An adaptive distributed dipole model for enhanced dynamic inversion in magnetic field and induction sensor

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**AN ADAPTIVE DISTRIBUTED DIPOLE MODEL FOR
ENHANCED DYNAMIC INVERSION IN MAGNETIC
FIELD AND INDUCTION SENSOR**

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**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

2014

**AN ADAPTIVE DISTRIBUTED DIPOLE MODEL FOR
ENHANCED DYNAMIC INVERSION IN MAGNETIC
FIELD AND INDUCTION SENSOR**

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ABSTRACT

A magnetic-field-based sensing/imaging system, being noninvasive, has been widely implemented for anomaly detection in various applications. The sensing/imaging performance is highly dependent on the modeling approach, which decides accuracy, efficiency and implementation. Distributed Multi-pole Model, as a semi-analytical modeling method, has been developed to simulate magnetic field from permanent magnet and direct current-carrying coils, but its application is limited to static field problems due to usage of scalar-potential-based governing equations. For sensing/imaging purpose, modeling of time-varying electromagnetic field plays a more critical role because it provides abundant information in both spatial and time domains.

The objective of this research is to develop a modeling approach for time-varying electromagnetic field and explore its capability for sensing/imaging purpose. The magnetic point dipole directly conceptualizes the physical field information (automatically satisfying Maxwell's equations) and offers a compact solution to account for the field variation. Then originated from vector potential formulas, the extended distributed multi-pole model is developed to account for AC source and further exploited to describe the inductive relation between two coils. The commonly seen integral form of the induction equation can be simply discretized based on extended distributed multi-pole models. Therefore real-time and robust mutual induction computation can be realized and orientation sensing schemes being developed. As a semi-analytical modeling method utilizing discretized source terms, computation for induction can be more efficient and simply applicable for various sensor configurations.

Another promising application is introduced as magnetic induction tomography (MIT), which utilizes compact measuring devices with moderate electromagnetic field, and thus could be a safer and portable imaging system for medical applications. With a number of point dipoles spanning the target domain and effectively characterizing the field from eddy current, a novel modeling approach is developed. Since there is a linear relation between the dipole moment and peripheral field variables, the forward mapping can be directly built, and the matrix for inversion is much more simplified. Compared to directly solving for conductivity, this inversion procedure for the dipole moment is proven to be fast-computing, which also provides guidance for an adaptive refining process.

While induction-based sensing and tomography has been developed with an easily implementable approach that discretizes the field problems from distributed dipole models, we expect the modeling and discretization method will have broader applications in matrix regularization and signal processing.

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NOMENCLATURE

Upper Characters

E	Electric field	I, J	Current, current density
J_f	Free current density	J₀	Excitation current density
D	Displacement current density	B	Magnetic flux density
H	Magnetic field intensity	A	Magnetic vector potential
V	Electric scalar potential	A	Cross-section area of a coil
P	Polarization	M_{ab}	Mutual inductance between coil a and coil b
R_j	Position vector of the <i>j</i> th dipole	T	Coordinate transformation matrix
R	Radius of the inner arc when a tube is bent		

Lower Characters

r	Position vector of the magnetic field	r'	Position vector of the current density
r	Radius of the coil	l	Length of the coil
r_m	Radius of each layer in the eDMP	l_m	Distance between the positive and negative layers
n	Number of dipoles in each layer	q	Number of layers in the eDMP model
p	Number of dipoles for coil a	k	Number of dipoles for coil b
m_j	Magnetic moment of the <i>j</i> th dipole		

Greek Characters

φ	Magnetic scalar potential	ρ	Charge density
σ	Electric conductivity	δ	Electric permittivity
μ	Magnetic permeability	χ_v	Volume magnetic susceptibility
γ	Aspect ratio of the coil geometry	Φ_{ab}	Magnetic flux in coil b excited by coil a
κ	Curvature of a bending tube	β	Inclination angle of a coil of oblique cylindrical shape
ω	Angular frequency		

I. INTRODUCTION

1.1 MOTIVATION

Electromagnetic field has been widely exploited for various sensing applications, including location detecting, motion sensing, material feature imaging, etc. Compared to sensors that rely on direct mechanical linkages, an electromagnetic sensor offers advantages such as contactlessness, compactness and easy implementation. It provides more abundant information than an optical sensor because the measured signal consists of magnitude, direction and even phase. Also, both excitation and detection devices can be simply customized and designed for specific applications. In order to fully make use of these advantages, an accurate modeling approach and a fast-computing sensing method are usually in demand.

Capable of integrating the benefits of both analytical and numerical modeling approaches, a semi-analytical model that utilizes the unit source terms has the potential to be fast and accurate at the same time. Originated from Single Dipole Model, Distributed Multi-pole model has been developed to account for permanent magnet or static electromagnet. Its applications on a PM-based motion sensor and spherical motor control have been studied and good performance is validated. Then in order to further explore the capability of this modeling approach, it is quite straightforward to tackle the problem of modeling of time-varying field.

One of the research areas that electromagnetic field has a high standing is biomedical imaging. Aiming to diagnose diseases and aid medical procedures, existing imaging instruments include X-ray computed tomography, medical ultrasonography, magnetic resonance imaging, etc. Although having their own advantages, none of these techniques perfectly satisfy all the requirements on the imaging resolution, devices compactness, and safety concerns, etc. In addition, due to their specific working

principles, each imaging technique provides different types of information about different areas of a human body, which indicates there are great potential and needs to develop new medical sensing/imaging systems that could be faster, safer or able to image the areas that current techniques cannot. Magnetic induction tomography (MIT), which utilizes the induction feature of electromagnetic field, is one rapidly developing technique in the last decades. It was initially proposed in 1990s, utilizing different capability of “induction” belonging to various materials. Images were generated based on passive electrical properties, conductivity, permittivity and permeability. By setting up a time-dependent electromagnetic field, eddy current would be generated inside those areas with nonzero conductivity, which produces a smaller, but distinct electromagnetic field. Magnitude and phase information of these eddy current bears a relation with the material feature. And the target of MIT is to abstract the material distribution information out of this perturbation field.

Due to limited imaging performance, there is no existing system of magnetic induction tomography in practical usage. Therefore, the motivation of this dissertation is to develop a novel modeling method for time-varying electromagnetic field and utilize it for induction-based sensing/imaging techniques.

1.2 BACKGROUND

In order to provide background knowledge about electromagnetic field modeling and its application for sensing as well as imaging, the following discussion is organized in three sections. The first section reviewed the existing studies on electromagnetic field modeling, and the second section discussed various existing biomedical sensing/imaging systems. Then the working principle of magnetic induction tomography was briefly explained, which emphasizes its difficulties and provides the questions for further discussion.

1.2.1 ELECTROMAGNETIC FIELD MODELING

Discussion of electromagnetic field could be systematized as electrostatics, magnetostatics, quasi-static fields and electromagnetic waves. The subject of electrostatics is phenomena involving time-independent distributions of electric charge and fields, whereas magnetostatics discusses the steady-state magnetic phenomena, which could be handled as analogy with electrostatics in most cases. It is noteworthy that at these static and steady situations, electric field and magnetic field are not coupled yet and can be discussed individually. As time-varying cases are considered, these two fields inevitably relate to each other and would be further treated as electromagnetic waves altogether when the propagation effect becomes significant. Discussion of time-dependent electromagnetic field without considering the propagation delay is referred as quasi-static problems, which applies at low frequencies so that the dimensions of the region of interest are small relative to the wavelength of the electromagnetic field. It is assumed that any change in the field is felt instantaneously across the region.

It has been a fascinating topic to express the electromagnetic phenomena in a mathematical form. Basic formulas and general governing equations have been derived [1-12], including Maxwell equations, Biot-Savart law, Gauss's law, and many other works by great scientists. And different boundary conditions would be set for various environment and applications. Therefore a beautiful set of equations can almost always be found to describe the concerned problem. But here comes the big contradiction between mathematical beauty and realistic application: not all of these equations can be solved analytically. In fact, if any at all, extremely complicated solutions would be generated by those neat-looking differential/integral equations under strict assumptions.

As the configuration and feature become more complicated, as well as realistic, analytical solution would become inapproachable.

Then with the astonishing development and universalization of computers, numerical methods [13-15] become more popular for modeling and due to the more and more user-friendly commercial software, people without strong background on mathematics or physics could solve an electromagnetic field modeling problem easily. Commonly used numerical methods include finite difference method (FDM) [16], finite element method (FEM) [17], and boundary element method (BEM) [18]. FDM discretizes the governing equations in their strong form and is the most straightforward way to obtain a discrete system equation. On the contrary, FEM and BEM utilize the weak form of governing equations and obtain the discrete equation based on intelligently generated meshes. While FEM discretizes the whole problem domain, BEM only deals with the boundary of the problem domain.

Analytical approaches have the advantages of efficiency and accuracy, while numerical techniques provide the convenience that always being solvable. In order to integrate these benefits, a Distributed Multiple-Pole (DMP) model [19] has been developed from a single-dipole model and utilized multiple source/sink terms to account for the shape of permanent magnet. It can provide accurate results for modeling the electromagnetic field from PM as well as static EM and has been utilized for control of spherical motors [20, 21].

1.2.2 EXISTING SENSING/IMAGING SYSTEMS

Sensing/imaging systems aim to provide information about the material feature, location and motion of a certain target. Motion sensors, as an essential component in many applications such as automation[22], robotics[23, 24], manufacturing machine and medical devices, etc., have been developed to monitor or control electrical and

mechanical systems. As growing techniques in integrated electronics and computer during past decades, many different kinds of sensors with various working principles including pedometers, accelerometers, optical and magnetic sensor etc., have been commercially available but still their research has been studied to improve performance [25]. Among these sensors, optical/imaging sensors for detecting motion/position have also been widely used since they offer visual identification [26] but their sampling rate is low and thus it is often limited in a fast dynamic system. An ultrasound and radar can be utilized to monitor wide area with high accuracy but is bulky and cannot be used with obstacle along the optical path [27]. Magnetic sensors have been utilized in numerous applications such as motion, orientation sensing and path tracking [28-31]. It is compact in size, easy at operation and capable of providing high sensitivity without mechanical contact. It is therefore suitable for small and safety-required measurement systems, such as biomedical applications.

Both static magnetic field [32, 33] and time-varying electromagnetic field [34, 35] have been used for sensing purpose. By comparison, magnetic sensor utilizing static magnetic field has the advantage of excitation-free, since no power supply or signal generator is needed. But due to the exact same cause, the permanent magnet based sensor is more difficult to control and customized because of its inability to be turned off or altered. On the contrary, both the excitation field and sensor configuration are more flexible when time-varying electromagnetic field is exploited for sensing purpose. Inductive signals are usually used to account for the geometric relation between various objects. Therefore design parameters including the excitation field pattern, frequency, and sensor geometry can be varied for optimum performance. Mutual inductance of various shaped coils with specific geometric relations has been studied [5] and many detailed problems [36-38] have been analyzed, but the computation process

often requires series of integral operations for various shapes and locations of coils. And any changes of the geometry in the existing solutions have to be evaluated again. A more general, accurate and fast-computing procedure is in demand to model the time-varying field first and then further utilized to calculate the inductance.

When the application is for biomedical environment, the sensing/imaging target is a human body. It is used not only for clinical purposes, such as medical procedures seeking to diagnose or examine disease, but also for medical science including the study of normal anatomy. And due to the involvement with human being, higher standards of accuracy, security and imaging performance are in demand.

Existing medical sensing/imaging systems can be classified into two categories based on the source of signals. Neuron activities inside human body produce magnetic fields themselves, which can be recorded and used to map the neuron activities, as in Magnetoencephalography (MEG) [39, 40], Electrocardiography (ECG) [41, 42] and Electroencephalography (EEG) [43]. This natural magnetic field is extremely weak and considerably smaller than the ambient magnetic noise, and the essential difficulty is the weakness of the signal relative to the sensitivity of the detectors. Thus these systems largely depend on superconducting quantum interference devices (SQUID) [44], which are sensitive enough to measure fields as low as 5×10^{-18} T within a few days of averaged measurements.

Another type of biomedical sensing/imaging technique, which includes computed tomography (CT) [45], medical ultrasonography [46], magnetic resonance imaging (MRI) [47, 48] and so on, utilizes external signals as excitation, measures the responding signal and shed light on the inner structure of the human body without causing damage. In this way, the signal to be detected is relatively large and can be controlled by excitation devices. Within this category, many different kinds of source

signals have been used. CT utilizes electromagnetic field with frequency from 3×10^{16} Hz to 3×10^{19} Hz, named as X-ray, and produces a large series of two-dimensional images taken around a single-axis of rotation. This volume of data can be manipulated in order to demonstrate various structures based on their ability to block the X-ray beam. It has been an important tool since invented and widely used over the last two decades. But as the radiation exposure increases, the potential risk of cancer would also increase, especially when the patient is already in weak health condition. Thus how to control the radiation dosage without sacrificing the imaging quality is an important issue.

Medical ultrasonography utilizes sound waves with a frequency between 2 and 18 MHz. Arc-shaped sound waves are produced from a piezoelectric transducer and travels into the body. Whenever there is a density changes in the body, e.g. blood cells in blood plasma, small structures in organs, etc., the sound wave is partially reflected. After the sonography scanner determines how long it takes the echo to be received from when the sound was transmitted and how strong the echo is, it can locate which pixel in the image to light up and to what extent. It is commonly used for visualizing subcutaneous body structures including tendons, muscles, joints, vessels and internal organs. Obstetric sonography is widely recognized by the public as a routine procedure during pregnancy. Although causing fewer safety concerns, resolution of ultrasonography is relatively low compared to CT and other imaging systems.

Another popular medical imaging system utilizing electromagnetic field, MRI, implements three different electromagnetic fields: a very strong static magnetic field (0.5-3.0 T) to polarize the resonant part; a weaker time-varying field for spatial encoding; and a weak radio-frequency (100 kHz to 1 THz) radiation for manipulating the resonant part to produce measurable signals. Because the proton of a hydrogen

atom, present in water molecules, is widely used as the “resonant part”, MRI achieves excellent soft-tissue contrast. Although MRI does not expose the patient to high energy radiation, it utilizes extremely powerful static magnetic field and therefore has strict restraints on patients with magnetic implants, such as pacemakers. Also the scan time is relatively long and patient could be required to lie in the MRI instrument for hours. The radio frequency radiation could heat the body to the point of hyperthermia and break down the protective mechanisms for heat control. The long imaging process could also be a serious problem for people with claustrophobia. Currently MRI is the most advanced biomedical imaging technique and also the most expensive one. An MRI instrument could cost up to millions of dollars.

Each of these systems can provide valuable information and tremendous help for doctors when operated for their applicable cases. This also drives scientists and engineers to enthusiastically develop new systems that could work for currently difficult situations.

1.2.3 MAGNETIC INDUCTION TOMOGRAPHY

As one of the developing techniques that have attracted a lot of attention, magnetic induction tomography reconstructs the material feature distribution by measuring the peripheral electromagnetic field. Similar working principle has been exploited in applications including nondestructive defeat testing (NDT)[49], treasure hunting[50] and metal detector[51], etc. Many studies [52-55] have been done in order to extend this technique for biomedical imaging purpose. Based on existing studies, comparison between existing medical imaging systems and MIT is shown in below.

Table 1.1 Comparison between medical imaging systems

Imaging system	Resolution	Radiation	Contact	Specialty	Costs (USD)
Ultrasonography	Less than 1mm	No	Yes	Internal organ	200
CT	Below 1 μ m	Yes	No	Bony structure	500
MRI	From mm to nm	No	No	Soft tissue	2000
MIT	Bigger than 1cm	No	No	Conductivity	None

MRI, as the most sensitive imaging system, can provide resolution around one millimeter with a common magnetic sensor; and high resolution MRI techniques (magnetic resonance microscopy and magnetic resonance force microscopy) can achieve resolution as low as nanometers with the help of highly sensitive magnetic sensor, such as SQUID (superconducting quantum interference device) and micro-fabricated cantilever. As mature imaging techniques, many approaches and assistant devices have been proposed and used for MRI, CT as well as ultrasonography to improve the image quality, reduce the computing time, etc. And since MIT is still at an early stage, its resolution is very low. However it does have the advantages of radiation free, contactless and generate images based on conductivity, which has not been utilized in other biomedical imaging techniques. There is still plenty of potential.

In order to examine the link between the measurements and material features, magnetic field needs to be analyzed and modeled first, which is usually a complicated process since electromagnetic field problem is not linear, even not analytic depending on the structure geometry.

1.3 RESEARCH SCOPE

Three subtasks are illustrated as below.

- Developing the modeling approach of time-varying electromagnetic field for anomaly detection/imaging purpose.
- Exploring the capability of magnetic-field-based motion/orientation sensor.
- Addressing the imaging problem for magnetic induction tomography system.

1.4 OUTLINE OF THIS DISSERTATION

The thesis is outlined as follows.

Chapter 1 begins with the motivation and background of the studies about electromagnetic field modeling and magnetic induction tomography. Research objectives and scopes are therefore emphasized.

Chapter 2 includes detailed reviews of existing researches on eddy current problem, modeling of magnetic induction tomography and practical system.

Chapter 3 proposes the extended DMP model in order to apply the semi-analytical approach for time-varying field. The model is developed based on vector potential formula and validated with induction computation. Computation results are presented.

Chapter 4 explores the modeling capability of the proposed approach for sensing applications and two applications are detailed: DMP model for PM based orientation sensor, and eDMP model for induction based orientation sensing. The latter study is also an introduction for distributed dipole model for magnetic induction tomography.

Chapter 5 further extends the application and develops the distributed dipole model for magnetic induction tomography. Forward mapping and inverse problem are discussed respectively. Adaptive discretization scheme is proposed.

Chapter 6 includes the sensitivity matrix study and further imaging performance evaluation for practical model. Simulation results from COMSOL software and experimental measurements from practical setups are shown. Reconstructed image are presented and discussed.

Finally, Chapter 7 summarizes and concludes this thesis and proposes the potential future researches about the extended DMP model and magnetic induction tomography.

II. REVIEW OF RELATED STUDIES

Sensing and imaging processes are both inverse problems by nature, which are usually handled based on the corresponding forward problems. For electromagnetic induction sensors, the forward problem is to compute the inductive signal due to various target features, locations and motion. Since the target of MIT is the material feature distribution, its forward mapping process would be to express the peripheral field as a function of material features and find the sensitivity function. As an important step, this forward modeling of the problem domain will largely affect the difficulty of inversion. Because of the nonlinearity and complexity of the problem, the sensitivity matrix for MIT is expected to be ill-conditioned and direct inversion is either impossible or would inevitably generate an unstable solution containing large errors. Therefore, regularization process is also inevitable.

A systematic review of related studies is elaborated in the following chapter. It is organized into three subjects: first, analysis and modeling of electromagnetic field for eddy current problem, which is the basis for further discussion; second is forward modeling and sensitivity matrix analysis for magnetic induction tomography, as a transition from classical eddy current problem to imaging problem; at last, existing magnetic induction tomography systems and experiment set-ups will be inspected.

2.1 EDDY CURRENT PROBLEM

Any technique utilizing electromagnetic field, no matter the application is to generate motion, detect feature variation or explore the closed structure, is confronted with a basic question: how to calculate the field variables. Because this is the beginning step to calculate any other concerned variables, such as forces, torques, and energy, etc. Basic formula descriptions and derivations can be found in many text books [4, 56-58]. A quasi-static field problem governs an important range of

applications including electrical circuit analysis, electromechanical devices and eddy current phenomena, which is the basis of imaging principle in induction tomography. Thus, modeling of electromagnetic field for eddy current problem will be discussed in this section.

An eddy current problem arises if a time-varying magnetic field is excited in a domain Ω including a body Ω_c made of conductive material, as in Fig. 2.1.

Fig. 2.1 Eddy current problem configuration

The nonconductive domain in Ω is referred as Ω_n . The outer boundary of Ω and the interface are expressed as Γ and Γ_{nc} . Ω_c could be single or multiple objects of homogenous conductivity surrounded by air, which is discrete conductivity distribution, or continuous inhomogeneous conductivity distribution. The discrete case is simpler and more commonly used in the existing studies.

Using complex phasor representation in eddy current problem is very common. Since eddy current problem is usually considered in the time harmonic condition, time differentiation can be replaced by a multiplication by $j\omega$ when the field vectors are all substituted by complex phasors. Then the Maxwell's equations can be expressed as below, where ω refers to the angular frequency.

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} \quad (2.1)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + j\omega \mathbf{D} \quad (2.2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.3)$$

$$\nabla \cdot \mathbf{D} = \rho_f \quad (2.4)$$

where $j\omega$ indicates time derivative; $\mathbf{J}_f$ refers to free current density, which includes contributions from excitation current density $\mathbf{J}_0$ and electric field inside the conductors; $j\omega \mathbf{D}$ represents displacement current density; ρ_f indicates free charge density.

Based on the quasi-static assumption, the displacement current term $j\omega \mathbf{D}$ can be neglected as in Eq. 2.2. This leads to static but discretely time-varying magnetic field in the non-conducting regions Ω_n and additional electric source, eddy current, present in Ω_c .

Constitutive equations are expressed as

$$\mathbf{B} = \mu \mathbf{H} \quad (2.5)$$

$$\mathbf{D} = \varepsilon \mathbf{E} \quad (2.6)$$

where μ refers to magnetic permeability and ε refers to electric permittivity.

Permeability can be expressed as $\mu = \mu_0 \mu_r = \mu_0 (1 + \chi_v)$, where μ_0 refers to magnetic constant $4\pi \times 10^{-7} \text{ Vs/(Am)}$, μ_r and χ_v refer to the relative permeability and volume magnetic susceptibility of the material. In this way, relation between magnetic flux density and magnetic field intensity can also be expressed as $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$, where magnetization $\mathbf{M} = \chi_v \mathbf{H}$. Because the main concern in an eddy-current problem is conductivity, the effect from magnetic features of the material is mostly neglected. It is common to just describe the relation between the magnetic flux density and magnetic field intensity as $\mathbf{B} = \mu_0 \mathbf{H}$. Permittivity has an analogous feature as

$\epsilon = \epsilon_0 \epsilon_r = (1 + \chi) \epsilon_0$, with ϵ_r and χ as the relative permittivity and electric susceptibility, and $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$, where $\mathbf{P}$ refers to the polarization. Notice that different terminology and symbols are used in various publications.

It is noticeable that although expressed as a combination of several terms in the form of $\mathbf{J}_f = \mathbf{J}_0 + \sigma \mathbf{E}$, $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$, they do not just refer to contributions from different sources at the same time. Each of these terms has its physical meaning and applicable environment, and for most of the realistic applications they will not be present simultaneously. Expressing as combination form is just for the purpose of formulating a more general equation.

Although Maxwell's equations (including differential form of Biot-Savart law, Gauss's law) contain in principle a description of electromagnetic phenomena, it cannot be directly utilized as governing equations because of the multiple coupled variables. Various forms of second-order PDEs can be developed depending on the chosen primary variable and the specific problem condition. Commonly used ones are listed in Table 2.1, where ϕ refers to the magnetic scalar potential, $\mathbf{T}$ refers to an impressed quantity with $\nabla \times \mathbf{T} = \mathbf{J}$; $\mathbf{A}$ refers to the magnetic vector potential, defined by $\nabla \times \mathbf{A} = \mathbf{B}$; V refers to the electric scalar potential, defined by $\mathbf{E} = -j\omega \mathbf{A} - j\omega \nabla V$. The usages of $\mathbf{A}$ - V formula, $\mathbf{T}$ - ϕ formula and $\mathbf{E}$ formula in existing studies are summarized in Table 2.2. It is assumed that excitation source is in the non-conducting region. To cope with the redundant degrees of freedom of potential terms, Gauge fixing can be used, with the mostly used one to be the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$.

Table 2.1 Commonly used formulas in eddy current problem

A-V formulation	$\nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} + j\omega\sigma\mathbf{A} + \sigma\nabla V - \mathbf{J}_0 = 0$ $\nabla \cdot \sigma(j\omega\mathbf{A} + \nabla V) = 0$
T-ϕ formulation	$\nabla \times \frac{1}{\sigma} \nabla \times \mathbf{T} - j\omega\mu\mathbf{H}_0 - j\omega\mu\mathbf{T} - j\omega\mu\nabla\phi = 0$ $\nabla \cdot \mu(\mathbf{H}_0 + \mathbf{T} + \nabla\phi) = 0$
E formulation	$\nabla \times \frac{1}{\mu} \nabla \times \mathbf{E} + j\omega\sigma\mathbf{E} + j\omega\mathbf{J}_0 = 0$

Table 2.2 Summary of existing studies

Formula	Analytical solution	Numerical solution
A-V	Gencer [59]: electrical impedance tomography, Kolyshkin [60]: eddy current analysis	Demerdash [67]: eddy current analysis, Nehl [66]: nondestructive detection, Ren [70]: eddy current analysis, Hariharan [71]: eddy current analysis, Tsuboi [74]: eddy current analysis
T-Φ	Harrington [62]: nondestructive detection	Biro [63]: eddy current analysis, Bouillault [68]: eddy current analysis
E	Pham [61]: magnetic induction tomography	Rucker [73]: eddy current analysis, Hiptmair [77]: eddy current analysis, Pham [76]: magnetic induction tomography, Borm [78]: eddy current analysis

When actually solving the problem, it is much easier and reasonable to rewrite the general equations in different domains respectively so that they can take a simpler form. Thus, describing the relations at interfaces and boundaries become very important, which are usually called boundary conditions. The field strength at artificially far boundaries is assumed to vanish. And the boundary conditions for the eddy-current problem can be given as below

$$\mathbf{H} \times \mathbf{n} = 0, \mathbf{B} \cdot \mathbf{n} = 0, \mathbf{E} \times \mathbf{n} = 0, \mathbf{D} \cdot \mathbf{n} = 0 \text{ on } \Gamma \quad (2.7)$$

$$\mathbf{H} \times \mathbf{n}, \mathbf{B} \cdot \mathbf{n}, \mathbf{E} \times \mathbf{n}, \mathbf{D} \cdot \mathbf{n} \text{ are continuous on } \Gamma_{nc} \quad (2.8)$$

where $\mathbf{n}$ is the outer normal unit vector. Term $\mathbf{B} \cdot \mathbf{n}$ has the physical meaning of the normal magnetic flux exiting the boundaries. Since flux is continuous, the normal components of flux density are continuous on each side of the boundary. And the work done in taking a unit electron round the typical infinitesimally small path consisting of the field lines and equipotential lines is zero. Thus it can be derived that in the absence of an interface current, the tangential components of $\mathbf{H}$ on each side of an interface are continuous. Magnetic field intensity $\mathbf{H}$ has an analogue meaning of electric field $\mathbf{E}$. Boundary conditions can also be derived into other forms, in terms of magnetic vector potential, electric scalar potential, etc.

