Unconditionally stable finite-difference time-domain methods for electromagnetic compatibility and antenna modeling

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Unconditionally Stable Finite-Difference Time-Domain Methods for Electromagnetic Compatibility and Antenna Modeling

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Abstract

This thesis presents advances in unconditionally stable finite-difference time-domain (FDTD) methods for electromagnetic compatibility and antenna modeling. The leapfrog alternating-direction-implicit (ADI) FDTD method is an unconditionally stable FDTD method that is more efficient than the conventional ADI-FDTD method. In this thesis, the stability and dispersion relation of the leapfrog ADI-FDTD method are discussed analytically. To enable modeling of general structures, the leapfrog ADI-FDTD method is extended to incorporate the Mur absorbing boundary condition, perfect electric conductor and lossy media. Applications of the leapfrog ADI-FDTD method for electromagnetic compatibility are demonstrated. The first application investigates the mutual coupling between two monopole antennas of different length mounted on a finite ground plane. For the next application, the field leakage of the leapfrog ADI-FDTD method for a shielded enclosure is determined. Detailed analysis reveals that the leapfrog ADI-FDTD method has substantial non-physical field leakage.

A novel method for the analysis of the divergence properties of the leapfrog ADI-FDTD method is introduced in this thesis. The analysis of divergence properties is based on the Fourier domain plane wave method for dispersion analysis but extended with additional procedure to determine the homogeneous solutions. The singular value decomposition is used to obtain the solutions of the homogeneous system in a

reliable manner. The analysis reveals that the leapfrog ADI-FDTD method is not divergence-free in the source-free region, as is the case for the conventional ADI-FDTD method.

To circumvent this issue, an unconditionally stable locally one-dimensional FDTD method with second-order temporal accuracy and complying divergence (LOD2-CD-FDTD) is proposed in this thesis. By extending the previous concept of achieving second-order temporal accuracy for the LOD2-FDTD method via implicit output processing, a novel explicit output processing that not only retains second-order temporal accuracy, but also complying divergence for the LOD2-CD-FDTD method is presented. To further enhance efficiency, the LOD2-CD-FDTD method is formulated into the fundamental LOD2-CD-FDTD method with efficient matrix-operator-free right-hand sides. This results in substantial reduction of floating point operations count and memory indexing. To solve open radiation and scattering problems, the LOD2-CD-FDTD method is extended to incorporate the convolutional perfectly matched layer. The LOD2-CD-FDTD method is divergence-complying in both the source region as well as the source-free region in a manner analogous to the conventional explicit FDTD method. Applications of the fundamental LOD2-CD-FDTD method for antenna modeling are demonstrated. Numerical results for the line-fed rectangular microstrip patch antenna and the strip-fed rectangular dielectric resonator antenna are provided.

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List of Abbreviations and Symbols

Abbreviations

ABC	Absorbing Boundary Condition
ADI	Alternating-Direction-Implicit
CD	Complying Divergence
CFL	Courant-Friedrich-Lewy
CFLN	Courant-Friedrich-Lewy Number
CPU	Central Processing Unit
CPW	Cell Per Wavelength
DRA	Dielectric Resonator Antenna
EM	Electromagnetic
EMC	Electromagnetic Compatibility
FADI	Fundamental Alternating-Direction-Implicit
FDTD	Finite-Difference Time-Domain
FLOD	Fundamental Locally One-Dimensional
GPU	Graphic Processing Unit

HPM	High Power Microwave
IED	Improvised Explosive Device
LHS	Left-Hand Side
LOD	Locally One-Dimensional
PEC	Perfect Electric Conductor
PMC	Perfect Magnetic Conductor
PML	Perfectly Matched Layer
RHS	Right-Hand Side
TE	Transverse Electric
TEM	Transverse Electromagnetic
TM	Transverse Magnetic
SSD	Solid State Devices
SVD	Singular Value Decomposition

Symbols

E_x, E_y, E_z	Electric Fields Components
H_x, H_y, H_z	Magnetic Fields Components

e_x, e_y, e_z	Auxiliary Electric Fields Components
h_x, h_y, h_z	Auxiliary Magnetic Fields Components
t, f, ω	Time, Frequency, Angular Frequency
Δt	Time Step Size
$\Delta x, \Delta y, \Delta z$	Spatial Step Size
c	Speed of Light in Free Space
$\epsilon, \epsilon_0, \epsilon_r$	Permittivity, Free Space Permittivity, Relative Permittivity
μ, μ_0, μ_r	Permeability, Free Space Permeability, Relative Permeability
σ, σ^*	Electric and Magnetic Conductivities

Chapter 1

Introduction

1.1 Motivation

Commercial aircrafts typically transports hundreds of passengers, and thus, it is of utmost importance that they operated reliably as designed under extreme conditions. The current generation of aircrafts has more efficient designs at the expense of increased complexity. To achieve fuel saving, and consequently longer flight range, composite materials with streamline aerodynamic shapes are used so as to reduce weight and drag of the aircraft.

These modern aircrafts also require an extensive amount of complex electronic systems to ensure proper flight operations. The electronic systems are usually densely packed and designed to operate very near to their limits. Furthermore, an aircraft usually has several antenna systems for radio communications with ground stations and satellites. More recently, Internet connectivity through WI-FI network is available on selected flights to enable passengers to access the Internet even while cruising several thousands of feet above sea-level. As a result, antenna systems

for data communications are further loaded onto the already complicated aircraft's electronic system. The various electronic systems may interfere with the proper operations of the aircraft and compromise the passengers safety. With such risk involved, it is thus important to understand how electromagnetic waves emitted either intentionally or unintentionally by the various electronic systems interacting with one another. This is a difficult task as such electromagnetic phenomena cannot be seen by the human eyes, and in many cases are not intuitive.

Fortunately, time-domain electromagnetic numerical methods allow one to visualize the propagation of electromagnetic waves through complex media, as well as the interaction of the electromagnetic waves with the physical environment. Thus, one can have a better understanding of the system, and subsequently apply appropriate modifications to the system accordingly. The conventional explicit finite-difference time-domain (FDTD) method is a widely adopted numerical technique for such analysis, as it implements Maxwell's equations in a straightforward and systematic approach [1].

However, the conventional FDTD method is an explicit method that is only conditionally stable. The time step size is restricted by minimum cell size of the simulation model. For practical models, the minimum cell size is usually determined by the resolution required to model the fine scale structures, and not by the highest frequency of interest. As a result, the time step size is usually very small, and a large number of iterations are required. Consequently, the overall simulation time increases with the complexity of the simulation model.

On the other hand, the number of iterations can be reduced by employing uncon-

ditionally stable FDTD methods [2]. Such methods are not constrained by the Courant-Friedrich-Lewy stability criterion, and therefore, the spatial mesh size can be reduced independently of the time step size. A suitable time step size can be chosen as long as it fulfills the required accuracy. This advantage makes unconditionally stable FDTD methods attractive as the overall simulation time is reduced. Therefore, development of efficient unconditionally stable FDTD methods is desirable as demands for efficiency-improved methods are ever increasing.

However, numerical properties for some of the unconditionally stable FDTD methods are still not yet fully understood. Numerical errors such as stability, dispersion, anisotropy, dissipation and divergence may affect the accuracy of the simulation result. In particular, the divergence property of the unconditionally stable FDTD method is not often studied. While it is known that the conventional explicit FDTD method enforces the divergence theorem implicitly, it is not directly clear if the unconditionally stable FDTD methods have such similar numerical property. Therefore, it is important to obtain an analysis method to investigate the divergence property of the unconditionally stable FDTD methods. Furthermore, the development of divergence-complying unconditionally stable FDTD methods, which do not incur spurious charges in a manner analogous to the conventional explicit FDTD method, is important as they approximate Maxwell's equations with less numerical errors.

1.2 Objectives

From the motivation highlighted in the previous section, the objective of this thesis is thus to propose and provide:

- Advances in unconditionally stable leapfrog ADI-FDTD method for modeling of general lossy structures by proposing formulations for open and closed boundaries and applications of the proposed method for electromagnetic compatibility to investigate its potential issues.
- Investigations of divergence property for commonly employed unconditionally stable FDTD methods. Such analysis method for the divergence property that can be readily applied to other FDTD methods is important for future development of FDTD methods.
- Advances in unconditionally stable FDTD method to incorporate divergence-complying properties and applications of the proposed method for antenna modeling to determine its accuracy and efficiency.

1.3 Major Contributions of the Thesis

The major contributions of this thesis related to the advancement of unconditionally stable FDTD methods are as follows.

The stability and dispersion relation of the leapfrog ADI-FDTD method have been derived analytically, and it is proven that the leapfrog ADI-FDTD method is unconditionally stable. To enable modeling of general structures, the leapfrog ADI-FDTD method has been extended to incorporate the Mur ABC, perfect electric conductor boundary condition and lossy media. Conductivity terms of lossy media have been incorporated into the leapfrog ADI-FDTD method in an analogous manner as the conventional explicit FDTD method since the leapfrog ADI-FDTD method is a perturbation of the conventional explicit FDTD method. The leapfrog ADI-FDTD method is also accelerated by implementing it on a low-cost graphic processing unit.

Applications of the unconditionally stable leapfrog ADI-FDTD method for electromagnetic compatibility have been demonstrated. For the first application, mutual coupling between two monopole antennas of different length mounted on a finite ground plane is investigated. The study of how an antenna behaves when placed in the vicinity of another antenna is important so as to ensure that radio-frequency interference is kept to a minimum. The resistive voltage source for the leapfrog ADI-FDTD method has been introduced to facilitate the analysis of the scattering parameters. For the next application, the field leakage of the leapfrog ADI-FDTD method for a shielded enclosure, i.e., non-penetrable target, has been investigated. Detailed analysis reveals that the leapfrog ADI-FDTD method has substantial non-physical field leakage. To overcome the field leakage issue for non-penetrable targets, a simple approach has been proposed.

A novel method for the analysis of the divergence properties has been introduced for the three-dimensional leapfrog ADI-FDTD method. The analysis of divergence properties is based on the Fourier domain plane wave method for dispersion analysis but extended with additional procedure to determine the homogeneous solutions. The singular value decomposition (SVD) is used to obtain the solutions of the homogeneous system in a reliable manner. There are two linearly independent null-modes in the homogeneous system corresponding to the repeated zero singular values. To provide more physical insights, proper linear combinations of the two null-modes are determined as transverse electric (TE) and transverse magnetic (TM) modes with respect to the direction of discrete numerical space wavevector. The divergence of electric fields is analyzed by examining the TM mode, while the divergence of magnetic fields is analyzed by examining the TE mode. The leapfrog ADI-FDTD method is not divergence-free in the source-free region, as is the case for the conventional ADI-FDTD method.

An unconditionally stable fundamental LOD-FDTD method with second-order temporal accuracy and complying divergence (LOD2-CD-FDTD) has been developed. While the conventional LOD-FDTD method does not have complying divergence, the LOD2-CD-FDTD method has complying divergence in a manner analogous to the conventional explicit FDTD method. By extending the previous concept of achieving second-order temporal accuracy for the LOD2-FDTD method via implicit output processing, a novel explicit output processing that not only retains second-order temporal accuracy, but also complying divergence for the LOD2-CD-FDTD method has been proposed. The output processing of the LOD2-CD-FDTD method consists only of explicit update equations, and does not require a tridiagonal solution that is necessary for the LOD2-FDTD method. The current source implementation for the LOD2-CD-FDTD method involves source-incorporation in only the first procedure. To further enhance efficiency, the LOD2-CD-FDTD method is formulated into the fundamental LOD2-CD-FDTD method with efficient matrix-operator-free right-hand sides. This results in substantial reduction of floating point operations count and memory indexing. To solve open radiation and scattering problems, the LOD2-CD-FDTD method has been extended to incorporate the CPML. Moreover, the output processing of the LOD2-CD-FDTD method is independent of the CPML media. The LOD2-CD-FDTD method is divergence-complying in both the source region as well as the source-free region in a manner analogous to the conventional explicit FDTD method.

Practical applications of the unconditionally stable fundamental LOD2-CD-FDTD method for antenna modeling have been demonstrated. Numerical results for the line-fed rectangular microstrip patch antenna and the strip-fed rectangular dielectric resonator antenna are provided. The formulation of the fundamental LOD2-CD-FDTD method is employed for these applications. Moreover, it has been extended

to incorporate the resistor voltage source and lumped resistor element, which is useful for antenna modeling. Finally, the fundamental LOD2-CD-FDTD method including resistive voltage source retains the advantages of the fundamental LOD2-CD-FDTD method as highlighted earlier.

1.4 Organization of the Thesis

The organization of this thesis is as follows. A literature review for unconditionally stable FDTD methods is presented in Chapter 2. Next, Chapter 3 derives the stability and dispersion relation of the leapfrog ADI-FDTD method analytically. Chapter 4 further extends the formulation of the leapfrog ADI-FDTD method to include the Mur ABC, perfect electric conductor boundary condition and lossy media. The leapfrog ADI-FDTD method is also accelerated by implementing it on a low-cost graphic processing unit. Some numerical examples are presented to validate the formulations.

Subsequently, Chapter 5 demonstrates application of the unconditionally stable leapfrog ADI-FDTD method for electromagnetic compatibility. For the first application, mutual coupling between two monopole antennas of different length mounted on a finite ground plane is presented. This chapter also investigates the field leakage of the leapfrog ADI-FDTD method for a shielded enclosure as an application example.

Chapter 6 introduces the analysis of the divergence properties for the leapfrog ADI-FDTD method. The leapfrog ADI-FDTD method is not divergence-free in the source-free region, as is the case for the conventional ADI-FDTD method.

Chapter 7 then introduces the LOD2-CD-FDTD method. Unlike the leapfrog ADI-FDTD method, the fundamental LOD2-CD-FDTD method has complying divergence, and thus does not have spurious charge in the source-free regions. The divergence-complying property of the fundamental LOD2-CD-FDTD method is also verified at the source region, by analyzing the electric charge deposited at the ends of a dipole radiator. In addition, the fundamental LOD2-CD-FDTD method is extended to include the CPML in order to model open boundaries efficiently.

Chapter 8 demonstrates practical applications of the unconditionally stable fundamental LOD2-CD-FDTD method for antenna modeling. The fundamental LOD2-CD-FDTD method is extended to incorporate the resistive voltage source. Numerical results for the line-fed rectangular microstrip patch antenna and the strip-fed rectangular dielectric resonator antenna are provided.

Finally, Chapter 9 presents the overall conclusion of this thesis and highlights some suggestions for future work.

Chapter 2

Literature Review

2.1 Overview

In this chapter, we provide a literature review for the more popular unconditionally stable FDTD methods. We shall focus on the ADI-FDTD method, LOD-FDTD method and the recently proposed leapfrog ADI-FDTD method. Meanwhile, the conventional explicit FDTD method has been extensively investigated, and many detailed references are available [3–7]. It is for this reason that the conventional explicit FDTD method will not be covered here.

2.2 Unconditionally Stable Finite-Difference Time-Domain Methods

The differential form of Maxwell's equations depicts four physical laws: the Faraday's law and Ampere's law are expressed using the curl operator, while the Gauss's laws for the electric field and magnetic field are expressed using the divergence oper-

ator. The FDTD method solves the Maxwell's equations in time-domain and is able to provide visualizations of the coupling mechanism of electromagnetic waves with the environment. However, the conventional FDTD method is an explicit method that is only conditionally stable. The time step size is restricted by the Courant-Fredrich-Lewy (CFL) stability criterion. The CFL condition is given by

$$\Delta t \leq \frac{1}{v_{max} \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}} \quad (2.2.1)$$

where v_{max} is the maximum phase velocity of the wave propagation in the medium. As such, the computed fields may grow without bound when time-marching proceeds with the CFL limit violated. To eliminate such a constraint, unconditionally stable implicit FDTD methods were proposed. The most popular is the ADI-FDTD method [8,9]. The conventional ADI-FDTD method is a class of split-step scheme with two procedures [10].

Another class of unconditionally stable implicit FDTD method is the LOD-FDTD method [11–13]. Similar to the conventional ADI-FDTD method, the conventional LOD-FDTD method is a two-procedure method with implicit field updating for both procedures. Moreover, each procedure is effectively reduced to a one-dimensional update equation locally. Therefore, the conventional LOD-FDTD method is more computationally efficient. While the conventional ADI-FDTD method is second-order temporal accurate, the conventional LOD-FDTD method has only first-order temporal accuracy.

The above mentioned unconditionally stable implicit FDTD methods have been recently simplified into the fundamental scheme [14–22]. The fundamental schemes

Figure 2.1: Electric and magnetic field components in a Yee's cell.

of the ADI-FDTD and LOD-FDTD methods are more efficient and simpler to implement, even though they are equivalent to the conventional methods and share similar numerical properties.

As the ADI-FDTD and LOD-FDTD methods are not restricted by the CFL stability criterion, their spatial mesh size can be reduced independently of the time step size. Thus, implicit FDTD methods are advantageous over explicit FDTD methods as they can model fine structures without increasing the simulation time. Moreover, these implicit FDTD methods utilize the Yee's cells of the conventional explicit FDTD method as shown in Fig. 2.1, where electric and magnetic fields are staggered in space.

To improve the efficiency of implicit FDTD methods in modeling fine structures, conformal ADI-FDTD methods have been proposed [23,24]. It overcomes the problem of staircase meshing by using distorted cells at the boundaries, and has better stability than other conformal FDTD methods. Alternatively, hybrid ADI-FDTD subgridding method can also be employed [25,26]. In this method, the conven-

Figure 2.2: Time-stepping schemes.

tional explicit FDTD method is used to model the coarse grid while the ADI-FDTD method is used to model the fine grid. ADI-FDTD method is unconditionally stable and can employ the larger time step size of the conventional explicit FDTD method. The time step size for the two methods is the same and there is no need for time interpolation when compared to FDTD subgridding methods [27].

However, the ADI-FDTD method is a split-step method while the conventional explicit FDTD method is a leapfrog method. Fig. 2.2 depicts the time-stepping schemes of the various methods. For the leapfrog methods, electric and magnetic fields are staggered in time and updated for one time step. Meanwhile, for the split-step methods, electric and magnetic fields are collocated in time and updated for half time steps. Thus, the split-step methods are updated more often than the leapfrog methods.

Recently, the leapfrog ADI-FDTD method was introduced to better synchronize the conventional explicit FDTD and ADI-FDTD methods [28–30]. As is the case of the ADI-FDTD method, the leapfrog ADI-FDTD method retains the usage of the

Yee's cells. Furthermore, the leapfrog ADI-FDTD method employs the same time-stepping scheme as the conventional explicit FDTD method, i.e., the electric and magnetic fields are staggered in time and not collocated in time. It is an implicit method that is derived from the conventional ADI-FDTD method. The leapfrog ADI-FDTD method is not constrained by the CFL stability criterion, and the spatial mesh size can be reduced independently of the time step size. It uses more memory than the conventional explicit FDTD method due to additional variables required to solve the tridiagonal matrix, but it can model open structures efficiently when the time step size is chosen to be adequately large [31, 32].

In the following subsections, we will briefly cover the unconditionally stable FDTD methods, namely, the ADI-FDTD method, LOD-FDTD method and leapfrog ADI-FDTD method.

2.2.1 ADI-FDTD Method

For simplicity, let us consider wave propagation in a medium with permittivity ϵ and permeability μ , and select the splitting matrix operators of Maxwell's equations as follows:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{\epsilon}\partial_y \\ 0 & 0 & 0 & \frac{1}{\epsilon}\partial_z & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\epsilon}\partial_x & 0 \\ 0 & \frac{1}{\mu}\partial_z & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\mu}\partial_x & 0 & 0 & 0 \\ \frac{1}{\mu}\partial_y & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.2.2a)$$

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{-1}{\epsilon}\partial_z & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{\epsilon}\partial_x \\ 0 & 0 & 0 & \frac{-1}{\epsilon}\partial_y & 0 & 0 \\ 0 & 0 & \frac{-1}{\mu}\partial_y & 0 & 0 & 0 \\ \frac{-1}{\mu}\partial_z & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{\mu}\partial_x & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.2.2b)$$

where ∂_x , ∂_y , ∂_z are the spatial difference operators for the first derivatives along x , y , z directions, respectively.

The conventional ADI-FDTD method comprises of two updating procedures in the main iterations:

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right) \mathbf{u}_{\text{ADI}}^{n+\frac{1}{2}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}_{\text{ADI}}^n + \frac{\Delta t}{2} \mathbf{s}^{n+\frac{1}{2}} \quad (2.2.3a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}_{\text{ADI}}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{A}\right) \mathbf{u}_{\text{ADI}}^{n+\frac{1}{2}} + \frac{\Delta t}{2} \mathbf{s}^{n+\frac{1}{2}} \quad (2.2.3b)$$

where

$$\mathbf{u} = \begin{bmatrix} E_x & E_y & E_z & H_x & H_y & H_z \end{bmatrix}^T \quad (2.2.4a)$$

$$\mathbf{s} = \begin{bmatrix} \frac{-1}{\epsilon}J_x & \frac{-1}{\epsilon}J_y & \frac{-1}{\epsilon}J_z & \frac{-1}{\mu}M_x & \frac{-1}{\mu}M_y & \frac{-1}{\mu}M_z \end{bmatrix}^T \quad (2.2.4b)$$

and $\mathbf{I}$ is the 6 x 6 identity matrix. Subscript “ADI” denotes the second-order temporal accurate ADI-FDTD method. Here, the symmetrical current source implementation is adopted [33–35]. It shall be highlighted that the field components for the conventional ADI-FDTD method are updated in alternating direction for the two procedures.

2.2.2 LOD-FDTD Method

Similarly, the conventional LOD-FDTD method comprises of two updating procedures in the main iterations:

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right) \mathbf{u}_{\text{LOD1}}^{n+\frac{1}{2}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{A}\right) \mathbf{u}_{\text{LOD1}}^n \quad (2.2.5a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}_{\text{LOD1}}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}_{\text{LOD1}}^{n+\frac{1}{2}} \quad (2.2.5b)$$

where subscript “LOD1” denotes the first-order temporal accurate LOD-FDTD method. Unlike the conventional ADI-FDTD method, the field components for the conventional LOD-FDTD method are updated in only one direction for each procedure. This allows for a more efficient implementation and is useful especially in parallel processing [36].

2.2.3 LOD2-FDTD Method

The conventional LOD-FDTD method is only of first-order temporal accuracy [12] as the matrix operators $\mathbf{A}$ and $\mathbf{B}$ do not commute. In order to attain second-order temporal accuracy, the conventional LOD2-FDTD method [37] which retains the two procedure updating scheme of the conventional LOD-FDTD method, but with shift in time indices was proposed as

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right) \mathbf{u}^{n+\frac{3}{4}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{A}\right) \mathbf{u}^{n+\frac{1}{4}} \quad (2.2.6a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}^{n+1\frac{1}{4}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}^{n+\frac{3}{4}} \quad (2.2.6b)$$

In addition, input processing for initial fields $\mathbf{u}^{t=0}$ may be performed at time step $n = 0$ as

$$\left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}\right) \mathbf{u}^{\frac{1}{4}} = \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{B}\right) \mathbf{u}^{t=0} \quad (2.2.6c)$$

and output processing at time step $n + 1$ reads

$$\left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{B}\right) \mathbf{u}_{\text{LOD2}}^{n+1} = \left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}\right) \mathbf{u}^{n+1\frac{1}{4}} \quad (2.2.6d)$$

where subscript “LOD2” denotes the second-order temporal accurate LOD-FDTD method. The input processing is to be invoked only once at the beginning and may be omitted for electromagnetic excitation problems with null initial fields. The output processing can be performed separately and in parallel - independent of the main iterations. Furthermore, the output field components are usually required at only some desired observation points and time interval. Hence, the output processing is to be invoked only when necessary. Note that (2.2.6c) and (2.2.6d) are implicit input and output processing.

It is worth mentioning that although the split-step approach may also achieve second-order temporal accuracy, it requires three updating procedures at every time step [38–41]. Furthermore, there are several variations of the LOD-FDTD method involving more than two updating procedures, but employing different splitting matrix operators as shown in (2.2.2) [42–44]. The LOD2-FDTD method only requires two updating procedures with output processing to be invoked only when necessary.

2.2.4 Leapfrog ADI-FDTD Method

The leapfrog ADI-FDTD method uses the Yee's cells, and it has been derived from the conventional ADI-FDTD method [28–30]. As mentioned before, the conventional ADI-FDTD method is a split-step approach with two procedures. By taking the previous time step of the second procedure in the conventional ADI-FDTD method and substituting the equations (cf. [29], eqn.(10)-(11)) into the tridiagonal implicit equations of the first procedure, one can obtain (2.2.7) and (2.2.8) in the following

1. Implicit update equations for $E^{n+\frac{1}{2}}$:

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \epsilon_x^{-1} \partial_y \mu_z^{-1} \partial_y\right) E_x^{n+\frac{1}{2}} &= \left(1 - \frac{\Delta t^2}{4} \epsilon_x^{-1} \partial_y \mu_z^{-1} \partial_y\right) E_x^{n-\frac{1}{2}} \\ &+ \Delta t \left(\epsilon_x^{-1} \partial_y H_z^n - \epsilon_x^{-1} \partial_z H_y^n \right) \end{aligned} \quad (2.2.7a)$$

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \epsilon_y^{-1} \partial_z \mu_x^{-1} \partial_z\right) E_y^{n+\frac{1}{2}} &= \left(1 - \frac{\Delta t^2}{4} \epsilon_y^{-1} \partial_z \mu_x^{-1} \partial_z\right) E_y^{n-\frac{1}{2}} \\ &+ \Delta t \left(\epsilon_y^{-1} \partial_z H_x^n - \epsilon_y^{-1} \partial_x H_z^n \right) \end{aligned} \quad (2.2.7b)$$

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \epsilon_z^{-1} \partial_x \mu_y^{-1} \partial_x\right) E_z^{n+\frac{1}{2}} &= \left(1 - \frac{\Delta t^2}{4} \epsilon_z^{-1} \partial_x \mu_y^{-1} \partial_x\right) E_z^{n-\frac{1}{2}} \\ &+ \Delta t \left(\epsilon_z^{-1} \partial_x H_y^n - \epsilon_z^{-1} \partial_y H_x^n \right) \end{aligned} \quad (2.2.7c)$$

2. Implicit update equations for H^{n+1} :

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \mu_x^{-1} \partial_y \epsilon_z^{-1} \partial_y\right) H_x^{n+1} &= \left(1 - \frac{\Delta t^2}{4} \mu_x^{-1} \partial_y \epsilon_z^{-1} \partial_y\right) H_x^n \\ &+ \Delta t \left(\mu_x^{-1} \partial_z E_y^{n+\frac{1}{2}} - \mu_x^{-1} \partial_y E_z^{n+\frac{1}{2}} \right) \end{aligned} \quad (2.2.8a)$$

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \mu_y^{-1} \partial_z \epsilon_x^{-1} \partial_z\right) H_y^{n+1} &= \left(1 - \frac{\Delta t^2}{4} \mu_y^{-1} \partial_z \epsilon_x^{-1} \partial_z\right) H_y^n \\ &+ \Delta t \left(\mu_y^{-1} \partial_x E_z^{n+\frac{1}{2}} - \mu_y^{-1} \partial_z E_x^{n+\frac{1}{2}} \right) \end{aligned} \quad (2.2.8b)$$

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \mu_z^{-1} \partial_x \epsilon_y^{-1} \partial_x\right) H_z^{n+1} &= \left(1 - \frac{\Delta t^2}{4} \mu_z^{-1} \partial_x \epsilon_y^{-1} \partial_x\right) H_z^n \\ &+ \Delta t \left(\mu_z^{-1} \partial_y E_x^{n+\frac{1}{2}} - \mu_z^{-1} \partial_x E_y^{n+\frac{1}{2}} \right) \end{aligned} \quad (2.2.8c)$$

where ϵ_ξ and μ_ξ are the permittivity and permeability of the medium, while ξ is the x , y , z directions, respectively. Note that the leapfrog ADI-FDTD method solves (2.2.7) and (2.2.8) implicitly and no explicit update is needed as in the conventional ADI-FDTD method. Thus, the leapfrog ADI-FDTD method has less equations to solve. The leapfrog ADI-FDTD method might also require less memory than the conventional ADI-FDTD method as it does not require intermediate field variables.

In addition, the leapfrog ADI-FDTD method updates electric and magnetic fields implicitly in one direction only. In particular, referring to (2.2.7a) and (2.2.8a), we can observe that E_x and H_x are updated only in the y -direction. Although the leapfrog ADI-FDTD method is derived from the conventional ADI-FDTD method, the leapfrog ADI-FDTD method does not retain the second procedure which alternates the direction of update.

Chapter 3

Stability and Dispersion Analysis of Leapfrog ADI-FDTD Method

3.1 Introduction

The leapfrog ADI-FDTD method is an unconditionally stable FDTD method that is not restricted by the CFL stability criterion. As such, the computed results do not grow without bound when time-marching proceeds even though the CFL limit is violated. Numerical examples highlighting the stability of the leapfrog ADI-FDTD method have been presented in [29,32,45–48]. Since it is an implicit FDTD method, it requires treatment of the boundary condition [49]. In [32], perfectly matched layer was implemented into the leapfrog ADI-FDTD method to model open structures. In [46], convolutional perfectly matched layer was implemented into the leapfrog ADI-FDTD method to simulate a bow-tie antenna. These examples demonstrated the stability of the leapfrog ADI-FDTD method only in numerical way with some boundary conditions. Furthermore, the dispersion relation of the leapfrog ADI-FDTD method is also not clear.

In this chapter, we present the rigorous analysis of the stability and dispersion for the 3-D leapfrog ADI-FDTD method. We reformulate the leapfrog ADI-FDTD method in the form similar to the conventional explicit FDTD method by introducing two auxiliary variables. The auxiliary variables serve as perturbations of the main field variables. The stability of the leapfrog ADI-FDTD method will be analyzed using the Fourier method. The eigenvalues of the Fourier amplification matrix are obtained analytically, and it is proven that the leapfrog ADI-FDTD method is unconditionally stable. In addition, the dispersion relation of the leapfrog ADI-FDTD method will be presented.

3.2 Reformulation of 3-D Leapfrog ADI-FDTD Method

In this section, we reformulate the leapfrog ADI-FDTD method into the form similar to the conventional explicit FDTD method by introducing two auxiliary variables e and h . Let us consider the 3-D wave propagation in a medium with permittivity ϵ and permeability μ . $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field component vectors

$$\mathbf{E} = \begin{bmatrix} E_x & E_y & E_z \end{bmatrix}^T, \quad \mathbf{H} = \begin{bmatrix} H_x & H_y & H_z \end{bmatrix}^T \quad (3.2.1)$$

The splitting matrix operators of Maxwell's equations may be selected as

$$\mathbf{A}_{12} = \begin{bmatrix} 0 & 0 & \epsilon^{-1}\partial_y \\ \epsilon^{-1}\partial_z & 0 & 0 \\ 0 & \epsilon^{-1}\partial_x & 0 \end{bmatrix}, \quad \mathbf{A}_{21} = \begin{bmatrix} 0 & \mu^{-1}\partial_z & 0 \\ 0 & 0 & \mu^{-1}\partial_x \\ \mu^{-1}\partial_y & 0 & 0 \end{bmatrix} \quad (3.2.2a)$$

$$\mathbf{B}_{12} = \begin{bmatrix} 0 & -\epsilon^{-1}\partial_z & 0 \\ 0 & 0 & -\epsilon^{-1}\partial_x \\ -\epsilon^{-1}\partial_y & 0 & 0 \end{bmatrix} \quad \mathbf{B}_{21} = \begin{bmatrix} 0 & 0 & -\mu^{-1}\partial_y \\ -\mu^{-1}\partial_z & 0 & 0 \\ 0 & -\mu^{-1}\partial_x & 0 \end{bmatrix} \quad (3.2.2b)$$

where ∂_x , ∂_y , ∂_z are the spatial difference operators for the first derivatives along x , y , z directions, respectively.

