On the dynamics of persistently degassing volcanoes from new physics-based approaches

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**ON THE DYNAMICS OF PERSISTENTLY DEGASSING
VOLCANOES FROM NEW PHYSICS-BASED
APPROACHES**

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SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

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**On the dynamics of persistently degassing
volcanoes from new physics-based approaches**

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Summary

Persistently degassing volcanoes are the most active sub-aerial volcanoes around the world. Their activity consists of passive degassing periods interspersed by eruptions every few months or years. I present three new physics-based studies that unravel the underlying dynamics of these volcanoes. First, I calculate the pressure changes induced by the gas release during quiescence. A key finding is that magma reservoirs can depressurize several MPa within the inter-eruptive timescales, which suggests that passive degassing can induce unrest episodes and eruptions. Second, I develop a method to monitor the high-frequency water vapor emissions using light scattering theory and analysis of digital images. I show that degassing of Erebus and Mayon volcanoes emerges as a fractal phenomenon. Third, I simulate the bubble dynamics of volcanic conduits to investigate the processes leading to phreatic eruptions. I found that they can be preceded by amplitude and frequency shifts in the gas emission time series. These studies give new insights on the dynamics of persistently degassing volcanoes, and help to improve the monitoring and interpretation of degassing time series.

Introduction

Persistently degassing volcanoes are some of the most active volcanoes around the world, and are characterized by the passive release of large amounts of gas during quiescent periods. There are many of these volcanoes spread worldwide (Fig. 1), and some well-known examples are Masaya (Nicaragua), Etna (Italy), Merapi (Indonesia), or Mayon (Philippines). Persistently degassing volcanoes show a fairly regular but unpredictable activity with quiescent degassing periods alternating with unrest episodes. These unrest episodes might reflect magma ascent towards the surface or another magmatic process (e.g., magma intrusion), and may in turn culminate in mildly explosive mid-to-small size eruptions (VEI 1-2), which typically occur every few months or years (Table 1). Every hundred years or so these volcanoes can also produce much larger explosive events (VEI > 3; e.g., Merapi in 2010; Mayon in 1814). The quiescence-unrest alternation may repeat during centuries [Siebert et al., 2010], for which persistently degassing volcanoes pose a permanent risk to the populations living around them, their life-supporting activities, and their assets. Larger eruptions are also a threat to the civil aviation. Hence, improving the forecast of the eruptions of persistently degassing volcanoes is a primary goal for volcanologists [Sparks, 2003].

Fig. 1. Map showing the location of some of the most well-known persistently degassing volcanoes around the world, i.e., volcanoes that release large amount of gas during quiescence without emitting other magmatic products.

Table 1. Mean inter-eruptive time of persistently degassing volcanoes.

Volcano*	Mean inter-eruptive time since 1900 (months)	Number of inter- eruptive times
Oshima	25 $\begin{smallmatrix} +35 \\ -22 \end{smallmatrix}$	27
Bromo	45 $\begin{smallmatrix} +47 \\ -42 \end{smallmatrix}$	24
Aso	12 $\begin{smallmatrix} +17 \\ -9 \end{smallmatrix}$	46
Slamet	32 $\begin{smallmatrix} +51 \\ -29 \end{smallmatrix}$	20
Poás	18 $\begin{smallmatrix} +25 \\ -15 \end{smallmatrix}$	24
Telica	29 $\begin{smallmatrix} +30 \\ -26 \end{smallmatrix}$	23
Merapi	30 $\begin{smallmatrix} +21 \\ -21 \end{smallmatrix}$	17
Langila	11 $\begin{smallmatrix} +8 \\ -8 \end{smallmatrix}$	18
Kilauea	18 $\begin{smallmatrix} +35 \\ -15 \end{smallmatrix}$	28
Manam	18 $\begin{smallmatrix} +26 \\ -15 \end{smallmatrix}$	13
White island	17 $\begin{smallmatrix} +13 \\ -13 \end{smallmatrix}$	12
Láscar	18 $\begin{smallmatrix} +17 \\ -15 \end{smallmatrix}$	14
Llaima	29 $\begin{smallmatrix} +25 \\ -25 \end{smallmatrix}$	16
Tangkubanparahu	94 $\begin{smallmatrix} +90 \\ -90 \end{smallmatrix}$	11
Asama	21 $\begin{smallmatrix} +32 \\ -18 \end{smallmatrix}$	36
San Cristóbal	13 $\begin{smallmatrix} +14 \\ -10 \end{smallmatrix}$	13
Etna	8 $\begin{smallmatrix} +7 \\ -5 \end{smallmatrix}$	35
Fuego	41 $\begin{smallmatrix} +49 \\ -38 \end{smallmatrix}$	16
Ulawun	21 $\begin{smallmatrix} +19 \\ -18 \end{smallmatrix}$	22
Mayon	57 $\begin{smallmatrix} +69 \\ -54 \end{smallmatrix}$	15
Bulusan	47 $\begin{smallmatrix} +40 \\ -40 \end{smallmatrix}$	10
Masaya	35 $\begin{smallmatrix} +50 \\ -32 \end{smallmatrix}$	14

* Volcanoes with at least 10 inter-eruptive periods recorded (third column) [Siebert et al., 2010]. Uncertainties in the second column are the standard deviations of the inter-eruptive times. For cases with standard deviation larger than the mean inter-eruptive time, we have chosen a negative uncertainty such that the minimum inter-eruptive time is at least three months. A minimum inter-eruptive time of three months is the criterion followed by Siebert et al. [2010] to consider that two eruptions are different.

Improving eruption forecasting requires unveiling the volcanic processes occurring during quiescent degassing and unrest episodes, for which monitoring of the amount, composition, and style of the gas emissions is fundamental [e.g., Symonds et al., 1994; Aiuppa et al., 2007]. Tracking the gas emissions can be performed nowadays in real-time by using remote-sensing techniques. For example, SO₂ fluxes can be routinely measured in real-time from remote distances by using ultraviolet cameras [e.g., Tamburello et al., 2013], or by determining the absorption of the ultraviolet radiation by the gas plume with instruments like DOAS (differential optical absorption spectrometers) [e.g., Oppenheimer and Kyle, 2008]. The ratios between SO₂ and other gases can be also measured by using systems like OP-FTIR (open-path Fourier transform infrared spectrometry) or Multi-GAS, even though their use is limited to easily accessible volcanoes where the instruments can be placed at the proximities of the crater vent [e.g., Burton et al., 2000; Shinohara, 2005]. The application of these real-time remote-sensing techniques reveals for example that persistently degassing volcanoes emit several thousand tonnes of gas per day, mainly H₂O (~ 50 - 90 wt%), CO₂ (~ 20 - 50 wt%) and SO₂ (~ 2 – 10 wt%), and at lesser extent other gases like HCl, HF, H₂, and H₂S [e.g., Burton et al., 2000; Kazahaya et al., 2002; McGonigle and Oppenheimer, 2003; Aiuppa et al., 2008; Martin et al., 2010].

Real-time gas monitoring, combined with theoretical and experimental studies, has allowed inferring many degassing-related processes occurring inside persistently degassing volcanoes. For example, variations of the gas flux and gas concentration ratios may occur because of magma motions towards the surface [e.g., Aiuppa et al., 2007], changes of convection rates inside the plumbing system [e.g., Shinohara et al., 2002],

permeability changes of the porous rocks covering the top of the volcanic conduit [e.g., Matthews et al., 1997], scrubbing by hydrothermal systems [e.g., Duffell et al., 2003], collapse of foams forming beneath geometrical obstacles [Jaupart and Vergnolle, 1988], complex bubble dynamics of the upper conduit [e.g., Manga, 1996; Pioli et al., 2012; Tamburello et al., 2013], or the spontaneous openings and closings of channel-like structures in non-Newtonian degassed magma [Divoux et al., 2011]. Despite the numerous studies on persistently degassing volcanoes from different approaches, there are still many unresolved questions. For example: (1) what is the effect of the large amount of gas lost during continuous passive gas emissions on the magma plumbing system? Does passive degassing have anything to do with their frequent eruptions, or is each eruption simply driven by new input of undegassed magma from depth? (2) Is it possible to make progress towards monitoring the cycles of the major component of the gas plume, i.e., water vapor, that has so far remained elusive? What does monitoring the water vapor tell us about the degassing behavior and associated processes of these volcanoes? And (3) how can we use the high-frequency degassing time series to infer the processes occurring in the volcanic conduit? Can we use the changes in the degassing patterns to anticipate eruptions or explosions? Here, I address these questions by developing new methods, theoretical models, and numerical schemes that give new insights on the overall dynamics of persistently degassing volcanoes, and help to improve the monitoring and interpretation of degassing time series.

The dissertation is organized in three chapters. Each chapter follows a research paper-like structure, and covers one of the three topics mentioned above. In the first chapter, I

present a theoretical model to calculate the pressure changes induced by passive degassing in volcanic magma reservoirs. The model is driven by the monitored gas fluxes, and a set of different scenarios is discussed depending on different geometrical and rheological parameters of the crust and magma. In the second chapter, I describe a method that allows determining the dominant frequencies of the water vapor emissions. The method is based on the scattering of the sunlight by the water droplets composing condensed volcanic plumes, and on the analysis of digital images. I show the validity of the method by means of some experiments with artificial condensed gas plumes, and then I use it to analyze the steam emissions from Erebus and Mayon volcanoes. In the third chapter, I present a new numerical model aiming to reproduce synthetic high-frequency degassing time series. The model is based on some of the results presented in the first chapter, and is validated with a natural high-frequency degassing time series from Mayon volcano, which is obtained with the method explained in the second chapter. Then, I use the model to investigate different scenarios that are thought to precede phreatic eruptions. At the end I have included the most significant findings and possible future works on the subject.

Chapter 1:

On depressurization of volcanic magma reservoirs by passive degassing

On depressurization of volcanic magma reservoirs by passive degassing

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Abstract

Many active volcanoes around the world alternate between episodes of unrest and mildly explosive eruptions, with quiescent periods dominated by abundant but passive gas emissions. These are the so-called persistently degassing volcanoes, and well-known examples are Mayon (Philippines) and Etna (Italy). Here, we develop a new lumped-parameter model to investigate by how much the gas released during quiescence can decrease the pressure within persistently degassing volcanoes. Our model is driven by the gas fluxes measured with monitoring systems, and takes into account the sizes of the conduit and reservoir, the viscoelastic response of the crust, the magma density change, the bubble exsolution and expansion at depth, and the hydraulic connectivity between reservoirs and deeper magma sources. A key new finding is that, for a vast majority of scenarios, passive degassing reduces the pressure of shallow magma reservoirs by several MPa in only a few months or years, that is, within the inter-eruptive timescales of persistently degassing volcanoes. Degassing-induced depressurization could be

responsible for the subsidence observed at some volcanoes during quiescence (e.g., at Satsuma-Iwojima and Asama, in Japan; Masaya, in Nicaragua; and Llaima, in Chile), and could play a crucial role in the onset and development of the physical processes which may in turn culminate in new unrest episodes and eruptions. For example, degassing-induced depressurization could promote magma replenishment, induce massive and sudden gas exsolution at depth, and trigger the collapse of the crater floor and reservoir roof.

1. Introduction

Some of the most active volcanoes around the world are the so-called persistently degassing volcanoes (e.g., Mayon, Philippines; Masaya, Nicaragua; Etna, Italy). Their activity consists of unrest episodes, sometimes culminating in mildly explosive eruptions, which alternate with periods of steady passive gas emissions lasting for a few months or years [e.g., Shinohara, 2008; Siebert et al., 2011]. The amount of gas released during these passive periods can be constrained by measuring the fluxes of the predominant volcanic gases (H_2O , CO_2 , and SO_2) in volcanic plumes and high temperature fumaroles [e.g., Stoiber et al., 1983; Symonds et al., 1994; Andres and Kasgnoc, 1998; Oppenheimer et al., 1998; Galle et al., 2002; Aiuppa et al., 2007; Sawyer et al., 2008; Johansson et al., 2009; Boichu et al., 2010; Galle et al., 2010]. These measurements show that persistently degassing volcanoes like Stromboli can release passively up to 6,000 - 12,000 t/day of gas [Allard et al., 1994], Etna emits up to 21,000 t/day [Aiuppa et al., 2008], and Masaya and

Satsuma-Iwojima volcanoes release about 14,000 - 16,000 t/day [Burton et al., 2000; Kazahaya et al., 2002; Martin et al., 2010].

Steady gas fluxes of several kilotonnes per day during months or years of quiescence have been explained through two different approaches. In the first approach, gas-magma separation occurs at shallow levels at the top of an open magma conduit. In this case, mean degassing rates are mostly limited by the rate at which volatiles are transported from the magma reservoir to shallow levels, which can occur via steady state magma convection in the conduit [e.g., Kazahaya et al., 1994; Stevenson and Blake, 1998; William-Jones et al., 2003; Harris et al., 2005; Burton et al., 2007; Shinohara and Tanaka, 2012]. In the second approach, gas-magma separation occurs at deep levels within a magma reservoir. In this case, mean degassing rates are mostly limited by the transport of volatiles within the reservoir [e.g., Simakin and Botcharnikov, 2001], the ascent velocity of bubbles through the magma [e.g., Vergnolle and Jaupart, 1986; Pioli et al., 2012], or the permeability of the magma conduit [e.g., Edmonds et al., 2001; Okumura et al., 2008; Divoux et al., 2011; Polacci et al., 2012]. Despite all these studies focused on passive degassing, only a few researchers have suggested that the gas lost during quiescence could decrease significantly the pressure within magma plumbing systems [Simakin and Botcharnikov, 2001; Iguchi et al., 2002].

Here, we develop a lumped-parameter model to calculate the magnitude of the pressure drops induced by passive degassing within persistently degassing volcanoes. Our model applies to volcanoes with open conduits and with shallow and deep gas-magma separation

levels, and it captures the long-term (from months to years) overall trend of the pressure change. Determining the pressure changes induced by passive degassing is crucial for a better understanding of the evolution of the magma plumbing systems, and for improving the interpretation of the monitoring signals of active volcanoes [Sparks, 2003].

2. Model approach and physical constraints

Calculating the pressure changes during passive degassing is not straightforward given the complex processes that occur within magmatic systems. For instance, bubbles nucleate and grow, ascend through the melt, and may accumulate beneath viscous magma layers or geometric barriers [e.g., Sparks, 1978; Patrick et al., 2011]. Viscous foams may form and collapse in the upper part of the conduit or at the reservoir roof, causing rise and fall of the magma level [Dibble, 1972; Jaupart and Vergnolle, 1988; Orr and Rea, 2012], and gas slugs may travel through the conduit generating long- and very long-period seismicity [e.g., Vergnolle and Jaupart, 1986; James et al., 2006]. Even more complexities arise if we consider degassing-induced crystallization [Lipman et al., 1985; Metrich et al., 2001], magma circulation transporting volatiles from the reservoir to the conduit [e.g., Shinohara, 2008], magma intrusions from deep sources to the reservoirs [e.g., Blake, 1981], or the response of the crust to pressure changes [e.g., Dzurisin, 2007]. Here we address the problem by considering the following conditions:

1) an idealized system in which a magma reservoir is connected to an open cylindrical conduit filled with magma (see Fig. 1) [e.g., Shinohara, 2008; Tanaka et al., 2008; Tanaka et al., 2009]. The depth of magma reservoirs ranges between 3-10 km [e.g., Dzurisin, 2007], and the radius of the conduit can vary from a few to up to one hundred meters (5-10 m, Stromboli; 40 - 60 m, Mt. St. Helens; 100 m, Satsuma-Iwojima [e.g., Pallister et al., 1992; Stevenson and Blake, 1998; Tanaka et al., 2009]). The volume of the magma reservoir of a given volcano is assumed to be on the same order or one order of magnitude larger than the volume of magma extruded in its largest eruptions [e.g., Dzurisin, 2007; Siebert et al., 2011]. Therefore, reservoir volumes of persistently degassing volcanoes likely range between 10^7 and 10^9 m³, even though a larger reservoir of 10^{10} m³ has also been proposed for Masaya [Walker et al., 1993].

Fig. 1. Sketch of a persistently degassing volcano illustrating the main features that we incorporate in the model. A magma reservoir is connected to a well-developed open conduit filled with magma up to somewhere close to the surface, such that the pressure in the reservoir is mainly hydrostatic during quiescence. Hence, weight changes within the conduit induced by degassing will cause pressure changes in the reservoir. The reservoir can be hydraulically connected to a deeper magma source through a cylindrical dyke, and gas finally escapes through a crater that can be covered or not by a permeable dome (e.g., Shinohara, 2008). For description of the parameters see Notation section.

2) the magma pressure within the reservoir is mainly magmastatic [e.g., Simakin and Botcharnikov, 2001]. Hence, the pressure change ΔP in the reservoir can be calculated from the mass change of magma within the conduit. For simplicity, we consider that the whole conduit is subjected to the same pressure change as the reservoir, and that the mass of gas inside the conduit is always much lower than the mass of bubble-free magma (liquid and solid phase). In the rest of the paper we refer to the bubble-free magma as melt, which is considered incompressible.

3) the melt inside the magma conduit is described as a mixture of two melts with different dissolved volatile content, and therefore with different densities [e.g., Stevenson and Blake, 1998; Kazahaya et al., 2002; Shinohara, 2008]. The maximum mass fraction of volatiles that can exsolve and escape from the melt n_x depends on the mechanism of degassing. If gas separation occurs in the reservoir, $n_x \approx n_r$, where n_r is the maximum mass fraction of water that can exsolve at the pressure of the reservoir. If gas separation occurs at shallow levels in the conduit, $n_x \approx n_c$, where n_c is the maximum mass fraction of water that can exsolve at the pressure of the upper part of the conduit. It is satisfied that $n_r < n_c \leq \alpha$, where α is the mass fraction of volatiles that were initially dissolved in the melt (from here on, we refer to the melt prior to gas exsolution as parent melt). We also assume that the mean rate of conversion from the parent melt to the fully degassed melt (i.e., melt that exsolves a mass fraction of volatiles given by n_x) equals the mean degassing rate $\hat{Q}$ [e.g., Kazahaya et al., 1994; Stevenson and Blake, 1998].

4) persistently degassing volcanoes release gas at a mean rate $\hat{Q}$ ranging between 1 and at least 10 kt/day [Allard et al., 1994; Burton et al., 2000; Kazahaya et al., 2002; Aiuppa et al., 2008; Martin et al., 2010]. Mean degassing rates during quiescence are considered constant, and thus we calculate the long-term (from months to years) pressure change in the reservoir due to steady gas loss [Shinohara, 2008]. Continuous gas emissions can in principle occur as long as the mass of volatiles released is lower than the mass of volatiles initially dissolved in the conduit-reservoir system.

5) the gas solubility depends only on pressure, and gas exsolution within the reservoir occurs at thermodynamic equilibrium according to Henry's law [e.g., Huppert and Woods, 2002]. We consider constant temperature T within the system, neglect crystal content in the reservoir, and calculate the mass of gas within the reservoir considering a given mean depth (Fig. 1). Gas expansion occurs in the reservoir and conduit following the ideal gas law. This allows us to account for the compressibility of bubble-bearing magmas.

6) the volume of melt decreases because of the exsolution of water during passive degassing [Simakin and Botcharnikov, 2001]. We neglect volume changes of melt that are due to the exsolution of other less abundant volatiles [e.g., Allard et al., 1994; Kazahaya et al., 2002].

7) the magmatic system is embedded in a half space with a Maxwell viscoelastic rheology characterized by a bulk modulus k and an effective viscosity μ [Jellinek and DePaolo, 2003]. This has been successfully used at volcanoes like Long Valley [Newman et al.,

2001] or Etna [Del Negro et al., 2009]. The typical value of the bulk modulus of pristine rocks is in the range of $k = 10^{10} - 10^{11}$ Pa [e.g., Blake, 1981; Touloukian et al., 1981], and the effective viscosities of the upper crust range between $\mu = 10^{17} - 10^{20}$ Pa s [Jellinek and DePaolo, 2003]. These values are assumed to be steady.

8) the reservoir can be permanently connected to a deeper magma source through a narrow cylindrical dyke (up to a few meters radius), which can lead to magma replenishment in the reservoir described by a well-developed Poiseuille flow [e.g., Pinel et al., 2010]. Note that this replenishment is not a requirement to sustain passive degassing as long as the mass of gas released is not larger than the mass of volatiles that were initially within the conduit-reservoir system [e.g., Shinohara, 2008; Simakin and Botcharnikov, 2001].

In the following we derive a time-dependent model that accounts for the mass change by degassing, volume change of the volcanic system, gas expansion at depth, magma density changes, and magma replenishment. With this model we perform a parametric analysis to assess the effect of passive degassing on the pressure evolution of magma plumbing systems within the inter-eruptive timescales of persistently degassing volcanoes (< 10 years). Then, we discuss the implications of our results.

3. Pressure change model for persistently degassing volcanoes during quiescence

The pressure change with time $\Delta P(t)$ within the magma reservoir can be calculated from the hydrostatic equation as (conditions 1 and 2):

$$\Delta P(t) = P(t) - P(t_0) = gL[\hat{\rho}(t) - \hat{\rho}(t_0)] \quad (1)$$

where $P(t)$ and $P(t_0)$ are the pressure in the reservoir at time t and at the reference time $t_0 = 0$, respectively, g is gravity, L is the column that comprises of the conduit length and the portion D that is within the reservoir (up to the mean depth of the reservoir, see Fig. 1), and $\hat{\rho}(t)$ and $\hat{\rho}(t_0)$ are the mean densities of the column of length L at time t and time t_0 , respectively. The mean density $\hat{\rho}(t)$ can be written as:

$$\hat{\rho}(t) = \frac{m_{g,c}(t) + m_{m,c}(t)}{\pi R(t)^2 L} \quad (2)$$

where $m_{g,c}(t)$ and $m_{m,c}(t)$ are the mass of gas and melt, respectively, within the column of length L and radius equal to the radius of the conduit $R(t)$ (Fig. 1). Note that, within the column of length L , magma level could move up and down depending on the bubble dynamics in the conduit [e.g., Jaupart and Vergnolle, 1988; Harris and Ripepe, 2007; Boichu et al., 2010; Patrick et al., 2011; Orr and Rea, 2012]. Replacing equation (2) in equation (1), we get:

$$\Delta P(t) = gL \left[\frac{m_{g,c}(t) + m_{m,c}(t)}{\pi R(t)^2 L} - \hat{\rho}(t_0) \right] \quad (3)$$

As we will show later, there are several terms needed to calculate $\Delta P(t)$ that depend on the rate of pressure change. Therefore, we take the time-derivative of equation (3) for convenience, which is given by:

$$\frac{d\Delta P(t)}{dt} = \frac{g}{\pi R(t)^2} \left\{ \frac{1}{m_{m,c}(t)} \left[\frac{dm_{g,c}(t)}{dt} + \frac{dm_{m,c}(t)}{dt} \right] - \frac{2}{R(t)} \frac{dR(t)}{dt} \right\} \quad (4)$$

We have assumed here that $m_{m,c}(t) \gg m_{g,c}(t)$ at any time (condition 2). Note that $m_{g,c}(t)$ must not be neglected prior to taking the derivative of equation (3) because we would omit the term $dm_{g,c}(t)/dt$, which in principle cannot be neglected when compared with $dm_{m,c}(t)/dt$. By replacing $m_{m,c}(t)$ by $\hat{\rho}_{m,c}(t)V_{m,c}(t)$ and $m_{g,c}(t)$ by $\hat{\rho}_{g,c}(t)V_{g,c}(t)$, where $\hat{\rho}_{m,c}(t)$ and $\hat{\rho}_{g,c}(t)$ are the mean densities of melt and gas inside the column, respectively, and $V_{m,c}(t)$ and $V_{g,c}(t)$ are the volumes of melt and gas inside the column, respectively, we can obtain:

$$\begin{aligned} \frac{d\Delta P(t)}{dt} = \frac{g}{\pi R(t)^2} \left\{ \frac{1}{\hat{\rho}_{m,c}(t)V_{m,c}(t)} \left[\hat{\rho}_{g,c}(t) \frac{dV_{g,c}(t)}{dt} + V_{g,c}(t) \frac{d\hat{\rho}_{g,c}(t)}{dt} \right. \right. \\ \left. \left. + \hat{\rho}_{m,c}(t) \frac{dV_{m,c}(t)}{dt} + V_{m,c}(t) \frac{d\hat{\rho}_{m,c}(t)}{dt} \right] - \frac{2}{R(t)} \frac{dR(t)}{dt} \right\} \quad (5) \end{aligned}$$

The volume of gas inside the column of length L can be also written as $V_{g,c}(t) = \pi R(t)^2 L - V_{m,c}(t)$. By replacing this in equation (5), and considering that $\hat{\rho}_{m,c}(t)$ is at least one order of magnitude larger than $\hat{\rho}_{g,c}(t)$, we get:

$$\begin{aligned} \frac{d\Delta P(t)}{dt} = & \frac{g \hat{\rho}_{m,c}(t) V_{m,c}(t)}{\pi R(t)^2} \left\{ \frac{1}{\hat{\rho}_{m,c}(t)} \frac{d\hat{\rho}_{m,c}(t)}{dt} + \frac{[\pi R(t)^2 L - V_{m,c}(t)]}{\hat{\rho}_{m,c}(t) V_{m,c}(t)} \frac{d\hat{\rho}_{g,c}(t)}{dt} \right. \\ & \left. + \frac{1}{V_{m,c}(t)} \frac{dV_{m,c}(t)}{dt} - \left[1 - \frac{\pi R(t)^2 L \hat{\rho}_{g,c}(t)}{\hat{\rho}_{m,c}(t) V_{m,c}(t)} \right] \frac{2}{R(t)} \frac{dR(t)}{dt} \right\} \quad (6) \end{aligned}$$

In the following we determine each of the derivatives of the right-hand side of equation (6).

3.1. Mean density of melt in the column ($\hat{\rho}_{m,c}(t)$)

The mean density of melt in the magma column $\hat{\rho}_{m,c}(t)$ is approximated by (condition 3):

$$\hat{\rho}_{m,c}(t) = \gamma_c(t) \rho_{c1} + (1 - \gamma_c(t)) \rho_{c2} \quad (7)$$

where ρ_{c1} is the density of the more degassed melt in the column and ρ_{c2} is the density of the less degassed (or fully undegassed) melt. The term $\gamma_c(t)$ is the volume fraction of the more degassed melt in the magma column, and it is by definition $\gamma_c(t) = V_{m,c1}(t)/V_{m,c}(t)$, where $V_{m,c1}(t)$ is the volume of the more degassed melt. From

equation (7), its derivative, and the definition of $\gamma_c(t)$, the term $d\hat{\rho}_{m,c}(t)/dt$ in equation (6) can be replaced by:

$$\frac{d\hat{\rho}_{m,c}(t)}{dt} = \Delta\rho_{1,2} \frac{d\gamma_c(t)}{dt} = \frac{\gamma_c(t)\Delta\rho_{1,2}}{V_{m,c}(t)} \left[\frac{1}{\gamma_c(t)} \frac{dV_{m,c1}(t)}{dt} - \frac{dV_{m,c}(t)}{dt} \right] \quad (8)$$

where $\Delta\rho_{1,2} = \rho_{c1} - \rho_{c2}$. The value of $\gamma_c(t)$, and hence the value of $\hat{\rho}_{m,c}(t)$, depends on the mechanism of degassing. For volcanoes in which gas-magma separation occurs in the reservoir and bubbles escape through a permeable degassed magma conduit [e.g., Pioli et al., 2012; Divoux et al., 2011], the volume fraction of the more degassed melt in the column is constant, and therefore $d\hat{\rho}_{m,c}(t)/dt \approx 0$ and $\hat{\rho}_{m,c}(t)$ is constant. For volcanoes in which gas-magma separation occurs at shallow levels via steady state convection in the conduit, $\gamma_c(t)$ is also roughly constant since the volume of the fully degassed melt sinking from the conduit to the reservoir is replaced continuously by less degassed melt rising from the reservoir to the conduit [e.g., Shinohara, 2008]. In this case, equation (8) is also zero and $\hat{\rho}_{m,c}(t)$ is constant as well.

An end-member case with a maximum change of $\hat{\rho}_{m,c}(t)$ during quiescence would be a volcano in which gas separation occurs at shallow levels without steady state convection in the conduit. In such a case, the system would not be gravitationally stable because the denser melt would be on top, but it might be a realistic possibility since the ability for convection also depends on the radius of the conduit and melt viscosity [e.g., Stevenson and Blake, 1998]. For this scenario, $V_{m,c1}(t)$, $\gamma_c(t)$, and $\hat{\rho}_{m,c}(t)$ increase progressively

with time as long as the whole conduit is not degassed. Below we show how to obtain the term $dV_{m,c1}(t)/dt$ of equation (8) for this end-member scenario.

By considering conditions 3 and 4, we can state that:

$$\hat{Q} = -n_x \rho_{nd} \left. \frac{dV_{undeg}}{dt} \right|_d \quad (9)$$

where n_x is the mass fraction of volatiles that can exsolve and escape from the melt at the gas-magma separation level, ρ_{nd} is the density of the parent undegassed melt, and the term $dV_{undeg}/dt|_d$ is the rate of volume change of undegassed melt in the conduit-reservoir system due only to degassing. This means that $dV_{undeg}/dt|_d$ does not account for the increase of undegassed melt because of replenishment. From mass conservation, it is also satisfied that:

$$\rho_x \frac{dV_{deg}(t)}{dt} = -\rho_{nd} \left. \frac{dV_{undeg}}{dt} \right|_d - \hat{Q} \quad (10)$$

where $\rho_x = \rho_{c1}$ when gas separation occurs in the conduit, and $\rho_x = \rho_r$ when gas separation occurs in the reservoir, such that ρ_r is the density of the degassed melt of the reservoir. In other words, ρ_x is the melt density at the separation level. The term $dV_{deg}(t)/dt$ is the rate of volume change of fully degassed melt (i.e., melt that has

exsolved a mass fraction of volatiles n_x) in the conduit-reservoir system. By combining equations (9) and (10), we get:

$$\frac{dV_{deg}(t)}{dt} = \frac{(1 - n_x)\hat{Q}}{n_x \rho_x} \quad (11)$$

Hence, if we impose that all the melt degasses and stagnates in the conduit, the term $dV_{m,c1}(t)/dt$ takes the form:

$$\frac{dV_{m,c1}(t)}{dt} = \frac{(1 - n_c)\hat{Q}}{n_c \rho_{c1}} \quad (12)$$

In such a case, $\gamma_c(t)$ can be calculated from:

$$\gamma_c(t) = \gamma_c(t_0) \frac{V_{m,c}(t_0)}{V_{m,c}(t)} + \frac{(1 - n_c)\hat{Q} t}{n_c \rho_{c1} V_{m,c}(t)} \quad \text{if } V_{m,c1}(t) \leq V_{m,c}(t) \quad (13)$$

$$\gamma_c(t) = 1 \quad \text{if } V_{m,c1}(t) > V_{m,c}(t) \quad (14)$$

which is obtained from the definition of $\gamma_c(t)$ and after integrating equation (12). Below we write equation (8) in a compact form that accounts for all the possible scenarios we have mentioned:

$$\frac{d\hat{\rho}_{m,c}(t)}{dt} = \frac{\gamma_c(t)\Delta\rho_{1,2}A^*}{V_{m,c}(t)} \left[\frac{(1 - n_c)\hat{Q}}{\gamma_c(t)n_c \rho_{c1}} - \frac{dV_{m,c}(t)}{dt} \right] \quad (15)$$

where A^* is an indicator variable whose value is 0 when $\hat{\rho}_{m,c}(t)$ and $\gamma_c(t)$ are constants, and 1 when the degassed melt accumulates in the column.

3.2. Density of gas in the column ($\hat{\rho}_{g,c}(t)$)

If we consider that the pressure change within the column is the same as the pressure change in the reservoir (condition 2), and that the mean density of gas within the column $\hat{\rho}_{g,c}(t)$ can be described with the ideal gas law (condition 5), the term $d\hat{\rho}_{g,c}(t)/dt$ of equation (6) is given by:

$$\frac{d\hat{\rho}_{g,c}(t)}{dt} = \frac{M_{H_2O}}{R_g T} \frac{d\Delta P(t)}{dt} \quad (16)$$

where M_{H_2O} is the molecular weight of water, T is the temperature of the magma, and R_g is the universal gas constant.

3.3. Volume of melt in the column ($V_{m,c}(t)$)

To calculate the rate of volume change of melt in the column $V_{m,c}(t)$, we begin with the volume of melt in the conduit-reservoir system $V_m(t)$:

$$V_m(t) = V_{m,r}(t) + V_{m,c}(t) - \pi R(t)^2 D \quad (17)$$

where $V_{m,r}(t)$ is the volume of melt in the reservoir and D the portion of the magma column which is within the reservoir. The last term of equation (17) is included to avoid accounting twice for the volume of melt inside the portion of magma column of length D (see Fig. 1). The derivative of $V_m(t)$ is:

$$\frac{dV_m(t)}{dt} = \frac{dV_{m,r}(t)}{dt} + \frac{dV_{m,c}(t)}{dt} - 2\pi R(t)D \frac{dR(t)}{dt} = \frac{dV_d(t)}{dt} + \frac{dV_{rep}(t)}{dt} \quad (18)$$

where $dV_d(t)/dt$ is the rate of volume decrease of melt in the conduit-reservoir system due to gas exsolution, and $dV_{rep}(t)/dt$ is the rate of volume increase of melt if there is replenishment. After rearranging terms, $dV_{m,c}(t)/dt$ in equation (6) can be replaced by:

$$\frac{dV_{m,c}(t)}{dt} = \frac{dV_d(t)}{dt} - \frac{dV_{m,r}(t)}{dt} + \frac{dV_{rep}(t)}{dt} + 2\pi R(t)D \frac{dR(t)}{dt} \quad (19)$$

In the next sub-sections we determine the first three derivatives of the right-hand side of the equation above.

3.3.1. Volume decrease of melt due to degassing ($V_d(t)$)

The rate of volume decrease of melt because of degassing can be found by rewriting equation (18) as:

$$\frac{dV_m(t)}{dt} = \frac{dV_d(t)}{dt} + \frac{dV_{rep}(t)}{dt} = \frac{dV_{deg}(t)}{dt} + \frac{dV_{undeg}(t)}{dt} \quad (20)$$

where $dV_{undeg}(t)/dt$ is the rate of volume change of the parent undegassed magma in the conduit-reservoir system due to degassing and replenishment. Hence, $dV_{undeg}(t)/dt$ can be written as:

$$\frac{dV_{undeg}(t)}{dt} = \left. \frac{dV_{undeg}(t)}{dt} \right|_d + \frac{dV_{rep}(t)}{dt} \quad (21)$$

The first-term of the right-hand side of equation (21) is the same as in equation (9), and with the second term we consider that the replenished magma is completely undegassed. By replacing equation (21) in equation (20), and using equations (9) and (11), we get:

$$\frac{dV_d(t)}{dt} = \frac{(1 - n_x)\hat{Q}}{n_x \rho_x} - \frac{\hat{Q}}{n_x \rho_{nd}} = -\frac{\hat{Q}}{\rho_w} \quad (22)$$

where we have defined $\rho_w = n_x \rho_x \rho_{nd} / [\rho_x - (1 - n_x) \rho_{nd}]$. For $n_x = \alpha$, $\rho_x = \rho_{c1}$, and by neglecting degassing-induced crystallization, the term ρ_w can be interpreted as the partial density of water dissolved in anhydrous melt [Simakin and Botcharnikov, 2001].

3.3.2. Volume of melt within the reservoir ($V_{m,r}(t)$)

The volume of melt within the reservoir $V_{m,r}(t)$ depends on the volume of gas that is exsolved at time t in the reservoir $V_{g,r}(t)$, and on the size of the reservoir itself $V_r(t)$. It can be written as:

$$V_{m,r}(t) = V_r(t) - V_{g,r}(t) \quad (23)$$

Therefore, the rate of volume change of melt in the reservoir $dV_{m,r}(t)/dt$ can be calculated from:

$$\frac{dV_{m,r}(t)}{dt} = \frac{dV_r(t)}{dt} - \frac{1}{\hat{\rho}_{g,r}(t)} \frac{dm_{g,r}(t)}{dt} + \frac{m_{g,r}(t)}{\hat{\rho}_{g,r}(t)^2} \frac{d\hat{\rho}_{g,r}(t)}{dt} \quad (24)$$

where $\hat{\rho}_{g,r}(t)$ and $m_{g,r}(t)$ are the mean density and mass of gas in the reservoir, respectively. In the following, we approximate the mass of gas in the reservoir by $m_{g,r}(t) = n_r(t) \rho_{nd} V_r(t)$. By assuming thermodynamic equilibrium, considering only

water vapor (condition 6), and neglecting crystal content, $n_r(t)$ is after Henry's law [e.g., Huppert and Woods, 2002]:

$$n_r(t) = \alpha - S [P(t_0) + \Delta P(t)]^{1/2} \quad \text{if } \alpha > S[P(t_0) + \Delta P(t)]^{1/2} \quad (25)$$

$$n_r(t) = 0 \quad \text{if } \alpha \leq S[P(t_0) + \Delta P(t)]^{1/2} \quad (26)$$

where $S = 4 * 10^{-6} \text{ Pa}^{-1/2}$ for water, and $P(t_0) = gL\hat{\rho}(t_0)$ is the initial pressure as in equation (1). The actual mass fraction of exsolved volatiles $n_r(t)$ in a magma reservoir could differ from the values given by equation (25) if gas-magma separation occurs at depth. For example, $n_r(t)$ could be larger if some bubbles accumulate progressively with time beneath the reservoir roof [Jaupart and Vergnolle, 1988], or could be lower if bubbles can escape from the reservoir very rapidly once they form (the so-called open-system degassing). By using equations (25) – (26) we will gain a first-order insight about the coupling between depressurization induced by degassing and gas exsolution in the reservoir. If we use again the ideal gas law (condition 5), the mean density of gas in the reservoir $\hat{\rho}_{g,r}(t)$ is given by:

$$\hat{\rho}_{g,r}(t) = \frac{[P(t_0) + \Delta P(t)]M_{H_2O}}{R_g T} \quad (27)$$

On the other hand, the volume change of the reservoir $V_r(t)$ with time can be described from the generalization of the Maxwell viscoelastic model (condition 7) as [Jellinek and DePaolo, 2003]:

$$\frac{dV_r(t)}{dt} \approx \frac{V_r(t)}{k} \frac{d\Delta P(t)}{dt} + \frac{V_r(t)\Delta P(t)}{\mu} \quad (28)$$

being k the bulk modulus of rocks, and μ the effective viscosity of the crust. Therefore, the time-derivative of the melt in the reservoir $dV_{m,r}(t)/dt$, after combining the derivatives of equation (25) - (27) with equations (28) and (24), is given by:

$$\begin{aligned} \frac{dV_{m,r}(t)}{dt} = & \left\{ \left[1 - \frac{n_r(t)\rho_{nd}}{\hat{\rho}_{g,r}(t)} \right] \frac{V_r(t)}{k} + \frac{\rho_{nd}V_r(t)S^2B^*}{2\hat{\rho}_{g,r}(t)(\alpha - n_r(t))} \right. \\ & \left. + \frac{n_r(t)\rho_{nd}V_r(t)M_{H_2O}}{\hat{\rho}_{g,r}(t)^2R_gT} \right\} \frac{d\Delta P(t)}{dt} \\ & + \left[1 - \frac{n_r(t)\rho_{nd}}{\hat{\rho}_{g,r}(t)} \right] \frac{V_r(t)}{\mu} \Delta P(t) \end{aligned} \quad (29)$$

where B^* is an indicator variable whose value is 0 if $\alpha \leq S[P(t_0) + \Delta P(t)]^{1/2}$, and thus when there are no bubbles in the reservoir ($n_r(t) = 0$), and 1 if $\alpha > S[P(t_0) + \Delta P(t)]^{1/2}$, and thus when there are bubbles in the reservoir ($n_r(t) > 0$).