Governing equations and boundary conditions introduced above are all in vector form, which would be decomposed into multiple scalar equations for calculation purpose. Analytical solutions can only be obtained for extremely simple models, and it is usually based on specific assumption that unknown variables have only one nonzero component, which reduces the vector equation to single scalar form equation. Gencer[59] simulated a circular object inside a circular coil as a two dimensional model when the object has two concentric layers of different conductivities. Originated from Maxwell equations and continuity equation, governing equation using electric scalar potential was formulated, which was in scalar form naturally. By equating the normal components of current density on the interface, boundary condition of the governing Poisson's equation was found to be Neumann boundary form. Then for two concentric domains of different conductivities, series solutions were provided respectively using Fourier series. Here presented equation is only applicable for a two-dimensional model. And in [60], series solution was obtained for a sphere with

varying conductivity and permeability profiles in three dimensional space. Spherical coordinate system was used, with the origin to be the center of conductive sphere. The problem domain was divided into three regions: the empty space, the outer spherical shell with varying electromagnetic feature and the inner ball with constant feature. Under the assumption of having only one nonzero component, the azimuth component, magnetic vector potential was used to form the governing equation in scalar form. And by introducing the associated Legendre function and Bessel function, the analytic solution was written for three regions respectively. Different material profiles were analyzed in the paper and computational impedance results were presented for different parameters. Pham [61] presented a governing equation in terms of electric field, which was validated to have only one nonzero component, and derived the series solution for nonconductive and conductive domains, respectively. Although the primary variables are different and the form of governing equations are slightly different, solutions mentioned above are all obtained by separation of the variables and reducing the complicated vector form partial differential equation (PDE) to simple ordinary differential equation (ODE). Their applications are restrained to specific geometry and specific material feature distribution, such as symmetric spherical or cylindrical structures.

When more general problems are dealt with, numerical methods are usually in demand. Unlike a continuous partial differential equation valid everywhere over the problem domain, which indicates the analytical solution as a function also valid everywhere, numerical solution can only be obtained for a finite number of points. Thus an additional numerical model/discretized model need to be established first, which consists of a sum of strategically chosen points to adequately represent the problem domain. A unifying principle for such technique can be found in the general

Moment Methods [62], in terms of which many specific approaches can be interpreted. With this method, the approximate solution is expressed as a finite summation of basis function. By introducing a series of weighting functions and defining the inner product operation, a matrix form equation can be formed. Detailed procedures and explanations are shown in Appendix a. One of the main tasks in any specific problem is the choice of basis function and weighting functions. They should both be linearly independent. Additional factors which affect the choice of basis function and weighting functions include the accuracy of solution desired, the ease of evaluation of the matrix elements, the size of the matrix that can be inverted, and the realization of a well-conditioned matrix.

Finite element method (FEM), which utilizes weak form governing equations, calculates the solutions for constituting nodes. It transfers the large-scale global problem into local problems by discretized the entire problem domain into elements and regularly cooperates with Moment Methods especially Galerkin's method [63]. Unknown variable within single element is assumed to be linear function of location coordinates, and then can be expressed as linear superposition of the specific value at its constituting nodes. Therefore basis functions are only dependent on the element geometry. Instead of globally strong formulations which can barely be satisfied, an integral form weighted equation which forces the governing equation to be fulfilled in an average sense over the problem domain is used, which is the fundamental working principle of FEM.

Many investigators have used magnetic vector potential formulation with finite element method for two dimensional fields [64-66], and then for two dimensional approximations of three dimensional problems, using symmetry properties. Demerdash [67] presented a three dimensional finite element formulation using vector potential for

the eddy-current problem in a finite-length aluminum bar of square cross-section, which was excited by a surrounding coil. Cartesian coordinate system was used. The permeability of the aluminum bar material assumed independent values in the x, y and z directions respectively and the conductivity was constant. After expanding the vector equation into three scalar PDEs, differential formulations for this class of eddy current were obtained. After boundary conditions were applied, it was solved using FEM and total eddy current power loss was calculated. As in [68], a hybrid method based on the $\mathbf{T}\text{-}\Phi$ formulation was used, which included FEM for conducting region and boundary integral method for free space. A hollow conducting sphere placed in a uniform field with sinusoidal variation in time was analyzed and the results showed agreement with analytical solutions. By introducing integral method, the main advantage of this method is the diminishing of the number of unknown and facility to treat open boundary problems. Biro [69] reviewed various un-gauged formulations of eddy current problems in terms of scalar and vector potentials. The vector potentials were approximated by eddy finite elements and the scalar potentials by nodal ones. Compared with using nodal elements to approximate vector potentials, presented approach avoided the singularities approximated at sharp corners. For adaptive mesh refinement, error estimators based on the principle of Ligurian and numerical discontinuity of field results were established in [70]. Mesh modification was done by so called “h method”, which ameliorates the mesh by decreasing the size of the element. Edge-division technique followed by moving the “max-weight” node was adopted for the division of elements.

Boundary element method (BEM) is another numerical computational method of solving linear PDEs, which requires the governing equations to be reformed as boundary integral equations first. The boundary element method attempts to fit the

given boundary conditions into the integral equation, rather than values throughout the space. This can be seen as one advantage of integral governing equations over differential governing equations. Only partial differential equations are not enough to describe the problem, since apparently they always give non-unique solutions while in reality there is always unique circumstance. Boundary and interface conditions have to be included and considered additionally to obtain accurate solution. On the contrary, integral equations can describe the problem solely since the boundary conditions are already included implicitly. The integral equation may also be regarded as an exact solution of the governing partial differential equation. In the post processing stage, the integral equation can be used again to calculate numerically the solution at any desired point in the solution domain.

Hariharan [71] presented two methods for a specific two-dimensional eddy-current problems, which was a wire carrying periodic current parallel to a uniform conducting cylinder. Both methods yielded integral solutions over the boundary of the conductor based on differentiate form Maxwell's equations and Green's theorem. The first method is exact while the second is an asymptotic solution. With two non-dimensional parameters to account for the material features, different cases were considered, including object with infinite conductivity and zero conductivity. Although the magnetic vector potential concept was not referred in the paper, the function introduced as the primary unknowns was proportional to the magnetic vector potential in physical sense. Maccamy [72] continued the study and extended this method to treat full three-dimensional situation, but it still cannot treat inhomogeneous or non-linearly magnetic materials.

Although different formulations would produce the same answers for one specific problem, they may differ in accuracy when implemented numerically and the

implementation will differ in complexity and computational cost. Various BEM formulations in terms of field variables were reviewed in [73]. And boundary element method using magnetic vector potential and electric scalar potential was described in [74], where tangential and normal components of the vector potential, tangential components of the magnetic flux density and an electric scalar potential on conductor surfaces were chosen as unknown variables. In [75], Misaki presented the boundary element method based on vector Green's theorem. Unknown electric field vectors and magnetic flux density vectors were assumed on the boundaries of two materials and unknown electric field vectors were assumed in the conductor regions. After determining these unknown vectors, 3-dimensional eddy current distributions were computed at every point in the conductors. The computation results were compared with a coupled circuit model and showed good agreement, but both tetrahedral and triangular elements were used in the discretizing process, which increased the computation effort.

Pham [61] presented a 3-D mathematical model in terms of electric field $\mathbf{E}$ for magnetic induction tomography system using boundary integral equations. The conductivity of the target area was written as $\sigma = \sigma_b + \Delta\sigma$, where σ_b was the background conductivity assuming to be constant within the whole target domain and $\Delta\sigma$ was the perturbation from it. Similarly, $\mu = \mu_b + \Delta\mu$, where $\Delta\mu$ was the perturbation from the constant background permeability μ_b .

The paper focused the forward problem based on σ_b and μ_b . With the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, the analytical partial differential governing equations were as below, derived from $\mathbf{A}$ formulation in eddy current free region and $\mathbf{A}$ -V formulation in eddy current region.

$$\nabla^2 \mathbf{A}_0 - j\omega\mu_b\sigma_b\mathbf{A}_0 = \mu_b\sigma_b\nabla V_0 \quad (2.9)$$

$$\nabla^2 \mathbf{A}_1 = -\mu_0 \mathbf{J}_s \quad (2.10)$$

where $\mathbf{A}_0$ and $\mathbf{A}_1$ were the magnetic vector potentials in the conductor and in the free space respectively, V_0 was the electric scalar potential in the conductor, ω was angular frequency, μ_0 was the magnetic permeability of the free space, $\mathbf{J}_s$ was the current density inside a coil. Here, the source in the free space region is considered only to be the applied current source $\mathbf{J}_s$, and the source in the conductive region is $\mathbf{J}_0 = \sigma_b \mathbf{E}_0$. And the boundary conditions in terms of magnetic vector potential were

expressed as $\mathbf{A}_0 = \mathbf{A}_1$ and $\frac{1}{\mu_b} \frac{\partial \mathbf{A}_0}{\partial n} = \frac{1}{\mu_0} \frac{\partial \mathbf{A}_1}{\partial n}$.

With the help of Green's functions, which can express the solution of partial differential equations as boundary integral form, potentials in the governing partial differential equations as Eq. 2.9 and Eq. 2.10 can be expressed in boundary integral solution, then deduced to surface integral expressions of $\mathbf{E}_0$, $\partial \mathbf{E}_0 / \partial \mathbf{n}$ and $\mathbf{E}_V$, electric field only due to the electric scalar potential. These equations can be discretized and linearized using the Boundary Element Method, which divide the boundary into a finite number of elements. Elementary boundary integral of Green functions, which is function of background material feature constitute the coefficients of the matrix form equations. In [76] Radial Basis Function approximation was further used to obtain the numerical results. After comparison, computational cost is favorable to that of boundary element method while accuracy is superior. Numerical issues as well as advantages and disadvantages of Radial Basis Function were discussed.

A FEM/BEM-coupled scheme based on edge elements is introduced in [77, 78], which utilized $\mathbf{E}$ formulation. The FEM part was used in target domain and the BEM

part, which was needed for the exterior vacuum domain, consists of boundary integral over the interface surface. The FEM part for the interior domain was coupled to the BEM part by using the interface conditions. And due to the fine triangulation of interface boundary, the BEM operators were dense, thus a compression technique [79] was applied to the boundary integral operators.

2.2 MODELING OF MIT

As previous discussed, many methods have been proposed in simulating and analyzing the eddy-current problem. As far as eddy current problem is generally concerned, discussion ends when field variables (electric field $\mathbf{E}$ or magnetic flux density $\mathbf{B}$) are computed, analytically or numerically. It can either be expressed as close-form when the geometry of structure is simplified, or solved with numerical techniques such as FEM and BEM for three dimensional arbitrary cases with complicated geometry. However, Magnetic Induction Tomography aims at sorting out the material property differences, no matter what variable is used to characterize this difference. Thus rather than field variables, sensitivities, which represent relation between material property and chosen variables, are the ultimate objective. The most straight-forward would be calculating twice for each transmitter-receiver combination, with and without perturbation, and then dividing the difference by the conductivity variation. But using this method requires a considerable amount of time and computation efforts. Although easy to understand and implement, it is not necessarily the best approach. Various studies have focused on building the theoretical model to calculate the sensitivities.

From mathematical point of view, this is a typical inverse problem that requires converting observed measurements into information about concerned system. The key problem is to select two variables, one accounting for material feature, and one for

measurements, so that their relation can be explicitly modeled and this model being invertible. This choice will decide the modeling difficulty, invertibility, computation efforts required, and practical concerns like resolution.

A proper study [80] presented by Gencer forms a mathematical basis for MIT process. First, forward modeling of the imaging system was operated by calculating the secondary magnetic field based on Biot-Savart law $\mathbf{B} = \frac{\mu_0}{4\pi} \int \left(\mathbf{J} \times \frac{\mathbf{R}}{R^3} \right) dV$, and constitutive equation $\mathbf{J} = \sigma \mathbf{E}$. It can be expressed explicitly as below after introducing the relation $\mathbf{E} = -j\omega \mathbf{A} - j\omega \nabla V$. Note that $j\omega$ indicates time derivative.

$$\mathbf{B}_s = -j \frac{\mu_0}{4\pi} \int \sigma (\omega \mathbf{A} + \nabla V) \times \frac{\mathbf{R}}{R^3} dV \quad (2.11)$$

Two assumptions were made: 1) Vector potential $\mathbf{A}$ remained unchanged when the conductive body was present, which make it easy to calculate for a known coil configuration and require no further computation with regard to material feature. This computation procedure only needs to be done once for a specific system. 2) Displacement current term is neglected due to its much smaller magnitude compared to conductive current.

It is noticeable from Eq. 2.11 that relation between the secondary magnetic field $\mathbf{B}_s$ and conductivity distribution σ is a rather complicated mapping since electric scalar potential is also a function of conductivity. Instead, relation between a first-order variation in the secondary magnetic fields and perturbation in the conductivity distribution can derived as Eq. 2.12,

$$\Delta \mathbf{B}_s = -j \frac{\mu_0}{4\pi} \left[\int \frac{\mathbf{R}}{R^3} \times \left(\omega \mathbf{A} + \nabla V_0 + \sigma_0 \frac{\partial(\nabla V)}{\partial \sigma} \right) \Delta \sigma dV \right] \quad (2.12)$$

where σ_0 refers the initial conductivity value, $\Delta\sigma$ refers to the perturbation conductivity and V_0 is simply the electric scalar potential for the initial conductivity distribution. Therefore the sensitivity term can be calculated independent of perturbation conductivity. This formulation is used for calculation of the sensitivity matrix and then proceeds to the inversion process.

For illustrative purpose, a cubic conductive body was divided into 500 cubic elements and conductivity of each element assumed to be constant, thus Eq. 2.12 was reformed as summation form of 500 elements. With 49 transmitter coil and 49 receiver coils being used, sensitivity matrix S was obtained as a 2401×500 matrix. It is noticeable that the matrix dimension does not indicate that the problem is over determined because the sensitivity matrix would be ill-conditioned. The pseudo-inverse of the sensitivity matrix was directly used in [81]. In order to handle severely ill condition, regularization techniques are usually implemented, which will be detailed in following sections.

Another approach that requires the excitation and detection coils to switch roles is more commonly used for magnetic induction tomography in current studies. Assuming two coils were present around the target domain, indicated as a and b , based on electromagnetic version of Tellegen's theorem the sensitivity term is related to the dot product of two field variable $\mathbf{E}$ and $\mathbf{H}$, one calculated for the scenario that coil a functions as excitation and coil b as detection, the other one for the scenario that coil b functions as excitation and coil a as detection. Because these two fields were considered to be unaffected by perturbation from conductive object, only two general eddy current problem need to be calculated.

Dyck [82] computed the sensitivity of induced voltage to perturbations both in sources and material parameters, which applied to systems with linear materials and

time harmonic source current. Consider two separate systems with possibly different material parameters, different source currents and/or different boundary conditions, which provide two fields $\mathbf{E}_1$ and $\mathbf{H}_2$. Taking the volume integral of the divergence of vector $\mathbf{E}_1 \times \mathbf{H}_2$, a general form of Tellegen's theorem can be obtained as Eq. 2.13 after proper derivations.

$$\int_V \left\{ \mathbf{E}_1 \bullet \left[(\sigma_2 + j\omega_2 \epsilon_2) \mathbf{E}_2 + \mathbf{J}_{s2} \right] + j\omega_1 \mu_1 \mathbf{H}_1 \bullet \mathbf{H}_2 \right\} dV = - \int_S \mathbf{E}_1 \times \mathbf{H}_2 \bullet \mathbf{n} dS \quad (2.13)$$

Then three systems with different physical characteristics were substituted into the Tellegen's theorem respectively; fields in the first system are used to compute the specified quantity of interest; in the second system material properties and source currents of system A are perturbed by a small amount, only needed for derivation; the third was an auxiliary system for computing the sensitivity of the specified quantity. Only system one and three need to be solved for the corresponding field information, which was directly substituted into the sensitivity equation.

The example problem in the paper aimed to determine the shape of a defect in an aluminum block based on measures from five coils situated above the block. The inverse problem solver converged to the correct defect in five iterations. The same working principle has been applied to MIT system [83-85] with weakly conductive material and low contrast.

Hollaus [83] presented the process of calculating the sensitivity matrix based on "reciprocity theorem", which was a different terminology for Tellegen's theorem for electromagnetic field in [82]. The induced voltage variation was equivalent to the multiplication of the sensitivity matrix by a vector with each entity representing the material feature changes of single element, including complex conductivity and permeability. And each entity of the sensitivity matrix was expressed as volume

integral of the dot product of field values over single element. Compared to finite difference method, reciprocity theorem only requires two eddy-current problems: one for the excitation and one for the receiver coil, which were simulated by Finite Element method with tetrahedral edge elements. The numerical example consisted of a conducting cylinder with a conducting sphere arranged in the center of the cylinder. Only one pair of excitation and sensing coil was tested. The excitation frequency was 500 kHz and small variations of the conductivity in the sphere were calculated within the range from -0.1 to 0.1S/m. A comparison of the results obtained by the reciprocity theorem and finite differences showed a satisfactory agreement as long as the perturbations were relatively small compared to the absolute values of the material properties. This method can be easily extended to multiple excitation coils as well as multiple receiver coils to facilitate an image reconstruction in 3D in magnetic induction tomography [84].

Ktistis [84] reduced the sensitivity equation further by neglecting the permeability since the permeability of biological tissue is very near to the permeability of air. And multiplication of two electric fields was replaced with magnetic vector potential for the empty sensor and current density due to the induced current inside the target. The former was calculated for the empty sensor by the Biot-Savart law and the latter was taken from the FEM model. By pointing out the exact solution of the inverse problem is neither necessary nor possible in an iterative image reconstruction technique, the author validates the use of approximate sensitivity coefficients matrix for reconstruction.

A circular array with eight coils and 266 mm diameter was modeled in a commercial FEM solver. With the excitation frequency and current to be 1A/10MHz, all coils were used as transmitters and receivers in turn. Results of FEM model and

linearized forward model showed good agreement. For image reconstruction, Tikhonov regularization using identity matrix was applied. Conductivities of the reconstructed object tend to be underestimated and perturbation cannot be detected when it is placed centrally. Many other regularization matrices and corresponding methods [86-88] have been developed to obtain accurate and dynamic solution for various research areas.

Cohen-Bacrie [89] presented both classical Tikhonov and variance uniformization technique. In order to improve the tradeoff between the quality of the images and the numerical complexity of the reconstruction method, the author first proposed a solution which is based on Tikhonov regularization, with identity matrix to be the regularization matrix; second introduce an original regularized reconstruction method in which the regularization matrix is determined by space-uniformization of the variance of the reconstructed conductivities. Due to the attenuation of the scattering the waves within the media, the measured signal contain a much richer information about the conductivity in the peripheral region than in the central region. Although reducing this effect, Tikhonov approach still applies the same amount of regularization on each element of the unknown object, regardless of its contribution to the measured data. Thus instead of $\delta^2 \|Lx - x^*\|_2^2$, a new form of $x^T R x$ is used as the regularization functional. Value of the uniform variance of the reconstruction and the truncation level were adjusted as the regularization parameters. Simulation is done for homogeneous circular objects with a single-point in-homogeneity and reconstructions were carried out with the variance uniformization technique and with classical Tikhonov regularization. Results show that with variance uniformization, the oscillations in the peripheral region were reduced and the presence of the peaks is

more obvious. Real data reconstructions also proved that variance uniformization clearly produces better contrast and resolution than classical Tikhonov regularization.

Casanova [90] also used new regularizing functional in order to preserve the edge. Started with classical Tikhonov method with regularization matrix to be the Jacobian matrix, the author stated that the quadratic penalty term, which is the regularization functional, would strongly penalize discontinuities and the estimated image vector yields blurred images of objects. Based on the basic idea that replace the quadratic penalty terms by other functions which allow for higher values of the gradients, different approaches have been suggested to select proper function for a new regularizing term, as reviewed in [91]. Here a deterministic half-quadratic algorithm named ARTUR was used, which introduced a second variable b that simultaneously mark the discontinuities and make easier the treatment of the optimization process. The quadratic term was replaced by inner products of the Jacobian matrix multiplied by model vector and auxiliary variable b , with potential functions φ to be the coefficients. One more regularization parameter were introduced as δ , a model-scaling parameter to control the values of the gradient above which a discontinuity is detected. This dual problem has such a structure that high values of the gradients corresponding to the edges are allowed in the optimization process by low values of the auxiliary variables b preserving in this way the discontinuities. Potential functions achieved as in [92] were used and both results of simulation as well as real data results proved that the Tikhonov regularization reconstruction, although having very good noise properties, smooth the image too much and fails to recover the object edges. On the contrary, ARTUR and ARTUR with non-negativity constraining recovers the edges much better; and the latter can also dampen the distortion of image when strong noise is present.

In [93], Silva implemented a Newton-Raphson-type algorithm for the inversion procedure, basic iteration step of which is described by the pseudo-inverse solution multiplied by a relaxation matrix. Several cases were simulated with the object to be located at different locations and of different material feature, and estimation accuracy behaves as expected regarding the noise level.

Scharfetter [94, 95] compared four different methods during the inversion process of MIT, including a different regularization matrix as the identity matrix and a defined neighboring matrix, variance uniformization approach and truncated SVD method. The inverse solver was tested with a 3-D model comprising a cylindrical conductor with two spherical in-homogeneities. With point spread functions defined to characterize the resolution, contrast/noise ratio to describe the image contrast, noise free data and noisy data with SNR of 44, 50 and 64 dB were implemented for the inversion procedure. A clear increase of the resolution with the distance from the center was observable, despite the inversion method. And the relative loss of resolution with the noise level was stronger in the center than in the periphery. CNR for TSVD as a function of the distance were almost independent on the noise level. And at a SNR of 50 dB, CNR depends strongly on the location of the perturbation; Tikhonov methods with identity matrix or neighboring matrix to be the regularization matrix yield highest CNR and Variance uniformization approach provided much poor contrast.

Vauhkonen [96] added a weighing matrix to the fitness term in the minimization function. The weighting matrix was a diagonal matrix having the inverses of the absolute voltage values on the diagonal, which means that small voltage values get bigger weighting in the reconstruction. Different cases were tested using both results of simulation and real measurements. Results from similar cases with simulations and real measurements showed agreement, which validated the forward model. The 2-D

images of the true conductivity distribution and reconstruction are qualitatively fairly similar.

Soleimani [85] reconstructed the image with a priori information that the reference conductivity is close to the exact conductivity. The iterative reconstruction algorithm starts with the initial conductivity distribution, and then the conductivity as well as Jacobian matrix is updated at each step.

2.3 EXISTING SYSTEMS

The ambient magnetic noise in an urban environment is on the order of 10^{-7} T. And the magnetic field caused by induced current is around $10^{-8} \sim 10^{-7}$ T, relatively bigger than the spontaneous field, but still quite weak. This leads to an essential problem of bio-magnetism, the weakness of signal relative to the sensitivity of the detectors, and to the competing environmental noise.

No practical instrument of Magnetic Induction Tomography has been applied in real life. But many researchers have built different systems to operate the MIT experiment, in order to simulate the real environment and optimize the sensing performance. A summary of existing setups has been shown in Table 2.3 and will be discussed in details as followed.

Table 2.3 Summary of existing MIT systems

Specific issue	Existing studies
Gradiometer sensor	Zheng [97], Riedel [98], Rosell-Ferrer [99], Scharfetter [55], Rosell [100]
Coil-orientation method	Watson [103], Gursoy [102],
Multiple excitation	Korjenevsky [53], Vauhkonen [96], Rosell-Ferrer [99]

Korjenevsky [53] built a system containing 16 inductor and detector coils and a corresponding number of oscillators and receivers combined in the transceiver

modules, as in Fig. 2.2(b). The inductors and detectors were reeled up in pairs on the 5cm cylindrical formers and placed along the 35cm diameter circle. The inductors consisted of two turns of wire and detectors consisted of four turns of wire. A cylindrical electromagnetic screen surrounded the workspace containing the coils and isolated it from external influence and from transceiver modules. Measurements were repeated with all inductors. The system operated at frequency 20 MHz and excitation signal in the inductor was approximately 60mA. Receivers connected to the detectors had differential inputs and included a mixer, which converts the signal to low frequency, a filter and an amplifier. Phase shifts between low frequency signals were digitally measured using microcontrollers. This type of tank-shaped system is a common choice for the magnetic induction system.

(a) Vauhkonen, 2008

(b) Korjenevsky, 2000

(c) Rosell-Ferrer, 2006

Fig. 2.2 Existing MIT systems

In Vauhkonen's work [96], 16 excitation coils were arranged circularly with a diameter of 31cm and 16 receiver coils with a diameter of 30cm, as in Fig. 2.2(a). Both excitation and receiver coils were mounted on the inner wall of the aluminum tank and the electronic modules were mounted on the outer wall.

There are two contributions to the signal detected by the sensing coil in magnetic induction tomography. The first is directly induced by the field from the excitation coil (the primary signal). The second is from the eddy currents induced within the material, which in turn produce their own magnetic field (the eddy-current or secondary signal). The primary signal is often very large in comparison to the eddy-current signal of interest, and is responsible for introducing noise into signal measurements. Minimizing the response of the sensors to the primary field while retaining responsiveness to the eddy-current field can therefore produce very significant improvements in the system's signal to noise ratio. Techniques for minimizing the response to the primary field in MIT systems may be categorized into two basic types: symmetry methods (gradiometer sensor) and orientation methods. In symmetry methods, two sensor coils are placed equal distances on either side of any axis of symmetry of the excitation coil. The coils are then connected in serial opposition to produce a gradiometer. Variations on this method include placing the sensor coil pair symmetrically about the plane of the excitation coil to produce an axial gradiometer [97, 98] and placing the sensor coil pair symmetrically about the center axis of the excitation coil to produce a planar gradiometer, as used in [99] and shown in Fig. 2.2(c).

