Radio resource management in heterogeneous wireless networks

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NANYANG TECHNOLOGICAL UNIVERSITY

**RADIO RESOURCE MANAGEMENT IN
HETEROGENEOUS WIRELESS NETWORKS**

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by

SUDARSHAN GURUACHARYA

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List of Abbreviations

3GPP	3rd Generation Partnership Project
3GPP2	3rd Generation Partnership Project 2
CDI	Channel Direction Information
CDMA	Code Division Multiple Access
CQI	Channel Quality Information
CRM	Conjecture-based Rate Maximization
CSG	Closed Subscriber Group
FAP	Femtocell Access Point
FDMA	Frequency Division Multiple Access
FMC	Fixed Mobile Convergence
FUE	Femto User Equipment
GGSN	Gateway GPRS Support Node
GPRS	General packet radio service
GSM	Global System for Mobile Communications
HBS	Home Base Station
HeNB	Home eNodeB (LTE)
HMS	HNB Management System
HNB	Home NodeB (3G)
HNB-GW	Home NodeB Gateway
HSPA	High Speed Packet Access
HSPA+	Evolved High Speed Packet Access

ICIC	Inter Cell Interference Coordination
IP	Internet Protocol
IWF	Iterative Water Filling
LTE	3GPP Long Term Evolution
LTE-A	3GPP Long Term Evolution Advanced
MBS	Macro Base Station
MPEC	Mathematical Programming with Equilibrium Constraint
MUE	Macro User Equipment
NE	Nash Equilibrium
OFDMA	Orthogonal Frequency-Division Multiple Access
PU2RC	Per-User Unitary Rate Control
QoS	Quality of Service
RNC	Radio Network Controller
RSRP	Reference Signal Received Power
SBS	Small Base Stations
SE	Stackelberg Equilibrium
SeGW	Security Gateway
SGSN	Serving GPRS Support Node
SINR	Signal to Interference plus Noise Ratio
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access
TSA	Two-user Suboptimal Algorithm
UE	User Equipment
UMA	Unlicensed Mobile Access
UMTS	Universal Mobile Telecommunications System
UTRAN	Universal Terrestrial Radio Access Network
W-CDMA	Wideband Code Division Multiple Access
WiMAX	Worldwide Interoperability for Microwave Access

Summary

Mobile traffic data has seen a growing demand from the consumers such that the wireless cellular network operators are finding it hard to meet this demand using conventional techniques of network enhancement. The typical network enhancement methods, which consist of increasing the radio spectrum or installing new base stations, are no longer scalable with the data demand. Hence, these traditional methods are neither technically viable nor economically feasible in the long term. This increase in demand has been forecasted to grow further in subsequent years, and the network operators need innovative paradigms to tackle this problem. smallcells (e.g., picocell, microcell, and femtocell) have been proposed as one of the solutions to address this issue. The basic aim of smallcells is to increase the areal spectral efficiency of the network by aggressive spectrum re-utilization and network densification using low power, short range base stations. The introduction of smallcells creates a heterogeneity where the available spectrum is shared by different base stations that have different characteristics. Thus, the deployment of smallcells themselves creates new technical challenges regarding its co-existence with the extant macrocells. In this regard, the heterogeneity in the network can be a cause for both conflict as well as cooperation. The smallcell's policy of aggressive spectrum reuse results in co-channel interference to both the macrocell as well as the smallcells themselves. This is likened to a social situation where each base station can make independent decisions to optimize their own performance parameters. Such social situations can be formally investigated by the methods of game theory.

This thesis addresses some of the important issues that the co-channel interference tends to raise in a heterogeneous network. The first part of our thesis considers the problem of downlink power control. The contrast in the transmit powers of macrocell and

smallcell gives rise to a natural hierarchy, which we formalize using a non-cooperative game model called the Stackelberg game. The smallcells and the macrocells in the network are assumed to compete with each other to maximize their capacity under transmit power constraints. The behavior of smallcells and macrocells is analyzed and power allocation algorithms are proposed to obtain an equilibrium solution. As a special case of this solution, it is shown that under the high interference condition, the macrocells and smallcells tend to allocate their power in mutually orthogonal channels. The performance of the system is compared to the case when such hierarchy is not considered. In the second part of our thesis, we apply the concepts of a network multiple-input-multiple-output (MIMO) system to smallcell networks. We propose a coalition formation game model to cluster the smallcell base stations so that they can perform cluster-wise joint beamforming. We take the recursive core as the solution concept of the coalition formation game. Three algorithms have been proposed and analyzed to obtain the solution, however only one of them is shown to have both a low complexity and a guaranteed stability. Analytical formulas are given to compute the average number of coalitions and the average size of coalitions that can form during such a coalition formation process. Finally, the last part of our thesis considers the problem of admission control in the uplink transmission of a smallcell network. An underlying non-cooperative power game is devised, based on which a coalition game is formulated by taking a suitable value function and payoff division function. A Markov model is constructed to obtain a stable coalition structure as the required solution of our access control problem.

In summary, our work investigates the three different issues arising from co-channel interference using game theory: power allocation, beamforming, and admission control. Some possible future investigations can be directed towards joint resource allocation (e.g., power, beamforming, and scheduling) and coalition formation for the smallcell network.

Chapter 1

Introduction

1.1 Background

Studies show that 85% of all data and 70% of all voice data from the mobile phones are generated indoors [2, 5]. However, due to the significant wall penetration loss of macro-cell signals at high frequency ranges (typically above 2 GHz) commonly used in systems like 3G and WiMAX, the macrocell signal quickly attenuates and deteriorates once the signal reaches indoors. Also, every multiple access technology (e.g., time division multiple access (TDMA), frequency division multiple access (FDMA), code division multiple access (CDMA), and orthogonal frequency division multiple access (OFDMA)) is inherently interference limited. This means that high service quality is achieved only when the user is close to the base station and when there are few simultaneous users served by the network. These requirements for good service contradicts the realistic network environment situations. Hence, the network often operates at a much lower capacity than its theoretical maximum [8]. Apart from the requirements of indoor voice data, there has been an explosion of mobile traffic data due to easy availability of wireless consumer devices such as smart phones and tablets. The mobile data traffic in 2011 was nearly twelve times the size of the entire global internet in 2000 [3]. In 2012, the global mobile data traffic grew 70%, with mobile video traffic consisting 51% of all the traffic [3]. Clearly, this statistics represents a shift from voice-centric, circuit-switched communication to data-centric, packet-switched communication. Therefore, imperfect network

coverage, especially in the interiors of houses and buildings, along with a demand for higher data rate and quality of service (QoS) for wireless networks is one of the problems of existing cellular systems.

The solution to these problems is to ensure i) good signal quality and ii) small number of simultaneous users per cell [4, 8]. The traditional method of enabling these two requirements consists of either increasing the spectrum of the network, adding new cells, or sectorizing the existing cells. Often all three methods are jointly utilized by the network operator with careful cell site selection, radio planning, and optimization. However, due to the proliferation of mobile communication devices and the demand for higher standards of services, the data traffic has seen a growing demand from the consumers. For instance, the overall global mobile data traffic is expected to grow from 0.9 Exabyte¹ per month in 2012 to 11.2 Exabyte per month by 2017, a thirteen fold increase between 2012 and 2017, with two-third of the data expected to be video by 2017 [3]. Fig. 1.1 shows the exponential data growth that is expected in the coming years. The rate of data demand has outpaced the ability of the network operators to deploy newer base stations or acquire new spectrum, as per the traditional network enhancement methods. The usual network enhancement methods, which take a lot of time and effort, are thus no longer scalable with the data demand and will eventually lead to a capacity bottleneck. Therefore, these traditional methods are neither technically viable nor economically feasible in the long term. Unfortunately, improvements in signal quality resulting from advances in coding, modulation, cognitive transmission, and multiple antennas are reaching their theoretical limits and are insufficient in crowded environments and at cell edges where performance can significantly degrade [4]. This leaves the network operators with few remaining strategies to increase the network's areal spectral efficiency. Since the data demand has been forecasted to grow further in subsequent years, the network operators need innovative paradigms to tackle this problem. Consequently, network operators are rethinking the conventional cellular topologies and are considering a new paradigm called heterogeneous networks. As such, smallcells (e.g., picocell, microcell, and femtocell) have been proposed as one of the solutions to address this issue.

¹1 Exabyte = 1000 Petabytes = 1 billion Gigabytes = 10^{18} bytes.

Figure 1.1: Forecast of mobile data traffic [3].

1.2 A Singaporean Story

Singapore is a densely urbanized island city state. The land for new building constructions is limited and does not come cheap. To accommodate a growing population, the new high rise residential buildings are usually built taller than the previous generations of buildings. This strategy maximizes the usage of available land. However, the residents in these new buildings, whose apartments are above the old skyline, often complain of poor cellular service quality to their network operator. The reason for this is that the macro base station antennas, which were installed atop previously tallest buildings, have now been dwarfed by these newer buildings. Also, the transmit antennas of these base stations are pointed below the horizon in order to concentrate the radio beams towards the mobile stations on the ground, which is presumed to be at lower height than the macro base station. Thus the residents in these new buildings that live higher than their nearest base station tend to have a weak signal reception. Often the strongest signal comes not from the nearest macro base station, but from a base station serving a neighboring cell. Therefore, the QoS experienced by the user is severely degraded. From the point of view of the network operators, it does not make any economic sense to deploy more macro base stations just to serve these few residents who complain of bad service.

Thus the only economically feasible solution for this problem is to deploy smallcells in these apartments so as to offload the macrocell traffic.²

1.3 What are Smallcells?

Smallcells have emerged as a new paradigm to address the network issues related to growing data rate demands that are difficult to solve by traditional methods. The Small Cell Forum defines smallcells as “an umbrella term for operator-controlled, low-powered radio access nodes, including those that operate in licensed spectrum and unlicensed carrier-grade Wi-Fi. Smallcells typically have a range from 10 meters to several hundred meters” [1]. Smallcells essentially try to increase the spectrum reuse by increasing the areal spectral efficiency. To do so the smallcells boost the coverage and spectrum reuse by cell size reduction and by network densification. Thus, more smallcells can operate within a given area than the macrocells, leading to capacity enhancement of the network.

Smallcells are short range (tens of meters), low power (around 15-20 dBm) and inexpensive (\$100-\$200) base stations, called small base stations, overlaid on the existing cellular network technology (2G, 3G, 4G, WiMAX). This small base station is connected with Internet Protocol (IP) based backhaul through a local broadband connection, such as DSL, cable, or fiber. Similar to a WiFi access point, a small base station is a low cost, simple plug-and-play device, allowing a massive deployment of small base stations. Therefore, the number of smallcells operating in a given area is expected to be much larger than the number of macrocells. But unlike WiFi, small base station operates in the licensed spectrum and reuses the network resources and data transport infrastructures used for wide area cellular networks and compatible with mobile stations designed for macrocells. Also unlike the traditional base stations, the small base stations can be deployed in an ad-hoc manner according to the consumer need without the traditional radio planning, site selection, deployment and maintenance by the operator. Hence, the network operator may have no control over the location of the smallcell setup. The smallcells are supposed to self-organize and autonomously adapt to different deployment

²The author is in debt to Mr. Bijay Nepal, who works for M1, for the information about the Singaporean situation.

scenarios, automatically integrating themselves into existing macrocell network. The smallcells working together with the traditional macrocells results in a heterogeneity in the network, and such a mixture is referred to as heterogeneous network.

Smallcells are broadly classified according to their cell size, which ultimately depends on the transmit power of the base station, as: femtocells, picocells, metrocells and microcells, with femtocell being the smallest and microcell being the largest. Any or all of these smallcells can be based on “femtocell technology,” which is a collection of standards, software, open interfaces, chips and know-how that have powered the growth of femtocells [1].

Smallcell technology has been helped by the ease, low cost, and extensive proliferation of wired broadband internet connection, along with concurrent development of 4G cellular standard that are orthogonal frequency division multiple access (OFDMA) and IP-based. Also, the relentless improvement in hardware and software technologies mean that the price of a smallcell base station can be brought within affordable limit. The papers [5]-[10] give good surveys of this field. Similarly, the books [11]-[14] give general background and cover comprehensive issues related to smallcell technology.

Some of the major benefits of using smallcells are:

1. *Load Sharing*: Smallcells will substantially reduce the traffic load handled by the macrocell, considering the fact that most calls are made by the indoor users. Also, cellular network will experience reduced congestion.
2. *Infrastructure cost reduction*: Since the smallcell traffic is carried over wired residential broadband connections, they are able to mitigate the shadowing of macrocell signals and offload macrocell traffic. For the operators, it saves the site lease cost, tower building cost, backhaul equipment cost, maintenance cost, and electricity cost.
3. *Signal quality enhancement*: Smallcells improve indoor coverage where macrocell signal can be weak. Thus, smallcells can provide high data rate and improved quality-of-services to the subscribers. At the same time, it lengthens the battery life of the mobile phones since the mobile phones do not need to communicate with a distant base station.

4. *Convergence of services*: Smallcells enable the convergence of landline and mobile services since the same hand held device can be used to access the broadband wireless connection indoors and outdoors.

Considering all these benefits, smallcells have been commercially deployed in many parts of the world. An estimated 1.7 million femtocell access points were deployed during the fourth quarter of 2010, and the number is expected to reach just under 49 million by 2014 [15]. The smallcells have been commercially deployed in USA by Sprint Nextel since 2008, Verizon since 2009, and AT&T since 2010. Similarly, in UK and Europe, Vodafone, SFR, and Optimus started to deploy smallcells since 2009. In Asia, smallcells have been launched by StarHub since 2008; SoftBank, China Unicom, and NTT DoCoMo since 2009; SingTel and KDDI since 2010. In just 2012, 33% of total mobile data traffic was offloaded onto the fixed network through Wi-Fi or femtocell, and this trend is expected to grow over the years [3]. Currently, the smallcell Forum [1] – formerly known as the Femto Forum – oversees much of the standardization issues related to smallcell network. Other standardization bodies such as ETSI, WiMAX, and 3GPP has been greatly influenced by the Small Cell Forum.

Despite these benefits, there are various technical challenges that arise due to the heterogeneity of network. These challenges have to be solved before smallcells can be commercially viable. Among the challenges, the commonly agreed areas that need to be addressed are (i) network architecture adopted by the smallcells, (ii) interference management, (iii) synchronization required to achieve successful handoffs between base stations, (iv) security and quality-of-service over third party backhaul, and (v) self-organization and autonomous operation [6].

1.4 Scope of the Thesis

The challenges inherent in the smallcell deployment can be classified according to whether they belong to the transport/network layer or to the physical layer where the radio access takes place. Some of the problems involved in transport/network layer are: the traffic load management between the macrocell and smallcells, the mobility management, and

backhaul congestion management. The physical layer, on the other hand, addresses the issues on interference mitigation and radio resource management. In this thesis we will focus on these latter class of issues, and in this section we will describe some of such issues that we will attempt to study in the later chapters.

The radio interference due to sharing of cellular frequency poses major problems for successful femtocell deployment. The interference is not only inter-tier, between the smallcells, but also more critically cross-tier, between macrocell and smallcells. It is crucial to address these interference issues as they can greatly degrade the performance of the network. While inter-tier interference between two smallcells can be weak, owing to the low transmit power of the smallcells and the double wall penetration losses, the cross-tier interference between macrocell and femtocell can be quite high. In downlink scenario, as shown in Fig. 1.2, this can happen when the signal reception of a faraway macro user equipment is hampered by multiple smallcells operating near that equipment. This aggregate influence by multiple smallcells can be quite severe. Similarly, in the uplink scenario, a macro user equipment may transmit at a very high power in order to communicate with the distant macro base station, which can create severe interference for the smallcells operating nearby that macro user equipment.

The problem is exacerbated by the fact that the smallcells can be randomly deployed by the subscribers and can have overlapping coverage. The fact that many smallcells can coexist within a single macrocells means that the centralized control and coordination must be abandoned for decentralized and uncoordinated approach. These two facts greatly amplify the complexity of the situation. In this thesis we address the following issues related to interference management: power control, beamforming, and user admission control.

1.4.1 Interference Management

1.4.1.1 Spectrum Reuse Modes

The simplest way to address this cross-tier interference is to have separate, dedicated channels for every smallcell and macrocell usage. This is the orthogonal channel assignment scheme, where each cell is given exclusive access to certain channels. Some popular

Figure 1.2: Cross-tier interference in downlink

orthogonal schemes are TDMA (time domain multiple access) and FDMA (frequency domain multiple access). While these schemes prevent cross-talk among smallcells and macrocell, they involve partitioning the available bandwidth among smallcells and macrocells, which inherently lead to some loss in the total achievable data rate of each cell. Bandwidth partitioning may not be a problem when the number of smallcells operating within a macrocell is small, as the interference free environment may improve the performance, compensating for the loss in bandwidth. However, as the number of smallcells increase, the portion of bandwidth allocated to each cell becomes smaller, thus reducing the data rate. The problem is enhanced by the fact that the location and the number of smallcells deployed inside a macrocell is random and cannot be controlled by the network operator. Thus, this solution does not scale well as the number of smallcells increase.

An alternative solution is to allow each cell to access the entire spectrum. This is the co-channel assignment scheme. This re-uses the same frequency for all the cells, increasing the spectral efficiency per area through spatial frequency reuse. However, it comes at the cost of co-channel interference, and thus the smallcells are required to dynamically adapt their transmission parameters according to the spectrum usage by the macrocell. This leads to cognitive radio like paradigms. Significant attention has been devoted to manage and mitigate interference in co-channel deployment. Some of the popular concepts that have been proposed to combat interference are beamforming

using smart antenna arrays, interference cancellation by coding techniques, and transmit power control techniques.

Since universal frequency reuse is practically difficult to achieve, a compromise is fractional frequency reuse. According to it, only a fraction of entire spectrum is reused by smallcell users, thus providing macrocell users cross-tier interference free bands. However, this begs a question: how much spectrum should the smallcell be allowed to reuse?

The challenge in tackling the interference management problem comes from the asymmetry between the uplink and downlink transmissions, coupled with lack of coordination, randomness and density of smallcell deployment, and differences in transmission properties among different tiers.

1.4.1.2 Power Control

For successful co-channel deployment, the transmit power control technique tries to optimize the available resources (bandwidth and power) in order to minimize the effect of interference on the performance of existing macrocell network. There are two basic questions that all power control techniques try to answer. They are: i) Which channel should be selected to allocate power to? ii) How much power should be allocated to the selected channel? The first question relates to channel assignment, whereas the second question relates to power assignment. Thus the orthogonal channel assignment scheme is a special case of power control technique. The answer to these questions depends on the system model assumed (single- or multi-carrier, channel access schemes, etc) as well as the objective (Shannon capacity, bit error rate, interference power, outage probability, etc) of optimization procedure.

1.4.1.3 Cooperative Beamforming

A “virtual” or “network” MIMO system is formed when multiple base stations cooperatively transmit their signals, enabling us to convert the otherwise interfering links into useful message carrying links. This technique gives us another method to deal with fading and interference. One of the ways that network MIMO derives its advantage is by transmit and receive beamforming. While MIMO techniques have been well studied

in the past and can be directly applied to network MIMO scenario, unlike conventional MIMO system, it is not known apriori how many base stations (and hence number of antennas) are going to cooperate to form such a network MIMO system. Thus, one of the challenges in network MIMO is to be able to cluster base stations into groups so as to treat each such group as a single virtual MIMO system. Such clustering of base stations is an important strategy as it allows local processing, reduces signalling overhead, and provides robustness to node failures.

1.4.1.4 User Admission Policies

A possible technique to reduce the cross-tier interference in the uplink is to enable the offloading of the macrocell traffic to the smallcells. The uplink interference caused by a macrocell user which is close to the smallcell can be eliminated by sharing some of the smallcell bandwidth with the macrocell user. However, since smallcells can be privately owned, the smallcell users are under no obligation to accept any foreign macrocell user into its service. Whereas, macrocell by contract must always accept the smallcell user into the macrocell. Thus, depending upon the availability of radio resources, we can envision three possible access policy by the smallcell: i) open access, where every incoming macro user is indiscriminately accepted into the smallcell, ii) closed access, where except for a small pre-registered minority, every macro user is barred from being served by the smallcell, and lastly iii) hybrid access, where some fraction of resource when free is made available to the macro users with lower priority than the smallcell users. While the closed access scheme can lead to high co-channel interference, the open access is the scheme that best enhances the network's capacity. However, from the smallcell user's perspective, such open access scheme can potentially lead to a degradation of its own performance. Thus, hybrid access scheme represents a compromise between the two schemes.

1.5 Organization of Thesis

The rest of the thesis is organized as follows:

- **Chapter 2:** This chapter gives the literature review on smallcells, in particular of femtocells.

- **Chapter 3:** This chapter considers the problem of downlink power allocation in an orthogonal frequency-division multiple access (OFDMA) cellular network with macrocells underlaid with smallcells. The small base stations in this case are the femto access points (FAPs). The FAPs and the macro-base stations (MBSs) in the network are assumed to compete with each other to maximize their capacity under power constraints. This competition is captured in the framework of a Stackelberg game with the MBSs as the leaders and the FAPs as the followers. The leaders are assumed to have foresight enough to consider the responses of the followers while formulating their own strategies. The Stackelberg equilibrium is introduced as the solution of the Stackelberg game, and it is shown to exist under some mild assumptions. The game is expressed as a mathematical program with equilibrium constraint (MPEC), and the best response for a one leader-multiple follower game is derived. The best response is also obtained when a quality-of-service constraint is placed on the leader. Orthogonal power allocation between leader and followers is obtained as a special case of this solution under high interference. These results are used to build algorithms to iteratively calculate the Stackelberg equilibrium, and a sufficient condition is given for its convergence. The performance of the system at the Stackelberg equilibrium is found to be much better than that at the Nash equilibrium.
- **Chapter 4:** In this chapter, we apply the concepts of network multiple-input-multiple-output (MIMO) to smallcell networks. To do so, the issue of imperfect channel state information (CSI) at the transmitter is considered when frequency-division duplexing is used, for which the feedback channel is limited. We first introduce a regret based learning approach to optimize the transmit beamforming parameters for the cases when the feedback channel is temporarily unavailable during deep fades. We then propose a coalition formation game model to cluster the smallcell base stations so that they can perform cluster-wise joint beamforming. We take the recursive core as the solution concept of the coalition formation game. To obtain the recursive core, we first consider a typical merge-split algorithm. However, we show that this algorithm can be unstable. Alternatively, we adopt the merge-only algorithm which guarantees the formation stability and show that

its outcome belongs to the recursive core. We also show split-only method has exponential complexity. Finally, we analyze the average number and the average size of coalitions that can form during such a coalition formation process. Numerical simulations are given to illustrate the behavior of the coalition formation among smallcell base stations.

- **Chapter 5:** This chapter considers the problem of access control in the uplink transmission of an OFDMA femtocell network with one macrocell and one femto-cell. An underlying noncooperative power game has been devised, based on which a coalition game is formulated by taking a suitable value function. Only two complementary coalitions are allowed to exist in order to reflect the set of user equipments connected to either the macro base station or the femto access point. The user equipments in the same coalition cooperate by operating on non-interfering sub-channels, while those in the complementary coalition are assumed to operate so as to cause maximum jamming. The value of a coalition is obtained as the max-min of utility sum of each user equipment in the given coalition. In the process, we also examine the optimal jamming strategy of the complementary coalition. Finally, we argue that the obtained value function cannot be super-additive. Since the super-additivity property is required for some of the solutions of cooperative game theory, we resort to the Shapley value solution, for which the super-additivity property is not required, to allocate the payoff to each user in a given coalition. Assuming that the user equipments want to be in the coalition that maximizes their payoff, we formulate a Markov model to obtain the stable coalition structure. We take these stable coalition structures as the required solution of our access control problem.
- **Chapter 6:** This chapter provides a summary of the results presented in this thesis and outlines a few issues which can be pursued as an extension of this research.

1.6 Summary of Contributions

Chapter 3: The main contributions of this chapter are as follows:

- We show the existence of Stackelberg equilibrium for a multiple leader-multiple follower game.
- We derive a simple, closed-form expression for unique best response of the leader in one leader-multiple follower game, given the channel allocation of the followers. A solution is also obtained when a QoS constraint is imposed on the leader. Orthogonal power allocation between the leader and the followers is shown as a special case of this solution under high interference condition.
- These results are used to build algorithms to iteratively calculate Stackelberg equilibrium, and a sufficient condition is given for its convergence under high interference.
- The idea of “pseudo-noise” is proposed, for the sake of visualization, that allows us to maintain the waterfilling interpretation.
- A practical recommendation for network deployment is given. More specifically, when the interference experienced is low, the macrocell utilizes the subchannel by treating the interference as ordinary noise; but when the interference is sufficiently high, the macrocell abandons the subchannel. Another possibility is as such: macrocell sets at least one subcarrier free of macrocell activity. Owing to the predictable, selfish nature and low transmit power of the femtocells, the femtocells’ operation will mostly be confined to these bands, thus allowing a relatively interference free operation for macrocell in the rest of the subchannels.

Chapter 4: The main contributions of this chapter are as follows:

- We explore a regret based learning method [199] to optimize the beamformers in finite rate feedback systems when the feedback is temporarily unavailable due to deep fades.
- We formulate a coalition formation game for the small base stations to establish network MIMO systems. Recursive core [202] is taken as the solution concept for such coalition formation game. To obtain the solution, the merge-split [212], split-only, and merge-only algorithms are considered. We demonstrate the instability of

the merge-split method when externalities are present. Thus an alternative idea of the merge-only algorithm is tailor-made for the wireless communication problem under study. We base our merge-only algorithm on an intuitive idea that except for the base stations in their immediate neighborhood, we do not expect far away base stations, which comprise of all the rest of the network, to be much affected by the behavior of a particular base station. Lastly, the split-only method is shown to have very exponential complexity.

- Finally, we perform a probabilistic analysis to calculate the average size and average number of the coalitions that are formed by the small base stations. This result can be used to obtain further analytical results as well as for network planning. For example, an a priori estimate of how many small base stations can cooperate to serve a user equipment can be used to inform the resource allocation and scheduling tasks.

Chapter 5: The main contributions of this chapter are as follows:

- The access control is modeled in terms of coalition formation game.
- Strategies for power allocation and jamming is investigated with respect to channel capacity.
- The solution to the access control problem is proposed to be the absorbing states of a Markov chain obtained using merge-split rule.

Chapter 2

Literature Review

The advancements in coding, modulation, multi-antenna system that can be used to enhance a wireless cellular system are reaching near theoretical limits of performance for an isolated system with a single base station. The relentless increase in mobile users and data demand means that the conventional network deployment techniques, like cell sectoring, enlarging the spectrum, or introducing new cells, will not be able meet this demand and will eventual lead to a bottleneck. Nevertheless, it is still possible to obtain further performance gains for wireless networks by considering a more flexible network topology that allows for network densification and aggressive spectrum reuse.

Heterogeneous networks, which utilizes a combination of diverse radio technologies and cell types, can be deployed to improve spectral efficiency per unit area. Heterogeneous networks consist of planned macro base stations, which typically transmit at high power, overlaid with several pico base stations, distributed antennas, femto base stations, and relays, which are low power nodes and are typically deployed in a relatively unplanned manner [16]. These low power nodes are deployed in areas with imperfect network coverage in the macro-only system and improve capacity in crowded areas with high traffic density [16]. While the installation of macro base stations is generally based on the tradition of careful site selection, radio planning, and optimization, the low power nodes are placed where they are expedient to do so, based only on a rough knowledge of coverage issues and traffic density (e.g. hot spots) in the network [16]. As such, the heterogeneous networks promote areal spectral efficiency via network densification and

spectrum reuse. Furthermore, in heterogeneous networks, smarter resource allocation and coordination among base stations, better base station selection strategies and more advanced techniques for efficient interference management can provide substantial gains in throughput and user experience as compared to a conventional approach of deploying cellular network infrastructure [16].

A smallcell is an umbrella term for operator-controlled, low-powered radio access nodes, including those that operate in licensed spectrum and unlicensed carrier-grade Wi-Fi [1]. Smallcells typically have a range from 10 meters to several hundred meters [1]. Due to their lower transmit power and smaller physical size, smallcells can offer flexible deployment schemes. Smallcells are broadly classified according to their cell size, which ultimately depends on the transmit power of the base station, as: femtocells, picocells, metrocells and microcells, with femtocell being the smallest and microcell being the largest [1]. Among the smallcells, femtocells enjoy an advantage of not requiring any site planning and maintenance by cellular operators. That is, they are connected to the core network through a last-mile Internet backhaul.

Femtocell is essentially a form of a cellular access point deployed by users. As a result, femtocell installation, organization, and configuration are some of the important issues in femtocell deployment. The installation of a femtocell should support plug-and-play, with minimum user intervention. In this regard, the femtocells are required to be able to completely self configure (i.e., selecting initial parameters including transmit power and frequency), self optimize (i.e., monitoring and adapting the resource usage) and self heal (i.e., resolve any problem by itself) for efficient integration within the mobile networks. To achieve these goals, challenges such as managing the radio interference, network synchronization, security, and reliability must be resolved [17].

In the following subsections, we provide a brief discussion regarding femtocell architecture and its types, access methods, femtocell convergence, and femtocell deployment, before going into key technical issues and state of the art.

2.1 Some Early Work Related to Femtocells

Some of the early simulation results for femtocells were presented in [52]-[60] showing the basic feasibility of the femtocell idea. In [52] preliminary simulations were performed

to study the performance degradation of macro users where femtocells were randomly deployed and shared the common spectrum. In [53] and [54], a power control method was introduced which ensured a constant femtocell radius in the downlink and a low pre-definable uplink performance impact to the macrocells. The issue regarding auto-configuration, public access, and performance of randomly deployed femtocells were also discussed and analyzed. In [55], the authors studied the service experienced using simulations and concluded that the deployment of femtocells caused only a small increase in call drop rates of a macrocell users in the worst case. In [56] the feasibility of modification and optimization of the UTRAN standard on the issues related to architecture, handover, and interference were considered for the support of femtocell deployment. In [57], the authors studied the tradeoffs between interference and bandwidth in closed and open access modes of femtocells. The authors concluded that the closed access schemes offered better service to femto users but introduced significant interference to macro users, while the open access with dedicated spectrum assignment can reduce interference concerns of macro users with some spectrum trade-off. In [58] a detailed radio propagation model was used in simulations to investigate the outage probability, spectral efficiency, and required radio link separation between macrocell and femtocells. In [59] intra-tier as well as inter-tier interference scenarios were studied for an OFDMA system with full frequency reuse. In [60] open access scheme in both dedicated-channel and co-channel femtocell deployment were studied with the view of investigating the cell selection metrics. They concluded that the capacity-based cell selection typically yields higher capacities for the users.

New mathematical models and analysis were proposed on two tiered network by [61, 62, 63, 64] for the uplink transmission. In particular, [61] introduced a metric called “operating contour” defined as the feasible combinations of average number of active macrocell users and femtocell base stations per cell site that satisfied a target outage constraint. Based on this metric, the authors provided capacity and outage probability analysis for uplink two-tiered CDMA system and concluded that time hopped CDMA and sectorized antennas allow greater densification compared to that of the two-tier network with dedicated spectrum and omni-directional femtocell antennas. In [62] the authors investigated the near-far effect due to cross-tier interference and concluded that femtocells with multiple antennas enhanced the coverage as well as the robustness against

this problem. In [63] the robust communication strategies were investigated in information theoretic terms and bounds as well as expected achievable sum-rates were given for macrocell users and femtocell users depending on the channel state of femto base station and macro base station link. In [64] the authors derived per-tier outage probability and capacity using simple mathematical models of cellular geometry by considering co-channel two-tier networks with outage constraints. They related their result with randomly deployed femtocell density and femtocell's transmit power.

Different approaches towards adaptive access control and mobility management were examined in [65, 66, 67, 28]. In particular, [65] showed that adaptive femtocell access policy which depends on the instantaneous load of the network is superior over closed or open access schemes. Thus, they advocated a hybrid access scheme. In [66] the authors described general issues regarding access control strategies for UMTS and LTE systems. In [67] the possibilities of open, closed, and hybrid access schemes were discussed and the performance of the network related to these schemes investigated under different traffic types. Finally, in [28] the authors made a comparison between open access and closed access schemes. The authors concluded that the best approach depends on the type of multiple access scheme used, whether it is orthogonal (like TDMA or OFDMA) or non-orthogonal (like CDMA). For TDMA/OFDMA networks, closed access scheme is preferable at high user densities; whereas for CDMA network, open access scheme is preferable.

The user assisted approaches to interference optimization were explored in [68], while interference management techniques for uplink and downlink based on High Speed Packet Access (HSPA) and its extensions were examined in [69, 70]. In particular, the authors of [69] proposed femtocell carrier selection, downlink transmit power self-calibration, and adaptive attenuation of the uplink transmit power of femtocell users in order to manage interference. The authors of [70] studied the relationship between closed-form outage probability function and uplink attenuation on femtocell systems, obtaining an optimal uplink attenuation factor to minimize the outage probability.

