Selective-actuation micro-positioning systems based on flexure parallel mechanisms

Pham, Huy Hoang

2005

Pham, H. H. (2005). Selective-actuation micro-positioning systems based on flexure parallel mechanisms. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/6272>

<https://doi.org/10.32657/10356/6272>

Nanyang Technological University

Downloaded on 26 Apr 2023 08:52:57 SGT

**SELECTIVE-ACTUATION MICRO-POSITIONING
SYSTEMS BASED ON FLEXURE PARALLEL
MECHANISMS**

PHAM HUY HOANG

**SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY**

2005

Selective-Actuation Micro-Positioning Systems Based On Flexure Parallel Mechanisms

Pham Huy Hoang

School of Mechanical and Aerospace Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirements for the degree
of Doctor of Philosophy

2005

Abstract

Possessing rigid architecture, no backlash, no error accumulation and compatibility with vacuum and clean environment, flexure parallel mechanisms (FPMs) are employed as precision positioning systems in optics, micro/nano scale metrology and manufacturing, semiconductor production and biology applications. In this work, the concept of selective actuation (SA) is proposed and elaborated for designing FPM to achieve decoupled end-effector motion of the mechanism by controlling separately the actuators, and hence, aids the control of decoupled motion axes to achieve precise positioning. This thesis focuses on the design and modeling methods of selective-actuation flexure parallel mechanisms that can be used as a micro-positioning system. The design of SA FPMs is realized through synthesis of macro-scale SA parallel mechanisms, conversion of the macro-scale SA parallel mechanisms into SA FPMs, and optimization of FPM parameters for resolution and stiffness. The premise of SA parallel mechanism synthesis is to obtain the Jacobian matrix of the mechanism in diagonal form based on screw theory. The optimization is carried out based on a number of performance indices including global resolution transmission scale, resolution uniformity and stiffness of the FPM. An improved pseudo rigid-body (PRB) model is proposed to give more precise estimation of FPM movement. In this improved version of PRB model, deformations of the flexure members are first computed using PRB method. Then these computed deformations are substituted back to the PRB model as second order effect for compensation. To illustrate the feasibility of the proposed design method, an SA FPM having three translational degrees of freedom is developed. Experimental results show that the prototype FPM can provide accurate decoupled linear motions and the resolution and stiffness performances of the FPM are very close to the designed values. As a generic FPM design method, this approach is also applied to the design of a 6-DOF dexterous SA FPM. Preliminary study of this mechanism has also carried out.

Acknowledgement

The author wishes to express his sincere gratitude for the research supervisor, Professor Chen I-Ming because of his continuous encouragement, support and guidance. His vision and broad knowledge play an important role in this research work. The author wishes to express thankfulness to Professor Yeo Song Huat for his assessment of the research works during three years. Grateful appreciation is also to Professor Yeh Hsien-Chi for his advice about the prototype development and the setup of experiments. The author would like to thank Professor David Butler for his highly valuable comments for the publication.

Acknowledgement also goes to Dr. Yang Guilin and Mr. Ho Hui Leong, Edwin from SIMTech for their encouragement, support and discussion during the early stage of the research. In addition, the author would like to thank the staffs of Robotic Research Center: Professor Gerald Seet, Mr. Lim Eng Cheng, Ms. Agnes Tan, Mr. You Kim San and Ms. Toh Yen Mei for providing him a very good research environment. The gratitude is extended to all his friends in Robotics Research Center, especially the members of Modular Robotic Research Group: Dr. Anjan Kumar Dash, Dr. Xing Shusong, Ms. Theingi, Mr. Jin Yan, Mr. Lim Chee Kian, Mr. Pham Cong Bang, Mr. Yan Liang, Mr. Mustafa Shabbir Kurbanhusen and Ms. Tang Xueyan for their helps.

The author would like to acknowledge Nanyang Technological University for scholarship during three years of Ph.D. study. This project is funded partially by Academic Research Fund Project RG 06/02 from Ministry of Education, Singapore and Innovation in Manufacturing Science and Technology Program under Singapore-MIT Alliance.

Finally, the author extends his deepest appreciation to his mother, wife and son for their love and encouragement.

Contents

Chapter 1: Introduction	1
1.1 Background	1
1.2 Aims of Research	3
1.3 Overview of Thesis	4
Chapter 2: Past Research	6
2.1 Flexure Mechanism Design	6
2.1.1 Flexure Joint and Flexure Chain Design	6
2.1.2 Flexure Parallel Mechanism Design	9
2.2 Modeling	15
2.3 Evaluation Criteria for Design	16
Chapter 3: Design Principle of Selective-Actuation Flexure Parallel Mechanism	18
3.1 Design Issues of SA FPM	19
3.2 Synthesis of Selective-Actuation Parallel Mechanism	21
3.2.1 Type Synthesis Using Screw Theory	22
3.2.2 Screw-Based Synthesis of Selective-Actuation Parallel Mechanism	25
3.2.3 Synthesis Examples	28
A. Selective-Actuation 3-DOF Translational Parallel Mechanism	28
B. Selective-Actuation 6-DOF Parallel Mechanism	31
3.3 Conceptual Design	36
3.3.1 Selective-Actuation 3-DOF Translational Flexure Parallel Mechanism	36
3.3.2 Selective-Actuation 6-DOF Flexure Parallel Mechanism	42

3.4 Summary	45
Chapter 4: Modeling of Flexure Parallel Mechanism	47
4.1 Pseudo Rigid-Body Modeling	48
4.2 Pseudo Rigid-Body Modeling with Consideration of Deformation	57
4.2.1 Fundamentals of PRB-D Model	58
4.2.2 Application to Selective-Actuation 3-DOF Translational FPM	63
4.3 Summary	72
Chapter 5: Evaluation of Positioning Resolution	74
5.1 Resolution Transmission Scale	75
5.2 Positioning Resolution Indices	78
5.2.1 Global Resolution Transmission Scale and Uniformity of Distribution	78
5.2.2 Computation of the Resolution Indices	79
5.2.3 Optimal Design Based on Global Resolution Indices	82
5.3 Case Studies	84
5.3.1 A 2-DOF Translational FPM	84
A. Kinematics and Workspace	85
B. Evaluation of Resolution	86
5.3.2 A Planar 3-DOF Flexure Parallel Mechanism	88
A. Kinematics	89
B. Workspace	92
C. Resolution Transmission Scale	92
5.2.3 Selective-Actuation 3-DOF Translational Flexure Parallel Mechanism	95
A. Workspace	95
B. Resolution Transmission Scale	96

5.4 Summary	98
Chapter 6: Stiffness Analysis	100
6.1 Stiffness Modeling and Stiffness Index	102
6.1.1 Stiffness Matrix of a Serial Connection	103
6.1.2 Stiffness Matrix of a Parallel Combination	105
6.1.3 Stiffness Index	107
6.2 Application of Stiffness Modeling	108
6.2.1 Stiffness Matrix of Double Linear Spring Stage	108
A. Stiffness Matrix of Linear Spring Stage	108
B. Stiffness of Double Linear Spring Stage	112
C. Finite Element Method and Experimental Validations	114
6.2.2 Stiffness Modeling of Flexure Parallel Mechanism	118
A. Expression of Compliance Matrices of Flexure Members in Base Frame	118
B. Compliance Matrix of Limb $A_{ij}B_{ij}C_{ij}$	121
C. Stiffness Matrix of 3-Flexure-Limb Groups	123
D. Compliance Matrix of FPM Limbs	125
E. Stiffness Matrix of Complete FPM	126
6.3 Actuation Stiffness	127
6.4 Summary	132
Chapter 7: Prototype Development and Experiments	134
7.1 Mechanical Design	135
7.1.1 Material Selection	135
7.1.2 Kinematic and Free Shape Design for Resolution Transmission Scale	137
7.1.3 Design for Dimension of Flexure Joints	139

7.1.4 Fabrication and Assembly	141
7.2 Control Implementation	143
7.3 Finite Element Modeling and Analysis	145
7.3.1 Finite Element Modeling	145
7.3.2 Modal Analysis	148
7.3.3 Dynamic Analysis	151
7.4 Experiments	154
7.4.1 PRB-D Model	154
7.4.2 Selective-Actuation	157
7.4.3 Resolution Determination	159
7.4.4 Repeatability	161
7.4.5 Stiffness Verification	163
7.4.6 Challenge of Hysteresis	168
7.5 Summary	171
Chapter 8: Conclusion and Future Study	172
8.1 Conclusion	172
8.2 Future Study	175
Appendix	177
Appendix A: Stiffness Matrix of Double Linear Spring Stage	177
Appendix B: Compliance Matrix of Flexure Hinge	179
Appendix C: Driving Forces of SA 3-DOF Translational FPM	181
Appendix D: Mechanical Drawings of Major Details	182
Bibliography	189

List of Figures

2.1	Flexure hinges.	7
2.2	Flexure prismatic joints.	7
2.3	Flexure universal joints.	8
2.4	Flexure spherical joints.	8
2.5	Multi-DOF flexure chains.	9
2.6	One-DOF micro-positioner [1].	9
2.7	Micro-gripper [36].	10
2.8	Planar translational FPM [38].	11
2.9	Complete design of XY θ stage mechanism [10, 11].	11
2.10	Compact θ_X - θ_Y -Z fine positioner [38].	12
2.11	Six-DOF fine motion Steward platform mechanism [44].	13
2.12	Low-cost six-axis nano-manipulation stage [20].	14
2.13	Six-axis hybrid nano-manipulator [45].	14
2.14	PRB modeling of a flexure hinge.	16
3.1	Systematical design of an SA FPM.	21
3.2	Definition of a screw.	22
3.3	Six-DOF parallel kinematic mechanism.	24
3.4	Limb i of a parallel mechanism.	26
3.5	Three-DOF translational parallel kinematic mechanism.	29
3.6	PRRR limb.	29
3.7	Selective-actuation 3-DOF translational parallel mechanism.	30

3.8	RPPPRR limb.	32
3.9	Selective-actuation 6-DOF dexterous parallel mechanism.	35
3.10	Flexure limb with three flexure hinges.	36
3.11	Three-flexure-limb group.	37
3.12	Substitution of three 3-flexure-limb groups by a passive 3-D frame.	38
3.13	Substitution of a prismatic joint by a double linear spring stage.	39
3.14	SA 3-DOF translational FPM.	40
3.15	Planar 3-DOF mechanism with R-R-R limbs.	41
3.16	RPPPRR flexure limb.	43
3.17	Two-D linear spring stage used as 2-D prismatic joint.	43
3.18	Assembly scheme of 6-DOF SA FPM.	44
3.19	Assembled SA 6-DOF FPM.	45
4.1	XY FPM and its PRB model.	49
4.2	Dimensions of flexure hinge and linear spring stage.	50
4.3	PRB model of SA 3-DOF translational FPM.	52
4.4	Three-flexure-limb group of the PRB model.	53
4.5	Dimensions of flexure hinge and double linear spring stage.	54
4.6	Vector close loop of limb j on 3-flexure-limb group i .	55
4.7	The establishment of PRB-D model of XY FPM.	60
4.8	Concept of PRB-D model.	62
4.9	Configuration of one limb $A_{ij}B_{ij}C_{ij}$.	64
4.10	Description of force/moment in different frames.	66
4.11	Static analysis corresponding to 3-flexure-limb group Z .	68

5.1	Task variables and actuated joint variables.	75
5.2	Principal axes of resolution ellipsoid.	77
5.3	Global Indices Sampling Algorithm (GISA).	81
5.4	Two-DOF translational FPM.	84
5.5	PRB model of 2-DOF FPM.	85
5.6	RT scale distribution of 2-DOF translational FPM.	87
5.7	Planar 3-DOF FPM.	88
5.8	PRB model of planar three-DOF FPM.	89
5.9	Closed-loop vector of one leg.	90
5.10	Free shapes of FPMs A and B.	94
5.11	Distribution of RT scale of FPM A and FPM B.	95
5.12	RT scale distribution with $Z = 0$.	97
6.1	Structure of flexure parallel mechanism.	102
6.2	Serial connection.	103
6.3	Parallel connection.	105
6.4	Linear spring stage.	109
6.5	Double linear spring stage used as a flexure prismatic joint.	113
6.6	FEM model of double linear spring stage.	115
6.7	Experiment on stiffness of double linear spring stage.	116
6.8	Relationship between displacement y and pushing force F_y .	117
6.9	Connection scheme of SA 3-DOF translational FPM.	118
6.10	Transformation from local frame to base frame.	119
6.11	Limb $A_{ij}B_{ij}C_{ij}$.	121

6.12	The i^{th} 3-flexure-limb group of the passive 3-D frame.	124
6.13	Stiffness distribution in X-Y plane ($Z = 0$).	127
7.1	Material index comparison of FPM materials.	137
7.2	Components of the SA 3-DOF translational FPM	141
7.3	Passive 3-D frame (a) and double linear spring stage (b).	142
7.4	Assembly of the FPM.	142
7.5	Assembled prototype of the FPM.	143
7.6	Feedback control schema.	144
7.7	Flow chart of control program.	145
7.8	FEM model of the SA 3-DOF translational FPM.	147
7.9	Selective-actuation error obtained by FEM modeling.	148
7.10	First mode shape.	149
7.11	Second mode shape.	150
7.12	Third mode shape.	150
7.13	End-effector position responses (X response).	152
7.14	End-effector position responses (X and Y responses).	153
7.15	End-effector position responses (X, Y and Z responses).	154
7.16	Experimental verification of the PRB-D model.	155
7.17	Comparison between PRB and PRB-D models (range 0-100 μm)	156
7.18	Comparison between PRB and PRB-D models.	156
7.19	Experimental verification of the selective actuation.	157
7.20	Slew motions dY and dZ versus the actuated motion in X-direction	158
7.21	Experimental process of resolution determination.	160

7.22	Responses of the end-effector position to step inputs 300, 20, 0.2 and 0.1 μm .	160
7.23	Maximum deviation in X-direction.	162
7.24	Standard deviation in X- direction.	163
7.25	Stiffness experiment of the FPM.	164
7.26	Stiffness K_{z-Fz} distribution with $Z = 0$.	166
7.27	Stiffness $K_{\alpha y-Fz}$ distribution with $Z = 0$.	167
7.28	Stiffness $K_{\alpha x-Mx}$ distribution with $Z = 0$.	167
7.29	Hysteresis evaluation.	169
7.30	Forward displacement and backward displacement ($Y = Z = 0$).	169
7.31	Driving force in forward and backward processes ($Y = Z = 0$).	170
B.1	Flexure hinge dimensions.	180

List of Tables

5.1	Resolution evaluation of a 2-DOF translational FPM using GISA.	88
5.2	Optimal design of the planar 3-DOF FPM.	93
5.3	Comparison between two free shapes of the planar 3-DOF FPM.	94
6.1	Stiffness of double linear spring stage.	117
7.1	Characteristics of materials for FPM.	135
7.2	Normalized properties of materials considered for FPM.	136
7.3	Results of optimization.	139
7.4	Dimension of flexure members.	140
7.5	Natural Frequencies.	149
7.6	Comparison of stiffness obtained in different manners.	165

List of Symbols and Abbreviations

$\boldsymbol{\varepsilon}$ or ε	Vector or amplitude of the deformation
τ	Resolution of the actuator
δ and S	Joint limit of a flexure hinge and a linear prismatic joint
a	A scalar quantity (a small character in italic face)
A	A point $\in \mathfrak{R}^3$
$\boldsymbol{A}$	A matrix (a capital character in bold and italic face)
$\boldsymbol{a}$	A vector (a small character in bold and italic face)
$\overrightarrow{AB}$	Vector pointing from point A to point B
$\boldsymbol{a} \cdot \boldsymbol{b}$ or $\boldsymbol{a} \cdot \boldsymbol{b}$	Dot product between vector $\boldsymbol{a}$ and vector $\boldsymbol{b}$
$\boldsymbol{a} \times \boldsymbol{b}$	Cross product between vector $\boldsymbol{a}$ and vector $\boldsymbol{b}$
$\boldsymbol{S}_1 \otimes \boldsymbol{S}_2$	Reciprocal product or orthogonal product between two screws $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$
$\boldsymbol{J}$, $\boldsymbol{J}_x$ and $\boldsymbol{J}_q$	Jacobian matrix, direct and inverse Jacobian matrices
$\boldsymbol{q}$	Vector of joint variables
$\boldsymbol{K}$ and $\boldsymbol{K}$	Stiffness and stiffness matrix
K_{AS}	Actuation stiffness
$\boldsymbol{C}$ and $\boldsymbol{C}$	Compliance and compliance matrix
$\boldsymbol{A}$ and $\boldsymbol{G}$	Amplification matrix of the displacements and rotation matrix
$d\boldsymbol{X}$	Vector of infinitesimal displacement
$d\boldsymbol{q}$	Differentiation of vector of joint variables
E and G	Young modulus and shear modulus of the material
R_{global}	Global resolution transmission scale ($R_{\text{global}} \in \mathfrak{R}$)
R_{max} and R_{min}	Maximal and minimal resolution transmission scale ($R_{\text{max}}, R_{\text{min}} \in \mathfrak{R}$)

U	Uniformity of resolution distribution ($U \in \Re$ and $1 \geq U > 0$)
RT scale	Resolution transmission scale
W	Workspace volume or area
FEM	Finite element method
FPM	Flexure parallel mechanism
SA	Selective actuation
DOF	Degree of freedom
GISA	Global Indices Sampling Algorithm
PRB model	Pseudo rigid-body model
PRB-D model	Pseudo rigid-body model with consideration of deformation
Fig.	Figure
Eq.	Equation

Chapter 1

Introduction

1.1 Background

In the past few years, there exist many needs of precision positioning devices in several fields such as (1) optics and metrology (stabilizing image, coupling optic fiber, positioning interferometer/mirror, measuring high-resolution topology profile, etc.), (2) precision mechanics and material engineering (adding new layer to smart structures, engraving in micro-scale, etc.), (3) semiconductors and microelectronics (aligning wafer/mask, processing micro-lithography, etc.), and (4) life science and biology (scanning microscopy, manipulating cell, etc.). The positioning systems are required to have resolution of sub-microns to some nanometers, high accuracy, high natural frequency and vacuum compatibility.

Flexure joints achieve the degree-of-freedom (DOF) from the elastic deformation at the slender/narrow position of the solid material structure, not from the sliding or rolling contacts as in the traditional joints. This kind of joint elements increases stiffness and

CHAPTER 1. INTRODUCTION.

natural frequencies of the jointing members, and rejects the backlash and the adverse effect of pressure angle existed in the traditional joints, and minimizes the delay and self-locking of the motion (or force) transmission. Flexure joints are compatible with vacuum working environment because they can work without lubrication. In addition, the working of flexure joints has no polluted discharge as the conventional joint emits metal powder caused by abrasion and wasted lubricant. Flexure joints also have drawbacks. The motion range of flexures is small in comparison with traditional joints with the same dimension because elastic deformation of flexure joints is limited by the strength of material. Flexure joints always incline to restore to the free shape (the initial shape). Thus, extra actuation load needs to overcome the elastic load inherited in the flexure joints when they deform. Therefore, mechanisms that use flexure joints to transmit motion/force are suitable for the environment of precision actions and can achieve very high resolution in a small workspace.

Some positioning systems are designed as serial stacks of single-degree-of-freedom stages. Owing to the serial arrangement, stacked stages have inadequacies such as lack of stiffness, large accumulated and amplified error [1-5]. To avoid such problems, parallel type mechanisms are proposed for many positioning systems. A parallel mechanism has a platform connected to the base by at least two kinematic chains. It possesses rigid architecture – an important factor for high repeatability and can limit the accumulation error. Moreover, in a parallel mechanism, actuators can be directly mounted on the base so as to reduce the vibration or dynamic error during the moving process. A flexure parallel mechanism (FPM) is a parallel mechanism using flexure joints instead of traditional joints to transmit motion/force.

Most of the recent positioning systems based on FPM provide coupled motions [6-18] or partially decoupled motions [19, 20]. Some of them can provide decoupled motion

CHAPTER 1. INTRODUCTION.

thanks to the control algorithm [18]. The motion of the end-effector along one DOF direction relies on the coordinated movement of all actuators through a control algorithm. In this work, the concept of selective actuation (SA) is proposed for FPM design. An FPM with SA characteristics is one that each actuator will effect only one-axis direction movement, i.e., the motion axes are decoupled. Furthermore, the actuators can be reconfigured on the system to obtain the desired axes of motion without affecting the work of other existing actuators. In this way, selective actuation rejects fully or eliminates partially the dependence of the end-effector motion on different axes of the actuators, and therefore, aids the decoupling of motion control. As the cost of precision actuators is relatively high compared to other subcomponents in a positioning system, it is possible to configure the SA FPM to have appropriate number of actuators to perform the task for cost effectiveness. Usually a precision positioning system is not reconfigurable, the selective actuation approach allows reconfiguration of the actuators in a precision stage mechanism. Hence, it will become an important design approach for miniaturized micro/nano manufacturing systems.

1.2 Aims of Research

Though researches on FPM have been seen in recent years, a number of issues are not satisfactorily addressed, namely:

- (1) Task-based design: Two questions are commonly asked: “How is the FPM design achieved?” and “Is there any other design matching the requirements?”.
- (2) Modeling of FPMs: Current FPM models are not accurate because of the ignorance of the deformation of flexure joints along directions not in the motion axis of flexure joints.

CHAPTER 1. INTRODUCTION.

- (3) Relationship between design parameters of FPM (dimensions and free shapes) and its resolution: At present, only post-design measures such as the control system and error compensation are implemented on the FPM system to obtain the desired resolution. There is no investigation on effect of the initial design on the resolution of the FPM.
- (4) Stiffness of FPM: The FPM stiffness is evaluated using the finite element method (FEM) or the reduced analytical method that do not allows the designer to observe the effect of dimensions and the stiffness of each flexure joint on the overall FPM stiffness.

The aim of this research is to develop a systematical design approach for selective-actuation micro-positioning systems based on flexure parallel mechanisms to address the above issues. Major tasks are:

- Synthesis of parallel mechanisms possessing selective-actuation using screw theory and conceptual designs of an SA 3-DOF translational FPM and an SA 6-DOF dexterous FPM.
- Formulation of FPM models considering deformation of flexure members in the FPM.
- Establishment of design criteria for FPM based on the structure of FPM, and development of computation procedure of those criteria for FPM optimal design.
- Prototype development for verification of SA principle with a 3-DOF translational FPM.

1.3 Overview of Thesis

The remaining chapters of this thesis are organized as follows. Chapter 2 reviews some past development in the areas of precision positioning systems, flexure mechanisms,

CHAPTER 1. INTRODUCTION.

modeling and design criteria. Chapter 3 addresses the conceptual design of SA FPM for kinematics point of view. Systematical design of a selective-actuation micro-positioning system based on flexure parallel mechanisms is proposed. Selective actuation kinematic mechanisms are synthesized by diagonalizing the Jacobian matrix established based on screw theory. In Chapter 4, static and kinematic analyses based on pseudo rigid-body model are combined together to obtain a new analytical modeling of FPM with consideration of deformation in flexure joints. Chapter 5 and Chapter 6 investigate the performance evaluation criteria of FPM including global resolution transmission scale, resolution uniformity, stiffness of the FPM and actuation stiffness used in the design of kinematic dimension, free shape and flexure member dimensions. The global resolution transmission scale and the uniformity of the resolution distribution are determined based on the transmission scale of infinitesimal motion from actuated joints to the end-effector that is computed based on the Jacobian matrix of the kinematic mechanism. The stiffness matrix of the FPM is obtained by joining the stiffness matrices of basic flexure members based on the way the flexure members are connected together in serial or parallel combination. The prototype development of an SA 3-DOF translational FPM is presented in Chapter 7 based on the study in previous chapters. This includes mechanical design of the FPM, FEM modeling and the control implementation. The performance of the prototype is also evaluated through experiments. The prototype FPM possesses a positioning resolution of $0.2\text{ }\mu\text{m}$. The current work is concluded in Chapter 8. A summary of the thesis contributions and possible future works is presented.

Chapter 2

Past Research

A review of past development and relevant literatures in area of FPM-based positioning systems is presented here. Several researches were done on the development of flexure joints, the design of FPM without selective-actuation characteristic, the FPM modeling and the performance evaluation criteria for the design of flexure mechanisms.

2.1 Flexure Mechanism Design

2.1.1 Flexure Joint and Flexure Chain Design

The circular flexure hinge (Fig. 2.1) was first studied in 1960's and applied to instruments in small sizes such as a pendulum pivot of miniaturized force-balance accelerometers [21]. To increase the compliance of the flexure hinges, many researchers work on development new flexure hinges with ellipse, parabolic, hyperbolic, elliptical and corner-filletted forms (Fig. 2.1) and their full stiffness/compliance modeling using analytical and FEM methods [22-28].

CHAPTER 2. PAST RESEARCH.

Fig. 2.1. Flexure hinges.

Linear spring stage – a parallelogram four-bar linkage using flexure hinges is used as a flexure prismatic joint (Fig. 2.2a) in the micro-motion stages [29, 30]. There exists a parasitic motion perpendicular to the working direction of the linear spring stage. To avoid that parasitic motion, two linear spring stages are combined to form a compound linear spring stage (Fig. 2.2b) [29].

Fig. 2.2. Flexure prismatic joints.

Two eccentric/concentric flexure hinges compound a flexure universal joint (Fig. 2.3) [21]. Also the spherical notch is employed as the flexure spherical joint (Fig. 2.4a) [21, 31]. Another spherical joint compounded by three concentric flexure hinges is proposed

CHAPTER 2. PAST RESEARCH.

in the early stage of our research to obtain the same compliance in three rotation axes. The joint shown in Fig. 2.4b is a serial chain of three flexure hinges whose rotation axes are mutually orthogonal and intersect at one point. These multi-rotation-axis joints are used for spatial FPMs.

Fig. 2.3. Flexure universal joints.

Fig. 2.4. Flexure spherical joints.

Flexure serial chains were designed as the multi-DOF flexure joint in order to minimize the assembly work and dimensions, for examples, the planar 3-DOF chain [32] and the 5-DOF chain [33] (Fig. 2.5) are used as the limbs of FPMs. Analytical models of developed flexure joints are verified on FEM.

CHAPTER 2. PAST RESEARCH.

Fig. 2.5. Multi-DOF flexure chains.

2.1.2 Flexure Parallel Mechanism Design

Several FPMs have been developed to fulfill the task requiring small motion and high accuracy. Some of them are described as follows.

The simplest single-DOF mechanism was developed to work as a micro-actuator [34], a one-DOF micro-positioner (Fig. 2.6) [1-4, 35] or a micro-gripper (Fig. 2.7) [36]. The motion of mechanisms is created by a piezo-actuator and amplified through a lever system. The mechanisms can achieve resolution of $0.05 \mu\text{m}$ and motion range of $200 \mu\text{m}$.

Fig. 2.6. One-DOF micro-positioner [1].

CHAPTER 2. PAST RESEARCH.

Fig. 2.7. Micro-gripper [36].

Most of multi-DOF FPMs utilized in the real world are planar monolithic FPMs with two or three DOFs [5-14, 37, 38]. Common structure of this kind of devices has slots and circular notches cut off from a plate by electro-discharge machine (EDM), laser or water-jet cutting machines. This creates a set of flexure joints and links or amplifier lever system that connects the platform to the base. The monolithic structure is only suitable for the planar FPM without complicated configuration.

Two typical monolithic FPMs are illustrated as follows. The planar translational FPM of Institute of Robotics Research – Ecole Polytechnique Federal de Lausanne (EPFL) with flexure hinges and flexure prismatic joints is used for positioning (Fig. 2.8) [38] and the 3-DOF planar stage with monolithic flexure structure is used as a planar ultra-precision positioner in deep ultraviolet lithography (Fig. 2.9) [10, 11]. The positioning stage possesses positioning resolution of $0.01\text{ }\mu\text{m}$ and rotational resolution of 0.057 arcsec within an area of $40\text{ }\mu\text{m} \times 40\text{ }\mu\text{m}$.

CHAPTER 2. PAST RESEARCH.

Fig. 2.8. Planar translational FPM [38].

Fig. 2.9. Complete design of XYθ stage mechanism [10, 11].

CHAPTER 2. PAST RESEARCH.

Several spatial FPMs were developed for positioning tasks [15-20, 39-45]. Due to the limit of fabrication, the spatial FPM is usually not designed as a monolithic structure but an assembly of a platform and limbs known as flexure kinematic chains. A flexure kinematics chain is a set of links that connected to each other by the notch hinges or thin plates. Several assembled notch hinges or thin plates form multi-DOF flexure revolute, universal or spherical joints in such a kinematic chain. Each limb in the spatial FPM is usually designed as monolithic structure in order to guarantee the accuracy of alignment between joints and links. Four examples of spatial FPMs are illustrated below.

The compact θ_X - θ_Y -Z fine positioner integrating suspension, actuator and sensors (Fig. 2.10) was designed by EPFL for micro-assembly application [38]. It has positioning range of 1 mm and resolution of 0.1 μm .

Fig. 2.10. Compact θ_X - θ_Y -Z fine positioner [38].

A 6-DOF fine-motion Stewart platform mechanism (Fig. 2.11) was developed based on a parallel-kinematics Stewart mechanism by Oiwa et al. [44]. The flexure prismatic joints of the struts are the double linear spring stages with embedded piezo elements

CHAPTER 2. PAST RESEARCH.

used for linear actuation. Culpepper et al. [20] developed a low-cost nano-manipulator driven by electromagnetic actuators (Fig. 2.12). The mechanism has a monolithic, compact and planar shape. Therefore, it is easily fabricated through planar manufacturing processes. The nano-manipulator was experimentally proved to possess resolution of 5 nm in a volume limited by $100\text{ nm} \times 100\text{ nm} \times 100\text{ nm}$ and motion error smaller than 0.2 % full scale in volume limited by $0.1\text{ mm} \times 0.1\text{ mm} \times 0.1\text{ mm}$. The nano-manipulator is claimed to be able to align fiber optic cables as well as micro-system components.

Fig. 2.11. Six-DOF fine motion Steward platform mechanism [44].

The 6-DOF nano-manipulation platform with large range, nanometer scale resolution and repeatability was built as the hybrid configuration of two 3-DOF FPMs: $X-Y-\theta_Z$ and $\theta_X-\theta_Y-Z$ (Fig. 2.13) [45]. The workspace of $0.14\text{ mm} \times 0.14\text{ mm} \times 8\text{ mm}$ and orientation ability of $2.4^\circ \times 2.4^\circ \times 7.6^\circ$ was achieved.

CHAPTER 2. PAST RESEARCH.

Fig. 2.12. Low-cost six-axis nano-manipulation stage [20].

Fig. 2.13. Six-axis hybrid nano-manipulator [45].

CHAPTER 2. PAST RESEARCH.

Overall speaking, except for the FPM developed by Culpepper et al. [20] having a monolithic structure that has no similar structure as the macro-scale spatial mechanism, other spatial FPMs have structure similar to their counterparts in the macro world. Although these designs have symmetric structures, the movement of the platform is generated through the coordination of various actuators. Each actuator has effect on motion axis other than its own movement.

2.2 Modeling

So far most of the FPMs are studied based on the pseudo rigid-body (PRB) model obtained when approximately replacing the FPM by a rigid-link mechanism with conventional joints model [1-3, 5, 7-12, 16-18, 35, 40, 41, 43]. The PRB model is only suitable for velocity and acceleration analyses, workspace, dynamic and stiffness modeling. When ignoring the deformation of flexure members in non-working directions, the PRB model cannot precisely express the motion of the FPM in the position kinematics. For instance, the rotation center of the PRB model (Fig. 2.14a) of the flexure hinge is not fixed but has a displacement due to the bending of the working flexure hinge. Thus, when building a model of FPM, the effect of flexure-member deformation in all directions on the motion of the end-effector must be considered.

In order to model flexure mechanism more precisely, Yi et al. has recently proposed the replacement of base-fixed flexure hinges in their planar 3-DOF FPM with three R-R-R limbs by a two-link chain that consists of a revolute joint with a torsion spring and a prismatic joint [14] (Fig. 2.14b). The actual direction of the displacement of the rotation center does not match the rotation angle. Therefore, the improved PRB model will be more accurate if two prismatic joints are used to express the displacement of the rotation center (Fig. 2.14c). However, this is only an improvement for modeling of active flexure

CHAPTER 2. PAST RESEARCH.

hinges. In addition, the elongation S , S_x and S_y and the bending α of the flexure hinge are actually tied together (S , S_x , S_y and α depend on reaction load at the flexure hinge). The replacement of a flexure hinge by a revolute joint and one or two prismatic joints is simply insufficient. Moreover, this replacement makes the modeling more complicated and unsuitable for spatial flexure mechanisms that have more joints and links than the planar flexure mechanism.

Fig. 2.14. PRB modeling of a flexure hinge.

2.3 Evaluation Criteria for Design

In the past, several design criteria on the FPM are proposed: the maximum range of motion [10, 11], the minimum of the cosine of pressure angles used as the

CHAPTER 2. PAST RESEARCH.

transmissibility of a 3-DOF positioning-orientation FPM [12], stiffness [14, 20], error budget [20]. To adapt the requirement of precision positioning, the FPM must be designed for high resolution and stiffness. However, no previous design criterion expresses the effect of the design on the resolution of the FPM. A method to evaluate the resolution of the FPM is necessary.

Stiffness is a very important factor of the FPM because it affects the precision. High stiffness allows the FPM to work with high speed and high repeatability, and to prevent the resonance of vibration from outside sources. In order to develop the evaluation criteria for the stiffness of the FPM, the stiffness matrix must be established in an analytical form. Previously analytical stiffness modeling of the FPM focused on the active stiffness in working DOF directions [2, 10, 11, 14, 15 and 46-48]. To establish the compliant matrix of the FPM, Castigliano's theory – the energy method was utilized to compute the deformation of flexure members and the displacement of the end-effector under unit external force [49]. The complexity of that method is not appropriate for the multi-DOF FPM with a complicate configuration. In addition, the method is used only for the partial stiffness in working directions (stiffness in DOF-motion direction) of the FPM. Kim et al. studied the stiffness of the flexure mechanism in all directions. However, the object of study is only a 1-DOF actuator based on the flexure mechanism [34]. For multi DOF mechanism, this is insufficient. Hence, it is necessary to construct the full stiffness matrix of the FPM as a function of design parameters.

