Building energy savings using high-albedo-high-emittance (cool) roof materials

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BUILDING ENERGY SAVINGS USING HIGH-ALBEDO-HIGH-EMITTANCE (COOL) ROOF MATERIALS

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Abstract

In the tropics, the earth surface receives abundant solar radiation annually contributing significantly to building heat gain and, thus, cooling demand. An effective method that can curb the heat gains through opaque surfaces could provide significant energy savings. This study investigates the effectiveness of “high-albedo-high-emittance (or cool) roof” on reducing heat gain through opaque surfaces into buildings. It is hypothesised that building heat gain can be reduced by increasing the albedo of opaque surfaces to reflect off the incident solar radiation. At the same time, by increasing the thermal emittance to emit off the absorbed heat (as infrared radiation) to outdoor. Computational simulations (calibrated using experiments) are conducted to compare the thermal performance of a cool concrete flat roof to that of a green roof and a thermal insulation roof under the tropical climate of Singapore. The cool roof provides higher annual net heat gain reduction of about 81-83% as compared to the green roof (about 60-75%) and the thermal insulation roof (about 68-80%).

A novel and general cool roof heat transfer (CRHT) model that is based on the spectral approximation method is proposed. The CRHT model provides very concise and easy-to-use expressions to evaluate the impact of cool coating on roof temperature, ceiling temperature, indoor air temperature and heat flux on single-skin roof (SSR, i.e., solid primary roof with no air-gap between roof layers) and double-skin roof (DSR, i.e., has an air-gap in between two solid roof layers). The model can handle transient outdoor and indoor boundary conditions as experienced by naturally ventilated buildings. The CRHT models are verified against analytical (the conduction transfer function) method and validated against experiments performed in real-scale apartments in Singapore.

On a sunny day, cool coating reduces the peak temperatures and daily (integrated) heat gain of a concrete flat SSR by up to 14.1°C and 0.74 kWh/m² (or 58%), respectively. The same cool coating reduces the peak temperatures and daily heat gain of a concrete flat DSR by up to 14.7°C and 0.21 kWh/m² (or 47%), respectively. A new roof thermal transfer value (RTTV) model is proposed based on the CRHT model to accurately assimilate the effect of solar reflectance changes (due to application of cool roof) into RTTV or equivalent models for air-conditioned buildings. The proposed RTTV model incorporates new formulations for modelling the equivalent-thermal resistance increment due to the solar reflectance effect on opaque roofs. The new model shows significant improvement in accurately capturing the cool roof effect in RTTV calculation compared to existing model. This study sheds new insights on cool roof performance in tropical climate. The proposed CRHT and RTTV models provide the foundation for accurate modelling of cool roof heat transfer characteristics. These proposed models are also applicable to other climates.

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Glossary

CRHT	Cool Roof Heat Transfer model
DSR	Double-skin roof
ETTV	Envelope Thermal Transfer Value (W/m^2)
OTTV	Overall Thermal Transfer Value (W/m^2)
RTTV	Roof Thermal Transfer Value (W/m^2)
$RTTV^A$	RTTV model based on method A (described in Table 2.3)
$RTTV^B$	RTTV model based on method B (described in Table 2.3)
R-value	Thermal resistance ($\text{m}^2\text{-K/W}$)
SSR	Single-skin roof
TMY	Typical Meteorological Year (includes weather data on an hourly basis)
U-value	Overall heat transfer coefficient ($\text{W/m}^2\text{-K}$)

Nomenclature

a, b	Intermediate coefficients
$(ach)_{gap}$	Air change per hour in air-gap (m^3/h)
$(ach)_i$	Air change per hour in indoor (m^3/h)
b_{gap}	Width of air-gap {m}
B^+, D^+, E^+, G^+	Intermediate variables
C_1, C_2	Intermediate constants
C^+	Intermediate variable
C_{SR}	Thermal specific heat of secondary roof (J/kg-K)
C_R	Thermal specific heat of primary roof (J/kg-K)
C_{air}	Thermal specific heat of air, (J/kg-K)
CF	Correction factor for solar heat gain through fenestration (includes the effect of orientation and pitch angle of roof)
CRHT	Cool roof heat transfer model
CTF	Conduction transfer function
D_h	Hydraulic diameter (m)
DSR	Double-skin roof
$ETTV$	Envelope Thermal Transfer Value (W/m^2)
$\hat{f}$	Roughness factor in Eq. (4.22a)
F	Intermediate variable
$\hat{F}_{C, floor}$	View factor of ceiling surface to the floor surface
$\hat{F}_{R, sky}$	View factor of primary roof surface to the sky
$\hat{F}_{SR, sky}$	View factor of secondary roof surface to the sky
g	Acceleration due to gravity, 9.81 (m/s^2)

Gr	Grashof number
h_i	Overall heat transfer coefficient for primary ceiling (W/m ² -K)
$h_{i,conv}$	Convective heat transfer coefficient for primary ceiling (W/m ² -K)
$h_{i,r}$	Radiative heat transfer coefficient for primary ceiling (W/m ² -K)
h_R	Overall heat transfer coefficient for primary roof (W/m ² -K)
$h_{R,conv}$	Convective heat transfer coefficient for primary roof (W/m ² -K)
$h_{R,r}$	Radiative heat transfer coefficient for primary roof (W/m ² -K)
h_{SR}	Overall heat transfer coefficient for secondary roof (W/m ² -K)
h_{SC}	Overall heat transfer coefficient for secondary ceiling (W/m ² -K)
I_{abs}	Absorbed incident radiation (W/m ²)
I_{in}	Incident solar radiation (W/m ²)
I^{IR}	Infrared radiation (W/m ²)
$I_{re,\lambda}$	Spectral reflected radiation (W/m ²)
$I_{in,\lambda}$	Spectral incident radiation (W/m ²)
$I_{e,\lambda}$	Spectral infrared radiation emitted by an opaque surface (W/m ²)
$I_{b,\lambda}$	Spectral infrared radiation emitted by a blackbody (W/m ²)
k	Thermal conductivity of original roof material (W/m-K)
k_{air}	Thermal conductivity of air (W/m-K)
k^{cool}	Thermal conductivity of cool layer or coating (W/m-K)
K_{cond}^{cool}	Conduction resistance of the cool layer or coating (m ² -K/W), l^{cool}/k^{cool}
K_{cond}	Conduction resistance of original opaque roof (m ² -K/W), L/k
K_i	Air film resistance of ceiling surface with indoor (m ² -K/W), $K_i = 1/h_i$
K_o	Air film resistance of roof surface with outdoor (m ² -K/W), $K_o = 1/h_R$

K^p	Equivalent thermal resistance increment due to increased reflectance (m^2 -K/W)
K^{incr}	Equivalent total thermal resistance increment on a roof (m^2 -K/W)
l^{cool}	Thickness of cool layer (m)
L or L_R	Thickness of primary roof (m)
L_{gap}	Air-gap height (m)
L_i	Distance from ceiling to indoor where adiabatic condition is assumed (m)
L_{SR}	Thickness of secondary roof (m)
LW	Long wave
m	Integer
N	Annual operating hours of the air-conditioning system (hour)
P	Cloud cover
q_{abs}	Absorbed solar flux (W/m^2)
q_C	Heat flux through the ceiling (W/m^2)
q_{cool}	Conduction heat flux through cool roof (W/m^2)
q_{conv}	Convective heat flux (W/m^2)
q_{org}	Conduction heat flux through original roof (W/m^2)
q_r	Radiative heat flux (W/m^2)
q_R	Conduction heat flux through primary roof (W/m^2)
$q_{R,\rho}$	Conduction heat flux through primary roof having reflectance of ρ , (W/m^2)
$q_{R,\rho=0}$	Conduction heat flux through primary roof having reflectance $\rho = 0$, (W/m^2)
q_{SR}	Conduction heat flux through secondary roof (W/m^2)

q_{system}	Air-conditioning system's output (W/m ²)
Q_{gain}	Daily heat gain (integrated-hourly downward conduction heat flux) (kWh/m ²)
RH	Relative humidity (%)
R_{cool}	R -value of cool roof (m ² -K/W), $R_{cool} = 1/U_{cool}$
R_{org}	R -value of original roof (m ² -K/W), $R_{org} = 1/U_{org}$
R_R	R -value of primary roof (m ² -K/W)
$R_{R,\rho=0}$	R -value of primary roof having reflectance of $\rho = 0$, (m ² -K/W)
$R_{R,\rho}$	R -value of primary roof having reflectance of ρ , (m ² -K/W)
$R_{R,\rho=0}^{new}$	R -value due to K^{incr} and $R_{R,\rho=0}$ (m ² -K/W)
RTTV	Roof Thermal Transfer Value (W/m ²)
$RTTV^{new}$	New $RTTV$ model (proposed in this study)
SC	Shading coefficient of fenestration
SKR	Skylight ratio of roof (skylight area / gross area of roof)
SSR	Single-skin roof
SW	Short wave
t	Time (hour)
T	Temperature (K)
T_c	Temperature of primary ceiling surface (K)
$\bar{T}_c$	Temperature of primary ceiling at time $t = 0$ (K)
T_{floor}	Temperature of floor (K)
$\bar{T}_{floor}$	Temperature of floor at time $t = 0$ (K)
T_{gap}	Temperature of air-gap (K)
$\bar{T}_{gap}$	Temperature of air-gap at time $t = 0$ (K)

T_{gap}^s	Simulated air-gap temperature (K)
$\tilde{T}_{gap}$	Mean air-gap temperature (K)
T_i	Indoor air temperature (K)
$\bar{T}_i$	Indoor air temperature of at time $t = 0$ (K)
T_o	Outdoor air temperature (K)
$\bar{T}_o$	Outdoor air temperature of at time $t = 0$ (K)
T_R	Temperature of primary roof surface (K)
$\bar{T}_R$	Temperature of primary roof at time $t = 0$ (K)
T_{SC}	Temperature of secondary ceiling surface (K)
$\bar{T}_{SC}$	Temperature of secondary ceiling at time $t = 0$ (K)
T_{SR}	Temperature of secondary roof surface (K)
$\bar{T}_{SR}$	Temperature of secondary roof at time $t = 0$ (K)
T_{sky}	Temperature of the sky (K)
$T_{Sol-air}$	Solar-air temperature obtained from TMY data (K)
$\tilde{T}_{Sol-air}$	Mean solar-air temperature (K)
TD_{eq}	Equivalent temperature difference for opaque roof in <i>RTTV</i> model (K)
TD_{eq}^{CRHT}	Equivalent temperature difference for opaque roof in <i>CRHT</i> model (K)
ΔT	Temperature difference between outdoor and indoor (K)
TMY	Typical Meteorological Year (includes weather data at hourly basis)
$U_{skylight}$	U -value of the skylight (W/m ² -K)
U_{ρ}^{new}	U -value of opaque roof after incorporating the effect of reflectance (W/m ² -K)
U_R	U -value of primary roof (W/m ² -K)

$U_{R,\rho=0}$	U -value of primary (opaque) roof having solar reflectance of $\rho = 0$, (W/m ² -K)
$U_{R,\rho}$	U -value of primary (opaque) roof having solar reflectance of ρ , (W/m ² -K)
U_{SR}	U -value of secondary roof (W/m ² -K)
U_{window}	U -value of window (W/m ² -K)
U^+	Intermediate variable
V	Wind speed (m/s)
VOL	Volume (m ³)
W	Width of roof in Eq. (4.23c), (m)
x, y	Space coordinates
X	Exterior coefficient in Eqs. (4.24) and (4.25)
Y	Cross coefficient in Eqs. (4.24) and (4.25)
Z	Interior coefficient in Eqs. (4.24) and (4.25)

Greek symbols

α	Solar absorptance ($0 < \alpha < 1$) for opaque surface $\alpha = 1 - \rho$
α_R	Solar absorptance of primary roof ($0 < \alpha_R < 1$)
α_{SR}	Solar absorptance of secondary roof ($0 < \alpha_{SR} < 1$)
α_{SR}^*	Thermal diffusivity of secondary roof (m ² /s)
α_R^*	Thermal diffusivity of primary roof (m ² /s)
β	Decrement factor ($0 \leq \beta \leq 1$)
ε	Thermal emittance ($0 < \varepsilon < 1$)
ε_C	Thermal emittance of primary ceiling ($0 < \varepsilon_C < 1$)
ε_i	Thermal emittance of indoor facing surfaces ($0 < \varepsilon_i < 1$)

ε'_λ	Spectral directional emittance ($0 < \varepsilon'_\lambda < 1$)
ε_{SR}	Thermal emittance of secondary roof ($0 < \varepsilon_{SR} < 1$)
ε_{SC}	Thermal emittance of secondary ceiling ($0 < \varepsilon_{SC} < 1$)
ε_R	Thermal emittance of primary roof ($0 < \varepsilon_R < 1$)
ε_{floor}	Thermal emittance of floor ($0 < \varepsilon_{floor} < 1$)
$\tilde{\theta}$	Zenith angle
$\tilde{\phi}$	Azimuth angle
ϕ	Tilt angle of the roof surface from horizontal plane
$\theta(t)$	Time dependent function
$\Phi(y)$	Basis function
Θ	Flux in Eqs. (4.24) and (4.25)
ψ	Time lag between harmonic of roof and ceiling temperatures
ρ	Solar reflectance ($0 < \rho < 1$)
ρ'_λ	Spectral directional reflectance ($0 < \rho'_\lambda < 1$)
ρ_λ	Spectral reflectance ($0 < \rho_\lambda < 1$)
ρ_{SR}^*	Mass density of secondary roof (kg/m^3)
ρ_R^*	Mass density of primary roof (kg/m^3)
ρ_{air}^*	Mass density of air (kg/m^3)
$\varepsilon_R I^{IR} / h_R$	Correction factor for infrared radiation, (K)
$\varepsilon_{SR} I^{IR} / h_{SR}$	Correction factor for infrared radiation (K),
σ	Stefan-Boltzman's constant ($\text{W/m}^2\text{-K}^4$), $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4$
δ	Volumetric thermal expansion coefficient of air ($1/\text{K}$)
$\mathcal{G}_{air}$	Dynamic viscosity of air (kg/m-s)

$\lambda_m^2, \mu^+, \nu^+, \eta^+$	Intermediate variables
w	Angular velocity (rad/h)
λ	Wavelength (or spectral)

Subscripts

C	Primary (opaque) ceiling
$cool$	Cool roof
$cond$	Conduction
$conv$	Convection
eq	Equivalent
gap	Air-gap
i	Indoor
in	Incident radiation
m	Integer
o	Outdoor
org	Original roof
opt	Optimum
out	Outgoing radiation
r	Radiation
R	Primary (opaque) roof
$sol-air$	Solar-air
SC	Secondary (opaque) ceiling
SR	Secondary (opaque) roof
λ	Wavelength (or spectral)

Superscripts

<i>Cool</i>	Cool layer
<i>e</i>	Experimental
<i>IR</i>	Infrared radiation
<i>incr</i>	Increment
<i>new</i>	New model (proposed in this study)
<i>Opt</i>	Optimum
+	Intermediate variable
<i>s</i>	Simulated
ρ	Solar reflectance

Publications Arising from this Thesis

Journal papers

Kishor T. Zingre, Man Pun Wan, Shanshan Tong, Hua Li, Victor W.-C. Chang, Swee Khian. Wong, Winston Boo Thian Toh and Irene Yen Leng Lee. “Modeling of cool roof heat transfer in tropical climate”, *Renewable Energy* 75;210-223, 2015.

Kishor T. Zingre, Man Pun Wan, Swee Khian. Wong, Winston Boo Thian Toh and Irene Yen Leng Lee “Modelling of cool roof performance for double-skin roofs in tropical climate”, *Energy*, 2015. (Accepted for publication)

Kishor T. Zingre, Man Pun Wan and Xingguo Yang, “A New Roof thermal transfer value (RTTV) calculation method for cool roofs”, *Energy*, 2015. DOI. 10.1016/j.energy.2014.12.030.

Shanshan Tong, Hua Li, Kishor T. Zingre, Man Pun Wan, Victor W.-C. Chang, Swee Khian. Wong, Winston Boo Thian Toh and Irene Yen Leng Lee. “Thermal performance of concrete roofs in tropical climate”, *Energy and Buildings* 76:392-401, 2014.

Conference papers

Kishor T. Zingre, Man Pun Wan, “An investigation of heat transfer characteristics of cool coatings for building roofs in tropical climate of Singapore”, *Proceedings of ASME Conference and Exposition, IMECE2013* November 15-21, paper #63999, San Diego, California, USA, 2013.

Kishor T. Zingre, Man Pun Wan, “Cool Roof Heat Transfer models for non-ventilated and ventilated roofs in tropical climate”, *Proceedings of ELSEVIER Conference, ISBE2014* January 28-30, paper #133, Doha, Qatar, 2014.

Kishor T. Zingre, Xingguo Yang and Man Pun Wan, “An investigation of the effect of cool coatings on its micro-climate and on the opposite buildings”, *Proceedings of ASME Conference and Exposition, IMECE2014* November 14-20, paper #69769, Montreal, Canada, 2014.

Kishor T. Zingre, Xingguo Yang, and Man Pun Wan, “Optimum air-gap height for double-skin roofs”, *Proceedings of ASME Conference and Exposition, IMECE2014* November 14-20, paper #40837, Montreal, Canada, 2014.

Kishor T. Zingre, Xingguo Yang, and Man Pun Wan, “An equivalent R-value increment due to cool roof coatings”, *Proceedings of ASME Conference and Exposition, IMECE2014* November 14-20, paper #40873, Montreal, Canada, 2014.

Chapter 1

Introduction

1.1 Motivation

Energy plays an important role in shaping Singapore's economic growth. However, the city-state mainly relies on foreign countries to fulfil its energy needs. About 82% of country's main sources of crude oil come from the Middle Eastern countries, while the remaining 18% from Vietnam, Malaysia, Australia and other countries (Tan et al., 2010). The electricity generation supplied to the country's national grid comes from 75.8% natural gas, 21.6% fuel oil and 2.6% refuse (Kua & Wong, 2012).

Reducing building heat gain for air-conditioned buildings and indoor thermal discomfort for naturally ventilated buildings are some of the main concerns of occupants in the tropical region. At the tropics, the earth surface receives abundant solar radiation (an annual-average of about 1680 kWh/m²) contributing significantly to the building heat gain (Wittkopf et al., 2012). Energy savings measures play a vital role in an industrialised and urbanised country like Singapore which endures deficiency of natural gas and oil for the production of electricity. Singapore is one of the highly developed countries in South East Asia situated in tropical zone at latitude 1.4°N and 104°E longitude (Chen & Chang, 2012). It is a small island city-state of 683 km² area with rapidly growing population from 2.3 - 2.6 million in 1980's (Goh et al., 1983; de Dear and Leow, 1990) to her current population of 5.2 million (Chong et al., 2013) and a projected population of 5.8 million in 5-10 years' time (Wong et al., 2003; Velasco and Roth, 2012). The speedy development of economic activities, scarcity of land resources and high population density of 4200 people per km² (Chong et al., 2013) has resulted in the rise of commercial as well as residential buildings such as multi-storey offices, retail complexes and public housing apartments.

According to a building energy study conducted by the Building and Construction Authority (BCA) of Singapore, the building sector consumes about 57% of the country's total electricity production (Chua et al., 2013). Air-conditioning of buildings alone consume about 60% of the electricity consumed by the building sector (Chua & Chou, 2011). This shows that about 34% of the country's total electricity production is being consumed for air-conditioning of the buildings alone (Chua & Chou, 2011). Thus, potential returns on building air-conditioning energy saving measures are numerous and at highest priority. The percentage of households in Singapore which installed air-conditioner (AC) units has significantly increased over a period of three decades from below 1 in 10 households in 1970-80's to nearly 7 in 10 households in 2000-10 (Wong et al., 2002; Chua & Chou, 2010) as illustrated in Fig. 1.1.

Fig. 1.1 AC units installation by households in Singapore (Chua & Chou, 2010).

This resulted in the increased demand for electricity generation rate and hence more import of the fossil fuels. The rapid increase in air-conditioning usage has led to

economic as well as environmental effects. The economic effects include more demand for electricity generation, more import of fossil fuel, more fossil fuel burning and rise in price of electricity bills. While, the environmental effects include increase in emission of greenhouse gases such as CO₂, NO, NO₂, CH₄ from refineries, resulting in increased outdoor air temperature, atmospheric reaction rate, ozone levels and Urban Heat Island (UHI) effect (Wong and Yu, 2005). The CO₂ emission for electricity generation in various countries is shown in Fig. 1.2.

Fig. 1.2 CO₂ emission factor for electricity generation in various countries (Tan et al., 2010).

Even though the CO₂ emission factor is comparatively lower in Singapore than other Asian countries like Taiwan, Malaysia, Thailand and South Korea, but being a small island city-state and the continuously increasing CO₂ emission rate has alerted the threat of Urban Heat Island effect (Rajagopalan et al., 2008).

Passive building cooling is a concept completely in line with the notion of sustainable buildings and it is an alternative to the mechanical air-conditioning systems. Chua &

Chou (2010) conducted computational simulations in a high-rise (12-storey), air-conditioned residential apartment building in Singapore and reported the distribution of air-conditioning loads of the building. They reported that the heat gains through the opaque envelope surfaces constitute about 30% (including 19% through walls and 11% through roof). It is hypothesised that the reduction in building heat gain through opaque surfaces can be achieved passively by increasing albedo (solar reflectance) to reflect off the incident solar radiation and by increasing emittance to emit off the absorbed heat to outdoor. In recent years, building surfaces with high solar reflectance and high emittance (or widely referred as “Cool Surfaces”) have attracted increasing attentions as a passive cooling solution for: reducing heat gain through opaque surfaces (Kolokotroni et al., 2013; Sproul et al., 2014), mitigation of UHI (Rajagopalan et al., 2008; Gaitani et al., 2011), enhancing COP of cooling system (Wray & Akbari, 2008) and cooling energy savings by lowering the temperature of outdoor air that is supplied to cool down the condenser unit located on rooftop (Wray & Akbari, 2008).

Since roofs receive more solar radiation than walls, these cool surfaces see more applications on roofs than walls, thus, are widely known as “cool roof”. In the U.S., cool roofs refer to the roof surfaces with solar reflectance (ρ) of ≥ 0.55 and thermal emittance (ε) of ≥ 0.75 (Cool Roof Rating Council, 1998). Solar Reflectance Index (*SRI*) is another metric in wide use for quantifying the coolness of surfaces using fixed black and white standards under fixed weather conditions and combined solar reflectance and emittance values (Akbari & Levinson, 2008). Standard black surface ($\rho = 0.05$ and $\varepsilon = 0.90$) refers to $SRI = 0$, standard white surface ($\rho = 0.80$ and $\varepsilon = 0.90$) refers to $SRI = 100$ and the cool roofs refer to the surfaces with $SRI \geq 78$ (for low-slope roofs) and 29 (for steep-slope roofs). Cool roofs are often achieved by adding a

layer of thin cool coating or single-ply membrane on top of the original primary (or base) roof. Some of the economic and environmental benefits of cool roof method are shown in Fig. 1.3.

Fig. 1.3 Economic and environmental benefits of cool roof (Taha, 1997).

It is hypothesised that the performance of a cool roof would be more prominent where incident solar radiation is abundant since the cool roof works on the principle of high solar reflectance when it is exposed to incident solar radiation. The tropical climate of Singapore receives abundant annual-average solar radiation compared to other locations (Sproul et al., 2014; Kolokotroni et al., 2013; Chan & Chow, 2013; Kolokotsa et al., 2012; Boixo et al., 2012; Chan, 2011) where the studies on cool roof have been mostly performed, as shown in Fig. 1.4. Air-conditioning of buildings is also sought throughout the year in the tropics, suggesting that the potential heating penalty (Zinzi & Agnoli, 2011) of cool roof does not exist in tropical climate applications.

Fig. 1.4 Annual-averaged solar radiation for various countries (Wittkopf et al., 2012).

Along with cool roof, there are several other potential methods such as thermal insulation, green roof and double-skin roof (DSR) for curbing heat gain into buildings through the opaque surfaces, as discussed below.

Thermal insulation: It works as a thermal barrier to heat gain into buildings (Ozel, 2012). It is often achieved by adding thermal insulation to the original primary roof material. During day time, the application of thermal insulation offers high thermal resistance to heat flux (from outdoor to indoor) through opaque building surfaces. While, during night time (when the heat transfer direction reverses), the heat flux from indoor to outdoor is hindered. This could cause indoor thermal discomfort for the naturally ventilated buildings with high internal heat sources. On the other hand, the thermal resistance offered by insulation materials is mainly because of air (thermal conductivity of 0.025 W/m-K at 25°C) which is a bad conductor of heat (Samah et al., 2012). However, as the temperature of the roof surface and nearby air increases, the

thermal resistance of air decreases, resulting increase in heat conduction through the insulated roof. At the same time, diurnal temperature cycles (experienced by the insulation material) develop thermal stresses within the insulation material, affecting its life span and durability. From economical perspective, in order to apply the thermal insulation, thickness of building envelope needs to be increased. This is quite difficult for the already existing buildings and even if it is possible for new building, it consumes large amount of material.

Green roof: It is a passive cooling method (Chan & Chow, 2013; Sailor, 2008) which works on the principle of evapotranspiration (due to moist soil and vegetation) and thermal insulation (due to thick soil layer). The vegetation layer absorbs the incident solar radiation. Some portions of which are utilised for photosynthesis processes (of vegetation), while some other portions are released to outdoor as evaporation. Green roof is achieved by installing (from bottom to top) a waterproofing membrane, followed by a thick layer (about 500-mm-thick) of moist soil and a layer of vegetation. The thick soil layer offers high conduction resistance (about $1.25 \text{ m}^2\text{-K/W}$) to heat gain (from outdoor to indoor) during day time. However, during night time (when the heat transfer direction reverses), the heat flux from indoor to outdoor is hindered. This could cause indoor thermal discomfort for naturally ventilated buildings with high internal heat sources. On the other hand, due to the weight and bulky nature, the installation of a green roof is difficult for old buildings which cannot endure the weight and also the buildings with sloped roof. The performance of a green roof is highly dependent on the moisture content of the soil layer and so it needs regular maintenance (Perez et al., 2012).

Double-skin roof (DSR): It is a passive cooling method (Biwole et al., 2008) which works on the principle of providing an air-gap between two solid roofs. DSR comprises two solid roofs (secondary roof on the top and primary roof at the bottom) separated by an air-gap. The secondary roof provides protection (to the primary roof) from the outdoor environment, and the airflow in the air-gap effectively takes away the heat to outdoor, resulting reduction in heat gain into the indoor environment through the primary roof. However, during night time (when the heat transfer direction reverses), the heat flux from indoor to outdoor is hindered due to the thermal resistance provided by the air-gap and secondary roof. This could cause indoor thermal discomfort for the naturally ventilated buildings with high internal heat sources. In general, ferro-cement material is commonly used as the secondary roof and concrete stumps are used to create air-gap between the secondary roof and primary roof (Wong & Li, 2007). Due to the weight of ferro-cement material (plus concrete stumps), the installation of DSR is difficult for old buildings which cannot endure the weight and the buildings with sloped roof.

Cool roof: It is a passive cooling method (Hernandez-Perez et al., 2014) which works on the principle of reflecting off the incident solar radiation during day time and emitting off the stored heat in the opaque material to outdoors when the sky is clear. It is often achieved by adding a layer of thin cool coating (about 0.5-mm-thick) or single-ply membrane on top of the original opaque primary roof surface. Even though, it emits most of the heat during night, its adverse impact on environment is less (compared to other methods) as the heat absorbed by the building material is less (Gaitani et al., 2011). The surface temperature of the cool surface does not change much and, so it avoids development of thermal stresses in the roof material. Other advantages of using the cool roof method includes the fact that it is easy to apply on

old and new buildings as well as flat and sloped surfaces unlike thermal insulation, green roof and DSR. In addition, cool roofs can be applied to the buildings with high internal heat sources, since the conduction resistance added by cool coating ($0.01 \text{ m}^2\text{-K/W}$) to the original primary roof is insignificant as compared to that added by thermal insulation, green roof and DSR (which are in the range $0.80 \text{ m}^2\text{-K/W}$ to $1.25 \text{ m}^2\text{-K/W}$). At the same time, the performance of cool roofs is prominent for climates that receive abundant solar radiation and where summer period is longer than winter such as the tropical climate of Singapore which receives abundant solar radiation (as shown in Fig. 1.4) and it is summer throughout the year (Dong et al., 2005). This highlights the suitability of cool roof for the tropical climate, buildings with high internal heat sources, old or new buildings (with any slope) and ease of installation as compared to thermal insulation, DSR and green roof methods.

As discussed, the thermal performances of cool roof, green roof and thermal insulation methods are highly climate dependent. Over the years, evaluation of performances of these methods have been mostly studied in Europe and the U.S. (Georgescu et al., 2014; Sproul et al., 2014; Zinzi & Agnoli, 2011). Cool roof has been increasingly recognised in Singapore. BCA Green Mark Scheme (BCA, 2011) has allotted 3 green mark points for using cool coatings in landed buildings and 1 to 7 green mark points for residential buildings (BCA, 2011). These facts indicate the gaining interest of the Singapore Government in adopting the cool roof. In South-East Asia, several large-scale projects on cool roof are being performed such as APEC (2013), IIPH (2013) and RDE (2013). Lawrence Berkeley National Laboratory (LBNL, 2012), has collaborated with various South-East Asian countries to perform cool roof studies, indicating the gaining popularity of cool roof in the region. However, no studies have been conducted yet in the tropical climate, resulting the thermal performances of these

methods and the effects of cool roof properties (solar reflectance and thermal emittance), green roof properties (evapotranspiration and soil layer) and thermal insulation on building heat gain are least understood for the tropical climate. Therefore, there is a great need of comprehensive studies to be conducted to investigate how effective is the performance of cool roof method as compared to thermal insulation and green roof method for the tropical climate.

Heat transfer through an opaque roof/wall is a mixed-mode heat transfer phenomenon. The roof/wall surface (exposed to outdoor) heat exchanges mainly consist of the radiation component (solar heat gain and thermal emission) and the convection component (heat loss/gain with surrounding air). A cool roof affects the radiation component by providing high radiation properties (solar reflectance and thermal emittance). In order to quantitatively analyse the impact of cool coating on the single-skin roof (SSR) heat transfer phenomenon, a robust heat transfer model is essential. Over the years, numerous analytical methods, such as response factor (RF) and conduction transfer function (CTF), and numerical methods, such as Finite Difference (FD) and Finite Element (FE) have been developed. In the implementations, the RF method requires long series of temperature histories or heat flux data. The CTF method requires a set of pre-calculated coefficients (Delcroix et al., 2013) which are tabulated for certain types of building materials and for specific climate conditions, limiting its potentials for general uses. The FD and FE methods overcome the drawback of the RF and CTF methods. However, due to the need for discretisation, these methods resolve the detailed temperatures distributions within the slab material which is unnecessary for air-conditioning load calculations and require a time step of 15 min or less to achieve sufficient accuracy. This results in demand for heavy computational resources. Therefore, a novel heat transfer model is essential for SSR

which 1) can model the effect of cool roof and provide very concise solution, 2) does not resolve the detailed temperature distribution within the slab and 3) can use higher time step (than that required by FD and FE methods).

