Parameters affecting estimation of unsaturated permeability of soils

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2015

Arezoo Rahimi. (2015). Parameters affecting estimation of unsaturated permeability of soils. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/62936>

<https://doi.org/10.32657/10356/62936>

PARAMETERS AFFECTING ESTIMATION OF UNSATURATED PERMEABILITY OF SOILS

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2015

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A thesis submitted to Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2015

Acknowledgment

Accomplishing major goals in life would not be possible without the courage, help and guidance of the people who are always there to give you a hand. Finishing this thesis is one such goal for me and I am delighted to thank the individuals who helped me through the process.

Firstly, I would like to thank my supervisor, Professor Harianto Rahardjo, for his invaluable support and guidance. Professor Rahardjo was always there to listen and give advice. He taught me how to ask questions and express my ideas. He showed me different ways to approach a research problem and the need to be persistent to accomplish any goal.

I would like to extend my gratitude to Professor D.G. Fredlund at the University of Saskatchewan, Canada, for his valuable advice and discussions. The advice and assistance of Associate Professor Leong Eng Choon for the research papers is also acknowledged.

The valuable knowledge and technical help from Mr. Vincent Heng Hiang Kim, Mr. Eugene Tan Hiap Guan and Mrs. Susie Lim-Ding in the laboratory are also truly appreciated

I thank my friends and colleagues at Nanyang Technological University for their kindness and understanding, with special thanks to Alfrendo Satyanaga Nio for his suggestions throughout this research. Thanks to everyone who helped me, even if their names are not mentioned here.

I also wish to thank my husband, Bahador, for his constant support and encouragement. Finally, I wish to thank my parents: their existence is a wonderful and irreplaceable gift.

The scholarship provided by A*STAR, the Agency for Science, Technology and Research, is also appreciated.

Abstract

Flow through the unsaturated zone of soil contributes to a variety of geotechnical and geo-environmental problems. Unsaturated permeability is the most important hydraulic property governing the flow process. Therefore, knowledge of unsaturated permeability is crucial in the analysis of the flow process in the unsaturated zone. Unsaturated permeability of soil can be directly measured in soil laboratories; however, this requires not only expensive equipment but also a well-trained technician. It is also a time consuming and tedious task. To overcome the high cost and challenges associated with direct measurement, permeability can be estimated by theoretical models from soil-water characteristic curve (SWCC). To date, there are numerous estimation models that can be used to obtain the permeability of unsaturated soil from SWCC. However, each model results in a different estimation curve. The reason for this difference is not well understood.

The objective of this study is to identify the controlling parameters that affect the estimation of unsaturated permeability of soils. The SWCC equation, relative permeability equation and measured SWCC data range were identified as possible controlling parameters. A matrix of unsaturated permeability estimation models was proposed by combining four SWCC and three relative permeability (k_r) equations to evaluate the effect of controlling parameters on estimation of unsaturated permeability of soils. The matrix of unsaturated permeability estimation models was evaluated using published data containing experimentally measured SWCC and permeability data. It was found that the range of SWCC measurements has the greatest effect on the estimated permeability functions, followed by the SWCC equation. The relative permeability equation has the least effect on variation in the estimation.

A triaxial permeameter system was modified to directly measure the unsaturated permeability of six different kaolin-sand soil mixtures in this study. Soil-water characteristic curve tests were conducted over a wide range of suctions for these soil mixtures. The measured SWCC data were used to estimate the unsaturated permeability of the soils. The estimated values of unsaturated permeability of the soils were validated using the directly measured unsaturated permeability data.

It was found from the results that the unsaturated permeability estimation models could reasonably estimate directly measured unsaturated permeability data when the full measurement of SWCC data covering a large range of suction were used. Even though, the full measurement of SWCC data needs less expensive equipment as compared to that of direct measurement of unsaturated permeability, the full measurement of SWCC data still requires a substantial testing time. Therefore, a procedure is proposed to estimate SWCC values at high suction using the grain size distribution and measured SWCC data at 100 kPa. The proposed procedure could reasonably estimate directly measured unsaturated permeability data. This procedure requires significantly shorter testing time and inexpensive equipment which are available in most of laboratories.

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List of symbols

A	Cross sectional area of a soil specimen
A_c	Cross sectional area of a cylindrical tube
A_1	Cross sectional area of a porous medium
a	A fitting parameter related to AEV
a_1	Cross section area of water reservoir
α	A fitting parameter related to the inverse of AEV
α_t	A parameter related to tortuosity factor of soil
AEV	Air-entry value of soil
b	A fitting parameter related to tortuosity
β	Cross sectional area of soil available for vapor flow per total area
c	Dimensionless constant depending on the geometry of the system
c'	Dimensionless constant depending on the geometry of the system
c_0	Dimensionless constant depending on the geometry of the system at saturation
D^{v*}	Diffusion coefficient of vapor through soil
D_g	Diameter of the grain
E	Total energy
e	Euler number
ε	Porosity
g	Gravitational acceleration
γ_w	Specific weight of water
h_c	Capillary height
h_w	Hydraulic head or total head
h_{wb}	Hydraulic head across bottom disk
h_{ws}	Hydraulic head across soil specimen
h_{wt}	Hydraulic head across bottom disk
h_1	Head of water at t_1
h_2	Head of water at t_2
Δh_w	Hydraulic head difference

η	Pore radius
i	Hydraulic gradient
i_T	Total hydraulic gradient
i_b	Hydraulic gradient across bottom disk
i_s	Hydraulic gradient across soil specimen
i_t	Hydraulic gradient across top disk
K	Intrinsic permeability
k	Hydraulic conductivity or permeability
k_r	Relative permeability
k_s	Saturated permeability
k_w	Unsaturated permeability with respect to water
k_T	Total unsaturated permeability
k_b	Permeability of bottom disk
k_t	Permeability of top disk
k_v	Vapor permeability
L_T	Total length of the system
L_b	Length of the bottom disk
L_s	Length of the soil specimen
L_t	Length of the top disk
L_a	Apparent or macroscopic path length
L_e	Actual or microscopic path length
M_1	A constant related to geometry of the pores and fluid properties
M_w	Mass of water at a selected point
m	A fitting parameter related to residual water content
M	Total number of pore sizes (intervals)
μ	Viscosity of a fluid
n	A fitting parameter related to the slope of SWCC
n_1	A fitting parameter in relative hydraulic conductivity equations
ϑ	Dummy variable of integration
p	Pressure

ΔP	Pressure difference across the contractile skin
π	Osmotic suction
Q	Volumetric flow rate
Q_{inflow}	Inflow volumetric flow rate
$Q_{outflow}$	Outflow volumetric flow rate
q	Volumetric flow rate per unit cross section per unit time
θ_r	Residual volumetric water content
θ_s	Saturated volumetric water content
θ_w	Volumetric water content at a given suction value
θ_{wL}	Lower limit of integration for volumetric water content
θ'_w	Derivative of SWCC
θ	Normalized volumetric water content
θ_n	Normalized volumetric water content
$RMSE$	Root mean square error
R	Universal gas constant
R_s	Radius of curvature of contractile skin
R_1	Radius of curvature of a curved membrane
R_2	Radius of curvature of a curved membrane
R_p	Radius of the pore created by three grains of the same size
r_0	Radius of a capillary tube
r_1	Radius of a pore
r_2	Radius of a pore
r	Distance from center of circular tube from concentric cylindrical surface
ρ_w	Density of water
ρ	Pore radius
S	Degree of saturation
S_n	Normalized degree of saturation
S_r	Residual degree of saturation
s	Surface area of a grain per unit bulk volume
T	Absolute temperature

T_s	Surface tension
t	Time
τ	Shear stress exerted by a fluid
u_a	Pore-air pressure
u_w	Pore-water pressure
$u_a - u_w$	Matric suction
$\bar{u}_v$	Partial pressure of pore-water vapor
$\bar{u}_{v0}$	Saturation pressure of water vapor over a flat surface of pure water at the same temperature
$\bar{u}_a$	Total pressure in the bulk air phase
u_{atm}	Atmospheric pressure
v	Average velocity in capillary tube
v_T	Total average velocity
v_b	Average velocity through bottom disk
v_s	Average velocity through soil
v_t	Average velocity through top disk
v_w	Flow velocity of water at a selected point
v	Velocity of fluid
u_{w0}	Specific volume of water or the inverse of the density of water
w	Gravimetric water content
W_v	Molecular weight of water vapor
ω_v	Molecular mass of water vapor
x	Number of observed data
y_i	A measured or observed value
$\hat{y}_i$	A predicted value by a model
y_{imin}	Minimum measured or observed value
y_{imax}	Maximum measured or observed value
y	Axis normal to the direction of the flow
y_1	Elevation of a selected point above the datum
ψ	Soil suction
ψ_r	Residual suction corresponding to residual water content

ψ_{aev} Suction corresponding to AEV of the soil

Chapter 1 Introduction

1.1 Background

Flow through the unsaturated zone of soil contributes to variety of geotechnical and geo-environmental problems. Unsaturated permeability is the most important hydraulic property governing the flow process. Therefore, knowledge of unsaturated permeability is crucial in the analysis of the flow process in the unsaturated zone.

Unsaturated permeability of soil can be directly measured in soil laboratories; however, this requires not only expensive equipment but also a well-trained technician. It is also a time consuming and tedious task (van Genuchten, 1980). To overcome the expenses and challenges associated with direct measurement, permeability can be predicted by estimation models. The estimation models usually combine knowledge of the soil-water characteristic curve (SWCC) with a flow equation and derive an equation to estimate the unsaturated permeability of the soil. This procedure is in fact an integration of SWCC with a relative permeability (k_r) equation. To date, there are numerous estimation models that can obtain the permeability of unsaturated soil from SWCC. However, each model will result in a different estimation curve and there is no one unified model that can be used for all soil types (Mualem, 1986). The reason for this variation is not well understood. Although numerous models have been developed by researchers to estimate the unsaturated permeability of soil, there have been very few studies investigating the controlling parameters that cause the variation between different models for the same soil.

1.2 Objectives

The main objective of this study is to identify the parameters affecting the estimation of unsaturated permeability using various models.

1.3 Scope

A schematic representation of this research is shown in Figure 1.1. The scope of the work in this research is as follows:

- 1- To study and review the literature to identify the parameters affecting the estimation of unsaturated permeability of soils.
- 2- To select/identify the SWCC equations, relative permeability equations and measured SWCC data range for evaluation.
- 3- To develop a matrix of unsaturated permeability estimation models as a standard procedure for the evaluation of the affecting parameters.
- 4- To study the literature for collating the soil database to evaluate the effect of identified parameters through the matrix of unsaturated permeability estimation models.
- 5- To evaluate the effect of controlling parameters identified in this study.
- 6- To propose a method for mitigating the effect of incomplete measured SWCC data ranges.
- 7- To prepare kaolin-sand mixtures for conducting experimental program
- 8- To conduct SWCC test over the wide suction range
- 9- To develop an automated Tempe cell set up for measuring SWCC of sandy soil mixtures
- 10- To modify a triaxial permeameter for direct measurement of unsaturated permeability
- 11- To validate the findings of the study against experimental results and select the best unsaturated permeability estimation model
- 12- To validate the proposed solution for mitigating the effect of incomplete measured SWCC data ranges against experimental results

Figure 1.1-Schematic representation of this research

1.4 Significance of research

- 1- The controlling parameters affecting estimation of unsaturated permeability of soils were identified and quantified through a matrix of unsaturated permeability estimation models that was developed by combining four SWCC and three relative permeability (k_r) equations.
- 2- A triaxial permeameter was modified and used to directly measure the unsaturated permeability of kaolin-sand mixtures. The soil-water characteristic curves of kaolin-sand mixtures were also measured over a wide suction range to estimate the unsaturated permeability of the soil mixtures.

- 3- A procedure was proposed to estimate SWCC values at high suction using the grain size distribution and measured SWCC data at 100 kPa. The estimated unsaturated permeabilities using the best-fit SWCCs by the proposed method estimated the directly measured unsaturated permeability data reasonably well. This procedure required significantly shorter testing time and inexpensive equipment.

1.5 Organization of thesis

This thesis is organized into seven chapters as follows:

Chapter 1 presents an introduction to the background, objectives, scope, significance of the research and outline of the research.

Chapter 2 presents a review of the literature related to unsaturated soil theory, the soil-water characteristic curve (SWCC), its measurement, estimation and best-fit equation. A review of the principles related to the flow of water through soil, permeability of soil, its measurement and relative permeability equations is provided. The existing unsaturated permeability estimation models are then described.

Chapter 3 presents applicable theories related to this research. A standard procedure for combining soil-water characteristic curve equations and relative permeability equations developed in this study is described (matrix of unsaturated permeability estimation models). A procedure to estimate SWCC data points beyond 100 kPa suction is proposed and presented.

Chapter 4 presents the research program used in the study. The research program includes the procedure for evaluating the affecting parameters of the estimation of unsaturated permeability of soils and the experimental program conducted.

Chapter 5 presents the results obtained from the study. The results of the affecting parameters of the estimation of unsaturated permeability of soils using published data from the literature are presented. The results of the experimental program, including kaolin-sand mixtures, SWCC and saturated and unsaturated permeability are presented.

Chapter 6 presents a discussion of the results obtained from the study.

Chapter 7 presents the conclusions drawn from the study as well as recommendations for future work.

Chapter 2 Literature review

2.1 Introduction

A review of literature related to this research is presented in this chapter. A brief introduction to unsaturated soil theory is presented first. A review of the soil-water characteristic curve (SWCC), its measurement, estimation and the best-fit equation is presented second. Next, a review of the principles related to the flow of water through soils, the permeability of soils, their measurement and estimation is presented. A review of the existing combined soil-water characteristic curve and relative permeability equations is presented lastly.

2.2 Unsaturated soil theory

The pore-space of saturated soil is filled with water (i.e., a wetting fluid), while the pore-space of unsaturated soil is simultaneously filled with water (i.e., a wetting fluid) and air (i.e., a non-wetting fluid). Unsaturated soils usually exist above the water table (vadose zone) with negative pore-water pressures, while saturated soils exist below the water table with positive pore-water pressures. The hydrological cycle (i.e., precipitation, infiltration, evaporation, evapotranspiration and runoff) causes the soil above the water table to change from a saturated state to an unsaturated state and vice versa. Figure 2.1 shows an unsaturated zone and pore-water pressure distribution lines. Precipitation and infiltration result in a downward flux from the ground surface, while evaporation and evapotranspiration result in an upward flux from the ground surface. A net upward flux across the ground surface results in de-saturation of the soil and, therefore, a decrease in pore-water pressure (see line 2 in Figure 2.1). On the other hand, a net downward flux across the ground surface results in saturation of the soil and, therefore, an increase in pore-water pressure (see line 3 in Figure 2.1). A net zero flux across the ground surface is an equilibrium condition which can be represented by hydrostatic pore-water pressure (see line 1 in Figure 2.1). The negative pore-water pressure in unsaturated soil is referred to as matric suction and will be explained in detail in the following sections.

