Modelling and analysis of bi-layer ceramic–metal protective structures

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Modelling and Analysis of Bi-Layer Ceramic–Metal Protective Structures

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Abstract

Technical ceramics due to their high specific hardness and compressive strength under pressure loading are attractive materials for armor structures. However, the low toughness of the ceramics is limiting their usage in practical designs. Adding a strong and tough plate at the back of the ceramic plate such as metal or polymer composite laminate, is one of the states of the art armor design that is extensively studied by researchers. Due to the cost of field experiments in defense industry, development of analytical, numerical/empirical models to estimate the ballistic velocity of armor systems by considering geometry of the projectile and target structure is gaining importance. Optimization of the armor design for either weight or thickness is also an important criterion. Furthermore, improving the toughness of the ceramic employing different means like adding metallic phases or pre-stressing them is of considerable interest.

In the present thesis, a semi-analytical model is presented for impact of flat-ended (blunt) hard projectiles against ceramic-metal armor. Comprehensive numerical simulations are performed based on which it is shown that the projectile residual velocity and BLV satisfy the replica scaling laws. An empirical equation for the BLV is proposed whereby the influence of projectile and armor geometrical parameters on the BLV is explored.

Experimental work is performed for validating the proposed semi-analytical model on alumina-aluminum armor system. A numerical model is then developed through matching with experimental results based on which variation of the length of the projectile with time and also with front plate and backing plate thicknesses is found.

A generalized empirical model to estimate the BLV, as a function of geometrical and material parameters for ceramic-metal bi-layer armor systems impacted by a blunt projectile, is proposed. Comparison between results obtained from the empirical model for the BLV of ceramic-metal armor impacted by blunt projectiles, with the available experimental data is carried out.

The proposed empirical BLV equations and the developed numerical model based on experimental results are employed for optimization of ceramic–metal armor systems, under blunt hard projectile impact, for given constraints of total thickness or areal density (mass per unit area) of the armor.

A new analytical model, accounting for erosion and plastic deformation mechanisms, is proposed for the conical projectile impact onto bi-layer ceramic–metal armor system, extended based on Recht’s model for the blunt projectile impact onto a monolithic armor. The residual length of the projectile and the time taken for the erosion and plastic deformation are found.

Finally, the effect of different types of pre-stress conditions such as radial, axial and hydrostatic on the penetration response of confined thick SiC ceramic armor under tungsten long rod impact is explored through AUTODYN[®] based numerical simulations. It is shown that pressure below and near to the impact site of the confined ceramic target at the locations where damage initiates is important in the transition behavior from dwell to penetration for targets of various pre-stress states and of different pre-stress ratios.

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To my Parents,
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without their love and sacrifice
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List of Abbreviations and Symbols

$\beta, \dot{\epsilon}_0, \rho_0, \sigma_{f_{\max}}^*, A, B, C, d_1, d_2, K, K_2, K_3, M, N, T^*$: JH-2 material model constants (see Table 3.2)

$\dot{\epsilon}_0, \rho_0, A', B', C', C_r, d_1', d_2', d_3', d_4', d_5', K, G, n, m, t_0, t_m$: JC material model constants (see Table 3.3)

$\beta, S_1, K, K_2, K_3, P_1, S_2, P_2, C, S_{f_{\max}}, \alpha, \epsilon_{f_{\max}}, P_3, T^*$: JH-1 material model constants (see Table 5.2)

Symbols	Definition
AD	Areal density
A_i	Cross sectional area of the cylindrical part of the projectile
A_e	Enlarged cross-sectional area
a, q	Constants in Lambert–Jonas model
a_1	Diameters of the hole at the rear of the aluminum backing plate
a_2	Diameters of the hole at the rear of the aluminum backing plate
C_{L_1}	Longitudinal wave velocity of the front plate
C_r	Specific heat
D	Damage parameter
D_1	Front plate diameter
D_2	Backing plate diameter
D_p	Projectile diameter
D'_p	Expanded projectile diameter
D_c	Inner diameter of the confinement
D_T	Diameter of the thick ceramic target
δ	Depth of the penetration of a fluid jet into a deformable target
$\Delta\epsilon_p$	Effective plastic strain during a cycle of integration
E_1	Bulk compression deformation energy per unit volume of the ceramic plate
E_2	Bulk stretching deformation energies per unit volume of the metallic plate
E	Young's modulus
E_{kb}	Kinetic energy of the fractured projectile after free collision
E_{kf}	Kinetic energy of the projectile-plugs after free collision
E_{pf}	Projectile fragmentation energy
$(E_{pf})_{bl}$	Projectile fragmentation energy under BLV impact
E_{tr}	Kinetic energy lost due to resistance of the armor plates
$(E_{tr})_{bl}$	Kinetic energy lost due to resistance of the armor plates under BLV impact
E'_{pf}	Fragmentation energy for additional fractured projectile at impact velocities higher than BLV
E_{pk}	Residual kinetic energy of fractured projectile material at the end of the impact process
E_r	Energy for overcoming the perforation resistance under BLV impact
ϵ_1	Compressive failure strain of the ceramic plate
ϵ_2	Tensile failure strain of the metallic plate
ϵ_p	Effective plastic strain
ϵ_p^f	Effective failure strain
$\dot{\epsilon}$	Effective strain rate
$\dot{\epsilon}^*$	Dimensionless strain rate
G	Shear modulus
H	Hardening modulus

<u>Symbols</u>	<u>Definition</u>
HEL	Hugoniot elastic limit
K_1	Bulk modulus of the front plate
K_2	Bulk modulus of the backing plate
K_{eq}	Equivalent bulk modulus
K_{IC}	Fracture toughness
K_t	Bulk modulus of the target material
k	Impedance matching parameter
k_{eq}	Equivalent impedance matching parameter
L_p	Projectile length
L_{pr}	Projectile residual length
L_i^b	Initial length of the blunt projectile
L	Length of the blunt projectile at the end of the erosion
l_b	Length of the elastic remainder of the blunt projectile at the end of the plastic deformation
L_i^c	Initial length of the conical projectile
l_1	Length of the cylindrical part of the conical projectile
l_2	Length of the the conical part of the conical projectile
L_i^c	Length of the conical projectile at the end of the erosion
L'	Length of the conical projectile at the end of the erosion
l'	Remaining length of the conical part of the conical projectile at the end of the erosion
l_c	Length of the elastic remainder of the conical projectile at the end of the plastic deformation
L_c	Inner length of the confinement
L_T	Length of the thick ceramic target
l	Length variable
λ	Mass per unit length of the projectile
M_p	Projectile mass
M_{pr}	Projectile residual mass
M_t	Total mass of the plate plugs
M_s	Sum of the masses of the projectile, ceramic front and metallic backing plate plugs
M_1	Mass of the ceramic front plate plug
M_2	Mass of the metallic backing plate plug
m'_p	Mass of the conical projectile at the end of the erosion
m	Mass variable
ν	Poisson's ratio
P_{HEL}	Pressure at the HEL
P_{max}	Maximum normal load per unit area
R_p	Projectile radius
R	Radius of the cylindrical part of the projectile
R_R	Radial pre-stress ratio
R_a	Axial pre-stress ratio
r''	Radius of the tip of the conical projectile when plastic deformation ends
r'	Radius of the tip of the conical projectile when erosion ends
r	Radial variable

Symbols	Definition
ρ	Material density
ρ_p	Density of the projectile
ρ_1	Density of the front plate
ρ_2	Density of the backing plate
ρ_t	Density of the target plate
ρ_{eq}	Equivalent density
σ	Equivalent stress
σ^*	Normalized equivalent stress
σ_f^*	Normalized fracture equivalent stress
σ_i^*	Normalized Intact equivalent stress
σ_{HEL}	Equivalent stress at the HEL
σ_m	Mean stress
σ_e	Dynamic elastic yield strength of the projectile
σ_2	Ultimate tensile strength (UTS) of the metallic plate
σ_{HEL_1}	Hugoniot elastic limit (HEL) strength of the ceramic plate
t''	Duration of the plastic deformation for conical projectile
t'	Duration of the erosion for conical projectile
t^d	Duration of the plastic deformation for blunt projectile
t^e	Duration of the erosion for blunt projectile
t	Temperature or time variable
T_1	Front plate thickness
T_2	Backing plate thickness
TT	Total thickness
U_p	Plastic wave velocity of the projectile
U_t	Plastic wave velocity in target
V_0	Impact velocity
V_f	Free impact final velocity
V_r	Projectile residual velocity
V_{bl}	Ballistic limit velocity
V_1	Volume of the ceramic front plug
V_2	Volume of the metallic backing plug
v_t, v_{teq}	Projectile–target interface velocity
$v_{r,b}^e$	Relative velocity between velocity of the elastic remainder of the blunt projectile and velocity of the interface during erosion
v_b^e	Velocity of the elastic remainder of the blunt projectile
v_b^e	Velocity of the elastic remainder of the blunt projectile during erosion
$v_{r,b}^d$	Relative velocity between velocity of the elastic remainder of the blunt projectile and velocity of the interface during plastic deformation
v_b^d	Velocity of the elastic remainder of the blunt projectile during plastic deformation
v_{b_0}	Initial velocity of the elastic remainder of the blunt projectile at the beginning of the plastic deformation

<u>Symbols</u>	<u>Definition</u>
$v_{r,c}^e$	Relative velocity between the velocity of the elastic remainder of the conical projectile and velocity of the interface during erosion
v_c^e	Velocity of the elastic remainder of the conical projectile during erosion
$v_{r,c}^d$	Relative velocity between velocity of the elastic remainder of the conical projectile and velocity of the interface during plastic deformation
v_{c_0}	Initial velocity of the elastic remainder of the conical projectile at the beginning of the plastic deformation
Y_p	Projectile static yield limit
Y	Static yield limit
y'	Eroded length of the conical part of the conical projectile
y	Length variable
<u>Abbreviations</u>	
AP	Armor-piercing
ALE	Arbitrary Lagrangian–Eulerian
APDS	Armor-piercing discarding-sabot
BLV	Ballistic limit velocity
DOP	Depth of penetration
EAP	Energy absorbing panel
EOS	Equation of state
ERA	Explosive reactive armor
Exp.	Experiment
FD	Full damage
FE	Field emission
FSI	Fluid structure interaction
GRP	Glass fiber reinforced polymer
HRC	Rockwell hardnesses
ID	Incipient damage
JH	Johnson-Holmquist
JC	Johnson-Cook
KFRP	Kevlar fiber reinforced polymer
KE	Kinetic energy
PVC	Polyvinyl chloride (PVC)
RHA	rolled homogeneous armor
SF	Surface failure
SPH	Smooth particle hydrodynamics
Sim.	Simulation
2D	Two-dimensional
3D	Three-dimensional
UTS	Ultimate tensile strength

Chapter 1 Introduction

1.1 Background and motivation

The quest for protective structures in armor industry, with the advent of everlasting threats endangering life of people, is unending. Great strides in modern armor technology demand for newer materials and also improved traditional materials. Armor as a cover, with the intention of dissipating impact/shock or blast energy against the kinetic energy of projectiles or impulse of blast energy has been utilized since the time of invention of sophisticated threats. Different kinds of classic Energy Absorbing Panels (EAPs) either monolithic panels or multi-layer (with or without functionally graded) materials or composites have been widely used in body armor (personnel armor), light armor (vehicular and aircraft armor) and heavy armor (tank armor). Figure 1.1 shows the evolution status of armors with different areal densities (mass per unit area) for protection against 0.5 caliber armor piercing (AP) projectile.

Figure 1.1 Evolution status of armor for protection against 0.50 caliber AP projectile [1]

Figure 1.2 shows the developments in armor systems and their performances over the last century. The utilization of ceramic materials as armor after 1960 opened a new subject of interest for research to explore.

Figure 1.2 Timeline of the evolution of the armor systems [2]

Bi-layer armor systems, comprising a hard ceramic front face and an energy absorbing metal or composite backing layer, result in a lighter design compared to monolithic metallic (like rolled homogeneous armor (RHA) steel) armor providing the same ballistic protection level, against armor-piercing (AP) projectiles, as shown in Figure 1.1 and increase the armor performance, as shown in Figure 1.2.

In general, factors that directly influence the ballistic limits of armor systems are areal density and thickness, geometry and velocity of projectile, angle of attack and armor material configuration. In addition, armor elastic modulus, strength and toughness are important properties that affect the projectile defeat mechanisms and influence the impact energy absorption of armor. In order to shield any material system from a high velocity projectile, kinetic energy and momentum of that projectile must be dealt with using some combination of three mechanisms: 1) absorption of the energy as heat and deformation in the target material, 2) rebound of the projectile and 3) gross deformation of the projectile. The last mechanism is the most efficient way for armor to defeat projectiles because most of the kinetic energy is absorbed in the destruction of the projectile itself and, with little rebound of the projectile, momentum transfer to the armor is minimized.

Ceramics possess two very important qualities that make them ideal candidates for armor materials: high hardness and low density [3-7], the former helps to have an

exceptional strength and the latter is useful for weight reduction which is a key point in armor design. However, ceramics, if used as monolithic layers against projectile impact, easily fracture due to their lack of toughness. Therefore, ductile metallic or composite layers have been used as the backing layer of the ceramic. In this way, ceramic layer function is to blunt and decelerate the projectile and the backing layer keeps the fractured ceramic in its place and absorbs the remnant energy of the projectile.

Residual velocity of the projectile and ballistic limit velocity (BLV) of the armor are two common measures of armor performance. Understanding the performance of the armor under impact is feasible by utilizing either analytical models or empirical models. Numerical simulation can conveniently provide a large amount of data to support empirical models and avoid the expensive experimental trials. Analytical models relating projectile residual velocity, impact velocity, and armor BLV have been extensively developed especially for monolithic metallic armor. However, similar empirical or analytical models for BLV of bi-layer armor systems comprising ceramics as front layer with metallic or composite backing layers are scarce. Numerical simulation techniques alternatively can, for development of empirical equations, be utilized for analyzing the effects of projectile and armor geometries on the BLV. Analytical or empirical equations for BLV, inherently considering armor and projectile geometries which can be also deployed for armor optimization purposes, are necessary. Besides, in the literature, analytical models on the residual length of the projectile mostly consider blunt projectiles without differentiating the effect of different nose geometry in interaction with the armor target. Analytical models specifically considering the nose geometry of the projectile and its deformation mechanisms would be beneficial in predicting the projectile behavior in interaction with the armor.

It is believed that the strength and ductility of a ceramic material may substantially depend on pressure, applied on the ceramic target, which is determined from the surface load, confinement and target geometry [8]. The effect of introducing ductility to ceramic armor has been of interest for decades [9] either by adding metal contents to ceramic leading to a class of materials, namely “cermets” or making graded armor like ceramic-metal armor, as explained above. Pre-stressing is another way of adding ductility to ceramic armor [10]. In the literature, there are not many studies on the modelling of pre-stressed ceramic armor while considering various pre-stress types and finding their

effect on the ballistic behavior of the confined thick ceramic armor. In view of the limited studies on the modelling of the pre-stressed ceramic armor, it would be significant to conduct studies which can be beneficial in understanding the key parameters for improving ballistic behavior of the confined thick ceramic armor under impact of long rod heavy metal projectiles and also can be a promising cost effective design tool for finding influences of material and geometrical parameters of the confined ceramic armor on its ballistic behavior.

1.2 Objectives

Analytical, numerical and experimental methods are utilized in the current research effort to improve the current state of the art for bi-layer ceramic–metal armor systems. Predominant research work is on the blunt projectile as a bench mark. The objectives of the present work are as follows:

- To develop a semi-analytical model relating projectile residual velocity, impact velocity and armor ballistic limit velocity (BLV) for ceramic–metal armor under impact of blunt hard projectile and experimentally validate the model against instrumented laboratory gas gun firing tests on alumina–aluminum armor system.
- To verify replica scaling laws on BLV of bi-layer armor and obtain an empirical BLV equation for specific armor system.
- To propose a generalized BLV equation supported by numerical simulations, for bi-layer ceramic–metal under plugging failure.
- To optimize the bi-layer armor for a given total thickness or areal density (weight) using the developed numerical and empirical BLV model.
- To develop analytical models for plastic deformation and erosion of blunt and conical projectiles impacting bi-layer ceramic–metal armor systems and compare the results of the proposed model with the literature experimental measurements.
- To investigate the influence of different pre-stress types on the ballistic performance of confined thick ceramic armor on BLV and also to explore interface dwell and defeat phenomenon.

1.3 Scope of the work

Pertinent review of the literature on the available models for thin and thick ceramic–metal armor systems revealed that the current state of the knowledge can be improved.

A semi-analytical model, concerning blunt projectile perforating bi-layer ceramic–metal armor at nominal impact velocities is developed considering two stages, namely (a) inelastic impact of the projectile into plate-plugs as if the plugs are entirely free from the plates and (b) energy transfer during perforation of the projectile–plugs combination into the armor.

A series of tests are performed for validating the proposed semi-analytical model. Hardened steel 4340 projectiles are used in the tests. Alumina 95% ceramic tiles are bonded, using Hysol EA 9309.3NA adhesive, to aluminum alloy 2024-T3 backing plates. A two-stage gas-gun is used for firings. A flash X-ray system is used for taking images of the impact event. The experiments are simulated. Simulation and experimental results are then used to validate the semi-analytical model results.

A generalized empirical BLV model is proposed considering momentum and energy balance during impact process for bi-layer ceramic–metal armor systems impacted by a flat-ended (blunt) tungsten projectile. The model is supported by the numerical simulations. Three ceramic materials as front layer and three materials as backing layer are considered in the numerical study. Impact simulations are performed for different combinations of armor geometries and materials systems with different projectile lengths. Numerical constants in the empirical model are obtained using least square fitting to the BLV data obtained from simulations.

An available model in literature, namely Recht’s model [91], for blunt projectile impact onto deformable monolithic target is extended for conical projectile, accounting for projectile erosion and plastic deformation mechanisms. The final length of the projectile and the time taken for the erosion and plastic deformation, are found.

Finally, effect of pre-stress on the dwell and interface defeat phenomenon were studied numerically modelling different pre-stress types with different intensities.

1.4 Thesis layout

Literature review on the current state of the topics on thin and thick ceramic armor is presented in Chapter 2. In Chapter 3, a semi-analytical model relating projectile residual velocity, impact velocity and armor ballistic limit velocity (BLV) and also an empirical model considering the influences of projectile and armor geometrical parameters on the BLV is presented for impact of blunt hard projectile against ceramic–metal armor based on which optimization of the armor system is performed. In Chapter 4, experimental work performed for validating the proposed semi-analytical model in Chapter 3 is presented together with a numerical model developed through matching with experimental data, based on which optimization of the armor system is carried out. In Chapter 5, a generalized empirical model to estimate the BLV, as a function of geometrical and material parameters for ceramic–metal bi-layer armor systems impacted by a blunt tungsten projectile is proposed using numerical simulations and the proposed empirical model is employed for optimization of alumina–aluminum armor system, as an example of ceramic–metal armor. In Chapter 6, an analytical model is developed for conical projectile impact onto bi-layer ceramic–metal armor based on an available analytical model available in literature for blunt projectile impact onto monolithic metallic armor. Finally, Chapter 7 deals with development of a reliable numerical model based on which relevant results on the topic of pre-stressing of confined thick ceramic armor are obtained.

Chapter 2 Literature Review

In this chapter, initially classification of armor problems based on target type, impact velocity and projectile type is presented. Status of the literature on analytical, empirical and numerical models for ballistic limit velocity (BLV) of (thin) armor systems is then reviewed; this part is the literature background for the work which will be presented in Chapters 3, 4 and 5. The overview of the models considering the deformation modes of blunt and conical projectiles onto ceramic–metal armor systems is then presented; this part is the background for the work which will be presented in Chapter 6. Finally, status of the literature on confined pre-stressed thick ceramic armor is reviewed; this part is the background for the work which will be presented in Chapter 7.

2.1 Classification of armor problems

In this section, the armor problems are divided to sub-categories considering: target type, impact velocity and projectile type, based on the classifications proposed by different authors.

2.1.1 Target type

Geometry (and specially thickness), material composition, penetrability are some of the parameters based on which the armor targets are classified.

Classification of the armor undergoing impact event based on their thickness is as follows [11]:

- 1) Thin, through which stress and deformation gradients do not exist.
- 2) Intermediate, for which the influence of rear surface on the deformation of the armor during whole (or almost whole) of the projectile motion, is considerable.
- 3) Thick, for which the influence of the distal boundary on the penetration process of the projectile is substantial only after considerable travel of the projectile into the target.
- 4) Semi-infinite, for which there is not any influence of the distal boundary on the penetration process.

Bodner [12] also, based on the thickness and considering the number of deformation modes of the armor target, classifies the armor as:

- 1) Thin, through which stress and deformation gradients do not exist and exhibiting only one deformation mode.
- 2) Moderately thick with the thickness approximately equal to diameter of the projectile and exhibiting multiple modes of failure.

Backman and Goldsmith [11] classified the armor target based on its material composition into (a) the man-made, like concrete, metals, composites, etc., and (b) naturally occurring, like soil, rock, etc. Penetrability is a measure of relative depth of penetration (DOP) in various armor target materials impacted by projectile of constant velocity and fixed geometrical parameters. Penetrability is related to the resistance (strength) of the different materials: low resistance materials like soils, moderately resistant materials like concrete, etc. As for the modelling of the armor behavior, it is assumed [11] that the response of the structure is localized, if the area of the impact is limited within multiple diameters of the projectile. For localized response, the effect of boundary conditions far from the impact area is usually considered negligible.

2.1.2 Projectile type

Zukas [13] gives a comprehensive characterization of the projectile, as listed in Table 2.1, considering basic shape, nose configuration, density, trajectory, impact condition, final condition of the shape and location. Mass, total length, nose shape (various degrees of “sharp” and “blunt”) and length, diameter and density are the parameters considered to determine the “ballistic limit” of a projectile. Backman and Goldsmith [11] defined the ballistic limit which is quoted here: Ballistic Limit is the average of two striking velocities, one of which is the highest velocity giving a partial penetration and the other of which is the lowest velocity giving a complete penetration. The three measures used the most in rating the resistance of armor are: (1) the army, (2) the protection, and (3) the navy ballistic limits. In these three measures, the criterion employed to define a perforation is different, as illustrated in Figure 2.1.

Table 2.1 Projectile characteristics [13]

<u>Geometry</u>		
<u>Basic shape:</u>	Solid rod Sphere Hollow shell Irregular Solid	<u>Nose Configuration:</u> Cone Ogive Hemisphere Right circular Cylinder
<u>Material</u>		
<u>Density:</u>	<u>Light-weight</u> Wood, plastics, ceramics, aluminum <u>Intermediate</u> Steel, copper <u>Heavy</u> Lead, tungsten	
<u>Flight characteristics</u>		
<u>Trajectory:</u>	Straight (stable) Curved (stable) Tumbling (unstable)	<u>Impact condition:</u> Normal Oblique
<u>Final condition</u>		
<u>Shape:</u>	Undeformed Plastically deformed Fractured Shattered	<u>Location:</u> Rebound Partial penetration Perforation

The three mentioned criteria are usually used in experiments to find the BLV. Backman and Goldsmith [11] emphasize that in the past “the army” or “the navy” criterion were used in performing experiments while the new experiments mostly emphasize on “the protection” criterion.

Figure 2.1 Definitions of perforation and partial penetration for defining the ballistic limit for (a) Army Ballistic Limit, (b) Protection Ballistic Limit and (c) Navy Ballistic Limit [11]

There are two effective parameters, namely obliquity and orientation, in determining the amount of energy absorbed by an armor target. Obliquity is the angle between the

velocity vector and the normal one into the target and orientation is the angle between the axis of symmetry of the projectile and velocity vector [11].

2.1.3 Impact velocity

The rate of the loading applied to a structure, specifies the type of analysis based on which the impact problem should be treated. For very low rate of loading, the response of the structure can be considered static and independent of time whereas in higher rates of loadings the response of the structure should be considered dynamic (strain rate dependent). Figure 2.2 [14] shows schematic diagram of strain rate regimes and the techniques used for obtaining them.

Figure 2.2 Schematic diagram of strain rate regimes (in reciprocal seconds) and the corresponding techniques for obtaining them [14]

In impact problems, one of the ways to classify the rate of loading applied on the armor target is considering the impact velocity of the projectile. Backman and Goldsmith [8] proposed different ranges for impact velocity as: (a) sub-ordnance range of 25-100 m/s, (b) ordnance range of 500-1300 m/s (conventional firearms perform in this velocity range), (c) ultra-ordnance range of 1300 to 3000 m/s, and (d) hypervelocity range of above 3000 m/s. The classification can be also done considering the projectile and target properties: (a) a range in which both the projectile and the target deforms only in their elastic regime, (b) a range in which plastic deformation occurs in the projectile and the target, (c) the dominant response is the one due to propagation of elastic, plastic and finally hydrodynamic stress waves beyond which shock wave response will occur, (d)

highest velocity range, so called hypervelocity range in which comminution and material phase changes occur.

2.2 Ceramic armor systems

A number of criteria must be considered when selecting materials for use in an armor system for protection against ballistic threats. These include the characteristics of the specific threats to be defeated, the allowable volume and weight parameters of the system, and the system cost [15].

Ceramic materials have played an important part in ballistic protection. Utilization of ceramics in armor application began in the 1960s in US Army for bullet-proof vests and seat-armor in helicopters [16, 17]. As effective protection and low weight are two main criteria in ballistic armor materials, high hardness and low density make ceramics ideal candidates for this technology. Ceramics, such as aluminum oxide (Al_2O_3 , also known as alumina), boron carbide (B_4C), titanium diboride (TiB_2) and silicon carbide (SiC) and its variants like silicon carbon boride (SiC-B) and silicon carbon nitride (SiC-N), are low-density high-hardness materials. Another attractive property of ceramics is that they possess high compressive strength which is suitable for ballistic applications [18]. In fact, ceramics meet the first four requisites of materials involved in armor design which are: low density to reduce the total weight of the armor; high bulk and shear moduli to avoid large deformations; high yield strength to preserve the armor resistance to failure; and high dynamic tensile strength to avoid material rupture when tensile waves appear; however as they are brittle (and weak in tension) with low fracture toughness [19], they extensively fragment due to the tensile waves generated by the compressive waves reflected from the free surfaces. Consequently, mixed armors made of ceramic plates and a more ductile backing plate (like metal or composite laminated plate) seem to form a very efficient shield against combining the high resistance of ceramic with the lightweight and ductility of the backing plate.

The function of the ceramic plate is to increase the ballistic efficiency of the armor by breaking the incoming projectile and decelerating it, but the ceramic material needs a backing plate to confine the ceramic fragments and to absorb the kinetic energy of the projectile and the ceramic rubble during target penetrations. The two materials work together, the backing plate confines the ceramic plate in the impact direction once it is

broken and fragmented, which increases the efficiency of the ballistic performance of the mixed armor system.

2.2.1 Failure mechanism of ceramic armor systems

The failure process of a ceramic armor system depends on its geometry, confinement, interfacial conditions and material properties [20]. A ceramic armor system may consist of either a thin ceramic plate supported by a backing ductile layer or a thick ceramic layer confined by a metallic ring or jacket. Chen et al. [21] described the failure mechanisms of these two configurations which are summarized below.

2.2.1.1 Failure of thin ceramic armor

Thin ceramic armor systems usually are used for defeating small caliber threats and machine guns in body armors, aircraft and light vehicle protection systems. A thin ceramic armor system comprises a front ceramic plate bonded to a ductile metal or composite fabric-based backing plate by a thin adhesive layer.

When a projectile impacts the ceramic front plate, compressive stress waves are produced in both ceramic plate and projectile. The compressive wave propagates across the thickness of the ceramic plate. The stress waves are partially reflected at the ceramic front-backing plate interface and partially transmitted into the backing plate, due to the mechanical impedance mismatch of the ceramic plate and the backing plate. As the mechanical impedance of the ceramic is typically higher than that of the backing plate, the reflected stress waves propagate in the tensile mode. Ceramic, being weak in tension, will fail due to the propagation of tensile stress waves exceeding the ultimate tensile strength of the ceramic. Therefore, the formation and propagation of the physical damage within the ceramic target are thereby controlled by the magnitude and direction of propagation of the tensile stress waves. While radial cracks are moving from the impact point toward the rear side of the ceramic plate, reflected tensile stress waves ascend toward the front surface of the ceramic plate which results in development of the cracked zones and its extension to the impact surface, as shown in Figure 2.3, in an upward tapering shape [20].

Figure 2.3 Damage zones in the ceramic tile, backing plate and a projectile (bullet) [20]

Projectile starts to penetrate the ceramic plate after full propagation of the conical damage zone and extension to the ceramic frontal surface and the projectile's head [22, 23]. Reijer [23] assumed that the fracture cone is fully formed at time, $t = 6T_1/C_{L_1}$ where T_1 is the thickness of the ceramic plate and C_{L_1} is the longitudinal wave velocity of the ceramic. In a typical ceramic plate, fracture conoid is formed within a few microseconds [24]. The time duration pertinent to this non-penetration period is termed *dwelt*. If the dwell period is prolonged, in turn, the time required for upward propagation of the damage zone to the free surface will be protracted and thereby limits the ability of the projectile to penetrate the ceramic target. The non-penetration phenomenon with a complete erosion of the projectile is called the *interface defeat* [25]. Complete failure (or complete perforation) of the ceramic tile occurs once the conical damage zone (or shatter cones) fully opens up onto the impact surface and the damaged area becomes comparable to the projectile cross-sectional area [20]. The backing plate also experiences compressive stress distributed over an expanded area its contact surface through the conical damaged zone of the ceramic front plate. If the impact velocity or the penetrator length is large enough to overcome the interface defeat, then a complete penetration will take place and the backing plate will also be subjected to deformation or failure.

2.2.1.2 Failure of thick ceramic armor

Confined thick ceramic armor is commonly used to defeat long rod heavy metal such as tungsten projectiles in heavy vehicle armors. For a heavy metal projectile long enough (i.e. with a high aspect ratio) impacting a thick armor, it takes relatively longer time for

the projectile to pass its kinetic energy to the impact area. In order to defeat the projectile under this impact condition, a thicker ceramic plate is needed as the projectile would continue to penetrate the target even though the ogive or some extent of the front end of the projectile may have eroded or fractured during the dwell. A thick ceramic, however, will continue to be shattered as a result of continuous penetration by the long projectile exerting compressive stress exceeding the compressive strength of the ceramic and also shear stress [26, 27]. The high compressive and shear stresses convert the ceramic material into a highly cracked interlocked state, known as *comminuted* state or *Mescall zone* [18, 28]. As the penetration into the ceramic continues, either the long rod projectile is fully eroded or the ceramic is perforated.

A common technique to contain the ceramic debris from scattering is to confine the comminuted ceramic in a metallic casing or cover. The advantage of this configuration is the extended time to total failure and the enhanced capacity of the ceramic target to erode the penetrator for a longer period of time, thereby diffusing its energy [20]. A confined fragmented zone can acquire sufficient kinetic energy from a long projectile that will erode the penetrator by a process of frictional shear flow if the frictional dissipation is greater than the wear resistance of the projectile material [29].

The details of effect of confinement on the ballistic resistance of different ceramics can be found in the works of Orphal et al. [5], Anderson Jr and Royal-Timmons [30], Anderson Jr et al. [31], Andersson et al. [32], Chen and Ravichandran [33, 34].

2.3 Constitutive and fracture models for metals and ceramics

In this section, the Johnson–Cook (JC) model for metals and Johnson–Holmquist (JH) for ceramics which are mostly used in numerical simulations in the current work, are described.

2.3.1 Johnson–Cook (JC) model

JC constitutive model

Johnson and Cook [35] developed a phenomenological model for metals which is the most commonly used in ballistic penetration studies. They subjected the material to a variety of tests like the Hopkinson bar test, quasi-static tension and torsion tests and derived an expression for the flow stress from the experimental data.

The JC constitutive model is relatively easy to calibrate since it allows isolation of the various effects. The flow stress is expressed, as an explicit function of strain hardening, strain rate hardening and thermal softening, as follows [35]:

$$\sigma = \left[A' + B' \varepsilon_p^n \right] \left[1 + C' \ln \dot{\varepsilon}^* \right] \left[1 - (T^*)^m \right] \quad (2.1)$$

where A' is the initial yield stress, B' is the strain hardening coefficient, ε_p is the effective plastic strain, n is the strain hardening exponent, $\dot{\varepsilon}^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}$ is the normalized strain rate in which $\dot{\varepsilon}$ is the effective plastic strain rate and $\dot{\varepsilon}_0$ is the reference strain rate, C' is the strain rate coefficient, m is the temperature softening exponent, $T^* = \frac{t - t_0}{t_m - t_0}$ is the normalized temperature in which t , t_0 and t_m are temperature, room temperature and melting temperature, respectively. The first term represents the classical power law hardening behavior through the effective plastic strain, the second term accounts for the strain rate dependence of flow stress and the third term accounts for decrease in flow stress due to thermal softening.

The JC constitutive model was used for computational modelling of ballistic impact on aluminum plates giving reliable results [36-38]. Hayhurst et al. [39] and Yaziv et al. [40] used this model to describe steel and tungsten behavior. Richards et al. [41] and Livingstone et al. [42] applied this model for the gilding metal jacket of the projectile and Preece et al. [43] used this model for the steel armor.

JC fracture model

The JC fracture model is defined as [44]:

$$D = \frac{\varepsilon_p}{\varepsilon_p^f} \quad (2.2)$$

where D is the damage parameter and ε_p^f is the effective failure strain (strain to fracture) given by [44]:

$$\varepsilon_p^f = \left[d'_1 + d'_2 \exp(d'_3 \sigma^*) \right] \left[1 + d'_4 \ln \dot{\varepsilon}^* \right] \left[1 + d'_5 T^* \right] \quad (2.3)$$

where $d'_1, d'_2, d'_3, d'_4, d'_5$ are damage constants, $\sigma^* = \frac{\sigma_m}{\sigma}$ is the pressure-stress ratio in which σ_m is the hydrostatic tension (average of the normal stresses), σ is the flow stress (equivalent stress) and $\dot{\epsilon}^*$ and T^* are same as what described in Eq. (2.1). The strain to fracture decreases as the hydrostatic tension, σ_m increases. It should be noted that for high values of hydrostatic tension ($\sigma^* > 1.5$), a different relationship is used [44].

2.3.2 Johnson–Holmquist (JH) model

One of the most widely used models for study of ceramics under ballistic impact is the Johnson-Holmquist (JH) model. Johnson and Holmquist developed three ceramic model [45, 46], namely JH-1, JH-2 and JHB models. In AUTODYN[®] [47], JH-1 and JH-2 models have been implemented. In LS-DYNA [48] also both these models are available to model brittle material behavior and generate agreeable results which therefore can provide good insight in the ceramic material response. JHB model which is an improved version of JH-1 model has not yet been implemented in any of these codes. JH constitutive models consist of: (a) strength model, (b) damage model and (c) pressure model.

