Towards the design of robust incentive mechanisms to address subjectivity and dishonesty problems in agent reporting

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**TOWARDS THE DESIGN OF ROBUST INCENTIVE
MECHANISMS TO ADDRESS SUBJECTIVITY AND
DISHONESTY PROBLEMS IN AGENT REPORTING**

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SCHOOL OF COMPUTER ENGINEERING

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Abstract

Agent reporting systems, such as reputation systems and crowdsourcing platforms, provide opportunities for human users represented as agents to report their ratings or opinions via electronic channels. However, their reports could vary and are difficult to be aggregated (subjectivity problem) and they may provide untruthful reports (dishonesty problem). The two problems have attracted considerable interests from both academic and industrial societies. The first objective of this thesis is to address the subjectivity and dishonesty problems through designing three incentive mechanisms where the agents are assumed to be rational. Furthermore, considering the existence of bounded rational agents in real systems, the second objective is to evaluate the robustness of incentive mechanisms against bounded rational agents.

Specifically, to address the subjectivity problem, a coalition formation game based reputation system (CONGRESS) is proposed for e-marketplaces. In CONGRESS, buyers with the same subjectivity are incentivized to form a separate *club* to build seller reputation based on the ratings collected from the buyers in the club. Consequently, the proposed model avoids suffering from the bad effect of the subjectivity problem. It has been theoretically proven that buyers with the same subjectivity have the incentive to form a club if a certain condition is satisfied. Furthermore, a set of experiments has been conducted to validate the proposed reputation system.

Secondly, to address the dishonesty problem, an incentive mechanism iMEMLI is proposed for e-marketplaces with limited inventory where the supply of sellers is less

than the demand of buyers. In iMEMLI, more honest buyers could achieve higher utility because they have the larger probability of being allocated to the limited products. The products of more honest sellers with higher reputation are offered at higher prices. Both theoretical analysis and experimental results have shown that the proposed incentive mechanism can promote buyer honesty in providing truthful ratings and seller honesty in delivering promised products.

Thirdly, to address both the subjectivity and dishonesty problems together, an incentive mechanism is proposed to elicit truthful opinions (iMET) from workers in crowdsourcing systems even when their truthful opinions are different and the majority workers behave untruthfully. iMET considers the truthful opinion difference between workers and models the workers' behaviors according to their provided opinions, and strategically rewards truthful opinions. Theoretical analysis and real-data based evaluations have shown the effectiveness of iMET in eliciting truthful opinions from workers.

Finally, a simulation framework is proposed to evaluate the robustness of incentive mechanisms against bounded rational agents taking non-equilibrium strategies. A formal robustness measure is defined, inspired by the studies of evolutionary game theory. The simulation framework bases on the evolutionary dynamics to empirically quantify the proposed robustness measure, which is validated by comparing the simulation results with analytical predictions based on an extended simplex analysis approach. The simulation framework is further implemented for evaluating the robustness of several incentive mechanisms against some untruthful strategies in reputation systems for e-marketplaces, including the proposed iMEMLI incentive mechanism for EMLI.

In summary, the proposed incentive mechanisms in this thesis are aimed to address the subjectivity and dishonesty problems even when the two problems coexist. The proposed simulation framework can evaluate and compare the robustness of the existing incentive mechanisms, providing insights towards the design of robust incentive mechanisms for agent-reporting systems.

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Chapter 1

Introduction

Agent reporting systems, such as reputation systems [16] and crowdsourcing platforms [117], have been widely used to assist human users represented as agents (e.g., buyers and task requesters, respectively) in making decisions by collecting truthful reports (i.e., ratings and opinions, respectively) from other agents via electronic channels. An agent reporting system in this research is referred to as an electronic platform or online community where one major activity of agents is to provide reports. The research presented is thus in the general area of multi-agent systems. This thesis focuses on designing incentive mechanisms to address the problems existing in these agent reporting systems and developing a framework to study the robustness of incentive mechanisms against bounded rational agents. Thus, the research presented is in the sub-field of game theory and mechanism design. Two specific types of agent reporting systems are studied, i.e., reputation systems and crowdsourcing platforms, and the background knowledge of the systems is introduced as follows.

The first type of agent reporting systems under study is the reputation systems for electronic marketplaces. E-marketplaces, such as Amazon¹ and eBay², provide Internet based platforms for buyers and sellers to conduct transactions [97]. Both buyers and sellers are able to gain benefits from this form of marketplaces and they could conduct

¹www.amazon.com

²www.ebay.com

transactions without time and location constraints through the Internet. Transactions happen every second and the buyers and sellers are distributed worldwide. On the one hand, sellers could save the cost of renting a store by posting their products or services online for free or at relatively low cost, and on the other hand, buyers are offered with a more convenient way to compare products or services rather than to spend lots of time on wandering through real stores. Due to these reasons, e-commerce has been rapidly growing over the last ten years. The analysts of Research and Markets forecast the global e-commerce market to grow at CAGR (Compound Average Growth Rate) of 13.54% over the period 2013-2018.³ Although e-commerce is gaining popularity, the lack of trust and reliability has been frequently cited to be one of the key factors that discourage buyers from participating in e-marketplaces [109, 36]. Since sellers and buyers are conducting transactions via the Internet without knowing one another in the physical world, buyers may experience unsatisfactory transactions which are caused by malicious sellers. Thus, the risk of buyers in their transactions increases, which hinders the growth of e-commerce.

Reputation systems have been shown to be an effective way to help buyers to select satisfactory sellers, by predicting sellers' future behaviors based on the ratings shared by buyers who have conducted transactions with the sellers in the past [70, 109, 71, 30, 32]. Reputation systems have two main roles: signaling honest sellers with high reputation scores, and penalizing dishonest sellers with low reputation scores [37, 122]. As a result, many buyers would be attracted to conduct transactions with honest sellers (those with high reputation scores) and meanwhile buyers are discouraged to conduct transactions with dishonest sellers (those with low reputation scores). Hence, reputation systems can promote seller honesty and protect buyers from conducting unsatisfactory transactions. However, the existing reputation systems still cannot effectively and efficiently promote honesty from sellers. According to the statistics from National Consumers League's

³http://www.researchandmarkets.com/research/8ss714/global_ecommerce

(NCL) fraud center⁴ in 2013, among the Internet scams, the percentage of general merchandize where goods are never delivered or misrepresented is 33.49%. The statistics show that the reputation systems in use are not very effective in promoting seller honesty. The major reasons causing this situation are that: (1) ratings provided by buyers about sellers could vary and are difficult to be aggregated (subjectivity problem), and (2) buyers may provide untruthful ratings (dishonesty problem). For example, the same items are received by two buyers within five shipping days. One buyer may provide a positive rating for this transaction to indicate that five-day shipping is acceptable and the seller satisfies her. Nonetheless, another buyer may provide a negative rating for the same transaction, because five-day shipping is too long to be accepted. In this case, the rating provided by the first buyer is misleading the second buyer, and vice versa. This problem in the literature of reputation systems is referred to as the subjectivity problem, which has been widely recognized by many researchers [36, 118, 16, 20]. The dishonesty problem happens when the first buyer who experiences a satisfactory transaction provides a negative rating, or the second buyer provides a positive rating, which are also called as “bad-mouthing” and “ballot stuffing”, respectively [36, 119]. Such ratings which do not truthfully reflect the behaviors of sellers hinder the effectiveness of reputation systems in helping buyers choose satisfactory sellers.

The second type of agent reporting systems is crowdsourcing platforms. Over the past five years, crowdsourcing has gained remarkable popularity as an effective technology for human-powered problem solving. In crowdsourcing, *task requesters* propose *tasks* to the crowd (i.e., the *workers*) to complete, which computers cannot efficiently solve. It has been widely applied in various domains such as image classification [2], interactive storytelling [55], and product categorization [29]. One of the most common applications of crowdsourcing is to find out the hidden states of the world by collecting opinions from

⁴<http://www.fraud.org>

a large number of workers. This type of tasks is referred to as *consensus tasks*. An example of crowdsourcing systems for consensus tasks is the *Galaxy Zoo* project⁵. It requires volunteer workers to label the shapes of galaxies in images captured through an automated astronomical survey using proprietary online tools. Based on the collected opinions, the consensus task requesters can form the final answers according to some opinion aggregation rules (e.g., the majority voting and trust-based fusion method [102]). In order to make sound decisions in consensus tasks, two important challenges need to be addressed: (1) how to elicit truthful opinions from workers, and (2) how to achieve the final answer through aggregating the collected opinions. This thesis concentrates on addressing the first challenge by designing an incentive mechanism to promote truthful opinions from workers for consensus tasks with multiple choice questions⁶. The truthful answers of workers regarding the questions in a consensus task are unknown to the task requester or the system. It is nearly impossible to verify whether the answers provided by workers are their truthful answers (dishonesty problem). Moreover, the truthful opinions of workers could be diverse (subjectivity problem).

Thus, the subjective problem and the dishonesty problem commonly exist in agent reporting systems, which will be addressed in this thesis.

1.1 Limitations of Existing Approaches

In reputation systems, in order to address the subjectivity problem, a few solutions [89, 85, 121, 107, 108] have been proposed. The solutions can mainly be classified into three types. The first type of solution requires buyers to express their experience using an objective ontology [89]. The buyers are required to understand the ontology and take efforts to represent their experience clearly using the ontology. In the second type of

⁵<http://zoo1.galaxyzoo.org>

⁶Consensus tasks involving open-ended questions are out of the scope of this thesis.

solution, a buyer evaluates the subjectivity difference with other buyers and selectively uses the ratings provided by them [121, 107]. Each buyer has to build the reputation of sellers, which results in the waste of computing or storage resources since similar buyers may hold similar seller reputation information. The third type of solution predicts or infers the ratings of buyers through learning buyer subjectivity over the attributes of sellers [21, 85]. The buyers are required to share additional detailed reviews and learn the correlations between buyer subjectivity and seller attributes. Thus, it is necessary to explore a new solution which can overcome the above shortcomings.

To address the dishonesty problem in reputation systems, many incentive mechanisms have been proposed to promote truthful (fair) ratings from buyers [37, 106, 120, 124, 82], as surveyed in Chapter 2. One common and implicit assumption of these approaches is that sellers can provide a large number of products compared to buyers' demand. However, in some realistic scenarios, the supply of sellers cannot satisfy the demand of all the buyers. For example, considering the dentist booking in U.S. as a marketplace, it has been observed that there are more dentist bookings than the number of services provided by the dentists [13]. Another example is the hotel booking system for a famous tourism destination during a peak season. It is often difficult to book a satisfactory hotel. Similar marketplaces also include second-hand online stores where some used and workable goods (e.g., textbooks) are often in short supply due to lower prices. There are two common properties of these marketplaces: (a) each seller has limited inventory, e.g., the number of dentists, rooms of a hotel, and used textbooks, to provide within a unit of time; (b) buyers compete with each other so as to purchase one piece of the inventory. Such an e-marketplace is defined as an *e-marketplace with limited inventory* (EMLI). More generally, this concept is applicable to an e-marketplace where only a few sellers could provide promised products. This thesis focuses on the e-marketplaces with limited inventory in the narrow case where the supply is less than the demand and leaves the general scenario for future investigation.

New challenges are imposed on promoting buyer and seller honesty in an EMLI. Sellers with limited inventory may behave maliciously in their transactions to gain more profit by not delivering promised products or by reducing the quality of delivered products, given that other sellers also hold limited inventory compared to buyer demand. Even though their reputation will be decreased due to negative ratings from the buyers cheated by them, the sellers may still be willing to increase their profit by sacrificing their reputation. The sellers could sell out their limited inventory even sustaining the low reputation. Therefore, in these e-marketplaces, reputation itself may not be sufficient to motivate sellers to behave honestly. Moreover, buyers may also have incentive to report untruthful ratings. After a successful transaction with a seller, the buyer knows that the particular seller is good. If the buyer provides a truthful (positive) rating about the seller, then other buyers considering the positive rating are more likely to conduct transactions with the good seller, which reduces the buyer's opportunity of doing business with the particular seller in the future, due to the limited inventory that the seller has. If the transaction is unsuccessful, reporting a truthful (negative) rating also reduces the buyer's opportunity of doing business with other good sellers because other buyers will be less likely to do business with the bad seller but with the other good sellers, after taking the buyer's advice. Thus, buyers may lose their chance to purchase products because of providing truthful ratings. In other words, in the EMLI, providing truthful ratings is costly for buyers. The existing incentive mechanisms seldom consider these costs imposed on providing truthful ratings.

In crowdsourcing systems, a number of incentive mechanisms have been proposed in an attempt to elicit truthful answers for consensus tasks by offering rewards to workers [68, 49, 111]. However, they often suffer from two main limitations. Firstly, most of them implicitly assume that all workers share the same truthful opinion about the questions in a consensus task (subjectivity problem), which is not always held in practice especially when subjective opinions are sought [110]. As a result, these mechanisms

cannot properly reward workers in providing truthful opinions which differ from other workers. Secondly, most of these mechanisms cannot promote truthful opinions when majority workers provide untruthful opinions (dishonesty problem) as the strategy of providing untruthful opinions is more beneficial than that of providing truthful opinions in this scenario. Thus, an incentive mechanism is motivated to be proposed for crowdsourcing systems where the subjectivity and dishonesty problems coexist.

As surveyed above, incentive mechanisms, based on game theoretical analysis, have been designed to promote the desired behavior from rational agents, such as truthfully reporting behaviors. However, in realistic scenarios, the agents with bounded rationality [103] may deviate from the desired equilibrium strategy, causing other rational agents to also deviate, and as a result the incentive mechanisms may fail to achieve the expected performance. Thus, the study of bounded rationality is increasingly important in the field of mechanism design. The existence of bounded rational agents has been widely recognized by many researchers [7, 18, 113], where agents may make mistakes due to reasons unknown to mechanism designers. Aiyer *et al.* [3] propose a filtering approach to eliminate the impact of naive Byzantine (irrational) agents who randomly deviate from equilibrium strategies. Moreover, the qualitative analysis of the robustness against various attacks has been proposed, such as [112], where a mechanism is claimed to be either *resistant* or *vulnerable* with respect to an attacking strategy. However, one common limitation of the existing approaches is that they cannot tell to what extent these incentive mechanisms are robust against non-equilibrium strategies. Measuring the robustness of mechanisms could be useful for ensuring the practical usability of incentive mechanisms and towards the design of more robust incentive mechanisms. This is the motivation of the proposed robustness evaluation framework in Chapter 6.

1.2 The Proposed Approaches

To address the subjectivity, dishonesty, and robustness problems, four works have been proposed in this thesis, which are summarized as follows:

- **A coalition formation game based reputation system**[59, 58]: As a novel solution to the subjectivity problem, a coalition formation game based reputation system (CONGRESS), has been proposed to incentivize agents to automatically form groups with other agents having the same subjectivity based on coalitional game theory. Specifically, in CONGRESS, a set of buyers with the same subjectivity can form a group which is called a *club*. In each club, the reputation system is built based on the ratings provided by the buyers in the club. The built reputation system can assist buyers in choosing more satisfactory sellers, and create *utility surplus* of the buyers which are charged by the club center in the form of administration fees. The utility surplus of a club (total administration fees) is then divided among the buyers in the club according to an allocation algorithm in one of the two specific ways, i.e., proportional and Shapley allocations. The club structure where buyers with the same subjectivity form a same club is proved to be the *optimal club structure*. Given the allocation algorithm, it has been theoretically and experimentally shown that the optimal club structure is stable (i.e., the buyers have incentive to form the optimal club structure) if certain conditions are satisfied.
- **An incentive mechanism for e-marketplaces with limited inventory** [63, 60]: To overcome the new challenges imposed by an EMLI in addressing the dishonesty problem as discussed in Section 1.1, an incentive mechanism iMEMLI is proposed. In iMEMLI, buyer honesty is measured by a *normalized proper scoring rule*, where a buyer can and only can gain maximal scores by providing truthful ratings. The higher score brings the buyer a larger expected utility. Seller honesty is

measured by the ratings provided by buyers (weighted based on the buyers' scores) so that honest sellers are able to gain high reputation. The products of sellers with higher reputation are offered with a higher price and the buyers with higher scores have more opportunities to conduct transactions with more reputable sellers. Theoretical analysis and experimental results prove that the proposed mechanism elicit truthfulness from both buyers and sellers, and they have no incentive to leave and re-register as new participants.

- **An incentive mechanism designed for eliciting truthful opinions in crowdsourcing:** An incentive mechanism is proposed to elicit truthful opinions from workers (iMET) in performing consensus tasks on crowdsourcing platforms. The mechanism contains an innovative technique to estimate the workers' honesty and the similarity between the truthful opinions of any two given workers. This additional dimension of information, coupled with the answers provided by each worker, is utilized to adapt the amount of *truthfulness reward* that each worker receives in answering each question. In addition, iMET also computes a *consistency reward* reflecting the compatibility between a worker's answer and the final aggregated answer which is achieved by an available opinion aggregation method. The rewards in the two components are combined in such a way that maintains incentive compatibility (i.e., workers have the incentive to provide truthful opinions) while minimizing the total cost incurred on the task requester. Through theoretical analysis, we prove the effectiveness of iMET in eliciting truthful opinions from workers especially when workers hold different truthful opinions and the majority of the workers provide untruthful opinions. Furthermore, iMET is evaluated in four real datasets, and compared with two state-of-the-art incentive mechanisms, showing its significant advantages over them.

- **A robustness evaluation framework of incentive mechanisms [62, 61]:**

Based on evolutionary game theory, the robustness of incentive mechanisms against non-equilibrium strategy is defined and a simulation based evaluation framework is proposed to empirically quantify the robustness. Specifically, (1) both a general robustness measure of a desired equilibrium and a practical measurement against a specific non-equilibrium strategy are defined; (2) a simulation framework is proposed to quantitatively evaluate the measurement, which has been validated in representative games; (3) the proposed framework is implemented in reputation systems of e-marketplaces to evaluate the robustness of incentive mechanisms against various types of non-equilibrium strategies.

1.3 Contributions

The contributions of the proposed approaches are summarized as follows:

- The subjectivity problem is addressed for reputation systems by proposing a reputation system based on the coalitional game theory.
- The dishonesty problem is addressed in EMLI considering the new challenges by designing an incentive mechanism (iMEMLI), where both buyers and sellers have the incentive to behave honestly.
- Both the subjectivity and dishonesty problems together are addressed in crowdsourcing systems by proposing an incentive mechanism (iMET).
- The robustness of incentive mechanisms against bounded rational agents is formally defined and empirically evaluated by a simulation based evaluation framework.

To summarize, this thesis aims to addressing the two challenging problems (subjectivity and dishonesty problems) in two scenarios (reputation systems of e-marketplaces and

crowdsourcing systems) through designing incentive mechanisms. The robustness study of incentive mechanisms in this thesis could serve as an evaluation tool to assist mechanism designers to choose and develop more robust incentive mechanisms, while addressing the subjectivity and dishonesty problems in different agent reporting systems.

1.4 Thesis Outline

Figure 1.1: The organization of the thesis

The organization of this thesis is outlined as in Figure 1.1. The thesis consists of 7 chapters. The first two chapters introduce the background knowledge, research problems, and related studies. Chapter 3 presents CONGRESS for addressing the subjectivity problem in reputation systems. Chapter 4 presents iMEMLI for e-marketplaces with limited inventory to address the dishonesty problem. Chapter 5 presents iMET to address the two problems simultaneously. All the mechanisms introduced in these three chapters and the existing incentive mechanisms assume that the agents are rational. To evaluate the robustness of incentive mechanisms against agents with bounded rationality, Chapter 6 presents a robustness evaluation framework. Finally, Chapter 7 concludes this thesis and proposes future work. Below is a summary of each of the rest chapters in this thesis:

- Chapter 2, **Literature review**: This chapter firstly introduces the basic concepts of reputation systems, highlights the subjectivity and dishonesty problems in reputation systems, and reviews the existing studies in addressing the two problems. It then surveys existing incentive mechanisms for both reputation systems and crowdsourcing platforms, and points out their shortcomings. Finally, related studies about bounded rationality of agents in incentive mechanisms are reviewed.
- Chapter 3, **Coalition formation game based reputation system (CONGRESS)**: In this chapter, the reputation system CONGRESS for e-marketplaces is proposed to address the subjectivity problem based on coalitional game theory. Two concrete allocations have been proposed to divide the utility surplus of *clubs*, i.e., the proportional and Shapley allocations. Theoretical analysis and experimental results confirm that CONGRESS can effectively address the subjectivity problem when certain conditions are satisfied.
- Chapter 4, **Incentive mechanism designed for EMLI (iMEMLI)**: In this chapter, an incentive mechanism iMEMLI has been proposed to elicit truthfulness from both buyers and sellers in EMLI. Theoretical and experimental results have shown that the proposed incentive mechanism can promote both buyer and seller honesty and discourage them to re-enter the system even when the inventory of sellers is limited.
- Chapter 5, **Incentive mechanism to elicit truthful opinions for crowdsourcing (iMET)**: The incentive mechanism iMET is proposed to promote truthful opinions from workers when their truthful opinions are different and the majority workers are dishonest. It consists of two components: truthfulness reward and consistency reward, which are introduced separately and further combined with respect to minimizing the total cost of task requesters. Theoretical analysis and real-data

based evaluations are provided to confirm the effectiveness of iMET in promoting truthful opinions from workers when subjectivity and dishonesty problems coexist.

- Chapter 6, **Robustness evaluation framework of incentive mechanisms**: It provides a measure to evaluate and compare the robustness of incentive mechanisms against bounded rational agents. Formal definitions of the robustness are provided based on the evolutionary game theory, and a simulation based evaluation framework is proposed to quantitatively measure the robustness. The validation and implementation of the simulation framework are also presented in this chapter.
- Chapter 7, **Conclusions and future work**: This chapter concludes the thesis and indicates potential future research directions.

Chapter 2

Literature Review

In this chapter, related studies about reputation systems in e-marketplaces are reviewed. Specifically, the concepts of trust and reputation as used in this thesis are defined, some computational models of trust and reputation are introduced, and the problems in reputation systems are pointed out including the subjectivity problem and dishonesty problem. Secondly, this chapter reviews the existing solutions to the subjectivity problem in reputation systems, clearly explains how these approaches can address the subjectivity problem, and analyzes their shortcomings. Thirdly, the incentive mechanisms designed for reputation systems to address the dishonesty problem are reviewed, including the new challenges in addressing this problem in e-marketplaces with limited inventory. Moreover, the existing incentive mechanisms to promote truthful opinions in crowdsourcing are surveyed and two common limitations are analyzed. Finally, it reviews the studies of bounded rationality and robustness evaluation in the literature of incentive mechanisms.

2.1 Reputation Systems for E-marketplaces

In e-marketplaces, such as Amazon and eBay, buyers have limited information about sellers' reliability or their product quality prior to conducting transactions with the sellers. The analysis of Akerloff in 1970 on the Lemon Markets [4] could be applied to e-marketplaces. The main issue in e-marketplaces is the information asymmetry between

buyers and sellers. Sellers fully know about their own trading behaviors in delivering their products, while buyers are uncertain about those sellers' behaviors. The information asymmetry further impacts the performance of e-marketplaces [73]. As buyers have limited knowledge about the quality of the products, they may avoid to offer a higher price for products with higher quality (the cost of products in high quality is always higher than that of products in low quality), but rather choose products with a lower price to reduce the risk of losing a lot. Consequently, products with higher quality may gradually disappear from the e-marketplaces [105]. In order to address the information asymmetry problem in e-marketplaces, trust and reputation systems are widely used in e-marketplaces. For the sake of completeness, the definitions of trust and reputation as used in this thesis are provided. Following it, several existing computational models of reputation systems are reviewed, along with some of their challenging problems.

2.1.1 Trust and Reputation Concepts

The terminologies of trust and reputation are widely used in a lot of research work, however, there is no standard definition. The definitions of trust and reputation are adopted from [36], where trust is defined according to the *meaning of trust* in [66] and reputation is derived from the *concise oxford dictionary* [98].

“Trust *is the extent to which one party is willing to depend on something or somebody in a given situation with a feeling of relative security, even though negative consequences are possible.*”

“Reputation *is the beliefs or opinions that are generally held about someone or something.*”

Compared with reputation which is a widespread belief, trust is a relatively subjective concept. However, both of these two terms are vague since their respective definitions use “relative” or “generally”. In the following discussions, the term “trust” is used to

describe the reliability property of other agents from the perspective of a single agent, and the term “reputation” is used to describe the same property from the perspective of a set of agents by aggregating their personal trust.

2.1.2 Reputation Models

In reputation systems, reputation value of a seller can be modeled in various ways. Three main computational models of trust and reputation in the literature are reviewed here.

2.1.2.1 Simple Aggregation

The simplest way of computing reputation of a seller is to use the number of positive ratings that the seller receives deducted by the number of negative ratings. One of the well known examples is the reputation system of eBay. The major advantage of this aggregation approach is that the computed reputation value is easily understandable for users. However, by getting more positive ratings in the future, sellers can easily cover their negative ratings received in the past.

A more advanced model is to compute a seller’s reputation value by calculating the weighted average of all ratings that the seller has received. The weight of a rating can be determined by many factors, such as the rater’s trustworthiness [121], the time when the rating was provided, etc.

2.1.2.2 Dempster-Shafer Theory

The reputation system proposed by Yu and Singh [115] constructs the trust relationship between buyers and sellers based on the Dempster-Shafer theory [90]. The distribution of buyers’ ratings is interpreted as a *basic probability assignment* over a *frame of discernment*. In addition, the Dempster-Shafer theory provides a rule to propagate and combine the ratings provided by different buyers.

2.1.2.3 Bayesian Theory

Ratings provided by buyers are also called as observations or evidences of sellers' behavior, and the ratings are determined by the outcomes of transactions. According to the Bayesian theory, the reputation of sellers can be modeled as a probability distribution over the set of possible outcomes of their transactions. If the outcomes of transactions are binary (e.g. successful or unsuccessful) [35], the Beta distribution is suitable to represent reputation of sellers. If the outcomes are multinomial [114, 34, 56, 57], the Dirichlet distribution can be used for capturing reputation of sellers.

2.1.3 Problems in Reputation Systems

Although several advanced and sound theories have been proposed for reputation modeling as summarized above, various problems still exist and may hinder the effectiveness and robustness of reputation systems. Many of these problems are due to the fact that seller reputation is modeled based on the ratings provided by buyers.

- **Subjectivity problem:** Buyers have different evaluation criteria or preferences in providing ratings. This can lead to the subjectivity problem in reputation systems [36, 118, 16]. The rating provided by a buyer may mislead other buyers. For example, the same products are received by two buyers in five shipping days. One buyer may provide a positive rating for this transaction, as five-day shipping is still acceptable for the first buyer. Nonetheless, the other buyer may provide a negative rating for the same transaction, because five-day shipping is too long to be accepted by her. In this case, the rating provided by the first buyer is misleading for the second buyer, and vice versa. Hence, the subjectivity problem renders reputation systems to work ineffectively.
- **Dishonesty problem:** There are mainly two types of unfair ratings, unfairly low ratings (or bad mouthing) and unfairly high ratings (or ballot stuffing) [15]. An

unfairly low rating is a negative rating provided by a buyer about a seller with whom the buyer actually has experienced a satisfactory transaction. Whereas, an unfairly high rating is a positive rating provided by a buyer about a seller with whom the buyer has experienced a dissatisfactory transaction. Unfair ratings would lead to reputation bias. Based on such a biased reputation system, buyers may not make proper purchasing decisions. Even worse, the biased reputation could discourage sellers from behaving honestly because the reputation system cannot accurately present their trustworthiness.

- **Lack of incentive in providing ratings:** Even though buyers can benefit from a reputation system which is built upon the ratings provided by them, buyers may still be reluctant to provide ratings. This is mainly because in most cases a buyer will not interact with a seller again after one transaction, therefore, the buyer cannot benefit directly from providing ratings towards the seller [36]. In consequence, few ratings may have been provided about sellers. The reputation of sellers cannot be accurately modeled based only on a small number of ratings.
- **Re-entry problem:** Since a dishonest seller will gain little utility because of being assigned with a low reputation score, the dishonest seller may leave and subsequently re-enter e-marketplaces under a different identity to whitewash his bad historical behaviors [33]. It is difficult for most reputation systems to distinguish the re-entry sellers from new registered sellers. The re-entry problem prevents reputation systems from effectively revealing dishonest sellers.
- **Other problems:** There are many other problems existing in reputation systems [33]. For example, the *abuse of good reputation* problem is caused by sellers who act honestly over a period to gain a high reputation score and then behave dishonestly to profit from the previously gained reputation. The *collusion* problem

is that a set of identities coordinate together to make significant influence on the reputation scores of a specific set of sellers. The *sybil attack* [32, 72] is that a single entity creates multiple pseudonym identities and provides multiple ratings to sellers. The *value imbalance exploitation* is that a seller provides a large number of high-quality and low-value products to gain a high reputation scores, but provides a small number of low-quality and high-value products to gain significant profit. The *discriminatory behavior* is that sellers provide good products to everyone except a few specific set of buyers who they “do not like” or provide exceptionally good products to a few selected buyers and average products to the rest [15].

In this thesis, we concentrate on addressing the subjectivity and dishonesty problems by proposing incentive mechanisms. With these mechanisms the problem of lacking of incentive in providing ratings does not exist any more. Besides, the re-entry problem will also be addressed in the incentive mechanism proposed for the e-marketplaces with limited inventory in Chapter 4. In fact, these problems also commonly exist in other agent reporting systems, such as the crowdsourcing platforms. We will address these problems by designing an incentive mechanism for crowdsourcing in Chapter 5.

2.2 Existing Solutions to the Subjectivity Problem in Reputation Systems

In the literature of reputation systems, three types of solution have been proposed to address the subjectivity problem in reputation systems for e-marketplaces. In the first type of solution [89], buyers objectively express their experiences using ontology. The ontology is designed by experts and consists of the fundamental concepts and properties of a specific domain. By using the ontology, buyer’s experience is represented as objective evidences. Then, a buyer receiving these objective evidences from other buyers will interpret personally according to their own preference or evaluation criteria. Thus, this

approach can avoid the bad effects of the subjectivity problem. The shortcoming of this approach is that it requires experts' effort to build an ontology and buyers' effort to clearly express their experience using the ontology. Since the ontology is domain-dependent, it is complex and costly to design an ontology for every domain. Moreover, buyers have different ability in learning the ontology. Given the ontology, it is difficult to ensure that all buyers can understand the ontology and even express their experience clearly by using it.

In the second type of solution, the problem is addressed by filtering out buyers with different subjectivity. In [121], buyers select like-minded buyers to be their advisors, and request advice from them to build the reputation of sellers. As like-minded buyers have similar subjectivity, the reputation system would not suffer from the subjectivity problem. In [107], communities are formed by the *super agents* who are buyers having high capability in exploring sellers. The super agents select similar buyers as their members and collect ratings from them to construct the reputation of sellers. Buyers evaluate the similarity with the super agents and aggregate the reputation provided by the super agents while taking into account the similarity. Thus, the adverse effect of the subjectivity problem could be effectively mitigated. However, this type of solution has two common shortcomings. The first shortcoming is that the modeling complexity exponentially grows as the number of agents increases, since each agent has to model every other agent. Another shortcoming is the low efficiency in computing the seller reputation. The agents with the same subjectivity would have to individually calculate the same seller reputation by wasting computational resources. These two problems will become significant when the e-marketplaces develop into large-scale systems.

The third type of solution is to re-interpret or convert the evaluations of advisors (i.e., other buyers). In BLADE [85], a buyer models sellers' properties and advisors' evaluation functions using a Bayesian learning approach. Then, the ratings received by the

buyer from an advisor could be re-interpreted so as to cater for the buyer's subjectivity. In SARC [21], the correlation between buyers' ratings and sellers' properties could be learned based on the ratings and detailed reviews provided by the buyers about sellers, and the weight of each property could be obtained through the regression analysis. In these two approaches, the ratings provided by an advisor are not directly aggregated by the receivers (buyers), but re-interpreted considering the subjectivity of the receivers, so the subjectivity problem is mitigated. However, the buyers are required to share their evaluation functions or additional detailed reviews, as well as to learn the subjectivity difference with other buyers. In Chapter 3, to address the above shortcomings, a reputation system is proposed based on the coalitional game theory [74] which incentivizes buyers to automatically form groups with buyers having the same subjectivity type.

2.3 Incentive Mechanisms in Reputation Systems

From the perspective of buyers, there are two types of incentive analyzed in the literature: one is for honest payment in post-payment e-marketplaces and the other is for honestly providing ratings in reputation systems.

In the first type of incentive for promoting buyer honesty, products are delivered first and buyers pay sellers after they receive the products (i.e., post-payment e-marketplaces). Since buyers may be dishonest by not paying money to sellers, the incentive for buyers in honest payment is necessary. In [9], Braynov proposes a trust revelation mechanism where buyers have the incentive to honestly reveal their trustworthiness of payment. The sellers can maximize their utility through manipulating the quality of the delivered products according to buyer trustworthiness that is revealed by buyers themselves. At the same time, the optimal quality that sellers determine to deliver can also maximize buyer utility if the buyers reveal their trustworthiness truthfully. Therefore, the mechanism can promote buyer honesty. The mechanism can work well in the case that sellers have no

preference over buyer honesty. However, it fails to incentivize buyers to truthfully reveal their trustworthiness when sellers prefer to conduct transactions with honest buyers. In this case, buyers may lose the chance to conduct transactions with sellers by honestly telling sellers that they are dishonest, which leads buyers to lose utility.

There are various works studying the second type of incentive. Several incentive mechanisms for eliciting truthful ratings from buyers have been proposed in the literature as summarized in Table 2.1. More specifically, Jurca et al. in [39, 43, 45, 46] propose

Table 2.1: The existing incentive mechanisms in promoting truthful ratings

Name of Incentive Mechanism	Shortcomings
Side-payment [39, 43, 45, 46]	Requires external reward; Neglects seller's strategic behavior; Not applicable for EMLI.
Trust-based [118, 123]	Ineffective for new buyers; Not applicable for EMLI.
Community-responsibility [5]	Requires the TTP; Not applicable for EMLI.
CONFESS [38, 40]; Credibility mechanism [80]	Wrong punishment; Not applicable for EMLI.
Wage-based [124]	Known cost function in P2P file sharing system.

a side-payment incentive mechanism where truthfully providing ratings is buyers' Nash equilibrium strategy. It is based on peer prediction method [68] by comparing the ratings provided by different buyers regarding the same sellers. In the side-payment mechanism, buyers can gain a certain amount of side payment if their ratings coincide with the refereed raters. The refereed rater could be a preceding buyer [45]. Since buyers would gain maximum expected side payment through providing truthful ratings, they have incentive to provide truthful ratings. In addition, Jurca et al. improve the incentive mechanism in [41] to compare buyer ratings with multiple refereed ratings so that the strategy of providing truthful ratings is the unique Nash equilibrium. Furthermore, Jurca et al. study the minimum side payment for eliciting truthful ratings in [42]. They also continue

to improve the side-payment mechanism with the aim of solving the collusion problem by proposing a collusion-resistant incentive compatible reward scheme in [44], where three pure lying strategies are considered, i.e., all positive ratings, all negative ratings and opposite ratings. The reward scheme could maximize buyer utility through solving a linear optimization problem, subject to certain constraints, such as the constraint that each buyer can gain a non-negative utility. Therefore, there exists a tradeoff in the side-payment mechanism between promoting buyer honesty and the budget to be paid. In the side-payment incentive mechanism, sellers are assumed to behave consistently and will not strategically conduct malicious behavior, such as discriminatory behavior.

Another approach which creates incentive for buyers to provide honest ratings is proposed in [118, 123], namely, a trust-based incentive mechanism. In the trust-based incentive mechanism, a community is organized by a buyer who selects the other trusted buyers to be her advisors. A honest buyer is more likely to be selected as advisors by many other buyers. Meanwhile, honest sellers prefer to behave honestly in the transactions with honest buyers, because honest buyers are helpful to reveal and propagate the trustworthiness of these honest sellers. Therefore, honest sellers may have incentive to attract honest buyers by offering lower prices for their products. As a result, buyers would have incentive to provide truthful ratings to gain more utility from transactions with those sellers. The advantage of this mechanism is that it does not require extra side payment to elicit truthful ratings from buyers. However, in trust-based incentive mechanism, sellers may dishonestly conduct transactions with new buyers who have not been widely accepted as advisors, which leads to the discriminatory problem.

Moreover, there are several incentive mechanisms considering the incentive of buyers in honestly providing ratings and the honesty of sellers. In the community-responsibility mechanism [5], buyers and sellers join online communities, respectively. Provided a transaction between a buyer and a seller, the buyer and the seller would evaluate the opponent

community's reputation. In their transaction, the buyer pays for the product and the seller delivers it. After the transaction, both of them are required to report the outcomes of their transaction. If the reports about the same transaction are different, then the trusted third parties (TTP) would determine which agent is honest and punishes the other agent. The punishment is imposed on the community of the punished agent instead of the individual agent. The punished community then can pass the punishment to the specific malicious agent or exert the agent out of the community. In order to achieve financial balance, the community will impose some amount of admission fees on buyers who join the community. Game theoretical analysis proves that the community-responsibility mechanism can promote honesty from both buyers and sellers. The disadvantage of this mechanism is that it requires the TTP to support its implementation, which may not be applicable in large-scale systems.

The "CONFESS" incentive mechanism [38, 40] is another approach to promote honesty from buyers and sellers simultaneously. According to the mechanism, both buyers and sellers are required to report the outcomes of their transactions. If the reports about the same transaction are different, then both the buyer and seller would be punished by the system. The advantage of the mechanism is that it does not rely on TTP. However, some agents will be wrongly punished. Whenever there are different reports about the same transaction, at least one of the two agents will be wrongly punished. Similar with the above-mentioned "CONFESS" incentive mechanism, in the credibility mechanism [80], both buyers and sellers are required to submit ratings about the outcomes of their transactions. The difference is that the punishment in [80] is imposed on the agent who has a lower credibility value (reputation of the agent). This incentive mechanism can effectively decrease the chance of wrong punishment. However, it raises another problem where the agent with high credibility may abuse her credibility by behaving dishonestly to gain profit.

In the existing incentive mechanisms reviewed above, there is a common and implicit assumption that sellers have a large number of products to sell compared to the demand of buyers, and sellers can gain more profit from sustaining a good reputation given truthful ratings provided by buyers. However, in e-marketplaces with limited inventory (EMLI) where the supply is in short, the sellers may not concern much about the amount of sales, thus the reputation itself may not be sufficient to motivate the sellers to be honest [84, 118]. In addition, as discussed in Section 1.1.1, buyers will have to bear additional cost for providing truthful ratings in an EMLI because of competition from other buyers on the limited inventory. Considering the competition cost and the property of an EMLI, the side-payment, credibility and trust based incentive mechanisms may fail to work. For the side-payment mechanism, the side-payment gained from providing truthful ratings is likely to be offset by the additional cost; for the credibility mechanism, the punishment over sellers by not allowing them to participate in the marketplaces for some periods will decrease the already limited products served in the marketplace, which is not applicable in an EMLI; for the trust-based incentive mechanism, honest sellers may lack the incentive to offer discount to honest buyers (advisors) as their limited products are likely to be sold out without the help of the honest advisors in propagating the seller's good reputation among their community.