Scharfetter [54] introduced a planar gradiometer as receiver, which consisted of two rectangular spiral coils with 7 turns each. The excitation coil was a solenoid with 17 turns of copper wire and radius of 47.5mm. A planar reference rectangular coil of 7 turns was used to provide the reference voltage for the phase detector and a simple

rectangular printed coil of 7 turns was used as a comparison to gradiometer. Distance between the excitation coil and planar gradiometer was 210mm. This coil system exhibits an anti-symmetric sensitivity with respect to the coil axis. In an ideally adjusted gradiometer the voltage induced in the absence of an object is zero because two parts of the coil experience the same magnetic flux. In this way, the “signal/carrier ratio” can be increased by the cancellation effect. As the induced current is much lower in the gradiometer than in the simple coil, an additional preamplifier was inserted with a gain of 100. By multiplying the signal voltage with the 90 degree phase shifted reference voltage, any phase deviation between them causes a non-vanishing voltage output.

This type of gradiometer coil system is examined in [55, 100]. The performance of a planar gradiometer as a sensing element was evaluated thoroughly, by comparing systems using a planar gradiometer and a simple coil. Theoretical sensitivity maps were derived from basic equations and compared with experimental data obtained at different frequencies. Different experiment environments were used, including brass sphere in empty space, spherical perturbation with conductivities in the physiological range (0.4-0.8 S/m) in null space, a plastic tank filled with saline and with conductive perturbations, etc. It is validated that the major advantage of a planar gradiometer is the bigger measured phase shift due to the cancellation factor. Also, compared to an outside back-off coil as reference, the gradiometer method provides higher mechanical stability since both coils are located in the same structure, for example, a printed circuit board. Moreover, this concept provides better electronic stability because it requires fewer components. And, the major problem of the planar gradiometer is its insensitivity to objects with axial symmetry.

A further technique for minimizing the response of the sensors to the primary field was described by Watson [101] and Gursoy [102] in which the sensor coil (termed a Bx sensor) is placed with its axis oriented normal to the primary field. No net primary flux passes through the sensor coil and it will therefore not detect the primary field. In [103], the performance of axial gradiometers and coil-orientation methods (Bx sensors) were compared. The sensitivity distribution with depth was calculated for a single Bx sensor and axial gradiometer. The results suggested that the Bx sensor provided better sensitivity at depth than the axial gradiometer. If only surface measurements are required, then the axial gradiometer appeared to be more suitable. Image was reconstructed from simulated data with/without noise and it displayed image distortion with the image compressed back towards the array.

In MIT, the location of the receivers affects the quality of the image reconstruction. As in [104], a fast deterministic algorithm was applied to obtain optimum receiver array designs for a given specific excitation. The design strategy was based on the iterative exclusion of receiver locations, which yield poor conductivity information, from the space spanning all possible locations until a feasible design is reached. Aiming at maximizing the degree of the independence between the rows of the sensitivity matrix in order to decrease the condition number of sensitivity matrix, the quality measure for a particular design was defined in terms of the N-dimensional angle between the rows of the sensitivity matrix. The more orthogonal the vectors are, the more independent information content they provide.

To reconstruct tomographic MIT images, multiple coil system are needed as in [53, 99]. And in [96], the measurement speed was improved by enabling a parallel readout of the total 16 measurement channels. The parallel measurements were carried out using high-quality audio sampling devices. The receiver modules consist of audio

sampling equipment with a sampling rate of 192 kHz in 24 bit resolution, a low noise amplifier, which amplifies the radio frequency measurement signal coming from the receiver coil, an analogue frequency mixer, which multiplies the amplified signal and excitation signal to an intermediate frequency, a low pass filter and then another amplifier.

III. EXTENDED DMP MODEL

3.1 OVERVIEW

It is emphasized in the last chapter that imaging reconstruction in MIT is still very underdeveloped. Many interesting works have been done. Non-phenomenological methods (methods that do not consider or utilize the knowledge of the underlying physics) such as Neural Network were used. And within the category of phenomenological methods, numerical models are usually generated based on differential or integral form equations that represent the physical problem. DMP model, as a semi-analytical approach, has been developed for simulating the magnetic field from PMs and show good accuracy as well as robustness. Here we'll extend the approach so that it can be applied for AC carrying coils therefore time-varying problems. Also, as mentioned in previous chapter, quasi-static assumption is commonly used when addressing the induction and eddy current problem. It is applicable when the length scale of the field is much smaller than the wavelength and will be used throughout the whole manuscript.

3.2 MODELING BASED ON VECTOR POTENTIAL

In order to approximate magnetic field as summation form based on static sources and sinks, the DMP model exploits the commonly used governing equation for static problem, the Laplace's equation in terms of scalar potential.

$$\nabla^2 \Phi = 0 \quad (3.1)$$

Relation between the potential and magnetic source/sink was therefore derived in the form of fundamental solution of Laplace's equation.

$$\Phi = \frac{m}{4\pi R} \quad (3.2)$$

where m indicates the strength of source or sink, and R indicates the distance from source/sink to computation location.

For time-varying problems, magnetic vector potential $\mathbf{A}$ is usually utilized instead of scalar potential due to the irrotational nature. And the governing equation can be expressed as below when using the Coulomb gauge that set the divergence of vector potential to be zero.

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J} \quad (3.3)$$

With the boundary condition that vector potential vanishes in infinity and the current source to be localized as well as finite, the unique solution of Eq. 3.3 can be expressed as below.

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' \quad (3.4)$$

where $\mathbf{J}$ indicates the current density at location $\mathbf{r}'$, V' indicates the volume that encloses the current source.

It is apparent a vector source is needed to correspond with the vector potential as well as field. The most straightforward speculation is point dipole with its magnetic dipole moment. Any arbitrary current distribution in space has a magnetic dipole moment, just like magnetic point dipole

$$\mathbf{m} = \frac{1}{2} \int \mathbf{r} \times \mathbf{J} d^3\mathbf{r} \quad (3.5)$$

where $\mathbf{r}$ is the position vector pointing from the origin of coordinate system to the location of the volume element, and $\mathbf{J}$ is the current density vector at that location.

To express the magnetic field in terms of the magnetic dipole moment, we begin with expressing the vector potential $\mathbf{A}$ [1].

Let $\frac{1}{|\mathbf{r} - \mathbf{r}'|} = f(\mathbf{r}')$, then based on Taylor series,

$$f(\mathbf{r}') = f(\mathbf{a}) + \nabla' f(\mathbf{a}) \bullet (\mathbf{r}' - \mathbf{a}) + \dots \quad (3.6)$$

Considering $\mathbf{a}$ to be zero, with

$$\nabla' \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \quad (3.7)$$

$\frac{1}{|\mathbf{r} - \mathbf{r}'|}$ can be expressed as

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{|\mathbf{r}|} + \frac{\mathbf{r} \bullet \mathbf{r}'}{|\mathbf{r}|^3} + \dots \quad (3.8)$$

Then magnetic vector potential can be expanded as

$$\begin{aligned} \mathbf{A} &= \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}|} d\mathbf{r}' + \frac{\mu_0}{4\pi} \int \frac{\mathbf{r} \bullet \mathbf{r}'}{|\mathbf{r}|^3} \mathbf{J}(\mathbf{r}') d\mathbf{r}' + O\left(\frac{1}{|\mathbf{r}|^3}\right) \\ &= \frac{\mu_0}{4\pi |\mathbf{r}|} \int \mathbf{J}(\mathbf{r}') d\mathbf{r}' + \frac{\mu_0}{4\pi |\mathbf{r}|^3} \int (\mathbf{r} \bullet \mathbf{r}') \mathbf{J}(\mathbf{r}') d\mathbf{r}' + O\left(\frac{1}{|\mathbf{r}|^3}\right) \end{aligned} \quad (3.9)$$

Considering the i th component of the vector potential $\mathbf{A}$,

$$A_i(\mathbf{r}) = \frac{\mu_0}{4\pi} \left[\frac{1}{r} \int J_i(\mathbf{r}') d\mathbf{r}' + \frac{1}{r^3} \sum_{m=i,j,k} \int r'_m r'_m J_i(\mathbf{r}') d\mathbf{r}' + O\left(\frac{1}{r^3}\right) \right] \quad (3.10)$$

where $r = |\mathbf{r}|$.

Observing the first term inside the parentheses, $J_i(\mathbf{r}')$ can be replaced with

$$\sum_{m=i,j,k} J_m \frac{\partial r'_i}{\partial r'_m}, \quad \text{since } \frac{\partial r'_i}{\partial r'_m} = 1 \text{ only when } m=i. \text{ Then this summary term is integrated}$$

by parts as in Eq. 3.11, where the first term vanishes because the source vanishes at

infinity and the second term vanishes because $\nabla \bullet \mathbf{J} = \sum_{m=i,j,k} \frac{\partial J_m}{\partial r'_m} = 0$. Note that this

follows from the quasi-static assumption that results $\nabla \bullet \mathbf{J} = -\frac{\partial \rho}{\partial t} = 0$.

$$\int J_i(\mathbf{r}') d\mathbf{r}'^3 = \int \sum_{m=i,j,k} J_m \frac{\partial r_i'}{\partial r_m'} d\mathbf{r}'^3 = \sum_{m=i,j,k} \int J_m r_i' d^2 r_{l \neq m} \bigg|_{r_m'=-\infty}^{r_m'=\infty} - \sum_{m=i,j,k} \int r_i' \frac{\partial J_m}{\partial r_m'} d\mathbf{r}'^3 \quad (3.11)$$

Then considering the second term in Eq. 3.10,

$$r_m' J_i = \frac{1}{2} (r_m' J_i + r_i' J_m) + \frac{1}{2} (r_m' J_i - r_i' J_m) \quad (3.12)$$

The first term in the equation above vanishes after integral,

$$\sum_{m=i,j,k} \int r_m' \frac{1}{2} (r_m' J_i + r_i' J_m) d\mathbf{r}'^3 = \sum_{m=i,j,k} \int r_m' \frac{1}{2} \left(r_m' \sum_{n=i,j,k} J_n \frac{\partial r_i'}{\partial r_n'} + r_i' \sum_{n=i,j,k} J_n \frac{\partial r_m'}{\partial r_n'} \right) d\mathbf{r}'^3 \quad (3.13)$$

Note that this has the similar form as Eq. 3.11 and will vanish due to the same reasons.

The remaining, second term of Eq. 3.12, can be connected with the magnetic dipole moment of current distribution $\mathbf{m} = \frac{1}{2} \int \mathbf{r} \times \mathbf{J} d^3 \mathbf{r}$. To prove this, we can start with the i th term of $\mathbf{m} \times \mathbf{r}$

$$\begin{aligned} (\mathbf{m} \times \mathbf{r})_i &= m_j r_k - m_k r_j = \frac{1}{2} r_k \int (r_k' J_i - r_i' J_k) d^3 \mathbf{r}' - \frac{1}{2} r_j \int (r_i' J_j - r_j' J_i) d^3 \mathbf{r}' \\ &= \frac{1}{2} \int (r_k r_k' J_i - r_k r_i' J_k - r_j r_i' J_j + r_j r_j' J_i) d^3 \mathbf{r}' \end{aligned} \quad (3.14)$$

And by substituting Eq. 3.12 into Eq. 3.10,

$$A_i(\mathbf{r}) = \frac{\mu_0}{4\pi} \left[\frac{1}{r^3} \sum_{m=i,j,k} \int r_m' \frac{1}{2} (r_m' J_i - r_i' J_m) d\mathbf{r}'^3 + O\left(\frac{1}{r^3}\right) \right] \quad (3.15)$$

Note here the first term in Eq. 3.12 is not shown since it has already been proved to vanish. Then by breaking the summary operator,

$$A_i(\mathbf{r}) = \frac{\mu_0}{4\pi r^3} \int \frac{1}{2} \left[r_i (r_i' J_i - r_i' J_i) + r_j (r_j' J_i - r_i' J_j) + r_k (r_k' J_i - r_i' J_k) \right] d\mathbf{r}'^3 + O\left(\frac{1}{r^3}\right)$$

$$\begin{aligned}
&= \frac{\mu_0}{4\pi r^3} \int \frac{1}{2} \left[r_j (r_j' J_i - r_i' J_j) + r_k (r_k' J_i - r_i' J_k) \right] d\mathbf{r}^3 + O\left(\frac{1}{r^3}\right) \\
&= \frac{\mu_0}{4\pi r^3} \int \frac{1}{2} (r_j r_j' J_i - r_j r_i' J_j + r_k r_k' J_i - r_k r_i' J_k) d\mathbf{r}^3 + O\left(\frac{1}{r^3}\right) \\
&= \frac{\mu_0 (\mathbf{m} \times \mathbf{r})_i}{4\pi r^3} + O\left(\frac{1}{r^3}\right)
\end{aligned} \tag{3.16}$$

Thus

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 (\mathbf{m} \times \mathbf{r})}{4\pi r^3} + O\left(\frac{1}{r^3}\right) \tag{3.17}$$

Since the second term above decreases rapidly as the distance increases, it will be neglected in the following equation. Note that this $\mathbf{m}$ refers to the magnetic dipole moment of current distribution, and then r in the denominator should be the distance from the center of current distribution to the field point. To avoid confusion, the equation is modified as below.

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 \mathbf{m} \times (\mathbf{r} - \mathbf{r}')}{4\pi |\mathbf{r} - \mathbf{r}'|^3} \tag{3.18}$$

where $\mathbf{r}$ refers to the location that magnetic field is calculated and $\mathbf{r}'$ represents the location of the magnetic dipole.

Similar to the DMP model for a PM, equally distributed magnetic poles were utilized in a number of layers and a set of poles were located to retain the shape of the coil and thus is capable of accounting for the magnetic field of various shaped coils. The overall magnetic field at any place can be expressed as a summation form of contributions from each magnetic dipole. The eDMP model is illustrated as in Fig. 3.1, in which the red arrows represent the point dipoles. For the eDMP model, radius and length of the coil are used for geometric reference, providing important information about where the dipoles should be located. Unlike PMs, voice coils as EMs usually take

a hollow shape due to its multi-layer winding. In order to obtain a radius that effectively reflects the field information, a switching radius r_s where the direction of magnetic flux is changed and thus calculated according to Eq. 3.19.

Fig. 3.1 Extended DMP model

$$r_s = r \text{ at } B_z = 0 \quad (3.19)$$

Without loss of generality, the eDMP model can be applied for any shaped coil and dipole arrangement needs to be changed according to the shape of the coil. However, for simplicity, a cylindrical coil in Fig. 3.1 is illustrated since it is commonly used in many applications and simple in demonstration.

For the eDMP model, radius and length of the coil are used for geometric reference, providing important information about where the dipoles should be located. The process of developing the distributed dipole model to simulate a coil can be seen as an optimization of magnetic field minimizing the errors between the model and the physical EM. Unknown model parameters include the number of layers (q), radius of each layer (r_m), distance between positive and negative layers (l_m), quantity of dipoles in each layer (n) and dipole moment ($\mathbf{m}$).

As shown in Fig. 3.1, the center of the coil is located at the origin of a coordinate system and magnetization of coil is parallel to the Z axis. A number of layers of dipoles,

represented by red arrows, are located inside the coil to approximate the field. The vector potential and then magnetic flux density $\mathbf{B}$ at any location can be expressed as sum of magnetic dipoles below.

$$\mathbf{A}(\mathbf{r}) = \sum_{j=1}^{q \times n} \frac{\mu_0 \mathbf{m}_j \times \mathbf{P}_j}{4\pi |\mathbf{P}_j|^3} \quad (3.20)$$

$$\mathbf{B}(\mathbf{r}) = \sum_{j=1}^{q \times n} \frac{\mu_0 \left[3(\mathbf{m}_j \cdot \mathbf{e}_j) \mathbf{e}_j - \mathbf{m}_j \right]}{4\pi |\mathbf{P}_j|^3} \quad (3.21)$$

where $\mathbf{P}_j = \mathbf{r} - \mathbf{R}_j$, $\mathbf{e}_j = \mathbf{P}_j / |\mathbf{P}_j|$; $\mathbf{r}$ indicates the location that magnetic field is calculated, $\mathbf{R}_j$ indicates the location of j th dipole, which can be expressed as $\mathbf{R}_j = (r_m \cos \frac{2\pi j}{n}, r_m \sin \frac{2\pi j}{n}, \pm \frac{l_m}{2})$ using modeling parameters; $\mathbf{m}_j$ is the magnetic dipole moment; $q \times n$ is the number of the dipoles.

It is assumed that current flows in the XY plane as circle as in Fig. 3.1, and then the Z component of vector potential vanishes based on Eq. 3.4. Also, the magnetic dipole moment is assumed to be along the Z axis since the coil axis is along this direction, which indicates $m_x = 0, m_y = 0$. Then the error between the eDMP from Eq. 3.19 and analytical field can be expressed as

$$E(\mathbf{r}) = \int_0^{2l} \left(|\mathbf{B}(\mathbf{r}) - \mathbf{B}'(\mathbf{r})|^2 \right) dz \quad (3.22)$$

where $\mathbf{B}(\mathbf{r})$ is from Eq. 3.21 and $\mathbf{B}'(\mathbf{r})$ is calculated from analytical model in Eq. 3.4 since it is easily computed along the Z-axis due to symmetry of the magnetic field.

$$\mathbf{B}'(\mathbf{r}) = \nabla \times \left(\frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' \right) \quad (3.23)$$

where $\mathbf{r}$ indicates the location that vector potential is calculated, $\mathbf{r}'$ indicates the location that current density exists. Besides, magnetic flux density in Eq. 3.22 can be replaced to others such as magnetic potential similar to [9].

To improve model accuracy and optimize modeling parameter, performance and efficiency, vector potential or magnetic flux density can be used as constraints to determine the model parameters. It can be expressed as below

$$\begin{bmatrix} \chi_1^T \\ \delta_1^T \\ \vdots \\ \chi_n^T \\ \delta_n^T \end{bmatrix} [\mathbf{m}] = \begin{bmatrix} A_{x1} \\ A_{y1} \\ \vdots \\ A_{xn} \\ A_{yn} \end{bmatrix} \quad (3.24)$$

where $\chi_{ij} = \frac{\mu_0 (y_{mj} - y_{fi})}{4\pi R}$, $\delta_{ij} = \frac{\mu_0 (x_{fi} - x_{mj})}{4\pi R}$; $\chi_i^T = [\chi_{i1} \cdots \chi_{in}]$,

$\delta_i^T = [\delta_{i1} \cdots \delta_{in}]$; $[\mathbf{m}] = [m_{z1} \cdots m_{zn}]^T$; m_j indicates the j th dipole and fi indicates the i th location that magnetic field being calculated; A_{xi} and A_{yi} ($i=1,2,\dots,n$) represent analytically calculated vector potential at selected locations.

Modeling accuracy in the eDMP model would be affected by the coil geometry, accounted mainly by parameters r_m and l_m as well as the number of dipoles, n and q . The general procedures of the eDMP can be summarized as below.

Step 1) Compute analytically the magnetic field from Eq. 3.4.

Step 2) Generate an initial set of spatial grid points (q and n).

Step 3) Formulate Eq. 3.19 and Eq. 3.20 in terms of the unknowns l_m and $\mathbf{m}$,

Step 4) Find l_m and $\mathbf{m}$ by minimizing Eq. 3.21 subject to the constraint Eq. 3.23. Error computed by Eq. 3.21 is saved.

Step 5) Check the error Eq. 3.21. If the accuracy requirement is not satisfied, increase q or n , and repeat from Step 3). Once the performance is good enough, the optimal parameters q, n, l_m and $\mathbf{m}$ obtained by minimizing Eq. 3.21 using Step 4).

3.3 MODEL DEMONSTRATION WITH INDUCTION

With eDMP model presented above, magnetic field information can be obtained at random locations. One of the main concerns when utilizing time-varying electromagnetic field is induction, which has been widely utilized for various industrial applications including non-destructive testing, metal detector and so on. In order to validate the modeling capability and performance of extended DMP model, mutual inductance between two coils arbitrarily oriented as in Fig. 3.2 is studied. Definition of mutual induction is illustrated as in Eq. 3.25.

Fig. 3.2 Two coils in their own coordinate systems

$$M_{ab} = \frac{\Phi_{ab}}{I_a} = \frac{\int (\mathbf{B} \cdot \mathbf{n}_b) dS_b}{I_a} = \frac{\int (\mathbf{B} \cdot I_b \mathbf{n}_b) dS_b}{I_a I_b} \quad (3.25)$$

where M_{ab} represents the mutual inductance between coil a and coil b; Φ_{ab} indicates the magnetic flux in coil b that excited by current I_a . Here I_b represents current flowing in coil b, which can be of any value.

The magnetic moment of a planar closed loop carrying current is defined as $\mathbf{m} = I\mathbf{A}\mathbf{n}$, where A represents the cross section area of loop, and $\mathbf{n}$ represents the direction vector of loop. While moment “ $\mathbf{m}$ ” here includes not only the source strength but also the coil cross-section area information, the multiple dipoles used in DMP model are considered only as source of electromagnetic field and there is no area multiplication in its moment. Thus, with eDMP models in Table 3.1, the mutual inductance in Eq. 3.24 can be reframed by summation form as below.

Table 3.1 Parameters of eDMP models

	Dipole quantity	Locations	Dipole moment
Coil a	p	$\mathbf{R}_{ai}$	m_a
Coil b	k	$\mathbf{R}_{bj}$	m_b

$$M_{ab} = \frac{A_b \sum_{j=1}^k \mathbf{m}_{bj} \bullet \mathbf{B}_{aj}}{I_a I_b} \quad (3.26)$$

where A_b indicates the cross-section area of coil b, subscript “ j ” represents the dipole index in expression $\mathbf{m}_{bj}$. And in expression $\mathbf{B}_{aj}$, it indicates magnetic flux density is calculated at the location that j th dipole of coil b resides.

Two examples of different applications are illustrated below.

1) Oriented coils

As in Fig. 3.2, each coordinate is placed at the center of each coil; center of the general coordinate system x_1, y_1, z_1 is set to be coincident with center of coil a and the other coordinate system x_2, y_2, z_2 is attached to coil b. From Eq. 3.20, magnetic flux

density $\mathbf{B}$ at any location can be calculated with eDMP model. Euler angles are usually used to describe the relation between two coordinates systems.

$$\mathbf{B}_{aj} = \frac{\mu_0}{4\pi} \sum_{i=1}^p \frac{3(\mathbf{m}_{ai} \bullet \mathbf{e}_{aj})\mathbf{e}_{aj} - \mathbf{m}_{ai}}{|\mathbf{P}_{aj}|^3} \quad (3.27)$$

where $\mathbf{e}_{aj} = \mathbf{P}_{aj} / |\mathbf{P}_{aj}|$; $\mathbf{P}_{aj} = \mathbf{R}_{bj} - \mathbf{R}_{ai}$; $\mathbf{R}_{bj} = \mathbf{T} \times [\tilde{\mathbf{R}}_{bj}] + [x_0 \ y_0 \ z_0]^T$; $\mathbf{R}_{bj}$

indicates the dipole coordinates of coil b in general coordinate system in vector form;

$[\tilde{\mathbf{R}}_{bj}]$ indicates the dipole coordinates of coil b in coordinate system $x_2-y_2-z_2$, which can

be directly obtained from eDMP model; x_0, y_0, z_0 indicate the coordinates of the center of coordinate system $x_2-y_2-z_2$ in the general coordinate system; $\mathbf{T}$ represents the transformation matrix which depends on the rotation axes utilized, and is usually described using Euler angles.

Do notice that dipole moments of eDMP model for coil b are along the coil axis z_2 , instead of z_1 axis of general coordinate system. Thus the x and y component of $\mathbf{m}_{bj}$ as in Eq. 3.26 could be nonzero. The transformed dipole moment can be described as below.

$$\mathbf{m}_{bj} = \mathbf{T} \times [0 \ 0 \ m_b]^T \quad (3.28)$$

2) Bended tubes

Figure 3.3 shows the bending scenario of a tube segment, around which two single layer coils were wound.

Fig. 3.3 Bending tubes

For formulation simplicity, coordinate system was always set to be on the central layer of coil a (the one with applying current) and the coil axis was along the z direction. In order to characterize the bending degree, R represents radius of the inner arc and κ represents the curvature. Unlike the oriented coil case, in which Euler angles are usually used when transformation matrix are needed to describe rotation, curvature is a better option to account for the orientation change of bending tube, since the arc distance between two coils remains the same, instead of direct distance. As in Fig. 3.3, the coil winding might tilted along with the bending tube, thus the shape of coil slightly changed and each layer of winding had different orientation. Although the coil deformation could be small, it has a major influence to the magnetic field pattern. In order to account this influence, the dipole direction was modified according to the bending situation of tube. $\mathbf{m}_{bj}$ and $\mathbf{m}_{ai}$ as in Eq. 3.26 are decided by both eDMP models and the tube bending

$$[\mathbf{m}_a] = \mathbf{T}_a \times [\mathbf{m}_a]_o, \quad [\mathbf{m}_b] = \mathbf{T}_b \times [\mathbf{m}_b]_o \quad (3.29)$$

where $\mathbf{T}_a = \begin{bmatrix} \cos(l_{ma}\kappa) & 0 & \sin(l_{ma}\kappa) \\ 0 & 1 & 0 \\ -\sin(l_{ma}\kappa) & 0 & \cos(l_{ma}\kappa) \end{bmatrix}$, representing the transformation matrix for

DMP model a;

$$\mathbf{T}_b = \begin{bmatrix} \cos(l\kappa \mp l_{mb}\kappa) & 0 & \sin(l\kappa \mp l_{mb}\kappa) \\ 0 & 1 & 0 \\ -\sin(l\kappa \mp l_{mb}\kappa) & 0 & \cos(l\kappa \mp l_{mb}\kappa) \end{bmatrix}, \text{ representing the transformation}$$

matrix for DMP model b;

$[m_a]_o$ and $[m_b]_o$ represent the original dipole coordinates that directly come from DMP models. l indicates the arc length between two coils, κ represents the curvature, l_m indicates the distance between positive and negative layers in DMP model.

For expression simplicity in the following explanation, we reform the equation Eq. 3.27 in the matrix form as below.

$$[B_{aj}] = \frac{\mu_0}{4\pi} \sum \mathbf{F} \times [m_{ai}] \quad (3.30)$$

where $[B_{aj}]$ represents the magnetic field expressed as three by one vector form;

$[m_{ai}]$ represents the dipole moment of i th dipole of coil a, expressed as three by one vector form;

$$\mathbf{F} = \begin{bmatrix} 3Px_j^2 - 1 & 3Py_jPx_j & 3Pz_jPx_j \\ 3Px_jPy_j & 3Py_j^2 - 1 & 3Pz_jPy_j \\ 3Px_jPz_j & 3Py_jPz_j & 3Pz_j^2 - 1 \end{bmatrix}, \text{ describing the relation between magnetic}$$

field and dipole moment in matrix form.