3.3.3. Volume of replenished melt ($V_{rep}(t)$)

If there is connectivity between the reservoir and deep magma sources through dykes [e.g., Blake, 1981], magma refilling from deep sources to the reservoir during quiescent periods is possible. If we assume a quasi-steady state approach (condition 8), the volumetric rate

of replenished magma can be approximated by the volumetric rate of a well-developed Poiseuille flow inside a dyke [e.g., Pinel et al., 2010]. Thus, we get:

$$\frac{dV_{rep}(t)}{dt} = \lambda(t)(P_s(t) - P(t) - P_{sr}) \quad (30)$$

where $P_s(t)$ is the pressure in the magma source, $P(t)$ is the pressure in the shallow reservoir as in equation (1), and P_{sr} is the hydrostatic pressure exerted on the source by the weight of magma inside the dyke. $\lambda(t)$ is the hydraulic strength of the connectivity between the shallow reservoir and the deep source, which depends on the geometry of the dyke and the rheology of the magma. The above equation can be also written as:

$$\frac{dV_{rep}(t)}{dt} = \lambda(t)(\Delta P_s(t) - \Delta P(t)) \quad (31)$$

where $\Delta P_s(t)$ is the overpressure of the source, and is defined as $\Delta P_s(t) = P_s(t) - P(t_0) - P_{sr}$. Equation (31) shows that magma refilling during quiescence at persistently degassing volcanoes can be induced by pressurizations of the deep magma source but also by depressurizations of the shallow reservoir. For a cylindrical dyke connecting the reservoir with the feeder source and a Newtonian magma rheology, the hydraulic strength is given by [e.g., Pinel et al., 2010]:

$$\lambda(t) = \frac{\pi R_d^4}{8M\mu_{nd}} \quad (32)$$

where R_d and M are the radius and the length of the dyke, respectively, and μ_{nd} is the viscosity of the replenished magma (undegassed melt). Magma cooling and crystallization can be important within narrow connecting dykes, which can vary the hydraulic strength $\lambda(t)$ by decreasing the dyke radius, increasing the magma viscosity, and inducing the onset of yield strength in the magma [e.g., Costa and Macedonio, 2003; Melnik and Sparks, 2005; Bruce and Huppert, 1989; Kavanagh and Sparks, 2011]. In turn, the cooling rate within the dyke varies with the pressure difference between the magma source and the reservoir [e.g., Bruce and Huppert, 1989], and thus the hydraulic strength also depends on $\Delta P(t)$. A detailed analysis of the coupling between thermal variations within the dykes, and variations of the hydraulic strength depending on $\Delta P(t)$, is beyond the scope of this paper. But we will use equations (30) – (32) to gain a first-order insight about the coupling between depressurization induced by degassing and magma replenishment.

3.4. Radius of the conduit ($R(t)$)

We calculate the radius change of the conduit by applying the Maxwell viscoelastic approach (equation (28)) to a cylinder (condition 7). In such a case, we get:

$$\frac{d\pi R(t)^2 L}{dt} \approx \frac{\pi R(t)^2 L}{k} \frac{d\Delta P(t)}{dt} + \frac{\pi R(t)^2 L \Delta P(t)}{\mu} \quad (33)$$

Neglecting the variations of the length L , we obtain:

$$\frac{dR(t)}{dt} \approx \frac{R(t)}{2k} \frac{d\Delta P(t)}{dt} + \frac{R(t) \Delta P(t)}{2\mu} \quad (34)$$

A more accurate formulation can be found in the literature for the elastic term of the deformation [e.g., Young and Budinas, 2002], but our simplification gives the same order of magnitude for the radius change. Hence, we use equation (34) for simplicity.

3.5. The final compact equation

A compact equation for the depressurization rate can be obtained by combining the above equations. If we replace equation (15) in equation (6), we get after rearranging terms:

$$\begin{aligned} \frac{d\Delta P(t)}{dt} = & \frac{g \hat{\rho}_{m,c}(t) V_{m,c}(t)}{\pi R(t)^2} \left\{ \frac{(1 - n_c) \hat{Q} \Delta \rho_{1,2} A^*}{n_c \rho_{c1} \hat{\rho}_{m,c}(t) V_{m,c}(t)} + \frac{[\pi R(t)^2 L - V_{m,c}(t)]}{\hat{\rho}_{m,c}(t) V_{m,c}(t)} \frac{d\hat{\rho}_{g,c}(t)}{dt} \right. \\ & + \frac{1}{V_{m,c}(t)} \left(1 - \frac{\gamma_c(t) \Delta \rho_{1,2} A^*}{\hat{\rho}_{m,c}(t)} \right) \frac{dV_{m,c}(t)}{dt} \\ & \left. - \left[1 - \frac{\pi R(t)^2 L \hat{\rho}_{g,c}(t)}{\hat{\rho}_{m,c}(t) V_{m,c}(t)} \right] \frac{2}{R(t)} \frac{dR(t)}{dt} \right\} \quad (35) \end{aligned}$$

The term $\gamma_c(t) \Delta \rho_{1,2} A^* / \hat{\rho}_{m,c}(t)$ of the above equation is at least one order of magnitude lower than one, given that $\Delta \rho_{1,2}$ is at least one order of magnitude lower than $\hat{\rho}_{m,c}(t)$ and $\gamma_c(t) \leq 1$. By taking this into account and replacing equation (19) in (35), we get:

$$\begin{aligned}
\frac{d\Delta P(t)}{dt} = & \frac{g \hat{\rho}_{m,c}(t) V_{m,c}(t)}{\pi R(t)^2} \left\{ \frac{(1 - n_c) \hat{Q} \Delta \rho_{1,2} A^*}{n_c \rho_{c1} \hat{\rho}_{m,c}(t) V_{m,c}(t)} + \frac{[\pi R(t)^2 L - V_{m,c}(t)]}{\hat{\rho}_{m,c}(t) V_{m,c}(t)} \frac{d\hat{\rho}_{g,c}(t)}{dt} \right. \\
& + \frac{1}{V_{m,c}(t)} \left[\frac{dV_d(t)}{dt} - \frac{dV_{m,r}(t)}{dt} + \frac{dV_{rep}(t)}{dt} \right] \\
& \left. - \left[1 - \frac{\pi R(t)^2 L \hat{\rho}_{g,c}(t)}{\hat{\rho}_{m,c}(t) V_{m,c}(t)} - \frac{\pi R(t)^2 D}{V_{m,c}(t)} \right] \frac{2}{R(t)} \frac{dR(t)}{dt} \right\} \quad (36)
\end{aligned}$$

Since $\hat{\rho}_{g,c}(t) \ll \hat{\rho}_{m,c}(t)$, $V_{m,c}(t)$ and $\pi R(t)^2 L$ are on the same order of magnitude, and $D \ll L$, the terms $\pi R(t)^2 L \hat{\rho}_{g,c}(t) / \hat{\rho}_{m,c}(t) V_{m,c}(t)$ and $\pi R(t)^2 D / V_{m,c}(t)$ are also at least one order of magnitude lower than one. Therefore, by neglecting these terms, and replacing equations (16), (22), (29), (31), and (34) in equation (36), we obtain after rearranging terms:

$$\frac{d\Delta P(t)}{dt} = \frac{C_1(t) + C_2(t) \Delta P(t)}{C_3(t)} \quad (37)$$

being:

$$C_1(t) = \left[\frac{(1 - n_c) \Delta \rho_{1,2} A^*}{n_c \rho_{c1} \hat{\rho}_{m,c}(t)} - \frac{1}{\rho_w} \right] \hat{Q} + \lambda(t) \Delta P_s(t) \quad (38)$$

$$C_2(t) = - \left\{ \left[1 - \frac{n_r(t) \rho_{nd}}{\hat{\rho}_{g,r}(t)} \right] \frac{V_r(t)}{\mu} + \lambda(t) + \frac{V_{m,c}(t)}{\mu} \right\} \quad (39)$$

$$C_3(t) = \frac{\pi R(t)^2}{g \hat{\rho}_{m,c}(t)} + \frac{V_{m,c}(t)}{k} + \left[1 - \frac{n_r(t) \rho_{nd}}{\hat{\rho}_{g,r}(t)} \right] \frac{V_r(t)}{k} + \frac{\rho_{nd} V_r(t) S^2 B^*}{2 \hat{\rho}_{g,r}(t) (\alpha - n_r(t))} + \left[\frac{n_r(t) \rho_{nd} V_r(t)}{\hat{\rho}_{g,r}(t)^2} - \frac{\pi R(t)^2 L - V_{m,c}(t)}{\hat{\rho}_{m,c}(t)} \right] \frac{M_{H_2O}}{R_g T} \quad (40)$$

The term $[\pi R(t)^2 L - V_{m,c}(t)] M_{H_2O} / [\hat{\rho}_{m,c}(t) R_g T]$ of equation (40) is also one order of magnitude lower than the term $\pi R(t)^2 / [g \hat{\rho}_{m,c}(t)]$ for typical magma temperatures.

Hence, the parameter $C_3(t)$ can be simplified to:

$$C_3(t) = \frac{\pi R(t)^2}{g \hat{\rho}_{m,c}(t)} + \frac{V_{m,c}(t)}{k} + \left[1 - \frac{n_r(t) \rho_{nd}}{\hat{\rho}_{g,r}(t)} \right] \frac{V_r(t)}{k} + \frac{\rho_{nd} V_r(t) S^2 B^*}{2 \hat{\rho}_{g,r}(t) (\alpha - n_r(t))} + \frac{n_r(t) \rho_{nd} V_r(t) M_{H_2O}}{\hat{\rho}_{g,r}(t)^2 R_g T} \quad (41)$$

The pressure change with time $\Delta P(t)$ can be calculated by integrating equation (37) from the values of melt density, the size of the conduit and magma reservoir, the mean flux of water vapor, the viscoelastic properties of the host-rock, the hydraulic strength of the connectivity between the reservoir and deeper magma sources, and the gas content in the reservoir (equations (38) – (41)).

3.6. Analytical solutions

A numerical scheme is required to solve equation (37) in a general case, even though a simple analytical solution can be obtained if we approximate $C_1(t) \approx C_1(t_0)$, $C_2(t) \approx C_2(t_0)$, and $C_3(t) \approx C_3(t_0)$. This can be done by assuming that $\lambda(t)$ and $\Delta P_s(t)$ are constant, and if the variations of the terms $n_r(t)$, $\hat{\rho}_{g,r}(t)$, $V_r(t)$, $R(t)$, $\hat{\rho}_{m,c}(t)$, and $V_{m,c}(t)$ are very small compared to their initial values. The terms $n_r(t)$ and $\hat{\rho}_{g,r}(t)$ can be approximated to $n_r(t_0)$ and $\hat{\rho}_{g,r}(t_0)$ if the magnitude of the pressure change $|\Delta P(t)|$ is at least one order of magnitude lower than the initial pressure of the reservoir $P(t_0)$ [see equations (25) and (27)]. This is satisfied for pressure changes of $|\Delta P(t)| \leq 10$ MPa when the mean reservoir depth is $L \geq 3 - 4$ km. The terms $V_r(t)$ and $R(t)$ can be replaced by $V_r(t_0)$ and $R(t_0)$, respectively, when $k \gg |\Delta P(t)|$ and $\mu \gg |\Delta P(t)| (t - t_0)$. This approximation is realistic for the typical values of bulk modulus $k \sim 10^{10}$ Pa, effective viscosities of $\mu \geq 10^{17}$ Pa s, pressure changes of $|\Delta P(t)| \leq 10$ MPa, and periods of passive degassing of $t - t_0 \leq 10$ years (see equations (28) and (34)). The term $\hat{\rho}_{m,c}(t)$ varies at most by an amount given by $\Delta \rho_{1,2}$ (see equation (7)), which in turn is at least one order of magnitude lower than $\hat{\rho}_{m,c}(t)$. The term $V_{m,c}(t)$ can be also replaced by $V_{m,c}(t_0)$ for the conditions cited above, what can be seen by combining equations (1) and (2), and considering that $m_{m,c}(t) \gg m_{g,c}(t)$ at any time (condition 2). In such a case, we can write $V_{m,c}(t) \approx \pi R(t)^2 [P(t_0) + \Delta P(t)] / g \hat{\rho}_{m,c}(t)$, and hence, $V_{m,c}(t)$ can be replaced by $V_{m,c}(t_0)$ when $R(t)$ and $\hat{\rho}_{m,c}(t)$ can be replaced by $R(t_0)$ and $\hat{\rho}_{m,c}(t_0)$, respectively, and when the pressure change $|\Delta P(t)|$ is at least one order of magnitude

lower than the initial pressure of the reservoir $P(t_0)$. Under these conditions, we obtain after integrating equation (37) that the pressure change $\Delta P(t)$ is given by:

$$\Delta P(t) = -\Delta P_\infty (1 - e^{-\Gamma t}) \quad (42)$$

where ΔP_∞ and Γ are two positive parameters given by $\Delta P_\infty = C_1(t_0)/C_2(t_0)$ and $\Gamma = -C_2(t_0)/C_3(t_0)$, respectively. We have taken into account that $t_0 = 0$. The term ΔP_∞ , which depends on the gas flux $\hat{Q}$, is a maximum value to which the pressure change tends asymptotically with time. In other words, ΔP_∞ is an upper bound and larger pressure changes (in absolute value) cannot be reached within inter-eruptive periods. Combining equation (42) with equations (28) and (34), and integrating, we get an analytical equation for the volume change of the reservoir $\Delta V_r(t)$ and for the radius change of the conduit $\Delta R(t)$:

$$\Delta V_r(t) = -V_r(t_0) \Delta P_\infty \left[\frac{t}{\mu} + \left(\frac{1}{k} - \frac{1}{\Gamma \mu} \right) (1 - e^{-\Gamma t}) \right] \quad (43)$$

$$\Delta R(t) = -\frac{R(t_0) \Delta P_\infty}{2} \left[\frac{t}{\mu} + \left(\frac{1}{k} - \frac{1}{\Gamma \mu} \right) (1 - e^{-\Gamma t}) \right] \quad (44)$$

where we have neglected second order terms, i.e., we have replaced $V_r(t)$ by $V_r(t_0)$ and $R(t)$ by $R(t_0)$ in the right-hand side of equations (28) and (34). We can also obtain an analytical expression which links the volume of replenished magma with the gas flux. By combining equations (31) and (42), we obtain:

$$V_{rep}(t) = \lambda(t_0) t [\Delta P_s(t_0) + \Delta P_\infty] + \frac{\lambda \Delta P_\infty}{\Gamma} (1 - e^{-\Gamma t}) \quad (45)$$

A simpler analytical expression for the pressure change can be obtained if the effective viscosity of the crust is very high ($\mu \rightarrow \infty$) and the hydraulic strength is very small ($\lambda \rightarrow 0$). In such a case, we retrieve from equation (42) an expression which applies to a volcano with only elastic response of the host-rock and as long as there is no magma intrusion:

$$\Delta P(t) = -[\Delta P_\infty \Gamma]^* t \quad (46)$$

being $[\Delta P_\infty \Gamma]^* = \Delta P_\infty \Gamma$ for $\mu \rightarrow \infty$ and $\lambda \rightarrow 0$. The term $[\Delta P_\infty \Gamma]^*$ can be used to get a first-order insight on the depressurization rates during passive degassing. For no connectivity between the reservoir and the deeper sources ($\lambda = 0$), no bubbles in the reservoir ($n_r(t) = 0$ and $B^* = 0$), volcanoes with reservoir volumes much larger than conduit volumes ($V_r(t) \gg V_{m,c}(t)$), and constant melt density in the column ($A^* = 0$ and $\hat{\rho}_{m,c}(t) = \hat{\rho}_{m,c}(t_0)$), depressurization by degassing is simply given by:

$$\Delta P(t) = - \frac{g \hat{\rho}_{m,c}(t_0) k \hat{Q} t}{\pi R(t_0)^2 \rho_w k + g \rho_w \hat{\rho}_{m,c}(t_0) V_r(t_0)} \quad (47)$$

4. Parametric analysis

In this section we explore the influence of the different parameters of our model for four scenarios that are thought to be applicable to persistently degassing volcanoes during quiescence: (1) we first describe the pressure change induced by passive degassing in a bubble-free reservoir that is not connected to deeper magma sources, and for a constant melt density in the column ($n_r(t) = 0$, $\lambda(t) = 0$, $\hat{\rho}_{m,c}(t) = \hat{\rho}_{m,c}(t_0)$). This would correspond to a scenario in which gas-magma separation occurs at low pressures via steady state convection in the conduit [e.g., Shinohara, 2008], as has been proposed at volcanoes like Izu-Oshima [Kazahaya et al., 1994], Mount St. Helens [Stevenson and Blake, 1998], Stromboli [Stevenson and Blake, 1998; Burton et al., 2007], Satsuma-Iwojima [Kazahaya et al., 2002], and Popocatépetl [Witter et al., 2005]. (2) Then, we analyze the case in which bubbles exsolve in a reservoir that is not connected to deeper magma sources, by keeping also constant the mean density of melt in the column ($n_r(t) \neq 0$, $\lambda(t) = 0$, $\hat{\rho}_{m,c}(t) = \hat{\rho}_{m,c}(t_0)$). This could correspond to a scenario in which gas-magma separation occurs at depth and bubbles can ascend and escape steadily through a vesicular magma conduit, as has been proposed at volcanoes like Stromboli [Jaupart and Vergnolle, 1988] and *Soufrière* Hills [Edmonds et al., 2001]. (3) We explore the coupling between depressurization by degassing and magma replenishment when there are no bubbles in the reservoir and for constant melt density in the column ($n_r(t) = 0$, $\lambda(t) \neq 0$, $\hat{\rho}_{m,c}(t) = \hat{\rho}_{m,c}(t_0)$). This scenario will give us some insights on how passive degassing may affect magma replenishment, which is thought to occur more or less frequently at persistently degassing volcanoes [e.g., Shinohara, 2008]. (4) Finally, we

explore the end-member scenario in which gas-magma separation occurs in the conduit but the degassed magma is not replaced by undegassed magma via convection. In this case we assume a bubble-free reservoir that is not connected to deeper magma sources ($n_r(t) = 0$, $\lambda(t) = 0$, $\hat{\rho}_{m,c}(t) \neq \hat{\rho}_{m,c}(t_0)$). This scenario will give us some insights on the maximum effects of the variations of the melt density in the conduit.

For all our calculations we use an andesitic magma composition at $T = 1000^\circ\text{C}$ and an oxygen fugacity at the Ni-NiO buffer. Melt densities and viscosities shown in the next figures are calculated at 2.5 kbar and for a given weight ratio of dissolved water by using MELTS thermodynamic algorithm [Ghiorso and Sack, 1995]. Using other reference pressures does not change the final results. The values of the parameters vary between those shown in the notation section.

4.1. Scenario 1: gas-free reservoir, no replenishment, and constant mean density of melt in the column ($n_r(t) = 0$, $\lambda(t) = 0$, $\hat{\rho}_{m,c}(t) = \hat{\rho}_{m,c}(t_0)$)

This scenario corresponds to a volcano in which gas exsolution and degassing occurs at low pressures via magma convection in the conduit [Shinohara, 2008]. First, we explore independently the influence of the gas flux, reservoir volume, effective viscosity of the crust, and conduit radius for small volcanic systems ($V_r(t_0) = 10^8 - 10^9 \text{ m}^3$) and for large volcanic systems ($V_r(t_0) = 10^9 - 10^{10} \text{ m}^3$). Then, we calculate the expected volume change of the reservoir $\Delta V_r(t)$ and radius change of the conduit $\Delta R(t)$ for a

viscoelastic, elastic, and rigid crust (i.e., with no response of the crust to pressure changes). Finally, we calculate the depressurization induced by degassing by accounting for the parameterization proposed by Stevenson and Blake [1998] for a convective magma column, which links the mean gas flux with the viscosity of the magma and the conduit radius.

4.1.1. Influence of the gas flux, reservoir volume, effective viscosity of the crust, and conduit radius

For small volcanic systems ($V_r(t_0) = 10^8 - 10^9 \text{ m}^3$), we find that underpressures of 5 - 10 MPa can be reached within a few months or years of quiescent degassing for most values of the parameters (Fig. 2a – 2d). The depressurization rate is roughly constant and proportional to the mean gas flux when the deformation is dominated by elastic effects (see Fig. 2a and equation (45)). Variations of the reservoir size within the range $10^8 - 10^9 \text{ m}^3$ do not have a major influence, and underpressures of 10 MPa are always reached after around one year of passive degassing (Fig. 2b). Whereas a pressure decrease of ~ 10 MPa is reached after around one year for $\mu \geq 10^{18} \text{ Pa s}$, it is about 10 MPa after two years for $\mu = 10^{17} \text{ Pa s}$. The conduit radius has a strong influence on the pressure change, and very small conduit radius leads to significant underpressures very quickly (Fig. 2d). For example, a volcano with 10 m conduit radius depressurizes 10 MPa in only a few months, whilst a volcano with 80 m conduit radius requires almost four years to reach the

same underpressure. The fact that the depressurization is larger for smaller conduit sections is consistent with the definition of pressure itself (weight over section).

For large volcanic systems ($V_r(t_0) = 10^9 - 10^{10} \text{ m}^3$), viscous effects of the crust become more important, such that the depressurization is not linear with time and is not proportional to the mean gas flux (compare Fig. 2a and Fig. 2e). The pressure evolution is very sensitive to the reservoir volume (contrary to the above scenario; compare Fig. 2b and Fig. 2f), and small differences in the range $10^9 - 10^{10} \text{ m}^3$ cause very significant changes in the depressurization rates. For example, magma reservoirs of $V_r(t_0) = 10^9 \text{ m}^3$ can decrease the pressure by about 10 MPa after around one and a half years, whereas a reservoir of $V_r(t_0) = 10^{10} \text{ m}^3$ needs around ten years to depressurize 7 MPa. The pressure evolution is also very sensitive to the effective viscosity of the crust (Fig. 2g), but even for very low effective viscosities ($\mu \sim 10^{17} \text{ Pa s}$) underpressures of about 2 MPa are reached in the typical inter-eruptive timescales. Very small conduit radius leads to lower depressurization rates than for smaller reservoirs, even though a pressure decrease of ~ 7 MPa can be reached after around two years for a conduit radius of 10 m (compare Fig. 2d and Fig. 2h).

Fig. 2. Models of pressure evolution at persistently degassing volcanoes for small ($V_r(t_0) = 10^8 - 10^9 \text{ m}^3$) and large ($V_r(t_0) = 10^9 - 10^{10} \text{ m}^3$) systems. Panels (a) and (e) explore depressurization for different gas fluxes; (b) and (f) for different reservoir sizes; (c) and (g) for different wall-rock viscosities; and (d) and (h) for different conduit radius. We have considered: $k = 10^{10} \text{ Pa}$, $\alpha = 3 \text{ wt\%}$; $n_x = n_c = 3 \text{ wt\%}$ (gas-magma separation at the upper conduit); $\rho_{c1} = 2670 \text{ kg/m}^3$; $\rho_{c2} = \rho_{nd} = 2430 \text{ kg/m}^3$; $\gamma_c(t_0) = 0.5$; and $L = 10 \text{ km}$. With these parameters, $n_r(t) = 0$. It is met that the mass of volatiles released in 10 years or up to $\Delta P = -10 \text{ MPa}$ is lower than the mass of volatiles initially dissolved in an undegassed conduit-reservoir system (i.e., $V_r(t_0)\rho_{nd}\alpha > \hat{Q}t$).

4.1.2. Influence of the rheology of the crust: deformation induced by passive degassing

We explore the change of the pressure, reservoir volume, and conduit radius for a rigid, elastic, and viscoelastic crust. Rigid crust accounts for the end-member case in which pressure changes in magma reservoirs do not deform the volcanic edifice. This could occur for example at highly inhomogeneous volcanic environments where deformation is mostly dominated by tectonic stresses [Battaglia and Segall, 2004]. We find that depressurization is linear with time for rigid and elastic wall-rocks (Fig. 3a), but rigid environments depressurize much faster than elastic ones (~ 8 MPa/year versus ~ 1 MPa/year for the parameters we have used in Fig. 3). In contrast, the depressurization rate in the viscoelastic regime decreases gradually with time, even though underpressures of several MPa can be still reached after a few years (~ 3 MPa after four years, Fig. 3a). The deformation of the reservoir and conduit is larger for the viscoelastic than for the elastic regime, but in both cases the volume change of the reservoir and the radius change of the conduit induced by passive degassing are on the order of $\sim 10^6 - 10^7$ m³ and ~ 10 mm, respectively, after a few years. These values represent a deformation of up to 0.1% for the parameters used in Figure 3b - 3c. An important feature we can extract from Figure 3 is that the lower the ability of a volcanic environment to deform, the larger the depressurization rates induced by degassing. This is because the volume decrease of the reservoir displaces melt into the conduit, which compensates part of the mass decrease of the magma column during degassing.

Fig. 3. Coupling between degassing-induced depressurization and conduit-reservoir deformation. **(a)** Pressure change with time with rigid (red), elastic (blue), and viscoelastic (green) host-rock. **(b)** volume change of the reservoir with rigid (red), elastic (blue), and viscoelastic (green) host-rock. **(c)** radius change of the conduit with rigid (red), elastic (blue), and viscoelastic (green) host-rock. For the rigid host-rock: $\mu \rightarrow \infty$; $k \rightarrow \infty$; for the elastic host-rock: $\mu \rightarrow \infty$; $k = 10^{10}$ Pa; and for the viscoelastic host-rock $\mu = 10^{18}$ Pa s; $k = 10^{10}$ Pa. For the rest of the parameters, we have used: $\hat{Q} = 4$ kt/day, $R(t_0) = 40$ m, $V_r(t_0) = 10^{10}$ m³; $\alpha = 3$ wt%; $n_x = n_c = 3$ wt% (gas-magma separation at the upper conduit); $\rho_{c1} = 2670$ kg/m³; $\rho_{c2} = \rho_{nd} = 2430$ kg/m³; $\gamma_c(t_0) = 0.5$; and $L = 10$ km. With these parameters, $n_r(t) = 0$.

4.1.3. Link between mean gas flux, magma viscosity, and conduit radius

Different types of convection models in a volcanic magma conduit have been used to explain the mean degassing rates of different volcanoes [e.g., Stevenson and Blake, 1998; Burton et al., 2007]. In Figure 4, we show the depressurization induced by degassing by assuming that the mean gas flux $\hat{Q}$ is related to the magma viscosity and the conduit radius through the simple parameterization proposed by Stevenson and Blake [1998]. This

parameterization has been used extensively [e.g., Kazahaya et al., 2002; Shinohara, 2008], and it states that the mean gas flux is given by $\hat{Q} \approx \pi(R^*)^2 \xi (g \Delta \rho_{1,2} R(t_0)^4 / \mu_1)$, where $R^* \approx 0.6$ is an experimental constant, and ξ is the Poiseuille number, which equals to 0.064 when $\mu_1 / \mu_{nd} > 12$, being μ_1 the viscosity of the fully degassed melt in the conduit, and μ_{nd} the viscosity of the undegassed melt (as in equation (32)). Note that this parameterization linking $\hat{Q}$ with $R(t_0)$ was obtained for a scenario in which the undegassed melt ascends through the conduit to occupy the space left by the denser degassed melt that sinks into the reservoir. In our model, undegassed melt can also rise through the conduit if the reservoir decreases its size, but we still use the same parameterization as a first-order approximation. We obtain that for a realistic mean gas flux between 1.5 and 7.5 kt/day, corresponding to relatively small radius between 10 and 15 m, depressurizations between 2 and 10 MPa are reached after less than five years, respectively (for the parameters used in Figure 4). The coupling of our analysis with simple convection models also shows that depressurization induced by degassing can be very significant.

Fig. 4. Depressurization by degassing when our model is coupled with the parameterization for magma convection in a conduit proposed by Stevenson and Blake (1998). Red lines correspond to different gas fluxes limited by the conduit radius, the density difference between degassed and undegassed melt, and the viscosity of the degassed melt (see text for details). For the rest of the parameters, we have used: $V_r(t_0) = 5 * 10^9 \text{ m}^3$, $\alpha = 3 \text{ wt\%}$, $n_x = n_c = 1.5 \text{ wt\%}$ (gas-magma separation at the upper conduit), $\mu_1 = 10^{4.9} \text{ Pa s}$, $\mu_{nd} = 10^{3.3} \text{ Pa s}$, $\rho_{c1} = 2530 \text{ kg/m}^3$, $\rho_{c2} = \rho_{nd} = 2430 \text{ kg/m}^3$, $\gamma_c(t_0) = 0.5$, $\mu = 10^{18} \text{ Pa s}$, $k = 10^{10} \text{ Pa}$, and $L = 10 \text{ km}$. With these parameters, $n_r(t) = 0$.

4.2. Scenario 2: gas exsolution in the reservoir, no replenishment, and constant mean density of melt in the column ($n_r(t) \neq 0$, $\lambda(t) = 0$, $\hat{\rho}_{m,c}(t) = \hat{\rho}_{m,c}(t_0)$)

This scenario can correspond to a volcano in which gas-magma separation occurs at depth and bubbles can ascend and escape steadily through a vesicular magma conduit. We found that the shallower the reservoir and the larger the volatile content, the lower the depressurization rate (Fig. 5). This is because there is a negative feedback between the

depressurization by degassing and the gas content within the reservoir. That is, a gradual and slow depressurization increases the gas content in the reservoir, which in turn pushes more melt into the conduit and tends to reduce the depressurization rate. Reservoirs between 3 - 4 km depth depressurize around 1 - 2 MPa after ten years of degassing (for the parameters of Fig. 5a - 5b), which contrasts with the fast depressurization of 3 MPa/year for bubble-free reservoirs at depths larger than 4 km if $\alpha = 4$ wt% (Fig. 5a), and larger than 10 km if $\alpha = 6$ wt% (Fig. 5b). The pressure decrease by degassing is independent of depth for bubble-free reservoirs. We can conclude that depressurization of at least a few MPa can be reached after a few years of passive degassing even if bubbles are present in the reservoir.

Fig. 5. Influence of the exsolution of bubbles in the reservoir on the pressure evolution during quiescence.

(a) Underpressure of the reservoir with time for $\alpha = 4\%$, and hence density $\rho_{nd} = 2370 \text{ kg/m}^3$. For $L = 3 \text{ km}$, $n_x = n_r = 2.6 \text{ wt\%}$, and $\rho_x = \rho_r = 2530 \text{ kg/m}^3$; for $L = 5 \text{ km}$, $n_x = n_r = 0$, and $\rho_x = \rho_{c1} = 2670 \text{ kg/m}^3$ (gas-magma separation in the upper conduit) (b) Underpressure of the reservoir with time for $\alpha = 6\%$, and hence densities $\rho_{nd} = 2260 \text{ kg/m}^3$. For $L = 4 \text{ km}$, $n_x = n_r = 2.1 \text{ wt\%}$, and $\rho_x = \rho_r = 2380 \text{ kg/m}^3$; for $L = 8 \text{ km}$, $n_x = n_r = 0.5 \text{ wt\%}$, and $\rho_x = \rho_r = 2290 \text{ kg/m}^3$; for $L = 10 \text{ km}$, $n_x = n_r = 0$ and $\rho_x = \rho_{c1} = 2670 \text{ kg/m}^3$ (gas-magma separation in the upper conduit). β_r is the gas volume fraction in the reservoir, which is $\beta_r = m_{g,r}(t_0) / \hat{\rho}_{g,r}(t_0) V_r(t_0)$. For the rest of the parameters, we have considered: $\hat{Q} = 4 \text{ kt/day}$, $\mu = 10^{18} \text{ Pa s}$; $k = 10^{10} \text{ Pa}$; $R(t_0) = 40 \text{ m}$, $V_r(t_0) = 10^9 \text{ m}^3$; $\gamma_c(t_0) = 1$ (completely degassed magma in the conduit). The initial pressure in the reservoir $P(t_0)$ is calculated as $P(t_0) = gL\rho_{c1}(1 - \beta_c(t_0))$, where $\beta_c(t_0)$ is the initial gas volume fraction in the magma column. The initial volume of melt in the column is $V_{m,c}(t_0) = \pi R(t_0)^2 L(1 - \beta_c(t_0))$. We have used $\beta_c(t_0) = 0.1$.

4.3. *Scenario 3: gas-free reservoir, magma replenishment, and constant mean density of melt in the column ($n_r(t) = 0$, $\lambda(t) \neq 0$, $\hat{\rho}_{m,c}(t) = \hat{\rho}_{m,c}(t_0)$).*

In this section we investigate how passive degassing may affect magma replenishment if there is a narrow dyke connecting the shallow reservoir with a deeper magma source. First, we analyze a case with constant hydraulic strength and pressure of the source equal to the hydrostatic pressure at time t_0 (see below equation (31)). Second, we explore a simple but more realistic scenario consisting in a variable hydraulic strength with time and an overpressurized deep magma source ($\Delta P_s > 0$).

4.3.1. *Constant hydraulic strength ($\lambda(t) = \text{constant} > 0$) and $\Delta P_s = 0$*

We find that the larger the dyke radius, the lower the depressurization rate (Fig. 6a). This occurs because of a negative feedback between depressurization by degassing and the replenishment of magma in the reservoir. That is, a gradual and slow depressurization increases the replenishment rate of magma into the reservoir (see equation (31)), which in turn pushes more melt into the conduit and tends to reduce the depressurization rate. For large enough dyke radius (more than 2 - 3 m for the parameters of Figure 6), the depressurization tends to zero. In such a case, the volcano would remain during quiescence in a state of ideal pressure balance between the volume change of melt by degassing and the magma volume gained by replenishment. The volume of replenished magma induced by passive degassing, and the rate of magma refilling, can reach values

on the order of $V_{rep} \sim 10^6 - 10^7 \text{ m}^3$ and $dV_{rep}/dt \sim 10 - 20 \text{ m}^3/\text{year}$, respectively (Fig. 6b - 6c). This finding implies that magma refilling of the reservoir can occur together with depressurization as opposed to pressurization because degassing can decrease the pressure more than what magma replenishment increases it.

4.3.2. Time-dependent hydraulic strength ($\lambda(t)$) and $\Delta P_s > 0$

A more realistic scenario accounts for changes of the hydraulic strength because the cooling of magma within narrow feeder dykes can be important, thus changing significantly the magma viscosity and the dyke radius. In fact, magma cooling and dyke closure depend on the temperature of the host-rock and the initial dyke radius, but also on the pressure difference between the source and the reservoir [e.g., Bruce and Huppert, 1989], and thus on $\Delta P(t)$. Analyzing the coupling between thermal variations within the dykes, the variations of the hydraulic strength, and $\Delta P(t)$, is beyond the scope of this paper, but in Figure 6d – 6f we assess the effect of depressurization by degassing on the replenishment rates if the reservoir and the deep source become suddenly connected. As we showed so far, the pressure of the shallow reservoir decreases a few MPa during passive degassing at a rate which depends on the gas flux (Fig. 6d). And once the reservoir and the deeper source become connected (after five years of quiescence in the example of Figure 6), magma moves from the source into the reservoir at a rate which is larger for larger gas fluxes (Fig. 6e - 6f). Neglecting depressurization by degassing can lead to underestimations of the volume of replenished magma (and replenishment rate) of

up to a factor two (for the parameters used in Figure 6). This simple scenario illustrates the important role that passive degassing can play in the magma intrusion dynamics beneath a volcano.

Fig. 6. Pressure evolution when the reservoir is hydraulically connected to a deeper magma source. **(a)** Depressurization for different dyke radius $R_d = 0, 1.2,$ and 2 m, for $\Delta P_s(t) = 0$, $\hat{Q} = 4$ kt/day, $R(t_0) = 40$ m, and $V_r(t_0) = 10^9$ m³. **(b)** Volume of replenished magma for the same parameters than in (a). **(c)** Rate of magma replenishment for the same parameters than in (a). **(d)** Pressure evolution for different values of the gas flux, $R_d = 1$ m, $\Delta P_s(t) = 10$ MPa, $R(t_0) = 50$ m, $V_r(t_0) = 5 \times 10^9$ m³. This is obtained by solving equation (37) numerically. **(e)** Volume of replenished magma for the same parameters than in (d). **(f)** Rate of magma replenishment for the same parameters than in (d). For the rest of the parameters, we have considered: $\mu_{nd} = 10^{4.2}$ Pa s, $k = 10^{10}$ Pa, $\mu = 10^{18}$ Pa s, $\alpha = 2$ wt%, $n_x = n_c = 2$ wt% (gas-magma separation at the upper conduit), $M = 5000$ m, $\rho_{c1} = 2670$ kg/m³, $\rho_{c2} = \rho_{nd} = 2500$ kg/m³, $\gamma_c(t_0) = 0.5$, $T = 1000$ °C, and $L = 6$ km. With these parameters, $n_r(t) = 0$.

