Optimization, dynamics analyses and trajectory tracking of aircraft perching maneuvers

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OPTIMIZATION, DYNAMICS ANALYSES AND TRAJECTORY TRACKING OF AIRCRAFT PERCHING MANEUVERS

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Abstract

This thesis presents optimization, parametric study, stability analysis and trajectory tracking of fixed-wing aircraft perching maneuvers, where gravity and aerodynamics forces are used simultaneously for climbing to decelerate and land with low velocity or negligible impact. A single-phase formulation, which minimizes the trajectory length of the maneuver, is proposed to simplify the optimization formulation and generate lower undershoot perching solutions, compared with the existing two-phase approach. A parametric study is carried out to identify some of the key aircraft geometric features and parameters having significant influence on the maneuver. The aircraft parameters are varied and optimal perching solutions are computed from which their influence on the maneuver is identified. Aerodynamics from changing aircraft configurations due to parameter changes is modeled using nonlinear vortex correction method coupled with Wagner's unsteady model. Stability analysis of the optimal perching trajectories is performed using contraction theory to analyze the effects of state perturbations. It is shown that the perching solutions are in general unstable and will diverge during the terminal phase. To avoid divergence, tracking of optimal trajectories is proposed. Sliding mode control technique is used to design the tracking controller, and its performance is validated through numerical simulations. The design of the controller is extended for tracking of a conventional aircraft landing maneuver by including lateral states into the model. The design of the controller is validated through numerical simulations, and its performance is compared with a PID controller.

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Chapter 1

Introduction

1.1 Background

Birds inspired humans to fly. It was, and will be man's continuing effort to learn from nature and use the knowledge for innovation and advancements in engineering. Aircraft perching is one such attempt to mimic nature and partly emulate the way bird's perch by landing with a near-zero velocity. Birds, using their flapping wings, generate the required lifting and propulsive forces for a wide range of flight conditions, even at low velocities. By changing the mode and frequency of flapping, they perform complex aerobatics and maneuvers that are highly difficult to be mimicked by a conventional fixed-wing aircraft. Their flapping capability enables them to perch adeptly with negligible impact, even in the presence of external winds. Aircraft wings, in this context, are fixed and fail to generate sufficient lift at low velocities to support the weight of the aircraft, which makes it not possible to perch exactly like birds. Bird-like perching can be realized along the lines of a cruise-to-hover maneuver [1, 2] using a high thrust-to-weight ratio ($(T/W)_{\max} > 1$) aircraft. However, this mode of landing is infeasible for low T/W ratio ($(T/W)_{\max} < 1$)

aircraft. To perch a low T/W ratio aircraft, birds approach to land on an elevated platform is adopted. A short climb maneuver is performed to convert its kinetic energy into potential energy [3]. A combination of aerodynamic forces and gravity decelerates the aircraft and brings down the velocity to near-zero at touchdown. This phase is called the climb-phase and the altitude gained during the climb is interpreted as undershoot [4]. Depending on the available power, aircraft may be required to dive (called the dive-phase) to reach the initial conditions of the climb phase. Fig. 1.1 gives an illustration of aircraft perching maneuver. Stills of various phases in landing of a bird are shown in Fig. 1.2.

Figure 1.1: Illustration of aircraft perching maneuver.

Figure 1.2: Stills of various phases in bird's perching maneuver.

[Source: http://images.betterphoto.com/0013/0305032143261blue_bird_landing.jpg]

Perching maneuver of a bird involves simultaneous generation of high lift and drag forces at low velocities. To maximize these two forces, birds use the principle of leading edge vortex (LEV) [3]. During landing, a vortex is created over the wing, past the leading edge, to create a pocket of low pressure region. This generates a large force perpendicular to the direction of chord, thus creating high lift and drag forces simultaneously, even at low angles-of-attack and velocities. Deceleration to zero velocity is achieved in several phases. Initial deceleration is achieved by keeping wings in a swept-back position, maintaining a low angle-of-attack and creating an LEV. Later, wings and tail are spread across, perpendicular to the direction of flow to create an airbrake and decelerate further. During the terminal phase, they either flap their wings at increased frequency or perform a short climbing to decelerate even more. Any remaining energy is finally absorbed by the bending legs after the landing impact.

Though most of the birds land normally on a flat ground as explained previously, few birds execute other interesting modes of landing. Humming birds are capable of hovering at a fixed position, and can take-off and land vertically. Some birds, like the Albatross, which spend most of the time gliding, do not use legs to hold their body weight, execute a controlled crash landing. Pelicans are capable of landing on water like amphibian airplanes. While touching down, they direct their feet forwards into the water, dragging and creating a brake force and come to rest. Some birds - similar to conventional airplanes - run for short distances during take-off and landing to increase lift forces.

Compared with a two-dimensional conventional landing maneuver [5], aircraft perching is different because of the additional climb-phase. The complete conventional landing maneuver, comprising a glide-path capture maneuver and a flare-path capture maneuver, can be considered equivalent to the dive phase in perching. In both conventional landing

and dive phase in perching, the aircraft starts from a level-flight trim state, descends to reduce the altitude, and pulls up at the end to bring the flight-path angle to zero. When the flight-path angle becomes zero, flaring in conventional landing terminates upon touchdown. However, in perching, flaring continues until the flight-path angle increases to a large value. The climb phase in perching can be viewed as aggressive or a continuation of flaring.

1.2 Motivation

The motivation to carry out research in perching mainly arises from several advantages the maneuver offers. Unmanned air vehicles (UAVs) are gaining prominence to conduct surveillance and reconnaissance operations. Most of the times, these missions require the aircraft to remain stationary and gather data. Quadcopters, which are capable of hovering, can be easily used for these operations. However, they have very low endurance and poor efficiency as most of the thrust generated is used to balance the weight. Fixed-wing aircraft, on the other hand, are quite efficient as thrust is used to counter the drag and lift is generated from the wings to balance weight. However, they do not have the capability to hover. If the fixed-wing aircraft is capable of perching, it can carry out the surveillance or reconnaissance operations without consuming any power to hover or fly. This can significantly increase the mission endurance.

Aircraft perching is a challenging problem as it requires precision landing and controllability at low speeds and high angles-of-attack. Autonomous perching requires a near-optimal robust controller to achieve high maneuverability. The problems like aerodynamic uncertainties and external wind disturbances further complicate the controller design. Air-

craft with perching capabilities can have superior performance during conventional landing. Perching can be interesting to aircraft design engineers as it add another dimension to the capabilities of aircraft and increases the flight envelope. All these advantages and challenges make aircraft perching an interesting problem, worthy of investigation.

1.3 Research Objective, Scope and Contributions

The objective of this research is to simplify the formulation of optimization of perching maneuvers, analyze the dynamics of the maneuver and design a controller for perching trajectory tracking.

The very first thought that comes from the idea of mimicking a bird landing maneuver is to design a flapping-wing aircraft for perching. Though it can be possible to perch using a flapping-wing aircraft, the scope of the thesis is restricted to a fixed-wing aircraft only. High powered aircraft ($(T/W)_{\max} \geq 1$) can perch on the lines of cruise to hover maneuver, but the idea is already ruled out, since the principle and mode of landing are different. Perching of low-powered aircraft ($(T/W)_{\max} < 1$) only is explored in this thesis. The work in this thesis is mainly confined to conventional, fixed-wing, low-powered aircraft.

A single-step formulation is proposed to simplify the computational procedure for perching trajectory optimization. Thrust vectoring is compared with aerodynamic vectoring and is shown to give least minimum undershoot perching trajectories. By creating an unsteady aerodynamic modeling tool based on nonlinear vortex correction method, parametric study of perching maneuvers is carried out to analyze to effects of different aircraft parameters on perching maneuver. Stability analysis of the maneuver is carried out using contraction theory and is shown to be unstable. Trajectory tracking is proposed

as solution to state perturbations and a sliding-mode based controller is designed. The perching trajectory tracking controller is modified and extended to design an automatic landing system for conventional aircraft.

1.4 Thesis Outline

The following chapters in this thesis are outlined as:

- Chapter 2 presents a brief review on the literature related to perching research and conventional automatic landing systems design.
- In Chapter 3, the formulation of perching problem into a trajectory optimization problem is presented. A single-step approach, which is computationally simple and capable of generating minimum undershoot perching trajectories, compared with existing two-step approach is proposed. This formulation is used to compute optimal perching solutions in the following chapters.
- Chapter 4 demonstrates the advantages of simple unconventional features like aerodynamic and thrust vectoring to reduce the spatial bounds of perching maneuvers.
- Chapter 5 studies the effects of different aircraft parameters on perching maneuvers. An unsteady aerodynamic modeling tool is designed based on nonlinear vortex correction method and perching trajectories are simulated for different aircraft parameters' variation.
- Chapter 6 carries out stability analysis of optimal perching trajectories and shows that the open-loop solutions from Chapter 3 are unstable. A sliding-mode based

controller is designed to address the problems of instability and state perturbations for autonomous perching.

- In Chapter 7, the sliding-mode controller designed for autonomous perching in Chapter 6 is modified and extended to design an automatic landing system for conventional aircraft landing maneuvers.
- Chapter 8 summarizes all the research carried out in this thesis and presents some directions which can extend the current research for real-time autonomous perching maneuvers.

Chapter 2

Literature Review

2.1 Overview

The literature available on aircraft perching is still sparse and can be categorized into three classes: numerical simulations, indoor experiments and perching mechanisms. First category mainly comprises the generation, dynamics analysis and control of perching trajectories, all of which are carried out numerically through simulations. Experimental demonstration of perching of micro air vehicles (MAVs), indoors, under Vicon motion capture cameras, falls under the second category. Last category, which has least dependence on perching dynamics, comprises different mechanisms as to how an UAV, either fixed-wing aircraft or quadcopter, holds the landing point at the end of the perching maneuver during touchdown. The following sections will provide a detailed review on the literature related to perching research work, available till date, followed by a small overview on different automatic landing systems design.

2.2 Perching Simulations

The most prominent work related to perching simulations is carried out by Wickenheiser and Garcia [6, 4]. In Ref. [4], the authors worked on an aerial regional-scale environmental survey Mars scout aircraft to study the advantages of morphing for perching maneuvers. The aircraft has unconventional configuration, with variable-incidence wing and rotating tail-boom features. Variable-incidence wing allows the aircraft to fly in low angles-of-attack flight regime, while maintaining high pitch-angle. It also allows the wing to remain in an attached flow region at high angles-of-attack, and simultaneously generate high drag from the fuselage. The rotating tail-boom allows the tail to position itself such that the wake generated aft the main wing does not affect the tail efficiency and therefore increase its moment control authority. To carry out perching simulations using this aircraft model, first, the aerodynamics is modeled using empirical and analytical methods [7]. The lifting-line theory is generalized to obtain the aerodynamic coefficients for different wing shapes in the pre-stall regime. In the post-stall regime, empirical methods - DATCOM and CFD tools - are used to generate the aerodynamic coefficients.

Figure 2.1: Wickenheiser's morphing aircraft model: A) Variable-incidence wing, B) Tail boom and C) Tail.

[Source: Ref. [4]]

After aerodynamic modeling, the perching trajectories are generated by optimization.

A two-step procedure is formulated to optimize climb and dive phases in the maneuver separately. In the first step, climb-phase is solved by minimizing the altitude gain. Initial conditions for this phase are unknown and are determined from the optimization solution. Terminal condition is a perched state. In the second step, dive-phase is optimized by minimizing the altitude loss. Though any cost function can be chosen, altitude loss is chosen to make it consistent with the cost function of climb-phase. Initial condition for dive-phase is a level-flight trim condition, which is determined from the optimization solution. Terminal condition is the initial condition of climb-phase. The solution is computed in two-step with an objective to minimize the undershoot. It is shown that inclusion of morphing features has increased flight handling [8], and reduced the spatial bounds to execute the perching maneuver. A parametric study is also carried out to study the role of maximum available thrust and static margin, center of gravity (CG) position with respect to neutral point. It is shown that increase in maximum available thrust, and decrease in static margin results in decrease of undershoot.

Another work on optimization of perching maneuvers is the research carried out by Alikhan et al. [9]. In their paper, inspired by the fruit bat's crabbed turn [10], an interesting three-dimensional perching strategy has been proposed by the authors. The perching strategy was to perform an aggressive three-dimensional momentum reversal maneuver and use high drag from large sideslip angles to achieve considerable reduction in velocity. Figs. 2.2 and 2.3 illustrate the fruit bat's crabbed turn and a simulated three-dimensional perching trajectory, respectively. Though large velocity reduction is achieved in this work, the aircraft still requires climbing if low-velocity perching is required.

Real-time perching of fixed-wing aircraft poses many challenges and problems. One of the problems is variations in initial/launch conditions. Assuming no aerodynamic uncer-

Figure 2.2: Illustration of fruit bat's crabbed turn. [Source: Ref. [10]]

Figure 2.3: Illustration of three-dimensional aircraft perching trajectory. [Source: Ref. [9]]

tainties or external disturbances, it is not possible to perch using the open-loop control inputs obtained from trajectory optimization. Trajectory optimization is computationally intensive and real-time generation of optimal perching trajectories is infeasible. To address the problem of variations in initial conditions during real-time perching, Meckstroth and Reich proposed a near-optimal trajectory selection method [11]. In this method, the shape of the perching trajectory is represented as a Bézier curve. The parameters representing the Bézier will be a function of initial conditions - energy and flight-path angle. Optimal

perching trajectories are generated for a wide range of initial conditions and the parameters of the Bézier curves are recorded. During real-time perching, based on the initial energy and flight-path angle, Bézier curve parameters are interpolated from the tabulated data set and a reference optimal perching trajectory is generated. Once a reference trajectory is generated, it can be tracked using any controller.

Regisford and Parry proposed a neighboring optimal control strategy to address the problem of perturbations in initial conditions [12]. In this approach, a nominal optimal perching trajectory is generated and optimal control inputs are determined. The initial conditions of the nominal perching solution are perturbed and an optimal trajectory, called neighboring optimal perching trajectory, is generated using the same cost functional used for nominal perching trajectory to perch at the same landing point with minimal velocity. Multiple perturbed initial conditions are generated and multiple neighboring optimal perching trajectories are computed. The control inputs of the neighboring optimal perching trajectories differ from the nominal control inputs. The difference between these control inputs is assumed to be a linear combination of the differences between state variables. For all the neighboring optimal perching trajectories, at each point along the trajectory, the differences between state variables and control inputs are calculated and a set of linear algebraic equations are formed in terms of the differences. The coefficients of the differences of the state variables are assumed to be constant and are determined from the least squares solution. Once the coefficients are determined, these can be used to design a controller, which computes corrections required in the control inputs based on the deviation of the state variables.

Choe and Hovakimyan designed a flight control system [13] to address the problems of time delays in the state feedback and aerodynamic uncertainties for autonomous perching

maneuvers. An optimal perching trajectory is generated for the chosen aircraft model and optimal control inputs are determined. The nominal control system, which commands the open-loop optimal control inputs, is augmented with an $\mathcal{L}_1$ adaptive controller. Based on the state feedback, error between reference and actual states is determined and is passed to the $\mathcal{L}_1$ controller. The $\mathcal{L}_1$ controller computes the correction required and adds to the nominal control signal. With the augmented controller, the MAV closely tracks its reference trajectory during autonomous perching and landing at the desired location is carried out with minimal deviation.

Moore and Tedrake simulated the possibility of perching on powerlines with a fixed-wing UAV [14]. Such capability enables the UAVs to recharge by a portable inductive power coupling, while carrying out a surveillance operation. This helps the UAVs to overcome the problem of short endurance due to limited power supply. The electricity passing through the powerline creates a magnetic field around it. By equipping the UAV with magnetometers, the location of the UAV with respect to the powerline can be determined. The setup provides a low cost and computationally inexpensive navigation to the perching UAV and assists it during the landing.

2.3 Perching Experiments

Research on perching experiments is mainly confined to its demonstration indoors under motion capture cameras. Though no experiments are carried out in this thesis, their literature is presented here for the sake of completeness. First among these comes the work done by Cory and Tedrake [15, 16]. In his experiments, Cory demonstrated perching of a light-weight glider, which is powerless with fixed-wing configuration and weighing 77

gms. Elevator deflection is the only control input to this aircraft. The glider is equipped with a receiver and a servo for deflecting the elevator. Reflectors are attached to the aircraft to determine its position and orientation using Vicon motion capture camera setup. The setup is capable of recording the flight trajectories in real-time. Using this setup, first, the aerodynamics of the glider is modeled. Glider is flown at different elevator deflections and the flight paths are recorded. From the flight-path histories, velocity and accelerations are determined. Aerodynamic forces are calculated from the accelerations and from which, aerodynamic coefficients are determined. Using few flight-test data, aerodynamic lift, drag and moment profiles are determined for different elevator settings. It was found that the aerodynamic model obtained is close to flat-plate aerodynamic model.

After aerodynamic modeling, an approximate optimal controller is designed to conduct perching experiments [17]. During the experiments, glider - attached with reflectors - is launched into the motion capture environment. Vicon setup tracks the flight motion, and transfers the position and orientation data to a PC with MATLAB running the control algorithm. The control algorithm computes the required elevator deflection, and transmits the control commands to the receiver on the glider through a transmitter connected to the PC. Fig. 2.4 shows the phases of the successful perching maneuver obtained with this setup. The controller, a short moment before landing, generates a pitch-up command by deflecting the elevator upwards. The sudden increase in angle-of-attack increases the drag and reduces the velocity to land with minimal impact.

Another notable work on perching experiments is the work carried out by Paranjape and Chung [18, 19]. The authors also carried out the experiments under motion capture cameras. A variable dihedral MAV is designed to achieve increased yaw control. The MAV is hand launched during the experiments and the variable-dihedral wings change

Figure 2.4: Cory’s experiment showing aircraft perching on a wire. [Source: Ref. [15]]

their angle to steer the vehicle towards the direction of landing point. After reaching a predetermined position, just before landing, an open-loop upward deflection of the elevator is commanded. Here also, the sudden increase in angle-of-attack creates an increase in the drag, which enables the aircraft to land with minimal impact. Fig. 2.5 shows successful demonstrations of perching on a chair and a hand using this vehicle.

Robertson and Reich designed a PID controller to demonstrate perching of a bio-mimetic remote controlled glider, experimentally [20]. The experiments were carried out

Figure 2.5: Paranjape's experiments showing successful demonstrations of MAV perching. [Source: Refs. [18, 19]]

using a Dreamflight[®] Alula glider, which has a wingspan of 0.9 m and a vertical tail, but no horizontal tail as shown in Fig. 2.6. The Air Force' Micro Air Vehicle Integration and

Figure 2.6: Glider used by Robertson and Reich for their perching experiment. [Source: Ref. [20]]

Figure 2.7: Robertson and Reich’s demo of their perching experiment. [Source: Ref. [20]]

Application Research Institute (μ AVIARI) indoor flight laboratory, which has a large room with 60 Vicon motion capture strobes and cameras, capable of recording flight position to within 1 *mm* accuracy, is used for the design and testing of the controller. Longitudinal and lateral states are controlled separately using decoupled PID controllers. The target perching trajectory was chosen to have a dive-phase and a climb-phase. The gains of the controllers are chosen such that tracking performance is reasonably well. The experimental setup and the designed controllers were able to perch the glider successfully. Fig. 2.7 shows a failed and successful demonstration of their perching experiments.