Using the matrix operators above, the tridiagonal implicit electric field update equations for the leapfrog ADI-FDTD method in (2.2.7) can be expressed as

$$\left(\mathbf{I} - \frac{\Delta t^2}{4} \mathbf{A}_{12} \mathbf{A}_{21} \right) \mathbf{E}^{n+\frac{1}{2}} = \left(\mathbf{I} - \frac{\Delta t^2}{4} \mathbf{A}_{12} \mathbf{A}_{21} \right) \mathbf{E}^{n-\frac{1}{2}} + \Delta t \left(\mathbf{A}_{12} + \mathbf{B}_{12} \right) \mathbf{H}^n \quad (3.2.3)$$

where $\mathbf{I}$ is the 3×3 identity matrix. We can rewrite (3.2.3) as

$$\mathbf{E}^{n+\frac{1}{2}} = \mathbf{E}^{n-\frac{1}{2}} + \frac{\Delta t^2}{4} \mathbf{A}_{12} \mathbf{A}_{21} \left(\mathbf{E}^{n+\frac{1}{2}} - \mathbf{E}^{n-\frac{1}{2}} \right) + \Delta t \mathbf{A}_{12} \mathbf{H}^n + \Delta t \mathbf{B}_{12} \mathbf{H}^n \quad (3.2.4a)$$

$$= \mathbf{E}^{n-\frac{1}{2}} + \Delta t \mathbf{A}_{12} \left[\frac{\Delta t}{4} \mathbf{A}_{21} \left(\mathbf{E}^{n+\frac{1}{2}} - \mathbf{E}^{n-\frac{1}{2}} \right) + \mathbf{H}^n \right] + \Delta t \mathbf{B}_{12} \mathbf{H}^n \quad (3.2.4b)$$

$$= \mathbf{E}^{n-\frac{1}{2}} + \Delta t \left(\mathbf{A}_{12} \mathbf{h}^n + \mathbf{B}_{12} \mathbf{H}^n \right) \quad (3.2.4c)$$

where we introduce the auxiliary variable

$$\mathbf{h}^n = \mathbf{H}^n + \frac{\Delta t}{4} \mathbf{A}_{21} \left(\mathbf{E}^{n+\frac{1}{2}} - \mathbf{E}^{n-\frac{1}{2}} \right). \quad (3.2.5)$$

Equation (3.2.4c) becomes the conventional explicit FDTD electric field update equation when $\mathbf{h}^n = \mathbf{H}^n$. It can be observed that the auxiliary variable $\mathbf{h}$ in (3.2.5) is a perturbation of $\mathbf{H}$. Next, we cast (3.2.4c) and (3.2.5) in a compact matrix form

as

$$\begin{bmatrix} \mathbf{I} & -\Delta t \mathbf{A}_{12} \\ -\frac{\Delta t}{4} \mathbf{A}_{21} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{E}^{n+\frac{1}{2}} \\ \mathbf{h}^n \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \Delta t \mathbf{B}_{12} \\ -\frac{\Delta t}{4} \mathbf{A}_{21} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{E}^{n-\frac{1}{2}} \\ \mathbf{H}^n \end{bmatrix}. \quad (3.2.6)$$

Similarly, the tridiagonal implicit magnetic field update equations for the leapfrog ADI-FDTD method in (2.2.8) can be expressed as

$$\left(\mathbf{I} - \frac{\Delta t^2}{4} \mathbf{B}_{21} \mathbf{B}_{12} \right) \mathbf{H}^{n+1} = \left(\mathbf{I} - \frac{\Delta t^2}{4} \mathbf{B}_{21} \mathbf{B}_{12} \right) \mathbf{H}^n + \Delta t \left(\mathbf{A}_{21} + \mathbf{B}_{21} \right) \mathbf{E}^{n+\frac{1}{2}}. \quad (3.2.7)$$

Following the procedure above, we can rewrite (3.2.7) as

$$\mathbf{H}^{n+1} = \mathbf{H}^n + \frac{\Delta t^2}{4} \mathbf{B}_{21} \mathbf{B}_{12} (\mathbf{H}^{n+1} - \mathbf{H}^n) + \Delta t \mathbf{A}_{21} \mathbf{E}^{n+\frac{1}{2}} + \Delta t \mathbf{B}_{21} \mathbf{E}^{n+\frac{1}{2}} \quad (3.2.8a)$$

$$= \mathbf{H}^n + \Delta t \mathbf{B}_{21} \left[\frac{\Delta t}{4} \mathbf{B}_{12} (\mathbf{H}^{n+1} - \mathbf{H}^n) + \mathbf{E}^{n+\frac{1}{2}} \right] + \Delta t \mathbf{A}_{21} \mathbf{E}^{n+\frac{1}{2}} \quad (3.2.8b)$$

$$= \mathbf{H}^n + \Delta t \left(\mathbf{A}_{21} \mathbf{E}^{n+\frac{1}{2}} + \mathbf{B}_{21} \mathbf{e}^{n+\frac{1}{2}} \right) \quad (3.2.8c)$$

where the auxiliary variable

$$\mathbf{e}^{n+\frac{1}{2}} = \mathbf{E}^{n+\frac{1}{2}} + \frac{\Delta t}{4} \mathbf{B}_{12} (\mathbf{H}^{n+1} - \mathbf{H}^n). \quad (3.2.9)$$

Equation (3.2.8c) becomes the conventional explicit FDTD magnetic field update equation when $\mathbf{e}^{n+\frac{1}{2}} = \mathbf{E}^{n+\frac{1}{2}}$. The auxiliary variable $\mathbf{e}$ in (3.2.9) is a perturbation of $\mathbf{E}$. Next, we cast (3.2.8c) and (3.2.9) in matrix form compactly as

$$\begin{bmatrix} \mathbf{I} & -\frac{\Delta t}{4} \mathbf{B}_{12} \\ -\Delta t \mathbf{B}_{21} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{e}^{n+\frac{1}{2}} \\ \mathbf{H}^{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\frac{\Delta t}{4} \mathbf{B}_{12} \\ \Delta t \mathbf{A}_{21} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{E}^{n+\frac{1}{2}} \\ \mathbf{H}^n \end{bmatrix}. \quad (3.2.10)$$

Equations (3.2.6) and (3.2.10) can be combined as

$$\begin{bmatrix} \mathbf{I} & -\Delta t \mathbf{A}_{12} & \mathbf{0} & \mathbf{0} \\ -\frac{\Delta t}{4} \mathbf{A}_{21} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} & \mathbf{I} & -\frac{\Delta t}{4} \mathbf{B}_{12} \\ -\Delta t \mathbf{A}_{21} & \mathbf{0} & -\Delta t \mathbf{B}_{21} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{E}^{n+\frac{1}{2}} \\ \mathbf{h}^n \\ \mathbf{e}^{n+\frac{1}{2}} \\ \mathbf{H}^{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \Delta t \mathbf{B}_{12} \\ -\frac{\Delta t}{4} \mathbf{A}_{21} & \mathbf{I} \\ \mathbf{0} & -\frac{\Delta t}{4} \mathbf{B}_{12} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{E}^{n-\frac{1}{2}} \\ \mathbf{H}^n \end{bmatrix}. \quad (3.2.11)$$

Solving (3.2.11) for $\mathbf{E}^{n+\frac{1}{2}}$ and $\mathbf{H}^{n+1}$ reduces symbolically to

$$\begin{bmatrix} \mathbf{E}^{n+\frac{1}{2}} \\ \mathbf{H}^{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & -\Delta t \mathbf{A}_{12} & \mathbf{0} & \mathbf{0} \\ -\frac{\Delta t}{4} \mathbf{A}_{21} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} & \mathbf{I} & -\frac{\Delta t}{4} \mathbf{B}_{12} \\ -\Delta t \mathbf{A}_{21} & \mathbf{0} & -\Delta t \mathbf{B}_{21} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} & \Delta t \mathbf{B}_{12} \\ -\frac{\Delta t}{4} \mathbf{A}_{21} & \mathbf{I} \\ \mathbf{0} & -\frac{\Delta t}{4} \mathbf{B}_{12} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{E}^{n-\frac{1}{2}} \\ \mathbf{H}^n \end{bmatrix}. \quad (3.2.12)$$

The electric and magnetic fields are staggered in time and updated in one full time step iteratively. Finally (3.2.12) can be expressed in matrix operator form compactly as

$$\begin{bmatrix} \mathbf{E}^{n+\frac{1}{2}} \\ \mathbf{H}^{n+1} \end{bmatrix} = \mathbf{G} \begin{bmatrix} \mathbf{E}^{n-\frac{1}{2}} \\ \mathbf{H}^n \end{bmatrix} \quad (3.2.13)$$

where

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22} \end{bmatrix}$$

and

$$\begin{aligned}
\mathbf{g}_{11} &= \mathbf{I}_3 \\
\mathbf{g}_{12} &= \Delta t \left(\mathbf{I}_3 - \frac{\Delta t^2}{4} \mathbf{A}_{12} \mathbf{A}_{21} \right)^{-1} (\mathbf{A}_{12} + \mathbf{B}_{12}) \\
\mathbf{g}_{21} &= -\Delta t \left(\mathbf{I}_3 - \frac{\Delta t^2}{4} \mathbf{B}_{21} \mathbf{B}_{12} \right)^{-1} (\mathbf{A}_{21} + \mathbf{B}_{21}) \\
\mathbf{g}_{22} &= \left[-\Delta t \left(\mathbf{I}_3 - \frac{\Delta t^2}{4} \mathbf{B}_{21} \mathbf{B}_{12} \right)^{-1} (\mathbf{A}_{21} + \mathbf{B}_{21}) \right] \left[\Delta t \left(\mathbf{I}_3 - \frac{\Delta t^2}{4} \mathbf{A}_{12} \mathbf{A}_{21} \right)^{-1} (\mathbf{A}_{12} + \mathbf{B}_{12}) \right] + \mathbf{I}_3.
\end{aligned}$$

3.3 Stability Analysis of Leapfrog ADI-FDTD Method

The stability of the leapfrog ADI-FDTD method will be analyzed using the Fourier (Von Neumann) method in this section [50, 51]. The matrix $\mathbf{G}$ in (3.2.13) is transformed to the spatial spectral domain with wavenumbers k_x , k_y and k_z [1] to obtain the Fourier amplification matrix $\mathbf{G}_F$. The eigenvalues of $\mathbf{G}_F$ will determine the stability of the leapfrog ADI-FDTD method. Equation (3.2.13) can be written in the Fourier domain as

$$\mathbf{U}_F^{n+1} = \mathbf{G}_F \mathbf{U}_F^n \quad (3.3.1a)$$

where

$$\mathbf{U}_F = \begin{bmatrix} E_{x0} e^{-j\omega \frac{\Delta t}{2}} & E_{y0} e^{-j\omega \frac{\Delta t}{2}} & E_{z0} e^{-j\omega \frac{\Delta t}{2}} & H_{x0} & H_{y0} & H_{z0} \end{bmatrix}^T \quad (3.3.1b)$$

$$\mathbf{G}_F = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{-j2bP_z}{\Omega_y} & \frac{j2bP_y}{\Omega_y} \\ 0 & 1 & 0 & \frac{j2bP_z}{\Omega_z} & 0 & \frac{-j2bP_x}{\Omega_z} \\ 0 & 0 & 1 & \frac{-j2bP_y}{\Omega_x} & \frac{-j2bP_x}{\Omega_x} & 0 \\ 0 & \frac{j2dP_z}{\Omega_y} & \frac{-j2dP_y}{\Omega_y} & \frac{g_{4,4}}{\Omega_x\Omega_y\Omega_z} & \frac{4bdP_xP_y}{\Omega_x\Omega_y} & \frac{4bdP_xP_z}{\Omega_y\Omega_z} \\ \frac{-j2dP_z}{\Omega_z} & 0 & \frac{-j2dP_x}{\Omega_z} & \frac{4bdP_xP_y}{\Omega_x\Omega_z} & \frac{g_{5,5}}{\Omega_x\Omega_y\Omega_z} & \frac{4bdP_yP_z}{\Omega_y\Omega_z} \\ \frac{j2dP_y}{\Omega_x} & \frac{-j2dP_x}{\Omega_x} & 0 & \frac{4bdP_xP_z}{\Omega_x\Omega_z} & \frac{4bdP_yP_z}{\Omega_x\Omega_y} & \frac{g_{6,6}}{\Omega_x\Omega_y\Omega_z} \end{bmatrix} \quad (3.3.1c)$$

$$g_{4,4} = b^3 d^3 P_x^2 P_y^2 P_z^2 + b^2 d^2 \left(P_x^2 P_y^2 - 3P_x^2 P_z^2 - 3P_y^2 P_z^2 \right) + bd \left(P_x^2 - 3P_y^2 - 3P_z^2 \right) + 1 \quad (3.3.1d)$$

$$g_{5,5} = b^3 d^3 P_x^2 P_y^2 P_z^2 + b^2 d^2 \left(P_y^2 P_z^2 - 3P_x^2 P_y^2 - 3P_x^2 P_z^2 \right) + bd \left(P_y^2 - 3P_x^2 - 3P_z^2 \right) + 1 \quad (3.3.1e)$$

$$g_{6,6} = b^3 d^3 P_x^2 P_y^2 P_z^2 + b^2 d^2 \left(P_x^2 P_z^2 - 3P_x^2 P_y^2 - 3P_y^2 P_z^2 \right) + bd \left(P_z^2 - 3P_x^2 - 3P_y^2 \right) + 1 \quad (3.3.1f)$$

$$P_\xi = \frac{-2 \sin \left(\frac{k_\xi \Delta_\xi}{2} \right)}{\Delta_\xi} \quad (3.3.1g)$$

$$\Omega_\xi = 1 + bdP_\xi^2, \quad \xi = x, y, z \quad (3.3.1h)$$

$$b = \frac{\Delta t}{2\epsilon}, \quad d = \frac{\Delta t}{2\mu} \quad (3.3.1i)$$

Here, P_ξ corresponds to the spatial difference operator ∂_ξ in the Fourier domain, and k is the wavenumber. $E_{\xi 0}$ and $H_{\xi 0}$ are the initial conditions of E_ξ and H_ξ fields respectively in the Fourier domain.

The eigenvalues of $\mathbf{G}_F$ are obtained as

$$\lambda_1 = \lambda_2 = 1 \quad (3.3.2a)$$

$$\lambda_3 = \lambda_4 = \frac{Q + \sqrt{-S^2}}{R} \quad (3.3.2b)$$

$$\lambda_5 = \lambda_6 = \lambda_3^* = \frac{Q - \sqrt{-S^2}}{R} \quad (3.3.2c)$$

where

$$R = \Omega_x \Omega_y \Omega_z = (1 + bdP_x^2)(1 + bdP_y^2)(1 + bdP_z^2) \quad (3.3.2d)$$

$$Q = b^3 d^3 P_x^2 P_y^2 P_z^2 - b^2 d^2 (P_x^2 P_y^2 + P_x^2 P_z^2 + P_y^2 P_z^2) - bd(P_x^2 + P_y^2 + P_z^2) + 1 \quad (3.3.2e)$$

$$S^2 = R^2 - Q^2 = 4bd(b^3 d^3 P_x^2 P_y^2 P_z^2 + 1) \left[P_x^2 + P_y^2 + P_z^2 + bd(P_x^2 P_y^2 + P_x^2 P_z^2 + P_y^2 P_z^2) \right]. \quad (3.3.2f)$$

Since R and Q are real numbers and (3.3.2f) is true, we can rewrite (3.3.2b) and (3.3.2c) as

$$\lambda = \frac{\sqrt{R^2 - S^2}}{R} \pm \frac{jS}{R} \quad (3.3.3)$$

Subsequently, the magnitude of the eigenvalues can be determined as

$$|\lambda| = \sqrt{\frac{R^2 - S^2}{R^2} + \frac{S^2}{R^2}} = 1. \quad (3.3.4)$$

The leapfrog ADI-FDTD method is thus unconditionally stable and non-dissipative as all the eigenvalues of $\mathbf{G}_F$ have unity magnitude.

It is worthwhile to mention that the eigenvalues of $\mathbf{G}_F$ for the leapfrog ADI-FDTD

method are repeated in complex conjugate pairs. Eigenvalues λ_1 and λ_2 correspond to the electrostatic and magnetostatic modes, while eigenvalues λ_3 to λ_6 correspond to the propagating modes. The eigenvalues of $\mathbf{G}_F$ for the leapfrog ADI-FDTD method are also the same as the eigenvalues of $\mathbf{G}_F$ for the conventional ADI-FDTD method. The stability analysis has also been extended for arbitrary spatial-order leapfrog ADI-FDTD method in [52].

3.4 Dispersion Analysis of Leapfrog ADI-FDTD Method

The dispersion relation of the leapfrog ADI-FDTD method can be analyzed by assuming the field to be a monochromatic wave with angular frequency ω [53–55]. The dispersion relation of the leapfrog ADI-FDTD method can be deduced from the eigenvalues of $\mathbf{G}_F$. Since the eigenvalues of the leapfrog ADI-FDTD method are the same as the conventional ADI-FDTD method, the dispersion relation of the leapfrog ADI-FDTD method is also the same as that of the conventional ADI-FDTD method and is given as

$$\sin^2(\omega\Delta t) =$$

$$\frac{4bd\left(b^3d^3P_x^2P_y^2P_z^2 + 1\right)\left[P_x^2 + P_y^2 + P_z^2 + bd\left(P_x^2P_y^2 + P_x^2P_z^2 + P_y^2P_z^2\right)\right]}{\left(1 + bdP_x^2\right)^2\left(1 + bdP_y^2\right)^2\left(1 + bdP_z^2\right)^2} \quad (3.4.1)$$

For illustration, the angular frequency ω is chosen to be 3 GHz with uniform cell size $\Delta = \Delta_x = \Delta_y = \Delta_z = \frac{\lambda}{50}$. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{3}}$ where c is the speed of light in freespace. We also denote CFLN as $\frac{\Delta t}{\Delta t_{CFL}}$.

Figure 3.1: Normalized phase velocity (v_p/c) vs. wave propagation angle ϕ for CFLN = 1, 5 and 10. (a) $\theta = 45^\circ$. (b) $\theta = 90^\circ$.

The numerical phase velocity is given as

$$v_p = \frac{\omega}{k} \quad (3.4.2)$$

where the wavenumber k can be obtained by solving (3.4.1) with the parameters given above. The normalized phase velocity (v_p/c) vs. wave propagation angle ϕ for CFLN = 1, 5 and 10 at $\theta = 45^\circ$ and 90° is plotted in Fig. 3.1. It is apparent that the numerical phase velocity error increases with CFLN and the numerical phase velocity is slightly anisotropic. Referring to Fig. 3.1(a), the normalized phase velocity approaches 1 for CFLN = 1, while the normalized phase velocity is approximately 0.975 for CFLN = 10. Referring to Fig. 3.1(b) for CFLN = 10, we can clearly observe that the normalized phase velocity varies with wave propagation angle ϕ , and it is maximum at wave propagation angle $\phi = 45^\circ$.

3.5 Conclusion

This chapter has presented the stability and dispersion analysis of the leapfrog ADI-FDTD method. The leapfrog ADI-FDTD method was reviewed and the implicit

update equations presented. The leapfrog ADI-FDTD method was reformulated in the form similar to conventional explicit FDTD method by introducing two auxiliary variables. The auxiliary variables serve as perturbations of the main fields variables. We analyzed the stability of the leapfrog ADI-FDTD method using the Fourier method. The eigenvalues of the Fourier amplification matrix are obtained analytically and it is proven that the leapfrog ADI-FDTD method is unconditionally stable. The dispersion relation of the leapfrog ADI-FDTD method was also presented. The leapfrog ADI-FDTD method has the same eigenvalues and dispersion relation as the conventional ADI-FDTD method.

The leapfrog ADI-FDTD method is unconditionally stable and employs the same leapfrog time-stepping scheme as the conventional explicit FDTD method. It should be useful for modeling of fine structures using hybrid subgridding methods [45, 47]. In the following chapter, extended formulations for the leapfrog ADI-FDTD method will be provided.

Chapter 4

Extended Formulations for Leapfrog ADI-FDTD Method

4.1 Introduction

In the last chapter, the numerical stability and dispersion relation of the leapfrog ADI-FDTD method were established. Since the leapfrog ADI-FDTD method is unconditionally stable, the maximum time step size that can be employed by the leapfrog ADI-FDTD method is no longer limited by the minimum cell size in the computational domain, i.e., the CFL stability criterion. However, the size of the computation domain is still usually restricted by the finite memory size of the computer. Therefore, some form of boundary condition is necessary so as to truncate the computation domain appropriately.

In order to solve open structures efficiently, absorbing boundary conditions (ABC) are employed to truncate the computation domain. The perfectly matched layer [56] has been previously implemented in the leapfrog ADI-FDTD method [32, 57]. How-

ever, such implementation is memory-intensive and complicated. On the other hand, the Mur ABC [58] is a simpler and more efficient alternative.

Furthermore, the leapfrog ADI-FDTD method in [45] was employed to model an electromagnetic bandgap waveguide filled with dielectric posts. The dielectric posts were modeled ideally. To further enhance the simulation model, we could incorporate more physical model parameter, such as the dielectric losses. However, the formulation of the leapfrog ADI-FDTD method for lossy media is not clear. Intuitively, to formulate the leapfrog ADI-FDTD method for lossy media, one should approach such derivation by first considering the conventional ADI-FDTD method for lossy media [8, 59]. However, such an approach would not result in the proper formulation for the leapfrog ADI-FDTD method for lossy media.

In this chapter, we extend the leapfrog ADI-FDTD method to include the Mur ABC, perfect electric conductor boundary condition and lossy media. The Mur ABC will be implemented for the TE_z leapfrog ADI-FDTD method. Meanwhile for completeness, the formulation of the leapfrog ADI-FDTD with PEC boundary condition by applying the image theory will be presented. Finally, we also consider the formulation of the leapfrog ADI-FDTD method for lossy media. The formulation presented in this chapter is not obtained from the *conventional* derivation of the leapfrog ADI-FDTD method for lossy media. The leapfrog ADI-FDTD method is then accelerated by implementing it on a low-cost graphic processing unit (GPU). Numerical results will be presented to validate the various formulations.

4.2 Formulation for Mur Absorbing Boundary Condition

In this section, we formulate the Mur ABC for the TE_z leapfrog ADI-FDTD method.

The time-domain update equations are given as

$$E_x^{n+\frac{1}{2}} = E_x^{n-\frac{1}{2}} + \Delta t \hat{\epsilon}_x^{-1} \partial_y H_z^n \quad (4.2.1a)$$

$$E_y^{n+\frac{1}{2}} = E_y^{n-\frac{1}{2}} - \Delta t \hat{\epsilon}_y^{-1} \partial_x H_z^n \quad (4.2.1b)$$

$$H_z^{n+1} = H_z^n + \Delta t \hat{\mu}_z^{-1} \left(\partial_y E_x^{n+\frac{1}{2}} - \partial_x E_y^{n+\frac{1}{2}} \right) \quad (4.2.1c)$$

where

$$\hat{\epsilon}_x = \epsilon_x - \frac{\Delta t^2}{4} \partial_y \mu_z^{-1} \partial_y \quad (4.2.2a)$$

$$\hat{\epsilon}_y = \epsilon_y \quad (4.2.2b)$$

$$\hat{\mu}_z = \mu_z - \frac{\Delta t^2}{4} \partial_x \epsilon_y^{-1} \partial_x \quad (4.2.2c)$$

Here, ∂_x and ∂_y corresponds to the spatial difference operators for the first derivatives along the x and y directions, respectively. Referring to (4.2.1) and (4.2.2), it can be observed that the E_x and H_z field components are updated implicitly, while the E_y field components are updated explicitly. The implicit updating of the field components may require the computation of out-of-domains terms in the tridiagonal matrix depending on the type of boundary condition implemented [49].

For the TE_z leapfrog ADI-FDTD method, both the electric and magnetic field components should be considered when formulating the Mur ABC, as the E_x and H_z field components are both updated implicitly. This is in contrast with the other unconditionally stable implicit FDTD methods, such as the efficient fundamental

ADI-FDTD and LOD-FDTD methods, where only the electric field components are update implicitly. For such methods, only the electric field components are necessary for the formulation of the Mur ABC [60,61].

For illustration, we consider the one-way wave equation along the x -direction:

$$\frac{\partial}{\partial t}U(x, t) = v \frac{\partial}{\partial x}U(x, t) \quad (4.2.3)$$

where U is the electric or magnetic field component and v is the wave velocity. Applying central difference approximation and time-averaging, we obtain the first-order Mur ABC (illustrated at the boundary $x = 0$) as

$$U|_{x=0}^{n+1} = U|_{x=1}^n + \frac{v\Delta t - \Delta x}{v\Delta t + \Delta x} \left(U|_{x=1}^{n+1} - U|_{x=0}^n \right) \quad (4.2.4)$$

Note that the indices for x and n needs to be replaced appropriately for electric and magnetic fields respectively. Equation (4.2.4) is the explicit Mur ABC updating equation and by performing some manipulation, we arrive at the implicit Mur ABC updating equation as

$$U|_{x=0}^{n+1} - \left(\frac{v\Delta t - \Delta x}{v\Delta t + \Delta x} \right) U|_{x=1}^{n+1} = U|_{x=1}^n - \left(\frac{v\Delta t - \Delta x}{v\Delta t + \Delta x} \right) U|_{x=0}^n \quad (4.2.5)$$

The first-order Mur ABC updating equation is then implemented for the TE_z leapfrog ADI-FDTD method by employing (4.2.5) for the implicit updating equations (4.2.1a) and (4.2.1c), and (4.2.4) for the explicit updating equation (4.2.1b). At the $i=0$ and $j=0$ boundary, the actual implementations for the update equations

are given as

$$E_x|_{j=0}^{n+\frac{1}{2}} - \left(\frac{v\Delta t - \Delta x}{v\Delta t + \Delta x} \right) E_x|_{j=1}^{n+\frac{1}{2}} = E_x|_{j=1}^{n-\frac{1}{2}} - \left(\frac{v\Delta t - \Delta x}{v\Delta t + \Delta x} \right) E_x|_{j=0}^{n-\frac{1}{2}} \quad (4.2.6)$$

$$E_y|_{i=0}^{n+\frac{1}{2}} = E_y|_{i=1}^{n-\frac{1}{2}} + \frac{v\Delta t - \Delta y}{v\Delta t + \Delta y} \left(E_y|_{i=1}^{n+\frac{1}{2}} - E_y|_{i=0}^{n-\frac{1}{2}} \right) \quad (4.2.7)$$

$$H_z|_{i=0}^{n+1} - \left(\frac{v\Delta t - \Delta x}{v\Delta t + \Delta x} \right) H_z|_{i=1}^{n+1} = H_z|_{i=1}^n - \left(\frac{v\Delta t - \Delta x}{v\Delta t + \Delta x} \right) H_z|_{i=0}^n \quad (4.2.8)$$

The Mur ABC formulation for the TE_z leapfrog ADI-FDTD method can be extended to the 3-D leapfrog ADI-FDTD method by using appropriate Mur ABC update equations accordingly.

4.2.1 Numerical Results

In this subsection, we validate the formulation of the Mur ABC for the leapfrog ADI-FDTD method through numerical simulations. The computation domain has a dimension of 80 x 80 grids with uniform cell size of $\Delta x = \Delta y = 1$ mm.

A magnetic sinusoid modulated Gaussian pulse excitation is located at the center of the computation domain as

$$M_z = \sin \left[2\pi f_0 (t - t_0) \right] e^{-\left(\frac{t-t_0}{\tau} \right)^2} \quad (4.2.9)$$

where $f_0 = 4$ GHz, $\tau = 180$ ps and $t_0 = 3\tau$. The observation point is located 20 cells away from the source in the y -direction. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{2}}$, where c is the speed of light in freespace and $CFLN = \frac{\Delta t}{\Delta t_{CFL}}$.

Figure 4.1: Reflection errors of the Mur ABC formulation for the TE_z leapfrog ADI-FDTD and conventional explicit FDTD method for various CFLN in time-domain. ©2012 IEEE.

The reflection error for the Mur ABC is evaluated by

$$R = 20 \log_{10} \left(\frac{|H - H_{ref}|}{\max |H_{ref}|} \right) \quad (4.2.10)$$

where H_{ref} is the reference value calculated in a grid large enough so that any reflection from the boundary is isolated.

Fig. 4.1 graphs the reflection errors of the Mur ABC implementation for the leapfrog ADI-FDTD method and the conventional explicit FDTD method in the time-domain. The CFLN for the leapfrog ADI-FDTD method is chosen as 1, 4 and 8, while the CFLN for the conventional explicit FDTD method is fixed at 1. It can be observed that the reflection errors of the Mur ABC for the leapfrog ADI-FDTD method is comparable to the conventional explicit FDTD method. The maximum reflection error for the leapfrog ADI-FDTD method is about -27 dB.

To investigate the late-time stability of the leapfrog ADI-FDTD method, a numerical simulation with CFLN = 10 and time step of 10 000 iterations has been performed.

Figure 4.2: Late-time simulation of the Mur ABC formulation for the TE_z leapfrog ADI-FDTD method with $CFLN = 10$. ©2012 IEEE.

Referring to Fig. 4.2, it is clear that the Mur ABC formulation of the TE_z leapfrog ADI-FDTD method is late-time stable.

4.3 Formulation for Perfect Electric Conductor Boundary Condition

In this section, we discuss the formulation of the leapfrog ADI-FDTD with PEC boundary condition by applying the image theory. For simplicity, we consider wave propagation in a medium with permittivity ϵ and permeability μ . The update equations for the leapfrog ADI-FDTD method (illustrated here using E_x and H_x field components) are given as

1. Implicit updating for $E_x^{n+\frac{1}{2}}$:

$$\begin{aligned}
& -\frac{\Delta t^2}{4\mu\epsilon\Delta y^2}E_x|_{i+\frac{1}{2},j-1,k}^{n+\frac{1}{2}} + \left(1 + \frac{\Delta t^2}{2\mu\epsilon\Delta y^2}\right)E_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} - \frac{\Delta t^2}{4\mu\epsilon\Delta y^2}E_x|_{i+\frac{1}{2},j+1,k}^{n+\frac{1}{2}} = \\
& -\frac{\Delta t^2}{4\mu\epsilon\Delta y^2}E_x|_{i+\frac{1}{2},j-1,k}^{n-\frac{1}{2}} + \left(1 + \frac{\Delta t^2}{2\mu\epsilon\Delta y^2}\right)E_x|_{i+\frac{1}{2},j,k}^{n-\frac{1}{2}} - \frac{\Delta t^2}{4\mu\epsilon\Delta y^2}E_x|_{i+\frac{1}{2},j+1,k}^{n-\frac{1}{2}} \\
& + \frac{\Delta t}{\epsilon\Delta y} \left(H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^n - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^n \right) \\
& - \frac{\Delta t}{\epsilon\Delta z} \left(H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^n \right)
\end{aligned} \tag{4.3.1}$$

2. Implicit updating for H_x^{n+1} :

$$\begin{aligned}
& -\frac{\Delta t^2}{4\mu\epsilon\Delta y^2}H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^{n+1} + \left(1 + \frac{\Delta t^2}{2\mu\epsilon\Delta y^2}\right)H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+1} - \frac{\Delta t^2}{4\mu\epsilon\Delta y^2}H_x|_{i,j+\frac{3}{2},k+\frac{1}{2}}^{n+1} = \\
& -\frac{\Delta t^2}{4\mu\epsilon\Delta y^2}H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^n + \left(1 + \frac{\Delta t^2}{2\mu\epsilon\Delta y^2}\right)H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^n - \frac{\Delta t^2}{4\mu\epsilon\Delta y^2}H_x|_{i,j+\frac{3}{2},k+\frac{1}{2}}^n \\
& + \frac{\Delta t}{\mu\Delta z} \left(E_y|_{i,j+\frac{1}{2},k+1}^{n+\frac{1}{2}} - E_y|_{i,j+\frac{1}{2},k}^{n+\frac{1}{2}} \right) \\
& - \frac{\Delta t}{\mu\Delta y} \left(E_z|_{i,j+1,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} \right)
\end{aligned} \tag{4.3.2}$$

Referring to (4.3.2), the H_x field components are solved implicitly using a tridiagonal matrix, and involve terms that are out-of-domain (e.g. $H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}$). Therefore, special consideration must be taken for these out-of-domain terms when implementing the boundary condition [49].