Similarly, for an OFDMA-based network, the interference coordination method was proposed in [71] where both the spectrum sensing and scheduling information were utilized to avoid strong co-channel, cross-tier interference. In [72] an autonomous component

carrier selection method was proposed for interference management, which allows each cell to select the best frequency configuration. Meanwhile, in [27] the authors conducted a coverage and interference analysis on an OFDMA based two-tiered network and provided few practical guidelines on spectrum allocation and interference mitigation.

Based on these early contributions, much of the standardizations and later day research issues have emerged.

2.2 Femtocell Architecture

For scalable, secure, and autonomous deployment of femtocells, femtocell architecture needs to be compliant with standards that define the interfaces with existing cellular network. Furthermore, the requirement of interoperability among multiple manufacturing vendors presents a need for the standard interface between the femtocells and the core network [20]. To overcome these issues, different femtocell architectures have been proposed particularly in the context of CDMA (UMTS/CDMA-2000) and OFDMA (LTE/LTE-A/WiMAX) by various standardization bodies. The 3rd Generation Partnership Project (3GPP) is the primary standardizing body for GSM, UMTS and LTE (formally, 3GPP Release 8 onwards) and LTE-Advanced (formally, 3GPP Release 10 onwards) [20]. The 3GPP became the pioneer in providing standards for femtocell architecture in the form of 3GPP TS 25.467 Release 9 [21], thus making it a reference architecture for the evolution of femtocell networks. In the following passage, we shall only describe the 3GPP UMTS femtocell architecture as a reference architecture.

As shown in Fig. 2.1, the proposed 3GPP UMTS femtocell architecture follows an access network-based approach, leveraging the existing *lu* interface (including *lu-CS* for circuit services and *lu-PS* for packet data services) into the core service network [17]. The 3GPP UMTS femtocell architecture defines two new network elements: the Home NodeB (HNB) and the Home NodeB Gateway (HNB-GW) which are equivalent to femtocell access point and femtocell network gateway, respectively [21]. The communication between these elements is performed via a new interface, i.e., *luh* as shown in Fig. 2.1. HNB Management System (HMS) and Security Gateway (SeGW) are also added to provide autonomous HNB management and to provide secure access to the backhaul

Figure 2.1: Femtocell architecture (based on [21]).

Internet Protocol (IP) network. However, all other network entities related to voice and data services follow the same architectural configuration of the proprietary 3G UMTS architecture, thus facilitating interoperability. The proposed standard 3GPP femtocell architecture includes specific functional entities as shown in Fig. 2.1 [21]:

- **Home NodeB (HNB):** HNBs are typical customer premise equipments equipped with NodeB transceiver that provides radio network controller (RNC) functionality. The basic function of HNB is to provide radio coverage for standard 3G handsets within a home over *Uu* interface. It also supports HNB registration to the existing mobile network.

- **HNB Gateway (HNB-GW):** HNB-GW is installed within an operator network to aggregate traffic from different scattered HNBs back into an existing core mobile service network through the standard *lu* interfaces. HNB-GW acts as a single RNC to the core mobile network which provides many-to-one relationship between HNBs and HNB-GW. This functionality of HNB-GW improves scalability of the femtocell network which also offloads the HNB-specific function from the core mobile network.
- **HNB Management System (HMS):** HMS provides cost effective management architecture to control and manage the femtocell devices. The main function of HMS is to provide the necessary provisioning of data required for configuring HNB remotely. For this, HMS uses *TR-069* standard, which is a traditional interface used for DSL modem configuration. HMS also performs location verification of HNB for authenticating the user location.
- **Security Gateway (SeGW):** SeGW provides a secure link between the HNB and the HNB-GW (over *luh*) and also between HNB and HMS using IPSec [23]. SeGW is responsible for providing the proper authentication of HNB for the core network. However, the SeGW is a logically separate entity and may be implemented either as a separate physical element or integrated into, for example, HNB-GW.

While the above architecture focuses on 3G UMTS, it provides a baseline for all other standards, including GSM, CDMA-2000, TD-SCDMA, LTE, LTE-A, and WiMAX solutions. The principle concept remains the same with certain changes in the terminologies specific to the underlying standards. For example, CDMA-2000 and WiMAX follow the architecture of the IP-based network by the 3GPP2 [24]. These include Voice over IP (VoIP) using Session Initiated Protocol (SIP), with RNC function integrated into the femtocell [17]. An overview of femtocell architecture that has been developed for 3GPP and 3GPP2 is summarized in [8, 22]. A major difference between 3GPP [21] and 3GPP2 [24] architecture is that 3GPP2 provides two distinct femtocell architecture for voice and packet data service [25]. However, 3GPP2 activities has essentially been discontinued and the focus is on LTE and LTE-A technologies which are also IP-based networks.

2.3 Femtocell Access Method

Apart from using co-channel access scheme or dedicated access scheme, femtocell access methods will also play an important role in mitigating the interference and improving the overall network performance. Spectrum access schemes (i.e., co-channel and dedicated) define the channel access scheme for femtocells, whereas femtocell access methods define rules and policies for accessing the femtocell by its subscribers (e.g., FUE) and other non-subscribers (e.g., MUE). Based on 3GPP standards, femtocell access schemes can be divided into following categories:

- **Closed Access Scheme:** In this access scheme, only a subset of users, defined by the femtocell owner, can connect to the femtocell [26]. For instance, in Fig. 2.2, FUE 1 can access the femtocell, while both MUE 1 and MUE 2 cannot, which is based on the access rule defined by the owner. This insures the user's reliability in terms of QoS, thus also providing secure connection within femtocell and with backhaul connection. However, due to closed subscriber group (CSG) policy (i.e., private access), strong cross-tier interference exists in closed access scheme. The power leaks from the femtocell will cause interference to nearby macrocell users, thus degrading their signal quality. However, efficient spectrum allocation algorithms using orthogonal frequency-division multiple access (OFDMA) subchannels can help to mitigate this interference [27, 28].
- **Open Access Scheme:** In this access scheme, all the users (e.g., FUE 1, MUE 1 and MUE 2 in Fig. 2.2) located close to femtocell are allowed to connect to femtocell when the macrocell coverage is low [26]. From the interference perspective, open access scheme reduces the cross-tier interference by letting the strong interferers such as nearby users to use the femtocell when necessary. Also, open access scheme is beneficial for operators to expand their network capacities by leveraging third party backhaul for free. However, this brings the concerns to femtocell owners whether to share their resources that may eventually degrade their QoS. As open access scheme provides unconditional access to femtocell by any legacy cellular users, security issue may arise due to this uncontrollable access. Moreover,

the number of handoffs and signaling will also increase drastically as a result of providing open access to femtocell. Recent studies show that open access scheme is preferable when the multiple channel access scheme is non-orthogonal, e.g., code division multiple access (CDMA) [28].

- **Hybrid Access Scheme:** Hybrid access scheme is a new access concept introduced in 3GPP Release 9 [29]. In this access scheme, a limited amount of the femtocell resources is available to all users (MUE 1 and MUE 2 in Fig. 2.2), while the rest is allocated for the femtocell owners (FUE 2) [26]. This access scheme combines both closed and open approaches to provide trade-off between the impact on the performance of femtocell owners and the level of access granted to other users. Therefore, hybrid access scheme requires fine tuning of the shared spectrum resources among the femtocell owners and other users. The main advantage of this scheme is that it solves the interference problems of closed access method without compromising the performance of femtocell owners. However, extensive research is still needed to apply hybrid access scheme to different deployment scenarios (e.g., how to allocate the resource between femtocell owner optimally).

To identify the existence of femtocell and its access method, 3GPP has defined a CSG identity (CSG ID) and CSG indicator [30, 31] for both UMTS and LTE systems. The presence/absence of the CSG ID in broadcast system helps to identify a cell as femtocell. Also, a combination of CSG indicator is used to identify whether a femtocell is closed/open/hybrid. The various combinations of CSG indicator and CSG ID proposed by 3GPP standard are shown in Fig. 2.2. A detailed summary of the ongoing effort and issues related to access control strategy covered by 3GPP for UMTS and LTE systems is provided in [66].

2.4 Integration of Femtocell into Core-Netork: Fixed Mobile Convergence

The fixed mobile convergence (FMC) is another emerging concept which allows seamless connectivity between fixed (e.g., local area network) and cellular networks [19]. The ultimate goal of FMC is to optimize transmission of all data, voice, and video communications

Figure 2.2: Access scheme for femtocell with CSG indicator and CSG ID (based on [26]).

to and among end users, regardless of their locations or devices. The unlicensed mobile access (UMA) (or universal mobile access) is the first 3GPP standardized technology enabling cellular (i.e., licensed) and WiFi (i.e., unlicensed) convergence [33]. However, UMA requires a new (dual mode) handset which should switch efficiently between cellular and WiFi network resulting in an additional cost [17]. On the contrary, femtocell-based deployment will work with the current handsets which only require installation of a new access point (i.e., FAP). Moreover, recent research studies show that UMA can be used to address the core network integration of femtocells into mobile core networks by providing a standard, scalable, IP-based interface [17, 34]. The principle idea is to combine all the functions of a radio access network and core network (e.g., RNC, GGSN, and SGSN) into a single network element, thus making the core network access independent [18]. This convergence architecture is often referred to as “collapsed stack” or “Base Station Router (BSR)” which is being considered in several standards (e.g., 3G UMTS, LTE, and WiMAX) [6].

In the following, we present specific integration of femtocell to existing and new technologies (e.g., 3G UMTS, LTE, and WiMAX).

2.4.1 Femtocell in 3GPP

The integration of femtocells into current cellular networks such as 3G UMTS/HSPA or into new 4G LTE is an active research topic which is being constantly pursued by standardization bodies such as 3GPP and 3GPP2. In 3GPP, the femtocells are called Home Node B's (HNB) for 3G UMTS/HSPA and Home eNode B's (HeNB) for 4G LTE. However, the H(e)NB functionality and interface are mostly the same. Moreover, for 3G networks based on W-CDMA air interface (i.e., UMTS/HSPA), femtocells can be integrated using collapsed stack architecture as stated by 3GPP standard group. For 3G CDMA-2000, IMS and SIP based approach [35] using full IP-based architecture is more desirable. This approach is adopted by 3GPP2 and is under discussion by 3GPP. For LTE, also an IP-based architecture, additional functionalities such as hybrid cell concept, inbound mobility, and access control, were added in 3GPP Release 9 [29]. However, LTE femtocell will have better advantage than its earlier 3G counterpart as LTE will use OFDMA instead of CDMA. Typically, OFDMA provides higher flexibility and robustness in avoiding intracell and multipath interference. This characteristic becomes beneficial while reducing cross-tier interference due to co-channel deployment [59, 37]. Also, OFDMA-based femtocell can exploit channel variation in both frequency and time domain unlike CDMA which can exploit channel variation only in time domain for interference avoidance [27].

2.4.2 Femtocell in WiMAX

WiMAX is a wireless network based on IEEE 802.16 standard that provides higher QoS and data service and is an alternative to cable and DSL [38]. However, due to the use of high frequency, WiMAX also suffers from poor indoor coverage. To overcome this, the IEEE 802.16m SDD (system description document) [39] introduced the concept of femtocell for WiMAX. Unlike 3G femtocell, WiMAX femtocell integration is much simpler since it follows the all IP based service. Depending on the type of application environment, various types of WiMAX femtocell can be developed. According to a recent study, WiMAX femtocell convergence provides improved performance in terms of higher throughput and smaller delay than those of other air interfaces such as UMTS [6, 40].

2.5 Market Infiltration

Femtocells have been attracting a lot of attention from the wireless network operators because of an extensive demand for better indoor voice and data traffic [17]. However, according to a recent market research, femtocell pricing will play an important role in successful femtocell deployment. The pricing issues and opportunity costs should be balanced in such a way that both the network operators and the consumers have an incentive to shift to femtocells [50]. Beside this, many standardization bodies (e.g., 3GPP and 3GPP2) are actively working to provide better solutions and approaches for successful femtocell deployment. A brief summary of standardization activities related to 3GPP and 3GPP2 is presented in [8, 22]. Small Cell Forum [1], formerly called Femto Forum, is another collaborative organization formed by operators, vendors, and content providers to promote and develop open standards for the femtocell deployment worldwide. Similarly, many vendors provide competitive solution to this growing industry, such as Alcatel Lucent, Huawei, ZTE, ip.Access, Airvana, Samsung, Ubiquisys, and Qualcomm, to name a few.

2.6 Issues in Femtocell

For the success of femtocell technology, many challenges have to be addressed. Foremost among them is that femtocells need to be low cost such that it favors a mass scale deployment. It should also meet all regulatory requirements of underlying implementing technology. Furthermore, it should be reliable and interoperable to handle interference and also support synchronization. In the following, we list some of the important issues related to the femtocell technology.

2.6.1 Femtocell Deployment Issues: Cost, Scalability and Security

For effective femtocell deployment, various technical and financial challenges should be tackled [5]. Basically, femtocell's use of IP connections (i.e., Internet Access) for backhaul

access creates network design and integration challenges that are different from the traditional network design considerations for the operators. The foremost task is to modify the backhaul system so as to make it capable of accepting and integrating the femtocells with full functionality. Moreover, the cost incurred in strategic deployment of a femtocell is also an important consideration for the adoption of the femtocell technology. In many ways, femtocells will offer savings to the operators in terms of capital and operation expenditures [18]. However, femtocell installation will incur a certain cost to the femtocell users since a femtocell user will be billed by both the cellular network provider as well as the Internet service provider. Therefore, proper incentive schemes (e.g., revenue share) should be provided to the femtocell users for adopting the femtocell service. Thus, the need for low cost femtocell development and implementation is a driving factor for the femtocell selection and its scalability. Due to advances in integrated technologies, the cost of a femtocell can be made very affordable (around \$100-\$200) [17]. Moreover, the operators can also provide competitive subscription plans for the femtocell users as an incentive to compete with the existing Wi-Fi market.

Another important concern related to the network integration is the issue of network security. As femtocells use IP connection to connect to the core network, security issues such as authentication and network intrusions become vital for network operators. Proper authentication and device provisioning should be provided to mitigate these issues. 3GPP user group has already added secured gateway (SeGW) and HNB management system (HMS) into standard femtocell architecture for providing necessary security and authentication for femtocells [21].

2.6.2 Interference Management

The radio interference (i.e., cross-tier and co-tier interference) due to the sharing of cellular frequency poses a major problem for femtocell deployment [17]. The spectrum allocation schemes such as dedicated deployment can mitigate this interference to some extent. Despite its low radiation power and shielding from the walls of buildings, a femtocell can still cause interference to the macrocell users and other femtocell users. These interferences are critical for femtocell as they degrade the network performance. The cross-tier interference exhibits a *dead zone* problem (especially in closed access scheme)

Figure 2.3: Cross-Tier and Co-Tier Interference Scenario for DL and UL.

when the high power macrocell user causes interference to nearby femtocells or when femtocell users cause unacceptable interference to the macrocell users [41]. Also, due to random placement of femtocells, co-tier interference can degrade the overall performance of the femtocell network.

Depending upon the mode of transmission and corresponding deployment scenario, the macro-femto interference can be classified into downlink (DL) and uplink (UL) interferences. Fig. 2.3 shows the DL and UL interference scenarios for a typical macro-femto network. The downlink interference can be caused by both MBS and FAP. During the macrocell downlink transmission, the transmit power from the MBS causes cross-tier interference to the nearby FUE receivers (FUE1 in Fig. 2.3) which are prominent for the femtocell users. Similarly, in the case of femtocell downlink transmission, the transmit power from the FAP (FAP1 in the figure) causes cross-tier interference to the nearby MUE (MUE1 in the figure) and also co-tier interference to adjacent FUE (FUE2 in the figure) in dense scenario. In the case of uplink transmission, the uplink signal from MUE and FUE causes interference to the nearby FAP receiver and MBS receiver. For macro-cell uplink scenario, the MUE (MUE2 in the figure) close to the femtocell or inside the femtocell coverage area causes cross-tier interference to the FAP receiver (FAP4 in the figure) when transmitting the uplink signal to the MBS. For femtocell uplink scenario, the FUE at cell edge (FUE3 in the figure) transmitting uplink signal to corresponding

FAP causes cross-tier interference to the overlaid MBS receiver and co-tier interference to the nearby neighbor FAP receiver (FAP4 in the figure). Both uplink and downlink interferences should be controlled to meet their respective QoS requirements.

Hence, there is a need for interference and resource management techniques to minimize various interference issues in femtocell environment. In the following section, we will focus on various interference and resource management issues and their corresponding approaches to mitigate/avoid the effect.

2.6.3 Spectrum Planning

Spectrum planning is an important issue in femtocell deployment due to the use of legacy licensed spectrum. Proper spectrum policies and access methods should be defined to effectively reuse the available spectrum. Also, unlike Wi-Fi technology, femtocell requires regulatory approval to use the licensed spectrum. As the spectrum and radio regulations vary in different countries, international agreements should be made for interoperable operations.

2.6.4 Synchronization

Network time synchronization is necessary between macrocells and femtocells to minimize multi-access interference and ensure a tolerable carrier offset [5, 27]. Due to the lack of centralized coordination between femtocells and macrocells, the synchronization is also required for the proper performance of handoffs. Moreover, providing high precision synchronization for low cost femtocells is a challenging issue, since the IP backhaul system for the femtocells inherits the packet jitter problem, thus creating a difficulty in obtaining a time reference [5]. Therefore, for femtocells, efficient synchronization solutions are being considered, such as using IEEE-1588 Precision Timing Protocol over IP or using GPS to synchronize with the macrocell [17].

2.7 Interference and Resource Management for OFDMA based Femtocell: State of the Art

2.7.1 Insights from Stochastic Geometrical Analysis

Using ideas from stochastic geometry, [74] showed that assuming open access and strongest cell selection, downlink heterogeneous, multi-tier deployments do not worsen the overall interference condition nor change the SINR statistics. They also derived a simple formula for the probability of coverage under both open and closed access schemes. Most crucially, they proved a counter-intuitive result that, under simplified conditions for open access scheme, the coverage probability is independent of the number and density of base stations, the number of tiers, the relative power levels, and the fading distribution. In [75] the analysis for SINR distribution for heterogeneous networks was extended to the case where there is a mixture of both the open access femtocells and closed access femtocells. The result was related to the relative densities and transmit powers of macro- and femtocells, fraction of femtocells adopting open versus closed access, and the condition of wireless channels.

2.7.2 Deployment Techniques

There are three possible frequency domain deployment scenarios (i.e., spectrum access schemes) for femtocells: (i) In dedicated channel deployment, macrocells and femtocells operate on different spectrum; but this increases bandwidth segmentation and may not be suited for open access femtocells. (ii) In the co-channel deployment, both the macrocell and femtocells operate over the same spectrum. This avoids bandwidth segmentation and is scalable for any system bandwidth; however interference issues in the case of closed access femtocells and hand-off issues in the case of open access femtocells are two of its challenges. (iii) Finally, there is the hybrid deployment scheme, which is also known as carrier aggregation approach. In the hybrid deployment scheme, the spectrum is divided into two parts: one part is used exclusively by macrocell while another part is shared by both the macrocell and the femtocells. However, this technique requires large available spectrum.

Due to analytical difficulties, much of the studies conducted in the deployment techniques have been done via simulations. The hybrid deployment has been studied in [76]-[81]. These studies investigated the effect of changing the size of the shared bandwidth on the performance of the system and different mechanisms to enable such a hybrid deployment scheme. They also studied different possible methods of adapting the size of the shared bandwidth.

2.7.3 Fractional Frequency Reuse

For the dedicated channel deployment, the fractional frequency reuse method essentially divides the available spectrum into several smaller subbands. A subband is assigned to a macrocell or a sector of a macrocell. Depending on the macrocell or the sector of a macrocell that a femtocell is located in, a subband which is different from that used by the macrocell or the macrocell sector is assigned to the femtocell. This prevents interference between the macro- and femtocells. However, since two nearby femtocells can utilize the same subbands, a proximity based spectrum partitioning scheme is often utilized to mitigate femto-to-femto interference. That is, two nearby femtocells are assigned different subbands, that are both different from the subband utilized by the common macrocell or the macrocell sector that they both reside in. Various adaptive fractional frequency reuse schemes have been proposed in [82]-[93] to mitigate the co-tier interference between femtocells as well as the cross-tier interference between femto- and macrocell. For instance, in [87] an interference graph is constructed and the subbands are assigned to each cell using a vertex coloring technique.

2.7.4 Cell Biasing and Range Expansion

Unlike a homogeneous network where all the base stations operate at similar levels of power, the transmit power disparity between a macro base station and femto base station means that a user equipment will most often be associated with the macrocell when the received signal strength is used as a metric for cell selection. Also the cross-tier interference will produce a dead zone around the cell boundary of a femtocell when the high power macrocell user causes interference to nearby femtocells or when femtocell users

cause unacceptable interference to the macrocell users. Cell biasing is the idea of providing a positive cell bias at the femtocells so that a user equipment is handed over from the macrocell to the femtocell whenever the difference in received signal strength of the macro and femtocells, as measured by the user equipment, falls below this bias value. The biasing of the femtocells effectively increases the cell coverage, and the user equipments need not connect to the base station with the strongest signal. Thus, the traffic can be offloaded to femtocells more effectively using cell range expansion, mitigating the dead zone problem.

In [94] the offloading benefits of range expansion were explored using simulations. In [95, 96] the downlink SINR analysis and the outage probability for biased cell association have been provided using stochastic geometrical analysis. A combination of cell range expansion and intercell interference coordination has been advocated in [9]. In [97] intercell interference in expanded regions in the context of uplink and downlink was discussed and expressions for range expansion bias were given.

2.7.5 Intercell Interference Coordination (ICIC)

For the case of an OFDMA-based femtocell network, the interference management techniques are more versatile considering the fact that OFDMA offers more parameters that can be controlled. Nevertheless, for co-channel deployment, mathematically optimal or near optimal resource allocation solutions are difficult to compute, are usually infeasible in practice, and are uneconomical to implement. As such the heuristic based intercell interference coordination (ICIC) technique has been much studied [100]-[106]. In this techniques, the macro base station informs the femto base station of its resource scheduling, so that the femto base station knows which subframes will not be utilized (also known as *almost blank subframes*) by the macrocell. The awareness of macro base station's interference pattern allows the femto base station to schedule user equipments in its coverage on subframes corresponding to the almost blank subframes of the macro base station, which is protected from the macrocell interference. Thus, ICIC essentially aims to lower the collision probability of resources scheduled by different base stations. The ICIC technique has been incorporated in LTE Rel-10 [99]. In LTE Rel-10, the ICIC technique is envisioned to be used in conjunction with cell range expansion.

In [100] a brief survey of ICIC was given and the potential of ICIC for OFDMA in terms of throughput, delay and mobile station energy consumption was studied. In [101] location based resource management is incorporated so as to maximize the spatial frequency reuse of femtocells. In [102] and [103], improvements over OFDMA/ICIC were proposed. The authors in [104] have proposed the use of belief propagation iteration to solve the interference coordination and resource allocation problems. In [105] a system wide simulation of LTE heterogeneous network has been performed. In [106] a survey on various ICIC implementation techniques for OFDMA based femtocells was given.

2.7.6 Power Control

Power control methods focus on adjusting the transmit powers of the macrocell and femtocells so as to minimize the cross-tier interference. Power control is an important strategy for co-channel deployment. The uplink power control of user equipment is much easier than the downlink problem. This is because the user equipment operates on a single subchannel, and there is also no heterogeneity of transmit power among the user equipments. As such the uplink power control has been studied in [107, 108]. In [107] a distributed utility based uplink power adaptation algorithm at femtocells was proposed, which reduces to Foschini-Miljanic algorithm under special cases. In [108] the transmit power of a femtocell user is controlled so that the cross-tier interference is less than a fixed threshold based on the noise and interference level experienced by the macro base station.

For downlink OFDMA-based femtocells, the transmit power control is invariably linked with the subchannel allocation problem. Also, it is assumed that the base stations have information about interference conditions in different subchannels. Power control has been approached using mathematical optimization techniques in [109]-[119], where the goal is to maximize a common network utility function, while at the same time ensuring the quality-of-service (QoS) to the base stations. In [120, 121] the power control has been approached using game theory. In this approach, each base station is regarded as a rational agent that selfishly tries to maximize their own utility. As such, the solution of the game is given by a Nash equilibrium. Game theoretic approach is particularly suited for distributed implementation.

2.7.7 Access Policies and Handoffs

Femtocell access control can be considered as one of the interference management techniques where the macrocell has the choice of offloading some of its traffic to the femtocells. Since the introduction of femtocells introduces heterogeneity in the cellular network, the effect of random placements and restricted access policy on the statistical characteristics of interference was investigated in [122, 123]. In [123] the authors examined the effect of soft handoff on the uplink user capacity of a two-tiered CDMA system. The authors proposed exact and approximate analytical methods to compute the uplink user capacity, and showed an increase in user capacity compared to hard handoff. The pros and cons of closed access, open access [28, 218], and hybrid access [213, 214, 215, 216, 217, 73] has been studied.

2.7.8 Self Organization

Since femtocells are user installed and supposed to be a plug-and-play operation, it is important that femtocells have self organizing capability. Considerable attention has been devoted to various aspects of this issue. In [128], the procedures for auto registration and authentication were considered. In [129, 130, 131, 132] auto channel selection, power adjustment and frequency assignment were investigated.

The idea of cognitive femtocell has also emerged over recent years, in which the femtocells are equipped with cognitive radio ability. Cognitive femtocells dynamically sense the spectrum used by the macrocell and opportunistically exploit any spectral vacancy by adjusting their transmission parameters [133]-[138].

Apart from these, there has also been a growing interest in power saving mechanisms for femtocell networks due to rising energy cost and increased environmental awareness [139, 140]. For this purpose, the idea of dynamic sleep (or idle) mode technique was explored in [141, 142, 143] to reduce the energy consumption.

2.7.9 Cooperative Communication

Apart from these, some advanced techniques where multiple base stations are allowed to cooperate has also been explored. The authors in [125, 198] studied the possibility of

interference alignment between the macrocell and femtocells. The interference cancellation techniques were studied by [126, 127]. A survey of cooperative communication has been given in [124, 61].

2.8 Conclusion

In this chapter, we described the motivation behind overlaying the traditional macrocells with smallcells, so as to create a heterogeneous network, in order to enhance the spectral efficiency per unit area via greater spectrum reuse and network densification, with particular reference to femtocells. Femtocells are the smallest of smallcells, with a typical cell range of 10 meters, and are envisioned to be plug-and-play devices deployed by customers, and are connected to the core network via IP-based backhaul. The femtocells offer a range of advantages in that they do not require site planning and maintenance by the network operators. For the customers, the femtocells offer a better quality-of-service. We have outlined different aspects of femtocells with respect to its architecture, access methods, integration into core-network, and various market and technical issues. The state of the art methods that are being investigated to manage interference were also described.

With this background, our work in the coming chapters is motivated by the co-channel deployment scheme, and we study the downlink power control, cooperative communication, and access control methods using game theoretic techniques. We are motivated to find distributed solutions to our problems. The downlink power control problem can also be related to the deployment technique, whereas the cooperative communication problem can also be related to self organization of the femtocell network.

Chapter 3

Hierarchical Competition for Downlink Power Allocation in OFDMA Femtocell Networks

The deployment of femto access points (FAPs) over a traditional cellular system creates a hierarchical overlay network. In this chapter, we address the issue of downlink transmit power control by each station (i.e., a femto-access point or a macro-base station) in an interference channel with a total power constraint when orthogonal frequency-division multiple access (OFDMA) is used with frequency reuse factor of one. Since FAPs are expected to operate on the same frequency band as that of macro BSs (MBSs), co-channel interference affects the overall performance of a network. Also, the aggregate interference to a macrocell user becomes a critical issue as the number of FAPs increases. This is despite the fact that FAPs are expected to use much lower transmit power than that of MBSs, and being deployed indoors, they have external walls of houses insulating the macrocell user from femtocell interference.

Assuming that every base station has an objective to maximize its capacity, the resulting hierarchical power control problem fits the natural framework of a Stackelberg game, or the leader-follower game. The MBSs are considered to be the leaders, whereas the FAPs are considered to be the followers in the leader-follower game. The game is divided into two sub-games such that the leaders and the followers not only compete

against the other group, but each player in the game also competes with other players of their own group. The solution of such a game is the Stackelberg equilibrium (SE).

The rest of the chapter is organized as follows. Section II describes the system model and assumptions, and the preliminaries on a Stackelberg game. The problem formulation and the solution for transmission power control are presented in Section III. Section IV presents the algorithm for power allocation by the leader and the algorithm to compute the Stackelberg equilibrium. The performance evaluation results are presented in Section V. Section VI states the conclusion.

3.1 Related Work

While several works have considered similar setting as ours and dealt with the Nash equilibrium (NE) of the resulting power control game [144]-[149], very few works have modeled the problem as a Stackelberg game. Much of the works [150]-[159] that has been done using Stackelberg game in power control problem involves pricing the data and power. Thus, these works take revenue as the utility function that is required to be maximized. It is worth mentioning that in [163, 164] the authors also considered a case similar to ours and proposed the SE as a solution concept. In [163], an analytical solution was obtained for one leader-one follower game with only two subchannels. In [164], the existence of SE was shown and an algorithm was proposed based on Lagrangian dual method [165] to obtain the equilibrium for one leader-one follower case. In [166], the authors showed the interference experienced by the leader as a linear function of its own power allocation and introduced the idea of conjectural equilibrium. It was shown that both the NE and the SE are special cases of conjectural equilibrium and an algorithm was proposed to reach the conjectural equilibrium. However, the inquiry is not pursued much further, and the question of convergence was not addressed.

3.2 System Model, Assumptions, and Preliminaries on Stackelberg Game

3.2.1 Network, Channel, and Transmission Models

Consider a system of transmitters made up of multiple MBSs and FAPs. We further assume that there exists a wired backhaul connecting the MBSs to the FAPs, which enables them to exchange relevant information required for transmit power allocation. Assuming a time synchronous downlink transmission model, in any given scheduled slot, each MBS/FAP serves exactly one subscribing receiver within its coverage area. Let the set of MBSs be given by $\mathcal{M} = \{\alpha_1, \dots, \alpha_M\}$, and the set of FAPs be given by $\mathcal{N} = \{\beta_1, \dots, \beta_N\}$.

We assume that OFDMA-based radio transmission is used by the MBSs and the FAPs. In all the cells, the number of subchannels are the same and the overlapping subchannels share the same spectrum (i.e., frequency reuse factor is one). We call such channels parallel Gaussian interference channels. For each transmitter $i \in \mathcal{M} \cup \mathcal{N}$, let the available spectrum be divided into a set of orthogonal channels $\mathcal{L} = \{1, \dots, L\}$. Each subchannel has a unit bandwidth. For any $i, j \in \mathcal{M} \cup \mathcal{N}$ and $k, l \in \mathcal{L}$, let $g_{ij}^{kl} \geq 0$ denote the link gain for channel l of transmitter j to channel k of receiver i . We assume that the channels are slow, flat fading channels. The transmitters estimate the channel gain based on the feedback obtained from the receivers, and a channel state is assumed not to change until the next time slot. We assume that the channel estimates are error free; thus transmitters have perfect channel information. The orthogonality of the subchannels of transmitter i assures that the link gain $g_{ii}^{kl} = 0$ for $k \neq l$. Also, for parallel Gaussian interference channels, the link gain $g_{ij}^{kl} = 0$ whenever $k \neq l$.

The noise is assumed to be additive, white Gaussian, with power n_i^k in channel k of receiver i . Let the transmit power of transmitter j in channel k be denoted by p_j^k . The interference and noise power as observed by receiver i in channel k is given by $\nu_i^k = \sum_{j \neq i} g_{ij}^k p_j^k + n_i^k$. Here we have dropped the double superscript in the channel gain to avoid redundancy. Normalizing the link gains and noise powers such that $g_{ii}^k = 1$ for

all $i \in \mathcal{M} \cup \mathcal{N}$ and $k \in \mathcal{L}$, the signal-to-interference-plus-noise ratio (SINR) of receiver i in channel k , which equals $g_{ii}^k p_i^k / \nu_i^k$, can be simplified to be $\frac{p_i^k}{\nu_i^k} = \frac{p_i^k}{\sum_{j \neq i} g_{ij}^k p_j^k + n_i^k}$.