2.4 Perching Mechanisms

Apart from perching aircraft dynamics and control, research on design of mechanisms for the aircraft to hold the landing point is also carried out. Again, this work is outside the scope of this thesis, but is presented here for the sake of completeness. In Ref. [21], Desbians et.al., experimentally demonstrated the concept of microspines, shown in Fig. 2.8, as landing mechanism for small UAVs to perch on a rough wall or vertical surface. The airplane is fitted with an ultrasonic range sensor and microspines, and is launched towards a wall with rough surface. Upon the detection of a wall by the range sensor, at a predefined distance, an open-loop pitch-up control command is given to the elevator. Aircraft pitches up and the fuselage makes a horizontal impact with the wall as shown in Fig. 2.9. During the impact, the spine mechanism absorbs all the kinetic energy, engages to the surface, and prevents the aircraft from bouncing back. The concept is extended in Ref. [22] to make the aircraft take-off vertically after perching.

Figure 2.8: Microspine engaging mechanism. [Source: Ref. [21]]

A few interesting concepts of perching mechanism have been presented in Ref. [23]. The most promising one among them is the design of a sticky-pad plane. A pusher MAV is mounted with a dispenser of sticky-pads on its nose. Each of the sticky-pads has a

Figure 2.9: Desbian’s aircraft perching on a vertical surface. [Source: Ref. [21]]

strong adhesive, rat glue. To perch, the MAV collides onto a vertical wall/surface made with its nose and a sticky-pad attaches to the wall. After perching, the MAV can carryout its mission without any requirement of additional power to hold on to the wall. After the mission, it can detach from the sticky-pad and take-off from the wall. The mechanism is passive and can be mounted on any light-weight airplane. The dispenser mechanism is designed to hold a stack of sticky-pads, which allows the vehicle to perch and disengage multiple times. Fig. 2.10 shows a plane with sticky-pads mounted on its nose and how it hangs after a successful perching maneuver.

Kovač et.al. designed a perching mechanism for MAVs to perch on walls of natural and man-made materials like trees and painted concrete facades of buildings [24]. The mechanism has two needles charged by a spring as shown in Fig. 2.11 and is attached to the nose. While perching, the MAV makes a head-on collision with the wall/surface, where a mechanical trigger releases the spring and the two needles snap forward to stick into the

Figure 2.10: Sticky-pad plane prototype hanging from a wall after perching. [Source: Ref. [23]]

wall/surface as shown in Fig. 2.12. The perching mechanism is designed to also include a simple detachment mechanism. The arms of the needles are connected with strings, which are attached to a DC motor. When actuated, the DC motor pulls the strings and the needles detach from the surface. This simple mechanism allows the MAV to perch and detach multiple times.

Figure 2.11: Kovač's spring mechanism for perching. [Source: Ref. [24]]

Figure 2.12: Illustration of functioning of Kovač's perching mechanism. [Source: Ref. [24]]

Doyle et al. designed an avian-inspired grasping foot for perching [25, 26]. The mechanism is an underactuated tendon-driven structure with multiple links connected by flexible joints as shown in Fig. 2.13. It is designed to resemble a bird's claw to perch on cylindrical

objects of various sizes or different flat surfaces. The foot has several toes linked to each other, which hold the cylinder when actuated. The gripping force provided by the toes prevent the vehicle from rotating after perching.

Figure 2.13: Demonstration of Doyle's perching mechanism. [Source: Refs. [25, 26]]

Mellinger et al. designed a gripper as shown in Fig. 2.14 for perching of quadrotors on rough surfaces [27]. The gripper has microspines, which can be actuated by a servo motor. After perching, the microspines are actuated to penetrate into the landing surface. The performance of the gripper is tested by attaching it to a quadcopter and landing on horizontal and vertical surfaces. The quadcopter is programmed to execute agile perching maneuvers to pitch upto 90° and land on vertical surfaces. Successful demonstration of the perching mechanism is shown in this paper. To address the issues of unsuccessful perching attempts, the quadcopter is also programmed to recover it quickly from falling when the gripper fails to engage.

Figure 2.14: Illustration of Mellinger’s gripper mechanism for perching. [Source: Ref. [27]]

2.5 Autonomous Conventional Aircraft Landing Systems

Conventional aircraft landing is considered to be one of the difficult maneuvers that aircraft routinely performs and its execution involves simultaneous and tight control of multiple aircraft states. Most of the landing procedures are carried out in two phases: glide-path capture and flare maneuver. Glide-path capture maneuver requires the aircraft to descend along a predefined straight line in the longitudinal landing plane with a constant negative flight-path angle ranging between -3° and -2° and heading directed toward the runway. Sometimes, the maneuver is performed in two steps with a slightly higher descent angle in the beginning and a lower descent angle in the latter step. In this phase, the aircraft is guided toward the specified straight path, and when this path has been reached, a steady descent is maintained by keeping all the state variables constant. Flare maneuver is initiated at the end of the glide-phase when the aircraft reaches around 30 – 50 ft altitude. During the flare maneuver, the aircraft is required to follow a curved path and reduce its descent rate to bring the flight-path angle to near-zero for a smooth touchdown and minimized impact on landing gears. Fig. 2.15 illustrates this typical landing maneuver

with the indication of the reference paths for its two phases.

Figure 2.15: Illustration of landing maneuver showing reference glide- and flare-paths.

Unlike autonomous cruise controls, which are employed widely in most of the commercial aircraft autopilots, autonomous landing systems are rarely used because of the high risks and reliability issues involved. High costs involved in flight testing and stringent flight regulations make it difficult to develop and validate the proposed controllers, thus rendering them impracticable [28]. Moreover, external disturbances, such as wind shear, gust, and downburst, complicate the landing controller design requirements further.

Various control techniques and approaches have been proposed for automatic aircraft landing problem. Shue and Agarwal proposed an automatic landing system using a mixed H_2/H_∞ control [29]. Their controller is designed based on the linearized model of the aircraft and limited to the longitudinal mode. Prasad and Pradeep employed feedback linearization technique to design a controller for automatic landing system [30]. Their study includes the ground effects, however it is confined to longitudinal motion only. Nho

and Agarwal [5] developed a proportional-derivative (PD) type of fuzzy logic controller for automatic landing system and tested it on simulations using linear and nonlinear aircraft models. Since, the fuzzy logic is modeled based on PD controller, the resulting performance is equivalent to it. A robust and fault tolerant controller for aircraft landing system that can handle external wind disturbances and control failures is developed by Liao et al. [31]. They employ H_2 control technique, and thus its application is limited to linearized aircraft models.

An unconventional and novel controller based on vision and neural networks is developed in [32, 33]. The landing maneuver in longitudinal mode is performed in a flight simulator by skilled pilots, and the visual cues, such as horizon, markers on the runway, runway centerline, etc., and the pilot control inputs are recorded. By considering the visual cues as *inputs* and the corresponding control inputs as *outputs*, the pilot action is modeled using neural network. The trained network is then used as an automatic landing controller. The concept is further extended to include lateral pilot motion by increasing the number of visual cues [34]. This approach, however, can only be reliable when the landing conditions fall within the range of trained data set, thus significant training data are required for increased reliability.

2.6 Summary

A brief overview on literature related to research on perching simulations, control, experiments and mechanisms is presented. Considering a few similarities between perching and conventional landing maneuvers, a short overview on designs of various autonomous landing systems for conventional aircrafts is also presented.

Chapter 3

Optimization Formulation of Perching Maneuvers

3.1 Overview

In this chapter, the formulation of aircraft perching problem into a trajectory optimization problem is presented. The need for perching trajectory optimization arises from the requirement of elevated landing platform during climbing. Two optimization formulations, based on single- and two-step procedures, are presented and compared. The two-phase optimization formulation is an existing procedure proposed by Wickenheiser and Garcia [4]. The formulation optimizes the two phases, dive and climb-phases, separately, by minimizing the altitude loss and gain, respectively, to generate perching trajectories with minimum undershoot. The two-step procedure has a complex iterative solution procedure and requires computation of multiple solutions that results in increased computational time. It can also result in suboptimal perching solutions, with larger undershoot, which will be discussed further in this chapter. To simplify the solution procedure, a single-phase

optimization formulation that minimizes the trajectory length of perching maneuver and computes the entire trajectory in a single step is proposed. The following chapter provides the details of the two formulations, solution procedures to transcribe the trajectory optimization problem into a nonlinear programming problem and compares the performances of the two formulations.

3.2 Background

The details of the aircraft model, aerodynamics and equations of motion used in the following perching simulations in this chapter are introduced in this section.

3.2.1 Aircraft Model

The aircraft model considered in this chapter is based on a commercially available remote-control aircraft - flash model. It has a conventional fixed-wing configuration with foam structure and two control surfaces: rudder and elevator. The propeller and motor are attached on the nose. Both wing and tail of the aircraft have rectangular planform with NACA 0012 airfoil cross sections. A picture of the reference aircraft model is presented in Fig. 3.1. The aircrafts mass and other geometric properties are given in Table 3.1.

3.2.2 Aerodynamics

The aerodynamics of this aircraft is taken from the model developed by Maqsood and Go [35], which is generated for similar aircraft geometry. In their model, low angle-of-attack aerodynamic data are obtained from low-speed wind-tunnel experiments using a 50% scaled model. The moderate angle-of-attack data are developed using vortex-lattice

Figure 3.1: Flash aircraft model.

[Source: <http://www.rcflight.se/modellflyg/rc-factory/modellflygplan.aspx>]

methods [36]. High angle-of-attack aerodynamics are generated through extrapolation of the moderate angle-of-attack aerodynamics. During perching simulations, a few simplifications are made to the model to simplify the computation. Slip-stream effects from the propeller on the tail are neglected and the tail is assumed to experience nominal airflow without any downwash effect. Unsteady aerodynamic effects are also ignored, however they can be approximated by the Wagner's model as presented in Chapter 5. The variations of lift, drag, and moment coefficients for the chosen model as functions of angles of attack and elevator deflections are presented in Fig. 3.2. It may be noticed that in Fig. 3.2, the effect of elevator deflection on the overall lift coefficient is asymmetric. The asymmetry arises because wing and elevator are treated as separate lifting surfaces having no influence on each other. The local angle-of-attack at the elevator is equal to the sum of geometric angle-of-attack and elevator deflection. The overall lift is equal to the sum of lift generated from these two surfaces. Since the elevator deflections are so high, its local angle-of-attacks will be in the nonlinear post-stall regime. As a result, the effect of elevator deflection is asymmetric on the overall lift coefficient. The profiles of the other

Quantity	Value
mass m	0.8 kg
chord c	0.25 m
span b	1 m
wing surface area S_{wing}	0.25 m^2
tail surface area S_{tail}	0.054 m^2
moment of inertia I_{yy}	0.1 $kg - m^2$
CG position x_{cg}	0.2 m
wing quarter-chord position $x_{c/4_{wing}}$	0.25 m
tail quarter-chord position $x_{c/4_{tail}}$	0.72 m
gravity g	9.8 m/sec^2
air density ρ	1.225 kg/m^3

Table 3.1: Aircraft mass and geometry properties

two aerodynamic coefficients show almost symmetric influence of elevator deflection.

3.2.3 Equations of Motion

The perching maneuver is confined to longitudinal plane of motion only. Lateral motions as well as lateral component forces and moments are assumed to have no contribution on the longitudinal motion of the aircraft. Optimization simulations are carried out based on the longitudinal aircraft dynamics. The three degrees-of-freedom longitudinal equations of motion in hybrid-axis frame of reference are obtained from Ref. [37] after the exclusion of lateral state variables and are given by

$$\dot{V} = \{T \cos \alpha - \frac{1}{2} C_D (\alpha, q, \delta_e) \rho V^2 S - mg \sin \gamma\} / m \quad (3.2.1a)$$

Figure 3.2: Lift, Drag and Moment coefficients at various angles of attack and elevator deflections.

$$\dot{\gamma} = \{T \sin \alpha + \frac{1}{2}C_L(\alpha, q, \delta_e) \rho V^2 S - mg \cos \gamma\} / mV \quad (3.2.1b)$$

$$\dot{\alpha} = q - \{T \sin \alpha + \frac{1}{2}C_L(\alpha, q, \delta_e) \rho V^2 S - mg \cos \gamma\} / mV \quad (3.2.1c)$$

$$\dot{q} = \frac{1}{2I_{yy}}C_M(\alpha, q, \delta_e) \rho V^2 S \quad (3.2.1d)$$

$$\dot{x} = V \cos \gamma \quad (3.2.1e)$$

$$\dot{h} = V \sin \gamma \quad (3.2.1f)$$

$$\dot{s} = V \quad (3.2.1g)$$

$$\theta = \gamma + \alpha \quad (3.2.1h)$$

where $V, \gamma, \alpha, \theta, q, x, h$ and s are aircraft state variables representing total flight-speed, flight-path angle, angle-of-attack, pitch angle, pitch-rate, horizontal position, altitude and trajectory length, respectively. T and δ_e are the control inputs from thrust and elevator

deflection, respectively. C_L, C_D and C_M are the lift, drag and moment coefficients of the aircraft, respectively, and are computed from aerodynamic lookup table.

3.3 Optimization Formulation

The two-phase and single-phase formulations of perching trajectory optimization problem are presented in this section. Solutions from the two approaches are compared to demonstrate the effectiveness of the single-phase formulation that gives lower minimum undershoot optimal perching trajectories.

3.3.1 Two-phase formulation

The two-phase formulation reported in [4] emphasizes minimizing the undershoot of the perching trajectory by dividing the optimization into two subproblems: climb phase and dive phase. The two phases are optimized separately, minimizing altitude gain and loss, respectively. The solutions of these two phases are then adjoined to get a complete trajectory. The climb phase is solved first using the cost function, which minimizes the altitude gain as follows:

$$J_{climb} = \min_{u \in [\delta_e(t), T(t)]} \int_0^{t_f} V \sin \gamma dt \quad (3.3.1)$$

To keep the cost function monotonic, the flight-path angle is constrained to be positive throughout this phase

$$\gamma(t)_{climb} \geq 0 \quad (3.3.2)$$

The terminal boundary condition is a perched state, for which the velocity is a low

value, sufficiently close to zero. This is prescribed as the following inequality constraint:

$$V(t_f)_{climb} \leq 0.6 \text{ m/sec} \quad (3.3.3)$$

To make the perching solutions realistic, the final pitch angle is constrained to be less than or equal to 90° . The remaining state variables are left free. The final position $(x(t_f), h(t_f))$ of perching can be either fixed or free, depending on the initial position of the dive phase. If the initial position of the dive phase is fixed, then the final position of the perching must be free, and vice versa. Constraining both of them in general leads to a suboptimal or infeasible formulation. In this chapter, the origin of the coordinate system is chosen as the initial position. After the perching solution has been obtained, in the presentation of the results, the trajectory is shifted such that the origin becomes the final position.

The onset of the climb phase is defined by the condition

$$\gamma(0)_{climb} = 0 \quad (3.3.4)$$

The other initial state variables are not constrained and are treated as unknowns to be determined from the optimization solution.

Once the solution to the climb phase has been computed, the dive phase is optimized.

Here, the altitude loss is chosen as the cost function to be minimized as follows:

$$J_{dive} = \min_{u \in [\delta_e(t), T(t)]} - \int_0^{t_f} V \sin \gamma dt \quad (3.3.5)$$

The flight-path angle in this case is constrained to a negative value to keep the cost function monotonic

$$\gamma(t)_{dive} \leq 0 \quad (3.3.6)$$

The terminal condition for the dive phase is the initial condition obtained from the optimal solution of the climb phase, which can be specified as a matching condition as

follows:

$$\underline{x}(t_f)_{dive} = \underline{x}(0)_{climb} \quad (3.3.7a)$$

$$\underline{u}(t_f)_{dive} = \underline{u}(0)_{climb} \quad (3.3.7b)$$

where $\underline{x} = [V, \gamma, \alpha, q, x, h, s, \theta]^T$ and $\underline{u} = [\delta_e, T]^T$ are the state and control input vectors, respectively.

The initial condition for the dive phase is a level-flight trim state, which can be either fixed to a particular trim state or be determined from the optimization solution. Constraining the initial state to a particular state will result in suboptimal formulation and may increase the minimum undershoot, therefore, it is chosen to be free here. The initial trim state is prescribed as equality constraints in the form of the following nonlinear algebraic equations:

$$\gamma = 0 \quad (3.3.8a)$$

$$q = 0 \quad (3.3.8b)$$

$$\{T \cos \alpha - \frac{1}{2}C_D(\alpha, q, \delta_e) \rho V^2 S - mg \sin \gamma\}/m = 0 \quad (3.3.8c)$$

$$\{T \sin \alpha + \frac{1}{2}C_L(\alpha, q, \delta_e) \rho V^2 S - mg \cos \gamma\}/mV = 0 \quad (3.3.8d)$$

$$\frac{1}{2I_{yy}}C_M(\alpha, q, \delta_e) \rho V^2 S = 0 \quad (3.3.8e)$$

The solutions for the two phases are then adjoined to form the complete solution for the perching trajectory.

3.3.2 Single-phase formulation

The objective of the single-phase approach is to simplify the perching trajectory optimization problem formulation, which leads to a simpler solution procedure and reduced

computational complexity. To achieve this, a monotonically varying cost function throughout the maneuver is required. The cost functions defined in the two-phase formulation (altitude loss and gain) are monotonic only in their respective phases and, therefore, they are not suitable to be used here. Alternatively, the trajectory length (s), which is identified to be monotonic throughout the maneuver, is chosen as the cost function in the single-phase optimization here. For this matter, s is included as a new state variable in the equations of motion, which can be computed by integrating the flight velocity with respect to time. Thus, the cost function for the single-phase formulation can be expressed as

$$J = \min_{\underline{u}} \int_0^{t_f} V dt \quad (3.3.9)$$

The terminal perched state boundary condition in this formulation is same as the one prescribed for the climb phase in Eqn. (3.3.3). To avoid having a suboptimal formulation, the initial boundary condition is chosen as a free level-flight trim state with constraints given in Eqn. (3.3.8). The free initial boundary condition for the two formulations in general leads to perching solutions with different initial states.