Chapter 3

Design Principle of Selective-Actuation Flexure Parallel Mechanism

This chapter discusses the design principle of flexure parallel mechanisms with selective-actuation features. The complete design process consists of three major steps including (1) conceptual design; (2) design of dimension and free shape for resolution and stiffness; (3) verification using finite element analysis. This chapter focuses on the conceptual design stage of FPMs with SA features. The conceptual design consists of type synthesis of parallel mechanisms possessing the required number of DOF, geometric arrangement of the structure to obtain the selective actuation and conversion of the kinematic mechanism into the flexure mechanism. Type synthesis of parallel mechanisms has been well studied. Therefore, this chapter will only briefly introduce the fundamentals of type synthesis of parallel mechanisms using screw theory, and concentrate on the effort of the geometrical arrangement for selective actuation and the conversion to flexure mechanisms.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

3.1 Design Issues of SA FPM

The performance quality of a micro-motion manipulator is evaluated through resolution and repeatability. Resolution represents the smallest motion that the manipulator can provide. This smallest controllable motion depends on (1) the ability to transmit the smallest movement of the actuator to the end-effector, and (2) the feedback system that can recognize the motion error of the end-effector. Therefore, the design of micro-motion manipulator must consider the kinematic design for resolution transmission and establishment of the feedback control system. Repeatability represents the error between a number of successive attempts to obtain the same position of the end-effector. This error is caused by deformation under dynamic load, structure vibration due to lack of stiffness and other unknown reasons. Consequently, the design for stiffness of flexure mechanisms must be included in the systematical design. Degrees of freedom and available motions of the end-effector are required based on the applications. The structure of the kinematic mechanism needs to be arranged to possess the selective actuation characteristics. Selective actuation permits the user to decouple the control of DOFs and to adjust the end-effector to achieve a precise position easily. Therefore, in the synthesis of the manipulator, the consideration of required DOFs and the selective actuation characteristics are the important issues. Last but not least, flexure joints are used to replace conventional rolling/sliding contact joints to avoid backlash and friction. To avoid parasitic motions caused by flexure joints, to simplify the flexure structure as well as to adapt the manufacturability, it is necessary to pay attention to the design of flexure joints and flexure structure in the overall design. As the structure of the FPM is continuous or semi-continuous, the outcome of the design needs to be verified using a closed-to-reality model – the finite element model before fabrication.

The systematical design of selective-actuation micro-positioning systems based on the

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

flexure parallel mechanism can be summarized as follows.

- (1) Conceptual design of the flexure parallel mechanism including type synthesis, selective-actuation synthesis and the flexure structure design.
- (2) Computational design of dimensions and free shapes of the flexure parallel mechanism for resolution transmissibility, stiffness, workspace and actuation stiffness.
- (3) Pre-fabrication verification of the design using finite element method.

The first step uses screw theory to enumerate all available kinematic parallel mechanisms adapting the requirement of DOFs and desired motions. The kinematic mechanism is then selected. Based on the Jacobian matrix of the selected mechanism, the links of the mechanism are geometrically arranged to achieve the selective actuation. Subsequently, the mechanism is converted into the corresponding flexure parallel mechanism by replacing groups of links and joints with flexure members. The second step determines the kinematic parameters and free shape of the flexure parallel mechanism in order to obtain the best resolution transmissibility and desired workspace. The dimensions of the flexure structures are determined for the desired stiffness as well as the actuation stiffness. The last step verifies the conceptual design and the approximately analytical computation using the numerical approach. This step allows us to simulate the designed mechanism, visualize the performance error due to the approximation and verify the analytical modeling used in the system control. The complete design of SA FPM can be summarized in Fig. 3.1.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

Fig. 3.1. Systematical design of an SA FPM.

3.2 Synthesis of Selective-Actuation Parallel Mechanism

Screw theory can be used to define the constraint systems of the limbs and the parallel mechanisms. Based on the constraint systems, the geometric conditions of the limbs are specified to construct the classes of parallel mechanisms. Hence, screw theory can be used for synthesis of parallel mechanisms [53-63]. FPM has small motions that can be

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

considered as instantaneous motions. The design of flexure type parallel mechanisms can employ screw theory that also describes the instantaneous kinematics of the mechanism. Jacobian matrices of the FPM can be established based on screw theory carrying information of the geometrical configuration of the mechanism [64, 65]. The information can be used to synthesize the SA FPM that requires a special form of the manipulator Jacobians.

Fig. 3.2. Definition of a screw.

3.2.1 Type Synthesis Using Screw Theory

A screw is defined by a straight line called screw axis with a scalar associated pitch λ and represented by six screw coordinates:

$$S = \begin{bmatrix} s \\ r \times s + \lambda s \end{bmatrix} = (S_1, S_2, S_3, S_4, S_5, S_6)^T, \quad S_i \in \Re, i = 1, 2, \dots, 6.$$

where s is a 3×1 unit vector pointing in the direction of the screw axis, r is the 3×1 position vector of any point on the screw axis with respect to the reference frame (body

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

frame) and $(\mathbf{r} \times \mathbf{s})$ defines the moment of the screw axis about the origin of the reference frame (Fig. 3.2).

The unit screw for a revolute joint is a zero-pitch screw directing along the rotation axis of the joint:

$$\mathbf{S}_0 = \begin{bmatrix} \mathbf{s} \\ \mathbf{r} \times \mathbf{s} \end{bmatrix}$$

The unit screw for a prismatic joint is an infinite-pitch screw directing along the translation axis of the joint:

$$\mathbf{S}_\infty = \begin{bmatrix} \mathbf{0} \\ \mathbf{s} \end{bmatrix}$$

A screw representing the instantaneous motion of a rigid body or the force and torque system acting on a rigid body is called a twist or a wrench respectively. The first three components of a twist/wrench represent the angular velocity/force. The remaining components of a twist/wrench represent the linear velocity/moment about the origin of the reference frame.

The screw $\mathbf{S}_r = (S_{r1}, S_{r2}, S_{r3}, S_{r4}, S_{r5}, S_{r6})^T$, where $S_{ri} \in \mathfrak{R}$ is called reciprocal to screw $\mathbf{S} = (S_1, S_2, S_3, S_4, S_5, S_6)^T$, where $S_i \in \mathfrak{R}$ if the reciprocal product (or orthogonal product defined by Tsai et al. [55, 64, 65]) is zero, i.e., $\mathbf{S} \otimes \mathbf{S}_r = S_4 S_{r1} + S_5 S_{r2} + S_6 S_{r3} + S_1 S_{r4} + S_2 S_{r5} + S_3 S_{r6} = 0$, where $\otimes$ is the operator of the reciprocal product. When a rigid body is in contact with the other rigid body, the twist system of this body is determined by the intersection of twists of contact points. The twist system represents the instantaneous motion the body can have. The wrench system of this body is determined as the union of the wrenches caused in the contacts. The wrench system refers to the constraints imposed on the body through the contact. The twist and wrench systems must be reciprocal as the virtual power developed by the wrench along the twist is equal to zero.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

In the parallel mechanism, the subject of concern is the end-effector motion. The reciprocity of screw systems can be used to determine the number of constraints of the limbs of the parallel mechanism imposing on the end-effector. Therefore, the available motions of the end-effector and the number of DOF of a parallel manipulator can be determined when studying the structure of limbs.

Fig. 3.3. Six-DOF parallel kinematic mechanism.

Type synthesis of parallel mechanisms using screw theory has been well studied in recent years [53-63]. Fang and Tsai [53] employed screw and reciprocal screw theory to analyze the geometric conditions of the limbs and enumerate the manipulators belonging to a class of identical-limb 4-DOF and 5-DOF parallel manipulators. Huang and Li [54] studied the synthesis of lower-mobility symmetrical parallel manipulators. Translating in-parallel actuated mechanisms are synthesized using screw algebra by Frisoli et al. [55]. Kong and Gosselin had contributions on type synthesis using screw theory. Among of those synthesis works are the classes of 3-DOF translational parallel manipulators

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

actuated by linear actuators [58] and the class of 3-DOF translational or spherical parallel manipulators [59, 60]. Recently, by combining a spherical and a translational mechanism into a single mechanism, Jin et al. [61, 62] proposed a three-identical-limb 6-DOF parallel manipulator actuated by three 2-DOF actuators (Fig. 3.3).

3.2.2 Screw-Based Synthesis of Selective-Actuation Parallel Mechanism

The relationship between the end-effector motion and the motions of actuated joints can be expressed using the instantaneous kinematics:

$$d\mathbf{X} = \mathbf{J}(\mathbf{q})d\mathbf{q},$$

where $d\mathbf{X} = [d\theta_x \ d\theta_y \ d\theta_z \ dx \ dy \ dz]^T$ is the vector of the infinitesimal end-effector motion; $d\mathbf{q} = [dq_1 \ dq_2 \ dq_3 \ dq_4 \ dq_5 \ dq_6]^T$ is the vector of infinitesimal joints variables and $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix.

A mechanism is considered as a selective-actuation mechanism when the infinitesimal end-effector motion along every DOF is driven by only one specified actuator. The influence between the set of joint variables $\{dq_1, dq_2, \dots, dq_6\}$ on the set of the infinitesimal DOF motions of the end-effector $\{d\theta_x, d\theta_y, d\theta_z, dx, dy, dz\}$ is desired to be an one-to-one mapping, i.e., Jacobian matrix $\mathbf{J}(\mathbf{q})$ has diagonal form.

Definition 3.1: Selective-actuation mechanism.

A mechanism possessing a diagonal-form Jacobian matrix is a selective-actuation mechanism.

We consider a parallel mechanism including m limbs. Each limb, without loss of generality, is an open-loop chain connecting the platform to the fixed base by l one-DOF joints (revolute/prismatic joint) and the actuated joints in one limb appear in the first g joints (Fig. 3.4). The total number of DOF of the mechanism is thus $(m \cdot g)$. We denote S_p

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

and $\dot{\mathbf{q}}$ as the instantaneous twist of the platform and the rate of joint variables respectively:

$$\mathbf{S}_P = (\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z, \dot{x}, \dot{y}, \dot{z})^T,$$

$$\dot{\mathbf{q}} = (\dot{q}_{11}, \dot{q}_{21}, \dots, \dot{q}_{g1}, \dot{q}_{12}, \dot{q}_{22}, \dots, \dot{q}_{g2}, \dots, \dot{q}_{1m}, \dot{q}_{2m}, \dots, \dot{q}_{gm})^T,$$

where

$\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z, \dot{x}, \dot{y}$ and $\dot{z}$ are the rotation and displacement rates of the end-effector.

$\dot{q}_{ji}$ is the intensity of the unit screw $\mathbf{S}_{ji}$ associated with the actuated joint j of limb i .

Fig. 3.4. Limb i of a parallel mechanism.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

As shown in [68, 69], the relationship between the instantaneous twist (displacement and rotation rate of the end-effector) and the actuated joint variables can be expressed as follows.

$$\mathbf{J}_x \mathbf{S}_P = \mathbf{J}_q \dot{\mathbf{q}}, \quad (3.1)$$

where $\mathbf{J}_x$ and $\mathbf{J}_q$ are the direct and inverse Jacobian matrices respectively.

The direct Jacobian matrix has the form:

$$\mathbf{J}_x = (\mathbf{J}_{x1}, \mathbf{J}_{x2}, \dots, \mathbf{J}_{xm})^T, \quad (3.2)$$

where

$\mathbf{J}_{xi} = [\mathbf{S}_{r1i}^T, \mathbf{S}_{r2i}^T, \dots, \mathbf{S}_{rgi}^T]^T$ is the matrix assembled from the unit screws $\mathbf{S}_{rji}$, ($j = 1, 2, \dots, g$ and $i = 1, 2, \dots, m$) that are reciprocal to all passive-joint screws in the limb i .

The inverse Jacobian matrix has the form:

$$\mathbf{J}_q = \text{diag}(\mathbf{J}_{q1}, \mathbf{J}_{q2}, \dots, \mathbf{J}_{qm}), \quad (3.3)$$

where

$$\mathbf{J}_{qi} = \begin{bmatrix} \mathbf{S}_{r1i} \otimes \mathbf{S}_{1i} & \mathbf{S}_{r1i} \otimes \mathbf{S}_{2i} & \cdots & \mathbf{S}_{r1i} \otimes \mathbf{S}_{gi} \\ \mathbf{S}_{r2i} \otimes \mathbf{S}_{1i} & \mathbf{S}_{r2i} \otimes \mathbf{S}_{2i} & \cdots & \mathbf{S}_{r2i} \otimes \mathbf{S}_{gi} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{S}_{rgi} \otimes \mathbf{S}_{1i} & \mathbf{S}_{rgi} \otimes \mathbf{S}_{2i} & \cdots & \mathbf{S}_{rgi} \otimes \mathbf{S}_{gi} \end{bmatrix}.$$

Assume that the mechanism is working in a singularity-free workspace, the direct Jacobian $\mathbf{J}_x$ is invertible. Equation (3.1) expressing the relationship of actuated joint variables and the DOF of the end-effector can be rewritten as:

$$\mathbf{S}_P = \mathbf{J}_x^{-1} \mathbf{J}_q \dot{\mathbf{q}}. \quad (3.4)$$

Lemma 3.1: *The product $\mathbf{J}_x^{-1} \mathbf{J}_q$ of a selective-actuation parallel mechanism is in diagonal form.*

In case that only one actuated joint appears in each limb ($g = 1$), from Eq. (3.3), we can

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

see that the inverse Jacobian $\mathbf{J}_q$ is always in diagonal form. Therefore, the above condition can be simplified as follows.

Lemma 3.2: *For a parallel mechanism with only one actuated joint in each limb, the condition of selective actuation requires only that the direct Jacobian matrix $\mathbf{J}_x$ is in diagonal form.*

Lemmas 3.1 and 3.2 can be used as guidelines to arrange the geometrical structure of the parallel mechanism resulted from type synthesis to achieve the selective actuation condition.

3.2.3 Synthesis Examples

Here we present synthesis design of two selective-actuation parallel mechanisms based on the 3-DOF translational parallel mechanism and the 6-DOF dexterous parallel mechanism obtained from type synthesis presented in [59] and in [61, 62] respectively.

A. Selective-Actuation 3-DOF Translational Parallel Mechanism

Kong and Gosselin [59] proposed a 3-DOF translational parallel mechanism using three identical limbs (Fig. 3.5). Each limb is an open-loop chain with one actuated prismatic joint and three passive revolute joints. Following the type synthesis, the axes of the prismatic joint and the revolute joints in a single limb must be parallel to each other [59]. Consider limb i of the mechanism (Fig. 3.6) and define an instantaneous reference frame (uvw), with its origin locates at P and u-, v-, w-axes parallel to the x-, y-, z- axes of the base frame, (xyz), located at origin O. The joint screws with respect to this instantaneous reference frame are determined as follows.

$$\mathbf{S}_{1i} = \begin{pmatrix} \mathbf{0} \\ s_{1i} \end{pmatrix}, \mathbf{S}_{2i} = \begin{pmatrix} s_{2i} \\ \vec{OB_i} \times s_{2i} \end{pmatrix}, \mathbf{S}_{3i} = \begin{pmatrix} s_{3i} \\ \vec{OC_i} \times s_{3i} \end{pmatrix}, \mathbf{S}_{4i} = \begin{pmatrix} s_{4i} \\ \vec{OD_i} \times s_{4i} \end{pmatrix}, \quad (3.5)$$

where $\mathbf{0}$ is a 3×1 zero vector and s_{ji} is the unit vector along the axis of prismatic joint j

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

when $j = 1$ or along the axis of revolute joint j when $j = 2, 3, 4$ of limb i .

Fig. 3.5. Three-DOF translational parallel kinematic mechanism.

Fig. 3.6. PRRR limb.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

Since $s_{1i} // s_{2i} // s_{3i} // s_{4i}$, the screw that is reciprocal to all screws except for S_{1i} can be defined as

$$S_{ri} = \begin{pmatrix} s_{1i} \\ \rightarrow \\ OP \times s_{1i} \end{pmatrix}, i = 2, 3, 4. \quad (3.6)$$

Substituting the reciprocal screws of three limbs defined in Eq. (3.6) into Eq. (3.2), we obtain the direct Jacobian matrix as

$$J_x = \begin{bmatrix} s_{11}^T \\ s_{12}^T \\ s_{13}^T \end{bmatrix}. \quad (3.7)$$

Fig. 3.7. Selective-actuation 3-DOF translational parallel mechanism.

Note that there is only one actuated joint in each limb. The condition of selective actuation of the mechanism can be simply stated as

Direct Jacobian J_x is a diagonal matrix

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

$$\begin{aligned} s_{11} &= (s_{11} \quad 0 \quad 0)^T \\ \Leftrightarrow s_{12} &= (0 \quad s_{12} \quad 0)^T \\ s_{13} &= (0 \quad 0 \quad s_{13})^T \end{aligned}$$

$\Leftrightarrow$ Three sliding paths are orthogonal to each other.

The mechanism is geometrically arranged as Fig. 3.7 with three mutually perpendicular axes of prismatic joints in order to obtain a selective-actuation 3-DOF translational parallel mechanism.

B. Selective-Actuation 6-DOF Parallel Mechanism

Jin et al. [61, 62] studied a synthesis of 6-DOF parallel mechanism with three identical limbs (Fig. 3.3). Each limb is an open-loop chain with two actuated joints: one prismatic joint and one revolute joint. The mechanism possesses some structure characteristics: all axes of the passive revolute joints intersect at one point in space, the axis of the actuated prismatic joint is perpendicular to the axes of corresponding passive prismatic joints, and the axes of three actuated revolute joints intersect at one point. This mechanism decouples the 3-DOF rotation and translation but not completely separate all the 6 DOFs.

We define an instantaneous coordinate frame, (uvw), with its origin locates at the intersecting point P of the axes of six passive revolute joints and u-, v-, w-axes parallel to the x-, y-, z- axes of the base frame, (xyz), located at origin O that is the intersecting point of the axes of three actuated revolute joints (Fig. 3.8). The joint screws of limb i with respect to this instantaneous reference frame are determined as follows.

$$\begin{aligned} S_{1i} &= \begin{pmatrix} s_{1i} \\ \vec{OA_i \times s_{1i}} \end{pmatrix}, S_{2i} = \begin{pmatrix} \mathbf{0} \\ s_{2i} \end{pmatrix}, S_{3i} = \begin{pmatrix} \mathbf{0} \\ s_{3i} \end{pmatrix}, S_{4i} = \begin{pmatrix} \mathbf{0} \\ s_{4i} \end{pmatrix}, \\ S_{5i} &= \begin{pmatrix} s_{5i} \\ \vec{OD_i \times s_{5i}} \end{pmatrix}, S_{6i} = \begin{pmatrix} s_{6i} \\ \vec{OE_i \times s_{6i}} \end{pmatrix}, \end{aligned} \quad (3.8)$$

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

where $\mathbf{0}$ is a 3×1 zero vector and s_{ji} is the unit vector along the axis of prismatic joint j when $j = 2, 3, 4$ or along the axis of revolute joint j when $j = 1, 5, 6$ of limb i .

Since the unit vector s_{1i} is perpendicular to the unit vectors s_{3i} and the unit vectors s_{4i} , and s_{5i} and s_{6i} intersects at point P, two screws S_{r1i} and S_{r2i} that are reciprocal to all screws except for S_{1i} and S_{2i} respectively are defined as

$$S_{r1i} = \begin{pmatrix} \mathbf{0} \\ s_{5i} \times s_{6i} \end{pmatrix}, S_{r2i} = \begin{pmatrix} s_{1i} \\ \vec{OP} \times s_{1i} \end{pmatrix}. \quad (3.9)$$

Fig. 3.8. RPPRR limb.

The reciprocity can be verified by taking the reciprocal product as below:

$$S_{r1i} \otimes S_{2i} = \mathbf{0} \cdot s_{2i} + (s_{5i} \times s_{6i}) \cdot \mathbf{0} = 0;$$

$$S_{r1i} \otimes S_{3i} = \mathbf{0} \cdot s_{3i} + (s_{5i} \times s_{6i}) \cdot \mathbf{0} = 0;$$

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

$$\mathbf{S}_{r1i} \otimes \mathbf{S}_{4i} = \mathbf{0} \cdot s_{4i} + (s_{5i} \times s_{6i}) \cdot \mathbf{0} = 0 ;$$

$$\mathbf{S}_{r1i} \otimes \mathbf{S}_{5i} = \mathbf{0} \cdot (\vec{\text{OD}}_i \times s_{5i}) + (s_{5i} \times s_{6i}) \cdot s_{5i} = 0 ;$$

$$\mathbf{S}_{r1i} \otimes \mathbf{S}_{6i} = \mathbf{0} \cdot (\vec{\text{OE}}_i \times s_{6i}) + (s_{5i} \times s_{6i}) \cdot s_{6i} = 0 ;$$

$$\mathbf{S}_{r2i} \otimes \mathbf{S}_{1i} = s_{1i} \cdot (\vec{\text{OA}}_i \times s_{1i}) + (\vec{\text{OP}} \times s_{1i}) \cdot s_{1i} = 0 ;$$

$$\mathbf{S}_{r2i} \otimes \mathbf{S}_{3i} = s_{1i} \cdot s_{3i} + (\vec{\text{OP}} \times s_{1i}) \cdot \mathbf{0} = 0 \quad (s_{1i} \perp s_{3i});$$

$$\mathbf{S}_{r2i} \otimes \mathbf{S}_{4i} = s_{1i} \cdot s_{4i} + (\vec{\text{OP}} \times s_{1i}) \cdot \mathbf{0} = 0 \quad (s_{1i} \perp s_{4i});$$

$$\begin{aligned} \mathbf{S}_{r2i} \otimes \mathbf{S}_{5i} &= s_{1i} \cdot (\vec{\text{OD}}_i \times s_{5i}) + (\vec{\text{OP}} \times s_{1i}) \cdot s_{5i} \\ &= s_{1i} \cdot (\vec{\text{PD}}_i \times s_{5i}) + s_{1i} \cdot (\vec{\text{OP}} \times s_{5i}) + (\vec{\text{OP}} \times s_{1i}) \cdot s_{5i}. \end{aligned}$$

Note that $\vec{\text{PD}}_i // s_{5i}$ and $s_{1i} \cdot (\vec{\text{OP}} \times s_{5i}) + (\vec{\text{OP}} \times s_{1i}) \cdot s_{5i} = 0$, hence $\mathbf{S}_{r2i} \otimes \mathbf{S}_{5i} = 0$, and similarly, $\mathbf{S}_{r2i} \otimes \mathbf{S}_{6i} = 0$.

Substituting the screws $\mathbf{S}_{1i}, \mathbf{S}_{2i}, \mathbf{S}_{3i}, \mathbf{S}_{4i}, \mathbf{S}_{5i}, \mathbf{S}_{6i}$ and reciprocal screws $\mathbf{S}_{r1i}, \mathbf{S}_{r2i}$ into Eq.

(3.2) and Eq. (3.3), we have:

$$\text{Inverse Jacobian matrix } \mathbf{J}_q = \text{diag}[s_{11}(s_{51} \times s_{61}), s_{12}(s_{52} \times s_{62}), s_{13}(s_{53} \times s_{63}), 1, 1, 1], \quad (3.10)$$

$$\text{Direct Jacobian matrix } \mathbf{J}_x = \begin{bmatrix} (s_{51} \times s_{61})^T & \mathbf{0}^T \\ (s_{52} \times s_{62})^T & \mathbf{0}^T \\ (s_{53} \times s_{63})^T & \mathbf{0}^T \\ (\vec{\text{OP}} \times s_{11})^T & s_{11}^T \\ (\vec{\text{OP}} \times s_{12})^T & s_{12}^T \\ (\vec{\text{OP}} \times s_{13})^T & s_{13}^T \end{bmatrix}. \quad (3.11)$$

Equation (3.10) shows that the inverse Jacobian matrix is in diagonal form. Therefore,

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

the selective-actuation condition can be simplified as the direct Jacobian matrix is in diagonal form, i.e.,

$$\begin{bmatrix} (s_{51} \times s_{61})^T \\ (s_{52} \times s_{62})^T \\ (s_{53} \times s_{63})^T \end{bmatrix} = \begin{bmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{bmatrix}. \quad (3.12)$$

$$\begin{bmatrix} s_{11}^T \\ s_{12}^T \\ s_{13}^T \end{bmatrix} = \begin{bmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{bmatrix}. \quad (3.13)$$

$$\begin{bmatrix} (\vec{OP} \times s_{11})^T \\ (\vec{OP} \times s_{12})^T \\ (\vec{OP} \times s_{13})^T \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (3.14)$$

Note that s_{ji} is a unit vector. Hence, Eqs. (3.12) and (3.13) can be respectively changed to:

$$(s_{51} \times s_{61}) = (1 \ 0 \ 0)^T; (s_{52} \times s_{62}) = (0 \ 1 \ 0)^T; (s_{53} \times s_{63}) = (0 \ 0 \ 1)^T. \quad (3.15)$$

$$s_{11} = (1 \ 0 \ 0)^T; s_{12} = (0 \ 1 \ 0)^T; s_{13} = (0 \ 0 \ 1)^T. \quad (3.16)$$

Equation (3.14) is satisfied when $\vec{OP} \perp s_{1i}$ ($i=1,2,3$) or $\vec{OP} = 0$. Equation (3.16) shows that unit vectors s_{11} , s_{12} and s_{13} are mutually perpendicular, hence the first case is impossible. Equation (3.14) is satisfied if and only if

$$\vec{OP} = 0. \quad (3.17)$$

The conditions of Eq. (3.15), Eq. (3.16) and Eq. (3.17) can be explained based on the geometrical arrangement of the mechanism respectively as follows.

(1) The axes of passive revolute joints of each limb are perpendicular together and

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

create a plane. That plane is perpendicular to other planes created in similarly manner by axes of passive revolute joints of the remaining limbs.

- (2) The axes of actuated prismatic joints (or actuated revolute joints) of the three limbs are perpendicular to one in other.
- (3) Points O and P coincide with each other. This condition guarantees decoupling of the translational motions.

Together with the conditions proposed by Jin et al. [61, 62], the above three conditions allow the 6-DOF dexterous parallel mechanism to provide six fully decoupled motion axes. The configuration of the SA 6-DOF parallel mechanism is illustrated in Fig. 3.9.

Fig. 3.9. Selective-actuation 6-DOF dexterous parallel mechanism.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

3.3 Conceptual Design**3.3.1 Selective-Actuation 3-DOF Translational Flexure Parallel Mechanism**

The kinematic diagram of the selective-actuation kinematic mechanism shown in Fig. 3.7 can be now converted into a flexure mechanism with flexure joints. The RRR limb of the mechanism can be replaced by a flexure limb with three flexure hinges as shown in Fig. 3.10.

Fig. 3.10. Flexure limb with three flexure hinges.

As the serial structure of this type of flexure limb does not have good stiffness, this design is equivalently replaced by a parallel 3-flexure-limb design as shown in Fig. 3.11. We call this design a 3-flexure-limb group that gives the same kind of planar 3-DOF motions required by the single R-R-R limb. The 3-flexure-limb group looks like a 3-DOF planar parallel mechanism with all revolute joints that are all passive compliant joints. Because the axes of the passive revolute joints of the three RRR limbs must be perpendicular to each other to form the parallel mechanism, planes of 3-flexure-limb

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

groups are perpendicular to each other as well in the construction of this FPM. To minimize assembly error, the three 3-flexure-limb groups are designed as a cubical monolithic structure called a passive 3-D frame (Fig. 3.12). The passive 3-D frame satisfies the manufacturing condition of machine tools such as EDM, water jet and laser cutting machines that can create the small hinges used in the flexure mechanism.

Fig. 3.11. Three-flexure-limb group.

The mechanism is supported by three actuated prismatic joints. The actuated prismatic joints must possess high rigidity in non-actuated directions. Therefore, the double linear spring is used as the actuated flexure prismatic joint in order to minimize deformations including bending, torsion and displacement along directions different from the

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

actuation direction. The double linear spring is rigidly mounted to the center of each of the 3-flexure-limb group of the passive 3-D frame as part of the complete limb of the FPM (Fig. 3.13). The fixed part of the double linear spring is fixed to the base.

Fig. 3.12. Substitution of three 3-flexure-limb groups by a passive 3-D frame.

The monolithic passive 3-D frame also serves as the platform or the end-effector of the

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

mechanism. Additional tool holding fixture will be integrated into the passive 3-D frame at later stage. The complete SA 3-DOF translational FPM is illustrated in the CAD drawing of Fig. 3.14.

Fig. 3.13. Substitution of a prismatic joint by a double linear spring stage.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

Fig. 3.14. SA 3-DOF translational FPM.

The configuration of a SA 3-DOF FPM also can be built when observing a 3-DOF $XY\theta_Z$ mechanism with three revolute joint (R-R-R) limbs (Fig. 3.15). If we let the outside part free and fix the center part to a linear actuator working along Z-direction (i.e. the center part can be move along Z-direction), the outside part now obtains four DOF (X, Y, Z and θ_Z). Three planar 3-DOF mechanisms are fixed together in one frame at the outside parts so that three mechanism planes are perpendicular to each other. If we let each center part of the frame move along the corresponding direction perpendicular to the side containing the center part, each outside part individually can perform four DOF motions. The DOF motions of X-side, Y-side and Z-side correspondingly are (Y, Z, θ_X , X), (Z, X, θ_Y , Y) and (X, Y, θ_Z , Z). The fourth DOF of each side is decoupled from the other DOF. Actually, the outside part of the frame possesses only three translation DOF that is the intersection of three above DOF set: $(Y, Z, \theta_X, X) \cap (Z, X, \theta_Y, Y) \cap (X, Y, \theta_Z, Z)$.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

$\theta_z, Z) = (X, Y, Z)$. The joints in the sides of the frame are passive, hence, the frame is called passive 3-D frame. Also the three DOF are decoupled. Those decoupled DOF guarantee the selective actuation of the mechanism.

Fig. 3.15 Planar 3-DOF mechanism with R-R-R limbs.

To obtain the SA 3-DOF translational FPM, the passive 3-D frame of the SA mechanism obtained above is built as a monolithic structure. The passive 3-D flexure frame has three mutually perpendicular sides. Each side is a 3-flexure-limb group that consists of a center part connected to the edge of the passive 3-D frame by three symmetrically located limbs. The V-shape limb with three hinges to create an R-R-R kinematics chain is called flexure limb. . The side of the passive 3-D frame possessing three flexure limbs is called 3-flexure limb group. The center part of each 3-flexure limb group of the passive 3-D frame is fixed to a flexure prismatic joint compounded by a double flexure linear slide. The dimensions of the double linear spring are determined to guarantee the very high torsion and bending stiffness and to allow the linear motion. Each flexure prismatic joint is actuated by a linear actuator. Three linear actuators provide the platform three decoupled translation motions following X-, Y- and Z- axes. The number

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

of DOF can be ranged from one to three by the number of linear actuators mounted on the mechanism. The one-piece structure of the passive 3-D frame avoids the error of perpendicular caused by the assembling work. Practically, there exist very small deformations in non-working axes of the flexure element. Those deformations can cause error of straightness. The symmetry of the 3-flexure-limb group of the passive 3-D frame is used to avoid error of straightness. The dimensions of the flexure elements are also determined so that the stiffness in non-working axis is maximized. Those design points guarantee the very small and acceptable error of straightness in the expected working range.

3.3.2 Selective-Actuation 6-DOF Flexure Parallel Mechanism

The conceptual design of a SA 6-DOF FPM is proposed based on the kinematic mechanism presented in Fig. 3.9. The RPPPRR limb of the mechanism is realized as a flexure limb including three flexure hinges, one linear spring stage and one 2-D flexure prismatic joint (Fig. 3.16). Two passive prismatic joints in Fig. 3.9 are combined into one 2-D flexure prismatic joint. To avoid error of the end-effector motion due to deformation along non-working directions of these passive prismatic joints, we use the 2-D linear spring stage built from four columns with two pairs of orthogonal corner-filletted hinges as a 2-D flexure prismatic joint (Fig. 3.17). One flexure hinge and one flexure spring stage is serially connected and actuated by two linear actuators to perform two actuated joints. The complete limb is designed as a monolithic structure and satisfies the manufacturability required by the cutting tools such as EDM or water jet machine. The monolithic structure of the limb avoids assembly error and allows three revolute joints to intersect and be orthogonal at a center (called center of three flexure hinges) as stated by the selective-actuation condition.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

Fig. 3.16. RPPRR flexure limb.

Fig. 3.17. Two-D linear spring stage used as 2-D prismatic joint.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

Three limbs are connected to a hollow-cube platform. The center of the cube is the end-effector. The position of connection interfaces is strictly arranged using positioning pins so that the flexure-hinge centers (Fig. 3.16) of three limbs coincide at the end-effector. The assembly scheme of the entire mechanism is shown in Fig. 3.18. Figure 3.19 illustrates the fully assembled selective-actuation 6-DOF flexure parallel mechanism.

Fig. 3.18. Assembly scheme of SA 6-DOF FPM.

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

Fig. 3.19. Assembled SA 6-DOF FPM.

3.4 Summary

A systematical design of selective-actuation flexure parallel mechanisms used as micro-motion positioning systems is proposed in this chapter. The design is divided into three steps including the conceptual design, the dimension and free shape design for resolution transmission and stiffness, and the verification using FEA software. This chapter presents the conceptual design stage of selective-actuation flexure parallel mechanism through two cases: a 3-DOF translational FPM and a 6-DOF dexterous FPM with three limbs. The conceptual design starts with type synthesis of the simplest parallel mechanisms matching the requirement of DOF. The condition of selective actuation is stated as the diagonal matrix form of Jacobian matrix of the mechanism. Based on the

CHAPTER 3. DESIGN PRINCIPLE OF SA FPM .

Jacobian matrix formulation using screw theory, the links and joints of the mechanism selected from type synthesis are geometrically arranged to satisfy the selective actuation condition. The links and joints of the selective-actuation parallel mechanisms are then realized as the flexure groups in order to obtain the desired selective-actuation flexure parallel mechanisms.

Chapter 4

Modeling of Flexure Parallel Mechanism

The flexure mechanism is usually described as a monolithic structure or a connected structure of multiple monolithic substructures. The flexure mechanism performs motion based on the compliance of flexure members. When studying flexure parallel mechanisms, the FEM is chosen because it can express accurately the motion of the flexure as well as rigid objects. However, the FEM modeling of flexure mechanisms is very complicated due to the structural geometry and its solution is computationally intensive. Therefore, it is not suitable to use this method for the control purpose. Furthermore, the FEM modeling does not permit us to observe the changes of stiffness and resolution following the dimension and free shape of the mechanism. This chapter presents two analytical modeling methods for the flexure mechanisms: the pseudo rigid-body model (PRB model) and the pseudo rigid-body model with consideration of deformation (PRB-D model). These two methods can be applied to analysis, design and position control of the flexure mechanisms. The PRB model is popularly used as the analytical model for the investigation of the flexure mechanism. In this study, we first

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

apply PRB model for the design and analysis of the FPM. Then we propose an improved modeling method – PRB-D for the consideration of the deformation of the flexure members. This method uses the flexure deformation to compensate the error due to the flexure characteristic. The PRB-D model is appropriate for the control purpose.