Figure 2.4 (a) and (b) show the JH-1 and JH-2 strength models, respectively and Figure 2.4 (c) and (d) show JH-2 damage and pressure models, respectively. JH-2 material strength and damage models are smooth analytical functions of pressure, whereas JH-1 strength and damage models are piecewise functions. In JH models, the failure affects the strength of the intact material differently. JH-1 and JHB models move from the intact strength curve to the failed strength curve instantaneously when $D = 1$, whereas JH-2 model allows for the gradual softening of the material as the damage progresses from 0 to 1 [49]. The JH models have been used to compute plate impact simulations and long rod penetrations on silicon carbide and aluminum nitride [49-51].

In order to get familiar with JH models, the JH-2 model, as an example, is briefly described in the next section.

Figure 2.4 (a) JH-1 strength model [50], (b) JH-2 strength model [49], (c) JH-2 damage model [49], (d) JH-2 pressure model [49]

2.3.2.1 JH-2 model

Strength model

Strength model, as shown in Figure 2.4 (b), relates the equivalent stress of the intact, partially or fully damaged (fractured) material to the pressure. The strength (von Mises equivalent stress) is dependent on the pressure, P , the dimensionless equivalent (total) strain rate, $\dot{\epsilon}^* = \dot{\epsilon} / \dot{\epsilon}_0$, and the damage, D . For undamaged material, $D = 0$; for partially damaged material, $0 < D < 1$; and for fully damaged (failed) material, $D = 1$.

The normalized equivalent stress of the ceramic [49] is given by:

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*) \quad (2.4)$$

where σ_i^* is the normalized intact equivalent stress, σ_f^* is the normalized fracture stress, and D is the damage parameter.

Normalized equivalent stresses (σ^* , σ_i^* and σ_f^*) are in general form of:

$$\sigma^* = \frac{\sigma}{\sigma_{HEL}} \quad (2.5)$$

where σ is the equivalent stress and σ_{HEL} is the equivalent stress at the Hugoniot elastic limit (HEL).

The equivalent stress is given by:

$$\sigma = \frac{1}{2} \sqrt{\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \right]} \quad (2.6)$$

where σ_x , σ_y , and σ_z are the three normal stresses, and τ_{xy} , τ_{xz} , and τ_{yz} are the three shear stresses. The normalized intact strength is given by:

$$\sigma_i^* = A(P^* + T^*)^N (1 + C \ln(\dot{\epsilon}^*)) \quad (2.7)$$

and the normalized fracture strength is given by:

$$\sigma_f^* = B(P^*)^M (1 + C \ln(\dot{\epsilon}^*)) \quad (2.8)$$

The normalized fracture strength can be limited by $\sigma_f^* \leq \sigma_{f_{\max}}^*$. The material constants

are A , B , C , M , N , T and $\sigma_{f_{\max}}^*$. The normalized pressure is $P^* = \frac{P}{P_{HEL}}$ in which

P is the actual pressure and P_{HEL} is the pressure at the HEL. The normalized maximum

tensile hydrostatic pressure is $T^* = \frac{T}{P_{HEL}}$ in which T is the maximum tensile hydrostatic

pressure the material can withstand. The dimensionless strain rate is $\dot{\epsilon}^* = \frac{\dot{\epsilon}}{\dot{\epsilon}_0}$ in which

$\dot{\epsilon}^*$ is the effective strain rate and $\dot{\epsilon}_0 = 1 \text{ s}^{-1}$ is the reference strain rate.

The effective strain rate is expressed as:

$$\dot{\varepsilon} = \frac{\sqrt{2}}{3} \sqrt{\left[\left(\dot{\varepsilon}_x - \dot{\varepsilon}_y \right)^2 + \left(\dot{\varepsilon}_x - \dot{\varepsilon}_z \right)^2 + \left(\dot{\varepsilon}_y - \dot{\varepsilon}_z \right)^2 + \frac{3}{2} \left(\dot{\gamma}_{xy}^2 + \dot{\gamma}_{xz}^2 + \dot{\gamma}_{yz}^2 \right) \right]} \quad (2.9)$$

where $\dot{\varepsilon}_x$, $\dot{\varepsilon}_y$ and $\dot{\varepsilon}_z$ are the three normal strain rates, and $\dot{\gamma}_{xy}$, $\dot{\gamma}_{xz}$ and $\dot{\gamma}_{yz}$ are the three shear stresses.

Damage model

Damage model, as shown in Figure 2.4 (c), renders the transition of the material from an intact state to a fracture state and defines the level of fracture.

The damage is expressed as [52]:

$$D = \sum \frac{\Delta \varepsilon_p}{\varepsilon_p^f} \quad (2.10)$$

where $\Delta \varepsilon_p$ is the effective plastic strain during a cycle of integration and $\varepsilon_p^f = f(P)$ is the plastic strain to fracture under a constant pressure, P given by:

$$\varepsilon_p^f = d_1 \left(P^* + T^* \right)^{d_2} \quad (2.11)$$

where d_1 and d_2 are constants and P^* and T^* are as defined in the previous section.

The material cannot undergo any plastic strain at $P^* = -T^*$.

Pressure model

Pressure model, as shown in Figure 2.4 (d), is a pressure-volume relationship that can include bulking (increase in pressure and/or volume).

The hydrostatic pressure, before fracture begins ($D=0$), is given by [52]:

$$P = K\mu + K_2\mu^2 + K_3\mu^3 \quad (2.12)$$

where K , K_2 , and K_3 are constants and $\mu = \frac{\rho}{\rho_0} - 1$ for current density, ρ and initial density, ρ_0 . For tensile pressure ($\mu < 0$), above equation is replaced by $P = K\mu$. After damage begins to accumulate ($D > 0$), bulking can occur. The effect of bulking is to

increase the pressure and/or to increase the volumetric strain. The bulking effect is included by adding an incremental pressure ΔP to pressure equation

$$P = K\mu + K_2\mu^2 + K_3\mu^3 + \Delta P \quad (2.13)$$

where pressure increment is determined from energy considerations.

2.3.2.2 Application of the JH models for ceramics

To date, different models have been suggested in the literature for constitutive behavior modelling of different ceramics. Table 2.2 summarizes the models used for material modelling of several ceramics.

Table 2.2 constitutive models used for some ceramics

Ceramic	Model	year
Alumina (Al_2O_3)	JH-2	1995 [31]
Silical float glass	JH-2	1995 [53]
Boron carbide (B_4C)	JH-2	1999 [52]
Boron carbide (B_4C)	JHB	2006 [54]
Boron carbide (B_4C)	JHB	2008 [55]
Aluminum nitride (AlN)	JH-2	2001 [49]
Aluminum nitride (AlN)	JHB	2003 [45]
Silicon carbide (SiC-B)	JH-1	2002 [50]
Silicon carbide (SiC-B)	JHB	2005 [51]

2.4 Computational modelling using hydrocodes

A computer program that is capable of computing strains, stresses, velocities and propagation of shock waves as a function of time and position is known as a hydrocode. In a hydrocode simulation, the response of a continuous media subjected to dynamic loading is governed by the conservation of mass, momentum and energy, and also the equation of state (EOS) and constitutive relation of the media. The EOS takes into account the effects of compressibility of the continuous media and is a function of internal energy and density, whereas the constitutive relation represents the relation between the evolutions of stress with strain.

In this work the hydrocode simulations on the impact of bi-layer ceramic–metal armor systems is performed using AUTODYN[®], a fully integrated and interactive hydrocode developed by Century Dynamics [56]. This code provides a number of fully coupled numerical processors including Lagrange, Euler, smooth particle hydrodynamics (SPH) and Arbitrary Lagrangian–Eulerian (ALE) [57]. In AUTODYN[®], the following

fundamental equations together with the initial and boundary conditions are solved using a finite difference scheme [58]:

1. Conservation of mass:

$$\frac{D\rho}{Dt} + \rho \frac{\partial v_i}{\partial x_i} = 0 \quad (2.14)$$

2. Conservation of momentum:

$$\rho \frac{Dv_i}{Dt} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i \quad (2.15)$$

3. Conservation of energy:

$$\rho \frac{De}{Dt} = \frac{\partial (\sigma_{ij} v_i)}{\partial x_j} - \frac{\partial q_i}{\partial x_i} + \rho S + \rho v_i b_i \quad (2.16)$$

4. Decomposition of stresses:

$$\sigma_{ij} = S_{ij} - \delta_{ij} P \quad (2.17)$$

5. Equation of state (EOS):

$$P = P(\rho, e) \quad (2.18)$$

where subscripts i and j can be 1, 2 or 3, ρ is material density, v_i are velocity components, x_i are spacial variables, σ_{ij} is stress tensor, S_{ij} is deviator tensor, $-\delta_{ij} P$ is the hydrostatic stress, q_i is heat flux vector, b_i is body force, e is internal energy per unit mass, $\frac{D}{Dt}$ is called convective derivative or material derivative defined as:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i} \quad (2.19)$$

2.4.1 Spatial discretization method

The spatial discretization is performed by representing the fields and structures of the problem using computational points in space, including mesh based and mesh-free techniques [59]. In this work, two spatial discretizing techniques in AUTODYN[®], Lagrange and SPH (Smooth Particles Hydrodynamics), are used to simulate the ballistic impact.

The Lagrange method of space discretization, in which the numerical grid moves and deforms with the material, is ideal for following the material motion and deformation in

regions for relatively low distortion, and possibly large displacement. Conservation of mass is automatically satisfied and material boundaries are clearly defined. The Lagrange method is most appropriate for representing solids like structures and projectiles. The advantages of the Lagrange method are computational efficiency and ease of incorporating complex material models. The disadvantage of Lagrange is that the numerical grid can become severely distorted or tangled in an extremely deformed region, which can lead to adverse effects on the integration time step and accuracy. However, these problems can be overcome to a certain extent by applying numerical techniques such as erosion and rezoning.

The mesh-free method of space discretization, SPH was used initially in astro-physics [60]. The SPH particles are not only interacting mass points but also interpolation points used to calculate the value of physical variables based on the data from neighboring SPH particles, scaled by a weighting function. Because there is no grid defined, the SPH method does not suffer from grid tangling in large deformation problems. Therefore, the SPH method is very useful to simulate material behavior subject to severe deformation and distortion, for example, in hypervelocity impact. However, the SPH method requires a sort of the particles in order to locate current neighboring particles, which makes the computational time per cycle more expensive than mesh based Lagrange techniques. This can make mesh-free methods less efficient than mesh based Lagrange methods with comparable resolution. In SPH, the basic steps used in each calculation cycle, as implemented in AUTODYN[®], are shown in Figure 2.5.

Figure 2.5 Computational cycle for SPH [39]

The calculation cycle is similar to that for a Lagrange zone, except for steps where a Kernel approximation is used. Kernel approximations are used to compute forces from spatial derivatives of stress and spatial derivatives of velocity required to compute strain rates [39].

2.4.2 Interaction (contact) in AUTODYN[®] [47]

The interactions between various parts in a model can be defined through the interaction dialogue panel define. AUTODYN[®] offers two types of Lagrange/Lagrange interaction (contact) methods, namely “Gap contact” and “Erosion”. In the simulations performed in the current thesis, the former has been used. In “Gap contact” method, each surface segment is surrounded by a contact detection zone. The dimension of this contact zone is called “gap size”. This means that initially the outside surfaces of the parts in the contact should be separated by the gap distance. Any node entering the contact zone of a surface segment is repelled by a force normal to the segment surface and proportional to the depth of penetration of the node into the contact detection zone. The gap size required for the interaction logic must be in the range $1/10^{\text{th}}$ to $1/2$ the dimension of the smallest element face of parts involved in interactions. For most models it is suggested to set the gap size close to the smallest allowed value. There are two different gap types, namely internal and external. The latter is strongly recommended because calculations using this option are more robust and efficient.

2.4.3 Erosion criterion

Numerical erosion is a technique used in Lagrange codes for handling heavily distorted cells (zones) happening in large deformation simulations, for instance, penetration problems. The zones are deleted (eroded) when a suitably defined effective strain exceeds a pre-set value, the erosion strain [61]. In AUTODYN[®] there are three effective strains, namely, effective plastic strain, incremental geometrical strain and instantaneous geometrical strain. Lagrangian elements are removed if a predefined strain (either effective plastic strain, incremental geometrical strain or instantaneous geometrical strain) exceeds a specified limit. When an element is removed from the calculation process in this way, the mass within the element can either be discarded or distributed to the corner nodes of the maintained elements. However the compressive strength and internal energy of the material within the element are lost whether or not the mass is

retained. Because of losses of internal energy, strength and (possibly) mass, care must be taken in using this option and erosion strain limits chosen so that elements are not discarded (eroded) until they are severely deformed and their compressive strength and/or mass are not likely to affect the overall results. If the retained inertia option is used in AUTODYN[®] and the four (2D) or eight (3D) elements around a particular node are discarded, the node become a free node. Free nodes are automatically added to the arrays of slave nodes in the impact-slide logic in AUTODYN[®]. Both effective plastic strain and incremental geometrical strain are non-decreasing functions of time and can be even large for a quite regular zone, namely if the zone is subjected to cyclic deformation. If that occurs, the zone might be eroded without reason. In order to avoid this drawback, “instantaneous geometrical strain” was introduced in AUTODYN[®], defined as:

$$\varepsilon_{inst} = \frac{2}{3} \sqrt{\left(\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2 \right) + 5 \left(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}\varepsilon_{33} + \varepsilon_{33}\varepsilon_{11} \right) - 3 \left(\varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{31}^2 \right)} \quad (2.20)$$

where ε_{ij} is strain tensor.

Erosion strain has no physical basis, its value is dependent on knowledge of the test result [62] and are primarily computational expedients to remove “bad” elements in order to continue the calculation [63].

2.5 Analytical, empirical and numerical models for ballistic limit velocity (BLV) of armor systems

Several researchers have developed analytical, empirical and numerical models to predict the projectile residual velocity and BLV of armor systems as their measure of performance. Analytical models for impact problems introduce simplifying assumptions into governing equations and reduce the problem to one- or two-dimensional simple equations. Empirical models are algebraic equations formulated based on a number of experimental or simulation data points. Numerical simulations can conveniently provide a large number of data to support empirical models and minimize the number of experiments that need to be conducted.

Analytical models relating projectile residual velocity, impact velocity and armor BLV have been extensively developed especially for monolithic metallic armor. Recht and

Ipson [64] proposed analytical models for normal impact of blunt and sharp projectiles of impact velocity, V_0 into thin metallic plates and proposed the residual velocity, V_r of the projectile as:

$$V_r = \frac{M_p}{M_p + M_t} (V_0^2 - V_{bl}^2)^{\frac{1}{2}}, \quad V_0 \geq V_{bl} \quad (2.21)$$

where M_p is the original mass of the projectile, M_t is the mass of the plug separated from the armor target, and V_{bl} is the ballistic limit velocity (BLV).

Lambert and Jonas [65] have proposed a unified form for residual velocity, V_r which is representative of other proposed models [64, 66, 67] as:

$$V_r = a(V_0^q - V_{bl}^q)^{\frac{1}{q}}, \quad V_0 \geq V_{bl} \quad (2.22)$$

where a , q and V_{bl} are determined by fitting the experimental data of V_0 and V_r [68-72].

Ipson and Recht [73], taking into account mass loss of the projectile by erosion, expressed the residual velocity of the projectile, V_r as:

$$V_r = \left[\left(\frac{M_p}{M_{pr} + M_t} \right) \left(\frac{M_p}{M_p + M_t} \right) \right]^{\frac{1}{2}} (V_0^2 - V_{bl}^2)^{\frac{1}{2}}, \quad V_0 \geq V_{bl} \quad (2.23)$$

where M_{pr} is the residual mass of the projectile after impact and remaining parameters are the same as those defined in Eq. (2.21).

Wen et al. [74] showed that impact velocity proposed by Recht and Ipson [64] confirms impact tests of projectiles with different nose shapes into both Kevlar fiber reinforced polymer (KFRP) monolithic laminates and also the sandwich panels consisting of glass fiber reinforced polymer (GRP) skins and Divinycell H130 polyvinyl chloride (PVC) foam core. However, similar analytical models for bi-layer armor systems comprising ceramics as front layer with metallic or composite backing layers are scarce. Shokrieh and Javadpour [75] proposed an analytical model for ceramic-composite armor, relating impact velocity, residual velocity and BLV which is the same as the model proposed by Recht and Ipson [64].

Analytical or empirical equations for BLV, inherently considering projectile and armor geometries are necessary to carry out the optimization of bi-layer ceramic–metal armor systems. Analytical models have been proposed by researchers either providing the details of the impact process like residual length and velocity of the projectile by dividing the impact process to different time stages [23, 76-78] or giving simplified equations for the BLV [79, 80]. Wang and Lu [62] presented an empirical model for the minimum kinetic energy needed to perforate a ceramic–metal armor based on experimental data. Numerical simulation techniques alternatively can, for development of empirical equations, be utilized for analyzing the effects of projectile and armor geometries on the BLV. Fawaz et al. [81] performed numerical simulations of normal and oblique impact on ceramic–metal armor and studied the energy and stress distribution within the impact zone. Cortés et al. [82] performed two-dimensional axisymmetric simulation of normal impact of ceramic–metal armor and compared their observation of projectile-target interaction with the experimental measurements of Reijer [23].

2.6 Models considering the deformation modes of blunt and conical projectiles onto ceramic–metal armor systems

It is known that the mechanisms involved in the impact of blunt and conical projectiles are extremely different [83]. The relative hardness of the projectile and target is of main importance in deformation of the projectile at different impact velocities while toughness is relatively of second importance [84]. Projectile with higher hardness compared to target will not deform while the one with lower hardness may fracture and/or deform. Xiao et al. [85] experimentally studied various deformation and fracture modes of steel projectiles with hardnesses of HRC 20 and 50 during impact onto hard steel plates at velocities in the range of 200-600 m/s. As the steel target plates are not fully rigid, the relative motion (velocity) of the projectile and target affects the deformation of the projectile. Rakvåg et al. [86] also studied the deformation and fracture modes of unhardened steel projectiles as well as ones with three hardnesses of HRC 19, 40 and 52 in impact onto a rigid target at velocities in the range of 100-350 m/s. They found a critical impact velocity for the three materials below which only

plastic deformation (mushrooming) of the projectile occurred, while above this critical velocity different types of fracture modes were observed.

Many researchers, mostly considering blunt projectiles, have studied the effect of nose shape on the impact load and/or penetration into the ductile targets [87, 88]. Table 2.3 lists the models/works studying projectile deformation/erosion in impact of a target.

Table 2.3 Summary of models/works studying projectile deformation in impact of a target

Type of work/model	Projectile material/nose shape	Target characteristic	Author(s), Reference	Remarks/Application
Analytical	Ductile/blunt	Rigid	Taylor [89]	Final deformed shape and dynamic yield strength of the projectile material is found.
Analytical	high-strength steel/blunt	Rigid	Lee and Tupper [90]	Modified Taylor's model [89], giving more realistic deformed shape profile
Analytical	high-strength steel/blunt	Monolithic non-rigid	Recht [91]	Extended version of Lee and Tupper model [90], considering plastic deformation and erosion of projectile
Analytical	Ductile/blunt	Ceramic-metal armor	Reijer [23]	Erosion and plastic deformation of the projectile were considered.
Numerical	Tungsten/blunt, hemispherical and conical	Hardened steel plate	Walker and Anderson Jr [88]	By the time of 80 μs , the original nose shape effect on deformation was gone.
Analytical	Ductile/blunt	Non-rigid	Zaera et al. [77]	Based on model proposed by Tate [92] and Alekseevskii [93], only considering erosion of projectile
			Anderson and Walker [25]	

For the case of impact on a rigid target, plastic deformation occurs at the impact side of the projectile at impact velocities lower than plastic wave velocity of projectile material [91]. Taylor [89], considering only impact velocities lower than plastic wave velocity of the projectile, U_p posited that projectile impact onto the rigid surface produces interface pressure which results in propagation of two plane elastic and plastic waves through the projectile toward its free end.

At higher impact velocities, erosion of projectile occurs. Erosion is a mechanism whereby material is physically separated from the projectile, therefore its momentum no longer contributes to armor perforation [23]. Zaera et al. [77] suggested to find equivalent diameter and length, for the case of non-cylindrical projectile impact, and used the equivalent projectile in their proposed model. Naik et al. [94] proposed an analytical energy-based model in case of a blunt projectile impact onto ceramic-composite armor and used Recht's model [91] for the prediction of deformation and erosion of the projectile.

2.7 Topics on long rod projectile impact on confined ceramic armor

2.7.1 Pre-stressing—a method of adding ductility to ceramic armor

It was mentioned that though ceramics are strong in compression, however they suffer from disintegration when loaded under tension. Wilkins et al. [9] discussed that adding metallic phase to the ceramic materials or use of graded armor inhibit early failure due to brittle fracture seen in most of ceramics experiencing tensile load in the course of impact event. Holmquist and Johnson [8] stated that the strength and ductility of a ceramic material may substantially depend on pressure, applied on the ceramic target, which is determined from the surface load, confinement, target geometry and wave propagation. Inclusion of confinement, as also mentioned in section 2.2.1.2, around ceramics results in keeping the damaged and failed ceramics in their place, thereby increasing ballistic behavior of ceramics which has been studied by many researchers [18, 34, 95-97]. The use of confinements to pre-stress the ceramics for further improving of ballistic behavior has also been explored [98-100]. The effect of pre-stressing ceramics in compression is increasing strength and ductility and also hindering early failure in tension [10]. Increase of ceramic strength can force the high velocity projectile to flow radially with no significant penetration, a phenomenon called *dwell* or *interface defeat* [101]. In the literature, dwell is usually referred to radial flow of the projectile on the surface of the target for some time (dwell time) followed by penetration into the ceramic while if no penetration occurs the phenomenon is termed as interface defeat. Dwell or interface defeat are important mechanisms during which the kinetic energy of the projectile is considerably reduced. Therefore, the methods to

produce and/or dwell or interface defeat have been of much interest in armor design in defeat of either small caliber threats or high kinetic energy (KE) projectiles.

Westerling et al. [102] investigated, via simulations, the influence of impact velocity and confinement with various thicknesses on the ballistic behavior of boron carbide targets impacted by long rod tungsten projectiles. They found out that at low impact velocities interface defeat occurred while at high impact velocities the penetration was steady and symmetrical and the influence of confinement on penetration velocity was small. Holmquist and Johnson [10] performed pre-stress modelling of thin and thick ceramic targets considering two levels of small and large pre-stress ratios and two pre-stress conditions of radial and hydrostatic. Impact simulation of radially pre-stressed thick targets at different impact velocities, performed by Holmquist and Johnson [10], do not completely match with experimental data [103] of the shrink-fitted (radially pre-stressed) targets. Quan et al. [104] also performed impact simulation of confined targets (without pre-stress); however, they compared their results with experimental data [103] of the pre-stressed target.

2.7.2 Sequence of mechanisms during transition from dwell to penetration for confined ceramic targets

In the literature, there are many researches on the transition from dwell to penetration into confined ceramic targets, based on which, a sequence of mechanisms, occurring during the transition from dwell to penetration is described in the following section.

2.7.2.1 Accumulation of damage

In the literature, transition from dwell to penetration has been widely attributed to the softening of the material or in another words accumulation of damage in comminuted region (or quasi-plasticity region [105]) underneath the impact site of the ceramic materials [101, 105-107]. This phenomenon has been observed also in long rod impact onto glass materials [108]. Behner et al. [107] stated that once damage is initiated, it propagates in a faster rate compared to rate of long rod penetration. Analytical models [103, 109] have been proposed for interaction of long rod metallic conical and cylindrical projectiles (for which perfect-fluid behavior is assumed) considering three levels for accumulation of damage: (a) incipient damage (ID) in which damage is initiated, at the point of maximum shear stress on the axis of symmetry below the

surface of the ceramic at impact site and moves toward impact site, (b) full damage (FD) in which damage reaches the surface of the ceramic and (c) surface failure (SF) in which a ring-shaped surface failure is initiated, as shown in Figure 2.6. It should be mentioned that material fails when damage parameter is equal to 1, i.e. $D=1$. For impact of a confined ceramic target by a cylindrical projectile, the transition from dwell to penetration occurs when the maximum normal load per unit area under the projectile, $P_{\max}$ varies in an interval with a lower and upper limit: $(1.3+1.03\nu)Y \leq P_{\max} \leq 2.85Y$ in which Y and ν are the yield strength and Poisson's ratio of the ceramic material.

Figure 2.6 Critical levels of damage and failure underneath the impact site of the ceramic impacted by a long rod conical projectile: (a) Incipient damage (ID), (b) full damage (FD) and (c) surface failure (SF), proposed by Lundberg et al. [110]

The damaged material, as shown in Figure 2.7, in the post-mortem examination of recovered ceramics after long rod impact, has been observed in the form of cone cracks, grain boundary micro-cracks and erosion of the material [111, 112].

Figure 2.7 Different forms of damages under the impact site of the polished SiC cross section recovered from impact of long rod tungsten projectile: (a) of 3.175 mm diameter and 1149 m/s impact velocity [111] and (b) 6.35 mm and 1600 m/s impact velocity [112]

2.7.2.2 Backflow of the projectile material

Renström [113] stated that the flow of a long rod projectile on the surface of a ceramic target during dwell or interface defat, is inertial (hydrodynamic) such that strength and compressibility of the projectile material can be ignored. Considering the fluid-like flow behavior of the long rod projectile on the ceramic target proposed by Renström [113], Uth and Deshpande [114] recently, conducted an experimental work, seeking the fluid-structure interaction of a liquid jet on a deformable target. They proposed and experimentally proved that impact of a jet of V_0 impact velocity, as shown in Figure 2.8 (a), much larger than penetration rate, onto a deformable target, results in the deformation of the target surface (dimpled shape of the target surface shown in Figure 2.8 (b)) and increase in the angle, θ between incoming and exiting fluid.

With the penetration of the jet into the target, as the deformation of the target surface increases (θ decreases), as shown in Figure 2.8 (b), the force applied on the surface of the target, given by $(1 + \cos \theta) \rho_{jet} v_0^2 A_{jet}$ (where ρ_{jet} is the material density of the jet, V_0 is the impact velocity and A_{jet} is the cross-sectional area of the jet), increases resulting in formation of a cavity, thereby amplifying the rate of penetration into the target.

Figure 2.8 Schematic view of the proposed fluid-structure interaction (FSI) mechanism by Uth and Deshpande [114] in impact of a fluid jet with velocity V_0 : (a) during the initial stages of the jet impact, the target surface is flat and the fluid spreads laterally and (b) as the jet deforms the target and penetrates a depth of δ into the target, it creates a dimple at the impact site; from the work of Uth and Deshpande [114]

Uth and Deshpande [114] concluded that in impact of long rod heavy metal projectiles onto ceramic targets, since a depth of penetration, like δ shown in Figure 2.8 (b), into the ceramic material, is needed before establishment of the backflow of the projectile material, ceramic materials due to their brittle nature inevitably experience damage accumulation before occurrence of the backflow and unsteady penetration.

Anderson Jr et al. [115], based on corroborative experiments showing uncertainty interval for dwell-penetration transition velocity for borosilicate glass, stated that transition from dwell to penetration is an unstable process, analogous to BLV experiments. This unstable process can be attributed to the unsteady fluid-structure interaction (FSI) mechanism proposed by Uth and Deshpande [114], described above.

2.7.3 Transition velocity

There is a critical impact velocity, namely transition velocity, at which dwell on the interface is no longer maintained and a transition between dwell and normal penetration occurs [103, 116]. It is speculated that this transition is related to the maximum surface load (strength) tolerable by the ceramic. Lundberg et al. [103] proposed a method to find a “velocity range” for which a transition occurs from dwell to penetration of the confined ceramic target. They compared their results with experiments performed using two projectile materials, namely tungsten and molybdenum, and five ceramic materials, namely boron carbide, titanium diboride, two types of silicon carbide and a polycrystalline diamond composite, radially pre-stressed with ratio of $R_r = 1.0035$ (radial pre-stress ratio, R_r , is defined as the ratio of the ceramic target initial diameter to confinement inner diameter). They concluded that there seems to be a unique transition velocity for each combination of projectile, target material and configuration. This implies that the upper limit and lower limit of the proposed velocity range in definition of transition velocity should be very close (or the same). The experiments [117] performed on a silicon carbide material, namely SiC-N, show the maximum impact velocity of 1512 m/s for which dwell occurred and the minimum impact velocity of 1502 m/s for which penetration occurred with the 10 m/s overlap between the two limits corresponding to the inaccuracy (of ± 5 m/s) in velocity measurements. Anderson et al. [32] studied the influence of pre-stress on the transition velocity of silicon carbide. They found that with mere applying of 200 MPa radial pre-stress, the transition velocity

increased from 1027 m/s for unconfined target (without pre-stress) to 1549 m/s for pre-stressed target.

Lundberg et al. [117] speculated that the fracture toughness may have more influence on transition velocity compared to hardness (or in another words, plastic flow) of the ceramic. Furthermore, pre-stressing of the ceramic can suspend the initiation and propagation of cracks in the course of impact event, thereby increasing the dwell–penetration transition velocity; however the effect of different pre-stress types on the transition velocity has not been studied yet.

2.7.4 Material and geometries of the components in confined ceramic armor as design parameters

Radial or hydrostatic pre-stress simulations performed by Holmquist and Johnson [10] shows that for a given ceramic/confinement/front and backing plug (cover plate) system, there exists a pre-stress value above which the peak confinement cannot be maintained; The reason is due to the excessive increase of the plastic strain beyond the bearable strain of the confinement. The implication is that material properties of the confinement (and also front and backing cover plates) are important design parameters, affecting the unique shape of the pressure profile at the ceramic-plug interface and also within the ceramic. It is believed that geometries of the confinement and front/backing cover plate should be also considered in design. The role of the cover plate is to attenuate the initial load, exerted on the ceramic after the projectile impact, to a quasi-static level [103]. Different geometries and materials and their effects on the ballistic behavior of the armor have been considered by researchers [117]. Change of the front cover plate material and geometries have shown considerable change in transition velocity and ballistic behavior of the armor [100, 103, 116-119]. Effect of material properties of the ceramic like hardness and toughness, on the behavior of the armor, have been also studied [117].

2.8 Summary

Analytical models relating projectile residual velocity, impact velocity, and armor BLV have been extensively developed especially for monolithic metallic armor. However, the similar empirical or analytical models for bi-layer ceramic–metal armor systems are scarce. Therefore, developing analytical and empirical models for understanding the

performance of the bi-layer ceramic–metal armor systems under impact is necessary. These models can be further utilized for armor optimization purposes.

Analytical models on the residual length of the projectile, available in the literature, mostly consider blunt projectiles without differentiating the effect of different nose geometries in interaction with the armor target. Analytical models specifically considering the nose geometry of the projectile and its deformation mechanisms would be beneficial in predicting the projectile behavior in interaction with the armor.

Studies on the modelling of pre-stressed ceramic armor while considering various pre-stress types and finding their effect on the ballistic behavior of the confined thick ceramic armor are rather limited in the literature. In view of this, it would be significant to conduct studies which can be beneficial in understanding the key parameters for improving ballistic behavior of the confined thick ceramic armor under impact of long rod heavy metal projectiles. This can be a promising cost effective design tool for finding influences of material and geometrical parameters of the confined ceramic armor.

In summary, the current thesis will focus on three main areas highlighted above, namely a) proposing analytical and/or empirical models for impact of bi-layer ceramic–metal armor systems, b) proposing analytical models considering the nose geometry of the projectile and its deformation mechanisms in impact of bi-layer ceramic–metal armor systems and finally c) proposing a robust numerical model for studying the effect of pre-stressed thick ceramic-metal armor on the interface defeat and dwell phenomenon.”

Chapter 3 A Semi-Analytical Model for Ballistic Impact on Bi-Layer Alumina–Aluminum Armor*

Residual velocity of the projectile and ballistic limit velocity (BLV) of the armor are the two common measures of armor performance. Analytical models relating projectile residual velocity, impact velocity and armor BLV have been extensively developed especially for monolithic metallic armor. However, similar analytical models for bi-layer armor systems comprising ceramics as front layer with metallic or composite backing layers have not been developed yet.

In this chapter, a numerical model for impact of alumina/aluminum armor with tungsten alloy projectile is initially established, using commercial explicit non-linear transient dynamic numerical code, AUTODYN® [47]. The numerical modeling approach is checked through comparison with available experimental data. The verified numerical model will be used to provide simulation data to develop a semi-analytical model relating the impact and residual velocity and BLV of the armor.

The influences of projectile and armor geometrical parameters on the BLV are then analyzed based on comprehensive numerical simulations performed using the verified numerical model. It is shown that the projectile residual velocity and BLV satisfy the replica scaling laws. An empirical equation is then proposed for the BLV. Finally, optimization of alumina–aluminum armor for given total thickness and weight constraints is performed based on the proposed BLV empirical equation, giving reasonable results similar to experimental measurements available in the literature.

3.1 Numerical modelling

Four ballistic impact tests performed by Gálvez and Paradela [16] are firstly simulated to determine valid numerical simulation and material parameters. In each test, a 20 mm armor-piercing discarding-sabot (APDS) projectile impacted at the nominal impact velocity of 1240 m/s and perforated a bi-layer armor composed of Al_2O_3 99.5% front plate and aluminum 5083-H111 backing plate.

*This Chapter was pre-dominantly published in “International Journal of Impact Engineering”.

Figure 3.1 shows a 20 mm APDS which is a sub-calibrated heavy projectile consisting of tungsten alloy core with ogive nose [120] and the diameter, length and mass of this projectile are 12 mm, 61.5 mm and 72 g, respectively.

Figure 3.1 A 20 mm APDS (a) complete projectile, (b) core and (c) eroded core [120]

The geometric parameters of the two plates, projectile residual velocity and residual length obtained from experimental measurements [16] and current simulations are listed in Table 3.1.

Table 3.1 Comparison of the current simulation results, for 20 mm APDS impacting bi-layer alumina–aluminum armors, with experimental results [16]

Test No.	T_1 (mm)	T_2 (mm)	L_{pr} (Exp.) (mm)	L_{pr} (Sim.) (mm)	V_r (Exp.) (m/s)	V_r (Sim.) (m/s)
1	20	10	22	26.8	930	1043
2	20	15	24 -27	26.1	930 - 960	1012
3	25	10	25	24.6	960	999
4	25	15	24	23.3	939	954

Two-dimensional (2D) axisymmetric simulations of these four tests were performed using AUTODYN[®]. As shown in Figure 3.2, the projectile and front plate were modelled in SPH domain while the backing plate was discretized using four node 2D axisymmetric Lagrangian elements with one integration point. SPH method is more effective to simulate the fragmentation behavior of tungsten projectile and alumina layer than Lagrangian method. The tungsten alloy core accounts for more than 99% of the mass of whole APDS projectile. Therefore, only tungsten alloy core was modelled in the simulations. Since axisymmetric conditions are used in the simulations, the armor plates were modelled as circular discs with a radius of 100 mm ensuring the ratio of diameter to thickness to be not less than 10 to avoid boundary effects. The armor plates were taken as fully clamped at the periphery.