4.4. Scenario 4: gas-free reservoir, no replenishment, and gradually increasing mean density of melt in the column ($n_r(t) = 0$, $\lambda(t) = 0$, $\hat{\rho}_{m,c}(t) \neq \hat{\rho}_{m,c}(t_0)$)

The maximum influence of the variation of the density of melt in the conduit $\hat{\rho}_{m,c}(t)$ can be explored by imposing a scenario in which only the magma conduit degasses, namely, gas-magma separation occurs at shallow levels but there is no convection replacing the degassed melt by undegassed melt from the reservoir. In such a case, we obtain that the depressurization is about three times slower than if the density is kept constant (2 MPa versus 6 MPa after five years of quiescence for the parameters used in Fig. 7a). In any case, we can conclude that the pressure in the reservoir can also decrease by several MPa during the inter-eruptive timescales if the volume fraction of degassed melt ($\gamma_c(t)$) and the mean density of melt in the column ($\hat{\rho}_{m,c}(t)$) increase during passive degassing (Fig. 7b – 7c).

Fig. 7. Maximum effect of the density changes of melt in the conduit. Blue lines correspond to the end-member scenario in which the mean density of melt in the conduit increases because the degassed melt at low pressures is not replaced via convection. Red lines correspond to a scenario in which the mean density of melt in the conduit is kept constant. **(a)** Underpressure of the reservoir with time. **(b)** Volume fraction of degassed melt in the magma column. **(c)** Mean density of melt in the magma column. For the parameters, we have considered: $\hat{Q} = 4 \text{ kt/day}$, $R(t_0) = 70 \text{ m}$, $V_r(t_0) = 10^9 \text{ m}^3$, $k = 10^{10} \text{ Pa}$, $\mu = 10^{18} \text{ Pa s}$, $L = 10 \text{ km}$, $\alpha = 6\%$, $n_x = n_c = 6 \text{ wt\%}$ (gas-magma separation at the upper conduit), $\gamma_c(t_0) = 0.2$, $\rho_x = \rho_{c1} = 2670 \text{ kg/m}^3$, $\rho_{c2} = \rho_{nd} = 2260 \text{ kg/m}^3$, $T = 1000 \text{ }^\circ\text{C}$.

5. Discussion

Previous studies investigated the mechanisms that can sustain passive degassing at persistently degassing volcanoes [e.g., Kazahaya et al., 1994; Stevenson and Blake, 1998; Edmonds et al., 2001; Harris et al., 2005; Shinohara and Tanaka, 2012], or the processes that lead to continuous pressurizations of magma reservoirs [e.g., Blake, 1981; Tait et al., 1989]. By contrast, we have performed a theoretical analysis that accounts for the mass decrease by degassing, and for different coupled processes that affect the pressure in the reservoir. On the one hand, degassing decreases the mass of melt within the conduit,

which tends to decrease the pressure at depth. On the other hand, a pressure decrease tends to reduce the volume of the system, nucleate and expand bubbles, and induce replenishment if the shallow reservoir and a deep magma source are hydraulically connected. These processes displace more magma and thus more mass into the conduit, such that the resulting pressure change is given by the mass decrease of melt within the conduit due to degassing, and the mass compensation due to the dynamic response of the system. For many scenarios, the mass moving into the conduit does not fully compensate the mass decrease by degassing, and the pressure in the reservoir decreases by several MPa in only a few months or years, i.e., within the inter-eruptive timescales.

Our lumped-parameter model predicts larger depressurization rates for larger gas fluxes, smaller volcanic systems, lower gas content in the magma reservoir, larger effective viscosities of the crust, and lower hydraulic strengths. In particular, if there are no bubbles in the reservoir and the connectivity between the reservoir and the feeder magma sources is low or null, the depressurization rate is constant for constant gas flux and rigid or elastic rheologies, whereas it decreases with time if the viscous effects of the crust become important. For small volcanic systems ($10^8 - 10^9 \text{ m}^3$), the depressurization rate can vary between 4 - 10 MPa/year, is not strongly sensitive to the volume of the magmatic system, and varies significantly with the conduit radius. For large volcanic systems ($10^9 - 10^{10} \text{ m}^3$), the depressurization is generally lower (up to $\sim 5 - 10 \text{ MPa}$ in several years), and is very sensitive to the reservoir volume and the effective viscosity of the crust. Gas exsolution in the reservoir tends to decrease the depressurization rate, even though underpressures of several MPa can be still reached after a few years. Our model also

shows that magma replenishment rates of volcanic reservoirs can be enhanced by degassing-induced depressurization, and that neglecting degassing leads to a significant underestimation of the volume of replenished magma. Even if the mean density of melt increases progressively with time, the depressurization can be on the order of several MPa during quiescence.

Various observations and geophysical data at several volcanoes are consistent with important mass decreases within the conduit and pressure drops during quiescent degassing. For instance, at Satsuma-Iwojima volcano (Japan), a crack opened at the summit of the volcanic edifice on June 1996, which was interpreted as a pressure decrease beneath the crater [Iguchi et al., 2002]; a very low density region, thought to be the result of collapsed material, was found up to 100 m below the crater floor by means of muon tomography [Tanaka et al., 2010]; and a low gravity anomaly has been recorded at the active cone [Shinohara and Tanaka, 2012]. At Mt. Asama (Japan), there were successive collapses of the crater floor after the eruption of 2004 ended and up to January 2006 [Urabe et al., 2006; Tanaka et al., 2010]; and a very low density region of up to 200 m was also found below the crater floor [Tanaka et al., 2008]. The very low density regions found beneath Satsuma-Iwojima and Asama craters could be consequence of a magma level drop in the conduit during quiescence. This is a possibility that is also predicted by our model. Indeed, if we neglect the short-term rise and fall of the magma level due to the complex bubble dynamics within the conduit [e.g., Dibble, 1972; Jaupart and Vergnolle, 1988; Orr and Rea, 2012], there is a correlation between the pressure change in the reservoir and the change of magma level. If for simplicity we neglect changes of the

radius, changes of the melt density, and we use that the mass of melt in the conduit is much larger than the mass of gas, we can get easily from equations (1) and (2) that the magma level drop is given by $|\Delta P(t)|/g\hat{\rho}_{m,c}$. Hence, magma level drops of 100 m, like in Satsuma-Iwojima, and 200 m, like in Asama, correspond to pressure drops of around 2.5 and 5 MPa, respectively. These values are consistent with the depressurization induced by degassing that we have obtained.

Another example of depressurization during passive degassing is found in Llaima volcano (Chile), where subsidence of the volcanic edifice on the order of 10^7 m^3 was recorded during three years and a half (from November 2003 to May 2007) before a fast inflation lasting for six months and the onset of an eruption [Bathke et al., 2011]. The subsidence is consistent with the volume decrease predicted for the reservoir by our model assuming elastic or viscoelastic rheology for the crust (see Fig. 2b). Factors such as the regional stress field and very shallow processes near pit craters may cause more complex deformations and may complicate the interpretation of data [Battaglia and Segall, 2004], and thus lack of deflation does not mean necessarily that there is no depressurization by degassing. That is the case of Masaya volcano (Nicaragua), where periods of passive degassing without deflation have been monitored, but recurrent collapses of the crater floor occur as consequence of cavernous structures located below [Rymer et al., 1998]. Microgravimetric data from Masaya also noted anomalies from 1993 to 1994 consistent with shallow mass decreases by gas loss [Rymer et al., 1998]; and another gravity decrease was recorded from 1997 to 1999 and previously attributed to changes in vesicularity of the upper conduit [William-Jones et al., 2003].

A question that arises naturally from our theoretical analysis is whether depressurization by passive degassing can trigger the onset of different processes that lead to new unrest episodes and eruptions. We propose below three processes we think could be triggered by degassing-induced depressurization after a few months or years of quiescence, and that require further study in the future.

First, degassing-induced depressurization could be responsible for the sudden replenishment discussed in Figure 6d – 6f if we consider that the magma initially filling the connecting dykes has a Bingham rheology because it is cold and crystal-rich [Melnik and Sparks, 2005]. In such a case, magma within the dykes acts as a solid plug as long as the pressure difference between the source and the shallow reservoir ($\Delta P_s(t) - \Delta P(t)$) does not exceed a threshold value, which could occur if the pressure in the reservoir decreases sufficiently after a few months or years of quiescence. The threshold pressure difference to unplug the dykes and initiate magma ascent can range between much less than 1 MPa and more than 30 MPa depending on the yield strength, the geometry of the dykes, cooling, and the crystal distribution [Melnik and Sparks, 2005].

Second, gradual depressurization by quiescent steady state degassing leads to gradual deepening of the exsolution level within the volcanic system, which can favor the transport of volatiles between different magma layers, and in turn affect the degassing rates [Simakin and Botcharnikov, 2001]. The deepening of the exsolution level also implies that the volume of magma able to exsolve volatiles depends on how the section of the conduit and reservoir vary with depth. For example, if the exsolution level moves

from the bottom of the conduit to the top of the reservoir we can expect a sudden increase of the volume of bubbly magma because of the larger cross-section of the reservoir. This in turn may lead to ascent of magma towards the surface and pressurizations at shallow depths which could induce explosions [e.g., Namiki and Manga, 2005].

Third, depressurization can induce cracking and sudden subsidence of the reservoir roof if the difference between the pressure of the reservoir ($P(t_0) + \Delta P(t)$) and the lithostatic pressure at reservoir depths (P_{Lit}) reaches the strength of rocks (σ) [Blake, 1981]. If we consider that initially the reservoir is in equilibrium with the surrounding crust ($P(t_0) = P_{Lit}$), the criteria for wall-rock failure would be simply $|\Delta P(t)| = \sigma$. The strength of rocks σ vary between 0.5 and 9 MPa when the rocks are subjected to tensile stresses [Gudmundsson, 2012], which are the stresses that develop around loaded cavities [Carter et al., 1991] or around reservoirs subjected to the load of a volcanic edifice [Pinel and Jaupart, 2005]. These values of the rock strength are consistent with the order of magnitude of the pressure changes caused by degassing. Degassing-induced depressurization can also induce crater collapse [Iguchi et al., 2002] and seismicity by poroelastic stressing, as it was proposed for the earthquakes that occurred near the Lacq gas field in France after gas extraction [Segall et al., 1994].

6. Conclusions

We have presented a new lumped-parameter model that captures the long-term (from months to years) pressure changes induced by passive degassing in volcanic magma reservoirs. Our model demonstrates that degassing-induced depressurization can reach values on the order of several MPa in the inter-eruptive timescales (< 10 years), which is consistent with the geophysical and geodetical observations (e.g., edifice deflation, very low density at the upper part of the conduit, or crater floor collapse) of several persistently degassing volcanoes. Our results suggest that the unrest episodes and eruptions occurring every few months or years at persistently degassing volcanoes could be triggered by a variety of physical processes that are promoted by degassing-induced depressurization during quiescence. Passive degassing could induce magma replenishment, sudden gas exsolution at depth, and cracking and sudden subsidence of the crater floor or reservoir wall-rock. The challenge for the future is to explore these processes through numerical and analog models.

Notation

Symbol	Description	Range
ΔP	Pressure change during quiescence	≤ 10 MPa (absolute value)
$\hat{Q}$	Total mean gas flux	1 – 10 kt/day
t	Time of passive degassing	≤ 10 years
R	Volcanic conduit radius	10 – 100 m
V_r	Volume of the reservoir	$10^7 - 10^{10}$ m ³
L	Length of the vertical column	3 – 10 km
α	Mass fraction of dissolved volatiles in parent melt	1– 6 wt%
k	Bulk modulus of rocks	$10^{10} - 10^{11}$ Pa
μ	Effective viscosity of the crust	$10^{17} - 10^{21}$ Pa s
g	Gravity	9.8 m/s ²
$\hat{\rho}$	Mean density of the magma column	1500 – 2500 kg/m ³
$\hat{\rho}_{m,c}$	Mean density of melt inside the magma column	2200 – 2800 kg/m ³
$\hat{\rho}_{g,c}$	Mean density of gas inside the magma column	≤ 200 kg/m ³
ρ_{c1}	Density of the fully degassed melt in the column	2500 – 2800 kg/m ³
ρ_{c2}	Density of partially degassed melt in the column	2200 – 2600 kg/m ³
$\Delta\rho_{1,2}$	Difference between ρ_{c1} and ρ_{c2}	50 – 400 kg/m ³
ρ_{nd}	Density of the parent undegassed melt	2200 – 2800 kg/m ³
ρ_r	Density of the melt that degasses in the reservoir	$\rho_r \leq \rho_{c1}$
ρ_x	Density of melt at the gas-magma separation level	$\rho_x = \rho_{c1}$ or $\rho_x = \rho_r$
$\hat{\rho}_{g,r}$	Mean density of gas in the reservoir	≤ 200 kg/m ³
n_c	wt% of water exsolved in the upper conduit	$\leq \alpha$
n_r	wt% of water exsolved in the reservoir	$n_r < n_c \leq \alpha$
n_x	wt% of water exsolved in the reservoir or conduit	$n_x = n_c$ or $n_x = n_r$
γ_c	Volume fraction of degassed melt in the magma column	0 – 1
μ_{nd}	Viscosity of the undegassed parent melt	$10^2 - 10^5$ Pa s
μ_1	Viscosity of the degassed melt in the conduit	$> \mu_{nd}$
R_d	Radius of the dyke	≤ 5 m
M	Length of the dykes	2 – 6 km
R^*	Empirical constant (Stevenson and Blake, 1998)	0.6

ζ	Poiseuille number (Stevenson and Blake, 1998)	0.064
M_{H_2O}	Molecular weight of water	18 g/mol
T	Temperature of the magma	900 – 1200 °C
S	Constant of the Henry's law for water	$4 * 10^{-6} \text{ Pa}^{-1/2}$
R_g	Universal gas constant	8.31 J / (K mol)
A^*	Indicator variable	0 or 1
B^*	Indicator variable	0 or 1
ΔP_∞	Maximum pressure change	
Γ	Parameter defined in equation (41)	
ρ_w	Parameter defined in equation (22)	
$m_{g,c}$	Mass of gas within the column of length L	
$m_{m,c}$	Mass of melt within the column of length L	
$m_{g,r}$	Mass of gas in the reservoir	
$V_{g,c}$	Volume of gas inside the column of length L	
$V_{g,r}$	Volume of gas that exsolves in the reservoir	
$V_{m,c}$	Volume of melt inside the column of length L	
$V_{m,c1}$	Volume of fully degassed melt inside the column of length L	
$dV_{undeg}/dt _d$	Volume change of undegassed melt in the system due to degassing	
dV_{deg}/dt	Volume change of fully degassed melt in the system	
dV_d/dt	Volume decrease of melt in the system due to gas exsolution	
V_m	Volume of melt in the conduit-reservoir	
$V_{m,r}$	Volume of melt in the reservoir	
V_{rep}	Volume of replenished melt	
D	Portion of the magma column which is within the reservoir	
β_r	Gas volume fraction in the reservoir	
β_c	Gas volume fraction in the magma column	
P_{Lit}	Lithostatic pressure at reservoir depths	
σ	Strength of rocks	
λ	Hydraulic strength	

Chapter 2:

**Fractal degassing from Erebus
and Mayon volcanoes revealed
by a new method to monitor H₂O
emission cycles**

Fractal degassing from Erebus and Mayon volcanoes revealed by a new method to monitor H₂O emission cycles

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Abstract

Many active volcanoes around the world release passively large amounts of gas between eruptions. Monitoring how these gas emissions fluctuate over time is crucial to infer the physical processes occurring within volcanic conduits and reservoirs. Here we report a new method to capture remotely the spectral properties of the emissions of H₂O, the major component of most volcanic plumes. The method is based on a new theoretical model that correlates the volcanogenic water content of condensed volcanic plumes with the intensity of the light scattered by the droplets moving with the gas. In turn, we show that light intensity of the plume, and thus steam pulses time series, can be obtained with a proper

analysis of digital images. The model is experimentally validated with controlled condensed plumes generated with an ultrasonic humidifier, and then the method is applied to the gas plumes of Erebus (Antarctica) and Mayon (Philippines) volcanoes. Our analysis reveals three main features: 1) H₂O time series are composed of numerous periodic components of finite duration; 2) some periodic components are common both in H₂O and SO₂ time series, but others are not; and 3) the frequency spectrum of the H₂O emissions follows a well-defined fractal distribution, that is, amplitude (Δ) and frequency (ν) are correlated by means of power-laws ($\Delta \propto \nu^\gamma$), with exponent $\gamma \approx -1$ both for Erebus and for Mayon. These findings suggest that quiescent degassing ultimately emerges from the complex coupling between different processes occurring within magma plumbing systems. Our method is ideal for real-time monitoring of high-frequency H₂O cycles at active volcanoes.

1. Introduction

Monitoring degassing cycles and gas puffing at active volcanoes during quiescence is valuable for volcanic surveillance because they are a proxy for the physical processes occurring within magma conduits and reservoirs [e.g., Ripepe et al., 2002; Allard et al., 2005; Tamburello et al., 2012]. Degassing cycles are currently monitored using remote methods as they allow obtaining time series at high sampling rates, and are safer than in-situ direct collection of volcanic gases [McGonigle and Oppenheimer, 2003]. For instance, Harris and Ripepe [2007] and Branan et al. [2008] analyzed gas puffing activity at

Stromboli and Masaya by measuring temperature changes of the vent with thermal infrared thermometers and thermal imagers; Oppenheimer et al. [2009] and Boichu et al. [2010] registered SO₂ cycles at Erebus volcano with spectroscopic techniques; and more recently, Tamburello et al. [2013] monitored SO₂ pulsations at Etna with UV cameras.

Despite the variety of information that can be obtained from the different remote sensing techniques to monitor degassing cycles and gas puffing, none of them can actually capture the spectral properties of the H₂O emissions, which is by far the major component of most volcanic plumes [e.g., Symonds et al., 1994; Shinohara, 2008]. This is mostly due to the difficulty of monitoring the water content of volcanic plumes, which is typically done by measuring simultaneously the SO₂ concentration with ultraviolet absorption spectrometers or UV cameras, and the molar ratio [H₂O]/[SO₂] with OP-FTIR (open-path Fourier transform infrared) spectrometers [e.g., Burton et al., 2000; Oppenheimer and Kyle, 2008]. An added limitation is that OP-FTIR technique can only supply reliable information when the spectrometer and an infrared lamp can be installed at opposite sides of the volcanic crater rim, since only close to the gas plume the infrared radiation absorbed by atmospheric water vapor is minimized [e.g., Burton et al., 2000; McGonigle and Oppenheimer, 2003; Sawyer et al., 2011]. Here, we aim to alleviate these difficulties and limitations by reporting a complementary method that allows monitoring H₂O pulsations from several kilometers afar from the plume. Contrary to the above mentioned OP-FTIR technique, whose measurement principle is the absorption of radiation by H₂O molecules, the method we propose is based on the light scattered by the micrometric water droplets that travel with the gas of condensed volcanic clouds. This approach allows us to capture

the spectral features of the H₂O emissions because the brightness of condensed plumes increases at each steam pulse, a fact that has been previously highlighted but not fully exploited [e.g., Matsushima and Shinohara, 2006; Harris and Ripepe, 2007].

In the next sections we first introduce a new theoretical model that correlates the brightness of condensed volcanic plumes with the water vapor emissions. We also show how to obtain a brightness time series with commercial digital cameras, and then, we validate the fundamentals of the model by performing some laboratory experiments. Later, we explain how to process a digital brightness time series, and we apply the method to capture for the first time the spectral properties of the steam emissions from Erebus and Mayon volcanoes. Finally, we discuss our results and the limitations of the method.

2. Theoretical model

The methodology we propose to track steam emission cycles applies to condensed volcanic clouds, which are commonly seen at active volcanoes during quiescence under many atmospheric conditions [Matsushima, 2005; Matsushima and Shinohara, 2006]. Volcanic clouds form when the hot volcanic gases (with temperatures of a few hundred degrees) leave the vent and quickly mix with the surrounding air, thus forming a saturated cooler mixture of volcanogenic water vapor, other less abundant volcanic gases (mostly CO₂ and SO₂), water vapor and dry air from the atmosphere, and micrometric water droplets [e.g., Gassó, 2008]. Water droplets, which are very light and hence travel with

the gas plume, act as secondary source of radiation by scattering the sunlight [e.g., Bohren and Huffman, 2004]. Therefore, the time-dependant luminance $I(t)$ received by a digital optical sensor pointing towards the volcanic gas plume can be written as:

$$I(t) = I_N(t) + I_b(t) \quad (1)$$

where $I_N(t)$ is the luminance of the radiation scattered by N water droplets of equal size, and $I_b(t)$ is the luminance corresponding to the background radiation, which is the radiation coming from the crater rocks and/or from the sky (whatever is behind the plume). A similar approach has been used to track turbulence in seeded flows, and is valid as long as the concentration of water droplets in the cloud is not high enough to absorb the background radiation [Roehle et al., 2000; Findeisen et al., 2005; Terhaar and Paschereit, 2012]. The term $I_N(t)$ can be written at first order approximation as [e.g., Bohren and Huffman, 2004]:

$$I_N(t) = N(t) f(t) \quad (2)$$

where we take into account that the number of water droplets $N(t)$ change with time due to the gas puffing activity, or due to atmospheric changes of relative humidity, pressure, or temperature [Matsushima and Shinohara, 2006]. The term $f(t)$ is the luminance scattered by a single water droplet, and it can be written according to Mie theory as a function of the luminance of the incident radiation (sunlight), the diameter of the droplet, the refraction index of water, the distance from the droplets to the observation point, and

the angle formed by Sun-plume-observer [e.g., Lock and Yang, 1991; Hergert and Wriedt, 2012]. Therefore, $f(t)$ varies with time depending on the position of the Sun along the day. By using equation (2) we assume single scattering, that is, the radiation scattered by the volcanic cloud is the sum of the radiation scattered by each water droplet, which is satisfied as long as the concentration of droplets is low enough [e.g., Bohren and Huffman, 2004].

The number of water droplets inside a given region of condensed plume $N(t)$ can be written in terms of the mass of volcanic water contained in that region $m_v(t)$. From mass conservation, we can state that the mass of volcanic water ($m_v(t)$) plus the mass of atmospheric water ($m_a(t)$) equals the mass of saturated water vapor ($m_{vap}^s(t)$) plus the mass of liquid water ($m_{liq}(t)$) from the droplets. In turn, the mass of liquid water is related to the number of droplets through $m_{liq}(t) = \hat{m}_d N(t)$, being $\hat{m}_d$ the mass of a droplet of average size, whereas the mass of atmospheric water is related to the mass of volcanic water through $m_a(t) = r(t)q(t)m_v(t)$, being $r(t)$ the mixing ratio between air and volcanic gases, and $q(t)$ the specific humidity of the atmosphere (mass of water vapor per unit mass of air) at crater levels. Hence, the number of water droplets within a region of the plume at a given time can be written after rearranging terms as:

$$N(t) = \frac{1 + r(t)q(t)}{\hat{m}_d} m_v(t) - \frac{m_{vap}^s(t)}{\hat{m}_d} \quad (3)$$

By combining equations (1) – (3), we can write the luminance $I(t)$ emitted from a given region of the plume as:

$$I(t) = \left[\frac{1 + r(t)q(t)}{\hat{m}_d} f(t) \right] m_v(t) + \left[I_b(t) - \frac{m_{vap}^s(t)}{\hat{m}_d} f(t) \right] \quad (4)$$

This equation shows that the luminance $I(t)$ of the plume can be used to monitor water vapor cycles $m_v(t)$ if the temporal variability of the other parameters is known and corrected. On the other hand, we propose to monitor the luminance $I(t)$ simply by calculating the brightness of digital images $I_d(t)$. These two parameters are proportional ($I_d(t) = \alpha I(t)$) if a gamma correction is not applied to the image (an operation that compensates some properties of human vision), and for a given shutter speed and aperture of the camera [Wüller and Gabele, 2007]. Therefore, equation (4) can be written as:

$$I_d(t) = a(t) m_v(t) + b(t) \quad (5)$$

where $a(t) = \alpha f(t)[1 + r(t)q(t)]/\hat{m}_d$, and $b(t) = \alpha[I_b(t) - m_{vap}^s(t)f(t)/\hat{m}_d]$. In turn, the digital brightness of a given region of interest $I_d(t)$ can be calculated using the following approximation [e.g., Bala and Braun, 2004]:

$$I_d(t) = \sum_i I_{i,d}(t) = \sum_i [0.2989 R_i(t) + 0.5870 G_i(t) + 0.1140 B_i(t)] \quad (6)$$

where $I_{i,d}(t)$ is the digital brightness of each pixel i of the region of interest, which is measured in digital units (D.U.) and vary between 0 (completely black) and 255 (completely white). The factors $R_i(t)$, $G_i(t)$, and $B_i(t)$ are the digital values of red, green, and blue color in RGB color space. Indeed, equation (6) represents a transformation of the image from RGB color space to grayscale by conserving the luminance [e.g., Bala and Braun, 2004].

3. Validation of the linear correlation between digital brightness and steam emissions

Our model suggests that steam emission time series $m_v(t)$ can be obtained indirectly by calculating the digital brightness $I_d(t)$ of the condensed plume with a sequence of images, and then correcting by the time variability of the terms $a(t)$ and $b(t)$ of equation (5). Below, we validate the linear correlation between $I_d(t)$ and $m_v(t)$ by performing experiments with an ultrasonic humidifier. Then, we show that our experimental results are extensible to actual volcanic plumes.

3.1. Experiments with condensed plumes

Equation (5) shows a linear positive correlation between the brightness of a digital image I_d and the mass of volcanic water within a region of the condensed plume m_v . To validate this relation, we generated a condensed cloud with an ultrasonic humidifier (model

Executive Standard, GB4706.48-2000, supplied by Bringnew. Pte. Ltd., Singapore). Ultrasonic humidifiers are based on the principle of atomization by high-frequency mechanical oscillations, which are induced by a piezoelectric transducer submerged in a water tank and excited with a periodic MHz signal (Fig. 1a). When the ultrasonic waves reach the water level, a fine mist of water droplets with a narrow size distribution ranging between 1 and 5 μm (on the same order of magnitude than water droplets within volcanic clouds formed immediately downwind the volcano; e.g., Gassó, [2008]) is atomized due to a combination of cavitation and capillary effects [e.g., Lozano et al., 2003]. These small droplets evaporate adiabatically by increasing the relative humidity of the air within the tank until it becomes saturated and forms a cloud. This cloud is then released from the tank through an opening, which was connected to a 35 cm long chimney with inner diameter of 19 mm that ended with a nozzle of inner diameter equal to 15 mm.

The humidifier was placed on a weighing scale to measure the weight variations with time, from which we calculated the atomization rate Q (kg of water released per second). Different atomization rates can be chosen by changing the voltage supplied to the piezoelectric transducer and thus by varying the amplitude of the ultrasonic waves [Lozano et al., 2003]. In turn, the atomization rate can be written as $Q = C_v S v$, where C_v is the concentration of water coming out from the humidifier (kg of water –equivalent to m_v – per cubic meter of plume), S is the section of the plume, and v is the velocity of the stream of droplets. If we normalize the atomization rate (for example, with respect to the maximum atomization rate reached in our experiments), and the section and velocity of the plume is fixed, it is satisfied that $Q^* = C_v^* = m_v^*$, where the symbol $*$ represents

normalized values. Thus, different values of m_v^* (or m_v) can be obtained simply by choosing different atomization rates. Constant section and velocity are achieved in our experiments with a fan located above the nozzle that suctions and stabilizes the stream of droplets (Fig. 1a).

The steady flux of droplets for a given atomization rate was recorded by taking 5000 images with an IP-camera (model BASLER acA2040-25gc, 2048x2048 pixels) and a short lens (model Computar 2/3" 8mm f1.4). During the experiments, the temperature and light intensity of the laboratory, as well as the shutter speed and aperture of the camera, were kept constant. Images were processed using MATLAB to obtain the digital brightness I_d of three different regions of interest (Fig. 1b) as explained in equation (6), and we calculated the mean value and standard deviation of all the images. We repeated this process with 21 different atomization rates ranging between 0 and 111 mg/s, which is the maximum achievable rate with this experimental setup (Table 1). A plot of digital brightness I_d versus normalized atomization rate $Q^* = m_v^*$ (Fig. 1c) shows very good linear correlations in the three regions of interest (with correlation coefficients $R^2 \geq 0.98$). However, the slope and the ordinate in the origin of the best fit lines increase with the observation height of the image. As we show below, this is due to different incident radiation with height.

Fig. 1. Experimental setup and results. **a)** Sketch of our experimental setup. An ultrasonic humidifier atomizes micrometric water droplets by means of high-frequency mechanical oscillations generated with a piezoelectric transducer. A vapor cloud, emulating a condensed volcanic plume, is released through a vertical chimney. The stream of water droplets is stabilized with a fan and recorded with an IP-camera. **b)** Example of the images we recorded in our experiments. This image corresponds to an atomization rate of $Q = 44.4 \pm 0.2$ mg/s. Green, red, and blue lines correspond to the three sections we analyze. Segments AB and $A'B'$ are used for analysis and correction of the brightness time series (see text for details). **c)** Digital brightness I_d in digital units (D.U.) versus normalized mass of water m_v^* . Green, red, and blue dots with error bars correspond to the measurements from the lines of the same color in panel b). The best fit lines are: $I_{d,green} = (11,700 \pm 400)m_v^* + (53,710 \pm 180)$ for the green region, $I_{d,red} = (10,000 \pm 200)m_v^* + (52,140 \pm 110)$ for the red region, and $I_{d,blue} = (7740 \pm 160)m_v^* + (49,690 \pm 80)$ for the blue region. R^2 is the correlation coefficient. **d)** Corrected digital brightness of green and red lines $I_{d,x}^c$, and digital brightness of blue line $I_{d,blue}$, versus m_v^* . These results confirm the linear correlation between the digital brightness of the condensed plume and the amount of water ejected by the humidifier.

Table 1. Experimental results of atomization rate and digital brightness.

Q (mg/s)	Q* = m _v * (x10 ⁻²)	I _d -blue- (x10 ² D.U.)	I _d -red- (x10 ² D.U.)	I _d -green- (x10 ² D.U.)
0	0	495 ± 6	519 ± 6	532 ± 7
4.573 ± 0.012	4.10 ± 0.03	504 ± 6	528 ± 6	542 ± 7
5.087 ± 0.018	4.57 ± 0.03	500 ± 5	524 ± 6	538 ± 6
5.81 ± 0.02	5.22 ± 0.04	496 ± 6	519 ± 6	533 ± 7
6.50 ± 0.02	5.84 ± 0.04	500 ± 6	525 ± 7	539 ± 7
8.32 ± 0.04	7.48 ± 0.06	502 ± 6	527 ± 7	542 ± 7
10.62 ± 0.02	9.54 ± 0.06	505 ± 6	531 ± 7	547 ± 7
13.75 ± 0.02	12.35 ± 0.08	508 ± 6	536 ± 7	553 ± 8
17.13 ± 0.17	15.39 ± 0.18	510 ± 6	538 ± 7	557 ± 9
22.57 ± 0.06	20.28 ± 0.14	512 ± 7	542 ± 8	562 ± 10
28.42 ± 0.15	25.5 ± 0.2	516 ± 7	548 ± 8	570 ± 9
32.37 ± 0.15	29.1 ± 0.2	520 ± 7	553 ± 8	577 ± 10
38.0 ± 0.2	34.1 ± 0.3	523 ± 8	557 ± 9	582 ± 11
44.4 ± 0.2	39.9 ± 0.3	528 ± 8	563 ± 10	589 ± 11
52.3 ± 0.2	47.0 ± 0.3	535 ± 8	572 ± 10	599 ± 12
58.2 ± 0.4	52.3 ± 0.5	540 ± 9	580 ± 11	607 ± 13
70.0 ± 0.4	62.9 ± 0.5	543 ± 8	583 ± 10	611 ± 11
79.6 ± 0.4	71.5 ± 0.6	553 ± 8	595 ± 10	624 ± 11
94.6 ± 0.5	85.0 ± 0.7	560 ± 9	603 ± 10	631 ± 12
102.3 ± 0.3	91.9 ± 0.6	571 ± 8	615 ± 10	642 ± 12
111.3 ± 0.7	100 ± 0.9	572 ± 8	616 ± 10	643 ± 12

3.2. Influence of the incident radiation

The mass of saturated vapor m_{vap}^s , the number of droplets N , and thus the total mass of water m_v crossing the three regions of interest of Figure 1b (blue, red, and green sections) should be the same for a given atomization rate. Hence, if our model is correct, the different slope and ordinate in the origin we have found in Figure 1c must be explained by a changing incident radiation on the plume with height (as a consequence of a source of radiation at the ceiling of the laboratory). To confirm this, we write the digital brightness measured in the three sections as follows:

$$I_{d,blue} = a_{blue} m_v + b_{blue} \quad (7)$$

$$I_{d,red} = a_{red} m_v + b_{red} \quad (8)$$

$$I_{d,green} = a_{green} m_v + b_{green} \quad (9)$$

where the subscripts blue, red and green refer to each one of the sections shown in Figure 1b. Note that here we do not account for the temporal variation of any parameter because the light intensity of the laboratory was constant. If we define $\Delta a_{x-blue} = a_x - a_{blue}$, and $\Delta b_{x-blue} = b_x - b_{blue}$, where x refers to red or green sections, we can write equations (8) and (9) as:

$$I_{d,x} = I_{d,blue} + [\Delta a_{x-blue} m_v + \Delta b_{x-blue}] \quad (10)$$

And rearranging terms, we get:

$$I_{d,x}^c = I_{d,x} - [\Delta a_{x-blue} m_v + \Delta b_{x-blue}] = I_{d,blue} \quad (11)$$

where $I_{d,x}^c$ is the corrected digital brightness of the red and green sections. This equation means that, if our model is consistent, the corrected digital brightness of the red and green sections should equal the digital brightness of the blue section. In the following we express Δa_{x-blue} and Δb_{x-blue} in terms of parameters that can be determined from the images.

If we consider that the mixing ratio r and the specific humidity q is the same at the three analyzed regions, we obtain from the definition of a that Δa_{x-blue} can be simply written as:

$$\Delta a_{x-blue} = \frac{\alpha[1 + rq]}{\hat{m}_d} \Delta f_{x-blue} \quad (12)$$

where $\Delta f_{x-blue} = f_x - f_{blue}$. On the other hand, if we consider that the mass of saturated vapor within the plume is the same in the three sections, we get from the definition of b that:

$$\Delta b_{x-blue} = \alpha \Delta I_{x-blue} - \frac{\alpha m_{vap}^s}{\hat{m}_d} \Delta f_{x-blue} \quad (13)$$

where $\Delta I_{x-blue} = I_x - I_{blue}$. By combining equations (11) - (13), considering again mass conservation, and taking into account that the number of droplets that crosses the three regions is the same, we finally obtain:

$$I_{d,x}^c = I_{d,x} - \Delta I_{d,x-blue}^{c,1} - \Delta I_{d,x-blue}^{c,2} \quad (14)$$

where $\Delta I_{d,x-blue}^{c,1} = \alpha N \Delta f_{x-blue}$ and $\Delta I_{d,x-blue}^{c,2} = \alpha \Delta I_{x-blue}$ are corrector terms that can be calculated from the digital images. The term $\Delta I_{d,x-blue}^{c,1}$ can be interpreted from equation (2) as the difference of the brightness emitted from the droplets crossing the red and green sections, and the droplets crossing the blue section. To determine this term, we first calculate the digital brightness of section $A'B'$ (Fig. 1b) in the three regions with equation (6) for all the atomization rates. Then, we subtract the digital brightness obtained in the same section $A'B'$ when the atomization rate is zero. In such a way, the background intensity is removed. Finally, we subtract the results obtained for the blue region in the previous steps to the results obtained for the red and green regions. The second corrector term $\Delta I_{d,x-blue}^{c,2}$ is the digital brightness difference between the background of the red and green sections, and the background of the blue section. To determine this term, we first calculate the digital brightness of section AB in the three regions with equation (6) when the atomization rate is zero, and then, we subtract the result obtained for the blue section to the results obtained for the red and green sections. Once these correction terms are applied to $I_{d,red}$ and $I_{d,green}$ (equation (14)), we find that the digital brightness of the

three regions versus the normalized atomization rate lies on the same straight line (Fig. 1d). This result shows the consistency of the model.

