

Essays in interactional dynamics of financial markets

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Essays in Interactional Dynamics of Financial Markets

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¹A paper coauthored with Weihong Huang based on this chapter has been accepted by Journal of Economic Behavior and Organization.

²A paper coauthored with Weihong Huang based on this chapter has been submitted for journal publication.

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⁴A paper coauthored with Weihong Huang and Huanhuan Zheng based on this chapter has been submitted for journal publication.

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Abstract

This thesis applies the deterministic dynamic model to investigate the interactions among markets given the background of financial globalization and market integration. By establishing market linkage, it not only proves theoretically, but also replicates numerically the existence of cross-correlation between two markets, one of the key quantitative measures of markets interaction. In modern financial markets, financial crisis always occurs from time to time. Usually, it is not isolated within one market, instead, it can propagate to other markets, causing contagion phenomena which exhibiting itself as large cross-correlation between markets. With the capability of capturing the feature of cross-correlation, this thesis is able to numerically demonstrate various patterns of financial crises with contagion behavior, showing the complexity of modern financial markets. By extending from two to multiple markets, market linkage connects individual markets into a market system, which exhibits new phenomena such as the formation of market clusters with market members sharing certain attributes.

Chapter 1 describes the research background and motivation of this thesis.

Chapter 2 develops a two-market heterogeneous agents model (HAM), which does not only prove in theory the existence of price co-movement but also replicate in simulation this typical characteristics, along with other well known stylized facts characterizing individual financial market. Moreover, theoretical analysis suggests meaningful implications for market opening policy. It is suggested that, in terms of financial stability, a relatively small market may not benefit from market linkage and market opening is essentially a double-edged sword.

Chapter 3 simplifies the model developed in Chapter 2 to simulate various patterns of financial crises with contagion behaviors. It is implied that financial crisis and its contagion could be endogenous, which supports scenario of over-valuation causing financial crises. In addition, the model shows that the financial system could be fragile in which small shock(s) hitting individual market's fundamental could cause financial crisis spreading to other market. This also supports scenario of external shock triggering financial crises.

Chapter 4 extends the model of Chapter 2 from the two markets to multiple markets. By numerical study, it is shown that the market system displays a new phenomenon in the chaotic regions, market cluster with members sharing the same sign of asset price deviation. This kind of cluster formation is similar to the concept of coupled map lattices (CML) in other disciplines such as Physics.

Chapter 5 applies Markov-regime switching technique to show the existence of inter-market traders whose trading decision is based on the fundamental value of foreign market under a two-market framework.

Chapter 6 concludes by summarizing the results and contributions of this thesis. It also points out the caveats and potential future researches.

Chapter 1

Introduction

Given the background of financial globalization, more and more markets are coupled and integrated into a market system. Individual market is no longer isolated such that movements in one market have an effect on other markets. Along with financial market integration comes markets co-movement or cross-correlation, which has been widely reported in empirical literature. Kenett et al. (2012) find that developed Western markets are highly correlated. Egert and Kocenda (2011) find strong correlation among returns of Germany, France and UK, even up to 0.9. In addition, strong co-movement was observed when financial crisis spread to many markets in the past decades. The contagion of the financial crisis can be quantified as high cross-correlation between markets. Preis et al. (2012) show that average correlation among DJIA members increases with market stress. Market interactions especially cross-correlation have played an important role in modern financial markets. Regrettably, very few theoretical models in literature have paid attention to this phenomenon, not to mention simulating the prices with these stylized facts.

Heterogeneous agents models (HAM) have been successful in modelling financial markets and replicating some of the stylized facts, such as bubbles and crashes, randomly switching bear and bull market episodes, excess volatility, volatility clustering and fat tails for returns distribution, see for instance, Lux (1995), Brock and LeBaron (1996), Lux (1998), Brock and Hommes (1998), Lux and Marchesi (2000), Chiarella and He (2001), Farmer and

Joshi (2002), Westerhoff (2004), Chiarella et al. (2006), Chiarella et al. (2007), He and Li (2008), Dieci and Westerhoff (2010), Gao and Li (2011) and Chen (2012).

Day and Huang (1990) introduce a stylized market maker framework in which two agent types, chartist and fundamentalist, invest in an asset market. Based on the excess demand of chartists and fundamentalists, a market maker updates price adaptively in each period with the principle of increasing price for positive demand and vice versa. The model is in discrete time and exhibits complicated, chaotic price fluctuations around a fundamental price with random switching between bear and bull market episodes. Instead of the market maker framework, Brock and Hommes (1998) apply the Walrasian equilibrium concept in heterogeneous agents model. In their model, micro-foundation is built on a fitness measure of the past realized profit. Agent aims to maximize investment profit and decides her supply and demand according to the chosen strategy. Market clears at the end of each period. This model is capable of explaining some stylized financial behaviors such as irregular switching among phases of price movements. Chiarella and He (2002) allow agents to have different risk attitudes and different expectation function of both first and second moments of the price distribution. Under Walrasian and market maker scenarios, it is found that heterogeneity of agents has a stabilized and destabilized double edged effect on asset prices.

The robustness and deterministic feature of HAM make it useful for many applications, especially policy related evaluation. He and Westerhoff (2005) investigate the effect of price limiter on price volatility. Huang et al. (2010) manage to simulate all three patterns of financial crisis documented in literature and conclude that financial crisis could be endogenous. Huang and Zheng (2012) study underlying mechanisms for various patterns of financial crisis using regime-dependent belief.

While majority of the HAM models focus on a single market, recent literatures have shed some light on multiple assets/markets. Bohm and Wenzelburger (2005) investigate the performance of efficient portfolios in a financial market with heterogeneous agents. Westerhoff and Dieci (2006) evaluate Keynes-Tobin financial transaction tax on price variability in a two-market framework. Dieci and Westerhoff (2010) examine a market system in which two stock markets are linked through foreign exchange market. It is found that upon

market interactions, stock markets may be destabilized while the stabilizing effect on the foreign exchange market and the whole market system can be observed. Compared to a single market model, a multiple market model involves more parameters, which make it more complicated for analysis. Coming along with multiple markets interactions, new research questions arise. The first research question is the price stability of the individual market and the whole market system. The relationship between individual market and the market system can provide meaningful policy recommendation for market opening. Another research question is cross-correlation between markets. A positive and large cross-correlation indicates price co-movement. A thorough understanding of cross-correlation is urged. So far, these two research questions are not fully addressed yet, more researches are needed to fill this research gap.

Chapter 2 intends to address the above mentioned two research questions theoretically based on the setup of Day and Huang (1990) and Westerhoff and Dieci (2006). There are two markets, both of which are populated with chartists and fundamentalists. Agents are inhomogeneous such that agents from different markets have different market demand strengths. There is no market barrier and investors are allowed to invest in any market. An investor has freedom in choices of market destination for investment as well as investment strategies to be used. The investment decision is related to a fitness measurement of price deviation from fundamental value. With the fitness measurement, composition of investors can be determined by a distribution function, inspired by Hommes (2001) and in spirit close to the one used in He and Westerhoff (2005). Upon market linkage/opening, there surfaces a new group of investors, inter-market traders whose investment in the local market is based on market condition of the other market. In contrast to the conclusion of Dieci and Westerhoff (2010) that stock markets may be destabilized while foreign exchange market is stabilized, Chapter 2 finds that stabilizing and destabilizing effect is not fixed for a market member. A market can be stabilized or destabilized, depending on market structure of individual market and its counterpart. Chapter 2 also proves the existence of cross-correlation between markets. In addition, it manages to replicate statistically significant cross-correlation numerically.

Chapter 3 aims to apply the concept of Chapter 2 into modelling financial crisis

contagion behavior. As the model of Chapter 2 incorporates the feature of endogenous investor composition, which complicates the model analysis by several parameters, to simplify the numerical calibration and capture the main driving mechanism, model of Chapter 2 is simplified with fixed investor composition. In view of the successful experience of Huang and Zheng (2012) in using regime-dependent belief to replicate financial crisis, Chapter 3 also makes use of regime-dependent belief in the model. Simulations are run to verify different scenarios of financial crises with contagion behaviors. From the point of view of endogeneity, Chapter 3 manages to capture the simultaneous crash behavior of US and UK stock markets during "Black Monday" in 1987 as well as other financial crisis patterns. All these financial crises occur without external shocks, which support the scenario of over-valuation causing financial crises. On the other hand, from the point of view of exogeneity, upon impact of permanent or temporary shock(s) on market member(s), financial crisis can arise and spread to the other market. Factors such as magnitude and sign of shock as well as duration of temporary shock are shown to play some roles in financial crises formation. This supports scenario of financial crisis triggered by external shocks. In addition, the result that financial crisis in one market triggered by shock causes similar crisis in the other market is analogous to the domino effect.

While Chapters 2 and 3 focus on two-market system, Chapter 4 examines a larger scale multiple-market system. In each market, a market maker faces excess demand of two groups of investors. The first group consists of fundamentalists and chartists. Their excess demand is derived from condition of domestic market. In contrast, excess demand of the second group, inter-market traders is based on the condition of the nearest neighboring markets. Compared to the model of Chapter 2, fundamentalists and chartists do not invest in foreign markets. The existence of the second group of investors captures the inter-market investment and connects all the market members to form a market system. The market maker function has a similar form of coupled map lattice (CML) phenomena documented in Physics. The coupling of the market system produces new phenomena of asset price deviation persistence enhancement in chaotic regions. If the effect of deviation persistence enhancement is large enough, market clusters emerge in which market members share the same sign of price deviation. Bifurcation and Lyapunov exponent studies are applied to

study this deviation persistence enhancement effect. By investigating the response of price of individual market to shock on one market member, coupling demonstrates stabilizing effect on the whole market system.

Investigating from the aspect of empirical study, Chapter 5 proposes a two-market empirical model with heterogeneous agents based on Chiarella et al. (2012). Using monthly data of French and US stock markets, the regression shows that individual markets have feature of two-regime switching process. By including inter-market traders whose trading decision is based on fundamental value of foreign market, the two-market model has a better capability in explaining both markets. The existence of inter-market traders implies that the two markets share some common set of factors, which provides foundation of market interactions, such as market co-movement. The regime-switching behavior of inter-market traders also suggests a contagion from France to US in the midst of subprime crisis, which exacerbates the crisis of US.

Chapter 6 concludes by summarizing the results and contributions of this thesis as well as pointing out the caveats and potential future researches.

Chapter 2

Modelling Regional Linkage of Financial Markets¹

2.1 Introduction

With the development of regional market integration, linkage among markets becomes stronger and common currency circulating within the region emerges. A typical example is the Euro, which serves as the transaction currency for all financial markets within the euro-zone. Examples of regional asset markets include but are not limited to the Shanghai and Shenzhen stock exchange markets in China, and NASDAQ and New York stock exchanges in the United States. The common transaction currency eliminates the need of currencies exchange so as to remove the impacts from the foreign exchange market. Along with financial market integration comes markets co-movement or cross-correlation, as it has been widely reported in empirical literature. Kenett et al. (2012) find that developed Western markets are highly correlated. Egert and Kocenda (2011) find strong correlation among returns of Germany, France and UK, even up to 0.9. In addition, strong co-movement

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was observed when financial crisis spread to many markets in the past decades. Preis et al. (2012) show that average correlation among DJIA members increases with market stress. Regrettably, there is no theoretical model in literature that can simulate the prices with these stylized facts.

Heterogeneous agents models (HAM) have so far managed to calibrate successfully some of the financial market stylized facts related to individual financial market, among which are large trading volume, cluster volatility, returns distribution with fat tails, and unpredictable asset returns with almost no autocorrelation. For reference, we cite Huang and Day (1993), Lux (1995), Lux (1998), Lux and Marchesi (2000), Brock and LeBaron (1996), Farmer and Joshi (2002), Chiarella and He (2001), Westerhoff (2004), Chiarella et al. (2006), Chiarella et al. (2007), He and Li (2008), Gao and Li (2011) and Chen (2012). There is a call to build a multi-market model with capability of not only replicating the stylized facts of prices co-movement, but also offering economically plausible explanations. Such model can enhance our understanding of the integrated financial system and further shed light on the study of the propagation mechanism of financial crisis. This research intends to fill up such a gap by building a simplest possible nonlinear dynamic HAM model. For this purpose, a market system composed of two markets linked by a common transaction currency is studied so that the investors are allowed to invest in both markets with no hassle of exchange rate. Such set up is shown to be able to replicate the typical characteristics of multi-markets system in addition to other well known stylized facts characterizing individual financial market.

Day and Huang (1990) introduce a stylized market maker framework in which two agent types, chartist and fundamentalist, invest in an asset market and a market maker updates price in each period. The model is in discrete time and exhibits complicated, chaotic price fluctuations around a fundamental price with random switching between bear and bull market episodes. Instead of the market maker framework, Brock and Hommes (1998) apply the Walrasian equilibrium concept in heterogeneous agents model. In their model, micro-foundation is built on a fitness measure, which is determined by the past realized profit. Every period, agent composition is determined by fitness measures. Agent aims to maximize investment profit and decides her supply and demand according to the chosen

strategy. Market clears at the end of each period. This model is capable of explaining some stylized financial behaviors such as irregular switching among phases of price movements. However, LeBaron (2006) argue that the market clearing Walrasian equilibrium in every period has limitations. One of the limitations is that it may not represent the continuous trading of financial market accurately. Nevertheless, the combination of market maker and micro-foundation based on fitness measure develops in later literature such as Westerhoff (2004), He and Westerhoff (2005), Westerhoff and Dieci (2006), He and Li (2008) and Huang et al. (2010).

Majority of the heterogeneous agents models focus on a single market or one risky asset with reference to one riskless asset. Recently, the idea of heterogeneous agents is extended to price dynamics of multi-asset within a market, or even to the interactional dynamics of multi-markets. For example, Bohm and Wenzelburger (2005) investigate the performance of efficient portfolios in a financial market in which heterogeneous investors including rational traders, noise traders, and chartists are active. Brock et al. (2009) introduce additional Arrow securities into the stylized evolutionary equilibrium model of Brock and Hommes (1998) and demonstrate that more hedging instruments may destabilize markets with heterogeneous agents and performance-based reinforcement learning. Westerhoff and Dieci (2006) develop a model in which chartists and fundamentalists invest in two speculative markets. The composition of investors varies according to profit fitness measurement. After stability conditions for the fundamental steady state are derived, the model generates a complex price dynamics resembling to actual speculative prices. Dieci and Westerhoff (2010) build up a three-market model in which two stock markets are linked via foreign exchange market. The foreign exchange market is populated with chartists and fundamentalists while the two stock markets have only fundamentalists. It is concluded that upon market interactions, stock markets may be destabilized while the stabilizing effect on the foreign exchange market and the whole market system can be observed.

This chapter follows the framework of Day and Huang (1990) and Westerhoff and Dieci (2006). There are two groups of investors. The first group consists of two types of investors, chartists and fundamentalists, who invest in two speculative markets with the same transaction currency based on condition of destination market. Each investor can choose

a chartist or fundamentalist strategy in each market. The difference between this chapter and Westerhoff and Dieci (2006) is that chartists or fundamentalists from different markets have different demand strengths. Another difference is the second group of investors, inter-market traders whose investment in local market is based on condition of foreign market. In addition, factor of market size/population is included to investigate its role. Theoretical analysis and simulations show that a market that is more stable initially will stabilize the market system while it is subjected to destabilizing effect from the market system. This mutual effect also applies to a market that is more unstable initially. Interpreting from the population size of individual market, a market with a smaller population has lesser influence on the market linkage.

This chapter is structured as follows. For the purpose of comparison, we start with in Section 2 a hypothesized case in which two regional markets are isolated with each other in the sense that the investors are not allowed to invest in the foreign market. Section 3 then explores the case when these two isolated markets are linked by allowing the investors from each market to invest in both. Theoretical analysis is carried out so that meaningful policy implications can be drawn. Section 4 provides various numerical simulations and verifies its capability to generate price series matching typical stylized facts documented in the literature, especially the price co-movement or cross-correlation. Section 5 concludes with the directions of future research.

2.2 Market Isolation

Following the market maker framework of Day and Huang (1990), we assume that a financial market is composed of three types of agents: chartists, fundamentalists and a market maker. Fundamentalists behave in a way that they sell over-priced asset and purchase under-priced one. In contrast, chartists simply assume the persistence of bullish and bearish market episodes in the short run. Following this expectation, they purchase the over-priced asset and sell the under-valued one.

There exist two regional stock markets, denoted by A and B . We first assume that these two markets are isolated so that investors are allowed to invest in their domestic

market only. The composition of chartists and fundamentalists among investors depends on market circumstance. Fundamentalists play roles of correcting market price and their composition would become larger for a larger price deviation, as demonstrated by Hommes (2001). A larger asset price deviation triggers more agents to rely on fundamentalist strategy based on the micro-foundation of fitness measures.

2.2.1 Hypothesized model

For the isolated market i with population n_i , $i = A$ or B , the log price of the asset at time t is denoted by $P_{i,t}$. The constant log fundamental value is denoted by F_i . For convenience, we define the log price deviation as

$$x_{i,t} \triangleq P_{i,t} - F_i. \quad (2.1)$$

Moreover, the state $x_{i,t} = \bar{x}_i = 0$ will be referred to as *the fundamental steady state* to indicate the fact of $P_{i,t} = F_i$.

For a chartist (c) or a fundamentalist (f) from market j , $j = A$ or B , the excess demand for the asset i is $D_{ij,t}^c$ and $D_{ij,t}^f$ respectively². To capture the facts that chartists purchase asset when the price deviation is positive and sell when it is negative while fundamentalists behave in an exactly opposite way, without loss of generality, we assume that the excess demand for chartist (fundamentalist) is positively (negatively) proportional to the price deviation. In other words, we have

$$D_{ij,t}^c = c_j \cdot x_{i,t} \text{ and } D_{ij,t}^f = -f_j \cdot x_{i,t},$$

where c_j and f_j reflect *the demand strength* of chartist and fundamentalist from market j , respectively.

Investors can choose to be either chartists or fundamentalists by comparing the relevant strategy fitness measures. According to the investment strategy of fundamentalists, if asset price deviation is large, chance of earning harvest (for *positive deviation*) or encountering investment opportunity (for *negative deviation*) increases. In other words,

²In the sequel, we shall adopt the same notation convention with the first subscript standing for the market asset demanded, the second subscript for market the investors originated from, and the superscript (c or f) for the type of investors.

more investors tend to adopt fundamentalist strategy when the asset price deviation $|x_{i,t}|$ becomes larger. Therefore, it is reasonable to assume that the strategy fitness measure of fundamentalists from market j to invest in market i at period t , denoted as $m_{ij,t}^f$, is an increasing function of magnitude of price deviation. For simplicity, we express $m_{ij,t}^f$ as

$$m_{ij,t}^f = |x_{i,t}|.$$

In contrast, the chartist faces increasing chance of investment loss (for *positive deviation*) or missing investment opportunity (for *negative deviation*) with price deviation. Fewer investors will adopt chartist strategy when the asset price deviation $|x_{i,t}|$ becomes larger. Therefore, the chartist strategy fitness measure to invest in market i , $m_{ij,t}^c$, should be a decreasing function of the magnitude of price deviation. Besides that, in the world of chartists, there are phenomena of support and resistance. Support (resistance) is a price level that may induce a net increase of buying (selling) to prevent price from further declining (increasing). Donaldson and Kim (1993) report empirical phenomena of “support” and “resistance” level in Dow Jones Industrial Average. As stock index approaches levels of support (resistance), stock sellers (buyers) become less aggressive with concern about a turn in the market. This implies that for a given price within support/resistance, investors will be more aggressive if asset price is far away from support/resistance. It is equivalent that, for a given asset price, investors will be more aggressive with a larger window width between support/resistance levels compared to a smaller one. In other words, a larger window width of support/resistance level will make chartists more confident, which implies a higher $m_{ij,t}^c$. Hence, $m_{ij,t}^c$ can be measured by:

$$m_{ij,t}^c = s_j - |x_{i,t}| = (\ln h_j) / \rho - |x_{i,t}|,$$

where ρ is a parameter and will be defined in the portion of investor composition; $h_j \geq 1$ is *chartist adjustment parameter* while $s_j = \ln h_j / \rho$ takes account of the effects of window width of support/resistance levels. It is noteworthy that, for such formulation, at the fundamental steady state, $x_{i,t} = 0$, the strategy fitness measure for chartist strategy is larger ($h_j > 1$) or equal to ($h_j = 1$) the one for fundamentalist. Such formulation reflects the fact that, in the eyes of speculators and trend followers, the financial market always fulfills with the investment opportunities, regardless whether it is in the bull trend or bear

trend. As h_j 's are related to supports/resistances that derived from common prevailing rules of technical analysis, we shall therefore assume that $h_A = h_B = h$ to reflect the facts that there is no informational asymmetry so that chartists from different markets can arrive at a similar h value³.

We shall see later that the chartist adjustment parameter h essentially enhances the chartist strength in the sense that the *relative strength in isolation* defined by

$$\beta_i \triangleq hc_i n_i / (f_i n_i) = hc_i / f_i, i = A, B, \quad (2.2)$$

plays an important role in determining the steady state of the financial markets.

Fitness measures affect investor composition in a way that investors are prone to adopt the strategy that has a comparatively higher fitness measure. The fitness measures in our paper is inspired by Hommes (2001) and in spirit close to the one used in He and Westerhoff (2005) for single market framework. Our fitness measures are simplified and can be directly applied to multi-market case in the sequel multi-market section. Following Brock and Hommes (1998), we define chartist and fundamentalist *compositions* in isolated market i respectively as:

$$W_{ii,t}^c \triangleq \frac{\exp(\rho m_{ii,t}^c)}{\exp(\rho m_{ii,t}^f) + \exp(\rho m_{ii,t}^c)} = \frac{h \cdot \exp(-\rho |x_{i,t}|)}{\exp(\rho |x_{i,t}|) + h \cdot \exp(-\rho |x_{i,t}|)},$$

$$W_{ii,t}^f \triangleq \frac{\exp(\rho m_{ii,t}^f)}{\exp(\rho m_{ii,t}^f) + \exp(\rho m_{ii,t}^c)} = \frac{\exp(\rho |x_{i,t}|)}{\exp(\rho |x_{i,t}|) + h \cdot \exp(-\rho |x_{i,t}|)}.$$

where ρ is the speed of switching. A larger ρ implies more investor will switch to fitter strategy.

The aggregate excess demand in market i , denoted by $D_{i,t}$, is contributed by both chartists and fundamentalists from the respective market:

$$D_{i,t} = W_{ii,t}^f D_{ii,t}^f n_i + W_{ii,t}^c D_{ii,t}^c n_i.$$

Following Day and Huang (1990), we assume the existence of a *market maker* who

³For the case that h_j , $j = A, B$, differ from each other, the main conclusions of this paper will not be altered except that the steady states and the relevant corresponding stability conditions may not be derived analytically. Since h_j affects the demand of chartist only, we choose to use chartist demand strength " c " to capture the difference among chartist.

updates the market price at each period adaptively with

$$P_{i,t+1} = P_{i,t} + a_i D_{i,t} \quad (2.3)$$

where a_i is the *price adjustment parameter* in market i .

2.2.2 Theoretical Implications

Substituting with relevant components, Eq. (2.3) leads to a nonlinear dynamics of $x_{i,t}$:

$$x_{i,t+1} = x_{i,t} + a_i \frac{h \cdot c_i n_i \exp(-\rho |x_{i,t}|) - f_i n_i \exp(\rho |x_{i,t}|)}{\exp(\rho |x_{i,t}|) + h \cdot \exp(-\rho |x_{i,t}|)} x_{i,t} \quad (2.4)$$

for $i = A$ or B .

We then arrive at the following conclusions.

Proposition 1 (a) *There exists a unique fundamental steady state $\bar{x}_i = 0$, which is stable if and only if $\beta^l < \beta_i < 1$ with $\beta^l = \max\{1 - 2(h + 1)/(a_i f_i n_i), 0\}$;*

(b) *There exist two nonfundamental steady states: $\bar{x}_i = \pm \frac{1}{2\rho} \ln \beta_i$, which are stable if $1 < \beta_i < \beta^u$ with $\beta^u = \exp(2(c_i + f_i)/(a_i c_i f_i n_i))$;*

(c) *A pitchfork bifurcation occurs at $\beta_i = 1$ while a flip bifurcation arises at $\beta_i = \beta^u$.*

Remark 2 *Since β_i defined in (2.2) is proportional to c_i and reciprocal to f_i , respectively, Proposition 1 essentially suggests that the chartists and fundamentalists exercise destabilizing and stabilizing effect in isolated market respectively so that the overall effect depends on their interaction. In particular, increasing c_i with f_i being fixed, the price orbit for Market i will go through a classical pattern of bifurcations, that is, from steady state to a sequence of periodic, quasi-periodic, and finally to irregular fluctuations, which will be illustrated by numerical simulations in the next section.*

2.3 Market Linkage

If the two stock markets open their markets to each other, investors are allowed to invest in both markets. We assume these two markets are traded with a common currency

in order to exclude the effect of exchange rate. The setup of the isolated market model can be easily extended to a two-market system.