Let $\mathbf{p}_i = (p_i^1, \dots, p_i^L)^T$ be the transmit power vector of transmitter i and $\boldsymbol{\nu}_i = (\nu_i^1, \dots, \nu_i^L)^T$ be the interference-plus-noise vector of receiver i . The power of each transmitter i is limited by the total power constraint such that $p_i^k \geq 0$ and $\sum_{k \in \mathcal{L}} p_i^k \leq \bar{p}_i$, for all $i \in \mathcal{N} \cup \mathcal{M}$ and $k \in \mathcal{L}$. Let $\mathcal{P}_i$ denote the set of all feasible power vectors of transmitter i as follows:

$$\mathcal{P}_i = \left\{ \mathbf{p}_i : p_i^k \geq 0, \sum_{k \in \mathcal{L}} p_i^k \leq \bar{p}_i \right\}. \quad (3.1)$$

We assume that each transmitter seeks to allocate its transmit power to maximize the total throughput. Each user considers the interference from other transmitters as additive white Gaussian. Therefore, given the power allocation vector, from Shannon's capacity formula for additive white Gaussian channels, the maximum data rate that user i can achieve is

$$C_i = C_i(\mathbf{p}_{\alpha_1}, \dots, \mathbf{p}_{\alpha_M}, \mathbf{p}_{\beta_1}, \dots, \mathbf{p}_{\beta_N}) = \sum_{k \in \mathcal{L}} \log_2 \left(1 + \frac{p_i^k}{\nu_i^k} \right). \quad (3.2)$$

3.2.2 Stackelberg Game Model

Stackelberg game, also known as the leader-follower game or the bi-level game, is an extension of non-cooperative game in which there is a group of players, called leaders, that have the privilege of making the first move, while the remaining players, called the followers, make their moves after the leaders. Therefore, a distinct hierarchy exists among the players, and the leaders can anticipate and take into consideration the behavior of the followers, before making their own moves. The followers cannot make this anticipation and thus can only react to the leaders' move.

Here we consider a game of complete and perfect information. The MBSs and the FAPs are the players of the game. The MBSs in set $\mathcal{M}$ belong to the upper level and are referred to as leaders, whereas the FAPs in set $\mathcal{N}$ belong to the lower level and are referred to as followers. Therefore, the total set of players in the Stackelberg game is $\mathcal{M} \cup \mathcal{N}$.

The strategy space of the leaders is given by $\mathcal{P}^{up} = \prod_{i \in \mathcal{M}} \mathcal{P}_i$, and the point in $\mathcal{P}^{up}$ is called a leader strategy. The leaders compete with each other in a non-cooperative manner to maximize their individual throughput, all the time anticipating the response of the followers. This sub-game is referred to as the upper sub-game, and its equilibrium is the upper sub-game equilibrium. After the leaders apply their strategies, the followers make their moves in response to the leaders' strategies. The strategy space of the followers is $\mathcal{P}^{low} = \prod_{i \in \mathcal{N}} \mathcal{P}_i$, and a point in $\mathcal{P}^{low}$ is called a follower strategy. The followers also compete with each other in a non-cooperative manner to maximize their own throughput. This sub-game is referred to as the lower sub-game, and its equilibrium is the lower sub-game equilibrium. Lastly, the strategy space of the entire game is given by the Cartesian product $\mathcal{P} = \mathcal{P}^{up} \times \mathcal{P}^{low}$.

Let us denote a best response function by $\mathbf{p}_i = \operatorname{argmax}_{\mathbf{p}_i} C_i(\mathbf{p}_i, \mathbf{p}_{-i}) = BR_i(\mathbf{p}_i, \mathbf{p}_{-i})$ such that it maximizes the i th user's capacity function subject to the power constraints. Also $-i$ denotes all the users in the set $\mathcal{M} \cup \mathcal{N}$ except user i .

We define the lower sub-game equilibrium as any fixed point $\mathbf{p}^{low*} = (\mathbf{p}_{\beta_1}^*, \dots, \mathbf{p}_{\beta_N}^*) \in \mathcal{P}^{low}$ such that $\mathbf{p}_i^* = BR_i(\mathbf{p}_i^*, \mathbf{p}_{-i}^*; \mathbf{p}^{up})$ where $\mathbf{p}^{up} \in \mathcal{P}^{up}$ is *fixed* but *arbitrary* leader strategy, for all $i \in \mathcal{N}$. Note that this definition is the same as the NE of the lower game. Since all users in the lower sub-game will myopically maximize their individual throughput, the best response $BR_i(\cdot)$ of each user in the sub-game will be given by the waterfilling function ((11) in [147]). Defining $BR^{low} \equiv (BR_{\beta_1}(\cdot), \dots, BR_{\beta_N}(\cdot))$, we can express the lower sub-game equilibrium as any fixed point of the system power space $\mathbf{p}^* \in \mathcal{P}$ such that

$$\mathbf{p}^* = BR^{low}(\mathbf{p}^*). \quad (3.3)$$

Note that the function $BR^{low}(\cdot)$ leaves the upper sub-game strategy unchanged.

We now define the upper sub-game equilibrium as any fixed point $\mathbf{p}^{up*} = (\mathbf{p}_{\alpha_1}^*, \dots, \mathbf{p}_{\alpha_M}^*) \in \mathcal{P}^{up}$ such that $\mathbf{p}_i^* = BR_i(\mathbf{p}_i^*, \mathbf{p}_{-i}^*; \mathbf{p}^{low*})$ where $\mathbf{p}^{low*} \in \mathcal{P}^{low}$ is an *equilibrium* follower strategy conditioned on the upper sub-game strategy, for all $i \in \mathcal{M}$. Equivalently, let $BR^{up} \equiv (BR_{\alpha_1}(\cdot), \dots, BR_{\alpha_M}(\cdot))$, then we can define the upper sub-game equilibrium as the fixed point $\mathbf{p}^{up*} \in \mathcal{P}^{up}$ such that $\mathbf{p}^{up*} = BR^{up}(\mathbf{p}^{up*}; BR^{low}(\mathbf{p}^{low*}; \mathbf{p}^{up*}))$. We can simplify the notations further and write the upper sub-game equilibrium in terms of system

power vector as any fixed point $\mathbf{p}^* \in \mathcal{P}$ such that

$$\mathbf{p}^* = BR^{up}(BR^{low}(\mathbf{p}^*)). \quad (3.4)$$

Note that although the function $BR^{up}(\cdot)$ accounts only for the upper sub-game strategy, the lower sub-game equilibrium strategy associated with each upper sub-game strategy needs to be computed as well.

The SE is defined as any fixed point $(\mathbf{p}^{up*}, \mathbf{p}^{low*}) = \mathbf{p}^* \in \mathcal{P}$, that satisfies (3.3) and (3.4). In other words, let the self map $BR : \mathcal{P} \rightarrow \mathcal{P}$ be a composition of two vector functions $BR \equiv BR^{up} \circ BR^{low}$. Then, we have the SE as any fixed point of the function BR , $\mathbf{p}^* = BR(\mathbf{p}^*)$ such that $\mathbf{p}^* = (\mathbf{p}^{up*}, \mathbf{p}^{low*})$.

3.2.3 Existence of Stackelberg Equilibrium

It is important to note that the best response function for the lower sub-game $BR^{low}(\cdot) \equiv (WF_{\beta_1}(\cdot), \dots, WF_{\beta_N}(\cdot))$, where $WF_i(\cdot)$ for each $i \in \mathcal{N}$ is given by the waterfilling function [147, Eq. 11], is a piecewise affine continuous function of $\mathbf{p}$ [147]. Assuming the continuity of the best response function of the upper sub-game BR^{up} , we can prove the existence of an SE using the Schauder fixed point theorem. For convenience, we state the Schauder fixed point theorem here as follows: Every continuous function from a convex compact subset $\mathcal{K}$ of a Banach space to $\mathcal{K}$ itself has a fixed point [167].

Proposition 1. *Assuming that the best response function of the upper sub-game is continuous, at least one Stackelberg equilibrium exists for the leader-follower power allocation game.*

Proof. Since the best response functions $BR^{up} : \mathcal{P} \rightarrow \mathcal{P}$ and $BR^{low} : \mathcal{P} \rightarrow \mathcal{P}$ are continuous functions, the composition of these two functions $BR = BR^{up} \circ BR^{low}$ is also continuous. It is also easy to see that $\mathcal{P}$ as defined by (3.1) is convex, closed, and bounded. Since SE is defined as any fixed point of $BR(\cdot)$, from the Schauder fixed point theorem, the above statement is proved. $\square$

Indeed, it can be shown that the lower sub-game equilibrium is in fact unique. It is straightforward to check that, for any fixed leader strategy in $\mathcal{P}^{up}$, the sum of capacities $\sum_{i \in \mathcal{N}} C_i$ for the lower game is diagonally strictly concave. Therefore, from [168], for any given leader strategy, there is at most one lower sub-game equilibrium. However, the uniqueness of the upper sub-game equilibrium may not be guaranteed.

3.3 Problem Formulation and Solution for Optimal Power Allocation

The solution of a Stackelberg game is given by the sub-game perfect NE, which is also referred to as SE in our context. In this section, the power allocation problem for a single leader and multiple follower scenario is formulated as a mathematical program with equilibrium constraint (MPEC) [169].

3.3.1 Single Leader-Multiple Follower Game with Sum Rate Maximization

Since a leader allocates its power to maximize its own data rate by considering the response of the followers to its transmission strategy, which is known to be the waterfilling algorithm, the best response of each leader $i \in \mathcal{M}$ can be found by solving the following optimization problem with equilibrium constraints:

$$\max_{\mathbf{p}_i} \quad \sum_{k=1}^L \log_2 \left(1 + \frac{p_i^k}{\sum_{j \neq i} g_{ji}^k p_j^k + n_i^k} \right) \quad (3.5)$$

$$\text{s.t.} \quad p_i^k \geq 0, \quad \sum_{k=1}^L p_i^k = \bar{p}_i \quad (3.6)$$

$$\mathbf{p}_j \in \operatorname{argmax}_{\mathbf{p}_j} \{C_j : p_j^k \geq 0, \sum_{k=1}^L p_j^k = \bar{p}_j\}, \quad \forall j \in \mathcal{N}. \quad (3.7)$$

Here the sub-problems (3.5) and (3.6) are the upper sub-problem while (3.7) is the lower sub-problem. The lower sub-problem (3.7) represents the equilibrium constraints.

In our case, the solution is obtained by parameterizing the lower sub-game NE of the corresponding followers by the leader's strategy.

The assumptions while deriving the results are as follows:

Assumption 1. *The leader has perfect knowledge of all the channel gains.*

Assumption 2. *The leader has the knowledge of the channel allocation strategy of all the corresponding followers.*

Assumption 3. *The followers' lower game converges to a unique and stable Nash equilibrium during iterative waterfilling process.*

Regarding the first assumption, it is possible for a leader to know the large-scale fading depending on the distance geometry of the transmitters and receivers. However, to account for fading due to multipath and shadowing effects, channel information exchange is required. However, we will see in the second discussion point of Section 3.3.1.3 how we can significantly relax this assumption. Regarding the second assumption, a macrocell receiver can sense the interference that it experiences in various subchannels and report to the corresponding macro BS. Furthermore, a leader can distinguish the transmission of its followers, allowing it to tell who is transmitting in which subchannel. This is a reasonable assumption if we are to maintain the hand-off capability among macro and femtocells. Regarding the third condition, it is sufficient for the spectral radius of the followers' channel gain matrix to be less than unity for the stability and convergence of iterative waterfilling process [147]. This condition essentially quantifies how large the co-channel interference between the followers is allowed to be. We can physically interpret this condition to mean that the interference between the followers should be small. When the femtocells are deployed indoor, the radio signals experience wall penetration loss before it can interfere with another femtocell. Thus this wall penetration loss along with sufficiently sparse enough geographical distribution of the femtocells can ensure that this condition will be satisfied.

In the following sections, by **active subchannel** we mean a subchannel in which a transmitter has allocated some positive power, by **inactive subchannel** we mean a

subchannel for which a transmitter has allocated zero power, and by **interfering sub-channel** we mean a subchannel in which there are more than one transmitter allocating some positive power.

The power allocation problem consists of two subproblems: “Which subchannels to choose to allocate power?” and, “How much power to be allocated over the chosen subchannels?” The idea behind the solution is to reduce the power allocation problem into an equivalent channel selection problem. Once the channel selection is known, the power allocated to the channels can be computed.

Let $\mathcal{A}_{f_i}$ and $\mathcal{A}_l$ denote the set of active subchannels of the follower $i \in \mathcal{N}$ and the leader, respectively. Define $\mathcal{A}_f = \cup_{i \in \mathcal{N}} \mathcal{A}_{f_i}$. Let $\mathcal{I} = \mathcal{A}_f \cap \mathcal{A}_l$ denote the set of interfering subchannels for the leader. Finally, let $\bar{\mathcal{I}} = \mathcal{A}_l \setminus \mathcal{I}$ denote the set of non-interfering subchannels for the leader. From here on, we shall denote the power allocated by the leader to be q^k and the co-channel gain of the i th follower’s transmitter to the leader’s receiver to be h_i^k on the k th subchannel.

3.3.1.1 Lower Problem

The lower problem of the i th follower is thus given as

$$\max_{\mathbf{p}_i} C_{f_i} = \max_{\mathbf{p}_i} \sum_{k \in \mathcal{L}} \log_2 \left(1 + \frac{p_i^k}{g_{il}^k q^k + \sum_{j \neq i} g_{ij}^k p_j^k + n_i^k} \right)$$

such that $\sum_{k=1}^L p_i^k = \bar{p}_i$ and $p_i^k \geq 0$. Here g_{il}^k is the co-channel gain from the leader’s transmitter to the i th follower’s receiver. Since the noise-plus-interference term, $\nu_i^k = g_{il}^k q^k + \sum_{j \neq i} g_{ij}^k p_j^k + n_i^k$, is treated as a constant by the follower, the objective function is concave. The solution of this optimization problem leads to the well-known waterfilling solution [170], given as

$$p_i^k = \begin{cases} 0, & \text{for } k \in \mathcal{L} \setminus \mathcal{A}_{f_i} \\ K_i - \nu_i^k, & \text{for } k \in \mathcal{A}_{f_i} \end{cases} \quad (3.8)$$

where $1/K_i$ is the positive Lagrange multiplier.

Finally, using (3.8) in the constraint $\sum_{k \in \mathcal{L}} p_i^k = \bar{p}_i$, we can have $K_i = \frac{1}{|\mathcal{A}_{f_i}|} (\bar{p}_i + \sum_{k \in \mathcal{A}_{f_i}} \nu_i^k)$, where $|\cdot|$ represents the cardinality of a set.

3.3.1.2 Upper Problem

Assuming that the leader has the information of set $\mathcal{A}_{f_i}$ as well as the channel gains, the upper problem of the leader is given as

$$\begin{aligned} \max_{\mathbf{q}} C_l &= \max_{\mathbf{q}} \sum_{k \in \mathcal{L}} \log_2 \left(1 + \frac{q^k}{\sum_{i \in \mathcal{N}} h_i^k p_i^k + n_l^k} \right) \\ &= \max_{\mathbf{q}} \left[\sum_{k \notin \mathcal{A}_f} \log_2 \left(1 + \frac{q^k}{n_l^k} \right) + \sum_{k \in \mathcal{A}_f} \log_2 \left(1 + \frac{q^k}{\sum_{i \in \mathcal{N}} h_i^k p_i^k + n_l^k} \right) \right] \end{aligned}$$

such that $\sum_{k=1}^L q^k = \bar{q}$ and $q^k \geq 0$. Here n_l^k is the noise power at the leader's receiver in k th subchannel. Since the noise plus interference term, $\sum_{i \in \mathcal{N}} h_i^k p_i^k + n_l^k$, is not treated as a constant by the leader, the second summation term of the objective function is non-concave. We can form the Lagrangian of the optimization problem as follows:

$$\Lambda = C_l - \frac{1}{W} \left(\sum_{k \in \mathcal{L}} q^k - \bar{q} \right),$$

where $1/W$ is the positive Lagrange multiplier constant which is the same for all the subchannels. For brevity, we have neglected the terms involving inequalities in the above Lagrangian, since they do not play any part in the later developments due to complementary slackness condition (see Appendix for discussion). From the first-order necessary condition for optimality, $\nabla_{\mathbf{q}} \Lambda = 0$, the solution can be obtained considering the following two cases.

For $k \notin \mathcal{A}_f$, as with the lower problem, the solution is given by the following water-filling equation:

$$q^k = \begin{cases} 0, & \text{for } k \in \mathcal{L} \setminus \mathcal{A}_l \\ W - n_l^k, & \text{for } k \in \bar{\mathcal{I}}. \end{cases} \quad (3.9)$$

For $k \in \mathcal{A}_f$, to ease the notations, from here on let us put the subchannel index k in the subscript, so that the leader's power allocation is q_k . Also, let n'_k denote the noise power at k th subchannel of the leader. Then, we have the following lemmas and their resulting proposition:

Lemma 1. *For $k \in \mathcal{A}_f$, let $F(q_k)$ denote the noise-plus-interference term of the leader as some function of q_k . Then, solution to $\nabla_{\mathbf{q}} \Lambda = 0$ is given by a fixed point equation*

$$q_k = \frac{W - F(q_k)}{1 + W \frac{F'(q_k)}{F(q_k)}} \quad (3.10)$$

where $F'(q_k)$ is the derivative of $F(q_k)$ with respect to q_k .

Proof. See Appendix. □

Lemma 2. For $k \in \mathcal{A}_f$, if $F(q_k) = a_k q_k + b_k$, where $b_k \neq 0$, then $q_k = 2(1 + 2a_k)W$ is the unique solution of (3.10) such that $W = -b_k/[4a_k(1 + a_k)]$.

Proof. See Appendix. □

Lemma 3. For $k \in \mathcal{A}_f$, if $F(q_k) = a_k q_k + b_k$, then $b_k > 0$ and $0 > a_k > -1$.

Proof. Observe that the noise-plus-interference term is always positive, (dropping the use of index k) $F(q) = \sum_{i \in \mathcal{N}} h_i p_i + n' = aq + b > 0$. Thus, $F(q = 0) = b > 0$. Also since the Lagrange multiplier is always positive, from Lemma 2, $W = -b/[4a(1 + a)] > 0$. Since $b > 0$, the expression $-b/[4a(1 + a)] > 0$ is valid if and only if $a < 0$ and $1 + a > 0$, i.e., $a > -1$. Thus, $0 > a > -1$. □

If there are $J \leq N$ interferers in the k th subchannel, then define the set of interfering followers for that subchannel $\mathcal{J} = \{\sigma_1, \sigma_2, \dots, \sigma_J\}$, such that $\mathcal{J} \subseteq \mathcal{N}$. We will drop the use of index k to simplify the notations, as it is understood that $k \in \mathcal{A}_f$. Since the followers treat the noise-plus-interference power as a constant, the power allocated by the followers given by the waterfilling function is as before:

$$p_i = K_i - \left(g_{il}q + \sum_{j \neq i} g_{ij}p_j + n_i \right), \quad \forall i, j \in \mathcal{J}.$$

These equations form a system of J simultaneous linear equations and can be written as follows:

$$p_i + \sum_{j \neq i} g_{ij}p_j = -g_{il}q + K_i - n_i, \quad \forall i, j \in \mathcal{J}.$$

Obviously, we cannot solve these simultaneous equations directly, since it would require the values of K_i , which need to be found numerically. A practical approach is to use the iterative waterfilling algorithm among the followers for a given leader power strategy. Thus, the NE of the lower sub-game would be the solution of the above simultaneous equations.

Proposition 2 (Linear interference). *Assuming that the equations*

$$p_i + \sum_{j \neq i} g_{ij} p_j = -g_{il} q + K_i - n_i, \quad \forall i, j \in \mathcal{J}.$$

are linearly independent, we can express p_i as $p_i = A_i q + B_i, \forall i \in \mathcal{J}$, where A_i and B_i are obtained by solving the following matrix equations:

$$\begin{bmatrix} g_{\sigma_1 \sigma_1} & \cdots & g_{\sigma_1 \sigma_J} \\ \vdots & \ddots & \vdots \\ g_{\sigma_J \sigma_1} & \cdots & g_{\sigma_J \sigma_J} \end{bmatrix} \begin{bmatrix} A_{\sigma_1} \\ \vdots \\ A_{\sigma_J} \end{bmatrix} = - \begin{bmatrix} g_{\sigma_1 l} \\ \vdots \\ g_{\sigma_J l} \end{bmatrix} \quad (3.11)$$

$$\text{and } \begin{bmatrix} g_{\sigma_1 \sigma_1} & \cdots & g_{\sigma_1 \sigma_J} \\ \vdots & \ddots & \vdots \\ g_{\sigma_J \sigma_1} & \cdots & g_{\sigma_J \sigma_J} \end{bmatrix} \begin{bmatrix} B_{\sigma_1} \\ \vdots \\ B_{\sigma_J} \end{bmatrix} = \begin{bmatrix} K_{\sigma_1} - n_{\sigma_1} \\ \vdots \\ K_{\sigma_J} - n_{\sigma_J} \end{bmatrix}. \quad (3.12)$$

Proof. See Appendix. □

The linear independence assumption in Proposition 2 was made to ensure that the channel gain matrix in the resulting equations (3.11) and (3.12) are non-singular and that the equations are mathematically guaranteed to have a solution. However, it is not necessary to make this statement explicit, since the Assumption 3 that the lower game converges to a unique and stable Nash equilibrium is sufficient to justify the assumption that the equations are linearly independent. The equations (3.11) and (3.12) essentially computes the lower game Nash equilibrium for given leader power. As stated previously, the Assumption 3 holds when the spectral radius of the followers' channel gain matrix is less than unity. This condition is also sufficient to ensure that the gain matrix is non-singular and hence linearly independent.

These equations (3.11) and (3.12) can also be derived as given in [166, Proposition 1]. In (3.12), $K_i - n_i > 0$ and so are $g_{\sigma_i \sigma_j} > 0$ for all $i, j \in \mathcal{J}$. Thus from [171], the sufficient condition that guarantees the existence of positive solution for B_i is $K_{\sigma_i} - n_{\sigma_i} > \sum_{j \neq i} g_{\sigma_i \sigma_j} (K_{\sigma_j} - n_{\sigma_j})$ for all $i, j \in \mathcal{J}$. Similarly, since $g_{\sigma_i l} > 0$ for all $i \in \mathcal{J}$, from [171], the sufficient condition that guarantees the existence of *negative* solutions for A_i is $g_{\sigma_i l} > \sum_{j \neq i} g_{\sigma_i \sigma_j} g_{\sigma_j l}$ for all $i, j \in \mathcal{J}$. Note that this condition also guarantees the existence of matrix inverse.

Proposition 3. *Given the active set of subchannels of the followers $\mathcal{A}_f$, the necessary condition required for a unique, optimal power allocation of the leader to maximize its Shannon capacity under total power constraint is given by*

$$q_k = \begin{cases} 0, & \text{for } k \in \mathcal{L} \setminus \mathcal{A}_l \\ W - n'_k, & \text{for } k \in \bar{\mathcal{I}} \\ 2(1 + 2a_k)W, & \text{for } k \in \mathcal{I} \end{cases} \quad (3.13)$$

depending on the set of interferers $\mathcal{J}$ present in k th subchannel, $a_k = \sum_{j=1}^J h_{\sigma_j} A_{\sigma_j}$, and W is given by

$$W = \frac{1}{|\bar{\mathcal{I}}| + \sum_{k \in \mathcal{I}} c_k} \left(\bar{q} + \sum_{k \in \bar{\mathcal{I}}} n'_k \right), \quad \text{where } c_k = 2(1 + 2a_k). \quad (3.14)$$

Proof. For $k \in \mathcal{A}_f$, dropping the use of index k , the interference experienced by the receiver of the leader is given by the function $F(q) = \sum_{i \in \mathcal{J}} h_i p_i + n'$. Substituting the expression of $p_i = A_i q + B_i$ from Proposition 2 into this expression for $F(q)$, we obtain $F(q) = aq + b$, where $a = \sum_{j=1}^J h_{\sigma_j} A_{\sigma_j}$ and $b = n' + \sum_{j=1}^J h_{\sigma_j} B_{\sigma_j}$. The linearity of $F(q)$ makes the upper subproblem quasi-concave. Thus, in linear interference case, a global maximum solution for the upper subproblem exists, and the solution can be obtained by the first order Karush-Kuhn-Tucker condition for optimality [172, Proposition 3.3.1]. Hence, from Lemma 1, Lemma 2, and Lemma 3, we obtain the power allocation to be $q = 2(1 + 2a)W$. Combining this result with (3.9) gives us (3.13). Lastly, the expression for W can be obtained by substituting the obtained q_k in the total power constraint $\sum_{k \in \mathcal{L}} q_k = \bar{q}$. $\square$

3.3.1.3 Discussion

1. The Proposition 3 shows that the best response function of the leader is a piece-wise linear function. Thus the continuity condition of Proposition 1 is satisfied, guaranteeing the existence of a Stackelberg equilibrium.

2. In Proposition 3, we have related the solution given by equation (3.13) to the parameter a_k , where a_k is described as a function of various channel gains and identity of channel users. However, we can also take a more operational view of a_k as being the rate of change of interference with respect to the leader's power in k -th subchannel, as given by the equation $F(q) = aq + b$. Thus, a practical way of computing this a_k , based only on directly measurable parameters and without resorting to any knowledge of the cross-channel gains, would be to take the approximation $a_k \approx \frac{\Delta F(q_k)}{\Delta q_k}$. This represents a significant loosening of our assumptions. Thus, there is no actual need to know cross-channel gains. We only need the local direct channel knowledge, which is possible using pilot carriers and with the assumption of the uplink-downlink channel reciprocity.
3. In practice, since we expect the macrocell receivers to sense the femtocell channel activities and report it to their serving BS, the sensing ability would be physically limited. Not all femtocells that are active within a macrocell domain may be detected. However the BS's reaction to the detected femtocell can still be exploited by undetected femtocells.
4. Since we have the constraint that $q_k \geq 0$, we assign $q_k = 0$ whenever $W - n'_k < 0$ for $k \in \bar{\mathcal{I}}$, or $a_k < -\frac{1}{2}$ for $k \in \mathcal{I}$. In other words, the set of active subchannels is given by $\mathcal{A}_l = \{k | W - n'_k > 0 \text{ or } c_k > 0\}$ where the range of positive values of $c_k \in (0, 2)$. Note that for one leader-one follower case, $a_k = -g_k h_k$. For one leader-two follower case, for any subchannel k , when $J = 1$, $a_k = -g_{\sigma_1 l} h_{\sigma_1}$, and when $J = 2$, $a_k = -\frac{h_{\sigma_1}(g_{\sigma_1 l} - g_{\sigma_2 l} g_{\sigma_1 \sigma_2}) + h_{\sigma_2}(g_{\sigma_2 l} - g_{\sigma_1 l} g_{\sigma_2 \sigma_1})}{1 - g_{\sigma_1 \sigma_2} g_{\sigma_2 \sigma_1}}$.
5. For any two subchannels $k_1, k_2 \in \bar{\mathcal{I}}$, if $n'_{k_1} > n'_{k_2}$, then $q_{k_2} > q_{k_1}$. That is, in the case of non-interfering subchannels, leader allocates more power in subchannels with lesser noise power. Also, for any $k_1, k_2 \in \mathcal{I}$, if $c_{k_1} > c_{k_2}$, then $q_{k_1} > q_{k_2}$. This means that, in the case of interfering sub-channels, leader allocates more power in subchannels with higher value of c_k . In later discussion point, we interpret this via the idea of "pseudo-noise."

6. Irrespective of the value of W , if $c_k > 1$ for $k \in \mathcal{I}$, then the power allocated to that interfering subchannel is greater than that of any non-interfering subchannel. This is because when $c_k > 1$ for any $k \in \mathcal{I}$, $q_k = c_k W > W \geq W - n'_{k \in \bar{\mathcal{I}}} = q_{k \in \bar{\mathcal{I}}}$.
7. For the purpose of visualization, it is convenient to introduce the idea of “pseudo-noise power” for the interfering subchannels. For any $k \in \mathcal{I}$, define the **pseudo-noise power**, denoted by $\tilde{n}'_k$, such that $c_k W = W - \tilde{n}'_k$. Thus, $\tilde{n}'_k = (1 - c_k)W$. Using this notion, allocating power over the interfering subchannels becomes analogous to waterfilling operation and we can interpret W as the “water level” for the entire set of subchannels. This means that leader allocates more power to subchannels with lesser “pseudo-noise power.” Since the power is allocated only to those subchannels for which $c_k \in (0, 2)$, we must have $\tilde{n}'_k \in (-W, W)$ for these active subchannels. Higher value of c_k means lower value of $\tilde{n}'_k$. Note that unlike the real noise power, the “pseudo-noise power” can be negative.

3.3.1.4 Illustration

Fig. 3.1 shows an example of the power allocated by the leader (based on equation (3.13)). Here we take $L = 32$ and $\bar{q} = 10$. The n'_k is taken to be a uniform random variable between the interval $[0, 1]$. Instead of calculating the parameter c_k , for the sake of illustration, assume that c_k varies uniformly in the interval $[-2, 2]$. The followers’ channel allocation is taken to be $\mathcal{A}_f = \{1, \dots, L/2\}$. In Fig. 3.1, the black bars represent either the noise or “pseudo-noise”, while the white bars represent allocated transmit power in various subchannels. The plot has been created by stacking the white allocated power bar over the black noise power bar. The subchannels from 1 to 16 have been sorted in an ascending order according to their equivalent “pseudo-noise” power, whereas the subchannels from 17 to 32 have been sorted according to their noise power. Note that the “pseudo-noise” level in subchannels 1 to 5 is negative, in contrast to actual noise power, so the white power bar takes the negative value as the stacking point. Hence the white bars denoting the allocated transmit power has overlapped the black bars. The use of this concept has allowed us to preserve the waterfilling interpretation for the interfering subchannels. Thus in the figure, for the subchannels from 1 to 16, power has been allocated such that

Figure 3.1: Leader's power allocation strategy.

$q_k = c_k W$, and for subchannels from 17 to 32, $q_k = W - n'_k$, which is waterfilling in proper sense. The allocated power is stacked over the noise power. Here the “water level” is $W = 0.537$.

3.3.2 Single Leader-Multiple Follower Game with QoS Constraint for Leader

In this section, we consider the effect of adding QoS constraint for the leader. It is emphasized that the SINR constraint for followers (femtocells) is not necessary to be enforced, unlike for the leader (macrocell). This is because the femtocells are expected to opportunistically exploit the “left-over” resource while the QoS for macrocell users is being met. This means that they are willing to negotiate their QoS for those users within femtocells, by virtue of adaptive modulation and coding technique (i.e., rate control).

The justification for excluding the QoS enforcement for femtocells is as follows: The femtocells have small coverage and few users share the femto resource. This means that the femto links are inherently more reliable and provide better service than that of the macro links. Thus the macro users are more vulnerable than the femto users. Adding QoS to the femtocell is straightforward, since it only requires that a femto BS allocate more power than some threshold at each subchannel. While this would guarantee a higher

SINR at the receiving femto user equipment, this usually also requires more transmit power at the femto BS. This in turn leads to a higher cross-interference induced to the macrocell. Our system model provides the so called “soft QoS” for the femtocells, where high performance can be achieved even when the QoS constraint is not imposed. Similar idea of “soft QoS” provisioning has also been considered in [173].

Let the threshold SINR $\theta_k \geq 0$ be given for every subchannel $k \in \mathcal{L}$. For a given subchannel $k \in \mathcal{A}_f$, the noise-plus-interference is given by the function $F(q_k) = a_k q_k + b_k$. Thus, the SINR constraint for the subchannel would be $q_k/F(q_k) \geq \theta_k$, or $q_k - \theta_k F(q_k) \geq 0$. Substituting the expression for $F(q_k)$, and solving for q_k , we obtain $(1 - \theta_k a_k)q_k - \theta_k b_k \geq 0$, i.e., $q_k \geq \theta_k b_k / (1 - \theta_k a_k)$. We can avoid the use of b_k by using the relationship between b_k and W as given in Lemma 2, which is also valid for any number of followers. Thus, the above condition becomes $q_k \geq -\frac{4\theta_k a_k(1+a_k)}{1-\theta_k a_k}W$. Note that the right hand side of the above inequality will be negative when $a_k < -1$. Since we also have the constraint $q_k \geq 0$, let us define $d_k = \max(0, -\frac{4\theta_k a_k(1+a_k)}{1-\theta_k a_k})$, which is constant for a given subchannel. We can thus express the above power bound as $q_k \geq d_k W$. Similarly, for subchannels $k \notin \mathcal{A}_f$, we have the SINR constraint $q_k/n'_k \geq \theta_k$, i.e., $q_k \geq \theta_k n'_k$.

Putting these two inequalities for $k \in \mathcal{A}_f$ and $k \notin \mathcal{A}_f$, we have the following power bound for QoS constraint satisfaction:

$$q_k \geq \begin{cases} \theta_k n'_k, & \text{for } k \notin \mathcal{A}_f \\ d_k W, & \text{for } k \in \mathcal{A}_f. \end{cases} \quad (3.15)$$

Here even with the inequality constraints (3.15), the solution as given by Proposition 3 will remain unchanged. The reason being that, the complementary slackness criteria of Karush-Kuhn-Tucker condition for the non-linear program ensures that the terms in the Lagrangian associated with the inequality will be zero. Thus this inequality does not play any active role in the later development of the equations.