As an illustration, we consider the commonly used PEC boundary condition. Implementation of more complex boundary condition follows in a similar manner. It is straightforward to impose the PEC boundary condition for the electric fields. We can simply define the tangential electric fields at the PEC boundaries to be zero. However, as mentioned above, the out-of-domain terms for the tangential magnetic fields are non-zero, and this is critical for the proper implementation of the PEC boundary condition. Note that this issue does not exist for the conventional ADI-FDTD method as the tangential magnetic fields are (usually) not updated implicitly,

and thus there are no such out-of-domain terms.

To circumvent this issue, we resort to the image theory. Since the H_x field components are updated in the y -direction, we consider the PEC at the $j = 0$ boundary. By applying the image theory, we can rewrite (4.3.2) as

$$\begin{aligned}
& \left(1 + \frac{\Delta t^2}{2\mu\epsilon\Delta y^2} - \frac{\Delta t^2}{4\mu\epsilon\Delta y^2}\right) H_x|_{i,\frac{1}{2},k+\frac{1}{2}}^{n+1} - \frac{\Delta t^2}{4\mu\epsilon\Delta y^2} H_x|_{i,\frac{3}{2},k+\frac{1}{2}}^{n+1} = \\
& \left(1 + \frac{\Delta t^2}{2\mu\epsilon\Delta y^2} - \frac{\Delta t^2}{4\mu\epsilon\Delta y^2}\right) H_x|_{i,\frac{1}{2},k+\frac{1}{2}}^n - \frac{\Delta t^2}{4\mu\epsilon\Delta y^2} H_x|_{i,\frac{3}{2},k+\frac{1}{2}}^n \\
& + \frac{\Delta t}{\mu\Delta z} \left(E_y|_{i,\frac{1}{2},k+1}^{n+\frac{1}{2}} - E_y|_{i,\frac{1}{2},k}^{n+\frac{1}{2}}\right) \\
& - \frac{\Delta t}{\mu\Delta y} \left(E_z|_{i,1,k+\frac{1}{2}}^{n+\frac{1}{2}}\right)
\end{aligned} \tag{4.3.3}$$

Note that the tangential electric field components at the $j = 0$ boundary are defined as zero. The update equations for the other magnetic field components can be written down by permuting the subscript indices accordingly.

Finally, it is also worth highlighting that the out-of-domain terms of the magnetic field updates must be treated carefully especially during actual implementation. Improper indexing and/or material assignment may potentially cause numerical instability. In addition, it is necessary to implement the PEC boundary condition properly for the PML as it is eventually terminated by PEC walls [56].

4.4 Formulation for Lossy Media

In the following section, we present the formulation of the leapfrog ADI-FDTD method for lossy media. Here, we consider wave propagation in a lossy medium with

permittivity ϵ , permeability μ , electric conductivity σ and magnetic conductivity σ^* .

Since the leapfrog ADI-FDTD method is a perturbation of the conventional explicit FDTD method [62], we can incorporate the conductivity terms of lossy media for the leapfrog ADI-FDTD method in an analogous manner as the conventional explicit FDTD method. Consequently, electric and magnetic losses are incorporated into the leapfrog ADI-FDTD method by means of introducing conductivity terms for lossy media at the right-hand-side of the update equations as follows:

1. Implicit updating for $E^{n+\frac{1}{2}}$:

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \epsilon_x^{-1} \partial_y \mu_z^{-1} \partial_y\right) E_x^{n+\frac{1}{2}} &= \left(1 - \frac{\Delta t^2}{4} \epsilon_x^{-1} \partial_y \mu_z^{-1} \partial_y\right) E_x^{n-\frac{1}{2}} \\ &+ \Delta t \epsilon_x^{-1} \left(\partial_y H_z^n - \partial_z H_y^n - \sigma_x E_x^n\right) \end{aligned} \quad (4.4.1a)$$

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \epsilon_y^{-1} \partial_z \mu_x^{-1} \partial_z\right) E_y^{n+\frac{1}{2}} &= \left(1 - \frac{\Delta t^2}{4} \epsilon_y^{-1} \partial_z \mu_x^{-1} \partial_z\right) E_y^{n-\frac{1}{2}} \\ &+ \Delta t \epsilon_y^{-1} \left(\partial_z H_x^n - \partial_x H_z^n - \sigma_y E_y^n\right) \end{aligned} \quad (4.4.1b)$$

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \epsilon_z^{-1} \partial_x \mu_y^{-1} \partial_x\right) E_z^{n+\frac{1}{2}} &= \left(1 - \frac{\Delta t^2}{4} \epsilon_z^{-1} \partial_x \mu_y^{-1} \partial_x\right) E_z^{n-\frac{1}{2}} \\ &+ \Delta t \epsilon_z^{-1} \left(\partial_x H_y^n - \partial_y H_x^n - \sigma_z E_z^n\right) \end{aligned} \quad (4.4.1c)$$

2. Implicit updating for H^{n+1} :

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \mu_x^{-1} \partial_y \epsilon_z^{-1} \partial_y\right) H_x^{n+1} &= \left(1 - \frac{\Delta t^2}{4} \mu_x^{-1} \partial_y \epsilon_z^{-1} \partial_y\right) H_x^n \\ &+ \Delta t \mu_x^{-1} \left(\partial_z E_y^{n+\frac{1}{2}} - \partial_y E_z^{n+\frac{1}{2}} - \sigma_x^* H_x^{n+\frac{1}{2}}\right) \end{aligned} \quad (4.4.2a)$$

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \mu_y^{-1} \partial_z \epsilon_x^{-1} \partial_z\right) H_y^{n+1} &= \left(1 - \frac{\Delta t^2}{4} \mu_y^{-1} \partial_z \epsilon_x^{-1} \partial_z\right) H_y^n \\ &+ \Delta t \mu_y^{-1} \left(\partial_x E_z^{n+\frac{1}{2}} - \partial_z E_x^{n+\frac{1}{2}} - \sigma_y^* H_y^{n+\frac{1}{2}} \right) \end{aligned} \quad (4.4.2b)$$

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \mu_z^{-1} \partial_x \epsilon_y^{-1} \partial_x\right) H_z^{n+1} &= \left(1 - \frac{\Delta t^2}{4} \mu_z^{-1} \partial_x \epsilon_y^{-1} \partial_x\right) H_z^n \\ &+ \Delta t \mu_z^{-1} \left(\partial_y E_x^{n+\frac{1}{2}} - \partial_x E_y^{n+\frac{1}{2}} - \sigma_z^* H_z^{n+\frac{1}{2}} \right) \end{aligned} \quad (4.4.2c)$$

Here, electric losses are implemented in (4.4.1), while magnetic losses are implemented in (4.4.2). Note that it is not proper to directly formulate the leapfrog ADI-FDTD method for lossy media by following strictly the previous derivation of the leapfrog ADI-FDTD method from the conventional ADI-FDTD method for lossy media. This is because of the additional (non-unity) coefficients incurred for the inclusion of the conductivity terms of lossy media. For the leapfrog ADI-FDTD method for lossless media, these coefficients are unity and permit the exact cancellation of the mixed derivative terms. However, the previous derivation of the leapfrog ADI-FDTD method [29] is no longer directly applicable for the leapfrog ADI-FDTD method for lossy media, as these coefficients are non-unity, and they do not eliminate the mixed derivative terms. Further discussion with detailed explanations of the above is provided in Appendix A.

As an illustration, we will present the implementation of electrically lossy media for the E_x field component. By taking time-averaging of the conductivity terms in

(4.4.1a), the update equation for E_x can be written as

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \epsilon_x^{-1} \partial_y \mu_z^{-1} \partial_y\right) E_x^{n+\frac{1}{2}} &= \left(1 - \frac{\Delta t^2}{4} \epsilon_x^{-1} \partial_y \mu_z^{-1} \partial_y\right) E_x^{n-\frac{1}{2}} \\ &+ \Delta t \epsilon_x^{-1} \left(\partial_y H_z^n - \partial_z H_y^n - \frac{\sigma_x}{2} E_x^{n+\frac{1}{2}} - \frac{\sigma_x}{2} E_x^{n-\frac{1}{2}} \right) \end{aligned} \quad (4.4.3)$$

and after some manipulation, we arrive at

$$\begin{aligned} \left(1 - \frac{\Delta t^2}{4} \epsilon_x^{-1} \partial_y \mu_z^{-1} \partial_y + \frac{\Delta t}{2} \epsilon_x^{-1} \sigma_x\right) E_x^{n+\frac{1}{2}} &= \left(1 - \frac{\Delta t^2}{4} \epsilon_x^{-1} \partial_y \mu_z^{-1} \partial_y - \frac{\Delta t}{2} \epsilon_x^{-1} \sigma_x\right) E_x^{n-\frac{1}{2}} \\ &+ \Delta t \epsilon_x^{-1} \left(\partial_y H_z^n - \partial_z H_y^n \right) \end{aligned} \quad (4.4.4)$$

The update equations for the other field components of the leapfrog ADI-FDTD method for lossy media can be written down by permuting the subscript indices accordingly. Similar to the conventional explicit FDTD method, time-averaging of the conductivity term for the leapfrog ADI-FDTD method is second-order temporal accurate.

Figure 4.3: Simulation setup for PEC cavity.

Figure 4.4: Time-domain E_z field component computed using conventional explicit FDTD, ADI-FDTD and leapfrog ADI-FDTD method for various CFLN. (a) Relative permittivity ϵ_r is 2 and electric conductivity σ is 0 S/m. (b) Relative permittivity ϵ_r is 2 and electric conductivity σ is 0.02 S/m. The lossy nature of the medium is clearly depicted by the decaying E_z waveform.

4.4.1 Numerical Results

In this subsection, we validate the formulation of the leapfrog ADI-FDTD method for lossy media through numerical simulations. The simulation setup for a PEC cavity meshed with $50 \times 30 \times 9$ uniform cells of size 1 mm is shown in Fig. 4.3. It is excited by a line Gaussian pulse, extended from bottom to top passing through the cavity center as

$$J_z = e^{-\left(\frac{t-t_0}{\tau}\right)^2}, \quad \tau = 150 \text{ ps}, \quad t_0 = 3\tau \quad (4.4.5)$$

The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{v_{max}\sqrt{3}}$, where v_{max} is the maximum phase velocity of wave propagation in the medium. We also denote $CFLN = \frac{\Delta t}{\Delta t_{CFL}}$. The observation point is located at cell (15,15,5).

Fig. 4.4(a) plots the time-domain E_z field component computed using the conven-

tional explicit FDTD method, ADI-FDTD method and leapfrog ADI-FDTD method for a lossless cavity. The CFLN for the conventional explicit FDTD method is 1, while the CFLN for the ADI-FDTD method and leapfrog ADI-FDTD method is 8. It is filled with a lossless ($\sigma = 0$ S/m) medium of relative permittivity $\epsilon_r = 2$. It can be observed that the E_z field component of the leapfrog ADI-FDTD method resonates in a similar manner as the conventional explicit FDTD method and ADI-FDTD method.

Next, to ascertain the formulation of the leapfrog ADI-FDTD method for the lossy media, we chose the electric conductivity σ as 0.02 S/m, while retaining the same relative permittivity ($\epsilon_r = 2$). Fig. 4.4(b) shows the time-domain E_z field component computed using the conventional explicit FDTD method, ADI-FDTD method and leapfrog ADI-FDTD method for a lossy cavity. As before, the CFLN for the conventional explicit FDTD method is 1, while the CFLN for the ADI-FDTD method and leapfrog ADI-FDTD method is 8. The lossy nature of the medium is clearly depicted by the decaying E_z waveform.

Referring to Fig. 4.4, we can also observe that the E_z field component of the leapfrog ADI-FDTD method differs slightly from the conventional explicit FDTD method and ADI-FDTD method during the source excitation period at time instant $t = 0.5$ ns. This is due to the difference in source excitation scheme of the various FDTD methods. Fig. 4.5 plots the time-domain E_z field component computed using the conventional explicit FDTD method, ADI-FDTD method and leapfrog ADI-FDTD method for CFLN = 1. It is clear that the various methods agree very well for CFLN = 1, and the discrepancies due to the different source excitation schemes are not longer visible. As mentioned earlier, the leapfrog ADI-FDTD method is a fully

Figure 4.5: Time-domain E_z field component computed using conventional explicit FDTD, ADI-FDTD and leapfrog ADI-FDTD method for CFLN = 1. (a) Relative permittivity ϵ_r is 2 and electric conductivity σ is 0 S/m. (b) Relative permittivity ϵ_r is 2 and electric conductivity σ is 0.02 S/m.

implicit FDTD method. Therefore, the current source of the leapfrog ADI-FDTD method is excited implicitly at half time step.

4.5 Numerical Examples

In the following, we present some numerical applications of the leapfrog ADI-FDTD method by employing the formulations discussed in the preceding sections. The first example describes the effects of electromagnetic wave propagation in lossy media, whereas the second example employs the Mur absorbing boundary condition to depict the high-pass filtering effect of a hollow rectangular metallic waveguide.

4.5.1 Ground Detection of Improvised Explosive Device

For a practical example, we consider an improvised explosive device (IED) buried into a ground filled with soil. IED is a threat to humans as they may explode and

incur injuries and even death to surrounding personnels. High power microwaves (HPM) may be employed to neutralize such threats by overloading the electronics of the the IED with high intensity electric fields [63]. Here, we examine the possible effects of such microwaves on an IED buried in soil.

The simulation setup for the IED buried in a ground filled with soil is shown in Fig. 4.6. The IED is modeled as a rectangular PEC box with $\sigma = 1\text{E}300$ S/m, and it is buried 5 cm below the surface of the soil. The relative permittivity ϵ_r of the soil is 4.4, while the loss tangent at 3 GHz is 0.046 [64]. Hence, the electric conductivity σ of the soil is 0.0338 S/m.

The source is excited in freespace by a sinusoidal modulated Gaussian pulse given by

$$J_z = \sin \left[2\pi f_0 (t - t_0) \right] e^{-\left(\frac{t-t_0}{\tau} \right)^2} \quad (4.5.1)$$

where $f_0 = 3$ GHz, $\tau = 0.5$ ns and $t_0 = 3\tau$. The computation domain has a dimension of 70 x 80 x 80 grids with uniform cell size of $\Delta = \Delta x = \Delta y = \Delta z = 2.5$ mm. It is truncated by a PML of ten cell thickness [57, 68].

Figure 4.6: Simulation setup for improvised explosive device (IED) buried in ground filled with soil.

Figure 4.7: (a) Received E_z fields component in freespace. (b) Received E_z field component in soil.

Fig. 4.7(a) presents the received E_z field component located in freespace at observation point A (51,40,60), while Fig. 4.7(b) shows the received E_z field component in the soil. It is located one cell above the IED at observation point B (35,40,36). The E_z field components in the two figures propagate almost the same distance away from the source, but in different media. Note the difference in scales between the two figures.

By comparing Figs. 4.7(a) and 4.7(b), it can be observed that the electric field component in the soil has lower magnitude than the electric field component in freespace. There are mainly two physical mechanism involved in this phenomena. Firstly, reflection occurs at the freespace-soil interface due to the difference in refractive index of the two media. Therefore, only a portion of the microwave would penetrate into the soil. Furthermore, such reflection is a function of incidence angles. For safety and practical reasons, the distance between the HPM source and the IED is usually quite large. Consequently, the incidence angle is large and reflection at the material interface is usually not negligible.

Figure 4.8: Magnitude of electric current density induced on the surface of the IED. (a) Conventional explicit FDTD method for CFLN = 1. (b) ADI-FDTD method for CFLN = 4. (c) Leapfrog ADI-FDTD method for CFLN = 4.

Secondly, the microwaves that penetrate into the soil are further attenuated due to the lossy nature of soil. Along the propagation in the lossy soil, power is absorbed by the soil and not by the IED. Note the IED is only buried quite shallow in the soil in this example. Thus, HPM systems might be more effective in neutralizing IED threats in freespace, rather than IED buried in soil or in general lossy media.

Fig. 4.8 shows the magnitude of the electric current density induced on the sur-

Table 4.1: Comparison of CPU time (seconds) and memory (MB) for the conventional explicit FDTD method, ADI-FDTD method and leapfrog ADI-FDTD method for ground detection of IED

Method	CFLN 1	CFLN 4	Memory (MB)
Conventional explicit FDTD	325	-	190
ADI-FDTD	898	225	217
Leapfrog ADI-FDTD	783	189	205

face of the IED for the conventional explicit FDTD method, ADI-FDTD method and leapfrog ADI-FDTD method. Fairly good agreement can be seen for the various methods. The maximum surface electric current density for the conventional explicit FDTD method, ADI-FDTD method and leapfrog ADI-FDTD method is $5.6\text{E-}3$ A/m, $6.0\text{E-}3$ A/m and $6.2\text{E-}3$ A/m, respectively.

Table 4.1 compares the CPU time (seconds) and memory (MB) for the conventional explicit FDTD method, ADI-FDTD method and leapfrog ADI-FDTD method for the ground detection of the IED. The simulations were performed using MATLAB. For $\text{CFLN} = 4$, the leapfrog ADI-FDTD method is more efficient than the ADI-FDTD method. When compared with the conventional explicit FDTD method, the leapfrog ADI-FDTD method requires 41% less time while the ADI-FDTD method requires 30% less time.

4.5.2 WR-62 Rectangular Waveguide

Transmission lines guide energy from one location to another location via the propagation of electromagnetic waves. The hollow rectangular metallic waveguide is one such transmission line that has high-power handling capability due to its large di-

mensions.

In this example, we employ the Mur absorbing boundary condition to terminate the ends of a WR-62 hollow rectangular metallic waveguide. The WR-62 rectangular waveguide operates in the Ku band from 12.4 to 18.0 GHz, and have a TE_{10} mode cutoff frequency of 9.486 GHz [65]. It does not support the TEM mode as it only has a single conductor. The rectangular waveguide is modeled by four PEC side-walls implemented using the image theory, while the two remaining walls at the opposite ends of the waveguide are terminated with the Mur ABC. It functions as the matched termination for the waveguide, and enables electromagnetic waves to propagate to an infinite extend beyond the waveguide.

The length of the rectangular waveguide is 101.6 mm. Its inner dimensions (width and height) is 15.8 mm by 7.9 mm. The computation domain consists of $400 \times 40 \times 10$ grids with uniform cell size of $\Delta x = 0.254$ mm, $\Delta y = 0.395$ mm and $\Delta z = 0.790$ mm. The Courant limit time step size is $\Delta t_{CFL} = \frac{1}{c\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}}$ where c is the speed of light in freespace and $CFLN = \frac{\Delta t}{\Delta t_{CFL}}$. A dominant TE_{10} mode is launched at the source plane located in the middle of the rectangular waveguide. The time-domain response of the hard-source excitation is a two-tone ramped sinusoidal waveform. The frequency of the first-tone F_1 is 3 GHz, while the frequency of the second-tone F_2 is 15 GHz.

Figs. 4.9 and 4.10 depict the field distribution of the rectangular waveguide for the leapfrog ADI-FDTD method with $CFLN = 1$ and 4, respectively. From the figures, it can be observed that the propagating mode is the dominant TE_{10} mode.

Figure 4.9: Field distribution of the rectangular waveguide TE_{10} mode for leapfrog ADI-FDTD method with $CFLN = 1$.

Figure 4.10: Field distribution of the rectangular waveguide TE_{10} mode for leapfrog ADI-FDTD method with $CFLN = 4$.

Fig. 4.11 plots the magnitude of E_z (dB) field component vs. frequency at the source plane for the conventional explicit FDTD method and leapfrog ADI-FDTD method. The CFLN for the leapfrog ADI-FDTD method is selected as 1 and 4, while the CFLN for the conventional explicit FDTD method is fixed at 1. Good agreement is obtain between the various methods. At the source plane, F_1 and F_2 can be clearly seen from the frequency-domain response.

The magnitude of E_z (dB) field component vs. frequency at the observation plane

Figure 4.11: Magnitude of E_z (dB) field component vs. frequency at the source plane for the conventional explicit FDTD method and leapfrog ADI-FDTD method.

Figure 4.12: Magnitude of E_z (dB) field component vs. frequency at the observation plane for the conventional explicit FDTD method and leapfrog ADI-FDTD method. It is located at the end of the rectangular waveguide near to the absorbing boundary.

for the conventional explicit FDTD method and leapfrog ADI-FDTD method is shown in Fig. 4.12. It is located at the end of the rectangular waveguide near to the absorbing boundary. Referring to the figure, the frequency-domain response shows that only F_2 remains while F_1 is attenuated. In other words, F_2 is the propagation mode, while F_1 is the evanescent mode, below the cutoff frequency where propagation is not possible. A hollow rectangular metallic waveguide is a high-pass filter, and thus it attenuates frequencies below its cutoff frequency. The results of the above simulations agrees with this well-known phenomena.

Table 4.2: Comparison of frequency (GHz) for the propagating mode of the WR-62 rectangular waveguide for the conventional explicit FDTD method and leapfrog ADI-FDTD method.

Method	CFLN 1	CFLN 4
Conventional explicit FDTD	15.029	-
Leapfrog ADI-FDTD	15.008	14.998
% Error	0.139	0.206

Moreover, there are some discrepancies among the various methods below the cut-off frequency of the WR-62 rectangular waveguide. This could be due to the Mur ABC been formulated to absorb plane waves, and thus it is unable to properly absorb evanescent waves of the dispersive waveguide [3]. Furthermore, the hard-source excitation of the leapfrog ADI-FDTD method for the rectangular waveguide have higher asymmetrical errors [66].

Next, we proceed to determine the error in the propagation mode of the WR-62 rectangular waveguide. Table 4.2 compares the frequency (GHz) for the propagation mode of the WR-62 rectangular waveguide for the conventional explicit FDTD method and leapfrog ADI-FDTD method. The error of the propagation mode of the WR-62 rectangular waveguide is still within reasonable limits. Therefore, the leapfrog ADI-FDTD method with Mur ABC is useful for quick analysis when properly employed.

4.6 GPU-Accelerated Leapfrog ADI-FDTD Method

4.6.1 Introduction

Graphic processing units (GPU) have recently been implemented for the unconditionally stable FDTD methods to further enhance their efficiencies [36, 67, 68]. Fig. 4.13 illustrates the typical architecture of the CPU and GPU. In general, GPUs have more cores than CPUs, and therefore they are more suitable for highly-parallel computational intensive applications such as the FDTD method [69].

Here, we further accelerate the leapfrog ADI-FDTD method by implementing it on a low-cost GPU. Numerical results will be presented to compare the efficiency and accuracy of the leapfrog ADI-FDTD method implemented on the GPU and CPU.

Figure 4.13: Typical architecture of the CPU and GPU. The GPU has more cores than the CPU. ©2013 IEEE.

4.6.2 GPU-Accelerated Leapfrog ADI-FDTD Method

Here, we consider wave propagation in a medium with permittivity ϵ and permeability μ . The time-domain implicit update equations for the leapfrog ADI-FDTD method in the x -direction are given as

$$E_x^{n+\frac{1}{2}} = E_x^{n-\frac{1}{2}} + \Delta t \hat{\epsilon}_x^{-1} \left(\partial_y H_z^n - \partial_z H_y^n \right) \quad (4.6.1a)$$

$$H_x^{n+1} = H_x^n + \Delta t \hat{\mu}_x^{-1} \left(\partial_z E_y^{n+\frac{1}{2}} - \partial_y E_z^{n+\frac{1}{2}} \right) \quad (4.6.1b)$$

where

$$\hat{\epsilon}_x = \epsilon_x - \frac{\Delta t^2}{4} \partial_y \mu_z^{-1} \partial_y \quad (4.6.1c)$$

$$\hat{\mu}_x = \mu_x - \frac{\Delta t^2}{4} \partial_y \epsilon_z^{-1} \partial_y \quad (4.6.1d)$$

The equations for other field components can be found in section 4.4. Referring to (4.6.1), it can be observed that the E_x field components are solved implicitly in the y -direction, and explicitly in x and z directions, respectively. Therefore, it is possible to parallelize E_x along the x and z directions by exploiting the multi-dimensional blocks and threads of the GPU. Furthermore, it is also possible to solve the E_x , E_y and E_z field components of the leapfrog ADI-FDTD method concurrently. This is because the electric fields update equations are independent of each other.

As mentioned above, the E_x field components of the leapfrog ADI-FDTD method are computed implicitly in the y -direction. To efficiently solve the tridiagonal implicit matrix equation, the Thomas algorithm is usually employed [70]. It factorizes the tridiagonal implicit matrix into the bidiagonal form, and subsequently solves the equation by using the forward and backward substitution procedures. However,

Table 4.3: Efficiency gain of the Leapfrog ADI-FDTD Method implemented on the GPU vs. CPU. ©2013 IEEE.

Domain Size	CPU timing (seconds)	GPU timing (seconds)	CPU/GPU
50 x 50 x 50	9.205	4.078	2.25
100 x 100 x 100	95.264	32.917	2.89
200 x 200 x 200	815.960	282.387	2.88

implementing the above procedure for the leapfrog ADI-FDTD method would require an additional intermediate solution. This intermediate solution has the same memory requirement as the field variable. This is in contrast with the fundamental ADI-FDTD method [14], where the intermediate solution of the Thomas algorithm may be reused, and hence does not require any additional memory. Therefore, the memory required for the GPU-accelerated leapfrog ADI-FDTD method may be comparable to the fundamental ADI-FDTD method. Moreover, the fundamental ADI-FDTD method only requires 42 floating point operations count [14], which is less than the leapfrog ADI-FDTD method, i.e., 66 floating point operations count.

4.6.3 Efficiency and Accuracy

In this subsection, we perform some numerical simulations to compare the efficiency and accuracy of the leapfrog ADI-FDTD method implemented on the GPU and CPU. The CPU platform is an Intel Xeon processor with clock speed of 2.4 GHz, while the GPU platform is a low-cost Nvidia Quadro 600 graphic card with 1 GB RAM and 96 CUDA parallel processing cores. The programs are compiled using Visual C++ and CUDA 4.0 [69] under Microsoft Windows 7 operating system.

We consider a perfect electric conductor cavity meshed with uniform cells. The size

of the simulation domain is chosen as 50^3 , 100^3 and 200^3 . Table 4.3 lists the efficiency gain of the leapfrog ADI-FDTD method implemented on the GPU vs. CPU. It can be observed that the leapfrog ADI-FDTD method implemented on the GPU is approximately 2 to 3 times faster than that implemented on the CPU.

Next, we proceed to determine the accuracy of the leapfrog ADI-FDTD method implemented on GPU by comparing with the conventional explicit FDTD method implemented on CPU. For our example, the dimension of the simulation domain is 100^3 with uniform cell size of $\Delta = \Delta x = \Delta y = \Delta z = 2$ mm. It is truncated by a perfectly matched layer of ten cell thickness [57]. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{3}}$, where c is the speed of light in freespace. We also denote CFLN = $\frac{\Delta t}{\Delta t_{CFL}}$.

The source is excited in freespace by a sinusoidal modulated Gaussian pulse given by

$$J_z = \sin \left[2\pi f_0 (t - t_0) \right] e^{-\left(\frac{t-t_0}{\tau} \right)^2} \quad (4.6.2)$$

where $f_0 = 3$ GHz, $\tau = 0.5$ ns and $t_0 = 3\tau$. The observation point is located at cell (75,75,75).

Fig. 4.14 plots the time-domain E_z field component computed using the conventional explicit FDTD method (CPU) and the leapfrog ADI-FDTD method (GPU). The CFLN for the leapfrog ADI-FDTD method is chosen as 1 and 4, while the CFLN for the conventional explicit FDTD method is fixed at 1. Good agreement can be observed for leapfrog ADI-FDTD method implemented on the GPU when comparing with the conventional explicit FDTD method implemented on the CPU.

Figure 4.14: Time-domain E_z field component computed using the conventional explicit FDTD method (CPU) and the leapfrog ADI-FDTD method (GPU) at cell (75,75,75). The CFLN for the leapfrog ADI-FDTD method is 1 and 4. ©2013 IEEE.

4.7 Conclusion

This chapter has extended the leapfrog ADI-FDTD method to include the Mur ABC, perfect electric conductor boundary condition and lossy media. Conductivity terms of lossy media have been incorporated into the leapfrog ADI-FDTD method in an analogous manner as the conventional explicit FDTD method since the leapfrog ADI-FDTD method is a perturbation of the conventional explicit FDTD method. Implementation of the leapfrog ADI-FDTD method for lossy media with special consideration for boundary condition has been provided.

In addition, the formulation of the Mur ABC for the TE_z leapfrog ADI-FDTD method has been presented. The first-order Mur updating equation is implemented for both the electric and magnetic field updates. Numerical results for the reflection errors and late-time stability was presented to validate the implementation of the Mur ABC for the leapfrog ADI-FDTD method.

Finally, this chapter has also presented the GPU-accelerated leapfrog ADI-FDTD method. Numerical results have been presented to compare the efficiency and accuracy of the leapfrog ADI-FDTD method implemented on the GPU and CPU.

The next chapter will be devoted to practical applications of the leapfrog ADI-FDTD method for electromagnetic compatibility modeling.

Chapter 5

Applications of Leapfrog ADI-FDTD Method for Electromagnetic Compatibility Modeling

5.1 Introduction

In this chapter, we illustrate applications of the leapfrog ADI-FDTD method for electromagnetic compatibility modeling. Mutual coupling between antennas and shielding of electronic devices are some important aspects of electromagnetic compatibility analysis which is of growing interest lately [71, 72].

Nowadays, it is increasingly common to find several antenna systems serving different applications on a single compact platform. Unfortunately, on such a complex

platform, one antenna system may interfere with another antenna system, and thus disrupt its intended function. Therefore, the study of how an antenna behaves when placed in the vicinity of another antenna, i.e., mutual coupling, is important to ensure that radio-frequency interference is kept to a minimum. As such, we will investigate the mutual coupling between two monopole antennas of different length mounted on a finite ground plane for the first application example.

Shielding is commonly employed to protect important electronic devices from unwanted/unintended radiation. This enables the electronic devices to function properly even in a harsh radio-frequency environment. Meanwhile, electronic devices are getting more complex and have higher clock speed. Consequently, they may generate more noise. Hence, it is also important that such electronic devices do not pollute the radio-frequency environment with stray emission by means of proper shielding. For the next application example, we implement the leapfrog ADI-FDTD method to investigate the field leakage of a non-penetrable target, i.e., shielded enclosure.

