Exploiting user interaction in interference-aware wireless networks

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**NANYANG
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UNIVERSITY**

**EXPLOITING USER INTERACTION IN
INTERFERENCE-AWARE WIRELESS NETWORKS**

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Abstract

Due to the shared nature of the wireless media, the wireless users in a network naturally interact with each other and thus affect the transmissions of each other. The aim of this thesis is to illustrate the crucial role of user interaction in designing efficient interference-aware communication schemes in wireless networks, from both user-centric and network-centric points of view. Specifically, this thesis investigates various key issues on exploring wireless user interaction, such as tradeoff analysis, co-existence criterion, algorithm complexity, and system capacity characterization, for wireless networks consisting of homogeneous and heterogeneous users, respectively. This thesis also demonstrates the usefulness of two mathematical tools, stochastic geometry and dynamic programming, and shows their effectiveness in designing efficient communication schemes in interference-aware wireless networks.

First, in an ad hoc network consisting of homogeneous wireless users, to guide the transmit interaction between users, this thesis proposes a transmission scheme with signal-to-interference-ratio (SIR) threshold based scheduling. This thesis considers a stochastic ad hoc network, where the locations of the transmitters and receivers as well as the channel fading between any transmitter and receiver are randomly distributed. By using tools from stochastic geometry, this thesis focuses on analyzing the effects of user interaction on the ad hoc network capacity. Specifically, this thesis adopts a homogeneous Poisson Point Process (PPP) to model the locations of transmitters that have the intention to transmit. Due to the SIR-threshold based scheduling, it is shown that the resulting point process formed by the transmitters that are allowed to transmit is a non-PPP in general. This thesis tackles the challenges on

analyzing the non-PPP based network, by proposing a new method to approximately characterizing the non-PPP network capacity. Such method is useful for analyzing performance of wireless networks with interacted transmitters. Moreover, this thesis reveals the exact/approximate dependence between the system performance and the system designing parameters, which gives designing insights for networks with SIR-threshold based scheduling.

Next, in a cognitive radio (CR) network consisting of secondary users (SUs) and primary users (PUs), this thesis assumes that the PUs are able to react to the SU's collision or interference by adapting its transmit power level and/or adjusting its channel access probability. Such PUs are referred to as reactive PUs, to differentiate from their non-reactive counterparts. This thesis proposes a 4-state Markov chain to model the channel occupancy of the reactive PU. Under a practical scenario where the PU's channel occupancy status is only partially known to the SUs, this thesis studies the opportunistic spectrum access (OSA) design for the SU, by adopting two methods to protect the reactive PU's transmission, respectively. One is the conventional short-term conditional collision probability (SCCP) constraint, and the other is a long-term PU throughput (LPUT) constraint. By exploiting a separation principle to separate the SU's channel selection from its spectrum sensor design and channel access decision on the selected channel without loss of optimality, an optimal OSA policy under the SCCP constraint and a suboptimal OSA policy under the LPUT constraint are obtained in this thesis, respectively. It is shown that the conventional SCCP constraint, which is effective in protecting the non-reactive PUs, is not able to properly protect the reactive PUs, while the LPUT constraint can provide sufficient protection for the reactive PU's transmission. Interesting tradeoff between SU throughput maximization and PU transmission protection are also revealed in this thesis.

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List of Symbols

$\mathbb{R}^2$	unbounded two-dimensional space
M	number of probing phases
γ_k	SIR threshold in the k -th probing phase
Φ_0	homogeneous PPP with density $\lambda_0 > 0$
l_{ij}	path loss between transmitter i and receiver j
h_{ij}	channel fading coefficient from transmitter i to receiver j
$\text{SIR}_i^{(k)}$	SIR at the i -th receiver in k -th probing phase
$\mathcal{P}_0^{p,M}$	successful transmission probability by the SIR-threshold based scheme with M probing phases
$\mathcal{P}_0^r$	successful transmission probability by the reference scheme
$\mathcal{C}^{p,M}$	spatial capacity by the SIR-threshold based scheme with M probing phases
$\mathcal{C}^r$	spatial capacity by the reference scheme
$\mathbb{A}_S$	set of primary channels
$a(t)$	SU's selected channel to sense in slot t
$\epsilon_a(t)$	PFA on channel a in slot t
$\delta_a(t)$	PM on channel a in slot t
$\mathbb{A}_\delta(a(t))$	set of feasible operating points on channel $a(t)$
$f_a(\theta, t)$	access probability on channel a in slot t with sensing result $\theta \in \{0, 1\}$
$K_a(t)$	feedback from the secondary receiver to its transmitter
$\mathbb{C}_S$	set of the PU's channel occupancy states

$\mathbb{A}$	SU's action space
$R_S(t)$	SU's instantaneous reward in slot t
$P_n(i j, A)$	transition probability from state j in slot t to state i in slot $t + 1$ over channel n with SU's action A
$\lambda_{nj}(t)$	probability that the occupancy state of channel n is $j \in \mathbb{C}_S$ in slot t
$\mathbf{\Lambda}(t)$	$\triangleq \{\lambda_{nj}(t)\}_{n \in \mathbb{A}_S, j \in \mathbb{C}_S}$, belief state of the POMDP
π_s	SU's sensing policy
π_δ	SU's sensor operating policy
π_c	SU's access policy
$\sigma_n(t)$	short-term conditional collision probability caused by SU on primary channel n in slot t
$R_{P,n}^o$	PU's normalized throughput on channel n
Υ_n	PU's benchmark throughput on channel n

List of Abbreviations

AP	access point
ARQ	automatic repeat request
CR	cognitive radio
CDF	cumulative distribution function
CSMA	carrier-sensing-multiple-access
ITC	interference temperature constraint
LPUT	long-term PU throughput
MDP	Markov decision process
OSA	opportunistic spectrum access
PFA	probability of false alarm
PGFL	probability generating functional
PM	probability of mis-detection
POMDP	partially observable Markov decision process
PPP	Poisson point process
PU	primary user
pdf	probability of density function
QoS	quality of service
ROC	receiver operating characteristic
SCCP	short-term collision probability
SIC	successive interference cancellation
SINR	signal-to-interference-plus-noise-ratio
SIR	signal-to-interference-ratio

SS	spectrum sharing
SU	secondary user
TPC	target-tracking power control

Chapter 1

Introduction

1.1 Motivation and Objectives

Due to the mismatch between the rapid growth of wireless communications and the physically limited frequency spectrum, wireless users in current wireless communication systems usually experience strong interference, which may severely degrade their communication quality. A long-standing pursuit in wireless communications is to effectively combat the interference between wireless users, so as to develop reliable and spectrum-efficient wireless communication systems.

Depending on the adopted spectrum access techniques by the wireless users, the interference level of a wireless network can be acquired from two types of information. One is the received interference power of the wireless users (e.g., in the spectrum sharing (SS) based cognitive radio (CR) network), and the other is the wireless user's experienced transmission collisions (e.g., in the carrier-sensing-multiple-access (CSMA) based wireless networks). While the received interference power of each wireless user gives the complete interference status of its local environment, the experienced transmission collisions only provide partial information on the interference status of its local environment. For example, for a transmitting user, if its caused interference to other concurrent transmitting wireless users is below a given threshold, its transmission is not detected as a collision by other users; otherwise, its transmission is

detected successfully and considered as a collision by other users [Q.ZhaoDSA]. As a result, *we consider both observations on received interference power and experienced transmission collisions as the wireless users' ability to be aware of the interference in the wireless network.*

In the interference-aware wireless networks, based on the observed interference status of the network, the wireless users can interact with each other by adapting their transmissions. Although the observed network interference status at one wireless user may change upon the transmissions of other wireless users, by properly coordinating the transmission interactions between the wireless users, the received interference level or the experienced collisions at each user can be well controlled. For example, in a multi-user wireless network, by adopting a proper power control scheme, the wireless users can combat the received interference by adjusting their transmit power level [2], such that the quality-of-service (QoS) requirement is satisfied at each wireless user. Also, in a CSMA based CR network, by designing effective spectrum access policies for the secondary users (SUs) under proper collision constraints to the primary users (PUs), the communication quality of both SUs and PUs can be satisfied at the same time [3]. Furthermore, user interaction is helpful for information exchange in the wireless network. By observing the change of the received interference or experienced collisions, each user is able to obtain more information on other users' transmission strategies and/or variation of the surrounding wireless environment. This is especially useful for transmission scheme design in a distributed wireless communication network. All these merits of exploiting user interaction make it a promising method for enhancing spectrum efficiency and improving system throughput in the interference-aware wireless network.

Although extensive work, such as the pioneer work by Foschini *et al.* [2], has been proposed to study user interaction in the interference-aware wireless networks, fundamental issues on user interaction, such as its effects on improving wireless network capacity and efficient exploitation of such effects for optimal transmission scheme design, are still largely unsolved. Consequently, the design of practical transmission schemes that exploits the potential of user interaction is left with lack of guidance.

The main objective of this thesis is to understand the fundamental effects of user interaction in the interference-aware wireless networks, and exploit these effects to design efficient interference-aware transmission schemes to improve wireless network performance, from both network-centric and user-centric points of view. Toward this objective, we consider two types of wireless networks. One is consisting of homogeneous users, and the other is consisting of heterogeneous users. For both types of wireless networks, we aim at the following targets.

- To effectively control and guide the transmission interactions between wireless users, so as to improve the wireless user coexistence.
- To reveal the effects of user interaction on improving spectrum efficiency and wireless network capacity.
- To design efficient interference-aware transmission schemes with low implementation and computational complexity.

Depending on whether the wireless network consists of homogeneous or heterogeneous users and/or the specific spectrum access techniques adopted by the wireless users, the specific transmission scheme design purposes as well as the effects of the user interaction may be different. In the following two subsections, we specifically introduce our research motivation and objectives in the wireless networks consisting of homogeneous and heterogeneous users, respectively.

1.1.1 Wireless Networks with Homogeneous Users

In wireless networks with homogeneous users, the wireless users are characterized by the same transmission properties, such as the transmission quality requirement, spectrum access priority, and/or channel fading statistics. Among all the wireless networks that consists of homogeneous users, the wireless ad hoc network attracts our attentions. Due to the lack of infrastructure, the wireless ad hoc network is under competitive and interference-dominant environment in nature. Thus, how to

effectively coordinate the wireless users' transmissions based on their local interference information becomes vital for improving the wireless ad hoc network performance. As a result, the wireless ad hoc network provides a proper platform for us to investigate the transmission interactions between the wireless users.

It is noted that in the traditional transmission schemes for an ad hoc network, each wireless user (transmitter) is enabled to independently decide whether to transmit over a particular channel based on its own willingness or channel strength (e.g., [4]-[10]). Although easy to be implemented, such independent transmission schemes do not consider the resulting user interactions in the wireless ad hoc networks due to the co-channel interference, and thus do not achieve high system performance in general cases. By exploring the information of signal-to-interference-ratio (SIR), more complex transmission schemes have been proposed recently to exploit the user interactions (e.g., [2], [11]-[13]). However, these studies either require each transmitter to know the channel information of all other transmitters, or are of high implementation complexity, and thus are not appropriate for practical wireless ad hoc networks, especially those of large scale.

On the other hand, due to the randomized location of each transmitter and the effects of channel fading, the system-level performance analysis is fundamentally important for the study of wireless ad hoc networks, though challenging. It came to our attention that stochastic geometry [14] provides a set of powerful tools for analyzing the network performance. Different from the *scaling laws* studied in [15], the network capacity can be explicitly characterized based on stochastic geometry. However, most of the existing work only studied the ad hoc network performance under an independent transmission scheme, by assuming the wireless users are distributed according to a homogeneous Poisson point process (PPP)[16]. To our best knowledge, there has been no existing work on studying the ad hoc network capacity with a SIR-based transmission scheme, where the user interactions are involved. The impacts of SIR-based transmissions, or the effects of the user interactions, are limitedly understood from the system-level point of view.

Therefore, for the research in wireless ad hoc networks, the principle goal is to

fill the void of wireless ad hoc network capacity characterization by an efficient SIR-based transmission scheme. Furthermore, based on the characterized ad hoc network capacity, we aim at revealing the effects of transmission interactions between the wireless users, so as to provide useful research insights for optimal ad hoc network design.

1.1.2 Wireless Networks with Heterogeneous Users

In wireless networks with heterogeneous users, the wireless users are distinguished from each other by their different transmission characteristics. As a typical wireless network with heterogeneous users, the CR network [17], which consists of PUs and SUs, attracts our attentions. In the CR networks, the PUs have absolute priority to access the spectrum, while the SUs are allowed to access only if their transmissions are guaranteed to not cause harmful interference to the PUs' transmissions. Since the PUs are usually unaware of the existence of the SUs and do not actively share their transmission information with the SUs, the SUs need to detect the PUs' transmission activities based on their received PU signals, so as to adjust their own transmissions. As a result, the CR network provides a proper platform for us to investigate the transmission reactions/interactions between the PUs/SUs.

There are two operation models for the SUs' spectrum access in CR networks [17], [18]. One is opportunistic spectrum access (OSA), and the other is spectrum sharing (SS). In the OSA-based CR, the SUs are only allowed to access the spectrum if the spectrum is sensed as idle, i.e., there is no PUs occupying the spectrum. Due to the physical limitation of the hardware, sensing errors are unavoidable in practice. Thus, one of the main design criterion in the OSA-based CR is to constrain the transmission collisions caused by the SUs to the PUs within an acceptable range. In contrast to the OSA-based CR, the SUs are allowed to transmit regardless of the PU's on/off status in the SS-based CR, provided that the resulting interference to PU is kept below a predefined threshold. Since the interference based user interaction is the main research focus in the wireless ad hoc networks, we concentrate on the transmission collision based user interaction in the CR networks and study the OSA-based CR

networks.

It is noted that most existing work on OSA-based CR assumed that the PUs are non-reactive to their experienced transmission collisions, and thus focused on controlling the SU's spectrum access reactions to the PU's transmissions (e.g., [19]-[25]). These studies usually modeled the PUs transmission over a particular channel as a 2-state on/off Markov chain with fixed state transition probabilities. The SU's optimal OSA actions are then obtained by solving the corresponding Markov decision process (MDP) or a partially observable Markov decision process (POMDP). To guarantee the PUs' transmission quality, a short-term collision probability (SCCP) constraint is often imposed to restrict the SU's instantaneous spectrum access actions in each transmission. However, such non-reactive PU model might not be practical since existing primary systems are mostly intelligent enough to adapt their transmissions upon experiencing collision or interference. For example, a PU may reduce its channel access probability when collision occurs in a CSMA based primary system. We refer to such PUs as *reactive PUs*, to differentiate from their non-reactive counterparts. It is worth noting that since the PUs are unaware of the existence of the SUs', they cannot distinguish the transmission collisions caused by other PUs from that by SUs. Thus, the reactive PU will adjust its spectrum access actions upon experiencing transmission collisions from both other PUs and SUs. As a result, unlike the CR network with non-reactive PUs, where only the SUs' reactions to the PU's transmission are considered, interesting interactions are observed between the PUs and the SUs in the CR network with reactive PU. This in turn requires a more careful design for the SU's OSA policy in the presence of reactive PUs. To our best knowledge, there has been no existing work addressing the SU's optimal OSA design in the presence of reactive PUs.

Therefore, for the research in the CR networks, the principle goal is to design effective OSA policy for the SUs in the presence of reactive PUs, so as to improve the CR network performance under the reactive PUs' transmission quality constraint. Moreover, we also aim to show some interesting new tradeoffs between SU throughput maximization and PU transmission protection in the practical interaction scenario.

1.2 Major Contributions of the Thesis

In this thesis, by investigating the transmission interactions between the wireless users, we design effective transmission schemes to improve user communication quality for both types of wireless networks with homogeneous and heterogeneous users. Moreover, in each type of wireless networks, we reveal the effects of transmission interactions, and show how to exploit such effects to improve the network throughput and/or spectrum efficiency.

First, for the wireless ad hoc networks consisting of homogeneous users, we use stochastic geometry to model and analyze the SIR-based transmission interactions between the wireless users. The main results are summarized as follows.

- *Novel SIR-threshold based scheduling scheme:* Unlike the conventional schemes without any scheduling, we propose a novel SIR-threshold based scheduling scheme for a single-hop wireless ad hoc network, to coordinate the transmission interactions between the wireless users. Specifically, each time frame is sequentially divided into multiple probing phases and one data transmission phase. By starting with initial probing, each transmitter iteratively decides to further probe or stay idle, depending on whether the estimated SIR in the proceeding probing is no smaller than a predefined threshold. It is shown that the proposed scheme is appropriate for distributed implementation in practical wireless ad hoc networks.
- *Stochastic modeling and analysis of the ad hoc network:* Though one can assume that the initial transmitters are deployed according to a homogeneous PPP as in most of the existing work, the SIR based scheduling makes the PPP no longer applicable to model the locations of retained transmitters in the subsequent probing and data transmission phases, which complicates the network performance analysis. Although challenging, we propose a new method to successfully characterize the non-PPP based network capacity, which is useful for analyzing performance of wireless networks with interacted transmitters.
- *Effects of the SIR thresholds on improving spatial capacity of the ad hoc network:*

First, for the proposed SIR-threshold based scheme with single-stage probing, we show that an optimal SIR threshold exists to maximize the network capacity, and the optimal SIR threshold is smaller than the required SIR level. Next, for the proposed scheme with multi-stage probing, we show that as long as the sequence of increasing SIR thresholds are properly decided over probing phases, the network capacity is assured to gradually improve over the number of probing phases.

- *Interesting ad hoc network performance tradeoff:* By analyzing the ad hoc network capacity over the initial transmitter density, we present a density-capacity tradeoff, which shows that a small initial transmitter density can help improve the network capacity, while a large one will introduce high interference level and thus reduce the network capacity. Moreover, since multi-stage probing can introduce non-ignorable time overhead, which reduces the network capacity, we also illustrate an interesting probing-capacity tradeoff over the probing phases.

Next, for the CR network in the presence of reactive PUs, we use discrete-time Markov chain to model the transmission collision based interactions between the PU and SU, and use dynamic programming to find the off-line optimal OSA policy for the SU. The main results are summarized as follows.

- *Markov chain based modeling and problem formulation with the reactive PU:* Considering the PU's reactions to the SU's transmission collisions, we propose a new 4-state discrete-time Markov chain model to describe the channel occupancy state of the reactive PU, which includes the conventional 2-state on/off model for the non-reactive PU as a special case. Moreover, based on the proposed model, we formulate the optimal OSA design for SU throughput maximization as a constrained finite-horizon POMDP problem.
- *SU's optimal OSA design under short-term protection to the reactive PU:* We first adopt the conventional SCCP constraint to protect reactive PU's transmission. We extend the *separation principle* proposed in [22] for the non-reactive PU case to the reactive PU case, and obtain the optimal OSA policy that can

be implemented efficiently. However, since the SCCP constraint does not take the PU's reactions to the SU's collisions into consideration, we analytically show that the reactive PU's throughput cannot be guaranteed under the SCCP constraint in general, which is different from the non-reactive PU case.

- *SU's optimal OSA design under long-term protection to the reactive PU:* We then apply a *long-term PU throughput* (LPUT) constraint, which is shown to be able to effectively protect reactive PU's transmissions. Although the resulting long-term constrained finite-horizon POMDP problem is very challenging to solve, we obtain the structure of the optimal OSA policy for the SU under the LPUT constraint. Moreover, we propose a suboptimal OSA policy for the SU with lower complexity, which is shown to guarantee the reactive PU's throughput.
- *Illustration of the transmission interaction effects between SU and PU:* We show that by properly coordinating the transmission interactions between the SU and reactive PU, the sum-throughput of the SU and PU and thus the spectrum efficiency can be effectively improved. Moreover, interesting tradeoffs between SU throughput maximization and PU transmission protection under the reactive PU model are also illustrated, by comparing with the counterparts under the non-reactive PU model.

1.3 Organization of the Thesis

The rest of this thesis is organized as follows: Chapter 2 provides an overview of the existing studies in the interference-aware wireless networks. Chapter 3 focuses on ad hoc networks consisting of homogeneous users and studies the effects of wireless user interaction on improving the ad hoc network capacity. Chapter 4 and Chapter 5 study the optimal OSA policy for the SU in the presence of reactive PU, where the SCCP and LPUT constraints are adopted to protect the reactive PU's transmissions, respectively. Finally, Chapter 6 concludes this thesis and presents future work.

Chapter 2

Background Review

User interactions are ubiquitous in wireless communications. Existing literature has exploited user interaction to improve wireless user coexistence, system throughput and spectrum efficiency. This chapter briefly reviews the progresses made on studying user interaction in wireless networks.

The rest of this chapter is organized as follows: Section 2.1 focuses on the wireless networks consisting of homogeneous users, where the popular transmission schemes and the existing work on network capacity analysis are introduced and discussed. Section 2.2 presents the existing work in the wireless network consisting of heterogeneous users, in particular, the CR network. The existing work on transmission scheme design as well as network capacity characterization for CR are both reviewed in Section 2.2. At last, we summarize this chapter in Section 2.3.

2.1 Wireless Networks with Homogeneous Users

Distributed transmission scheme design has become the main approach to exploit the interactions between homogeneous wireless users in the past decade. A typical example is given by the iterative power control scheme, where the transmit power level of each user is updated iteratively based on its local channel information (e.g., the received interference power from other users). In the following, we first introduce

several classic iterative power control schemes. Then we review the existing work on determining and characterizing capacity of a wireless network by applying distributed transmission schemes.

2.1.1 Iterative Power Control Schemes

This subsection reviews classic iterative power control schemes in the existing literature. These power control schemes provide good examples to effectively coordinate the transmission interactions between wireless users in a distributed manner.

One of the most popular iterative power control schemes is the *target-tracking* power control (TPC), which drives the signal-to-interference-plus-noise-ratios (SINRs) of all wireless users toward their target levels. Consider a wireless network with N users. Denote $\hat{\gamma}_i$ as the target-SINR of the i -th user, $i \in \{1, \dots, N\}$. Denote $\mathbf{p}(k) = \{p_1(k), \dots, p_N(k)\}$ as the transmit power vector in the k -th iteration, $k = \{0, 1, \dots\}$, where $p_i(k)$ is the i -th user's transmit power in the k -th iteration. Denote g_{ij} as the path gain from transmitter i to receiver j , with $j \in \{1, \dots, N\}$. Assuming the path gains do not change over iterations, we have $I_i(\mathbf{p}(k)) = \sum_{j \neq i, j=1}^N p_j(k)g_{ji}$ as the received interference power at the i -th receiver in the k -th iteration. Each user i updates its transmit power level in the $(k+1)$ -th iteration based on a power-update function $f_i(\cdot)$, which is a function of the power vector in the k -th iteration, i.e.,

$$p_i(k+1) = f_i(\mathbf{p}(k)). \quad (2.1)$$

In [2], Foschini and Miljanic proposed a famous power-update function, given by

$$f_i(\mathbf{p}(k)) = \hat{\gamma}_i \times \frac{I_i(\mathbf{p}(k)) + \sigma^2}{g_{ii}}, \quad (2.2)$$

where σ^2 is the power of the background noise at each receiver. We say a system is *feasible*, if there exists a fixed power vector $\mathbf{p}^*$, such that the desired SINRs of all users can be retained at the same time at $\mathbf{p}^*$. It has been shown in [2] and [11] that if and only if the system is *feasible*, the Foschini-Miljanic algorithm can guarantee

power convergence to $\mathbf{p}^*$ either synchronously or asynchronously, regardless of the initial power level of each user. Moreover, the fixed power vector $\mathbf{p}^*$ provides the minimum aggregate transmit power of all users. As a result, $\mathbf{p}^*$ is the solution to the following optimization problem:

$$\begin{aligned} \min_{\mathbf{p}} \quad & \sum_{i=1}^N p_i \\ \text{s.t.} \quad & \frac{p_i g_{ii}}{I_i(\mathbf{p}) + \sigma^2} \geq \hat{\gamma}_i, \quad \forall i \in \{1, 2, \dots, N\}. \end{aligned}$$

However, when the system is not feasible, the Foschini-Miljanic algorithm renders all users to increase their transmit power levels in each iteration to track the target SINRs, and thus causes power divergence. In [26] and [27], the authors solved the power divergence problem in infeasible systems by adopting a *constrained* TPC, i.e.,

$$p_i(k+1) = \min \left\{ \bar{p}_i, \hat{\gamma}_i \times \frac{I_i(\mathbf{p}(k)) + \sigma^2}{g_{ii}} \right\}, \quad (2.3)$$

where $\bar{p}_i$ is the maximum allowable transmit power level of user i . Although power convergence is guaranteed in both feasible and infeasible systems by using (2.3), some users cannot be well supported, if the achieved SINRs lower than their target-SINRs. Moreover, all non-supported users transmit at their maximum power $\bar{p}_i$. Thus, the non-supported users have to inefficiently consume their energy and cause unnecessary interference to other users. It is noted that in [28], the authors proposed a TPC algorithm with temporary removal and feasibility check to gradually remove the infeasible users from the transmitting user set in a distributed manner. Specifically, if a user's required transmit power level for reaching the target-SINR in the current iteration exceeds its maximum allowable transmit power level, it is temporarily removed from the transmitting user set; and it can be re-included in the set, if such required transmit power level goes below a prescribed threshold in a later iteration. It has been shown in [28] that there exists at least one fixed-power vector by applying the proposed power control algorithm; and as in the Foschini-Miljanic algorithm, the fixed-power vector guarantees the minimum aggregate transmit power of all users.

Besides TPC schemes, iterative power control algorithms with other designing purposes have also been studied in the literature. For example, an *opportunistic* power control was proposed in [30] to improve the system throughput by using tools from game theory [29]. Different from the TPC counterpart, where the wireless user increases its transmit power level to compensate for the link loss, the wireless user with opportunistic power control adopts a higher transmit power level when the channel is good and vice versa. It has been shown in [30] that the opportunistic power control can achieve a tenfold throughput increase when compared with the target-tracking counterpart.

In the next two subsections, we focus on reviewing various approaches for network capacity determination and characterization by applying the distributed transmission schemes.

2.1.2 Network Capacity Determination

This section presents the existing approaches for determining the network capacity in the literature. *Capacity* of a wireless network is usually defined as the number of wireless users that can transmit successfully at the same time (e.g., [31]-[37]). Assuming the wireless users are arbitrarily deployed in the network, the wireless network capacity maximization problem has been studied in the recent years, where both interference graph based and SINR based several interference models have been applied.

By using interference graph to model the interference between wireless users, a user's transmission is successful if and only if there is no concurrent transmission from its neighbors in the graph. Under this model, the capacity maximization problem is equivalent to the *maximum independent set* problem in the graph theory [38]. By placing proper restrictions on the structure of the graph, a number of solutions to the capacity maximization problem have been proposed (e.g. [31]-[33]). However, it is well-known that the graph-based methods may not be able to capture the essence of interference effects in many practical scenarios [39], [40]. Thus, a more practical *SINR-based model* has received a great deal of attentions, where a user's transmission

is successful if the received SINR at its receiver is larger than or equal to a prescribed threshold. However, it has been shown in [35] that, based on the SINR-based model, the capacity maximization problem in the worst case instances is NP-hard. As a result, a number of recent work has focused on designing algorithms to approximate the maximum achievable capacity of a wireless network. In the following, we review the existing approximation algorithms that can be implemented in a distributed manner under the SINR-based model.

In [35]-[37], the authors studied the distributed algorithms for maximizing the wireless network capacity by adopting a game theoretic model, where each wireless user receives positive, negative, and zero payoff for transmitting successfully, transmitting unsuccessfully, and not transmitting, respectively. Moreover, two methods for power assignment were considered in [35]-[37]: one is arbitrary power assignment, where each user may select different power levels to increase the capacity; and the other is uniform power assignment, where each user adopts the same transmit power level. For both methods, it has been shown in [35] that if the wireless users adopt a probabilistic transmission strategy, a mixed Nash equilibrium exists. Furthermore, for the case of arbitrary power assignments, the achieved *average* capacity at the mixed Nash equilibrium is within a factor of $\mathcal{O}(\Delta^{2\alpha})$ of the optimum (i.e., the maximum achievable capacity), where $\alpha > 2$ is the path loss exponent, and $\Delta = l_{\max}/l_{\min}$, with $l_{\max}$ and $l_{\min}$ being the longest and shortest distance between any transmitter and receiver in the network, respectively. However, [35] did not provide any algorithmic results. In [36], the authors converted the structural result in [35] to an algorithmic result, by using the concepts from *no-regret* algorithms (e.g., [41]). Specifically, the authors showed that if each user adopts a no-regret algorithm to interact with each other, the network can iteratively reach to a status similar to that at a mixed Nash equilibrium in [35], i.e., within a factor of $\mathcal{O}(\Delta^{2\alpha})$ of the optimum. Furthermore, under a more general assumption with $\alpha > 0$, the authors in [37] improved the performance bounds in [36] based on the no-regret algorithms. Specifically, for the case with arbitrary power assignment, the converged average capacity is within a factor of $\mathcal{O}(\log \Delta)$ of the optimum, which is improved exponentially as compared to that

in [36]. For the case of uniform power assignment, the converged average capacity is within a factor of $\mathcal{O}(1)$ of the optimum, which is the best possible bound that could be obtained.

2.1.3 Network Capacity Characterization

This section presents the existing work on characterizing the wireless network capacity. Due to the difficulty in characterizing the network interference, it is generally challenging to analyze/characterize the capacity of a wireless network. Notably, Gupta and Kumar's pioneer work on *scaling laws* quantified the increase of the capacity region volume over the number of transmitters in an ad hoc network [15]. However, such results cannot characterize the effects of various system designing parameters, e.g., the transmitter density, the transmission distance, and the transmitted power level, on the capacity of a network; and thus [15] lacks the complete insight into wireless network design problems. Recently, stochastic geometry [14] has caught wide attentions due to its ability to characterize the network performance [42]. According to [4], stochastic geometry can help to obtain, if not completely exact, asymptotically exact dependence between network performance (e.g., network capacity, outage probability) and various network designing parameters. We thus focus on the exploiting tools from stochastic geometry to analyze/characterize the wireless network. In the following, we briefly introduce historical background of applying stochastic geometry in wireless communications, and then review the existing work based on stochastic geometry on studying the wireless networks.

Historical Background

The approach of applying stochastic geometry to study wireless communications, despite of the wide interest raised recently, could be traced back more than 100 years from today. Assuming an infinitely large network, where infinite number of wireless transmitters and receivers are randomly deployed, the received interference

at a receiver can be modeled as a *shot noise process*, which is given by

$$I(x) = \sum_{i=-\infty}^{i=+\infty} g(x - X_i), \quad (2.4)$$

in one dimension $\mathbb{R}$. Specifically, in (2.4), $g(x)$ is the impulse response of a linear and time-invariant system at the point $x \in \mathbb{R}$ and $\{X_i\}$ is a stationary Poisson point process (PPP) on $\mathbb{R}$. More generally, in the two-dimensional space $\mathbb{R}^2$, by letting x denote the location of a point on $\mathbb{R}^2$, we accordingly have the stationary PPP $\{X_i\}$ on $\mathbb{R}^2$. Suppose $g(x)$ is only determined by the Euclidean norm $\|x\|$. We can then use $g(x)$ to represent the path loss model in wireless networks. Consequently, without considering the fading effects, the resulting $I(x)$ in (2.4) gives the aggregated interference to a point locating at $x \in \mathbb{R}^2$. Historically, the first study on shot noise processes could date back to the pioneer work in 1909 by Campbell [43], who focused on characterizing the mean and variance of a shot noise process and then was followed by Schottky in 1918 [44]. From 1940 through 1970, a number of investigations have been done by Rice (e.g., [45]), whose focus was on the distribution of $I(x)$. In 1990, Lowen and Teich considered Power law shot noise [46], where $g(x)$ is assumed to follow a decaying power-law form. Due to its ability to model the fading effects, power law shot noise is very relevant to the study in wireless communications.