2.3.1 Main model

Upon market opening, investors have more investment options with a combination of market and agent strategy choices. Each investor from market j will compare four strategy fitness measures, $m_{ij,t}^k$, $i = A$ or B and $k = c$ or f , to make an investment decision. The composition of investors originating from market j is thus defined by.

$$W_{ij,t}^k \triangleq \frac{\exp\left(\rho m_{ij,t}^k\right)}{\sum_i \sum_k \exp\left(\rho m_{ij,t}^k\right)}.$$

We categorize fundamentalists and chartists into one group. Excess demands on market i due to this group are

$$D_{i,t} = n_A W_{iA,t}^c D_{iA,t}^c + n_A W_{iA,t}^f D_{iA,t}^f + n_B W_{iB,t}^c D_{iB,t}^c + n_B W_{iB,t}^f D_{iB,t}^f \quad (2.5)$$

where

$$D_{ij,t}^c = c_j \cdot x_{i,t} \text{ and } D_{ij,t}^f = -f_j \cdot x_{i,t}$$

for $i, j = A$ or B .

Due to the existence of many common economic factors (such as macro-economic environment and same financial regulations) as well as non-economic factors (such as political environment, infrastructure, social institutions, language, etc.) in a regional economic zone, there surfaces a second group of investors, inter-market traders whose excess demand in one market is based on conditions of the other market. De Jong et al. (2009) report the existence of this type of investors empirically. In our model, excess demand of inter-market traders at market i is treated as a function of excess demand of the first group of investors in the other market, $D_{-i,t}$. This is because excess demand of the first group of investors reflect the condition of the other market. In this setup, inter-market traders need information of excess demand on the other market. Given the rapid development of information technology and financial market, the access of such information is not unrealistic at all. The former, information technology, improves data analysis and transmission. The

latter can be exemplified by stock exchange mergers and acquisitions. Recent example is the acquisition of New York Stock Exchange (NYSE) by Inter-Continental Exchange (ICE). Information technology and financial market development make it feasible to share transaction information among markets to better reveal market trends. Market makers of each market adaptively adjust market prices based on excess demand of both groups of investors. Specifically, we have

$$\left. \begin{aligned} P_{A,t+1} &= P_{A,t} + s_{A1}D_{A,t} + s_{A2}D_{B,t} \\ P_{B,t+1} &= P_{B,t} + s_{B1}D_{B,t} + s_{B2}D_{A,t} \end{aligned} \right\}, \quad (2.6)$$

where s_{i1} and s_{i2} ($i = A$ or B) are the demand coefficients of the two groups of investors. $s_{A2}D_{B,t}$ and $s_{B2}D_{A,t}$ are due to inter-market traders in Markets A and B , respectively.

For a better understanding of the strength of market linkage, we recast Eq. 2.6 into

$$\left. \begin{aligned} P_{A,t+1} &= P_{A,t} + a_A ((1 - g_A) D_{A,t} + g_A D_{B,t}) \\ P_{B,t+1} &= P_{B,t} + a_B ((1 - g_B) D_{B,t} + g_B D_{A,t}) \end{aligned} \right\} \quad (2.7)$$

where, for $i = A$ and B , $a_i \triangleq (s_{i1} + s_{i2})$ is the *price adjustment speed* in market i while $g_i \triangleq s_{i2}/(s_{i1} + s_{i2})$ is the *weightage of inter-market traders*.

Without loss of generality, it is assumed that $g_i < 0.5$ to reflect the fact that the first group of investors with reference to local market conditions will always play the dominant role in price movement. This can be justified by Preis et al. (2013) that investment strategies based on U.S. data are more successful in U.S. markets. This assumption is equivalent to

$$g_A + g_B < 1, \quad (2.8)$$

a condition that is indispensable to derive the stability conditions in Proposition 3.

2.3.2 Steady State and Stability

Noticing that $x_i = P_i - F_i$, the regional market linkage given by Eq. 2.7 can be alternatively expressed as a two-dimensional discrete dynamical system.

$$\left. \begin{aligned} x_{A,t+1} &= x_{A,t} + a_A ((1 - g_A) D_{A,t} + g_A D_{B,t}) \\ x_{B,t+1} &= x_{B,t} + a_B ((1 - g_B) D_{B,t} + g_B D_{A,t}) \end{aligned} \right\} \quad (2.9)$$

The steady states for this system with arbitrary a_A and a_B can be derived and whose stability can be discussed.

However, we shall confine ourselves to a more economically meaningful situation in which $a_A = a_B = a$. For a company listed in both markets, when excess demands in two markets are identical ($D_{A,t} = D_{B,t}$), $a_A \neq a_B$ will result in different asset prices and create arbitrage opportunity. Hence, the very assumption $a_A = a_B = a$ is essential and indispensable to exclude the arbitrage opportunities due to unequal price updating mechanism.

For the simplicity of the expression, we define the *relative strength of chartists* as

$$\beta_{A+B} \triangleq h \cdot (c_A n_A + c_B n_B) / (f_A n_A + f_B n_B). \quad (2.10)$$

The steady states of two markets, denoted as (\bar{x}_A, \bar{x}_B) , and the corresponding stability conditions can be summarized as the following

Proposition 3 (a) *There exists a fundamental steady state $(\bar{x}_A, \bar{x}_B) = (0, 0)$, which is stable if and only if $\beta^l < \beta_{A+B} < 1$ with $\beta^l = \max\{1 - 4(h + 1) / (a \cdot (f_A n_A + f_B n_B)), 0\}$.*

(b) *There exist four nonfundamental steady states: $(\bar{x}_A, \bar{x}_B) = \left(\pm \frac{1}{2\rho} \ln \beta_{A+B}, \pm \frac{1}{2\rho} \ln \beta_{A+B}\right)$, which are stable if $1 < \beta_{A+B} < \beta^u$ with*

$$\beta^u = \exp[4 \cdot (1 / (c_A n_A + c_B n_B) + 1 / (f_A n_A + f_B n_B)) / a].$$

(c) *Mixed fundamental and nonfundamental steady states $(\bar{x}_A, \bar{x}_B) = \left(\pm \frac{1}{2\rho} \ln \beta_{A+B}, 0\right)$ or $\left(0, \pm \frac{1}{2\rho} \ln \beta_{A+B}\right)$ are all unstable.*

Proof. detailed proof and clarification is provided in Appendix. ■

Remark 4 *Once the two regional markets are open to each other, they will synchronize to be in either the fundamental (if $\beta_{A+B} < 1$) or the nonfundamental (if $\beta_{A+B} > 1$) steady states concurrently. The possibility for one market being in the fundamental steady state while the other in a nonfundamental one will never exist since all such steady states are unstable. Moreover, if one market ends up either with regular (irregular) periodic cycles or chaos, the other will do the same.*

Similar to the case in isolation, the pivotal factor for stability of the market system is β_{A+B} , the relative strength of chartists. If $\beta_{A+B} < 1$, the two-market system is stable in the fundamental steady state. With $\beta_{A+B} > 1$, the system deviates from the fundamental steady state and stabilizes in one of the nonfundamental steady states. Further increasing β_{A+B} will violate the stability condition so that the system then experiences a series of state transitions such as periodic, quasi-periodic and even chaos. These facts can be numerically illustrated.

2.3.3 Linkage Effect and Policy Implications

Following the convention of Dieci and Westerhoff (2010), a market is said to be stabilized if the range of price deviation(s) is reduced. Formally, we have

Definition 5 *The market linkage system defined in Eq. 2.9 is said to be stabilized if either*

- i) the steady state changes from one of non-fundamental states to the fundamental state; or*
- ii) the range of price fluctuations is reduced.*

As the stability of the market linkage system is determined by the relative strength of the investors from both markets, individual market's intrinsic characteristics may be overwritten. It is possible that an initially stable isolated market may be destabilized and an initially unstable isolated market may be stabilized by market linkage.

The linkage effect can be illustrated by a phase space diagram, as given in Fig. 2.1, where a default set of parameters is adopted⁴. Three phases: Fundamental (I), Non-

⁴To preserve continuity and unity, a default parameter set ($a_A = a_B = 2.131$, $c_A = 1$, $c_B = 0.5$, $f_A = f_B = 1$, $F_A = F_B = 0$, $g_A = g_B = 0.46$, $h = 1$, $\rho = 60$ and $n_A = n_B = 1$) will be adopted unless is otherwise specified.

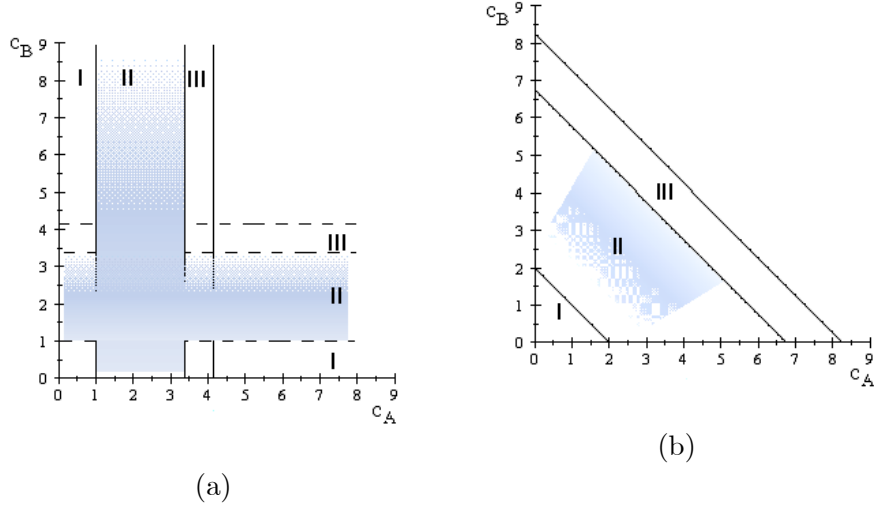


Figure 2.1: Phase space c_B vs c_A for fundamental (I), nonfundamental (II) and two-period (III) states. (a) isolated markets: vertical and horizontal lines separate different states for Markets A and B, respectively. (b) market linkage established: phase boundaries of market system are determined jointly by c_A and c_B .

fundamental (II) and Two-period (III) states, are illustrated in this phase space diagram. When markets are isolated, each market falls into any of the three phases independently of the other market. c_B and c_A are independent of each other in the diagram. Once the market linkage is set up, c_B and c_A are related to form the phase space as β_{A+B} is jointly determined by both of them. Regions of market state changes. For illustration, at point $(c_A, c_B) = (0.5, 4)$, Markets A and B are in fundamental steady state and two-period orbit state, respectively, when they are isolated. With market linkage, the market system transforms into the case in which both markets are in one of the nonfundamental steady states. In addition, mixed steady state with fundamental and nonfundamental market members does not exist.

The comparison of stabilities between market linkage and its isolation counterparts can be understood from β_{A+B} and its isolation counterparts β_A and β_B . It has been shown that chartists have a destabilizing effect on the market system so that a larger β_{A+B} leads the market system to a more unstable state. However, it can be seen from the definition

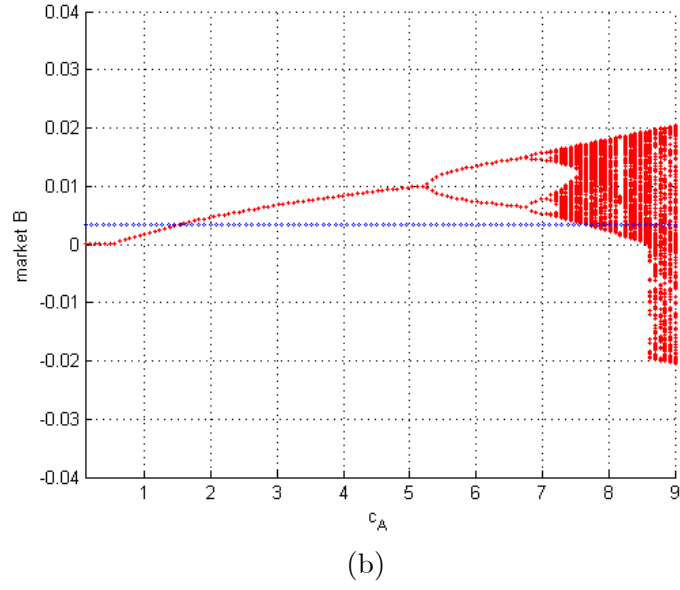
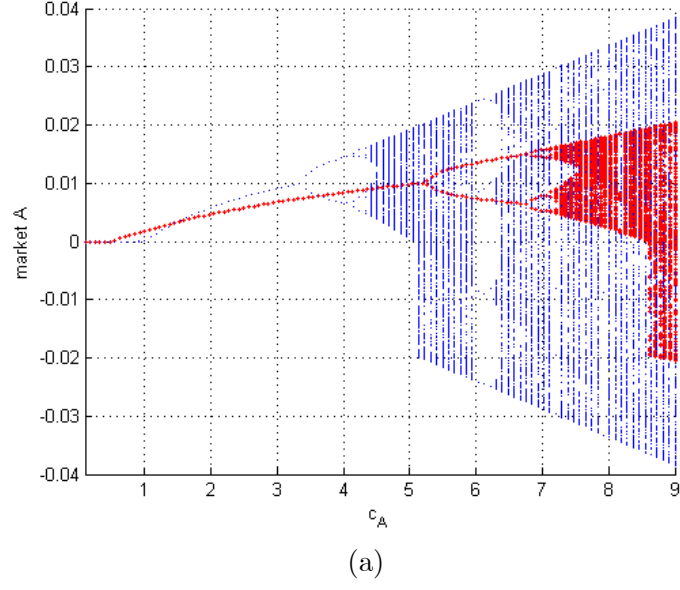


Figure 2.2: Bifurcation diagram comparison between isolated market and market linkage with c_A as a bifurcation parameter, given fixed $c_B = 1.5$. If $c_A < c_B$, Market A is destabilized while Market B is stabilized; if $c_A > c_B$, Market A is stabilized while Market B is destabilized.

(2.10) that, if $\beta_A < \beta_B$, we have

$$\beta_A = hc_A n_A / (f_A n_A) \leq \beta_{A+B} \leq hc_B n_B / (f_B n_B) = \beta_B,$$

Therefore, the market linkage system is always relatively more stable in comparison to one isolated market and also relatively more unstable in comparison to the other market. It is analogous to a zero-sum game in the sense that, with market linkage, *one market experiencing stabilizing effect will definitely imply the other to be suffering from a destabilizing effect.*

Fig. 2.2 illustrates such a linkage effect with a bifurcation diagram overlapped with its isolation counterpart, where the chartist strength of Market A, c_A , is taken as the bifurcation parameter while other parameters remain the same as specified in the default sets. The blue color diagrams are for the case of market isolation while red color ones are for the case of market linkage. Given condition $c_B = 1.5$ and $f_B = 1$, Market B is in the nonfundamental steady state and is not affected by the changes of chartist strength in Market A when it is isolated. But, once market linkage is established, each market can experience both stabilizing and destabilizing effects. Market A is destabilized from the fundamental steady state into the nonfundamental steady state for $c_A \in (0.5, 1.5)$ and then stabilized for $c_A > 1.5$. In contrast, originally in the nonfundamental steady state, Market B is no longer non-reactive to the change of c_A . Instead, it experiences the similar bifurcation state transitions with Market A. In particular, it is stabilized for $c_A < 1.5$ and destabilized for $c_A > 1.5$. From this illustration, we can see that as long as $c_A < c_B$, Market A is destabilized while Market B is stabilized by market linkage and vice versa for $c_A > c_B$. It is shown that an initially relatively stable market can stabilize the market system while it is subjected to destabilizing effect from the market system.

The linkage effect can also be further examined from the perspective of investors' population. The market with a larger population will have a larger impact on the market linkage. Without loss of generality, we use Market B's population (n_B) to investigate the effect of individual market population on the resulted market system β_{A+B} . It can be derived that

$$\frac{d\beta_{A+B}}{dn_B} = f_A \cdot f_B \cdot n_A \cdot (\beta_B - \beta_A) / (f_A \cdot n_A + f_B \cdot n_B)^2.$$

This implies that if Market B is relatively unstable compared to Market A ($\beta_B > \beta_A$), increasing population of Market B increases β_{A+B} ($d\beta_{A+B}/dn_B > 0$) as well as the price deviations under the market linkage. Conversely, if Market B is relatively stable compared to Market A ($\beta_B < \beta_A$), increasing population of Market B decreases β_{A+B} ($d\beta_{A+B}/dn_B < 0$) as well as price deviations under the market linkage. The limit of n_B is that β_{A+B} will converge to β_B as

$$\lim_{n_B \rightarrow \infty} \beta_{A+B} = \beta_B$$

The discussions in this section can be summarized formally as

Proposition 6 *When market linkage is established, an initially relatively stable market applies stabilizing effect on the market system but suffers from destabilizing effect from market linkage. The market with relatively larger population has relatively larger influence on the market linkage. Increasing population of one market will stabilize(destabilize) the market system if that market is initially relatively stable (unstable) in isolation.*

Policy implication from the linkage effect is straightforward. Under financial liberalization, a small market might have less influence on the financial stability given the fixed investor's strength of both markets. It might be subjected to the influence of a large market and lose its intrinsic properties.

2.4 Price Co-movement and other Stylized Facts

As per Cont (2001), Lux and Ausloos (2002) and Westerhoff and Dieci (2006), real world speculative markets have following characteristics: (1) prices have random switching bearish and bullish episodes; (2) volatility cluster phenomena are observed in which high-volatility events tend to cluster in time; (3) the distribution of returns has fat tails; (4) daily return autocorrelation tends to be insignificant; (5) absolute daily returns exhibit strong autocorrelation. Besides the above stylized fact for a single market, empirical studies have also shown the existence of cross-correlation between markets.

2.4.1 Theoretical Results

Theoretically, it can be shown that

Proposition 7 *When two regional markets share a common currency and other underlying economic and non-economic factors, there exists cross-correlation between returns of the two markets.*

Proof. According to Lux and Ausloos (2002), return r can be defined as log price changes, that is,

$$r_{i,t+1} = P_{i,t+1} - P_{i,t}.$$

Two-market system Eq. 2.7 is then simplified to

$$\left. \begin{aligned} r_{A,t+1} &= a_A \cdot ((1 - g_A) D_{A,t} + g_A D_{B,t}) \\ r_{B,t+1} &= a_B \cdot ((1 - g_B) D_{B,t} + g_B D_{A,t}) \end{aligned} \right\}$$

Notice that $D_{A,t}$ and $D_{B,t}$ themselves can be expressed as the functions of $r_{A,t}$ and $r_{B,t}$, we are able to get a recursive dynamic system for the market returns as

$$\left. \begin{aligned} r_{A,t+1} &= F(r_{A,t}, r_{B,t}) \\ r_{B,t+1} &= G(r_{A,t}, r_{B,t}) \end{aligned} \right\}$$

In fact, simple mathematical manipulation reveals a nonlinear correlation relationship between the two returns as

$$r_{A,t+1} = \alpha_A \cdot r_{B,t+1} + \gamma_A \cdot D_{A,t},$$

where $\alpha_A = a_A g_A / (a_B (1 - g_B))$ and $\gamma_A = a_A a_B (1 - g_A - g_B) / (a_B (1 - g_B))$.

In other words, two returns are correlated and the two markets exhibit the typical price co-movement. ■

Remark 8 *Given cross-correlation or price co-movement, both markets will experience simultaneous high and low returns. Looking at return trajectories of both markets, simultaneous high and low volatility should be observed. These expectations will be confirmed by the simulations followed.*

Table 2.1: Summary statistics of returns

Variable	Kurtosis	Skewness	Mean	S.D.	Median	Min	Max
Return A	3.650	-0.003	0	0.146	0	-0.396	0.396
Return B	3.649	-0.002	0	0.146	0	-0.396	0.396

2.4.2 Calibrated Simulations

We first use the chaos region parameters setting without noise ($a_A = a_B = 1$, $c_A = 53.68$, $h = 25$, $\sigma_A = \sigma_B = 0$) and initial prices ($x_{A,0} = 0.00011$ and $x_{B,0} = 0.0001$) to conduct the simulations. 15,000 observations of each market are generated. For clearer illustration, log price movements of last 200 time steps are plotted to check the random switching of market episodes. Return time series are plotted to demonstrate the volatility clustering phenomena.

Fig. 2.3 plots log price trajectories. It demonstrates features of the random switching of bullish and bearish market episodes. Most of the time, the two markets have similar price movement, especially in a period with large volatility, although prices of the two markets are not the same for most of the time. This period of large volatility corresponds to the volatility clustering which is observed in the return trajectories with a larger time scale of 1500 time steps in Fig. 2.4. In addition, simultaneous high and low volatility is found in both markets.

Fig. 2.5 compares return distribution to the normally distributed one for each market. Fat tails are observed from the return distribution, which indicates that there are more extreme returns compared to the normal distribution. Table 2.1 lists down the descriptive statistics.

Fig. 2.6 investigates the behavior of autocorrelation. For both market members, autocorrelation of returns tend to be insignificant across lags except for the first few lags, while the ones of absolute returns are significant and decrease slowly with lags. These autocorrelation behaviors are similar to Zhu et al. (2009).

Fig. 2.7 examines cross-correlation of return between the two market members. At 95% confidence interval, cross-correlation exists across lags, especially for lag zero. The strong cross-correlation ($\rho_{AB} = 1$) at lag zero explains the co-movement of the log price

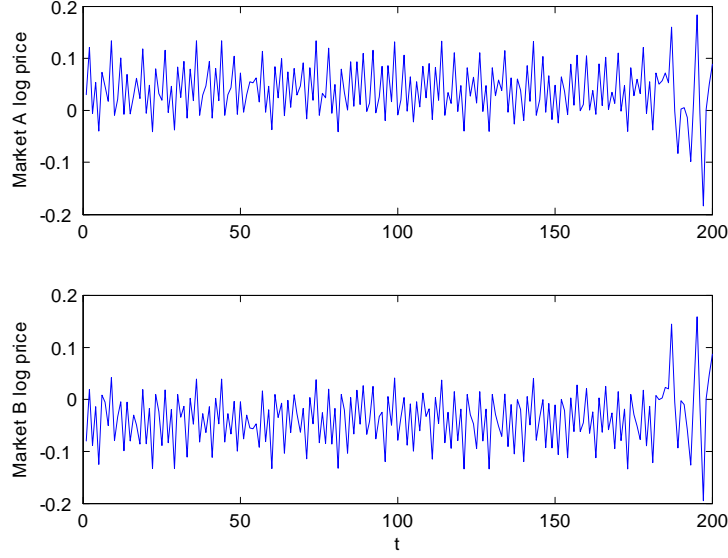


Figure 2.3: Trajectories of last 200 steps, similar price movement with different price values.

trajectories and simultaneous high and low volatility in both markets.

It can be verified that our model is robust to small noise in the sense that similar statistics results can be obtained, even if a white noise is added to the market linkage system Eq. 2.7.

The above stylized facts demonstrate our model's capability to generate some of the most important stylized facts observed in financial markets.

2.5 Conclusion

A two-market heterogeneous agents model is developed in this chapter. Each market has two groups of investors. The first group of investors consists of chartists and fundamentalists who are inhomogeneous across markets. Market linkage is established by allowing chartists and fundamentalists to invest in each market. Aware of capital movement of the investors and common factors underlying the two markets, a second group of investors, inter-market traders, surface upon market linkage. Excess demand of inter-market traders in one market is based on conditions of the other market. Individual market maker updates

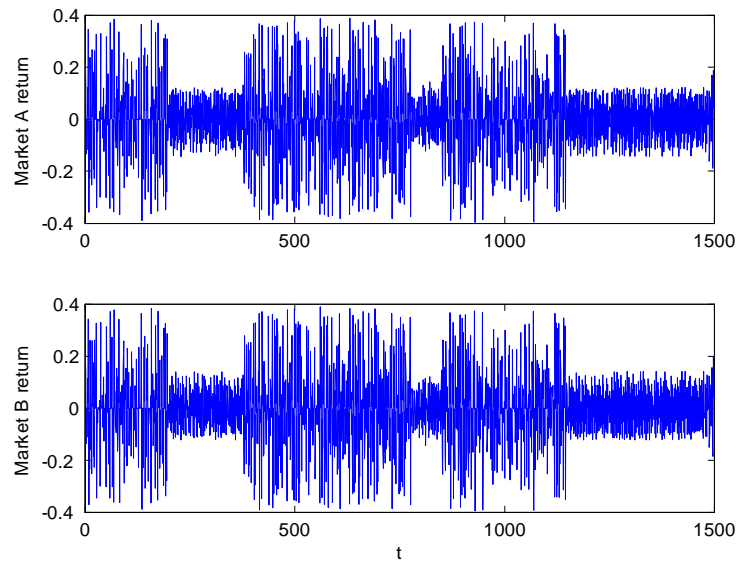


Figure 2.4: Volatility clustering, two markets with similar high and low volatilities.

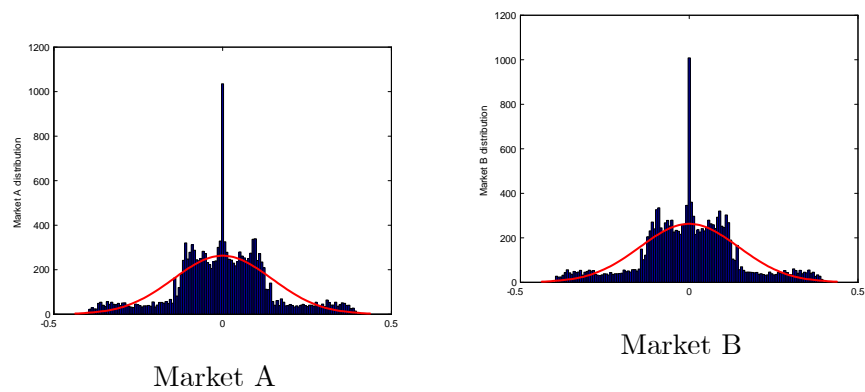


Figure 2.5: "Fat tails" of returns distribution with kurtosis larger than 3.