When the tube bended and location of coil b changed, mutual inductance calculation can be conducted as below.

$$M_{ab} = \frac{A_b}{I_a I_b} \sum_{j=1}^k \sum_{i=1}^p [m_{bj}]_o^T \times \mathbf{T}_b^T \mathbf{F} \mathbf{T}_a \times [m_{ai}]_o \quad (3.31)$$

where $[m_{bj}]_o$ indicates the original coordinates of j th dipole of coil b;

$[m_{ai}]_o$ indicates the original coordinates of i th dipole of coil a;

$\mathbf{T}_a$ and $\mathbf{T}_b$ represent the transformation matrix as in Eq. 3.29 ;

$\mathbf{F}$ describes the relation between magnetic field and dipole moment as in Eq. 3.30, but now a function of curvature .

$\begin{bmatrix} m_{bj} \end{bmatrix}_o$ and $\begin{bmatrix} m_{ai} \end{bmatrix}_o$ are only decided by eDMP models, thus independent of tube bending. We regard the mutual inductance as a function of bending, which is described by the middle three terms. Because the axes of two coils are initially along the z axis, there is only one nonzero term in the dipole moment matrixes. Thus Eq. 3.31 can be further simplified as below.

$$M_{ab} = \frac{A_b}{I_a I_b} \sum_{j=1}^k \sum_{i=1}^p m_{bj} S_{ij} m_{ai} \quad (3.32)$$

where S_{ij} can be calculated analytically as the last term of $\mathbf{T}_b^T \mathbf{F} \mathbf{T}_a$.

3.4 ILLUSTRATIVE APPLICATIONS AND DISCUSSIONS

In order to validate the proposed eDMP model for time varying filed, the magnetic field is calculated using both eDMP model and single dipole model, and then compared with the analytical solution. Two examples are shown for demonstration purpose: coils with various aspect ratios and a coil with customized shape. The modeling accuracy and efficiency will be discussed respectively.

3.4.1. EXAMPLE 1: CYLINDRICAL COILS WITH VARIOUS ASPECT RATIOS

A number of single layer coils with different aspect ratio ($\gamma=2r/l$) are compared to validate the model. The Single Dipole model is well known and commonly used for many applications due to simplicity. In order to examine how the coil geometry affects the modeling accuracy, the coil length is fixed at 7mm, which corresponds to 20 turns winding with wire of 0.3mm diameter. Although theoretically the coil length would be

6mm as the multiplication of turns by the wire diameter, actual coils are made for experiments and it is found that the coil length is slightly bigger due to manufacture error. Coils with different radius, 0.5cm, 1cm, and 2cm, are compared. The eDMP model parameter and simulation results are shown as below.

Table 3.2 eDMP model parameters

Coil geometry	eDMP model
$r=0.5cm$ $l=0.7cm$ 20 turns	$q=2, n=6$ $l_m = 0.3cm$ $r_m = 0.25cm$ $ \mathbf{m} = 2.11e-4 Am^2$
$r=1cm$ $l=0.7cm$ 20 turns	$q=1, n=6$ $l_m = 0$ $r_m = 0.61cm$ $ \mathbf{m} = 0.0018 Am^2$
$r=2cm$ $l=0.7cm$ 20 turns	$q=1, n=6$ $l_m = 0$ $r_m = 1.46cm$ $ \mathbf{m} = 0.0048 Am^2$

By defaults, one layer of dipoles resides at the positive side as in coordinate system of Fig. 3.1, and the other layer resides at the negative side, each layer consisting of 6 dipoles. And $l_m = 0$ in Table 3.2 indicates only one layer of dipoles resides at the central surface of coil and the layer consists of 6 dipoles. This difference is automatically generated after the optimization operation and apparently depends on the aspect ratio of EMs.

(a) $\gamma = 1.4$

(b) $\gamma = 2.8$

(c) $\gamma = 5.6$

Fig. 3.4 Simulation results of different aspect ratio

For coil (a), the maximum modeling error using Single Dipole (SD) model is 12% and that of eDMP is 5.8%, which indicates improved modeling accuracy. As the aspect ratio increasing as 2.8 and 5.6 in Fig. 3.4 (b) and (c) respectively, the modeling error using SD model is getting larger and cannot simulate the magnetic field of flat coils. On the contrary, the eDMP model provides a modeling discrepancy of 9.34% and

13.41% for the coil (b) and (c) respectively due to its capability of accounting for the coil shape. The actual magnetic field is also measured and shown in Fig. 3.4 to validate the analytical model and eDMP model, similar to each other. The measured magnetic field can also be used as the reference data for eDMP model in Eq. 3.21 to improve accuracy if necessary.

3.4.2. EXAMPLE 2: COIL OF CUSTOMIZED SHAPE

The eDMP model can be applied for coils of any shape and the dipole arrangement needs to be changed accordingly. Figure 3.5 shows a coil of oblique cylindrical shape, the inclined angle of which is represented as β . The central axis of the coil is along the z axis, but its polarization as an electromagnet is shown using red arrow and has an inclination angle with the z axis. Its cross-section surface is also shown as a projection into the inclined plate and unlike the normal cylinder, the cross-section surface is an ellipse with a major axis r_1 and a minor axis r_2 . In order to account for this specific coil geometry, the eDMP model is arranged that the dipole moments are along the direction of coil polarization and the dipole locations correspond to the coil geometry.

Fig. 3.5 An oblique cylindrical coil

The coil geometry is decided by the coil length l , inclination angle β , major axis r_1 and minor axis r_2 of the cross-section area. And the eDMP model parameters include the number of layers (q), major and minor radius ($r_{m,1}$, $r_{m,2}$) of each layer, distance between positive and negative layers (l_m), quantity of dipoles in each layer (n) and dipole moment ($\mathbf{m}$). With these parameters, the modeling is conducted in the same procedures described in chapter 3.2. By minimizing the discrepancy between the simulation field and the reference data, the model parameters can be calculated. The geometry parameters of a demonstration example are shown in Table 3.3 and the modeling results are shown in Fig. 3.6.

Table 3.3 Geometry and model parameters of an oblique cylindrical coil

Coil geometry	eDMP model
$r_1 = 5mm, r_2 = 2.5mm$	$q = 4, n = 6$
$\beta = 60^\circ$	$r_{m,1} = 2.5mm, r_{m,2} = 1.25mm$
$l = 8mm$	$l_m = [2.6mm \quad 4.2mm]$
	$m = [0.3141 \times 10^{-4} \quad 0.6094 \times 10^{-4}] Am^2$

Fig. 3.6 Simulation results of an oblique cylindrical coil

The simulation results shown above were calculated at a radical distance of 2.6mm and the z coordinates ranged from zero to twice the coil length. Numerical

solutions from COMSOL are also included for comparison purpose. An intel quad-core desktop computer with 16G RAM and 3GHz CPU is used for computation. The minor fluctuations in the COMSOL results are due to the limited computation power and a finer mesh of the problem domain cannot be solved. It is seen that the general trend and magnitude are consistent between the analytical solution, numerical results from COMSOL and simulated results with the eDMP model. The discrepancy between the analytical solutions and eDMP modeling results are bigger when the $2z/l$ is smaller than one. And when comparing the numerical results from COMSOL and simulation results based on eDMP model, it is apparent that the eDMP model provides much smoother results and lesser computation efforts.

IV. MAGNETIC FIELD BASED MOTION SENSOR

4.1 OVERVIEW

DMP model has been extended for time-varying electromagnetic field in last chapter. With the scalar-potential-governed DMP model and the vector-potential-governed eDMP model, most of electromagnetic field utilizing system can be therefore modeled. In order to explore the capability of proposed modeling approach for sensing application, two examples are studied as below: DMP model for PM based motion sensing, and eDMP model for induction based orientation sensing.

4.2 DMP MODEL FOR PM BASED ORIENTATION SENSOR

Generally, two different types of magnetic sources have been used in a measurement system to generate the reference signal of magnetic field [105, 106], which is usually a function of orientation; namely, static magnetic field generated by a permanent magnet (PM), and periodic excitation from an electromagnet (EM) controlled by alternative currents with varying frequencies and phases. For both sources being used, magnetic field variations caused by motions of the object need to be analyzed and solved inversely to obtain the location and orientation. When periodic excitation sources are used, phase and magnitude differences between the excitation signal and detected signal are mainly concerned. And the fundamental measurement principle is the eddy-current effect introduced by the conductive property of the object, which is usually ill-posed and nonlinear, relatively difficult to be solved inversely. On the contrary, a PM as the field source reduces computational burdens and provides relatively strong magnetic fields. Thus, the PM becomes more attractive as excitation sources in simple motion tracking applications [28, 107, 108].

Various methods have been proposed to solve the inverse problem [105], but it is still a challenging issue because specific analytical formula and constraints have to be

considered for each problem. Single dipole [109] has been used for analytical calculation to model the magnetic field since it is mathematically simple and able to provide a closed-form solution in the three dimensional space. When the magnet is attached to the rotating object and rotates with respect to a point, the magnetic field is varying nonlinearly, but it is periodic and can be illustrated with certain pattern. The shape of the rotating magnetic field modeled by a single dipole located at the origin has known as an ellipsoid in three dimensional space [109]. In other words, the shape of the measured magnetic field using a three axes magnetic sensor becomes the ellipsoid. Similarly, the shape using a sensor with single and two axis measurement can be deduced to be a line and an ellipse respectively. However, the method in [109] is only valid for a needle-shape of PMs and a circular hollow coil since the single dipole model can be only validated for those shapes, which generally makes the method limited. To eliminate these drawbacks, the DMP method is here utilized to accurately estimate the excited magnetic field in three dimensional spaces. The method can effectively account for any shaped PM or EM located at arbitrary position yet in a circular motion.

4.2.1 TRACKING STRATEGY

Figure 4.1(a) shows a schematic design of an orientation measurement system consisting of a cylindrical PM located at P_m and a three-axis magnetic sensor located at P.

Fig. 4.1. Excited magnetic field of a rotating magnet

The cylindrical PM is commonly used for designing an actuator and a measurement system due to low price and easy installation. For this reason, only cylindrical PM is considered here, but other shapes can be extended without loss of generality. The PM is rotating with respect to the origin with the distance offset h . The magnetic field of the PM at the location $\mathbf{P}$ is varying upon geometric parameters (shape and aspect ratio, etc) of the PM and its magnetization etc. Dominant parameters affecting the field variation and the sensing performance can be selected as follows:

- 1) shape and strength of a PM;
- 2) position/distance (h) of the PM with respect to the origin;
- 3) orientation $\mathbf{q}$ of the permanent magnet;
- 4) position $\mathbf{P}$ of the magnetic sensor;
- 5) orientation $\mathbf{q}_s$ of the sensor;

Generally, the orientation of the PM in Fig. 4.1(a) is described using Euler angles α , β and γ , but due to the symmetry of the magnetization along the axis of the cylindrical

PM, only two rotations can be measured (the rotation angle γ with respect to the z axis is immeasurable). First, the XYZ coordinate rotates around the X axis, denoted here as angle α ; then β around y' axis in the $Xy'z'$. After these two rotations, the xyz coordinate is considered to be the frame fixed at the PM, the orientation of which can be described as $\mathbf{q} = [\alpha \ \beta]^T$.

Due to the circular motion of the magnet, the shape of the field can be similarly assumed as an ellipsoid-like shape shown in Fig. 4.1(b) and further characterized by three primary axes $\mathbf{B}_{\max}$, $\mathbf{B}_{\min}$ and $\mathbf{B}_{\text{int}}$ simply denoted here as $\mathbf{B}_i$ ($i=1,2$, and 3). The ellipsoid of the magnetic field remains same as long as the sensor maintains similar distance and pointing to the PM regardless of a sensor location but the orientation of the ellipsoid will rotate along with the sensor location. Based on DMP model, the magnitude of the field at any location can be analytically formulated as:

$$\bar{\mathbf{B}}(x, y, z) = \mathbf{B}\mathbf{B}^T \quad (4.1)$$

Then the ellipsoid can be approximately formulated with the longest axis as the strongest magnetic field $\mathbf{B}_{\max}$ and the shortest axis as the weakest magnetic field $\mathbf{B}_{\min}$ as below respectively:

$$\mathbf{B}_{\max} = \mathbf{u}_{\max}(\bar{\mathbf{B}})\sqrt{\lambda_{\max}(\bar{\mathbf{B}})} \quad (4.2)$$

$$\mathbf{B}_{\min} = \mathbf{u}_{\min}(\bar{\mathbf{B}})\sqrt{\lambda_{\min}(\bar{\mathbf{B}})} \quad (4.3)$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues of $\bar{\mathbf{B}}$. Similarly, $\mathbf{u}_{\max}$ and $\mathbf{u}_{\min}$ are the unit eigenvectors along the longest and the shortest axes, respectively. In addition, the orientation of the intermediate axis $\mathbf{u}_{\text{mid}}$ can be computed from $\mathbf{u}_{\max}$ and $\mathbf{u}_{\min}$ due to the feature of an ellipsoid three primary axes of which should be orthogonal to each other as follows:

$$\mathbf{u}_{\max} \bullet \mathbf{u}_{\text{int}} = \mathbf{u}_{\min} \bullet \mathbf{u}_{\text{int}} = 0 \quad (4.4)$$

Once the direction of the intermediate axis is known, the magnitude of the corresponding vector can be also calculated from the DMP method. An approximated ellipsoid is further configured to account for the excited magnetic field.

$$\mathbf{B}_r = \mathbf{B}_c(h) + \Gamma(\mathbf{q}_s)\mathbf{B}_q(\mathbf{q}) \quad (4.5)$$

$$B_{q1} = |\mathbf{B}_1| \sin(\alpha + \tilde{\alpha}) \cos(\beta + \tilde{\beta}) \quad (4.6)$$

$$B_{q2} = |\mathbf{B}_2| \cos(\alpha + \tilde{\alpha}) \cos(\beta + \tilde{\beta}) \quad (4.7)$$

$$B_{q3} = |\mathbf{B}_3| \sin(\beta + \tilde{\beta}) \quad (4.8)$$

where $\mathbf{B}_r (\in R^{3 \times 1})$ is the magnetic field expressed in the inertial XYZ coordinate frame;

$\mathbf{B}_c$ is the center of the ellipsoid, which is mainly a function of the distance offset h due to the asymmetric field; $\Gamma(\mathbf{q}_s)$ is the coordinate transformation matrix from the sensor frame to the inertial frame; $\mathbf{B}_q(\mathbf{q})$ is the magnetic field expressed in the sensor frame;

$|\mathbf{B}_i|$ ($i=1,2,3$) indicates $|\mathbf{B}_{\max}|$, $|\mathbf{B}_{\text{int}}|$ and $|\mathbf{B}_{\min}|$; $[\tilde{\alpha} \ \tilde{\beta}]$ are orientation of the ellipsoid with respect to the sensor frame which can be found from the eigenvectors in Eq. 4.2-4.3. Once the $\mathbf{B}$ -field is obtained, the position and orientation of the sensor can be further optimized to maximize sensing performance. The sensing performance can be defined as *traceable motion range* as below.

$$\eta = f(\mathbf{P}) = \frac{A_e}{A} \quad (4.9)$$

where $A = \int \mathbf{B}(\mathbf{P}, \mathbf{q}) d\mathbf{q}$; $A_e = \int \mathbf{B}_e(\mathbf{P}, \mathbf{q}) d\mathbf{q}_e$; A is the surface of magnetic field $\mathbf{B}$ on an entire ellipsoid in Eq. 4.5-4.8; A_e is an effective area of the magnetic field $\mathbf{B}_e$ the sensor can measure in rotation range $\mathbf{q}_e$ of the PM.

Measurable magnetic field can be illustrated using a three axes magnetic sensor shown in Fig. 4.1(c). Assuming the magnetic sensor is isotropic for all axes, contour of the measurable field then becomes a hollow sphere, the outer radius of which is the maximum range of measurement $\tilde{\mathbf{B}}_{\max}$ and the inner radius is $\tilde{\mathbf{B}}_{\min}$. Then, the hollow sphere (referred here as measurable field) can be superimposed into the ellipsoid (excited magnetic field generated by a rotating permanent magnet) to visually identify an effective measurable range of the given design. In the region of $\mathbf{B} < \tilde{\mathbf{B}}_{\min}$, the output signal of the sensor becomes low resolution; and vice versa, the sensor is saturated in the region of $\mathbf{B} > \tilde{\mathbf{B}}_{\max}$. Once the magnetic field has been analyzed and compared to the capability of the sensor, the sensitivity as below, which is the gradient of the magnetic field with respect to the rotation angles, is then computed for the accuracy of the performance.

$$\mathbf{S} = \Gamma(\mathbf{q}_s)^{-1} \left[\partial \mathbf{B}_r(\mathbf{q}) / \partial \mathbf{q} \right]^T \quad (4.10)$$

Given the desired resolution of a tracking system $\hat{\mathbf{q}}_{\min} = [\hat{\alpha}_{\min} \quad \hat{\beta}_{\min}]^T$, some range of motion may not be detectable since the magnetic field variation with respect to $\hat{\mathbf{q}}_{\min}$ is smaller than the physical resolution $\tilde{\mathbf{B}}_{\min}$ of the magnetic sensor. It is only valid and detectable when the magnetic field variation is bigger than the minimum resolution $\tilde{\mathbf{B}}_{\min}$, which indicates the sensitivity should be larger than the minimum sensitivity $S_r (= \tilde{B}_{\min} / \hat{q}_{\min})$ for the given sensor. Thus, in order to optimize the sensing performance, the effective magnetic field $\mathbf{B}_e$ (the sensor can measure) should be subjected to both measurability and sensitivity requirements given below respectively. Equation 4.11 indicates the effective magnetic field $\mathbf{B}_e$ must be within the measurable range of the magnetic sensor in use and Eq. 4.12 indicates the magnetic

field variation with respect to the desired angle resolution must be larger than the sensitivity of the magnetic sensor in use. Both conditions must be simultaneously satisfied to measure the orientation of the PM as follows:

$$\mathbf{B}_e \subset \left\{ \mathbf{B} \mid (\mathbf{B}_{\min} > \tilde{B}_{\min}) \cap (\mathbf{B}_{\max} < \tilde{B}_{\max}) \right\} \quad (4.11)$$

$$\mathbf{B}_e \subset \left\{ \mathbf{B} \mid S > S_r \right\} \quad (4.12)$$

An optimized design for the given magnetic sensor can be obtained from Eq. 4.11 and 4.12. However, even after performing the optimization and maximizing the traceable motion range, the design may not meet the desired tracking coverage or accuracy requirement. Then, the method can be extended adding more sensors in the same procedure until a set of sensors satisfies the desired performance. General procedures to optimize the sensor design can be summarized as follows:

- Given a PM as the magnetic source, excited magnetic field $\mathbf{B}$ can be calculated from DMP model.
- Primary axes of an ellipsoid can be characterized based on Eq. 4.1-4.4 to approximately configure the magnetic field in Eq. 4.5-4.8. The compact formulation of the ellipsoid can be used to account for the performance of the sensing system.
- With measurability, resolution and sensitivity of the sensor from Eq. 4.10, constrains in Eq. 4.11 and 4.12 are applied to the ellipsoid-like magnetic field. The traceable motion range in Eq. 4.9 is computed.
- The optimal location for the magnetic sensor can be found, and certain iterations of calculation might be needed for locating more sensors. The design of the sensing system can be further optimized accordingly.

4.2.2 ILLUSTRATIVE EXAMPLES AND SIMULATION

Two illustrative examples are shown here to provide a better understanding of the method. First, a cylindrical PM is rotating at the origin; the other is that the PM is rotating with respect to the origin with certain distance offset. Both magnetic fields generated by the PM and measured by a sensor are visually compared to find an optimized location of the sensor to estimate the orientation of the PM.

1) Cylindrical PM at the Origin

Figure 4.2 shows that the cylindrical PM is placed at the origin of the reference frame and rotating with respect to the origin (coincident to the center of rotation $\mathbf{P}_m=(0, 0, 0)$). A three-axes magnetic sensor are placed at $\mathbf{P}=(5, 0, 0)$ along with the $x_s y_s z_s$ coordinate parallel to the reference coordinate so that its axis points to the origin. The PM is modeled using the DMP method and parameters of the magnetic sensor are detailed in Table 4.1. The magnetic field of the rotating PM measured by sensor is analytically calculated from DMP model and approximated by Eq. 4.1-4.8 respectively. The results of each ellipsoid are shown in Fig. 4.2(a) and 4.2(b), the coordinate of which indicating the $\mathbf{B}$ -fields and their representative parameters detailed in Table 4.2.

Table 4.1 Parameters of DMP model for PM and magnetic sensor

PM (inch, Tesla)	Dia=0.5; Length=0.5; Br=1.35 T
DMP	n=6; k=1 ; $m_i=[-0.2204; 0.6115] * 1.0e-004 \text{ Am}^2$
3-Axis magnetic sensor (B_x, B_y, B_z)	
Measurable range	-7.3mT~7.3mT
Resolution	$\pm 10\mu\text{T}$

Table 4.2 Primary axes in XYZ coordinate

Primary axes (T)	Corresponding rotation angle set
$\mathbf{B}_{\max} = (3.1240e-4, 0, 0)$	$\alpha=0, \beta=90$
$\mathbf{B}_{\min} = (0, 1.5639e-4, 0)$	$\alpha=90, \beta=180$
$\mathbf{B}_{\text{int}} = (0, 0, 1.5639e-4)$	$\alpha=180, \beta=360$

(a) Magnetic field by DMP method

(b) Approximated magnetic Field

Fig. 4.2. Magnetic fields measured by a sensor

The magnetic field is symmetric since the PM is at the origin ($h=0$) and thus the center $\mathbf{B}_c$ of the ellipsoid is at the origin. The sensor is here located on the X axis, which indicates the X component of the magnetic field is not affected when the PM rotates around the X axis. If the sensor keeps pointing to the PM at the origin while the distance (between the sensor and the PM) varies, the orientation of the ellipsoid will maintain same but the size of the ellipsoid will varies as the magnetic field. The ellipsoid $\mathbf{B}_r$ and the sensitivity in the XYZ frame can be obtained as follows.

$$\mathbf{B}_r = \begin{bmatrix} B_{\max} \sin \beta \\ B_{\min} \sin \alpha \cos \beta \\ B_{\text{int}} \cos \alpha \cos \beta \end{bmatrix} \quad (4.13)$$

$$\frac{\partial \mathbf{B}_r}{\partial \mathbf{q}} = \begin{bmatrix} 0 & B_{\max} \cos \beta \\ -B_{\min} \cos \alpha \cos \beta & B_{\min} \sin \alpha \sin \beta \\ B_{\text{int}} \sin \alpha \cos \beta & B_{\text{int}} \cos \alpha \sin \beta \end{bmatrix} \quad (4.14)$$

Figure 4.3 shows the sum of the squared error between the excited magnetic field in Fig. 4.2(a) and the approximated ellipsoid in Fig. 4.2(b). Each primary axis of the $\mathbf{B}$ -field approximately coincides with the X, Y and Z axis respectively due to the motion

symmetry. Thus, the **B**-field can be approximated in the closed form of an ellipsoid in Eq. 4.13-4.14. The maximum discrepancy is 6×10^{-14} T.

Fig. 4.3. Sum of squared error

The error is caused mainly due to the shape of PM. The aspect ratio of the PM is one and cannot be accurately modeled as a single dipole model. However, the comparison still shows an excellent agreement indicating the rotating magnetic field forms the ellipsoid.

In the example, only distance between the cylindrical magnet and sensor $|\mathbf{P}|$ needs to be optimized due to the symmetry of the system including the PM rotating at the origin. Even when the sensor is placed at another location, as long as the distance between the magnet and sensor does not change, the shape of the ellipsoid will remain same. The orientation of the ellipsoid can be simply calculated by coordinate transformation. Thus, the measurability in Eq. 4.11 is considered to account for magnetic strength within the sensor capability, which can be directly related to the distance range where magnetic fields are measurable. Then, the sensitivity in Eq. 4.12 with given sensor specifications will restrain the field variation along with the rate of change in motion and completes the measurable range on the ellipsoid.

The minimum and maximum field measured by the sensor located at $\mathbf{P}=(5, 0, 0)$ (given in Table 4.2) can be compared to the sensor capacity and both belong to the measurable range from -7.3mT to 7.3mT, which indicates the sensor already satisfied measurability in Eq. 4.11. Figure 4.4 shows the corresponding traceable rotation angles $\mathbf{q}_e$ in a unit sphere when the sensitivity requirement S_r in Eq. 4.12 is set to be $1e-4$ (T/rad). The *traceable motion range* becomes $\eta=30\%$. Only 30% of motion, the color mapped area, can be traceable and the other, non-covered area, indicates the sensor cannot measure the orientation of the PM. To maximize the sensible rotation range, measurability and sensitivity requirements in Eq. 4.11 and 4.12 are analyzed separately.

Fig. 4.4. Sensible rotation range

The design of the sensor can be here optimized by the genetic algorithm firstly defining feasible solution domain by comparing the ellipsoid of the magnetic field and the sphere given by sensor capacity. Then, the design variables (location and orientation of the sensor) are repeatedly evaluated and compared in the domain to maximize the measurement performance. Figure 4.5 shows that measurability restrains on the sensor capacities. The magnetic field in the form of ellipsoid is overlapped with two spheres referring to sensor capabilities. Each sphere indicates the homogenous magnetic field measured by the sensor; namely, the outer sphere represents the maximum magnetic

field the sensor can measure and the inner represents the minimum respectively. Thus, the sensor is only able to measure the region between two spheres. The sensor is saturated in the region of the ellipsoid outside the sphere. To minimize such undesired regions, two different locations of the sensor $\mathbf{P} = (2, 0, 0)$ and $(12, 0, 0)$ are considered as feasible solution domain as shown in Fig. 4.6 respectively; Figure 4.6(a) shows the entire ellipsoid belongs to the outer sphere. The ellipsoid has contact with the sphere and meets the upper range 7.3mT of the measurability as the sensor moves closer to the PM at $\mathbf{P} = (2, 0, 0)$; and Fig. 4.6(b) the inner sphere belong to the ellipsoid, containing the sphere referring to the given sensor resolution $10\mu\text{T}$ as in Table 4.1, when $\mathbf{P} = (12, 0, 0)$. The results show the magnetic field is measurable in the distance range (2~12) by the given sensor. In addition, the sensitivity in Eq. 4.14 proportionally increases with the magnetic field strength and thus, the measurable range is maximized when the sensor is placed at $\mathbf{P} = (2, 0, 0)$ as shown in Fig. 4.7(a). The measurable orientation range becomes an entire sphere except the rotation along the x-axis and *traceable motion range* $\eta = 96.7\%$.