Moreover, for research in wireless networks, stochastic geometry has been applied to analyze interference at least since 1978 [47]. In early 1990s, Sousa and Silvester first recognized the aggregate interference distribution as Lévy stable in an ad hoc network [48]-[50]. As two classic mathematical books on stochastic geometry, [14] (with first edition in 1987) and [16] were published around that time. In the late 1990's, by characterizing aggregate co-channel interference, Ilow and Hatzinakos studied the random effects of wireless channel variations in [51]. In addition, Baccelli *et al.* considered hierarchical networks and studied mobility management in cellular networks, based on Poisson Voronoi tessellations [52]-[54].

Finally, we note that there have been some very useful tutorials and monographs published recently to apply stochastic geometry to analyze wireless network, such as

the tutorial article on this topic for a JSAC special issue by Haeggi *et al.* [42], the tutorial article for network interference characterization in Poisson fields [55], the two-volume monograph by Baccelli and Blaszczyzyn [5], [56], the monograph by Haenggi and Ganti [4], and the monograph by Weber and Andrews [6].

Capacity Characterization

In the existing literature, mathematical tools from stochastic geometry, especially the PPP, have been widely adopted to analyze ad hoc networks [7]-[10], [58], the cellular systems [59]-[64], femtocells [65]-[67], relay networks [68], heterogeneous networks [69]-[72], and many other types of wireless systems.

As a metric to measure the network capacity, *transmission capacity*, defined as the average number of successful transmitters per unit area, subject to a given outage probability [8], has attracted great research attentions. Typically, in [7], Weber *et al.* studied the effects of fading, channel inversion and *channel-threshold scheduling* on ad hoc networks, where by adopting the channel-threshold scheduling, a transmission is scheduled if the direct channel strength between the transmitter and its intended receiver is acceptably strong. Based on the PPP model, both upper and lower bounds of the transmission capacity were provided in [7] for various transmission strategies. In [8], Weber *et al.* surveyed and unified a number of work on studying transmission capacity, and discussed the effectiveness of transmission capacity on capturing the essential effects of various wireless techniques on the network capacity. Moreover, a fractional power control was proposed in [9], where the transmit power of each user is determined by a normalized channel quality with an exponent up to $-s$, with $s \in [0, 1]$. It was shown in [9] that the outage probability is minimized by using the optimal exponent $s^* = 0.5$. In [63], by proposing a new framework to model the downlink cellular network based on the PPP, Andrews *et al.* obtained closed-form transmission capacity expression for a cellular network. We also note that some practical issues on applying stochastic geometry to analyze wireless networks, such as the fitness of the mathematical models to the real network deployments, have been discussed in, e.g., [67] and [71].

Other performance metrics have also been adopted to measure the network capacity. For example, Baccelli *et al.* in [56] studied an Aloha type access control mechanism for large mobile, multihop, wireless networks using a performance metric called *spatial density of progress*, which is defined as the mean number of meters progressed by all transmissions taking place per unit surface area. Moreover, Baccelli *et al.* in [10] also studied the channel-threshold scheduling scheme as that in [7], and measured the achieved network performance under various metrics, such as the *spatial density of successful transmissions*, i.e., the mean number of successful transmissions per unit area, and the *spatial density of transport*, i.e., the mean number of bit-meters transported per second per unit and space. In addition, different from most of the existing work that used PPPs to model the locations of the transmitters, [75] adopted a *Matern hard-core process* to model the correlated locations of the transmitters in a carrier-sensing-multiple-access (CSMA) network. This is mainly because that the PPP assumes independence between the transmitters, and thus is not able to model the *interactions* between transmitters in the CSMA network.

Although stochastic geometry is able to model and characterize the interference distribution in a wireless network, the resultant interference usually follows *stable distributions* [76], which generally do not have closed-form expressions for probability density functions (pdfs) or cumulative distribution functions (CDFs). Thus, approximation methods are usually adopted in the literature to obtain tractable expressions for the network capacity. For example, the authors in [7] and [8] obtained a tight lower bound and several upper bounds to the transmission capacity in the PPP-based networks. Specifically, the tight lower bound was obtained by approximating the aggregate interference generated by all interfering nodes with that caused by *dominant* interfering nodes; and the upper bounds were obtained by properly applying Markov, Chebychev, and/or Chernoff inequalities [6] to lower bound the interference. Moreover, it is generally more difficult to analyze/characterize the non-PPP networks than the PPP counterparts. It is noted that in [57], [64] and [75], by using a PPP wireless network to approximate the non-PPP wireless network with the same transmitter density, problems on optimal guard zone design in an ad hoc network, analytical

modeling of uplink cellular network, and capacity characterization of IEEE 802.11 CSMA networks have been properly solved, respectively. In addition, by letting some system designing parameters go to infinity or zero, asymptotic transmission capacity for non-PPP ad hoc networks have been studied in [73] and [74].

2.2 Wireless Networks with Heterogeneous Users

In wireless networks, heterogeneous users are distinguished by their different communication characteristics, such as different types of traffic loads or different priorities to access the spectrum. The CR network is a typical wireless network consisting of heterogeneous users, where the CR users, or the so-called secondary users (SUs), are able to dynamically access the dedicated channels of primary users (PUs). Specifically, with the cognitive capability and reconfigurability [77]-[79], the SUs are able to identify and acquire useful information from the surrounding wireless environment, and thereby intelligently adapt their transmission parameters in response to the change of the environment [80]. Accordingly, CR has been recognized as one promising solution to resolving the spectrum scarcity versus spectrum underutilization paradox in wireless communications, which is caused by the current static spectrum management policies [1], [17] and [18].

Various issues on CR have been widely-studied in the literature, such as the spectrum sensing [81]-[86], resource allocation for single-antenna [87]-[90] and multi-antenna [91]-[94] CR networks, impacts of sensing errors on the CR design [19], [22], sensing-access tradeoff [95],[96], and spectrum-aware routing [97], [98]. Mathematical tools like convex optimization [99], game theory [29], dynamic programming [100], and Markov decision process (MDP) [101] have been extensively adopted for designing efficient transmission schemes for SUs in the CR networks and have been discussed in the tutorial/survey papers, like [1], [102]-[104]. We have also noted several very helpful books on CR, including [105] by Hossain *et al.* and [106] by Chen *et al.*, where fundamentals and development of CR are addressed.

In the following, we introduce basic operation models in CR, and then present

the existing transmission schemes designed for CR networks. At last, we review the existing work on characterizing CR network capacity.

2.2.1 Cognitive Radio Operation Models

In the literature, the SUs are usually assumed to adopt one of the following two operation models to access the PU channels: opportunistic spectrum access (OSA) and spectrum sharing (SS). Fig. 2.1 illustrates the OSA-based model. In this model, the SU usually adopts efficient spectrum sensing techniques to check whether the PU is occupying the frequency band of interest or not, i.e., to detect the PU's transmission on-off status. Suppose the sensing results indicate that there is no PU occupying the sensed band with a high probability, i.e., a "spectrum hole" [107] is detected. The SU then reconfigures its radio parameters, including the operating frequency, transmit power and modulation scheme, and accesses the sensed band. While doing so, the SU is required to frequently monitor the its working frequency band and quickly vacate this band if active PUs are detected. This operation model was first proposed by Mitola in [77] and [78], under the name *spectrum pooling*. It is noted that the OSA-based model is also referred to as *spectrum overlay* [1], [108], or the interweave paradigm [109]. In contrast to the OSA-based model, as shown in Fig. 2.2, for the SS-based CR, the SU is allowed to transmit over the frequency band regardless of the PUs on/off status, as long as the SU is able to control its transmission such that the resultant PU transmission degradation is tolerable. Thus, under this model, the SUs need to predict the received aggregate interference at the PU receiver. The SS-based model is also referred to as *spectrum underlay* [1] or *concurrent spectrum access* [104] in the literature.

Under both OSA and SS models, most existing work focused on studying the SU's transmission schemes in the presence of *non-reactive* PUs, such as the Television transmitters in IEEE 802.22 standard [110], which maintains their spectrum access decisions even when collision or interference occurs. Since the PU does not react to the SU's transmissions, the SUs can only rely on their own sensing results to design transmission schemes. Due to the limited available PU transmission information, it

Figure 2.1: OSA model for CR: SU dynamically accesses the spectrum holes to transmit.

Figure 2.2: SS model for CR: PU and SU are transmitting at the same time over the same frequency f_1 .

is usually difficult for the SUs to design transmission schemes of high performance. On the other hand, it is also noted that increasingly more PUs in current wireless communication systems are able to intelligently adapt their transmission upon receiving interference/collisions. For example, a PU may increase transmit power to compensate the link loss due to the received interference. Alternatively, it may reduce the channel access probability when collision occurs in a CSMA-based primary system. In this thesis, we refer to such PUs as *reactive* PUs, to differentiate from their non-reactive counterparts. Since the existence of the SUs is oblivious to the PUs, and also due to PU's higher priority to access the channels, the PUs do not provide their transmission information to the SUs. In the CR networks with reactive PUs, the SU is able to obtain more PU transmission information by observing the PU's transmission adjustment, such as a boosted power level. However, due to the complicated transmission interactions between the PU and SU, it is still challenging for the SU to design an efficient transmission scheme in the presence of reactive PUs.

In the following two subsections, we review the existing work in OSA and SS-based CR, respectively, where in each subsection the existing studies in the presence of both non-reactive and reactive PUs are discussed.

2.2.2 Opportunistic Spectrum Access based Cognitive Radio

For OSA-based CR, two competing goals need to be addressed at the same time: the "spectrum holes" unused by the PUs should be optimally explored by the SUs to maximize their throughput, whereas the probability of the SU's transmission collision with undetected active PUs should be minimized. To achieve the optimal tradeoffs between SU throughput maximization and PU collision minimization, it is important to design efficient OSA schemes for the SU to decide *where, when and how* to access the spectrum. In the following, we focus on the existing work in OSA-base CR with non-reactive PUs, and then review that with reactive PUs.

Figure 2.3: Channel occupancy model for the non-reactive PU.

Opportunistic Spectrum Access with Non-reactive Primary Users

Most existing work on OSA design has focused on non-reactive PUs. As shown in Fig. 2.3, a two-state Markov Chain, with fixed state transition probabilities α_0 and β_0 , has been widely adopted to model the non-reactive PU's transmission on-off status over a particular frequency band. Under this model, various issues on OSA design have been investigated, such as the methods for detecting spectral sources in multi-band CR [111] and the defending method against jamming attack [112]. Moreover, assuming the SU is only able to sense a certain part of the spectrum at each time due to hardware limitations, the authors in [19] proposed a partially observable Markov decision process (POMDP) framework to design the optimal OSA policy. Due to "the curse of dimensionality", POMDP problems are in general computationally prohibitive to solve [113], [114]. Zhao *et al.* in [20] formulated the design of SU's optimal sensing policy as a POMDP and proposed a myopic sensing policy, which maximizes SU's average reward over a finite horizon. In [21], Chen *et al.* proposed a threshold-based optimal spectrum sensing and accessing policy, which maximizes SU's throughput during its battery lifetime. However, both [20] and [21] assumed that sensing errors are negligible.

In [22], Chen *et al.* considered a practical scenario with the presence of sensing errors in a time-slotted primary system. The OSA design for the SU in [22] is specified by three policies: 1) a sensing policy, which decides the SU's sensed channels in each time slot; 2) a sensor operating policy, which decides the sensor operating point, i.e., the probability of mis-detection (PM) and the probability of false alarm (PFA), over the sensed channels; and 3) an access policy, which decides the probability of accessing the sensed channels based on the sensing results. Due to the sensing errors,

the SU may wrongly access a busy PU channel and cause intolerable interference to the PU's transmission. To guarantee the PU's transmission quality, Chen *et al.* proposed a *short-term conditional collision probability (SCCP)* constraint for protecting PUs from SU's collisions, which ensures that on every channel and in each time slot, the PU experiences collisions from the SU for no more than a small fraction of the transmission time. Consequently, the OSA design in [22] was formulated as a finite horizon *constrained* POMDP problem, which maximizes SU's total throughput over the finite horizons subject to the SCCP constraint. To reduce the complexity of solving the constrained finite horizon POMDP problem, [22] further proposed a *separation principle*, to separately design sensing policy from the sensor operating policy and the access policy, without loss of optimality. According to [22], the optimal OSA design for the SU has the following properties:

- Both the optimal sensor operating policy and the optimal access policy are stationary, i.e., the SU's actions decided by these policies in each time slot do not change over the state of the underlying POMDP.
- For the optimal sensor operating policy, the optimal PM and optimal PFA are on the optimal receiver operating characteristic (ROC) curve¹.
- For the optimal access policy, the SU should always trust the spectrum sensing result even though there may exist sensing errors. Specifically, if the sensing results show that the sensed channel is not being occupied by any PU, the SU should access this channel with probability 1; otherwise, the SU should access this channel with probability 0.

Following the work in [22], a great deal of studies have adopted SCCP constraint to protect the non-reactive PUs' transmissions. For example, a POMDP problem similar to that in [22], subject to the SCCP constraint, was considered in [23] for unslotted PU systems. An online OSA algorithm learning PU's signal statistics was

¹Given the maximum allowable PFA, denoted by ϵ , the smallest achievable PM, denoted by δ^* , can be attained by the optimal Neyman-Pearson detector [115]. By varying ϵ over $[0, 1]$, the resultant δ^* and ϵ pairs form the optimal ROC curve.

proposed in [24], also under the SCCP constraint. Moreover, Li *et al.* in [25] showed that when the SCCP constraints over time are tight, the optimal OSA policy can be implemented as a simple memoryless policy with periodic channel sensing.

Opportunistic Spectrum Access with Reactive Primary Users

Compared to the work for OSA design with non-reactive PUs, the work considering reactive PUs for OSA-based CR is very limited. It is noted that a CSMA-based reactive PU system has been considered in [3]. In this system, if a PU has intention to access a particular channel, it senses this channel to check whether it is idle. If the sensing results indicate that this channel is busy, the PU backs off its transmission; otherwise, it accesses this channel with a prescribed probability. Suppose that the PU decides to access this channel. Upon experiencing collisions from other concurrent transmissions, the PU halves its spectrum access probability; and after a successful transmission, the PU resets its spectrum access probability to the original prescribed value. It has been emphasized in [3] that due to the PU's reactive nature, its communication quality could be seriously degraded by the SU's concurrent transmissions over the same channel. Moreover, the SU's transmission over an idle PU channel may still trigger the PU's transmission backoff. This occurs when a PU has intention to transmit over a channel, which is not being used by any other PUs but occupied by a SU. Since the PU is not able to distinguish the SU's transmission from other PUs' transmissions, it sees this channel as being busy and thus backs off its own transmission. The authors in [3] studied the performance of different SU spectrum access policies in such CSMA-based reactive PU system; however, [3] did not address the optimal OSA design.

2.2.3 Spectrum Sharing based Cognitive Radio

In SS-based CR, the SUs are allowed to transmit simultaneously with the PUs over the same frequency bands. The major design objective for SS-based CR is thus to effectively control the SU's transmission, such that the resulting transmission degradation to the PU is within a tolerable margin, while the SU's own communication

requirement can be achieved at the same time. In the following, we review the existing work for SS-based CR with non-reactive PUs, and then focus on that with reactive PUs.

Spectrum Sharing with Non-Reactive Primary Users

As in the case for OSA-based CR, most existing work for SS-based CR focused on SU's transmission scheme design with non-reactive PUs. In the literature, interference temperature constraint (ITC) has been widely adopted to protect the non-reactive PU's transmission for SS-based CR. By adopting ITC, the SUs' resultant total interference to the PU must be kept below a prescribed threshold [87], [88], [116], [117]. The effectiveness of the ITC in protecting non-reactive PUs has been investigated in [92], where the ITC constraint has been shown to be able to guarantee the PU's maximum throughput loss due to the SU's interference. Moreover, by exploiting multi-antennas, [92] focused on the SU's optimal transmit spatial spectrum designing, which maximizes the secondary transmission capacity, subject to the ITC constraint at each PU receiver. In [118], the authors studied an interesting phenomenon of interference diversity, by studying and comparing the transmission scheme with an average ITC over different fading states and the transmission scheme with only a peak ITC. In [119], the authors formulated an optimization problem of joint subchannel and transmission power allocation, and obtained efficient scheme that can allocate the subchannels and the transmission power for the SUs in a distributed way. An auction-based SS was proposed in [120], by adopting the ITC constraint. In [121], the authors considered the SU's transmission scheme design under the SU's minimum quality of service (QoS) constraint and the ITC for protecting PU, and studied the case where all the SU links can be supported and the case where only part of the SU links can be supported, based on tools from convex optimization and game theory, respectively. We also note that some practical issues on implementing ITC for CR have been discussed in [122].

Moreover, there has been work investigating other methods to protect the non-reactive PUs' transmissions. For example, the authors in [123] exploited PU's on-off status and channel state information, and proposed a PU ergodic capacity loss constraint to protect the PU's transmission. In [124], by eavesdropping the PU's automatic repeat request (ARQ) feedback, the SUs can control their resultant interference at the PU receiver and thus protect the PU's transmission in terms of the PU's average rate budget. Compared to the work with a fixed ITC, these work has exploited additional information on the PU transmission and thus are able to achieve higher PU and SU spectrum sharing throughput; however, more time overheads are involved in [123] and [124] for practical implementation.

Spectrum Sharing with Reactive Primary Users

There has been some existing work studying SS-based CR with reactive PUs. Due to the PU's reaction to the received interference, a fixed ITC may not be able to effectively protect the reactive PU's transmissions. In [122], [127], and [128], the PU's average performance loss constraint has been adopted to protect the reactive PU's transmissions in SS-based CR. It is noted that such constraint was also adopted to protect the non-reactive PUs' transmissions in [123] and [124]. Moreover, as illustrated in [125], by exploiting the PU's reactive nature, the SUs are able to obtain the SU-to-PU channel gain with the absence of a dedicated PU feedback channel, based on an interesting *hidden power-feedback loop*: if the PU reacts upon receiving SU's interference, the SU will receive a power-boosted PU signal that is easier to detect. Following [125], [126] proposed a proactive sensing scheme and a sequential transmit power adaptation strategy to exploit spectrum opportunities in the SS-based CR. In [122], the author extended the work in [125] and designed active learning and supervised transmission schemes. In addition, the ARQ-based reactive PUs have been considered in, e.g., [127] and [128], for the SS based CR. Under the assumption that SU has full knowledge of PU's buffer state and ARQ state, the authors in [127] adopted a Markov process based model to determine SU's optimal transmission policy over an *infinite* horizon, which maximizes SU's long-term average throughput subject

to PU's long-term throughput loss. As shown in [127], the SU's optimal transmission policy is stationary and thus can be obtained by solving a linear program. Online algorithms have also been proposed in [128] for the cases where only partial and/or noisy observations of PU's buffer state and ARQ state are available to the SU.

2.2.4 Cognitive Radio Network Capacity Characterization

This subsection reviews the existing work on characterizing the CR network capacity based on stochastic geometry [129]-[132]. It is noted that only limited work on this topic exists in the literature; and all the existing work considered the CR networks with non-reactive PUs.

In [129] and [130], the authors studied the *spectrum-sharing transmission capacity*, which satisfies the outage probabilities required by each of the coexisting networks. Specifically, [129] studied the capacity trade-off between two coexisting networks, under the cases of spectrum underlay, spectrum overlay and successive interference cancellation (SIC). For small target outage probability, [129] proved that the transmission capacities of the coexisting networks satisfy a linear equation, whose coefficients depend on the spectrum sharing method and whether SIC is applied. In [130], the authors addressed the coexisting issue of multiple networks in both cases of spectrum underlay and spectrum overlay. By proposing a fair coexistence constraint, [130] derived the optimal spatial densities and optimal transmission power ratios of the coexisting networks in terms of the sum of the transmission capacity of these networks, under both scenarios with or without the proposed constraint. In [131], the author first modeled the locations of the PUs and SUs as two independent PPPs. By applying a policy that a SU can transmit only if it is outside the PU's exclusion regions, the author then showed that the point process formed by the active SUs become a *Poisson hole process*, which is difficult to tackle with. Thus, two methods were proposed in [131] to quantify the interference and outage of the CR network. One is to derive bounds for the interference by using Laplace transforms. The other is to approximate the actual interference by that in a Poisson cluster process, by which the performance has shown to be tight. In [132], the authors discussed CR network

modeling based on stochastic geometry, where a CSMA-based CR network based on Matern point process was considered. Moreover, to obtain tractable results, [132] adopted a PPP-based network to approximate the Matern point process based CR network.

2.3 Summary

This chapter reviews the background and existing work on studying user interaction for both wireless networks consisting of homogeneous or heterogeneous users. We summarize this chapter as follows:

- For the transmission scheme design in the wireless networks consisting of homogeneous users, a great number of work has been proposed since 1990s. These studies have provided good examples on how to effectively coordinate the transmit interactions between wireless users, for achieving required SINR levels or improving system throughput. However, most of the existing schemes are of high implementation or computational complexity.
- For the capacity analysis in the wireless networks consisting of homogeneous users, plenty of work has been proposed to characterize the wireless network capacity based on stochastic geometry. However, most of the existing work focuses on the simple independent transmission schemes. Very limited work has considered a more practical transmission scheme, where users interactions are involved. Moreover, there has been no existing work study the network capacity by a SIR-based transmission scheme, to our best knowledge.
- For the transmission scheme design in CR networks, a great number of work has been proposed in the last decade to improve the coexistence between PUs and SUs in both OSA- and SS-based CR networks. While most of the existing work focused on a CR network with non-reactive PUs, some recent work has been proposed to study transmission scheme design in SS-based CR with reactive PUs. However, only limited work has been proposed for the OSA-based CR

with reactive PUs; and there has been no existing work addressing the optimal OSA design in the presence of reactive PUs, to our best knowledge.

- For the capacity analysis in CR networks, some recent work has been proposed to characterize the capacity of a CR network with non-reactive PUs, by using mathematical tools from stochastic geometry. However, there has been no existing work addressing the CR network capacity analysis in the presence of reactive PUs, to our best knowledge.

In the following chapters, we first consider wireless network consisting of homogeneous users, and focus on studying the effects of user interaction on the network capacity, by using mathematical tools from stochastic geometry. We then consider CR networks and focus on designing optimal OSA policy for the SUs in the presence of reactive PUs, by using mathematical tools from POMDP and dynamic programming.

Chapter 3

Spatial Capacity in Ad Hoc Networks

Wireless *ad hoc* networks have emerged as a promising technology that can provide seamless communication between wireless users (transmitter-receiver pairs) without relying on any pre-existing infrastructure. In such networks, the wireless users communicate with each other in a distributed manner. Due to the lack of centralized coordinators to coordinate the transmissions among the users, the wireless ad hoc network is under competitive and interference-dominant environment in nature. Thereby, efficient transmission schemes for transmitters to effectively schedule/adapt their transmissions are appealing for system performance improvement, and thus have attracted wide research attentions in the past decade. This chapter considers a large-scale ad hoc network consisting of homogeneous wireless users, and proposes the efficient signal-to-interference-ratio (SIR) threshold based scheduling schemes, by exploiting the transmission interactions between the users. By applying the proposed scheme, this chapter focuses on investigating the system-level performance of the ad hoc network, and studies the effects of user interactions on improving the ad hoc network capacity.

The rest of this chapter is organized as follows: Section 3.1 presents the motivation of studying the ad hoc network capacity by the SIR-based scheduling scheme. Section

3.2 presents the designed SIR-threshold based scheduling scheme and the network model based on stochastic geometry. Sections 3.3 focuses on the spatial capacity of the ad hoc network by the proposed scheme with single-stage probing. Sections 3.4 extends the study to the proposed scheme with multi-stage probing, and investigates the effects of the SIR thresholds on improving the spatial capacity of the ad hoc network. Finally, we conclude this chapter in Section 3.5.

3.1 Problem Motivation

Traditionally, in an ad hoc network, each transmitter is enabled to independently decide whether to transmit over a particular channel based on its own willingness or channel strength (e.g., [4]-[10]), and the transmission rate of each user can be maximized by finding an optimal transmission probability or an optimal channel strength threshold, respectively. Although easy to be implemented, such independent transmission schemes do not consider the resulting user interactions in the wireless ad hoc networks due to the co-channel interference, and thus do not achieve high system performance in general cases. Therefore, more complex transmission schemes have been proposed to exploit the user interactions by exploring the information of signal-to-interference-ratio (SIR). For example, by iteratively adapting the transmit power level based on the estimated SIR, the Foschini-Miljanic algorithm [2] assures zero outage probability and/or minimum aggregate power consumption for uplink transmission in a cellular network. In [11], Yates has studied power convergence conditions for such iterative power control algorithms. Moreover, there have been some recent studies (e.g. [12] and [13]) that extend the Foschini-Miljanic algorithm to the wireless ad hoc network through joint scheduling and power control transmission schemes. In addition, by adapting the transmission probability depending on the received SIR, [133] has studied various random access schemes to improve the system throughput and/or the user fairness. However, these schemes either require each transmitter to know the channel information of all other transmitters, or are of high implementation complexity, and thus are not appropriate for practical large-scale

wireless ad hoc networks.

On the other hand, due to the randomized location of each transmitter and the effects of channel fading, the system-level performance analysis is fundamentally important for the study of wireless ad hoc networks. It is noted that Gupta and Kumar in [15] studied *scaling laws*, which quantified the increase of the volume of capacity region over the number of transmitters in ad hoc networks. Moreover, to determine the set of active transmitters that can yield maximum aggregate Shannon capacity in the network, the authors in [35]-[37] addressed the *capacity maximization* problem for an arbitrary wireless ad hoc network. However, the authors in [15] and [35]-[37] did not address the *capacity characterization* in a wireless ad hoc network and thus can only provide limited insight for ad hoc network design problem. It came to our attention that under the assumption that the transmitters are deployed according to a homogeneous Poisson point process (PPP) [16], the exact/approximate capacity of a wireless ad hoc network under various independent transmission schemes, such as Aloha-based random transmission [8], channel-inversion based power control [9], and channel-threshold based scheduling [7], [10], can all be successfully characterized by using advanced tools from stochastic geometry [14]. However, limited work has studied the system-level performance of an ad hoc network with a SIR-based transmission scheme, where the user interactions are involved. It is noted that [134] studied probability-based scheduling, where each transmitter independently adjusts its current transmission probability based on the received SIR in the proceeding iteration. However, [134] only addressed the convergence property of the probability-based scheduling, by using stochastic geometry and submodule game. To our best knowledge, there has been no existing work on studying the network capacity with a SIR-based transmission scheme. The impacts of SIR-based transmissions, or the effects of the user interactions, are limitedly understood from the system-level point of view.

A principle goal of the study in this chapter is to fill the void of wireless network capacity characterization by an efficient SIR-based transmission scheme. To this end, we propose a novel SIR-threshold based scheduling scheme for a single-hop large-scale

Figure 3.1: Illustration of the SIR-threshold based transmission in P-Phase k .

wireless ad hoc network. We consider a probe-and-transmit protocol, where multi-stage probings are sequentially performed to gradually determine the transmitters that are allowed to transmit data in each slot. Specifically, we assume there are in total M probing phases in each slot, $1 \leq M < \infty$, which are sequentially labeled as P-Phase 0, P-Phase 1, ..., and P-Phase $M - 1$, and label the data transmission phase as D-Phase. As illustrated in Fig. 3.1, if the feedback SIR from receiver i in Prob $k - 1$, $1 \leq k \leq M - 1$, is no smaller than a pre-defined threshold, transmitter i decides to transmit in P-Phase k ; otherwise, to improve the system throughput as well as save its own energy, transmitter i stays idle in the remaining time of the slot as in [135], so as to let other transmitters that have higher SIR levels re-contend the current transmission opportunity. Since each transmitter only requires direct-channel SIR feedback from its intended receiver for limited times, the proposed scheme is appropriate for *distributed implementation* in practical wireless ad hoc networks. We characterize the wireless ad hoc network capacity with a metric called *spatial capacity*, which gives the average number of successful transmitters per unit area, for any given initial transmitter density. We aim at closed-form capacity characterization and maximization by exploring the SIR-threshold based transmission.

As will be shown in the later sections, for the SIR-threshold-based scheduling scheme, though one can use a homogeneous PPP to model the stochastic locations of the transmitters in the initial probing phase, we find that due to the iterative SIR-based scheduling, the PPP model is no longer applicable to model the locations of the retained transmitters in all the subsequent P-Phases or D-Phase. We note there

has been existing work (e.g., [57], [64], and [132]) that successfully characterized the non-PPP based network capacity, by approximating the underlying non-PPP with a homogeneous PPP of the same density. However, besides the resulting non-PPP transmitter distribution as in the existing work, the SIR distributions in all the P-Phases and the D-phase in our work are also strongly correlated. Thus, for a good spatial capacity characterization, we need to consider both non-PPP transmitter distribution and the strongly-correlated SIR distributions in all probing and data transmission phases, which makes our work more challenging.

3.2 System Model and Performance Metric

This section focuses on developing the system model and defining the performance metric. We consider single-hop communication for a stochastic time-slotted ad hoc network. In the following, we first describe the considered transmission schemes. We then develop the network model based on stochastic geometry. At last, we define the spatial capacity as our performance metric.

3.2.1 Transmission Schemes

We focus on the proposed scheme with SIR-threshold based scheduling. For comparison, we also consider a reference scheme without any transmission scheduling (or user interactions). The reference scheme and/or its variants have been widely adopted in the literature, e.g., [4]-[10]. By comparing the resultant ad hoc network capacity by the proposed scheme with that by the reference scheme, we can explicitly obtain the effects of the user interactions on the ad hoc network capacity. For both transmission schemes, we assume that all transmitters transmit in a synchronized time-slotted manner. We also assume that all transmitters transmit at the same power level,¹ which is normalized to be unity for convenience.

¹In general, each transmitters can transmit at different power levels by iteratively adjusting its transmit power based on the feedback SIR information, as in [2] or [11]. However, in the thesis, we mainly focus on SIR-based transmission scheduling and thus restrict transmit power adaptation to be binary for simplicity.

SIR-Threshold Based Scheme

Based on the probe-and-transmit protocol, in each time slot, M probing phases with $1 \leq M < \infty$ are sequentially implemented before the data transmission phase. We assume M is a pre-given parameter and its effects will be studied later in Section 3.4.2. Moreover, as shown in Fig. 3.1, we denote the duration of a time slot and a probing phase as T and τ , respectively, with $\tau \ll T$, such that $M\tau < T$, as in [135]. By normalizing over T , the *effective data transmission time* in a time slot is obtained as $\frac{T-M\tau}{T}$, which reduces linearly over M [136]. Furthermore, we assume if a transmitter transmits probing signals in a probing phase, its intended receiver is able to measure the received signal power over the total interference power, i.e., the SIR, and feeds it back to the transmitter at the end of the probing phase. For simplicity, we assume that the feedback is perfect and the SIR value is exactly known at the transmitter. Based on the feedback SIR level of its own channel, each transmitter can iteratively perform the threshold-based transmission decision in each P-Phase or D-Phase, for which the details are given as follows:

- In the initial probing phase, i.e., P-Phase 0, to initialize the communication between each transmitter and receiver pair, all transmitters independently transmit probing signals to their intended receivers. Each receiver then estimates the channel amplitude and phase (for possible coherent communication in the subsequent P-Phases and D-Phase), and measures the received SIR of the probing signal. Each transmitter receives the feedback SIR from its intended receiver at the end of P-Phase 0.
- In each of the remaining probing phases from P-Phase 1 to P-Phase $M - 1$, by exploiting the feedback SIR in the proceeding probing phase, each transmitter decides whether to transmit in the current probing phase with a predefined SIR threshold. Specifically, suppose a transmitter transmits in P-Phase $k - 1$, $1 \leq k \leq M - 1$. As shown in Fig. 3.1, if the feedback SIR in P-Phase $k - 1$ is larger than or equal to the predefined SIR-threshold, denoted by $\gamma_k \geq 0$ for P-Phase k , the transmitter continues its transmission in P-Phase k and

thus receives the feedback SIR in P-Phase k ; otherwise, to improve the system throughput as well as save its energy, the transmitter decides not to transmit any more in the remaining time of this slot and will seek another transmission opportunity in the next slot, so as to let other transmitters that have higher SIR levels to re-contend the current transmission opportunity.