Figure 2.1-Illustration of the unsaturated zone above the water table (vadose zone) (from Fredlund and Rahardjo, 1993)

2.2.1 Phases of unsaturated soil

Unsaturated soil has traditionally been considered a three-phase system that includes solids, water and air. Fredlund and Morgenstern (1977) introduced the air-water interface (i.e., contractile skin) as an additional phase for unsaturated soil due to its different properties from ordinary water (Terzaghi, 1943). Therefore, unsaturated soil can be considered a four-phase system for stress state analysis (see Figure 2.2a). However, for volume-mass relationships, unsaturated soil can be considered a three-phase system (see Figure 2.2b) due to the thinness of the contractile skin, which is in the order of 1.5-2 molecular diameters of water (Israelachvili, 1992).

Figure 2.2-Phase system of unsaturated soil: (a) four-phase (b) three-phase (from Fredlund and Rahardjo, 1993)

2.2.2 Surface tension

The unbalanced forces of water molecules in the contractile skin cause a tensile force that acts tangential to the contractile skin to be in the equilibrium condition. This property of the contractile skin is called surface tension(T_s). The surface tension causes the contractile skin to act like an elastic membrane subjected to different pressures on each side. The membrane has a concave curvature towards higher pressures in order to be in equilibrium. The results of force equilibrium in the vertical direction can be written according to Equation 2.1.

$$\Delta P = \frac{T_s}{R_s} \quad \text{Equation 2.1}$$

where:

R_s is the radius of curvature of contractile skin

ΔP is the pressure difference across the contractile skin

Young-Laplace equation can be used to obtained pressure difference across the curved membrane as follows (Fredlund and Rahardjo, 1993):

$$\Delta P = T_s \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \text{Equation 2.2}$$

where:

R_1 and R_2 are the radii of curvature of a curved membrane

If the radius of curvature is the same in all directions (i.e., $R_1=R_2=R_s$), Equation 2.2 will result in Equation 2.3:

$$\Delta P = \frac{2T_s}{R_s} \quad \text{Equation 2.3}$$

2.2.3 Matric suction

The contractile skin in unsaturated soil is subject to two different pressures: pore-air pressure (u_a), on one side and pore-water pressure (u_w), on the other side (Fredlund and Rahardjo, 1993). The difference between these two pressures, ($u_a - u_w$) can be calculated from Equation 2.3 and written as Equation 2.4:

$$u_a - u_w = \frac{2T_s}{R_s} \quad \text{Equation 2.4}$$

The pressure difference ($u_a - u_w$), across the contractile skin (i.e., air-water interface) is called matric suction. Matric suction is usually associated with the capillary phenomenon caused by surface tension of water. The capillary height can be calculated in unsaturated soil by assuming that a capillary tube represents a pore of unsaturated soil according to Equation 2.5:

$$h_c = \frac{2T_s}{\rho_w g R_s} \quad \text{Equation 2.5}$$

where:

h_c is the capillary height

ρ_w is the water density

g is the gravitational acceleration

R_s is the radius of curvature of the meniscus (i.e., contractile skin)

Figure 2.3 shows a schematic diagram of a capillary tube and its corresponding pore-water pressure distribution.

Figure 2.3-Schematic diagram of capillarity in a capillary tube (from Fredlund and Rahardjo, 1993)

Pore-water pressure at the meniscus (point C in Figure 2.3) can be calculated as follows:

$$u_w = -\rho_w g h_c \quad \text{Equation 2.6}$$

Since pore-air pressure is equal to atmospheric pressure, which is zero, the following equation can be written:

$$u_a - u_w = \rho_w g h_c \quad \text{Equation 2.7}$$

According to Equation 2.7, the matric suction ($u_a - u_w$), in unsaturated soil becomes a positive value. It should be noted that substitution of Equation 2.5 into Equation 2.7 will result in Equation 2.4, which expresses matric suction in terms of surface tension.

2.2.4 Soil suction

Soil suction usually refers to the free energy state of soil water and it can be measured in terms of the partial vapor pressure of the soil water (Edlefsen and Anderson, 1943). The combination of Gibbs free energy with the capillary equation will result in Kelvin equation (Fredlund and Rahardjo, 1993) as follows:

$$\psi = -\frac{RT}{v_{w0}\omega_v} \ln\left(\frac{\bar{u}_v}{\bar{u}_{v0}}\right) \quad \text{Equation 2.8}$$

where:

ψ is the soil suction or total suction (kPa)

R is the universal gas constant (i.e., 8.31432 J/(mol K))

T is the absolute temperature (K)

v_{w0} is the specific volume of water or the inverse of the density of water (m³/kg)

ω_v is the molecular mass of water vapor (kg/kmol)

$\bar{u}_v$ is the partial pressure of pore-water vapor (kPa)

$\bar{u}_{v0}$ is the saturation pressure of water vapor over a flat surface of pure water at the same temperature (kPa)

The term $\left(\frac{\bar{u}_v}{\bar{u}_{v0}}\right)$ is also called relative humidity, RH (%). Therefore, based on the Kelvin equation, soil suction can be quantified in terms of relative humidity. The soil suction that can be calculated from relative humidity is usually called total suction and has two components: matric suction and osmotic suction. The following equation shows this relationship:

$$\psi = (u_a - u_w) + \pi \quad \text{Equation 2.9}$$

where:

π is the osmotic suction

Osmotic suction is associated with dissolved salts in soil water. The decrease in relative humidity due to the presence of salt in soil water is referred to as osmotic suction (Hillel, 1982).

2.3 Soil-water characteristic curve

The soil-water characteristic curve (SWCC) is defined as the relationship between the amount of water in the soil and the soil suction (Fredlund, 2002). SWCC has been recognized as the key unsaturated soil property required for seepage, shear strength and volume change analyses (Barbour, 1998; Fredlund, 2000; Fredlund et al., 2012). The amount of water in the soil can be quantified by several terms such as gravimetric water content(w), volumetric water content(θ_w) and degree of saturation(S)(Fredlund et al., 2012). The gravimetric water content is computed based on the mass of water and soil solids, while the volumetric water content and degree of saturation are calculated based on volumes. The SWCC of soil is usually expressed as the relationship between volumetric water content and soil suction. However, it can also be expressed using any of the terms used to define the water content of the soil. It should be noted that if the volume change of the soil is negligible, SWCC will be the same for all the defined terms for water content (Fredlund, 2002). A dimensionless water content or normalized water content can also be used for expressing the water content of the soil in SWCC (Fredlund, 2002). As the soil suction increases, the amount of water in the soil decreases, which is a drying or desorption curve of SWCC. On the other hand, as the soil suction decreases, the amount of water increases, which is a wetting or adsorption curve of SWCC. However, the drying curve is usually different from the wetting curve and, therefore, SWCC is subject to hysteresis as explained in Section 2.3.4. Figure 2.4 shows a typical soil-water characteristic curve.

Figure 2.4-Typical soil-water characteristic curve of a soil

The soil-water characteristic curve is usually plotted in such a way that the x-axis is soil suction and the y-axis is water content. Soil suction is usually shown on a logarithmic scale and water content is shown on a decimal scale. As shown in the figure, the SWCC usually has two distinct points: the air-entry value (AEV) and the residual state of the soil. The air-entry value of the soil (AEV) is the matric suction where air starts to enter the largest pores in the soil and the soil starts to desaturate. The residual state is where a large suction change is required to remove additional water from the soil (Fredlund and Xing, 1994). However, these descriptions are quite vague and there are no theoretical definitions (Fredlund et al., 2012). The common practice for determining the air-entry value and residual water content is the graphical method (Fredlund and Xing, 1994; Vanapalli et al., 1998) and there are no independent procedures for determining them (van Genuchten, 1980).

2.3.1 Measurement of soil-water characteristic curve

Measurement of the soil-water characteristic curve requires measurement of the soil water content at different suction values. The equipment used for measurement of SWCC can

be divided into equipment that controls matric suction and equipment that controls total suction (Fredlund, 2002). The equipment or pressure chambers used for the application of matric suction to the soil specimen must have a high air-entry ceramic disk. The axis translation technique (Hilf, 1956) is usually used for the application of matric suction and the maximum value of matric suction that can be applied to the soil specimen depends on the air-entry value of the ceramic disk. For instance, a 1-bar ceramic disk can be used to control matric suction up to 100 kPa. It should be noted that pressure chambers can be used to control matric suction up to 1500 kPa. The soil specimen is usually weighed at each matric suction value and the water content of the soil is obtained. For controlling the total suction of the soil, equipment that can control the relative humidity of an enclosed environment is usually used, e.g., a vacuum desiccator. The relative humidity can then be converted to total suction using the Kelvin equation as explained in Section 2.2.4. A detailed description for the determination of SWCC is provided in ASTM D6836-02 (ASTM, 2003). The common practice is to measure the desorption curve of SWCC since the adsorption curve of SWCC is usually difficult to measure (Fredlund, 2002; Fredlund, 2006). The SWCC of a soil is affected by the soil's texture and grain sizedistribution (Azam et al., 2013). Figure 2.5 shows typical soil-water characteristic curves for clayey, silty and sandy soils. As can be seen from the figure, soils with a higher percentage of fine particles have a higher initial water content and a higher water content at a given suction value (Hillel, 1982; Fredlund and Xing, 1994; Vanapalli et al., 1999; Yang et al., 2004; Iyer et al., 2013). It should be noted that the AEV of a soil typically increases with an increase in the fine content of the soil. The stress history can also affect the SWCC of a soil. For instance, a higher confining pressure will result in a higher AEV and a gentler slope of SWCC (Vanapalli et al., 1996; Ng and Pang, 2000; Lee et al., 2005; Thu et al., 2007).

Figure 2.5-Typical SWCCs for clayey, silty and sandy soils

2.3.2 Estimation of soil-water characteristic curve

Soil-water characteristic curve measurement can be a time-consuming task. It also requires precise and expensive equipment. Therefore, estimating the SWCC of a soil would greatly reduce the time and cost of conducting the test (Fredlund et al., 2002; Jaafar and Likos, 2011; Mohammadi and Vanclooster, 2011). The term Pedo-Transfer Function (PTF) has been used to describe SWCC estimation functions that relate basic soil properties to yield a soil property function (Bouma, 1989; Wösten et al., 2001).

The estimation algorithms for SWCC have been classified into four major approaches by Johari et al. (2006) as follows:

- 1) In the first approach, the water contents at different matric suction values are estimated statistically (Gupta and Larson, 1979; Saxton et al., 1986). The water contents and soil properties are then correlated at a selected suction value using regression analysis followed by a curve-fitting procedure.

- 2) In the second approach, regression analysis is used to correlate soil properties with the fitting parameters of an analytical equation representing the SWCC (Ahuja et al., 1985; Zapata, 1999; Chin et al., 2010).
- 3) In the third approach, the SWCC is estimated using a physics-based conceptual model (Arya and Paris, 1981; Kovács, 1981; Smettem and Gregory, 1996; Fredlund, 2002; Hunt and Gee, 2002; Aubertin et al., 2003) that involves the conversion of a grain size distribution into a pore-size distribution.
- 4) In the fourth approach, artificial intelligence methods, such as neural networks, genetic programming and other machine learning methods (Pachepsky et al., 1996; Johari et al., 2006; Garg et al., 2014) that involve training the model with an existing database and using the model to estimate the SWCC of other soils, are used.

It should be noted that, although PTF can be used to estimate the SWCC of a soil, it still has limitations due to the underlying assumptions in the method. Therefore, the estimated SWCC may not be the same as the measured one.

2.3.3 Best-fit equations for soil-water characteristic curve

To date, numerous closed-form empirical equations have been proposed to best-fit the soil-water characteristic curve data of an unsaturated soil. These equations usually use the least-squares regression method to best-fit the measured data. The equations can be divided into two groups: equations with two fitting parameters and equations with three fitting parameters (Sillers and Fredlund, 2001; Fredlund et al., 2012). Table 2.1 shows some of the two and three fitting parameter best-fit equations for SWCC of unsaturated soil as described by Sillescu and Fredlund (2001). As can be seen from the table, equations with two fitting parameters have one parameter (a), which is related to the air-entry value (AEV) of the soil, and another parameter (n), which is related to the rate of desaturation of the soil which, in turn, is related to the pore-size distribution of the soil. For equations with three fitting parameters, the third parameter (m), is related to the asymmetry of the fitted curve and allows the low matric suction range near the AEV to have a shape independent from the high matric suction range near the residual matric suction (Fredlund, 2006). Therefore, equations with three fitting parameters provide more flexibility for best-fit analysis (Fredlund et al., 2012). In this study, three fitting parameter

equations were selected as the best-fit SWCC equations. These equations are described in detail in the following sections.

Table 2.1-Best-fit equations for soil-water characteristic curves

Reference	Equation	Fitting parameters
Gardner (1958)	$\theta_w = \frac{\theta_s}{1 + a\psi^n}$	a and n
Brooks and Corey (1964)	$\theta_w = \theta_s \text{ for } \psi \leq \psi_{AEV}$ $\theta_w = \theta_s \left(\frac{a}{\psi}\right)^n \text{ for } \psi > \psi_{AEV}$	a and n
Brutsaert (1966)	$\theta_w = \theta_r + \frac{(\theta_s - \theta_r)}{1 + \left(\frac{\psi}{a}\right)^n}$	a and n
McKee & Bumb (1984)	$\theta_w = \theta_r + (\theta_s - \theta_r) \exp\left(-\frac{a - \psi}{n}\right)$	a and n
McKee & Bumb (1987)	$\theta_w = \theta_r + \frac{(\theta_s - \theta_r)}{1 + \exp\left(\frac{a - \psi}{n}\right)}$	a and n
van Genuchten (1980)	$\theta_w = \theta_r \frac{(\theta_s - \theta_r)}{[1 + (a\psi)^n]^m}$	a, n and m
Fredlund and Xing (1994)	$\theta_w = \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)}\right] \frac{\theta_s}{\left[\ln\left[e + \left(\frac{\psi}{a}\right)^n\right]\right]^m}$	a, n and m

where:

a and a are the fitting parameters related to the air-entry value (AEV) of the soil

e is Euler's number, 2.71828

m is the fitting parameter related to the residual water content

n is the fitting parameter related to the slope of SWCC or rate of de saturation of the soil

ψ is the soil suction

θ_s is the saturated volumetric water content

θ_w is the volumetric water content at a given suction value

θ_r is the volumetric water content at a residual suction value

$C(\psi) = 1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)}$ is a correction factor that forces the volumetric water content to zero at a matric suction of 1 GPa

ψ_r is the suction related to the residual volumetric water content

2.3.3.1 Fredlund and Xing (1994)

Fredlund and Xing (1994) is a continuous closed-form best-fit SWCC equation written according to Equation 2.10.

$$\theta_w = C(\psi) \frac{\theta_s}{\left[\ln \left(e + \left(\frac{\psi}{a} \right)^n \right) \right]^m} \quad \text{Equation 2.10}$$

where:

θ_w is the volumetric water content

θ_s is the saturated volumetric water content

a is the fitting parameter related to the air-entry value of the soil (kPa)

n is the fitting parameter related to the slope of the SWCC

m is the fitting parameter related to the residual water content of the soil

e is Euler's number, 2.71828

ψ is the soil suction, (kPa)

$$C(\psi) = 1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)}$$

where:

ψ_r is the suction related to the residual volumetric water content

The correction factor in the Fredlund and Xing (1994) equation forces the volumetric water content to zero at a suction value of 10^6 kPa.