Over the years, investigations on the thermal performance of DSR were mainly carried out by performing experiments (Fracastoro et al., 1997; Soubdhan et al., 2005), while few studies (Gomez & Galvez, 2013; Dimoudi et al., 2006; Yew et al., 2013) were devoted to the formulation of heat transfer models. Numerical methods such as FD and FE have been developed and implemented for the thermal analysis of transient heat transfer through the DSR (Dimoudi et al., 2006; Yew et al., 2013; Lai et al., 2008; Chang et al., 2008). However, the previous models (Dimoudi et al., 2006; Yew et al., 2013; Lai et al., 2008; Chang et al., 2008) were formulated based on the assumption of constant indoor air temperature boundary condition, limiting their applicability for naturally ventilated buildings. In order to investigate the thermal performance of DSR alone and a combination of cool roof + DSR for naturally ventilated buildings, and since 86% of the country's total population is living in naturally ventilated residential buildings (Chua & Chou, 2010), a robust heat transfer model is required that can handle transient outdoor and indoor boundary conditions. At the same time, the model 1) should model the effect of cool roof and provide very concise solution, 2) does not resolve the detailed temperature distribution within the slab and 3) uses higher time step (than that required by FD and FE methods).

A number of countries/regions in tropical/sub-tropical Asia (including Hong Kong, India, Sri Lanka, Thailand, Indonesia, Malaysia, Philippines and Pakistan) adopted the approach of using Overall Thermal Transfer Value (*OTTV*) (Building Energy Code of Pakistan, 1990; Chan & Chow, 2013; Devgan et al., 2010), or Envelope Thermal Transfer Value (*ETTV*), or Roof Thermal Transfer Value (*RTTV*) (Code on Envelope

Thermal Performance for Buildings in Singapore, 2008) in Singapore, to evaluate and regulate the thermal performance of the envelopes of air-conditioned buildings. Either *OTTV* or *ETTV* or *RITV* is essentially the annual-average heat gain through the building envelope under the typical climate conditions of the location concerned. Methods to estimate these models (*OTTV* or *ETTV* or *RITV*) currently adopted by numerous South East Asian countries have different inherent limitations in accurately evaluating the thermal performance of cool roofs. These existing methods either use a fixed value to represent the solar reflectance effect or assume a linear-correlation between annual-average conduction heat gain and solar absorptance, which are shown to be inaccurate. Therefore, a new method is essential to accurately assimilate the effect of solar reflectance of the opaque roof/wall surface in predicting the thermal performance of air-conditioned buildings with cool roof.

In order to fill up the above discussed knowledge gaps, the knowledge hierarchy of this work is drawn (as shown in Fig. 1.5). The dash-line rectangles (with grey background) are the knowledge gaps to be filled by this study, while the dash-line rectangles (with white background) are the knowledge available from existing literature. The ultimate goal of this work (as shown by the solid-double-line rectangle on top) is to investigate the heat gain reduction through opaque roof/wall surfaces using cool roof in tropical climate and comparison with green roof, thermal insulation and DSR. Correspondingly, the objectives and scope of this work are presented as follows.

Fig. 1.5 The knowledge hierarchy of this work.

1.2 Objectives and scope

The objectives and scope of this work are

- 1) to investigate the thermal performance of cool roof in tropical climate and make comparisons to thermal insulation and green roof methods. In order to achieve this objective, computational simulations (using experimentally calibrated computational model) will be performed on a real-scale air-conditioned building in tropical climate of Singapore (Chapter 3),
- 2) to analyse the thermal performance of SSR (with cool roof) against DSR (with and without cool roof) for naturally ventilated buildings. In order to achieve this objective, cool roof heat transfer (CRHT) models will be formulated (using spectral approximation method) for SSR (Chapter 4) and DSR (Chapter 6) that can handle transient outdoor and indoor boundary conditions as experienced by naturally ventilated buildings. The CRHT models will be verified against analytical method (the

CTF) and validated against real-scale measurements on two identical configured, side-by-side naturally ventilated apartments test buildings, and

3) to assimilate the effects of opaque surface solar reflectance changes (due to application of cool roof) into roof thermal transfer value (RTTV or equivalent) model for air-conditioned buildings. In order to achieve this objective, a new RTTV model will be formulated with new formulation for equivalent R -value increment due to the increased solar reflectance of opaque roofs using the CRHT model (for SSR). The new formulation will be incorporated into U -value of the heat conduction gain component. The new RTTV model will be validated against computational simulations and experiments on a real-scale air-conditioned building in Singapore (Chapter 5).

1.3 Thesis outline

Chapter 1 introduces motivation for this research. This chapter provides background information about the importance of building air-conditioning energy savings in tropical climate, followed by a review on potential building cooling methods such as cool roof, thermal insulation, green roof and double-skin roof. The objectives, scope and thesis outline are also presented.

Chapter 2 discusses the characteristics and working mechanisms of cool roof method, followed by detailed energy balance at a cool roof surface. This chapter also provides a comprehensive literature survey and research gap in modelling the cool roof heat transfer for SSR and DSR of naturally ventilated buildings, and various approaches adopted in different countries to incorporate the effect of solar reflectance of opaque surfaces on annual-averaged heat gain for air-conditioned buildings.

Chapter 3 presents computational simulations (using experimentally calibrated computational model) on an air-conditioned building to investigate the thermal performance of cool roof, green roof and thermal insulation in tropical climate. The

effects of individual component such as solar reflectance, thermal emittance, evapotranspiration, soil layer and thermal insulation on annual net heat gain through opaque roofs and cooling energy savings are also investigated.

Chapter 4 presents formulation of the CRHT model for SSR (using the spectral approximation method) that can handle transient outdoor and indoor boundary conditions and its verification and experimental validation. Results of the effect of cool roof on the conduction heat flux, surface temperatures and indoor air temperature are presented and discussed for commonly used concrete and metal roofs. Furthermore, the effect of wind speed on the performance of cool roof is discussed.

Chapter 5 presents a novel method to assimilate the effects of opaque surface solar reflectance changes (due to application of cool roof) on annual-averaged heat gain for air-conditioned buildings. In this chapter, formulation of a new RTTV model (based on the proposed novel method) and its computational and experimental validation are presented. Discussions on the practicality of the new RTTV model are also presented.

Chapter 6 presents formulation of the CRHT model for DSR (using the spectral approximation method) that can handle transient outdoor and indoor boundary conditions and its experimental validation. Results on the thermal performance of cool DSR on the conduction heat flux, surface temperatures and indoor air temperature for naturally ventilated buildings are discussed. Impact of cool roof on DSR and SSR are studied in tropical and Mediterranean climates.

Chapter 7 presents key results of the thermal performance of cool roof, green roof and thermal insulation in tropical climate. Also, the accuracy imparted by the proposed new RTTV model and the thermal performance of cool roof for SSR and DSR are presented. Subsequently, some recommendations for future work are provided.

Chapter 2

Literature Review

This chapter is intended to provide a comprehensive literature survey of the existing studies on modelling the cool roof heat transfer for single-skin roof (SSR) and double-skin roof (DSR). It also discusses the currently adopted methods by various South East Asian countries to assimilate the effects of opaque surface solar reflectance changes (due to application of cool roof) into annual-average heat gain prediction models. Firstly, the characteristics of cool roof are presented, followed by a discussion on energy balance at a cool roof surface. Finally, the research gaps in 1) modelling the cool roof heat transfer for SSR, 2) modelling the cool roof heat transfer for DSR, 3) modelling the effect of solar reflectance changes of opaque surfaces into annual-average heat gain prediction models, and 4) evaluation of thermal performance of cool roof against green roof and thermal insulation in tropical climate, are discussed.

2.1 Characteristics of cool roof

(a) Solar reflectance (ρ): Solar reflectance is a measure of ability of a surface to reflect the incident solar radiation over a hemisphere (0 to 180°) and at wavelengths of the solar spectrum. It is measured on a scale of 0 to 1. It designates the weighted average of spectral directional reflectance as given by Eq. (2.1)

$$\text{Solar reflectance } (\rho) = \frac{\int_{300}^{2500} \int_0^{2\pi} \int_0^{\pi/2} \rho'_\lambda I_{in,\lambda} \cos(\tilde{\theta}) \sin(\tilde{\theta}) d\tilde{\phi} d\tilde{\theta} d\lambda}{\int_{300}^{2500} \int_0^{2\pi} \int_0^{\pi/2} I_{in,\lambda} \cos(\tilde{\theta}) \sin(\tilde{\theta}) d\tilde{\phi} d\tilde{\theta} d\lambda} \quad (2.1)$$

where ρ'_λ is the spectral directional reflectance, $I_{in,\lambda}$ is the spectral incident radiation, $\tilde{\theta}$ is the zenith angle and $\tilde{\phi}$ is the azimuth angle.

For an opaque diffuse surface, solar reflectance is given by Eq. (2.1a)

$$\text{Solar reflectance } (\rho) = \frac{\int_{300}^{2500} \rho_{\lambda} I_{in,\lambda} d\lambda}{\int_{300}^{2500} I_{in,\lambda} d\lambda} \quad (2.1a)$$

where ρ_{λ} is the spectral reflectance.

(b) Thermal emittance (ε): Thermal emittance is a measure of the ability of a surface to emit the absorbed heat in all possible directions and at all wavelengths. It specifies how well a surface emits heat away from itself as compared to a black body operating at the same temperature. It is measured on a scale of 0 to 1.

It designates the weighted average of spectral directional emittance integrated over all wavelengths (0 to ∞) and over a hemisphere (0 to 180°).

$$\text{Thermal emittance } (\varepsilon) = \frac{\int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} \varepsilon'_{\lambda} I_{e,\lambda} \cos(\tilde{\theta}) \sin(\tilde{\theta}) d\tilde{\phi} d\tilde{\theta} d\lambda}{I_{b,\lambda}} \quad (2.2)$$

where ε'_{λ} is the spectral directional emittance, $I_{e,\lambda}$ is radiation emitted at wavelength λ , $I_{b,\lambda}$ is spectral blackbody emission.

For an opaque diffuse surface, thermal emittance is given by Eq. (2.2a)

$$\text{Thermal emittance } (\varepsilon) = \frac{\int_0^{\infty} \varepsilon_{\lambda} I_{e,\lambda} d\lambda}{I_{b,\lambda}} \quad (2.2a)$$

where ε_{λ} is the spectral emittance.

These two radiation properties (ρ and ε) affect the temperature of a surface. If a surface with high ρ and high ε is exposed to solar radiation, it will have a lower surface temperature compared to a similar surface with low ρ and low ε values (Uemoto et al. 2010) as described in Fig. 2.1.

Fig. 2.1 Working principle of cool roof (Santamouris et al., 2011).

2.2 Energy balance at a cool roof surface

2.2.1 Interaction between electromagnetic (EM) waves and material surface

All materials are comprised of molecules and atoms which contain electrons. The electrons have a natural frequency at which they tend to resonate. When an EM wave (having the same natural frequency) impinges upon an atom, the electrons of that atom set into vibrational motion. If an EM wave of a given frequency strikes an atom with electrons having the same vibrational frequencies, then those electrons absorb the quantum energy of the EM wave and transform it through the energy gap (from initial state to final state) and set to vibrational motion. During its vibration, the electrons interact with neighboring atoms in such a manner as to convert its vibrational energy into thermal energy. Subsequently, the EM wave with that given frequency is absorbed by the material. The selective absorption of EM waves by a particular material occurs because the selected frequency of the EM wave matches the (energy gap) frequency at which electrons in the atoms of that material travel from the initial state to the final

state. Since different atoms and molecules have different natural frequencies of vibration, they selectively absorb different frequencies of EM waves.

Reflection and transmission of EM waves occur because the frequencies of the EM waves do not match with the natural frequencies of vibration of the objects. When EM waves of these frequencies strike an object, the electrons vibrate for brief periods of time with small amplitudes of vibration, instead of vibrating in resonance at large amplitude. Then the energy is re-emitted as an EM. If the object is transparent, the vibrations of the electrons are passed on to neighboring atoms through the bulk of the material and re-emitted on the opposite side of the object. Such frequencies of EM waves are said to be transmitted. If the object is opaque, then the vibrations of the electrons are not passed from atom to atom through the bulk of the material. Rather the electrons of atoms on the material's surface vibrate for short periods of time and then re-emit the energy as a reflected EM wave. Such frequencies of light are said to be reflected.

Due to the internal interactions, the excited electrons (during absorption, reflection and transmission) lower their energy before making a downward transition to initial state. In the downward transition, the electrons emit another EM wave of lower energy and longer wavelength, resulting long wave heat emission. According to Planck's law, the spectral distribution of emissive power of a black body is given by

$$I_{b,\lambda} = \frac{2 \pi 0.6 \times 10^{-16}}{\lambda^5 \left(\exp\left(\frac{0.014}{\lambda T}\right) - 1 \right)}. \quad (2.1)$$

While according to Wien's displacement law, the spectral distribution of emissive power of black body per unit wavelength, peaks at the wavelength $\lambda_{\max}$ is given by

$$\lambda_{\max} T = 2898 \text{ } \mu\text{m-K}. \quad (2.2)$$

It can be observed from Fig. 2.2 that with the increase in absolute temperature, $\lambda_{\max}$ decreases (shifts towards left).

Fig. 2.2 Black body emission as a function of wavelength for various absolute temperatures (McAdams, 1954).

2.2.2 Heat transfer processes at the cool roof surface

The incident solar radiation is the thermal radiation (electromagnetic waves) emitted by the sun at surface temperature of about 5800K. The thermal radiation spectrum of the sun, also called as solar radiation spectrum stretches from 200 nm – 2500 nm (Shen et al., 2011; Xiwei, 1996; Li et al., 2010; Zubielewicz et al., 2012) as described in Table 2.1. It consists of ultra-violet radiation, visible radiation and near-infrared radiation which are called short waves (SW). The wavelength range of each region is shown in Table 2.1. The thermal infrared radiation spectrum of the objects on the earth surface ranges from about (2500 nm < λ < 100000 nm) which are called long waves (LW). These are the electromagnetic waves emitted by a surface at temperature lower than 800K. It constitutes far-infrared (2500 nm < λ < 20000 nm) to radio waves (20000 nm < λ < 100000 nm). Atmospheric greenhouse gases such as NH₃, CH₄, CO₂,

H₂O, O₃ have less affinity to the high energy short waves, but have high affinity to low energy long waves as described in Table 2.2. The influence of UV (high energy) radiation is responsible for the creation of radicals and free electrons which when interacts with the skin can damage the skin cells. The selective absorption in the VIS radiation is responsible for the change in chromaticity and does not heat up a body. While, an opaque building surface exposed to incident solar radiation is heated up by absorbing the NIR (as shown in Table 2.1). The surface temperature of an opaque building material (roof/wall) directly exposed to solar radiation can be determined by considering the balance between incident radiation and outgoing radiation at that surface.

Table 2.1 Spectral distribution of solar energy reaching the earth surface (Zubielewicz et al., 2012).

Electromagnetic solar spectrum	Wavelength range		Solar energy reaching the earth surface (%)		Remarks
UV	UV-C	200 – 280 nm	-		Absorbed by ozone layer
	UV-B	280 – 320 nm	5%		Penetrate epidermal layer of skin and causes sun burn
	UV-A	320 – 400 nm			Penetrate epidermis of skin and cause skin damage
VIS	400 – 720 nm		42%		Causes change in chromaticity (colour)
NIR	720 – 1000 nm		26	53%	On absorption, converts into heat energy
	1000- 1500 nm		15		
	1500- 2500 nm		11		

UV = Ultra-violet, VIS = Visible, NIR = Near-infrared radiation

The incident radiation includes short waves in the wavelength range of 200 nm – 2500 nm (SW_{in} i.e., insolation and the radiation reflected by other objects on the earth

surface) and long waves with wavelength > 2500 nm (LW_{in} i.e., thermal radiation emitted by clouds and other objects on the earth surface). The outgoing radiation includes short waves (SW_{out} i.e., reflected radiation by the surface during day time) and long waves (LW_{out} i.e., thermal radiation emitted by the surface to the sky and other objects on the earth surface) as described in Fig. 2.3.

Table 2.2 Electromagnetic wavelength, energy reaching earth and absorbing gases (AboulNaga and Abdrabboh, 2000)

Electromagnetic Waves	Wavelength (nm)	% Energy reaching the earth surface (Solar flux)	Absorbing atmospheric gases
X-rays	100 – 200 nm	0.0644	NH ₃ , CH ₄ , CO ₂ , H ₂ O, O ₂
UV + VIS	200 – 350 nm	0.46	O ₃
VIS	350 – 450 nm	5.63	-
VIS	450 – 850 nm	48.77	O ₃ , H ₂ O
VIS + NIR	850 – 1000 nm	16.98	H ₂ O
NIR	1000 – 1600 nm	18.92	H ₂ O
NIR	1600 – 1950 nm	4.81	H ₂ O
NIR	1950 – 2120 nm	1.24	H ₂ O
NIR + FIR	2120 – 2610 nm	2.41	H ₂ O
FIR	2610 – 9500 nm	0.84	H ₂ O, NH ₃ , CH ₄ , CO ₂

The absorption of NIR by an opaque surface, converts the solar energy into heat energy resulting rise in temperature of the surface. Part of the absorbed thermal energy is stored in the opaque material due to its thermal storage capacity and some other part is lost to outdoor by emission and convection. The remaining is conducted into the building. The heat lost to outdoor by emission involves radiative heat transfer to the

nearby objects on the earth surface and to the sky due to the temperature difference between the roof surface (T_R) and the sky (especially when the sky is clear). The sky temperature (T_{sky}) is the temperature of a hypothetical black hemisphere absorbing the same radiation flux emitted by an opaque surface on the earth as absorbed by the sky. From Eqs. (3.3) – (3.5), T_{sky} depends on the factors such as outdoor air temperature, emittance of the sky and cloud cover.

Fig. 2.3 Schematic diagram showing heat balance processes at cool roof (not-to-scale).

Heat transfer through roof/wall is a mixed-mode heat transfer phenomenon. The roof/wall surface heat exchanges mainly consist of the radiation component (solar heat gain and thermal emission) and the convection component (heat exchanging with surrounding air). A cool coating affects the radiation component by providing high solar reflectance and high emittance. Thus absorbed solar heat is minimised and the pumping away of any heat that is absorbed or gained from internal sources and stored in building walls and roof mass is enhanced maximising the thermal emission. The

cool coating layer adds resistance to the heat conduction through the roof/wall, which is coupled with the surface heat exchanges. In order to quantitatively analyse the impact of cool coating on the roof/wall heat transfer phenomenon, a robust cool roof heat transfer model is essential.

2.3 Research gap

2.3.1 Modelling of cool roof heat transfer for single-skin roof (SSR)

Over the years, numerous analytical methods, such as response factor (RF) and conduction transfer function (CTF), and numerical methods, such as Finite Difference (FD) and Finite Element (FE) have been developed. These methods have been implemented in building energy simulations programs, such as EnergyPlus (Gentle et al., 2011), TRNSYS (Zinzi & Agnoli, 2012; Kolokotsa et al., 2012), eQUEST (Kolokotroni et al., 2013), BEopt (Tabares-Velasco et al., 2012), roof/wall savings calculators (Roof savings calculator; DOE cool roof calculator; Building load calculation software), building load calculation software (Building energy simulation software; Building energy calculator), and CTF and RF coefficient calculators (Harris & McQuiston, 1988). In these implementations, RF method requires long series of history temperatures or heat flux data. CTF method requires a set of pre-calculated coefficients (Delcroix et al., 2013; Tabares-Velasco et al., 2012; Iu and Fisher, 2004) which are tabulated for certain types of building envelope surfaces and for specific climate conditions, limiting its potentials for general uses. FD and FE methods overcome the drawbacks of RF and CTF methods. However, FD and FE methods use low-order polynomials, and require small grid size in the slab material and small time step (15 min or less) to achieve sufficient accuracy (Tabares-Velasco et al., 2012). This leads to heavy computational resources. Due to the need for discretisation, these

methods resolve the detailed temperatures distributions within the slab material which is unnecessary for air-conditioning load calculations.

The spectral approximation method (Zhuang & Ren, 2014; Oliveira & Azevedo, 2014; Gamba & Haack, 2014), on the other hand, uses orthogonal functions or higher-order (trigonometric) polynomials to solve the partial differential equation for heat conduction. This leads to a system of equations that is very easy to handle and substantially reduces the complexity of calculations. It does not require a set of pre-calculated coefficients, history heat flux data or the use of tables. At the same time, it does not require small grid size in the slab material (hence no need to calculate the detailed temperatures within the slab material) and a time step of one hour can be used (Podowitz et al., 2014). Despite the numerous advantages of the spectral approximation method, it has not yet been explored to develop a heat transfer model for the analysis of heat transfer through opaque building surfaces.

In Chapter 4, a novel and general cool roof heat transfer (CRHT) model is proposed for SSR or solid primary roof (no air-gap between roof layers) applied with cool coating on the roof surface based on the spectral approximation method. It should be noted that the CRHT model is also applicable to opaque solid wall surfaces. The CRHT model can handle transient outdoor and indoor boundary conditions as experienced by naturally ventilated buildings. The CRHT model is advantageous over other existing building energy simulation models since the final solution of the CRHT model is very concise and, therefore, much easier to apply.

The proposed CRHT model (for SSR) is further used to 1) formulate a novel method to assimilate the effects of opaque surface solar reflectance changes (due to application of cool roof) on annual-average heat gain for air-conditioned buildings (as discussed in

Section 2.3.2) and 2) incorporate the effect air-gap between two solid roof layers i.e., DSR (as discussed in Section 2.3.3).

2.3.2 Envelope thermal performance prediction models for AC buildings

A number of countries/regions in tropical/sub-tropical Asia (including Hong Kong, India, Sri Lanka, Thailand, Indonesia, Malaysia, Philippines and Pakistan) adopted the approach of using Overall Thermal Transfer Value (*OTTV*) (Building Energy Code of Pakistan, 1990; Chan & Chow, 2013; Sustainable Energy Authority of Sri Lanka, 2010; Chirarattananon & Taveekun, 2004; Sukri et al., 2012; Devgan et al., 2010), or Envelope Thermal Transfer Value (*ETTV*) (Code on Envelope Thermal Performance for Buildings in Singapore, 2008) in Singapore, to evaluate and regulate the thermal performance of the envelopes of air-conditioned buildings. Either *OTTV* or *ETTV* is essentially the annual-averaged heat gain through the building envelope under the typical climate conditions of the location concerned. Among these *OTTV* or *ETTV* models, there are mainly three methods (as summarised in Table 2.3) in dealing with the impact of solar reflectance of opaque surfaces. 1) The “Current” method: The models adopted by Singapore and Pakistan (Code on Envelope Thermal Performance for Buildings in Singapore, 2008; Building Energy Code of Pakistan, 1990) employ the method of using a fixed value of higher equivalent temperature difference (TD_{eq}) to include the effect of solar reflectance of opaque surfaces on annual-average heat gain. This is rather obviously a deficiency since solar reflectance varies from one surface to another, especially in cases of cool surfaces which have large differences in solar reflectance compared to conventional surfaces. 2) Method A: The models adopted by Hong Kong, Sri Lanka, Thailand, Indonesia, Malaysia and Philippines (Chan & Chow, 2013; Sustainable Energy Authority of Sri Lanka, 2010; Chirarattananon & Taveekun, 2004; Sukri et al., 2012) employ the method of directly

multiplying solar absorptance (α) of opaque surfaces with the component of heat conduction gain. This treatment can still be problematic since, by doing so, it assumes that the annual-average heat gain is linearly proportional (correlation coefficient of 1.00) to solar absorptance which is not the case as reported by Chan & Chow (2013) and Devgan et al. (2010). Computational studies by Chan & Chow (2013) in Hong Kong and Devgan et al. (2010) in India showed that the correlation coefficient lies in the range 0.91 to 0.95. 3) Method B: Devgan et al. (2010) observed that the second degree polynomial (instead of linear correlation) can more accurately correlate the data and hence proposed empirical models for different wall orientations, except for roof. This shows that only two methods (Method 1 and Method 2) of the existing three methods are applicable to roofs.

In Singapore, Roof Thermal Transfer Value (*RTTV*) model is prescribed for air-conditioned, non-residential buildings as guidelines for thermal performance of roofs (Code on Envelope Thermal Performance for Buildings in Singapore, 2008). The current *RTTV* model perceives the fixed value of solar reflectance of opaque roof surface which could be problematic when it is used to estimate the *RTTV* of cool roofs. Cool roofs are often achieved by adding a layer of thin coating or single-ply membrane on top of the original opaque structural roof. The current *RTTV* model could incorporate the additional conduction resistance of the cool layer simply by adding the conduction resistance to the *R*-value of the opaque roof assembly. However, in the current *RTTV* model, there is no provision to vary the effect of solar reflectance on heat transfer to capture the impact of high solar reflectance of cool layer.

Table 2.3 Summary of different methods adopted to incorporate the impact of solar reflectance of opaque surfaces.

Method (Model)	Parameter of the component of heat conduction gain through opaque portion		Remarks	Applicable to climate	Source
	TD_{eq}	U^{new} or $TD_{eq} U^{new}$			
new* ($RTTV^{new}$)	Fixed value for a given orientation	Incorporation of K^{incr} and K_{cond}^{cool} into $U_{\rho=0}$ i.e., $\frac{1}{U_{\rho}^{new}} = \frac{1}{U_{\rho=0}} + K_{cond}^{cool} + K^{incr}$	1. Perceives the effect of varying α . 2. Applicable to roofs and walls in any climate.	All climates	This study
Current+ ($RTTV$)	Fixed value for a given orientation	No change i.e., $TD_{eq} U^{new} = TD_{eq} U = \text{constant}$	Does not perceive the effect of varying α .	Singapore, Pakistan.	(Code on Envelope Thermal Performance for Buildings in Singapore, 2008; Building Energy Code of Pakistan, 1990)
Method A ($RTTV^A$)	Fixed value for a given orientation	Multiplication by α i.e., $U^{new} = \alpha U$ or $TD_{eq} U^{new} = TD_{eq} (\alpha U)$	Assumes correlation coefficient of 1.00.	Hong Kong, Sri-Lanka, Thailand, Philippines, Indonesia, Malaysia.	(Chan & Chow, 2013; Sustainable Energy Authority of Sri Lanka, 2010; Chirarattananon & Taveekun, 2004; Sukri et al., 2012)
Method B ($RTTV^B$)	Fixed value for a given orientation	Empirical models obtained by second degree polynomial fitting for the building walls in India. $TD_{eq} U^{new} = -4.2(\alpha U)^2 + 30(\alpha U)$ for North wall $TD_{eq} U^{new} = -4.5(\alpha U)^2 + 40.5(\alpha U)$ for South wall $TD_{eq} U^{new} = -4.5(\alpha U)^2 + 37.4(\alpha U)$ for East wall $TD_{eq} U^{new} = -4.5(\alpha U)^2 + 41.9(\alpha U)$ for West wall	1. No empirical model for roof. 2. Applicable to walls only and limited to the climate of India.	India	(Devgan et al., 2010)

*new proposed method in this study.

⁺Current $RTTV$ model is modified to formulate the $RTTV^{new}$ model in this study.

In Chapter 5, a novel method is proposed to incorporate the effects of surface solar reflectance of the opaque portion of roofs into RTTV estimation using the CRHT model (for SSR). This method is generally applicable to *OTTV*, *ETTV*, *RTTV* models or any similar models and is not limited by climate conditions or building materials. A new *RTTV* model is formulated based on this novel method in Chapter 5. In order to incorporate the effect air-gap between two solid roof layers on thermal analysis, the previously formulated CRHT model (for SSR) is modified (as discussed in Section 2.3.3).

2.3.3 Modelling of cool roof heat transfer for double-skin roof (DSR)

Over the years, investigations on the thermal performance of DSR were mainly carried out by performing experiments (Fracastoro et al., 1997; Soubdhan et al., 2005), while several studies (Gomez & Galvez, 2013; Dimoudi et al., 2006; Yew et al., 2013; Lai et al., 2008; Chang et al., 2008; Miranville et al., 2003; Shoda et al., 1981; Nelson & Wood, 1989; Palomino, 1995; Zhai et al., 2005) were devoted to formulate heat transfer models. Some of the heat transfer models were developed for steady state conditions (Gomez & Galvez, 2013), while some others were for transient outdoor and constant indoor conditions. Numerical methods such as FD, FE and FV methods have been developed and implemented for the thermal analysis of transient heat transfer through the DSR (Dimoudi et al., 2006; Yew et al., 2013; Lai et al., 2008; Chang et al., 2008; Miranville et al., 2003; Shoda et al., 1981; Nelson & Wood, 1989; Palomino, 1995; Zhai et al., 2005). However, FD, FE and FV methods require small grid size and small time step (15 min or less) to achieve sufficient accuracy (Tabares-Velasco et al., 2012), leading to heavy computational resources. Due to the need for discretisation, these methods resolve the detailed temperatures distributions within the slab material which is unnecessary for air-conditioning load calculations. At the same time, the

previous models (Dimoudi et al., 2006; Yew et al., 2013; Lai et al., 2008; Chang et al., 2008; Miranville et al., 2003; Shoda et al., 1981; Nelson & Wood, 1989; Palomino, 1995; Zhai et al., 2005) were formulated based on the assumption of constant indoor air temperature boundary condition, limiting their applicability for naturally ventilated buildings. In order to investigate the thermal performance of DSR alone and a combination of cool roof + DSR for naturally ventilated buildings, and since 86% of Singapore's total population is living in naturally ventilated residential buildings (Chua & Chou, 2010), a robust heat transfer model is required that can handle transient outdoor and indoor boundary conditions.

In Chapter 6, a novel CRHT model is proposed for DSR (using spectral approximation method) that can handle transient outdoor and indoor boundary conditions as experienced by naturally ventilated buildings. The CRHT model is advantageous over other existing building energy simulation models since the final solution of the CRHT model is very concise and, therefore, much easier to apply. In contrast, the existing building energy simulation models are quite complicated to apply to DSRs due to the difficulties in geometry modelling and setting up boundary conditions for the air-gap region.