- In the D-Phase, similar to the SIR-threshold based scheduling from P-Phase 1 to P-Phase $M - 1$, if a transmitter transmits in P-Phase $M - 1$ and its feedback SIR in P-Phase $M - 1$ is larger than or equal to the predefined threshold, denoted by $\gamma_M \geq 0$ for the data transmission phase, the transmitter sends data to its intended receiver; otherwise, the transmitter remains silent in the rest time of this slot. The data transmission is successful if the SIR at the receiver is larger than or equal to the required SIR level, denoted by $\beta > 0$.

It is worth noting that to design a scalable transmission scheme in a large-scale wireless ad hoc network, we assume all the transmitters adopt the same number of probing phases M , and the same SIR threshold γ_k , $1 \leq k \leq M$, for each transmission decision. The impacts of M and γ_k 's on improving the network capacity will be analyzed globally in the later sections.

Reference Scheme

To compare with the proposed scheme with SIR-based scheduling, we consider a reference scheme with no transmission scheduling. In each time slot, we assume all transmitters transmit directly to their intended receivers in an independent manner. Thus, the transmitters do not interact with each other by applying the reference scheme. Note that the considered reference scheme or its variants have been widely studied in the literature (e.g., [4]-[10]). It is easy to find that the effective data transmission time for the reference scheme is 1.² The data transmission is successful

²It is worth pointing out that for the reference scheme, an initial training is needed prior to data transmission for the receiver to estimate the channel for coherent communication, similar to the initial probing of the proposed scheme with $M = 1$, but without the SIR feedback to the transmitter. Here, we have assumed that such training incurs a negligible time overhead as compared to each slot duration.

if the SIR at the receiver is larger than or equal to the required SIR level β as the proposed scheme. Moreover, by implementing an initial probing phase before the data transmission, the reference scheme can be improved to be a proposed scheme with single-stage probing.

Advantages of the SIR-Threshold Based Scheme

The advantages of the proposed SIR-threshold based scheduling scheme are summarized as follows.

- Different from the reference scheme, the proposed SIR-threshold based scheme considers the resulting user interactions in the wireless ad hoc networks due to the co-channel interference. Thus, the proposed scheme is expected to achieve higher network capacity, if the transmission parameters (e.g., M and γ_k) are properly designed.
- Different from other transmission schemes where the user interactions are considered, such as [2], [11]-[13], the proposed scheme not only enables each transmitter to exploit its local information for deciding whether to transmit, but also can be operated with scalability. As a result, the proposed scheme can be efficiently implemented in a distributed manner in the large-scale ad hoc network.

Since the proposed scheme is simple to be implemented and effective in exploiting user interaction, we adopt it for analyzing the impact of user interactions on improving wireless ad hoc network capacity.

3.2.2 Network Model

In the next, we develop the network model based on stochastic geometry. For both considered transmission schemes, we focus on single-hop communication in one particular time slot.

For both schemes, we assume that all initial transmitters that have the intention to transmit in a time slot are independently and uniformly distributed in the

unbounded two-dimensional plane $\mathbb{R}^2$. We thus model the locations of all initial transmitters by a homogeneous PPP, denoted by Φ_0 , of density λ_0 . Each transmitter has one intended receiver, which is assumed to be uniformly distributed on a circle of radius d meters (m) centered at the transmitter. We denote the locations of the i -th transmitter and its intended receiver as x_i , with $x_i \in \Phi_0$, and r_i (not included in Φ_0), respectively. The path loss between the i -th transmitter and the j -th receiver is given by $l_{ij} = |x_i - r_j|^{-\alpha}$, where $\alpha > 2$ is the path-loss exponent. We use h_{ij} to denote the distance-independent channel fading coefficient from transmitter i to receiver j . We assume flat Rayleigh fading, where all h_{ij} 's are independent and exponentially distributed random variables with unit mean. We also assume that h_{ij} 's do not change within one time-slot. We denote the SIR at the i -th receiver as $\text{SIR}_i^{(0)}$, which is given by

$$\text{SIR}_i^{(0)} = \frac{h_{ii}d^{-\alpha}}{\sum_{x_j \in \Phi_0, j \neq i} h_{ji}l_{ji}}. \quad (3.1)$$

Note that for the reference scheme without transmission scheduling, $\text{SIR}_i^{(0)}$ gives the received SIR level at the i -th receiver for the data transmission of transmitter i . As a result, in the reference scheme, the data transmission of transmitter i is *successful* if $\text{SIR}_i^{(0)} \geq \beta$ is satisfied.

Unlike the reference scheme, in the proposed scheme, $\text{SIR}_i^{(0)}$ only gives the received SIR level at the i -th receiver in the initial probing phase P-Phase 0. We then denote the point process formed by the retained transmitters in P-Phase k with $1 \leq k \leq M-1$, or the data transmission phase with $k = M$, as Φ_k . We also denote $\text{SIR}_i^{(k)}$ as the received SIR at the i -th receiver in Φ_k . Clearly, we have $\Phi_k = \{x_i \in \Phi_{k-1} : \text{SIR}_i^{(k-1)} \geq \gamma_k\}$, where the number of transmitters in Φ_k is reduced as compared to that in Φ_{k-1} . Thus, it is easy to verify that $\text{SIR}_i^{(k)} \geq \text{SIR}_i^{(k-1)}$ for any given $\gamma_k \geq 0$, $\forall i \in \Phi_{k-1} \cap \Phi_k$. Moreover, similar to $\text{SIR}_i^{(0)}$, for any Φ_k , $k \in \{1, \dots, M\}$, we can express $\text{SIR}_i^{(k)}$ as

$$\text{SIR}_i^{(k)} = \frac{h_{ii}d^{-\alpha}}{\sum_{x_j \in \Phi_k, j \neq i} h_{ji}l_{ji}}, \quad k \in \{1, \dots, M\}. \quad (3.2)$$

It is worth noting that due to the SIR-based scheduling, the transmitters are not retained independently in Φ_k . Thus, unlike $\text{SIR}_i^{(0)}$ in (3.1), which is determined by the homogeneous PPP Φ_0 , $\text{SIR}_i^{(k)}$ in (3.2) is determined by the *non-PPP* Φ_k in general [14]. For the proposed scheme, the data transmission of transmitter i is *successful* if $\text{SIR}_i^{(k-1)} \geq \gamma_k$, $\forall k \in \{1, \dots, M\}$, and $\text{SIR}_i^{(M)} \geq \beta$ are all satisfied.

3.2.3 Spatial Capacity

Due to the stationarity of the homogeneous PPP Φ_0 , it is easy to verify that Φ_k , $\forall k \in \{1, \dots, M\}$, is also stationary [10]. We thus consider a typical pair of transmitter and receiver. Without loss of generality, we assume that the typical receiver is located at the origin. The typical pair of transmitter and receiver is named pair 0, i.e., $i = 0$. Denote the *successful transmission probability* of the typical pair in the D-Phase of the proposed scheme with M probing phases or the reference scheme as $\mathcal{P}_0^{p,M}$ or $\mathcal{P}_0^r$, respectively. We thus have

$$\mathcal{P}_0^{p,M} = P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M, \text{SIR}_0^{(M)} \geq \beta). \quad (3.3)$$

$$\mathcal{P}_0^r = P(\text{SIR}_0^{(0)} \geq \beta). \quad (3.4)$$

We adopt *spatial capacity* as our performance metric, which is defined as the spatial density of successful transmissions, or more specifically the average number of transmitters with successful data transmission per unit area. Considering the effective data transmission time in a time slot, we thus define the spatial capacity by the proposed scheme with M probing phases and the reference scheme as $\mathcal{C}^{p,M}$ and $\mathcal{C}^r$, respectively, given by

$$\mathcal{C}^{p,M} \triangleq \frac{T - M\tau}{T} \lambda_0 \mathcal{P}_0^{p,M}, \quad (3.5)$$

$$\mathcal{C}^r \triangleq \lambda_0 \mathcal{P}_0^r. \quad (3.6)$$

First, we look at the reference scheme. From (3.1) and (3.4), we obtain

$$\begin{aligned}
\mathcal{P}_0^r &= P(h_{00} \geq \beta d^\alpha h_{j0} l_{j0}) \\
&= E_{\Phi_0, \{h_{j0}\}} \left[\prod_{j \in \Phi_0, j \neq 0} e^{-\beta d^\alpha h_{j0} l_{j0}} \right] \\
&\stackrel{(a)}{=} E_{\Phi_0} \left[\prod_{j \in \Phi_0, j \neq 0} \frac{1}{1 + \beta d^\alpha l_{j0}} \right] \\
&\stackrel{(b)}{=} \exp \left(-2\pi \lambda_0 \int_0^\infty \left(1 - \frac{1}{1 + \beta d^\alpha u^{-\alpha}} \right) u \, du \right)
\end{aligned} \tag{3.7}$$

where (a) follows by taking expectation over each h_{j0} , which holds since fading is assumed to be independent of the point process, and all h_{j0} 's are independent of each other. Moreover, (b) follows by applying probability generating functional (PGFL) of the PPP³ and then changing the rectangular coordinates to the polar coordinates. From (3.7), by letting $v = \left(\frac{u}{\beta^{1/\alpha} d} \right)^2$, we obtain the following proposition.

Proposition 3.2.1. *The successful transmission probability in the reference scheme is*

$$\mathcal{P}_0^r = \exp(-\pi \lambda_0 d^2 \beta^{\frac{2}{\alpha}} \rho), \tag{3.8}$$

where $\rho = \int_0^\infty \frac{1}{1+v^{\alpha/2}} dv$. When $\alpha = 4$, we have $\rho = \frac{\pi}{2}$. Furthermore, from (3.6) and (3.8), we obtain the spatial capacity $\mathcal{C}^r$ by the reference scheme as

$$\mathcal{C}^r = \lambda_0 \exp(-\pi \lambda_0 d^2 \beta^{\frac{2}{\alpha}} \rho). \tag{3.9}$$

Remark. Since the network interference level in the D-Phase increases over the initial transmitter density λ_0 , we find that $\mathcal{P}_0^r$ in (3.8) monotonically decreases over λ_0 as expected. However, it is observed from (3.9) that unlike $\mathcal{P}_0^r$, the spatial capacity $\mathcal{C}^r$ does not vary monotonically over λ_0 , since $\mathcal{C}^r$ can be benefited by increasing λ_0 if the resulting interference is acceptable. It is also observed from (3.8) and (3.9) that both

³Let $f(x) : \mathbb{R}^d \rightarrow [0, \infty)$ be measurable. The PGFL of a point process Θ in $\mathbb{R}^d$ is defined as $E \left[\prod_{x \in \Theta} f(x) \right]$. For a homogeneous PPP Φ with density $\lambda > 0$, $E \left[\prod_{x \in \Phi} f(x) \right] = \exp \left(-\lambda \int_{x \in \mathbb{R}^d} (1 - f(x)) dx \right)$ [4].

$\mathcal{P}_0^r$ and $\mathcal{C}^r$ exponentially vary over the transmitter-receiver distance d and the required SIR level β , due to the Rayleigh fading. Moreover, both $\mathcal{P}_0^r$ and $\mathcal{C}^r$ monotonically decrease d , due to the reduced signal power received at the receiver, and decrease over β .

Next, we focus on the proposed scheme. However, unlike the reference scheme, which is determined by the homogeneous PPP Φ_0 , the proposed scheme is jointly determined by Φ_0 and a sequence of non-PPPs $\{\Phi_k\}$, $1 \leq k \leq M$, where the resulting SIR distributions are correlated. Therefore, it is very difficult to analyze/characterize the spatial capacity of the proposed scheme with M probing phases. To start up, in the next section, we focus on a simple case with single-stage probing ($M = 1$) for some insightful results.

3.3 Proposed Scheme with Single-Stage Probing

In this section, we consider the proposed scheme with single-stage probing, i.e., $M = 1$. In this case, there is only one round of SIR-based scheduling, which is implemented with the threshold γ_1 . For notational simplicity, for the case of $M = 1$, we omit the superscript M and use $\mathcal{P}_0^p$ and $\mathcal{C}^p$ to represent the successful transmission probability and the spatial capacity of the typical transmitter, respectively. Based on (3.3), the successful transmission probability for the case of $M = 1$ is reduced to

$$\mathcal{P}_0^p = P(\text{SIR}_0^{(0)} \geq \gamma_1, \text{SIR}_0^{(1)} \geq \beta) \quad (3.10)$$

$$= P(\text{SIR}_0^{(0)} \geq \gamma_1)P(\text{SIR}_0^{(1)} \geq \beta | \text{SIR}_0^{(0)} \geq \gamma_1). \quad (3.11)$$

Moreover, when $M = 1$, the effective data transmission time for the proposed scheme is $\frac{T-\tau}{T}$. Since $\tau \ll T$, we assume the single-stage probing overhead is negligible; and thus, the effective data transmission time becomes 1 as the reference scheme. Consequently, based on (3.5), we can express the spatial capacity $\mathcal{C}_0^p$ as

$$\mathcal{C}^p = \lambda_0 \mathcal{P}_0^p. \quad (3.12)$$

Furthermore, by substituting (3.11) to (3.12), we can express $\mathcal{C}^p$ alternatively as

$$\begin{aligned}\mathcal{C}^p &= \lambda_0 P(\text{SIR}_0^{(0)} \geq \gamma_1) P(\text{SIR}_0^{(1)} \geq \beta | \text{SIR}_0^{(0)} \geq \gamma_1) \\ &= \lambda_1 P(\text{SIR}_0^{(1)} \geq \beta | \text{SIR}_0^{(0)} \geq \gamma_1)\end{aligned}\quad (3.13)$$

where $\lambda_1 = \lambda_0 P(\text{SIR}_0^{(0)} \geq \gamma_1)$ is the density of Φ_1 in the D-Phase, with $\lambda_1 \leq \lambda_0$. Based on Proposition 3.2.1, by replacing β with γ_1 , it is easy to find that

$$\lambda_1 = \lambda_0 \exp\left(-\pi \lambda_0 d^2 \gamma_1^{\frac{2}{\alpha}} \rho\right). \quad (3.14)$$

In the following two subsections, we compare the spatial capacity of the two considered schemes, and characterize $\mathcal{C}^p$ for the proposed scheme.

3.3.1 Spatial Capacity Characterization: $\gamma_1 = 0$ and $\gamma_1 \geq \beta$

In this subsection, we compare the spatial capacity of the proposed scheme with that of the reference scheme. We then characterize the spatial capacity $\mathcal{C}^p$ for the proposed scheme and obtain closed-form expressions for the cases of $\gamma_1 = 0$ and $\gamma_1 \geq \beta$.

First, from (3.6) and (3.12), to compare $\mathcal{C}^p$ and $\mathcal{C}^r$, the key is to compare $\mathcal{P}_0^p$ and $\mathcal{P}_0^r$. In the reference scheme, denote the total interference power received at the typical receiver as $I_0 = \sum_{x_i \in \Phi_0, i \neq 0} h_{i0} l_{i0}$. In the proposed scheme, the received total interference power at the typical receiver in P-Phase 0 is thus I_0 , while that in the D-Phase is given by $I_1 = \sum_{x_i \in \Phi_1, i \neq 0} h_{i0} l_{i0}$. For any $\gamma_1 \geq 0$, we have $I_0 \geq I_1$ since $\Phi_1 \subseteq \Phi_0$, and thus $\text{SIR}_i^{(1)} \geq \text{SIR}_i^{(0)}$. As a result, by changing over the value of $\gamma_1 \in [0, \infty)$, we obtain the following proposition.

Proposition 3.3.1. *Given the required SIR level $\beta > 0$, for any $\gamma_1 \in [0, \infty)$, we have*

$$\begin{cases} \mathcal{C}^p > \mathcal{C}^r, & \text{if } 0 < \gamma_1 < \beta \text{ (conservative transmission regime)} \\ \mathcal{C}^p = \mathcal{C}^r, & \text{if } \gamma_1 = 0 \text{ or } \gamma_1 = \beta \text{ (neutral transmission regime)} \\ \mathcal{C}^p < \mathcal{C}^r, & \text{if } \gamma_1 > \beta \text{ (aggressive transmission regime).} \end{cases} \quad (3.15)$$

Proof. Please refer to Appendix A. □

Remark. Compared to the spatial capacity of the reference scheme, Proposition 3.3.1 shows that for the proposed scheme with SIR-threshold based scheduling, due to the reduced interference level in the D-Phase, the spatial capacity is improved in the conservative transmission regime with $0 < \gamma_1 < \beta$. However, in the case of the aggressive transmission regime with $\gamma_1 > \beta$, where the transmitters that are able to transmit successfully in the data transmission phase may also be removed from transmission, the retained transmitters in the D-Phase are overly reduced. Consequently, the spatial capacity is reduced in the aggressive transmission regime. It is also noted that in the neutral transmission regime with $\gamma_1 = 0$ or $\gamma_1 = \beta$, the spatial capacity is identical for the two schemes. At last, it is worth noting that Proposition 3.3.1 holds regardless of the specific channel fading distribution and/or transmitter location distribution.

Next, we characterize the spatial capacity $\mathcal{C}^p$ for the proposed scheme with $M = 1$. We focus on deriving the successful transmission probability $\mathcal{P}_0^p$ in (3.10). Unlike $\mathcal{P}_0^r$ in (3.4), which is given by the marginal CCDF of $\text{SIR}_0^{(0)}$ taken at value β , $\mathcal{P}_0^p$ is given by the joint CCDF of $\text{SIR}_0^{(0)}$ and $\text{SIR}_0^{(1)}$ taken at values (γ_1, β) . In the following, we consider three cases $\gamma_1 = 0$, $\gamma_1 \geq \beta$, and $0 < \gamma_1 < \beta$, and find closed-form spatial capacity expressions for both cases of $\gamma_1 = 0$ and $\gamma_1 \geq \beta$.

Specifically, for the simple case with $\gamma_1 = 0$, we can infer from Proposition 3.3.1 directly that $\mathcal{C}^p = \mathcal{C}^r$, which is given in (3.9). For the case of $\gamma_1 \geq \beta$, since $\text{SIR}_0^{(1)} > \text{SIR}_0^{(0)}$, we have $P(\text{SIR}_0^{(1)} \geq \beta | \text{SIR}_0^{(0)} \geq \gamma_1) = 1$. According to (3.11), we thus obtain $\mathcal{P}_0^p = P(\text{SIR}_0^{(0)} \geq \gamma_1)$ in this case. By replacing β with γ_1 in Proposition 3.2.1, we further obtain that $\mathcal{P}_0^p = \exp(-\pi\lambda_0 d^2 \gamma_1^{\frac{2}{\alpha}} \rho)$. As a result, based on (3.12), we can express $\mathcal{C}^p$ for the case of $\gamma_1 \geq \beta$ as

$$\mathcal{C}^p = \lambda_0 \exp(-\pi\lambda_0 d^2 \gamma_1^{\frac{2}{\alpha}} \rho). \quad (3.16)$$

Similar to $\mathcal{C}^r$ given in (3.9), it is observed that $\mathcal{C}^p$ for both cases of $\gamma_1 = 0$ and $\gamma_1 \geq \beta$ does not vary monotonically over λ_0 , but monotonically decreases over the distance d between each transmitter and receiver pair. Moreover, unlike $\mathcal{C}^r$ and $\mathcal{C}^p$ for $\gamma_1 = 0$, $\mathcal{C}^p$ for $\gamma_1 \geq \beta$ is not related to the required SIR level β any more, since all the retained

transmitters in the D-Phase meet the condition $\text{SIR}_i^{(1)} \geq \beta$ in this case.

However, for the case of $0 < \gamma_1 < \beta$, $\mathcal{P}_0^p$ cannot be simply expressed by a marginal CCDF of $\text{SIR}_0^{(0)}$ as in the above two cases. Moreover, from (3.10), due to the correlation between $\text{SIR}_0^{(0)}$ and $\text{SIR}_0^{(1)}$ as well as the underlying non-PPP Φ_1 that determines $\text{SIR}_0^{(1)}$, it is very difficult, if not impossible, to find an exact expression of $\mathcal{P}_0^p$ and thus $\mathcal{C}^p$ in this case. As a result, in the next subsection, we focus on finding a tight approximate to $\mathcal{C}^p$ with a tractable expression for the case of $0 < \gamma_1 < \beta$.

3.3.2 Spatial Capacity Characterization: $0 < \gamma_1 < \beta$

This subsection focuses on approximating the spatial capacity of the proposed scheme for the case of $0 < \gamma_1 < \beta$. We first propose a new approximate approach for $\mathcal{C}^p$ and obtain an integral-based expression. Next, to find a closed-form expression for $\mathcal{C}^p$, we further approximate the integral-based expression obtained by the proposed approach. At last, we apply the conventional approximate approach in the literature and discuss its approximate performance. The details of the three approximate approaches are given as follows.

Proposed Approximation

From (3.10), to find a good approximate to $\mathcal{P}_0^p$ and thus $\mathcal{C}^p$, the key is to find a good approximate to the joint SIR distributions in Φ_0 and Φ_1 . Since $\Phi_1 \subseteq \Phi_0$, we first divide the initial PPP Φ_0 into two *disjoint non-PPPs*: one is Φ_1 , and the other is its complementary set $\Phi_1^c = \Phi_0 - \Phi_1$, which is the point process formed by the non-retained transmitters in the data transmission phase. We denote the density of Φ_1^c as $\lambda_1^c = \lambda_0 - \lambda_1$. Clearly, Φ_1 and Φ_1^c are mutually dependent. Denote the received SIR level at the typical receiver in Φ_1^c as $\text{SIR}_0^{(1,c)} = h_{00}d^{-\alpha} / \sum_{i \in \Phi_1^c} h_{i0}l_{i0}$. Since $\Phi_1 \cup \Phi_1^c = \Phi_0$ and $\Phi_1 \cap \Phi_1^c = \emptyset$, we have $1/\text{SIR}_0^{(0)} = 1/(\text{SIR}_0^{(1)} + \text{SIR}_0^{(1,c)})$. As a result, (3.10) can be equally represented by using the joint distributions of $\text{SIR}_0^{(1)}$ and $\text{SIR}_0^{(1,c)}$.

Next, we state an assumption, based on which we can use a homogeneous PPP to approximate Φ_1 and Φ_1^c , respectively, such that the existing results on PPP interference distribution in the literature can be applied to approximate the joint distributions

of $\text{SIR}_0^{(1)}$ and $\text{SIR}_0^{(1,c)}$.

Assumption 1. *In the proposed scheme with $M = 1$, the transmitters are retained independently in the D-Phase, with probability $P(\text{SIR}_0^{(0)} \geq \gamma_1)$.*

Remark. *By applying Assumption 1, we denote the resulting point processes formed by the retained and non-retained transmitters in the D-Phase as $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, respectively. Clearly, both $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$ are homogeneous PPPs; and the density of $\hat{\Phi}_1$ or $\hat{\Phi}_1^c$ is the same as that of Φ_1 or Φ_1^c , respectively. It is worth noting that from [57], [64], [75] and [132], the homogeneous PPPs $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$ provide good approximations to the non-PPP Φ_1 and Φ_1^c , respectively; and such good approximation performance are also verified in Appendix B.*

Since the two homogeneous PPPs $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$ are disjoint, they are *independent* of each other [14]. Denote $\hat{I}_1 = \sum_{i \in \hat{\Phi}_1} h_{i0} l_{i0}$ and $\hat{I}_1^c = \sum_{i \in \hat{\Phi}_1^c} h_{i0} l_{i0}$ as the received interference power at the typical receiver in $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, respectively. We then use $f_{\hat{I}_1}(x_1)$ and $f_{\hat{I}_1^c}(x_2)$ to denote the probability density functions (pdfs) of $\hat{I}_1$ and $\hat{I}_1^c$, respectively. The following lemma gives the general interference pdf in a homogeneous PPP-based network with Rayleigh fading channels, which is a well-known result in the literature (e.g., [4]).

Lemma 3.1. *For any homogeneous PPP of density $\lambda \geq 0$, if the channel fading is Rayleigh distributed, the pdf of the received interference I at the typical receiver is given by*

$$f_I(x) = \frac{1}{\pi x} \sum_{i=1}^{\infty} \frac{(-1)^{i+1} \Gamma(1+2i/\alpha) \sin(2\pi i/\alpha)}{i!} \left(\frac{\lambda \pi^2 2/\alpha}{x^{2/\alpha} \sin(2\pi/\alpha)} \right)^i. \quad (3.17)$$

Moreover, when $\alpha = 4$, (3.17) can be further expressed in a simpler closed-form as

$$f_I(x) = \frac{\lambda}{4} \left(\frac{\pi}{x} \right)^{3/2} \exp \left(-\frac{\pi^4 \lambda^2}{16x} \right). \quad (3.18)$$

As a result, based on Lemma 3.1, by substituting $\lambda = \lambda_1$ to (3.17) and (3.18), we can obtain $f_{\hat{I}_1}(x_1)$ for the cases of general α and $\alpha = 4$, respectively. Similarly, with $\lambda =$

λ_1^c , from (3.17) and (3.18) we can obtain $f_{\hat{I}_1^c}(x_2)$ for general α and $\alpha=4$, respectively. Therefore, by approximating Φ_1 and Φ_1^c by $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, respectively, we can well approximate the joint distribution of $\text{SIR}_0^{(1)}$ and $\text{SIR}_0^{(1,c)}$ based on the interference pdfs $f_{\hat{I}_1}(x_1)$ and $f_{\hat{I}_1^c}(x_2)$, and thereby obtain an integral-based good approximate to $\mathcal{P}_0^p$ in the following proposition.

Proposition 3.3.2. *The successful transmission probability by the proposed scheme for the case of $0 < \gamma_1 < \beta$ is well approximated as*

$$\mathcal{P}_0^p \approx \int_0^\infty e^{-h_{00}} \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) \int_0^{\frac{h_{00}}{\gamma_1 d^\alpha} - x_1} f_{\hat{I}_1^c}(x_2) dx_2 dx_1 dh_{00}. \quad (3.19)$$

The exact approximate error is decided by the performance error of approximating the non-PPPs Φ_1 and Φ_1^c with $\hat{\Phi}_1$ or $\hat{\Phi}_1^c$, respectively.

Proof. Please refer to Appendix C. □

Since we have shown that it is of good performance by approximating the non-PPPs Φ_1 and Φ_1^c with $\hat{\Phi}_1$ or $\hat{\Phi}_1^c$, respectively, study on the exact approximate error of (3.19) is left as our future work.

Finally, by multiplying λ_0 with the right-hand side of (3.19), we obtain an integral-based approximate to $\mathcal{C}^p$ for the case of $0 < \gamma_1 < \beta$ as

$$\mathcal{C}^p \approx \lambda_0 \int_0^\infty e^{-h_{00}} \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) \int_0^{\frac{h_{00}}{\gamma_1 d^\alpha} - x_1} f_{\hat{I}_1^c}(x_2) dx_2 dx_1 dh_{00}. \quad (3.20)$$

Note that the proposed approximate approach considers the correlation between $\text{SIR}_0^{(0)}$ and $\text{SIR}_0^{(1)}$, and only adopts PPP-based approximation to approximate Φ_1 and Φ_1^c by $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, respectively. Since it has been shown in Appendix B as well as the literature (e.g., [57], [64], [75], [132]) that such PPP-based approximation can provide good/tight approximate to the corresponding non-PPP, the proposed approximate approach is able to provide good spatial capacity approximate to $\mathcal{C}^p$ for the case of $0 < \gamma_1 < \beta$ in general.

Closed-form Approximation for (3.20)

Although the spatial capacity expression obtained in (3.20) is easy to integrate, it is not of closed-form. Thus, based on (3.19), we focus on finding a closed-form approximate to $\mathcal{P}_0^p$ and thus $\mathcal{C}^p$. We first increase the upper limit of $f_{\hat{I}_c}(x_2)$ in (3.19) from $\gamma_1 d^\alpha - x_1$ to $\gamma_1 d^\alpha$ to obtain an upper bound for the right-hand side of (3.19). Then by properly lower-bounding the obtained upper bound based on Chebyshev's inequality [137], we obtain a closed-form approximate to $\mathcal{P}_0^p$, which is shown in the following proposition.

Proposition 3.3.3. *Based on the integral-based expression given in (3.19), a closed-form approximate to $\mathcal{P}_0^p$ for the case of $0 < \gamma < \beta$ is obtained as*

$$\mathcal{P}_0^p \approx \exp(-\pi \lambda_1 d^2 \beta^{\frac{2}{\alpha}} \rho) \exp(-\pi \lambda_1^c d^2 \gamma_1^{\frac{2}{\alpha}} \rho). \quad (3.21)$$

Proof. Please refer to Appendix D. □

From (3.12), (3.14) and (3.21), we obtain a closed-form approximate to spatial capacity of the proposed scheme for the case of $0 < \gamma_1 < \beta$ as

$$\begin{aligned} \mathcal{C}^p \approx & \lambda_0 \times \exp\left(-\pi \lambda_0 d^2 \gamma_1^{\frac{2}{\alpha}} \rho\right) \times \exp\left[-\pi \lambda_0 \exp(-\pi \lambda_0 d^2 \gamma_1^{\frac{2}{\alpha}} \rho) d^2 \beta^{\frac{2}{\alpha}} \rho\right] \\ & \times \exp\left[\pi \lambda_0 \exp(-\pi \lambda_0 d^2 \gamma_1^{\frac{2}{\alpha}} \rho) d^2 \gamma_1^{\frac{2}{\alpha}} \rho\right]. \end{aligned} \quad (3.22)$$

Conventional Approximation

It is noted that the conventional approximate approach in the literature (e.g., [57], [64], [75], [132]), which only focuses on dealing with the non-PPP Φ_1 , can often yield a closed-form expression. Thus, in the following, we apply the conventional approximate approach and discuss its approximate performance to $\mathcal{C}^p$.

First, since only the performance in Φ_1 is concerned by the conventional approximate approach, it takes $P(\text{SIR}_0^{(1)} \geq \beta)$ as the successful transmission probability of the typical transmitter in the D-Phase. Next, the non-PPP Φ_1 is approximated by the homogeneous PPP $\hat{\Phi}_1$ under Assumption 1. We denote the received SIR at the typical

receiver in $\hat{\Phi}_1$ as $\text{SIR}_0^{(1)} = h_{00}d^{-\alpha} / \sum_{i \in \hat{\Phi}_1} h_{i0}l_{i0}$. Thus, $P(\text{SIR}_0^{(1)} \geq \beta)$ is approximated by $P(\text{SIR}_0^{(1)} \geq \beta)$. At last, by adopting the product of λ_1 and $P(\text{SIR}_0^{(1)} \geq \beta)$ as an approximate to the spatial capacity $\mathcal{C}^p$, a closed-form approximate to $\mathcal{C}^p$ for the case of $0 < \gamma_1 < \beta$ is obtained as

$$\mathcal{C}^p \approx \lambda_1 \times P(\text{SIR}_0^{(1)} \geq \beta) \quad (3.23)$$

$$\stackrel{(a)}{=} \lambda_0 \exp(-\pi \lambda_0 d^2 \gamma_1^{\frac{2}{\alpha}} \rho) \exp\left[-\pi \lambda_0 \exp(-\pi \lambda_0 d^2 \gamma_1^{\frac{2}{\alpha}} \rho) d^2 \beta^{\frac{2}{\alpha}} \rho\right] \quad (3.24)$$

where (a) follows by Proposition 3.2.1 and (3.14). Note that since $\lambda_1 = \lambda_0 \times P(\text{SIR}_0^{(0)} \geq \gamma_1)$, we can rewrite (3.23) as $\mathcal{C}^p \approx \lambda_0 \times P(\text{SIR}_0^{(0)} \geq \gamma_1) P(\text{SIR}_0^{(1)} \geq \beta)$ under the conventional method. However, according to the definition of $\mathcal{C}^p$ for $M = 1$, which is given in (3.10) and (3.12), we have $\mathcal{C}^p = \lambda_0 \times P(\text{SIR}_0^{(0)} \geq \gamma_1, \text{SIR}_0^{(1)} \geq \beta)$, where the distribution of $\text{SIR}_0^{(1)}$ is strongly dependent on that of $\text{SIR}_0^{(0)}$ as $\Phi_1 \subseteq \Phi_0$. As a result, the conventional approximate approach only focuses on the PPP-based approximate to Φ_1 , but ignores the dependence between Φ_0 and Φ_1 . Therefore, (3.23) does not hold for representing, or reasonably approximating, the spatial capacity of the proposed scheme. In addition, by comparing (3.22) and (3.24), it is observed that for the case of $0 < \gamma_1 < \beta$, given any $\lambda_0 > 0$ and $d > 0$, the closed-form spatial capacity obtained based on the proposed approach is always outperformed that by the conventional approach.