The wage-based incentive mechanism [124] for P2P file sharing reputation systems, where agents report ratings for other agents about file downloading/sharing transactions, explicitly considers the effort or cost exerted by agents, i.e., time and hardware resources consumed in reporting ratings, as part of agents' payoffs. The authors model the process of sharing ratings as a reporting game between a reporter and a querist where the payoff of the reporter contains the cost function in reporting ratings. The optimal wage contract (payment protocol) is designed through solving an optimization problem so as to minimize the total payment subjective to some constraints, which is similar to [44]. However,

the cost caused by the competition between agents is still not included in the model. Considering the competition cost in an EMLI, the result of the optimization problem may not be directly applicable since the solution of the problem strictly relies on the specific form of the cost function. Moreover, the competition cost in sharing ratings is difficult to be explicitly and quantitatively modeled. Thus, a new incentive mechanism needs to be designed specifically for an EMLI.

For an EMLI, the way that a seller could increase its profit is to sell every single product at a higher price. In the proposed mechanism in Chapter 4, honest sellers are offered higher prices for their products than that for dishonest sellers. This idea of the price premium is well supported by economics studies. Empirical evidence reveals that prices of products sold by honest sellers are generally higher [65, 96]. Buyers' purchase intention of buying products from high reputation sellers would not be affected by the price premium, as the risk of the buyers will be decreased by conducting transactions with those reputable sellers [12, 28]. A dynamic pricing agent [84] is designed to increase product prices when sellers have limited inventory, following the rule of supply and demand in the economic theory. Their purpose is to maximize the utility of sellers. In contrast, the proposed mechanism increases product prices to create incentive for sellers to be honest.

One of the limited studies about EMLI in the literature [75, 76] considers the impact of the limited inventory provided by sellers in the design of the optimal reporting strategy for buyers. Because the amount of products provided by sellers is limited, buyers are in competition to purchase the products and their optimal reporting strategy is to only provide truthful ratings to other honest buyers (advisors) and untruthful ratings to dishonest buyers. The optimal strategy can effectively balance between the competition among buyers and the discovery of good sellers. It also indicates that truthful ratings in an EMLI are challenging to be elicited from all the buyers. The proposed incentive mechanism in Chapter 4 aims to address these challenges.

2.4 Incentive Mechanisms in Crowdsourcing

There has been growing interest in designing incentive mechanisms in crowdsourcing with respect to offering payment to workers so as to maximize the utility of task requesters. One direction is to use auction based mechanisms, such as [27, 94, 95], where workers bid their cost in performing tasks. The “truthful incentive” in their analysis is referred to as the property of the mechanism that the workers have the incentive to bid their true private cost in completing the task. The mechanisms work based on the assumption that the workers will honestly fulfill their obligations once they are allocated with the tasks. Another direction is to encourage workers to behave truthfully when performing the tasks, which is the focus of the work presented in Chapter 5. In this thesis, we focus on one of the most common applications of crowdsourcing which is to find out the hidden states of the world by collecting opinions from a large number of workers. This type of tasks is referred to as consensus tasks, such as the Galaxy Zoo project (<http://zool.galaxyzoo.org>).

Most of the existing incentive mechanisms in promoting truthful ratings in reputation systems as reviewed above can be adopted by crowdsourcing systems to promote truthful opinions from workers. Moreover, the property of fairness in rewarding workers has also attracted the interests of many researchers. To improve the fairness of the peer prediction method, a consensus prediction approach has been proposed [47] [49]. Under this approach, a worker is rewarded according to the importance of her opinion (i.e., how much the final result will change if her opinion were to be removed). The consensus prediction approach can arguably achieve better fairness in the sense that a competent worker will not receive a low reward as a result of being compared with a less competent worker’s opinion under this approach. But how to define a worker’s competence remains an unresolved difficult problem, and when the majority workers behave untruthfully, the workers may not keep their incentive to be truthful.

In addition, these mechanisms assume that the workers hold the same truthful opinion (i.e., subjectivity) which is not valid in crowdsourcing especially when subjective opinions are sought. In an attempt to accommodate the situation where the workers have diverse truthful opinions, the shadow peer prediction approach is proposed [111] to elicit truthful ratings of products. Under this approach, a worker is asked to report her prior belief and a binary rating before and after she receives a product, respectively. Then the shadow post belief is calculated based on the reported prior belief and rating. The worker receives a reward according to strictly proper scoring rules based on her reported prior belief, calculated shadow post belief, and the product rating from another worker. This mechanism is only applicable in scenarios with binary ratings. Furthermore, it is difficult for a worker to report her prior belief before she actually looks into the consensus task. Another issue with most of the above mentioned mechanisms is that they cannot promote truthful opinions when majority workers provide untruthful opinions as the strategy of providing untruthful opinions is more beneficial than that of providing truthful opinions in this scenario. To promote truthful ratings from agents when majority agents provide untruthful ratings, in the side payment mechanism [37], the rating of an agent is probabilistically compared with the ratings provided by previously identified trustworthy agents.

In general, existing incentive mechanisms are effective only in environments where all the workers share the same truthful opinion and the majority workers behave truthfully. However, in practical conditions, especially in the consensus tasks involving workers from diverse background, the conditions as mentioned above are not always satisfied. Chapter 5 aims to fill in the gaps to elicit truthful opinions from workers even when the workers' truthful opinions are different and the majority workers behave untruthfully.

2.5 Robustness Study of Incentive Mechanisms

In the literature of incentive mechanism design, increasing attention has been paid to designing practical incentive mechanisms which could achieve the desired outcome while relaxing the assumption that every player is rational (e.g., [1][23][7]). These studies aim to design mechanisms for players with bounded rationality. However, the reasons for a player being bounded rational could include unknown external utility and emotional factors [103, 24], and it is nearly impossible to motivate them to behave “rationally”. In Chapter 6, from a practical perspective, our study focuses on evaluating the behavior of rational players when bounded rational players choose non-equilibrium strategies.

The robustness problem is also referred to as the implementation problem when some of the players are “faulty” in the sense that they fail to act optimally [18]. The k -FTNE is defined to serve as a stronger equilibrium concept than Nash equilibrium, where each non-faulty player would sustain the designed strategy as long as other non-faulty players sustain the designed strategy, regardless of the strategy of up to k faulty players. Another relevant concept in game theory is k - t robust Nash equilibrium (NE) [23]. An equilibrium is t -immune if it can be tolerant up to t irrational agents. An equilibrium is k -resilient if all the rational agents in a coalition of size up to k ($k \geq 1$) will not deviate from the equilibrium strategy. The limitation of the k - t robustness study is that it only provides an abstract characterization of a robust equilibrium, but does not provide an operational approach to quantitatively evaluate mechanism robustness.

To improve the robustness and protect incentive mechanisms from the influence of bounded rational players, a crowd filtering approach has been proposed. In [3], a fault tolerance model named BAR is used to filter out players who randomly deviate from the desired equilibrium strategies. However, the model fails to filter out players with complex strategies. Since it is difficult to insulate the impact of every non-equilibrium strategy, it is necessary to study the robustness of mechanisms.

Qualitative analysis of the robustness of incentive mechanisms has been proposed for different non-equilibrium strategies. Witkowski *et al.* [112] analyze the conditions when a mechanism is robust against various types of collusive attacks. In contrast, the study in Chapter 6 aims to provide a quantitative measurement of robustness, which is especially useful for comparing incentive mechanisms used in practice. Moreover, the existing incentive mechanisms are designed based on different methodologies, but there is no measurement to justify which one is more robust when they are implemented in real agent reporting systems. Therefore, Chapter 6 proposes a robustness measure and a simulation based evaluation framework. The proposed simulation framework is implemented to evaluate and compare the robustness of several incentive mechanisms as well as the proposed mechanism in Chapter 4 against various non-equilibrium strategies.

2.6 Summary

To summarize, this chapter extensively reviews related studies in the literature of agent reporting. Specifically, we point out the two challenging problems in reputation systems and crowdsourcing platforms where agents report their opinions, i.e., the subjectivity and dishonesty problems. Different approaches have been proposed to address the subjectivity problem, but they often require additional efforts from agents. Various incentive mechanisms have been proposed to provide incentives for agents to truthfully report their opinions (that is the dishonesty problem). However, most of them do not consider the subjectivity issue in agent reporting. All these issues will be addressed by our proposed incentive mechanisms that will be presented in subsequent chapters. Furthermore, most of the incentive mechanisms (including our own) assume that agents behave rationally, which overlook the robustness issue. In this thesis, we provide the first attempt to quantitatively evaluate the robustness of incentive mechanisms, with the intention of drawing some insights that might be helpful towards the design of a robust incentive mechanism.

Chapter 3

CONGRESS: A Coalition Formation Game based Reputation System

Game theory is a theoretic tool to analyze the behavior of self-interested players, which has been widely used in e-marketplaces [104]. A game can be either cooperative or non-cooperative [74]. The coalition formation game (CFG) is one type of cooperative games which can be further categorized into two main classes: transferable CFG where the coalition value can be dividable and allocated among members in a certain way, and nontransferable CFG where the coalition value is indivisible. The coalition formation game as employed in this work belongs to the transferable utility CFG. The coalition formation games have been popular in various applications of computer science, such as grid computing [22], sensor networks [86] and task allocation [93], etc., where a set of agents cooperate aiming to achieve high utility or system performance. In e-commerce, the coalition formation game has been adopted to the design the group-buying schemes [67], where buyers from the same geographical location purchase items together by forming a coalition to save shipping cost and also enjoy the quantity discount offered by sellers. In this chapter, a COalition formatioN based REputation SyStem CONGRESS is proposed to address the subjectivity problem in reputation systems. In Section 3.1, a hybrid architecture of reputation systems is developed and CONGRESS belonging to this architecture is then proposed by mapping the components of the system with a coalition formation

game. In Section 3.2, the utility surplus of clubs in different types are formalized, and an allocation algorithm is proposed for a club to allocate its utility surplus among buyers in the club. The club structure is analyzed in Section 3.3, and buyer strategies are discussed in Section 3.4. Experimental results are presented in Section 3.5. Section 3.6 finally concludes this section.

3.1 The CONGRESS Model

In this section, the architecture of the proposed CONGRESS is firstly abstracted. Its model is then proposed by mapping its components with those of a coalition formation game.

3.1.1 The Architecture of CONGRESS

To give a general overview of the proposed model, the architecture of the proposed model is shown on the right hand side of Figure 3.1, which is developed from the two main types of reputation systems in the literature [36] presented on the left hand side of the same figure. Figure 3.1 presents how ratings and reputation scores are communicated between agents in reputation systems. In centralized reputation systems, the information about the performance of a given agent is collected as a rating from other agents who have direct experience with that agent. The central authority (reputation center) aggregates all the ratings to derive a reputation score of every agent and makes all the scores public in the reputation system. The reputation scores are continuously updated as a function of the received ratings. The agents in the system can consider the scores of other agents to decide whether or not to conduct a transaction with a particular agent. In Figure 3.1, agents A and b are potentially conducting transactions, and they consider the scores of each other provided by the reputation center which are derived based on the ratings provided by the agents about A or b in their past transactions. In distributed reputation

Figure 3.1: The architecture of CONGRESS

systems, there is no centralized authority to collect ratings. Instead, the ratings can be recorded by each agent. An agent in a potential transaction is required to find the agents having direct experience with the target agent and request their ratings to compute the reputation score of the target agent. Based on the two main architectures of reputation systems, we propose a hybrid architecture to which the proposed CONGRESS belongs. A subset of agents in the systems can autonomously form a *club*, and the club is managed by a distributed authority (club center) which has the same role of the reputation center in the standard centralized reputation systems.¹ When an agent decides whether to conduct

¹The number of club centers is equal to the number of clubs formed by the agents.

a transaction with another agent, the agent considers the target agent's reputation score which is based on the ratings provided by all the agents of the club and only publicly known to the agents in the club. The hybrid reputation systems have the potential to address the subjectivity problem if the agents with the same subjectivity form a club.

3.1.2 The Model of CONGRESS

This section focuses on building seller reputation for buyers in e-marketplaces and propose CONGRESS for this specific scenario in the hybrid architecture. A buyer requests an item, i.e., a product or a service, from a seller, and the seller provides the item according to his capability and honesty². After receiving the item, the buyer evaluates the performance of the corresponding seller based on her subjective criteria (preference), and provides a binary rating³ ('*negative*' or '*positive*'). The buyer's subjectivity is represented by a vector specifying the threshold value of the buyer regarding each attribute of the item. Specifically, if the difference between the received attribute value and the promised attribute value of every attribute is no larger than the threshold specified in her subjectivity vector for that attribute, the buyer provides a '*positive*' rating towards the seller; otherwise, a '*negative*' rating is provided. A subset of buyers could form a club which is managed by a *club center*. The club center then collects and aggregates the ratings provided by the buyers in its club to build seller reputation. The reputation information can only be accessed by the buyers in the club. It is noted that the reputation of a seller could be different in different clubs. In addition, the club center calculates the utility surplus created by the reputation system for buyers in their transactions, and charges them administration fees in the same amount. In order for the system to be trusted by

²It would be interesting to study the strategically malicious behavior of sellers in trading with buyers having different subjectivity. In order to concentrate on the proposed CONGRESS to address the subjectivity problem, this study simplifies the complex scenario by assuming that the seller honesty is an objective property to be evaluated.

³The proposed model can also be applied when the ratings have n degrees, where the subjectivity of buyers will be represented as an $n - 1$ dimensional matrix.

the participants (buyers and sellers), it is important to make sure that the budget of the system is balanced, meaning that the system itself will not make profit out of it. So, it is necessary to give the admission fees back to the buyers. The collected administration fees are then divided by the club center among the buyers of the club according to an allocation algorithm

The proposed CONGRESS is then modeled by mapping its components (buyers and clubs) with those in a coalition formation game [74] which is defined as follows.

Definition 1. [*Coalition Formation Game (CFG)*] *A coalition formation game is represented by a tuple (N, h) , where N denotes a set of players and h denotes a characteristic function. A subset of N forms a coalition, denoted by M , and the set of all the coalitions is denoted by 2^N . The characteristic function $h: 2^N \mapsto \mathbb{R}$ specifies the value of every coalition, and $h(M)$ is the coalition value of coalition M .*

Table 3.1: The mappings from CONGRESS to CFG

Components in CONGRESS	Concepts in CFG
B : A set of buyers	N : A set of players
C : A club	M : A coalition
$v(C)$: Utility surplus of C	$h(M)$: Coalition value of M
v : Utility surplus function	h : Characteristic function

The mappings from the components of CONGRESS to those of CFG are shown in Table 3.1. The buyer set B in the proposed CONGRESS model corresponds with the player set N in CFG. A subset of buyers forms a club which is denoted by C , corresponding with a coalition M in CFG. The reputation system built based on the ratings provided by the buyers of a club can assist the buyers of the club to conduct satisfactory transactions, which results in utility surplus for the club. The utility surplus of the club C denoted by $v(C)$ is the total utility surplus created by all the buyers of the club C during a period, which corresponds with the coalition value $h(M)$ in CFG. In

addition, the utility surplus function v , which specifies the utility surplus of every club, corresponds with the characteristic function h in CFG.

To elaborate the CONGRESS model, all its components are described in details in the following sections. In Section 3.1.2.1 and Section 3.1.2.2, the models of sellers and buyers are provided, respectively, where the capability and dishonest behaviors of sellers and subjectivity of buyers are formalized. In Section 3.1.2.3, the process of forming clubs is introduced.

3.1.2.1 Model of sellers

When a seller s_i publishes an item, the seller is required to report his promised attribute values regarding θ independent attributes (such as price, delivery time, etc.) to the market mediator. The set of attributes is denoted by vector $\mathbf{x} = [x_1, \dots, x_\theta]$. For each attribute x_i in $\mathbf{x}$, its value is normalized between 0 and 1: 0 represents the worst and 1 represents the best⁴. The promised quality of a seller s_i is then denoted by vector $\mathbf{p}_i = [p_i^1, p_i^2, \dots, p_i^\theta]$ where $p_i^k \in [0, 1]$ for $k = 1, 2, \dots, \theta$ is the value of attribute x_k . The promised quality vector $\mathbf{p}_i$ reflects the capability of seller s_i in supplying the item and the needs of buyers. Given two sellers s_1 and s_2 whose promised quality vectors are $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively, if $p_1^k \geq p_2^k$ for all x_k and $p_1^k > p_2^k$ for an attribute x_k , then seller s_1 *dominates* seller s_2 , denoted by $\mathbf{p}_1 \succ \mathbf{p}_2$. If $p_1^k = p_2^k$ for all x_k , then sellers s_1 and s_2 are equivalent, denoted by $\mathbf{p}_1 = \mathbf{p}_2$. The sum of the promised attribute values of seller s_i is the seller's *promised grand quality*, denoted by $P_i = \sum_{k=1}^{\theta} p_i^k$.

Proposition 1. *Given two sellers with the same grand quality, none of them is dominated by the other seller.*

Proof. Assume that the quality vectors of two sellers are $\mathbf{p}_1 = [p_1^1, p_1^2, \dots, p_1^\theta]$ and $\mathbf{p}_2 = [p_2^1, p_2^2, \dots, p_2^\theta]$, and seller s_1 dominates seller s_2 , i.e., $\mathbf{p}_1 \succ \mathbf{p}_2$. Thus $p_1^k \geq p_2^k$ for all x_k in

⁴For the scenario where the smaller attribute value is the better, such as price, the smallest attribute value is converted to 1.

$\mathbf{x}$ and there exists at least one attribute x_k such that $p_1^k > p_2^k$. It can be derived that the grand quality of s_1 is strictly larger than that of s_2 , i.e., $P_1 > P_2$, which contradicts with the assumption that the sellers have the same promised grand quality. $\square$

Since buyers will prefer to conduct transactions with dominant sellers, resulting in that the dominated sellers gradually disappear from e-marketplaces, in this model, we assume that all the sellers have the same grand quality, i.e., $P_i = P$ for all seller s_i where P is a constant.

In a transaction, seller s^i actually supplies an item with quality vector $\mathbf{q}_i = [q_i^1, q_i^2, \dots, q_i^\theta]$ where $\mathbf{q}_i \prec \mathbf{p}_i$ or $\mathbf{q}_i = \mathbf{p}_i$.⁵ The seller is called as an *honest seller*, if the delivered quality conforms with the promised quality, i.e., $\mathbf{q}_i = \mathbf{p}_i$, otherwise the seller is regarded as a *dishonest seller*. The *honesty* of a seller is taken as the probability in delivering items with quality $\mathbf{q}_i = \mathbf{p}_i$. The *reputation* of a seller is taken as the probability in conducting satisfactory transactions with buyers. It is worth noting that the reputation concept here is subjective, since the meaning of satisfactory transactions could be different for the buyers with different subjectivity types which is to be discussed in the next section. The sum of the supplied attribute values is *supplied grand quality*, denoted by $Q_i = \sum_{k=1}^{\theta} q_i^k$. We also assume that the supplied grand quality of sellers are the same, i.e., $Q_i = Q$ for each seller s_i where $Q < P$.

3.1.2.2 Model of buyers

When a buyer b_j receives an item from seller s_i whose supplied quality vector is $\mathbf{q}_i$, the buyer calculates the quality difference vector $\mathbf{d}_i = [d_i^1, d_i^2, \dots, d_i^\theta]$, where $d_i^k = q_i^k - p_i^k$ is a value in $[-1, 0]$. Based on vector $\mathbf{d}_i$, buyer b_j provides a binary rating for seller s_i , i.e., *positive* if the transaction is satisfactory and *negative* otherwise. The subjectivity of buyer b^j is represented by a vector $\mathbf{r}_j = [r_j^1, r_j^2, \dots, r_j^\theta]$ where $r_j^k \in [-1, 0]$ serves as the

⁵The current study does not consider the over-promised part of a quality value.

evaluation criteria of the buyer in evaluating attribute x_i . Buyer b_j can only feel satisfied if $d_i^k \leq r_j^k$ for every attribute x_k in $\mathbf{x}$, i.e., $\mathbf{d}_i \prec \mathbf{r}_j$ or $\mathbf{d}_i = \mathbf{r}_j$. For example, if $r_j^k = 0$, the buyer b_j feels satisfied regarding attribute x_k only when the received attribute value q_i^k is equal to the promised value q_i^k ; if $r_j^1 = -1$, buyer b_j always feels satisfied regarding attribute x_k even when she receives attribute value 0 which should be 1 as promised. Thus, a large value of r_j^k (r_j^k approaches to 0) indicates a high requirement that buyer b_j holds with respect to attribute x_k . Given two buyers b_1 and b_2 with their subjectivity being $\mathbf{r}_1$ and $\mathbf{r}_2$, if $\mathbf{r}_1 \succ \mathbf{r}_2$, then buyer b_1 dominates b_2 . The sum of a subjectivity is *grand subjectivity*, i.e. $R_j = \sum_{k=1}^{\theta} r_j^k$. To avoid the discrimination of sellers over the buyers with a dominated subjectivity type by intentionally delivering low quality items, it is assumed that there does not exist a dominated subjectivity type by assuming that all the buyers have the same grand subjectivity R where $R = Q - P < 0$ (or $Q = P + R$) to ensure that the buyers are able to experience satisfactory transactions and have a reasonable tolerance regarding the sellers, where $R > Q - P$ means that the buyers cannot be satisfied by sellers and $R < Q - P$ indicates the buyers are too tolerant.

Proposition 2. *All the buyers will experience satisfactory transactions with an honest seller, regardless of their subjectivity.*

Proof. If a seller s_i is honest, then the supplied quality vector is equal to what is promised in his transactions, i.e., $\mathbf{q}_i - \mathbf{p}_i$. Thus the quality difference vector is an all-zero vector, i.e., $d^i = [0, 0, \dots, 0]$. In this case, $\mathbf{d}_i \succ \mathbf{r}_j$ or $\mathbf{d}_i = \mathbf{r}_j$ for all buyer $b_j \in B$. Therefore, all the buyers will experience satisfactory transactions with an honest seller. $\square$

Based on Proposition 2, honest sellers cannot receive different ratings since all the buyers should provide the same “positive” ratings regardless of their evaluation criteria or subjectivity. Thus, the subjectivity problem does not exist when the sellers are honest. In this model, the sellers in consideration are dishonest by supplying items with supplied grand quality Q where Q is less than the promised grand quality P .

Proposition 3. *Given that $Q = P + R$, the identical items provided by a seller cannot guarantee that buyers with different subjectivity types experience satisfactory transactions.*

Proof. Suppose two identical items are received by two buyers whose subjectivity types are $\mathbf{r}_1$ and $\mathbf{r}_2$, where $\mathbf{r}_1 \neq \mathbf{r}_2$ and $\sum_{k=1}^{\theta} r_1^k = R$ and $\sum_{k=1}^{\theta} r_2^k = R$. Assuming that a seller s_i supplies the identical items in quality $\mathbf{q}_i$ where $\sum_{k=1}^{\theta} q_i^k = Q$, both of the two buyers feel satisfied. Then, seller s_i 's quality difference vector $\mathbf{d}_i$ (i.e., $\mathbf{d}_i = \mathbf{q}_i - \mathbf{p}_i$) should satisfy $\mathbf{d}_i \succ \mathbf{r}_1$ and $\mathbf{d}_i \succ \mathbf{r}_2$. Since $\sum_{k=1}^{\theta} d_i^k = Q - P = R$, then $\mathbf{d}_i = \mathbf{r}_1$ and $\mathbf{d}_i = \mathbf{r}_2$, resulting in that $\mathbf{r}_1 = \mathbf{r}_2$ which contradicts with the assumption that $\mathbf{r}_1$ and $\mathbf{r}_2$ are different. Therefore, a seller cannot conduct satisfactory transactions with buyers having different subjectivity types. $\square$

In other words, in the proposed model, a type of sellers can only conduct satisfactory transactions with buyers with a specific subjectivity type, and there are no common sellers who can satisfy buyers with different subjectivity types.

Next the utility of buyers is formalized. A buyer b_j can achieve α ($\alpha > 0$) utility from a satisfactory transaction and lose β ($\beta > 0$) utility from an unsatisfactory transaction. Thus, the utility of buyer b_j in a transaction can be described as a step function:

$$u_{ji} = \begin{cases} \alpha & \text{when } \mathbf{d}_i = \mathbf{r}_j, \\ -\beta & \text{when } \mathbf{d}_i \prec \mathbf{r}_j. \end{cases} \quad (\text{Eq. 3.1})$$

Here, since we assume that $\sum_{k=1}^{\theta} d_i^k = Q - P = R$ which is equal to the grand subjectivity, there does not exist the case $\mathbf{d}_i \succ \mathbf{r}_j$. If a buyer b_j experiences a satisfactory transaction with seller s_i , then a *positive* rating is provided by the buyer, and otherwise a *negative* rating is provided. The shared ratings are managed by the club center and are aggregated together with the ratings provided by other buyers in the same club. In order to ensure a finite number of buyers' subjectivity types or sellers' quality vectors, the domain of every attribute is uniformly discretized into m units. Thus, for θ attributes, the number of subjectivity types and quality vectors is less than m^{θ} when removing the dominant types and satisfying $Q = P + R$.

3.1.2.3 Clubs

When a buyer registers in the system at the first time, the system automatically assigns a specific size of memory to record the ratings provided by the buyer. When a buyer's memory is full, the system replaces the oldest rating with the new coming one. The content of the memory cannot be revised/updated by the buyer after being written. In addition, each buyer has a revisable memory cell which is used for the buyer to specify her club index. All the club indices are public in the system. Each club index is assigned with an agent to manage the club and the agent is referred as a *club center*. For a set of buyers with the same club index, the corresponding club center (instead of buyers themselves for the reason of information privacy) builds the reputation system of sellers by aggregating the ratings in the memory of the buyers in the club. If a buyer leaves her current club, the club center will not be able to access the ratings in the buyer's memory any longer. This is because if the buyer leaves the current club, she cannot enjoy the benefit of providing ratings to the club anymore. For a club which is composed by a single buyer (singleton), the seller reputation is calculated based on only the ratings provided by this buyer.

A club center performs two tasks. Firstly, it gathers the ratings provided by the buyers in its club, and aggregates them based on a reputation computation model. In the literature, various reputation models have been proposed [36], which can be adopted. Secondly, it calculates the utility surplus of each buyer in its club and charges buyers administration fees in the same amount. After that, it divides the collected administration fees among the buyers of its club.

3.2 Utility Surplus of Clubs

In this section, the utility surplus of various types of clubs is formalized, and the utility surplus collection and allocation algorithm is also specified.

Given Eq. 3.1, a buyer b_j 's expected utility in a transaction can be calculated as $\omega_j\alpha - (1 - \omega_j)\beta$, where ω_j is the probability of buyer b_j in conducting a satisfactory transaction.

If buyer b_j accumulates a lot of useful transaction experience, then the buyer tends to choose a satisfactory seller. This probability of conducting a satisfactory transaction should increase as transaction experience grows, and the increase rate gets smaller with the growing of transaction experience. Finally, the satisfactory probability will converge to 100% when the transaction experience becomes sufficiently large enough. Based on this inspiration, a function $P_r(z)$ is defined to describe the probability of buyers in conducting satisfactory transactions, where z is the amount of transaction experience⁶ that a buyer can access. The function $P_r(z)$ is described as follows:

$$P_r(z) = \begin{cases} f(z) & z < 0, \\ \omega_0 & z = 0, \\ g(z) & z > 0, \end{cases} \quad (\text{Eq. 3.2})$$

where $f(0) = g(0) = \omega_0$. The diagram of $P_r(z)$ is shown in Figure 3.2. The $\omega_0 \in$

Figure 3.2: The diagram of function $P_r(z)$

$(0, 1)$ is the probability of a buyer in conducting satisfactory transactions without any direct or indirect experience. $z > 0$ means that certain amount of useful experience

⁶Transaction experience of a buyer is not equivalent to her transaction number, and in the experiments, transaction experience is an exponential function of transaction number.

has been accumulated by the buyer to conduct satisfactory transactions, and $z < 0$ indicates that misleading (inappropriate) experience has been accumulated which leads to unsatisfactory transactions. When $z \in (0, +\infty)$, function $g(z)$ should be concave, by satisfying $\frac{dg(z)}{dz} > 0$, $\frac{d^2g(z)}{dz^2} < 0$, and $\lim_{z \rightarrow +\infty} g(z) = 1$. When $z \in (-\infty, 0)$, function $f(z)$ should be convex by satisfying $\frac{df(z)}{dz} > 0$, $\frac{d^2f(z)}{dz^2} > 0$, and $\lim_{z \rightarrow -\infty} f(z) = 0$. The concavity and convexity of $f(z)$ and $g(z)$ have been empirically studied in [96]. Thus, the function $P_r(z)$ is in a ‘S’ shape, which is visualized in Figure 3.4 of Section 3.5.

Therefore the probability of a buyer b_j of conducting a satisfactory transaction ω_j can be calculated by $P_r(z = e_j)$ where e_j is the direct transaction experience of buyer b_j , and the probability of the buyer in conducting an unsatisfactory transaction is then equal to $1 - \omega_j = 1 - P_r(z = e_j)$. When buyer b_j joins club C_i , the club center is then able to access the ratings of the buyers to update the seller reputation of the club, and the buyer is able to access the seller reputation information, which contains e_j^i (direct and indirect) transaction experiences. Consequently, the satisfactory transaction probability of b_j becomes $P_r(z = e_j^i)$ which results in the utility surplus of the club. According to the subjectivity of buyers in clubs, the clubs can be classified into two types: *pure clubs* if all the buyers in the clubs have the same subjectivity, and *mixed clubs* otherwise. The utility surplus of the two types of clubs is formalized as follows.

3.2.1 Utility Surplus of a Pure Club

Let b_j be a buyer in pure club C_i . Based on her direct transaction experience e_j , buyer b_j can achieve the expected utility u_j in a transaction period, which is defined as follows:

$$\begin{aligned} u_j &= \gamma_j [P_r(e_j)\alpha - (1 - P_r(e_j))\beta] \\ &= \gamma_j P_r(e_j)(\alpha + \beta) - \gamma_j \beta, \end{aligned} \tag{Eq. 3.3}$$

where $\gamma_j \in \mathbb{N}_+$ is the transaction rate of b_j , denoting the expected number of transactions that b_j conducts in a period. The value of γ_j can be obtained by observing the transaction history of buyer b_j .

When b_j becomes a member of a pure club C_i , the amount of experience that b_j can access becomes e_j^i which is the sum experience of buyers in club C_i , i.e., $e_j^i = \sum_{b_k \in C_i} e_k$. It can be noticed that $e_j^i \geq e_j$. Thus, the probability of b_j in conducting a satisfactory transaction becomes $P_r(e_j^i)$ and the expected utility of b_j in club C_i can be described as:

$$u_j^i = \gamma_j P_r(e_j^i)(\alpha + \beta) - \gamma_j \beta, \quad (\text{Eq. 3.4})$$

by replacing e_j in Eq. 3.3 with e_j^i .

By comparing Eq. 3.4 with Eq. 3.3, one can derive the utility surplus a_j^i created by buyer b_j in club C_i as follows:

$$a_j^i = u_j^i - u_j = \gamma_j (P_r(e_j^i) - P_r(e_j))(\alpha + \beta). \quad (\text{Eq. 3.5})$$

Therefore, the utility surplus of club C_i is calculated as the sum of the utility surplus created by every buyer in the club, which is denoted by $v(C_i)$ as follows:

$$v(C_i) = \sum_{b_k \in C_i} a_k^i. \quad (\text{Eq. 3.6})$$

3.2.2 Utility Surplus of a Mixed Club

Suppose C_i is a mixed club, consisting of buyers in m different types of subjectivity ($\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_m$). The mixed club C_i can also be denoted by $C_i = \{C_i^k | k = 1, 2, \dots, m\}$, where C_i^k is a set of buyers with subjectivity type $\mathbf{r}_k$. Suppose $b_j \in C_i$ is a member of C_i^1 , if the buyers in C_i^1 form a separate pure club, the utility surplus of C_i^1 is $v(C_i^1)$ according to Eq. 3.6. Considering that C_i^1 is a part of C_i , the ratings provided by buyers with a different subjectivity type, e.g., the buyers in C_i^2 , will mislead b_j , as their satisfactory seller sets are not intersected as shown in Proposition 3. The amount of useful transaction experience for b_j being in C_i can be calculated as:

$$e_j^i = \sum_{b_k \in C_i^1} e_k - \sum_{b_k \in C_i, b_k \notin C_i^1} e_k. \quad (\text{Eq. 3.7})$$

Obviously, the amount of accessible transaction experience of b_j in the mixed club C_i is less than that in her pure club C_i^1 . Thus, the utility surplus created by b_j in the mixed club C_i is less than in the pure club C_i^1 . Consequently, it can be derived that the utility surplus of C_i is less than the sum of the utility surplus created by the same set of buyers through forming their pure clubs, i.e.,

$$v(C_i) < \sum_{k=1}^m v(C_i^k). \quad (\text{Eq. 3.8})$$

Therefore, forming a mixed club is dominated by splitting it into several pure clubs. Especially, when $e_j^i < e^j$, the utility surplus a_j^i is negative, meaning that buyer b_j will lose utility by joining the mixed club C_i .

In the next section, the implementation of the utility surplus collection and division in the proposed CONGRESS is specified.

3.2.3 Collection and Allocation of Utility Surplus

For a (pure or mixed) club C_i , its club center first collects the utility surplus from buyers of the club at the end of each transaction period in the form of administration fees, and then allocates u_i^j to each buyer $b_j \in C_i$. A general procedure is described as in Algorithm 1. Especially, if buyer b_j in a mixed club C_i contributes negative utility surplus, i.e., $a_j^i < 0$, the administration fee is negative, which means the club center compensates buyer b_j for her own fault in joining the mixed club C_i . To avoid this effect, the club center stops performing the utility surplus collection and allocation in this case.

Thus, the overall utility of buyer b_j by joining club C_i is:

$$U_j^i = u_j^i - a_j^i + u_i^j = u_j + u_i^j. \quad (\text{Eq. 3.9})$$

Two specific ways of calculating u_i^j (Line 12) are proposed in the following two sections, i.e., proportional and Shapley allocations, respectively.

Input : C_i , a club containing at least two buyers;
 α , profit from a satisfactory transaction;
 β , loss from an unsatisfactory transaction;
 γ^j , transaction rate of buyer b_j ;
Output: w_i^j , utility surplus allocated to b_j in C_i ;

```

1  Flag = 0;
2  foreach  $b_j \in C_i$  do
3      Calculates  $a_j^i$  according to Eq. 3.5;
4      if  $a_j^i < 0$  then
5          Flag = 1;
6          Break;
7  if Flag = 0 then
8      foreach  $b_j \in C_i$  do
9          Charge administration fee  $a_j^i$  from  $b_j$ ;
10     Calculate  $v(C_i)$  according to Eq. 3.6;
11     foreach  $b_j \in C_i$  do
12         Allocate  $b_j$  with  $w_i^j$ ;
    
```

Algorithm 1: Utility surplus collection and division in CONGRESS

3.2.3.1 Proportional Allocation

A buyer b_j in a club C_i , the allocated utility surplus w_i^j is proportional to b_j 's transaction experience, and w_i^j can be calculated as:

$$w_i^j = \frac{e_j}{\sum_{b_k \in C_i} e_k} v(C_i), \quad (\text{Eq. 3.10})$$

where e_j is b_j 's transaction experience. Thus, the overall utility of buyer b_j by joining in club C_i becomes:

$$U_j^i = u_j + \frac{e_j}{\sum_{b_k \in C_i} e_k} \sum_{b_k \in C_i} a_k^i. \quad (\text{Eq. 3.11})$$

3.2.3.2 Shapley Allocation

In Shapley allocation, the value of u_i^j is equal to the Shapley value [91] of buyer b_j , formalized as follows:

$$\phi_j(v, C_i) = \sum_{\substack{\forall C_i^k \subset C_i \\ b_j \notin C_i^k, b_j \in C_i}} \frac{(n_i - n_i^k - 1)! n_i^k!}{n_i!} [v(C_i^k \cup \{b_j\}) - v(C_i^k)], \quad (\text{Eq. 3.12})$$

where n_i is the number of buyers in club C_i and n_i^k is the number of buyers in the subset $C_i^k \in C_i$. Then, the overall utility of b_j in C_i is given by:

$$\begin{aligned} U_j^i &= u_j^i - a_j^i + \phi_j(v, C_i) \\ &= u_j + \phi_j(v, C_i). \end{aligned} \quad (\text{Eq. 3.13})$$

The Shapley value is well-known to be able to capture the marginal contributions of individuals for improving the performance (utility surplus) of a club or community. The contribution of a buyer in a pure club C_i can be analyzed in two aspects. Suppose there are two buyers in a pure club, b_j and b_k ($j \neq k$). On one hand, b_j 's transaction experience can help b_k to make a better decision and achieve higher probability in conducting satisfactory transactions. Thus, the utility surplus a_k^i of buyer b_k is created due to the help of buyer b_j . On the other hand, buyer b_j also contributes the chance of using b_k 's transaction experience so as to create a_j^i .⁷ Therefore, for buyer b_j being in a club, her Shapley value ϕ_j is strictly positive (there does not exist a dummy [91]) if she ever shares ratings to the club center or conducts transactions.

To compare the two allocations, the distinctive characteristics of them are emphasized as follows:

- The proportional allocation only considers buyers' transaction experience where the utility surplus allocated to a buyer b_j is proportional to her direct transaction experience e_j . Thus, the more experience a buyer has, the higher utility surplus

⁷The transaction experience of b_j herself cannot contribute to a_j^i according to Eq. 3.5.

the buyer can create, which is straightforward and easily understandable. The computational complexity is linear. As each buyer b_j is allocated with the same size of memory, buyers will be allocated with the same utility surplus u_i^j in the long run, regardless of b_j 's transaction rate γ^j and the satisfactory transaction probability function $P_r(z)$. Eventually, the proportional allocation can lead to the *strict core partition* as will be discussed in Section 3.3.

- The Shapley allocation divides the utility surplus to buyers based on their Shapley values or marginal contributions. The contribution of a buyer includes two parts created by the shared ratings and the conducted transactions, respectively. The computational complexity is equal to the complexity of calculating the Shapley values, which can be obtained with linear complexity by adopting an approximation approach based on randomization [11]. According to the additivity property [11] of the Shapley value, it can be further divided as

$$\phi_j(v, C_i) = \phi_j^1(v, C_i) + \phi_j^2(v, C_i), \quad (\text{Eq. 3.14})$$

where $\phi_j^1(v, C_i)$ is the marginal contribution of b_j due to her shared ratings, and $\phi_j^2(v, C_i)$ is the marginal contribution of b_j due to her conducted transactions. The Shapley allocation is a relatively more fair way to allocate the utility surplus of a club, which is the advantage of this allocation. However, it may not always result in a strict core partition – a strong stability concept, but could lead to another weaker stability concept, i.e., a *strict Nash partition*.

Therefore, the two allocations can serve as the solutions of different *stability* concepts as defined in Section 3.3.

3.3 Club Structure Analysis

Given the utility surplus collection and division in Algorithm 1, every buyer should join a club which maximizes her own utility. A club structure is then formed, which is a partition of the set of buyers B and denoted by $\Pi = \{C_i | \bigcup_i C_i = B, C_i \cap C_k = \emptyset \text{ for } i \neq k\}$. In this section, the stability property of the club structure is analyzed. The *optimal club structure* and two stability concepts are defined: *strict core partition* and *strict Nash partition*. After that, the conditions that the optimal club structure is a strict core partition or a Nash partition are discussed, given the proportional and Shapley allocations.

From the viewpoint of system designers, a particular club structure is desired, which maximizes the total utility surplus of the system.