5.2 Mutual Coupling between Monopole Antennas

As mentioned before, antennas when placed in close proximity of one another may interfere with the proper operations of the antenna systems. In particular, the mutual coupling between antennas, which is the amount of energy one antenna picks up when the other antenna is transmitting, is an important parameter for the analysis of electromagnetic compatibility. However, mutual coupling between antennas is usually difficult to analyze analytically [73], and more often than not, numerical methods such as the FDTD method is preferred [74].

To analyze the mutual coupling between antennas, the scattering parameters of the network must first be determined. For the leapfrog ADI-FDTD method, the matched load terminations for the ports of the scattering matrix can be obtained via the resistor voltage source. In the section to follow, we will cover the resistive voltage source for the leapfrog ADI-FDTD method.

5.2.1 Resistive Voltage Source for Leapfrog ADI-FDTD Method

We begin this subsection by casting the leapfrog ADI-FDTD method with resistive voltage source into the form similar to the conventional explicit FDTD method. First, we review the leapfrog ADI-FDTD method for lossy media employing the time-averaging scheme with permittivity ϵ , permeability μ , electric conductivity σ and magnetic conductivity σ^* . The time-domain implicit update equations can be expressed compactly as follows:

$$E_x^{n+\frac{1}{2}} = \tilde{\epsilon}_x E_x^{n-\frac{1}{2}} + \Delta t \hat{\epsilon}_x^{-1} \left(\partial_y H_z^n - \partial_z H_y^n \right) \quad (5.2.1a)$$

$$E_y^{n+\frac{1}{2}} = \tilde{\epsilon}_y E_y^{n-\frac{1}{2}} + \Delta t \hat{\epsilon}_y^{-1} \left(\partial_z H_x^n - \partial_x H_z^n \right) \quad (5.2.1b)$$

$$E_z^{n+\frac{1}{2}} = \tilde{\epsilon}_z E_z^{n-\frac{1}{2}} + \Delta t \hat{\epsilon}_z^{-1} \left(\partial_x H_y^n - \partial_y H_x^n \right) \quad (5.2.1c)$$

$$H_x^{n+1} = \tilde{\mu}_x H_x^n + \Delta t \hat{\mu}_x^{-1} \left(\partial_z E_y^{n+\frac{1}{2}} - \partial_y E_z^{n+\frac{1}{2}} \right) \quad (5.2.2a)$$

$$H_y^{n+1} = \tilde{\mu}_y H_y^n + \Delta t \hat{\mu}_y^{-1} \left(\partial_x E_z^{n+\frac{1}{2}} - \partial_z E_x^{n+\frac{1}{2}} \right) \quad (5.2.2b)$$

$$H_z^{n+1} = \tilde{\mu}_z H_z^n + \Delta t \hat{\mu}_z^{-1} \left(\partial_y E_x^{n+\frac{1}{2}} - \partial_x E_y^{n+\frac{1}{2}} \right) \quad (5.2.2c)$$

where the coefficients are defined as

$$\hat{\epsilon}_x \equiv \left(\epsilon_x - \frac{\Delta t^2}{4} \partial_y \mu_z^{-1} \partial_y + \frac{\Delta t}{2} \sigma_x \right) \quad (5.2.3a)$$

$$\hat{\epsilon}_y \equiv \left(\epsilon_y - \frac{\Delta t^2}{4} \partial_z \mu_x^{-1} \partial_z + \frac{\Delta t}{2} \sigma_y \right) \quad (5.2.3b)$$

$$\hat{\epsilon}_z \equiv \left(\epsilon_z - \frac{\Delta t^2}{4} \partial_x \mu_y^{-1} \partial_x + \frac{\Delta t}{2} \sigma_z \right) \quad (5.2.3c)$$

$$\hat{\mu}_x \equiv \left(\mu_x - \frac{\Delta t^2}{4} \partial_y \epsilon_z^{-1} \partial_y + \frac{\Delta t}{2} \sigma_x^* \right) \quad (5.2.3d)$$

$$\hat{\mu}_y \equiv \left(\mu_y - \frac{\Delta t^2}{4} \partial_z \epsilon_x^{-1} \partial_z + \frac{\Delta t}{2} \sigma_y^* \right) \quad (5.2.3e)$$

$$\hat{\mu}_z \equiv \left(\mu_z - \frac{\Delta t^2}{4} \partial_x \epsilon_y^{-1} \partial_x + \frac{\Delta t}{2} \sigma_z^* \right) \quad (5.2.3f)$$

and

$$\tilde{\epsilon}_x \equiv \left(\epsilon_x - \frac{\Delta t^2}{4} \partial_y \mu_z^{-1} \partial_y - \frac{\Delta t}{2} \sigma_x \right) / \hat{\epsilon}_x \quad (5.2.4a)$$

$$\tilde{\epsilon}_y \equiv \left(\epsilon_y - \frac{\Delta t^2}{4} \partial_z \mu_x^{-1} \partial_z - \frac{\Delta t}{2} \sigma_y \right) / \hat{\epsilon}_y \quad (5.2.4b)$$

$$\tilde{\epsilon}_z \equiv \left(\epsilon_z - \frac{\Delta t^2}{4} \partial_x \mu_y^{-1} \partial_x - \frac{\Delta t}{2} \sigma_z \right) / \hat{\epsilon}_z \quad (5.2.4c)$$

$$\tilde{\mu}_x \equiv \left(\mu_x - \frac{\Delta t^2}{4} \partial_y \epsilon_z^{-1} \partial_y - \frac{\Delta t}{2} \sigma_x^* \right) / \hat{\mu}_x \quad (5.2.4d)$$

$$\tilde{\mu}_y \equiv \left(\mu_y - \frac{\Delta t^2}{4} \partial_z \epsilon_x^{-1} \partial_z - \frac{\Delta t}{2} \sigma_y^* \right) / \hat{\mu}_y \quad (5.2.4e)$$

$$\tilde{\mu}_z \equiv \left(\mu_z - \frac{\Delta t^2}{4} \partial_x \epsilon_y^{-1} \partial_x - \frac{\Delta t}{2} \sigma_z^* \right) / \hat{\mu}_z \quad (5.2.4f)$$

Notice that the update equation of the leapfrog ADI-FDTD method is similar to the conventional explicit FDTD method except for the re-definition of ϵ as $\hat{\epsilon}$ and $\tilde{\epsilon}$, and μ as $\hat{\mu}$ and $\tilde{\mu}$.

Moreover, the leapfrog ADI-FDTD method for lossy media can be derived from the conventional forward-backward ADI-FDTD method for lossy media. Unfortunately, previous studies on the forward-backward ADI-FDTD method reveal that it has non-physical field leakage when modeling non-penetrable targets, i.e., perfect electric conductor or perfect magnetic conductor [75]. Consequently, the accuracy of

electromagnetic waves within such non-penetrable targets may be compromised as the fields leak into the target in a non-physical manner. In the later portion of this chapter, we shall investigate the field leakage of the leapfrog ADI-FDTD method for non-penetrable targets.

For now, we continue with our discussion on the resistive voltage source for the leapfrog ADI-FDTD method. Referring to the equations above for the leapfrog ADI-FDTD method for lossy media, we can incorporate the lumped resistor element in the z -direction with an effective electric conductivity $\sigma_z = \frac{\Delta_z}{\Delta_x \Delta_y R_z}$.

Similarly, for the z -directed resistive voltage source, we incorporate the source in the update equation as follows:

$$E_z^{n+\frac{1}{2}} = \tilde{\epsilon}_z E_z^{n-\frac{1}{2}} + \Delta t \hat{\epsilon}_z^{-1} \left(\partial_x H_y^n - \partial_y H_x^n - J_z^n |_{\text{RVS}} \right) \quad (5.2.5)$$

where

$$J_z^n |_{\text{RVS}} = \frac{V_s^n}{\Delta_x \Delta_y R_s} \quad (5.2.6)$$

and V_s is the source voltage excitation, while R_s is the source resistance. Here, subscript “RVS” denotes resistive voltage source. The formulations of the leapfrog ADI-FDTD method with resistive voltage source for the other directions can be obtained in a similar manner. In addition, the leapfrog ADI-FDTD method including lumped elements can be found in [76] and will not be repeated here.

Figure 5.1: Geometry of monopole antennas mounted on ground plane.

5.2.2 Investigation of Mutual Coupling between Monopole Antennas

In the following, we investigate the mutual coupling between two monopole antennas of different length mounted on a finite ground plane. Similar analysis has been studied previously in [77]. The resistive voltage source formulation discussed in the previous section will be applied to make possible the extraction of the scattering parameters of the monopole antennas.

The dimension of the computation domain is $184 \times 136 \times 80$ grids with uniform cell size of $\Delta = \Delta x = \Delta y = \Delta z = 4.25$ mm. It is further truncated by a ten cell perfectly matched layer [57,68]. In all numerical simulations, the PML parameters are defined as $\sigma_{\zeta_{\max}} = \sigma_{\text{opt}} = 10.61$ S/m, $\kappa_{\zeta_{\max}} = 15$, $\alpha_{\zeta} = 0.02$ and $m = 3$. The Courant limit time step size is $\Delta t_{\text{CFL}} = \frac{\Delta}{c\sqrt{3}}$ where c is the speed of light in freespace. We also denote $\text{CFLN} = \frac{\Delta t}{\Delta t_{\text{CFL}}}$.

The monopole antennas are fed by a resistive voltage source with source impedance

of 50Ω located at the gap-feed between the monopole antenna and the ground plane. The source excitation is a wide-band Gaussian pulse

$$J_z = e^{-\left(\frac{t-t_0}{\tau}\right)^2} \quad (5.2.7)$$

where $\tau = 14$ ps and $t_0 = 3\tau$. A single run of the various FDTD methods was performed to extract the broad-band frequency-domain results.

The geometry of the monopole antennas mounted on the ground plane is depicted in Fig. 5.1. The configuration consists of two monopole antennas with different lengths, and hence different operating frequencies. The length of shorter monopole antenna is 76.5 mm, while the length of longer monopole antenna is 127.5 mm. Thus, the operating frequency of the shorter monopole antenna is 980 MHz, whereas the operating frequency of the longer monopole antenna is 558 MHz. The size of the finite ground plane is 612 mm by 408 mm. The shorter monopole antenna is mounted 153 mm and 102 mm away from the lower edge, while the longer monopole antenna is mounted 153 mm and 204 mm away from the opposite edge.

5.2.3 Numerical Results

In this subsection, we investigate the mutual coupling between the two monopole antennas by analyzing its scattering parameters. First, we determine the S_{11} of the shorter monopole antenna, followed by the S_{12} of the shorter monopole antenna in order to determine the mutual coupling (isolation) between the antennas. As illustrated in Fig. 5.1, the two monopole antennas have different lengths and are not identical. Thus, the scattering matrix is not symmetrical, i.e., $S_{11} \neq S_{22}$. In addition, the monopole antennas are passive, and therefore, the scattering matrix is

Figure 5.2: $|S_{11}|$ (dB) of the shorter monopole antenna computed using the conventional explicit FDTD method, FADI-FDTD method, leapfrog ADI-FDTD method, and HFSS. The CFLN of the FDTD methods is 1.

reciprocal, i.e., $S_{12} = S_{21}$.

For the purpose of evaluating the scattering matrix, we denote the shorter monopole antenna as port 1 and the longer monopole antenna as port 2. The port impedance of the resistive voltage source for both antennas is defined as 50Ω . The S_{11} is determined by employing port 1 of the shorter monopole antenna as both the transmitting as well as receiving port. On the other hand, the S_{12} is obtained by exciting port 1 of the shorter monopole antenna as the transmitting port, and employing port 2 of the longer monopole antenna as the receiving port. When port 1 is transmitting, port 2 is terminated with a matched load of 50Ω to avoid reflections. Thus, the resistive voltage source at port 2 has no voltage excitation, and it is effectively a lumped resistor element.

Fig. 5.2 plots the $|S_{11}|$ (dB) of the shorter monopole antenna computed using the conventional explicit FDTD method, FADI-FDTD method, leapfrog ADI-FDTD method and HFSS. The CFLN for the FDTD methods is 1. Good agreement can be

Figure 5.3: $|S_{11}|$ (dB) of the shorter monopole antenna computed using the conventional explicit FDTD method, FADI-FDTD method and leapfrog ADI-FDTD method. The CFLN for the FADI-FDTD method and leapfrog ADI-FDTD method is 4.

observed between the conventional explicit FDTD method, FADI-FDTD method, leapfrog ADI-FDTD method and HFSS. In addition, the shorter monopole antenna resonants near the predicted operating frequency (980 MHz), while commercial software package ANSYS HFSS provides an independent verification of the FDTD simulated results.

Fig. 5.3 presents the $|S_{11}|$ (dB) of the shorter monopole antenna computed using the conventional explicit FDTD method, FADI-FDTD method and leapfrog ADI-FDTD method. The CFLN for the conventional explicit FDTD method is 1, while the CFLN for the FADI-FDTD method and leapfrog ADI-FDTD method is 4. With the increase in CFLN, the agreement of the FADI-FDTD method and leapfrog ADI-FDTD method with the conventional explicit FDTD method degrades. Referring to Fig. 5.3, the error of the leapfrog ADI-FDTD method is noticeable especially for higher frequencies. For the leapfrog ADI-FDTD method, the first resonant frequency of the shorter monopole antenna is still acceptable within reasonable limits. However, it is not true for the third resonant frequency of the shorter monopole

Figure 5.4: $|S_{12}|$ (dB) of the short monopole antenna computed using the conventional explicit FDTD method, FADI-FDTD method, leapfrog ADI-FDTD method and HFSS. The CFLN of the FDTD methods is 1.

antenna for the leapfrog ADI-FDTD method, as it has already degraded beyond acceptable limits. There is no longer a distinct resonant around the frequency of interest. On the other hand, the third resonant frequency of the shorter monopole antenna for the FADI-FDTD method is clearly visible. It is merely frequency-shifted from the predicted value of the conventional explicit FDTD method.

Fig. 5.4 presents the $|S_{12}|$ (dB) of the shorter monopole antenna computed using the conventional explicit FDTD method, FADI-FDTD method, leapfrog ADI-FDTD method and HFSS. The CFLN for the FDTD methods is 1. The mutual coupling between the monopole antennas is the strongest at around 800 MHz to 1 GHz, and drops to below -30 dB for most part of the frequency spectrum. Thus, there is good isolation between the two monopole antennas, and electromagnetic compatibility is not a concern for the configuration. The monopole antennas are able to coexist and function properly without causing interference to one another.

Fig. 5.5 shows the $|S_{12}|$ (dB) of the shorter monopole antenna computed using

Figure 5.5: $|S_{12}|$ (dB) of the short monopole antenna computed using the conventional explicit FDTD method, FADI-FDTD method and leapfrog ADI-FDTD method. The CFLN for the FADI-FDTD method and leapfrog ADI-FDTD method is 4.

Table 5.1: Comparison of CPU time (seconds) and memory (MB) for the conventional explicit FDTD method, FADI-FDTD method and leapfrog ADI-FDTD method for analysis of mutual coupling between two monopole antennas

Method	CFLN 1	CFLN 4	Memory (MB)
Conventional explicit FDTD	3318	-	332
FADI-FDTD	9954	2489	422
Leapfrog ADI-FDTD	9041	2223	362

the conventional explicit FDTD method, FADI-FDTD method and leapfrog ADI-FDTD method. The CFLN for the unconditionally stable FDTD methods is 4. The leapfrog ADI-FDTD method has considerable errors for high CFLN at higher frequencies. Beyond 3 GHz, the response of the leapfrog ADI-FDTD method is clearly not within acceptable limits. This is because the computation domain is spatially under-sampled for high CFLN.

Table 5.1 compares the CPU time (seconds) and memory (MB) for the conventional explicit FDTD method, FADI-FDTD method and leapfrog ADI-FDTD method for

the analysis of mutual coupling between the two monopole antennas. For CFLN = 4, the ADI-FDTD method requires 24% less time than the conventional explicit FDTD method, while the leapfrog ADI-FDTD method requires 32% less time than the conventional explicit FDTD method.

5.3 Field Leakage of Shielded Enclosure

The leapfrog ADI-FDTD method has been applied to efficiently analyze scattered waves in lossy media [78] as well as wave propagation in electromagnetic bandgap structures [32]. These analyses, and many others, usually involve targets and/or materials that allow electromagnetic waves to either penetrate or propagate.

However, analyses of the leapfrog ADI-FDTD method on non-penetrable targets, i.e., shielded enclosure, which are often of interest for electromagnetic compatibility problems, have not been performed. Such analysis is necessary and important as it has been previously determined that non-physical field leakage may occur for non-penetrable targets [75].

In the following sections, the field leakage of the leapfrog ADI-FDTD method for non-penetrable targets will be investigated. Next, we propose a simple approach to overcome the non-physical field leakage of the leapfrog ADI-FDTD method for non-penetrable targets. Finally, numerical results will be presented to validate the proposed approach.

5.3.1 Field Leakage of Leapfrog ADI-FDTD Method for Non-Penetrable Targets

In this subsection, we investigate the field leakage of the leapfrog ADI-FDTD method for non-penetrable targets via numerical simulations. For this purpose, we consider a hollow spherical conducting shell located at the center of the computation domain as shown in Fig. 5.6. The outer radius of the spherical shell is 25 mm, while the inner radius of the spherical shell is 10 mm. The non-penetrable conducting shell is modeled using perfect electric conductor. It is theoretically well-known that there should be no electromagnetic waves in the hollow freespace region of the spherical conducting shell as it is totally enclosed and shielded.

The simulation domain has a dimension of 100^3 with uniform cell size of $\Delta = \Delta x = \Delta y = \Delta z = 1$ mm. It is truncated by a perfectly matched layer of 10 cell thickness [57]. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{3}}$, where c is the speed of light in freespace. We also denote $CFLN = \frac{\Delta t}{\Delta t_{CFL}}$. The source is excited in freespace by a Gaussian pulse given by

$$J_z = J_0 e^{-\left(\frac{t-t_0}{\tau}\right)^2} \quad (5.3.1)$$

where $\tau = 60$ ps and $t_0 = 3\tau$. The signal amplitude J_0 is 1×10^{12} A/m².

For comparison purposes, the conventional explicit FDTD method is also simulated to determine its field leakage. Fig. 5.7 presents the snap-shot of the electric field at time instant $t = 0.962$ ns for the conventional explicit FDTD method and leapfrog ADI-FDTD method for $CFLN = 1$. As expected, no electric field leakage is observed in the hollow spherical conducting shell for the conventional explicit FDTD method.

Figure 5.6: Geometry of hollow spherical conducting shell. The outer radius is 25 mm and the inner radius is 15 mm.

Furthermore, the outer boundaries of the spherical shell can be clearly seen, while the inner hollow freespace region cannot be identified.

On the other hand, substantial amount of electric field leakage can be observed in the hollow spherical conducting shell for the leapfrog ADI-FDTD method. Electric fields within the conducting shell are non-zero and the inner hollow freespace region can be clearly identified. This is a non-physical phenomenon as electromagnetic waves should not be able to leak into a non-penetrable target.

5.3.2 Proposed Approach

To overcome the field leakage issue of the leapfrog ADI-FDTD method, we hereby propose a different approach for the leapfrog ADI-FDTD method for non-penetrable targets. Recall that Ampere's law for lossy media is given as [79]

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E} = j\omega\epsilon\left(1 - \frac{j\sigma}{\omega\epsilon}\right)\mathbf{E} \quad (5.3.2)$$

Figure 5.7: Snap-shot of electric field at time instant $t = 0.962$ ns. (a) Conventional explicit FDTD method for $CFLN = 1$. (b) Leapfrog ADI-FDTD method for $CFLN = 1$. Electric field leakage can be observed for the leapfrog ADI-FDTD method in the hollow spherical conducting shell.

where for good conductors σ is $\gg \omega\epsilon$. The ideal case of the good conductor is the perfect electric conductor, where the skin depth is zero. Thus, electromagnetic waves are not able to penetrate into the target and there is total reflection at the air-conductor interface.

Meanwhile, for perfect or lossless dielectric σ is $\ll \omega\epsilon$. When ϵ approaches ∞ , the intrinsic impedance of a perfect dielectric $\eta = \sqrt{\frac{\mu}{\epsilon}}$ approaches zero. The reflection coefficient at the air-dielectric interface is

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0 - 377}{0 + 377} = -1 \quad (5.3.3)$$

Thus, the reflection coefficient (magnitude and phase) of the perfect dielectric at the air-dielectric interface is the same as that of the perfect electric conductor at the air-conductor interface. This suggests that the perfect dielectric may be regarded as a good substitute to the perfect electric conductor for non-penetrable targets. It

should be emphasized that although the basic theory of the two approaches is different, they achieve the same objective, i.e., total reflection at the material interface.

5.3.3 Numerical Validation

To validate the proposed approach, we perform numerical simulations with the hollow spherical conducting shell as before. For convenience, we designate the proposed approach as the infinite- ϵ method and the previous approach as the infinite- σ method.

Fig. 5.8 illustrates the snap-shot of the electric field at time instant $t = 0.962$ ns for the infinite- ϵ leapfrog ADI-FDTD method. Here, the CFLN is increased to 4 in order to exploit its unconditionally stable advantage. It is clear that there is no electric field leakage in the hollow spherical conducting shell for the infinite- ϵ leapfrog ADI-FDTD method. The outer boundaries of the spherical shell can be clearly seen, while the inner hollow freespace region cannot be identified. This phenomenon is similar to the conventional explicit FDTD method as shown previously.

The time-domain E_z field component computed using the conventional explicit FDTD method, infinite- σ and infinite- ϵ leapfrog ADI-FDTD methods at various observation points is shown in Fig. 5.9. The observation points are located in freespace at the front and back of the hollow spherical conducting shell. Good agreement can be observed for the various methods. This indicates that both approaches of the leapfrog ADI-FDTD method for non-penetrable targets produces comparable results for scattered fields as there is total reflection at the material interface.

Figure 5.8: Snap-shot of electric field at time instant $t = 0.962$ ns for the infinite- ϵ leapfrog ADI-FDTD method for CFLN = 4. No electric field leakage is observed in the hollow spherical conducting shell for the proposed approach.

Next, we further analyze the field leakage by examining the electric fields within the hollow spherical conducting shell. Fig. 5.10 compares the time-domain E_z field component computed using the infinite- σ leapfrog ADI-FDTD method and the infinite- ϵ leapfrog ADI-FDTD method at various observation points. The first observation point is located one-cell within the surface of the hollow spherical conducting shell, while the second observation point is located in freespace in the middle of the hollow spherical conducting shell. The conventional explicit FDTD method is not included here as the electric field is zero.

Referring to Fig. 5.10(a), it can be seen that the infinite- σ leapfrog ADI-FDTD method exhibits electric field leakage. Moreover, as shown in Fig. 5.10(b), the electric field component of the infinite- σ leapfrog ADI-FDTD method appears to resonant and its electric field strength is actually larger than that at the first observation point. This is because the hollow spherical conducting shell is a spherical cavity.

Figure 5.9: Time-domain E_z field component computed using the conventional explicit FDTD method, infinite- σ and infinite- ϵ leapfrog ADI-FDTD methods at various observation points. (a) Electric field recorded at the front of the spherical shell at cell (20,50,50). (b) Electric field recorded at the back of the spherical shell at cell (80,50,50). Good agreements can be observed for the various methods.

While it may not be obvious in Fig. 5.10(a), it is worthwhile to note that the E_z field component of the infinite- σ leapfrog ADI-FDTD method still has some extremely-small field leakage one-cell within the surface of the hollow spherical conducting shell. To gain an insight into the root cause of this residual field leakage, we analyze the update equation of the infinite- σ leapfrog ADI-FDTD method in the following.

If we define ϵ in (5.2.1)-(5.2.4) as ∞ , the update equation for E_x becomes

$$E_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} = E_x|_{i+\frac{1}{2},j,k}^{n-\frac{1}{2}} \quad (5.3.4)$$

Equation (5.3.4) states simply that for a perfect dielectric with extremely-large ϵ , the electric field computed using the infinite- ϵ leapfrog ADI-FDTD method at the current time step is the same as the previous time step.

However, we should also be mindful that the leapfrog ADI-FDTD method involves

Figure 5.10: Time-domain E_z field component computed using the infinite- σ leapfrog ADI-FDTD method and infinite- ϵ leapfrog ADI-FDTD method at various observation points. (a) Electric field recorded one-cell within the surface of the spherical shell at cell (26,50,50). (b) Electric field recorded in freespace in the middle of the spherical shell at cell (50,50,50).

solving a tridiagonal system, which has a larger spatial stencil than the conventional explicit FDTD method. Thus, it is also necessary to consider one-cell before the material interface, where the standard update equations are employed. Consequently, the upper term of the tridiagonal system one-cell before the material interface will also update the electric field component within the material accordingly. Furthermore, there is no condition enforcing the electric fields within the non-penetrable target to be zero.

Fortunately, the electric field leakage does not propagate beyond one-cell into the material. Due to (5.3.4), the electric fields of the inner cells are not modified and will remain as zero throughout. Thus, an *equivalent* perfect electric conductor is obtained from this cell onwards. In practice, the “one-cell” skin depth of the infinite- ϵ leapfrog ADI-FDTD method should not pose any issue for non-penetrate targets when modeled with appropriate thickness.

5.4 Conclusion

This chapter has illustrated applications of the leapfrog ADI-FDTD method for electromagnetic compatibility modeling. Mutual coupling between two monopole antennas of different length mounted on a finite ground plane has been investigated using the leapfrog ADI-FDTD method. The study of how an antenna behaves when placed in the vicinity of another antenna is important so as to ensure that radio-frequency interference is kept to a minimum.

Finally, this chapter has also investigated the field leakage of the leapfrog ADI-FDTD method for non-penetrable targets. A shielded enclosure is used as an application example for the non-penetrable target. Detailed analysis reveals that the leapfrog ADI-FDTD method has substantial non-physical field leakage. To overcome the field leakage issue for non-penetrable targets, a simple approach has been proposed. Numerical results have been presented to validate the proposed approach.

Chapter 6

Analysis of Divergence Properties for the Leapfrog ADI-FDTD Method

6.1 Introduction

The numerical stability and dispersion properties of the leapfrog ADI-FDTD method were analyzed and found to be similar to the conventional ADI-FDTD method in Chapter 3. Unlike the conventional explicit FDTD method, the conventional ADI-FDTD method is not divergence-free in the source-free region [80]. The divergence of fields in the conventional ADI-FDTD method may introduce spurious charges. For the complex envelope ADI-FDTD method, these spurious charges are time-harmonic and produce secondary radiation [81].

In [29], it was claimed that the leapfrog ADI-FDTD method is divergence-free. Such

claim was made by analogy of the update equations between the leapfrog ADI-FDTD method and the conventional explicit FDTD method. However, the leapfrog ADI-FDTD method has dispersion properties of the conventional ADI-FDTD method and not of the conventional explicit FDTD method. Therefore, a more rigorous analysis of the divergence properties for the leapfrog ADI-FDTD method is necessary.

In this chapter, we present the analysis of the divergence properties for the leapfrog ADI-FDTD method. Section 6.2 presents the analysis of the divergence properties for the leapfrog ADI-FDTD method. The analysis is based on the Fourier domain plane wave method for the dispersion analysis but extended with additional procedure to determine the homogeneous solutions. The singular value decomposition (SVD) will be used to obtain the solutions of the homogeneous system in a reliable manner. There are two linearly independent null-modes in the homogeneous system corresponding to the repeated zero singular values. To provide more physical insights, proper linear combinations of the two null-modes are determined as TE and TM modes with respect to the direction of discrete numerical space wavevector. The divergence of the electric fields is analyzed by examining the TM mode, while the divergence of the magnetic fields is analyzed by examining the TE mode. In Section 6.3, numerical results for the divergence properties of the leapfrog ADI-FDTD method are presented and compared with the conventional ADI-FDTD method. Some numerical experiments are performed to illustrate the existence of divergent waves for the leapfrog ADI-FDTD method in Section 6.4.

6.2 Analysis of Divergence Property

In this section, we present the analysis of the divergence properties for the 3-D leapfrog ADI-FDTD method. The analysis is based on the Fourier domain plane wave method for the dispersion relation but extended with additional procedure to determine the homogeneous solutions. Assuming monochromatic waves with angular frequency ω , the fields can be written in complex exponential form as (noting the leapfrog stepping in time)

$$E_x^{n-\frac{1}{2}} = E_{x0} e^{j\omega(n-\frac{1}{2})\Delta t} \quad (6.2.1a)$$

$$E_y^{n-\frac{1}{2}} = E_{y0} e^{j\omega(n-\frac{1}{2})\Delta t} \quad (6.2.1b)$$

$$E_z^{n-\frac{1}{2}} = E_{z0} e^{j\omega(n-\frac{1}{2})\Delta t} \quad (6.2.1c)$$

$$H_x^n = H_{x0} e^{j\omega n\Delta t} \quad (6.2.1d)$$

$$H_y^n = H_{y0} e^{j\omega n\Delta t} \quad (6.2.1e)$$

$$H_z^n = H_{z0} e^{j\omega n\Delta t}. \quad (6.2.1f)$$

The time-stepping relation of the leapfrog ADI-FDTD method can be expressed in matrix operator form compactly as

$$\begin{bmatrix} \mathbf{E}^{n+\frac{1}{2}} \\ \mathbf{H}^{n+1} \end{bmatrix} = \mathbf{G}_F \begin{bmatrix} \mathbf{E}^{n-\frac{1}{2}} \\ \mathbf{H}^n \end{bmatrix} \quad (6.2.2)$$

where $\mathbf{G}_F$ is the 6 x 6 matrix derived previously in Chapter 3. Introducing the

initial field vector of (6.2.1) as

$$\mathbf{U}_0 = \begin{bmatrix} E_{x0} e^{-j\omega \frac{\Delta t}{2}} \\ E_{y0} e^{-j\omega \frac{\Delta t}{2}} \\ E_{z0} e^{-j\omega \frac{\Delta t}{2}} \\ H_{x0} \\ H_{y0} \\ H_{z0} \end{bmatrix}, \quad (6.2.3)$$

we can express (6.2.2) as

$$\left(e^{j\omega \Delta t} \mathbf{I}_6 - \mathbf{G}_F \right) \mathbf{U}_0 = 0 \quad (6.2.4)$$

where $\mathbf{I}_6$ is the 6 x 6 identity matrix. Let us denote the matrix on the left hand side of (6.2.4) as

$$\mathbf{\Lambda} = \left(e^{j\omega \Delta t} \mathbf{I}_6 - \mathbf{G}_F \right) \quad (6.2.5)$$

For a non-trivial solution of (6.2.4), its determinant should be zero as follows:

$$\det(\mathbf{\Lambda}) = 0 \quad (6.2.6)$$

The wavenumber k that satisfies (6.2.6) is solved by using appropriate zero searching routine. Then, the homogeneous solutions can be obtained for (6.2.4) with angular frequency w and wavenumber k . We resort to the singular value decomposition (SVD) to obtain the solutions of the homogeneous system in a reliable manner. The SVD factorization of the 6 x 6 matrix $\mathbf{\Lambda}$ is given by

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{U}_{SVD} \end{bmatrix} \begin{bmatrix} \mathbf{W}_{SVD} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{SVD}^H \end{bmatrix} \quad (6.2.7)$$

Figure 6.1: Logarithm of singular values w_ξ for the 3-D leapfrog ADI-FDTD method. ©2012 IEEE.

where the superscript “ H ” is the Hermitian transpose operator and

$$\mathbf{U}_{SVD} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{u}_4 & \mathbf{u}_5 & \mathbf{u}_6 \end{bmatrix} \quad (6.2.8a)$$

$$\mathbf{W}_{SVD} = \text{diag}(\begin{bmatrix} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{bmatrix}) \quad (6.2.8b)$$

$$\mathbf{V}_{SVD} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 & \mathbf{v}_5 & \mathbf{v}_6 \end{bmatrix} \quad (6.2.8c)$$

$\mathbf{u}_\xi$ and $\mathbf{v}_\xi$ for $\xi = 1, 2, \dots, 6$ are 6×1 column vectors. The scalar elements w_ξ are the singular values of $\mathbf{\Lambda}$. The field vector $\mathbf{U}_0$ corresponds to the nullspace of $\mathbf{\Lambda}$ and it can be obtained from the columns of $\mathbf{V}_{SVD}$ whose corresponding elements of w_ξ are zero. Note that $\mathbf{U}_{SVD}$ and $\mathbf{V}_{SVD}$ are unitary matrices.