3.3. Applicability of our experimental results to volcanic plumes

Our experiments confirm a linear correlation between the water ejected by the ultrasonic humidifier and the digital brightness of the condensed plume. This implies that the absorption of the background light intensity by the droplets, and the multiple scattering within the cloud, can be neglected. This result can be directly extrapolated to actual volcanic clouds if the fraction of light blocked by the natural volcanic plumes (both the light coming from the background and the light scattered by the droplets) is lower than in the experimental plume. We check this condition by comparing the maximum column droplet density (i.e., the maximum number of droplets per unit area), which is given by CD , where C is the concentration of droplets in the part of the plume where the brightness is measured (number of droplets N per cubic meter of plume), and D is the plume diameter. The concentration C can be determined by writing equation (3) as follows:

$$C = \frac{[1 + rq]C_v - C_{vap}^s}{\hat{m}_d} \quad (15)$$

where C_v is the concentration of the water ejected by the humidifier or the volcano (mass of water m_v per cubic meter of plume), and C_{vap}^s is the concentration of saturated water

vapor in the condensed plume (mass of saturated water m_{vap}^s per cubic meter of plume). The term $\hat{m}_d$ can be estimated from $\hat{m}_d \approx (4/3)\pi r_d^3 \rho_l$, where r_d is the radius of an average size water droplet and ρ_l is the density of liquid water. The terms C_v and C_{vap}^s can be estimated from $C_v = Q/Sv$ and $C_{vap}^s = q_{vap}^s \rho$, where Q is the water flux released by the humidifier or the volcano, S and v are the section and velocity of the condensed plume, respectively, q_{vap}^s is the specific humidity of the plume (mass of saturated water vapor per unit mass of gas mixture), and ρ is the partial density of the gas mixture in the plume (mass of gas mixture per cubic meter of plume), which can be approximated to the gas density (mass of gas mixture per cubic meter of gas within the plume) if we neglect the volume of condensed water versus the volume of gases. In turn, the specific humidity q and q_{vap}^s can be written as [e.g., Fleagle and Businger, 1980]:

$$q = \frac{\varepsilon e_s(T_{air}) RH}{p - e_s(T_{air})(1 - \varepsilon)RH} \quad (16)$$

$$q_{vap}^s = \frac{\varepsilon e_s(T_p)}{p - e_s(T_p)(1 - \varepsilon)} \quad (17)$$

where $\varepsilon = M_w/M_d \approx 0.622$, with M_d the molecular mass of dry air (~ 28.97 g/mol) and M_w the molecular mass of water (~ 18.02 g/mol). Note that here we neglect the content of other much less abundant gases in the volcanic plume like SO_2 and CO_2 . The terms $e_s(T_{air})$ and $e_s(T_p)$ are the saturation vapor pressure at the temperature of the air T_{air} and at the temperature of the plume T_p , respectively, RH is the relative humidity of the air, and

p is the pressure at crater levels. The saturation vapor pressure can be estimated for a given temperature T from the corrected Clausius-Clapeyron equation [Koutsoyiannis, 2012]:

$$e_s(T) = A_1 e^{A_2(T-A_3)/T} \left(\frac{A_3}{T}\right)^{A_4} \quad (18)$$

where $A_1 = 611.657$, $A_2 = 24.921$, $A_3 = 273.16$, and $A_4 = 5.06$. With these values of the constants, both the temperature T and the saturation pressure $e_s(T)$ are in SI units. Finally, the mean plume temperature T_p is related to the mixing ratio and air temperature through [e.g., Matsushima and Shinohara, 2006]:

$$T_p = \frac{C_w T_w + r C_{air} T_{air}}{C_w + r C_{air}} \quad (19)$$

where C_w and T_w are the specific heat capacity of water vapor and the volcanic gas temperature, respectively, and C_{air} and T_{air} are the specific heat capacity and temperature of the air, respectively. From now on we consider $C_w \approx 2.0$ kJ/kg K and $C_{air} \approx 1.0$ kJ/kg K [e.g., Fleagle and Businger, 1980]. By combining equations (15) - (19), we can estimate the concentration of droplets both in our experiments and in volcanic plumes.

In our experiments, the maximum flux reached was $Q \approx 111$ mg/s, the section of the plume is given by the diameter of the nozzle from which the droplets exit the humidifier ($D = 15$ mm), and the velocity is on the order of $v \sim 5$ cm/s (estimated by comparing

images). For $T_w \sim T_{air} \sim 20\text{ }^{\circ}\text{C}$ (ultrasonic humidifiers create cool fog, so we consider room temperature), $p \sim 1\text{ atm}$, $r_d \sim 5\text{ }\mu\text{m}$, $\rho_l \sim 1000\text{ kg/m}^3$, $RH \sim 60 - 70\%$, $r \sim 10 - 500$, and calculating ρ from an ideal gas approach, we obtain that the maximum droplet concentration of our experiments is on the order of $C \sim 10^7 - 10^8\text{ drops/cm}^3$. Hence, the column droplet density in our experiments reaches values in the range $CD \sim 10^7 - 10^8\text{ drops/cm}^2$. The same calculations are done for volcanic plumes, and we obtain that the column droplet density reaches values in the range $CD \sim 10^6 - 10^8\text{ drops/cm}^2$ for many scenarios (Table 2). For example, for volcanic gas temperatures of $T_w \sim 500\text{ }^{\circ}\text{C}$, atmospheric temperature $T_{air} \sim 15\text{ }^{\circ}\text{C}$, mixing ratio $r \sim 20$, relative humidity $RH \sim 30\%$, steam flux $Q = 100\text{ kg/s}$, ascent velocities $v \sim 2\text{ m/s}$, and plume diameters $D \sim 20\text{ m}$, we obtain that the column droplet density is $CD \sim 3.2 \cdot 10^8\text{ drops/cm}^2$. Therefore, we conclude that the column droplet density, at least in diluted volcanic plumes, is at most on the same order of magnitude than the column droplet density we have reached in our experiments, and thus we can assure the linear correlation between the brightness of condensed volcanic clouds and its volcanogenic water content.

Table 2. Estimation of the column droplet density in condensed volcanic plumes.

Parameters and units	Value	Range ^{*1}				
Gas temperature T_w (°C)	500	350-650	---	---	---	---
Air Temperature T_{air} (°C)	15	---	5-25	---	---	---
Mixing ratio r	20	---	---	15-200	---	---
Relative humidity RH (%)	30	---	---	---	10-90	---
Steam flux Q (kg/s)	100	---	---	---	50-200	---
Plume diameter D (m)	20	---	---	---	---	5-25
Plume velocity v (m/s)	2	---	---	---	---	0.5-4
Plume temperature T_p (°C) ^{**2}	59	45-72	50-68	20-72	---	---
Gas density ρ (kg/m ³) ^{**3}	0.66	0.63-0.69	0.64-0.68	0.64-0.75	---	---
Droplet concentration C ($\cdot 10^6$ drops/cm ³)	0.16	0.02-0.24	0.09-0.21	0.03-0.47	0.15-0.20	0.002-0.49
Column droplet density CD ($\cdot 10^8$ drops/cm ²)	3.2	0.5-4.8	1.9-4.2	0.6-9.5	3.0-4.1	0.05-9.7
					1.2-25	0.05-23

^{*1} The parameters with the symbol --- are not varied and their values are given in the second column. ^{**2} The temperature of the plume is calculated with equation (19). ^{**3} The gas density is calculated by assuming and ideal gas approach from the temperature of the plume T_p and atmospheric pressure $p \sim 1 \text{ atm}$. As in the experimental plume, we consider $r_d \sim 5 \mu m$ [Gassó, 2008] and $\rho_l \sim 1000 \text{ kg/m}^3$.

4. Processing method of the brightness time series of a volcanic plume

The variability of the terms $a(t)$ and $b(t)$ of equation (5) must be corrected in a brightness time series to ensure that a brightness change is only due to a vapor emission change. As we stated above, we use: $a(t) = \alpha f(t)[1 + r(t)q(t)]/\hat{m}_d$ and $b(t) = \alpha[I_b(t) - m_{vap}^s(t)f(t)/\hat{m}_d]$. The term $f(t)$ accounts for the scattering of each water droplet, and varies gradually during the day with the incident solar radiation on the plume, the angle Sun-plume-observer, and the mean water droplet size [e.g., Lock and Yang, 1991; Han et al., 1994; Zuidema and Hartmann, 1995; Gassó, 2008; Hergert and Wriedt, 2012]. The term $I_b(t)$ accounts for the background radiation of the region of interest that is analyzed, and thus also varies with the position of the Sun during the day. The variability of the mass of saturated water vapor $m_{vap}^s(t)$ depends on the temperature and pressure, and thus depends indirectly on the temperature of the volcanic gas, temperature, pressure, and relative humidity of the atmosphere, and mixing ratio between air and volcanic gases [e.g., McGonigle et al., 2004; Matsushima, 2005; Matsushima and Shinohara, 2006]. The volcanic gas temperature depends on magma temperature, and thus we consider it constant during quiescence, at least over short timescales of hours or days; temperature, pressure, and relative humidity of the atmosphere (and hence specific humidity $q(t)$ as well -see equation (16)-) fluctuate on a diurnal timescale [e.g., McGonigle et al., 2004]; and the mixing ratio $r(t)$ depends on the entrainment of air into the plume, which can be considered constant with time as long as a buoyant quiescent plume is geometrically stable, that is, as long as there are no important changes of wind speed and direction at crater heights [e.g., Bursik, 2001; Suzuki and Koyaguchi, 2010].

Therefore, at least for stable buoyant plumes, the terms $a(t)$ and $b(t)$ are expected to show long-term trends that in principle can be corrected. Short-term irregular variations of $a(t)$ and $b(t)$ can also occur if an atmospheric cloud, or a part of the downstream gas plume, interposes between the Sun and the section of plume that is analyzed. In the following, we describe a strategy to correct long-term trends and irregular perturbations in a brightness time series:

1. *Choice of regions of interest.* We calculate the digital brightness according to equation (6) in three regions of the images: (R1) in a section of the plume, which accounts for the temporal variation of the background, incident light, weather conditions at crater level, and steam emissions; (R2) outside the plume but with the same background, which accounts for the temporal variation of the background; and (R3) in the crater rim right below the section of the plume, which accounts for irregular and gradual temporal variations of the incident solar light on the crater.

2. *Light change correction.* We divide the brightness values at (R1), (R2), and (R3) by the number of pixels of each region, and remove the mean of the resulting time series. Then, we subtract the mean-removed time series of regions (R2) and (R3) to the mean-removed time series of (R1). By doing this, we are removing irregular and gradual trends induced by the changing incident light, the trend induced by the changing background, and even possible perturbations induced by instabilities of the digital camera. With this approach, we use brightness changes of (R3) as a proxy for the changes of the incident solar light on

(R1), which is realistic if both regions are spatially very close, and as long as the Sun is not on the opposite side of the volcano from the position of the camera.

3. *Weather change correction.* Whatever trend remains in the brightness time series of the region (R1) should be only the consequence of changes in the atmospheric conditions at crater levels. By assuming these changes are long-term (with diurnal or semi-diurnal periodicity), detrending can be done by fitting a low order polynomial to the time series, and then subtracting the best fit to the brightness data. This undesirable long-term trend could be also removed by filtering the data with a high-pass filter.

For simplicity, we suggest standardizing (i.e., normalizing) the resulting time series of the region (R1) by subtracting the mean value of the time series to each datum and dividing by the standard deviation. This is not a requirement, but it produces a dimensionless time series which allows a direct comparison with the normalized time series of other volcanic gases like SO_2 (as we show in the next section). Normalization does not alter the spectral content of the signal.

5. Case studies

Below we determine the spectral features of the steam emissions from two volcanoes. First, we analyze the steam pulses from Mount Erebus by computing the digital brightness from a movie of the gas plume recorded on December 26, 2006, by Boichu et al. [2010].

This brightness time series is compared with an SO₂ time series that was measured during the same period. Second, we analyze the steam cycles of Mount Mayon by computing the digital brightness time series corresponding to a set of digital images we took on March 4, 2014.

5.1. Erebus volcano

Mount Erebus is a basaltic stratovolcano located in Ross Island, Antarctica, which is known for its convective lava lake persisting since 1972. Lava lake activity, characterized by passive gas emissions with average SO₂ flux on the order of 60 t/day, is interspersed by more or less frequent minor explosions, and by larger Strombolian paroxysms occurring occasionally [e.g., Calkins et al., 2008; Davies et al., 2008; Johnson et al., 2008; Sweeney et al., 2008].

Digital images of the condensed gas plume were extracted from a 30 minutes movie that was recorded on December 26, 2006 (movie supplied by M. Boichu -personal communication-, and partially published in Boichu et al. [2010]). The movie was split into frames, and one frame per second of shooting was analyzed (sampling frequency of 1 Hz). With these frames, we calculated the digital brightness of a region of interest (R1) that crosses the volcanic plume above the crater rim (red section of Fig. 2a), which in turn coincides with the section where Boichu et al. [2010] measured the SO₂ column amounts (molecules of SO₂/cm²). To apply the correction strategy explained in the previous section,

we also calculated the brightness time series in two auxiliary regions (R2) and (R3): region (R2) is centered in the sky close to the plume, and region (R3) is centered in the upper left part of the crater rim (green and blue polygons of Fig. 2a, respectively).

Local peaks and troughs of the brightness time series of the region (R1) are in general well correlated with those corresponding to the SO_2 time series (e.g., time window 500 – 1000 s in Fig. 2b). However, there is also an increasing trend of the brightness with time that is not present in the SO_2 data, and that is not likely due to a gradual increase of the water emissions because it is also observed in the regions (R2) and (R3) of the image (Fig. 2c). A sudden brightness drop is also observed in (R2) and (R3) at $t \sim 1620$ s, which could be due to a fault in the power supply of the camera. These gradual and irregular changes of the brightness time series are removed from (R1) as explained in the light change correction step of the previous section, and then a first order polynomial is fitted to the resulting time series to make the weather change correction (Fig. 2d). The slope of the fitting straight line is very low, thus suggesting that atmospheric parameters like temperature and relative humidity remained almost invariant during the thirty minutes shooting that we analyzed. The standardized and corrected digital brightness time series, and the standardized SO_2 column amounts time series, are now very similar (Fig. 2e).

Fig. 2. Digital brightness analysis of the plume of Erebus volcano, and comparison with SO₂ time series. **a)** Image of the movie recorded at Erebus volcano on December 26, 2006 (movie partially published by Boichu et al., 2010. Complete movie by courtesy of M. Boichu, personal communication). Red line is the region (R1) of interest where we perform the digital brightness analysis. Green and blue regions are the (R2) and (R3) auxiliary regions, respectively, that we use to correct the brightness time series. **b)** Standardized SO₂ (black) and raw brightness (red) time series. **c)** Mean removed digital brightness in digital units (D.U.) of the (R1), (R2), and (R3) regions. **d)** Mean removed digital brightness of the region (R1) after the light change correction. Grey line is the first-order polynomial that best fit to the brightness curve, and which is used for the weather change correction (slope = -1.6×10^{-5} , and ordinate in the origin = 0.01). **e)** Comparison between standardized SO₂ time series (black) and standardized and corrected brightness time series (red).

The wavelet transform analysis of the time series shows that there are not steady periodic pulses in the whole time series, and reflects that steam emissions actually emerge from a mix of finite pulses with different periodicities. In particular, we identified two main periodic bands (Fig. 3a). Band 1 contains the more energetic component, which appears between 500 – 1400 s of the time series with periodicities in the range $\sim 100 - 250$ s. Band 2 is less energetic, and the periodicity is in the range $\sim 500 - 650$ s. Some short-

duration periodic components with periodicities lower than 100 s also appear irregularly through the time series. The fast-Fourier transform shows that digital brightness, and thus steam emissions, follows a well-defined fractal (power-law) distribution in a wide range of frequencies of the spectrum. That is, $\Delta \sim \nu^\gamma$, where Δ is the amplitude of the pulse, ν is the frequency, and γ is a constant which in this case is $\gamma \approx -1.0$ (Fig. 3b).

The wavelet transform of the SO₂ signal also shows the bands 1 ($\sim 100 - 250$ s) and 2 ($\sim 500 - 650$ s), but there is a third band appearing at least between 500 – 1300 s with periodicities in the range $\sim 300 - 450$ s (Fig. 3c). The fractal distribution is not so evident for the SO₂ time series because there is a clear deviation from a power-law distribution for periodicities lower than ~ 12 s (Fig. 3d), even though the best fit for larger periodicities also gives an exponent of $\gamma \approx -1.0$. Power-law deviation at high frequencies is also observed in the fast-Fourier transform of the brightness time series, but only for periodicities lower than ~ 4 s (compare Fig. 3b and 3d).

Fig. 3. Spectral features of the H₂O and SO₂ emissions from Erebus volcano. **a)** Wavelet transform of the brightness time series. Colors represent normalized amplitude, and rectangles indicate the more characteristic periodic components of the time series. **b)** Fast-Fourier transform of the brightness time series (red), and best-fit line (black). **c)** Wavelet transform of the SO₂ time series (data from Boichu et al. [2010]). **d)** Fast-Fourier transform of the SO₂ time series (red), and best-fit line for periodicities larger than 12 s (black). Wavelet (Morlet algorithm) and fast-Fourier transforms are calculated by using MATLAB. In a) and c), the cone of influence (part of the spectrum that is affected by edge effects) is removed. In b) and d), γ is the slope of the best-fit line.

5.2. *Mayon volcano*

Mayon is a basaltic-andesitic stratovolcano located to the north of Legazpi city, more than 300 km south-east of Manila, the capital city of the Philippines. This volcano is known by its almost perfect conical shape that extends from the sea level up to almost 2500 m height, and its activity is characterized by persistent passive degassing (with average SO₂ fluxes around 100 – 500 t/day, according to the reports published online in <http://www.phivolcs.dost.gov.ph> by the Philippine Institute of Volcanology and Seismology). Passive degassing is interspersed by mildly-explosive eruptions occurring every few years, and large explosive eruptions occur every 100 years or so [e.g., Siebert et al., 2011].

Digital images of Mayon crater and plume were taken with an IP-camera (model BASLER acA2040-25gc, 2048x2048 pixels, with lens Tamron A08N SP AF200-500mm F/5-6.3) located at the volcano observatory, which is about 12 km south-east of the crater. One image per second was taken continuously during 106 minutes from 6:50 am to 8:36 am on March 4, 2014, period during which the plume rose vertically. We chose a region (R1) right in the crater rocks (red section of Fig. 4a) and an auxiliary region right below the crater (blue polygon of Fig. 4a). The choice of (R1) right in the crater rocks is advantageous because the perturbation of the plume by wind changes is minimized, and the auxiliary region represents both (R2) and (R3) since the crater rocks are the background of (R1) in this case.

The standardized raw brightness data shows an increasing trend of the brightness with time (Fig. 4b). This is also observed in the auxiliary region of the image (Fig. 4c), thus suggesting that is not related to a long-term increase of the water vapor emissions. Irregular brightness changes are also observed in (R2) (= (R3)), maybe because of stability problems of the camera. The trend of (R2) (= (R3)) is removed twice to the time series of (R1), and then a first order polynomial is fitted to the resulting data to make the weather change correction (Fig. 4d). The slope of the fit is slightly negative, suggesting a possible slight increase of temperature and decrease of relative humidity during the studied period, which is realistic during dawn [e.g., McGonigle et al., 2004]. The standardized and corrected digital brightness time series shows a quite regular behavior of the steam emissions (Fig. 4e).

Fig. 4. Digital brightness analysis of the plume of Mayon volcano. **a)** Summit of Mayon volcano as viewed from the observation point. Red line is the region (R1) of interest where we perform the digital brightness analysis. Blue region (R2) (= (R3)) is the auxiliary region we use to correct the brightness time series for light and weather change. **b)** Standardized raw brightness time series. **c)** Mean removed digital brightness in digital units (D.U.) of the (R1) and (R2) (= (R3)) regions. **d)** Mean removed digital brightness of the region (R1) after the light change correction. Grey line is the first-order polynomial that best fit to the brightness curve, and which is used for the weather change correction (slope = -5.6×10^{-6} , and ordinate in the origin = 0.02). **e)** Standardized and corrected brightness time series.

The wavelet transform of the brightness signal shows four characteristic bands (Fig. 5a). Band 1, in the range $\sim 100 - 500$ s, contains short-duration periodic components appearing intermittently. Band 2 contains a quite stable periodic component (~ 1 hour) in the range $\sim 600 - 900$ s, even though the amplitude gradually decreases with time. Band 3 also contains a very stable periodic signal in the range $\sim 1200 - 1600$ s and, contrary to band 2, the amplitude gradually increases with time. One more periodic band is observed

around 2000 s, even though it is close to the limit detectable with the wavelet transform. Just as at Erebus, the wavelet transform shows that there is no steady pulse during the whole time series, since, although there are long-duration periodic components (~ 1 hour), their amplitudes change gradually. The frequency spectrum of the brightness time series of Mayon also follows a well-defined fractal (power-law) distribution with exponent $\gamma \approx -1.0$ (Fig. 5b).

Fig. 5. Spectral features of the H_2O emissions from Mayon volcano. **a)** Wavelet transform of the brightness time series. Colors represent normalized amplitude, and rectangles indicate the more characteristic periodic components of the time series. The cone of influence (part of the spectrum that is affected by edge effects) is removed. **b)** Fast-Fourier transform of the brightness time series (red), and best-fit line (black). The parameter γ is the slope of the best-fit line. Wavelet (Morlet algorithm) and fast-Fourier transform are calculated by using Matlab.

6. Discussion

6.1. Limitations and uncertainties of the brightness method

The main limitation of our method is that it can be only applied when the volcanic plume saturates, a condition that, however, is very common at many active volcanoes under typical atmospheric conditions at crater heights [Matsushima, 2005; Matsushima and Shinohara, 2006]. Complications can arise from external perturbations that can contaminate the brightness time series with undesired trends. However, these trends can be corrected because they follow well-defined long-term cycles, such as the semidiurnal and diurnal variations of pressure, temperature, and humidity [e.g., Zimmer and Erzinger, 2003; McGonigle et al., 2004], and the possible daily change of the water droplet size [e.g., Han et al., 1994; Zuidema and Hartmann, 1995]. Other long-term trends, like those caused by the changing incident radiation on the plume due to the changing position of the Sun along the day, can be also corrected by analyzing different auxiliary regions of the digital images. The same strategy can be used to correct short-term perturbations of brightness time series, like those caused by instabilities of the camera, or by clouds interposing between the Sun and the analyzed region of the plume.

Other processes, like wind changes at crater heights, could induce uncontrolled and fast variations of the mixing ratio between air and volcanic gas r [e.g., Bursik, 2001]. Short-term changes of the mixing ratio can modify the temperature of the condensed plume T_p , the concentration of droplets C with time, and thus the brightness measurements. For

example, for the parameters of Table 2 (column 2), we obtain by using equation (19) that the temperature of the plume T_p varies around 30 °C when the mixing ratio varies from $r = 18$ to $r = 50$. This variation of the mixing ratio would induce a change of the water droplet concentration in the range $(1.3 - 3.1) \cdot 10^5 \text{ cm}^3$, whose mean value and absolute uncertainty is $(2.2 \pm 0.9) \cdot 10^5 \text{ cm}^3$. This represents a relative uncertainty on the order of 40%, which is a realistic maximum value for the brightness measurements of stable ascending plumes since the amplitude of fumarole gas temperatures is typically on the order of a few tens of degrees [e.g., Connor et al., 1993; Byrdina et al., 2012], like the variation of the condensed plume temperature we have considered above. Uncertainty of 40% is on the same order of magnitude than the uncertainty typically assigned to the SO_2 gas fluxes measured with absorption spectrometers [e.g., Boichu et al., 2010].

Another factor that could increase the brightness uncertainty is the variation of the molar ratio $[\text{H}_2\text{O}]/[\text{HCl}]$ in the plume because the dissolution of HCl in water droplets affects the saturation vapor pressure, and hence can modify the mass of saturated water and the number of water droplets itself [Matsushima and Shinohara, 2006]. This ratio has been found to remain roughly constant during quiescence at volcanoes like Masaya [Burton et al., 2000], and thus we do not take into account this possibility. In any case, short-term sporadic perturbations are not expected to obscure the overall spectral features of sufficiently long brightness time series [e.g., Boichu et al., 2010].

6.2. Interpretation of the degassing patterns from Erebus and Mayon volcanoes

In the following, we discuss the processes that could explain the overall results we have obtained from Erebus and Mayon volcanoes. That is: periodic gas emissions with finite duration; a periodic component in the band $\sim 300 - 450$ s that appears in the SO₂ time series of Erebus but that is absent in the brightness time series; and the fractal distribution of the frequency spectra.

6.2.1. Periodic gas emissions with finite duration

Several mechanisms could explain periodic patterns during quiescence. For example, pulsed entrance of magma into the lava lake could explain gas fluctuations at Erebus [Oppenheimer et al., 2009]; the non-Newtonian rheology of magmas can lead spontaneously to the alternation between bubbly regime, with quasi-periodic release of gas, and channeling regime, in which bubbles percolate through the magma column [Divoux et al., 2011]; and the presence of geometrical or rheological barriers in the magma plumbing system (e.g., reservoir roof [Jaupart and Vergnolle, 1988], cavities in the conduit [Boichu et al., 2010], or viscoelastic layers formed beneath the magma level [Patrick et al., 2011]) can lead to the cyclical formation and collapse of foams. Although these mechanisms could explain periodic gas emissions, a model able to reproduce completely the spectral features of volcanic degassing time series is still missing.

6.2.2. *Difference between the brightness and SO₂ wavelet transforms of Erebus volcano*

The corrected brightness and SO₂ time series are in general very similar (Fig. 2e), thus suggesting that the molar ratio [H₂O]/[SO₂] during the passive degassing period analyzed varies very little. However, the spectral analysis reveals that the SO₂ time series contains a periodic component in the band 300 – 450 s (band 3) that is absent in the brightness time series. This component lasts for at least 700 s, which suggests that it is not the result of short-term effects like changes in the mixing ratio or vent gas temperature. Another possibility could be that water droplets do not follow the same path as the molecules of SO₂ during plume ascent, but it does not seem feasible in this frequency range because bands 1 and 2, with higher and lower periodicities than band 3, appear very clearly in both time series (Fig. 3). Therefore, band 3 likely reflects a real volcanic process. We propose that band 3 is only present in the SO₂ time series because it is induced by the collapse of gas foams formed beneath very deep geometrical or rheological barriers, whereas bands 1 and 2 are present in both time series because they are induced by the formation and collapse of shallow foams. This is supported by the fact that most of the water exsolves at very low pressures (depths < 1 km; e.g., Liu et al., [2005]; Gonnermann and Manga, [2012]), and thus we can expect lower H₂O content in the foams accumulating beneath very deep barriers than in the foams accumulating beneath very shallow barriers. The possibility of gas emissions sourced at different depths was also proposed by Oppenheimer and Kyle [2008].

6.2.3. *Fractal frequency spectra*

The fast-Fourier transform analysis of the brightness time series of Erebus and Mayon volcanoes reveals that the larger the amplitude of the steam pulses, the lower the frequency. This feature likely reflects a volcanic process, instead of atmospheric, since turbulent eddies only affect very high frequencies (periodicities lower than ~ 40 s [Tamburello et al., 2013]). For example, the frequency spectra could reflect the presence of geometrical or rheological barriers of different sizes because the larger the section of the barrier where bubbles accumulate, the larger the volume of the foam, and the lower the collapsing frequency (as it can be inferred from the experiments performed by Jaupart and Vergnolle, [1989]). However, the problem is likely much more complex because fractal (power-law) frequency spectra emerge from the coupling between the different parts that compose a dynamic system which can be in a self-organized critical state [e.g., Bak et al., 1987; Bak, 1996; Turcotte, 1999]. Hence, we propose that Erebus and Mayon volcanoes are self-organized critical systems, such that degassing patterns arise spontaneously from the coupling between bubble ascent through the conduit [Vergnolle and Jaupart, 1986], magma convection in the conduit [e.g., Shinohara, 2008], or changes of conduit permeability [Michaut et al., 2009; Divoux, 2011]. Experimental and numerical research is required to understand the conditions that determine the exponent of the power-law. Self-organized critical behavior was already proposed for the gas pistoning activity of Kilauea volcano after the analysis of seismic tremor signals [Chouet and Shaw, 1991].

Contrary to the frequency spectrum of the brightness time series of both volcanoes, which show well-defined power-law distributions along a wide range of frequencies, the spectrum of the SO₂ time series of Erebus clearly deviates from a power-law for periodicities lower than ~ 12 s. We propose this can be consequence of an external perturbation. The gas molecules, which is what is monitored with absorption spectrometers or UV cameras, have a mass many orders of magnitude lower than the mass of a micrometric water droplet, which is what we monitor with the brightness method (the weight of a molecule of SO₂ is on the order of 10⁻²⁵ kg, whereas the weight of micrometric water droplets is on the order of 10⁻¹³ kg). Because of this much lower mass, the inertia of the SO₂ molecules during plume ascent is much lower than the inertia of the water droplets, and thus it is feasible that the local wind of the crater area perturbs more the motion of the molecules than the motion of the droplets. This is patent for high frequencies (low periodicities) because they are less energetic.

7. Conclusions

We have reported a new method to capture remotely the spectral features of the H₂O volcanic emissions, which is by far the major component of most volcanic plumes. This can be done simply by analyzing digital images since there is a correlation between the brightness of condensed volcanic clouds and its volcanogenic water content. Our method is based on the light scattered by the micrometric droplets moving with the plume, and it allows real-time monitoring of the steam cycles with low maintenance, is low cost since it

only requires a commercial digital camera to measure brightness changes and a laptop for data processing, the sampling rate can be very large (limited by the specs of the digital camera), data processing is relatively straightforward, can be used far away from the volcanic vent with proper amplification lens (making it especially useful for hardly accessible volcanoes), and external disturbances caused by wind changes are minimized since brightness can be measured right in the crater area. Hence, digital brightness analysis can become an ideal method for real-time monitoring of steam cycles at many active volcanoes.

We have applied the brightness method to two digital movies from Erebus and Mayon volcanoes. The brightness time series of Erebus contains two finite but well-defined periodic components in the bands 100 – 250 s and 500 – 650 s. These two periodic components are also present in the SO₂ time series published by Boichu et al. [2010], in addition to a third component in the range 300 – 450 s. The brightness time series of Mayon contains two long-duration (> 1 hour) periodic components, but with changing amplitude, in the bands 600 – 900 s and 1200 – 1600 s. Another band around 2000 s and some very short-lived components with periodicities below 500 s are also present. On the other hand, the frequency spectra of the brightness time series of both volcanoes reveal that steam emissions exhibit fractal behavior in a wide range of frequencies. These findings suggest that quiescent degassing cannot be understood from a unique steady periodic mechanism, but that it emerges as a self-organized critical phenomenon from the complex coupling of different processes.

Chapter 3:

Shifts of volcanic degassing patterns as possible precursor of phreatic eruptions

Shifts of volcanic degassing patterns as possible precursor of phreatic eruptions

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Abstract

Phreatic eruptions are explosions occurring when the pressure beneath volcanic craters increases because of gas over-accumulation. Despite the small size of phreatic eruptions compared to magmatic or phreatomagmatic eruptions, they pose a risk to scientists and tourists because of the lack of clear precursors. For example, a small phreatic event occurred at Mayon volcano (Philippines) on 7 May 2013 without previous warning, which killed five tourists that were close to the summit. Another unexpected phreatic eruption occurred on 27 September 2014 at Ontake volcano (Japan), which killed tens of people. Here, we present a new numerical model to explore how the high-frequency gas emission patterns reflect the processes that are thought to trigger phreatic eruptions at persistently degassing volcanoes with magma located high in the conduit. Our model combines a lumped-parameter model for the transfer of gases through a permeable porous dome, a

lumped-parameter model for the pressure accumulation beneath the crater, and a three-dimensional agent-based model that captures the bubble dynamics of the upper part of the magma conduit. A key new finding is that the processes triggering gas explosions, i.e., groundwater infiltration through the volcanic edifice and dome plugging, induce shifts of amplitudes and frequencies in the high-frequency degassing time series. These shifts can be quantified by determining the standard deviation of the degassing time series (Δ) and the best fit line of their frequency spectra (ζ), which systematically vary with the overpressure beneath the crater. Our numerical analysis suggests that the monitoring of Δ and ζ from high-frequency degassing time series can be used to track gas pressure variations beneath volcanic craters, which could help to forecast phreatic eruptions.

1. Introduction

The spectacular technical development of remote-sensing methods during the last decades has marked a turning point in the surveillance of active volcanoes. Currently, it is possible to detect simultaneously, safely, in real time, with good precision and accuracy, and with high sampling rates, the deformation of volcanic edifices [e.g., Dzurisin, 2007], seismic events [e.g., Chouet, 2003], gravity and temperature anomalies [e.g., Carbone and Greco, 2007; Spampinato et al., 2011], and/or variations of the gas emission rates and compositions [e.g., Symonds et al., 1996; Aiuppa et al., 2007]. The combined analysis of these data allows identifying magma ascents towards the surface, and thus anticipating possible magmatic eruptions in the short-term. However, other volcanic events, like

phreatic eruptions, appear to occur without clear previous warnings and precursors [Barberi et al., 1992; Mordret et al., 2010].

Phreatic eruptions, as defined by Barberi et al. [1992], are gas explosions in which all the solid ejecta are fragments of pre-existing rocks, and which occur when the gas pressure below volcanic craters increases dramatically up to values that the cap-rocks are unable to withstand. Even though these eruptions are generally minor events compared to magmatic or phreatomagmatic eruptions, they pose a substantial risk to population, tourists, and scientists. For example, a recent phreatic event occurred without warning at Mount Mayon (Philippines) on 7 May 2013, and killed five tourists that were close to the summit at the moment of the explosion. Another phreatic eruption occurred on 27 September 2014 at Ontake volcano (Japan) that caused over thirty victims. Many other near-miss eruptions have occurred at other very touristic volcanoes like Merapi (Indonesia) or Masaya (Nicaragua). The gas over-accumulation beneath the crater floors of persistently degassing volcanoes like Mayon, Merapi, or Masaya, i.e., volcanoes that release large amounts of gas during quiescence and that are thought to have magma located high in the volcanic edifice [Shinohara, 2008], has been suggested to occur because of two possible processes [Barberi et al., 1992]: (1) the infiltration of meteoric water into the magmatic system, which generates large amounts of steam [e.g., Duffell et al., 2003], and (2) the plugging of the dome by mineral precipitation, fracture closure by gravitational compaction, or wall rock collapse [e.g., Matthews et al., 1997]. In this study, we investigate how these processes that may lead to phreatic eruptions affect persistent degassing, and try to identify any potential precursory change that might be recorded in

the high-frequency gas emission time series. We do this analysis from a numerical approach by coupling a lumped parameter model for the gas flow through a porous dome, a lumped parameter model for the pressure accumulation beneath the crater, and a three-dimensional agent-based model to simulate the bubble flow in a vertical magma conduit. Understanding how gas accumulation beneath the crater affects the gas emission cycles at the surface is paramount towards a potential improved forecasting of phreatic eruptions.

In the next section we first describe the fundamentals of our model. Then, we validate it by comparing our results with experiments from the literature. We also compare the spectral properties of our numerical degassing time series with those from monitored steam emissions at Mount Mayon. Later, we perform a series of numerical experiments to explore the influence of water infiltration and dome plugging on the high-frequency degassing cycles. Finally, we discuss our results and their application for volcano monitoring.

2. Model description

Let us consider that the shallower magma plumbing system of persistently degassing volcanoes (i.e., volcanoes releasing large amounts of gas during quiescence) is composed of a magma reservoir connected to a partially open conduit [e.g., Shinohara, 2008]. We also assume that the reservoir-conduit system comprises of four different regions during quiescence (Fig. 1). The first and deepest region includes the shallow magma reservoir

and the conduit up to the gas-separation level. Here, gas bubbles are very small and cannot move separately from the magma, and hence volatiles are transported to shallower levels via magma convection [e.g., Kazahaya et al., 1994; Stevenson and Blake, 1998]. The second region extends from the gas-separation level to the top of the magma column (magma level or magma head), where separated two-phase flow occurs [e.g., Vergnolle and Jaupart, 1986; Pioli et al., 2012]. The third region is a gas chamber on top of the magma level, where gas accumulates before being released from the volcano. The existence of a gas chamber is suggested by the subsidence of the crater floors of volcanoes like Masaya and Asama [Rymer et al., 1998, Urabe et al., 2006], by the very low density layers found by means of muon tomography beneath the craters of Asama and Satsuma-Iwojima [Tanaka et al., 2008; Tanaka et al., 2010], and by the huge decrease of melt volume during degassing, which tends to decrease the mean magma level after a few months or years of quiescence [Chapter 1]. The fourth and upper part of the conduit is a permeable dome through which the volatiles are finally released to the atmosphere [e.g., Shinohara, 2008].

Fig.1. Sketch of the conduit of a persistently degassing volcano during quiescence with the main features considered in our model. $Q_{out}(t)$ is the gas mass rate or gas output at the surface with time, L is the thickness of the porous solid dome, κ_1 and κ_2 are the viscous and inertial permeability of the dome, respectively, A is the section of the dome (assumed to be equal to the section of the conduit), $D(t)$ and $P_{ch}(t)$ are the thickness and pressure of the gas chamber, respectively, $H(t)$ and $\varepsilon(t)$ are height and gas holdup of the magma column over the gas separation level, respectively, and R is the conduit radius. Brown and orange colors represent degassed and undegassed melt, respectively.

Given the above structure of the conduit, we hypothesize that the fluctuations of the gas emission rates during quiescence reflect pressure changes of the gas chamber located between the magma level and the dome. In turn, these pressure changes during persistent degassing are caused by successive ups and downs of the magma head, which behaves like the piston of a syringe. To test this hypothesis, we have developed a model that

includes: (1) a lumped-parameter model for the gas migration through a porous dome (uppermost region of the conduit), (2) a lumped-parameter model for the pressure evolution of the gas chamber (third region of the conduit), and (3) a three-dimensional two-phase flow agent-based model that simulates the bubble column dynamics over the gas-separation level (second region of the conduit). The supply of volatiles from the first to the second region of the conduit is assumed to occur at a constant rate via steady-state convection. With this model we aim to simulate the high-frequency degassing signals (periodicities lower than 4 – 5 minutes), which are thought to be caused by the complex bubble dynamics in the upper conduit [e.g., Tamburello et al., 2013]. The analysis of other lower frequency signals caused, for example, by changing convection rates, or by the intermittent collapse of magma foams [e.g., Jaupart and Vergnolle, 1988; Shinohara et al., 2002; Boichu et al, 2010; Tamburello et al., 2013], is beyond the scope of this paper.