Definition 2. [Optimal Club Structure] *The optimal club structure, denoted by Π^* , is a partition of buyers such that the total utility surplus is maximized:*

$$\Pi^* = \underset{\Pi}{\operatorname{argmax}} \sum_{C_i \in \Pi} v(C_i). \quad (\text{Eq. 3.15})$$

Proposition 4. *In CONGRESS, the optimal club structure Π^* is the partition of buyers where all the buyers with a same subjectivity type form a pure club.*

Proof. Suppose a buyer b_j with subjectivity type $\mathbf{r}_i$ joins a club C_i . The utility surplus created by buyer b_j is $a_j^i = \gamma_j(\alpha + \beta)(P_r(e_j^i) - P_r(e_j))$. If C_i contains all the buyers with the same subjectivity with b_j , then e_j^i is maximized which results in that $P_r(e_j^i) - P_r(e_j)$ is maximized. Consequently, a_j^i is maximized. When each buyer contributes her maximum a_j^i , the total utility surplus of the club structure can also be maximized. Hence the statement in Proposition 4 follows. $\square$

It is noted that the optimal club structure Π^* does not suffer from the subjectivity problem. The stability of Π^* then becomes essential to be discussed. The stability

concepts in the analysis are *strict core partition* [86] and *strict Nash partition* [10] as defined as follows.

Definition 3. [*Strict Core Partition*] A strict core partition Π is a partition of the buyers where there does not exist a club $C_{i'} \notin \Pi$ such that the utility of every buyer in $C_{i'}$ is no less than that in the club $C_i \in \Pi$ and at least one buyer can achieve strictly higher utility.

Such a club $C_{i'}$ in Definition 3 is referred to as a *deviation* of club structure Π .

Definition 4. [*Strict Nash Partition*] A strict Nash partition Π is a partition of buyers where there does not exist a buyer in any club $C_i \in \Pi$ such that the buyer is able to achieve higher utility by joining another club $C_{i'} \in \Pi$.

A strict core partition describes a stable club structure where no group of buyers will prefer to form a deviation, whereas the strict Nash partition requires that no buyer is willing to unilaterally deviate from the club structure. Thus, the strict core partition is a stronger stability concept than the strict Nash partition.

Proposition 5. *Given the proportional or Shapley allocation algorithm, the optimal club structure Π^* is a strict Nash partition.*

Proof. Suppose a buyer b_j is a member of a pure club $C_i \in \Pi^*$ where all the buyers with the same subjectivity as b_j are also in C_i . There are three actions from which buyer b_j can choose:

- (A₁) Staying in Club C_i to achieve utility U_j^i .
- (A₂) Joining another club $C_{i'} \in \Pi^*$, and achieve utility $U_j^{i'}$.
- (A₃) Forming a singleton $C_{i''}$ by herself, and achieve utility $U_j^{i''}$.

By taking action (A_3) , buyer b_j forms a club by herself and $U_j^{i''} = u_j$ which is no more than U_j^i when she chooses action (A_1) according to Eq. 3.11 and Eq. 3.13 in the proportional and Shaley allocations respectively. Thus, action (A_3) is always dominated by (A_1) .

By taking action (A_2) , as b_j joins another club $C_{i'} \in \Pi^*$ such that the club $C_{i'}$ becomes the set $C_{i'} \cup \{b^j\}$, then the newly formed club is a mixed club. Firstly, if $a_j^{i'} < 0$ or $a_k^{i'} < 0$ for a buyer $b_k \in C_{i'}$, then the club center of $C_{i'}$ stops performing the utility surplus collection and allocation, resulting in $u_j^i = 0$. Meanwhile, buyer b_j is misled worse and achieves even less utility than in club $C_{i''}$. Thus buyer b_j will prefer to choose action (A_1) rather than action (A_2) in this case. Secondly, if $a_j^{i'} \geq 0$ and $a_k^{i'} \geq 0$ for any buyer $b_k \in C_{i'}$, then the utility surpluses created by the buyers in $C_{i'}$ are non-negative but less than that in the club except b_j according to Eq. 3.7. In proportional allocation, the buyers $b_k \in C_{i'}$ will choose to leave and form a new club, leaving buyer b_j being a singleton (A_3) again. In Shapley allocation, $\phi_j^1(v, C_{i'})$ in Eq. 3.14 is negative as the experience shared by b_j is misleading for other buyers in $C_{i'}$, which results in lower marginal contribution than that staying club C_i (A_1) .

Therefore, the optimal club structure Π^* is a strict Nash partition where every buyer takes action (A_1) . □

The following analysis focuses on the conditions that the optimal club structure Π^* is a strict core partition. Given the proportional allocation, the following three propositions show three conditions: a sufficient, and a necessary and sufficient conditions that Π^* is a strict core partition, and a sufficient condition that Π^* is not a strict core partition. Moreover, when the Shapley allocation is applied, a sufficient condition that Π^* is a strict core partition has been proven.

Given a club C_i , $\bar{e}_i$ denotes the average transaction experience of the buyers of club C_i , i.e., $\bar{e}_i = \frac{\sum_{b_k \in C_i} e_k}{n_i}$ where n_i is the number of buyers in the club. The average satisfactory

transaction probability of the buyers in club C_i is further denoted by $\bar{\omega}_i = \frac{\sum_{b_j \in C_i} P_r(e_j)}{n_i}$.

Proposition 6. *Given the proportional allocation and that buyers have the same transaction rate γ , the optimal club structure Π^* is a strict core partition if all the buyers have the same transaction experience.*

Proof. In the optimal club structure Π^* , it is known that the buyers with the same subjectivity will form only one club. Let $b_j \in C_i$ where $C_i \in \Pi^*$. It is assumed that there is a deviation $C_{i'} \notin \Pi^*$ and $b_j \in C_{i'}$. The deviation $C_{i'}$ is a subset of C_i , i.e., $C_{i'} \subset C_i$, since a mixed club is dominated by splitting it into several pure clubs. Thus, the number of buyers in $C_{i'}$ is less than in C_i , i.e., $n_{i'} < n_i$.

If $e_j = \bar{e}_i$ for all $b_j \in C_i$, then $\bar{e}_i = \bar{e}_{i'}$. The utility of b_j gained in C_i and $C_{i'}$ is denoted as U_j^i and $U_j^{i'}$, respectively.

$$U_j^i = u_j + \gamma(\alpha + \beta)(P_r(n_i e_j) - P_r(e_j)), \quad (\text{Eq. 3.16})$$

$$U_j^{i'} = u_j + \gamma(\alpha + \beta)(P_r(n_{i'} e_j) - P_r(e_j)). \quad (\text{Eq. 3.17})$$

Since $n_{i'} < n_i$ and $P_r(\cdot)$ is an increasing function as defined in Eq. 3.2, it can be derived that $U_j^i > U_j^{i'}$. Therefore, $C_{i'}$ cannot be a deviation of Π^* , which contradicts with the initial assumption at the beginning of this proof. In conclusion, there does not exist a deviation in this case, and the optimal club structure Π^* is a strict core partition. $\square$

Proposition 7. *Given the proportional allocation and that buyers have the same transaction rate γ , the optimal club structure Π^* is not a strict core partition if there exists a buyer $b_j \in C_i$ where $C_i \in \Pi^*$ such that*

$$\frac{P_r(e_j)}{\bar{\omega}_i} \bar{e}_i > e_j > e_k, \quad (\text{Eq. 3.18})$$

where e_k is the maximum value of the set $\left\{ e_j \mid \frac{(n_i-1)P_r(n_i \bar{e}_i - e_j)}{n_i \bar{e}_i - e_j} = \frac{P_r(n_i \bar{e}_i)}{\bar{e}_i} \right\}$.

Proof. In this proof, Π^* is shown not to be a strict core partition by demonstrating the existence of a deviation $C_{i'}$ where $C_{i'}$ is the set of buyers in club C_i except buyer b_j whose transaction experience e_j satisfies Eq. 3.18.

Let $C_{i'}$ be the club formed by the buyers in C_i except buyer b_j . For any buyer $b_k \in C_{i'}$, the utility of b_k in club C_i and $C_{i'}$ can be calculated as U_k^i and $U_k^{i'}$, respectively.

$$U_k^i = \frac{\gamma e_k(\alpha + \beta)}{n_i \bar{e}_i} \{n_i P_r(n_i \bar{e}_i) - n_i \bar{\omega}_i\}, \quad (\text{Eq. 3.19})$$

$$U_k^{i'} = \frac{\gamma e_k(\alpha + \beta)}{n_i \bar{e}_i - e_j} \{(n_i - 1)P_r(n_i \bar{e}_i - e_j) - n_i \bar{\omega}_i + P_r(e_j)\}. \quad (\text{Eq. 3.20})$$

The difference between $U_k^{i'}$ and U_k^i is calculated as follows:

$$\begin{aligned} U_k^{i'} - U_k^i = & \gamma e_k(\alpha + \beta) \left\{ \frac{(n_i - 1)P_r(n_i \bar{e}_i - e_j)}{n_i \bar{e}_i - e_j} - \frac{P_r(n_i \bar{e}_i)}{\bar{e}_i} \right\} \\ & + \gamma e_k(\alpha + \beta) \left\{ \frac{\bar{\omega}_i}{\bar{e}_i} - \frac{n_i \bar{\omega}_i - P_r(e_j)}{n_i \bar{e}_i - e_j} \right\}. \end{aligned} \quad (\text{Eq. 3.21})$$

Since $P_r(z)$ is concave when z is positive (z is always positive in pure clubs), then the first order derivation of $\frac{dP_r(z)}{dz}z - P_r(z)$ is negative:

$$\frac{d\left(\frac{dP_r(z)}{dz}z - P_r(z)\right)}{dz} = \frac{d^2 P_r(z)}{dz^2} < 0. \quad (\text{Eq. 3.22})$$

Given $\left\{ \frac{dP_r(z)}{dz}z - P_r(z) \right\}_{z=0} < 0$, it can be derived that

$$\frac{dP_r(z)}{dz}z - P_r(z) < 0, \quad (\text{Eq. 3.23})$$

for $z > 0$. As a result,

$$\begin{aligned} & d \left\{ \frac{(n_i - 1)P_r(n_i \bar{e}_i - e_j)}{n_i \bar{e}_i - e_j} - \frac{P_r(n_i \bar{e}_i)}{\bar{e}_i} \right\} / de_j \\ & = \frac{n_i - 1}{(n_i \bar{e}_i - e_j)^2} \left\{ P_r(z) - \frac{dP_r(z)}{dz}z \right\} > 0, \end{aligned} \quad (\text{Eq. 3.24})$$

where $z = n_i \bar{e}_i - e_j$.

By satisfying $e_j > e_k$ as in Eq. 3.18, it can be derived that the first part of Eq. 3.21 on the left hand side is positive based on Eq. 3.24. Moreover, according to the condition $\frac{P_r(e_j)}{\bar{\omega}_i} \bar{e}_i$ as in Eq. 3.18, the second part of Eq. 3.21 on the left hand side is also positive. Therefore, $U_k^{i'} > U_k^i$ which means that club $C_{i'}$ is a deviation of Π^* , and consequently the optimal club structure Π^* is not a strict core partition. $\square$

It is possible that there does not exist a buyer satisfying Eq. 3.18, and in that case, the buyers in a club cannot form a derivation by forming a new club except one of them.

Proposition 8. *Given the proportional allocation and that buyers have the same transaction rate γ , the optimal club structure Π^* is a strict core partition if and only if there does not exist a subset $C_{i'} \subset C_i$ such that*

$$\frac{P_r(n_{i'}\bar{e}_{i'}) - \bar{\omega}_{i'}}{\bar{e}_{i'}} > \frac{P_r(n_i\bar{e}_i) - \bar{\omega}_i}{\bar{e}_i}, \quad (\text{Eq. 3.25})$$

for all $C_i \in \Pi^*$.

Proof. For any club $C_i \in \Pi^*$, b_j is a buyer in club C_i . According to Proposition 4, the buyers with the same subjectivity with b_j should also be in club C_i . The utility of b_j in club C_i can be calculated as

$$U_j^i = u_j + \frac{e_j}{\sum_{b_k \in C_i} e_k} \sum_{b_k \in C_i} a_k^i = u_j + \frac{e_j}{n_i \bar{e}_i} \sum_{b_k \in C_i} a_k^i. \quad (\text{Eq. 3.26})$$

For any $C_{i'} \subseteq C_i$, the utility of b_j by joining $C_{i'}$ is

$$U_j^{i'} = u_j + \frac{e_j}{n_{i'} \bar{e}_{i'}} \sum_{b_k \in C_{i'}} a_k^{i'}. \quad (\text{Eq. 3.27})$$

The difference between U_j^i and $U_j^{i'}$ is calculated as

$$\begin{aligned} U_j^i - U_j^{i'} &= \frac{e_j}{n_i \bar{e}_i} \sum_{b_k \in C_i} a_k^i - \frac{e_j}{n_{i'} \bar{e}_{i'}} \sum_{b_k \in C_{i'}} a_k^{i'} \\ &= \frac{\gamma e_j}{n_i \bar{e}_i} \sum_{b_k \in C_i} \{(\alpha + \beta)(P_r(n_i \bar{e}_i) - P_r(e_k))\} \\ &\quad - \frac{\gamma e_j}{n_{i'} \bar{e}_{i'}} \sum_{b_k \in C_{i'}} \{(\alpha + \beta)(P_r(n_{i'} \bar{e}_{i'}) - P_r(e_k))\} \\ &= (\alpha + \beta) \gamma e_j \left\{ \frac{P_r(n_i \bar{e}_i) - \bar{\omega}_i}{\bar{e}_i} - \frac{P_r(n_{i'} \bar{e}_{i'}) - \bar{\omega}_{i'}}{\bar{e}_{i'}} \right\}. \end{aligned} \quad (\text{Eq. 3.28})$$

Since for all $C_i \in \Pi^*$ there does not exist a subset $C_{i'} \subset C_i$ satisfying Eq. 3.25, then $U_j^i - U_j^{i'} \geq 0$. If $U_j^i - U_j^{i'} > 0$, then Π^* is a strict core partition. If $U_j^i - U_j^{i'} = 0$ for all $b_j \in C_{i'}$, according to the definition of the strict core partition, Π^* is still a strict core partition as there does not exist a buyer whose utility is strictly higher in $C_{i'}$ than in C_i .

Conversely, suppose Π^* is a strict core partition, and then $U_j^i \geq U_j^{i'}$ for all $b_j \in C_{i'}$ where $C_{i'} \subset C_i$. Then it can be derived that $\frac{P_r(n_{i'}\bar{e}_{i'}) - \bar{\omega}_{i'}}{\bar{e}_{i'}} \leq \frac{P_r(n_i\bar{e}_i) - \bar{\omega}_i}{\bar{e}_i}$ which contradicts to Eq. 3.25. In other words, there does not exist such a deviation $C_{i'}$. $\square$

Proposition 9. *Given the Shapley allocation and that buyers have the same transaction rate γ , the optimal club structure Π^* is a strict core partition if the following condition is satisfied:*

$$\frac{dP_r(z)/dz|_{z=n_{i'}\bar{e}_{i'}+e_j}}{dP_r(z)/dz|_{z=n_{i'}\bar{e}_{i'}}} > \frac{n_{i'}}{n_{i'} + 1}, \quad (\text{Eq. 3.29})$$

for all $C_{i'} \subset C_i$ where $b_j \notin C_{i'}$ $b_j \in C_i$ and $C_i \in \Pi^*$.

Proof. First of all, a function is constructed as follows:

$$f(n, z) = (n + 1)P_r(z + e_j) - (n)P_r(z), \quad (\text{Eq. 3.30})$$

where $P_r(\cdot)$ is a satisfactory transaction probability function. The first order partial derivatives with respect to n and z can be calculated, respectively:

$$\frac{\partial f(n, z)}{\partial n} = P_r(z + e_j) - P_r(z), \quad (\text{Eq. 3.31})$$

$$\frac{\partial f(n, z)}{\partial z} = (n + 1)\frac{dP_r(z)}{dz} - n\frac{dP_r(z)}{dz}, \quad (\text{Eq. 3.32})$$

where $\frac{\partial f(n, z)}{\partial n} > 0$ as $P_r(\cdot)$ is an increasing function, and $\frac{\partial f(n, z)}{\partial z} > 0$ according to Eq. 3.29.

Therefore, it can be claimed that for two subsets $C_{i'}$ and $C_{i''}$ of $C_i \in \Pi^*$ where $C_{i''} \subset C_{i'}$, $C_{i'} \subset C_i$, and a buyer $b_j \notin C_{i'}$, the following equation will be satisfied:

$$(n_{i''} + 1)P_r(n_{i''}\bar{e}_{i''} + e_j) - n_{i''}P_r(n_{i''}\bar{e}_{i''}) < (n_{i'} + 1)P_r(n_{i'}\bar{e}_{i'} + e_j) - n_{i'}P_r(n_{i'}\bar{e}_{i'}). \quad (\text{Eq. 3.33})$$

In other words, the following equation is satisfied:

$$v(C_{i''} \cup \{b_j\}) - v(C_{i''}) < v(C_{i'} \cup \{b_j\}) - v(C_{i'}). \quad (\text{Eq. 3.34})$$

Therefore, the club formation problem becomes a convex game as defined in [92]. According to Theorem 7 in [92], the Shapley value is in the core, implying that buyers

in club $C_i \in \Pi^*$ can achieve no less utility than that in any subset of C_i . Therefore, the buyers with the same subjectivity are likely to form club C_i (grand club). To conclude, the optimal club structure Π^* is a strict core partition. $\square$

To explicitly explain the condition in Proposition 9, an example is provided here. Suppose $P_r(z)$ to be an exponential function in a general form (the same as that in the experimental settings) when $z > 0$:

$$P_r(z) = 1 - \zeta \exp(-\eta z), \quad (\text{Eq. 3.35})$$

where $\zeta > 0$ and $\eta > 0$. It's diagram is shown as in Figure 3.3.

Figure 3.3: The diagram of Eq. 3.35

The condition in Proposition 9 can be rewritten as

$$\frac{dP_r(z)/dz|_{z=n_{i'}\bar{e}_{i'}+e_j}}{dP_r(z)/dz|_{z=n_{i'}\bar{e}_{i'}}} = \zeta \eta \exp(-\eta e_j) > \frac{n_{i'}}{n_{i'} + 1}. \quad (\text{Eq. 3.36})$$

Since $\frac{n_{i'}}{n_{i'}+1} < \frac{n_i}{n_i+1}$ due to the fact that $C_{i'}$ is a subset of C_i and $n_{i'} < n_i$, then the condition can be interpreted that $\zeta \eta \exp(-\eta \times e_j) > \frac{n_i}{n_i+1}$ holds for all $b_j \in C_i$ where $C_i \in \Pi^*$. Given that $\zeta \eta \exp(-\eta \times e_j)$ decreases as e_j grows, the maximum transaction experience e_i^{max} of buyers in club $C_i \in \Pi^*$ should satisfy the following equation:

$$\zeta \eta \exp(-\eta e_i^{max}) > \frac{n_i}{n_i + 1}, \quad (\text{Eq. 3.37})$$

such that Π^* is a strict core partition.

Thus, the condition in Proposition 9 requires that the maximum transaction experience of buyers is upper bounded where the bound is determined by the number of buyers with the same subjectivity. Alternatively, the number of buyers with a same subjectivity type should also be upper bounded by a value which is determined by the maximum transaction experience of the buyers with the subjectivity.

3.4 Buyer Strategy Analysis

The club structure is formed by the buyers, and their strategies in forming clubs are directly impacted by the utility that they can achieve in the formed club structure. This section is to analyze buyer strategies in two cases where Π^* is a strict core partition and Π^* is not a strict core partition, respectively. The strategies of buyers include:

- (S_1) Forming a singleton by herself.
- (S_2) Joining a mixed club that contains buyers with different types of subjectivity.
- (S_3) Joining one of the existing clubs which contains buyers with the same subjectivity.
- (S_4) Forming a new club with other buyers with the same subjectivity.

Proposition 10. *The strategies (S_1) and (S_2) are dominated strategies.*

Proof. When a buyer b_j forms a singleton C_i , the utility surplus of the club is zero, according to Eq. 3.6. For any club $C_{i'}$ which is formed by b_j and another buyer b_k ($e_k > 0$) with the same subjectivity, the utility surplus of $C_{i'}$ is positive, since $a_j^{i'} > 0$. Thus, strategy (S_1) forming a singleton is dominated by strategy (S_3). Forming a mixed club is always dominated by forming a new club with the buyers in the mixed club (S_4) as analyzed in Section 3.2.2. Therefore, both strategies (S_1) and (S_2) are dominated strategies. □

For strategies (S_3) and (S_4) , the behaviors of buyers will be different according to whether Π^* is a strict core partition, as analyzed in the following two cases.

3.4.1 Case 1: Π^* is a Strict Core Partition

When the condition in Proposition 8 or Proposition 6 is satisfied, Π^* is a strict core partition.

Proposition 11. *If the optimal club structure Π^* is a strict core partition, then strategy (S_3) is a dominant strategy.*

Proof. Since the optimal club structure Π^* is a strict core partition, then each buyer can achieve their maximum utility by forming a club with all the buyers with the same subjectivity. Therefore, strategy (S_3) joining the existing club which contains buyers with the same subjectivity is their dominant strategy. $\square$

3.4.2 Case 2: Π^* is not a Strict Core Partition

When the optimal club structure Π^* is a strict core partition, each buyer would choose the pure dominant strategy (S_3) , as discussed in Proposition 11. However, in Case 2, the buyers take mixed strategies. The following proposition proves the strategies S_3 and S_4 coexist when using proportional allocation.

Proposition 12. *If the optimal club structure Π^* is not a strict core partition and the utility surplus is proportionally allocated, then there are separate sets of buyers taking strategy (S_3) and strategy (S_4) .*

Proof. Suppose C_i contains all the buyers with a same subjectivity type and there is only one deviation $C_{i'} \subset C_i$. When all the buyers with the same subjectivity form a club, the total utility surplus can be maximized, which is $n_i P_r(n_i \bar{e}_i) - n_i \bar{\omega}_i$. Since $C_{i'}$ is a deviation, then $\frac{n_{i'} P_r(n_{i'} \bar{e}_{i'}) - n_{i'} \bar{\omega}_{i'}}{n_{i'} \bar{e}_{i'}} > \frac{n_i P_r(n_i \bar{e}_i) - n_i \bar{\omega}_i}{n_i \bar{e}_i}$, indicating that the utility surplus

of club $C_{i'}$ is larger than that created by the same set of buyers in club C_i . The set of buyers in C_i but not in $C_{i'}$ is denoted by $C_{i''}$. Since $v(C_{i'}) + v(C_{i''}) < v(C_i)$, then the utility surplus of $C_{i''}$ is less than the overall allocated utility surplus created by the same set of buyers in C_i , i.e., $\frac{e_j}{\sum_{b_k \in (C_{i''})} e_k} v(C_{i''}) < \frac{e_j}{\sum_{b_k \in C_i} e_k} v(C_i)$ where b_j is any buyer in $C_{i''} \subset C_i$. Suppose all the buyers initially join a club which is indexed by L_1 . Then the set of buyers of $C_{i'}$ prefer to leave and form a new club indexed by L_2 , leaving $C_{i''}$ to be indexed by L_1 . However, the buyers of $C_{i''}$ can achieve less utility and they then turn to join the club indexed by L_2 , resulting in the formed club to still be C_i by changing its index from L_1 to L_2 . This process goes on and on. Therefore, the set of buyers of $C_{i'}$ always takes strategy (S_3) , whereas the set buyers of $C_{i''}$ prefers to take strategy (S_4) . Due to the existence of the two strategies in the system, the club structure does not sustain a stable state. $\square$

3.5 Experimental Validation

This section first evaluates the proposed CONGRESS through conducting experiments. The hypotheses are (1) buyers have the incentive to form pure clubs; (2) buyers have the incentive to share ratings; (3) the optimal club structure is a strict core partition when some conditions are satisfied. The experimental results show that these hypotheses are true. To demonstrate the advantages of the proposed model, CONGRESS is then compared with the Beta Reputation System (BRS) [35].

3.5.1 Experimental Settings

In the experiments, sellers provide items to buyers and the items are evaluated through four attributes, i.e. $\theta = 4$. The value domain of each attribute is discretized into two equal units, i.e., $m = 2$ and an attribute value is either 0.5 or 1. The value domain of buyer subjectivity is also discretized into two units, either -0.5 or 0. The grand promised

quality is $P = 4$ (i.e., θ times maximal attribute value 1) and the grand supplied quality is $Q = 3$ (allowing two attributes to be delivered at the attribute value 0.5). Thus, the grand subjectivity is $R = Q - P = -1$. There are 6 types of sellers and 6 types of buyers, as illustrated in Tables 3.2 and 3.3. According to Table 3.2, the promised quality of sellers is the same, which is $[1, 1, 1, 1]$ and the supplied qualities have two attribute values to be lower than the promised ones. The types of buyer subjectivity have two attributes allowing their received values lower than the promised ones by 0.5. The number of buyers in each type is 10, and the number of sellers in each type is also 10. In each transaction period, a buyer requests an item at probability $\gamma = 0.5$, and a seller can deliver items to multiple buyers. The memory of a buyer can store 500 ratings. The profit of buyers from a satisfactory transaction is $\alpha = 2$ and the loss of buyers from an unsatisfactory transaction is $\beta = 1$.

Table 3.2: The types of sellers

Seller Quality Vector	p	q
Type 1	[1,1,1,1]	[1, 1, 0.5, 0.5]
Type 2	[1,1,1,1]	[1, 0.5, 1, 0.5]
Type 3	[1,1,1,1]	[0.5, 1, 1, 0.5]
Type 4	[1,1,1,1]	[1, 0.5, 0.5, 1]
Type 5	[1,1,1,1]	[0.5, 1, 0.5, 1]
Type 6	[1,1,1,1]	[0.5, 0.5, 1, 1]

Table 3.3: The types of buyers

Buyer Subjectivity Type	r
Type 1	[0, 0, -0.5, -0.5]
Type 2	[0, -0.5, 0, -0.5]
Type 3	[-0.5, 0, 0, -0.5]
Type 4	[0, -0.5, -0.5, 0]
Type 5	[-0.5, 0, -0.5, 0]
Type 6	[-0.5, -0.5, 0, 0]

The reputation score of a seller s_i is calculated in a club C_j according to the Beta

Reputation Model [35] so as to be fairly compared with it in Section 3.5.2.4:

$$R_j^i = \frac{P_j^i + 1}{P_j^i + N_j^i + 2}, \quad (\text{Eq. 3.38})$$

where P_j^i and N_j^i are the numbers of satisfactory and unsatisfactory transactions happening between the buyers in club C_j and s_i , respectively.

The seller selection process includes two steps. The buyers in club C_j firstly select the sellers whose reputation is no less than $\max_{s_i}\{R_j^i - \epsilon\}$, where $\epsilon = 0.5$. Secondly, buyers choose a seller from the above selected set and the probability of a seller been chosen is proportional to his reputation score:

$$\frac{P_s(s_i)}{P_s(s_k)} = \frac{R_j^i}{R_j^k}. \quad (\text{Eq. 3.39})$$

The satisfactory transaction probability $f(z)$ when $z > 0$ is gained by fitting the simulated results when buyers conduct transaction in the specified settings to ensure that the function is properly set, which is experimentally shown to be in the following general form:

$$f(z) = 1 - \zeta \exp(-\eta z), \quad (\text{Eq. 3.40})$$

where $z = n^\lambda$. The satisfactory transaction probability in the experiments is shown in Figure 3.4, where x-axis is the number of transactions and y-axis is the average satisfactory transaction probability. It is observed that the fitted curve can well capture the simulated points where the parameters of the fitted curve are $\zeta = 0.823$, $\eta = 5.526e - 7$, and $\lambda = 3.243$, with the fitting confidence being 0.99. In the following experiments, the $f(z)$ is set according to these parameters. In Figure 3.4, when $z = 0$, $f(z) = 0.177$ which is determined by the sellers in the system where a buyer is expected to achieve a satisfactory transaction at 17.7% with no direct or indirect transaction experience. In addition, it is interesting to observe that the satisfactory transaction probability is almost 1 when the transaction number is about 140. Considering that there are 6×10 sellers

in the experiments, the number 140 indicates that a single buyer is expected to conduct 140 transactions with the 60 sellers (2.33 transactions per seller) so as to achieve fully satisfactory transactions.

Figure 3.4: The satisfactory transaction probability function used in the experiments

3.5.2 Experimental Results

A set of experiments have been conducted using the above settings, and the experimental results are shown in this section. The incentive of buyers to form pure clubs and share their ratings is firstly validated in Sections 3.5.2.1 and 3.5.2.2, respectively. Secondly, the stability properties of CONGRESS as theoretically analyzed in Section 3.3 are verified through case study. Thirdly, the proposed CONGRESS is compared with the Beta Reputation System [35] to demonstrate its efficacy in addressing the subjectivity problem.

3.5.2.1 Incentive to Join Pure Clubs

Three scenarios are simulated. In the first scenario, buyers form pure clubs with buyers having the same subjectivity. In the second scenario, the buyers randomly form clubs (including mixed clubs) and each club can contain maximum 10 buyers so as to be comparable with the first scenario where the maximum number of buyers in each club is 10.

In the third scenario, all the buyers form singletons. The average satisfactory transaction probability of buyers is shown in Figure 3.5 where x-axis is the average transaction number of buyers, and y-axis is the average satisfactory transaction probability of buyers who have the corresponding number of transactions.

Figure 3.5: The satisfactory transaction probability of buyers in different scenarios

Figure 3.5 shows that the buyers can gain significantly higher satisfactory transaction probability by forming pure clubs in the first scenario than the other two scenarios. The third scenario where buyers form singleton by themselves can achieve higher satisfactory transaction probability than forming mixed clubs, making the second scenario to be the worse. Moreover, the third scenario achieves almost 1 when the transaction number is about 140 which is 10 times as large as that in the first scenario, which is exactly the number of buyers in the pure clubs. In other words, the satisfactory transactio probability of a buyer in a pure club is equal to what the buyer can achieve when she accumulates the same amount of transaction experience as the sum transaction experience of the buyers of the club.

3.5.2.2 Incentive to Share Ratings

Another three scenarios adopting CONGRESS are simulated. In the first scenario, the buyers share all their ratings (100% ratings). In the second scenario, the buyers share

their 50% ratings. In the third scenario, the buyers never share their ratings. The utility surplus of a club is allocated by its club center to the buyers of the club according to the proportional allocation⁸, as shown in Figure 3.6.

Figure 3.6: The average utility of buyers by sharing different percentages of ratings

It can be observed that the utility of the buyers in all the three scenarios initially decreases (lower than 0) and then increases as they accumulate transaction experience. Since the initial satisfactory transaction probability is too low (only about 0.2 according to Figure 3.4). This causes the buyers to lose profit, i.e. $\alpha \times 0.2 - \beta \times 0.8 < 0$, and hence their utility firstly decreases. The trend (first decreasing and then increasing) of the buyers' utility exactly reflects the fact that the satisfactory transaction probability increases with their transaction experience. Figure 3.6 shows that buyers in the first scenario can achieve the largest utility by sharing all their ratings among the three scenarios. According to the allocation algorithm, by sharing all their ratings, buyers can be allocated with the largest utility surpluses, which is the main reason for the phenomenon observed.

⁸Similar results could be achieved when using Shapley allocation.

3.5.2.3 Stability Verification

In this experiment, 10 buyers with the same subjectivity type are considered and the number of clubs formed by them is examined. When there exists multiple subjectivity types, the clubs formed by the buyers in each subjectivity type will have the same observation since mixed clubs are dominated. Initially, each buyer forms a singleton and the number of clubs is 10 (there are 10 club indexes, and the number of clubs is the number of indices except the empty ones). It has been demonstrated that if the optimal club structure is strict core partition, the number of clubs formed by the 10 buyers converges to 1 and otherwise the number of clubs should not stabilize at a constant value.

Table 3.4: The transaction numbers of buyers in four cases to verify the club stability

Case	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}
1	10	10	10	10	10	10	10	10	10	10
2	10	10	10	10	10	10	10	10	10	50
3	10	10	10	10	10	10	10	10	10	100
4	10	10	10	10	10	10	100	100	100	50

The 10 buyers with the same subjectivity type form clubs for 100 iterations by assuming there is no new transaction occurs. Four different cases are studied and the number of transactions of each buyer is listed as in Table 3.4. In the first case, each buyer has the same number of transactions, and the 10th buyer has 50 and 100 transactions in the 2nd and 3rd cases, respectively. The fourth case involves 6 buyers having 10 transactions and 4 buyers having 100 transactions. Each of the cases displayed in Table 3.4 satisfies or does not satisfy the conditions in Propositions 6 to 9 in Section 3.3 so as to verify the theoretical results. In each case, the experiment is conducted by using both the proportional and Shapley allocations separately. The results are shown in Figure 3.7.

In case 1 as shown in Figure 3.7(a), the number of clubs converges to 1, i.e., all the buyers prefer to form a grand club, when the utility surplus is allocated by either

Figure 3.7: The club number vs. iterations

the proportional or Shapley allocation. The reason of this observation is that case 1 satisfies the conditions described in Proposition 6 and Proposition 9 where the optimal club structure is a strict core partition which incentivizes buyers to take pure strategy (S_3) according to Proposition 11. In case 2 as shown in Figure 3.7(b), the number of clubs stabilizes at 1 when using the proportional allocation, but switches between 1 and 2 when using the Shapley allocation. Case 2 satisfies the conditions in Proposition 7 such that the optimal club structure is a strict core partition and buyers take pure strategy (S_3) to form the grand club according to Proposition 11. However case 2 does not satisfy the

condition in Proposition 9, and the club number is dynamically changing between 1 and 2, which indicates the existence of two strategies (S_3 and S_4) based on Proposition 12. In case 4 shown in Figure 3.7(d) which does not satisfy the conditions in both Proposition 8 and Proposition 9, the club number changes more frequently than cases 2 and 3 shown in Figure 3.7(b) and (c), and it switches between 1 and 3. The reason is that there are more than two deviations, resulting in that the club structure changes complexly and frequently.

3.5.2.4 Comparative Results

To present the effectiveness of CONGRESS in addressing the subjectivity problem, two systems adopting CONGRESS and the Beta Reputation System (BRS) are compared. The number of buyers in each subjectivity type (keeping the number of sellers constant) is varied and the average satisfactory transaction probability of buyers in the two systems as the number of transactions increases is shown in Figure 3.8 .

Figure 3.8: The satisfactory transaction probability of CONGRESS and BRS

First of all, the buyers in CONGRESS can achieve higher satisfactory transaction probability than that in BRS, as shown in Figures 3.8 (a) and (b). The reason is that

buyers in BRS are misled by the ratings provided by the buyers with different types of subjectivity. Moreover, by comparing the CONGRESS in Figure 3.8(a) and Figure 3.8(b), it can be observed that the satisfactory transaction probability increases a little earlier in Figure 3.8(a) than that in Figure 3.8(b). The reason of this observation is that the larger number of buyers in each subjectivity type is helpful for them to accumulate transaction experience faster so as to make a better decision in choosing satisfactory sellers. Moreover, the fluctuation of the curve for BRS in Figure 3.8(b) is greater than that in Figure 3.8(a), but the two curves have the same expected satisfactory transaction probability, because the number of buyers with the same subjectivity is smaller ($5 < 10$) and the effect of misleading becomes more sensitive. Therefore, the proposed CFG based reputation system can outperform BRS by effectively avoiding the impact of the subjectivity problem.

3.6 Summary

In this chapter, a coalition formation game based reputation system (CONGRESS) has been proposed to address the subjectivity problem, by incentivizing the buyers with different types of subjectivity to form separate clubs. The components of the proposed model are mapped with those of the coalition formation game. In CONGRESS, buyers can form clubs and the seller reputation is built within clubs by aggregating the ratings provided by the buyers of the clubs. The utility created by the reputation information of a club is formalized as the utility surplus of the club, which is collected by the club center in the form of administration fees. Furthermore, the collected administration fees are divided according to two allocations, i.e., proportional and Shapley allocations. The club structure and buyer strategies are then analyzed. As it is desired that buyers with the same subjectivity form the same club, which is proven to be the optimal club structure, our analysis focuses on this club structure. It has also been proven that the

optimal club structure can be a strict core partition or a Nash partition and buyers have incentives to form the desirable club structure, if specific conditions are satisfied. Finally, a set of experiments are conducted to evaluate CONGRESS and demonstrate its effectiveness in addressing the subjectivity problem. The advantages of the proposed CONGRESS are summarized as follows: (1) The subjectivity problem has been addressed in a distributed way. The clubs are formed by buyers autonomously in the proposed model, and the clubs are managed by distributed club centers, which allows the system to be flexibly applied in a real e-marketplace. (2) It can avoid the shortcomings of the existing solutions to the subjectivity problem. Specifically, CONGRESS does not use the modeling approaches to filter or reinterpret the ratings provided by buyers, thus it can avoid complex modeling. Moreover, the ratings are aggregated within clubs and the agents with the same subjectivity can benefit from their common reputation system, which greatly decreases the resources needed in calculating seller reputation. (3) It is based on game theory. The coalition formation game has been well studied in the literature of game theory [74] and it can provide the strong theoretical support.

Chapter 4

iMEMLI: An Incentive Mechanism for E-marketplaces with Limited Inventory

This chapter focuses on addressing the dishonesty problem in reputation systems in the scenario where the inventory is in short supply, i.e. an *e-marketplace with limited inventory* (EMLI). The challenges are in two-fold: (a) for sellers who aim to maximize their profit, they may intentionally conduct dishonest transactions since the limited products are likely to be sold out regardless of their reputation; (b) for buyers who intend to gain the limited products, they may provide untruthful ratings to mislead other buyers. To address these issues, an incentive mechanism is proposed to promote buyer and seller honesty for EMLI, named iMEMLI. In Section 4.1, our system environment is specified by explicitly listing the assumptions and clearly defining the concept of e-marketplaces with limited inventory. Section 4.2 presents the proposed incentive mechanism. Section 4.3 is devoted to the theoretical analysis of the properties of the proposed mechanism. In Section 4.4, a set of experiments are conducted to evaluate the efficacy of the proposed mechanism. Finally, Section 6.5 summarizes this chapter.

4.1 Assumptions

For the purpose of simplicity and clarity in presenting the proposed incentive mechanism, the following assumptions are made:

- (a) *No difference between promised products*: In the e-marketplaces, the product promised by different sellers are assumed to be identical so that buyers will have the same experience in their transactions if the products are honestly delivered by different sellers. In other words, the different transaction results are due to difference in the seller honesty, rather than the difference between the products. For example, the digital products provided by different retailers are indifferent for buyers as long as they have the same functionality [25]. The buyers in such marketplaces could experience differently even if the products are identical, since some sellers behave honestly by delivering the required products on time and others may lingeringly or never deliver. By setting this assumption, in this chapter, the study focuses on the impact of seller honesty on the transaction results, by neglecting the context information of the products. When the products are not identical, the proposed mechanism should be separately implemented for each type of identical products.
- (b) *No subjectivity in providing ratings*: Buyers have the same evaluation criteria in providing ratings. Specifically, given a transaction where sellers behave honestly, any honest buyer will provide a positive rating and any dishonest buyer on the other hand will provide a negative rating, and vice versa.
- (c) *One-inventory per transaction period*: Each seller can only serve one piece of product within a transaction period. It requires the length of the transaction period to be properly determined so that the number of products provided by a seller within a period is no more than one. For example, a dentist can only make one diagnosis within an hour, and in this case, the period is an hour.

- (d) *Buyers are able to afford the price of the required products:* The candidate buyers in the proposed mechanism are assumed to have the capability to pay for the required products. In other words, the budget of buyers is out of the consideration. Several techniques can guarantee the achievement of this assumption. One is to allow buyers to submit their maximum acceptable prices along with their request, and the system would filter out those buyers whose acceptable prices are lower than the offered price. Another technique is to publicize the maximum offered price such that only buyers who accept the price could become the candidate buyers. In the analysis, it has been shown that the price of a product is upper bounded.
- (e) *Sellers are able to deliver the promised products:* Sellers in the mechanism have the capability of conducting a satisfactory transactions. In other words, sellers can manipulate their behavior or control their honesty in the transactions.