4.1 Pseudo Rigid-Body Modeling

The pseudo rigid-body model concept was proposed by Howell and Midha [66] when finding the way to reuse theory of rigid-body mechanics to investigate the flexure mechanism. In the PRB model, the device is assumed to be a mechanism with rigid links possessing the same number of degrees of freedom as the original flexure mechanism. In addition, this rigid-link mechanism can create the same force/moment or achieve the same motions as the original mechanism. In other word, the PRB model is an equivalent rigid-link mechanism of the flexure mechanism to replace the original flexure mechanism in the analysis and design. In PRB model, the flexure characteristic of flexure hinge that has circular/conic/corner filleted geometry is represented by the torsional spring with stiffness cited in [21, 29- 31, 49]. For flexure hinge of other geometries, the stiffness of torsional spring can be determined using elliptical integral. The establishment of a PRB model of an FPM is demonstrated in two examples.

Example 1: Pseudo Rigid-Body Model of the XY FPM

Several slots and circular notches are cut off from a plate to form a close-loop serial connection of four linear springs that can be used as an XY FPM (Fig. 4.1).

(1) Recognize flexure joints:

The flexure members of the XY FPM are grouped to form four linear spring stages 1X, 2X, 1Y and 2Y used as prismatic joints where 1 and 2 denote the 1st and 2nd limbs and X and Y denote X- and Y- axes.

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

The joint characteristics are expressed by the stiffness along working axes (K_x and K_y) and the joint limits (S_x and S_y). Here, the flexure hinges are in circular geometry, hence, following [29, 30] the stiffness can be approximated:

$$K_x = K_y = \frac{8Eet^{5/2}}{9\pi L^2 r^{1/2}}, \quad (4.1)$$

where L is the length of each limb of linear spring stages and e , t and r are the thickness of plate, the inter-hole space and the common radius of notches respectively (Fig. 4.2) and E is the Young's modulus of the material of the FPM.

Fig. 4.1. XY FPM and its PRB model.

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

Fig. 4.2. Dimensions of flexure hinge and linear spring stage.

The joint limit of the linear spring stage is determined based on the maximum rotation angle δ of a flexure hinge. It is the deflection angle when the maximum stress reaches the yield stress S_y . The maximum stress σ_{max} occurs at the outermost location of the hinge section with minimum thickness t and can be expressed as:

$$\sigma_{max} = \frac{M \frac{t}{2}}{I}, \quad (4.2)$$

where $I = \frac{et^3}{12}$ is the moment of inertia of the smallest section of the hinge and $M = K_\alpha \delta$ is the moment making the flexure hinge of torsional stiffness K_α rotate an angle δ .

Substituting I and M into Eq. (4.2), the maximum stress can be determined as:

$$\sigma_{max} = \frac{6K_\alpha \delta}{et^2}. \quad (4.3)$$

When the deflection angle of the flexure hinge reaches the maximum value, the maximal stress equals to the yield stress, i.e.,

$$\sigma_{max} = S_y. \quad (4.4)$$

Substituting Eq. (4.3) into Eq. (4.4), we have:

$$\delta = \frac{et^2}{6K_\alpha} S_y. \quad (4.5)$$

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

The range of the flexure hinge can be computed by substituting K_α cited from [21] into Eq. (4.5):

$$\delta = \frac{3\pi}{4} \frac{S_y}{E} \sqrt{\frac{r}{t}}. \quad (4.6)$$

Therefore, the joint limit of the linear spring stage is:

$$S_x = S_y = L\delta = \frac{3L\pi}{4} \frac{S_y}{E} \sqrt{\frac{r}{t}}. \quad (4.7)$$

(2) Substitute flexure joints with traditional joints:

The flexure joints 1X, 2X, 1Y and 2Y are substituted with traditional prismatic joints that can perform the motions along X- and Y- axes respectively. The rigid links are connected together not only by joints but also by some springs expressing the compliance of the flexure members.

(3) Assign the dimension, joint limit and the stiffness of spring:

The axes of two prismatic joints connected to the base are the actuation lines and converge at the end-effector. The linear stiffness of the springs is assigned the values computed in Eq. (4.1). The range of prismatic joints is given by Eq. (4.7). The obtained 2-DOF mechanism is the PRB model of the XY FPM and can be used in the analysis and design instead of the original flexure mechanism.

Example 2: PRB model of the selective-actuation 3-DOF translational FPM

In this example, the PRB model of the SA 3-DOF translational FPM proposed in Chapter 3 is presented for further investigation. The PRB model of the FPM is obtained by replacing the flexure hinges of the passive 3-D frame with the passive revolute joints A_{ij} , B_{ij} and C_{ij} , where $i = 1, 2$ or 3 denotes respectively the 3-flexure-limb groups Z, X or Y of the passive 3-D frame and $j = 1, 2$ or 3 denotes the flexure limb in each group

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

(Figs. 4.3 and 4.4). The double linear spring stages are replaced with the actuated prismatic joints. Each 3-flexure-limb group is treated as a 3-DOF planar parallel mechanism with three 3-revolute-joint (R-R-R) limbs (Fig. 4.4). Moreover, a torsional spring of stiffness K_α (α represents θ or ψ) is placed between two rigid links in both sides of each passive revolute joint in order to express the elastic characteristic of the original flexure hinge. Similarly, a linear spring of stiffness K_s is placed between the moving and stationary elements of each actuated prismatic joint in order to express the elastic characteristic of the original double linear spring stage.

Fig. 4.3. PRB model of SA 3-DOF translational FPM.

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

Fig. 4.4. Three-flexure-limb group of the PRB model.

The elastic characteristics of flexure joints are represented by the stiffness of springs and the motion range as follows:

Passive flexure hinges:

The torsional stiffness of circular flexure hinge is defined as:

$$K_{\alpha} = \frac{2Eet^{5/2}}{9\pi r^{1/2}} \text{ (cited from [21])}, \quad (4.8)$$

where e , t , r and E are described in Eq. (4.1) and illustrated in Fig. 4.2.

The joint limit δ is given by Eq. (4.6)

Actuated double linear spring stage:

The linear stiffness K_S of the double linear spring stage is determined as twice of the stiffness of a single linear spring stage, therefore,

$$K_S = \frac{16Eest_S^{5/2}}{9\pi l^2 r_S^{1/2}}, \quad (4.9)$$

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

where l is the length of limbs of double linear spring stages, and e_s , t_s and r_s are respectively the thickness of plate, the inter-hole space and the common radius of notches of the flexure hinges in the double linear spring stage (Fig. 4.5).

Fig. 4.5. Dimensions of flexure hinge and double linear spring stage.

The joint limit of the double linear spring stage is determined as

$$S = l\delta_s = \frac{3l\pi}{4} \frac{S_y}{E} \sqrt{\frac{r_s}{t_s}}, \quad (4.10)$$

where δ_s is the limit of the rotation angle of the flexure hinges in the double linear spring stage and is determined by Eq. (4.6).

Denote $\mathbf{X} = (X, Y, Z)$ and $\mathbf{x} = (x, y, z)$ as the displacement vector of the end-effector and the vector of actuated joint variables, we have

$$\begin{pmatrix} \dot{X} & \dot{Y} & \dot{Z} \end{pmatrix}^T = \mathbf{J} \begin{pmatrix} \dot{x} & \dot{y} & \dot{z} \end{pmatrix}^T, \quad (4.11)$$

where Jacobian matrix $\mathbf{J}$ is an identity matrix.

To determine all joint variables (passive and actuated), we use the following notations:

- Subscript index “ $_{ij}$ ” denotes the limb j on 3-flexure-limb group i and subscript index “ $_{i0}$ ” denotes the initial value of a quantity of limb j on 3-flexure-limb group i . For example, θ_{120} represents the initial angle between the X-axis and link $A_{12}B_{12}$ of limb 2 on 3-flexure-limb group Z ($i = 1$).

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

- In the PRB model of each 3-flexure-limb group of the FPM, (u, v, w) denotes a coordinate system according to the subscript index “ i ” and following to the cyclic conditions.

$i = 1$: u, v and w represent for x, y and z respectively.

$i = 2$: u, v and w represent for y, z and x respectively.

$i = 3$: u, v and w represent for z, x and y respectively.

Now consider limb j on 3-flexure-limb group i of the PRB model (Fig. 4.6), the vector close-loop equation of the limb is an addition of the vectors:

$$\vec{O_i C_{ij}} = \vec{O_i A_{ij}} + \vec{A_{ij} B_{ij}} + \vec{B_{ij} C_{ij}} . \quad (4.12)$$

Similarly denote B_{ij0} and C_{ij0} as the initial positions of joints B_{ij} and C_{ij} of limb j on 3-flexure-limb group i , and u, v as the displacements of point C_{ij} along u -, v - axes, the equation becomes

$$A_{ij} \vec{B_{ij0}} + B_{ij0} \vec{C_{ij0}} + u + v = A_{ij} \vec{B_{ij}} + B_{ij} \vec{C_{ij}} . \quad (4.13)$$

Fig. 4.6. Vector close loop of limb j on 3-flexure-limb group i .

Projecting Eq. (4.13) on two axes u and v , we have

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

$$\begin{aligned} a \cos \theta_{ij0} + b \cos \psi_{ij0} + u &= a \cos \theta_{ij} + b \cos \psi_{ij}, \\ a \sin \theta_{ij0} + b \sin \psi_{ij0} + v &= a \sin \theta_{ij} + b \sin \psi_{ij}, \end{aligned} \quad (4.14)$$

where a, b are the lengths of links $A_{ij}B_{ij}$, $B_{ij}C_{ij}$ and θ_{ij0} , ψ_{ij0} , θ_{ij} , ψ_{ij} are the initial and current position angles of links $A_{ij}B_{ij}$, $B_{ij}C_{ij}$ respectively.

Define two intermediate variables c and d as

$$\begin{aligned} c &= a \cos \theta_{ij0} + b \cos \psi_{ij0} + u, \\ d &= a \sin \theta_{ij0} + b \sin \psi_{ij0} + v. \end{aligned} \quad (4.15)$$

Solving Eq. (4.14), we obtain passive joint variables as follows

$$\begin{aligned} \theta_{ij} &= \gamma + \cos^{-1} \left(\frac{c^2 + d^2 + a^2 - b^2}{2a\sqrt{c^2 + d^2}} \right), \\ \psi_{ij} &= 2\pi + \gamma - \cos^{-1} \left(\frac{c^2 + d^2 + b^2 - a^2}{2b\sqrt{c^2 + d^2}} \right), \end{aligned} \quad (4.16)$$

where γ is given as

$$\cos \gamma = \frac{c}{\sqrt{c^2 + d^2}} \text{ and } \sin \gamma = \frac{d}{\sqrt{c^2 + d^2}}. \quad (4.17)$$

Differentiating Eq. (4.14) and solving the equation, we obtain the passive joint displacement rates:

$$\begin{aligned} \dot{\theta}_{ij} &= \frac{\cos \psi_{ij}}{a \sin(\psi_{ij} - \theta_{ij})} \dot{u} + \frac{\sin \psi_{ij}}{a \sin(\psi_{ij} - \theta_{ij})} \dot{v}, \\ \dot{\psi}_{ij} &= \frac{-\cos \theta_{ij}}{b \sin(\psi_{ij} - \theta_{ij})} \dot{u} + \frac{-\sin \theta_{ij}}{b \sin(\psi_{ij} - \theta_{ij})} \dot{v}. \end{aligned} \quad (4.18)$$

The passive joint variables are used to compute the flexure load in static analysis and to determine the workspace or global design indicators in the optimal design of the FPM.

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

4.2 Pseudo Rigid-Body Modeling with Consideration of Deformation

The flexure mechanism can be studied effectively using the PRB model in problems without the need of very high accuracy such as static force, workspace and velocity analysis. However, the PRB model ignores the deformation of the flexure joints in non-working direction such as the displacement of the rotation center and the bending of flexure hinges. Those deformations cause the difference between the real kinematic configuration of the flexure mechanism and the PRB model, and subsequently cause error in the modeling of the flexure mechanism. That error can be considered small and neglected in the design or analysis of flexure mechanisms. However, it causes error in the precision position control of the mechanism. Therefore, when controlling the flexure mechanism, the PRB model can be used only for approximation. Recent efforts of the PRB model improvement using more joints and links to express the deformation [14] make the model more complicated when the number of joints and links increases. In addition, this improvement needs the establishment of a function that expresses the constraint between the displacement of added members and the loads. Therefore, it is very difficult to apply this improvement to the spatial flexure mechanism.

Here we present an improvement on the PRB model with consideration of deformation of flexure members, which is termed the PRB-D model. The main features of the PRB-D model are: (1) determining the deformations of flexure members that have the effect on the end-effector motion, (2) transmitting those deformations to the end-effector and (3) compensating them in the modeling of the PRB model.

The deformation of flexure members is determined based on reaction forces and moments resulted from the static analysis using PRB model. Hence, the model is still simple and realizable. In the meantime, the model expresses more accurately the motion

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

of flexure mechanisms due to the consideration of deformation of flexure members and can be used for the precision position control of the mechanism.

4.2.1 Fundamentals of PRB-D Model

Assume that the desired position of the end-effector and the computed actuated joint variables are respectively specified by vector $\mathbf{X}$ and vector $\mathbf{x}$. Kinematics based on the PRB model leads to $\mathbf{X}$ as a function of $\mathbf{x}$ denoted as

$$\mathbf{X} = \mathbf{H}(\mathbf{x}). \quad (4.19)$$

Suppose that there exist n flexure members that their deformation has an effect on the end-effector motion. The i^{th} flexure member has the deformation denoted by the vector $\boldsymbol{\varepsilon}_i$. This deformation vector is transmitted to the end-effector by transformation matrix $\mathbf{G}_i$ and causes effect on the end-effector motion evaluated by the vector $\mathbf{E}_i$ of the individual deformation effect of the i^{th} flexure member.

$$\mathbf{E}_i = \mathbf{G}_i \boldsymbol{\varepsilon}_i, \quad (4.20)$$

Assume that the flexure mechanism complies with the linear deformation and superposition principles, the actual motion of the end-effector is the combined effect of the PRB modeling and vector $\mathbf{E}_t$ that is defined as the accumulated effect of the vectors $\{\mathbf{E}_i\} (i = 1, 2, \dots, n)$, so we have

$$\mathbf{X} = \mathbf{H}(\mathbf{x}) + \mathbf{E}_t, \quad (4.21)$$

where

$$\mathbf{E}_t = \sum_{i=1}^n \mathbf{E}_i. \quad (4.22)$$

The deflection $\boldsymbol{\varepsilon}_i$ of the i^{th} flexure member is caused by reaction forces and moments applied to that particular member. The reactions are integrated into one vector $\mathbf{F}_i$. Hence,

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

$$\varepsilon_i = \mathbf{K}_i^{-1} \mathbf{G}_{fi} \mathbf{F}_i, \quad (4.23)$$

where $\mathbf{K}_i$ is the stiffness matrix of the i^{th} flexure member, $\mathbf{G}_{fi}$ is the matrix transforming the vector $\mathbf{F}_i$ from the frame whose axes are parallel to the axes of the base frame to the local frame of the flexure member (Two frames have the same origin) and the vector $\mathbf{F}_i$ is determined from the static force analysis based on the PRB model.

The combined effect of the deformations of n flexure members on the end-effector then can be expressed as

$$\mathbf{E}_t = \sum_{i=1}^n \mathbf{G}_i \mathbf{K}_i^{-1} \mathbf{G}_{fi} \mathbf{F}_i. \quad (4.24)$$

Substituting Eq. (4.24) into Eq. (4.21), we have the PRB-D model of the flexure mechanism given as

$$\mathbf{X} = \mathbf{H}(\mathbf{x}) + \sum_{i=1}^n \mathbf{G}_i \mathbf{K}_i^{-1} \mathbf{G}_{fi} \mathbf{F}_i. \quad (4.25)$$

The PRB-D model of the XY FPM illustrated in Fig. 4.7 is established for clarity. The linear spring stages 1X and 2Y have effect on the end-effector motion and are numbered as flexure member 1 and 2 respectively. Here we have $\mathbf{X} = (X, Y)^T$ and $\mathbf{x} = (x, y)^T$ where (X, Y) and (x, y) are the position of the end-effector and the actuated variables respectively. The kinematics based on the PRB model leads only to $\mathbf{X} = \mathbf{x}$.

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

Fig. 4.7. The establishment of PRB-D model of XY FPM.

The deformation of flexure members and their effect on the end-effector motion are described for each flexure member as below.

Flexure member 1: The reaction causing the deformation of the 1st flexure member is

$$\mathbf{F}_1 = (F_{x1}, F_{y1}, 0, 0, 0, M_{z1})^T. \quad (4.26)$$

The axes of the base frame and the local frame of the 1st flexure member are parallel. Hence, the force transformation matrix $\mathbf{G}_{f1}$ is an identity matrix. The stiffness matrix $\mathbf{K}_1$ of the linear spring stage 1X is given in Appendix A. The deformation of the 1st flexure member is determined as

$$\boldsymbol{\varepsilon}_1 = \mathbf{K}_1^{-1} \mathbf{G}_{f1} \mathbf{F}_1 = \mathbf{K}_1^{-1} \mathbf{F}_1. \quad (4.27)$$

The out-of-plane deformation, if any, has no effect on the end-effector motion. The bending around z_1 -axis of the flexure member causes the displacement of the end-effector along X-direction. The linear deformations along x_1 - and y_1 - axes cause the

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

displacement of the end-effector along X- and Y- axes respectively. Therefore, the transformation matrix of the deformation of the 1st flexure member is

$$\mathbf{G}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & L \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.28)$$

where L is the length of the limb of the linear spring stage (Fig. 4.2).

Substituting Eq. (4.27) and Eq. (4.28) into Eq. (4.20), the effect of the deformation of the 1st flexure member on the end-effector motion is

$$\mathbf{E}_1 = \mathbf{G}_1 \boldsymbol{\varepsilon}_1 = \mathbf{G}_1 \mathbf{K}_1^{-1} \mathbf{F}_1. \quad (4.29)$$

Flexure member 2: The reaction causing the deformation of the 2nd flexure member is

$$\mathbf{F}_2 = (F_{x2}, F_{y2}, 0, 0, 0, M_{z2})^T. \quad (4.30)$$

The local frame ($x_2 y_2 z_2$) of the 2nd flexure member is obtained when rotating the base frame ($X Y Z$) at the origin an angle of 90° around Z-axis. Hence, the force transformation matrix $\mathbf{G}_{f2}$ is

$$\mathbf{G}_{f2} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.31)$$

The stiffness matrix $\mathbf{K}_2$ of the linear spring stage 2Y is given in Appendix A and the deformation of the 2nd flexure member is determined as

$$\boldsymbol{\varepsilon}_2 = \mathbf{K}_2^{-1} \mathbf{G}_{f2} \mathbf{F}_2. \quad (4.32)$$

The linear deformations along x_2 - and y_2 - axes of the flexure member cause the displacement of the end-effector along Y-axis and opposite to X-axis respectively. The bending around z_2 -axis causes the displacement of the end-effector along Y-direction. Therefore, the transformation matrix of the deformation of the 2nd flexure member is

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

$$\mathbf{G}_2 = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & L \end{bmatrix}. \quad (4.33)$$

Substituting Eq. (4.32) and Eq. (4.33) into Eq. (4.20), the effect of the deformation of the 2nd flexure member on the end-effector motion is

$$\mathbf{E}_2 = \mathbf{G}_2 \boldsymbol{\varepsilon}_2 = \mathbf{G}_2 \mathbf{K}_2^{-1} \mathbf{G}_{f2} \mathbf{F}_2. \quad (4.34)$$

The accumulated effect of deformation of flexure members on the end-effector motion is given as

$$\mathbf{E}_t = \mathbf{E}_1 + \mathbf{E}_2 = \mathbf{G}_1 \mathbf{K}_1^{-1} \mathbf{F}_1 + \mathbf{G}_2 \mathbf{K}_2^{-1} \mathbf{G}_{f2} \mathbf{F}_2. \quad (4.35)$$

Fig. 4.8. Concept of PRB-D model.

This accumulated effect is added to the PRB model to obtain the PRB-D model of the XY FPM:

$$\mathbf{X} = \mathbf{x} + \mathbf{E}_t = \mathbf{x} + \mathbf{G}_1 \mathbf{K}_1^{-1} \mathbf{F}_1 + \mathbf{G}_2 \mathbf{K}_2^{-1} \mathbf{G}_{f2} \mathbf{F}_2 \quad (4.36)$$

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

The PRB-D model of a flexure mechanism can be summarized in the chart shown in Fig. 4.8. The PRB-D model needs extensive computations because it involves the static analysis and all the flexure members that have effect on the end-effector motion. Fortunately, the flexure mechanisms used for ultra-precision manipulation usually have simple structure with limited number of flexure members and not all flexure members have effect on the end-effector motion.

4.2.2 Application to the Selective-Actuation 3-DOF Translational FPM

This subsection establishes the kinematic modeling using the PRB-D model of the SA 3-DOF translational FPM whose conceptual design is given in Chapter 3. This modeling is used as an open-loop transfer function in the position control of the end-effector of the FPM driven by controlled-position actuators. As seen in the PRB model illustrated in Fig. 4.3, the deformations of the flexure hinges A_{ij} , B_{ij} and C_{ij} in limbs $A_{ij}B_{ij}C_{ij}$ ($i, j = 1, 2, 3$) have effect on the end-effector motion. Hence, the number of flexure members whose deformations have effect on the end-effector motion is $n = 27$. Here we denote $\mathbf{X} = (X, Y, Z)^T$ and $\mathbf{x} = (x, y, z)^T$, where (X, Y, Z) and (x, y, z) are the position of the end-effector and the actuated variables respectively. The kinematics based on the PRB model leads to $\mathbf{X} = \mathbf{x}$. Therefore, the PRB-D model of the designed FPM can be described as

$$\begin{aligned} (X, Y, Z)^T = (x, y, z)^T + \sum_{i=1}^3 \sum_{j=1}^3 \left(\mathbf{G}_{A_{ij}} \mathbf{K}_{A_{ij}}^{-1} \mathbf{G}_{fA_{ij}} \mathbf{F}_{A_{ij}} \right. \\ \left. + \mathbf{G}_{B_{ij}} \mathbf{K}_{B_{ij}}^{-1} \mathbf{G}_{fB_{ij}} \mathbf{F}_{B_{ij}} + \mathbf{G}_{C_{ij}} \mathbf{K}_{C_{ij}}^{-1} \mathbf{G}_{fC_{ij}} \mathbf{F}_{C_{ij}} \right). \end{aligned} \quad (4.37)$$

Consider limb $A_{ij}B_{ij}C_{ij}$ on 3-flexure-limb group i (Fig. 4.9). The effect of the deformations of the flexure hinges A_{ij} , B_{ij} and C_{ij} on the end-effector motion is described as below. The linear and rotation deformations are represented by ε or ε_α respectively.

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

Fig. 4.9. Configuration of one limb $A_{ij}B_{ij}C_{ij}$.

Flexure hinge A_{ij} : Among the deformations of the flexure hinge along or around the axes of the local frame (x_l, y_l, z_l) attached to the flexure hinge, the linear deformation $\varepsilon_{zA_{ij}}$ along z_l -axis and the rotation bending $\varepsilon_{\alpha x A_{ij}}, \varepsilon_{\alpha y A_{ij}}$ around x_l, y_l - axes have effect on the end-effector motion along w -axis of the base frame. The deformation $\varepsilon_{zA_{ij}}$ is added directly to the accumulated effect, whereas the bending $\varepsilon_{\alpha x A_{ij}}$ and $\varepsilon_{\alpha y A_{ij}}$ are multiplied by $[a \cos(\psi_{ij} - \theta_{ij}) + b]$ and $a \sin(\psi_{ij} - \theta_{ij})$ respectively and then added to the accumulated effect. Therefore, the transformation matrices of the flexure-hinge deformation to the end-effector are obtained as follows:

$$\mathbf{G}_{A_{1j}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & a \cos(\psi_{1j} - \theta_{1j}) + b & a \sin(\psi_{1j} - \theta_{1j}) & 0 \end{bmatrix},$$

$$\mathbf{G}_{A_{2j}} = \begin{bmatrix} 0 & 0 & 1 & a \cos(\psi_{2j} - \theta_{2j}) + b & a \sin(\psi_{2j} - \theta_{2j}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.38)$$

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

$$\mathbf{G}_{A_{3j}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & a \cos(\psi_{3j} - \theta_{3j}) + b & a \sin(\psi_{3j} - \theta_{3j}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Flexure hinge B_{ij} : The linear deformation $\varepsilon_{zB_{ij}}$ along z_l -axis is added directly to the accumulated effect on the end-effector motion along w -axis of the base frame, whereas the bending $\varepsilon_{\alpha yB_{ij}}$ around y_l -axis is multiplied by the length b of link $B_{ij}C_{ij}$ and then added to the accumulated effect. The transformation matrices of flexure hinges B_{ij} are:

$$\mathbf{G}_{B_{1j}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & b & 0 \end{bmatrix}, \mathbf{G}_{B_{2j}} = \begin{bmatrix} 0 & 0 & 1 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.39)$$

$$\mathbf{G}_{B_{3j}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Flexure hinge C_{ij} : Only the linear deformation $\varepsilon_{zC_{ij}}$ of the flexure hinge along z_l -axis has effect on the end-effector motion along w -axis of the base frame and it is added directly to the accumulated effect. Hence, the transforming matrices are:

$$\mathbf{G}_{C_{1j}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{G}_{C_{2j}} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.40)$$

$$\mathbf{G}_{C_{3j}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The stiffness matrices $\mathbf{K}_{A_{ij}}$, $\mathbf{K}_{B_{ij}}$ and $\mathbf{K}_{C_{ij}}$ of flexure hinges A_{ij} , B_{ij} and C_{ij} whose deformations have effect on the end-effector motion are given in Appendix B.

The local frame $(x_l \ y_l \ z_l)$ defined on the flexure hinge is obtained by rotating the base frame $(u \ v \ w)$ an angle ψ_{ij} around w -axis (Fig. 4.10). Hence, the flexure hinges on one 3-

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

flexure-limb group have identical matrix changing the description of the reaction from the base frame to the local frame as shown below.

3-flexure-limb group Z: ($i = 1$) u -, v - and w -axes represent x -, y - and z -axes of the base frame respectively.

$$\mathbf{G}_{fA1j} = \mathbf{G}_{fB1j} = \mathbf{G}_{fC1j} = \begin{bmatrix} \cos \psi_{1j} & \sin \psi_{1j} & 0 & 0 & 0 & 0 \\ -\sin \psi_{1j} & \cos \psi_{1j} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \psi_{1j} & \sin \psi_{1j} & 0 \\ 0 & 0 & 0 & -\sin \psi_{1j} & \cos \psi_{1j} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.41)$$

Fig. 4.10. Description of force/moment in different frames.

3-flexure-limb group Y: ($i = 3$) u -, v - and w -axes represent z -, x - and y -axes of the base frame respectively.

$$\mathbf{G}_{fA3j} = \mathbf{G}_{fB3j} = \mathbf{G}_{fC3j} = \begin{bmatrix} \cos \psi_{3j} & 0 & -\sin \psi_{3j} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin \psi_{3j} & 0 & \cos \psi_{3j} & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \psi_{3j} & 0 & -\sin \psi_{3j} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin \psi_{3j} & 0 & \cos \psi_{3j} \end{bmatrix}. \quad (4.42)$$

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

3-flexure-limb group X: ($i = 2$) u-, v- and w-axes represent y-, z- and x-axes of the base frame respectively.

$$\mathbf{G}_{fA_{2j}} = \mathbf{G}_{fB_{2j}} = \mathbf{G}_{fC_{2j}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos \psi_{2j} & \sin \psi_{2j} & 0 & 0 & 0 \\ 0 & -\sin \psi_{2j} & \cos \psi_{2j} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \psi_{2j} & \sin \psi_{2j} \\ 0 & 0 & 0 & 0 & -\sin \psi_{2j} & \cos \psi_{2j} \end{bmatrix}. \quad (4.43)$$

Assume that the links, the center, and the frame of the 3-flexure-limb groups are rigid and the loads are distributed symmetrically on the limbs. Suppose that driving forces $\mathbf{F}_x = (F_x, 0, 0)^T$, $\mathbf{F}_y = (0, F_y, 0)^T$ and $\mathbf{F}_z = (0, 0, F_z)^T$ are exerted on the actuated joints of the X-, Y- and Z-directions respectively in order to move the end-effector from the initial position to a position expressed in the base frame by (X, Y, Z) .

Reactions affecting motion along Z –axis: ($i = 1$) The deformation of the flexure hinges in the 3-flexure-limb group Z along the Z-direction is caused by the reaction force along Z-axis and the reaction moments about X- and Y-axes. Due to the stiffness of the double linear spring stage in the Z-direction, the force exerting on the center O_1 of the 3-flexure-limb group Z is $P_z = (F_z - K_S z)$, where K_S and z are the linear stiffness and the linear motion along Z-direction of the double linear spring stage respectively. Figure 4.11 illustrates the static analysis of the links $A_{11}A_{12}A_{13}$, $A_{11}B_{11}$ and $B_{11}C_{11}$.

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

Fig.4.11. Static analysis corresponding to 3-flexure-limb group Z.

The analysis can be solved similarly for the remaining links of the 3-flexure-limb group Z. The reaction forces along Z-axis and the reaction moments about X- and Y-axes of the flexure hinges A_{1j} , B_{1j} and C_{1j} are:

$$\begin{aligned}
 R_{A_{1j}}^z &= R_{B_{1j}}^z = R_{C_{1j}}^z = \frac{1}{3}(F_z - K_S z), \\
 M_{A_{1j}}^x &= -R_{A_{1j}}^z r \sin \gamma_j = -\frac{1}{3}(F_z - K_S z) r \sin \gamma_j, \\
 M_{A_{1j}}^y &= R_{A_{1j}}^z r \cos \gamma_j = \frac{1}{3}(F_z - K_S z) r \cos \gamma_j, \\
 M_{B_{1j}}^x &= M_{A_{1j}}^x + R_{B_{1j}}^z a \sin \theta_{1j} = \frac{1}{3}(F_z - K_S z)(-r \sin \gamma_j + a \sin \theta_{1j}), \\
 M_{B_{1j}}^y &= M_{A_{1j}}^y - R_{B_{1j}}^z a \cos \theta_{1j} = \frac{1}{3}(F_z - K_S z)(r \cos \gamma_j - a \cos \theta_{1j}),
 \end{aligned} \tag{4.44}$$

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

$$\begin{aligned} M_{C1j}^x &= M_{B1j}^x + R_{C1j}^z b \sin \psi_{1j} = \frac{1}{3}(F_z - K_S z)(-r \sin \gamma_j + a \sin \theta_{1j} + b \sin \psi_{1j}), \\ M_{C1j}^y &= M_{B1j}^y - R_{C1j}^z b \cos \psi_{1j} = \frac{1}{3}(F_z - K_S z)(r \cos \gamma_j - a \cos \theta_{1j} - b \cos \psi_{1j}), \end{aligned}$$

where $r = O_1A_{11} = O_1A_{12} = O_1A_{13}$, θ_{1j} and ψ_{1j} are the passive joint variables and angles $\gamma_1 = 7\pi/6$ (330°), $\gamma_2 = -\pi/6$ (-30°) and $\gamma_3 = \pi/2$ (90°) specify the position of vectors $\vec{O_1A_{11}}$, $\vec{O_1A_{12}}$ and $\vec{O_1A_{13}}$ with respect to X-axis respectively.

Reactions affecting motion along X- axis: ($i = 2$)

$$\begin{aligned} R_{A2j}^x &= R_{B2j}^x = R_{C2j}^x = \frac{1}{3}(F_x - K_S x), \\ M_{A2j}^y &= -R_{A2j}^x r \sin \gamma_j = -\frac{1}{3}(F_x - K_S x)r \sin \gamma_j, \\ M_{A2j}^z &= R_{A2j}^x r \cos \gamma_j = \frac{1}{3}(F_x - K_S x)r \cos \gamma_j, \end{aligned} \tag{4.45}$$

$$\begin{aligned} M_{B2j}^y &= M_{A2j}^y + R_{B2j}^x a \sin \theta_{2j} = \frac{1}{3}(F_x - K_S x)(-r \sin \gamma_j + a \sin \theta_{2j}), \\ M_{B2j}^z &= M_{A2j}^z - R_{B2j}^x a \cos \theta_{2j} = \frac{1}{3}(F_x - K_S x)(r \cos \gamma_j - a \cos \theta_{2j}), \end{aligned}$$

$$\begin{aligned} M_{C2j}^y &= M_{B2j}^y + R_{C2j}^x b \sin \psi_{2j} = \frac{1}{3}(F_x - K_S x)(-r \sin \gamma_j + a \sin \theta_{2j} + b \sin \psi_{2j}), \\ M_{C2j}^z &= M_{B2j}^z - R_{C2j}^x b \cos \psi_{2j} = \frac{1}{3}(F_x - K_S x)(r \cos \gamma_j - a \cos \theta_{2j} - b \cos \psi_{2j}), \end{aligned}$$

Reactions affecting motion along Y- axis: ($i = 3$)

$$\begin{aligned} R_{A3j}^y &= R_{B3j}^y = R_{C3j}^y = \frac{1}{3}(F_y - K_S y), \\ M_{A3j}^z &= -R_{A3j}^y r \sin \gamma_j = -\frac{1}{3}(F_y - K_S y)r \sin \gamma_j, \\ M_{A3j}^x &= R_{A3j}^y r \cos \gamma_j = \frac{1}{3}(F_y - K_S y)r \cos \gamma_j, \end{aligned} \tag{4.46}$$

$$\begin{aligned} M_{B3j}^z &= M_{A3j}^z + R_{B3j}^y a \sin \theta_{3j} = \frac{1}{3}(F_y - K_S y)(-r \sin \gamma_j + a \sin \theta_{3j}), \\ M_{B3j}^x &= M_{A3j}^x - R_{B3j}^y a \cos \theta_{3j} = \frac{1}{3}(F_y - K_S y)(r \cos \gamma_j - a \cos \theta_{3j}), \end{aligned}$$

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

$$M_{C3j}^z = M_{B3j}^z + R_{C3j}^y b \sin \psi_{3j} = \frac{1}{3}(F_y - K_S y)(-r \sin \gamma_j + a \sin \theta_{3j} + b \sin \psi_{3j}),$$

$$M_{C3j}^x = M_{B3j}^x - R_{C3j}^y b \cos \psi_{3j} = \frac{1}{3}(F_y - K_S y)(r \cos \gamma_j - a \cos \theta_{3j} - b \cos \psi_{3j}),$$

The reaction vectors of the flexure hinges are:

$$\begin{aligned} F_{Aij} &= (R_{Aij}^x, R_{Aij}^y, R_{Aij}^z, M_{Aij}^x, M_{Aij}^y, M_{Aij}^z)^T, \\ F_{Bij} &= (R_{Aij}^x, R_{Bij}^y, R_{Bij}^z, M_{Bij}^x, M_{Bij}^y, M_{Bij}^z)^T, \quad i, j=1, 2, 3. \quad (4.47) \\ F_{Cij} &= (R_{Cij}^x, R_{Cij}^y, R_{Cij}^z, M_{Cij}^x, M_{Cij}^y, M_{Cij}^z)^T, \end{aligned}$$

Substituting Eq. (4.47) into Eq. (4.37), we have:

$$(X, Y, Z)^T = (x, y, z)^T + (E_x, E_y, E_z)^T. \quad (4.48)$$

The deformation along X-, Y- and Z- axes are determined as follows.