2.3.4 Thermal performance of cool roof, green roof, thermal insulation in tropical climate

Along with cool roof, there are several other potential methods such as green roof (which works on the principle of evapotranspiration and thermal resistance) and thermal insulation (which works on the principle of providing thermal resistance) for curbing heat gain into buildings through opaque roof surfaces. The thermal performances of these methods are highly climate dependent. Over the years, evaluation of performances of these methods have been mostly studied in Europe and

the U.S. (Georgescu et al., 2014; Zinzi & Agnoli, 2011; Sproul et al., 2014; Hideki & Masakazu, 2007). However, no studies have been conducted yet in the tropical climate, resulting the thermal performances of these methods and the effect of individual components such as solar reflectance, thermal emittance, evapotranspiration and thermal insulation on annual net heat gain are least understood for the tropical climate.

In Chapter 3, computational simulations are performed (using experimentally calibrated computational model) on an air-conditioned building to investigate the thermal performance of cool roof, green roof and thermal insulations in tropical climate. The effects of individual component such as solar reflectance, thermal emittance, evapotranspiration and thermal insulation on annual net heat gain through opaque roofs are also investigated for the tropical climate.

Chapter 3

Thermal Performance of Various Roofing Methods for Air-conditioned Buildings in Tropical Climate

3.1 Introduction

Over the years, evaluations of performances of cool roof, green roof and thermal insulation methods have been mostly conducted in Europe and the U.S. (Georgescu et al., 2014; Sproul et al., 2014; Zinzi & Agnoli, 2011). However, no studies have been performed yet in the tropical climate, resulting in the effects of roof surface radiation properties, green roof properties and thermal insulation being least understood for tropical climates. In this chapter, computational simulations (calibrated by measurements) are performed on an air-conditioned, single-storey building to investigate the thermal performance of cool roof in the tropical climate and comparison with green roof and thermal insulation.

3.2 Computational simulation

Computational simulations (using experimentally calibrated model) are performed on an air-conditioned building in the tropical climate of Singapore. The computational simulation program, EnergyPlus (version 6.0.0 (Gentle et al., 2011; Boixo et al., 2012)) is used to estimate the annual heat gain, annual heat loss and annual net heat gain for various roofing methods. The program allows very detailed inputs, including: Typical Meteorological Year (TMY) weather data, building geometry, building construction materials, HVAC system and building schedules. The TMY weather data includes hourly solar radiation, outdoor air temperature, wind speed and relative humidity. The background information about the EnergyPlus program and the development of computational model of the test building (which is required as an input to the program) is explained as follows.

3.2.1 EnergyPlus program

The EnergyPlus program (which is the successor of BLAST and DOE-2) facilitates an ability to input extensive building details and provides results which are quite sensitive to the change in various parameters like building construction materials, building envelope surfaces, air-conditioning system and weather conditions. EnergyPlus is a building energy analysis and thermal load simulation program which uses fundamental heat balance principles to calculate the air-conditioning energy required for cooling or heating of a building. EnergyPlus can model heating, cooling, lightings, vegetation, and other energy flows within buildings as well as energy flows between a building and the outdoor environment through infiltration, glazing, walls and roof.

EnergyPlus requires input as building information (as shown in Fig. 3.1) which includes building construction, materials, geometry, geographical location, internal loads, occupants, equipment and lightings, and TMY weather data of the surrounding area where the building is located and boundary conditions of the building surfaces.

Fig. 3.1 Working strategy of EnergyPlus program.

The output of the program includes inside/outside temperatures of building envelope surfaces, conduction heat gain/loss, solar heat gain, air-conditioning energy consumption for heating/cooling, convective/radiative heat transfer coefficients for indoor and outdoor as illustrated in Fig. 3.1. The annual conduction heat transfer per unit area is calculated using the inside and outside surface temperatures and thermophysical properties of the material. The algorithm of the program is shown as a flowchart in Fig. 3.2.

Fig. 3.2 Flowchart showing algorithm of EnergyPlus program.

3.2.1.1 Governing equations

EnergyPlus uses fundamental heat balance principles to estimate the heat flux through an opaque exterior surface to the building. The program assumes that all the opaque roof/wall surfaces are gray, diffuse and having uniform surface temperature. In the calculation of indoor radiative heat transfer among interior surfaces, it is assumed that air is transparent to long wave radiation and it is well-stirred so it has a same temperature throughout the indoor environment.

Heat balance at an opaque exterior surface

In order to estimate the conduction heat flux from an opaque exterior surface to an interior surface, the program considers heat balance at the exterior surface as given by Eq. (3.1),

Absorbed solar radiation flux + Thermal radiation flux exchange with outdoor + Convective flux exchange with outdoor – Conduction heat flux into the roof/wall = 0

$$q_{abs} + q_r + q_{conv} - q_R = 0. \quad (3.1)$$

Now, considering the roof as an exterior surface (as shown in Fig. 2.2),

Absorbed solar radiation flux by the roof + Thermal radiation flux exchange between roof and outdoor + Convective flux exchange between roof and outdoor – Conduction heat flux into the roof = 0

$$I_{in} \alpha_R - h_{R,r} (T_R - T_o) - h_{R,conv} (T_R - T_o) + k \left(\frac{\partial T}{\partial y} \right)_{y=0} = 0 \quad (3.2)$$

$$\text{where } h_{R,r} = \varepsilon_R \widehat{F}_{R,sky} (T_R^2 + T_{sky}^2) (T_R + T_{sky}) \frac{T_R - T_{sky}}{T_R - T_o} \quad (3.2a)$$

The sky temperature (T_{sky}) is estimated by implementing the sky model (Eqs. (3.3) – (3.5)),

$$T_{sky} = 0.0552 \varepsilon_{sky} (T_o)^{1.5} \text{ for clear sky days or} \quad (3.3)$$

$$T_{sky} = \varepsilon_{sky} T_o \text{ for cloudy days} \quad (3.4)$$

$$\varepsilon_{sky} = \left(0.79 + 0.76 \ln \left(\frac{T_o}{273} \right) \right) (1 + 0.022N - 0.004P^2 - 0.0003P^3). \quad (3.5)$$

Heat balance at an opaque interior surface

In order to estimate the combined convective and radiative heat exchange between the interior surfaces to the indoor environment, the program considers heat balance at the interior surface (ceiling/wall) as given by Eq. (3.6),

$$q_R + q_r + q_{conv} = 0. \quad (3.6)$$

Conduction heat flux into the roof/wall + Thermal radiation flux exchange with indoor
+ Convective flux exchange with indoor = 0

Now, considering the ceiling as an interior surface (as shown in Fig. 2.2),

Conduction heat flux into the roof + Thermal radiation flux exchange between ceiling
and indoor + Convective flux exchange between ceiling and indoor = 0

$$-k \left(\frac{\partial T}{\partial y} \right)_{y=L} - h_{C,r} (T_C - T_i) - h_{C,conv} (T_C - T_i) = 0. \quad (3.7)$$

Heat balance in indoor environment

The air-conditioning energy to be supplied to a building (indoor) to maintain the air temperature at a predefined set temperature, EnergyPlus considers heat balance on the indoor air as given by Eq. (3.8),

$$L_i \rho_{air}^* C_{air} \left(\frac{dT_i}{dt} \right) = h_i [T_C - T_i] + \sum_{wall} h_i [T_{wall} - T_i] + \sum_{window} U_{window} [T_o - T_i] + h_i [T_{floor} - T_i] + \eta^+ [T_o - T_i] + q_{system} \cdot \quad (3.8)$$

$$\text{where } \eta^+ = \frac{(ach)L_i \rho_{air}^* C_{air}}{3600 \times VOL_i}$$

$h_i [T_C - T_i]$ = Heat flux exchange between ceiling surface and indoor air (W/m²)

$\sum_{wall} h_i [T_{wall} - T_i]$ = Sum of all heat flux from inside wall surfaces to indoor air (W/m²)

$\sum_{window} U_{window} [T_o - T_i]$ = Sum of all heat flux through windows (W/m²)

$h_i [T_{floor} - T_i]$ = Heat flux exchange between floor surface and indoor air (W/m²)

q_{system} = Air-conditioning system's output (W/m²)

$L_i \rho_{air}^* C_{air} \left(\frac{dT_i}{dt} \right)$ = Energy stored in the indoor air (W/m²)

If the air capacitance is neglected, $C_{air} = 0$, the steady-state system output must be

$$-q_{system} = h_i [T_C - T_i] + \sum_{wall} h_i [T_{wall} - T_i] + \sum_{window} U_{window} [T_o - T_i] + h_i [T_{floor} - T_i] + \eta^+ [T_o - T_i]$$

EnergyPlus calculates q_{system} i.e., air-conditioning energy required to maintain the indoor at predefined set point temperature T_i .

3.2.1.2 Boundary conditions

All the outer (exterior) building surfaces are assigned outdoor boundary conditions i.e., exposed to wind, rain and solar radiation. All the interior building surfaces are assigned indoor boundary conditions and the indoor air temperature is set to T_i .

3.2.2 Computational model development

As the EnergyPlus program provides quite limited user interface and currently does not provide three-dimensional modelling tools to create model of the building to be simulated. An additional software i.e., GoogleSketchUp is used to create the building

model combined with OpenStudio plug-in. A computational model of a real-scale air-conditioned, single-storey test building with a solid flat concrete roof (100-mm-thick concrete + 10-mm-thick plaster on the ceiling) is developed using Google SketchUp (version 1.0.7) with OpenStudio plug-in. Figure 3.3 shows the three-dimensional model of the test building located in Nanyang Technological University, Singapore. The test building is of rectangular shape having total opaque roof surface area of about 40 m^2 (without any skylight surface). Two windows, two doors and an exhaust fan are located on the south facing wall. The opaque concrete building roof is applied with a cool coating (0.50-mm-thick when dried) having a solar reflectance of 0.74 and thermal emittance of 0.90.

Fig. 3.3 GoogleSketchUp model of the test building.

In order to include the shading effect caused by the nearby objects such as buildings, window overhangs and trees, shading elements are utilised. The thermophysical

properties of the building materials which are required as inputs are obtained from the manufacturers and the EnergyPlus material dataset (2013) as shown in Table 3.1.

Table 3.1 Thermophysical properties of the test building materials.

Building material	Thermal conductivity (W/m-K)	Thickness (mm)	Conduction resistance (m ² -K/W)	Density (kg/m ³)	Specific heat (J/kg-K)
Air ^{e+}	0.03	-	-	1.23	1008
Concrete ^{e+}	0.65	100 ^a	0.15	2450	840
Cool coating ^a	0.05 ^a	0.5 ^a	0.01 ^a	1053 ^a	**
Glazing ^{e+}	0.70	3.5 ^a	0.01	-	-
Plastered concrete block ^{e+}	1.10	125 ^a	0.12	800	920
Plaster ^{e+}	0.25	10 ^a	0.04	850	1000
Wood ^{e+}	0.15	50 ^a	0.33	608	1630

^a Thermophysical properties are obtained from the manufacturers.

^{e+} Thermophysical properties are taken from the EnergyPlus material dataset (2013).

**Thermal mass, mC_p is 0 J/K due to negligible mass of coating.

The solar reflectance and thermal emittance properties of the cool coating are obtained from on-site measurements (described in Section 3.2.2 and Table 3.5), whereas the thermal conductivity, mass density and specific heat of the cool coating are provided by the manufacturer (CPG Consultants Pte. Ltd.). The TMY weather data file for the Changi airport weather station in Singapore is obtained from the available EnergyPlus weather dataset (2013). Detailed physical and operational information about the test building are shown in Table 3.2. In order to model the effect of cool roof, green roof and thermal insulation, separate models are developed for each roofing method as explained in Section 3.2.1.1.

Table 3.2 Physical details of the test building.

General information	
Location	Nanyang Technological University, Singapore
Surface area of West facing wall	11 m ²
Surface area of East facing wall	11 m ²
Surface area of South facing wall	38 m ²
Surface area of North facing wall	38 m ²
Surface area of roof	40 m ²
Terrain	Urban
Latitude and longitude	1.4°N and 104.0°E
Elevation	35 m above sea level
Building materials	
Walls	Plastered concrete block ^a (125-mm-thick)
Roof	Cool coating ^a (0.5-mm-thick) + concrete ^a (100-mm-thick) + plaster ^a (10-mm-thick), SKR = 0
Windows	Glazing ^a (3.5-mm-thick) + air ^a (13-mm-thick) + Glazing ^a (3.5-mm-thick)
Floor	Concrete block ^a (300-mm-thick) and acoustic tile ^a (20-mm-thick)
Doors	Wood ^a (50-mm-thick)
Building service	
Air-conditioning system	Inverter ^a (Single Split Type): On ^b / Off ^c
	Indoor set temperature: 24°C
	Cooling capacity: 2.5 kW
	COP: 3.33, Power consumption: 0.75 kW
	Annual operating hours: 3012 hours
Internal loads	No. of people ^b : 4, No. of people ^c : 0 Lighting level ^b : 432 W, Lighting level ^c : 0 W Electric equipment ^b : a desktop (250 W) and an exhaust fan (25 W)

	Electric equipment ^c : a desktop (0 W) and an exhaust fan (0 W) Ventilation rate: 0.30 ACH, Infiltration rate: 0.90 ACH
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^a Building construction material information obtained from the manufacturers.

^b Schedule during the weekdays (Monday to Friday), operating hours: 07:00 – 19:00.

^c Schedule during the weekends (Saturday and Sunday) and holidays.

3.2.2.1 Modelling of green roof

EnergyPlus program features a validated (Sailor et al., 2008) one-dimensional green roof model (Material: Roof Vegetation) which is used to model the effect of green roof in this study. The green roof model [developed by Sailor (2008)] takes into account the evapotranspiration of the vegetation layer and soil layer, the transient soil thermal properties and, radiative and convective heat exchanges with outdoor (EnergyPlus Engineering Reference, 2013). Thermophysical properties of the green roof are shown in Table 3.3.

Table 3.3 Thermophysical properties of green roof.

Parameter	Value
Height of plant	0.5 m
Leaf area index	1.2
Leaf solar reflectance	0.10
Leaf thermal emittance	0.86
Minimum stomatal resistance	120 s/m
Max volumetric moisture content of soil	0.32
Min volumetric moisture content of soil	0.01
Initial volumetric moisture content of soil	0.15
Density of soil	960 kg/m ³
Specific heat of soil	1500 J/kg-K
Thermal conductivity of soil	0.4 W/m-K
Soil layer thickness	0.5 m

3.2.2.2 Modelling of cool roof

The effect of the 0.5-mm-thick cool coating layer is modelled in EnergyPlus by assigning the solar reflectance of 0.74 and the thermal emittance of 0.90 of the cool coating and the combined conduction resistance, $0.20 \text{ m}^2\text{-K/W}$ (adding $K_{cond} = 0.19 \text{ m}^2\text{-K/W}$ and $K_{cond}^{cool} = 0.01 \text{ m}^2\text{-K/W}$) to the concrete roof. The radiation properties of the cool coating were obtained from the actual measurements on test building roof (as discussed in Section 3.2.2).

3.2.2.3 Modelling of thermal insulation

Extruded polystyrene insulation, which is commonly employed for the roofs and walls in tropical climate (Ozel, 2012), is used to model the effect of thermal insulation. Thermophysical properties of extruded polystyrene insulation are shown in Table 3.4. Table 3.4 Thermophysical properties of extruded polystyrene insulation (ASHRAE, 2009).

Properties	Value
Thermal conductivity	0.03 W/m-K
Thickness	0.05 m
Density	24 kg/m ³
Specific heat	1590 J/kg-K
Roughness	Medium

3.2.3 Test building measurements for computational model calibration

The computational model developed using EnergyPlus is calibrated by experimental measurements to compare the simulation results for the cool roof surface temperature and the cool ceiling surface temperature with the measurements. The measurements were conducted in the test building for a week in April 2013. Figure 3.4 illustrates the schematic diagram of the experimental set up.

The cool coating thickness was measured using a coating thickness gauge (Elcometer, A456CSF11) which was calibrated to an accuracy of $\pm 1\%$ reading using the zero offset calibration method compatible with ISO 19840 (2012). In the calibration process, the thickness gauge probe was initially placed on the original roof surface and the thickness was set to zero (as reference). Then a set of calibration standard samples (provided with the instrument) were placed on the original surface and measured by the thickness gauge. The offset between the thickness gauge readings and the actual thicknesses of the calibration standards were adjusted to match with the calibration standards. The cool coating thickness measurements were made at five locations by directly placing the calibrated thickness gauge on the cool coated roof surface.

Fig. 3.4 Schematic diagram illustrating the experimental set up for cool concrete roof of the test building (not-to-scale).

The incident solar radiation received by the roof surface was monitored by an upward-facing (towards the sky) pyranometer (Kipp and Zonen, CMP 11) mounted at 1.5 m

above the roof surface. The pyranometer was factory calibrated to an accuracy of $\pm 1.4\%$ reading. The detailed calibration procedure of CMP 11 pyranometer can be found in Kipp and Zonen, CMP 11 manual. The surface temperatures of the roof and the ceiling were monitored by surface-type resistance temperature detectors (RTDs) encapsulated in stainless steel probes of 4 mm in diameter and 25 mm in length (Omega, 4 wire RTDs). The calibrated RTDs were installed at five points (to check the homogeneity) of each roof and the ceiling surfaces such that the sensor portion of RTDs was covered with an insulation tape followed by another reflective tape on top. The solar reflectance and thermal emittance properties of the reflective tape were 0.73 ± 0.02 and 0.88 ± 0.02 , respectively. These values are almost the same as that of the cool roof (see Table 3.5). The reflective tape and insulation tape prevented the sensor tip from being affected by solar radiation such that the sensor tip measured the actual temperature of the cool roof surface. The RTDs were calibrated to an accuracy of $\pm 0.5^\circ\text{C}$ prior to the experiment using a stirred silicon oil bath thermal calibrator (Tempsens Instruments (I) Pvt. Ltd, Calsys -40/200) for the temperature range of 20°C - 40°C (as experienced by the cool concrete roof on a sunny day). The solar reflectance and thermal emittance of both original and cool roof surfaces were measured using a portable Solar Spectrum Reflectometer (Devices and Services, SSR-6) compatible with the ASTM standard C1549 (2009) and a portable Emissometer (Devices and Services, AE1-RD1) compatible with the ASTM standard E408 (2013), respectively. Both instruments were calibrated using the factory suggested calibration procedures prior to the experiment. The Reflectometer was calibrated to an accuracy of ± 0.001 using the set of calibration standard samples (a black-body cavity, mirror and ceramic tiles) supplied with the instrument. The Emissometer was calibrated to an accuracy of ± 0.01 using the two calibration standard samples, a high thermal emittance standard (ε

= 0.88) and a low thermal emittance standard ($\varepsilon = 0.06$), supplied with the instrument. The slide method was used to measure the thermal emittance of the original and cool roof surfaces. In the slide method, the detector head of the calibrated Emissometer was directly placed on the roof surface and allowed about a minute for the detector reading to reach a near steady value. Then the detector was slid several inches across the roof surface to a different spot without breaking contact with the roof surface. Solar reflectance and thermal emittance measurements were made at five locations on the roof surfaces to check the homogeneity. Table 3.5 summarises the radiation properties of the original and cool roof surfaces.

Table 3.5 Radiation properties of the original and cool concrete roof surfaces.

Roof surface	Solar reflectance (ρ)	Thermal emittance (ε)
Original	0.10	0.86
Cool	0.74	0.90

Note: Homogeneity of ρ and ε measurements at five locations was found to be ± 0.02 units.

Inside the test building, at the centre point, a standalone indoor air temperature recorder (MadgeTech, TransiTemp-II) was installed at the height of about 1.6 m above the floor to measure the indoor RH and air temperature. The recorder was factory calibrated to the accuracy of $\pm 0.5^\circ\text{C}$ temperature and $\pm 3.5\%$ RH. The detailed specifications and calibration procedure of TransiTemp-II recorder can be found in MadgeTech. TransiTemp-II manual. The outdoor air temperature and wind speed were monitored using a wireless weather station (Scientific Sales Inc., WeatherHawk 916) installed at the roof of the building. The weather station was factory calibrated to the accuracy of $\pm 0.5^\circ\text{C}$ temperature and ± 0.3 m/s wind speed. The detailed specifications and calibration procedure of WeatherHawk 916 weather station can be found in Scientific Sales Inc., WeatherHawk 916 manual. Incident solar radiation, surface

temperatures, indoor air temperature, outdoor air temperature and wind speed were monitored and recorded at 1-minute intervals. The specifications of all the instrumentations are shown in the *Appendix-1*. The mass density, specific heat and thermal conductivity of the building materials were obtained from the manufacturers and the EnergyPlus material dataset (2013), as shown in Table 3.1.

3.3 Results

3.3.1 Computational model calibration

In the test building, the infiltration rate is unknown. In order to obtain a proper infiltration rate, computational simulations are run for the same week in April (during which the test building measurements were conducted) under different infiltration rate settings. Figure 3.5 shows the matching between measurements and simulation data after calibration.

Fig. 3.5 Comparison of computational simulated temperatures (after calibration) against measured temperatures for the cool roof and cool ceiling.

The simulation results obtained under each infiltration rate setting are compared to the measurements results. The roof surface temperature and the ceiling surface temperature are compared in this calibration exercise since these two parameters are used in the subsequent calculation of heat gain through the roof. An infiltration rate of 0.90 ACH gives the best matching between simulation and measurement data with errors fall within 5%. The infiltration rate of 0.90 ACH is used in all subsequent computational simulations. The calibrated model is further used to investigate the effect of solar reflectance and thermal emittance on annual heat gain, annual heat loss and annual net heat gain reduction (as discussed in Section 3.3.2).

3.3.2 Effect of radiation properties (solar reflectance and thermal emittance)

In this analysis, simulations are performed on an original roof (low- ε) ($\varepsilon = 0.10$, $\rho = 0.10$), an original roof ($\varepsilon = 0.86$, $\rho = 0.10$) and a cool roof ($\varepsilon = 0.90$, $\rho = 0.74$). The calibrated cool roof model is used for this analysis. Figure 3.6 shows the annual heat gain, annual heat loss and annual net heat gain (summation of annual heat gain and loss) through the three roofs. The simulations are performed for common range of R -value roofs ($0.25 \text{ m}^2\text{-K/W}$ to $1.00 \text{ m}^2\text{-K/W}$). Table 3.6 compares the effect of radiation properties on annual net heat transfer.

Fig. 3.6 Comparison of annual heat gain, annual net heat gain and annual heat loss for cool roof, original roof and original roof (low-ε).

The difference between annual net heat gains for original roof and original roof (low-ε) shows the effect of thermal emittance increment of 0.76 (from 0.10 to 0.86) on annual net heat gain reduction. The difference between annual net heat gain for original roof and cool roof shows the effect of solar reflectance 0.64 (from 0.10 to 0.74) on annual net heat gain reduction. It can be observed (from Table 3.6) that the effect of solar reflectance increment is more prominent (about 30-33% higher) than the effect of thermal emittance increment in the tropical climate. The difference between annual net heat gains for original roof (low-ε) and cool roof shows the combined effect of thermal emittance increment of 0.76 (from 0.10 to 0.86) and solar reflectance increment of 0.64 (from 0.10 to 0.74) on annual net heat gain reduction. Table 3.6 shows that the combined effect of solar reflectance and thermal emittance brings in

additional annual net heat gain reduction by 10-15% as compared to that by solar reflectance (increment by 0.64) alone.

Table 3.6 Comparison of effect of radiation properties on annual net heat transfer.

Reduction in parameter	Original roof (low- ϵ)	Original roof	Cool roof
Annual net heat gain	Reference	50-53%	90-92%
	-	Reference	80-83%

3.3.3 Effect of green roof

The green roof method provides passive cooling due to two components: 1) conduction resistance offered by the soil layer (about 500-mm-thick) and 2) evapotranspiration offered by the moisture content of the soil and vegetation. In order to investigate the effect of each component, simulations are performed using the green roof model discussed in Section 3.2.1.1. In this analysis, simulations are performed on an original roof and two green roof cases: 1) green roof (100% moist soil), and 2) green roof (0% moist soil). In each green roof case, the soil layer is 500-mm-thick, and the thermal emittance of soil layer and vegetation is 0.86. It can be observed (from Fig. 3.7) that both green roof cases significantly reduce the annual net heat gain as compared to the original roof.

Fig. 3.7 Comparison of annual heat gain, annual net heat gain and annual heat loss for original roof and green roofs (two cases).

Table 3.7 compares the effect of green roofs on annual net heat transfer. The comparison between the two green roof cases shows the effect of evapotranspiration component alone. The increase in annual net heat gain (due to the increased moisture content of the green roof from 0% to 100%) is because the increased moisture content reduces the annual heat gain by about 1.4 kWh/m² (or 3.8%), as can be observed in Fig. 3.7. However (for the R -value of 0.48 m²-K/W), the annual heat loss by almost 10 times (from -6.3 kWh/m² to -0.6 kWh/m²), resulting in the annual net heat gain for 100% moist green roof being higher than that for 0% moist green roof. The comparison between original roof and green roof (0% moist soil) indicates the effect of conduction resistance due to the soil layer alone. While the comparison between original roof and green roof (100% moist soil) shows the combined effect of conduction resistance (due to the soil layer) and evapotranspiration.

Table 3.7 Comparison of effect of green roof on annual net heat transfer.

Reduction in parameter	Original roof (low- ε)	Green roof (100% moist soil)	Green roof (0% moist soil)
Annual net heat gain	Reference	55-75%	62-80%
	-	Reference	9-13%

3.3.4 Effect of thermal insulation

Thermal insulations provide additional conduction resistance to the original roof. Extruded polystyrene insulation (50-mm-thick) is simulated (as discussed in 3.2.1.3). In this analysis, simulations are performed on the original roof and the thermal insulation. Figure 3.8 and Table 3.8 show the effect of insulation on annual net heat gain reduction through the original roofs.

Fig. 3.8 Comparison of annual heat gain, annual net heat gain and annual heat loss for insulation (50-mm-thick) and original roof.

Table 3.8 Comparison of effect of insulation on annual net heat transfer.

Reduction in parameter	Original roof	Insulation (50-mm-thick)
Annual net heat gain	Reference	68-85%

3.3.5 Comparison of effect of radiation properties, green roof properties and thermal insulation

Figure 3.9 shows the annual net heat gain reduction due to the increment in solar reflectance (from 0.10 to 0.74), increment in thermal emittance (from 0.10 to 0.86), increment in combined solar reflectance (from 0.10 to 0.74) + thermal emittance (from 0.10 to 0.86), soil layer (500-mm-thick without moisture content), combined evapotranspiration + soil layer (500-mm-thick with 100% moisture content), and extruded polystyrene insulation (50-mm-thick). It can be observed (from Fig. 3.9) that the combined increment in solar reflectance (by 0.64) and thermal emittance (by 0.76) provides the highest annual net heat gain reduction (about 90-92%) for all R -values. Followed by, the increment in solar reflectance (by 0.64) alone provides about 81-83% annual net heat gain reduction for all R -values. While, the increment in thermal emittance (by 0.76) alone provides about 50-52% annual net heat gain reduction for all R -values. However, the annual net heat gain reduction provided by combined evapotranspiration + soil resistance reduces from 75% to 55%, by soil resistance alone reduces from 80% to 62%, and by extruded polystyrene insulation (50-mm-thick) reduces from 85% to 68%. This shows that the percentage annual net heat gain reduction provided by the increment in radiation properties almost remains constant, while that due to increment/addition of conduction resistance and green roof properties decreases with the increase in R -value of the roof, as can be observed from Fig. 3.9.

Fig. 3.9 Annual net heat gain reduction brought by the increment in radiation properties, green roof properties and thermal insulation.

3.3.6 Comparison of effect of radiation properties, green roof properties and thermal insulation on annual cooling energy savings

Figure 3.10 shows the annual cooling energy consumptions for the original roof ($\varepsilon = 0.86$, $\rho = 0.10$), original roof (low- ε) ($\varepsilon = 0.10$, $\rho = 0.10$), cool roof ($\varepsilon = 0.90$, $\rho = 0.74$), green roof (100% moist soil), green roof (0% moist soil) and insulation (50-mm-thick) for common R -value roofs in tropical climate. Table 3.9 compares the effect of radiation properties, green roof properties and insulation on annual cooling energy consumption. It can be observed (from Table 3.9) that with reference to the original roof (low- ε), the insulation provides the highest cooling energy savings, followed by both green roof cases, cool roof and least for the original roof.

Table 3.9 Comparison of the effect of radiation properties, green roof properties and insulation on annual cooling energy consumption.

Reduction in parameter	Original roof (low- ε)	Original roof	Cool roof	Green roof (100% moist soil)	Green roof (0% moist soil)	Insulation (50-mm-thick)
Annual cooling energy consumption	Reference	32-48%	77-84%	80-90%	78-88%	81-91%

Fig. 3.10 Annual cooling energy consumption for different roofing methods.

3.4 Discussion on performance of cool roof, green roof and thermal insulation in various climates

Figure 3.11 shows the comparison of annual heat gain reduction due to cool roof, green roof and thermal insulation in four climates: wet tropical climate, Mediterranean climate, hot and dry climate and semi-arid climate. Annual heat gain data for Mediterranean climate is obtained from Zinzi & Agnoli (2011), while that for hot and

dry climate of New Delhi and semi-arid climate of Beijing are obtained by performing simulations of the test building for the TMY weather data of those climate (EnergyPlus weather dataset, 2013). The same properties (as given in Zinzi & Agnoli (2011)) are used for original roof (solar reflectance of 0.25 and thermal emittance of 0.90), cool roof (solar reflectance of 0.80 and thermal emittance of 0.90), green roof (100% moist soil layer with 120-mm-thickness) and thermal insulation (30-mm-thick polystyrene).

Fig. 3.11 Comparison of performance of cool roof, green roof and thermal insulation in wet tropical climate, Mediterranean climate, hot and dry climate and semi-arid climate.

It can be observed (from Fig. 3.11) that the annual heat gain reduction provided by cool roof is about 55-65%, by thermal insulation is about 40-50%, and by green roof is about 25-35%. This shows that the annual heat gain reduction provided by cool roof is highest for all the four climates, followed by thermal insulation and green roof.

3.5 Summary

The thermal performance of cool roof (solar reflectance of 0.74 and thermal emittance of 0.90) is investigated by performing computational simulations on an air-conditioned, single-storey building in tropical climate. The computational model was calibrated using experimental measurements. The performance of cool roof is evaluated with other potential roofing methods like thermal insulation (50-mm-thick extruded polystyrene) and green roof. It is found that for the commonly used roofs in tropical climate (R -value in the range $0.25 \text{ m}^2\text{-K/W}$ to $1.00 \text{ m}^2\text{-K/W}$), the annual net heat gain reduction provided by increment/addition of

- combined solar reflectance (from 0.10 to 0.74) and thermal emittance (from 0.10 to 0.86) is about 92% to 90%,
- solar reflectance (from 0.10 to 0.74) is about 83% to 81%,
- thermal emittance (from 0.10 to 0.86) is about 53% to 50%,
- green roof (100% moist soil) is about 75% to 55%,
- green roof (0% moist soil) is about 80% to 62% and
- extruded polystyrene insulation (50-mm-thick) is about 85% to 68%.