The single-phase formulation circumvents the matching problem between the final state of the dive phase and the initial state of the climb phase, which is a significant advantage. In such a formulation, constraint on the flight-path angle during the maneuver is not required. Note that, although undershoot is not directly assessed, the cost function (3.3.9) minimizes it indirectly.

3.4 NLP Transcription

Solutions to the optimal perching maneuver problems formulated in the previous section are computed using pseudospectral methods [38]. This approach transcribes the dynamic optimization problem [39] into a nonlinear programming (NLP) problem [40] by the parameterization of the control and state vectors. DYNOPT toolbox [41], developed for MATLAB environment, is used to carry out the transcription. During the transcription, the total maneuver time is divided into finite number of equal time intervals. In each interval, the state and control profiles are approximated by Lagrangian polynomials [42] in non-dimensional time. The end points of the time interval and the roots of the shifted Legendre polynomial [43] are selected as nodes/collocation points for the Lagrangian polynomials. The state and control values at these nodes are unknown and are treated as the decision variables. An advantage of this approach is that bounds on the state and control variables can be directly imposed on the nodal values/decision variables. To generate smooth and feasible state and control profiles, the connectivity and slope continuity between the adjacent polynomials are maintained as equality constraints.

To understand the details of this transcription procedure, let us consider a trajectory optimization problem with Meyer-type cost function to be minimized

$$J = \min_{\underline{u}, \underline{p}} [G(\underline{x}(t_f), \underline{p}, t_f)] \quad (3.4.1)$$

where $\underline{u}$ in the control input vector, $\underline{p}$ are parameters, t_f is the final time and $\underline{x}(t_f)$ is the final state.

The aircraft's equations of motion, equality and inequality constraints, and bounds on parameters, state and control variables can be expressed as

$$\dot{\underline{x}}(t) = f(t, \underline{x}(t), \underline{u}(t), \underline{p}) \quad (3.4.2a)$$

Figure 3.3: Finite element collocation of control and state profiles for $K = 2$.

$$\underline{h}(t, \underline{x}(t), \underline{u}(t), \underline{p}) = 0 \quad (3.4.2b)$$

$$\underline{g}(t, \underline{x}(t), \underline{u}(t), \underline{p}) \leq 0 \quad (3.4.2c)$$

$$\underline{x}(t)^L \leq \underline{x}(t) \leq \underline{x}(t)^U \quad (3.4.2d)$$

$$\underline{u}(t)^L \leq \underline{u}(t) \leq \underline{u}(t)^U \quad (3.4.2e)$$

$$\underline{p}^L \leq \underline{p} \leq \underline{p}^U \quad (3.4.2f)$$

where $\underline{x}$, $\underline{u}$ and $\underline{p}$ are state, control and parameter vectors, respectively. h and g are set of nonlinear equality and inequality constraints, respectively. $\underline{x}^L$, $\underline{u}^L$ and $\underline{p}^L$ are lower bounds of state, control and parameters vectors, respectively. $\underline{x}^U$, $\underline{u}^U$ and $\underline{p}^U$ are upper bounds of state, control and parameters vectors, respectively.

The state and control profiles for i^{th} element, $\zeta_i \leq t \leq \zeta_{i+1}$ are approximated by Lagrangian polynomials of order $K + 1$ as

$$x_K(t) = \sum_{j=0}^K x_{ij} \phi_j(t); \quad \phi_j(t) = \prod_{k=0, j}^K \frac{(t - t_{ik})}{(t_{ij} - t_{ik})} \quad (3.4.3a)$$

$$u_K(t) = \sum_{j=1}^K u_{ij} \theta_j(t); \quad \theta_j(t) = \prod_{k=1, j}^K \frac{(t - t_{ik})}{(t_{ij} - t_{ik})} \quad (3.4.3b)$$

where $i = 1, \dots, NE$, NE is the number of elements each profile is discretized into, $K + 1$ is the order of the polynomial, x_{ij} and u_{ij} are the collocation coefficients, t is time, t_{ij} are the nodes.

Nodal values, x_{ij} and u_{ij} , become the decision variables. With transformation $t_{ik} = \zeta_i + \Delta\zeta_i \tau_i$, and substitution of parameterized equations, Eqn. (3.4.3), into the state equa-

tions results in a set of algebraic equations called residual equations, which become equality constraints as

$$\Delta\zeta_i r(t_{ik}) = \sum_{j=0}^K x_{ij} \dot{\phi}_j(\tau_k) - \Delta\zeta_i f(t_{ik}, x_{ik}, u_{ik}, p) = 0 \quad (3.4.4)$$

where $i = 1, \dots, NE$, $j = 0, \dots, K$, $k = 1, \dots, K$

The nodes, τ_k are chosen as roots of the shifted Legendre polynomial of order $K + 1$, which lie in the interval $[0, 1]$, and are given by

$$P_{K+1} = \frac{1}{(K+1)!} \frac{d^{K+1}}{dx^{K+1}} [(x^2 - x)^n] \quad (3.4.5)$$

The choice of Lagrangian polynomial as basis function enables to maintain the bounds on state and control variables easily due to the condition that at nodes, the value of the Lagrangian polynomial is equal to the nodal value

$$x_K(t_{ij}) = x_{ij} \quad (3.4.6)$$

To keep the profiles smooth, continuity and connectivity constraints are imposed between the elements

$$x_{i,K} = x_{i+1,0} \quad (3.4.7a)$$

$$u_{i,K} = u_{i+1,0} \quad (3.4.7b)$$

$$\dot{x}_{i,K} = \dot{x}_{i+1,0} \quad (3.4.7c)$$

$$\dot{u}_{i,K} = \dot{u}_{i+1,0} \quad (3.4.7d)$$

for $i = 1, \dots, NE - 1$.

At this point, it should be noted that the original DYNOPT toolbox does not maintain connectivity and slope continuity for control profiles. However, changes are made in the original code to ensure that control profiles satisfy these constraints. With this

transcription, the optimization problem is transcribed into an NLP problem as

$$J = \min_{x_{ij}, u_{ij}, \Delta\zeta_i, \underline{p}} [G(\underline{x}(t_f), \underline{p}, t_f)] \quad (3.4.8)$$

with initial and terminal conditions transcribed as

$$x_{10} - \underline{x}(0) \quad (3.4.9a)$$

$$x_f - x_K^{NE}(\zeta_{NE+1}) = 0 \quad (3.4.9b)$$

The residual constraints are transcribed as

$$r(t_{ik}) = 0 \quad i = 1, \dots, NE \quad k = 1, \dots, K \quad (3.4.10)$$

The bounds on nodal values at collocation points are transcribed as

$$x_{ij}^L \leq x_{ij} \leq x_{ij}^U \quad i = 1, \dots, NE \quad j = 1, \dots, K \quad (3.4.11a)$$

$$u_{ij}^L \leq u_{ij} \leq u_{ij}^U \quad i = 1, \dots, NE \quad j = 1, \dots, K \quad (3.4.11b)$$

The time interval equality constraint is transcribed as

$$\Delta\zeta_i = \Delta\zeta_{i+1} \quad i = 1, \dots, NE - 1 \quad (3.4.12)$$

The time constraint for which the total time is equal to the sum of individual time intervals is transcribed as

$$\sum_{i=1}^{NE} \Delta\zeta_i - \zeta_{total} = 0 \quad (3.4.13)$$

The bound constraints on parameters remain same as

$$\underline{p}^L \leq \underline{p} \leq \underline{p}^U \quad (3.4.14)$$

The nonlinear equality and inequality constraints are transcribed as

$$\underline{h}(t_{ij}, x_{ij}, u_{ij}, \underline{p}) = 0 \quad (3.4.15a)$$

$$\underline{g}(t_{ij}, x_{ij}, u_{ij}, \underline{p}) \leq 0 \quad (3.4.15b)$$

After the transcription, the NLP problem is solved by using `fmincon` function available in MATLAB with interior point algorithm [44, 45]. During all the following optimization simulations, the maneuver time is divided into eight equal intervals. Both state and control input profiles in each time interval are approximated by fifth-order Lagrangian polynomials. The roots of third-order shifted Legendre polynomials are chosen as the collocation points.

3.5 Optimization Results

Optimal perching trajectories computed using both single- and two-phase formulations are compared. In both formulations, the final boundary condition is a perched state and the initial boundary condition is a free level-flight trim state given by Eqs. (3.3.3) and (3.3.8), respectively. During the maneuver, the state variables are constrained to be between their lower and upper limits as given in Table 3.2. The control variables are subjected to both saturation and rate limits specified in Table 3.3. A set of optimal perching trajectories and their corresponding state profiles and control commands computed using the two optimization formulations are shown in Figs. (3.4), (3.5) and (3.6), respectively.

The two-phase optimization is expected to yield a lower undershoot perching solution than the single-phase one because altitude gain is minimized directly in the scheme. Fig. (3.4) shows the typical result obtained, which indicates that the single-phase optimization leads to lower undershoot. The reason for the larger undershoot from the two-phase formulation is due to the matching step given by Eqn. (3.3.7). In the two-phase formulation, the climb phase is solved first, which commands high lift and drag at the beginning to rapidly increase the flight-path angle and reduce the velocity. To meet these high lift and

State variable (units)	Lower limit	Upper limit
V (m/sec)	0	25
γ (rad)	$-\pi/2$	$\pi/2$
α (rad)	0	$\pi/2$
q (rad/sec)	-1	1
x (m)	0	100
h (m)	-20	20
s (m)	0	200

Table 3.2: Bounds on state variables

Control variable (units)	Lower limit	Upper limit	Rate limit (units/sec)
δ_e (deg)	-30	30	± 30
T/W	0	0.4	$\pm 1/W$

Table 3.3: Bounds and rate constraints on control variables

drag requirements, the climb-phase optimization solution yields a moderately large velocity and high angle-of-attack state as the initial state. This initial state can be infeasible and not reachable during the dive phase. To guarantee its reachability, additional bounds are imposed on the initial state variables of the climb phase [6]. These bounds are not known a priori and they are determined by trial and error procedure. For a chosen set of bounds, the climb phase is solved and then the solution to the dive phase is attempted. If the dive phase fails to reach the initial state given by the climb-phase solution, the bounds are adjusted and the solutions to the two phases are computed again. This procedure is repeated until a reachable initial state to the climb phase is obtained. The maximum limits on angle of attack and velocity play conflicting roles in the tuning of the bounds.

Figure 3.4: Optimal trajectories of perching maneuvers

If the maximum angle-of-attack limit is reduced, the initial velocity necessarily increases to compensate for the loss of lift from the lift coefficient reduction, which results in an increase in undershoot. If angle-of-attack limit is raised too high, the initial state may become unreachable. Also, if both limits are lowered, the climb may become infeasible. The determination of bounds for a feasible perching solution is very iterative and thus often leads to a suboptimal choice of limits that results in the climb solutions having an increased initial velocity and a larger undershoot compared with single-phase solutions. The single-phase formulation, on the other hand, does not impose additional bounds during the maneuver. Although the undershoot is not minimized directly in the single-phase optimization, the minimization of trajectory length optimizes the undershoot indirectly, leading to lower undershoot than in the two-phase formulation.

The better performance of the single-phase formulation can be observed from the other factors also. During the climb phase, when the flight-path angle becomes sufficiently high, gravity is nearly in the opposite direction of velocity and contributes largely to the deceleration. At this stage, an increase in drag enlarges the overall deceleration and

Figure 3.5: State histories of perching trajectories

reduces the undershoot. In the two-phase optimization solution, the poststall drag is not used efficiently and, therefore, the resulting climb trajectory has a slightly increased undershoot. In the case of the single-phase optimization solution, the aircraft quickly

Figure 3.6: Control histories of perching trajectories

enters the poststall regime to increase the drag at this stage.

The suboptimal solution to the climb phase also has its effect on the dive phase. To reach the increased initial velocity of the climb phase, the aircraft need to dive more, extending the dive phase and increasing the overall spatial bound of the perching trajectory. The results also show an interesting observation on the initial state for the two perching solutions. Although the initial condition for the dive phase is prescribed as a free state to be determined from the optimization solution in both formulations, the two solutions yield the same trim state corresponding to the maximum thrust value. At maximum thrust, the aircraft is at its maximum feasible level-flight velocity. By initiating the dive phase from a high-velocity state, the relatively large velocity required to initiate the climb phase can be reached quickly and the dive phase can be shortened.

Based on the advantages offered in terms of reduced undershoot, efficient use of post-stall aerodynamics, and simplified computational procedure, the single-phase formulation can be considered as a good choice for the optimization of the perching maneuver. Other potential cost function candidates, such as minimum time and minimum absolute altitude change, which are also monotonic throughout the maneuver, have also been explored for the single-phase formulation, but they are not presented here for being suboptimal in terms of minimum undershoot. The minimum time optimization results have almost the same undershoot as minimum trajectory length, whereas the others tried have larger undershoot.

3.6 Summary

The formulation of the perching problem into a trajectory optimization problem is simplified using the proposed single-phase optimization approach. The single-phase formulation, with its simpler computational procedure, makes effective use of poststall aerodynamics and produces lower undershoot optimal perching trajectories as compared with the two-phase formulation. The complex iterative solution procedure in the two-phase formulation that results in suboptimal perching solutions can be avoided by using the single-phase formulation. All the solutions are computed by transcribing the optimization problem into an NLP problem by pseudospectral methods with the help of DYNOPT toolbox, and are solved using interior point algorithm.

Chapter 4

Perching Trajectory Optimization

Using Aerodynamic And Thrust

Vectoring

4.1 Overview

An optimization formulation for reducing the spatial bounds of perching maneuver has been presented in Chapter 3. For a given fixed-wing aircraft model, optimal control inputs can be computed to perch the aircraft with minimum undershoot. To further minimize the undershoot, unconventional design features and configurations, like morphing, which expand the flight envelope, can be used. Wickenheiser and Garcia used a morphing aircraft to optimize the perching maneuvers [4]. The wing and tail boom are given rotational freedom to gain increased maneuverability at high angles-of-attack. This morphing concept has been successfully shown to reduce the minimum undershoot and spatial bounds of the perching trajectories. A similar concept using a variable-incidence wing feature has

been used by Maqsood and Go to optimize cruise-to-hover and hover-to-cruise transition maneuvers [35]. The outcome is a considerable reduction in control effort and transition time. Bourdin et al. present the concept of variable-cant winglets to vary the dihedral angle of the outboard wing section [46]. The design feature resulted in increased yaw control of an aircraft. Similar feature, but with flexible variable-dihedral wings, is studied to maneuver a tailless micro aerial vehicle by Paranjape et al. [47]. The use of variable-sweep morphing wing has been studied by Chakravarthy et al. to improve the longitudinal performance of an aircraft [48]. Abdulrahim and Lind tested the variable gull-wing morphing to achieve increased longitudinal flight handling and stability [49].

Most of the performance improvements proposed in literature are based on morphing techniques. It is definitely enticing to use one or more of these techniques to improve the performance of a perching-capable aircraft and reduce its spatial bounds. Morphing techniques alter the aerodynamics of the aircraft through shape modifications. Their effectiveness depends on their effects to the aerodynamic forces and moments, which are scaled by the square of the velocity. However, in the final phase of perching, the aircraft maneuvers in a low-velocity flight regime. In such condition, morphing features, excluding flapping, can become ineffective and fail to deliver the same advantages they offer at high velocities. This renders them to be of no advantage for their use in perching maneuvers. Thrust vectoring, on the other hand, is relatively independent of flight velocity and can be effective even at low velocities. It appears to be a viable alternative to morphing for perching maneuvers. With this motivation, a comparative study is attempted to identify the merits of thrust vectoring over morphing for perching maneuvers.

In this chapter, two unconventional features, aerodynamic and thrust vectoring, are studied to minimize the spatial requirements of the perching maneuver to allow the air-

craft to execute the climb-phase with reduced minimum undershoot. The aerodynamic vectoring, which is also called as variable-incidence wing feature, gives the pitch-rotational freedom to the wing. The wing in this case can adjust its local angle-of-attack to provide additional control over the magnitude and direction of the lift force, hence increase aircraft maneuverability. The thrust vectoring gives directional freedom to the thrust axis. It provides additional control over the direction and magnitude of the resultant of aerodynamic and thrust forces. Using the two features and their combinations, four different aircraft configurations are evaluated:

- *Aerodynamic Vectoring*
- *Thrust Vectoring*
- *Coupled Aerodynamic and Thrust Vectoring*
- *Uncoupled Aerodynamic and Thrust Vectoring*

The aerodynamic and thrust vectoring features are used individually for the first two configurations. For the third configuration, both features are assumed to be coupled following a certain relationship between the wing-incidence and the thrust angle. For the last configuration, the two features are used simultaneously with the wing-incidence and thrust angle allowed to vary independently. Optimal perching trajectories are computed for all these configurations and the results are compared with the corresponding fixed-wing perching maneuvers. It is shown that these unconventional configurations are able to reduce the minimum undershoot. In addition, the optimization results also lead to someinsight into the dynamics of the maneuver.

The following sections in this chapter are organized as follows. In Section 4.2, the details of the aircraft model, aerodynamics, and equations of motion used to carry out

the perching simulations are provided. Optimization results of different unconventional configurations are presented, compared, and discussed in Section 4.3. Finally, the chapter ends with some concluding remarks in Section 4.4.

4.2 Aircraft Model and Aerodynamics

4.2.1 Aircraft Model Description

The geometry, dimensions and configuration of the aircraft model used in this chapter are based on a commercially available light-weight aircraft model. The model has a conventional design configuration with a rectangular planform wing, a nose-attached propeller, and aft-located tail. Its mass, moment of inertia and other important geometric properties are listed in Table 4.1. The model is modified with two unconventional features: the aerodynamic and thrust vectoring discussed earlier, illustrated in Fig. 4.1.

Figure 4.1: Illustration of aerodynamic and thrust vectoring features.

The variable-incidence wing feature allows the wing to vary its pitch angle relative to the fuselage axis. As complete spanwise rotation of the wing is not possible due to structural obstructions from the fuselage, the feature is implemented as a fixed inboard

Quantity	Value
Aircraft mass, m	0.8 kg
Pitch Inertia, I_{yy}	0.1 $kg - m^2$
Wing span, b	1 m
Fixed inboard wing span, b_{in}	0.25 m
Rotatable outboard wing span, b_{out}	0.75 m
Mean aerodynamic chord, c	0.25 m
Wing area, S_w	0.25 m^2
Tail surface area, S_t	0.054 m^2
Air density, ρ	1.225 kg/m^3
Gravitational acceleration, g	9.8 m/s^2

Table 4.1: Aircraft parameters and their values.

wing and rotatable outboard wing. The inboard wing section experiences the same angle-of-attack as the fuselage. The outboard wing section rotates about its quarter-chord line along the span and it can adjust its local angle-of-attack. The angle between the fuselage and the chord line of the outboard wing is called wing-incidence (i_w) and is treated as a new control input. In a way, the variable-incidence wing feature greatly decouples the aerodynamic forces of wing and fuselage. This creates a possibility for simultaneous generation of high lift from the outboard wing and high drag from the inboard wing and fuselage, which is desired for perching. The other unconventional feature, the thrust vectoring, allows the thrust to be pointed in the desired direction. The angle between the thrust axis and the fuselage axis is called thrust angle (δ_T) and is also an additional control input. It is assumed that the thrust vectoring does not cause any changes to

the aerodynamics of the aircraft. The details of the aerodynamic modeling process are presented in the following subsection.