2.1. Gas migration through the permeable dome

The gas emitted through a permeable dome can be related to the pressure inside the gas chamber $P_{ch}(t)$ by means of the extended Darcy-Forchheimer equation for porous media [Darcy, 1856; Forchheimer, 1901; Polubarinova-Kochina, 1962; Hassanizade and Grey, 1987; Rust and Cashman, 2004; Zhu et al., 2014], which in terms of the gas mass flux or gas outflux $Q_{out}(t)$ can be written as:

$$\frac{P_{ch}(t)^2 - P_{atm}^2}{2P_{atm}L} = \frac{\mu_g Q_{out}(t)}{\kappa_1 A \rho_{atm}} + \frac{Q_{out}(t)^2}{\kappa_2 A^2 \rho_{atm}} + \frac{\gamma}{A \rho_{atm}} \frac{dQ_{out}(t)}{dt} \quad (1)$$

where t is time, P_{atm} is the atmospheric pressure at the exit vent, L is the thickness of the porous dome, μ_g is the gas viscosity, A is the cross-sectional area of the dome, ρ_{atm} is the volcanic gas density at atmospheric pressure, γ is a coefficient which accounts for transient inertia forces, and κ_1 and κ_2 are the viscous and inertial permeability of the dome, respectively. The left-hand side of the above equation is the pressure gradient through the dome, where an ideal gas is considered [e.g., Katz et al., 1959; Firoozabadi and Katz, 1979]. The first term on the right-hand side of the above equation was proposed by Darcy [1856] from steady-state experiments under viscous regime (low Reynolds numbers), the second term is a non-linear correction proposed by Forchheimer [1901] to extend Darcy's equation to the inertial regime (moderate to high Reynolds numbers), which has been previously used to understand gas escape during magma ascent [e.g., Rust and Cashman, 2004; Bouvet de Maisonneuve et al., 2009; DeGruyter et al., 2012], and the third term is the correction proposed by Polubarinova-Kochina [1962] to account for transient effects, which in principle must not be neglected in our study given the intermittent nature of passive degassing [e.g., Hall et al., 1995].

2.2. Pressure in the gas chamber

By considering that volcanic gases are ideal, assuming a cylindrical conduit, and neglecting as a first order approximation the compressibility of melt (bubble-free magma) and host-rock, the pressure in the gas chamber at a given time t can be simply written as $P_{ch}(t) = R_g T m_{ch}(t) / \pi R^2 M D_{ch}(t)$, where R_g is the ideal gas constant, T is the gas temperature, $m_{ch}(t)$ is the mass of gas inside the gas chamber, R is the radius of the conduit, M is the molecular weight of the gas, and $D_{ch}(t)$ is the thickness of the gas layer, which depends on the position of the magma level with time. By taking the time derivative of this equation, we obtain:

$$\frac{dP_{ch}(t)}{dt} = \frac{R_g T}{\pi R^2 M} \frac{Q_{in}(t) - Q_{out}(t)}{D_{ch}(t)} - \frac{P_{ch}(t)}{D_{ch}(t)} \frac{dD_{ch}(t)}{dt} \quad (2)$$

Here we have used mass conservation to write $dm_{ch}(t)/dt = Q_{in}(t) - Q_{out}(t)$, where $Q_{in}(t)$ is the mass rate of gas entering the gas chamber. In turn, it is satisfied that $D_{ch}(t) + H(t) = D_{ch,0} + H_0$, where $H(t)$ is the thickness of the bubbly magma column over the gas-separation level (see Fig. 1), and H_0 and $D_{ch,0}$ are defined as the thicknesses of the magma column and gas layer, respectively, if the magma over the gas-separation level is bubble-free. If we assume that the volume of melt over the gas-separation level remains roughly constant, the thickness of the magma column can be written as $H(t) = H_0 / [1 - \varepsilon(t)]$, where $\varepsilon(t)$ is the gas holdup or mean gas volume fraction over the gas separation level. Hence, the thickness of the gas layer can be expressed in terms of H_0 and

$D_{ch,0}$ as $D_{ch}(t) = D_{ch,0} - [\varepsilon(t)/(1 - \varepsilon(t))]H_0$. By combining these relations with equation (2), we finally obtain:

$$\begin{aligned} \frac{dP_{ch}(t)}{dt} = & \frac{R_g T}{\pi R^2 M} \frac{[Q_{in}(t) - Q_{out}(t)][1 - \varepsilon(t)]}{D_{ch,0}[1 - \varepsilon(t)] - H_0 \varepsilon(t)} \\ & + \frac{H_0 P_{ch}(t)}{[D_{ch,0}[1 - \varepsilon(t)] - H_0 \varepsilon(t)][1 - \varepsilon(t)]} \frac{d\varepsilon(t)}{dt} \end{aligned} \quad (3)$$

Equation (3) shows that calculating the pressure evolution of the gas chamber, and hence determining the gas flux cycles at the surface from equation (1), requires modeling the dynamics of the two-phase bubble column to determine how $Q_{in}(t)$ and $\varepsilon(t)$ vary with time. Note that $\varepsilon(t)$ depends on the gas volume inside the magma column, which in turn depends on the pressure of the gas chamber as well.

2.3. Agent-based model of a bubble column

Agent-based models are used to simulate the overall behavior of complex systems, that is, dynamic systems composed by many autonomous entities (agents) that evolve with time interacting with each other. This approach consists in providing each agent with some local rules that capture the fundamental physics of the problem, such that the collective behavior of the system as a whole emerges spontaneously after some time iterations. Despite the power and success of agent-based methods to predict complex patterns in

fields as diverse as social sciences [e.g., Macy and Willer, 2002], economy [e.g., Parker et al., 2003], and ecology [e.g., Grimm et al., 2006], its use in multi-phase fluid dynamics has not proliferated. So far, only Herrero et al. [1996] proposed a simple two-phase model that allows capturing some properties of bubble columns. However, bubbles were treated as two-dimensional disks, and bubble expansion during ascent through the column was not considered, such that the validation of the model by comparison with actual experiments is quite controversial. The model from Herrero et al. [1996] seems to have gone unnoticed until Marcel et al. [2011] used the same ideas to simulate boiling processes in square-section channels. Other agent-based models have been proposed, but they are restricted to the simulation of Taylor bubbles in pure slug flow [e.g., Barnea and Taitel, 1993; Mayor et al., 2008]. Below, we extend the model developed by Herrero et al. [1996] to simulate the behavior of three-dimensional cylindrical bubble columns emulating the separated two-phase flow region of volcanic conduits. Bubbles are treated as spheres with a volume equivalent to the volume of the actual distorted bubbles, and they can displace, expand by depressurization during ascent, breakup, coalesce, and escape from the magma column. We also account for the possible formation of gas slugs, which can occur in volcanic conduits because conduit lengths, gas expansion, and high melt viscosity, favor the coalescence of small bubbles [e.g., Pioli et al., 2012]. In the following, we describe the local rules applying on each bubble at each time iteration.

2.3.1. Introduction of bubbles at the bottom of the column (gas-separation level)

At each time step i , a new spherical bubble is placed at the bottom (gas-separation level) of a vertical cylindrical column, which contains a volume of liquid given by $\pi R^2 H_0$. Each new bubble is identified with a group of six variables: its mass m_1 , the three spatial coordinates of its center of mass, which coincide with the center of the sphere $(x_{1,i}, y_{1,i}, z_{1,i})$, its density $\rho_{1,i}$, and its radius $r_{1,i}$. The mass is constant and given by $m_1 = Q/N$, where Q and N are the gas mass flux and the number of bubbles introduced in the domain per second, respectively. Both Q and N are input parameters of the model. The vertical coordinate of the new bubble is imposed to be always $z_{1,i} = r_{1,i}$, whereas the horizontal coordinates $x_{1,i}$ and $y_{1,i}$ take random values but satisfying that $x_{1,i}^2 + y_{1,i}^2 \leq (R - r_{1,i})^2$. The bubble density is determined by assuming that the pressure in the column is mostly hydrostatic (i.e., we neglect friction and acceleration contributions to the pressure gradient [Pioli et al., 2012]), and that the mass of gas is always negligible compared to the mass of liquid. In such a case, the gas pressure inside the bubble is $P_{1,i} = P_{ch,i} + \rho_{liq} g H_0 + 2\sigma/r_{1,i}$, where $P_{ch,i}$ is the pressure of the gas chamber as above, ρ_{liq} is the liquid density, g is gravity, σ is the surface tension, and we have considered that $H_0 \gg r_{1,i}$. For typical gas-magma surface tensions [Proussevitch and Kutolin, 1986], and considering that bubble radii at the gas-separation level are larger than 0.1 mm, the term $2\sigma/r_{1,i}$ of the above equation can be neglected because it is tiny compared to $P_{ch,i}$, whose value is at least on the order of the atmospheric pressure [Vergniolle and Jaupart, 1986; Sparks, 1978]. Thus, after considering an ideal gas approach, the density of the new

bubble is simply calculated from $\rho_{1,i} = M(P_{ch,i} + \rho_{liq}gH_0)/R_gT$. Finally, the radius of the new spherical bubbles is calculated from $r_{1,i} = (3m_1/4\pi\rho_{1,i})^{1/3}$. The pressure oscillations during quiescent degassing are expected to be much lower than the mean pressure of the gas chamber $\hat{P}_{ch}$. Thus, we can replace above $P_{ch,i} \approx \hat{P}_{ch}$.

2.3.2. Expansion during ascent

Gas expands during its ascent towards the surface, which in turn affects the velocity, coalescence, and breakup of bubbles. This is significant for volcanoes because conduit lengths of several kilometers result in large hydrostatic pressure changes. As above, we assume that the pressure in the column is mainly hydrostatic, and we neglect the mass of gas versus the mass of liquid and the contribution of the surface tension. Hence, the pressure inside the second lowest bubble in the column can be written as $P_{2,i} = P_{ch,i} + \rho_{liq}g(H_o - z_{2,i}) + (4g\rho_{liq}r_{1,i}^3/3R^2)$, where the third term arises from the presence of the first bubble below, whose radius is $r_{1,i}$. In the same manner, the gas pressure inside the third lowest bubble in the column can be determined from $P_{3,i} = P_{ch,i} + \rho_{liq}g(H_o - z_{3,i}) + (4g\rho_{liq}r_{1,i}^3/3R^2) + (4g\rho_{liq}r_{2,i}^3/3R^2)$. Applying this reasoning, the pressure inside all the bubbles of the column are calculated from:

$$P_{n,i} = P_{ch,i} + \rho_{liq}g(H_o - z_{n,i}) + \frac{4g\rho_{liq}}{3R^2} \sum_{k=1}^{n-1} r_{k,i}^3 \quad (4)$$

where n is the umpteenth bubble counted from the bottom of the column and k is a summation index. After calculating the pressure inside each bubble, the bubble density is calculated from the ideal gas law as $\rho_{n,i} = MP_{n,i}/R_gT$, and then the radius of the bubbles are recalculated by using $r_{n,i} = (3m_{n,i}/4\pi\rho_{n,i})^{1/3}$. The mass of the umpteenth bubble $m_{n,i}$ remains invariant during expansion, and, in the case of Taylor bubbles, $r_{n,i}$ represents the radius of a spherical bubble whose volume is equivalent to the volume of the slug. By following this strategy, we recalculate densities and radii for all the bubbles in the column. On the other hand, the expansion of a certain bubble induces an upwards displacement of the bubbles above it [e.g., Mayor et al., 2008]. To account for this, we recalculate the position of each bubble n by adding the term $\sum_{k=1}^{n-1} \Delta V_k / \pi R^2$ to its vertical coordinate $z_{n,i}$, where ΔV_k is the volume change of each bubble k located below the bubble n .

2.3.3. Displacement of spherical bubbles

Each bubble of the column is imposed to move upwards a given distance determined by its buoyancy, and a small arbitrary distance d in a random direction to account for the chaotic and uncontrolled turbulent motion. Thus, the spatial coordinates of the center of mass of a given bubble (x_n, y_n, z_n) change according to:

$$x_{n,i} = x_{n,i-1} + d \sin\Phi_{n,i} \cos\theta_{n,i} \quad (5)$$

$$y_{n,i} = y_{n,i-1} + d \sin\Phi_{n,i} \sin\theta_{n,i} \quad (6)$$

$$z_{n,i} = z_{n,i-1} + d \cos\Phi_{n,i} + v_{n,i}\Delta t \quad (7)$$

where the subscripts i and $i - 1$ refer to the values of the variables in the current and previous time step, respectively. The variables $\Phi_{n,i}$ and $\theta_{n,i}$ are two random numbers ranging between 0 and 2π that represent a polar and azimuthal angle, respectively, measured in a reference system fixed on the center of the bubble n , the term Δt is the time step between the two time iterations given by $\Delta t = 1/N$, and the variable $v_{n,i}$ is the upwards velocity of the bubble. By following drift-flux models, we express $v_{n,i}$ in terms of the terminal ascent velocity of the bubble moving in a stagnant liquid $u_{n,i}$, plus a correction term which is proportional to the superficial gas velocity q [e.g., Zuber and Findlay, 1965; Wallis, 1969; Ruzicka et al., 2001; Pioli et al., 2012]:

$$v_{n,i} = u_{n,i} + cq \quad (8)$$

where c is an empirical constant that varies between $\sim 1.2 - 1.4$ for turbulent fluids and is ~ 2.3 for laminar flows [e.g., Pioli et al., 2012]. The superficial gas velocity is determined from the gas mass flux and the radius of the cylindrical conduit as $q = Q/\rho_1\pi R^2$. The terminal ascent velocity of a bubble $u_{n,i}$ has been extensively studied both experimentally and theoretically. Here we use the theoretical formulation proposed by Baz-Rodríguez et al. [2012] because it fits very well with experimental data in a wide range of Reynolds

numbers, and because it is expressed in terms of the radius of a spherical bubble with volume equivalent to the actual distorted bubble [Levich, 1962; Baz-Rodríguez et al., 2008]. In such a case, $u_{n,i}$ is given by:

$$u_{n,i} = \left[\left(\frac{1}{u_{n,i,a}} \right)^2 + \left(\frac{1}{u_{n,i,b}} \right)^2 \right]^{-1/2} \quad (9)$$

where the terms $u_{n,i,a}$ and $u_{n,i,b}$ are:

$$u_{n,i,a} = \frac{\Delta\rho_{n,i} g r_{n,i}^2}{9\mu_{liq}} \left[1 + 9.376 \frac{\mu_{liq}}{g^{1/2} r_{n,i}^{3/2} \Delta\rho_{n,i}} \right]^{1/2} \quad (10)$$

$$u_{n,i,b} = \left[\frac{3\sigma}{2\rho_{liq} r_{n,i}} + \frac{g r_{n,i} \Delta\rho_{n,i}}{\rho_{liq}} \right]^{1/2} \quad (11)$$

The variable $\Delta\rho_{n,i}$ is the density difference between the bubble n and the liquid, which depends on the pressure and thus on the depth of the bubble itself, and μ_{liq} is the liquid viscosity.

2.3.4. Displacement of gas slugs

The local displacement rules we have proposed so far (equations (5) – (11)) apply to bubbles whose radii are smaller than a certain maximum value which is close to the

conduit radius R . If a bubble radius overcomes this maximum because of coalescence and expansion by depressurization, a gas slug or Taylor bubble forms in the conduit, thus changing the two-phase flow regime [e.g., Vergnolle and Jaupart, 1986; Pioli et al., 2012]. We consider that a Taylor bubble forms when $r_i \geq R - \lambda$, where λ is the equilibrium thickness of the film of liquid that falls around a slug. According to the experiments of Llewellyn et al. [2012], λ can be calculated from:

$$\lambda = R [0.204 + 0.123 \tanh(2.66 - 1.15 \log_{10} N_f)] \quad (12)$$

where N_f is the dimensionless inverse viscosity, which is defined as $N_f = \rho_{liq} \sqrt{8gR^3} / \mu_{liq}$. This expression is valid as long as the dynamics of the Taylor bubble is controlled by inertial and viscous forces and not by interfacial tension forces, which is fulfilled when the dimensionless Eotvos number (EO) is larger than 40 [Viana et al., 2003; Llewellyn et al., 2012]. The spatial coordinates of the center of mass of a Taylor bubble change at each time step according to:

$$x_{n,i} = 0 \quad (13)$$

$$y_{n,i} = 0 \quad (14)$$

$$z_{n,i} = z_{n,i-1} + v_{n,i} \Delta t \quad (15)$$

where $z_{n,i-1}$ is the vertical component of the center of mass of the slug in the previous time step. In the transition from spherical to Taylor bubble, the center of mass of the gas

slug coincides with the center of mass of the previous sphere. In turn, we consider for simplicity that Taylor bubbles have cylindrical shape [e.g., White and Beardmore, 1962] with radius $R - \lambda$ and height $4r_{n,i}^3/[3(R - \lambda)^2]$, where $r_{n,i}$ represents the radius of a spherical bubble whose volume is equivalent to the volume of the Taylor bubble. The velocity of the center of mass $v_{n,i}$ of a Taylor bubble is calculated by combining empirical expressions that account for wake effects caused by other Taylor bubbles [Pinto and Campos, 1996], with a correction term proportional to the superficial gas velocity [e.g., van Hout et al., 2002]:

$$v_{n,i} = \left[4.24 - 11.4 \frac{l}{l_{max}} \right] u_{n,i} + cq \quad \text{if } \frac{l}{l_{max}} \leq 0.24 \quad (16)$$

$$v_{n,i} = \left[2.01 - 1.96 \frac{l}{l_{max}} + 0.95 \left(\frac{l}{l_{max}} \right)^2 \right] u_{n,i} + cq \quad \text{if } 0.24 < \frac{l}{l_{max}} < 1 \quad (17)$$

$$v_{n,i} = u_{n,i} + cq \quad \text{if } \frac{l}{l_{max}} \geq 1 \quad (18)$$

where l is the distance that separates a given gas slug from another gas slug placed above, l_{max} is the maximum separation between two slugs below which wake effects are not negligible, and $u_{n,i}$ represents the ascent velocity of an isolated Taylor bubble. The value of l_{max} can be also estimated from the dimensionless inverse viscosity as follows [Pinto and Campos, 1996]:

$$l_{max} = (2.92 + 0.0095 N_f) R \quad \text{if } 100 < N_f < 500 \quad (19)$$

$$l_{max} = (1.384 + 0.0158 N_f)R \quad \text{if } 500 \leq N_f < 1500 \quad (20)$$

$$l_{max} = 25R \quad \text{if } N_f \geq 1500 \quad (21)$$

On the other hand, the ascent velocity of isolated Taylor bubbles $u_{n,i}$ can be calculated from [e.g., Dumitrescu, 1943; Davies and Taylor, 1950]:

$$u_{n,i} = Fr\sqrt{2gR} \quad (22)$$

The parameter Fr is the Froude dimensionless number, which for $Eo > 40$ can be also expressed in terms of the dimensionless inverse viscosity as [Llewellyn et al., 2012]:

$$Fr = 0.34 \left[1 + \left(\frac{31.08}{N_f} \right)^{1.45} \right]^{-0.71} \quad (23)$$

2.3.5. Breakup

Bubble breakup occurs because of the collision of random turbulent eddies on the bubble surface [e.g., Valentas et al., 1966]. In particular, breakup is considered to occur if the Weber dimensionless number associated to a bubble overcomes a critical value which depends on the bubble deformation and flow pattern [Hinze, 1955]. Given the complex and ultimately random nature of bubble breakup, here we follow the same approach as

Herrero et al. [1996] and assume that a bubble n splits in two daughter bubbles when the following condition is met:

$$\beta \frac{r_n^2}{r_n^2 + r_{sta}^2} > P_{breakup} \quad (24)$$

where β is the breakup coefficient ($0 \leq \beta \leq 1$), r_n is the radius of the original bubble, r_{sta} is a tuning coefficient representing a maximum radius for which a bubble is stable, and $P_{breakup}$ is a random parameter ($0 < P_{breakup} < 1$). Equation (24) accounts for the empirical observation that the larger the radius of a bubble, the larger the probability of the bubble to split [e.g., Hinze, 1955]. The sum of the masses of the two daughter bubbles equals the mass of the original spherical bubble, and if we consider that the density of the daughter and original bubbles is the same because they are practically at the same depth in the column, it is met that:

$$r_{n_a}^3 + r_{n_b}^3 = r_n^3 \quad (25)$$

where r_{n_a} and r_{n_b} represent the radii of the two daughter bubbles n_a and n_b , respectively. In addition, it has been shown experimentally that the size of the daughter bubbles can be similar or very different [Hesketh et al., 1991]. Thus, we calculate the radius of the bubbles n_a and n_b from:

$$r_{n_a} = r_n f_{n_a}^{1/3} \quad (26)$$

$$r_{n_b} = r_n (1 - f_{n_a})^{1/3} \quad (27)$$

where f_{n_a} is the breakage volume fraction, which is also a random number ($0 < f_{n_a} < 1$). Equation (27) is obtained from equations (25) and (26). We assume that the centers of the daughter bubbles are located at random but symmetrical positions with respect to the center of the original bubble. The distances between the centers of the daughter bubbles and the original one are $r_{n_a} + d$ and $r_{n_b} + d$, respectively, and thus the center of the daughter bubbles are separated by a distance of $r_{n_a} + r_{n_b} + 2d$.

2.3.6. Coalescence

We assume that two bubbles coalesce when they come into contact [Herrero et al., 1996]. In the case of spherical bubbles, coalescence occurs when the distance between their centers is smaller than the sum of their radius. In the case of Taylor bubbles, coalescence occurs when the distance between their centers is smaller than the sum of the half heights of each bubble. In the case of one Taylor bubble and one spherical bubble, coalescence occurs when the distance between their centers is smaller than the sum of the half height of the Taylor bubble and the radius of the spherical bubble. When the above conditions are satisfied, the two original bubbles, with masses m_{n_a} and m_{n_b} , are replaced by a new one whose mass is $m_n = m_{n_a} + m_{n_b}$. The density of the new and the two original bubbles are approximately the same, and hence the radius of the new bubble r_n (or the

equivalent radius for the case of a gas slug) is simply calculated from $r_n = (r_{n_a}^3 + r_{n_b}^3)^{1/3}$. This is analogous to equation (25), but now r_{n_a} and r_{n_b} represent the radius of the two original bubbles. The center of the new “coalesced” bubble is imposed to coincide with the center of mass of the two original bubbles.

2.3.7. Confinement in the domain and bubble bursting

Because of bubble displacement, gas expansion, breakup, and coalescence, spherical bubbles could partially or completely exit from the space delimited by the walls of the conduit. When this occurs, bubbles are reintroduced in the domain by moving them in radial direction (relative to the cylinder) and attaching them to the walls of the conduit [Herrero et al., 1996]. On the other hand, bubble bursting is imposed when a bubble comes into contact with the liquid surface (i.e., magma head). With this approach we are neglecting the possibility that bubbles get trapped temporally in viscoelastic magma layers formed beneath the magma level [e.g., Patrick et al., 2010; Orr and Rea, 2012].

2.4. Solution algorithm of the model

The iterative procedure of our model is as follows:

- (a) Values for the gas mass flux, pressure in the gas chamber, and gas holdup are given as initial conditions (e.g., $Q_{out}(t = 0) = 0$, $P_{ch}(t = 0) = P_{atm}$, $\varepsilon(t = 0) = 0$). We also give a value for H_0 and D_0 , and choose the gas mass flux at the input of the column Q and the number of bubbles per second N .
- (b) A new bubble is added at the bottom of the column (gas-separation level) as explained in section 2.3.1.
- (c) The density and radius of the bubbles are recalculated by following the expansion strategy explained in section 2.3.2.
- (d) Bubbles are imposed to move according to equations (5) – (11) if they are spherical, or according to (13) – (23) if they are cylindrical (gas slugs).
- (e) A random parameter P_b is associated to each bubble, which splits in two daughter bubbles if condition (24) is satisfied.
- (f) Coalescence between bubbles can occur according to the rules explained in section 2.3.6.
- (g) Bubbles outside the domain are reintroduced into the conduit as explained in section 2.3.7. If two or more bubbles result in contact again, we go back to step (f).
- (h) All the bubbles overtaking the liquid level are moved into the gas chamber and thus removed from the magma column. The mass of the bubbles $\Delta m_{in,i}$ that move into the gas chamber each time step is determined.
- (i) The new liquid level is determined according to $H_{i+1} = H_0 + (4/3R^2) \sum_n r_n^3$, where r_n refers to the radii of the bubbles that are below the level H_i . The gas volume fraction is also calculated from $\varepsilon_{i+1} = [H_{i+1} - H_0]/H_{i+1}$.

(j) The new pressure in the gas-chamber $P_{ch,i+1}$ is calculated from the discretized version of equation (3). By considering that $Q_{in,i} = \Delta m_{in,i} / \Delta t$, and following a forward difference scheme, $P_{ch,i+1}$ is given by:

$$P_{ch,i+1} = P_{ch,i} \left[1 + \frac{H_0 (\varepsilon_{i+1} - \varepsilon_i)}{[D_{ch,0} (1 - \varepsilon_i) - H_0 \varepsilon_i] [1 - \varepsilon_i]} \right] + \Delta t \left[\frac{R_g T}{\pi R^2 M} \frac{(\Delta m_{in,i} / \Delta t - Q_{out,i}) (1 - \varepsilon_i)}{D_{ch,0} (1 - \varepsilon_i) - H_0 \varepsilon_i} \right] \quad (28)$$

(k) The outflux $Q_{out,i+1}$ is determined from the discretized version of equation (1), which is:

$$Q_{out,i+1} = Q_{out,i} + \frac{A \rho_{atm} \Delta t}{\gamma} \left[\frac{(P_{ch,i+1}^2 - P_{atm}^2)}{2 P_{atm} L} - \frac{\mu_g Q_{out,i}}{\kappa_1 A \rho_{atm}} - \frac{Q_{out,i}^2}{\kappa_2 A^2 \rho_{atm}} \right] \quad (29)$$

(l) We go back to (b) for the next time step.

3. Validation and comparison with a natural degassing time series

Below we evaluate the extent to which our model can realistically simulate high-frequency volcanic degassing time series. First, we show that the gas holdup $\varepsilon(t)$ properly reflects the different flow regimes observed at open bubble columns ($P_{ch}(t) = P_{atm}$)

containing high-viscosity liquids. We also show that the numerical time-averaged gas holdup ($\bar{\varepsilon}$) and the superficial gas velocity q follow the functional correlation that has been found experimentally [e.g., Kantarcy et al., 2005]. Second, we show that our model reproduces the main spectral features observed in the high-frequency degassing time series of persistently degassing volcanoes during quiescence, i.e., episodes of periodic degassing with finite duration, and power-law spectra in a certain range of frequencies [e.g., Boichu et al., 2010; Tamburello et al. 2013; Chapter 2].

3.1. Flow patterns in open bubble columns ($P_{ch}(t) = P_{atm}$)

In our simulations, bubbly flow dominates (Fig. 2a) for superficial gas velocities lower than $q \approx 0.2$ m/s. This regime is characterized by low values of the gas holdup ($\varepsilon(t) \sim 5\%$; Fig. 2b) with low standard deviations ($std(\varepsilon) < 2\%$; Fig. 2c). When the superficial gas velocity is in the range $q \approx 0.2 - 0.9$ m/s, the system evolves within the so-called transition regime, in which disperse bubbles alternate with Taylor bubbles forming from time to time (Fig. 2a). The gas holdup is larger in average, it increases if a Taylor bubble forms in the column, and decreases suddenly when the slug bursts and escapes from the liquid ($\varepsilon(t) \approx 20 - 40\%$; Fig. 2b). The rises and falls of the column caused by the formation and escape of slugs increases the dispersion of the gas holdup values ($std(\varepsilon) = 2 - 6\%$; Fig. 2c). At the end of the transition regime, the dispersion of the gas holdup decreases again (Fig. 2c) until Taylor bubbles prevail in the column. For superficial gas

velocities in the range $q \approx 0.9 - 1.8$ m/s, pure slug flow regime dominates (Fig. 2a). The gas holdup is larger ($\varepsilon(t) \approx 45 - 55\%$; Fig. 2b), and the dispersion gradually decreases ($std(\varepsilon) = 3\% - 4\%$; Fig. 2c). For superficial gas velocities larger than $q \approx 1.8$ m/s, the gas holdup shows very fast and large oscillations ($\varepsilon(t) \approx 0 - 80\%$; Fig. 2b), and the dispersion increases dramatically ($std(\varepsilon) \sim 22\%$; Fig. 2c). This behavior was also found by Pioli et al. [2012] in their experiments with air bubbles and glucose syrup, and marks the onset of the churn flow regime (Fig. 2a). The different flow regimes we have simulated are consistent with the experimental results obtained in bubble columns containing high-viscosity liquids [e.g., Schumpe and Deckwer, 1982; Shah et al., 1982; Kantarcy et al., 2005; Pioli et al., 2012]. Using other tuning parameters (i.e., other values for d , β , and r_{sta}) changes the values of q that separate the different flow regimes, but do not alter the main features we have described above.

The different flow patterns are also evident on a log-log plot of the time-averaged gas holdup $\hat{\varepsilon}$ and superficial gas velocity q (Fig. 2d). Universal correlations that predict the value of $\hat{\varepsilon}$ do not exist because it depends on many variables, such as the physicochemical properties of the gas and liquid, column dimensions, pressure and temperature, concentration of impurities in the liquid phase, and even the type of sparger that generates the bubbles at the bottom of the column [e.g., Kantarcy et al., 2005]. However, numerous studies have shown that, for a given experimental design, and in the bubbly and slug regime, $\hat{\varepsilon}$ can be expressed in terms of the superficial gas velocity q according to the law $\hat{\varepsilon} \propto q^\eta$. The exponent η is an empirical constant that has been found to range between 0.1 and 1.2 in the bubbly flow regime [e.g., Kawase et al., 1992]. It has also been found that η

is lower in the slug than in the bubbly regime, even though the exact values are not well documented [Schumpe and Deckwer, 1982; Shah et al., 1982]. There is very good agreement between our numerical simulations and the empirical law, with exponents $\eta \approx 0.60$ and $\eta \approx 0.32$ for the bubbly and slug regime, respectively (Fig. 2d). It is worth noting that Hikita et al. [1980] found that many of the experimental results from the literature performed in bubbly regime can be fitted with exponent $\eta \approx 0.58$. From the onset of the churn regime, the time-averaged gas holdup tends to a constant value ($\sim 54\%$), a fact that has been also observed experimentally [Pioli et al., 2012]. Hence, we feel confident that our model describes properly the variation of the gas holdup $\varepsilon(t)$ in bubble columns for increasing input gas flux.

Fig. 2. Simulation of flow patterns in bubble columns. **(a)** Snapshot of our simulations in the bubbly, transition, slug, and churn regime. **(b)** Gas holdup $\varepsilon(t)$ versus time for different input gas fluxes: $Q = 0.001$ kg/s (red, bubbly regime), $Q = 0.013$ kg/s (black, transition regime), $Q = 0.07$ kg/s (green, slug regime), and $Q = 0.18$ kg/s (blue, churn regime). **(c)** Standard deviation of $\varepsilon(t)$ versus superficial gas velocity (logarithm). **(d)** Time-averaged gas holdup $\hat{\varepsilon}$ (logarithm) versus superficial gas velocity q (logarithm). Black dots are the time-averaged values of $\varepsilon(t)$ when steady-state is reached, and error bars are the standard deviations. Red and green lines are the best fits (both with R-squared coefficient of 0.99) in the bubbly and slug regime, respectively. We have considered: $R = 0.1$ m, $H_0 = 1$ m, $\rho_{liq} = 1230$ kg/m³, $\sigma = 0.064$ N/m, $\mu_{liq} = 0.15$ Pa s (this density, surface tension, and viscosity are typical for glycerol 85%), $M = 0.029$ kg/mol (molecular weight of air), $N = 100$, $T = 20$ °C, $P_{ch}(t) = P_{atm} = 101,325$ Pa (open column), $d = 0.001$, $c = 1.2$, $\beta = 0.05$, $r_{sta} = 0.1$ m.

3.2. Comparison with natural observations

We now compare the spectral features of a natural high-frequency degassing time series with those that we have simulated numerically (Fig. 3). For the sake of computational cost, we do not use actual values of the parameters in the simulations, but we use a system which is scaled to volcanic conduits. Scaling is achieved by preserving geometric and dynamic similarity to volcanic conduits above the gas separation level [e.g., Pioli et al., 2012]. Geometric similarity is ensured by preserving the ratios $\hat{H}/R$, $\hat{D}/\hat{H}$, $\hat{D}/L$, and $\hat{r}/R$, where $\hat{H}$ and $\hat{D}$ are the mean values of $H(t)$ and $D(t)$ during persistent degassing, and $\hat{r}$ is the mean bubble radius in the column. Dynamic similarity of the magma column is ensured by preserving the following independent dimensionless numbers [Seyfried and Freundt, 2000; Llewellyn et al., 2012]: Morton, which is the ratio of viscous to surface tension forces ($Mo = g\mu_{liq}^4/\rho_{liq}\sigma^3$); Eötvös, which is the ratio of buoyancy to surface tension forces ($Eo = 4\rho_{liq}gR^2/\sigma$); and Froude, which is the ratio of inertial to gravitational forces (Fr , as given by equation (23)). Note that Eo and Fr are calculated here for Taylor bubbles and hence they represent maximum values, but the dynamic similarity is also ensured for smaller bubbles because we preserve the ratio $\hat{r}/R$. Dynamic similarity of the porous dome is achieved by scaling the terms of equation (1). By comparing the Forchheimer and Darcy's terms, i.e., taking the ratio of inertial to viscous forces, we obtain the so-called Forchheimer dimensionless number [Ruth and Ma, 1993; Rust and Cashman, 2004; Zeng and Grigg, 2006], which is given by $Fo = \kappa_1\hat{Q}_{out}/\kappa_2\mu_g A$, where $\hat{Q}_{out}$ is the time-averaged gas mass outflux. From now on, we consider as a first order approximation that $\kappa_2 = 4.72 \cdot 10^{14} \kappa_1^{1.87}$, an empirical correlation found by Rust

and Cashman [2004] from the analysis of several crystal-poor samples from Medicine Lake volcano (USA). Thus, the Forchheimer number can be written as $Fo = \hat{Q}_{out} / (4.72 \cdot 10^{14} \kappa_1^{0.87} \mu_g A)$. On the other hand, by scaling the Darcy and pressure gradient terms of equation (1), we define another dimensionless number given by $\Gamma = 2 \mu_g \hat{Q}_{out} L / (\kappa_1 A \rho_{atm} P_{atm})$. The coefficient γ appearing in the Polubarinova-Kochina's term in equation (1) is not known for porous volcanic rocks. Thus, it is considered here as another tuning coefficient of the model, and its influence is discussed in the next section. The values of the geometric and dynamic dimensionless numbers estimated for persistently degassing volcanoes during quiescence, and the range of values used in our simulations in this and the next section, are summarized in Tables 1 and 2.

The natural time series we show in Figure 3a was obtained during a quiescent degassing period of Mayon volcano (Philippines) with the brightness method [Chapter 2], which allows capturing the cycles of the water vapor emissions. Raw data are filtered with a median filter of order 120 to capture only periodicities lower than 2 - 3 minutes [e.g., Moore et al., 1993], which is within the periodicities that have been suggested to be induced by the bubble dynamics of the upper conduit [e.g., Tamburello et al., 2013]. The numerical degassing time series, which is obtained assuming bubbly regime, shows as complex behavior as the natural data, with continuous ups and downs of different amplitudes associated to the burst of bubbles of different sizes at the top of the magma column (Fig. 3b). Both time series contain periodic components that switch spontaneously to other periodicities every some time (e.g., around $t \sim 350$ s in Fig. 3c and around $t \sim 35$ s in Fig. 3d). They also have in common a fractal frequency spectrum [Chapter 2],

i.e., amplitude and frequency of the oscillations are related by means of a power-law in a certain range of frequencies (with exponent $\xi^* \sim -1$ in this case; Fig. 3e - 3f). Although the timescales of the oscillations of the two time series differ as consequence of the scaling of the numerical experiments, our simulations show the same dynamics as the high-frequency gas emissions observed at Mayon volcano. Hence, our model captures properly the spectral features of the high-frequency gas emissions of persistently degassing volcanoes during quiescence. Some animations of the simulations showing simultaneously the gas emissions, gas holdup in the magma column, and pressure fluctuations in the gas chamber, can be found as supplementary material on the following webpage: www.tgirona.com.

Table 1. Geometric dimensionless numbers for persistently degassing volcanoes during quiescence and in our numerical simulations.

	$\hat{H}/R$	$\hat{D}/\hat{H}$	$\hat{D}/L$	$\hat{r}/R$
Volcanoes ^a	$10^0 - 10^3$	$0 - 10^0$	$0 - 10$	$10^{-4} - 10^{-1}$
Numerical	$\sim 40 - 50$	$\sim 0.3 - 0.6$	$\sim 0.6 - 5$	$10^{-2} - 10^{-1}$

^aThese values are estimated by considering $\hat{H} = 10^2 - 10^4 \text{ m}$ [e.g., Shinohara, 2008]; $R = 10 - 10^2 \text{ m}$ [Chapter 1]; $\hat{D} = 0 - 10^2 \text{ m}$ [Tanaka et al., 2008; Tanaka et al., 2009]; $L \sim 10 - 10^2 \text{ m}$ [e.g., Taisne and Jaupart, 2008]; and bubble radius ranging from $r \sim 10^{-2} \text{ m}$ (maximum size of bubbles retained in solidified lava flows or ejecta [e.g., Pioli et al, 2012]) to $r \sim R$ (for Taylor bubbles).

Table 2. Dynamic dimensionless numbers for persistently degassing volcanoes during quiescence and in our numerical simulations.

	Mo	EO	Fr	FO	Γ
Volcanoes ^a	$10^6 - 10^{18}$	$10^7 - 10^9$	$0.07 - 0.34$	$10^{-5} - 10^4$	$10^{-2} - 10^7$
Numerical	$2.5 \cdot 10^{15}$	$3.9 \cdot 10^8$	0.33	$10^{-5} - 10^{-2}$	$10^{-2} - 10^0$

^aThese values are estimated by considering $\mu_{liq} = 10^2 - 10^5 \text{ Pa s}$, $\rho_{liq} \sim 2500 \text{ kg/m}^3$, $\sigma \sim 0.4 \text{ N/m}$, $\mu_g \sim 10^{-5} \text{ Pa s}$, $R = 10 - 10^2 \text{ m}$, $Q = 10 - 100 \text{ kg/s}$ [Chapter 1], $\kappa_1 = 10^{-17} - 10^{-9} \text{ m}^2$, which are representative of highly fractured shallow domes [e.g., Jaupart and Allègre, 1991; Edmonds et al., 2003; Bouvet de Maisonneuve et al., 2009], $L \sim 10 - 10^2 \text{ m}$ [e.g., Taisne and Jaupart, 2008], $P_{atm} = 101,325 \text{ Pa}$, $T = 600 \text{ }^\circ\text{C}$, and $A \sim \pi R^2$.