Fig. 6.1 plots the logarithm of singular values w_ξ of a typical $\mathbf{\Lambda}$ for the 3-D leapfrog ADI-FDTD method. The singular values are repeated in pairs, with the last pair of w_5 and w_6 being (practically) zero. The corresponding column vectors $\mathbf{v}_5$ and $\mathbf{v}_6$ of

$\mathbf{V}_{SVD}$ are the two null-modes of $\mathbf{\Lambda}$, i.e.,

$$\mathbf{\Lambda} \mathbf{v}_5 = 0 \quad (6.2.9a)$$

$$\mathbf{\Lambda} \mathbf{v}_6 = 0 \quad (6.2.9b)$$

Null-modes $\mathbf{v}_5$ and $\mathbf{v}_6$ of (6.2.8c) are the eigenvectors of (6.2.5) for (repeated) eigenvalue zero, as the dimension of the nullspace of matrix $\mathbf{\Lambda}$ is two [82]. In addition, any linear combination of $\mathbf{v}_5$ and $\mathbf{v}_6$ would also satisfy the homogeneous system in (6.2.4), i.e.,

$$\mathbf{\Lambda} \left(a \mathbf{v}_5 + b \mathbf{v}_6 \right) = 0 \quad (6.2.10)$$

for any scalars a and b . Therefore, there are infinite possible solutions for the field vector $\mathbf{U}_0$ being the linear combination of null-modes $\mathbf{v}_5$ and $\mathbf{v}_6$ as

$$\mathbf{U}_0 = a \mathbf{v}_5 + b \mathbf{v}_6 \quad (6.2.11)$$

The divergence properties of field vectors in the discrete numerical space can be investigated conveniently using the discrete del operator in the Fourier domain defined by

$$\nabla_d = -j \mathbf{k}_d, \quad \mathbf{k}_d = \left[k_{dx} \ k_{dy} \ k_{dz} \right]^T \quad (6.2.12)$$

where $\mathbf{k}_d$ is the discrete numerical space wavevector. Note that k_d approaches the continuous physical space wavenumber k when the spatial discretization Δ approaches zero. The subscript “d” denotes discrete numerical space. Applying such operator for the divergence of fields amounts to taking the dot product of $\mathbf{k}_d$ and

$\mathbf{U}_0$, or individually $\mathbf{E}_0$ and $\mathbf{H}_0$ as (omitting “ $-j$ ” factor)

$$\mathbf{k}_d \cdot \mathbf{U}_0 \implies \begin{cases} \mathbf{k}_d \cdot \mathbf{E}_0 \\ \mathbf{k}_d \cdot \mathbf{H}_0 \end{cases} \quad (6.2.13)$$

With infinite possible solutions of (6.2.11) available, we expect that there will also be an infinite number of dot products in (6.2.13). Some of these dot products may not reveal the complete divergence properties of the leapfrog ADI-FDTD method. For instance, certain $\mathbf{U}_0$ may give rise to $\mathbf{k}_d \cdot \mathbf{E}_0 = 0$, while another one may lead to $\mathbf{k}_d \cdot \mathbf{H}_0 = 0$, etc. Therefore, it is more meaningful to choose the scalars a and b in (6.2.11) properly such that (6.2.13) yields the dot products with some physical insights.

To that end, we recall that in isotropic media, the 3-D Maxwell’s equations with six field components allow propagation of transverse electromagnetic (TEM) modes. The TEM modes do not have any field components along the direction of wave propagation. In general, the six field components can be decomposed into transverse electric (TE) and transverse magnetic (TM) modes with respect to the direction of wave propagation. The TE mode does not have any electric field component along the direction of wave propagation, while the TM mode does not have any magnetic field component along the direction of wave propagation. However, note that the direction of wave propagation in the continuous physical space is generally not parallel to that in the discrete numerical space, i.e., $\mathbf{k} \neq \mathbf{k}_d$ [83]. Therefore, it is more general to decompose the fields into TE and TM modes with respect to certain direction of interest (not necessary the direction of wave propagation). In our case here for divergence analysis, it is more insightful to select $\mathbf{k}_d$ as the direction

of interest so that the decomposition reads

$$\text{TE}_{\mathbf{k}_d} \text{ mode : } \mathbf{k}_d \cdot \mathbf{E}_0^{\text{TE}} = 0, \mathbf{k}_d \cdot \mathbf{H}_0^{\text{TE}} \neq 0 \quad (6.2.14a)$$

$$\text{TM}_{\mathbf{k}_d} \text{ mode : } \mathbf{k}_d \cdot \mathbf{H}_0^{\text{TM}} = 0, \mathbf{k}_d \cdot \mathbf{E}_0^{\text{TM}} \neq 0. \quad (6.2.14b)$$

As the special cases of (6.2.14), $\text{TE}_{\mathbf{k}_d}$ mode with zero magnetic field component along $\mathbf{k}_d$ and $\text{TM}_{\mathbf{k}_d}$ mode with zero electric field component along $\mathbf{k}_d$ will constitute the $\text{TEM}_{\mathbf{k}_d}$ modes. When they exist (e.g. for the conventional explicit FDTD method), these $\text{TEM}_{\mathbf{k}_d}$ modes feature the divergence-free property of fields in the source-free region. Henceforth, we shall employ the above decomposition of TE and TM modes for analysis of the divergence of electric and magnetic fields for the 3-D leapfrog ADI-FDTD method.

To analyze the divergence of the electric fields, one can examine the $\text{TM}_{\mathbf{k}_d}$ mode, which is determined by proper linear combination of null-modes $\mathbf{v}_5$ and $\mathbf{v}_6$ such that the magnetic field component is zero along the $\mathbf{k}_d$ direction. For convenience, let us partition the 6×1 column vector $\mathbf{v}_\xi$ into two 3×1 column vectors for electric ($\mathbf{v}_\xi^e$) and magnetic ($\mathbf{v}_\xi^h$) field components as

$$\mathbf{v}_\xi = \begin{bmatrix} \mathbf{v}_\xi^e \\ \mathbf{v}_\xi^h \end{bmatrix} \quad (6.2.15)$$

Next, we substitute the $\mathbf{v}_\xi^h$ part of (6.2.11) into (6.2.14b) and obtain

$$\mathbf{k}_d \cdot \left(a \mathbf{v}_5^h + b \mathbf{v}_6^h \right) = 0. \quad (6.2.16)$$

To determine the $\text{TM}_{\mathbf{k}_d}$ mode in a reliable manner, we consider the following cases

depending on the relation between $|\mathbf{k}_d \cdot \mathbf{v}_5^h|$ and $|\mathbf{k}_d \cdot \mathbf{v}_6^h|$:

$$\text{Case 1 : } |\mathbf{k}_d \cdot \mathbf{v}_5^h| > |\mathbf{k}_d \cdot \mathbf{v}_6^h| \quad (6.2.17a)$$

$$\text{Case 2 : } |\mathbf{k}_d \cdot \mathbf{v}_5^h| < |\mathbf{k}_d \cdot \mathbf{v}_6^h| \quad (6.2.17b)$$

$$\text{Case 3 : } |\mathbf{k}_d \cdot \mathbf{v}_5^h| = |\mathbf{k}_d \cdot \mathbf{v}_6^h| > 0 \quad (6.2.17c)$$

$$\text{Case 4 : } |\mathbf{k}_d \cdot \mathbf{v}_5^h| = |\mathbf{k}_d \cdot \mathbf{v}_6^h| = 0. \quad (6.2.17d)$$

For case 1, the $\text{TM}_{\mathbf{k}_d}$ mode can be obtained as

$$\begin{bmatrix} \mathbf{E}_0^{\text{TM}} \\ \mathbf{H}_0^{\text{TM}} \end{bmatrix} = b \left(\mathbf{v}_6 - \frac{\mathbf{k}_d \cdot \mathbf{v}_6^h}{\mathbf{k}_d \cdot \mathbf{v}_5^h} \mathbf{v}_5 \right) \quad (6.2.18)$$

where b can have arbitrary value which will not affect the final result when normalized. For case 2, we write the $\text{TM}_{\mathbf{k}_d}$ mode as

$$\begin{bmatrix} \mathbf{E}_0^{\text{TM}} \\ \mathbf{H}_0^{\text{TM}} \end{bmatrix} = a \left(\mathbf{v}_5 - \frac{\mathbf{k}_d \cdot \mathbf{v}_5^h}{\mathbf{k}_d \cdot \mathbf{v}_6^h} \mathbf{v}_6 \right) \quad (6.2.19)$$

for any arbitrary scalar a . Either (6.2.18) or (6.2.19) is applicable for case 3. When both $\mathbf{k}_d \cdot \mathbf{v}_5^h = 0$ and $\mathbf{k}_d \cdot \mathbf{v}_6^h = 0$ for case 4, $\mathbf{U}_0$ of (6.2.11) with arbitrary a and b may be regarded as the $\text{TM}_{\mathbf{k}_d}$ mode.

In a similar manner, the divergence of the magnetic fields can be analyzed by examining the $\text{TE}_{\mathbf{k}_d}$ mode accordingly with

$$\mathbf{k}_d \cdot (a \mathbf{v}_5^e + b \mathbf{v}_6^e) = 0 \quad (6.2.20)$$

and considering the dual cases of (6.2.17) (with $\mathbf{v}_5^h$ and $\mathbf{v}_6^h$ replaced by $\mathbf{v}_5^e$ and $\mathbf{v}_6^e$):

$$\text{Case 1'} : |\mathbf{k}_d \cdot \mathbf{v}_5^e| > |\mathbf{k}_d \cdot \mathbf{v}_6^e| \quad (6.2.21a)$$

$$\text{Case 2'} : |\mathbf{k}_d \cdot \mathbf{v}_5^e| < |\mathbf{k}_d \cdot \mathbf{v}_6^e| \quad (6.2.21b)$$

$$\text{Case 3'} : |\mathbf{k}_d \cdot \mathbf{v}_5^e| = |\mathbf{k}_d \cdot \mathbf{v}_6^e| > 0 \quad (6.2.21c)$$

$$\text{Case 4'} : |\mathbf{k}_d \cdot \mathbf{v}_5^e| = |\mathbf{k}_d \cdot \mathbf{v}_6^e| = 0. \quad (6.2.21d)$$

In particular, the $\text{TE}_{\mathbf{k}_d}$ mode can be determined reliably as

$$\begin{bmatrix} \mathbf{E}_0^{\text{TE}} \\ \mathbf{H}_0^{\text{TE}} \end{bmatrix} = b \left(\mathbf{v}_6 - \frac{\mathbf{k}_d \cdot \mathbf{v}_6^e}{\mathbf{k}_d \cdot \mathbf{v}_5^e} \mathbf{v}_5 \right) \quad (6.2.22)$$

for case 1' with arbitrary b and

$$\begin{bmatrix} \mathbf{E}_0^{\text{TE}} \\ \mathbf{H}_0^{\text{TE}} \end{bmatrix} = a \left(\mathbf{v}_5 - \frac{\mathbf{k}_d \cdot \mathbf{v}_5^e}{\mathbf{k}_d \cdot \mathbf{v}_6^e} \mathbf{v}_6 \right) \quad (6.2.23)$$

for case 2' with arbitrary a . Similar arguments apply for case 3' using either (6.2.22) or (6.2.23), and for case 4' using $\mathbf{U}_0$ of (6.2.11) with arbitrary a and b for the $\text{TE}_{\mathbf{k}_d}$ mode.

For completeness, we highlight the key difference between the analysis of the divergence properties for the 2-D and 3-D leapfrog ADI-FDTD method. From the 3-D leapfrog ADI-FDTD method, one can reduce to either 2-D TE_z or 2-D TM_z leapfrog ADI-FDTD method, where there are only three instead of six field components in (6.2.3). Therefore, $\mathbf{\Lambda}$ for the 2-D leapfrog ADI-FDTD method involves only a 3 x 3 matrix, i.e.,

$$\mathbf{\Lambda} \mathbf{U}_0 = \left(e^{j\omega\Delta t} \mathbf{I}_3 - \mathbf{G}_F \right) \mathbf{U}_0 = 0 \quad (6.2.24)$$

where $\mathbf{I}_3$ is the 3 x 3 identity matrix. The SVD factorization of matrix $\mathbf{\Lambda}$ for the 2-D leapfrog ADI-FDTD method is given as

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{U}_{SVD} \end{bmatrix} \begin{bmatrix} w_1 & & \\ & w_2 & \\ & & w_3 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{SVD}^H \end{bmatrix} \quad (6.2.25)$$

and it does not yield repeated singular values w . The homogeneous solution of $\mathbf{\Lambda}$ can be obtained directly from $\mathbf{v}_3$ subjected to an arbitrary scalar a (no more b as there is only one null mode). Hence, the divergence of the electric fields can be analyzed using the 2-D TE_z leapfrog ADI-FDTD method, while the divergence of the magnetic fields can be analyzed using the 2-D TM_z leapfrog ADI-FDTD method.

6.3 Numerical Results

In this section, we employ the SVD along with TE_{k_d} and TM_{k_d} decomposition to analyze the divergence properties of the 3-D leapfrog ADI-FDTD method. The divergence of the electric and magnetic fields for the leapfrog ADI-FDTD method will be compared with the conventional ADI-FDTD method and the conventional explicit FDTD method. To standardize the comparison among the methods, $\mathbf{E}_0$ and $\mathbf{H}_0$ are normalized to their unit vectors $\hat{\mathbf{E}}_0$ and $\hat{\mathbf{H}}_0$ respectively. $\mathbf{k}_d$ is also normalized by k to be

$$\mathbf{k}_{dn} = \mathbf{k}_d/k. \quad (6.3.1)$$

In our numerical analysis, the angular frequency ω is chosen to be 3 GHz with uniform cell size $\Delta = \Delta_x = \Delta_y = \Delta_z$. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{3}}$ where c is the speed of light in freespace. We also denote CFLN = $\frac{\Delta t}{\Delta t_{CFL}}$.

6.3.1 Divergence of $\hat{E}_0$

The divergence of $\hat{E}_0$ is analyzed by examining the $|\mathbf{k}_{\text{dn}} \cdot \hat{\mathbf{E}}_0^{\text{TM}}|$ dot product magnitude for the TM_{k_d} mode (with $\mathbf{k}_d \cdot \mathbf{H}_0^{\text{TM}} = 0$). Figs. 6.2 and 6.3 plot the divergence of $\hat{E}_0$ vs. propagation angle ϕ for the leapfrog ADI-FDTD method, conventional ADI-FDTD method and conventional explicit FDTD method with $\text{CFLN} = 1$, $\Delta = \frac{\lambda}{20}$ at $\theta = 45^\circ$ and 90° respectively. As expected, the divergence of $\hat{E}_0$ is zero for the conventional explicit FDTD method at all propagation angles θ and ϕ . However, the divergence of $\hat{E}_0$ for both leapfrog and conventional ADI-FDTD methods is not zero in general except only at specific propagation angles. Furthermore, as is evident in these figures, the divergence of $\hat{E}_0$ is different for the leapfrog ADI-FDTD method and the conventional ADI-FDTD method. Referring to Fig. 6.2, along the diagonal grid direction ($\theta = 45^\circ$), the divergence of $\hat{E}_0$ for both leapfrog and conventional ADI-FDTD methods is not zero at all propagation angles ϕ . It can be observed that the maximum magnitudes of the divergence of $\hat{E}_0$ at $\theta = 45^\circ$ for both methods occur at the same propagation angle ϕ and they have approximately the same value of 2.1×10^{-3} . However, the minimum magnitudes of the divergence of $\hat{E}_0$ at $\theta = 45^\circ$ for both methods do not occur at the same ϕ . Moreover, the minimum magnitudes of the divergence of $\hat{E}_0$ at $\theta = 45^\circ$ for the leapfrog ADI-FDTD method is higher than the conventional ADI-FDTD method. Referring to Fig. 6.3, the divergence of $\hat{E}_0$ at $\theta = 90^\circ$ for the leapfrog ADI-FDTD method appears to be a mirror image of that for the conventional ADI-FDTD method at propagation angles $\phi = 45^\circ$ and 135° .

Fig. 6.4 compares the divergence of $\hat{E}_0$ vs. propagation angle ϕ for the leapfrog ADI-FDTD method and the conventional ADI-FDTD method with different CFLN and $\Delta = \frac{\lambda}{20}$ at $\theta = 45^\circ$. The CFLN is selected to be 2, 5 and 10. The conventional

Figure 6.2: Divergence of $\hat{E}_0$ vs. propagation angle ϕ for leapfrog ADI-FDTD, ADI-FDTD and FDTD with CFLN = 1, $\Delta = \frac{\lambda}{20}$ at $\theta = 45^\circ$. ©2012 IEEE.

Figure 6.3: Divergence of $\hat{E}_0$ vs. propagation angle ϕ for leapfrog ADI-FDTD, ADI-FDTD and FDTD with CFLN = 1, $\Delta = \frac{\lambda}{20}$ at $\theta = 90^\circ$. ©2012 IEEE.

explicit FDTD method is not included as it is unstable for CFLN > 1. From Fig. 6.4, one can observe clearly that the divergence of $\hat{E}_0$ increases with CFLN for both methods. Notice also that with a particular CFLN, the maximum magnitude of the divergence of $\hat{E}_0$ for the leapfrog ADI-FDTD method is about the same as that for the conventional ADI-FDTD method. With CFLN = 5, the maximum magnitude of the divergence of $\hat{E}_0$ is 0.05, while with CFLN = 10, the maximum magnitude is approximately 0.23.

Figure 6.4: Divergence of $\hat{E}_0$ vs. propagation angle ϕ for leapfrog ADI-FDTD and ADI-FDTD with CFLN = 2, 5 and 10, $\Delta = \frac{\lambda}{20}$ at $\theta = 45^\circ$. ©2012 IEEE.

Figure 6.5: Divergence of $\hat{E}_0$ vs. propagation angle ϕ for leapfrog ADI-FDTD and ADI-FDTD with CFLN = 5, $\Delta = \frac{\lambda}{20}$, $\frac{\lambda}{50}$ and $\frac{\lambda}{100}$ at $\theta = 90^\circ$. ©2012 IEEE.

Fig. 6.5 shows the divergence of $\hat{E}_0$ vs. propagation angle ϕ for the leapfrog ADI-FDTD method and the conventional ADI-FDTD method with CFLN = 5 and different cell size Δ at $\theta = 90^\circ$. The cell size Δ is chosen as $\frac{\lambda}{20}$, $\frac{\lambda}{50}$ and $\frac{\lambda}{100}$. It can be seen that the divergence of $\hat{E}_0$ may be decreased by refining the mesh.

Figure 6.6: Divergence of $\hat{H}_0$ vs. propagation angle ϕ for leapfrog ADI-FDTD, ADI-FDTD and FDTD with CFLN = 1, $\Delta = \frac{\lambda}{20}$ at $\theta = 45^\circ$. ©2012 IEEE.

6.3.2 Divergence of $\hat{H}_0$

The divergence of $\hat{H}_0$ is analyzed by examining the $|\mathbf{k}_{\text{dn}} \cdot \hat{\mathbf{H}}_0^{\text{TE}}|$ dot product magnitude for the TE_{k_d} mode (with $\mathbf{k}_d \cdot \mathbf{E}_0^{\text{TE}} = 0$). Figs. 6.6 and 6.7 plot the divergence of $\hat{H}_0$ vs. propagation angle ϕ for the leapfrog ADI-FDTD method, conventional ADI-FDTD method and conventional explicit FDTD method with CFLN = 1, $\Delta = \frac{\lambda}{20}$ at $\theta = 45^\circ$ and 90° respectively. As before, the divergence of $\hat{H}_0$ is also zero for the conventional explicit FDTD method at all propagation angles θ and ϕ . Thus, we have reasserted the divergence-free properties of the conventional explicit FDTD method, which are shown in [4] but not in the same way as we have presented in this thesis.

Referring to Fig. 6.6, the divergence of $\hat{H}_0$ at $\theta = 45^\circ$ for both leapfrog and conventional ADI-FDTD methods is again not zero at all propagation angles ϕ . On the other hand, it is interesting to see from Fig. 6.7 that the divergence of $\hat{H}_0$ at $\theta = 90^\circ$ for the leapfrog ADI-FDTD method yields approximately the same magnitude and overlaps that for the conventional ADI-FDTD method.

Figure 6.7: Divergence of $\hat{H}_0$ vs. propagation angle ϕ for leapfrog ADI-FDTD, ADI-FDTD and FDTD with CFLN = 1, $\Delta = \frac{\lambda}{20}$ at $\theta = 90^\circ$. ©2012 IEEE.

Figure 6.8: Divergence of $\hat{H}_0$ vs. propagation angle ϕ for leapfrog ADI-FDTD and ADI-FDTD with CFLN = 2, 5 and 10, $\Delta = \frac{\lambda}{20}$ at $\theta = 45^\circ$. ©2012 IEEE.

Fig. 6.8 compares the divergence of $\hat{H}_0$ vs. propagation angle ϕ for the leapfrog ADI-FDTD method and the conventional ADI-FDTD method with different CFLN and $\Delta = \frac{\lambda}{20}$ at $\theta = 45^\circ$. As before, the CFLN is selected to be 2, 5 and 10. The divergence of $\hat{H}_0$ increases with CFLN for both methods. With a particular CFLN, the maximum magnitude of the divergence of $\hat{H}_0$ for the leapfrog ADI-FDTD method is about the same as that for the conventional ADI-FDTD method.

Fig. 6.9 shows the divergence of $\hat{H}_0$ vs. propagation angle ϕ for the leapfrog ADI-

Figure 6.9: Divergence of $\hat{H}_0$ vs. propagation angle ϕ for leapfrog ADI-FDTD and ADI-FDTD with CFLN = 5, $\Delta = \frac{\lambda}{20}$, $\frac{\lambda}{50}$ and $\frac{\lambda}{100}$ at $\theta = 90^\circ$. ©2012 IEEE.

FDTD method and the conventional ADI-FDTD method with CFLN = 5 and cell size Δ of $\frac{\lambda}{20}$, $\frac{\lambda}{50}$ and $\frac{\lambda}{100}$ at $\theta = 90^\circ$. It is apparent that the divergence of $\hat{H}_0$ may be reduced when the mesh density is increased.

From the above, it is seen that the leapfrog ADI-FDTD method is not divergence-free in the source-free region, as is the case for the conventional ADI-FDTD method. Furthermore, both methods have different divergence properties in general.

6.4 Numerical Experiments

In this section, we perform some numerical experiments to illustrate the existence of divergent waves for the 3-D leapfrog ADI-FDTD method. Let us consider an air-filled PEC cavity of 400 x 400 x 400 uniform cells with size of $\Delta = \Delta_x = \Delta_y = \Delta_z = \frac{\lambda}{20}$. A sinusoidal modulated Gaussian pulse excitation is located at the center of the computation domain as

$$J_z = \sin \left[2\pi f_0 \left(t - t_0 \right) \right] e^{-\left(\frac{t-t_0}{\tau} \right)^2} \quad (6.4.1)$$

Figure 6.10: Divergence of electric fields (dB) in the xy -plane at $k = 200$ at time instant $t = 0.23$ ns. (a) Leapfrog ADI-FDTD method with CFLN = 1. (b) ADI-FDTD method with CFLN = 1. (c) Leapfrog ADI-FDTD method with CFLN = 4. (d) ADI-FDTD method with CFLN = 4. The divergence of electric field increases with CFLN. ©2012 IEEE.

where

$$f_0 = 15 \text{ GHz}, \tau = 60 \text{ ps}, t_0 = 2\tau.$$

Fig. 6.10 shows the distribution of the divergence of electric fields (dB) in the xy -plane at $k = 200$ for CFLN = 1 and 4, at time instant $t = 0.23$ ns for the leapfrog

ADI-FDTD method and conventional ADI-FDTD method. The results for the conventional explicit FDTD method are not presented as it is divergence-free, and the snapshots are simply plots of uniform low intensity (blue) color. Note that the figure shows a region of the computation domain that does not include the side walls.

At this time instant, the wave front has not reached the outer boundary domain, and is free from reflection errors. From the figure, it is clear that divergent waves exist for the leapfrog ADI-FDTD method, as well as for the conventional ADI-FDTD method in the near-field region. These regions have high intensity (red) of divergence. Furthermore, the divergence of the E field increases with CFLN.

6.5 Conclusion

This chapter has presented the analysis of the divergence properties for the leapfrog ADI-FDTD method. The amplification matrix of the leapfrog ADI-FDTD method has been formulated in the Fourier domain. The analysis of divergence properties is based on the Fourier domain plane wave method for dispersion analysis but extended with additional procedure to determine the homogeneous solutions. The SVD has been used to obtain the solutions of the homogeneous system in a reliable manner. There are two linearly independent null-modes in the homogeneous system corresponding to the repeated zero singular values.

To provide more physical insights, proper linear combinations of the two null-modes have been determined as TE and TM modes with respect to the direction of discrete numerical space wavevector. Numerical results for the divergence properties of the leapfrog ADI-FDTD method have been presented and compared with the conven-

tional ADI-FDTD method.

The leapfrog ADI-FDTD method is not divergence-free in the source-free region, as is the case for the conventional ADI-FDTD method. Furthermore, both methods have different divergence properties in general. The divergence of fields may result in intrinsic grid capacitance due to the absence of charge dynamics in the Yee cells [84].

Chapter 7

Fundamental LOD-FDTD Method with Second-Order Temporal Accuracy and Complying Divergence

7.1 Introduction

The conventional explicit FDTD method employs second-order central-difference in both time and space to solve the Maxwell's curl equations. While due to the manner in which the field components are structured in the Yee's cell and the numerical space-derivative operations of the scheme, the conventional explicit FDTD method implicitly enforces the Gauss's laws for the electric and magnetic fields [4]. Hence, the conventional explicit FDTD method conforms with the full set of Maxwell's equations, and is a true 3-D Maxwell's solver.

While the conventional explicit FDTD method has complying divergence, the conventional ADI-FDTD method does not have complying divergence [80, 81] as the Maxwell's curl equations are being enforced based on locally inconsistent scheme. Although the conventional ADI-FDTD method uses the same Yee's cell as the conventional explicit FDTD method, the spatial-differencing operations of the former method do not inherently enforce the Gauss's laws for the electric and magnetic fields. Recently, the conventional ADI-FDTD method has been reformulated into the more efficient leapfrog ADI-FDTD method. However, analysis of the divergence properties of the leapfrog ADI-FDTD method shows that this variant of the ADI-FDTD method still does not have complying divergence [85].

The LOD-FDTD method is another class of FDTD method that is free from the CFL stability criterion [17, 37]. Several variations of the LOD-FDTD method have been recently formulated. For instance, the LOD2-FDTD method in [37] has second-order temporal accuracy, while the LOD-FDTD method in [12] may have higher-order spatial accuracy. Similarly, the LOD-FDTD methods, which we have discussed in Chapter 2, also does not have complying divergence. Spurious charges may exist in the source-free regions when the method is not divergence-complying. Consequently, there would be additional grid capacitance in such regions.

In this chapter, we introduce the unconditionally stable fundamental locally one-dimensional finite-difference time-domain method with second-order temporal accuracy and complying divergence (denoted as LOD2-CD-FDTD) for 3-D Maxwell's equations. The update procedures for a family of LOD-FDTD methods that employ similar splitting matrix operators have been present before in Chapter 2. Section 7.2 extends the previous concept of achieving second-order temporal accuracy for the

LOD2-FDTD method via implicit output processing, and propose a novel, explicit output processing that not only retains second-order temporal accuracy, but also complying divergence for the LOD2-CD-FDTD method. The current source implementation for the LOD2-CD-FDTD method involves source-incorporation in only the first procedure. To further enhance efficiency, the LOD2-CD-FDTD method is formulated into the fundamental LOD2-CD-FDTD method with efficient matrix-operator-free right-hand side (RHS). Subsequently, detailed implementation for the fundamental LOD2-CD-FDTD method will be presented. Analytical proof for the second-order temporal accuracy of the LOD2-CD-FDTD method is provided. In Section 7.3, numerical results and examples will be presented to validate the divergence-complying property of the LOD2-CD-FDTD method. In Section 7.4, the divergence-complying property of the LOD2-CD-FDTD method will be investigated at the source region by analyzing the electric charge deposited at the ends of a dipole radiator. Finally, Section 7.5 extends the fundamental LOD2-CD-FDTD method by incorporating the convolutional perfectly matched layer (CPML) absorbing boundary condition.

7.2 LOD2-FDTD Method with Complying Divergence

In this section, we present the LOD2-CD-FDTD method along with the current source implementation for 3-D Maxwell's equations. We consider the wave propagation in a medium with permittivity ϵ and permeability μ , and select the splitting matrix operators (repeated here for convenience) of Maxwell's equations as follows:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{\epsilon}\partial_y \\ 0 & 0 & 0 & \frac{1}{\epsilon}\partial_z & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\epsilon}\partial_x & 0 \\ 0 & \frac{1}{\mu}\partial_z & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\mu}\partial_x & 0 & 0 & 0 \\ \frac{1}{\mu}\partial_y & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (7.2.1a)$$

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{-1}{\epsilon}\partial_z & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{\epsilon}\partial_x \\ 0 & 0 & 0 & \frac{-1}{\epsilon}\partial_y & 0 & 0 \\ 0 & 0 & \frac{-1}{\mu}\partial_y & 0 & 0 & 0 \\ \frac{-1}{\mu}\partial_z & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{\mu}\partial_x & 0 & 0 & 0 & 0 \end{pmatrix} \quad (7.2.1b)$$

where ∂_x , ∂_y , ∂_z are the spatial difference operators for the first derivatives along x , y , z directions, respectively.