Figure 3.2 Two-dimensional (2D) axisymmetric numerical model

After mesh sensitivity analysis in which five different SPH particle and Lagrangian element sizes were considered, the SPH particle size used was $1/12^{\text{th}}$ the projectile radius. The size of the square Lagrangian element was chosen to be the same as the size of the SPH particle in the central area (within the radius of 30 mm) of the backing plate. The element size becomes coarse gradually in radial direction towards the clamped edge of the plate. The Lagrangian element nodes and edges and SPH particles at the interface of the front plate and backing plate were tied together using the “JOIN module” in AUTODYN[®]. Separation of the two plates occurs when the SPH particles close to the interface fail based on ceramic material damage model. Friction between the parts in contact was neglected.

3.1.1 Material models and constants

The JH-2 ceramic constitutive model, described in section 2.3.2, was used for Al_2O_3 99.5%. Its material model constants are listed in Table 3.2 and its Hugoniot elastic limit (HEL) is 8.3 GPa [121].

The JC constitutive and fracture models, described in section 2.3.1, were used for the projectile (tungsten alloy) and backing plate (aluminum 5083-H116) materials. The JC model constants for aluminum alloy and tungsten are listed in Table 3.3.

Table 3.2 JH-2 material constants for Al₂O₃ 99.5% in the simulations [31]

Constants with units	Values
Density, ρ_0 (Kg/m ³)	3890
Bulk modulus, K (GPa)	231
Pressure constant, K_2 (GPa)	-160
Pressure constant, K_3 (GPa)	2774
Shear modulus, G (GPa)	152
Intact strength constant, A	0.88
Intact strength exponent, N	0.64
Strain rate constant, C	0.007
Fracture strength constant, B	0.28
Fracture strength exponent, M	0.6
Hydro tensile limit, T^* (GPa)	-0.26
Normalized maximum fractured strength, $\sigma_{f_{\max}}^*$	1
Reference strain rate, $\dot{\epsilon}_0$ (s ⁻¹)	1
Damage constant, d_1	0.01
Damage exponent, d_2	0.7
Bulking factor, β	1

Table 3.3 Material constants for tungsten alloy and aluminum 5083-H116 [38, 122]

Constants with units	Tungsten alloy	Al 5083-H116
Density, ρ_0 (Kg/m ³)	17600	2700
Bulk modulus, K (GPa)	310	58.33
Shear modulus, G (GPa)	160	26.92
Static yield strength, A (GPa)	1.506	0.167
Strain hardening constant, B' (GPa)	0.177	0.596
Strain hardening exponent, n	0.12	0.551
Strain rate constant, C'	0.016	0.001
Reference strain rate, $\dot{\epsilon}_0$ (s ⁻¹)	1.0	1.0
Thermal softening exponent, m	1.0	0.859
Reference temperature, t_0 (K)	300	300
Melting temperature, t_m (K)	1723	900
Specific heat, C_i (J/Kg·K)	134	910
Damage constant, d_1'	0	0.0261
Damage constant, d_2'	0.33	0.263
Damage constant, d_3'	-1.5	-0.349
Damage constant, d_4'	0	0.247
Damage constant, d_5'	0	16.8

The modified JC constitutive/fracture model in the work of Clausen et al. [38] is different from JC model available in AUTODYN[®]. Experimental data points of equivalent plastic fracture strain with respect to strain rate for aluminum 5083-H116 is

available in [4]. The strain rate parameter d'_4 in the JC fracture model of aluminum 5083-H116, needed for our simulation in AUTODYN[®], was calculated using least square fitting method of JC fracture model to the experimental data points provided in [38].

In the Lagrangian domain, erosion algorithm was used to remove the elements experiencing large distortions. In the simulations it was observed that aluminum 5083-H116 elements, though experiencing large distortions, were not yet fully failed. Therefore, geometric erosion strain of 3 was chosen for aluminum 5083-H116 in order to keep the strength of the backing plate and match with the experimental measurements.

3.1.2 Comparison of current simulation results with experimental data [16]

The projectile residual lengths and velocities calculated from current simulations are presented in Table 3.1 and compared with the experimental measurements. It can be seen that the agreement is good except for the test No. 1. Experimentally measured residual length in the test 1 may be in error as the total target plate thickness of test No. 1 is less than the other cases implying that the residual length should be larger under the same impact velocity. However, the experimentally measured residual length was 22 mm which is smaller than that in the other experimental cases.

It can be concluded based on the good correlation observed between current simulation results and experimental measurements that the numerical model and material parameters presented can be utilized for simulation of the ballistic experiments in which a tungsten projectile impacts bi-layer alumina–aluminum armor.

3.2 Semi-analytical model for projectile residual velocity

A number of numerical simulations were performed, using the material parameters and SPH–Lagrange domain discretization rules mentioned in section 3.1, to investigate the alumina–aluminum armor ballistic performance. The complicated geometry of the 20 mm APDS core is simplified as a cylindrical projectile of 12 mm diameter and 36 mm length with equivalent mass. The projectile residual velocities for all simulations were measured for subsequent analyses and discussions.

The projectile residual velocities for one combination of projectile and bi-layer ceramic–metal armor plates (each with thickness of 15 mm) at nine different impact velocities are shown in Figure 3.3. It can be seen that the metal–ceramic armor has a specific characteristic in its energy absorption. When the impact velocity is 545 m/s, the projectile residual velocity is zero. However, when the impact velocity increases by 5 m/s to 550 m/s, the projectile residual velocity increases significantly (from 0 to 58 m/s) implying that the BLV is between 545 m/s and 550 m/s.

Figure 3.3 Variation of projectile residual velocity with impact velocity

The variation of projectile residual velocity with impact velocity, shown in Figure 3.3, in impact of ceramic–metal armor is similar to that of monolithic metallic armor. Børvik et al. [123] have shown this variation for monolithic metallic armor. The Lambert–Jonas approximation [65] (Eq. (2.22)) for monolithic armor was used to accurately fit a curve through the simulation data points for this bi-layer armor case. The BLV (V_{bl}) can be considered as a constant and predicted based on the least square method for best fitting to the simulation results. In this way, the V_{bl} was calculated to be 546.3 m/s for the above projectile–armor combination (lying between 545 m/s and 550 m/s). This example shows the accuracy of the Lambert–Jonas approximation in determining V_{bl} .

For each projectile–armor combination in the present simulations, the projectile residual-impact velocity relationships were all successfully described using Lambert–

Jonas approximation [65]. Another three sets of cases were also shown in Figure 3.3 for comparison and will be used in the following discussion. The constants a , q and V_{bl} in Eq. (2.22), used for fitting in Figure 3.3, are listed in Table 3.4.

Table 3.4 Constants of Lambert–Jonas equation [65] (Eq. (2.22)) obtained in the current work for curve fitting in Figure 3.3

Front (T_1) and backing (T_2) plate thickness (mm)	a	q	V_{bl} (m/s)
5	0.93	2.14	208
10	0.97	2.09	419
15	0.94	2.07	546
20	1.03	1.72	698

Lambert–Jonas approximation [65] can only be used with a large number of experimental or simulation data. Therefore, it is necessary to create an analytical model for perforation of ceramic–metal armor to simplify the relative calculations.

3.2.1 Derivation of the semi-analytical model

Since the impact process of ceramic–metal armor is a complex phenomenon including different mechanisms such as wave propagation and reflection, fragmentation, friction, fracture, erosion, etc., proposing a single theoretical model taking into account all the critical aspects of the process is complicated. However, simple analytical models considering the seminal mechanisms in the process can successfully predict the features of the impact phenomenon [62]. Consider the normal impact perforation in which a blunt projectile perforates a ceramic–metal armor as shown in Figure 3.4.

Figure 3.4 Sequence of events in projectile impact onto ceramic–metal armor

Description of perforation process and assumptions based on the simulation observations are as follows:

1. The projectile core typically comprises a hard material, such as tungsten alloy or hardened steel, presenting relatively low plasticity. When the projectile with diameter of D_p impacts the ceramic plate, high stress waves induced in the projectile result in a radial expansion and fragmentation of the projectile tip as shown in Figure 3.4 (a) and (b). Projectile erosion may continue to take place, depending on the resistance of the armor experienced by the projectile. The diameter of expanded portion of the projectile keeps constant at D'_p till the end of impact process as shown in Figure 3.4 (c). The remaining intact projectile with residual mass of M_{pr} perforates the bi-layer armor plates. The residual projectile mass will decrease as the impact velocity increases. Fragmentation and plastic deformation of hard projectiles are typical phenomena and have been seen in the experiments [124, 125]. Fragmentation of the projectile depends on impact velocity, resistance of the armor, brittleness of the projectile material, etc. As an example, for impact of hard steel projectile into different combinations of bi-layer ceramic–metal armor, projectile core is deformed and fractured due to the hardness of the ceramic [125].
2. In the ceramic front plate, a fracture conoid can be formed in front of the projectile a few microseconds after impact due to the stress wave propagation, reflection and interaction, as shown in Figure 3.4 (a). Afterwards, the projectile pushes the ceramic conoid and backing plate and attempts to penetrate through the comminuted ceramic. The backing plate deflects under the applied load, and thus creates space for the comminuted ceramic to move as well. As the projectile continues its penetration, the impact load is spread over an increasingly smaller area of the backing plate by the deteriorating fracture conoid, as shown in Figure 3.4 (b). At the end, the backing plate reaches its limits of energy absorption and failure becomes evident. It is assumed that there are two cylindrical plugs ejected from the ceramic and backing plate by the eroded projectile which passes through the armor (see shown in Figure 3.4 (c)). The two plugs will have a diameter, D'_p equivalent to that of the expanded portion of the projectile and the lengths which are same as the thicknesses of the two plates, respectively. In the highly inelastic impact process, the

projectile-plugs combination is considered to have the same residual velocity.

In the whole perforation process, only the two plate-plugs are involved in the interaction with projectile throughout. Two major processes reduce the velocity of the projectile as it perforates the armor. Acceleration of the plate-plug masses is accomplished at the expense of momentum transferred from the projectile. Shear at the backing plate-plug periphery, plastic deformation of the backing plate and fragmentation of projectile and ceramic result in deceleration of the projectile-plugs combination. Therefore, a semi-analytical model concerning blunt projectile perforating ceramic-metal armor at nominal impact velocities can be developed as follows:

- (a) First consider inelastic impact of the projectile into the plate-plugs as if the plugs are entirely free from the plates, i.e., inelastic impact between projectile and two plugs (free collision). The final velocity of these three masses will be same since the impact is considered to be completely inelastic. The projectile mass is considered constant in this procedure. The momentum transfer can be written as:

$$M_p V_0 = (M_p + M_t) V_f \quad (3.1)$$

The free impact final velocity, V_f can be determined by Eq. (3.1).

The kinetic energy of projectile-plugs combination after the free collision is:

$$E_{kf} = \frac{1}{2} \frac{M_p^2}{M_p + M_t} V_0^2 \quad (3.2)$$

The difference between initial and final kinetic energy is the energy dissipated in the plastic deformation of projectile-plugs combination.

- (b) Now consider energy transfer during perforation of the projectile-plugs combination into the armor. The projectile-plugs combination with kinetic energy of E_{kf} attempts to defeat the armor plates and pass through with a residual velocity. Another portion of energy is used for the projectile fragmentation. The energy transfer can be written as:

$$E_{kf} = \frac{1}{2} (M_{pr} + M_t) V_r^2 + E_{tr} + E_{pf} + E_{pk} \quad (3.3)$$

E_{tr} can be considered to include two parts: ceramic material fragmentation energy due to shear resistance at the periphery of backing plate-plug and plastic deformation energy for the rest of the backing plate.

Kinetic energy of fractured projectile material after free collision, E_{kb} can be written as:

$$E_{kb} = \frac{1}{2} (M_p - M_{pr}) \left(\frac{M_p}{M_p + M_t} \right)^2 V_0^2 \quad (3.4)$$

In the energy transfer process, it is assumed that the kinetic energy of the fractured projectile is initially used for defeating the armor plates resistance and breaking up itself. If E_{kb} is not enough to defeat the armor, then $E_{pk} = 0$ and the kinetic energy in the remaining intact projectile will be used in defeating the armor. On the contrary, the fractured projectile material will have a residual velocity (i.e., $E_{pk} > 0$) and all the kinetic energy of the intact projectile will be transferred to that of projectile-plugs combination. Therefore, Eq. (3.3) can be written as:

$$E_{kf} = \frac{1}{2} (M_{pr} + M_t) V_r^2 + E_{tr} + E_{pf}, \quad \text{for } E_{kb} < E_{tr} + E_{pf} \quad (3.5)$$

$$E_{kf} - E_{kb} = \frac{1}{2} (M_{pr} + M_t) V_r^2, \quad \text{for } E_{kb} \geq E_{tr} + E_{pf} \quad (3.6)$$

At the minimum perforation velocity, i.e., V_{bl} , residual velocity, V_r is equal to 0. Substitution of Eq. (3.2) into Eq. (3.5), at $V_0 = V_{bl}$ leads to Eq. (3.7) for the energy required to overcome the perforation resistance:

$$E_r = (E_{tr})_{bl} + (E_{pf})_{bl} = \frac{1}{2} \frac{M_p^2}{M_p + M_t} V_{bl}^2 \quad (3.7)$$

E_r is assumed to be constant value for a specific projectile–armor combination at any impact velocity.

It must be noted that the fractured projectile mass at impact velocities higher than BLV is larger than that at BLV. The projectile fragmentation energy at any velocity higher than BLV can be written as:

$$E_{\text{pf}} = (E_{\text{pf}})_{\text{bl}} + E'_{\text{pf}} \quad (3.8)$$

The accumulated projectile fragmentation energy is defined as part of the “internal energy” in AUTODYN[®]. It was observed that the projectile internal energy increases with increasing impact velocity for a given projectile–armor combination. The differences of the projectile internal energy and initial energy for impact under BLV and higher impact velocities were also calculated. It was observed that the increasing internal energy is much less than increasing initial energy for any impact velocity higher than BLV. Hence, the fragmentation energy, E'_{pf} can be neglected in Eq. (3.8).

If Eqs. (3.4) and (3.7) are substituted into Eqs. (3.6) and (3.5), respectively, and solved for the projectile–plugs combination residual velocity, the following expressions are obtained:

$$V_r = \left[\frac{M_p^2}{(M_{\text{pr}} + M_t)(M_p + M_t)} (V_0^2 - V_{\text{bl}}^2) \right]^{\frac{1}{2}}, \quad \text{for } \frac{M_p - M_{\text{pr}}}{M_p + M_t} < \left(\frac{V_{\text{bl}}}{V_0} \right)^2 \quad (3.9)$$

$$V_r = \frac{M_p}{M_p + M_t} V_0, \quad \text{for } \frac{M_p - M_{\text{pr}}}{M_p + M_t} \geq \left(\frac{V_{\text{bl}}}{V_0} \right)^2 \quad (3.10)$$

Conditions in Eqs. (3.9) and (3.10) are simplified forms of $E_{\text{kb}} < E_r$ and $E_{\text{kb}} \geq E_r$, respectively. Equation (3.9) has the same form as the model proposed by Ipson and Recht [73] (Ipson–Recht model) for monolithic metallic armor impact. Equation (3.10) ensures V_r to be less than impact velocity, a condition which may be dissatisfied if the Ipson–Recht model [73] is used for ceramic–metal armor.

The masses of the projectile (before impact), M_p , residual projectile (at the end of impact), M_{pr} , and armor plug M_t , in Eqs. (3.9) and (3.10) can be expressed as:

$$M_p = \frac{1}{4} \rho_p \pi D_p^2 L_p; \quad M_{\text{pr}} = \frac{1}{4} \rho_p \pi D_p^2 L_{\text{pr}}; \quad M_t = \frac{1}{4} \pi D_p'^2 (\rho_1 T_1 + \rho_2 T_2) \quad (3.11)$$

The equation of M_t is a general expression for total plate plugs mass and is equal to just the front plate plug mass when there is no backing plate plug sheared from the backing plate which is mostly under stretching.

The difficulty in using Eqs. (3.9) and (3.10) lies in the requirement of pre-determined BLV, V_{bl} , projectile residual length, L_{pr} and diameter of the expanded portion of the projectile, D'_p . In the present work, AUTODYN[®] simulation data of specific impact cases is used to obtain these values of V_{bl} , L_{pr} and D'_p for specific constitutive material parameters of alumina and aluminum. From the simulation data, the expanded projectile diameters, D'_p seemed to be close to $1.1D_p$. Equations (3.9) and (3.10) were used to calculate the projectile residual velocities at different impact velocities for four projectile–armor combinations shown in Figure 3.3. For each case, E_{kb} was first calculated using Eq. (3.7) to be compared with E_r in Eq. (3.7). Equations (3.9) and (3.10) were used respectively for $E_{kb} < E_r$ and $E_{kb} \geq E_r$. For the case of 15 mm thick front plate and 15 mm thick backing plate at impact velocity of 1300 m/s, Eq. (3.10) was used to calculate projectile residual velocity. In other cases, Eq. (3.9) was used. In addition, formation of shear plug in backing plate was checked in simulations for each case: when $T_1 = T_2 = 10$ mm, no backing plate plugs were observed. Therefore, M_t in Eq. (3.11) for these cases excluded the $\rho_2 T_2$ term. The complete M_t equation in Eq. (3.11) was used in the calculation of plug masses for all the remaining cases. Figure 3.3 shows that the residual velocities obtained from simulations compare well with those obtained from semi-analytical model.

3.3 Geometrical scale influence on residual velocity

From the viewpoint of dimensional consideration, the performance of the ceramic–metal armor is not only dependent on the geometrical dimensions but also material parameters of the projectile, front plate and backing plate.

Replica scale model is a model in which the geometrical dimensions are scaled uniformly, while the materials and impact velocity are kept constant. In general, it is believed that strain rate and fracture toughness are the terms that cannot be kept constant between replica-scaled model and prototype in penetration problems. Based on Buckingham Pi theorem [127], for the model to reproduce the prototype response, all the non-dimensional terms (generally referred to as Pi terms) must remain invariant. For a geometrical scale factor, ζ (the size of the replica model relative to the prototype), it

can be shown that the scale factor for stress, strain rate and fracture toughness would be 1, $1/\zeta$ and $\sqrt{\zeta}$, respectively. Therefore, in a sub-scale model (for example $\zeta = 1/5$), the strain rate will be higher than the prototype. As for some materials, strength (resistance to plastic flow) is increased with increase in strain rate, the Pi terms depending on plastic flow will be distorted [128]. Fracture toughness also will be lower in sub-scale model compared to the prototype which may result in dissimilar failure processes in the model and the prototype. Therefore, researchers have attempted to quantify the magnitude of strain-rate or fracture toughness effects on the response of penetration into specific (replica-scaled) armor system impacted by specific projectile at specific impact velocity. Two examples are described here:

Lundberg et al. [121] studied the influence of scale on penetration of long tungsten projectiles of 1.5 km/s and 2.5 km/s impact velocities onto replica-scaled unconfined steel-backed alumina targets. They showed that there is a tendency for normalized penetration to increase slightly with scale which may be attributed to the influence of strain rate and fracture toughness. They showed that the laws of replica-scaling hold with sufficient degree of accuracy to justify scaled-down experiments.

Anderson et al. [128] showed, using rate dependent material models in numerical simulations, that the normalized depth of penetration of tungsten-alloy long-rod projectile into semi-infinite armor steel target at 1.5 km/s is approximately 5% less, for the sub-scale model of $\zeta = 1/10$, than the prototype (strain-rate effect for the depth of penetration is only 5%). They also found that after perforating the finite-thickness armor steel target, the prototype projectile residual velocity and its normalized residual length are both almost 5% higher than for the model projectile ($\zeta = 1/10$), implying that the sub-scale model is stronger due to strain rate effect.

In this section, an attempt is made to quantify the deviation from replica scaling, if any, for impact of tungsten alloy projectile onto bi-layer Al_2O_3 99.5%–aluminum 5083-H111 armor system. Dimensional analysis, similar to the one carried out by Lundberg et al. [121], was performed. The geometries of the projectile, ceramic front plate and metal backing plate are specified by six parameters, viz. the length (or thickness) and planar diameter of the projectile (L_p, D_p), the front plate (T_1, D_1) and the backing plate

(T_2, D_2) . The three materials are characterized by their densities, ρ and parameters representing their mechanical properties such as Young's modulus, E , shear modulus, G , static yield limits, Y , hardening modulus, H and fracture toughness, K_{IC} . In order to take into account the strain rate effect for tungsten and aluminum and alumina, threshold strain-rate, $\dot{\epsilon}_0$, should be also considered. For considering conversion of kinetic energy to heat due to plastic work, specific heat, C_r is also taken into account. These parameters are summarized in Table 3.5 in which the subscripts p, 1 and 2 are referred to the projectile, front plate and backing plate, respectively.

Table 3.5 Parameters characterizing projectile, front plate and backing plate

Quantity	Unit	Parameter
Impact velocity	m.s^{-1}	V_0
Lengths (or Thickness)	m	L_p, T_1, T_2
Planar diameters	m	D_p, D_1, D_2
Densities	Kg.m^{-3}	ρ_p, ρ_1, ρ_2
Young's modulus	Pa	E_p, E_1, E_2
Shear modulus	Pa	G_p, G_1, G_2
Static yield limits	Pa	Y_p, Y_1, Y_2
Hardening modulus	Pa	H_p, H_1, H_2
Fracture toughnesses	$\text{Pa.m}^{1/2}$	$K_{IC,p}, K_{IC,1}, K_{IC,2}$
Strain rate thresholds	s^{-1}	$\dot{\epsilon}_{0p}, \dot{\epsilon}_{01}, \dot{\epsilon}_{02}$
Specific heats	$\text{J.kg}^{-1}.\text{K}^{-1}$	C_{rp}, C_{r1}, C_{r2}
Temperatures	K	t_p, t_1, t_2

In order to establish dimensionless analysis, all the parameters are expressed in terms of D_p, V_0, ρ_p , and t_p as follows:

$$\frac{V_r}{V_0} = f \left(\frac{T_1}{D^*}, \frac{T_2}{D^*}, \frac{L_p}{D^*}, \frac{D_1}{D^*}, \frac{D_2}{D^*}, \frac{\rho_1}{\rho^*}, \frac{\rho_2}{\rho^*}, \frac{E_p}{\sigma^*}, \frac{E_1}{\sigma^*}, \frac{E_2}{\sigma^*}, \frac{G_p}{\sigma^*}, \frac{G_1}{\sigma^*}, \frac{G_2}{\sigma^*}, \frac{Y_p}{\sigma^*}, \frac{Y_1}{\sigma^*}, \frac{Y_2}{\sigma^*}, \frac{H_p}{\sigma^*}, \frac{H_1}{\sigma^*}, \frac{H_2}{\sigma^*}, \frac{K_{IC,p}}{K_{IC}^*}, \frac{K_{IC,1}}{K_{IC}^*}, \frac{K_{IC,2}}{K_{IC}^*}, \frac{\dot{\epsilon}_{0,p}}{\dot{\epsilon}_0^*}, \frac{\dot{\epsilon}_{0,1}}{\dot{\epsilon}_0^*}, \frac{\dot{\epsilon}_{0,2}}{\dot{\epsilon}_0^*}, \frac{C_{rp}}{C_r^*}, \frac{C_{r,1}}{C_r^*}, \frac{C_{r,2}}{C_r^*} \right) \quad (3.12)$$

where $D^* = D_p$, $\rho^* = \rho_p$, $\sigma^* = \rho_p V_0^2$, $K_{IC}^* = \rho_p V_0^2 D_p^{\frac{1}{2}}$, $\dot{\epsilon}_0^* = V_0/D_p$ and $C_r^* = V_0^2/t_p$.

Plastic work increases the bulk temperature of the material, which can lead to thermal softening. Thermal conductivity effects can be ignored in the analysis, as there is little time during impact process for heat conduction [127]. For the constant impact velocity

and fixed materials, the Pi terms including Young's modulus, E , shear modulus, G , static yield limits, Y , hardening modulus, H remain unvaried. Therefore, Eq. (3.12) can be reduced to:

$$\frac{V_r}{V_0} = f\left(\frac{T_1}{D^*}, \frac{T_2}{D^*}, \frac{L_p}{D^*}, \frac{D_1}{D^*}, \frac{D_2}{D^*}, \frac{K_{IC,p}}{K_{IC}^*}, \frac{K_{IC,1}}{K_{IC}^*}, \frac{K_{IC,2}}{K_{IC}^*}, \frac{\dot{\epsilon}_{0,p}}{\dot{\epsilon}_0^*}, \frac{\dot{\epsilon}_{0,1}}{\dot{\epsilon}_0^*}, \frac{\dot{\epsilon}_{0,2}}{\dot{\epsilon}_0^*}\right) \quad (3.13)$$

Equation (3.13) shows that the dimensionless parameters, except those including the fracture toughness, K_{IC} , and strain rate threshold, $\dot{\epsilon}_0$, remain constant when the dimensions are scaled uniformly. As mentioned before, when the geometrical dimensions are scaled, the model response in terms of strength (strain rate effect) or failure process (fracture toughness effect) can be affected. This fact is also discussed in [121, 129]. Therefore, replica scaling can be expected to be valid, if only fracture toughness and strain rate have negligible influence on the perforation process. According to the above discussion, deviations from such behavior would be due to improper scaling of fracture toughness and strain rate or other parameters which have been overlooked in Table 3.5.

Two sets of simulations (S_1 – S_6) were performed and their details are listed in Table 3.6.

Table 3.6 Parameters of impact simulations with same geometrical ratio

Set	1			2		
No.	S_1	S_2	S_3	S_4	S_5	S_6
Impact velocity (m/s)	800	800	800	650	650	650
Projectile diameter (mm)	6	9	12	6	12	15
Projectile length (mm)	18	27	36	18	36	45
Front plate thickness (mm)	10	15	20	5	10	12.5
Front plate diameter (mm)	50	75	100	50	100	125
Backing plate thickness (mm)	10	15	20	7.5	15	18.75
Backing plate diameter (mm)	50	75	100	50	100	125

In each set, there were three cases with same impact velocities and same geometrical dimension ratios for the projectile, front plate and backing plate. In each set, there were three cases with same impact velocities and same geometrical dimension ratios for the projectile, front plate and backing plate. The JC constitutive and fracture models were used for the projectile (tungsten alloy) and backing plate (aluminum 5083-H116) materials and the JH-2 model was used for Al_2O_3 99.5%. The details of the material models used can be found in section 3.1.1. Results for three different cases in the same

set are expected to be same if replica scaling is valid. The variation of residual velocity, V_r with projectile diameter, D_p for the three cases under the same impact velocities in each set, is shown in Figure 3.5.

Figure 3.5 Variation of projectile residual velocity with projectile diameter for the same geometrical ratio

It is apparent that the residual velocity remains constant for each set which support the validity of replica scaling, implying that the effects of strain rate or fracture toughness on the perforation process can be considered negligible. It must be emphasized that the above remarks, apply specifically to the case of tungsten-alloy projectile impacting bi-layer Al_2O_3 99.5%–aluminum 5083-H111 armor system, with specified impact velocities and geometrical scale factors listed in Table 3.6.

In the following, it is discussed that the proposed semi-analytical model satisfies the replica scaling laws.

Substituting Eq. (3.11) into Eqs. (3.9) and (3.10), V_r can be written as:

$$V_r = \left[\frac{V_0^2 - V_{bl}^2}{\left(\frac{L_{pr}}{L_p} + \frac{D_p'^2 (\rho_1 T_1 + \rho_2 T_2)}{D_p^2 \rho_p L_p} \right) \left(1 + \frac{D_p'^2 (\rho_1 T_1 + \rho_2 T_2)}{D_p^2 \rho_p L_p} \right)} \right]^{\frac{1}{2}}, \text{ for } \frac{M_p - M_{pr}}{M_p + M_t} < \left(\frac{V_{bl}}{V_0} \right)^2 \quad (3.14)$$

$$V_r = \frac{1}{1 + \frac{D_p'^2 (\rho_1 T_1 + \rho_2 T_2)}{D_p^2 \rho_p L_p}}, \text{ for } \frac{M_p - M_{pr}}{M_p + M_t} \geq \left(\frac{V_{bl}}{V_0} \right)^2 \quad (3.15)$$

Consider Eqs. (3.14) and (3.15) under conditions of constant impact velocity, materials, and geometrical ratios. There are still V_{bl} , L_{pr}/L_p and D'_p/D_p which need to be explored in V_r to satisfy replica scaling laws. It was shown, based on simulation results, that scaling laws can be used for the projectile residual velocity for the same materials at same impact velocity. Therefore, BLV which is a special case of impact velocity with zero residual velocity for projectile–ceramic–metal armor combinations with same materials and geometrical dimension ratios can be extrapolated to behave the same. The ratio of projectile residual length to original length, L_{pr}/L_p were measured for these six cases and shown in Figure 3.6.

Figure 3.6 Variation of L_{pr}/L_p with D_p for the same geometrical ratio

A phenomenon similar to the residual velocity was found for L_{pr}/L_p implying that L_{pr}/L_p remains constant for three cases in the same set. The ratio of D'_p/D_p was also seen to be the same for the three cases in the same set. Therefore, discussions about V_{bl} , L_{pr}/L_p and D'_p/D_p ensure that Eqs. (3.9) and (3.10) satisfy the scaling laws. The semi-

analytical model for residual velocity expressed by Eqs. (3.9) and (3.10) is further shown to be suitable for characterizing the perforation of the ceramic–metal armor impacted by blunt hard projectile.

3.4 Empirical model for ballistic limit velocity (BLV)

Optimization of armor plates dimensions is a typical issue for armors with constant materials. In such problems, the selections of optimum values of armor plates dimensions, such as T_1 and T_2 for a constant armor total thickness, $TT = T_1 + T_2$ or a constant areal density (mass per unit area), $AD = \rho_1 T_1 + \rho_2 T_2$ are of interest. In order to carry out optimization analysis, it is necessary to create an equation for BLV in which projectile and armor dimensions are independent variables.

From an engineering viewpoint, a new formulation for the BLV of alumina/aluminum armors impacted by a blunt tungsten projectile is suggested supported by the simulations. In the simulations, the BLV for each projectile–armor combination was estimated using least squares method based on a number of projectile residual velocity at different impact velocities and Lambert–Jonas equation [65] (Eq. (2.22)).

Prior discussions indicated that the BLV satisfies replica scaling laws. Therefore, BLV can be expressed as follows:

$$\frac{V_{bl}}{\sqrt{Y_p/\rho_p}} = g\left(\frac{T_1}{D_p}, \frac{T_2}{D_p}, \frac{L_p}{D_p}, \frac{D_1}{D_p}, \frac{D_2}{D_p}\right) \quad (3.16)$$

or in explicit form it can be written as:

$$\frac{V_{bl}}{\sqrt{Y_p/\rho_p}} = g_1\left(\frac{T_1}{D_p}, \frac{T_2}{D_p}\right) \cdot g_2\left(\frac{L_p}{D_p}\right) \cdot g_3\left(\frac{D_1}{D_p}, \frac{D_2}{D_p}\right) \quad (3.17)$$

where g_1 , g_2 and g_3 are three sub-functions presented for T_1/D_p and T_2/D_p combinations, L_p/D_p , D_1/D_p and D_2/D_p combinations, respectively. The term $\sqrt{Y_p/\rho_p}$ was introduced in Eq. (3.16) to give a dimensionless expression of V_{bl} and is equal to 292.5 m/s for the tungsten projectile (Y_p and ρ_p have the fundamental units of $M\tau^{-2}L^{-1}$ and ML^{-3} , respectively, in which M , τ and L are the fundamental units of

mass, time and length, respectively. So, $\sqrt{Y_p/\rho_p}$ has the fundamental unit of L/τ ($\sqrt{M\tau^{-2}L^{-1}/ML^{-3}} = \sqrt{L^2/\tau^2}$) which is the same as the fundamental unit of velocity).

For finding the form of the function g , numerical simulations were carried out. The projectile and armor materials were kept constant. The blunt projectile diameter, D_p was kept constant at 12 mm and other geometrical variables, namely, T_1 , T_2 , L_p , D_1 and D_2 were varied. When considering each sub-function, g_i the variables in other sub-functions were held constant.

3.4.1 Effect of T_1/D_p and T_2/D_p on BLV

Although T_1/D_p and T_2/D_p were considered simultaneously in sub-function g_1 , it is convenient to analyze the influence of each one, separately. Four series of cases were simulated to illustrate the influence of T_1/D_p and T_2/D_p on the BLV. L_p/D_p , D_1/D_p and D_2/D_p were kept constant as 3, 16.67 and 16.67, respectively. In the set 1, T_1/D_p varied from 0 to 3.33 for a constant T_2/D_p of 0.83. In the set 2, T_2/D_p varied from 0 to 3.33 for a constant T_1/D_p of 1.25. The BLV for the two sets are shown in Figure 3.7.

Figure 3.7 Variation of BLV with normalized front and backing plate thicknesses, T_1/D_p and T_2/D_p ($L_p/D_p = 3$, $D_1/D_p = D_2/D_p = 16.67$)

It can be observed that BLV increases non-linearly with increasing T_1/D_p and T_2/D_p for constant T_2/D_p and T_1/D_p , respectively. It is supposed that power-law functions relate the BLV to T_1/D_p and T_2/D_p . The determination of intercepts on the vertical axis varying with T_1/D_p and T_2/D_p are necessary for complete functional representation of g_1 . For this purpose, the impact response of monolithic alumina and aluminum plates with blunt tungsten projectile were simulated in sets 3 and 4. The T_1/D_p and T_2/D_p varied from 0 to 3.33. Figure 3.8 shows the BLV for these monolithic alumina and aluminum plates. It is confirmed that aluminum plate has higher BLV than monolithic alumina plate with the same thickness. It was supposed that the curves in Figure 3.8 can be also expressed using power functions.

Figure 3.8 BLV versus T_1/D_p and T_2/D_p for monolithic alumina and aluminum plate
 $(L_p/D_p = 3, D_1/D_p = D_2/D_p = 16.67)$

In summary, sub-function g_1 representing the variation of BLV with T_2/D_p and T_1/D_p is here approximated as:

$$g_1 \left(\frac{T_1}{D_p}, \frac{T_2}{D_p} \right) = a_1 \cdot \left(\frac{T_1}{D_p} \right)^{a_2} + a_3 \cdot \left(\frac{T_1}{D_p} \right)^{a_4} \left(\frac{T_2}{D_p} \right)^{a_5} + a_6 \cdot \left(\frac{T_2}{D_p} \right)^{a_7} \quad (3.18)$$

where a_1 to a_7 are fitting constants.

3.4.2 Effect of L_p/D_p on BLV

Consider the influence of L_p/D_p on BLV for determination of sub-function g_2 in Eq. (3.17). Five cases were simulated in which constant ratios of $T_1/D_p = T_2/D_p = 0.83$ and $D_1/D_p = D_2/D_p = 16.67$ were used and L_p/D_p were varied. BLV for five different L_p/D_p were determined and shown in Figure 3.9. It is obvious that BLV decreases with increasing L_p/D_p and can be supposed to be expressed using power function as:

$$g_2\left(\frac{L_p}{D_p}\right) = b_1\left(\frac{L_p}{D_p}\right)^{b_2} \quad (3.19)$$

where b_1 and b_2 are constants.