Based on the assumptions, the e-marketplaces with limited inventory is formally defined as follows.

Definition 5. [*The e-marketplace with limited inventory (EMLI)*] Given a set of sellers $\mathbf{S}$ each of whom provides the same product and a set of buyers $\mathbf{B}$ each of whom demands one piece of the products. An e-marketplace satisfying the condition, $|\mathbf{S}| < |\mathbf{B}|$, is called an e-marketplace with limited inventory.

4.2 The iMEMLI Mechanism

The e-marketplaces employing the proposed mechanism runs periodically. At the beginning of each transaction period, sellers post the products that they want to sell and buyers post the requests specifying the products that they want to buy. The e-marketplace center gathers together the sellers who sell no-difference products and the buyers who want to buy those products. The products are in short supply, i.e. the marketplace is an EMLI.

Thus some buyers would not be able to gain the products that they want to buy. For those no-difference products, their prices will then be determined by the e-marketplace center and be allocated to some buyers. At the end of each transaction period, buyers who have been successfully allocated products can provide ratings in $[0, 1]$ about sellers from whom the buyers bought products, reflecting the honesty of sellers in their transaction, i.e. the satisfactory level of a buyer towards a seller by comparing the actually experienced transaction with what the seller promised.

Figure 4.1: The main components of the proposed iMEMLI

As the central component of the e-marketplace, iMEMLI is composed of a *normalized proper scoring rule* to model buyer honesty, a *reputation model* to evaluate seller honesty, a *pricing algorithm*, and an *allocation algorithm*, as shown in Figure 4.1. More specifically, in the proposed incentive mechanism, buyer honesty is measured by a *score* and seller honesty is measured by the *reputation*, which are updated after each transaction period. Buyer score will be updated after the buyer submits a rating, according to the normalized proper scoring rule. The normalized proper scoring rule ensures that truthful ratings provided by buyers are awarded the maximum expected scores. Seller reputation is calculated by the reputation model which aggregates ratings of the seller provided by buyers. The pricing algorithm sets higher prices for products provided by sellers with

higher reputation. The allocation algorithm ranks buyers according to their scores, and allocates products of honest sellers to buyers with high scores. As a result, sellers prefer to behave honestly in delivering promised products to achieve higher prices for their products, and buyers intend to provide truthful ratings in order to receive products from honest sellers. Therefore, the proposed incentive mechanism can promote honesty from both buyers and sellers.

4.2.1 Modeling Buyer Honesty

In this section, a class of *normalized proper scoring rules* is proposed to measure buyer honesty, where buyers providing truthful ratings about sellers will be able to gain the maximum scores.

Given a binary event with two outcomes e and $\hat{e}$, p is the actual probability of the outcome e happening and the actual probability of $\hat{e}$ happening is $1 - p$. Let x be a predicted probability of p . If the outcome of the event happens to be e , then the agent having predicted the probability as x will be rewarded the scores $\mathbf{S}(x)$, whereas if the outcome happens to be $\hat{e}$, the agent will be rewarded $\mathbf{S}(1 - x)$. The expected amount of the rewarded scores is calculated as $E(\mathbf{S}, x, p) = p\mathbf{S}(x) + (1 - p)\mathbf{S}(1 - x)$. The scoring function $\mathbf{S}(x)$ is called a proper scoring rule, if and only if $E(\mathbf{S}, p, p) \geq E(\mathbf{S}, x, p)$ and the equality is true only when $x = p$ [19]. Based on the concept of the proper scoring rule, the normalized proper scoring rule is extensively defined. The reason why proper scoring rules cannot be directly used in the proposed mechanism is that the scores are used to measure buyer honesty which should be comparable and range from 0 to 1, even when the scores are gained from transactions with sellers having different honesty levels in delivering products.

Definition 6. Normalized Proper Scoring Rule $\hat{\mathbf{S}}$ *Given a proper scoring rule $\mathbf{S}$, $Max(p) = \max_x E(\mathbf{S}, x, p)$ and $Min(p) = \min_x E(\mathbf{S}, x, p)$, a normalized proper scoring rule is defined as $\hat{\mathbf{S}}(x) = \frac{\mathbf{S}(x) - Min(p)}{Max(p) - Min(p)}$.*

From Definition 6, normalized proper scoring rules are bounded in $[0, 1]$. It is also essentially observed that they inherit the properties of the proper scoring rules, that is $E(\hat{\mathbf{S}}, p, p) \geq E(\hat{\mathbf{S}}, x, p)$, and equality is true only when $x = p$.

In order to further explain the definition, an example is given here. Suppose that a buyer is to predict the honesty of one seller in delivering promised products. A quadratic scoring rule $\mathbf{S}(x) = -2x^2 + 4x - 1$ [19] is adopted, and the maximum and minimum expected scores are calculated as: $Max(p) = 2p^2 - 2p + 1$ and $Min(p) = \min\{1 - 2p, 2p - 1\}$. According to Definition 6, the normalized proper scoring rule is:

$$\hat{\mathbf{S}}(x) = \frac{2x - (x^2 + (1 - x)^2) - \min\{1 - 2p, 2p - 1\}}{2p^2 - 2p + 1 - \min\{1 - 2p, 2p - 1\}}. \quad (\text{Eq. 4.1})$$

In the proposed mechanism, the honesty of a seller s in delivering promised products is modeled by the seller's reputation R_s , which will be introduced in detail in the next section. Thus, the probability of s being dishonest¹ is $1 - R_s$. At the end of the current transaction period t , a buyer b involved in the transaction with seller s can provide a rating indicating the honesty of the seller in the transaction. Once the rating is given, the buyer's score towards seller s will be updated, according to Eq. 4.2. In consequence, the buyer's overall scores towards all sellers will also be updated.

More formally, $\bar{r}_b^s(t) \in [0, 1]$ is calculated as the expected value of the distribution of the ratings provided by a buyer b towards a seller s till the transaction period t . The buyer b 's score towards seller s is measured as:

$$R_b^s(t) = R_s(t - 1)\hat{\mathbf{S}}(\bar{r}_b^s(t)) + (1 - R_s(t - 1))\hat{\mathbf{S}}(1 - \bar{r}_b^s(t)), \quad (\text{Eq. 4.2})$$

where $\hat{\mathbf{S}}$ is a normalized proper scoring rule, and $R_s(t - 1)$ is the reputation of seller s up to the previous transaction period $t - 1$. The total number of ratings given by b towards

¹The reputation of sellers is considered to be equivalent to and interchangeable with the probability of the sellers in providing products honestly.

s is denoted as $N_b^s(t)$. By weighted average of the scores towards different sellers, the buyer b 's overall score is:

$$R_b(t) = \frac{\sum_{s \in S} R_b^s(t) \times N_b^s(t)}{\sum_{s \in S} N_b^s(t)}, \quad (\text{Eq. 4.3})$$

where S is the set of sellers whom the buyer b has done transactions with before and provided ratings for.

Proposition 13. *Given a seller s whose reputation is $R_s(t-1)$ up to the previous transaction period, a buyer b who buys a product from s can achieve the maximum amount of scores by providing truthful ratings, if (1) $\hat{\mathbf{S}}$ is a normalized proper scoring rule, and (2) $R_s(t-1)$ truly reflects the honesty of s in delivering promised products.*

Proof. Measured by Eq. 4.2, the amount of scores that buyer b can achieve by providing ratings towards seller s is $R_s(t-1)\hat{\mathbf{S}}(\bar{r}_b^s(t)) + (1 - R_s(t-1))\hat{\mathbf{S}}(1 - \bar{r}_b^s(t))$. Based on the properties of the normalized proper scoring rule $\hat{\mathbf{S}}$, buyer b 's score is in fact equivalent to $E(\hat{\mathbf{S}}, \bar{r}_b^s(t), R_s(t-1))$, where $\bar{r}_b^s(t)$ is the expectation of the distribution of the ratings provided by buyer b towards seller s . Given the condition that the ratings provided by buyer b towards seller s are truthful, $\bar{r}_b^s(t)$ can thus truly reflect the honesty of seller s in delivering promised products, that is $\bar{r}_b^s(t) = R_s(t-1)$. Then, one has that $E(\hat{\mathbf{S}}, R_s(t-1), R_s(t-1)) \geq E(\hat{\mathbf{S}}, \bar{r}_b^s(t), R_s(t-1))$ and the equality is true only when $\bar{r}_b^s(t) = R_s(t-1)$. In another word, when $\bar{r}_b^s(t) = R_s(t-1)$, $E(\hat{\mathbf{S}}, \bar{r}_b^s(t), R_s(t-1))$ achieves the maximum value, i.e. the buyer b achieves the maximum score.

Measured by Eq. 4.2, the amount of scores that buyer b can achieve by providing ratings towards seller s is $R_s(t-1)\mathbf{S}'(\bar{r}_b^s(t)) + (1 - R_s(t-1))\mathbf{S}'(1 - \bar{r}_b^s(t))$. Based on the properties of the normalized proper scoring rule $\mathbf{S}'$, buyer b 's score is in fact equivalent to $E(\mathbf{S}', \bar{r}_b^s(t), R_s(t-1))$, where $\bar{r}_b^s(t)$ is the expectation of the distribution of the ratings provided by buyer b towards seller s . Given the condition that the ratings provided by

buyer b towards seller s are truthful, $\bar{r}_b^s(t)$ can thus truly reflect the honesty of seller s in delivering promised products, that is $\bar{r}_b^s(t) = R_s(t - 1)$. From the properties of $\mathbf{S}'$, it can be known that $E(\mathbf{S}'R_s(t - 1), R_s(t - 1)) \geq E(\mathbf{S}', \bar{r}_b^s(t), R_s(t - 1))$ and the equality is true only when $\bar{r}_b^s(t) = R_s(t - 1)$. In another words, when $\bar{r}_b^s(t) = R_s(t - 1)$, $E(\mathbf{S}', \bar{r}_b^s(t), R_s(t - 1))$ achieves the maximal value, i.e. the buyer b achieves the maximal score. Proposition 1 holds. $\square$

4.2.2 Modeling Seller Honesty

The honesty of a seller s is modeled by aggregating the ratings provided by buyers (who have conducted transactions with s), considering the buyers' honesty in providing them. More formally, at the end of the transaction period t , given the expected value $\bar{r}_b^s(t)$ of the distribution of a buyer b 's ratings towards the seller s , buyer b 's score $R_b(t)$ measured by Eq. 4.3, and the number of transactions $N_b^s(t)$ between buyer b and seller s , the reputation value (in $[0, 1]$) of seller s can be calculated as:

$$R_s(t) = \mathbf{F}(R_s(t - 1), N_{b \in B}^s(t), R_{b \in B}(t - 1), \bar{r}_{b \in B}^s(t)), \quad (\text{Eq. 4.4})$$

where B is the set of buyers whom the seller s has conducted transactions with before and received ratings from, and $R_s(t - 1)$ is seller reputation at the end of the previous transaction period $t - 1$. $\mathbf{F}$ is a reputation model which can truly measure seller honesty in delivering promised products. The specific form of $\mathbf{F}$ is not introduced, since it is application dependent and many reputation modeling approaches have already been proposed in the literature, such as [70, 109, 116].

4.2.3 Pricing and Allocating Products

In the mechanism, the pricing algorithm determines the prices for the products provided by sellers and the allocation algorithm assigns the limited products to some of the buyers. It assumes that buyers' valuation of the products is a random value chosen in an interval

$[V_*, V^*]$ where V^* and V_* are the maximum and minimum valuation of buyers towards the products provided by sellers, respectively. It also assumes that sellers have the same cost C of producing the non-difference products, and $V_* > C$ making sure that the products are worthy of being produced.

As sellers in the EMLI generally lack of the incentive to behave honestly even with reputation systems employed, because reputation information about sellers cannot impose competition among sellers in the EMLI. Sellers with relatively low reputation can still have the chance to do business with buyers because of the limited available products in the markets. The consequence is that sellers will decrease the probability of delivering the products (also reputation) to the point where buyers' utility is minimized (i.e. approaches 0) and at the same time maximize their own profit. In the mechanism, the pricing algorithm associates sellers' profit with their behavior. More specifically, it offers higher prices to products of sellers with higher reputation. In this way, it creates incentives for sellers to behave honestly. At the same time, the pricing algorithm makes sure that buyers can gain positive utility which motivates the buyers to participate in the EMLI.

In the pricing algorithm, product prices are determined by a pricing function $\mathbf{P}(R_s)$, where R_s is seller reputation modeled by Eq. 4.4. The pricing function should satisfy some basic requirements: (a) $\mathbf{P}(R_s) > 0$ for $R_s \in (0, 1]$; (b) $\mathbf{P}(0) = 0$; (c) $\mathbf{P}(\delta) = C$; (d) $\frac{d\mathbf{P}(R_s)}{dR_s} > 0$; (e) $\mathbf{P}(R_0) = R_0 \times C$. Requirement (a) ensures that the price set for sellers with positive reputation is still positive. In the extreme case where sellers never deliver products at all, the price for the sellers' products should be set 0 as in requirement (b). In requirement (c), δ is a reputation value set by the proposed mechanism so that the price of products provided by sellers with reputation δ is exactly equal to the cost C , and δ is called *cost-price* reputation. Also, the price should increase with seller reputation by satisfying requirement (d) (i.e. it should be a monotonically increasing

function), because sellers with higher reputation bear higher expected cost for delivering the promised products. Since $\mathbf{P}(0) = 0$ and $\mathbf{P}(\delta) = C$, there should exist a reputation value R_0 so that $\mathbf{P}(R_0) = R_0 \times C$ as in requirement (e), according to the continuity property of the pricing function $\mathbf{P}(R_s)$. Thus, when a seller's reputation $R_s = R_0$, the seller's profit would be $\mathbf{P}(R_0) - R_0C = 0$. In other words, R_0 is the minimum reputation that sellers can gain non-negative profit and sellers with reputation lower than R_0 will not be profitable. R_0 is also called *zero-profit* reputation, which is set by the mechanism designer. The purpose of setting the zero-profit reputation is to disappoint those sellers who intend to take advantage of the limited inventory situation by behaving dishonestly since sellers with reputation lower than R_0 will generally leave the market.

To come up with a proper but simple pricing function, a linear function for $\mathbf{P}(R_s)$ is firstly tested, however it is impossible to satisfy all the basic requirements listed above. Thus, a quadratic function is chosen which is in the general form $\mathbf{P}(R_s) = \alpha R_s^2 + \beta R_s + \gamma$. Given requirement (b), it can be calculated that $\gamma = 0$. Given requirements (c) and (e), it can be derived that $\alpha = \frac{C(1-\delta)}{\delta(\delta-R_0)}$ and $\beta = \frac{C(\delta^2-R_0)}{\delta(\delta-R_0)}$. According to requirement (d), one can also derive that $2\alpha R_s + \beta > 0$, which can be satisfied by setting the constraint $\delta \geq \sqrt{R_0}$. The pseudo code summary of the pricing algorithm is shown in Algorithm 2.

Input : S , a set of sellers offering the products;
 R_s , reputation of a seller $s \in S$ before the current transaction period;
 C, δ, R_0 , which are introduced above;

Output: P , the price for a seller's product;

```

1  $\alpha = \frac{C(1-\delta)}{\delta(\delta-R_0)}$ ;
2  $\beta = \frac{C(\delta^2-R_0)}{\delta(\delta-R_0)}$ ;
3 foreach  $s \in S$  do
4    $P_s = \mathbf{P}(R_s) = \alpha R_s^2 + \beta R_s$ ;
```

Algorithm 2: The pricing algorithm in iMEMLI

In addition, the proposed pricing algorithm has two nice properties. The first property is that buyers' utility is positive when R_0 and δ are set so as to satisfy the conditions in

Proposition 14 in Section 4.3. The second property is that buyers who conduct transactions with sellers having higher reputation will be able to gain larger utility even though the prices are higher due to the lower risks in conducting transactions with those reputable sellers (see Proposition 16). Therefore, buyers are willing to buy products from sellers with higher reputation (see Proposition 16). Due to the first property and the fact that not all buyers can be allocated with products (limited inventory), the proposed allocation algorithm ensures that honest buyers (i.e. buyers with larger scores) will have higher probabilities of being allocated with products. Due to the second property, it requires to ensure that honest buyers will also likely be allocated with products provided by sellers with higher reputation, so that honest buyers will be able to gain more profit. These create incentives for buyers to behave honestly by providing truthful ratings.

Input : B , buyers who want to buy products;
 S , sellers offering the products;
 η , the exploration factor;

Output: Allocation of products to buyers;

- 1 $S_g \leftarrow$ Randomly choose $1 - \eta$ percentage of S (products);
- 2 $S_r \leftarrow$ The rest η percentage of S (products);
- 3 Sort S_g based on seller reputation in descending order;
- 4 Sort B based on buyer scores in descending order;
- 5 **foreach** $s \in S_g$ **do**
- 6 Allocate product of s to top-ranked buyer $b \in B$;
- 7 Remove b from B ;
- 8 **foreach** $s \in S_r$ **do**
- 9 Allocate product of s to random buyer $b \in B$;
- 10 Remove b from B ;

Algorithm 3: The allocation algorithm in iMEMLI

Following the two properties of the pricing algorithm, an allocation algorithm is presented and its pseudo code is summarized in Algorithm 3. More specifically, the algorithm sets an *exploration factor* $\eta \in [0, 1]$. All the available products are randomly divided into

two sets: S_g taking $1 - \eta$ percentage and S_r taking the other η percentage of the products. S_g will be allocated to the most honest buyers, i.e. the buyers with the largest scores. To be specific, these products are sorted according to their sellers' reputation in a descending order. The buyers are also ranked in a descending order according to their scores. The products are then allocated to the buyers one by one according to the orders, so that the products of sellers with higher reputation are allocated to the buyers with larger scores (see Lines 5-7 in Algorithm 3). The computational complexity is not more than $o(n^2)$ where n is the large number of buyers and sellers in the system. Note that each buyer is allocated with one product in each transaction period. S_r will be randomly allocated to some buyers (excluding the buyers who have been allocated with the products in S_g) (see Lines 8-10 in Algorithm 3). The parameter η provides a chance for the system to "explore" honest buyers among the new buyers and the existing buyers who are not recognized.

4.2.4 Initialization to Avoid the Whitewashing Attack

This section discusses how to initialize reputation values and scores for new sellers and buyers, and at the same time avoid the whitewashing attack where buyers and sellers with low scores and reputation values leave the marketplace and re-register as new identities to clean their bad history, respectively.

For a new seller, since the system has no additional information to judge the seller's honesty, initializing the reputation for the seller is tricky. If the initial reputation is set too high, existing sellers with lower reputation values will prefer to leave the marketplace and re-register as new sellers. On the other hand, if the initial reputation is set too low (e.g. lower than R_0), new honest sellers will be discouraged to participate into the system because their initial profit is negative according to the pricing algorithm. In the mechanism, the initial seller reputation is set as δ . New honest sellers will gain zero profit

in their first transactions. If they act honestly in their transactions, they will be able to gain positive profit in their future transactions. However, existing sellers with reputation lower than δ may leave and re-enter the market. To prevent this phenomenon, some amount of membership fee $M = N_0(M_1 + M_2) = N_0C$ is imposed on new sellers, where $M_1 = (1-R_0)C$, $M_2 = R_0C$, and N_0 is the minimum number of transactions conducted by sellers so that the system can model the sellers' reputation within a required confidence level σ or error rate ε . The value of N_0 can be determined by the Chernoff Bound Theorem [70] as follows:

$$N_0 = \left\lceil -\frac{1}{2\varepsilon^2} \ln\left(\frac{1-\sigma}{2}\right) \right\rceil, \quad (\text{Eq. 4.5})$$

where $\lceil \cdot \rceil$ is a function to convert a value to the closest integer that is larger than the value. For simplicity, the seller reputation $R_s(t-1)$ up to the transaction period t is denoted by R_s by omitting the $t-1$. Similarly, the buyer score is denoted by R_b . When sellers decide to leave the marketplace, the amount M_r will be returned back to the sellers, which is:

$$M_r = \mathbf{M}(R_s) = \begin{cases} N_0(M_1 + M_2) & R_s > \delta, \\ (\frac{R_s - R_0}{\delta - R_0})^2 N_0 M_1 + N_0 M_2 & R_0 \leq R_s \leq \delta, \\ 0 & R_s < R_0. \end{cases} \quad (\text{Eq. 4.6})$$

According to the proposed mechanism, the profit of the seller s in a transaction period t can be formalized as:

$$U_s = \mathbf{U}_s(R_s) = P_s - R_s C, \quad (\text{Eq. 4.7})$$

where P_s is the price calculated by Algorithm 2 for seller s , and C is the cost of seller s in producing the promised product. A buyer b 's utility of carrying out a transaction with seller s can be formalized as:

$$U_b^s = \mathbf{U}_b(V_b^s, R_s) = R_s V_b^s - P_s, \quad (\text{Eq. 4.8})$$

where V_b^s is buyer b 's valuation for the product provided by seller s .

If a seller s whose actual reputation R_s is in (R_0, δ) performs the whitewashing attack, the reputation of s is initialized to δ at the cost of $M - M_r$. The profit of s in the first N_0 transactions after re-entry is $\sum_{t=1}^{N_0} \mathbf{U}_s(R_s(t)) \doteq \sum_{t=1}^{N_0} \mathbf{P}(R_s(t)) - N_0 R_s C + \mathbf{M}(R_s) - M < N_0[\mathbf{P}(\delta) - R_s C] + \mathbf{M}(R_s) - M$. If the seller s stays in the marketplace, the profit gained in the N_0 transactions would be $\sum_{t=1}^{N_0} \mathbf{U}_s(R_s(t)) = N_0[\mathbf{P}(R_s) - R_s C]$. Since $M = N_0 C$ and $\sum_{t=1}^{N_0} [\mathbf{U}_s^r(R_s(t)) - \mathbf{U}_s(R_s)] < N_0[\mathbf{M}(R_s)/N_0 - \mathbf{P}(R_s)]$ when $R_s \in (R_0, \delta)$, it concludes that $\mathbf{M}(R_s)/N_0 - \mathbf{P}(R_s) = [\delta(\delta - R_0) + R_0(1 - \delta)](R_s - R_0)(R_s - \delta) < 0$. It means that sellers with reputation $R_s \in (R_0, \delta)$ will gain smaller profit by re-entering. When $R_s < R_0$, $\sum_{t=1}^{N_0} \mathbf{U}_s^r(R_s(t)) < N_0(-R_s C) \leq 0$. Therefore, sellers with reputation $R_s < R_0$ cannot gain positive utility by re-entry. Thus, with the properly determined membership fee as designed in this section, the sellers whose honesty is lower than the initial reputation value do not have the incentive to perform the whitewashing attack. Since sellers with reputation higher than δ are obviously unwilling to perform the whitewashing attack by being initialized with δ , the system can effectively get rid of the whitewashing attack conducted by sellers.

For new buyers, their scores are initialized as zero. They still have the opportunity to be allocated with products because the allocation algorithm (Algorithm 3) randomly allocates η percentage of products to buyers. Since the scores assigned to buyers are non-negative according to Eq. 4.7, any buyers who want to buy products would never be willing to clean their scores by re-entering the e-marketplaces.

It is worthy to note that the designed membership fee can incentivize sellers to keep staying in the system, but cannot prevent the propose mechanism suffering from the exit problem where the agents would be dishonest in their last several transactions and then permanently disappear from the system. In the literature of trust and reputation systems, no effective and efficient solution to the exit problem has been proposed, which is also beyond the scope of this research.

4.2.5 An Example

To illustrate iMEMLI well, a simple example is used to demonstrate the whole working process. Suppose there are ten buyers b_1 to b_{10} , each of whom demands one product in each period. There are six sellers s_1 to s_6 , each of whom can supply one such product in each period. The system parameters are set as: $C = 1$, $V_* = 2$, $V^* = 3$, $\delta = 0.85$, $R_0 = 0.6$, $\eta = 0.2$, and $N_0 = 3$ (calculated by Eq. 4.5 where confidence level σ is 0.5 and error rate ε is 0.5). After conducting a transaction, each of the buyers is required to provide a binary rating. Among the ten buyers, the first five buyers (b_1 to b_5) are honest in providing ratings, and the other five (b_6 to b_{10}) share 50% of their ratings honestly and the other 50% dishonestly (opposite to the true ratings). For sellers, the first two sellers (s_1 and s_2) always provide promised products and the second two sellers (s_3 and s_4) provide promised products at 70% ($> R_0$, $< \delta$), and the last two sellers provide promised product at 50% ($< R_0$). The above described settings of buyers and sellers are shown in the first two columns of Table 4.1.

Initially, the scores of buyers are set as 0 and the reputation values of sellers as $\delta = 0.85$. Each of the sellers is required to deposit $M = N_0 C = 3$ when they register into the system. Seller reputation is first bootstrapped² by only allowing honest buyers (b_1 to b_5) to participate into the system for some periods. Suppose that seller reputation of the six sellers becomes 0.98, 0.98, 0.69, 0.69, 0.49 and 0.48, after bootstrapping, respectively. Dishonest buyers (b_6 to b_{10}) are allowed to participate into the system. Several periods later, it assumes the score for the first five buyers is 0.95 and the other five buyers 0.45. The buyer score and seller reputation are recorded in the third column in Table 4.1.

At the beginning of the next period, each of the buyers requests a product from sellers. The system then runs Algorithm 2 to determine the price for sellers and executes

²The reason of bootstrapping the reputation of sellers is to accurately initialize the sellers' reputation such that it can truthfully reflect the sellers' honesty.

Table 4.1: An example to illustrate iMEMLI

Agents	Honesty	Buyer Scores/ Seller Reputation	Buyer Valuation	Utility/ Profit	Expected Utility of Each Honesty Level
b_1	1	0.95	2.37	1.13	1.03
b_2	1	0.95	2.65	1.41	
b_3	1	0.95	2.47	0.97	
b_4	1	0.95	2.68	0.86	
b_5	1	0.95	2.35	0.79	
b_6	0.5	0.45	2.22	1.10	0.22
b_7	0.5	0.45	2.22	0	
b_8	0.5	0.45	2.60	0	
b_9	0.5	0.45	2.46	0	
b_{10}	0.5	0.45	2.47	0	
s_1	1	0.98	—	0.24	0.24
s_2	1	0.98	—	0.24	
s_3	0.7	0.69	—	0.03	0.03
s_4	0.7	0.69	—	0.03	
s_5	0.5	0.49	—	-0.05	-0.05
s_6	0.5	0.48	—	-0.05	

Algorithm 3 to allocate the products to some of the buyers. According to Algorithm 2 ($P_s = \alpha R_s^2 + \beta R_s$, where $\alpha = 0.71$ and $\beta = 0.58$ calculated given the system parameters), the prices for the sellers are 1.24, 1.24, 0.73, 0.73, 0.45, and 0.44, respectively. In Algorithm 3, the set of sellers are randomly divided into two subsets: $S_r = \{s_3\}$ and $S_g = \{s_1, s_2, s_4, s_5, s_6\}$. For S_g , the sellers are sorted based on their reputation in a descending order (randomly break the tie), and the ranked order is assumed to be: s_1, s_2, s_4, s_5, s_6 . Meanwhile, the buyers are also sorted based on their scores in a descending order, and the ranked result is supposed to be: b_1 to b_{10} . The system first allocates S_g to buyers, according to the ranks. Specifically, the product provided by s_1 is allocated to b_1 , and the product provided by s_2 is allocated to b_2 , etc. Secondly, the system allocates S_r randomly among the buyers who have not been allocated with any product, i.e. b_6 to b_{10} , and $s_3 \in S_r$ is allocated to one buyer, supposing b_6 . Assuming that the buyers have different valuation towards the product, which follows a Gaussian distribution (2.5,

0.2)³ in the interval $[V_*, V^*]$, the valuations of the buyers are assumed to be as displayed in the fourth column of Table 4.1. Given the allocation, the utility of buyers and sellers is calculated according to Eq. 4.8 and Eq. 4.7, which are listed as in the fifth column of Table 4.1. In the sixth column, the average utility for each type of buyers and sellers is calculated. It shows that the average utility of honest buyers and dishonest buyers are 1.03 and 0.22, respectively. The average profit of the first two sellers is 0.24, and the second two sellers is 0.03, where the profit of the four sellers are positive and increase as their honesty increases ($0.24 > 0.03 > 0$). Nevertheless, the average profit of the last two sellers is -0.05 (< 0), as their honesty is lower than the zero-profit reputation R_0 .

In this example, if a buyer, e.g. b_6 , leaves the system and registers under a new identity, then his score (0.45) will be initialized as 0. The allocation algorithm will then rank the buyer at the last position so that the buyer ends up with less chance to get the products from S_g . For the first two sellers, if they leave, the system will return all the deposit membership fee ($Mr = M$) since their reputation is greater than the initial reputation ($R_s > \delta$). However, their reputation will be initialized as $\delta = 0.85$ which are lower than their current reputation values when they re-register as new sellers. Thus, they will lose profit by reentering. For the second two sellers, their reputation is lower than the initialized reputation but larger than R_0 . If they leave, the system will return a part of the membership fee $M_r = 1.95 < M$ and they lose 1.05. When they re-register, their reputation is also initialized as δ . However, the profit gained from reentering is less than $N_0(1 - \delta)C = 0.45$ which cannot cover their loss ($0.45 < 1.05$) of leaving⁴. For the last two sellers, they will lose all the membership fee when they leave. After reentering the system, the estimated gain is still negative. Therefore, both buyers and sellers in the system have no incentive to conduct the whitewashing attack.

³The mean of the distribution is equal to $(V_* + V^*)/2 = 2.5$, and the standard deviation is 0.2.

⁴After N_0 transactions, the reputation value will truthfully reflect the honesty of the seller.

4.3 Analysis of the Mechanism

In this section, the properties of IMELI are analyzed in terms of individual rationality, incentive compatibility, social welfare, and robustness against various of collusive attacks. Moreover, the semantic meanings of parameters and the practicability of the proposed mechanism are summarized.

4.3.1 Individual Rationality

Individual rationality is a property describing an individual's incentive to participate in a marketplace. It shows that both buyers and sellers can obtain positive utility in the system. In addition, the price is upper bounded for buyers and lower bounded for honest sellers, which assists both buyers and sellers in deciding whether to participate into the e-marketplaces in advance.

Proposition 14. *The utility of a buyer gained in a transaction with a seller s is positive, if (a) the seller reputation satisfies $R_s > R_0$, and (b) δ satisfies the following inequality:*

$$\delta > \max \left\{ \sqrt{R_0}, \frac{V_* R_0 - 2C + \sqrt{(V_* R_0 - 2C)^2 + 4C(V_* - C)(2 - R_0)}}{2(V_* - C)} \right\}. \quad (\text{Eq. 4.9})$$

The profit of a seller is positive if his reputation is greater than R_0 , and the profit is negative if her reputation is less than R_0 , given that the inequality Eq. 4.9 is satisfied.

Proof. Firstly the rationality of buyers is proven. According to Algorithm 2, the price of the product provided by seller s is calculated by the quadratic function: $P_s = \alpha R_s^2 - \beta R_s$.

Then, the utility function of buyer b who is allocated the product is calculated as:

$$\begin{aligned} U_b(V_b^s, R_s) &= R_s V_b^s - P_s \geq R_s V_* - P_s \\ &= U_b(V_*, R_s) \\ &= R_s V_* - \frac{C(1-\delta)}{\delta(\delta-R_0)} R_s^2 - \frac{C(\delta^2-R_0)}{\delta(\delta-R_0)} R_s \\ &= \frac{V_* \delta(\delta-R_0) - C(\delta^2-R_0)}{\delta(\delta-R_0)} R_s - \frac{C(1-\delta)}{\delta(\delta-R_0)} R_s^2. \end{aligned}$$

The derivative of $\mathbf{U}_b(V_*, R_s)$ with respect to R_s is calculated as:

$$\begin{aligned}
\frac{d\mathbf{U}_b(V_*, R_s)}{dR_s} &= \frac{V_*\delta(\delta-R_0)-C(\delta^2-R_0)}{\delta(\delta-R_0)} - 2\frac{C(1-\delta)}{\delta(\delta-R_0)}R_s \\
&> \frac{V_*\delta(\delta-R_0)-C(\delta^2-R_0)}{\delta(\delta^2-R_0)} - 2\frac{C(1-\delta)}{\delta(\delta-R_0)} \\
&= \frac{V_*\delta(\delta-R_0)-C(\delta^2-R_0)-2C(1-\delta)}{\delta(\delta-R_0)} \\
&= \frac{(V_*-C)\delta^2+(2C-V_*R_0)\delta+(CR_0-2C)}{\delta(\delta-R_0)}.
\end{aligned}$$

It could be observed that the numerator of the right-hand side of Eq. 4.10, $(V_* - C)\delta^2 + (2C - V_*R_0)\delta + (CR_0 - 2C)$ is greater than 0 when $\delta = 1$, and less than 0 when $\delta = R_0$. Since the numerator is a continuous quadratic function of variable δ and $V_* - C > 0$, it can be claimed that there exists $\delta_0 = \frac{V_*R_0-2C+\sqrt{(V_*R_0-2C)^2+4C(V_*-C)(2-R_0)}}{2(V_*-C)} \in (R_0, 1)$ ensuring $(V_* - C)\delta^2 + (2C - V_*R_0)\delta + (CR_0 - 2C) > 0$. Therefore, if $\delta > \max\{\sqrt{R_0}, \delta_0\}$, then $\frac{d\mathbf{U}_b(V_*, R_s)}{dR_s} > 0$ for $R_s \in [R_0, 1]$. In addition, $\mathbf{U}_b(V_*, R_0) = R_0V_* - \mathbf{P}(R_0) = R_0(V_* - C) > 0$, due to requirement (e) for $\mathbf{P}(R)$ and $V_* > C$. Therefore, buying products from sellers with reputation larger than R_0 will bring positive utility for buyers.

In Algorithm 3, $1 - \eta$ percentage of products are allocated to buyers according to their honesty levels, i.e. the products of the sellers with the highest reputation are allocated to the buyer who have highest score, and the other η percentage of the products are randomly allocated among the remaining buyers. In this way, the expected reputation of sellers who conduct transaction with more honest buyers will be higher, which results in higher expected buyer utility.

Next, the rationality of sellers is proven. According to Eq. 4.7 and Algorithm 2, the profit of a seller s can be presented as:

$$\begin{aligned}
U_s &= \frac{C(1-\delta)}{\delta(\delta-R_0)}R_s^2 + \frac{C(\delta^2-R_0)}{\delta(\delta-R_0)}R_s - R_sC \\
&= \frac{C(1-\delta)}{\delta(\delta-R_0)}R_s^2 + \frac{R_0C(\delta-1)}{\delta(\delta-R_0)}R_s \\
&= \frac{C(1-\delta)}{\delta(\delta-R_0)}R_s(R_s - R_0).
\end{aligned}$$

If the seller's reputation R_s is greater than R_0 , the utility of the seller s will be positive, and vice versa, since $\delta < 1$ and $\delta > \sqrt{R_0} > R_0$ according to Proposition 14. $\square$

In the proposed mechanism, the buyers can always achieve positive utility given the two conditions being satisfied, which motivates them to participate into the marketplaces. The first condition requires that the buyers should conduct transactions with sellers whose reputation is larger than R_0 , and the second condition provides a guide to set a proper δ . The interpretation of the parameters is summarized in Section 4.3.5. Meanwhile, the sellers whose reputation is larger than R_0 have incentives to participate in the system to achieve the positive profit. Nevertheless, those sellers whose reputation is less than R_0 are discouraged to appear in the system or have to improve their honesty to at least R_0 .

In addition, considering the assumptions (d) and (e) where buyers are able to afford the price for a product and sellers are able to deliver the promised products, the following proposition shows that the price for a product is upper bounded for buyers and the price is lower bounded for honest sellers, which can assist buyers with limited budget and rational sellers to join the marketplace.

Proposition 15. *The upper bound of the price for a product is $\frac{C}{R_0}$, the lower bound of the price for a product provided by an honest seller is $\frac{C}{\delta}$ if Eq. 4.9 is satisfied, where C is the production cost,*

Proof. In this proof, it firstly shows that the price of a product for buyers is upper bounded. As the price for a product provided by a seller is determined by the function $\mathbf{P}(R_s) = \alpha R_s^2 + \beta R_s$. The maximum price is $P_b^{max} = \alpha + \beta$ due to the fact that $\alpha = \frac{C(1-\delta)}{\delta(\delta-R_0)} > 0$, and $\beta = \frac{C(\delta^2-R_0)}{\delta(\delta-R_0)} > 0$ when Eq. 4.9 is satisfied. Since $\delta > \sqrt{R_0}$, the following equation can be obtained:

$$\begin{aligned} P_b^{max} &= \alpha + \beta = \frac{C(1+\delta^2-\delta-R_0)}{\delta(\delta-R_0)} \\ &= \frac{C(1-\delta)^2+C(\delta-R_0)}{\delta(\delta-R_0)} = \frac{C(1-\delta)^2}{\delta(\delta-R_0)} + \frac{C}{\delta} \\ &< \frac{C(1-\delta)^2}{\delta(\delta-\delta^2)} + \frac{C}{\delta} = \frac{C(1-\delta)+C\delta}{\delta^2} = \frac{C}{\delta^2}. \end{aligned}$$

According to Eq. 4.9, $\delta^2 > R_0$. Therefore, $\frac{C}{\delta^2} < \frac{C}{R_0}$, i.e. the price is upper bounded by $\frac{C}{R_0}$.

Next, it proves that the price for honest sellers is lower bounded. For an honest seller ($R_s = 1$), the price for his product is determined by Algorithm 2. Thus $P_s^{min} = \mathbf{P}(1) = \alpha + \beta$. Since $\delta > \sqrt{R_0} > \delta^2$, it can be obtained that:

$$\begin{aligned} P_s^{min} &= \alpha + \beta = \frac{C(1+\delta^2-\delta-R_0)}{\delta(\delta-R_0)} \\ &= \frac{C(1-\delta)^2+C(\delta-R_0)}{\delta(\delta-R_0)} > \frac{C}{\delta}, \end{aligned}$$

which means that the price for the products provided by the honest sellers is always greater than $\frac{C}{\delta}$. In other words, the price of the products provided by an honest seller is lower bounded by $\frac{C}{\delta}$. $\square$

Since the prices of products are upper bounded by $\frac{C}{R_0}$, the budget of buyers is required to be at least $\frac{C}{R_0}$. To simplify the proposed model and focus on the incentive problem, the budget constraint problem is neglected by setting the assumption (d) in Section 4.1. However, this upper bound indicates that this assumption is not too strict to be satisfied. The case where some of the buyers are unable to afford the price ($< \frac{C}{R_0}$) is still valuable to be studied in the future work as discussed in Section 7.2.2. In addition, since the range of price for the products provided by honest sellers is between $\frac{C}{\delta}$ and $\frac{C}{R_0}$ where $\frac{C}{R_0} > \frac{C}{\delta} > C$, the profit of honest buyers could always be positive and they have the incentives to participate, and keep behaving honestly in the proposed incentive mechanism.

4.3.2 Incentive Compatibility

The incentive compatibility in the analysis is a property where each individual has the incentives to be honest, provided that others are also honest. The incentive of buyers and sellers to be honest is shown in this section, respectively.

