describe and analyze the AM effect of the vibration transfer path due to the relative motion between meshing location and the fixed sensor. However, the actual transfer path window function of the planetary gearbox is unknown. The results derived in this thesis using Fourier series analysis is robust to the different possible transfer path window functions as the only prerequisite is that the window function is periodic at f_{carrier} .
- (3) The AM-FM model presented in this thesis predicts the dominant components as observed in the experimentally obtained vibration spectrum, which is measured using a fixed sensor attached to the stationary ring gear. These dominant frequency components are expected in frequency regions in the immediate neighborhoods of the gear meshing frequency and only at planet carrier rotational harmonics that are integer multiples of the number of equi-spaced planets.
- (4) As a result of the ability of extended AM-FM vibration signal model to describe the phase cancellation of vibration contribution from individual planet-ring and planet-sun gear meshes, it provides a deeper understanding of the observed vibration spectrum from healthy and faulty planetary gearboxes. This deeper understanding of the vibration spectrum measured by sensor fixed to stationary gear explains why the components such as meshing frequency and fault sideband components that may appear in the fixed-axis gear pair, sometimes do not appear at corresponding locations in the observed experimental spectrum of the

planetary gear system. This is due to the phase cancellation of vibration contributions from individual planets that occurs due to the planet phase relationships defined by the planets' angular positions with respect to the sun gear. Such deeper understanding is important to provide more specific and accurate predication when the fault exists on the ring gear or sun gear.

Measured vibration signals from usual industrial gearboxes usually exhibit small fluctuations in speed and load under nominal operating conditions. Under these circumstances, the sidebands components resulting from faulty vibration signals due to fault induced modulations are often difficult to extract accurately from an ambiguous/blurred measured vibration spectrum due to the limited frequency resolution and small fluctuations in the operating speed of the machine. Therefore, a new time domain approach, which combines the fast dynamic time warping (Fast DTW) as well as the correlated kurtosis (CK) techniques, to detect gear faults and identify their location from measured vibration signal has been presented in this thesis. The proposed algorithms presented in this thesis are illustrated and validated by using dynamic simulations, experiments on a 4 KW laboratory planetary gearbox test rig and monitoring. The simulation and experimental results indicate that it provides an effective and easy implementation of time domain approach to detect gear faults and identify its corresponding locations in practical gearbox operation conditions. Such analytical model and diagnostic method can assist in feature extraction algorithm for early detection of gearbox failures from a measured vibration spectrum, which are often contaminated with several other components resulting from noise and other processes. A manuscript describing

this novel technique in detail has been published in *Journal of Sound and Vibration*.

In view of these conclusions, the main contributions of this research to the state of-the-art can be listed as follows:

- i. A mathematical AM-FM (amplitude modulated/frequency modulated) signal model is developed to simulate the distinct vibration spectrum of planetary gearboxes operating under healthy and faulty conditions;
- ii. Fourier series analysis based on this mathematical signal model explains the measured vibration pattern to identify additional frequency components that depend on the geometry of the planetary gear-set and the location of the local faults;

A new time domain approach that combines dynamic time warping and correlated kurtosis is proposed to detect gear faults and identify their location in this thesis for assisting in development of robust fault alarm system.

6.2 Future works

Future work will focus on the following issues:

- I. Knowledge of the dynamic behaviors of planetary gears is crucial for developing monitoring strategies. Although the two-dimensional, spur planetary gears with equally spaced planets have been studied in this thesis, usually the industrial planetary gear sets employ helical gears that require modeling of their three-dimensional motion, three-dimensional gear mesh interface, and the gear-shaft bodies supported by bearings at arbitrary locations along the shafts especially when the planetary gear-

set operates around their resonance region. The lumped parameter model obtained for such three dimensional planetary gear-set will have significantly more degrees of freedom.