Fig. 4.5. Superimposition of spheres and ellipsoid

Fig. 4.6. Measurability of sensor

The orientation of the sensor $\mathbf{q}_s$ affects the sensor performance in particular for the sensitivity in Eq. 4.10. At the given distance $|\mathbf{P}|=5$, the sensitivity in Eq. 4.12 along with Eq. 4.10 is applied to different sensor orientation to maximize Eq. 4.9. The optimized orientation of the sensor is found to be $\mathbf{q}_s = \pi[1/3 \ 1/3]$, resulting in the performance $\eta=65.33\%$ shown in Fig. 4.7(b). Compared to the result of $\eta=30\%$ pointing to the PM (shown in Fig. 4.4), the performance is dramatically improved.

Fig. 4.7. Sensible range

2) Effect of Distance Offset on the Performance ($h \neq 0$)

The cylindrical PM is placed at distance $h=2$ inch from the origin and rotating with respect to the origin while the parameters of the PM and sensor are the same in Table 4.1.

The magnetic field at $\mathbf{P} (5, 0, 0)$ is computed by the DMP method and further approximated to an ellipsoid as shown in Fig. 4.8(a) and 4.8(b) respectively. It can be recognized that the field from the DMP method is not an exact ellipsoid but can be approximated by an ellipsoid. As shown in Fig. 4.8(a), the ellipsoid is still axis-aligned since the sensor orientation and location remain the same as the previous example in Fig. 4.2 but its center is not at the origin, which is caused by the asymmetry of the magnetic field since PM is not at the origin. The center of the ellipsoid can be thus calculated from the DMP method as below.

$$\mathbf{B}_{ci} = \text{mean} \{ \max(\mathbf{B}) + \min(\mathbf{B}) \} \quad (4.15)$$

Fig. 4.8. Magnetic fields measured by a sensor

Figure 4.9 shows the error between the magnetic field of the DMP method and approximated field by the ellipsoid. Compared to Fig. 4.3, error in this design greatly increases due to the large variation of magnetic fields during rotation. The maximum error is below $1.5\mu\text{T}$ at $\beta = \pi / 6$ and $5\pi / 6$.

Fig. 4.9. Sum of squared error

Fig. 4.10. Superimposition of hollow sphere and ellipsoid magnetic field

As explained previously, Fig. 4.10 shows the measurability in Eq. 4.11 by superimposing the ellipsoid into the hollow sphere which represents the measurable range of the sensor. The primary axes of the ellipsoid rotate with respect to the center $\mathbf{B}_c$ while the center of the hollow sphere is still at the origin. The maximum and minimum of the magnetic field become $(B_{\max} + B_{cx})$ and $(B_{\max} - B_{cx})$ respectively. Thus, the magnetic field is measurable if Eq. 4.16 is satisfied.

$$\mathbf{B}_e \subset \left\{ \mathbf{B} \mid ((B_{\max} - B_{cx}) > \tilde{B}_{\min}) \cap ((B_{cx} + B_{\max}) < \tilde{B}_{\max}) \right\} \quad (4.16)$$

The sensitivity of the magnetic field with respect to rotation is the same in Eq. 4.14 and $S_r = 1\text{e-}4\text{T/rad}$. Then, the coverage of sensible rotation range $\mathbf{q}_c$ is shown in Fig. 4.11(a), resulting in the performance $\eta=84.8\%$. Figure 4.11(b) shows the maximized measurable range by shifting the center of the ellipsoid. The magnetic field strength increases with distance offset h and the sensitivity can be maximized at $h=3$, the upper level that meets the measurability requirement. Compared to the range in Fig. 4.11(a), it detects larger range, $\eta=96.0\%$.

Fig. 4.11. Coverage of sensible rotation range

Table 4.3 Effect of distance offset on the shape of the ellipsoid

h (inch)	$\mathbf{B}_c$ (T)	$B_{\max}$ (T)	$B_{\min}$ (T)	$B_{\max} / B_{\min}$
0	[0; 0; 0]	3.1e-4	1.55e-4	2
1	[2.15e-4; 0; 0]	3.95e-4	2.3e-4	1.70
2	[6.57e-4; 0; 0]	7.9e-4	5.51e-4	1.43
3	[0.0023; 0; 0]	0.0025	0.0019	1.3
4	[0.0187; 0; 0]	0.0197	0.0159	1.24

Table 4.3 shows the magnetic field to investigate the effect of the distance offset h on the center shift and shape of ellipsoid when $\mathbf{P}=(5 \ 0 \ 0)$. Since magnetic sensor is placed on the X axis, the center is also moving along the X axis. In addition, the center of the ellipsoid is moving far from the origin as the distance increases and shape becomes sphere so that the sensor cannot measure the field beyond the certain distance offset h . For example, at $h=4$, the maximum magnitude of the field become 0.0384 T ($=B_{\max} + B_{cx}$

0.0187+0.0197 from Table 4.3), which exceeds the measurable range (-7.3mT~7.3mT) and the magnetic sensor becomes saturated. Similar, the effect of distance between origin and sensor can also be calculated when the distance h is fixed.

4.2.3 EXPERIMENTAL RESULTS

The prior illustrative example is experimentally demonstrated here to validate the design of the sensing system. As shown in Fig. 4.12, the cylindrical PM and a three-axis magnetic sensor, MFS-3A, are placed at the origin and at 5 inch away from the origin respectively. Three-axis magnetic sensor is mounted on the x axis of the inertial frame parallel to the sensor frame. The axis of cylindrical PM is along the z axis therefore it is shown as a circle. In order to generate the full motion range, rotation around the y axis is conducted using a DC motor, the constant speed of which is 4.01rad/s, and rotation around the x axis is realized by rotating the magnetic sensor around the x axis. The full motion range is shown as a sphere.

Fig. 4.12. Diagram of experiment setup

The experimental parameters including the PM and the sensor as well as the DMP model for the magnetic field are detailed in Table 4.4. The sensitivity of each axis of the

sensor is calibrated prior to the experiment. The conversion constants of each axis from an electric voltage output to a magnetic field can be computed by the least square method as a linear regression in Table 4.4. With a sampling rate of 4010 Hz, the magnetic fields of three orthogonal axes between the experimental data and the DMP model are compared in Fig. 4.13.

Table 4.4 Parameters of experimental setup and DMP model

PM (inch, Tesla) DMP	Dia=0.01 m; Length=0.01 m; Br=1.46 Tesla n=6; k=1 ; $m_i=[-0.1477;0.4100] * 1.0e-004 \text{ Am}^2$
Specification of MFS-3A	
Manufacturer	Ametes
Field range	$\pm 7.3 \text{ mT}$
Resolution	$\pm 10 \mu\text{T}$
Conversion constant of three-axes output(volt/T)	
x/y/z axis	253 / 283 / 270
Data acquisition system	
NI USB-6251	A/D 16 bit; Maximum sampling rate 1.25MS/s;

Fig. 4.13. Magnetic fields and sensor calibration

Once the sensor has been calibrated, it can be shown the magnetic field at the sensor location shapes an ellipsoid as shown in Fig. 4.14. First, three angles ($0, \pi/4, \pi/2$) of the PM along the x axis are inclined, and then rotates around the corresponding y axis for a whole revolution.

Fig. 4.14. Analytical magnetic field at $\alpha=0,45,90$

The magnetic fields measured along three orthogonal axes are compared to those of the DMP model in Fig. 4.15. The maximum error among three axes is 6% at $\alpha=\pi/4$. The errors and offsets in particular for $\alpha=\pi/4$ are expected mainly due to the sensor alignment and the PM location error. Similar to the ellipsoid of the magnetic fields, Fig. 4.16 shows the gradient of the magnetic field with respect to β to compute the sensitivity and traceable motion range. The largest error is occurred at $\alpha=\pi/4$ and $\pi/2$ and the misalignment is expected to cause the error since both α and β are involved.

Based on the **B**-field in Fig. 4.15 and its gradient in Fig. 4.16, the traceable motion range in Eq. 4.9 can be computed with the desired sensitivity 1.0×10^{-5} T/rad. Both analytical and experimental traceable motion ranges at $\alpha=0, \pi/4$ and $\pi/2$ are compared in Fig. 4.17. The error is mainly occurred at the boundaries in traceable motion range due to deviation of the magnetic fields from experimental measurements and analytical solutions. The errors of each angle are 1.65%, 8.8% and 4.19% respectively. The maximum error is 8.8% at $\alpha=\pi/4$ since the error in the PM location causes the non-coincident rotation centers and then asynchronous variations of all three components of magnetic field.

Fig. 4.15. Comparisons of magnetic field

Fig. 4.16. Comparisons on gradient of magnetic fields

(a) 3-D comparison

(b) $\alpha=0$

(c) $\alpha=45$

(d) $\alpha=90$

Fig. 4.17. 3-D plot of traceable motion range and comparison plot

4.3 EDMP MODEL FOR INDUCTION BASED ORIENTATION SENSING

When EM is used as excitation source, magnetic inductance has become the fundamental of sensing techniques in numerous applications such as motion sensing [110, 111], orientation sensing, machine path tracking [112], and so on. In particular, it has been increasing attention in orientation measurement from many industries as well as biomedical applications since it offers high sensitivity without mechanical contact. Moreover, it can be simple and compact in design and easy at operation, suitable for small and precise measurement system. However, it has difficulties in design and

development since it requires a good understanding of magnetic field, precise estimation and computation in real time [113, 114].

Mutual inductance of various shaped coils at their geometric relation has been studied and pre-computed results and mathematical relation has been tabulated in [6]. The computation requires series of integral operations for various shapes and locations of coils. However, it cannot include all situations in various configurations so that any change in the existing form/solution has to be evaluated again. Although these formulas and results in tables are usually analytical but computation is still time consuming and tedious, computing process is neither simple nor efficient. In addition, it could cause ambiguity problem since various approximations have to be used. Apparently, it is better to have a more general, accurate and fast-computing method to characterize time varying magnetic field.

Time varying magnetic field has been studied analytically and numerically by numerous approaches [5]. For simplified problems with extreme assumptions, series solutions were usually obtained and able to provide reasonable simulation results. However, for more realistic and applicable models, numerical approaches were implemented, including finite element method, boundary element method, and so on. Although with the rapid development of computer industry, numerical methods can provide quite accurate results in a short time, analytical modeling is still preferred due to simple and easily implementable, especially for calculating many magnetic field information such as magnetic force, torque, and inductance [37].

With the extended DMP (eDMP) model, inductance calculation has been discussed by the relation between inductance and magnetic energy in the previous chapter. Here both simulation and experimental results of various coils and their

configuration are compared to demonstrate the modeling accuracy and computational efficiency.

4.3.1 INDUCTANCE CALCULATION AND COMPARISON WITH GROVER'S METHOD

Magnetic inductance of various shaped coils at their geometric relation has been studied and pre-computed results and mathematical relation has been tabulated in [6]. Grover's method [115], as most widely used approach for computing induction, requires series of integral operations for various shapes and locations of coils. However, it cannot include all situations in various configurations so that any change in the existing form/solution has to be evaluated again. Although these formulas and results in tables are usually analytical but computation is still time consuming and tedious, computing process is neither simple nor efficient, especially when the coil is randomly oriented or located. In addition, it could cause ambiguity problem since various approximations have to be used. On the contrary, the eDMP model for coils at random location can be simply obtained and summation formula as in Eq. 3.19 and Eq. 3.20 can always be calculated easily.

Coil configuration as examples in [116] is used for comparing mutual inductance, which include different combinations of thin wall solenoid and filamentary circular coil.

1) Thin wall solenoid and filamentary circular coil

Two coils are modeled using the eDMP method first, and the resulting parameters are shown as below.

Table 4.5 eDMP model parameters

	Coil geometry	DMP model
Coil a	A solenoid with 6cm radius, 12cm height, and 120 turns of winding.	$l_m = 5.88\text{cm}$, $r_m = 3\text{cm}$, $ \mathbf{m} = 1.8825\text{Am}^2$
Coil b	A circular coil with radius to be 5cm.	$l_m = 0$, $r_m = 2.5\text{cm}$ $ \mathbf{m} = 0.0303\text{Am}^2$

Fig. 4.18. Simulation results

Maximum modeling error of coil *a* is 3.66% and that of coil *b* is 6.8%. Two coils are located coaxially and the distance between two centers is 12cm as in Fig. 4.19. Then the orientation of circular coil is rotated around y axis. The mutual inductance is calculated using the proposed approach and compared with results provided in [116] as Fig. 4.20, in which the maximum error is 9.8%. Good agreement has been achieved and the discrepancy is a combination effect of the approximation error of mutual inductance formula as Eq. 3.26 and the modeling error of eDMP model, as shown in Fig. 4.18.

Fig. 4.19. Multi and single wound coils

Fig. 4.20. Mutual inductance simulation results

2) Two thin wall solenoids

Mutual inductance of two solenoids are implemented, one similar to the solenoid in the previous example, and the other as coil b is detailed in Table 4.6. Simulation results are also shown in Fig. 4.21 and the maximum error is 10.2%.

Table 4.6 DMP model parameters

	Coil geometry	DMP model
Coil a	a solenoid with radius to be 6cm, height to be 12cm, and the number of turns to be 120	$r_m = 3cm$, $l_m = 5.88cm$ $ \mathbf{m} = 1.8825 Am^2$
Coil b	a solenoid with radius to be 5cm, height to be 4cm, and the number of turns to be 60	$l_m = 0$, $r_m = 2.5cm$ $ \mathbf{m} = 1.5645 Am^2$

Fig. 4.21. Field simulation results of coil b

Two coils are located coaxially and distance between two centers is 12cm. Then orientation of coil b is rotated around y axis as in Fig. 4.22. The mutual inductance is compared with published results as in Fig. 4.23. The maximum error is 8.9%.

Fig. 4.22. Multi and multi wound coils

Fig. 4.23. Mutual inductance simulation results

4.3.2 EXPERIMENTS AND DISCUSSION

Computation of electromagnetic induction between multiple coils has been discussed in chapter three. Here the experiment results will be presented.

1) Coils wound around tube

In order to validate the influence of deformed coil while tube bending, both simulations were performed to show its difference as below.

Fig. 4.24. Comparison of simulation results

2) Coil Experiments

In order to further demonstrate the eDMP method, the experiment in Fig. 4.25 is simply set up. The coil *a* is fixed at the center, but the other coil is adjustable and can be rotated around the center. The base structure and rotating plate are made of non-magnetic conductor, transparent acrylic. The experiment parameters are summarized in Table 4.7.

Fig. 4.25. Coils for mutual inductance

Table 4.7 Experiment parameters

Frequency Current		10kHz 0.5A(RMS)
Centre coil	Diameter	2cm
	Turns	40
	Resistance	0.5Ω

Three types of coils with different aspect ratio (γ) are used as in Fig. 4.26. Measurements were conducted at different orientation angles and compared to the simulation. In addition, the conventional single dipole (SD) model is also used for further comparison and calculates the mutual inductance.

(a) Coil 1 ($\gamma=1.4$)

(b) Coil 2 ($\gamma = 2.8$)

(c) Coil 3 ($\gamma = 5.7$)

Fig. 4.26. Coils in various designs

a) Coil 1: 1cm diameter, 20 turns

It can be seen in Fig. 4.27 that when the orientation is bigger than 60 degree, the signal becomes quite small as 0.005V, which is almost at the same magnitude as noise. As the orientation angle further increases and the signal further decreases, noise

becomes dominant, indicating the designed coil can be operated in a range smaller than 60 degree.

Fig. 4.27. Simulation results comparison for Coil 1

Within the measurable range, the discrepancy between the eDMP results and experimental results is reduced to 29.4% of the discrepancy due to the single dipole model.

b) Coil 2: 2cm diameter, 20 turns

Coil configuration is plotted in Fig. 4.25(b), similar to Coil 1 but different size and number of turns.

Fig. 4.28. Simulation results comparison for Coil 2

It is apparent that the eDMP modeling provides better performance than the single dipole model when being used to compute the mutual inductance. The maximum error is only 0.004V, which is 3.6% of the error due to single dipole model. In order to measure

higher output from the sensing coil and improve the signal and noise ratio, an amplifier circuit with gain to be 100 was implemented in the experiment. However, due to the accumulating heat during the experiment, the amplifier gain shifted with the resistance variation and introduced more error into the measurements.

c) Coil 3: 4cm diameter, 20 turns

As shown in Fig. 4.29, the errors are getting bigger as the coils become flat and multiple wound. However, the error is still much smaller as compared to the single dipole model.

Fig. 4.29. Simulation results comparison for Coil 3

The maximum difference between the experimental results and eDMP modeling results is 0.02V, 7.75% of the difference between the experimental results and the single dipole modeling results. Increasing error compared to previous case is due to the modeling error for big aspect ratio coil.

3) Tube experiment

In order to simulate medical stomach tube with 5mm diameter, acrylic cylindrical block with 5mm diameter was used in this experiment. Coil was made using 0.8mm diameter wire. The experiment was conducted with different coil arc distance as well as path curvature as in Fig. 4.30. In the following expression, “model a” indicates

DMP model that does not account for coil shape change with the tube bending; and “model b” indicates DMP model modified to account for coil shape change.

Fig. 4.30. Tube experiment set-up

(a) Comparison of experiment and simulation

(b) Comparison of different simulation models

Fig. 4.31. Results of arc distance to be 2cm

(a) Comparison of experiment and simulation

(b) Comparison of different simulation models

Fig. 4.32. Results of arc distance to be 2.5cm

Based on Fig. 4.31(a) and Fig. 4.32(a), it is ambiguous which simulation model matches the experiment results better. The differences were considerably small and could be caused by experiment error. But when angle range (10-100 degree) was examined closer as in Fig. 4.31(b) and Fig. 4.32(b), model-a showed decreasing tendency and model b showed increasing tendency, which made model-b share the same trend with experiment results. It is expected that the bigger discrepancy in Fig. 4.31(a) is due the small distance between two coils that makes the experiment conduct more difficult. The coils have to be forced to stay in position and tend to contact each other, which would introduce unwanted noise. This is confirmed by the comparison results shown in Fig. 4.32(a), in which the distance between two coils are 2.5cm and the experimental results are consistent with the modeling results.

A general and fast-computing method has been presented to calculate mutual inductance between coils. Time varying magnetic field of the coils is first characterized by the extended DMP model and then mutual inductance is further computed by discretizing the integral relation with the help of discontinuously located dipoles. The method proposed a summation form for estimating the inductive effect. Simulation for various cases proved the modeling accuracy of proposed method. Experiment set-up and preliminary results were presented.

Besides the computing efficiency and accuracy, the proposed method does not demand any limitation on the coil configuration or design. Unlike the existing look-up table, eDMP can be easily extended and used for modeling of irregular-shaped coils, even conductors of any shape or specific material. For future research, proposed method could be applied for path tracking of medical intubation process, conductivity-based medical imaging and so on. The human-friendly nature of inductive phenomenon indicates huge potential for its application on human welling. By introducing and

developing real-time computation approach, its current utilization in medical field can be further improved.

V. DISTRIBUTED DIPOLE MODEL FOR MIT

Extended DMP model has been proposed in previous chapters to simulate the electromagnetic field from time-varying current sources and its application on induction computation has also been validated. From this chapter, the modeling scheme with distributed dipoles will be further explored and utilized for the magnetic induction tomography.

Review of existing Magnetic Induction Tomography (MIT) studies in chapter II has shown that the mechanism/model used to connect the measurements with the material feature inside target domain would significantly affect the imaging performance. As an inverse problem by nature, the exact solution of induction tomography cannot be obtained by direct inversion due to nonlinearity; iterative calculations are to be expected. Therefore the mapping function need to be updated often and computation efforts significantly increase. If a linkage term can be used to disintegrate this nonlinear relation between the material feature and measurements, the computation process can be simplified.

5.1 OVERVIEW

Physical magnetic dipole exists only in the form of the quantum-mechanical spin associated with particles such as electrons, but theoretical magnetic dipole can be simply regarded as the source of unit magnetic field. Then elementary magnetic field can be seen as generated by a point dipole. By introducing the magnetic moment term, which itself includes an integral operator with the integrand to be function of current density and location, linear form equations can be obtained to calculate magnetic field information, as Eq. 3.19 and 3.20.

For magnetic induction tomography, Biot-Savart law [80] has been widely used for developing the forward modeling equation and a perturbation model was used to

formulate the matrix form equation after discretizing the problem domain. This type of mapping creates difference images, requires prior information and the relative changes to be small, and inevitably causes accumulation error for real-time imaging. On the contrary, if dipole moment is used to account for the eddy current, and the relation between magnetic field and magnetic moment can be described in linear form, there is no need to use the perturbation formula. Instead, absolute imaging can be directly obtained by considering the magnetic moment in Eq. 3.21 as the quantity corresponding to the eddy current distribution. Therefore, it is reasonable to represent the imaging domain with a group of distributed point dipoles, each with different moment and located at different place, each indicating the eddy current strength in its belonging area. This is the motivation of distributed dipole model for magnetic induction tomography.

5.2 INDUCED DIPOLE MOMENT AND CONDUCTIVITY

The imaging objective of magnetic induction tomography is material features, including conductivity, permeability and permittivity. So far conductivity distribution is the main concern since it contributes most to the measurements. Here, the relation between dipole moment and conductivity will be studied.

As in Fig. 5.1 (a), when examining single dipole $\mathbf{m}_i$, which is represented by red point with arrow, it accounts for eddy current inside a small conductive area V_i , conductivity value σ_i within the element assumed constant. Blue circle line indicates the direction of eddy current $\mathbf{J}_i$ and red arrow indicates the direction of dipole i , which is dependent on $\mathbf{J}_i$. A simple linear relation can be formulated between the induced current density and electric field as $\mathbf{J}_i = \sigma_i \mathbf{E}_i$. Therefore, magnetic moment of corresponding point dipole can be expressed as Eq. 5.1.

Fig. 5.1 Single dipole accounting for eddy current

$$\mathbf{m}_i = \frac{1}{2} \sigma_i \int \mathbf{r} \times \mathbf{E}(\mathbf{r}) dV_i \quad (5.1)$$

where $\mathbf{m}_i$ represents the magnetic moment of i th dipole; σ_i represents the conductivity value within the specific element; V_i represents the volume of element.

In the illustrative equation of electric field $\mathbf{E} = -\nabla V - \frac{d\mathbf{A}}{dt}$, the first term accounts for contribution from net charge and second term accounts for contributions from time varying magnetic field. Since here we discuss about eddy current and time-varying electromagnetic field, effect of latter term is dominant compared to previous term. And magnetic vector potential can be calculated from the eDMP model of the excitation coil as Eq. 5.2, then substituted into Eq. 5.1.

$$\mathbf{A}_i = \frac{\mu_0}{4\pi} \sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \quad (5.2)$$

$$\mathbf{m}_i = -\frac{1}{2} \sigma_i \int \mathbf{r} \times \frac{d\mathbf{A}}{dt} dV_i = -\frac{\mu_0}{8\pi} \sigma_i \frac{d \left(\int \mathbf{r} \times \sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} dV_i \right)}{dt} \quad (5.3)$$

where n indicates the quantity of dipoles in eDMP model for excitation coil and j indicates j th dipole in this model; i indicates the imaging dipole that being discussed as

in Eq. 5.1; $\mathbf{P}_{ij}$ represents the vector pointing from j th dipole to i th dipole.

When distributed dipoles are used to represent the imaging area, a discrete model can be shown as in Fig. 5.1(b). It is reasonable to approximate the volume integral with summation form as below

$$\mathbf{m}_i = -\frac{\mu_0}{8\pi} \sigma_i \frac{d \left(\sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right) \right)}{dt} \quad (5.4)$$

where k indicates surrounding dipoles as in Fig. 5.1(b); l indicates the quantity of surrounding dipoles, which is seven in illustrative example; $\mathbf{r}_{ik}$ represents the vector pointing from i th dipole to k th dipole.

In sum, when distributed dipoles are used to model the eddy current environment, a discretized formula as Eq. 5.4 can be used to approximate the relation between dipole moment and conductivity. Therefore it is reasonable to use dipole moment as the imaging term for magnetic induction tomography.

5.3 FORWARD MAPPING WITH DIPOLES

5.3.1 DISTRIBUTED DIPOLE MODEL

An illustrative dipole distribution model is configured as below to simulate the eddy current environment with inhomogeneous conductivity distribution. The excitation coil was shown with its corresponding eDMP model, in which the red arrow indicates the point dipole as well as its direction. The dot line describes the boundary of cross-section surface of the target domain and blue dots randomly distributed represent the dipoles accounting for eddy current. Measurements were obtained along the dot line.

Fig. 5.2 Illustrative configuration of distributed dipole model

When forward modeling is considered, locations of dipoles accounting for excitation coil, magnetic field measurements locations and dipoles representing image domain are known information, which are referred as $\mathbf{r}_{s1}, \mathbf{r}_{s2}, \dots, \mathbf{r}_{sn}; \mathbf{r}_{f1}, \mathbf{r}_{f2}, \dots,$

$\mathbf{r}_{iq}; \mathbf{r}_{m1}, \mathbf{r}_{m2}, \dots, \mathbf{r}_{mp}$, n and p indicating the quantity of point dipoles accounting for excitation coil and eddy current, q for quantity of measurement points.