Figure 3.9 Variation of BLV with normalized projectile length, L_p/D_p
 $(T_1/D_p = T_2/D_p = 0.83, D_1/D_p = D_2/D_p = 16.67)$

3.4.3 Effect of D_1/D_p and D_2/D_p on BLV

The effects of D_1/D_p and D_2/D_p on BLV were considered and incorporated together in sub-function g_3 . D_1/D_p and D_2/D_p have different influences on the BLV of the ceramic-metal armor due to the different behaviors of ceramic and metallic materials under impact. Relevant discussions are presented below, in which D_1/D_p and D_2/D_p were both limited to larger than 1, which is the case for most engineering problems:

The ceramic material has low plasticity and most fragmentation takes place locally in the central cone area. The material out of the cone is supposed to have much less influence on the armor performance than that in the cone when the radial boundary is fixed. In order to verify this hypothesis, the BLV for one series of cases were determined using simulations shown in Figure 3.10. In these cases, the same projectile with $L_p/D_p = 3$ and same backing plate with $T_2/D_p = 0.83$ and $D_2/D_p = 16.67$ were used while the D_1/D_p (for the front plate) was varied.

Figure 3.10 BLV versus D_1/D_p ($L_p/D_p = 3$, $T_1/D_p = T_2/D_p = 0.83$, $D_2/D_p = 16.67$)

As shown in Figure 3.10, the BLV decreased linearly when D_1/D_p increased from 1 to a limit value $(D_1/D_p)_L$. This phenomenon is analyzed by comparing the impact processes of two cases with D_1/D_p of 1.83 and 10 under the same impact velocity. Two perforation sequence images at two different instances for the two cases are shown in Figure 3.11. At the instance of $33 \mu s$, as shown in Figure 3.11 (a), the backing plate deformations for the two cases are different and the deformation in the case with smaller D_1/D_p can restrict the material between projectile and backing plate to expand laterally more effectively compared to the one with larger D_1/D_p . This will result in keeping more fractured ceramic and projectile material in front of the intact projectile and produce a larger interaction area in the ceramic and backing plate interface as shown in

Figure 10 consists of two panels, (a) and (b), showing the evolution of a plasma jet. Panel (a) is at $t = 33 \mu s$ and shows a jet with a central core (green) and a surrounding sheath (blue). The jet is elongated and has a slightly irregular shape. Panel (b) is at $t = 129 \mu s$ and shows the jet after a perturbation, with a more complex, irregular structure. The jet is now more elongated and has a more pronounced central core. Both panels include labels for the diameter ratio $D_1/D_p = 1.83$ and $D_1/D_p = 10$.

The limit value of D_1 is marked in the numerical model of one case in Figure 3.12. In this case, D_1/D_p is 4.33 which is a little larger than $(D_1/D_p)_L$. It can be observed that this limit value is approximately equal to the maximum diameter of the fracture cone.

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was observed that the ratio of maximum BLV difference to the minimum constant BLV (BLV for data points larger than $(D_1/D_p)_L$) is 6% which is relatively small value implying that the effect of D_1 on the BLV is not very significant.

The backing metal plate can represent high plasticity under ballistic impact. It can deform in a region with large planar dimension especially at impact velocities close to BLV. The influence of monolithic metal plate planar dimension has been investigated by many researchers [130-133]. The BLV generally increases linearly with the increase of plate planar dimension until a certain value depending on the thickness of the plate [134]. Beyond this value, the effect of the plate planar dimension on the BLV diminishes. It is supposed in the present work that BLV increases (not necessarily linearly) with increase of D_2/D_p . One series of simulations were performed and their BLV are as shown in Figure 3.13. In these cases, the same projectile with $L_p/D_p = 3$ impacted on the front ceramic plate with same $T_1/D_p = 0.83$ and backing plate with $T_2/D_p = 0.83$. The D_2/D_p and D_1/D_p were the same and varied in a range larger than $(D_1/D_p)_L$ to eliminate the influence of D_1/D_p .

Figure 3.13 BLV versus D_2/D_p ($L_p/D_p = 3$, $T_1/D_p = T_2/D_p = 0.83$)

The solid curve in Figure 3.13, consists of two parts determined by fitting the simulation data. The two parts intersect at a D_2/D_p limit value, $(D_2/D_p)_L$. The dash curve was extrapolated by the same function used for the first part of the solid curve and has no practical significance because it is for D_2/D_p between 0 and 1. However, it illustrates the intercept of 0 on the vertical axis which is the BLV when D_2/D_p and corresponding D_1/D_p are both 0. The BLV increases with increasing D_2/D_p from 1 to a limit value $(D_2/D_p)_L$ and for the D_2/D_p larger than $(D_2/D_p)_L$, the BLV kept a constant maximum value. For all the values of D_2/D_p , the ratio of the maximum BLV difference to the maximum constant BLV is 24.5%. In addition, the value of $(D_2/D_p)_L$ was observed to be much larger than $(D_1/D_p)_L$. Therefore, it can be concluded that the D_2/D_p is more effective on BLV than D_1/D_p .

Considering the fact that D_1/D_p should not be larger than D_2/D_p and according to the variation trend of BLV with D_1/D_p and D_2/D_p , sub-function g_3 can be expressed as:

$$g_3\left(\frac{D_1}{D_p}, \frac{D_2}{D_p}\right) = \left[c_1 \frac{D_1}{D_p} + c_2 \left(\frac{D_2}{D_p}\right)^{c_3} \right] \cdot \left(\frac{D_2}{D_p}\right)^{c_4}, 1 < \frac{D_1}{D_p} \leq \left(\frac{D_1}{D_p}\right)_L, 1 < \frac{D_2}{D_p} \leq \left(\frac{D_2}{D_p}\right)_L \quad (3.20)$$

where c_1 to c_4 are fitting constants.

In Eq. (3.20), D_1/D_p and D_2/D_p are respectively replaced by $(D_1/D_p)_L$ and $(D_2/D_p)_L$ when they are respectively larger than $(D_1/D_p)_L$ and $(D_2/D_p)_L$. From the dimensional viewpoint, the $(D_1/D_p)_L$ and $(D_2/D_p)_L$ should be influenced by T_1/D_p , T_2/D_p and L_p/D_p . The relationship between the two former physical quantities and the three latter geometric parameters were not sought to be determined in the present work considering the fact that the determination of $(D_1/D_p)_L$ and $(D_2/D_p)_L$ needs a large number of additional data and the results are only suitable for the current armor considered in this work.

3.4.4 Determination of parameters for simplified empirical model

Function g can be obtained through curve fitting using least square method from BLV simulation data. In the fitting procedure, D_1/D_p and D_2/D_p were both chosen as 20 which ensure that D_1/D_p and D_2/D_p have no influence on BLV. There are two reasons for neglecting sub-function g_3 . The first one is that the maximum BLV differences are not large relative to BLV values, especially for D_1/D_p variation. The second reason is that the planar dimensions of armor are considered to be so large that its influence on BLV is little in general engineering problems. The obtained expression, from curve fitting of BLV simulation data (60 data points with various T_1/D_p , T_2/D_p and L_p/D_p) to function g (while sub-function g_3 was neglected), relating T_1/D_p , T_2/D_p and L_p/D_p to BLV of alumina–aluminum armor is given by:

$$\frac{V_{bl}}{\sqrt{\frac{Y_p}{\rho_p}}} = \left[0.44 \left(\frac{T_1}{D_p} \right)^{1.1} + 1.04 \left(\frac{T_1}{D_p} \right)^{0.68} \left(\frac{T_2}{D_p} \right)^{0.62} + 1.38 \left(\frac{T_2}{D_p} \right)^{0.44} \right] \cdot \left(\frac{L_p}{D_p} \right)^{-0.57} \quad (3.21)$$

The range of application for the Eq. (3.21) is as follows:

$$0 \leq \frac{T_1}{D_p} \leq 1.67, \quad 0 \leq \frac{T_2}{D_p} \leq 1.67, \quad 1.5 \leq \frac{L_p}{D_p} \leq 4.5 \quad (3.22)$$

In the following section, armor optimizations based on Eq. (3.21), for given total thickness, $TT = T_1 + T_2$ or areal density (mass per unit area), $AD = \rho_1 T_1 + \rho_2 T_2$ are discussed, assuming that Eq. (3.21) is valid over an extended range of TT and AD that is given by Eq. (3.22).

3.5 Optimization of the armor based on the proposed empirical model

The BLV of alumina–aluminum armor under blunt tungsten projectile impact were calculated using Eq. (3.21) for four different constant TT and AD , shown in Figure 3.14. The diameter and length of the projectile used in calculations are 12 mm and 36 mm, respectively.

The curves shown in Figure 3.14 (a) present the typical BLV variation with T_1/TT . Maximum BLV for a given TT (the peak points for each curve in Figure 3.14 (a)) corresponding to an optimum ratio of T_1/TT can be calculated.

Figure 3.14 BLV as a function of the (a) normalized thickness and (b) normalized areal density of the front layer

A polynomial curve, shown in Figure 3.14 (a) was fitted to the optimum points (maximum BLV and optimum ratio of T_1/TT) expresses as:

$$V_{bl} = 445690 \left(\frac{T_1}{TT} \right)^6 - 546090 \left(\frac{T_1}{TT} \right)^5 + 232662 \left(\frac{T_1}{TT} \right)^4 - 32245 \left(\frac{T_1}{TT} \right)^3 - 1863 \left(\frac{T_1}{TT} \right)^2 + 983 \left(\frac{T_1}{TT} \right) + 72 \quad (3.23)$$

For $T_1/TT = 1$ or 0 viz. the armor only consists of a monolithic alumina or aluminum layer, shown in Figure 3.14, Eq. (3.21) predicts higher BLV for monolithic aluminum layer compared to monolithic alumina layer with the same thickness. The same behavior was observed in the experiments of Mayseless et al. [135], for BLV of monolithic aluminum and alumina plates of same areal density (AD). By way of an example, when $TT = 15$ mm, the maximum BLV is 307 m/s and the optimum T_1/TT is 0.41 (corresponding to optimum front plate thickness, $T_1 = 6.15$ mm). The variation of maximum BLV and optimum T_1/TT with TT in the range of 0 – 40 mm, are shown in Figure 3.15 (a). The optimum values of T_1/TT increases with increase of TT and the

increasing trend becomes gradually less significant and nears a constant value. The maximum BLV was observed to increase linearly with increasing TT in the range considered.

Similar phenomena for BLV are observed in Figure 3.14 (b) and Figure 3.15 (b) when AD is kept constant. $AD_1 = \rho_1 T_1$, denotes the areal density of front plate, where ρ_1 and T_1 are density and thickness of the front plate, respectively. Same trend for BLV for alumina–aluminum armor with constant areal density, is observed in experiments performed by Lee and Yoo [136].

Figure 3.15 (a) Optimum T_1/TT and maximum BLV as a function of TT and (b) optimum AD_1/AD and maximum BLV as a function of AD

A polynomial curve was fitted to the optimum points (maximum BLV and optimum AD_1/AD) shown in Figure 3.14 (b) and its expression is given by:

$$V_{bl} = 635176.6 \left(\frac{AD_1}{AD} \right)^6 - 774855.5 \left(\frac{AD_1}{AD} \right)^5 + 346902 \left(\frac{AD_1}{AD} \right)^4 - 62328 \left(\frac{AD_1}{AD} \right)^3 + 2591 \left(\frac{AD_1}{AD} \right)^2 + 753 \left(\frac{AD_1}{AD} \right) + 91.8 \quad (3.24)$$

Consider the case of an areal density of $AD = 90 \text{ kg/m}^2$, a maximum BLV of 511 m/s is obtained corresponding to optimum front plate areal density, $AD_1 = 41.4 \text{ kg/m}^2$.

Although the constants in Eq. (3.21) are limited to the specific materials in the present work, the general form of the Eq. (3.21) and the conclusions drawn based on that, can be supposed to be used for BLV of ceramic–metal armors impacted by a hard projectile.

Optimization analysis for other fixed projectile–ceramic–metal systems can similarly be performed.

The optimization of alumina 95%–aluminum alloy 2024-T3 armor system impacted by hardened steel 4340 projectile will be discussed in Chapter 4.

3.6 Summary

A semi-analytical model relating the hard projectile residual velocity, BLV and impact velocity for ceramic–metal armor was presented. It was shown that the proposed model agrees well with the numerical simulations results that have been verified by experiments. Dimensional analysis was used together with numerical simulations to show that, for a given ceramic–metal armor with the same geometric ratios, projectile residual velocity and BLV of the armor remain constant, viz. they satisfy replica scaling laws. Based on the verified numerical simulations, the influence of armor and projectile geometries on the BLV was discussed and an empirical equation considering the effects of those variables was proposed. The effects of large variations of D_1/D_p and D_2/D_p on the BLV are smaller than those of T_1/D_p , T_2/D_p and L_p/D_p . Based on the replica scaling laws of projectile residual velocity and BLV, an empirical equation for BLV was obtained which was used for alumina–aluminum armor optimization. The empirical BLV equation can be used for alumina–aluminum armor optimization and its form can be assumed to be suitable for general ceramic–metal armor under blunt hard projectile impact.

Chapter 4 Validation of Semi-Analytical Model

In chapter 3, a semi-analytical model (Eqs. (3.9) and (3.10)) was proposed for the impact of ceramic–metal bi-layer armor system impacted by hard blunt projectile, relating residual velocity, V_r impact velocity, V_0 and ballistic limit velocities (BLV), V_{bl} . A series of tests were planned for validating the proposed model. Experiments details along with verified simulations are explained in the ensuing section. The results obtained are then discussed with reference to the effect of front plate and backing plate thickness on the residual length and velocity of the projectile. Finally the optimal thickness and mass designs are explored for the tested material system.

4.1 Experimental work

Hardened steel 4340 projectiles of 52 HRC hardness, 7.56 mm nominal diameter and 30.54 nominal length were used in the tests. Alumina 95% ceramic tiles of 100 mm×100 mm planar dimension were bonded, using Hysol EA 9309.3NA adhesive, to aluminum alloy 2024-T3 backing plates of 160 mm×160 mm planar dimension. Figure 4.1 shows the setting of the target system.

Aluminum plates were cut from two big plates of 6.2 mm×1250 mm×2500 mm and 10 mm×900 mm×2000 mm dimensions and then milled off to different required thicknesses. The thicknesses of the aluminum plates after milling were measured using ultrasound technique at four different sections and their average values were considered as the thickness of the plates. Four rectangular steel confinements were used for fully clamping the four sides of the ceramic tile. An aluminum plate was put at distance from back of the target as “projectile collector”. Since the projectile is very hard its length will not change considerably after impacting this aluminum plate. This fact was proven by comparing the measured residual length obtained from images and the one obtained from “collector plate”. A two-stage gas-gun was used for firings. A problem in the test set-up was the presence of yaw angle after firing of the projectile which is very difficult to control and eliminate, as it depends on many parameters like length to diameter ratio of the projectile, pressure of each stage of the gas gun, vacuum pressure, etc.

Figure 4.1 Setting of the target system

An eight channels 150 KV Field Emission (FE) flash X-ray system was used for taking images of the impact event. Four of the eight channels, herein called channels 1, 2, 3 and 4, were used to take image in a plane, herein called plane 1. The other four channels, channels 5, 6, 7 and 8, were used to take image in a plane, herein called plane 2, which is perpendicular to the plane 1. Therefore, two-dimensional (2D) images can be obtained in two perpendicular planes out of which the yaw angle, if any, could be measured.

As the impact velocity, residual length and residual velocity of the projectile were required for current research to be measured from the taken images, channels 1, 2, 5 and 6 were used to take image of the projectile before impacting the target and channels 3, 4, 7 and 8 were used to take image of the projectile after perforating the target. Two circuits, made of thin aluminum foils carefully wrapped with two pieces of papers, were connected to pulse generators as triggers, as shown in Figure 4.1, for X-ray recording system. One of the triggers was taped on the ceramic tile face. Once the aluminum foil circuit is impacted and broken, a trigger pulse is generated.

The difficulty in taking image was in setting the delay time, which was calculated based on guessed impact and residual velocity, for different channels of the X-ray system. The impact and residual velocity were estimated intuitively considering the applied pressure of the gas gun and the thickness of the membrane used in between the first and second

stage tubes of the gas gun. Therefore, it happened sometimes that no images were taken due to inappropriate time delay setting for the channels.

Figure 4.2 shows details of test No. 14. The two X-ray images in planes 1 and 2 are shown in Figure 4.2 (e) and (f).

Figure 4.2 Test No. 14: (a) ceramic front tile after impact, (b) projectile and aluminum plug separated from backing plate beside each other, plug placed on top of the projectile, front and rear view of the plug, respectively from left to right, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by channels 1, 2, 3, 4 and (f) X-ray image taken by channels 5, 6, 7, 8

As seen in Figure 4.2 (e), two images were taken by channels 1 and 2, but no images were taken by channels 3 and 4. The bright strip behind the target in the X-ray images is due to occurrence of many exposures in the image taking system.

In the cases that no image was taken by X-ray system, projectile length was measured after taking out the projectile from the “collector plate”. In some cases, like in test No. 20 and 30, as shown in Figure A.6 (b) and Figure A.12 (b), respectively, in Appendix A,

the projectile was fractured during penetration. Therefore, the total measured length of the projectile was considered as the sum of the length of the two parts of the projectile. Details of the tests including ceramic tile thickness, T_1 , aluminum plate thickness, T_2 , projectile diameter, D_p , length, L_p , and mass, M_p , yaw angles of the projectile before impact in plane 1, θ_1 , and in plane 2, θ_2 , impact velocity, V_0 are listed in Table 4.1. As shown in Figure 4.3 (a) and (b), if the projectile head before impacting the target is higher than normal impact line, the yaw angle is considered as positive (+ve) and otherwise is negative (-ve). For test No 14, as shown in Figure 4.2 (e) and (f), both yaw angles are considered as negative. For test No. 12 shown in Figure 4.3 (a) and (b), θ_1 is negative while θ_2 is positive. For the other tests, same method was applied for denoting the yaw angle in Table 4.1.

Table 4.1 Details of the experiments

Test No.	T_1 (mm)	T_2 (mm)	D_p (mm)	L_p (mm)	M_p (g)	θ_1	θ_2	V_0 (m/s)
7	6.1	6	7.56	30.54	10.65	+6.3°	+6.5°	650
9	6.12	6	7.56	30.48	10.63	+2.9°	+7.3°	655
12	6.14	6.03	7.56	30.54	10.65	-10°	+5.6°	830
13	6.1	5.87	7.56	30.54	10.65	-9.9°	+1.1°	712
14	6.04	5.96	7.56	30.52	10.64	-5°	-4.1°	493
16	6.16	5.9	7.56	30.54	10.65	-4°	+0.9°	820
17	8.96	6	7.56	30.54	10.65	-5.2°	-1.8°	775
18	8.98	5.97	7.56	30.48	10.63	0°	+5.2°	845
20	9.04	6	7.56	30.54	10.65	-9°	+4.1°	676
21	6.12	4.07	7.56	30.54	10.65	+2.5°	-1.5°	819
22	6.14	5.4	7.56	30.45	10.62	-5.5°	+12.9°	786
23	6.14	6.88	7.54	30.58	10.61	-8.4°	+1.3°	860
24	6.12	8.25	7.56	30.54	10.65	+1.1°	+0.2°	834
29	9.04	5.91	7.56	30.54	10.65	-6.8°	-0.9°	948
30	9.08	5.94	7.56	30.54	10.65	-4°	+3.3°	982
31	9.04	5.87	7.56	30.45	10.62	-6.1°	-7.3°	1062

It should be noted that in proposing semi-analytical model for BLV in Chapter 3, normal impact of the bi-layer ceramic-metal armor was considered whereas the practical experiments had yaw. Therefore, the strategy in this chapter is to find reliable numerical model with appropriate material constitutive models after matching the simulation results of residual projectile velocity and length with experimental measurements. The results obtained from numerical model simulations for normal impact cases and

experiments with negligible yaw angle are then used for validation of the semi-analytical model.

4.1.1 Numerical modelling of the experiments and comparison of the results

Three-dimensional (3D) simulations of the tests (with yaw angles for projectile and other details of the armor target) were performed using AUTODYN[®]. The projectile was modelled using SPH particles and front and backing plate were discretized using Lagrangian elements. Planar size of 150 mm×150 mm was used for both front and backing plate. The central planar square zone of 60 mm was modelled using element size of 0.6 mm. The element size is gradually increased toward the periphery of the plates. The element size of 0.6 mm or close to 0.6 mm in the thickness direction were used for different plate thicknesses as all the plate thicknesses were not divisible by 0.6 mm element size. The particle size of 0.65 mm was used for projectile in all the simulations. Instantaneous geometric erosion strain of 1 was used for both front and backing plate in simulations. Figure 4.3 shows material location for test No. 12 in planes 1 and 2 and meshing of the target.

Figure 4.3 Numerical model for test No. 12: (a) material location in plane 1, (b) material location in plane 2 and (c) meshing of the target

The four sides at the periphery of the front and backing plates were clamped (zero displacement and rotation in three directions). The front and backing plate were tied using “JOIN module” in AUTODYN[®]. The alumina front plate in the experiments in which the yaw angle was not very big, was not shattered. This can be seen in Figure 4.2 (a). Since the front plate is not shattered during impact and is adhered to the backing

plate through the impact event, joining the front and backing plate is valid assumption. The Johnson-Holmquist-2 (JH-2) model constants for Al₂O₃ 95% proposed by Zhenqi [137] were used in simulations for ceramic front tiles, listed in Table 4.2.

Table 4.2 Material model constants for Al₂O₃ 95% [137]

Constants with units	
Density, ρ_0 (kg/m ³)	3741
<u>EOS</u>	<u>Polynomial</u>
Bulk modulus, K (GPa)	184.56
Pressure constant, K_2 (GPa)	185.87
Pressure constant, K_3 (GPa)	157.54
<u>Strength model</u>	<u>JH-2</u>
Shear modulus, G (GPa)	120.34
Hugoniot elastic limit (HEL) (GPa)	6
Intact strength constant, A	0.889
Intact strength exponent, N	0.764
Strain rate constant, C	0.0045
Fracture strength constant, B	0.29
Fracture strength exponent, M	0.53
Normalized maximum fractured strength, $\sigma_{f_{max}}^*$	1
<u>Fracture model</u>	<u>JH-2</u>
Normalized hydrostatic tensile limit, T^* (GPa)	-0.3
Damage constant, d_1	0.005
Damage exponent, d_2	1
Bulking factor, β	1

The JC constitutive and fracture model parameters used for aluminum alloy 2024-T3 and hardened steel 4340 are listed in Table 4.3. Details of the JH-2 and JC models can be found in section 2.3. The residual velocity, V_r and length, L_{pr} of the projectile, major and minor axis length of the elliptical hole at the rear of the aluminum backing plate, a_1 and a_2 (see Figure 4.2 (c)) in experiments (Exp.) and the ones obtained from simulations (Sim.) are listed in Table 4.4. Residual velocities in test No. 7, 12, 14, 16 and 30 could not be measured as no images were taken by X-ray system after impact. As revealed in Table 4.4, the results obtained from simulations except case No. 20 are in fairly good agreement with the ones of experiments. The reason for difference of 106 m/s between residual velocity of simulation and experiment is due to the typical behavior of armors near their ballistic limit.

Table 4.3 Material model constants for aluminum 2024-T3 [138, 139] and steel 4340 [122, 140]

Constants with units	Al 2024-T3	Steel 4340
Density, ρ_0	2785	7770
<u>EOS</u>	<u>Shock</u>	<u>Linear</u>
Bulk modulus, K (GPa)	-	159
Gruneisen constant	2	-
Parameter C_1 (m/s)	5328	-
Parameter S_1	1.338	-
Specific heat, C_r (J/Kg·K)	874.9	477
<u>Strength model</u>	<u>JC</u>	<u>JC</u>
Shear modulus, G (GPa)	26.92	77
Static yield strength, A' (GPa)	0.167	0.950
Strain hardening constant, B' (GPa)	0.596	0.725
Strain hardening exponent, n	0.551	0.375
Strain rate constant, C'	0.001	0.015
Thermal softening exponent, m	0.859	0.625
Melting temperature, t_m (K)	893	1793
Reference strain rate, $\dot{\epsilon}_0$	1	1
<u>Fracture model/criterion</u>	<u>JC</u>	<u>JC</u>
Damage constant, d'_1	0.112	-0.8
Damage constant, d'_2	0.123	2.1
Damage constant, d'_3	1.5	-0.5
Damage constant, d'_4	0.007	0.002
Damage constant, d'_5	0	0.61

Table 4.4 Comparison of results obtained from simulations and experiments

<i>Test No.</i>	V_0 (m/s)	V_r (Exp.) (m/s)	V_r (Sim.) (m/s)	L_{pr} (Exp.) (mm)	L_{pr} (Sim.) (mm)	a_1 (Exp.) (mm)	a_1 (Sim.) (mm)	a_2 (Exp.) (mm)	a_2 (Sim.) (mm)
7	650	-	341	23.48	23.80	14	13.70	14.5	14.25
9	655	351	364	23.36	24.10	14	13.57	16	13.84
12	830	-	559	22.44	23.23	16	14.52	16.5	14.60
13	712	430	406	24.52	24.27	17	15.65	19	17.51
14	493	-	112	25.86	25.83	12	13.16	12.5	12.58
16	820	-	581	24	23.98	15.5	16.55	15	16
17	775	370	333	20	20.70	14	13.38	13.5	14.66
18	845	463	444	21.16	20.54	13	13.50	14	13.70
20	676	0	106	23.70	22.02	-	14.25	-	14.56
21	819	649	636	24.78	25.72	16.5	14.35	17.5	15.30
22	786	532	534	23	23.40	17.5	15.70	19	15.46
23	860	580	601	20.38	21.20	18	16.40	16	17.50
24	834	532	539	23.32	23.40	13	14.97	14	15.13
29	948	605	580	19.38	20.20	19	17.13	17	18.71
30	982	-	637	19.90	20.10	15	16.45	16	17.92
31	1062	772	732	21.56	20.68	19.5	17.30	19	16.72

Figure 4.4 shows the bulged rear side of the backing plate in experiment No. 20 implying that the projectile did not perforate the backing plate. Therefore, it is expected that impact velocity in this experiment is near the BLV of the target. It was observed [141-144] that near ballistic limit of different armor systems, the tangent of residual velocity as a function of impact velocity is very steep, implying that increase of residual velocity is much faster than increase of the impact velocity. This fact was also observed in current simulations: if the impact velocities in two different simulations for the case No. 20 (with details specified in Table 4.1) are set as 655 m/s and 660 m/s, then the residual velocity will be 0 and 64 m/s, respectively. It is clear that the impact velocity of 655 m/s in simulations corresponding to zero residual velocity is close to the experimental one of 676 m/s for case No. 20. It will be shown in section 4.2.1 that the BLV for the armor system with front plate of 9 mm thickness and backing plates of 6 mm thickness under normal impact is 590 m/s.

Figure 4.4 Bulged side of the aluminum backing plate in test No. 20: (a) rear view (b) top view

Finally, it can be concluded that the numerical model reliably matches experimental results. Therefore, the developed numerical model can be used for normal impact simulations for verification of proposed semi-analytical model in Chapter 3.

4.2 Validation of the semi-analytical model

4.2.1 Comparison of analytical model, simulation and experimental results

Two sets of the targets with front alumina 95% ceramic plate thicknesses and aluminum alloy 2024-T3 backing plate normally impacted with blunt steel projectile of 7.56 mm

diameter and 30.54 mm length were simulated. As can be seen in Figure 4.2 (b) and Figure A.10 (b) (in Appendix A), the diameter of the plug separated from the aluminum backing plate is in the range of 11 mm to 12.5 mm which, in other words, is in the range of $1.45D_p$ to $1.71D_p$. Therefore, based on this observation, for current projectile–armor system, revised value of $1.5D_p$ (instead of $1.1D_p$ used in Chapter 3) is assumed as the diameter of the ceramic front and aluminum backing plate for calculating the total plate plugs mass, M_t in Eq. (3.11). The thicknesses of the plugs are assumed as the initial thickness of the plates; the same assumption was used in Eq. (3.11).

Details of the results obtained from simulations and semi-analytical model (Eqs. (3.9) and (3.10)) are listed in Table 4.5. Percentage of the error between residual velocities obtained from the model and simulations are also included in Table 4.5.

Table 4.5 Projectile residual length and velocity obtained from simulations and residual velocity obtained from semi-analytical model for two armor systems of 6 mm and 9 mm front plate and fixed 6 mm backing plate thicknesses

Set No.	T_1 (mm)	T_2 (mm)	V_0 (m/s)	L_{pr} (Sim.) (mm)	V_r (Sim.) (m/s)	V_r (Model) (m/s)	Error % for V_r
S ₁	6	6	435 (BLV)	26.37	0	0	0
S ₂	6	6	440	26.32	61	51	16.4
S ₃	6	6	493	25.81	158	180	13.9
S ₄	6	6	575	25.18	275	293	6.5
S ₅	6	6	655	24.96	384	383	0.2
S ₆	6	6	712	24.90	457	442	3.3
S ₇	6	6	820	24.55	580	547	5.7
S ₈	6	6	950	24.536	732	665	9.1
S ₉	6	6	1200	23.471	994	875	11.9
S ₁₀	9	6	590 (BLV)	24.1	0	0	0
S ₁₁	9	6	595	24.07	75	56	25.3
S ₁₂	9	6	650	23.27	212	201	5.2
S ₁₃	9	6	775	21.64	383	379	1
S ₁₄	9	6	845	21.20	467	459	1.7
S ₁₅	9	6	948	21.11	603	564	6.4
S ₁₆	9	6	982	20.93	645	598	7.3
S ₁₇	9	6	1062	20.75	738	674	8.7
S ₁₈	9	6	1200	20.556	912	801	12.2
S ₁₉	9	6	1300	20.293	1018	879	13.6

The simulation sets S₃, S₅, S₆ and S₇ in Table 4.5 are respectively counterparts for test No. 14, 9, 13 and 16 given in Table 4.1. The simulation sets S₁₃, S₁₄, S₁₅, S₁₆ and S₁₇ are respectively counterparts for test No. 17, 18, 29, 30 and 31 given in Table 4.1. Besides numerical simulations were performed on set No. S₂, S₄, S₈, S₉, S₁₈ and S₁₉ for the

purpose of adding more data points to the $V_r - V_0$ curves and set No. S₁ and S₁₀ are simulated for finding BLV of the two target sets. In order to find the BLV, impact simulations with impact velocities of 5 m/s interval were performed to find the maximum impact velocity such that the residual velocity of the projectile is zero and the head of the projectile does not come out from rear of the backing plate. The material models used are the same as those listed in Table 4.2 and Table 4.3 and same planar size and element size for the target and particle size for the projectile as described in section 4.1.1 was used in the simulations.

In analytical model calculations, the conditions expressed in Eqs. (3.9) and (3.10) were checked to see which equation should be used. Therefore, Eq. (3.10) was used for the sets S₉ and S₁₉ and for all the other sets Eq. (3.9) is used for calculating the residual velocity, V_r .

Figure 4.5 shows the comparison between semi-analytical model and simulations for the variation of the residual velocity of the projectile with impact velocity for the two armor system sets. Experimental data points with yaw (not exactly normal impacts) for armor of nominal thicknesses of 9 mm and 6 mm for front and backing plate, respectively, are also included in Figure 4.5 (see Table 4.1 for exact values of thicknesses and yaw angle). The inclusion of these experimental measurements is not for the sake of direct comparison, however to show that experimental measurements, though having minor yaw, lie on a curve close to the one of the semi-analytical model and simulations for normal impact cases.

It can be seen that the results obtained from the proposed model correlate well with those of simulations and also experimental measurements, in the velocity range below 1000 m/s. For higher velocities, the results of the model start to deviate from simulations. The reason is probably due to the simplifications in the semi-analytical model such as considering shear plugging separation mechanism for both front and backing plate with same diameter which may not be suitable for higher velocity ranges.

Figure 4.5 Comparison between semi-analytical model and simulations for variation of residual velocity of the projectile with impact velocity for two ceramic-metal armor systems of 6 mm and 9 mm front plate thicknesses and fixed 6 mm backing plate thicknesses

4.2.2 Comparison with experimental work of Anderson Jr [145]

Anderson Jr [145] performed experiments on two bi-layer armor systems composed of boron carbide (B_4C) front plate of 2500 kg/m^3 density and 5.08 mm and 6.35 mm thicknesses and aluminum alloy 6061-T6 backing plate of 2700 kg/m^3 density and 6.35 mm fixed thickness impacted by armor-piercing (APM2) projectile with initial mass and length of 10.74 g and 35.3 mm, respectively. APM2 projectile has an ogive nose consists of a brass jacket, lead filler and a hard still core. Though the proposed semi-analytical model is for blunt projectile which is different from APM2 projectile, the purpose of the comparison is to see the trend of the two results. Figure 4.6 shows comparison between the results obtained from semi-analytical model and experiments performed by Anderson Jr [145]. It is assumed in the calculations of the analytical model that projectile nose becomes blunt after impact and diameter of the aluminum plug is considered as 1.5 times the diameter of the projectile. The same value for the aluminum plug diameter was considered in section 4.2.1. Impact and residual velocities together with residual length of the projectile for different cases were extracted from the images of the work of Anderson Jr [145], as shown in Figure A.14 in Appendix A. It should be noted that extracted residual lengths were not very accurate and has some errors due to digitization of data from the image, nonetheless it suffices for the purpose

of comparison of the results. It can be seen that within the experimental scatter and considering the sources of error for extracting data from the images and also difference between projectile nose shapes used in the experiments and considered in semi-analytical model, the measured residual velocities are in reasonable agreement with the proposed semi-analytical model.

Figure 4.6 Comparison between results of current semi-analytical model and experiments of Anderson Jr [145] for two bi-layers B_4C – aluminum 6061-T6 armor systems of 5.08 mm and 6.35 mm ceramic plate thickness and fixed backing plate of 6.35 mm thickness impacted by APM2 projectile

4.3 Effect of front plate and backing plate thicknesses on residual length and velocity of the projectile

In this section, effect of front ceramic and metallic backing plate thicknesses on residual length and velocity of the projectile are discussed based on the simulation results. The material model constants used are the same as those listed in Table 4.2 and Table 4.3 and same planar size and element size for the target and particle size as described in section 4.1.1 was used in the simulations.

Effect of backing plate thickness on residual velocity and residual length of the projectile for two ceramic-metal armor systems with front plate of fixed 6 mm and 9 mm thicknesses under fixed normal impact velocities of 820 m/s and 982 m/s, respectively, together with relevant experimental data points are shown in Figure 4.7. It

can be seen that residual velocity and length of the projectile both decrease as the backing plate thickness is increased.