4.2.2 Aerodynamics

The same aircraft model with the same aerodynamic vectoring feature has been used in Ref. [35] to study aircraft transition maneuvers. The aerodynamics used for simulations in this chapter is obtained from the same paper with slight modifications, which include the exclusion of unsteady aerodynamics, three-dimensional and propeller slip-stream effects. Though unsteady aerodynamic effects are ignored, they can be modeled as approached in Chapter 5. In the model, the low angle-of-attack aerodynamics is obtained from wind tunnel tests using a 50% scaled model at a Reynold's number of around 150000. The moderate angle-of-attack aerodynamics is obtained using vortex lattice method (VLM). The high angle-of-attack aerodynamics is approximated by extrapolating the moderate angle-of-attack aerodynamics. The variations of sectional lift and drag coefficients for different angles-of-attack are plotted in Fig. 4.2.

Figure 4.2: Sectional lift and drag coefficients.

Depending on the local angle-of-attack, the lift and drag coefficients are determined

separately for the inboard wing, outboard wing and tail sections. These coefficients are used to calculate the sectional lift and drag forces and are integrated to give the total aircraft lift, drag, and moment.

The perching simulations in this chapter are also confined to longitudinal plane of motion only. The same equations of motion presented in the previous chapter are used in the following simulations.

4.3 Optimization using Aerodynamic- and Thrust-Vectoring

4.3.1 Aerodynamic Vectoring

With the inclusion of the aerodynamic vectoring feature, the longitudinal equations of motion remain same, but the lift, drag and moment coefficients additionally depend on wing-incidence, which can be expressed as:

$$C_L = C_L(\alpha, q, \delta_e, i_w) \quad (4.3.1)$$

$$C_D = C_D(\alpha, q, \delta_e, i_w) \quad (4.3.2)$$

$$C_M = C_M(\alpha, q, \delta_e, i_w) \quad (4.3.3)$$

Optimal perching trajectories are computed with the variable-incidence wing feature for different $(T/W)_{\max}$ ratios: 0.35, 0.40 and 0.45. The outboard wing is allowed to vary its wing-incidence in the range of $[-90^\circ, 90^\circ]$. At the beginning of the maneuver, the wing-incidence is assumed zero.

$$i_w(0) = 0 \quad (4.3.4)$$

The effective local angle-of-attack of the outboard wing is always bounded by its limits

through the following constraint

$$0 \leq \alpha(t) + i_w(t) \leq \pi/2 \quad (4.3.5)$$

The computed optimal perching trajectories are plotted in Fig. 4.3. For comparison, the optimal trajectories of the fixed-wing perching maneuvers are also plotted in the figure.

The time histories of the wing-incidence are shown in Fig. 4.4.

Figure 4.3: Optimal perching trajectories for both fixed-wing and variable-incidence aircraft configurations with different $(T/W)_{\max}$ ratios.

Figure 4.4: Control histories of wing-incidence for different $(T/W)_{\max}$ ratios.

It can be seen that the aerodynamic vectoring reduces both the undershoot and spatial bounds of the maneuver. For low $(T/W)_{\max}$, where perching is executed in two phases, the wing-incidence quickly becomes negative to achieve faster transition to descent mode.

After the aircrafts flight-path angle becomes sufficiently negative, the wing-incidence tries to come back to zero. When the aircraft almost reaches the maximum velocity and the pull-up maneuver is initiated, the wing-incidence becomes negative again. For large $(T/W)_{\max}$ the wing-incidence monotonically becomes negative, which brings the outboard wing to pre-stall regime and increases lift. To gain better insight into the dynamics, the variation of the local angle-of-attack for outboard wing is also plotted and shown in Fig. 4.5. It can be seen that the local angle-of-attack is almost constant for most of the time. Referring to the plot of the lift coefficient in Fig. 3.2, it can be confirmed that this angle corresponds to the maximum lift coefficient. This shows that the aircraft tries to pull-up fast by maximizing lift to execute the perching maneuver with less spatial requirements. The need for a large lift for perching calls for an efficient wing design capable of generating more lift. The use of aerodynamic vectoring has been shown to be an advantage for perching aircraft and should be considered as an improved design feature.

Figure 4.5: Time histories of effective local angle-of-attack on outboard wing section for different $(T/W)_{\max}$ ratios.

4.3.2 Thrust Vectoring

By rotating the thrust axis, the resulting thrust can be pointed to the desired direction in the longitudinal plane. If there is an offset between the thrust axis and the aircraft's center of gravity (C.G.), additional moment will be generated. In this chapter, no such offset is assumed and thus, thrust axis always passes through C.G. The equations of motion get modified slightly to the following form:

$$\dot{V} = \frac{1}{m} \left[T \cos(\alpha + \delta_T) - \frac{1}{2} C_D(\alpha, q, \delta_e) \rho V^2 S - mg \sin \gamma \right] \quad (4.3.6)$$

$$\dot{\gamma} = \frac{1}{mV} \left[T \sin(\alpha + \delta_T) + \frac{1}{2} C_L(\alpha, q, \delta_e) \rho V^2 S - mg \cos \gamma \right] \quad (4.3.7)$$

$$\dot{\alpha} = q - \frac{1}{mV} \left[T \sin(\alpha + \delta_T) + \frac{1}{2} C_L(\alpha, q, \delta_e) \rho V^2 S - mg \cos \gamma \right] \quad (4.3.8)$$

$$\dot{q} = \frac{1}{2} C_M(\alpha, q, \delta_e) \rho V^2 S \quad (4.3.9)$$

$$\dot{x} = V \cos \gamma \quad (4.3.10)$$

$$\dot{h} = V \sin \gamma \quad (4.3.11)$$

$$\dot{s} = V \quad (4.3.12)$$

where δ_T is the thrust angle with respect to the longitudinal body axis. The thrust angle is bounded between its minimum and maximum values through the following constraint

$$\delta_{T_{\min}} \leq \delta_T(t) \leq \delta_{T_{\max}} \quad (4.3.13)$$

At the initial time, the thrust angle is set to zero.

$$\delta_T(0) = 0 \quad (4.3.14)$$

The optimal trajectories achieved using thrust vectoring are computed for different $(T/W)_{\max}$: 0.35, 0.40 and 0.45. The resulting trajectories are plotted along with the fixed-wing results in Fig. 4.6. The variations of the thrust angle are plotted in Fig. 4.7.

Figure 4.6: Optimal perching trajectories for fixed-wing and thrust-vectoring aircraft configurations with different $(T/W)_{\max}$ ratios.

Figure 4.7: Control histories of thrust angle for different $(T/W)_{\max}$ ratios.

It can be seen that the thrust vectoring also reduces the spatial bounds of perching maneuver. The time histories of thrust angle in Fig. 4.7 show that for the single-phased trajectories, the thrust angle increases continuously and almost monotonically immediately after the perching maneuver is initiated. For the two-phased trajectories, this thrust angle is nearly stationary around zero in the beginning, and starts to increase only after pull-up maneuver is initiated. The increase in thrust angle is also continuous during the climb-phase. During the terminal phase, the value of thrust angle is about 90° . Thrust has a sine component in the direction of lift. The increase in the thrust angle or magnitude increases the thrust component that complements the lift needed to do a quick pull-up. To

maximize the benefit from this thrust component, the thrust angle necessarily increases at a faster rate for larger $(T/W)_{\max}$ values, which results in a swift execution of the climb-maneuver and reduced undershoot. This can be confirmed from the undershoot values which are reduced by about 40%, 45% and 55% for perching trajectories with $(T/W)_{\max}$ of 0.35, 0.40 and 0.45, respectively, from their fixed-wing counterparts. It can also be seen that the thrust angles are mostly confined to between 0° and 90° . To check for the effects of the thrust angle limits, the perching trajectories are computed with three different bounds, $[-\pi, \pi]$, $[-\pi/2, \pi/2]$ and $[0, \pi/2]$, using the same $(T/W)_{\max}$ value of 0.35. The histories of thrust angle obtained are plotted in Fig. 4.8. The variation of thrust angle is similar in most of the perching duration, except during the initial and terminal phases. The resulting trajectories are also observed to almost coincide with one another. To summarize, the thrust vectoring also reduces the spatial requirements of the perching maneuver, while the thrust angle can be restricted between 0° and 90° with no significant difference in the results.

Figure 4.8: Control histories of thrust angle for different bounds $((T/W)_{\max} = 0.35)$.

4.3.3 Coupled Aerodynamic and Thrust Vectoring

After evaluating the two features individually, for further optimization of the perching maneuvers, it is of interest to use them simultaneously to further reduce the spatial bounds. With this motivation, two scenarios are framed: the coupled and uncoupled use of aerodynamic and thrust vectoring features. For the coupled aerodynamic and thrust vectoring case, the wing-incidence and thrust angles are assumed to change at the same rate and have the same angle. Such configuration is possible for example if two propellers are present and both are fixed to the outboard wings. The first scenario is tested through trajectory optimization simulations for different $(T/W)_{\max}$: 0.35, 0.40 and 0.45. The computed optimal trajectories are plotted in Fig. 4.9 and time histories of the wing-incidence and thrust angle are shown in Fig. 4.10.

Figure 4.9: Optimal perching trajectories for fixed-wing and coupled aerodynamic and thrust vectoring configurations with different $(T/W)_{\max}$ ratios.

The comparison of the optimal perching trajectories in Fig. 4.9 confirms the obvious guess that the coupled aerodynamic and thrust vectoring configuration also reduces the spatial bounds of perching maneuvers as compared to the fixed-wing case. Reductions in undershoot of about 25%, 23% and 37% are achieved for $(T/W)_{\max}$ values of 0.35, 0.40 and 0.45, respectively. The wing-incidence/thrust angle is close to zero or negative at

Figure 4.10: Control histories of wing-incidence and thrust angles for coupled aerodynamic and thrust vectoring configuration with different $(T/W)_{\max}$ ratios.

first and then suddenly starts to increase rapidly. During the initial phase of the perching maneuver, the aircraft velocity is large and high lift is feasible. The component of the thrust in the lift direction isn't large enough to match with the lift produced by wings. Also, large angle of wing-incidence beyond stall reduces the lift. To avoid such loss of lift, the wing-incidence is initially negative and close to zero. When the velocity reduces to a small value, the lift drops and the aerodynamic vectoring becomes ineffective. At this stage the thrust vectoring effect becomes more prominent. The thrust angle starts to increase rapidly to substitute the loss of lift with the thrust component.

4.3.4 Uncoupled Aerodynamic and Thrust Vectoring

The second scenario considered is uncoupled aerodynamic and thrust vectoring feature. In this case, the wing-incidence and thrust angle become two additional and independent control inputs. The computed optimal trajectories using this feature are plotted along with the fixed-wing trajectories in Fig. 4.11. It shows that the additional freedom in the two control inputs reduces the spatial bounds of perching maneuver further. 45%, 45% and 52% of reductions are achieved in undershoot for $(T/W)_{\max}$ values of 0.35, 0.40

and 0.45, respectively. The time histories of wing-incidence and thrust are plotted in Fig. 4.12. The control histories of the thrust angle are almost similar to the results computed with thrust vectoring alone in Fig. 4.7. In contrast, the wing-incidence is not similar to the aerodynamic vectoring results in Fig. 4.4 due to shorter maneuver time. The wing-incidence histories are more similar to the coupled aerodynamic and thrust vectoring results in Fig. 4.10.

Figure 4.11: Optimal perching trajectories for fixed-wing and uncoupled aerodynamic and thrust vectoring configurations with different $(T/W)_{\max}$ ratios.

Figure 4.12: Control histories of wing-incidence and thrust angles for uncoupled aerodynamic and thrust vectoring configurations with different $(T/W)_{\max}$ ratios.

Finally, the optimal trajectories of all the four unconventional configurations and the fixed-wing design for $(T/W)_{\max}$ of 0.35 and 0.40 are plotted in Fig. 4.13 for a comparison.

The uncoupled aerodynamic and thrust vectoring configuration achieves a perching trajectory with minimum undershoot. Fixed-wing configuration has the maximum undershoot. The trajectories computed with the thrust vectoring only come very close to the ones using the uncoupled aerodynamic and thrust vectoring feature. In terms of the minimum undershoot, the uncoupled aerodynamic and thrust vectoring is superior to the remaining configurations and is closely followed by the thrust vectoring configuration. At this point, it may appear counter intuitive that the undershoot of coupled aerodynamic and thrust vectoring is larger than the one achieved with thrust vectoring as in the former's case both the unconventional configurations are active. This is because, in coupled aerodynamic and thrust vectoring configuration, the advantages of thrust vectoring are constrained by the coupling. Though the thrust angle can be increased to high values as required for quick pull-up and climbing, it also increases the local angle-of-attack, drag and reduces the velocity, making it impossible to sustain climbing and perch. To achieve successful perching, the thrust and variable-incidence angles are constrained to low values during the initial phase of the maneuver, resulting in larger undershoot for coupled aerodynamic and thrust vectoring configuration. The aerodynamic vectoring can be effective and superior to the thrust vectoring only if the available thrust is very low. In addition, the aerodynamic vectoring becomes ineffective during the terminal phase of perching when velocity is small. Based on the merits observed, these comparisons suggest that the thrust vectoring can be considered as the better option for the optimization of the perching maneuvers.

Figure 4.13: Comparison of all unconventional configurations' optimal perching trajectories with fixed-wing configuration trajectories ($(T/W)_{\max} = 0.35$ and 0.40).

4.4 Summary

A comparative study is presented on the use of the aerodynamic and thrust vectoring features for optimization of perching maneuvers. Four different configurations: aerodynamic vectoring only, thrust vectoring only, coupled aerodynamic and thrust vectoring, and uncoupled aerodynamic and thrust vectoring are studied and the results are compared with the corresponding fixed-wing perching trajectories. Perching trajectories with minimum undershoot can be achieved using the uncoupled aerodynamic and thrust vectoring configuration, closely trailed by the thrust vectoring only configuration. The aerodynamic vectoring becomes ineffective during the low-velocity terminal flight phase and does not contribute to a substantial reduction in undershoot. The thrust-vectoring-only configuration, owing to its simplicity and ability to generate near minimum undershoot perching trajectories, can be effective throughout the maneuver and is recommended as a good design feature for optimization of the perching maneuvers.

Chapter 5

Parametric Study of Perching

Maneuvers

5.1 Overview

Aircraft perching requires elevation of landing platform due to the climbing involved. Though elevation is unwanted, it cannot be avoided. To alleviate this requirement, perching trajectories are optimized to minimize the undershoot and spatial bounds of the maneuver. By formulating the perching problem into a trajectory optimization problem, optimal control inputs can be determined to perch with minimum undershoot. Minimum undershoot is not only a complex function of control inputs, but it also depends on aircraft features like wing shape and geometry, stability, available power, etc. These features can be represented quantitatively by parameterization into a finite set of parameters. For a given parameters set, optimal perching solution can be computed. However, change of a certain aircraft parameter affects the optimal perching solution, and can result in either increase or decrease of undershoot. Depending on the role of the parameter, the effect can

be major or minor. If the parameters playing significant role in perching can be identified and are allowed to vary in their valid range, better understanding and insight into the dynamics of the perching maneuver can be gained. It can also act as guideline to optimize the design of a perching aircraft and achieve better performance during the maneuver.

A very basic parametric study of perching maneuver has been presented by Wickenheiser and Garcia [4]. Only two parameters, maximum available thrust and static margin, are examined. It is shown that high available thrust and positive static margin minimize the undershoot. However, optimal perching solutions can be influenced by other aircraft parameters also, and it is important to identify them as well. With this motivation, a parametric study of perching maneuver is attempted in this chapter.

The methodology of the parametric study is to vary different aircraft parameters individually, compute optimal perching solutions, observe the trajectories and identify the role of the parameters. To carry out the parametric study, first, an aircraft model, based on a commercially available hobby airplane, is chosen as the reference aircraft configuration. Parameters of interest, and those that are assumed to affect the perching maneuver, are selected. Variations in some of these parameters alter the aircraft geometry, and modify the aerodynamics. To determine the aerodynamics for changing aircraft configurations, a modeling tool is developed based on nonlinear vortex correction method [50]. Nonlinear vortex correction method (NVCM) is a modified version of vortex lattice method (VLM), which can handle post-stall aerodynamic phenomena and estimate the steady-state lift, drag and moment of the aircraft for the given state and geometry. Unsteady aerodynamics is modeled using state-space representation [51] of Wagner's function [52]. During the parametric investigation, each of the selected parameters is varied in its feasible range, and optimal perching trajectories are computed. For quantitative comparison, undershoot and

cost function of the trajectories are chosen as measures or merit functions. These values are plotted against the changing parameter and a trend is captured. The trend reflects the effect of variation of that parameter on perching maneuver. This framework is crafted and followed to carry out the parametric investigation in this chapter.

5.2 Framework

The framework to carry out the parametric study is developed in this section. Details of the reference aircraft model, development of aerodynamic modeling tool using NVCM and reference optimal perching solutions are explained in the following subsections.

5.2.1 Baseline Aircraft Model

The baseline aircraft model considered in this chapter is very similar to the one chosen in Chapter 3 for perching simulations. The aircraft has a conventional fixed-wing configuration, with propeller attached to the nose, and tail located aft the wing. The model's mass, inertia and other geometric properties are given in the Table 5.1.

5.2.2 Aerodynamic Modeling

During parametric study, changes in some of the parameters cause modifications to the aircraft geometry and thereby the aerodynamics. To model the changing aerodynamics for altering geometric configurations, a low fidelity aerodynamic modeling tool, NVCM [50], is used.

Quantity	Value
mass m	0.8 kg
chord c	0.25 m
span b	1 m
wing surface area S_{wing}	0.25 m^2
tail surface area S_{tail}	0.04 m^2
moment of inertia I_{yy}	0.1 $kg - m^2$
CG position x_{cg}	0.0 m
wing quarter-chord position $x_{c/4_{wing}}$	-0.0625 m
tail quarter-chord position $x_{c/4_{tail}}$	-0.71 m
gravity g	9.8 m/sec^2
air density ρ	1.225 kg/m^3

Table 5.1: Reference aircraft's mass and geometry properties.