4.3.2.1 Buyer incentive

Proposition 16. *The utility of a buyer in conducting transactions with a seller s increases as the seller's reputation R_s increases, if (a) $R_s > R_0$, and (b) Eq. 4.9 is satisfied.*

Proof. According to the proof of Proposition 14, the first order derivation of the buyer utility function is positive when $R_s > R_0$, due to requirement (e) for $\mathbf{P}(R_s)$ and $V_* > C$. Therefore, buying products from sellers with higher reputation will bring larger utility for buyers. $\square$

The conditions in the proposition are the same as those in Proposition 14, which indicates that buyers could achieve more utility by conducting transactions with sellers having higher reputation in spite of the higher price charged for the sellers' products. The main reason of this phenomenon is that the higher price of the products provided by more honest sellers is covered by the benefit that is obtained from the transaction with the honest sellers.

Proposition 17. *The utility of a buyer who provides truthful ratings is no less than that of a buyer who provides untruthful ratings.*

Proof. According to the proof of Proposition 14, the first order derivation of the buyer utility function is positive when $R_s > R_0$, due to requirement (e) for $\mathbf{P}(R_s)$ and $V_* > C$. Therefore, buying products from sellers with higher reputation will bring larger utility for buyers. $\square$

From the perspective of buyers, they can obtain more utility from transactions with sellers who have higher reputation, and meanwhile, they prefer to provide truthful ratings rather than untruthful ratings in order to achieve the larger utility. In short, the proposed mechanism promotes buyer honesty in providing truthful ratings.

4.3.2.2 Seller incentive

Proposition 18. *Sellers have the incentives to improve their reputation by delivering promised products.*

Proof. For a seller with the reputation R_s , the seller utility function is $\mathbf{U}_s(R_s) = \frac{C(1-\delta)}{\delta(\delta-R_0)}R_s(R_s-R_0)$ (refer to Eq. 4.10). The derivation of the seller utility $\mathbf{U}_s(R)$ with respect to the seller reputation R_s is:

$$\frac{d\mathbf{U}_s(R_s)}{dR_s} = \frac{C(1-\delta)}{\delta(\delta-R_0)}(2R_s - R_0). \quad (\text{Eq. 4.10})$$

Since a rational seller existing in the system should sustain a reputation value which is greater than R_0 (refer to Proposition 14), it has $\frac{d\mathbf{U}_s(R_s)}{dR_s} > 0$. By improving reputation, the seller can increase his utility of selling the products at higher prices. Therefore, sellers have the incentive to improve their reputation. Moreover, since buyers are more willing to provide truthful ratings (refer to Proposition 17), sellers can only improve their reputation by behaving honestly, i.e. delivering promised products. $\square$

Therefore, from the perspective of sellers, it is profitable for the sellers to be honest by delivering the promised products to build up their reputation. In addition, rational sellers will not let their reputation become less than R_0 otherwise their utility will become negative. To conclude, the proposed mechanism promotes seller honesty in delivering promised products.

4.3.3 Social Welfare

The above analysis is discussed from the perspective of individuals (sellers and buyers), and now the social welfare is analyzed to evaluate the efficacy of the proposed mechanism. It would consider the social welfare created by the buyers and sellers in their current transactions and future transactions. For the buyers without any allocated products, they would not create loss or significant gain for the social welfare. Since the products are limited, some buyers will not be allocated with products in any way. Among these buyers, some of them who are dishonest and indicated by low score are deserved to be punished by not being allocated with the products. Thus, the social welfare created

by them is not negative, but slightly positive by avoiding the limited products being allocated to these dishonest buyers. The rest of these buyers may be honest often and have achieved relative high scores, but not so high to be allocated with products. The social welfare created by these agents is also zero, as the products have been allocated to more honest buyers or buyers with the same level of honesty. For this reason, the social value of of all the agents who are not allocated with products are set as zero.

For a transaction happening between a seller s and a buyer b , the current social utility brought by them would be the sum of their utility. Since an honest buyer b always shares truthful ratings to reveal the honesty of sellers, which is helpful for the system to discover honest sellers, the system will obtain certain future utility from the transactions conducted with the honest buyers (with a higher score). Therefore, the buyer score has the same effect as the *discount rate* in calculating the future social welfare, i.e. the higher the discount rate (buyer score) is, the more future social welfare could be expected. The total social welfare, including the future social welfare, is then equivalent to the welfare in the current transaction period multiplied by an additional factor $\frac{1}{1-R_b}$. Thus, the social welfare of a transaction happening between seller s and buyer b can be formalized as:

$$\mathbf{W}(R_s, R_b) = (\mathbf{U}_s(R_s) + \mathbf{U}_b(R_s)) \frac{1}{1-R_b} = R_s(V_b^s - C) \frac{1}{1-R_b}, \quad (\text{Eq. 4.11})$$

where V_b^s is the valuation of the product provided by s , which is uniformly distributed between V_* and V^* .

Proposition 19. *The proposed iMEMLI can increase the total social welfare as defined in Eq. 4.11.*

Proof. According to Eq. 4.11, the total social welfare can increase if the social welfare increases in each transaction. The first order derivation of the W with respect to R_s and R_b can be calculated as:

$$\frac{d\mathbf{W}(R_s, R_b)}{dR_s} = (V_b^s - C) \frac{1}{1-R_b} > 0, \quad (\text{Eq. 4.12})$$

and

$$\frac{d\mathbf{W}(R_s, R_b)}{dR_b} = R_s(V_b^s - C) \frac{1}{(1 - R_b)^2} > 0. \quad (\text{Eq. 4.13})$$

Thus, the social welfare gets improved if seller s and buyer b increase their reputation and scores, respectively. According to Proposition 17 and Proposition 18, both sellers and buyers have incentive to improve their honesty. Therefore, the total social welfare of the system will be increased. $\square$

The intuition behind this proposition is that the social welfare will increase as both buyers and seller have the incentive to improve their honesty. Both the dishonest buyers and dishonest sellers are harmful for the social welfare. Since untruthful ratings provided by dishonest buyers will discourage sellers to be honest and the dishonest sellers will increase the number of unsatisfactory transactions, the total utility created by the system will be decreased. In addition, as both buyers and sellers in the proposed system are motivated to behave honestly, the system will generally become a safer transaction platform, resulting in that more and more honest buyers and sellers will be attracted by the system in the long run.

In addition, supposing that the valuations of the products for buyers follow a certain distribution which is independent of the buyer honesty, it can be shown that the social welfare achieved by the proposed mechanism is actually no less than the free-trading marketplace where the buyers and sellers interact freely and have the same chance to conduct transactions.

Proposition 20. *The social welfare of iMEMLI is no less than that of the free-trading e-marketplace.*

Proof. The expected social welfare of the system using iMEMLI and a free-trading e-marketplace in a transaction are denoted as w_1 and w_2 . Given that the valuation of

products for buyers and the honesty of buyers are independent, the expected value of the social welfare is calculated as described in Eq. 4.11 as:

$$E(\mathbf{W}(R_s, R_b)) = E\left(R_s(V_b - C) \frac{1}{1 - R_b}\right) = E(V_b - C) \times E\left(\frac{R_s}{1 - R_b}\right), \quad (\text{Eq. 4.14})$$

where $E(\cdot)$ denotes the expected value of a variable.

According to the proposed allocation algorithm, the products are divided into two parts. The first part of products (taking for $1 - \eta$) are allocated to buyers with highest scores and the second part of products (taking for η) are randomly allocated among buyers. Then it obtains that $w_1 = (1 - \eta)w_g + \eta w_r$, where w_g is the expected social welfare in the first part of transactions and w_r is the expected social welfare in the second part of transactions. It can be concluded that $w_r = w_2$ due to the fact that the second part of products are allocated in the same way as in the free-trading marketplace. To compare w_g and w_2 , they are further described as $w_g = E_g(V_b - C) \times E_g(\frac{R_s}{1 - R_b})$ and $w_2 = E_2(V_b - C) \times E_2(\frac{R_s}{1 - R_b})$. Considering that the proposed allocation algorithm is independent of the buyers' valuations and the valuations are independent of the honesty, it can be concluded that $E_g(V_b - C) = E_2(V_b - C)$. Thus, it only needs to compare $E_g(\frac{R_s}{1 - R_b})$ and $E_2(\frac{R_s}{1 - R_b})$.

Since the products are randomly allocated to buyers in the free-trading e-marketplace, then $E_2(\frac{R_s}{1 - R_b}) = E_2(R_s)E_2(\frac{1}{1 - R_b})$. According to the proposed allocation, the products provided by sellers with higher reputation will be allocated to buyers with higher scores. In this case, $E_g(\frac{R_s}{1 - R_b})$ will achieve its maximum value which is no less than $E_2(R_s)E_2(\frac{1}{1 - R_b})$. Thus, it can be derived that $w_g \geq w_2$. Therefore, $w_2 \geq w_1$. To conclude, the social welfare of the proposed system is no less than the free-trading e-marketplace. $\square$

In this proposition, the social welfare of the proposed system is compared with that of the free-trading e-marketplace to show the efficiency of the proposed mechanism. The

main reason that the proposed mechanism can achieve higher social welfare than that in the free-trading marketplace is that it satisfies honest buyers and sellers simultaneously, which creates higher future benefit for the system.

4.3.4 Robustness against Attacks

In Sections 4.2.4 and 4.2.5, iMEMLI has been shown to be an effective approach against the whitewashing attack. This section discusses the robustness of the mechanism against other general types of attacks. Two types of attacks are considered: rational and irrational attacks. The agents conducting rational attacks aim to maximize their utility by behaving dishonestly. The agents who conduct irrational attacks, on the other hand, attack the system regardless of the cost. Firstly, the rational attacks are categorized and the robustness of the proposed mechanism against each of them is analyzed. Secondly, it analyzes how the proposed mechanism could be impacted by irrational buyers and sellers, respectively.

Rational attacks are categorized and analyzed as follows.

- (a) *Buyer uncoordinated attack*: A buyer performs malicious behavior in order to maximize his utility, without coordinating with other buyers or sellers. In iMEMLI, buyer's possible malicious behavior is to report untruthful ratings. Each buyer has incentive to provide truthful ratings so as to achieve a high score given that the seller reputation is accurate as claimed in Proposition 13. If seller reputation is not accurate, then buyers may lose the incentive to provide ratings honestly. However, in this case, as long as the ranking of buyers in terms of their honesty is accurate, buyers are still incentivised to report honestly, since it is the order instead of the scores used in the proposed allocation algorithm.
- (b) *Seller uncoordinated attack*: A seller delivers low quality products or does not deliver in order to maximize his utility, without coordinating with other sellers or buyers.

As analyzed in Section 4.3.2.2, each seller has incentive to improve his reputation so that a higher price can be achieved for his products. If buyers report truthful ratings, then the seller can only improve his expected reputation by improving his honesty in delivering products. In other words, a rational seller has no incentive to conduct the seller uncoordinated attack.

(c) *Buyer coordinated attack*: A set of buyers coordinate their malicious behavior to maximize their group utility. When buyers coordinately provide untruthful ratings to a seller such that the price of the products provided by the seller becomes low, the utility of the buyer group will increase. However, the allocation is determined by the system which makes it impossible for the group of buyers to always conduct transactions with the attacked seller. Thus, the proposed mechanism can effectively weaken the impact of the buyer coordinated attack. Nevertheless, as a large number of the buyers (even all the buyers) form a group to attack a seller, the seller becomes non-immune to the attack. Therefore, iMEMLI can be robust against the buyer coordinated attack to a great extent.

(d) *Seller coordinated attack*: A set of sellers form a group and cooperatively conduct malicious behavior to maximize their group utility. For an uncoordinated seller, as analyzed above, he has no incentive to misbehave. However, for a group of sellers, such honesty incentive may be influenced. For example, the colluding sellers can cooperatively choose not to deliver products to a particular buyer (discriminated buyer) but deliver promised products to all other buyers. As a result, when the discriminated buyer truthfully provides negative ratings, the system will assign him a low score which mitigate the influence of the buyer's ratings. However, the allocation algorithm cannot always assign the attacked buyer to one of the collusive sellers. As the attacked buyer will be treated fairly by other sellers out of the attacking group, it

becomes harder for the small group of sellers to attack some particular buyers. As the number of collusive sellers increases, the particular buyer will be more often attacked. Then, the truthful ratings from the buyer will be significantly discounted. The group of sellers will continually attack the buyer to gain more profit without losing their high reputation. Finally, the buyer will lose his incentive to report truthful ratings. Therefore, iMEMLI is robust against the seller coordinated attack to some extent.

- (e) *Buyer-seller coordinated attack*: A set of buyers and a set of sellers form a group and coordinate their behavior to maximize their group utility. Buyer-seller coordinated attack contains both buyer coordinated attack and seller coordinated attack. Because iMEMLI is only robust against buyer coordinated attack and seller coordinated attack to some extent, separately, the proposed mechanism is unable to achieve perfect performance against the buyer-seller coordinated attack.

Next the irrational attacks are discussed. The irrational attackers are different from rational agents, as their behavior is not motivated by the utility achieved in the mechanism. The reasons of the irrationality of agents are complicated and hard to illustrate in a general logic [83]. The analysis will focus on how the irrational agent will impact the efficiency of iMEMLI in promoting honesty from other rational agents, where the irrational agents are separately discussed.

- (a) *Irrational buyers*: Irrational buyers are buyers who attack the system by providing untruthful ratings regardless of the cost. They can be newly registered or disguised by the existing sellers, and their behavior can be in a coordinated or uncoordinated way. When new irrational buyers enter the system, the system initializes their scores to zero. As in Algorithm 3, the parameter η indicates the probability that buyers with low score (including new buyers) are allocated with products. The value of η could be set close to 0 in a marketplace when the set of honest buyers is stable. In that case,

the new irrational buyers will have little chance to be allocated with products and provide ratings. However, for a newly formed marketplace, the initial value of η needs to be set greater than 0. In this case, new irrational buyers have a greater chance to attack the system. In other situations, between the two extreme cases, new irrational buyers have some influence in the system but the impact is limited. For the existing irrational buyers, who have already earned high scores, can effectively manipulate seller reputation by providing untruthful ratings. However, after providing untruthful ratings, the attackers' scores will decrease, and later untruthful ratings provided by them become less influential. Thus, a bounded number of influential untruthful ratings can be provided by an existing irrational buyer through sacrificing his score, and the seller reputation would suffer from bounded impact. Therefore, iMEMLI can mitigate the impact of the existing irrational buyers to some extent.

- (b) *Irrational sellers*: Irrational sellers are those who deliver low quality products or do not deliver at all. The irrational sellers can also be newly registered sellers or disguised by the existing sellers in a coordinated or uncoordinated way. When irrational sellers conduct malicious behavior, rational buyers will provide truthful negative ratings which can decrease the sellers' reputation value. The decreased seller reputation can reflect the irrational seller's honesty. According to Proposition 17, the incentives of buyers will not be impacted. As the existence of some number of irrational sellers cannot affect the honesty incentive of rational buyers, iMEMLI can be robust against irrational sellers to some extent.

To conclude, iMEMLI is more robust against uncoordinated attacks than coordinated attacks, and it is still robust against coordinated and irrational attacks to some extent. In a realistic scenario, the attacks can be sophisticated and complex, and the proposed mechanism still cannot achieve perfect robustness against all types of attacks. In the

future work, the various type of attacks mentioned herein will be considered, for the design of a robust incentive mechanism.

4.3.5 Parameters

According to the analysis in Sections 4.3.1 and 4.3.2, the parameters, i.e. R_0 , δ and η , can impact the performance of iMEMLI. The semantic meaning of each parameter and their possible configurations are explained as follows:

- (a) *Zero-profit reputation R_0* : In the pricing algorithm, $R_0 \in [0, 1]$ represents the seller reputation value at which sellers earn zero profit. Sellers prefer a smaller R_0 to easily gain positive profit. However, buyers prefer a higher R_0 , since the upper bound of the price for a product is $\frac{C}{R_0}$ (refer to Proposition 15), meaning that a higher R_0 can decrease the bound. In setting R_0 , the system should balance the preferences of sellers and buyers. To achieve a good balance, three main factors should be considered. The first factor is the risk borne by buyers and sellers. R_0 should be set so that both the risks borne by buyers and sellers are balanced at the same level (the same expected marginal profit). The second factor is the importance of buyers and sellers. The system should set R_0 to favor the more important side (buyers or sellers) where the importance degree is determined by the system organizers or designers. The third factor is the relationship between supply and demand. The system should increase R_0 when the supply increases and decrease when the demand increases. When those factors are conflict with each other, the system should preferably consider more significant factors.
- (b) *Cost-price reputation δ* : In the pricing algorithm, $\delta \in [0, 1]$ represents the seller reputation value with which a seller will be assigned the price that is equal to the cost C for his products. Sellers prefer a smaller δ , because the lower bound of the

price for sellers is $\frac{C}{\delta}$ (refer to Proposition 15). However, δ cannot be too low, since buyer incentive computability requires a lower bound for δ as in Eq. 4.9 (refer to Proposition 16). Therefore, δ is set to be equal to the right-hand side of Eq. 4.9.

- (c) *Exploration factor η* : In the allocation algorithm, $\eta \in (0, 1)$ represents the percentage of products that are randomly allocated among buyers (including new buyers). First, η should be positive, due to the following two reasons: (a) η provides a chance for new buyers to be allocated with products and allows the system to discover honest buyers from those newcomers; (b) a positive η can also allow buyers with low scores to gain high scores by providing truthful ratings, instead of having to whitewash their scores by reentering the system. In addition, η determines the strength of incentive provided for the existing buyers. A lower η means that a larger proportion of products will be allocated to honesty buyers, which provides higher incentives for the existing honest buyers to report truthful ratings. Therefore, in order to set a proper η , the system should balance between the existing honest buyers and new buyers, as well as dishonest buyers. For a newly formed e-marketplace, η should be set relatively high to attract and discover more new honest buyers. While for a stable system with few newcomers, η should be set as a small positive value.

4.3.6 Practicability

The proposed iMEMLI is a centralized system to promote buyer and seller honesty for EMLI. The price of products is determined by the pricing algorithm considering the sellers' reputation, and the matching between products and buyers is implemented by the allocation algorithm based on the buyers' score. However, iMEMLI is a practical incentive mechanism in the following aspects:

- iMEMLI is consistent with the general phenomenon in the real world and the design of many existing (incentive) mechanisms. Firstly, the idea of providing

price premium to honest sellers is well supported in the literature. It is generally observed that the price of products provided by sellers with higher reputation is also higher in many empirical studies of e-marketplaces [12, 65]. In this sense, the proposed pricing algorithm is a natural realization and extension of this observed phenomenon.

Secondly, iMEMLI allocates limited products to honest buyers so that dishonest buyers have less chance to conduct transactions, which has the similar spirit with several existing incentive mechanisms. More specifically, for the honest buyers, they can gain more utility which is similar to the side-payment incentive mechanism where the extra side-payment will be rewarded to honest buyers [37]; for the dishonest buyers, the probability of conducting transactions is decreased, which is similar to the credibility mechanism where these dishonest buyers will not be allowed to conduct transactions for a period of time [81]. Moreover, the effect of the pricing and allocation algorithms has certain similarity with the double auction mechanism where sellers “ask” by specifying the cost in producing the products and buyers “bid” by providing their valuation towards the products. From the perspective of communication complexity, iMEMLI only requires both buyers and sellers to submit their requests (bid or ask). This does not involve new information, compared with a typical auction mechanism. The double auction mechanism, based on the costs and valuations, determines which pairs of buyers and sellers are able to conduct transitions and how much money the buyer or seller should pay or be paid, respectively, such that the products are purchased by the desirable buyers. In iMEMLI the reputation of sellers and scores of buyers are modeled. The reputation and scores are then taken into account to determine the allocation of the limited products and their prices. Through the allocation algorithm, the products provided by the sellers with highest reputation are allocated to buyers with highest scores

(desired buyers) at the price determined by the pricing algorithm. For this reason, an auction-based design of incentive mechanism will be investigated in future work as explained in Section 7.

Thirdly, the centralized design is common among the existing incentive mechanisms, such as Trunit [51], trust-based mechanism [14], the incentive mechanism in [123], etc.

- As proved in Section 4.3.1, both buyers and sellers have incentives to participate into the system. In iMEMLI, the honest sellers are rewarded with higher prices for their products and honest buyers gain the limited products provided by the honest sellers. In this case, honest buyers and sellers will be motivated to participate into the system. Since the existence of these buyers and sellers, more buyers and sellers will participate too. Moreover, the incentives for buyers and sellers will motivate dishonest buyers and sellers change their behavior so as to achieve more benefit. Thus, it realizes a virtuous circle in the proposed system where more and more buyers and sellers will join the system and they are also motivated to become more and more honest.
- The social welfare of iMEMLI is increasing and no less than the free-trading e-marketplace (proved in Proposition 10). The allocation algorithm tries to match products from honest sellers to honest buyers in the proposed mechanism. The social welfare also includes the future profit of the system by considering the honesty of buyers and sellers in transactions. Once buyers and seller have incentives to improve their honesty, the social welfare can be absolutely increased due to the increase of satisfactory transaction ratio. The way of bringing the honest sellers and buyers in a transaction further increases the future benefit of the system, making the social welfare of iMEMLI is no less than the free-trading e-marketplace. In order

to further improve the social welfare, the valuations of products for buyers will be considered in the allocation algorithm in the future work. Since the valuation is private information, it may be necessary to apply the auction theory to incentivize buyers to reveal their true valuation [52], which will be an important research direction to optimize the social welfare as discussed in Section 7.2.2.

4.4 Experimental Evaluation

In this section, a set of experiments are carried out to evaluate iMEMLI. The hypotheses are (1) both buyers and sellers have the incentive to behave honestly; (2) the designed membership fee can discourage sellers from conducting the whitewashing attack; (3) the proposed incentive mechanism can perform well regardless of sellers inventory. In the following experiments, first of all, the incentives created by the proposed mechanism are validated in both static and dynamic settings. In the static setting, there is no new seller or new buyer participating in the marketplace during the process of experiments. While in the dynamic setting, some new sellers and buyers join the marketplace periodically. Secondly, iMEMLI is shown to be able to discourage sellers from conducting the whitewashing attack, by initializing the reputation of a new seller as δ and applying the membership fee designed in Section 4.2.4. Finally, iMEMLI is compared with the side-payment mechanism [37], in marketplaces with unlimited (EMUL) and limited inventory (EMLI), respectively. The side-payment mechanism and other mechanisms are commonly facing the new challenges discussed in Section 2.2 when they are applied in EMLI. In the comparison, the new challenges in promoting honesty in EMLI is explicitly shown. Moreover, the side-payment mechanism is a representative mechanism among the existing incentive mechanism. Therefore, the proposed mechanism is compared with the side-payment mechanism. The experimental results show that the three hypotheses are true.

4.4.1 Experimental Settings

The e-marketplace environment in the experiments involves sellers and buyers conducting transactions. The total number of (same) products provided by the sellers is less than that of the buyers' demand, i.e. a market with the limited inventory. The parameters are set as follows: $R_0 = 0.6$, $\delta = 0.85$ (satisfying Proposition 16), the cost in producing promised product $C = 1$, the minimal valuation of buyers towards the product $V_* = 2$, the maximum valuation of buyers towards the product $V^* = 2.5$, and the exploration factor $\eta = 0.1$. A set of simulations with various settings have been implemented and the results are similar, so it only shows the results of one specific setting as described above.

In the simulations, if a seller behaves honestly in one transaction, the seller would deliver a quality product. Otherwise, the seller delivers a product with 50% quality. The seller reputation model used in the experiments is

$$R_s(t) = \lambda R_s(t-1) + (1-\lambda) \frac{\sum_{b \in B} N_b^s(t) \times R_b(t-1) \times \bar{r}_b^s(t)}{\sum_{b \in B} N_b^s(t) \times R_b(t-1)}, \quad (\text{Eq. 4.15})$$

where $\lambda = 0.5$ is the learning rate, N_b^s the number of ratings provided by buyer b for seller s and $\bar{r}_b^s(t)$ the expected value of the ratings provided by buyer b for seller s . For a buyer, if he behaves honestly, he provides 1 for sellers who deliver quality products and 0.5 for sellers who deliver products with 50% quality. If the buyer is dishonest, he provides 1 for sellers who deliver products with 50% quality and 0.5 for those who have delivered quality products.

4.4.2 Experimental Results

The experimental results in static and dynamic scenarios are shown in the Sections 4.4.2.1 and 4.4.2.2, respectively. In Section 4.4.2.3, the robustness of iMEMLI against the whitewashing attack is demonstrated. Finally, the comparison results are shown in Section 4.4.2.4.

4.4.2.1 Static scenario

In the static setting, iMEMLI is shown to be able to promote honesty from both buyers and sellers. In the beginning of the simulation, the system is bootstrapped by only allowing 80 honest buyers who are labeled by buyer identities (ID) from 1 to 80, and 40 sellers labeled by seller identities (ID) from 1 to 40, to conduct transactions in the first 1000 transactions. Among the 40 sellers, there are five types of honesty in delivering products and each type includes 8 sellers, i.e the sellers labeled by ID from 1 to 8 deliver quality products at 100% and the sellers labeled by ID from 9 to 16 deliver quality product at 90%, etc. The reputation and profit of the sellers is shown in Figure 4.2.

Figure 4.2: Seller reputation and profit vs. seller honesty during the bootstrapping stage

In Figure 4.2, the probability of sellers in delivering quality products is displayed using the left y axis in both Figures 4.2(a) and 4.2(b). One can observe that seller reputation reflects their probability of honest delivery in Figure 4.2(a). Seller average profit in selling one product also follows the same trend with the probability of honest delivery as shown in Figure 4.2(b). During the bootstrapping stage, all sellers are assigned relatively accurate reputation, and sellers with higher reputations can gain more profit.

After the bootstrapping, another 9000 transaction periods are conducted, where another 320 (dishonest) buyers labeled by buyer ID from 81 to 400 join the system. These

320 buyers have five types of honesty in providing ratings and each type includes 80 buyers, i.e. the buyers labeled by ID from 81 to 160 provide truthful ratings at 90%, and the buyers labeled from 161 to 240 provide truthful ratings at 80%, etc. The reputation and average profit of the sellers in selling a product (seller ID 1 to 40) is shown in Figure 4.3 and the score and utility of buyers (buyer ID 1 to 400) is shown in Figure 4.4.

Figure 4.3: Seller reputation and profit vs. seller honesty in the static setting

Figure 4.4: Buyer score and utility vs. buyer honesty in the static setting

Figure 4.3 shows how seller reputation and profit change with respect to their probability in delivering quality products. It can be observed in Figure 4.3(a) that seller reputation follows the same trend as the probability of honest delivery. But it is slightly

influenced by untruthful ratings compared with that during the bootstrapping stage. The average profit in selling a product also follows the same trend. The sellers with higher honesty are able to gain more profit, as shown in Figure 4.3(b). Therefore, Figure 4.3 shows that sellers have the incentive to deliver quality products. Figure 4.4 shows the relationships between buyer scores, total utility and the probability of honest ratings. One can observe that the expected buyer score decreases as the buyer probability of honest ratings reduces in Figure 4.4(a). In addition, the fluctuation of the score increases, since buyers with lower scores have less opportunities to be allocated with the products of honest sellers which results in a high fluctuation. In Figure 4.4(b), it can be observed that the total utility of an honest buyer is greater than that of a dishonest buyer. It is because, $1 - \eta = 90\%$ percentage of products ($90\% \times 40 = 36$) are allocated to honest buyers (with high scores) according to Algorithm 3 and the other η percentage products ($10\% \times 40 = 4$) are randomly allocated among other buyers ($400 - 36 = 364$, including all dishonest buyers), resulting in that many dishonest buyers do not have any chance to do business with sellers.

Moreover, in Figure 4.4(b), one can observe that honest buyers (those buyers whose honesty is 1) have much higher chance to conduct transactions than dishonest buyers (honesty is less than 1), since the utility of dishonest buyers is close to zero indicating that the dishonest buyers are rarely allocated with products. Indicated in Eq. 4.11, the social welfare is higher as the buyer score increases in the transactions. Thus, iMEMLI can achieve an improved social welfare by allocating most products to honest buyers.

4.4.2.2 Dynamic scenario

In the dynamic setting, it allows new buyers and sellers to join the marketplace during the simulation. In order to maintain the system being an EMLI, the system allows 1 new seller and 10 new buyers to participate into the system at the same time. After the

bootstrapping stage, 5 new sellers participate into the system along with 50 new buyers in every 100 transaction periods, where the honesty of the the new sellers have two types (delivering quality products at 100% and 60%) and the honesty of the buyers have five types (providing truthful ratings at 100%, 90%, 80%, 70%, 60%). After 400 transaction periods, 20 new sellers and 200 new buyers participate into the system. The new sellers are labeled by ID from 40 to 60. Specifically, the sellers labeled by seller ID from 41 to 50 deliver quality products at 100% and the other sellers labeled by ID from 51 to 60 deliver quality products at 60%. The new buyers are labeled by buyer ID from 401 to 600 and among every 50 buyers each type of buyers takes for 10. For example in the first 50 buyers from 401 to 450, the buyers labeled by 401 to 410 provide truthful ratings at 100%, and the buyers labeled by 411 to 420 provide truthful ratings at 90%, etc. The buyer honesty and seller honesty are displayed by using the left y axis in Figure 4.5 and Figure 4.6. After such a dynamic process, another 1000 transaction periods are simulated in the static setting to observe seller profit and reputation. The obtained results are shown as in Figure 4.5 and Figure 4.6.

Figure 4.5: Seller reputation and profit vs. seller honesty in the dynamic setting

Figure 4.5 shows seller reputation and profit in selling one product (60 sellers in total). The new honest sellers can gain the same reputation and profit as the honest

Figure 4.6: Buyer score and utility vs. buyer honesty in the dynamic setting

sellers who previously exist in the system. These results are shown in Figures 4.5(a) and 4.5(b), respectively. It means that honest sellers can always gain higher reputation and more profit no matter when they join the e-marketplace than that of dishonest sellers. In addition, more honest buyers gain higher scores and more utility, as shown in Figures 4.6(a) and 4.6(b). Therefore the incentives of buyers and sellers in behaving honestly are still maintained when new sellers and buyers dynamically join into the e-marketplace. In other words, for honest sellers or buyers, regardless of whether they are new or existing ones, they always have incentives to be honest in the proposed system. To conclude, iMEMLI ensures the sustainability of the e-marketplace by allowing new sellers and new buyers to participate into the e-marketplace, and the proposed mechanism still works well in such a dynamic environment.

4.4.2.3 Evaluation against the whitewashing attack

In this simulation, three sellers s_1 , s_2 and s_3 are considered whose reputation is $R_1 = 1$, $R_2 = 0.7$ and $R_3 = 0.55$, respectively. The total profit of each seller is recorded. The simulation are repeated for 10 times and the average results are shown in Figures 4.7(a) to (c). The profit loss of the sellers with different reputation is shown in Figure 4.7(d).

Figure 4.7: The performance of iMEMLI in re-entry scenarios

In Figure 4.7(a), it can be observed that s_1 gains positive profit. Seller s_2 also gains positive profit which is less than that of s_1 , and s_3 gains negative profit. It shows that sellers with reputation less than $R_0 = 0.5$ has the incentive to leave the marketplace due to the negative profit. Figure 4.7(b) shows how the mechanism performs without membership fee. The sellers s_2 and s_3 , whose reputation is less than initial reputation $\delta = 0.85$, have the incentive to exit and re-participate again to gain temporarily higher profit, by comparing the two lines Rep=0.7 and Rep=0.5 in Figure 4.7(b) with that in 4.7(a) (as the profit of the two sellers increases a bit after the reentry point.). When membership fee is applied, as shown in Figure 4.7(c), all the three sellers will lose profit if they leave and re-enter the marketplace, by comparing them with Figure 4.7(a). Figure

4.7(d) captures the profit loss of sellers with different reputation if they perform re-entry in iMEMLI. It can be observed that the profit loss follows a unimodal function with the peak 0 at $\delta = 0.85$. Thus, all the sellers would always gain non-positive profit by performing reentry. To conclude, with the membership fee, sellers do not have the incentive to perform the whitewashing attack.

4.4.2.4 Comparison results

In this section, iMEMLI is compared with one representative mechanism, i.e. the side-payment mechanism [37], in e-marketplaces with unlimited inventory (EMUI) and limited inventory (EMLI), respectively. The settings for the EMLI is similar to that described in Section 4.4.1. In the EMUI, each seller can supply 20 products within a period⁵ where the products from honest sellers could satisfy the demand. The sellers and buyers are divided into two groups: the first half always behaves honestly (100%) and the second half behave honestly at 60%. The two marketplaces are repeatedly run for 10 times and the average results are presented. The profit of sellers and the utility of buyers in the EMUI and EMLI are presented in Figure 4.8 and Figure 4.9, respectively.

Figure 4.8: The performance of iMEMLI and the side-payment mechanism in EMUI

⁵In the experiment for the EMUI, a period is 1/20 of that for EMLI.

In EMUI, the profit of honest sellers is higher than that of the dishonest sellers when applying both iMEMLI and the side-payment mechanism as shown in Figure 4.8(a), which means that the sellers have the incentive to be honest when they can supply unlimited inventory in the marketplace. In the side-payment mechanism the honest sellers can sell more products and in iMEMLI the honest sellers can sell their products at a higher price, which is the source of their incentive. In Figure 4.8(b), the honest buyers in iMEMLI can obtain more utility than that of dishonest buyers, which is similar to the side-payment incentive mechanism where the extra side-payment will be rewarded to honest buyers. The extra side-payment offered to honest buyers is the cost that the system owners have to bear. This in fact limits the applicability of the side-payment mechanism. In iMEMLI, the utility of honest buyers is just slightly higher than that of dishonest buyers due to the following two reasons: (1) the inventory is unlimited resulting in that all the buyers could obtain products, and (2) the products from honest sellers are more likely to be allocated to buyers at a higher price which weaken the advantage of these honest buyers. Therefore, in EMUI, iMEMLI, the same as the side-payment mechanism, can promote honesty from buyers and sellers, but provide weaker incentive for buyers than that provided by side-payment mechanism. Note that the stronger incentive provided by the side-payment mechanism bears cost to the marketplace operator because the marketplace has to pay additional side-payment to honest buyers. In contrast, the marketplace operating iMEMLI does not need to pay any additional money to honest buyers, achieving the property of budget balance.

In EMLI where each seller has limited inventory, the sellers in the side-payment mechanism could achieve more utility by behaving dishonestly. In iMEMLI, the limited products provided by honest sellers can be sold at higher prices, which provides incentive for sellers to be honest. For buyers, in the side-payment mechanism, the utility of honest buyers is almost the same as that of dishonest buyers. Even though honest buyers

Figure 4.9: The performance of iMEMLI and the side-payment mechanism in the EMLI

can gain more side-payment, the truthful ratings provided by them can increase the competition for the products provided by honest sellers which nearly offsets the achieved higher side-payment. Comparatively, buyers in iMEMLI can achieve larger utility by being honest.

By comparing the results in Figure 4.8 and Figure 4.9, one can conclude that the proposed mechanism can work in both the two types of e-marketplaces, especially in the EMLI where the side-payment mechanism may fail to work.

4.5 Summary

In this chapter, an incentive mechanism named iMEMLI has been proposed to promote seller and buyer honesty in the e-marketplaces with limited inventory (EMLI). More specifically, a pricing algorithm has been proposed to set high prices for products provided by honest sellers which motivates sellers to be honest, and an allocation algorithm has been proposed to allocate quality products to honest buyers, motivating buyers to be honest. Theoretical and empirical evaluations have been conducted to confirm the efficacy of the proposed mechanism.

Chapter 5

iMET: An Incentive Mechanism to Elicit Truthful Opinions For Crowdsourcing

Crowdsourcing is a popular new technology to harness collective human intelligence for tasks that are not suitable for computer automation completion, such as consensus tasks which seek opinions from workers to make sound decisions. To motivate workers to participate in these tasks and provide their truthful opinions, incentive mechanisms have been proposed to offer the workers rewards. However, most existing incentive mechanisms are based on the unrealistic assumption that workers share the same truthful opinion of the tasks, which is not always held in practice. Moreover, they may not promote truthful opinions from workers when the majority workers are providing untruthful opinions.

To resolve these limitations, an incentive mechanism, named as iMET, is proposed to elicit truthful opinions in consensus tasks on crowdsourcing platforms. Section 5.1 presents the proposed incentive mechanism iMET. The theoretical analysis of iMET is given in Section 5.2. Section 5.3 presents the experimental results of evaluating iMET using real-world datasets and demonstrates its advantages over two state-of-the-art mechanisms. Some potential practical issues are discussed in Section 5.4. Finally, Section 5.5 summarizes this chapter.

5.1 The iMET Mechanism

The consensus tasks in the system are in the form of multiple-choice questions (MCQs)¹. The framework of iMET is illustrated in Figure 5.1. Under iMET, a set of *control questions* are inserted into the consensus task question set from the task requester before they are published in the crowdsourcing system. The control questions are the ones to which the workers have the same truthful answers (i.e., opinions) known to the system. An example of such a question can be “How many trees are there in the photo?” with options “1”, “2”, “3”, and “4”. The questions to which the task requester is really soliciting answers are referred to as *non-control questions*. The control and non-control questions should be of the same type to ensure that the workers cannot differentiate between them when answering the whole set of questions. For example, if the non-control questions are image labelling questions, then the control questions should also be image labelling questions. The combined questions as well as their alternative options are randomly sorted to become a task for each worker, and workers can provide answers to the combined questions to earn rewards.

Figure 5.1: An overview of the proposed iMET

¹In the following descriptions, the MCQs are simply called as questions.

Once a task is closed (e.g., when the stipulated deadline has been reached), iMET computes the reward that each worker receives for each of the non-control questions in two components. The first component rewards the *truthfulness* of answers. A worker's *honesty* is estimated based on her answers to the control questions. The *similarity* between each pair of workers's truthful opinions² is estimated based on their answers to the non-control questions as well as their honesty. The truthfulness reward is based on the estimated honesty, similarity, and the collected answers of all workers to the non-control questions. The second component rewards the *consistency* of workers' answers with the final answers which are achieved by an available opinion aggregation method. The final reward is a weighted combination of the rewards in the above components. The weight is dynamically adjusted to achieve incentive compatibility for promoting truthful answers while minimizing the total reward amount paid by the task requester.

5.1.1 Definitions and Assumptions

Suppose a task consists of n questions including n^c control MCQs and n^{-c} non-control MCQs, where $n^c + n^{-c} = n$. Each question has k options. The honesty of a worker, and the similarity of two workers' truthful opinions are defined as follows.

Definition 7 (Honesty). *The honesty of a worker is the probability that she provides a truthful answer to a question.*

The honesty of a worker i is denoted by h_i . The actual h_i of a worker i in performing a specific task is unknown but it can be estimated as:

$$\hat{h}_i = \frac{n_i^c}{n^c}, \quad (\text{Eq. 5.1})$$

where n_i^c is the number of control questions to which worker i provides the same answers as those known to the system.

²The truthful opinions of workers do not evolve or update over time due to the fact that the workers perform the task only once.

Definition 8 (Similarity). *The similarity of two workers' truthful opinions is the percentage of the control questions for which they share the same answers.*

For a pair of workers i and j , their truthful opinion similarity is denoted s_{ij} and it can be estimated as:

$$\hat{s}_{ij} = \begin{cases} \left(\frac{n_{ij}^c}{n^c} - \frac{1}{k}\right) \frac{(k-1)^2}{(1-\hat{h}_i)(1-\hat{h}_j)} + \frac{1}{k} & \hat{h}_i \neq \frac{1}{k} \text{ and } \hat{h}_j \neq \frac{1}{k}, \\ \frac{1}{k} & \hat{h}_i = \frac{1}{k} \text{ or } \hat{h}_j = \frac{1}{k}, \end{cases} \quad (\text{Eq. 5.2})$$

where n_{ij}^c is the number of non-control questions to which i and j provide the same answers, and $\hat{h}_i$ and $\hat{h}_j$ are the estimated honesty value of the two workers as in Eq. 5.1. The derivation of Eq. 5.2 is justified later in this section. The proposed mechanism works under the following assumptions.