The performance of cool roof, green roof and thermal insulation are compared for four climates: wet tropical climate, Mediterranean climate, hot and dry climate and semi-arid climate. It is observed that the annual heat gain reduction provided by cool roof is about 55-65%, by thermal insulation is about 40-50%, and by green roof is about 25-35%. This shows that the annual heat gain reduction provided by cool roof is highest for all the four climates, followed by thermal insulation and green roof.

Chapter 4

Modelling of Cool Roof Heat Transfer for Single-Skin Roofs

A part of this chapter is presented in the paper:

Kishor T. Zingre, Man Pun Wan, Shanshan Tong, Hua Li, Victor W.-C. Chang, Swee Khian Wong, Winston Boo Thian Toh and Irene Yen Leng Lee. “Modelling of cool roof heat transfer in tropical climate”, *Renewable Energy* 75:210-223, 2015.

4.1 Introduction

Over the years, numerous analytical methods (RF and CTF), and numerical methods (FD and FE) have been developed for thermal analysis of solid or single-skin roofs (SSR). In the implementations, the RF method requires long series of temperature histories or heat flux data. While the CTF method requires a set of pre-calculated coefficients (Delcroix et al., 2013) which are tabulated for certain types of building materials and for specific climate conditions, limiting its potentials for general uses. The FD and FE methods overcome the drawbacks of RF and CTF methods. However, due to the need for discretisation, these methods resolve the detailed temperatures distributions within the slab material which is unnecessary for air-conditioning load calculations and require time step of 15 min or less to achieve sufficient accuracy. This results in demand for heavy computational resources. In this chapter, a novel and general heat transfer model is proposed for SSR applied with cool coating on the roof top surface based on the spectral approximation method. The proposed model is applicable to all opaque surfaces and can handle transient outdoor and indoor boundary conditions as experienced by naturally ventilated buildings. The proposed model does not require small grid size in the slab material (hence no need to calculate the detailed temperatures within the slab material) and a time step of one hour can be used. The proposed model is verified against the CTF method and further validated against experiments performed on real-scale apartments in Singapore.

4.2 Methodology

4.2.1 Cool roof heat transfer (CRHT) model formulation

Building roofs experience transient heat transfer as the intensity of solar radiation and ambient air temperature vary continuously. In a naturally ventilated building, the indoor air temperature also varies with the changing environments and the internal heat loads. In this derivation, it is assumed that the roof material has constant thermophysical (conductivity, specific heat) and radiation properties (solar reflectance, thermal emittance). The roof material is homogeneous with constant material property (mass density). In the case of multi-layered roof, good contacts between different layers of the roof material with negligible interfacial resistance are assumed. The indoor air is assumed to be well-mixed and there are no internal heat gains inside the apartment. Hourly-averaged values of overall (combined convection and radiation) indoor heat transfer coefficient (h_i) for ceiling and overall outdoor heat transfer coefficient (h_R) for roof are used (Kontoleon & Eumorfopoulou, 2008; Khalid, 2003) in this study since the inputs are hourly-averaged values.

4.2.1.1 Problem formulation and boundary conditions

Consider an opaque SSR applied with a cool coating (or “cool roof”) having its top surface exposed to solar radiation as shown in Fig. 2.2 (Chapter 2). The incident radiation includes short waves in the wavelength range of 200 nm – 2500 nm (SW_{in} , i.e., insolation and the radiation reflected by other objects on the earth surface) and long waves with wavelength >2500 nm (LW_{in} , i.e., thermal radiation emitted by clouds and other objects on the earth surface). The outgoing radiation includes SW_{out} (i.e., reflected radiation by the roof surface during day time) and LW_{out} (i.e., thermal radiation emitted by the roof surface to the sky and other objects on the earth surface). The radiation balance at the cool roof surface is given as follows:

Absorbed radiation by the cool roof surface (I_{abs}) = Incident radiation on the cool roof surface ($I_{in} = SW_{in} + LW_{in}$) - Outgoing radiation from the cool roof surface ($I_{out} = SW_{out} + LW_{out}$).

The diurnal heat transfer through the cool roof is assumed to be one-dimensional i.e., in the direction perpendicular to the exposed surface (along the thickness). Assuming that the coordinate direction perpendicular to the roof surface is y as shown in Fig. 2.2, the governing differential equation for conduction heat transfer within the cool roof yields

$$\frac{\partial T}{\partial t} = \alpha_R^* \frac{\partial^2 T}{\partial y^2} \quad (4.1)$$

$$\text{where } \alpha_R^* = \frac{k_R}{\rho_R^* C_R}$$

The homogeneous-linear partial differential Eq. (4.1) can be solved using an initial condition in time and two boundary conditions in space. The Neumann boundary condition on the cool roof surface ($y = 0$) is

$$\begin{aligned} q(y=0, t) &= -k \left(\frac{\partial T}{\partial y} \right)_{y=0} = I_{in} \alpha_R - h_R (T_R - T_o) - \varepsilon_R I^{IR} \\ &= h_R \frac{I_{in} \alpha_R}{h_R} - h_R (T_R - T_o) - h_R \frac{\varepsilon_R I^{IR}}{h_R} = h_R \left(\frac{I_{in} \alpha_R}{h_R} - (T_R - T_o) - \frac{\varepsilon_R I^{IR}}{h_R} \right) \\ &= h_R \left(\frac{I_{in} \alpha_R}{h_R} + T_o - \frac{\varepsilon_R I^{IR}}{h_R} - T_R \right) \\ &= h_R (T_{sol-air}^e - T_R) \end{aligned} \quad (4.2)$$

$$\text{where } T_{sol-air}^e = \frac{I_{in} \alpha_R}{h_R} + T_o - \frac{\varepsilon_R I^{IR}}{h_R}, \quad h_R = h_{R,conv} + h_{R,r} \cdot \frac{T_R - T_{sky}}{T_R - T_o} \quad (4.2a)$$

and the Neumann boundary condition on the cool ceiling surface ($y = L$) is

$$q_c(y=L, t) = -k \left(\frac{\partial T}{\partial y} \right)_{y=L} = h_i (T_c - T_i) \quad (4.3)$$

and initial condition is steady state solution of Eq. (4.1) at $t = 0$.

4.2.1.2 Solution method for the CRHT model

The spectral approximation method (Zhuang & Ren, 2014; Oliveira & Azevedo, 2014; Gamba & Haack, 2014; Guo & Zhang, 2013; Podowitz et al., 2014) is a useful numerical method to solve partial differential equations. The solution of a partial differential equation using spectral approximation method is given as the summation of products of higher-order polynomial with time dependent coefficients.

An approximate solution of the governing differential Eq. (4.1) is given by

$$T(y, t) = \sum_{m=0}^{\infty} \Phi(y) \cdot \theta(t) \quad (4.4)$$

$$T(y, t) = \sum_{m=0}^{\infty} \left[a_m \cos\left(\frac{m\pi y}{L}\right) + b_m \sin\left(\frac{m\pi y}{L}\right) \right] \theta(t) \quad (4.5)$$

To find the expression for $\theta(t)$, partial derivatives of Eq. (4.5) are taken with respect to t and y and substituted in Eq. (4.1) giving Eq. (4.8)

$$\frac{\partial T(y, t)}{\partial t} = \sum_{m=0}^{\infty} \frac{\partial \theta(t)}{\partial t} \cdot \left[a_m \cos\left(\frac{m\pi y}{L}\right) + b_m \sin\left(\frac{m\pi y}{L}\right) \right] \quad (4.6)$$

$$\frac{\partial^2 T(y, t)}{\partial y^2} = \sum_{m=0}^{\infty} \theta(t) \cdot \left[-\frac{m^2 \pi^2}{L^2} a_m \cos\left(\frac{m\pi y}{L}\right) - \frac{m^2 \pi^2}{L^2} b_m \sin\left(\frac{m\pi y}{L}\right) \right] \quad (4.7)$$

$$\sum_{m=0}^{\infty} \frac{\partial \theta(t)}{\partial t} \cdot \left[a_m \cos\left(\frac{m\pi y}{L}\right) + b_m \sin\left(\frac{m\pi y}{L}\right) \right] = \sum_{m=0}^{\infty} -\alpha_R^* \frac{m^2 \pi^2}{L^2} \cdot \theta(t) \cdot \left[a_m \cos\left(\frac{m\pi y}{L}\right) + b_m \sin\left(\frac{m\pi y}{L}\right) \right] \quad (4.8)$$

Re-arranging Eq. (4.8), it becomes

$$\sum_{m=0}^{\infty} \left(\frac{\partial \theta(t)}{\partial t} + \frac{\alpha_R^* \pi^2 m^2}{L^2} \theta(t) \right) \left[a_m \cos\left(\frac{m\pi y}{L}\right) + b_m \sin\left(\frac{m\pi y}{L}\right) \right] = 0 \quad (4.9)$$

Since the functions $\left\{ a_m \cos\left(\frac{m\pi y}{L}\right) + b_m \sin\left(\frac{m\pi y}{L}\right) \right\}_{m=0}^{\infty}$ are orthogonal, Eq. (4.9) can be simplified to the ordinary differential equation

$$\frac{\partial \theta(t)}{\partial t} + \frac{\alpha_R^* \pi^2 m^2}{L^2} \theta(t) = 0 \quad (4.10)$$

$$\text{The solution of Eq. (4.10) is } \theta(t) = A e^{-\lambda_m^2 t} \quad (4.11)$$

where $\lambda_m^2 = \frac{\alpha_R^* \pi^2 m^2}{L^2}$ and A is an arbitrary constant.

Substituting the value of $\theta(t)$ in Eq. (4.5), it becomes

$$T(y, t) = \sum_{m=0}^{\infty} \left[a_m \cos\left(\frac{m\pi y}{L}\right) + b_m \sin\left(\frac{m\pi y}{L}\right) \right] A e^{-\lambda_m^2 t} \quad (4.12)$$

where coefficients a_m and b_m are

$$a_m = \frac{2}{L} \int_0^L (c_1 y + c_2) \cdot \cos\left(\frac{m\pi y}{L}\right) dy \quad \text{and} \quad b_m = \frac{2}{L} \int_0^L (c_1 y + c_2) \cdot \sin\left(\frac{m\pi y}{L}\right) dy \quad (4.13)$$

Applying the initial (steady state) condition to Eq. (4.1), it yields

$$T(y, 0) = c_1 y + c_2 \quad (4.14)$$

where c_1 and c_2 can be obtained by considering an arbitrary uniform temperature field (Ozel, 2012). Hence the final solution of Eq. (4.1) which satisfies the Neumann boundary conditions at the roof and ceiling surfaces and the initial condition is given as

$$T(y, t) = \left[\left(B \cdot \cos\left(\frac{m\pi y}{L}\right) - D \cdot \sin\left(\frac{m\pi y}{L}\right) \right) \left(E \cdot \frac{y}{L} - G \cdot \left(1 - \frac{y}{L}\right) \right) \right] e^{-\lambda_m^2 t} \quad (4.15)$$

$$\text{where } B^+ = \sum_{m=0}^{\infty} \left[\frac{2}{m^2 \pi^2} (T(L,0) - T(0,0)) (\cos m\pi - 1) \right], \quad D^+ = \sum_{m=0}^{\infty} \left[\frac{2}{m\pi} T(L,0) (\cos m\pi - 1) \right]$$

$$E^+ = \left[\frac{\frac{h_i}{k} T_i}{\frac{4}{L} T(L,0) + \frac{4h_i}{\pi^2 k} (T(L,0) - T(0,0))} \right], \quad G^+ = \left[\frac{\frac{h_R}{k} T_{sol-air}^s}{\frac{4}{L} T(L,0) + \frac{4h_R}{\pi^2 k} (T(L,0) - T(0,0))} \right]. \quad (4.15a)$$

$T_{sol-air}^s$ in Eq. (4.15a) is a function of outdoor conditions (solar radiation, outdoor air temperature and wind speed, as shown in Eq. (4.2a)) for successive 24-hour cycles (Ozel, 2012) and hence, it changes with the daily outdoor conditions. $T_{sol-air}^s$ can be mathematically expressed as Eq. (4.16)

$$T_{sol-air}^s = \tilde{T}_{sol-air} + T_{sol-air}^+ \cdot \cos(\omega t - \psi) \quad (4.16)$$

$$\text{where } T_{sol-air}^+ = \sqrt{\left(\frac{1}{12} \sum_{t=1}^{24} T_{sol-air}^e \cdot \cos(\omega t) \right)^2 + \left(\frac{1}{12} \sum_{t=1}^{24} T_{sol-air}^e \cdot \sin(\omega t) \right)^2},$$

$$\tan(\psi) = \frac{\left(\frac{1}{12} \sum_{t=1}^{24} T_{sol-air}^e \cdot \sin(\omega t) \right)}{\left(\frac{1}{12} \sum_{t=1}^{24} T_{sol-air}^e \cdot \cos(\omega t) \right)}.$$

T_i in Eq. (4.15a) is calculated using the heat balance in indoor environment, as given by Eq. (4.17). As mentioned, it is assumed that the indoor air is well-mixed and there are no internal heat gains inside the apartment.

$$L_i \rho_{air}^* C_{air} \left(\frac{dT_i}{dt} \right) = h_i [T_C - T_i] + \sum_{wall} h_i [T_{wall} - T_i] + \sum_{window} U_{window} [T_o - T_i] + h_i [T_{floor} - T_i] + \eta^+ [T_o - T_i]. \quad (4.17)$$

$$\text{where } \eta^+ = \frac{(ach) L_i \rho_{air}^* C_{air}}{3600 \times VOL_i}$$

By solving Eq. (4.17) with steady state initial condition, it gives

$$(T_i)_{cool} = \frac{(h_i)_{cool} \left[(T_C)_{cool} + \sum_{wall} T_{wall} + T_{floor} \right] + \left(\sum_{window} U_{window} + \eta^+ \right) T_o}{(\mu)_{cool}} - \left[h_i \left(\bar{T}_C + \sum_{wall} \bar{T}_{wall} + \bar{T}_{floor} \right) + \left(\sum_{window} U_{window} + \eta^+ \right) \bar{T}_o \right]$$

$$-\bar{T}_i(\mu) \Big|_{cool} \cdot \frac{e^{\frac{-I}{L_i \rho_{as}^* C_{air}}}}{(\mu)_{cool}}. \quad (4.18)$$

where $\mu_i^+ = h_i + \sum_{wall} h_i + \sum_{window} U_{window} + \eta_i^+$.

In order to analyse the cooling impact (of applying a cool coating) on the indoor air temperature, a simplified Eq. (4.18a) is derived by subtracting $(T_i)_{cool}$ from $(T_i)_{org}$. In Eq. (4.18a), $F_1 < 1$ entails positive cooling impact or reduction in the indoor air temperature, $F_1 > 1$ implies negative cooling impact or increase in the indoor air temperature and $F_1 = 1$ implies no cooling impact on the indoor air temperature by applying a cool coating.

$$T_i^{cooling_impact} = (T_c)_{org} \left(\frac{h_i}{\mu^+} \right)_{org} \cdot (1 - F_1) \quad (4.18a)$$

where $F_1 = \frac{(T_c)_{cool}}{(T_c)_{org}}$ and assuming $(\mu^+)_{org} \cong (\mu^+)_{cool}$.

Substituting Eq. (4.15) in Eq. (4.3) and re-arranging, the surface temperature of cool ceiling can be obtained as given by Eq. (4.19)

$$(T_c)_{cool} = \left[h_i \cdot \left(\sum_{wall} T_{wall} + T_{floor} \right) + \left(\sum_{window} U_{window} + \eta^+ \right) T_o - \left(h_i \cdot \left(\bar{T}_c + \sum_{wall} \bar{T}_{wall} + \bar{T}_{floor} \right) + \left(\sum_{window} U_{window} + \eta^+ \right) \bar{T}_o - \bar{T}_i(\mu^+) \right) \Big|_{cool} \right] \cdot \left(1 - \frac{U}{h_i} \right)_{cool} \\ + \left[\frac{U}{h_i} \cdot (\tilde{T}_{sol-air} + \beta \cdot T_{sol-air} \cos(wt - \psi - \phi)) \right]_{cool}. \quad (4.19)$$

In order to analyse the cooling impact on the ceiling temperature, a simplified equation Eq. (4.19a) is derived by subtracting $(T_c)_{cool}$ from $(T_c)_{org}$. In Eq. (4.19a), $F_2 < 1$ implies positive cooling impact or reduction in the ceiling temperature, $F_2 > 1$ implies negative cooling impact or increase in the ceiling temperature and $F_2 = 1$ implies no cooling impact on the ceiling temperature by applying a cool coating.

$$T_C^{cooling_impact} = \left(\tilde{T}_{sol-air} + \beta T_{sol-air} \cos(wt - \psi - \phi) \right)_{org} \left[\left(\frac{U}{h_i} \right)_{org} - \left(\frac{U}{h_i} \right)_{cool} F_2 \right] \quad (4.19a)$$

$$\text{where } F_2 = \frac{\left(\tilde{T}_{sol-air} + \beta T_{sol-air} \cos(wt - \psi - \phi) \right)_{cool}}{\left(\tilde{T}_{sol-air} + \beta T_{sol-air} \cos(wt - \psi - \phi) \right)_{org}}$$

$$\frac{1}{U_{cool}} = \frac{1}{h_i} + \frac{1}{h_R} + \frac{L}{k} + \frac{l^{cool}}{k^{cool}} \quad (4.19b)$$

Substituting Eq. (4.15) in Eq. (4.2) and re-arranging, the surface temperature of cool roof can be obtained as given by Eq. (4.20)

$$(T_R)_{cool} = \left(\frac{\tilde{T}_{sol-air} + T_{sol-air} \cos(wt - \psi)}{\left(1 + \frac{1}{\nu^+} \right)} + \frac{\left[T_i \left(1 - \frac{U}{h_i} \right) + \frac{U}{h_i} \tilde{T}_{sol-air} \right]}{(1 + \nu^+)} \right)_{cool} \quad (4.20)$$

$$\text{where } \nu^+ = \frac{h_i L}{k}$$

In order to analyse the cooling impact on the roof temperature, a simplified equation Eq. (4.20a) is derived by subtracting $(T_R)_{cool}$ from $(T_R)_{org}$. In Eq. (4.20a), $F_3 < 1$ implies positive cooling impact or reduction in the roof temperature, $F_3 > 1$ implies negative cooling impact or increase in the roof temperature and $F_3 = 1$ implies no cooling impact on the roof temperature by applying a cool coating.

$$T_R^{cooling_impact} = (T_{sol-air}^s)_{org} \left(\frac{1}{\left(1 + \frac{1}{\nu^+} \right)_{org}} - \frac{F_3}{\left(1 + \frac{1}{\nu^+} \right)_{cool}} \right) + (T_c)_{org} \left(\frac{1}{\left(1 + \nu^+ \right)_{org}} - \frac{F_1}{\left(1 + \nu^+ \right)_{cool}} \right) \quad (4.20a)$$

$$\text{where } F_3 = \frac{\left(\tilde{T}_{sol-air} + T_{sol-air} \cos(wt - \psi) \right)_{cool}}{\left(\tilde{T}_{sol-air} + T_{sol-air} \cos(wt - \psi) \right)_{org}}$$

The transient heat flux through cool roof can be obtained as given by Eq. (4.21)

$$q_{cool} = (U(1 + \nu^+))_{cool} \left(T_{sol-air}^s - \left[T_i \left(1 - \frac{U}{h_i} \right) + \frac{U}{h_i} \tilde{T}_{sol-air} \right] \right)_{cool} \quad (4.21)$$

In order to analyse the cooling impact on the heat flux by applying a cool coating, a simplified Eq. (4.21a) is derived by subtracting q_{cool} from q_{org} . In Eq. (4.21a), $F_4, F_5 < 1$ implies positive cooling impact or reduction in the heat flux, $F_4, F_5 > 1$ implies negative cooling impact or increase in the heat flux and $F_4, F_5 = 1$ implies no cooling impact on the heat flux by applying a cool coating.

$$q^{cooling_impact} = (U(1 + \nu^+))_{org} [(T_{sol-air}^s)_{org} \cdot (1 - F_4) - (T_c)_{org} \cdot (1 - F_5)] \quad (4.21a)$$

$$\text{where } F_4 = \frac{(U(1 + \nu^+)T_{sol-air}^s)_{cool}}{(U(1 + \nu^+)T_{sol-air}^s)_{org}}, \quad F_5 = \frac{(U(1 + \nu^+)T_c)_{cool}}{(U(1 + \nu^+)T_c)_{org}}$$

The CRHT model, i.e., Eq. (4.21), is a generalised heat transfer model that can be applied to solid roofs/walls in any climate condition. For validation of the CRHT model, an experimental study was carried out.

4.2.2 Experimental setup for validation of the CRHT model

An experimental study was carried out in a high-rise (12-storey), naturally ventilated residential building with a 100-mm-thick concrete (plus 10-mm-thick plaster on the ceiling) solid flat roof in Singapore on a sunny day (with clear sky) in February, 2012. The selected residential building is one of the public housing buildings developed by the Singapore government. In Singapore, about 86% of the total population resides in public housing buildings (Chua & Chou, 2010), in which the majority are high-rise apartment buildings. Two identically configured side-by-side unoccupied apartments located on the top-most storey were used for the experiments. Each of the two test apartments had the same roof area of about 90 m². White-coloured cool coating (0.50-mm-thick when dried) was applied on the roof surface of one of the two apartments while the other apartment's roof was remained uncoated (original) as the reference for comparison. Figure 4.1 shows the schematic diagram of the experimental set up. The cool coating thickness was measured using a coating thickness gauge (Elcometer,

A456CSFI1) which was calibrated to an accuracy of $\pm 1\%$ reading using the zero offset calibration method compatible with ISO 19840 (2012). In the calibration process, the thickness gauge probe was initially placed on the original roof surface and the thickness was set to zero (as reference). Then a set of calibration standard samples (provided with the instrument) were placed on the original surface and measured by the thickness gauge. The offset between the thickness gauge readings and the actual thicknesses of the calibration standards was adjusted to match with the calibration standards. The cool coating thickness measurements were made at five locations by directly placing the calibrated thickness gauge on the coated roof surface. The cool coating thickness homogeneity of these five locations was ± 0.01 mm.

The solar radiation received by the roof surfaces was monitored by a pyranometer (Kipp and Zonen, CMP 11) facing upwards (towards the sky) mounted at 1.5 m above the roof surface. The pyranometer was factory calibrated to an accuracy of $\pm 1.4\%$ reading. The detailed calibration procedure of CMP 11 pyranometer can be found in Kipp and Zonen, CMP 11 manual. The surface temperatures of the roof and the ceiling were monitored by surface-type resistance temperature detectors encapsulated in stainless steel probes of 4 mm in diameter and 25 mm in length (Omega, 4 wire RTDs). The calibrated RTDs were installed at five points (to check the homogeneity) of the each roof and the ceiling surfaces such that the sensor portion of RTDs was covered with an insulation tape followed by another reflective tape on top. The reflective tape and insulation tape prevented the sensor tip from being affected by solar radiation such that the sensor tip measured the actual temperature of the surfaces. The RTDs were calibrated to an accuracy of $\pm 0.5^\circ\text{C}$ prior to the experiment using a stirred silicon oil bath thermal calibrator (Tempsens Instruments (I) Pvt. Ltd, Calsys -40/200) for the temperature range of 15°C - 65°C (as experienced by the concrete roof on a

sunny day). In the calibration process, the silicon oil bath was controlled to the calibration temperature in the thermal calibrator. The RTD sensors were inserted into the silicon oil bath. The temperature readings by the RTD sensors were compared against the oil bath temperature reading from the thermal calibrator to produce a calibration curve for each RTD sensor.

Fig. 4.1 Schematic diagram showing the experimental set up for original and cool concrete roofs (not-to-scale).

The outdoor air temperature and wind speed were monitored using a wireless weather station (Scientific Sales Inc., WeatherHawk 916) installed at the roof of the experimental building. The weather station was factory calibrated to the accuracy of $\pm 0.5^\circ\text{C}$ temperature and ± 0.3 m/s wind speed. The detailed specifications and calibration procedure of WeatherHawk 916 weather station can be found in Scientific Sales Inc. WeatherHawk 916 manual.

Inside each apartment at the centre point, a standalone indoor air temperature and relative humidity (RH) recorder (MadgeTech, TransiTemp-II) was installed at a height of 1.4 m above the floor to measure the indoor RH and air temperature. The recorder was factory calibrated to the accuracy of $\pm 0.5^{\circ}\text{C}$ temperature and $\pm 3.5\%$ RH. The detailed calibration procedure of TransiTemp-II recorder can be found in MadgeTech. TransiTemp-II manual. The specifications of all the instrumentations are shown in the *Appendix-1*.

The solar reflectance and thermal emittance of both the roof surfaces were measured using a portable solar spectrum reflectometer (Devices and Services, SSR-6) compatible with the ASTM standard C1549 (2009) and a portable emissometer (Devices and Services, AE1-RD1) compatible with the ASTM standard E408 (2013), respectively. Both instruments were calibrated using the factory suggested calibration procedures prior to the experiment. The reflectometer was calibrated to an accuracy of ± 0.001 using the set of calibration standard samples (a black-body cavity, mirror and ceramic tiles) supplied with the instrument. In the calibration process, the reflectometer was powered on to warm up for approximately half an hour. The calibration standard samples were then placed alternately on the reflectance head to measure their solar reflectance values. The offset between the solar reflectance readings and the actual solar reflectance of the calibration standards was adjusted using the electronic module until the reflectometer readings matched with the calibration standards. The reflectance head of the calibrated reflectometer was then directly placed on the original and cool roof surfaces to measure their solar reflectance values.

The emissometer was calibrated to an accuracy of ± 0.01 using the two calibration standard samples, a high thermal emittance standard ($\varepsilon = 0.88$) and a low thermal

emittance standard ($\varepsilon = 0.06$), supplied with the instrument. The slide method, which is commonly used to measure thermal emittance of the materials that cannot be placed on the heat sink and the materials with low thermal conductivity, was used to measure the thermal emittance. In the calibration process, both the high and low thermal emittance standard samples were placed side-by-side on the roof surface. In order to ensure that both the standard samples were at the same surface temperature, several drops of water were put on the surface before placing the standard samples. The detector head, which had been switched on and warmed up for approximately half an hour, was placed alternately on both standard samples to get the thermal emittance readings. The gain and the offset of the emissometer were adjusted until the thermal emittance readings matched with both standard samples. In the slide method, the detector head of the calibrated emissometer was directly placed on the roof surface and allowed about a minute for the detector reading to reach a near steady value. Then the detector was slid several inches across the roof surface to a different spot without breaking contact with the roof surface. Solar reflectance and thermal emittance measurements were made at five locations on each roof to check the homogeneity. The readings difference obtained at these five locations for both solar reflectance and thermal emittance were within ± 0.02 units. The mass density, specific heat and thermal conductivity of the roof materials were obtained from various references (Samah, 2012; Ozel, 2012; Duffie & Beckman, 1991).

Solar radiation, surface temperatures, indoor RH, indoor air temperature, outdoor air temperature and wind speed were monitored at 1-minute intervals. Figures 4.2 (a) and 4.2 (b) show the hourly-averaged values and standard deviations (shown as error bar) of these measurements that were estimated from the 1-minute raw data. In Fig. 4.2 (a), the original and cool solar-air temperatures were estimated (using Eq. (4.2a)) from the

measurement data of solar radiation, solar reflectance, thermal emittance, outdoor air temperature and wind speed. Estimating from the instrumentation errors of the pyranometer, reflectometer, emissometer and weather station, the error of the ‘measured’ solar-air temperatures was $\pm 0.9^\circ\text{C}$. The measured solar-air temperatures were approximated using Fourier series (Eq. (4.16)) which were required as an input to the CRHT model. The parameters involved in the solar-air temperature estimations are shown in Table 4.1. The solar reflectance and thermal emittance of the original and cool concrete roofs were obtained from the measurements.

Fig. 4.2 (a) Hourly-averaged solar radiation, outdoor air temperature, sky temperature and solar-air temperature measurements (with Fourier series simulations). Error bars show 1 standard deviation of the 1-minute measurements in each hour.

Table 4.1 Parameters involved in solar-air temperature calculations for various roof materials.

Roof	Solar reflectance (ρ)	Thermal emittance (ε)	$\tilde{T}_{sol-air}$ (°C)	$T_{sol-air}$ (°C)	ψ (radian)
Original concrete	0.30	0.84	31.6	18.4	3.53
Cool concrete	0.74	0.90	24.9	7.6	3.63
Original galvanised steel (metal)	0.60*	0.30*	26.3	9.9	3.59
Cool galvanised steel (metal)	0.74	0.90	24.9	7.6	3.63

*Radiation properties are taken from Suehrcke et al. (2012) and ASHRAE (2009).

Figure 4.2 (b) shows the hourly-averaged values of wind speed, overall outdoor heat transfer coefficient h_R and overall indoor heat transfer coefficient h_i . Equation (4.22) shows that h_R combines the effects of $h_{R,conv}$ and $h_{R,r}$. The correlations for $h_{R,conv}$ is taken from (Clear et al., 2003) and for $h_{R,r}$ is taken from (Suehrcke et al., 2008). Equation (4.22a) is applicable for the estimation of $h_{R,conv}$ for any roof surface and is not sensitive to wind direction nor roughness of the roof surface. In case of wind speed (V) < 4.88 m/s, $h_{R,conv}$ (Eq. (4.22a)) assumes a relatively large constant value of 5.6 W/m²-K to represent the natural convection portion of the total convection coefficient between the roof surface and outdoor air. Equations (4.22b) – (4.22d) are applicable for the estimation of $h_{R,r}$. The sky temperature (i.e., the temperature of a hypothetical black hemisphere absorbing the same radiation flux emitted by roof surface as absorbed by the sky (Samah et al., 2012)) was estimated using Eq. (4.22d). The h_R values for the original and cool concrete roofs were estimated using the measured roof surface temperatures, outdoor air temperature, wind speed and thermal emittance. Estimating from the instrumentation errors of the surface temperature sensors, weather station and Emissometer, the error of the ‘measured’ h_R was ± 1.4 W/m²-K.

$$h_R = h_{R,conv} + h_{R,r} \cdot \frac{T_R - T_{sky}}{T_R - T_o} \quad \text{for a roof surface directly facing the sky,} \quad (4.22)$$

$$\text{where } h_{R,conv} = \frac{k_{air}}{D_h} 0.15 Ra^{1/3} + k_{air} \hat{f} 0.0296 Re^{4/5} Pr^{1/3} \quad (4.22a)$$

$$h_{R,r} = \varepsilon_R \cdot \sigma \cdot \hat{F}_{R,sky} (T_R^2 + T_{sky}^2) \cdot (T_R + T_{sky}), \quad (4.22b)$$

$$\hat{F}_{R,sky} = 0.5(1 + \cos \phi) \quad (\hat{F}_{R,sky} = 1 \text{ for a horizontal surface}), \quad (4.22c)$$

$$T_{sky} = 0.0552 \varepsilon_{sky} (T_o)^{1.5} \text{ for clear sky days or} \quad (4.22d)$$

$$T_{sky} = \varepsilon_{sky} T_o \text{ for cloudy days} \quad (4.22e)$$

$$\varepsilon_{sky} = \left(0.79 + 0.76 \ln \left(\frac{T_o}{273} \right) \right) \cdot (1 + 0.022 P - 0.004 P^2 - 0.0003 P^3) \quad (4.22f)$$

Fig. 4.2 (b) Hourly-averaged wind speed (V) and h_R for original and cool concrete roofs. Error bars show 1 standard deviation of the 1-minute measurements in each hour.