- II. Compound faults refer to the kind of failure wherein several faults occur simultaneously in the gearbox with their features getting coupled together. Due to the complexity of gearbox operational condition, the faults emerged in practical gearboxes often appear as the compound faults. Compound faults are highly detrimental to mechanical parts and algorithms to detect such compound faults are not well developed. Because several faults modes are mixed together and interfere with each other, extraction of fault features more challenging. Classification techniques; such as the support vector machine, artificial neural networks and genetic algorithm; will be studied to discriminate the failure modes and assess their severity.

In condition monitoring system, prognosis is an important component that possesses the ability to predict accurately and precisely the future condition and remaining useful life of a failing component. Therefore, algorithms for damage level estimation, damage propagation and the remaining useful life estimation are crucial topics to study further.

References

- [1] Anonymous, "Gearboxes-the Achilles heel of wind power engineering (Special Focus Article)," *Windpower Monthly*, vol. 21, 2005.
- [2] David M. Blunta, Jonathan A. Keller, "Detection of a fatigue crack in a UH-60A planet gear carrier using vibration analysis vibration analysis," *Mechanical Systems and Signal Processing*, vol. 20, pp. 2095-2111, 2006.
- [3] Wilson Q. Wang, Fathy Ismail, M. Farid Golnaraghi, "Assessment of gear damage monitoring techniques using vibration measurements," *Mechanical Systems and Signal Processing*, vol. 15, pp. 905-922, 2001.
- [4] William D. Mark, J. A. Hines, "Stationary transducer response to planetary-gear vibration excitation with non-uniform planet loading," *Mechanical Systems and Signal Processing*, vol. 23, pp. 1366-1381, 2009.
- [5] David G. Lewicki, Kelsen E. LaBerge, Ryan T. Ehinger, Jason Fetty, "Planetary Gearbox Fault Detection Using Vibration Separation Techniques," *NASA/TM-2011-217127*, 2001.
- [6] J. McNames, "Fourier series analysis of epicyclic gearbox vibration," *ASME Journal of Vibration and Acoustics*, vol. 124, pp. 150-152, 2002.
- [7] P. D. McFadden, J. D. Smith, "An explanation for the asymmetry of the modulation sidebands about the tooth meshing frequency in epicyclic gear vibration," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 199, pp. 65-70, 1985.
- [8] S. Abouel-seoud, I. Ahmed, M. Khalil, "An Experimental Study on the Diagnostic Capability of Vibration Analysis for Wind Turbine Planetary Gearbox," *International Journal of Modern Engineering Research*, vol. 2, pp. 667-675, 2012.
- [9] Bin Zhang, Khawaja T., Patrick R., Vachtsevanos G., "Application of blind deconvolution de-noising in failure prognosis," *IEEE Transactions of Instrumentation and Measurement*, vol. 58, 2009.
- [10] Chaochao Chen, George Vachtsevanos, Marcos E.Orchard, "Machine remaining useful life prediction: An integrated adaptive neuro-fuzzy and high-order particle filtering approach," *Mechanical Systems and Signal Processing*, vol. 28, pp. 597-607, 2012.
- [11] Luisa F. Villa, Renones Anibal, Jose R. Peran, Luis J. de Miguel, "Angular resampling for vibration analysis in wind turbines under non-linear speed fluctuation," *Mechanical Systems and Signal Processing*, vol. 25, pp. 2157-2168, 2011.
- [12] Hong Liu, Jaspreet Singh Dhupia, Sheng Shuangwen, "An explanation of frequency features enabling detection of faults in equally-spaced planetary gearbox.", *Mechanism and Machine Theory*, vol. 73, pp. 169-183, 2014.
- [13] Hong Liu, Jaspreet Singh Dhupia, "A time domain approach to diagnose gearbox faults based on measured vibration signals ", *Journal of Sound and Vibration*, vol. 333, issue 7, pp. 2164–2180, 2014.
- [14] P. Lynwander, "Gear Drive Systems," *Mechanical Engineering*, 1983.