Let $q_k = e_k + \theta_k n'_k$ for $k \notin \mathcal{A}_f$ and $q_k = e_k + d_k W$ for $k \in \mathcal{A}_f$, where e_k is the incremental power allocated to k th subchannel. Then, modifying (3.13), from the necessary optimality condition, we have

$$e_k = \begin{cases} 0, & \text{for } k \in \mathcal{L} \setminus \mathcal{A}_l \\ W - (1 + \theta_k)n'_k, & \text{for } k \in \bar{\mathcal{I}} \\ (c_k - d_k)W, & \text{for } k \in \mathcal{I} \end{cases} \quad (3.16)$$

where we re-define $\mathcal{A}_l = \{k|e_k > 0\}$.

Finally, we obtain the expression for W using the total power constraint as follows:

$$\begin{aligned}
\bar{q} &= \sum_{k \in \mathcal{L}} e_k + \sum_{k \notin \mathcal{A}_f} \theta_k n'_k + \sum_{k \in \mathcal{A}_f} d_k W \\
&= \sum_{k \in \bar{\mathcal{I}}} (W - (1 + \theta_k) n'_k) + \sum_{k \in \mathcal{I}} (c_k - d_k) W + \sum_{k \notin \mathcal{A}_f} \theta_k n'_k + \sum_{k \in \mathcal{A}_f} d_k W \\
&= (|\bar{\mathcal{I}}| + \sum_{k \in \mathcal{I}} c_k + \sum_{k \in \mathcal{A}_f \setminus \mathcal{I}} d_k) W - \sum_{k \in \bar{\mathcal{I}}} n'_k + \sum_{k \notin \mathcal{A}_l \cup \mathcal{A}_f} \theta_k n'_k \\
\therefore W &= \frac{\bar{q} + \sum_{k \in \bar{\mathcal{I}}} n'_k - \sum_{k \notin \mathcal{A}_l \cup \mathcal{A}_f} \theta_k n'_k}{|\bar{\mathcal{I}}| + \sum_{k \in \mathcal{I}} c_k + \sum_{k \in \mathcal{A}_f \setminus \mathcal{I}} d_k}. \tag{3.17}
\end{aligned}$$

We can check that when $\theta_k = 0$ for all $k \in \mathcal{L}$, (3.16) and (3.17) reduce to the case without QoS constraint.

For $k \in \bar{\mathcal{I}}$, we can interpret the equation $e_k = W - (1 + \theta_k) n'_k$ as so called “mercury/waterfilling” operation. Here we first allocate power to each subchannel so that the QoS constraint $\theta_k n'_k$ is satisfied. This is akin to mercury filling. We then allocate the incremental power e_k over noise plus QoS power. This is akin to waterfilling, which floats over mercury. However, for $k \in \mathcal{I}$, an easy analogy is not available for the incremental power allocation. However, we can express the condition $e_k = (c_k - d_k) W$ in terms of a_k , using the definition of c_k and d_k , as follows: $e_k = 2W \left[\frac{1+a_k(2+\theta_k)}{1-\theta_k a_k} \right]$. For illustration, in Fig. 3.2 we use the same setting as for Fig. 3.1 of Section III.A, but with $\theta_k = 0.5$ for all $k \in \mathcal{L}$. The grey bars in subchannels 17 to 32 represents the mercury filling.

Proposition 4. *Given the threshold SINR $\theta_k \geq 0$, and the set of active subchannels of all the followers $\mathcal{A}_{f_i}$, the unique, optimal power allocation with QoS constraints for the leader is given as follows:*

$$e_k = \begin{cases} 0, & \text{for } k \in \mathcal{L} \setminus \mathcal{A}_l \\ W - (1 + \theta_k) n'_k, & \text{for } k \in \bar{\mathcal{I}} \\ 2W \left[\frac{1+a_k(2+\theta_k)}{1-\theta_k a_k} \right], & \text{for } k \in \mathcal{I} \end{cases} \tag{3.18}$$

where W is given by (3.17) and $\mathcal{A}_l$ is re-defined as $\mathcal{A}_l = \{k|e_k > 0\}$.

Proof. As discussed above. □

Figure 3.2: Leader's power allocation strategy with QoS constraints.

For one leader–one follower case, $a_k = -g_k h_k$. Substituting this expression in (3.18), we have for $k \in \mathcal{I}$,

$$e_k = 2W \left[\frac{1 - g_k h_k (2 + \theta_k)}{1 + \theta_k g_k h_k} \right].$$

If e_k is to be strictly positive, then it is required that $1 - g_k h_k (2 + \theta_k) > 0$. Thus, the relationship between $g_k h_k$ and θ_k is given by the condition: $g_k h_k < 1/(2 + \theta_k)$. Note that this condition is not different from our previous condition without the QoS constraint, $g_k h_k < 1/2$.

3.4 Algorithms for Leader's Power Allocation and Stackelberg Equilibrium

3.4.1 Algorithm for Leader's Power Allocation

Given the channel allocation of the follower, we have the formula for power allocation of the leader as given by (3.13) and that of W by (3.14). Once we know the channel allocation for the leader, the use of these formulas makes the power allocation a trivial task. Thus, finding a proper channel allocation for the leader becomes a combinatorial problem. In this section, we develop an algorithm to select the channels using the binary

search technique [174]. It is based on the fact that the channel allocation $\mathcal{I} = \{k \in \mathcal{A}_f : c_k > 0\}$ does not change with W . Hence, the sum $\sum_{k \in \mathcal{I}} c_k$ that appears in the calculation of W remains constant. Thus, it only remains to select $\bar{\mathcal{I}}$ such that the allocated power satisfies the power constraints. This is done by using the binary search technique given in Algorithm 1. We can also incorporate the QoS constraints by a slight modification of this algorithm.

In detail, Algorithm 1 tries to determine the set of active subchannels $\mathcal{A}_l$ for the leader over which it can allocate power. The set $\mathcal{A}_l$ comprises of $\mathcal{I}$ (the set of interfering subchannels, where the followers too have allocated some power) and $\bar{\mathcal{I}}$ (the set of non-interfering subchannels). Thus, making $\mathcal{A}_l = \mathcal{I} \cup \bar{\mathcal{I}}$. Our task is to determine these two sets $\mathcal{I}$ and $\bar{\mathcal{I}}$.

Finding $\mathcal{I}$ is easy since we just need to search over the set of subchannels $\mathcal{A}_f$ over which the followers are transmitting their power and for which the computed value of c_k is positive. Thus, we have $\mathcal{I} = \{k \in \mathcal{A}_f | c_k > 0\}$.

To find $\bar{\mathcal{I}}$ we need to search over the set subchannels $\mathcal{L} \setminus \mathcal{A}_f$ that are free of interference from the followers. For this purpose, binary search method is applied to find the right number of subchannels from $\mathcal{L} \setminus \mathcal{A}_f$ so as to satisfy the total power constraint. In short, the binary search method halves the search space at each iteration, until the required answer is found. First the set $\mathcal{L} \setminus \mathcal{A}_f = \{1, \dots, |\mathcal{L} \setminus \mathcal{A}_f|\}$ is re-arranged in an ascending order of its noise power. The variables lo and hi are indexes indicating two of the subchannels in this re-arranged set $\mathcal{L} \setminus \mathcal{A}_f$. Initially, $lo = 1$ and $hi = |\mathcal{L} \setminus \mathcal{A}_f|$ to indicate the first and last subchannels of this set $\mathcal{L} \setminus \mathcal{A}_f$ over which the search is to take place. Another index mid is computed to indicate the subchannel which is in the middle of the range of lo and hi . We take the required set of interference free subchannel over which the leader's power is to be allocated to be $\bar{\mathcal{I}} = \{1, \dots, mid\}$ and check if this selection of $\bar{\mathcal{I}}$ is successful. Ideally, the value of the water-level W and the noise power at mid , n'_{mid} , should be the same. However, if the computed value of the water-level W is higher than the noise power at mid , $W > n'_{mid}$, then we need to expand our selected set $\bar{\mathcal{I}}$ by increasing the value of mid . If, however, the computed value of the water-level W is lower than the noise power at mid , $W < n'_{mid}$, then we need to contract our selected set $\bar{\mathcal{I}}$ by decreasing the value of mid . The value of mid is changed likewise, until the value of allocated power converges.

Algorithm 1 Power Allocation for Leader

- 1: Given $\mathcal{A}_f$, find $\mathcal{I} = \{k \in \mathcal{A}_f : c_k > 0\}$
 - 2: Arrange all $k \notin \mathcal{A}_f$ in ascending order such that $n'_1 \leq n'_2 \leq \dots \leq n'_{|\mathcal{L} \setminus \mathcal{A}_f|}$
 - 3: Set $lo = 1$ and $hi = |\mathcal{L} \setminus \mathcal{A}_f|$
 - 4: **repeat**
 - 5: **if** $|\mathcal{A}_f| = L$ or $mid = 0$ **then**
 - 6: Calculate $W = \bar{q} / \sum_{k \in \mathcal{I}} c_k$
 - 7: Allocate $q_k = 0$ for $k \in \mathcal{L} \setminus \mathcal{A}_f$
 - 8: Allocate $q_k = \max(0, c_k W)$ for $k \in \mathcal{A}_f$
 - 9: Terminate
 - 10: **end if**
 - 11: Set $mid = \lfloor lo + (hi - lo)/2 \rfloor$
 - 12: Set $\bar{\mathcal{I}} = \{1, \dots, mid\}$ so that $\bar{\mathcal{I}} \subseteq \mathcal{L} \setminus \mathcal{A}_f$
 - 13: Calculate $W = (\bar{q} + \sum_{k \in \bar{\mathcal{I}}} n'_k) / (|\bar{\mathcal{I}}| + \sum_{k \in \mathcal{I}} c_k)$
 - 14: Allocate $q_k = \max(0, W - n'_k)$ for $k \in \mathcal{L} \setminus \mathcal{A}_f$
 - 15: Allocate $q_k = \max(0, c_k W)$ for $k \in \mathcal{A}_f$
 - 16: **if** $W < n'_{mid}$ **then**
 - 17: Update $hi = mid - 1$
 - 18: **else**
 - 19: Update $lo = mid + 1$
 - 20: **end if**
 - 21: **until** q converges
-

3.4.1.1 Computational Complexity

The worst case performance of Algorithm 1 is log-linear, $\mathcal{O}(L \log_2 L)$, which is due to the sorting operation¹ required in Step 2. This is comparable to ordinary waterfilling algorithm which has the same worst-case computational complexity. In fact, the Algorithm 1 degenerates into the ordinary waterfilling algorithm when there is no interfering subchannels.

Here we compare our algorithm with those proposed in [164, 166]. For a single update of the leader's power allocation, the worst case computational complexity of the two-user suboptimal algorithm (TSA) [164] and the [166, Algorithm 1] of the conjecture-based rate maximization (CRM) algorithm are $\mathcal{O}(T_1 T_2 L \bar{q} / \Delta_q)$ and $\mathcal{O}(T_3 T_4 \bar{q} / \Delta_q)$ respectively, where $T_1, \dots, T_4$ are the number of iterations required until a tolerable solution is obtained and Δ_q is the power granularity. Here the $T_1, \dots, T_4$ are independent of the number of subchannels L . Therefore, to compare these algorithms with the our leader's power allocation method given by Algorithm 1, we first model the power granularity by $\Delta_q = \bar{q} / L^\alpha$, where $\alpha > 0$. Thus, this makes the worst case computational complexity of the TSA to be $\mathcal{O}(T_1 T_2 L^{1+\alpha})$ and [166, Algorithm 1] of CRM to be $\mathcal{O}(T_3 T_4 L^{1+\alpha})$.

Note that in the TSA, we need to evaluate the objective function of the leader every time we consider changing a single grain of power in the leader's power allocation. In the expression for the computational complexity of the TSA, it is implicitly assumed that evaluating the leader's objective function incurs zero cost, or at most a constant cost. This, however, is not true. Computing the leader's objective function involves conducting the waterfilling operation by the follower, which has $\mathcal{O}(L \log_2 L)$ computation complexity. Thus, if we account for the waterfilling step by the follower for every grain of power changed by the leader, the actual computational complexity of the TSA becomes $\mathcal{O}(T_1 T_2 L^{2+\alpha} \log_2 L)$. Such a cost is not incurred by our Algorithm 1 or the the [166, Algorithm 1] of CRM, since these two algorithms compute a projected value of follower's interference, instead of measuring the actual interference. Thus, the TSA is inherently slower than the [166, Algorithm 1].

¹Most commonly used sorting methods such as the merge sort algorithm has log-linear worst case complexity. Here we have assumed that the best sorting algorithm has log-linear worst case complexity.

Table 3.1: The leader's computational complexity for different algorithms

Algorithm	Computational Complexity
Exhaustive search	$\mathcal{O}((L^{1+\alpha} \log_2 L)^L)$
Alg. 1	$\mathcal{O}(L \log_2 L)$
TSA	$\mathcal{O}(T_1 T_2 L^{2+\alpha} \log_2 L)$
[166, Algorithm 1] of CRM	$\mathcal{O}(T_3 T_4 L^{1+\alpha})$
WF	$\mathcal{O}(L \log_2 L)$

Now, it can easily be shown that the function $f(L) = \frac{\log_2 \log_2 L}{\log_2 L}$ is a concave function, has a maxima at $L = 2^e \approx 6.58$ with $f(2^e) \approx 0.53$, and that $f(2) = 0$ and $\lim_{L \rightarrow \infty} f(L) = 0$. This implies that for any value of $\alpha > 0$, there exists a value of L^* such that for every value of $L > L^*$, $\alpha > \frac{\log_2 \log_2 L}{\log_2 L}$. In other words, this means that for any $\alpha > 0$, it must be the case that $L^{\alpha+1} > L \log_2 L$ for every value of $L > L^*$. Thus, our proposed Algorithm 1 has lesser complexity than the [166, Algorithm 1], and by this virtue, it has less complexity than the TSA as well.

Table 3.1 summarizes the computational complexities of these three algorithms. For the sake of comparison, we have also included the computational complexity of the exhaustive search method and the waterfilling method (WF). It should be noted that the computational complexity given for the TSA and the exhaustive search method is only for the case with one leader and one follower. However, the computational complexities of WF, Algorithm 1, and [166, Algorithm 1] are the same even for the case with multiple followers.

3.4.2 Algorithm to Compute Stackelberg Equilibrium

Proposition 2 only gives a partial answer, and it has to be taken in conjunction with Algorithm 2 to obtain the Stackelberg equilibrium. Proposition 2 makes an assumption that the followers' channel allocation does not change. But since any changes in leader's response can potentially cause a change in followers' channel allocation, we have to run the lower sub-game to check for any such changes. This naturally leads to an iterative method of finding the Stackelberg equilibrium as given in Algorithm 2.

Algorithm 2 Computing Stackelberg Equilibrium

- 1: **repeat**
 - 2: Suppress any limit cycle if detected
 - 3: Allocate power for the followers until they reach their sub-game Nash equilibrium
 - 4: Allocate power for the leader given the channel allocation of the followers
 - 5: **until** Convergence is achieved
-

Here we propose to compute the SE of one leader-multiple follower game using the iterative procedure given in Algorithm 2. First, the followers take turn allocating power until they reach their sub-game NE. The leader senses the channel allocation of the followers, and allocates its own power based on the sensed information. However, the leader's power allocation can change the followers' power allocation. After the leader allocates its power, the followers again update the power until they reach sub-game NE. This in turn can change the leader's power allocation again. This process of allocating power by the followers and the leader is repeated until the algorithm converges. The order of iteration among the leader and followers is irrelevant for the steady-state outcome of this game.

Although a sufficient condition for convergence was given in the Corollary 1 of Proposition 5 in terms of the parameter a_k , the Algorithm 2 is not guaranteed to converge under any general condition. A possible way to deal with this issue is to limit the number of iterations to some maximum number. Alternatively, we can examine the path that the state of the system assumes at every iteration of the algorithm. We can examine the stability of the algorithm by looking at the binary channel allocation vector, which is a vector of ones and zeros: one if power is allocated over a subchannel, zero otherwise. If the channel allocation vector does not change in the course of running the algorithm, then the algorithm has converged. In our own experience with the algorithm, we found that the system often eventually fluctuates among two states. Thus, if X_1 and X_2 are two possible states of channel allocation vector, then over time the channel allocation vector will alternate between these two states as $\dots \rightarrow X_1 \rightarrow X_2 \rightarrow X_1 \rightarrow X_2 \rightarrow \dots$. This is because of the two-tier nature of the problem. Thus, we say that the system has entered a limit cycle. Both these states represent a possible solution to our problem. However, we can suppress this behavior by making sure that only one of the two vectors is chosen.

In general, simulation studies for one leader and one follower show that the iterative algorithm converges to a limit cycle (see Fig. 3.3). It is observed that the period of the limit cycle is at most two. We can suppress this limit cycle by generating a vector that cancels out the cycle, thus forcing the system to choose only one solution. Let $X_l^{(n)}$ be a binary channel allocation vector of the leader at n th iteration. Then, the change in the channel allocation is given by $\epsilon^{(n)} = X_l^{(n)} \oplus X_l^{(n-1)}$, where $\oplus$ is modulo-2 addition. We can detect that the system enters the limit cycle when $\epsilon^{(n)} = \epsilon^{(n-1)}$. The cycle can be suppressed, once it is detected, by making the leader adopt the allocation vector $\tilde{X}_l^{(n)} = X_l^{(n)} \oplus \epsilon^{(n)}$. Thus the future iterations will always have $X_l^{(n+1)} = \tilde{X}_l^{(n)} \oplus \epsilon^{(n)} = X_l^{(n)} \oplus \epsilon^{(n)} \oplus \epsilon^{(n)} = X_l^{(n)}$.

Figure 3.3: Max norm of successive difference of system power vector.

3.4.3 Convergence

Here we provide a sufficient condition for the Algorithm 2 to be convergent under high interference.

Proposition 5. *If $\min_{k \in \mathcal{A}_f} \tilde{n}'_k > W$, then $\mathcal{I} = \emptyset$.*

Proof. Assume $\mathcal{I} \neq \emptyset$. Since by definition $\mathcal{I} = \mathcal{A}_f \cap \mathcal{A}_l$, there must exist a $k' \in \mathcal{A}_f$ such that $q_{k'} > 0$. From Proposition 2, $q_{k'} = W - \tilde{n}'_{k'} > 0$, i.e., $\tilde{n}'_{k'} < W$. But since $\tilde{n}'_{k'} \geq \min_{k \in \mathcal{A}_f} \tilde{n}'_k > W$, this is a contradiction. $\square$

Corollary 1. *It is sufficient that $\max_{k \in \mathcal{A}_f} a_k < -1/2$ for the leader to adopt an orthogonal power allocation strategy and for Algorithm 2 to converge.*

Proof. From Proposition 4, the condition arises when $\min_{k \in \mathcal{A}_f} \tilde{n}'_k > W$. Putting $\tilde{n}'_k = (1 - c_k)W$, this inequality becomes, $\max_{k \in \mathcal{A}_f} c_k < 0$. Again, since $c_k = 2(1 + 2a_k)$, we can further simplify the condition as $\max_{k \in \mathcal{A}_f} 2(1 + 2a_k) < 0$, i.e., $\max_{k \in \mathcal{A}_f} a_k < -1/2$. Since under this condition the leader no longer causes any interference to the followers, $\mathcal{I} = \emptyset$, the updates to Algorithm 2 will come to an end. $\square$

This means that if the value of a_k is bounded within the interval $(-1, -1/2)$ for all subchannels over which the followers cause interference, then the Algorithm 2 will converge. Intuitively, this happens when there is insufficient total power $\bar{q}$ such that the power allocated to the non-interfering subchannels, and hence the “water level” W , is not high enough to “spill over” to the interfering subchannels. When there is only one follower, $a_k = -g_k h_k$, thus the condition reduces to $\min_{k \in \mathcal{A}_f} g_k h_k > 1/2$. This is equivalent to the condition that $g_k h_k > 1/2$ for all $k \in \mathcal{A}_f$.

3.4.4 Discussion

1. The key to the success of this algorithm seems to lie in high co-channel gain condition and low transmit power of the followers. This allows the orthogonal channel allocation to take hold. So, when co-channel gain is high, the leader avoids transmitting over some subchannels which can cause large interference, thus effectively creating “holes” in the spectrum. The follower then exploits these “holes” created by the leader by allocating most of their power in these “holes”. If the followers’ total power is relatively small compared to that of the leader’s, the power allocated by the followers in these “holes” is not sufficient to “overflow” to other subchannels. Thus the operation of the followers is confined to just these “holes” from which it cannot escape. Hence, the loss in bandwidth of the macro BS is compensated by its freedom from interference. For the success of this scheme, it is necessary to have a small transmit power of the followers compared to the surrounding interference environment. This criterion is readily satisfied by femtocell network.

2. Based on these results, a practical recommendation for network deployment can be given. One possibility is as such: When the interference experienced is low, the macrocell utilizes the subchannel by treating the interference as ordinary noise; but when the interference is sufficiently high, the macrocell abandons the subchannel. Another possibility is as such: macrocell sets at least one subcarrier free of macrocell activity. Owing to the predictable, selfish nature and low transmit power of the femtocells, the femtocells' operation will mostly be confined to these bands, thus allowing a relatively interference free operation for macrocell in the rest of the subchannels.

3.4.5 Extension to multiple-leader case

In the situation with multiple leaders, this combinatorial problem of channel selection is solved by each leader as given by Algorithm 1. In this case, the leader tries to predict the behavior of the followers, but assumes the action of other leaders to be fixed. The interference due to other leaders' action is incorporated as the fixed background noise n' . The leaders iterate the response among themselves until they reach an upper equilibrium. Once the upper equilibrium is reached, the followers play their Nash game to arrive at the lower equilibrium. Thus the process of finding the lower and upper equilibrium is iterated until the whole process converges.

3.5 Numerical Evaluation

To obtain the performance of SE, the leader uses Algorithm 1 whereas the followers use waterfilling algorithm. The solution is determined using Algorithm 2. Here $L = 100$ and the total power of a follower, the FAP, is taken to be 20 dB below the total power of the leader, the macro BS. We assume $\bar{q} = 30$ dBm and $\bar{p} = 10$ dBm. The noise power in each subchannel is taken to be -120 dBm. Each subchannel is assumed to undergo independent and identically distributed Rayleigh fading. For each subchannel, the Rayleigh random variable is modeled as the modulus of circularly symmetrical complex Gaussian (CSCG) distribution. The instantaneous channel gain is obtained by squaring

Figure 3.4: Ratio of capacities at Stackelberg and Nash equilibrium when average co-channel gain is 0.5.

the Rayleigh random variable, thus giving a chi-square distribution. The average gain of a subchannel is controlled by changing the variance of the CSCG distribution. Each plot is based on 50,000 random channel gain samples. Lastly, during the iterative updates of the leader and follower's power, an averaging process was conducted. That is, if $q(t)$ is the leader's power vector at t iterate, then in the next iterate $t + 1$, the leader's power was updated such that $q(t + 1) = \alpha q_c + (1 - \alpha)q(t)$, where q_c is the computed power vector for $t + 1$ iterate and $\alpha \in (0, 1)$. Likewise for the follower's power allocation. This ensures that the updates to the powers are smooth.

To compare the performance of the system at SE with the NE, we plot the cumulative distribution function (CDF) of the ratio between the capacity obtained under SE and NE for the leader and the follower (Figs. 3.4-3.7). We are particularly interested in examining the performance under high interference conditions. However, under these conditions, the iterative waterfilling algorithm (IWF) used to reach NE may not always converge. In such cases, for the sake of comparison, the IWF algorithm is run up to 100 iterations, after which the value of the last iteration is taken to be representative of the NE.

Figure 3.5: Ratio of capacities at Stackelberg and Nash equilibrium when average co-channel gain is 0.25.

Table 3.2: Comparison between Stackelberg and Nash equilibrium

	Avg. co-ch. gain	Tot. Cap.	Leader	Follower	When lead. above NE	When foll. above NE
1 L - 1 F	0.5	28.07 %	32.45 %	55.9 %	36.2 %	88.1 %
	0.25	34.62 %	39.20 %	22.27 %	42.3 %	64.45 %
1 L - 2 F	0.5	4.22 %	1.73 %	220.98 %, 222.1 %	23.83 %	243.82 %, 244.45 %
	0.25	11.48 %	13.63 %	55.6 %, 55.53 %	30.09 %	103 %, 104 %

Table 3.2 summarizes the results in Figs. 3.4-3.7. The first two rows present the results of one leader-one follower system (i.e., labeled as 1 L - 1 F), while the rest present the results of one leader-two followers system (i.e., labeled as 1 L - 2 F). For each case, the average direct channel gain is maintained to be unity, while the average co-channel gain is kept to be 0.5 and 0.25. The changes in average performance when SE is adopted over NE are shown as percentages. The third, fourth, and fifth columns show the increase in average total system capacity, leader's capacity, and followers' capacity, respectively. The sixth and seventh columns show the conditional increase in average capacity of the leader and followers, respectively, when they are doing better than that at NE.

From the Table 3.2, we observe that there is an overall improvement for both the

Figure 3.6: Ratio of capacities at Stackelberg and Nash equilibrium when average co-channel gain is 0.5.

leader and the followers. For the leader, its performance improves with lower average co-channel gain, while it is just the opposite for the followers. Also, increasing the number of followers degrades the improvement of leader's performance. This is due to the increased bandwidth occupied by the followers.

Fig. 3.8 shows the plots for two leaders and two followers. Here the average co-channel gain is kept to be 0.25. Not surprisingly, the followers benefit more than the leaders. Also, the performance of the leaders suffers due to the loss of bandwidth as well as the introduction of another leader with high transmit power, whose interference is not mitigated.

3.5.1 Comparison Between Different Algorithms

We now compare our algorithm with the two-user suboptimal algorithm (TSA) proposed in [164, Algorithm 1] and the conjecture-based rate maximization algorithm (CRM) proposed in [166, Table IV]. We consider the case with one leader and one follower. To expedite the run time, we assume $\bar{q} = 23.0103$ dBm, $\bar{p} = 10$ dBm, and $L = 20$. For each plot, 1000 random channel instances were considered. As before, all the algorithms are

Figure 3.7: Ratio of capacities at Stackelberg and Nash equilibrium when average co-channel gain is 0.25.

compared with respect to the NE obtained via IWF. The figures have been clipped at $C_{se}/C_{ne} = 5$ so as to highlight the differences between the algorithms with respect to the leader's performance. In both the Fig. 3.9 and Fig. 3.10, we observe that all three algorithms provide a better performance for the leader compared to the IWF algorithm.

More specifically, in the Fig. 3.9, when the average co-channel gain is 0.25, we observe that the leader's performance is less than or equal to its performance at the NE for less than 5 % of the time for all three algorithms. Whereas, for the follower, the performance is less than or equal to its performance at NE for less than 55 %, 10 %, and 50 % for Algorithm 1, TSA, and CRM respectively.

Similarly, in the Fig. 3.10, when the average co-channel gain is 0.5, we observe that the leader's performance is less than or equal to its performance at the NE for less than 5 % of the time for Algorithm 1 and TSA. Interestingly, for the CRM algorithm, the leader's performance is equal to that of the NE for 11.8 % of the time, but the performance is never below the NE. Likewise, for the follower, the performance is less than or equal to its performance at NE for less than 31.4 % in the case of Algorithm 1. For the CRM algorithm, the follower's performance is the same as that of NE for 97.9 % of the time. Interestingly, for the follower, the TSA algorithm always outperforms the NE.

Figure 3.8: Ratio of capacities at Stackelberg and Nash equilibrium when average co-channel gain is 0.25.

The comparison between these algorithms as presented in Fig. 3.9 and Fig. 3.10 has been summarized in Table 3.3. As before, the percentages show the amount of change in the performance over the NE when a particular algorithm is adopted. Notice that in terms of the total system capacity, the performance of Algorithm 2 and CRM is not very different. Similarly, for the leader's performance, the CRM algorithm is slightly better than the Algorithm 2. Likewise, for the follower's performance, the Algorithm 2 gives significantly better performance than the CRM algorithm. Interestingly, in all three categories of performance, the TSA significantly outperforms both Algorithm 2 and CRM.

The differences in the performance between the three algorithms can be accounted by the following facts: The CRM algorithm first computes the leader's power allocation based on the linear interference model by considering a local approximation of the original leader's problem as given in [166, (11)]. However, there can be a mismatch between the problems. As such, a line search is adopted to scan the interval between the previous power allocation vector and the newly computed power allocation vector. A leader's power is updated whenever an allocation is obtained that gives a better performance.

Table 3.3: Comparison between Algorithm 2, TSA, and CRM for one leader and one follower case

Algorithm	Avg. co-ch. gain	Tot. Cap.	Leader	Follower
Alg. 2	0.25	46.32 %	50.45 %	53.89 %
TSA		73.78 %	70.89 %	241.71 %
CRM		47.86 %	56.85 %	25.23 %
Alg. 2	0.5	45.72 %	49.53 %	99.79 %
TSA		63.61 %	62.76 %	296.81 %
CRM		43.39 %	52.44 %	1.12 %

In our implementation of CRM algorithm, we adopted the backtracking method for the line search. Thus, the added complexity of the line search allows for a slightly better performance.

In the case of TSA algorithm, it attempts to approximately evaluate the Lagrangian dual of the leader's power allocation problem by locally optimizing the Lagrangian at each subchannel. For fixed dual variable, the algorithm searches for the optimal q^k while keeping $q^1, \dots, q^L$ fixed, iterating over the subchannels until it converges to a local maximum of the Lagrangian. The algorithm updates the dual variable by bisection search and repeats the above procedure until convergence. The reason why this algorithm outperforms both our proposed algorithm and the CRM algorithm is that, during the search for the local maxima of the Lagrangian of the leader's power allocation problem at each subchannel, it accesses the actual changes in the leader's objective function at every changes to its power allocation. This is in contrast to our algorithm and the CRM algorithm where the changes in the follower's power allocation, and hence its effects on the leader's objective function, is anticipated but not actually accessed. In practice, we found the TSA algorithm requires a higher run time than the CRM algorithm.

Thus in terms of performance, our proposed algorithm is quite comparable to the TSA and CRM algorithms, but at a reduced computational complexity.

Figure 3.9: Ratio of capacities using Algorithm 2, TSA, and CRM as compared to NE when average co-channel gain is 0.25.

3.6 Chapter Summary

This chapter has addressed the problem of allocating downlink transmit power over OFDMA cellular networks comprising of macrocells underlaid with low power, small coverage femtocells. The objective of each femtocell is to maximize its individual capacity under power constraints. Unlike femto base stations, the macrocell base stations are assumed to have enough information to predict the response of the femtocells for given macrocell power profile. Therefore, a Stackelberg game has been formulated with the macrocell base stations as the leaders and the femtocell base stations as the followers. The femtocells are assumed to use an iterative waterfilling strategy among themselves to reach a sub-game Nash equilibrium. Thus the entire game is given as a mathematical program with equilibrium constraint (MPEC). Analysis of this problem has shown that under high interference condition, the leader prefers not to transmit in the same subchannel as the followers. An iterative algorithm has been presented to reach the Stackelberg equilibrium, and a sufficient condition was given for its convergence. Numerical results have shown that significant performance gain can be achieved by both the macrocell as well as the femtocells, at Stackelberg equilibrium when compared to Nash equilibrium

Figure 3.10: Ratio of capacities using Algorithm 2, TSA, and CRM as compared to NE when average co-channel gain is 0.5.

even under high cross-tier interference condition. The proposed algorithm has also been compared with two other algorithms.

Based on these results, a practical recommendation for network deployment can be given during high interference condition. It is recommended that the macro base station sets at least one subcarrier free of macrocell interference. Owing to the predictable, selfish nature and low transmit power of the femtocells, the femtocells' operation will mostly be confined to these bands, thus allowing a relatively interference free operation for macro base station in the rest of the subchannels.