Figure 4.7 Variation of (a) residual velocity and (b) residual length of the projectile with backing plate thickness for two ceramic-metal armor systems with front plate of 6 mm and 9 mm thicknesses under noraml impact velocities of 820 m/s and 982 m/s, respectively, together with some experimental measurements

The synergistic roles of ceramic front and backing plate has been the reason of success and their long term use in armor protective industry in defeating hard projectiles. The ceramic front plate due to its high hardness and compressive strength erodes and/or deforms the head of the projectile. On the other hand, backing plate will support the ceramic front plate, keeping the fractured ceramic in its place and delaying its premature failure in tension, therefore, projectile will face the ceramic front for longer time. Besides, backing plate will be effective in decreasing the remnant kinetic energy of the projectile, since its hardness is usually less than the ones of hard projectile. In the normal impact of armor system with fixed ceramic front plate thickness, the reason for continuous decrease of the residual length of the projectile with increase of the backing plate thickness is that as the thickness of backing plate increases, ceramic material has a stronger support in its rear side and therefore projectile will be eroded and/or deformed for longer time. It is clear that with increase of the backing plate thickness, the residual velocity of the projectile is decreased, as shown in Figure 4.7 (a).

Effect of front plate thickness on residual velocity and residual length of the projectile for two ceramic-metal armor systems with backing plate of fixed 6 mm and 9 mm thicknesses under fixed noraml impact velocities of 820 m/s and 982 m/s, respectively, are shown in Figure 4.8. It can be seen that residual velocity and length of the projectile

both decrease as the front plate thickness is increased. It is known that upon impact the compressive wave propagates through the thickness of the ceramic plate and is reflected back as tensile wave from the interface of ceramic-metallic backing plate, thereby damaging the ceramic under tension loading. As the front plate thickness increases (while the backing plate thickness is fixed) it takes longer time for the compressive wave to reach the rear side of the ceramic plate, therefore, ceramic plate keeps its intact form for a longer time and continues to erode the projectile for longer time, thereby continuously decreasing the residual velocity and length of the projectile, as shown Figure 4.8 (a) and (b).

Figure 4.8 Variation of (a) residual velocity and (b) residual length of the projectile with front plate thickness for two ceramic-metal armor systems with backing plate of 6 mm and 9 mm thicknesses under noraml impact velocities of 820 m/s and 982 m/s, respectively

Temporal variation of the residual length of the projectile for armor systems of 6 mm fixed front ($T_1 = 6$, variable backing plate thickness) and fixed backing plate ($T_2 = 6$, variable front plate thickness) thicknesses under noraml impact velocity of 820 m/s are shown in Figure 4.9 (a) and (b), respectively. It can be seen that armor systems with fixed front plate thickness of 6 mm and backing plate thicknesses of 0 mm, 1 mm and 4 mm in Figure 4.9 (a) have lower final residual length compared to their counterparts (with fixed backing plate thickness of 6 mm thickness and front plate thicknesses of 0 mm, 1 mm and 4 mm) in Figure 4.9 (b). It should be noted that the lower residual length of a projectile impacting an armor implies that the armor comparatively is stronger. The ratio, T_1/TT (ratio of the front plate thickness, T_1 over the total thickness, TT) for armor systems with fixed front plate thickness of 6 mm and backing plate thicknesses of 0

mm, 1 mm and 4 mm are 0, 0.86 and 0.6, respectively, compared to that of 0, 0.14 and 0.4 for their counterparts with fixed backing plates. However, armor systems with fixed front plate thickness of 6 mm and backing plate thicknesses of 10 mm, 15 mm and 20 mm in Figure 4.9 (a) (corresponding to T_1/TT ratios of 0.38, 0.29 and 0.23) have longer final residual length compared to their counterparts with fixed backing plates in Figure 4.9 (b) (with T_1/TT ratios of 0.62, 0.71 and 0.77).

Figure 4.9 Temporal variation of residual length of the projectile for ceramic-metal armor systems with (a) fixed front plate of 6 mm thickness and variable backing plate thicknesses and (b) fixed backing plate of 6 mm thickness and variable front plate thicknesses under normal impact velocity of 820 m/s

By way of an example, for armor of 6 mm front plate thickness and 15 mm backing plate thickness ($TT = 21$), the reduction of the length of the projectile (to the length of 20.7 mm) ends after $36 \mu s$ from the beginning of the impact while for armor of 15 mm front plate thickness and 6 mm backing plate thickness the reduction of the length of the projectile (to the length of 18.7 mm) ends $48 \mu s$, implying that the latter is stronger as it hinders the projectile for a longer time resulting in shorter length and lower kinetic energy of the projectile. The same behavior is seen for armor systems of 9 mm fixed front ($T_1 = 9$, variable backing plate thickness) and fixed backing plate ($T_2 = 9$, variable front plate thickness) thicknesses under normal impact velocity of 982 m/s, shown in Figure 4.10 (a) and (b), respectively. Armor systems with fixed front plate thickness of 9 mm and backing plate thicknesses of 0 mm, 4 mm and 6 mm in Figure 4.10 (a) have lower final residual length compared to their counterparts with fixed backing plate thickness in Figure 4.10 (b). However, armor systems with fixed front plate thickness of

9 mm and backing plate thicknesses of 12 mm and 20 mm in Figure 4.10 (a) have longer final residual length compared to their counterparts in Figure 4.10 (b).

Figure 4.10 Temporal variation of residual length of the projectile for ceramic-metal armor systems with (a) fixed front plate of 9 mm thickness and variable backing plate thicknesses and (b) fixed backing plate of 9 mm thickness and variable front plate thicknesses under normal impact velocity of 982 m/s

Therefore, as discussed in Chapter 3, there exists an optimum ratio T_1/TT for a given TT (or AD_1/AD for a given areal density, AD) such that the armor is the strongest.

In the next section, optimization of alumina 95%–aluminum alloy 2024-T3 armor system impacted by hardened steel 4340 projectile under normal impact is discussed.

4.4 Optimization of alumina 95%–aluminum 2024-T3 armor system

Optimization of alumina 95%–aluminum alloy 2024-T3 armor system with total thicknesses (TT) of 2 mm, 12 mm and 15 mm and areal densities (AD) of 6.39 kg/m², 22.28 kg/m² and 50.46 kg/m² under normal impact of hardened blunt steel 4340 projectile were performed using AUTODYN[®] numerical simulations. Same planar size and element size for the target and particle size for the projectile as described in section 4.1.1 was used for TT of 12 mm and 15 mm and AD of 22.28 kg/m² and 50.46 kg/m². For TT of 2 mm and areal AD of 6.39 kg/m², element size of 0.3 mm to 0.6 mm (depending on the thicknesses of the front or backing plates) for the armor target and particle size of 0.5 mm for the projectile was used in the simulations.

It should be borne in mind that analysis of armors with small TT or AD , though seemingly not very practical, is helpful to find the optimum design of the armor and study its behavior in a broader range.

The BLV for each armor was found from impact simulations with impact velocities of 5 m/s interval for armors with $TT = 12$ mm, $TT = 15$ mm and $AD = 50.46$ kg/m²; the interval of 2 m/s was used for armors with $TT = 2$ mm, $AD = 6.39$ kg/m² and $AD = 22.28$ kg/m². In the region of maximum BLV for any given TT or AD , impact velocities of 1 m/s or 2 m/s interval were used in order to rigorously find the maximum BLV. The results of the simulations shown in Figure 4.11 (a) and (b) are the variations of the ballistic limit velocity (BLV) with T_1/TT for three given TT values and AD_1/AD for three given AD values, respectively. It can be seen that for any given TT or AD , there exists an optimum ratio T_1/TT corresponding to a maximum BLV: for given TT of 2 mm, 12 mm and 15 mm, the optimum ratios of T_1/TT are 0.25, 0.52 and 0.57, respectively; For given AD of 6.39 kg/m², 22.28 kg/m² and 50.46 kg/m², the optimum ratios of AD_1/AD are 0.29, 0.62 and 0.62, respectively; (the three optimum ratios of AD_1/AD correspond to optimum T_1/TT ratios of 0.24, 0.55 and 0.55, respectively).

Figure 4.11 Variation of the BLV with (a) T_1/TT for three given TT and (b) AD_1/AD for three given AD

It is clear that for small TT and AD , the optimum normalized ceramic plate thickness and areal density ratios are small and for larger TT and AD , they are close to each other, nearing a constant value of 0.55 and 0.62, respectively. The same behavior was

observed in section 3.5. The reason for this variable trend can be found from the experimental work of Mayseless et al. [135].

Figure A.15 in Appendix A, from the experimental work of Mayseless et al. [135], shows the variation of ballistic limit velocity (BLV) with areal density (AD) for five armor systems, namely, monolithic metal armor, aluminum alloy 2024-O, monolithic ceramic armor, alumina 85%, bi-layer alumina 85%–aluminum alloy 6061-T6 armor, bi-layer alumina 85%–steel SAE 4130 armor and bi-layer alumina 85%–aluminum alloy 2024-O armor, impacted normally by a 12.7 mm diameter hard-steel projectile with 60° conical tip. It can be seen in Figure A.15 that at low impact velocities (small AD), monolithic aluminum alloy 2024-O armor, has higher BLV than the other four armor with same areal densities. Mayseless et al. [135], based on their experimental results shown in Figure A.15, concluded that at low impact velocities, a monolithic metallic plate can be more efficient than either monolithic ceramic plate or some bi-layer ceramic-metal armor system. It implies that in small AD (or TT) range, the ceramic plate is not so effective as a metallic plate. Physical interpretation for this fact is that, in small AD (or TT) range, ceramics easily disintegrate without absorbing much energy compared to metals which can absorb more kinetic energy of the projectile. Therefore, in the small range of AD (or TT), there exists an optimum monolithic metallic armor system or bi-layer ceramic–metal armor system composed of thinner ceramic front plate, T_1 and comparatively thicker backing plate, T_2 thicknesses.

For the reason mentioned above, the optimum ratios of T_1/TT and AD_1/AD , respectively for bi-layer armors of $TT = 2$ mm and $AD = 6.39$ kg/m², are considerably smaller than those of armors with larger TT and AD .

In larger AD (or TT) range, however, ceramic plate, is more effective, as it has the support of the metallic backing plate and even after disintegration, it is kept in its place; therefore, the synergistic role of the ceramic front and metallic backing plates results in blunting the projectile and reducing its kinetic energy.

4.5 Summary

A series of tests were conducted for examining the semi-analytical model proposed in Chapter 3. Since there was the problem of yaw in the experiments, an attempt was made to simulate the experiments performed and after finding reliable material and numerical model parameters, normal impact simulations were carried out. Based on the validated numerical model, normal impact simulations were performed and the new results were accordingly presented. The results obtained from semi-analytical model are in good agreement with the ones of simulations and also experimental measurements.

The proposed semi-analytical model is of great importance. Based on these results, a new method can be proposed for quick estimate of ballistic limit velocity (BLV) of the bi-layer ceramic–metal armor systems with acceptable accuracy: by only performing two or three penetration tests and measuring impact and residual velocities of the projectile and its residual length and then substituting the measured values in the proposed semi-analytical equations (Eqs. (3.9) and (3.10)) to find the best BLV (which satisfies the equation). In this way it is possible to avoid the large expenses and tedious repetitions of experiments to find the BLV of bi-layer armor.

Based on the validated numerical model, effect of front plate and backing plate thickness on residual length and velocity of the projectile was studied. Finally, optimization of bi-layer alumina 95%–aluminum alloy 2024-T3 armor system was numerically performed and a discussion based on the available literature was presented.

Chapter 5 A Generalized Empirical Model for the Ballistic Limit of Bi-Layer Ceramic–Metal Armor*

In this chapter, a generalized empirical model to estimate the BLV is proposed for bi-layer ceramic–metal armor systems impacted by a flat-ended (blunt) tungsten projectile, considering momentum and energy balance during impact process as a function of geometrical and material parameters. The proposed model is supported by numerical simulations performed using AUTODYN[®] [47]. Three ceramic materials as front layer, namely: alumina (Al_2O_3 96%), boron carbide (B_4C) and silicon carbide (SiC) and three materials as backing layer, namely: aluminum alloy (Al 5083H116), steel 4340 and titanium alloy (Ti6Al4V) were considered in the numerical study. Material constitutive and fracture model constants are first validated [146] through comparison of simulations, performed in AUTODYN[®] and available experimental data in literature. Impact simulations are performed for different combinations of armor geometries and material systems with different projectile lengths; the combination of these variables are chosen based on orthogonal array approach [147]. The constants in the empirical model are obtained using least square fitting to the BLV data obtained from simulations for various armor systems. Comparison between results obtained from the empirical model for BLV of ceramic–metal armor, with available limited experimental data is carried out. Finally, the empirical model is employed for optimization of alumina–aluminum armor, as an example of ceramic–metal armor system. Total thickness or areal density (mass per unit area) of the armor was minimized through the optimization and maximum BLV was found for each case.

5.1 Material constitutive and fracture models constants

Proper material constitutive and fracture models constants were validated through comparison of simulations with the depth of penetration (DOP) experimental work of Reaugh et al. [148] on alumina and steel armor system with tungsten projectile. First, in order to find proper material constants for tungsten and steel, DOP simulation for impact

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of tungsten projectile into the hardened steel 4340 blocks was compared with experimental results. DOP simulation of target, composed of square tiles bonded, using adhesive Stycast 1266, onto the face of square steel backing plate, was then performed. The projectile used in all the experiments is a right-circular cylinder of tungsten sinter-alloy W2 of 25.4 mm length and 6.35 mm diameter.

5.1.1 Material model constants for steel 4340 and tungsten alloy

In order to find proper material constants for tungsten and steel, DOP simulation for impact of tungsten projectile into the 4340 steel hardened blocks is compared with the experiment [148]. Equation of states (EOS), Johnson-Cook (JC) strength and fracture model (described in section 2.3.1) constants used in the simulation for tungsten and steel are given in Table 5.1. For tungsten, density is changed from 17600 Kg/m³ to 18360 Kg/m³, the one used in experiment. Instantaneous geometric erosion strain of 3 is used for steel 4340 in simulations.

5.1.2 Details of numerical modelling

Based on the symmetry, all the impact modelling was done in two-dimensional (2D) axisymmetric domain. Projectile was modelled using smooth particle hydrodynamics (SPH) particles with size of 0.3 mm. Steel block with diameter of 152 mm and height of 64 mm was modelled using quadrilateral solid Lagrangian elements with size of 0.3 mm in the central region within diameter of 58 mm and height of 30 mm and mesh size becoming gradually coarser in the outer region.

The boundary condition of the back side of the steel block in the impact direction was clamped. In the AUTODYN[®] simulations of present work, the external gap interaction (see section 2.4.2) was used for considering the contact between different parts. The minimum gap size between two different Lagrangian parts (modelled using Lagrangian elements) without joining was determined to be 1/10th of the minimum element size (default value).

The numerically simulated DOP of 25.2 mm at an impact velocity of 1340 m/s matched quite well with the experimentally measured DOP of 27 mm. Hence, the assumed material properties for tungsten alloy and steel are deemed to be appropriate.

Table 5.1 Material model constants and EOS and for aluminum alloy 5083H116 [38, 149], Ti6Al4V [150, 151], tungsten [122, 152], steel 4340 [44] and adhesive, Stycast 1266 [153, 154]

Material/Constants	Al 5083H116	Ti6Al4V	Tungsten alloy	Steel 4340	Stycast 1266
Density, ρ_0 (Kg/m ³)	2700	4428	18360	7830	1120
<u>EOS</u>	<u>Linear</u>	<u>Shock</u>	<u>Shock</u>	<u>Linear</u>	<u>Shock</u>
Bulk modulus, K (GPa)	58.33	-	-	159	-
Gruneisen constant	-	1.23	1.54	-	1.13
Reference temperature, t_0 (K)	300	300	300	300	300
Parameter C_1 (m/s)	-	5130	4029	-	1782
Parameter S_1	-	1.028	1.237	-	1.493
Specific heat, C_r (J/Kg·K)	910	580	134	477	-
<u>Strength model</u>	<u>JC</u>	<u>JC</u>	<u>JC</u>	<u>JC</u>	<u>Elastic</u>
Shear modulus, G (GPa)	26.92	41.9	160	77	1.185
Static yield strength, A' (GPa)	0.167	1.098	1.506	0.792	-
Strain hardening constant, B' (GPa)	0.596	1.092	0.177	0.51	-
Strain hardening exponent, n	0.551	0.93	0.12	0.26	-
Strain rate constant, C'	0.001	0.014	0.016	0.014	-
Reference strain rate, $\dot{\epsilon}_0$	1	1	1	1	-
Thermal softening exponent, m	0.859	1.1	1	1.03	-
Melting temperature, t_m (K)	893	1878	1723	1793	-
<u>Fracture model/criterion</u>	<u>JC</u>	<u>JC</u>	<u>JC</u>	<u>JC</u>	<u>Maximum stress</u>
Damage constant, d'_1	0.0261	-0.09	0	0.05	-
Damage constant, d'_2	0.263	0.25	0.33	3.44	-
Damage constant, d'_3	-0.349	-0.5	-1.5	-2.12	-
Damage constant, d'_4	0.247	0.014	0	0.002	-
Damage constant, d'_5	16.8	3.87	0	0.61	-
Principal stress (GPa)	-	-	-	-	0.059

5.1.3 Material model constants for aluminum 5083H116, Ti6Al4V and Stycast 1266

The JC constitutive and fracture model parameters for aluminum 5083H116 and Ti6Al4V are given in Table 5.1. Validation of material model constants for aluminum alloy 5083-H116 was performed in section 3.1. Geometric instantaneous erosion strain of 3 is used for aluminum alloy 5083-H116 and Ti6Al4V in simulations.

The parameters for elastic strength model and Mie–Gruneisen EOS of adhesive, Stycast 1266 are listed in Table 5.1. Geometric erosion strain of 1.5 was used for adhesive.

5.1.4 Material model validation for three ceramic materials

DOP simulations of three ceramic materials, alumina (Al_2O_3 96%), boron carbide (B_4C) and silicon carbide (SiC), bonded using adhesive Stycast 1266, to steel blocks were performed. For each material, comparison of DOP simulation results with experiments [148] are presented.

It should be noted that Reaugh et al. [148] used Al_2O_3 96% in their experiments, however since the model constants for this material are not available in literature, the JH-2 model constants for Al_2O_3 95% proposed by Zhenqi [137] were used in simulations with the assumption that the difference for the two materials is not considerable.

Johnson-Holmquist-1 (JH-1) strength and fracture model (described in section 2.3.1) constants used for SiC and JH-2 strength and fracture model constants used for Al_2O_3 95% and B_4C are listed in Table 5.2.

Table 5.2 Material model and EOS constants for Al_2O_3 95% [137], B_4C [52] and SiC [50, 126]

Material	Al_2O_3 95%	B_4C	SiC
Density, ρ_0 (kg/m ³)	3750	2516	3215
<u>EOS</u>	<u>Polynomial</u>	<u>Polynomial</u>	<u>Polynomial</u>
Bulk modulus, K (GPa)	184.56	233	220
Pressure constant, K_2 (GPa)	185.87	-593	361
Pressure constant, K_3 (GPa)	157.54	2800	0
Parameter T (GPa)	184.56	233	220
<u>Strength model</u>	<u>JH-2</u>	<u>JH-2</u>	<u>JH-1</u>
Shear modulus, G (GPa)	120.34	197	193.5
Hugoniot elastic limit (HEL) (GPa)	6	19	11.7
Intact strength constant, A	0.889	0.927	Intact strength constant, S_1 (GPa) 7.1
Intact strength exponent, N	0.764	0.67	Intact strength constant, P_1 (GPa) 2.5
Strain rate constant, C	0.0045	0.005	Intact strength constant, S_2 (GPa) 12.2
Fracture strength constant, B	0.29	0.7	Intact strength constant, P_2 (GPa) 10
Fracture strength exponent, M	0.53	0.85	Strain rate constant, C 0.009
Normalized maximum fractured strength, $\sigma_{f_{\max}}^*$	1	0.09	Maximum fracture strength, $S_{f_{\max}}$ 1.3
			Failed strength constant, α 0.4
<u>Fracture model</u>	<u>JH-2</u>	<u>JH-2</u>	<u>JH-1</u>
Normalized hydrostatic tensile limit, T^* (GPa)	-0.3	-0.26	Normalized hydrostatic tensile limit, T^* (GPa) -0.75
Damage constant, d_1	0.005	0.01	Damage constant, $\varepsilon_{f_{\max}}$ 0.8
Damage constant, d_2	1	0.5	Damage constant, P_3 (GPa) 99.75
Bulking constant, β	1	1	Bulking constant, β 1

5.1.4.1 Mesh size and boundary condition

Two-dimensional (2D) axisymmetric DOP simulations were performed. Ceramics were square tiles with side of 102 mm, bonded using adhesive of 0.3 mm thickness, onto the face of square steel backing plates with side of 152 mm, thickness 64 mm and hardness of 35 HRC. Projectile and central part of the ceramic front plate within diameter of 31 mm were modelled using SPH particles of 0.3 mm size. The remaining part of the ceramic front plate was modelled using quadrilateral solid elements. Geometric erosion strain of 1 is used for ceramics for the part modelled using elements. Steel block of 102 mm diameter and 64 mm height was modelled using quadrilateral solid elements of 0.3 mm size in the central region within diameter of 31 mm and height of 24 mm. Element size in steel block becomes gradually coarser from central region toward back side of the block and also circumference of the block, matching the element size of the ceramic part. The rear side of the steel block in the impact direction was clamped. Figure 5.1 shows discretized parts involved in impact simulation.

Figure 5.1 Two-dimensional (2D) axisymmetric model with finite element discretization

5.1.4.2 Comparison between simulations and experiments

DOP simulation results for three ceramic materials with three sub-sets for each ceramic having different thicknesses are compared with experimental results [148] and listed in Table 5.3. The results show that simulation results agree fairly well with experiments within the scatter band. Percentage of the error between DOP obtained from the experiments and simulations are also included in Table 5.3. It should be noted that high

percentage of error like 32.5% in Table 5.3 is due to the fact that the DOP itself is small value (2 mm).

Table 5.3 Comparison of DOP experimental measurements [148] with current simulation results

Ceramic	Tile thickness (mm)	Impact velocity (m/s)	DOP (Sim.) (mm)	DOP (Exp.) (mm)	Error % for DOP
SiC	10	1370	11.65	13.9	16.1
	15.1	1360	2.65	2	32.5
	20	1370	0	0	0
Al ₂ O ₃ 95%	10.5	1350	14.45	14.5	0.3
	20.6	1360	7.8	8.7	10.3
	31.7	1720	5.77	6.4	9.8
B ₄ C	10.4	1280	12.5	13.3	6
	15.2	1220	3.79	3.8	0.3

Figure 5.2 shows DOP simulation of Al₂O₃ 95% at different times from the beginning of projectile impact, with velocity of 1350 m/s, till the projectile is stopped in steel block.

Figure 5.2 Depth of penetration (DOP) simulation into Al₂O₃ 95% backed by steel block at (a) 1 μs , (b) 5 μs , (c) 16 μs , (d) 30 μs , (e) 43 μs and (f) 52 μs , from beginning to end of impact

5.2 Empirical model for the ballistic velocity limit (BLV)

In previous section, material model constants for three ceramic and three metal materials were presented. In this section, an empirical model is proposed for BLV of ceramic–metal bi-layer armor for the normal impact by a blunt tungsten projectile. Numerical simulations are performed to find BLV (impact velocity for which the residual velocity after impact is zero and without exit of the projectile front from the rear side of the backing plate), for different ceramic–metal bi-layer targets. Empirical model constants are determined, using least square fitting to the BLV data obtained from simulations.

Figure 5.3 shows simplified interaction of a hard projectile at the beginning and end of the impact process. It is assumed that two cylindrical plugs of different radii are separated from the front and backing plates through the impact process, as shown in Figure 5.3 (b). It should be noted that in the current model, fragmentation, erosion and plastic deformation of the projectile have not been considered and mass of the projectile is not changed throughout of the impact process.

Figure 5.3 Simplified interaction of the projectile with the armor at the (a) beginning and (b) end of the impact process

The normal impact of blunt projectile into bi-layer armor is considered in two stages:

- 1) Momentum conservation: It is assumed that after impact of projectile with mass of M_p and impact velocity of v_{bl} (BLV), a ceramic front plug mass of M_1 and a metal

backing plug with mass of M_2 are separated and afterwards, projectile, ceramic plug and metal plug will have same velocity, v_f given by:

$$v_f = \frac{M_p v_{bl}}{M_s} \quad (5.1)$$

where $M_s = M_p + M_1 + M_2$ is the sum of the masses of the projectile, ceramic front and metallic plate plugs.

- 2) Energy transfer: Kinetic energy of projectile–ceramic plug–metal plug is dissipated through energy lost in compression of the ceramic plug and stretching of the backing plug considered as

$$\frac{1}{2} M_s v_f^2 = b_1 E_1 V_1 + b_2 E_2 V_2 + \left[c_1 \left(\frac{T_2}{R_p} \right)^{c_2} E_1 V_1 + c_3 \left(\frac{T_1}{R_p} \right)^{c_4} E_2 V_2 \right] \quad (5.2)$$

where E_1 and E_2 are the bulk deformation energies per unit volume of ceramic (under compression) and backing metallic plate (under stretching), respectively and V_1 and V_2 are volumes of ceramic front plug and metallic backing plug, respectively.

On the right hand side of the Eq. (5.2), three energy terms are considered: first term for the effect of monolithic ceramic design, second term for the effect of monolithic metallic armor and third term (in brackets) for the effect of ceramic–metal bi-layer armor. It should be noted that for the case of monolithic ceramic or metal armor, the last term in Eq. (5.2) will be zero. Therefore, the model is applicable not only for bi-layer ceramic–metal armor but also monolithic ceramic or metal armor.

Substituting v_f from Eq. (5.1) into Eq. (5.2), we have:

$$\frac{1}{2} \frac{M_p^2}{M_s} v_{bl}^2 = b_1 E_1 V_1 + b_2 E_2 V_2 + \left[c_1 \left(\frac{T_2}{R_p} \right)^{c_2} E_1 V_1 + c_3 \left(\frac{T_1}{R_p} \right)^{c_4} E_2 V_2 \right] \quad (5.3)$$

where $M_s = M_p + M_1 + M_2$ is sum of the masses of the projectile, $M_p = \rho_p \pi L_p R_p^2$ ceramic plug, $M_1 = \rho_1 \pi T_1 (\xi_1 R_p)^2$ and metallic backing plug, $M_2 = \rho_2 \pi T_2 (\xi_2 R_p)^2$, respectively in which ρ_p , R_p and L_p are density, radius and length of the projectile, respectively, T_1 and ρ_1 are thickness and density of the ceramic front plug, respectively

and T_2 and ρ_2 are thickness and density of the metallic backing plug, respectively and $\xi_1, \xi_2, b_1, b_2, c_1, c_2, c_3, c_4$ are fitting constants.

Proposing empirical forms in Eq. (5.3) for bulk deformation energies: $E_1 = \sigma_{\text{HEL}_1} \varepsilon_1$ in which σ_{HEL_1} is Hugoniot elastic limit (HEL) strength of ceramic plate, ε_1 is compressive failure strain of ceramic, $E_2 = \sigma_2 \varepsilon_2$ in which σ_2 is the ultimate tensile strength (UTS) of metallic plate, ε_2 is tensile failure strain of metallic plate and considering volumes of ceramic front and metallic backing plugs as: $V_1 = \pi T_1 (\xi_1 R_p)^2$ and $V_2 = \pi T_2 (\xi_2 R_p)^2$, respectively, results in an expression for v_{bl} given by:

$$v_{\text{bl}} = \frac{\sqrt{2\pi M_s}}{M_p} \sqrt{\sigma_{\text{HEL}_1} \varepsilon_1 T_1 \left((\xi_1 R_p)^2 \right) \left(b_1 + c_1 \left(\frac{T_2}{R_p} \right)^{c_2} \right) + \sigma_2 \varepsilon_2 T_2 \left((\xi_2 R_p)^2 \right) \left(b_2 + c_3 \left(\frac{T_1}{R_p} \right)^{c_4} \right)} \quad (5.4)$$

5.2.1 Determination of empirical constants using simulations

The constants in the BLV empirical model in Eq. (5.4) were found using least square fitting to the two-dimensional (2D) axisymmetric BLV simulation data. Front and backing plates were of 200 mm diameter. Thicknesses of the front and backing plates were varied from 5 mm to 15 mm in steps of 2.5 mm. Length of the projectile was varied from 24 mm to 54 mm in steps of 6 mm. Adhesive Stycast 1266 with thickness of 0.5 mm was modelled using quadrilateral solid elements to join front and backing plate. Projectile and central part of the ceramic front plate within diameter of 100 mm were modelled using SPH particles with size of 0.5 mm. The remaining part of ceramic front plate and the whole backing plate were modelled using quadrilateral solid elements. The element size of 0.5 mm was chosen for the backing plate in the central part diameter of within 100 mm. Element size becomes coarser gradually in remaining region. The boundary condition for the circumference of the front and backing plates were clamped.

Based on orthogonal array technique, 36 sets of different combinations of ceramic front and metal backing plate materials and thicknesses and different projectile lengths were chosen and their respective BLV were calculated using simulations [146] and listed in Table 5.4.

Table 5.4 Different sets of ceramic and metal materials and thicknesses, projectile lengths and their BLV found from numerical simulations

No.	Ceramic layer	Metallic layer	Ceramic plate thickness (mm)	Metallic plate thickness (mm)	Projectile length (mm)	BLV (m/s)
1	SiC	Al5083H116	5	5	24	250
2	B ₄ C	Ti6Al4V	10	10	24	660
3	Al ₂ O ₃ 95%	Steel 4340	20	20	24	1505
4	B ₄ C	Ti6Al4V	5	5	36	215
5	Al ₂ O ₃ 95%	Steel 4340	10	10	36	550
6	SiC	Al5083H116	20	20	36	765
7	SiC	Steel 4340	5	10	54	450
8	B ₄ C	Al5083H116	10	20	54	540
9	Al ₂ O ₃ 95%	Ti6Al4V	20	5	54	250
10	Al ₂ O ₃ 95%	Ti6Al4V	5	20	24	725
11	SiC	Steel 4340	10	5	24	485
12	B ₄ C	Al5083H116	20	10	24	750
13	Al ₂ O ₃ 95%	Al5083H116	5	10	36	275
14	SiC	Ti6Al4V	10	20	36	840
15	B ₄ C	Steel 4340	20	5	36	600
16	B ₄ C	Steel 4340	5	20	54	740
17	Al ₂ O ₃ 95%	Al5083H116	10	5	54	180
18	SiC	Ti6Al4V	20	10	54	365
19	SiC	Al5083H116	7.5	7.5	30	310
20	B ₄ C	Ti6Al4V	12.5	12.5	30	720
21	Al ₂ O ₃ 95%	Steel 4340	15	15	30	940
22	B ₄ C	Ti6Al4V	7.5	7.5	42	360
23	Al ₂ O ₃ 95%	Steel 4340	12.5	12.5	42	640
24	SiC	Al5083H116	15	15	42	500
25	SiC	Steel	7.5	12.5	48	635
26	B ₄ C	Al5083H116	12.5	15	48	530
27	Al ₂ O ₃ 95%	Ti6Al4V	15	7.5	48	305
28	Al ₂ O ₃ 95%	Ti6Al4V	7.5	15	30	585
29	SiC	Steel 4340	12.5	7.5	30	635
30	B ₄ C	Al5083H116	15	12.5	30	660
31	Al ₂ O ₃ 95%	Al5083H116	7.5	12.5	42	360
32	SiC	Ti6Al4V	12.5	15	42	650
33	B ₄ C	Steel 4340	15	7.5	42	630
34	B ₄ C	Steel 4340	7.5	15	48	700
35	Al ₂ O ₃ 95%	Al5083H116	12.5	7.5	48	245
36	SiC	Ti6Al4V	15	12.5	48	500

It was mentioned in previous section that the model is applicable for both monolithic and bi-layer armor. Therefore, BLV simulations of monolithic ceramic and metal armors were also performed and used together with previous 36 sets for finding empirical model constants.

Figure 5.4 shows least square fitting of the BLV results obtained from empirical model (Eq. (5.4)) to the ones obtained from simulation of monolithic and bi-layer armor. The

empirical constants obtained were: $\xi_1 = 1.21$, $\xi_2 = 2.51$, $b_1 = 20.96$, $b_2 = 2.26$, $c_1 = 66.65$, $c_2 = 1.12$, $c_3 = 1.30$, $c_4 = 0.55$.

Figure 5.4 Least square fitting of BLV results from empirical model to simulations for different projectile–armor sets

Material model constants for three ceramics and three metals, used to calculate BLV obtained from empirical model fitted to BLV simulation data in Figure 5.4, are listed in Table 5.5.

Table 5.5 Material constants for SiC [155, 156], B₄C [155, 156], Al₂O₃ 95%[155, 156], Al 5083-H116 [141, 157], Ti6Al4V [158] and steel 4340 [159] used in empirical model calculations

Material	Density (kg/m ³)	HEL strength, σ_{HEL_1} (GPa)	Compressive failure strain, ϵ_1	UTS, σ_2 (MPa)	Tensile failure strain, ϵ_2
SiC	3215	9.86	0.00125	-	-
B ₄ C	2510	12.3	0.0012	-	-
Al ₂ O ₃ 95%	3750	4.12	0.0008	-	-
Al5083H116	2700	-	-	317	0.16
Ti6Al4V	4428	-	-	950	0.14
Steel 4340	7830	-	-	1070	0.18

5.2.2 Comparison between results of current empirical model and experiments [160]

Wilkins [160] performed experiments using blunt projectile made of Allegheny steel 609 of 24 mm length, 7.62 mm diameter and average Rockwell hardness (HRC) of 55. In these experiments, different ceramic front plates and fixed aluminum alloy 6061-T6 backing metal plate were used.

Material model constants for three ceramics and also Al 6061-T6 backing, used in the current proposed empirical model calculations are provided in Table 5.6. Due to lack of data, compressive failure strain for alumina 85% and 99.5% was considered as 0.0004 and 0.001, respectively.

Table 5.6 Material constants used for alumina 85% [155, 156, 160], B₄C [155, 156], high purity alumina (99.5%) [155, 156, 160] and Al 6061-T6 [157, 160, 161] in empirical model calculations

Material	Density (kg/m ³)	HEL strength, σ_{HEL_1} (GPa)	Compressive failure strain, ε_1	UTS, σ_2 (MPa)	Tensile failure strain, ε_2
Alumina 85%	3430	3	0.0004	-	-
B ₄ C	2510	12.3	0.0012	-	-
Alumina 99.5%	3920	5.7	0.001	-	-
Al 6061-T6	2703	-	-	310	0.17

Figure 5.5 shows the comparison of experimental results performed by Wilkins [160] and the current empirical BLV model estimates. It can be seen that for armor with B₄C front ceramic plate the prediction of model and experiment are close and for the two other armors, the results obtained from the current model follow the same trend as the one of experiments.