3.3.3 Numerical Results

Numerical results are presented in this subsection. According to the method described in [14], we generate a spatial Poisson process, in which the transmitters are placed uniformly in a square of $[0\text{m}, 600\text{m}] \times [0\text{m}, 600\text{m}]$. To take care of the border effects, we focus on sampling the transmitters that locate in the interim square of $[200\text{m}, 400\text{m}] \times [200\text{m}, 400\text{m}]$. We calculate the spatial capacity as the average of the network capacity over 2000 independent network realizations, where for each network realization, the network capacity is evaluated as the ratio of the number of successful transmitters in the sampling square to the square area of $4 \times 10^2 \text{m}^2$. Unless otherwise specified, in

this subsection, we set $\alpha = 4$, $\beta = 2.5$, and $d = 10\text{m}$. We also observe by simulation that similar performance can be obtained by using other parameters.

In the following, we first validate our analytical results on the spatial capacity of the proposed scheme and the reference scheme without scheduling. To highlight the spatial capacity improvement performance of the proposed scheme, we then compare the spatial capacity achieved by the proposed scheme with that by two existing distributed scheduling schemes: one is the probability-based scheduling in [134], and the other is the channel-threshold based scheduling in [7] and [10]. At last, we consider a more practical scenario with SIR estimation and feedback errors, and show the effects of the SIR errors on the spatial capacity of the proposed scheme.

Validation of the Spatial Capacity Analysis

We validate our spatial capacity analysis in Section 3.3.1 and Section 3.3.2 for both proposed and reference schemes.

Fig. 3.2 shows the spatial capacity versus the SIR threshold γ_1 , for both the reference scheme without transmission scheduling and the proposed scheme with SIR-based scheduling. We set the initial transmitter density as $\lambda_0 = 0.0025/\text{m}^2$ in both schemes. The analytical spatial capacity of the reference scheme is given in (3.9). By comparing the simulation results for the proposed scheme with the analytical results for the reference scheme, we observe that $\mathcal{C}^r$ is constant over γ_1 as expected. We also observe that 1) when $\gamma_1 < \beta$, $\mathcal{C}^p > \mathcal{C}^r$; 2) when $\gamma_1 = 0$ or $\gamma_1 = \beta$, $\mathcal{C}^p = \mathcal{C}^r$; and 3) when $\gamma_1 > \beta$, $\mathcal{C}^p < \mathcal{C}^r$. This is in accordance with our analytical results in Proposition 3.3.1. Moreover, for the proposed scheme, we adopt (3.9) and (3.16) as the analytical spatial capacity for the cases of $\gamma_1 = 0$ and $\gamma_1 \geq \beta$, respectively, and observe that the analytical results of the spatial capacity fit well to the simulation counterparts. Furthermore, for the case of $0 < \gamma_1 < \beta$ of the proposed scheme, where only approximate expressions for the spatial capacity are available, we compare the approximate performance of the three approximate approaches given in Section 3.3.2. It is observed that the integral-based expression by the proposed approximate approach, given in (3.20), provides a tight approximate to $\mathcal{C}^p$ for the case of $0 < \gamma_1 <$

Figure 3.2: Spatial capacity against γ_1 with $\lambda_0 = 0.0025$ and $\beta = 2.5$.

β . In addition, as a cost of expressing in closed-form, (3.22) is not as tight as (3.20), but (3.22) still provides a close approximate to $\mathcal{C}^p$ for the case of $0 < \gamma_1 < \beta$. At last, it is observed that the closed-form expression given in (3.24) by the conventional approximate approach cannot properly approximate $\mathcal{C}^p$ for the case of $0 < \gamma_1 < \beta$ as expected.

Fig. 3.3 shows the spatial capacity versus the initial transmitter density λ_0 when $\gamma_1 < \beta$. We set $\gamma_1 = 0.6$. For the proposed scheme, similar to the case in Fig. 3.2, we observe tight and close approximates are provided by (3.20) and (3.22), respectively, based on the proposed approximate approach, while improper approximate is provided by (3.24) based on the conventional approximate approach. Moreover, it is observed that the spatial capacity of the proposed scheme is always larger than that of the reference scheme, given in (3.9), for all values of λ_0 , which is as expected from Proposition 3.3.1 since $\gamma_1 < \beta$ in this example. Furthermore, for both the proposed and reference schemes, we observe an interesting *density-capacity tradeoff*: by increasing λ_0 , the spatial capacity first increases due to more available transmitters,

Figure 3.3: Spatial capacity against λ_0 . $\gamma_1 = 0.6$. $\beta = 2.5$.

but as λ_0 exceeds a certain threshold, it starts to decrease, due to the more dominant interference effect. Thus, to maximize the spatial capacity, under the system scenario set in Fig. 3.3, the optimal λ_0 should be set as $0.003/m^2$.

Performance Comparison with Existing Distributed Schemes

We consider two existing distributed scheduling schemes for performance comparison. The first scheme is the iterative probability-based scheduling as in [134]. Denote the transmission probability for transmitter i in P-Phase k , $1 \leq k \leq M - 1$, and the D-Phase as $\phi_i^{(k)}$ or $\phi_i^{(M)}$, respectively. For any $k \in \{1, \dots, M\}$, [134] sets $\phi_i^{(k)} = \min\left(\frac{\text{SIR}_i^{(k-1)}}{\beta}, 1\right)$. Intuitively, [134] provides a simple and proper way to iteratively adjust the transmission probability $\phi_i^{(k)}$. The second scheme is the channel-threshold based scheduling with single-stage probing as in [7] and [10], where the received interference power is not involved in the transmission decision and each transmitter decides to transmit in the D-Phase if its direct channel strength in P-Phase 0 is no smaller than a predefined threshold γ'_1 , i.e., $h_{ii} \geq \gamma'_1$. For a fair comparison, we

Figure 3.4: Spatial capacity comparison with existing distributed scheduling schemes. $\gamma_1 = 0.4$. $\beta = 2.5$.

consider single-stage probing with $M = 1$ for all the proposed SIR-threshold based scheme, the probability-based scheduling in [134], and the channel-threshold based scheduling in [7] and [10].

Fig. 3.4 shows the spatial capacities achieved by the proposed scheme, the probability-based scheduling, the channel-threshold based scheduling, and the reference scheme without scheduling. To clearly show the effects of involving interference in the transmission decision for the proposed scheme, we set $\gamma'_1 = \gamma_1 = 0.4$ for the channel-threshold based scheduling. We obtain the spatial capacity of the channel-threshold based scheduling by applying its exact expression given in [10]. Due to the lack of an exact spatial capacity expression for the probability-based scheduling, we obtain its spatial capacity by simulation. We list our observations from Fig. 3.4 as follows:

- *SIR based schemes v.s. channel-threshold based scheme:* It is observed that

by adapting the transmission decision to the SIR, the achieved spatial capacities by both the proposed scheme and the probability-based scheduling are always higher than that by the channel-threshold based scheduling, where the interference information is not exploited. Moreover, the spatial capacity of the channel-threshold based scheduling is smaller than that of the reference scheme when λ_0 is small, and becomes larger when λ_0 is sufficiently large. This is in sharp contrast to the cases of the proposed scheme and the probability-based scheduling, which always guarantee capacity improvement over the reference scheme without scheduling.

- *SIR-threshold based scheduling v.s. probability-based scheduling:* It is interesting to observe that although both the proposed scheme and the probability-based scheduling adapt the transmission decision to the SIR, the achieved spatial capacity by the former scheme is always higher than that by the latter one in this simulation. This is because that the proposed scheme assures the improvement of the successful transmission probability of each retained transmitter in the D-Phase, while the probability-based scheduling only assures such improvement with some probability. Moreover, it is observed that the optimal initial transmitter density that maximizes the spatial capacity of the proposed scheme is $\lambda_0^* = 0.0036$, which is larger than that for the probability-based scheduling locating at $\lambda_0^* = 0.0026$.

Note that for the proposed scheme, a lower SIR threshold γ_1 allows more transmitters to retain in the D-Phase, so as to have a second chance to transmit. Thus, by comparing the simulation results of the proposed scheme in Fig. 3.4 with that in Fig. 3.3, it is observed that the achieved optimal spatial capacity over λ_0 with $\gamma_1 = 0.4$ in Fig. 3.4 is larger than that with $\gamma_1 = 0.6$ in Fig. 3.3. In addition, for all the considered schemes in Fig. 3.4, we observe a density-capacity tradeoff, which is similar to that in Fig. 3.3.

Effects of the SIR Estimation and Feedback Errors

We consider a more practical scenario, where SIR estimation and feedback errors exist in the implementation of the proposed scheme, and show the effects of the SIR errors on the spatial capacity. We assume the SIR estimation and feedback errors follow zero-mean Gaussian distributions with variance σ_{est}^2 and σ_{fed}^2 , respectively. By further assuming that the two types of SIR errors are mutually independent, the sum of both SIR errors at transmitter i , denoted by n_i , follows zero-mean Gaussian distribution with variance $\sigma^2 = \sigma_{est}^2 + \sigma_{fed}^2$. Thus, in the presence of SIR errors, the feedback SIR level at transmitter i in P-Phase 0 is $\text{SIR}_i^{(0)} + n_i$. Moreover, if the feedback SIR level $\text{SIR}_i^{(0)} + n_i \geq \gamma_1$ for a given SIR threshold γ_1 in P-Phase 1, transmitter i decides to transmit in P-Phase 1; otherwise, it decides to be idle in the remaining time of this time slot. Similar to its counterpart without SIR errors in Fig. 3.3 and Fig. 3.4, the spatial capacity with SIR errors is calculated as an average value over all the transmitters' random locations, the random fading channels, as well as the random SIR errors.

Fig. 3.5 numerically shows the spatial capacities of the proposed scheme in both cases with and without SIR errors. We set $\sigma^2 = 10^{-2}$ and $\gamma_1 = 0.4$ in this example. It is observed from Fig. 3.5 that when λ_0 is small, due to the resultant small interference in the network, each receiver i feeds back a sufficiently high SIR level $\text{SIR}_i^{(0)}$ to its associated transmitter, such that n_i has a small probability to affect the transmitter's decision. Thus, we observe that when λ_0 is small, the spatial capacity with SIR errors is tight to that without SIR errors. However, as λ_0 increases, due to the decreased $\text{SIR}_i^{(0)}$ at each transmitter i , the transmitters become more easily affected by the SIR errors n_i when deciding whether to transmit based on $\text{SIR}_i^{(0)} + n_i \geq \gamma_1$. It is noted that when λ_0 is sufficiently large, the average SIR level at each transmitter becomes very small; and even if $\text{SIR}_i^{(0)} \geq \gamma_1$ for transmitter i , $\text{SIR}_i^{(0)}$ is close to γ_1 with a large probability. Thus, under the case with zero-mean Gaussian distributed error n_i , for the transmitters with $\text{SIR}_i^{(0)} \geq \gamma_1$ in the SIR error-free case, it is more likely that these transmitters become $\text{SIR}_i^{(0)} + n_i < \gamma_1$ than $\text{SIR}_i^{(0)} + n_i \geq \gamma_1$ in the SIR error-involved case. Similarly, we can easily find that for the transmitters with

Figure 3.5: Effects of the SIR errors on the spatial capacity of the proposed scheme. $\gamma_1 = 0.4$. $\beta = 2.5$.

$\text{SIR}_i^{(0)} < \gamma_1$ in the SIR error-free case, it is also more likely that these transmitters maintain $\text{SIR}_i^{(0)} + n_i < \gamma_1$ than $\text{SIR}_i^{(0)} + n_i \geq \gamma_1$ in the SIR error-involved case. Thus, the number of transmitters with $\text{SIR}_i^{(0)} + n_i < \gamma_1$ in the SIR error-involved case is larger than that with $\text{SIR}_i^{(0)} < \gamma_1$ in the SIR error-free case in general. Hence, as compared to the case without SIR errors, more transmitters will be refrained from transmitting in the D-Phase in the case with SIR errors, which improves the successful transmission probability in the D-Phase due to the reduced interference. As a result, it is interesting to observe from Fig. 3.5 that when the initial transmitter density λ_0 increases to some significant point, the spatial capacity with SIR errors becomes slightly higher than that without SIR errors; and their gap slowly increases over λ_0 after this point. Therefore, inaccurate SIR may even help improve the SIR-based scheduling performance in more interference-limited regime, which makes the proposed design robust to SIR errors.

3.4 Proposed Scheme with Multi-Stage Probing

In this section, we consider the proposed scheme with multi-stage probing, i.e., $M > 1$. In this case, M probing phases are sequentially implemented to gradually decide the transmitters that are allowed to transmit in the data transmission phase. Unlike the case with $M = 1$, where the single-stage overhead $M\tau = \tau \ll T$ is negligible, the multi-stage overhead $M\tau$ with $M > 1$ may not be negligible. In the following, we first study the impact of multiple SIR thresholds on the spatial capacity by extending Proposition 3.3.1 for the case of $M = 1$ to the case of $M > 1$. We then investigate the effects of the multi-stage probing overhead on the spatial capacity.

3.4.1 Impact of SIR Thresholds

From (3.3) and (3.5), the spatial capacity of the proposed scheme is determined by the values of SIR thresholds as well as the time overhead $M\tau$ for probing. To focus on the impact of the SIR thresholds, in this subsection, we assume $M\tau$ is negligible and thus have

$$\mathcal{C}^{p,M} = \lambda_0 P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M, \text{SIR}_0^{(M)} \geq \beta) \quad (3.25)$$

where the distributions of $\text{SIR}_0^{(k)}$'s, $0 \leq k \leq M$, are mutually dependent and all the Φ_k 's, $1 \leq k \leq M$, are non-PPPs in general. It is also noted that for any $1 \leq k \leq M$, we have $\Phi_k \subseteq \Phi_{k-1}$ for $\gamma_k \geq 0$. Thus, the network interference level in Φ_k is reduced, as compared to that in Φ_{k-1} . As a result, by extending Proposition 3.3.1 for the case of $M = 1$, we obtain the following proposition for the case of $M > 1$.

Proposition 3.4.1. *Consider two proposed schemes with arbitrary $M - 1$ and M probing phases, respectively, $M > 1$. Suppose the two schemes adopt the same SIR threshold $\gamma_k \geq 0$ in each Φ_k , $\forall k \in \{1, \dots, M - 1\}$. Then given $\beta > 0$, by varying the SIR threshold $\gamma_M \in [0, \infty)$ in the D-Phase for the proposed scheme with M probing*

phases, we have the following relationship between $\mathcal{C}^{p,M}$ and $\mathcal{C}^{p,M-1}$ based on (3.25):

$$\begin{cases} \mathcal{C}^{p,M} > \mathcal{C}^{p,M-1}, & \text{if } \gamma_{M-1} < \gamma_M < \beta \text{ (conservative transmission regime)} \\ \mathcal{C}^{p,M} = \mathcal{C}^{p,M-1}, & \text{if } 0 \leq \gamma_M \leq \gamma_{M-1} \text{ or } \gamma_M = \beta \text{ (neutral transmission regime)} \\ \mathcal{C}^{p,M} < \mathcal{C}^{p,M-1}, & \text{if } \gamma_M > \beta \text{ (aggressive transmission regime)}. \end{cases} \quad (3.26)$$

Proof. Please refer to Appendix E. □

Remark. Similar to the case of Proposition 3.3.1, in Proposition 3.4.1, in the conservative transmission regime with $\gamma_{M-1} < \gamma_M < \beta$, we obtain improved spatial capacity; in the aggressive transmission regime with $\gamma_M > \beta$, we obtain reduced spatial capacity; and in the neutral transmission region with $0 \leq \gamma_M \leq \gamma_{M-1}$ or $\gamma_M = \beta$, we obtain unchanged capacity. Moreover, based on the fact that the conservative transmission decision is beneficial for improving the spatial capacity of the proposed scheme, we obtain the following corollary, which gives a proper method to set the values of all the SIR-thresholds, such that the improvement of spatial capacity over the number of probing phases is assured.

Corollary 3.4.1. For a proposed scheme with $M > 1$ probing phases with negligible overhead, if the designed SIR thresholds are properly increased as $0 < \gamma_1 < \dots < \gamma_M < \beta$, the resulting spatial capacity $\mathcal{C}^{p,M}$ increases with the number of probing phases M .

It is worth noting that based on (3.25), for a given $\lambda_0 > 0$, $\mathcal{C}^{p,M}$ is only determined by the successful transmission probability $\mathcal{P}_0^{p,M}$, given in (3.3). Thus, both Proposition 3.4.1 and Corollary 3.4.1 also apply for $\mathcal{P}_0^{p,M}$.

In the next, we provide a numerical example with $M = 2$ to further discuss the impact of SIR thresholds on the spatial capacity. In this example, we set $\alpha = 4$, $\beta = 2$, and $d = 10$. Fig. 3.6 shows the corresponding spatial capacity over $\gamma_1 \in [0, \beta]$ and $\gamma_2 \in [0, \beta]$. It is observed from Fig. 3.6 that if $0 \leq \gamma_2 \leq \gamma_1$, the spatial capacity achieved at (γ_1, γ_2) remains unchanged over γ_2 ; and if $\gamma_2 > \gamma_1$, the spatial capacity achieved at (γ_1, γ_2) is always larger than that achieved at $(\gamma_1, 0)$. Apparently, this is

Figure 3.6: Spatial capacity against γ_1 and γ_2 for $M = 2$.

in accordance with Proposition 3.4.1. Moreover, among all the points over $\gamma_1 \in [0, \beta]$ and $\gamma_2 \in [0, \beta]$, such trend is more obviously observed for small γ_1 and small γ_2 . In addition, it is also observed that the spatial capacity varies much faster over γ_1 than over γ_2 , and when γ_1 is sufficiently large, the resulting spatial capacity does not change much over γ_2 . As a result, the SIR threshold γ_1 plays a more critical role in determining the spatial capacity than γ_2 , since γ_1 determines how many transmitters can have a second chance to contend the transmission opportunity. Furthermore, it is observed that *to achieve a higher spatial capacity, it is preferred to start with a small $\gamma_1 > 0$, and then set $\gamma_2 < \beta$ with an increasing step-size, i.e., $\gamma_1 - 0 < \gamma_2 - \gamma_1$.* As shown in this example, the maximum spatial capacity is achieved at $\gamma_1 = 0.15$ and $\gamma_2 = 0.8$; and the spatial capacity at $\gamma_1 = 0.15$ reduces very slowly over $\gamma_2 \in [0.8, \beta]$.

Figure 3.7: Spatial capacity against the number of probing phases.

3.4.2 Effects of Multi-Stage Overhead of Probing

In this subsection, we assume the multi-stage overhead $M\tau$ for probing is not negligible and study the effects of $M\tau$ on the spatial capacity. In this case, it is easy to find from (3.5) that the effective data transmission time is reduced over the probing-stage number M , which reduces the spatial capacity. On the other hand, from Corollary 3.4.1, under the constraint that $0 < \gamma_1 < \dots < \gamma_M < \beta$, the successful transmission probability increases over M . As a result, from (3.5), there exists a probing-capacity tradeoff over M under the condition that $0 < \gamma_1 < \dots < \gamma_M < \beta$. In the following, we illustrate the probing-capacity tradeoff by a numerical example (see Fig. 3.7).

In this example, we show the spatial capacity of the proposed scheme over the number of probing phases M . We set $\alpha = 4$, $\beta = 2$, $d = 10$, and the time slot duration $T = 1$ second (s). We consider two cases with $\tau = 0$ s and $\tau = 0.04$ s, respectively, where the time overhead for probing is zero for the former case and non-negligible for the latter one. For both cases, as enlightened by Fig. 3.6, we start with $\gamma_1=0.01$

and gradually increase γ_k , $2 \leq k \leq M$, based on $\gamma_k = \gamma_{k-1} + 0.01k$, which gives an increasing step-size with $\gamma_k - \gamma_{k-1} < \gamma_{k+1} - \gamma_k$. To ensure $\gamma_M < \beta$, the maximum allowable M is obtained as 19. As shown in Fig. 3.7, it is observed that the spatial capacity increases over M for the case with $\tau = 0$, which validates Corollary 3.4.1. Moreover, for the case with $\tau = 0.04$, the probing-capacity tradeoff is observed as expected: *the spatial capacity first increases over M , due to the improved performance of successful transmission probability, but after $M = 10$, the spatial capacity begins to decrease over M , due to the more dominant effects of the reduced data transmission time.*

3.5 Summary

This chapter considers an ad hoc network consisting of homogeneous wireless users, and studies the effects of the user interaction on improving the ad hoc network capacity. A transmission scheme with SIR-threshold based scheduling is proposed to exploit the transmit interactions between wireless users. We model the transmitter locations in the initial probing phase as a homogeneous PPP, and find the SIR-based scheduling makes the PPP assumption no longer hold in the following probing and data transmission phases. First, we studied the proposed scheme with single-probing phase. We showed that if the SIR threshold γ_1 satisfies $0 < \gamma_1 < \beta$, the proposed scheme strictly outperforms the reference scheme in terms of spatial capacity. We proposed a new method to characterize the spatial capacity, which is useful for analyzing performance of wireless networks with interacted transmitters. We also showed an interesting density-capacity tradeoff by numerical examples. Next, we extended our study to the multi-probing phase. We showed that if the sequence of increasing SIR thresholds are properly set, the spatial capacity is guaranteed to increase over the number of probing phases. Furthermore, we analyzed the impact of multi-stage probing overhead and presented a probing-capacity tradeoff in scheduling design.

Chapter 4

Opportunistic Spectrum Access with Short-term Protection to Reactive Primary Users

By enabling the secondary users (SUs) to access the unoccupied channels of the primary users (PUs) in a cognitive radio (CR) network, opportunistic spectrum access (OSA) is regarded as one promising solution to resolving the spectrum scarcity versus spectrum underutilization paradox in wireless communications [1], [17], [18]. To design optimal OSA strategies for the SU, two competing goals are addressed at the same time: the “spectrum holes” unused by the PUs should be optimally explored by the SUs to maximize their throughput, whereas the probability of the SU’s transmission collision with undetected active PUs should be minimized. In this chapter, we focus on the *reactive PUs*, which are able to adapt their transmissions upon experiencing transmission collisions or receiving interference. We adopt the conventional short-term conditional collision probability (SCCP) constraint, which is effective to protect the non-reactive PUs, to protect the reactive PUs. By exploiting the reaction/interaction between the reactive PUs and SUs, we study the optimal OSA policy for the SUs in the presence of reactive PUs.

The rest of this chapter is organized as follows: Section 4.1 presents the motivation of studying the OSA-based CR network in the presence of reactive PUs. Section 4.2 describes the channel occupancy model for reactive PUs in a CR network. Section 4.3 formulates the OSA design under the reactive PU model as a constrained partially observable Markov decision process (POMDP) problem. Section 4.4 develops the optimal OSA policy based on a separation principle, and further studies the ineffectiveness of the SCCP constraint for protecting reactive PUs. Section 4.5 presents numerical results on the reactive PU's and SU's throughput by adopting the optimal OSA policy under the SCCP constraint. Finally, Section 4.6 summarizes this chapter.

4.1 Problem Motivation

Due to hardware limitations, the SU is only able to sense a certain part of the spectrum at each time; and such limited sensing results may be further corrupted by the sensing errors. Thus, the transmission collisions caused by the SU to the PU are unavoidable in practice. Considering such partially observable environment for the SU, the authors in [19] proposed a POMDP framework to design the optimal OSA for the SU. However, due to “the curse of dimensionality”, POMDP problems are in general computationally prohibitive to solve [113], [114]. Zhao *et al.* in [20] studied the optimality of the myopic sensing policy for the SU, which aims to optimize the SU's instantaneous reward, rather than the long-term reward base on the POMDP. Although the myopic policy is of much lower complexity to derive and is optimal in some spacial cases, it is not optimal for the (constrained) POMDP problem in general. Chen *et al.* in [21] proposed a threshold-based optimal spectrum sensing and accessing policy, which can maximize SU's throughput during its battery lifetime. However, both [20] and [21] assumed that sensing errors are negligible. It is noted that in the prior work of [22], based on a discrete-time 2-state Markov Chain, Chen *et al.* addressed the sensing errors in the OSA design, and formulated the OSA design for the SU as a finite-horizon constrained POMDP problem. This problem maximizes SU's

long-term throughput subject to the SCCP constraint to protect the PU's instantaneous transmission in each horizon. Moreover, [22] proposed a separation principle to significantly reduce the complexity of solving the constrained finite horizon POMDP problem. From [22], the SCCP constraint ensures that the PU experiences collisions from the SU within an acceptable fraction of the transmission time, and thus is able to provide effective protection to PU's transmissions. Following [22], the SCCP constraint has been widely adopted in the existing studies on OSA. For example, a similar POMDP problem subject to the SCCP constraint was considered in [23] for unslotted PU systems. An online OSA algorithm by learning PU's signal statistics was proposed in [24] under the SCCP constraint. Li *et al.* in [25] showed that when the SCCP constraints over time are tight, the optimal OSA policy can be implemented as a simple memoryless policy with periodic channel sensing. Moreover, unlike [22], [25] considered a continuous-time Markovian on-off process to model the PU's channel occupancy status and focuses on an infinite-horizon constrained POMDP formulation, where a stationary optimal policy generally exists.

Most existing work on OSA with time-slotted PUs, including the aforementioned one, has assumed a *non-reactive* PU model. As shown by Fig. 2.3 in Chapter 2, the non-reactive PU's transmission over a particular channel is modeled as a 2-state on/off Markov chain with fixed state transition probabilities. Similar non-reactive PU assumptions can also be found in the experimental based work on OSA with unslotted PUs, such as [138] and [139]. Although greatly simplifying the OSA design, the non-reactive PU model might not be practical since existing wireless systems are mostly intelligent enough to adapt their transmissions upon experiencing collision or interference. For example, a PU may increase transmit power to compensate the link loss due to the received interference. Alternatively, it may reduce the channel access probability when collision occurs in a carrier sensing multiple access (CSMA) based primary system. We refer to such PUs as *reactive PUs*, to differentiate from their non-reactive counterparts. Due to PUs' unawareness of the existence of the SUs, the PUs may not be able to differentiate the transmission collisions caused by other PUs from that by SUs. Thus, if the SU's OSA policy is not carefully designed, the SU may

take advantage of the PU's reactions to reduce its channel access probability, so as to improve the SU's own network throughput. This can cause more serious PU system performance degradation, as compared to that under the non-reactive PU model.

It is worth noting that there has been recent work that addressed reactive PUs for OSA and/or spectrum sharing (SS) based CR networks. In contrast to OSA, with SS, SU is allowed to transmit regardless of the PU's on/off status, provided that the resulting interference to PU is kept below a predefined threshold. In [125], the authors proposed a *hidden power-feedback loop* for the CR: If PU is reactive and reacts upon receiving SU's interference, SU will receive a power-boosted PU signal that is easier to detect. Following [125], [126] proposed a proactive sensing scheme and a sequential transmit power adaptation strategy to exploit spectrum opportunities in the SS based CR. In [122], the author extended the work in [125] and designed active learning and supervised transmission schemes. Automatic retransmission request (ARQ) based reactive PUs have been considered in, e.g., [127] and [128], for the SS based CR. Under the assumption that SU has full knowledge of PU's buffer state and ARQ state, the authors in [127] adopted a Markov process based model to determine SU's optimal transmission policy over an infinite horizon, which maximizes SU's long-term average throughput subject to PU's long-term throughput loss. As shown in [127], the SU's optimal transmission policy is stationary and thus can be obtained by solving a linear program. Online algorithms have also been proposed in [128] for the cases where only partial and/or noisy observations of PU's buffer state and ARQ state are available to the SU. Compared with the existing work for SS based CR, the work considering reactive PUs for OSA based CR is very limited. It is noted that a CSMA-based reactive PU model has been proposed in [3] to investigate the performance of different SU access policies; however, [3] did not address the optimal OSA design.

Therefore, in this chapter, we focus on the OSA-based CR network in the presence of reactive PUs, and use dynamic programming and POMDP to investigate the optimal OSA policy for the SU. As our preliminary step toward studying the OSA-based CR with reactive PUs, how to develop a simple but reasonable reactive PU model, which captures the essence of the PU's reactions to the experienced collisions is our

first concern. In addition, we are also interested to study how to effectively protect the reactive PU's transmissions. Since the conventional SCCP constraint is able to effectively protect the non-reactive PU's transmissions, a natural question is whether it can effectively protect the reactive PU's transmissions. Moreover, it is also interesting to see if there are any differences between the SU's OSA policies in the presence of reactive PUs and non-reactive PUs. All these issues are challenging to be addressed and important for understanding the interactions between the reactive PU and SU.

In this chapter, we adopt the conventional SCCP constraint to protect the reactive PU's transmissions and study the optimal OSA policy for the SU. Then based on the obtained OSA policy for the SU, we examine whether the reactive PU's transmissions can be effectively protected as its non-reactive PU counterparts. In addition, we are also interested in studying the tradeoff between the reactive PU's transmission protection and SU's throughput maximization. To start with, in the next section, we introduce our proposed model on the reactive PU's channel occupancy state and the SU's sequential OSA decisions.

4.2 System Model

We consider a primary network with N reactive PUs. We assume each reactive PU is preassigned a dedicated channel and the traffic carried by each channel is assumed to be *independent* from each other. We model the channel occupancy of each reactive PU using a 4-state discrete-time Markov chain, which will be specified later. We consider the scenario where a SU is trying to access the unoccupied channels of the PUs. To avoid the trivial case where the SU is located very far from the PUs, such that it can transmit regardless of the PUs' on/off status, we assume the SU is close to the PUs and it is able to detect the PU's transmissions. Moreover, we assume synchronized time-slotted transmission for all the PUs and SU as in [19]-[22], and [24]. Suppose the SU is always backlogged with data packets over a finite number of time horizons. We assume the time horizons consist of T time slots, with $1 < T < \infty$ under the practical consideration of the SU's traffic loads. As a result, the SU has the intention

Figure 4.1: Time slot structure of the SU's OSA.

to transmit and thus access the primary channels in each slot t , $t \in \{1, \dots, T\}$. As shown in Fig. 4.1, at the beginning of each slot, the SU first decides which channel out of the N primary channels to sense and how to sense (e.g., how to set the sensor operating point). Based on the sensing result, the SU decides whether to access the sensed channel. If the SU transmits in a slot, it will receive a one-bit feedback from its receiver, which indicates whether its transmission in the current slot is successful or not. In the following, we give the detailed description on modeling of the reactive PU's channel occupancy and the SU's OSA access.

4.2.1 Reactive Primary User

Fig. 2.3, given in Chapter 2, shows a typical example of the channel occupancy model for the conventional non-reactive PU, which has been adopted in prior work, e.g., [1], [19], [20]-[25]. In this model, the primary traffic over a given channel is approximated by a two-state discrete-time Markov chain with states '0' and '1' denoting whether the channel is busy or idle, respectively. The PU's state changes slot by slot according to transition probabilities α_0 and β_0 shown in Fig. 2.3. Clearly, this model is not able to reflect the state of reactive PUs, for which the state transition depends on the SU's past access decisions. For example, the reactive PU usually reduces its channel access probability if a collision occurs and increases such probability when the environment becomes friendly again (no collision is observed). To reflect PU's reactive behaviors in practice, we propose an enhanced channel occupancy model.