- [15] Lai Wuxing, Peter W. Tse, Zhang Guicai, Shi Tielin, "Classification of gear faults using cumulants and radial basis function network," *Mechanical Systems and Signal Processing*, vol. 18, pp. 381-389, 2004.
- [16] Chaari Fakher, Walid Baccar, Abbes S. Mohamed, Haddar Mohamed, "Effect of spalling or tooth breakage on gearmesh stiffness and dynamic response of a one-stage spur gear transmission," *European Journal of Mechanics, A/Solids*, vol. 27, pp. 691-705, 2008.
- [17] Rui Man, Yushu Chen, Qingjie Cao, "Research on dynamics and fault mechanism of spur gear pair with spalling defect," *Journal of Sound and Vibration*, vol. 331, pp. 2097-2109, 2012.
- [18] BinQiang Chen, ZhouSuo Zhang, YanYang Zi, ZhengJiaHe, ChuangSun, "Detecting of transient vibration signatures using an improved fast spatial-spectral ensemble kurtosis kurtogram and its applications to mechanical signature analysis of short duration data from rotating machinery," *Mechanical Systems and Signal Processing*, 2013.
- [19] Jimeng Li, Xufeng Chen, Zhengjia He, "Adaptive stochastic resonance method for impact signal detection based on sliding window," *Mechanical Systems and Signal Processing*, vol. 36, pp. 240-255, 2013.
- [20] J. Lin. M. J. Zuo, "Gearbox fault diagnosis using adaptive wavelet filter," *Mechanical Systems and Signal Processing*, vol. 17, pp. 1259-1269, 2003.
- [21] Tomasz Barszcz, Robert B. Randall, "Application of spectral kurtosis for detection of a tooth crack in the planetary gear of a wind turbine," *Mechanical Systems and Signal Processing*, vol. 23, pp. 1352-1365, 2009.
- [22] P. Velez. M. Maatar, "A mathematical model for analyzing the influence of shape deviations and mounting errors on gear dynamic behaviour," *Journal of Sound and Vibration*, vol. 191, pp. 629-660, 1996.
- [23] D. Felten, "Understanding Bearing Vibration Frequencies," www.easa.com, 2003.
- [24] T. A. Harris, "Rolling Bearing Analysis," Wiley-Interscience, SBN 0-471-35457-0, 4th Edition., 2000.
- [25] Robert B. Randall, Jerome Antoni, "Rolling element bearing diagnostics—A tutorial," *Mechanical Systems and Signal Processing*, vol. 25, pp. 485-520, 2011.
- [26] Robert B. Randall, "Detection and diagnosis of incipient bearing failure in helicopter gearboxes," *Engineering Failure Analysis*, vol. 11, pp. 177-190, 2004.
- [27] N. Sawalhi, R. B. Randall, "Vibration response of spalled rolling element bearings: Observations, simulations and signal processing techniques to track the spall size," *Mechanical Systems and Signal Processing*, vol. 25, pp. 846-870, 2011.
- [28] M. Copping, "Vibration Analysis Reporting - Bearing Failure Stages & Responses."
- [29] John Piotrowski, "The Importance of Shaft Alignment.", by Maintenance Resources, Inc.
- [30] Mehmet Akar, "Detection of a static eccentricity fault in a closed loop driven induction motor by using the angular domain order tracking