Figure 5.5 Comparison between results obtained from current empirical model and experiments [160]

It is important to bear in mind that the tungsten alloy blunt projectile was used in our simulations, based on which the empirical model fitting constants were found, while the

blunt projectile used in Wilkins experiments [162] (reported by Holmquist et al. [160]) was made of hard steel. The reason for using the experiments of Wilkins [162] is that there were not found any experiments in the literature in which tungsten alloy blunt projectile is used for impact of bi-layer ceramic–metal armor. Therefore, we do not expect exact match between our model and these experiments. If exact material properties used in simulations and model calculations are known and same projectile material are used in the experiments and the model, better agreement between results may be achieved.

5.3 Armor optimization of fixed material system

Optimization consideration of armor plates dimensions is a typical issue for armors with fixed material systems. In such problems, the selections of optimum values of armor plates dimensions, such as T_1 and T_2 for a given constraint of constant total armor thickness ($TT = T_1 + T_2$) or a constant areal density ($AD = \rho_1 T_1 + \rho_2 T_2$) are of interest. In the following section, examples of armor optimization based on Eq. (5.4), for a given total thickness (TT) or areal density (AD) are presented.

5.3.1 Alumina–aluminum bi-layer armor optimization

As an example for optimization of bi-layer armor using proposed empirical model, the BLV of alumina 99.5%–aluminum 5083H116 bi-layer armor under blunt tungsten alloy projectile impact were calculated using Eq. (5.4) for given constraints of TT or AD .

Material properties used here in optimization analysis for alumina 99.5% ceramic plate, aluminum 5083H116 metallic plate and tungsten alloy projectile are: Hugoniot elastic limit (HEL) strength of alumina 99.5%, $\sigma_{\text{HEL}_1} = 5.77$ GPa, compressive failure strain of alumina 99.5%, $\varepsilon_1 = 0.0008$, density of alumina 99.5%, $\rho_1 = 3890$ kg/m³, ultimate tensile strength (UTS) of aluminum, $\sigma_2 = 317$ MPa, tensile failure strain of aluminum, $\varepsilon_2 = 0.16$, density of aluminum, $\rho_2 = 2700$ kg/m³ and tungsten alloy projectile diameter, $D_p = 12$ mm, length, $L_p = 36$ and density, $\rho_p = 18360$ kg/m³.

The curves shown in Figure 5.6 (a) and (b) present the typical BLV variation with T_1/TT for given TT and AD_1/AD ($AD_1 = \rho_1 h_1$ denotes the areal density of front

plate) for given AD , respectively. Similar trends for BLV, as function of T_1/TT or AD_1/AD to ones observed in section 3.5 and section 04.4, and experiments on bi-layer ceramic–metal armor [136, 161, 163] on alumina–aluminum material system, were obtained for impact of tungsten alloy projectile into alumina 99.5%–aluminum 5083H116 armor system. For $T_1/TT = 1$ or 0 ($AD_1/AD = 1$ or 0) viz. the armor only consists of either monolithic alumina or aluminum plate, shown in Figure 5.6 (a) and (b), Eq. (5.4) predicts higher BLV for monolithic aluminum plate compared to monolithic alumina plate with the same TT (or AD). The same behavior was observed in the experiments of Mayseless et al. [135], for BLV of monolithic aluminum and alumina plates with given AD .

Figure 5.6 BLV as a function of the (a) normalized thickness and (b) normalized areal density of the front plate

It can be seen that with variation of T_1/TT or AD_1/AD from 0 to 1, the BLV increases to a maximum value and then decreases. Maximum BLV for a given TT or AD corresponding to an optimum T_1/TT or AD_1/AD can be calculated. By way of an example, when $TT = 15$ mm, the maximum BLV is 251 m/s corresponding to optimum front plate thickness, T_1 of 3.7 mm; when $AD = 90$ kg/m², the maximum BLV of 465 m/s is obtained corresponding to optimum front plate areal density of $AD_1 = 23.8$ kg/m². Polynomials were fitted to the optimum points (maximum BLV and optimum T_1/TT or AD_1/AD) and the expressions, shown in Figure 5.6 (a) and (b), are respectively as follows:

$$V_{bl} = 75927876 \left(\frac{T_1}{TT} \right)^8 + 6280062 \left(\frac{T_1}{TT} \right)^7 - 49687828 \left(\frac{T_1}{TT} \right)^6 + 24727643 \left(\frac{T_1}{TT} \right)^5 - 5386178 \left(\frac{T_1}{TT} \right)^4 + 629266 \left(\frac{T_1}{TT} \right)^3 - 43020 \left(\frac{T_1}{TT} \right)^2 + 2306 \left(\frac{T_1}{TT} \right) + 18 \quad (5.5)$$

$$V_{bl} = 2370789 \left(\frac{AD_1}{AD} \right)^5 - 1525007 \left(\frac{AD_1}{AD} \right)^4 + 387759 \left(\frac{AD_1}{AD} \right)^3 - 45246 \left(\frac{AD_1}{AD} \right)^2 + 3145 \left(\frac{AD_1}{AD} \right) + 20 \quad (5.6)$$

Optimum T_1/TT , AD_1/AD ratios and corresponding maximum BLV as a function of TT and AD are shown in Figure 5.7 (a) and (b), respectively, assuming that Eq. (5.4) is valid over an extended range of TT and AD .

Figure 5.7 (a) Optimum T_1/TT and maximum BLV as a function of TT and (b) optimum AD_1/AD and maximum BLV as a function of AD

5.3.2 Discussion

In the literature, there are a few works which focus on optimization of ceramic–metal bi-layer armor system design and analysis. Hetherington [163] analytically carried out optimization of “alumina 95%–aluminum alloy 5083-O” armor system, based on Florence model [79], for fixed areal density (AD) and found specific optimum ratio of $T_1/TT = 0.74$. He also performed experiments on alumina–aluminum armor system with fixed $AD = 50 \text{ kg/m}^2$ impacted with 7.62 AP projectile and found optimum ratio of around 0.71 for T_1/TT .

Wang and Lu [161] performed optimization of alumina 94%–aluminum alloy 6061-T6 armor system, based on Florence model [79], for fixed total thickness (TT) and found specific optimum ratio of $T_1/TT = 0.77$. They also performed experiment for alumina–aluminum armor system with fixed $TT = 12$ mm impacted with 7.62 AP projectile and estimated optimum ratio of $T_1/TT \approx 0.75$.

Lee and Yoo [136] carried out simulations and experiments on four alumina–aluminum alloy 5083 system with fixed AD impacted with steel blunt projectile. Figure 5.8 (a) shows the variation of BLV with front to backing plate thickness ratio, T_1/T_2 for four bi-layer alumina–aluminum systems with areal density of $AD = 90.88$ kg/m² [136]; simulations and experiments performed by Lee and Yoo [136]. The experiments of Lee and Yoo [136] are plotted again in Figure 5.8 (b) to show the variation of BLV with the ratio of front plate areal density over total areal density, AD_1/AD .

Figure 5.8 Variation of ballistic limit velocity (BLV) (from the work of Lee and Yoo [136]) (a) with front to backing plate ratio, T_1/T_2 for bi-layer alumina–aluminum of constant areal density $AD = 90.88$ kg/m² impacted at normal incidence by hard-steel blunt projectile of 12.5 mm diameter and (b) re-graphed as a function of ratio of front plate areal density over total areal density, AD_1/AD

Details of the calculated data points in Figure 5.8 (b) can be found in Appendix B. It is conjectured from Figure 5.8 (b) that the optimum ratio of AD_1/AD exists in the range of 0.43 to 0.69. For alumina 95%–aluminum alloy 2024-T3 with given AD of 22.28 kg/m² and 50.46 kg/m², the optimum ratio of $AD_1/AD = 0.62$ was found in section 4.4.

Ben-Dor et al. [80] performed optimization of alumina 85%–aluminum alloy 6061-T6 armor system under impact of blunt projectile based on their improved Florence model and found fixed optimum ratio of 0.77 for both T_1/TT (for given TT) and AD_1/AD (for given AD) and compared their results with experimental work of Wilkins [162] which is alumina 85%–aluminum alloy 6061-T6 armor system under the impact of conical hard steel projectile.

It is believed that the optimal ratio for bi-layer ceramic–metal armor system depends on the material properties of its ceramic and metallic plates and also geometry and material properties of the projectile.

In section 3.5, the trends (an inverted U curve) were obtained for optimal normalized thickness T_1/TT and normalized areal density AD_1/AD ratios for impact of tungsten alloy blunt projectile onto alumina 99.5%–aluminum 5083H116 armor system over an extended ranges of TT and AD , respectively. Similar trends are observed in section 5.3.1 for the same armor material/projectile system and in section 4.4 for the impact of hardened steel blunt projectile onto alumina 95%–aluminum alloy 2024-T3 armor system. In section 4.4, for impact of hardened steel blunt projectile onto alumina 95%–aluminum alloy 2024-T3 armor system, the optimal T_1/TT and AD_1/AD ratios of almost 0.55 and 0.62 were found, respectively.

The optimization case studies under given total thickness (TT) or areal density (AD) constraints are listed in Table 5.7. For high range of TT and AD , the optimal T_1/TT and AD_1/AD ratios of almost 0.5 was found in section 3.5, while the corresponding optimal values found in section 5.3.1, were 0.35 and 0.3, respectively.

The possible reasons for the observed difference are explained as follows:

Firstly, an extended range over the initial range of TT and AD based on which the semi-analytical model (used in section 3.5) and empirical model (used in section 5.3.1) were considered for extracting optimal data which itself is an approximation method. Secondly, the results obtained in section 04.4, is based on validated numerical simulations in which approximations like application of erosion strain in removing highly distorted elements were used. Thirdly, it should be borne in mind that the

proposed generalized empirical model, Eq. (5.4) which itself does not consider dissipation of energy due to erosion or plastic deformation of the projectile though there was a failure criterion for this, is based on an approximate method of finding empirical constants through matching the BLV obtained from the model with those from impact simulations of different armor material system. Nonetheless, the predictions of the numerical, semi-analytical and empirical models are reasonable in the sense that it can produce trends similar to ones observed in experiments and roughly estimate the optimal ratios. Further work needs to be done to comprehensively cover the damage in the projectile in the optimization of TT and AD for BLV.

Table 5.7 Optimization case studies in Chapters 3 to 5

5.4 Summary

In this chapter, generalised empirical model for BLV was proposed by considering different armor material systems based on the momentum and energy balances. Material constitutive and fracture models were validated through comparison of numerical simulations, performed in AUTODYN[®], with available experimental measurements in the literature. These models were used in the impact simulations of armor systems designed based on “Design of Experiments”: different combinations of projectiles with different lengths and also armor with different thicknesses and materials from three

ceramic materials and three metal materials were chosen based on orthogonal array technique and their respective BLV were calculated using AUTODYN[®] simulations. The empirical constants in the model were obtained using least square fitting to BLV results obtained from simulations. Comparison of the proposed empirical model results with available experimental measurements from the literature was discussed. Optimization of a ceramic–metal bi-layer armor system using empirical model was performed and similar trends, to those available in literature and also those obtained in Chapter 3 and Chapter 4 were captured. Finally, the possible reasons for the minor differences between optimal ratios obtained from different models were discussed.

Chapter 6 Erosion and Plastic Deformation of Blunt and Conical Projectiles Impact

In this chapter, Recht's model [91] accounting for erosion and plastic deformation mechanisms of blunt projectile impact onto deformable monolithic target is first reviewed and partly revised. This model is then extended for conical projectile impact onto bi-layer ceramic–metal armor systems. The residual length of the projectile and the time taken for the erosion and plastic deformation, are found. Comparison between these results obtained from the models with literature experiments for both blunt and conical projectile impact on ceramic–metal armor are presented. The model predictions of the blunt and conical projectile residual length through the time are in good agreement with the experimental measurements.

6.1 Recht's model [91] for blunt projectile

Recht [91] considered a blunt projectile of initial length, L_i^b and velocity V_0 impacting onto a monolithic ductile (bi-linear elastic plastic) target, as shown in Figure 6.1. He considered two modes of deformation for the projectile, namely erosion and plastic deformation.

Figure 6.1 Schematic of blunt projectile impact onto a monolithic target

Recht [91] assumed that upon impact, the projectile–target interface velocity achieves an initial constant value, v_i due to impedance match between projectile and target, as given by:

$$v_t = \frac{v_0}{1 + \frac{\rho_t U_t}{\rho_p U_p}} \quad (6.1)$$

where ρ_p and ρ_t are densities of projectile and monolithic target plate, respectively and U_t is the plastic wave velocity in the target given by:

$$U_t = \sqrt{\frac{K_t}{\rho_t}} \quad (6.2)$$

where K_t is the bulk modulus of the target. Equation (6.1) is based on the pressure balance on either side of the projectile–target interface. The value for v_0 depends on whether erosion starts first upon the impact or plastic deformation occurs (without occurrence of erosion), which will be specified through the chapter wherever it is needed. The impedance matching parameter is defined as [91]:

$$k = 1 + \frac{\rho_p U_p}{\rho_t U_t} \quad (6.3)$$

The ratio of the mechanical impedance of projectile, $\rho_p U_p$ to that of the target, $\rho_t U_t$ has been added to 1 in Eq. (6.3), to consider for material properties effect. It can be understood from Eqs. (6.2) and (6.3) that for a rigid target, $U_t = \infty$ and $k = 1$.

For a bi-layer ceramic–metal armor (composed of front and backing plates of T_1 and T_2 thicknesses, respectively) interacting with projectile, a bulk material is considered with (equivalent) material properties like density and bulk modulus related to properties of single layers. A new equivalent impedance matching parameter is defined as:

$$k_{eq} = 1 + \frac{\rho_p U_p}{\rho_{eq} U_{eq}} \quad (6.4)$$

where $U_{eq} = \sqrt{K_{eq} / \rho_{eq}}$ is the plastic wave velocity in the bi-layer target in which ρ_{eq} is the equivalent density of the bi-layer armor, based on equivalent mass, given by:

$$\rho_{eq} = \frac{\rho_1 T_1 + \rho_2 T_2}{T_1 + T_2} \quad (6.5)$$

and K_{eq} is the equivalent bulk modulus for the bi-layer ceramic–metal armor, considering volume ratio of each plate, given by:

$$K_{eq} = \frac{\frac{T_1 + T_2}{\frac{T_1}{K_1} + \frac{T_2}{K_2}}}{\frac{T_1}{K_1} + \frac{T_2}{K_2}} \quad (6.6)$$

where K_1 and K_2 are the bulk modulus of front and backing plates, respectively. A detailed description for finding equivalent Young's modulus can be found in [164]. Therefore projectile–target interface velocity for bi-layer armor target is given by:

$$v_{t_{eq}} = \frac{v_0}{1 + \frac{\rho_{eq} U_{eq}}{\rho_p U_p}} \quad (6.7)$$

Henceforth, it is assumed that the target in Recht's model [91] is a bi-layer ceramic–metal armor. Therefore equivalent matching parameter is used in the equations.

6.1.1 Erosion of blunt projectile

Upon occurrence of the erosion, projectile material may flow laterally on the surface of the target without much penetration into the ceramic layer, a phenomenon called *dwell*. The reason for this is that it takes time till intact ceramic layer fragments and let the projectile penetrate into the target. Consider a blunt projectile, shown in Figure 6.2 (a), with initial length, L_i^b radius, R and impact velocity, V_0 .

Figure 6.2 Blunt projectile (a) before impact, $t=0$, (b) at times $0 < t < t^e$, during erosion and (c) at the end of erosion, $t = t^e$

Relative velocity, $v_{r,b}^e$, during erosion process is defined as $v_{r,b}^e = v_b^e - v_{t_{eq}}$ in which v_b^e is the velocity of the elastic remainder of the blunt projectile and $v_{t_{eq}}$ is the velocity of the projectile–target interface. For the case of impact on a non-rigid target, erosion occurs when $V_0/k_{eq} > U_p$, where k_{eq} is the equivalent impedance matching parameter (given by Eq. (6.4)), and V_0/k_{eq} is the initial value of the relative velocity, $v_{r,b}^e$. In this case the plastic wave cannot propagate away from the impact interface and stands at very short distance above the impact interface [91]. The interface pressure is high enough so that the projectile material going through the shock wave erodes away laterally from the impact zone, accompanied with heating and probably melting of the projectile material. This phenomenon can be observed in the X-ray photographs taken by Reijer [23] and Wilkins [9]. Once erosion occurs, the equation of motion, at time t , for the uneroded portion of the projectile, with length l and mass m is given by*:

$$-A_i \sigma_e = m \frac{dv_{r,b}^e}{dt} \quad (6.8)$$

where σ_e , is the dynamic elastic yield strength of the projectile and mass m is given by:

$$m = A_i l \rho_p \quad (6.9)$$

where ρ_p and $A_i (= \pi R^2)$ are the density the cross-sectional area of the projectile.

The incremental length, dl is given by:

$$dl = -v_{r,b}^e dt \quad (6.10)$$

Considering Eqs. (6.8) to (6.10) and integrating $v_{r,b}^e$ and l with initial values of V_0/k_{eq} and L_i^b , respectively, $v_{r,b}^e$ is found as:

$$v_{r,b}^e = \sqrt{\left(\frac{V_0}{k_{eq}}\right)^2 + 2 \frac{\sigma_e}{\rho_p} \ln \frac{l}{L_i^b}} \quad (6.11)$$

*Correct form of Eq. (6.8) is $-A_i \sigma_e = m \frac{dv_b^e}{dt}$, but for the interface velocity, v_{int} , given in Eq. (6.7) which is independent of time ($dv_{t_{eq}}/dt = 0$), we have $dv_{r,b}^e/dt = d(v_b^e - v_{t_{eq}})/dt = dv_b^e/dt$.

6.1.1.1 Variation of length of blunt projectile during erosion

Equating $v_{r,b}^e$ to its final value U_p in Eq. (6.11), the projectile length at the end of erosion, L as shown in Figure 6.2 (c), can be expressed as:

$$L = L_i^b \exp \left\{ - \left[\frac{\rho_p}{2\sigma_e} \right] \left[\left(\frac{V_0}{k_{eq}} \right)^2 - U_p^2 \right] \right\} \quad (6.12)$$

Substituting Eq. (6.11) into Eq. (6.10) and integrating l and t with initial values of L_i^b and 0, respectively, variation of length of the projectile with time is evaluated as:

$$t = - \int_{L_i^b}^L \frac{dl}{\sqrt{\left(\frac{V_0}{k_{eq}} \right)^2 + 2 \frac{\sigma_e}{\rho_p} \ln \frac{l}{L_i^b}}} \quad (6.13)$$

Equation (6.13) can be evaluated numerically. By varying the upper limit of the integral, l in Eq. (6.13), in the range $L_i^b < l \leq L$ (where L was found in Eq. (6.12)), variation of the length of the projectile with time is found. The erosion will end at time t^e when length of the projectile reduces to L . Once the value of $v_{r,b}^e$, decelerating through erosion, decreases to U_p , plastic deformation of the remaining length of the projectile initiates with velocity of v_{b_0} and for this case $v_{b_0} = v_0 = k_{eq} U_p$.

6.1.2 Plastic deformation of blunt projectile

Upon impact, an elastic compressive wave first propagates upward into the projectile with stress intensity equal to the dynamic elastic yield strength, σ_e , of the projectile and velocity equal to longitudinal wave velocity, $C_p = \sqrt{E_p / \rho_p}$ (E_p and ρ_p are Young's modulus and density of the projectile material, respectively). This results in decrease of the particle velocity, through reflection of the wave from free end of the projectile, each time by an increment $\Delta v = \sigma_e / \rho_p C_p$; this is followed by a plastic wave of a much lower velocity, namely plastic wave velocity, U_p which reduces the particle velocity to the projectile–target interface velocity. After completion of erosion, the shortened projectile, as shown in Figure 6.3 (a), will be of length L , radius R and velocity $v_{b_0} = k_{eq} U_p$ and undergoes plastic deformation.

Figure 6.3 Blunt projectile (a) at the beginning of plastic deformation (after erosion ended), $t = t^e$, (b) at times $t^e < t < t^d$, during deformation and (c) at the end of deformation, $t = t^d$

At any instant of time, $t^e < t < t^d$, only the material within the increment dx , as shown in Figure 6.3 (b), undergoes plastic deformation, while the permanent deformed (enlarged) part of the projectile moves with interface velocity; the projectile–target interface has velocity of $v_{t_{eq}}$ (given by Eq. (6.7)) and elastic remainder of the projectile has velocity of v_b^d .

Recht [91] assumed that plastic wave velocity for projectile, U_p is constant. The length of the elastic remainder of the projectile, l , shown in Figure 6.3 (b), is given by:

$$l = L - U_p t \quad (6.14)$$

which results in the relationship between incremental length and time as:

$$dl = -U_p dt \quad (6.15)$$

and the mass of the elastic remainder, m_l is expressed as:

$$m_l = A_l l \rho_p \quad (6.16)$$

where ρ_p and $A_l (= \pi R^2)$ are the density the cross-sectional area of the projectile.

The equation of motion for mass m_l is given by:

$$-A_l \sigma_e = m_l \frac{dv_b^d}{dt} \quad (6.17)$$

Regarding the equation of motion, Recht [91] posits, without any further explanation, that for this case, the well-known Newton's second law of motion $-F = \frac{d(m_l v_b)}{dt}$ (in which $F = A_i \sigma_e$) does not apply.

An explanation for this, which is solely the opinion of the author and probably not the reasoning of Recht [91], is given here: The Newton's second law of motion may be written as $-F = \left(m_l \frac{dv_b}{dt} + v_b \frac{dm_l}{dt} \right) = \rho_p A_0 \left(l \frac{dv_b}{dt} + v_b \frac{dl}{dt} \right)$. Substituting $\frac{dl}{dt} = -v_{r,b}$ (in which $v_{r,b} = v_b - v_{teq}$) in the equation of motion results in $-F = \rho_p A_0 \left(l \frac{dv_b}{dt} - v_b v_{r,b} \right)$, which suggests that part of the force acts to decelerate the projectile, $F_1 \propto -\frac{dv_b}{dt}$, while part of the force acts to increase the velocity of the projectile, $F_2 \propto v_b v_{r,b}$ which is physically not possible. However, it should be borne in mind that Newton's second law is applicable to a system within a boundary. In the current case considered by Recht [91], the boundary that encloses the mass is changing – a case of moving boundary. The second term in the equation, $\frac{dm_l}{dt}$ is independent of the applied force; thus, the Newton's second law of motion should be applied over the moving boundary; i.e. $F = -m_l \frac{dv_b}{dt}$ where the velocity v_b is defined in an inertia frame – a reference frame of constant velocity.

Combining Eqs. (6.15) to (6.17) and integrating with initial values of v_{b_0} and 0 for v_b^d and t , respectively, results in the equation for v_b^d as a function of time given by:

$$v_b^d = v_{b_0} + \frac{\sigma_e}{\rho_p U_p} \ln \left(1 - \frac{U_p t}{L} \right) \quad (6.18)$$

The relative velocity, $v_{r,b}^d$ between the elastic remainder of the projectile, v_b^d and projectile–target interface, v_{teq} then is expressed as:

$$v_{r,b}^d = v_{b_0} + \frac{\sigma_e}{\rho_p U_p} \ln \left(1 - \frac{U_p t}{L} \right) - v_{teq} \quad (6.19)$$

Substituting Eq. (6.7) into (6.19) results in:

$$v_{r,b}^d = \frac{v_0}{k_{eq}} + \frac{\sigma_e}{\rho_p U_p} \ln \left(1 - \frac{U_p t_e}{L} \right) \quad (6.20)$$

It should be noted that if the impact velocity is high enough ($V_0 > k_{eq} U_p$) such that first erosion occurs at the end of which $v_{r,b} = U_p$, followed by the plastic deformation, then $v_{b_0} = v_0 = k_{eq} U_p$ (where $v_{b_0} = U_p - v_{t_{eq}} = k_{eq} U_p$); if the impact velocity, V_0 , is not high enough ($V_0 < k_{eq} U_p$) for occurrence of erosion, plastic deformation of the original length of the projectile starts upon impact and for this case $v_{b_0} = v_0 = V_0$ [91].

6.1.2.1 Temporal variation of non-deformed length of blunt projectile during plastic deformation

Plastic deformation ends when the velocity of the elastic remainder of the projectile, v_b^d is equal to the velocity of the projectile–target interface, $v_{t_{eq}}$. In other words, the relative velocity between the projectile and interface, $v_{r,b}^d$ at the end of the plastic deformation is zero. Equating Eq. (6.20) to zero, time to plastic deformation, t_d , can be found as:

$$t_d = \frac{L}{U_p} \left(1 - \exp \left(\frac{-v_0 \rho_p U_p}{k_{eq} \sigma_e} \right) \right) \quad (6.21)$$

and the length of the elastic remainder of the projectile, l_b as shown in Figure 6.3 (c), is found by substituting Eq. (6.21) into Eq. (6.14), given by:

$$l_b = L \exp \left(\frac{-v_0 \rho_p U_p}{k_{eq} \sigma_e} \right) \quad (6.22)$$

Length of the remainder at any time between beginning to end of the plastic deformation, $t^e < t < t^d$, is found from Eq. (6.14).

6.1.2.2 Radius and length of deformed portion of blunt projectile

During the plastic deformation process, passing of the plastic wave through any cross-sectional area, $A_i (= \pi R^2)$ results in expansion of projectile material, as shown in

Figure 6.3 (b), which is a constant volume process, to a new cross-sectional area, A_e behind the plastic wave. The enlarged cross-sectional area, A_e will move with the same velocity as the interface velocity, $v_{t_{eq}}$. Conservation of mass during plastic deformation (at times, $t^e < t < t^d$), results in an expression for the relative velocity, $v_{r,b}^d$ as a function of cross-sectional area ratio, A_i/A_e given by [91]:

$$v_{r,b}^d = U_p \left(1 - \frac{A_i}{A_e} \right) \quad (6.23)$$

Substituting Eq. (6.20) into Eq. (6.23) results in an expression for A_i/A_e given by:

$$\frac{A_i}{A_e} = 1 - \frac{v_0}{k_{eq} U_p} - Q \ln(1 - Jt) \quad (6.24)$$

where $J = U_p/L$ and $Q = \sigma_e / \rho_p U_p^2$.

The radius of the enlarged cross-section at any time $t^e < t < t^d$, can be found from Eq. (6.24). Since the projectile penetrates through a hole in the target, the enlarged radius of the deformed part of the projectile is often sheared off. Recht [91] considered, based on the typical experimental data, the radius at which material is sheared off to be 1.25 times the radius of the projectile. He assumed that for enlarged radius less this value, shear mass loss is not occurred.

The material under deformation within the increment dx , shown in Figure 6.3 (b), is given by [91]:

$$dx = (U_p - v_{r,b}^d) dt \quad (6.25)$$

Combining Eqs. (6.23) to (6.25) and then integrating with the end points for x from 0 to x_b and for t from 0 to t_d , the final length of the deformed portion of the projectile, x_b , shown in Figure 6.3 (c), is given by:

$$\frac{x_b}{L} = \left(J - \frac{v_0}{k_{eq} L} + \frac{Q U_p}{L} \right) t_d + Q(1 - Jt_d) \ln(1 - Jt_d) \quad (6.26)$$

where t_d is duration of the plastic deformation (Eq. (6.21)) and L is the length of the projectile at the beginning of plastic deformation which is either equal to the one given

by Eq. (6.12) for the case when plastic deformation occurs after erosion or is equal to L_i^b (initial length of the projectile) for the case when plastic deformation of the projectile initiates from the beginning of the impact (without occurrence of erosion). The final length of the projectile at the end of the deformation, (at time $t = t^e + t^d$), shown in Figure 6.3 (c), will be $l_b + x_b$, after which the length of the projectile remains constant.

In the above analysis, so far the concept of equivalent armor has been added to the Recht's model [91]. This model is extended for conical projectile in ensuing section.

6.2 Erosion and deformation models for conical projectile

6.2.1 Erosion of conical projectile

Consider the conical projectile, shown in Figure 6.4 (a), with an initial impact velocity of V_i . The radial and axial variables are r and y , respectively.

Figure 6.4 Conical projectile (a) before impact, $t = 0$, (b) at times $0 < t < t'$, during erosion and (c) at the end of erosion, $t = t'$

The radius of the conical head is r_1 and R is the radius of the cylindrical part. The lengths of the cylindrical and conical portion are l_1 and l_2 , respectively, giving an initial length of the projectile as $L_i^c = l_1 + l_2$.

If the relative velocity between the elastic remainder of the projectile and the impact interface, $v_{r,c}^e$ is greater than U_p , erosion occurs. At time t' that erosion stops at radius r' ($r_1 < r' < R$), the length y' ($0 < y' < l_2$) out of the projectile length has been eroded.

The length of the elastic remainder of the projectile, l , at times $0 < t < t'$, is given by:

$$l = L_i^c - v_{r,c}^e t \quad (6.27)$$

Therefore the change in length, dl is given by:

$$dl = -v_{r,c}^e dt \quad (6.28)$$

Suppose that at any time $0 < t < t'$ in Figure 6.4 (b), after impact, the length y out of the length l_2 has been eroded. Therefore, remainder length is $l = l_1 + l_2 - y$. The relationship among incremental lengths dl , dy and incremental radial variable, dr is given by:

$$dy = -dl = \frac{l_2}{R - r_1} dr \quad (6.29)$$

The mass of the elastic remainder of the projectile, m' at time, t is given by:

$$m' = \pi \left(R^2 l_1 + \frac{l_2}{3(R - r_1)} (R^3 - r^3) \right) \rho_p \quad (6.30)$$

The equation of motion for the elastic portion of the projectile is given by:

$$-\pi r^2 \sigma_e = m' \frac{dv_{r,c}^e}{dt} \quad (6.31)$$

where $v_{r,c}^e$ is the relative velocity between the elastic remainder of the blunt projectile, v_c^e and the projectile–target interface, $v_{t_{eq}}$.

Combining Eqs. (6.29) to (6.31) and integrating with initial values of V_0/k_{eq} and r_1 for $v_{r,c}^e$ and r , respectively, $v_{r,c}^e$ is found as a function of r given by:

$$v_{r,c}^e = \sqrt{\left(\frac{V_0}{k_{eq}} \right)^2 + \frac{2\sigma_e}{\rho_p} \ln \left(\frac{R^2 (R(3l_1 + l_2) - 3r_1 l_1) - l_2 r^3}{(R - r_1) (R^2 (3l_1 + l_2) + l_2 (Rr_1 + r_1^2))} \right)} \quad (6.32)$$

6.2.1.1 Temporal variation of conical projectile length during erosion

The radius $r' (< R)$, shown in Figure 6.4, wherein erosion has stopped, can be found from Eq. (6.32), by equating $v_{r,c}^e$ to U_p and solving for r' . Substituting Eq. (6.32) into Eq. (6.28) while incorporating Eq. (6.29) and integrating with initial values of r_1 and 0 for r and t , respectively, variation of length of the projectile with time is found as:

$$t = \frac{l_2}{R - r_1} \int_{r_1}^{r'} \frac{dr}{\sqrt{\left(\frac{V_0}{k_{eq}}\right)^2 + \frac{2\sigma_e}{\rho_p} \ln \left(\frac{R^3(3l_1 + l_2) - 3R^2l_1r_1 - l_2r^3}{(R - r_1)(R^2(3l_1 + l_2) + l_2(Rr_1 + r_1^2))} \right)}} \quad (6.33)$$

Equation (6.33) can be solved numerically. By varying the upper limit of the integral, r in Eq. (6.33), in the range $r_1 < r \leq r'$, variation of the radius of the projectile with time is found and corresponding eroded projectile length will be $\frac{l_2}{R - r_1}(r - r_1)$. Length of the projectile at the end of the erosion, as shown Figure 6.4 (c), will be $L' = L_i^c - y'$ in which $y' = \frac{l_2}{R - r_1}(r' - r_1)$ is eroded length of the projectile.

When erosion stops at time t' , plastic deformation of the remaining length of the projectile, with velocity $v_{c_0} = v_0 = k_{eq}U_p$ initiates. It should be borne in mind that if $r' > R$, it means that the conical part has been fully eroded followed by erosion of the cylindrical part. In this case, initial relative velocity which should be used in Eqs. (6.11), (6.12) and (6.13) for analysis of erosion for the cylindrical part is not V_0/k_{eq} anymore and instead, $v_{r,c}^e|_{r=R}$ should be used, meaning that erosion of the cylindrical part will be continued with relative velocity of the conical part at $r = R$, $v_{r,c}^e|_{r=R}$, obtained from Eq. (6.32).

6.2.2 Plastic deformation of conical projectile

After completion of erosion (at time $t = t'$), the shortened projectile, as shown in Figure 6.5 (a), will have the velocity of v_{c_0} , length of L' found in section 6.2.1.1 and mass of m'_p given by:

$$m'_p = \pi \left(R^2 l_1 + \frac{l_2}{3(R-r_1)} (R^3 - r'^3) \right) \rho_p \quad (6.34)$$

Figure 6.5 Conical projectile (a) at the beginning of plastic deformation (after erosion ended), $t = t'$, (b) at times $t' < t < t''$, during deformation and (c) at the end of deformation, $t = t''$

Plastic deformation of the conical part of the projectile occurs during time interval $t' < t < t''$, as shown in Figure 6.5 (b), while r changes in the range $r'_1 < r < R$ during this period. The length of the elastic remainder of the projectile, l at times $t' < t < t''$, is given by:

$$l = L' - U_p t \quad (6.35)$$

and the mass of the elastic portion of the projectile, m'_l is given by:

$$m'_l = \pi \left(R^2 l' + \frac{l'}{3(R-r')} (R^3 - r'^3) \right) \rho_p \quad (6.36)$$

where $l' = l_2 - y'$ and r' and y' were found in section 6.2.1.1.

The equation of motion for mass m'_l is given by:

$$-\pi r^2 \sigma_e = m'_l \frac{dv_c^d}{dt} \quad (6.37)$$

Combining Eqs. (6.29), (6.36), (6.37) and also $dl = -U_p dt$ (from Eq. (6.35)) and integrating the result with initial value of v_{c_0} and r' for v_c^d and r , respectively, one can find the velocity of the elastic remainder of the projectile, v_c^d , expressed as:

$$v_c^d = v_{c_0} + \frac{\sigma_e}{\rho_p U_p} \ln \left(\frac{R^3 (3l_1 + l') - 3R^2 l_1 r' - l' r^3}{(R - r') (R^2 (3l_1 + l') + l' (Rr' + r'^2))} \right) \quad (6.38)$$

The relative velocity, $v_{r,c}^d$ between the the elastic remainder of the projectile, v_c^d and projectile–target interface, $v_{t_{eq}}$ then is found as:

$$v_{r,c}^d = v_{c_0} + \frac{\sigma_e}{\rho_p U_p} \ln \left(\frac{R^3 (3l_1 + l') - 3R^2 l_1 r' - l' r^3}{(R - r') (R^2 (3l_1 + l') + l' (Rr' + r'^2))} \right) - v_{t_{eq}} \quad (6.39)$$

Substituting Eq. (6.7) into (6.39) results in:

$$v_{r,c}^d = \frac{v_{c_0}}{k_{eq}} + \frac{\sigma_e}{\rho_p U_p} \ln \left(\frac{R^3 (3l_1 + l') - 3R^2 l_1 r' - l' r^3}{(R - r') (R^2 (3l_1 + l') + l' (Rr' + r'^2))} \right) \quad (6.40)$$

For the case that the plastic deformation occurs after erosion, $v_{c_0} = v_0 = k_{eq} U_p$; if erosion does not occur upon impact then plastic deformation of the conical part of the projectile initiates for which $v_{c_0} = v_0 = V_0$.