3.7 Appendix

3.7.1 Proof of Lemma 1

From $\nabla_q \Lambda = 0$, we have

$$\begin{aligned}
& \frac{d}{dq_k} \log \left(1 + \frac{q_k}{F(q_k)} \right) - \frac{1}{W} = 0 \\
\text{or, } & \frac{1}{1 + \frac{q_k}{F(q_k)}} \frac{F(q_k) - q_k F'(q_k)}{[F(q_k)]^2} = \frac{1}{W} \\
\text{or, } & \frac{1}{q_k + F(q_k)} \frac{F(q_k) - q_k F'(q_k)}{F(q_k)} = \frac{1}{W} \\
\text{or, } & 1 - q_k \frac{F'(q_k)}{F(q_k)} = \frac{1}{W} (q_k + F(q_k)) \\
\text{or, } & q_k \left(\frac{1}{W} + \frac{F'(q_k)}{F(q_k)} \right) = 1 - \frac{F(q_k)}{W} \\
\therefore & q_k = \frac{W - F(q_k)}{1 + W \frac{F'(q_k)}{F(q_k)}}.
\end{aligned}$$

3.7.2 Proof of Lemma 2

Since $F(q_k) = aq_k + b$ and $F'(q_k) = a$, from (3.10) we have

$$\begin{aligned}
q_k &= \frac{W - aq_k - b}{1 + W \frac{a}{aq_k + b}} \\
\text{or, } & q_k \left(1 + a + \frac{Wa}{aq_k + b} \right) = W - b \\
\text{or, } & (1 + a)aq_k^2 + [(1 + a)b + Wa]q_k = (W - b)(aq_k + b) \\
\text{or, } & (1 + a)aq_k^2 + [(1 + a)b + Wa - (W - b)a]q_k - (W - b)b = 0 \\
\text{or, } & (1 + a)aq_k^2 + (1 + 2a)bq_k - (W - b)b = 0 \\
\therefore & q_k^2 + \frac{(1 + 2a)b}{(1 + a)a}q_k - \frac{(W - b)b}{(1 + a)a} = 0. \tag{3.19}
\end{aligned}$$

The unique solution of (3.19) is given by its double root,

$$q_k = -\frac{(1 + 2a)b}{2(1 + a)a}. \tag{3.20}$$

Furthermore, for the existence of the double root, we also have the condition that the discriminant of (3.19) be zero. Thus using this condition,

$$\begin{aligned}
& \left[\frac{(1+2a)b}{(1+a)a} \right]^2 + \frac{4(W-b)b}{(1+a)a} = 0 \\
\text{or, } & (1+2a)^2 b^2 + 4(1+a)a(W-b)b = 0 \\
\text{or, } & (1+4a+4a^2)b^2 + 4(a+a^2)(Wb-b^2) = 0 \\
\text{or, } & b^2 + 4ab^2 + 4a^2b^2 + 4abW + 4a^2bW - 4ab^2 - 4a^2b^2 = 0 \\
\text{or, } & b^2 + 4a(1+a)bW = 0 \\
\therefore & b[b + 4a(1+a)W] = 0.
\end{aligned}$$

Since $b \neq 0$, we have $b + 4aW(1+a) = 0$. Hence

$$W = -\frac{b}{4a(1+a)}. \quad (3.21)$$

Using (3.21), we can eliminate b in (3.20), and write the solution in terms of W as $q_k = 2(1+2a)W$.

3.7.3 Proof of Proposition 2

For the sake of notational simplicity as well as for the purpose of elucidation, let us first assume that the number of followers $N = 2$. However, the readers should be notified that the method developed in this section loses none of its generality by this assumption and is valid for any number of followers.

To start with, let us consider the set of interfering subchannels $\mathcal{I}$ for the leader, given by the set relationship $\mathcal{I} = \mathcal{A}_l \cap (\mathcal{A}_{f_1} \cup \mathcal{A}_{f_2})$, where $\mathcal{A}_{f_1}$ and $\mathcal{A}_{f_2}$ are the set of active subchannels of two followers. The set $\mathcal{I}$ can be divided into three disjoint sets: $\mathcal{A}_l \cap \mathcal{A}_{f_1} \cap (\mathcal{L} \setminus \mathcal{A}_{f_2})$, $\mathcal{A}_l \cap \mathcal{A}_{f_2} \cap (\mathcal{L} \setminus \mathcal{A}_{f_1})$, and $\mathcal{A}_l \cap \mathcal{A}_{f_1} \cap \mathcal{A}_{f_2}$. The first two cases represent the instances when there is only one interferer in the subchannel. The method of allocating power for such cases is the same as that described in the previous section. Thus it suffices for us to focus only on the third case when both the followers interfere each other in the subchannel.

Here we will drop the use of index k to simplify the notations, as it is considered understood that $k \in \mathcal{A}_l \cap \mathcal{A}_{f_1} \cap \mathcal{A}_{f_2}$. For a subchannel $k \in \mathcal{A}_l \cap \mathcal{A}_{f_1} \cap \mathcal{A}_{f_2}$, let p_1 , p_2 , and

q denote the transmit power allocated by the two followers and the leader, respectively. Also, g_{1l} and g_{2l} are the co-channel gains from the leader's transmitter to the two followers' receiver. g_{12} and g_{21} are co-channel gains from one follower's transmitter to the other follower's receiver. h_1 and h_2 are the co-channel gains of the followers' transmitter to the leader's receiver. Lastly, n_1 , n_2 , and n_l are the noise powers at the receivers of the two followers and the leader, respectively. Thus, noise-plus-interference power at the receiver of the first and second followers will be $g_{1l}q + g_{12}p_2 + n_1$ and $g_{2l}q + g_{21}p_1 + n_2$, respectively, while at the receiver of the leader, it will be $h_1p_1 + h_2p_2 + n_l$.

Since the followers treat the noise-plus-interference power as a constant, we have the power allocated by the two followers given by the waterfilling function as before, i.e.,

$$\begin{aligned} p_1 &= K_1 - (g_{1l}q + g_{12}p_2 + n_1) \\ p_2 &= K_2 - (g_{2l}q + g_{21}p_1 + n_2). \end{aligned} \quad (3.22)$$

Here (3.22) is a system of simultaneous linear equations and can be re-expressed as follows:

$$\begin{aligned} p_1 + g_{12}p_2 &= -g_{1l}q + K_1 - n_1 \\ g_{21}p_1 + p_2 &= -g_{2l}q + K_2 - n_2. \end{aligned} \quad (3.23)$$

Assuming that (3.23) are linearly independent, we can solve them using Cramer's rule. We thus have the expressions of p_1 and p_2 in terms of q as follows:

$$p_1 = \frac{1}{\Delta} \begin{vmatrix} -g_{1l}q + K_1 - n_1 & g_{12} \\ -g_{2l}q + K_2 - n_2 & 1 \end{vmatrix} \quad (3.24)$$

and

$$p_2 = \frac{1}{\Delta} \begin{vmatrix} 1 & -g_{1l}q + K_1 - n_1 \\ g_{21} & -g_{2l}q + K_2 - n_2 \end{vmatrix} \quad (3.25)$$

where

$$\Delta = \begin{vmatrix} 1 & g_{12} \\ g_{21} & 1 \end{vmatrix}.$$

Obviously, we cannot solve these simultaneous equations directly using Cramer's rule, since to do so would require the values of K_1 and K_2 , which themselves needs to be numerically determined. A practical approach to obtaining the solution is to use the

iterative waterfilling algorithm among the followers for a given leader power strategy. Thus, a Nash equilibrium of the lower sub-game would be the solution of (3.23).

Using the elementary properties of determinant, we can simplify the expressions for p_1 and p_2 in (3.24) and (3.25) as follows:

$$\begin{aligned} p_1 &= A_1 q + B_1 \\ p_2 &= A_2 q + B_2 \end{aligned} \tag{3.26}$$

where

$$A_1 = \frac{1}{\Delta} \begin{vmatrix} -g_{1l} & g_{12} \\ -g_{2l} & 1 \end{vmatrix} \quad B_1 = \frac{1}{\Delta} \begin{vmatrix} K_1 - n_1 & g_{12} \\ K_2 - n_2 & 1 \end{vmatrix}$$

and

$$A_2 = \frac{1}{\Delta} \begin{vmatrix} 1 & -g_{1l} \\ g_{21} & -g_{2l} \end{vmatrix}, \quad B_2 = \frac{1}{\Delta} \begin{vmatrix} 1 & K_1 - n_1 \\ g_{21} & K_2 - n_2 \end{vmatrix}.$$

Inspecting the above expressions for A_1 and A_2 , it is apparent that they are the solutions of the matrix equation

$$\begin{bmatrix} 1 & g_{12} \\ g_{21} & 1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} -g_{1l} \\ -g_{2l} \end{bmatrix}. \tag{3.27}$$

Similarly, for B_1 and B_2 , we have

$$\begin{bmatrix} 1 & g_{12} \\ g_{21} & 1 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} K_1 - n_1 \\ K_2 - n_2 \end{bmatrix}. \tag{3.28}$$

Now the interference experienced by the receiver of the leader is given by the function $F(q) = h_1 p_1 + h_2 p_2 + n_l$. Using the expressions of p_1 and p_2 obtained in (3.26) in the above equation, we get $F(q) = aq + b$ where $a = h_1 A_1 + h_2 A_2$ and $b = h_1 B_1 + h_2 B_2 + n_l$.

Chapter 4

Dynamic Coalition Formation for Network MIMO in Small Cell Networks

In a wireless communications system, interference imposes a constraint on the ability to reuse spectral resources, while fading limits the reliability and the coverage of communication. Conventional mitigation techniques involve diversity and relaying schemes, spectral partitioning, and scheduling methods. Recently, a cooperative model for transmission and reception has been proposed (see [175] and references therein). That is, base stations are allowed to cooperate with each other to form a single “virtual” or “network” MIMO system, thus converting interfering links into useful message carrying links. The possible advantages from such a system are derived by coordinating suitable scheduling, power allocation, transmit and receive beamforming, and coding strategies.

In addition to network MIMO, the concept of small cells (e.g., micro-, pico-, and femtocells) has also been introduced to enhance the network performance [176]. A conventional approach to improve the network capacity and coverage is to split cells by deploying more base stations. A more economically feasible approach is to overlay low power and low cost base stations on the existing cellular network infrastructure (2G, 3G, and LTE) [5]. A small cell base station (SBS) is connected with a wired or wireless back-

haul through a local broadband connection. Similar to a Wi-Fi access point, the SBS is a low cost, simple plug-and-play device, which is installed as customers deem fit [5, 6, 7].

However there are some practical issues that need to be addressed for the network MIMO, as outlined in [175]. The first issue is the scalability of signaling overhead for joint processing as the network grows larger. The second problem occurs when we consider constrained backhaul and imperfect channel state information (CSI). The third issue is related to the clustering of base stations to achieve the optimal performance. Instead of forming a single virtual MIMO system of all the base stations in the system, clustering divides the base stations into smaller groups and treats each group as a virtual MIMO system. Such clustering of base stations is an important strategy as it allows local processing, reduces signalling overhead, and provides robustness to node failures.

In this chapter, we seek to combine the idea of network MIMO with downlink small cells. The network MIMO technique is particularly well suitable for small cells. In particular, when universal frequency reuse for the small cells is considered, excessive intercell interference may result from the random deployment of small cells. To avoid such interference, it is better if the small cells can cooperate with each other to form a network MIMO system. In this case, the coordination among cooperative small cells can be performed by the macro base station, as long as the macro base station knows which small cell is cooperating with which small cell. This reduces the signaling overhead of the network MIMO typically encountered during such data sharing process. Thus the small cells now only need to share their CSI.

In this chapter, we consider the network MIMO system integrated with small cells [177, 178]. Under a finite rate feedback system, the SBSs have to autonomously optimize the transmit beamformer, as well as form the network MIMO systems distributively. Here the main purpose of forming a network MIMO system is to mitigate the effects of intercell interference and shadow fading via cooperative beamforming. However, allowing all the SBSs to form a single network MIMO system may result in suboptimal performance, especially when the SBSs are concerned about their own performance.

Fig. 4.1 shows the framework for the integrated network MIMO and small cell networks. Firstly, we describe the signal model and beamforming operations, which constitute the physical layer of our small cell networks. We consider two cases for the

Figure 4.1: Interaction among beamformer selection, regret based learning, and coalition formation among SBSs.

beamforming, i.e., when a feedback channel is present, and when a feedback channel is temporarily absent. For the case with the feedback channel, we adopt a space division multiple access technique known as per-user unitary rate control (PU²RC) [193]. On the other hand, for the case when the feedback channel is temporarily absent, the historical information is used for regret based learning approach to determine the beamforming parameters. The PU²RC gives a deterministic method to select a beamformer from the beamforming codebook, whereas the regret based learning approach gives a probabilistic method. The achievable data rate, based on either of the beamforming schemes, is then used by the SBSs to form coalitions that constitute network MIMO systems. We discuss three algorithms for the coalition formation, namely merge-split method and merge-only method. For the sake of completeness, the split-only method is also briefly discussed.

The rest of the chapter is organized as follows. Section 4.1 reviews related work. Section 4.2 presents the system model (i.e., network and signal models). Section 4.3 describes the coalition formation game. Section 4.4 presents the coalition formation algorithms (i.e., merge-split, merge-only, and split-only algorithms) and their probabilistic analysis. Section 4.5 shows the performance evaluation of the coalition formation process. Section 4.6 concludes the chapter.

4.1 Related Work

4.1.1 Channel Quantization and Beamforming

Unlike beamforming for the case of time division duplexing (TDD) where perfect channel side information can be assumed based on channel reciprocity (see [179]-[183]), beamforming is formally equivalent to channel quantization when only finite rate feedback is assumed as in frequency-division duplexing (FDD). Much work has been done on channel quantization for limited feedback systems for the case of single-cell single-user system [184]. Much less attention has been devoted for single-cell multi-user cases [185]. The field of inquiry is still wide open when it comes to cooperating multi-cell multi-user channel quantization [186, 187, 188]. The regret based learning approach has previously been used for beamforming in [204] in the context of multi-cell single-user system.

4.1.2 Clustering

Instead of forming a single network MIMO system of all base stations in the network, clustering divides the base stations into smaller groups and treats each group as a network MIMO system. Such clustering of base stations is an important strategy as it allows local processing, reduces signalling overhead, and provides robustness to node failures. During base station clustering, we partition the set of SBSs with the view of optimizing some system parameters. The number of possible partitions for a given number of SBSs is given by the Bell number. The Bell numbers can be generated by the recursion $\mathcal{B}_{n+1} = \sum_{k=0}^n \binom{n}{k} \mathcal{B}_k$ with initialization $\mathcal{B}_0 = 1$ by convention [194]. The first few Bell numbers are 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, and so on. The number of partitions quickly becomes very large as the number of base stations increases. Thus, an exhaustive search of all these possibilities quickly becomes infeasible. An alternative method is desired for distributed networks. The problem of clustering in network MIMO systems with single antenna base stations has been investigated in [195, 196] where a coordinated zero-forcing beamforming technique is applied. Our clustering method is similar to theirs in that it involves merging base stations to form bigger groups. Once a group is formed, it is not allowed to break. Nonetheless, the metric they have used to decide whether a

Figure 4.2: Schematic diagram of two cooperating SBSs serving two UEs with feedback channels.

base station will join a group in order to form a network MIMO is based on the average data rate of the group. This metric can be unfair to some users, as over-achieving users can outweigh under-achieving users during the averaging process. So long as the average performance of the overall cluster improves, a user can be included in a cluster even if the user's individual performance degrades by being a member of that cluster. Since the SBSs can be privately owned, a user can be unwilling to cooperate if it degrades its own performance, while benefiting others. Thus, instead of average performance, our approach is based on the vector of data rates obtained by each user. We show that the solution concept corresponds to the idea of recursive core of cooperative game theory. The idea of a merge-split algorithm was used for coalition formation for joint beamforming in [197]. The idea of recursive core has previously been applied to the interference alignment problem in [198].

4.2 System Model

4.2.1 System Model and Assumptions

Consider the downlink multiple-input-single-output (MISO) system using the FDD mode. Let $\mathcal{Q} = \{1, \dots, Q\}$ be the set of small cell base stations (SBSs) operating in a common

spectrum. Each SBS has N antennas serving a single scheduled user equipment (UE) equipped with a single antenna at a given time. The UE is identified by the same label as its corresponding SBS. We further assume that a two-way zero delay control channel is present between any two SBSs. Thus, the SBSs are allowed to cooperate with each other so as to form a virtual, super base station and jointly transmit data over the combined channel.

As shown in Fig. 4.2, a feedback channel is present between each corresponding pair of UE and SBS. The feedback channel is assumed to be a finite rate over-the-air control channel that has zero delay and is error free. The channel state is assumed to be known perfectly at the UE. The UE constructs a joint channel vector by concatenating the direct and indirect links of the cooperating SBSs. A pre-defined beamformer codebook is constructed using random vector quantization. The random quantization vectors are independently chosen from an isotropic distribution on a hypersphere [189, 190]. The codebooks are assumed to be synchronized between the UEs and the cooperating SBSs. After the UE has measured the joint channel of the virtual base station, the UE sends the indices of a chosen column from a chosen beamformer matrix in the beamforming codebook to the SBS as the channel direction information (CDI). Along with this, the UE also sends the signal-to-interference-plus-noise-ratio (SINR) as the channel quality information (CQI) to the transmitter. The CDI and CQI are shared and used by the cooperating SBSs to select the beamformer using a space division multiple access scheme known as the PU²RC method [191, 192] as outlined in [193]. The PU²RC selects a beamformer from the codebook that best matches with the channel vector of each UE and schedules the UEs that have the best SINR. In the case that the feedback channel is temporarily absent, e.g., due to deep fades, and the CDI and CQI are not available, the regret based learning approach is used for beamformer selection from a fixed codebook.

4.2.2 Signal Model

We shall refer to a group of cooperating SBSs as a *coalition*. Let $\Pi(\mathcal{Q})$ be the set of all partitions of $\mathcal{Q}$. For the partition $\pi_{\mathcal{Q}} \in \Pi(\mathcal{Q})$, a set of coalitions of SBSs is denoted by $\pi_{\mathcal{Q}} = \{\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_{|\pi_{\mathcal{Q}}|}\}$, such that $\mathcal{S}_s \cap \mathcal{S}_r = \emptyset$ for any $s \neq r$. One such partition $\pi_{\mathcal{Q}}$ is

referred to as a *coalition structure* of $\mathcal{Q}$.¹ With Q SBSs, the number of possible coalition structures, and hence the size of $\Pi_{\mathcal{Q}}$, is given by the Bell number $\mathcal{B}_Q$. Every coalition $\mathcal{S} \in \pi_{\mathcal{Q}}$ will be equivalent to a multiuser network MIMO system with the total of $N|\mathcal{S}|$ transmitting antennas serving $|\mathcal{S}|$ simultaneously scheduled UEs operating within the coalition through $|\mathcal{S}|$ independent data streams. Each UE in $\mathcal{S}$ is subject to co-channel interference from the SBSs in other coalitions $\pi_{\mathcal{Q}} \setminus \mathcal{S}$ (i.e., inter-coalitional interference), as well as interference caused by the signals intended for other UEs in the same coalition $\mathcal{S}$ (i.e., intra-coalitional interference). We can model the downlink channel as follows²:

$$y_{s_i} = \sqrt{\frac{P}{N}} \mathbf{h}_{ss_i}^\dagger \mathbf{x}_s + \sqrt{\frac{P}{N}} \sum_{\substack{r=1 \\ r \neq s}}^{|\pi_{\mathcal{Q}}|} \mathbf{h}_{rs_i}^\dagger \mathbf{x}_r + z_{s_i}, \quad (4.1)$$

where $\mathbf{x}_r$ is a $N|\mathcal{S}_r| \times 1$ complex signal vector from coalition- r , $\mathbf{h}_{rs_i}$ is an $N|\mathcal{S}_r| \times 1$ complex channel vector from coalition- r to UE- i of coalition- s , y_{s_i} represents the received signal at the UE- i in coalition- s , and z_{s_i} is the additive white Gaussian noise with variance σ^2 . The transmit power P of an SBS is assumed to be equally distributed among each antenna for each SBS. Each channel is assumed an aggregate of distance attenuation, shadowing, and flat fast fading. The precoder design of coalition- s is given by $\mathbf{x}_s = \sum_{i=1}^{|\mathcal{S}_s|} x_{s_i} \mathbf{w}_{s_i}$, where x_{s_i} is scalar representation of the signal intended for UE- i and $\mathbf{w}_{s_i}$ is an $N|\mathcal{S}_s| \times 1$ complex precoding vector for UE- i . We assume $\mathbb{E}[|x_{s_i}|^2] = 1$. The SINR of UE- i in coalition- s is given by

$$\text{SINR}_{s_i} = \frac{|\mathbf{w}_{s_i}^\dagger \mathbf{h}_{ss_i}|^2}{\sum_{j \neq i}^{|\mathcal{S}_s|} |\mathbf{w}_{s_j}^\dagger \mathbf{h}_{ss_i}|^2 + \sum_{r \neq s}^{|\pi_{\mathcal{Q}}|} \sum_{j=1}^{|\mathcal{S}_r|} |\mathbf{w}_{rj}^\dagger \mathbf{h}_{rs_i}|^2 + N\sigma^2/P}. \quad (4.2)$$

Here in the denominator, the first term $\sum_{j \neq i}^{|\mathcal{S}_s|} |\mathbf{w}_{s_j}^\dagger \mathbf{h}_{ss_i}|^2$ is the intra-coalitional interference, whereas the second term $\sum_{r \neq s}^{|\pi_{\mathcal{Q}}|} \sum_{j=1}^{|\mathcal{S}_r|} |\mathbf{w}_{rj}^\dagger \mathbf{h}_{rs_i}|^2$ is the inter-coalitional interference, both of which we assume to be additive white Gaussian. Lastly, the background noise σ^2 is scaled by N/P .

¹For the rest of this chapter, we use “partition” and “coalitional structure” interchangeably.

² $(\cdot)^\dagger$ denotes conjugate transpose.

4.2.3 Beamformer Selection

4.2.3.1 With Feedback Using PU²RC

Assuming that the channel is known perfectly at the receiver, each UE concatenates the direct channel of its associated SBS and the cross-channels of SBSs in the same coalition so as to form a joint channel vector. The UE then encodes and feeds back its channel state information to its associated SBS as two parameters: CDI and CQI. The SBSs then share the obtained feedback information with other SBSs in the same coalition.

Given B feedback bits per UE, a pre-defined beamforming codebook $\mathcal{W}_s = \{\mathbf{W}_s^{(1)}, \dots, \mathbf{W}_s^{(2^B)}\}$ is used for CDI quantization. This codebook is assumed to be known a priori by all the SBSs and the UEs of a given coalition- s . We consider the random orthogonal beamforming approach to construct such a codebook, in which the codebook consists of 2^B sets of $N|\mathcal{S}_s| \times N|\mathcal{S}_s|$ dimensional orthonormal matrices. Let $\mathbf{w}_{s_n}^{(m)}$ denote the n -th column of the m -th orthonormal matrix $\mathbf{W}_s^{(m)} = [\mathbf{w}_{s_1}^{(m)}, \dots, \mathbf{w}_{s_{N|\mathcal{S}_s|}}^{(m)}]$ in the codebook $\mathcal{W}_s$ of coalition- s . The CDI feedback from a UE- i consists of the indices (m, n) of a matrix and its column $\mathbf{w}_{s_n}^{(m)}$ chosen from the precoder codebook $\mathcal{W}_s$ according to some fixed criteria. The CQI consists of the SINR measured at UE- i when $\mathbf{w}_{s_n}^{(m)}$ is used. In this chapter, we adopt the method described in [193] to obtain the feedback CDI and CQI. The final data rate of user i in coalition s is given by u_{s_i} and the data rate of the entire coalition is given by $u_s = \sum_{s_i \in \mathcal{S}_s} u_{s_i}$.

1. Each user- i of coalition- s computes the expected SINR for all possible vectors $\mathbf{w}_{s_n}^{(m)}$ in the codebook $\mathcal{W}_s$ as

$$\text{SINR}_{s_i, n}^{(m)} = \frac{\|\mathbf{h}_{ss_i}\|^2 |\tilde{\mathbf{h}}_{ss_i}^H \mathbf{w}_{s_n}^{(m)}|^2}{\|\mathbf{h}_{ss_i}\|^2 \sum_{j=1, j \neq n}^N |\tilde{\mathbf{h}}_{ss_i}^H \mathbf{w}_{s_j}^{(m)}|^2 + \sum_{r \neq s}^{|\pi \mathcal{Q}|} \sum_{j=1}^{|\mathcal{S}_r|} |\mathbf{w}_{r_j}^H \mathbf{h}_{rs_i}|^2 + \sigma^2}, \quad (4.3)$$

where $\tilde{\mathbf{h}}_{ss_i}$ is a unit-norm channel direction vector.

2. The CDI report of the user- i in coalition- s is determined as

$$(m_{s_i}, n_{s_i}) = \underset{m, n}{\operatorname{argmax}} \text{SINR}_{s_i, n}^{(m)} \quad (4.4)$$

and the CQI report of the user- i in coalition- s is

$$\eta_{s_i} = \text{SINR}_{s_i, n_{s_i}}^{(m_{s_i})}. \quad (4.5)$$

3. The CQI is assumed to be known perfectly at the base station without quantization. Based on these recommendation sent by all the users, the base station coalition selects the best user set and transmit beamforming matrix.
4. The set of users who select $\mathbf{w}_{s_n}^{(m)}$ as their CDI index is $\mathcal{I}_{s_n}^{(m)}$, where $\mathcal{I}_{s_n}^{(m)} = \emptyset$ indicates that the corresponding vector is not selected by any users. Among 2^B orthonormal matrices in the codebook, the index of the best orthonormal matrix is obtained as

$$\bar{m}_s = \operatorname{argmax}_m \sum_{n=1}^{N|\mathcal{S}|} \log \left(1 + \max_{s_i \in \mathcal{I}_{s_n}^{(m)}} \eta_{s_i} \right) \quad (4.6)$$

which will be used as a transmit beamforming matrix. The set of active transmit beamforming vectors, and hence the scheduled users, is $\Phi = \{1 \leq n \leq |\mathcal{S}_s| : \mathcal{I}_{s_n}^{(\bar{m})}\}$.

5. Finally, the data rate of user i in coalition s is given by

$$u_{s_i} = \log_2 \left(1 + \max_{s_i \in \mathcal{I}_{s_n}^{(\bar{m})}} \eta_{s_i} \right) \quad (4.7)$$

while the data rate of the entire coalition is

$$u_s = \sum_{s_i \in \mathcal{S}_s} u_{s_i}. \quad (4.8)$$

4.2.3.2 Without Feedback Using Regret Based Learning

Consider the case when the feedback from the UEs to the SBSs is temporarily unavailable, which can happen during deep shadow fading of the feedback channel. The aforementioned selection of beamforming vector using the feedback method is not applicable. Thus we need to randomly assign the transmit beamforming vectors from the columns of matrix $\mathbf{W}_s = [\mathbf{w}_{s_1}, \dots, \mathbf{w}_{s_{N \cdot |\mathcal{S}_s|}}]$ to the UE. In the absence of CQI and CDI information, a better option is to use a learning method whereby the SBSs learn by a trial-and-error method to use a better beamforming scheme.

In practice, we expect the feedback channel to be present during much of the period of operation. Thus, the SBSs have access to the data rate information for much of the time. The SBSs can exploit their history of data rate, obtained when the feedback is

present, by concurrently constructing a probabilistic strategy to select beamformers for periods during which the feedback channel becomes temporarily unavailable.

In the regret based learning [199] for beamformer selection, each coalition of SBSs forming a network MIMO system, as given by partition $\pi = \{\mathcal{S}_1, \dots, \mathcal{S}_{|\pi|}\}$, is assumed to be an agent that engages in a long-term optimization process. Accordingly, the decisions regarding beamformers are made based on accumulated history of its data rate values. The action space of each coalition- s consists of a *fixed* beamforming codebook $\mathcal{W}_s = \{\mathbf{W}_s^{(1)}, \dots, \mathbf{W}_s^{(|2^B|)}\}$ with 2^B matrices. Each UE under the network MIMO system is assigned a column of a chosen m -th beamforming matrix $\mathbf{W}_s^{(m)}$ according to some fixed pre-defined assignment rule. Let the utility of UE- i in coalition- s be

$$u_{s_i} = \log_2(1 + SINR_{s_i}), \quad (4.9)$$

where $SINR_{s_i}$ is as given in (4.2). The utility obtained by each coalition is then taken to be the sum of data rate of the UEs as given by (4.8).

As shown in Algorithm 3, each coalition compares the time-average of its utility metric observations $\tilde{u}_s(n)$ at n -th time instant. The observations are obtained by constantly changing its actions following a particular probabilistic selection strategy vector $p_s = [p(\mathbf{W}_s^{(1)}), \dots, p(\mathbf{W}_s^{(|2^B|)})]$. Also, the observations are obtained for the case where the coalition- s would have played the same action in all previous stages, while the other coalitions use their current probabilistic strategies p_{-s} . Here $-s$ denotes strategies of all coalitions except coalition- s . The regret of coalition- s for not having played action $\mathbf{W}_s^{(m)}$ from $n = 1$ up until time t is calculated as

$$r_{s, \mathbf{W}_s^{(m)}}(t) = \frac{1}{t} \sum_{n=1}^t \left(u_s(\mathbf{W}_s^{(m)}, \mathbf{W}_{-s}(n)) - \tilde{u}_s(n) \right). \quad (4.10)$$

The coalitions behave in such a way that they prefer to choose an action that might give high regrets if that particular action is not chosen, yet allowing a non-zero probability that any action can be considered for play. This behavioral rule can be modeled by the probability distribution $\beta_s(\mathbf{r}_s^+(t))$ defined as follows:

$$\beta_s(\mathbf{r}_s^+(t)) \in \operatorname{argmax}_{p_s \in \Delta(\mathcal{W}_s)} \left(\sum_{\mathbf{W}_s \in \mathcal{W}_s} p_{s, \mathbf{W}_s} r_{s, \mathbf{W}_s}(t) + \kappa_s H(p_s) \right), \quad (4.11)$$

Algorithm 3 Regret based beamformer selection algorithm.

- 1: Initialize actions $p_s(0)$ and regrets $r_s(0)$.
 - 2: Set learning rates γ_s , ζ_s , λ_s , and ϵ_s such that $\gamma_s = \gamma$, $\zeta_s = \zeta$, $\lambda_s = \lambda$ for each coalition- s
 - 3: **repeat**
 - 4: $t \leftarrow t + 1$
 - 5: **for** each coalition- s **do**
 - 6: Estimate the utility function of a coalition- s $\hat{u}_{s,W_s^m}(t)$:

$$\hat{u}_{s,W_s^m}(t) = \hat{u}_{s,W_s^m}(t-1) + \gamma_s(t) \mathbb{I}_{\{W_s(t)=W_s^m\}} (\tilde{u}_k(t) - \hat{u}_{s,W_s^m}(t))$$
 - 7: Compute the instantaneous regret $r_{s,W_s^m}(t)$:

$$r_{s,W_s^m}(t) = r_{s,W_s^m}(t-1) + \lambda_s(t) (\hat{u}_{s,W_s^m}(t) - r_{s,W_s^m}(t) \tilde{u}_k(t))$$
 - 8: Update the selection probability $p_{s,W_s^m}(t)$:

$$p_{s,W_s^m}(t) = p_{s,W_s^m}(t-1) + \zeta_s(t) (\beta_{s,W_s^m}(\mathbf{r}_s(t)) - p_{s,W_s^m}(t-1))$$
 - 9: **end for**
 - 10: **until** convergence
-

where $\mathbf{r}_s^+(t)$ denotes a vector of positive regrets and $\Delta(\mathcal{W}_s)_s$ represents the set of all probability distributions over the elements of the finite set of $\mathcal{W}$. The temperature $\kappa_s > 0$ represents the interest of coalition- s to choose other actions rather than those maximizing the regret to improve the estimations of the regret vectors. The H represents the Shannon entropy function of the randomized strategy p_s . Given a probability measure $p_1, \dots, p_N$ over a set of N elements, H is given by $H(p_1, \dots, p_N) = -\sum_{n=1}^N p_n \log_2(p_n)$. The unique solution to the right hand side of the continuous and strictly concave optimization problem (4.11) is written as $\beta_s(\mathbf{r}_s^+(t)) = (\beta_{s,\mathbf{w}_{s_1}}(\mathbf{r}_s^+(t)), \dots, \beta_{s,\mathbf{w}_{s_{2^B}}}(\mathbf{r}_s^+(t)))$, where for all $\mathcal{S}_s \in \pi_Q$ and for all $m \in \{1, \dots, 2^B\}$, we have

$$\beta_{s,\mathbf{w}_{s_m}}(\mathbf{r}_s^+(t)) = \exp(\kappa_s r_{s,\mathbf{w}_{s_m}}^+(t)) / \sum_{\mathbf{w}_s \in \mathcal{W}_s} \exp(\kappa_s r_{s,\mathbf{w}_s}^+(t)). \quad (4.12)$$

In the Algorithm 3, we can interpret the learning rates $\gamma(t)$, $\lambda(t)$, $\zeta(t)$ as the user controlled factors that govern the rate at which the relevant parameters including utility, regret, and probability related to different strategies are updated. In a way, they are

analogous to velocity or momentum of a particle which governs how fast the particle will move. For the regret-based learning algorithm, the learning rates $\gamma(t)$, $\lambda(t)$, $\zeta(t)$ should be chosen such that the following three conditions are satisfied:

$$\begin{aligned}
(i) \quad & \lim_{T \rightarrow \infty} \sum_{t=0}^T \gamma(t) = +\infty, \quad \lim_{T \rightarrow \infty} \sum_{t=0}^T \lambda(t) = +\infty, \quad \lim_{T \rightarrow \infty} \sum_{t=0}^T \zeta(t) = +\infty \\
(ii) \quad & \lim_{T \rightarrow \infty} \sum_{t=0}^T \gamma^2(t) < +\infty, \quad \lim_{T \rightarrow \infty} \sum_{t=0}^T \lambda^2(t) < +\infty, \quad \lim_{T \rightarrow \infty} \sum_{t=0}^T \zeta^2(t) < +\infty, \\
(iii) \quad & \lim_{t \rightarrow \infty} \frac{\lambda(t)}{\gamma(t)} = 0, \quad \lim_{t \rightarrow \infty} \frac{\zeta(t)}{\lambda(t)} = 0.
\end{aligned}$$

The first condition implies that the series formed of these parameters are diverging, where as the second condition implies that the series are square convergent. Lastly, the third condition implies that the values for λ become smaller faster than that of γ , and that the value of ζ become smaller faster than that of λ . All these three condition are satisfied when the learning rates are chosen such that $\gamma = 1/t^a$, $\lambda = 1/t^b$, $\zeta = 1/t^c$, where t is the iteration and $a > b > c$ so that they fall within the interval $(0, 1)$.