Deformation along Z-axis: ($i = 1$)

$$\begin{aligned} E_z &= \sum_{j=1}^3 \{ C_{33A1j} R_{A1j}^z + a C_{53A1j} \sin(\psi_{1j} - \theta_{1j}) R_{A1j}^z + C_{33B1j} R_{B1j}^z + C_{33C1j} R_{C1j}^z \\ &\quad + b C_{53B1j} R_{B1j}^z + C_{44A1j} [a \cos(\psi_{1j} - \theta_{1j}) + b] (\cos \psi_{1j} M_{A1j}^x + \sin \psi_{1j} M_{A1j}^y) \\ &\quad + [C_{35A1j} + C_{55A1j} a \sin(\psi_{1j} - \theta_{1j})] (-\sin \psi_{1j} M_{A1j}^x + \cos \psi_{1j} M_{A1j}^y) \\ &\quad + (C_{35B1j} - b C_{55B1j}) (-\sin \psi_{1j} M_{B1j}^x + \cos \psi_{1j} M_{B1j}^y) \}. \end{aligned}$$

where $C_{33A1j}, C_{33B1j}, C_{53B1j}, C_{35B1j}, C_{55B1j}$ and C_{33C1j} ($j = 1, 2, 3$) are the terms of the compliance matrix of the passive flexure hinges A_{1j}, B_{1j} and C_{1j} respectively and are given in Appendix B.

Substituting Eq. (4.45) into E_z , we have:

$$\begin{aligned} E_z &= \frac{F_z - K_S z}{3} \sum_{j=1}^3 \{ [C_{33A1j} + a C_{53A1j} \sin(\psi_{1j} - \theta_{1j}) + C_{33B1j} + b C_{53B1j} + C_{33C1j} \\ &\quad + [C_{35A1j} + C_{55A1j} a \sin(\psi_{1j} - \theta_{1j})] r \sin(\psi_{1j} - \gamma_j) \\ &\quad - C_{44A1j} a \cos(\psi_{1j} - \theta_{1j}) r \sin(\psi_{1j} - \gamma_j) \\ &\quad + (C_{35B1j} + b C_{55B1j}) [r \cos(\psi_{1j} - \gamma_j) - a \cos(\psi_{1j} - \theta_{1j})] \}. \end{aligned}$$

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

Assume that the end-effector motions are small and $dX \approx X$, $dx \approx x$, $dY \approx Y$, $dy \approx y$ and $dZ \approx Z$, $dz \approx z$. Hence, the driving force F_z is computed in Appendix C as:

$$F_x = K_{AS_{x-Fx}} x + K_{AS_{y-Fx}} y + K_{AS_{z-Fx}} z.$$

Therefore, the deformation along Z-axis is:

$$E_z = k_{x-Fz} x + k_{y-Fz} y + k_{z-Fz} z,$$

where

$$\begin{aligned} k_{Fz} = & \sum_{j=1}^3 \{ \{ C_{33A_{1j}} + aC_{53A_{1j}} \sin(\psi_{1j} - \theta_{1j}) + C_{33B_{1j}} + bC_{53B_{1j}} + C_{33C_{1j}} \\ & + [C_{35A_{1j}} + C_{55A_{1j}} a \sin(\psi_{1j} - \theta_{1j})] r \sin(\psi_{1j} - \gamma_j) \\ & - C_{44A_{1j}} a \cos(\psi_{1j} - \theta_{1j}) r \sin(\psi_{1j} - \gamma_j) \\ & + (C_{35B_{1j}} + bC_{55B_{1j}}) [r \cos(\psi_{1j} - \gamma_j) - a \cos(\psi_{1j} - \theta_{1j})] \}, \end{aligned} \quad (4.49)$$

$$k_{x-Fz} = k_{Fz} K_{AS_{x-Fz}}, \quad k_{y-Fz} = k_{Fz} K_{AS_{y-Fz}}, \quad k_{z-Fz} = k_{Fz} (K_{AS_{z-Fz}} - K_S).$$

The deformations along the remaining directions are similarly determined as below.

Deformation along X-axis: ($i = 2$)

$$E_x = k_{x-Fx} x + k_{y-Fx} y + k_{z-Fx} z,$$

where

$$\begin{aligned} k_{Fx} = & \sum_{j=1}^3 \{ \{ C_{33A_{2j}} + aC_{53A_{2j}} \sin(\psi_{2j} - \theta_{2j}) + C_{33B_{2j}} + bC_{53B_{2j}} + C_{33C_{2j}} \\ & + [C_{35A_{2j}} + C_{55A_{2j}} a \sin(\psi_{2j} - \theta_{2j})] r \sin(\psi_{2j} - \gamma_j) \\ & - C_{44A_{2j}} a \cos(\psi_{2j} - \theta_{2j}) r \sin(\psi_{2j} - \gamma_j) \\ & + (C_{35B_{2j}} + bC_{55B_{2j}}) [r \cos(\psi_{2j} - \gamma_j) - a \cos(\psi_{2j} - \theta_{2j})] \}, \end{aligned} \quad (4.50)$$

$$k_{x-Fx} = k_{Fx} (K_{AS_{x-Fx}} - K_S), \quad k_{y-Fx} = k_{Fx} K_{AS_{y-Fx}}, \quad k_{z-Fx} = k_{Fx} K_{AS_{z-Fx}}.$$

Deformation along Y-axis: ($i = 3$)

$$E_y = k_{x-Fy} x + k_{y-Fy} y + k_{z-Fy} z,$$

where

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

$$\begin{aligned}
 k_{Fy} = & \sum_{j=1}^3 \{C_{33}A_{3j} + aC_{53}A_{3j} \sin(\psi_{3j} - \theta_{3j}) + C_{33}B_{3j} + bC_{53}B_{3j} + C_{33}C_{3j} \\
 & + [C_{35}A_{3j} + C_{55}A_{3j} a \sin(\psi_{3j} - \theta_{3j})] r \sin(\psi_{3j} - \gamma_j) \\
 & - C_{44}A_{3j} a \cos(\psi_{3j} - \theta_{3j}) r \sin(\psi_{3j} - \gamma_j) \\
 & + (C_{35}B_{3j} + bC_{55}B_{3j}) [r \cos(\psi_{3j} - \gamma_j) - a \cos(\psi_{3j} - \theta_{3j})]\},
 \end{aligned} \quad (4.51)$$

$$k_{x-Fy} = k_{Fy} K_{AS_{x-Fy}}, \quad k_{y-Fy} = k_{Fy} (K_{AS_{y-Fy}} - K_S), \quad k_{z-Fy} = k_{Fy} K_{AS_{z-Fy}}.$$

Hence, Eq. (4.48) becomes:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{bmatrix} 1 + k_{x-Fx} & k_{y-Fx} & k_{z-Fx} \\ k_{x-Fy} & 1 + k_{y-Fy} & k_{z-Fy} \\ k_{x-Fz} & k_{y-Fz} & 1 + k_{z-Fz} \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad (4.52)$$

where coefficients $k_{x-Fx}, k_{y-Fx}, k_{z-Fx}, k_{x-Fy}, k_{y-Fy}, k_{z-Fy}, k_{x-Fz}, k_{y-Fz}$ and k_{z-Fz} are determined using Eq. (4.49), Eq. (4.50) and Eq. (4.51).

Equation (4.52) is the PRB-D model of the SA 3-DOF translational FPM. It has a linear form by assuming small motion of the end-effector.

4.3 Summary

In this chapter, a modeling method for studying flexure mechanisms is presented based on the traditional PRB model. This model is established by replacing flexure members with equivalent rigid links, traditional joints and springs like the PRB model. The dimension and spring constant are determined based on the structure of respective flexure members. To express precisely the motion of the flexure mechanism, the deformation of flexure members are computed based on the PRB model and then compensated in the joint variables found in the inverse kinematics and position control, or the task variables (the position of the end-effector) in the forward kinematics. The

CHAPTER 4. MODELING OF FLEXURE PARALLEL MECHANISM.

proposed pseudo rigid-body model with consideration of deformation allows us to predict precisely the end-effector motion, and therefore, can be used as an open-loop transfer function of the position control of the mechanism.

Chapter 5

Evaluation of Positioning Resolution

The resolution of an FPM depends on many factors such as resolution of actuators, the feedback control system, stiffness of the FPM and transmission of resolution/infinitesimal motion from the actuator to the end-effector. Most of previous researches in FPM focused on post-design measures such as (1) the improvement of the pseudo rigid-body model [14], (2) the development and improvement of feedback control by managing the error budget distribution [8] and (3) the error prediction and the modification of the control [11]. Such kind of improvement on positioning resolution will reach a limit caused by the stiffness of the mechanism and resolution transmission. This chapter studies the second factor – the resolution transmission. The influence of the configuration of an FPM on the resolution transmission is investigated. The resolution indices used to evaluate FPM positioning are proposed. The indices can also be used as criteria for optimal design of the FPM.

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

5.1 Resolution Transmission Scale

Consider a non-redundant FPM with n degrees of freedom and let $d\mathbf{q} = (dq_1, dq_2, \dots, dq_n)^T$ be a vector of the infinitesimal displacements of n actuated joints and $d\mathbf{X} = (dX_1, dX_2, \dots, dX_n)^T$ be a vector of the infinitesimal motions (displacements and/or orientations) of the end-effector (Fig. 5.1). Let $\mathbf{J}$ be the $n \times n$ Jacobian matrix that transfers the infinitesimal motions from actuated joints to the end-effector. This relationship can be described as

$$d\mathbf{X} = \mathbf{J} d\mathbf{q}. \quad (5.1)$$

Fig. 5.1. Task variables and actuated joint variables.

The limit of the positioning resolution of the FPM is the smallest available motion of the end-effector when only one of the actuators drives with the smallest controllable motion that is called resolution of the actuator. Denote the resolution of the actuator as τ , then: $dq_1 = \tau$, or $dq_2 = \tau$, ..., or $dq_n = \tau$. Hence, we have

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

$$d\mathbf{q}^T d\mathbf{q} = dq_1^2 + dq_2^2 + \dots + dq_n^2 = \tau^2. \quad (5.2)$$

Since the workspace of the FPM is usually small and can be assumed singularity-free, Eq. (5.1) and Eq. (5.2) lead to:

$$d\mathbf{X}^T (\mathbf{J}\mathbf{J}^T)^{-1} d\mathbf{X} = \tau^2. \quad (5.3)$$

As there is no singularity, Jacobian $\mathbf{J}$ has full rank n , and can be written in Singular Value Decomposition (SVD) form [67], i.e., there exist two $n \times n$ orthogonal matrices $\mathbf{A}$ and $\mathbf{B}$ such that:

$$\mathbf{J} = \mathbf{A}\mathbf{D}\mathbf{B}, \quad (5.4)$$

where the $n \times n$ matrix $\mathbf{D}$ has diagonal form with rank n and the diagonal terms are the singular values $\sigma_1, \sigma_2, \dots, \sigma_n$ of Jacobian $\mathbf{J}$:

$$\mathbf{D} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_n). \quad (5.5)$$

Substituting Eq. (5.4) and Eq. (5.5) into $\mathbf{J}\mathbf{J}^T$ and considering the orthogonality of matrix $\mathbf{B}$, we have:

$$\mathbf{J}\mathbf{J}^T = \mathbf{A}\mathbf{D}\mathbf{B}(\mathbf{A}\mathbf{D}\mathbf{B})^T = \mathbf{A}\mathbf{D}\mathbf{D}^T \mathbf{A}^T, \quad (5.6)$$

where

$$\mathbf{D}\mathbf{D}^T = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2).$$

Thus, Eq. (5.3) becomes

$$d\mathbf{X}^T (\mathbf{A}\mathbf{D}\mathbf{D}^T \mathbf{A}^T)^{-1} d\mathbf{X} = d\mathbf{X}^T \mathbf{A} (\mathbf{D}\mathbf{D}^T)^{-1} \mathbf{A}^T d\mathbf{X} = \tau^2. \quad (5.7)$$

We consider matrix $\mathbf{A}^T$ as the $n \times 1$ matrix whose terms are the row vectors of $\mathbf{A}^T$:

$$\mathbf{A}^T = (\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)^T, \quad (5.8)$$

where $\mathbf{a}_i$ is the i^{th} row vector of matrix $\mathbf{A}^T$.

Vector $d\hat{\mathbf{X}} = \mathbf{A}^T d\mathbf{X} = (d\hat{X}_1, d\hat{X}_2, \dots, d\hat{X}_n)^T$ is defined as the infinitesimal motion of the end-effector described in a frame with the set of unit vectors $\{\hat{\mathbf{k}}_i\}$, where:

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

$$d\hat{X}_i = a_i dX \quad \forall i = 1, 2, \dots, n, \quad (5.9)$$

$$\hat{k}_i = \frac{A^T J k_i^T}{\|A^T J k_i^T\|} \quad \forall i = 1, 2, \dots, n,$$

and $\{k_i\}$ is the set of unit vectors of the actuated joint space.

The unit vectors $\{\hat{k}_i\}$ point to the directions of the smallest available motions of the end-effector. Now Eq. (5.7) can be rewritten as:

$$d\hat{X}^T (DD^T)^{-1} d\hat{X} = \frac{d\hat{X}_1^2}{\sigma_1^2} + \frac{d\hat{X}_2^2}{\sigma_2^2} + \dots + \frac{d\hat{X}_n^2}{\sigma_n^2} = \tau^2. \quad (5.10)$$

Equation (5.10) describes the resolution ellipsoid as its principal axes follow the directions $\{\hat{k}_i\}$ of the smallest available motions of the end-effector (Fig. 5.2).

Fig. 5.2. Principal axes of resolution ellipsoid.

From Eq. (5.10), the limit of the smallest controllable motions that the end-effector can perform along the principal axes are $\sigma_1\tau$, $\sigma_2\tau$, ..., $\sigma_n\tau$, which are the resolution of the

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

end-effector along those axes and the singular values are the scale of resolution transmission.

The resolution of the end-effector is proportional to the singular values and varies along the direction of the motion. Therefore, the average scale of resolution transmission at a particular point can be defined as follows.

Definition 5.1: Resolution transmission (RT) scale R is the geometric mean of the singular values σ_i of the Jacobian matrix.

$$R = \sqrt[n]{\sigma_1 \sigma_2 \dots \sigma_n} . \quad (5.11)$$

Equation (5.6) leads to $\det(\mathbf{J}\mathbf{J}^T) = \det[\mathbf{A}^T(\mathbf{D}\mathbf{D}^T)\mathbf{A}] = \det(\mathbf{A}^T)\det(\mathbf{D}\mathbf{D}^T)\det(\mathbf{A})$. Due to the orthogonality of $\mathbf{A}$, we have $\det(\mathbf{A})\det(\mathbf{A}^T) = \det(\mathbf{A}\mathbf{A}^T) = \det(\mathbf{I}) = 1$, therefore

$$\det(\mathbf{J}\mathbf{J}^T) = \det(\mathbf{D}\mathbf{D}^T) = \sigma_1^2 \sigma_2^2 \dots \sigma_n^2 .$$

Hence, the RT scale can be written as

$$R = \sqrt[2n]{\det(\mathbf{J}\mathbf{J}^T)} . \quad (5.12)$$

5.2 Positioning Resolution Indices

5.2.1 Global Resolution Transmission Scale and Uniformity of Distribution

Since Jacobian $\mathbf{J}$ is configuration dependent, the RT scale R as well as resolution of the end-effector are the local performance measures valid at a certain pose only. It is necessary to develop the indices to evaluate the resolution of the FPM in the entire workspace. The global value of RT scale is thus a suitable index for the overall performance evaluation.

Definition 5.2: Global resolution transmission scale is an average value of RT scale in the entire workspace, i.e.,

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

$$R_{\text{global}} = \frac{\int_W R dw}{\int_W dw}, \quad (5.13)$$

where W is the workspace and dw is an infinitesimally small volume in the workspace in which the RT scale R can be considered as a constant.

In addition to the average RT scale in the workspace, variation of positioning resolution in the workspace is also important. Thus, we have another measure to evaluate the uniformity of the positioning resolution.

Definition 5.3: Uniformity of resolution is the ratio of the minimum and the maximum values of the RT scale, i.e.,

$$U = \frac{R_{\min}}{R_{\max}}, \quad (5.14)$$

where $R_{\min}$ and $R_{\max}$ are the minimal and the maximal values of the RT scale within the entire workspace W .

The global RT scale evaluates the average transmissibility of the small motions from the actuators to the end-effector, whereas the uniformity measures variation of the resolution within the workspace. Both indices have positive values. However, the global RT scale R should be kept small. The uniformity U should be kept close to unity in order to obtain the uniform resolution performance.

5.2.2 Computation of the Resolution Indices

Both indices, the global RT scale R and the uniformity U , require integration over the entire workspace. Unlike robot manipulators that the workspace can be obtained by simply sweeping all of its joints, the workspace of the FPM is limited by its configuration, range of actuation and limited motion due to stiffness and rigidity of the

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

flexure members. Hence, obtaining the workspace of the FPM is necessary before calculating the resolution measures. The workspace W of the FPM is bounded by three workspaces:

$$W = W_1 \cap W_2 \cap W_3, \quad (5.15)$$

where

W_1 is the workspace determined by the architecture reachability:

$$W_1 = \{P(x, y, z) \mid \det(\mathbf{J}) \neq 0\}; \quad (5.16)$$

W_2 is the workspace determined by the actuator range:

$$W_2 = \{P(x, y, z) \mid \forall i=1, 2, \dots, n: q_i \leq S\}, \quad (5.17)$$

here the actuated joint variables $\{q_i\}$ are resulted from the inverse kinematics, and S is the actuator range.

W_3 is the workspace based on the limited motion of flexure members:

$$W_3 = \{P(x, y, z) \mid S_{\max} < S_y\}, \quad (5.18)$$

here $S_{\max}$ and S_y are the maximum stress in the flexure members and the yield stress of the material of the FPM respectively.

The maximal stress does not exist at the same position within the structure of the FPM when the end-effector moves within the workspace. It is difficult to describe the closed-form boundary equation of workspace W_3 . Consequently, the integration over the workspace based on closed-form description of the boundary cannot be guaranteed. Therefore, a numerical approach is proposed here to determine the workspace and the global indices. An algorithm termed the Global Indices Sampling Algorithm (GISA) is proposed here for the computation of the workspace and global indices. This method is based on the sampling method (originated from Monte Carlo method).

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

Fig. 5.3. Global Indices Sampling Algorithm (GISA).

The features of GISA are the partition of a covering volume A into small sampling pieces $\{v_i\}$ and the piece-by-piece test in order to know whether the piece v_i belongs to the workspace. The dimension of the samples is determined based on the required accuracy. The samples that belong to the workspace are added to workspace variable W of the algorithm and the local RT scales at those samples are added to the summing

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

variable $SumR$. Samples are compared to identify the maximal and the minimal values of RT scale. After all of the sampling pieces are examined, the workspace is obtained and stored in variable W . The summing variable $SumR$ is divided by the number of samples identified within the workspace in order to obtain the global RT scale R_{global} . The uniformity U is determined based on the minimal and the maximal values of the local RT scales.

The principle of GISA is illustrated in Fig. 5.3. The problem of the workspace and global index determination now becomes a process of checking the reachability of the pose of the end-effector in W_1 , W_2 and W_3 , i.e., whether the pose of the end-effector satisfies Eq. (5.16), Eq. (5.17) or Eq. (5.18). The algorithm is suitable not only for the computation of the workspace, the global RT scale and the uniformity of the resolution distribution but also for the computation of other global type of indices for the flexure mechanism.

5.2.3 Optimal Design Based on Global Resolution Indices

The proposed global indices and the computation algorithm GISA can be applied to the optimal design of the FPM based on the criteria of positioning resolution. As decreasing RT scale leads to higher positioning resolution due to the proportion of the resolution of the end-effector to RT scale. In the optimal design of FPM, the RT scale needs to be minimized. However, the RT scale is a function of the product of singular values of the Jacobian matrix. The singular values are proportional to the inverse of force amplification scale [68], i.e., the smaller the singular values are the more actuation force needs to move the FPM. In order to satisfy the conditions of precision motion as well as the small driving forces, the RT scale should be as close to isotropy (close to 1) as possible.

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

The resolution performance of the mechanism is usually evaluated based not only on the average resolution, but also on the uniformity of resolution distribution. The uniformity is desired to be close to the value of one in order to limit the variety of resolution. The uniformity can be set as a constraint in the optimal design of FPM when a lower limit of the uniformity is prescribed based on the desired quality of the designed mechanism. The workspace volume of the FPM is another constraint of the optimal design of FPM. The minimal available workspace volume is specified by the requirement of the application.

Definition 5.4: Free shape of the flexure mechanism is the shape of the mechanism when all flexure members are in their original and non-deflected shapes.

The free shape of an FPM specifies the point around which the FPM works. Therefore, the free shape has effect on the global indices. In the contrary, dimensions of the kinematic mechanism of the FPM have effect on the Jacobian matrix and the local RT scale. The free shape and dimensions of the FPM are considered as the design variables. Based on the compromise between the resolution and driving forces and the constraints imposed on the resolution distribution and the workspace, the optimal design problem of the FPM can be formulated as follows.

Objective:

Minimize the difference between global resolution transmission scale and unity.

$$|R_{\text{global}} - 1|. \quad (5.19)$$

Constraints:

- The uniformity of resolution distribution must be higher than an allowable value U_{min} .

$$U \geq U_{\text{min}}. \quad (5.20)$$

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

- The workspace volume of the FPM must be larger than an allowable minimum value $W_{\min}$.

$$W \geq W_{\min}. \quad (5.21)$$

Design parameters:

The variables are the normalized dimensions of the PRB model and the free shape of the FPM. The free shape is represented by the parameters that describe the initial position of the rigid links in the PRB model of the FPM.

5.3 Case Studies

To demonstrate usage of the proposed indices and the optimal design of configuration of FPM, three examples are presented in this section.

5.3.1 A 2-DOF Translational FPM

A monolithic 2-DOF FPM illustrated in Fig. 5.4 can be used as an X-Y positioning system. Parasitic rotation about the z-axis is avoided using a parallelogram structure. The mechanism actuated by two linear actuators located on parallel guides can perform two DOF motions along x- and y- axes:

Fig. 5.4. Two-DOF translational FPM.

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

A. Kinematics and workspace

The FPM is investigated using the PRB model shown in Fig. 5.5. The link lengths of the model are defined to be the distances between the centers of the flexure hinges. The hinges are replaced by the revolute joints and the torsional springs. The derivation of the kinematics and workspace has been discussed [69-71]. The position of the platform can be determined by the position of reference point B (x, y). To establish the closed-loop vector equation, we denote two scalars x_A and x_F as the actuated joint variables corresponding to actuated joints A and F respectively. The four scalars x, y, φ_A and φ_E are defined as the task variables and the passive joint variables corresponding to joints A and B respectively. The initial values of passive joint variables φ_{A0} and φ_{F0} represent the free shape of the FPM. The dimensions of the rigid links of the PRB model are defined as: $a = AD = BC$, $b = AB = CD$, $c = CE$, $d = EF$, $f = FG$. The inverse kinematics solutions of the actuated joint variables x_A and x_F , and the passive joint variables φ_A and φ_E in terms of the task variables x and y are

Fig. 5.5. PRB model of 2-DOF FPM.

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

$$\varphi_A = \sin^{-1}(y/b), \quad \varphi_E = 2\pi - \sin^{-1}[(y-f+c)/d], \quad (5.22)$$

$$x_A = x - \sqrt{b^2 - y^2}, \quad x_F = x + a + \sqrt{d^2 - (y-f+c)^2}.$$

The Jacobian is obtained from the kinematic analysis:

$$\mathbf{J} = \begin{pmatrix} 1 & \tan \varphi_A \\ 1 & \tan \varphi_E \end{pmatrix}. \quad (5.23)$$

The complete workspace of the FPM is the intersection of permissible working areas of the platform based on the architecture reachability (W_1), actuator range (W_2) and limited rotation angle (W_3). These areas are analytically represented by the constraints:

$$W_1 = \left\{ (x, y) \mid \begin{cases} b^2 - y^2 \geq 0 \\ d^2 - (y-f+c)^2 \geq 0 \end{cases} \right\}; \quad (5.24)$$

$$W_2 = \left\{ (x, y) \mid \begin{cases} x^2 + y^2 \geq b^2 \\ (x-S)^2 + y^2 \leq b^2 \\ (x-x_{F\min}-S+a)^2 + (y-f+c)^2 \geq d^2 \\ (x-x_{F\min}+a)^2 + (y-f+c)^2 \leq d^2 \end{cases} \right\}, \quad (5.25)$$

where $x_{F\min}$ is the smallest value of x_F and S is the motion range of the linear actuators.

$$W_3 = \{(x, y) \mid y_{\min} \leq y \leq y_{\max}\}, \quad (5.26)$$

and

$$\begin{aligned} y_{\min} &= \max \{ b \sin(\varphi_{A0} - \delta_A), f - c - d \sin(\varphi_{E0} + \delta_E) \}, \\ y_{\max} &= \min \{ b \sin(\varphi_{A0} + \delta_A), f - c - d \sin(\varphi_{E0} - \delta_E) \}, \end{aligned}$$

where φ_{A0} and φ_{E0} are the initial values of φ_A and φ_E respectively, and δ_A and δ_E are the joint limits of flexure hinges of A and E respectively. These joint limits are determined by Eq. (4.6) presented in Chapter 4.

B. Evaluation of resolution

Substituting Eq. (5.23) into Eq. (5.12), the local resolution transmission scale becomes

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

$$R = \sqrt{\frac{\sin(\varphi_A - \varphi_E)}{\cos \varphi_A \cos \varphi_E}}. \quad (5.27)$$

Applying the local RT scale of Eq. (5.27) and the workspace expression of Eq. (5.24), Eq. (5.25) and Eq. (5.26) to the GISA, the dimensions and the free shape of the 2-DOF FPM and the results of the computation are given in Table 5.1.

If the actuators with a resolution of $\tau = 0.6 \mu\text{m}$ are used to drive the mechanism, the theoretical average resolution would be $\tau R_{\text{global}} = 0.61 \mu\text{m}$. Figure 5.6 illustrates the distribution of RT scale over the entire workspace. The RT scale remains unchanged when the FPM moves along the X-axis. The resolution of the FPM becomes worse when the FPM moves up. The variation of resolution over the entire workspace is

$$\frac{\max(|R - R_{\text{global}}|)}{R_{\text{global}}} 100\% = 27.72\%. \quad (5.28)$$

Fig. 5.6. RT scale distribution of 2-DOF translational FPM.

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

Table 5.1. Resolution evaluation of a 2-DOF translational FPM using GISA.

Dimensions	$(x_{Fmin} - a) = 220 \text{ mm}, b = 130 \text{ mm}$ $(f - c) = 8 \text{ mm}, d = 115 \text{ mm}$
Free shape	$\varphi_{A0} = 25^\circ$ $\varphi_{F0} = 300^\circ$
Motion range of linear actuators	$S = 14 \text{ mm}$
Computation results	Workspace $W = 134.2 \text{ mm}^2$ Global resolution transmission scale $R_{global} = 1.01$ Uniformity of resolution $U = 0.63$

Fig. 5.7. Planar three-DOF FPM.

5.3.2 A Planar 3-DOF Flexure Parallel Mechanism

The resolution transmission scales of two FPMs of the same dimensions but with different free shapes are studied for the effect of the free shape on the overall performance of the FPM. An $XY\theta_z$ -FPM actuated by three linear actuators is shown in

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

Fig. 5.7. Three linear actuators are arranged on parallel guides to let the platform move along x- or y- axis or rotate around z-axis. This mechanism can be used as a positioning and orientation system.

Fig. 5.8. PRB model of planar three-DOF FPM.

A. Kinematics

The PRB model of the FPM is shown in Fig. 5.8. We assume that the flexure hinges in the FPM are of the same dimensions. Therefore, their elastic characteristics are represented by the same torsional spring K . Assume that the origin of the base system O_0xy is located at the initial position of platform O_0 . The shape of the platform is a symmetrical triangle with distance r from the platform center O to each vertex. The angles of line pointing from the vertex to the center of the platform A_1O , A_2O , and A_3O with respect to A_1A_2 are $\alpha_1 = 30^\circ$, $\alpha_2 = 150^\circ$ and $\alpha_3 = 270^\circ$ respectively. The platform connects to the base with three legs A_iB_i ($i = 1, 2, 3$) of the same length l . The end link

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

of leg i has an initial position at B_{i0} and the current position at B_i . The distance between parallel guides of linear actuators is h .

We denote $O(x, y)$ and θ as the current position and orientation of the platform, ψ_i as the posture angle of each leg A_iB_i ($i = 1, 2, 3$), and vector s_i as the motion provided by actuator i . The free shape of the FPM is represented by two angles θ_0 and ψ_{10} which are the initial values of θ and ψ_1 . The posture angles of A_1O , A_2O and A_3O are $\theta_1 = \theta + \alpha_1$, $\theta_2 = \theta + \alpha_2$ and $\theta_3 = \theta + \alpha_3$. The vector equation of each leg (Fig. 5.9) can be written as

$$\vec{OO_0} + \vec{O_0B_{i0}} + \vec{B_{i0}B_i} = \vec{OA_i} + \vec{A_iB_i}, \quad (i = 1, 2, 3). \quad (5.29)$$

Fig. 5.9. Closed-loop vector of one leg.

Rewriting Eq. (5.29) into the component equations and differentiating them with respect to time, we obtain

$$\begin{aligned} \dot{x} + r\dot{\theta}\sin\theta_i + h\dot{\psi}_i\sin\psi_i &= \dot{s}_i, \\ \dot{y} + r\dot{\theta}\cos\theta_i + h\dot{\psi}_i\cos\psi_i &= 0. \end{aligned} \quad (5.30)$$

The kinematic equation is formulated by combining Eq. (5.30) of all three legs:

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

$$\begin{bmatrix} 1 & 0 & r \sin \theta_1 & l \sin \psi_1 & 0 & 0 \\ 0 & 1 & r \cos \theta_1 & l \cos \psi_1 & 0 & 0 \\ 1 & 0 & r \sin \theta_2 & 0 & l \sin \psi_2 & 0 \\ 0 & 1 & r \cos \theta_2 & 0 & l \cos \psi_2 & 0 \\ 1 & 0 & r \sin \theta_3 & 0 & 0 & l \sin \psi_3 \\ 0 & 1 & r \cos \theta_3 & 0 & 0 & l \cos \psi_3 \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\psi}_1 \\ \dot{\psi}_2 \\ \dot{\psi}_3 \end{pmatrix} = \begin{pmatrix} \dot{s}_1 \\ 0 \\ \dot{s}_2 \\ 0 \\ \dot{s}_3 \\ 0 \end{pmatrix}. \quad (5.31)$$

Solving Eq. (5.31), we have

$$\begin{pmatrix} \dot{x}, \dot{y}, \dot{\theta} \end{pmatrix}^T = \mathbf{J}(\dot{s}_1, \dot{s}_2, \dot{s}_3)^T, \quad (5.32)$$

where Jacobian $\mathbf{J}$ is a 3×3 matrix with the terms:

$$\begin{aligned} J_{11} &= \frac{2 \cos \psi_1}{D} [\sin \psi_2 \sin(\psi_3 - \theta_3) - \sin \psi_3 \sin(\psi_2 - \theta_2)], \\ J_{12} &= \frac{2 \cos \psi_2}{D} [\sin \psi_3 \sin(\psi_1 - \theta_1) - \sin \psi_1 \sin(\psi_3 - \theta_3)], \\ J_{13} &= \frac{2 \cos \psi_3}{D} [\sin \psi_1 \sin(\psi_2 - \theta_2) - \sin \psi_2 \sin(\psi_1 - \theta_1)], \\ J_{21} &= \frac{2 \cos \psi_1}{D} [\cos \psi_2 \sin(\psi_3 - \theta_3) - \cos \psi_3 \sin(\psi_2 - \theta_2)], \\ J_{22} &= \frac{2 \cos \psi_2}{D} [\cos \psi_3 \sin(\psi_1 - \theta_1) - \cos \psi_1 \sin(\psi_3 - \theta_3)], \\ J_{23} &= \frac{2 \cos \psi_3}{D} [\cos \psi_1 \sin(\psi_2 - \theta_2) - \cos \psi_2 \sin(\psi_1 - \theta_1)], \\ J_{31} &= \frac{2 \cos \psi_1 \sin(\psi_2 - \psi_3)}{rD}, \quad J_{32} = \frac{2 \cos \psi_2 \sin(\psi_3 - \psi_1)}{rD}, \\ J_{33} &= \frac{2 \cos \psi_3 \sin(\psi_1 - \psi_2)}{rD}, \end{aligned} \quad (5.33)$$

and

$$\begin{aligned} D &= \cos(\psi_1 + \psi_2 - \psi_3 - \theta_1) - \cos(\psi_1 - \psi_2 + \psi_3 - \theta_1) - \cos(\psi_1 + \psi_2 - \psi_3 - \theta_2) + \\ &\quad \cos(\psi_1 - \psi_2 - \psi_3 + \theta_2) + \cos(\psi_1 - \psi_2 + \psi_3 - \theta_3) - \cos(\psi_1 - \psi_2 - \psi_3 + \theta_3). \end{aligned}$$

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

B. Workspace

The workspace of the mechanism is determined based on the inverse kinematics solution and the strength of fabrication material. Rewriting Eq. (5.29) into two component equations and solving them, we have

$$\sin \psi_i = \frac{y - O_0 B_{i0y} - r \sin \theta_i}{l},$$

$$s_i = x - O_0 B_{i0x} - r \cos \theta_i - l \cos \psi_i. \quad (5.34)$$

Let S , Δ_θ , δ_θ and δ_ψ denote the range of the linear actuators, the range of platform orientation and the joint limits of the flexure hinges at A_i and B_i ($i = 1, 2, 3$) respectively. The workspace of the FPM can be determined by the intersection of areas W_1 (configuration), W_2 (actuator range) and W_3 (strength of material) as follows:

$$W_1 = \left\{ (x, y) \left| \begin{array}{l} \forall \theta \in [\theta_0 - \frac{\Delta_\theta}{2}, \theta_0 + \frac{\Delta_\theta}{2}] \\ \left| \frac{y - O_0 B_{i0y} - r \sin \theta_i}{l} \right| \leq 1 \end{array} \right. \quad i = 1, 2, 3 \right\},$$

$$W_2 = \left\{ (x, y) \left| \begin{array}{l} \forall \theta \in [\theta_0 - \frac{\Delta_\theta}{2}, \theta_0 + \frac{\Delta_\theta}{2}] \quad i = 1, 2, 3 \\ |x - O_0 B_{i0x} - r \cos \theta_i - l \cos \psi_i| \leq \frac{S}{2} \end{array} \right. \right\}, \quad (5.35)$$

$$W_3 = \left\{ (x, y) \left| \begin{array}{l} \forall \theta \in [\theta_0 - \frac{\Delta_\theta}{2}, \theta_0 + \frac{\Delta_\theta}{2}] \\ |\psi_i - \psi_{i0}| \leq \delta_\psi \\ |\theta_i - \theta_{i0}| \leq \delta_\theta \end{array} \right. \quad i = 1, 2, 3 \right\}.$$

C. Resolution transmission scale

The elements in Jacobian $\mathbf{J}$ of Eq. (5.33) do not have the same units. Elements in the last row of Jacobian matrix have the scale of length r of the platform to obtain a non-dimensional RT scale. The GISA is employed to determine the global scale and the

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

uniformity of resolution of the planar 3-DOF FPM.