- analysis method," *Mechanical Systems and Signal Processing*, vol. 34, pp. 173-182, 2013.
- [31] Luisa F.Villa, Anibal Renones, Jose R. Peran, Luis J. de Miguel, "Statistical fault diagnosis based on vibration analysis for gear test-bench under non-stationary conditions of speed and load," *Mechanical Systems and Signal Processing*, 2011.
 - [32] Bob Boyle, "Tracking the causes of coupling failure," *PlanetServices*, 2008.
 - [33] Troy Feese, Charles Hill, "Guidelines for preventing torsional vibration problems in reciprocating machinery," presented at *Gas Machinery Conference*, 2002.
 - [34] M. Williamson, "Controlling Gearbox Contamination," by Machinery Lubrication Inc., 2002.
 - [35] P. D. Mcfadden, "Detecting fatigue cracks in gears by amplitude and phase demodulation of the meshing vibration," *Journal of Vibration, Acoustics, Stress and Reliability in Design*, vol. 108, pp. 165-170, 1986.
 - [36] William D. Mark, Hyungdae Lee, Romano Patrick, Joseph D. Coker, "A simple frequency-domain algorithm for early detection of damaged gear teeth," *Mechanical Systems and Signal Processing*, vol. 24, pp. 2807-2823, 2010.
 - [37] Wenyi Wang, Albert K. Wong, "Autoregressive model-based gear fault diagnosis," *Journal of Vibration and Acoustics*, 2002.
 - [38] Jianming Yang, "Vibration analysis on multi-mesh gear-trains under combined deterministic and random excitations," *Mechanism and Machine Theory*, vol. 59, pp. 20-33, 2013.
 - [39] Wenyi Wang, "Early detection of gear tooth cracking using the resonance demodulation technique," *Mechanical Systems and Signal Processing*, vol. 15, pp. 887-903, 2001.
 - [40] Zhihong Man, Wenyi Wang, Suiyang Khoo, Juliang Yin, "Optimal sinusoidal modelling of gear mesh vibration signals for gear diagnosis and prognosis," *Mechanical Systems and Signal Processing*, vol. 33, pp. 256-274, 2012.
 - [41] Victor Girondin, Komi Midzodzi Pekpe, Herve Morel, Jean-Philippe Cassar, "Bearings fault detection in helicopters using frequency readjustment and cyclostationary analysis," *Mechanical Systems and Signal Processing*, vol. 38, pp. 499-514, 2013.
 - [42] Walter Bartelmus, Radoslaw Zimroz, Mohamed Haddar, "Modelling of gearbox dynamics under time-varying nonstationary load for distributed fault detection and diagnosis," *European Journal of Mechanics A/Solids* vol. 29, pp. 637-646, 2010.
 - [43] Robert B. Randall, "Rolling element bearing diagnostics—A tutorial," *Mechanical Systems and Signal Processing*, vol. 25, pp. 485-520, 2011.
 - [44] Chuan Li, Ming Liang, "Time–frequency signal analysis for gearbox fault diagnosis using a generalized synchrosqueezing transform," *Mechanical Systems and Signal Processing*, 2011.
 - [45] Jinde Zheng, Junsheng Cheng, Yu Yang, "Generalized empirical mode decomposition and its applications to rolling element bearing fault diagnosis," *Mechanical Systems and Signal Processing* , vol 40, pp. 136-153.