The outcome of the regret based learning converges almost surely to the correlated equilibrium solution of a non-cooperative inter-coalitional game [199]. In particular, all coalitions of SBSs compete to maximize their individual utility by optimizing their beamformer strategies. The references [200, 201] also provide good discussion on the convergence of this algorithm. In short, the convergence of this algorithm can be studied based on the notion of stochastic approximation. The convergence of such stochastic approximation itself can be established as a consequence of the law of large numbers or as an asymptotic state of a dynamic system described by ordinary differential equations. Convergence is guaranteed when the aforementioned three conditions on the learning rates are met. In the next section, the SBSs will use the utility obtained from (4.7) or (4.9) based on the availability of the feedback CSI to form their coalitions.

4.3 Coalition Formation Game Formulation

In this section, we formulate the coalition formation game for SBSs to form network MIMO systems. First, we note that the data rate obtained by one SBS cannot be

transferred to another base station. This is known as *non-transferable utility*. Second, the rate that a coalition is able to achieve depends not only on their beamforming strategies, but also on the SBSs that are not within that coalition, and how they organize themselves into their own coalitions. This is known as *externality*. Therefore, this coalition formation game is a coalitional game with *non-transferable utility* (NTU) in a partition form with an *externality* [212].

4.3.1 Game Formulation

Definition 1. A coalitional game in a partition form with non-transferable utility is defined by a pair $(\mathcal{Q}, \nu)$, where $\mathcal{Q}$ is the set of players (i.e., SBSs), and ν is a partition function $\nu : \Pi(\mathcal{Q}) \rightarrow \mathbb{R}^{\mathcal{Q}}$ that maps, for every partition $\pi_{\mathcal{Q}}$ and coalition $\mathcal{S} \subseteq \mathcal{Q}$, $\mathcal{S} \in \pi_{\mathcal{Q}}$, to a closed convex subset of $\mathbb{R}^{|\mathcal{S}|}$ that contains the payoff vectors that players in $\mathcal{S}$ can achieve.

For a given partition $\pi_{\mathcal{Q}} \in \Pi_{\mathcal{Q}}$, we define the payoff of an SBS as its average data rate when the SBS belongs to the coalition $\mathcal{S} \in \pi_{\mathcal{Q}}$ as follows³:

$$x_i(\mathcal{S}, \pi_{\mathcal{Q}}) = \mathbb{E}[u_{s_i}(t)] \quad (4.13)$$

where $u_{s_i}(t)$ is as given in Section 4.2.3 for time instant t . Since cooperative beamforming is performed in order to mitigate long-term shadow fading, the payoff in (4.13) essentially assesses the long-term effects, while the signal fluctuations due to short-term effects are averaged out. We consider average data rate as the payoff instead of the instantaneous data rate due to the following reasons: 1) we are using random beamforming technique, 2) it makes the payoff immune to fading effects, 3) it allows the SBS to make a decision based on long-term effects of joining a coalition, 4) different UEs may be scheduled for the downlink by an SBS, and 5) since an average value is unique, each partition is mapped to a unique payoff vector. Thus the decisions of an SBS are not based on instantaneous payoff, but rather on long term average payoff it obtains by being in a particular coalition of a partition.

³Note that x_i is the i -th component of the partition function ν

Lastly, for any two payoff vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^Q$, we write $\mathbf{x} >_{\mathcal{S}} \mathbf{y}$ to indicate that $x_i \geq y_i$ for all $i \in \mathcal{S} \subset Q$ and for at least one $j \in \mathcal{S}$, we have $x_j > y_j$. We say that $\mathbf{y}$ is *Pareto dominated* by $\mathbf{x}$ via $\mathcal{S}$. The *outcome* of the game is defined by a pair $(\mathbf{x}, \pi_Q)$, where $\mathbf{x}$ is the payoff vector of all players due to partition π_Q . The set of all undominated outcomes is called the *core*.

4.3.2 Recursive Core

The SBSs' payoff given in (4.13) depends on how the SBSs partition themselves into coalitions. As such, we will use the concept of *recursive core* [202] to analyze this game. The analysis is based on the assumption on how the *residual players* will react if some players unite and deviate to form their own coalition. Such *deviating players* may themselves further split into smaller coalitions. We first define the *residual game* as follows:

Definition 2 (Residual game [202]). *Let (Q, ν) be a coalitional game among SBSs. If $\mathcal{S}$ is the set of deviators and $\pi_{\mathcal{S}}$ is their partition, then let $\mathcal{R} \equiv Q \setminus \mathcal{S}$ denote the set of residual players. Given the deviation $\pi_{\mathcal{S}}$, the residual game $(\mathcal{R}, \nu_{\pi_{\mathcal{S}}})$ is the partition form game over the player set $\mathcal{R}$, with partition function $\nu_{\pi_{\mathcal{S}}}$, that assigns for every partition $\pi_{\mathcal{R}}$ of $\mathcal{R}$ and every coalition $\mathcal{C} \subseteq \mathcal{R}$, $\mathcal{C} \in \pi_{\mathcal{R}}$, to a closed convex subset of $\mathbb{R}^{|\mathcal{C}|}$ that contains the payoff vectors that players in $\mathcal{C}$ can achieve, $\nu_{\pi_{\mathcal{S}}} : 2^{|\mathcal{R}|} \times \Pi(\mathcal{R}) \rightarrow \mathbb{R}^{|\mathcal{C}|}$, i.e., $\nu_{\pi_{\mathcal{S}}} : \mathcal{C}, \pi_{\mathcal{R}} \mapsto \nu(\mathcal{C}, \pi_{\mathcal{R}} \cup \pi_{\mathcal{S}})$.*

Here we have $\nu_{\pi_{\mathcal{S}}}(\mathcal{C}, \pi_{\mathcal{R}}) \equiv \nu(\mathcal{C}, \pi_{\mathcal{R}} \cup \pi_{\mathcal{S}})$. Once the residual players are known, we can give the post-deviation payoff for the deviating coalitions. This post-deviation payoff is the payoff of the players in the deviating coalitions in a partition form, together with the *preferred* residual partition taken into account. Exactly, what constitutes as preferred residual partitions depends on particular assumptions made by the deviators regarding the resultant behaviour of the residual players after the deviation process. Therefore, the deviating players decide which residual outcomes to take into account. A deviation is profitable only if it is profitable for all the deviating coalitions.

Example: Let us have five players $\{1, 2, 3, 4, 5\}$ such that players 4 and 5 deviate to form the coalition $\{4, 5\}$. Here the residual players are $\{1, 2, 3\}$ and the possible

residual partitions that they can form are $\{\{1\}, \{2\}, \{3\}\}$, $\{\{1, 2\}, \{3\}\}$, $\{\{1\}, \{2, 3\}\}$, $\{\{1, 3\}, \{2\}\}$ and $\{\{1, 2, 3\}\}$. The payoff for each residual player is assigned by the partition function $\nu_{\{4,5\}}$, where the deviating partition $\{4, 5\}$ is held fixed.

We can see from the definition that the residual game is essentially another smaller game in a partition form embedded inside the original game. The residual game will be solved in the same way as the original game, so long as the partition of the residual game is consistent with the initial game and the deviators' partition is held fixed. Thus it becomes possible to recursively obtain the solution of the entire game. The core of the recursive game is called *recursive core*. Since each partition is mapped to a unique payoff vector, it is sufficient to specify the partitions belonging in the core. In the following, the core of a game $(\mathcal{Q}, \nu)$ is denoted by $C(\mathcal{Q}, \nu)$.

Definition 3 (Recursive core [202]). *The recursive core is defined inductively as follows:*

1. *Trivial game:* Given a game $(\mathcal{Q}, \nu)$, the core of a game with $\mathcal{Q} = \{1\}$ consists of the trivial partition: $C(\{1\}, \nu) = \{\{1\}\}$.
2. *Inductive assumption:* Given the core $C(\mathcal{R}, \nu)$ for every game with $|\mathcal{R}|$ players, where $|\mathcal{R}| < Q$, we define dominance for a game of $Q = |\mathcal{Q}|$ players. Let $A(\mathcal{R}, \nu)$ denote the assumption⁴ about the residual game $(\mathcal{R}, \nu)$. If $C(\mathcal{R}, \nu) \neq \emptyset$, then $A(\mathcal{R}, \nu) = C(\mathcal{R}, \nu)$. Otherwise, $A(\mathcal{R}, \nu) = \Omega(\mathcal{R}, \nu)$ is the set of all possible outcomes.
3. *Dominance:* The partition $\pi_{\mathcal{Q}}$ is dominated via coalition $\mathcal{S}$ forming partition $\pi_{\mathcal{S}}$ if for all assumptions $\pi_{\mathcal{Q} \setminus \mathcal{S}} \in A(\mathcal{Q} \setminus \mathcal{S}, \nu_{\pi_{\mathcal{S}}})$, there exists an outcome $((\mathbf{y}_{\mathcal{S}}, \mathbf{y}_{\mathcal{Q} \setminus \mathcal{S}}), \pi_{\mathcal{S}} \cup \pi_{\mathcal{Q} \setminus \mathcal{S}}) \in \Omega(\mathcal{Q}, \nu)$ such that $\mathbf{y} >_{\mathcal{S}} \mathbf{x}$.
4. *Core:* The core of a game of Q players is the set of undominated partitions, denoted by $C(\mathcal{Q}, \nu)$.

⁴We will distinguish the *assumption* set using italics so as to avoid confusing this word with its ordinary usage.

Note that the *assumption* set of game $(\mathcal{R}, \nu)$ consists of partitions of $\mathcal{R}$ that the residual players can play against the set of deviating players $\mathcal{S} \equiv \mathcal{Q} \setminus \mathcal{R}$. This set is obtained by asking the question, “What can the deviating players expect the residual players to do after they deviate?” It is reasonable to assume that the residual players will play their core strategies. However, since the core can be empty, the core’s existence is too strict assumption. In the case that the core is empty, every possible partition of $\mathcal{R}$, as given by the Bell number, must be considered for the *assumption* set. Checking the dominance relationship for such *assumption* set in step 3 of the definition of the recursive core is not a trivial task. Nonetheless, the size of the *assumption* set can be reduced by using game specific assumptions regarding the behavior of the residual players.

It is not necessary to assume that the residual players will engage in an intelligent retaliatory behavior, as we often do expect in non-cooperative game. This can indeed form a framework to analyze the residual players’ behavior, but it is not a pre-requisite. In our case, the anticipation of how the residual players react is not based on their intelligent retaliatory behavior but rather on a particular system wide constraint imposed on the behavior of the players which is known a-priori to all players. In our later development of the merge-only algorithm, the *assumption* set is given by the status quo residual partition. Although this constraint may seem artificial in the wider context of game theory, it makes practical sense in the context of our wireless communication problem. The reason being that the behavior of a particular base station is likely to affect only those base stations around its immediate neighborhood, while the rest of the network remains unaffected. Furthermore, even if this assumption was not in place, it would have been unreasonable to assume that the deviating base stations know a-priori all the payoffs of all possible partitions of residual players. This is because, computing the payoff for a particular partition, let alone all possible partitions, itself is not a trivial task. Therefore, it is extremely difficult for the deviating base stations to predict the “best” residual partition that the residual players will prefer. Thus a “reasonable” assumption is to consider that the residual players will maintain their status quo.

Thus based on the *assumption* set of $(\mathcal{R}, \nu)$, the coalition $\mathcal{S}$ tries to arrange itself into a partition $\pi_{\mathcal{S}}$ that is undominated by the current partition π . Thus this recursive approach accounts for the possible responses by $\mathcal{R}$ to the coalition structure $\pi_{\mathcal{S}}$ formed by $\mathcal{S}$.

4.4 Coalition Formation Algorithms

We now propose an algorithm for coalition formation. A possible straightforward method is to solve the following optimization problem: $\max_{\pi_{\mathcal{Q}} \in \Pi(\mathcal{Q})} \sum_{\mathcal{S}_s \in \pi_{\mathcal{Q}}} \mathbb{E}[u_s]$, where $\mathbb{E}[u_s]$ is the expected data rate of the coalition- s . Since the size of the set of partitions $|\Pi(\mathcal{Q})|$ is given by the Bell number, an exhaustive search is not possible in practice. For instance, when there are just fifteen SBSs, [194] gives the Bell number $\mathcal{B}_{15} = 1,382,958,545$. A possible centralized approach might be to use randomized search techniques like simulated annealing or genetic algorithm. However, these approaches take very long time to converge and may not be suitable for real-time implementation. Therefore we shall give a distributed algorithm inspired by game theoretical method of coalition formation. While distributed methods are more convenient for practical implementation, they do not necessarily lead to optimal result. Instead, stability becomes the crucial issue of such distributed method.

4.4.1 Merge-Split Method

The game theoretic approach to the coalition formation is based on certain behavioral assumptions of the players. The most general approach to coalition formation as given in the game theoretic literature [212] is the merge-split process. In the merge-split process, the players are allowed to split from its current coalition, and merge with another coalition, so long as split and merge actions lead to Pareto improvement in the new coalition. Thus, if we have a partition given by $\pi = \{\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n\}$, a player $q \in \mathcal{S}_s$ splits from $\mathcal{S}_s$ and joins $\mathcal{S}_r$ to form a new partition π' if $x_q(\mathcal{S}_r \cup \{q\}, \pi') > x_q(\mathcal{S}_s, \pi)$ and $x_j(\mathcal{S}_r \cup \{q\}, \pi') \geq x_j(\mathcal{S}_r, \pi)$, $\forall j \in \mathcal{S}_r$. The $x_i(\cdot, \cdot)$ is the average payoff obtained from (4.13). Hence, π is dominated by π' via $\mathcal{S}_r \cup \{q\}$, which is written as $\pi <_{\mathcal{S}_r \cup \{q\}} \pi'$.

While the SBSs can split from a coalition and join another coalition so as to obtain better payoff, such a merge-split process can lead to cycles, and hence system instability. A game has a cycle of length n if and only if there exists $\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n$ in partitions $\pi^{(1)}, \pi^{(2)}, \dots, \pi^{(n)}$, respectively, such that $\pi^{(1)} <_{\mathcal{S}_2} \pi^{(2)} <_{\mathcal{S}_3} \dots <_{\mathcal{S}_n} \pi^{(n)} <_{\mathcal{S}_1} \pi^{(1)}$. It can be shown that the length of such a cycle should be at least $n \geq 3$.

Proposition 6. Let $\mathcal{Q} = \{1, 2, \dots, Q\}$ be a set of players in a game $(\mathcal{Q}, \nu)$, and let their payoffs be given by a vector $\mathbf{x} = [x_1, x_2, \dots, x_Q]$ for partition π . There is a game with cycle of length $n = Q$ such that $\pi^{(1)} <_{S_2} \pi^{(2)} <_{S_3} \dots <_{S_n} \pi^{(n)} <_{S_1} \pi^{(1)}$ if the payoffs $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(n)}$ are related as follows:

$$\begin{aligned} x_1^{(1)} &< x_1^{(2)} < x_1^{(3)} < \dots < x_1^{(n)}, \\ x_2^{(n)} &< x_2^{(1)} < x_2^{(2)} < \dots < x_2^{(n-1)}, \\ x_3^{(n-1)} &< x_3^{(n)} < x_3^{(1)} < \dots < x_3^{(n-2)}, \\ &\dots \\ x_Q^{(2)} &< x_Q^{(3)} < x_Q^{(4)} < \dots < x_Q^{(1)}. \end{aligned}$$

Proof. Construct

$$\begin{aligned} \pi^{(1)} &= \{\{1\}, \{2\}, \{3\}, \{4\}, \dots, \{Q\}\}, \\ \pi^{(2)} &= \{\{1, 2\}, \{3\}, \{4\}, \dots, \{Q\}\}, \\ \pi^{(3)} &= \{\{2\}, \{1, 3\}, \{4\}, \dots, \{Q\}\}, \\ \pi^{(4)} &= \{\{2\}, \{3\}, \{1, 4\}, \dots, \{Q\}\}, \\ &\dots, \\ \pi^{(n)} &= \{\{1\}, \{2\}, \{3\}, \{4\}, \dots, \{1, Q\}\}. \end{aligned}$$

Then we have from the condition on payoffs $\pi^{(1)} <_{\{1,2\}} \pi^{(2)} <_{\{1,3\}} \dots <_{\{1,Q\}} \pi^{(n)} <_Q \pi^{(1)}$. Here the number of inequalities is Q , thus the length of the cycle is $n = Q$. $\square$

Corollary 2. *There exists a game for which the merge-split process is unstable.*

Proof. Following Proposition 6, we consider the game in a partition form with externality with the set of outcomes $(\mathbf{x}, \pi)$ given as follows: $([10, 20, 30], \{\{1\}, \{2\}, \{3\}\})$, $([20, 30, 10], \{\{1, 2\}, \{3\}\})$, and $([30, 10, 20], \{\{2\}, \{1, 3\}\})$. It is conceivable for the following transitions to occur: $[10, 20, 30] <_{\{1,2\}} [20, 30, 10] <_{\{1,3\}} [30, 10, 20] <_{\{3\}} [10, 20, 30]$. Thus, the system is unstable. $\square$

4.4.2 Merge-only Method

From Corollary 2, the merge-split process can lead to unstability for the coalitional games with externality. Since guaranteed stability is a much desired feature of any algorithm, we consider a variation of the merge-split process called the merge-only process. We introduce two simplifying assumptions which allows us to constrain the number of possible partitions.

Assumption 4 (Commitment). *SBSs remain committed to the coalition they decide to join.*

Assumption 5 (Status quo). *SBSs maintain their status quo as other SBSs are forming coalitions.*

As discussed in Section 4.3.2, both these assumptions model the behavior of the SBSs. These assumptions about the behaviour of the SBSs not only simplify the design of the algorithm, but also helps to lower the signaling overhead. An SBS may not easily switch away from a coalition due to some inherent switching cost. These assumptions reflect the reluctance of SBS to increase such switching costs. In [202], the second assumption is referred to as the δ -approach.

We adopt the merge-only algorithm, where any two coalitions can decide to merge with each other to form a bigger coalition. However, once a merger has been performed, the SBSs in the bigger coalition cannot split away into smaller coalitions. Also, the second assumption means that when the merger is performed, the SBSs, which do not involve in the merger, will not react and remain in the same coalition.

For the merge-only method, we have the following algorithm:

1. We initialize the coalition structure to be all singletons. A singleton is a coalition with only one SBS as its member.
2. At any given time, when we have a coalition structure π , the coalitions in π will take turn to “negotiate” with another coalition and check if a merger is possible, thus forming a new coalition structure π' . During the negotiation, the payoff of the SBSs in the newly formed coalition structure is calculated. The negotiation is performed in two phases:

- (a) The two concerned coalitions temporarily agree to join together to form a bigger coalition. The other coalitions maintain their status quo.
 - (b) The newly formed coalition structure π' is maintained for some fixed trial period, at the end of which the two concerned coalitions compute the time averaged data rate for each of its SBSs.
3. Based on the calculated payoff, if the two concerned coalitions decide to merge, then the new partition π' is retained. Otherwise, we revert to the previous partition π . Let $\mathcal{R}$ and $\mathcal{S}$ be any two coalitions in a given coalition structure π . Then, a merger will take place if the joint coalition $\mathcal{T} = \mathcal{R} \cup \mathcal{S}$ in the new coalition structure π' is such that every SBS in $\mathcal{T}$ is not worse off than it was in the original coalition, with at least one SBS doing strictly better after the merger. That is,

$$\mathbf{x}(\mathcal{T}, \pi') >_{\mathcal{T}} \mathbf{y}(\mathcal{R} \cup \mathcal{S}, \pi) \quad (4.14)$$

for the merger, where $\mathbf{x}(\mathcal{T}, \pi')$ is the payoff vector of SBSs in $\mathcal{T}$ under partition π' , while $\mathbf{y}(\mathcal{R} \cup \mathcal{S}, \pi)$ is the payoff vector of SBSs in $\mathcal{R}$ and $\mathcal{S}$ under partition π .

4. The negotiation process is iterated for every pair of coalition in the new partition π' .

The merge-only method can be performed in two loops. Given a partition $\pi^{(t)} = \{\mathcal{S}_1, \dots, \mathcal{S}_n\}$ at time t , the outer loop selects and fixes a coalition $\mathcal{S}_s \in \pi^{(t)}$, while the inner loop selects another coalition $\mathcal{S}_r \in \pi^{(t)}$, such that $r \neq s$, as a potential candidate for $\mathcal{S}_s$ to merge with. Two possibilities are

1. If coalitions $\mathcal{S}_s$ and $\mathcal{S}_r$ decide not to merge, then the partition in the next time instant $t + 1$ will remain the same, i.e., $\pi^{(t+1)} \leftarrow \pi^{(t)}$. In the next time instant $t + 1$, the coalition $\mathcal{S}_s$ will continue to check other remaining coalitions $\mathcal{S}_k \in \pi^{(t+1)} \setminus \mathcal{S}_r$, such that $k \neq s$.
2. In the case that $\mathcal{S}_s$ and $\mathcal{S}_r$ decide to merge, then in the next time instant $t + 1$, the newly merged coalition will be given the same label as the coalition that initiated the merging process, i.e., $\mathcal{S}_s \leftarrow \mathcal{S}_s \cup \mathcal{S}_r$. This operation reduces the number of

coalitions in $\pi^{(t)}$ by unity. Therefore, the other coalitions in $\pi^{(t)} \setminus \{\mathcal{S}_s, \mathcal{S}_r\}$ are re-labeled in order to reflect the fact that there are now only $|\pi^{(t+1)}| = |\pi^{(t)}| - 1$ coalitions in the new partition $\pi^{(t+1)}$.

This way, $\mathcal{S}_s$ will finish negotiating with all other coalitions of the partition in the inner loop. The further iterations of the inner loop may or may not result in further mergers. Once the inner loop terminates and the new iteration of the outer loop begins, coalition $\mathcal{S}_s$ will no longer enter into any negotiations with other coalitions. Let the coalition structure be given by $\pi^{(t')}$ during this new iteration of the outer loop. Now another coalition apart from $\mathcal{S}_s$ is chosen from $\pi^{(t')}$ and the same process repeats. The newly chosen coalition does not need to enter into further negotiations with those coalitions, such as $\mathcal{S}_s$, that have already completed this inner loop process. The implementation of this process is given in Algorithm 4.

The maximum number of negotiations is given by the series $(Q-1) + (Q-2) + \dots + 2 + 1 = Q(Q-1)/2$. Thus the worst case complexity of the merge-only algorithm is $\mathcal{O}(Q^2)$, which happens when no SBS is willing to merge with any other SBS, thus resulting in all coalitions being singletons. Likewise, the minimum number of negotiations that must be performed is $Q-1$. Thus the best case complexity of the merge-only algorithm is $\mathcal{O}(Q)$, which happens when all the SBSs agree to merge and form a single grand coalition (i.e., a coalition with all SBSs as members). Lastly, if we assume that there is a probability p for any two SBSs to cooperate and form a coalition, then the average number of negotiations is given by the series $(Q-1) + (Q-1)(1-p) + (Q-1)(1-p)^2 + \dots + (Q-1)(1-p)^{Q-2} \leq (Q-1)[1 + (1-p) + (1-p)^2 + \dots] = (Q-1)/p$. Next, we will show that for large values of Q , this probability p can be approximated by $\frac{1}{Q}(\ln Q)^{1-1/\ln Q}$. Thus the average case complexity is given by $\mathcal{O}(Q^2(\ln Q)^{-(1-1/\ln Q)})$. We have the following proposition:

Proposition 7. *The maximum, the minimum, and the average number of negotiations required before the termination of the merge-only algorithm is $Q(Q-1)/2$, $Q-1$, and $\mathcal{O}(Q^2(\ln Q)^{-(1-1/\ln Q)})$, respectively.*

Proposition 8. *Given the Assumptions 4 and 5, the final outcome of the merge-only algorithm belongs to the recursive core.*

Algorithm 4 Implementation of merge-only algorithm.

```
1: Initialize:  $\pi_0 = \pi^{ch} = \{\mathcal{S}_1, \dots, \mathcal{S}_Q\}$ ,  $\pi^{sel_1} = \pi^{sel_2} = \pi^{rem} = \emptyset$ ,  $flag_1 = 1$ 
2: while  $flag_1 = 1$  do
3:    $\pi^{sel_1} \leftarrow \pi^{sel_1} \cup \pi^{ch}(1)$ 
4:    $\pi^{rem} \leftarrow \pi^{ch} \setminus \pi^{ch}(1)$ 
5:   Set  $flag_2 \leftarrow 1$  if  $\pi^{rem} \neq \emptyset$ , else set  $flag_2 \leftarrow 0$ 
6:   while  $flag_2 = 1$  do
7:      $\pi^{sel_2} \leftarrow \pi^{sel_2} \cup \pi^{rem}(1)$ 
8:     Check if  $\pi^{ch}(1)$  and  $\pi^{rem}(1)$  decides to merge. Let coalition structure be  $\pi^{(t+1)}$ .
       Set  $merge \leftarrow 1$  if merger takes place, else  $merge \leftarrow 0$ 
9:     if  $merge = 0$  then
10:       $\pi^{rem} \leftarrow \pi^{rem} \setminus \pi^{sel_2}$ 
11:    end if
12:    if  $merge = 1$  &  $\pi^{rem} \neq \emptyset$  then
13:      Relabel coalitions in  $\pi^{rem}$  so that  $|\pi^{(t+1)}| = |\pi^{(t)}| - 1$ 
14:       $\pi^{sel_2} \leftarrow \emptyset$ 
15:    end if
16:    Set  $flag_2 \leftarrow 0$  if  $\pi^{rem} = \emptyset$ 
17:  end while
18:   $\pi^{ch} \leftarrow \pi^{(t+1)} \setminus \pi^{sel_1}$ 
19:  Set  $flag_1 \leftarrow 0$  if  $\pi^{ch} = \emptyset$ 
20: end while
```

Proof. Let $\pi^{(t)} = \{\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n\}$ be the partition of players at time t . Let the players in any two coalitions $\mathcal{S}_s$ and $\mathcal{S}_r$ such that $s \neq r$ in $\pi^{(t)}$ deviate, so that the set of deviating players is $\mathcal{S} = \mathcal{S}_s \cup \mathcal{S}_r$. The set of residual players is $\mathcal{R} = \mathcal{Q} \setminus \mathcal{S}$. As per Assumption 5, the residual players maintain their status quo. Consequently, the *assumption* set of the deviating players regarding the outcomes of the residual game will have only a single entity, i.e., $A(\mathcal{R}, \nu_{\pi_{\mathcal{S}}}) = \pi_{\mathcal{R}} = \pi^{(t)} \setminus \{\mathcal{S}_s, \mathcal{S}_r\}$.

As per Assumption 4, due to the commitment of the deviating players (i.e., players in $\mathcal{S}_s$ and $\mathcal{S}_r$) to their respective coalitions, the players do not split apart to form smaller coalitions. As a result, there are only two possible partitions for the deviating players. Let us denote the partition as $\pi_{\mathcal{S}}^1 = \{\mathcal{S}_s, \mathcal{S}_r\}$, which is the status quo of the deviating players, and $\pi_{\mathcal{S}}^2 = \{\mathcal{S}_s \cup \mathcal{S}_r\}$, which is the merged coalition of the deviating players. Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be the payoff vectors of the players in partitions $\pi_{\mathcal{S}}^1 \cup \pi_{\mathcal{R}}$ and $\pi_{\mathcal{S}}^2 \cup \pi_{\mathcal{R}}$, respectively, where $\pi_{\mathcal{R}}$ is the partition of the residual players. We now need to check the dominance relationship between two possible outcomes, i.e., $(\mathbf{x}_1, \pi_{\mathcal{S}}^1 \cup \pi_{\mathcal{R}})$ and $(\mathbf{x}_2, \pi_{\mathcal{S}}^2 \cup \pi_{\mathcal{R}})$ with respect to $\mathcal{S}$. If one of them is dominant over the other, the new partition will be

$$\pi^{(t+1)} = \begin{cases} \pi_{\mathcal{S}}^2 \cup \pi_{\mathcal{R}}, & \text{if } \mathbf{x}_2 >_{\mathcal{S}} \mathbf{x}_1, \\ \pi_{\mathcal{S}}^1 \cup \pi_{\mathcal{R}}, & \text{otherwise.} \end{cases}$$

Hence, we can establish a relationship between $\pi^{(t)}$ and $\pi^{(t+1)}$ via a set of players $\mathcal{S}^{(t)} = \mathcal{S}_s \cup \mathcal{S}_r$, such that $s \neq r$, as $\pi^{(t+1)} \geq_{\mathcal{S}^{(t)}} \pi^{(t)}$. Here by $>_{\mathcal{S}^{(t)}}$ we mean $\pi^{(t)}$ is dominated via $\mathcal{S}^{(t)}$ forming partition $\pi_{\mathcal{S}^{(t)}}$ so as to form the new partition $\pi^{(t+1)}$. Likewise, $\pi^{(t+1)} = \pi^{(t)}$ means $\pi^{(t)}$ is undominated via $\mathcal{S}^{(t)}$ forming partition $\pi_{\mathcal{S}^{(t)}}$ so that the partition at time $t+1$ remains the same as $\pi^{(t)}$. By checking all possible pairs of coalitions, we can establish a chain of relationship as $\pi^{(T)} \geq_{\mathcal{S}^{(T-1)}} \dots \geq_{\mathcal{S}^{(t)}} \pi^{(t)} \geq_{\mathcal{S}^{(t-1)}} \dots \geq_{\mathcal{S}^{(0)}} \pi^{(0)}$.

We now claim that by the end of this process we will have an undominated outcome $\pi^{(T)}$. That is, $\pi^{(T)}$ cannot improve upon itself. To prove our claim, let there be a partition $\pi^{(T+1)}$ such that $\pi^{(T+1)} \geq_{\mathcal{S}^{(T)}} \pi^{(T)}$. For $\pi^{(T+1)} >_{\mathcal{S}^{(T)}} \pi^{(T)}$, this will require any two coalitions in $\pi^{(T)}$ to merge together to get an undominated payoff vector. This means that one more pair of coalitions in $\pi^{(T)}$ needs to be checked. However, we assume

that all possible pairs have already been checked, which is a contradiction. Therefore, $\pi^{(T+1)} = \pi^{(T)}$. Thus by the end of the process, we will have an undominated outcome $\pi^{(T)}$, which by Definition 3 belongs to the recursive core. $\square$

4.4.3 Split-Only Method

The purpose of Algorithm 4 is to cluster the base stations in a low complexity distributive manner using the merge-only method. The alternatives to this are merge-split method and split-only method. As demonstrated in Proposition 6 and its subsequent Corollar 2, the merge-split method is not guaranteed to converge for cases with externality. Here we will briefly discuss the split-only method for the sake of completeness. In a split-only strategy, we proceed from a grand coalition and only allow the splitting operations. However, a short analysis will demonstrate that this procedure is more complex compared to the merge-only method. The players in a grand coalition $\mathcal{Q}$ must first determine how they might split into two coalitions. The number of ways that any set $\mathcal{S}$ can be partitioned into two subsets is given by the Stirling number of the second kind $S(|\mathcal{S}|, 2)$. The Stirling number of second kind can be written in terms of binomial coefficients as given by the formula $S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$ [205]. For $n = |\mathcal{S}|$ and $k = 2$, the formula simplifies to $S(|\mathcal{S}|, 2) = 2^{|\mathcal{S}|-1} - 1$. For a grand coalition $\mathcal{Q}$, the number of comparisons required in the worst case is $2^{Q-1} - 1$. Since any further splitting process will involve a subset with smaller number of SBSs than $\mathcal{Q}$, the worst case complexity of split-only method is $\mathcal{O}(2^{Q-1})$. Thus we see that the complexity of the split-only method is exponential with respect to the number of SBSs.