The new model is composed of two levels, namely, Level 0 and Level 1. The reactive PU is assumed to have a higher probability to access the channel when it is in Level 0 than in Level 1. As a result, the enhanced channel occupancy model

Figure 4.2: Channel occupancy model for the reactive PU.

becomes a four-state Markov chain as shown in Fig. 4.2, with each level having two states (busy or idle). For convenience, we use 2 bits to represent the total 4 states for the reactive PU. The first bit denotes the level and the second bit denotes whether the channel is busy ('0') or idle ('1').

In each time-slot, the state of a reactive PU evolves according to the state and the SU's action in the previous slot. Suppose that initially the PU is in Level 0 with transition probabilities α_0 and β_0 , as shown in Fig. 2.3. If there is no SU accessing the channel, or the SU accesses when the PU is at state '01', no collision occurs and the PU will stay in Level 0. However, if the SU accesses when the PU's state is '00', the PU will react to the resulted collision by transiting from Level 0 to Level 1, with probability α_1 to state '11' and probability $1 - \alpha_1$ to state '10'. We assume $\alpha_1 \geq \alpha_0$ to reflect the reduced probability that the reactive PU accesses the channel in Level 1 than in Level 0. When the PU transits to Level 1, it will stay in this level if the SU continues to access the channel. However, if the SU does not access the channel and the PU is at state '10', the PU observes no collision and thus conceives that the environment has become friendly for its transmission. As a result, the PU increases its probability to access the channel by returning to Level 0, with transition probabilities α_0 to state '01' and $1 - \alpha_0$ to state '00', respectively. Since the reactive PU's state transitions are related to the SU's actions at state '00' or '10', the

corresponding transition probabilities are conditioned on the SU's action as shown in Fig. 4.2. Moreover, notice that when the PU's state is '01' or '11', the state transition probabilities are not affected by the SU's actions. This is because no collision occurs if the PU does not attempt to transmit. We assume $\beta_1 \geq \beta_0$ to be consistent with the reactive PU's more willingness to access in Level 0 than in Level 1. Note that when $\alpha_1 = \alpha_0$ and $\beta_1 = \beta_0$, the proposed 4-state channel occupancy model for the reactive PU reduces to the conventional 2-state counterpart in Fig. 2.3 for the non-reactive PU.

It is worth pointing out that our proposed two-level Markov chain model is a basic model that captures essential reactions of the PU subject to the SU's collisions; and therefore it can be generalized to specify more complicated reactions of the PU (e.g., random transmission backoff in CSMA) by appropriately setting the transition probabilities in each level and/or increasing the number of levels in the model.

4.2.2 Opportunistic Spectrum Access of Secondary User

We assume that the SU can only select one channel for sensing in each time-slot due to hardware limitations, and the sensing result over the selected channel may not be the PU's actual state due to sensing errors. As shown in Fig. 4.1, the SU makes a sequence of decisions in each slot as follows. At the beginning of slot t , $t \geq 1$, the SU transmitter selects a channel $a(t) \in \mathbb{A}_S$ to sense, where $\mathbb{A}_S = \{1, 2, \dots, N\}$ denotes the set of channels. Supposing $a(t) = a$, the SU then decides the sensor operating point to sense channel a , which is determined by the probability of false alarm (PFA) $\epsilon_a(t) \in [0, 1]$ and the probability of mis-detection (PM) $\delta_a(t) \in [0, 1]$. A feasible operating point must be confined by the optimal receiver operating characteristic (ROC) curve and the line determined by $1 - \delta_a(t) = \epsilon_a(t)$. The set of all feasible operating points is denoted by $\mathbb{A}_\delta(a(t))$. Based on the sensing result $\Theta_a(t) \in \{0, 1\}$, the SU decides a pair of access probabilities $(f_a(0, t), f_a(1, t)) \in [0, 1]^2$ for this channel, where $f_a(\theta, t)$ is the access probability on channel a in slot t with $\Theta_a(t) = \theta$. Denoting $\Phi_a(t) \in \{0(\text{not access}), 1(\text{access})\}$ as the SU's access action on channel a in slot t ,

$f_a(\theta, t)$ is expressed by the following conditional probability

$$f_a(\theta, t) = P\{\Phi_a(t) = 1 | \Theta_a(t) = \theta\}. \quad (4.1)$$

At the end of slot t , the SU transmitter receives error free feedback $K_a(t) \in \{0, 1\}$ from the SU receiver, where $K_a(t) = 1$ means that the SU's information is transmitted successfully, and $K_a(t) = 0$ represents that the SU transmits but the SU receiver fails to receive the transmitted information due to that the PU is busy and hence a collision occurs. Note that if the SU does not transmit, the SU transmitter will not receive any feedback. For the ease of representation, we assume that this case is also represented by $K_a(t) = 0$. Note that $K_a(t)$ is for the SU transmitter and receiver to maintain their decision synchronization [22].

4.3 Partially Observable Environment

As described in Section 4.2.2, in each time-slot, the SU selects one channel for sensing and thus is unable to observe the PUs' states in the other $N - 1$ channels. Even for the case of $N = 1$, the SU may not be able to obtain the PU's actual state due to sensing errors. This renders the PUs' states are only *partially* observable at the SU over time. Thus, we adopt a POMDP model to design the SU's OSA. In this section, we describe the POMDP and formulate the SU's optimal OSA design as a constrained POMDP problem.

4.3.1 Partially Observable Markov Decision Process based Model

A POMDP in general consists of the following elements [114]: a set of time-slots $\{1, \dots, T\}$, where T is called the horizon, and a set of system states (with transition probabilities), actions, observations (with observation probabilities) and rewards, for each of the time-slots. In this subsection, we formulate the POMDP model for the SU's OSA by specifying these elements.

Specifically, we consider a finite-horizon POMDP with $T < \infty$. Each system state in the POMDP is denoted by an N -element vector, with each element representing one PU's state at its assigned channel. For brevity, we represent the states in Fig. 4.2, namely, 00, 01, 10, 11, using 0, 1, 2, 3, respectively, and denote $\mathbb{C}_S = \{0, 1, 2, 3\}$ as the set of the states. Since $|\mathbb{C}_S| = 4$, there are in total 4^N POMDP system states. Denote the action space of the POMDP as $\mathbb{A} = \left\{ \left(a(t), (\epsilon_a(t), \delta_a(t)), (f_a(0, t), f_a(1, t)) \right) : a(t) \in \mathbb{A}_S, (\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a(t)), (f_a(0, t), f_a(1, t)) \in [0, 1]^2 \right\}$. Let the observation in the POMDP be $K_a(t) \in \{0, 1\}$ in slot t . Suppose that $A(t) = A$, where $A = (a(t) = a, (\epsilon_a(t), \delta_a(t)), (f_a(0, t), f_a(1, t)))$. The observation probability is then denoted as $U_A(k|i) \triangleq P\{K_a(t) = k|i, A\}$ with $k = \{0, 1\}$, which represents the conditional probability of observing $K_a(t) = k$ given that the SU's action is A and the PU's state over selected channel a is $i, i \in \mathbb{C}_S$. Let $I(a, t) = 0$ and $I(a, t) = 1$ represent channel a being busy and idle in slot t , respectively. Denote $1_{[x]}$ as an indicator function, which equals 1 if x is true and 0 otherwise. Note that $K_a(t) = I(a, t)\Phi_a(t)$. By applying a derivation similar to that in [22], we obtain the following result:

$$U_A(k|i) = \begin{cases} 1_{[I(a,t)=1]}g_a(t), & \text{if } k = 1 \\ 1 - U_A\{k = 1|i\}, & \text{if } k = 0 \end{cases} \quad (4.2)$$

where

$$g_a(t) \triangleq P\{\Phi_a(t) = 1|a(t) = a, I(a, t) = 1\} \quad (4.3)$$

$$= \epsilon_a(t) \times f_a(0, t) + (1 - \epsilon_a(t)) \times f_a(1, t) \quad (4.4)$$

is the SU's conditional access probability on channel a , given that the PU is idle on this channel in slot t and the SU selects channel a to sense. If the SU transmits successfully in slot t , i.e., the observation $K_a(t) = 1$, it will obtain a unit throughput. Denote the instantaneous reward of the SU in slot t by $R_S(t)$. By assuming a unit bandwidth ($B = 1$) for each channel, we have

$$R_S(t) = K_a(t) \times B = K_a(t). \quad (4.5)$$

Figure 4.3: State transition probabilities in the reactive PU model.

At the end of each slot, the POMDP system moves to the next state from the current state according to the POMDP state transition probability. Since the PUs' states over different channels evolve *independently*, the POMDP system state transition probability is obtained as the product of each PU's state transition probability. We thus focus on the state transition probability for a given channel $n \in \mathbb{A}_S$. Let $i, j \in \mathbb{C}_S$ and denote $P_n(i|j, A)$ as the transition probability from state j in slot t to state i in slot $t + 1$ over channel n under the SU's action $A = (a(t) = a, (\epsilon_a(t), \delta_a(t)), (f_a(0, t), f_a(1, t)))$. If $n \neq a$, the SU does not select channel n for sensing. Fig. 4.3(a) shows the state transition probabilities for this case, where we use superscript n to denote channel n . These probabilities are easily obtained from

Fig. 4.2 with the SU's access action being *not access*. If $n = a$, the SU selects channel a to sense (and probably access). Fig. 4.3(b) shows the state transition probabilities for this case. If the state is '01' or '11', the transition probabilities are independent of the SU's action; otherwise, according to Fig. 4.2, they are subject to the SU's access action $\Phi_a(t)$. Note that given the PU's state on channel a , $\Phi_a(t)$ is determined in probability by $(\epsilon_a(t), \delta_a(t))$ and $(f_a(0, t), f_a(1, t))$. Based on Fig. 4.2, with the fact that $P_a(i|j = 2, A) = P_a(i|j = 0, A), \forall i \in \mathbb{C}_S$, we thus obtain the following transition probabilities:

$$\begin{cases} P_a(i = 0|j = 0, A) = P_a(i = 0|j = 2, A) = (1 - \mu_a(t)) \times (1 - \alpha_0), \\ P_a(i = 1|j = 0, A) = P_a(i = 1|j = 2, A) = (1 - \mu_a(t)) \times \alpha_0, \\ P_a(i = 2|j = 0, A) = P_a(i = 2|j = 2, A) = \mu_a(t) \times (1 - \alpha_1), \\ P_a(i = 3|j = 0, A) = P_a(i = 3|j = 2, A) = \mu_a(t) \times \alpha_1, \end{cases}$$

where

$$\mu_a(t) \triangleq P\{\Phi_a(t) = 1|a(t) = a, I(a, t) = 0\} \quad (4.6)$$

$$= (1 - \delta_a(t)) \times f_a(0, t) + \delta_a(t) \times f_a(1, t) \quad (4.7)$$

is the SU's conditional access probability on channel a , given that the PU is busy on this channel in slot t and the SU selects channel a to sense.

Belief on POMDP States

In the POMDP model, the system states are not exactly known at the SU. However, based on the SU's previous actions and observations, a belief on the POMDP system state can be obtained. The belief is defined as the conditional probability distribution over all possible POMDP system states given the history of the SU's actions and observations. As shown in [113], the belief on the POMDP system state is a *sufficient statistic* for the design of optimal actions. For our model, the POMDP system state consists of the PUs' states over independent channels. The SU's belief on the POMDP system state is thus given by the belief on the PU's 4 possible states over N channels.

Hence, we adopt a $4 \times N$ matrix $\mathbf{\Lambda}(t) \triangleq \{\lambda_{nj}(t)\}_{n \in \mathbb{A}_S, j \in \mathbb{C}_S}$ as the *belief state* of the POMDP, where the element $\lambda_{nj}(t)$ represents the conditional probability that the state of channel $n \in \mathbb{A}_S$ is $j \in \mathbb{C}_S$ in slot t , given the SU's decision and observation history. We have $\sum_{j \in \mathbb{C}_S} \lambda_{nj}(t) = 1$ for $t \in \{1, \dots, T\}$. Clearly, the space of the POMDP belief states is $[0, 1]^{4 \times N}$. The belief state is updated slot by slot based on the SU's previous actions and observations. Suppose that channel a is selected in slot t . The belief on states in slot $t + 1$ is updated as follows.

- If $n \neq a$, the updated belief on the state is not affected by the SU's action. We thus have

$$\lambda_{nj}(t+1) = \sum_{i \in \mathbb{C}_S} \lambda_{ni}(t) P_n(j|i, A), \quad \forall j \in \mathbb{C}_S. \quad (4.8)$$

- If $n = a$, the updated belief on the state is related to the SU's action and is obtained according to the observation $K_a(t) = k$ via Bayes rule [113] as

$$\lambda_{aj}(t+1) = \frac{\sum_{i \in \mathbb{C}_S} \lambda_{ai}(t) P_a(j|i, A) U_A(k|i)}{\sum_{i=0}^3 \lambda_{ai}(t) U_A(k|i)}, \quad \forall j \in \mathbb{C}_S. \quad (4.9)$$

4.3.2 Policy Description

The OSA design for the SU is given by a sensing policy π_s , a sensor operating policy π_δ and an access policy π_c . Specifically, the sensing policy specifies a sequence of functions as $\pi_s \triangleq \{d_1^s, d_2^s, \dots, d_T^s\}$, where d_t^s in slot t maps a belief state $\mathbf{\Lambda}(t)$ to the channel $a(t) \in \mathbb{A}_s$ selected to sense for this slot. Given the selected channel $a(t) = a$, the sensor operating policy specifies a sequence of functions as $\pi_\delta \triangleq \{d_1^\delta, d_2^\delta, \dots, d_T^\delta\}$, where d_t^δ in slot t maps $\mathbf{\Lambda}(t)$ to a feasible sensor operating point $(\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a(t))$ for this slot. Similarly, the access policy is specified as $\pi_c \triangleq \{d_1^c, d_2^c, \dots, d_T^c\}$. Given the sensing result $\Theta(t) \in \{0, 1\}$ in slot t , d_t^c maps $\mathbf{\Lambda}(t)$ to the access probabilities $(f_a(0, t), f_a(1, t)) \in [0, 1]^2$ in slot t .

4.3.3 Problem Formulation

The optimal OSA design $\{\pi_s^*, \pi_\delta^*, \pi_c^*\}$ is obtained by solving a constrained POMDP problem, which maximizes the SU's expected reward over T slots subject to various constraints to protect the PUs' transmission. Specifically, the objective of the POMDP problem is to obtain

$$\{\pi_s^*, \pi_\delta^*, \pi_c^*\} = \arg \max_{\pi_s, \pi_\delta, \pi_c} E_{\{\pi_s, \pi_\delta, \pi_c\}} \left\{ \sum_{t=1}^T R_S(t) | \mathbf{\Lambda}(1) \right\}, \quad (4.10)$$

where $\mathbf{\Lambda}(1)$ is the initial belief state in slot $t = 1$. The elements in $\mathbf{\Lambda}(1)$ are set according to the stationary distribution of the underlying Markov chain, under the assumption that the PU is at Level 0 initially, i.e., $\lambda_{n0}(1) = \frac{1-\beta_0^n}{1+\alpha_0^n-\beta_0^n}$, $\lambda_{n1}(1) = 1 - \lambda_{n0}(1)$, and $\lambda_{n2}(1) = \lambda_{n3}(1) = 0$, $\forall n \in \mathbb{A}_S$. Suppose that the SU selects channel a in slot t . Given $\mathbf{\Lambda}(t)$, from (4.5), the SU's expected reward in slot t over all possible PU's states on channel a and the SU's observations is obtained as

$$E_{\{\pi_s, \pi_\delta, \pi_c\}} \left\{ R_S(t) | \mathbf{\Lambda}(t) \right\} = (\lambda_{a1} + \lambda_{a3}) \times g_a(t). \quad (4.11)$$

In this section, we consider the conventional SCCP constraint to protect the re-active PU transmissions, for which, the details will be given in Section 4.4.

4.4 Optimal Policy

As in [22], the short-term conditional collision probability on channel $n \in \mathbb{A}_S$ is defined as

$$\sigma_n(t) \triangleq P\{\Phi_n(t) = 1 | I(n, t) = 0\}, \quad \forall n \in \mathbb{A}_S, \quad \forall t \in \{1, \dots, T\}. \quad (4.12)$$

By imposing a fixed constraint ζ for the SCCP in each slot t , we obtain the SCCP constraint as

$$\sigma_n(t) \leq \zeta, \quad \forall n \in \mathbb{A}_S, \quad \forall t \in \{1, \dots, T\}. \quad (4.13)$$

The SCCP constraint has been widely adopted in the literature, e.g., [22]-[25], to protect the PU's transmission. As in (4.13), the SCCP constraint ensures that on every channel, the PU experiences collisions from the SU for no more than ζ fraction of the transmission time. Thus, the PU's throughput under the SU's OSA is at least $100 \times (1 - \zeta)$ percentage of that without presence of the SU, if the PU is non-reactive. However, the effectiveness of the SCCP constraint in protecting the reactive PU's transmission remains unaddressed yet in the literature. In this section, we adopt the conventional SCCP constraint and design the optimal OSA policy for the SU under the reactive PU model. By adopting the optimal OSA policy, we show that the SCCP constraint is not able to provide effective protection to the reactive PU's transmission.

Suppose $a(t) = a$. With a derivation similar to that in [22], we obtain from (4.13) that

$$\sigma_n(t) = \begin{cases} (1 - \delta_a(t)) \times f_a(0, t) + \delta_a(t) \times f_a(1, t), & \text{if } n = a. \\ 0, & \text{if } n \neq a. \end{cases} \quad (4.14)$$

Note that if $n = a$, $\sigma_a(t)$ has the same expression as $\mu_a(t)$ in (4.7). Using (4.10) and (4.14), the OSA design for the SU under the SCCP constraint is formulated as

$$\begin{aligned} (\text{P1}) : \quad & \max_{\pi_s, \pi_\delta, \pi_c} E_{\{\pi_s, \pi_\delta, \pi_c\}} \left\{ \sum_{t=1}^T R_S(t) | \mathbf{\Lambda}(1) \right\} \\ & \text{s.t. } \sigma_n(t) \leq \zeta, \quad \forall n \in \mathbb{A}_S, \forall t \in \{1, \dots, T\}. \end{aligned}$$

According to the principle of dynamic programming [140], (P1) can be decoupled into T subproblems without loss of optimality. Each subproblem is to find a value function $V_t(\mathbf{\Lambda}(t))$, $1 \leq t \leq T$, which represents the SU's maximum expected reward that can be obtained from slot t to slot T under the SCCP constraint, given the belief state $\mathbf{\Lambda}(t)$. Given the SU's action $A(t) = A$ and the belief state $\mathbf{\Lambda}(t)$, the reward in slot t , $1 \leq t \leq T - 1$, consists of two parts: the SU's expected immediate reward $E_A\{R_S(t) | \mathbf{\Lambda}(t)\}$ and the SU's maximum expected future reward $E_A\{V_{t+1}(\mathbf{\Lambda}(t+1)) | \mathbf{\Lambda}(t)\}$ over all possible PU's states on channel a and the SU's observations in slot t , where $\mathbf{\Lambda}(t+1)$ is updated from $\mathbf{\Lambda}(t)$ according to (4.8) and (4.9).

In the last slot $t = T$, the SU's expected immediate reward alone is the value function $V_T(\mathbf{\Lambda}(T))$. By averaging over all possible PUs' states and the SU's observations $K_a(t) = k \in \{0, 1\}$ and maximizing over all SU's actions $A \in \mathbb{A}$, we obtain the value functions expressed as

$$V_t(\mathbf{\Lambda}(t)) = \max_{A \in \mathbb{A}} \sum_{k=0}^1 \sum_{i=0}^3 \lambda_{ai}(t) U_A(k|i) [k + V_{t+1}(\mathbf{\Lambda}(t+1))], \quad 1 \leq t \leq T-1. \quad (4.15)$$

$$V_T(\mathbf{\Lambda}(T)) = \max_{A \in \mathbb{A}} \sum_{k=0}^1 \sum_{i=0}^3 \lambda_{ai}(T) U_A(k|i) \times k, \quad t = T. \quad (4.16)$$

By computing the value functions given in (4.15) and (4.16) recursively backward in time and searching over all possible actions $A(t) \in \mathbb{A}$ in each slot, we can find the optimal policy $\{\pi_s^*, \pi_\delta^*, \pi_c^*\}$ for (P1) that maximizes the SU's expected reward over T slots, i.e., $V_1(\mathbf{\Lambda}(1))$ in (4.15), under the SCCP constraint. However, (4.15) and (4.16) are generally intractable and computationally prohibitive due to the infinite and uncountable action space $\mathbb{A}$ [22].

4.4.1 Separation Principle

In [22], a *separation principle* was proposed to obtain the optimal policy for an OSA design problem similar to (P1), but under the non-reactive PU model. It is shown in [22] that with this method, the sensing policy can be separately designed from the sensor operating policy and the access policy. The optimal sensor operating policy π_δ^* and the optimal access policy π_c^* over any selected channel a for sensing is obtained by maximizing $g_a(t)$ given in (4.4), subject to the SCCP constraint in slot t . Since the action space of $\mathbb{A}_S$ is finite and countable, the optimal sensing policy π_s^* is then obtained by standard dynamic programming techniques, given π_δ^* and π_c^* .

For the ease of presentation, we define $Q_t(\mathbf{\Lambda}(t)|A)$ as the SU's maximum expected reward that can be obtained from slot t to slot T , given the SU's action $A \in \mathbb{A}$ in

Table 4.1: The non-monotonicity of $Q_t(\mathbf{\Lambda}(t)|A)$ with respect to $g_a(t)$ under the reactive PU model. ($\alpha_0^a = 0.5, \beta_0^a = 0.5, \alpha_1^a = 0.9, \beta_1^a = 0.9$).

Cases \ Actions	$f_a(0, 1)$	$f_a(1, 1)$	$\epsilon_a(1)$	$\delta_a(1)$	$g_a(1)$	$Q_1(\mathbf{\Lambda}(1) A)$
<i>case 1</i>	0	0.5	0.5	0.5	0.25	0.675
<i>case 2</i>	0	0.6	0.5	0.5	0.3	0.71
<i>case 3</i>	0	0.6	0.5	0.1	0.3	0.662

slot t and the belief state $\mathbf{\Lambda}(t)$, i.e.,

$$Q_t(\mathbf{\Lambda}(t)|A) = \sum_{i=0}^3 \sum_{k=0}^1 \lambda_{ai}(t) U_A(k|i) [k + V_{t+1}(\mathbf{\Lambda}(t+1))], \quad 1 \leq t \leq T-1. \quad (4.17)$$

$$Q_T(\mathbf{\Lambda}(T)|A) = \sum_{i=0}^3 \sum_{k=0}^1 \lambda_{ai}(t) U_A(k|i) \times k, \quad t = T. \quad (4.18)$$

Then we have

$$V_t(\mathbf{\Lambda}(t)) = \arg \max_{A \in \mathbb{A}} Q_t(\mathbf{\Lambda}(t)|A), \quad 1 \leq t \leq T. \quad (4.19)$$

As shown in [22], the main reason why the separation principle holds under the non-reactive PU model is that over any selected channel a at slot t , $Q_t(\mathbf{\Lambda}(t)|A)$ strictly increases with $g_a(t)$ given in (4.4). However, under the proposed reactive PU model, since the PU's channel access probabilities in the future slots depend on the SU's current action decision, $Q_t(\mathbf{\Lambda}(t)|A)$ is generally not monotonically increasing with $g_a(t)$. A simple example with $T = 2$ and $N = 1$ is shown in TABLE 4.1 validate this observation. In TABLE 4.1, we consider three cases, where the SU has different sensor operating points and spectrum access probabilities over channel a in slot $t = 1$. We compute $g_a(1)$ and $Q_1(\mathbf{\Lambda}(1)|A)$ based on (4.4), (4.17) and (4.18) in slot $t = 1$ and compare them under these cases. It is shown that both $g_a(1)$ and $Q_1(\mathbf{\Lambda}(1)|A)$ in *case 2* are larger than that in *case 1*; however, the SU's actions in *case 3* give a larger $g_a(1)$ but a smaller $Q_1(\mathbf{\Lambda}(1)|A)$, compared to *case 1*. Hence, $Q_t(\mathbf{\Lambda}(t)|A)$ does not necessarily increase with $g_a(t)$. As a result, the proof in [22] for the separation

principle does not apply to our problem at hand under the reactive PU model.

Interestingly, as shown in the following theorem, a separation principle similar to that in [22] is applicable under the reactive PU model without any loss of optimality. This is true mainly due to the fact that the SCCP constraint in (P1) only depends on π_δ and π_c .

Theorem 4.4.1. *The SU's optimal OSA policy for (P1) under the reactive PU model and the SCCP constraint is obtained by the following two steps:*

- *Step 1: Determine the optimal sensor operating policy π_δ^* and the optimal access policy π_c^* . Specifically, in slot t , supposing $a(t) = a$, the optimal policies of π_δ^* and π_c^* are given by*

$$\begin{cases} \delta_a^*(t) = \zeta, \\ \epsilon_a^*(t) \text{ is on the optimal ROC curve corresponding to } \zeta, \\ f_a^*(0, t) = 0, \\ f_a^*(1, t) = 1. \end{cases} \quad (4.20)$$

- *Step 2: Apply the optimal policies π_δ^* and π_c^* in Step 1 to obtain the optimal sensing policy π_s^* by solving the following unconstrained POMDP:*

$$\pi_s^* = \arg \max_{\pi_s} E_{\pi_s} \left\{ \sum_{t=1}^T R_S(t) | \mathbf{\Lambda}(1), \pi_\delta^*, \pi_c^* \right\}. \quad (4.21)$$

Proof. Please refer to Appendix F.¹ □

Remark. Since the optimal action decisions, $\delta_a^*(t)$, $\epsilon_a^*(t)$, $f_a^*(0, t)$, and $f_a^*(1, t)$, given in (4.20), are independent of the sensing policy, the optimal sensing policy π_s^* can be separately designed (as shown in Step 2 of Theorem 4.4.1). Since all the optimal actions $\delta_a^*(t)$, $\epsilon_a^*(t)$, $f_a^*(0, t)$, and $f_a^*(1, t)$ are time-invariant, the optimal policies π_δ^* and π_c^* are independent of belief states. With $f_a^*(0, t) = 0$ and $f_a^*(1, t) = 1$, it follows that

¹Note that by setting $\alpha_1^n = \alpha_0^n$ and $\beta_1^n = \beta_0^n$, $\forall n \in \mathbb{A}_s$, the proposed proof for the separation principle also holds for the non-reactive PU model.

the SU always trusts the spectrum sensing result even though there may exist sensing errors, i.e., the SU accesses channel a in slot t with probability 1 when the sensing result is $\Theta_a(t) = 1$, and with probability 0 when the sensing result is $\Theta_a(t) = 0$.

From [22] and Theorem 4.4.1, the optimal sensor operating policy π_δ^* and the optimal access policy π_c^* for the SU are the same under both reactive and non-reactive PU models, due to the same SCCP constraint that is adopted. However, since the PU can change its channel access probability under the reactive PU model, which affects the SU's expected instantaneous reward, given in (4.11), the optimal sensing policy π_s^* under the reactive PU model is different from that under the non-reactive PU model. Since there is no closed-form expression for π_s^* in (4.21), we will numerically show such difference later in Section 4.5.3.

It is also worth noting that although the unconstrained POMDP problem in (4.21) is usually of high complexity to solve due to the curse of dimensionality, the optimal policy π_s^* is essentially an *off-line* policy [19], as explained in the following: given $\Lambda(1)$, π_δ^* , and π_c^* , from (4.8) and (4.9), it is easy to find all the possible belief states $\Lambda(t)$ in slot t , $1 \leq t \leq T$, corresponding to the possible observations and channel selections. Solving the unconstrained POMDP problem in (4.21) gives the optimal channel decision for each possible belief state in each slot, i.e., the optimal sensing policy, which can be stored in e.g., a table. Then during the SU's OSA, based on its actual belief state in each slot, it only needs to check the table to obtain the optimal channel that should be selected to sense. Moreover, the action space of (4.21) is accountable and finite, which largely reduces the computational complexity to find the optimal sensing policy. In addition, we also note that efficient POMDP solver softwares (e.g., [142]) have been developed and widely applied to solve the unconstrained POMDP problems. Compared to the conventional recursive calculation (e.g., by backward induction) in dynamic programming, by exploiting the properties of the value functions, the POMDP solvers can substantially reduce the computational complexity [142].

4.4.2 Ineffectiveness for Protecting Reactive Primary Users

With the optimal OSA policy for the SU under the conventional SCCP constraint, in this subsection, we examine whether the SCCP constraint is an effective method for protecting reactive PU's transmission. Unlike the non-reactive PU which can maintain its channel access probability, the reactive PU changes its channel access probability upon experiencing transmission collisions. This in turn changes the reactive PU's expected throughput in each slot. We thus measure the PU's transmission quality based on its achievable throughput over all T slots. In the following, we first give the PU's benchmark throughput, and then analytically study whether the SCCP constraint is effective to protect the reactive PU's transmission.

We first derive the PU's benchmark throughput on each channel. Denote the PU's throughput on channel n in slot t by $R_{P,n}(t)$. If the PU on channel n accesses the assigned channel in slot t , $1 \leq t \leq T$, and transmits successfully, it will obtain a unit throughput. Given the current belief state $\mathbf{\Lambda}(t)$ and the SU's OSA policies π_s , π_δ , and π_c , it is then easy to obtain that the PU's expected throughput on channel n over all possible states in slot t is

$$E_{\pi_s, \pi_\delta, \pi_c} \{R_{P,n}(t) | \mathbf{\Lambda}(t)\} = P\{I(n, t) = 0\} \times (1 - \sigma_n(t)), \quad (4.22)$$

where $P\{I(n, t) = 0\} = \lambda_{n0}(t) + \lambda_{n2}(t)$ is the PU's access probability on channel n in slot t , and $\sigma_n(t)$ is given in (4.14). By summing the PU's throughput over T slots and dividing the sum by T , the PU's normalized throughput on channel n , denoted by $R_{P,n}^o$, is given as

$$R_{P,n}^o = \frac{1}{T} \times E_{\pi_s, \pi_\delta, \pi_c} \left\{ \sum_{t=1}^T R_{P,n}(t) | \mathbf{\Lambda}(1) \right\}, \quad \forall n \in \mathbb{A}_S. \quad (4.23)$$

Note that under the non-reactive PU model, the PU's channel access probability is independent of the SU's spectrum access policy and thus remains the same in each slot. Specifically, from the stationary distribution of the underlying Markov chain, we have $P\{I(n, t) = 0\} = \frac{1 - \beta_0^n}{1 + \alpha_0^n - \beta_0^n}$, $n \in \mathbb{A}_S$, $t \in \{1, \dots, T\}$. Since $\sigma_n(t) \leq \zeta$, from

(4.22), we obtain PU's minimum achievable expected throughput in slot t on channel n as $\frac{1-\beta_0^n}{1+\alpha_0^n-\beta_0^n} \times (1-\zeta)$, where $1-\zeta$ is the minimum probability that the SU does not collide with the PU in slot t on channel n . Then according to (4.23), we obtain PU's minimum achievable normalized throughput on channel n under the non-reactive PU model as

$$\Upsilon_n = \frac{1-\beta_0^n}{1+\alpha_0^n-\beta_0^n} \times (1-\zeta), \quad \forall n \in \mathbb{A}_S. \quad (4.24)$$

Taking Υ_n as the benchmark throughput for PU on channel n , we say that the PU system is under *effective protection* if $R_{P,n}^o \geq \Upsilon_n, \forall n \in \mathbb{A}_S$, is guaranteed. Clearly, the non-reactive PU is under effective protection by using the SCCP constraint.

Next, we show that under the SCCP constraint, the reactive PU's normalized throughput is not always larger than or equal to the benchmark throughput. We consider two cases. One is the single-channel case with $N = 1$, and the other is the multi-channel case with $N > 1$.

For the case of $N = 1$ with $n = a$, since the SU always selects to sense channel a and probably accesses it, $N = 1$ can be considered the PU's *worst case* transmission. The following shows that the SCCP constraint is not able to provide effective protection for PU's transmission with $N = 1$.