5.2.2.1 Nonlinear Vortex Correction Method

In NVCM, similar to VLM, the lifting surfaces like wing and tail are discretized into finite quadrilateral sections called panels as shown in Fig. 5.1. To each of these panels, a horseshoe vortex of unknown strength is attached at the quarter-chord line as shown in Fig. 5.2. In VLM, the strengths of the bound vortices are determined such that no-flow-through boundary condition is satisfied at the control points, which are located at the center of the three-fourth chord lines on each panel. In NVCM, the strengths of the bound vortices are determined such that the lift from the circulations is matched to that from the two-dimensional aerodynamic coefficient table across all panels. The aerodynamic lookup table can be easily created from wind-tunnel experiments and is also valid in post-stall

Figure 5.1: Wing discretized into panels.

Figure 5.2: Illustration of horseshoe vortices bounded to panels.

regimes. Since, perching involves post-stall aerodynamics, the capability of NVCM to solve post-stall aerodynamics is clearly advantageous.

To model the aerodynamics of a perching aircraft using NVCM, let us consider a vortex grid superimposed with free stream. The resultant velocity at any point in space is a summation of the velocity induced by bound vortices and the free stream velocity. The magnitude of the induced velocity is determined by the strengths of bound vortices, which are unknown. Let Γ_n be the strength of the horseshoe vortex bounded to a panel, numbered n , along its quarter-chord line with end points $A(x_{1n}, y_{1n}, z_{1n})$ and $B(x_{2n}, y_{2n}, z_{2n})$. The two trailing vortices from A and B , denoted by $A\infty$ and $B\infty$, extend to infinity in the direction parallel to the free stream. The velocity induced by the bound vortex segment

AB at a point (x, y, z) in the space is given by

$$\vec{V}_{AB} = \frac{\Gamma_n}{4\pi} \frac{\vec{r}_1 \times \vec{r}_2}{|\vec{r}_1 \times \vec{r}_2|^2} \left[\vec{r}_0 \cdot \left(\frac{\vec{r}_1}{r_1} - \frac{\vec{r}_2}{r_2} \right) \right] \quad (5.2.1)$$

where

$$\vec{r}_0 = \overrightarrow{AB} = (x_{2n} - x_{1n})\hat{i} + (y_{2n} - y_{1n})\hat{j} + (z_{2n} - z_{1n})\hat{k}$$

$$\vec{r}_1 = (x - x_{1n})\hat{i} + (y - y_{1n})\hat{j} + (z - z_{1n})\hat{k}$$

$$\vec{r}_2 = (x - x_{2n})\hat{i} + (y - y_{2n})\hat{j} + (z - z_{2n})\hat{k}$$

The velocities induced by the trailing vortices $A\infty$ and $B\infty$ are given by the following equations

$$\vec{V}_{A\infty} = \frac{\Gamma_n}{4\pi} \left(\frac{(z - z_{1n})\hat{j} + (y_{1n} - y)\hat{k}}{(z - z_{1n})^2 + (y - y_{1n})^2} \right) \left[1.0 + \frac{x - x_{1n}}{\sqrt{(x - x_{1n})^2 + (y - y_{1n})^2 + (z - z_{1n})^2}} \right] \quad (5.2.2)$$

$$\vec{V}_{B\infty} = -\frac{\Gamma_n}{4\pi} \left(\frac{(z - z_{2n})\hat{j} + (y_{2n} - y)\hat{k}}{(z - z_{2n})^2 + (y - y_{2n})^2} \right) \left[1.0 + \frac{x - x_{2n}}{\sqrt{(x - x_{2n})^2 + (y - y_{2n})^2 + (z - z_{2n})^2}} \right] \quad (5.2.3)$$

The resultant induced velocity is a total of the three velocities, and is directly proportional to the vortex strength

$$\vec{V} = \vec{V}_{AB} + \vec{V}_{A\infty} + \vec{V}_{B\infty} = \vec{C}_n \Gamma_n \quad (5.2.4)$$

If the point (x, y, z) is the center of the quarter-chord line of panel m , the induced velocity due to horseshoe vortex attached to panel n , denoted by $\vec{V}_{m,n}$, can be expressed as

$$\vec{V}_{m,n} = \vec{C}_{m,n} \Gamma_n \quad (5.2.5)$$

If N is the total number of panels, the velocity induced by all the vortices, $\vec{V}_m$, is given by the summation

$$\vec{V}_m = u_m \hat{i} + v_m \hat{j} + w_m \hat{k} = \sum_{n=1}^N \vec{C}_{m,n} \Gamma_n \quad (5.2.6)$$

The local angle-of-attack at the panel m is given by

$$\alpha_m = \tan^{-1} \left(\frac{w_m}{u_m} \right) \quad (5.2.7)$$

The forces at the panel from airfoil data are given by

$$dL_{m_{Lookup}} = \frac{1}{2} C_L(\alpha_m) \rho \vec{V}^2 dA_m \quad (5.2.8)$$

$$dD_{m_{Lookup}} = \frac{1}{2} C_D(\alpha_m) \rho \vec{V}^2 dA_m \quad (5.2.9)$$

where $\vec{V}$ is the local velocity given by

$$\vec{V} = \vec{V}_\infty + \vec{V}_m \quad (5.2.10)$$

ρ is the density, dA_m is the surface area of the panel m . $C_L(\alpha_m)$ and $C_D(\alpha_m)$ are aerodynamic coefficients from lookup table.

The lift at the panel from Kutta-Joukowski theorem is given by

$$dL_{m_{VLM}} = \rho \vec{V} \Gamma_m dS_m \quad (5.2.11)$$

where dS_m is the length the vortex is bounded to the panel.

The lifts from Kutta-Joukowski theorem and aerodynamic table should match with each other as

$$dL_{m_{VLM}} = dL_{m_{Lookup}} \quad (5.2.12)$$

The matching conditions at all the N panels, together, can be represented as

$$dL_{1_{VLM}} = dL_{1_{Lookup}} \quad (5.2.13a)$$

$$dL_{2_{VLM}} = dL_{2_{Lookup}} \quad (5.2.13b)$$

.....

$$dL_{N_{VLM}} = dL_{N_{Lookup}} \quad (5.2.13c)$$

The preceding equations are functions of strengths of bound vortices and can be expressed as the following nonlinear algebraic equations

$$F(\Gamma_1, \Gamma_2, \dots, \Gamma_N) = \begin{bmatrix} F_1(\Gamma_1, \Gamma_2, \dots, \Gamma_N) \\ F_2(\Gamma_1, \Gamma_2, \dots, \Gamma_N) \\ \vdots \\ F_N(\Gamma_1, \Gamma_2, \dots, \Gamma_N) \end{bmatrix} = \begin{bmatrix} dL_{1_{VLM}} - dL_{1_{Lookup}} \\ dL_{2_{VLM}} - dL_{2_{Lookup}} \\ \vdots \\ dL_{N_{VLM}} - dL_{N_{Lookup}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (5.2.14)$$

Solution to the above equations can be computed by any root finding scheme like Newton-Raphson method [53].

5.2.2.2 Unsteady Aerodynamics

The lift, drag and moment coefficients estimated by NVCN are valid for steady-state aerodynamics only. To account for unsteadiness, Wagner's unsteady aerodynamic model [52] is used. The Wagner's function is given as

$$\phi(t) = 1 - A_1 e^{-b_1 \tau} - A_2 e^{-b_2 \tau} \quad (5.2.15)$$

where

$$\tau = \frac{2V_\infty t}{c} \quad (5.2.16)$$

In the previous two equations, c is the chord length, t is the time, and V_∞ is the free stream velocity. The numerical values of A_1 , b_1 , A_2 and b_2 are taken from Jones' two-pole

approximation [54, 55] to the Wagner's function. The Wagner's function relates effective and actual angles-of-attack as

$$\alpha_{eff} = \alpha\phi(t) \quad (5.2.17)$$

where α_{eff} and α are effective and actual angles-of-attack.

For changing angles-of-attack, these two are related by Leishman and Nguyen's state-space model [51]. The model defines two internal state variables, x_1 and x_2 , whose dynamics are given by

$$\dot{x}_1 = x_2 \quad (5.2.18)$$

$$\dot{x}_2 = -0.01375 \left(\frac{2V}{c}\right)^2 x_1 - 0.3455 \left(\frac{2V}{c}\right) x_2 + \alpha_{3/4}(t) \quad (5.2.19)$$

where $\alpha_{3/4}$ is the angle-of-attack at three-fourth chord line.

The effective angle-of-attack is determined by using

$$\alpha_{eff}(t) = 0.006825 \left(\frac{2V}{c}\right)^2 x_1(t) + 0.10805 \left(\frac{2V}{c}\right) x_2(t) + \alpha_{3/4}(t) \quad (5.2.20)$$

During perching simulations, aerodynamic forces and moment are estimated based on effective angle-of-attack, not the actual angle-of-attack.

5.2.2.3 Reference Aircraft Model's Aerodynamics

To demonstrate the performance of NVCM, it is applied to estimate the aerodynamics of the reference aircraft model chosen for parametric study. The aircraft has NACA 0012 airfoil cross-section for its wing and tail. Sectional aerodynamics of this airfoil is obtained experimentally from wind-tunnel experiments by Maqsood and Go [35], and is taken here as the reference aerodynamics for the NVCM. The profiles of lift and drag coefficients for the chosen airfoil are plotted in Fig. 5.3.

Figure 5.3: Sectional aerodynamics of NACA 0012 wing.

NVCM is applied to obtain the steady-state aerodynamics of the reference aircraft model. The estimated lift, drag and moment coefficients for different angles-of-attack and elevator deflections are plotted in Fig. 5.4.

From Fig. 5.4, it can be seen that lift coefficient decreases and increases with upward and downward elevator deflections, respectively. The change in lift and moment coefficients with elevator deflections is consistent with the expected behavior, where upward and downward deflections of elevator should result in decrease and increase in lift coefficients, respectively and positive and negative moment coefficients, respectively. The change in elevator deflection seems to have little effect on drag coefficient. Though it is not noticeable from the plot, drag coefficient has its least value when the elevator deflection is zero, which is also an expected behavior. One of the main advantages of NVCM is that it can be used for in-flight determination of aerodynamics, especially for changing geometry during the maneuver.

Figure 5.4: Aerodynamics of the reference aircraft model estimated using NVCM.

5.2.3 Equations of Motion

The state-space model, representing the unsteady aerodynamics, is appended to the longitudinal aircraft equations of motion [56], resulting in the following nonlinear, first-order, ordinary differential equations

$$\dot{V} = \{T \cos \alpha - \frac{1}{2}C_D(\alpha_{eff}, \delta_e) \rho V^2 S - mg \sin \gamma\} / m \quad (5.2.21a)$$

$$\dot{\gamma} = \{T \sin \alpha + \frac{1}{2}C_L(\alpha_{eff}, \delta_e) \rho V^2 S - mg \cos \gamma\} / mV \quad (5.2.21b)$$

$$\dot{\alpha} = q - \{T \sin \alpha + \frac{1}{2}C_L(\alpha_{eff}, \delta_e) \rho V^2 S - mg \cos \gamma\} / mV \quad (5.2.21c)$$

$$\dot{q} = \frac{1}{2I_{yy}}C_M(\alpha_{eff}, \delta_e) \rho V^2 S \quad (5.2.21d)$$

$$\dot{x} = V \cos \gamma \quad (5.2.21e)$$

$$\dot{h} = V \sin \gamma \quad (5.2.21f)$$

$$\dot{s} = V \quad (5.2.21g)$$

$$\dot{x}_1 = x_2 \quad (5.2.21h)$$

$$\dot{x}_2 = -0.01375 \left(\frac{2V}{c} \right)^2 x_1 - 0.3455 \left(\frac{2V}{c} \right) x_2 + \alpha_{3/4}(t) \quad (5.2.21i)$$

where

$$\theta = \gamma + \alpha \quad (5.2.22a)$$

$$\alpha_{eff}(t) = 0.006825 \left(\frac{2V}{c} \right)^2 x_1(t) + 0.10805 \left(\frac{2V}{c} \right) x_2(t) + \alpha_{3/4}(t) \quad (5.2.22b)$$

for which $V, \gamma, \alpha, \theta, q, x, h$ and s are aircraft state variables, total flight-velocity, flight-path angle, angle-of-attack, pitch angle, pitch-rate, horizontal position, altitude and trajectory length, respectively. x_1 and x_2 are the unsteady aerodynamic parameters. T and δ_e are control inputs, thrust and elevator deflection, respectively. α_{eff} is the effective angle-of-attack due to unsteady aerodynamics. C_L, C_D and C_M are lift, drag and moment coefficients, dependent on aircraft geometry and effective angle-of-attack, respectively.

5.2.4 Reference Perching Solutions

Optimal perching solution is computed for the baseline aircraft model presented in Table 5.1 using the NVCM based aerodynamic model and single-phase formulation proposed in Chapter 3. During the simulation $(T/W)_{\max}$ is chosen to be 0.2. The computed flight-path of the perching solution is shown in Fig. 5.5. Corresponding state, unsteady parameters and control profiles are plotted in Figs. 5.6, 5.7 and 5.8.

Undershoot and trajectory length of the flight trajectory in Fig. 5.5 are obtained as 13.36 and 56.16 m , respectively. During parametric study, when the aircraft parameters

Figure 5.5: Optimal trajectory of the the reference perching solution.

are varied and new perching solutions are computed, the trajectories are compared with the reference aircraft model's perching solution. Both undershoot and trajectory length are normalized with those corresponding to the reference perching solution.

5.3 Parametric Study Results and Discussion

Parametric study is carried out on trajectory constraints like terminal-velocity, maximum available thrust and angle-of-attack, design parameters like aspect ratio, static margin and sweep angle, and aerodynamic parameters like zero-lift drag. Their results are presented and discussed in this section as follows:

5.3.1 Terminal Velocity Constraint

Constraints are part of the optimization formulation and their changes affect the perching solution. During the formulation of perching trajectory optimization problem, terminal velocity is expressed as a fraction of initial velocity, and is given as inequality constraint

Figure 5.6: Aircraft state profiles of the reference perching solution.

to the optimization problem in the form of following equation

$$V(t_f) \leq 0.05V(0) \quad (5.3.1)$$

Figure 5.7: Unsteady aerodynamic state profiles of the reference perching solution.

Figure 5.8: Control profiles of the reference perching solution.

Though it is obvious that undershoot and trajectory length reduce with increase in this fraction, the parameter is studied for quantitative purpose. Increase in this fraction implies that aircraft is allowed to perch with high velocities. The fraction is chosen as 5 % in the reference optimal perching solution and other parametric study results. To find the impact of undershoot reduction achievable through increase of this fraction, the parameter is varied from 0.0 to 0.2 in steps of 0.02. Results of normalized undershoot and cost function are presented in Fig. 5.9.

Figure 5.9: Variation of normalized undershoot and trajectory length with respect to terminal velocity fraction.

The plots in Fig. 5.9 show that as the fraction increases, both undershoot and trajectory length decrease monotonically. At around 20 %, both undershoot and trajectory length are

reduced by 30 %. The decrease in undershoot and cost function with increase in terminal velocity is due to the reduction in the amount of kinetic energy required to be converted to potential energy. Since moderate increase in terminal velocity greatly reduces the spatial bounds of the maneuver, design of the perching aircraft can be made slightly stronger to absorb larger landing impact and perch with significantly reduced undershoot.

5.3.2 Maximum Available Thrust

Maximum available thrust or thrust-to-weight ratio, $((T/W)_{\max})$ is determined by the selection of the engine that powers the aircraft. To study the effect of $(T/W)_{\max}$ on perching, it is varied from 0.15 to 0.30 in steps of 0.01, and optimal trajectories are computed. For $(T/W)_{\max}$ below 0.15, perching solutions are infeasible, and hence, the results are not presented. Normalized undershoot and cost function obtained from variation of this parameter are presented in Fig. 5.10.

Figure 5.10: Variation of normalized undershoot and trajectory length with respect to maximum available thrust.

The plots in Fig. 5.10 show that both undershoot and trajectory length reduce with increase in $(T/W)_{\max}$. However, the reduction and variation in undershoot is less than 5 %. High $(T/W)_{\max}$ allows the aircraft to fly at high angles-of-attack and low velocities.

As a result, perching can be initiated at low velocities. The decrease in initial velocity with increase in available thrust reduces the total kinetic energy required to be converted to potential energy, and thereby the undershoot during perching. Though cost function should reduce monotonically with $(T/W)_{\max}$, it is observed to reach saturation at around 0.25. Beyond 0.25, perching trajectories are identified to remain almost similar. Though high available thrust can balance the weight of the aircraft, the elevator doesn't have sufficient moment control power to trim at high angles-of-attack. Hence, perching trajectories remained similar beyond certain $(T/W)_{\max}$ value.

5.3.3 Maximum Angle-of-Attack

Maximum angle-of-attack, $\alpha_{\max}$, is usually a limitation due to the availability of high angle-of-attack aerodynamic data. Its nominal value for the reference perching solution is 90° . To understand the role played by maximum angle-of-attack in perching, it is varied from 10° to 90° in steps of 5° and optimal perching solutions are computed. Variations of normalized undershoot and cost function with change in maximum angle-of-attack are presented in the second two plots of Fig. 5.11.

Figure 5.11: Variation of normalized undershoot and trajectory length with respect to maximum angle-of-attack.

The results show that as the maximum angle-of-attack increases, undershoot reduces, and reaches saturation at around 20° , where lift coefficient is close to its maximum. Beyond 20° , the reduction is very minor. Similar trend is observed with cost function as well, but the saturation is reached at a slower rate beyond 20° . During perching, angle-of-attack is optimally adjusted to generate high lift to sustain the altitude climb. It is not high and drag will not be at its maximum. Aircraft enters high angle-of-attack regime during the terminal phase of climb maneuver only when flight-path angle is large. At this stage, velocity is low, and the contribution of aerodynamic drag to the total deceleration force becomes insignificant. As a result, high angles-of-attack aerodynamics doesn't have major influence on the perching maneuver. The findings are contrary to the experimental results by Aditya [19] and Cory [15], in which the gliders were pitched to nearly 90° just before landing. In their experiments, only drag was used for the deceleration, and gravity is least exploited. Drag, alone, cannot decelerate an aircraft to a low velocity state. Due to the low weights of their gliders, the impact during landing was insignificant. This landing strategy cannot be used if the aircraft weight is high, which is the case in this paper as the landing impact would be higher. To perch with low-velocity, both gravity and drag are to be used. This is an important finding since high angles-of-attack aerodynamic data is usually not available for most of the UAVs. By constraining the the angle-of-attack to moderate values, perching can still be accomplished with only marginal increase in undershoot.

5.3.4 Aspect Ratio

Aspect ratio, (b^2/s) , is a crucial parameter that has significant effect on the overall aerodynamics of the aircraft. It is the ratio between square of wing-span and surface area.

Figure 5.12: Variation of normalized undershoot and trajectory length with respect to aspect ratio.