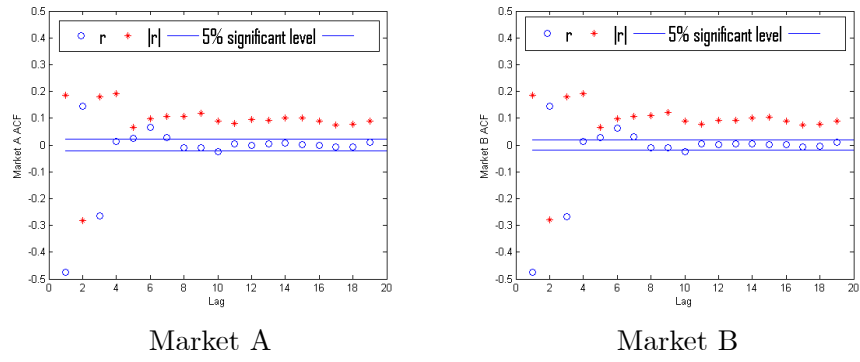


Figure 2.6: Autocorrelations vs lags, absolute returns are significant while returns are insignificant.

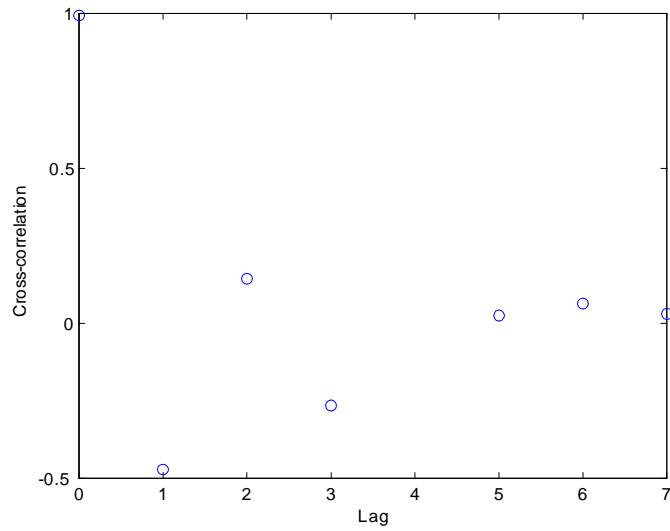


Figure 2.7: Significant cross-correlation vs lags.

price for her market based on excess demands of both groups of investors. Existence of price co-movement/cross-correlation between markets is proved.

By establishing market linkage, individual market's intrinsic dynamic properties may be overwritten. A market that is more stable initially in isolation will exert stabilizing effect on the market system while it will be subjected to destabilizing effect from the resultant market system. In addition, a market with a larger population has larger influence over the resulted assets prices of the market system. This market linkage can provide policy implication for financial market opening. In a world consisting of a small market and a large market (or market agglomeration), if the small market is stable compared to the large one, market opening of the small market will cause the small market to be destabilized. Small market will benefit from market opening only if it is unstable originally compared to the large market. This example indicates that market opening is a double-edged sword. Decision of market opening should be based on the impact assessment on internal and external markets.

Lastly, numerical simulations demonstrate the model's capability to generate some of the stylized facts of speculative financial market, especially the cross-correlation between markets. To our best knowledge, very few HAM models are capable of generating the significant cross-correlation effect. Schmitt and Westerhoff (2013) also manage to replicate this stylized facts. Our model can be useful to study multi-market financial system as cross-correlation should become more and more evident given the current trend of financial integration.

With market opening and financial market integration, investors enjoy lower transaction cost and more investment opportunities. However, we should be aware of the other side of the coin. Market coupling and cross-correlation between markets should become larger, especially during global events such as financial crisis. More markets tend to synchronize and are impacted by negative shocks and there will be less chance of risk diversification. If the process of financial market integration is inevitable, further studies of market cross-correlation are urged to have an insightful understanding of market interactions. One of the future research directions can be behavioral explanation for market correlation from aspect of market data rather than statistical fitting, as exemplified by Feng et al. (2012).

Appendix: Proof of Proposition 3

For a two-dimension system given by Eq. (2.9), where $D_{A,t}$ and $D_{B,t}$ are defined in Eq. 2.5, if the system reaches a steady state (\bar{x}_A, \bar{x}_B) , the following identities hold true:

$$0 = (1 - g_A) D_{A,t} + g_A D_{B,t},$$

$$0 = g_B D_{A,t} + (1 - g_B) D_{B,t},$$

which imply that $D_{A,t} = D_{B,t} = 0$ due to $g_A + g_B \neq 1$ implied by inequality Eq. 2.8.

Substituting the details, we see that

$$D_{A,t} = \frac{h(c_A n_A + c_B n_B) \exp(-\rho |x_{A,t}|) - (f_A n_A + f_B n_B) \exp(\rho |x_{A,t}|)}{h(\exp(-\rho |x_{A,t}|) + \exp(-\rho |x_{B,t}|)) + \exp(\rho |x_{A,t}|) + \exp(\rho |x_{B,t}|)} x_{A,t} = 0,$$

$$D_{B,t} = \frac{h(c_A n_A + c_B n_B) \exp(-\rho |x_{B,t}|) - (f_A n_A + f_B n_B) \exp(\rho |x_{B,t}|)}{h(\exp(-\rho |x_{A,t}|) + \exp(-\rho |x_{B,t}|)) + \exp(\rho |x_{A,t}|) + \exp(\rho |x_{B,t}|)} x_{B,t} = 0,$$

together suggest that the steady state can be a *fundamental one*

$$(\bar{x}_A, \bar{x}_B) = (0, 0),$$

or a *nonfundamental type*:

$$(\bar{x}_A, \bar{x}_B) = (\pm \frac{1}{2\rho} \ln \beta_{A+B}, \pm \frac{1}{2\rho} \ln \beta_{A+B}),$$

or a *mixed type*

$$(\bar{x}_A, \bar{x}_B) = (\pm \frac{1}{2\rho} \ln \beta_{A+B}, 0) \text{ or } (0, \pm \frac{1}{2\rho} \ln \beta_{A+B}),$$

where $\beta_{A+B} = h(c_A n_A + c_B n_B) / (f_A n_A + f_B n_B) > 1$ is imposed for the existence of non-fundamental steady state for individual market.

The Jacobian matrix evaluated at the fundamental steady state $(\bar{x}_A, \bar{x}_B) = (0, 0)$ turns out to be

$$\mathcal{J}_f = \begin{bmatrix} 1 + a(1 - g_A) A_0 & a g_A A_0 \\ a g_B A_0 & 1 + a(1 - g_B) A_0 \end{bmatrix}$$

where

$$A_0 = \frac{h(c_A n_A + c_B n_B) - (f_A n_A + f_B n_B)}{2(h + 1)},$$

a pair of eigenvalues is yielded:

$$\left. \begin{aligned} \lambda_1 &= 1 + a \frac{h(c_A n_A + c_B n_B) - (f_A n_A + f_B n_B)}{2(h+1)} (1 - g_A - g_B) \\ \lambda_2 &= 1 + a \frac{h(c_A n_A + c_B n_B) - (f_A n_A + f_B n_B)}{2(h+1)} \end{aligned} \right\}.$$

The stability conditions of $|\lambda_j| < 1$, $j = 1, 2$, demand that

$$\beta^l < \beta_{A+B} < 1,$$

where $\beta^l = \max\{1 - 4(h+1)/(a \cdot (f_A n_A + f_B n_B)), 0\}$.

Similarly, at the nonfundamental steady state $(\bar{x}_A, \bar{x}_B) = (\pm \frac{1}{2\rho} \ln \beta_{A+B}, \pm \frac{1}{2\rho} \ln \beta_{A+B})$, the Jacobian matrix take the form of

$$\mathcal{J}_n = \begin{bmatrix} 1 + a(1 - g_A) A_1 & ag_A A_1 \\ ag_B A_1 & 1 + a(1 - g_B) A_1 \end{bmatrix},$$

where

$$A_1 = \frac{-(c_A n_A + c_B n_B)(f_A n_A + f_B n_B)}{2[(f_A n_A + f_B n_B) + (c_A n_A + c_B n_B)]} \ln \frac{h(c_A n_A + c_B n_B)}{f_A n_A + f_B n_B},$$

which yields a pair of eigenvalues:

$$\left. \begin{aligned} \lambda_1 &= 1 + a(1 - g_A - g_B) A_1 \\ \lambda_2 &= 1 + a A_1 \end{aligned} \right\}.$$

The corresponding stability condition is $1 < \beta_{A+B} < \beta^u$ with

$$\beta^u = \exp[4 \cdot (1/(c_A n_A + c_B n_B) + 1/(f_A n_A + f_B n_B)) / a].$$

Finally, for the mixed steady state given by $(\bar{x}_A, \bar{x}_B) = (\pm \frac{1}{2\rho} \ln \beta_{A+B}, 0)$, the Jacobian matrix becomes

$$\mathcal{J}_m = \begin{bmatrix} 1 + a(1 - g_A) A_2 & ag_A A_3 \\ ag_B A_2 & 1 + a(1 - g_B) A_3 \end{bmatrix},$$

where

$$\begin{aligned} A_2 &= -\frac{\rho\gamma [h(c_A n_A + c_B n_B) \exp(-\rho\gamma) + (f_A n_A + f_B n_B) \exp(\rho\gamma)]}{h(\exp(-\rho\gamma) + 1) + \exp(\rho\gamma) + 1} < 0, \\ A_3 &= \frac{h(c_A n_A + c_B n_B) - (f_A n_A + f_B n_B)}{h(\exp(-\rho\gamma) + 1) + \exp(\rho\gamma) + 1} > 0, \end{aligned}$$

and $\gamma \triangleq \frac{1}{2\rho} \ln \beta_{A+B}$ with $\beta_{A+B} > 1$.

The eigenvalues are

$$\lambda_{1,2} = 1 + \frac{1}{2}a \left((1 - g_A) A_2 + (1 - g_B) A_3 \pm \sqrt{((1 - g_A) A_2 + (1 - g_B) A_3)^2 + 4A_3 |A_2| (1 - g_A - g_B)} \right).$$

Due to $g_A + g_B < 1$ indicated in Eq. 2.8, we have $\lambda_1 > 1$ regardless of the sign of $(1 - g_A) A_2 + (1 - g_B) A_3$. Hence, this mixed steady state is not stable. By the symmetricity of the system, the mixed steady state $(\bar{x}_A, \bar{x}_B) = (0, \pm \frac{1}{2\rho} \ln \beta_{A+B})$ is unstable either.

Note: in deriving the stability at the fundamental equilibrium $(\bar{x}_A, \bar{x}_B) = (0, 0)$, there might be concerns of non-continuous derivatives. We release this concern by the proof below.

$$\frac{dx_{A,t+1}}{dx_{A,t}} = 1 + a_A \left((1 - g_A) \frac{dD_{A,t}}{dx_{A,t}} + g_A \frac{dD_{B,t}}{dx_{A,t}} \right),$$

By defining

$$\left. \begin{aligned} A &= h(c_A n_A + c_B n_B) \exp(-\rho |x_{A,t}|) - (f_A n_A + f_B n_B) \exp(\rho |x_{A,t}|), \\ B &= h(c_A n_A + c_B n_B) \exp(-\rho |x_{B,t}|) - (f_A n_A + f_B n_B) \exp(\rho |x_{B,t}|), \\ Z &= h(\exp(-\rho |x_{A,t}|) + \exp(-\rho |x_{B,t}|)) + \exp(\rho |x_{A,t}|) + \exp(\rho |x_{B,t}|), \end{aligned} \right\}.$$

we have

$$\left. \begin{aligned} D_{A,t} &= \frac{h(c_A n_A + c_B n_B) \exp(-\rho |x_{A,t}|) - (f_A n_A + f_B n_B) \exp(\rho |x_{A,t}|)}{h(\exp(-\rho |x_{A,t}|) + \exp(-\rho |x_{B,t}|)) + \exp(\rho |x_{A,t}|) + \exp(\rho |x_{B,t}|)} x_{A,t} = \frac{A}{Z} x_{A,t}, \\ D_{B,t} &= \frac{h(c_A n_A + c_B n_B) \exp(-\rho |x_{B,t}|) - (f_A n_A + f_B n_B) \exp(\rho |x_{B,t}|)}{h(\exp(-\rho |x_{A,t}|) + \exp(-\rho |x_{B,t}|)) + \exp(\rho |x_{A,t}|) + \exp(\rho |x_{B,t}|)} x_{B,t} = \frac{B}{Z} x_{B,t}. \end{aligned} \right\}.$$

Then,

$$\left. \begin{aligned} \frac{dD_{A,t}}{dx_{A,t}} &= \frac{A}{Z} + x_{A,t} \frac{d}{dx_{A,t}} \left(\frac{A}{Z} \right), \\ \frac{dD_{B,t}}{dx_{A,t}} &= x_{B,t} \frac{d}{dx_{A,t}} \left(\frac{B}{Z} \right). \end{aligned} \right\}.$$

It can be shown that at $(\bar{x}_A, \bar{x}_B) = (0, 0)$, $\frac{d}{dx_{A,t}} \left(\frac{A}{Z} \right)$ and $\frac{d}{dx_{A,t}} \left(\frac{B}{Z} \right)$ are finite. Hence

$$\left. \begin{aligned} \frac{dD_{A,t}}{dx_{A,t}}|_{x_{A,t} \rightarrow 0^+} &= \frac{dD_{A,t}}{dx_{A,t}}|_{x_{A,t} \rightarrow 0^-} = \frac{A}{Z}, \\ \frac{dD_{B,t}}{dx_{A,t}}|_{x_{A,t} \rightarrow 0^+} &= \frac{dD_{B,t}}{dx_{A,t}}|_{x_{A,t} \rightarrow 0^-} = 0. \end{aligned} \right\}.$$

Similarly, $\frac{dD_{A,t}}{dx_{B,t}}$ and $\frac{dD_{B,t}}{dx_{B,t}}$ can be verified with the same procedure. Hence, There is no non-continuous derivatives at the fundamental equilibrium $(\bar{x}_A, \bar{x}_B) = (0, 0)$ due to the absolute value function.

Chapter 3

Modelling Contagion of Financial Crises¹

3.1 Introduction

Along the history of financial market development, financial crisis is one of the perennial phenomena, in which large decline of asset price is observed. Usually, financial crisis is not isolated within one market. Instead, it has contagion effect: financial crisis originating from one market spreads to other markets causing simultaneous or sequential crises. Kindleberger and Aliber (2005) devote two chapters to document this contagion effect in their book. One of the examples of financial crisis contagion is the "Black Monday" of US stock market in October 19, 1987. In that day, other national stock markets such as UK experienced nearly simultaneous sharp decline. Technically, contagion can be measured by cross-correlation. Preis et al. (2012) show that average cross-correlation among DJIA (Dow Jones Industrial Average) members increases with market stress. Manconi et al. (2012) argue that investors with liquidity constraint play a role in propagating crisis from

¹A paper coauthored with Weihong Huang based on this chapter has been submitted for journal publication.

securitized to corporate bonds during subprime crisis. Allen and Moessner (2012) identify flight to liquidity and safety as a common features in propagation of financial crises in 1931 and 2008. Nevertheless, working mechanisms of financial crisis and its contagion are not fully understood yet. Given development of financial market integration and globalization, impact and depth of financial crisis contagion, if any, should become even more severe and deserve more attention as markets become more closely linked.

Following the financial crisis grouping of Rosser (2000), we classify financial crises into sudden crisis, smooth crisis and disturbing crisis according to the depth and length. In sudden crisis, price falls precipitately from peak to bottom in a short period. The "Black Monday" of Dow Jones Industrial Average Index (DJIA) in October 1987 is one of this kind. In smooth crisis, price decreases smoothly from peak to bottom in prolonged period with a persistent trend. The decline of the DJIA during the great depression is a typical example. In between sudden crisis and smooth crisis is disturbing crisis, in which price fluctuates disturbingly with a declining tendency. This can be exemplified by the drop of the DJIA during the crash in October 1929. These three patterns of financial crisis are exemplified in Fig. 3.1.

The development of heterogenous agents models (HAM) has provided a tool to investigate financial crises. Day and Huang (1990) setup the stylized framework of market maker and generate randomly switching bear and bull market episodes. The transition from bull to bear market episode mimics a sudden financial crisis. He and Westerhoff (2005) also investigate sudden crisis and evaluate policy of price limiters. Chiarella et al. (2003) investigate smooth crisis. Huang et al. (2010) manage to simulate all three patterns of financial crises. Their model indicates that both fundamentalists and chartists could contribute to financial crises and hence financial crises could be endogenous. Huang and Zheng (2012) generalize regime-dependent beliefs and regime-switching dynamics to examine the triggering mechanisms for all the three financial crisis patterns. For more literature study of HAM model, we cite, in particular, Lux (1995), Brock and LeBaron (1996), Brock and Hommes (1998), Lux and Marchesi (2000), Chiarella and He (2001), Farmer and Joshi (2002), Westerhoff and Reitz (2005), Hommes et al. (2005), Gao and Li (2011) and Chen (2012).

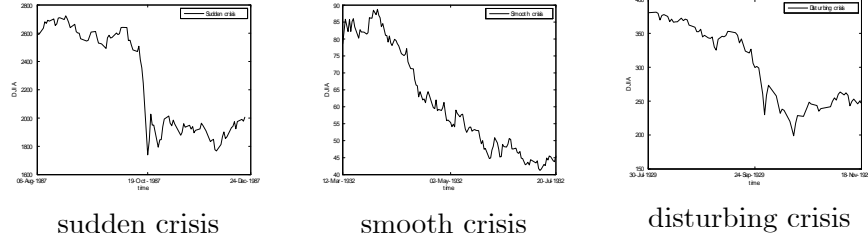


Figure 3.1: Three categories of financial crisis.

All the above literatures are related to one single market. To investigate the contagion behavior of financial crisis, a multi-market model is required. Academia has pointed out the direction of multi-market model. As pioneers, Brock et al. (2009) develop multi-asset model by introducing additional Arrow securities into the stylized evolutionary equilibrium model of Brock and Hommes (1998) and demonstrate that more hedging instruments may destabilize markets. Dieci and Westerhoff (2010) build up a three-market model in which two stock markets are linked via foreign exchange market. The two stock markets have only fundamentalists while the foreign exchange market is populated with chartists and fundamentalists. It is concluded that upon market interactions, stock markets may be destabilized while foreign exchange market and the whole market system can be stabilized relatively. De Jong et al. (2009) investigate stock markets of Hong Kong and Thailand during 1997 Asian crisis. In addition to the typical fundamentalists and chartists, they introduce into each market a third type of traders, inter-market traders, whose demand function is similar to that of chartists in foreign market. They also prove the existence of these inter-market traders empirically.

Inspired by De Jong et al. (2009) and Chapter 2, this chapter extends Huang and Zheng (2012)'s model to multi-market framework and include inter-market traders whose decision of trading in domestic market is based on fundamental value and chartists activities of foreign market. We mainly use simulations to demonstrate bursts of financial crisis and the contagion effect under different scenarios. From the point of view of endogeneity, we manage to capture the simultaneous crash behavior of US and UK stock markets during

"Black Monday" in 1987 as well as other financial crisis patterns. On the other hand, from the point of view of exogeneity, upon impact of permanent or temporary shock(s) on one market, financial crisis can arise and spread to other market. Factors such as magnitude of shock and duration of temporary shock also affect patterns of financial crisis. Lastly, numerical evaluation verifies capability of our model to match stylized facts of financial markets.

The rest of the chapter is organized as follows. Section 2 describes the dynamic two-market model and its steady state. Section 3 focuses on crisis behavior matching and simulating financial crisis under different scenarios. Section 4 matches stylized facts. Lastly, Section 5 concludes the chapter.

3.2 Theoretical Model

This section develops a two-market model with coupling mechanism market maker framework. Besides that, steady state and its stability conditions are derived.

3.2.1 Model Setup

We develop a two-market nonlinear model in this section. There are two markets A and B . $p_{j,t}$ is price per share of risky asset of market j ($= A$ or B) at period t . Each market is populated with three kinds of investors (agents): fundamentalists (f), chartists (c), and inter-market traders (i).

For a fundamentalist from j ($= A$ or B), her demand for asset j , $D_{j,t}^f$ ², is simply defined as

$$D_{j,t}^f = v_{j,t} (F_j - p_{j,t}) \quad (3.1)$$

where F_j is fundamental value of market j and $v_{j,t} > 0$ is convergence speed following the definition of Day and Huang (1990). $v_{j,t}$ is a bimodal function with modes near or at bottoming price $u_1 F_j$ ($u_1 < 1$) and topping price $u_2 F_j$ ($u_2 > 1$). The bimodal implies that convergence speed becomes high when price deviates too much from the fundamental value.

²In the sequel, we shall adopt the same notation convention with the subscript j standing for market, and the superscript k for type of traders.

Without loss of generality, we assume

$$v_{j,t} = (p_{j,t} - u_1 F_j)^d (u_2 F_j - p_{j,t})^d$$

with $d < 0$. With this setting, fundamentalists buy in asset when its price is below F_j and sell it out vice versa.

For a chartist, she applies technical analysis and divides price domain P_j into n regimes

$$P_j = \cup_{l=1}^n P_{j,l} = [\bar{p}_{j,0}, \bar{p}_{j,1}) \cup [\bar{p}_{j,1}, \bar{p}_{j,2}) \cup \dots \cup [\bar{p}_{j,n-1}, \bar{p}_{j,n}]$$

where $\bar{p}_{j,l}$ ($l = 1, 2, \dots, n$) represents thresholds of price regime l . It can also be interpreted as different support/resistance in technical analysis. The width of each price regime is equal to λ , i.e., $\lambda = \bar{p}_{j,l} - \bar{p}_{j,l-1}$. By conducting technical analysis, a reference price $p_{j,t}^c$ is derived by averaging the top and bottom thresholds of a price regime, in which the current price $p_{j,t}$ falls in. That is

$$p_{j,t}^c = \frac{\bar{p}_{j,l-1} + \bar{p}_{j,l}}{2}, \text{ if } p_{j,t} \in [\bar{p}_{j,l-1}, \bar{p}_{j,l}).$$

Given the reference price, a chartist from market j has demand for asset j , $D_{j,t}^c$, as determined by

$$D_{j,t}^c = \tau (p_{j,t} - p_{j,t}^c) \quad (3.2)$$

where τ is the demand parameter of a chartist. This asset demand function captures the behavior of chartists that they buy in asset when its price is above chartist reference price and sell out vice versa.

In this two-market model, we assume the number of each type of investor, k ($= c, f, i$), in individual market, j ($= A, B$), w_j^k is fixed. Then, excess demand in market j due to fundamentalists and chartists, $D_{j,t}$, is derived

$$D_{j,t} = w_j^f D_{j,t}^f + w_j^c D_{j,t}^c.$$

In the context of globalization and financial integration, news/innovation in one market member can transmit to other markets. The effect of transmission can be positive or negative. For example, upon increasing of oil price, stock market of oil producing country will go up while the stock market of oil consuming country will go down. This is a negative correlation. On the other hand, it is possible to have positive correlation. Economic growth

in one country increases demand for foreign goods. As a result, stock markets of both countries will increase together. Aware of this economic and financial connection, the third type of traders, inter-market traders trade based on foreign market conditions. The existence of this group of traders has been proven by De Jong et al. (2009) empirically. The group of inter-market traders is not homogeneous. While some of them look at the fundamental value of foreign market and have excess demand similar to that of fundamentalists in foreign market, the rest of them rely on chartist analysis in foreign market and have excess demand similar to that of chartists in foreign market. For simplicity, we assume excess demand of inter-market traders in market j is proportional to excess demand of fundamentalists and chartists in foreign market, $D_{-j,t}$:

$$D_{j,t}^i = w_j^i D_{-j,t}.$$

Given the excess demand of three types of traders, market maker in market j updates her price adaptively as

$$p_{j,t+1} = p_{j,t} + a [D_{j,t} + w_j^i D_{-j,t}] \quad (3.3)$$

where a is adjustment speed of price.

Based on Eq. 3.3, specifically, we have a two-dimensional deterministic market system:

$$\begin{cases} p_{A,t+1} = p_{A,t} + a [D_{A,t} + w_A^i D_{B,t}], \\ p_{B,t+1} = p_{B,t} + a [D_{B,t} + w_B^i D_{A,t}]. \end{cases}$$

3.2.2 Steady State

For theoretical analysis, steady state and its corresponding stability conditions are addressed in this subsection. Steady state of the two-market system is denoted by (\hat{p}_A, \hat{p}_B) .