Proposition 4.4.1. *With the optimal OSA policy in (4.20), for $N = 1$ and $T > 1$, the reactive PU's normalized throughput on channel a is strictly smaller than the benchmark throughput, i.e., $R_{P,a}^o < \Upsilon_a$.*

Proof. Please refer to Appendix G. □

For the case of $N > 1$, since the SU can select one from N channels to sense, $R_{P,n}^o, \forall n \in \mathbb{A}_n$, will be at least equal to that in the worst case with $N = 1$. However, under the reactive PU model with $T > 1$, since $R_{P,a}^o < \Upsilon_a$ for $N = 1$, it is difficult to analyze whether the SCCP constraint is an effective PU protection method for $N > 1$. As will be shown later by simulations in the next section, the SCCP constraint is not sufficient to guarantee the benchmark throughput of all the N reactive PUs when $N > 1$. Thus, the SCCP constraint is not able to provide effective protection to the PU transmissions.

4.5 Numerical Results

Numerical examples are presented in this section. By adopting the same SCCP constraint, we compare the SU's and the PU's throughput under the reactive PU model with that under the non-reactive PU model. For the case under the reactive PU model, we apply the SU's optimal OSA policy obtained in Theorem 4.4.1. For the case under the non-reactive PU model, we apply the SU's optimal OSA policy obtained in [22]. It is worth noting that the problem formulation in [22] is the same as that in (P1), except the underlying PU channel occupancy model. Thus, by comparing our results with that in [22], the effects of the transmission interactions between the reactive PU and the SU as well as the ineffectiveness of the SCCP constraint in protecting the non-reactive PU's transmissions can both be properly illustrated.

4.5.1 System Scenario

We consider an energy detection based spectrum sensor for the SU, where we model the background noise and the received PU signal as independent white Gaussian processes. Denote M as the number of PU signal measurements, and η_n as the decision threshold for channel $n \in \mathbb{A}_S$. Denote $\kappa_{n,0}^2$ and $\kappa_{n,1}^2$ as the power of the noise and the received PU's signal on channel n , respectively. By the Neyman-Pearson (NP) criterion, we obtain the PFA and PM in slot $t \in \{1, \dots, T\}$ as [115]:

$$\delta_n(t) = \gamma\left(\frac{M}{2}, \frac{\eta_n(t)}{2(\kappa_{n,0}^2 + \kappa_{n,1}^2)}\right), \quad \epsilon_n(t) = 1 - \gamma\left(\frac{M}{2}, \frac{\eta_n(t)}{2\kappa_{n,0}^2}\right)$$

where

$$\gamma(a, m) = \frac{1}{\Gamma(m)} \times \int_0^a t^{m-1} e^{-t} dt$$

is the incomplete gamma function [141]. Under the SCCP constraint, the optimal decision threshold $\eta_n^*(t)$ of the energy detector in slot t is selected such that $\delta_n(t) = \zeta$. Unless otherwise mentioned, we set $\kappa_{n,0}^2 = 0$ dB, $\kappa_{n,1}^2 = 5$ dB, $\forall n \in \mathbb{A}_S$, and $M = 30$. In the following, we consider two cases, one is single-channel case, and the other is multi-channel case, where in each case, we show the SU's and PU's throughput in

non-reactive PU model and reactive PU model, respectively.

4.5.2 Single-Channel Case

We first consider the single-channel case with $N = 1$. We set $\alpha_0^a = 0.1$, $\beta_0^a = 0.2$, $\alpha_1^a = 0.9$, and $\beta_1^a = 0.95$ for the reactive PU model. The non-reactive PU model is then obtained equivalently by setting $\alpha_1^a = \alpha_0^a = 0.1$ and $\beta_1^a = \beta_0^a = 0.2$ in the reactive PU model. According to the stationary distribution of the underlying Markov chain, the PU's initial channel access probability in slot $t = 1$ is obtained as $\frac{1-\beta_0^a}{1-\beta_0^a+\alpha_0^a} = 0.889$ for both models. We consider two cases: $\zeta = 0.05$ and $\zeta = 0.1$. From (4.24), we obtain the PU's benchmark throughput $\Upsilon_a = 0.846$ when $\zeta = 0.05$, and $\Upsilon_a = 0.8$ when $\zeta = 0.1$. In the following, for each case, we show the SU's normalized throughput, the PU's normalized throughput, and the sum of SU's and PU's normalized throughput, under both the reactive and non-reactive PU models.

Fig. 4.4 shows the SU's normalized throughput subject to the SCCP constraint with $N = 1$. For each case of $\zeta = 0.05$ and $\zeta = 0.1$, we consider three scenarios: 1) under the reactive PU model, where the SU adopts the optimal sensor operating policy π_δ^* and the optimal spectrum access policy π_c^* , as shown in Section 4.4.1; 2) under the non-reactive PU model, where the SU adopts the optimal OSA policy given in [22]; 3) for the case of model mismatch, which occurs when the SU implements the optimal access policies assuming the non-reactive PU model, but actually under the reactive PU model. It is observed that for both cases with $\zeta = 0.05$ and $\zeta = 0.1$, the SU achieves higher throughput in the reactive PU model than in the non-reactive PU model. This is mainly because that the reactive PU reduces but the non-reactive PU remains the channel access probability after a collision with the SU occurs. Since the non-reactive PU remains the same channel access probability over time, when $N = 1$, the expected channel access opportunities are unchanged over time for the SU. As a result, the SU's throughput is a constant under the non-reactive PU model. It is also observed that the SU's throughput under the non-reactive PU model remains the same in both cases of $\zeta = 0.05$ and $\zeta = 0.1$; however, the SU's throughput under the reactive PU model is higher in the case of $\zeta = 0.1$ than that in the case of $\zeta = 0.05$ due

Figure 4.4: SU's normalized throughput under the SCCP constraint. $N = 1$.

to the more relaxed SCCP constraint. Moreover, for the scenario of model mismatch, it is observed that the SU obtains the same throughput as that under the reactive PU model, i.e., model mismatch does not affect the SU's throughput. The reason for it is that the optimal policies π_δ^* and π_c^* can be shown to be the same under both reactive and non-reactive PU models; thus, since $N = 1$, the optimal policy π_s^* is also the same.

Fig. 4.5 compares the PU's normalized throughput under the reactive PU model and the non-reactive PU model with $N = 1$. According to Section 4.4.2, the benchmark throughput, which remains as a constant over time in both cases of $\zeta = 0.05$ and $\zeta = 0.1$, is actually the non-reactive PU's normalized throughput under the SCCP constraint in the single-channel case. Thus, the non-reactive PU is effectively protected by the SCCP constraint. However, under the reactive PU model, it is observed that under the SCCP constraint, the PU's normalized throughput is lower than the benchmark throughput Υ_a in both cases of $\zeta = 0.05$ and $\zeta = 0.1$ if $T > 1$; and thus the PU is not protected properly, which is in accordance with Proposition 4.4.1.

Figure 4.5: PU's normalized throughput under the SCCP constraint. $N = 1$.

Furthermore, the PU's throughput loss is more substantial in the case of $\zeta = 0.1$ than in the case of $\zeta = 0.05$, since the PU allows more collisions when ζ is larger.

Fig. 4.6 shows the sum of SU's and PU's normalized throughput under the reactive PU model as well as the non-reactive PU model with $N = 1$. It is observed that, in both cases of $\zeta = 0.05$ and $\zeta = 0.1$, the sum-throughput is higher in the reactive PU model than in the non-reactive PU model. This implies that in the reactive PU model, although the PU may reduce its channel access probability, the sum-throughput can also be improved due to the SU's throughput improvement, which has been shown in Fig. 4.4. It is also observed that under both models, the sum-throughput is higher in the case of $\zeta = 0.05$ than that in the case of $\zeta = 0.1$.

4.5.3 Multi-Channel Case

Next, we consider the multi-channel case by assuming $N = 3$. The reactive PU model is given by four vectors $\alpha_0 = (0.1, 0.1, 0.05)$, $\beta_0 = (0.1, 0.2, 0.6)$, $\alpha_1 = (0.9, 0.9, 0.9)$, and $\beta_1 = (0.95, 0.95, 0.95)$. Similar to the single-channel case, for the multi-channel

Figure 4.6: Sum throughput under the SCCP constraint. $N = 1$.

case, the non-reactive PU model is obtained by setting $\alpha_1 = \alpha_0 = (0.1, 0.1, 0.05)$ and $\beta_1 = \beta_0 = (0.1, 0.2, 0.6)$. According to the stationary distribution of the underlying Markov chain, the PU's initial channel access probabilities in slot $t = 1$ are obtained as $(0.9, 0.889, 0.889)$ under both models. Since the results of the SU's and the PU's throughput with $\zeta = 0.1$ are similar to those with $\zeta = 0.05$, we only show the case of $\zeta = 0.05$ in this subsection. From (4.24), the PU's benchmark throughput is obtained as $\Upsilon = (\Upsilon_1, \Upsilon_2, \Upsilon_3) = (0.855, 0.846, 0.846)$ with $\zeta = 0.05$.

Fig. 4.7 shows the SU's normalized throughput subject to the SCCP constraint when $N = 3$. As in Fig. 4.4 for the single-channel case, we also consider three scenarios, i.e., 1) under the reactive PU model, 2) under the non-reactive PU model, and 3) the model mismatch case. We observe that the SU achieves higher throughput under the reactive PU model than under the non-reactive PU model, which is similar to the single-channel case shown in Fig. 4.4. In addition, compared with the single-channel case of $\zeta = 0.05$ in Fig. 4.4, we observe that the SU achieves higher throughput in the multi-channel case under both models. This is because that when $N > 1$, the

Figure 4.7: SU's normalized throughput under the SCCP constraint. $N = 3$.

SU has more flexibility in selecting channels that are more likely to be unoccupied to access. Furthermore, for the model mismatch case, different from that under the single-channel case, where there is no throughput loss, we observe throughput loss occurs when $T \geq 3$ under the multi-channel case. This is because that when $N > 1$, the SU's optimal channel sensing policy π_s^* under the reactive PU model is different from that under the non-reactive PU model.

Fig. 4.8 shows the PU's normalized throughput on each channel under the SCCP constraint with $N = 3$, under both the reactive and non-reactive PU models. Similar to that under the single-channel case, since the benchmark throughput Υ_n of channel n , $\forall n \in \{1, 2, 3\}$, is actually the non-reactive PU's normalized throughput on channel n under the SCCP constraint, the non-reactive PU on each channel is effectively protected by the SCCP constraint. Under the reactive PU model, it is observed that the PU's normalized throughput on channel 1 is higher than the benchmark throughput Υ_1 and remains as 0.9 over T , while the PU's normalized throughput on channel 2 and channel 3 vary over T . This indicates that the SU only selects channel

Figure 4.8: PU's normalized throughput under the SCCP constraint. $N = 3$.

2 and channel 3 to access in this example. We also observe that the PU's throughput on channel 2 is always larger than its benchmark throughput Υ_2 . However, when $T > 2$, the PU's throughput on channel 3 decreases over T and becomes lower than the benchmark throughput Υ_3 . This indicates that the SU selects channel 3 to access in most of the slots. Thus, the PU on channel 2 is protected, while the PU on channel 3 is not protected properly. As a result, as we discussed in Section 4.4.2, the SCCP constraint is in general not able to provide effective protection to all the reactive PUs when $N > 1$.

Fig. 4.9 shows the sum of SU's and PU's normalized throughput on each channel. We observe that under both reactive and non-reactive PU models, the sum-throughput on channel 1 remains as a constant over T , which is equal to the PU's benchmark throughput 0.9 on channel 1, while the sum-throughput on channel 2 and channel 3 vary over T . This implies that under both models, the SU only selects channel 2 and channel 3 to access, which is in accordance with Fig. 4.8 for reactive PUs. From the model mismatch case of Fig. 4.7, the specific channel selected to

Figure 4.9: Sum throughput under the SCCP constraint. $N = 3$.

access in each time slot under the non-reactive PU model is different from that under the reactive PU model, thus it is observed that the sum-throughput on channel 2 and channel 3 are different under both models. Moreover, under the multi-channel case, the SU has more options to select channels to access. Suppose the SU accesses channel a in the previous slot but channel b in the current slot, where $a \neq b$, due to the reactive PU's action on reducing the channel access probability on channel a for the current slot, the reactive PU experiences more throughput loss than the non-reactive PU on channel a , which does not change its channel access probability. As a result, we observe that the sum-throughput under the reactive PU model is lower than that under the non-reactive PU model on channel 2 when $T > 3$ and on channel 3 when $1 < T < 5$, due to the reactive PU's larger throughput loss. However, it is observed that when $T > 5$, the sum-throughput on channel 3 under the reactive PU model is higher than that under the non-reactive PU model. This implies that when $T > 5$, under the reactive PU model, the SU selects channel 3 to access in

most of the time slots, and thus largely improves its own throughput by taking advantage of the PU's reactions, which is similar to the single-channel case. As a result, the sum-throughput under the reactive PU model is improved due to the SU's large throughput improvement for often accessing this channel when $T > 5$.

4.6 Summary

This chapter investigates the SU's optimal OSA policy design in the presence of reactive PUs, under a practical scenario where the PU's channel occupancy status is only partially known to the SU. A 4-state discrete-time Markov chain model is proposed to describe the channel occupancy states of a reactive PU. This model is able to capture the essential reactions of the PU subject to the SU's transmit collisions. The optimal OSA design for the SU's throughput maximization is formulated as a finite-horizon POMDP problem, under the conventional SCCP constraint to protect the reactive PU's transmissions. Moreover, by extending the *separation principle* proposed in [22] for the non-reactive PU case to the reactive PU case, an optimal OSA policy is obtained, which can be implemented efficiently. Due to the same SCCP constraint adopted, we show that the optimal sensor operating policy and the optimal access policy for the SU under the reactive PU model are the same as that under the non-reactive PU model, respectively. However, since the reactive PU can change its channel access probability upon experiencing transmission collisions, the optimal sensing policy for the SU under the reactive PU model is different from that under the non-reactive PU model. Moreover, unlike the non-reactive PU case, we analytically show that the reactive PU's average throughput over T slots cannot be guaranteed by adopting the optimal OSA policy under the SCCP constraint in general; and thus a new PU protection method is needed for the reactive PU.

Chapter 5

Opportunistic Spectrum Access with Long-term Protection to Reactive Primary Users

In Chapter 4, we have showed that the conventional SCCP constraint, which is effective to protect the non-reactive PU's transmissions, is not sufficient to protect the reactive PU's transmissions. In this chapter, we propose a new method to protect the reactive PU's transmissions, which is referred to as long-term PU throughput (LPUT) constraint. We show that the LPUT constraint can provide effective protection to the reactive PUs. By adopting the same system model as that in Chapter 4, we study the optimal OSA design for the SU under the LPUT constraint.

The rest of this chapter is organized as follows: Section 5.1 focuses on presenting the motivation of applying the LPUT constraint to protect the reactive PU's transmissions. Section 5.2 formulates the OSA design under the LPUT constraint as a finite-horizon long-term constrained POMDP problem. Since the proposed problem is very challenging to solve, to start with, we first study the single-channel case in Section 5.3. We obtain the optimal OSA policy structure as well as a suboptimal policy with lower computational complexity under this case. Then in Section 5.4, we extend the suboptimal policy for the single-channel case to the multi-channel case.

Numerical results on the PU's and SU's throughput are showed in Section 5.5. Finally, Section 5.6 summarizes this chapter.

5.1 System Model and Effective Protection

Since the SU's optimal OSA policy under the conventional SCCP constraint in Chapter 4 cannot effectively protect the reactive PU's transmissions, this chapter aims at finding the optimal OSA policy for the SU that can simultaneously maximize the SU's throughput and effectively protect the reactive PU's transmissions.

To be in accordance with the study in Chapter 4, this chapter adopts the same system topology, the same reactive PU channel occupancy model, and the same SU's sequential OSA decisions as that in Section 4.2, respectively. Moreover, we also use POMDP to model the SU's OSA, where the POMDP is defined as the same as that in Section 4.3. We adopt the same objective function for the SU's OSA as that given by (4.10).

In the following, we focus on finding a new method to effectively protect the reactive PU. As have explained in Chapter 4, since the SCCP constraint does not consider the PU's reactions on decreasing/increasing its channel access probability upon experiencing transmission collisions, it fails to effectively protect the reactive PU in the long term. One simple way to reflect the PU's transmission reactions is to improve the SCCP constraint by using a time-varying constraint ζ_t , $t \in \{1, \dots, T\}$, to replace the fixed ζ , given in (4.13). To guarantee the benchmark throughput of the PU, ζ_t needs to vary over the PU's channel access probability, which is affected by the SU's OSA actions that need to be determined. Thus, it is very difficult to find a proper sequence of $\{\zeta_t\}$, such that the improved SCCP constraint can effectively guarantee the PU's benchmark throughput on each channel.

As a result, instead of focusing on the instantaneous collision probability constraint that is difficult to provide long-term protection to the PU, we focus on finding a long-term constraint to protect the reactive PU. Directly from the definition of the PU's

benchmark throughput, we propose a LPUT constraint, which is formulated as

$$R_{P,n}^o \geq \Upsilon_n, \quad \forall n \in \mathbb{A}_S, \quad (5.1)$$

where $R_{P,n}^o$ is defined in (4.23) and Υ_n is given in (4.24). Unlike the SCCP constraint in (4.13), from (4.23), the LPUT constraint takes into account the PU's reaction to the SU's collision in each slot. Clearly, if the LPUT constraint is satisfied by the SU's OSA policy, the PU's achieved long-term average throughput is guaranteed to be no smaller than its benchmark throughput. Thus, the LPUT constraint in (5.1) is able to provide effective protection to the reactive PU transmissions.

It is noted that the LPUT constraint has been widely adopted in the SS-based CR systems, to guarantee that the PU's transmission quality is always above a predefined threshold regardless of the PU's on/off status [123]-[128]. In contrast, in the OSA-based CR systems, due to the simplicity and effectiveness of the SCCP constraint in protecting the non-reactive PU transmissions, the complicated LPUT constraint has not been used to protect PU transmissions, to our best knowledge. Moreover, as will be specified in the next section, under the assumption that the SU are backlogged with data packets in a period of T slots, $1 < T < \infty$, we formulate the OSA design problem under the LPUT constraint as a finite-horizon long-term constrained POMDP problem. Such problem is more difficult to solve than the infinite-horizon POMDP problem in, e.g., [127] and [128] for the SS-based CR, where the SU's optimal policy is generally stationary.

5.2 Problem Formulation

Suppose the SU is always backlogged with data packets in a period of T slots. As in Chapter 4, we assume T is finite, i.e., $1 < T < \infty$, under the practical consideration of the SU's traffic loads. Thus, the SU has the intention to transmit in each slot t , $t \in \{1, \dots, T\}$. By using (4.10) and (5.1), the OSA design under the LPUT constraint

is then formulated as

$$\begin{aligned}
 (\text{P2}) : \max_{\pi_s, \pi_\delta, \pi_c} & \quad E_{\pi_s, \pi_\delta, \pi_c} \left\{ \sum_{t=1}^T R_S(t) | \Lambda(1) \right\} \\
 \text{s.t.} \quad & R_{P,n}^o \geq \Upsilon_n, \quad \forall n \in \mathbb{A}_S.
 \end{aligned}$$

To our best knowledge, there is no existing work that addresses the *finite-horizon long-term constrained* POMDP problem as that in (P2). As the problem has infinite and unaccountable action space, it is challenging to find the optimal policy $\{\pi_s^*, \pi_\delta^*, \pi_c^*\}$ for it. Note that for (P1) studied in Chapter 4, we have proposed a separation principle to design π_s^* separately from π_δ^* and π_c^* , since the SCCP constraint in (P1) is only related to π_δ^* , and π_c^* . However, the LPUT constraint in (P2) is determined by all the three policies. Thus, π_s^* is generally *not independent* of π_δ^* and π_c^* , which implies that (P2) cannot be solved optimally by the separation principle. However, a suboptimal policy for (P2) can be found based on the separation principle. To start with, we first focus on the single-channel case with $N = 1$ for (P2), where only π_δ^* and π_c^* need to be determined. We then propose the suboptimal policy for the general multi-channel case with $N > 1$ by extending the results from $N = 1$ to $N > 1$ based on the separation principle.

5.3 Optimal Policy: Single-Channel Case

5.3.1 Optimal Policy Structure

For (P2) in the single-channel case with $n = a$, we have

$$\begin{aligned}
 (\text{P2-S}) : \max_{\pi_\delta, \pi_c} & \quad E_{\pi_\delta, \pi_c} \left\{ \sum_{t=1}^T R_S(t) | \Lambda(1) \right\} \\
 \text{s.t.} \quad & R_{P,a}^o \geq \Upsilon_a,
 \end{aligned}$$

with $\left\{ \left((\epsilon_a(t), \delta_a(t)), (f_a(0, t), f_a(1, t)) \right) : (\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a), (f_a(0, t), f_a(1, t)) \in [0, 1]^2 \right\}$ as the action space. To simplify (P2-S), in this subsection, we first propose an equivalent problem to (P2-S), namely, (P2-S-1), through which we find the optimal OSA policy structure. Based on the optimal OSA policy structure, we reduce (P2-S-1) to another problem (P2-S-2) with a significantly reduced action space. To facilitate our analysis, we first present the following proposition.

Proposition 5.3.1. *Under the case of $N = 1$, for any given SU's OSA policy $\pi = \{\pi_\delta, \pi_c\}$, where we have the resultant PU's normalized throughput $R_{P,a}^o > \Upsilon_a$ and the SU's reward $E_\pi \left\{ \sum_{t=1}^T R_S(t) | \Lambda(1) \right\}$, we can find another policy $\pi' = \{\pi'_\delta, \pi'_c\}$, which guarantees that the resultant PU's normalized throughput $R_{P,a}^{o'} = \Upsilon_a$ and the SU's reward $E_{\pi'} \left\{ \sum_{t=1}^T R_S(t) | \Lambda(1) \right\} > E_\pi \left\{ \sum_{t=1}^T R_S(t) | \Lambda(1) \right\}$.*

Proof. Please refer to Appendix H. □

According to Proposition 5.3.1, the optimal policy $\pi^* = \{\pi_\delta^*, \pi_c^*\}$ is selected to ensure that $R_{P,a}^o = \Upsilon_a$. Thus, (P2-S) is *equivalent* to

$$\begin{aligned} \text{(P2-S-1)} : \max_{\pi_\delta, \pi_c} \quad & E_{\pi_\delta, \pi_c} \left\{ \sum_{t=1}^T R_S(t) | \Lambda(1) \right\} \\ \text{s.t.} \quad & R_{P,a}^o = \Upsilon_a. \end{aligned}$$

Proposition 5.3.2. *The structure of the optimal policy $\pi^* = \{\pi_\delta^*, \pi_c^*\}$ for (P2-S-1) is given as follows.*

$$\begin{cases} \delta_a^*(t) \text{ is in general time-variant and to be determined,} \\ \epsilon_a^*(t) \text{ is on the optimal ROC curve corresponding to } \delta_a^*(t), \\ f_a^*(0, t) = 0, \\ f_a^*(1, t) = 1. \end{cases} \quad (5.2)$$

Proof. Please refer to Appendix I. □

Remark. As shown in (5.2), the optimal spectrum access policy π_c^* for (P2-S-1) is the same as that for (P1), and the optimal sensor operating point $(\delta_a^*(t), \epsilon_a^*(t))$ for

(P2-S-1) is on the optimal ROC curve as that for (P1). However, different from the time-invariant case for (P1), the optimal PM decision for (P2-S-1) is in general time-varying. As proved in Appendix I, $\delta_a^*(t)$ for (P2-S-1) is related to the current belief state $\mathbf{\Lambda}(t)$ and thus needs to be determined adaptively over time. This indicates that the spectrum sensor design plays a crucial role in protecting the reactive PU's transmission under the LPUT constraint.

By applying (4.23) and (5.2) and without loss of optimality, (P2-S-1) is reduced to

$$\begin{aligned} \text{(P2-S-2)} : \max_{\pi_\delta} \quad & E_{\pi_\delta} \left\{ \sum_{t=1}^T R_S(t) | \mathbf{\Lambda}(1), \pi_c^* \right\} \\ \text{s.t.} \quad & \frac{1}{T} E_{\pi_\delta} \left\{ \sum_{t=1}^T R_{P,a}(t) | \mathbf{\Lambda}(1), \pi_c^* \right\} = \Upsilon_a, \end{aligned}$$

where $(\delta_a(t), \epsilon_a(t))$ determined by π_δ is on the optimal ROC curve. Thus, to find the optimal policy π_δ^* for (P2-S-2), we only need to search the action space of $\{\delta_a(t) : \delta_a(t) \in [0, 1]\}$, which is greatly reduced over that of (P2-S-1).

Since (P2-S-2) is reduced from (P2-S-1) and (P2-S-1) is equivalent to (P2-S), substituting the optimal π_δ^* for (P2-S-2) into (5.2) yields the optimal OSA policy for (P2-S-1) and thus (P2-S). Hence, in the following, we focus on solving (P2-S-2).

However, finding π_δ^* for (P2-S-2) is of high complexity, mainly due to the following two reasons: 1) the infinite and unaccountable action space of (P2-S-2), and 2) the non-deterministic POMDP belief state transitions. As the complexity due to the first reason is obvious, here we explain the complexity due to the second reason. From (4.15) and (4.16), to maximize the SU's throughput in (P2-S-2), i.e., to find $V_1(\mathbf{\Lambda}(1))$ under the LPUT constraint, we need to obtain $V_t(\mathbf{\Lambda}(t))$ for all $t \in \{2, \dots, T\}$. As shown in (4.9), given $\mathbf{\Lambda}(t)$ and the SU's OSA actions in slot t , 2 possible belief states in slot $t+1$ exist with non-zero probability, corresponding to the 2 possible observations, respectively. Thus, for the case of $N = 1$, given the initial belief state $\mathbf{\Lambda}(1)$ and the SU's OSA policies π_δ and π_c^* , the complexity of computing $V_1(\mathbf{\Lambda}(1))$ is $\mathcal{O}(2^T)$, which is not scalable with T .

In the following, we focus on designing a suboptimal policy for (P2-S-2) that can meet the LPUT constraint. We use the method given in Appendix G to calculate the PU's normalized throughput, which is similar to the SU's throughput calculation in (4.17) and (4.18). However, also due to the non-deterministic POMDP belief state transitions, it is of exponentially increased complexity over time to find a policy that can meet the LPUT constraint. Motivated by [114], we note that the complexity due to the non-deterministic POMDP belief state transitions is reducible, by converting the POMDP into an equivalent MDP with deterministic state transitions. In the following subsections, we first construct the equivalent MDP, and then based on the MDP, we propose a suboptimal policy for (P2-S-2), which satisfies the LPUT constraint in (P2-S-2).

5.3.2 Equivalent Markov Decision Process

In this subsection, we first convert the POMDP for (P2-S-2) into an MDP with deterministic state transitions, and reformulate the LPUT constraint in (P2-S-2) based on the MDP. We then show that if a policy satisfies the MDP-based LPUT constraint, it will also satisfy the POMDP-based counterpart in (P2-S-2).

An MDP in general consists of the following elements [101]: a set of time-slots $\{1, \dots, T\}$, a set of system states (with transition probabilities), actions and rewards, for each of the time-slots. In the following, we formulate the MDP model for the SU's OSA by specifying these elements according to [114]. Specifically, for the single-channel case with $n = a$, the MDP state in slot t , $1 \leq t \leq T$, is denoted by a 4-element vector $\boldsymbol{\Omega}(t) = \{\omega_{ai}(t)\}_{i \in \mathcal{C}_S}$, where $\omega_{ai}(t) \in [0, 1]$ is the conditional probability that the reactive PU is at the i -th state on channel a in slot t , given the SU's action history. Note that the MDP state space, given by $[0, 1]^4$, is the same as the POMDP belief state space, given in Section 4.3.1, for $N = 1$. We assume that $\boldsymbol{\Omega}(1) = \boldsymbol{\Lambda}(1)$ in the initial slot $t = 1$. Based on the current MDP state $\boldsymbol{\Lambda}(t)$ in slot t , the SU selects an action $A(t) = ((\epsilon_{a,\mathcal{M}}(t), \delta_{a,\mathcal{M}}(t)), (f_{a,\mathcal{M}}(0, t), f_{a,\mathcal{M}}(1, t)))$ for OSA, where $(\epsilon_{a,\mathcal{M}}(t), \delta_{a,\mathcal{M}}(t)) \in \mathbb{A}_\delta(a)$ is the sensor operating point and $(f_{a,\mathcal{M}}(0, t), f_{a,\mathcal{M}}(1, t)) \in [0, 1]^2$ is the channel access probability. Thus, the MDP action space is the same

as the POMDP action space for $N = 1$. We then follow the optimal OSA policy structure in (5.2) and set

$$\begin{cases} (\epsilon_{a,\mathcal{M}}(t), \delta_{a,\mathcal{M}}(t)) \text{ locates on the optimal ROC curve,} \\ f_{a,\mathcal{M}}(0, t) = 0, \\ f_{a,\mathcal{M}}(1, t) = 1, \end{cases} \quad \forall t \in \{1, \dots, T\}, \quad (5.3)$$

where in slot t , the SU only needs to determine the PM action $\delta_{a,\mathcal{M}}(t)$. Denote $P_a(\mathbf{\Omega}(t+1)|\mathbf{\Omega}(t), \delta_{a,\mathcal{M}}(t))$ as the MDP state transition probability from state $\mathbf{\Omega}(t) = \{\omega_{ai}(t)\}_{i \in \mathbb{C}_S}$ in slot t to state $\mathbf{\Omega}(t+1) = \{\omega_{ai}(t+1)\}_{i \in \mathbb{C}_S}$ in slot $t+1$ on channel a , given the SU's selected PM action $\delta_{a,\mathcal{M}}(t)$ in slot t . From Fig. 4.3(b), $\omega_{ai}(t+1) = \sum_{j=0}^3 \omega_{aj}(t) P_a(i|j, \delta_{a,\mathcal{M}}(t))$, $i, j \in \mathbb{C}_S$, where $P_a(i|j, \delta_{a,\mathcal{M}}(t))$ is the state transition probability $P_a(i|j, A)$ given in Fig. 4.3(b), with A reduced to $\delta_{a,\mathcal{M}}(t)$ by applying (5.3). Thus, we obtain the MDP state transition probability as

$$\begin{aligned} & P_a(\mathbf{\Omega}(t+1)|\mathbf{\Omega}(t), \delta_{a,\mathcal{M}}(t)) \\ &= \begin{cases} 1, & \text{if } \omega_{ai}(t+1) = \sum_{j=0}^3 \omega_{aj}(t) P_a(i|j, \delta_{a,\mathcal{M}}(t)), \forall i \in \mathbb{C}_S. \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (5.4)$$

From (5.4), the MDP state transition is deterministic. That is, given $\delta_{a,\mathcal{M}}(t)$ selected at MDP state $\mathbf{\Omega}(t)$ in slot t , there is only one possible MDP state $\mathbf{\Omega}(t+1)$ in slot $t+1$.

Denote the PU's throughput in slot t on channel a by $R_{P,a}^{\mathcal{M}}(t)$, which is

$$R_{P,a}^{\mathcal{M}}(t) = (\omega_{a0}(t) + \omega_{a2}(t)) \times (1 - \delta_{a,\mathcal{M}}(t)). \quad (5.5)$$

The PU's normalized throughput on channel a over T slots is thus $\frac{1}{T} \sum_{t=1}^T R_{P,a}^{\mathcal{M}}(t)$. With the benchmark throughput Υ_a , given in (4.24), the MDP-based LPUT constraint is formulated as

$$\frac{1}{T} \sum_{t=1}^T R_{P,a}^{\mathcal{M}}(t) = \Upsilon_a. \quad (5.6)$$

Note that due to the deterministic MDP state transitions, with a complexity

analysis similar to that in Section 5.3.1, it is easy to find that the complexity in computing $\sum_{t=1}^T R_{P,a}^{\mathcal{M}}(t)/T$ under the MDP policy is $\mathcal{O}(T)$, which is substantially reduced as compared to that based on the POMDP.