For i th image dipole, its contribution to the magnetic flux density at location $\mathbf{r}_{fh}$ can be expressed as

$$\mathbf{B}_{ih} = \frac{\mu_0}{4\pi} \left(\frac{3(\mathbf{m}_i \cdot (\mathbf{r}_{fh} - \mathbf{r}_{mi}))(\mathbf{r}_{fh} - \mathbf{r}_{mi})}{|\mathbf{r}_{fh} - \mathbf{r}_{mi}|^5} - \frac{\mathbf{m}_i}{|\mathbf{r}_{fh} - \mathbf{r}_{mi}|^3} \right) \quad (5.5)$$

For each measurement point, the induced field includes contribution from each imaging dipole. In this way, a q by p matrix can be formulated by summing up the contributions from all the dipoles.

$$\begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \mathbf{G}_{13} & \cdots & \mathbf{G}_{1p} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \mathbf{G}_{23} & \cdots & \mathbf{G}_{2p} \\ \mathbf{G}_{31} & \mathbf{G}_{32} & \mathbf{G}_{33} & \cdots & \mathbf{G}_{3p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{G}_{q1} & \mathbf{G}_{q2} & \mathbf{G}_{q3} & \cdots & \mathbf{G}_{qp} \end{bmatrix} \times \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \\ \vdots \\ \mathbf{m}_p \end{bmatrix} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \mathbf{B}_3 \\ \vdots \\ \mathbf{B}_q \end{bmatrix} \quad (5.6)$$

Notice that all the terms in formula above are vector, which indicates the actual matrix should be $3q$ by $3p$. In order to reduce the unknowns, relation between the dipole moment and conductivity is examined again here.

$$\begin{aligned} \mathbf{m}_i &= -\frac{\mu_0}{8\pi} \sigma_i \frac{d \left(\sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right) \right)}{dt} \\ &= -\frac{\mu_0 \omega \sigma_i}{8\pi} \left(\sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right) \right) \hat{j} \\ &= -\frac{\mu_0 f}{4} \sigma_i \left(\sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right) \right) \hat{j} \end{aligned} \quad (5.7)$$

where ω and f represent the angular frequency and frequency of excitation signal respectively; $\hat{j}$ here indicates imaginary component, as a result of time derivative accounting for induction. In the time domain, we can see from Eq. 5.7 that there is 90 degree phase difference between image dipoles and dipoles accounting for excitation signal. And only magnitude of image dipole contains information about conductivity.

When direction of $\mathbf{m}_i$ is considered, term $\sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right)$ is fully

responsible and can be computed without prior knowledge about material feature in the image area. Hence, vector equation Eq. 5.6 can be reduced as below

$$\begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \mathbf{G}_{13} & \cdots & \mathbf{G}_{1p} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \mathbf{G}_{23} & \cdots & \mathbf{G}_{2p} \\ \mathbf{G}_{31} & \mathbf{G}_{32} & \mathbf{G}_{33} & \cdots & \mathbf{G}_{3p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{G}_{q1} & \mathbf{G}_{q2} & \mathbf{G}_{q3} & \cdots & \mathbf{G}_{qp} \end{bmatrix} \times \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_p \end{bmatrix} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \mathbf{B}_3 \\ \vdots \\ \mathbf{B}_q \end{bmatrix} \quad (5.8)$$

where

$$\mathbf{G}_{ih} = \frac{\mu_0}{4\pi} \left(\frac{3(\hat{\mathbf{m}}_i \bullet (\mathbf{r}_{fh} - \mathbf{r}_{mi}))(\mathbf{r}_{fh} - \mathbf{r}_{mi})}{|\mathbf{r}_{fh} - \mathbf{r}_{mi}|^5} - \frac{\hat{\mathbf{m}}_i}{|\mathbf{r}_{fh} - \mathbf{r}_{mi}|^3} \right)$$

$$\hat{\mathbf{m}}_i = \frac{\sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right)}{\left| \sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right) \right|}$$

In actual application, most commonly used sensor is simple winding coils, which are single axis sensors. And the simplest configuration is to locate the coils along the radial direction of the problem domain and obtain numerous measurements by rotating the coils. Thus it is better to express the overall magnetic flux density vector in spherical coordinate. In order to do the sum operation directly, each vector $\mathbf{B}_i$ is broken down to x, y, z terms first. By transferring the vector form matrix equation, a simpler form can be rewritten as

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & \cdots & G_{1p} \\ G_{21} & G_{22} & G_{23} & \cdots & G_{2p} \\ G_{31} & G_{32} & G_{33} & \cdots & G_{3p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{q1} & G_{q2} & G_{q3} & \cdots & G_{qp} \end{bmatrix} \times \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_p \end{bmatrix} = \begin{bmatrix} Br_1 \\ Br_2 \\ Br_3 \\ \vdots \\ Br_q \end{bmatrix} \quad (5.9)$$

where $G_{ij} = \frac{Gx_{ij}x_{fi} + Gy_{ij}y_{fi} + Gz_{ij}z_{fi}}{\sqrt{x_{fi}^2 + y_{fi}^2 + z_{fi}^2}}$, $Br_i = \frac{\mathbf{B}_i \bullet \mathbf{r}_{fh}}{|\mathbf{r}_{fh}|}$.

This matrix form equation is the forward mapping that will be used for further imaging problem. It is apparent that by replacing the current density with dipole moment as target variable, the sensitivity matrix becomes solely dependent on geometries and excitation coil. Unlike many existing model, no eddy current problem need to be computed in prior.

In order to compute for conductivity images, another sensitivity term can be extracted from Eq. 5.4.

$$m_i = S_i \sigma_i, \text{ where } S_i = -\frac{\mu_0}{8\pi} \frac{\sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right)}{\left\| \sum_{k=1}^l \mathbf{r}_{ik} \times \left(\sum_{j=1}^n \frac{\mathbf{m}_j \times \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|^3} \right) \right\|} \quad (5.10)$$

And Eq. 5.9 can be derived into

$$\begin{bmatrix} G_{11}S_1 & G_{12}S_2 & G_{13}S_3 & \cdots & G_{1p}S_p \\ G_{21}S_1 & G_{22}S_2 & G_{23}S_3 & \cdots & G_{2p}S_p \\ G_{31}S_1 & G_{32}S_2 & G_{33}S_3 & \cdots & G_{3p}S_p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{q1}S_1 & G_{q2}S_2 & G_{q3}S_3 & \cdots & G_{qp}S_p \end{bmatrix} \times \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \vdots \\ \sigma_p \end{bmatrix} = \begin{bmatrix} Br_1 \\ Br_2 \\ Br_3 \\ \vdots \\ Br_q \end{bmatrix} \quad (5.11)$$

5.3.2 VALIDATION OF DIPOLE DIRECTION

In order to validate the discretized form of induced dipole moment especially its direction, computation of sensitivity matrix is done with MATLAB and illustrative model was shown as below.

Fig. 5.3 distributed dipole model in MATLAB

The imaging domain is a circle area with 0.1m radius, which is represented by red circle. 300 measurements were obtained at equally located positions along the circle. Black dots indicated the dipole locations, which were generated by MATLAB “pdetool” function. Then we can consider these black dots as an initial distributed dipole model, in which dipoles are randomly and uniformly located. Quantity of dipoles was 148, which could be simply modified by refinement function in MATLAB.

Since dipole moment is used to account for eddy current, analysis on eddy current can be done first. As stated before, formula $\mathbf{E} = -\frac{d\mathbf{A}}{dt}$ is used to compute electric field from eDMP model for excitation coil. Since eddy current is simply multiplication of electric field with conductivity, unit eddy current plot is shown as below when single excitation coil is located at (0, 0.11m) and uniform conductivity is assumed.

Fig. 5.4 Unit eddy current plot

Eddy current vector was plotted using “quiver” function in MATLAB, represented by blue arrows. It indicated the vector direction with arrows and length of the arrows indicates the magnitude of vector. Fig. 5.4(b) showed that in general, the polarity of eddy current is against that of excitation coil. And Fig. 5.4(c) indicated that eddy current magnitude drop dramatically along the y axis.

After computing unit eddy current, direction of dipole moments can be decided according to Eq. 5.4. Normalized direction vector is calculated and the results are shown as below, in which blue arrows indicate the dipole moment direction and approximately oppose the magnetic field from excitation coil. This coincides with its inductive nature.

Fig. 5.5 Direction of induced dipole moment

We can utilize this direction information to reduce the quantity of unknowns about dipole moment. A more simple assumption would be to assume dipole moment always pointing to the excitation coil. Comparison results will be presented later.

5.3.3 SENSITIVITY MATRIX STUDY

According to Eq. 5.9 and Eq. 5.11, sensitivity matrix can be calculated respectively, when considering dipole moment or conductivity as the imaging variable. Study of sensitivity matrix can be done without any actual measurements, either from simulation or experiment. Because by breaking down the matrix step by step, what might happen after the inversion can be exactly predicted without actually solving the inverse problem. Therefore theoretical imaging capability of sensitivity matrix can be discussed here.

(1) Effect of multiple excitation sources

First step to analyze the sensitivity matrix is singular value decomposition. Considering distributed model as Fig. 5.3, condition numbers of the matrixes are

shown as below, in which horizontal axis represents the k th singular value and vertical axis indicates the magnitude of k th singular value in decibel.

Fig. 5.6 Matrix condition

It is seen that condition number slightly increases when conductivity is directly considered to be imaging target. This is because while sensitivity matrix in Eq. 5.9 is the discretization form of only one integral formula, which relates the imaging dipole moment $\mathbf{m}_i$ to surrounding magnetic field, sensitivity matrix in Eq. 5.11 contains information about two processes: 1) excitation coil generates exciting magnetic field; 2) eddy current generates a secondary magnetic field. When the sensitivity matrix include two nonlinear processes, the matrix condition becomes worse.

When these two processes are considered individually, the latter that signal go from eddy current to sensors can be controlled by increasing the quantity of measurements, optimizing sensor location and so on. Effect of sensor system configuration has been widely discussed in existing studies. When it comes to excitation process, we could utilize pair of sources on the opposite sides of imaging area and the response in the central area can be increased while outer area barely affected. This could improve the uniformity of eddy current response in the overall imaging area. Unit eddy current plot when two excitation coils are used is shown as below.

(a) 3D plot

(b) xz view

(c) yz view

Fig. 5.7 Unit eddy current plot of two excitation coils

When comparing eddy current plot above with Fig. 5.4, especially Fig. 5.7(c) and Fig. 5.4(c), the eddy current response along y axis is more uniformly distributed. The induced eddy current in central area has doubled while that in outer area remains the same. Imaging performance can be therefore improved.

(2) Imaging performance evaluation

Various parameters have been designed to evaluate the sensitivity matrix and provide information about imaging performance. Resolution matrix is introduced to provide the images for each voxel perturbation and describes the response of an imaging system to a point source or point object. The degree of blurring of the point object is a measure for the quality of imaging system.

By definition, resolution matrix $\mathbf{R}$ can be calculated as below.

$$\mathbf{R}=\mathbf{G}^{-1}\mathbf{G} \quad (5.12)$$

where $\mathbf{G}^{-1}$ represents the inversed sensitivity matrix and $\mathbf{G}$ represents the sensitivity matrix. In the ideal case that $\mathbf{G}$ could be exactly inversed, resolution matrix would become identity matrix, which means the image is exactly reconstructed. Then relation between the original conductivity perturbation $\Delta\sigma$ and reconstructed conductivity perturbation $\Delta\sigma^*$ can be expressed mathematically as

$$\Delta\sigma^*=\mathbf{R}\Delta\sigma \quad (5.13)$$

Here we can see each column of resolution matrix is the image of a single voxel perturbation. In the practical case, various regularization methods will be used, therefore resolution matrix will be affected by not only the modeling approach, but also the inversion techniques.

Three types of criteria can be defined based on resolution matrix: amplitude response (AP), position error (PE) and resolution. Amplitude response is defined as the maximum amplitude of PSF divided by the associated voxel area, which is decided by setting specific threshold on reconstructed value. Position error is calculated as the discrepancy between the center of region of interest and the original position of the target, which is used to measure the extent of dislocation of PSF peak from the target position. Resolution is calculated based on the ratio of the area of region of interest to the area of entire target area and used to measure the smallest visible object. In general, it is desirable for AR, PE, and resolution to be constant, while PE and resolution should be as small as possible for any target position.

In order to evaluate these parameters along the radial direction due to the axisymmetry of imaging model, we divide the radius into five ranges, 0-0.02m, 0.02m-0.04m, 0.04-0.06m, 0.06-0.08m, 0.08m-0.1m. And an average value is obtained for each range. Three different models are computed: 1). dipole moments are assumed

to point to the excitation coil location; 2) dipole moment directions are calculated based on unit eddy current; 3) dipole moment directions are calculated based on unit eddy current when two excitation coils are used.

Fig. 5.8 Amplitude response

Fig. 5.9 Resolution

Fig. 5.10 Position error

It is apparent that amplitude response increases with the radius, resolution decreases with radius, and position error decreases with the radius, all indicating better imaging quality at the outer area. This is due to the better penetration of excitation signal at the outer area. Therefore when two excitation coils are located on the opposite side of imaging area, we can see the amplitude response improved in Fig. 5.8, indicating by black line, and resolution reduced in Fig. 5.9, position error reduced in Fig. 5.10.

5.4 ISSUES RELATED TO INVERSE PROBLEM

After forward modeling the Magnetic Induction Topography problem as a discrete equation in matrix multiplication form, inverse problem need to be considered. For illustrative purpose, it is a common expression to first relate physical parameters $\tilde{x}$ to collected measurements $\tilde{d}$. Forward mapping can be expressed as $G(\tilde{x}) = \tilde{d}$. Depending on different cases, $\tilde{d}$ may be a function of time and/or space, and $\tilde{x}$ could also be a continuous function describing a physical environment. When $\tilde{x}$ and $\tilde{d}$ are functions, we typically refer to G as an operator, which could be an ordinary differential operator, partial differential operator or integral. For most practical cases, we determine a finite number (n) of parameters to define a model $\tilde{x}$. Then we can express the model parameters as an n -element vector $\mathbf{x}$. Similarly, if there are a finite number of data points then we can express the data as an m -element vector $\mathbf{d}$. Then a discretized formula is rewritten as

$$\mathbf{G} \mathbf{x} = \mathbf{d} \quad (5.14)$$

where $\mathbf{G}$ is an m by n matrix; $\mathbf{x}$ is an n by 1 vector; and $\mathbf{d}$ is an m by 1 vector.

Afterwards, discrete inverse problem is to find $\mathbf{x}$ given $\mathbf{d}$. A critical issue is that actual observations always contain some amount of noise. It may come from neglected

influences during forward modeling, numerical calculation error, or background noise during measurement. We can thus envision data as generally consisting of noiseless observations from a “perfect” experiment $\mathbf{d}_{true}$ plus a noise component η .

$$\mathbf{d} = \mathbf{d}_{true} + \eta = \mathbf{G} \times \mathbf{x}_{true} + \eta \quad (5.15)$$

where $\mathbf{d}_{true}$ exactly satisfies Eq. 5.14 for $\mathbf{x}$ equal to the true model $\mathbf{x}_{true}$. It is often the case that a solution that is influenced by even small noise can have little or no correspondence to $\mathbf{x}_{true}$. Another key issue is that there are commonly an infinite number of models aside from $\mathbf{x}_{true}$ that fit the data perfectly.

Therefore inverse problem solving is far more than simply finding mathematically acceptable answers to the governing equation.

5.4.1 ILL-CONDITIONED PROBLEM

It is essential to characterize what solution has been obtained, how good it is in terms of physical plausibility and fit to the data, and how consistent it is with other physical constraints. Essential issues that must be considered include solution existence, solution uniqueness and instability.

- Existence. There may be no model that exactly fits the data. This can occur in practice because the mathematical model of the system’s physics is approximate or the data contain large noise.
- Uniqueness. If exact solutions do exist, they may not be unique, even for an infinite number of exact data points. Non-uniqueness is a characteristic of rank-deficient discrete linear inverse problems because the matrix $\mathbf{G}$ has a nontrivial null space. By superposition, any linear combination of these null space models can be added to a particular model that satisfies Eq. 5.15 and not change the fit to the data. An important issue with problems that have

non-unique solutions is that an estimated model may be significantly smoothed or otherwise biased relative to the true model.

- **Instability.** The process of computing an inverse solution often is extremely unstable in that a small change in measurement [e.g., a small η in Eq. 5.15] can lead to an enormous change in the estimated model. Inverse problems where this situation arises are referred to as ill-conditioned. And it is commonly possible to stabilize the inversion process by imposing additional constraints that bias the solution, a process that is generally referred to as regularization.

First, to ensure the existence of solution, we can see that in the mathematics sense, only square and full rank matrix is invertible. In practical, the sensitivity matrix can be square matrix by controlling the observations quantity. But it is rarely of full rank. The simplest and most direct way to handle this issue is to refine the model and increase the measurements. Then the sensitivity matrix is not square matrix any more. If the matrix is full rank after including more data, the normal inversion formulation can be expressed as below.

$$\mathbf{x} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{d} \quad (5.16)$$

where $\mathbf{G}^T \mathbf{G}$ is a n by n matrix and can only be inverted when the sensitivity matrix $\mathbf{G}$ is full rank.

The rank of sensitivity matrix is dependent on the problem nature and cannot be improved despite the measurements quantity. In order to study the case that sensitivity matrix is rank-deficient, it is necessary to introduce the concept of singular value decomposition of a matrix, where an m by n matrix $\mathbf{G}$ is factored into

$$\mathbf{G} = \mathbf{U} \mathbf{S} \mathbf{V}^T \quad (5.17)$$

where $\mathbf{U}$ is an m by m orthogonal matrix that $\mathbf{U}\mathbf{U}^T = \mathbf{I}$; $\mathbf{V}$ is an n by n orthogonal matrix that $\mathbf{V}\mathbf{V}^T = \mathbf{I}$; $\mathbf{S}$ is an m by n diagonal matrix with nonnegative diagonal elements called singular values. The singular values along the diagonal of $\mathbf{S}$ are customarily arranged in decreasing size, $s_1 \geq s_2 \geq \dots \geq s_{\min(m,n)} \geq 0$. Note that for rank-deficient matrix, some of the singular values may be zero. If only the first p singular values are nonzero, we partition $\mathbf{S}$ as $\mathbf{S} = \begin{bmatrix} \mathbf{S}_p & 0 \\ 0 & 0 \end{bmatrix}$, where $\mathbf{S}_p$ is a p by p diagonal matrix composed of the positive singular values. Then we can simplify the SVD of $\mathbf{G}$ into its compact form

$$\mathbf{G} = \mathbf{U}_p \mathbf{S}_p \mathbf{V}_p^T \quad (5.18)$$

where $\mathbf{U}_p$ denotes the first p columns of $\mathbf{U}$, $\mathbf{V}_p$ denotes the first p columns of $\mathbf{V}$.

Therefore a generalized inverse of $\mathbf{G}$ can be computed even when $\mathbf{G}$ is not of full rank, which is named Moore-Penrose Pseudo-inverse $\mathbf{G}^*$.

$$\mathbf{G}^* = \mathbf{V}_p \mathbf{S}_p^{-1} \mathbf{U}_p^T \quad (5.19)$$

Then Pseudo-inverse solution is defined to be

$$\mathbf{x}^* = \mathbf{G}^* \mathbf{d} = \mathbf{V}_p \mathbf{S}_p^{-1} \mathbf{U}_p^T \mathbf{d} \quad (5.20)$$

Compared to the normal inversion equation $\mathbf{x} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{d}$, it is obvious that Eq. 5.20 has a desirable property that $\mathbf{x}^*$ would always exist.

By comparing the features of $\mathbf{G}$ and corresponding generalized inverse matrix $\mathbf{G}^*$, insights of the solution uniqueness and fitness to data can be provided.

$$\mathbf{G}\mathbf{x}^* = \mathbf{U}_p \mathbf{S}_p \mathbf{V}_p^T \mathbf{V}_p \mathbf{S}_p^{-1} \mathbf{U}_p^T \mathbf{d} = \mathbf{U}_p \mathbf{U}_p^T \mathbf{d} \quad (5.21)$$

Apparently the data cannot be fit exactly. And also, no matter how many data points are used, the solution is not unique because the last $n-p$ entities of model $\mathbf{x}$ are always multiplied by zero during the computation. This would lead to two major

problems, non-fitness and non-uniqueness. In order to incorporate further information about the desired solution and stabilize the problem, regularization process is needed.

5.4.2 REGULARIZATION TECHNIQUES

Although many types of information about the solution can be used to account for the solution stability, the dominating approach used is to require that the norm of the solution be small. Then the estimated true model can be expressed as

$$x_{\delta,L} = \arg \min \left\{ \|Gx - d\|_2^2 + \delta^2 \|L(x - x^*)\|_2^2 \right\} \quad (5.22)$$

where $\|Gx - d\|_2^2$ represents the fitness; $\delta^2 \|Lx - x^*\|_2^2$ represents the side constraint, which is also called regularization functional in some studies; x^* refers to the initial estimate of the solution, which is set to be zero in many cases; δ is named regularization parameter and L is named regularization matrix. Regularization parameter establishes a commitment between fitness to the data and the amount of applied regularization. Let $\|Gx - d\|_2^2 + \delta^2 \|Lx\|_2^2$ equals to zero, which can also be expressed as matrix form

$$(\mathbf{G}\mathbf{x} - \mathbf{d})^T (\mathbf{G}\mathbf{x} - \mathbf{d}) + (\delta\mathbf{L}\mathbf{x})^T (\delta\mathbf{L}\mathbf{x}) = 0 \quad (5.23)$$

An explicit form of solution can be expressed as

$$\mathbf{x}_{\lambda,L} = (\mathbf{G}^T \mathbf{G} + \delta^2 \mathbf{L}^T \mathbf{L})^{-1} \mathbf{G}^T \mathbf{d} \quad (5.24)$$

This method is called Tikhonov regularization, which is the most widely used method to solve an inverse problem.

Least squares problem Eq. 5.23 can also be expressed as $\min \left\| \begin{pmatrix} G \\ \delta L \end{pmatrix} x - \begin{pmatrix} d \\ \delta Lx^* \end{pmatrix} \right\|_2$

, which can be considered as circumventing the ill conditioning $\mathbf{G}$ by introducing a

new problem with a well-conditioned coefficient matrix $\begin{pmatrix} G \\ \delta L \end{pmatrix}$ with full rank. When matrix rank is considered, a different way to treat the ill-conditioning of $\mathbf{G}$ is to derive a new problem with a well-conditioned rank deficient coefficient matrix. A fundamental result about rank deficient matrices, which can be derived from the SVD of $\mathbf{G}$, is the closest rank- k approximation $\mathbf{G}_k$. It can be obtained by truncating the SVD expansion

$$\mathbf{G}_k = \mathbf{U}_k \mathbf{S}_k \mathbf{V}_k^T \quad (5.25)$$

where $\mathbf{U}_k$ denotes the first k columns of $\mathbf{U}$, $\mathbf{V}_k$ denotes the first k columns of $\mathbf{V}$, $\mathbf{S}_k$ is a k by k diagonal matrix composed of the first k singular values. Thus the solution can be expressed as $\mathbf{x}_k = \mathbf{V}_k \mathbf{S}_k^{-1} \mathbf{U}_k^T \mathbf{d}$.

Tikniov regularization and truncated SVD method, as most commonly used treatment for ill-conditioned problem, will be used to obtain the inversed solution in the next chapter.

5.5 ADAPTIVE DISCRETIZATION SCHEME

As previously explained, distributed dipoles have been used as sources terms in order to discretize the problem. Although the imaging performance is not solely dependent on the dipoles quantity, it is expected a denser distribution of dipoles could provide better resolution. Random distribution of the dipoles is only suitable for initial computation. Adaptive discretization is in demand for reduction of the computation efforts when iterative computation is conducted.

When the imaging purpose is to obtain the contour of a conductive object in a nonconductive background, areas with higher inverted dipole moments indicate that more attention is required here. It is reasonable to use it as a guideline for further

refinement. There is a monotonic relation between the conductivity and the representing dipole moment since material with higher conductivity would generate stronger eddy current and then contributes to larger dipole moment. And refining process with the help of dipole moments would be more efficient, since the discretization procedure can be the most time-consuming part and could be a major obstacle for generating real-time images. The adaptive discretization scheme can be described mathematically as Eq. 5.26.

Fig. 5.11 Refinement scheme

$$\mathbf{r}_{m_ADD} = \frac{\mathbf{r}_{mi} + \mathbf{r}_{mk}}{2} \quad \text{where } m_i \geq \text{threshold} \quad (5.26)$$

where $\mathbf{r}_{m_ADD}$ represents the location of added dipoles; $\mathbf{r}_{mi}$ and $\mathbf{r}_{mk}$ represents the location of i th dipole and its surrounding dipole, as indicated by red points in Fig. 5.11. Blue points indicate the refined dipole locations. And the threshold is usually set according to the maximum value of reconstructed dipole moments.

Therefore the sensitivity matrix can be expressed as below.

$$\begin{bmatrix} \tilde{\mathbf{G}}_{INI} & \tilde{\mathbf{G}}_{ADD} \end{bmatrix} \times \begin{bmatrix} \tilde{\mathbf{m}}_{INI} \\ \tilde{\mathbf{m}}_{ADD} \end{bmatrix} = \tilde{\mathbf{B}}_r \quad (5.27)$$

$$\text{where } \tilde{\mathbf{G}}_{INI} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & \cdots & G_{1p} \\ G_{21} & G_{22} & G_{23} & \cdots & G_{2p} \\ G_{31} & G_{32} & G_{33} & \cdots & G_{3p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{q1} & G_{q2} & G_{q3} & \cdots & G_{qp} \end{bmatrix}, \quad \tilde{\mathbf{m}}_{INI} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_p \end{bmatrix}, \quad \tilde{\mathbf{B}}_r = \begin{bmatrix} Br_1 \\ Br_2 \\ Br_3 \\ \vdots \\ Br_q \end{bmatrix}, \quad \tilde{\mathbf{G}}_{ADD} \text{ can be}$$

calculated according to $\mathbf{r}_{m_ADD}$ as Eq. 5.26.

VI. ILLUSTRATION AND DISCUSSION

6.1 OVERVIEW

In order to elaborate and demonstrate the method presented in the previous chapter, illustrative model consist of excitation coils, an imaging area and conductive objects is simulated and the field information is used for inversion. The sensitivity matrix study has been done in chapter 5.3.3 and the imaging performance has been examined based on the resolution matrix. Although the “imaging” in the mathematical and computational sense can be analyzed without real measurements, the development of an actual imaging system requires consideration on practical issues including noisy environment, measuring instrumentation and so on. In order to examine the imaging system in practical context, experimental setups are built and actual measurements are conducted.

6.2 SIMULATION RESULTS

COMSOL Multiphysics was used to simulate the model and study the magnetic field of eddy current environment.

6.2.1 MAGNETIC FIELD INFORMATION FROM COMSOL

Fig. 6.1 Simulation model in COMSOL

From the model as above, magnetic field information in both real and imaginary component can be obtained. First, edge current feature was used to simulation excitation coil, which is applied with 20A current of 100 kHz. The conductivity of a spherical object with 3cm radius was set to be 100 S/m and represented the interested target. Magnetic flux density was obtained surrounding the imaging area, which was represented by red circular line in Fig. 6.2. Conductive object was located along the axis of excitation coil and 5cm away from the center of imaging area. Three hundred measurements were acquired at equally distributed locations.