Figure 4.2 (b) also shows the hourly-averaged values of overall indoor heat transfer coefficient h_i for the original and cool concrete ceilings. Equation (4.23) shows that h_i combines the effects of $h_{i,conv}$ and $h_{i,r}$ (Anderson, 1981). The h_i values were estimated by Eqs. (4.23) – (4.23c) using the measured ceiling surface temperatures, floor surface temperatures, indoor air temperatures and thermal emittance values. The correlation for $h_{i,conv}$ (Eq. (4.23a), see details in *Appendix-4*, A-1) is applicable to the ceilings of naturally ventilated buildings (Awbi, 1998). The correlation for $h_{i,r}$ (Eq. (4.23b)) is based on the assumptions that the thermal emittance of all the indoor facing opaque surfaces (including ceiling, inner walls and floor) is the same (Anderson, 1981). Accounting for the errors of the instruments used in the current study, the accuracy in the measured h_i was $\pm 0.3 \text{ W/m}^2\text{-K}$.

$$h_i = h_{i,conv} + h_{i,r} \quad (4.23)$$

where

$$h_{i,conv} = (0.704/D_h^{0.601})(\Delta T)^{0.133}, \quad (4.23a)$$

$$h_{i,r} = \frac{\sigma \cdot \varepsilon_i}{4(2 - \varepsilon_i)} (1 + \hat{F}_{C,floor})(T_C + T_{floor})^3, \quad (4.23b)$$

$$\hat{F}_{C,floor} = \sqrt{1 + \frac{L_i^2}{W^2}} - \frac{L_i}{W} \quad (\hat{F}_{C,floor} = 1 \text{ for } L_i \ll W). \quad (4.23c)$$

The correlation for $h_{i,conv}$ (Eq. (4.23a)) is applicable to the ceilings of naturally ventilated buildings (Awbi, 1998). While, the correlation for $h_{i,r}$ (Eq. (4.23b)) is based on the assumptions that the thermal emittance of all the indoor facing opaque surfaces (including ceiling, inner walls and floor) is same (Anderson, 1981). Due to the instrumentation errors, the accuracy in the ‘measured’ h_i was $\pm 0.3 \text{ W/m}^2\text{-K}$.

4.3 Results

4.3.1 Verification and validation of the CRHT model

The CRHT model verification was carried out against analytical method (the conduction transfer function (CTF)) and validation against real-scale experiments, similar to the approach suggested in ASHRAE Standard 140 (Tabares-Velasco et al., 2012).

The CTF method defines the conduction heat flux as a linear function of the current and past surface temperatures and past heat fluxes using four set of CTF coefficients (Tabares-Velasco et al., 2012; EnergyPlus Engineering References, 2013). The CTF coefficients (which involve the interior, exterior, cross and flux terms) represent the material's thermal response as determined by its material properties (and independent of weather data). The state-space method (EnergyPlus Engineering References, 2013) was used for the estimation of CTF coefficients of the roof of the test building using ASHRAE toolkit (ASHRAE, 2009) and then utilised in conjunction with the monitored weather data of a sunny day (shown in Figs. 4.2 (a) and 4.2 (b)). The thermophysical properties of the roof of the test building are given in Table 4.2. These properties were not measured on site but were taken from the literature or data provided by the manufacturers.

It can be observed (from Table 4.3) that the totals of exterior coefficients, cross coefficients and interior coefficients are equal. At the same time, the ratios given by Eq. (4.25) are also satisfied by the CTF coefficients (see Table 4.3) and are equal to U -value of the cool roof of the test building i.e., $2.44 \text{ W/m}^2\text{-K}$, hence both the criterion (as given by Eqs. (4.24) and (4.25)) are satisfied by the CTF coefficients.

Table 4.2 Thermophysical properties of various materials.

Roof material	Thermal conductivity (W/m-K)	Thickness (mm)	Conduction resistance (m ² -K/W)	Density (kg/m ³)	Specific heat (J/kg-K)
Air	0.025	-	-	1.27	1008
Plaster*	0.533	10	0.019	1568	840
Concrete*	1.442	100	0.069	2400	880
Galvanised steel ⁺⁺	15.3	8	0.001	4800	500
Cool coating	0.045*	0.50	0.011*	1053*	**

*Properties are taken from the manufacturers.

⁺⁺ Properties are taken from ASHRAE (2009).

**Thermal mass, mC_p is 0 J/K due to negligible mass of coating.

The verified CTF coefficients were then utilised in conjunction with the measured weather data, heat transfer coefficients and temperature data on a sunny day (as shown in Figs. 4.4 (a), 4.4 (b) and 4.4 (c)). The thermophysical properties of the roof materials of the test building are given in Table 4.2. These properties were taken from the literature (Suehrcke et al., 2012; ASHRAE, 2009) or data provided by the manufacturers.

$$\sum_{j=0}^p X_j = \sum_{j=0}^p Y_j = \sum_{j=0}^p Z_j \quad (4.24)$$

$$\frac{\sum_{j=0}^p X_j}{1 - \sum_{j=0}^p -\Theta_j} = \frac{\sum_{j=0}^p Y_j}{1 - \sum_{j=0}^p -\Theta_j} = \frac{\sum_{j=0}^p Z_j}{1 - \sum_{j=0}^p -\Theta_j} = U \quad (4.25)$$

Figure 4.2 (a) shows the measured hourly-averaged solar radiation, outdoor air temperature and the solar-air temperatures for both roofs on the day of the experiment (a sunny day). On that day, the peak hourly-averaged solar radiation (940 W/m²) occurred at 13:00 hours, sunrise occurred at 07:00 hours and sunset occurred at 19:00

hours. The measured solar reflectance and thermal emittance of both roofs are shown in Table 4.1.

Table 4.3 CTF coefficients for the test building roof ($U = 2.44 \text{ W/m}^2\text{-K}$).

CTF term index (j)	Exterior coefficient, X_j ($\text{W/m}^2\text{-K}$)	Cross coefficient, Y_j ($\text{W/m}^2\text{-K}$)	Interior coefficient, Z_j ($\text{W/m}^2\text{-K}$)	Flux, Θ_j
0	1.14E+01	9.91E-02	3.56E+01	0
1	-1.25E+01	5.02E-01	-3.04E+00	7.34E-01
2	1.91E+00	1.46E-01	2.26E-01	-3.99E-02
3	-1.96E-02	1.73E-03	-7.21E-04	1.33E-05
4	2.71E-06	1.57E-07	6.12E-08	-4.29E-12
5	-5.05E-13	1.87E-14	-6.41E-15	-9.92E-19
^a Total	$\sum_{j=0}^p X_j = 7.48\text{E-}01$	$\sum_{j=0}^p Y_j = 7.48\text{E-}01$	$\sum_{j=0}^p Z_j = 7.48\text{E-}01$	$\sum_{j=0}^p \Theta_j = 6.94\text{E-}01$
^b Total / $(1 - \sum_{j=0}^p \Theta_j)$	2.44E+00	2.44E+00	2.44E+00	

^aCriteria given in Eq. (4.24).

^bCriteria given in Eq. (4.25).

CRHT model predictions of ceiling temperature using Eq. (4.19), roof temperature using Eq. (4.20) and conduction heat flux using Eq. (4.21) were compared with the CTF estimation results for original and cool concrete roofs. As one of the inputs for the CRHT model, the 24-hour solar-air temperature profiles were obtained from Fourier series simulation (as shown in Fig. 4.2 (a)). The correction factor for infrared radiation ($\varepsilon_R I^R / h_R$) in Eq. (4.2a) takes into account the infrared radiation emitted by a building surface to the sky (which is usually cooler than building surfaces) and surrounding objects. For horizontal surfaces those emit infrared radiation only to the

sky, ASHRAE recommends the correction factor be given value of 3.9°C (ASHRAE, 2005). Thus solar-air temperature is cooler by 3.9°C for horizontal surfaces due to the emitted infrared radiation to the cooler sky. Therefore, the correction factor for infrared radiation in Eq. (4.2) was assumed to be 3.9°C for flat roofs. The values of h_i and h_R for the original and cool concrete roofs were obtained from Fig. 4.2 (b). Thermophysical properties of the materials that were required for the CRHT model are shown in Table 4.2.

Figure 4.3 shows the comparisons between CRHT predictions and CTF estimation for hourly-averaged roof and ceiling surface temperatures as well as conduction heat flux through the original and cool concrete roofs. It can be observed that the CRHT predicted profiles of roof and ceiling surface temperatures and conduction heat flux almost follow the same trends as those estimated by the CFT method (for the respective parameter) and the values are considerably close to each other. The maximum difference between the CRHT prediction and the CTF estimation for all the three (roof surface temperature, ceiling surface temperature and conduction heat flux) parameters are within 6%.

The validation of the CRHT model predictions was performed by comparing ceiling temperature using Eq. (4.19), roof temperature using Eq. (4.20), conduction heat flux using Eq. (4.21) and indoor air temperature using Eq. (4.18) with the experimental results for original and cool concrete roofs. Figure 4.4 (a) shows the comparisons between CRHT predictions and experimental measurements of hourly-averaged ceiling and roof surface temperatures as well as conduction heat flux through the concrete roofs. The error bars show 1 standard deviation of the 1-minute measurements in each hour. The ‘measured’ heat flux through the roofs was estimated from the measured roof and ceiling surface temperatures.

Fig. 4.3 Comparison of CRHT predictions with conduction transfer function (CTF) method for hourly-averaged roof temperature, ceiling temperature and conduction heat flux through original vs. cool concrete roofs.

Estimating from the instrumentation error of the surface temperature sensors, the error of the ‘measured’ heat flux was $\pm 3.3 \text{ W/m}^2$. The CRHT model can generally capture the ceiling and the roof temperature profiles with reasonable accuracy. The difference between CRHT prediction and measured hourly-averaged ceiling temperature is $\pm 1.1^\circ\text{C}$ and for roof temperature is $\pm 1.5^\circ\text{C}$. The higher discrepancy between CRHT prediction and measurement found in roof temperature, compared to ceiling

temperature, may be because the actual h_R varied more significantly with the transient outdoor conditions than h_i .

Fig. 4.4 (a) Comparison of CRHT predictions with measurements of hourly-averaged roof temperature, ceiling temperature and conduction heat flux through original vs. cool concrete roofs. Error bars show 1 standard deviation of the 1-minute measurements in each hour.

4.3.2 Impact of cool coating on indoor air temperature, surface temperatures and heat gain

Figure 4.4 (a) also depicts the impact of cool coating on roof temperature, ceiling temperature and conduction heat flux (due to the temperature difference between roof

and ceiling) through the concrete roof. The cool coating, which increases the solar reflectance of concrete roof from 0.30 (original) to 0.74, keeps the cool roof and cool ceiling temperatures lower than the original counterpart almost for the whole day. During day time, the peak cool roof temperature is lower by up to 14.1°C and the cool ceiling temperature is lower by up to 8.3°C than the original counterpart. While during night time, the cool roof temperature is about 2.6°C lower and the cool ceiling temperature is about 4.3°C lower than the original counterpart. The impact of cool coating is higher for cool ceiling than cool roof during night time. This is because (in second half of the day i.e., period after peak solar irradiation occurred at 13:00 hours) the original roof cools down faster than the cool roof (due to the higher temperature difference between the original roof and surrounding). However, the original ceiling cools down comparatively slower than the original roof (due to thermal mass) and remain at higher temperature than cool ceiling due to large heat stored in (original roof's) thermal mass during day time. This results in, during night time, higher temperature difference is observed between the original ceiling and the cool ceiling, as compared to that between the original roof and the cool roof. The reduction in cool roof temperature (during day time) leads to lower heat gain into the building through the cool roof as compared to the original roof. Cool roof has a peak heat gain reduction of 0.12 kW/m² (or 57%) compared to the original roof during day time. During night time the heat flow direction reverses. Even though the cool coating gives a higher thermal emittance to the cool roof (as shown in Table 4.1), the original roof still shows a higher heat loss rate compared to the cool roof. This is because the original ceiling temperature remains higher than that of the cool ceiling during night time due to the larger amount of heat stored in the thermal mass from daytime heat gain, resulting in a larger temperature difference between the roof and the surroundings, giving rise to the

higher heat emission rate found in the original roof compared to the cool roof. Integrating the downward heat flux profile with respect to time for the day using Eq. (4.26) (by using Simpson's 3/8 rule as discussed in Appendix-7),

$$Q_{gain} = \int_{daily} q dt, \quad \text{where } q \text{ is the downward heat flux through roof} \quad (4.26)$$

the daily gains through the two roofs are estimated. The original roof has 1.28 kWh/m² of daily heat gain, while the cool roof has only 0.54 kWh/m². The cool coating leads to a daily heat gain reduction of 0.74 kWh/m² (or 58%). Figure 4.4 (b) shows the ceiling heat flux (using Eq. (4.3)) and indoor air temperatures (using Eq. (4.18)) for the original and cool concrete roofs. It can be observed that the cool coating keeps the indoor air temperature lower than that beneath the original roof almost throughout the day. The cool coating provides maximum indoor air temperature reduction of 3.1°C at 16:00 hours, due to the reduction in the ceiling heat flux gain of 2.95 W/m². In order to comprehend the performance of cool coating at different locations, the results of this study are compared with that of the previous studies for concrete roofs (as shown in Table 4.4). It is evident that the impact of cool coating in tropical climate seems to be more prominent compared to other locations.

Table 4.4 Impact of cool coating on concrete roofs at different locations.

Source	Location (Climate zone)	Roof material	Increment in solar reflectance ($\Delta\rho$)	Reduction in peak roof temperature (°C)
This study	Singapore (Wet tropical)	Concrete	0.44	14.1
Kolokotsa et al. (2012)	Crete (Mediterranean)	Concrete	0.69	20
Miller et al. (2005)	Atlanta (humid subtropical)	Concrete	0.52	14
Kolokotroni et al. (2013)	London (Marine west coast)	Concrete	0.50	7.7

Fig. 4.4 (b). Comparison of CRHT predictions with measurements of hourly-averaged indoor temperature and ceiling heat flux for original vs. cool concrete and metal roofs. Error bars (in concrete) show 1 standard deviation of the 1-minute measurements in each hour.

Other than concrete roofs, about 30% of the buildings in Singapore have metal roof with no insulation such as industrial (workshops) buildings (Central Steering Committee for Housing and Real Estate Market Policy, 2010). The roof temperature, ceiling temperature and conduction heat flux through an 8-mm-thick galvanised steel (metal) roof are also predicted using the CRHT model with the same environmental conditions and cool coating as those used in Figs. 4.3 and 4.4 (a). The prediction results are shown in Fig. 4.5. The cool coating, which increases the solar reflectance of

the metal roof from 0.60 (original) to 0.74, keeps the cool roof and cool ceiling temperature lower than the original counterpart almost for the whole day. During day time, the peak cool roof and cool ceiling temperatures are lower by up to 5.4°C than the original counterpart. Cool coating reduces the peak heat gain during day time by 0.10 kW/m² (or 23%). Similar to the concrete roof, the heat flow direction reverses during night time, i.e., heat flows upward from ceiling to roof and losses to the surroundings. Even though the cool coating gives a much higher thermal emittance to the surface (0.90 for cool roof and 0.30 for original roof), the original metal roof still shows a slightly higher heat flow rate compared to the cool roof during night time. This is because the addition of cool coating increases the *R*-value (including overall indoor and outdoor heat transfer coefficients) of the metal roof from 0.162 m²-K/W to 0.183 m²-K/W i.e., about 12% increment. In terms of conduction resistance (which is used to estimate the conduction heat flux through both metal roofs), the cool coating adds about 11 times higher conduction resistance than the metal roofs' original conduction resistance (see Table 4.2). The increased conduction resistance due to the cool coating hinders the upward heat flow through the cool metal roof. The heat storage effect due to thermal mass as seen in the concrete roof plays a negligible role in the metal roof due to the fact that both the metal roof (8-mm-thick) and the cool coating (0.50-mm-thick) have very small thermal mass compared with the 100-mm-thick concrete (plus 10-mm-thick plaster) roof. Integrating the heat flux profile with respect to time for the day using Eq. (4.26), the daily heat gains through the two metal roofs are estimated. The original metal roof has 2.15 kWh/m² of daily heat gain, while the cool roof has only 1.41 kWh/m². The cool coating leads to a daily heat gain reduction of 0.74 kWh/m² on the metal roof. Fig. 4.4 (b) shows the ceiling heat flux (using Eq. (4.3)) and indoor air temperatures (using Eq. (4.18)) for the original and

cool metal roofs. The cool coating provides maximum indoor air temperature reduction of 3.7°C at 13:00 hours, due to the reduction in the ceiling heat flux gain of 3.59 W/m^2 .

Fig. 4.5 CRHT prediction results for hourly-averaged roof temperature, ceiling temperature and conduction heat flux through original Vs. cool galvanised steel (metal) roofs.

4.4 Discussion on the impact of wind speed on cool roof's performance

In addition to solar reflectance and R -value of the original roof, the heat gain reduction brought by cool coatings is largely affected by h_R , which combines the effects of roof convective heat transfer coefficient, $h_{R,conv}$ and roof radiative heat transfer coefficient, $h_{R,r}$ as shown in Eq. (4.22). The correlation for $h_{R,conv}$ suggests that the wind speed that the roof is subjected to could also have an impact on the heat gain reduction.

The reduction in daily heat gain through the concrete and metal roofs brought by cool coatings under different wind speeds (2, 6 or 10 m/s) is predicted by the CRHT model and the results are presented in Fig. 4.6. Constant wind speed throughout the day (2, 6 or 10 m/s), and thus constant h_R , was used in this investigation. It shows that the reduction in daily heat gain brought by the addition of cool coating drops as wind speed increases, meaning that cool coating provides more heat gain reduction at lower wind speed. It is because as the wind speed increases (higher h_R) the roof surface gets cooler due to the increased convective heat transfer. This reduces the amount of heat gain reduction by the cool coating since part of that has already been achieved by the higher wind speed. Similar observations are found in both concrete and metal roof.

Fig. 4.6 Impact of daily-average wind speed (V) or h_R on reduction in daily heat gain brought by cool coatings.

The drop in daily heat gain reduction by cool coating as wind speed increases becomes less significant for lower solar reflectance. It is because the heat gain by absorbing the

incident solar radiation becomes more significant at lower solar reflectance and the impact of increasing wind speed (increase h_R) on heat gain becomes relatively less significant than that at high solar reflectance. This can also be seen in Fig. 4.6 which shows the prediction results of heat gain reduction for solar reflectance = 0.74 and solar reflectance = 0.90 for both roofs.

4.5 Summary

A novel and general cool roof heat transfer (CRHT) model capable of incorporating transient outdoor and indoor boundary conditions is developed using spectral approximation method. The proposed CRHT model is applicable to all solid opaque surfaces (walls and roof). The CRHT model was verified against the Conduction Transfer Function method and was further validated against experiments performed in a high-rise residential building with 100-mm-thick concrete (plus 10-mm-thick plaster) flat roof in Singapore. Furthermore, simplified expressions are presented to evaluate the impacts of cool coating on roof temperature, ceiling temperature, indoor air temperature and heat flux. On a sunny day, cool coating reduces the peak roof temperature by up to 14.1°C on the concrete roof. This results in a reduction of peak heat gain by up to 0.12 kW/m² (or 57%), peak indoor air temperature by up to 3.1°C and a reduction of daily heat gain by 0.74 kWh/m² (or 58%). The same cool coating reduces the peak roof temperature by up to 5.4°C on an 8-mm-thick galvanised steel (metal) roof. This leads to a reduction of peak heat gain by up to 0.10 kW/m² (or 23%), peak indoor air temperature by up to 3.7°C and a reduction of daily heat gain by 0.74 kWh/m² (or 34%). The reduction in daily heat gain brought by cool coating drops as the roof is exposed to higher wind speeds.

The conclusions provided in this study are based on the results obtained on a sunny day in Singapore. These results may vary at different weather conditions.

Chapter 5

A New Method for Calculation of Roof Thermal Transfer Value (RTTV) for Cool Roofs

A part of this chapter is presented in the paper:

Kishor T. Zingre, Man Pun Wan, and Xingguo Yang “A New RTTV (Roof Thermal Transfer Value) Calculation Method for cool roofs”, *Energy*, 2015, DOI. 10.1016/j.energy.2014.12.030.

5.1 Introduction

Methods to estimate *OTTV* or *ETTV* or *RTTV* currently adopted by numerous South East Asian countries have different inherent limitations in accurately evaluating the thermal performance of cool roofs. These existing methods either use a fixed value to represent the solar reflectance effect or assume a linear-correlation between annual-averaged conduction heat gain and solar absorptance, which are shown to be inaccurate. In this chapter, a novel method is proposed to incorporate the effects of surface solar reflectance of the opaque portion of roofs into RTTV estimation using the CRHT model (formulated in Chapter 4). The proposed method is generally applicable to *OTTV*, *ETTV*, *RTTV* models or any similar models and is not limited by climate conditions or building materials. A new *RTTV* model is formulated based on the proposed method.

5.2 RTTV formulations

5.2.1 Limitations of the Current RTTV model

In Singapore, the Roof Thermal Transfer Value (*RTTV*) model is prescribed for air-conditioned, non-residential buildings as guidelines for thermal performance of roofs (Code on Envelope Thermal Performance for Buildings in Singapore, 2008). The Current *RTTV* model (Code on Envelope Thermal Performance for Buildings in Singapore, 2008) takes into account the three basic components of heat gain through

roofs, i.e., the heat conduction gain through the opaque portion of the roof, the heat conduction gain through the skylight portion of the roof and the solar radiation gain through the skylight portion of the roof. According to the Current *RTTV* model (denoted by “*RTTV*”), the annual-averaged heat gain through an opaque roof (having skylight) is given by Eq. (5.1)

$$RTTV = TD_{eq}(1 - SKR)U_R + \Delta T(SK R)U_{skylight} + SF(SK R)(CF)(SC). \quad (5.1)$$

The first term on the right hand side of Eq. (5.1) is the component of heat conduction gain through the opaque portion of the roof, which can be expressed as

$$TD_{eq}(1 - SKR)U_R = \frac{\int_{1 \text{ year}} q_R dt}{\text{Annual operating hours of the air - conditioning system}}. \quad (5.2)$$

The second and third terms on the right hand side of Eq. (5.1) are the components of heat gain through the skylight portion of the roof.

Chou & Chang (1996) carried out a parametric study and obtained the coefficients of each particular heat gain component in Eq. (5.1) and proposed the following final expression for *RTTV*,

$$RTTV = 12.5(1 - SKR)U_R + 4.8(SK R)U_{skylight} + 485(SK R)(CF)(SC). \quad (5.3)$$

From Eq. (5.3), it can be noticed that the component of heat conduction gain through the opaque portion of the roof depends on the *U*-value (overall heat transfer coefficient) of opaque portion of roof, $U_{op-roof}$. The *U*-value of an opaque roof can be expressed as Eq. (5.4)

$$\frac{1}{U_R} = \frac{1}{h_i} + \frac{1}{h_o} + \frac{L}{k} = K_i + K_o + K_{cond} = R_R. \quad (5.4)$$

The heat transfer coefficients, h_R for the opaque roof top surface and h_i for the ceiling surface, can be obtained using Eqs. (A-1) - (A-2d) (Clear et al., 2003; Karadag, 2009) given in the *Appendix-5*. The effect of thermal emittance (ε) is already incorporated in the Current *RTTV* model as shown in Eq. (A-2b).

Comparing Eq. (5.3) with Eq. (5.1), it can be observed that a higher equivalent temperature difference (TD_{eq}) value (12.5 K) is used for the opaque portion than the ΔT value (4.8 K) used for the skylight portion. This is to include the heating effect of incident solar radiation on the opaque roof. Using a fixed higher TD_{eq} value to address the incident solar radiation effect on the opaque roof has inherent limitations since solar heat gain varies with the solar reflectance of the opaque roof surface. A similar comment is made by Devgan et al. (2010). With such limitation, the Current *RTTV* model could be problematic when it is used to estimate the RTTV of cool roofs. Cool roofs are often achieved by adding a layer of thin coating or single-ply membrane on top of the original opaque structural roof. The Current *RTTV* model could incorporate the additional conduction resistance of the cool layer simply by adding the conduction resistance to the R -value (reciprocal of U -value) of the opaque roof assembly. However, in the Current *RTTV* model, there is no provision to vary the effect of solar reflectance on heat transfer to capture the effect of high solar reflectance of cool layer.

5.2.2 Formulation of the new *RTTV* model

This formulation considers an opaque solid roof applied with a cool layer (having uniform thickness) on the roof top. In case of multi-layered roof, the interfacial resistance between the different layers of the roof material are assumed to be negligible. There is no air-gap between the roof layers. The equivalent thermal resistance of the cool roof (opaque original roof + cool layer on top) depends on three

factors: the R -value of the original roof, the solar reflectance (ρ) of the cool layer and the climate conditions that affects the air-film thermal resistances on the roof top and ceiling boundary layers.

5.2.2.1 Modelling method

In this proposed method, the impact of cool layer on opaque roof heat transfer is incorporated in two ways: 1) the reduction in heat gain due to the increased roof surface solar reflectance (due to the addition of the cool layer). Such effect is considered as an ‘equivalent thermal resistance increment’ (denoted by K^{incr}). 2) The increment of the conduction resistance due to the addition of a cool layer (denoted by K_{cond}^{cool}), e.g., a layer of cool coating or cool membrane. Since the effect of emittance is already inside the Current $RTTV$ model (Eq. (A-2b)), therefore, no modification to the Current $RTTV$ model for the emittance effect is needed.

A new $RTTV$ model (denoted by $RTTV^{new}$), which incorporates the effect of K^{incr} and K_{cond}^{cool} , is proposed as shown in Eqs. (5.5) and (5.6),

$$RTTV^{new} = 12.5(1 - SKR)U_R^{new} + 4.8(SK R)U_{skylight} + 485(SK R)(CF)(SC) \quad (5.5)$$

$$\begin{aligned} \text{where } \frac{1}{U_R^{new}} &= \frac{1}{U_{R,\rho=0}} + K_{cond}^{cool} + K^{incr} \\ &= (K_i + K_o)_{\rho=0} + K_{cond} + K_{cond}^{cool} + K^{incr} \\ &= R_R^{new} \end{aligned} \quad (5.6)$$

Since the cool layer only applies on the opaque portion of the roof, the second and third terms on the right hand side of the $RTTV^{new}$ model (Eq. (5.5)) remain the same as the Current $RTTV$ model (Eq. (5.3)). The conceptual difference made to the $RTTV^{new}$ model, as compared to the Current $RTTV$ model [Eqs. (5.3) and (5.4)], is the definition of the U -value of the opaque portion of the roof, as graphically demonstrated in Fig.

5.1. The $RTTV^{new}$ model incorporates K^{incr} and K_{cond}^{cool} in the estimation of equivalent thermal resistance (R_R^{new}). K_{cond}^{cool} should be readily available as basic material property of the cool layer. K^{incr} mainly varies with three factors, the R -value of the original roof, the solar reflectance (ρ) values of the original roof (before cool layer is added) and the cool layer, and the local climate conditions. However, original roofs come with many different R -values and solar reflectance values, which lead to huge complications in K^{incr} determination. In order to simplify K^{incr} determination, we introduce the idea of representing the original roof by an imaginary roof, which has exactly the same constructions as the original roof but has a roof surface ρ of 0 (here, the U -value of this imaginary roof is denoted as $U_{R,\rho=0}$, as shown in Eq. (5.6)). With this definition, the expression for determining K^{incr} for any cool roof (with finite roof surface ρ) is derived using the previously formulated $CRHT$ model (derived in Chapter 4), as given by Eq. (5.7)

$$K^{incr} = [K_{cond}^{cool} + K_{cond}]. \left(\frac{1}{F} - 1 \right) + \frac{1}{F} \cdot \left[(K_i + K_o)_\rho - (K_i + K_o)_{\rho=0} \right] \quad (5.7)$$

Detailed procedure for the estimation of K^{incr} using the $CRHT$ model is given in the *Appendix-5*. Equation (5.7) gives the K^{incr} that needs to be added on the imaginary roof (in addition to K_{cond}^{cool}) in order to have the same annual-averaged heat gain as that through the cool roof. Equation (5.7) shows that K^{incr} depends on the solar reflectance of the cool layer (embedded in F , Eqs. (A-6) – (A-6b)) and the conduction resistances of the original roof (K_{cond}) and the cool layer (K_{cond}^{cool}), air film resistances of the ceiling surface (K_i) and the roof surface (K_o). The F parameter (Eqs. (A-6) – (A-6b)), which is required to determine K^{incr} (Eq. (5.7)), is estimated using the $CRHT$ model (given in *Appendix*) and the annual data.

Fig. 5.1 Schematic diagram demonstrating the conceptual difference between the Current and new $RTTV$ models (not-to-scale).

5.2.2.2 Equivalent thermal resistance increment for common roofs in tropical climate

Equation (5.7) suggests that K^{incr} needs to be pre-determined in order to use the $RTTV^{new}$ model. K^{incr} varies with $R_{R,\rho=0}$ values and climate conditions. In order to determine K^{incr} , F is estimated using the $CRHT$ model (Eq. (A-6b)) and input parameters listed in Table 5.1.