Fig. 3. Comparison between natural and numerical high-frequency degassing time series. **(a)** High-frequency H₂O emissions time series of Mayon volcano obtained with the brightness method [Chapter 2] during a 700 s experiment (from 6:30 am on 2 March 2014). A median filter of order 120 is applied to the raw data to capture periodicities of up to 2 – 3 minutes [e.g., Moore et al., 1993]. The time series is normalized (the mean is removed to the data and the result is divided by the standard deviation). **(b)** Normalized numerical degassing time series obtained with our model. **(c) – (d)** Wavelet transform (Morlet algorithm) of the natural and numerical time series, respectively. **(e) – (f)** Fast-Fourier transform of the natural and numerical time series, respectively. Both of them show a fractal (power-law) distribution in a

certain range of frequencies with exponent $\xi^* \sim -1$. Wavelet and fast-Fourier transforms are calculated with MATLAB. Note that the timescale of the natural and numerical oscillations is different because of scaling. In our simulations, we have considered: $Q = 0.002$ kg/s, $R = 0.18$ m, $A = \pi R^2$, $H_0 = 6$ m, $D_0 = 2$ m, $L = 0.2$ m, $\rho_{liq} = 2500$ kg/m³, $\sigma = 10^{-5}$ N/m, $\mu_{liq} = 5$ Pa s, $\kappa_1 = 10^{-10}$ m², $\gamma = 10^4$ kg/m³, $\mu_g = 10^{-5}$ Pa s, $M = 0.018$ kg/mol (molecular weight of H₂O, the major component of most volcanic plumes), $N = 10$, $T = 600$ °C, $Q_{out}(t = 0) = 0$, $P_{ch}(t = 0) = P_{atm} = 101,325$ Pa, $\varepsilon(t = 0) = 0$, $d = 0.002$, $c = 2$, $\beta = 0.1$, $r_{sta} = 0.09$ m. With these values of the parameters, the dimensionless numbers are within the range shown in Tables 1 and 2.

4. Numerical experiments

In this section we perform a set of numerical experiments to explore the different processes that are thought to induce phreatic eruptions in volcanoes with a magma head high in the conduit [Barberi et al., 1992]. First, we explore how a degassing time series changes when groundwater infiltrates through the volcanic edifice and comes into contact with the magma of the conduit. Second, we explore the influence of the dome permeability, which could change during persistent degassing because of mineral precipitation, fracture closure, and crater wall rock collapse [Matthews et al., 1997]. In these numerical experiments we account for the bubbly and transition regimes, and we obviate the possibility of pure slug and churn flow since they are not expected to occur during quiescence. We also explore how the length of the dome affects our results. Given that our numerical experiments are scaled to volcanic conduits (Tables 1 and 2), the results we obtain in this section are extensible to actual volcanic systems.

4.1. Groundwater infiltration

We analyze two possible scenarios of groundwater infiltration (Fig. 4). In the first one, groundwater penetrates through the volcanic edifice and evaporates somewhere inside the magma column, thus generating extra steam bubbles that mix with the volcanogenic bubbles. In the second scenario, meteoric water penetrates somewhere into the gas chamber, which evaporates when it comes into contact with the magma head. In such a case, meteoric water affects the mass of gas and thickness of the gas chamber, but without generating extra bubbles in the magma column.

Fig. 4. Illustration of the two scenarios of groundwater infiltration that we explore with our model. **(a)** Water-penetration level is inside the two-separated flow region of the conduit, such that an extra amount of water vapor bubbles are formed in the bubble column. **(b)** Water-penetration level is in the gas chamber. Water evaporates when it contacts the magma head and affects the mass of gas and thickness of the gas chamber, but without adding new bubbles into the magma column.

4.1.1. Groundwater penetration in the magma column (Fig. 4a)

For simplicity we assume that gas separation occurs at the water penetration level. Thus, more or less groundwater penetration can be analyzed by changing the values of the input gas flux Q . We note this scenario is also consistent with an increase of the supply of volatiles at shallow depths, even though both possibilities are conceptually different. Input gas flux Q affects the steady-state flow pattern of the bubbly column, thus directly affecting the gas oscillations at the surface, gas holdup over the gas separation level, and pressure fluctuations of the gas chamber. For large enough input fluxes, Taylor bubbles form in the conduit, which induces abrupt pressure increase and holdup decrease during bubble bursting. Gas emissions fluctuate by following pressure and holdup oscillations, even though the time series is shifted and smoothed depending on the coefficient γ appearing in the transient term of equation (1).

An evident consequence of groundwater infiltration at depth (or increase of convection rates and supply of volatiles) is that the amplitudes and frequencies of the gas oscillations, and the overpressure of the gas chamber $\Delta\hat{P}_{ch}$ (defined here as the difference between the time-averaged pressure of the gas chamber and the atmospheric pressure), are larger for larger Q (Fig. 5a). The amplitude and frequency shifts can be quantified with the standard deviation of the degassing time series (λ), and the slope of the best fit line of its frequency spectrum (ξ). For example, for an input gas flux of $Q = 0.3$ g/s, and thus overpressure $\Delta\hat{P}_{ch} = 4.6$ kPa, it is satisfied that $\lambda \approx 5.4 \cdot 10^{-3}$ g/s, and $\xi \approx -0.89$, whereas for a gas flux $Q = 3$ g/s, and thus overpressure $\Delta\hat{P}_{ch} = 39.5$ kPa, it is satisfied that $\lambda \approx 27.8 \cdot$

10^{-3}g/s , and $\xi \approx -1.41$ (Fig. 5a – 5b). Note that, for simplicity, we have just calculated here the best fit line of the whole frequency spectrum, not only from the region where the spectra follow a clear power-law distribution. We find that the parameter $\mathcal{A}$ always increases with the overpressure $\Delta\hat{P}_{ch}$, even though there is a clear change of trend. For low pressures ($< 30 - 80$ kPa; Fig. 5c), $\mathcal{A}$ follows a concave down curve up to an inflexion point, from where the standard deviation of the time series increases much faster for the same pressure increase. In addition, for the same gas input flux Q , it is met that the smaller the dome length L , the smaller the overpressure $\Delta\hat{P}_{ch}$ and the larger the standard deviation $\mathcal{A}$. On the other hand, ξ decreases with Q and $\Delta\hat{P}_{ch}$ until it reaches a minimum ($\sim 20 - 40$ kPa, depending on the length dome; Fig. 5d), from where it increases gradually. We also find that the shorter the dome, the lower the minimum value of ξ , and the lower the value of the overpressure where this minimum is reached.

Fig. 5. Influence of groundwater infiltration into the magma column on degassing time series. **(a)** Degassing time series obtained with our model for two input gas fluxes Q (red and blue). **(b)** Frequency spectra (fast-Fourier transform) of the degassing time series of caption (a). Continuous lines are the best fits of the whole spectra, whose exponent -or slope in the logarithmic plot- is ξ . **(c)** Standard deviation of the steady degassing time series (Λ) as function of the overpressure ($\Delta \hat{P}_{ch}$), and for three dome lengths (L). **(d)** Slope ξ as function of $\Delta \hat{P}_{ch}$ for different L . Continuous lines at captions (c) and (d) are the best fit polynomials (3rd and 5th order, respectively), which show the trend of the results of the model (dots). For the rest of parameters: $Q = 0.05 - 20 \text{ g/s}$ (captions (c) and (d)), $\kappa_1 = 10^{-10} \text{ m}^2$, $R = 0.2 \text{ m}$, $A = \pi R^2$, $H_0 = 9 \text{ m}$, $D_0 = 4 \text{ m}$, $L = 5 \text{ m}$ (captions (a) and (b)), $\rho_{liq} = 2500 \text{ kg/m}^3$, $\sigma = 10^{-5} \text{ N/m}$, $\mu_{liq} = 5 \text{ Pa s}$, $\gamma = 2 \cdot 10^4 \text{ kg/m}^3$, $\mu_g = 10^{-5} \text{ Pa s}$, $M = 0.018 \text{ kg/mol}$ (molecular weight of H_2O), $N = 10$, $T = 600^\circ \text{C}$, $Q_{out}(t = 0) = 0$, $P_{ch}(t = 0) = P_{atm} = 101,325 \text{ Pa}$, $\varepsilon(t = 0) = 0$, $d = 0.001$, $c = 2$, $\beta = 0.1$, $r_{sta} = 0.1 \text{ m}$. Dimensionless numbers are within the range shown in Tables 1 and 2.

4.1.2. Groundwater penetration in the gas chamber (Fig. 4b)

This scenario can be explored by considering that the mass rate of gas entering the gas chamber $Q_{in,i}$ is given by the mass of bubbles moving from the bubble column $\Delta m_{in,i}$ at each time step (see section 2.4, step (j)) plus a mass rate ΔM which accounts for the water vapor that forms when groundwater enters in contact with the magma head and evaporates. For simplicity we consider that the temperature of this water vapor is the same than the gas temperature in the whole system and, as previously, the thickness of the gas chamber is determined from the volume fraction of bubbles inside the magma column. We find that ΔM also affects overpressure of the gas chamber, although its influence on the amplitudes and frequencies of the gas oscillations is not as evident as in the previous scenario (Fig. 6a). For example, for $\Delta M = 3$ g/s, and thus overpressure $\Delta \hat{P}_{ch} = 170$ kPa, it is satisfied that $\lambda \approx 42.1 \cdot 10^{-3}$ g/s, and $\xi \approx -1.40$, whereas for $\Delta M = 20$ g/s, and thus overpressure $\Delta \hat{P}_{ch} = 581$ kPa, it is satisfied that $\lambda \approx 32.0 \cdot 10^{-3}$ g/s, and $\xi \approx -1.34$ (Fig. 6a – 6b). However, there are trends that can be identified as function of the overpressure of the gas chamber. For example, the standard deviation λ tends to increase with $\Delta \hat{P}_{ch}$ until a maximum (at overpressures around 200 kPa; Fig. 6c), and then it decreases gradually. We also find that, for the same mass rate ΔM , the larger the dome length L , the lower λ and the larger $\Delta \hat{P}_{ch}$. On the other hand, ξ shows a linear increasing trend with $\Delta \hat{P}_{ch}$ (Fig. 6d). Even though there is significant dispersion in the data, the slope ξ tends to be slightly lower for shorter domes at the same overpressure.

Fig. 6. Influence of groundwater infiltration into gas chamber on degassing time series. **(a)** Degassing time series obtained with our model for two infiltration mass rates ΔM (red and blue). **(b)** Frequency spectra (fast-Fourier transform) of the degassing time series of caption (a). Continuous lines are the best fits to the whole spectra, whose exponent -or slope in the logarithmic plot- is ξ . **(c)** Standard deviation of the steady degassing time series (Λ) as function of the overpressure ($\Delta \hat{P}_{ch}$), and for three dome lengths (L). **(d)** Slope ξ as function of $\Delta \hat{P}_{ch}$ for different L . Continuous lines at captions (c) and (d) are the best fit polynomials (3rd and 1st order, respectively), which show the trend of the results of the model (dots). We have used: $Q = 0.01 \text{ g/s}$, $\Delta M = 0.05 - 20 \text{ g/s}$ (captions (c) and (d)), $L = 3 \text{ m}$ (captions (a) and (b)). The rest of parameters are the same as Figure 5.

4.2. Dome plugging

A porosity change of the dome can be analyzed by varying the permeability κ_1 . As we noted above, we consider that κ_2 is given by the empirical relationship found by Rust and Cashman [2005]. As for the scenarios of groundwater infiltration, we find that a permeability change of the dome also modifies the overpressure of the gas chamber $\Delta\hat{P}_{ch}$, and the amplitudes and frequencies of the gas oscillations (Fig. 7a). For example, for a Darcy permeability of $\kappa_1 = 2 \cdot 10^{-10} \text{ m}^2$, and thus overpressure $\Delta\hat{P}_{ch} = 4.7 \text{ kPa}$, it is satisfied that the standard deviation of the steady-state time series is $\lambda \approx 33.8 \cdot 10^{-3} \text{ g/s}$, and the slope of the power-law distribution is $\xi \approx -1.39$, whereas for a permeability $\kappa_1 = 5 \cdot 10^{-11} \text{ m}^2$, and thus overpressure $\Delta\hat{P}_{ch} = 17.5 \text{ kPa}$, it is satisfied that $\lambda \approx 12.1 \cdot 10^{-3} \text{ g/s}$, and $\xi \approx -1.11$ (Fig. 7a – 7b). The parameter λ decreases systematically with $\Delta\hat{P}_{ch}$ by following a very well-defined power-law $\lambda = a_1 \Delta\hat{P}_{ch}^{a_2}$, where a_1 and a_2 are constants independent of the dome thickness (Fig. 7c). However, for the same permeability κ_1 , the larger the dome length L , the lower λ and the larger $\Delta\hat{P}_{ch}$. The slope ξ shows an increasing trend with $\Delta\hat{P}_{ch}$, even though there is much data dispersion at low overpressures (Fig. 7d)

So far we have considered γ as a tuning parameter because its dependence with κ_1 in volcanic rocks has not been studied. However, previous experiments in granular media have shown that the magnitude of γ usually increases with decreasing porosity and permeability κ_1 [Hall et al., 1995]. Therefore, we could expect that γ increases during dome plugging and pressurizations. On the other hand, the larger the values of γ , the

lower the amplitude and frequency of the oscillations (Fig. 8), and thus we can expect that amplitudes and frequencies could decrease even more with $\Delta\hat{P}_{ch}$ than what we have shown in Figure 7c.

Fig. 7. Influence of dome permeability on degassing time series. **(a)** Degassing time series obtained with our model for two permeabilities κ_1 (red and blue). **(b)** Frequency spectra (fast-Fourier transform) of the degassing time series of caption (a). Continuous lines are the best fits to the whole spectra, whose exponent -or slope in the logarithmic plot- is ξ . **(c)** Standard deviation of the steady degassing time series (Λ) as function of the overpressure ($\Delta\hat{P}_{ch}$), and for three dome lengths (L). **(d)** Slope ξ as function of $\Delta\hat{P}_{ch}$ for different L . Continuous lines at captions (c) and (d) are the best fit power-law and polynomial (2nd order), respectively, which show the trend of the results of the model (dots). The power-laws of caption (c) are roughly the same for the three dome lengths. We have used: $\kappa_1 = 5 \cdot 10^{-11} - 10^{-9} \text{ m}^2$ (captions (c) and (d)), $Q = 1 \text{ g/s}$, $L = 3 \text{ m}$ (captions (a) and (b)). The rest of parameters are the same as Figure 5.

Fig. 8. Influence of parameter γ on degassing time series. We have used $Q = 1$ g/s, $L = 3$ m, and the rest of the parameters are the same as Figure 5.

5. Discussion and conclusion

Previous studies have shown that changes in the volcanic gas emissions can be used to track magma ascent towards the surface [e.g., Symonds et al., 1996; Aiuppa et al., 2007]. In addition, we show that degassing patterns also reflect the processes that can induce phreatic eruptions at volcanoes that have a magma head high in the conduit [Barberi et al. 1992]. We infer this from a numerical model that couples the transfer of gases through a permeable dome, the gas pressure beneath the crater, and the bubble dynamics of the upper magma conduit. In particular, our model considers the magma column in the

conduit as the piston of a syringe, such that the gas pressure of the gas chamber beneath the crater varies with the rises and falls of the magma level. In turn, the position of the magma level depends on the bubble dynamics of the magma column, and indirectly on the gas outflux and permeability of the dome. Our synthetic degassing time series are comparable to natural high-frequency degassing time series, which are interpreted to be the result of the bubble dynamics in the upper conduit (periodicities of up to a few minutes [e.g., Tamburello et al., 2013]). For example, our model reproduces the spontaneous switch of periodic degassing components, and the fractal frequency spectra [Chapter 2].

Despite the complexity of volcanic degassing time series, our model suggests that there are some easily measurable parameters that can be used as a proxy for the pressure changes related to phreatic eruptions. For example, we have found in our numerical experiments that the standard deviation ($\mathcal{A}$) of high-frequency degassing time series, which can be considered as an indicator of the amplitude of the gas oscillations, increases continuously with the mean overpressure beneath the crater ($\Delta\hat{P}_{ch}$) caused by water infiltration into the bubbly magma column. On the contrary, $\mathcal{A}$ tends to decrease with $\Delta\hat{P}_{ch}$ after reaching a maximum value when water infiltrates directly beneath the crater. During dome plugging, $\mathcal{A}$ decreases continuously with $\Delta\hat{P}_{ch}$ by following a power-law which is independent of the dome length. Other characteristic parameters of degassing time series, like the slope of the best-fit line of their frequency spectra (ξ), also show systematic trends with $\Delta\hat{P}_{ch}$. In particular, ξ tends to increase with overpressurization beneath the dome, at least from a certain pressure value.

Our results could have direct implications for the interpretation of volcano monitoring data. Gas accumulations beneath volcanic craters can be monitored with gravimeters placed at the summit area, although the installation of these systems can be dangerous and is limited to easily accessible volcanoes like Stromboli [Carbone et al., 2012]. Hence, for inaccessible volcanic craters like that of Mayon, the analysis of gas emission patterns might be the only way to track the onset of processes leading to phreatic eruptions. For example, if the average flux of SO_2 remains roughly constant with time during quiescence, a gradual decrease of the standard deviation λ of the high-frequency SO_2 time series [e.g., Tamburello et al., 2013] would imply a gradual permeability decrease of the dome, and thus a gradual pressurization beneath the crater. On the other hand, a roughly constant average emission of SO_2 , accompanied by a gradual increase of λ of the high-frequency time series of the steam emissions [Chapter 2], could imply a gradual pressurization by increasing infiltration of groundwater into the magma column. In a case in which water infiltrates suddenly and massively, increases of λ could be impossible to detect before the explosion. The idea of using detectable amplitude changes of time series to forecast volcanic events is also used in volcano seismology [e.g., Carbone et al., 2012].

Our work also shows the feasibility of using agent-based models to reproduce the main properties of the complex volcanic degassing time series. This has not been possible to do so far, in part due to the computational cost of more traditional numerical methods to simulate bubble dynamics. The agent-based approach deals with critical processes like bubble coalescence and breakup in a probabilistic manner, and the complex behavior of

the system as a whole emerges spontaneously after the combination of the different empirically tested rules applying on each bubble. More complexities, like the presence of crystals and the non-Newtonian rheology of magmas [Divoux et al., 2011], the accumulation of bubbles beneath viscoelastic layers at the top of the magma column [e.g., Patrick et al., 2010; Orr and Rea, 2012], and the effects of the walls on the bubble motion [e.g., Pioli et al., 2012], must be taken into account in the future to evaluate their influence on the overall bubble dynamics and in the degassing time series. Our study also opens the door to investigate numerically other degassing related processes that have remained elusive so far, like the accumulation and release of bubbles forming foams beneath geometrical barriers [e.g., Jaupart and Vergnolle, 1988; Boichu et al., 2010], which can induce Strombolian eruptions. Agent-based numerical approaches can help to understand the degassing signals that are registered from monitoring systems at the surface, and thus could help improving the forecasting of volcanic processes.

Notation

Symbol	Description
t	Time
P_{ch}	Pressure in the gas chamber
P_{atm}	Atmospheric pressure
Q_{out}	Gas outflux
L	Thickness of the porous dome
κ_1	Viscous permeability of the dome
κ_2	Inertial permeability of the dome
γ	Coefficient that accounts for transient inertia forces in equation (1)
A	Cross section of the dome
ρ_{atm}	Volcanic gas density at atmospheric pressure
μ_g	Gas viscosity
R_g	Ideal gas constant
T	Gas temperature
m_{ch}	Mass of gas inside the gas chamber
D_{ch}	Thickness of the gas chamber
$D_{ch,0}$	Thickness of the gas chamber at the time $t = 0$
H	Thickness of the bubbly magma column over the gas separation level
H_0	Thickness of the bubbly magma column over the gas separation level at $t = 0$
R	Radius of the conduit
M	Molecular weight of gas
Q_{in}	Mass rate of gas entering in the gas chamber
ε	Gas holdup or mean gas volume fraction of the magma column
i	Time step
m_n	Mass of a given bubble n
x_n, y_n, z_n	Spatial coordinates of the center of a given bubble n
ρ_n	Density of a given bubble n
r_n	Radius of a given bubble n
Q	Gas mass flux imposed at the bottom of the column (input parameter)
N	Number of bubbles per second (input parameter)
P_n	Pressure inside a given bubble n

ρ_{liq}	Liquid density
g	Gravity
σ	Surface tension
k	Summation index to account for the bubbles located below than a bubble n
ΔV_k	Volume change of a given bubble k because of expansion during ascent
d	Arbitrary small distance that bubbles displace (tuning parameter)
Φ_n, θ_n	Random angles to determine the direction in which a bubble n displaces
Δt	Time step
v_n	Upwards velocity of a bubble n
u_n	terminal ascent velocity of the bubble (spherical or Taylor)
c	Empirical constant
q	Superficial gas velocity
$\Delta \rho_n$	Density difference between the bubble n and the liquid
μ_{liq}	Liquid viscosity
λ	Equilibrium thickness of the film of liquid falling around a slug
N_f	Dimensionless inverse viscosity
l	Distance separating a given gas slug from another gas slug placed above
l_{max}	Maximum separation between two slugs below which wake effects are non-negligible
β	Breakup coefficient (tuning parameter)
r_{sta}	Maximum radius for which a bubble is stable (tuning parameter)
$P_{breakup}$	Random parameter used for breakup
r_{na}, r_{nb}	Radius of two daughter bubbles after breakup, or two original bubbles before coalescence
f_{na}	Breakage volume fraction
Δm_{in}	Mass of bubbles moving from bubble column into the gas chamber each time step
ΔM	Mass rate of groundwater evaporating in the gas chamber
$\hat{\varepsilon}$	Time-averaged gas holdup
$std(\varepsilon)$	Standard deviation of the gas holdup
η	Exponent of the power-law that correlates $\hat{\varepsilon}$ and q
$\bar{H}$	Mean value of H during persistent degassing
$\bar{D}_{ch}$	Mean value of D_{ch} during persistent degassing
$\hat{r}$	Mean bubble radius in the column
Mo	Morton dimensionless number
EO	Eötvös dimensionless number
Fr	Froude dimensionless numbers
Fo	Forchheimer dimensionless number

Γ	Dimensionless number comparing the Darcy and pressure gradient terms of equation (1)
A	Standard deviation of the degassing time series
ν	Frequency of the signal with maximum amplitude in the degassing time series
ξ	Slope of the best fit line of the frequency spectrum of the degassing time series
ξ^*	Slope of the part of the frequency spectrum that follows a power law distribution
$\Delta\hat{P}_{ch}$	Overpressure of the gas chamber
$\hat{P}_{ch}$	Mean pressure of the gas chamber

Conclusions and prospects for future work

There are tens of eruptions every year of persistently degassing volcanoes. Many of them occur in highly populated areas, and their activities are responsible for significant societal and economic problems. It is thus very important to improve the knowledge of the processes that govern their eruptions and to try to make progress about forecasting their activity. With these objectives in mind, I have addressed three interrelated topics that deal with persistently degassing volcanoes using mainly a physics-based approach:

First, I have developed a new theoretical model that allows determining the pressure changes induced by passive degassing during quiescence. With this model I have found that quiescent degassing can depressurize shallow magma reservoirs by up to tens of MPa in the typical inter-eruptive timescales of these volcanoes. This suggests that degassing-induced depressurization could be responsible for triggering different physical processes that eventually lead to unrest episodes and eruptions. This research opens new questions to investigate in the future, like how passive degassing can induce cyclical magma replenishment in the shallow reservoir, or up to which extent the gradual displacement of the gas exsolution level during quiescence can promote the sudden formation of bubbles and overpressure. This model should also enable to make new synthetic time series of different parameters that are expected to vary during depressurization (e.g., deflation of the edifice) and which could be compared to natural monitoring data.

Second, I have made progress in the monitoring of the water vapor emissions, one of the major challenges in volcano monitoring, by developing an innovative method that combines notions from the light scattering theory with the analysis of digital images. With this approach, it is now possible to determine the spectral features of the water vapor emissions by measuring the digital brightness. I propose two paths for progress in this context. On the one hand, the brightness method could be automatized at volcano observatories to monitor the steam emission patterns in real-time. This could help to understand the dynamics of magma degassing, in particular when combined with simultaneous measurements of other gases like SO₂. On the other hand, my research opens the possibility of developing more complex systems able to measure the absolute values of the steam flux. This could be done by combining the brightness measurements with real-time tracking of the position of the Sun along the day, and with simultaneous measurements of the atmospheric parameters at crater levels (relative humidity, pressure, and temperature).

Third, I have produced for the first time a series of synthetic volcanic gas emission cycles by simulating the bubble dynamics of the upper conduit with an agent-based model. These simulations show that the pressure changes preceding the onset of phreatic eruptions might be indeed accompanied by a shift in the high-frequency degassing patterns. This suggests that a real-time monitoring of the gas emitted can help to improve the forecast of phreatic eruptions. From the research point of view, future work should be focused on extending the model to more complex situations (e.g., non-Newtonian rheology for magmas and bubble accumulation beneath geometrical barriers), and determining

experimentally how a transient pressure affects the gas flow through the permeable rocks of the crater. From the monitoring point of view, I suggest to monitor shifts in the high-frequency degassing patterns as a new tool to improve the forecasting of phreatic eruptions.

My work sheds new insights on the behavior of persistently degassing volcanoes, how to improve gas monitoring, and the interpretation of degassing patterns. Advances in these fields are crucial towards improved eruption forecasting of the most active volcanoes around the world.

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Appendices

Auxiliary material of Chapter 1:

On depressurization of volcanic magma reservoirs by passive degassing

Description:

The following are Matlab scripts to plot Figures 2 - 7. The definition and values of the parameters, and other details used in the model, are written in the scripts.

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Script to plot FIG.2 of the manuscript:
%
% Girona, T., F. Costa, C. Newhall, and B. Taisne (2014), "On
depressurization of volcanic magma reservoirs by passive degassing".
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL  A
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%Constants
L=10000; %mean reservoir
depth
Mh2o=0.018; %molecular weight
of water
Rg=8.3145; %universal gas
constant
T=1273; %temperature in
Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust
k=1e10; %elastic modulus
of the crust
S=4e-6; %Henry's constant
Q=[2000 4000 6000];Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.03; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2430; %density of the
parent melt
rho_c1=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.03; %mass fraction of
volatiles exsolved in the upper conduit
Drho=rho_c1-rho_c1; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level

```

```

rho_x=rho_cl; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=1e5; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=7e6; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_cl+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0); B=1; if nr_0<=0; nr_0=0; B=0; end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e8; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_cl*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA*(time*365*24*60*60-t0))); DPa1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA*(time*365*24*60*60-t0))); DPa2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA*(time*365*24*60*60-t0))); DPa3=DP/1e6;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL  B
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%Constants
L=10000; %mean reservoir
depth %molecular weight
Mh2o=0.018;
of water %universal gas
Rg=8.3145; %universal gas
constant %temperature in
T=1273; %temperature in
Kelvin %gravity
g=9.81; %number pi
pi=3.1415; %viscosity of the
mu=1e18; %viscosity of the
crust %elastic modulus
k=1e10; %elastic modulus
of the crust %Henry's constant
S=4e-6; %gas flux
Q=4000; Q=Q*1000/(24*60*60);

%%Densities
alpha=0.03; %mass fraction of
volatiles dissolved in the parent melt %density of the
rho_nd=2430; %density of the
parent melt %density at the
rho_c1=2670; %density of the
top of the conduit %density of the
rho_c2=rho_nd; %density of the
less degassed melt in the conduit %mass fraction of
n_c=0.03; %mass fraction of
volatiles exsolved in the upper conduit %density
Drho=rho_c1-rho_c1; %density
difference %if A=0 ->
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level %density of melt
rho_x=rho_c1; %density of melt
at the separation level %partial density
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=1e5; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength

```

```

DPs=7e6; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0); B=1; if nr_0<=0; nr_0=0; B=0; end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=[1e8 5e8 1e9]; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0))); DPb1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0))); DPb2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA(3)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(3)*(time*365*24*60*60-t0))); DPb3=DP/1e6;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL C
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%Constants
L=10000; %mean reservoir
depth

```

Mh2o=0.018;	%molecular weight
of water	
Rg=8.3145;	%universal gas
constant	
T=1273;	%temperature in
Kelvin	
g=9.81;	%gravity
pi=3.1415;	%number pi
mu=[1e17 1e18];	%viscosity of the
crust	
k=1e10;	%elastic modulus
of the crust	
S=4e-6;	%Henry's constant
Q=4000;Q=Q*1000/(24*60*60);	%gas flux
%%Densities	
alpha=0.03;	%mass fraction of
volatiles dissolved in the parent melt	
rho_nd=2430;	%density of the
parent melt	
rho_c1=2670;	%density at the
top of the conduit	
rho_c2=rho_nd;	%density of the
less degassed melt in the conduit	
n_c=0.03;	%mass fraction of
volatiles exsolved in the upper conduit	
Drho=rho_c1-rho_c1;	%density
difference	
A=0;	%if A=0 ->
steady-state degassing -> no density change of melt in	the conduit
nx=n_c;	%mass fraction of
volatiles exsolved at the gas-melt separation level	
rho_x=rho_c1;	%density of melt
at the separation level	
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd);	%partial density
of water dissolved in the melt	
%%Replenishment	
Rd=0;	%dyke radius
M=5000;	%dyke length
mu_m=1e5;	%magma viscosity
lambda=8*Rd^4/(8*M*mu_m);	%hydraulic
strength	
DPs=7e6;	%overpressure of
the source	
%%Initial conditions	
gamma_c0=0.5;	%mass fraction of
fully degassed melt in the conduit	
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2;	%mean density of
melt in the conduit	
beta_c0=0.1;	%gas volume
fraction in the conduit at t=0;	

```

P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e8; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPS;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0./mu+lambda+Vm_c0./mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0)));DPc1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0)));DPc2=DP/1e6;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL D
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%Constants
L=10000; %mean reservoir
depth
Mh2o=0.018; %molecular weight
of water
Rg=8.3145; %universal gas
constant
T=1273; %temperature in
Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust
k=1e10; %elastic modulus
of the crust
S=4e-6; %Henry's constant

```



```

Q=4000;Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.03; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2430; %density of the
parent melt
rho_c1=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.03; %mass fraction of
volatiles exsolved in the upper conduit
Drho=rho_c1-rho_c1; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=1e5; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=7e6; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=[10 50 80]; %radius of the
conduit
Vm_c0=pi*R_0.^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e8; %initial volume
of the reservoir

%%Parameters

```

```

C1=((1-n_c)*Drho*A/(n_c*rho_cl*rho_mc0)-1/rho_w)*Q+lambda*DPS;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0.^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0.^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0)));DPd1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0)));DPd2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA(3)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(3)*(time*365*24*60*60-t0)));DPd3=DP/1e6;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL E
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%Constants
L=10000; %mean reservoir
depth %depth
Mh2o=0.018; %molecular weight
of water
Rg=8.3145; %universal gas
constant
T=1273; %temperature in
Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust
k=1e10; %elastic modulus
of the crust
S=4e-6; %Henry's constant
Q=[2000 4000 6000];Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.03; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2430; %density of the
parent melt
rho_cl=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.03; %mass fraction of
volatiles exsolved in the upper conduit

```

```

Drho=rho_c1-rho_c1; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=1e5; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=7e6; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0); B=1; if nr_0<=0; nr_0=0; B=0; end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e9; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;

```

```

DP=(-DPinf(1)*(1-exp(GAMMA*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA*(time*365*24*60*60-t0)));DPe1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA*(time*365*24*60*60-t0)));DPe2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA*(time*365*24*60*60-t0)));DPe3=DP/1e6;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL F
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

%%Constants
L=10000; %mean reservoir
depth
Mh2o=0.018; %molecular weight
of water
Rg=8.3145; %universal gas
constant
T=1273; %temperature in
Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust
k=1e10; %elastic modulus
of the crust
S=4e-6; %Henry's constant
Q=4000;Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.03; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2430; %density of the
parent melt
rho_cl=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.03; %mass fraction of
volatiles exsolved in the upper conduit
Drho=rho_cl-rho_c1; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_cl; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment

```

```

Rd=0; %dyke radius
M=5000; %dyke length
mu_m=1e5; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=7e6; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=[1e9 5e9 1e10]; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
*Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
(nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0)));DPf1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0)));DPf2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA(3)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(3)*(time*365*24*60*60-t0)));DPf3=DP/1e6;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL G
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

%%Constants
L=10000; %mean reservoir
depth
Mh2o=0.018; %molecular weight
of water
Rg=8.3145; %universal gas
constant
T=1273; %temperature in
Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=[1e17 1e18 1e20]; %viscosity of the
crust
k=1e10; %elastic modulus
of the crust
S=4e-6; %Henry's constant
Q=4000;Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.03; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2430; %density of the
parent melt
rho_cl=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.03; %mass fraction of
volatiles exsolved in the upper conduit
Drho=rho_cl-rho_c1; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_cl; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=1e5; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=7e6; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit

```

```

rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2;           %mean density of
melt in the conduit
beta_c0=0.1;                                           %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0);                           %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end     %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T);                               %density of
volatiles exsolved in the reservoir at t=0
R_0=40;                                                %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0);                         %initial volume
of melt in the conduit
Vr0=5e9;                                              %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0./mu+lambda+Vm_c0./mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0)));DPg1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0)));DPg2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA(3)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(3)*(time*365*24*60*60-t0)));DPg3=DP/1e6;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL H
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%Constants
L=10000;                                              %mean reservoir
depth
Mh2o=0.018;                                          %molecular weigth
of water
Rg=8.3145;                                           %universal gas
constant
T=1273;                                              %temperatue in
Kelvin
g=9.81;                                              %gravity
pi=3.1415;                                           %number pi

```

```

mu=1e18;                                %viscosity of the
crust
k=1e10;                                  %elastic modulus
of the crust
S=4e-6;                                  %Henry's constant
Q=4000;Q=Q*1000/(24*60*60);              %gas flux

%%Densities
alpha=0.03;                              %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2430;                              %density of the
parent melt
rho_cl=2670;                              %density at the
top of the conduit
rho_c2=rho_nd;                            %density of the
less degassed melt in the conduit
n_c=0.03;                                 %mass fraction of
volatiles exsolved in the upper conduit
Drho=rho_cl-rho_c1;                       %density
difference
A=0;                                       %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c;                                   %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_cl;                             %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0;                                     %dyke radius
M=5000;                                  %dyke length
mu_m=1e5;                                 %magma viscosity
lambda=8*Rd^4/(8*M*mu_m);                %hydraulic
strength
DPs=7e6;                                  %overpressure of
the source

%%Initial conditions
gamma_c0=0.5;                             %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_cl+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1;                              %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0);                %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T);                   %density of
volatiles exsolved in the reservoir at t=0
R_0=[10 50 80];                           %radius of the
conduit

```



```

Vm_c0=pi*R_0.^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e9; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0.^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0.^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0)));DPh1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0)));DPh2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA(3)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(3)*(time*365*24*60*60-t0)));DPh3=DP/1e6;

%% PLOTS
figure(1)
hFig = figure(1);
set(gcf,'PaperPositionMode','auto')
set(hFig, 'Position', [0 0 1000 1000])

subplot(4,2,1)
plot(time,DPa1,'r',time,DPa2,'r',time,DPa3,'r','LineWidth',2);
axis([0, 10, -10, 0]);
set(gca,'FontSize',12,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);
title('V_r(t_0) = 10^8 - 10^9 m^3','fontsize',14)

subplot(4,2,3)
plot(time,DPb1,'r',time,DPb2,'r',time,DPb3,'r','LineWidth',2);
axis([0, 10, -10, 0]);
set(gca,'FontSize',12,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);

subplot(4,2,5)
plot(time,DPC1,'r',time,DPC2,'r','LineWidth',2);

```

```

axis ([0, 10, -10, 0]);
set(gca,'FontSize',12,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);

subplot(4,2,7)
plot(time,DPd1,'r',time,DPd2,'r',time,DPd3,'r','LineWidth',2);
axis ([0, 10, -10, 0]);
set(gca,'FontSize',12,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);

subplot(4,2,2)
plot(time,DPe1,'r',time,DPe2,'r',time,DPe3,'r','LineWidth',2);
axis ([0, 10, -10, 0]);
set(gca,'FontSize',12,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);
title('V_r(t_0) = 10^9 - 10^1^0 m^3','fontsize',14)

subplot(4,2,4)
plot(time,DPf1,'r',time,DPf2,'r',time,DPf3,'r','LineWidth',2);
axis ([0, 10, -10, 0]);
set(gca,'FontSize',12,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);

subplot(4,2,6)
plot(time,DPg1,'r',time,DPg2,'r',time,DPg3,'r','LineWidth',2);
axis ([0, 10, -10, 0]);
set(gca,'FontSize',12,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);

subplot(4,2,8)
plot(time,DPh1,'r',time,DPh2,'r',time,DPh3,'r','LineWidth',2);
axis ([0, 10, -10, 0]);
set(gca,'FontSize',12,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Script to plot FIG.3 of the manuscript:
%
% Girona, T., F. Costa, C. Newhall, and B. Taisne (2014), "On
depressurization of volcanic magma reservoirs by passive degassing".
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```
clear all
```

```

%%Constants
L=10000; %mean reservoir
depth
Mh2o=0.018; %molecular weight
of water
Rg=8.3145; %universal gas
constant
T=1273; %temperature in
Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=[1e18 1e30 1e30]; %viscosity of the
crust
k=[1e10 1e10 1e30]; %elastic modulus
of the crust
S=4e-6; %Henry's constant
Q=4000;Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.03; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2430; %density of the
parent melt
rho_c1=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.03; %mass fraction of
volatiles exsolved in the upper conduit
Drho=rho_c1-rho_c1; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

```

```

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=1e5; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=7e6; %overpressure of
the source
n=5000;

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0); B=1; if nr_0<=0; nr_0=0; B=0; end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=1e10; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0./mu+lambda+Vm_c0./mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0./k+(1-nr_0*rho_nd/rho_gr0)...
*Vr0./k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
(nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0))); DP1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0))); DP2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA(3)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(3)*(time*365*24*60*60-t0))); DP3=DP/1e6;

```