2.3.3.2 Fredlund & Xing (1994) with correction factor $C(\psi) = 1$

Leong and Rahardjo (1997) reviewed all of the SWCC equations and concluded that the Fredlund & Xing (1994) equation gives the most satisfactory fit for a wide range of soils

over the entire matric suction range. They recommended that $C(\psi) = 1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)}$

should be assumed to be equal to unity. Therefore, the equation takes the following form:

$$\theta_w = \frac{\theta_s}{\left[\ln \left(e + \left(\frac{\psi}{a} \right)^n \right) \right]^m} \quad \text{Equation 2.11}$$

Leong and Rahardjo (1997) concluded that $C(\psi) = 1$ will not affect the initial portion of the SWCC, which is relatively important. In addition, the computational effort for determining the fitting parameters can be reduced.

2.3.3.3 van Genuchten (1980)

van Genuchten (1980) proposed Equation 2.12 to best-fit the SWCC measured data for estimating the unsaturated permeability of soils:

$$\theta_w = \theta_r + \frac{(\theta_s - \theta_r)}{[1 + (\alpha\psi)^n]^m} \quad \text{Equation 2.12}$$

where:

θ_w is the volumetric water content

θ_s is the saturated volumetric water content

θ_r is the residual volumetric water content

α, n and m are fitting parameters

2.3.3.4 van Genuchten (1980) with $m=1-1/n$

In order to derive a closed-form solution for estimating the unsaturated permeability of soils using the Mualem (1976) relative permeability equation, van Genuchten (1980) substituted parameter (m) from Equation 2.12 with $(m=1-1/n)$, as shown in Equation 2.13.

$$\theta_w = \theta_r + \frac{(\theta_s - \theta_r)}{[1 + (\alpha\psi)^n]^{1-\frac{1}{n}}} \quad \text{Equation 2.13}$$

2.3.4 Hysteresis of soil water characteristic curve

The soil-water characteristic curve is a relationship between the water content and suction of a soil. As explained in Section 2.3, the relationship can be obtained by either drying (i.e., increasing suction) an initially saturated soil or by wetting (i.e., decreasing suction) a dry soil. However, the resulting curves are not identical and soil that has been dried has a higher water content at a given matric suction than soil that has been wetted. This phenomenon is called hysteresis (Miller and Miller, 1955a; Miller and Miller, 1955b; Hillel, 1982; Fredlund and Rahardjo, 1993). The hysteresis effect is related to factors such as non-uniformity of pore-geometry (Hillel, 1982), contact angle (Lu and Likos, 2004), entrapped air (Fredlund and Rahardjo, 1993) and swelling, shrinkage or aging phenomena of soil (Hillel, 1982). The non-uniformity of pore-geometry is usually considered the most important factor causing hysteresis. It can be explained by capillary theory and is usually referred to as the ink bottle phenomenon.

Consider a pore in the soil structure with a radius of r_1 bonded by pores with a smaller radius of r_2 (see Figure 2.6). During the drying process, once the suction of the soil reaches $\psi_2=C/r_2$ (where C is a factor related to the geometry of the pore), according to Equation 3.22 the pore with a radius of r_2 will be emptied. During the wetting process, the pore with a radius of r_1 will be filled with water first, which requires a suction value of $\psi_1=C/r_1$, which is lower than $\psi_2=C/r_2$. Therefore, at a given suction value, the soil has a lower water content in the wetting branch of SWCC than in the drying branch of SWCC.

Figure 2.6-Ink bottle effect in unsaturated soil

2.4 Flow in soils

The energy state of the soil water is one of the most important physical soil properties (Baver et al., 1972). It can be computed at a selected point according to Equation 2.14:

$$E = M_w g y_1 + \frac{M_w u_w}{\rho_w} + \frac{M_w v_w^2}{2} \quad \text{Equation 2.14}$$

where:

E is the total energy

M_w is the mass of water at a selected point

g is the gravitational acceleration

y_1 is the elevation of a selected point above the datum

u_w is the pore-water pressure at a selected point

ρ_w is the density of water

v_w is the flow velocity of water at a selected point

When the total energy is expressed per unit weight of water, the result is called the potential or hydraulic head, in accordance with Equation 2.15.

$$h_w = y_1 + \frac{u_w}{\rho_w g} + \frac{v_w^2}{2g} \quad \text{Equation 2.15}$$

where:

h_w is the hydraulic head or total head

The third term, which is the velocity head, is usually ignored due to its negligible magnitude compared to the other two terms.

Water flows towards a position of lower energy or from regions of higher hydraulic head towards regions of lower hydraulic head; thus, energy is usually required to overcome the viscosity's resistance to the flow.

Flow in soil is a special case of the larger problem of fluid flow in porous media (Childs, 1969); therefore, the general nature of fluid flow in narrow tubes is described first. The flow of a real fluid is associated with viscosity, which is what causes the fluid's resistance to flow (Hillel, 1982). A Newtonian fluid will follow Newton's law of viscosity (Falkovich, 2011) as follows:

$$\tau = \mu \left(\frac{dv}{dy} \right) \quad \text{Equation 2.16}$$

where:

τ is the shear stress exerted by a fluid

μ is the fluid's viscosity

v is the velocity of a fluid

y is the axis normal to the direction of the flow

dv/dy is the velocity gradient perpendicular to the direction of shear

Consider flow in a straight cylindrical tube as shown in Figure 2.7. The free body of a concentric cylinder of fluid in a tube, as shown in Figure 2.8, is considered.

Figure 2.7-Schematic of a fluid in a circular tube (from Massey and Ward-Smith, 2012)

Figure 2.8-Schematic of a free body of a concentric cylinder of fluid in a circular tube
(from Massey and Ward-Smith, 2012)

If a fluid such as water wants to move from point 1 to point 2, there will be a pressure gradient to overcome the frictional force on the concentric cylindrical surface. Therefore, shear stress(τ) can be written according to Equation 2.17:

$$\tau = -\left(\frac{\partial p}{\partial x}\right)\left(\frac{r}{2}\right) \quad \text{Equation 2.17}$$

where:

p is pressure

$\partial p / \partial x$ is the pressure gradient in the flow direction (x direction)

r is the distance from the center of the circular tube to the concentric cylindrical surface

By substituting Equation 2.16 into Equation 2.17 and considering $y = r_0 - r$, the following equation is obtained:

$$-\mu \left(\frac{\partial v}{\partial r} \right) = - \left(\frac{\partial p}{\partial x} \right) \left(\frac{r}{2} \right) \rightarrow \partial v = \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) r dr \quad \text{Equation 2.18}$$

By integrating Equation 2.18 with respect to r and the assumption that $v = 0$ at $r = r_0$, the following equation is obtained:

$$v = \frac{1}{4\mu} \left(- \frac{\partial p}{\partial x} \right) (r_0^2 - r^2) \quad \text{Equation 2.19}$$

where:

r_0 is the radius of a capillary tube

The volumetric rate of flow (Q), at any cross section at a radius of r_0 is calculated by integrating $dq = v dA_c = \frac{1}{4\mu} \left(- \frac{\partial p}{\partial x} \right) (r_0^2 - r^2) (2\pi r) dr$ according to Equation 2.20.

$$Q = \frac{\pi}{8\mu} \left(- \frac{\partial p}{\partial x} \right) r_0^4 \quad \text{Equation 2.20}$$

where:

q is the volumetric flow rate per cross sectional area

A_c is the cross sectional area of a cylindrical tube

Equation 2.20 is known as Poiseuille's law and indicates that the volume rate of the flow is proportional to the pressure drop per unit distance and the fourth power of the radius of the tube. Thus, the average velocity (v), over the cross section (A_c) of the tube can be calculated as follows:

$$v = \frac{Q}{A_c} = \frac{\frac{\pi}{8\mu} \left(- \frac{\partial p}{\partial x} \right) r_0^4}{\pi r_0^2} = \frac{1}{8\mu} \left(- \frac{\partial p}{\partial x} \right) r_0^2 \quad \text{Equation 2.21}$$

It should be noted that the Poiseuille equation is valid for laminar flows.

The pore-structure of a soil is usually considered a series of tubes with various sizes and the overall flow rate of water through soil is assumed to be equal to the sum of flow rates through individual tubes (Baver et al., 1972; Hillel, 1982; Marshall and Holmes, 1988). However, the flow channels in soil are more complicated than uniform and smooth tubes and are very irregular and tortuous. Therefore, the microscopic details of flow are not considered and flow in soil is considered as macroscopic.

2.4.1 Permeability of soils

Henry Darcy (1856) was the first to quantify the flow of water through a porous medium. He measured the flow rate of water through saturated sand filters and discovered that the volume rate of flow(Q), is directly proportional to the cross section, A_1 , of the porous medium and hydraulic head difference between the inlet and outlet of water, ($\Delta h_w = h_{wi} - h_{wo}$), and inversely proportional to the length of the medium(L), according to the following equation:

$$Q \propto A_1 \Delta h_w / L \quad \text{Equation 2.22}$$

The term $\Delta h_w / L$ is usually called the hydraulic gradient(i), and is the cause or driving potential of the flow. If the volume rate of flow is considered per unit cross section (A_1) per unit time (t), the flux (q) can be calculated. A proportionality factor relates all of the parameters together according to the following equation:

$$q = \frac{Q}{A_1 t} = k \Delta h_w / L \quad \text{Equation 2.23}$$

The proportionality factor(k) is usually called the permeability or hydraulic conductivity and contains all the microscopic details of flow including tortuosity, pore-size distribution and fluid properties through a porous medium (i.e., soil). If the fluid properties are considered explicitly in permeability(k), the intrinsic permeability of a porous medium(K) can be obtained according to the following equation:

$$k = \rho_w g K / \mu \quad \text{Equation 2.24}$$

Flow in either saturated or unsaturated soil can be explained by driving potentials, as explained in Section 2.4.

2.4.2 Saturated permeability of soils

As explained in Section 2.4, the driving potential for flow of water through a saturated soil is the hydraulic gradient. Whenever there is a hydraulic head difference in a saturated soil, water will flow towards the lower hydraulic head. The pore-space of a saturated soil is always filled with water and, as a result, the permeability is constant (Marshall and Holmes, 1988) and is only a function of the void ratio (Lambe and Whitman, 1979). Thus, the permeability of a saturated soil can be calculated by means of Darcy's law and the Hagen-Poiseuille equation.

2.4.3 Unsaturated permeability of soils

The driving potential for an unsaturated soil has ever been considered to be the water content gradient, matric suction gradient or hydraulic head gradient (Fredlund and Rahardjo, 1993). However, the water content and matric suction gradient do not have a physical basis, and therefore, the hydraulic head gradient is the only driving potential in an unsaturated soil. Water will flow from high hydraulic head regions towards low hydraulic head regions. The pore-space of an unsaturated soil is filled with both water and air and, as a result, the permeability is not only a function of the void ratio but also of the amount of water present in the pore-space (Fredlund and Rahardjo, 1993). Therefore, the permeability of an unsaturated soil is not constant. When soil suction increases in an unsaturated soil, the amount of water in the soil decreases and, as a result, the permeability will decrease. Therefore, the permeability of an unsaturated soil is not a constant but a function of its suction or volumetric water content. It should be noted that although permeability is not constant, Darcy's law can still be applied for unsaturated soils (Buckingham, 1907; Richards, 1931; Childs and Collis-George, 1950). The decrease in the unsaturated permeability of soils can be several orders of magnitude as a result of an increase in soil suction or a decrease in volumetric water content (Fredlund et al., 2012).

2.4.4 Measurement of permeability of soils

2.4.4.1 *Saturated permeability of soils*

There are two methods for measuring the saturated permeability of soils in the laboratory, namely the constant-head and falling-head test methods. In the constant-head method, the

inflow or outflow volume rate is measured at an applied constant hydraulic gradient. The permeability can then be computed using Darcy's law according to Equation 2.25.

$$k_s = \frac{QL_s}{\Delta h_w At} \quad \text{Equation 2.25}$$

where:

Q is the inflow or outflow volume rate of flow

L_s is the length of the soil specimen

Δh_w is the hydraulic head difference

A is the cross sectional area of the specimen

t is time

In the falling-head method, the inflow and outflow volume rates are measured at an applied hydraulic gradient that changes with time; the permeability is computed using Darcy's law and the continuity of water (i.e., inflow volume rate equals outflow volume rate) is computed according to Equation 2.26.

$$k_s = \frac{a_1 L}{At} \ln \left(\frac{h_1}{h_2} \right) \quad \text{Equation 2.26}$$

where:

a_1 is the cross section area of the water reservoir

h_1 is the water head at t_1

h_2 is the water head at t_2

It should be noted that both tests can be conducted in a rigid wall or flexible wall permeameter apparatus. The detailed procedures for all tests are given in the ASTM standard shown in Table 2.2.

Table 2.2-ASTM standard for saturated permeability tests

Test method, apparatus	Standard
Constant-head, rigid wall permeameter	D2434
Constant-head, flexible wall permeameter	D5084
Falling head, flexible wall permeameter	D5084

2.4.4.2 Unsaturated permeability of soils

The unsaturated permeability test can be conducted directly in the laboratory by measuring the volume rate of flow through voids in the soil specimen. The measurement can be done using either steady-state or unsteady-state methods. When measuring the unsaturated permeability of a soil, a high-air entry ceramic disk is usually used as a contractile skin to separate the wetting fluid (i.e., water) from the non-wetting fluid (i.e., air) as explained in Section 2.2.1. Due to the head loss across the high-air entry ceramic disk, ASTM D7664 specifies that the saturated permeability of the high air-entry ceramic disk must be at least two orders of magnitude higher than the saturated permeability of the soil specimen. Therefore, it is not necessary to consider the impedance of the high air-entry ceramic disk in the analysis of the data. However, if the ASTM requirement cannot be satisfied, a head loss across the high air-entry ceramic disk must be taken into account for computation of the permeability of the soil specimen. It should be noted that the steady-state method for low flow rates usually requires a long time period and an accurate volume change measurement should be conducted (Klute, 1965; Klute, 1972; Olson and Daniel, 1981). Table 2.3 summarizes some of the various apparatuses developed for measuring the unsaturated permeability of soils using the steady-state method.