- [46] J. R. Mathews, "Acoustic emission," *Gordon and Breach Science Publishers Inc, ISSN 0730-7152*, 1983.
- [47] Blitz J., Simpson G., "Ultrasonic Methods of Non-Destructive Testing," *Springer-Verlag New York, LLC*, 1991.
- [48] N. Tandon, A. Choudhury, "A review of vibration and acoustic measurement methods for the detection of defects in rolling element bearings," *Tribology International*, vol. 32, pp. 469-480, 1999.
- [49] Babak Eftekharnjad, D. Mba, "Seeded fault detection on helical gears with acoustic emission," *Applied Acoustics*, vol. 70, pp. 547-555, 2009.
- [50] Tim Toutountzakis, Tan Chee Keong, David Mba, "Application of acoustic emission to seeded gear fault detection," *NDT&E International*, vol. 38, pp. 27-36, 2005.
- [51] D. Mba, Raj B.K.N. Rao, "Development of Acoustic Emission Technology for Condition Monitoring and Diagnosis of Rotating Machines; Bearings, Pumps, Gearboxes, Engines and Rotating Structures," *The Shock and Vibration Digest*, vol. 38, pp. 3-16, 2006.
- [52] Chee Keong Tan, D. Mba, "Limitation of Acoustic Emission for Identifying Seeded Defects in Gearboxes," *Journal of Nondestructive Evaluation*, vol. 24, 2005.
- [53] Chinmaya Kar, A. R. Mohanty, "Multistage gearbox condition monitoring using motor current signature analysis and Kolmogorov–Smirnov test," *Journal of Sound and Vibration*, vol. 290, pp. 337-368, 2006.
- [54] N. Feki, G. Clerc, Ph. Vexé, "An integrated electro-mechanical model of motor-gear units—Applications to tooth fault detection by electric measurements," *Mechanical Systems and Signal Processing*, 2011.
- [55] Chinmaya Kar, A. R. Mohanty, "Monitoring gear vibrations through motor current signature analysis and wavelet transform," *Mechanical Systems and Signal Processing*, vol. 20, pp. 158-187, 2006.
- [56] Nicole J. Kessissoglou, Zhongxiao Peng, "Integrating Vibration and Oil Analysis for Machine Condition Monitoring," *Proceedings of the International Conference on Condition Monitoring*.
- [57] S. Ebersbach, Z. Peng, N. J. Kessissoglou, "The investigation of the condition and faults of a spur gearbox using vibration and wear debris analysis technique," *Wear*, vol. 260, pp. 16-24, 2006.
- [58] Yong-Sung Choi, Hyang-Kon Kim, Chung-Seog Choi, Kyung-Sup Lee, "On-line Condition Monitoring and Diagnostics of Power Distribution Equipment," *the Proceedings of the 2008 International Conference on Condition Monitoring and Diagnosis*.
- [59] M.T. Khabou, N. Bouchaala, F. Chaari, T. Fakhfakh, "Study of a spur gear dynamic behavior in transient regime," *Mechanical Systems and Signal Processing*, vol 25, pp. 3089–3101, 2011.
- [60] W. Bartelmus, "Gearbox damage process," presented at *9th International Conference on Damage Assessment of Structures (DAMAS 2011)*.
- [61] M. Chandra, R. Langari, "Gearbox degradation identification using pattern recognition techniques," presented at *2006 IEEE International Conference on Fuzzy Systems*.
- [62] N. Saravanan, V.N.S. Kumar Siddabattuni, "A comparative study on classification of features by SVM and PSVM extracted using Morlet

- wavelet for fault diagnosis of spur bevel gear box," *Expert Systems with Applications*, vol. 25, pp. 1351-1366, 2008.
- [63] K. I. R. N. Saravanan, "Incipient gear box fault diagnosis using discrete wavelet transform (DWT) for feature extraction and classification using artificial neural network (ANN)," *Expert Systems with Applications*, vol. 37, pp. 4168-4181, 2010.
 - [64] Zhixiong Li, Xinpeng Yan, Chengqing Yuan, Zhongxiao Peng, Li Li, "Virtual prototype and experimental research on gear multi-fault diagnosis using wavelet-autoregressive model and principal component analysis method," *Mechanical Systems and Signal Processing*.
 - [65] Ian Howard, Shengxiang Jia, Jiande Wang, "The dynamic modelling of a spur gear in mesh including friction and a crack," *Mechanical Systems and Signal Processing*, vol. 15, pp. 831-853, 2001.
 - [66] D. C. H. Yang, Z. S. Sun, "A rotary model for spur gear dynamics," *ASME Journal of Mechanical Transmissions and Automation in Design*, vol. 107, pp. 529-535, 1985.
 - [67] S. Du, R. B. Randall, D. W. Kelly, "Modelling of spur gear mesh stiffness and static transmission error."
 - [68] Zhang Jidong, Hong Liu, Jaspreet Singh Dhupia, "Gear Fault Detection in Planetary Gearbox using Stator Current Measurement of AC Motors," presented at *2012 ASME Dynamic Systems and Control Conference*.
 - [69] J. Ma, C. J. Li, "Gear defect detection through model-based wideband demodulation of vibrations," *Mechanical Systems and Signal Processing*, vol. 10, pp. 653-665, 1996.
 - [70] G.L. McDonald, Q. Zhao, Ming J. Zuo, "Maximum correlated kurtosis deconvolution and application on gear tooth chip fault detection," *Mechanical Systems and Signal Processing* vol. 33, pp. 237-255, 2012.
 - [71] S. Braun, "The synchronous (time domain) average revisited," *Mechanical Systems and Signal Processing*, vol. 25, pp. 1087-1102, 2011.
 - [72] P. Senin, "Dynamic time warping algorithm review."
 - [73] D. Zhen, T. Wang, F. Gu, A. D. Ball, "Fault diagnosis of motor drives using stator current signal analysis based on dynamic time warping," *Mechanical Systems and Signal Processing*, vol. 34, pp. 191-202, 2013.
 - [74] A. Jablonski, T. Barszcz, "Validation of vibration measurements for heavy duty machinery diagnostics," *Mechanical Systems and Signal Processing*, vol. 38, pp. 248-263, 2013.
 - [75] Qingbo He, Xiangxiang Wang, "Time-frequency manifold correlation matching for periodic fault identification in rotating machines," *Journal of Sound and Vibration*, vol. 332, pp. 2611-2626, 2013.
 - [76] J. Antoni, "The spectral kurtosis: a useful tool for characterising non-stationary signals," *Mechanical Systems and Signal Processing*, vol. 20, pp. 282-307, 2006.
 - [77] Farag K. Omar, A. M. Gaouda, "Dynamic wavelet-based tool for gearbox diagnosis," *Mechanical Systems and Signal Processing*, vol. 26, pp. 190-204, 2012.
 - [78] F. Chaari, T. Fakhfakh, M. Haddar, "Dynamic Analysis of a Planetary Gear Failure Caused by Tooth Pitting and Cracking," 2006.