6.2.2.1 Temporal variation of non-deformed length of conical projectile during plastic deformation

For the projectile, at the end of the plastic deformation, shown in Figure 6.5 (c), the relative velocity, Eq. (6.40), is zero, i.e. $v_{r,c}^d = 0$ from which one can find the radius, r'' wherein plastic deformation stops.

At any time between beginning to end of the plastic deformation, $t' < t < t''$, shown in Figure 6.5 (b), the length of the elastic remainder, from Eq. (6.35), will be

$l_c = L' - \frac{l'(r-r')}{R-r'}$, where r varies in the range $r' < r \leq r''$. The time, t'' at which deformation stops, will be $t'' = \frac{l'(r''-r')}{U_p(R-r')}$.

It should be noted that for the plastic deformation analysis in this section, it was assumed that $r'' < R$ meaning that plastic deformation stops before the length l_2 undergoes full plastic deformation. If it was found that $r'' > R$, the whole conical part has undergone plastic deformation and analysis should be continued with the deformation of the cylindrical part, then v_{b_0} used in Eqs. (6.18) and (6.19) for the analysis of plastic deformation of the cylindrical part is equal to v_c^d (Eq. (6.38)) at $r = R$, i.e. $v_{b_0} = v_c^d|_{r=R}$, meaning that plastic deformation of the cylindrical part will be continued with velocity of the elastic remainder at the end of the plastic deformation of the conical part; therefore v_0/k_{eq} is substituted with $(v_c^d|_{r=R}) - v_{t_{eq}}$ in Eqs. (6.21), (6.22), (6.24) and (6.26) and $v_{t_{eq}}$ has the same initial value used in the plastic deformation of the conical part.

6.2.2.2 Radius and length of deformed portion of conical projectile

Similar to Eq. (6.23), at any cross-section of radius, r , ($r' < r < R$), shown in Figure 6.5 (b), and area $A_r = \pi r^2$, the relationship below holds:

$$v_{r,c}^d = U_p \left(1 - \frac{A_r}{A_e} \right) \quad (6.41)$$

where A_e is the deformed (enlarged) cross-sectional area (at times $t' < t < t''$).

Substituting Eq. (6.39) into Eq. (6.41) results in an expression for A_r/A_e as:

$$\frac{A_r}{A_e} = 1 - \frac{v_0}{k_{eq}U_p} - Q \ln \left(\frac{R^3(3l_1 + l') - 3R^2l_1r' - l'r^3}{(R-r')(R^2(3l_1 + l') + l'(Rr' + r'^2))} \right) \quad (6.42)$$

where $Q = \sigma_e / \rho_p U_p^2$, r' is the radius of the conical tip and l' is the length of the conical part of the projectile at the beginning of the plastic deformation. The radius of the enlarged cross-section at any time $t' < t < t''$, can be found from Eq. (6.42).

The material under deformation within the increment dx , shown in Figure 6.5 (b), is given by:

$$dx = (U_p - v_{r,c}^d) dt \quad (6.43)$$

Combining Eqs. (6.29), (6.41) to (6.43) and then integrating with the end points for dx from 0 to x_c and for dr from r' to r , the length of the deformed portion of the projectile, x_c shown in Figure 6.5 (c), is given by:

$$\frac{x_c}{l'} = \frac{r'' - r'}{R - r'} \left(1 - \frac{v_0}{k_{eq} U_p} \right) - \frac{Q}{R - r'} \int_{r'}^{r''} \ln \left(\frac{R^3 (3l_1 + l') - 3R^2 l_1 r' - l' r^3}{(R - r') (R^2 (3l_1 + l') + l' (Rr' + r'^2))} \right) dr \quad (6.44)$$

where r'' is the radius wherein plastic deformation stops and l' is the length of the conical part at the beginning of the plastic deformation. The integral in Eq. (6.44) needs to be evaluated numerically.

6.3 Comparison of results of model with experiments

The comparison between results obtained from the proposed models, for impacts of blunt and conical projectiles on ceramic–metal armor and experiments is presented in following sections.

6.3.1 Blunt projectile

Reijer [23] performed experiments using blunt steel projectile of 28 Rockwell hardness (HRC) and 7 g mass impacting bi-layer high purity alumina–aluminum alloy 6061-T6 armor. Figure 6.6 (a) shows schematic of a blunt projectile impacting a bi-layer target.

Figure 6.6 Schematic of (a) blunt and (b) conical projectile impact onto bi-layer armor target

Fixed alumina thickness, T_1 of 8.1 mm and variable aluminum alloy 6061-T6 thicknesses, T_2 of 4 mm, 6 mm and 8 mm were used in the experiments [23]. Table 6.1 lists some properties of the steel (projectile material), alumina and aluminum 6061-T6.

Table 6.1 Properties of the steel projectile, Hilcox 973 high purity alumina and aluminum alloy 6061-T6 [23, 160, 165]

Material	Steel	Alumina	Aluminum 6061-T6
Projectile radius (mm)	3	-	-
Projectile length (mm)	31.5	-	-
Density (kg/m^3)	7800	3810	2705
Bulk modulus (GPa)	-	242	72.8
Young's modulus (GPa)	-	378	73.5
Longitudinal wave speed (m/s)		10820	-

Reijer [166] found the projectile dynamic yield strength, σ_e , of 1.7 GPa by fitting polynomials to experimental projectile rear-end positions, which is close to the values of 1.9 GPa and 1.965 GPa given by Roseberg and Tsaliah [167] and Recht [91], respectively, for steels of 30 HRC.

The value of 1.7 GPa was used for σ_e in our calculations. Plastic wave velocity, $U_p = 500$ m/s was used for the projectile; this value was used by Zhang et al. [78] for the same material. Reijer [23] provided the raw data for variation of length of the projectile through time obtained from X-ray photographs. The original projectiles together with the remaining one collected after the impact experiments performed by Reijer [23] are shown in Figure 6.7. It can be seen in Figure 6.7 that the deformed radius

of the projectile in some cases partly and in some others completely has been sheared off.

Figure 6.7 Original projectile and the one at the end of the impact obtained from the impact experiments performed by Reijer [23] using blunt projectile of different impact velocities (V_0) onto high purity alumina–aluminum alloy 6061-T6 armor composed of fixed 8.1 mm front plate thickness (T_1) and different backing plate thicknesses (T_2)

Figure 6.8 shows the variation of projectile length with time obtained from the current model compared with experimental measurements of Reijer [23]. It can be seen that the model can predict the final length of the projectile very well. The cases shown in Figure 6.8 (e) and (f) though having same armor system, the residual length of the projectile for case (f) with impact velocity of 1091 m/s is not shorter than that of the case in Figure 6.8 (e) with impact velocity of 995 m/s, which is contrary to the expectation. One possible reason for this discrepancy could be the presence of flaws in the ceramic resulting in degradation of the strength of the ceramic in the course of interaction with projectile during impact.

Figure 6.8 Comparison of the results obtained from current model and experimental measurements [166] for blunt projectile impact with different velocities on high purity alumina–aluminum alloy 6061-T6 armor composed of 8.1 mm front plate thickness and backing plate thickness of: (a) and (b) 4 mm, (c) and (d) 6 mm, and (e) and (f) 8 mm thicknesses, respectively

Figure 6.9 shows the length, specified by vertical axis and radius, specified by horizontal axis, of the projectiles at the end of the impact calculated based on the current model (without considering shear mass loss). The solid line (without solid circles)

shows the non-deformed length and the one with solid circles shows the deformed part of the projectile. The solid circles at any cross-section relates to the gradual formation of the deformed radius from the beginning to end of the deformation.

Figure 6.9 Calculated length of the projectile at the end of the impact obtained from the current model for blunt projectile impact with different velocities onto high purity alumina–aluminum alloy 6061-T6 armor system composed of 8.1 mm front plate thickness and backing plate thickness of: (a) and (b) 4 mm, (c) and (d) 6 mm, and (e) and (f) 8 mm thicknesses, respectively

Figure 6.10 shows the original and final projectile after impact obtained from current model calculations. It can be seen that except for the case with impact velocity of 1091 m/s, the correlation between the original and final projectile in Figure 6.10 compared with the experimental counterparts in Figure 6.7 is very good.

Figure 6.10 Original projectile and the final one obtained from the current model for blunt projectile impact with different velocities onto high purity alumina–aluminum alloy 6061-T6 armor system composed of 8.1 mm front plate thickness and backing plate thickness of: (a) and (b) 4 mm, (c) and (d) 6 mm, and (e) and (f) 8 mm thicknesses, respectively

6.3.2 Conical projectile

Figure 6.6 (b) schematically shows a conical projectile impacting a bi-layer target. Wilkins [162] performed impact experiments using conical projectiles made of Allegheny steel 609 of 54-56 HRC, cone angle of 55° and mass of 8.12 g [168]. Impact on two bi-layer armors composed of ceramics, namely, beryllium oxide (BeO) and boron carbide (B_4C) backed by aluminum alloy 6061-T6 are chosen for comparison.

Figure 6.11 [25] shows an example of the the X-ray images of impact on bi-layer 7.24 mm B_4C –6.35 mm aluminum alloy 6061-T6 armor system, performed by Wilkins [162] and reproduced by Anderson Jr and Walker [25].

Figure 6.11 X-ray images for steel projectile of 701 m/s velocity impacting 7.24 mm B_4C –6.35 mm aluminum alloy 6061-T6 system at (a) $1.8 \mu s$, (b) $3.8 \mu s$, (c) $8.9 \mu s$, (d) $11.8 \mu s$, (e) $15.8 \mu s$, (f) $19.8 \mu s$, (g) $25.2 \mu s$ and (h) $35.5 \mu s$ from the experiments of Wilkins [162] reported by Anderson Jr and Walker [25]

As the author could not find the explicit changes of length of the projectile with time through the impact event for the experiments performed by Wilkins [162], an attempt was made to compare the calculated length and diameter of the projectile in the course of impact event at the same times as shown in Figure 6.11 for qualitative comparison purpose.

It should be noted that Andeson Jr and Walker [25] reported the length of $L_i^c = 28.1$ mm and mass of 8.13 g for the 0.30 caliber conical projectile (with a cone angle of 55°); Holmquist et al. [160] reported the length of $L_i^c \simeq 29$ mm, having the mass of 8.32 g same to a 0.30 caliber blunt projectile of $L_i^b \simeq 24$ mm length; and Wilkins [168] reported the mass of 8.12 for the 0.30 caliber conical projectile.

For the current model, a 0.30 caliber conical projectile of 8.12 g mass, 7790 kg/m^3 density, 55° cone angle, $r_1 = 0.1$ mm initial cone radius and $R = 3.81$ mm final cone radius, was considered for which the length is $L_i^c = 28.75$ mm, composed of lengths of the cylindrical part, $l_1 = 19.80$ mm and conical part, $l_2 = 8.95$ mm, respectively (corresponding to a 0.30 caliber blunt projectile of 8.12 g mass and $L_i^b = 22.86$ mm or 0.9 inch). Properties of the steel projectile, B₄C and BeO ceramics are listed in Table 6.2 and those of aluminum backing plate are listed in Table 6.1.

Table 6.2 Properties of the steel projectile [168] and ceramics [9] used in experiments of Wilkins [162]

Material	Steel	BeO	B ₄ C
Projectile mass (g)	8.12	-	-
Density (kg/m^3)	-	2840	2500
Bulk modulus (GPa)	-	185	196
Young's modulus (GPa)	-	376	422
Longitudinal wave velocity (m/s)	-	11500	13300

It is expected, as Recht [91] postulated, that projectile material of 54-56 HRC, with higher dynamic yield strength compared to the one of 28 HRC used in the experiments of Reijer [23], has comparatively higher plastic wave velocity. Plastic wave velocity, $U_p = 600$ m/s was chosen for the projectile. Dynamic yield strength, $\sigma_e = 2.1$ GPa was used for the projectile; a value used by Anderson Jr and Walker [25] for the same projectile.

Figure 6.12 shows the projectile length, calculated based on the current model (without considering shear mass loss), at the same times as the X-ray image has been taken in the experiments [162] shown in Figure 6.11.

Figure 6.12 Calculated length of the projectile at (a) $1.8 \mu s$, (b) $3.8 \mu s$, (c) $8.9 \mu s$, (d) $11.8 \mu s$, (e) $15.8 \mu s$, (f) $19.8 \mu s$, (g) $25.2 \mu s$ and (h) $35.5 \mu s$ (same times as the those of Figure 6.11) obtained from the current model for impact of a conical projectile of 701 m/s velocity impacting 7.24 mm B₄C–6.35 mm aluminum alloy 6061-T6 system

Figure 6.13 shows the length, specified by vertical axis and radius, specified by horizontal axis, of the projectiles at the same times that X-ray image has been taken in the experiments [162] shown in Figure 6.11, calculated based on the current model (without considering shear mass loss).

The solid line (without solid circles) shows the non-deformed length and the one with solid circles show the deformed part of the projectile. The solid circles at any cross-section relate to the gradual formation of the deformed radius from the beginning to end of the deformation. It can be seen that the tip of the projectile undergoes plastic deformation from the beginning of the impact and gradually the radius and length of the deformed part increases. Once the whole conical part is deformed, the plastic deformation of the cylindrical part initiates, the radius of the deformed part is decreased

and its length is increased. When the radius of the deformed part in the cylindrical region of the projectile reaches the radius of the projectile, plastic deformation ends.

A careful observation on Figure 6.11 (c), (d) and (e) at times $8.9\mu s$, $11.8\mu s$ and $15.8\mu s$ reveals the presence of very thin rings of deformed material with radii larger than the radius of the projectile ejected laterally out of the projectile. The results obtained from the model shown in Figure 6.12 also reflects the same behavior for the observed deformed material. It can be seen that the behavior of the projectile in the course of impact obtained from the current model agrees qualitatively well with the one observed in experiments of Wilkins [162], shown in Figure 6.11.

In order to have a clearer observation of the sequence of plastic deformation, Figure 6.13 with magnified deformed part for the projectile is shown in Figure 6.14 wherein the vertical axis has a higher resolution for the deformed length and for the remaining non-deformed length there is a break in the axis and also in the projectile itself.

Figure 6.13 Calculated length of the projectile at (a) $1.8 \mu s$, (b) $3.8 \mu s$, (c) $8.9 \mu s$, (d) $11.8 \mu s$, (e) $15.8 \mu s$, (f) $19.8 \mu s$, (g) $25.2 \mu s$ and (h) $35.5 \mu s$ (same as the ones of Figure 6.11) obtained from the current analytical model for impact of a conical steel projectile with velocity of 701 m/s onto bi-layer 7.24 mm B₄C–6.35 mm aluminum alloy 6061-T6 system

Figure 6.14 Magnified tip of the conical projectile at (a) $1.8 \mu s$, (b) $3.8 \mu s$, (c) $8.9 \mu s$, (d) $11.8 \mu s$, (e) $15.8 \mu s$, (f) $19.8 \mu s$, (g) $25.2 \mu s$ and (h) $35.5 \mu s$ obtained from the current analytical model for impact of a conical steel projectile with velocity of 701 m/s onto bi-layer 7.24 mm B_4C -6.35 mm aluminum alloy 6061-T6 system

Figure 6.15 shows X-ray images taken by Wilkins [9] at different times, for steel projectile of 731.5 m/s velocity impacting bi-layer 6.35 mm BeO–6.35 mm aluminum alloy 6061-T6 armor system.

Figure 6.15 X-ray images for steel projectile of 731.5 m/s velocity impacting bi-layer 6.35 mm BeO–6.35 mm aluminum alloy 6061-T6 armor system at different times from the experiments of Wilkins [9]

Figure 6.16 shows the length, specified by vertical axis and radius (without considering shear mass loss), specified by horizontal axis, of the projectiles at the same times as X-ray images [9], shown in Figure 6.15. It can be seen that the results obtained from the current model, shown in Figure 6.16, correlate qualitatively well with the ones in the experiments of Wilkins [9] in Figure 6.15.

Figure 6.16 Calculated length of the projectile at (a) $2 \mu s$, (b) $5 \mu s$, (c) $10 \mu s$, (d) $15.1 \mu s$, (e) $20 \mu s$, (f) $25.1 \mu s$, (g) $30 \mu s$ and (h) $40 \mu s$ (same as the ones of Figure 6.15) obtained from the current analytical model for impact of a conical steel projectile with velocity of 731.5 m/s onto bi-layer 6.35 mm BeO–6.35 mm aluminum alloy 6061-T6 system

6.4 Discussion

It was assumed in the current proposed model that the projectile undergoes erosion and/or plastic deformation without fracture. However, it has been shown [85] that projectile can show different deformation and fracture modes of behavior through the impact. The results of experiments, shown in Appendix A, using hardened 4340 steel projectile of 52 HRC in impact of bi-layer alumina 95%–aluminum alloy 2024-T3 armor system, also show that the projectile does not accommodate much ductile behavior and in some cases fractures through the impact. This can be compared with the behavior of the Allegheny steel 609 of 54-56 HRC used in impact experiments onto different bi-layer ceramic–metal armor systems performed by Wilkins [9, 162] showing clear ductile behavior. This difference in behavior can be attributed to the carbon contents of the two projectiles, implying that the hardness and toughness of the projectile, both can affect the projectile behavior.

The plastic wave velocity is generally defined as $U_p = \sqrt{\frac{1}{\rho} \frac{\partial \sigma}{\partial \varepsilon}}$, in which $\frac{\partial \sigma}{\partial \varepsilon}$ is the tangent modulus in the region of plastic behavior of the material. Recht [91], same as Taylor [89], assumed bi-linear engineering stress-strain behavior, for which plastic wave velocity of the projectile material is constant. If stress-strain behavior of the projectile material is far from the bi-linear behavior, the results obtained from the model can deviate from the experiments.

In the Recht's model [91] (Eqs. (6.23) and (6.41)) one dimensional motion of the waves through the projectile is assumed without considering the inertia (rigidity) of the lateral motion of the deformed material. As for the equation of motion of the projectile, Recht [91] assumes that the constant stress equal to the dynamic yield strength of the projectile, σ_e is applied on the cross-section undergoing erosion (Eq. (6.8)) and/or plastic deformation (Eq. (6.17)). An alternative method which can be explored is using a pressure term which is not necessarily constant. Hopkinson [169] considered the plastic deformation through impact of lead bullets onto heat treated steel rod targets (which can be considered almost rigid). Hopkinson [169] showed that the total pressure at the cross-sectional area of the projectile is the sum of two terms: (a) the term λV^2 , in which λ is mass per unit length of the section under deformation and V is the velocity of that

section (which is continuously decreasing through the impact) and (b) the pressure term which is proportional to the cross-sectional area of the projectile; the former term is interpreted as hydrodynamic pressure for resisting the momentum of the section of the projectile, if considered as fluid, and the latter considers the rigidity of the projectile which results in gradual retardation of the deformed part rather than instantaneous stoppage of the section, as it would if projectile was quite fluid. It should be noted that for a deformable target material, the velocity, V in the hydrodynamic pressure, λV^2 should be substituted by relative velocity between the elastic remainder of the projectile and projectile–target interface velocity. However, as the mass per unit length, λ for a conical projectile is not constant compared to the constant one of the blunt projectile, application of pressure instead of constant dynamic yield strength of the projectile in the equation of motion, results in a more complicated scenario which is left for future work.

It should be kept in mind that the impedance matching factor proposed by Recht [91] (Eq. (6.3)), does not take into account the layering of the target, as Recht [91] considered the target as half space and ignored the wave dispersion in it. The constant equivalent matching factor, k_{eq} proposed in the current work, is an approximate method in which the effect of layering of the target has been incorporated; the purpose for this approximation has been to somehow consider the effect of various thicknesses of the layers on the impact process. As from the beginning of impact, the strength of the front layer is degraded, thereby reducing the effective thickness with the penetration of projectile into the front plate, it is expected that the equivalent matching factor, which has been considered constant in the current model, should be changed through time. It is known that failure mechanisms of the ceramic front plate of brittle nature and metallic backing plate of ductile nature are very different. It is believed that if the current model is incorporated into a model considering the failure mechanisms of the front and backing plate and also temporal variation of the plate thicknesses such as the one proposed by Zaera [77], then the effect of layering can be separated from the impedance matching factor.

6.5 Summary

In this chapter, Recht's model [91] was reviewed and extended accounting for erosion and plastic deformation mechanisms of blunt and conical projectile impact onto bi-layer

ceramic–metal armor systems. The projectile length, including deformed and non-deformed part and the time taken for the erosion and plastic deformation, were determined. These results showed a good comparison with literature experiments for both blunt and conical projectile impact on ceramic–metal armor. Finally, the assumptions considered in the model were discussed and an alternative method for considering the force applied on the cross-sectional area of the projectile was suggested.

Chapter 7 Influence of Different Pre-Stress Types on Ballistic Performance of Confined Ceramic

In this chapter, a new method for simulation of pre-stressed ceramic targets under high velocity impact, using explicit software, AUTODYN[®], is proposed. Validation of the proposed numerical model is performed through comparison of simulated results with the available experimental data in the literature. Impact simulations of confined ceramic targets with and without pre-stress of three different types, namely radial, axial and hydrostatic, with varied amounts are carried out to find the effect of pre-stressing on the ballistic behavior of the thick ceramic armor.

7.1 Experimental details [103]

Lundberg and co-authors [103] carried out long rod impact experiments on confined pre-stressed thick SiC target. The SiC target was confined by a tube made of maraging steel (Mar 350) with front and rear plugs made of tempered steel (SIS 2541-3, comparable to AISI/SAE 4340) and impacted by tungsten rod, as shown in Figure 7.1.

Figure 7.1 Geometries of long rod, confinement and SiC ceramic [10]

The pre-stress was applied on the SiC target through shrink fitting, by heating the tube to about 475°C before inserting the ceramic target. The front and rear plugs were mounted after cooling and locked to the tube by threads and welding. The inner diameter, D_C and length, L_C of the tube confinement were both 20 mm and its

thickness, T_c was 4 mm. The cylindrical tungsten rod projectile was of 2 mm diameter, 80 mm length and made to impact the target at different velocities. The SiC target diameter, D_T and length, L_T were set as different values to produce desired pre-stress types.

The numerical model, material constants and pre-stress modelling technique are described in the following sections.

7.1.1 Numerical model

Two-dimensional (2D) axisymmetric models of the confined SiC target and tungsten rod were created in AUTODYN[®] software as shown in Figure 7.2. The SiC target was discretized using four nodes 2D axisymmetric Lagrangian elements of 0.15 mm size with one integration point. The long tungsten rod projectile was modelled using SPH method with particle size of 0.125 mm. The front and rear plugs were divided into two parts with same thickness of 4 mm in the axial direction: the inner part, close to the SiC target, was modelled using SPH and the outer part was modelled using Lagrangian elements. The two parts of plugs were joined together at their interface using “JOIN module” in AUTODYN[®]. According to the actual target structure, the tube was also joined with front and rear plugs, while the SiC target was just in contact with the confinement and plugs without any join. There is an important advantage to use SPH method for discretizing inner parts of the plugs rather than Lagrangian method. The Lagrangian elements must be eroded under large distortion leading to a void with zero pressure allowing the surrounding material to expand into it with further pressure loss [170]. The ceramic material strength and failure characteristics are pressure dependent. Therefore, the elements erosion in the plug nearby SiC target can lead to the lower material strength exposing it to unrealistic damage initiation. This phenomenon will not occur if SPH domain is located nearby the SiC target. The SPH particle size in the inner plug part is 0.125 mm and Lagrangian elements in the outer plug part is 0.2 mm except for the two layers of elements nearby the SPH part. The axial element dimension for these two layers is 0.1 mm to make the layer, joined with SPH part, finer.

Figure 7.2 Two-dimensional (2D) axisymmetric model for the projectile and the target

The square Lagrangian element size is 0.2 mm in tube except for the four positions, A, B, C and D as marked in Figure 7.2. In the positions A and D, the size of the elements is same as that of the elements at the same axial position in the corresponding plug part. In position B and C, the 0.2 mm square element was divided into two elements in axial direction, one axial dimension is 0.06 mm and the other is 0.14 mm. By discretizing elements in this way, the edges of plug part modelled by SPH and SiC target can be located corresponding to different elements in the tube. Otherwise, the elements in the tube will distort drastically during the simulation.

In the AUTODYN[®] simulations, the external gap interaction (see section 2.4.2) was used for defining the contact between the parts which were not joined to each other in the experiments of Lundberg et al. [103], namely between ceramic and plugs and between ceramic and tube. The minimum gap size between two different Lagrangian parts used was 1/10th of the minimum element size (default value). The minimum gap size between a Lagrangian part and a SPH part used was half the SPH particle size.

7.1.2 Modelling of the pre-stress

A numerical technique for pre-stressing SiC target was developed in AUTODYN[®]. The pre-stress is produced while the pre-compressed SiC cylinder interacts with the confinement. This basic mechanism is similar to that proposed by Holmquist and Johnson [10]. The numerical technique used to apply pre-stress is performed in six steps:

- 1) Target components (SiC target, steel Mar 350 tube and two steel 4340 plugs) are created. Damping parameters are set for subsequent static process. These four components are put in separated positions from each other, therefore, the numerical contacting and joining settings for components interaction are not required in this step. The initial dimension(s) of SiC target is larger than the confinement inner size(s) for producing pre-stress. The radial ratio, $R_r = D_T/D_C$ is defined as the ratio of the SiC target initial diameter, D_T , to confinement inner diameter, D_C and axial ratio, $R_a = L_T/L_C$ is the ratio of the SiC target initial length, L_T to confinement inner length, L_C , respectively. Larger values of R_r and R_a results in higher pre-stress applied on the target. Different combinations of radial and axial ratio can produce different pre-stress types in the SiC target. In other words, since the confinement inner diameter and length are constant, the initial SiC target diameter and length can control the pre-stress value and type.
- 2) For radial (axial) pre-stress, an inward velocity boundary condition of -2 m/s is set on the radial (axial) periphery of SiC target. Meanwhile, the movement of SiC in the axial (radial) direction during the compression process was fixed. It should be noted, as Holmquist and Johnson [10] also stated, the radial (axial) pre-stress is not purely radial (axial) due to the Poisson's effect resulting in the creation of some axial (radial) stress. For hydrostatic pre-stress, first axial (axial velocity while radial motion of the target is fixed) and then radial (radial velocity while axial motion of the target is fixed) loading is applied. Therefore, hydrostatic pre-stress condition is obtained by using same values of R_r and R_a , i.e. $R_r = R_a$. Because of the confinement deformation, the hydrostatic pre-stress is not purely hydrostatic. The SiC target is compressed till its edge(s) reach the position which is slightly smaller than the tube inner dimensions; since the compression process (downward velocity boundary condition) is done dynamically, the static damping parameter is set to be 0.001 in order to eliminate the stress oscillations. The boundary condition (radial or axial velocity) is then changed from -2 m/s to 0 m/s (all SiC edges are fixed) and simulation is run until the pressure in the SiC target becomes uniform. The pressure variation at the center of the SiC target in this step is shown in Figure 7.3 for the radially pre-stressed SiC target of $R_r = 1.0035$ radial ratio. The radial ratio chosen to

be $R_r = 1.0035$ which is same as the one used in the experiments of Lundberg et al. [103].

Figure 7.3 Temporal variation of pressure measured at the center of the radially pre-stressed SiC target, of $R_r = 1.0035$ radial ratio, during pre-stressing

- 3) All the target components are put in their correct position.
- 4) Releasing process: All the boundary conditions on the SiC target are removed and all the components are activated (before final impact simulation) and the model is run for a short time, thereby SiC target increases in size in axial and/or radial direction until it comes in contact with the inner surface of the confinement and pre-stress is obtained. Static damping is set to 0.16 at the beginning of this step and gradually during time intervals is decreased. In this process (and subsequent steps), the quadratic and linear viscosity parameters in SPH solver option is set to be 1 and 5, respectively, in order to eliminate the stress oscillations.
- 5) Static damping parameter is gradually reduced to 0, because the long rod projectile in subsequent projectile impact step cannot move if static damping value is larger than 0.
- 6) Initial projectile impact velocity is set and impact simulation is performed.

7.1.3 Material models and validation of numerical simulations

Quan et al. [104] indicated that the JH-1 approach for material failure is more accurate than the JH-2 approach for SiC. The JH-1 model, described in section 2.3.2, was used for SiC and its constants are listed in Table 7.1.

Table 7.1 JH-1 material and model constants for SiC

Material/Constants	SiC
Density, ρ_0 (Kg/m ³)	3215
Bulk modulus, K (GPa)	220
Pressure constant, K_2 (GPa)	361
Pressure constant, K_3 (GPa)	0
Bulking factor, β	1.0
Shear modulus, G (GPa)	193
Hugoniot elastic limit (<i>HEL</i>) (GPa)	11.7
Intact strength constant, S_1 (GPa)	7.1
Intact strength constant, P_1 (GPa)	2.5
Intact strength constant, S_2 (GPa)	12.2
Intact strength constant, P_2 (GPa)	10.0
Strain rate constant, C	0.009
Maximum fracture strength, $S_{\max}^f$ (GPa)	1.3
Failed strength constant, α	0.4
Normalized hydrostatic tensile limit, T^* (GPa)	-0.75
Principal tensile failure stress (GPa)	1.3
Damage constant, $\varepsilon_{\max}^f$	0.5
Damage constant, P_3 (GPa)	99.75
Fracture energy (J/m ²)	37.3

In the simulations performed by Holmquist and Johnson [10] and Quan et al. [104], the two JH-1 model constants, namely, the maximum strength of the failed material $S_{\max}^f$ and the maximum failure strain $\varepsilon_{\max}^f$ (or the damage constant ϕ), were obtained by matching the numerical predictions with the experimental results of Lundberg et al. [103]. It should be noted that $\varepsilon_{\max}^f$ and ϕ are related to each other according to JH-1 model equations, therefore, only one of them needs to be found [104]. Since the maximum failure strain $\varepsilon_{\max}^f$ is used in the AUTODYN[®] for JH-1 model, ϕ was not

considered. These parameters $S_{\max}^f$ and $\varepsilon_{\max}^f$ are also validated here using the same method:

The values of the maximum strength of the failed material $S_{\max}^f$ and the maximum failure strain $\varepsilon_{\max}^f$ can be determined by matching the numerical penetration depths into the SiC target with those from the two experiments [103] in which the SiC target was under tungsten rod impacts of 1645 m/s and 2175 m/s velocities. The modified value for $\varepsilon_{\max}^f = 0.5$ was determined and $S_{\max}^f = 1.3$ GPa was used in simulations. The JC constitutive and fracture models, described in section 2.3.1, were used for the tungsten projectile [122] and steel 4340 plugs [44] with material constants listed in Table 7.2.

Table 7.2 JC material and model constants for tungsten alloy [122] and steel 4340 [44]

Material/Constants	Tungsten alloy	Steel 4340
Density, ρ_0 (Kg/m ³)	17600	7830
<u>EOS</u>	<u>Shock</u>	<u>Linear</u>
Bulk modulus, K (GPa)	285	159
Gruneisen constant	1.54	-
Reference temperature, t_0 (K)	300	300
Parameter C_1 (m/s)	4029	-
Parameter S_1	1.237	-
Specific heat, C_t (J/Kg·K)	134	477
<u>Strength model</u>	<u>JC</u>	<u>JC</u>
Shear modulus, G (GPa)	160	77
Static yield strength, A' (GPa)	1.506	0.75
Strain hardening constant, B' (GPa)	0.177	0.51
Strain hardening exponent, n	0.12	0.26
Strain rate constant, C'	0.016	0.014
Reference strain rate, $\dot{\varepsilon}_0$	1	1
Thermal softening exponent, m	1	1.03
Melting temperature, t_m (K)	1723	1793
<u>Fracture model/criterion</u>	<u>JC</u>	<u>JC</u>
Damage constant, d_1^f	0	0.05
Damage constant, d_2^f	0.33	3.44
Damage constant, d_3^f	-1.5	-2.12
Damage constant, d_4^f	0	0.002
Damage constant, d_5^f	0	0.61

Some of the constants were modified slightly from original published values [44, 122] to the values of the materials used in experiments [103]. In doing so, the density, yield strength and bulk modulus of the tungsten alloy were changed to 17600 kg/m³, 1.2 GPa

and 285 GPa, respectively. The yield strength for the steel 4340 plugs was set to 0.75 GPa.

The elastic-perfectly plastic constitutive model and effective plastic strain fracture model were used for steel Mar 350 tube [103, 104]. The constants used for steel tube are given in Table 7.3. Geometric erosion model is selected for removing the elements with large distortion. Erosion strain used for SiC, steel 4340 and steel Mar 350 are all equal to 2. As the projectile was modelled using SPH method, no erosion model was needed for modelling the tungsten alloy projectile.

Table 7.3 Material and model constants for steel Mar 350 [103, 104]

Material/Constants	Steel Mar 350
Density, ρ_0 (Kg/m ³)	8080
Bulk modulus, K (GPa)	140
Shear modulus, G (GPa)	77
Yield stress (GPa)	2.6
Failure strain	0.4

For numerical model validation, three representative cases from the test number 230, 226 and 234 [103] were selected for comparing the experimental measurements with simulation results. The impact velocities in these three cases are 1410 m/s, 1645 m/s and 2175 m/s, respectively. The radial and axial ratios are chosen to be $R_r = 1.0035$ and $R_a = 1$, respectively, which is the same as the radially pre-stressed SiC target in the experiments [103].

7.2 Comparison of simulation results of radially pre-stressed ceramic with the experimental measurements

The comparison of penetration into radially pre-stressed SiC target of radial pre-stress ratio $R_r = 1.0035$, as a function of time obtained from simulations and experimental measurements [103] is shown in Figure 7.4. The results obtained from simulations of Holmquist and Johnson [10] are also presented in Figure 7.4 for comparison purpose.

It can be seen that the results obtained from current simulations are in excellent agreement with experimental measurements for all the three velocities, whereas

computational results of Holmquist and Johnson [10] are matched well only with the case of 2175 m/s.

Figure 7.4 The comparison of penetration into radially pre-stressed SiC target of radial pre-stress ratio $R_r = 1.0035$, as a function of time impacted at three velocities of 1410 m/s, 1645 m/s and 2175 m/s, obtained from current simulations and experimental measurements [103] together with simulation results from Holmquist and Johnson model [10]

In the following section the influence of pre-stress on interface dwell time is presented.

7.3 Radial pre-stress influence on ballistic behavior of confined SiC target and interface dwell time

Material damage for the SiC targets without pre-stress ($R_r = R_a = 1$) and also with radial pre-stress ratio of $R_r = 1.0035$, under three different impact velocities, namely 1410 m/s, 1645 m/s and 2175 m/s, are shown in Figure 7.5.