Table 2.3-Apparatuses for measuring unsaturated permeability by the steady-state method

Author	Remark	Schematic of apparatus	Note
Klute (1965)	Suction up to 90 kPa Rigid wall		Shrinkage of sample and separation from the wall of the permeameter
Fleureau and Taibi (1994)	Suction up to 80 kPa k_w up to 10^{-8} m/s k_a up to 10^{-8} m/s Rigid wall
Barden and Pavlakis (1971)	k_w 10^{-12} - 10^{-10} m/s k_a 10^{-8} - 10^{-3} m/s Flexible wall		Good contact between the rubber membrane and the soil and control of stress state variable pressure
Huang , Fredlund et al. (1998)	k_w 10^{-11} to 10^{-8} m/s Flexible wall		Volume change of soil specimen can be measured

As already mentioned, unsaturated permeability can be measured by the unsteady-state method in the laboratory as well as in the field. The main advantage of the unsteady-state

method is that the test can be completed in a short period of time. Unsteady-state methods include the instantaneous profile method (Richards and Weeks, 1953; Hamilton et al., 1981; Meerdink et al., 1996; Krisdani et al., 2009; Li et al., 2009), single or multistep outflow method (Gardner, 1956; Kool et al., 1985; Šimůnek et al., 1998; To-Viet et al., 2013), absorption method (Bruce and Klute, 1956) and sorptivity method (Klute and Dirksen, 1986). However, there are many semi-empirical assumptions associated with these methods that are difficult to verify, and so the results are questionable.

2.4.5 Estimation of unsaturated permeability of soils

Direct measurement of unsaturated permeability of soils requires expensive and precise equipment, well-trained technicians and tremendous amounts of time (van Genuchten, 1980; Fredlund et al., 1994; Agus et al., 2005; Cihan et al., 2009). As the water content of a soil decreases, the unsaturated permeability of the soil may decrease by several orders of magnitude; however, there is no apparatus that can measure such a wide range of values (Fredlund et al., 2012). Therefore, several estimation methods have been developed to predict the unsaturated permeability of soils. These estimation methods are usually derived from the flow equation in a capillary tube, as explained in Section 2.4, considering the appropriate boundary conditions. These flow equations usually express the relative permeability, k_r , of soils as defined by Equation 2.27 as a function of normalized water content, $\theta_n = \frac{\theta_w - \theta_r}{\theta_s - \theta_r}$, or normalized degree of saturation, $S_n = \frac{S - S_r}{1 - S_r}$, of the soil.

$$k_r = \frac{k_w}{k_s} \quad \text{Equation 2.27}$$

where:

k_w is the unsaturated permeability with respect to water

k_s is the saturated permeability

The estimation models use knowledge about the soil-water characteristic curve and SWCC data to estimate the unsaturated permeability of soils. These estimation models are called relative permeability equations in this study. A brief history of the development of estimation models (i.e., relative permeability equations) is described here.

A group of investigators assumed that a porous medium or soil is analogous to a bundle of uniform and parallel circular capillary tubes. By means of the Hagen-Poiseuille equation and Darcy's law, as explained in Sections 2.4, equations have been proposed for the relative permeability, k_r , of unsaturated soils. Table 2.4 shows a summary of some proposed equations based on the aforementioned assumption.

Table 2.4-Proposed models based on the uniform pore-size model

Uniform pore-size model -analogy of bundle of uniform and parallel circular capillary tubes			
Equation	Author	Note	Assumption
$k_r = S_n^{n_1}$	Averjanov (1950)	$n_1 = 3.5$	Stagnant air surrounded by annulus of water
$k_r = S^2$	Yuster (1951)		Moving air surrounded by annulus of water

where:

n_1 is the fitting parameter

Another group of investigators combined the uniform pore-size model with the concept of hydraulic radius, which replaced pore-radius in the Hagen-Poiseuille equation. The hydraulic-radius was defined as the ratio of pore volume to particle area (Blake, 1922; Kozeny, 1927; Fair and Hatch, 1933). Table 2.5 shows a summary of some proposed equations based on these assumptions.

Table 2.5-Proposed models based on the uniform pore-size model and hydraulic radius

Uniform pore-size model -analogy of a bundle of uniform and parallel circular capillary tubes with the concept of hydraulic radius			
Equation	Author	Note	Assumption
$k_s = c\varepsilon^3/s^2$	Kozeny (1927)		Flux equals the average velocity in the pores times the porosity of the medium
$k_s = c' \left(\frac{L_a}{L_e} \right) \varepsilon^3/s^2$	Carmen (1937; 1956)		Introduces the concept of tortuosity
$k_r = S_n^3$	Irmay (1954)		Applies the Kozeny model to unsaturated soils
$k_r = S_n^{n_1}$	Wyllie and Spangler (1952)	$n_1 = 2 - 5$	Limits the applicability of the uniform pore-size model

where:

ε is the porosity of the medium

s is the surface area of a grain per unit bulk volume

c is the dimensionless constant depending on the geometry of the system

c' is the dimensionless constant depending on the geometry of the system

c_0 is the dimensionless constant depending on the geometry of the system at saturation

L_a is the apparent or macroscopic path length

L_e is the actual or microscopic path length

Moving forward, the uniform pore-size model was improved by another group of researchers. They assumed that a porous medium consists of a number of parallel portions each with a uniform pore-size. The unsaturated permeability was computed from the uniform pore-size model as mentioned above for each portion and then integrated over all portions. The pore-size distribution or portions are usually obtained from the soil-water characteristic curve of the soil. Table 2.6 shows a summary of equations based on the above assumption.

Table 2.6-Proposed models based on parallel capillary tubes

Parallel model			
Equation	Author	Note	Assumption
$k_s = \varepsilon T_s^2 \int_{S=0}^{S=1} c \frac{dS}{\psi^2}$	Purcell (1949)		Applied for a saturated porous medium
$k_w = \varepsilon T_s^2 \int_{S=0}^{S=1} c \frac{dS}{\psi^2}$	Burdine et al. (1950)		Applied for an unsaturated porous medium
$k_r = \frac{\int_0^S dS/\psi^2}{\int_0^1 dS/\psi^2}$	Gates and Templeaar Lietz (1950)		Applied for unsaturated soils, change of integration limits and calculating relative hydraulic conductivity
$k_r = \frac{c \int_0^S dS/\psi^2}{c_0 \int_0^1 dS/\psi^2}$	Wyllie and Spangler (1952)		Proposed $\frac{c}{c_0}$ tortuosity factor is related to degree of saturation and proposed to be determined from electrical resistivity
$k_r = \frac{\int_0^S dS/\psi^{2+b}}{\int_0^1 dS/\psi^{2+b}}$	Fatt and Dykstar (1951)	$b = 1$	Tortuosity is proportional to $\frac{1}{r^b}$
$k_r = S_n^2 \frac{\int_0^S dS/\psi^2}{\int_0^1 dS/\psi^2}$	Burdine (1953)		Tortuosity is a factor of effective degree of saturation
$k_r = S_n^2 \frac{\int_0^S dS/\psi^2}{\int_0^1 dS/\psi^2}$	Wyllie and Gardner (1958)		Applied “cutting and rejoining” theory to the parallel model
$k_r = S_n^{n_1}$	Rowe (1960)	$n_1 = 2 - 4$	Applied “cutting and rejoining” theory to the parallel model

where:

c_0 is a dimensionless constant depending on the geometry of the system at saturation

T_s is the surface tension of the contractile skin

b is a fitting parameter related to tortuosity

These models overestimated the unsaturated permeability at high suction values; therefore, the tortuosity concept was introduced to compensate for the poor fit between the measured and predicted values. Mualem (1986) classified these two groups (i.e., uniform pore-size models and parallel models) as macroscopic models. The main criticism of these models is that the effect of pore-size distribution is neglected (Childs and Collis-George, 1950; Brooks and Corey, 1964).

The parallel model was further improved upon by the “cutting and rejoining” theory. In “cutting and rejoining” theory, the parallel capillary tubes are cut at a cross section normal to the flow direction and rejoined. This is done to account for the random distribution of pore-sizes in the direction of the flow. The pore-size distribution can be obtained from the soil-water characteristic curve of the soil. Table 2.7 shows a summary of proposed equations based on the above assumption.

Table 2.7-Proposed models based on cutting and rejoining theory

Series-Parallel model			
Equation	Author	Note	Assumption
$k_w(\theta_w)$ $= M_1 \left[\int_{\rho=R_{min}}^{\rho=R(\theta_w)} \int_{\eta=R_{min}}^{\eta=R(\theta_w)} \rho^2 f(\rho) f(\eta) d\eta d\rho \right.$ $\left. + \int_{\rho=R_{min}}^{\rho=R(\theta_w)} \int_{\eta=R_{min}}^{\eta=R(\theta_w)} \eta^2 f(\eta) f(\rho) d\rho d\eta \right]$	Childs and Collis-George (1950)		<p>The volume flow rate in each single tube with two sections in series is governed by the smaller diameter</p> <p>There is no by-passing of several pores</p>
$K = \frac{\varepsilon^2}{8m_1^2} (r_1^2 + 3r_2^2 + 5r_3^2 + (2m_1 - 1)r_{m_1}^2)$	Marshall (1958)		The volume flow rate is controlled by a cross section of necks connecting the pores
$K = \frac{\varepsilon^{4/3}}{8m_1^2} (r_1^2 + 3r_2^2 + 5r_3^2 + (2m_1 - 1)r_{m_1}^2)$	Millington and Quirk (1959)		The available area for flow is assumed to be equal to $n^{2/3}$.
$k_r(\theta_w)$ $= \sum_{i=0}^{l-1} \frac{[2(l-i)+1]}{\psi_i^2} \bigg/ \sum_{i=0}^{m_1-1} \frac{[2(m_1-i)+1]}{\psi_i^2}$	Kunze et al. (1968)		
$k_r(S_n) = S_n^{n_1} \left[\int_{\theta=0}^S \frac{dS}{\psi} \bigg/ \int_{\theta=0}^1 \frac{dS}{\psi} \right]^2$	Mualem (1976)	$d = 0.5$	Pore space is assumed to be a pair of capillary tubes with lengths proportional to their radius
$k_r(S_n) = S_n^{n_1} \left[\int_{\theta=0}^S \frac{dS}{\psi} \bigg/ \int_{\theta=0}^1 \frac{dS}{\psi} \right]^\beta$	Assouline (2001, 2004)		β is related to soil properties

where:

ρ and η are the radii of the pore

$f(\eta), f(\rho)$ are the function distributions that describe the pore space of the medium

M_1 is a constant related to the geometry of pores and fluid properties

b is the fitting parameter related to tortuosity

m_1 is the total number of pore sizes (intervals)

r_{m_1} is the pore size of interval m

l is the number of intervals corresponding to θ_w

β is the parameter related to soil properties

Mualem (1986) classified this group of estimation models as statistical models since they take the pore-size distribution into account by means of the “cutting and rejoining” concept. The pore-size distribution is considered to be a soil-water characteristic curve that also reflects the physical properties of the soil. These models may underestimate relative permeability at low moisture contents (Brutsaert, 1966); however, they seem to be good for practical use due to the fewer empirical factors needed and also appear to be more theoretical.

Mualem and Dagan (1978) generalized the Childs and Collis-George and Burdine and Mualem models as statistical models. However, the Burdine model is a macroscopic model which incorporates the random distribution of pore-size by means of a tortuosity factor. These three models (i.e., relative permeability equations) are considered relative permeability equations in this study. The equations are described in detail in the following sections.

2.4.5.1 Relative permeability equation of Childs and Collis-George (1950)

Childs and Collis-George (1950) were the first to consider the random distribution of pore-sizes in developing a relative permeability equation by means of the “cutting and rejoining” concept (Brutsaert, 1966). The original equation of Childs and Collis-George is as follows:

$$k_w(\theta_w) = M \left[\int_{\rho=R_{min}}^{\rho=R(\theta_w)} \int_{\eta=R_{min}}^{\eta=R(\theta_w)} \rho^2 f(\rho) f(\eta) d\eta d\rho + \int_{\rho=R_{min}}^{\rho=R(\theta_w)} \int_{\eta=R_{min}}^{\eta=R(\theta_w)} \eta^2 f(\eta) f(\rho) d\rho d\eta \right]$$

The equation was written in integration form (Mualem, 1976) and generalized by Mualem and Dagan (1978) according to Equation 2.28, which is used in this study.

$$k_r(\theta) = S_n^{n_1} \int_0^\theta \frac{(\theta - \vartheta)}{\psi^{2+b}} d\vartheta \bigg/ \int_0^{\theta_s} \frac{(\theta_s - \vartheta)}{\psi^{2+b}} d\vartheta \quad \text{Equation 2.28}$$

where:

b and n_1 are related to the tortuosity factor.

2.4.5.2 Relative permeability equation of Burdine (1953)

As was explained in Section 2.4.5, the Burdine equation is a macroscopic model that includes the random distribution of the pore-space by a tortuosity factor. The generalized form of the equation was given by Mualem and Dagan (1978) according to Equation 2.29, which is used in this study.

$$k_r(\theta) = S_n^{n_1} \int_0^\theta \frac{d\theta}{\psi^{2+b}} / \int_0^{\theta_s} \frac{d\theta}{\psi^{2+b}} \quad \text{Equation 2.29}$$

2.4.5.3 Relative permeability equation of Mualem (1976)

The Mualem equation is a statistical equation that incorporates the random distribution of the pore-space by the “cutting and rejoining concept”. It includes a correction factor accounting for partial correlation between the pores of the cut faces. However, since there is no independent procedure for determination of the considered correction factor, a factor similar to the tortuosity factor of the Burdine model was assumed in this equation. The generalized form of the equation was given by Mualem and Dagan (1978) according to Equation 2.30.

$$k_r(\theta) = S_n^{n_1} \left[\int_0^\theta \frac{d\theta}{\psi^{1+b}} / \int_0^{\theta_s} \frac{d\theta}{\psi^{1+b}} \right]^2 \quad \text{Equation 2.30}$$

2.4.5.4 Emergence of tortuosity factor

The concept of tortuosity was first introduced in the parallel models due to their overestimation of unsaturated permeability at high suction values. The Gates and Tempelaar-Lietz (1950) equation was developed for unsaturated media from the Purcell (1949) equation. However, due to overestimation of their equation at high suction values, they suspected that the tortuosity of the flow increases as the degree of saturation decreases; however, they did not provide any solution for the problem. The overestimation is due to the assumption that the pore sizes in the sequence are completely dependent on one another and the cross section of each flow channel is uniform over its whole length. Wyllie and Spangler (1952) proposed their equation and introduced $\frac{c}{c_0}$ as

the ratio of tortuosity, and then proposed to calculate it from electrical resistivity measurements. Burdine (1953) concluded that $\frac{c}{c_0} = S_n^2$ and proposed his equation. Tortuosity is a vague concept and it arises from a poor fit between calculated and measured unsaturated permeability data. The tortuosity factor becomes worse when there is no basis for the functional form of it (Hunt et al., 2013). The concept of “cutting and rejoining” was first introduced by Childs and Collis-George (1950) to develop the equation; however, the concept has also been used in some parallel models (Rowe, 1960; Wyllie and Gardner, 1958). The model developed by Wyllie and Gardner (1958) was identical to the Burdine (1953) equation and incorporated the concept of tortuosity by means of statistical consideration (i.e., “cutting and rejoining”), which probably sounded more theoretical than empirical. The series-parallel models, which were developed based on the “cutting and rejoining” concept, may underestimate the unsaturated permeability at low moisture contents (Brutsaert, 1966). However, these models are good for practical use as they require fewer empirical factors and sound more theoretical. Although the parallel models sound simple to use, in practice they require complicated additional measurements.