Its nominal value for the reference aircraft model is 4. To find the role of aspect ratio on perching, it is varied from 3 to 10 in steps of 0.5. As changes in aspect ratio modify wing dimensions, chord length is adjusted to keep the wing area constant. Wing's leading edge position is also kept unchanged. Very high increase in aspect ratio can result in structural issues like wing design feasibility. However, it is assumed that wing design is feasible till the maximum aspect ratio, 10. Another issue that occurs due to high aspect ratio is increase in wing flexibility, which leads to deformations like bending and twisting of the wing and loss of lift. Since aeroelastic effects are outside the scope of this thesis, it is assumed that the wing is rigid and no such deformations occur. Parametric study results of aspect ratio can be found in Fig. 5.12.

The results show that both undershoot and trajectory length reduce and saturate with increase in aspect ratio. High lift is desired during the initial phase of climb maneuver for rapid velocity pitching. Aircraft dives to increase its velocity and generate large lift force. As aspect ratio increases, downwash reduces, quality of lift distribution improves and lift increases. With increase in available lift from increased aspect ratio, flight-path angle turns up swiftly and climb-phase shortens, resulting in undershoot reduction. Availability of high lift also eliminates the requirement for a dive-phase, resulting in cost function

reduction. For large aspect ratios, more than 40 % reduction in undershoot is observed. One particular observation that can be made for this result is that high lift generation is desirable for perching. Conventional lift enhancement devices like flaps can be used to increase the lift coefficient and reduce the spatial bounds of perching trajectory.

5.3.5 Static Margin

Static margin is the location of neutral point relative to the C.G. position along body x-axis of the aircraft. It is an indication for the longitudinal stability of the aircraft, where negative and positive values imply stability and instability, respectively. For the reference aircraft model, C.G. is located 0.12 *m* ahead of neutral point. To study the effect of static margin, C.G. position relative to neutral point is varied from 0.13 *m* to -0.05 *m*, in steps of -0.01 *m* and optimal perching trajectories are computed. Results of normalized undershoot and cost function are shown in first two plots of Fig. 5.13.

Figure 5.13: Variation of normalized undershoot and trajectory length with respect to static margin.

The plots show that as C.G. moves rearward, undershoot reduces, reaches a minimum and then starts to increase. There is an optimal location for C.G. position, aft neutral point, at which undershoot and cost function have a lowest minimum. Close location of

C.G. to the neutral point reduces the pitch control effort required for the maneuver, and results in reduced undershoot trajectories. For large C.G. offsets, control effort increases, and large undershoot and trajectory length are required to perch the aircraft.

5.3.6 Sweep

Wing sweep, (Λ), is the rotation of wing in the xy-plane of aircraft. For the reference aircraft model, sweep angle is 0° . It is varied from -40° to 20° , in steps of 2.5° , where positive and negative sweep angles indicate rearward and forward sweep, respectively.

The parametric study results of wing sweep are presented in Fig. 5.14.

Figure 5.14: Variation of normalized undershoot and trajectory length with respect to sweep angle.

The results show that as wing sweep increases from -40° , undershoot reaches a minimum value at around -35° and then increases. Dive-phase disappears and cost function doesn't show significant variation for large negative sweep angles. But, for sweep angles greater than -5° , dive-phase appears and cost function increases. Slight increase in undershoot is observed with sweep angle as well during the transition from two-phased perching to single-phased perching at around -2.5° . Wing sweep mostly affects the downwash distribution and causes changes to the induced drag of the aircraft. Rearward and forward

sweep reduce and increase induced drag, respectively. Perching requires high lift and drag forces. With increased availability of drag force from forward sweep, perching trajectory shortens. Through sweep adjustment, a reduction of about 15% and 30% can be achieved for undershoot and cost function, respectively.

5.3.7 Zero-lift Drag

Zero-lift drag coefficient is a design parameter, which is basically determined by the shape and surface roughness of the aircraft. Usually, aircraft are designed to keep this parameter as low as possible. Since perching utilizes drag for deceleration, the parameter is studied for its effect on the maneuver. The parameter is varied from 0 to 0.04 in steps of 0.0025. Results of normalized undershoot and cost function are presented in Fig. 5.15.

Figure 5.15: Variation of normalized undershoot and trajectory length with respect to zero-lift drag coefficient.

The results show that with increase in zero-lift drag coefficient, both undershoot and cost function increase. The variation in undershoot is less than 2 %, but cost function has shown around 50 % reduction. During perching, high lift is desired during initial phase of climb maneuver to quickly point the velocity vector upwards, opposite to the direction of gravity. To generate high lift, aircraft is required to fly at high velocities, but with

increase in drag, the power to fly at high speeds reduces. To initiate climbing with high speed, aircraft performs dive maneuver, which results in increase in spatial bounds.

5.4 Summary

A framework, comprising NVCM based aerodynamic modeling tool and perching trajectory optimization formulation to systematically analyze the effects of various aircraft parameters in perching maneuver, is developed. The parameterization of geometric design features and use of NVCM solver for inflight determination of aerodynamic forces and moment has simplified the optimization formulation and trajectory computation process. Parametric study shows that cost function is significantly influenced by all the parameters. Undershoot is significantly effected by aspect ratio and terminal velocity and maximum angle-of-attack constraints. The study shows that dive-phase is more sensitive to parametric changes. Availability of large control moment and resultant aerodynamic force are desirable for optimal perching. Relaxation of terminal velocity constraint greatly reduces the undershoot. High angle-of-attack aerodynamics is not critical and can be discarded for very low-velocity perching. Effects of passive and dynamic three-dimensional wing morphing observed in birds can also be studied using the framework developed in this paper.

Chapter 6

Trajectory Tracking of Perching Maneuvers

6.1 Overview

The motivation for carrying out the stability analysis arises from practical problems like external disturbances that prevent the aircraft from initiating the perching maneuver at the same initial condition determined by the optimization solution. If the initial conditions deviate from the optimal initial conditions, it is important to know whether these perturbations lead to the trajectory that diverges or converges to the optimal trajectory. Here, the stability of the perching maneuver is considered as the convergence of the neighboring trajectories to the optimal trajectories. The tools and framework required to carry out this stability analysis are borrowed from the contraction theory. It is shown that optimal perching trajectories are in general unstable and neighboring solutions will diverge from the nominal solution in the presence of state perturbations. To address this problem, a scheme for tracking the optimal trajectories is proposed. Sliding mode control technique,

which is robust and efficient in handling highly nonlinear systems, is used in the design of the tracking controller. The objective of the controller is to correct the state errors and bring the aircraft to its nominal optimal trajectory. Precise landing at the desired location is very crucial in perching and hence, the tracking is carried out not with respect to time, but with respect to spatial domain, which is based on the horizontal position relative to the landing point. The performance of the controller is then validated through numerical simulations.

6.2 Stability Analysis

The optimization formulation presented in Chapter 3 gives only open-loop type control inputs to execute the perching maneuver. Given that, in practice, it will be difficult to initiate the maneuver exactly with the same initial condition determined by the optimization solution and to anticipate the constant presence of uncertainties and external disturbances, it is important to examine whether deviations in state variables at any instant will lead to trajectory that converges to or diverges from the nominal optimal trajectory. The stability analysis results will also determine the control strategy to be employed for a successful perching maneuver. Contraction theory is used for the stability analysis here.

6.2.1 Contraction Theory

Contraction theory provides the necessary tools to analyze the stability properties of transient trajectories for linear or nonlinear systems in state-space form [57, 58]. This theory is inspired from the concepts in fluid mechanics, and is derived using elementary tools from continuum mechanics and differential geometry. In the conventional approach of stability

analysis using small perturbation theory, equilibrium points of the nonlinear system are determined first and the state-space is linearized around them. Stability is defined whether the trajectories originating from the state-space neighboring the equilibrium points converge to them or not by losing the information on initial conditions. In simple terms, stability is defined only for equilibrium points; for non-equilibrium points or trajectories, stability is undefined. Contraction analysis generalizes this notion of stability, by analyzing it differentially. Stability is defined whether two neighboring trajectories converge or not. The treatment of stability of one system trajectory with respect to another is called as incremental form. By this generalization, convergence around non-equilibrium points or trajectories can also be analyzed. One of the main advantages of contraction analysis is that the knowledge of equilibrium points or nominal motion is not required. Conventional stability analysis can be considered as a special case, where the trajectory of interest is constant. To derive the necessary conditions for determining/analyzing the stability of a trajectory using contraction analysis, let us consider a nonlinear system in state-space form as

$$\dot{\underline{x}} = f(\underline{x}, t) \quad (6.2.1)$$

where $\underline{x}$ is an n -dimensional vector, t is time, and f is a nonlinear vector field. The first variation of the preceding equation is given by

$$\delta \dot{\underline{x}} = \delta f = \frac{\partial f}{\partial \underline{x}} \delta \underline{x} \quad (6.2.2)$$

Define a state-dependent virtual transformation as

$$\delta \underline{z} = \Theta(\underline{x}, t) \delta \underline{x} \quad (6.2.3)$$

where $\Theta(\underline{x}, t)$ is a nonsingular transformation matrix. With this transformation, the virtual dynamics in $\delta \underline{z}$ can be expressed as

$$\delta \dot{\underline{z}} = F(\underline{x}, t) \delta \underline{z} \quad (6.2.4)$$

where

$$F(\underline{x}, t) = \left(\dot{\Theta} + \Theta \frac{\partial f}{\partial \underline{x}} \right) \Theta^{-1}$$

The definition of contraction is as follows: Given the system $\dot{\underline{x}} = f(\underline{x}, t)$, a region in state space is called a contraction region with respect to a positive-definite metric $M(\underline{x}, t) = \Theta^T \Theta$ if F as given earlier is strictly negative definite, that is,

$$F(\underline{x}, t) = \left(\dot{\Theta} + \Theta \frac{\partial f}{\partial \underline{x}} \right) \Theta^{-1} \leq -\beta I \quad (6.2.5)$$

or, equivalently,

$$\frac{\partial f^T}{\partial \underline{x}} M + \dot{M} + M \frac{\partial f}{\partial \underline{x}} \leq -2\beta M \quad (6.2.6)$$

where β is a strictly positive constant.

The matrices in Eqns. (6.2.5) and (6.2.6) are called the generalized and symmetric form of the Jacobians, respectively. If this Jacobian is negative definite, semidefinite, or indefinite, the region is said to be contracting, semicontracting, or indifferent, respectively, with respect to Θ or M . Contraction implies convergence of the neighboring trajectories to the given trajectory, which implies that the given trajectory is stable.

The definition of contraction can be conversely stated as follows: A region in state space is contracting if there exists a positive-definite metric M such that Eqn. (6.2.6) is satisfied. If the existence of the metric can be proven, then the transient stability of the given trajectory is guaranteed. Thus, the problem of determining the stability of a

trajectory becomes the problem of searching for the metric M that satisfies Eqn. (6.2.6). The existence of a metric can be established through the optimization-based solution procedure presented by Aylward et al. [59]. In their procedure, the metric elements are represented as multivariate polynomials in terms of the state variables $\underline{x}$ and unknown coefficients as

$$M = \begin{bmatrix} p_{11}(\underline{x}) & p_{12}(\underline{x}) & \dots & p_{1n}(\underline{x}) \\ p_{21}(\underline{x}) & p_{22}(\underline{x}) & \dots & p_{2n}(\underline{x}) \\ \dots & \dots & \dots & \dots \\ p_{n1}(\underline{x}) & p_{n2}(\underline{x}) & \dots & p_{nn}(\underline{x}) \end{bmatrix} \quad (6.2.7)$$

where n is the number of state variables. The symmetric form of generalized Jacobian is represented as

$$R(x) = \frac{\partial f^T}{\partial \underline{x}} M + M \frac{\partial f}{\partial \underline{x}} + \dot{M} \quad (6.2.8)$$

The metric existence can be proven by determining the unknown polynomial coefficients, such that $y^T M y$ and $-y^T R y$ are positive definite. Here, y is a vector with monomial elements of state variables. For a given Jacobian $\frac{\partial f}{\partial \underline{x}}$, the positive definiteness of $y^T R y$ can be evaluated by solving for its feasibility as a sum of squares problem [60] with positive-definite constraint on the metric. If a sum of squares solution is feasible, the metric exists and the state space is contracting. Otherwise, the state space is noncontracting. This procedure provides a semi-analytic approach to determine the existence of the metric and the stability of transient trajectory. If the metric existence is feasible throughout the trajectory, the maneuver is stable. Its inexistence at any point implies instability.

6.2.2 Stability Analysis of Perching Trajectory

Consider the aircraft equations of motion in state-space form given by Eqns. (3.2.1). Excluding the kinematic state variables, the state vector comprising only the dynamic state variables can be expressed as

$$\underline{x} = [V, \gamma, \alpha, q]^T \quad (6.2.9)$$

The Jacobian $J = \frac{\partial f}{\partial \underline{x}}$ can be computed numerically along the optimal perching trajectory. In this work, because the aircraft equations of motion are highly nonlinear and the Jacobian J is available numerically, the optimization procedure for determining the existence of the metric outlined in the preceding subsection is implemented numerically. Instead of multivariate polynomials, constants are used for the metric elements. Negative definiteness of $R(\underline{x})$ is evaluated numerically by computing its maximum eigenvalue. If $R(\underline{x})$ is not negative definite, the metric elements are computed to minimize the maximum eigenvalue with positive-definite constraint on metric M . If the maximum eigenvalue ceases to become negative, the metric existence is infeasible; otherwise, the metric exists. To avoid an incorrect conclusion on the metric existence, the computation procedure is implemented in two steps. In the first step, the genetic algorithm [61] is used to compute the metric elements to obtain a reasonable solution. Using this solution as an initial guess, the maximum eigenvalue is further minimized by using sequential quadratic programming [62] in the second step. The search for the metric elements is terminated once $R(\underline{x})$ becomes negative definite or the maximum eigenvalue ceases to go below zero.

For the optimal perching trajectory obtained in the preceding section, during the 3.931 s maneuver, it is found that the metric existence is feasible in time intervals $[0, 0.216 \text{ s}]$, $[0.334 \text{ s}, 2.378 \text{ s}]$ and $[2.791 \text{ s}, 2.811 \text{ s}]$. For the other time intervals, the metric

Figure 6.1: Time histories of perturbation errors of angle-of-attack and velocity.

existence is infeasible. To confirm these results, simulations are carried out to compute the resulting flight trajectory using the same optimal control inputs, but with perturbed initial conditions. The time histories of errors between the perturbed and optimal trajectories of velocity and angle-of-attack state variables are plotted in Fig. 6.1. To indicate the stable and unstable intervals in the trajectory, contracting and non-contracting regions are denoted by solid (—) and dashed (---) linetypes, respectively.

Except for the short interval between 0.216 and 0.334 s, the metric existence is feasible and the trajectory converges to the nominal one until 2.3782 s. The perturbed angle-of-attack history converges to the nominal one relatively quickly (in about 0.5 s). The convergent trend of the perturbed velocity to the nominal velocity history is also observed, but at a much slower rate due to the relatively slow translational dynamics. After 2.37 s, the existence of the metric is not feasible and the trajectory diverges from the optimal solution. As a result, the perturbed velocity history never fully converges to the nominal one, and both perturbed angle of attack and velocity start to diverge from their nominal trajectories, as can be observed from Fig. 6.1. Because of this divergence, the open-loop

optimal perching trajectories can be considered unstable and cause the aircraft to deviate from its desired landing point.

6.3 Trajectory Tracking

The analysis in the preceding section has shown that the optimal perching trajectories are not stable and the perturbed trajectories tend to diverge from the optimal one. Real-time calculation of a new optimal trajectory from its perturbed state is not generally possible due to the large computational time required. Under these circumstances, the optimal open-loop-type control inputs obtained previously cannot be relied on to perch the aircraft at the desired location, and corrections to control inputs in response to state perturbations are typically required. In this paper, this is addressed by formulating the aircraft perching problem into a trajectory tracking problem as follows: Given an optimal perching trajectory and perturbed states, determine the control inputs to correct the perturbations and track the optimal trajectory. Though aerodynamic uncertainties and external disturbances are present and need to be accounted for in real-time perching, they are assumed to be absent; only state perturbations are considered. To design a controller for this task, the sliding-mode control technique, which is inherently robust in application to highly nonlinear systems, is chosen.

6.3.1 Sliding Mode Controller Design

Sliding-mode control (SMC) is a form of variable structure control generally used for nonlinear systems to follow a desired trajectory [63, 64]. By defining a set of sliding functions with the desired trajectory as a solution, a stable control law can be derived.

To design a sliding-mode controller for the perching problem, let us consider the aircrafts longitudinal equations of motion given in Eqn. (3.2.1):

$$\dot{\underline{x}} = f(\underline{x}, \underline{u}) \quad (6.3.1)$$

where $\underline{x} = [V, \gamma, \alpha, q, x, h]^T$ and $\underline{u} = [\delta_e, T]^T$.

To apply SMC, the system is required to be affine in the control input. However, the aircraft equations of motion for the current aerodynamic model are nonlinear in control inputs. To address this problem, the equations of motion are linearized about its control position as

$$\dot{\underline{x}} = f(\underline{x}, \underline{u} + \Delta \underline{u}) \approx f(\underline{x}, \underline{u}) + \left(\frac{\partial f}{\partial \underline{u}} \right)_{\underline{x}, \underline{u}} \Delta \underline{u} \quad (6.3.2)$$

With this linearization, change in the control inputs becomes the control inputs to be computed from SMC, not the actual control inputs.

Two control inputs, elevator deflection and thrust, are used to define two sliding functions. Elevator deflection mostly affects the rotational dynamics (angle-of-attack, pitch, and pitch rate). Thrust mostly affects the translational dynamics (velocity and flight-path angle). The angle of attack and flight-path angle mostly determine the contribution of the aerodynamic and gravity forces, respectively. Because perching requires deceleration resulting from these forces to achieve a successful landing, these two variables are chosen to be tracked using the following sliding functions:

$$\underline{s} = \begin{bmatrix} (\dot{\alpha} - \dot{\alpha}_d(x)) + \lambda_\alpha (\alpha - \alpha_d(x)) \\ (\dot{\gamma} - \dot{\gamma}_d(x)) + \lambda_\gamma \left(\gamma - \gamma_d(x) - \tan^{-1} \left(\frac{h_d(x) - h}{k_h} \right) \right) \end{bmatrix} \quad (6.3.3)$$

where λ_α , λ_γ and k_h are positive real numbers, $\dot{\alpha}_d(x)$, $\dot{\gamma}_d(x)$, $\alpha_d(x)$, $\gamma_d(x)$ and $h_d(x)$ are state variables interpolated from the optimal perching trajectory with respect to horizontal

distance. The term $\tan^{-1}\left(\frac{h_d(x)-h}{k_h}\right)$ is introduced to correct for the altitude difference, which makes it a cascaded control structure with altitude tracking as outer loop and flight-path tracking as inner loop.