An optimal design of the planar 3-DOF FPM is carried out based on the above computations. The result of the optimization is listed in Table 5.2. The dimensions of the optimized mechanisms are $l = 80$ mm, $h = 70$ mm and $r = 70$ mm. The influence of the free shape on the resolution of the FPM can be seen when comparing two FPMs with the same dimension: FPM A resulted from the optimal design and FPM B with a free shape of $\theta_0 = -20^\circ$ and $\psi_{10} = 45^\circ$ (Fig. 5.10).

Table 5.2. Optimal design of the planar 3-DOF FPM.

Inputs	$r = 70$ mm, $U_{\min} = 0.5$ and $W_{\min} = 70$ mm ² Actuator range: $S = 10$ mm, orientation range: $\Delta\theta = 5^\circ$ Joint limits: $\delta_\theta = \delta_\psi = 6^\circ$ (flexure hinges A_i and B_i have the same dimension).
Outputs	Normalized dimensions: $l/r = 1.143$ and $h/r = 0.987$ Free shape $\theta_0 = 0^\circ$, $\psi_{10} = 25^\circ$ Workspace $W = 95.6$ mm ² , Global RT scale $R_{\text{global}} = 1.063$ Uniformity of resolution $U = 0.64$

Comparison of the two FPMs is summarized in Table 5.3. The FPM A possesses a larger workspace, better and more uniform resolution than the FPM B (Fig. 5.11). This figure shows that the free shape of the FPM has strong influence on the performances of the FPM such as workspace, resolution transmission scale and variation of resolution distribution. The RT scale of the FPM varies around a certain value, which is the RT scale of the kinematic mechanism of the FPM at the position specified by the free shape. In the meantime, RT scales are very different in the workspace of the kinematic mechanism of the FPM. Therefore, the free shape takes the decisive role in the selection

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

of the FPM possessing the best global RT scale among the FPMs of the same dimension. Moreover, the slow changing rate of the RT scale at the free-shape position will lead to the high uniformity.

Fig 5.10. Free shapes of FPMs A and B.

Table 5.3. Comparison between two free shapes of planar 3-DOF FPM.

	FPM A	FPM B
Workspace area $W \text{ (mm}^2\text{)}$	95.6	81.2
Global RT scale R_{global}	1.063	1.968
Uniformity U	0.7	0.54
Maximum RT scale R_{max}	1.31	2.74
Minimal RT scale R_{min}	0.93	1.48
Resolution variety $\frac{\max(R - R_{\text{global}})}{R_{\text{global}}} 100 \text{ (%)}$	23.24	39.23

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

Fig. 5.11. Distribution of RT scale of FPM A and FPM B.

5.2.3 Selective-Actuation 3-DOF Translational Flexure Parallel Mechanism

This case studies the resolution of the SA 3-DOF translational FPM whose Jacobian matrix is formulated using PRB-D model. The conceptual design of the FPM is mentioned in Chapter 3 (Fig. 3.13) and its PRB-D model is studied in Chapter 4 (Eq. 4.52) and [71, 72].

A. Workspace

The workspace of the SA 3-DOF translational FPM can be obtained by solving the inverse kinematics of the FPM in Chapter 4 (Eq. 4.16). The workspace of the FPM is the intersection of volumes W_1 (configuration), W_2 (actuator range) and W_3 (strength of material). The reachable volume W_1 is determined based on the solvable condition of the inverse kinematics:

$$W_1 = \left\{ (x, y, z) : \left| \frac{c^2 + d^2 + a^2 - b^2}{2a\sqrt{c^2 + d^2}} \right| \leq 1 \quad \text{and} \quad \left| \frac{c^2 + d^2 + b^2 - a^2}{2b\sqrt{c^2 + d^2}} \right| \leq 1 \quad \forall i, j = 1, 2, 3 \right\}, \quad (5.36)$$

where

a and b are the dimensions of links A_iB_i and B_iC_i respectively,

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

$$c = a \cos \theta_{ij0} + b \cos \psi_{ij0} + u, d = a \sin \theta_{ij0} + b \sin \psi_{ij0} + v,$$

and $u \equiv x, v \equiv y$ (if $i = 1$), $u \equiv y, v \equiv z$ (if $i = 2$), $u \equiv z, v \equiv x$ (if $i = 3$).

The reachable volume due to actuation range W_2 is determined as:

$$W_2 = \{(x, y, z) \mid x \leq S; y \leq S; z \leq S\}, \quad (5.37)$$

where S is the range of linear actuator.

The reachable volume due to strength of material W_3 is determined by the limited deflection of flexure joints:

$$W_3 = \left\{ (x, y, z) \mid \begin{array}{l} x \leq \delta_s; y \leq \delta_s; z \leq \delta_s \\ |\psi_{ij} - \psi_{ij0}| \leq \delta_\psi \\ |\theta_{ij} - \theta_{ij0}| \leq \delta_\theta \end{array} \quad \forall i, j = 1, 2, 3 \right\}, \quad (5.38)$$

where δ_θ , δ_ψ and δ_s are the joint limits of the passive flexure hinges A_{ij} , B_{ij} and the actuated flexure prismatic joint (the double linear spring stage) respectively.

B. Resolution transmission scale

According to the PRB-D model, the actual motion of the end-effector is the combined effect of the PRB modeling and the deformation of flexure members as shown in Eq. (4.52). The motion of the FPM is very small. So, Eq. (4.52) can be written as $(dX, dY, dZ)^T = \mathbf{J} (dx, dy, dz)^T$, where $\mathbf{J}$ is the Jacobian of the mechanism

$$\mathbf{J} = \begin{bmatrix} 1 + k_{x-Fx} & k_{y-Fx} & k_{z-Fx} \\ k_{x-Fy} & 1 + k_{y-Fy} & k_{z-Fy} \\ k_{x-Fz} & k_{y-Fz} & 1 + k_{z-Fz} \end{bmatrix}. \quad (4.52)$$

We then apply GISA to compute the global value of RT scale and the uniformity of resolution. Figure 5.12 illustrates the distribution of RT scale over the X-Y space when the Z-position is fixed at the initial position. We can observe from Fig. 5.12 that the variation of resolution increases when the end-effector moves away from the initial

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

posture significantly. The RT scale changes from 7.0 at initial position to 22.5 at the farthest boundary point. When the end-effector moves away from both the x- and y-axes, the RT scale increases faster (from 7.0 to 22.5) than it moves only along either the x- or y- axis (from 7.0 to 10.5). The significant variation of the RT scale is due to decrease of the stiffness of the FPM at the boundary of the workspace. The increase in deformation of flexure members of the FPM causes the controllable motion of the end-effector to increase, i.e., the resolution becomes worse and the RT scale increases.

Fig. 5.12. RT scale distribution with $Z = 0$.

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

This case study also shows the differences in the RT scale using PRB and PRB-D models. The PRB model of the SA 3-DOF translational FPM leads to an identity Jacobian matrix. Therefore, the RT scale based on the PRB model is always unity and the positioning resolution of the FPM remains unchanged and is equal to the resolution of actuators. However, Jacobian matrix in the PRB-D model of the FPM is not a constant matrix and the RT scale varies over the workspace as seen in Fig. 5.12. The deformation of the flexure members itself is the reason of this difference. Therefore, the actual positioning resolution of the FPM also varies.

5.4 Summary

This chapter investigated the resolution transmission from actuators to the end-effector. Indices expressing the influence of dimensions and free shape of the FPM on the resolution transmission were proposed. The resolution transmission at one particular pose of the FPM is studied through the establishment of the local RT scale based on the Jacobian matrix $\mathbf{J}$. The average RT scale over the entire workspace of the FPM and the ratio between the minimum and the maximum of the local RT scale are defined as evaluation criteria: the global RT scale and the uniformity that can be used to evaluate globally the resolution of the FPM. Resolution of the actuator and the global RT scale specify the lower limit and the uniformity of resolution of the end-effector. These criteria and other previously studied global criteria such as the workspace volume are computed using the GISA algorithm based on the sampling method. The proposed criteria allow the designer to predict the average and the distribution of resolution of the FPM. The influence of dimensions and free shape of FPM on resolution can be observed using the proposed criteria. Therefore, the designer can obtain the FPM with desired resolutions by adjusting the dimension and free shape of the mechanism. In other words,

CHAPTER 5. EVALUATION OF POSITIONING RESOLUTION.

the proposed criteria can be used as an objective measure for the resolution performance of FPMs.

Chapter 6

Stiffness Analysis

Flexure parallel mechanisms designed with high stiffness in all six DOFs possess good dynamic characteristics such as high natural frequencies and good repeatability. High stiffness also prevents vibration resonance and can resist deformation under external forces as well as perform precise motion. Currently stiffness of the flexure mechanism is studied by two methods: the FEM modeling (Zhang et al. [27], Schotborgh [28], Xu et al. [29] and Woronko et al. [47]) and the reduced analytical modeling (Yi et al. [14], Paros and Weisbord [21], Lobontiu [23, 24, 27, 31, 49], Wu et al. [25], Tseitlin [26], Xu et al. [29], Henein et al. [30], Hsiao [32], Kim [34], Alici et al. [73]). It takes a lot of time and computation resource to establish the analytical relationship between stiffness of flexure mechanism and the flexure member dimension and the free shape of flexure mechanism using finite element method. Therefore, despite being an accurate approach, FEM modeling is only suitable for performance verification of the mechanism before fabrication and assembly. The analytical method is used to investigate an individual flexure joint separated from the mechanism [21, 23-27, 29-32, 49]. The reduced

CHAPTER 6. STIFFNESS ANALYSIS.

analytical method is recently proposed for the investigation of the stiffness of flexure mechanisms [14, 34, 73]. Reduced stiffness modeling assumes that (1) stiffness in non-working directions of flexure joints and links are infinite and only stiffness in working directions of the end-effector is modeled; (2) external forces applied to the end-effector follow only the direction of the degrees of freedom. With those assumptions, the modeling is more suitable for the planar FPMs. This chapter presents the analytical formulation of the 6×6 stiffness matrix of the FPM when considering stiffness of the mechanism as well as flexure members in all directions. The transformation of the infinitesimal motions from joints to the end-effector that is often used in the study of stiffness and kinematics of robotic mechanisms [64, 74-78] is employed to formulate the compliance/stiffness matrices of the FPMs. In stiffness analysis of robotic mechanisms, mechanical transmission mechanisms and servo systems are assumed as the main sources of compliance in order to simplify the problem. In the case of FPMs, the compliance of the mechanism is mainly originated from the compliance of the flexure members. In addition, the flexure joint in the FPM is considered as a “6-DOF” joint, i.e., the compliance exists in all directions of the joints unlike the “rigid” DOF in the robotic mechanisms. Recently, Hale et al. [79] uses transformation matrices to reflect contact stiffness matrices to the parallel system of a kinematic coupling. This idea can be applied to the FPM stiffness modeling. The proposed modeling regards the influence of all flexure members on the general stiffness of the mechanism. Therefore, stiffness index for FPM design can be defined based on this model. The stiffness index allows one to select and distribute the stiffness of flexure members to obtain the desired stiffness of the mechanism.

The second issue discussed in this chapter is the actuation stiffness that is the resistance of the structure that the actuator must overcome in order to drive the flexure mechanism.

CHAPTER 6. STIFFNESS ANALYSIS.

The study on actuation stiffness allows us to select the appropriate actuator with sufficiently large force to drive the flexure mechanism.

6.1 Stiffness Modeling and Stiffness Index

The stiffness of an FPM is defined as the forces that cause a unit deflection of the end-effector. It can be determined as the inverse of the compliance - the end-effector deflection due to a unit load at the end-effector. The structure of an FPM can be described as the parallel combination of flexure serial chains and flexure hybrid structures (Fig. 6.1), where the flexure serial chain is a serial connection of basic flexure members, and the flexure hybrid structure is a serial connection of flexure chains and parallel combinations of basic flexure members.

$$\text{FPM} = \left\{ \begin{array}{l} \text{serial chains} = \{ \text{basic flexure members} \} \\ \text{hybrid structures} = \left\{ \begin{array}{l} \{ \text{basic flexure members} \} \\ \{ \text{basic flexure members} \} \end{array} \right\} \end{array} \right\}$$

serial connection

serial connection

parallel combination

serial connection

parallel combination

Fig. 6.1. Structure of flexure parallel mechanism.

Consequently the purposes of the stiffness modeling are: (1) to establish the stiffness matrices of the basic flexure members, and (2) to put together the stiffness matrices of the basic flexure members/serial or parallel structure in a serial connection/parallel combination in order to obtain the stiffness matrix of the higher serial/parallel structure. The first task is well studied in the recent researches [21, 23-27, 29-32, 49]. The second task is presented in the following subsections.

CHAPTER 6. STIFFNESS ANALYSIS.

6.1.1 Stiffness Matrix of a Serial Connection

Suppose that a flexure serial chain consists of n flexure members (links/joints) (Fig. 6.2). Denote $\mathbf{K}_i$ and $\mathbf{C}_i$ as the local stiffness and compliance matrices of the i^{th} flexure member that are established in the local frame attached to the flexure member. Denote $\mathbf{J}_i$ as the 6×6 Jacobian matrix relating the 6×1 vector of elastic deformation $\mathbf{X}_i$ of the i^{th} flexure member to the 6×1 motion vector $\Delta \mathbf{X}_i$ of the tip of the chain. We make the following assumptions to express the full stiffness in all directions:

- All flexure members contribute to the end-effector compliance based on the superposition principle.
- All flexure joints are assumed to have six DOFs and can transmit infinitesimal linear displacements and rotation about axes of the local frame.
- The stiffness or compliance of flexure members in six directions is considered.

Fig. 6.2. Serial connection.

The Jacobian matrix $\mathbf{J}_i$ cannot be determined using the ordinary kinematic analysis assumptions that links have infinite rigidity in all directions and joints have infinite rigidity in directions not along their DOFs. Here the Jacobian is determined based on the

CHAPTER 6. STIFFNESS ANALYSIS.

superposition of deformations in a serial chain. It is assumed that the remaining parts of the chain other than the i^{th} flexure member become rigid. The matrix $\mathbf{J}_i^T$ relates the 6×1 external force vector $\mathbf{F}$ at the tip to the 6×1 reaction force vector $\mathbf{F}_i$ at the i^{th} flexure member. Thus we have

$$\Delta \mathbf{X}_i = \mathbf{J}_i \mathbf{X}_i, \quad (6.1)$$

$$\mathbf{F}_i = \mathbf{J}_i^T \mathbf{F}. \quad (6.2)$$

The deformation at the tip caused by the deformation of the flexure members is

$$\mathbf{X} = \sum_{i=1}^n \Delta \mathbf{X}_i = \sum_{i=1}^n \mathbf{J}_i \mathbf{X}_i. \quad (6.3)$$

The deformation at the tip can be expressed as the product of the external force $\mathbf{F}$ and the compliance matrix $\mathbf{C}$ of the chain. Because of superposition, it can be expressed as the summation of the products of the reaction force $\mathbf{F}_i$ and local compliance matrix $\mathbf{C}_i$. Hence,

$$\mathbf{C}\mathbf{F} = \sum_{i=1}^n \mathbf{J}_i \mathbf{C}_i \mathbf{F}_i = \sum_{i=1}^n \mathbf{J}_i \mathbf{C}_i \mathbf{J}_i^T \mathbf{F}. \quad (6.4)$$

The compliance of the serial chain at the tip can be found as

$$\mathbf{C} = \sum_{i=1}^n \mathbf{J}_i \mathbf{C}_i \mathbf{J}_i^T. \quad (6.5)$$

Let $\mathbf{J}$ and $\mathbf{C}^*$ be the $6 \times 6n$ Jacobian matrix and $6n \times 6n$ local compliance matrix of the serial chain,

$$\mathbf{J} = [\mathbf{J}_1 \mathbf{J}_2 \cdots \mathbf{J}_n], \quad (6.6)$$

$$\mathbf{C}^* = \text{diag}(\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_n). \quad (6.7)$$

Equation (6.5) can be described as a 6×6 matrix

$$\mathbf{C} = \mathbf{J} \mathbf{C}^* \mathbf{J}^T. \quad (6.8)$$

CHAPTER 6. STIFFNESS ANALYSIS.

The stiffness matrix $\mathbf{K}$ of the serial chain is the inverse of the compliance matrix $\mathbf{C}$:

$$\mathbf{K} = (\mathbf{J}^T)^{-1} \mathbf{C}^{*-1} \mathbf{J}^{-1}. \quad (6.9)$$

6.1.2 Stiffness Matrix of a Parallel Combination

A parallel structure can be considered as an infinite rigid platform supported by n limbs that can be serial chains or hybrid structures of serial chains and other parallel combinations (Fig. 6.3). The platform and tips of limbs will have the same angular displacements. However, the linear displacement of the reference point of the platform is the result of both linear and angular displacements of the tip of each limb:

$$\alpha_x = \alpha_{x_i}, \quad \alpha_y = \alpha_{y_i}, \quad \alpha_z = \alpha_{z_i},$$

$$x = x_i + r_{iz}\alpha_{y_i} - r_{iy}\alpha_{z_i}, \quad y = y_i - r_{iz}\alpha_{x_i} + r_{ix}\alpha_{z_i}, \quad z = z_i + r_{iy}\alpha_{x_i} - r_{ix}\alpha_{y_i},$$

where

$(x, y, z, \alpha_x, \alpha_y, \alpha_z)^T$ and $(x_i, y_i, z_i, \alpha_{x_i}, \alpha_{y_i}, \alpha_{z_i})^T$ are the displacement vectors of the platform at the reference point and the tip of the i^{th} limb respectively.

$\mathbf{r}_i = (r_{ix}, r_{iy}, r_{iz})^T$ is the vector pointing from the tip of the i^{th} limb to the reference point.

Fig. 6.3. Parallel connection.

CHAPTER 6. STIFFNESS ANALYSIS.

Therefore,

$$\mathbf{X} = \mathbf{A}_1 \mathbf{X}_1 = \mathbf{A}_2 \mathbf{X}_2 = \cdots = \mathbf{A}_n \mathbf{X}_n, \quad (6.10)$$

where $\mathbf{X}$ and $\mathbf{X}_i$ are, respectively, the 6×1 vectors of linear and angular displacements of the reference point and the tip of the i^{th} limb ($i = 1, 2, \dots, n$); $\mathbf{A}_i$ is the amplification matrix of the displacement that

$$\mathbf{A}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & r_{iz} - r_{iy} \\ 0 & 1 & 0 & -r_{iz} & 0 & r_{ix} \\ 0 & 0 & 1 & r_{iy} - r_{ix} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6.11)$$

The 6×1 vector of force and moment $\mathbf{F}$ applied to the reference point of the platform is distributed to the tip of each limb as a 6×1 vector of force and moment $\mathbf{F}_i$.

$$\mathbf{F} = \sum_{i=1}^n \mathbf{A}_{Fi} \mathbf{F}_i, \quad (6.12)$$

where $\mathbf{A}_{Fi}$ is the transformation matrix when the vector of force and moment $\mathbf{F}_i$ is moved from the tip of the i^{th} limb to the reference point and

$$\mathbf{A}_{Fi} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & r_{iz} - r_{iy} & 1 & 0 & 0 & 0 \\ -r_{iz} & 0 & r_{ix} & 0 & 1 & 0 \\ r_{iy} - r_{ix} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6.13)$$

Let $\mathbf{K}$ and $\mathbf{K}_i$ be the stiffness matrices of the parallel combination and the i^{th} limb respectively. Expressing Eq. (6.12) in stiffness and deformation terms and noting Eq. (6.10), we have

$$\mathbf{KX} = \sum_{i=1}^n \mathbf{A}_{Fi} \mathbf{K}_i \mathbf{X}_i = \sum_{i=1}^n \mathbf{A}_{Fi} \mathbf{K}_i \mathbf{A}_i^{-1} \mathbf{X}. \quad (6.14)$$

CHAPTER 6. STIFFNESS ANALYSIS.

The stiffness becomes

$$\mathbf{K} = \sum_{i=1}^n \mathbf{A}_{Fi} \mathbf{K}_i \mathbf{A}_i^{-1} = \mathbf{A}_F^* \mathbf{K}^* \mathbf{A}^*, \quad (6.15)$$

where

$$\mathbf{A}_F^* = [\mathbf{A}_{F1} \ \mathbf{A}_{F2} \ \cdots \ \mathbf{A}_{Fn}], \ \mathbf{K}^* = [\mathbf{K}_1 \ \mathbf{K}_2 \ \cdots \ \mathbf{K}_n]^T, \ \mathbf{A}^* = [\mathbf{A}_1^{-1} \ \mathbf{A}_2^{-1} \ \cdots \ \mathbf{A}_n^{-1}] \quad (6.16)$$

6.1.3 Stiffness Index

The stiffness matrix of the FPM obtained based on the above method has the following form:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_I & \mathbf{K}_{II} \\ \mathbf{K}_{III} & \mathbf{K}_{IV} \end{bmatrix}. \quad (6.17)$$

The submatrices $\mathbf{K}_I$, $\mathbf{K}_{II}$, $\mathbf{K}_{III}$ and $\mathbf{K}_{IV}$ are

$$\mathbf{K}_I = \begin{bmatrix} K_{x-F_x} & K_{y-F_x} & K_{z-F_x} \\ K_{x-F_y} & K_{y-F_y} & K_{z-F_y} \\ K_{x-F_z} & K_{y-F_z} & K_{z-F_z} \end{bmatrix} \text{ (N/m): displacement-force stiffness,} \quad (6.18)$$

$$\mathbf{K}_{II} = \begin{bmatrix} K_{\alpha_x-F_x} & K_{\alpha_y-F_x} & K_{\alpha_z-F_x} \\ K_{\alpha_x-F_y} & K_{\alpha_y-F_y} & K_{\alpha_z-F_y} \\ K_{\alpha_x-F_z} & K_{\alpha_y-F_z} & K_{\alpha_z-F_z} \end{bmatrix} \text{ (N/rad): orientation-force stiffness,} \quad (6.19)$$

$$\mathbf{K}_{III} = \begin{bmatrix} K_{x-M_x} & K_{y-M_x} & K_{z-M_x} \\ K_{x-M_y} & K_{y-M_y} & K_{z-M_y} \\ K_{x-M_z} & K_{y-M_z} & K_{z-M_z} \end{bmatrix} \text{ (Nm/m): displacement-torque stiffness,} \quad (6.20)$$

$$\mathbf{K}_{IV} = \begin{bmatrix} K_{\alpha_x-M_x} & K_{\alpha_y-M_x} & K_{\alpha_z-M_x} \\ K_{\alpha_x-M_y} & K_{\alpha_y-M_y} & K_{\alpha_z-M_y} \\ K_{\alpha_x-M_z} & K_{\alpha_y-M_z} & K_{\alpha_z-M_z} \end{bmatrix} \text{ (Nm/rad): orientation-torque stiffness,} \quad (6.21)$$

CHAPTER 6. STIFFNESS ANALYSIS.

where subscripts F , M and α denote the force, the moment and the rotation about an axis respectively. For instances, $K_{\alpha_y-F_z}$ expresses the stiffness relating the force along z-axis and the rotation about y-axis; K_{z-M_y} expresses the stiffness relating the moment about y-axis and the linear displacement along z-axis.

The stiffness matrices have different units and their values have different effect on the deformation as well as the accuracy of the flexure mechanism. The decrement of any element in the submatrices will weaken the FPM. Therefore, we define three indices based on the element with the smallest value in the submatrices to evaluate the ability of the mechanism to prevent rotation and linear deformations caused by external force or torque. The indices are defined as:

$$K_{1\min} = \min_{\substack{i=1 \\ j=1}}^3 |K_I[i, j]|, \quad (6.22)$$

$$K_{2\min} = \min_{\substack{i=1 \\ j=1}}^3 |K_{IV}[i, j]|, \quad (6.23)$$

$$K_{3\min} = \min_{\substack{i=1 \\ j=1}}^3 \{ |K_{II}[i, j]|, |K_{III}[i, j]| \}. \quad (6.24)$$

In principle, all three stiffness indices need to be maximized in the optimal design.

6.2 Application of Stiffness Modeling

6.2.1 Stiffness Matrix of Double Linear Spring Stage

A. Formulation of Stiffness Matrix of Linear Spring Stage

A linear spring was modeled by Xu et al. [29] based on the virtual work principle. The modeling did not show the stiffness along non-working directions. The modeling is not

CHAPTER 6. STIFFNESS ANALYSIS.

accurate because of the assumption that only the torsional stiffness of the flexure hinges takes part in the stiffness of the linear spring. This section presents the stiffness of a linear spring along all directions.

Fig. 6.4. Linear spring stage.

A linear spring shown in Fig. 6.4 can be considered as a parallel structure with an infinite rigid platform $B_1 B_2$ and two similar limbs $A_1 B_1$ and $A_2 B_2$. Each limb $A_i B_i$ ($i = 1, 2$) is a serial chain with two flexure hinges A_i , B_i and one link $A_i B_i$. The compliance matrices C_A and C_B of flexure hinges A_i and B_i are established in [21] and presented in Appendix B. The compliance matrix C_{AB} of link $A_i B_i$ ($i = 1, 2$) is determined as the compliance matrix of a cantilever with one fixed end and one free end:

$$C_{AB} = \begin{bmatrix} C_{AB11} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{AB22} & 0 & 0 & 0 & C_{AB26} \\ 0 & 0 & C_{AB33} & 0 & C_{AB35} & 0 \\ 0 & 0 & 0 & C_{AB44} & 0 & 0 \\ 0 & 0 & C_{AB53} & 0 & C_{AB55} & 0 \\ 0 & C_{AB62} & 0 & 0 & 0 & C_{AB66} \end{bmatrix}, \quad (6.25)$$

CHAPTER 6. STIFFNESS ANALYSIS.

where

$$\begin{aligned} C_{AB11} &= \frac{l_{AB}^3}{3EI_x}, C_{AB22} = \frac{l_{AB}^3}{3EI_y}, C_{AB26} = -\frac{l_{AB}^2}{2EI_z}, C_{AB33} = \frac{l_{AB}^3}{3EI_z}, \\ C_{AB35} &= \frac{l_{AB}^2}{2EI_y}, C_{AB44} = \frac{l_{AB}}{GI_p}, C_{AB53} = \frac{l_{AB}^2}{2EI_z}, C_{AB55} = \frac{l_{AB}}{EI_y}, \\ C_{AB62} &= -\frac{l_{AB}^2}{2EI_y}, C_{AB66} = \frac{l_{AB}}{EI_z}. \end{aligned} \quad (6.26)$$

and l_{AB} is the length of link A_iB_i and I_x , I_y and I_z are the moments of inertia of the cross section respectively, E and G are the Young's modulus and shear modulus of the material.

The local compliance matrices of flexure members are assembled to create the local compliance 18×18 matrix of the limb as follows.

$$\mathbf{C}_{limb}^* = diag(\mathbf{C}_A, \mathbf{C}_{AB}, \mathbf{C}_B). \quad (6.27)$$

The Jacobian matrices corresponding to the flexure members are determined as

$$\mathbf{J}_A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & l \\ 0 & 0 & 1 & 0 & -l & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } \mathbf{J}_{AB} = \mathbf{J}_B = \mathbf{I}. \quad (6.28)$$

These matrices are concatenated together as a 6×18 matrix:

$$\mathbf{J}_{limb} = [\mathbf{J}_A \quad \mathbf{J}_{AB} \quad \mathbf{J}_B]. \quad (6.29)$$

The compliance matrix of limb A_iB_i is thus:

CHAPTER 6. STIFFNESS ANALYSIS.

$$\mathbf{C}_{\text{limb}} = \mathbf{J}_{\text{limb}} \mathbf{C}_{\text{limb}}^* \mathbf{J}_{\text{limb}}^T = \begin{bmatrix} C_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 & 0 & C_{26} \\ 0 & 0 & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & C_{53} & 0 & C_{55} & 0 \\ 0 & C_{62} & 0 & 0 & 0 & C_{66} \end{bmatrix}. \quad (6.30)$$

The elements of the compliance matrix $\mathbf{C}_{\text{limb}}$ are

$$\begin{aligned} C_{11} &= C_{A11} + C_{AB11} + C_{B11}, \\ C_{22} &= C_{A22} + lC_{A26} + lC_{A62} + l^2C_{AB66} + C_{AB22} + C_{B22}, \\ C_{33} &= C_{A33} - lC_{A35} + lC_{A53} - l^2C_{A55} + C_{AB33} + C_{B33}, \\ C_{44} &= C_{A44} + C_{AB44} + C_{B44}, \quad C_{55} = C_{A55} + C_{AB55} + C_{B55}, \\ C_{66} &= C_{A66} + C_{AB66} + C_{B66}, \quad C_{26} = C_{A26} + lC_{A66} + C_{AB26} + C_{B26}, \\ C_{35} &= C_{A35} + lC_{A55} + C_{AB35} + C_{B35}, \quad C_{53} = C_{A53} - lC_{A55} + C_{AB53} + C_{B53}, \\ C_{62} &= C_{A62} + lC_{A66} + C_{AB62} + C_{B62}. \end{aligned} \quad (6.31)$$

C_{ABij} , C_{Aij} and C_{Bij} denote the elements in the i^{th} row and j^{th} column of compliance matrices $\mathbf{C}_{AB}$, $\mathbf{C}_A$, and $\mathbf{C}_B$ of link AB and flexure hinges A and B respectively. The stiffness matrix of one limb is

$$\mathbf{K}_{\text{limb}} = \mathbf{C}_{\text{limb}}^{-1}. \quad (6.32)$$

The structure of the linear spring has symmetry. Hence, point O at the center of the platform is selected as the reference point. The vectors pointing from the tips B_i of limbs A_iB_i to the reference point O are

$$\mathbf{r}_{A_1B_1} = \vec{B_1O} = (0, -d, 0)^T, \quad \mathbf{r}_{A_2B_2} = \vec{B_2O} = (0, d, 0)^T, \quad (6.33)$$

where d is half of the length of the platform of the linear spring stage.

CHAPTER 6. STIFFNESS ANALYSIS.

Applying $\mathbf{r}_{A_1B_1}$ and $\mathbf{r}_{A_2B_2}$ to Eq. (6.11), we have the amplification matrices of the two limbs A_1B_1 and A_2B_2 as follows.

$$\mathbf{A}_{A_1B_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & d \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -d & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_{A_2B_2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -d \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6.34)$$

Also applying $\mathbf{r}_{A_1B_1}$ and $\mathbf{r}_{A_2B_2}$ to Eq. (6.13), we have the force transformation matrices of the tips of two limbs A_1B_1 and A_2B_2 as follows.

$$\mathbf{A}_{FA_1B_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -d & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ d & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_{FA_2B_2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & d & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -d & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6.35)$$

The stiffness matrix of the linear spring stage is

$$\mathbf{K}_{ls} = \mathbf{A}_{Fls}^* \mathbf{K}_{ls}^* \mathbf{A}_{ls}^*, \quad (6.36)$$

where

$$\mathbf{A}_{Fls}^* = [\mathbf{A}_{FA_1B_1} \ \mathbf{A}_{FA_2B_2}], \ \mathbf{K}_{ls}^* = [\mathbf{K}_{limb} \ \mathbf{K}_{limb}]^T, \ \mathbf{A}_{ls}^* = [\mathbf{A}_{A_1B_1}^{-1} \ \mathbf{A}_{A_2B_2}^{-1}]. \quad (6.37)$$

B. Stiffness Matrix of Double Linear Spring Stage

A double linear spring stage has a parallel and symmetric structure with two similar linear spring stages described in Fig. 6.5. The stiffness matrix of the double linear spring stage is given by

$$\mathbf{K}_S = 2\mathbf{K}_{ls}. \quad (6.38)$$

Hence, we have

CHAPTER 6. STIFFNESS ANALYSIS.

$$\mathbf{K}_S = \begin{bmatrix} K_{S11} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{S22} & 0 & 0 & 0 & K_{S26} \\ 0 & 0 & K_{S33} & 0 & K_{S35} & 0 \\ 0 & 0 & 0 & K_{S44} & 0 & 0 \\ 0 & 0 & K_{S53} & 0 & K_{S55} & 0 \\ 0 & K_{S62} & 0 & 0 & 0 & K_{S66} \end{bmatrix}, \quad (6.39)$$

where

$$\begin{aligned} K_{S11} &= \frac{4}{C_{11}}, \quad K_{S22} = \frac{4C_{66}}{C_{22}C_{66} - C_{26}C_{62}}, \quad K_{S26} = \frac{4C_{26}}{C_{26}C_{62} - C_{22}C_{66}}, \\ K_{S33} &= \frac{4C_{55}}{C_{33}C_{55} - C_{35}C_{53}}, \quad K_{S35} = \frac{4C_{35}}{C_{35}C_{53} - C_{33}C_{55}}, \\ K_{S44} &= 4 \frac{C_{35}C_{53} - C_{33}C_{55} - d^2 C_{44}C_{55}}{C_{35}C_{44}C_{53} - C_{33}C_{44}C_{55}}, \\ K_{S53} &= \frac{4C_{53}}{C_{35}C_{53} - C_{33}C_{55}}, \quad K_{S55} = \frac{4C_{33}}{C_{33}C_{55} - C_{35}C_{53}}, \\ K_{S62} &= \frac{4C_{62}}{C_{26}C_{62} - C_{22}C_{66}}, \quad K_{S66} = 4 \frac{C_{11}C_{22} - d^2 C_{26}C_{62} + d^2 C_{22}C_{66}}{C_{11}C_{22}C_{66} - C_{11}C_{26}C_{62}}. \end{aligned} \quad (6.40)$$

Fig. 6.5. Double linear spring stage used as a flexure prismatic joint.

CHAPTER 6. STIFFNESS ANALYSIS.

Definition 6.1: Linear stiffness of a linear spring stage is the stiffness along the working direction of the linear spring stage.