Different values have been proposed for the fitting parameters of n_1 and b in the generalized Equation 2.28, Equation 2.29 and Equation 2.30 used in this study. For instance, values of 0, 2 and 4/3 have been proposed for parameter n_1 by Childs and Collis-George (1950), Burdine (1953) and Millington and Quirk (1961), respectively. It should be noted that some negative values have been reported for parameter n_1 (Kosugi, 1999; Schaap and Leij, 2000), which physically means that the flow paths are straighter than straight (Hunt et al., 2013). Therefore, it can be seen that the tortuosity factor is only a scaling factor to compensate for the poor fit of the estimation models to the measured data and depends on the soil database under consideration.

2.5 Combination of soil-water characteristic curve and relative permeability equations

As was explained in Section 2.4.5, the flow equations usually express the relative permeability, k_r , of soils as defined according to Equation 2.27 as a function of normalized water content, $\theta_n = \frac{\theta_w - \theta_r}{\theta_s - \theta_r}$, or normalized degree of saturation, $S_n = \frac{S - S_r}{1 - S_r}$, of the soil. Therefore, knowledge of volumetric water content as a function of soil suction (i.e., soil-water characteristic curve) is required in order to obtain the unsaturated

permeability of soils. As a result, the soil-water characteristic curve equation (i.e., volumetric water content as a function of soil suction) is combined with the relative permeability equation and the required integrations are then performed. The result of this combination and integration is the estimated unsaturated permeability, which could be a closed-form equation or a non-closed form equation. Some closed-form equations that can predict the unsaturated permeability of soils have been developed (Brooks and Corey, 1964; Brutsaert, 1966; Mualem, 1986; van Genuchten, 1980). It should be noted that, in some cases, a closed-form solution does not exist and the integration should be performed numerically (Fredlund et al., 1994).

The combination of the four soil-water characteristic curve equations and the three relative permeability equations in this study forms a matrix as shown in Table 2.8.

Table 2.8-Combined equations for estimation of unsaturated permeability

Relative permeability equation		Childs and Collis-George	Burdine	Mualem
		C	B	M
Fredlund and Xing (1994)	F	(Fredlund et al., 1994)	N.A.	N.A.
Fredlund and Xing (1994) with $C(\psi) = 1$	F, $C(\psi) = 1$	N.A.	N.A.	N.A.
van Genuchten (1980)	V	N.A.	N.A.	N.A.
van Genuchten (1980) with $m=1-1/n$	V, $m = 1 - 1/n$	N.A.	N.A.	(van Genuchten, 1980)

As can be seen from the matrix, there are 12 possible combinations for estimating the unsaturated permeability of soils. Two of the combinations have been proposed by other researchers, as shown in the matrix by the researchers' names; the rest of the combinations have not yet been considered, as shown in the matrix by N.A. (i.e., not available). The N.A. combinations are proposed in this study.

In the following sections, the available combinations proposed by Fredlund et al. (1994) and van Genuchten (1980) are described. These two models are used as comparative models later in the study.

2.5.1 Fredlund et al. (1994) unsaturated permeability estimation model

Fredlund et al. (1994) used the SWCC equation from work by Fredlund and Xing (1994), Equation 2.10 and the relative permeability equation by Childs and Collis-George (CCG) (1950) (Equation 2.28) to develop a non-closed form equation to estimate the unsaturated permeability of soils. The integration form of the CCG equation suggested by Mualem (1976) was used in their model development. Equation 2.31 shows the Fredlund et al. (1994) equation. It should be noted that this model is named F&X-1994 in this study (see Table 3.2).

$$k_r(\psi) = \frac{\int_{\psi}^{\psi_r} \frac{\theta(\vartheta) - \theta(\psi)}{\vartheta^2} \theta'(\vartheta) d\vartheta}{\int_{\psi_{aev}}^{\psi_r} \frac{\theta(\vartheta) - \theta_s}{\vartheta^2} \theta'(\vartheta) d\vartheta} \quad \text{Equation 2.31}$$

where:

ϑ is the dummy variable of integration representing suction

θ'_w is the derivative of the SWCC equation (i.e., $\theta_w = C(\psi) \frac{\theta_s}{\left[\ln \left(e + \left(\frac{\psi}{a} \right)^n \right) \right]^m}$)

ψ_r is the suction corresponding to the residual water content

ψ_{aev} is the air-entry value of the soil under consideration

Performing the required integrations did not lead to a closed-form solution (Fredlund et al., 1994). Therefore, a numerical integration was performed on a logarithmic scale to avoid the numerical difficulties of integration over the entire suction range from ψ_{aev} to 10^6 kPa. As a result, the equation takes the following form:

$$k_r(\psi) = \frac{\int_{\ln(\psi)}^b \frac{\theta(e^\vartheta) - \theta_\psi}{e^\vartheta} \theta'(e^\vartheta) d\vartheta}{\int_{\ln(\psi_{aev})}^b \frac{\theta(e^\vartheta) - \theta_s}{e^\vartheta} \theta'(e^\vartheta) d\vartheta} \quad \text{Equation 2.32}$$

where:

$b = \ln(1000000)$

The numerical procedure used to solve the integration is provided in Appendix A.

2.5.2 van Genuchten (1980) unsaturated permeability estimation model

van Genuchten (1980) used a modified form of his own SWCC equation (Equation 2.13) and the relative permeability equation by Mualem (1976) (Equation 2.30) to derive a closed form equation to estimate the unsaturated permeability of soils. By limiting parameter (m) as equal to $(1-1/n)$, the following closed form equation for estimating unsaturated permeability was derived:

$$k_r(\psi) = \frac{\{1 - (\alpha\psi)^{n-1}[1 + (\alpha\psi)^n]^{-m}\}^2}{[1 + (\alpha\psi)^n]^{m/2}} \quad (m = 1 - \frac{1}{n}) \quad \text{Equation 2.33}$$

In addition, van Genuchten (1980) combined another modified form of his own SWCC equation with the relative permeability equation by Burdine (1953) (i.e., Equation 2.29) and proposed the following closed-form equation to estimate the unsaturated permeability of soils:

$$K_r(\psi) = \frac{1 - (\alpha\psi)^{n-2}[1 + (\alpha\psi)^n]^{-m}}{[1 + (\alpha\psi)^n]^{2m}} \quad (m = 1 - \frac{2}{n}) \quad \text{Equation 2.34}$$

It should be noted that, Equation 2.33 is the estimation model used in this study as a comparative model and is named VG-1980 (see Table 3.2).

2.6 Measured soil-water characteristic curve data range

As was explained in Section 2.5, the SWCC equation is required to obtain an unsaturated permeability estimation model. Usually, the SWCC is measured in the laboratory and an equation is used to best-fit the data to get a continuous function over the entire suction range. The obtained SWCC function is then used to estimate the unsaturated permeability function. Therefore, the estimated permeability functions are directly dependent on the measured SWCC data. It has been suggested that a complete soil-water characteristic curve could improve the estimated unsaturated permeability function (Kunze et al., 1968). However, there is a need to quantitatively investigate the effect of the SWCC measurement range on the estimation of the unsaturated permeability function. Therefore, the effect of SWCC measurement ranges associated with commonly used laboratory equipment (for the measurement of SWCC) on unsaturated permeability is also investigated in this study.

2.7 Summary of chapter

Unsaturated permeability is the most important hydraulic property governing the flow process. Therefore, knowledge of unsaturated permeability is crucial in the analysis of the flow process in the unsaturated zone. Unsaturated permeability of soil can be directly measured in soil laboratories; however, direct measurement of unsaturated permeability of soils requires expensive and precise equipment, well-trained technicians and tremendous amounts of time (van Genuchten, 1980; Fredlund et al., 1994; Agus et al., 2005; Cihan et al., 2009). Therefore, several estimation methods have been developed to predict the unsaturated permeability of soils from soil-water characteristic curve (SWCC). The estimation is in fact an integration of SWCC with a relative permeability (k_r) equation. To date, there are numerous estimation models that can obtain the permeability of unsaturated soil from SWCC. However, each model will result in a different estimation curve (Mualem, 1986). The reason for this variation is not well understood. Although numerous models have been developed by researchers to estimate the unsaturated permeability of soil, there have been very few studies investigating the controlling parameters that cause the variation between different models for the same soil. In this study, best-fit SWCC equation, relative permeability equation and measured SWCC data ranges were identified as main controlling parameters causing variation between different estimation models. Four different best-fit SWCC equations, three relative permeability equations and four different measured SWCC data ranges are considered to study the parameters affecting estimation of unsaturated permeability of soils.

Chapter 3 Theory

3.1 Introduction

In this chapter, theories applicable to the study and theoretical developments are presented. First, the proposed matrix of unsaturated permeability estimation models is presented. The procedure for the development of the proposed combination models is presented next. Subsequently, the assumptions made for the combined models are presented. The procedure for the estimation of SWCC data points in a higher suction range is presented lastly.

3.2 Matrix of unsaturated permeability estimation models

As was explained in Section 2.5, combination of any pair of best-fit soil-water characteristic curve (SWCC) equation and relative permeability equation (k_r) would result in an unsaturated permeability estimation model. The selected best-fit soil-water characteristic curve (SWCC) equations and the relative permeability (k_r) equations as explained in Section 2.5 form a matrix as shown in Table 3.1.

Table 3.1-Matrix of unsaturated permeability estimation models

Relative permeability equation Soil-water characteristic curve equation		Childs and Collis-George	Burdine	Mualem
		C	B	M
Fredlund and Xing (1994)	F	(Fredlund et al., 1994)	N.A.	N.A.
Fredlund and Xing (1994) with $C(\psi) = 1$	F, $C(\psi) = 1$	N.A.	N.A.	N.A.
van Genuchten (1980)	V	N.A.	N.A.	N.A.
van Genuchten (1980) with $m=1-1/n$	V, $m = 1 - 1/n$	N.A.	N.A.	(van Genuchten, 1980)

As can be seen from Table 3.1, the combination of Fredlund and Xing (1994) and Childs and Collis-George (1950) has been suggested by Fredlund et al. (1994) as a non-close

form equation (see Section 2.5.1). The combination of van Genuchten (1980) with $m=1-1/n$ and Mualem (1976) has been suggested by van Genuchten (1980) as a closed form equation (see Section 2.5.2). However, the rest of the combinations as shown by N.A. have not been considered in any literature. Therefore, the combination of all the best-fit SWCC and relative permeability equations pairs are considered in this study as a matrix of unsaturated permeability estimation models. The matrix is shown in Table 3.2.

Table 3.2-Matrix of unsaturated permeability estimation models

Relative permeability equation		Childs and Collis-George	Burdine	Mualem
Soil-water characteristic curve equation		C	B	M
Fredlund and Xing (1994)	F	(Fredlund et al., 1994) FCM	FBM	FMM
Fredlund and Xing (1994) with $C(\psi) = 1$	F, $C(\psi) = 1$	FCM ($C(\psi) = 1$)	FBM ($C(\psi) = 1$)	FMM ($C(\psi) = 1$)
van Genuchten (1980)	V	VCM	VBM	VMM
van Genuchten (1980) with $m=1-1/n$	V, $m = 1 - 1/n$	VCM ($m=1-1/n$)	VBM ($m=1-1/n$)	(van Genuchten, 1980) VMM ($m=1-1/n$)

It should be noted that this matrix is created as the standard procedure to study the effect of SWCC and k_r equations on the estimation of unsaturated permeability of soils. The unsaturated permeability estimation models combined here are named based on the SWCC and k_r equations used in the model. For instance, FMM means that the model was resulted from the combination of Fredlund and Xing's best-fit SWCC equation and Mualem relative permeability equation. The capital M at the end of each name stands for model. Therefore, FMM refers to the Fredlund and Xing-Mualem model. The rest of the names can be interpreted according to above description.

3.3 Procedure of combining SWCC and k_r equations

Soil-water characteristic curve is represented by any function which describes the relationship between water content(w) and suction (ψ) of a soil. If volumetric water content(θ_w) of a soil is expressed as a function of suction, the relationship could be described according to Equation 3.1.

$$\theta_w = f(\psi) \quad \text{Equation 3.1}$$

The inverse form of Equation 3.1 could describe suction(ψ) of a soil as a function of volumetric water content(θ_w), according to Equation 3.2.

$$\psi = f^{-1}(\theta_w) = g(\theta_w) \quad \text{Equation 3.2}$$

The relationship between Equation 3.1 and Equation 3.2 is shown in Figure 3.1.

Figure 3.1-Soil-water characteristic curve

A generalized equation according to Equation 3.3 can be used to express the relative permeability(k_r) of soil as a function of volumetric water content(θ_w), as follows:

$$k_r(\theta_w) = \left[\frac{\int_{\theta_{wL}}^{\theta_w} f(\psi) d\theta_w}{\int_{\theta_{wL}}^{\theta_s} f(\psi) d\theta_w} \right]^d \quad \text{Equation 3.3}$$

where:

θ_{wL} is the lower limit of integration for volumetric water content

d is a parameter which varies according to the model.

Equation 3.3 can be transformed to a form that describes the relative permeability(k_r) of soil as a function of suction. Substituting of Equation 3.4 into Equation 3.3 and transforming the integration limits to $(\psi(\theta_w), \theta_w)$, $(\psi(\theta_s), \theta_s)$ and $(\psi(\theta_{wL}), \theta_{wL})$ points would result in Equation 3.5 which describes the relative permeability (k_r) of soil as a function of suction (see Figure 3.2).

$$\theta_w = f(\psi) \rightarrow d\theta_w = f'(\psi)d\psi \quad \text{Equation 3.4}$$

where:

$f'(\psi)$ is derivative of $f(\psi)$

$$k_r(\psi) = \left[\int_{\psi(\theta_{wL})}^{\psi(\theta_w)} f(\psi)f'(\psi)d\psi / \int_{\psi(\theta_{wL})}^{\psi(\theta_s)} f(\psi)f'(\psi)d\psi \right]^d \quad \text{Equation 3.5}$$

It can be shown if $\theta_w = f(\psi)$ and $\psi = f^{-1}(\theta_w) = g(\theta_w)$, the $k_r(\psi) = k_r(\theta_w)$ as described later in Sections 3.3.2, 3.3.3 and 3.3.4. It should be noted that all the theoretical developments were conducted for the drying curve of soil-water characteristic curves.