To derive a stable control law to track the desired trajectory, defined by $\underline{s} = 0$, consider a positive-definite Lyapunov candidate function

$$V = \frac{1}{2} \underline{s}^T \underline{s} \quad (6.3.4)$$

Differentiating Eqn. (6.3.4) with respect to time gives

$$\dot{V} = \underline{s}^T \dot{\underline{s}} \quad (6.3.5)$$

To drive $\underline{s}$ towards the solution $\underline{s} = 0$, $\dot{V}$ must be negative-definite. This is achieved by defining a reaching-law as

$$\dot{\underline{s}} = -(Q\underline{s} + K\mathbf{sat}(\underline{s})) \quad (6.3.6)$$

where

$$\mathbf{sat}(\mathbf{s}) = \begin{cases} \mathbf{s}/\epsilon & \text{if } \mathbf{abs}(\mathbf{s}) \leq \epsilon, \\ \mathbf{sgn}(\mathbf{s}) & \text{if } \mathbf{abs}(\mathbf{s}) > \epsilon. \end{cases} \quad (6.3.7)$$

Q and K are positive-definite diagonal matrices, $\mathbf{sgn}$ indicates the sign function, and ϵ is half the width of the boundary layer around the sliding surface in which the sign function can be approximated by a linear function. The first term in the reaching law, $-Qs$, ensures smooth and asymptotic convergence of all the sliding functions towards the solution. The second term in the reaching law, $-K\mathbf{sat}(\underline{s})$, addresses bounded uncertainty in the system, but can result in chattering. The use of the sat function instead of the $\mathbf{sgn}$ function inside the boundary layer is to minimize chatter [65], which sliding-mode controllers often suffer

from. By this approximation, the discontinuous form of switching dynamics in Eqn. (6.3.6) due to discontinuity in the **sgn** function is replaced with a continuous one. This removes the root for the discontinuity in the control law and alleviates chatter. Different choice of values for the parameters in Q and K matrices results in different types of solutions.

By chain rule, $\dot{\underline{s}}$ in Eqn. (6.3.6) can be expanded as

$$\dot{\underline{s}} = \frac{\partial \underline{s}}{\partial \underline{x}} \dot{\underline{x}} = \frac{\partial \underline{s}}{\partial \underline{x}} \left(f(\underline{x}, \underline{u}) + \left(\frac{\partial f}{\partial \underline{u}} \right)_{\underline{x}, \underline{u}} \Delta \underline{u} \right) = - (Q \underline{s} + K \mathbf{sat}(\underline{s})) \quad (6.3.8)$$

Solving the preceding equation for $\Delta \underline{u}$ gives

$$\Delta \underline{u} = - \left(\frac{\partial \underline{s}}{\partial \underline{x}} \frac{\partial f}{\partial \underline{u}} \right)^{-1} \left[\frac{\partial \underline{s}}{\partial \underline{x}} f(\underline{x}, \underline{u}) + Q \underline{s} + K \mathbf{sat}(\underline{s}) \right] \quad (6.3.9)$$

The control command is then updated as

$$\underline{u}_c = \underline{u} + \Delta \underline{u} \quad (6.3.10)$$

6.3.2 Simulation Results

To validate the resulting controller, simulations are carried out to track the optimal perching trajectory generated previously with $(T/W)_{\max} = 0.4$. Two different sets of initial states are generated by perturbing the initial conditions. In one set, the altitude is raised by 2 m, and in the other set, it is lowered by 2 m. Remaining state variables are perturbed randomly within 10% of their nominal values. The thrust available for the tracking aircraft is chosen to be $(TW)_{\max} = 0.5$, slightly greater than the maximum available thrust used for computing the reference trajectory. The increased thrust is to provide extra power to the tracking aircraft to improve its tracking performance and increase the success rate of landing. If this extra thrust is not provided, the aircraft starting below the nominal trajectory will not have sufficient energy to climb and reach the landing point. The aircraft

Figure 6.2: Perturbed trajectories tracking the nominal trajectory using SM controller: — nominal, -- and - . perturbed.

starting above the nominal trajectory has no problem because it does not need any extra energy to climb. With the chosen optimal trajectory and perturbed initial conditions, the control inputs are computed numerically using the designed controller to track the reference trajectory. The equations of motion are solved using Euler method [42], with a constant time-step of 0.01 s. The choice of time-step is verified by decreasing it further and solving the equations of motion. By checking that the solution does not vary, the time-step is chosen to be 0.01 s. The gains in the control law are determined by trial and error such that the performance of the controller is satisfactory. The computed trajectories, state histories, and control inputs are plotted in Figs. 6.2, 6.3 and 6.4. To ensure the feasibility of the control inputs, they are subjected to rate limits given in Table 3.3. Because exact landing at the final position is difficult, a tolerance of 10 cm is allowed. When the final position of the tracking trajectory is within the radial distance of this tolerance, the controller stops and the simulation is terminated.

Simulation results in Fig. 6.2 show that successful tracking of reference trajectory is achieved from both perturbed initial conditions. Convergence to the reference trajectory

Figure 6.3: State histories tracking the nominal trajectory using SM controller: — nominal, -- and - . perturbed.

is achieved within a horizontal range of 1015 m . One of the requirements of the controller is to not exceed the optimal undershoot of the reference trajectory. It can be observed that the undershoot of the controlled trajectories does not exceed the optimal undershoot. The final position for the controlled trajectories is also within the allowed 10 cm radial tolerance. Fig. 6.3 shows that the state variables to be tracked, flight-path angle and angle of attack, quickly converge to their nominal solutions within a horizontal range of 1520 m . The other state variables, although not controlled directly, also converge to the nominal solutions, albeit at a slower rate than then tracked variables. Interestingly, convergence of elevator deflection history to its nominal history can also be observed in Fig. 6.4. Such convergence is not seen in the thrust input. Chattering, a performance-

Figure 6.4: Control commands for tracking the nominal trajectory using SM controller: — nominal, -- and -·-. perturbed.

degrading phenomenon associated with sliding-mode controllers, is greatly minimized by the use of the **sat** function in the reaching law. As a result, smooth elevator deflections and thrust inputs are obtained. These successful tracking results from different perturbed initial conditions demonstrate the effectiveness of the designed controller.

6.4 Summary

Contraction analysis indicates that the optimal perching trajectories are unstable in the presence of state perturbations, the trajectories will in general diverge from its nominal trajectory. A trajectory tracking scheme utilizing sliding mode control approach has been proposed as a solution to address this problem. With proper choice of the gains, sliding functions, and reaching law, the controller is proved to be effective. The simulations performed indicate that successful tracking of the optimal solutions without exceeding the optimal undershoot can be achieved.

Chapter 7

Extension of Trajectory Tracking

Controllers to Conventional Aircraft

Landing Maneuvers

7.1 Overview

The sliding-mode controller, developed in the previous chapter, has been successfully applied for tracking perching trajectories in the longitudinal plane of motion. The applicability of this controller is extended in this chapter to address a conventional landing maneuver problem by considering lateral dynamics and making slight modifications in the control law design. The reference flight-path changes to as required for the conventional landing. In addition to altitude, tracking of lateral positions is also carried out with respect to horizontal spatial position in X -direction. The desired flight-path angle still remains same as the required angle plus a correction due to altitude offset. The desired lateral state variables are defined as functions of offsets in lateral position. The additional

control inputs, aileron and rudder, are used to track the desired lateral state variables, which indirectly track the reference flight-path. Since different automatic landing systems are already designed based on different controllers, the performance of the sliding-mode is verified against a conventional PID controller.

The following sections in this chapter are organized as follows. Section 7.2 presents a detailed introduction to the conventional aircraft landing problem. Section 7.3 explains the design of sliding mode controller and the derivation of the control law for the aircraft landing maneuver. In Section 7.4, the design of a PID landing controller is formulated for comparison purposes. Simulation results, validating the controller design, are presented and discussed in Section 7.5. Chapter summary is presented in Sec. 7.6 .

7.2 Problem Formulation

The objective in this chapter is to extend the applicability of perching trajectory tracking controller for autonomous aircraft landing maneuver. A reference flight path, comprising the glide-capture and flare phases, is specified to be tracked by the aircraft to achieve the desired automatic landing. The glide trajectory is defined as a straight line with a negative flight-path angle that lies on the longitudinal landing plane, consisting the touchdown point and centerline of the runway.

Consider an inertial coordinate system as shown in Fig. 7.1, with the x, y and z axes pointing toward the runway, vertically downwards, and rightwards, respectively. In this coordinate system, the reference glide-path can be mathematically expressed by the following equations:

$$Z - Z_0 = - (X - X_0) \tan \gamma_{glide} \quad (7.2.1)$$

Figure 7.1: Illustration of landing maneuver showing reference glide- and flare-paths.

$$Y = Y_0 \quad (7.2.2)$$

where (X_0, Y_0, Z_0) is the initial point of glide-phase maneuver and γ_{glide} is the descent angle (constant in this case). Without loss of generality, the origin can be placed at the starting point of the reference glide-path. In such case, Eqs. (7.2.1) and (7.2.2) are reduced to

$$Z = -X \tan \gamma_{glide} \quad (7.2.3)$$

$$Y = 0 \quad (7.2.4)$$

The aircraft is required to descend along this straight line steadily until the altitude to initiate the flare maneuver is reached, which is approximately $30ft$ above the ground. During the flare maneuver, the aircraft is specified to reduce its descent rate exponentially and reach a near zero flight-path angle by tracking an exponential curve. The reference

flare-path with an exponential-curve profile is defined in the following form:

$$h_f = h_{f0}e^{(-t/\tau)} \quad (7.2.5)$$

where h_f is the desired altitude at time t , h_{f0} is the initial altitude at which the flare maneuver is initiated, and τ is a constant indicating the rate of descent.

The straight line during the glide-phase and the exponential curve during the flare maneuver lie on the same longitudinal landing plane and are treated as the reference trajectory that needs to be tracked. Autonomous aircraft landing maneuver is assumed to be initiated with large offsets in initial lateral and longitudinal positions from the reference glide-path trajectory. The functions of the landing controller are to (a) compute the required control inputs to steer the aircraft toward the longitudinal plane of landing, (b) descend along the glide-path steadily, and (c) execute the flare maneuver smoothly.

A nonlinear six degrees-of-freedom aircraft model from the High Angle-of-Attack Research Vehicle (HARV) [66] is used to simulate the landing maneuver. Mass, inertia and other geometric properties of this aircraft are given in Table 7.1. The complete equations of motion representing the aircraft dynamics are given in 7.2.1. The usual aircraft controls, i.e. thrust, and deflections of ailerons, elevators, and rudders, are used as the control inputs. Saturation and rate limits of the control inputs are provided in Table 7.2. The aerodynamic model of the aircraft is available in a tabulated form for angles-of-attack ranging from $-14^\circ - 90^\circ$ with four degrees interval. The aerodynamic force and moment coefficients are expressed in linearized form in terms of stability derivatives, state, and control variables as given in 7.2.2. Given an angle-of-attack, the stability derivatives can be obtained from the aerodynamics lookup table using certain interpolation scheme to compute the aerodynamic coefficients. In the current study, aerodynamic model uncer-

tainties and external disturbances are not considered. The control laws obtained using the sliding mode control design approach are validated using this aircraft model.

Quantity	Value
Wing span, b	37.42 ft
Mean aerodynamic chord, c	11.52 ft
Gravitational acceleration, g	32.0 ft/s^2
Roll Inertia, I_{xx}	22789 $slug - ft^2$
Pitch Inertia, I_{yy}	176809 $slug - ft^2$
Yaw Inertia, I_{zz}	191744 $slug - ft^2$
Aircraft mass, m	1128.09 $slug$
Wing area, S	400 ft^2
Maximum thrust, T_m	16000 lb
Air density, ρ	0.00258 $slug/ft^3$

Table 7.1: Aircraft parameters and their values

Control Input	Saturation Limit	Rate Limit
Aileron, δ_a	(-35,35) deg	± 100 deg/sec
Elevator, δ_e	(-25,10) deg	± 40 deg/sec
Rudder, δ_r	(-30,30) deg	± 82 deg/sec
Throttle, η	(0,1)	None

Table 7.2: Control inputs saturation and rate limits.

7.2.1 Equations of Motion

The mathematical model of the aircraft used in the present study is represented by the following first-order differential equations [37]:

$$\begin{aligned}
\dot{V} &= \frac{1}{m} \left[T_m \eta \cos \alpha \cos \beta - \frac{1}{2} C_D (\alpha, q, \delta e) \rho V^2 S - mg \sin \gamma \right] \\
\dot{\alpha} &= q - (p \cos \alpha + r \sin \alpha) \tan \beta \\
&\quad - \frac{1}{mV \cos \beta} \left[T_m \eta \sin \alpha + \frac{1}{2} C_L (\alpha, q, \delta e) \rho V^2 S - mg \cos \mu \cos \gamma \right] \\
\dot{\beta} &= \frac{1}{mV} \left[-T_m \eta \cos \alpha \sin \beta + \frac{1}{2} C_Y (\beta, p, r, \delta e, \delta a, \delta r) \rho V^2 S + mg \sin \mu \cos \gamma \right] \\
&\quad + (p \sin \alpha - r \cos \alpha) \\
\dot{\gamma} &= \frac{1}{mV \cos \gamma} \left[T_m \eta \sin \alpha + \frac{1}{2} C_L (\alpha, q, \delta e) \rho V^2 S - mg \cos \mu \cos \gamma \right] \cos \mu \cos \gamma \\
&\quad - \frac{\sin \mu \cos \gamma}{mV \cos \gamma} \left[-T_m \eta \cos \alpha \sin \beta + \frac{1}{2} C_Y (\beta, p, r, \delta e, \delta a, \delta r) \rho V^2 S + mg \sin \mu \cos \gamma \right] \\
\dot{p} &= \frac{I_y - I_z}{I_x} qr + \frac{1}{2I_x} \rho V^2 S b C_l (\beta, p, r, \delta e, \delta a, \delta r) \\
\dot{q} &= \frac{I_z - I_x}{I_y} pr + \frac{1}{2I_y} \rho V^2 S c C_m (\alpha, q, \delta e) \\
\dot{r} &= \frac{I_x - I_y}{I_z} pq + \frac{1}{2I_z} \rho V^2 S b C_n (\beta, p, r, \delta e, \delta a, \delta r) \\
\dot{\phi} &= p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{X} &= V [\cos \alpha \cos \beta \cos \theta \cos \psi + \sin \beta (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\
&\quad + \sin \alpha \cos \beta (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)] \\
\dot{Y} &= V [\cos \alpha \cos \beta \cos \theta \sin \psi + \sin \beta (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) \\
&\quad + \sin \alpha \cos \beta (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)] \\
\dot{Z} &= -V [\cos \alpha \cos \beta \sin \theta - \sin \beta \sin \phi \cos \theta - \sin \alpha \cos \beta \cos \phi \cos \theta] \\
\dot{h} &= -\dot{Z}
\end{aligned}$$

Wind axes orientation angles, μ and γ , are related as follows:

$$\begin{aligned}\sin \gamma &= \cos \alpha \cos \beta \sin \theta - \sin \beta \sin \phi \cos \theta - \sin \alpha \cos \beta \cos \phi \cos \theta \\ \sin \mu \cos \gamma &= \sin \theta \cos \alpha \sin \beta + \sin \phi \cos \theta \cos \beta - \sin \alpha \sin \beta \cos \phi \cos \theta \\ \cos \mu \cos \gamma &= \sin \theta \sin \alpha + \cos \alpha \cos \phi \cos \theta\end{aligned}$$

7.2.2 Aerodynamic Model

The aerodynamic model of the aircraft in terms stability derivatives, state, and control variables is given by:

$$\begin{aligned}C_L &= C_{L_0} + C_{L_q} \left(\frac{qc}{2V} \right) + C_{L_{\delta_e}} \delta_e \\ C_D &= C_{D_0} + C_{D_q} \left(\frac{qc}{2V} \right) + C_{D_{\delta_e}} \delta_e \\ C_m &= C_{M_0} + C_{m_q} \left(\frac{qc}{2V} \right) + C_{m_{\delta_e}} \delta_e \\ C_l &= C_{l_\beta} \beta + C_{l_p} \left(\frac{pb}{2V} \right) + C_{l_r} \left(\frac{rb}{2V} \right) + C_{l_{\delta_a}} \delta_a + C_{l_{\delta_e}} \delta_e + C_{l_{\delta_r}} \delta_r \\ C_n &= C_{n_\beta} \beta + C_{n_p} \left(\frac{pb}{2V} \right) + C_{n_r} \left(\frac{rb}{2V} \right) + C_{n_{\delta_a}} \delta_a + C_{n_{\delta_e}} \delta_e + C_{n_{\delta_r}} \delta_r \\ C_Y &= C_{Y_\beta} \beta + C_{Y_p} \left(\frac{pb}{2V} \right) + C_{Y_r} \left(\frac{rb}{2V} \right) + C_{Y_{\delta_a}} \delta_a + C_{Y_{\delta_e}} \delta_e + C_{Y_{\delta_r}} \delta_r\end{aligned}$$

The stability derivatives in the above expansions are assumed to be functions of angle-of-attack only and are obtained using interpolation of the aerodynamic look-up table.

7.3 Sliding Mode Controller Design

The controller design for the aircraft landing is formulated into a trajectory tracking problem. Consider a general autonomous form of nonlinear equations

$$\dot{\underline{x}} = f(\underline{x}, \underline{u}) \quad (7.3.1)$$

where $\underline{x}$ and $\underline{u}$ are the state and control-input vectors, respectively. For the chosen aircraft model, the equations of motion are nonlinear, but affine in control input and can be expressed as

$$\dot{\underline{x}} = A(\underline{x}) + B(\underline{x})\underline{u} \quad (7.3.2)$$

The first step in designing a sliding mode controller is to define a set of sliding functions with size equal to the number of available control inputs

$$\underline{s} = [s_1, s_2, \dots, s_{m-1}, s_m]^T \quad (7.3.3)$$

Usually, differential equations representing the error dynamics are chosen as sliding functions in the form

$$s_i = \left(\frac{d}{dt} + \lambda_i \right)^{k-1} e_i \quad i = 1, 2, \dots, m \quad (7.3.4)$$

where e_i is the error, λ_i is a positive number, and k is the least possible integer such that the matrix $\frac{\partial \underline{s}}{\partial \underline{x}} B(\underline{x})$ is non-singular.