- [79] Zhipeng Feng, Ming Liang, Fulei Chu, "Recent advances in time–frequency analysis methods for machinery fault diagnosis: A review with application examples," *Mechanical Systems and Signal Processing*.
- [80] M. Inalpolat, A. Kahraman, "A theoretical and experiment investigation of modulation sidebands of planetary gear sets," *Journal of Sound and Vibration*, vol. 323, pp. 677-696, 2009.
- [81] A. Kahraman, "Load sharing characteristics of planetary transmissions," *Mechanism and Machine Theory*, vol. 29, pp. 1151-1165, 1994.
- [82] Irving Girsang, Jaspreet Singh Dhupia, Eduard Muljadi, Mohit Singh, Lucy Pao, "Gearbox and drivetrain models to study dynamic effects of modern wind turbines," Presented at *IEEE Energy Conversion Congress & Expo, ECCE 2013*.
- [83] Jian Lin, Robert G. Parker, "Analytical characterization of the unique properties of planetary gear free vibration," *Journal of Vibration and Acoustics*, vol. 121, pp. 316-321, 1999.
- [84] Xinghua Wu, Robert G. Parker, "Modal Properties of Planetary Gears With an Elastic Continuum Ring Gear," *Journal of Applied Mechanics*, vol. 75, pp. 1-12, 2008.
- [85] Vijaya Kumar Ambarisha, Robert G. Parker, "Nonlinear dynamics of planetary gears using analytical and finite element models," *Journal of Sound and Vibration*, vol. 302, pp. 577-595, 2007.
- [86] Tristan M. Ericson, Robert G. Parker, "Planetary gear modal vibration experiments and correlation against lumped-parameter and finite element models," *Journal of Sound and Vibration*, vol. 332, pp. 2350-2375, 2013.
- [87] M. Inalpolat, A. Kahraman, "A dynamic model to predict modulation sidebands of a planetary gear set having manufacturing errors," *Journal of Sound and Vibration*, vol. 329, pp. 371-393, 2010.
- [88] X. Gu, Ph. Velex, "A dynamic model to study the influence of planet position errors in planetary gears," *Journal of Sound and Vibration*, vol. 331, pp. 4554-4574, 2012.
- [89] Yi Guo, Robert G. Parker, "Dynamic modeling and analysis of a spur planetary gear involving tooth wedging and bearing clearance nonlinearity," *European Journal of Mechanics, A/Solids*, vol. 29, pp. 1022-1033, 2010.
- [90] Woohyung Kim, Ji Yeong Lee, Jintai Chung, "Dynamic analysis for a planetary gear with time-varying pressure angles and contact ratios," *Journal of Sound and Vibration*, vol. 331, pp. 883-901, 2012.
- [91] R. G. Parker, Jin Lin, "Mesh phasing relationships in planetary and epicyclic gears," *Journal of Mechanical Design*, vol. 126, pp. 365-370, 2004.
- [92] P. D. McFadden, "Window Functions for the calculation of the time domain averages of the vibration of the individual planet gears and sun gear in an epicyclic gearbox," *Journal of Vibration and Acoustics* vol. 116, 1994.
- [93] Hong Liu, Jaspreet Singh Dhupia, "Vibration Signal Modulation of Equally Spaced Planetary Gear-set with Gear Tooth Faults," presented at the *Eighth International Conference on Structural Dynamics*, 2011.