4.4.4 Probabilistic Analysis of Coalition Formation

The exact number and size of the coalitions that form during the coalition formation process depends on many physical factors such as the geometry of the layout of the SBSs, number of feedback bits, number of transmit antennas, channel conditions, and so on. While it is difficult to tell at the outset how many such coalitions are likely to form, we can make a probabilistic estimate of such figures. Here we will perform the probabilistic analysis of the coalition formation process using two different methods: the

first is based on counting arguments and is valid regardless of the algorithm used to form the coalitions, while the second is based on probabilistic modeling of the merge-only algorithm.

4.4.4.1 Method Based on Counting

For Q SBSs, the number of ways of forming partitions such that there are exactly k , where $1 \leq k \leq Q$, nonempty coalitions is given by the Stirling number of second kind denoted by $S(Q, k)$ [205]. The Stirling number of second kind can be generated using the recurrence relation $S(Q + 1, k) = S(Q, k - 1) + kS(Q, k)$ with initialization $S(1, 1) = 1$ and $S(Q, k) = 0$ for $k > Q$ or $k < 1$. Note that $S(Q, 1) = S(Q, Q) = 1$ for any Q . The total number of possible partitions, as given by the Bell number, is related to the Stirling number of second kind by summation $\sum_{k=1}^Q S(Q, k) = \mathcal{B}_Q$.

From a purely combinatorial point of view, we can assign a probability to the number of coalitions that can form in a partition π_Q . The probability that exactly k coalitions are formed is given by $\Pr\{|\pi_Q| = k\} = \frac{S(Q, k)}{\mathcal{B}_Q}$. Using the recurrence relation of the Stirling number of second kind, we can prove the following identities (see Appendix): $\sum_{k=1}^Q kS(Q, k) = \mathcal{B}_{Q+1} - \mathcal{B}_Q$, and $\sum_{k=1}^Q k^2S(Q, k) = \mathcal{B}_{Q+2} - 2\mathcal{B}_{Q+1}$.

Using these identities, we can show that the expected number of coalitions is given by

$$\mathbb{E}\{|\pi_Q|\} = \sum_{k=1}^Q \frac{kS(Q, k)}{\mathcal{B}_Q} = \frac{\mathcal{B}_{Q+1}}{\mathcal{B}_Q} - 1, \quad (4.15)$$

whereas its second moment is given by

$$\mathbb{E}\{|\pi_Q|^2\} = \sum_{k=1}^Q \frac{k^2S(Q, k)}{\mathcal{B}_Q} = \frac{\mathcal{B}_{Q+2}}{\mathcal{B}_Q} - \frac{2\mathcal{B}_{Q+1}}{\mathcal{B}_Q}. \quad (4.16)$$

Thus the variance of the number of coalitions is given by

$$\text{Var}\{|\pi_Q|\} = \mathbb{E}\{|\pi_Q|^2\} - (\mathbb{E}\{|\pi_Q|\})^2 = \frac{\mathcal{B}_{Q+2}}{\mathcal{B}_Q} - \left(\frac{\mathcal{B}_{Q+1}}{\mathcal{B}_Q}\right)^2 - 1. \quad (4.17)$$

Since the expression for the expected number of coalitions is in terms of Bell numbers, the formula is very unwieldy. We can make a large number approximation as given in [206, 207, 208]. As Q becomes very large, the central limit theorem applies, and the distribution

for the random variable $Z = [|\pi_Q| - \mathbb{E}\{|\pi_Q|\}]/\sqrt{\text{Var}\{|\pi_Q|\}}$ can be approximated by the standard normal distribution. The estimate is given by the value where the distribution peaks. Thus we get an approximation for the mean value as follows [208]:

$$\ln \mathbb{E}\{|\pi_Q|\} \sim \ln Q - \ln \ln Q + \frac{\ln \ln Q}{\ln Q} + \mathcal{O}\left(\frac{1}{\ln Q^2}\right). \quad (4.18)$$

Consequently, the second order approximation of $\mathbb{E}\{|\pi_Q|\}$ gives $\mathbb{E}\{|\pi_Q|\} \sim \frac{Q}{\ln Q}$, while the third order approximation yields $\mathbb{E}\{|\pi_Q|\} \sim Q(\ln Q)^{-(1-1/\ln Q)}$. With this formula, we can now find the average number of SBSs in a coalition as $\mathbb{E}\{|\mathcal{S}|\} = \frac{Q}{\mathbb{E}\{|\pi_Q|\}}$. Since the discrete distribution has been approximated by a continuous distribution, we can replace Q by continuous terms of cell density and area. Thus we have the following proposition:

Proposition 9. *Let ρ be the uniform SBS density and α be some large area under consideration, then the average number of coalitions is given by $\ln \mathbb{E}\{|\pi_Q|\} \sim \ln(\rho\alpha) - \ln \ln(\rho\alpha) + \frac{\ln \ln(\rho\alpha)}{\ln(\rho\alpha)} + \mathcal{O}\left(\frac{1}{\ln(\rho\alpha)^2}\right)$ and the average size of coalitions is given by $\mathbb{E}\{|\mathcal{S}|\} = \rho\alpha/\mathbb{E}\{|\pi_Q|\}$.*

For instance, when $q = 5$, $\mathcal{B}_5 = 52$, $\mathcal{B}_6 = 203$ and $\mathcal{B}_7 = 877$. Therefore the expected number of coalitions is 2.90 and the variance of this estimate is 1.62. Also, the approximate estimate is $5/\ln 5 = 3.1$. This fits quite well with the previous estimate for such a low number. Fig. 4.3 shows how well the approximations fit the exact formula given by (4.15).

Similarly, the probability that any two randomly chosen SBSs will agree to cooperate is given by $p = \frac{\mathbb{E}\{|\mathcal{S}|\}}{Q} = \frac{1}{\mathbb{E}\{|\pi_Q|\}} \sim \frac{1}{Q}(\ln Q)^{1-1/\ln Q}$. Note that as $Q \rightarrow \infty$, then $p \rightarrow 0$. Thus, in merge-only method, when an SBS initiates a negotiation with $Q-1$ other SBSs, the probability that k SBSs will join and form a coalition with the initiator is given by binomial distribution, $\Pr\{k \text{ SBSs cooperate}\} = \binom{Q-1}{k} p^k (1-p)^{Q-k-1}$. Similarly, if we have exactly $|\pi_Q| = n$ coalitions, then the number of SBSs in each coalition is given by multinomial distribution as follows: $\Pr\{|\mathcal{S}_1| = a_1, \dots, |\mathcal{S}_n| = a_n\} = \frac{Q!}{a_1! \dots a_n!} \theta_1^{a_1} \dots \theta_n^{a_n}$, where a_s is the number of SBSs in coalition- s such that $\sum_{s=1}^n a_s = Q$ and $\theta_s = p(1-p)^{s-1}$. Thus the mean size of coalition- s is $\mathbb{E}\{|\mathcal{S}_s|\} = Q\theta_s = Qp(1-p)^{s-1}$.

Figure 4.3: Comparison between the exact versus approximate clustering formula.

4.4.4.2 Method Based on Modeling of Merge-Only Algorithm

Let there be Q SBSs and let p be the probability that any two SBSs will merge with each other. We can consider the merge-only algorithm in the following equivalent way: Here we first take a single SBS, which forms a single coalition. Let the second SBS arrive and negotiate with the first SBS. It will cooperate with the first SBS with probability p . Otherwise, if the negotiations fail, the second SBS will create its own coalition with probability $1 - p$. Now let the third SBS arrive and negotiate with the first SBS and second SBS in sequence. It can cooperate with the first SBS with probability p , or disagree with the first SBS and join the second SBS with probability $p(1 - p)$. If both the negotiations fail, then the third SBS will create its own coalition with probability $(1 - p)^2$. Proceeding this way, if $n - 1$ coalitions are present, then the probability that a new n -th coalition will be created when a new SBS arrives for the negotiation process is given by $\Pr\{n|n - 1\} = (1 - p)^{n-1}$. This process continues until all Q SBSs have arrived. Thus the probability that there are exactly n coalitions is given by the chain rule of conditional probability $\Pr\{n\} = \Pr\{n|n - 1\}\Pr\{n - 1|n - 2\} \cdots \Pr\{2|1\}\Pr\{1\} = (1 - p)^{n-1} \cdot (1 - p)^{n-2} \cdots (1 - p) \cdot 1 = (1 - p)^{\sum_{i=1}^{n-1} i} = (1 - p)^{n(n-1)/2}$. Thus the expected

number of coalitions is given by $\mathbb{E}\{|\pi_{\mathcal{Q}}|\} = \sum_{i=1}^Q i(1-p)^{i(i-1)/2}$. Such a process is also known as the Chinese restaurant process [209]. Thus we have the following proposition:

Proposition 10. *Let p be the probability that any two SBS cooperates with each other, then the average number of coalitions formed by merge-only algorithm is given by $\mathbb{E}\{|\pi_{\mathcal{Q}}|\} = \sum_{i=1}^Q i(1-p)^{i(i-1)/2}$.*

4.4.5 Heuristic Improvements

In the clustering process, we assumed that all the cells within a network negotiates with every other cells in that network, regardless of their distance of separation. If the network is big and spans over a large geographical area, then this becomes computationally burdensome. Note that the signaling overhead is of utmost concern in cooperative communication. It is one of the reasons why we do not pursue the merge-split algorithm, since that algorithm can be unstable as shown in Proposition 6 and its subsequent Corollary 2. It is for this practical reason that we adopt the merge-only algorithm which is guaranteed to terminate after a finite number of steps. If the signal overhead is proportional to the number of negotiations, then the worst case signal overhead is quadratically related to the number of base stations. Also, although communication delays are always present in an actual system, zero-delay condition was taken as an idealization of reality. This is a good approximation when the network is confined to a relatively small geographical area. However, if the the network is spread over large geographical area, there can be large delays which can cause synchronization problems.

We can lower the worse case upper bound if we introduce additional heuristics to the merge-only algorithm. If we focus the clustering of cells within a relatively small geographical area, then such computation, delay, and signal overhead problem will not be a big threat. This heuristic makes sense because only nearby cells are likely to cooperate with each other, as is shown by simulation studies as well as by intuitive physical consideration. Therefore, during the process of clustering, it is enough to allow each cell to negotiate with their neighboring cells within a small fixed area. How big the neighborhood should be made can be determined by using the approximation given in equation (4.18) and Proposition 9 as follows: Given the total number of cells Q , we first compute

the average cluster size $\mathbb{E}\{|\mathcal{S}|\}$ using (4.18) as $\mathbb{E}\{|\mathcal{S}|\} = Q/\mathbb{E}\{|\pi_{\mathcal{Q}}|\} \sim (\ln Q)^{1-1/\ln Q}$. From Proposition 9 we can then use the cell density ρ to compute the neighborhood size $\alpha_0 = \mathbb{E}\{|\mathcal{S}|\}/\rho$ required to construct this average sized cluster. Also, since each base station is expected to negotiate with only $\mathbb{E}\{|\mathcal{S}|\} \sim (\ln Q)^{1-1/\ln Q}$, the expected number of negotiations is now upper bounded by $\mathcal{O}(Q \ln Q)$.

Thus the implications of restricting the focus of cooperation to a smaller, local area not only improves the computational complexity but also helps in reducing the signaling overhead and makes the system more immune to synchronization problems caused by non-zero communication delays between the base stations, which is always present in a realistic system. Lastly, since the algorithm is based on long term payoff, it is run only once, or very infrequently in response to changes in the network topology. Therefore, the signaling cost incurs only once, or very infrequently.

4.5 Numerical Results

In the following sections, we evaluate the performance of the algorithms and analytical results that we have derived so far. The SBSs are assumed to be uniformly distributed within a square area with sides of 50 meters. In the simulations, small-scale fading between transmitter- i to receiver- j is assumed to be given by complex Gaussian distribution $CN(0, \mathbf{R}_{ij})$, where the correlation matrix is spatially uncorrelated, $\mathbf{R}_{ij} \propto \mathbf{I}$. The pathloss at distance d meters is taken as $PL(d) = 127 + 30 \log_{10}(d)$ dB. The standard deviation of log-normal shadow fading is taken as 4 dB in small-cell coverage, whereas the noise variance $\sigma^2 = -120$ dBm. Each SBS has one UE associated with it. The UE is located within 5 meters of the SBS. Lastly, the transmit power of the SBS is assumed to be 30 dBm. These parameters have been summarized in Table 4.1.

4.5.1 Effect of Distance

To illustrate how the coalition formation performs, we first plot the variation of the average data rate with respect to the separation distance between two small cell base stations (SBSs) if they both form a single coalition $\{1, 2\}$ and if they act as singletons

Table 4.1: Simulation parameters

Parameter	Value
Area	$50 \times 50\text{m}^2$
Small-scale fading model	$CN(0, \mathbf{R}_{ij})$
Spatial correlation matrix ($\mathbf{R}_{ij}$)	$\propto \mathbf{I}$
Pathloss model	$PL(d) = 127 + 30 \log_{10}(d)$ dB
Standard deviation of log-normal shadow fading	4 dB
Noise variance (σ^2)	-120 dBm
No. of UE per SBS	1
Location of UE from SBS	5 m
Transmit power of SBS	30 dBm

$\{\{1\}, \{2\}\}$. The distance between the two SBSs is varied from 5 to 70 meters. We assume $B = 6$ bits of feedback, and two transmit antennas for each SBS. The average data rate is obtained by using 1000 instances of random channel realizations. Fig. 4.4 shows that the performance of the singletons improves with increasing distance of separation, and becomes stable around 4.1 bps/Hz/coalition. This is because with longer distance, the two SBSs cause less interference to each other. On the other hand, the performance of the coalition remains almost the same at around 3.3 bps/Hz/coalition. As a result, when the SBSs are far apart, there is no gain in forming a coalition. When the SBSs are near to each other, in this case within 15 meters, the performance, if they form a coalition, is much better than that of singletons. From this, we observe that coalition formation is profitable only when the SBSs are close enough to each other. The merge-only algorithm decides when to form singletons and when to form a coalition.

4.5.2 Effect of Feedback Bits

Next, we examine the effect of the feedback bits to the performances of two SBSs when they act as a coalition and as singletons. We fix the distance between two SBSs at 20 meters, and change the number of feedback bits from 1 to 11. All other parameters are the same as before. Fig. 4.5 shows that the performances of both the coalition and singleton increase with increasing number of feedback bits. The performance of singletons

Figure 4.4: Average data rate of singletons and coalition when the distance between two SBSs is varied and the feedback $B = 6$ bits.

becomes saturated with increasing number of feedback bits. This is in contrast to the performance of the coalition, which constantly improves as the number of feedback bits is increased. This is because increasing the feedback bits increases the size of joint beamformer codebook. Thus there is more chances of finding a beamformer that matches with the channel. At some point, with 8 bits, the performance of the coalition becomes better than that of singletons. Again, the merge-only algorithm chooses the best among the two options. However, it should be noted that increasing the number of feedback bits increases the computational complexity of the PU²RC method, which finds the best beamformer in the codebook via exhaustive search method.

4.5.3 Effect of Transmit Antennas

Now we evaluate the performance of two SBSs when the number of transmit antennas is varied. We again fix the distance between two SBSs at 20 meters and consider 6 bits for the feedback channel. All other parameters are the same as before. In Fig. 4.6, we vary the number of transmit antennas from 2 to 10. As expected at this distance, with two transmit antennas, the singletons outperform the coalition. As the number of transmit

Figure 4.5: Average data rate of singletons and coalition when the number of feedback bits is varied.

antennas increases, the performances of both the singletons and the coalition decrease, with the coalition always underperforming the singletons. We also see that the curves are approaching some steady value as the number of transmit antennas increases. The reason for the decrease in performance is that the number of feedback bits is fixed. As the number of transmit antennas increases, not only does the inter-stream interference increase, but also the fixed number of bits are used to quantize a larger channel vector. This leads to a degraded quantization, thereby degrading the performance.

4.5.4 Effect of Location of Small Cell Base Stations

We now consider the effect of location of the SBSs on the merge-only algorithm. We consider a 50-by-50 square meter area where 6 SBSs are located as shown in Fig. 4.7. Each UE (shown as dots) associated with an SBS (shown as triangles) is also randomly placed within 7 meters of the SBS. Each SBS is equipped with 2 antennas, and we assume that there are 6 bits for feedback. The final coalition structure obtained from the merge-only algorithm is $\{\{1, 2\}, \{3\}, \{4\}, \{5, 6\}\}$. In Fig. 4.7, the coalitions are shown in ellipses. Clearly, the grand coalition is not formed. Also, the SBSs form coalitions

Figure 4.6: Average data rate of singletons and coalition when the number of transmit antennas is varied and feedback $B = 6$ bits.

based on their geographical location. Specifically, the SBSs close to each other will have higher chance to form a coalition. We observe that the long term average data rate of this final coalition structure achieves 6.5 bps/Hz/coalition, while that for singletons is 5.6 bps/Hz/coalition. Thus the performance improvement is 0.9 bps/Hz/coalition.

4.5.5 Effect of Number of Small Cell Base Stations

Here we evaluate the effect of increasing the number of SBSs, uniformly distributed in a 50-by-50 square meter area, on the performance of a merge-only algorithm. To do so, we consider the average performance of an ensemble of 100 random geometric patterns of SBSs located in the given area. We assume that there are 6 bits for feedback and 2 transmit antennas per SBS. During the negotiations process for coalition formation, the foresighted method takes 1000 samples to obtain the average payoffs, where as the myopic method takes only 1 sample. Fig. 4.8 shows that the average data rate of the entire network monotonically decreases with increasing number of SBSs, implying that

Figure 4.7: Coalition formation among six SBSs when feedback $B = 6$ bits.

the system is interference limited. We also observe that the change in performance is decreasing. Thus, we can infer that the performance is approaching a steady value as the number of SBSs increases. In the foresighted case, the rate obtained by coalition formation is always greater than the rate obtained by singletons. As the SBSs increase, for the foresighted case, we notice that the performance gap between the singletons and coalition increases. This is because with larger number of SBSs, the merge-only algorithm can yield bigger coalitions, leading to better performance relative to singletons. However, in the myopic case, we observe that the performance of singletons is better than the coalitions. This is because the decisions based on single observation can lead to bad long term consequences. Also, we observe that foresighted coalition formation does much better than the myopic case.

4.5.6 Effect of Distance and Codebook Size on Regret Based Learning

We consider the effect of using regret based learning when the feedback channel is temporarily unavailable from a UE to an SBS. As before, we first examine the performance of two SBSs when the distance increases. We construct an action space of a fixed codebook

Figure 4.8: Average data rate when the number of SBSs is varied when feedback $B = 6$ bits.

of beamformers that can be represented by 6 bits for each coalition. Thus there are 64 beamformer matrices in the codebook. These beamformers are randomly selected based on a probability distribution that is iteratively updated according to the regret based learning as shown in Algorithm 3 for each coalition. We took the learning parameters of Algorithm 3 such that $\gamma = 1/t^a$, $\lambda = 1/t^b$, $\zeta = 1/t^c$, where t is the iteration and $a > b > c$ so that they fall within the interval $(0, 1)$. For the simulation, we took $a = 0.8$, $b = 0.7$ and $c = 0.6$. Fig. 4.9 shows that the performance of singletons improves with increasing distance. Beyond 30 meters, the rate of improvement is smaller, and the data rate becomes saturated at 1.3 bps/Hz/coalition. On the other hand, the performance of coalitions remains almost the same regardless of the distance at 1 bps/Hz/coalition. We observe that the performance of singletons is better than that of a coalition right after 10 meters of distance. This is in contrast to the earlier case with feedback shown in Fig. 4.4 for which the overtaking point is 15 meters. Also, the overall performances of both the singletons and coalitions using regret learning shown in Fig. 4.9 are much lower than their performance when a feedback channel is available as shown in Fig. 4.4. This is because the Algorithm 3 selects the beamformer from the codebook in a probabilistic manner. In contrast, when the feedback is available, the feedback information is exploited by PU²RC

to search for the best beamformer in the codebook. Nevertheless, we can observe that it is more profitable to form a coalition when the SBSs are close to each other.

Figure 4.9: Average data rate of singletons and coalition when the distance between two SBSs is varied using regret based learning.

Similarly, we evaluate the impact of the size of the codebook on the performance. In the presence of feedback, the size of the codebook is given by 2^B , where B is the number of feedback bits. However, for the regret based learning case, the size of the codebook is arbitrarily pre-defined, and each element in the codebook can be represented by B bits. Thus more number of representing bits means a larger codebook. We consider 2 SBSs separated at 20 meters. As expected from Fig. 4.9, the singletons do better than the coalition when the SBSs are separated at 20 meters. However, we notice that there is no significant change in the performances of the singletons and the coalition, which achieve 1.2 bps/Hz/coalition and 1 bps/Hz/coalition, respectively. This highlights the fact that changing the size of the action space of the coalitions for the regret based learning in Algorithm 3 does not lead to performance improvement. This is in contrast to Fig. 4.5 where there is considerable improvement in performance when more feedback bits are available. The reason for this result is that, despite having a larger codebook, the probabilistically selected beamformers may not always match with the channel. This

is different from the PU²RC method when the feedback channel is available and an exhaustive search is performed for matching beamformer over the codebook.

4.5.7 Probability of Cooperation, Average Number of Coalitions, and Average Coalition Size

Lastly, we evaluate how accurately the a priori estimation of the average number of coalitions compare with actual results obtained from the merge-only algorithm. Fig. 4.10 shows that the probability of cooperation as given by the third order approximation fits relatively well with that of the merge-only algorithm. Similarly, Fig. 4.11 shows that the average number of coalitions obtained from the merge-only algorithm is in reasonable agreement with those of the third order approximation as well as the Chinese restaurant process. For a large number of SBSs, the results of the merge-only algorithm also match the exact predicted value as given by the ratio of Bell numbers. Finally, Fig. 4.12 shows the average coalition size as the number of SBSs increases. We observe that the curve given by the third order approximation and the Chinese restaurant process fits reasonably well with that obtained from the merge-only algorithm. The curve given by the exact estimation also follows the data curve obtained from the merge-only algorithm as the number of SBSs becomes very large.

4.6 Chapter Summary

This chapter has studied coordinated beamforming in the context of small cell networks. The cases when the feedback channel is present and absent have been considered. When the feedback is present, the PU²RC method has been used, whereas when the feedback is absent, the regret based learning has been applied. We have formulated the coalition formation game. It has been shown that the merge-split algorithm could be unstable, while the split-only algorithm has exponential worst case complexity. As a result, the merge-only algorithm has been adopted which guarantees the stability at quadratic worst case complexity. The recursive core has been considered as the solution of coalition formation among SBSs. In addition, we have presented the analysis of coalition formation

Figure 4.10: Probability of cooperation between any two SBSs when the number of SBSs is varied.

to estimate the average number of coalitions and the average size of coalitions. Simulations have been performed to investigate the behavior of the merge-only algorithm and to verify the analysis.

4.7 Appendix

For the first identity, we have from the summation of the recurrence relation

$$\begin{aligned}
\sum_{k=1}^n kS(n, k) &= \sum_{k=1}^n S(n+1, k) - \sum_{k=1}^n S(n, k-1) \\
&= \sum_{k=1}^{n+1} S(n+1, k) - S(n+1, n+1) - \sum_{k=1}^{n+1} S(n, k-1) + S(n, n) \\
&= \sum_{k=1}^{n+1} S(n+1, k) - \sum_{k=1}^{n+1} S(n, k-1) \quad \because S(n, n) = S(n+1, n+1) = 1 \\
\therefore \sum_{k=1}^n kS(n, k) &= \mathcal{B}_{n+1} - \mathcal{B}_n
\end{aligned} \tag{4.19}$$

Figure 4.11: Average number of coalitions when the number of SBSs is varied.

For the second identity, we have

$$\begin{aligned}
\sum_{k=1}^n k^2 S(n, k) &= \sum_{k=1}^n k S(n+1, k) - \sum_{k=1}^n k S(n, k-1) \\
&= \sum_{k=1}^{n+1} k S(n+1, k) - (n+1) S(n+1, n+1) - \sum_{k=1}^n k S(n, k-1) \\
&= (\mathcal{B}_{n+2} - \mathcal{B}_{n+1}) - (n+1) \cdot 1 - \sum_{k=1}^n k S(n, k-1)
\end{aligned} \tag{4.20}$$

Here the second summation gives

$$\begin{aligned}
\sum_{k=1}^n k S(n, k-1) &= \sum_{k=1}^{n-1} (k+1) S(n, k) \\
&= \sum_{k=1}^{n-1} k S(n, k) + \sum_{k=1}^{n-1} S(n, k) \\
&= \sum_{k=1}^n k S(n, k) - n S(n, n) + \sum_{k=1}^n S(n, k) - S(n, n) \\
&= (\mathcal{B}_{n+1} - \mathcal{B}_n) - n + \mathcal{B}_n - 1 \\
&= \mathcal{B}_{n+1} - (n+1)
\end{aligned} \tag{4.21}$$

Figure 4.12: Average coalition size when the number of SBSs is varied.

Thus putting this value in the previous summation, we get

$$\begin{aligned}
 \sum_{k=1}^n k^2 S(n, k) &= (\mathcal{B}_{n+2} - \mathcal{B}_{n+1}) - (n+1) - [\mathcal{B}_{n+1} - (n+1)] \\
 \therefore \sum_{k=1}^n k^2 S(n, k) &= \mathcal{B}_{n+2} - 2\mathcal{B}_{n+1}
 \end{aligned} \tag{4.22}$$

Chapter 5

Access Control via Coalitional Power Game

The techniques of game theory has been successfully utilized to solve various issues pertaining to communications, such as distributed resource allocation, routing, congestion control, power control, and spectrum sharing [210]. In the noncooperative game theory, the Nash equilibrium is often taken as the solution concept. It is assumed that the players in such noncooperative game seek to maximize their own self interest and do not cooperate with other players by forming coalitions or alliances. However, it is of interest to know what would happen if we allow such cooperation and coalitions to form, since the cooperation can have potential benefits. This chapter attempts to make such an analysis where a coalition form game is constructed from an underlying noncooperative game. Also in the process, we hope to find a novel approach towards access control. Some good surveys of such cooperative games are found in [211, 212] in the context of communication networks.

Our investigation is motivated by a situation in smallcell networks. In smallcell networks, low power and low cost base stations, also known as femto-access points, are overlaid on the existing cellular network technology (2G, 3G, WiMAX) [5]. This femto-access point is connected with IP backhaul through a local broadband connection, such as DSL, cable, or fiber. For such smallcells it is natural to ask what its access scheme should be. One possibility is to have a closed access scheme where only few users can

connect to the smallcell. Another option is to have an open access scheme where all customers of the operator are allowed to make use of any smallcell. Lastly, a hybrid access scheme is also possible, where a limited fraction of smallcell resource is available to all users. Related work in this area has been conducted by [213]-[217] via system level simulations of different access schemes. A more analytical approach has been taken by [218, 28] based on the transmitter-receiver geometry.

In this chapter we take a new analytical approach toward access control. More specifically, an uplink orthogonal frequency division multiple access (OFDMA) communication system is considered, where a set of user equipments (UE) tries to link with any two common access points (i.e., either the mobile base station or the femto-access point). It is more convenient to study such user association in terms of uplink rather than downlink. This is because the downlink user association in heterogeneous network is much more challenging due to transmit power disparity between the transmitting base stations (e.g., macro base station and femto base station). However, for the case of the uplink, it is safe to assume that the uplink transmit power of all mobile user equipments are the same. We first define an underlying noncooperative power game. It then becomes natural to divide the set of uplink UEs into two coalitions, depending on which access point they want to connect to. By “cooperation between transmitters of the same coalition”, we mean that they refrain from transmitting in the same frequency band as other UEs, thus reducing the interference they can cause to each other. Here we ask a question: which coalitions are likely to form, thus allowing us to know which UEs will cooperate with which UE? To get to the answer, we define an appropriate value function for a given coalition. We resort to the standard method of defining the value function of a coalitional game based on max-min method. In the process, we investigate the optimal jamming strategy of complementary coalition. We then show that the value function that we have defined cannot be super-additive. The non-super-additivity of the value function reflects the physical situation where, if the UEs of the same coalition is located far away from a given receiver, then the orthogonal channel allocation may not be the best strategy for them in an interference-limited environment. Instead, it may make more sense if they break away from their current coalition and join another coalition represented by a receiver closest to it. Mathematically, this also means that we have to dismiss some of

the standard cooperative game solutions, such as the core, which rely on super-additivity of the value function. Thus we resort to using Shapley value as the preferred solution concept. This Shapley value is then used to examine the formation of different coalition structures via merging and splitting of the UEs from one coalition to another. The stable solutions for such coalition formation process is considered as the required solution of an access control problem.

The rest of the chapter is divided into following sections: Section II gives the system model and assumptions. Section III makes the analysis of the coalition form game. Section IV shows how Shapley values can be used to examine coalition structure. Lastly section V gives a numerical example, while section VI concludes the chapter.

5.1 System Model and Assumptions

Without any loss of generality, consider a system with two access points $\{s, t\}$ (which in the context of smallcell network would represent a macro base station and a femto-access point) that operates over a common set of $\mathcal{L} = \{1, \dots, L\}$ orthogonal frequency bands. Let there be $\mathcal{N} = \{1, \dots, N\}$, where $N \geq 2$, uplink UEs trying to connect with any one of the given access points. Each of the UEs can use any of the frequency bands to transmit its data. Let g_{is}^k denote the channel gain between UE i and access point s on subchannel k . Likewise, g_{it}^k denotes the channel gain between UE i and access point t on subchannel k . The channel gain may depend on distance attenuation, antenna gain, and the random fading effects. Also, let n be the variance of additive white Gaussian noise at each access point, which for simplicity we assume to be constant over all the subchannels. Let us designate the transmit power of a UE $i \in \mathcal{N}$ on a k -th subchannel to be p_i^k , such that it follows the constraints $\sum_{k \in \mathcal{L}} p_i^k \leq \bar{p}_i$ and $0 \leq p_i^k$, where $\bar{p}_i$ is the total transmit power of i . Thus, let $\mathcal{P}_i = \{(p_i^1, \dots, p_i^L) : \sum_{k \in \mathcal{L}} p_i^k \leq \bar{p}_i, 0 \leq p_i^k\}$ be the set of all feasible power vectors of UE $i \in \mathcal{N}$.

Let $\mathcal{S}$ and $\mathcal{T}$ denote the two sets of UEs that have been admitted by access points s and t , respectively, such that $\mathcal{S} \cap \mathcal{T} = \emptyset$ and $\mathcal{S} \cup \mathcal{T} = \mathcal{N}$. We shall call these sets *coalitions*. The UEs have the ability to defect from one coalition to another.

Assumption 6. *Once the UEs are admitted into a coalition, they do not interfere with the transmission of another UE in the same coalition.*

Since we deal with the uplink UEs gaining access to an access point, when a user is associated with a particular access point, the access point assigns certain resource block to the UE. It should be the case that these resource blocks assigned to different UEs associated with a particular access point be non-overlapping. This assumption reflects this fact. That is, as a gesture of cooperation, each UEs in a given coalition transmit over non-overlapping subchannels. In other words, in a given coalition, each subchannel is assigned to only a single UE belonging to that coalition.

Thus, the signal-to-interference-plus-noise ratio (SINR) at access point s is $\frac{g_{is}^k p_i^k}{n + \sum_{j \in \mathcal{T}} g_{js}^k p_j^k}$. Similarly, the SINR at access point t is $\frac{g_{it}^k p_i^k}{n + \sum_{j \in \mathcal{S}} g_{jt}^k p_j^k}$. The utility of each UE $i \in \mathcal{N}$ is taken to be the maximum rate at which it can transfer its data to its respective access point, $u_i = \sum_{k \in \mathcal{L}} \log(1 + \frac{g_{is}^k p_i^k}{n + \sum_{j \in \mathcal{T}} g_{js}^k p_j^k})$, if $i \in \mathcal{S}$, or $u_i = \sum_{k \in \mathcal{L}} \log(1 + \frac{g_{it}^k p_i^k}{n + \sum_{j \in \mathcal{S}} g_{jt}^k p_j^k})$, if $i \in \mathcal{T}$. Finally, let $\mathcal{U} = (u_1, \dots, u_N)$ denote the utility vector of the system.

During a noncooperative game, the utility of a UE is calculated such that except for the given UE, all other UEs are assumed to be in the complementary coalition. That is when UEs do not cooperate $u_i(\mathcal{S}) = \nu(\mathcal{S} = \{i\})$ and $u_i(\mathcal{T}) = \nu(\mathcal{T} = \{i\})$, where $\nu(\cdot)$ is the value function of coalitional game, as defined in the next section. Thus the actions of a UE is confined to just two moves, either join $\mathcal{S}$ or join $\mathcal{T}$. A noncooperative solution would be to join the coalition that gives the highest payoff. Thus $\mathcal{S} = \{i | u_i(\mathcal{S}) > u_i(\mathcal{T})\}$, and similarly $\mathcal{T} = \{i | u_i(\mathcal{T}) > u_i(\mathcal{S})\}$. In such a case two UEs may superficially belong to the same coalition, but there is no provision for cooperation. Thus the solution is overtly pessimistic. A better solution can be obtained by allowing cooperation between UEs.