For an aircraft model, state and control input vectors are given by

$$\underline{x} = [\alpha, \beta, \gamma, V, p, q, r, \phi, \theta, \psi]^T \quad (7.3.5)$$

$$\underline{u} = [\eta, \delta_a, \delta_e, \delta_r]^T \quad (7.3.6)$$

With throttle, aileron, elevator and rudder deflections available as active control inputs, four sliding functions can be defined ($m = 4$). Though the dynamics are coupled, the four individual control inputs show their maximum influence on translational, roll, pitch, and yaw motions, respectively. By choosing flight-path angle (γ), roll angle (ϕ), angle-of-attack

(α), and yaw angle (ψ) as variables of interest, the error vector can be defined as

$$\underline{e} = \begin{bmatrix} \gamma - \gamma_d \\ \phi - \phi_d \\ \alpha - \alpha_d \\ \psi - \psi_d \end{bmatrix} \quad (7.3.7)$$

where subscript d denotes the desired state variables.

With the choice of the error vector, $k = 2$ in Eq. 7.3.4. By expanding the error dynamics equations, the sliding functions are obtained as follows

$$\underline{s} = \begin{bmatrix} s_\gamma \\ s_\phi \\ s_\alpha \\ s_\psi \end{bmatrix} = \begin{bmatrix} (\dot{\gamma} - \dot{\gamma}_d) + \lambda_\gamma (\gamma - \gamma_d) \\ (\dot{\phi} - \dot{\phi}_d) + \lambda_\phi (\phi - \phi_d) \\ (\dot{\alpha} - \dot{\alpha}_d) + \lambda_\alpha (\alpha - \alpha_d) \\ (\dot{\psi} - \dot{\psi}_d) + \lambda_\psi (\psi - \psi_d) \end{bmatrix} \quad (7.3.8)$$

where λ_γ , λ_ϕ , λ_α and λ_ψ are positive constants. However, the actual reference trajectory the aircraft is required to track are given as in Eqs. 7.2.3 and 7.2.4, which can be expressed as

$$Y = 0 \quad (7.3.9)$$

$$Z = Z_d(X) \quad (7.3.10)$$

To make the above reference flight path as solution to the sliding functions, the spatial position errors are transcribed into the desired state variables given in Eq. 7.3.7 through the following transformation

$$\begin{bmatrix} \gamma_d \\ \phi_d \\ \alpha_d \\ \psi_d \end{bmatrix} = \begin{bmatrix} \gamma_{ref} + \tan^{-1}((Z - Z_d)/k_\gamma) \\ \tan^{-1}(Y/k_\phi) \\ \alpha_{ref} \\ \tan^{-1}(Y/k_\psi) \end{bmatrix} \quad (7.3.11)$$

where γ_{ref} is the flight-path angle of the reference trajectory, which remains constant during the glide-path capture maneuver and is equal to the descent angle, i.e.

$$\gamma_{ref} = \gamma_{glide} \quad (7.3.12)$$

During the flare maneuver, the flight-path angle (γ_{ref}) is represented by

$$\gamma_{ref} = -\sin^{-1}(h_f/\tau V) \quad (7.3.13)$$

where τ is a constant. The terms $\tan^{-1}((Z - Z_d)/k_\gamma)$ and $\tan^{-1}(Y/k_\phi)$ are mainly to correct the offsets in vertical and lateral positions from the reference trajectory, respectively. The term $\tan^{-1}(Y/k_\psi)$ is to maintain a constant heading toward the runway. α_{ref} is the angle-of-attack of the reference trajectory and can usually be kept constant.

To move the aircraft toward the reference flight-path, define a positive definite Lyapunov functional in terms of sliding functions as

$$V = \frac{1}{2} \underline{s}^T \underline{s} \quad (7.3.14)$$

Taking the time derivative of the above equation results in

$$\dot{V} = \underline{s}^T \dot{\underline{s}} \quad (7.3.15)$$

The errors between the actual and reference trajectories can be reduced by forcing the time derivative of the Lyapunov function to be negative, i.e. $\dot{V} < 0$, which can be ensured by a proper selection of reaching law as follows

$$\dot{\underline{s}} = \frac{\partial \underline{s}}{\partial \underline{x}} \dot{\underline{x}} = -K \underline{s} \quad (7.3.16)$$

where K is a positive definite diagonal matrix with appropriate dimensions.

The substitution of Eq. 7.3.2 into Eq. 7.3.16 results in

$$\frac{\partial \underline{s}}{\partial \underline{x}} [A(\underline{x}) + B(\underline{x}) \underline{u}] = -K \underline{s} \quad (7.3.17)$$

By rearranging the terms in Eq. 7.3.17, the following control law can be obtained:

$$\underline{u} = - \left(\frac{\partial \underline{s}}{\partial \underline{x}} B(\underline{x}) \right)^{-1} \left[\frac{\partial \underline{s}}{\partial \underline{x}} A(\underline{x}) + K \underline{s} \right] \quad (7.3.18)$$

The control law computes the required control inputs to achieve the desired landing maneuver. The computed control inputs are subject to saturation and rate limits. It should be noted that same form of control law is used for the glide-path capture and flare maneuvers, but the gains are changed appropriately to achieve a satisfactory performance.

7.4 PID Controller Design

PID controllers are widely employed in many control problems due to their simplicity, low computational complexity, and robustness. They have also been successfully applied to different aircraft control and maneuvering problems, including landing. PID control can be a good choice if the dynamics of the system, especially the nonlinearity, is not available or poorly modeled. However, for aircraft motion, whose dynamics is readily available, PID control usually yields suboptimal performance as it does not exploit the known nonlinearity in its control architecture. In order to show the effectiveness and merits of the designed sliding mode controller, the aircraft landing problem is also approached with a conventional PID control, which is used for many control related problems. The results obtained from the two techniques are then compared and discussed in the next Section.

In the PID control, the control input vector is expressed as a linear combination of the errors, error integrals and error rates between the desired and the actual state variables.

Mathematically, the input vector in PID control is defined as

$$\underline{u} = K_P \underline{e} + K_I \int_0^t \underline{e} dt + K_D \dot{\underline{e}} \quad (7.4.1)$$

where $\underline{u} = [\eta, \delta_a, \delta_e, \delta_r]^T$ is the control input vector. K_P , K_I and K_D are diagonal matrices with positive constants. $\underline{e}$ is the error vector given by

$$\underline{e} = \begin{bmatrix} Z - Z_d \\ \phi - \phi_d \\ \alpha - \alpha_d \\ \psi - \psi_d \end{bmatrix} \quad (7.4.2)$$

where Z_d is the desired altitude. The other desired state variables, ϕ_d , α_d and ψ_d , are as given in Eq. 7.3.11.

The gains in K_P , K_I and K_D will be different for the glide-path capture and the flare maneuvers. They are tuned manually until a satisfactory tracking performance is obtained.

7.5 Results and Discussion

Some aircraft landing simulations using the HARV aircraft model are performed employing both the SMC and PID-based automatic controllers. Euler method [42] of integration, with a time-step of 0.001 s, is used to solve the equations of motion. For proper comparison, the same reference landing trajectory is defined for both the controllers. In the glide-phase, the reference trajectory is defined in the form of a straight line in the longitudinal landing plane with -2.5° glide-path angle. The landing maneuvers are initiated at the altitude of 1500 ft. The aircraft starts from a level-flight trim condition 2000 ft behind the origin, with 100 ft offset in lateral position. This corresponds to the initial position of $(X(0), Y(0), Z(0)) = (-2000, 100, 0)$. The initial heading is in the direction of the runway

centerline and is parallel to it. To initiate this landing with large offsets in spatial position, the aircraft is allowed to fly steadily for 10 s and cross the origin by a few hundred feet. The glide path capture maneuver is initiated after 10 s, where the automatic controllers take over and start executing the control inputs. The gains in both the controllers are tuned by trial and error method until satisfactory tracking performances are achieved. The control inputs are computed from the control laws formulated in Eqs. (7.3.18) and (7.4.1) and are adjusted for the rate and saturation limits before the execution. After the initiation of the maneuver, the aircraft starts heading towards the reference glide-path and aligning with it. The state histories, trajectories, and control histories of the simulated glide-path capture maneuvers obtained for both types of controllers are plotted in Figs. 7.2, 7.3, and 7.4, respectively.

Figure 7.2: State histories during the glide-path capture maneuver.

Figure 7.3: Three and two dimensional trajectories showing the aircraft's glide-phase landing.

Figure 7.4: Control histories using SMC and PID methods during the glide-path capture maneuver.

The first plot in Fig. 7.3 shows the achieved three-dimensional trajectories of the aircraft, aligning with the reference glide-path. It can be seen that aircraft is converged to the longitudinal landing plane smoothly with negligible overshoot and it remains on the reference trajectory after the convergence in both cases. However, the settling time with the PID controller is much slower as compared to the SMC. The second plot in Fig. 7.3 shows a portion of the projected aircraft trajectories on the vertical plane and the reference glide-path trajectory. As soon as the glide-path capture maneuver is initiated, the aircraft's projected trajectory using the SMC reaches the reference glide-path in a relatively short time with an altitude adjustment of about 150 ft. With the PID controller, the time

taken is considerably longer. Though faster settling time can be achieved by changing the gains in the PID control law, but it is usually accompanied by an excessive control activity, which is not achievable practically. The state histories of the aircraft during the glide-phase are shown in Fig. 7.2. All the aircraft state variables are constant during the level-flight phase for first 10 s. When the glide-path capture maneuver is initiated at the 10th s, it can be seen that most of the state variables reached their steady-state values corresponding to the reference glide-path within 20 s in the case of the SMC. The variables ϕ and ψ take relatively longer time to settle due to the large initial lateral-position offset, but they also reach their steady-state values in about 70 s, well before the flare maneuver is initiated. In the case of the PID controller, the state variables take more time to reach the steady-state. Also, for this particular gains selected, the transient is oscillatory, unlike the near asymptotic convergence achieved using the SMC. This comparison indicates the the SMC leads to a superior performance than the PID control in the glide-phase. The fast convergence of the aircraft to the reference trajectory indicates the proposed controller's capability to handle significant lateral position offset. Fig. 7.4 shows the time histories of control inputs to capture the reference glide-path. The left-hand side plot in Fig. 7.4 shows the deflection histories of the three control surfaces and the right-hand side plot shows the throttle time history. Just as observed in state variables, the control inputs also reach the steady-state values smoothly. Control surface deflections in the SMC case show larger variations as compared to the PID control case, thereby enabling faster convergence. The throttle history of the SMC settles to the steady-state value in relatively shorter time, while the PID controller's throttle history shows oscillatory response and takes considerably longer time to achieve steady-state. At this point, it is important to observe the chatter-free control inputs obtained from the SMC. Generally, SMC suffers

from chattering problems and high control activity, once the system trajectory or the sliding functions come close to the solution. However, in this case the reaching law formulated in Eq. (7.3.16) prevents the occurrence of the chatter due to the continuous form of switching dynamics used. During the sliding-phase, when the system has reached the reference trajectory, the controller gives constant control inputs. High frequency switching is also avoided here because of the selected reaching law. The complete glide-path capture maneuver lasts for about 150 s in this case. In general, the maneuver time can vary for different descent angles and aircraft models used.

During the flare maneuver, which is initiated at 30 ft altitude, the aircraft is to track an exponential curve to reduce the descent rate exponentially and achieve near zero flight-path angle at touchdown. Eq. (7.2.5) forms the reference trajectory to be tracked. To achieve smooth transition from the glide-path capture maneuver to the flare maneuver, τ is chosen such that the descent rate remains the same at the transition point. Matching descent rates at the junction of the two phases can be mathematically expressed as

$$V \sin \gamma_{glide} = -h_{f0}/\tau \quad (7.5.1)$$

Thus, τ can be obtained from the above formulation using

$$\tau = -h_{f0}/V \sin \gamma_{glide} \quad (7.5.2)$$

The same SMC and PID control laws, derived for the glide-path capture maneuver, are used during the flare maneuver with only their gains changed. State histories, trajectories and control input profiles obtained for the simulated flare maneuver are plotted in Figs. 7.5, 7.6, and 7.7, respectively.

Unlike the glide maneuver, the flare phase is unsteady and lasts for a few seconds only. Fig. 7.6 shows the simulated flare maneuver obtained using both controllers and

Figure 7.5: State histories during the flare-path capture maneuver.

the reference trajectory to track. Both controllers are able to track the reference flare trajectory closely. A slight oscillatory behavior in altitude can be observed for the solution obtained using the PID controller. The state histories of flare maneuver are shown in

Figure 7.6: Two dimensional trajectory showing the aircraft flare-path capture maneuver.

Figure 7.7: Control histories during the flare-path capture maneuver.

Fig. 7.5. All the state variables corresponding to the lateral motion of the aircraft can be observed to be zero for both controllers. This is because all the lateral position errors have been corrected during the glide-path capture maneuver. Hence, no lateral correction is required during this phase, confining the flare maneuver to the longitudinal landing plane of motion only. The state histories corresponding to the SMC show smooth transients, but the ones corresponding to the PID control are oscillatory.

Fig. 7.7 shows the control inputs computed for the flare maneuver. It can be seen that the control input deflections of aileron and rudder are zero for both the controllers as no lateral position errors are present. Also here, smooth elevator deflection and throttle variation are observed from the SMC, without any chatter and high control activity. An oscillatory elevator and throttle inputs are observed from the PID controller. The flare maneuver simulation is terminated once the aircraft has reached 2 *ft* altitude.

Overall, the simulations performed indicates successful implementation of the SMC method with better performance than the PID controller in both the glide-path and flare-path capture maneuvers. The ability of the SMC to handle the nonlinear aircraft dynamics in landing is clearly observed. All the control inputs are computed simultaneously without

the need for linearization or longitudinal-lateral decoupling in the design. Considering the near asymptotic tracking performance and ease in handling nonlinear aircraft model, the SMC appears to be a very suitable choice for automatic landing application.

7.6 Summary

The trajectory tracking controller, developed for perching maneuvers, has been extended to design an automatic landing system for conventional aircraft landing. The landing maneuver problem is formulated into a trajectory tracking problem by transcribing the spatial position errors with respect to the reference flight-path into the desired state variables of the sliding functions. The choice of sliding functions, selection of reaching laws, and derivation of control laws are validated using numerical simulations. Convergence from a level-flight trim state to the reference glide-path can be achieved with a good tracking performance. Accurate tracking of the reference flare-path is achieved and the flight-path angle is brought to near zero at touchdown. Chattering and high control activity, which are two critical problems of sliding mode control, are avoided by appropriately selecting the reaching law. Successful implementation of the sliding mode controller for autonomous aircraft landing maneuvers is further emphasized by comparing the simulation results with the ones obtained using a conventional PID controller under the same conditions. The results indicate that the sliding mode control outperforms the PID controller for this application.

Chapter 8

Conclusions

8.1 Conclusions

Few conclusions are made based on the results obtained in the previous chapters

8.1.1 Conclusions related to optimization of perching maneuvers

The formulation of the perching problem into a trajectory optimization problem is simplified using the proposed single-phase optimization approach. The single-phase formulation, with its simpler computational procedure, makes effective use of poststall aerodynamics and produces lower undershoot optimal perching trajectories as compared with the two-phase formulation. The complex iterative solution procedure in the two-phase formulation that results in suboptimal perching solutions can be avoided by using the single-phase formulation.

Perching trajectories are optimized using aerodynamic and thrust vectoring features. Four different configurations: aerodynamic vectoring only, thrust vectoring only, coupled aerodynamic and thrust vectoring, and uncoupled aerodynamic and thrust vectoring are

studied and the results are compared with the corresponding fixed-wing perching trajectories. Perching trajectories with minimum undershoot can be achieved using the uncoupled aerodynamic and thrust vectoring configuration, closely trailed by the thrust vectoring only configuration. The aerodynamic vectoring becomes ineffective during the low-velocity terminal flight phase and does not contribute to a substantial reduction in undershoot. The thrust-vectoring-only configuration, owing to its simplicity and ability to generate near minimum undershoot perching trajectories, can be effective throughout the maneuver and is recommended as a good design feature for optimization of the perching maneuvers.

8.1.2 Conclusions related to parametric study of perching maneuvers

A framework, comprising NVCM based aerodynamic modeling tool and perching trajectory optimization formulation, to systematically analyze the role of various aircraft parameters in perching maneuver, is developed. The parameterization of geometric design features and use of NVCM solver for inflight determination of aerodynamic forces and moment has simplified the optimization formulation and trajectory computation process. Parametric study shows that cost function is significantly influenced by all the parameters. Undershoot is significantly effected by aspect ratio, terminal velocity and maximum angle-of-attack constraints. The study reveals that dive-phase is more sensitive to parametric changes. Availability of large control moment and resultant aerodynamic force are desirable for optimal perching. Relaxation of terminal velocity constraint greatly reduces the undershoot. High angle-of-attack aerodynamics is not critical and can be discarded for very low-velocity perching.

8.1.3 Conclusions related to stability analysis of perching maneuvers

Stability analysis of perching maneuvers is carried out using contraction theory to analyze the behavior of perching trajectories in case of state perturbations. It is found that the optimal perching trajectories are unstable in the presence of state perturbations and the trajectories will in general diverge from its nominal trajectory.

8.1.4 Conclusions related to tracking of perching maneuvers

A trajectory tracking scheme using the sliding-mode control approach has been proposed as a solution to address the problem of trajectory instabilities due to state perturbations. With proper choice of the gains, sliding functions, and reaching law, the controller is proved to be effective. The simulations performed indicate that successful tracking of the optimal solutions without exceeding the optimal undershoot can be achieved.

8.1.5 Conclusions related to tracking of conventional landing maneuvers

The perching tracking controller's design is extended to design an automatic landing system for a six degrees-of-freedom nonlinear aircraft model. Accurate tracking of the reference flare-path is achieved and the flight-path angle is brought to near zero at touchdown. Chattering and high control activity, which are two critical problems of sliding mode control, are avoided by appropriately selecting the reaching law. Successful implementation of the sliding mode controller for autonomous aircraft landing maneuvers is further emphasized by comparing the simulation results with the ones obtained using a conventional PID

controller under the same conditions. The results indicate that the sliding mode control outperforms the PID controller for this application.

8.2 Future work

Some of the following topics can be carried out to continue the work presented in this thesis

- The controller proposed for perching trajectory tracking in this thesis is robust only to address perturbed initial conditions. The robustness of the controller can be extended to address aerodynamic uncertainties and external disturbances. Other controllers can also be explored for perching trajectory tracking.
- Real-time computation of optimal perching trajectories is difficult. Singular perturbation control provides a possibility to design approximate optimal controller for perching and can be explored.
- The nonlinear aerodynamic modeling tool created for parametric study allows for inflight determination of aerodynamic forces and moment. This tool can be used to optimize perching trajectories for flapping and morphing wings.
- Real-time outdoor perching is still an open problem. By integrating aircraft state estimation with the perching controller in an autopilot, real-time perching can be demonstrated.
- The sliding-mode controller designed for an automatic landing system of conventional aircraft also does not address the issues related to aerodynamic uncertainties, time-delay and external disturbances. The controller design can be extended to address these problems, and make it more applicable for real-time landing.

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