Proposition 9 *Individual steady state \hat{p}_A and \hat{p}_B can be implicitly determined separately by $D_{A,t} = 0$ and $D_{B,t} = 0$, where*

$$\begin{cases} D_{A,t} = w_A^f v_{A,t} (F_A - \hat{p}_A) + w_A^c \tau (\hat{p}_A - p_{A,t}^c), \\ D_{B,t} = w_B^f v_{B,t} (F_B - \hat{p}_B) + w_B^c \tau (\hat{p}_B - p_{B,t}^c). \end{cases}$$

Remark 10 Convergence speed $v_{A,t}$ is a function of price $p_{A,t}$; chartist reference price $p_{A,t}^c$ is exogenously determined by technical analysis price levels $\bar{p}_{j,l-1}$ and $\bar{p}_{j,l}$, in between which $p_{A,t}$ locates; hence, $D_{A,t}$ is a function of its corresponding price $p_{A,t}$. If condition $D_{A,t} = 0$ is satisfied, \hat{p}_A can be implicitly determined and it is possible to have multiple solutions due to the multiple price regimes in chartists analysis. Thanks to the symmetry of A and B , the same analysis applies to market B .

Detailed proofs are provided in Appendix A.

3.3 Financial Crisis Simulations

In this section, we will apply the two-market model to simulate simultaneous crash behavior from two points of view: endogeneity and exogeneity with external shocks. Different views correspond to different scenarios of causes of financial crisis. Investigating from the point of view of endogeneity, we simulate different patterns of financial crises, including the sudden crisis with empirical reference to "Black Monday" of US and UK stock markets in 1987. On the other hand, from the point of view of exogeneity, permanent and temporary shocks are employed to evaluate their role in financial crisis.

3.3.1 Crisis from Endogeneity

In the morning of October 19, 1987, crash began in Far Eastern markets and then spread to Europe and US. During that day, DJIA dropped by 22.6%, the largest one-day percentage drop in history. And that day is called as "Black Monday". There are various versions of explanation for this crash, such as programming trading, over-valuation and market psychology. Our intension is to simulate a two-market crisis with contagion phenomena. If a model manages to simulate the contagion phenomena, at least it can provide a tool to understand partly, if not all, the crisis. Two markets DJIA (US) and FTSE 100 (UK) are used as reference. Sample period is from 08-01-1987 to 12-29-1987. A common set of parameters are defined for subsequent simulations unless the parameters are specified. This common set of parameters and conditions for subsequent simulations

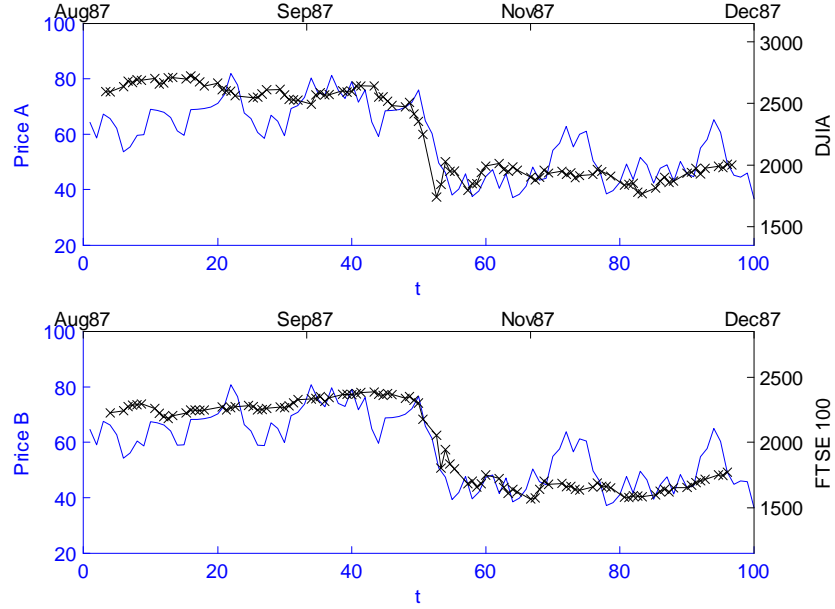
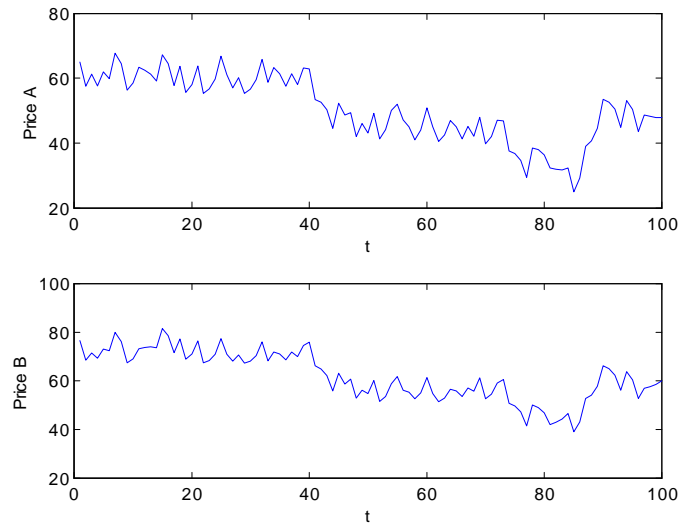


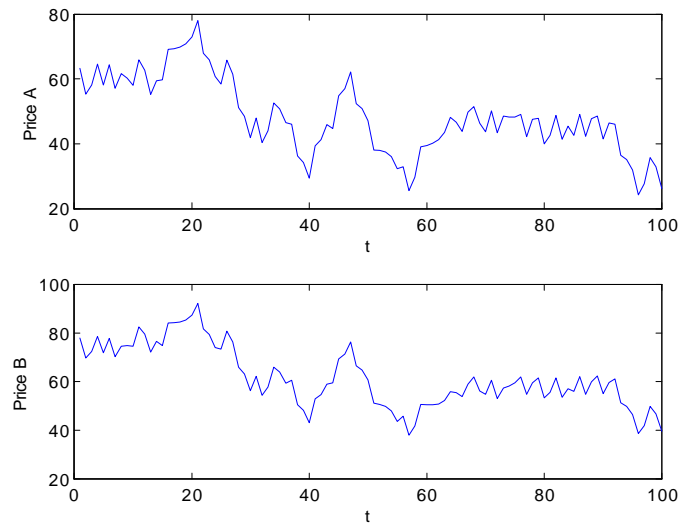
Figure 3.2: Crisis matching. Blue color is for simulation while the black color with makers is for real indexes DJIA and FTSE 100.

are provided in Appendix B. With initial prices condition "endo-sudden", our deterministic two-market model manages to mimic market trend of the two indexes during the crisis – a simultaneous sudden drop of asset prices (Fig. 3.2).

Besides sudden crisis, just by changing initial prices, our model also manages to produce patterns of smooth crisis and disturbing crisis with contagion behavior: both markets have similar trends although individual market values are not the same. Smooth crisis is produced with "endo-smooth". Both markets evolve without large fluctuations till time step 40 and then decline gradually to bottom around time step 85. During this declining process, both markets lose around 50% of their initial market values (Fig. 3.3.a). In contrast, given another set of initial prices "endo-disturbing", disturbing crisis emerges in both markets. From initial prices, both markets climb and reach their peaks at time step 21. After that, prices drop dramatically till time step 40 and then rebound. However, the rebound is temporary and prices drop to even lower bottoms at time step 58. From peak to bottom, each market loses around 60% of its value (Fig. 3.3.b).



(a)



(b)

Figure 3.3: Crisis with contagion behavior: (a) smooth crisis (b) disturbing crisis.

Simulations of this subsection for three types of financial crisis have a common feature that peaks of both markets' prices before crisis are well above fundamental values $F_i = 50$. Over-valuation causes dramatic adjustment of market without external force, which supports the scenario that over-valuation causes financial crisis. These simulations demonstrate the capability of the model to a certain extent to explain financial crisis and its contagion behavior. Similar to the conclusion of Huang et al. (2010) that financial crisis can be endogenous, in this case, financial crisis and its contagion effect occur without external force and are endogenous.

3.3.2 Crisis from Exogeneity

In real world, innovations continue to emerge and financial markets always encounter shocks affecting market fundamental. Such kinds of shocks can be technological innovation, macro-economics fluctuation and so on. In this subsection, simulations are conducted to evaluate the possibility of financial crisis induction by shocks to market fundamental. Market fundamental values F_A or F_B are no longer constant as in previous subsection. A shock can change F_A or F_B permanently or temporarily. Time frame of financial crisis usually is short in unit of days or months. The time window for this paper's study is 100 time steps. When a shock is in effect for a period longer than the time window, it is treated as permanent. Similarly, when a shock lasts less than the time window, it is treated as temporary. The purpose of the simulation is to verify whether a small shock to market fundamental value can cause dramatic price change as well as whether crisis contagion is possible.

Permanent shock

A permanent shock can arise from sources like changes in fiscal policy, such as a decrease in government expenditure, as well as some critical market events. Calvo (2012) argues that the collapse of Lehman Brothers triggers sub-prime crisis of 2008 as market conjectures that other large financial institutions might not be bailed out from then on and falls into panic. This subsection demonstrates that a small permanent shock in the

fundamental value of one market can induce drastic drop in asset price – financial crisis. Also, this financial crisis can spread to the other market. Depending on the magnitude of the shock, different patterns of crisis can be induced.

With condition "permanent-shock", reference price trajectories of markets A and B are created: fundamental values F_A and F_B do not change their values 50 and $p_{A,t}$ and $p_{B,t}$ fluctuate with range 25 ($= 65 - 40$)³ and 30 ($= 80 - 50$), respectively. At time step 30, a permanent shock hits F_B and F_B reduces its value by 1 and changes to 49, i.e. a 2% reduction in fundamental value of market B while F_A is not affected. This shock does not cause an immediate effect on both markets. However, after around 15 time steps, both markets experience a price drop. In terms of magnitude, $p_{A,t}$ still fluctuate approximately within the same price range with the reference price. In contrast, although $p_{B,t}$ experiences similar up and down trends with $p_{A,t}$, its adjustment is severe with a lower bottom value 40. Range of price fluctuation for market B has been increased by 30%, from 30 ($= 80 - 50$) to 40 ($= 80 - 40$) (Fig. 3.4.a).

To confirm the result, the same setting is utilized except the magnitude of the shock changed to 2, a 4% reduction in F_B . This time, a more severe drop in asset price of market B occurs. Disturbing crisis occurs in market B . $p_{B,t}$ drops from 80 to 30 such that price fluctuation range increases by 60%. Meanwhile, similar disturbing crisis is also observed in market A with fluctuation range increased more than 100%, from 25 ($= 65 - 40$) to 55 ($= 65 - 10$) (Fig. 3.4.b). Here, a 4% small shock in one market's fundamental value can trigger financial crisis spreading to both market with price fluctuation range increased more than 60%.

Shocks are not always negative. What will over-valued markets response to a positive shock? At time step 30, a permanent 0.6% increase in F_B from 50 to 50.3 causes both markets to adjust at first. After that, both markets are pushed up to reach new peaks, followed by smooth crisis. During the smooth crisis, $p_{B,t}$ drops from 82 to 22, with price fluctuation range increased by 100%, from 30 to 60 ($= 82 - 22$); $p_{A,t}$ drops from 80 to 20, with price range increased more than 100%, from 25 to 60 (Fig. 3.4.c). Here, even a positive shock can trigger a crisis by booming up larger assets bubbles which collapse

³peak of price - bottom of price

eventually. As a comparison, magnitude of the positive shock on F_B is increased to 1%, i.e., F_B increases from 50 to 50.5. Surprisingly, no crisis is observed this time. Prices of both markets fluctuate within the reference range (Fig. 3.4.d). In an over-valued market, depending on the magnitude, a positive shock on market fundamental has different possible consequences. It may push market to a higher peak upon which market self-correction is triggered and a crisis occurs. On the other hand, it can be absorbed within normal market fluctuations.

In a market system, each market can encounter shocks simultaneously. It has been shown that it is possible for a shock in one market to cause a financial crisis across markets. What will happen if market members encounter shocks simultaneously? Can shocks in different markets cancel out each other? At step 30, F_A decreases by 1.8% from 50 to 49.1 while F_B increases by 0.6% from 50 to 50.3. Contrasting to Fig. 3.4.c for a 0.6% increase in F_B , the smooth crisis disappears. Instead, both markets fluctuate within the reference range and no crisis is triggered (Fig. 3.5.a). In this case, shocks hitting individual market cancel out each other. If we increase the magnitude of shock in F_A to 3%, that is, F_A decreases from 50 to 48.5, smooth crisis occurs in both markets. $p_{A,t}$ decreases from 65 to 10, with fluctuation range increased more than 100%, from 25 to 55 ($= 65 - 10$). Similarly, fluctuation range of $p_{B,t}$ also increases by 100%, from 30 to 60 ($= 80 - 20$) (Fig. 3.5.b).

These demonstrations show that in a closely correlated financial world, a small shock, either positive or negative, in one market can create a financial crisis spreading to the other market. Shocks with different magnitudes have different impacts. Simultaneous shocks in market members can cause a financial crisis or cancel out each other without causing dramatic market reactions.

Temporary shock

Temporary shocks can be due to short term policy changes or psychological fluctuations caused by rumors. Individual company is the common entity hit by rumors. In September 8, 2008, United Airline's stock price plummeted more than 75%, from prior day's close \$12.3 to a low of \$3. The crash was caused by a rumor of an erroneous report

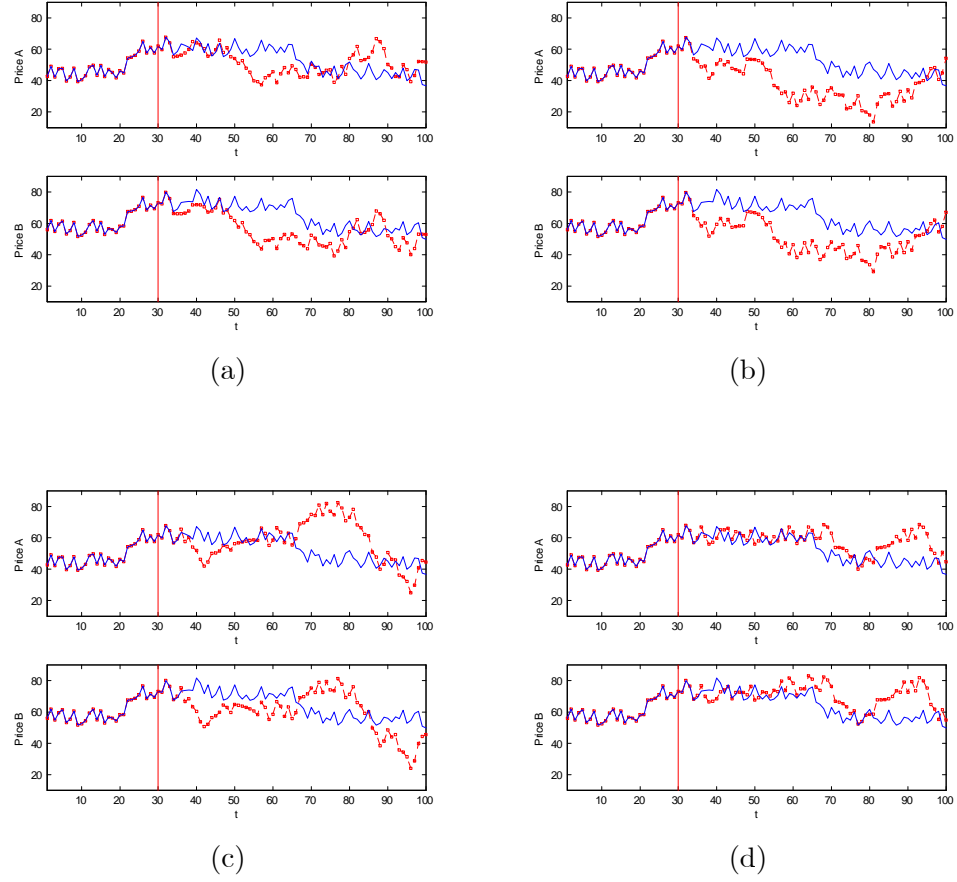


Figure 3.4: Impact of permanent shock. Blue color is reference trajectory while red marker represents situation in which F_B is hit by a permanent shock at step 30, highlighted by a vertical line. (a) F_B reduces by 2%. (b) F_B reduces by 4%. (c) F_B increases by 0.6%. (d) F_B increases by 1%.

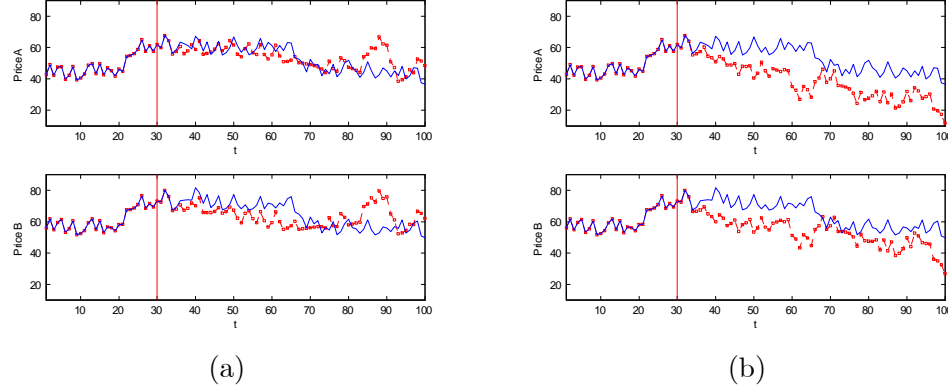


Figure 3.5: Impact of simultaneous permanent shocks. F_B increases by 0.6%. (a) F_A decreases by 1.8%. (b) F_A decreases by 3%.

claiming bankruptcy of the company. Even to a bigger scale of market level, rumors still could trigger financial crises. Kindleberger and Aliber (2005) discuss several cases of rumor triggering crisis. One of the examples is "Black Friday" of May 11, 1866 due to rumors of the Prussian-Austrian war. We demonstrate that a financial crisis could be triggered by temporary shock in this subsection.

Condition "temp-shock" is applied to create reference trajectories. Similar to the cases of permanent shock, $p_{A,t}$ and $p_{B,t}$ fluctuate with ranges of 25 ($= 65 - 40$) and 30 ($= 80 - 50$), respectively. At time step 30, F_B is hit by a shock and changes its value from 50 to 47, a 6% reduction. Since the shock is temporary, F_B recovers to its previous level 50 at time step 34. The duration of shock is 4 time steps. Meanwhile, market A is free of shock. There is no much change in the new price trajectories of both markets compared to the reference ones (Fig 3.6.a). However, if duration of the temporary shock to F_B is extended to a longer time, i.e. 6 time steps such that F_B recovers to its previous level 50 at time step 36, disturbing financial crisis occurs in both markets. Fluctuation ranges of markets A and B increase by 40% and 30% (Fig 3.6.b). As a comparison, we switch to a positive shock and evaluate its effect. At time step 30, a positive shock impacts F_A so that F_A increases from 50 to 51.4, a 2.8% increment. If F_A recovers to 50 at time step 34, no major effect is found on new price trajectories (Fig 3.6.c). However, if F_A recovers at time

step 36, disturbing crisis arises in market A with price range increased by 80%, from 25 to 45 ($= 65 - 20$). At the same time, market B also experiences similar crisis, with price range increased by 100%, from 30 to 60 ($= 80 - 20$) (Fig 3.6.d). Hence, even for a temporary shock in one market's fundamental value, depending on its duration, it is possible to cause financial crises in the two markets.

Extending to the case of simultaneous temporary shocks, both F_A and F_B encounter shocks simultaneously at step 30 and both recover to original value 50 at step 36. During the shock effective period, F_B decreases by 6% from 50 to 47. If F_A increases by 1.4% from 50 to 50.7 temporarily, disturbing financial crisis develops in both markets. Fluctuation range of $p_{A,t}$ increases by more than 100%, from 25 to 60 ($= 80 - 20$). Similarly, fluctuation range of $p_{B,t}$ increases by 100%, from 30 to 60 ($= 80 - 20$) (Fig. 3.7.a). If magnitude of positive shock in F_A is increased to 3% so that F_A increases from 50 to 51.5, both markets fluctuate comparably with the reference trajectories and no financial crisis is observed (Fig. 3.7.b). These results show that in an over-valued market system, simultaneous temporary shocks hitting individual market can produce different results, depending on the magnitude of shocks.

In above simulations of permanent and temporary shocks, at the time fundamental value of individual market is affected by a shock, prices of both markets are above their market fundamental values. Financial crisis in both markets is triggered by shock(s). It is implied that over-valuation does not always cause a financial crisis. This supports scenario that a financial crisis is attributed to market shock. In addition, the result that financial crisis triggered by shock in one market causes similar change in the other market is resemblance to domino effect in which a change causes a similar change nearby. In a world of over-valued and closely linked financial markets, certain macro-economic changes or market shocks in market member(s) can lead to over-adjustment and even disasters to all markets. These changes or shocks can be permanent or temporary. In this sense, financial markets are fragile. Policy implication is that policy to remove asset price bubbles must be designed with deliberation. Otherwise, adverse consequence might be caused.

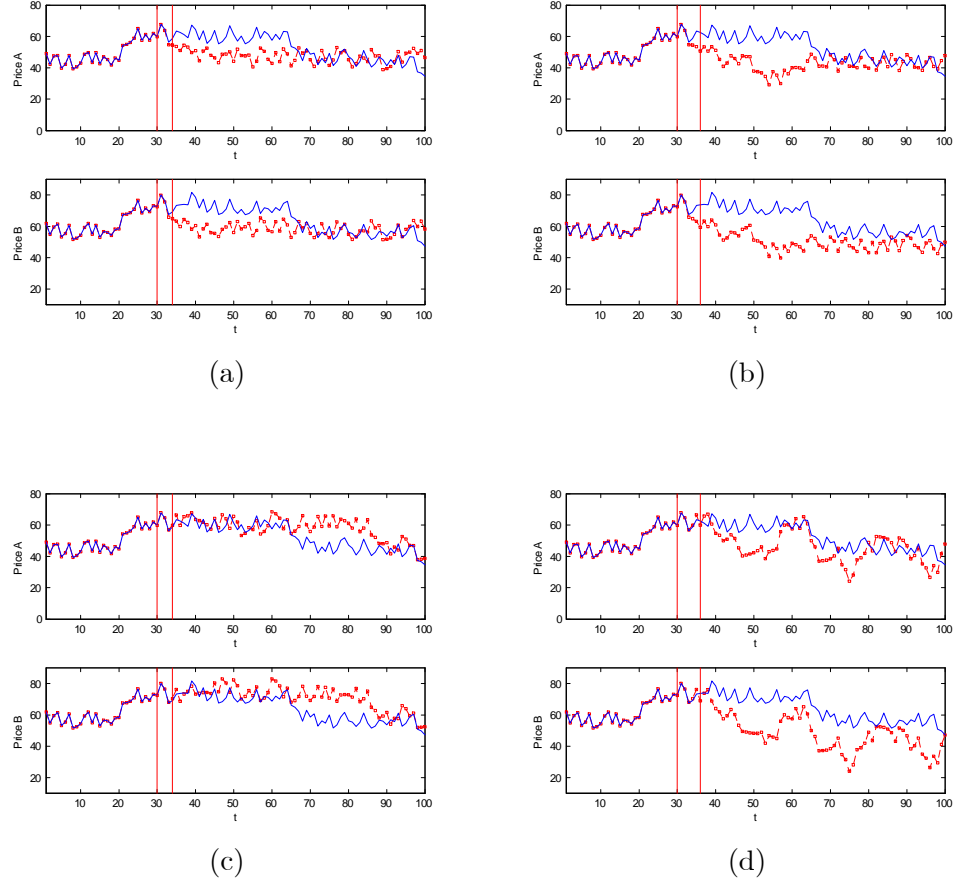


Figure 3.6: Impact of temporary shock on one market's fundamental value. In between two vertical lines is effective period of shock. In (a) and (b), F_B reduces by 6% at time step 30. (a). F_B recovers to original value 50 at time step 34. (b). F_B recovers at time step 36. In (c) and (d), F_A increases by 2.8% at time step 30. (c) F_A recovers to original value 50 at time step 34. (d). F_A recovers at time step 36.

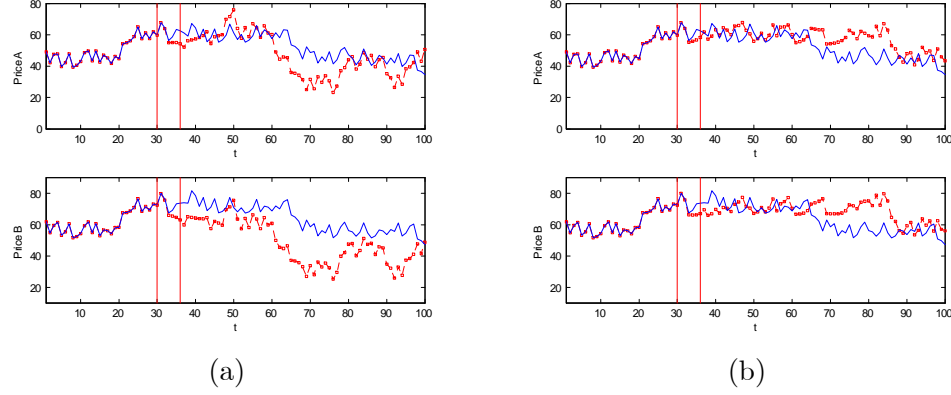


Figure 3.7: Impact of simultaneous temporary shocks. F_B reduces by 6% and effective in between step 30 and 36. (a) F_A increases by 1.4%. (b) F_A increases by 3%.