Assumption 1. *The honesty and truthful answers of a worker are independent and they are assumed to be constant during a task.*

Honesty is an inherent property of a worker and indicates her likelihood to give a truthful answer. It is thus reasonable to assume that the two properties are independent and do not change within one task. For example, for a question, a worker's truthful answer is "A". Her honesty is the probability that the worker will provide "A" as her answer.

Assumption 2. *When a worker provides an untruthful answer to a question, she randomly chooses an option from the set of all possible answers, excluding her own truthful answer.*

We assume that there is no easy way for workers to resort the questions and options in a certain order so that cooperation among workers is not possible.

Based on Assumptions 1 and 2, a worker's likelihood to provide truthful or untruthful answers depends on her inherent honesty. It is worth pointing out that the similarity of a

pair of workers' truthful opinion is independent of their honesty according to Assumption 1, and Eq. 5.2 just provides a way to estimate it given that only $\frac{n_{ij}^{-c}}{n^{-c}}$, $\hat{h}_i$ and $\hat{h}_j$ are observed. Eq. 5.2 is derived as follows.

Let $\mathcal{H}_i$ and $\mathcal{H}_j$ denote the states that workers i and j provide truthful answers, respectively, and $\mathcal{D}_i$ and $\mathcal{D}_j$ denote the states that they provide untruthfully answers, respectively. There exist four possible combination cases (i.e., $\mathcal{H}_i$ and $\mathcal{H}_j$, $\mathcal{H}_i$ and $\mathcal{D}_j$, $\mathcal{D}_i$ and $\mathcal{H}_j$, $\mathcal{D}_i$ and $\mathcal{D}_j$). We calculate the probability that their answers a_i^q and a_j^q regarding a non-control question q are the same for the four cases, respectively, as follows.

$$\begin{aligned} Pr(a_i^q = a_j^q | \mathcal{H}_i, \mathcal{H}_j) &= s_{ij}; \\ Pr(a_i^q = a_j^q | \mathcal{H}_i, \mathcal{D}_j) &= \frac{1-s_{ij}}{k-1}; \\ Pr(a_i^q = a_j^q | \mathcal{D}_i, \mathcal{H}_j) &= \frac{1-s_{ij}}{k-1}; \\ Pr(a_i^q = a_j^q | \mathcal{D}_i, \mathcal{D}_j) &= \frac{k+s_{ij}-2}{(k-1)^2}. \end{aligned}$$

Given i and j 's honesty h_i and h_j , we have

$$\begin{aligned} Pr(\mathcal{H}_i, \mathcal{H}_j) &= h_i h_j; \\ Pr(\mathcal{H}_i, \mathcal{D}_j) &= h_i (1 - h_j); \\ Pr(\mathcal{D}_i, \mathcal{H}_j) &= (1 - h_i) h_j; \\ Pr(\mathcal{D}_i, \mathcal{D}_j) &= (1 - h_i)(1 - h_j). \end{aligned}$$

Then the probability that i 's and j 's answers are the same is denoted by α and we have the following equation:

$$\begin{aligned} \alpha = & Pr(\mathcal{H}_i, \mathcal{H}_j) \times Pr(a_i^q = a_j^q | \mathcal{H}_i, \mathcal{H}_j) + \\ & Pr(\mathcal{H}_i, \mathcal{D}_j) \times Pr(a_i^q = a_j^q | \mathcal{H}_i, \mathcal{D}_j) + \\ & Pr(\mathcal{D}_i, \mathcal{H}_j) \times Pr(a_i^q = a_j^q | \mathcal{D}_i, \mathcal{H}_j) + \\ & Pr(\mathcal{D}_i, \mathcal{D}_j) \times Pr(a_i^q = a_j^q | \mathcal{D}_i, \mathcal{D}_j). \end{aligned} \tag{Eq. 5.3}$$

By solving Eq. 5.3, the solution of s_{ij} can be obtained that

$$s_{ij} = \begin{cases} (\alpha - \frac{1}{k}) \frac{(k-1)^2}{(1-kh_i)(1-kh_j)} + \frac{1}{k} & h_i \neq \frac{1}{k} \text{ and } h_j \neq \frac{1}{k}, \\ \frac{1}{k} & h_i = \frac{1}{k} \text{ or } h_j = \frac{1}{k}, \end{cases} \tag{Eq. 5.4}$$

where $h_i = \frac{1}{k}$ or $h_j = \frac{1}{k}$, i or j randomly provides answers regardless of their truthful opinions. The percentage of the same answers α is expected to be $\frac{1}{k}$, and s_{ij} is set as $\frac{1}{k}$. Thus, when α , h_i , and h_j are estimated by $\frac{n_{ij}^{-c}}{n^{-c}}$, $\hat{h}_i$, and $\hat{h}_j$, respectively, Eq. 5.2 is obtained.

5.1.2 Truthfulness Reward

The truthfulness rewarding component operates by comparing the worker's answer a_i^q to a question q with each worker j 's answer a_j^q to the same question. According to the comparison result between a_i^q and a_j^q , the tentative reward t_{ij}^q that i receives is determined by one of the two rewarding schemes ($\mathcal{M}_1$ and $\mathcal{M}_2$) as shown in Table 5.1 and 5.2, respectively, where “1” represents a unit of reward and “ $\frac{1}{k-1}$ ” represents $\frac{1}{k-1}$ unit of reward³.

Table 5.1: Rewarding scheme $\mathcal{M}_1$

		Worker j			
		Option 1	Option 2	...	Option k
Worker i	Option 1	1	0	...	0
	Option 2	0	1	...	0
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	Option k	0	0	...	1

Table 5.2: Rewarding scheme $\mathcal{M}_2$

		Worker j			
		Option 1	Option 2	...	Option k
Worker i	Option 1	0	$\frac{1}{k-1}$	...	$\frac{1}{k-1}$
	Option 2	$\frac{1}{k-1}$	0	...	$\frac{1}{k-1}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	Option k	$\frac{1}{k-1}$	$\frac{1}{k-1}$	...	0

It could be observed in $\mathcal{M}_1$ that worker i gets the reward of 1 when her answer a_i^q is the same as j 's answer a_j^q . In $\mathcal{M}_2$, worker i gets the reward of $\frac{1}{k-1}$ when her answer is different from j 's answer. Compared to $\mathcal{M}_1$, the positive reward of $\mathcal{M}_2$ is discounted by $\frac{1}{k-1}$ since worker i provides her truthful answer in only one of the $k-1$ cases where i and j 's answers are different.

The detailed truthfulness rewarding procedure is specified as in Algorithm 4.

³A unit of reward corresponds to a certain amount of money which is determined by task requesters.

Input	k : Number of options of question q ; $\mathcal{M}_1$: The first rewarding scheme; $\mathcal{M}_2$: The second rewarding scheme; h_j : The honesty of worker j ; s_{ij} : The similarity of workers i and j 's truthful opinions; I^q : The set of workers answering question q ;
Output	t_i^q : The reward of worker i for answering the question q ;

```

1 Initialize  $t_{ij}^q = 0$  for every  $j \in I^q$ ;
2 for  $j \in I^q, j \neq i$  do
3   if  $h_j \geq \frac{1}{k}$  and  $s_{ij} \geq \frac{1}{k}$  then
4      $t_{ij}^q$  is assigned to be 0 or 1 according to  $\mathcal{M}_1$ ;
5   if  $h_j < \frac{1}{k}$  and  $s_{ij} < \frac{1}{k}$  then
6      $t_{ij}^q$  is assigned to be 0 or 1 according to  $\mathcal{M}_1$ ;
7   if  $h_j \geq \frac{1}{k}$  and  $s_{ij} < \frac{1}{k}$  then
8      $t_{ij}^q$  is assigned to be 0 or  $\frac{1}{k-1}$  according to  $\mathcal{M}_2$ ;
9   if  $h_j < \frac{1}{k}$  and  $s_{ij} \geq \frac{1}{k}$  then
10     $t_{ij}^q$  is assigned to be 0 or  $\frac{1}{k-1}$  according to  $\mathcal{M}_2$ ;
11 return  $t_i^q = \frac{\sum_{j \in I^q, j \neq i} t_{ij}^q}{|I^q| - 1}$ ;

```

Algorithm 4: The truthfulness reward of worker i in answering question q

In Algorithm 4, t_{ij}^q is the tentative reward that worker i achieves in comparing her answer a_i^q with worker j 's answer a_j^q , which is initialized to be zero for every $j \in I^q$, where I^q is the set of workers providing answers to the question q (**Line 1**). For each worker j , given j 's honesty h_j and similarity s_{ij} , t_{ij}^q is assigned according to $\mathcal{M}_1$ or $\mathcal{M}_2$ (**Line 2-15**). Specifically, worker i receives a tentative reward of 1 when workers i and j 's answers are the same and the conditions to trigger scheme $\mathcal{M}_1$ are satisfied. The conditions to trigger $\mathcal{M}_1$ are: (1) worker j 's honesty, estimated through Eq. 5.1, is not less than $\frac{1}{K}$ (i.e., $h_j \geq \frac{1}{K}$) and the truthful opinion similarity between i and j , estimated through Eq. 5.2, is not less than $\frac{1}{k}$ (i.e., $s_{ij} \geq \frac{1}{k}$) (**Line 3-5**), or (2) $h_j < \frac{1}{k}$ and $s_{ij} < \frac{1}{k}$ (**Line 6-8**). On the other hand, worker i receives a tentative reward of $\frac{1}{k-1}$ when workers i and j 's answers are different and the conditions to trigger scheme $\mathcal{M}_2$ are satisfied. The conditions to trigger $\mathcal{M}_2$ are: (1) $h_j \geq \frac{1}{k}$ and $s_{ij} < \frac{1}{k}$ (**Line 9-11**), or (2) $h_j < \frac{1}{k}$ and

$s_{ij} \geq \frac{1}{k}$ (**Line 12-14**). Finally, the truthfulness reward T_i^q of worker i in answering q is the average of the tentative rewards achieved by comparing with all other workers in I^q (**Line 16**). In the experiments, the value h_j and s_{ij} are estimated according to Eq. 5.1 and Eq. 5.2, respectively.

Intuitively speaking, when j 's honesty is high and the similarity of i 's and j 's truthful opinions is high, or when j 's honesty is low and the similarity of i 's and j 's truthful opinions is low, i is expected to provide the same answer as j if i is providing her truthful answer. In these cases, i should get reward when her answer is the same as j 's answer. On the other hand, when j 's honesty is low and the similarity of i 's and j 's truthful opinions is high, or when j 's honesty is high and the similarity of i 's and j 's truthful opinions is low, i is expected to provide a different answer from j if i is providing her truthful answer. Therefore, i should get rewards when her answer is different from j 's answer. The setting of the threshold value $\frac{1}{k}$ for j 's honesty h_j and the similarity s_{ij} in the conditions to trigger $\mathcal{M}_1$ or $\mathcal{M}_2$ will be discussed in Section 5.2.

5.1.3 Consistency Reward

The consistency rewarding component operates by comparing worker i 's answer a_i^q to question q with the final answer, a_q^* for question q following an available opinion aggregation method. For each question q , the consistency reward that worker i can achieve is c_i^q which is calculated as follows:

$$c_i^q = \begin{cases} 1, & \text{if } a_i^q = a_q^*, \\ 0, & \text{otherwise.} \end{cases} \quad (\text{Eq. 5.5})$$

It is noted that the consistency reward depends on the underlying opinion aggregation method used for deriving final answers in the consensus tasks. The purpose of the consistency reward is to bring the compatibility of the truthfulness reward with the existing crowdsourcing platforms where certain static rewarding schemes are already adopted.

For example, in the Amazon Mechanical Turk (MTurk), all the answers accepted by task requesters (through qualification test before participating in tasks) are uniformly rewarded, and in the Baidu crowdsourcing platform (test.baidu.com), the workers are rewarded according to their accuracy ratio by comparing with final answers aggregated through majority voting. These static rewarding schemes may give higher rewards to workers providing untruthful opinions, but they cannot be fully abandoned due to the regulations of the existing crowdsourcing platforms. In these cases, if the rewards offered in the incentive mechanisms and the original ones are not combined properly, the workers may lose the incentive to provide truthful opinions.

The consistency rewarding component could be based on any of the available aggregation methods [6, 48, 102]. Since the aggregation method is not the focus of this work, we do not discuss how to improve the soundness of the final answer. In the experiments, we use the majority voting method to derive the final answer for each question.

5.1.4 The Final Combined Reward

The truthfulness and consistency rewards for worker i 's answer a_i^q are combined to obtain her final reward r_i^q for answering question q :

$$r_i^q = \omega \times t_i^q + (1 - \omega) \times c_i^q, \quad (\text{Eq. 5.6})$$

where ω ($0 \leq \omega \leq 1$) is the combination weight which is determined by solving the following linear optimization problem:

$$\begin{aligned} & \text{minimize} && \sum_i \sum_q r_i^q, \\ & \text{subject to} && (\sum_q r_i^q - \sum_q r_j^q)(h_i - h_j) > 0 \\ & && \forall i, j \in I \text{ and } h_i \neq h_j, \end{aligned} \quad (\text{Eq. 5.7})$$

where I is the set of workers participating in the consensus task. It can be solved by many well-studied methods, such as simplex methods and interior-point methods, with polynomial computational complexity[101].

It is worth noting that: (1) rewarding truthfulness and consistency are two independent dimensions in iMET. It might happen that a worker receives a high t_i^q but a low c_i^q , or the opposite. But the constraint in (Eq. 5.7) ensures that the total reward r_i^q is higher for a worker with a higher honesty, thereby providing the incentive for workers to be truthful; (2) iMET is not limited to the specific objective function in use, i.e., to minimize the budget of the task requester. Other objective functions can be used.

5.2 Theoretical Analysis

As the proposed mechanism is an incentive mechanism to elicit truthful opinions from workers, we focus on analyzing whether it is more beneficial for workers to provide their truthful opinions in the proposed mechanism.

Proposition 21. *Providing truthful answers is a dominant strategy of workers in the truthfulness rewarding component.*

Proof. As t_i^q is the average of the tentative reward t_{ij}^q in comparing with each worker j who answers the question q , in this proof, we will prove that the expected value of t_{ij}^q monotonically increases with i 's honesty. Similar to the derivation of Eq. 5.2, we can derive the expected value of t_{ij}^q for worker i in providing an answer to question q by comparing her answer with the answer provided by worker j when scheme $\mathcal{M}_1$ is triggered, as follows.

$$\begin{aligned}
\mathbf{E}(t_{ij}^q) &= Pr(\mathcal{H}_i, \mathcal{H}_j) \times Pr(a_i^q = a_j^q | \mathcal{H}_i, \mathcal{H}_j) \times 1 + \\
&\quad Pr(\mathcal{H}_i, \mathcal{D}_j) \times Pr(a_i^q = a_j^q | \mathcal{H}_i, \mathcal{D}_j) \times 1 + \\
&\quad Pr(\mathcal{D}_i, \mathcal{H}_j) \times Pr(a_i^q = a_j^q | \mathcal{D}_i, \mathcal{H}_j) \times 1 + \\
&\quad Pr(\mathcal{D}_i, \mathcal{D}_j) \times Pr(a_i^q = a_j^q | \mathcal{D}_i, \mathcal{D}_j) \times 1 \\
&= h_i h_j s_{ij} + h_i (1 - h_j) \left(\frac{1 - s_{ij}}{k - 1} \right) + (1 - h_i) h_j \left(\frac{1 - s_{ij}}{k - 1} \right) + \\
&\quad (1 - h_i)(1 - h_j) \left(\frac{k + s_{ij} - 2}{(k - 1)^2} \right).
\end{aligned} \tag{Eq. 5.8}$$

The first order derivative of Eq. 5.8 with respect to h_i is:

$$\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_i} = \frac{(k h_j - 1)(k s_{ij} - 1)}{(k - 1)^2}. \tag{Eq. 5.9}$$

We can see that $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_i} \geq 0$ when $h_j \geq \frac{1}{k}$ and $s_{ij} \geq \frac{1}{k}$, or when $h_j < \frac{1}{k}$ and $s_{ij} < \frac{1}{k}$. $\mathbf{E}(t_{ij}^q)$ monotonically increases with h_i beyond the borderline points at $s_{ij} = \frac{1}{k}$ and $h_j = \frac{1}{k}$. As $\mathcal{M}_1$ is triggered at the exact same conditions according to Algorithm 4, worker i has the incentive to provide more truthful answers to achieve higher expected reward when using $\mathcal{M}_1$ regardless of whether other workers are providing truthful answers, i.e. providing truthful answers is the dominant strategy of workers.

Similarly, when scheme $\mathcal{M}_2$ works, the expected value of t_{ij}^q can be calculated as follows:

$$\begin{aligned} \mathbf{E}(t_{ij}^q) = & h_i h_j (1 - s_{ij}) \times \left(\frac{1}{k-1}\right) + \\ & h_i (1 - h_j) \left(1 - \frac{1-s_{ij}}{k-1}\right) \times \left(\frac{1}{k-1}\right) + \\ & (1 - h_i) h_j \left(1 - \frac{1-s_{ij}}{k-1}\right) \times \left(\frac{1}{k-1}\right) + \\ & (1 - h_i) (1 - h_j) \left(1 - \frac{k+s_{ij}-2}{(k-1)^2}\right) \times \left(\frac{1}{k-1}\right). \end{aligned} \quad (\text{Eq. 5.10})$$

The first order deviation of Eq. 5.10 with respect to h_i is:

$$\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_i} = \frac{(1 - k h_j)(k s_{ij} - 1)}{(k - 1)^3}. \quad (\text{Eq. 5.11})$$

We can get $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_i} \geq 0$ when $h_j \geq \frac{1}{k}$ and $s_{ij} < \frac{1}{k}$ or when $h_j < \frac{1}{k}$ and $s_{ij} \geq \frac{1}{k}$, meaning that $\mathbf{E}(t_{ij}^q)$ monotonically increases with h_i when $h_j \geq \frac{1}{k}$ and $s_{ij} < \frac{1}{k}$, or $h_j < \frac{1}{k}$ and $s_{ij} > \frac{1}{k}$ beyond the borderline points at $s_{ij} = \frac{1}{k}$ and $h_j = \frac{1}{k}$, which are the exact conditions for $\mathcal{M}_2$ to be triggered. As a result, worker i also has the incentive to provide truthful answers to achieve high expected rewards when using $\mathcal{M}_2$ no matter how other workers provide their answers.

In all, when either $\mathcal{M}_1$ or $\mathcal{M}_2$ is triggered, providing truthful answers is the dominant strategy of workers. □

Proposition 22. *The truthfulness rewarding component has a unique Nash equilibrium⁴ where each worker adopts her dominant strategy, i.e. providing truthful answers.*

⁴A Nash equilibrium is a strategy profile, in which no player/worker has incentive to change her strategy [54].

Proof. Since each worker has the same dominant strategy, which makes the equilibrium composed by the dominant strategy be the unique Nash equilibrium. $\square$

From an intuitive point of view, the conditions for $\mathcal{M}_1$ or $\mathcal{M}_2$ to be triggered can be explained as follows. When $h_j = \frac{1}{k}$, the probability of the worker j providing the truthful answer to a question is equal to that of providing any other untruthful answer, making it has the same effect as the case where worker j randomly provides an answer for the question. When $s_{ij} = \frac{1}{k}$, the probability that worker i and j have the same truthful answer to a question q is $\frac{1}{k}$. We also know that if j randomly provides the answer to the question, then the probability that worker i 's truthful answer and worker j 's provided answer are the same is expected to be $\frac{1}{k}$. Therefore, the case that $s_{ij} = \frac{1}{k}$ has the same effect as worker j randomly chooses an answer to the question. Thus, there is no difference for worker i between providing truthful and untruthful answers when $h_j = \frac{1}{k}$ or $s_{ij}^q = \frac{1}{k}$. The cases beyond the above scenarios are discussed as follows.

- $h_j > \frac{1}{k}$ and $s_{ij} > \frac{1}{k}$: In this case, the probability of worker j in providing her truthful answer is higher than that of providing any of the other $k - 1$ untruthful answers, and worker i 's truthful answer is more likely to be the same as worker j 's truthful answer than any of worker j 's untruthful answers. Then worker i 's truthful answer is more likely to be the same as the answer provided by worker j . Therefore, worker i should be more likely to be rewarded when $a_i^q = a_j^q$, which is exactly achieved by triggering $\mathcal{M}_1$.
- $h_j < \frac{1}{k}$ and $s_{ij} < \frac{1}{k}$: In this case, the probability of worker j in providing one of her untruthful answers is higher than that of providing her truthful answer, and worker i 's truthful answer is more likely to be different from worker j 's truthful answer, and hence same as worker j 's provided answer. Therefore, worker i should be more likely to be rewarded when $a_i^q = a_j^q$, which can be achieved by triggering $\mathcal{M}_1$.

- $h_j > \frac{1}{k}$ and $s_{ij} < \frac{1}{k}$: In this case, the probability of worker j in providing her truthful answer is higher than that of providing any of the other $k - 1$ untruthful answers, and worker i 's truthful answer is more likely to be different from worker j 's truthful answer. Then worker i 's truthful answer is more likely to be different from the answer provided by worker j . Therefore, worker i should be more likely to be rewarded when $a_i^q \neq a_j^q$. This can be achieved by triggering $\mathcal{M}_2$.
- $h_j < \frac{1}{k}$ and $s_{ij} > \frac{1}{k}$: In this case, the probability of worker j in providing one of her untruthful answers is higher than that of providing her truthful answer, and worker i 's truthful answer is more likely to be worker j 's truthful answer, and hence different from worker j 's provided answer. Therefore, worker i should be more likely to be rewarded when $a_i^q \neq a_j^q$, achieved by triggering $\mathcal{M}_2$.

To assist the better understanding of Propositions 21 and 22, an example is provided. Suppose there are four workers answering a question q which has binary options ($k = 2$). Workers 1 and 3 always provide their truthful answers about the question, i.e., $h_1 = h_3 = 1$, and workers 2 and 4 randomly draw an option among the $k - 1$ options excluding their truthful answers, i.e., $h_2 = h_4 = 0$. Since $k = 2$, workers 2 and 4 would always choose the option which is not their truthful answers. Workers 1 and 2 share the same truthful answers (Option 1) and workers 3 and 4 share the same truthful answers (Option 2). The similarity s_{ij} between any pair of workers's truthful opinions is 0 except $s_{12} = s_{21} = s_{34} = s_{43} = 1$. Therefore, the answer provided by worker 1 (Option 1) is different from that provided by worker 2 who provides her untruthful answer, and also different from that provided by worker 3 who provides her truthful answer. For each worker, by comparing with each of the other three workers, the truthfulness rewards are listed in Table 5.3.

For instance, the truthfulness rewards t_{21}^q and t_{31}^q are highlighted in **bold** in Table 5.3.

Table 5.3: An example: the truthfulness rewards for a single question

$i \backslash$	1	2	3	4
t_{i1}^q	-	0	1	0
t_{i2}^q	1	-	1	0
t_{i3}^q	1	0	-	0
t_{i4}^q	1	0	1	-
t_i^q	1	0	1	0

- t_{21}^q : The tentative truthfulness reward of worker 2 by comparing with worker 1. Since $h_1 = 1$ and $s_{21} = 1$, rewarding scheme $\mathcal{M}_1$ is triggered. As their provided answers are different and the mechanism assigns $t_{21}^q = 0$ according to $\mathcal{M}_1$.
- t_{31}^q : The tentative truthfulness reward of worker 3 by comparing with worker 1. Since $h_1 = 1$ and $s_{31} = 0$, rewarding scheme $\mathcal{M}_2$ is triggered. As their provided answers are different and the mechanism assigns $t_{31}^q = 1$ according to $\mathcal{M}_2$.

By averaging the rewards gained in comparing each other worker, the truthfulness reward for each worker is listed in the last row of Table 5.3. It can be observed that the rewards of worker 1 and 3 who provide their truthful answers are higher than those of worker 2 and 4 who provide their untruthful answers, respectively, i.e., $t_1^q > t_2^q$, and $t_3^q > t_4^q$.

The existence of ω in combining truthfulness and consistence rewards is proven in the following proposition.

Proposition 23. *There exists at least one ω satisfying the constraints of the optimization problem as defined in Eq. 5.7.*

Proof. According to Proposition 21, T_i^q increases as h_i grows. We can then derive that $(\sum_q t_i^q - \sum_q t_j^q)(h_i - h_j) > 0$ when $h_i \neq h_j$. It is exactly the case of setting $\omega = 1$. Therefore, there exists at least one solution of Eq. 5.7 as $\omega = 1$. $\square$

As it can be seen that the value $\frac{1}{k}$ serves as the thresholds of j 's honesty h_j and similarity s_{ij} in the conditions to trigger either $\mathcal{M}_1$ or $\mathcal{M}_2$, we then further study the truthfulness reward when $h_i = \frac{1}{k}$, $h_j = \frac{1}{k}$, or $s_{ij} = \frac{1}{k}$.

Proposition 24. *When $h_i = \frac{1}{k}$ or $h_j = \frac{1}{k}$ or $s_{ij} = \frac{1}{k}$, $\mathbf{E}(t_{ij}^q) \equiv \frac{1}{k}$.*

Proof. Suppose $\mathcal{M}_1$ is triggered and let $h_i = \frac{1}{k}$. We can calculate $\mathbf{E}(t_{ij}^q)$ according to Eq. 5.8.

$$\mathbf{E}(t_{ij}^q) = \frac{1}{k}h_js_{ij} + \frac{1}{k}(1-h_j)\frac{1-s_{ij}}{k-1} + (1-\frac{1}{k})h_j\frac{1-s_{ij}}{k-1} + (1-\frac{1}{k})(1-h_j)\frac{k+s_{ij}-2}{(k-1)^2} = \frac{1}{k}$$

Similarly, when $h_i = \frac{1}{k}$ and $\mathcal{M}_2$ is triggered, it can be derived that the expected tentative truthfulness reward of worker i is also equal to $\frac{1}{k}$, i.e., $\mathbf{E}(t_{ij}^q) = \frac{1}{k}$.

Next, let us consider the case that $h_j = \frac{1}{k}$. According to Eq. 5.4, $s_{ij} = \frac{1}{k}$, thus $\mathcal{M}_1$ is triggered. Since $h_j = \frac{1}{k}$, the probability of each option being answered by worker j is exactly equal to $\frac{1}{k}$. Then given an answer a_i^q provided by worker i to the question, the expected truthfulness reward $\mathbf{E}(t_{ij}^q) = \frac{1}{k} \times 1$ where $\frac{1}{k}$ is the probability of their answers are the same and 1 is the amount of reward when their answers are the same in $\mathcal{M}_1$.

Finally, we prove that $\mathbf{E}(t_{ij}^q) = \frac{1}{k}$ when $s_{ij} = \frac{1}{k}$ and $\mathcal{M}_1$ is triggered.

$$\begin{aligned} \mathbf{E}(t_{ij}^q) &= h_i h_j \frac{1}{k} + h_i(1-h_j)\frac{1-\frac{1}{k}}{k-1} + (1-h_i)h_j\frac{1-\frac{1}{k}}{k-1} + (1-h_i)(1-h_j)\frac{k+\frac{1}{k}-2}{(k-1)^2} \\ &= h_i \frac{1}{k} + (1-h_i)\frac{1}{k} \\ &= \frac{1}{k}. \end{aligned}$$

Similarly, when $s_{ij} = \frac{1}{k}$ and $\mathcal{M}_2$ is triggered, we can derive $\mathbf{E}(\mathbf{T}_{ij}^q) = \frac{1}{k}$. □

Intuitively speaking, this proposition suggests that a worker can achieve the reward of $\frac{1}{k}$ when she just answers questions with a random option, i.e., her honesty is $\frac{1}{k}$. Based on Proposition 21 which indicates that workers can achieve more rewards by providing more truthful answers, a worker can achieve at least $\frac{1}{k}$, if she could provide more truthful answers than provides random answers. But the exact value of t_{ij}^q is also influenced

by the honesty of worker j and the similarity of their truthful opinions. We will show that there is a fixed reward $\frac{1}{k}$ when a worker provides answers randomly, i.e., $h_i = \frac{1}{k}$ in the experiments carried out in the next section. This specific amount of reward could effectively convince the workers to participate in the task with an expectation of their rewards (no less than $\frac{1}{k}$) in providing truthful answers to the questions in the task.

5.3 Experimental Evaluation

We carried out experiments to assess the effectiveness of iMET in eliciting truthful opinions from workers. The hypothesis is that the workers have the incentive to provide their truthful answers even when (1) they have different truthful answers (different subjectivity) and (2) the majority of the workers provide untruthful answers. The experimental results validate that the above hypothesis is true.

5.3.1 Datasets and Experimental Settings

Though some datasets are available to study consensus tasks (e.g., the Galaxy Zoo project), they cannot be used in our experiments due to the lack of the information about whether the provided answers are the workers' truthful answers. Thus, we collected our own datasets as in Table 5.4, where k is the number of options for each question, n^c is the number of control questions, and $n^{\neg c}$ is the number of non-control questions.

More specifically, we first collected the truthful answers from 45 workers in answering questions. For each (control or non-control) question, there are k different images and the workers are asked to provide their truthful answers about which one is clearer from their personal point of view. We supervised the workers to ensure that they provided their truthful answers. There are about 10 questions to which all the workers provide the same answers in each of the four datasets. These questions are used as the control questions in our experiments.

Table 5.4: The descriptions of the datasets used in the experiments

Dataset	k	n^c	n^{-c}
Data-2	2	11	90
Data-2S	2	10	50
Data-4	4	10	90
Data-4S	4	9	50

Figure 5.2: The truthful opinion similarity distribution of the datasets

Through analyzing the collected datasets, we have the truthful opinion similarity distribution by comparing every pair of workers ($C_{45}^2 = 990$ pairs in total), as shown in Figure 5.2. It can be seen that there exists significant differences among their truthful

Figure 5.3: The honesty settings

opinions. Since the questions in the first two datasets have the same number of options ($k = 2$), we name them as “Data-2” and “Data-2S”. Similarly, we call the other two datasets where each question has four options ($k = 4$) as “Data-4” and “Data-4S”. In Data-2, it can be observed in Figure 5.2(a) that most of the similarity values fall between 0.5 and 0.9 with the mean value at around 0.65, and 1% of the similarity values are smaller than the similarity threshold $\frac{1}{k} = 0.5$. In Data-4, the similarity values are mostly between 0.2 and 0.5, and 6% of the similarity values are below the similarity threshold $\frac{1}{k} = 0.25$ as shown in Figure 5.2(c). In Data-2S and Data-4S, the answers for questions are more diverse than in Data-2 and Data-4 as shown in Figures 5.2(b) and (d) and about 50% similarity values fall below $\frac{1}{k}$.

After collecting the real datasets, we conduct experiments in the following way. Firstly, we have four honesty settings where the average honesty of the workers is high-level (the average honesty of the workers is 0.64), medium-level (the average honesty of the workers is 0.50), low-level (the average honesty of the workers is 0.36), and ultra-low-level (the average honesty of the workers is 0.20), respectively, as shown in Figure 5.3.

In the experiments, each worker is randomly associated with a honesty value according to the specific honesty setting. Then, we simulate the provided answers of each worker according to the honesty assigned to her and her collected truthful answers. For example, if a worker's honesty is 0.6, then in the experiments, the answers to 60% questions use the answers in the datasets (truthful answers), and the answers to the other 40% questions are randomly changed to one of the options which are different from their truthful answers stored in the datasets.

In the experiments, we first validate iMET in the aspects of eliciting truthful opinions in the four experimental settings in the four datasets. Then iMET is compared with two state-of-the-art incentive mechanisms – the static reward mechanism used by test.baidu.com, and the side payment (SP) mechanism [37]⁵. To achieve high statistical accuracy, we conduct the experiments for 100 times and present the average results in the next section.

5.3.2 iMET Evaluation Results

In this experiment, we first separately evaluate the performance of the truthfulness and consistency rewarding components of iMET in various experimental settings. Then we examine the average total reward of the mechanism for workers in answering each question in the settings.

5.3.2.1 Truthfulness Reward

For each of the four honesty settings shown in Figure 5.3, the average truthfulness rewards of workers in answering a question are presented in Figure 5.4, where x-axis shows the honesty of the workers.

⁵iMET is not compared with the consensus prediction approach [49] as it is inherently the same as SP except that it achieves better fairness by giving more competent workers more rewards. But a worker's competence is difficult to define. We do not compare with the shadow peer prediction approach [111] as a worker cannot tell her truthful opinion on which image is clearer until she sees the images.

Figure 5.4: The truthfulness reward of workers in various settings

In Figures 5.4(a) to (d), it can be observed that the workers with higher honesty are able to gain more rewards regardless of the honesty settings. The truthfulness reward of workers whose honesty is $\frac{1}{k}$ is about $\frac{1}{k}$ (as proven in Proposition 24).

In Figures 5.4(a) to (d), it can also be observed that the workers whose honesty is higher than $\frac{1}{k}$ (i.e., $h_i > \frac{1}{k}$) can achieve higher rewards in Data-2 than in Data-2S, and also achieve higher rewards in Data-4 than in Data-4S. The opposite trend can be observed when the workers's honesty is less than $\frac{1}{k}$ (i.e., $h_i < \frac{1}{k}$). This observation can

be explained as follows. According to Eq. 5.8 and Eq. 5.10 in the proof of Proposition 21, the first order deviation of $\mathbf{E}(t_{ij}^q)$ with respect to s_{ij} is,

$$\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial s_{ij}} = \begin{cases} \frac{(kh_i-1)(kh_j-1)}{(k-1)^2} & \text{when } \mathcal{M}_1 \text{ is triggered,} \\ \frac{(1-kh_i)(kh_j-1)}{(k-1)^3} & \text{when } \mathcal{M}_2 \text{ is triggered.} \end{cases} \quad (\text{Eq. 5.12})$$

Thus, for workers whose honesty is greater than $\frac{1}{k}$ (i.e., $h_i > \frac{1}{k}$), $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial s_{ij}}$ is positive when (1) $\mathcal{M}_1$ is triggered and $h_j > \frac{1}{k}$, or (2) $\mathcal{M}_2$ is triggered and $h_j < \frac{1}{k}$. $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial s_{ij}}$ is negative when (1) $\mathcal{M}_2$ is triggered and $h_j > \frac{1}{k}$, or (2) $\mathcal{M}_1$ is triggered and $h_j < \frac{1}{k}$. That $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial s_{ij}}$ is positive means worker i 's expected truthfulness reward increases with the similarity between i and other workers' truthful opinions in the system, and that $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial s_{ij}}$ is negative means worker i 's expected truthfulness reward decrease with the similarity between i and other workers' truthful opinions in the system. The frequencies of $\mathcal{M}_1$ and $\mathcal{M}_2$ being triggered under the above analyzed conditions are shown in Table 5.5, and the symbols (i.e., “+” and “−”) in the second row is the sign of $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial s_{ij}}$ when $h_i > \frac{1}{k}$ under different conditions. The relative high frequency values have been highlighted in **bold**. We could observe that the sign “+” happens more often than the sign “−” (i.e., the sum of the third and fifth columns is greater than the sum of the fourth and sixed columns regarding each setting and each dataset), thus the truthfulness reward of workers whose honesty $h_i > \frac{1}{k}$ increases as the similarity grows from Data-2 to Data-2S or from Data-4 to Data-4S. For the workers whose honesty $h_i < \frac{1}{k}$, the sign of $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial s_{ij}}$ is opposite to the ones shown in Table 5.5, and their truthfulness rewards show the opposite trend.

Moreover, comparing the truthfulness rewards of workers in Data-2 in the four honesty settings in Figures 5.4(a) to (d), the truthfulness rewards of workers whose honesty is greater than $\frac{1}{k}$ is higher in high-level honesty than medium-level honesty setting, and is also higher in ultra-low-level honesty setting than in low-level honesty setting. The truthfulness rewards of the workers whose honesty is smaller than $\frac{1}{k}$ shows the opposite

Table 5.5: The frequency of $\mathcal{M}_1$ and $\mathcal{M}_2$ triggered in different conditions

	Dataset	$h_j \geq \frac{1}{k}$		$h_j < \frac{1}{k}$	
		$s_{ij} \geq \frac{1}{k}$ $\mathcal{M}_1$	$s_{ij} < \frac{1}{k}$ $\mathcal{M}_2$	$s_{ij} \geq \frac{1}{k}$ $\mathcal{M}_2$	$s_{ij} < \frac{1}{k}$ $\mathcal{M}_1$
Sign of $\frac{\partial \mathbf{E}(T_{ij}^q)}{\partial s_{ij}}$ when $h_i > \frac{1}{k}$		+	−	+	−
Sign of $\frac{\partial \mathbf{E}(T_{ij}^q)}{\partial h_j}$ when $h_i > \frac{1}{k}$		+	+	−	−
High-level	Data-2	0.6186	0.1001	0.2187	0.0627
	Data-2S	0.4827	0.2390	0.1722	0.1061
	Data-4	0.6689	0.1889	0.0873	0.0549
	Data-4S	0.5123	0.3582	0.0761	0.0534
Medium-level	Data-2	0.4605	0.1104	0.3257	0.1034
	Data-2S	0.3865	0.1678	0.2761	0.1696
	Data-4	0.5435	0.2358	0.1324	0.0883
	Data-4S	0.4471	0.3360	0.1256	0.0913
Low-Level	Data-2	0.3078	0.0653	0.5303	0.0966
	Data-2S	0.2597	0.1079	0.3954	0.2371
	Data-4	0.3659	0.1905	0.2720	0.1716
	Data-4S	0.3280	0.2360	0.2515	0.1845
Ultra-low-level	Data-2	0.1286	0.0161	0.8156	0.0398
	Data-2S	0.0995	0.0429	0.5276	0.3299
	Data-4	0.1805	0.1060	0.4373	0.2762
	Data-4S	0.1620	0.1147	0.4129	0.3105

trend. This observation could be explained as follows. According to Eq. 5.8 and Eq. 5.10 in the proof of Proposition 21, the first order deviation of $\mathbf{E}(t_{ij}^q)$ with respect to h_j is,

$$\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j} = \begin{cases} \frac{(kh_i-1)(ks_{ij}-1)}{(k-1)^2} & \text{when } \mathcal{M}_1 \text{ is triggered,} \\ \frac{(1-kh_i)(ks_{ij}-1)}{(k-1)^3} & \text{when } \mathcal{M}_2 \text{ is triggered.} \end{cases} \quad (\text{Eq. 5.13})$$

For workers whose honesty is greater than $\frac{1}{k}$, (i.e., $h_i > \frac{1}{k}$), $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j}$ is positive when (1) $\mathcal{M}_1$ is triggered and $s_{ij} > \frac{1}{k}$, or (2) $\mathcal{M}_2$ is triggered and $s_{ij} < \frac{1}{k}$. $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j}$ is negative when (1) $\mathcal{M}_2$ is triggered and $s_{ij} > \frac{1}{k}$, or (2) $\mathcal{M}_1$ is triggered and $s_{ij} < \frac{1}{k}$. That $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j}$ is positive means worker i 's expected truthfulness reward increases with the honesty of other workers in the system, and that $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j}$ is negative means worker i 's expected truthfulness reward decreases with the honesty of other workers in the system. The sign of $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j}$ when

$h_i > \frac{1}{k}$ under the above conditions are shown in the third row of Table 5.5. According to the triggering frequency of $\mathcal{M}_1$ and $\mathcal{M}_2$ in Table 5.5, it can be observed that the sign “+” happens more often than the sign “−” by using Data-2 in high-level honesty setting and medium-level honesty setting (i.e., $0.6186 + 0.2187 > 0.1001 + 0.0627$ in high-level honesty setting and $0.4605 + 0.3257 > 0.1104 + 0.1034$ in medium-level honesty setting). Thus, workers whose honesty is higher than $\frac{1}{k}$ can achieve more truthfulness rewards in high-level honesty setting than in medium-level honesty setting because the average honesty h_j is higher in high-level honesty setting and $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j}$ is more often positive. In low-level and ultra-low-level honesty settings, the sign “−” happens more often than the sign “+” (i.e., $0.5303 + 0.0966 > 0.3078 + 0.0653$ in low-level honesty setting and $0.8156 + 0.0398 > 0.1286 + 0.0161$ in ultra-low-level honesty setting). Hence, workers whose honesty is higher than $\frac{1}{k}$ can achieve less truthfulness rewards in low-level honesty setting than in ultra-low-level honesty setting where the average honesty h_j is lower and $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j}$ is more often negative. For workers whose honesty is less than $\frac{1}{k}$, $\frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_j}$ presents the opposite sign, resulting in the opposite trend. The above analysis can also be applied to explain the observations in the other three datasets.