```

DV=-
Vr0*DPinf(1)*((time*365*24*60*60)/mu(1)+(1/k(1)+1/(GAMMA(1)*mu(1)))*(1-
exp(GAMMA(1)*(time*365*24*60*60))));DV1=DV/1e6;
DV=-
Vr0*DPinf(2)*((time*365*24*60*60)/mu(2)+(1/k(2)+1/(GAMMA(2)*mu(2)))*(1-
exp(GAMMA(2)*(time*365*24*60*60))));DV2=DV/1e6;
DV=-
Vr0*DPinf(3)*((time*365*24*60*60)/mu(3)+(1/k(3)+1/(GAMMA(3)*mu(3)))*(1-
exp(GAMMA(3)*(time*365*24*60*60))));DV3=DV/1e6;

DR=-
(R_0/2)*DPinf(1)*((time*365*24*60*60)/mu(1)+(1/k(1)+1/(GAMMA(1)*mu(1)))*
(1-exp(GAMMA(1)*(time*365*24*60*60))));DR1=DR*1e3;
DR=-
(R_0/2)*DPinf(2)*((time*365*24*60*60)/mu(2)+(1/k(2)+1/(GAMMA(2)*mu(2)))*
(1-exp(GAMMA(2)*(time*365*24*60*60))));DR2=DR*1e3;
DR=-
(R_0/2)*DPinf(3)*((time*365*24*60*60)/mu(3)+(1/k(3)+1/(GAMMA(3)*mu(3)))*
(1-exp(GAMMA(3)*(time*365*24*60*60))));DR3=DR*1e3;

%% PLOTS
figure(1)
hFig = figure(1);
set(gcf, 'PaperPositionMode', 'auto')
set(hFig, 'Position', [0 0 1200 300])

subplot(1,3,1)
plot(time,DP1,'g',time,DP2,'b',time,DP3,'r','LineWidth',3);
axis([0, 10, -10, 0]);
set(gca, 'FontSize',14, 'XTick',[0 2 4 6 8 10], 'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);
xlabel('Time (years)','fontsize',14);

subplot(1,3,2)
plot(time,DV1,'g',time,DV2,'b',time,DV3,'r','LineWidth',3);
axis([0, 10, -20, 0]);
set(gca, 'FontSize',14, 'XTick',[0 2 4 6 8 10], 'YTick',[-20 -15 -10 -5 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaV_r (* 10^6 m^3)','fontsize',14);

subplot(1,3,3)
plot(time,DR1,'g',time,DR2,'b',time,DR3,'r','LineWidth',3);
axis([0, 10, -30, 0]);
set(gca, 'FontSize',14, 'XTick',[0 2 4 6 8 10], 'YTick',[-30 -25 -20 -15 -
10 -5 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaR (mm)','fontsize',14);
xlabel('Time (years)','fontsize',14);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Script to plot FIG.4 of the manuscript:
%
% Girona, T., F. Costa, C. Newhall, and B. Taisne (2014), "On
depressurization of volcanic magma reservoirs by passive degassing".
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all

%%Constants
L=10000; %mean reservoir
depth %molecular weight
Mh2o=0.018; %universal gas
of water %temperature in
Rg=8.3145; Kelvin
constant %gravity
T=1273; %number pi
Kelvin %viscosity of the
g=9.81; %elastic modulus
pi=3.1415; %Henry's constant
mu=1e18;
crust
k=1e10;
of the crust
S=4e-6;

%%Densities
alpha=0.03; %mass fraction of
volatiles dissolved in the parent melt %density of the
rho_nd=2430; parent melt
rho_c1=2530; %density at the
top of the conduit %density of the
rho_c2=rho_nd; less degassed melt in the conduit
n_c=0.015; %mass fraction of
volatiles exsolved in the upper conduit %density
Drho=rho_c1-rho_c2; difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level %density of melt
rho_x=rho_c1; at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

```

```

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=10^4.9; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=7e6; %overpressure of
the source
n=5000;

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=[10 12 15]; %radius of the
conduit
Vm_c0=pi*R_0.^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e9; %initial volume
of the reservoir

%%Convection following parameterization by Stevenson and Blake (1998)
R_exp=0.6;
Ps=0.064;
Q=pi*R_exp^2*Ps*g*Drho*R_0.^4/mu_m;

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0.^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
*Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
(nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0.^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0)));DP1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0)));DP2=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA(3)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(3)*(time*365*24*60*60-t0)));DP3=DP/1e6;

```

```

%% PLOTS
figure (1)
hFig = figure(1);
set(gcf, 'PaperPositionMode', 'auto')
set(hFig, 'Position', [0 0 500 400])

%subplot(1,3,1)
plot(time,DP1,'r',time,DP2,'r',time,DP3,'r','LineWidth',3);
axis ([0, 10, -10, 0]);
set(gca, 'FontSize',14, 'XTick',[0 2 4 6 8 10], 'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);
xlabel('Time (years)','fontsize',14);

```



```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Script to plot FIG.5 of the manuscript:
%
% Girona, T., F. Costa, C. Newhall, and B. Taisne (2014), "On
depressurization of volcanic magma reservoirs by passive degassing".
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```
clear all
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL  A
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

%%Constants
L=[3000 5000]; %mean reservoir
depth %molecular weight
Mh2o=0.018; %universal gas
of water %temperature in
Rg=8.3145; Kelvin
constant %gravity
T=1273; %number pi
Kelvin %viscosity of the
g=9.81; %elastic modulus
pi=3.1415;
mu=1e18;
crust %Henry's constant
k=1e10; %gas flux
of the crust
S=4e-6;
Q=4000;Q=Q*1000/(24*60*60);

%%Initial conditions
alpha=0.04; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2370; %density of the
parent melt %density at the
rho_c1=2670; %density of the
top of the conduit %mass fraction of
rho_c2=rho_nd; %mean density of
less degassed melt in the conduit %gas volume
gamma_c0=1;
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2;
melt in the conduit %initial pressure
beta_c0=0.1;
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0);
of the reservoir

```

```

nr_0=alpha-S*sqrt(P0);B=[1 1];B(nr_0<=0)=0;nr_0(nr_0<=0)=0;    %mass
fraction of exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T);                                     %density of
volatiles exsolved in the reservoir at t=0
R_0=40;                                                       %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0);                                %initial volume
of melt in the conduit
Vr0=1e9;                                                       %initial volume
of the reservoir
mgr_0=nr_0*rho_nd*Vr0;                                        %mass of gas in
the reservoir
beta_r=(mgr_0*100)./(rho_gr0*Vr0);                           %gas volume
fraction in the reservoir (in %)

%%Densities
n_c=0.04;                                                       %mass fraction of
volatiles exsolved in the upper conduit
Drho=rho_c1-rho_c2;                                           %density
difference
A=0;                                                           %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=[nr_0(1) alpha]; if nx==0;nx=alpha;end                    %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=[2530 2670];                                           %density of melt
at the separation level. It is obtained with MELTS algorithm (Ghiorso
and Sack, 1995)
rho_w=nx.*rho_x*rho_nd./(rho_x-(1-nx)*rho_nd);              %partial density
of water dissolved in the melt

%%Replenishment
Rd=0;                                                           %dyke radius
M=5000;                                                         %dyke length
mu_m=1e5;                                                       %magma viscosity
lambda=8*Rd^4/(8*M*mu_m);                                     %hydraulic
strength
DPs=7e6;                                                         %overpressure of
the source
n=5000;

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1./rho_w)*Q+lambda*DPs;
C2=-((1-nr_0.*rho_nd./rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0./k+(1-nr_0.*rho_nd./rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B./(2.*rho_gr0.*(alpha-nr_0))+...
    (nr_0.*rho_nd.*Vr0./rho_gr0.^2-(pi*R_0^2.*L-
Vm_c0)./rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

```

```

%%Analytical solution
time=0:0.01:50;
DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0)));DP1=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0)));DP2=DP/1e6;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANEL B
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%Constants
L=[4000 8000 10000]; %mean reservoir
depth %molecular weight
Mh2o=0.018;
of water %universal gas
Rg=8.3145;
constant %temperature in
T=1273;
Kelvin %gravity
g=9.81;
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust %elastic modulus
k=1e10;
of the crust %Henry's constant
S=4e-6; %gas flux
Q=4000;Q=Q*1000/(24*60*60);

%%Initial conditions
alpha=0.06; %mass fraction of
volatiles dissolved in the parent melt %density of the
rho_nd=2260; parent melt %density at the
rho_c1=2670; top of the conduit %density of the
rho_c2=rho_nd; less degassed melt in the conduit %mass fraction of
gamma_c0=1; fully degassed melt in the conduit %mean density of
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; melt in the conduit %gas volume
beta_c0=0.1; fraction in the conduit at t=0; %initial pressure
P0=g*L*rho_mc0*(1-beta_c0); of the reservoir %mass
nr_0=alpha-S*sqrt(P0);B=[1 1];B(nr_0<=0)=0;nr_0(nr_0<=0)=0; fraction of exsolved volatiles in the reservoir at t=0 %density of
rho_gr0=P0*Mh2o/(Rg*T); volatiles exsolved in the reservoir at t=0

```

```

R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=1e9; %initial volume
of the reservoir
mgr_0=nr_0*rho_nd*Vr0; %mass of gas in
the reservoir
beta_r=(mgr_0*100)./(rho_gr0*Vr0); %gas volume
fraction in the reservoir (in %)

%%Densities
n_c=0.04; %mass fraction of
volatiles exsolved in the upper conduit
Drho=rho_c1-rho_c2; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=[nr_0(1) nr_0(2) alpha]; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=[2380 2290 2670]; %density of melt
at the separation level. It is obtained with MELTS algorithm (Ghiorso
and Sack, 1995)
rho_w=nx.*rho_x*rho_nd./(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=1e5; %magma viscosity
lambda=8*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=7e6; %overpressure of
the source
n=5000;

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1./rho_w)*Q+lambda*DPs;
C2=-((1-nr_0.*rho_nd./rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0./k+(1-nr_0.*rho_nd./rho_gr0)...
*Vr0/k+rho_nd*Vr0*S^2*B./(2.*rho_gr0.*(alpha-nr_0))+...
(nr_0.*rho_nd.*Vr0./rho_gr0.^2-(pi*R_0^2.*L-
Vm_c0)./rho_mc0)*Mh2o/(Rg*T);
DPinf=C1./C2;
GAMMA=C2./C3;
DP0=0;
t0=0;

%%Analytical solution
time=0:0.01:50;

```

```

DP=(-DPinf(1)*(1-exp(GAMMA(1)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(1)*(time*365*24*60*60-t0)));DP3=DP/1e6;
DP=(-DPinf(2)*(1-exp(GAMMA(2)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(2)*(time*365*24*60*60-t0)));DP4=DP/1e6;
DP=(-DPinf(3)*(1-exp(GAMMA(3)*(time*365*24*60*60-
t0)))+DP0*exp(GAMMA(3)*(time*365*24*60*60-t0)));DP5=DP/1e6;

%%PLOT
figure (1)
hFig = figure(1);
set(gcf, 'PaperPositionMode', 'auto')
set(hFig, 'Position', [0 0 1000 300])

subplot(1,2,1)
plot(time,DP1,'r',time,DP2,'r','LineWidth',2);
axis ([0, 10, -10, 0]);
set(gca, 'FontSize',14,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
ylabel('\DeltaP (MPa)','fontsize',14);
xlabel('Time (years)','fontsize',14);

subplot(1,2,2)
plot(time,DP3,'r',time,DP4,'r',time,DP5,'r','LineWidth',2);
axis ([0, 10, -10, 0]);
set(gca, 'FontSize',14,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
ylabel('\DeltaP (MPa)','fontsize',14);
xlabel('Time (years)','fontsize',14);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Script to plot FIG.6 of the manuscript:
%
% Girona, T., F. Costa, C. Newhall, and B. Taisne (2014), "On
depressurization of volcanic magma reservoirs by passive degassing".
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANELS A, B, C
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Gas flux = 4 kt/day

%%Constants
L=6000; %mean reservoir
depth %molecular weight
Mh2o=0.018; %universal gas
of water %constant
Rg=8.3145; %temperature in
constant %Kelvin
T=1273; %gravity
Kelvin %number pi
g=9.81; %viscosity of the
pi=3.1415; %elastic modulus
mu=1e18; %Henry's constant
crust %gas flux
k=1e10;
of the crust
S=4e-6;
Q=4000;Q=Q*1000/(24*60*60);

%%Densities
alpha=0.02; %mass fraction of
volatiles dissolved in the parent melt %density of the
rho_nd=2500; %density at the
parent melt %density of the
rho_c1=2670; %density of the
top of the conduit %mass fraction of
rho_c2=rho_nd; %density
less degassed melt in the conduit %mass fraction of
n_c=0.02; %density
volatiles exsolved in the upper conduit or reservoir %density
Drho=rho_c1-rho_c2; %density
difference %if A=0 ->
A=0; %steady-state degassing -> no density change of melt in the conduit
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level

```

```

rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=0; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=1e9; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Numerical scheme
tmaxyears=10; %years
tmax=tmaxyears*365*24*60*60; %seconds
steps=10000; %number of times steps
Dt=tmax/steps; %value of Dt

DVrep=[0];
DP(1)=0;
Vrep(1)=0;
m=1;

```

```

for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPS-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPS-DP(m+1));

    C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPS;
    C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);

    m=m+1;
end
DP1=DP/1e6;
Vrep1=Vrep/1e6;
DVrep1=(DVrep/1e6)*365*24*60*60;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Gas flux = 4 kt/day

%%Constants
L=10000; %mean reservoir depth
Mh2o=0.018; %molecular weight of water
Rg=8.3145; %universal gas constant
T=1273; %temperature in Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the crust
k=1e10; %elastic modulus of the crust
S=4e-6; %Henry's constant
Q=4000;Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.02; %mass fraction of volatiles dissolved in the parent melt
rho_nd=2500; %density of the parent melt
rho_c1=2670; %density at the top of the conduit
rho_c2=rho_nd; %density of the less degassed melt in the conduit
n_c=0.02; %mass fraction of volatiles exsolved in the upper conduit or reservoir
Drho=rho_c1-rho_c2; %density difference
A=0; %if A=0 -> steady-state degassing -> no density change of melt in the conduit

```



```

nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=1.2; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma
viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=0; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=1e9; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Numerical scheme
tmaxyears=10; %years
tmax=tmaxyears*365*24*60*60; %seconds
steps=10000; %number of times steps
Dt=tmax/steps; %value of Dt

DVrep=[0];

```

```

DP(1)=0;
Vrep(1)=0;
m=1;
for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPs-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPs-DP(m+1));

    C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
    C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);

    m=m+1;
end
DP4=DP/1e6;
Vrep4=Vrep/1e6;
DVrep4=(DVrep/1e6)*365*24*60*60;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Gas flux = 4 kt/day

%%Constants
L=10000; %mean reservoir
depth
Mh2o=0.018; %molecular weight
of water
Rg=8.3145; %universal gas
constant
T=1273; %temperature in
Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust
k=1e10; %elastic modulus
of the crust
S=4e-6; %Henry's constant
Q=4000;Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.02; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2500; %density of the
parent melt
rho_c1=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.02; %mass fraction of
volatiles exsolved in the upper conduit or reservoir
Drho=rho_c1-rho_c2; %density
difference

```

```

A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=2; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=0; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0); B=1; if nr_0<=0; nr_0=0; B=0; end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=40; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=1e9; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
*Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
(nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Numerical scheme
tmaxyears=10; %years
tmax=tmaxyears*365*24*60*60; %seconds
steps=10000; %number of times steps
Dt=tmax/steps; %value of Dt

```

```

DVrep=[0];
DP(1)=0;
Vrep(1)=0;
m=1;
for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPs-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPs-DP(m+1));

    C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
    C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);

    m=m+1;
end
DP5=DP/1e6;
Vrep5=Vrep/1e6;
DVrep5=(DVrep/1e6)*365*24*60*60;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PANELS D, E, F
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Gas flux = 0

%%Constants
L=10000; %mean reservoir
depth %molecular weight
Mh2o=0.018; %universal gas
of water %temperature in
Rg=8.3145; Kelvin
constant %gravity
T=1273; %number pi
Kelvin %viscosity of the
g=9.81; %elastic modulus
pi=3.1415; %Henry's constant
mu=1e18; %gas flux
crust
k=1e10;
of the crust
S=4e-6;
Q=0000;Q=Q*1000/(24*60*60);

%%Densities
alpha=0.02; %mass fraction of
volatiles dissolved in the parent melt %density of the
rho_nd=2500; parent melt %density at the
rho_c1=2670; top of the conduit %density of the
rho_c2=rho_nd; less degassed melt in the conduit

```

```

n_c=0.02; %mass fraction of
volatiles exsolved in the upper conduit or reservoir
Drho=rho_c1-rho_c2; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=1e7; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=50; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e9; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
*Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
(nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Numerical scheme
tmaxyears=10; %years

```

```

tmax=tmaxyears*365*24*60*60;      %seconds
steps=10000;                       %number of times steps
Dt=tmax/steps;                     %value of Dt

DVrep=[0];

%%Analytical
DP(1)=0;
Vrep(1)=0;
m=1;
for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPs-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPs-DP(m+1));

    if i>round(steps/2)
        Rd=1;                                %dyke radius
        lambda=8*Rd^4/(8*M*mu_m);            %hydraulic strength
    end

    C1=((1-n_c)*Drho*A/(n_c*rho_cl*rho_mc0)-1/rho_w)*Q+lambda*DPs;
    C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);

    m=m+1;
end
DP6=DP/1e6;
Vrep6=Vrep/1e6;
DVrep6=(DVrep/1e6)*365*24*60*60;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Gas flux = 2 kt/day

%%Constants
L=10000;                                %mean reservoir
depth
Mh2o=0.018;                             %molecular weight
of water
Rg=8.3145;                               %universal gas
constant
T=1273;                                  %temperature in
Kelvin
g=9.81;                                  %gravity
pi=3.1415;                                %number pi
mu=1e18;                                  %viscosity of the
crust
k=1e10;                                  %elastic modulus
of the crust
S=4e-6;                                  %Henry's constant
Q=2000;Q=Q*1000/(24*60*60);             %gas flux

```

```

%%Densities
alpha=0.02; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2500; %density of the
parent melt
rho_c1=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.02; %mass fraction of
volatiles exsolved in the upper conduit or reservoir
Drho=rho_c1-rho_c2; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=1e7; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0); B=1; if nr_0<=0; nr_0=0; B=0; end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=50; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e9; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);

```

```

C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Numerical scheme
tmaxyears=10; %years
tmax=tmaxyears*365*24*60*60; %seconds
steps=10000; %number of times steps
Dt=tmax/steps; %value of Dt

DVrep=[0];

%%Analytical
DP(1)=0;
Vrep(1)=0;
m=1;
for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPs-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPs-DP(m+1));

    if i>round(steps/2)
        Rd=1; %dyke radius
        lambda=8*Rd^4/(8*M*mu_m); %hydraulic strength
    end

    C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
    C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);

    m=m+1;
end
DP7=DP/1e6;
Vrep7=Vrep/1e6;
DVrep7=(DVrep/1e6)*365*24*60*60;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Gas flux = 3 kt/day

%%Constants
L=10000; %mean reservoir
depth %molecular weight
Mh2o=0.018; %molecular weight
of water %universal gas
Rg=8.3145; %universal gas
constant

```



```

T=1273; %temperature in Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the crust
k=1e10; %elastic modulus
of the crust
S=4e-6; %Henry's constant
Q=3000;Q=Q*1000/(24*60*60); %gas flux

%%Densities
alpha=0.02; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2500; %density of the
parent melt
rho_c1=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
n_c=0.02; %mass fraction of
volatiles exsolved in the upper conduit or reservoir
Drho=rho_c1-rho_c2; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=1e7; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0

```

```

R_0=50; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=5e9; %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_cl*rho_mc0)-1/rho_w)*Q+lambda*DPS;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Numerical scheme
tmaxyears=10; %years
tmax=tmaxyears*365*24*60*60; %seconds
steps=10000; %number of times steps
Dt=tmax/steps; %value of Dt

DVrep=[0];

%%Analytical
DP(1)=0;
Vrep(1)=0;
m=1;
for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPS-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPS-DP(m+1));

    if i>round(steps/2)
        Rd=1; %dyke radius
        lambda=8*Rd^4/(8*M*mu_m); %hydraulic strength
    end

    C1=((1-n_c)*Drho*A/(n_c*rho_cl*rho_mc0)-1/rho_w)*Q+lambda*DPS;
    C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);

    m=m+1;
end
DP8=DP/1e6;
Vrep8=Vrep/1e6;
DVrep8=(DVrep/1e6)*365*24*60*60;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

%Gas flux = 4 kt/day

%%Constants
L=10000; %mean reservoir
depth %molecular weight
Mh2o=0.018;
of water %universal gas
Rg=8.3145;
constant %temperature in
T=1273;
Kelvin %gravity
g=9.81;
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust %elastic modulus
k=1e10;
of the crust %Henry's constant
S=4e-6; %gas flux
Q=4000; Q=Q*1000/(24*60*60);

%%Densities
alpha=0.02; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2500; %density of the
parent melt %density at the
rho_cl=2670;
top of the conduit %density of the
rho_c2=rho_nd;
less degassed melt in the conduit %mass fraction of
n_c=0.02; %density
volatiles exsolved in the upper conduit or reservoir
Drho=rho_cl-rho_c2;
difference %if A=0 ->
A=0; the conduit
steady-state degassing -> no density change of melt in
nx=n_c; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_cl; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=1e7; %overpressure of
the source

%%Initial conditions
gamma_c0=0.5; %mass fraction of
fully degassed melt in the conduit

```

```

rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2;           %mean density of
melt in the conduit
beta_c0=0.1;                                           %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0);                           %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end    %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T);                               %density of
volatiles exsolved in the reservoir at t=0
R_0=50;                                                %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0);                         %initial volume
of melt in the conduit
Vr0=5e9;                                              %initial volume
of the reservoir

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPS;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Numerical scheme
tmaxyears=10;                                         %years
tmax=tmaxyears*365*24*60*60;                         %seconds
steps=10000;                                         %number of times steps
Dt=tmax/steps;                                       %value of Dt

DVrep=[0];

%%Analytical
DP(1)=0;
Vrep(1)=0;
m=1;
for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPS-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPS-DP(m+1));

    if i>round(steps/2)
        Rd=1;                                         %dyke radius
        lambda=8*Rd^4/(8*M*mu_m);                   %hydraulic strength
    end

    C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPS;
    C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);

```

```

        m=m+1;
end
DP9=DP/1e6;
Vrep9=Vrep/1e6;
DVrep9=(DVrep/1e6)*365*24*60*60;

time=0:tmaxyears/steps:tmaxyears;

%% PLOTS
figure (1)
hFig = figure(1);
set(gcf, 'PaperPositionMode', 'auto')
set(hFig, 'Position', [0 0 1200 600])

subplot(2,3,1)
plot(time,DP1,'r',time,DP4,'r',time,DP5,'r','LineWidth',3);
axis ([0, 10, -10, 0]);
set(gca, 'FontSize',14,'XTick',[0 2 4 6 8 10],'YTick',[-10 -8 -6 -4 -2 0])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);
xlabel('Time (years)','fontsize',14);

subplot(2,3,2)
plot(time,Vrep1,'r',time,Vrep4,'r',time,Vrep5,'r','LineWidth',3);
axis ([0, 10, 0, 30]);
set(gca, 'FontSize',14,'XTick',[0 2 4 6 8 10],'YTick',[0 10 20 30])
xlabel('Time (years)','fontsize',14);
ylabel('V_r_e_p (*10^6 m^3)','fontsize',14);
xlabel('Time (years)','fontsize',14);

subplot(2,3,3)
plot(time,DVrep1,'r',time,DVrep4,'r',time,DVrep5,'r','LineWidth',3);
axis ([0, 10, 0, 3]);
set(gca, 'FontSize',14,'XTick',[0 2 4 6 8 10],'YTick',[0 1 2 3])
xlabel('Time (years)','fontsize',14);
ylabel('dV_r_e_p/dt (*10^6 m^3/year)','fontsize',14);
xlabel('Time (years)','fontsize',14);

subplot(2,3,4)
plot(time,DP6,'r',time,DP7,'r',time,DP8,'r',time,DP9,'r','LineWidth',3);
axis ([0, 10, -10, 10]);
set(gca, 'FontSize',14,'XTick',[0 2 4 6 8 10],'YTick',[-10 -5 0 5 10])
xlabel('Time (years)','fontsize',14);
ylabel('\DeltaP (MPa)','fontsize',14);
xlabel('Time (years)','fontsize',14);

```

```

subplot(2,3,5)
plot(time,Vrep6,'r',time,Vrep7,'r',time,Vrep8,'r',time,Vrep9,'r','LineWi
dth',3);
axis ([0, 10, 0, 30]);
set(gca,'FontSize',14,'XTick',[0 2 4 6 8 10],'YTick',[0 10 20 30])
xlabel('Time (years)','fontsize',14);
ylabel('V_r_e_p (*10^6 m^3)','fontsize',14);
xlabel('Time (years)','fontsize',14);

subplot(2,3,6)
plot(time,DVrep6,'r',time,DVrep7,'r',time,DVrep8,'r',time,DVrep9,'r','Li
neWidth',3);
axis ([0, 10, 0, 8]);
set(gca,'FontSize',14,'XTick',[0 2 4 6 8 10],'YTick',[0 2 4 6 8])
xlabel('Time (years)','fontsize',14);
ylabel('dV_r_e_p/dt (*10^6 m^3/year)','fontsize',14);
xlabel('Time (years)','fontsize',14);

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Script to plot FIG.7 of the manuscript:
%
% Girona, T., F. Costa, C. Newhall, and B. Taisne (2014), "On
depressurization of volcanic magma reservoirs by passive degassing".
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% BLUE (degassed melt stagnates in the conduit) %%%

clear all

%%Constants
L=10000; %mean reservoir
depth
Mh2o=0.018; %molecular weight
of water
Rg=8.3145; %universal gas
constant
T=1273; %temperature in
Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust
k=1e10; %elastic modulus
of the crust
S=4e-6; %Henry's constant
Q=4000;Q=Q*1000/(24*60*60); %gas flux

%%Initial conditions
alpha=0.06; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2260; %density of the
parent melt
rho_c1=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
gamma_c0=0.2; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_c1+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0

```

```

R_0=70; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=1e9; %initial volume
of the reservoir

%%Densities
n_c=0.06; %mass fraction of
volatiles exsolved in the upper conduit or reservoir
Drho=rho_c1-rho_c2; %density
difference
A=1; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c/nr_0; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=rho_c1/%2290; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=0; %overpressure of
the source

%%Parameters
C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPs;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;

%%Numerical scheme
tmaxyears=10; %years
tmax=tmaxyears*365*24*60*60; %seconds
steps=10000; %number of times steps
Dt=tmax/steps; %value of Dt

DVrep=[0];

%%Analytical
DP(1)=0;
Vrep(1)=0;
Vmc(1)=Vm_c0;
Vmc1(1)=gamma_c0*Vm_c0;

```



```

rho_mc(1)=rho_mc0;
DV(1)=0;
gamma_c(1)=gamma_c0;
m=1;
for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPs-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPs-DP(m+1));
    DV(m)= -Vr0*DPinf*((m*365*24*60*60)/mu+(1/k+1/(GAMMA*mu))*(1-
exp(GAMMA*(m*365*24*60*60)))));

    Vmc(m+1)=Vmc(i)+Dt*(-Q/rho_w-((1-
nr_0*rho_nd/rho_gr0)*Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-
nr_0))+nr_0*rho_nd*Vr0*Mh2o/(rho_gr0^2*Rg*T))*(DP(m+1)-DP(m))/Dt-(1-
nr_0*rho_nd/rho_gr0)*Vr0*DP(m)/mu);

    Vmc1(m+1)=((1-n_c)*Q/(n_c*rho_c1))*Dt+Vmc1(m);
    gamma_c(m+1)=gamma_c0*Vm_c0/Vmc(m+1)+(1-
n_c)*Q*Dt*m/(n_c*rho_c1*Vmc(m+1));

    if Vmc1(m+1)>=Vmc(m+1);gamma_c(m+1)=1;A=0;end
    rho_mc(m+1)=gamma_c(m+1)*rho_c1+(1-gamma_c(m+1))*rho_c2;

    C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc(m+1))-1/rho_w)*Q+lambda*DPs;

    m=m+1;
end
DP1=DP/1e6;
Vrep1=Vrep/1e6;
DVrep1=(DVrep/1e6)*365*24*60*60;
gamma_c1=gamma_c;
rho_mc1=rho_mc;

%%%%%%%%%%%%% RED (degassed melt does not stagnate in the conduit) %%%

%%Constants
L=10000; %mean reservoir
depth %depth
Mh2o=0.018; %molecular weight
of water %of water
Rg=8.3145; %universal gas
constant %constant
T=1273; %temperature in
Kelvin %Kelvin
g=9.81; %gravity
pi=3.1415; %number pi
mu=1e18; %viscosity of the
crust %crust
k=1e10; %elastic modulus
of the crust %of the crust
S=4e-6; %Henry's constant
Q=4000;Q=Q*1000/(24*60*60); %gas flux

```

```

%%Initial conditions
alpha=0.06; %mass fraction of
volatiles dissolved in the parent melt
rho_nd=2260; %density of the
parent melt
rho_cl=2670; %density at the
top of the conduit
rho_c2=rho_nd; %density of the
less degassed melt in the conduit
gamma_c0=0.2; %mass fraction of
fully degassed melt in the conduit
rho_mc0=gamma_c0*rho_cl+(1-gamma_c0)*rho_c2; %mean density of
melt in the conduit
beta_c0=0.1; %gas volume
fraction in the conduit at t=0;
P0=g*L*rho_mc0*(1-beta_c0); %initial pressure
of the reservoir
nr_0=alpha-S*sqrt(P0);B=1;if nr_0<=0;nr_0=0;B=0;end %mass fraction of
exsolved volatiles in the reservoir at t=0
rho_gr0=P0*Mh2o/(Rg*T); %density of
volatiles exsolved in the reservoir at t=0
R_0=70; %radius of the
conduit
Vm_c0=pi*R_0^2*L*(1-beta_c0); %initial volume
of melt in the conduit
Vr0=1e9; %initial volume
of the reservoir

%%Densities
n_c=0.06; %mass fraction of
volatiles exsolved in the upper conduit or reservoir
Drho=rho_cl-rho_c2; %density
difference
A=0; %if A=0 ->
steady-state degassing -> no density change of melt in the conduit
nx=n_c/nr_0; %mass fraction of
volatiles exsolved at the gas-melt separation level
rho_x=2670;%2290; %density of melt
at the separation level
rho_w=nx*rho_x*rho_nd/(rho_x-(1-nx)*rho_nd); %partial density
of water dissolved in the melt

%%Replenishment
Rd=0; %dyke radius
M=5000; %dyke length
mu_m=10^4.2; %magma viscosity
lambda=pi*Rd^4/(8*M*mu_m); %hydraulic
strength
DPs=0; %overpressure of
the source
n=5000;

%%Parameters

```

```

C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc0)-1/rho_w)*Q+lambda*DPS;
C2=-((1-nr_0*rho_nd/rho_gr0)*Vr0/mu+lambda+Vm_c0/mu);
C3=(pi*R_0^2/(g*rho_mc0))+Vm_c0/k+(1-nr_0*rho_nd/rho_gr0)...
    *Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-nr_0))+...
    (nr_0*rho_nd*Vr0/rho_gr0^2-(pi*R_0^2*L-Vm_c0)/rho_mc0)*Mh2o/(Rg*T);
DPinf=C1/C2;
GAMMA=C2/C3;
DP0=0;
t0=0;
cont=0;

%%Numerical scheme
tmaxyears=10; %years
tmax=tmaxyears*365*24*60*60; %seconds
steps=10000; %number of times steps
Dt=tmax/steps; %value of Dt

DVrep=[0];

%%Analytical
DP(1)=0;
Vrep(1)=0;
Vmc(1)=Vm_c0;
Vmc1(1)=0;
rho_mc(1)=rho_mc0;
DV(1)=0;
m=1;
for i=1:steps
    DP(m+1)=DP(m)+(C1+C2*DP(m))*Dt/C3;
    Vrep(m+1)=Vrep(m)+lambda*(DPS-DP(m+1))*Dt;
    DVrep(m+1)=lambda*(DPS-DP(m+1));
    DV(m)= -Vr0*DPinf*((m*365*24*60*60)/mu+(1/k+1/(GAMMA*mu))*(1-
exp(GAMMA*(m*365*24*60*60))));

    Vmc(m+1)=Vmc(i)+Dt*(-Q/rho_w-((1-
nr_0*rho_nd/rho_gr0)*Vr0/k+rho_nd*Vr0*S^2*B/(2*rho_gr0*(alpha-
nr_0))+nr_0*rho_nd*Vr0*Mh2o/(rho_gr0^2*Rg*T))*(DP(m+1)-DP(m))/Dt-(1-
nr_0*rho_nd/rho_gr0)*Vr0*DP(m)/mu);

    Vmc1(m+1)=((1-n_c)*Q/(n_c*rho_c1))*Dt+Vmc1(m);
    gamma_c(m+1)=gamma_c0;

    rho_mc(m+1)=gamma_c(m+1)*rho_c1+(1-gamma_c(m+1))*rho_c2;

    C1=((1-n_c)*Drho*A/(n_c*rho_c1*rho_mc(m+1))-1/rho_w)*Q+lambda*DPS;

    m=m+1;
end
DP2=DP/1e6;
Vrep2=Vrep/1e6;
DVrep2=(DVrep/1e6)*365*24*60*60;
gamma_c2=gamma_c;

```

```

rho_mc2=rho_mc;

time=0:tmaxyears/steps:tmaxyears;

%% PLOTS
figure (1)
hFig = figure(1);
set(gcf, 'PaperPositionMode', 'auto')
set(hFig, 'Position', [0 0 1300 300])

subplot(1,3,1)
plot(time,DP1,'b',time,DP2,'r','LineWidth',3);
axis ([0, 10, -10, 0]);
set(gca, 'FontSize',14, 'XTick',[0 2 4 6 8 10], 'YTick',[-10 -8 -6 -4 -2 0])
ylabel('\DeltaP (MPa)', 'fontsize',14);
xlabel('Time (years)', 'fontsize',14);

subplot(1,3,2)
plot(time,gamma_c1,'b',time,gamma_c2,'r','LineWidth',3);
axis ([0, 10, 0, 1]);
set(gca, 'FontSize',14, 'XTick',[0 2 4 6 8 10], 'YTick',[0 0.2 0.4 0.6 0.8 1])
ylabel('\gamma', 'fontsize',14);
xlabel('Time (years)', 'fontsize',14);

subplot(1,3,3)
plot(time,rho_mc1,'b',time,rho_mc2,'r','LineWidth',3);
axis ([0, 10, 2300, 2700]);
set(gca, 'FontSize',14, 'XTick',[0 2 4 6 8 10], 'YTick',[2300 2400 2500 2600 2700])
ylabel('\rho_m_c (kg/m^3)', 'fontsize',14);
xlabel('Time (years)', 'fontsize',14);

```


Auxiliary material of Chapter 2:

Fractal degassing from Erebus and Mayon volcanoes revealed by a new method to monitor H₂O emission cycles

Description:

The following is the Matlab code developed to make the analysis of the digital images of volcanic plumes. This script plots Figure 4.

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Script to make the analysis of the digital images of Mayon volcano
%
% Fig. 4
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all

d=dir('..\tests_velocity\*.tiff'); %directory where images are saved
im=imread(d(1).name);imshow(im);

x2=[181 255]; %coordinates of region (R1)
y2=[240 241];

h2 = impoly(gca,[x2(1),y2(1); x2(2),y2(1);x2(2),y2(2); x2(1),y2(2)]);
M2 = ~h2.createMask(); %it creates a matrix of zeros inside the
polygon and a matrix of ones outside
setColor(h2,'red');
addNewPositionCallback(h2,@(p) title(mat2str(p,3)));
fcn = makeConstrainToRectFcn('impoly',get(gca,'XLim'),get(gca,'YLim'));
setPositionConstraintFcn(h2,fcn)
M2=uint8(M2);
Mask2=M2;Mask2(M2==1)=0;Mask2(M2==0)=1;
Inew2 = im.*repmat(Mask2,[1,1,3]);
imcut2=Inew2;
I2=double(rgb2gray(imcut2));
I2(find(I2==0))=[];

x3=[150 300]; %coordinates of region (R2) (=(R3))
y3=[290 320];
h3 = impoly(gca,[x3(1),y3(1); x3(2),y3(1);x3(2),y3(2); x3(1),y3(2)]);
M3 = ~h3.createMask(); %it creates a matrix of zeros inside the
polygon and a matrix of ones outside
setColor(h3,'blue');
addNewPositionCallback(h3,@(p) title(mat2str(p,3)));
fcn = makeConstrainToRectFcn('impoly',get(gca,'XLim'),get(gca,'YLim'));
setPositionConstraintFcn(h3,fcn)
M3=uint8(M3);
Mask3=M3;Mask3(M3==1)=0;Mask3(M3==0)=1;
Inew3 = im.*repmat(Mask3,[1,1,3]);
imcut3=Inew3;
I3=double(rgb2gray(imcut3));
I3(find(I3==0))=[];

setColor(h2,'red');
setColor(h3,'blue');

figsave = figure('Position', [100, 100, 300, 600]);