3.4 Stylized Facts

In this section, we calibrate our two-market model to match the stylized facts. According to Cont (2001), Lux and Ausloos (2002) and Westerhoff and Dieci (2006), real world speculative markets have following characteristics: (1) volatility cluster phenomena in which high-volatility events tend to cluster in time; (2) distribution of returns with fat tails; (3) insignificant autocorrelation for daily return; (4) strong autocorrelation for absolute daily returns. Besides the above stylized fact for a single market, empirical studies already show the existence of correlation between markets. For example, Egert and Kocenda (2011) find strong correlation among returns of Germany, France and UK, up to 0.9. This cross-correlation has been replicated in Chapter 2.

We denote $r_{j,t}$ for return of market j (A, B) at time step t . $r_{j,t}$ is defined as

$$r_{j,t} = \ln p_{j,t} - \ln p_{j,t-1}$$

To calibrate the stylized facts, condition "stylized-facts" are applied to generate 10,000 periods price trajectories and the corresponding returns are calculated. It is shown that individual market manages to match the typical stylized facts. Both markets exhibit volatility cluster in their return trajectories. Besides that, both markets experience the same large and small volatilities most of the time (Fig. 3.8). Distributions of return have fat tails

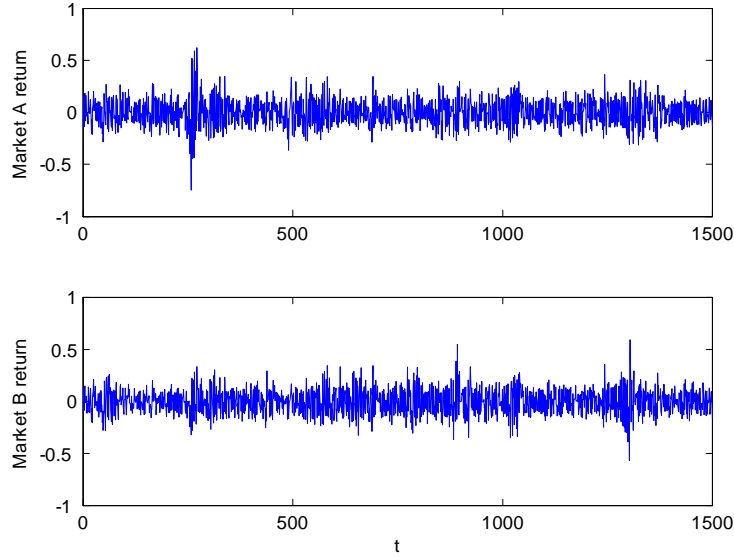


Figure 3.8: Volatility clustering

and kurtosis of markets A and B are 4.0 and 4.9, respectively (Fig. 3.9). Autocorrelation of return is insignificant across lags generally while autocorrelation of absolute returns is significant (Fig. 3.10). These stylized facts calibrations are similar to Zhu et al. (2009). In addition, at 95% confidence interval, there are significant cross-correlation for different lags. Especially at lag zero, the cross-correlation is up to 0.7, which implies prices comovement of the two markets and explains the similar volatility patterns of both markets (Fig. 3.11).

3.5 Conclusion

This chapter proposes a behavioral two-market model. In addition to typical traders such as fundamentalists and chartists, we introduce into the model a third type of traders, inter-market traders whose trading decision are based on condition of foreign market, given the background of globalization and financial integration. The existence of inter-market traders captures the common factors underlying and the linkage between the two markets. Market makers of individual market adjust prices according to excess demand

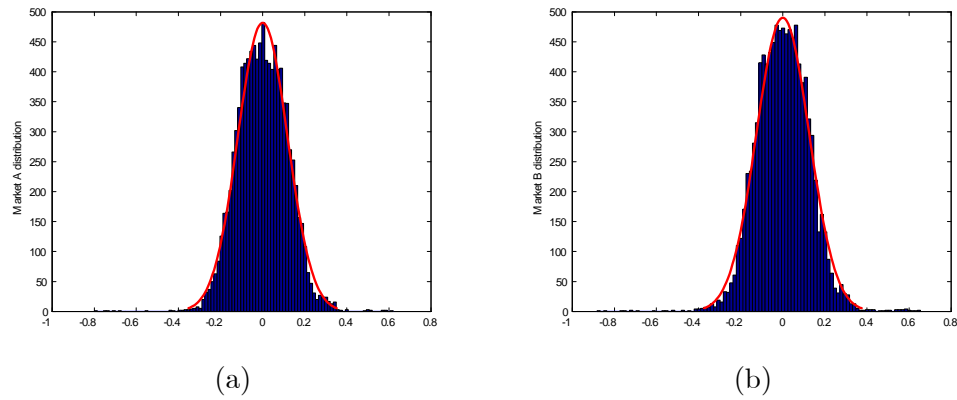


Figure 3.9: Distribution of returns. (a) market A. (b) market B.

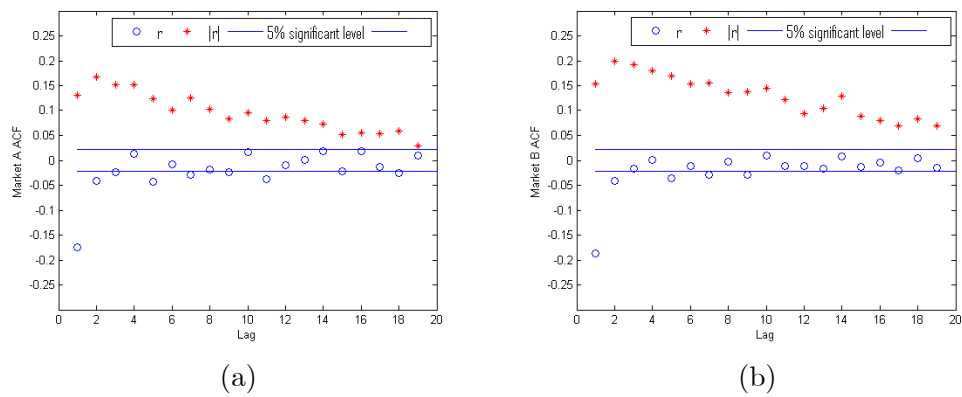


Figure 3.10: Autocorrelation of returns and absolute returns. (a) market A. (b) market B.

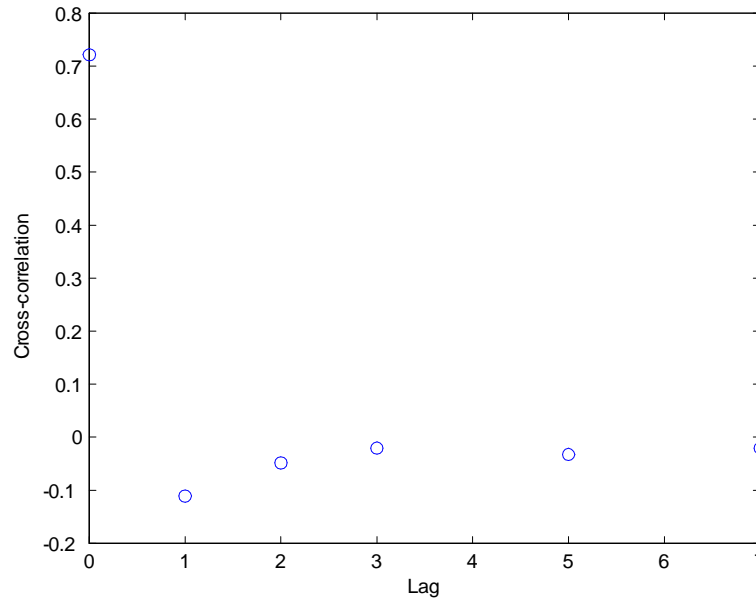


Figure 3.11: Significant cross-correlation in 95% confidence interval.

of the three types of traders. The main purpose of this chapter is to simulate financial crisis within two-market framework from points of view of endogeneity and exogeneity so that causes of financial crisis could be explored for different scenarios.

In terms of endogeneity, we manage to simulate different patterns of financial crisis across two markets endogenously, especially sudden crisis with empirical reference to "Black Monday" of US and UK stock markets in 1987. These simulations imply that financial crisis and its contagion could occur endogenously. As all our simulated financial crises occur at price level above market fundamental levels, they provide support to the scenario of market over-valuation causing a financial crisis. In terms of exogeneity, shocks are introduced to fundamental value of individual market. Depending on the magnitude, sign and duration of the shocks, different patterns of financial crisis could be triggered. Similar to simulations of endogeneity, reference prices without shock(s) have over-value prices in some periods. However, financial crisis only occurs when shock(s) hit individual market. This supports scenario of financial crisis triggered by external shock(s). In addition, the fact that a financial crisis in one market triggered by shock causes a similar crisis in the other market

is analogous to the domino effect.

In matching stylized facts, in addition to volatility clustering, fat tails, insignificant autocorrelation of return and significant autocorrelation of absolute return, we also manage to calibrate cross-correlation, which is exclusive to multi-market model. Cross-correlation can match with empirical phenomena of prices comovement among markets, especially contagion effect of the financial crisis.

Financial crisis involves a lot of aspects such as macro-economics and financial markets. Although a single model might not fully capture all the factors underlying financial crisis, a model that is more closed to realistic world to provide more intuition should be more robust in understanding financial crisis. The current model is limited by the simplicity of fixed investor composition. Features such as endogenous investor composition based on some evolutionary fitness should be included into the model for future research!

Appendix A

For a two-dimension system:

$$\begin{cases} p_{A,t+1} = p_{A,t} + a [D_{A,t} + w_A^i D_{B,t}], \\ p_{B,t+1} = p_{B,t} + a [D_{B,t} + w_B^i D_{A,t}]. \end{cases}$$

where

$$\begin{cases} D_{A,t} = w_A^f v_{A,t} (F_A - \hat{p}_A) + w_A^c \tau (\hat{p}_A - p_{A,t}^c), \\ D_{B,t} = w_B^f v_{B,t} (F_B - \hat{p}_B) + w_B^c \tau (\hat{p}_B - p_{B,t}^c). \end{cases}$$

At equilibrium prices (\hat{p}_A, \hat{p}_B) , we have

$$\begin{cases} 0 = a [D_{A,t} + w_A^i D_{B,t}], \\ 0 = a [D_{B,t} + w_B^i D_{A,t}]. \end{cases}$$

(\hat{p}_A, \hat{p}_B) can be determined implicitly from:

$$D_{A,t} = 0 \text{ and } D_{B,t} = 0.$$

Table 3.1: Conditions of financial crisis simulations

common set of parameters	$d = -0.25$, $u_1 = -0.2$, $u_2 = 2$, $a = 0.1375$, $\lambda = 15.02$, $w_A^f = w_B^f = 2$, $w_A^c \tau = w_B^c \tau = 5.8$, and $w_A^i = w_B^i = 0.818$
endo-sudden	$p_{A,0} = 64.2691$ and $p_{B,0} = 64.6191$
endo-smooth	$p_{A,0} = 64.9957$ and $p_{B,0} = 76.5895$
endo-disturbing	$p_{A,0} = 63.2693$ and $p_{B,0} = 77.9186$,
permanent-shock	$p_{A,0} = 42.6906$ and $p_{B,0} = 55.9131$
temp-shock	$p_{A,0} = 49.0616$ and $p_{B,0} = 61.9338$
stylized-facts	$a = 0.186$, $w_A^i = w_B^i = 0.44$, $p_{A,0} = 54.1100$ and $p_{B,0} = 57.0915$

Appendix B

Parameters setting for all the simulations are listed in Table 3.1. A common set of parameters are used for each simulation scenario unless the individual parameter is specified.

Chapter 4

Heterogeneous agents in multi-markets: a coupled map lattices approach¹

4.1 Introduction

In 1981, Kaneko discovered the spatiotemporal pattern of coupled map lattices (CML) when he started a simulation in which a chain of logistic maps is utilized. In the chaotic regions, each logistic map couples to nearby ones. Discrete time evolutions display spatiotemporal pattern in which values of logistic maps are either greater or less than fixed point value. CML has been expanded into fields of spatiotemporal chaos and pattern formation, biology, mathematics, engineering and so on. For further understanding of CML, we cite here in particular, Kaneko (1986), Kaneko (1989a), Kaneko (1989b), Kaneko (1992), Kaneko (1995) and Ouchi and Kaneko (2000).

¹A paper coauthored with Weihong Huang based on this chapter has been published in Mathematics and Computers in Simulation.

In contrast to the "efficient market hypothesis" with strong assumptions of rational investors and completed information access, heterogeneous agents models (HAM) release these two strong assumptions with bounded rational investors and limited information. So far, HAM models have managed to replicate some of the stylized facts of financial markets, such as bubbles and crashes, randomly switching bear and bull market episodes, excess volatility, volatility clustering and fat tails for returns distribution. For reference, we cite, for instance, Brock and Hommes (1998), Westerhoff (2004), Chiarella et al. (2006), Chiarella et al. (2007), Chiarella and He (2001), Dieci and Westerhoff (2010), He and Li (2008), Gao and Li (2011) and Chen (2012). In addition to the above mentioned stylized facts matching, Chapter 2 further contributes to matching of cross-correlation. Based on the market maker framework of Day and Huang (1990) and two-market model of Westerhoff and Dieci (2006), Chapter 2 models coupling function of market maker's price adjustment by weighting excess demand of two different groups of investors. The market system displays market pooling phenomenon and strong cross-correlation.

This chapter extends the framework of Chapter 2 to multiple markets through CML mechanism. To segregate and investigate the effect from coupling, fundamentalists and chartists can only invest in their home market while inter-market traders are still active. Market makers update price based on the total excess demand of all investors. With this setup, market cluster or enhancement on persistence of asset price deviation is observed. The effect becomes prominent in the chaotic interval, where market cluster with market members sharing the same sign of deviation is formed in the spatio-temporal diagram. Market clusters can regroup if market member is hit by shock in asset price. Viewing from the point of price trajectories, coupling effect can stabilize market members with smaller fluctuation compared to the case of isolation.

This chapter is structured as follows. Section 2 describes the details of the model setup and proposes a coupling market maker framework in a multi-market system. In Section 3, with numerical bifurcation study and Lyapunov exponent plots as reference, deviation spatio-temporal diagrams are plotted to demonstrate the deviation enhancement effect. To understand this enhancement phenomenon, phase diagram and Lorenz plot are utilized. Also, external shocks are employed to investigate the market cluster pattern stabil-

ity. Lastly, Section 4 concludes the chapter and suggests possible future research direction.

4.2 Model Setup

For the isolated market i , asset price at time t is denoted by $P_{i,t}$. The fundamental value is treated as constant and is denoted by F_i . For convenience, we define the price deviation $x_{i,t} = P_{i,t} - F_i$. For a chartist (c) or a fundamentalist (f) in market i , their excess demand for the asset i is $D_{i,t}^c$ and $D_{i,t}^f$, respectively. These excess demands are linear functions of price deviation, for simplicity.

$$D_{i,t}^c = b_c x_{i,t}, D_{i,t}^f = b_f (-x_{i,t})$$

where b_c and b_f are the strength of demand of chartists and fundamentalists. Chartists purchase over-valued asset and sell under-value one while fundamentalists behave in an opposite way. The distribution compositions of chartists and fundamentalists in market i are $W_{i,t}^c$ and $W_{i,t}^f$, respectively. When the price deviation is larger, more investors will become fundamentalists and the proportion of fundamentalists is larger. Investors distribution composition is determined according to:

$$W_{i,t}^c = \frac{h \exp(-|x_{i,t}|)}{\exp(|x_{i,t}|) + h \exp(-|x_{i,t}|)}, W_{i,t}^f = \frac{\exp(|x_{i,t}|)}{\exp(|x_{i,t}|) + h \exp(-|x_{i,t}|)}$$

where $h \geq 1$ is chartist distribution parameter, proportional to the confidence level of chartist. A large h indicates that chartists are more confident. The setup of the investor composition is inspired by Hommes (2001) that fraction of technical trader (chartists) decreases if price further deviates from fundamental value. It also simplifies the analysis.

We first categorize fundamentalists and chartists as a group of investors as they trade based on the local market conditions. With the individual excess demand and investor compositions, the excess demand to market i due to this group of investors, $D_{i,t}$, can be derived:

$$D_{i,t} = W_{i,t}^f D_{i,t}^f + W_{i,t}^c D_{i,t}^c$$

For the case of market isolation, in response to the excess demand, market maker of market i updates next period's price.

$$P_{i,t+1} = P_{i,t} + aD_{i,t}$$

where a is the price adjustment coefficient.

Expressed in the price deviation form, the price updating process can be expressed as:

$$\begin{aligned} x_{i,t+1} &= x_{i,t} + aD_{i,t} \\ &= x_{i,t} + \frac{ah \exp(-|x_{i,t}|)b_c x_{i,t}}{\exp(|x_{i,t}|) + h \exp(-|x_{i,t}|)} - \frac{a \exp(|x_{i,t}|)b_f x_{i,t}}{\exp(|x_{i,t}|) + h \exp(-|x_{i,t}|)} \end{aligned} \quad (4.1)$$

Above is the price dynamics for individual market i in isolation. It shows price change $\Delta P_{i,t+1} = P_{i,t+1} - P_{i,t}$ is a function of excess demand of fundamentalists and chartists. In case of multi-market, here comes the second group of investors, inter-market traders whose excess demand in market i is based on conditions of other markets. As excess demands of fundamentalists and chartists in other markets reflect conditions of other markets, excess demand of inter-market traders in market i can be represented as a function of excess demands of fundamentalists and chartists in other markets. For simplicity, we assume inter-market traders in market i trade with reference to adjacent markets $i - 1$ and $i + 1$. Hence, price change in market i in multi-market case can be modelled as:

$$\Delta P_{i,t+1} = a \left[(1 - g) D_{i,t} + g \left(\frac{D_{i+1,t} + D_{i-1,t}}{2} \right) \right]$$

That is,

$$x_{i,t+1} = x_{i,t} + a \left[(1 - g) D_{i,t} + g \left(\frac{D_{i+1,t} + D_{i-1,t}}{2} \right) \right] \quad (4.2)$$

where g is coupling parameter with range $0 \leq g < 0.5$. $(1 - g)$ and g are coefficients for market influence of the first and second group of investors in market i . Excess demand of inter-market traders is positively proportional to the average of excess demand of fundamentalists and chartists in adjacent markets. This setup captures the underlying assumption that adjacent markets share various common factors such as macro-economics conditions.

4.3 Result

If we ignore deviation magnitude and focus on the sign of deviation, the deviations can be categorized into four types: persistently positive deviation, persistently negative deviation, alternate deviations, and diminishing to zero (the fundamental value). If the deviations fluctuate within either the positive value domain or the negative value domain, the signs of the deviations do not change. They are defined as persistently positive deviations or persistently negative deviations, respectively. Once the deviations fluctuate alternately between the positive and negative domains, they are not persistent and are defined as alternate deviations. In contrast, there are cases where deviation can diminish to zero—the fundamental value state. We are interested with the deviation persistence under cases of isolation and coupling effect which implies market linkage. The persistence of the deviation is related to the price deviation dynamics, which involves dynamical stability and bifurcation study such as switching between positive and negative regions. Hence, stability of the isolated market is investigated, followed by bifurcation illustration and other numerical demonstrations for multi-market cases.

4.3.1 Isolated market

For the isolated stock market i , Eq. 4.1,

$$x_{i,t+1} = x_{i,t} + \frac{ah \exp(-|x_{i,t}|)b_c x_{i,t}}{\exp(|x_{i,t}|) + h \exp(-|x_{i,t}|)} - \frac{a \exp(|x_{i,t}|)b_f x_{i,t}}{\exp(|x_{i,t}|) + h \exp(-|x_{i,t}|)}$$

the steady state and stabilities properties is derived:

Proposition 11 (a) *There exists a unique fundamental steady state $\bar{x}_i = 0$, which is stable if and only if $\max\{1 - 2(h + 1)/(ab_f), 0\} < b_c \cdot h/b_f < 1$;*

(b) *There are two nonfundamental steady states: $\bar{x}_i = \pm \ln \sqrt{b_c \cdot h/b_f}$ given $b_c \cdot h/b_f > 1$, they are stable if $b_c \cdot h/b_f < \gamma$, where $\gamma = \exp\left(\frac{2(b_c + b_f)}{a \cdot b_c \cdot b_f}\right)$.*

(c) *A pitchfork bifurcation occurs at $b_c \cdot h/b_f = 1$*

Proof of proposition 11 is provided in Appendix.

We concentrate on the bifurcation diagram as it relates the sign of deviation to parameter region. For all the numerical simulations, a common set of parameters is defined: $a = 2.131$, $b_f = 1$, and $h = 1$. Fig. 4.1.a.1 reports the isolated market bifurcation diagram with initial value above the fundamental value, in which b_c is the bifurcation parameter. Bifurcation diagrams with initial values above and below fundamental value are mirror-symmetric with respect to the fundamental value and hence diagram with initial value below fundamental value is omitted here. When $b_c < 1$, fundamental steady state is stable. By increasing b_c , the attractor experiences nonfundamental steady state and a sequence of period-doubling bifurcations to chaotic states. Depending on the initial conditions, chaotic intervals locate either above or below the fundamental state. When $b_c < 5.051$, deviations take place either in the positive or negative region, persistent deviations in our definition. After that, deviation starts to wander across both positive and negative regions. These portions are characterized by intrinsic fluctuations and erratic switching between positive and negative regions—alternate deviations. These alternate deviation portions demonstrate market clusters under coupling effect in the sequel as the coupling effect can change the bifurcation property and restrict these portions into either positive or negative regions. In addition, the attractor experiences a transition from chaotic state to 4-orbit state across positive and negative regions, followed by chaotic fluctuation again. The interval of this 4-orbit is $5.97 < b_c < 6.225$. This interval can be found with negative Lyapunov exponent as in Fig. 4.1.a.2. In a short summary, an increase of the chartist strength destabilizes the market.

The above states could correspond to the intrinsic states of the real world economics. The concept is similar to regimes in the Markov-regime switching process, which has been intensively studied in the literature. A typical example is the double-period cycle pattern. The economic illustration for this cycle pattern could be the boom and burst periods of Chiarella et al. (2012).

4.3.2 Multi-markets

To investigate the effect of coupling on deviation persistence, we study a system with 200 markets with price dynamics Eq. 4.2. The market system is close such that mar-

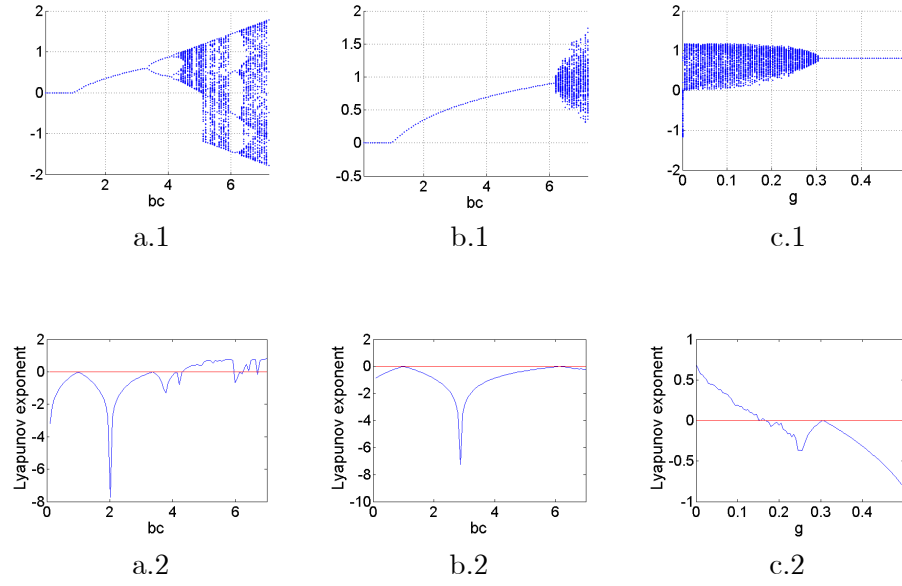


Figure 4.1: Bifurcation curves and Lyapunov exponents with initial value of $x_i > 0$. Rows 1 and 2 are bifurcation curves and Lyapunov exponents respectively. (a.1) and (a.2). $g = 0$ and b_c varying. (b.1) and (b.2) $g = 0.4$ and b_c varying. (c.1) and (c.2) $b_c = 5.0533333$ and g varying.

kets 2 and 200 are adjacent to market 1. After randomization, deviation evolvments of each market are plotted in the deviation sign spatio-temporal plots. The y axis is market i up to 200 and the x axis is the time step of the evolvment. Colors of green, violet and red represent positive, zero and negative deviation, respectively, regardless of the magnitudes. In this numerical demonstration, b_c and g are changed to study their effect on the deviation persistence. The demonstration is divided into two portions by the region-crossing point $b_c = 5.051$. Fig. 4.2 shows the deviation persistence for cases where $b_c < 5.051$. In this range, deviations converge to zero or are persistent in either the disjointed deviation regions. When $b_c < 1$, all the markets eventually converge to the fundamental steady states. The introduction of coupling effect, $g = 0.4$, prolongs the time needed to return to the fundamental states. The time steps taken to converge to fundamental steady state under isolation and coupling effect are around 130 and 650, respectively (Fig. 4.2.a and 4.2.b). In case of $1 < b_c < 5.051$, deviations locate in the persistent regions. The signs of deviations follow the ones determined by initial randomization. It seems the spatio-temporal plots under coupling effect are not significantly different from the isolated counterparts as persistence is observed visually for both cases (Fig. 4.2.c-4.2.f). However, if the same initial randomized values are evaluated with different values of g , both of the signs and magnitudes of deviations can be changed. That will be discussed in the subsequent subsection of deviation distribution study.