5.3.2.2 Consistency Reward

In Figure 5.5, the average consistency reward for answering a question that a worker with the honesty value shown in x-axis can achieve in various settings is presented.

In high-level honesty setting, the workers with higher honesty can achieve more consistency rewards in all the four datasets as shown in Figure 5.5 (a). However, when the average honesty of the workers decreases to ultra-low-level honesty settings, the consistency reward curves present decreasing gradients as workers’ honesty increases in the datasets as shown in Figure 5.5(d). It is because iMET does not consider the workers’ honesty when rewarding the consistency of their answers. Therefore, the effectiveness

Figure 5.5: The correctness reward of workers in various settings

of consistency reward in promoting truthful answers from workers actually relies on the following two conditions: (1) the workers share comparatively similar truthful opinion; (2) the average honesty of the workers is relatively high.

5.3.2.3 Final Combined Reward

The final combined reward of workers under the four honesty settings could be achieved by combining the truthfulness reward in Figure 5.4 and consistency reward in Figure

5.5. The optimal ω (the solution of Eq. 5.7) is 1 for all the four datasets in the four honesty settings. It shows that the proposed truthfulness component in iMET could achieve the minimum average rewards for each question and promote truthful incentive simultaneously. The average total rewards for each question q in different datasets in different settings are shown in Table 5.6. It can be seen that the total reward $\sum_i r_i^q$ for a question is largest in the high-level honesty setting of all the four datasets. It indicates that it is more costly to promote truthfulness when the system is dominated by workers providing their truthful opinions, as those workers are deserved to be offered with high reward.

Table 5.6: The average total rewards for each question

	Dataset	High-level	Medium-level	Low-level	Ultra-low-level
$\sum_i r_i^q$	Data-2	24.79	22.51	20.26	15.92
	Data-2S	23.51	22.54	21.58	20.35
	Data-4	13.76	12.71	11.55	10.55
	Data-4S	12.50	12.21	11.44	10.54

In summary, iMET can encourage workers to provide their truthful opinions in various experimental settings by always giving more rewards to workers with higher honesty. In the experiments, the weight is zero which makes the second component insignificant or unnecessary. It is determined by the collected datasets, the optimization objective and constraints in Eq. 5.7. These results also demonstrate that the opinion aggregation method used in the experiments (majority voting) is ineffective in promoting truthful opinions from workers in the scenario where the workers have diverse truthful opinions. However, it will become significant when the optimization objective is changed to consider more factors (only the total cost is considered in Eq. 5.7). It also bridges the proposed truthfulness reward and the existing crowdsourcing platforms where the constraints could be changed to satisfy the existing regulations. For example, in MTurk, the constraints should include that each worker achieves at least a certain amount of rewards β by

successfully submitting her answers, i.e., $r_i^q \geq \beta$ for any answered question q of worker i , where β is specified by the task requester.

5.3.3 Comparison Evaluation

We compare iMET with the following two mechanisms:

1. The static incentive mechanism used by test.baidu.com (Baidu): It rewards workers based on their accuracy ratio (i.e., the percentage of a worker's answers being consistent with the majority ones). Specifically, if 80% of a worker's answers are accurate, then she receives 5 rewards for every 5 questions; if 60% – 80% of the worker's answers are accurate, she receives 3 rewards for every 5 questions; otherwise, she receives nothing.
2. The side-payment mechanism (SP) [37]: It rewards a worker by comparing her answer to a question with another worker's answer to the same question. If their answers are the same, then each worker receives reward 1; otherwise, they receive nothing.

We use *average incentive strength*, Ψ , to measure the effectiveness of the three mechanisms in eliciting truthful opinions. It is calculated as:

$$\Psi = \text{avg}_{h_i \neq h_j} \left\{ \frac{\overline{r}_i^q - \overline{r}_j^q}{h_i - h_j} \right\}, \quad (\text{Eq. 5.14})$$

where $\overline{r}_i^q$ is the average reward that worker i achieves in answering a question. The range of Ψ is $[-1, 1]$. A greater positive value suggests a stronger incentive in eliciting truthful opinions. A negative value means an opposite incentive that encourages workers to provide untruthful opinions.

As the existing mechanisms suffer from two major limitations that they cannot promote truthful opinions from workers when (1) the truthful opinions of workers are different, and (2) the majority workers provide untruthful opinions, we first investigate the

effectiveness of the mechanisms in eliciting truthful opinions when they are exposed to each limitation scenario and then we examine them in a combined limitation scenario.

Firstly, to show the effectiveness of the mechanisms in eliciting truthful opinions when workers have different truthful opinions, we compare the average reward that a worker can achieve in Data-2S and Data-4S which consist of questions for which the workers' have more diverse truthful answers as shown in Figures 5.2(b) and (d). The honesty setting is high-level to decrease the influence of untruthful opinions. In Figure 5.6(a) and (b), it can be seen that the workers providing more truthful opinions (i.e., the workers with higher honesty) can always achieve more rewards by using iMET. SP almost loses its incentive by giving nearly the same rewards to all workers, and Baidu cannot work by presenting a fluctuating curve or constant zero when $k = 2$ and $k = 4$, respectively.

Secondly, Figures 5.6(c) and (d) show the average reward in the ultra-low honesty setting by using Data-2 and Data-4. We set that all the 45 workers share one particular worker's truthful opinion to remove the impact of different truthful opinions. The y-axis presents the average reward of workers in the 45 simulations where each worker is chosen as that particular worker. Here, the setting where "the majority workers provide untruthful answers" means that more than $1 - \frac{1}{k}$ percentage of the provided answers are untruthful answers⁶. In low-level honesty setting, the average honesty of workers is 0.3578, leading to that 64.22% answers are untruthful answers. We have $1 - \frac{1}{2} < 64.22\% < 1 - \frac{1}{4}$, making the low-level honesty setting not qualify to be used in demonstrating the effectiveness of the mechanisms in the scenario where the majority workers provide untruthful answers when $k = 4$. Therefore, we use the ultra-low-level honesty setting where the average honesty of workers is 0.2022 resulting in that the percentage of untruthful answers is 79.78% which takes for the "majority" ($79.78\% > 1 - \frac{1}{k}$ where $k = 2$ or $k = 4$). In Figures 5.6(c)

⁶According to Assumption 3, when untruthful answers provided by the workers for a question take more than $1 - \frac{1}{k}$, the percentage of each untruthful answer is more than $\frac{1}{k}$, which is greater than the percentage of truthful answer and becomes the majority.

Figure 5.6: The comparison results of the three mechanisms

and (d), it can be seen that iMET presents an obvious stronger incentive by giving more rewards to workers with higher honesty, than the other two mechanisms. For Baidu, due to its majority based aggregation rule, it gives more rewards to the majority workers providing untruthful opinions in Figure 5.6(c). Baidu provides zero reward to all the workers in Figure 5.6(d) due to the fact that the maximum accuracy ratio of workers when $k = 4$ is less than 60% which is the minimum accuracy to achieve positive rewards. In SP, a worker providing truthful opinions may not get reward when her answers are compared with the answers of a worker providing untruthful opinions. As SP has two equilibria, the majority of workers providing untruthful opinions may lead the strategy of providing untruthful opinions to become more beneficial than that of providing truthful opinions. Thus, SP presents a negative incentive when the majority of workers provide untruthful opinions as shown in Figures 5.6(c) and (d).

Thirdly, we compare the three mechanisms when workers have different truthful opinions and the majority of workers provide untruthful answers in Data-2 and Data-4 under the ultra-low-level honesty setting⁷. Figures 5.6(e) and (f) show that iMET is the only one maintaining positive incentive among the three mechanisms. Baidu and SP even present negative incentives in Figure 5.6(e). Specifically, when k is 4 in Figure 5.6 (f), the truthful opinions of workers are more diverse than in Figure 5.6 (e), resulting in that SP and Baidu provide almost same rewards for workers regardless of their honesty.

Table 5.7 shows the average incentive strength of the three mechanisms in the six scenarios corresponding to Figure 5.6(a) to (f), respectively. It can be seen that iMET can always achieve the largest positive incentive strength in the six scenarios as highlighted in **bold** in Table 5.7, meaning that the workers are most encouraged to provide truthful opinions to gain more rewards under iMET. For the scenario of workers holding various

⁷Since SP and Baidu cannot effectively promote truthful opinions in Data-2S and Data-4S under the high-level honesty setting as shown in Figures 5.6(a) and (b), they cannot achieve better performance in the two datasets under ultra-low-level honesty setting. It would be unfair to compare them with iMET in these two datasets. Due to this consideration, we choose Data-2 and Data-4.

truthful opinions (i.e., the scenario corresponding to Figures 5.6(a) and (b) where $k = 2$ and $k = 4$, respectively), Baidu and SP both present a positive incentive strength, but smaller than that by using iMET. When majority workers behave untruthfully (i.e., the scenarios corresponding to Figures 5.6(c) to (f) where $k = 2$ and $k = 4$, respectively), SP and Baidu both present non-positive incentives, meaning that they cannot encourage workers to provide truthful opinions.

Table 5.7: The average incentive strength of the three mechanisms in various settings

	Settings	Baidu	SP	iMET
$k = 2$	Figure 5.6(a): Various Truthful Opinions	0.1011	0.0216	0.1460
	Figure 5.6(c): Majority Being Dishonest	-1.0905	-0.5858	0.7146
	Figure 5.6(e): The Combined Scenario	-0.8968	-0.1936	0.4736
$k = 4$	Figure 5.6(b): Various Truthful Opinions	0.002	0.0150	0.0454
	Figure 5.6(d): Majority being dishonest	0.0000	-0.0636	0.1752
	Figure 5.6(f): The combined scenario	0.0000	-0.0079	0.1206

In Table 5.7, it is interesting to notice that when the truthful opinions of the workers are more diverse (from the scenario of Figure 5.6(c) to the scenario of Figure 5.6(e) when $k = 2$, and from the scenario of Figure 5.6(d) to the scenario of Figure 5.6(f) when $k = 4$), the incentive strength by using Baidu or SP actually increases, except Baidu being constant zero when $k = 4$. The reason behind this observation is that Baidu uses majority voting to decide the final answers and rewards the workers according to the extent of their answers being consistent with the final answers. When there exist diverse truthful opinions among workers and the majority workers provide untruthful answers, the workers providing truthful answers may possibly get rewards when their truthful opinion is different from the majority workers, and the workers providing untruthful answers may possibly not get rewards when their truthful opinion is different from the majority workers. Therefore, a slightly increase of incentive strength can be observed. A smaller negative incentive strength is presented by using SP when there exist diverse truthful opinions among workers and majority workers provide untruthful

answers because a worker's truthful answer and another worker's untruthful answer may be the same when their truthful opinions are different. Then the worker providing the truthful answer may get reward in some cases, resulting in smaller negative incentive strength in the scenarios of Figures 5.6(e) and (f) than that in Figures 5.6(c) and (d), respectively.

To summarize, according to the experimental results, it can be seen that iMET can elicit truthful answers and outperform the existing mechanisms, especially when workers have various truthful opinions and the majority workers provide untruthful answers.

5.4 Discussion

We have proposed an incentive mechanism to elicit truthful opinions in consensus tasks on crowdsourcing platforms. In this section, some practical and potential issues are discussed.

5.4.1 How to Choose the Control Questions?

In our mechanism, we insert the control questions to estimate the honesty of a worker in providing her truthful answers, which is further used to estimate the truthful opinion similarity between two workers. Therefore, the control questions need to be carefully designed. The control questions can be selected from other tasks of the same type, assuming that the answers for the questions are common knowledge [79]. Another way to get the control questions is to distribute the candidate control questions to some pre-identified trustworthy workers, and consider the questions with the highly consistent answers (e.g., the same answers provided by at least 90% trustworthy workers) as the control questions.

5.4.2 Collusive Behaviors of Workers

In our mechanism, we assume that the workers answer questions independently. But in reality, some workers may collude in an attempt to increase their rewards. Though this problem may be reduced by presenting the questions and options in a random order to each worker, other methods may be needed in some cases. A possible solution is to filter out some workers' answers through analyzing their behavior patterns, e.g., the time spent on answering each question and mouse trajectories [126].

5.4.3 Dynamics of Workers' Honesty

In this work, we assumed that the honesty of a worker does not change during a task. In reality, this assumption may not be valid. For example, a worker might provide truthful answers at the beginning, then gets tired soon and starts to provide random answers. We can address this case by dividing a task into several smaller parts and distribute the control questions uniformly among these parts. Then the worker's honesty can be separately estimated in different parts, and she can be rewarded accordingly. The task can also be designed with fewer questions.

5.4.4 Setting the Number of Options

According to the experimental results in Figure 5.6 and Table 5.7, iMET can achieve a larger positive incentive strength with $k = 2$ than with $k = 4$. This observation implies that the value of k affects the incentive strength, which could be explained as follows.

The incentive strength of the mechanism can be defined as $\Phi = \frac{\partial \mathbf{E}(t_{ij}^q)}{\partial h_i}$ which is compatible with Eq. 5.14. The first order derivation of Φ with respect to k is calculated as follows when $\mathcal{M}_1$ and $\mathcal{M}_2$ are triggered, respectively:

$$\frac{\partial \Phi}{\partial k} = \begin{cases} \frac{(kh_j-1)(1-s_{ij})+(ks_{ij}-1)(1-h_j)}{(k-1)^3} & \text{when } \mathcal{M}_1 \text{ is triggered,} \\ -\frac{(1-ks_{ij})(kh_j-1)+(1-ks_{ij})(1-h_j)+(kh_j-1)(1-s_{ij})}{(k-1)^4} & \text{when } \mathcal{M}_2 \text{ is triggered.} \end{cases} \quad (\text{Eq. 5.15})$$

The sign of $\frac{\partial \Phi}{\partial k}$ has been explicitly presented in Table 5.8, where “ > 0 ” means that incentive strength increases as k increases, and “ < 0 ” means that incentive strength decreases as k increases.

Table 5.8: The sign of $\frac{\partial \Phi}{\partial k}$ under different conditions

	$s_{ij} > \frac{1}{k}$	$s_{ij} < \frac{1}{k}$
$h_j > \frac{1}{k}$	> 0	< 0
$h_j < \frac{1}{k}$	> 0	< 0

It can be observed that $\frac{\partial \Phi}{\partial k} > 0$ when $s_{ij} > \frac{1}{k}$, and $\frac{\partial \Phi}{\partial k} < 0$ when $s_{ij} < \frac{1}{k}$. When the workers hold very diverse truthful opinions (satisfying $s_{ij} < \frac{1}{k}$), task requesters are suggested to decrease the number of options so as to achieve high incentive strength. When the workers hold similar truthful opinions (satisfying $s_{ij} > \frac{1}{k}$), task requesters should increase the number of options to achieve high incentive strength. It still necessary to notice that the similarity of the same pair of workers’ truthful opinions intends to become different when the number of options is too many, which results in that $s_{ij} < \frac{1}{k}$ where $\frac{\partial \Phi}{\partial k} < 0$. Therefore, task requesters should choose a suitable tradeoff between incentive strength of the system and the number of options provided for questions in the tasks.

5.5 Summary

In this chapter, we have presented a novel incentive mechanism to elicit truthful opinions (iMET) in consensus tasks on crowdsourcing platforms. It advances the state-of-the-art in two significant ways. Firstly, it relaxes the assumption that the workers participating in a consensus task share the same truthful opinion. In this way, iMET is more suitable for applications in practical crowdsourcing systems where workers can be from various backgrounds. Secondly, iMET works well even when the majority of workers provide untruthful opinions, which is a neglected scenario in most existing incentive mechanisms.

Experimental results show that iMET can effectively elicit truthful opinions and significantly outperforms existing mechanisms when diverse truthful opinions exist among workers and/or the majority of them provide untruthful opinions.

Chapter 6

Robustness Evaluation Framework of Incentive Mechanisms

In game theoretical analysis of incentive mechanisms, all players in the system are assumed to be rational. Since it is likely that mechanism participants in the real world may not be fully rational, such mechanisms may not work as effectively as in the idealized settings for which they were designed. For example, in an incentive mechanism for reputation systems where the equilibrium strategy for each agent is to provide fair/truthful ratings, the attacking agents would provide unfair ratings that may lead the rational agents deviate from their equilibrium strategy. Therefore, it is important to evaluate the robustness of incentive mechanisms against various types of agents with irrational or bounded rational behaviors. Such evaluations would provide us with the information needed to choose mechanisms with desired properties in real environments. In this chapter, a general robustness measure is proposed, inspired by the studies of evolutionary game theory, for measuring the resilience of a desired equilibrium against a non-equilibrium strategy. A simulation framework is then proposed based on evolutionary dynamics to empirically quantify the robustness. The proposed simulation framework is validated by comparing the simulated results with the analytical predictions based on a modified simplex analysis approach. Finally, the proposed simulation framework is implemented for evaluating the robustness of the incentive mechanisms in reputation

systems for electronic marketplaces. The results from the implementation show that the evaluated mechanisms have high robustness against a certain non-equilibrium strategy, but could be vulnerable to another strategy, indicating the need for designing a more robust incentive mechanism for reputation management to have secure e-marketplaces.

The organization of this chapter is as follows. In Section 6.1, two formal robustness measures for incentive mechanisms are defined when the populations are infinite and finite, respectively. In Section 6.2, a simulation framework for evaluating the robustness is proposed. The validation of the simulation framework in representative games is presented in Section 6.3, and the implementation and evaluation of the simulation framework in e-marketplaces are presented in Section 6.4. Finally, Section 6.5 concludes this chapter.

6.1 Robustness Measure

Incentive mechanism design is a sub-field of game theory that considers how to implement the desired, e.g., socially efficient, solutions by motivating rational players to truthfully disclose their private information [31, 53]. Players' equilibrium strategy profile¹ is called a desired strategy profile (equilibrium) which leads to the desired outcome of the mechanism, assuming that the players are rational. Players who instead adopt a non-equilibrium strategy is regarded as *bounded rational* players. This chapter studies the robustness of incentive mechanisms when some players are bounded rational.

The proposed robustness measure of incentive mechanisms is inspired by the concept of evolutionary stable strategy (ESS) studied in evolutionary game theory — the application of game theory to biology [99, 88, 83] — to study the evolution of strategies in the population(s) of individuals who are competing with each other for survival and

¹For ease of analysis, it assumes a single equilibrium for a mechanism. The following analysis can be extended to handle multiple equilibria.

reproduction. A strategy is evolutionarily stable if a population of individuals using the strategy will not deviate from it even when the population is invaded or infiltrated by a small fraction of mutants, i.e., individuals using an alternative (non-equilibrium) strategy. In an incentive mechanism, if the desired equilibrium strategy is an ESS, the mechanism will be robust against a sufficiently small fraction of bounded rational invaders. Furthermore, an incentive mechanism is more robust than another if its desired equilibrium is robust against a larger fraction of invaders. Based on this intuition, a general quantitative definition of robustness is proposed with respect to the desired equilibrium.

An evolutionary framework of agent populations is used in this study. Within this framework an incentive mechanism in a game with n players $I = \{1, \dots, n\}$ is modeled by n populations: for each player position², there is a large population of individuals (agents), and each such individual is following a pure strategy. Let $S_i = \{s_1^i, \dots, s_{m_i}^i\}$ be the set of pure strategies available to the individuals of population i and $\Delta_i = \{x_i \in \mathbb{R}^{m_i} \mid \sum_{s_j^i \in S_i} x_i(j) = 1, x_i(j) \geq 0, j = 1, \dots, m_i\}$ be the set of possible strategy profiles for population i , i.e., each $x_i(j)$ corresponds to the fraction of individuals in the population i playing strategy $s_j^i \in S_i$. Note that x_i is formally equivalent to a mixed strategy for the player $i \in I$ in the n -player game. The combination of n population profiles is $x \in \Theta$ where $\Theta = \times_{i \in I} \Delta_i$. Suppose $\epsilon \in (0, 1)$ proportion of the population i invade the mechanism by playing a different profile $y_i \in \Delta_i$. As a result, the new population profile for population i becomes $x_i^\epsilon = \epsilon \cdot y_i + (1 - \epsilon)x_i$. If the set of payoff functions for all populations is denoted as $u = \{u_1, \dots, u_n\}$ where the payoff of an agent in population i taking strategy s_j^i is $u_i(s_j^i, x_{-i})$. The average payoff of the rational players in the population i before and after the invasion can be expressed as $u_i(x_i, x_{-i}) = \sum_{j=1}^{m_i} x_i(j)u_i(s_j^i, x_{-i})$ and $u_i(x_i, x_{-i}^\epsilon) = \sum_{j=1}^{m_i} x_i(j)u_i(s_j^i, x_{-i}^\epsilon)$ respectively and the average payoff of the mutant invaders in population i is $u_i(y_i, x_{-i}^\epsilon) = \sum_{j=1}^{m_i} y_i(j)u_i(s_j^i, x_{-i}^\epsilon)$. Now, a general mechanism

²When the game is symmetric and the positions of the n players are equivalent, a single population could represent all the players who interact with each other.

can be denoted as $\mathcal{M} = \{I, \Theta, u\}$ and the robustness of a desired equilibrium can be defined as follows.

Definition 9. *[Robustness of a desired equilibrium] Given an incentive mechanism $\mathcal{M} = \{I, \Theta, u\}$ with a desired equilibrium $x \in \Theta$, the robustness of x is R such that*

$$R = \arg \max_{\epsilon \in [0,1]} u_i(x_i, x_{-i}^\epsilon) > u_i(y_i, x_{-i}^\epsilon), \forall y \in \Theta(y \neq x), \forall i \in I. \quad (\text{Eq. 6.1})$$

In other words, the robustness of a desired equilibrium is the maximum proportion of invaders such that the desired equilibrium strategy is still the best strategy for rational players in each population.

Observation 1. *The robustness of a strictly dominant strategy equilibrium is 1.*

Proof. Given that the desired equilibrium x is a strictly dominant strategy equilibrium, $u_i(x_i, x_{-i}^\epsilon) > u_i(y_i, x_{-i}^\epsilon)$ in Eq. 6.1 would be always true for all $\epsilon \in [0, 1]$. Thus the robustness of the equilibrium is 1, i.e., $R = 1$. $\square$

However, the computational cost of formally analyzing the robustness measure in Definition 9 makes such analysis infeasible. First, the size of the population corresponding to each player position is assumed to be infinite in the definition so as to model all possible mixed strategies of the player. The payoff of a population is thus difficult to calculate through aggregating all individuals' utilities. Second, the strategy of the mutants $y \neq x$ is continuous and it may be impossible to verify Eq. 6.1 for all $y \in \Theta$. Moreover, the size of players I in a realistic game could be very large, making the robustness measure computationally prohibitive. Given the difficulty of formally analyzing the robustness of incentive mechanisms, finite populations is considered to be able to realistically measure the robustness of incentive mechanisms.

Moreover, the mutants or invaders in a realistic mechanism always attack the mechanism using several mature techniques or strategies, e.g., constant attack and whitewashing

attack in reputation systems [36]. It is also observed that these attacks are typically conducted by a single player. Thus, it is practical to measure the robustness of an incentive mechanism against representative types (and their combinations) of mutants in a single population.

The robustness measure in Definition 9 is noted to be a static concept. A mechanism is always implemented in real time and players could dynamically adjust their strategies as time goes on, which could be modeled by the well-known replicator dynamics (RD) model [78, 77, 26, 17] — a representative model for group dynamics — for capturing long-range evolutionary dynamics with natural selection forces. If agents using the desired equilibrium strategy could achieve higher payoff (fitness) in a population, then they are more likely to reproduce and produce offsprings which use the same strategy (replicator) and eventually through this process the strategy distribution of the population will converge to a stable equilibrium profile.

Figure 6.1: Evaluating robustness based on replicator dynamics (RD)

An evolutionary dynamics based practical robustness measure is proposed, which evaluates the robustness of an equilibrium against any specific non-equilibrium strategy (see Figure 6.1). In this approach, agents adopting the non-equilibrium strategy $y_k \neq x_k$ are gradually added to the original *rational population* (population without mutants) and the resultant populations repeatedly interact under the mechanism. The strategy distribution of the rational populations evolves through RD until convergence. Successively more mutants are added until the population converges to a distribution which is different from the desired equilibrium.

Definition 10. *[Robustness against a non-equilibrium strategy] The robustness of an incentive mechanism $\mathcal{M}$, with desired equilibrium strategy profile x , against a non-equilibrium strategy y_k , used by bounded-rational invaders, is*

$$R(N, y_k) = \frac{M}{N + M}, \quad (\text{Eq. 6.2})$$

where N is the finite population size for each player position and M is the maximum number of bounded rational invaders such that the converged strategy profile, under RD, does not deviate from the desired equilibrium.

It is assumed that each player position has the same population size and the robustness of an incentive mechanism is a function of the population size and the non-equilibrium strategy. The robustness of a mechanism with different population sizes and against different non-equilibrium strategies will be evaluated. It is to be expected that the accuracy of the robustness measure estimation increases with the population size. Therefore, the population size N is gradually increased until the estimated mechanism robustness does not change for any larger populations and use this converged value as the accurate robustness estimation of the mechanism.

6.2 Simulation Framework

In this section, a simulation-based framework is proposed to calculate the robustness of an incentive mechanism. The proposed framework is based on replicator dynamics [69] and is used to study the dynamics of the equilibrium strategy distribution in populations that include agents using a non-equilibrium strategy. Strategies which can garner higher payoffs become more popular while ineffective strategies are gradually eliminated through the replicator dynamics process. According to Definition 10, the agents are gradually added by taking a specific non-equilibrium strategy and the population evolve

over time until convergence. It stops inserting more agents when the equilibrium strategies are abandoned by the rational populations in the converged population profile after a sufficiently large number of evolutionary dynamics steps. The proposed simulation framework is different from the Monte Carlo simulation [8] which is based on statistical sampling to determine the properties of some phenomenon (or behavior) and mostly used in physical sciences, computer graphics, mathematics, etc.

Recall that an incentive mechanism is denoted by $\mathcal{M}(I, \Theta, u)$. The population size for every player position is N , and the pure strategy set of player $i \in I$ is $S_i = \{s_1^i, \dots, s_{m_i}^i\}$. The desired equilibrium is $x = (x_1, \dots, x_n)$ where x_i is the desired equilibrium strategy of population i . The robustness of a mechanism $\mathcal{M}$ against a non-equilibrium strategy y_k is evaluated using Algorithm 5.

In Algorithm 5, the three parameters used in the simulation framework are set (Line 1). The number of undesired agents is initialized by 1, i.e., $M = 1$ (Line 2), then gradually increases the number M until the rational agents deviate from their desired equilibrium profile x in at least $\varepsilon = \frac{1}{T} = 0.02$, i.e., in 2% of the simulations (Line 3-17). Due to the randomness involved in the evaluation framework (Line 10-13), the evolutionary process is repeated for $T = 50$ times, and in each round, the strategy profile of each population i evolves (Line 8-13) until it converges. The “ $\bar{x}$ is not converged” (Line 8) in the framework means that the strategy profile $\bar{x}$ is still not stable enough according to the convergence criteria specified in Section 6.3.2. The variable λ records the number of times out of T rounds that the rational agents deviate from their equilibrium strategy ($\bar{x} \not\approx x$), i.e., the converged strategy profile deviates from the desired equilibrium x as indicated by $\max_{i \in I, j \in \{1, \dots, m_i\}} |\bar{x}_i(j) - x_i(j)| > 0.01$ (Line 14-15). After completing T repetitions, the percentage of the times that the rational populations deviate from the desired equilibrium is calculated (Line 16). The probability $\varepsilon = \frac{1}{T}$ is to specify the extent to which the deviation is considered acceptable, which is further explained in Proposition

<div style="margin-bottom: 10px;"> Input : $\mathcal{M}$: incentive mechanism under evaluation; n : number of players in $\mathcal{M}$; N: population size for each player; x : desired equilibrium; y_k: non-equilibrium strategy of mutants in population k; </div> <div style="margin-bottom: 10px;"> Output: $R(N, y_k)$, Robustness of $\mathcal{M}$ against y_k; </div> <div> <ol style="list-style-type: none"> 1 Initialize $T = 50$; $\varepsilon = \frac{1}{T}$; $\xi = 0$; 2 Set $M = 1$; 3 repeat 4 For each player j, set N agents taking the desired strategy x_j; 5 Add M undesired agents taking strategy y_k in population k; 6 for $Times = 1 \rightarrow T$ do 7 $\lambda = 0$; $\bar{x} = x$; 8 while $\bar{x}$ is not converged do 9 Run $\mathcal{M}$ in the n populations; 10 for $i = 1 \rightarrow n$ do 11 Calculate average payoff $u_i(s_j^i, \bar{x}_{-j})$ for each strategy $s_j^i \in S_i$; 12 Generate new generation to replace population i; 13 $\bar{x} =$ the strategy profile of rational populations; 14 if $\bar{x} \not\approx x$ then 15 $\lambda = \lambda + 1$; 16 $\xi = \frac{\lambda}{T}$; $M = M + 1$; 17 until $\xi \geq \varepsilon$; 18 return $R(N, y_k) = \frac{M-1}{N+M-1}$; </div>

Algorithm 5: Evaluating the robustness of an incentive mechanism $\mathcal{M}$ against a non-equilibrium strategy y_k .

26. If $\xi < \varepsilon$, it is concluded that the rational populations could still sustain their desired equilibrium when invaded by M agents using the non-equilibrium strategy y_k , and M increases by 1. This continues until the deviation of the rational agent strategies in the converged population becomes too frequent ($\xi \geq \varepsilon$). Then, the robustness of the mechanism against y_k is returned as $R(N, y_k) = \frac{M-1}{N+M-1}$ (Line 18), as per Definition 10.

The new population for each player (Line 10-13) will be generated after each evolutionary dynamics step based on the Moran process [69, 77], a well-known evolutionary dynamics process for finite populations. More specifically, each population i with N rational agents is divided into k_i subsets where agents in each set j take pure strategy

$s_j^i \in S_i$. The size of the subsets are $N_1, \dots, N_{m_i}$, and the average payoff of the set j is $u_i(s_j^i, \bar{x}_{-i})$. If subset j is empty, i.e., $N_j = 0$, $u_i(s_j^i, \bar{x}_{-i}) = 0$. In each step, an agent from each population i is randomly chosen to be replaced by a new agent using strategy s_j^i with probability

$$P_i(s_j^i) = \frac{N_j \times u_i(s_j^i, x_{-i})}{\sum_{l=1}^{m_i} N_l \times u_i(s_l^i, x_{-i})}, \quad (\text{Eq. 6.3})$$

and the other $N - 1$ agents in the new generation inherit the strategies of the $N - 1$ agents (excluding the selected agent) one by one. Thus, a strategy s_j^i in population i is more likely to be taken by more individuals in the next generation when the proportion of agents taking s_j^i is larger and average payoff $u_i(s_j^i, x_{-i})$ is higher.

Note that a strategy s_j^i that becomes extinct (N_j drops to 0) will never re-appear as the probability of another agent adopting that strategy $P_i(s_j^i)$ also becomes 0. To avoid the system converging to homogeneous states with all agents using the same non-optimal strategy, it is necessary to use mutation to re-introduce extinct strategies.

For each population i , if the strategy set $S_i^d = \{s_j^i | N_j = 0, j = 1, \dots, m_i\}$ is not empty, the framework will run the mutation process. First, one strategy s_j^i is randomly chosen from the ones appearing in the population with probability $P_i(s_j^i)$, and uniformly choose another strategy $s_{j'}^i$ in the distinct strategy set S_i^d . Then the strategy of an agent from subset j is changed to be $s_{j'}^i$ with probability

$$F_i(s_j^i \rightarrow s_{j'}^i) = \frac{1}{e^{\rho(u_i(s_j^i, x_{-i}) - u_i(s_{j'}^i, x_{-i}))} + 1}, \quad (\text{Eq. 6.4})$$

where ρ is a positive constant, representing the intensity of mutation [77]. When $\rho < 1$, $F_i(s_j^i \rightarrow s_{j'}^i)$ converges to 0.5 which produces high neutral fluctuation. When $\rho \rightarrow +\infty$, $F_i(s_j^i \rightarrow s_{j'}^i)$ becomes a step function where the probability of the strategy $s_{j'}^i$ appearing is 1 if $u_i(s_j^i, x_{-i}) < u_i(s_{j'}^i, x_{-i})$, and 0 otherwise. The extensive evaluations with different ρ values have been conducted and the results show that ρ does not significantly influence the evolution of the population profile. It has been verified that the evaluated robustness

is not sensitive to the value of ρ in the Prisoner's Dilemma game, as presented in Figure 6.3. It is due to the fact that a strategy could only reappear in the population through the mutation process when the strategy has already been extinct. Therefore, ρ is set to be 1 in the simulations without exception.

The robustness of mechanisms is related to the population size N . However, for special settings, the robustness $R(N, y_k)$ could be independent of population size N as shown in Proposition 25. Generally speaking, $R(N, y_k)$ increases as the population size grows and finally converges, as shown in Proposition 26.

Proposition 25. *There exists $\varepsilon = G(N)$ where $\frac{dG(N)}{dN} < 0$ such that the evaluated robustness of a mechanism is independent of the population size N .*

Proof. The proposition is first proven in a special case and then we show how to generalize this result. Consider a special mechanism for a symmetric game in which each player has two pure strategies s_1 and s_2 . The desired equilibrium strategy is pure strategy s_1 denoted by $x = [1, 0]$. The other strategy s_2 is $y = [0, 1]$. The population containing N rational agents whose strategy profile is x and M bounded rational agents whose strategy profile is y such that the expected utility of agents taking s_1 in the rational population is $a = u(x, \delta \cdot y + (1 - \delta)x)$ and the utility of taking s_2 is $b = u(y, \delta \cdot y + (1 - \delta)x)$ for any $\delta \in (0, 1)$.

Among the rational agents, the number of agents taking s_1 is denoted by N_1 , and the number of agents taking s_2 by N_2 . Initially, $N_1 = N$ and $N_2 = 0$. According to the Moran process in generating a new population, only one agent could change its strategy in each step. Suppose that the probability of the number of desired strategy taker N_1 changing from i to j after each step is $T_{i,j}$. $T = \{T_{i,j}\}$ is a $(N + 1) \times (N + 1)$ tridiagonal

matrix where

$$\begin{aligned}
 T_{0,1} &= 0; \\
 T_{0,0} &= 1; \\
 &\dots \quad \dots \\
 T_{i,i-1} &= \frac{i}{N} \cdot \frac{(N-i)b}{i \cdot a + (N-i)b}; \\
 T_{i,i+1} &= \frac{N-i}{N} \cdot \frac{i \cdot a}{i \cdot a + (N-i)b}; \\
 T_{i,i} &= 1 - T_{i,i-1} - T_{i,i+1}; \\
 &\dots \quad \dots \\
 T_{N,N-1} &= 0; \\
 T_{N,N} &= 1.
 \end{aligned} \tag{Eq. 6.5}$$

Next it shows that when $\varepsilon = G(N) = \frac{1}{N+1}$, the robustness of the mechanism is independent of the population size N . The dynamic process has two absorbing states (when the population profile becomes one of these, the dynamic process will converge [17]): $N_1 = 0$ and $N_1 = N$. It then calculates the probability, f_i , of the population converging to state $N_1 = 0$ when starting at state $N_1 = i$, which is recursively defined as, $\forall i \in \{1, \dots, N-1\}$,

$$f_i = T_{i-1,i} \cdot f_{i-1} + T_{i,i} \cdot f_i + T_{i+1,i} \cdot f_{i+1}. \tag{Eq. 6.6}$$

It is noted that $f_0 = 1$ and $f_N = 0$, and hence $f_1, \dots, f_{N-1}$ can be calculated by solving these $N-1$ linear equations [77]:

$$f_i = \begin{cases} \frac{i}{N} & \eta = 1, \\ \frac{1 - (\frac{1}{\eta})^i}{1 - (\frac{1}{\eta})^N} & \eta > 1, \end{cases} \tag{Eq. 6.7}$$

where $\eta = \frac{b}{a}$. When $\eta \rightarrow 1$, it can be derived that $\lim_{\eta \rightarrow 1} f_i = \frac{i}{N}$, and it is how the value f_i is obtained when $\eta = 1$. Since f_1 and f_{N-1} will be utilized in the following proof, they are specified as follows:

$$f_1 = \begin{cases} \frac{1}{N} & \eta = 1, \\ \frac{1 - \frac{1}{\eta}}{1 - (\frac{1}{\eta})^N} & \eta > 1, \end{cases} \quad \text{and} \quad f_{N-1} = \begin{cases} \frac{N-1}{N} & \eta = 1, \\ \frac{1 - (\frac{1}{\eta})^{N-1}}{1 - (\frac{1}{\eta})^N} & \eta > 1. \end{cases} \tag{Eq. 6.8}$$

The value f_1 is also called the *fixation probability* that a single, bounded rational agent using y makes all the other $N-1$ rational agents using x to eventually choose strategy

y . Conversely, the probability that a single agent using x makes all the other $N - 1$ agents using y to eventually use strategy x , is denoted by f'_1 which could be calculated by $1 - f_{N-1}$.

It can be derived that $f_1 < \frac{1}{N}$ when $\eta < 1$, and $\frac{\partial f_1}{\partial \eta} > 0$.³ Since $\lim_{\eta \rightarrow 1} \frac{1 - \frac{1}{\eta}}{1 - (\frac{1}{\eta})^N} = \frac{1}{N}$ when N is a constant, and $\frac{\partial[(1 - \frac{1}{\eta})/(1 - \frac{1}{\eta^N})]}{\partial \eta} > 0$, f_1 is a strictly increasing function of η . Similarly, it can be derived that f'_1 is a strictly decreasing function of η . Intuitively, as $\eta = \frac{b}{a}$ increases, the strategy y can obtain higher utility compared to strategy x , making it more likely that all agents take y , i.e., f_1 increases with η , and correspondingly, it is less likely that all agents adopt x , i.e., f'_1 decreases with η .