After finding F , K^{incr} is estimated using Eq. (5.7) and $RTTV^{new}$ is estimated using Eqs. (5.7) and (5.8). K^{incr} for a common range of $R_{R,\rho=0}$ (0.25 to 3.00 m²-K/W) are estimated and summarised in Table 5.2.

Table 5.1 Input parameters for the CRHT model [Eqs. (A-3) and (A-4)].

Parameters	Value	Source
h_i	4.1 W/m ² -K for air-conditioned buildings (using Eq. (A-1)) Typical h_i values change negligibly with roof surface ρ .	(EnergyPlus Engineering References, 2013)
h_R	24.5 W/m ² -K for tropical climate [using Eqs. (A-2) - (A-2d)]. Typical h_R values range from 21.5 (for $\rho = 0.1$) to 27.5 (for $\rho = 0.9$) W/m ² -K but varying h_R value has very little impact on K^{incr} according to Eq. (5.7).	(EnergyPlus Engineering References, 2013; Clear et al., 2003; Smith, 2009; Gentle et al., 2013)
T_i	24°C	
Correction factor for infrared radiation ($\varepsilon_R I^R / h_R$)	3.9 K [in Eq. (A-3c)]	(Kaska & Yumrutas, 2008)
$T_{sol-air}$, $\tilde{T}_{sol-air}$, T_o , V	Typical Meteorological Year (TMY) of Singapore.	(EnergyPlus Weather Dataset, 2013)

Linear interpolations can be used to calculate K^{incr} for any other roofs having $R_{R,\rho=0}$ within this range. It should be noted that the method proposed to develop the $RTTV^{new}$ model and determination of K^{incr} is generally applicable to different climate conditions, building materials as well as other similar models (such as $OTTV$ or $ETTV$). Here, for demonstration purposes, a broad set of K^{incr} values are derived based on the tropical climate of Singapore. Similarly, K^{incr} can be estimated for any other climate conditions simply by using the TMY data of the concerned climate as input to the $CRHT$ model (as shown in Table 5.1).

Table 5.2 K^{incr} for common roofs in tropical climate ($0.25 \leq R_{R,\rho=0} \leq 3.00$ m²-K/W).

Solar reflectance of roof (ρ)	K^{incr} (m ² -K/W)				
	$R_{R,\rho=0} = 0.25$ m ² -K/W	$R_{R,\rho=0} = 0.48^+$ m ² -K/W	$R_{R,\rho=0} = 1.00$ m ² -K/W	$R_{R,\rho=0} = 2.00$ m ² -K/W	$R_{R,\rho=0} = 3.00$ m ² -K/W
0.10 ⁺	0.02	0.04	0.08	0.15	0.23
0.20	0.04	0.08	0.17	0.34	0.51
0.30	0.07	0.14	0.29	0.58	0.87
0.40	0.11	0.22	0.46	0.92	1.38
0.50	0.17	0.35	0.71	1.41	2.12
0.60	0.28	0.54	1.12	2.24	3.36
0.70	0.49	0.97	1.98	3.96	5.95
0.74 ⁺	0.78	1.52	3.07	6.14	9.20
0.80	1.19	2.33	4.69	9.37	14.09
0.90	3.13	5.95	11.95	23.92	35.88

⁺Properties of the test building roof.

With Table 5.2, the RTTV of any roof having $\rho \leq 0.90$ and $R_{op-roof,\rho=0}$ between 0.25 to 3.00 m²-K/W in Singapore's tropical weather condition can be quickly estimated using the $RTTV^{new}$ model (Eq. (5.5)) without the need of going through the *CRHT* modelling procedure (described in Section 5.2.2.1 and the *Appendix-5*) again.

5.3 Validation of the new *RTTV* model

5.3.1 Computational model development

Since RTTV is essentially the annual-averaged thermal transfer through a roof, experimental validation of the $RTTV^{new}$ model requires at least a full-year measurement on a series of identical test roofs with different surface solar reflectance, which is beyond the available resources of the current study. This study employed

computational modelling as the means of validation. The computational model was calibrated against a one-week measurement conducted in a test building located in Singapore.

Computational validation of the $RTTV^{new}$ model is carried out by simulating a concrete built, air-conditioned, single-storey test building with a solid flat roof (100-mm-thick concrete + 10-mm-thick plaster on the ceiling) located in Nanyang Technological University, Singapore. The test building is of rectangular shape having a total roof surface area of about 40 m² (without any skylight surface). Two windows, two doors and an exhaust fan are located on the south facing wall. A commercially available white cool coating (0.5-mm-thick when dried) having a solar reflectance of 0.74 and thermal emittance of 0.90 (with measurement accuracies of ± 0.02 described in Section 3.2) was applied on its roof. It is the same building, which was modelled to investigate the thermal performance of various roofing methods in tropical climate, as discussed in Chapter 3. Detailed computational model development is discussed in Chapter 3 (Section 3.2.1).

5.3.2 Test building measurements for computational model calibration

The computational model developed using EnergyPlus was calibrated by experimental measurements to compare the simulation results for the cool roof surface temperature and the cool ceiling surface temperature with the measurements. The measurements were conducted in the test building for a week in April, 2013. The test building measurement and results of calibration are discussed in detail in Chapter 3 (Section 3.2.2 and 3.3.1, respectively).

5.4 Results on the validation of the new *RTTV* model

The validity of the $RTTV^{new}$ model is tested by comparing its RTTV estimations with computational simulations of the test building roof (concrete roof with $R_{R,\rho=0} = 0.48$ m²-K/W). The comparisons include the original roof ($\rho = 0.10$) and a series of cool roof cases (a 0.5-mm-thick cool coating layer added on top of the imaginary roof, $0.20 \leq \rho \leq 0.90$) to cover the range of solar reflectance up to 0.90 (a hypothetical super-performance cool coating). RTTV estimations made by the Current RTTV model are also compared to illustrate its deficiencies in capturing the solar reflectance effect.

5.4.1 Equivalent thermal resistance increment (K^{incr}) for the test building roof

The value of K^{incr} for the test building cool roof ($R_{R,\rho=0} = 0.48$ m²-K/W) with cool coating of 0.74 solar reflectance is 1.52 m²-K/W, which can be obtained directly from Table 5.2. The K^{incr} value can then be substituted into the $RTTV^{new}$ model [Eqs. (5.6) and (5.5)] for estimation of equivalent R -value (R_R^{new}) of the test building cool roof and also its RTTV. The thickness and conduction resistance of the cool coating used in the current test building is much smaller than that of the original roof. Therefore, K_{cond}^{cool} is neglected in this estimation. However, in cases of thinner and more conductive roofs, K_{cond}^{cool} will become more significant and cannot be neglected. In order to demonstrate the effect of roof solar reflectance on the equivalent R -value of the roof, the values of K^{incr} , brought to the test building roof due to the addition of cool coatings with different solar reflectance values are shown in Fig. 5.2.

Fig. 5.2 K^{incr} brought to the test building roof by cool coatings with different solar reflectance values.

It can be observed that K^{incr} increase with the solar reflectance of the cool coatings (a phenomenon that the Current $RTTV$ model cannot capture). At higher solar reflectance values (0.6 or above), K^{incr} becomes the dominant contributor to the equivalent R -value of the test building roof. This highlights the importance of the proposed $RTTV^{new}$ model, especially for high-reflectance roofs.

5.4.2 Comparison between $RTTV$ estimations and computational simulations

$RTTV$ estimations of the test building roof using the Current $RTTV$ [Eqs. (5.3) and (5.4)] and $RTTV^{new}$ models [Eqs. (5.5) and (5.6)] are compared to the computational simulations of annual-averaged heat gains in Fig. 5.3. Estimations and simulations are done for roof solar reflectance values range from 0.10 to 0.90. The simulated annual-averaged heat gain is obtained by integrating the conduction heat flux through the roof

for the whole year which is then divided by the annual operating hours of the air-conditioning system (obtained from the system's operation schedule shown in Table 3.2).

Figure 5.3 shows that in the case of original roof ($\rho = 0.10$), both the Current *RTTV* model and the *RTTV*^{new} model give RTTV estimations that are very similar to the annual-averaged heat gain predicted by computational simulation. This suggests that the Current *RTTV* model gives fairly accurate RTTV estimates at this low ρ . However, the estimates by *RTTV*^{new} model differ from those by the Current *RTTV* model at $\rho \geq 0.20$ (the 'cool roof' cases). As pointed out earlier, the Current *RTTV* model is not sensitive to change in solar reflectance of opaque roof surface. Such limitation of the Current *RTTV* model is reflected in Fig. 6 that the estimated RTTV remains constant in all ρ (from $\rho = 0.10$ to 0.90). This is apparently problematic especially for higher solar reflectance values which the Current *RTTV* model exhibits significant over estimations of RTTV. For the particular test roof used in this study, the error in RTTV estimated by the Current *RTTV* model (as compared to the predictions by computational simulation) increases from 2.2% at $\rho = 0.10$ to an over-estimation of more than 20 times at $\rho = 0.90$. At $\rho = 0.74$, which is the solar reflectance of the cool coating used in the calibration measurements, the Current *RTTV* model over-estimates the RTTV by more than 3 times.

On the other hand, the *RTTV*^{new} model is able to capture the effect of roof solar reflectance on RTTV. The RTTV estimated by the *RTTV*^{new} model closely follows the annual-averaged heat gain value predicted by computational simulations in all roof solar reflectance values (from $\rho = 0.10$ to 0.90) with a maximum error of about 12%. This highlights the significant accuracy improvement imparted by the *RTTV*^{new} model

in the prediction of the thermal performance of cool roofs applied on air-conditioned buildings in the tropical climate.

Fig. 5.3 Estimated RTTV of the test building roof at different roof solar reflectance values.

The $RTTV^{new}$ model has significant practical implications in places where RTTV, or other index of similar nature, is used for regulatory purposes. For instance, in Singapore, the BCA Green Mark pre-requisite requirements (BCA, 2008) states that a heavy weight (weight $> 230 \text{ kg/m}^2$) opaque roof (the category of roofs that the test building roof belongs to) without skylight shall not exceed a maximum U -value of $1.2 \text{ W/m}^2\text{-K}$ (or R -value shall not be lower than $0.83 \text{ m}^2\text{-K/W}$). Using the new R -value calculation method employed in the $RTTV^{new}$ model, it suggests that such requirement can be achieved by adding a cool coating with solar reflectance of about 0.50 on the test building roof. However, if the method of R -value calculation in the Current $RTTV$

model is used, it gives an R -value estimate of $0.49 \text{ m}^2\text{-K/W}$ (almost 41% less than the estimation by the $RTTV^{\text{new}}$ model) if the same cool coating is applied on the test building roof, meaning that it still fails to comply with the BCA Green Mark pre-requisite requirements (BCA, 2008).

5.5 Discussion

Figure 5.3 shows that using a fixed value to represent the roof surface solar reflectance effect in RTTV estimation, as employed by the Current $RTTV$ model, is clearly a deficiency. The proposed $RTTV^{\text{new}}$ model provides much more accurate RTTV estimations for high solar reflectance roofs by incorporating the solar reflectance effect using the $CRHT$ model. As reviewed in Chapter 3 (Table 3.2), there is another method adopted by Hong Kong, Sri Lanka, Thailand, Indonesia, Malaysia and Philippines (Chan & Chow, 2013; Sustainable Energy Authority of Sri Lanka, 2010; Chirarattananon & Taveekun, 2004; Sukri et al., 2012). This method (denoted as “Method A” here) directly multiplies the solar absorptance ($1 - \rho$) with the component of heat conduction gain through the opaque portion of the building envelope. In order to investigate the accuracy of Method A compared to proposed $RTTV^{\text{new}}$, an additional set of RTTV estimations of the same test building roof is performed using the $RTTV^A$ model. The results are plotted in Fig. 5.3.

Due to its definition, the $RTTV^A$ model predicts that RTTV is linearly proportional (correlation coefficient of 1.00) to solar absorptance ($1 - \rho$). However, computational simulation predictions and $RTTV^{\text{new}}$ model (in Fig. 5.3) shows that the correlation between RTTV (or annual-averaged heat gain) and solar absorptance is non-linear and is better approximated by a second-order polynomial (similar to that suggested by Chan & Chow (2013) and Devgan et al. (2010).

5.6 Summary

A new $RTTV$ ($RTTV^{new}$) model capable of assimilating the effect of solar reflectance (ρ) of the opaque roof surface is proposed to provide accurate thermal performance prediction of air-conditioned buildings with high solar reflectance roofs (e.g., cool roofs). The $RTTV^{new}$ model is developed by formulating an expression for equivalent thermal resistance increment due to the increased solar reflectance of the opaque roof (K^{incr}) using the $CRHT$ model and incorporating it into U -value of the component of heat conduction gain through the opaque portion of the roof. This proposed new method overcomes the deficiencies of the existing methods (i.e., they either use a fixed higher equivalent temperature difference value or assume a linear correlation between annual-averaged heat gain and solar absorptance) in capturing the solar reflectance effect while retaining the simple and easy-to-use characteristics. The method is generally applicable to other similar models (such as $OTTV$ or $ETTV$) and is not limited by climate conditions or building materials.

The accuracies of the three models: Current $RTTV$ (use a fixed higher equivalent temperature difference value), $RTTV^A$ (assume a linear correlation between annual-averaged heat gain and solar absorptance) and the proposed $RTTV^{new}$ model, in estimating $RTTV$ in are tested by comparing against computational simulations of an air-conditioned single-storey test building under tropical climate. The computational model was calibrated against experiments. The Current $RTTV$ model shows obvious limitations in capturing the solar reflectance effect as the error increases with roof solar reflectance (from about 3% at $\rho = 0.10$ to an over-estimation of more than 20 times at $\rho = 0.90$). Estimations by the $RTTV^A$ model also show significant deviation from computational simulation predictions. The proposed $RTTV^{new}$ model is the most accurate among the three models for opaque roof surfaces of all solar reflectance.

Chapter 6

Modelling of Cool Roof Heat Transfer for Double-Skin Roofs

A part of this chapter is presented in the paper:

Kishor T. Zingre, Man Pun Wan, Swee Khian Wong, Winston Boo Thian Toh and Irene Yen Leng Lee. “Modelling of cool roof performance for double-skin roofs in tropical climate”, *Energy*. Accepted for publication.

6.1 Introduction

Previously developed heat transfer models (for transient thermal analysis of DSR) assumed the transient outdoor and constant indoor boundary conditions, limiting their applicability for naturally ventilated buildings. Over the years, numerical methods such as FD, FE and FV methods have been developed and implemented to formulate the heat transfer models for DSR. However, these methods require small grid size and small time step (15 min or less) to achieve sufficient accuracy (Tabares-Velasco et al., 2012), leading to heavy computational resources. Due to the need for discretisation, these methods resolve the detailed temperatures distributions within the slab material which is unnecessary for air-conditioning load calculations. In this chapter, a novel heat transfer model is proposed for DSR (having a cool coating on top surface) based on the spectral approximation method. The proposed model can handle transient outdoor and indoor boundary conditions as experienced by naturally ventilated buildings. The proposed model is advantageous over other existing building energy simulation models since the final solution is very concise and, therefore, much easier to apply. In contrast, the existing building energy simulation models are rather complicated to apply to DSRs due to the difficulties in geometry modelling and setting up boundary conditions for the air-gap region.

6.2 Methodology

6.2.1 Cool roof heat transfer (CRHT) model for DSR

In this derivation, it is assumed that the roof materials (in the primary and secondary roofs) have constant thermophysical (conductivity and specific heat) and radiation properties (solar reflectance and thermal emittance). The roof materials are homogeneous with constant material property (mass density). In case of multi-layered roof, the contacts between different layers are assumed to have negligible interfacial resistance. The thickness of the roof is assumed to be much smaller than the length and breadth such that the heat transfer through the roof is one-dimensional, i.e., in the direction perpendicular to the exposed surface, y direction as shown in Fig. 6.1. There are no internal heat sources inside the air gap and airflow is unidirectional (along the x -direction only). Hourly-averaged values of overall (combined convection and radiation) heat transfer coefficients (for the double-skin roof surfaces) are used (Kontoleon & Eumorfopoulou, 2008; Khalid, 2003) in this study since the inputs are hourly-averaged values.

6.2.1.1. Formulation for secondary roof

Consider an opaque secondary roof applied with a cool coating (having uniform thickness throughout) on the top surface and exposed to incident radiation as shown in Fig. 6.1. The radiation balance at the top surface of cool secondary roof is given as: Net absorbed radiation by the cool secondary roof (I_{abs}) = Incident radiation on the cool secondary roof (I_{in}) - Outgoing radiation from the cool secondary roof (I_{out}). The governing differential equation for conduction heat transfer in the cool secondary roof can be expressed as the following homogeneous-linear partial differential Eq. (6.1)

$$\frac{\partial T}{\partial t} = \alpha_{SR}^* \frac{\partial^2 T}{\partial y^2}, \quad (6.1)$$

where $\alpha_{SR}^* = k_{SR} / \rho_{SR}^* C_{SR}$.

Fig. 6.1 Schematic diagram showing heat transfer processes at a cool double-skin roof (not-to-scale).

Equation (6.1) can be solved using an initial condition in time and two boundary conditions in space. The Neumann boundary condition on the cool secondary roof surface ($y = 0$) is

$$\begin{aligned}
 q(y=0, t) &= -k_{SR} \left(\frac{\partial T}{\partial y} \right)_{y=0} = I_{in} \cdot \alpha_{SR} - h_{SR} (T_{SR} - T_o) - \varepsilon_{SR} I^{IR} \\
 &= h_{SR} \frac{I_{in} \cdot \alpha_{SR}}{h_{SR}} - h_{SR} (T_{SR} - T_o) - h_{SR} \frac{\varepsilon_{SR} I^{IR}}{h_{SR}} = h_{SR} \left(\frac{I_{in} \cdot \alpha_{SR}}{h_{SR}} + T_o - \frac{\varepsilon_{SR} I^{IR}}{h_{SR}} - T_{SR} \right) \\
 &= h_{SR} (T_{sol-air}^e - T_{SR})
 \end{aligned} \tag{6.2}$$

where $T_{sol-air}^e = \frac{I_{in} \cdot \alpha_{SR}}{h_{SR}} + T_o - \frac{\varepsilon_{SR} I^{IR}}{h_{SR}}$ and $\frac{\varepsilon_{SR} I^{IR}}{h_{SR}}$ is the correction factor for radiation.

(6.2a)

The Neumann boundary condition on the secondary ceiling surface ($y = L_{SR}$) is

$$q(y = L_{SR}, t) = -k_{SR} \left(\frac{\partial T}{\partial y} \right)_{y=L_{SR}} = h_{SC}(T_{SC} - T_{gap}) \quad (6.3)$$

While, the initial condition is the steady state solution of Eq. (6.1) at $t = 0$. Table 6.1 shows the expressions to estimate the convection and radiation heat transfer coefficients for double-skin roof surfaces (Kontoleon & Eumorfopoulou, 2008; Khalid, 2003).

Table 6.1 Convection and radiation heat transfer coefficients for double-skin roof surfaces.

Surface	Convection heat transfer coefficient	Radiation heat transfer coefficient	Overall heat transfer coefficient
Secondary roof*	$h_{R,conv} = \frac{k_{air}}{D_h} 0.15 Ra^{1/3} + k_{air} \hat{f} 0.0296 Re^{4/5} Pr^{1/3}$	$h_{SR,r} = \varepsilon_{SR} \sigma \hat{F}_{SR,sky} (T_{SR}^2 + T_{sky}^2) \cdot (T_{SR} + T_{sky})$ $\hat{F}_{SR,sky} = 0.5(1 + \cos \phi)$ $T_{sky} = 0.06(T_o)^{1.5}$	$h_{SR} = h_{SR,conv} + h_{SR,r} \cdot \frac{T_{SR} - T_{sky}}{T_{SR} - T_o}$
Secondary ceiling ⁺ and Primary roof ⁺	$h_{SC,conv} = h_{R,conv} = \frac{Nu \cdot k_{air}}{D_h}$ where $D_h = (2b_{gap} L_{gap}) / (b_{gap} + L_{gap})$ $Nu = 1.78(Gr)^{0.133}$ (for $10^9 < Gr < 10^{11}$) $Gr = \frac{(\rho_{air}^*)^2 g(\delta)(D_h)^3 \Delta T}{(\rho_{air})^2}$	$h_{SC,r} = (\varepsilon_{SC} \varepsilon_R / (\varepsilon_{SC} + \varepsilon_R - \varepsilon_{SC} \varepsilon_R)) \cdot \sigma (T_{SC}^2 + T_R^2) \cdot (T_{SC} + T_R)$	$h_{SC} = h_{SC,conv} + h_{SC,r} \cdot \frac{T_{SC} - T_R}{T_{SC} - T_{gap}}$
Primary ceiling ⁺⁺	$h_{i,conv} = (0.704 / D_h^{0.601}) \cdot (T_C - T_i)^{0.133}$	$h_{R,r} = (\varepsilon_{SC} \varepsilon_R / (\varepsilon_{SC} + \varepsilon_R - \varepsilon_{SC} \varepsilon_R)) \cdot \sigma (T_{SC}^2 + T_R^2) \cdot (T_{SC} + T_R)$	$h_R = h_{R,conv} + h_{R,r} \cdot \frac{T_R - T_{SC}}{T_R - T_{gap}}$
		$h_{i,r} = \frac{\sigma \cdot \varepsilon_i}{4 \times (2 - \varepsilon_i)} (1 + \hat{F}_{C,floor}) (T_C + T_{floor})^3$ $\hat{F}_{C,floor} = \sqrt{1 + \frac{L_i^2}{W^2}} - \frac{L_i}{W}$	$h_i = h_{i,conv} + h_{i,r}$

*Expressions are obtained from Clear et al. (2003) and EnergyPlus Engineering References (2013).

⁺Expressions are obtained from Tchinda (2008).

⁺⁺Expressions are obtained from Awbi (1998) and Anderson (1981).

The spectral approximation method (Zhuang & Ren, 2014; Oliveira & Azevedo, 2014; Gamba & Haack, 2014; Guo & Zhang, 2013; Podowitz et al., 2014) is used to solve the transient heat conduction equation [Eq. (6.1)]. It is an emerging numerical method to solve partial differential equations. The method provides the solution of a partial differential equation as summation of product of higher-order polynomial with time dependent coefficients. Detailed procedure for the solution of Eq. (6.1) using the spectral approximation method is given in the *Appendix-6*.

The final solution of Eq. (6.1) which satisfies the Neumann boundary conditions at secondary roof and secondary ceiling surfaces and the initial condition is given as

$$T(y,t) = \left[\left(B_1^+ \cos\left(\frac{m\pi y}{L_{SR}}\right) - D_1^+ \sin\left(\frac{m\pi y}{L_{SR}}\right) \right) \left(E_1^+ \frac{y}{L_{SR}} - G_1^+ \left(1 - \frac{y}{L_{SR}}\right) \right) \right] e^{-\lambda_m^2 t} \quad (6.4)$$

where $B_1^+, D_1^+, E_1^+, G_1^+$ are given in Appendix. Substituting Eq. (6.4) into Eq. (6.3) and re-arranging, the surface temperature of secondary ceiling can be expressed as

$$T_{SC} = \left[h_{SC}(T_R - \bar{T}_R) + \eta_{gap}^+(T_o - \bar{T}_o) - h_{SC}\bar{T}_{SC} \right] \frac{1}{\mu_{gap}^+} \left(1 - \frac{U_{SR}}{h_{SC}} \right) + \left[\frac{U_{SR}}{h_{SC}} (T_{sol-air}^s) \right]. \quad (6.5)$$

$$\text{where } \frac{1}{U_{SR}} = \frac{1}{h_{SC}} + \frac{1}{h_{SR}} + \frac{L_{SR}}{k_{SR}} + \frac{l_{coating}}{k_{coating}}, \quad \eta_{gap}^+ = \frac{(ach)_{gap} L_{gap} \rho_{air}^* C_{air}}{3600 \times VOL_{gap}}, \quad \mu_{gap}^+ = h_{SC} + \eta_{gap}^+ + h_R. \quad (6.5a)$$

The term $l_{coating}/k_{coating}$ in Eq. (6.5a) is the conduction resistance of the cool coating layer. This term equals to 0 in case of original secondary roof. Substituting Eq. (6.4) into Eq. (6.2) and re-arranging, the surface temperature of secondary roof can be obtained as given by Eq. (6.6)

$$T_{SR} = \frac{T_{sol-air}^s}{\left(1 + \frac{1}{\nu_{SR}^+}\right)} + \frac{T_{gap}^s \left(1 - \frac{U_{SR}}{h_{SC}}\right) + \frac{U_{SR}}{h_{SC}} \tilde{T}_{sol-air}}{(1 + \nu_{SR}^+)} \quad (6.6)$$

$$\text{where } \nu_{SR}^+ = \frac{h_{SC} L_{SR}}{k_{SR}}. \quad (6.6a)$$

The transient conduction heat flux through the secondary roof can be obtained using Eq. (6.7)

$$q_{SR} = U_{SR} (1 + \nu_{SR}^+) \left(T_{sol-air}^s - \left[T_{gap}^s \left(1 - \frac{U_{SR}}{h_{SC}}\right) + \frac{U_{SR}}{h_{SC}} \tilde{T}_{sol-air} \right] \right). \quad (6.7)$$

In Eq. (6.7), $T_{sol-air}^s$ and $\tilde{T}_{sol-air}$ are affected by the solar reflectance of the secondary roof surface. The expressions for $T_{sol-air}^s$ and $\tilde{T}_{sol-air}$ are given in the *Appendix-6*.

6.2.1.2 Formulation for air gap

The heat transfer inside the air gap mainly occurs by thermal emission (between the secondary ceiling and the primary roof) and by convection (between the secondary ceiling and air gap, and the primary roof and air gap). The governing differential equation for heat balance inside the air gap is given by Eq. (6.8),

Change in internal energy of air gap = heat gain by air gap – heat loss by air gap

$$L_{gap} \rho_{air}^* C_{air} \left(\frac{\partial T_{gap}}{\partial t} \right) = h_{SC} (T_{SC} - T_{gap}) - C^+ \left(\frac{\partial T_{gap}}{\partial x} \right) - \frac{C^+}{L_{gap}} (T_{gap} - T_o) - h_R (T_{gap} - T_R) \quad (6.8)$$

$$\text{where } C^+ = \frac{\rho_{air}^* (ach)_{gap} C_{air}}{3600 \times b_{gap}} \quad (6.8a)$$

The variation of T_{gap} along x -direction is given by Tchinda (2008)

$$C^+ \left(\frac{\partial T_{gap}}{\partial x} \right) = [U^+ (T_{gap} - T_o)] \quad \text{where } U^+ = \frac{h_{SC} h_R (h_{SC} + 2h_R)}{(h_R + h_{SC})^2 - (h_{SC})^2} \quad (6.9)$$

Applying boundary condition (at $x = 0$, $T_{gap} = T_o$) to Eq. (6.9), it gives

$$T_{gap} = T_o \left(e^{\frac{U^+}{C^+} x} \right) \quad (6.10)$$

Putting Eq. (6.10) into Eq. (6.8) and applying the initial (steady state) condition, it yields

$$T_{gap}(x, t) = \left(1 - e^{\frac{-(h_{SC} + h_R + \frac{C^+}{L_{gap}}) t}{L_{gap} C_{air} \rho_{air}^*}} \right) \left(\frac{h_R T_R + h_{SC} T_{SC} + U^+ T_o \left(e^{\frac{U^+}{C^+} x} \right) + \frac{C^+}{L_{gap}} T_o}{h_{SC} + h_R + \frac{C^+}{L_{gap}}} \right) + \left(e^{\frac{-(h_{SC} + h_R + \frac{C^+}{L_{gap}}) t}{L_{gap} C_{air} \rho_{air}^*}} \right) \left(\frac{h_R T_R - h_{SC} T_{SC} - U^+ T_o \left(e^{\frac{U^+}{C^+} x} \right) - \frac{C^+}{L_{gap}} T_o}{h_{SC} + h_R - \frac{C^+}{L_{gap}}} \right) \quad (6.11)$$

T_{gap} is a function of outdoor conditions, as shown in Eq. (6.11)). T_{gap}^s can be mathematically expressed by Eq. (6.12),

$$T_{gap}^s = \bar{T}_{gap} + T_{gap}^+ \cdot \cos(\omega t - \psi) \quad (6.12)$$

$$\text{where } T_{gap}^+ = \sqrt{\left(\frac{1}{12} \sum_{t=1}^{24} T_{gap} \cdot \cos(\omega t) \right)^2 + \left(\frac{1}{12} \sum_{t=1}^{24} T_{gap} \cdot \sin(\omega t) \right)^2}, \quad \tan(\psi) = \frac{\left(\frac{1}{12} \sum_{t=1}^{24} T_{gap} \cdot \sin(\omega t) \right)}{\left(\frac{1}{12} \sum_{t=1}^{24} T_{gap} \cdot \cos(\omega t) \right)}. \quad (6.13)$$

The model for air gap temperature (T_{gap}^s) couples the secondary and primary roof formulations.

6.2.1.3 Formulation for primary roof

The formulation for the primary roof uses an approach similar to the cool secondary roof. The only differences are that for the roof surface, the solar-air temperature (in cool secondary roof) is replaced by the air gap temperature (in primary roof). For the

ceiling surface, the air gap temperature (in cool secondary roof) is replaced by indoor air temperature (in primary roof).

The governing differential equation for conduction heat transfer in the primary roof can be expressed as

$$\frac{\partial T}{\partial t} = \alpha_R^* \frac{\partial^2 T}{\partial y^2} \quad (6.14)$$

$$\text{where } \alpha_R^* = \frac{k_R}{\rho_R^* C_R} \quad (6.14a)$$

Equation (6.14) can be solved using two boundary conditions in space [Eqs. (6.15) and (6.16)] and an initial condition in time. The Neumann boundary condition on the primary roof surface ($y = 0$) is

$$q(y = 0, t) = -k_R \left(\frac{\partial T}{\partial y} \right)_{y=0} = h_R (T_{gap} - T_R) \quad (6.15)$$

and the Neumann boundary condition on the primary ceiling surface ($y = L_R$) is

$$q(y = L_R, t) = -k_R \left(\frac{\partial T}{\partial y} \right)_{y=L_R} = h_C (T_C - T_i) \quad (6.16)$$

and initial condition is steady state solution of Eq. (6.14) at $t = 0$.