The coupling effect becomes apparent when b_c is larger than the region-crossing point 5.051 (Fig. 4.3). Before b_c exceeds 5.97, the isolated markets are in turbulent states with varying deviation signs. In these markets, the duration that deviation remains in the same domain decreases with b_c (Fig. 4.3.a, 4.3.c and 4.3.e). The introduction of coupling effect enhances the deviation persistence. Market clusters with the same deviation sign are formed. Each cluster is deviation persistent. The coupling effect is not always dominant as some of the clusters become unstable with "defect" emerging when b_c increases (Fig. 4.3.b, 4.3.d and 4.3.f). When $5.97 < b_c < 6.225$, the isolated markets enter the state of 4-period orbit; the spatio-temporal plot shows a regular sign-switching pattern. The coupling effect still boosts the deviation persistence by increasing the duration of deviation in the same region. It disturbs the regular pattern into a turbulent state, with some intermittent market

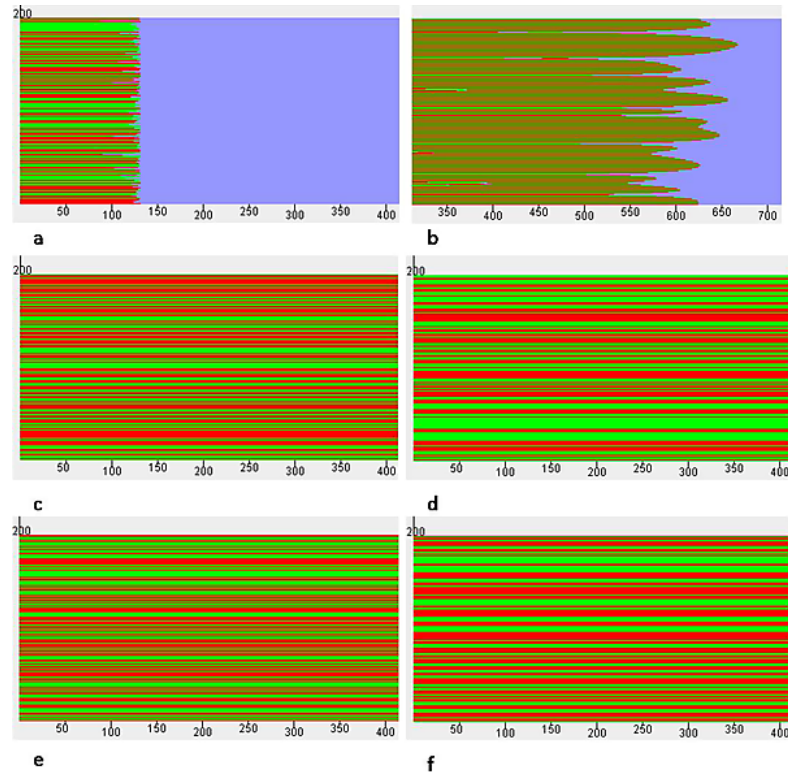


Figure 4.2: Coupling effect before the region-crossing point. Columns 1 & 2 have $g = 0$ & $g = 0.4$: (a) & (b) $b_c = 0.9$. (c) & (d) $b_c = 3.33$. (e) & (f) $b_c = 3.4$.

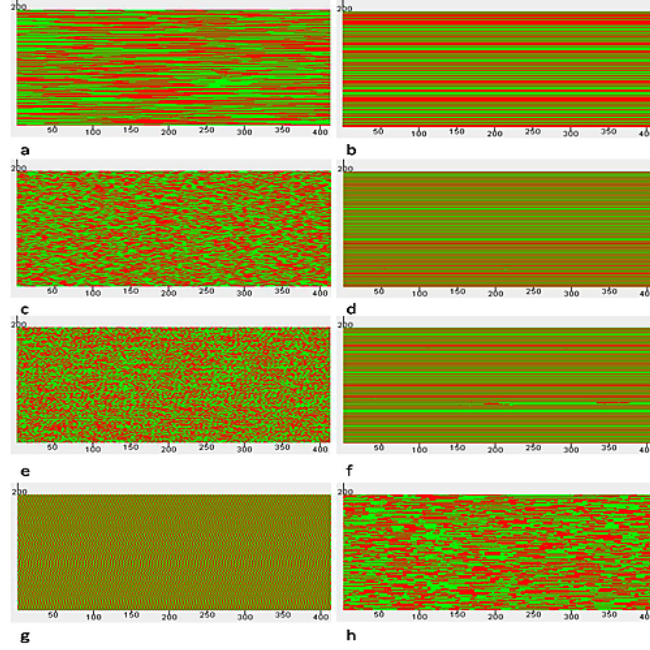


Figure 4.3: Coupling effect after the region-crossing point. Columns 1 & 2 have $g = 0$ & $g = 0.4$, respectively: (a) & (b) $b_c = 5.0533333$. (c) & (d) $b_c = 5.2$. (e) & (f) $b_c = 5.3$. (g) & (h) $b_c = 6.0$.

clusters growing and diminishing (Fig. 4.3.g and 4.3.h). Here, the phrase "turbulent state" is used for the situation in which markets switch irregularly among different intrinsic boom and burst market states.

To convince that deviation persistence is enhanced by coupling effect such as $g = 0.4$, we plot bifurcation and Lyapunov exponent for market 100 within a market system of 200 market members. For bifurcation diagram with respect to chartist strength b_c , those portion covering both positive and negative regions now is restricted within positive region (Fig. 4.1.b.1). This is consistent with the deviation enhancement and implies that market clusters will emerge in Fig. 4.3.h given sufficient evolvement time. For Lyapunov exponent with respect to b_c , those portions used to be chaotic are no more chaotic, with Lyapunov exponent less than zero (Fig. 4.1.b.2). Stabilizing effect by coupling is implied from this Lyapunov exponent plot.

After demonstrating the deviation persistence enhancement effect from the coupling parameter g , the coupling effect is further investigated by varying g in the region-

crossing chaotic state, given $b_c = 5.0533333$. Without any coupling effect, $g = 0$, the isolated markets are in turbulent state (Fig. 4.4.a). The turbulent state transforms into a less turbulent one after a small coupling effect $g = 0.005$ is introduced into the system. Under the influence of weak coupling effect, there is a continual process in which clusters are destroyed by growing "defect" and new clusters emerge (Fig. 4.4.b). By increasing g to 0.04825, more stable clusters are formed. The size of each cluster changes slightly with time as evidenced by the contrast of the plot at different time steps (Fig. 4.4.c). Further increasing g to 0.4 and eventually 0.5, the clusters are stable with small fixed sizes (Fig. 4.4.d and 4.4.e). By varying the strength of coupling effect, different spatio-temporal patterns appear. When the coupling effect is weak, market clusters are unstable. The unstable clusters can display a dynamic process of "defect" creation or cluster size change. In case the coupling effect is strong, coupling effect dominates the regions switching chaotic process; stable market clusters with persistent deviation signs appear. The effect of coupling strength g can be demonstrated with bifurcation and Lyapunov exponent plots of 100th market with respect to g . By increasing g , bifurcation transits from chaos to non-chaos regions (Fig. 4.1.c.1). Consistently, Lyapunov exponent also transits from positive to negative (Fig. 4.1.c.2).

4.3.3 Deviation Lorenz plot study

It has been shown that coupling effect tends to form clusters in terms of the same sign of deviation. Enquiries about the magnitude of deviation arise. Will the magnitude of deviation be constant? If not, is there any distribution pattern? To answer these questions, we plot Lorenz plots: $x_{i,t+1}$ vs $x_{i,t}$. One common set of initial random numbers is created to generate the 200 markets data. 50,000 iterations of evolution are computed. The last 1000 iterations data from markets 4, 5, and 6 are used to plot the Lorenz distribution diagrams, where blue, red and green colors represent markets 4, 5, and 6, respectively. In Fig. 4.5, sub-figures of column one are phase diagrams of isolated market for different chartist strengths b_c . Consistent with column one, column two is the corresponding Lorenz plots of isolated markets 4, 5, and 6. Loci of the Lorenz plots match the orbits in the corresponding phase diagrams. In contrast, column three plots the same Lorenz plots under coupling effect. When $b_c = 4.2$, each isolated market experiences 4-period orbit dynamics

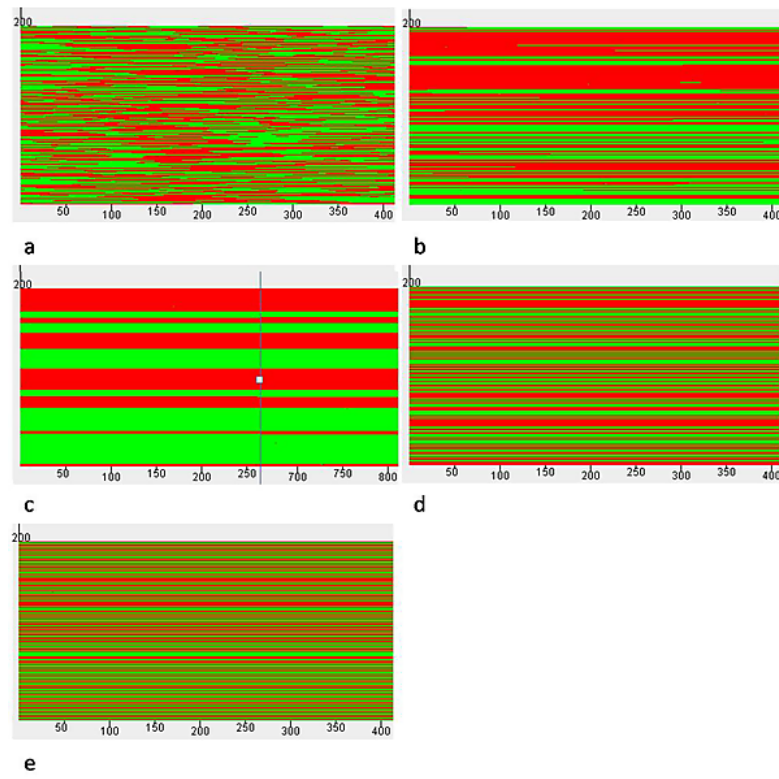


Figure 4.4: Effect of g given $b_c = 5.0533333$. (a) $g = 0$. (b) $g = 0.005$. (c) $g = 0.04825$. (d) $g = 0.4$. (e) $g = 0.5$.

within the disjointed deviation regions. The introduction of coupling effect $g = 0.4$ does not alter the periods of the orbit dynamics. However, the distribution loci have been changed slightly, besides the reversal of deviation sign of green-colored market 6 (Fig. 4.5.a.2 and 4.5.a.3). A 3-dimensional plot of the three markets provide another angle to visualize the distribution change due to coupling effect. Sign reversal of market 6 is also observed in Fig. 4.6.a.1 and 4.6.a.2. When $b_c = 5.2$, in the region where the spatio-temporal plot shows turbulent state for isolated market as deviations wander across positive and negative regions, distributions of the isolated markets converge to strange attractors. Once coupling effect $g = 0.4$ is applied, the shapes of the strange attractors are changed and data points of individual markets are segregated within either disjointed regions. This explains why the coupling effect can enhance the deviation persistence (Fig. 4.5.b.2, 4.5.b.3, 4.6.b.1 and 4.6.b.2). In case b_c is increased to 6.0, the isolated markets converge to the 4-period orbit distribution across positive and negative regions. This 4-period circulation translates into a regular sign-switching pattern in the deviation spatio-temporal plot Fig. 4.3.g in the above demonstration. The application of coupling effect $g = 0.4$ destroys the 4-period pattern distribution and creates strange attractors covering all the four quadrants in the distribution diagram. (Fig. 4.5.c.2, 4.5.c.3, 4.6.c.1 and 4.6.c.2). These strange attractors justify the turbulent defects pattern in the spatio-temporal plot Fig. 4.3.h.

4.3.4 Single disturbance

Coupling effect enhances the formation of market clusters especially in chaotic intervals where deviation wanders across positive and negative regions. Stability of the resulted market structure is still ambiguous. To address this concern, we simulate by introducing a shock s to one of the markets and check the clusters activities. Given market conditions $b_c = 5.08$ and $g = 0.4$, a single shock s hits the 100^{th} market site at time step t such that $x_{100,t} = s$. The corresponding deviation spatio-temporal diagrams are plotted in Fig. 4.7. If the shock s is not large enough, it seems only the adjacent markets are affected. The affected markets may change the sign of deviation or result in new clusters. After adjustment for some periods, the whole market system again shows stable cluster pattern. With a larger shock, more markets are involved in adjustment with a longer adjusting

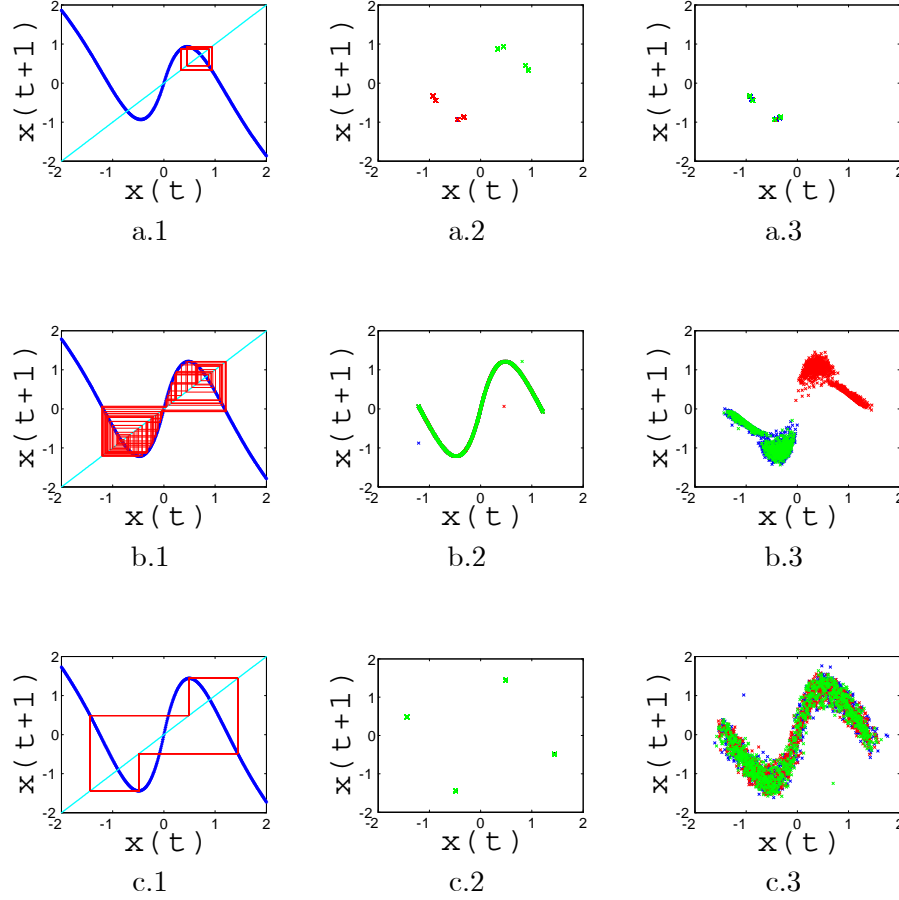
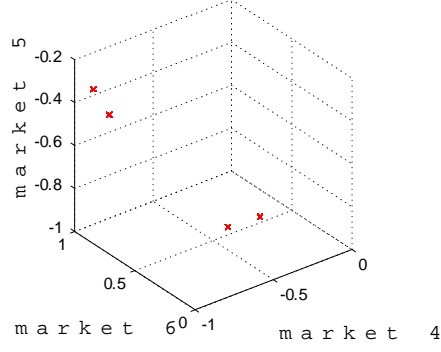
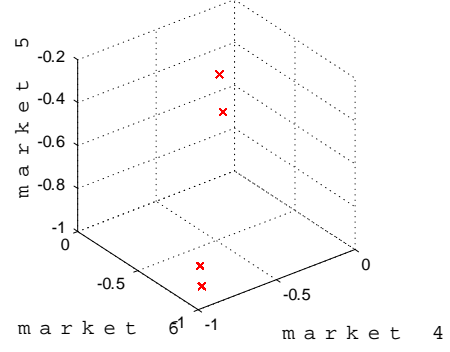


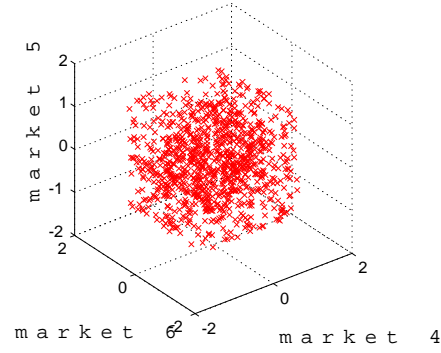
Figure 4.5: Isolated market phase diagrams and Lorenz plots: x_{t+1} vs x_t . Columns 1 to 3 are phase diagrams ($g = 0$), isolated market distributions ($g = 0$), and distributions with coupling effect ($g = 0.4$). For the distribution plots, Blue, red and green colors represent markets 4, 5, and 6, respectively. (a.1) - (a.3) $b_c = 4.2$. (b.1) - (b.3) $b_c = 5.2$. (c.1) - (c.3) $b_c = 6.0$.



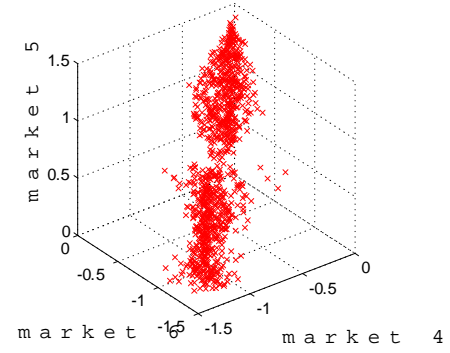
a.1



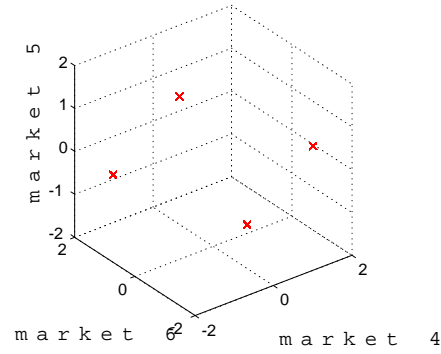
a.2



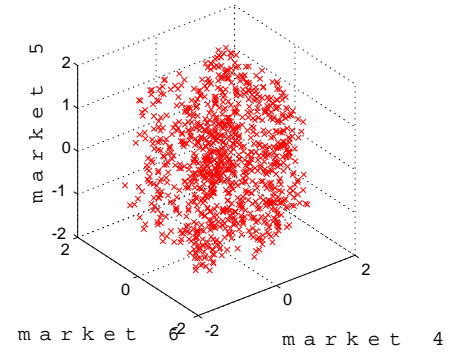
b.1



b.2



c.1



c.2

Figure 4.6: 3-dimensional plots in price deviation. Vertical axis is market 5. Columns 1 and 2 have $g = 0$ and $g = 0.4$ respectively: (a.1) and (a.2) $b_c = 4.2$. (b.1) and (b.2) $b_c = 5.2$. (c.1) and (c.2) $b_c = 6.0$.

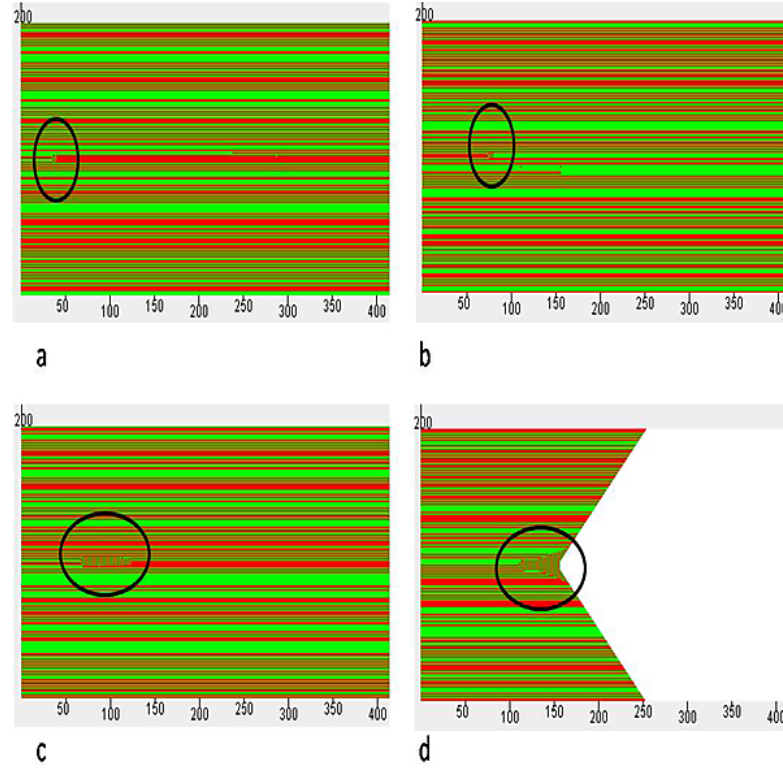


Figure 4.7: Cluster pattern upon external shocks s hitting the 100^{th} market. (a) $s = -10$, after a short adjustment, a new cluster is formed. Adjustment is highlighted by a circle. (b) $s = -12$, a new stable cluster is formed. Also, there is an adjustment in cluster below the impacted market. (c) $s = -14$, more markets adjust and the adjustment time is longer. (d) $s = -16$, Market-collapse spreads to the whole market system.

time required (Fig. 4.7.a-4.7.c). Once s reaches certain strength, coupling effect no longer stabilizes the cluster pattern. Instead, market-collapse in which prices diverge to out of bounds spreads from the impacted market to the whole market system through coupling effect and eventually all markets collapse. An avalanche appears. (Fig. 4.7.d).

Based on the above disturbance analysis, when a shock is not large enough, it seems market members far away from the shock originating market are not affected as their signs of deviation do not change. To verify whether these market members are affected or not, investigation in terms of magnitude is necessary. We conduct the magnitude analysis with procedure: First, create a common set of initial random numbers for the 200-market system at condition $b_c = 5.08$. Second, after 50,000 rounds of evolvement, the last time step values are denoted as step 1, $x_{i,1}$, for analysis. A small shock $s = 0.001$ is introduced

to the 100th market site such that

$$x_{100,1} = x_{100,1} + s$$

Third, deviations of the respective original and disturbed market system are recorded down for the next 400 periods. Denote the original and perturbed deviations as $x_{i,t}^o$ and $x_{i,t}^p$. Next, subtract $x_{i,t}^o$ from $x_{i,t}^p$ to get the difference pattern $d_{i,t}$ with filter $s = 0.001$. $d_{i,t}$ can be expressed as below:

$$d_{i,t} = \begin{cases} x_{i,t}^p - x_{i,t}^o & \text{if } |x_{i,t}^p - x_{i,t}^o| \geq s \\ 0 & \text{if } |x_{i,t}^p - x_{i,t}^o| < s \end{cases} \quad (4.3)$$

Lastly, plot $d_{i,t}$ in spatio-temporal diagrams, in which green, white and red colors represent positive, zero and negative difference. Fig. 4.8 reports the difference patterns for different coupling strength g . Based on Fig. 4.8, the shock propagation can be categorized into two modes: diffusion and localization. Fig. 4.8 row one shows the diffusive shock propagation when coupling strength g is small. The diffusive propagation speed increases with coupling strength since the corresponding time required for the disturbance to reach the whole system decreases. If coupling strength is increased, the other propagation mode – localization – emerges: the disturbance is confined in a zone and does not disappear with time (Fig. 4.8.b.1). Row two shows a mixture of the two modes. The localized zones can be observed visually. The difference pattern shows an irregular mixture of propagation modes. Our simulation results are similar to the finding of Kaneko (1986) except the irregular propagation patterns in Fig. 4.8.b.2.