The linear stiffness of the double linear spring stage is also the stiffness of the linear spring accompanying with a prismatic joint in the PRB model of the double linear spring stage. It is element K_{S22} of the stiffness matrix $\mathbf{K}_S$. Assuming that the rigidity of link A_iB_i is infinite and the flexure hinges A_i and B_i have the same dimensions, the linear stiffness K_{S22} can be simplified as follows.

$$K_{S22} = \frac{4C_{66}}{C_{22}C_{66} - C_{26}C_{62}} = \frac{8C_{A66}}{l^2 C_{A66}^2 + 4(C_{A66}C_{A33} - C_{A62}C_{A26})}, \quad (6.41)$$

Substituting elements C_{A26} , C_{A33} , C_{A62} and C_{A66} (Appendix B) into Eq. (6.41), we have:

$$K_{S22} = \frac{16Eet^{5/2}}{9\pi \left[l^2 + \frac{8}{9} \frac{E}{G} \left[1 - \frac{2.57}{\pi} \left(\frac{t}{r} \right)^{1/2} \right] t^2 \right] r^{1/2}}, \quad (6.42)$$

where e , t , r and l are the dimensions of flexure hinges (Appendix B) and the double linear spring stage (Fig. 6.5). E and G are the Young's modulus and shear modulus of the material of the double linear spring stage.

Note that the element K_{S22} formulated by Xu et al. [29] is based on the PRB model and the virtual work principle:

$$K_{S22}^* = \frac{16Eet^{5/2}}{9\pi l^2 r^{1/2}}, \quad (6.43)$$

which is used in the reduced stiffness modeling of the flexure-based mechanism. The difference of Eq. (6.42) and Eq. (6.43) will be verified by the experiment in next section.

C. Finite Element Method and Experimental Validations

The stiffness calculation of the double linear spring stage is verified using FEM modeling and experiments. ANSYS software is used to establish the FEM model of the

CHAPTER 6. STIFFNESS ANALYSIS.

double linear spring stage and to determine the deformation under static load. Figure 6.6 illustrates the FEM model of a double linear spring stage using the 3-D 10-node solid element SOLID 92. Force F_y was applied to the moving link and displacement y of the moving link is computed. The applied force and the displacement are compared with analytical results obtained by Xu's model [29] and our proposed model as well as experimental result.

Fig. 6.6. FEM model of double linear spring stage.

An experiment is set up to validate the proposed stiffness model. Figure 6.7 shows the experimental setup on the linear stiffness of the double linear spring stage. A linear actuator Picomotor is employed to drive the moving link. The pushing force F_y is measured by a Nano43 force/torque transducer (ATI) rigidly attached to the rear surface of the moving link. The displacement y of the moving link is measured using an

CHAPTER 6. STIFFNESS ANALYSIS.

inductive probe Mahr 1318. The force increment rate of the probe is 0.04 N/mm that is very small in comparison with the magnitude of the pushing force F_y . Therefore, deformation of the probe stand can be neglected. The ratio between the force and the displacement is the linear stiffness $K_S = F_y/y$.

Fig. 6.7. Experiment on stiffness of double linear spring stage.

Figure 6.8 illustrates the displacement of the moving link of the double linear spring stage and the applied force obtained by four different methods: Xu's model, our models, FEM analysis and the experiment. The figure shows that there exists a discrepancy between Xu's approach, Eq. (6.43), and the remaining three approaches. The accuracy of our model decreases when the displacement reaches the end of the motion range. Table 6.1 summarizes the stiffness result of the four methods. Our model has an error under

CHAPTER 6. STIFFNESS ANALYSIS.

5% and is more accurate than Xu's model. Hence, the proposed modeling is more suitable for the investigation of the FPM.

Fig. 6.8. Relationship between displacement y and pushing force F_y .

Table 6.1. Stiffness of double linear spring stage.

Model	Xu's model [29]	Proposed model	FEM	Experiment
K_{S22} (N/mm)	129.42	126.31	120.7	122.6
Error (%)	5.6	3.02	1.55	-

CHAPTER 6. STIFFNESS ANALYSIS.

6.2.2 Stiffness Modeling of Flexure Parallel Mechanism

The stiffness of a selective-actuation 3-DOF translational FPM described in Chapter 3 is analyzed here using the proposed method. Figure 6.9 shows the assembly scheme of the FPM with the construction of the limbs, the 3-flexure-limb group of the passive 3-D frame and the complete FPM. Based on this scheme, we establish the stiffness matrices of the components including: (1) the limbs $A_{ij}B_{ij}C_{ij}$ and the double linear spring stages, (2) the 3-flexure-limb group, (3) the limbs in x-, y- and z-directions of the FPM, and (4) the complete FPM.

Fig. 6.9. Connection scheme of SA 3-DOF translational FPM.

A. Expression of Compliance Matrices of Flexure Members in Base Frame

Limb $A_{ij}B_{ij}C_{ij}$ ($i, j = 1, 2, 3$) includes three flexure hinges A_{ij} , B_{ij} and C_{ij} and the two links $A_{ij}B_{ij}$ and $B_{ij}C_{ij}$ as shown in Fig. 6.10. The compliance matrices of the flexure members are previously presented. These matrices are obtained based on the local frame whose axes are in the same plane but not parallel with the axes of the base frame. In order to compute the deflection of the flexure member, the vector of reaction forces and

CHAPTER 6. STIFFNESS ANALYSIS.

moments need to be expressed in the local frame. The deflection of the flexure member expressed in the local frame must be expressed in a frame whose axes are parallel to the axes of the base frame in order to derive the compliance matrix of the limb. The expressions are simply changes of the coordinate system between the local frame of the flexure member and the frame obtained by a rotation of the local frame about its axis perpendicular to the plane of the 3-flexure-limb group containing the limb. Therefore, the two expression processes have the same rotation matrix given as follows.

Fig. 6.10. Transformation from local frame to base frame.

Flexure members in 3-flexure-limb group Z ($i = 1$): u -, v - or w -axis represents x -, y - or z -axis.

$$\mathbf{G}_1 = \begin{bmatrix} \cos \psi_{1j} & \sin \psi_{1j} & 0 & 0 & 0 & 0 \\ -\sin \psi_{1j} & \cos \psi_{1j} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \psi_{1j} & \sin \psi_{1j} & 0 \\ 0 & 0 & 0 & -\sin \psi_{1j} & \cos \psi_{1j} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6.44)$$

CHAPTER 6. STIFFNESS ANALYSIS.

Flexure members in 3-flexure-limb group Y ($i = 3$): u-, v- or w-axis represents z-, x- or y-axis.

$$\mathbf{G}_3 = \begin{bmatrix} \cos \psi_{3j} & 0 & -\sin \psi_{3j} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin \psi_{3j} & 0 & \cos \psi_{3j} & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \psi_{3j} & 0 & -\sin \psi_{3j} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin \psi_{3j} & 0 & \cos \psi_{3j} \end{bmatrix}. \quad (6.45)$$

Flexure members in 3-flexure-limb group X ($i = 2$): u-, v- or w-axis represents y-, z- or x-axis.

$$\mathbf{G}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos \psi_{2j} & \sin \psi_{2j} & 0 & 0 & 0 \\ 0 & -\sin \psi_{2j} & \cos \psi_{2j} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \psi_{2j} & \sin \psi_{2j} \\ 0 & 0 & 0 & 0 & -\sin \psi_{2j} & \cos \psi_{2j} \end{bmatrix}. \quad (6.46)$$

The reaction $\mathbf{F}_i^b$ at flexure member i expressed in the base frame is now expressed in the local frame as

$$\mathbf{F}_i = \mathbf{G}_i \mathbf{F}_i^b. \quad (6.47)$$

The flexure member deflection expressed in the local frame is

$$d\mathbf{X}_i = \mathbf{C}_i \mathbf{F}_i = \mathbf{C}_i \mathbf{G}_i \mathbf{F}_i^b. \quad (6.48)$$

This deflection is expressed in the base frame as follows.

$$d\mathbf{X}_i^b = \mathbf{G}_i^{-1} d\mathbf{X}_i = \mathbf{G}_i^{-1} \mathbf{C}_i \mathbf{G}_i \mathbf{F}_i^b. \quad (6.49)$$

The compliance matrix of a flexure member expressed in the base frame is

$$\mathbf{C}_{bi} = \mathbf{G}_i^{-1} \mathbf{C}_i \mathbf{G}_i, \quad (6.50)$$

where $\mathbf{C}_i$ is the compliance matrix of the flexure member expressed in the local frame.

CHAPTER 6. STIFFNESS ANALYSIS.

The compliance matrices of the flexure hinges A_{ij} , B_{ij} and C_{ij} and the links $A_{ij}B_{ij}$ and $B_{ij}C_{ij}$ are expressed in the base frame using Eq. (6.50) as:

$$\begin{aligned} C_{A_{ij}}^b &= G_i^{-1} C_{A_{ij}} G_i, \quad C_{B_{ij}}^b = G_i^{-1} C_{B_{ij}} G_i, \quad C_{C_{ij}}^b = G_i^{-1} C_{C_{ij}} G_i, \\ C_{A_{ij}B_{ij}}^b &= G_i^{-1} C_{A_{ij}B_{ij}} G_i, \quad C_{B_{ij}C_{ij}}^b = G_i^{-1} C_{B_{ij}C_{ij}} G_i, \end{aligned} \quad (6.51)$$

where $C_{A_{ij}}$, $C_{B_{ij}}$, $C_{C_{ij}}$, $C_{A_{ij}B_{ij}}$ and $C_{B_{ij}C_{ij}}$ are the compliance matrices expressed in the local frame of the flexure hinges A_{ij} , B_{ij} and C_{ij} , and the links $A_{ij}B_{ij}$ and $B_{ij}C_{ij}$ of the limb $A_{ij}B_{ij}C_{ij}$.

B. Compliance Matrix of the Limb $A_{ij}B_{ij}C_{ij}$

Limb $A_{ij}B_{ij}C_{ij}$ of the i^{th} 3-flexure-limb group ($i, j = 1, 2$ or 3) can be considered as a serial chain of five flexure members including three flexure hinges A_{ij} , B_{ij} and C_{ij} , and two links $A_{ij}B_{ij}$ and $B_{ij}C_{ij}$ (Fig. 6.11). The compliance matrices of flexure members are now concatenated to form the local compliance matrix of the limb as follows.

$$C_{\text{limb } ij}^* = \text{diag}(C_{A_{ij}}^b, C_{A_{ij}B_{ij}}^b, C_{B_{ij}}^b, C_{B_{ij}C_{ij}}^b, C_{C_{ij}}^b). \quad (6.52)$$

Fig. 6.11. Limb $A_{ij}B_{ij}C_{ij}$.

CHAPTER 6. STIFFNESS ANALYSIS.

The Jacobian matrix of the limb is determined as

$$\mathbf{J}_{A_{ij}B_{ij}C_{ij}} = [\mathbf{J}_{A_{ij}} \mathbf{J}_{A_{ij}B_{ij}} \mathbf{J}_{B_{ij}} \mathbf{J}_{B_{ij}C_{ij}} \mathbf{J}_{C_{ij}}], \quad (6.53)$$

where the Jacobian matrices of flexure members are established based on the summation of the transmitted deformation of flexure members (hinges A_{ij} , B_{ij} , C_{ij} and links $A_{ij}B_{ij}$ and $B_{ij}C_{ij}$) at point C_{ij} of limb $A_{ij}B_{ij}C_{ij}$.

Denote vectors:

$$\mathbf{r}_{A_{ij}} = A_{ij} \vec{C_{ij}} = (r_{A_{ij}}^x, r_{A_{ij}}^y, r_{A_{ij}}^z)^T, \quad \mathbf{r}_{B_{ij}} = B_{ij} \vec{C_{ij}} = (r_{B_{ij}}^x, r_{B_{ij}}^y, r_{B_{ij}}^z)^T, \quad (6.54)$$

where

For 3-flexure-limb group Z ($i = 1$):

$$\begin{aligned} r_{A_{1j}}^x &= a \cos \theta_{1j} + b \cos \psi_{1j}, & r_{A_{1j}}^y &= a \sin \theta_{1j} + b \sin \psi_{1j}, & r_{A_{1j}}^z &= 0, \\ r_{B_{1j}}^x &= b \cos \psi_{1j}, & r_{B_{1j}}^y &= b \sin \psi_{1j}, & r_{B_{1j}}^z &= 0. \end{aligned} \quad (6.55)$$

For 3-flexure-limb group X ($i = 2$):

$$\begin{aligned} r_{A_{2j}}^y &= a \cos \theta_{2j} + b \cos \psi_{2j}, & r_{A_{2j}}^z &= a \sin \theta_{2j} + b \sin \psi_{2j}, & r_{A_{2j}}^x &= 0, \\ r_{B_{2j}}^y &= b \cos \psi_{2j}, & r_{B_{2j}}^z &= b \sin \psi_{2j}, & r_{B_{2j}}^x &= 0. \end{aligned} \quad (6.56)$$

For 3-flexure-limb group Y ($i = 3$):

$$\begin{aligned} r_{A_{3j}}^z &= a \cos \theta_{3j} + b \cos \psi_{3j}, & r_{A_{3j}}^x &= a \sin \theta_{3j} + b \sin \psi_{3j}, & r_{A_{3j}}^y &= 0, \\ r_{B_{3j}}^z &= b \cos \psi_{3j}, & r_{B_{3j}}^x &= b \sin \psi_{3j}, & r_{B_{3j}}^y &= 0. \end{aligned} \quad (6.57)$$

The Jacobian matrices of flexure members are given as follows.

Flexure hinge A_{ij} :

CHAPTER 6. STIFFNESS ANALYSIS.

$$\mathbf{J}_{A_{ij}} = \begin{bmatrix} 1 & 0 & 0 & 0 & r_{A_{1j}}^z & -r_{A_{ij}}^y \\ 0 & 1 & 0 & -r_{A_{ij}}^z & 0 & r_{A_{ij}}^x \\ 0 & 0 & 1 & r_{A_{ij}}^y & -r_{A_{ij}}^x & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6.58)$$

Link $A_{ij}B_{ij}$ and flexure hinge B_{ij} :

$$\mathbf{J}_{A_{ij}B_{ij}} = \mathbf{J}_{B_{ij}} = \begin{bmatrix} 1 & 0 & 0 & 0 & r_{B_{ij}}^z & -r_{B_{ij}}^y \\ 0 & 1 & 0 & -r_{B_{ij}}^z & 0 & r_{B_{ij}}^x \\ 0 & 0 & 1 & r_{B_{ij}}^y & -r_{B_{ij}}^x & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6.59)$$

Link $B_{ij}C_{ij}$ and flexure hinge C_{ij} :

$$\mathbf{J}_{B_{ij}C_{ij}} = \mathbf{J}_{C_{ij}} = \mathbf{I}. \quad (6.60)$$

Following Eq. (6.8), the compliance matrix of the limb is determined as:

$$\mathbf{C}_{\text{limb}_{ij}} = \mathbf{J}_{A_{ij}B_{ij}C_{ij}} \mathbf{C}_{\text{limb}_{ij}}^* \mathbf{J}_{A_{ij}B_{ij}C_{ij}}^T. \quad (6.61)$$

The stiffness matrix of the limb is the inverse of the compliance matrix

$$\mathbf{K}_{\text{limb}_{ij}} = \mathbf{C}_{\text{limb}_{ij}}^{-1}. \quad (6.62)$$

C. Stiffness Matrix of 3-Flexure-Limb Groups

Limbs $A_{i1}B_{i1}C_{i1}$, $A_{i2}B_{i2}C_{i2}$, and $A_{i3}B_{i3}C_{i3}$ connect in parallel to form the i^{th} 3-flexure-limb group of the passive 3-D frame (Fig. 6.12). Let the center of the passive 3-D frame, O, be the reference point of the parallel combination. Denote vectors pointing from the tip C_{ij} of the limbs to the reference point O:

CHAPTER 6. STIFFNESS ANALYSIS.

$$\mathbf{r}_{ij} = \vec{C_{ij}O} = (r_{ij}^x, r_{ij}^y, r_{ij}^z)^T, \quad (6.63)$$

where the coordinates of the vectors are determined as follows.

Fig. 6.12. The i^{th} 3-flexure-limb group of the passive 3-D frame.

For 3-flexure-limb group Z ($i = 1$):

$$r_{1j}^x = -a \cos \theta_{1j} - b \cos \psi_{1j} - r \cos \gamma_j, \quad r_{1j}^y = -a \sin \theta_{1j} - b \sin \psi_{1j} - r \sin \gamma_j, \quad r_{1j}^z = c,$$

where $r = O_1A_{1j}$ and angles $\gamma_1 = 210^\circ$, $\gamma_2 = -30^\circ$ and $\gamma_3 = 90^\circ$ denote the position of

vectors $\vec{O_1A_{11}}$, $\vec{O_1A_{12}}$ and $\vec{O_1A_{13}}$ respectively.

For 3-flexure-limb group X ($i = 2$):

$$r_{2j}^y = -a \cos \theta_{2j} - b \cos \psi_{2j} - r \cos \gamma_j, \quad r_{2j}^z = -a \sin \theta_{2j} - b \sin \psi_{2j} - r \sin \gamma_j,$$

$$r_{2j}^x = c.$$

CHAPTER 6. STIFFNESS ANALYSIS.

For 3-flexure-limb group Y ($i = 3$):

$$r_{3j}^z = -a \cos \theta_{3j} - b \cos \psi_{3j} - r \cos \gamma_j \quad , \quad r_{3j}^x = -a \sin \theta_{3j} - b \sin \psi_{3j} - r \sin \gamma_j \quad ,$$

$$r_{3j}^y = c \quad .$$

Applying these vectors to Eq. (6.11) and Eq. (6.13), we have

$$A_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & r_{ij}^z & -r_{ij}^y \\ 0 & 1 & 0 & -r_{ij}^z & 0 & r_{ij}^x \\ 0 & 0 & 1 & r_{ij}^y & -r_{ij}^x & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad , \quad (6.64)$$

$$A_{Fij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & r_{ij}^z & -r_{ij}^y & 1 & 0 & 0 \\ -r_{ij}^z & 0 & r_{ij}^x & 0 & 1 & 0 \\ r_{ij}^y & -r_{ij}^x & 0 & 0 & 0 & 1 \end{bmatrix} \quad . \quad (6.65)$$

Applying A_{ij} and A_{Fij} and K_{limbij} to Eq. (6.15), we obtain the stiffness matrix of the i^{th} 3-flexure-limb group of the passive 3-D frame:

$$K_{\text{group}_i} = \sum_{j=1}^3 A_{Fij} K_{limbij} A_{Fij}^{-1} \quad . \quad (6.66)$$

D. Compliance Matrix of FPM Limbs

The platform of the FPM connects to the base by three limbs. Each FPM limb is a serial connection of one 3-flexure-limb group and the double linear spring stage. The thickness of the 3-flexure-limb group is considered negligible in comparison with the width of the passive 3-D frame. Therefore, Jacobian matrices of the linear spring and the 3-flexure-

CHAPTER 6. STIFFNESS ANALYSIS.

limb group are identity matrices I . The compliance matrix of the serial chain is hence given as:

$$C_{\text{limb}_i} = \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} C_{S_i} & O \\ O & K_{\text{group}_i}^{-1} \end{bmatrix} \begin{bmatrix} I \\ I \end{bmatrix} = C_{S_i} + K_{\text{group}_i}^{-1}, \quad (6.67)$$

where K_{group_i} and C_{S_i} are the stiffness matrix of the i^{th} 3-flexure-limb group and the compliance matrix of the double linear spring stage respectively.

Taking the inverse of compliance matrix C_{limb_i} , we get the stiffness matrix of the serial chain K_{limb_i} .

E. Stiffness Matrix of the Complete FPM

The three limbs – the subassemblies of the double linear spring stage and one 3-flexure-limb group are assembled in parallel to obtain the complete SA FPM. The tips of the limbs coincide at the center O of the passive 3-D frame. Using the center as the reference point of the parallel assemblage, the amplification matrices and the force transformation matrices of assemblage become the identity matrices. Consequently, the stiffness matrix of the complete FPM is the summation of the stiffness matrices of three subassemblies:

$$K = \sum_{i=1}^3 K_{\text{limb}_i}. \quad (6.68)$$

Stiffness distributions within the workspace in the X-Y plane ($Z = 0$) of elements of the stiffness matrix K : $K_{x-Fx} = K_{11}$, $K_{y-Fy} = K_{22}$ and $K_{z-Fz} = K_{33}$, which are the end-effector stiffness along X-, Y- and Z- directions respectively under applied forces F_x , F_y and F_z are shown in Fig. 6.13. From the figure we learn that the magnitude of stiffness K_{z-Fz} is symmetrical with respect to X=Y plane. Its magnitude decreases when the end-effector

CHAPTER 6. STIFFNESS ANALYSIS.

move away from the initial position and the decrement rate increases when the end-effector moves away from both X- and Y-axes. The magnitudes of K_{x-Fx} and K_{y-Fy} are generally similar to the distribution of K_{z-Fz} except that K_{x-Fx} (or K_{y-Fy}) increases when the end-effector moves only away from Y- (or X-) axis.

Fig. 6.13. Stiffness distribution in X-Y plane ($Z = 0$): (a) K_{z-Fz} ; (b) K_{x-Fx} ; (c) K_{y-Fy} .

6.3 Actuation Stiffness

To drive a flexure mechanism, the actuators need to provide actuation force/torque large enough to overcome not only the external load exerted on the end-effector but also the flexure load caused by the elastic characteristics of the flexure joints. The actuation

CHAPTER 6. STIFFNESS ANALYSIS.

force/torque due to elasticity increases when the end-effector moves away from its free shape position (initial position).

Definition 6.2: Actuation stiffness matrix of the n -DOF FPM is the $n \times n$ matrix expressing the increasing rate of the driving forces along the motion DOF of the end-effector.

The actuation stiffness matrix of the SA 3-DOF translational FPM is now formulated. The virtual work δW of the end-effector load, the driving forces and the flexure potential energy of the FPM kept at a certain position is zero, i.e.,

$$\delta W = \mathbf{F}^T d\mathbf{x} + \mathbf{P}^T d\mathbf{X} + \delta V = 0, \quad (6.69)$$

where

$\mathbf{F} = (F_x, F_y, F_z)^T$ is the driving forces;

$\mathbf{P} = (P_x, P_y, P_z)$ is the end-effector load;

δV is the differential of the potential energy V accumulated at the flexure joints.

The potential energy V of the flexure joints can be computed as

$$V = \frac{1}{2} \sum_{i=1}^3 \left\{ K_{S_i} (w - w_0)^2 + \sum_{j=1}^3 \left[K_{A_{ij}} (\theta_{ij} - \theta_{ij0})^2 + K_{C_{ij}} (\psi_{ij} - \psi_{ij0})^2 + K_{B_{ij}} (\psi_{ij} - \theta_{ij} - \psi_{ij0} + \theta_{ij0})^2 \right] \right\}, \quad (6.70)$$

where $K_{A_{ij}}$, $K_{B_{ij}}$ and $K_{C_{ij}}$ are the torsional stiffness of the flexure hinges A_{ij} , B_{ij} and C_{ij} respectively; K_{S_i} is the linear stiffness of the flexure prismatic joints of the limbs of the FPM; $\theta_{ij}, \theta_{ij0}$ and ψ_{ij}, ψ_{ij0} are respectively the current and initial angles of passive joints A_{ij} and B_{ij} or C_{ij} ; and $u \equiv x, v \equiv y, w \equiv z$ if $i = 1$; $u \equiv y, v \equiv z, w \equiv x$ if $i = 2$; $u \equiv z, v \equiv x, w \equiv y$ if $i = 3$.

Without loss of generality, we consider that there is no load at the end-effector.

Substituting Eq. (6.70) in to Eq. (6.69), we have

CHAPTER 6. STIFFNESS ANALYSIS.

$$F_x dx + F_y dy + F_z dz + \sum_{i=1}^3 \left\{ K_{S_i} (w - w_0) + \sum_{j=1}^3 \left[K_{A_{ij}} (\theta_{ij} - \theta_{ij0}) d\theta_{ij} + K_{C_{ij}} (\psi_{ij} - \psi_{ij0}) d\psi_{ij} + K_{B_{ij}} (\psi_{ij} - \theta_{ij} - \psi_{ij0} + \theta_{ij0}) (d\psi_{ij} - d\theta_{ij}) \right] \right\} = 0. \quad (6.71)$$

From Eq. (4.18) we have the partial differentials of passive joint angles

$$a_{ij} = \frac{\partial \theta_{ij}}{\partial u} = \frac{\cos \psi_{ij}}{a \sin(\psi_{ij} - \theta_{ij})}, \quad b_{ij} = \frac{\partial \theta_{ij}}{\partial v} = \frac{\sin \psi_{ij}}{a \sin(\psi_{ij} - \theta_{ij})}, \quad \frac{\partial \theta_{ij}}{\partial w} = 0, \quad (6.72)$$

$$c_{ij} = \frac{\partial \psi_{ij}}{\partial u} = \frac{-\cos \theta_{ij}}{a \sin(\psi_{ij} - \theta_{ij})}, \quad d_{ij} = \frac{\partial \psi_{ij}}{\partial v} = \frac{-\sin \theta_{ij}}{a \sin(\psi_{ij} - \theta_{ij})}, \quad \frac{\partial \psi_{ij}}{\partial w} = 0.$$

Therefore,

$$d\theta_{ij} = \frac{\partial \theta_{ij}}{\partial u} du + \frac{\partial \theta_{ij}}{\partial v} dv = a_{ij} du + b_{ij} dv, \quad d\psi_{ij} = \frac{\partial \psi_{ij}}{\partial u} du + \frac{\partial \psi_{ij}}{\partial v} dv = c_{ij} du + d_{ij} dv. \quad (6.73)$$

Substituting Eq. (6.73) into Eq. (6.71), we have

$$\begin{aligned} & F_x dx + F_y dy + F_z dz + K_{S_1} (z - z_0) dz + K_{S_2} (x - x_0) dx + K_{S_3} (y - y_0) dy \\ & + \sum_{j=1}^3 \{ K_{A_{1j}} (\theta_{1j} - \theta_{1j0}) (a_{1j} dx + b_{1j} dy) + K_{C_{1j}} (\psi_{1j} - \psi_{1j0}) (c_{1j} dx + d_{1j} dy) \\ & + K_{B_{1j}} (\psi_{1j} - \theta_{1j} - \psi_{1j0} + \theta_{1j0}) [(c_{1j} - a_{1j}) dx + (d_{1j} - b_{1j}) dy] \} \\ & + \sum_{j=1}^3 \{ K_{A_{2j}} (\theta_{2j} - \theta_{2j0}) (a_{2j} dy + b_{2j} dz) + K_{C_{2j}} (\psi_{2j} - \psi_{2j0}) (c_{2j} dy + d_{2j} dz) \\ & + K_{B_{2j}} (\psi_{2j} - \theta_{2j} - \psi_{2j0} + \theta_{2j0}) [(c_{2j} - a_{2j}) dy + (d_{2j} - b_{2j}) dz] \} \\ & + \sum_{j=1}^3 \{ K_{A_{3j}} (\theta_{3j} - \theta_{3j0}) (a_{3j} dz + b_{3j} dx) + K_{C_{3j}} (\psi_{3j} - \psi_{3j0}) (c_{3j} dz + d_{3j} dx) \\ & + K_{B_{3j}} (\psi_{3j} - \theta_{3j} - \psi_{3j0} + \theta_{3j0}) [(c_{3j} - a_{3j}) dz + (d_{3j} - b_{3j}) dx] \} = 0 \end{aligned} \quad (6.74)$$

Separating the element with dx , dy and dz , we can find the actuation forces as follows.

CHAPTER 6. STIFFNESS ANALYSIS.

Actuation force F_x :

$$\begin{aligned} F_x = & -K_{S2}(x - x_0) - \sum_{j=1}^3 [K_{A1j}(\theta_{1j} - \theta_{1j0})a_{1j} + K_{C1j}(\psi_{1j} - \psi_{1j0})c_{1j} \\ & + K_{B1j}(\psi_{1j} - \theta_{1j} - \psi_{1j0} + \theta_{1j0})(c_{1j} - a_{1j}) + K_{A3j}(\theta_{3j} - \theta_{3j0})b_{3j} \\ & + K_{C3j}(\psi_{3j} - \psi_{3j0})d_{3j} + K_{B3j}(\psi_{3j} - \theta_{3j} - \psi_{3j0} + \theta_{3j0})(d_{3j} - b_{3j})]. \end{aligned} \quad (6.75)$$

The motion of the flexure mechanism is very small in comparison with the conventional rigid-link mechanism. We can assume that

$$x - x_0 \approx dx, \quad y - y_0 \approx dy, \quad z - z_0 \approx dz, \quad \theta_{ij} - \theta_{ij0} \approx d\theta_{ij}, \quad \psi_{ij} - \psi_{ij0} \approx d\psi_{ij}.$$

With the above assumption, Eq. (6.75) can be rewritten as

$$\begin{aligned} F_x = & -K_{S2}dx - \sum_{j=1}^3 [K_{A1j}a_{1j}d\theta_{1j} + K_{C1j}c_{1j}d\psi_{1j} + K_{B1j}(d\psi_{1j} - d\theta_{1j})(c_{1j} - a_{1j}) \\ & + K_{A3j}b_{3j}d\theta_{3j} + K_{C3j}d_{3j}d\psi_{3j} + K_{B3j}(d\psi_{3j} - d\theta_{3j})(d_{3j} - b_{3j})]. \end{aligned} \quad (6.76)$$

Substituting Eq. (6.73) into Eq. (6.76), we have

$$\begin{aligned} F_x = & -K_{S2}dx - \sum_{j=1}^3 \{K_{A1j}(a_{1j}^2dx + a_{1j}b_{1j}dy) + K_{C1j}(c_{1j}^2dx + c_{1j}d_{1j}dy) \\ & + K_{B1j}[(c_{1j} - a_{1j})^2dx + (c_{1j} - a_{1j})(d_{1j} - b_{1j})dy] + K_{A3j}(b_{3j}^2dx + a_{3j}b_{3j}dz) \\ & + K_{C3j}(d_{3j}^2dx + c_{3j}d_{3j}dz) + K_{B3j}[(d_{3j} - b_{3j})^2dx + (c_{3j} - a_{3j})(d_{3j} - b_{3j})dz]\}. \end{aligned} \quad (6.77)$$

Gathering the terms with dx , dy and dz in Eq. (6.77) into three groups, we have

$$\begin{aligned} F_x = & -\{K_{S2} + \sum_{j=1}^3 [K_{A1j}a_{1j}^2 + K_{C1j}c_{1j}^2 + K_{B1j}(c_{1j} - a_{1j})^2 + K_{A3j}b_{3j}^2 \\ & + K_{C3j}d_{3j}^2 + K_{B3j}(d_{3j} - b_{3j})^2]\}dx - \sum_{j=1}^3 [K_{A1j}a_{1j}b_{1j} + K_{C1j}c_{1j}d_{1j} \\ & + K_{B1j}(c_{1j} - a_{1j})(d_{1j} - b_{1j}) + K_{A3j}a_{3j}b_{3j} + K_{C3j}c_{3j}d_{3j} \\ & + K_{B3j}(c_{3j} - a_{3j})(d_{3j} - b_{3j})]dy - \sum_{j=1}^3 [K_{A3j}a_{3j}b_{3j} + K_{C3j}c_{3j}d_{3j} \\ & + K_{B3j}(c_{3j} - a_{3j})(d_{3j} - b_{3j})]dz. \end{aligned} \quad (6.78)$$

Denoting the terms as follows

CHAPTER 6. STIFFNESS ANALYSIS.

$$\begin{aligned}
 K_{AS_{x-Fx}} &= K_{S2} + \sum_{j=1}^3 [K_{A1j} a_{1j}^2 + K_{C1j} c_{1j}^2 + K_{B1j} (c_{1j} - a_{1j})^2 + K_{A3j} b_{3j}^2 \\
 &\quad + K_{C3j} d_{3j}^2 + K_{B3j} (d_{3j} - b_{3j})^2], \\
 K_{AS_{y-Fx}} &= \sum_{j=1}^3 [K_{A1j} a_{1j} b_{1j} + K_{C1j} c_{1j} d_{1j} + K_{B1j} (c_{1j} - a_{1j})(d_{1j} - b_{1j})], \quad (6.79) \\
 K_{AS_{z-Fx}} &= \sum_{j=1}^3 [K_{A3j} a_{3j} b_{3j} + K_{C3j} c_{3j} d_{3j} + K_{B3j} (c_{3j} - a_{3j})(d_{3j} - b_{3j})],
 \end{aligned}$$

the actuation force F_x can be computed by

$$F_x = K_{AS_{x-Fx}} dx + K_{AS_{y-Fx}} dy + K_{AS_{z-Fx}} dz. \quad (6.80)$$

Actuation forces F_y and F_z :

Similarly we can obtain the actuation forces F_y and F_z as follows.

$$F_y = K_{AS_{x-Fy}} dx + K_{AS_{y-Fy}} dy + K_{AS_{z-Fy}} dz, \quad (6.81)$$

$$F_z = K_{AS_{x-Fz}} dx + K_{AS_{y-Fz}} dy + K_{AS_{z-Fz}} dz,$$

where

$$\begin{aligned}
 K_{AS_{y-Fy}} &= K_{S3} + \sum_{j=1}^3 [K_{A1j} b_{1j}^2 + K_{C1j} d_{1j}^2 + K_{B1j} (d_{1j} - b_{1j})^2 + K_{A2j} a_{2j}^2 \\
 &\quad + K_{C2j} c_{2j}^2 + K_{B2j} (c_{2j} - a_{2j})^2], \\
 K_{AS_{z-Fz}} &= K_{S1} + \sum_{j=1}^3 [K_{A1j} b_{1j}^2 + K_{C1j} d_{1j}^2 + K_{B1j} (d_{1j} - b_{1j})^2 + K_{A3j} a_{3j}^2 \\
 &\quad + K_{C3j} c_{3j}^2 + K_{B3j} (c_{3j} - a_{3j})^2], \quad (6.82)
 \end{aligned}$$

$$\begin{aligned}
 K_{AS_{z-Fy}} &= K_{AS_{y-Fz}} = \sum_{j=1}^3 [K_{A2j} a_{2j} b_{2j} + K_{C2j} c_{2j} d_{2j} \\
 &\quad + K_{B2j} (c_{2j} - a_{2j})(d_{2j} - b_{2j})],
 \end{aligned}$$

$$K_{AS_{x-Fy}} = K_{AS_{y-Fx}}.$$

$$\text{Therefore, } (F_x, F_y, F_z)^T = \mathbf{K}_{AS}^* (dx, dy, dz)^T = \mathbf{K}_{AS}^* \mathbf{J}^{-1} (dX, dY, dZ)^T, \quad (6.83)$$

CHAPTER 6. STIFFNESS ANALYSIS.

where $\mathbf{J}$ is the Jacobian matrix of the FPM and

$$\mathbf{K}_{AS}^* = \begin{bmatrix} K_{AS_{x-Fx}} & K_{AS_{y-Fx}} & K_{AS_{z-Fx}} \\ K_{AS_{x-Fy}} & K_{AS_{y-Fy}} & K_{AS_{z-Fy}} \\ K_{AS_{x-Fz}} & K_{AS_{y-Fz}} & K_{AS_{z-Fz}} \end{bmatrix}.$$

Because the motion of the flexure mechanism is very small, we can assume that $dX \approx X$, $dY \approx Y$ and $dZ \approx Z$. Equation (6.83) can be rewritten as follows:

$$(F_x, F_y, F_z)^T = \mathbf{K}_{AS}(X, Y, Z)^T, \quad (6.84)$$

where $\mathbf{K}_{AS}$ is defined as the actuation stiffness matrix of the flexure mechanism that expresses the increasing rate of driving forces versus the end-effector motion:

$$\mathbf{K}_{AS} = \mathbf{K}_{AS}^* \mathbf{J}^{-1}. \quad (6.85)$$

The determination of actuation stiffness matrix allows one to determine the actuation force large enough to move the FPM inside its workspace. The actuators are then selected based on the required actuation force.