By extending the previous concept of achieving second-order temporal accuracy for the LOD2-FDTD method via implicit output processing, we hereby propose novel, explicit output processing that not only retains second-order temporal accuracy, but also complying divergence for the LOD2-CD-FDTD method. We have

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right) \mathbf{u}^{n+\frac{1}{2}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{A}\right) \mathbf{u}^n + \Delta t \mathbf{s}^{n+\frac{1}{2}} \quad (7.2.2a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}^{n+\frac{1}{2}} \quad (7.2.2b)$$

where

$$\mathbf{s} = \begin{bmatrix} \frac{-1}{\epsilon} J_x & \frac{-1}{\epsilon} J_y & \frac{-1}{\epsilon} J_z & \frac{-1}{\mu} M_x & \frac{-1}{\mu} M_y & \frac{-1}{\mu} M_z \end{bmatrix}^T \quad (7.2.3)$$

for the main iterations. For null initial fields, $\mathbf{u}^{n+1} = \mathbf{u}_{\text{LOD1}}^{n+1}$ and $\mathbf{u}^{n+\frac{1}{2}} = \mathbf{u}_{\text{LOD1}}^{n+\frac{1}{2}}$. On the other hand, for nonzero initial fields $\mathbf{u}^{t=0}$, implicit input processing needs to be invoked (only once) at time step $n = 0$ as

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B} \right) \mathbf{u}^0 = \mathbf{u}^{t=0} \quad (7.2.4)$$

The explicit output processing at time step $n + 1$ is

$$\mathbf{u}_{\text{CD}}^{n+1} = \left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B} \right) \mathbf{u}^{n+1} \quad (7.2.5)$$

where the subscript “CD” denotes complying divergence.

The update equations for the main iterations of the LOD2-CD-FDTD method in (7.2.2) are simply those of the conventional LOD1-FDTD method. Furthermore, the current implementation of the LOD2-CD-FDTD method involves source-incorporation in the first procedure only, similar to the concise current source implementation [35]. It is also worthwhile to mention that $\mathbf{u}_{\text{CD}}^{n+1}$ is the final field output from the explicit output processing (7.2.5), while $\mathbf{u}^{n+1}$ is the field variable for the main iterations.

7.2.1 Fundamental LOD2-CD-FDTD Method

To further enhance efficiency, we formulate the LOD2-CD-FDTD method into the fundamental form (denoted as FLOD2-CD-FDTD), with its right-hand sides free of

matrix operators $\mathbf{A}$ and $\mathbf{B}$, by introducing auxiliary variable

$$\mathbf{v} = \begin{bmatrix} e_x & e_y & e_z & h_x & h_y & h_z \end{bmatrix}^T \quad (7.2.6)$$

We manipulate (7.2.2) to yield:

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}\right) \mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^n + \frac{\Delta t}{2}\mathbf{s}^{n+\frac{1}{2}} \quad (7.2.7a)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n \quad (7.2.7b)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}\right) \mathbf{v}^{n+1} = \mathbf{u}^{n+\frac{1}{2}} \quad (7.2.7c)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^{n+\frac{1}{2}} \quad (7.2.7d)$$

Notice that the RHS of (7.2.7) are all matrix-free as compared to (7.2.2). This results in substantial reduction of floating point operations (flops) count and memory indexing. Furthermore, (7.2.4) is already in the fundamental implicit form (matrix-operator free RHS), while (7.2.5) is already in the fundamental explicit form (matrix-operator free LHS). Therefore, no further simplification is required for these equations.

7.2.2 Implementation of FLOD2-CD-FDTD Method

We now present the actual implementation of the FLOD2-CD-FDTD method in detail. For additional savings of operations and memory, we have omitted the auxiliary fields h 's by absorbing them into the physical fields H 's. The main iterations comprise two updating procedures as follows

First substep from n to $n + \frac{1}{2}$:

1. Auxiliary (implicit) updating for $e^{n+\frac{1}{2}}$

$$\frac{1}{2}e_x^{n+\frac{1}{2}} - \frac{\Delta t^2}{8\mu\epsilon}\partial_y^2 e_x^{n+\frac{1}{2}} = E_x^n + \frac{\Delta t}{2\epsilon}\partial_y H_z^n - \frac{\Delta t}{2\epsilon}J_x^{n+\frac{1}{2}} - \frac{\Delta t^2}{4\mu\epsilon}\partial_y M_z^{n+\frac{1}{2}} \quad (7.2.8a)$$

$$\frac{1}{2}e_y^{n+\frac{1}{2}} - \frac{\Delta t^2}{8\mu\epsilon}\partial_z^2 e_y^{n+\frac{1}{2}} = E_y^n + \frac{\Delta t}{2\epsilon}\partial_z H_x^n - \frac{\Delta t}{2\epsilon}J_y^{n+\frac{1}{2}} - \frac{\Delta t^2}{4\mu\epsilon}\partial_z M_x^{n+\frac{1}{2}} \quad (7.2.8b)$$

$$\frac{1}{2}e_z^{n+\frac{1}{2}} - \frac{\Delta t^2}{8\mu\epsilon}\partial_x^2 e_z^{n+\frac{1}{2}} = E_z^n + \frac{\Delta t}{2\epsilon}\partial_x H_y^n - \frac{\Delta t}{2\epsilon}J_z^{n+\frac{1}{2}} - \frac{\Delta t^2}{4\mu\epsilon}\partial_x M_y^{n+\frac{1}{2}} \quad (7.2.8c)$$

2. Explicit updating for $E^{n+\frac{1}{2}}$

$$E_\xi^{n+\frac{1}{2}} = e_\xi^{n+\frac{1}{2}} - E_\xi^n, \quad \xi = x, y, z \quad (7.2.9)$$

3. Explicit updating for $H^{n+\frac{1}{2}}$

$$H_x^{n+\frac{1}{2}} = H_x^n + \frac{\Delta t}{2\mu}\partial_z e_y^{n+\frac{1}{2}} - \frac{\Delta t}{\mu}M_x^{n+\frac{1}{2}} \quad (7.2.10a)$$

$$H_y^{n+\frac{1}{2}} = H_y^n + \frac{\Delta t}{2\mu}\partial_x e_z^{n+\frac{1}{2}} - \frac{\Delta t}{\mu}M_y^{n+\frac{1}{2}} \quad (7.2.10b)$$

$$H_z^{n+\frac{1}{2}} = H_z^n + \frac{\Delta t}{2\mu}\partial_y e_x^{n+\frac{1}{2}} - \frac{\Delta t}{\mu}M_z^{n+\frac{1}{2}} \quad (7.2.10c)$$

Second substep from $n + \frac{1}{2}$ to $n + 1$:

1. Auxiliary (implicit) updating for e^{n+1}

$$\frac{1}{2}e_x^{n+1} - \frac{\Delta t^2}{8\mu\epsilon}\partial_z^2 e_x^{n+1} = E_x^{n+\frac{1}{2}} - \frac{\Delta t}{2\epsilon}\partial_z H_y^{n+\frac{1}{2}} \quad (7.2.11a)$$

$$\frac{1}{2}e_y^{n+1} - \frac{\Delta t^2}{8\mu\epsilon}\partial_x^2 e_y^{n+1} = E_y^{n+\frac{1}{2}} - \frac{\Delta t}{2\epsilon}\partial_x H_z^{n+\frac{1}{2}} \quad (7.2.11b)$$

$$\frac{1}{2}e_z^{n+1} - \frac{\Delta t^2}{8\mu\epsilon}\partial_y^2 e_z^{n+1} = E_z^{n+\frac{1}{2}} - \frac{\Delta t}{2\epsilon}\partial_y H_x^{n+\frac{1}{2}} \quad (7.2.11c)$$

2. Explicit updating for E^{n+1}

$$E_{\xi}^{n+1} = e_{\xi}^{n+1} - E_{\xi}^{n+\frac{1}{2}}, \quad \xi = x, y, z \quad (7.2.12)$$

3. Explicit updating for H^{n+1}

$$H_x^{n+1} = H_x^{n+\frac{1}{2}} - \frac{\Delta t}{2\mu} \partial_y e_z^{n+1} \quad (7.2.13a)$$

$$H_y^{n+1} = H_y^{n+\frac{1}{2}} - \frac{\Delta t}{2\mu} \partial_z e_x^{n+1} \quad (7.2.13b)$$

$$H_z^{n+1} = H_z^{n+\frac{1}{2}} - \frac{\Delta t}{2\mu} \partial_x e_y^{n+1} \quad (7.2.13c)$$

Input processing at $n = 0$ (only for nonzero initial fields $E^{t=0}$ and $H^{t=0}$) :

1. Implicit updating for E^0

$$E_x^0 - \frac{\Delta t^2}{4\mu\epsilon} \partial_z^2 E_x^0 = E_x^{t=0} - \frac{\Delta t}{2\epsilon} \partial_z H_y^{t=0} \quad (7.2.14a)$$

$$E_y^0 - \frac{\Delta t^2}{4\mu\epsilon} \partial_x^2 E_y^0 = E_y^{t=0} - \frac{\Delta t}{2\epsilon} \partial_x H_z^{t=0} \quad (7.2.14b)$$

$$E_z^0 - \frac{\Delta t^2}{4\mu\epsilon} \partial_y^2 E_z^0 = E_z^{t=0} - \frac{\Delta t}{2\epsilon} \partial_y H_x^{t=0} \quad (7.2.14c)$$

2. Explicit updating for H^0

$$H_x^0 = H_x^{t=0} - \frac{\Delta t}{2\mu} \partial_y E_z^0 \quad (7.2.15a)$$

$$H_y^0 = H_y^{t=0} - \frac{\Delta t}{2\mu} \partial_z E_x^0 \quad (7.2.15b)$$

$$H_z^0 = H_z^{t=0} - \frac{\Delta t}{2\mu} \partial_x E_y^0 \quad (7.2.15c)$$

Output processing at $n + 1$ (only whenever and wherever necessary) :

1. Explicit updating for E_{CD}^{n+1}

$$E_{x\text{CD}}^{n+1} = E_x^{n+1} + \frac{\Delta t}{2\epsilon} \partial_z H_y^{n+1} \quad (7.2.16a)$$

$$E_{y\text{CD}}^{n+1} = E_y^{n+1} + \frac{\Delta t}{2\epsilon} \partial_x H_z^{n+1} \quad (7.2.16b)$$

$$E_{z\text{CD}}^{n+1} = E_z^{n+1} + \frac{\Delta t}{2\epsilon} \partial_y H_x^{n+1} \quad (7.2.16c)$$

2. Explicit updating for H_{CD}^{n+1}

$$H_{x\text{CD}}^{n+1} = H_x^{n+1} + \frac{\Delta t}{2\mu} \partial_y E_z^{n+1} \quad (7.2.17a)$$

$$H_{y\text{CD}}^{n+1} = H_y^{n+1} + \frac{\Delta t}{2\mu} \partial_z E_x^{n+1} \quad (7.2.17b)$$

$$H_{z\text{CD}}^{n+1} = H_z^{n+1} + \frac{\Delta t}{2\mu} \partial_x E_y^{n+1} \quad (7.2.17c)$$

As mentioned earlier, the input processing is to be invoked only once at the beginning and may often be omitted for null initial fields, while the output processing may be performed independent of the main iterations at only some specific observation locations when field data are required.

It is interesting to note that the output processing of the LOD2-CD-FDTD method consists only of explicit update equations, and does not require a tridiagonal solution that is necessary for the LOD2-FDTD method. Therefore, the output processing for the former is in general more efficient.

7.2.3 Temporal Order of LOD2-CD-FDTD Method

Next, we proceed to determine the temporal order of the LOD2-CD-FDTD method. The main iterations of the LOD2-CD-FDTD method comprise the LOD1-FDTD update equations with n iterations, i.e.,

$$\mathbf{G}_{\text{LOD1}}^n = \left[\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B} \right)^{-1} \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{B} \right) \left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A} \right)^{-1} \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{A} \right) \right]^n \quad (7.2.18)$$

where $\mathbf{G}_{\text{LOD1}}$ is the amplification matrix of the LOD1-FDTD method. Subsequently (7.2.18) can be expanded as

$$\begin{aligned} \mathbf{G}_{\text{LOD1}}^n = \mathbf{I} + n\Delta t(\mathbf{A} + \mathbf{B}) + \left(\frac{n\Delta t^2}{2} \right) \left[n\mathbf{A}^2 + (n-1)\mathbf{AB} + (n+1)\mathbf{BA} + n\mathbf{B}^2 \right] \\ + O(\Delta t^3) \end{aligned} \quad (7.2.19)$$

which shows only the first-order temporal accuracy. The proof of (7.2.19) by mathematical induction is provided in Appendix B. Next, we perform input processing and some manipulation as

$$\begin{aligned} \mathbf{G}_{\text{LOD1}}^n \left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B} \right)^{-1} = \mathbf{I} + \Delta t \left[n\mathbf{A} + \left(n + \frac{1}{2} \right) \mathbf{B} \right] + \frac{\Delta t^2}{2} \left[n^2 \mathbf{A}^2 + n^2 \mathbf{AB} \right. \\ \left. + (n^2 + n) \mathbf{BA} + \left(n^2 + n + \frac{1}{2} \right) \mathbf{B}^2 \right] + O(\Delta t^3) \end{aligned} \quad (7.2.20)$$

Finally by performing output processing, we can obtain the amplification matrix of the LOD2-CD-FDTD method

$$\begin{aligned} \mathbf{G}_{\text{CD}}^n &= \left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B} \right) \mathbf{G}_{\text{LOD1}}^n \left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B} \right)^{-1} \\ &= \mathbf{I} + n\Delta t(\mathbf{A} + \mathbf{B}) + \frac{n^2 \Delta t^2}{2} (\mathbf{A}^2 + \mathbf{AB} + \mathbf{BA} + \mathbf{B}^2) + O(\Delta t^3) \end{aligned} \quad (7.2.21)$$

Figure 7.1: Normalized norm errors vs. p of geometric time subdivision ($\Delta t = \Delta t_{CFL}/2^p$) for the LOD2-CD-FDTD, LOD1-FDTD method and ADI-FDTD method. ©2013 IEEE.

Equation (7.2.21) agrees with the analytical solution of $e^{(\mathbf{A}+\mathbf{B})\Delta t}$ throughout the Δt^2 terms. Thus, the LOD2-CD-FDTD method is accurate to second-order in time.

To show the temporal order of the LOD2-CD-FDTD method numerically, we chose the cells per freespace wavelength (CPW) as 50 and 500 at 3 GHz. Fig. 7.1 plots the normalized norm errors vs. p of geometric time subdivision ($\Delta t = \Delta t_{CFL}/2^p$) for the LOD2-CD-FDTD method and LOD1-FDTD method. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{3}}$, where c is the speed of light in freespace. It can be seen that the norm errors of the LOD2-CD-FDTD method decrease with second order, while the norm errors of the LOD1-FDTD method decrease with first order.

The LOD2-CD-FDTD method is an improved extension of the LOD1-FDTD method, which is an unconditionally stable FDTD method since the magnitudes of the eigenvalues of $\mathbf{G}_{\text{LOD1}}$ are unity. The input and output processing of the LOD2-CD-FDTD method perform simple additional arithmetic operations that are never iterated in the main iterations, and as such will not cause the field components to grow without

Table 7.1: Comparison Of Main Features Of Various FDTD Methods. ©2013 IEEE.

Method	Unconditionally Stable	Complexity	Total (RHS) M/D + A/S	Field Array	Temporal Order	Complying Divergence
Conventional explicit FDTD	No	Low	30	6	2 nd	Yes
ADI-FDTD	Yes	Moderate High	102	9	2 nd	No
LOD1-FDTD	Yes	Moderate Low	72	9	1 st	No
LOD2-FDTD	Yes	Moderate Low	72	9	2 nd	No
Fundamental LOD2-CD-FDTD	Yes	Low	42	9	2 nd	Yes

bound. Indeed from (7.2.21), $(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B})$ and its inverse are not raised to n . It is also interesting to note that $\mathbf{G}_{\text{CD}}^n$ and $\mathbf{G}_{\text{LOD1}}^n$ are related simply by similarity transformation. Thus, the LOD2-CD-FDTD method will also be unconditionally stable in an analogous manner to the LOD1-FDTD method. The stability condition of the LOD2-CD-FDTD method is the same as the LOD1-FDTD method.

Table 7.1 presents the comparison of the main features of the conventional explicit FDTD method, ADI-FDTD method, LOD1-FDTD method, LOD2-FDTD method and FLOD2-CD-FDTD method. It is clear that the FLOD2-CD-FDTD method has most of the main features of the conventional explicit FDTD method while being unconditionally stable. In addition, the table also includes the total flops count for multiplications/divisions (M/D) and additions/subtractions (A/S) required for one complete time step of various FDTD methods. Among the unconditionally stable FDTD methods, it is apparent that the FLOD2-CD-FDTD method has the least flops count. This flops reduction is due to the fundamental form of the FLOD2-CD-FDTD method, with its RHS free of matrix operators. Furthermore, the memory requirement of the FLOD2-CD-FDTD method is still the same as the ADI-FDTD method and LOD1-FDTD method. Note that the flops count does not include the

computation cost of solving the tridiagonal matrix. Detailed discussion for the above may be found in [14].

7.3 Numerical Results

In this section, we present numerical results to compare and validate the divergence-complying property of the LOD2-CD-FDTD method for 3-D Maxwell's equations. The Gauss's law for the electric field in differential form states that

$$\nabla \cdot \mathbf{D} = \nabla \cdot \epsilon \mathbf{E} = \rho_v \quad (7.3.1)$$

where $\mathbf{D}$ is the electric flux density and ρ_v is the volume charge density. In the absence of charge, (7.3.1) states that the divergence of the electric flux density is zero. To examine the divergence of electric flux density, we employ the finite-difference method to (7.3.1) as [86]

$$\nabla \cdot \epsilon \mathbf{E} = \partial_x(\epsilon E_x) + \partial_y(\epsilon E_y) + \partial_z(\epsilon E_z). \quad (7.3.2)$$

7.3.1 Numerical Validation

To validate the divergence-complying property of the LOD2-CD-FDTD method, we simulate an air-filled perfect electric conductor cavity meshed with 50 x 30 x 9 uniform cell of size 2 mm each. A point source excited by a differentiated Gaussian pulse is located at the center of the cavity as

$$J_z = \frac{t - t_0}{\tau} e^{-\left(\frac{t-t_0}{\tau}\right)^2} \quad (7.3.3)$$

Figure 7.2: Time-domain E_z field component computed using conventional explicit FDTD, LOD2-CD-FDTD, ADI-FDTD and LOD1-FDTD for CFLN = 1. Good agreement is shown between the LOD2-CD-FDTD method and the conventional explicit FDTD method. ©2013 IEEE.

where

$$\tau = 150 \text{ ps}, t_0 = 3\tau.$$

The divergence of electric flux density is computed numerically using (7.3.2), and converted to a decibel scale referenced to 1 Coulomb per meter cube (C/m^3) as

$$\nabla \cdot \hat{\mathbf{D}} \text{ (dB)} = 10 \log_{10} \left(\frac{\nabla \cdot \mathbf{D}}{1 \text{ C}/\text{m}^3} \right). \quad (7.3.4)$$

We compare the divergence of $\hat{\mathbf{D}}$ for the LOD2-CD-FDTD method, conventional explicit FDTD method, ADI-FDTD method and LOD1-FDTD method. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{3}}$ where c is the speed of light in freespace. We also denote $\text{CFLN} = \frac{\Delta t}{\Delta t_{CFL}}$.

Fig. 7.2 graphs the time-domain E_z field component computed using the conventional explicit FDTD method, LOD2-CD-FDTD method, ADI-FDTD method and LOD1-FDTD method for CFLN = 1. Here, the output processing is performed at

only one location (observation point) for all time steps as

$$E_{z\text{CD}}|_{25,15,5\frac{1}{2}}^{n+1} = E_z|_{25,15,5\frac{1}{2}}^{n+1} + \frac{\Delta t}{2\epsilon\Delta y} \left(H_x|_{25,15\frac{1}{2},5\frac{1}{2}}^{n+1} - H_x|_{25,14\frac{1}{2},5\frac{1}{2}}^{n+1} \right) \quad (7.3.5)$$

Good agreement is shown between the LOD2-CD-FDTD method and the conventional explicit FDTD method. However, the E_z field component of the ADI-FDTD method and the LOD1-FDTD method differs from the conventional explicit FDTD and LOD2-CD-FDTD method.

During the source excitation period, the E_z field component is around its peak value at time $t = 0.446$ ns, while at time $t = 1.232$ ns the source excitation has already died off. Henceforth, we shall sample the electric fields at time $t = 0.446$ ns to investigate the divergence of $\hat{\mathbf{D}}$ in the presence of a source. Likewise, to investigate the divergence of $\hat{\mathbf{D}}$ in the absence of a source, we sample the electric fields at time $t = 1.232$ ns. As discussed previously, the presence of a source would indicate the existence of a corresponding charge density.

Fig. 7.3 shows the divergence of $\hat{\mathbf{D}}$ for the conventional explicit FDTD method, LOD2-CD-FDTD method, ADI-FDTD method and LOD1-FDTD method in the grid regions around the source point for CFLN = 1 at time $t = 0.446$ ns. The divergence of $\hat{\mathbf{D}}$ is plotted in the xy -plane at $k = 5$. The peak in the middle of the grid indicates the source excitation location, while the rest of the grids are the source-free regions. Here, the output processing is performed at only one time step (observation time instant) in the xy -plane at $k = 5$ and is given by

$$E_{z\text{CD}}|_{i,j,5\frac{1}{2}}^{n+1=116} = E_z|_{i,j,5\frac{1}{2}}^{n+1=116} + \frac{\Delta t}{2\epsilon\Delta y} \left(H_x|_{i,j+\frac{1}{2},5\frac{1}{2}}^{n+1=116} - H_x|_{i,j-\frac{1}{2},5\frac{1}{2}}^{n+1=116} \right) \quad (7.3.6)$$

Figure 7.3: Divergence of $\hat{\mathbf{D}}$ for CFLN = 1 at time $t = 0.446$ ns (presence of source excitation). (a) Conventional explicit FDTD. (b) LOD2-CD-FDTD. (c) ADI-FDTD. (d) LOD1-FDTD. The divergence of $\hat{\mathbf{D}}$ for the LOD2-CD-FDTD method agrees well with the conventional explicit FDTD method. ©2013 IEEE.

Referring to Fig. 7.3, it is clear that the LOD2-CD-FDTD method has similar divergence-complying property as the conventional explicit FDTD method. At the source excitation location, the divergence of $\hat{\mathbf{D}}$ for both LOD2-CD-FDTD and conventional explicit FDTD methods is approximately - 58 dB, while at the source-free regions, the divergence of $\hat{\mathbf{D}}$ is about - 250 dB. Such level of divergence of $\hat{\mathbf{D}}$ for the source-free regions is good enough subjected to numerical noise floor.

Figure 7.4: Divergence of $\hat{\mathbf{D}}$ for CFLN = 8 at time $t = 0.446$ ns (presence of source excitation) (a) LOD2-CD-FDTD. (b) ADI-FDTD. (c) LOD1-FDTD. The divergence-complying property of the LOD2-CD-FDTD method is still evident even at higher CFLN. ©2013 IEEE.

On the other hand, the divergence of $\hat{\mathbf{D}}$ for the ADI-FDTD method and the LOD1-FDTD method is significantly higher than the conventional explicit FDTD method and the LOD2-CD-FDTD method in the source-free regions. Thus, it is evident that the LOD2-CD-FDTD method has complying divergence, while the ADI-FDTD and LOD1-FDTD methods do not have complying divergence.

It is of interest to ascertain the divergence-complying property of the LOD2-CD-

Figure 7.5: Divergence of $\hat{\mathbf{D}}$ for CFLN = 8 at time $t = 1.232$ ns (absence of source excitation) (a) LOD2-CD-FDTD. (b) ADI-FDTD. (c) LOD1-FDTD. ©2013 IEEE.

FDTD method at higher CFLN. To that end, we chose CFLN = 8 and exclude the conventional explicit FDTD method (as it is unstable for CFLN > 1). Fig. 7.4 shows the snapshot of the divergence of $\hat{\mathbf{D}}$ for the LOD2-CD-FDTD method, ADI-FDTD method and LOD1-FDTD method in the grid regions around the source for CFLN = 8 at time $t = 0.446$ ns. The divergence-complying property of the LOD2-CD-FDTD method is still evident even at higher CFLN. However, the divergence of $\hat{\mathbf{D}}$ for the ADI-FDTD and LOD1-FDTD methods increases slightly at the source-free regions for CFLN = 8.

Next, we illustrate the divergence-complying property of the LOD2-CD-FDTD method in the absence of source for $\text{CFLN} = 8$. Fig. 7.5 presents the snapshot of the divergence of $\hat{\mathbf{D}}$ for the LOD2-CD-FDTD method, ADI-FDTD method and LOD1-FDTD method in the grid regions around the source point for $\text{CFLN} = 8$ at time $t = 1.232$ ns.

Referring to Fig. 7.5(a), it can be observed that the divergence-complying property of the LOD2-CD-FDTD method remains the same even when the source excitation has died off. At the source excitation location, the divergence of $\hat{\mathbf{D}}$ for the LOD2-CD-FDTD method is approximately -104 dB. This static charge is deposited onto the Yee's cell by the source excitation and remains in the grid. Although it is not shown here, the conventional explicit FDTD method (for $\text{CFLN} \leq 1$) also displays similar phenomenon [84].

Upon comparing Figs. 7.5(b) and 7.5(c), we note that the divergence of $\hat{\mathbf{D}}$ for the ADI-FDTD method is lower than the LOD1-FDTD method. For such methods, spurious charges exist even in the source-free regions. These spurious charges may affect the accuracy of simulations especially for problems involving capacitances or passive lumped elements [87].

7.3.2 Divergence Analysis

In this subsection, we present further analysis of the divergence property to compare the divergence of electric field for the LOD2-CD-FDTD method, conventional explicit FDTD method, ADI-FDTD method and LOD1-FDTD method. For brevity, details of the analysis have been omitted but may be found in Chapter 6.

Fig. 7.6 plots the divergence of $\hat{E}_0$ (dB) vs. propagation angle ϕ for the conventional explicit FDTD method, LOD2-CD-FDTD method, ADI-FDTD method and LOD1-FDTD method for CFLN = 1 and CPW = 20. Here, propagation angles θ and ϕ are defined using the spherical coordinate system (θ = elevation angle and ϕ = azimuthal angle). From the figures, it can be observed that the ADI-FDTD method and LOD1-FDTD method are not divergence-free in the source-free regions. On the other hand, the divergence of $\hat{E}_0$ for the LOD2-CD-FDTD method is much lower than the ADI-FDTD method and LOD1-FDTD method. From the divergence analysis as well as numerical validation in the previous subsection, we have shown that the LOD2-CD-FDTD method is divergence-complying in a manner analogous to the conventional explicit FDTD method.

Figure 7.6: Divergence of $\hat{E}_0$ (dB) vs. propagation angle ϕ for the conventional explicit FDTD, LOD2-CD-FDTD, ADI-FDTD and LOD1-FDTD method for CFLN = 1 and CPW = 20. (a) $\theta = 30^\circ$ (b) $\theta = 60^\circ$ (θ = elevation angle and ϕ = azimuthal angle). ©2013 IEEE.

Figure 7.7: Time-domain E_z field component computed using the analytical method, conventional explicit FDTD method and LOD2-CD-FDTD method. ©2013 IEEE.

7.3.3 Accuracy and Efficiency

To demonstrate the accuracy of the LOD2-CD-FDTD method, we simulate an air-filled cavity meshed with $50 \times 50 \times 50$ uniform cells of size 2 mm each. It is initialized with TE_{111} mode fields at time $t = 0$ and the observation point is located at cell (31,31,26). Fig. 7.7 depicts the time-domain E_z field component computed using the analytical method, conventional explicit FDTD method and LOD2-CD-FDTD method. The CFLN for the conventional explicit FDTD method is 1, while the CFLN for the LOD2-CD-FDTD method is 4. The LOD2-CD-FDTD method agrees well with the analytical solution of the TE_{111} mode fields.

Next, we compare the efficiency of various FDTD methods based on the CPU time taken to simulate a cavity of $100 \times 100 \times 100$ uniform cells. The simulations are terminated at the same time instant $t = \frac{20000 \Delta t_{CFL}}{CFLN}$. The CFLN is chosen as 1 and 4. Hence, the number of iterations are 20000 and 5000, respectively. The programs have been compiled using Microsoft Visual C++ under Microsoft Win7 (32-Bit) operating system, and run on a platform of 3.1 GHz Intel I5 processor with 4 GB RAM.

Table 7.2: CPU Time (seconds) of Various FDTD Methods. ©2013 IEEE.

Method	CFLN 1	CFLN 4
Conventional explicit FDTD	1738	-
ADI-FDTD	6049	1520
LOD1-FDTD	5127	1289
Fundamental LOD2-CD-FDTD	4278	1067

Table 7.2 compares the CPU time (seconds) taken by the conventional explicit FDTD method, ADI-FDTD method, LOD1-FDTD method and FLOD2-CD-FDTD method. As expected, the unconditionally stable FDTD methods require more CPU time than the conventional explicit FDTD method for CFLN = 1. However for CFLN = 4, the FLOD2-CD-FDTD method requires 38.6% less time than the conventional explicit FDTD method, and 17.2% less time than the LOD1-FDTD method.

7.3.4 Numerical Example

In this example, we simulate a bow-tie antenna which may be useful for the detection of breast cancer [88]. The computation domain consists of 100 x 100 x 50 uniform cells with size $\Delta = \Delta_x = \Delta_y = \Delta_z = 1$ mm. It is terminated by a ten cell perfectly matched layer absorbing boundary condition, which will be discussed in the later part of this chapter.

Fig. 7.8 illustrates the configuration of the bow-tie antenna. It is fed at its apex with a sinusoidal modulated Gaussian pulse excitation given by

$$J_x = \sin \left[2\pi f_0 \left(t - t_0 \right) \right] e^{-\left(\frac{t-t_0}{\tau} \right)^2} \quad (7.3.7)$$

Figure 7.8: Configuration of bow-tie antenna.

where

$$f_0 = 2 \text{ GHz}, \tau = 150 \text{ ps}, t_0 = 3\tau.$$

The CFLN for the conventional explicit FDTD method is 1, while the CFLN for the implicit FDTD methods is 8. Fig. 7.9 compares the divergence of $\hat{\mathbf{D}}$ for the bow-tie antenna at time $t = 0.962 \text{ ns}$ for the conventional explicit FDTD method, LOD2-CD-FDTD method, ADI-FDTD method and LOD1-FDTD method. It can be observed from the figure that charges are only deposited onto the surface of the bow-tie antenna for the LOD2-CD-FDTD method and the conventional explicit FDTD method. Moreover, it is interesting to note that the geometry of the bow-tie antenna can be clearly seen.

However, significant amount of charge can still be found in the source-free regions surrounding the bow-tie antenna for the ADI-FDTD method and LOD1-FDTD method. It is not clear where the boundaries of the bow-tie antenna are, as the charges on the bow-tie antenna are almost the same as the surrounding source-free regions.

Figure 7.9: Divergence of $\hat{\mathbf{D}}$ for the bow-tie antenna at time $t = 0.962$ ns. (a) Conventional explicit FDTD with $\text{CFLN} = 1$. (b) LOD2-CD-FDTD with $\text{CFLN} = 8$. (c) ADI-FDTD with $\text{CFLN} = 8$. (d) LOD1-FDTD with $\text{CFLN} = 8$. The geometry of the bow-tie antenna can be clearly seen for both conventional explicit FDTD and LOD2-CD-FDTD methods. ©2013 IEEE.

For the conventional explicit FDTD method and LOD2-CD-FDTD method, the divergence of $\hat{\mathbf{D}}$ in freespace is about 100 dB lower than the divergence of $\hat{\mathbf{D}}$ on the surface of the bow-tie antenna. On the other hand, for the ADI-FDTD method and LOD1-FDTD method, the divergence of $\hat{\mathbf{D}}$ in freespace about 20 cells away from the boundary of the bow-tie antenna is only 20 dB lower than the divergence of $\hat{\mathbf{D}}$ on the surface of the bow-tie antenna.