When a shock s has magnitude less than the avalanche level, the coupling market system can absorb the shock and disperses to other markets. In this sense, it can be conjectured that coupling has stabilizing effect. To verify this conjecture, set $b_c = 5.08$, a common set of initial random numbers for the 200-market is adopted to the market system with and without coupling effect, that is $g = 0$ and $g = 0.4$ respectively. At time step 40, a shock s hits 100th market such that its deviation $x_{100,40} = -2.29198$. Time series data of the adjacent markets are plotted. Without coupling, each market is isolated with

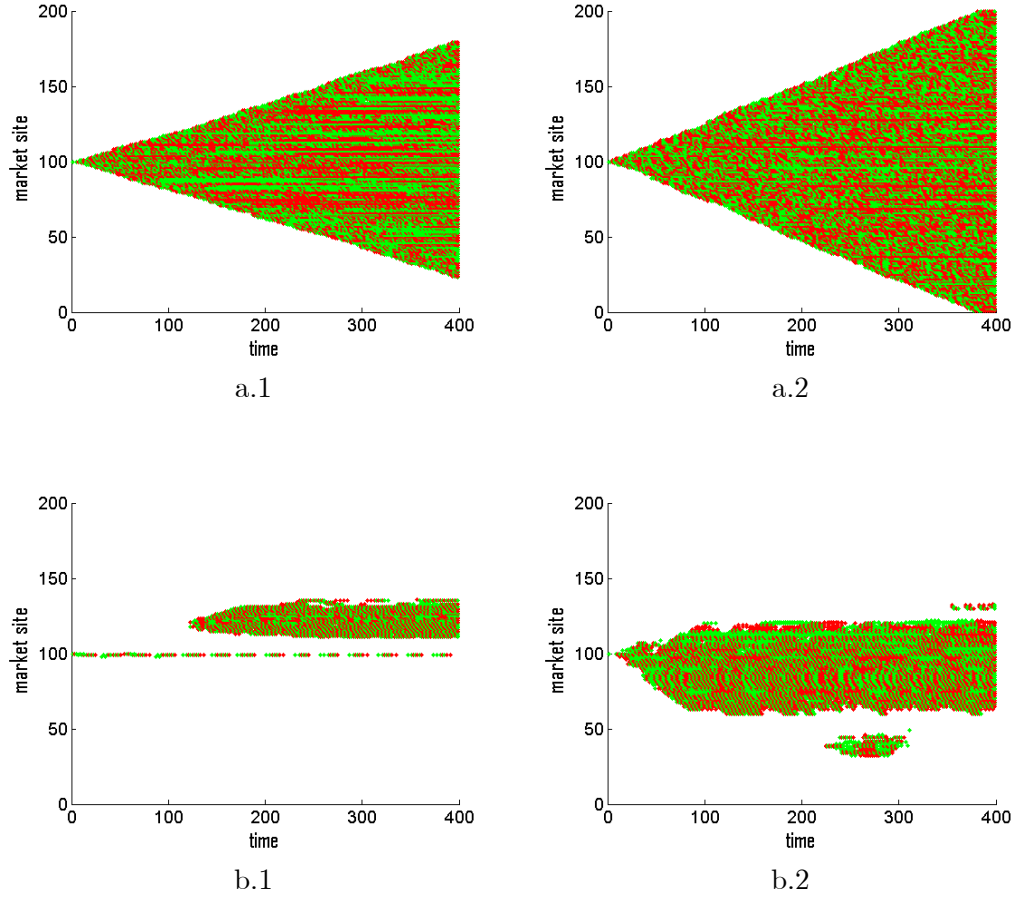


Figure 4.8: Shock propagation upon a shock $s = 0.001$ hitting the 100^{th} market. (a.1) $g = 0.01$. (a.2) $g = 0.06$. (b.1) $g = 0.08$, localization mode. (b.2) $g = 0.47$, irregular pattern.

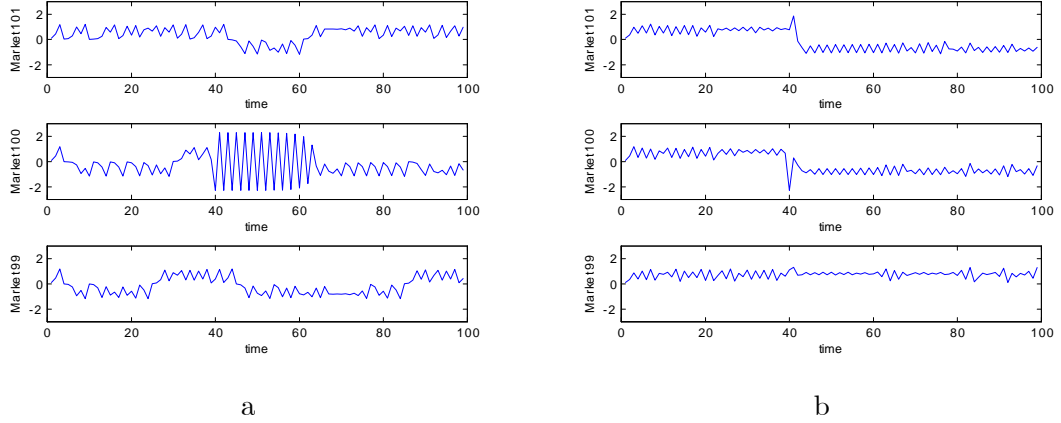


Figure 4.9: Prices response to shock comparison for coupling effect. (a) no coupling effect. (b) coupling effect $g = 0.4$.

price deviation switching randomly between positive and negative values. At time step 40, market 100 is hit by a shock and fluctuates dramatically. It takes around 20 time steps for market 100 to recover to its normal fluctuation path. As there is no market connection, other markets are not affected (Fig. 4.9.a). For the case of coupling, each market evolves in a way such that the price deviation is always positive or negative. At time step 40, market 100 is hit with the same shock such that $x_{100,40} = -2.29198$. It takes market 100 less than 5 time steps to stabilize to normal fluctuation in the region of negative values. The adjacent markets 99 and 101 are impacted by the shock, especially market 101, which takes a similar time steps of market 100 to recover from the shock (Fig. 4.9.b).

Coupled market system of this chapter is inspired by our more and more inter-related real world. Markets are connected by various economic and financial activities. Through these connections, mutual interactions arise: a market can transmit its shocks outside and ease the adverse effect it faces through stabilizing force from other markets; meanwhile, it also has to be disturbed by shocks propagating from other markets. Overall, market connection could have stabilizing effect on shocks. However, once severity of a shock from individual market exceeds certain level, shock propagation through market connection could cause breakdown of the whole market system. Policy implication is that market opening is not always beneficial to individual market.

4.4 Conclusion

This chapter examines an asset market system consisting of multi-markets. Each market has a market maker and two groups of investors. Consisting of fundamentalists and chartists, the first group trades based on the condition of local market. In contrast, the second group of investors, inter-market traders invests based on conditions of other markets. A coupling market maker framework is proposed: market maker updates market price based on a weighted excess demands of the two groups of investors. The weight of excess demand is coupling parameter g . With the introduction of coupling effect g , duration of deviation remaining in either the "disjointed" positive or negative regions increases and persistent deviation appears. Market cluster with market members sharing the same sign of deviation becomes apparent in the original chaotic interval characterized by erratic switching between positive and negative regions. This deviation enhancement effect is robust for different parameters such as the price adjustment coefficient a and the number of markets n . It can also be found by using the agent composition function of He and Westerhoff (2005).

Check the isolated market phase diagram and distribution plots, it is found that coupling effect tends to segregate the distribution into quadrant I or III, that is, either the "disjointed" regions. This explains why the duration of deviation is enhanced. After the market cluster is established with coupling effect, enquiry about its stability arises. A series of disturbances are introduced to one of the markets. From the perspective of market clusters, when the disturbance is weak, only the adjacent markets are affected for adjustment and a new market cluster pattern is formed; if the disturbance is large enough, market system avalanche is generated from the initially impacted market. Next, we investigate the deviation magnitude difference created by the disturbance. Even if the disturbance is weak, disturbance can propagate to the market system with propagation modes of diffusion or localization, or the mixture of the two modes. Lastly, time series data of shock response shows ability of coupling effect to stabilize market member hit by a shock. Policy implication implied by disturbance simulation is that market opening has the dual stabilizing and destabilizing effects. Stabilizing effect refers to small shock absorption by the whole market system while destabilizing concerns for market system breakdown due to large shock from market member.

The goal of this chapter is to introduce coupling effect as a bridge for heterogeneous agents multi-market interactions. Numerical experiments have demonstrated the deviation persistence enhancement effect. More efforts of both numerical and theoretical works are still needed to further explore this area. Possible directions can be the application of coupling in financial markets.

Appendix

For the isolated stock market i Eq. 4.1,

$$x_{i,t+1} = x_{i,t} + \frac{ah \exp(-|x_{i,t}|)b_c x_{i,t}}{\exp(|x_{i,t}|) + h \exp(-|x_{i,t}|)} - \frac{a \exp(|x_{i,t}|)b_f x_{i,t}}{\exp(|x_{i,t}|) + h \exp(-|x_{i,t}|)}$$

Set \bar{x}_i as steady state and solve, we get $\bar{x}_i = 0$ or $\bar{x}_i = \pm \ln \sqrt{b_c \cdot h/b_f}$ if $b_c \cdot h/b_f > 1$.

Evaluated at $\bar{x}_i = 0$, $\frac{dx_{i,t+1}}{dx_{i,t}} = 1 + a \frac{hb_c - b_f}{h+1}$, to be stable, we need $\max\{1 - 2(h + 1)/(ab_f), 0\} < b_c \cdot h/b_f < 1$.

At $\bar{x}_i = \pm \ln \sqrt{b_c \cdot h/b_f}$, $\frac{dx_{i,t+1}}{dx_{i,t}} = 1 - ab_c b_f \frac{\ln h \frac{b_c}{b_f}}{b_c + b_f}$, stability condition required that $1 < b_c \cdot h/b_f < \gamma$, where $\gamma = \exp\left(\frac{2(b_c + b_f)}{a \cdot b_c \cdot b_f}\right)$

Chapter 5

Estimating heterogeneous agents behavior in a two-market financial system¹

5.1 Introduction

Given the background of globalization and financial market integration, market co-movement or cross-correlation has attracted attention of researchers long ago and become a more and more obvious phenomenon in recent years. One of the common place to show co-movement is stock markets, either within stock markets such as market component, or between different stock markets. Markets co-movement has been widely reported in empirical literature. Egert and Kocenda (2011) find strong correlation among returns of Germany, France and UK, even up to 0.9. Kenett et al. (2012) find that developed Western markets are highly correlated. In addition, strong co-movement was observed when financial

¹A paper coauthored with Weihong Huang and Huanhuan Zheng based on this chapter has been submitted for journal publication.

crisis spread to various markets in the past decades. Preis et al. (2012) show that average correlation among DJIA (Dow Jones Industrial Average) members increases with market stress.

Heterogeneous agents models (HAM) have proven to be successful in explaining financial markets theoretically (e.g. Day and Huang (1990), Brock and Hommes (1998), Lux (1998), Chiarella et al. (2003), Hommes et al. (2005), He and Westerhoff (2005), Dieci and Westerhoff (2010), Huang et al. (2010)). This inspires the empirical investigation using HAM. Westerhoff and Reitz (2005) shows behavioral heterogeneity in US corn market. Frijns et al. (2010) verify that traders with different beliefs about volatility are active in option market. Manzan and Westerhoff (2007) and De Jong et al. (2010) find the existence of heterogeneous traders in foreign exchange markets. Lux (2012) uses agent-based model to estimate the opinion formation of German investors. Boswijk et al. (2007) and Chiarella et al. (2012) estimate behavioral heterogeneity in S&P 500 using techniques of nonlinear least square and Markov regime switching respectively. In the paper of Chiarella et al. (2012), they show the existence of boom and bust two states. The bust state is characterized by depressing price movements and high volatility.

However, all the above mentioned empirical HAM literatures are related to single market model. Interactions among financial markets, including market co-movement, are not addressed. De Jong et al. (2009) constitutes an exception. They investigate stock markets of Hong Kong and Thailand during the 1997 Asian crisis. In addition to the typical fundamentalists and chartists, they innovatively introduce into each market a third type of traders, internationalists, whose demand function in the domestic market is based on the chartist analysis in foreign market. These three types of traders play their roles during the crisis. The inclusion of internationalists provides an indication of the cross-correlation between the two markets and captures the contagion effect during the crisis.

Inspired by De Jong et al. (2009) and Chiarella et al. (2012), this chapter proposes a two-market model to study stock markets of France and US represented by CAC 40 and DJIA and estimates the trading behaviors of fundamental group, chartist group and inter-market traders using monthly data from January 2000 to April 2013. After verifying that individual markets have the feature of two-state regime switching, we include inter-market

traders to form a two-market model. Excess demand of inter-market traders in one market is based on market fundamental value of foreign market. It is found that fundamental group, chartist group and inter-market traders exist in both markets. It is implied that price adjustments in both markets have a common set of factors in terms of fundamental values of the two markets, providing a foundation of markets co-movement or cross-correlation.

The rest of the chapter is organized as follows. In Section 2, we develop the methodology of two-state regime switching empirical models for the single market and two-market frameworks and discuss the data used in this paper. Section 3 presents the regression results for the two frameworks and shows that the two-market framework with inter-market traders has a better capability in explaining the two markets. Lastly, Section 4 concludes the chapter.

5.2 Methodology and Data

5.2.1 Methodology

We develop a two-market asset pricing model with stock markets of France and US in this section. There are two groups of traders in each market: fundamental group and chartist one. They trade based on market fundamental value and chartist reference value respectively. Following Brock and Hommes (1998), there are trend chasers and contrarians for each group. A trend chaser believes that the trend of price deviation from the reference value will continue while a contrarian holds an opposite one. That is, a contrarian believes that the price deviation will be reversed. The third group of investors is noise traders. Fundamental value of the market can be derived from real economic conditions. Fundamental group is assumed to have access to this fundamental value and treats it as trading reference while chartist group and noise traders do not have the information due to their trading nature or information cost. Chartists and noise traders trade based on market conditions and historical prices. They believe the market has high and low two reference prices and follows a two-state Markov regime switching process. Depending on the Markov state n ($n = 1$ or 2), chartists trade based on the reference prices v_t while noise trades decide their

order e_t following a normal distribution $N(0, \sigma_{n,t}^2)$. Conditional volatility $\sigma_{n,t}^2$ is regime-dependent. Details of this regime-dependent properties will be discussed later. Moreover, the last group of traders is inter-market traders, whose trading in one market is based on the fundamental value of the other market. We first describe the trading strategies of the all types of traders and their excess demand as well as the price adjustment functions that relate price to excess demands.

Fundamental group

In market j ($j = \text{Fr}$ denoting France or US), fundamental group (f) is assumed to know the information of fundamental value, u_t^j . Based on the price deviation from the fundamental value $p_{t-1}^j - u_t^j$, excess demand function of investors follows a rule as:

$$D_{f,t}^j = b_f^j (p_{t-1}^j - u_t^j), \quad (5.1)$$

where b_f^j is the demand coefficient of fundamental group. There are trend chasers and contrarians in this group. A trader is called fundamental trend chaser if $b_f^j = b_{ft}^j > 0$ or fundamental contrarian if $b_f^j = -b_{fc}^j < 0$. A fundamental trend chaser buys (sells) by assuming that positive (negative) price deviation will further continue from the previous period. In contrast, a fundamental contrarian invests in an opposite way with a belief that price will return back to fundamental value. Hence, a fundamental contrarian sells (buys) given positive (negative) price deviation from fundamental value.

Following Chiarella et al. (2012), fundamental value u_t^j is derived based on static Gordon growth model of Gordon and Shapiro (1956) as well as Fama and French (2002),

$$u_t^j = d_t^j \frac{1 + g^j}{y^j}, \quad (5.2)$$

where d_t^j is dividend flow, g^j is the average growth rate of dividend, and y^j is the average dividend yield.

Chartist group

Instead of using the fundamental value, chartists rely on reference prices (or benchmark prices) v_t^j for their trading decision. v_t^j is related to historical price and market be-

liefs. Given current price p_{t-1}^j and reference price v_{t-1}^j , chartist price deviation is denoted as $p_{t-1}^j - v_{t-1}^j$. Based on the chartist deviation price, demand of chartists is expressed as

$$D_{c,t}^j = b_c^j \left(p_{t-1}^j - v_{t-1}^j \right), \quad (5.3)$$

where b_c^j is the demand coefficient of chartists.

All chartists share the same belief of reference price v_t^j . The difference among them is captured by the sign of b_c^j . A trader is named chartist trend chaser if $b_c^j = b_{ct}^j > 0$ and a chartist contrarian if $b_c^j = -b_{cc}^j < 0$. A chartist trend chaser believes that price will deviate further away from the current reference price. In contrast, a chartist contrarian believes that price deviation from current reference price will be reduced or even reversed.

As mentioned earlier, v_{t-1}^j is state dependent, switching stochastically between two states $s_t^j \in S = \{1, 2\}$. The dynamics behind the switching process can be captured by transition probabilities

$$P(s_t = l | s_{t-1} = k) = P_{l,k} \quad (5.4)$$

for $k, l \in S$. $P_{l,k}$ indicates the probability of a switch from state (regime) k to state (regime) l . The switching probabilities are assumed as constants and should satisfy constraints of $0 \leq P_{l,k} \leq 1$ and $\sum_{l=1}^2 P_{l,k} = 1$ for $k = 1, 2$. The state s_t^j can be estimated by a filter estimation/Markov regime switching model through the market prices. Technical details regarding Markov regime switching can be found in Hamilton (1994). As the reference prices are state contingent on the states, the regime dependent v_t^j is given by

$$v_t^j = \begin{cases} v_{1,t}^j, & s_t^j = 1, \\ v_{2,t}^j, & s_t^j = 2. \end{cases} \quad (5.5)$$

Noise traders

Noise traders do not rely on a fundamental value or price pattern to trade. However, their trading behavior is affected by the state of regime switching process. Information of the market state can be revealed by various news media. Demand of noise traders is ex-

pressed as

$$D_{n,t}^j = e_t^j = \begin{cases} N\left(0, \left(\sigma_{1,t}^j\right)^2\right), & s_t^j = 1, \\ N\left(0, \left(\sigma_{2,t}^j\right)^2\right), & s_t^j = 2. \end{cases} \quad (5.6)$$

that is, the mean of demand of noise trader is zero; but variance is state dependent.

Market maker

We denote the composition of fundamental trend chasers, fundamental contrarians, chartist trend chasers, chartist contrarians, and noise traders in market j ($j = \text{Fr}$ or US) by ω_{ft}^j , ω_{fc}^j , ω_{ct}^j , ω_{cc}^j and ω_n^j . Market maker of market j collects excess demand of all types of traders to update price of market j . Price impact function in a single market model can be expressed as

$$\begin{aligned} \Delta p_t^j &= p_t^j - p_{t-1}^j \\ &= \delta^j \left[\begin{aligned} &\omega_{ft}^j b_{ft}^j (p_{t-1}^j - u_t^j) - \omega_{fc}^j b_{fc}^j (p_{t-1}^j - u_t^j) + \omega_{ct}^j b_{ct}^j (p_{t-1}^j - v_{t-1}^j) \\ &- \omega_{cc}^j b_{cc}^j (p_{t-1}^j - v_{t-1}^j) + \omega_n^j e_t^j \end{aligned} \right] \\ &= \alpha^j (p_{t-1}^j - u_t^j) + \beta^j (p_{t-1}^j - v_{t-1}^j) + \varepsilon_t^j \end{aligned} \quad (5.7)$$

where $\alpha^j = \delta^j (\omega_{ft}^j b_{ft}^j - \omega_{fc}^j b_{fc}^j)$, $\beta^j = \delta^j (\omega_{ct}^j b_{ct}^j - \omega_{cc}^j b_{cc}^j)$, and $\varepsilon_t^j = \delta^j \omega_n^j e_t^j$. Noise term ε_t^j still has state-dependent distribution

$$\varepsilon_t^j = \begin{cases} N\left(0, \left(\sigma_1^j\right)^2\right), & s_t^j = 1, \\ N\left(0, \left(\sigma_2^j\right)^2\right), & s_t^j = 2. \end{cases} \quad (5.8)$$

where $\sigma_1^j = \delta^j \omega_n^j \sigma_{1,t}^j$, and $\sigma_2^j = \delta^j \omega_n^j \sigma_{2,t}^j$. Therefore, each market undergoes a price updating process with regime-dependent mean and variance. Specifically,

$$\begin{cases} \Delta p_t^{Fr} = \alpha^{Fr} (p_{t-1}^{Fr} - u_t^{Fr}) + \beta^{Fr} (p_{t-1}^{Fr} - v_{t-1}^{Fr}) + \varepsilon_t^{Fr}, \\ \Delta p_t^{US} = \alpha^{US} (p_{t-1}^{US} - u_t^{US}) + \beta^{US} (p_{t-1}^{US} - v_{t-1}^{US}) + \varepsilon_t^{US}. \end{cases} \quad (5.9)$$

Inter-market traders

The innovation of this paper is that we introduce to each market a new group of traders, inter-market traders, whose trading decision is based on information of fundamental value of the other market. Demand of inter-market traders in market j is assumed to be

$$D_{i,t}^j = b_i^j (p_{t-1}^k - u_t^k) + b_s^j \Delta S_t,$$

where b_i^j is demand coefficient of inter-market traders; k is the market other than j ; and b_s^j is demand coefficient for the change of exchange rate that is denoted by $\Delta S_t = S_t - S_{t-1}$. Adding the demand of inter-market traders into the single market model to form the two-market model, we get

$$\begin{aligned} \Delta p_t^j &= p_t^j - p_{t-1}^j \\ &= \alpha^j (p_{t-1}^j - u_t^j) + \beta^j (p_{t-1}^j - v_{t-1}^j) + \delta^j b_i^j (p_{t-1}^k - u_t^k) + \delta^j b_s^j \Delta S_t + \varepsilon_t^j \\ &= \alpha^j (p_{t-1}^j - u_t^j) + \beta^j (p_{t-1}^j - v_{t-1}^j) + \gamma^j (p_{t-1}^k - u_t^k) + \lambda^j \Delta S_t + \varepsilon_t^j, \end{aligned}$$

where $\gamma^j = \delta^j b_i^j$ and $\lambda^j = \delta^j b_s^j$. Specifically, for two markets France and US, we have the two-market price updating model as

$$\begin{cases} \Delta p_t^{Fr} = \alpha^{Fr} (p_{t-1}^{Fr} - u_t^{Fr}) + \beta^{Fr} (p_{t-1}^{Fr} - v_{t-1}^{Fr}) + \gamma^{Fr} (p_{t-1}^{US} - u_t^{US}) + \lambda^{Fr} \Delta S_t + \varepsilon_t^{Fr}, \\ \Delta p_t^{US} = \alpha^{US} (p_{t-1}^{US} - u_t^{US}) + \beta^{US} (p_{t-1}^{US} - v_{t-1}^{US}) + \gamma^{US} (p_{t-1}^{Fr} - u_t^{Fr}) + \lambda^{US} \Delta S_t + \varepsilon_t^{US}. \end{cases} \quad (5.10)$$

5.2.2 Data

We use data from Bloomberg including indexes and dividend of both CAC 40 (France) and DJIA (US), Consumer Price Index (CPI) and exchange rate denoted by Euro/US dollar from January 2000 to April 2013. All the indexes and dividend are discounted by CPI to get the real values for evaluation in this paper.

Real stock price and calculated real fundamental value are compared in Fig. 5.1 for both France and US. In most of the time, stock price does not equal to fundamental value

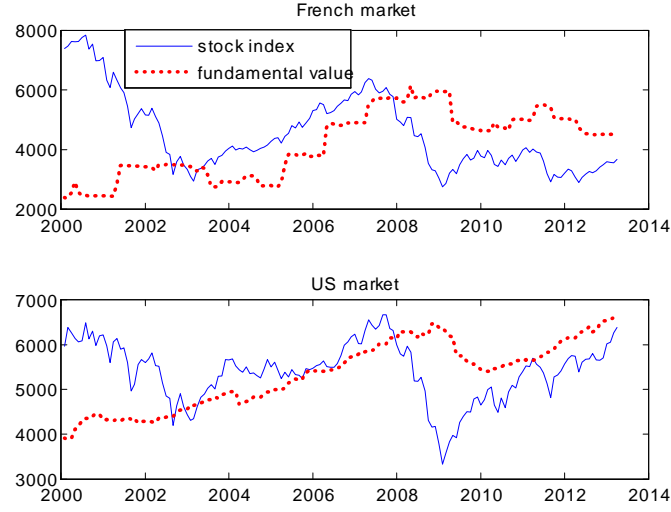


Figure 5.1: Real stock price and fundamental value of French and US stock markets.

Table 5.1: Statistics summary, sample period from January 2000 to April 2013.

variable	p_t^{Fr}	d_t^{Fr}	g_t^{Fr}	u_t^{Fr}	p_t^{US}	d_t^{US}	g_t^{US}	u_t^{US}	S_t
Mean	4551.30	124.33	0.005	4175.52	5412.66	123.43	0.003	5327.21	1.22
Std. dev.	1277.50	32.86	0.048	1103.43	659.99	17.42	0.012	752.22	0.19
Min	2754.65	70.84	-0.174	2379.17	3328.54	90.30	-0.064	3897.40	0.85
Max	7845.21	183.08	0.255	6148.50	6667.11	153.76	0.053	6636.19	1.58

for both markets. In addition, the two markets have similar co-movement between stock price and fundamental value. For both markets, prices are above fundamental value before 2003, mainly during the "dot com" bubble period. After that, prices and fundamental values rise together till 2007, when subprime crisis occurs. The effect of the crisis is to push prices below fundamental values. The difference between the two markets is reflected in the period after the crisis. Price of France remains stagnant and below the fundamental value while price and fundamental value of US recover almost to the pre-crisis level. Table 5.1 summarizes the statistics. It is shown that France has larger standard errors with its variables and tends to have larger monthly fluctuation.