Fig. 6.2 Object at location 1

The measurement points start from the nearest end to excitation coil and continue for a whole round. Due to the phase difference between eddy current as well as its corresponding field and excitation field, the magnetic flux density measured was always complex number. Here the real and imaginative parts are plotted individually as in Fig. 6.3. The peaks in the real component plots indicate the location of excitation coil. Since the axis of excitation coil is along y axis, B_x show opposite sign on different sides of y axis and B_y show single peak with maximum value at the starting point. And when considering imaginative component, which represents field information that has a phase difference with the excitation field, it is reasonable to say this part of information account for magnetic field due to eddy current. Because the object was located as in Fig. 6.2, peaks in B_x and B_y are in the same location with real component. But for B_x , imaginative component decreases when real component increases, and vice versa. For B_y , imaginative component showed the same sign with real component although excitation coil and conductive object, both as source of magnetic field, are at different side of measurement location.

(a) Real component

(b) Imaginary component

Fig. 6.3 Magnetic flux density

These results show the opposing nature of inductive field and confirm that the imaginary component indeed, represents the effect of eddy current.

When one more excitation coil is added on the opposite side of imaging area, the magnetic field results are shown as below.

(a) Real component

(b) Imaginary component

Fig. 6.4 Magnetic flux density

It is shown above that real component of magnetic flux density contains two peaks now, due to two excitation coils. But the imaginary component approximately remain the same, which indicates that eddy current inside the object is mainly due to the one near excitation coil. Adding one excitation coil would change the sensitivity matrix as explained in last section, but it does not necessarily affect the eddy current since the conductive object is only near to one excitation coil.

If the object location is shifted as below and still use single excitation coil, the results will also differ.

Fig. 6.5 Object at location 2

It is shown below that when object location has been shifted, real component of magnetic flux density does not change significantly. This confirms that the real component mainly reflect the influence of excitation field. And by examining the imaginative component, it is apparent the peak has shifted to around the object location.

(a) Real component

(b) Imaginative component

Fig. 6.6 Magnetic flux density

In conclusion, the imaginary component of magnetic flux density contains information about the object location. However, in experiments, it is difficult to directly measure the imaginary component of magnetic flux density. This part of information is usually extracted from the phase information of measured signal. Therefore phase angle is calculated based on the simulation results shown above.

Fig. 6.7 Phase information of the magnetic flux density

6.2.2 INITIAL SOLUTION

Three hundred equally located peripheral field values were obtained according to Fig. 6.5. With the initial distributed model, the inverted result is shown as below and compared with exact conductivity distribution, indicated by red circle. The object location can be reconstructed.

Fig. 6.8 Reconstructed image

When two excitation coils are used, results are shown as below.

Fig. 6.9 Reconstructed images

6.2.3 ADAPTIVE REFINED SOLUTION

After obtaining the initial solution of dipole moment distributions, overall examination provide the area that requires further attention. A threshold was set to be fifth percent, which indicates the dipoles with moment value bigger than half the maximum value would be highlighted, as in Fig. 6.10(a). And according to the refinement scheme explained in Chapter 5.5, the surrounding area would be populated with more dipoles, as in Fig. 6.10(b).

Fig. 6.10 Refinement of distributed dipole model

With the refined model, the reconstructed 2D image is shown as below.

Fig. 6.11 Refined solution

By comparison with Fig. 6.9(b), it is visible that the refined solution provides better shape reconstruction performance.

6.3 EXPERIMENT WITH LOW FREQUENCY EXCITATION

6.3.1 EXPERIMENT SETUP

A preliminary experiment system has been designed as in Fig. 6.12. Both excitation component and sensing component are installed on slide structure so that they can be adjusted and fixed with screws, as shown in Fig. 6.12(a). This design enables both the excitation component and sensing component to be movable at the vertical direction. The mechanical scanning structure is fabricated using acrylic plastic sheet of various thicknesses ranging from 4-10mm. These slide structures are installed on the rail structures as in Fig. 6.12(b), which enable the excitation and sensing component adjustable at the radical direction. A fish tank is put in Fig. 6.12(c) to indicate the target imaging area, surrounded by a mechanical rotating device. It is expected that the wires from the exciters will tangle along with the rotation. In order to prevent the wires from obstructing the imaging area, a gear and belt structure is used to drive the rotating device instead of directly installing the DC motor at the rotation

center. The motor has a maximum rotational speed of 30rpm and can be adjusted by varying voltage.

Both exciters and sensors are cylindrical coils with acrylic core, as in Fig. 6.12(d). Parameters of experiment are detailed as in Table 6.1. In order to obtain both phase and magnitude information, RF lock-in amplifier model SR844 from Stanford Research Systems is used to analyze the signal. NI USB-6341 is used for data acquisition and LM1875 audio power amplifier is used for driving the excitation coil. The instrumental specifications are detailed as below.

Table 6.1 Parameters of low frequency experiment

DC motor		Maximum supply: 12V, 30 rpm
SR844 Lock-in Amplifier		Frequency range: 25kHz to 200MHz
NI USB-6341 Data Acquisition		16 analog inputs, 500kS/s
LM1875 Audio Power Amplifier		Bandwidth 70kHz, Output power 30w Current capability: 4A
Radius of target imaging area		10cm
Excitation coil	Diameter	2cm
	Turns	20
	Resistance	0.8Ω
	Applied current	100kHz, 0.9A(RMS)
Sensing coil	Diameter	2cm
	Turns	20
	Resistance	0.8Ω

(a) design graph

(b) top view

(c) actual structure

(d) coils as exciter/sensor

Fig. 6.12 Experimental set up for magnetic induction tomography

6.3.2 MEASUREMENTS AND DISCUSSION

As shown in Fig. 6.12(a), two sensing coils are fixed on the rotating vertical slides that are always on the opposite side of imaging area. Two exciting coils are installed respectively on the slides attached to the base. The rotation speed of DC motor can be adjusted by supply voltage and set to be 6s/round approximately. Measurements from two sensing coils can be recorded at the same time. And then experiment was conducted again with an aluminum block placed inside the imaging domain, near excitation coil.

Sensing coils were manually adjusted to be at the same location for each round of experiment. For comparison purpose, synchronization between different rounds is an important issue. As raw data, measurements were recorded for 20 seconds after the motor started rotating. Because the measured magnetic field should be at maximum value when the sensor is nearest to the excitation coil, it is reasonable to identify the starting point of a revolution with the peaks in raw data. As in Fig. 6.13-6.16, measurements were captured for approximately three revolutions and each revolution included two peaks due to two excitation coils.

Fig. 6.13 Measurements from Sensor *a* when no object present

Fig. 6.14 Measurements from Sensor *b* when no object present

Fig. 6.15 Measurements from Sensor *a* when object is present

Fig. 6.16 Measurements from Sensor *b* when object is present

By comparing Fig. 6.13 and Fig. 6.14, it can be seen when no object is present, signal variations from sensor *a* and sensor *b* were exactly opposite. This is reasonable because measurements from two sensors can be considered to have a 180 degree phase difference since they are on the opposite side of imaging domain. Do notice phase difference here does not refer to the measured phase information from lock-in amplifier, but the measurement sequence during one revolution. And when conductive object is present, it is difficult to directly deduce the object location from Figures 6.15 and 6.16, although disturbance in signal can be observed.

Imaginary component of magnetic flux density has been used as data for image reconstruction in Chapter 6.2, but it is impossible to abstract from practical measurements. Therefore lock-in amplifier is used to obtain the magnitude and phase information of signal. And phase information may as well represent the effect from eddy current, which indicates the location of conductive object. Hence, the phase information is used to replace data B_r of the imaging problem described as in Eq. 5.9 and 5.11. The reconstructed images were shown as Fig. 6.17-6.18.

As mentioned in Chapter 6.2, COMSOL simulation provides magnetic field information using complex values. And measurements from practical experiments are described using phase and magnitude. Although it is difficult to directly compare them,

reconstructed image as shown in Fig. 6.9(b) has the similar pattern with images in Figures 6.17 and 6.18.

Fig. 6.17 Conductivity image based on Sensor *a*

Fig. 6.18 Conductivity image based on Sensor *b*

6.4 EXPERIMENT WITH HIGH FREQUENCY EXCITATION

The previous experiment was conducted with 100 kHz excitation and an aluminum block was used as the imaging object. When low conductivity object is present, high frequency excitation is required so that the eddy current signal can be large enough for measurements.

6.4.1 EXPERIMENT SETUP

It is expected that high frequency experiment will be more sensitive to position error and electromagnetic noise. Thus, instead of simple DC power supply, PWM signal is generated by NI PXIe-6368 data acquisition instrument and used for position control. Aluminum sheet of 0.5mm thickness is used to shield the whole structure from electromagnetic noise. In order to realize the excitation/measurement with single instrument and minimize the synchronization error, a two-port vector network analyzer NI PXIe-5632 is used. Since high frequency excitation is used, the impedance of the excitation coil at the operating frequency can be much larger than its resistance. If the excitation coil consists of too few turns, the mutual inductance between the excitation coil and sensing coil can be too low that inductive signal is too small; if the excitation coil consists of too many turns, its impedance becomes too high and the excitation current is reduced with limited power supply. Here an air-core cylindrical coil with 2cm diameters and 35 turns winding is used as the excitation coil. The experiment is conducted with 10 MHz excitation and the current is approximately 1.3A. The modified experimental setup is detailed as in Table 6.2 and Fig. 6.19.

Table 6.2 Parameters of high frequency experiment

DC motor		Power supply: 3.5V
Position control with PWM signal		44Hz, Duty cycle: 0.048~0.078
NI PXIe-5632		8.5 GHz full two-port vector network analyzer
THS4502 Fully differential amplifier		Bandwidth 370MHz, Slew Rate: 2800 V/ μ s
Radius of target imaging area		10cm
Excitation coil	Diameter	2cm
	Turns	35
	Resistance	1.2 Ω
	Applied current	10MHz, 1.3A(RMS)
Sensing coil	Diameter	2cm
	Turns	20
	Resistance	0.8 Ω

(a) Acrylic structure

(b) Shielding with aluminum sheet

(c) Imaging area

(d) Measurement instrument

Fig. 6.19 Setup for high frequency experiment

As mentioned in previous chapter, a gear and belt structure was used to prevent the wire blocking the imaging area. However, after conducting the experiments, it is found that the asymmetric structure as Fig. 6.12 wobbles during the rotation, which would cause extra experimental error. Therefore the mechanical structure has been modified as Fig. 6.19(a). The excitation coils are installed on the inner layer and would be rotating along with the circular table. The sensing coils on the outer layer are fixed. By making

space under the rotation table, the wires from the excitation coil can be connected to the measuring instrument without affecting the imaging area. Also, a bearing is installed under the rotation table and the DC motor is installed on top of the rotating device. The imaging area is shown in Fig. 6.19(c).

With a wide bandwidth of 8.5GHz, the vector network analyzer as in Fig. 6.19(d) compare the signal from port 2 with the signal from port 1 and describe the differences using S-parameters, S_{11} , S_{22} , S_{12} and S_{21} . Here the signal from port 1 is used as the excitation source and a fully differential amplifier THS4502 is used to drive the excitation coil, as explained in Fig. 6.20. In order to increase the signal/noise ratio, voltage amplifier is used for the sensing coil and the amplified signal is connected to port 2 of the vector network analyzer. Therefore, only S_{21} will be examined here.

Fig. 6.20 Diagram of instrumentation

6.4.2 MEASUREMENTS AND DISCUSSION

In the low frequency experiments, measurements have been obtained for the rotation case and the phase information was directly used for inversion. When high frequency excitation is used, the noise would become much more significant than the low frequency experiments. In order to analyze the experiment setup and examine the signal variation carefully, measurements are obtained first when the excitation coil and sensing coil are installed at fixed locations.

(1) Static system results

With the experimental diagram shown in Fig. 6.21, recordings of the linear magnitude and phase of S_{21} were taken at 1-cm intervals. To reduce noise, 1500 samples were taken for every measurement and averaged for three seconds. According to existing studies [117, 118], the variations caused by an object with high conductivity are mainly shown as the magnitude changes when high frequency excitation is applied. This is due to the skin effect, which cause the eddy current to flow only within a small penetration depth and generate strong opposing magnetic field.

Fig. 6.21 Diagram of experiment

As a highly sensitive instrument, vector network analyzer requires transmission calibration every time before conducting the experiment, which sets the calibrated scenario as a reference. In ideal calibrated state, the linear magnitude of S_{21} would be 1 and the phase would be zero. After calibrating the instrument when no object is present, a plot of the magnitude changes in percentage is shown in Fig. 6.22 when the imaging object is present and moved away from exciter/sensor.

Fig. 6.22 Magnitude changes versus object position

With an aluminum block as the imaging target, the magnitude change is as high as 5.1% when the sample is close to both the excitation coil and sensing coil. The reason of negative changes is that the eddy current induced in the aluminum block would be an opposing field compared to the field from excitation coil. Thus their effects on the sensing coil would be opposing too. And as the distance increases, the magnitude change decreases due to the reduced eddy current effect. A 5% (by weight) saline water is also used as the imaging target and the magnitude changes are shown in Fig. 6.22. As expected, the magnitude change is too small to reflect the location changes, since the eddy current effect would be much smaller than the aluminum block. The phase changes versus the object location is shown as Fig. 6.23 and it is observed that the phase changes systematically vary as a function of the distance.

Fig. 6.23 Phase changes versus object position

As shown in Fig. 6.21, the measurement in the sensing coil is a combination of inductive signals due to the excitation coil and the eddy current in the conductive object. Since the exciter is closer to the sensor and the applied signal is much bigger than the eddy current effect, it can be very difficult to distinguish the variation caused by the conductive object, especially with low conductivity. Therefore a different setup is used to relatively increase the inductive signal due to the eddy current. As shown in Fig. 6.24, two sets of exciter/sensor are installed on opposite sides of the target area. A differential amplifier is used to subtract the signal from two sensors when object is moved along the horizontal axis at 2-cm intervals. Again, for single measurement, 1500 samples were taken and averaged for three seconds.

Fig. 6.24 Diagram of experiment

The magnitude changes due to the aluminum block are shown in Fig. 6.25 and it is apparent the variation is increased comparing to Fig. 6.22. The changes of the baseline could be due to the calibration error.

Fig. 6.25 Magnitude changes versus object position

(2) Rotational system results

As shown in Fig. 6.19, two exciting coils are fixed on the rotating vertical slides that are always on the opposite side of imaging area and rotating with the circular table. Two sensing coils are installed on the slides attached to the base. The experiments are conducted with and without the imaged object, which is a 5 cm aluminum block.

For comparison purpose, synchronization and location between different rounds of

experiments are important. The PWM signal controlled motor is set to start from the same location. But since the recording program for VNA has varying preset time, it is difficult to match the starting time of recording with the starting time of rotation exactly. So the measurements during about three rounds of rotations are recorded, and exact starting point of one period can be observed based on the periodic signals. Before each experiment when no object is present, the VNA instrument is calibrated therefore the magnitude is set to be 1 and phase angle is set to be 0. As seen in Fig. 6.26(a) and Fig. 6.27(a), the phase angle is zero at the starting point. Magnitude in Fig. 6.26(b) starts from below 1, but by observing the periodic signal which shows starting value of second round as 1, it is reasonable to deduce the synchronization error exists here. Magnitude in Fig. 6.27(b) starts from approximately 1. Do notice this value does not mean the actual measured voltage, therefore magnitude wise, cannot be compared with the low frequency results as Fig.6.13-14. But it is seen that Fig. 6.26 shows consistent variation trend with Fig. 6.13. Also, when signal from sensor *a* and sensor *b* are used as two inputs of a differential amplifier, the measurements during three rounds rotation are shown as Fig. 6.28 and Fig. 6.31.

Fig. 6.26 Measurements from Sensor *a* when no object present

(a) Phase

(b) Magnitude

Fig. 6.27 Measurements from Sensor *b* when no object present

(a) Phase

(b) Magnitude

Fig. 6.28 Measurements from (Sensor *a* – Sensor *b*) when no object present

(a) Phase

(b) Magnitude

Fig. 6.29 Measurements from Sensor *a* when object is present

(a) Phase

(b) Magnitude

Fig. 6.30 Measurements from Sensor *b* when object is present

(a) Phase

(b) Magnitude

Fig. 6.31 Measurements from (Sensor *a* – Sensor *b*) when object is present

(3) Imaging results

After obtaining one round measurements from Fig. 6.26-31 and subtracted the signal when no object is present from the signal when the object is present, the results are used as the data of the imaging problem described in Eq. 5.9 and 5.11. The reconstructed images are shown as below.

Fig. 6.32 Conductivity image based on Sensor *a*

Fig. 6.33 Conductivity image based on (Sensor *a* – Sensor *b*)

Fig. 6.32 and 6.33, in which the red circles indicate the actual object, shows approximately similar pattern, but the image based on the differential signal from sensor *a* and sensor *b* is slightly better since Fig. 6.33 shows a mistaken high value spot on the opposite side.

(4) Measurements and inverted results for saline water

An aluminum block has been used as the imaging target for the rotational experiments presented above. When low conductivity object like saline water is used as

the imaging target, the signal is much weaker and more unstable especially during the rotation. The rotational experiments have been conducted, however no reasonable results have been obtained due to lack of consistency. Therefore, the exciters and the circular table for placing the objects as in Fig. 6.19(a) are manually rotated to 18 equally distributed locations. For each location, approximately 1500 measurement samples are obtained within 3 seconds and averaged. The results are shown as in Fig. 6.34 and no obvious variation can be seen whether the saline bottle is present or not. The static experiment has shown the phase variation is a function of the distance between the saline bottle and the excitation coil/sensing coil, as in Fig. 6.23. But the variation is so small and may not be able to detect when the sensing coil is moving farther from the excitation coil.

Fig. 6.34 Measurements from sensor *a*

One of the possible solutions for increasing the measured signal is to use a higher excitation signal therefore increase the eddy current, but the effect can be limited since the inductive signal directly due to the excitation would be increased at the same time. The inductive signal due to the eddy current would still be relatively small and fail to be abstracted. To deal with this, two excitation coils with opposite polarity are installed as in Fig 6.35. When the sensor is installed in the vertically symmetrical plate between two

excitation coils, ideally the magnetic flux that goes through the sensor would be mainly along the vertical direction due to symmetric cancellation. This is validated by actual measurements that the signal in the sensor can be reduced to about 15% when the exciter as in Fig. 6.35 is used instead of single excitation coil. Of course the excitation field on the other side of the exciter, where the eddy current would be generated when the conductive object is present, would also have the cancelling effect in the vertically symmetrical plate. But since it is expected the imaging target will be of a more random shape and at a random location, the reduction of the eddy current would be relatively minor.

Fig. 6.35 Improved exciter

With this set of exciter, the measurements are obtained again at 18 equally distributed locations. For each location, approximately 1500 measurement samples are obtained within 3 seconds and averaged. The experiments are conducted and the measurements are shown as in Fig. 6.36-37. A trend can be observed after subtracting the no object measurements from the measurements with saline bottle inside: the magnitude starts with negative value and gradually increases to the approximately symmetric positive value; the phase starts with positive value and gradually decreases to the approximately symmetric negative value. The varying range is $-0.3\% \sim 0.3\%$ for magnitude and $-8 \sim 8$ for phase angle.

(a) Magnitude

(b) Phase

Fig. 6.36 Measurements

(a) Magnitude

(b) Phase

Fig. 6.37 Substraction results

Due to the limited measurement quantities, only 18 dipoles in a circular pattern are used to represent the imaging area, represented using red dot in Fig. 6.38(b) and dipole 1 is nearest to the location of saline bottle, indicated with black circle in Fig. 6.38(b). The inverted results and the interpolated image are both shown in Fig. 6.38.

(a) Inverted dipole moment

(b) Interpolated 2D image

Fig. 6.38 Inverted results and interpolated image

It can be seen that the inverted dipole moment reflected the location of the saline bottle correctly. But the 2D image is rather coarse because the measurement quantity is limited. In the future, the imaging system will be improved that it is capable of pausing intermittently and recording the measurements for further averaging purpose. It is expected better images can be obtained with more available measurements.

VII. CONCLUSION AND FUTURE WORK

7.1 ACCOMPLISHMENTS AND CONTRIBUTION

This research developed a novel sensing-oriented approach to model electromagnetic fields and its applications in orientation sensing and magnetic induction tomography. Three specific contributions have been made:

I. Time-varying electromagnetic field modeling and induction computation

Magnetic elementary fields have been utilized to account for the time-varying field by introducing multiple dipoles with magnetic moment. By utilizing vector-potential-based governing equation instead of scalar form, application of this approach is largely broadened from physics and mathematical basis. And as a semi-analytical method that provides close-form solutions for time-varying magnetic field, this model integrates the advantage of both analytical and numerical methods. It can be computed much more efficiently without sacrificing the accuracy.

The modeling results have been shown and compared with solutions that are either analytically calculated or experimentally measured. And then, a novel induction computation approach has been developed by utilizing the extended DMP model. With the help of discrete source term of dipole moments, it has been shown that complex integral formula to calculate mutual induction can be computed by finite summation and reduced computation effects dramatically while maintaining accuracy.

II. Magnetic field based motion sensor

With DMP model simulating magnetic field from PM and the extended DMP model for time-varying electromagnetic field, two magnetic-field-based motion sensors have been demonstrated.

- For a PM based motion sensor, the eDMP model has been utilized to characterize varying magnetic field at any point. As a single dipole is rotating,

the resultant magnetic field has been shown as an ellipsoid. The same pattern was validated for the DMP model and tracking strategy has been developed for the PM attached to the moving object. By utilizing the sensor resolution and measurable field range as constraints, an optimization scheme for sensor system configuration has been presented.

- For an induction based orientation sensor, modeling accuracy of the eDMP model has been shown by comparison with Grover's formulas. Two different practical setups have been designed for experimental validation: first, a cylindrical coil was rotated around another one, which can be used whenever PM based motion sensor is applicable and has the advantage of simple customization; and second, tubes with wound coils were bended with different curvatures, which has the potential to aid medical intubation procedures by detecting its orientation.

III. Modeling of magnetic induction tomography

Since modeling of electromagnetic field has been developed and implementation on induction sensing validated, the eDMP model has been further utilized to develop magnetic induction tomography. Still as source term, distributed dipoles were considered as the imaging target and the inverse problem were discussed according to a novel model for magnetic induction tomography. Sensitivity matrix analysis has been conducted based on the imaging performance and the effect of multiple excitation sources. Due to the nonlinear nature of the problem and ill-condition of sensitivity matrix, difficulty with inversion has been addressed and iterative computations were to be expected as in Chapter V. Because the distributed dipoles accounted for eddy current inside conductive object, it indicated the area that needs further inspection and was reasonably used as a guiding criterion for adaptive refinement. Numerical

simulation using commercial software, COMSOL Multiphysics, has been used to simulate the eddy current environment. Magnetic field information was examined first and then used as data for the inverse problem. A simple magnetic induction tomography system was also built to test for practical applications. Reconstructed images based on both simulation and experimental results were presented in Chapter VI.

In summary, this research proposed a discrete modeling method of time-varying electromagnetic field and exploited its potential of computing interactions between various field sources. The modeling performance has been proved to be both efficient and accurate; also the applications in induction sensor and tomography system have been demonstrated. Fast computation of mutual inductance between arbitrarily located coils has been demonstrated and a novel forward mapping method has been illustrated for magnetic induction tomography.

7.2 FUTURE WORK

Although the novelty and contributions of this research has been summarized as in Chapter 7.1, there are still many difficulties being exposed in this research area. Especially for magnetic induction tomography, adaptive discretization scheme presented here are only for simple implementation. More detailed study is required to provide better imaging performance generally. Also, the current experiment set up provides quite noisy measurements during continuous rotation and the design of exciters and sensors are to be optimized for better operation.

Future developments and research directions are expected as below.

- The extended DMP model for an induction based orientation sensor can be implemented for medical intubation procedures. In order to explore the potential usage and sensing performance, more detailed modeling of coils

with various geometry need to be done and examined. Design optimization is expected in order to find the most suitable sensor structure for the specific application. Background study about intubation path and existing sensors will also be done for the integrity of the research.

- As the geometry of the conductivity distribution becomes more complex, which is the case for a realistic problem, adaptive refinement procedure would become more indispensable. Further research would be done on more detailed refinement approaches considering specific imaging problems.

- The exciting/sensing coil configuration and arrangement need to be carefully analyzed and then reinforced after further experiments. The ultimate objective is to develop a stably functional system that provides real-time satisfying images, but the current research is just the beginning and many challenging tasks wait to be solved. For instance, various methods of reducing background signal have already been investigated in existing studies but the signal noise ratio is still poor. Also, due to the ill-conditioned nature of the eddy current problem, the inversion quality of the sensitivity matrix has always been bad and various regularization techniques have been developed. However, the imaging performance of magnetic induction tomography is still limited. With the eDMP model as a useful tool to simulate the magnetic field from multiple sources, it is possible to design the excitation field in much more details that more dynamic imaging procedures can be realized.

- Current regularization techniques are mostly discussed in regards to pure mathematical theories about matrix condition. Considering a specific

induction-based imaging problem, further work would be focused on search for a method that can better integrate and utilize physical information provided by distributed dipole models, also for signal processing purpose.

Appendix a. Procedures of solving linear PDEs with moment methods

Consider an inhomogeneous equation

$$L(f) = g \quad (\text{a.1})$$

where L is a linear operator, which can be an integral operator, Laplace operator, etc; g is known and f is to be determined. Let f be expanded in a series of functions $f_1, f_2, f_3, \dots$ in the domain of L , as

$$f = \sum_n a_n f_n \quad (\text{a.2})$$

where a_n is constant and f_n is usually called basis function. For exact solutions, Eq. a.2 is usually an infinite summation and the f_n form a complete set of basis functions. For approximate solutions, it is usually a finite summation.