Proposition 5.3.3. *Given an MDP policy $\pi_{\delta,\mathcal{M}}^*$, which specifies a PM decision $\delta_{a,\mathcal{M}}^*(t)$ in slot t , $1 \leq t \leq T$, we construct a POMDP policy π_{δ}^* for (P2-S-2), where the corresponding PM decision in each slot t is $\delta_a^*(t) = \delta_{a,\mathcal{M}}^*(t)$. If the MDP-based LPUT constraint in (5.6) is satisfied under $\pi_{\delta,\mathcal{M}}^*$, the POMDP-based LPUT constraint in (P2-S-2) is also satisfied under π_{δ}^* .*

Proof. Please refer to Appendix J. □

5.3.3 Suboptimal Policy

In this subsection, by studying the MDP-based LPUT constraint, we derive a suboptimal policy for (P2-S), such that the LPUT constraint in (P2-S) is satisfied. In the following, we first give a sufficient condition for satisfying the MDP-based LPUT constraint, based on which, we propose an MDP-based policy $\pi_{\delta,\mathcal{M}}^*$, which can satisfy the MDP-based LPUT constraint. Based on $\pi_{\delta,\mathcal{M}}^*$ and Proposition 5.3.3, we then obtain a suboptimal policy for (P2-S-2), such that the LPUT constraint in (P2-S-2) is satisfied. Finally, we obtain the suboptimal policy for (P2-S) by substituting the suboptimal policy for (P2-S-2) into the optimal OSA policy structure in (5.2).

A sufficient condition for satisfying MDP-based LPUT constraint

From (5.6), to satisfy the LPUT constraint, we take $\Upsilon_a \times T$ as the PU's throughput requirement over all T slots in the MDP. We then denote $X_{P,a}(t)$ as the PU's throughput requirement from slot t to slot T in the MDP and have

$$X_{P,a}(1) \triangleq \Upsilon_a \times T, \quad t = 1, \quad (5.7)$$

$$X_{P,a}(t) \triangleq X_{P,a}(t-1) - R_{P,a}^{\mathcal{M}}(t-1), \quad \forall t \in \{2, \dots, T\}. \quad (5.8)$$

Given the PU's obtained throughput $R_{P,a}^M(t-1)$ in slot $t-1$, from (5.8) and by calculating backward in time, we observe that if $X_{P,a}(t)$ in slot t is satisfied, $X_{P,a}(t-1)$ in slot $t-1$ is achieved. Thus, we can easily show that if $X_{P,a}(T)$ in the last slot T is satisfied, the PU's throughput requirement $X_{P,a}(1)$ is achieved, i.e., the LPUT constraint given in (5.6) is met. Note that $X_{P,a}(T)$ can be satisfied by selecting $\delta_{a,\mathcal{M}}(T)$ such that $X_{P,a}(T) = (\omega_{a0}(T) + \omega_{a2}(T)) \times (1 - \delta_{a,\mathcal{M}}(T))$. Since $\delta_{a,\mathcal{M}}(T) \in [0, 1]$, the following inequality is obtained as a *sufficient condition* for satisfying (5.6):

$$0 \leq X_{P,a}(T) \leq \omega_{a0}(T) + \omega_{a2}(T). \quad (5.9)$$

MDP-based policy $\pi_{\delta,\mathcal{M}}^*$

The MDP-based policy $\pi_{\delta,\mathcal{M}}^*$ is given by the PM actions $\{\delta_{a,\mathcal{M}}(1), \dots, \delta_{a,\mathcal{M}}(T)\}$. In the following, we derive the minimum required PM, denoted by $\delta_{a,\mathcal{M}}^L(t)$, and the maximum allowable PM, denoted by $\delta_{a,\mathcal{M}}^U(t)$, in slot t , such that (5.9) is satisfied if the SU selects $\delta_{a,\mathcal{M}}(t) \in [\delta_{a,\mathcal{M}}^L(t), \delta_{a,\mathcal{M}}^U(t)]$, $\forall t \in \{1, \dots, T\}$.

From (5.7) and (5.8), to ensure $X_{P,a}(T) \geq 0$, we need $X_{P,a}(t) \geq 0$ in all the previous slots with $t \in \{1, \dots, T-1\}$. By substituting $X_{P,a}(t) \geq 0$ to (5.8) and using (5.5), we obtain $\delta_{a,\mathcal{M}}(t) \geq 1 - \frac{X_{P,a}(t)}{\omega_{a0}(t) + \omega_{a2}(t)}$, $1 \leq t \leq T-1$. When $t = T$, if $X_{P,a}(T) \leq \omega_{a0}(T) + \omega_{a2}(T)$ is satisfied, we only need to set $\delta_{a,\mathcal{M}}(T) = 1 - \frac{X_{P,a}(T)}{\omega_{a0}(T) + \omega_{a2}(T)}$ to satisfy $X_{P,a}(T)$. Thus, we obtain

$$\delta_{a,\mathcal{M}}^L(t) = \max \left(0, 1 - \frac{X_{P,a}(t)}{\omega_{a0}(t) + \omega_{a2}(t)} \right), \quad 1 \leq t \leq T. \quad (5.10)$$

Proposition 5.3.4. *To ensure $X_{P,a}(T) \leq \omega_{a0}(T) + \omega_{a2}(T)$, the SU's PM $\delta_{a,\mathcal{M}}(t)$ selected in slot t needs to satisfy the following inequality:*

$$\delta_{a,\mathcal{M}}(t) \leq \frac{m_1(t)}{m_4(t)} + \frac{\omega_{a1}(t) \times m_2(t) + \omega_{a3}(t) \times m_3(t) - X_{P,a}(t)}{(\omega_{a0}(t) + \omega_{a2}(t)) \times m_4(t)} \quad (5.11)$$

where

$$\begin{cases} m_1(t) = 1 + (1 - \alpha_0^a) \times m_1(t+1) + \alpha_0^a \times m_2(t+1), \\ m_2(t) = (1 - \beta_0^a) \times m_1(t+1) + \beta_0^a \times m_2(t+1), \\ m_3(t) = (1 - \beta_1^a) \times m_1(t+1) + \beta_1^a \times m_3(t+1), \\ m_4(t) = 1 + (\alpha_1^a - \alpha_0^a) \times m_1(t+1) + \alpha_0^a \times m_2(t+1) - \alpha_1^a \times m_3(t+1), \end{cases} \quad (5.12)$$

for $1 \leq t < T$ and

$$\begin{cases} m_1(T) = 1, \\ m_2(T) = 0, \\ m_3(T) = 0, \\ m_4(T) = 1, \end{cases} \quad (5.13)$$

for $t = T$.

Proof. The proof is based on mathematical induction and by computing backward in time. It is easy to obtain (5.11) holds in slot $t = T$. Now suppose (5.11) holds in slot $t+1 \leq T$ with $m_j(t+1)$, $j \in \mathbb{C}_S$, given in (5.12), if $t < T-1$, or in (5.13), if $t = T-1$. Then the following inequality must hold, otherwise, $\delta_{\mathcal{M}}(t+1)$ can be shown to be negative:

$$X_P(t+1) \leq (\omega_0(t+1) + \omega_2(t+1)) \times m_1(t+1) + \omega_1(t+1) \times m_2(t+1) + \omega_3(t+1) \times m_3(t+1).$$

By substituting (5.4) and (5.8) into the above inequality, after some algebra, we find that (5.11) still holds in slot t with $m_j(t)$, $j \in \mathbb{C}_S$, given in (5.12). Proposition 5.3.4 is thus proved. $\square$

With Proposition 5.3.4, the SU's maximum allowable PM $\delta_{a,\mathcal{M}}^U(t)$ is obtained as

$$\delta_{a,\mathcal{M}}^U(t) = \min \left(1, \frac{m_1(t)}{m_4(t)} + \frac{\omega_{a1}(t) \times m_2(t) + \omega_{a3}(t) \times m_3(t) - X_{P,a}(t)}{(\omega_{a0}(t) + \omega_{a2}(t)) \times m_4(t)} \right). \quad (5.14)$$

We only need to compute $m_1(t)$, $m_2(t)$, $m_3(t)$ and $m_4(t)$, given in (5.12) and (5.13), to find $\delta_{a,\mathcal{M}}^U(t)$ in each slot t . Clearly, it is only of complexity of $\mathcal{O}(T)$ to find $\delta_{a,\mathcal{M}}^U(t)$.

Clearly, if $\delta_{a,\mathcal{M}}(t) \in [\delta_{a,\mathcal{M}}^L(t), \delta_{a,\mathcal{M}}^U(t)]$ in slot t is selected, (5.9) is guaranteed and thus the MDP-based LPUT constraint given in (5.6) is satisfied. We then propose the MDP-based policy $\pi_{\delta,\mathcal{M}}^*$ by specifying

$$\delta_{a,\mathcal{M}}^*(t) = \delta_{a,\mathcal{M}}^L(t) + \psi(t) \times (\delta_{a,\mathcal{M}}^U(t) - \delta_{a,\mathcal{M}}^L(t)), \quad (5.15)$$

where $\delta_{a,\mathcal{M}}^*(t) \in [\delta_{a,\mathcal{M}}^L(t), \delta_{a,\mathcal{M}}^U(t)]$ is guaranteed by selecting $\psi(t) \in [0, 1]$ in slot t .

POMDP-based suboptimal policy

We first consider the POMDP-based problem (P2-S-2). From Proposition 5.3.3, by setting $\delta_a^*(t) = \delta_{a,\mathcal{M}}^*(t)$, $1 \leq t \leq T$, we find a suboptimal policy π_δ^* for (P2-S-2), which satisfies the LPUT constraint in (P2-S-2).

Next, we consider the original POMDP-based problem (P2-S). Note that (P2-S-2) is reduced from (P2-S-1) without loss of optimality and (P2-S-1) is equivalent to (P2-S). Thus, by substituting the suboptimal spectrum sensor operating policy π_δ^* into the optimal OSA policy structure in (5.2), we obtain a *suboptimal OSA policy* for both (P2-S-1) and (P2-S) as

$$\begin{cases} \delta_a^*(t) = \delta_{a,\mathcal{M}}^L(t) + \psi(t) \times (\delta_{a,\mathcal{M}}^U(t) - \delta_{a,\mathcal{M}}^L(t)), \text{ where } \psi(t) \in [0, 1], \\ \epsilon_a^*(t) \text{ is on the best ROC curve corresponding to } \delta_a^*(t), \\ f_a^*(0, t) = 0, \\ f_a^*(1, t) = 1, \end{cases} \quad (5.16)$$

Since π_δ^* satisfies the LPUT constraint in (P2-S-2), the suboptimal OSA policy in (5.16) satisfies the LPUT constraint in (P2-S-1) and thus (P2-S) with equality.

5.4 Optimal Policy: Multi-Channel Case

At last, we consider the general case with $N > 1$ for (P2). In this case, although the spectrum sensing policy π_s generally depends on the sensor operating policy π_δ and the spectrum access policy π_c , a suboptimal policy for (P2) can be obtained by

separately designing π_s from π_δ and π_c , i.e., applying a separation principle. Based on the results for $N = 1$, a suboptimal policy for $N > 1$ is proposed as follows.

- Step 1: On each channel $n \in \mathbb{A}_S$, the SU selects the sensor operating point and the spectrum access probabilities in slot t according to π_δ^* and π_c^* , as given in (5.16).
- Step 2: Apply π_δ^* and π_c^* to obtain the SU's sensing policy π_s^* , which determines the channel to be sensed in slot t , by solving the following unconstrained POMDP:

$$\pi_s^* = \arg \max_{\pi_s} E_{\pi_s} \left\{ \sum_{t=1}^T R_S(t) | \mathbf{\Lambda}(1), \pi_\delta^*, \pi_c^* \right\}. \quad (5.17)$$

Similar to the case for solving the unconstrained POMDP in (4.21), the off-line policy π_s^* can be obtained by searching the accountable and finite action space of the POMDP or using the efficient POMDP solver software.

The sensor operating policy π_δ^* and the spectrum access policy π_c^* in (5.16) are provided such that the PU's normalized throughput on channel n , $n \in \mathbb{A}_S$, equals the benchmark throughput Υ_n , if the SU always selects this channel to sense. When $N > 1$, the SU has the option to select one from N channels to sense. Thus, the PU's normalized throughput on channel n will be at least Υ_n . Hence, the LPUT constraint in (P2) over each channel is satisfied by the proposed suboptimal policy.

5.5 Numerical Results

In this section, we show the SU's and PU's throughput under the proposed reactive PU model. The system is set under the same scenario as that given in Section 4.5.1. We study both single-channel and multi-channel cases. For both cases, we compare the SU's and reactive PU's throughput obtained under the SCCP and LPUT constraint, respectively. Such comparison can properly show the effectiveness of the LPUT constraint to protect the reactive PU's transmissions, and illustrate the interesting tradeoff between PU transmission protection and SU throughput maximization.

Figure 5.1: Reactive PU's normalized throughput. $N = 1$.

Single-Channel Case

We first study the PU's and the SU's performance in the single-channel case. As in Section 4.5.2, we set $\alpha_0^a = 0.1$, $\beta_0^a = 0.2$, $\alpha_1^a = 0.9$, and $\beta_1^a = 0.95$, and thus obtain the PU's initial channel access probability in slot $t = 1$ as $\frac{1-\beta_0^a}{1-\beta_0^a+\alpha_0^a} = 0.889$. We also consider two cases: $\zeta = 0.05$ and $\zeta = 0.1$, as in Section 4.5.2, where we obtain the PU's benchmark throughput $\Upsilon_a = 0.846$ when $\zeta = 0.05$, and $\Upsilon_a = 0.8$ when $\zeta = 0.1$.

We focus on the reactive PU model and compare the PU's and the SU's normalized throughput under the SCCP constraint and the LPUT constraint in both cases of $\zeta = 0.05$ and $\zeta = 0.1$. Specifically, under the SCCP constraint, the SU adopts the optimal sensor operating policy π_δ^* and the optimal spectrum access policy π_c^* , as shown in Section 4.4.1, while under the LPUT constraint, the SU adopts the suboptimal sensor operating policy π_δ^* with $\psi(t) = 0.8$ and the optimal access policy π_c^* , as shown in Section 5.3.3.

Figure 5.2: SU's normalized throughput under the reactive PU model. $N = 1$.

Fig. 5.1 compares the PU's normalized throughput with the benchmark throughput. According to Section 4.4.2, the benchmark throughput, which remains as a constant over time in both cases of $\zeta = 0.05$ and $\zeta = 0.1$, is actually the non-reactive PU's normalized throughput under the SCCP constraint in the single-channel case. Thus, the non-reactive PU is effectively protected by the SCCP constraint. However, under the reactive PU model, it is observed that if the SU adopts the SCCP constraint, the PU's normalized throughput is lower than the benchmark throughput Υ_a in both cases of $\zeta = 0.05$ and $\zeta = 0.1$ if $T > 1$; and thus the PU is not protected properly, which is in accordance with Proposition 4.4.1. Furthermore, the PU's throughput loss is more substantial in the case of $\zeta = 0.1$ than in the case of $\zeta = 0.05$, since the PU allows more collisions when ζ is larger. On the other hand, if the SU adopts the LPUT constraint, we observe that the PU's normalized throughput is equal to the benchmark throughput in both cases, and thus the PU is protected as expected.

Fig. 5.2 shows the corresponding SU's normalized throughput under the reactive

PU model. We observe that the SCCP constraint leads to a higher SU throughput than the LPUT constraint in both cases of $\zeta = 0.05$ and $\zeta = 0.1$. This is because the SU can exploit the PU's reaction to get more throughput under the SCCP constraint. It is also observed from Fig. 5.2 that, unlike the case under the SCCP constraint, the SU's throughput under the LPUT constraint does not always increase over T . For example, the SU's throughput obtained with $T = 5$ is lower than that with $T = 4$. In addition, by comparing the SU's normalized throughput in both cases, we observe that the SU achieves higher throughput with $\zeta = 0.1$ as compared to $\zeta = 0.05$. This is consistent with the PU's higher throughput loss when $\zeta = 0.1$ as shown in Fig. 5.1. To evaluate the performance of the proposed suboptimal OSA policy for the SU, in the following, we propose an *upper bound* of the SU's normalized throughput for the single-channel case, under the constraint that the PU must achieve the benchmark throughput. By noticing the fact that unit throughput is the maximum normalized throughput that a channel can provide, the upper bound is given by the difference between the unit throughput and the PU's benchmark throughput on the channel. Since the SU's throughput loss due to sensing errors is not considered, the upper bound is higher than the SU's maximum normalized throughput when the PU achieves the benchmark throughput. As shown in Fig. 5.2, we compare the SU's normalized throughput with the upper bound in both cases of $\zeta = 0.05$ and $\zeta = 0.1$ under both the SCCP and LPUT constraints. It is observed that the SU's normalized throughput under the suboptimal policy is always lower than the upper bound under the LPUT constraint. However, under the SCCP constraint, after $T = 3$ in both cases of $\zeta = 0.05$ and $\zeta = 0.1$, the SU's normalized throughput becomes higher than the *upper bound*, since the PU's achievable throughput deviates from the benchmark throughput.

Fig. 5.3 compares the sum of SU's and PU's normalized throughput under the SCCP and LPUT constraints in both cases of $\zeta = 0.05$ and $\zeta = 0.1$, where the reactive PU model is considered. It is observed from Fig. 5.3 that, in both cases, the LPUT constraint leads to a higher sum-throughput than the SCCP constraint. It is worth pointing out that, although the sum-throughputs under the SCCP and LPUT constraints are close, the individual portions of the SU's and PU's throughput, as

Figure 5.3: Sum throughput under the reactive PU model. $N = 1$.

shown in Fig. 5.1 and Fig. 5.2, are very different under these two constraints.

Multi-Channel Case

Next, we consider the multi-channel case by assuming $N = 3$. Since the performances of SU's and PU's throughput under the reactive PU model in the multi-channel case are similar to those in the single-channel case, respectively, we only consider the reactive PU model in this case. Moreover, we use the same PU model parameters at that in Section 4.5.3. Specifically, we set the reactive PU model by four vectors $\alpha_0 = (0.1, 0.1, 0.05)$, $\beta_0 = (0.1, 0.2, 0.6)$, $\alpha_1 = (0.9, 0.9, 0.9)$, and $\beta_1 = (0.95, 0.95, 0.95)$, where the PU's initial channel access probabilities in slot $t = 1$ are obtained as $(0.9, 0.889, 0.889)$. Since the results of the SU's and the PU's throughput with $\zeta = 0.1$ are similar to those with $\zeta = 0.05$, we only show the case of $\zeta = 0.05$ under the reactive PU model. From (4.24), the PU's benchmark throughput is obtained as $\Upsilon = (\Upsilon_1, \Upsilon_2, \Upsilon_3) = (0.855, 0.846, 0.846)$ with $\zeta = 0.05$. Similar to the single-channel case, the SU adopts π_δ^* with $\psi(t) = 0.8$.

Figure 5.4: Reactive PU's normalized throughput under the LPUT constraint. $N = 3$.

Fig. 5.4 shows the PU's normalized throughput under the LPUT constraint. In Fig. 4.8, we have showed the PU's normalized throughput under the SCCP constraint, by using the same PU model parameters and same value of ζ . It is observed that similar to Fig. 4.8 under the SCCP constraint, the PUs on channels 1 and 2 are both properly protected by the LPUT constraint, since their normalized throughput are larger than their respective benchmark throughput Υ_1 and Υ_2 , respectively. Different from Fig. 4.8, where the PU's normalized throughput on channel 3 is not guaranteed to meet the benchmark throughput Υ_3 , we observe from Fig. 5.4 that the throughput under the LPUT constraint is higher than Υ_3 , i.e., the PU on channel 3 is protected properly. This shows that the LPUT constraint provides effective protection to all the reactive PUs.

Fig. 5.5 compares the SU's normalized throughput under the SCCP and LPUT constraints. Similar to the single-channel case shown in Fig. 5.2, the SU achieves higher throughput under the SCCP constraint than under the LPUT constraint. Compared with the single-channel case of $\zeta = 0.05$ in Fig. 5.2, we observe that the

Figure 5.5: SU's normalized throughput under the LPUT constraint. $N = 3$.

SU achieves higher throughput in the multi-channel case under both the SCCP and LPUT constraints. This is because when $N > 1$, the SU has more flexibility in selecting channels that are more likely to be unoccupied to access.

Fig. 5.6 compares the sum of SU's and PU's normalized throughput on each channel under the SCCP and LPUT constraints. It is already shown in Fig. 4.8 (SCCP constraint) and Fig. 5.4 (LPUT constraint) that the SU only accesses channel 2 or channel 3 and does not access channel 1. Therefore, we observe from Fig. 5.6 that the sum-throughput on channel 1 under both constraints remains unchanged over time, which is equal to the PU's normalized throughput on channel 1. It is also observed from Fig. 5.6 that, on channel 2 and channel 3, the LPUT constraint leads to a higher sum-throughput than the SCCP constraint, which is similar to the single-channel case, as shown in Fig. 5.3.

Figure 5.6: Sum throughput under the reactive PU model. $N = 3$.

5.6 Summary

To effectively protect the reactive PU's transmissions, this chapter investigates the optimal OSA design under the LPUT constraint. The optimal OSA policy structure for the PU's *worst case* transmission with $N = 1$ is obtained and it reveals that the spectrum sensor design plays a crucial role in effectively protecting PU's transmissions. To reduce the complexity of designing an effective spectrum sensor, the POMDP with non-deterministic belief state transitions is converted into an equivalent MDP with deterministic state transitions. Based on the reformulated LPUT constraint based on the MDP, a suboptimal OSA policy with lower implementation complexity is proposed, which is shown to guarantee the reactive PU's throughput. Moreover, by applying the separation principle studied in Chapter 4, the suboptimal policy for the case of $N = 1$ is extended to the general case of $N > 1$. At last, the tradeoff between the SU throughput maximization and the reactive PU transmission protection is illustrated by simulation.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

This thesis illustrates the importance of exploiting user interaction to improve user coexistence, spectrum efficiency and system throughput in interference-aware wireless networks. POMDP and dynamic programming techniques are shown to be useful for solving problems involving user interaction, and stochastic geometry is shown to be an effective mathematical tool for characterizing the wireless network performance. The research contributions presented in this thesis are summarized as follows:

- *Chapter 3:* A transmission scheme with SIR-threshold based scheduling has been proposed, to efficiently coordinate the transmission interactions between the homogeneous users in a large-scale wireless ad hoc network. We start with the proposed scheme with single-stage probing. We show that if the transmission interactions can be well controlled with a SIR threshold $\gamma_1 \in (0, \beta)$, the spatial capacity by the proposed scheme is strictly larger than that by the reference scheme; otherwise, the spatial capacity of the proposed scheme may be smaller than that of the reference scheme. We also propose a new method to successfully characterize the spatial capacity of the proposed scheme, which is useful for analyzing performance of wireless networks with interacted transmitters. Next, we extend our studies to the case with multi-stage probing. We give

the condition under which the spatial capacity of the proposed scheme can be gradually improved over the probing-stage number. Considering the multi-stage probing overhead, we also investigate an interesting probing-capacity tradeoff.

- *Chapter 4:* The optimal OSA design for the SU in the presence of reactive PUs has been addressed under a practical scenario where the PU's channel occupancy status is only partially known to the SUs. First, a new 4-state discrete-time Markov chain model has been proposed to describe the channel occupancy status of each reactive PU. Such model is able to capture the essential reactions of the PUs subject to the SU's transmit collisions. Next, by adopting the conventional SCCP constraint to protect reactive PU's transmission as in [22], the optimal OSA design for the SU has been formulated as a finite-horizon short-term constrained POMDP problem. We obtain the optimal OSA policy by extending the *separation principle* proposed in [22] for the non-reactive PU case to the reactive PU case. Moreover, since the optimality proof of the separation principle in [22] does not apply to the OSA design problem under the reactive PU model, we propose a more general new proof, which hold for both reactive PU and non-reactive PU cases. At last, we analyze whether the SCCP constraint is effective to protect the reactive PU's transmissions. However, unlike the non-reactive PU case, we analytically show that the reactive PU's long-term average throughput requirement cannot be satisfied by the SCCP constraint in general.
- *Chapter 5:* The LPUT constraint has been adopted to effectively protect the reactive PU's transmission. Accordingly, the optimal OSA design for the SU has been reformulated as a finite-horizon long-term constrained POMDP problem. First, for PU's worst case transmission with $N = 1$, the optimal OSA policy structure has been obtained, which reveals the crucial role of spectrum sensor design in effectively protecting the reactive PU's transmissions. Next, by converting the POMDP with non-deterministic belief state transitions into an equivalent MDP with deterministic state transitions, a suboptimal OSA policy with lower computational complexity has been proposed for the case of

$N = 1$, which is shown to guarantee the reactive PU's throughput under this case. Moreover, based on the separation principle, the suboptimal policy for the case of $N = 1$ has been extended to the general case of $N > 1$. It has been shown that the reactive PU's throughput on each channel is guaranteed by the proposed suboptimal policy. At last, we numerically illustrate the interesting tradeoffs between SU throughput maximization and PU transmission protection.

6.2 Future Work

Some research directions, which can be explored in the future, are presented as follows:

- *Spatial capacity of an ad hoc network by enhanced SIR-threshold based transmission schemes:* It is interesting to enhance the SIR-threshold based transmission scheme and study the ad hoc network capacity by the enhanced schemes. Two possible methods are presented here to enhance the scheme: 1) instead of using a universal SIR threshold, we allow each wireless users to adopt different SIR thresholds in each iteration in the enhanced scheme, so as to improve the user fairness; 2) instead of using a uniform transmit power level, we can also allow the active wireless users to adjust their transmit power levels in each iteration, so as to improve the system throughput. The challenges of both of the two enhanced schemes lie at analyzing/characterizing the outage probability and/or the network capacity. New approximation methods may need to be developed, by studying the properties of the point process formed in the data transmission phase. Moreover, to achieve more complete work of addressing the ad hoc network capacity by a SIR-based scheme, it is also interesting to analyze the exact performance error of using a homogeneous PPP to approximate a non-PPP of the same density in the future work.
- *More practical OSA design in the presence of reactive PUs:* By relaxing the assumption that each licensed channel is assigned to one PU, or the assumption

that the SU can only sense one channel at each time, more practical OSA policy can be obtained for the SU. Specifically, by assuming that multiple PUs may access the same licensed channel at the same time, the possible collisions among PUs must be considered. Unaware of the existence of the SU, each PU needs to react to the transmit collisions from other PUs as well as that from the SUs. The OSA policy adopted by the SU affects not only the PUs' access actions on certain channels, but also the PUs' decision on which coordination policy they should adopt to avoid collisions between themselves. In addition, by assuming that each SU is able to sense multiple channels at each time, more system states and actions are involved in the underlying POMDP, which largely complicates the OSA design. Computational complexity reduction is a challenging task for designing an efficient OSA policy. A Multi-armed bandit formulation, as introduced in [143], may help to derive an efficient OSA policy under these more practical assumptions.

- *Capacity analysis of a CR network with reactive PUs:* As have mentioned in Chapter 2, there has been no existing work studying CR network capacity in the presence of reactive PUs, to our best knowledge. Although challenging, such work is important, as it can provide useful guidance to develop effective OSA policies for the SUs in practice. The Matern point processes, as introduced in [75], may help to model and analyze such CR system. Moreover, the approximation method introduced in this thesis or the asymptotic methods introduced in [73] and [74] can be exploited to derive tractable results.

Appendix A

Proof of Proposition 3.3.1

By expressing $\text{SIR}_0^{(0)} = h_{00}d^{-\alpha}/I_0$ and $\text{SIR}_0^{(1)} = h_{00}d^{-\alpha}/I_1$, based on (3.4), (3.6), (3.11) and (3.12), we have

$$\frac{\mathcal{C}^p}{\mathcal{C}^r} = \frac{P(h_{00} \geq \gamma_1 d^\alpha I_0) \times P(h_{00} \geq \beta d^\alpha I_1 | h_{00} \geq \gamma_1 d^\alpha I_0)}{P(h_{00} \geq \beta d^\alpha I_0)}. \quad (\text{A.1})$$

In the following, we compare $\frac{\mathcal{C}^p}{\mathcal{C}^r}$ with 1 by varying $\gamma_1 \in [0, \infty)$. Clearly, when $\gamma_1 = 0$, $\frac{\mathcal{C}^p}{\mathcal{C}^r} = 1$. Next, we consider the case of $\gamma_1 \geq \beta$. Since $I_0 \geq I_1$, if $\gamma_1 \geq \beta$, we obtain $P(h_{00} \geq \beta d^\alpha I_1 | h_{00} \geq \gamma_1 d^\alpha I_0) = 1$. Moreover, for the non-negative and continuous random variables h_{00} and I_0 , it is easy to find that if $\gamma_1 > \beta$, $P(h_{00} \geq \gamma_1 d^\alpha I_0) < P(h_{00} \geq \beta d^\alpha I_0)$, and if $\gamma_1 = \beta$, $P(h_{00} \geq \gamma_1 d^\alpha I_0) = P(h_{00} \geq \beta d^\alpha I_0)$. As a result, from (A.1), if $\gamma_1 > \beta$, $\frac{\mathcal{C}^p}{\mathcal{C}^r} < 1$, and if $\gamma_1 = \beta$, $\frac{\mathcal{C}^p}{\mathcal{C}^r} = 1$. At last, we consider the case of $0 < \gamma_1 < \beta$. In this case, we have $P(h_{00} \geq \gamma_1 d^\alpha I_0 | h_{00} \geq \beta d^\alpha I_0) = 1$, or equivalently,

$$\frac{P(h_{00} \geq \gamma_1 d^\alpha I_0) \times P(h_{00} \geq \beta d^\alpha I_0 | h_{00} \geq \gamma_1 d^\alpha I_0)}{P(h_{00} \geq \beta d^\alpha I_0)} = 1. \quad (\text{A.2})$$

Moreover, since $\gamma_1 \neq 0$ in this case, we have $F_{I_1}(x) > F_{I_0}(x)$, $\forall x > 0$, where $F_{I_0}(\cdot)$ and $F_{I_1}(\cdot)$ denote the cumulative distribution functions (CDFs) of I_0 and I_1 , respectively. It is then easy to verify that $P(h_{00} \geq \beta d^\alpha I_1 | h_{00} \geq \gamma_1 d^\alpha I_0) > P(h_{00} \geq \beta d^\alpha I_0 | h_{00} \geq \gamma_1 d^\alpha I_0)$, for which, by multiplying $\frac{P(h_{00} \geq \gamma_1 d^\alpha I_0)}{P(h_{00} \geq \beta d^\alpha I_0)}$ on both sides and based on (A.2), we

have

$$\frac{P(h_{00} \geq \gamma_1 d^\alpha I_0) \times P(h_{00} \geq \beta d^\alpha I_1 | h_{00} \geq \gamma_1 d^\alpha I_0)}{P(h_{00} \geq \beta d^\alpha I_0)} > 1.$$

That is, $\frac{c^p}{c^r} > 1$. Proposition 3.3.1 thus follows.

Appendix B

Validation on Poisson Approximation

From [14], a simple point process on $\mathbb{R}^2$ is fully characterized by its *void probability* on an arbitrary compact subset K of $\mathbb{R}^2$. Thus, to evaluate the approximation performance of $\hat{\Phi}_1$ for Φ_1 , we compare the void probability of $\hat{\Phi}_1$ and Φ_1 on arbitrary compact subset K . It is known that for the homogeneous PPP $\hat{\Phi}_1$, the void probability in a compact set K is

$$P(\hat{\Phi}_1(K) = 0) = \exp(-\lambda_1 v(K)), \quad (\text{B.1})$$

where $v(K)$ is the area of set K in $\mathbb{R}^2$ [14]. In Fig. B.1, we compare the void probabilities of $\hat{\Phi}_1$ and Φ_1 over compact set K of different sizes. We set $\alpha = 4$, $\beta = 2.5$, $\gamma_1 = 1.5$, and $\lambda_0 = 0.0018$. It is observed that the void probabilities of Φ_1 are always close to that of $\hat{\Phi}_1$; and thus $\hat{\Phi}_1$ is a tight approximation to Φ_1 . We also examine the interference distributions in both Φ_1 and $\hat{\Phi}_1$. As shown in Fig. B.2, where we plot the histogram of the interference received at the typical receiver in Φ_1 and that in $\hat{\Phi}_1$, the approximated interference distribution in $\hat{\Phi}_1$ follows the shape of the actual one in Φ_1 , as expected. Similar performance on the void probabilities and interference distribution can also be obtained by using other parameters. Therefore, the homogeneous PPP $\hat{\Phi}_1$ provides tight approximation to the non-Poisson point process Φ_1 .