Figure 3.2-Integration limits for $k_r(\theta_w)$ and $k_r(\psi)$

In general, soil-water characteristic curve which is described by $\theta_w = f(\psi)$ function can be divided into finite number of intervals along the volumetric water content (θ_w) axis. The relationship between each two subsequent points on the curve can be described according to Equation 3.6 which is a linear equation.

$$\theta_w = f(\psi) = \theta_{wi} + \frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)}(\psi - \psi_i) \quad \text{Equation 3.6}$$

Figure 3.3 shows the division of $\theta_w = f(\psi)$ curve into finite number of intervals (i.e., N points).

If Equation 3.6 is substituted into Equation 3.5 and a numerical integration is performed, a series would be obtained as an unsaturated permeability estimation model. This unsaturated permeability model expresses the relative permeability as a function of suction ($k_r(\psi)$). If the relative permeability is multiplied by saturated permeability of the soil, the unsaturated permeability of the soil as a function of suction would be obtained ($k_w(\psi)$).

Figure 3.3-Division of soil-water characteristic curve along the volumetric water content and suction axis

The division of the curve can also be done along the suction axis of the soil-water characteristic curve. The relationship between each two subsequent points on the curve can be described according to Equation 3.7 which is a linear equation (see Figure 3.3).

$$\psi = g(\theta_w) = \psi_i + \frac{(\psi_{i+1} - \psi_i)}{(\theta_{w_{i+1}} - \theta_{w_i})} (\theta_w - \theta_{w_i}) \quad \text{Equation 3.7}$$

If Equation 3.7 is substituted into Equation 3.3 and a numerical integration is performed, a series would be obtained as an unsaturated permeability estimation model. This unsaturated permeability model expresses the relative permeability as a function of volumetric water content ($k_r(\theta_w)$). If the relative permeability is multiplied by saturated permeability of the soil, the unsaturated permeability of the soil as a function of volumetric water content would be obtained ($k_w(\theta_w)$).

3.3.1 Assumptions made in the combined models

1) All the models considered in this study were developed based on actual volumetric water content (θ_w) and not on effective water content ($\theta = \theta_w - \theta_r$). The residual volumetric water content (θ_r) is required in order to compute the effective water content. Residual volumetric water content is defined qualitatively as the water content below which a large increase in suction is required to remove additional water (Fredlund et al., 1994). However, there is no theoretical definition for this parameter. The common practice for determining the residual water content is by the graphical method (Fredlund and Xing, 1994; Vanapalli et al., 1998) and there is no independent procedure for determining the residual water content (van Genuchten, 1980). If an effective volumetric water content (normalized) is used in estimating the unsaturated permeability of soil, the value of unsaturated permeability (k_r), at residual volumetric water content (θ_r) would be zero. The physical meaning of zero value of unsaturated permeability at residual water content implies no further flow at volumetric water contents below residual water content (Prunty, 2003). However, an increase in suction above residual suction would result in lower volumetric water content of the soil which contradicts the meaning of $k_r(\theta_r) = 0$. It should be noted that estimation models use effective volumetric water content, $\theta = \theta_w - \theta_r$ for estimation of unsaturated permeability of soils which results in $k_r(\theta_r) = 0$ (Fredlund et al., 1994). Therefore, the unsaturated permeability estimation models presented in this study were based on actual volumetric water content and not on the normalized

volumetric water content. It should be noted that the soil suction corresponding to zero volumetric water content was considered to be 10^6 kPa which can be supported by thermodynamic consideration (Richards, 1965). It has been experimentally shown that zero volumetric water content for various soils is at a value slightly below 10^6 kPa (Croney and Coleman, 1961). The θ_r in SWCC equation of van Genuchten (1980) is treated as one of the fitting parameters of the equation and is suggested to be determined by least square curve-fitting procedure.

2) Parameters b and n_1 were introduced into Equation 2.28, Equation 2.29 and Equation 2.30 to provide relative permeability equations with more flexibility (Mualem and Dagan, 1978). However, these two parameters need to be determined empirically from measured data. Therefore, the value of these two parameters depends on the soil database under consideration. For instance, values of 0, 2 and 0.5 were proposed for parameter n_1 by Childs Collis-George (1950), Burdine (1953) and Mualem (1976), respectively, based on their soil database. As the objective of this paper was to investigate the variation between different models fairly independently from soil database, correction factor (or tortuosity factor as explained in Section 2.4.5.4) was not considered and the value of zero was considered for parameter b and n_1 in Equation 2.28, Equation 2.29 and Equation 2.30. Therefore, Equation 2.28, Equation 2.29 and Equation 2.30 take the following forms:

Childs and Collis-George (C)

$$k_r(\theta_w) = \int_0^{\theta_w} \frac{(\theta_w - \vartheta)}{\psi^2} d\vartheta / \int_0^{\theta_s} \frac{(\theta_s - \vartheta)}{\psi^2} d\vartheta \quad \text{Equation 3.8}$$

Burdine (B)

$$k_r(\theta_w) = \int_0^{\theta_w} \frac{d\theta_w}{\psi^2} / \int_0^{\theta_s} \frac{d\theta_w}{\psi^2} \quad \text{Equation 3.9}$$

Mualem (M)

$$k_r(\theta_w) = \left[\int_0^{\theta_w} \frac{d\theta_w}{\psi} / \int_0^{\theta_s} \frac{d\theta_w}{\psi} \right]^2 \quad \text{Equation 3.10}$$

By incorporating Equation 3.4, the selected relative permeability equations in this study would be transformed to the form of Equation 3.5.

Equation 3.11, Equation 3.12 and Equation 3.13 are transformed form of Equation 3.8, Equation 3.9 and Equation 3.10, respectively, as follows:

Childs and Collis-George (C)

$$k_r(\psi) = \int_{\psi(\theta_w)}^{\psi(\theta_{wL})} \frac{(\theta_w(\psi) - \theta(\vartheta))}{\vartheta^2} f'(\vartheta) d\vartheta \bigg/ \int_{\psi(\theta_s)}^{\psi(\theta_{wL})} \frac{(\theta_s - \theta(\vartheta))}{\vartheta^2} f'(\vartheta) d\vartheta \quad \text{Equation 3.11}$$

Burdine (B)

$$k_r(\psi) = \int_{\psi(\theta_w)}^{\psi(\theta_{wL})} \frac{f'(\psi) d\psi}{\psi^2} \bigg/ \int_{\psi(\theta_s)}^{\psi(\theta_{wL})} \frac{f'(\psi) d\psi}{\psi^2} \quad \text{Equation 3.12}$$

Mualem (M)

$$k_r(\psi) = \left[\int_{\psi(\theta_w)}^{\psi(\theta_{wL})} \frac{f'(\psi) d\psi}{\psi} \bigg/ \int_{\psi(\theta_s)}^{\psi(\theta_{wL})} \frac{f'(\psi) d\psi}{\psi} \right]^2 \quad \text{Equation 3.13}$$

3) The relative permeability equations used in this study which stem from derivation of flow equation in a capillary tube considering the appropriate boundary conditions, are known to underestimate the unsaturated permeability of soil in the lower water content range (Tuller and Or, 2001; Peters and Durner, 2008). The underestimation of unsaturated permeability at higher suction values (i.e., lower water content) may result in numerical instabilities. Therefore, a lower limit was assumed for the estimated unsaturated permeability in this study. Vapor permeability which is computed based on modified form of Fick's law was assumed as a lower limit of unsaturated permeability. The lower limit of permeability or vapor permeability is computed according to Equation 3.14(Ebrahimi-B et al., 2004; Saito et al., 2006; Peters and Durner, 2008).

$$k_v = \gamma_w D^m \quad \text{Equation 3.14}$$

where:

k_v is vapor permeability, (m/s)

γ_w is the specific weight of water(kN/m³)

$$D^m = \frac{(\bar{u}_a + \bar{u}_v)gW_v\bar{u}_vD^{v*}}{\bar{u}_a\gamma_wRT\rho_w}, (\text{m/s})/(\text{kN/m}^3)$$

$\bar{u}_a$ is the total pressure in the bulk air phase, $\bar{u}_a = u_a + u_{atm}$, kPa

u_a is the pore-air pressure, kPa

u_{atm} is the atmospheric pressure, 101.325 kPa

$\bar{u}_v$ is the partial pressure of water vapor, kPa

g is the gravitational acceleration, m/s^2

W_v is the molecular weight of water vapor, 18.016 kg/kmol

D^{v*} is the diffusion coefficient of vapor through soil, $D^{v*} = \frac{\alpha\beta D^v W_v}{RT}$ (kg.m/kN.s)

D^v is the molecular diffusivity of vapor in air, $0.229 \times 10^{-4(1+T/273.15)1.75}$, m^2/s

α_t is the tortuosity factor of the soil, $\alpha_t = \beta^{2/3}$

β is the cross sectional area of soil available for vapor flow per total area, $\beta = (1 - S)\varepsilon$

S is the degree of saturation

ε is the porosity of soil

R is the universal gas constant, 8.314 J/(mol.K)

T is temperature, K

ρ_w is the density of water, kg/m^3

The computed vapor permeability was normalized with respect to the saturated permeability of the soil and it was considered as relative vapor permeability.

3.3.2 Childs and Collis-George based estimation models

In order to obtain Childs and Collis-George based estimation models as a function of suction, Equation 3.6 is substituted into Equation 3.11 and the numerical integration is performed. Equation 3.15 presents the procedure of numerical integration along the suction axis.

$$\begin{aligned}
 k_r(\psi) &= \int_{\psi(\theta_w)}^{\psi(\theta_{wL})} \frac{(\theta_w(\psi) - \theta(\vartheta))}{\vartheta^2} f'(\vartheta) d\vartheta / \int_{\psi(\theta_s)}^{\psi(\theta_{wL})} \frac{(\theta_s - \theta(\vartheta))}{\vartheta^2} f'(\vartheta) d\vartheta \\
 &= \sum_{i=1}^M \int_{\psi_i}^{\psi_{i+1}} \frac{\left(\theta_w(\psi) - \left(\frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)} (\vartheta - \psi_i) + \theta_{wi} \right) \right)}{\vartheta^2} \frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)} d\vartheta \\
 &/ \sum_{i=1}^N \int_{\psi_i}^{\psi_{i+1}} \frac{\left(\theta_w(\psi) - \left(\frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)} (\vartheta - \psi_i) + \theta_{wi} \right) \right)}{\vartheta^2} \frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)} d\vartheta \\
 &= \sum_{i=1}^M \frac{-(\theta_{wi} - \theta_{wi+1})(\theta_w(\psi_i - \psi_{i+1}) + \theta_{wi}\psi_{i+1} - \theta_{wi+1}\psi_i)}{\vartheta(\psi_i - \psi_{i+1})^2} - \frac{(\theta_{wi} - \theta_{wi+1})^2}{(\psi_i - \psi_{i+1})^2} \ln(\vartheta) \Big|_{\psi_i}^{\psi_{i+1}} \\
 &/ \sum_{i=1}^N \frac{-(\theta_{wi} - \theta_{wi+1})(\theta_w(\psi_i - \psi_{i+1}) + \theta_{wi}\psi_{i+1} - \theta_{wi+1}\psi_i)}{\vartheta(\psi_i - \psi_{i+1})^2} - \frac{(\theta_{wi} - \theta_{wi+1})^2}{(\psi_i - \psi_{i+1})^2} \ln(\vartheta) \Big|_{\psi_i}^{\psi_{i+1}}
 \end{aligned} \tag{Equation 3.15}$$

$$\begin{aligned}
 k_r(\psi) &= \sum_{i=1}^M \left[\theta_w \frac{\theta_{wi+1} - \theta_{wi}}{\psi_i \psi_{i+1}} - \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right) \left(\frac{\theta_{wi+1}}{\psi_{i+1}} - \frac{\theta_{wi}}{\psi_i} \right) + \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right)^2 \ln \left(\frac{\psi_i}{\psi_{i+1}} \right) \right] \\
 &/ \sum_{i=1}^N \left[\theta_s \frac{\theta_{i+1} - \theta_{wi}}{\psi_i \psi_{i+1}} - \left(\frac{\theta_{wi+1} - \theta_w}{\psi_i - \psi_{i+1}} \right) \left(\frac{\theta_{wi+1}}{\psi_{i+1}} - \frac{\theta_{wi}}{\psi_i} \right) + \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right)^2 \ln \left(\frac{\psi_i}{\psi_{i+1}} \right) \right]
 \end{aligned}$$

The result of the integration is a series which describes the relative permeability of the soil as a function of suction whose multiplication with saturated permeability of the soil would result in unsaturated permeability of the soil as a function of suction.

As was explained in Section 3.3, if $\theta_w = f(\psi)$ and $\psi = f^{-1}(\theta_w) = g(\theta_w)$, the $k_r(\psi) = k_r(\theta_w)$. If Equation 3.7 is substituted into Equation 3.8 and the numerical integration is performed, a series would be obtained which describes the relative permeability as a function of volumetric water content. Equation 3.16 presents the procedure of numerical integration along the volumetric water content. As can be seen from both Equation 3.15 and Equation 3.16, $k_r(\psi) = k_r(\theta_w)$.

$$\begin{aligned}
&= \sum_{i=1}^M \int_{\theta_{wi}}^{\theta_{wi+1}} \frac{(\theta_w - \vartheta) d\vartheta}{\left(\psi_i - \frac{(\psi_i - \psi_{i+1})}{(\theta_{wi+1} - \theta_w)} (\vartheta - \theta_{wi}) \right)^2} \bigg/ \sum_{i=1}^N \int_{\theta_{wi}}^{\theta_{wi+1}} \frac{(\theta_w - \vartheta) d\vartheta}{\left(\psi_i - \frac{(\psi_i - \psi_{i+1})}{(\theta_{wi+1} - \theta_{wi})} (\vartheta - \theta_{wi}) \right)^2} \\
&= \sum_{i=1}^M -(\theta_{wi} - \theta_{i+1})^2 \frac{\left[\frac{\theta_{wi+1} \psi_i - \theta_{wi} \psi_{i+1}}{-\theta_{wi} \psi_{i+1} + \theta_{wi+1} \psi_i + \vartheta (\psi_{i+1} - \psi_i)} + \ln \left(\frac{\theta_{wi} \psi_{i+1} - \theta_{wi+1} \psi_i + \vartheta (\psi_i - \psi_{i+1})}{\theta_{wi} \psi_{i+1} - \theta_{wi+1} \psi_i + \vartheta (\psi_i - \psi_{i+1})} \right) \right]}{(\psi_i - \psi_{i+1})^2} \bigg|_{\theta_{wi}}^{\theta_{wi+1}} \\
&/ \sum_{i=1}^N -(\theta_{wi} - \theta_{wi+1})^2 \frac{\left[\frac{\theta_{wi+1} \psi_i - \theta_{wi} \psi_{i+1}}{-\theta_{wi} \psi_{i+1} + \theta_{wi+1} \psi_i + \vartheta (\psi_{i+1} - \psi_i)} + \ln \left(\frac{\theta_{wi} \psi_{i+1} - \theta_{wi+1} \psi_i + \vartheta (\psi_i - \psi_{i+1})}{\theta_{wi} \psi_{i+1} - \theta_{wi+1} \psi_i + \vartheta (\psi_i - \psi_{i+1})} \right) \right]}{(\psi_i - \psi_{i+1})^2} \bigg|_{\theta_{wi}}^{\theta_{wi+1}}
\end{aligned} \tag{Equation 3.16}$$

$$\begin{aligned}
k_r(\theta_w) &= \sum_{i=1}^M \left[\theta_w \frac{\theta_{wi+1} - \theta_{wi}}{\psi_i \psi_{i+1}} - \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right) \left(\frac{\theta_{wi+1}}{\psi_{i+1}} - \frac{\theta_{wi}}{\psi_i} \right) + \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right)^2 \ln \left(\frac{\psi_i}{\psi_{i+1}} \right) \right] \\
/ \sum_{i=1}^N &\left[\theta_s \frac{\theta_{wi+1} - \theta_{wi}}{\psi_i \psi_{i+1}} - \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right) \left(\frac{\theta_{wi+1}}{\psi_{i+1}} - \frac{\theta_{wi}}{\psi_i} \right) + \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right)^2 \ln \left(\frac{\psi_i}{\psi_{i+1}} \right) \right]
\end{aligned}$$

Childs and Collis-George based estimation models are shown in first column of Table 3.2 as FCM, FCM (C (ψ) =1), VCM and VCM (m=1-1/n).