Substituting Eq. a.2 into Eq. a.1, and using the linearity of L , we have

$$\sum_n a_n L(f_n) = g \quad (\text{a.3})$$

It is assumed that a suitable inner product $\langle f, g \rangle$ has been defined for the problem. Now define a set of weighting functions, or testing functions $w_1, w_2, w_3, \dots$ in the range of L , and take the inner product of Eq. a.3 with each w_m . The result is

$$\sum_n a_n \langle w_m, Lf_n \rangle = \langle w_m, g \rangle \quad (\text{a.4})$$

This set of equations can be rewritten in matrix form as

$$[l_{mn}][a_n] = [g_m] \quad (\text{a.5})$$

where

$$[l_{mn}] = \begin{bmatrix} \langle w_1, Lf_1 \rangle & \langle w_1, Lf_2 \rangle & \dots & \langle w_1, Lf_n \rangle \\ \langle w_2, Lf_1 \rangle & \langle w_2, Lf_2 \rangle & \dots & \langle w_2, Lf_n \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle w_m, Lf_1 \rangle & \langle w_m, Lf_2 \rangle & \dots & \langle w_m, Lf_n \rangle \end{bmatrix}$$

$$[a_n] = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \quad [g_m] = \begin{bmatrix} \langle w_1, g \rangle \\ \langle w_2, g \rangle \\ \vdots \\ \langle w_m, g \rangle \end{bmatrix}$$

If the matrix $[l_{mn}]$ is nonsingular and can be directly inverted, then $[a_n]$ can be given by

$$[a_n] = [l_{mn}]^{-1} [g_m] \quad (\text{a.6})$$

And the solution of f is given by Eq. a.2. For concise expression of this result, define the matrix of functions

$$[f_n] = [f_1 \quad f_2 \quad \dots \quad f_n] \quad (\text{a.7})$$

Then the solution can be written as

$$f = [f_n][a_n] = [f_n][l_{mn}]^{-1} [g_m] \quad (\text{a.8})$$

This solution may be exact or approximate, depending upon the choice of the basis function and weighting function. The particular choice $w_m = f_n$ is known as Galerkin's method.

REFERENCES

- [1] P. Lorrain, D. R. Corson, and F. Lorrain, *Electromagnetic Fields and Waves: Including Electric Circuits*: W.H. Freeman & Company, 1988.
- [2] H. E. Knoepfel, *Magnetic Fields: A Comprehensive Theoretical Treatise for Practical Use*: Wiley, 2008.
- [3] B. S. Guru, and H. R. Hiziroglu, *Electromagnetic Field Theory Fundamentals*: Cambridge University Press, 2004.
- [4] J. D. Jackson, *Classical electrodynamics*, 3 ed., 1999.
- [5] P. L. K. J. Binns, C. Trowbridge, *The analytical and numerical solution of electric and magnetic fields*: Wiley, 1992.
- [6] C. G. Someda, *Electromagnetic Waves*: Taylor & Francis, 1998.
- [7] D. K. Cheng, *Field and Wave Electromagnetics*: Addison-Wesley, 1989.
- [8] J. Schwinger, L. L. Deraad, K. Milton, and W. Tsai, *Classical Electrodynamics*: Perseus Books, 1998.
- [9] D. J. Griffiths, *Introduction to Electrodynamics*: Pearson Education, 2013.
- [10] A. S. Mahajan, and A. A. Rangwala, *Electricity and Magnetism*: Tata McGraw-Hill, 1988.
- [11] D. M. Cook, *The Theory of the Electromagnetic Field*: Dover Publications, 1975.
- [12] T. L. Chow, *Introduction to Electromagnetic Theory: A Modern Perspective*: Jones and Bartlett Publishers, 2006.
- [13] K. H. Huebner, *The Finite Element Method for Engineers*: Wiley, 2001.
- [14] A. Taflove, and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*: Artech House, 2005.
- [15] M. N. O. Sadiku, *Numerical Techniques in Electromagnetics*, Second Edition: Taylor & Francis, 2000.
- [16] A. R. Mitchell, and D. F. Griffiths, *The finite difference method in partial differential equations*: Wiley, 1980.
- [17] D. W. Pepper, and J. C. Heinrich, *Finite Element Methods: Basic Concepts And Applications: Basic Concepts and Applications*: Taylor & Francis, 1992.
- [18] M. Schanz, and O. Steinbach, *Boundary Element Analysis: Mathematical Aspects and Applications*: Springer, 2007.
- [19] K.-M. Lee, and H. Son, "Distributed multipole model for design of permanent-magnet-based actuators," *IEEE Transactions on Magnetics*, vol. 43, pp. 3904-3913, 2007.
- [20] K.-M. Lee, K. Bai, and J. Lim, "Dipole models for forward/inverse torque computation of a spherical motor," *IEEE/ASME Transactions on Mechatronics*, vol. 14, pp. 46-54, 2009.
- [21] H. Son, and K.-M. Lee, "Distributed Multipole Models for Design and Control of PM Actuators and Sensors," *Mechatronics, IEEE/ASME Transactions on*, vol. 13, no. 2, pp. 228-238, 2008.
- [22] Hong Z. Tan, Lynne A. Slivovsky, Alex Pentland, "A Sensing Chair Using Pressure Distribution Sensors," *Mechatronics, IEEE/ASME Transactions on*, vol. 6, no. 3, pp. 261-268, 2001.
- [23] Mobedi, B., Nejat G., "3-D Active Sensing in Time-Critical Urban Search and Rescue Missions," *Mechatronics, IEEE/ASME Transactions on*, vol. 17, no. 6, pp. 1111-1119, 2012.

- [24] Menciassi A., Eisinberg A., Carrozza M.C., Dario P., "Force sensing microinstrument for measuring tissue properties and pulse in microsurgery," *Mechatronics, IEEE/ASME Transactions on*, vol. 8, no. 1, pp. 10-17, 2003.
- [25] C. C. Yang, and Y. L. Hsu, "A review of accelerometry-based wearable motion detectors for physical activity monitoring," *Sensors*, 2010.
- [26] Z. Sun, G. Bebis, and R. Miller, "On-road vehicle detection: a review," *IEEE transactions on pattern analysis and machine intelligence*, vol. 28, no. 5, pp. 694-711, 2006.
- [27] W. J. Fleming, "Overview of automotive sensors," *IEEE Sensors Journal*, vol. 1, no. 4, pp. 296-308, 2001.
- [28] D. V. Lee, and S. A. Velinsky, "Analysis and experimental verification of a three-dimensional noncontacting angular motion sensor," *IEEE/ASME Transactions on Mechatronics*, vol. 12, no. Copyright 2008, The Institution of Engineering and Technology, pp. 612-622, 2007.
- [29] N. Shentu, H. Zhang, Q. Li, H. Zhou, R. Tong, and X. Li, "A Theoretical Model to Predict Both Horizontal Displacement and Vertical Displacement for Electromagnetic Induction-Based Deep Displacement Sensors," *Sensors*, vol. 12, no. 1, pp. 233-259, 2011.
- [30] S. M. Donecker, T. A. Lasky, and B. Ravani, "A mechatronic sensing system for vehicle guidance and control," *IEEE/ASME Transactions on Mechatronics*, vol. 8, no. 4, pp. 500-510, 2003.
- [31] S. Xue, N. He, and Z. Long, "Electromagnetic Field Analysis and Modeling of a Relative Position Detection Sensor for High Speed Maglev Trains," *Sensors*, vol. 12, no. 5, pp. 6447-6462, 2012.
- [32] S. Foong, K.M. Lee, and K. Bai, "Harnessing Embedded Magnetic Fields for Angular Sensing With Nano-degree Accuracy," *Mechatronics, IEEE/ASME Transactions on*, vol. 17, pp. 687-696, 2012.
- [33] F. Wu and H.S. Son, "Optimization of Measuring Magnetic Fields for Position and Orientation Tracking," *Mechatronics, IEEE/ASME Transactions on*, vol. 16, pp. 440-448, 2011.
- [34] N. Vu Huy and K. Won-Jong, "Design and Control of a Compact Lightweight Planar Positioner Moving Over a Concentrated-Field Magnet Matrix," *Mechatronics, IEEE/ASME Transactions on*, vol. 18, pp. 1090-1099, 2013.
- [35] F. Wu and H.S. Son, "Inductance calculation for orientation/position by extended DMP model," in *Advanced Intelligent Mechatronics (AIM)*, 2013 IEEE/ASME International Conference on, 2013, pp. 774-779.
- [36] S. Babic, F. Sirois, C. Akyel, and C. Girardi, "Mutual Inductance Calculation Between Circular Filaments Arbitrarily Positioned in Space: Alternative to Grover's Formula," *Magnetics, IEEE Transactions on*, vol. 46, pp. 3591-3600, 2010.
- [37] T. C. John, "Inductance Calculations for Noncoaxial Coils Using Bessel Functions," *Magnetics, IEEE Transactions on*, vol. 43, pp. 1023-1034, 2007.
- [38] E. Joy, A. Dalal, and P. Kumar, "The Accurate Computation of Mutual Inductance of Two Air Core Square Coils with Lateral and Angular Misalignments in a Flat Planar Surface," *Magnetics, IEEE Transactions on*, vol. 50, 2014.
- [39] H. Fukuda, M. Odagaki, O. Hiwaki, A. Kodabashi, and T. Fujimoto, "Brain activity during bilateral rapid alternate finger tapping measured with magnetoencephalography," *Journal of Applied Physics*, vol. 105, no. 7, pp. 07-313, 2009.
- [40] S. M. Plis, J. S. George, S. C. Jun, J. Pare-Blagoev, D. M. Ranken, C. C. Wood, and D. M. Schmidt, "Modeling spatiotemporal covariance for

- magnetoencephalography or electroencephalography source analysis,” *Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)*, vol. 75, no. 1, 2007.
- [41] O. Y. Mirnyuk, and A. Y. Loskutov, “Detection of cardiac pathologies using dimensional characteristics of RR intervals in electrocardiograms,” *Biophysics*, vol. 51, no. 1, pp. 115-19, 2006.
- [42] K. Hnatkova, M. Malik, P. Kulakowski, and A. J. Camm, “Wavelet analysis of signal-averaged electrocardiograms. I. Design, clinical assessment, and reproducibility of the method,” *Annals of Noninvasive Electrocardiology*, vol. 5, no. 1, pp. 4-19, 2000.
- [43] D. M. Goldenholz, S. P. Ahlfors, M. S. Hamalainen, D. Sharon, M. Ishitobi, L. M. Vaina, and S. M. Stufflebeam, “Mapping the signal-to-noise-ratios of cortical sources in magnetoencephalography and electroencephalography,” *Human Brain Mapping*, vol. 30, no. 4, pp. 1077-86, 2009.
- [44] M. A. Espy, A. N. Matlachov, R. H. Kraus, and J. B. Betts, "SQUID noise as a function of temperature: a survey of three SQUID's," *Applied Superconductivity, IEEE Transactions on*, vol. 13, no. 2, pp. 731-4.
- [45] Z. Jun, L. Yang, Z. Tiange, and W. Ge, "Overview of multi-source CT systems and methods," *Proc. SPIE*, vol. 8, 2010.
- [46] U. Elchalal, N. Yanai, D. V. Valsky, H. Y. Sela, Y. Erez, S. Yagel, and M. Nadjari, “Application of 3-dimensional ultrasonography to imaging the fetal anal canal,” *Journal of Ultrasound in Medicine*, vol. 29, no. 8, pp. 1195-1201, 2010.
- [47] M. A. Balafar, A. R. Ramli, M. I. Saripan, and S. Mashohor, “Review of brain MRI image segmentation methods,” *Artificial Intelligence Review*, vol. 33, no. 3, pp. 261-274, 2010.
- [48] M. Bozzali, and A. Cherubini, “Diffusion tensor MRI to investigate dementias: a brief review,” *Magnetic Resonance Imaging*, vol. 25, no. 6, pp. 969-77, 2007.
- [49] Y. Deng, and X. Liu, “Electromagnetic Imaging Methods for Nondestructive Evaluation Applications,” *Sensors*, vol. 11, no. 12, pp. 11774-11808, 2011.
- [50] K. Krueger, W. R. Scott Jr, and J. H. McClellan, "Location and orientation estimation of buried targets using electromagnetic induction sensors," *Proceedings of SPIE*, vol. 8357, 2012.
- [51] S. P. McKenna, and J. R. McKenna, “Modeling and analysis of the response of a triaxial, frequency-domain electromagnetic induction sensor to a buried linear conductor,” *Geophysics*, vol. 75, no. 1, pp. F1-F14, 2010.
- [52] H. Griffiths, W. R. Stewart, and W. Gough, “Magnetic Induction Tomography: A Measuring System for Biological Tissues,” *Annals of the New York Academy of Sciences*, vol. 873, no. 1, pp. 335-345, 1999.
- [53] A. Korjenevsky, V. Cherepenin, and S. Sapetsky, “Magnetic induction tomography: experimental realization,” *Physiological Measurement*, vol. 21, no. 1, pp. 89-94, 2000.
- [54] H. Scharfetter, H. K. Lackner, and J. Rosell, “Magnetic induction tomography: hardware for multi-frequency measurements in biological tissues,” *Physiological Measurement*, vol. 22, no. 1, pp. 131-146, 2001.
- [55] H. Scharfetter, “Planar gradiometer for magnetic induction tomography (MIT): theoretical and experimental sensitivity maps for a low-contrast phantom,” *Physiological Measurement*, vol. 25, no. 1, pp. 325-325, 2004.
- [56] H. Feshbach, and P. M. Morse, "Methods of theoretical physics," McGraw-Hill Interamericana, 1953.
- [57] J. A. Tegopoulos, and E. Kriezis, *Eddy currents in linear conducting media*: Elsevier, 1985.

- [58] K. J. Binns, P. Lawrenson, and C. Trowbridge, *The analytical and numerical solution of electric and magnetic fields*: Wiley, 1992.
- [59] N. G. Gencer, M. Kuzuoglu, and Y. Z. Ider, "Electrical impedance tomography using induced currents," *IEEE Transactions on Medical Imaging*, vol. 13, no. 2, pp. 338-350, 1994.
- [60] A. A. Kolyshkin, and R. Vaillancourt, "Series solution of an eddy-current problem for a sphere with varying conductivity and permeability profiles," *Magnetics, IEEE Transactions on*, vol. 35, pp. 4445-4451, 1999.
- [61] M. H. Pham, and A. J. Peyton, "A model for the forward problem in magnetic induction tomography using boundary integral equations," *IEEE Transactions on Magnetics*, vol. 44, pp. 2262-2267, 2008.
- [62] R. F. Harrington, and J. L. Harrington, *Field Computation by Moment Methods*: Oxford University Press, 1996.
- [63] O. Biro, and K. Preis, "Finite element analysis of 3-D eddy currents," *Magnetics, IEEE Transactions on*, vol. 26, no. 2, pp. 418-423, 1990.
- [64] N. Demerdash, and T. Nehl, "Use of numerical analysis of nonlinear eddy current problems by finite elements in the determination of parameters of electrical machines with solid iron rotors," *Magnetics, IEEE Transactions on*, vol. 15, pp. 1482-1484, 1979.
- [65] S. J. Salon, and H. B. Hamilton, "Calculation of Induced Field Current and Voltage in Solid Rotor Generators," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-97, pp. 1918-1924, 1978.
- [66] T. Nehl, and N. Demerdash, "Application of finite element eddy current analysis to nondestructive detection of flaws in metallic structures," *Magnetics, IEEE Transactions on*, vol. 16, pp. 1080-1082, 1980.
- [67] N. A. Demerdash, T. W. Nehl, F. A. Fouad, O. A. Mohammed, and R. H. Miller, "Solution of eddy current problems using three dimensional finite element complex magnetic vector potential," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-101, pp. 4222-4229, 1982.
- [68] F. Bouillault, Z. Ren, and A. Razek, "Calculation of 3D eddy current problems by a hybrid T-Omega method," *Magnetics, IEEE Transactions on*, vol. 26, pp. 478-481, 1990.
- [69] O. Biro, "Edge element formulations of eddy current problems," *Computer Methods in Applied Mechanics and Engineering*, vol. 169, pp. 391-405, 1999.
- [70] C. Li, Z. Ren, and A. Razek, "An approach to adaptive mesh refinement for three-dimensional eddy-current computations," *Magnetics, IEEE Transactions on*, vol. 30, pp. 113-117, 1994.
- [71] S. I. Hariharan, and R. C. MacCamy, "Integral equation procedures for eddy current problems," *Journal of Computational Physics*, vol. 45, pp. 80-99, 1982.
- [72] R. C. MacCamy, and E. Stephan, "Solution procedures for three-dimensional eddy current problems," *Journal of Mathematical Analysis and Applications*, vol. 101, pp. 348-379, 1984.
- [73] W. M. Rucker, R. Hoschek, and K. R. Richter, "Various BEM formulations for calculating eddy currents in terms of field variables," *Magnetics, IEEE Transactions on*, vol. 31, pp. 1336-1341, 1995.
- [74] H. Tsuboi, and M. Tanaka, "Three-dimensional eddy current analysis by the boundary element method using vector potential," *Magnetics, IEEE Transactions on*, vol. 26, pp. 454-457, 1990.

- [75] T. Misaki, and H. Tsuboi, "Computation of 3-dimensional eddy current problems by using boundary element method." *Magnetics, IEEE Transactions on*, vol. 12, pp. 2227-2230, 1985.
- [76] M. H. Pham, and A. J. Peyton, "Computation of 3-D Sensitivity Coefficients in Magnetic Induction Tomography Using Boundary Integral Equations and Radial Basis Functions," *Magnetics, IEEE Transactions on*, vol. 44, pp. 2268-2276, 2008.
- [77] R. Hiptmair, "Symmetric coupling for eddy current problems," *SIAM Journal on Numerical Analysis*, vol. 40, pp. 41-65, 2002.
- [78] S. Borm, and J. Ostrowski, "Fast evaluation of boundary integral operators arising from an eddy current problem," *Journal of Computational Physics*, vol. 193, pp. 67-85, 2004.
- [79] W. Hackbusch, and S. Borm, "H2-matrix approximation of integral operators by interpolation." *Journal Applied Numerical Mathematics*, vol. 43, no. 1-2, pp. 129-143, 2002.
- [80] N. G. Gencer, and M. N. Tek, "Electrical conductivity imaging via contactless measurements," *IEEE Transactions on Medical Imaging*, vol. 18, pp. 617-627, 1999.
- [81] N. G. Gencer, and M. N. Tek, "Electrical conductivity imaging via contactless measurements," *IEEE Transactions on Medical Imaging*, vol. 18, no. 7, pp. 617-627, 1999.
- [82] D. N. Dyck, D. A. Lowther, and E. M. Freeman, "A method of computing the sensitivity of electromagnetic quantities to changes in materials and sources." *Magnetics, IEEE Transactions on*, vol. 30, no. 5, pp. 3415-3418.
- [83] K. Hollaus, and et al., "Fast calculation of the sensitivity matrix in magnetic induction tomography by tetrahedral edge finite elements and the reciprocity theorem," *Physiological Measurement*, vol. 25, no. 1, pp. 159, 2004.
- [84] C. A. Ktistis, D W; Peyton, A J, "Calculation of the forward problem for absolute image reconstruction in MIT," *Physiological Measurement*, 2008, p. S455.
- [85] M. Soleimani, and W. R. B. Lionheart, "Absolute conductivity reconstruction in magnetic induction tomography using a nonlinear method," *IEEE Transactions on Medical Imaging*, vol. 25, no. 12, pp. 1521-30, 2006.
- [86] M. Hanke, "Regularization with differential operators. An iterative approach," *Numerical Functional Analysis and Optimization*, vol. 13, pp. 523, 1992.
- [87] R. D. Fierro, G. H. Golub, P. C. Hansen, and D. P. O'Leary, "Regularization by truncated total least squares," *SIAM Journal on Scientific Computing*, vol. 18, pp. 1223-1241, 1997.
- [88] T. Kitagawa, S. Nakata, and Y. Hosoda, "Regularization using QR factorization and the estimation of the optimal parameter," *BIT*, vol. 41, pp. 1049-1058, 2001.
- [89] C. Cohen-Bacrie, Y. Goussard, and R. Guardo, "Regularized reconstruction in electrical impedance tomography using a variance uniformization constraint," *Medical Imaging, IEEE Transactions on*, vol. 16, pp. 562-571, 1997.
- [90] R. Casanova, and e. al., "MIT image reconstruction based on edge-preserving regularization," *Physiological Measurement*, vol. 25, pp. 195, 2004.
- [91] P. Charbonnier, L. Blanc-Feraud, G. Aubert, and M. Barlaud, "Deterministic edge-preserving regularization in computed imaging," *Image Processing, IEEE Transactions on*, vol. 6, pp. 298-311, 1997.
- [92] D. Geman, and C. Yang, "Nonlinear image recovery with half-quadratic regularization," *Image Processing, IEEE Transactions on*, vol. 4, pp. 932-946, 1995.

- [93] R. Casanova, A. Silva, and A. R. Borges, "A quantitative algorithm for parameter estimation in magnetic induction tomography," *Measurement Science and Technology*, vol. 15, no. 7, pp. 1412-1419, 2004.
- [94] H. Scharfetter, K. Hollaus, J. Rosell-Ferrer, and R. Merwa, "Single-step 3-D image reconstruction in magnetic induction tomography: theoretical limits of spatial resolution and contrast to noise ratio," *Annals of Biomedical Engineering*, vol. 34, pp. 1786-1798, 2006.
- [95] R. Merwa, K. Hollaus, P. Brunner, and H. Scharfetter, "Solution of the inverse problem of magnetic induction tomography (MIT)," *Physiological Measurement*, vol. 26, no. 2, pp. 241-50, 2005.
- [96] M. Vauhkonen, M. Hamsch, and C. H. Igney, "A measurement system and image reconstruction in magnetic induction tomography," *Physiological Measurement*, vol. 29, pp. 445-454, 2008.
- [97] X. Zheng, L. Haijun, H. Wei, H. Chuanhong, S. Xiaodong, and Z. Zhanglong, "A multi-channel magnetic induction tomography measurement system for human brain model imaging," *Physiological Measurement*, vol. 30, pp. 175-186, 2009.
- [98] C. H. Riedel, and e. al., "Planar system for magnetic induction conductivity measurement using a sensor matrix," *Physiological Measurement*, vol. 25, pp. 403, 2004.
- [99] J. Rosell-Ferrer, R. Merwa, P. Brunner, and H. Scharfetter, "A multifrequency magnetic induction tomography system using planar gradiometers: data collection and calibration," *Physiological Measurement*, vol. 27, pp. 271-280, 2006.
- [100] J. Rosell, R. Casanas, and H. Scharfetter, "Sensitivity maps and system requirements for magnetic induction tomography using a planar gradiometer," *Physiological Measurement*, vol. 22, pp. 121-130, 2001.
- [101] S. Watson, A. Morris, R. J. Williams, H. Griffiths, and W. Gough, "A primary field compensation scheme for planar array magnetic induction tomography," *Physiological Measurement*, vol. 25, pp. 271, 2004.
- [102] D. Gursoy, and H. Scharfetter, "The effect of receiver coil orientations on the imaging performance of magnetic induction tomography," *Measurement Science and Technology*, vol. 20, no. 10, pp. 105505, 2009.
- [103] S. Watson, C. H. Igney, O. Dössel, R. J. Williams, and H. Griffiths, "A comparison of sensors for minimizing the primary signal in planar-array magnetic induction tomography," *Physiological Measurement*, vol. 26, pp. S319, 2005.
- [104] D. Gursoy, and H. Scharfetter, "Optimum receiver array design for magnetic induction tomography," *IEEE Transactions on Biomedical Engineering*, vol. 56, pp. 1435-1441, 2009.
- [105] S. Tan, B. J. Roth, and J. P. Wikswo, Jr., "The magnetic field of cortical current sources: the application of a spatial filtering model to the forward and inverse problems," *Electroencephalography and Clinical Neurophysiology*, vol. 76, pp. 73-85, 1990.
- [106] Y. Liang, I. M. Chen, Y. Guilin, and L. Kok-Meng, "Analytical and experimental investigation on the magnetic field and torque of a permanent magnet spherical actuator," *IEEE/ASME Transactions on Mechatronics*, vol. 11, pp. 409-19, 2006.
- [107] H. Chao, L. Mao, S. Shuang, Y. Wan'an, Z. Rui, and M. Q. H. Meng, "A Cubic 3-Axis Magnetic Sensor Array for Wirelessly Tracking Magnet Position and Orientation," *IEEE Sensors Journal*, vol. 10, pp. 903-913, 2010.

- [108] V. Schlageter, P. A. Besse, R. S. Popovic, and P. Kucera, "Tracking system with five degrees of freedom using a 2D-array of Hall sensors and a permanent magnet," *Sens. Actuators A, Phys. (Switzerland)*, vol. 92, no. 1-3, pp. 37-42.
- [109] E. Paperno, I. Sasada, and E. Leonovich, "A new method for magnetic position and orientation tracking," *Magnetics, IEEE Transactions on*, pp. 1938-1940, 2001.
- [110] T. Cuong, and M. M. Trivedi, "3-D Posture and Gesture Recognition for Interactivity in Smart Spaces," *Industrial Informatics, IEEE Transactions on*, vol. 8, no. 1, pp. 178-187, 2012.
- [111] H. Ghasemzadeh, and R. Jafari, "Physical Movement Monitoring Using Body Sensor Networks: A Phonological Approach to Construct Spatial Decision Trees," *Industrial Informatics, IEEE Transactions on*, vol. 7, no. 1, pp. 66-77, 2011.
- [112] V. Roberge, M. Tarbouchi, and G. Labonte, "Comparison of Parallel Genetic Algorithm and Particle Swarm Optimization for Real Time UAV Path Planning," *Industrial Informatics, IEEE Transactions on*, vol. 9, no. 1, pp. 132-141, 2012.
- [113] L. LoBello, and E. Toscano, "An Adaptive Approach to Topology Management in Large and Dense Real-Time Wireless Sensor Networks," *Industrial Informatics, IEEE Transactions on*, vol. 5, no. 3, pp. 314-324, 2009.
- [114] N. Motoi, M. Ikebe, and K. Ohnishi, "Real-Time Gait Planning for Pushing Motion of Humanoid Robot," *Industrial Informatics, IEEE Transactions on*, vol. 3, no. 2, pp. 154-163, 2007.
- [115] F. W. Grover, *Inductance calculations: working formulas and tables*: Courier Dover Publications, 2004.
- [116] S. I. Babic, and C. Akyel, "Calculating Mutual Inductance Between Circular Coils With Inclined Axes in Air," *Magnetics, IEEE Transactions on*, vol. 44, pp. 1743-1750, 2008.
- [117] H.-Y. Wei, and A. J. Wilkinson, "Design of a Sensor Coil and Measurement Electronics for Magnetic Induction Tomography," *Instrumentation and Measurement, IEEE Transactions on*, vol. 60, no. 12, pp. 3853-3859, 2011.
- [118] H. Griffiths, "Magnetic induction tomography," *Measurement Science and Technology*, vol. 12, pp. 1126-1131, 2001.