An approximate solution of Eq. (6.14) can be derived using the similar procedure that was adopted for cool secondary roof, i.e., Eqs. (A-1) to (A-11) in the *Appendix-6*. It yields

$$T(y, t) = \left[\left(B_2^+ \cos\left(\frac{m\pi y}{L_R}\right) - D_2^+ \sin\left(\frac{m\pi y}{L_R}\right) \right) \left(E_2^+ \frac{y}{L_R} - G_2^+ \left(1 - \frac{y}{L_R}\right) \right) \right] e^{-\lambda_m^2 t} \quad (6.17)$$

where $B_2^+, D_2^+, E_2^+, G_2^+$ are given in Appendix. Substituting Eq. (6.17) into Eq. (6.16) and re-arranging, the surface temperature of primary ceiling can be obtained as given by Eq. (6.18)

$$T_C = \left[h_C \left(\sum_{wall} T_{wall} + T_{floor} \right) + \left(\sum_{window} U_{window} + \eta_i^+ \right) T_o - \left(h_C \left(\bar{T}_C + \sum_{wall} \bar{T}_{wall} + \bar{T}_{floor} \right) + \left(\sum_{window} U_{window} + \eta_i^+ \right) \bar{T}_o - \bar{T}_i (\mu_i^+) \right) \right] \frac{1}{\mu_i^+} \left(1 - \frac{U_R}{h_C} \right) + \left[\frac{U_R}{h_C} \cdot (\tilde{T}_{gap} + \beta \cdot T_{gap}^s \cos(\omega t - \psi)) \right]. \quad (6.18)$$

Substituting Eq. (6.17) into Eq. (6.15) and re-arranging, the surface temperature of primary roof can be obtained as given by Eq. (6.19)

$$T_R = \frac{T_{gap}^s}{\left(1 + \frac{1}{\nu_R^+}\right)} + \frac{\left[T_i \left(1 - \frac{U_R}{h_C}\right) + \frac{U_R}{h_C} \tilde{T}_{gap} \right]}{(1 + \nu_R^+)} \quad (6.19)$$

$$\text{where } \nu_R^+ = \frac{h_C L_R}{k_R}. \quad (6.19a)$$

The transient conduction heat flux through the primary roof can be obtained as expressed by Eq. (6.20)

$$q_R = U_R (1 + \nu_R^+) \left(T_{gap}^s - \left[T_i \left(1 - \frac{U_R}{h_C}\right) + \frac{U_R}{h_C} \tilde{T}_{gap} \right] \right). \quad (6.20)$$

It should be noted that the CRHT model is a generalised heat transfer model that can be applied to double-skin roofs in any climate condition. For validation of the CRHT model, experimental measurements were carried out in a residential apartment building in Singapore.

6.2.2 Experimental setup for validation of the CRHT model

Experimental measurements were performed in a naturally ventilated residential building with double-skin roofs (DSRs) in Singapore. The measurements were monitored on the same sunny day on which the SSR test was conducted. Therefore, the experimental data shown in Fig. 4.2 (a) (in Chapter 4) is common to both SSR and DSR tests. The DSR consisted of a 30-mm-thick ferrocement secondary roof, followed by an air-gap of 220-mm height and a 100-mm-thick concrete (plus 10-mm-thick plaster towards the ceiling) primary roof. Two identically configured unoccupied apartments located on the top-most storey (separated by distance of about 20 m) were used for the experimental measurements, as shown in Fig. 6.2.

Fig. 6.2 Schematic diagram showing the experimental set up for original and cool double-skin roofs (not-to-scale).

Each of the two test apartments had the same roof area of about 90 m^2 . White-coloured cool coating (0.50-mm-thick when dried) was applied on the secondary roof of one of the two test apartments while another apartment's secondary roof was remained uncoated (original) as the reference for comparison. The cool coating was eco-friendly organic water based and it was applied by spraying technique in two layers. The primary roof of both the apartments remained in the original condition. The schematic diagram of the experimental set up is shown in Fig. 6.2.

The measurement protocol of the DSR test was similar to that for the SSR test, as discussed in Section 4.2.2 (Chapter 4). Incident solar radiation, surface temperatures, indoor RH, indoor air temperature, outdoor air temperature and wind speed were monitored at 1-minute intervals. Hourly-averaged values and standard deviations of these measurements were estimated from the 1-minute raw data and are shown in Figs. 4.2 (a) and 6.3. The original and cool solar-air temperatures in Fig. 4.2 (a) are estimated (using Eq. (6.2a)) from the measurement data of incident solar radiation, solar reflectance and thermal emittance of secondary roof, outdoor air temperature and wind speed.

Fig. 6.3 Hourly-averaged outdoor wind speed and heat transfer coefficients for surfaces of original and cool DSRs. Error bars show 1 standard deviation of the 1-minute measurements in each hour.

Estimating from the instrumentation errors of the pyranometer, reflectometer, emissometer and weather station, the error of the ‘measured’ solar-air temperatures was $\pm 0.9^{\circ}\text{C}$. The overall heat transfer coefficients, which combines the effects of convection heat transfer coefficient and radiation heat transfer coefficient (as shown in Table 6.1), were estimated for the secondary and primary roof and ceiling surfaces of original and cool DSRs using the measured surface temperatures, outdoor air temperature, wind speed and thermal emittance values. Estimating from the instrumentation errors of the surface temperature sensors, weather station and emissometer, the maximum error of the ‘measured’ overall heat transfer coefficients was $\pm 0.6 \text{ W/m}^2\text{-K}$.

6.3 Results

6.3.1 Experimental validation of the CRHT model

The measured hourly-averaged outdoor air temperature, solar radiation and the solar-air temperatures for cool and original secondary roofs on a sunny day in tropical climate are shown in Fig. 4.2 (a) (in Chapter 4).

CRHT model predictions of secondary ceiling temperature using Eq. (6.5), secondary roof temperature using Eq. (6.6), conduction heat flux through secondary roof using Eq. (6.7), primary ceiling temperature using Eq. (6.18), primary roof temperature using Eq. (6.19), indoor air temperature using Eq. (A6-16) and conduction heat flux through primary roof using Eq. (6.20) were compared with the experimental results for original and cool DSRs. The solar-air temperature profiles for the cool and original DSRs, which were required as one of the inputs to the CRHT model, were obtained from Fourier series simulation [using Eq. (6.2a) and Eq. (A6-13)] of the measured solar-air temperatures (shown in Fig. 4.2 (a)).

The radiation properties of the original and cool DSRs were obtained from the measurements as shown in Table 6.2. The correction factor for infrared radiation ($\varepsilon_{SR} I^{IR}/h_{SR}$) in Eq. (6.2a) takes into account the infrared radiation emitted by the secondary roof surface to the sky and surrounding objects. The correction factor for infrared radiation in Eq. (6.2) was assumed to be 3.9°C for flat secondary roofs (Ozel, 2012; Duffie & Beckman, 1991; Clear et al., 2003; Suehrcke et al., 2008, ASHRAE, 2005). The hourly-averaged values of overall heat transfer coefficients (as shown in Fig. 6.3) for secondary roof, secondary ceiling, primary roof and primary ceiling of the original and cool DSRs were estimated using the experimental data and implementing the formulations given in Table 6.1. From the experimental data and using Table 6.1,

the Grashof number for the air-gap was found to be in the range $10^9 < Gr < 10^{10}$. Due to the instrumentation errors, the maximum error in the ‘measured’ overall heat transfer coefficients was $\pm 0.6 \text{ W/m}^2\text{-K}$. Table 6.3 shows the thermophysical properties of the secondary and primary roof materials that were required as an input to the CRHT model. These properties were taken from the literature (Suehrcke et al., 2012; ASHRAE, 2009) or data provided by Housing and Development Board (HDB), Singapore.

Table 6.2 Radiation properties of cool and original double-skin roofs and single-skin roofs.

Roof case		Roof surface	Solar reflectance (ρ)	Thermal emittance (ε)
Double-skin roof	Cool	Secondary roof	0.74	0.90
		Primary roof	0.30	0.84
	Original	Secondary roof	0.30	0.84
		Primary roof	0.30	0.84
Single-skin roof	Cool	Primary roof	0.74 ^a	0.90 ^a
	Original	Primary roof	0.30	0.84

^a Obtained from Chapter 4.

Homogeneity of ρ and ε measurements at five locations: ± 0.02 units.

Table 6.3 Thermophysical properties of DSR materials.

Roof material	Thermal conductivity (W/m-K)	Thickness (mm)	Conduction resistance ($\text{m}^2\text{-K/W}$)	Density (kg/m^3)	Specific heat (J/kg-K)
Air	0.025	-	-	1.27	1008
Plaster ⁺	0.533	10	0.019	1568	840
Concrete ⁺	1.442	100	0.069	2400	880
Ferrocement ⁺	0.447	30	0.067	2000	840
Cool coating	0.045 ⁺	0.50	0.011	1053 ⁺	**

⁺ Thermophysical properties are taken from the manufacturers.

** Thermal mass, mC_p is 0 J/K due to negligible mass of coating.

Figure 6.4 (a) shows the comparisons between CRHT predictions and experimental measurements of hourly-averaged secondary roof temperatures, secondary ceiling temperatures and conduction heat flux through the secondary roofs. While, Fig. 6.4 (b) shows the comparisons between CRHT predictions and experimental measurements of hourly-averaged primary roof temperature, primary ceiling temperature and conduction heat flux through the primary roofs. The error bars show 1 standard deviation of the 1-minute measurements in each hour. The ‘measured’ conduction heat flux through the secondary roofs were estimated from the measured secondary roof and secondary ceiling temperatures, while that through the primary roofs were estimated from the measured primary roof and primary ceiling temperatures. Estimating from the instrumentation error of the surface temperature sensors, the error of the ‘measured’ heat flux was $\pm 3.4 \text{ W/m}^2$. The CRHT model can generally capture the secondary roof, secondary ceiling, primary roof, primary ceiling and indoor air temperature profiles with reasonable accuracy. The maximum difference between CRHT prediction and measured hourly-averaged secondary roof, secondary ceiling, primary roof, primary ceiling and indoor air temperature is $\pm 2.2^\circ\text{C}$.

Fig. 6.4 (a) Comparison of CRHT predictions with measurements of hourly-averaged secondary roof temperature, secondary ceiling temperature and conduction heat flux through original Vs. cool secondary roofs. Error bars show 1 standard deviation of the 1-minute measurements in each hour.

Fig. 6.4 (b) Comparison of CRHT predictions with measurements of hourly-averaged primary roof temperature, primary ceiling temperature and conduction heat flux through original vs. cool primary roofs. Error bars show 1 standard deviation of the 1-minute measurements in each hour.

6.3.2 Cool roof effect on surface temperatures, heat gain and indoor environment

6.3.2.1 Cool roof effect for the secondary roof

Figure 6.4 (a) also depicts the effect of cool coating on secondary roof temperature, secondary ceiling temperature and conduction heat flux through the ferrocement secondary roof (due to the temperature difference between secondary roof and

secondary ceiling). The cool coating, which increases the solar reflectance of ferrocement secondary roof from 0.30 (original) to 0.74, keeps the cool secondary roof temperature and cool secondary ceiling temperature lower than the original counterpart during the day time. The cool secondary roof temperature is lowered by up to 14.7°C and the cool secondary ceiling temperature is lowered by up to 11.9°C than the original counterpart. The reduction in secondary roof temperature leads to lower heat gain into the air-gap through the cool secondary roof as compared to the original secondary roof. Cool secondary roof has a peak time heat gain reduction of 0.23 kW/m² (or 63%) compared to the original secondary roof during day time. The heat flux through the secondary roof is in the downward direction during the day time, however at night time, the heat flux direction reverses. This is due to the fact that secondary roof temperature is higher than the secondary ceiling temperature during day time, but lower than the secondary ceiling temperature during night time. The daily heat gains into air-gap are estimated by integrating the downward heat flux for both secondary roofs with respect to time using Eq. (6.21) (using Simpson's 3/8 rule as discussed in Appendix-7),

$$Q_{gain} = \int_{daily} q dt, \quad \text{where } q \text{ is the downward heat flux} \quad (6.21)$$

The original secondary roof has 1.79 kWh/m² of daily heat gain, while the cool secondary roof has only 0.71 kWh/m². This shows that the cool coating leads to a daily heat gain reduction of 1.08 kWh/m² (or 60%) through the ferrocement secondary roof into the air-gap.

6.3.2.2 Cool roof effect for the primary roof and indoor environment

Figure 6.4 (b) also depicts the effect of cool coating on the primary roof temperature, primary ceiling temperature and conduction heat flux through the concrete primary

roof (due to temperature difference between primary roof and primary ceiling). The cool coating (applied on the secondary roof) keeps the cool primary roof temperature and cool primary ceiling temperature lower than the original counterpart throughout the 24-hours. The cool primary roof temperature is lowered by up to 4.7°C and the cool primary ceiling temperature is lowered by up to 3.5°C than the original counterpart. The reduction in primary roof temperature leads to lower heat gain into the building through the cool primary roof as compared to the original primary roof. Cool primary roof has a peak time heat gain reduction of 0.03 kW/m^2 (or 43%) compared to the original primary roof during day time. During night time the heat flow direction reverses, i.e., heat flows upward from primary ceiling to primary roof. As the thermal emittance of both cool and original primary roofs is same (see Table 6.2), the heat emission through both the primary roofs is also almost same. Integrating the heat flux (downward) profile with respect to time for the day using Eq. (6.21), the daily heat gains through both primary roofs are estimated. The original primary roof has 0.45 kWh/m^2 of daily heat gain, while the cool primary roof has only 0.24 kWh/m^2 . The cool coating leads to a daily heat gain reduction of 0.21 kWh/m^2 (or 47%) through the primary roof. Figure 6.4 (c) shows the hourly-averaged primary ceiling heat flux [using Eq. (6.16)] and indoor air temperatures [using Eq. (A6-16)] for the original and cool DSRs. It can be observed that the cool DSR keeps the indoor air temperature lower than that beneath the original DSR almost throughout the 24-hours. The cool coating provides peak indoor air temperature reduction of 2.4°C .

Fig. 6.4 (c) Comparison of CRHT predictions with measurements of hourly-averaged indoor air temperature and ceiling heat flux for original vs. cool primary roofs. Error bars show 1 standard deviation of the 1-minute measurements in each hour.

6.4 Discussion

6.4.1 Thermal performance of cool roof on SSR compared to DSR in tropical climate

The thermal performance of cool roof as a solution for curbing heat gain through the primary roof (heat gain into indoor environment) is compared to DSR using the proposed CRHT model for DSR and the CRHT model for SSR developed in in Chapter 4. Four roof cases, namely original SSR (used as the baseline for comparison), cool SSR (Original SSR + cool coating on primary roof), Original DSR and Cool DSR (Original DSR + cool coating on secondary roof) are modelled in this comparison. Table 6.2 shows the radiation properties of the Cool and Original SSRs and DSRs used in this analysis. As an input to the CRHT model, the solar-air temperature

profiles (shown in Fig. 4.2 for the sunny day) are used for the cool and original roof surfaces. The hourly-averaged values of overall heat transfer coefficients (as shown in Fig. 6.3) are used for the Original and Cool DSRs, while that for the Original and Cool SSRs are taken from Fig. 4.2 (b). The thermophysical properties of the roof materials (in the primary and secondary roofs) as shown in Table 6.3 are used as an input to the CRHT model. Figure 6.5 (a) show the primary roof temperature, primary ceiling temperature and conduction heat flux through the primary roof on the sunny for the four roof cases.

Fig. 6.5 (a) Comparison of original SSR, original DSR, cool SSR and cool DSR for hourly-averaged primary roof temperature, primary ceiling temperature and conduction heat flux through the primary roof.

Figure 6.5 (b) shows the indoor air temperature profiles for the four roof cases. Key quantities for comparisons are summarised in Table 6.4.

Fig. 6.5 (b) Comparison of original SSR, original DSR, cool SSR and cool DSR for hourly-averaged primary ceiling heat flux and indoor air temperature.

Table 6.4 Comparison of thermal performance of four roof cases (with baseline to ‘Original SSR’) for the tropical climate of Singapore.

Day	Reduction in parameter with baseline to ‘Original SSR’	Roof case			
		Original SSR (baseline)	Cool SSR	Original DSR	Cool DSR
Sunny	Peak time T_R (°C)	0	12.2	13.3	18.0
	Peak time T_C (°C)	0	5.9	5.6	9.1
	Peak time T_i (°C)	0	3.8	3.2	5.6
	Peak time heat flux , q_R (kW/m ²)	0	0.13 (or 59%)	0.17 (or 77%)	0.20 (or 89%)

Daily heat gain, daily heat loss and net daily heat gain of the four roof cases are compared in Fig. 6.6. Combining the benefits of cool roof and DSR, Cool DSR shows the most reduction in daily heat gain (by 85% on the sunny day) compared to the baseline Original SSR among all roof cases. The Original DSR (reduces by 71%) performs slightly better than Cool SSR (reduces by 59%) in reducing daily heat gain. However, when net daily heat gain (daily heat gain during day time – daily heat loss during night time) is considered, Cool SSR (reduces by 62%) performs almost equally as Original DSR (reduces by 62%). This is because in the both DSR cases, the daily heat losses are substantially lower than those in the SSR cases.

Fig. 6.6 Comparison of daily heat transfer through the primary roofs of original SSR, cool SSR, original DSR and cool DSR.

It suggests that the DSR provides good thermal insulation to the primary roof against heat gain during day time but it also hinders heat loss from the primary roof during night time. For tropical region, such as Singapore, where cooling is always sought, the insulation effect of DSR could cause cooling energy penalty at night time. In contrast

Cool SSR provides less day time heat gain reduction than original DSR but it allows more heat loss during night time, resulting in a net daily heat gain reduction almost equal to that found in Original DSR.

The heat transfer performance of the four roof cases from a yearly perspective was investigated by modelling of annual heat transfer (heat gain, heat loss and net heat gain) using the CRHT model. The typical meteorological year (TMY) weather data of Singapore is obtained from (EnergyPlus Weather Dataset) for this analysis. Figure 6.7 shows the annual heat transfer prediction of the four roof cases in Singapore and in Athens, Greece (which will be discussed in the following Section 6.4.2). Figure 6.7 shows that, for Singapore, the comparisons between the four roof cases in terms of yearly performance are very similar to the trends observed in Figure 6.6.

Fig. 6.7 Comparison of annual heat transfer through the primary roofs of original SSR, cool SSR, original DSR and cool DSR in the tropical climate of Singapore and Mediterranean climate of Athens, Greece.

Cool DSR shows the most reduction (82%) in annual heat gain compared to the baseline Original SSR. Original DSR (66% reduction) outperforms Cool SSR (54% reduction) in reducing annual heat gain. However, Cool SSR (56% reduction) performs almost equally as Original DSR (56% reduction) when net annual heat gain (annual heat gain – annual heat loss) is concerned. It suggests that the analysis on the sunny day is rather representative to the yearly performance in Singapore.

6.4.2 Thermal performance of cool roof on SSR compared to DSR in seasonal climate

In order to provide a wider perspective of the performance of cool roof and DSR and to evaluate the wider applicability of the proposed CRHT models for other climates, modelling of thermal performances of all four roof cases for the subtropical Mediterranean climate of Athens, Greece is conducted, covering a full year weather cycle. The same four roof cases (as those in Section 6.4.1) are modeled and, thus, the same radiation and thermophysical properties (as shown in Tables 6.2 and 6.3) are used for this analysis. The TMY weather data of subtropical Mediterranean climate of Athens is taken from (EnergyPlus Weather Dataset) and the solar-air temperature profiles are estimated using Eqs. (6.2a) and (A6-13).

Figure 6.7 shows that, in terms of performance in reducing annual heat gain, the trend is very similar to that in Singapore (tropical climate). Cool DSR shows the most reduction in annual heat gain, compared to the baseline Original SSR, followed by Original DSR and Cool SSR. The SSR cases allow substantially more annual heat loss from primary roof compared to the DSR cases. This could, again, largely attribute to the extra insulation provided by the secondary roof and air gap in the DSR cases. However, since Athens receives much less yearly solar radiation than Singapore and there are several cold months in a year in Athens, annual heat loss through primary

roofs in Athens is much more substantial than Singapore. This contributes to the lowest net annual heat gain in Cool SSR in Athens, lower than both DSR cases. The seasonal effect in Athens is further illustrated (in Fig. 6.8) in which modelling results are further broken down into hot months and cold months. In contrast to Singapore where cooling is sought throughout the year, the insulation effect provided by DSR in preventing night time heat loss could become a benefit during the cold months in Athens.

Fig. 6.8 Comparison of seasonal heat transfer through the primary roof of original SSR, cool SSR, original DSR and cool DSR for the Mediterranean climate of Athens, Greece.

6.5 Summary

A novel cool roof heat transfer (CRHT) model for double-skin roof (DSR) is proposed to investigate the thermal performance of combined cool roof + DSR in the tropical climate. The CRHT model is capable of handling the transient outdoor and indoor boundary conditions as experienced by naturally ventilated residential buildings. The

CRHT model, which is generally applicable to any climate conditions, was validated against experiments in two identically configured, naturally ventilated apartments with flat DSR on a residential apartment building with concrete flat DSR in Singapore. Reasonable agreement between the CRHT model predictions and experimental measurement results is found.

The CRHT predictions showed that adding a cool coating on the DSR reduces the peak secondary roof temperature by up to 14.7°C, primary roof temperature by up to 4.7°C, the indoor air temperature by up to 2.4°C and daily heat gain by up to 0.21 kWh/m² (or 51%) compared to the original counterpart on a sunny day in the tropical climate of Singapore. Due to the rather minor variations in climate conditions throughout the year in tropical climate, the observations in daily heat gain/loss reductions by cool roof or DSR are fairly representative to the yearly projection. The thermal performances of original and cool double-skin roofs are compared against original and cool single-skin roofs using the CRHT models for the tropical climate of Singapore and subtropical Mediterranean climate of Athens, Greece. Among the four roof cases, the cool double-skin roof is found to be most effective in terms of reducing day time heat gain (with reference to ‘original single-skin roof’), followed by original double-skin roof and then cool single-skin roof. Cool single-skin roof becomes almost equal effective as original double-skin roof when net heat gain reduction is concerned. The extra insulation effect of the double-skin roof provides good thermal insulation against day time heat gain but it also hinders night time heat loss. This could become a night time cooling energy penalty in the tropical Singapore but could become a benefit in colder climate, such as during the cold months in Athens, Greece.

Chapter 7 Conclusions and Suggestions for Future Work

7.1 Conclusions

In this study, thermal performance of cool roof (solar reflectance, $\rho = 0.74$ and thermal emittance, $\varepsilon = 0.90$) is investigated in tropical climate by performing computational simulations (calibrated using experiments) on an air-conditioned, single-storey test building in Singapore. The performance of cool roof is compared with other potential roofing methods like thermal insulation (50-mm-thick extruded polystyrene) and green roof. It is observed that cool roof provides the higher annual net heat gain reduction as compared to thermal insulation and green roof for common R -value roofs ($0.25 \text{ m}^2\text{-K/W}$ to $1.00 \text{ m}^2\text{-K/W}$) in the tropical climate. The increment of ε by 0.76 (from 0.10 to 0.86) reduces the annual net heat gain by 50-53%, the increment of ρ by 0.64 (from 0.10 to 0.74) reduces the annual net heat gain by 81-83%, the combined increment of ε by 0.76 and ρ by 0.64 reduces the annual net heat gain by 90-92%, the addition of green roof with 100% moist soil layer (500-mm-thick) reduces the annual net heat gain by 75-55% and the addition of extruded polystyrene insulation (50-mm-thick) reduces the annual net heat gain by 80-68%. This shows that the percentage annual net heat gain reduction provided by cool roof almost remains same with the increase in R -value of the roof, while that provided by thermal insulation and green roof decreases.

The performance of cool roof, green roof and thermal insulation are also investigated for four climates: wet tropical climate, Mediterranean climate, hot and dry climate and semi-arid climate. It is observed that the annual heat gain reduction provided by cool roof is about 55-65%, by thermal insulation is about 40-50%, and by green roof is about 25-35% for these four climates. This shows that cool roof provides higher annual heat gain reduction compared to green roof and thermal insulation.

Further investigations on cool roof are carried out by formulating heat transfer models that provide very concise and easy to use expressions to evaluate the impacts of cool coating on roof temperature, ceiling temperature, indoor air temperature and heat flux. A novel and general cool roof heat transfer (CRHT) model capable of incorporating transient outdoor and indoor boundary conditions (as experienced by naturally ventilated buildings) is developed using spectral approximation method. The proposed CRHT model is applicable to all opaque single-skin roof (SSR) or solid roof/wall surfaces (no air-gap in between layers). The CRHT model was verified against the Conduction Transfer Function method and was further validated against experiments performed in two identically configured, side-by-side, naturally-ventilated apartments with 100-mm-thick concrete (plus 10-mm-thick plaster) flat roofs in Singapore. White-coloured cool coating, providing a solar reflectance of 0.74, was applied on one of the two roofs and another roof remained original (solar reflectance of 0.30). The CRHT predictions show that on a sunny day, cool coating reduces the peak roof temperature by up to 14.1°C, peak indoor air temperature by up to 3.1°C and daily heat gain by 0.74 kWh/m² (or 58%) on the concrete roof. On an 8-mm-thick galvanised steel (metal) roof (original solar reflectance of 0.60), the same cool coating reduces peak roof temperature by up to 5.4°C, peak indoor air temperature by up to 3.7°C and daily heat gain by 0.74 kWh/m² (or 34%).

Furthermore, a novel and general CRHT model is proposed for cool double-skin roof (or cool DSR) to investigate the thermal performance of combined passive cooling methods i.e., cool roof + DSR in the tropical climate. The CRHT model is capable of handling the transient outdoor and indoor boundary conditions as experienced by naturally ventilated residential buildings. The CRHT model was validated against experiments in two identically configured, naturally ventilated apartments with flat

DSR on a residential building in Singapore. The DSR consisted of a 30-mm-thick ferrocement secondary roof, an air-gap of 220-mm height and a 100-mm-thick concrete (plus 10-mm-thick plaster towards the ceiling). White-coloured cool coating, providing a solar reflectance of 0.74 was applied on the secondary roof of one of the two apartments. Another apartment's secondary roof remained original with a solar reflectance of 0.30. The CRHT predictions showed that on a sunny day, the cool coating on DSR reduces the peak secondary roof temperature by up to 14.7°C, secondary ceiling temperature by up to 11.9°C, primary roof temperature by up to 4.7°C, primary ceiling temperature by up to 3.5°C, the indoor air temperature by up to 2.4°C and daily heat gain by up to 0.21 kWh/m² (or 47%) compared to the original counterpart on a sunny day in tropical climate.

Thermal performances of original DSR, cool DSR, original SSR and cool SSR are also analysed using the proposed CRHT models (for SSR and DSR). It is observed that comparing to the original SSR (as reference), the cool SSR provides daily heat gain reduction of 0.74 kWh/m² (or 58%), the original DSR provides daily heat gain reduction of 0.83 kWh/m² (or 65%) and the cool DSR provides daily heat gain reduction of 1.05 kWh/m² (or 82%). In terms of daily net heat gain, comparing to the original SSR (as reference), the cool SSR provides reduction of 0.34 kWh/m² (or 81%), the original DSR provides reduction of 0.15 kWh/m² (or 36%) and the cool DSR provides daily heat gain reduction of 0.37 kWh/m² (or 88%).

A new *RTTV* (*RTTV*^{new}) model capable of assimilating the effect of solar reflectance (ρ) of the opaque roof surface is proposed to provide accurate thermal performance prediction of air-conditioned buildings with high solar reflectance roofs (e.g., cool roofs). The *RTTV*^{new} model is developed by formulating an expression for equivalent

thermal resistance increment due to the increased solar reflectance of the opaque roof (K^{incr}) using the *CRHT* model and incorporating it into U -value of the component of heat conduction gain through the opaque portion of the roof. This proposed new method overcomes the deficiencies of the existing methods (i.e., they either use a fixed higher equivalent temperature difference value or assume a linear correlation between annual-averaged heat gain and solar absorptance) in capturing the solar reflectance effect while retaining the simple and easy-to-use characteristics. The method is generally applicable to other similar models (such as *OTTV* or *ETTV*) and is not limited by climate conditions or building materials.

The accuracies of the three models: Current *RTTV* (use a fixed higher equivalent temperature difference value), $RTTV^A$ (assume a linear correlation between annual-averaged heat gain and solar absorptance) and the proposed $RTTV^{new}$ model, in estimating *RTTV* in are tested by comparing against computational simulations of an air-conditioned single-storey test building under tropical climate. The computational model was calibrated against experiments. The Current *RTTV* model shows obvious limitations in capturing the solar reflectance effect as the error increases with roof solar reflectance (from about 3% at $\rho = 0.10$ to an over-estimation of more than 20 times at $\rho = 0.90$). Estimations by the $RTTV^A$ model also show significant deviation from computational simulation predictions. The proposed $RTTV^{new}$ model is the most accurate among the three models for opaque roof surfaces of all solar reflectance.

7.2 Suggestions for future work

Despite the above work accomplished, some more investigations are still needed to understand the effect of cool roof in tropical climate. The author would like to

recommend the following future directions to extend and supplement the current research work:

7.2.1 Effect of ageing on cool roof's performance

Cool roof is prone to ageing due to the environmental factors such as deposition of soot and dust, exposure to sunlight, rainfall and thermal cycles, and growth of fungi (Bretz & Akbari, 1997). The ageing process deteriorates cool coating's radiation, mechanical and chemical properties, e.g., solar reflectance, thermal emittance, tensile strength, colour, roughness, and dirt resistance, leading to degradation of cool coating's thermal performance (Berdahl et al., 2008; Bretz & Akbari, 1997). The environmental factors vary from climate to climate, so different climate zones, would observe different ageing processes of cool coatings. Investigating the role of these environmental factors in ageing process (in tropical climate) would help to understand the degradation of cool roof, so that necessary measures could be taken to maintain the high performance level of cool roofs. Therefore, a natural ageing study was performed to investigate the effect of ageing on radiation properties of cool roof as explained below.

Preliminary natural ageing study

In order to investigate the effect of ageing on the performance of roof materials applied with cool coatings in the tropical climate, Ferro-cement (which is a commonly used roof material in Singapore as a secondary roof in the double-skin roof) was selected as a base material. The Ferro-cement roof material was cut to a sample size of 65-mm (length) $\times$ 65-mm (width) $\times$ 30-mm (thick) so as to accommodate on UV/Vis/NIR LAMBDA 950 Spectrophotometer (as shown in *Appendix-1*) for the measurement of solar reflectance and portable Emissometer for the measurement of thermal emittance. The Ferro-cement samples applied with cool coating were placed

on a flat horizontal rack on a roof in Nanyang Technological University for 3 years. The radiation properties of the samples were measured after the ageing of 1 year, 2 years and 3 years. The spectral solar reflectance profiles of the samples are shown in Fig. 7.1. Over the period of 3 years of ageing, the solar reflectance of the cool coating reduced from 0.80 (New), 0.73 (after 1 year), 0.67 (after 2 years) to 0.56 (after 3 years). However, the thermal emittance remained almost unchanged with 0.87.