3.3.3 Combination of Burdine based models

In order to obtain Burdine based estimation models as a function of suction, Equation 3.6 is substituted into Equation 3.12 and the numerical integration is performed. Equation 3.17 presents the procedure of numerical integration along the suction axis. The result of the integration is a series which describes the relative permeability of the soil as a function of suction and multiplication of saturated permeability of the soil would result in unsaturated permeability of the soil as a function of suction.

$$\begin{aligned}
k_r(\psi) &= \int_{\psi(\theta_w)}^{\psi(\theta_{wL})} \frac{f'(\psi) d\psi}{\psi^2} \bigg/ \int_{\psi(\theta_s)}^{\psi(\theta_{wL})} \frac{f'(\psi) d\psi}{\psi^2} \\
&= \sum_{i=1}^M \int_{\psi_i}^{\psi_{i+1}} \frac{\frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)} d\psi}{\psi^2} \bigg/ \sum_{i=1}^N \int_{\psi_i}^{\psi_{i+1}} \frac{\frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)} d\psi}{\psi^2} \\
&= \sum_{i=1}^M \frac{\frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)}}{-\psi} \bigg|_{\psi_i}^{\psi_{i+1}} \bigg/ \sum_{i=1}^N \frac{\frac{(\theta_{wi+1} - \theta_{wi})}{(\psi_{i+1} - \psi_i)}}{-\psi} \bigg|_{\psi_i}^{\psi_{i+1}}
\end{aligned} \tag{Equation 3.17}$$

$$k_r(\psi) = \frac{\sum_{i=1}^M \frac{\theta_{wi+1} - \theta_{wi}}{\psi_i \psi_{i+1}}}{\sum_{i=1}^N \frac{\theta_{wi+1} - \theta_{wi}}{\psi_i \psi_{i+1}}}$$

If Equation 3.7 is substituted into Equation 3.9 and the numerical integration is performed, a series would be obtained which describe the relative permeability as a function of volumetric water content. Equation 3.18 presents the procedure of numerical integration along the volumetric water content. As can be seen from both Equation 3.17 and Equation 3.18, $k_r(\psi) = k_r(\theta_w)$. Multiplication of Equation 3.18 with saturated permeability of the soil would result in unsaturated permeability of the soil as a function of volumetric water content.

$$k_r(\theta_w) = \frac{\int_0^{\theta_w} \frac{d\theta_w}{\psi^2}}{\int_0^{\theta_s} \frac{d\theta_w}{\psi^2}}$$

$$\frac{\sum_{i=1}^M \int_{\theta_{wi}}^{\theta_{wi+1}} \frac{d\theta_w}{\left(\psi_i - \frac{(\psi_i - \psi_{i+1})}{(\theta_{wi+1} - \theta_{wi})} (\theta_w - \theta_{wi}) \right)^2}}{\sum_{i=1}^N \int_{\theta_{wi}}^{\theta_{wi+1}} \frac{d\theta_w}{\left(\psi_i - \frac{(\psi_i - \psi_{i+1})}{(\theta_{wi+1} - \theta_{wi})} (\theta_w - \theta_{wi}) \right)^2}} \quad \text{Equation 3.18}$$

$$\frac{\sum_{i=1}^M - \frac{(\theta_{wi} - \theta_{wi+1})^2}{(\psi_i - \psi_{i+1})(\theta_w(\psi_i - \psi_{i+1}) + \theta_{wi}\psi_{i+1} - \theta_{wi+1}\psi_i)} \Big|_{\theta_i}^{\theta_{i+1}}}{\sum_{i=1}^N - \frac{(\theta_{wi} - \theta_{wi+1})^2}{(\psi_i - \psi_{i+1})(\theta(\psi_i - \psi_{i+1}) + \theta_{wi}\psi_{i+1} - \theta_{wi+1}\psi_i)}}$$

$$k_r(\theta_w) = \frac{\sum_{i=1}^M \frac{\theta_{wi+1} - \theta_{wi}}{\psi_i \psi_{i+1}}}{\sum_{i=1}^N \frac{\theta_{wi+1} - \theta_{wi}}{\psi_i \psi_{i+1}}}$$

Burdine based estimation models are shown in first column of Table 3.2 as FBM, FBM (C(ψ)=1), VBM and VBM (m=1-1/n).

3.3.4 Combination of Mualem based models

In order to obtain Mualem based estimation models as a function of suction, Equation 3.6 is substituted into Equation 3.13 and the numerical integration is performed. Equation 3.19 shows the procedure of numerical integration along the suction axis. The result of the integration is a series which describes the relative permeability of the soil as a

function of suction whose multiplication with saturated permeability of the soil would result in unsaturated permeability of the soil as a function of suction.

$$\begin{aligned}
 k_r(\psi) &= \left[\int_{\psi(\theta_w)}^{\psi(\theta_{wL})} \frac{f'(\psi)d\psi}{\psi} / \int_{\psi(\theta_s)}^{\psi(\theta_{wL})} \frac{f'(\psi)d\psi}{\psi} \right]^2 \\
 &= \left[\sum_{i=1}^M \int_{\psi_i}^{\psi_{i+1}} \frac{\frac{(\theta_{wi+1}-\theta_{wi})}{(\psi_{i+1}-\psi_i)} d\psi}{\psi} / \sum_{i=1}^N \int_{\psi_i}^{\psi_{i+1}} \frac{\frac{(\theta_{wi+1}-\theta_{wi})}{(\psi_{i+1}-\psi_i)} d\psi}{\psi} \right]^2 \\
 &= \left[\sum_{i=1}^M \frac{(\theta_{wi} - \theta_{wi+1}) \ln(\psi)}{(\psi_i - \psi_{i+1})} \Big|_{\psi_i}^{\psi_{i+1}} / \sum_{i=1}^N \frac{(\theta_{wi} - \theta_{wi+1}) \ln(\psi)}{(\psi_i - \psi_{i+1})} \Big|_{\psi_i}^{\psi_{i+1}} \right]^2 \quad \text{Equation 3.19}
 \end{aligned}$$

$$\begin{aligned}
 k_r(\psi) &= \left[\sum_{i=1}^M \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right) \ln \left(\frac{\psi_i}{\psi_{i+1}} \right) / \sum_{i=1}^N \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right) \ln \left(\frac{\psi_i}{\psi_{i+1}} \right) \right]^2
 \end{aligned}$$

If Equation 3.7 is substituted into Equation 3.10 and the numerical integration is performed, a series would be obtained which describe the relative permeability as a function of volumetric water content. Equation 3.20 shows the procedure of numerical integration along the volumetric water content. As can be seen from both Equation 3.19 and Equation 3.20, $k_r(\psi) = k_r(\theta_w)$. Multiplication of Equation 3.20 with saturated permeability of the soil would result in unsaturated permeability of the soil as a function of volumetric water content.

$$\begin{aligned}
 k_r(\theta_w) &= \left[\int_{\theta_{wL}}^{\theta_w} \frac{d\theta_w}{\psi} / \int_{\theta_{wL}}^{\theta_s} \frac{d\theta_w}{\psi} \right]^2 = \left[\frac{\sum_{i=1}^M \int_{\theta_{wi}}^{\theta_{wi+1}} \frac{d\theta_w}{\left(\psi_i - \frac{(\psi_i - \psi_{i+1})}{(\theta_{wi+1} - \theta_{wi})} (\theta_w - \theta_{wi}) \right)}}{\sum_{i=1}^N \int_{\theta_{wi}}^{\theta_{wi+1}} \frac{d\theta_w}{\left(\psi_i - \frac{(\psi_i - \psi_{i+1})}{(\theta_{wi+1} - \theta_{wi})} (\theta_w - \theta_{wi}) \right)}} \right]^2 \\
 &= \left[\frac{\sum_{i=1}^M \frac{(\theta_{wi} - \theta_{wi+1}) \ln \left(\frac{\theta_w (\psi_i - \psi_{i+1}) + \theta_{wi} \psi_{i+1} - \theta_{wi+1} \psi_i}{(\psi_i - \psi_{i+1})} \right)}{\left(\psi_i - \frac{(\psi_i - \psi_{i+1})}{(\theta_{wi+1} - \theta_{wi})} (\theta_w - \theta_{wi}) \right)}}{\sum_{i=1}^N \frac{(\theta_{wi} - \theta_{wi+1}) \ln \left(\frac{\theta_w (\psi_i - \psi_{i+1}) + \theta_{wi} \psi_{i+1} - \theta_{wi+1} \psi_i}{(\psi_i - \psi_{i+1})} \right)}}{\left(\psi_i - \frac{(\psi_i - \psi_{i+1})}{(\theta_{wi+1} - \theta_{wi})} (\theta_w - \theta_{wi}) \right)}} \right]^2 \quad \text{Equation 3.20} \\
 k_r(\theta_w) &= \left[\sum_{i=1}^M \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right) \ln \left(\frac{\psi_i}{\psi_{i+1}} \right) / \sum_{i=1}^N \left(\frac{\theta_{wi+1} - \theta_{wi}}{\psi_i - \psi_{i+1}} \right) \ln \left(\frac{\psi_i}{\psi_{i+1}} \right) \right]^2
 \end{aligned}$$

Mualem based estimation models are shown in first column of Table 3.2 as FMM, FMM ($C(\psi) = 1$), VMM and VMM ($m = 1 - 1/n$).

3.4 Procedure for estimating SWCC data points in higher suction range

As mentioned in Section 2.6, a complete soil-water characteristic curve can improve the estimated unsaturated permeability function (Kunze et al., 1968). In most laboratories, SWCC can be measured up to 100 kPa by a Tempe cell. However, the complete measurement of SWCC in a laboratory is an expensive and time-consuming task. Therefore, the scope of this study is to propose a procedure for estimating SWCC data points beyond 100 kPa.

Soil is an assembly of discrete grains of various shapes and sizes. The grain size distribution groups the soil grains into size ranges based on the relative proportion by weight of each size range. Figure 3.4 shows the grain sizedistribution curve (GSD) of a typical soil. The random assembly of soil grains creates a specific pore-structure for that soil. The soil-water characteristic curve of a soil is greatly dependent on its pore-structure, as was explained in Section 2.3.1, and the pore-structure of a soil is usually considered as a series of capillary tubes with various sizes (see Section 2.4.5).

Figure 3.4-Grain sizedistribution of a typical soil with idealized dense packing conditions

One of the possible assemblies of a soil with an idealized dense packing condition is shown in Figure 3.4. The pore space created by the soil grains can be seen in the figure. The radius of the pore can be computed using knowledge of the grain sizes creating that pore. The pore radius is related to suction, which is equivalent to the capillary height of that pore using the Young-Laplace equation (i.e., Equation 2.5). It is assumed that the pore will be emptied once the suction applied to the soil increases beyond its equivalent suction. The soil is considered fully saturated when its pores are completely filled with water. Once the suction applied to the soil has increased beyond the suction equivalent to the largest pore size, the soil starts to desaturate.

In a soil with an idealized dense packing condition, for any grain size available in the grain size distribution curve, the assembly of three soil grains of that size (i.e., the same size) would create the largest possible pore size for this grain size and the smaller grain sizes. It should be noted that the assembly of that particular grain size with its smaller grains would result in smaller pore sizes. Therefore, the largest pore that is created by the assembly of three grains of the same size would be emptied at its equivalent suction as computed by the Young-Laplace equation. The smaller pores created by the assembly of that grain size with its smaller grain sizes would remain saturated at that suction. This means that, for any grain size, the percent finer, PF (%), could be related to the water content at the equivalent suction of that grain size. In other words, the water content for that grain size reflects that the smaller pores remain fully saturated at the equivalent suction.

If the soil grains are assumed to have a spherical shape with an idealized dense packing condition similar to that shown in Figure 3.5, the radius of the pore can be computed according to Equation 3.21, which is known as Descartes' theorem.

$$\frac{1}{R_p} = \frac{6 + 4\sqrt{3}}{D_g} \quad \text{Equation 3.21}$$

where:

D_g is diameter of the grain

R_p is radius of the pore created by three grains of the same size

Figure 3.5-Idealized dense packing condition of soil grains

The capillary height of the corresponding pore can be expressed in terms of the radius of that pore according to the Young-Laplace equation as follows:

$$h_c = \frac{2T_s}{\rho_w g R_p} \quad \text{Equation 3.22}$$

where:

h_c is the capillary height

T_s is the surface tension

ρ_w is the water density

g is the gravitational acceleration

R_p is the radius of the pore

The equivalent suction(ψ), corresponding to each available grain diameter in the grain sizedistribution curve of the soil can be computed according to Equation 3.23, which is obtained by substituting Equation 3.21 into Equation 3.22.

$$\psi = (12 + 8\sqrt{3}) \frac{T_s}{\rho_w g D_g} \quad \text{Equation 3.23}$$

where:

ψ is the soil suction, (m)

Therefore, the x-axis of the grain sizedistribution curve (i.e., diameter) can be converted to equivalent suction(ψ), using Equation 3.23.