```

```

i=0;
for m=3000:1:length(d)-1
    i=i+1;

    im=imread(d(m+1).name); %load image

    I1=double(rgb2gray(im)); %conversion to grayscale
    Inew2 = im.*repmat(Mask2,[1,1,3]);
    imcut2=Inew2;
    I2=double(rgb2gray(imcut2));
    I2(find(I2==0))=[];
    intens_r(i)=sum(sum(I2)); %digital brightness of (R1). Equation (6)

    Inew3 = im.*repmat(Mask3,[1,1,3]);
    imcut3=Inew3;
    I3=double(rgb2gray(imcut3));
    I3(find(I3==0))=[];
    intens_b(i)=sum(sum(I3))*numel(I2)/numel(I3); %digital brightness of
(R2)=(R3) for Mayon). Equation (6)

    time(i)=m-3000;
end

%% Correction
intens_r_or=intens_r/numel(intens_r)-mean(intens_r/numel(intens_r));

%PANNEL B
intens_r=(intens_r-mean(intens_r))/std(intens_r); %standardized
brightness without correction
time=time-time(1);

%PANNEL C
intens_b=(intens_b/numel(intens_b)-mean(intens_b/numel(intens_b)));
intens_sun=2*intens_b; %factor 2 because (R2) = (R3)
filter_intens_r=intens_sun; %detrending (light change correction)

%PANNEL D
intens_r_filt1=intens_r_or-filter_intens_r;
p=polyfit(time,intens_r_filt1,1); %fit to low order polynomial
(weather change correction)
trend=p(2)+p(1)*time;

%PANNEL E
intens_r_filt2=(intens_r_filt1-mean(intens_r_filt1))/std(intens_r_filt1);
intens_r_filt2=detrend(intens_r_filt2);

%% PLOTS
figure(1)
hFig = figure(1);
set(gcf, 'PaperPositionMode', 'auto')
set(hFig, 'Position', [0 0 1400 500])

```



```

%PANNEL A
subplot('position',[0.05 0.07 0.4 0.88])
im=imread(d(5000).name);
imshow(im)
h2 = impoly(gca,[x2(1),y2(1); x2(2),y2(1);x2(2),y2(2); x2(1),y2(2)]);
h3 = impoly(gca,[x3(1),y3(1); x3(2),y3(1);x3(2),y3(2); x3(1),y3(2)]);
setColor(h2,'red');
setColor(h3,'blue');

%PANNEL B
subplot('position',[0.5 0.6 0.19 0.35])
plot(time,intens_r,'r','linewidth',2.5)
xlabel('Time (s)','fontsize',12);
ylabel('Standardized uncorrected time series','fontsize',12);
axis([0 time(end) -4 4])
set(gca,'FontSize', 12);

%PANNEL C
subplot('position',[0.76 0.6 0.19 0.35])
hold on
plot(time,intens_r_or,'r',time,intens_b,'b','linewidth',2.5)
hold off
xlabel('Time (s)','fontsize',12);
ylabel('Mean-removed brightness per pixel (D.U.)','fontsize',12);
axis([0 time(end) -0.4 0.4])
set(gca,'FontSize', 12);
box on

%PANNEL D
subplot('position',[0.5 0.1 0.19 0.35])
hold on
plot(time,intens_r_filt1,'r','linewidth',2.5)
plot(time,trend,'color',[0.5 0.5 0.5],'linewidth',2.5)
hold off
xlabel('Time (s)','fontsize',12);
ylabel('Mean-removed brightness per pixel (D.U.)','fontsize',12);
axis([0 time(end) -0.4 0.4])
set(gca,'FontSize', 12);
box on

%PANNEL E
subplot('position',[0.76 0.1 0.19 0.35])
plot(time,intens_r_filt2,'r','linewidth',2.5)
xlabel('Time (s)','fontsize',12);
ylabel('Standardized corrected time series','fontsize',12);
axis([0 time(end) -4 4])
set(gca,'FontSize', 12);
box on

%% SAVING IN .TXT
fileID = fopen('values.txt','w');
for i=1:length(intens_r)
    fprintf(fileID,'%8.4f\r\n',intens_r_filt2(i));
end
fclose(fileID);

```

Auxiliary material of Chapter 3:

Shifts of volcanic degassing patterns as possible precursor of phreatic eruptions

Description:

Matlab script of the numerical model.

```

%% AGENT-BASED MODEL OF VOLCANIC DEGASSING FOR ACTIVE VOLCANOES
%% Tarsilo Girona, 5 September,2014

clear all

%% Geometry / Initial conditions
R=0.2; %radius of the cylinder
length_conduit=13; %length of the cylinder = L + D
H_0=9; %initial liquid level
H=H_0; %liquid level
D0=length_conduit-H_0; %thickness of the gas chamber at t=0
time=0; %initial real time
Q_out(1)=0; %initial outflux=0
dimensionlesstime=0; %dimensionless time

%% Physical parameters
rho_liq=2500; %density of the liquid
T=600+273.15; %temperature of magma and
gas
Patm=101325; %pressure outside (exit vent)
P_ch=Patm; %initial pressure in the gas
chamber
M=0.018; %molecular weigth of water
Rg=8.3145; %ideal gas constant
rho_gas_0=Patm*M/(Rg*T); %density of gas at 1 atm and
temperature T assuming ideal gas
g=9.81; %gravity
mu_liq=5; %viscosity of liquid
sigma=1e-5; %surface tension
Q=0.001; %gas flow in kg/s
N=10; %number of bubbles per
second
tau=1/N; %time step

Nf=rho_liq*sqrt(8*g*R^3)/mu_liq; %dimensionless inverse
viscosity
if Nf<500 %lmax: maximum distance
below which there is interaction between Taylor bubbles %lmax depends on the value
    lmax=(2.92+0.0095*Nf)*R;
of Nf
else if Nf>=500 && Nf<1500
    lmax=(1.384+0.0158*Nf)*R;
else
    lmax=25*R;
end
end

lambda=R*(0.204+0.123*tanh(2.66-1.15*log10(Nf))); %equilibrium
thickness of the slug. Expression from Llewellyn et al. (2012)
Fr=0.34*(1+(31.08/Nf)^1.45)^-0.71; %Froude number, used
for the velocity of slugs. Expression from Llewellyn et al. (2012)
L=3; %length of the dome
mu_gas=1e-5; %gas viscosity
K1=1e-10; %viscous (Darcian)
permeability (units: m^2)

```

```

K2=4.72e+14*K1^1.87; %inertial
(Forchheimer) permeability (units: m)
A=pi*R^2; %section of
permeable part
tau_real=1; %factor to convert
to real time. Being = N --> sampling time = 1s
eps=0; %initial gas volume

%% Tuning parameters
gamma=2e4; %coefficient of the transient term of the
generalized Darcy-Forchheimer equation
d=0.001; %distance of the random motion
c=2; %similar to distribution factor (e.g., Zuber and
Findlay, 1965)
beta=0.1; %breakup coefficient 0 - 1
r_sta=R/2; %maximum radius for which a bubbles is stable (for
breakup)
alpha_burst=1; %bursting coefficient 0 - 1
r_burst=0; %maximum radius for which a bubbles is stable
beneath the magma surface (for bursting)

%% Definition of the six identifying parameters
x=[];y=[];z=[]; %coordinates of the position of the bubbles. x,y,z -
> position of the center.
r=[]; %radius of the bubbles (effective radius for the
case of Taylor bubbles)
rho_gas=[]; %density of each bubble
mass=[]; %mass inside each bubble
sup_velocity=[];

%% Create figure
figure(1)
hFig = figure(1);
set(gcf, 'PaperPositionMode', 'auto')
set(hFig, 'Position', [20 0 600 600])

DQ_in=0;
%% Main loop
for K1=[5e-11 6e-11 7e-11 8e-11 9e-11 1e-10 2e-10 3e-10 4e-10 5e-10 6e-
10 7e-10 8e-10 9e-10 1e-9];%[0.001 0.002 0.003 0.004 0.005 0.006 0.007
0.008 0.009 0.010 0.011 0.012 0.013 0.014 0.015 0.016 0.017 0.018 0.019
0.020 0.022 0.024 0.026 0.028 0.03 0.032 0.034 0.036 0.038 0.040 0.045
0.05 0.055 0.06 0.065 0.07 0.075 0.08 0.085 0.09 0.095 0.1 0.12 0.15
0.16 0.18 0.2 0.22 0.24 0.26 0.28 0.3]
    t=1;
    create_direct=1;
    K2=4.72e+14*K1^1.87; %inertial
    (Forchheimer) permeability (units: m)
    while time(end)<=1500

        %% New bubble at the bottom of the domain (gas-separation level)
        mass(end+1)=Q/N;

```

```

rho_gas(end+1)=M*(P_ch(end)+rho_liq*g*H_0)/(Rg*T);
r(end+1)=(3*mass(end)/(4*pi*rho_gas(end)))^(1/3);
x(end+1)=cos(2*pi*rand)*R*rand;
y(end+1)=sin(2*pi*rand)*R*rand;
z(end+1)=r(end);
sup_velocity(end+1)=Q/(rho_gas(end)*pi*R^2);    %gas superficial
velocity (with the density at the bottom)

%% Expansion
r_old=r;DV=0;z_old=z;
for j=1:length(x)
    P=P_ch(end)+g*rho_liq*(H_0-
z_old(j))+(4*g*rho_liq/(3*R^2))*sum(r_old(find(z_old<z_old(j))).^3);
    rho_gas(j)=P*M/(Rg*T);
    r(j)=(3*mass(j)/(4*pi*rho_gas(j)))^(1/3);
    DV(j)=(4*pi/3)*(r(j)^3-r_old(j)^3);
end
for j=1:length(x)
    DZ=sum(DV(find(z_old<z_old(j))))/(pi*R^2);
    z(j)=z(j)+DZ;
end

%% Displacement
zold=z;
random=rand(1,length(x));randomlb=rand(1,length(x));v=[];
for j=1:length(x)
    if r(j)<R-lambda %spherical bubbles
        Drho=rho_liq-rho_gas(j);
        Vtpot=(1/36)*Drho*g*(2*r(j))^2/mu_liq;
        Vt1=Vtpot*(1+(0.73667*(g*2*r(j))^0.5)/Vtpot)^0.5;
Vt2=((3*sigma/(rho_liq*2*r(j)))+g*2*r(j)*Drho/(2*rho_liq))^0.5;
        v(j)=1/sqrt(1/Vt1^2+1/Vt2^2);

        x(j)=x(j)+d*sin(2*pi*randomlb(j))*cos(2*pi*random(j));
        y(j)=y(j)+d*sin(2*pi*randomlb(j))*sin(2*pi*random(j));

z(j)=z(j)+d*cos(2*pi*randomlb(j))+tau*(v(j)+c*sup_velocity(end));    %the
last term is to incorporate ascent velocity
    else %Taylor bubbles. I account for wake effects
        vector_position_bubbles_above=find(z>z(j));
        radius_bubbles_above=r(vector_position_bubbles_above);
%radius of the bubbles that are above the current bubble

radius_taylor_bubbles_above=radius_bubbles_above(find(radius_bubbles_abo
ve>=R-lambda)); %determine Taylor bubbles above
        position_taylor_bubbles_above=[];
        for i=1:length(radius_taylor_bubbles_above)
position_taylor_bubbles_above(i)=z(find(r==radius_taylor_bubbles_above(i
)));
        end

```

```

position_closest_taylor_bubble_above=min(position_taylor_bubbles_above);
%coordinate z of the closest slug above
    if numel(position_closest_taylor_bubble_above)==0;
        v(j)=Fr*sqrt(2*g*R);
    else
        distance_slugs=position_closest_taylor_bubble_above-z(j)-
2*(r(find(z==position_closest_taylor_bubble_above))^3+r(j)^3)/(3*(R-
lambda))^2;
        if distance_slugs/lmax<=0.24; v(j)=(4.24-
11.4*distance_slugs/lmax)*Fr*sqrt(2*g*R);end
        if distance_slugs/lmax>0.24 && distance_slugs/lmax<1;
v(j)=(2.01-
1.96*distance_slugs/lmax+0.95*(distance_slugs/lmax)^2)*Fr*sqrt(2*g*R);en
d
        if distance_slugs/lmax>=1; v(j)=Fr*sqrt(2*g*R);end
    end
    x(j)=0;
    y(j)=0;
    z(j)=z(j)+tau*(v(j)+c*sup_velocity(end));
end
end

%% Binary bubble breakup

m=1;rem=[];x1=[];x2=[];y1=[];y2=[];z1=[];z2=[];r1=[];r2=[];mass1=[];mass
2=[];rho_gas1=[];rho_gas2=[];
    random=rand(1,length(x));
    random2=rand(1,length(x));%0.1+0.8*rand(1,length(x));
    phi=rand(1,length(x)); %polar angle, to allocate the new
bubbles
    theta=rand(1,length(x)); %azhimutal angle
    for j=1:length(x)
        Prob=beta*r(j)^2/(r(j)^2+r_sta^2); %breakup
probability for a given bubble
        if r(j)<R-lambda && Prob>rand
            rem(m)=j;
            bre_fraction=random2(j); %breakage volume
fraction
            %bre_fraction=0.5;
            r1(m)=bre_fraction^(1/3)*r(j); %radius of one
daughter bubble
            r2(m)=(1-bre_fraction)^(1/3)*r(j); %radius of the other
daughter bubble
            rho_gas1(m)=rho_gas(j);
            rho_gas2(m)=rho_gas(j);
            mass1(m)=(4/3)*pi*r1(m)^3*rho_gas1(m);
            mass2(m)=(4/3)*pi*r2(m)^3*rho_gas2(m);

            x1(m)=x(j)+(r1(m)+d)*sin(2*pi*phi(j))*cos(2*pi*theta(j));
            x2(m)=x(j)-(r2(m)+d)*sin(2*pi*phi(j))*cos(2*pi*theta(j));
            y1(m)=y(j)+(r1(m)+d)*sin(2*pi*phi(j))*sin(2*pi*theta(j));
            y2(m)=y(j)-(r2(m)+d)*sin(2*pi*phi(j))*sin(2*pi*theta(j));
            z1(m)=z(j)+(r1(m)+d)*cos(2*pi*phi(j));

```

```

        z2(m)=z(j)-(r2(m)+d)*cos(2*pi*phi(j));

        m=m+1;
    end
end

x(rem)=[ ];y(rem)=[ ];z(rem)=[ ];r(rem)=[ ];mass(rem)=[ ];rho_gas(rem)=[ ];
    x=[x x1 x2];y=[y y1 y2];z=[z z1 z2];r=[r r1 r2];mass=[mass mass1
mass2]; rho_gas=[rho_gas rho_gas1 rho_gas2];

%% Coalescence
aux=1;j=1;
while aux>0
    j=1;
    while j<=length(x)
        m=1;rem=[ ];
        for i=1:length(x)
            if r(i)<R-lambda && r(j)<R-lambda %coalescence
between 2 bubbles
                if sqrt((x(j)-x(i))^2 + (y(j)-y(i))^2 + (z(j)-
z(i))^2)< (r(j)+r(i)) && sqrt((x(j)-x(i))^2 + (y(j)-y(i))^2 + (z(j)-
z(i))^2)>0
                    rem(m)=i;
                    x(j)=(x(j)*mass(j)+x(i)*mass(i))/(mass(j)+mass(i));
                    y(j)=(y(j)*mass(j)+y(i)*mass(i))/(mass(j)+mass(i));
                    z_old=z(j);
                    z(j)=(z(j)*mass(j)+z(i)*mass(i))/(mass(j)+mass(i));
                    r(j)=(r(j)^3+r(i)^3)^(1/3);

                    mass(j)=mass(j)+mass(i);
                    m=m+1;
                end

                else if z(j)>z(i) && r(i)>=R-lambda && r(j)>=R-lambda
%coalescence between 2 slugs
                    if z(j)-z(i)< 2*r(j)^3/(3*(R-
lambda)^2)+2*r(i)^3/(3*(R-lambda)^2)
                        x(j)=0;
                        y(j)=0;

z_old=z(j);z(j)=(z(j)*mass(j)+z(i)*mass(i))/(mass(j)+mass(i));
                        r(j)=(r(j)^3+r(i)^3)^(1/3);

                        mass(j)=mass(j)+mass(i);
                        rem(m)=i;
                        m=m+1;
                    end

                    else if z(i)>z(j) && r(i)>=R-lambda && r(j)>=R-lambda
%coalescence between 2 slugs
                        if z(i)-z(j)< 2*r(i)^3/(3*(R-
lambda)^2)+2*r(j)^3/(3*(R-lambda)^2)
                            x(j)=0;

```

```

y(j)=0;

z_old=z(j);z(j)=(z(j)*mass(j)+z(i)*mass(i))/(mass(j)+mass(i));
r(j)=(r(j)^3+r(i)^3)^(1/3);

mass(j)=mass(j)+mass(i);
rem(m)=i;
m=m+1;
end
else if z(j)>z(i) && r(j)>=R-lambda && r(i)<R-lambda
%coalescence between slug (up) + bubble (down)
if z(j)-z(i)< 2*r(j)^3/(3*(R-lambda)^2)+r(i)
x(j)=0;
y(j)=0;

z_old=z(j);z(j)=(z(j)*mass(j)+z(i)*mass(i))/(mass(j)+mass(i));
r(j)=(r(j)^3+r(i)^3)^(1/3);

mass(j)=mass(j)+mass(i);
rem(m)=i;
m=m+1;
end
else if z(j)<z(i) && r(i)>=R-lambda && r(j)<R-lambda
%coalescence between slug (up) + bubble (down)
if z(i)-z(j)< 2*r(i)^3/(3*(R-lambda)^2)+r(j)
x(j)=0;
y(j)=0;

z_old=z(j);z(j)=(z(j)*mass(j)+z(i)*mass(i))/(mass(j)+mass(i));
r(j)=(r(j)^3+r(i)^3)^(1/3);

mass(j)=mass(j)+mass(i);
rem(m)=i;
m=m+1;
end
else if z(j)<z(i) && r(j)>=R-lambda && r(i)<R-lambda
%coalescence between slug (down) + bubble (up)
if z(i)-z(j)< 2*r(j)^3/(3*(R-lambda)^2)+r(i)
x(j)=0;
y(j)=0;

z_old=z(j);z(j)=(z(j)*mass(j)+z(i)*mass(i))/(mass(j)+mass(i));
r(j)=(r(j)^3+r(i)^3)^(1/3);

mass(j)=mass(j)+mass(i);
rem(m)=i;
m=m+1;
end
else if z(i)<z(j) && r(i)>=R-lambda && r(j)<R-lambda
%coalescence between slug (down) + bubble (up)
if z(j)-z(i)< 2*r(i)^3/(3*(R-lambda)^2)+r(j)
x(j)=0;
y(j)=0;

z_old=z(j);z(j)=(z(j)*mass(j)+z(i)*mass(i))/(mass(j)+mass(i));

```



```

        r(j)=(r(j)^3+r(i)^3)^(1/3);

        mass(j)=mass(j)+mass(i);
        rem(m)=i;
        m=m+1;
    end
end
end
end
end
end
end
end
end

x(rem)=[ ];y(rem)=[ ];z(rem)=[ ];r(rem)=[ ];mass(rem)=[ ];rho_gas(rem)=[ ];aux
=0;rem=[ ];j=j+1;
end

%confinement
for j=1:length(x)
    while r(j)<R-lambda && sqrt(x(j)^2+y(j)^2)+r(j)>R
        x(j)=1*((R-r(j))/sqrt(x(j)^2+y(j)^2))*x(j);
        y(j)=1*((R-r(j))/sqrt(x(j)^2+y(j)^2))*y(j);
    end
    if r(j)<R-lambda && z(j)<r(j);z(j)=r(j);end
    if r(j)>=R-lambda && z(j)<2*r(j)^3/(3*(R-
lambda)^2);z(j)=2*r(j)^3/(3*(R-lambda)^2);end
end

    for j=1:length(x)
        for i=1:length(x)
            if r(i)<R-lambda && r(j)<R-lambda && sqrt((x(j)-x(i))^2 +
(y(j)-y(i))^2 + (z(j)-z(i))^2)< (r(j)+r(i)) && sqrt((x(j)-x(i))^2 +
(y(j)-y(i))^2 + (z(j)-z(i))^2)> 0
                aux=aux+1;
            end
        end
    end
end

end

%% remove bubbles over the level, calculate delta_m, and calculate
new level
aux=1;delta_m_in=0;
while aux>0
    m=1;rem=[ ];

    for j=1:length(x)
        if r(j)<R-lambda && H(end)-z(j)<r(j)

```

```

        if
alpha_burst*r(j)^2/(r(j)^2+r_burst^2)>rand;rem(m)=j;m=m+1;else;z(j)=H(en
d)-r(j);end
        end
        if r(j)>=R-lambda && H(end)-z(j)<2*r(j)^3/(3*(R-lambda)^2)
        if
alpha_burst*r(j)^2/(r(j)^2+r_burst^2)>rand;rem(m)=j;m=m+1;else;z(j)=H(en
d)-2*r(j)^3/(3*(R-lambda)^2);end
        end
        if r(j)>=R-lambda && H(end)-z(j)<2*r(j)^3/(3*(R-lambda)^2)
        if
H(end)>=0.98*length_conduit;rem(m)=j;m=m+1;else;z(j)=H(end)-
2*r(j)^3/(3*(R-lambda)^2);end
        end
        end

        delta_m_in=delta_m_in+sum(mass(rem));

x(rem)=[];y(rem)=[];z(rem)=[];r(rem)=[];mass(rem)=[];rho_gas(rem)=[];
%remove identifying variables
        aux=0;
        for j=1:length(x)
                if r(j)<R-lambda && H(end)-z(j)<r(j);aux=1;end
                if r(j)>=R-lambda && H(end)-z(j)<2*r(j)^3/(3*(R-
lambda)^2);aux=1;end
        end
        end
        DQ_in(end+1)=delta_m_in;
        H(end+1)=H_0+sum((4/3)*pi*r.^3)/(pi*R^2); %calculate new level
position
        time(t+1)=time(t)+tau*tau_real; %new time --> converte to
real time

        %% calculation of the gas flux and pressure change in the gas chamber
        eps=(H-H_0)./H; %gas volume fraction
        A1=(1+H_0*(eps(end)-eps(end-1)))/((D0*(1-eps(end-1))-H_0*eps(end-
1))*(1-eps(end-1)));
        A2=(Rg*T/(pi*R^2*M))*(delta_m_in/tau-Q_out(end))*(1-eps(end-
1))/(D0*(1-eps(end-1))-H_0*eps(end-1));
        P_ch(end+1)=P_ch(end)*A1+tau*A2;
        %P_ch(end+1)=Patm;

        A3=(P_ch(end)^2-Patm^2)/(2*Patm*L);
        A4=mu_gas/(K1*A*rho_gas_0);
        A5=1/(K2*A^2*rho_gas_0);
        Q_out(end+1)=Q_out(end)+(A*rho_gas_0*tau/gamma)*(A3-A4*Q_out(end)-
A5*Q_out(end)^2);

        %% plot
        if mod(t,500)==0;%mod(t,round((1/tau)))==0
                for j=1:length(x)

```

```

if r(j)<R-lambda
    [x_aux,y_aux,z_aux]=sphere(50);
    x_aux=r(j)*x_aux;y_aux=r(j)*y_aux;z_aux=r(j)*z_aux;
    x_aux=x_aux+x(j);y_aux=y_aux+y(j);z_aux=z_aux+z(j);
    x_aux=real(x_aux);y_aux=real(y_aux);z_aux=real(z_aux);

    subplot('position',[-0.05 0.1 0.2 0.8])
    surf(x_aux,y_aux,z_aux);shading interp;
    colormap([1 1 1])
    shading interp; camlight right; lighting phong; %view(60,30)
    axis equal
    grid on
    axis([-R R -R R 0 length_conduit])
    box off
    hold on
    title('3D conduit')
else
    [x_slug,y_slug,z_slug]=cylinder;
    x_slug=x_slug*(R-lambda);x_slug=real(x_slug);
    y_slug=y_slug*(R-lambda);y_slug=real(y_slug);
    z_slug=z_slug-0.5;
    z_slug=z_slug*4*r(j)^3/(3*(R-lambda)^2);
    z_slug=z_slug+z(j);z_slug=real(z_slug);

    subplot('position',[-0.05 0.1 0.2 0.8])
    surf(x_slug,y_slug,z_slug);shading interp;
    hold on
    colormap([1 1 1])
    %view(20,15)
    shading interp; camlight right; lighting phong; %view(60,30)
    axis equal
    grid on
    axis([-R R -R R 0 length_conduit])
    box off
    hold on
    title('3D conduit')
end
end

```

```

[x_cond,y_cond,z_cond]=cylinder;x_cond=x_cond*R;y_cond=y_cond*R;z_cond=z
_cond*length_conduit;
surf(x_cond,y_cond,z_cond);shading interp;
colormap([0 0 1])
view(20,15)
set(gca, 'Visible','off')
[xunit yunit]=circle(0,0,R);
level_col=fill3(xunit,yunit,H(end)*ones(1,length(xunit)),'r');
set(level_col,'EdgeColor','None');
[xunit yunit]=circle(0,0,R);
level_col=fill3(xunit,yunit,zeros(1,length(xunit)),'r');
set(level_col,'EdgeColor','None');
alpha(0.1);
hold off

```

```

dimensionlesstime=time*Fr*sqrt(2*g*R)/H_0;
Fo=Q_out/(4.72e14*K1^0.87*mu_gas*pi*R^2);
subplot('position',[0.2 0.74 0.75 0.2])
Q_out2=(Q_out-Q)/std(Q_out);
plot(time,Q_out/mean(Q_out),'k','LineWidth',1)
xlabel('Time (s)','fontsize',13);
ylabel('Q_o_u_t(t) / Q','fontsize',13);
axis([0 time(end) 0 1.05*max(Q_out/mean(Q_out))])
set(gca,'FontSize',12)
box on

subplot('position',[0.2 0.42 0.75 0.2])
void_fraction=100*(H-H_0)./H;

plot(time,void_fraction,'k','LineWidth',1)
xlabel('Time (s)','fontsize',13);
ylabel('\epsilon(t) (%)','fontsize',13);
axis([0 time(end) 0 1.05*max(void_fraction)])
set(gca,'FontSize',12)
box on

subplot('position',[0.2 0.10 0.75 0.2])
plot(time,(P_ch)/Patm,'k','LineWidth',1)
xlabel('Time (s)','fontsize',13);
ylabel('P_c_h(t) / P_a_t_m','fontsize',13);
axis([0 time(end) 0.99*min(P_ch/Patm) 1.01*max(P_ch/Patm)])
set(gca,'FontSize',12)
box on

%%save figures and save to files
set(hFig,'Color',[0.8 0.8 0.8])

set(hFig,'Color',[1 1 1])
set(hFig,'InvertHardCopy','off');
saveas(hFig,sprintf('figure_%.2f.jpg',time(end)))
if create_direct==1;mkdir(['gamma_' num2str(gamma)]);end
create_direct=2;
loc=['..\experiments_changing_permeability_gamma_dome_3\gamma_'
num2str(gamma)];
movefile(sprintf('figure_%.2f.jpg',time(end)),loc)

filetoopen_Q_out=['..\experiments_changing_permeability_gamma_dome_3\gamma_' num2str(gamma) '\Q_out.txt'];

filetoopen_holdup=['..\experiments_changing_permeability_gamma_dome_3\gamma_' num2str(gamma) '\holdup.txt'];

filetoopen_pressure=['..\experiments_changing_permeability_gamma_dome_3\gamma_' num2str(gamma) '\pressure.txt'];
results_Q_out=fopen(filetoopen_Q_out,'w');
results_holdup=fopen(filetoopen_holdup,'w');

```

```

results_pressure=fopen(filetoopen_pressure,'w');

for k=1:length(P_ch)
    fprintf(results_Q_out,'%f',Q_out(k));
    fprintf(results_Q_out,'\t');
    fprintf(results_Q_out,'%f',time(k));
    fprintf(results_Q_out,'\r\n');

    fprintf(results_holdup,'%f',eps(k));
    fprintf(results_holdup,'\t');
    fprintf(results_holdup,'%f',time(k));
    fprintf(results_holdup,'\r\n');

    fprintf(results_pressure,'%f',P_ch(k));
    fprintf(results_pressure,'\t');
    fprintf(results_pressure,'%f',time(k));
    fprintf(results_pressure,'\r\n');
end
fclose(results_Q_out);
fclose(results_holdup);
fclose(results_pressure);
end
pause(0.01)
t=t+1;
end

Morton=g*mu_liq^4/(rho_liq*sigma^3);
Eotvos=4*rho_liq*g*R^2/sigma;
Froud=Fr;
Forchheimer=K1*Q/(K2*mu_gas*A);
GAMMA=2*L*mu_gas*Q/(K1*A*rho_gas_0*Patm);
H_R=mean(H(200:end))/R;
D_H=mean(length_conduit-H(200:end))/mean(H(200:end));
D_L=mean(length_conduit-H(200:end))/L;
r0_R=r(end)/R;

filetoopen_numbers=['..\experiments_changing_permeability_gamma_dome_3\gamma_' num2str(gamma) '\Dimensionless_numbers.txt'];
results_numbers=fopen(filetoopen_numbers,'w');
fprintf(results_numbers,'Input flux: %d ',Q);
fprintf(results_numbers,'\r\n');
fprintf(results_numbers,'\r\n');
fprintf(results_numbers,'Dimensionless numbers: ');
fprintf(results_numbers,'\r\n');
fprintf(results_numbers,'Morton: %d ',Morton);
fprintf(results_numbers,'\r\n');
fprintf(results_numbers,'Eotvos: %d ',Eotvos);
fprintf(results_numbers,'\r\n');
fprintf(results_numbers,'Froud: %d ',Froud);
fprintf(results_numbers,'\r\n');
fprintf(results_numbers,'Forchheimer: %d ',Forchheimer);
fprintf(results_numbers,'\r\n');

```

```

fprintf(results_numbers, 'GAMMA: %d ', GAMMA);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'H_R: %d ', H_R);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'D_H: %d ', D_H);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'D_L: %d ', D_L);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'r0_R: %d ', r0_R);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Parameters:');
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Height without bubbles: %d ', H_0);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Conduit length: %d ', length_conduit);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Radius of the conduit: %d ', R);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Thickness of the gas chamber: %d ', D0);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Temperature of the gas: %d ', T);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Atmospheric pressure: %d ', Patm);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Molecular weight: %d ', M);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Liquid density: %d ', rho_liq);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Liquid viscosity: %d ', mu_liq);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Surface tension: %d ', sigma);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Bubbles per second: %d ', N);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Length of the dome: %d ', L);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Gas viscosity: %d ', mu_gas);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Viscous (Darcy) permeability: %d ', K1);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Inertial (Forchheimer) permeability: %d
', K2);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Transient coefficient: %d ', gamma);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Distance of the random motion: %d ', d);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Distribution factor: %d ', c);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Breakup coefficient: %d ', beta);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Maximum stable radius for breakup: %d
', r_sta);
fprintf(results_numbers, '\r\n');
fprintf(results_numbers, 'Bursting coefficient: %d ', alpha_burst);

```

```

    fprintf(results_numbers, '\r\n');
    fprintf(results_numbers, 'Maximum stable radius for bursting: %d', r_burst);
    fclose(results_numbers);

    %%reinitialize parameters to use new value of influx Q
    time=[];time=0;dimensionlesstime=0; %initial real
time
    eps=[];eps=0; %initial
gas volume fraction = 0

    x=[];y=[];z=[]; %coordinates of the position of the bubbles.
x,y,z -> position of the center.
    r=[]; %radius of the bubbles (effective radius for the
case of Taylor bubbles)
    rho_gas=[]; %density of each bubble
    mass=[]; %mass inside each bubble
    sup_velocity=[];

    H=[];H=H_0;
    DQ_in=[];DQ_in=0;
    Q_out=[]; Q_out=0;
    P_ch=[];P_ch=Patm;

    %% Create figure
    figure(1)
    hFig = figure(1);
    set(gcf, 'PaperPositionMode', 'auto')
    set(hFig, 'Position', [20 0 600 600])

end %end for Q

```

Presentations in international meetings and additional works

Description:

Abstracts presented at IAVCEI2013, NEMOH workshop 2013, and EGU2014.
Publication in G-cubed (Girona and Costa, 2013).

Can passive degassing induce volcanic crisis?

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Openly-degassing volcanoes (e.g. Asama, Japan; Mayon, Philippines; Etna, Italy) usually erupt every few months or years. During quiescence, they passively emit several thousands of tonnes of gas per day (mainly water vapour). Such gas mass withdrawal leads to a volume change of melt because the volatiles were previously dissolved in it. On the other hand, there are evidences showing that openly-degassing volcanoes have well developed conduits with the head of the magma column close to the summit. Thus, a volume decrease of magma could decrease the mean liquid level in the conduit and may, in turn, decrease significantly the pressure at depth. This reasoning suggests that passive degassing may play an important role in the pressure evolution of magma plumbing systems. However, the problem is much more complex when we consider a response of the host-rock to pressure changes and hydraulic connection between deeper sources and shallow reservoirs. In such a case, there is a feedback between degassing-induced underpressure, closure of the conduit and reservoir, and magma replenishment. We have modelled these processes and we have found that underpressures up to some tens of MPa may easily occur during the observed repose timescales. Continuous magma intrusions through permanent hydraulic connections are induced by passive degassing without the need of a pressurized magma source. Intermittent replenishment can also be explained via passive degassing and successive openings and closures of the feeding dykes. These pressure decreases may also cause physical destabilizations of the chamber wall-rock and massive bubble nucleation and growth in the reservoir. The imbalance created by the large amount of gas released during repose may incite the frequent volcanic unrest and eruptions of openly-degassing volcanoes. This "top-down" process contrasts with the commonly proposed "bottom-up" processes of magma intrusions in shallow reservoirs driven by unspecified mechanisms occurring at depth. Coupling of top-down models with real time monitoring of total gas fluxes may open new doors for anticipating volcanic crisis.

DIPRA: A new user-friendly program to determine the timescales of magmatic processes from diffusion modelling of multiple elements in olivine

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One recent advance in igneous petrology has been to be able to quantify the timescales and rates of magmatic processes. This provides a lot of valuable information about a wide range of phenomena, such as the residence times of magmas in shallow reservoirs before eruptions. A methodology to determine these timescales is modelling the concentration of different elements in minerals, which is based on chemical diffusion within the crystals. However, despite the good knowledge of diffusion coefficients and the ease of measurements of chemical gradients, the use of this technique is not widespread yet. This could be due in part to the lack of a user-friendly tool that allows modelling the chemical zonings in crystals easily. We have developed an interface-based computer program (DIPRA: Diffusion Process Analysis) that allows modelling intuitively the diffusion of Fe, Mg, Mn, Ni, and Ca in olivine (one of the most widespread minerals in igneous rocks) by performing an automatic and quick fit to the natural chemical profiles. DIPRA accounts for most variables that affect diffusivity, such as temperature, oxygen fugacity, and anisotropy, and initial and boundary conditions can be chosen as complex as the user requires. The program also calculates the uncertainties of the diffusion time based on the uncertainty of the data and temperature. DIPRA is very versatile and, besides being a working tool for petrologists, it could also be useful for teaching kinetics of chemical re-equilibration and timescales of magmatic processes in higher education courses. The program can be downloaded from www.tgirona.com.

**Dynamics of crystal-rich magma
conduits: a preliminary study with
lattice-Boltzmann methods**

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Studying the magma dynamics within volcanoes is important to improve the interpretation of monitoring signals and understand volcanic eruption cycles. The analysis of natural rocks collected from volcanic eruptions at the Earth surface reveals that a large amount of the magma stored beneath volcanoes is crystal-rich because of degassing and cooling processes. These crystal-rich magmas may form the so-called mushy zones within volcanic conduits and reservoirs, whose behaviour can be reproduced by assuming a non-Newtonian rheology characterized by a yield stress. The question of how these mushy zones may alter and regulate the magma ascent towards the surface has been little studied so far. We address this issue by developing a new lattice-Boltzmann code to model the motion of Bingham materials through 2D channels. Our code is more stable than other lattice-Boltzmann schemes developed to simulate Bingham flows, and we use it to study the response of the Bingham material to a pressure perturbation in a wide range of dimensionless numbers. Our preliminary results show that: 1) mushy zones act as a cap within the conduit if the pressure difference between the inlet and the outlet of the channel is not larger than a critical value; 2) there is a pressure loss for large yield stresses, namely, pressure changes do not transmit efficiently through crystal-rich magma layers; 3) sudden and sharp pressure changes develop within the Bingham layer till the steady-state is reached. These preliminary results show that mushy zones affect significantly the magma dynamics by altering the evolution and transmission of the pressure beneath volcanoes. In the future, we aim to develop 2-phase lattice-Boltzmann codes to make a detailed analysis of the magma ascent when a crystal-free magma interacts with a crystal-rich magma.

Passive degassing during quiescence: a trigger of volcanic unrest

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Unveiling what initiates a volcanic unrest is crucial for making progress towards improved eruption forecasts. Many active volcanoes, like Mayon (Philippines), Masaya (Nicaragua), and Etna (Italy), show a characteristic pattern: they alternate unrest episodes, sometimes culminating in strombolian-vulcanian eruptions, with quiescent periods dominated by abundant but passive gas emissions. These quiescent periods typically last between several months or a few years, and the mean passive gas fluxes (mainly of H_2O , CO_2 , and SO_2) are on the order of several kilotonnes per day. Volcanoes with these features are the so-called openly degassing volcanoes. Here, we investigate if the duration of the quiescent periods of openly-degassing volcanoes is related to the gas released passively. We address this issue by developing a mass-balance lumped-parameter model that allows calculating how passive degassing affects the pressure at depth. Our model is driven by the gas fluxes measured with monitoring systems, and accounts for different conduit-reservoir sizes, the rheology of the crust, the bubble exsolution and expansion at depth, and the hydraulic connectivity between reservoirs and deeper magma sources. We find that, for a vast majority of scenarios, openly degassing volcanoes can depressurize several MPa in only a few months or years, coinciding with the inter-eruptive timescales. The depressurization rate is larger for larger gas fluxes, smaller magmatic systems, larger effective viscosities of the host-rock, deeper magma reservoirs, lower magma viscosity, and narrower dykes connecting the feeder magma sources with the reservoir. Our model also predicts that pressure drops of 5 - 10 MPa, considered critical for the evolution of volcanic systems, can occur after the volcanoes have released passively between 10^4 - 10^6 tonnes of SO_2 . This mass loss of SO_2 matches with the average mass of SO_2 released passively between eruptions by a dataset of 22 openly-degassing volcanoes. Thus, our results suggest that many of the unrest episodes at openly-degassing volcanoes are causally linked to the gas emitted during quiescence. We propose that degassing-induced depressurization is a trigger of different physical processes, such as magma replenishment and fractures, which can culminate in unrest episodes and eventual eruption. This study opens the possibility to use the total mass of gas released during quiescence as a new proxy to anticipate unrest at openly-degassing volcanoes.