Fig. 7.1 Effect of ageing on the solar reflectance of cool coating.

Furthermore, the effect of change in the solar reflectance (due to ageing) on the surface temperature was studied by performing a carbon soot deposition test. In the test, six Ferro-cement samples (applied with cool coating) were loaded with different concentration of carbon soot (over the cool coating surface) to have the samples with different solar reflectance (in the range 0.45 to 0.80). On a sunny day, all the samples were placed exposed to the sun and the surface temperatures were measured. Figure 7.2 shows the effect of solar reflectance on the surface temperature using a thermal imager infrared camera (FLIR E60) as shown in *Appendix-1*. It can be observed that

the surface temperature increases by 2.9°C (from 39.6°C to 42.5°C) with the reduction in solar reflectance from 0.80 to 0.56. The effect of ageing on the annual heat gain is shown in Fig. A-4 (*Appendix-3*).

There are further studies to be performed to investigate the main factors causing ageing of coatings in the tropical climate of Singapore. The CRHT models proposed in this study for SSR (Chapter 4) and for DSR (Chapter 6) assume invariant nature of radiation properties of cool coating. However, in reality these properties change over the period of outdoor exposure (as shown in the ageing study). This may lead to inaccurate estimation of thermal performance of cool roof over a long period of time. Therefore, it is recommended to modify the CRHT models in future studies.

Fig. 7.2 Effect of solar reflectance on the surface temperature.

7.2.2 Micro-climatic impact of cool roof

An experimental study by Rajagopalan et al. (2004) highlighted the occurrence of urban heat island (UHI) effect in Singapore. The use of cool roof for opaque building

surfaces can be a possible solution to mitigate the UHI effect. It is suggested to investigate the micro-climatic impact of using cool roof for the tropical climate of Singapore.

7.2.3 Life cycle cost analysis of cool roof, green roof, thermal insulation and DSR

This study investigated the thermal performance of cool roof in comparison to green roof, thermal insulation and DSR. However, in order to determine the most cost-effective method among these, it is suggested to perform a comprehensive life cycle cost analysis.

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Appendix-1 Instrumentation

No .	Parameter	Location	Instrument	Brand/Specification
1	Surface temperature	Roof	RTD and Data loggers	RTD (J-Quest) Data-loggers (Spectra-Teknik)
		Ceiling		Specifications Temperature range: -50 to +150°C Accuracy: $\pm 0.5^{\circ}\text{C}$ Stability: less than 0.2°C drift/year
2	Indoor air temperature and Relative humidity	Indoor environment	TransiTemp-II RH-Recorder	MADGE TECH (18003-26) TransiTemp (18003-27)
		Specifications Temperature range: -40 to $+80^{\circ}\text{C}$ Temperature resolution: 0.1°C Temperature accuracy: 0.5°C Humidity range: 0 to 95% RH Humidity resolution: 0.1% RH Humidity accuracy: $\pm 5\%$ RH
3	Thermal image	Roof	Thermal imager	FLIR (Model: E60)
		Specifications Field of view/ Minimum focus distance: $25^{\circ} \times 19^{\circ} / 0.4 \text{ m}$ Spatial resolution: 1.36 rad Image frequency: 60 Hz Temperature range: -20°C to $+120^{\circ}\text{C}$, Accuracy: $\pm 2\%$ of reading IR resolution: 320×240 pixels

4	Solar reflectance measurement (ASTM C1549)	Original roof	Portable Reflectometer	Devices and Services (Model: ER version 6)
		Cool roof
		Specifications Resolution: 0.001 units Repeatability: ±0.003 units Accuracy: ±0.002 and standard deviation of 0.005 for spectral calibration tile Drift: ±(1% of reading + 0.003)/hour	
5	Emittance measurement (ASTM E408)	Original roof	Emissometer	Devices and Services (Model: AE1)
		Cool roof
		Specifications Output: 2.4 millivolts nominal with sample emittance of 0.9 and sample temperature of 25°C Repeatability: ± 0.01 units Linearity: within ± 0.01 units. Time Constant: 10 seconds, nominal (time to reach 63% of final value.	
6	Incident solar radiation (ASTM E1918)	CMP 11 Pyranometer		Kipp and Zonen Pyranometer (CMP 11) Data Logger (LOGBOX)
				Specifications Spectral range: 285 nm to 2800 nm Sensitivity: 7 to 14 μV/W/m² Response time: < 5s Non-stability (change/year): < 0.5% Non-linearity (0 to 1000 W/m²): < 0.2%
	Weather	Outdoor air temperature	WeatherHawk	Scientific sales, Inc.
Relative humidity				
Dew temperature				

7	parameter	Barometer pressure	916	(WeatherHawk)
		Wind direction		
		Wind speed		
		Incident solar radiation		
		Rain fall
		Specifications Rain gauge: 0 to 5 inches Temperature: -20 to 60 °C Relative Humidity (RH): 0 to 100 % Solar irradiance: 0 to 111 watts per square meter(w/m ²) Wind Speed/Direction: 0 to 125 mph/ 5 to 355 degrees	
Surface reflectance (ASTM E903)	UV/Vis/NIR Spectrometer (surface reflectance of samples)	PerkinElmer (Lambda 950)
	Specifications Wavelength range: 175 nm - 3300 nm Wavelength Accuracy: ±0.08nm UV/Vis NIR Resolution of ≤ 0.20nm UV/Vis Resolution of ≤ 0.05nm Spectral bandwidth 0.05 to 5 nm for UV/Vis, 0.2 to 20 nm for NIR Integrating sphere -150mm PMT and InGaAs Module High performance UV/Vis/NIR double beam Double monochromator Ratio Recording Spectrometer pre-aligned tungsten halogen lamp and deuterium lamp.		

Appendix-2 Effect of coating colour on the roof heat transfer

Figure A-2 shows the distribution of spectral solar reflectance for various coating colours over the solar spectrum (300 nm - 2500 nm). It can be observed that the spectral solar reflectance varies with the cool coating colour. For instance, the white cool coating has integrated solar reflectance (ρ) = 0.74, the blue cool coating has ρ = 0.60 and the purple cool coating has ρ = 0.56. The effect of coating colour on the roof heat transfer are shown in Fig. A-3, which is estimated using the CRHT model (developed in Chapter 4).

Fig. A-2 Spectral solar reflectance for different cool coating colours.

Fig. A-3 Annual heat gain through roofs having different cool coating colours.

Appendix-3 Effect of weathering on the roof heat transfer

Weathering tends to reduce the solar reflectance of the cool coating due to the accumulation of the dust and microbial growths on the coating surface. However, the extent of deterioration in the solar reflectance depends on the local weather conditions. Figure A-4 shows that with the reduction in solar reflectance of the cool coating (due to ageing), the annual heat gain increases. The solar reflectance values of the aged samples (shown in Fig. A-4) are obtained from the ageing study discussed in Section 7.2.1.

Fig. A-4 Effect of ageing on the annual heat gain through cool roof.

Appendix-4 Derivation of heat transfer coefficients

A-1. Derivation of $h_{i,conv}$ for naturally ventilated buildings and comparison with ASHRAE (2009)

An experimental study was performed by Awbi (1998) in a real-size chamber (2.78 m $\times$ 2.78 m $\times$ 2.3 m high). In the experiment, the ceiling temperature was varied in the range 15°C to 45°C (as experienced by the ceilings of naturally ventilated buildings). Using the experimental data for the heated ceiling in the chamber and applying the heat transfer calculations, it gives Eq. (A4-1)

$$Nu = 2.376(Gr)^{0.133} \quad (A4-1)$$

$$\text{where } Gr = \frac{(\rho_{air}^*)^2 g \delta (D_h)^3 \Delta T}{(\mathcal{G}_{air})^2} \text{ and } Nu = \frac{h_{i,conv} D_h}{k_{air}} \quad (A4-2)$$

Substituting Eqs. (A4-2) in (A4-1), it gives

$$\frac{h_{i,conv} D_h}{k_{air}} = 2.376 \left(\frac{(\rho_{air}^*)^2 g \delta (D_h)^3 \Delta T}{(\mathcal{G}_{air})^2} \frac{C_{air} \mu_{air}}{k_{air}} \right)^{0.133} \quad (A4-3)$$

Using the properties of air, it gives the expression for $h_{i,conv}$

$$h_{i,conv} = \frac{0.704}{D_h^{0.601}} (\Delta T)^{0.133} \quad (A4-4)$$

The $h_{i,conv}$ estimations (for $D_h = 2.78$ m) using Eq. (A4-4) and ASHRAE (2009) (non-dimensional correlation for natural convection from a horizontal surface) are compared in Fig. A-5. It can be observed that the ASHRAE (2009) results are generally greater than those given by Eq. (A4-4). This discrepancy could be because $h_{i,conv}$ given in ASHRAE is based upon the experiments using small free-edge heated plates, thus the air movement over a surface in a room is different from that over a small free-edge heated plate.

Fig. A-5 Comparison of $h_{i,conv}$ between Awbi (1998) and ASHRAE (2009).

A-2. Derivation of $h_{i,conv}$ for air-conditioned buildings

Karadag (2009) computationally derived the convective heat transfer coefficient for the ceilings of air-conditioned buildings. The study was performed at a constant floor temperature ($T_{\text{floor}} = 25^{\circ}\text{C}$). The ceiling and wall temperatures were varied in the range 0 to 25°C and 28 to 36°C , respectively. In the analysis, simulations were performed for three different room dimensions ($3 \times 3 \times 3$ m, $4 \times 3 \times 4$ m and $6 \times 3 \times 4$ m).

The natural convection heat transfer correlation for the ceilings of air-conditioned buildings (Karadag, 2009) is given by,

$$Nu = 1.45(Gr)^{0.22} \quad (\text{A4-5})$$

$$\text{where } Gr = \frac{(\rho_{air}^*)^2 g \delta (D_h)^3 \Delta T}{(\mathcal{G}_{air})^2} \text{ and } Nu = \frac{h_{i,conv} D_h}{k_{air}} \quad (\text{A4-6})$$

$$\frac{h_{i,conv} D_h}{k_{air}} = 1.45 \left(\frac{(\rho_{air}^*)^2 g \delta (D_h)^3 \Delta T}{(\mathcal{G}_{air})^2} \right)^{0.22} \quad (\text{A4-7})$$

Using the properties of air, it gives the expression for $h_{i,conv}$

$$h_{i,conv} = 3.1(\Delta T)^{0.22} \quad (\text{A4-8})$$

A-3. Derivation of radiative heat transfer coefficient ($h_{i,r}$)

Anderson (1981) considered a hot ceiling surface at temperature T_C and a cold floor surface at temperature T_{floor} . Since $(T_C - T_{\text{floor}}) \ll T_C$ and T_{floor} , the side walls were assumed to be at a uniform temperature $T_{\text{wall}} = \frac{1}{2} (T_C + T_{\text{floor}})$. (A4-9)

$$\text{Taking } T_C^4 - T_{\text{floor}}^4 = 4T_{\text{wall}}^3 (T_C - T_{\text{floor}}) \quad (\text{A4-10})$$

$$\text{and } T_C^4 - T_{\text{wall}}^4 = T_{\text{wall}}^4 - T_{\text{floor}}^4 = 2T_{\text{wall}}^3 (T_C - T_{\text{floor}}). \quad (\text{A4-11})$$

The radiation coefficient across the ceiling and floor is given by

$$h_{i,r} = 2\sigma T_{\text{wall}}^3 \frac{\varepsilon_i (1 + \hat{F}_{C,\text{floor}})}{1 + \rho \hat{F}_{C,\text{floor}}} \quad (\text{A4-12})$$

For most of building materials ε_i is usually in the range 0.90 to 0.95. However, the value of ε_i will not normally be known precisely so that it is reasonable to take

$\rho \hat{F}_{C,\text{floor}} \sim 1 - \varepsilon_i$ for all $\hat{F}_{C,\text{floor}}$ (provided that ε_i is large) so that Eq. (A4-12) becomes,

$$h_{i,r} = \frac{\sigma \varepsilon_i}{4(2 - \varepsilon_i)} (1 + \hat{F}_{C,\text{floor}}) (T_C + T_{\text{floor}})^3 \quad (\text{A4-13})$$

$$\text{where } \hat{F}_{C,\text{floor}} = \sqrt{1 + \frac{L_i^2}{W^2}} - \frac{L_i}{W} \quad (\hat{F}_{C,\text{floor}} = 1 \text{ for } L_i \ll W). \quad (\text{A4-14})$$

Appendix-5 Supplementary data for Chapter 5

The heat transfer coefficients, h_R for the opaque roof top surface and h_i for the ceiling surface can be obtained using Eqs. (A5-1) - (A5-2d) (Karadag, 2009; Clear et al., 2003)

$$h_i = 3.1 (\Delta T)^{0.22} \quad \text{for the ceiling of air-conditioned buildings,} \quad (\text{A5-1})$$

$$h_R = h_{R,c} + h_{R,r} \cdot \frac{T_R - T_{sky}}{T_R - T_o} \quad \text{for an opaque roof directly facing the sky,} \quad (\text{A5-2})$$

where

$$h_{R,conv} = \frac{k_{air}}{D_h} 0.15 Ra^{1/3} + k_{air} \hat{f} 0.0296 Re^{4/5} Pr^{1/3} \quad (\text{A5-2a})$$

$$h_{R,r} = \varepsilon \sigma \cdot \hat{F}_{sky} (T_R^2 + T_{sky}^2) \cdot (T_R + T_{sky}), \quad (\text{A5-2b})$$

$$\hat{F}_{sky} = 0.5(1 + \cos \phi) \quad (\hat{F}_{sky} = 1 \text{ for a horizontal opaque roof}), \quad (\text{A5-2c})$$

$$T_{sky} = 0.0552(T_o)^{1.5} \quad \text{for clear days or}$$

$$T_{sky} = T_o \quad \text{for cloudy days.} \quad (\text{A5-2d})$$

Equation (A5-1) is generally applicable for the estimation of h_i of ceiling surfaces of air-conditioned buildings (Karadag, 2009). Equation (A5-2a) is applicable for estimation of outdoor convective heat transfer coefficient ($h_{R,conv}$) for any opaque roof surfaces and is not sensitive to wind direction (Clear et al., 2003). Equations (A5-2b) – (A5-2d) are applicable for estimation of outdoor radiative heat transfer coefficient ($h_{R,r}$). The view factor of the opaque roof to the sky ($\hat{F}_{sky}$) and the sky temperature (T_{sky}) can be estimated using Eqs. (A5-2c) and (A5-2d), respectively.

Formulation of K^{incr} using the previously developed CRHT model

The heat flux through both the cool roof (ρ) and the imaginary roof ($\rho = 0$) surfaces can be determined by the Cool Roof Heat Transfer (CRHT) model (derived in Chapter 4) for transient outdoor and constant indoor boundary conditions (since RTTV estimation applies for the air-conditioned buildings only). The *CRHT* model is applicable to solid roofs i.e., no air-gap between roof layers. The heat flux through the imaginary roof surface is given by

$$q_{R,\rho=0} = U_{R,\rho=0} \cdot (TD_{eq}^{CRHT})_{\rho=0}, \quad (A5-3)$$

$$\text{where } (TD_{eq}^{CRHT})_{\rho=0} = \left(T_{sol-air} - \left[T_i \left(1 - \frac{U_{R,\rho=0}}{h_R} \right) + \frac{U_{R,\rho=0}}{h_R} \tilde{T}_{sol-air} \right] \right)_{\rho=0}, \quad (A5-3a)$$

$$\frac{1}{U_{R,\rho=0}} = \left(\frac{1}{h_i} + \frac{1}{h_R} \right)_{\rho=0} + \frac{L}{k}, \quad (A5-3b)$$

$$T_{sol-air} = \frac{I_{in}\alpha}{h_R} + T_o - \frac{\mathcal{E}^{IR}}{h_R} \quad (\alpha = 1 \text{ for imaginary roof}) \quad (A5-3c)$$

$$U_{R,\rho=0} = 1/R_{R,\rho=0}, \quad R_{R,\rho=0} = (K_i + K_o)_{\rho=0} + K_{cond}. \quad (A5-3d)$$

In parallel, the heat flux through the cool roof surface is given by

$$q_{R,\rho} = U_{R,\rho} \cdot (TD_{eq}^{CRHT})_{\rho}, \quad (A5-4)$$

$$\text{where } (TD_{eq}^{CRHT})_{\rho} = \left(T_{sol-air} - \left[T_i \left(1 - \frac{U_{R,\rho}}{h_R} \right) + \frac{U_{R,\rho}}{h_R} \tilde{T}_{sol-air} \right] \right)_{\rho}. \quad (A5-4a)$$

$$\frac{1}{U_{R,\rho}} = \left(\frac{1}{h_i} + \frac{1}{h_R} \right)_{\rho} + \frac{L}{k} \quad (A5-4b)$$

In Eqs. (A5-3) and (A5-4), h_R , $U_{R,\rho}$, $T_{sol-air}$ and $\tilde{T}_{sol-air}$ are functions of roof surface solar reflectance (Suehrcke et al., 2008).

In order to estimate the equivalent total thermal resistance increment, K^{incr} (as shown in Eq. (5.7)), due to the addition of the cool layer, the $U_{R,\rho=0}$ in Eq. (A5-3) is replaced by $U_{R,\rho=0}^{new}$ such that the annual-averaged heat gain through the imaginary roof becomes equal to the annual-averaged heat gain through the cool roof. Therefore, Eq. (A5-3) becomes

$$q_{R,\rho=0} = U_{R,\rho=0}^{new} (TD_{eq}^{CRHT})_{\rho=0} \quad (A5-5)$$

$$\text{where } U_R^{new} = \frac{1}{R_R^{new}}, \quad (A5-5a)$$

$$R_R^{new} = (K_i + K_R)_{\rho=0} + K_{cond} + K_{cond}^{cool} + K^{incr} \quad (A5-5b)$$

Equating the annual-averaged heat gains through the cool roof (ρ) and the imaginary roof ($\rho=0$), it becomes

$$\frac{1}{N} \cdot \int_{1 \text{ year}} q_{R,\rho} dt = \frac{1}{N} \cdot \int_{1 \text{ year}} q_{R,\rho=0} dt \quad (A5-5c)$$

The integrals in Eq. (A5-5c) can be estimated by summing the upward and downward conduction heat flux through the roofs for the whole year. Substituting Eqs. (A5-4) and (A5-5) into Eq. (A5-5c), it yields

$$U_{R,\rho} \left(\int_{1 \text{ year}} (TD_{eq}^{CRHT})_{\rho} dt \right) = U_R^{new} \left(\int_{1 \text{ year}} (TD_{eq}^{CRHT})_{\rho=0} dt \right) \quad (A5-5d)$$

Re-arranging Eq. (A5-5d), it becomes

$$\frac{\int_{1 \text{ year}} (TD_{eq}^{CRHT})_{\rho} dt}{\int_{1 \text{ year}} (TD_{eq}^{CRHT})_{\rho=0} dt} = \frac{U_R^{new}}{U_{R,\rho}} \quad (A5-5e)$$

Now, consider
$$\frac{\int_{1 \text{ year}} (TD_{eq}^{CRHT})_{\rho} dt}{\int_{\rho=0} (TD_{eq}^{CRHT})_{\rho=0} dt} = F \quad (\text{A5-6})$$

From Eqs. (A5-5e) and (A5-6), $\frac{U_R^{new}}{U_{R,\rho}} = F$ and

from Eqs. (A5-3d) and (A5-5a),
$$\frac{U_R^{new}}{U_{R,\rho}} = \frac{R_{R,\rho}}{R_R^{new}} \quad (\text{A5-6a})$$

Therefore $\frac{R_{R,\rho}}{R_R^{new}} = F$ where $F \leq 1$ (A5-6b)

Substituting Eqs. (A5-3d) and (A5-5b) into Eq. (A5-6b), it yields

$$(K_i + K_o)_{\rho} + K_{cond} = F \cdot [(K_i + K_o)_{\rho=0} + K_{cond} + K_{cond}^{cool} + K^{incr}] \quad (\text{A5-7})$$

Re-arranging Eq. (A5-7), it gives the expression for K^{incr} (as shown in Eq. (5.7)).

Appendix-6 Supplementary data for Chapter 6

Solution of heat conduction equation (6.1) using the spectral approximation method

An approximate solution of the governing differential Eq. (6.1) using spectral approximation method is given by

$$T(y,t) = \sum_{m=0}^{\infty} \Phi(y) \cdot \theta(t) \quad (\text{A6-1})$$

$$T(y,t) = \sum_{m=0}^{\infty} \left[a_m \cos\left(\frac{m\pi y}{L_{SR}}\right) + b_m \sin\left(\frac{m\pi y}{L_{SR}}\right) \right] \cdot \theta(t) \quad (\text{A6-2})$$

To find the expression for $\theta(t)$, partial derivatives of Eq. (A6-2) are taken with respect to t (Eq. (A6-3)) and with respect to y (Eq. (A6-4)), and then substituted in Eq. (6.1), it yields Eq. (A6-5)

$$\frac{\partial T(y,t)}{\partial t} = \sum_{m=0}^{\infty} \frac{\partial \theta(t)}{\partial t} \cdot \left[a_m \cos\left(\frac{m\pi y}{L_{SR}}\right) + b_m \sin\left(\frac{m\pi y}{L_{SR}}\right) \right] \quad (\text{A6-3})$$

$$\frac{\partial^2 T(y,t)}{\partial y^2} = \sum_{m=0}^{\infty} \theta(t) \cdot \left[-\frac{m^2 \pi^2}{L_{SR}^2} a_m \cos\left(\frac{m\pi y}{L_{SR}}\right) - \frac{m^2 \pi^2}{L_{SR}^2} b_m \sin\left(\frac{m\pi y}{L_{SR}}\right) \right] \quad (\text{A6-4})$$

$$\sum_{m=0}^{\infty} \frac{\partial \theta(t)}{\partial t} \cdot \left[a_m \cos\left(\frac{m\pi y}{L_{SR}}\right) + b_m \sin\left(\frac{m\pi y}{L_{SR}}\right) \right] = \sum_{m=0}^{\infty} -\alpha_{SR}^* \frac{m^2 \pi^2}{L_{SR}^2} \cdot \theta(t) \cdot \left[a_m \cos\left(\frac{m\pi y}{L_{SR}}\right) + b_m \sin\left(\frac{m\pi y}{L_{SR}}\right) \right] \quad (\text{A6-5})$$

Re-arranging Eq. (A6-5), it becomes

$$\sum_{m=0}^{\infty} \left(\frac{\partial \theta(t)}{\partial t} + \frac{\alpha_{SR}^* \pi^2 m^2}{L_{SR}^2} \theta(t) \right) \cdot \left[a_m \cos\left(\frac{m\pi y}{L_{SR}}\right) + b_m \sin\left(\frac{m\pi y}{L_{SR}}\right) \right] = 0 \quad (\text{A6-6})$$

Since the functions $\left\{ a_m \cos\left(\frac{m\pi y}{L_{SR}}\right) + b_m \sin\left(\frac{m\pi y}{L_{SR}}\right) \right\}_{m=0}^{\infty}$ are orthogonal, Eq. (A6-6) can

be simplified to the ordinary differential equation

$$\frac{\partial \theta(t)}{\partial t} + \frac{\alpha_{SR}^* \pi^2 m^2}{L_{SR}^2} \theta(t) = 0 \quad (\text{A6-7})$$

The solution of Eq. (A6-7) is $\theta(t) = A.e^{-\lambda_m^2 t}$ (A6-8)

where $\lambda_m^2 = \frac{\alpha_{SR}^* \pi^2 m^2}{L_{SR}^2}$ and A is an arbitrary constant.

Substituting the value of $\theta(t)$ in Eq. (A6-2), it gives

$$T(y, t) = \sum_{m=0}^{\infty} \left[a_m \cos\left(\frac{m\pi y}{L_{SR}}\right) + b_m \sin\left(\frac{m\pi y}{L_{SR}}\right) \right] . A.e^{-\lambda_m^2 t} \quad (\text{A6-9})$$

where the coefficients a_m and b_m are

$$a_m = \frac{2}{L_{SR}} \int_0^{L_{SR}} (c_1 y + c_2) . \cos\left(\frac{m\pi y}{L_{SR}}\right) dy \quad \text{and} \quad b_m = \frac{2}{L_{SR}} \int_0^{L_{SR}} (c_1 y + c_2) . \sin\left(\frac{m\pi y}{L_{SR}}\right) dy \quad (\text{A6-10})$$

Applying the initial (steady state) condition to Eq. (6.1), it yields

$$T(y, 0) = c_1 y + c_2 \quad (\text{A6-11})$$

where c_1 and c_2 can be obtained by considering an arbitrary uniform temperature field.

Hence the final solution of Eq. (6.1) which satisfies the Neumann boundary conditions at the secondary roof and secondary ceiling surfaces and the initial condition is given by Eq. (6.4).

Intermediate variables in Eq. (6.4)

$$\text{where } B_1^+ = \sum_{m=0}^{\infty} \left[\frac{2}{m^2 \pi^2} (T(L_{SR}, 0) - T(0, 0)) (\cos m\pi - 1) \right], \quad D_1^+ = \sum_{m=0}^{\infty} \left[\frac{2}{m\pi} T(L_{SR}, 0) (\cos m\pi - 1) \right]$$

$$E_1^+ = \left[\frac{\frac{h_{SC}}{k_{SR}} T_{gap}}{\frac{4}{L_{SR}} T(L_{SR}, 0) + \frac{4h_{SC}}{\pi^2 k_{SR}} (T(L_{SR}, 0) - T(0, 0))} \right], \quad G_1^+ = \left[\frac{\frac{h_{SR}}{k_{SR}} T_{sol-air}^s}{\frac{4}{L_{SR}} T(L_{SR}, 0) + \frac{4h_{SR}}{\pi^2 k_{SR}} (T(L_{SR}, 0) - T(0, 0))} \right]. \quad (\text{A6-12})$$

$T_{sol-air}^s$ in Eq. (A6-12) is a function of outdoor conditions (solar radiation, outdoor air temperature and wind speed, as shown in Eq. (6.2a)). $T_{sol-air}^s$ can be mathematically expressed by Eq. (A6-13),

$$T_{sol-air}^s = \widetilde{T}_{sol-air} + T_{sol-air}^+ \cos(wt - \psi) \quad (A6-13)$$

$$\text{where } T_{sol-air}^+ = \sqrt{\left(\frac{1}{12} \sum_{t=1}^{24} T_{sol-air}^e \cdot \cos(wt)\right)^2 + \left(\frac{1}{12} \sum_{t=1}^{24} T_{sol-air}^e \cdot \sin(wt)\right)^2}, \quad \tan(\psi) = \frac{\left(\frac{1}{12} \sum_{t=1}^{24} T_{sol-air}^e \cdot \sin(wt)\right)}{\left(\frac{1}{12} \sum_{t=1}^{24} T_{sol-air}^e \cdot \cos(wt)\right)} \quad (A6-13a)$$

Intermediate variables in Eq. (6.17)

$$\begin{aligned} \text{where } B_2^+ &= \sum_{m=0}^{\infty} \left[\frac{2}{m^2 \pi^2} (T(L_R, 0) - T(0, 0)) (\cos m\pi - 1) \right], \\ D_2^+ &= \sum_{m=0}^{\infty} \left[\frac{2}{m\pi} T(L_R, 0) (\cos m\pi - 1) \right] \\ E_2^+ &= \left[\frac{\frac{h_C}{k_R} T_i}{\frac{4}{L_R} T(L_R, 0) + \frac{4h_C}{\pi^2 k_R} (T(L_R, 0) - T(0, 0))} \right], \quad G_2^+ = \left[\frac{\frac{h_R}{k_R} T_{gap}^s}{\frac{4}{L_R} T(L_R, 0) + \frac{4h_R}{\pi^2 k_R} (T(L_R, 0) - T(0, 0))} \right]. \end{aligned} \quad (A6-14)$$

T_{gap}^s in Eq. (A6-14) can be obtained from Eq. (6.12). While, T_i in Eq. (A6-14) can be estimated using heat balance in indoor environment, as given by Eq. (A6-15)

$$L_i \rho_{air}^* C_{air} \left(\frac{dT_i}{dt} \right) = h_C [T_C - T_i] + \sum_{wall} h_i [T_{wall} - T_i] + \sum_{window} U_{window} [T_o - T_i] + h_i [T_{floor} - T_i] + \eta_i^+ [T_o - T_i] \quad (A6-15)$$

$$\text{where } \eta_i^+ = \frac{(ach)_i L_i \rho_{air}^* C_{air}}{3600 \times VOL_i} \quad (A6-15a)$$

Since the experimental apartments were unoccupied, the heat balance in Eq. (A6-15) does not include internal heat loads. By solving Eq. (A6-15) with steady state initial condition, it yields

$$\begin{aligned} T_i &= \frac{(h_C) \left[T_C + \sum_{wall} T_{wall} + T_{floor} \right] + \left(\sum_{window} U_{window} + \eta_i^+ \right) T_o}{(\mu_i^+)} - \left[h_C \left(\bar{T}_C + \sum_{wall} \bar{T}_{wall} + \bar{T}_{floor} \right) + \left(\sum_{window} U_{window} + \eta_i^+ \right) \bar{T}_o \right. \\ &\quad \left. - \bar{T}_i (\mu_i^+) \right] \frac{e^{\frac{-t}{\mu_i^+}}}{(\mu_i^+)}. \end{aligned} \quad (A6-16)$$

$$\text{where } \mu_i^+ = h_C + \sum_{wall} h_i + \sum_{window} U_{window} + \eta_i^+ \quad (A6-16a)$$

Appendix-7 Estimation of daily heat gain using Simpson's 3/8 rule

The daily heat gain (or area under the downward heat flux profile) is estimated using Simpson's 3/8 rule as shown by Eqs. (A7-1a) and (A7-1b),

$$Q_{gain} = \int_{daily} q dt, \quad \text{where } q \text{ is the downward heat flux through roof} \quad (A7-1a)$$

$$= \frac{3h}{8} \left\{ q(t_0) + 3 \sum_{i=1,4,7,\dots}^{n-2} q(t_i) + 3 \sum_{i=2,5,8,\dots}^{n-1} q(t_i) + 2 \sum_{i=3,6,9,\dots}^{n-3} q(t_i) + q(t_n) \right\} \quad (A7-1b)$$

where spacing interval, $h = \frac{t_n - t_o}{n}$, $q(t_o)$ is the first point (on the downward heat flux profile, shown in Fig. A-6) and $q(t_n)$ is the n^{th} point (on the downward heat flux profile).

Fig. A-6 Estimation of daily heat gain using Simpson's 3/8 rule.