For e.g., from our numerical example section, we see that $u_1(\mathcal{S}) = 69.76$ and $u_1(\mathcal{T}) = 0.149$; while $u_2(\mathcal{S}) = 1.55$ and $u_2(\mathcal{T}) = 40.51$; and $u_3(\mathcal{S}) = 0.45$ and $u_3(\mathcal{T}) = 14.51$. Thus by noncooperative method, $\mathcal{S} = \{1\}$ and $\mathcal{T} = \{2, 3\}$ and the payoff obtained by each players is $u_1^* = 69.76$, $u_2^* = 40.51$, and $u_3^* = 14.24$. This payoff is much inferior compared to stable cooperative solution given by the same coalition structure $\omega_7 = (\{1\}\{2, 3\})$: $\phi_1(\omega_7)^* = 69.76$, $\phi_2(\omega_7)^* = 73.61$, and $\phi_3(\omega_7)^* = 47.34$.

5.2 Coalitional Form

The coalitional form of an n -person game is given by the pair $(\mathcal{N}, \nu)$, where $\mathcal{N}$ is the set of players (which for us is the set of UEs) and ν is a real-valued function, called the characteristic function of the game, defined on the set, 2^N , of all coalitions (subsets of N) satisfying i) $\nu(\emptyset) = 0$, and ii) (super-additivity) if $\mathcal{S}_1$ and $\mathcal{S}_2$ are any disjoint coalitions ($\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$), then $\nu(\mathcal{S}_1) + \nu(\mathcal{S}_2) \leq \nu(\mathcal{S}_1 \cup \mathcal{S}_2)$ [219]. The quantity $\nu(\mathcal{S})$ can be considered as the value, or worth, that a coalition $\mathcal{S}$ is guaranteed to obtain without external assistance, if its members act together as a unit. Since we are dealing with an uplink case where the access points have the authority over the UEs, we assume that this value can be arbitrarily divided by the access point among the UEs belonging to its coalition, thus making it a game with transferable utility. It should be noted that while the super-additivity of the value function is a standard assumption, some of the solution concepts of coalitional game (such as Shapley value) do not rely on this assumption.

Given a strategic form of an n -person non-cooperative power game, $(\mathcal{N}, \mathcal{P}, \mathcal{U})$, transforming the game from strategic form to coalitional form involves specifying the value, $\nu(\mathcal{S})$, for each coalition $\mathcal{S} \in 2^N$. The usual way to assign a characteristic function to a strategic form game is to define $\nu(\mathcal{S})$ for each $\mathcal{S} \in 2^N$ as the value of the two person zero-sum game obtained when the coalition $\mathcal{S}$ acts as one player and the complementary coalition, $\mathcal{N} \setminus \mathcal{S} = \mathcal{T}$, acts as the other player, where the payoff to $\mathcal{S}$ is the sum of the payoffs to the players in $\mathcal{S}$: $\sum_{i \in \mathcal{S}} u_i(p_1, \dots, p_N)$ [219]. Thus

$$\begin{aligned} \nu(\mathcal{S}) &= \max_{p_i \in \mathcal{S}} \min_{p_j \in \mathcal{T}} \sum_{i \in \mathcal{S}} u_i(p_1, \dots, p_N) \\ \text{s.t. } &\sum_{k \in \mathcal{L}} p_i^k \leq \bar{p}_i, \quad 0 \leq p_i^k, \quad \forall i \in \mathcal{N} \end{aligned} \tag{5.1}$$

where the players in $\mathcal{S}$ jointly choose p_i for $i \in \mathcal{S}$, and players in $\mathcal{T}$ choose p_j for $j \in \mathcal{T}$. In this chapter, we focus on obtaining this value function from an underlying non-cooperative game.

5.2.1 Value Function

Since we first need to find the minimum and then the maximum of the utility sum, we can divide the problem of obtaining the value function into two logical steps.

5.2.1.1 Minimization

We first concern ourself with the minimization problem of the form

$$\begin{aligned} & \min_{p_j \in \mathcal{T}} \sum_{i \in \mathcal{S}} u_i \\ \text{s.t. } & \sum_{k \in \mathcal{L}} p_j^k \leq \bar{p}_j, \quad 0 \leq p_j^k, \quad \forall j \in \mathcal{T}. \end{aligned} \quad (5.2)$$

Using the Lagrange multiplier technique (see Appendix A), we find the unique optimal strategy of the complementary coalition $\mathcal{T}$ to be such that the noise-plus-interference is maintained as some constant depending on the subchannels of the coalition $\mathcal{S}$. That is, for every $j \in \mathcal{T}$,

$$\sum_{j \in \mathcal{T}} g_{js}^k p_j^k + n = c_T^k, \quad \forall k \in \mathcal{L}, \quad (5.3)$$

where c_T^k is a constant that depends on the size of complementary coalition $\mathcal{T}$.

Note the striking similarity with the waterfilling equation. Also notice the lack of g_{is}^k terms in the solution, where $i \in \mathcal{S}$. This is because this is also the required condition for the uniqueness of the solution. If uniqueness of the solution is not desired, then the solution will also depend on the channel gain of the users in coalition $\mathcal{S}$. It should be noted that uniqueness is always a desired feature of a mathematical solution. Here, while the uniqueness condition is not required, it is nonetheless much desired since there will be no ambiguity in the implementation of the equation. Thus, the (5.3) has been derived by imposing the uniqueness requirement for the solution.

5.2.1.2 Maximization

We now try to solve the maximization problem in the form

$$\begin{aligned} & \max_{p_i \in \mathcal{S}} \left[\min_{p_j \in \mathcal{T}} \sum_{i \in \mathcal{S}} u_i \right] \\ \text{s.t. } & \sum_{k \in \mathcal{L}} p_i^k \leq \bar{p}_i, \quad 0 \leq p_i^k, \quad \forall i \in \mathcal{S}. \end{aligned} \quad (5.4)$$

From our previous result we know that the noise-plus-interference due to UEs in $\mathcal{T}$ is constant over all subchannels. Also, since a subchannel is assigned to a single UE in

$\mathcal{S}$, the strategy of i -th UE in $\mathcal{S}$ would be the same as that of waterfilling strategy, i.e.,

$$p_i^k = \left[\frac{1}{\lambda_i} - \frac{c_T^k}{g_{is}^k} \right]^+, \quad \forall i \in \mathcal{S}, \quad \text{where } [x]^+ = \max(0, x). \quad (5.5)$$

Thus we can write the associated value of the coalition as

$$\nu(\mathcal{S}) = \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{A}_i} \log(1 + g_{is}^k p_i^k / c_T^k), \quad (5.6)$$

where $\mathcal{A}_i \cap \mathcal{A}_j = \emptyset$, for all $i, j \in \mathcal{S}$, and c_T^k is some constant associated with the complementary coalition $\mathcal{T}$. Here $c_T^k \geq n$, with equality only when $\mathcal{T} = \emptyset$.

5.2.2 Jamming Strategy

We have seen that the optimal strategy of a complementary coalition should satisfy (5.3). To achieve this condition, a successive waterfilling method can be adopted, where each UE takes turn allocating power one after another and does not change its transmit power once it is already allocated.

Analysis: Let $\mathcal{T} = \{1, 2\}$ be two UEs in the complementary coalition. Then according to the successive waterfilling scheme, let UE 1 allocate power first, followed by UE 2. When UE 1 does waterfilling, it must satisfy $n + g_{1s}^k p_1^k = c_1^k$, where $c_1^k = 2g_{1s}^k / \lambda_1$. Next, UE 2 does the waterfilling. So, $n + g_{1s}^k p_1^k + g_{2s}^k p_2^k = c_{1,2}^k$ and $p_2^k = \frac{1}{g_{2s}^k} (c_{1,2}^k - (n + g_{1s}^k p_1^k)) = \frac{1}{g_{2s}^k} (c_{1,2}^k - c_1^k)$. Since $p_2^k \geq 0$, we conclude that $c_{1,2}^k \geq c_1^k$. Thus adding more UEs in the complementary coalition increases the value of c_T^k , i.e. $c_{T+1}^k \geq c_T^k$, where T is the number of UEs in $\mathcal{T}$.

More generally, let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two complementary coalitions (not necessarily disjoint). If they separately attempt to jam the communication of coalition $\mathcal{S}$, then they both must separately satisfy the conditions $n + \sum_{i \in \mathcal{T}_1} g_{is}^k p_i^k = c_{\mathcal{T}_1}^k$ and $n + \sum_{i \in \mathcal{T}_2} g_{is}^k p_i^k = c_{\mathcal{T}_2}^k$. When these two coalitions combine to form a joint coalition, $\mathcal{T}_1 \cup \mathcal{T}_2$, then the joint

coalition need to satisfy

$$\begin{aligned}
c_{T_1 \cup T_2}^k &= n + \sum_{i \in T_1 \cup T_2} g_{is}^k p_i^k \\
\text{or, } c_{T_1 \cup T_2}^k &= n + \sum_{i \in T_1} g_{is}^k p_i^k + \sum_{i \in T_2} g_{is}^k p_i^k - \sum_{i \in T_1 \cap T_2} g_{is}^k p_i^k \\
\text{or, } c_{T_1 \cup T_2}^k &= (n + \sum_{i \in T_1} g_{is}^k p_i^k) + (n + \sum_{i \in T_2} g_{is}^k p_i^k) \\
&\quad - (n + \sum_{i \in T_1 \cap T_2} g_{is}^k p_i^k) \\
\therefore c_{T_1 \cup T_2}^k &= c_{T_1}^k + c_{T_2}^k - c_{T_1 \cap T_2}^k.
\end{aligned} \tag{5.7}$$

Thus if $T_1 \cap T_2 = \emptyset$, then $c_{T_1 \cup T_2}^k = c_{T_1}^k + c_{T_2}^k - n$. Also since $c_T^k \geq n$, with equality only when $T = \emptyset$, it is obvious that $c_{T_1 \cup T_2}^k - c_{T_1}^k = c_{T_2}^k - n \geq 0$; thus $c_{T_1 \cup T_2}^k \geq c_{T_1}^k$.

5.2.3 Non-Super-Additivity of the Value Function

Here we evaluate the property of the value function as we have defined in (5.6). It is easily seen that the first condition requiring $\nu(\emptyset) = 0$ readily holds.

Now the value function is said to be super-additive if $\mathcal{S}_1$ and $\mathcal{S}_2$ are any disjoint coalitions ($\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$) and $\nu(\mathcal{S}_1) + \nu(\mathcal{S}_2) \leq \nu(\mathcal{S}_1 \cup \mathcal{S}_2)$. One consequence of super-additivity is that all the UEs would prefer to form a single grand coalition. In our context, this means that all the UEs in set $\mathcal{N}$ will prefer to transmit to a same access point. This is because, super-additivity of the value function guarantees that the value of two disjoint coalition is at least as good when they work together as when they work apart. But this physically does not make sense, because the preference for a particular access point is governed in part by the distance relationship between the UEs and the access points. If the two access points are separated by a considerable distance, then UEs that cluster around either of the access point will cause very little interference to the other coalition and will not have any incentive to defect from its current coalition. Thus a UE which is nearer to an access point s would prefer to link to that access point than to the other access point t . Thus the value function as defined by (5.6) cannot be super-additive for every possible cases.

However it is a different matter to investigate for which cases the value function is super-additive.

5.2.4 Shapley Value

After finding the value of a coalition, we require a method of dividing the value among the players in that coalition. However, since the value cannot be super-additive for all cases, solutions like the core that rely on the super-additivity and characterizes the stability of the grand coalition is not implementable. Thus we select the Shapley value, which does not rely on super-additivity assumption, as the solution of the cooperative game [219]. A preferable property about Shapley value is that it can be used as a solution of a smaller game with only a subset of players taken from the grand coalition [220].

The Shapley value function, ϕ , is a function that assigns to each possible characteristic function of an n -person game, ν , an n -tuple, $\phi(\nu) = (\phi_1(\nu), \phi_2(\nu), \dots, \phi_N(\nu))$ of real numbers. The $\phi_i(\nu)$ represents the worth or value of player i in the game with characteristic function ν and is defined by the following axioms of fairness:

1. *Efficiency.* $\sum_{i \in \mathcal{N}} \phi_i(\nu) = \nu(\mathcal{N})$.
2. *Symmetry.* If i and j are such that $\nu(\mathcal{S} \cup \{i\}) = \nu(\mathcal{S} \cup \{j\})$ for every coalition $\mathcal{S}$ not containing i and j , then $\phi_i(\nu) = \phi_j(\nu)$.
3. *Dummy Axiom.* If i is such that $\nu(\mathcal{S}) = \nu(\mathcal{S} \cup \{i\})$ for every coalition $\mathcal{S}$ not containing i , then $\phi_i(\nu) = 0$.
4. *Additivity.* If u and v are characteristic functions, then $\phi(u + v) = \phi(u) + \phi(v)$.

There exists a unique function that satisfies all these fairness axioms, and that function is given by:

$$\phi_i(\nu) = \sum_{\substack{\mathcal{S} \subset \mathcal{N} \\ i \in \mathcal{S}}} \frac{(|\mathcal{S}| - 1)!(n - |\mathcal{S}|)!}{n!} [\nu(\mathcal{S}) - \nu(\mathcal{S} - \{i\})]. \quad (5.8)$$

This gives the average marginal contribution made by i when it joins a random coalition $\mathcal{S}$ and take this value as the “fair” payoff allocation among the players within a coalition.

5.3 Coalition Formation

Since the value function is not always super-additive, a grand coalition may not always yield an optimal payoff. Let the ordered pair $\omega = (\mathcal{S}, \mathcal{T})$ define the coalition structure which is a bi-partition of the set $\mathcal{N}$. Let $\phi(\omega)$ denote the Shapley values of the players associated with the given coalition structure ω . Given a current set of coalitions, the players can join or split from any coalition in order to increase their individual payoffs. Thus a player i belonging to $\mathcal{T}$ would split from $\mathcal{T}$ and join $\mathcal{S}$ if it leads to a strict increase in its payoff, while at the same time, does not lead to any decrease in the payoff of players already in $\mathcal{S}$:

$$\begin{aligned} \phi_i(\mathcal{S} \cup i) &> \phi_i(\mathcal{T}), \quad i \in \mathcal{T}, \\ \text{and } \phi_j(\mathcal{S} \cup i) &\geq \phi_j(\mathcal{S}), \quad \forall j \in \mathcal{S}. \end{aligned} \tag{5.9}$$

A given coalition structure ω^* is *stable* if it shows both the internal as well as external stability [221]. Internal stability implies that, given a coalition, no player in this coalition has any incentive to leave this coalition and act alone (non-cooperatively as a singleton), i.e., $\phi_i(\omega) \geq u_i^*$, where u_i^* is the utility at noncooperative solution. External stability implies that, in a given partition, no player can improve its payoff by switching its current coalition and join another one. The stable coalitions corresponds to the ergodic set of a Markov chain defined by the state space of all the coalition structures. The transition from one state to another takes place based on the merge/split decision taken by the players. The singleton ergodic set would give a set of absorbing states. Such stable coalitions are exactly the required solution to our access control problem.

5.4 Numerical Example

Consider a set of three UEs $\mathcal{N} = \{1, 2, 3\}$. We have considered the following parameters: $L = 12$, $\bar{p}_i = 10$ dBm for all UEs $i \in \mathcal{N}$, and $n = -90$ dBm. We assume that all the UEs and access points are located along the x-axis. As shown in the Fig. 5.1, let the access point s be at origin, and the access point t be located at 100 meters away from the origin. Let the three UEs be at 30, 90, and 120 meters away from the origin, respectively.

Figure 5.1: Layout of the user equipments and the base stations

Let the antenna gain be $g_A = 17.95$ dB. We let the random fading in each subchannel, $\tilde{g}_{is}^k$, be given by independent and identical Rayleigh random variable with average gain of -10 dB. Thus, we model the channel gain at each subchannel k by $g_A \tilde{g}_{is}^k / d_{is}^4$, where d is the distance between the UE i and access point s . Same thing goes for access point t . Finally, the subchannels are divided evenly among the UEs within a coalition.

An instance of this channel was simulated. We perform the power allocation according to equations (5.3) and (5.5) of Section 5.2.1. Successive waterfilling as discussed in Section 5.2.2 was used to implement equation (5.3) while the usual waterfilling was used to implement equation (5.5). This way the values of different possible coalitions were calculated. Based on these calculated values of the coalitions, Shapely value of each UE, which we take as the payoff of each UE under different possible coalition structures ω , was computed using equation (5.8), as given in Table 5.1.

The payoff matrix, as given in Table 5.1, is used to construct the transition graph from one coalition structure to another coalition structure based on the merge-split rule given in (5.9). Here the coalition structures ω are represented as the nodes or vertices of the transition graph. Any two nodes are connected to each other if there is a player in that coalition structure that can deviate from one coalition and join another coalition thus forming a new coalition structure. Such user initiated transitions can continue until the system reaches a coalition structure from which no such transition is possible. These are the absorbing states of the transition graph, and we take these absorbing states as the solution of access control problem.

We first note that looking at the coalitional values given in Table 5.1, $\nu(\mathcal{S})$ and $\nu(\mathcal{T})$, it becomes apparent that they are not super-additive. For instance, $\nu(\mathcal{S} = \{1\}) = 69.76$

and $\nu(\mathcal{S} = \{3\}) = 0.45$, while $\nu(\mathcal{S} = \{1, 3\}) = 53.52$, which is clearly less than the sum $\nu(\mathcal{S} = \{1\}) + \nu(\mathcal{S} = \{3\}) = 53.97$. Secondly, we notice that the coalition structures ω_2 , ω_3 , and ω_5 are immediately disqualified due to the negative payoff obtained by one of the players. That is, at least one of the players will always have an incentive to join a different coalition.

Using the remaining coalition structures, we can construct a transition diagram as given in Fig. 5.2 with the coalition structures as the nodes. The nodes are connected if any player has an incentive to merge or split from one coalition to another coalition as given by the merge-split rule in (5.9). For e.g., in $\omega_6 = (\{2\}\{1, 3\})$, we have $\phi(\omega_6) = (0.24, 1.55, 14.34)$, while in $\omega_4 = (\{1, 2\}\{3\})$ we have $\phi(\omega_4) = (70.37, 2.16, 14.24)$. Since UE 1 gets a higher payoff in ω_4 than in ω_6 without downgrading the payoff of UE 2, the UE 1 in ω_6 has an incentive to split from $\mathcal{T} = \{1, 3\}$ and join $\mathcal{S} = \{2\}$ to form ω_4 . Thus $(\{2\}\{1, 3\}) \rightarrow (\{1, 2\}\{3\})$.

From inspecting the graph, we see that ω_7 is the absorbing state. Thus this coalition structure is stable, and is the required solution for our access control problem. Thus in $\omega_7 = (\{1\}, \{2, 3\})$ the UE-1 is associated with receiver s , which is located at the origin, while UE-2 and UE-3 are associated with receiver t , which is located 100 meters away from the origin on the x-axis.

5.5 Practical Issues

The chapter is primarily concerned with general mathematical modeling of the access control problem and exploring theoretically possible scenarios, rather than focus on practical deployment algorithms. The main idea of this chapter is to take the absorbing states of a coalition structure transition graph as the solution of the user association problem. While this may seem a reasonable theoretical starting point, actually finding this absorbing state can be difficult. If a central controller knows all the possible coalition structures, transitions, and payoff, then it is possible to enumerate all the states, construct a transition graph, and determine the stable state. While we have explicitly constructed such a transition graph using the payoff matrix in our example in Section 5.4 as in a centralized manner, in practice the payoffs corresponding to all possible coalition structures are not

Table 5.1: Payoff Matrix for Three UE Coalition Game

Coalition Structure	$\nu(\mathcal{S})$	$\nu(\mathcal{T})$	$\phi_1(\omega)$	$\phi_2(\omega)$	$\phi_3(\omega)$
$\omega_1 = (\{1, 2, 3\}, \emptyset)$	177.94	0	102.92	42.53	32.47
$\omega_2 = (\{2, 3\}, \{1\})$	0.96	0.149	0.149	1.03	-0.07
$\omega_3 = (\{1, 3\}, \{2\})$	53.52	40.51	61.41	40.51	-7.89
$\omega_4 = (\{1, 2\}, \{3\})$	72.54	14.24	70.37	2.16	14.24
$\omega_5 = (\{3\}, \{1, 2\})$	0.45	22.21	-9.07	31.28	0.45
$\omega_6 = (\{2\}, \{1, 3\})$	1.55	14.58	0.24	1.55	14.34
$\omega_7 = (\{1\}, \{2, 3\})^*$	69.76	120.96	69.76	73.61	47.34
$\omega_8 = (\emptyset, \{1, 2, 3\})$	0	256.43	42.21	115.58	98.63

Figure 5.2: Markov Chain of Coalition Structures

known a-priori. Thus it is impractical to assume that we can construct such a graph for all cases. One possibility is to start from an arbitrary coalition structure and allow users to freely associate or disassociate themselves with any base station according to the merge-split rule given in (5.9). If this process is performed for a sufficient time, then it will converge to an absorbing state. However, it is an open question if such a process is guaranteed to converge to an absorbing state.

Currently, much of the practical user association is based on received-signal-strength (RSS). Our analysis in this chapter helps to create possibly new ways of user associations, by changing the definition of the utility function, which here has been taken to be the user capacity, or other payoff functions apart from Shapely value. Hopefully, the RSS method can be subsumed under this general theoretical outlook.

5.6 Chapter Summary

In this chapter, we have modeled the uplink admission control problem of OFDMA smallcell network as a noncooperative game, which we mapped into coalition form game, using max-min approach. We showed that the resulting value function cannot be super-additive for all cases. The concept of Shapley value was used to allocate payoff to each user. From this we obtained the stable coalition as solutions to our admission control problem.

5.7 Appendix

The Lagrangian of the problem is

$$\Lambda = \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{L}} \log \left(1 + \frac{g_{is}^k p_i^k}{n + \sum_{j \in \mathcal{T}} g_{js}^k p_j^k} \right) + \sum_{j \in \mathcal{T}} \lambda_j \left(\sum_{k \in \mathcal{L}} p_j^k - \bar{p}_j \right),$$

and the first order optimality condition is $\nabla_{p_j} \Lambda = 0$ for all $j \in \mathcal{T}$. That is,

$$\frac{\partial \Lambda}{\partial p_j^k} = 0 = \sum_{i \in \mathcal{S}} \sum_{k' \in \mathcal{L}} \frac{\partial}{\partial p_j^k} \log \left(1 + \frac{g_{is}^{k'} p_i^{k'}}{n + \sum_{j' \in \mathcal{T}} g_{j's}^{k'} p_{j'}^{k'}} \right) + \lambda_j.$$

Here

$$\frac{\partial}{\partial p_j^k} \log \left(1 + \frac{g_{is}^{k'} p_i^{k'}}{n + \sum_{j' \in \mathcal{T}} g_{j's}^{k'} p_{j'}^{k'}} \right) = 0 \quad \text{if } k' \neq k,$$

thus the equation simplifies to

$$\sum_{i \in \mathcal{S}} \frac{\partial}{\partial p_j^k} \log \left(1 + \frac{g_{is}^k p_i^k}{n + \sum_{j' \in \mathcal{T}} g_{j's}^k p_{j'}^k} \right) + \lambda_j = 0.$$

Now, in accordance to our assumption, a given subchannel k is assigned to only one of $i \in \mathcal{S}$. This means that $p_{i'}^k = 0$ for all other i' different from i (i.e. $i' \neq i$). Thus the expression further simplifies to

$$\frac{\partial}{\partial p_j^k} \log \left(1 + \frac{g_{is}^k p_i^k}{n + \sum_{j' \in \mathcal{T}} g_{j's}^k p_{j'}^k} \right) + \lambda_j = 0.$$

Putting $x = n + \sum_{j' \in \mathcal{T}} g_{j's}^k p_{j'}^k$ and taking the partial derivative with respect to p_j^k , we obtain the following quadratic equation:

$$\begin{aligned} \frac{g_{is}^k g_{js}^k p_i^k}{x^2 \left(1 + \frac{g_{is}^k p_i^k}{x} \right)} &= \lambda_j \\ \text{or, } x^2 + g_{is}^k p_i^k x - \frac{g_{is}^k g_{js}^k p_i^k}{\lambda_j} &= 0. \end{aligned}$$

Solving the quadratic equation for the unique value of x , we obtain $x = -g_{is}^k p_i^k / 2$ and the discriminant as zero, i.e.,

$$\begin{aligned} (g_{is}^k p_i^k)^2 + 4 \frac{g_{is}^k g_{js}^k p_i^k}{\lambda_j} &= 0 \\ \text{or, } g_{is}^k p_i^k \left(g_{is}^k p_i^k + \frac{4g_{js}^k}{\lambda_j} \right) &= 0. \end{aligned}$$

Since $g_{is}^k p_i^k \neq 0$, we have $g_{is}^k p_i^k + 4g_{js}^k / \lambda_j = 0$, i.e., $g_{is}^k p_i^k = -4g_{js}^k / \lambda_j$. Substituting this in the solution for x , we get $x = 2g_{js}^k / \lambda_j$. Putting back the expression for x , we obtain our desired result,

$$n + \sum_{j' \in \mathcal{T}} g_{j's}^k p_{j'}^k = 2g_{js}^k / \lambda_j. \quad (5.10)$$

We should note that for a given subchannel k , the (5.10) should be valid for every $j \in \mathcal{T}$. Thus $2g_{js}^k / \lambda_j = c_T^k$ is a constant for given k regardless of $j \in \mathcal{T}$. Here $c_T^k \geq n$, with equality only when $\mathcal{T} = \emptyset$.

Chapter 6

Summary and Future Works

6.1 Summary of Contributions

The research contributions presented in this thesis can be summarized as follows:

- *Chapter 3:* This chapter addressed the problem of allocating downlink transmit power over OFDMA cellular networks comprising of macrocells underlaid with low power, small coverage small cells. Stackelberg game was used to model the hierarchical competition between the macrocell base stations and the smallcell base stations. The game was presented as a mathematical program with equilibrium constraint (MPEC). Analysis of this problem has shown that under high interference condition, the leader prefers not to transmit in the same subchannel as the followers. Analysis is also performed for the case when there is a QoS constraint on the macrocell. Algorithms were provided for the power allocation and for the Stackelberg equilibrium. A sufficient condition was given for its convergence. Numerical simulations were used to compare the cases with and without hierarchy in the game.
- *Chapter 4:* This chapter has considered clustering methods for coordinated beamforming in the context of small cell networks. Two different methods for beamformer selection is presented for the case when the feedback is present and when

the feedback is temporarily absent. A coalitional game in partition form with non-transferable utility is formulated to cluster the base stations for joint beamforming. Three coalition formation approaches are considered. It has been shown that the merge-split algorithm could be unstable, while split-only method is shown to have exponential worst case complexity. As a remedy, the merge-only algorithm has been adopted which guarantees the stability at a quadratic worst case complexity. The result of the merge-only algorithm is shown to be an outcome of a recursive game. In addition, we give the analytical formulas to estimate the average number of coalitions and the average size of coalitions. Simulations have been performed to investigate the behavior of the merge-only algorithm and to verify the analysis.

- *Chapter 5:* In this chapter, we modeled the uplink admission control problem of OFDMA femtocell network as a noncooperative game, which we mapped into coalition form game, using max-min approach. A suitable payoff division function was used to allocate payoff to each user. From this, we construct a Markov model whose absorbing state represents the stable coalition which is considered as the solutions to the admission control problem.

6.2 Future Works

Some of the issues which can be addressed in the future research are as follows:

6.2.1 Imperfect Sensing of the Femtocells

In Chapter 3, we assumed that the macrocell user is able to perfectly sense the transmission of all the femtocells within the macrocell. In practice, the macrocell user can have only limited sensing ability, thus limiting the number of femtocells that the macrocell user can sense around its neighborhood. Intuitively, this local sensing effect should lead to a global performance improvement, since a distant femtocell that does not cause any interference to the macrocell user can also exploit the spectrum “holes” created by the macrocell. We can examine how this imperfection will affect the system performance.

6.2.2 Paradox of All Leader Game

In Chapter 3, we assumed that some of the base stations are leaders, while some are followers. However, paradoxically, if every player becomes a leader, then the game will degenerate into a Nash game, negating any network benefit that leader-follower game might have incurred. Thus instead of letting every player becomes the leader, we might consider allowing every player to be foresighted in their decision making by considering the interference measurement in their decision making process. It is expected that such foresighted decision making by all the players will yield a performance better than both Nash equilibrium and the Stackelberg equilibrium.

6.2.3 Extension to MIMO-OFDM

In Chapter 4, we considered OFDMA system where each transmitter and receiver has only single antenna. The Stackelberg game theoretic analysis can be extended to the case for MIMO-OFDM. Power allocation in a MIMO system is complicated by the fact that it is accompanied by beamforming issues. The power allocation method will depend on the beamforming criteria that has been chosen. For instance, with full channel state knowledge at both the receiver and the transmitter, the channel capacity maximizing beamforming technique involves performing the singular value decomposition of the channel matrix, with the singular values being the allocated power. Whether the linear interference model so crucial for much of the theoretical derivation in this chapter still holds for MIMO-OFDM case is an open question.

6.2.4 Exploiting the Analytical Formulas for Coalition Formation

In Chapter 4, we derived the analytical formulas for the average number of coalitions and the average size of coalitions that can form during coalition formation process. This information can be combined with stochastic geometric techniques and exploited to make apriori estimate of the amount of resources that needs to be allocated (i.e., resource scheduling problem). It can also be used to obtain analytical performance measure for joint coalition formation and resource allocation process.

6.2.5 Coalition Formation with Resource Allocation

In Chapter 4, we assumed that the uniform power allocation is used to simplify the study of coalition formation process, focusing instead on joint beamforming. The next logical step would be to study joint power allocation and user scheduling along with the coalition formation process. Also, we had assumed that the network MIMO transmits the same data from different base stations, thus making it a single flow MIMO system. However, we can extend to a more general case of a multi-flow problem.

Apart from this, it is also possible to explore other schemes of coalition formation. In particular, in this work we assumed that two coalitions do not overlap. That is, a player can belong to only one coalition. However, for the case of network MIMO, it might be possible to consider the case where overlapping coalitions are permissible. In such a case, it is possible that the merge-split algorithm is stable.

6.2.6 Hybrid Access Control

In Chapter 5, we only considered the open access scheme. We can extend the study to a hybrid access control scheme where an outside macrocell user is allowed to connect so long as the quality-of-service to the small cell users is not degraded. The situation is complicated by the fact that only a few macro cell users are allowed into a small cell. Also, there is an asymmetry in the access. That is, a small cell user can always access the macro cell service, whereas not all macro cell users are allowed into the small cell. It is not apparent how such a situation should be handled using the game theoretical techniques.

6.2.7 Distributed Learning for Coalition Formation

In Chapters 4 and 5, we focused on deterministic rules to form coalitions. A promising area of future research is to search for distributed probabilistic methods for coalition formation. Unlike heuristic methods (e.g., simulated annealing and genetic algorithms) that need to be implemented by some central entity, the process should be distributed and thus involve some form of learning mechanism. Also the method should not involve

enumerating the entire state space, since it is too large to be accomplished. Modification of regret-based learning method can be a suitable candidate for the task.

List of Publications

- **Sudarshan Guruacharya**, Dusit Niyato, Mehdi Bennis, and Dong In Kim, “Dynamic Coalition Formation for Network MIMO in Small Cell Networks,” *IEEE Trans. Wireless Commun.*, Sept. 2013. (to appear)
- **Sudarshan Guruacharya**, Dusit Niyato, Dong In Kim, and Ekram Hossain, “Hierarchical Competition for Downlink Power Allocation in OFDMA Femtocell Networks,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 4, pp. 1543–1553, Apr. 2013.
- Tran Kien Thuc, **Sudarshan Guruacharya**, and Dusit Niyato, “Robust bandwidth allocation in wireless mesh network,” *IEEE GLOBECOM*, pp. 1704–1709, 2012.
- **Sudarshan Guruacharya**, Dusit Niyato, and Dong In Kim, “Access control via coalitional power game,” *IEEE WCNC*, pp. 2824–2828, 2012 2011.
- M. Bennis, **Sudarshan Guruacharya**, and Dusit Niyato, “Distributed Learning Strategies for Interference Mitigation in Femtocell Networks,” *IEEE GLOBECOM*, pp. 1–5, 2011.
- **Sudarshan Guruacharya**, Dusit Niyato, Ekram Hossain, and Dong In Kim, “Hierarchical Competition in Femtocell-Based Cellular Networks,” *IEEE GLOBECOM*, pp. 1–5, 2010.

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