5.3 Estimation Results

5.3.1 Model estimation with single market framework

We estimate the single market framework defined in e.q. (5.9) based on maximum likelihood method coded by Perlin (2012). As a beginning, separate regressions are run to check the existence of fundamental and chartist groups of traders. Detailed estimation results are presented in Table 5.2. For both markets, the two switching regions for noise traders are statistically significant, implying that both markets have regime-switching behaviors. For France, coefficients of fundamental group are statistically insignificant while it is significant with negative value for chartist group, indicating that contrarians dominate the chartist groups. Chartist group has only one regime as v_2 is insignificant. For US, the separate regressions show the existence of fundamental and chartist groups with significant coefficients. Chartist group has two switching regimes with reference values v_1 and v_2 . Both coefficients for fundamental group (α) and chartist group (β) are negative, suggesting contrarians dominating both groups. Traders of the two groups believe that price will move towards their trading reference values. To make it fly in the ointment, the coefficients of fundamental group and the second reference value of chartists become insignificant when regression for US is run on both fundamental and chartist groups. One of the reasons of this insignificance is multicollinearity between the fundamental and chartist groups, similar to the case of S&P 500 studied in Chiarella et al. (2012).

Another reason for the insignificance of fundamental group in both market is the missing variable: fundamental value of the other market and the exchange rate. The result is reported in the sequel subsection.

5.3.2 Model estimation with two-market framework

In the two-market framework e.q. (5.10), each market includes both price deviation from fundamental value of the other market and the exchange rate for regression. Estimation results are reported in columns 2 and 3 of Table 5.3. Coefficients of fundamental group

Table 5.2: Estimation result of individual group of traders under single market framework, sample period from January 2000 to April 2013. *, **, and *** represent significance at 10%, 5% and 1% level. P-value is in parenthesis.

variables	Fundamental group only		Chartist group only		Fundamental and chartist groups	
	France	US	France	US	France	US
α	-0.0008 (0.923)	-0.0422** (0.038)			0.0209 (0.169)	-0.0117 (0.64)
β			-0.0098** (0.013)	-0.0666*** (0.008)	-0.0332* (0.058)	-0.0569 (0.104)
v_1			9468.81 (0.127)	6199.37** (0.002)	5630.32** (0.014)	6315.99** (0.059)
v_2			-10320.91 (0.248)	4313.53** (0.037)	-253.46 (0.913)	4251.04 (0.190)
σ_1	161.66*** (0.000)	161.48*** (0.000)	145.05*** (0.000)	142.94*** (0.000)	148.56*** (0.000)	140.16*** (0.000)
σ_2	328.06*** (0.000)	296.47*** (0.000)	295.59*** (0.000)	282.81*** (0.000)	300.04*** (0.000)	284.31*** (0.000)
$P_{1,1}$	0.9853** (0.017)	0.9813** (0.039)	0.9503*** (0.007)	0.9523*** (0.009)	0.9580*** (0.009)	0.9495*** (0.008)
$P_{1,2}$	0.0417 (0.905)	0.0461 (0.902)	0.0971 (0.798)	0.0834 (0.82)	0.0940 (0.812)	0.0843 (0.818)
LL	-1075.13	-1070.02	-1067.23	-1064.41	-1066.44	-1064.28
AIC	16.10	15.95	16.09	15.95	16.09	15.96

become significant for both markets, implying that missing variable is one of the causes of the insignificance of fundamental group in both markets under the single market framework. The value for France is positive while it is negative for US. This means trend chasers dominate the fundamental group in France while it is contrarians who dominate the group in US. On average, traders based on fundamental value in France believe the price deviation from the fundamental value will increase while those in US believe price will reverse back to the fundamental value.

The more interesting part is the result for the coefficients of inter-market traders, γ and λ . Coefficients of inter-market traders with respect to fundamental value of foreign market, γ , are significant for both markets, suggesting the existence of such kind of inter-market traders. Investment behaviors of inter-market traders in the two markets are different as the coefficient for France is negative while it is positive for US. According to this result, the trading of inter-market traders in France is negatively correlated with the price deviation of US from its fundamental value while in US, it is positively correlated with the price deviation of France from its fundamental value. One of the possible explanations for this phenomenon is that inter-market traders in France use the price signal of US to gauge France and trade contrarily while inter-market traders in US react to the price movement of France in a manner of positive feedback. In this sense, given unchanged in fundamental values of the two markets, if price of US increases such that price deviation is positive, inter-market traders in France treat it as an alert and will sell to push price of France down. In contrast, for a negative price deviation in France, inter-market traders in US are discouraged and will sell to push price of US down. If we go one more step, we can find that the contrary behaviors of inter-market traders in the two markets form a stabilizing mechanism for prices in both markets. An increase of US price leads to a price decreasing in France, which will in turn push down the price of US. The similar stabilizing mechanism can be deduced for France vice versa.

The other coefficient of inter-market traders is respect to exchange rate, λ . As exchange rate is expressed as Euro/US dollar, a positive change in the exchange rate indicates that US dollar appreciates or Euro depreciates. λ for France is marginally significant with p-value 0.112. The positive value suggests that inter-market traders will buy in French

asset upon the depreciation of Euro. The effect of exchange rate in US is different. λ for US is significant with a positive value, implying that appreciation of US dollar attracts inter-market traders into the US stock market.

Coefficients for the two-regime noise traders are statistically significant, indicating the existence of regime switching even under the two-market model. Although the chartist reference values in US are insignificant, this may be still due to the multicollinearity. Chartist regime 2 reference values are smaller in both markets. As the chartist groups behave like contrarians, they will sell to push the prices down towards the chartist regime 2 reference values. In this sense, regime 2 can be labelled as a bust period for both markets. Notice that, noise traders always have larger standard errors in regime 2 for both markets. This result of bust period with larger volatility is similar to the finding of Chiarella et al. (2012).

5.3.3 Regime dependent of inter-market traders

The existence of inter-market traders suggests that individual markets are subject to influence of the other markets. When the influence of foreign market becomes stronger, markets contagion can be observed, especially during depressed periods. The question in this paper will be whether the influence of foreign markets is constant or not. We evaluate different combinations of coefficients with regime switching feature. The most fitted results are reported in columns 4 and 5 of Table 5.3. For both markets, coefficients of exchange rate, λ , are significant and do not have regime switching behavior. The main finding is that coefficients of fundamental value of foreign markets, γ , are significant with regime switching behavior. Inter-market traders in regime 2 have larger coefficients than those in regime 1. In other words, when domestic stock markets have larger fluctuation in price movements, they become more vulnerable to the influence from foreign markets. As the regime 2 reference value of chartist group, v_2 for France is always insignificant, we treat chartist group of France as non-regime-switching. While for US, we still treat the chartist group with regime switching even though it is insignificant in this case due to possible cause of multicollinearity.

Table 5.3: Estimation result under two-market framework, sample period from January 2000 to April 2013. *, **, and *** represent significance at 10%, 5% and 1% level. P-value is in parenthesis.

variables	Inter-market traders without regime switching		Inter-market traders with regime switching	
	France	US	France	US
α	0.0911*** (0.000)	-0.131*** (0.006)	0.0841*** (0.000)	-0.1158** (0.033)
β	-0.036** (0.044)	-0.0231 (0.512)	-0.0536** (0.015)	-0.0165 (0.642)
γ_1	-0.1405*** (0.003)	0.0517** (0.020)	-0.0694** (0.048)	0.0403* (0.094)
γ_2			-0.2559*** (0.000)	0.0822*** (0.002)
λ	759.69 (0.112)	1427.76*** (0.003)	1105.09** (0.019)	1464.50*** (0.003)
v_1	4850.34** (0.025)	4132.94 (0.620)	4445.90** (0.013)	7126.94 (0.545)
v_2	310.02 (0.897)	8426.74 (0.314)		-2628.16 (0.826)
σ_1	144.61*** (0.000)	145.78*** (0.000)	161.76*** (0.000)	147.99*** (0.000)
σ_2	294.31*** (0.000)	279.93*** (0.000)	308.98*** (0.000)	271.58*** (0.000)
$P_{1,1}$	0.9843** (0.019)	0.9751** (0.016)	0.9756** (0.003)	0.9875** (0.013)
$P_{1,2}$	0.0429 (0.906)	0.0631 (0.849)	0.0715 (0.857)	0.0401 (0.904)
LL	-1059.92	-1059.30	-1065.39	-1055.58
AIC	16.05	15.92	16.13	15.89

Transition probabilities describe the switching between the two regimes. We use the transition probabilities from the regression results of this subsection to discuss. In France, $P_{1,1}^{Fr} = 0.9756$, implying that once in regime 1, France will remain in regime 1 in the next period with probability 0.9756 and the expected duration of regime 1 will be $1/(1 - 0.9756) = 41$ months. $P_{1,2}^{Fr} = 0.0715$ means $P_{2,2}^{Fr} = 0.9285$, suggesting the expected duration of regime 2 to be $1/(1 - 0.9285) = 14$ months. Similarly, in US, $P_{1,1}^{US} = 0.9875$, the market will remain in regime 1 with the expected duration of $1/(1 - 0.9875) = 80$ months. Probability to remain in state 2 $P_{2,2}^{US} = 1 - P_{1,2}^{US} = 0.9599$, with an expected duration of 25 months. In both markets, regime 2 has a shorter duration and investors face relatively shorter depressed periods. Based on the entire sample prices, we can calculate the smoothed probabilities at each period (algorithm details can be found from Kim and Nelson (1999)). Probabilities for regimes 1 and 2 are plotted in Fig. 5.2. In the timeframe when "dot com" bubble bursts before 2003, both markets are in the bust regime 2 state. From then on, both markets are in regime 1 state and enjoy the booming period till the occurrence of subprime crisis, in which both markets fall in regime 2 state again. After the subprime crisis, regime 1 starts to gain control and dominate. The switching regimes match the market episodes well. In addition, the state evolvments are similar for both markets, suggesting some commonality underlying the two markets. Notice that influence of foreign markets becomes stronger in regime 2 while both markets fall in regime 2 in the similar timeframes, contagion may occur between the two markets in regime 2.

To investigate the roles of different groups of traders, we plot their excess demands in Fig. 5.3. During the burst of "dot com" bubble before 2003, inter-market traders and chartist group are main sellers to push price of France down. In the origin of "dot com" bubble, US, it is fundamental and chartist groups who are net sellers to push price down. As both markets are in regime 2 in which coefficients of inter-market traders become larger, inter-market traders are major players during this period. The over-valued US causes the large sales of inter-market traders in France.

During the booming period before the subprime crisis in 2008, price of France is mainly driven up by fundamental group while it is inter-market traders and chartist group who are main drivers for price increment in US. During the subprime crisis before 2010,

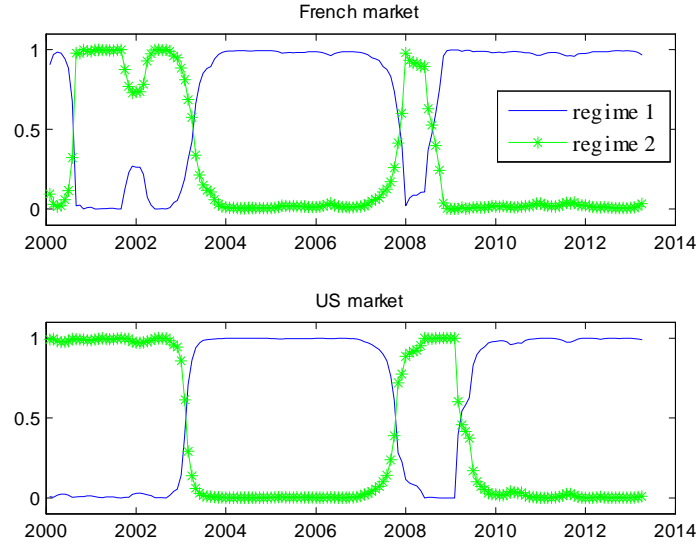


Figure 5.2: Smoothed probability of switching regimes for French and US stock markets, Green color with markers is for state 2.

fundamental group is main seller while inter-market traders are main buyers in France. In US, by dominating the purchasing of fundamental group, inter-market traders and chartist group are main sellers to push price down. During the crisis, US has a longer period in regime 2. Although the subprime crisis spreads from US to other markets, at the beginning of 2008, the plunge of France causes the panic selling of inter-market traders in US, which exacerbates the crisis of US. There is a contagion from France to US. The behaviors of inter-market traders in the two markets imply that inter-market traders shift capital from US to France during the crisis.

Beginning in 2010 when Euro Debt crisis emerges, inter-market traders and chartist group continue to buy in French stock. However, their efforts are offset by the selling force of fundamental group and hence the price of French market does not recover to the pre-crisis level. In US, domestic investors fundamental and chartist groups are main buyers to push the price of US up. Facing the depressed price of France, inter-market traders in US keep on selling, causing price to drop intermittently. Overall, if there is no negative external impact from the Euro Debt crisis, the price of US stock market could have even reached a higher level. Although the performance of France is depressed, inter-market capitals continue

to flow into Europe in view of booming US stock market and investment opportunity in Europe.

5.3.4 Out-of-sample forecast

To verify the value and the forecasting capability of the model, we conduct an out-of-sample forecast practice. We first use the two-market model with regime switching inter-market traders to estimate the parameters based on sample periods from January 2000 to April 2011. The estimated parameters are used to forecast the prices in the rest of the periods. That is, we use 136 data points to forecast 23 periods. The predicted prices match the actual prices quite well, especially the US. This indicates the capability of the model to explain the two stock markets.

5.4 Conclusion

Given the context of globalization and financial integration, interactions among different markets, such as cross-correlation, surface out and become an important phenomenon. However, most of the existing empirical heterogeneous agents literatures focus on a single market while a single market model might not capture these kinds of market interactions.

Following the methodology of Chiarella et al. (2012), this chapter first demonstrates the regime switching features for the monthly price changes of stock markets of France and US under single market framework even though the fundamental groups are seemingly statistically insignificant. By including inter-market traders whose trading decision is based on the fundamental value of the foreign market and foreign exchange, the fundamental groups in both markets become statistically significant, suggesting that missing variable renders the insignificance of regression under the single market framework. Further investigations show that inter-market traders are regime dependent with respect to the fundamental value of foreign markets. The regime switching behavior of inter-market traders suggests a contagion from France to US in the midst of subprime crisis, which exacerbates the crisis of US.

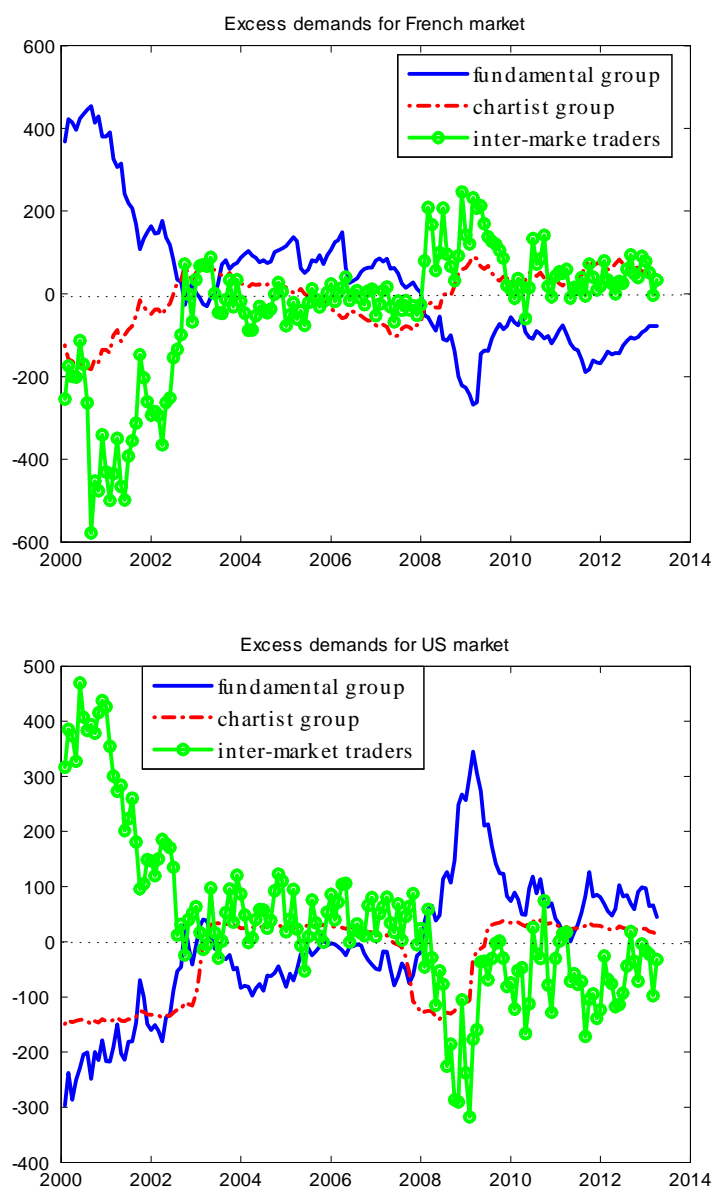


Figure 5.3: Excess demands of traders for French and US stock markets.

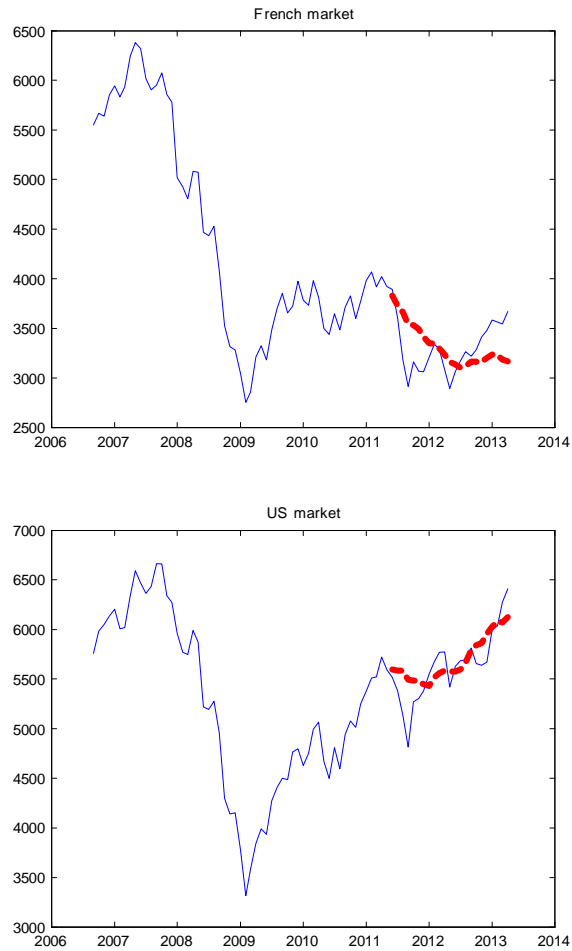


Figure 5.4: Out-of-sample forecast for French and US stock markets. The bold dash-lines are the forecasted prices.

The existence of inter-market traders implies that condition in one market can affect other markets. It also reveals a channel of market interactions. Fundamental values of both markets are common factors of price changes in individual markets. This provides a behavioral explanation for inter-market phenomena such as markets co-movement or cross-correlation. In the context of financial integration, individual market cannot isolate itself from the market system or just focuses solely on innovations and market state in its own market. Market players need to look at a bigger picture including other markets. This is because innovations/shocks in other markets might eventually affect its market even though there might be no direct impact from those innovation/shocks.

Chapter 6

Conclusion

Price co-movement between financial markets as well as financial crisis contagion can be quantified by high cross-correlation. They are manifestations of financial markets integration, underlying which is economic integration. Financial markets integration has double-edged effect. On the one hand, it reduces investment barriers, increases market liquidity and overall improves market efficiency. On the other hand, the interconnected market system may generate unexpected bad results. One of the extreme cases is financial crisis contagion causing many markets to go into a tailspin and consequently huge loss to investors. With the current development of globalization, interactions involving multiple markets will become increasingly apparent. Although heterogeneous agents models have been successful in explaining financial markets with various insightful theoretical models and empirical studies, regrettably, there is still a lack of thorough understanding of multiple-market interaction. This thesis intends to investigate the interactional dynamics of financial markets and to shed some light on the ongoing research in this topic.

Chapter 2 develops a two-market heterogeneous agents model. Each market has a market maker and two groups of investors. The first group consists of chartists and fundamentalists who are inhomogeneous across markets. The established market linkage allows investors of first group to invest in each market. It also promotes the emergence of the second group, inter-market traders who trade in the domestic market with reference to excess

demand of foreign market. The rationale for inter-market traders is the free movement of capital and common factors underlying the two markets. Market maker updates price for her market by taking account of excess demands of both groups of investors. Existence of price co-movement/cross-correlation between markets is proved.

By establishing market linkage, individual market's intrinsic dynamic properties may be overwritten. A market that is more stable initially in isolation will exert stabilizing effect on the market system while it will be subjected to destabilizing effect from the resultant market system. In addition, a market with a larger population has a larger influence over the resulted assets prices of the market system. This market linkage can provide policy implication for financial market opening. In a world consisting of a small market and a large market (or market agglomeration), if the small market is stable compared to the large market, market opening of the small market will cause the small market to be destabilized. Small market will benefit from market opening only if it is unstable originally compared to the large market. This example indicates that market opening is a double-edged sword. Decision of market opening should be based on the impact assessment on internal and external markets.

Similar to Chapter 2, Chapter 3 also manages to replicate numerically statistically significant cross-correlation as well as other typical financial market stylized facts. While Chapter 2 focuses on theoretical aspect, Chapter 3 mainly uses numerical method to demonstrate financial crises with contagion behavior. The key point of the chapter is to simulate financial crisis within two-market framework from points of view of endogeneity and exogeneity so that causes of financial crisis could be explored for different scenarios.

In terms of endogeneity, Chapter 3 manages to simulate different patterns of financial crisis across two markets endogenously, which imply that financial crisis and its contagion could occur endogenously. As all the simulated financial crises occur at price level above market fundamental levels, they support scenario of market over-valuation causing a financial crisis. In terms of exogeneity, shocks are introduced to fundamental value of individual market. Depending on the magnitude, sign and duration of shocks, different patterns of financial crisis could be triggered. Without the external shock(s), financial crisis does not occur even though there are periods of market over-valuation. This supports scenario

of financial crisis triggered by an external shock. In addition, the fact that a financial crisis in one market triggered by shock causes a similar crisis in the other market is analogous to the domino effect.

While Chapters 2 and 3 work on two-market framework, Chapter 4 studies multi-market system. While applying the model of Chapter 2, Chapter 4 simplifies the setup by assuming that fundamentalists and chartists just invest in their home markets. The connection among market members is established by inter-market traders. It is shown numerically that market clusters in which market members share the same sign of price deviation appear even in the chaotic interval. To check the stability of the market clusters, disturbances are introduced to one of the market members. If the disturbance is weak, it seems only adjacent markets are affected for adjustment and a new market cluster pattern is formed; if the disturbance is large enough, market system avalanche is generated from the initially impacted market. However, from the point of view of deviation magnitude, even if the disturbance is weak, disturbance still can propagate to the market system with propagation modes of diffusion or localization, or the mixture of the two modes. That is, the magnitudes of price deviation of all market members change due to small shock on one market member eventually. Lastly, price trajectory of shock response shows ability of coupling effect to stabilize market member hit by a shock. Policy implication implied by disturbance simulation is that market opening has the dual effect of stabilizing and destabilizing effects. Stabilizing effect refers to small shock absorption by the whole market system while destabilizing concerns for breakdown of the market system due to large shock from market member.

Inter-market traders play a key role in this thesis. Chapter 5 proves the existence of such kind of investor in the real world. By utilizing the monthly price changes of stock markets of France and US, it is shown that inter-market traders exist in both markets, whose trading decision is based on the fundamental value of the foreign market and foreign exchange rate. Further investigations show that inter-market traders are regime dependent with respect to the fundamental value of foreign markets. The regime switching behavior of inter-market traders suggests a contagion from France to US in the midst of subprime crisis, which exacerbates the crisis of US. The existence of inter-market traders implies

that condition in one market can affect other markets. It also reveals a channel of market interactions. Fundamental values of both markets are common factors of price changes in individual markets. This provides a behavioral explanation for inter-market phenomena such as markets co-movement or cross-correlation.

6.1 Caveats and Extensions

This thesis focuses on the interactional dynamics of financial markets in which inter-market investors play an essential role. Although these models are useful in explaining theoretically and replicating numerically cross-correlation, they are subject to some limitations.

First, foreign exchange market is missing in this thesis for the purpose of simplicity. Today, a large volume of international financial market transactions still involve foreign exchange. The role of foreign exchange in multi-market interactions needs to be explored.

Second, the coupling factor or weightage of inter-market investors is fixed. It would be desired to have an endogenous weightage of inter-market investors determined by economic factors or profit performance. The trade off is to make the model more complicated for analysis.

Third, all market members share the same coupling factor or weightage of inter-market investors in Chapter 4. Even if it is too complicated to have endogenous weightage of inter-market investors, it is possible for different markets to have different weightage given the intrinsic attributes of each market.

6.2 Future Research

Three main strands of this thesis can be further explored in the short future.

First, we would like to incorporate foreign exchange market into analysis to get a complete picture, as like Dieci and Westerhoff (2010).

Second, for the model of Chapter 4, further research can be implemented to evaluate different weightages of inter-market investors to reflect the realistic fact that some

markets have large influence on other markets.

Third, one of the values of studies like this thesis is to detect and avoid crisis. It would be worthwhile to calibrate or estimate for the documented financial crisis and then investigate whether the model has capability to predict or warn future financial crisis.¹

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