- [94] William D. Mark, "Stationary transducer response to planetary-gear vibration excitation II: Effects of torque modulations," *Mechanical Systems and Signal Processing*, vol. 23, pp. 2253-2259, 2009.
- [95] Patrick R., Orchard M. E., Bin Zhang, Koelemay M.D., Kacprzynski G.J., Ferri A.A., Vachtsevanos G., "An integrated approach to helicopter planetary gear fault diagnosis and failure prognosis," *Autotestcon 2007 IEEE*, 2007.
- [96] Cheon-Jae Bahk, Robert G. Parker, "Analytical solution for the nonlinear dynamics of planetary gears," *Journal of Computational and Nonlinear Dynamics*, vol. 6, 2011.
- [97] Shiyun Wang, Mina Huo, Ge Zhang, Jianping Liu, Yimin Song, Shuqian Cao, Yuhu Yang, "Effect of mesh phase on wave vibration of spur planetary ring gear," *European Journal of Mechanics, A/Solids*, vol. 30, pp. 820-827, 2011.
- [98] R. Errichello, J. Muller, "Gearbox Reliability Collaborative Gearbox 1 Failure Analysis Report," *NEREL/SR-5000-530262*.
- [99] S. Sheng, "Wind turbine gearbox condition monitoring round robin study – vibration analysis," *NREL/TP-5000-54530*.
- [100] W. Bartelmus, "Editorial statement," *Mechanical Systems and Signal Processing*, vol. 38, pp. 1-4, 2013.
- [101] Xiyang Wang, Villiam Makis, Ming Yang, "A wavelet approach to fault diagnosis of a gearbox under varying load conditions," *Journal of Sound and Vibration* vol. 329, pp. 1570-1585, 2010.
- [102] J. R. Ottewill, M. Orkisz, "Condition monitoring of gearboxes using synchronously averaged electric motor signals," *Mechanical Systems and Signal Processing* vol. 38, pp. 482-498, 2013.
- [103] Paul D. Samuel, D. J. Pines, "Vibration separation methodology for planetary gear health monitoring," *Smart Structures and Materials 2000*.
- [104] S. Salvador, "Fast DTW: Toward accurate dynamic time warping in linear time and space," *Intelligent Data Analysis*, vol. 11, pp. 561-580, 2007.
- [105] W. Bartelmus, F. Chaari, R. Zimroz, M. Haddar, "Vibration signal models for fault diagnosis of planetary gearboxes," *Journal of Sound and Vibration* vol. 331, pp. 4919-4939, 2012.
- [106] W. Bartelmus, "Mathematical modelling and computer simulations as an aid to gearbox diagnostics," *Mechanical Systems and Signal Processing*, vol. 15, pp. 855-871, 2001.
- [107] Hong Liu, Jaspreet Singh Dhupia, "A Time-domain Fault Detection Method based on an Electrical Machine Stator Current Measurement for Planetary Gear-sets," presented at *the 2013 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*.