7.4 Electric Charge Deposition on Dipole Antenna

7.4.1 Introduction

So far, we have ascertain that the ADI-FDTD and LOD1-FDTD methods are not divergence-complying in the source-free regions. On the contrary, the LOD2-CD-FDTD method is divergence-complying in the source-free regions in a manner similar to the conventional explicit FDTD method. In this section, we continue to investigate the divergence-complying property of the LOD2-CD-FDTD method at the source region, by analyzing the electric charge deposited at the ends of a dipole radiator [84].

7.4.2 Continuity Equation - Current and Charge

The continuity equation of Maxwell's equations provides the necessary link between the electric current density and the electric charge density. In differential form, it states that

$$\nabla \cdot \mathbf{J} = -\partial_t p_v \quad (7.4.1)$$

where ∂_t is the temporal difference operator. The divergence of the electric current density, on the right-hand side of (7.4.1), can be evaluated using the finite-difference method

$$\nabla \cdot \mathbf{J} = \partial_x J_x + \partial_y J_y + \partial_z J_z \quad (7.4.2)$$

Consider a non-trivial case where $\mathbf{J} \neq 0$ at the ends of a dipole radiator, i.e., freespace to PEC boundary. The electric current density in freespace is zero, while

the electric current density on the surface of the dipole radiator is nonzero. Thus referring to (7.4.2), it is expected that $\nabla \cdot \mathbf{J}$ is nonzero at the ends of the dipole radiator. Hence, $\partial_t p_v$ is also nonzero. For source excitations such as the Gaussian pulse where the time-integral of the source waveform has nonzero DC component, electric charges may be deposited at the ends of the dipole radiator for the various FDTD methods.

7.4.3 Numerical Illustration

To illustrate the electric charge deposition at the ends of a dipole radiator, we perform some numerical simulations for the LOD2-CD-FDTD method. The computation domain consists of 80 x 80 x 80 uniform cells of size $\Delta = \Delta x = \Delta y = \Delta z = 1$ m. It is truncated by a PML of ten cell thickness. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{3}}$ where c is the speed of light in freespace and $CFLN = \frac{\Delta t}{\Delta t_{CFL}}$. A Gaussian pulse excitation is placed at the center of the dipole radiator (40,40,40) as

$$J_z = e^{-\left(\frac{t-t_0}{\tau}\right)^2} \quad (7.4.3)$$

where

$$\tau = 40 \text{ ns}, \quad t_0 = 3\tau$$

To determine the total electric charge at the end of the dipole radiator, we employ the integral form of Gauss's Law for electric field as follows:

$$\epsilon_0 \oint_S \mathbf{E} \cdot d\mathbf{s} = \iiint_V p_v dv = Q_{\text{enclosed}} \quad (7.4.4)$$

Figure 7.10: Electric charge deposited at the ends of the dipole radiator as a function of time computed using the conventional explicit FDTD method, LOD2-CD-FDTD method and ADI-FDTD method for $CFLN = 1$.

Fig. 7.10 depicts the electric charge deposited at the ends of the dipole radiator as a function of time computed using the conventional explicit FDTD method, LOD2-CD-FDTD method and ADI-FDTD method for $CFLN = 1$. The electric current waveform is normalized to the peak electric charge. Referring to Fig. 7.10, it can be seen that when the electric current changes with time, the electric charge also changes with time correspondingly. Moreover, after time $t = 200$ ns when the electric current is constant, the electric charge also remains constant. There is no time rate-of-change of electric charge and it remains at the same level as before.

Fig. 7.11 plots the electric charge deposited at the ends of the dipole radiator as a function of time computed using the conventional explicit FDTD method, LOD2-CD-FDTD method and ADI-FDTD method. The $CFLN$ for the conventional explicit FDTD method is 1, while the $CFLN$ for the LOD2-CD-FDTD method and ADI-FDTD method is 4. The electric charge deposited at the ends of the dipole radiator for the LOD2-CD-FDTD method and ADI-FDTD method decreases when $CFLN$ is increased. However, the reduction is much more significant for the ADI-FDTD method than the LOD2-CD-FDTD method.

Figure 7.11: Electric charge deposited at the ends of the dipole radiator as a function of time computed using the conventional explicit FDTD method, LOD2-CD-FDTD method and ADI-FDTD method for various CFLN.

Table 7.3: Electric charge (nC) deposited at the end of a dipole radiator for various FDTD methods for CFLN = 1 and 4.

Method	CFLN 1	CFLN 4
FDTD	28.66	-
LOD2-CD-FDTD	27.65	21.17
ADI-FDTD	25.25	9.769

Table 7.3 further examines the electric charge deposited at the end of the dipole radiator for the various FDTD methods. From the table, it is clear that the LOD2-CD-FDTD method has fairly good agreement with the conventional explicit FDTD method. Therefore, the LOD2-CD-FDTD method is divergence-complying in both the source region as well as the source-free region in a manner similar to the conventional explicit FDTD method.

On the other hand, the ADI-FDTD method has much larger error than the LOD2-CD-FDTD method especially at higher CFLN. For the ADI-FDTD method with CFLN = 4, the electric charge deposited at the end of the dipole radiator is about three times less than the conventional explicit FDTD method.

7.5 Convolutional Perfectly Matched Layer for Fundamental LOD2-CD-FDTD Method

7.5.1 Introduction

In the preceding sections, an unconditionally stable LOD-FDTD method with second-order temporal accuracy and complying divergence (LOD2-CD-FDTD) has been successfully developed for 3-D Maxwell's equations. However, in order to solve open radiation and scattering problems, the computation domain must be truncated by a suitable absorbing boundary condition. The convolutional perfectly matched layer (CPML) is commonly employed for such unbounded problems due to its effectiveness and simplicity [56, 90].

So far, the CPML formulation for the LOD2-CD-FDTD method has been presented for the numerical examples without explanation. In the following, we extend the LOD2-CD-FDTD method by incorporating the CPML. To further enhance efficiency, the LOD2-CD-FDTD method with CPML is formulated into the fundamental form with the simplest and most concise RHS free of matrix operators. The explicit output processing of the LOD2-CD-FDTD method has complying divergence, and it is also independent of the CPML media. Numerical results will be presented to validate the formulation.

7.5.2 CPML for LOD2-CD-FDTD Method

In this subsection, we present the LOD2-CD-FDTD method with CPML for 3-D Maxwell's equations. The electromagnetic waves are assumed to propagate in a medium with permittivity ϵ and permeability μ . We define the splitting matrix

operators $\mathbf{A}$ and $\mathbf{B}$ as

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{\epsilon \kappa_y} \partial_y \\ 0 & 0 & 0 & \frac{1}{\epsilon \kappa_z} \partial_z & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\epsilon \kappa_x} \partial_x & 0 \\ 0 & \frac{1}{\mu \kappa_z} \partial_z & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\mu \kappa_x} \partial_x & 0 & 0 & 0 \\ \frac{1}{\mu \kappa_y} \partial_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7.5.1a)$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{-1}{\epsilon \kappa_z} \partial_z & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{\epsilon \kappa_x} \partial_x \\ 0 & 0 & 0 & \frac{-1}{\epsilon \kappa_y} \partial_y & 0 & 0 \\ 0 & 0 & \frac{-1}{\mu \kappa_y} \partial_y & 0 & 0 & 0 \\ \frac{-1}{\mu \kappa_z} \partial_z & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{\mu \kappa_x} \partial_x & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7.5.1b)$$

where ∂_x , ∂_y , ∂_z are the spatial difference operators for the first derivatives along x , y , z directions, respectively. In addition, κ is the stretching variable of the CPML region. For non-PML regions, κ is simply set to 1, and matrix operators $\mathbf{A}$ and $\mathbf{B}$ then becomes the same as that defined previously in Section 7.2.

The updating procedures for the LOD2-CD-FDTD method with CPML may be

compactly expressed as

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}\right) \mathbf{u}^{n+\frac{1}{2}} = \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{A}\right) \mathbf{u}^n + \Delta t \mathbf{W}_A \Psi^n \quad (7.5.2a)$$

$$\Psi^{n+\frac{1}{2}} = \mathbf{C} \Psi^n + \mathbf{D} \mathbf{u}^{n+\frac{1}{2}} \quad (7.5.2b)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B}\right) \mathbf{u}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{B}\right) \mathbf{u}^{n+\frac{1}{2}} + \Delta t \mathbf{W}_B \Psi^{n+\frac{1}{2}} \quad (7.5.2c)$$

$$\Psi^{n+1} = \mathbf{C} \Psi^{n+\frac{1}{2}} + \mathbf{D} \mathbf{u}^{n+1} \quad (7.5.2d)$$

for the main iterations, where

$$\mathbf{u} = \begin{bmatrix} E_x & E_y & E_z & H_x & H_y & H_z \end{bmatrix}^T \quad (7.5.3)$$

are the main field variables. The CPML terms are defined as follows:

$$\mathbf{W}_A = \begin{bmatrix} \frac{1}{\epsilon} \Theta_A & \mathbf{O}_{3 \times 6} \\ \mathbf{O}_{3 \times 6} & \frac{1}{\mu} \Theta_A \end{bmatrix}, \quad \mathbf{W}_B = \begin{bmatrix} \frac{1}{\epsilon} \Theta_B & \mathbf{O}_{3 \times 6} \\ \mathbf{O}_{3 \times 6} & \frac{1}{\mu} \Theta_B \end{bmatrix} \quad (7.5.4)$$

$$\Theta_A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \Theta_B = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (7.5.5)$$

$$\Psi = \begin{bmatrix} \Psi_E & \Psi_H \end{bmatrix}^T \quad (7.5.6a)$$

$$\Psi_E = \begin{bmatrix} \psi_{exy} & \psi_{exz} & \psi_{eyz} & \psi_{eyx} & \psi_{ezx} & \psi_{ezy} \end{bmatrix}^T \quad (7.5.6b)$$

$$\Psi_H = \begin{bmatrix} \psi_{hxz} & \psi_{hxy} & \psi_{hyx} & \psi_{hyz} & \psi_{hzy} & \psi_{hzx} \end{bmatrix}^T \quad (7.5.6c)$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_e & \mathbf{O}_{6 \times 6} \\ \mathbf{O}_{6 \times 6} & \mathbf{C}_h \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{O}_{6 \times 3} & \mathbf{D}_e \\ \mathbf{D}_h & \mathbf{O}_{6 \times 3} \end{bmatrix} \quad (7.5.7)$$

$$\mathbf{C}_e = \begin{bmatrix} c_y & 0 & 0 & 0 & 0 & 0 \\ 0 & c_z & 0 & 0 & 0 & 0 \\ 0 & 0 & c_z & 0 & 0 & 0 \\ 0 & 0 & 0 & c_x & 0 & 0 \\ 0 & 0 & 0 & 0 & c_x & 0 \\ 0 & 0 & 0 & 0 & 0 & c_y \end{bmatrix}, \quad \mathbf{C}_h = \begin{bmatrix} c_z & 0 & 0 & 0 & 0 & 0 \\ 0 & c_y & 0 & 0 & 0 & 0 \\ 0 & 0 & c_x & 0 & 0 & 0 \\ 0 & 0 & 0 & c_z & 0 & 0 \\ 0 & 0 & 0 & 0 & c_y & 0 \\ 0 & 0 & 0 & 0 & 0 & c_x \end{bmatrix} \quad (7.5.8)$$

$$\mathbf{D}_e = \begin{bmatrix} 0 & 0 & d_y \partial_y \\ 0 & d_z \partial_z & 0 \\ d_z \partial_z & 0 & 0 \\ 0 & 0 & d_x \partial_x \\ 0 & d_x \partial_x & 0 \\ d_y \partial_y & 0 & 0 \end{bmatrix}, \quad \mathbf{D}_h = \begin{bmatrix} 0 & d_z \partial_z & 0 \\ 0 & 0 & d_y \partial_y \\ 0 & 0 & d_x \partial_x \\ d_z \partial_z & 0 & 0 \\ d_y \partial_y & 0 & 0 \\ 0 & d_x \partial_x & 0 \end{bmatrix} \quad (7.5.9)$$

$$c_\zeta = e^{-\left(\frac{\sigma_\zeta}{\kappa_\zeta} + \alpha_\zeta\right) \frac{\Delta t}{2\epsilon}} \quad (7.5.10)$$

$$d_\zeta = \frac{\sigma_\zeta}{\kappa_\zeta (\sigma_\zeta + \kappa_\zeta \alpha_\zeta)} (c_\zeta - 1) \quad (7.5.11)$$

where $\mathbf{O}_{p \times q}$ is the null matrix with $p \times q$ dimension. c_ζ and d_ζ are the update coefficients for Ψ within the CPML region.

For nonzero initial fields $\mathbf{u}^{t=0}$, implicit input processing needs to be invoked (only

once) at time step $n = 0$ as

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}^0 = \mathbf{u}^{t=0} \quad (7.5.12)$$

Moreover, the explicit output processing at time step $n + 1$ is

$$\mathbf{u}_{\text{CD}}^{n+1} = \left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B}\right) \mathbf{u}^{n+1} \quad (7.5.13)$$

where the subscript “CD” denotes complying divergence.

It is worthwhile to highlight that Ψ is not required for the input and output processing. This is because, for input processing, we only need to initialize the fields within the computation domain (non-PML region). Furthermore, the input processing may often be omitted for null initial fields. Similarly, the explicit output processing is performed independent of the main iterations at only some specific observation locations (non-PML regions) whenever the field data are required.

In addition, the field variables of the LOD2-CD-FDTD method are updated in one-dimensional locally for each procedure. Hence, $\mathbf{W}_A$ and $\mathbf{W}_B$ are used for the first and second updating procedures, respectively. On the other hand, the field variables of the ADI-FDTD method are updated in alternating direction for both procedures. Thus, $\mathbf{W}$ is employed for both procedures for the ADI-FDTD method [68].

7.5.3 CPML for Fundamental LOD2-CD-FDTD Method

To further improve efficiency, we formulate the LOD2-CD-FDTD method with CPML into the fundamental form. Equation (7.5.2a) can be expressed as

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}\right) \mathbf{u}^{n+\frac{1}{2}} = \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{A}\right) \mathbf{u}^n + \Delta t \mathbf{W}_A \Psi^n \quad (7.5.14a)$$

$$= -\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}\right) \mathbf{u}^n + \Delta t \mathbf{W}_A \Psi^n + 2 \mathbf{u}^n \quad (7.5.14b)$$

where (7.5.14b) can be manipulated to read:

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}\right) \left(\mathbf{u}^{n+\frac{1}{2}} + \mathbf{u}^n\right) = 2 \mathbf{u}^n + \Delta t \mathbf{W}_A \Psi^n \quad (7.5.15)$$

Here, we introduce an auxiliary variable

$$\mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^{n+\frac{1}{2}} + \mathbf{u}^n \quad (7.5.16)$$

Subsequently, (7.5.2a)-(7.5.2b) of the first substep can be rewritten as

$$\left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \mathbf{A}\right) \mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^n + \frac{\Delta t}{2} \mathbf{W}_A \Psi^n \quad (7.5.17a)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n \quad (7.5.17b)$$

$$\Psi^{n+\frac{1}{2}} = \mathbf{C} \Psi^n + \mathbf{D} \mathbf{u}^{n+\frac{1}{2}} \quad (7.5.17c)$$

By repeating the above procedures for (7.5.2c)-(7.5.2d), the updating procedures of the second substep are

$$\left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \mathbf{B}\right) \mathbf{v}^{n+1} = \mathbf{u}^{n+\frac{1}{2}} + \frac{\Delta t}{2} \mathbf{W}_B \Psi^{n+\frac{1}{2}} \quad (7.5.18a)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^{n+\frac{1}{2}} \quad (7.5.18b)$$

$$\Psi^{n+1} = \mathbf{C} \Psi^{n+\frac{1}{2}} + \mathbf{D} \mathbf{u}^{n+1} \quad (7.5.18c)$$

Figure 7.12: Time-domain E_z field component computed using the conventional explicit FDTD method with CPML and the FLOD2-CD-FDTD method with CPML. ©2013 IEEE.

The main iterations (7.5.17)-(7.5.18) with implicit input processing (7.5.12) and explicit output processing (7.5.13) form the efficient FLOD2-CD-FDTD method with CPML. Notice that the RHS of (7.5.17)-(7.5.18) are all matrix-operator free, i.e., no matrix operators $\mathbf{A}$ and $\mathbf{B}$. As a result, the total arithmetic operations of the fundamental LOD2-CD-FDTD method is significantly reduced.

7.5.4 Numerical Results

In this section, we examine the performance of the FLOD2-CD-FDTD method with CPML through numerical simulations. The dimension of the computation domain is $42 \times 42 \times 42$ grids with uniform cell size of $\Delta = \Delta x = \Delta y = \Delta z = 1$ mm. Furthermore, the computation domain also includes ten cells of CPML so as to absorb the impinging electromagnetic waves. For subsequent simulations, the CPML parameters are defined as $\sigma_{\zeta_{\max}} = \sigma_{opt} = 10.61$ S/m, $\kappa_{\zeta_{\max}} = 15$, $\alpha_{\zeta} = 0.08$ and $m = 4$.

A sinusoid modulated Gaussian pulse excitation is located at the center of the

Figure 7.13: Reflection errors (dB) as a function of time for the conventional explicit FDTD method with CPML and the FLOD2-CD-FDTD method with CPML. ©2013 IEEE.

computation domain as

$$J_z = \sin \left[2\pi f_0 (t - t_0) \right] e^{-\left(\frac{t-t_0}{\tau} \right)^2} \quad (7.5.19)$$

where $f_0 = 3.175$ GHz, $\tau = 160$ ps and $t_0 = 3\tau$. The observation point is located one cell away from the PML interface. The Courant limit time step size is $\Delta t_{CFL} = \frac{\Delta}{c\sqrt{3}}$ where c is the speed of light in freespace and $CFLN = \frac{\Delta t}{\Delta t_{CFL}}$. The CFLN for the FLOD2-CD-FDTD method is chosen as 1 and 4, while the CFLN for the conventional explicit FDTD method is fixed at 1.

The reflection error for the CPML is evaluated by

$$R = 20 \log_{10} \left(\frac{|E - E_{ref}|}{\max |E_{ref}|} \right) \quad (7.5.20)$$

where E_{ref} is the reference value calculated in a large computation domain such that any reflection from the boundary is negligible.

Fig. 7.12 plots the time-domain E_z field component computed using the conventional explicit FDTD method and FLOD2-CD-FDTD method. Good agreement can be observed between the various methods. Fig. 7.13 graphs the reflection errors (dB) as a function of time for the conventional explicit FDTD method and FLOD2-CD-FDTD method. It can be observed that the reflection errors of the FLOD2-CD-FDTD method increases with CFLN, demonstrating the trade-off of accuracy and efficiency.

7.6 Conclusion

This chapter has presented an unconditionally stable fundamental LOD2-CD-FDTD method with second-order temporal accuracy and complying divergence for 3-D Maxwell's equations. By extending the previous concept of achieving second-order temporal accuracy for the LOD2-FDTD method via implicit output processing, we have proposed a novel, explicit output processing that not only retains second-order temporal accuracy, but also complying divergence for the LOD2-CD-FDTD method. To solve open radiation and scattering problems, the LOD2-CD-FDTD method has also been extended to incorporate the CPML. The output processing of the LOD2-CD-FDTD method consists only of explicit update equations, and does not require a tridiagonal solution that is necessary for the LOD2-FDTD method. Moreover, the output processing of the LOD2-CD-FDTD method is independent of the CPML media. To further enhance efficiency, the LOD2-CD-FDTD method has been formulated into the fundamental LOD2-CD-FDTD method with efficient matrix-operator-free right-hand side.

The LOD2-CD-FDTD method is divergence-complying in both the source region

as well as the source-free region in a manner analogous to the conventional explicit FDTD method. Numerical results and examples have been presented to validate the divergence-complying property of the LOD2-CD-FDTD method. In the next chapter, we will illustrate practical applications of the LOD2-CD-FDTD method for antenna modeling.

Chapter 8

Applications of Fundamental LOD2-CD-FDTD Method for Antenna Modeling

8.1 Introduction

In the following, we illustrate practical applications of the fundamental LOD2-CD-FDTD method for antenna modeling. We begin this chapter by analyzing a line-fed rectangular microstrip patch antenna, followed by a strip-fed rectangular dielectric resonator antenna. The formulation of the fundamental LOD2-CD-FDTD method presented in the previous chapter will be employed for these applications. In addition, this chapter includes the resistor voltage source for the fundamental LOD2-CD-FDTD method required for the simulation of the dielectric resonator antenna. Numerical results such as scattering parameters and radiation patterns will be provided.

Figure 8.1: Geometry of line-fed rectangular microstrip patch antenna.

8.2 Analysis of Line-Fed Rectangular Microstrip Patch Antenna

For the first example, we apply the fundamental LOD2-CD-FDTD method with CPML to determine the scattering parameter of a line-fed rectangular microstrip patch antenna. The geometry of the line-fed rectangular microstrip patch antenna is illustrated in Fig. 8.1, and it has been studied previously in [91]. The relative permittivity of the dielectric substrate ϵ_r for the microstrip patch antenna is 2.2.

The dimension of the computation domain is 80 x 120 x 26 grids with cell size $\Delta x = 0.389$ mm, $\Delta y = 0.400$ mm and $\Delta z = 0.265$ mm. The Courant limit time step size is $\Delta t_{CFL} = \frac{1}{c\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}}$ where c is the speed of light in freespace. We also define CFLN as $\frac{\Delta t}{\Delta t_{CFL}}$. All of the simulations are performed with $\frac{2000}{CFLN}$ number of iterations. A Gaussian pulse excitation is located in the substrate region between the feeding line and the ground plane as

$$J_z = e^{-\left(\frac{t-t_0}{\tau}\right)^2} \quad (8.2.1)$$

Figure 8.2: E_z field distribution underneath the microstrip patch antenna at various time instant. (a) $t = 0.416$ ns. (b) $t = 1.057$ ns.

where $\tau = 15$ ps and $t_0 = 3\tau$.

In addition, the computation domain is truncated in the lower z -direction by a PEC, which also serves as the ground plane of the microstrip patch antenna. The remaining outer boundaries are truncated by the PML. Moreover, the substrate and feeding line of the microstrip patch antenna extends all the way into the PML regions in order to achieve the matched line condition. Here, the PML parameters are defined as $\sigma_{\zeta_{\max}} = \sigma_{opt} = 10.61$ S/m, $\kappa_{\zeta_{\max}} = 15$, $\alpha_{\zeta} = 0.02$ and $m = 3$.

8.2.1 Numerical Results

Next, we analyze the performance of the line-fed rectangular microstrip patch antenna for the LOD2-CD-FDTD method. The S_{11} is evaluated via

$$|S_{11}| \text{ (dB)} = 20 \log_{10} \frac{|V_1^-|}{|V_1^+|} \quad (8.2.2)$$

Figure 8.3: $|S_{11}|$ (dB) of the line-fed rectangular microstrip patch antenna computed using the conventional explicit FDTD method, fundamental LOD2-CD-FDTD method and HFSS.

where V_1^- is the outward traveling waves at port 1, while V_1^+ is the inward traveling waves at port 1. As mentioned earlier, the PML absorber that terminates the microstrip line feed functions as a matched load for port 1. The reference plane of port 1 is located between the source plane and the PML. The microstrip line supports the quasi-TEM mode, and thus the quasi-static approximation may be used to determine V directly from the corresponding electric fields.

Fig. 8.2 illustrates the E_z field distribution underneath the microstrip patch antenna at time instant $t = 0.416$ ns and $t = 1.057$ ns. It can be observed that the electric fields are confined mostly under the microstrip patch, and the electric fields at the outer boundaries are properly truncated by the PML.

Fig. 8.3 plots the $|S_{11}|$ (dB) of the line-fed rectangular microstrip patch antenna computed using the conventional explicit FDTD method and fundamental LOD2-CD-FDTD method for $CFLN = 1$. In addition, commercial software package ANSYS HFSS is used to validate the FDTD results. Good agreement can be observed

Figure 8.4: $|S_{11}|$ (dB) of the line-fed rectangular microstrip patch antenna computed using the conventional explicit FDTD method for CFLN = 1 and fundamental LOD2-CD-FDTD method for CFLN = 1, 4, 6 and 8.

between the conventional explicit FDTD method, fundamental LOD2-CD-FDTD method and HFSS. Fig. 8.4 shows the $|S_{11}|$ (dB) of the line-fed rectangular microstrip patch antenna computed using the conventional explicit FDTD method for CFLN = 1 and the fundamental LOD2-CD-FDTD method for various CFLN for a narrower frequency range. The CFLN of the LOD2-CD-FDTD method is chosen as 1, 4, 6 and 8. As expected, the error of the LOD2-CD-FDTD method increases with CFLN. In the next section, we will discuss the accuracy of the LOD2-CD-FDTD method in more detail.

8.2.2 Accuracy and Efficiency

The line-fed rectangular microstrip patch antenna has a measured resonant frequency of 7.5 GHz [91]. It is thus appropriate to use the measured resonant frequency of the line-fed rectangular microstrip patch antenna as the reference for subsequent error calculations. Table. 8.1 compares the resonant frequency (GHz) of the line-fed rectangular microstrip patch antenna computed using the conventional explicit FDTD method and the fundamental LOD2-CD-FDTD method for various

Table 8.1: Comparison of resonant frequency (GHz) of the line-fed rectangular microstrip patch antenna computed using the conventional explicit FDTD method and the fundamental LOD2-CD-FDTD method for various CFLN.

Method	CFLN 1	CFLN 4	CFLN 6	CFLN 8
Conventional Explicit FDTD	7.513	-	-	-
Fundamental LOD2-CD-FDTD	7.501	7.352	7.204	7.045
% error	0.15	2.14	4.11	6.22

Table 8.2: CPU Time (seconds) of the conventional explicit FDTD method and the fundamental LOD2-CD-FDTD method for determining the $|S_{11}|$ (dB) of the line-fed rectangular microstrip patch antenna for various CFLN.

Method	CFLN 1	CFLN 4	CFLN 6	CFLN 8
Conventional Explicit FDTD	22.87	-	-	-
Fundamental LOD2-CD-FDTD	63.75	16.09	11.58	8.00
% speedup	-	26.96	49.36	64.75

CFLN. As shown previously in Fig. 8.4, the resonant frequency of the microstrip patch antenna decreases with increasing CFLN. Referring to Table. 8.1, the error in the resonant frequency of the microstrip patch antenna for the fundamental LOD2-CD-FDTD method is only 6% for CFLN = 8. Nonetheless, the overall response of the line-fed rectangular microstrip patch antenna is still preserved.

Next, we determine the efficiency of the fundamental LOD2-CD-FDTD method for the line-fed rectangular microstrip patch antenna. The programs have been compiled using Intel C++ under Microsoft Win7 (64-Bit) operating system, and run on a platform of 3.1 GHz Intel I5 Quad-core processor with 4 GB RAM. The output processing of the fundamental LOD2-CD-FDTD method was performed for the electric fields at all locations. Thus, further speedup can be achieved if the

output processing is only performed at the desired observation points. Table. 8.2 presents the CPU time (seconds) taken by the conventional explicit FDTD method and the fundamental LOD2-CD-FDTD method for determining the $|S_{11}|$ (dB) of the line-fed rectangular microstrip patch antenna for various CFLN. Referring to Table. 8.2, the speedup achieved by the fundamental LOD2-CD-FDTD method for determining the scattering parameter of the microstrip patch antenna is almost 64% for CFLN = 8. Finally, it is worthwhile to note that the programs are general-purpose and are not optimized for any particular geometry. The programs for the conventional explicit FDTD method and the fundamental LOD2-CD-FDTD method requires 23 MB and 36 MB of memory, respectively.

8.3 Analysis of Strip-Fed Rectangular Dielectric Resonator Antenna

In first example, the dielectric substrate and feeding line of the rectangular microstrip patch antenna extends into the PML region, and absorbs the outgoing electromagnetic waves propagating along the microstrip line. Thus, the transmission line is effectively infinitely long in extend, and we can determine the scattering parameters of the network by analyzing its traveling waves.

However, for some practical geometry, such a configuration may not always be possible. It might not be feasible to terminate the transmission line by a PML absorber, and therefore an interior port is necessary. One such commonly used interior port is the resistive voltage source [5].

In the following, we will first derive the resistive voltage source formulation for the fundamental LOD2-CD-FDTD method, in order to achieve a non-reflective (matched) source. The fundamental LOD2-CD-FDTD method including resistive voltage source will then be applied to analyze a dielectric resonator antenna.

8.3.1 Resistive Voltage Source for Fundamental LOD2-CD-FDTD Method

We begin this section by presenting the fundamental LOD2-CD-FDTD method for lossy media. Here, we employ the time-averaging scheme where the conductivity terms are averaged between two time indices in both procedures [75]. The main iterations of the fundamental LOD2-CD-FDTD method for lossy media are

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A} + \frac{\Delta t}{8}\mathbf{L}\right) \mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^n + \frac{\Delta t}{2} \mathbf{s}^{n+\frac{1}{2}} \quad (8.3.1a)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n \quad (8.3.1b)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{B} + \frac{\Delta t}{8}\mathbf{L}\right) \mathbf{v}^{n+1} = \mathbf{u}^{n+\frac{1}{2}} \quad (8.3.1c)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^{n+\frac{1}{2}} \quad (8.3.1d)$$

with explicit output processing at time step $n + 1$ as

$$\mathbf{u}_{\text{CD}}^{n+1} = \left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B} + \frac{\Delta t}{4}\mathbf{L}\right) \mathbf{u}^{n+1} \quad (8.3.2)$$

where

$$\mathbf{s} = \begin{bmatrix} \frac{-1}{\epsilon}J_x & \frac{-1}{\epsilon}J_y & \frac{-1}{\epsilon}J_z & \frac{-1}{\mu}M_x & \frac{-1}{\mu}M_y & \frac{-1}{\mu}M_z \end{bmatrix}^T \quad (8.3.3)$$

The diagonal loss matrix containing the electric and magnetic conductivity terms is

expressed as

$$\mathbf{L} = \text{diag} \left(\left[\frac{\sigma_x}{\epsilon_x}, \frac{\sigma_y}{\epsilon_y}, \frac{\sigma_z}{\epsilon_z}, \frac{\sigma_x^*}{\mu_x}, \frac{\sigma_y^*}{\mu_y}, \frac{\sigma_z^*}{\mu_z} \right] \right) \quad (8.3.4)$$

Circuit elements such as the resistor, capacitor and inductor have been incorporated into the conventional explicit FDTD method by discretizing the lumped element's voltage-current relation using the finite-difference method. For many practical applications (especially antenna modeling), the most useful and commonly used lumped circuit element is perhaps the lumped resistor element. The lumped resistor element for the fundamental LOD2-CD-FDTD method can be readily obtained from the fundamental LOD2-CD-FDTD method for lossy media by making some minor modifications to the coefficients of the update equation.

Assuming a z -directed lumped resistor element, the effective electric conductivity σ_z of the cell can be related to the resistance R_z of the lumped element by

$$\sigma_z = \frac{\Delta_z}{\Delta_x \Delta_y R_z} \quad (8.3.5)$$

Similarly for a z -directed resistive voltage source, the electric current density J_z may be replaced by

$$J_z = \frac{I_z}{\Delta_x \Delta_y} = \frac{V_z}{\Delta_x \Delta_y R_z} \quad (8.3.6)$$

where V_z is the excitation source voltage and R_z is the internal source resistance. The formulations for the x and y directed lumped resistor element and resistive voltage source can be obtained in a similar manner.