At the impact velocity of 1410 m/s, at time $t = 36\mu s$, there is no penetration into the SiC for the two target conditions (with and without pre-stress). This phenomenon is referred to as interface defeat [101]. There is more damage in the SiC for target without pre-stress, particularly at the interface of the SiC and rear plug.

For the impact velocities of 1645 m/s and 2175 m/s at times $t = 36\mu s$ and $t = 18\mu s$, respectively, the SiC targets with radial pre-stress exhibit less amount of penetration and

smaller extent of damage, either at the impact side or at the rear side of the target, compared, at the same times, to the SiC targets without pre-stress. The main reason for the difference in penetration is due to the amount of dwell time that occurs.

Figure 7.5 Damage contours in the confined SiC targets (a) without pre-stress ($R_r = R_a = 1$) and (b) with radial pre-stress ($R_r = 1.0035$), under impact velocities of 1410 m/s at $t=36 \mu s$, 1645 m/s at $t=36 \mu s$ and 2175 m/s at $t=18 \mu s$, respectively

For the impact velocity of 2175 m/s, at time $t = 18\mu s$, as shown in Figure 7.5, the penetration into SiC target without pre-stress is larger by 1.17 mm than that for SiC target with pre-stress. However, the difference of penetration into the SiC targets with and without pre-stress is less than that for impact velocity of 1645 m/s. It can be also seen in Figure 7.5, at time $t = 18\mu s$, that for the impact velocity of 2175 m/s the amount of damage is more in SiC target without pre-stress than the one in target with radial pre-stress.

Figure 7.6 shows the influence of the radial pre-stress ratio (R_r) variation on the dwell time and penetration into the confined ceramics under fixed impact velocity of 1645 m/s. It should be noted that the dwell time in the current work includes two times: the time for front plug (cover plate) penetration plus the time during which the long rod flows radially on the SiC without penetrating it.

Figure 7.6 Influence of radial pre-stress ratio, R_r variation on the dwell time and penetration into the confined SiC target under fixed impact velocity of 1645 m/s

It can be seen in Figure 7.6 that the SiC target with radial pre-stress of $R_r = 1.0035$ dwells for about $3\mu s$ longer than the SiC target without pre-stress ($t = 16\mu s$ vs. $t = 19\mu s$) justifying the larger penetration observed for the latter case, under the impact

velocity of 1645 m/s shown in Figure 7.5 at time $t = 36\mu s$. The dwell time increases with increase of radial pre-stress value, thereby decreasing the penetration depth into the SiC target. For pre-stressed targets with radial pre-stress ratios, $R_r = 1.0035$ and $R_r = 1.01$, full penetration occurs. For pre-stressed target with $R_r = 1.015$, projectile is stuck in the SiC target after around $50\mu s$ and for pre-stressed target with $R_r \geq 1.02$ no penetration into the SiC target occurs.

It should be noted that there are several positions where SiC material damage is larger for targets with pre-stress at these three impact velocities than targets without pre-stress. These positions mainly are at the interface of SiC and tube. These larger damages are introduced by the pressure or stress concentration at these positions during pre-stressing and are not due to long rod projectile impact. These material damages seem to have negligible influence on the long rod projectile penetration.

As a conclusion, the radial pre-stress can improve the confined ceramic target ability of resisting damage and penetration under long rod projectile impact. It can be deduced from experimental results that for target without pre-stress, at impact velocities at which there is interface defeat (like 1410 m/s) or there is no dwell (like 2175 m/s), the effect of adding radial pre-stress on the target performance is small.

7.4 Effect of different pre-stress types on the ballistic behavior

Figure 7.7 shows the temporal variation of the penetration into confined SiC target of different pre-stress conditions: “no pre-stress” (confined SiC target without pre-stress for which $R_r = R_a = 1$), radial ($R_r = 1.0035$), axial ($R_a = 1.0035$) and hydrostatic ($R_r = R_a = 1.0035$), impacted with three different velocities: 1410 m/s, 1645 m/s and 2175 m/s. It can be seen that at impact velocity of 1410 m/s, the confined SiC target, irrespective of the pre-stress conditions (types), is not penetrated as the velocity is not high enough to overcome the strength of the target.

The high kinetic energy for the projectile of 2175 m/s impact velocity is the reason for earlier penetration and higher rate of penetration (tangent of DOP versus time) compared to the ones of 1645 m/s.

Figure 7.7 Temporal variation of penetration into SiC target of different pre-stress types: “no pre-stress” ($R_r = R_a = 1$), radial ($R_r = 1.0035$), axial ($R_a = 1.0035$) and hydrostatic ($R_r = R_a = 1.0035$), under three different impact velocities: 1410 m/s, 1645 m/s and 2175 m/s

At impact velocity of 2175 m/s, penetration initiates almost at the same time (around 7 μs) and, irrespective of the pre-stress types. At times greater than around 14 μs (for the same impact velocity of 2175 m/s), during which the projectile has already penetrated into the SiC target, the influence of various pre-stress condition on the rate of penetration can be distinguished: “no pre-stress” and axial show almost same rate of penetration; radial and hydrostatic penetration also show almost same rate of penetration, though lower than the one for the “no-pre-stress” and axial types. The kinetic energy of the long rod projectile of 2175 m/s impact velocity at the beginning of the penetration into the SiC target is so high that none of the pre-stress types show effective role in hindering the projectile; however at later times when long rod projectile has lost some kinetic energy, the influence of the radial and hydrostatic pre-stress types on the rate of penetration, is more obvious, implying that these two pre-stress types are more effective, compared to axial and “no pre-stress” types, in retarding the long rod projectile. This fact is also clearly seen for the cases with impact velocity of 1645 m/s.

At impact velocity of 1645 m/s, penetration into SiC targets with axial pre-stress and “no pre-stress” starts at around 15 μs and 16 μs , respectively and penetration continues for “axial” type with higher rate compared to the one of “no pre-stress” type; for SiC

targets with radial and hydrostatic pre-stress types penetration starts at around $19 \mu s$ and continues with a slightly lower rate for hydrostatic pre-stress when compared to radial pre-stress case. The behavior observed in Figure 7.7 for SiC targets of different pre-stress types can be explained by studying the pressure contours in the targets, as explained by Holmquist and Johnson model [10].

Figure 7.8 shows the pressure contours in the SiC targets (confinements are not shown) with different pre-stress types: (a) “no pre-stress” ($R_r = R_a = 1$), (b) axial ($R_a = 1.0035$), (c) radial ($R_r = 1.0035$) and (d) hydrostatic ($R_r = R_a = 1.0035$). The axes of symmetries (which are parallel to impact direction) are also shown in Figure 7.8.

Figure 7.8 Pressure contours in the SiC targets (confinements are not shown) with different pre-stress types: (a) “no pre-stress” ($R_r = R_a = 1$), (b) axial ($R_a = 1.0035$), (c) radial ($R_r = 1.0035$) and (d) hydrostatic ($R_r = R_a = 1.0035$)

It can be seen in Figure 7.8 (b) that along the axis of symmetry, the pressure at the surface of the SiC (SiC-plug interfaces) is negative (tension) while pressure in the radial direction is positive (compression). This is due to the smaller initial geometry of the SiC in axial direction and deformation of the confinements (front, rear plugs and tube) through axial pre-stressing such that the top and rear plug are not in full contact with ceramic at the SiC-plug interfaces along the axis of symmetry, thereby decreasing the pressure at this surface; while at the radial direction the tube deforms such that pressure at the SiC-tube interface increases. The presence of negative pressure at the SiC-plug interfaces along the axis of symmetry, in Figure 7.8 (b), compared to zero pressure for the SiC target without pre-stress, in Figure 7.8 (a), justifies the higher rate of penetration for the SiC target with axial pre-stress than that of without pre-stress.

Figure 7.8 (c) shows that for SiC target with radial pre-stress, the pressure within the SiC target along the axis of symmetry is smaller compared to the one observed in Figure 7.8 (d) for the SiC target with hydrostatic pre-stress, justifying the larger rate of penetration through the SiC target with radial pre-stress, observed in Figure 7.7, compared to the rate of penetration under hydrostatic pre-stress. The difference in the observed pressure is due to the initial geometry of the SiC in radial and/or axial directions and deformation of the confinements through pre-stressing.

Figure 7.9 shows damage contours in the confined SiC targets of four different pre-stress types, namely “no pre-stress ($R_r = R_a = 1$), axial ($R_a = 1.0035$) radial ($R_r = 1.0035$) and hydrostatic ($R_r = R_a = 1.0035$), under impact velocity of 1645 m/s at three different times, $18 \mu s$, $25 \mu s$ and $35 \mu s$. It can be seen that at times $t = 18 \mu s$ and $t = 25 \mu s$, the damage at the impact side and also at the rear side of the SiC target without pre-stress is smaller when compared to the one for the SiC target with axial pre-stress. Larger extent of damage in the SiC target with axial pre-stress is due to the presence of negative pressure at both the SiC-front plug and SiC-rear plug interface resulting in the failure of the SiC target as it is weak in tension. At times $t = 18 \mu s$, in Figure 7.9, no damage is observed in the rear sides of the SiC targets with radial and hydrostatic pre-stress; however the damage zone in the SiC target with radial pre-stress

is comparatively smaller which is due to the presence of larger pressure at the SiC-front plug interface compared to the target with hydrostatic pre-stress.

Figure 7.9 Damage contours in confined SiC targets of four different pre-stress types: (a) “no pre-stress” ($R_r = R_a = 1$), (b) axial ($R_a = 1.0035$), (c) radial ($R_r = 1.0035$) and (d) hydrostatic ($R_r = R_a = 1.0035$) under impact velocity of 1645 m/s at different times: $18 \mu s$, $25 \mu s$ and $35 \mu s$, respectively

At times $t = 25 \mu s$ and $t = 35 \mu s$, in Figure 7.9, the extent of the damage and the penetration into the SiC target with radial pre-stress is larger than the ones under hydrostatic pre-stress due to the comparatively lower pressure within the target with

radial pre-stress, observed in Figure 7.8 (c), compared to the one with hydrostatic pre-stress, observed in Figure 7.8 (d).

The behavior of the confined SiC target of different pre-stress types can be reasonably explained considering the pressure at the interface and within the SiC targets. The pressure distribution affects both dwell on the impact surface and rate of penetration within the ceramic target.

7.5 Mechanisms occurring during transition from dwell to penetration

The mechanisms occurring during transition from dwell to penetration in sequence were reviewed in section 2.7.2. The accumulation of damage and softening of the ceramic material into a comminuted state before occurrence of penetration and backflow of the projectile material on the deformed surface of the ceramic material is seen in Figure 7.9 (c) and (d).

Figure 7.10 and Figure 7.11 show in more detail the transition period from dwell to penetration for three confined ceramic targets with different pre-stress types, namely “no pre-stress” ($R_r = R_a = 1$), axial ($R_a = 1.0035$), radial ($R_r = 1.0035$) and (d) hydrostatic ($R_r = R_a = 1.0035$), under impact velocity of 1645 m/s, respectively. The sequence of mechanisms shown in Figure 7.10 and Figure 7.11 are compatible with the available literature described in section in section 2.7.2. It can be seen that damage, once initiated below the impact site of the SiC material, propagates towards impact site with a rate faster than penetration rate, as also stated by Behner et al. [107]. The damage, once reached the impact site at the surface of the ceramic material, propagates into the ceramic material in both depth and radial direction with a fast rate probably due to the reason that the force exerted on the surface of the ceramic material is amplified with increase in the ceramic surface curvature resulted gradually in backflow of the projectile material.

The different temporal variations of the shape of the damage contours and various rate of propagation of the extent of the damage during transition from dwell to penetration for the confined SiC targets of different pre-stress types, shown in Figure 7.10 and Figure 7.11, can be reasonably justified, in the same manner explained for Figure 7.9,

based on the intensity of the pressure contours at the impact site and within the confined SiC target.

Figure 7.10 Damage contours in confined SiC targets of two different pre-stress types: (a) “no pre-stress” ($R_r = R_a = 1$) and (b) axial ($R_a = 1.0035$), under impact velocity of 1645 m/s

Figure 7.11 Damage contours in confined SiC targets of two different pre-stress types: (a) radial ($R_r = 1.0035$) and (b) hydrostatic ($R_r = R_a = 1.0035$), under impact velocity of 1645 m/s

7.6 Threshold velocity

A concept of “threshold velocity” is herein defined, as the minimum velocity for which no interface dwell occurs on the ceramic interface before penetration. Threshold velocity can be interpreted as “no dwell-transition velocity”. Experiments have shown the presence of such “no dwell-transition velocity” for SiC targets [118]. Threshold velocity, analogous to ballistic limit velocity (BLV) for thin armor under impact of small caliber threats, can be used as a measure for ballistic strength of the confined thick ceramic targets. Threshold velocity defined here can be used as a measure of efficiency for different pre-stress types with varying intensities in improving the ballistic behavior of the confined ceramic armor. Such a measure of has not been used in the literature.

Variation of the threshold velocity with applied axial, radial or hydrostatic pre-stress for confined SiC targets is shown in Figure 7.12. The peak threshold velocities for axial, radial and hydrostatic conditions are 1780 m/s, 1895 m/s and 1915 m/s, respectively.

Figure 7.12 Variation of threshold velocity with axial, radial or hydrostatic pre-stresses of different ratios

Figure 7.13 shows the pressure contours for radially pre-stresses confined SiC targets (confinements are not shown) of different radial pre-stress ratios. It can be seen that the

pre-stress increases up to radial ratio of $R_r = 1.02$ and for radial ratios above $R_r = 1.02$, the pressure drops in both radial and axial directions at the SiC–tube and SiC–plugs interfaces, respectively, due to the confinements deformation. By way of an example, in order to justify the observed trend in Figure 7.12, Figure 7.14 shows both the pressure contour and effective plastic strain contour in radially pre-stressed confined SiC targets for three different radial pre-stress ratios, namely $R_r = 1.015$, 1.02 and 1.025 . For $R_r = 1.02$, the SiC target has the largest pressure in the radial direction. The concentration of the plastic strain on the four corners of the target due to the “pinching” effect (as also seen in the work of Holmquist and Johnson [10]) of the SiC target is evident Figure 7.14.

The plastic strain value increases for radial pre-stress, R_r higher than 1.02 , such that steel tube displaces away from the SiC (as discussed in section 7.4), effect of confinements (both tube and plugs) on the SiC target decreases and pressure drops in radial direction at the SiC–tube interface and also in axial direction at the SiC–plugs interface. The pressure-drop in both radial and axial direction, due to the excessive deformation of the steel tube, is observed in Figure 7.13 (f) under $R_r = 1.035$ radial pre-stress ratio. The largest pressure in the radial direction of the SiC target under $R_r = 1.02$ radial pre-stress ratio, fortifies, like a backing support, the pressure at the impact site of the SiC target, thereby increasing the threshold velocity to the maximum value.

The trends observed for axially and hydrostatically SiC targets of different pre-stress ratios can be explained, in the same way as described for radially pre-stressed cases with the focus on confinement deformation and pressure contour in the pre-stressed target.

As seen in Figure 7.9 to Figure 7.11, during impact simulation of the pre-stressed targets, there are locations along the axis of symmetry below the impact site wherein damage initiates first. Gauge points were set at these locations before doing pre-stressing and pressure obtained at the end of pre-stressing (before impact simulation) at these gauge points were examined.

Figure 7.13 Pressure contour in the radially pre-stressed SiC targets (confinements are not shown) with different ratios: (a) $R_r = 1.01$, (b) $R_r = 1.015$, (c) $R_r = 1.02$, (d) $R_r = 1.025$, (e) $R_r = 1.03$ and (f) $R_r = 1.035$

Figure 7.14 Pressure contour and effective plastic strain contour in radially pre-stressed confined SiC targets (confinements are also shown) with different pre-stress ratios: (a) $R_r = 1.015$, (b) $R_r = 1.02$ and (c) $R_r = 1.025$

Figure 7.15 shows the variation of pressure at a gauge point which is firstly fully damaged during transition from dwell to penetration, on the axis of symmetry below the impact site of the SiC targets axially, radially or hydrostatically pre-stressed with different axial pre-stress ratios. It is observed from Figure 7.12 that pre-stressed targets of different pre-stress ratios imitate similar trend as shown in Figure 7.15, implying that pressure (obtained from pre-stressing) below and near to the impact site of the confined SiC target at the locations which initially experience damage is important in the transition behavior from dwell to penetration.

Figure 7.15 Variation of pressure at a gauge points, firstly fully damaged during transition from dwell to penetration, on the axis of symmetry below the impact site of the SiC target axially, radially or hydrostatically pre-stressed with different pre-stress ratios

Checking the pressure contours for hydrostatically pre-stressed targets revealed that the pressure below the SiC-plug interface (where damage initiates), except for the target of 1.0035 pre-stress ratio ($R_r = R_a = 1.0035$) with small difference to one of radially pre-stressed ($R_r = 1.0035$), is higher compared to the counterpart radially pre-stressed SiC targets. This implies that hydrostatic pre-stress is more effective than radial pre-stress (and also axial pre-stress) in improving the ballistic behavior of the confined thick ceramic armor. The reason for higher threshold velocity for radially pre-stressed SiC target of $R_r = 1.0035$ pre-stress ratio compared to hydrostatically pre-stressed SiC target of $R_r = R_a = 1.0035$ pre-stress ratio is that, as shown in Figure 7.8 (c) and (d), the

radially pre-stressed target had higher pressure near the impact site of the SiC target compared to the hydrostatically pre-stressed target. Figure 7.16 shows the pressure contours for hydrostatically pre-stressed SiC targets of different values.

Figure 7.16 Pressure contour in the hydrostatically pre-stressed SiC targets with different ratios: (a) 1.01, (b) 1.015, (c) 1.02, (d) 1.025 and (e) 1.03

Comparison of pressure contours in Figure 7.13 for radially pre-stressed targets and their counterparts in Figure 7.16 for hydrostatically pre-stressed ones, justifies the higher threshold velocity observed in Figure 7.12 for hydrostatically pre-stressed targets of ratios higher than $R_r = R_a = 1.0035$ compared to the radially pre-stressed ones of ratios higher than $R_r = 1.0035$.

7.7 Optimization of pre-stress

The influences of material and geometries of the confined ceramic armor components, as design parameters, on the ballistic behavior of the confined ceramic armor were reviewed in section 2.7.4. Section 7.4 show that using different ceramic/confinement geometric ratios, used to obtain the three pre-stress types, different pressure contours are formed in the ceramic-plug interface and within the ceramic influencing the duration of dwell on the ceramic-plug interface and rate of penetration within the ceramic target.

An interesting optimization problem in impact of heavy metal long rod projectiles onto confined thick ceramic armor is finding, utilizing different radial, R_r and axial, R_a pre-stress ratios (values) and confinement materials, the optimized pressure contour, resulting in the longest dwell time on the ceramic-plug interface and lowest rate of penetration within the ceramic target.

7.8 Summary

In this chapter, a method for the simulation of pre-stressed ceramic targets under high velocity impact, using explicit software, AUTODYN[®], was proposed. Impact simulations of targets without pre-stress and also with different pre-stress conditions were carried out. The impact simulation results of pre-stressed targets are in good agreement with available experimental data and explain different mechanisms occurring during penetration of long rod heavy metal projectiles into confined SiC targets. Pressure distribution affects: (a) dwell on the impact surface at the impact site and (b) rate of penetration within the ceramic armor. Damage accumulation below the impact site of the ceramic target and backflow of the projectile material on the surface of the ceramic material, occurring during transition from dwell to penetration into the confined ceramic target were shown using numerical model. A concept of threshold velocity was

proposed as a measure of efficiency for different pre-stress types and magnitudes in improving the ballistic behavior of the confined SiC target.

Chapter 8 Conclusions and Recommendations

8.1 Summary and conclusions

This thesis pre-dominantly presents numerical simulation investigations on the ballistic limit velocity (BLV) of bi-layer ceramic–metal armor structures. The studies are supported by analytical and limited experimental results. It was shown that the optimal BLV depends on various material properties and geometrical parameters. The summary of the current research work and conclusions drawn are as follows:

1. Numerical simulations for bi-layer alumina 99.5%–aluminum alloy 5083-H111 armor system impacted by 20 mm APDS, using explicit non-linear transient dynamic numerical code, AUTODYN[®], were carried out and verified against the literature experimental data. Analytical models, like Recht-Ipson model, relating projectile residual velocity, impact velocity and armor BLV have been extensively developed in the literature especially for monolithic metallic armor. Similar model, using a semi-analytical approach, was developed in the current thesis for bi-layer ceramic–metal armor.

It was shown that the projectile residual velocity and BLV obtained satisfy the replica scaling laws. The influences of projectile and armor geometrical parameters on the BLV were investigated and the following conclusions were drawn:

- BLV increases non-linearly with increasing dimensionless front (backing) plate thickness, while backing (front) plate thickness is fixed.
- BLV decreases non-linearly with increasing dimensionless length of the projectile, while armor geometrical parameters are fixed.
- BLV decreases linearly when dimensionless front plate diameter (planar dimension) increases (while backing plate diameter is fixed) from 1 to a limit value after which BLV remains constant.

Besides, the BLV increases with increasing dimensionless backing plate diameter from 1 to a limit value and then the BLV keeps a constant maximum value; during this analysis, front and backing plate diameter are

kept same and larger than a limit value for elimination of front plate diameter effect on variation of backing plate diameter.

Based on the above analysis, an empirical equation for BLV was proposed. Finally, optimization of alumina 99.5%–aluminum alloy 5083-H111 armor for given total thickness and areal density (mass per unit area) constraints was performed based on the proposed BLV empirical equation, giving reasonable results similar to experiments available in the literature.

2. A series of impact tests were carried out using hardened steel 4340 projectiles, alumina 95% ceramic tiles bonded, using adhesive Hysol EA 9309.3NA, to aluminum alloy 2024-T3. The tests were performed to validate the proposed semi-analytical BLV model (Eqs. (3.9) and (3.10)). Inherently, there was presence of yaw angle after firing of the projectile which was very difficult to control and eliminate. Since the proposed semi-analytical model is for normal impact of the bi-layer ceramic-metal armor whereas the experiments had yaw, an attempt was made to find reliable numerical and material model after matching the simulations with experimental cases. The results obtained from normal impact simulations together with experimental cases with small yaw angles and also other experiments available in literature were used for validating the semi-analytical model. It was shown that the results obtained from the proposed model correlate well with those based on simulation and also experimental measurements, in the velocity range less than 1000 m/s. For higher velocities, the results of the model starts to deviate from simulations probably due to the simplifications inherent in the semi-analytical model such as considering shear plugging separation mechanism for both front and backing plate with same diameter which may not be suitable for higher ranges.

Effect of front plate and backing plate thicknesses on residual length and velocity of the projectile were captured based on the validated numerical model:

- The residual velocity and length of the projectile both decrease as the backing plate thickness (while front plate thickness is fixed) or front plate thickness (while backing plate thickness is fixed) is increased.
- Similarly the residual velocity and length of the projectile both decrease as the front plate thickness is increased.

Temporal variation of projectile residual length in impact of bi-layer alumina 99.5%–aluminum alloy 5083-H111 armor systems of fixed front or backing plate thickness shows the presence of an optimum front to backing plate thickness ratio. Optimization of bi-layer alumina 95%–aluminum alloy 2024-T3 armor system was performed for a given total thickness or areal density (mass per unit area) of the armor system. The significance of the proposed semi-analytical model (Eqs. (3.9) and (3.10)) is quick estimation of ballistic limit velocity (BLV) of the bi-layer ceramic–metal armor system, compared to Lambert–Jonas approximation which needs a large number of experimental or simulation data.

3. A generalized empirical model to estimate the BLV, considering momentum and energy balance during impact process, as a function of geometrical and material parameters for ceramic–metal bi-layer armor systems impacted by a flat-ended (blunt) tungsten projectile was proposed, supported by numerical simulations. Impact simulations were performed for different combinations of armor geometries and materials systems with different projectile lengths; the combination of these variables were chosen based on orthogonal array approach. For the bi-layer armor systems the front and backing layer choices were: Al_2O_3 96%, B_4C , SiC and Al5083H116, steel 4340, Ti6Al4V, respectively. Material constitutive and fracture model constants were first validated through comparison of simulations, performed in AUTODYN[®] and available experimental data in literature. Numerical constants in the empirical model were obtained using least square fitting to the BLV data obtained from the simulation of various armor systems. Comparison between results obtained from the empirical model for BLV of ceramic–metal armor impacted by blunt projectiles, with limited experimental data (in which the projectile material was not same as the one of empirical model) show a trend similar to one of experiments for BLV as a function of ceramic plate thickness. It is believed that if exact material properties used in simulations and model calculations are known, better agreement between results may be achieved. The empirical model was employed for the optimization of bi-layer alumina 99.5%–aluminum 5083H116 armor system under blunt tungsten alloy projectile impact, for a given total thickness or areal density.

4. The optimization problems solved for impact of blunt hard projectile on the bi-layer ceramic–metal armor systems based on the proposed numerical and empirical models rendered results analogous to the corroborative experimental works in the literature. The salient features of these experiments are as follows:

- (a) Armor consisting of monolithic metallic plate has higher BLV compared to one composed of monolithic ceramic plate of same areal density (or same thickness).
- (b) For a given total thickness (or areal density) constraint, there is a maximum BLV and a unique optimum thickness ratio (areal density ratio).
- (c) At low impact velocities, a monolithic metallic plate can be more efficient than either monolithic ceramic plate or some ceramic-metal armor system (implying that in small areal density or total thickness range, the ceramic plate is not as effective as metallic plate; therefore in the small areal density or total thickness range, there exists an optimum monolithic metallic armor system or bi-layer ceramic–metal armor system composed of thinner ceramic front plate and comparatively thick backing plate).

It was shown that the optimum thickness ratio (or areal density ratio) increases with increase of given total thickness (or areal density) of the bi-layer armor system and the increasing trend becomes gradually less significant and finally approaches a constant value.

- (d) Recht's analytical model for erosion and plastic deformation mechanisms of blunt projectile impact onto deformable monolithic target was modified by taking into account the equivalent impedance factor for the ceramic–metal armor. This model was then extended for conical projectile impact. Equations for the final length of the projectile and the time taken for the erosion and plastic deformation were proposed. Comparison between the results obtained from the models with literature experimental results for both blunt and conical projectile impact on ceramic–metal armor revealed good agreements. Finally, the assumptions considered in the model were discussed and an alternative method beside the one used for considering the force applied on the cross-sectional area of the projectile was suggested.

(e) A numerical model comprised of six steps for simulation of pre-stressed ceramic targets under high velocity impact, using explicit software, AUTODYN[®], was developed. Validation of the proposed numerical model was performed through comparison of results with the available experimental data in the literature. Impact simulations of confined ceramic targets with and without pre-stress were carried out. Effect of pre-stress in radial, axial and hydrostatic states with different intensities on dwell and interface defeat were explored. Two mechanisms, namely damage accumulation below the impact site of the ceramic target and backflow of the projectile material on the surface of the ceramic material, occurring during transition from dwell to penetration into the confined ceramic target, were shown using numerical model. Finally, “threshold velocity” was defined as a measure of efficiency of different pre-stress types with varying intensities in improving the ballistic behavior of the confined ceramic target. The observed trend for the threshold velocity was justified through the deformation of confinements under different pre-stress conditions and also by examining the pressure obtained from pre-stressing at locations on the axis of symmetry below the impact site of the SiC targets wherein damage initiates. The pressure below and near the impact site of the confined SiC target is important in the transition behavior from dwell to penetration for targets of various pre-stress types and of different pre-stress ratios. Furthermore, hydrostatic pre-stress is more effective than radial pre-stress (and also axial pre-stress) in improving the ballistic behavior of the confined thick ceramic armor.

8.2 Recommendations for future work scope

The following works are suggested for further research:

- An experimental program can be planned to clarify the basis of the increase of optimum thickness ratio (or areal density ratio) with increase of total thickness (or areal density) of the bi-layer ceramic–metal armor system and justify that the increasing trend approaches a constant value.
- The influence of material properties of the ceramic and metallic plates used in bi-layer ceramic–metal armor system and also geometries and material properties of the projectile on the optimal thickness ratio of the armor system should be explored using a comprehensive experimental work.

- As for the analytical modelling of projectile impact onto ceramic armor systems, variable pressure applied on the cross-sectional area of the projectile can be used in the equation of motion of the projectile (Eqs. (6.8) and (6.17)) instead of using constant projectile dynamic yield strength.
- The proposed analytical model for the impact of blunt and conical projectiles onto ceramic armor system should be incorporated into a model considering failure mechanisms of the front and backing plate. In doing so, temporal variation of the front and backing plate thicknesses (effect of layering) can be separated from the impedance matching factor.
- Optimization problem in long rod heavy metal projectile impact onto confined thick ceramic armor can be explored, using the numerical model developed. In doing so, different radial and axial pre-stress ratios (values) and confinement materials are used to find the optimized pressure contour which results in the longest dwell time on the ceramic-plug interface and lowest rate of penetration within the ceramic target.
- The efficiency of various pre-stress types and values in improving ballistic behavior of thin ceramic armor can be studied using the numerical model developed.

List of Journal Publications and Conferences

Journal publications:

1. A. Serjouei, I. Sridhar, W. E. Hua, On the design of bi-layer armor materials. *Solid State Phenomena*, 2012. 185: p. 48-50.
2. R. Chi, A. Serjouei, I. Sridhar, G. E. B. Tan, Ballistic impact on bi-layer alumina/aluminum armor: A semi-analytical approach. *International Journal of Impact Engineering*, 2013. 52(0): p. 37-46.
3. A. Serjouei, R. Chi, I. Sridhar, G. E. B. Tan, An empirical model for the ballistic limit of bi-layer ceramic/metal armour. *Procedia Engineering*, 2014. 75(0): p. 14-18.
4. A. Serjouei, R. Chi, Z. Zhang, I. Sridhar “Experimental validation of BLV model on ceramic-metal bi-layer armor”, submitted to *International Journal of Impact Engineering*, under revision.
5. A. Serjouei, R. Chi, I. Sridhar, G. E. B. Tan, Optimal design of ceramic-metal armor system based on empirical BLV model, submitted to *Journal of Materials & Design*, under revision.
6. R. Chi, A. Serjouei, I. Sridhar, Influence of radial, axial and hydrostatic pre-stresses on ballistic behavior of confined thick ceramic armor (manuscript preparation for submission to *International Journal of Impact Engineering*).

Conferences:

1. R. Chi, A. Serjouei, I. Sridhar, G. E. B. Tan, Geometrical influences on the ballistic limit of bi-layer armor materials, in 4th International Conference on Design and Analysis of Protective Structures (DAPS), June 19-22, 2012, Jeju, South Korea.
2. A. Serjouei, R. Chi, I. Sridhar, Pre-stress effect on the ballistic behavior of ceramic armor: Numerical approach, in 3rd International Conference on Computational Modeling of Fracture and Failure of Materials and Structures, 5-7 June, 2013, Prague, Czech Republic.
3. A. Serjouei, R. Chi, I. Sridhar, “An empirical model for the ballistic limit of ceramic/metal bi-layer armor”, The 7th International Conference on Materials for Advanced Technologies (ICMAT), 30th June to 5th July, 2013, Singapore.

Appendix A

Figure A.1 Test No. 7: (a) ceramic front tile after impact, (b) projectile after impact, (c) hole dimension in aluminum backing plate, (d) top view of the target and the bulged hole at the back of the aluminum plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

(e)

(f)

Figure A.2 Test No. 9: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

Figure A.3 Test No. 12: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

(e)

(f)

Figure A.4 Test No. 13: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

(e)

(f)

Figure A.5 Test No. 16: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

Figure A.6 Test No. 20: (a) ceramic front tile, (b) projectile and its broken part after impact beside each other and placed on each other from the fractured surface, respectively, from left to right, (c) front view and top view of the backing plate rear side bulged after impact and (d) front view (90° rotated) of the backing plate bulged rear side, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

(e)

(f)

Figure A.7 Test No. 21: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

(e)

(f)

Figure A.8 Test No. 22: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

(e)

(f)

Figure A.9 Test No. 23: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

Figure A.10 Test No. 24: (a) ceramic front tile after impact, (b) projectile and aluminum plug separated from backing plate after impact, respectively, from left to right, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

(e)

(f)

Figure A.11 Test No. 29: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

Figure A.12 Test No. 30: (a) ceramic front tile after impact, (b) projectile and its broken part after impact, (c) X-ray image taken by tubes 1, 2, 3, 4 and (d) X-ray image taken by tubes 5, 6, 7, 8

(a)

(b)

(c)

(d)

(e)

(f)

Figure A.13 Test No. 31: (a) ceramic front tile after impact, (b) projectile after impact, (c) and (d) hole dimensions in aluminum backing plate, (e) X-ray image taken by tubes 1, 2, 3, 4 and (f) X-ray image taken by tubes 5, 6, 7, 8

Figure A.14 shows residual velocity, V_r , and residual length, L_{pr} , of APM2 projectile in impact of two bi-layer boron carbide (B_4C)–aluminum 6061-T6 armor systems with front plate thickness of 5.08 mm and 6.35 mm and fixed backing plate thickness of 6.35 mm, from experiments performed by Anderson Jr [145].

Figure A.14 (a) Residual velocity and (b) residual length as a function of impact velocity for two armor systems with front plate thickness of 5.08 mm and 6.35 mm and fixed backing plate thickness of 6.35 mm [145] (courtesy of C. E. Anderson Jr)

Figure A.15 (a) shows the variation of ballistic limit velocity (BLV) with areal density, AD , from the experimental work performed by Mayseless et al. [135].

Figure A.15 Variation of BLV with areal density (AD) for two monolithic and three bi-layer armor systems impacted at normal incidence by hard-steel projectile of 12.7 mm diameter and 60° conical tip [135]

Appendix B

Lee and Yoo [136] performed simulations and experiments on four alumina–aluminum alloy 5083 systems (namely C-1 to C-4) with fixed areal density (AD) impacted with steel projectile. Lee and Yoo [136] only provided the densities of the front and backing plates, namely 3380 kg/m^3 and 2260 kg/m^3 , ratio of the front plate to and backing plate thicknesses, T_1/T_2 and maximum thicknesses of the front and backing plates used in their experiments, namely 25 mm and 25.4 mm, respectively. Considering the various alumina types in the database provided by Holmquist et al. [160], it is conjectured that the purity of this alumina used in experiments of Lee and Yoo [136] should be equal or less than 85%. With some calculations, the thicknesses of the front plate and backing plate and areal density for the four bi-layer armor are found, given in Table B.1, together with front to backing plate ratios, T_1/T_2 and front plate areal density over total areal density, AD_1/AD .

Table B.1 Details of the four armors in experiemnts of Lee and Yoo [136]

Configuration	T_1 (mm)	T_2 (mm)	T_1/T_2	AD_1/AD
C-1	9.90	25.40	0.39	0.37
C-2	18.43	23.33	0.79	0.69
C-3	11.59	7.03	1.65	0.43
C-4	25	6.33	3.95	0.93

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