6.4 Summary

In this chapter, an analytical approach is presented to obtain the stiffness matrix of an FPM. We show how to concatenate stiffness matrices of flexure members to obtain that of a larger serial chain or parallel structure. The stiffness matrix of a complete FPM is then derived by putting together the stiffness matrices of basic flexure members according to a scheme based on the FPM configuration. The analytical modeling allows the designer to formulate the stiffness of a flexure mechanism as a function of variables describing the dimensions and the free shape of the mechanism. The stiffness indices are obtained for the evaluation as well as the design of the mechanism. The modeling is demonstrated in two cases: a flexure prismatic joint based on a double linear spring stage

CHAPTER 6. STIFFNESS ANALYSIS.

and an SA 3-DOF translational FPM. The FEM analysis and experiments are carried out for the first case. The results confirm that the proposed approach is accurate enough for the stiffness analysis of FPMs. In addition, this chapter studies the relationship between driving forces and the motion of the end-effector through actuation stiffness. The actuation stiffness can be applied to the selection of the actuators used in FPM to provide force large enough to overcome the elastic characteristics of the flexure mechanism.

Chapter 7

Prototype Development and Experiments

This chapter presents the development of a selective-actuation 3-DOF translational flexure parallel mechanism that provides three decoupled translation micro-motions along three orthogonal axes. The work includes the design of the FPM, the pre-fabrication finite element analysis and the experimental verification. To illustrate the systematical design of FPMs introduced in previous chapters, the design of the prototype is presented in three steps: (1) the conceptual design of the general form, (2) the free shape design and kinematic synthesis for resolution transmission and (3) the dimension design of the flexure joints for stiffness. Then, FEM performance evaluation and dynamical analysis of the designed mechanism are carried out before fabrication. The development of the prototype is accompanied by a series of experiments to evaluate the selective actuation, the PRB-D model proposed to control the FPM, the stiffness and the hysteresis existing in the mechanism. In addition, the actual resolution and the repeatability of the FPM are determined in the experiments.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

7.1 Mechanical Design

The conceptual design of the SA 3-DOF translational FPM has been discussed in Chapter 3. In this section, we present the material selection, the kinematic and free shape design and the design of flexure-joint dimensions.

7.1.1 Material Selection

The material used for the FPM requires good static and dynamic characteristics [81] (Table 7.1).

Table 7.1: Characteristics of materials for FPM.

Behavior	Static characteristics	Dynamic characteristics
Mechanical	High stiffness- E High strength- S_y Large deflection range- S_y/E	Low deflection under its own weight or avoidance vibration noise- E/ρ Large acceleration- S_y/ρ
Thermal	Low expansion of the constrained body conducted by a steady heat flow - k/Ω Low stress in heavily clamped body conducted by a steady heat flow- $k/\Omega E$ Low thermal expansion- $1/\Omega$ Low stress generated in the constrained body due to thermal expansion- $1/\Omega E$	High rate of thermal distribution- $k/C\rho$ Low expansion of non-uniform temperature- $C\rho/\Omega$

(E , S_y , ρ , k , Ω and C are Young's modulus, yield stress, density, thermal conductivity, thermal expansion coefficient and specific heat respectively)

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Though polyethylene, polypropylene and e-glass composite (reinforced plastic using e-glass) possess high deflection range index (S_y / E) [82, 83], they are not appropriate for the FPM because of their nonlinear elasticity, high creep and variability of mechanical property. E-glass composite is not suitable either due to its high cost and low manufacturability. Six materials including aluminum 7075, aluminum alloy (duralumin), titanium Ti-13, beryllium copper, bronze 90/10 and stainless steel are considered and compared as material of the FPM. We normalize the properties of the above materials by dividing the properties of the materials by the corresponding properties of a model material proposed by Smith and Chetwynd [81]. The model material has properties on the good side of typical in all criteria. The normalized properties of the compared materials and the properties of the model material [81-83] are shown in Table 7.2 and are used to determine the material indices. The bar chart of Fig. 7.1 illustrates the comparison of the material based on the computed material indices.

Table 7.2: Normalized properties of materials considered for FPM.

Material	E	S_y	ρ	k	Ω	C
Aluminum (7075)	0.35	0.4	0.7	1.6	3.4	1.2
Aluminum alloy (Duralumin)	0.35	1	0.7	1	3.3	1.2
Titanium (Ti-13)	0.6	2.3	1.1	0.15	1.3	0.7
Beryllium copper	0.65	2.5	2.1	0.65	2.4	0.45
Bronze (90/10)	0.65	2	2.2	0.35	2.4	0.5
Stainless steel	0.926	1.8	1.95	0.6	2.1	0.483
Model material	200 GPa	300 MPa	4000 Kg/m ³	$7 \cdot 10^{-6}$ 1/°K	150 W/m°K	750 J/Kg°K

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

The bar chart shows that aluminum alloy (duralumin), beryllium copper and titanium (Ti-13) possess good mechanical characteristic such as large deflection, high strength and large acceleration. Beside of this, their characteristics on thermal behaviors including small thermal expansion, small stress due to thermal expansion and low expansion due to non-uniform temperature are very good. Hence, they are the good candidates for the FPM based on their static and dynamic behaviors. Considering manufacturability and cost, aluminum alloy (duralumin) was finally chosen as the material of the FPM.

Fig. 7.1. Material index comparison of FPM materials.

7.1.2 Kinematic and Free Shape Design for Resolution Transmission Scale

The kinematic parameters of the FPM are optimized for isotropy of global resolution transmission scale R_{global} in order to obtain good motion resolution and small actuating

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

load. The constraint of the uniformity U and workspace volume W are included in the optimization in order to achieve a workspace large enough to cover the requirement task and the uniformity of the resolution within the workspace. The kinematic and free shape design is formulated as an optimization problem in Chapter 5:

- **Objective:** Minimize $|R_{\text{global}} - 1|$. (7.1)

- **Constraints:** $U \geq U_{\min}$ and $W \geq W_{\min}$, (7.2)

where $U_{\min}$ and $W_{\min}$ are the allowable lower bounds of the uniformity and workspace volume respectively.

- **Design variables:**

- The kinematic dimensions: normalized lengths $k_a = a/h$ and $k_b = b/h$ of the links $A_{ij}B_{ij}$ and $B_{ij}C_{ij}$. (Fig. 4.4)
- The free shape of the FPM: initial position angles θ_{i10} and ψ_{i10} of links $A_{i1}B_{i1}$ and $B_{i1}C_{i1}$.

There are only four design variables. In addition, the optimization function and the constraints are not in analytical form but in numerical form. Therefore, the direct search method is appropriate for the solution. To reduce the computation resource and to obtain the convergence solution regardless of non-quadratic form of the optimization function, the Powell's conjugate direction method is used for the optimization. In this method, the new searching direction is created based on the result of the previous searching solutions.

The result of the optimization is shown in Table 7.3. The computed kinematic dimensions are $a = k_a.r = 34.86$ mm and $b = k_b.r = 40.26$ mm.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Table 7.3. Results of optimization.

Inputs	Initial parameters: $h = 60$ mm, Range of actuator $S = 5$ mm Allowable lower limits: $U_{\min} = 0.6$, $W_{\min} = 64$ mm ³
Outputs	Normalized dimension: $k_a = a/h = 0.581$, $k_b = b/h = 0.671$ Free shape: $\theta_{i10} = 80^\circ$, $\psi_{i10} = 165^\circ$ Global RT scale: $R_{\text{global}} = 15.64$ Uniformity $U = 0.603$ Workspace volume $W = 99.1$ mm ³

To adapt the manufacturability of the flexures, the kinematic dimensions are actually chosen as $a = 35$ mm and $b = 40$ mm. Therefore, the designed FPM has:

Global RT scale: $R_{\text{global}} = 15.6$

Uniformity $U = 0.61$

Workspace volume $W = 99.34$ mm³

Global RT scale in horizontal plane with $Z = 0$: $R_{Z=0} = 6.1$

7.1.3 Design for Dimension of Flexure Joints

The dimensions of the flexure joints of the FPM are now determined in an optimization for stiffness. The FPM provides three translational motions with selective actuation feature. Therefore, the stiffness index $K_{\min}$ proposed in Chapter 6 is used as the optimization function and is maximized in the optimal design. Besides, the actuation stiffness K_{AS} is chosen based on the load capacity of the selected actuator. The Picomotor with a maximum load of 22 N is used as the actuator. The range of motion is around 1 mm. Therefore, the actuation stiffness is desired to be smaller than 22 N/mm.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

This limit of the actuation stiffness is set as a constraint in the optimal design. With these requirements, the design problem can be stated as:

- **Objective:** Maximize stiffness index K_{1min} .
- **Constraint:** Actuation stiffness $K_{AS} \leq 22$ N/mm.
- **Design Variables:** Normalized dimensions of flexure hinges of the passive 3-D frame $k_t = t/e$, $k_r = r/e$ and of the double linear spring stage $k_{tS} = t_S/e_S$, $k_{rS} = r_S/e_S$.

The result of the optimal design is shown in Table 7.4. Actuator Picomotor 8301 with maximum load of 22 N and motion resolution of 0.03 μm is chosen as the actuator of the FPM.

Table 7.4. Dimension of flexure members.

Inputs	<p>Material: Yield stress $S_y = 500$ MPa, Young's modulus $E = 71$ GPa</p> <p>Plate thickness of:</p> <ul style="list-style-type: none"> - Double linear spring: $e_S = 20$ mm - Hinges: $e = 6$ mm
Outputs	<p>Stiffness:</p> <ul style="list-style-type: none"> - Flexure hinges of the passive 3-D frame: $K_\theta = K_\psi = 1240$ N.mm/rad - Double linear spring stage: $K_S = 0.25$ N/mm <p>Dimension of flexure members:</p> <ul style="list-style-type: none"> - Prismatic joint: $t_S = 0.5$ mm, $r_S = 5$ mm, $l_S = 22$ mm - Flexure hinges of limbs: $t = 0.4$ mm, $r = 3.5$ mm <p>Stiffness $K_{1min} = 126.4$ N/mm; Actuation stiffness $K_{AS} = 21.3$ N/mm</p>

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

7.1.4 Fabrication and Assembly

Based on the kinematic dimensions, free shapes and flexure joint dimensions, the mechanical design of the FPM is carried out. The detail drawings can be found in Appendix D. Components of the mechanism are shown in Fig. 7.2. The moving link of the FPM including the passive 3-D frame 1 and the three double linear spring stages 2, 3 and 4 connects to the base through the fixed part consisting of the fixed base 10 and the two supports 9 and 10. The passive 3-D frame is a compact structure of three mutually orthogonal planar 3-DOF flexure parallel mechanisms as shown in Fig. 7.3a. The passive 3-D frame is firmly attached to the three double linear spring stages used as the flexure prismatic joints. The double linear spring stage can be seen in Fig. 7.3b.

Fig. 7.2. Components of the SA 3-DOF translational FPM

1- Passive 3-D frame; 2, 3 and 4- Double linear spring stages; 5- Platform; 6- Column;
7- 3-D mirror; 8 and 9-Supports; 10- Fixed base.

Platform 5 is attached to the passive 3-D frame. Column 6 is used to support the platform. The 3-D mirror 7 is located on the platform and used as the reflector in the

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

experimental measurement using interferometry system or in the close-loop control system using optical sensor. Figure 7.4 illustrates the assembling scheme and Fig. 7.5 is the assembled prototype of the FPM.

Fig. 7.3. Passive 3-D frame (a) and double linear spring stage (b).

Fig. 7.4. Assembly of the FPM.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.5. Assembled prototype of the FPM.

7.2 Control Implementation

A feedback control system for the FPM is developed. The control diagram is shown in Fig. 7.6. The desired position $(X, Y, Z)^T$ of the end-effector is the input. The PRB-D model is used to compute the actuated variables $(x, y, z)^T$ of the actuated prismatic joints. These actuated variables are then used to compute the numbers of step n_x , n_y and n_z of the Picomotors corresponding to X-, Y- and Z-axes. The control program outputs those numbers of steps to the drivers of the Picomotors. The Picomotors drive the end-effector of the FPM to the position $(X_a, Y_a, Z_a)^T$. This actual position is measured using the

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

optical sensors Keyence LC-2430 (Keyence) with a resolution of $0.02 \mu\text{m}$ and a range of measurement of 1 mm. After sending the command to activate the Picomotor, the control program takes a delay time $t_{\text{delay}} = 0.003 \text{ s}$ before sending a command to the sensor to require the data reading. The delay time is determined as the inverse of the first natural frequency of the FPM in order to obtain the stable data. After receiving the actual position $(X_a, Y_a, Z_a)^T$ from the sensor controller, the control program performs the comparison between the actual and the desired positions in order to compensate the error.

Fig. 7.6. Feedback control schema.

The control program is coded using Visual C++ based on the flow chart illustrated in Fig. 7.7.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.7. Flow chart of control program.

7.3 Finite Element Modeling and Analysis

7.3.1 Finite Element Modeling

Before fabrication of the mechanism, FEM model of the FPM is established using ANSYS (Fig. 7.8). Two elements eight-node Brick SOLID 45 and ten-node Tet SOLID

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

92 are combined to mesh the solid model of the FPM [84-86]. The meshing process is divided into two steps: a coarse step for the large body and a fine step for areas of flexure hinges. The coarse step uses Brick SOLID45 elements to reduce the number of nodes and save computer memory. The fine step uses Tet SOLID92 elements to reduce error, to mesh the sharp corner areas, and to refine the mesh. The elements on the model have different sizes. The sizes of the elements are gradually decreased towards the hinges in order to reduce the number of nodes and computation time, and to guarantee the accuracy of the model. For analysis, the material is assumed to be an aluminum-alloy-like linearly elastic material with Young's modulus of 71GPa, Poisson's ratio of 0.31 and density of 2700 kg/m³.

The static analysis based on the FEM modeling is performed to verify the selective actuation of the FPM. Three double linear spring stages are used as the prismatic joints. The base of the three prismatic joints are assigned a displacement constraints $X = Y = Z = 0$. The moving link of prismatic joint X is assigned displacements ranging from 0.5 μm to 1000 μm when the moving link of the prismatic joint Z is assigned displacement constraint $Z = 0$. According to the selective-actuation feature, the motion of the end-effector along the Z-direction is theoretically zero. Three reference points are selected on the FPM for measuring errors of the selective actuation. The selective-actuation error dZ of the end-effector is obtained by taking the average of the displacement of three reference points. The static analysis is carried out with the gravity force in the Z-direction. The measurement is repeated three times with the moving link position of prismatic joint Y at 0 μm , 200 μm and 400 μm from the initial position respectively. The selective actuation error dY is also measured in the static analysis repeated under the condition that the Y-prismatic joint is assigned zero displacement and the Z-prismatic joint is positioned at 0 μm , 200 μm and 400 μm from the initial position respectively.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

The result in Fig. 7.9 shows that the error of selective actuation below is 3.9%. The error can be ignored in the first 100 μm of the displacement. The gravity force as well as the direction of the actuation has little influence on the error of selective actuation.

Fig. 7.8. FEM model of the SA 3-DOF translational FPM.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.9. Selective-actuation error obtained by FEM modeling.

7.3.2 Modal Analysis

Modal analysis of the FPM is performed to determine the dynamic characteristics including mode shapes and respective natural frequencies of the mechanism. The first ten natural frequencies are shown in Table 7.5. The first natural frequency is used to set the delay time t_{delay} of the data reading command in the control program. Delay time t_{delay} must be larger than a period of 0.002 seconds corresponding to the first natural frequency of the FPM. Figures 7.10, 7.11 and 7.12 illustrate the mode shapes of the mechanism corresponding to the first three natural frequencies. The first mode shape corresponds to the vibration along the motion direction of the moving link of the prismatic joint. The second mode corresponds to the vibration of three limbs of the 3-flexure-hinge group of the passive 3-D frame and the moving link of the prismatic joint

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

in the working direction of this joint. The third mode relates the vibration of the passive 3-D frame. The remaining modes are the combination among the first three modes.

Table 7.5. Natural Frequencies.

Mode shape	Natural frequency (Hz)	Mode shape	Natural frequency (Hz)
1	493.09	6	1836.4
2	616.96	7	2573.8
3	1000.05	8	2842.4
4	1419.5	9	2929.9
5	1599.6	10	3216.5

Fig. 7.10. First mode shape.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.11. Second mode shape.

Fig. 7.12. Third mode shape.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

7.3.3 Dynamic Analysis

The transient dynamic analysis is carried out to show the response of the end-effector position under the actuation of time-dependent load. The reduced mode of ANSYS allows us to form a set of master nodes that characterize the behavior of the model and the motion of the remaining nodes (slaves) is subsequently computed based on the motion of the masters [86]. Therefore, the stiffness matrix can be simplified. The reduced mode is chosen here to solve the dynamic problem in order to save considerable computation time and consumed memory. In the meantime, we still guarantee the accuracy of the solution by selecting the suitable number and location of the master nodes [86]. The transient dynamic analysis consists of following steps:

- Set the constraints and master nodes used for the reduced analysis.
- Set the Integration Time Step (time step of the numerical integration) $ITS = 0.0001$ seconds based on the condition of resolving the motion of the structure ($ITS \leq 1/20f$) [86], where $f = 493.09$ Hz is the frequency of the first mode shape.
- Apply and solve for each load step, expand the solution, and review the results.

The dynamic response of the end-effector is studied in three situations without the consideration of the damping factor:

Case 1: A step force of 36N is applied to the moving link of the X-prismatic joint along the X-direction, and drops to zero at $t = 0.4$ sec. The response is shown in Fig. 7.13. It is observed that there exist several attenuations before the response settles down.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.13. End-effector position responses (X response).

Case 2: This case is used to study the decoupled characteristic of the mechanism. A step force of -19 N is applied to the moving link of the Y-prismatic joint since $t = 0$ sec. Another step force of 19 N is then applied to the moving link of the X-prismatic joint at $t = 0.5$ sec. Both step forces drop at $t = 1$ sec. The responses are recorded in Fig. 7.14. It can be seen that the force in the X-direction has minor effect (2.8 % of the desired position in overshoot and less than 2 % of the desired position in settled position) on the motion along the Y-direction. Likewise, the force along the Y-direction has negligible effect on the motion along the X-direction. The effect is observed as oscillations around the settled position.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.14. End-effector position responses (X and Y responses).

Case 3: This case shows minimal interaction among the responses to the step actuation simultaneously at three prismatic joints. The step forces of 45, 40 and 35 N are applied to the moving links of the X-, Y- and Z-prismatic joints simultaneously in the period from $t = 0$ sec to $t = 1$ sec. This setup produces error of 11.5 % of the desired position in the overshoot and less than 1% of the desired position in the settled position (Fig 7.15). This interaction can be compensated using the feedback control.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.15. End-effector position responses (X, Y and Z responses).

7.4 Experiments

7.4.1 PRB-D Model

This experiment illustrates the advantage of the PRB-D model. The solutions of the kinematics of the FPM based on the PRB-D and PRB models are used as the inputs of the open-loop control system to activate the end-effector along the Y- or Z- direction. The actual motion of the end-effector is measured by an inductive probe Mahr 1318 with a resolution of $0.1 \mu\text{m}$, increase of measuring force of 0.04N/mm and measurement range from -1.6 to $+1.6 \text{ mm}$ (Fig. 7.16). The increment rate of the measuring force is very small (0.04 N/mm). Therefore, the deformation of probe's stand is negligible and the measuring result can be considered as the motion of the end-effector only. The error of the modeling is defined as the difference between the measured displacements of the end-effector using PRB-D or PRB models and the desired displacement. The errors of

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

PRB-D and PRB models are illustrated in Fig. 7.17 and Fig. 7.18. The error of the PRB model increases very fast even when the desired displacement of the end-effector is smaller than $10\text{ }\mu\text{m}$ whereas the errors of the PRB-D model at this position is negligible (Fig. 7.17). In the motion range of $400\text{ }\mu\text{m}$, the error produced by the PRB-D model is only $1/3$ of that by the PRB model (Fig. 7.18). The effect of the gravity force on the accuracy of the modeling can be seen by the difference between the error of dY and dZ . The difference is small and can be ignored.

Fig. 7.16. Experimental verification of PRB-D model.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.17. Comparison between PRB and PRB-D models (range 0-100 μm)

(a)- range 0 - 5 μm ; (b)- range 5 - 100 μm

Fig. 7.18. Comparison between PRB and PRB-D models.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

7.4.2 Selective-Actuation

Experimental verification of the selective-actuation feature of the FPM is carried out. The error of the selective actuation is defined as the slew motions of the end-effector in the non-working actuator directions. The experiment is conducted to verify the slew motion in Y-axis and in Z-axis.

Fig. 7.19. Experimental verification of the selective actuation.

Slew motion in Y-axis: The X-direction actuator provides the end-effector a series of displacements along X-direction ranged from 0.5 μm to 1000 μm while the Y-direction actuator stops at the initial position all the time. The optical sensor Keyence LC-2430 (resolution of 0.02 μm and measurement range of 1 mm) is used to measure the displacement of the end-effector along Y-direction (Fig. 7.19). The laser beam emitted

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

from the laser source in the sensor is reflected at the 3-D mirror and goes back to the receiver of the sensor. The difference of the reflected and the emitted beams produce the displacement information. The experiment is conducted three times with the Z-prismatic joint positioned at a distance of 0 μm , 200 μm and 400 μm far from the initial position. The slew motion of the end-effector in non-actuated Y-direction is shown in Fig. 7.20(a).

Fig. 7. 20. Slew motions dY and dZ versus actuated motion in X-direction

Slew motion in Z-axis: The experiment is repeated with the roles of the Y- and Z-direction actuators interchanged. The slew motion of the end-effector in non-actuated Z-direction is shown in Fig. 7.20(b).

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

The experimental error is higher than that resulted from FEM modeling. This may be caused by undesired error in the fabrication and the assembly. The error is closed to zero and can be ignored in the first 10 μm . When the motion of the end-effector along the X-direction is in the range from 10 μm to 125 μm , the error is limited by 5 % of the actuated motion in X-direction. The trend of experimental error and the effect of gravity match those obtained from FEM analysis, i.e., the gravity has minor influence on the selective actuation.

7.4.3 Resolution Determination

The resolution of the FPM is theoretically computed based on the resolution transmission scale. This experiment is to determine the resolution of the FPM and to validate the method of resolution evaluation through the resolution transmission scale.

The FPM with a feedback control system (Fig. 7.19) provides a series of small motions along X- and Y- directions at $Z = 0$. The range of the motion is gradually reduced and the response of the end-effector position is recorded to determine the smallest controllable motion of the FPM. To record the response of the end-effector motion, the control program repeatedly sends data reading commands to the sensor controller (Fig. 7.21). The received data of the end-effector position is stored in a file. The responses of the end-effector position to the position commands with ranges of 0.1, 0.2, 20 and 300 μm are illustrated in Fig. 7.22. Resolution of motion in plane $Z = 0$ of the FPM equipped a feedback system is experimentally determined as 0.2 μm whereas the average resolution predicted by the design is $\tau R_{\text{global}} = 0.18 \mu\text{m}$ (resolution of the actuator - Picomotor $\tau = 0.03 \mu\text{m}$ and the global RT scale at $Z = 0$: $R_{Z=0} = 6.1$).

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7. 21. Experimental process of resolution determination.

Fig. 7.22. Responses of end-effector position to step inputs 300, 20, 0.2 and $0.1 \mu\text{m}$.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

7.4.4 Repeatability

This experiment is carried out to investigate the positioning repeatability of the FPM. We repeat 20 times of the same action: actuate the FPM with a designated X-displacement and measure the actual displacement X_i ($i = 1 - 20$) by optical sensor LC2430 (Fig. 7.19). The repeatability of the FPM is evaluated in two manners:

- Maximum deviation in percentage:

$$\varepsilon_{\max} = \frac{\max_{i=1-N} \{|X_i - X_{\text{mean}}|\}}{X_{\text{mean}}} \times 100\% , \quad (7.3)$$

- Standard deviation:

$$\sigma_X = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (X_i - X_{\text{mean}})^2} , \quad (7.4)$$

where

$N = 20$ is the number of the same action

$$X_{\text{mean}} = \frac{1}{N} \sum_{i=1}^N X_i \text{ is the mean of values } X_i.$$

The experiment is repeated for several designated X-displacements ranging from 1 to 200 μm . The repeatability evaluation can be seen in Fig. 7.23 and Fig. 7.24. The range of motion of the FPM affects the repeatability. The repeatability in forward and backward directions is also different. The standard deviation changes unstably and is smaller than 0.2 μm when the designated motion is smaller than 20 μm . With a motion range of 200 μm , the standard deviation σ_X increases with the actuator displacement. The maximum deviation is observed under 3.5% (forward) and 2.5% (backward) of the designated motion. From control perspective, the error of any large motion can be compensated using the feedback control and leads to the performance of a small motion. In addition, the high repeatability of the small motion allows the FPM to perform the

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

same motion of small range, and thus enhances the controlability of the small motion and repeatability corresponding to the small motion that is critical to the evaluation of the device. Hence, we can conclude that the repeatability of the FPM is satisfactory for obtaining the good resolution. The repeatability in the remaining direction is the same due to the symmetrical design of the FPM.

Fig. 7.23. Maximum deviation in X-direction.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.24. Standard deviation in X- direction.

7.4.5 Stiffness Verification

The experiment investigates the stiffness of the FPM and verifies the proposed stiffness modeling. The FPM is rigidly mounted on an X-Y sliding table (Fig. 7.25). A Picomotor is mounted on a rigid box connected to the base of the table through a Mini40 Force/Torque Transducer (ATI). The ball head of Picomotor is in contact with the platform of the FPM along the Z-direction. The stiffness of the FPM is determined in two steps stated as follows.

Stiffness K_{z-F_z} and $K_{\alpha_y-F_z}$: The screw of Picomotor exerts a static force F_z measured by the transducer upon the center O of the platform. The displacements z_O of the center O

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

and z_{P_1} of point P_1 of the platform located on X-axis and $OP_1 = p_1$ are measured by an inductive probe Mahr 1318 and used to determine the stiffness K_{z-Fz} along the Z-direction and K_{α_y-Fz} about the Y-axis based on Eq. (7.5).

$$K_{z-Fz} = F_z / z_O, \quad K_{\alpha_y-Fz} = p_1 F_z / (z_{P_1} - z_O). \quad (7.5)$$

Fig. 7.25. Stiffness experiment of the FPM.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Stiffness $K_{\alpha x-Mx}$: The base of the FPM is located at a distance q from the position of the first experiment along the Y-direction of the XY table. The distance q and the static force F_z exerted by the Picomotor create a moment M_x applying to the end-effector of the FPM. The displacements z_O and z_{P_2} (point P_2 located on Y-axis and $OP_2 = p_2$) can be used to determine the stiffness $K_{\alpha x-Mx}$ as follows.

$$K_{\alpha x-Mx} = \frac{p_2 q F_z}{(z_{P_2} - z_O)}. \quad (7.6)$$

The experiment is carried out at a set of the end-effector positions. The stiffness obtained from experiment is shown in Table 7.6 and compared with those obtained from the analytical method and FEM modeling of the FPM. The difference between the results obtained from analytical modeling and FEM modeling is quite small (<7.7%). The errors of the stiffness resulted from the analytical modeling in comparison with the experimental results are smaller than 8%.

Table 7.6. Comparison of stiffness obtained in different manners.

Model Stiffness	Analytical		FEM		Experiment
	Value	Error (%)	Value	Error (%)	
K_{z-Fz} (N / mm)	139.6	7.78	148.28	1.94	151.22
$K_{\alpha y-Fz}$ (N / rad)	3968.3	8.12	4306.2	4.13	4133.3
$K_{\alpha x-Mx}$ (Nm / rad)	541.1	5.1	535.6	4.02	514.9

The experimental and computational results of stiffness are then used to establish the distribution of stiffness K_{z-Fz} (Fig. 7.26), $K_{\alpha y-Fz}$ (Fig. 7.27) and $K_{\alpha x-Mx}$ (Fig. 7.28) in the working range of the FPM at $Z = 0$.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Due to symmetrical design of the FPM, K_{z-Fz} should have symmetrical distribution with respect to the $Y = X$ line of the plane. Its value should decrease when the end-effector moves away from the origin as shown in Fig. 7.26a. Experiment indicated slightly different distribution from the theoretical one in Fig. 7.26b.

The remaining stiffness $K_{\alpha y-Fz}$ and $K_{\alpha x-Mx}$ do not have symmetry properties. And they do not have significant changes when the end-effector moves away from the Y-axis. However, experimental results show a slightly different distribution when the stiffness $K_{\alpha y-Fz}$ and $K_{\alpha x-Mx}$ change as the end-effector moves away from the Y-axis. Nevertheless, this difference is very small and can be ignored. The experimental data confirm that the proposed modeling is accurate and reliable for evaluating the stiffness of the flexure mechanism.

Fig. 7.26. Stiffness K_{z-Fz} distribution with $Z = 0$.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.27. Stiffness $K_{\alpha y-F_z}$ distribution with $Z = 0$.

Fig. 7.28. Stiffness $K_{\alpha x-M_x}$ distribution with $Z = 0$.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

7.4.6 Challenge of Hysteresis

We also devised an experiment to show the hysteresis phenomenon in the FPM. The experimental set up is shown in Fig. 7.29. A Nano43 Force/Torque transducer (ATI) is mounted to the rear surface of the moving link of the X-direction prismatic joint. The transducer is placed between the ball head of the X-direction Picomotor and the moving link of the X-direction prismatic joint in order to measure the driving force. The Picomotor drives the active prismatic joint through the transducer. The displacement of the end-effector is measured by a charge-coupled device (CCD) camera sensor system LS7010MT (Keyence) with resolution of $0.06\text{ }\mu\text{m}$. The experiment is carried out in the following manners.

- (1) **Homing:** The end-effector is controlled to return to the initial position.
- (2) **Forward:** The control program is provided with N -step input to let the end-effector move along the X-axis (input = N). We observe and record the driving force provided by the X-direction actuator and the position of the end-effector until the command execution is completed.
- (3) **Backward:** The control program is provided with N -step input to let the end-effector move backward against the initial position (input = $-N$). Similarly to the forward step, we observe and record the position and driving force of the X-direction actuator until the moving-back command is completed.

The three steps are repeated with number of steps N ranging from 1000 to 10000. Collecting data from forward and backward steps, hysteresis phenomenon in the FPM can be observed clearly in the displacement diagram (Fig. 7.30) and force diagram (Fig. 7.31). The difference of the home position in forward and backward steps is $28.68\text{ }\mu\text{m}$ as shown in Fig. 7.30 with $N = 10000$. This difference will create difficulty in the homing process as well as the control of the FPM.

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Fig. 7.29. Hysteresis evaluation.

Fig. 7.30. Forward displacement and backward displacement ($Y = Z = 0$).

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

Figure 7.31 shows the hysteresis effect on the driving force. The data is collected when observing the end-effector in the range from 90 μm to 100 μm in the experiment with $N = 2000$. The driving force in the backward step is always smaller than in the forward step.

The similar hysteresis phenomenon can be seen in the motions along Y- or Z- directions. For controlling the FPM, we must investigate the hysteresis effect caused by the flexure structure and by the piezoelectric material used in Picomotor in order to have the correct compensation.

Fig. 7.31. Driving force in forward and backward processes ($Y = Z = 0$).

CHAPTER 7. PROTOTYPE DEVELOPMENT AND EXPERIMENTS.

7.5 Summary

The systematical design of flexure parallel mechanisms proposed in Chapter 3 is illustrated in a case study – the SA 3-DOF translational FPM. The performance of the designed mechanism is verified by FEM modeling before fabrication. A feedback control system based on the optical sensors is then implemented on the FPM. The experiments are setup to evaluate the proposed modeling method and to validate the design approach in the performances including stiffness, selective-actuation and resolution. The experimental data shows that there exist a small and negligible difference between the actual performance and the one predicted by the design. The experiment also shows the advantage of the PRB-D modeling of the FPM versus the recent modeling. The hysteresis phenomenon is shown as the challenges in the future investigation of the flexure-mechanism control.

Chapter 8

Conclusion and Future Study

8.1 Conclusion

This thesis aims to develop a design methodology for selective-actuation micro-positioning systems using flexure parallel mechanisms. The design includes three steps: conceptual design, design of free shape, kinematic dimension and flexure dimension, and pre-fabrication verification using finite element method. Major topics addressed are: (1) conceptual design emphasizing on synthesis of selective-actuation parallel mechanisms and the corresponding flexure mechanisms; (2) pseudo rigid-body modeling with consideration of deformation of the flexure members; (3) design of kinematic dimensions and free shape for resolution transmission; (4) determination of flexure member dimensions for desired stiffness and the actuation stiffness of the system; (5) implementation of a selective-actuation 3-DOF translational flexure parallel mechanism. The conceptual design of a selective-actuation flexure parallel mechanism includes generation of kinematic parallel mechanisms matching the required number of degrees-

CHAPTER 8. CONCLUSION.

of-freedom, synthesis for selective-actuation and conversion of kinematic mechanisms to the flexure mechanisms. Since type synthesis has been well studied in the past, we concentrate on the selective-actuation synthesis that is analytically expressed by the Jacobian matrix in diagonal form. The Jacobian matrix is established using screw theory to indicate the effect of link and joint arrangement on the elements in Jacobian matrix. Based on the diagonal form condition, links and joints of the parallel mechanism are geometrically arranged in order to obtain the selective-actuation characteristic. The links and joints are then grouped and converted to the corresponding flexure member groups according to the following requirements: minimal deformation effect on the end-effector motion, minimal assembly error and parasitic motion, monolithic structure, rigidity and manufacturability in order to obtain the desired flexure parallel mechanism with the property of selective actuation. The conceptual design is illustrated with two flexure parallel mechanisms with selective-actuation capability: 3-DOF translational and 6-DOF dexterous motion stages.