Figure 8.5: Geometry of strip-fed rectangular dielectric resonator antenna.

Therefore, the lumped resistor element and resistive voltage source for the fundamental LOD2-CD-FDTD method is obtained by modifying the update coefficients of the fundamental LOD2-CD-FDTD method for lossy media appropriately. Hence, the fundamental LOD2-CD-FDTD method including resistive voltage source retains the advantages of the fundamental LOD2-CD-FDTD method as discussed in the previous chapter. In the subsequent sections, we shall apply the resistive voltage source formulation of the fundamental LOD2-CD-FDTD method to model and analyze a strip-fed rectangular dielectric resonator antenna.

8.3.2 Analysis of Rectangular Dielectric Resonator Antenna

For the application of the fundamental LOD2-CD-FDTD method including resistive voltage source, we model a strip-fed rectangular dielectric resonator antenna to analyze its scattering parameter and radiation properties. Fig. 8.5 presents the geometry of the strip-fed rectangular dielectric resonator antenna. The rectangular dielectric resonator antenna has the same dimension as that of [92]. The relative permittivity of the dielectric material ϵ_r of the dielectric resonator antenna is 9.8.

The computation domain consists of $92 \times 122 \times 102$ grids with cell size $\Delta x = 0.715$ mm, $\Delta y = 0.508$ mm and $\Delta z = 0.500$ mm. It is also truncated by ten cells of PML on all six outer boundaries in order to absorb the outgoing electromagnetic waves. For subsequent simulations, the PML parameters are defined as $\sigma_{\zeta_{\max}} = \sigma_{opt} = 10.61$ S/m, $\kappa_{\zeta_{\max}} = 15$, $\alpha_{\zeta} = 0.02$ and $m = 3$.

A resistive voltage source with source impedance of 50Ω is located at the end of the feeding strip just above the ground plane. It is excited with a Gaussian pulse waveform given by

$$J_z = e^{-\left(\frac{t-t_0}{\tau}\right)^2} \quad (8.3.7)$$

where $\tau = 30$ ps and $t_0 = 3\tau$. The Courant limit time step size is $\Delta t_{CFL} = \frac{1}{c\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}}$ where c is the speed of light in freespace. We also define CFLN $= \frac{\Delta t}{\Delta t_{CFL}}$. The simulations are terminated at time instant $t = \frac{6000 \Delta t_{CFL}}{\text{CFLN}}$.

8.3.3 Numerical Results

Next, we analyze the scattering parameter of the strip-fed rectangular dielectric resonator antenna. In contrast with the previous example, the S_{11} for this example is determined as follows:

$$S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \quad (8.3.8)$$

where Z_0 is the source impedance and

$$Z_{in} = \frac{V(\omega)}{I(\omega)} \quad (8.3.9)$$

Time-domain voltage and current are measured at the strip-feed gap and Fourier transformed into the frequency domain. It should be noted that phasor $V(\omega)$ of the

Figure 8.6: $|S_{11}|$ (dB) of the strip-fed rectangular dielectric resonator antenna computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for $CFLN = 1$. The dual resonant TE_{111}^y and TE_{113}^y modes can be clearly seen.

conventional explicit FDTD method is time-shifted by $e^{-j\omega\frac{\Delta t}{2}}$ in order to compensate for the leapfrog staggering in time.

Fig. 8.6 presents the $|S_{11}|$ (dB) of the strip-fed rectangular dielectric resonator antenna computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for $CFLN = 1$. Referring to the figure, the resonant TE_{111}^y (3.5 GHz) and TE_{113}^y (4.3 GHz) modes can be clearly seen. The dual resonant TE_{111}^y and TE_{113}^y modes broaden the impedance bandwidth of the strip-fed rectangular dielectric resonator antenna [92].

The $|S_{11}|$ (dB) of the strip-fed rectangular dielectric resonator antenna computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method is depicted in Fig. 8.7. The CFLN for the conventional explicit FDTD method is 1, while the CFLN for the LOD1-FDTD method and LOD2-CD-FDTD method is 4. Good agreement is shown between the LOD2-CD-FDTD method and the conventional explicit FDTD method even at large CFLN. On the

Figure 8.7: $|S_{11}|$ (dB) of the strip-fed rectangular dielectric resonator antenna computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for various CFLN.

Table 8.3: Comparison of 10-dB impedance bandwidth for the strip-fed rectangular dielectric resonator antenna computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for various CFLN.

Method	CFLN 1	CFLN 4
Conventional Explicit FDTD	35.23	-
Fundamental LOD1-FDTD	22.03	13.33
Fundamental LOD2-CD-FDTD	35.53	37.79

other hand, the accuracy of the LOD1-FDTD method degrades with increasing CFLN. The following paragraph will elaborate on the accuracy of the LOD2-CD-FDTD method and LOD1-FDTD method, as compared to the conventional explicit FDTD method.

Table. 8.3 summaries the 10-dB impedance bandwidths of the strip-fed rectangular dielectric resonator antenna computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for various CFLN. The impedance bandwidth of the strip-fed rectangular dielectric resonator antenna

Figure 8.8: Radiation pattern (dBi) of the TE_{111}^y mode at 3.5 GHz for $\Phi = 0^\circ$ computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for $CFLN = 1$. (a) Co-polar. (b) Cross-polar.

computed using the conventional explicit FDTD method for $CFLN = 1$ and the LOD2-CD-FDTD method for $CFLN = 1$ and 4 are in good agreement. The error of the LOD2-CD-FDTD method is only 7% for $CFLN = 4$, while the error of the LOD1-FDTD method is almost 61% for $CFLN = 4$. Referring to Fig. 8.7, it can be seen that the second resonant TE_{113}^y mode of the strip-fed rectangular dielectric resonator antenna computed using the LOD1-FDTD method has lower levels as compared to the conventional explicit FDTD method and the LOD2-CD-FDTD method. This cause the impedance bandwidth of the strip-fed rectangular dielectric resonator antenna computed using the LOD1-FDTD method to become narrower as its return loss decreases to less than 10-dB. As mentioned in the previous chapter, the LOD1-FDTD method is only first-order temporal accurate, while the conventional explicit FDTD method and the LOD2-CD-FDTD method are second-order temporal accurate. This could have lead to the difference in accuracies between the various FDTD methods.

Figure 8.9: Radiation pattern (dBi) of the TE_{111}^y mode at 3.5 GHz for $\Phi = 0^\circ$ computed using the conventional explicit FDTD method for $CFLN = 1$, LOD1-FDTD method and LOD2-CD-FDTD method for $CFLN = 4$. (a) Co-polar. (b) Cross-polar.

Figure 8.10: Radiation pattern (dBi) of the TE_{111}^y mode at 3.5 GHz for $\Phi = 90^\circ$ computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for $CFLN = 1$. (a) Co-polar. (b) Cross-polar.

Next, we proceed to analyze the radiation properties of the strip-fed rectangular dielectric resonator antenna. To take into account the variation of the impedance bandwidth for various CFLN, the realized gain is used in this section [93, 94].

Figure 8.11: Radiation pattern (dBi) of the TE_{111}^y mode at 3.5 GHz for $\Phi = 90^\circ$ computed using the conventional explicit FDTD method for CFLN = 1, LOD1-FDTD method and LOD2-CD-FDTD method for CFLN = 4. (a) Co-polar. (b) Cross-polar.

Figure 8.12: Radiation pattern (dBi) of the TE_{113}^y mode at 4.3 GHz for $\Phi = 0^\circ$ computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for CFLN = 1. (a) Co-polar. (b) Cross-polar.

Figs. 8.8 to 8.11 plot the radiation pattern (dBi) of the TE_{111}^y mode at 3.5 GHz for $\Phi = 0^\circ$ and 90° computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method. The CFLN is selected as 1 and 4. Good agreement can be seen for the various methods for CFLN = 1. For $\Phi = 0^\circ$,

Figure 8.13: Radiation pattern (dBi) of the TE_{113}^y mode at 4.3 GHz for $\Phi = 0^\circ$ computed using the conventional explicit FDTD method for $CFLN = 1$, LOD1-FDTD method and LOD2-CD-FDTD method for $CFLN = 4$. (a) Co-polar. (b) Cross-polar.

Figure 8.14: Radiation pattern (dBi) of the TE_{113}^y mode at 4.3 GHz for $\Phi = 90^\circ$ computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method for $CFLN = 1$. (a) Co-polar. (b) Cross-polar.

the peak gain of the rectangular dielectric resonator antenna computed using the conventional explicit FDTD method is 5.755 dBi at $\Theta = -5^\circ$. Similarly, the peak gain computed using the LOD2-CD-FDTD method and LOD1-FDTD method for $CFLN = 4$ is 5.801 dBi and 5.598 dBi, respectively.

Figure 8.15: Radiation pattern (dBi) of the TE_{113}^y mode at 4.3 GHz for $\Phi = 90^\circ$ computed using the conventional explicit FDTD method for $CFLN = 1$, LOD1-FDTD method and LOD2-CD-FDTD method for $CFLN = 4$. (a) Co-polar. (b) Cross-polar.

Table 8.4: Error (%) of the peak gain for the strip-fed rectangular dielectric resonator antenna for $CFLN = 4$.

Method	TE_{111}^y	TE_{113}^y
Fundamental LOD1-FDTD	2.7280	11.836
Fundamental LOD2-CD-FDTD	0.7993	0.6548

Figs. 8.12 to 8.15 plot the radiation pattern (dBi) of the TE_{113}^y mode at 4.3 GHz for $\Phi = 0^\circ$ and 90° computed using the conventional explicit FDTD method, LOD1-FDTD method and LOD2-CD-FDTD method. As before, the CFLN is chosen as 1 and 4. For $\Phi = 0^\circ$, the peak gain of the rectangular dielectric resonator antenna computed using the conventional explicit FDTD method is 6.108 dBi at $\Theta = -12^\circ$. Similarly, the peak gain computed using the LOD2-CD-FDTD method and LOD1-FDTD method for $CFLN = 4$ is 6.068 dBi and 5.385 dBi, respectively.

Table. 8.4 presents the error (%) of the peak gain for the strip-fed rectangular

dielectric resonator antenna for $CFLN = 4$ computed using the LOD2-CD-FDTD method and LOD1-FDTD method. Here, the reference peak gain is computed using the conventional explicit FDTD method for $CFLN = 1$. Referring to the table, it can be observed that the errors of the LOD2-CD-FDTD method for both resonant modes are quite small and are of the same order of magnitude. On the other hand, for the LOD1-FDTD method, the error of TE_{113}^y mode is approximately 4 times larger than the TE_{111}^y mode. This could be because the TE_{113}^y mode is of a higher frequency and thus suffers from more dispersion errors.

8.4 Conclusion

This chapter has illustrated practical applications of the fundamental LOD2-CD-FDTD method for antenna modeling. Numerical results for the line-fed rectangular microstrip patch antenna and the strip-fed rectangular dielectric resonator antenna have been presented.

The formulation of the fundamental LOD2-CD-FDTD method described in the previous chapter has been employed for these applications. Moreover, it has also been extended to incorporate the resistor voltage source and lumped resistor element, which is useful for antenna modeling. Similar to the ADI-FDTD method including resistor lumped element [87], the fundamental LOD2-CD-FDTD method including resistor lumped element is unconditionally stable as it only involves lossy media. This has been validated through numerous numerical simulations. Finally, the fundamental LOD2-CD-FDTD method including resistive voltage source retains the advantages of the fundamental LOD2-CD-FDTD method as discussed earlier.

Chapter 9

Conclusion and Future Work

9.1 Conclusion

The presented contributions of this thesis related to the advancement of unconditionally stable FDTD methods are summarized as follows.

A review of the more popular unconditionally stable FDTD methods has been provided in Chapter 2. From this review, it is determined that most unconditionally stable FDTD methods have two or more updating procedures. To resolve this issue, an unconditionally stable leapfrog ADI-FDTD method which employs the same time-stepping scheme as the conventional explicit FDTD method was recently proposed. However, proper analysis and development of the leapfrog ADI-FDTD method was lacking. Thus, Chapter 3 analytically analyses the stability of the leapfrog ADI-FDTD method, and proves that it is indeed unconditionally stable.

Chapter 4 further develops the leapfrog ADI-FDTD method by extending it to include the Mur ABC, perfect electric conductor boundary condition and lossy media.

It has also been accelerated by implementing it on a low-cost graphic processing unit. Some numerical examples have been provided to validate the formulations. From these examples, it has been shown that the leapfrog ADI-FDTD method requires less CPU time and memory as compared to the ADI-FDTD method. The unconditionally stable leapfrog ADI-FDTD method, which employs the same time-stepping scheme as the conventional explicit FDTD method, is more efficient than the two-procedure ADI-FDTD method as it involves less equations and does not have any intermediate field variables.

To demonstrate the efficiency of the leapfrog ADI-FDTD method for practical applications, it has been applied to two electromagnetic compatibility problems in Chapter 5. Mutual coupling between two monopole antennas of different length mounted on a finite ground plane has been investigated using the leapfrog ADI-FDTD method. As before, the leapfrog ADI-FDTD method requires less CPU time and memory. However, the leapfrog ADI-FDTD method has considerable errors for high CFLN at higher frequencies, while the errors of the ADI-FDTD method are still within acceptable limits under similar conditions. Results of the mutual coupling between the two monopole antennas have also been validated using commercial software package HFSS. In addition, the field leakage of a shielded enclosure has been investigated for the leapfrog ADI-FDTD method. Detailed analysis reveals that the leapfrog ADI-FDTD method has substantial non-physical field leakage, and subsequently a simple approach has been proposed to overcome this issue. From these application examples, it is obvious that while the leapfrog ADI-FDTD method may be more efficient than the ADI-FDTD method, it may also have more issues when applied to practical applications.

Next, Chapter 6 clarifies the uncertainty of whether the leapfrog ADI-FDTD method is divergence-free or not by developing an analysis method to determine its divergence property. Using the proposed analysis method, it can be concluded that the leapfrog ADI-FDTD method is not divergence-free in the source-free region, as is the case for the ADI-FDTD method.

To overcome the issue of divergence for unconditionally stable FDTD methods, Chapter 7 then proposed an unconditionally stable FDTD method which has comply divergence. The LOD2-CD-FDTD method is divergence-complying in both the source region as well as the source-free region in a manner analogous to the conventional explicit FDTD method. Moreover, the proposed method also has second-order temporal accuracy.

Practical applications of the LOD2-CD-FDTD method to determine its accuracy have been demonstrated in Chapter 8 for antenna modeling. The trade-off between accuracy and efficiency for the LOD2-CD-FDTD method has been discussed for the line-fed rectangular microstrip patch antenna. Results of the microstrip patch antenna have also been validated using commercial software package HFSS. Furthermore, when applied for the strip-fed rectangular dielectric resonator antenna, the second-order temporal accurate LOD2-CD-FDTD method is more accurate than the first-order temporal accurate LOD1-FDTD method.

In summary, this thesis has analyzed and extended the unconditionally stable leapfrog ADI-FDTD method and found that it is not divergence-complying. Subsequently, an unconditionally stable LOD2-CD-FDTD method with comply divergence was proposed. The LOD2-CD-FDTD method can employ time step size larger than that

allowed by the CFL stability criterion, and thus achieves speedup in computational time when a sufficiently large time step is chosen.

9.2 Future Work

The presented contributions in this thesis can be further investigated and extended as follows.

The leapfrog ADI-FDTD method is attracting more attention recently due to its unconditional stability and leapfrog time-stepping scheme. However, research and development is still lacking in areas such as the complex envelope [95–97], spherical grid [98] and nonorthogonal grid [99, 100]. The spherical leapfrog ADI-FDTD method could be a suitable candidate to replace the spherical ADI-FDTD method, which has late-time instability issues [98]. Meanwhile, the nonorthogonal leapfrog ADI-FDTD method will be useful for problems involving complex geometries.

While unconditionally stable FDTD methods, such as the leapfrog ADI-FDTD method and the fundamental LOD2-CD-FDTD method are fast and efficient for solving most computational electromagnetic problems, they do have some shortcomings, which is inherited from the conventional explicit FDTD method. Essentially, the FDTD method is a volume-based method, and it discretizes the entire computation domain. For some problem of interest, the geometry may be electrically large but consisting mainly of freespace. Hence, it is computationally inefficient to analyze such problems using only the FDTD methods, as most of the resources, i.e., memory and time, are spent computing the propagation of electromagnetic waves in freespace. High frequency methods, such as the PO and GO, can be hybridized with the unconditionally stable FDTD methods to reap the advantages of

both methods [101,102]. For other problems, the geometry may contain several fine details, which may be difficult to model accurately using only the FDTD methods. Thus, hybrid MoM/unconditionally stable FDTD methods can be further investigated for better modeling flexibility [103,104]. Other volume-based methods, such as the finite-volume, finite-element, or meshless method may also be suitable for hybridization [105–108]. Efficient implementation of these hybrid methods could be studied so as to further reduce computation resource required.

Over the past few years, several schemes have been proposed to further enhance the accuracy of unconditionally stable FDTD methods. It has been determined that numerical dispersion is the main cause of the error, and thus, efforts have been focused on reducing numerical dispersion. A straightforward approach is to optimize the numerical dispersion by adding artificial anisotropy parameters [109,110]. An alternative approach is to employ higher order spatial methods [10,111,112]. While the numerical dispersion of higher order spatial methods have been rigorously studied, most of these studies are analytical and few examples have been shown [12,50,52]. This is because higher order spatial methods have larger stencils, and thus it is not as straightforward to implement the spatial derivative as before. While the Thomas algorithm can be employed to solve the tridiagonal matrix for second-order spatial accurate methods, higher order spatial accurate methods require more complex matrix solutions. The compact method has been employed for the fourth-order spatial accurate methods to improve the computational efficiency [113–116]. However, extension to inhomogeneous media has not been studied yet. Higher order temporal methods [117,118] as well as the (2,4) parameter optimized methods [119,120] are some other ways of improving the accuracy of unconditionally stable FDTD methods. The various methods listed above have been developed independently, and it is not clear how they can be employed concurrently to enhance the solution of a

problem. For instance, the problem can be divided into multiple domains, and each domain employs the most appropriate scheme to reduce its numerical dispersion. Adaptive methods can also be used to refine higher order spatial derivatives and/or artificial anisotropy parameters when necessary. The efficient implementation of such unconditionally stable FDTD methods with multiple solvers may be useful for engineering applications.

Appendix A

Discussion on the derivation of the leapfrog-ADI FDTD method for lossy media

In this section, we discuss the derivation of the leapfrog ADI-FDTD method for lossy media, by following strictly the previous derivation of the leapfrog ADI-FDTD method from the conventional ADI-FDTD method for lossy media.

For simplicity, let us consider a medium with permittivity ϵ , permeability μ and electric conductivity σ . The magnetic conductivity σ^* is not considered here, but may be appended accordingly, if necessary. The update equation for the E_x field component of the conventional ADI-FDTD method in the first procedure for electrically lossy media is given as

$$E_x^{n+\frac{1}{2}} = \frac{\left(1 - \frac{\Delta t \sigma}{4\epsilon}\right)}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} E_x^n + \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \left(\partial_y H_z^{n+\frac{1}{2}} - \partial_z H_y^n\right) \quad (10.2.1)$$

where the time-averaging method is adopted. Likewise, the update equation for the H_z field components of the conventional ADI-FDTD method in the first procedure can be expressed as

$$H_z^{n+\frac{1}{2}} = H_z^n + \frac{\Delta t}{2\mu} \left(\partial_y E_x^{n+\frac{1}{2}} - \partial_x E_y^n \right). \quad (10.2.2)$$

The tridiagonal implicit update equation for E_x can be obtained by substituting (10.2.2) into (10.2.1) as

$$\begin{aligned} E_x^{n+\frac{1}{2}} = & \frac{\left(1 - \frac{\Delta t\sigma}{4\epsilon}\right)}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} E_x^n + \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} \partial_y \left[H_z^n + \frac{\Delta t}{2\mu} \left(\partial_y E_x^{n+\frac{1}{2}} - \partial_x E_y^n \right) \right] \\ & - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} \left(\partial_z H_y^n \right) \end{aligned} \quad (10.2.3)$$

and after some manipulations, the tridiagonal implicit E_x update equation for the conventional ADI-FDTD method for the first procedure reads

$$\begin{aligned} E_x^{n+\frac{1}{2}} - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} \frac{\Delta t}{2\mu} \partial_y^2 E_x^{n+\frac{1}{2}} = & \frac{\left(1 - \frac{\Delta t\sigma}{4\epsilon}\right)}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} E_x^n + \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} \partial_y H_z^n \\ & - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} \frac{\Delta t}{2\mu} \partial_y \partial_x E_y^n - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} \partial_z H_y^n. \end{aligned} \quad (10.2.4)$$

To formulate the leapfrog ADI-FDTD method for lossy media, we first consider the second procedure of the conventional ADI-FDTD method, but at the previous time step. The E_x update equation can then be written as

$$E_x^n = \frac{\left(1 - \frac{\Delta t\sigma}{4\epsilon}\right)}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} E_x^{n-\frac{1}{2}} + \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t\sigma}{4\epsilon}\right)} \left(\partial_y H_z^{n-\frac{1}{2}} - \partial_z H_y^n \right) \quad (10.2.5)$$

while the H_z update equation can be expressed as

$$H_z^{n-\frac{1}{2}} = H_z^n - \frac{\Delta t}{2\mu} \left(\partial_y E_x^{n-\frac{1}{2}} - \partial_x E_y^n \right). \quad (10.2.6)$$

Next, we substitute (10.2.6) into (10.2.5), and obtain

$$\begin{aligned}
E_x^n = & \frac{\left(1 - \frac{\Delta t \sigma}{4\epsilon}\right)}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} E_x^{n-\frac{1}{2}} + \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \partial_y \left[H_z^n - \frac{\Delta t}{2\mu} \left(\partial_y E_x^{n-\frac{1}{2}} - \partial_x E_y^n \right) \right] \\
& - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \partial_z H_y^n
\end{aligned} \tag{10.2.7}$$

Finally, we substitute (10.2.7) into the implicit tridiagonal E_x update equation (10.2.4) of the first procedure to arrive at

$$\begin{aligned}
E_x^{n+\frac{1}{2}} - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \frac{\Delta t}{2\mu} \partial_y^2 E_x^{n+\frac{1}{2}} = & \frac{\left(1 - \frac{\Delta t \sigma}{4\epsilon}\right)}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \left\{ \frac{\left(1 - \frac{\Delta t \sigma}{4\epsilon}\right)}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} E_x^{n-\frac{1}{2}} + \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \partial_y H_z^n \right. \\
& - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \frac{\Delta t}{2\mu} \partial_y^2 E_x^{n-\frac{1}{2}} + \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \frac{\Delta t}{2\mu} \partial_y \partial_x E_y^n - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \partial_z H_y^n \left. \right\} \\
& + \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \partial_y H_z^n - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \frac{\Delta t}{2\mu} \partial_y \partial_x E_y^n - \frac{\frac{\Delta t}{2\epsilon}}{\left(1 + \frac{\Delta t \sigma}{4\epsilon}\right)} \partial_z H_y^n
\end{aligned} \tag{10.2.8}$$

Referring to (10.2.8), it is clear that the mix partial derivatives $(\partial_y \partial_x E_y^n)$ terms does not cancel off due to the additional (non-unity) coefficient $\frac{1 - \frac{\Delta t \sigma}{4\epsilon}}{1 + \frac{\Delta t \sigma}{4\epsilon}}$. This cancellation is essential for the formulation of the leapfrog ADI-FDTD method. Thus, the aforementioned derivation in this section does not directly leads to the leapfrog ADI-FDTD method for lossy media.

Appendix B

Prove of Temporal Order of LOD1ⁿ by Mathematical Induction

We first assume that

$$\begin{aligned} & \left[\left(I - \frac{\Delta t}{2} B \right)^{-1} \left(I + \frac{\Delta t}{2} B \right) \left(I - \frac{\Delta t}{2} A \right)^{-1} \left(I + \frac{\Delta t}{2} A \right) \right]^n = \\ & I + n\Delta t(A + B) + \left(\frac{n\Delta t^2}{2} \right) \left[nA^2 + (n-1)AB + (n+1)BA + nB^2 \right] + O(\Delta t^3) = \\ & I + n\Delta t(A + B) + \frac{\Delta t^2}{2} \left[n^2 A^2 + n(n-1)AB + n(n+1)BA + n^2 B^2 \right] + O(\Delta t^3) \quad (10.2.9) \end{aligned}$$

is correct. Next, we proceed to find n+1:

$$\begin{aligned} & \left[\left(I - \frac{\Delta t}{2} B \right)^{-1} \left(I + \frac{\Delta t}{2} B \right) \left(I - \frac{\Delta t}{2} A \right)^{-1} \left(I + \frac{\Delta t}{2} A \right) \right]^{n+1} = \\ & \left[\left(I - \frac{\Delta t}{2} B \right)^{-1} \left(I + \frac{\Delta t}{2} B \right) \left(I - \frac{\Delta t}{2} A \right)^{-1} \left(I + \frac{\Delta t}{2} A \right) \right]^n \\ & \times \left[\left(I - \frac{\Delta t}{2} B \right)^{-1} \left(I + \frac{\Delta t}{2} B \right) \left(I - \frac{\Delta t}{2} A \right)^{-1} \left(I + \frac{\Delta t}{2} A \right) \right] = \\ & \left\{ I + n\Delta t(A + B) + \left(\frac{n\Delta t^2}{2} \right) \left[nA^2 + (n-1)AB + (n+1)BA + nB^2 \right] + O(\Delta t^3) \right\} \\ & \times \left\{ I + \Delta t(A + B) + \frac{\Delta t^2}{2} (A^2 + B^2 + 2BA) + O(\Delta t^3) \right\} = \\ & I + (n+1)\Delta t(A + B) + \frac{\Delta t^2}{2} \left[(n+1)^2 A^2 \right. \\ & \left. + n(n+1)AB + (n+1)(n+2)BA + (n+1)^2 B^2 \right] + O(\Delta t^3) \quad (10.2.10) \end{aligned}$$

It is clear (10.2.9) can be obtained from (10.2.10) by replacing n with $n+1$. Thus, by proved of mathematical induction, the assumption that (10.2.9) is correct is valid.

Appendix C

Implementation of FDTD Method Using Solid-State Drive

Computational electromagnetics have progressed rapidly in recent years due to intensive research and development of numerous fast and efficient numerical methods. This is partly also due to advancements in computer hardware technology. The finite-difference time-domain (FDTD) method is a simple and elegant numerical method that can readily adapt to such hardware advancements [7]. It solves Maxwell's equations in three-dimensional (3-D) space, and discretizes the entire computational domain using the Yee cells [1]. For large computational domain, significant amount of memory is needed to store the electric and magnetic field variables.

Fig. 10.1 estimates the memory required vs. domain size for the 3-D FDTD method. As expected, the memory required increases when the domain size grows. This estimation uses double-precision floating-point data format, and does not include the memory required for additional features such as the perfectly matched layer, dispersive media and near-to-far-field transformation etc [4]. Hence, the actual memory

required for practical problems is usually more than the estimate given. This in turn reduces the size of the problem that may be solved by the FDTD method.

The electric and magnetic field variables of the FDTD method are usually stored as data on random access memory (RAM). While large amount of RAM is available on expensive workstations or computer clusters, such facilities may not always be available to the end user. On the other hand, low-cost personal computers typically have limited amount of RAM.

Meanwhile, the hard-disk drive (HDD) is an essential component of a personal computer, and it often has plenty of data storage space. The data is stored magnetically on mechanically rotating platters. Such mechanical movement requires much time and consequently increases the data access time of the HDD.

Recently, solid-state drives (SSD) are increasingly being used as alternative forms of medium-to-large capacity data storage drives. Non-volatile flash memory which does not require any mechanical movement, stores data electronically in the SSD. Thus, the SSD has faster data access speed than the HDD.

Figure 10.1: Memory required vs. domain size for 3-D FDTD method.

Figure 10.2: Corresponding C++ and memory-mapped file functions for memory allocation and deallocation.

Here, we consider the implementation of the FDTD method using the SSD on low-cost personal computers with limited amount of RAM. Memory-mapped files will be used to access the data on the storage drives.

Memory-Mapped Files

The typical structure of a FDTD code written in C++ consists of the pre-processor directives and a main function. In the main function, we declare and allocate memory for the field variables in RAM by using the malloc function. Next, the field variables are iteratively updated by using the Yee's algorithm. After obtaining the final solution, the memory allocated to the field variables is no longer required, and may be deallocated using the free function.

The aforementioned is only applicable for field variables allocated in RAM. When the amount of RAM available on the hardware platform is not adequate, one can exploit the large-capacity storage of the SSD. To that end, we employ the memory-mapped files of Windows operating system to use the SSD as memory for the field

Figure 10.3: Configuration of hardware platform

variables [121]. Fig. 10.2 shows the corresponding C++ and memory-mapped file functions for memory allocation and deallocation. Instead of allocating memory in RAM for the field variables, files are created on the SSD by using the `CreateFile` function. Next, the files created on the SSD are then mapped to the field variables by using the `CreateFileMapping` and `MapViewOfFile` functions. The field variables can then be accessed normally as before, while the operating system's virtual-memory manager handles the file access to the SSD in the background. Therefore, no change is required for the computational section of the code. After obtaining the final solution, instead of deallocating memory in RAM, the files created on the SSD are unmapped and deleted. Here, we use the `UnmapViewOfFile`, `CloseHandle` and `DeleteFile` functions. In order to invoke the above functions, the Windows header must be included in the pre-processor definition.

It should be noted that memory-mapped file functions are also available on Linux operating system. While the functions may have slightly different names, they perform the same basic operations [122].

Table 10.1: CPU Time for FDTD Simulations using SSD and HDD

Domain	Memory (GB)	SSD timing	HDD timing	HDD/SSD
600^3	9.66	6.86	32.36	4.7
800^3	22.89	17.17	86.30	5.0

Numerical Results

In this section, we perform some numerical simulations to compare the efficiency of SSD and HDD. The configuration of the hardware platform of the low-cost personal computer is shown in Fig. 10.3. The processor is an Intel I5 quad-core with clock speed of 3.1 GHz and 4 GB of RAM. The operating system is Windows 7. The HDD is a 500 GB Seagate with 7200 revolutions-per-minute. It is the primary boot-up drive where the operating system is installed. The SSD is a 180 GB Intel 330. It is configured as the secondary drive. Memory-mapped files are used for both the SSD and HDD.

We consider a perfect electric conductor cavity meshed with uniform cells. The domain size is chosen such that the memory required for the simulation exceeds the memory available on the hardware platform. Referring to Table 10.1, the 600^3 cavity requires almost 10 GB of memory. It is clear that such a simulation if performed without using SSD or HDD, is not possible with 4 GB of RAM on the hardware platform.

The CPU time (hours) taken for the simulations are shown in Table 10.1. The SSD is about 4 to 5 times faster than the HDD. Furthermore, for this configuration, the SSD is used as the secondary drive. The performance of the SSD may be better if it is configured as the primary drive where the operating system is installed.

Author's Publications

Journal:

1. T. H. Gan and E. L. Tan, "Design of broadband circular polarization truncated horn antenna with single feed," *Progress In Electromagnetics Research C*, Vol. 24, pp. 197-206, Oct. 2011.
2. T. H. Gan and E. L. Tan, "Stability and dispersion analysis for three-dimensional (3-D) leapfrog ADI-FDTD method," *Progress In Electromagnetics Research M*, Vol. 23, pp. 1-12, Jan. 2012.
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