Figure 6.2: The state transition of the population

Considering the mutation process, it assumes a strategy x (or y) can be re-introduced into the population even after x (or y) became extinct. For a population where all agents use x at state $N_1 = N$, the probability of it evolving to the state $N_1 = 0$ (considering one mutator) is f_1 and the probability of the population evolving from $N_1 = 0$ to $N_1 = N$ is f'_1 . This state transition could be described as in Figure 6.2. Then, the probability that the population where all agents use x evolve to a population where all agents use y is

$$\begin{aligned} \xi(N, \eta) &= f_1 - f_1 f'_1 + f_1(f_1 f'_1) - f_1 f'_1(f_1 f'_1) + \dots \\ &= f_1 + f_1(f_1 f'_1) + f_1(f_1 f'_1)^2 \dots - f_1 f'_1 - f_1 f'_1(f_1 f'_1) - f_1 f'_1(f_1 f'_1)^2 - \dots \\ &= \frac{f_1 - f_1 f'_1}{1 - f_1 f'_1}. \end{aligned} \tag{Eq. 6.9}$$

From Eq. 6.8 and Eq. 6.9 and using $\lim_{\eta \rightarrow 1} f'_1 = \frac{1}{N}$, one can have $\xi(N, \eta)|_{\eta=1} = \frac{1}{N+1}$. It can be derived that

$$\frac{\partial \xi(N, \eta)}{\partial \eta} = \frac{\frac{\partial f_1}{\partial \eta}(1 - f'_1) - \frac{\partial f'_1}{\partial \eta} f_1(1 - f_1)}{(1 - f_1 f'_1)^2} > 0. \tag{Eq. 6.10}$$

³It does not consider the case of $\eta < 1$ in Eq. 6.8 as in that case $f_1 < \frac{1}{N}$, which is a very small value for a large population size and hence equilibrium strategies are very likely be robust against one or few invaders.

In Algorithm 5, ε represents the maximum acceptable rate of deviation from the desired strategy. The algorithm increases M until $\xi(N, \eta) = \varepsilon$. If $\varepsilon = \frac{1}{N+1}$, the condition becomes $\xi(N, \eta) = \frac{1}{N+1}$. Since $\xi(N, \eta)$ is a strictly increasing function of η , it could be satisfied only when $\eta = 1$, i.e., $b = a$, which is independent of the population size N . Therefore, in the above special case, there exists a function $G(N) = \frac{1}{N+1}$ such that the robustness is independent of the population size N , as $\varepsilon = G(N)$ where $\frac{dG(N)}{dN} < 0$.

When there are more than two pure strategies in the population, the robustness of the mechanism will still be independent of N by treating s_2 in the above proof as the non-equilibrium pure strategy such that the agents taking the non-equilibrium strategy could achieve the highest payoff among all the non-equilibrium pure strategies, and $\eta = \frac{b}{a}$ where b is the payoff of taking this strategy and a is the payoff of taking the desired equilibrium strategy. For a general mechanism with $n > 2$ players, it could be easily derived that such $G(N)$ still exists, since the population for each player could evolve in the same way as that in the special case. $\square$

However, it is more reasonable to set ε as a constant since ε represents the acceptable rate of deviation from the desired strategy.

Proposition 26. *When $\varepsilon = \varepsilon_0$ is a constant, the robustness of a mechanism increases with the population size N where $N > \frac{1}{\varepsilon_0} + 1$, and finally converges at a value when the agents of a population taking a non-equilibrium strategy can achieve $\frac{\varepsilon_0}{1-\varepsilon_0}$ times payoff more than that of the desired equilibrium strategy.*

Proof. In Algorithm 5, it stops increasing M when the rate of deviation from the desired strategy, $\xi(N, \eta) = \frac{\lambda}{T}$, becomes more than ε . For a constant ε_0 , if the robustness of a mechanism increases with the population size, then $\xi(N, \eta)$ should be a decreasing function of the population size N .

Given that $N > \frac{1}{\varepsilon_0}$, it can be obtained that $\eta > 1$ when $\xi(N, \eta) = \varepsilon_0$ since $\frac{\partial \xi(N, \eta)}{\partial \eta} > 0$ and $\xi(N, 1) = \frac{1}{N+1} < \varepsilon_0$. Since

$$\begin{aligned}\frac{\partial f_1}{\partial N} &= \frac{(\frac{1}{\eta})^N \ln(\frac{1}{\eta})(1-\frac{1}{\eta})}{(1-(\frac{1}{\eta})^N)^2} = \frac{\eta^{N-1} \ln(\eta)(1-\eta)}{(1-\eta^N)^2}, \\ \frac{\partial f'_1}{\partial N} &= \frac{\eta^N \ln(\eta)(1-\eta)}{(1-\eta^N)^2},\end{aligned}\tag{Eq. 6.11}$$

then $\frac{\partial f'_1}{\partial N} = \eta \frac{\partial f_1}{\partial N} < 0$. According to Eq. 6.9, it can be derived that

$$\begin{aligned}\frac{\partial \xi(N, \eta)}{\partial N} &= \frac{\frac{\partial f_1}{\partial N}(1-f'_1) - \frac{\partial f'_1}{\partial N} f_1(1-f_1)}{(1-f_1 f'_1)^2} \\ &= \frac{1}{(1-f_1 f'_1)^2} \frac{\partial f_1}{\partial N} (1-f'_1 - \eta f_1 + \eta f_1^2) \\ &= \frac{(1-f_1)(1-f'_1)}{(1-f_1 f'_1)^2} \frac{\partial f_1}{\partial N} < 0.\end{aligned}\tag{Eq. 6.12}$$

Thus, for a constant ε_0 , the robustness is a decreasing function of the population size N where $N > \frac{1}{\varepsilon_0} + 1 > 1$.

The convergence of the robustness value is next shown as the population size approaches infinity. Since $\frac{\partial \xi(N, \eta)}{\partial N} < 0$ and $\lim_{N \rightarrow +\infty} \frac{\partial \xi(N, \eta)}{\partial N} = 0$, then $\xi(N, \eta)$ is a decreasing function of N and $\lim_{N \rightarrow +\infty} \xi(N, \eta) = 1 - \frac{1}{\eta}$. Given that $\varepsilon = \varepsilon_0$, it can be derived that $\eta = \frac{1}{1-\varepsilon_0}$. In other words, the robustness of an incentive mechanism will converge to a constant as population size approaches infinity when η satisfies the condition $\eta = \frac{1}{1-\varepsilon_0}$. This means that there exists a non-equilibrium strategy whose payoff is $\frac{\varepsilon_0}{1-\varepsilon_0}$ times more than that of the desired equilibrium strategy. $\square$

It has been shown that the robustness of an incentive mechanism is an increasing function of the population size and is bounded from below, provided that ε is a constant. As ε is set to a very large value (e.g. 1), then the framework would allow the non-equilibrium to achieve a payoff of $\frac{\varepsilon_0}{1-\varepsilon_0} \rightarrow +\infty$ times than the equilibrium strategy when the population is large. To avoid this singularity in the following simulations, $\varepsilon = \frac{1}{T} = 0.02$ which is a sufficiently small but measurable value⁴, and will gradually increase the population size until the robustness value converges. It will be observed that by setting

⁴There are two considerations for choosing an appropriate value for the parameter ε : (1) ε should be no less than its measurable accuracy $\frac{1}{T}$; (2) it is better to use a smaller value for ε .

ε to a small value the evaluation framework will converge at nearly the same robustness value as the ones analyzed by the extended simplex analysis in the next section.

6.3 Validation of the Framework on Representative Games

In this section, the proposed simulation framework is validated by comparing the simulated results with the analytical ones in simple representative games. The simplex analysis approach is used to analytically evaluate the robustness of equilibrium strategies.

6.3.1 Extended Simplex Analysis Approach

For ease of presentation, it considers a 2-player asymmetric game⁵ to introduce the simplex analysis which is a well-known approach for analyzing evolutionary dynamics, and it is extended to calculate the robustness of some representative games.

The strategy sets of the two players are $S_1 = \{s_1^1, \dots, s_{m_1}^1\}$ and $S_2 = \{s_1^2, \dots, s_{m_2}^2\}$, and their payoff matrices are $\mathbf{A}_1$ and $\mathbf{A}_2$ which are $m_1 \times m_2$ and $m_2 \times m_1$ matrix, respectively:

$$\mathbf{A}_1 = \begin{bmatrix} a_{11}^1 & \cdots & a_{1m_2}^1 \\ \vdots & \ddots & \vdots \\ a_{m_11}^1 & \cdots & a_{m_1m_2}^1 \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} a_{11}^2 & \cdots & a_{1m_1}^2 \\ \vdots & \ddots & \vdots \\ a_{m_21}^2 & \cdots & a_{m_2m_1}^2 \end{bmatrix} \quad (\text{Eq. 6.13})$$

The population profile of population 1 is $x_1 = (x_1(1), \dots, x_1(m_1))$ where $x_1(j)$ is the proportion of agents in this population using strategy $s_j^1 \in S_1$ and $\sum_{j=1}^{m_1} x_1(j) = 1$. Similarly, the population profile of population 2 is $x_2 = (x_2(1), \dots, x_2(m_2))$.

The evolutionary dynamics of the populations is then the dynamics of x_1 and x_2 . Since the population profile x_i for population i is a strategy distribution satisfying $\sum_{j=1}^{m_i} x_i(j) = 1$, x_i is also called as a *simplex* and this analysis approach is thus called the

⁵For a symmetric game played in a single population, it could be considered as two replicated populations.

simplex analysis. The evolutionary dynamics of the simplex is described by the replicator dynamics [77, 88, 100]:

$$\nabla x_1 = x_1 \left[(\mathbf{A}_1 x_2 - \frac{x_1^T \mathbf{A}_1 x_2}{m_1}) \right], \quad (\text{Eq. 6.14})$$

$$\nabla x_2 = x_2 \left[(\mathbf{A}_2 x_1 - \frac{x_2^T \mathbf{A}_2 x_1}{m_2}) \right], \quad (\text{Eq. 6.15})$$

where ∇x_1 and ∇x_2 denote the gradient vector of the population profiles at x_1 and x_2 , respectively. These equations show that the proportion of agents taking a certain pure strategy in a population increases if and only if the agents could achieve higher utility than the average payoff of the population, and vice versa.

According to Eq. 6.14 and Eq. 6.15, the gradient at each point in the simplex space can be calculated. Starting from a particular point, a smooth trajectory can be generated by moving a small distance in the calculated gradient, until reaching a fixed point which is called an *attractor*. Each attractor attracts the points around it.

Let x_i^* be an attractor of the population i where $i = 1$ or 2 . Since the attractor is the point to which the points around it will converge, the following equations should be satisfied:

$$\begin{aligned} \nabla x_i &= \mathbf{0}_{1 \times m_i} & x_i &= x_i^*, \\ \nabla x_i \cdot (x_i^* - x_i) &> 0 & x_i &\in x_i^* \pm \epsilon, \end{aligned} \quad (\text{Eq. 6.16})$$

where $x_i^* \pm \epsilon$ represents the point set around x_i^* in the space.

Next, this approach is extended to analytically evaluate the robustness of an incentive mechanism. One population is denoted by i and the invaders are added to population i taking non-equilibrium strategy y_i , and the other population is denoted by j . It evaluates the maximum proportion of invaders such that the population profile of rational agents x is an attractor x^* . Based on this intuition, the following modifications are made on the basic simplex analysis approach:

1. Update the population profile of population i from x_i to

$$\hat{x}_i = \{(1 - R)x_i, R \cdot y_i\}, \quad (\text{Eq. 6.17})$$

by separately listing the strategy profiles of rational population i and the bounded rational strategy, where R is the proportion of the bounded rational agents. The desired equilibrium⁶ becomes $\hat{x}_i^* = \{(1 - R)x_i^*, R \cdot y_i\}$.

2. Update the payoff matrix of the population j ($j \neq i$) from $\mathbf{A}_j$ to $\hat{\mathbf{A}}_j$ by adding m_i columns to describe the payoff of the population j interacting with the bounded rational agents taking y_i . Then $\hat{\mathbf{A}}_j$ is a $m_j \times 2m_i$ matrix.
3. Calculate $\nabla \hat{x}_i = [\nabla x_i, \mathbf{0}_{1 \times m_i}]$, and $\nabla x_j = x_j [(\hat{\mathbf{A}}_j \hat{x}_i - \frac{x_j^T \hat{\mathbf{A}}_j \hat{x}_i}{m_j})]$.
4. Calculate solution set $\Omega = \{R\}$ of Eq. 6.17 satisfying:

$$\nabla \hat{x}_i = \mathbf{0}_{1 \times 2m_i}, \nabla x_j = \mathbf{0}_{1 \times m_j}, \quad (\text{Eq. 6.18})$$

and one of the following $m_i + m_j$ equations

$$\begin{aligned} \frac{\partial \nabla \hat{x}_i}{\partial \hat{x}_i(k)} &= 0, \text{ where } k = 1, \dots, m_i, \\ \frac{\partial \nabla x_j}{\partial x_j(k)} &= 0, \text{ where } k = 1, \dots, m_j. \end{aligned} \quad (\text{Eq. 6.19})$$

5. Calculate the robustness value:

$$R(\mathcal{M}, y_i) = \begin{cases} \max\{\Omega\} & \Omega \neq \emptyset, \\ 1 & \Omega = \emptyset. \end{cases} \quad (\text{Eq. 6.20})$$

The robustness computed using the simplex analysis is exactly the one defined in Definition 10 when the population size is infinite. Thus, the simulated robustness should converge to this analytical value as the population size N becomes sufficiently large.

It worth noticing that the extended simplex analysis approach can only be used to evaluate the robustness of stylized games which have explicit payoff matrixes. For

⁶It assumes that the strategy of bounded rational agents cannot evolve as they are not rational.

complex real mechanisms, the payoff matrix is not explicitly available, resulting in that this analytical approach may not be able to be generally applied. Thus, we validate the proposed simulation frame on several representative games which could be analyzed using the extended simplex analysis approach.

6.3.2 Evaluation on Representative Games

In this section, the proposed simulation framework is evaluated by comparing the simulated results with the analytical ones calculated by the simplex analysis approach in representative games.

First of all, the parameters used in the simulation framework are discussed. Then experiments of symmetric coordinated game is conducted where both α and β in its payoff matrix are set to be 1 with population size $N = 400, 600$, and 800 , respectively. The evaluated robustness values are listed in Table 6.1. For the same population size (e.g., $N = 400$), the robustness of the mechanism increases as ε increases, and for the same ε the robustness value increases as the population size increases as shown in Proposition 26. In the following evaluation framework, $\varepsilon = \frac{1}{T} = 0.02$. Moreover, to justify that a population profile has converged, the convergence criterion is set to be $\max_{j=1}^L (\max |\bar{x}_{(t)} - \bar{x}_{(t-j)}|) \leq \vartheta$ where $t > L$ and $\bar{x}_{(t)}$ is the population profile of the population at step t , which includes two parameters L and ϑ . This criterion specifies that the maximum difference of the converged profile of the population at step t with that over the previous L time steps is no more than a threshold ϑ , and the two parameters should be set according to the requirements of different applications. $L = 100$ and $\vartheta = 1e^{-2}$ in the following experiments.

The first game is the *Prisoner's Dilemma*, where each player has two pure strategies: *Cooperation* and *Defection*. Its payoff matrix is shown in the matrix on the left-hand side of Figure 6.3, where $d > b > c > a$, $2b > a + d$. The population profile is $x = (x(1), x(2))$

Table 6.1: Robustness of the coordinated game ($\alpha = 1, \beta = 1$) with different N and ϵ

N	$\epsilon = 0.1$	$\epsilon = 0.3$	$\epsilon = 0.9$
400	$325/(325+N) = 0.448$	$330/(330+N) = 0.452$	$375/(375+N) = 0.484$
600	$525/(525+N) = 0.467$	$532/(532+N) = 0.470$	$576/(576+N) = 0.490$
800	$725/(725+N) = 0.475$	$735/(735+N) = 0.479$	$779/(779+N) = 0.493$

where $x(1)$ denotes the proportion of population taking strategy *cooperation*. In this game, there is one unique dominant Nash equilibrium where all players take strategy defection, denoted by $x^* = (0, 1)$. The non-equilibrium strategy is cooperation. Based on Observation 1 and Eq. 6.18 and Eq. 6.19, the robustness of the game against this non-equilibrium strategy is 1. Figure 6.3 shows that the simulated robustness converges to 1. ρ is also set as different values (0.2, 0.5, 0.8 and 1), and the simulation results are exactly the same (the curves for different ρ overlap with each other), proving that the simulation results are not sensitive to the parameter ρ .

Figure 6.3: Validation of the simulation framework on the prisoner's dilemma game

The second game evaluated is a *Symmetric Coordination game*. The pure strategy set for each player is $\{Left, Right\}$ and the payoff matrix is shown at the left side of Figure 6.4. Both players gain the same payoff if they choose the same strategy, i.e., α if both choose *Left* and β if both choose *Right*. Thus, two pure strategy Nash equilibria exist. Assume that $\{Left, Left\}$ is the desired equilibrium, rational agents would always choose

Left given that other agents are also rational and choose *Left*, i.e., $x^* = \{1, 0\}$. The bounded rational agents, instead, would choose any other strategy but *Left*. If bounded rational agents always choose *Right*, i.e., $y = \{0, 1\}$, the robustness of the game against this non-equilibrium strategy is $\frac{\alpha}{\alpha+\beta}$ according to Eq. 6.18 – Eq. 6.20. This game is implemented in the proposed simulation framework by varying α and β values. Figure 6.4 shows that the simulated robustness could always converge to the analytical results when the population size is large enough.

		Player II	
		Left	Right
Player I	Left	α α	0 0
	Right	0 0	β β

Figure 6.4: Validation of the simulation framework on the symmetric coordinated game

The third game for validation is *Battle of Sexes game*. There are two players: *Man* and *Woman*, and each man or woman has two strategies, *Boxing* and *Ballet*. The payoff matrix of the players is shown in the matrix on the left side of Figure 6.5. A man prefers to watch *Boxing* and a woman prefers *Ballet*. In this game, a pair of individuals is randomly chosen to show up sites for either boxing or ballet. If the two individuals have the same sex or different sites, they will gain nothing. Otherwise, a pair of man and woman showing up at the same site, they will gain α or β ($\alpha > \beta$), i.e., when both choose *Boxing*, the man agent gains α and the woman agent gains β ; when both choose *Ballet*, the man agent gains β and the woman agent gains α .

The population profile is denoted by $x = (x(1), x(2), x(3), x(4))$ where $x(1)$ is the proportion of men who choose *Boxing*, $x(2)$ is the proportion of men who choose *Ballet*,

Figure 6.5: Validation of the simulation framework on the battle of sexes game

and so on. The sum of $x(1)$ and $x(2)$ is the proportion of men, denoted by ω , and the sum of x_3 and x_4 is women, which is $1 - \omega$. It assumes that the desired equilibrium (attractor) is $x^* = (\omega, 0, 1 - \omega, 0)$ when both man agents and woman agents choose *Boxing*. The non-equilibrium strategy of players is *Ballet*. Based on the simplex analysis, the robustness of the game should be $((1 - \omega)\alpha)/[(1 - \omega)\alpha + \beta]$. As one can observe in Figure 6.5, simulation robustness converges to the theoretical one as population size is large enough.

6.4 Implementation of the Framework in Evaluating Incentive Mechanisms

The proposed simulation framework is applied to evaluate and compare the robustness of five incentive mechanisms in reputation systems for e-marketplaces to promote truthful ratings from buyers. Specifically, Jurca [37] proposes a *side-payment mechanism* where truthfully providing ratings is the buyers' optimal strategy. This mechanism can maximize buyer utility by providing truthful ratings if every other buyer reveals his truthful ratings. Jurca further proposes another version of this mechanism to minimize the budget required to reward buyers, called *min-budget side-payment mechanism* [37]. The *credibility mechanism* [81] requires both buyers and sellers to submit ratings about the outcome of their transactions. If there is a disagreement, both of them are punished and pre-

vented from conducting transactions for several periods, because such a case signals that one of them is lying. The *trust-based mechanism* [123] is built on a buyer social network where reputable buyers (advisors) are more likely to be chosen as advisors by many other buyers. Sellers offer more attractive products or lower prices to satisfy those reputable buyers who will be helpful in propagating the sellers' good reputation. Buyers are thus incentivized to provide truthful ratings so as to become reputable. The details about these four incentive mechanisms can be found in Chapter 2. The proposed iMEMLI in Chapter 4 is also implemented. All the five incentive mechanisms are designed to address the dishonesty problem in reputation systems. However, CONGRESS in Chapter 3 aims to address the subjectivity problem and iMET is specifically designed for crowdsourcing. Thus they are not comparable with those five incentive mechanisms and we do not implement CONGRESS and iMET in this section.

In the simulation framework, a set of buyers and sellers with the proportion 10:1, interact with each other periodically. In each period, each buyer chooses one seller to conduct a transaction by considering the sellers' reputation, i.e., the probability of a seller being chosen is proportional to his reputation value. After the transaction, the buyer (and/or seller) has a chance to rate the transaction. The provided ratings are collected and processed to update seller trustworthiness (and/or buyer social network). At the end of each period, the population then evolves according to Algorithm 5, i.e., a buyer is randomly selected to choose his strategy based to the expected payoff of each existing strategy and also randomly choose another buyer to mutate towards one of the extinct strategies according to Eq. 6.4. The strategy set of a rational buyer contains two strategies: the desired equilibrium strategy and the non-equilibrium strategy⁷. The parameters of the five incentive mechanisms are set according to their suggested values as in the corresponding papers [37, 81, 123]. Especially, in the trust-based incentive

⁷The strategy profile of rational buyers could be various, and we assume that the rational buyers would not investigate the strategies not existing in the system.

mechanism, the maximum number of advisors for a buyer is set to $1/4$ of the whole buyer population.

Figure 6.6: Implementation of the simulation framework for evaluating incentive mechanisms in reputation systems for e-marketplaces

Four non-equilibrium strategies (i.e. attacks) adopted by irrational agents [37, 50] are considered: (1) *random attack*, y_1 , where a buyer randomly reports ratings for the transactions with sellers; (2) *constant attack*, y_2 , where a buyer always reports the opposite ratings for transactions; (3) *whitewashing attack*, y_3 , where a buyer first provides untruthful ratings, then leaves the system but re-enters using a new identity; (4) *collusive*

attack, y_4 , where a set of buyers form a group to conduct transactions with trustworthy sellers (top 50%) and report negative ratings for them. To measure the robustness of a mechanism against a specific attack, the simulation framework each time takes into account two strategies, i.e., the desired equilibrium strategy (truthfully reporting strategy) $x = (1, 0)$ and the simulated attacking strategy $y_k = (0, 1)$ ($k \in \{1, 2, 3, 4\}$). In the simulated scenarios, the population size, N , is equal to the number of rational buyers who take the desired strategy x , starting from 1. The irrational buyers taking y_k , in a number of M starting from 1, then gradually enter into the system to be combined with the N rational buyers. The combined population then evolve according to the simulation framework (Algorithm 5).

The robustness results are shown in Figure 6.6. Figure 6.6(a) shows that all the five mechanisms are totally robust against agents that report randomly. As the proportion of these attackers increases, the utility of buyers in taking any single non-equilibrium strategy however cannot increase due to their random ratings.

Figure 6.6(b) shows the measured robustness against the constant attack. iMEMLI can achieve the highest robustness of 1, since the agents taking the constant attacking strategy gain low scores so that they have a little chance to conduct transactions with honest sellers. The trust-based mechanism is the second and the min-budget side-payment mechanism performs the worst. In the trust-based mechanism, buyers choose other honest buyers (advisors) to join their social network. The constant attackers have little chance to enter rational buyers' network (i.e., gain high reputation), thus cannot significantly affect the behavior of rational buyers. Due to the fact that the attacking agents can conduct transactions with sellers, the robustness of trust-based mechanism is less than iMEMLI. In the credibility mechanism, as the proportion of the constant attackers increases, rational sellers would firstly deviate from equilibrium strategy, which further causes rational buyers to deviate. Thus, its robustness is only slightly higher

than 0.5. In the side-payment mechanism, ratings provided by buyers are compared with the majority of buyers' ratings. When the majority becomes dishonest, the rational buyers will also deviate. Its robustness is thus around 0.5. The min-budget side-payment mechanism is less robust than the side-payment mechanism, because the former sacrifices the robustness property in order to achieve the minimal side-payment imposed on the e-marketplace owner.

Figure 6.6(c) shows the robustness against the whitewashing attack. iMEMLI can achieve the robustness as high as 1. It is because the initial score and reputation of new comers effectively harm the benefit of the agents taking the whitewashing attack as shown in Section 4.2.4. The two side-payment mechanisms achieve higher robustness than the other two mechanisms, and the credibility mechanism bears the lowest robustness. The side-payment mechanisms have little reliance on the experience or history of buyers, resulting in the similar robustness as that against the constant attack in Figure 6.6(b). Further, in the credibility mechanism, if a punished agent that cannot conduct transactions re-enters the system with a new identity, the punishment becomes invalid and fails to effectively incentivize agents to be honest, resulting in the lowest robustness.

Figure 6.6(d) shows robustness against the collusive attack. iMEMLI matches buyers and sellers according to Algorithm 3, so that the collusive agents cannot conduct transactions with trustworthy sellers as often as they attempt in other mechanisms, resulting in that iMEMLI achieves the higher robustness than any of the other four mechanisms. The credibility mechanism achieves the second highest robustness but the trust-based mechanism performs the worst. The credibility mechanism punishes agents by not allowing them to conduct transactions for several periods, which effectively decreases the impact of the collusive attackers. In the trust-based mechanism, collusive attackers can provide untruthful ratings, add each other in the social network, and become reputable. These "reputable" attackers will then gain a high utility, causing rational buyers easily to deviate from equilibrium strategy. From Figure 6.6, it can be observed that the robustness of

the incentive mechanisms decreases as attacks become delicate (e.g., whitewashing and collusive attacks are more sophisticated than random and constant attacks). This trend was also predicted by other researchers [37].

6.5 Summary

In this chapter, a simulation framework has been proposed to evaluate the robustness of incentive mechanisms against bounded rational strategies. Specifically, we formally define the robustness of incentive mechanisms are proposed in the presence of bounded rational players. Secondly, a general simulation framework is proposed based on the evolutionary dynamics, and the influence of its parameter settings on the robustness measure is also analyzed. Thirdly, the proposed simulation framework is validated in several representative games by comparing the evaluated outcomes with theoretical predictions based an extended simplex approach. Finally, the simulation framework is applied to evaluate and compare the robustness of several incentive mechanisms proposed for reputation systems in e-marketplaces against different undesired strategies. The results show that the robustness of an incentive mechanism decreases as the non-equilibrium strategy becomes complicated, and indicate the need for more robust incentive mechanisms for reputation systems.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

This chapter concludes the completed research so far *towards the design of robust incentive mechanisms to address subjectivity and dishonesty problems in agent reporting*. To address the subjectivity problem in reputation systems, a coalition formation game based reputation system, named as CONGRESS, has been proposed in Chapter 3. To address the dishonesty problem in e-marketplaces with limited inventory, an incentive mechanism named as iMEMLI has been proposed to promote buyer and seller honesty in Chapter 4. For crowdsourcing systems, Chapter 5 proposes an incentive mechanism iMET to promote truthful opinions by addressing the truthfulness and subjectivity problems simultaneously. The above three incentive mechanisms have been summarized in Table 7.1 including their limitations which lead to the future work in Section 7.2. Finally, Chapter 6 defines the robustness of incentive mechanisms against non-equilibrium strategies and proposes a simulation based evaluation framework to empirically quantify the robustness.

More specifically, in Chapter 3, in order to address the subjectivity problem, a CFG-based reputation system CONGRESS has been proposed. The proposed approach does not require buyers to elaborately learn advisors' subjectivity, and also avoids the redundant reputation computation. It provides incentives for buyers with the same subjectivity

Table 7.1: Summary of the proposed incentive mechanisms

Name	Addressed problems	Applied systems	Limitations
CONGRESS	Subjectivity problem	E-marketplaces	Cannot ensure a strict core partition; Consider pure subjectivity types.
iMEMLI	Dishonesty problem	E-marketplaces	Not consider the subjectivity problem; Several unrealistic assumptions.
iMET	Subjectivity problem Dishonesty problem	Crowdsourcing	Ineffective to workers' complex strategies; Not applicable to uncertain answers.

to automatically cooperatively construct seller reputation so as to cope with the subjectivity problem. More specifically, in the proposed approach, each buyer is able to join a club and share ratings within the club. The club center aggregates buyers' ratings to build the reputation of sellers which guides the buyers in the club to conduct transactions and creates utility surplus. The utility surplus of the club is further divided among the buyers of the club. The desired club structure is the optimal club structure where buyers with different subjectivity types form separated clubs. Two allocation algorithms have been proposed, which are proportional allocation and Shaley allocation. Theoretically and empirically results have shown that buyers have the incentives to form the optimal club structure when certain conditions are satisfied. However, the proposed allocation algorithms cannot guarantee the formation of a strict core partition, and each buyer is restricted to join only one club so as to form non-overlapped club structure.

In Chapter 4, an incentive mechanism iMEMLI has been proposed to promote buyer and seller honesty in EMLI. In the proposed mechanism, buyer honesty in providing fair ratings is measured by a normalized proper scoring rule, where a buyer can only gain the maximum score by providing truthful ratings. Seller honesty is measured by ratings provided by buyers (weighted by the buyers' scores) so that honest sellers are able to gain high reputation values. The pricing algorithm ensures that the products of sellers with higher reputation are offered at higher prices. The allocation algorithm allocates honest buyers with the limited products at a higher probability. Therefore, iMEMLI promotes buyer and seller honesty. Theoretical analysis and experiential results have

proven that the proposed mechanism induces honesty from both buyers and sellers. How the proposed mechanism addresses the re-entry problem has also been discussed. The limitations of iMEMLI are that it does not consider subjectivity problem and several unrealistic assumptions constraint its applicability.

In Chapter 5, an incentive mechanism iMET is proposed to elicit truthful opinions in consensus tasks, where both the subjectivity and dishonesty problems are addressed together. It relaxes the assumption that the workers participating in a consensus task share the common truthful opinions (no subjectivity problem), and can promote truthfulness even when the majority of workers provide untruthful opinions. Experimental results demonstrate that iMET is incentive compatible in rewarding truthful opinions, and it significantly outperforms the existing mechanisms when diverse truthful opinions exist and the majority of workers behave untruthfully. The limitations of iMET are that it may be ineffective when dishonest workers take complex untruthfully reporting strategy such as collusive strategy, and workers are assumed to be certain about their truthful answers.

In Chapter 6, a quantitative robustness evaluation approach is proposed to study the robustness of incentive mechanisms against bounded rational agents performing non-equilibrium strategies. The formal definitions of a robustness measure are provided for incentive mechanisms in the presence of bounded rational players. A general simulation framework based on the evolutionary dynamics is proposed, and the influence of its parameter settings on the robustness measure is also discussed. The proposed simulation framework is then validated by comparing its simulated results with those theoretically predicted in several representative games. The proposed simulation framework is implemented for reputation systems in e-marketplaces to evaluate and compare the robustness of several incentive mechanisms against different non-equilibrium strategies. The results show that the robustness of an incentive mechanism decreases as the non-equilibrium

strategy becomes sophisticated, and indicate the improvement directions of the existing incentive mechanisms. For example, the credibility incentive mechanism [81] requires to be improved by enhancing the robustness against the whitewashing attack. The proposed framework has potential to be extended in many aspects as specified in Section 7.2.4

In all, the goal of the research in this thesis is to design a robust incentive mechanism that can simultaneously address the subjectivity and dishonesty problems. We offer a set of incentive mechanisms that can cope with the problems, especially the iMET mechanism is capable of addresses the two problems together. Our robustness evaluation framework proposed in Chapter 6 can serve as a basis for reaching the goal by providing a tool to quantify the robustness measure. Through evaluating the robustness of several existing incentive mechanisms, several insights have been drawn that might be useful for the design of a robust incentive mechanism in the future.

7.2 Future Work

Based on the current work, the future research is planned in four aspects.

7.2.1 Extension of CONGRESS

In CONGRESS as proposed in Chapter 3, the proportional and Shapley allocations have been studies. In future work, other allocation methods will be explored and the properties associated with them will also be studied, so as to find a set of allocation methods such that the optimal club structure is always a strict core partition. In addition, the club structure in the current work is non-overlapped by constraining that there does not exist a dominated buyer or seller. It would be interesting to extend the model based on the overlapped coalition formation game [127], allowing buyers to join multiple clubs which contain buyers with dominant subjectivity types.

7.2.2 Extension of iMEMLI

For iMEMLI, some of the assumptions listed in Section 4.1 will be relaxed. For example, by relaxing assumption (b), the subjectivity of buyers in providing ratings will be considered. In other words, it would be ideal if both the subjectivity and dishonesty problems can be addressed at the same time in reputation systems for e-marketplaces. However, it is difficult to simultaneously address both the two problems in the design of incentive mechanisms, because different ratings provided by buyers for a target seller can be caused by buyer dishonesty or buyer subjectivity. It is hard to distinguish what causes the difference in agent reporting. The iMET mechanism proposed in Chapter 5 can achieve this goal because it is assumed that the subjectivity difference of agents can be properly measured. However, it is challenging to do so in reputation systems for e-marketplaces, unless we can identify a set of sellers with known honesty. In future work, one possible research direction is to integrate CONGRESS proposed in Chapter 3 that specifically addresses the subjectivity problem and iMEMLI in Chapter 4 that specifically addresses the dishonesty problem, so that both of the problems can be effectively addressed at the same time for e-marketplaces.

The returning rate of buyers and sellers is also an potential factor to be considered in designing incentive mechanisms for EMLI, where the agents would discriminately behave dishonestly against agents with the low returning ratio. In addition, designing the incentive for new sellers to join the e-marketplaces with limited inventory is also an interesting direction for the study of EMLI.

Moreover, an auction based incentive mechanism is planed for EMLI to further increase the social welfare and relax the assumptions, based on the proposed reputation-aware continuous double auction in [64]. Given that the social welfare of the proposed mechanism is not maximized (even though no less than the free-trading marketplace), it is possible to improve the social welfare by allowing buyers to bid their valuations towards

products and sellers to reveal their costs in producing the products [125]. Meanwhile, we could relax assumption (d) by allowing buyers to report their budget. The auction based mechanism will adapt accordingly to consider the buyer budget information, e.g. by not allocating a product to a buyer whose budget is smaller than the price of the product.

7.2.3 Extension of iMET in Crowdsourcing

The proposed iMET mechanism will be developed by considering strategic untruthful behaviors, such as collusive attacks where some workers collude together to increase the chance of their answers being rewarded. A possible way to address this issue is to filter out some workers' answers through analyzing their behavior patterns, such as the time spent on answering each question and mouse trajectories [126]. Integrating workers' confidence in designing incentive mechanisms is also another future direction [87]. Furthermore, the confidence of workers in answering questions is a potential factor to improve the proposed incentive mechanism.

7.2.4 Extension of the Robustness Evaluation Framework

The proposed simulation framework could be enhanced in several aspects. Firstly, it can be used to search for the worst non-equilibrium strategy. In the proposed simulation framework, it is designed to find the robustness of an incentive mechanism against a specific non-equilibrium strategy. It is a natural extension to search for the worst attacking strategy where the incentive mechanism under evaluation exhibits the lowest robustness value. It then becomes an optimization problem to minimize the robustness in a space composed by all possible non-equilibrium strategies. Secondly, the robustness evaluation framework can be extended so as to allow rational players to form coalitions and conduct cooperative strategy. A coalition could be formed by k rational players who coordinate their actions to increase the combined utility of the coalition, considering the

possibility that the players exchange side-payments within the formed coalition. The rational coalition is different from the collusive attack evaluated in Section 6.4 where the bounded rational players form coalitions to attack the system for gaining more utility. By checking the evolution of rational non-collusive players, the proposed framework can be extended to calculate the largest ratio of collusive agents such that non-collusive players still sustain equilibrium strategies. Finally, the robustness of many other incentive mechanisms in the literature will be studied through the proposed simulation framework. After conducting lots of experiments, the key factors that impact the robustness of incentive mechanisms can be analyzed, with the aim of designing a more robust incentive mechanism by carefully considering those factors.

Author's Publications

Journals Articles

- 1 **Yuan Liu**, Jie Zhang, Quanyan Zhu. CONGRESS: *Coalition Formation Game-based Reputation System*. IEEE Transactions on Computational Social Systems, 2014. (in submission)
- 2 **Yuan Liu**, Siyuan Liu, Jie Zhang, Han Yu, Cyril Leung, Chunyan Miao. *iMET: An Incentive Mechanism to Elicit Truthful Opinions in Crowdsourcing*. ACM Transactions on Intelligent Systems and Technology, 2014. (in submission)
- 3 **Yuan Liu**, Jie Zhang, Bo An, Sandip Sen. *Measuring Robustness of Incentive Mechanisms via Simulation and Its Implementation in Reputation Systems for E-marketplaces*. Journal of Autonomous Agents and Multi-Agent Systems (JAAMAS), 2014. (Prepare to submit)
- 4 **Yuan Liu**, Jie Zhang. *An Incentive Mechanism for E-marketplaces with Limited Inventory*. Journal of Electronic Commerce Research and Applications (ECRA), 13(2):110-127, 2013.
- 5 Zeinab Noorian, Jie Zhang, **Yuan Liu**, Stephen Marsh, and Michael Fleming. *Trust-Oriented Buyer Strategies for Seller Reporting and Selection in Competitive Electronic Marketplaces*. Journal of Autonomous Agents and Multi-Agent Systems (JAAMAS), November 2013

Conference Papers

- 1 **Yuan Liu**, Jie Zhang, Han Yu, and Chunyan Miao. *Reputation-aware Continuous Double Auction*. In Proceedings of 28th AAAI Conference on Artificial Intelligence, Student Poster Program, 2014.
- 2 **Yuan Liu**, Siyuan Liu, Hui Fang, Jie Zhang, Han Yu, and Chunyan Miao. *RepRev: Mitigating the Negative Effects of Misreported Ratings*. In Proceedings of 28th Proceedings of the 28th AAAI Conference on Artificial Intelligence, Student Poster Program, 2014.
- 3 **Yuan Liu**, Jie Zhang, An Bo, and Sandip Sen. *A Practical Robustness Measure of Incentive Mechanisms*. In Proceedings of the 13th International Conference on Autonomous Agents and Multiagent Systems (AAMAS), 2014.
- 4 **Yuan Liu**, and Jie Zhang. *Robustness Evaluation of Incentive Mechanisms*. In Proceedings of the 12th International Conference on Autonomous Agents and Multiagent Systems (AAMAS), pp.1293-1294, 2013.
- 5 **Yuan Liu**. *Towards the Design of a Robust Incentive Mechanism for E-Marketplaces with Limited Inventory*. In Proceedings of the 12th International Conference on Autonomous Agents and Multiagent Systems (AAMAS), Doctorial Consortium, pp. 1441-1442, 2013.
- 6 **Yuan Liu**, Jie Zhang, and Qin Li. *An Incentive Mechanism to Promote Honesty in E-marketplaces with Limited Inventory*. In Proceedings of the 6th IFIP WG 11.11 International Conference on Trust Management (IFIPTM), pp. 209-216, 2012.
- 7 **Yuan Liu**, Jie Zhang, and Qin Li. *Design of an Incentive Mechanism to Promote Honesty in E-marketplaces with Limited Inventory*. In Proceedings of the 14th Annual International Conference on Electronic Commerce (ICEC), pp. 54-61, 2012.

- 8 **Yuan Liu**, Qin Li, and Jie Zhang. *Coalition Formation Game Based Reputation System*. In the ACM EC Workshop on Incentives and Trust in E-Commerce (WIT-EC), Valencia, Spain, 2012.
- 9 **Yuan Liu**, Jie Zhang, and Quanyan Zhu. *Design of a Reputation System based on Dynamic Coalition Formation*. In Proceedings of the 3rd International Conference on Social Informatics (SocInfo), pp. 135-144, 2011.

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