Traditionally, by solving the statics of FPM based on pseudo rigid-body model, we obtain the reaction forces at flexure joints, and subsequently the deformation of flexure members. The deformations considered as infinitesimal motions are transmitted to the end-effector. The inclusion of transmitted deformations to the end-effector motion resulted from the kinematics of the mechanism (pseudo rigid-body model) provides a more accurate modeling of the flexure mechanism. We call this method pseudo rigid-body model with consideration of deformation of flexure members or PRB-D in short. This improved modeling method that is experimentally verified allows us to compute more precisely the end-effector motion when the joints are actuated. Therefore, using this model as the plant function in the position control of the system, the settling time is

CHAPTER 8. CONCLUSION.

shortened, the oscillation around the desired position reduced and the mechanism can go for higher resolution.

Ignoring deformation caused by the nature of flexure members, the resolution of a flexure parallel system is referred as the smallest available motion provided by the actuator and the transmissibility of infinitesimal motions from actuators to the end-effector. The transmissibility factor has connection with the kinematic structure of the flexure parallel mechanism. The transmission scale $R = \sqrt[n]{\det(\mathbf{J}\mathbf{J}^T)}$ varies over the workspace. Therefore, the resolution performance indices, namely the global resolution transmission scale over the workspace and the uniformity of resolution, are proposed as general evaluation criteria. Global Indices Sampling Algorithm (GISA) are proposed to compute the global criteria and used for the optimal design of kinematic dimensions and free shape of the flexure mechanism.

High stiffness of the flexure parallel mechanism could prevent the mechanism from dynamic errors and vibration noise so as to achieve high repeatability. The stiffness matrix of the FPM is formed from the assembly of stiffness matrices of basic flexure members. The elements of the stiffness matrix are then used to establish the stiffness performance index for the design of the FPM. Actuation stiffness representing the increment of actuator driving forces according to the end-effector motion is also considered. Actuation stiffness is used as a guideline to choose appropriate actuators for the FPM. The dimensions of flexure members are determined in the optimization for stiffness with actuation stiffness as the constraints.

To illustrate that the systematical design works, the selective-actuation 3-DOF translational FPM proposed in the conceptual design is fabricated and tested. The control program is coded and the feedback system using optical laser sensors is implemented in order to control the developed mechanism. The FPM can provide decoupled

CHAPTER 8. CONCLUSION.

translational motions with resolution of 0.2 μm . The FPM achieves the repeatability with deviation of 0.2 μm for the motion of 20 μm . Verification of the system properties like resolution, selective actuation, repeatability and stiffness by experiments shows the viability of the design method. Experimental results also validate the indices used to evaluate resolution transmission scale and stiffness for the design. The experiment also shows the challenge of hysteresis on the control of the mechanism.

8.2 Future Study

The following research directions are suggested to extend the understanding of selective-actuation micro positioning system based on flexure parallel mechanisms.

- (1) *Development of dexterous 6-DOF flexure parallel mechanism with selective actuation feature for further study:* The conceptual design of the mechanism has been proposed in the thesis. The remaining studies include the determination of dimensions and free shape, and fabrication.
- (2) *Control of flexure mechanisms:* Hysteresis is observed as a challenge in controlling FPM. Modeling that separates hysteresis caused by the actuators and by the nature of material and mechanism structure using Preisach or Prandtl-Ishlinskii [87, 88] models allows the designer to have the correct compensation for each type of hysteresis. Therefore, the entire system can obtain a better resolution as well as accuracy. Another problem in the control of FPM is the error analysis and calibration. An analysis of errors caused by fabrication allows us to build a compensation strategy for the modeling function, thus giving better resolution as well as accuracy. Real time control of the FPM is also very important. Using strain gauges to measure deformation at the flexure members which contributes to the

CHAPTER 8. CONCLUSION.

end-effector motion can reduce the computation time in the PRB-D modeling and shorten adaptive time.

- (3) *Dynamic modeling and Synthesis for decoupled dynamics*: Generally, the dynamic differential equation of a flexure mechanism is $M\ddot{q} + C\dot{q} + Kq = \tau$, where M , C , K , q and τ are inertial matrix, damping matrix, stiffness matrix, generalized coordinates and generalized external forces respectively. The formulation of dynamics equations allows the designer to investigate the relationship between dynamic behaviors (natural frequencies, mode shapes and dynamic responses) and the configuration (free shape and dimension) of the flexure parallel mechanism. This investigation is important for dynamic synthesis and design. For robust performance of a positioning system, decoupled dynamics will be useful. Synthesis for decoupled dynamics involves the relocation of actuators and the selection of link mass and inertia in order to obtain the inertia matrix M in diagonal form. However, the synthesis for decoupled dynamics is difficult and, sometime, cannot be done. In this case, it is necessary to establish the decoupling motion control algorithm.
- (4) *Multiobjective optimization*: So far the indices of resolution and stiffness are used separately for resolution and stiffness optimizations. Stiffness and resolution are interrelated in the FPM in general. Therefore, a multiobjective optimization can be employed based on both criteria in order to obtain an FPM with better performance.

Appendix A:

Stiffness Matrix of Double Linear Spring Stage

$$\mathbf{K}_S = \begin{bmatrix} K_{S11} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{S22} & 0 & 0 & 0 & K_{S26} \\ 0 & 0 & K_{S33} & 0 & K_{S35} & 0 \\ 0 & 0 & 0 & K_{S44} & 0 & 0 \\ 0 & 0 & K_{S53} & 0 & K_{S55} & 0 \\ 0 & K_{S62} & 0 & 0 & 0 & K_{S66} \end{bmatrix}, \quad (\text{A.1})$$

where

$$\begin{aligned} K_{S11} &= \frac{4}{C_{11}}, \quad K_{S22} = \frac{4C_{66}}{C_{22}C_{66} - C_{26}C_{62}}, \quad K_{S26} = \frac{4C_{26}}{C_{26}C_{62} - C_{22}C_{66}}, \\ K_{S33} &= \frac{4C_{55}}{C_{33}C_{55} - C_{35}C_{53}}, \quad K_{S35} = \frac{4C_{35}}{C_{35}C_{53} - C_{33}C_{55}}, \\ K_{S44} &= 4 \frac{C_{35}C_{53} - C_{33}C_{55} - d^2C_{44}C_{55}}{C_{35}C_{44}C_{53} - C_{33}C_{44}C_{55}}, \\ K_{S53} &= \frac{4C_{53}}{C_{35}C_{53} - C_{33}C_{55}}, \quad K_{S55} = \frac{4C_{33}}{C_{33}C_{55} - C_{35}C_{53}}, \\ K_{S62} &= \frac{4C_{62}}{C_{26}C_{62} - C_{22}C_{66}}, \quad K_{S66} = 4 \frac{C_{11}C_{22} - d^2C_{26}C_{62} + d^2C_{22}C_{66}}{C_{11}C_{22}C_{66} - C_{11}C_{26}C_{62}}. \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} C_{11} &= C_{A11} + C_{AB11} + C_{B11}, \\ C_{22} &= C_{A22} + lC_{A26} + lC_{A62} + l^2C_{AB66} + C_{AB22} + C_{B22}, \\ C_{33} &= C_{A33} + lC_{A35} + lC_{A53} + l^2C_{A55} + C_{AB33} + C_{B33}, \\ C_{44} &= C_{A44} + C_{AB44} + C_{B44}, \quad C_{55} = C_{A55} + C_{AB55} + C_{B55}, \\ C_{66} &= C_{A66} + C_{AB66} + C_{B66}, \quad C_{26} = C_{A26} + lC_{A66} + C_{AB26} + C_{B26}, \\ C_{35} &= C_{A35} + lC_{A55} + C_{AB35} + C_{B35}, \quad C_{53} = C_{A53} + lC_{A55} + C_{AB53} + C_{B53}, \end{aligned}$$

APPENDIX.

$$C_{62} = C_{A62} + lC_{A66} + C_{AB62} + C_{B62}$$

and C_{Aij} , C_{Bij} , C_{Bij} are the elements of the compliance matrices of flexure hinges A, B and link AB.

The investigated double linear spring stage has the stiffness matrix as below.

$$\mathbf{K}_S = \begin{bmatrix} 3.8E5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 126.31 & 0 & 0 & 0 & -0.2E5 \\ 0 & 0 & 1E6 & 0 & 6.2E5 & 0 \\ 0 & 0 & 0 & 14.6E6 & 0 & 0 \\ 0 & 0 & 6.2E5 & 0 & 16.5E6 & 0 \\ 0 & -0.2E5 & 0 & 0 & 0 & 2.2E7 \end{bmatrix},$$

where the units are N/mm, Nmm/rad, Nmm/mm or N/rad and the dimensions of the double linear spring stage are given in Appendix D.

The stiffness matrix $\mathbf{K}_L$ of the linear spring stage is considered as half of the stiffness matrix $\mathbf{K}_S$ of the double linear spring stage:

$$\mathbf{K}_L = \frac{1}{2} \mathbf{K}_S. \quad (\text{A.3})$$

APPENDIX.

Appendix B

Compliance Matrix of Flexure Hinge

The compliance matrix of a flexure hinge is

$$\mathbf{C}_A = \mathbf{C}_B = \begin{bmatrix} C_{A11} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{A22} & 0 & 0 & 0 & C_{A26} \\ 0 & 0 & C_{A33} & 0 & C_{A35} & 0 \\ 0 & 0 & 0 & C_{A44} & 0 & 0 \\ 0 & 0 & C_{A53} & 0 & C_{A55} & 0 \\ 0 & C_{A62} & 0 & 0 & 0 & C_{A66} \end{bmatrix}, \quad (\text{B.1})$$

where

$$\begin{aligned} C_{A11} &= \frac{\pi(r/t)^{1/2} - 2.57}{Ee}, \\ C_{A22} &= \frac{9\pi r^{5/2}}{2Eet^{5/2}} + \frac{\pi(r/t)^{1/2} - 2.57}{Ge}, \\ C_{A26} &= C_{A62} = \frac{9\pi r^{3/2}}{2Eet^{5/2}}, \\ C_{A33} &= \frac{12\pi r^2}{Ee^3} \left[(r/t)^{1/2} - 1/4 \right] + \frac{\pi(r/t)^{1/2} - 2.57}{Ge}, \end{aligned}$$

(B.2)

$$\begin{aligned} C_{A35} &= C_{A53} = \frac{12r}{Ee^3} \left[\pi(r/t)^{1/2} - 2.57 \right], \\ C_{A44} &= \frac{9\pi r^{1/2}}{8Get^{5/2}}, \\ C_{A55} &= \frac{12}{Ee^3} \left[\pi(r/t)^{1/2} - 2.57 \right], \end{aligned}$$

APPENDIX.

$$C_{A66} = \frac{9\pi r^{1/2}}{2Et^{5/2}},$$

and e , t and r are respectively thickness of plate, inter-hole space and common radius of flexure hinge, and E and G are the Young's modulus and shear modulus of material.

Fig. B.1. Flexure hinge dimensions.

The stiffness matrix of a flexure hinge is $\mathbf{K} = \mathbf{C}^{-1}$.

The stiffness and compliance matrices of flexure hinges A_{ij} , B_{ij} , C_{ij} , of the passive 3-D frame and the flexure hinges of the linear spring stages are determined based on Eq. (B.1).

APPENDIX.

Appendix C

Driving Forces of SA 3-DOF Translational FPM

The driving forces are obtained from the actuation stiffness analysis in Chapter 6.

$$\begin{aligned} F_x &= K_{AS_{x-Fx}} dx + K_{AS_{y-Fx}} dy + K_{AS_{z-Fx}} dz, \\ F_y &= K_{AS_{x-Fy}} dx + K_{AS_{y-Fy}} dy + K_{AS_{z-Fy}} dz, \\ F_z &= K_{AS_{x-Fz}} dx + K_{AS_{y-Fz}} dy + K_{AS_{z-Fz}} dz, \end{aligned} \quad (C.1)$$

, where:

$$\begin{aligned} K_{AS_{x-Fx}} &= K_{S2} + \sum_{j=1}^3 [K_{A1j} a_{1j}^2 + K_{C1j} c_{1j}^2 + K_{B1j} (c_{1j} - a_{1j})^2 + K_{A3j} b_{3j}^2 \\ &\quad + K_{C3j} d_{3j}^2 + K_{B3j} (d_{3j} - b_{3j})^2], \end{aligned}$$

$$K_{AS_{y-Fx}} = \sum_{j=1}^3 [K_{A1j} a_{1j} b_{1j} + K_{C1j} c_{1j} d_{1j} + K_{B1j} (c_{1j} - a_{1j})(d_{1j} - b_{1j})],$$

$$K_{AS_{z-Fx}} = \sum_{j=1}^3 [K_{A3j} a_{3j} b_{3j} + K_{C3j} c_{3j} d_{3j} + K_{B3j} (c_{3j} - a_{3j})(d_{3j} - b_{3j})],$$

$$\begin{aligned} K_{AS_{y-Fy}} &= K_{S3} + \sum_{j=1}^3 [K_{A1j} b_{1j}^2 + K_{C1j} d_{1j}^2 + K_{B1j} (d_{1j} - b_{1j})^2 + K_{A2j} a_{2j}^2 \\ &\quad + K_{C2j} c_{2j}^2 + K_{B2j} (c_{2j} - a_{2j})^2], \end{aligned}$$

$$\begin{aligned} K_{AS_{z-Fz}} &= K_{S1} + \sum_{j=1}^3 [K_{A1j} b_{1j}^2 + K_{C1j} d_{1j}^2 + K_{B1j} (d_{1j} - b_{1j})^2 + K_{A3j} a_{3j}^2 \\ &\quad + K_{C3j} c_{3j}^2 + K_{B3j} (c_{3j} - a_{3j})^2], \end{aligned}$$

$$\begin{aligned} K_{AS_{z-Fy}} &= K_{AS_{y-Fz}} = \sum_{j=1}^3 [K_{A2j} a_{2j} b_{2j} + K_{C2j} c_{2j} d_{2j} \\ &\quad + K_{B2j} (c_{2j} - a_{2j})(d_{2j} - b_{2j})], \end{aligned}$$

$$K_{AS_{x-Fy}} = K_{AS_{y-Fx}}. \quad (C.2)$$

APPENDIX.

Appendix D

Mechanical Drawings of Major Details

APPENDIX.

Notes:

- Coordinates of points are described in attached text file/papper coordinates.txt
- The limb is described in drawing 1d: 3-side frame (limb)

Nanyang Technological University			
Title : 3-SIDE FRAME (coordinates of points)		Drawn: Pham Huy Hoang	
Dwg. No.: 1c	Scale: 2:1	Date: 23 May 2003	
Material: Aluminium alloy 7075 T6 heat treated	Proj: third angle	All dimesion in mm	

APPENDIX.

List of point coordinates of the passive 3-D frame

! point I	!Points A2I	!Points B3i
xi=0 mm	xa21=5.65 mm	xb31=24.7 mm
yi=0 mm	ya21=-7.53 mm	yb31=8.66 mm
! point M, N	xa22=9.35 mm	xb32=17.3 mm
xM=0 mm	ya22=-1.13 mm	yb32=8.66 mm
yM= -3 mm	xa23=16.93 mm	xb33=17.3 mm
xN=0 mm	ya23=-5.5 mm	yb33=-0.09 mm
yN= 3 mm	xa24=13.23 mm	xb34=24.7 mm
-----	ya24=-11.91 mm	yb34=-0.09 mm
! Coordinates of points Aij	!Points B2i	!Points C3i
(i=1-3, j=1-4) (unit: mm):	xb21=-4.85 mm	xc31=24.7 mm
!Points A1i	yb21=-25.72 mm	yc31=29.66 mm
xa11=-9.35 mm	xb22=-1.15 mm	xc32=17.3 mm
ya11=-1.13 mm	yb22=-19.31 mm	yc32=29.66 mm
xa12=-5.65 mm	xb23=-8.73 mm	xc33=17.3 mm
ya12=-7.53 mm	yb23=-14.94 mm	yc33=33.16 mm
xa13=-13.23 mm	xb24=-12.43 mm	xc34=24.7 mm
ya13=-11.91 mm	yb24=-21.35 mm	yc34=33.16 mm
xa14=-16.93 mm	!Points C2i	-----
ya14=-5.5 mm	xc21=13.34 mm	!Points: P, Q, R, S, T, U, V, W
!Points B1i	yc21=-36.2 mm	xT=-50 mm
xb11=-19.85 mm	xc22=17.04 mm	yT=44 mm
yb11=17.06 mm	yc22=-29.81 mm	xW=-50 mm
xb12=-16.15 mm	xc23=20.07 mm	yW=-50 mm
yb12=10.65 mm	yc23=-31.56 mm	xV=44 mm
xb13=-8.57 mm	xc24=16.37 mm	yV=-50 mm
yb13=15.03 mm	yc24=-37.97 mm	xU=44 mm
xb14=-12.27 mm	!Points A3i	yU=44 mm
yb14=21.44 mm	xa31=3.7 mm	xP=-41.07 mm
!Points C1i	ya31=8.66 mm	yP=33.16 mm
xc11=-38.04 mm	xa32=-3.7 mm	xS=-37.37 mm
yc11=6.56 mm	ya32=8.66 mm	yS=-37.97 mm
xc12=-34.34 mm	xa33=-3.7 mm	xR=41.07 mm
yc12=0.15 mm	ya33=17.41 mm	yR=-31.56 mm
xc13=-37.37 mm	xa34=3.7 mm	xQ=41.07 mm
yc13=-1.6 mm	ya34=17.41 mm	yQ=33.16 mm
xc14=-41.07 mm		
yc14=4.81 mm		

APPENDIX.

APPENDIX.

APPENDIX.

APPENDIX.

Bibliography

- [1] Chang S. H. and Li S. S. A high resolution long travel friction-drive micropositioner with programmable step size. Review of Scientific Instruments 1999; 70 (6): 2776-2782.
- [2] Yang R., Jouaneh M. and Schweizer R. Design and characterization of a low-profile micropositioning stage. Precision Engineering 1996; 18: 20-29.
- [3] Her I. and Chang J. C. A linear scheme for the displacement analysis of micropositioning stages with flexure hinges. ASME Machine Elements and Machine Dynamics 1994; 71: 517-525.
- [4] Pernet E. and Clavel R. Ultra-accurate actuator with long travel. In: SPIE Proceedings of Conference on Microrobotics and Microsystem Fabrication. Pittsburgh: USA; 1998. 3202: pp. 54-65.
- [5] Gao P., Swei S. M. and Yuan Z. A new piezodriven precision micropositioning stage utilizing flexure hinges. Nanotechnology 1999; 10: 394-398.
- [6] Yamakawa K., Furutani K. and Mohri N. XYZ-stage for scanning probe microscope by using parallel mechanism. In: Proceedings of the ASME Design Engineering Technical Conferences. Las Vegas: USA; 1999. pp. 1-6.
- [7] Gao P. and Swei S.M. A six-degree-of-freedom micro-manipulator based on piezoelectric translators. Nanotechnology 1999; 10: 447-452.
- [8] Chang S. H., Tseng C. K. and Chien H.C. An ultra-precision $XY\theta_z$ piezo-micropositioner – Part I: Design and analysis. IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control 1999; 46(4): 897-905.

BIBLIOGRAPHY.

- [9] Chang S. H., Tseng C. K. and Chien H. C. An ultra-precision $XY\theta_z$ piezo-micropositioner – Part II: Experiment and performance. IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control 1999; 46(4): 906-912.
- [10] Ryu J. W., Gweon D. G. and Moon K. S. Optimal design of a flexure hinge based $XY\theta$ wafer stage. Precision Engineering 1997; 21: 18-28.
- [11] Ryu J. W., Lee S. Q., Gweon D. G. and Moon K. S. Inverse kinematic modeling of a coupled flexure hinge mechanism. Mechatronics 1999; 9: 657-674.
- [12] Horie M., Kubo S. and Kamiya D. Characteristics of the 3-DOF planar positioning-orientation mechanisms with links and large-deflective hinges. In: Proceedings of the International Conference on Mechatronics Technology (ICMT 2002). Kytakyushu: Japan; 2002. pp. 475-480.
- [13] Lee C. W. and Kim S. W. An ultra precision stage for alignment of wafers in advanced microlithography. Precision Engineering 1997; 21: 113-122.
- [14] Yi B. J., Chung G. B., Na H. Y., Kim W. K. and Suh I. H. Design and experiment of a 3-DOF parallel micromechanism utilizing flexure hinges. IEEE Transactions on Robotics and Automation 2003; 19(4): 604-612.
- [15] Koseki Y., Tanikawa T., Koyachi N. and Arai T. Kinematic analysis of translational 3-DOF micro parallel mechanism using matrix method. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2000. pp. 786-792.
- [16] Han C. S., Hudgens J. C., Tesar D. and Tracer A. E. Modeling, synthesis, analysis and design of high resolution micromanipulator to enhance robot accuracy. In: Proceedings of IEEE/RSJ International Workshop on Intelligent Robots and Systems IROS. Osaka: Japan; 1991. pp. 1157-1162.

BIBLIOGRAPHY.

- [17] Hudgens J. C. and Tesar D. A fully-parallel six degree-of-freedom micromanipulator: kinematic analysis and dynamic model. In: Proceeding of 20th Biennial ASME Mechanisms Conference. Kissimmee, Florida: USA; 1988. pp. 29-37.
- [18] Zhang W. J., Zou J., Watson L. G., Zhao W., Zong G. H. and Bi S. S. The constant-Jacobian method for kinematics of a three-DOF planar micro-motion stage. Journal of Robotics Systems 2002; 19(2): 63-72.
- [19] Seugling R., LeBrun T., Smith S. and Howard L. P. A six-degree-of-freedom precision motion stage. Review of Scientific Instruments 2002; 73(6): 2462-2468.
- [20] Culpepper M. L. and Anderson G. Design of a low-cost nano-manipulator which utilizes a monolithic, spatial compliant mechanism. Precision Engineering 2004; 28: 469-482.
- [21] Paros J. M. and Weisbord L. How to Design Flexure Hinges. Machine Design 1965; 37: 151-156.
- [22] Lobontiu N., Paine J. S. N., Garcia E. and Goldfarb M. Corner-filletted flexure hinges. ASME Journal of Mechanical Design 2001; 123: 346-352.
- [23] Lobontiu N., Paine J. S. N., O'Malley E. and Samuelson M. Parabolic and hyperbolic flexure hinges: flexibility, motion precision and stress characterization based on compliance closed-form equations. Precision Engineering 2002; 26: 183-192.
- [24] Lobontiu N., Paine J. S. N., Garcia E. and Goldfarb M. Design of symmetrical conic-section flexure hinges based on closed-form compliance equations. Mechanism and Machine Theory 2002; 37: 477-498.
- [25] Wu Y. and Zhou Z. Design calculations for flexure hinges. Review of Scientific Instruments 2002; 73(8): 3102-3106.
- [26] Tseytlin Y. Notch flexure hinges: an effective theory. Review of Scientific Instruments 2002; 73(9): 3363-3368.

BIBLIOGRAPHY.

- [27] Zhang S. and Fasse E. A finite-element-based method to determine the spatial stiffness properties of a notch hinge. *ASME Journal of Mechanical Design* 2001; 123: 141-147.
- [28] Schotborgh W. O., Kokkeler F., Tragter H. and Van Houten F. Dimensionless design graphs for flexure elements and a comparison between three flexure elements. *Precision Engineering* 2004; In press, Corrected proof.
- [29] Xu G. and Qu L. Some analytical problems of high performance flexure hinge and micro-motion stage design. In: *Proceedings of the IEEE International Conference on Industrial Technology*. 1996. pp. 771-775.
- [30] Henein S., Bottinelli S. and Clavel R. Parallel spring stages with flexures of micrometric cross-sections. *SPIE Microrobotics and Microsystem Fabrication*. Pittsburgh: USA; 1998; 3202: 209-220.
- [31] Lobontiu N. and Paine J. Design of circular cross-section corner-filletted flexure hinges for three-dimensional compliant mechanisms. *ASME Journal of Mechanical Design* 2002; 124: 479-484.
- [32] Hsiao F. Z. and Lin T.W. Analysis of a novel flexure hinge with three degrees of freedom. *Review of Scientific Instruments* 2001; 72(2): 1565-1573.
- [33] Hale L. C. Principles and techniques for designing precision machines. Ph.D. Thesis. 1999. Massachusetts Institute of Technology, Department of Mechanical Engineering. Boston, USA.
- [34] Kim J. H., Kim S. H. and Kwak Y. K. Development of a piezoelectric actuator using a three-dimensional bridge-type hinge mechanism. *Review of Scientific Instruments* 2003; 74(5): 2918-2924.

BIBLIOGRAPHY.

- [35] Furukawa E., Mizuno M. and Doi T. Development of a flexure-hinged translational mechanism driven by two piezoelectric stacks. JSME International Journal - Series C 1995; 38(4): 743-748.
- [36] Center for Intelligent Mechatronics. <http://fourier.vuse.vanderbilt.edu/cim/projects>.
- [37] Slocum A. and Awatar S. Fabrication, assembly and testing of a new X-Y flexure stage with substantially zero parasitic error motions. Technical Report. <http://www.mit.edu/~shorya/www/phd>.
- [38] Microtechnique. <http://microtechnique.epfl.ch>. Ecole Polytechnique Federal De Lausanne.
- [39] Portman V., Sandler B. and Zahavi E. Rigid 6x6 parallel platform for precision 3-D micro-manipulation: theory and design application. IEEE Transaction on Robotics and Automation 2000; 16(6): 629 - 643.
- [40] Lee K. M. and Arjunan S. A three degrees of freedom micro-motion in-parallel actuated manipulator. , IEEE Transaction on Robotics and Automation 1991; 7: 634-641.
- [41] Ohya Y., Arai T., Mae Y., Inoue K. and Tanikawa T. Development of 3-DOF finger module for micro manipulation. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 1999. pp. 894-899.
- [42] Canfield S. L., Beard J. W., Lobontiu N., O'Malley E., Samuelson M. and Paine J. Development of a spatial compliant manipulator. International Journal of Robotics and Automation 2002; 17(1): 63-71.
- [43] Wang S. C., Hikita H., Kubo H., Zhao Y. S., Huang Z. and Ifukube T. Kinematics and dynamics of a 6 degree-of-freedom fully parallel manipulator with elastic joints. Mechanism and Machine Theory 2003; 38(5): 439-461.
- [44] OIWA Laboratory. <http://oiwa.eng.shizuoka.ac.jp>. Shizuoka University.

BIBLIOGRAPHY.

- [45] Zuo L., Kartik M., Prakazh M., Landsiell N., Culpepper M. and Slocum A. Design of six-axis compliant mechanism for nano manipulator. Technical report 2003. MIT.
- [46] Dwarakanath T. A., Crane C. and Duffy J. Design of an in-parallel compliant force-torque sensor. Conference on Recent Advances in Robotics. Florida: USA; 1999. Session 2.
- [47] Woronko A., Huang J. and Altintas Y. Piezoelectric tool actuator for precision machining on conventional CNC turning centers. Precision Engineering 2003; 27(4): 335-345.
- [48] Smith S. T. Flexure elements of elastic mechanisms. Netherlands 1994. Gordon and Breach Science Publisher.
- [49] Lobontiu N. Compliant mechanisms: design of flexure hinges. 2004. CRC Press.
- [50] Carricato M. and Castelli V. P. Singularity-free fully-isotropic translational parallel mechanisms. International Journal of Robotics Research 2002; 21(2): 161-174.
- [51] Jin Q. and Yang T. Synthesis and analysis of a group of 3-degree-of-freedom partially decoupled parallel manipulators. ASME Journal of Mechanical Design 2004; 126: 301-306.
- [52] Gao F., Li W., Jin Z. and Zhao H. New kinematic structures for 2-, 3-, 4- and 5- DOF parallel manipulator designs. Mechanism and Machine Theory 2002; 37: 1395-1411.
- [53] Fang Y. and Tsai L. W. Structure synthesis of a class of 4-DOF and 5-DOF parallel manipulators with identical limb structures. International Journal of Robotics Research 2002; 21(9): 799-810.
- [54] Huang Z. and Li Q. C. General methodology for type synthesis of symmetrical lower-mobility parallel manipulators and several novel manipulators. International Journal of Robotics Research 2002; 21(2): 131-145.

BIBLIOGRAPHY.

- [55] Frisoli A., Checcacci D., Salsedo F. and Bergamasco M. Synthesis by screw algebra of translating in-parallel actuated mechanisms. International Symposium on Advances in Robot Kinematics. Piran Portoroz: Slovenia; 2000. pp.1-8.
- [56] Kim D. and Chung W. K. Analytical formulation of reciprocal screws and its application to nonredundant robot manipulators. ASME Journal of Mechanical Design 2003; 125: 159-164.
- [57] Zhao T. S., Dai J. S. and Huang Z. Geometric analysis of over constraint parallel manipulators with three and four degrees of freedom. JSME International Journal-Series C 2002; 45(3): 730-740.
- [58] Kong X. and Gosselin C. M. A class of 3-DOF translational parallel manipulators with linear input-output equations. In: Proceeding of the Workshop on Fundamental Issues and Future Research Directions for Parallel mechanisms and Manipulators. Quebec: Canada; 2002. pp. 25-32.
- [59] Kong X. and Gosselin C. M. Type synthesis of 3-DOF translational parallel manipulators based on screw theory. ASME Journal of Mechanical Design 2004; 126: 83-92.
- [60] Kong X. and Gosselin C. M. Type synthesis of 3-DOF spherical parallel manipulators based on screw theory. ASME Journal of Mechanical Design 2004; 126: 101-108.
- [61] Jin Y., Chen I-M. and Yang G. Mobility and singularity analysis of a selective actuated parallel mechanism. 15th CISM-IFTOMM Symposium on Robot Design, Dynamics and Control (ROMANSY). Montreal: Canada; 2004.
- [62] Jin Y., Chen I-M. and Yang G. Structure synthesis and singularity analysis of a parallel manipulator based on selective actuation. IEEE International Conference on Robotics and Automation (ICRA). New Orleans: USA; 2004. pp. 4534-4538.

BIBLIOGRAPHY.

- [63] Tsai L. W. The Jacobian analysis of a parallel manipulator using reciprocal screws. Institute for Systems Research Technical Research Report 1998; TR98-34.
- [64] Tsai L. W. Robot analysis-The mechanics of serial and parallel manipulators. New York , USA 1999. John Wiley & Sons, Inc.
- [65] Hao F. and McCarthy M. Condition for line-based singularities in spatial platform manipulators. Journal of Robotics System 1998; 15(1): 43-55.
- [66] Howell L. Compliant Mechanism. 2001. John Wiley & Sons, Inc.
- [67] Griffel D. Linear algebra and its applications – Volume 2. Chichester, UK, 1989. Ellis Horwood.
- [68] Kim H. S. and Choi Y. J. Forward/Inverse force transmission capability analyses of fully parallel manipulators. IEEE Transaction on Robotics and Automation 2001. 17(4): 526-531.
- [69] Pham H-H. and Chen I-M. Kinematics, workspace and static analyses of two degree-of-freedom flexure parallel mechanism. In: Proceedings of the International Conference on Control, Automation, Robotics and Vision. Singapore; 2002. pp. 968-973.
- [70] Pham H-H., Chen I-M. and Yeh H-C. Micro-motion selective-actuation XYZ flexure parallel mechanism: design and modeling. Journal of Micromechatronics 2005; 3(1):51-73.
- [71] Pham H-H. and Chen I-M. Evaluation of resolution of flexure parallel mechanisms for ultra precision manipulation. Review of Scientific Instruments 2004; 75(9): 3016-3024.
- [72] Pham H-H. and Chen I-M. Design and modeling of a selective-actuation XYZ flexure parallel mechanism. 9th International Symposium on Experimental Robotics ISER04. Singapore; 2004.

BIBLIOGRAPHY.

- [73] Alici G. and Shirinzadeh B. Kinematics and stiffness analyses of a flexure-jointed planar micromanipulator system for a decoupled compliant motion. In: Proceedings of the IEEE/RSJ International on Intelligent Robots and Systems. Las Vegas, Nevada: USA; 2003. pp. 3282-3287.
- [74] Gosselin C. M. Stiffness mapping for parallel manipulators. IEEE Transactions on Robotics and Automation 1990; 6(3): 377-382.
- [75] Gosselin C. M. and Zhang D. Stiffness analysis of parallel mechanisms using lumped model. International Journal of Robotics and Automation 2002; 17(1): 17-27.
- [76] Kim W. K., Lee J. Y. and Yi B. J. Analysis for a planar 3 degree-of-freedom mechanism with actively adjustable stiffness characteristics. In: Proceedings of IEEE International Conference on Robotics and Automation. Albuquerque. New Mexico: USA; 1997. pp. 2663-2670.
- [77] Ceccarelli M. and Carbone G. A stiffness analysis for CaPaMan (Casino parallel manipulator). Mechanism and Machine Theory 2002; 37(5): 427-439.
- [78] Simaan N. and Shoham M. Stiffness synthesis of a variable geometry six-degrees-of-freedom double planar parallel robot. International Journal of Robotics Research 2003; 22(9): 757-775.
- [79] Hale L. C. and Slocum A. Optimal design techniques for kinematics couplings. Precision Engineering 2001; 25: 114-127.
- [80] Pham H-H. and Chen I-M. Stiffness modeling of flexure parallel mechanism. Precision Engineering 2005; 29: 467-478.
- [81] Smith S. T. and Chetwynd D. G. Foundations of ultra-precision mechanism design. 1994. Gordon and Breach.
- [82] Metal and Material for Research and Industry. <http://www.goodfellow.com>.

BIBLIOGRAPHY.

- [83] Materials Processing Center. <http://metalcasting.auburn.edu/data/data.html>. Auburn University. Alabama, USA.
- [84] ANSYS Inc. Structural Analysis Guide. 1994. ANSYS Release 5.7, SAS IP, Inc.. Canonsburg.
- [85] Benzley S., Perry E., Merkley K., Clark B. and Sjaardana G. A comparison of all hexagonal and all tetrahedral finite element meshes for elastic and elasto-plastic analysis. In: Proceedings of 4th International Meshing Roundtable SNL. Albuquerque, New Mexico: USA; 1995. pp. 179-192.
- [86] ANSYS Inc. Advanced Analysis Techniques Guide. 1994. ANSYS Release 5.3, SAS IP, Inc.. Canonsburg.
- [87] Kuhnen K. Modeling, identification and compensation of complex hysteresis nonlinearities a modified Pradtl-Ishlinskii approach. European Journal of Control 2003; 9(4): 407-418.
- [88] Kuhnen K. and Janocha H. Inverse feedforward controller for complex hysteresis nonlinearities in smart-material systems. Control and Intelligent System 2001; 29(3): 55-63.