Figure B.1: Void probability of Φ_1 and $\hat{\Phi}_1$. Figure B.2: Interference distributions in Φ_1 and $\hat{\Phi}_1$.

Appendix C

Proof of Proposition 3.3.2

Under Assumption 1, we obtain two independent PPPs $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, with $\hat{\Phi}_1 \cup \hat{\Phi}_1^c = \Phi_0$ and $\hat{\Phi}_1 \cap \hat{\Phi}_1^c = \emptyset$. Since from (3.10), it follows that

$$\begin{aligned}\mathcal{P}_0^p &= P(\text{SIR}_0^{(0)} \geq \gamma_1, \text{SIR}_0^{(1)} \geq \beta) \\ &= P\left(\sum_{i \in \Phi_0, i \neq 0} h_{i0} l_{i0} \leq \frac{h_{00}}{\gamma_1 d^\alpha}, \sum_{i \in \Phi_1, i \neq 0} h_{i0} l_{i0} \leq \frac{h_{00}}{\beta d^\alpha}\right),\end{aligned}$$

we have

$$\begin{aligned}\mathcal{P}_0^p &\approx P\left(\left(\sum_{i \in \hat{\Phi}_1, i \neq 0} h_{i0} l_{i0} + \sum_{i \in \Phi, i \neq 0_1^c} h_{i0} l_{i0}\right) \leq \frac{h_{00}}{\gamma_1 d^\alpha}, \sum_{i \in \hat{\Phi}_1} h_{i0} l_{i0} \leq \frac{h_{00}}{\beta d^\alpha}\right) \\ &= P\left(\hat{I}_1 + \hat{I}_1^c \leq \frac{h_{00}}{\gamma_1 d^\alpha}, \hat{I}_1 \leq \frac{h_{00}}{\beta d^\alpha}\right).\end{aligned}$$

Due to the independence of $\hat{\Phi}_1$ and $\hat{\Phi}_1^c$, $\hat{I}_1$ is independent of $\hat{I}_1^c$. Given h_{00} , we have

$$\begin{aligned}&P\left(\hat{I}_1 + \hat{I}_1^c \leq \frac{h_{00}}{\gamma_1 d^\alpha}, \hat{I}_1 \leq \frac{h_{00}}{\beta d^\alpha} \middle| h_{00}\right) \\ &= \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) \int_0^{\frac{h_{00}}{\gamma_1 d^\alpha} - x_1} f_{\hat{I}_1^c}(x_2) dx_2 dx_1.\end{aligned}\tag{C.1}$$

By integrating (C.1) over the (exponential) distribution of h_{00} , we obtain (3.19). Proposition 3.3.2 thus follows.

Appendix D

Proof to Proposition 3.3.3

In this proof, we first derive an upper bound for the right-hand side of (3.19), and then by properly lower-bounding the obtained upper bound, we give a tractable approximate to $\mathcal{P}_0^p$.

First, in (3.19), by increasing the upper limit of $f_{\hat{I}_1^c}(x_2)$, i.e., $\gamma_1 d^\alpha - x_1$, to $\gamma_1 d^\alpha$, the tight approximation of $\mathcal{P}_0^p$ is upper-bounded as

$$\begin{aligned}\mathcal{P}_0^p &\approx \int_0^\infty e^{-h_{00}} \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) \int_0^{\frac{h_{00}}{\gamma_1 d^\alpha} - x_1} f_{\hat{I}_1^c}(x_2) dx_2 dx_1 dh_{00} \\ &< \int_0^\infty e^{-h_{00}} \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) \int_0^{\frac{h_{00}}{\gamma_1 d^\alpha}} f_{\hat{I}_1^c}(x_2) dx_2 dx_1 dh_{00}.\end{aligned}\tag{D.1}$$

Next, denote $Y_1(h_{00}) = \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) dx_1$ and $Y_2(h_{00}) = \int_0^{\frac{h_{00}}{\gamma_1 d^\alpha}} f_{\hat{I}_1^c}(x_2) dx_2$. We can rewrite (D.1) as

$$\int_0^\infty e^{-h_{00}} \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) \int_0^{\frac{h_{00}}{\gamma_1 d^\alpha}} f_{\hat{I}_1^c}(x_2) dx_2 dx_1 dh_{00} = E[Y_1(h_{00})Y_2(h_{00})].\tag{D.2}$$

Note that both $Y_1(h_{00})$ and $Y_2(h_{00})$ are monotonically increasing over h_{00} . Thus, according to the Chebyshev's inequality [137], the right-hand side of (D.2) can be

lower-bounded as

$$E[Y_1(h_{00})Y_2(h_{00})] \geq E[Y_1(h_{00})]E[Y_2(h_{00})]. \quad (\text{D.3})$$

For $E[Y_1(h_{00})]$ in (D.3), by integrating $Y_1(h_{00})$ over the exponential distributed h_{00} , we obtain that

$$\begin{aligned} E[Y_1(h_{00})] &= \int_0^\infty e^{-h_{00}} \int_0^{\frac{h_{00}}{\beta d^\alpha}} f_{\hat{I}_1}(x_1) dx_1 dh_{00} \\ &= P\left(0 \leq \hat{I}_1 \leq \frac{h_{00}}{\beta d^\alpha}\right) \\ &\stackrel{(a)}{=} \exp(-\pi \lambda_1 d^2 \beta^{\frac{2}{\alpha}} \rho), \end{aligned} \quad (\text{D.4})$$

where (a) is obtained based on Proposition 3.2.1, by replacing λ_0 with λ_1 . Similarly, we can obtain that

$$E[Y_2(h_{00})] = \exp(-\pi \lambda_1^c d^2 \gamma_1^{\frac{2}{\alpha}} \rho). \quad (\text{D.5})$$

Finally, by substituting (D.4) and (D.5) into the right-hand side of (D.3) and then adopting the resulting right-hand side of (D.3) to approximate $\mathcal{P}_0^p$, we can obtain a tractable approximate to $\mathcal{P}_0^p$ for the case of $0 < \gamma_1 < \beta$ as in (3.21). Proposition 3.3.3 is thus proved.

Appendix E

Proof to Proposition 3.4.1

From (3.25), we have

$$\begin{aligned} \mathcal{C}_0^{p,M} &= \lambda_0 \times P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M) \\ &\quad \times P(\text{SIR}_0^{(M)} \geq \beta | \text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M). \end{aligned} \quad (\text{E.1})$$

Thus, based on (3.25) and (E.1), we have

$$\begin{aligned} \frac{\mathcal{C}_0^{p,M}}{\mathcal{C}_0^{p,M-1}} &= \frac{P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M)}{P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \beta)} \\ &\quad \times P(\text{SIR}_0^{(M)} \geq \beta | \text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M). \end{aligned} \quad (\text{E.2})$$

Since both proposed schemes adopt the same SIR thresholds γ_k for any $k \in \{1, \dots, M-1\}$, the distributions of these $\text{SIR}_0^{(k)}$'s are the same for both proposed schemes and thus do not affect the ratio of $\frac{\mathcal{C}_0^{p,M}}{\mathcal{C}_0^{p,M-1}}$. Hence, in the following, we focus on the distribution $\text{SIR}_0^{(M)}$ by varying $\gamma_M \in [0, \infty)$, and compare $\frac{\mathcal{C}_0^{p,M}}{\mathcal{C}_0^{p,M-1}}$ with 1. With a proof similar to that of Proposition 3.3.1, it is easy to verify that

- if $0 \leq \gamma_M \leq \gamma_{M-1}$, the distribution of $\text{SIR}_0^{(M)}$ is the same as that of $\text{SIR}_0^{(M-1)}$; and thus we have $\frac{\mathcal{C}_0^{p,M}}{\mathcal{C}_0^{p,M-1}} = 1$;
- if $\gamma_M \geq \beta$, $P(\text{SIR}_0^{(M)} \geq \beta | \text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M) = 1$, and $P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M) = 1$.

$\gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M) \leq P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M)} \geq \beta)$, where “=” holds when $\gamma_M = \beta$. Thus, from (E.2), if $\gamma_M > \beta$, $\frac{\mathcal{C}_0^{p,M}}{\mathcal{C}_0^{p,M-1}} < 1$, and if $\gamma_M = \beta$, $\frac{\mathcal{C}_0^{p,M}}{\mathcal{C}_0^{p,M-1}} = 1$;

- if $\gamma_{M-1} < \gamma_M < \beta$, we have

$$\begin{aligned} \frac{\mathcal{C}_0^{p,M}}{\mathcal{C}_0^{p,M-1}} &> \frac{P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M)}{P(\text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \beta)} \\ &\quad \times P(\text{SIR}_0^{(M-1)} \geq \beta | \text{SIR}_0^{(0)} \geq \gamma_1, \dots, \text{SIR}_0^{(M-1)} \geq \gamma_M) = 1. \end{aligned}$$

Proposition 3.4.1 thus follows.

Appendix F

Proof of Theorem 4.4.1

First, we present the following lemma.

Lemma F.1. $(\epsilon_a^*(t), \delta_a^*(t))$ and $(f_a^*(0, t), f_a^*(1, t))$ given in (4.20) are the optimal solutions to the problem

$$\begin{aligned} \max_{\substack{(\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a(t)), \\ (f_a(0, t), f_a(1, t)) \in [0, 1]^2}} C_1 \times \mu_a(t) + C_2 \times g_a(t) + C_3 \\ \text{s.t.} \quad \mu_a(t) \leq \zeta. \end{aligned}$$

where C_1 , C_2 , and C_3 are constants with $C_1 \geq 0$, $C_2 \geq 0$, $g_a(t)$ is given in (4.4), and $\mu_a(t)$ is given in (4.7).

Proof. In [22], the authors proved that (4.20) is the optimal solution to the problem

$$\begin{aligned} \max_{\substack{(\epsilon_a(t), \delta_a(t)) \in \mathbb{A}_\delta(a(t)), \\ (f_a(0, t), f_a(1, t)) \in [0, 1]^2}} g_a(t) \\ \text{s.t.} \quad \mu_a(t) \leq \zeta. \end{aligned}$$

By applying (4.20), the constraint function $\mu_a(t)$ achieves ζ with equality. Since $C_1 \geq 0$, $C_2 \geq 0$, when the solution is given by (4.20), the objective function is maximized. Lemma F.1 thus follows. $\square$

With Lemma F.1, we now prove the separation principle. By mathematical induction, in the following, we prove that (4.20) is the SU's optimal action decision for (P1) in each slot t , $1 \leq t \leq T$, and with (4.20), the value function $V_t(\mathbf{\Lambda}(t))$ in slot t is of the following form:

$$V_t(\mathbf{\Lambda}(t)) = D_t \times (\lambda_{a0}(t) + \lambda_{a2}(t)) + F_t \times \lambda_{a1}(t) + H_t \times \lambda_{a3}(t) + z_t, \quad 1 \leq t \leq T \quad (\text{F.1})$$

where $D_t \geq 0$, $H_t \geq F_t \geq 0$, and z_t is independent of $\mathbf{\Lambda}(t)$.

Specifically, in the last slot $t = T$, suppose that channel a is selected to sense. From (4.18), we then have $Q_T(\mathbf{\Lambda}(T)|A) = (\lambda_{a1}(T) + \lambda_{a3}(T)) \times g_a(T)$. Since $\lambda_{a1}(T) + \lambda_{a3}(T) \geq 0$, from Lemma F.1, it is easy to verify that (4.20) is the SU's optimal action decision in slot T , such that $Q_T(\mathbf{\Lambda}(T)|A(T))$ is maximized subject to the SCCP constraint. Applying (4.20) to $Q_T(\mathbf{\Lambda}(T)|A)$, we obtain $V_T(\mathbf{\Lambda}(T)) = (\lambda_{a1}(T) + \lambda_{a3}(T)) \times (1 - \epsilon_a^*(T))$, which follows the form given in (F.1).

Now suppose that in slot t , channel a is selected to sense, and (4.20) is the SU's optimal action decision in slot t with value function $V_t(\mathbf{\Lambda}(t))$ given by (F.1). Next, supposing $a(t-1) = n$ in slot $t-1$, given $\mathbf{\Lambda}(t-1)$, we derive the optimal action decision in slot $t-1$ and the value function $V_{t-1}(\mathbf{\Lambda}(t-1))$ in the following two cases:

- *Case 1: $n \neq a$.* According to (4.8) and (4.17) and after some algebra, we obtain $Q_{t-1}(\mathbf{\Lambda}(t-1)|A(t-1)) = (\epsilon_n(t-1)f_n(0, t-1) + (1 - \epsilon_n(t-1))f_n(1, t-1)) \times (\lambda_{n1}(t-1) + \lambda_{n3}(t-1)) + V_t(\mathbf{\Lambda}(t))$. Since from (4.8), $\mathbf{\Lambda}(t)$ is independent of the SU's action $A(t-1)$, $V_t(\mathbf{\Lambda}(t))$ is treated as a constant. Then according to Lemma F.1, (4.20) is the SU's optimal action to maximize $Q_{t-1}(\mathbf{\Lambda}(t-1)|A(t-1))$ subject to the SCCP constraint. Applying (4.20) to $Q_{t-1}(\mathbf{\Lambda}(t-1)|A(t-1))$ yields that $V_{t-1}(\mathbf{\Lambda}(t-1)) = (1 - \epsilon_a^*(t-1)) \times (\lambda_{n1}(t-1) + \lambda_{n3}(t-1)) + V_t(\mathbf{\Lambda}(t))$. Clearly, $V_{t-1}(\mathbf{\Lambda}(t-1))$ follows the form given in (F.1).
- *Case 2: $n = a$.* Similarly to *Case 1*, according to (4.9) and (4.17), we can obtain the expression of $Q_{t-1}(\mathbf{\Lambda}(t-1)|A(t-1))$, in which $V_t(\mathbf{\Lambda}(t))$ is related to the SU's action $A(t-1)$. By adopting the same method as in *Case 1*, it is easy to verify that (4.20) is the SU's optimal action and the resultant

$V_{t-1}(\mathbf{\Lambda}(t-1))$ follows the form given in (F.1), with $D_{t-1} = \zeta[H_t\alpha_1^a - D_t\alpha_1^a - F_t\alpha_0^a + D_t\alpha_0^a] + F_t\alpha_0^a + D_t(1 - \alpha_0^a) \geq 0$, $F_{t-1} = F_t\beta_1^a + D_t - D_t\beta_1^a + 1 - \epsilon_a^*(t-1) \geq 0$, $H_{t-1} = H_t\beta_1^a + D_t - D_t\beta_1^a + 1 - \epsilon_a^*(t-1) \geq 0$ and $z_{t-1} = z_t$. Obviously, $H_{t-1} \geq F_{t-1}$.

Hence, (4.20) is the SU's optimal action in each slot under the SCCP constraint. From (4.20), the optimal sensor operating point and the optimal access probabilities are constant and independent from the channel selected to sense. As a result, we can separately design the optimal spectrum sensing policy as shown in Theorem 4.4.1, without loss of optimality. Theorem 4.4.1 is thus proved.

Appendix G

Proof of Proposition 4.4.1

For the case of $N = 1$ with $n = a$, there is only one PU and SU pair sharing one channel and the SU's policies are reduced to be π_δ and π_c . Given the current belief state $\mathbf{\Lambda}(t)$, we use $G_t(\mathbf{\Lambda}(t)|\pi)$ to denote the PU's throughput on channel a from slot t to slot T under the SU's policy $\pi = \{\pi_\delta, \pi_c\}$. Similar to $Q_t(\mathbf{\Lambda}(t)|A)$ for the SU in (4.17) and (4.18), from (4.22) and (4.23) and with the fact that $\sigma_a(t) = \mu_a(t)$, $t \in [0, 1]$, for $N = 1$, we have

$$G_t(\mathbf{\Lambda}(t)|\pi) = (\lambda_{a0}(t) + \lambda_{a2}(t))(1 - \mu_a(t)) + \sum_{i=0}^3 \sum_{k=0}^1 \lambda_{ai} U_A(k|i) G_{t+1}(\mathbf{\Lambda}(t+1)|\pi),$$

(G.1)

for $1 \leq t \leq T - 1$.

$$G_T(\mathbf{\Lambda}(T)|\pi) = (\lambda_{a0}(T) + \lambda_{a2}(T))(1 - \mu_a(T)),$$

(G.2)

for $t = T$.

It is easy to find that $G_1(\mathbf{\Lambda}(1)|\pi) = E_\pi\{\sum_{t=1}^T R_{P,a}(t)|\mathbf{\Lambda}(1)\}$, which is the PU's throughput on channel a over all T slots. Thus, $R_{P,a}^o = G_1(\mathbf{\Lambda}(1)|\pi)/T$.

From (G.1) and (G.2), to find $G_1(\mathbf{\Lambda}(1)|\pi)$, we need to compute $G_t(\mathbf{\Lambda}(t)|\pi)$ for all $t \in \{1, \dots, T\}$. We consider PU's throughput from slot t to slot T under two cases. One is with the SU's optimal policy for (P1) under the reactive PU model. The other is with the SU's optimal policy in [22], which is under the SCCP constraint and under

the non-reactive PU model. For notational convenience, we denote PU's throughput from slot t to slot T obtained in the former case by G_t , and denote that in the latter case by G'_t . Note that $G'_1/T = \Upsilon_a$. We take G'_1 as a reference and show that $G_1 < G'_1$. Since the proof is similar to that in Appendix A, in the following, we only provide the proof sketch.

Based on mathematical induction and by computing backward in time from (G.1) and (G.2), it is easy to find that in slot t , $\forall t \in \{1, \dots, T\}$ and $T > 1$, G_t is of the following form:

$$G_t = (\lambda_{a0}(t) + \lambda_{a0}(t)) \times q_t + \lambda_{a1}(t) \times w_t + \lambda_{a3}(t) \times m_t, \quad (\text{G.3})$$

where the coefficients q_t , w_t and m_t are time-varying and depend on $(\alpha_0^a, \beta_0^a, \alpha_1^a, \beta_1^a)$. Since the SU's optimal policies under the two cases are the same, given $\mathbf{\Lambda}(1)$, the belief states $\mathbf{\Lambda}(t)$ under two cases in each slot are also the same. Moreover, given $\mathbf{\Lambda}(t)$, G'_t has the similar expression as G_t in slot t , but with different coefficients, which are denoted by q'_t , w'_t , and m'_t . Note that by reducing α_1^a and β_1^a to $\alpha_1^a = \alpha_0^a$ and $\beta_1^a = \beta_0^a$, the reactive PU model is reduced to the non-reactive counterpart. Thus, by doing so, in each slot, the coefficients of G_t , i.e., q_t , w_t , and m_t , are reduced to those of G'_t , i.e., q'_t , w'_t , and m'_t , respectively. Based on mathematical induction, we find that in slot t , $q_t < q'_t$, $w_t \leq w'_t$ and $m_t < m'_t$. Thus, we obtain $G_t < G'_t$, $\forall t \in \{1, \dots, T\}$.

Hence, we have $G_1/T < G'_1/T$, i.e., for (P1) and under the SU's optimal policy, the PU's normalized throughput $R_{P,a}^o < \Upsilon_a$. Proposition 4.4.1 is thus proved.

Appendix H

Proof of Proposition 5.3.1

We first study the PU's throughput under the policy π and construct policy π' based on π . Suppose under the policy π , the PU's throughput from slot $t = 1$ to slot $t = T - 1$ is $E_\pi\{\sum_{t=1}^{T-1} R_{P,a}(t)|\mathbf{\Lambda}(1)\} = \rho_a$, and the PU's throughput in the last slot $t = T$ is u_T . From (4.22), we have

$$(\lambda_{a0}(T) + \lambda_{a2}(T)) \times (1 - \mu_a(T)) = u_T, \quad (\text{H.1})$$

where $\mu_a(T)$ is given in (4.7). Note that under policy π , $E_\pi\{\sum_{t=1}^T R_{P,a}(t)|\mathbf{\Lambda}(1)\} > \Upsilon_a \times T$. Thus, $u_T > \Upsilon_a \times T - \rho_a$.

The policy π' is constructed based on π . We let the decision functions, as described in Section 4.3.2, from slot $t = 1$ to slot $t = T - 1$ of policy π' be the same as those of policy π . We thus have $E_{\pi'}\{\sum_{t=1}^{T-1} R_{P,a}(t)|\mathbf{\Lambda}(1)\} = \rho_a$. Different from policy π , under the policy π' in the last slot $t = T$, we let $E_{\pi'}\{R_{P,a}(T)|\mathbf{\Lambda}(T)\} = \Upsilon_a \times T - \rho_a$ by selecting actions $\delta'_a(T) = 1 - \frac{\Upsilon_a \times T - \rho_a}{\lambda_{a0}(T) + \lambda_{a2}(T)}$, $\epsilon'_a(T)$ be the one on the optimal ROC curve corresponding to $\delta'_a(T)$, $f'_a(0, T) = 0$, and $f'_a(1, T) = 1$. Note that since $u_T > \Upsilon_a \times T - \rho_a$, from (H.1), it is clear that $0 \leq \delta'_a(T) < 1$. That is, we select feasible actions such that $(\lambda_{a0}(T) + \lambda_{a2}(T)) \times (1 - \mu'_a(T)) = \Upsilon_a \times T - \rho_a$.

Next, we show that $E_\pi\{\sum_{t=1}^T R_S(t)|\mathbf{\Lambda}(1)\} < E_{\pi'}\{\sum_{t=1}^T R_S(t)|\mathbf{\Lambda}(1)\}$. Since the decision functions under policies π and π' are the same from slot $t = 1$ to slot $t = T - 1$, the following equation holds for the SU's expected throughput from slot $t = 1$ to slot

$t = T - 1$:

$$E_\pi \left\{ \sum_{t=1}^{T-1} R_S(t) | \mathbf{\Lambda}(1) \right\} = E_{\pi'} \left\{ \sum_{t=1}^{T-1} R_S(t) | \mathbf{\Lambda}(1) \right\}. \quad (\text{H.2})$$

Thus, to compare the SU's expected throughput over T slots under policies π and π' , we only need to compare the SU's expected rewards in the last slot $t = T$. It is easy to find that the belief states $\mathbf{\Lambda}(T)$ in slot T under both policies are the same. In the following, we compute an upper bound on the SU's expected reward in slot $t = T$ under the policy π , which is denoted as $E_\pi^U \{ R_S(T) | \mathbf{\Lambda}(T) \}$ with $E_\pi^U \{ R_S(T) | \mathbf{\Lambda}(T) \} \geq E_\pi \{ R_S(T) | \mathbf{\Lambda}(T) \}$, and show that $E_{\pi'} \{ R_S(T) | \mathbf{\Lambda}(T) \} > E_\pi^U \{ R_S(T) | \mathbf{\Lambda}(T) \}$. Denote the SU's actions in slot $t = T$ that achieve $E_\pi^U \{ R_S(T) | \mathbf{\Lambda}(T) \}$ under the constraint given in (H.1) by $\delta_a^U(T)$, $\epsilon_a^U(T)$, $f_a^U(0, T)$, and $f_a^U(1, T)$. To find these actions, we need to solve an optimization problem, which is to maximize $(\lambda_{a1}(T) + \lambda_{a3}(T)) \times g_a^U(T)$ subject to $\mu_a^U(T) = 1 - \frac{u_T}{\lambda_{a0}(T) + \lambda_{a2}(T)}$. According to Lemma F.1, it is easy to find that $(\delta_a^U(T), \epsilon_a^U(T))$ is on the optimal ROC curve with $\delta_a^U(T) = 1 - \frac{u_T}{\lambda_{a0}(T) + \lambda_{a2}(T)}$, $f_a^U(0, T) = 0$, and $f_a^U(1, T) = 1$. Since $u_T > \Upsilon_a \times T - \rho_a$, thus $\delta_a'(T) > \delta_a^U(T)$. Correspondingly, $\epsilon_a'(T) < \epsilon_a^U(T)$. From (4.4), it is obvious that $g_a'(T) > g_a^U(T)$. Thus, given $\mathbf{\Lambda}(T)$ and from (4.11), we have $E_{\pi'} \{ R_S(T) | \mathbf{\Lambda}(T) \} > E_\pi^U \{ R_S(T) | \mathbf{\Lambda}(T) \} \geq E_\pi \{ R_S(T) | \mathbf{\Lambda}(T) \}$. From (H.2), we have $E_{\pi'} \left\{ \sum_{t=1}^T R_S(t) | \mathbf{\Lambda}(1) \right\} > E_\pi \left\{ \sum_{t=1}^T R_S(t) | \mathbf{\Lambda}(1) \right\}$. Proposition 5.3.1 is thus proved.

Appendix I

Proof of Proposition 5.3.2

Suppose that π_δ^* and π_c^* are the SU's optimal policies for (P2-S-1). Denote $c^*(t)$, $1 \leq t \leq T$, as the resultant PU's throughput in slot t under π_δ^* and π_c^* . That is, given $\Lambda(t)$ in slot t , we have $E_{\pi_\delta^*, \pi_c^*} \{R_{P,a}(t) | \Lambda(t)\} = c^*(t)$ and $\sum_{t=1}^T c^*(t) = \Upsilon_a \times T$. Thus, if we have found $c^*(t)$, $1 \leq t \leq T$, we can adopt the following short-term protection for PU's transmission for (P2-S-1), without loss of optimality:

$$E_{\pi_\delta, \pi_c} \{R_{P,a}(t) | \Lambda(t)\} = c^*(t), \quad \forall t \in \{1, \dots, T\}. \quad (\text{I.1})$$

Then from (4.22) and (I.1), under the availability assumption of $c^*(t)$, (P2-S-1) is equivalent to

$$\begin{aligned} (\text{D-1}) : \max_{\pi_\delta, \pi_c} \quad & E_{\pi_\delta, \pi_c} \left\{ \sum_{t=1}^T R_S(t) | \Lambda(1) \right\} \\ \text{s.t.} \quad & \sigma_a(t) = 1 - \frac{c^*(t)}{P\{I(a, t) = 0\}}, \quad \forall t \in \{1, \dots, T\}. \end{aligned}$$

Since the formulation is similar to (P1) with $N = 1$, from Appendix A, the optimal solutions for (D-1) are

$$\begin{cases} \delta_a^*(t) = 1 - \frac{c^*(t)}{P\{I(a,t)=0\}}, \\ \epsilon_a^*(t) \text{ is on the optimal ROC curve corresponding to } \delta_a^*(t), \\ f_a^*(0, t) = 0, \\ f_a^*(1, t) = 1, \end{cases} \quad (\text{I.2})$$

Since (D-1) is equivalent to (P2-S-1), (I.2) is also the optimal solutions for (P2-S-1). Furthermore, with a proof similar to that in Appendix B, it is easy to show that if $\frac{c^*(t)}{P\{I(a,t)=0\}}$ is a constant over time t , the resultant PU's throughput over T slots will be smaller than $\Upsilon_a \times T$, which is contrary to the fact that $\sum_{t=1}^T c^*(t) = \Upsilon_a \times T$. Thus, $\frac{c^*(t)}{P\{I(a,t)=0\}}$ is not a constant over time t . Hence, the optimal PM decision $\delta_a^*(t)$ is time-varying and needs to be adaptively selected based on $\mathbf{\Lambda}(t)$ over time. Proposition 5.3.2 is thus proved.

Appendix J

Proof of Proposition 5.3.3

Firstly, we introduce some new notations for the POMDP. According to the complexity analysis for (P2-S-2) in Section 5.3.1, given $\mathbf{\Lambda}(1)$ and the SU's POMDP policy π_δ for (P2-S-2), 2^{t-1} possible belief states could occur with non-zero probability in slot t , $\forall t \in \{1, \dots, T\}$. We use $\mathbf{\Lambda}^b(t) = (\lambda_{a0}^b(t), \lambda_{a1}^b(t), \lambda_{a2}^b(t), \lambda_{a3}^b(t))$, $b \in \{1, \dots, 2^{t-1}\}$ to denote these possible belief states in slot t and use $h^b(t)$ to denote the occurrence probability of the belief state $\mathbf{\Lambda}^b(t)$, where $h^b(t)$ is determined by the SU's action decision history and observation history in the previous $t-1$ slots and $\sum_{b=1}^{2^{t-1}} h^b(t) = 1$. We denote the SU's PM decision on channel a for belief state $\mathbf{\Lambda}^b(t)$ as $\delta_a^b(t)$. Next, under the MDP policy $\pi_{\delta, \mathcal{M}}^*$ and the POMDP policy π_δ^* , we give the following lemma, based on which, Proposition 5.3.3 can be proved.

Lemma J.1. *Given $\mathbf{\Lambda}(1) = \mathbf{\Omega}(1)$, the relationship between the POMDP belief states and the MDP state in each slot is*

$$\omega_{aj}(t) = \sum_{b=1}^{2^{t-1}} h^b(t) \times \lambda_{aj}^b(t), \quad \forall j \in \mathbb{C}_S, \quad \forall t \in \{1, \dots, T\}. \quad (\text{J.1})$$

Proof. We use mathematical induction to prove this lemma. Since $\mathbf{\Omega}(1) = \mathbf{\Lambda}(1)$, (J.1) holds when $t = 1$. Suppose (J.1) holds in slot t , $t > 1$, by applying (4.2) to compute $h^b(t+1)$ and applying (4.9) and (5.4) to update the POMDP belief state and the MDP state, respectively, after some algebra, we find that (J.1) still holds in slot $t+1$.

Lemma J.1 is thus proved. $\square$

We now prove Proposition 5.3.3. By computing over all the possible belief states in slot t , the PU's throughput under the POMDP policy $\pi_{\delta, \mathcal{M}}^*$ in slot t is

$$E_{\pi_{\delta, \mathcal{M}}^*} \{R_{P,a}(t) | \mathbf{\Lambda}(1)\} = \sum_{b=1}^{2^{t-1}} h^b(t) \times (\lambda_{a0}^b(t) + \lambda_{a2}^b(t)) \times (1 - \delta_{a, \mathcal{M}}^*(t)). \quad (\text{J.2})$$

Under the MDP policy π_{δ}^* , suppose $\sum_{t=1}^T R_{P,a}^{\mathcal{M}}(t)/T = \Upsilon_a$ is satisfied. Then under the POMDP policy $\pi_{\delta, \mathcal{M}}^*$, from Lemma J.1, we find that (J.2) is equal to (5.5) under $\pi_{\delta, \mathcal{M}}^*$, i.e., $E_{\pi_{\delta}^*} \{R_{P,a}(t) | \mathbf{\Lambda}(1)\} = R_{P,a}^{\mathcal{M}}(t)$ in each slot t . As a result, by summing the PU's throughput over all T slots, we have $E_{\pi_{\delta}^*} \{ \sum_{t=1}^T R_{P,a}(t) | \mathbf{\Lambda}(1) \} = \Upsilon_a$. Proposition 5.3.3 thus follows.

Author's Publications

- Y. L. Che, R. Zhang, and Y. Gong, “Study on spatial capacity in ad-hoc networks with SIR threshold based scheduling,” In review in *IEEE Trans. Wireless Commun.*, revised.
- Y. L. Che, R. Zhang, and Y. Gong, “On design of opportunistic spectrum access in the presence of reactive primary users,” *IEEE Trans. Commun.*, vol. 61, no. 7, pp. 2678-2691, May 2013.
- Y. L. Che, R. Zhang, and Y. Gong, “On spatial capacity in ad-hoc networks with threshold based scheduling,” in *Proc. IEEE Int. Symp. Info. Theory (ISIT)*, Jul. 2013.
- Y. L. Che, R. Zhang, and Y. Gong, “Opportunistic spectrum access for cognitive radio in the presence of reactive primary users,” in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2011.

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