The percent finer, PF (%), of that grain size could represent the percentage of smaller pores corresponding to that grain size. Therefore, the water content at any grain size can be computed according to Equation 3.24.

$$wc = \frac{PF(\%) \times wc_{Sat}}{100} \quad \text{Equation 3.24}$$

where:

wc is the water content of the soil at any computed suction equivalent to grain diameter

wc_{Sat} is the saturated water content of the soil

A procedure for estimating SWCC data points in higher suction ranges, specifically those beyond suction of 100 kPa, that uses actual measurement of SWCC at 100 kPa and the grain size distribution curve is proposed in this study based on the following steps:

- 1- The diameter corresponding to a suction of 100 kPa is computed according to Equation 3.23 and is called $D_{(100)}$
- 2- The percent finer(PF), corresponding to $D_{(100)}$ is determined from the GSD curve and multiplied by the saturated water content of the soil as a first estimation of water content, $wc_{(100)}$, at 100 kPa
- 3- The first estimate of water content at 100 kPa, $wc_{(100)}$, is scaled to the measured water content of the soil at 100 kPa, $wc_{(m)}$, and the scaled factor is equal to

$$S_f = \frac{wc_{(100)}}{wc_{(m)}}$$

- 4- The equivalent suction, $\psi_{(X_n)}$, of the available grain diameters, $D_{(X_n)}$, smaller than $D_{(100)}$ is computed according to Equation 3.23
- 5- The percent finer, PF, corresponding to $D_{(X_n)}$ smaller than $D_{(100)}$ is multiplied by the saturated water content of the soil as a first estimation of water content, $w_{c(X_n)}$, at the computed suctions, $\psi_{(X_n)}$
- 6- The first estimation of water content, $w_{c(X_n)}$, at the computed suctions, $\psi_{(X_n)}$, is scaled by dividing it by the scaled factor, S_f

Figure 3.6 shows the grain size distribution curve with grain diameters smaller than $D_{(100)}$ and Figure 3.7 shows the transformation of the grain size distribution curve to SWCC data points, $(\psi_{(X_n)}, w_{c(X_n)})$, beyond 100 kPa.

Figure 3.6-Grain sizedistribution curve with grain sizes beyond the $(D(100), PF(100))$ point

Figure 3.7-Transformed grain sizes to SWCC data points beyond 100 kPa

The proposed procedure is similar to the third approach (a physics-based conceptual model) for estimating SWCC as explained in Section 2.3.2, which involves the conversion of the grain sizedistribution into a pore-size distribution(Arya and Paris, 1981; Kovács, 1981; Smettem and Gregory, 1996; Hunt and Gee, 2002; Aubertin et al.,2003). However, the proposed procedure is independent of the soil database. It should be noted that the proposed procedure is applicable for suctions beyond 100 kPa.

3.5 Summary of chapter

Combination of any pair of best-fit soil-water characteristic curve (SWCC) equation and relative permeability equation (k_r) would result in an unsaturated permeability estimation model. The four selected best-fit soil-water characteristic curve (SWCC) equations and three the relative permeability (k_r) equations as explained in Chapter 2 form a matrix as shown in Table 3.1. There are 12 possible combinations for estimating the unsaturated permeability of soils. Two of the combinations have been proposed by other researchers, as shown in the matrix by the researchers' names; the rest of the combinations have not yet been considered, as shown in the matrix by N.A. (i.e., not available). The N.A. combinations are developedin this study (Table 3.2). It should be noted that this matrix is created as a standard procedure to study the effect of SWCC and k_r equations on the estimation of unsaturated permeability of soils. As mentioned in Section 2.6, a complete soil-water characteristic curve can improve the estimated unsaturated permeability

function (Kunze et al., 1968). In most laboratories, SWCC can be measured up to 100 kPa by a Tempe cell. However, the complete measurement of SWCC in a laboratory is an expensive and time-consuming task. Therefore, a procedure for estimating SWCC data points beyond 100 kPa is proposed.

Chapter 4 **Research program**

4.1 Introduction

An overview of this research was shown in Figure 1.1. The outline of the research program conducted consists of steps 4, 5, 6, 7, 8 and 9, as shown in Figure 1.1.

Firstly, the procedure for the estimation of unsaturated permeability of soil for selected soil databases is presented in Section 4.2. An evaluation of the matrix of unsaturated permeability estimation models for the selected soil database (as explained in Section 4.2) is presented in Section 4.3. Evaluations of the effects of the k_r equations, SWCC equations and measured SWCC data ranges on the estimation of unsaturated permeability are then presented in Sections 4.4, 4.5 and 4.6, respectively. An evaluation of the proposed procedure for estimating SWCC data points in a higher suction range is presented in Section 4.7. Finally, the experimental program conducted to validate the results of this study is presented in Section 4.8.

4.2 Procedure for estimation of unsaturated permeability of soil

A soil database was selected from the literature consists of twenty sets of published data. Each soil had experimentally measured SWCC and $k(\psi)$ data as shown in Table 4.1.

Table 4.1-Soil database used for evaluation of effect of SWCC and k_r equations

No.	Reference	Soil name	Soil number	k_s (m/s)
1	(Elrick and Bowman, 1964)	Guelph Loam	S1	3.917×10^{-6}
2	(van Genuchten, 1980)	Beit Netofa Clay	S2	9.491×10^{-9}
3	(Moore, 1939)	Yolo light clay	S3	1.230×10^{-7}
4	(Brooks and Corey, 1964)	Touchet silt loam	S4	3.507×10^{-5}
5	(Brooks and Corey, 1964)	Columbia Sandy loam	S5	N.A.
6	(Brooks and Corey, 1964)	Hygiene Sandstone	S6	1.250×10^{-5}
7	(Richards, 1952)	Superstition Sand	S7	1.830×10^{-5}
8	(van Genuchten, 1980)	Silt Loam	S8	5.741×10^{-7}
9	(Meerdink and Benson, 1996)	Wenatchee Silty Clay	S9	2.200×10^{-9}
10	(Meerdink and Benson, 1996)	Live Oak Red Clay	S10	3.200×10^{-8}
11	(Ng and Leung, 2011)	CDT	S11	2.52×10^{-7}
12	(Rassam and Williams, 1999)	Mine Tailings	S12	9.19×10^{-7}
13	(Samingan et al., 2003)	UP-1 (Residual soil)	S13	1.21×10^{-8}
14	(Samingan et al., 2003)	UP-2 (Residual soil)	S14	6.65×10^{-8}
15	(Samingan et al., 2003)	UP-3 (Residual soil)	S15	6.25×10^{-10}
16	(Samingan et al., 2003)	UP-4 (Residual soil)	S16	9.48×10^{-7}
17	(Valiantzas, 2011)	Weld Silty Clay	S17	5.671×10^{-6}
18	(Valiantzas, 2011)	Fine Sand	S18	1.250×10^{-5}
19	(Valiantzas, 2011)	Volcanic Sand	S19	8.102×10^{-5}
20	(Van Genuchten and Nielsen, 1985)	G.E. No.2 Sand	S20	N.A.

The measured SWCC data of each soil was best-fit by Equation 2.10, Equation 2.11, Equation 2.12 and Equation 2.13 by minimizing the residual sum of squares (SSE) according to Equation 4.1.

$$SSE = \sum_{i=1}^X (\theta_{wi} - \hat{\theta}_{wi}) = 0 \quad \text{Equation 4.1}$$

where:

θ_w is the measured volumetric water content

$\hat{\theta}_w$ is the estimated volumetric water content by best-fit SWCC equation

X is the number of measured data points

i is a counter

The unsaturated permeabilities were then estimated according to Equation 3.15, Equation 3.17 and Equation 3.19 for each best-fit SWCC. As a result, twelve unsaturated permeability curves as a function of suction were estimated for each of the soils presented here. The lower limit of unsaturated permeability (or vapor permeability) was computed in accordance with Equation 3.14. According to Equation 3.14, each SWCC equation would result in one vapor permeability curve. Therefore, for each of the four SWCC equations used in this study, one vapor permeability curve was obtained. This means that, four vapor permeability curves were obtained for each of the soils in the database. The computed vapor permeabilities were then normalized with respect to the saturated permeability of the soil.

Figure 4.1 shows the process for estimation the unsaturated permeability of soils by the matrix of unsaturated permeability estimation method.

Figure 4.1-Process of unsaturated permeability estimation

4.3 Evaluation of matrix of unsaturated permeability estimation models

The matrix of unsaturated permeability estimation models (i.e., Table 3.2) was evaluated for the soil database shown in Table 4.1. The combined estimation models in this study along with the two existing models of F&X-1994 and VG-1980 were evaluated. The root mean square error (RMSE) is used as a statistical measure to evaluate the fitness of the estimation models. The root-mean-square error (RMSE) is defined according to Equation 4.2.

$$RMSE = \sqrt{\sum_{i=1}^{i=X} (\log(k_{ri}) - \log(\hat{k}_{ri}))^2} \quad \text{Equation 4.2}$$

where

k_r is the measured unsaturated permeability value

$\hat{k}_r$ is the estimated unsaturated permeability value

X is the number of measured data points

i is a counter

4.4 Evaluation of effect of k_r equation on estimation of unsaturated permeability

In order to evaluate the effect of relative permeability(k_r) equation on estimation of unsaturated permeability of soil, all the unsaturated permeability estimation models (see Table 3.2) were categorized into SWCC equations category. Therefore, there are four groups namely, Fredlund and Xing (1994), Fredlund and Xing (1994) with $C(\psi) = 1$, van Genuchten (1980) and van Genuchten (1980) with $m=1-1/n$ based models in SWCC category. The estimated unsaturated permeability values by the estimation models in each group of the category (i.e., SWCC category) were compared. For instance, the estimated unsaturated permeability values by VCM, VMM and VBM models in van Genuchten (1980) group were compared. The comparison was done according to Equation 4.3.

$$I_D = \log\left(\frac{\max(E(k_r(\psi)))}{\min(E(k_r(\psi)))}\right) \quad \text{Equation 4.3}$$

where:

I_D is difference index

$\max(E(k_r(\psi)))$ is maximum estimated unsaturated permeability by models in one group

$\min(E(k_r(\psi)))$ is minimum estimated unsaturated permeability by models in one group

It should be noted that the difference index, I_D , was computed at the last measured suction value of the SWCC data used for best-fit exercise.

4.5 Evaluation of effect of SWCC equation on estimation of unsaturated permeability

In order to evaluate the effect of best-fit SWCC equation on estimation of unsaturated permeability of soil, all the unsaturated permeability estimation models were categorized into k_r equations category. Therefore, there are three groups namely, Childs Collis-George, Burdine and Mualem based models in k_r category. The estimated unsaturated permeability values by the estimation models in each group of the category (i.e., k_r category) were compared. For instance, the estimated unsaturated permeability values by FCM, FCM, $C(\psi) = 1$, VCM and VCM ($m=1-1/n$) models in Childs Collis-George group were compared. The comparison was done according to Equation 4.3.

It should be noted that the difference index, I_D , was computed at the last measured suction value of the SWCC data used for best-fit exercise.

4.6 Evaluation of effect of measured SWCC data ranges on estimation of unsaturated permeability

In order to investigate the effect of SWCC measurement range on the estimation of unsaturated permeability, data of seven soils from Table 4.1 which had SWCC measurements over a wide suction range were selected. These soils are shown in Table 4.2. The ranges of the measured soil-water characteristic curves of these soils varied from zero to as high as 10000 kPa. The ranges of the directly measured unsaturated permeability data of these soils were up to 300 kPa. The maximum suction values that SWCC and unsaturated permeability(k_w) were directly measured and the saturated

permeability(k_s) value for each soil are tabulated in Table 4.2. The USCS classification of each soil is also given in the table.

Table 4.2-Soil database with wider range of SWCC measurement

Soil Name	Soil Type (USCS)	Saturated Permeability k_s (m/s)	Maximum Measured Suction for SWCC (kPa)	Maximum Measured Suction for k_w (kPa)	Reference
UP-1	ML	1.21×10^{-8}	10000	300	(Samingan et al., 2003)
UP-2	MH	6.65×10^{-8}	10000	300	(Samingan et al., 2003)
UP-3	CL	6.25×10^{-10}	10000	300	(Samingan et al., 2003)
UP-4	SC	9.48×10^{-7}	1000	200	(Samingan et al., 2003)
Beit Netofa Clay	N.A.	9.49×10^{-9}	1500	95	(van Genuchten, 1980)
Wenatchee Silty Clay	CL-ML	2.2×10^{-9}	1500	4500	(Meerdink and Benson, 1996)
Live Oak Red Clay	MH	3.2×10^{-8}	1500	2400	(Meerdink and Benson, 1996)

Four different ranges of SWCC measurement were considered in this study, namely DR1, DR2, DR3 and DR4 as shown in Figure 4.2.

The ranges of SWCC measurement were selected on the basis of the maximum suction capacity of the available laboratory equipment for measuring the soil-water characteristic curve (Li et al., 2009). As can be seen in Figure 4.2, DR1 is the range of SWCC measurement of 0.1-100 kPa which can be measured by Tempe pressure cells. DR2 is the range of SWCC measurement of 0.1-500 kPa which can be measured by 5-bar pressure plate. DR3 is the range of SWCC measurement of 0.1-1500 kPa which can be measured by 15-bar pressure plate. DR4 is the range of SWCC measurement of 0.1-10000 kPa which can be measured using salt solutions.

Figure 4.2-Selected ranges of SWCC measurements

The detailed explanation on measurement of SWCC can be found in Section 2.3.1. In order to conduct the study, the measured SWCC data were intentionally bounded within these different suction ranges. For instance, for DR1, the measured SWCC data within the range of 0.1-100 kPa were considered, while the SWCC data beyond 100 kPa were omitted in the best-fit exercise. Since, there were four best-fit SWCC equations (i.e., Fredlund and Xing (1994), Fredlund and Xing (1994) with $C(\psi) = 1$, van Genuchten (1980) and van Genuchten (1980) with $m=1-1/n$) and four ranges of SWCC measurements (or three in some cases), for every soil in Table 4.2, 16 (or 12) best-fit SWCCs were obtained. The unsaturated permeabilities were then estimated according to Equation 3.15, Equation 3.17 and Equation 3.19 for each best-fit SWCC.

4.7 Evaluation of procedure for estimating SWCC data points in higher suction range

The proposed method in Section 3.4 for estimating the soil-water characteristic curve data points in a higher suction range was evaluated for a selected soil database. The soil database was selected from the SoilVision (2006) database. SoilVision (2006) is a large soil database consisting of over 6000 soils that includes details of the grain sizedistribution, texture, bulk density, porosity, specific gravity, saturated volumetric water content, SWCC and saturated and unsaturated coefficient of permeability. As

described in Section 3.4, knowledge of the grain sizedistribution (GSD) and soil-water characteristic curve (SWCC) are required for the estimation procedure. There are 879 soils in the SoilVision database that contain both SWCC and grain sizedistribution data. However, only 145 of these soils fulfil all of the criteria for selection in this study. The criteria for the selection of the soil database are as follows:

- 1- The SWCC measurements must be available from saturations to at least one suction point beyond 100 kPa.
- 2- The SWCC measurements must be available for suctions around 100 kPa (i.e., 90-110 kPa).
- 3- The GSD must be available for grain diameters ($D_{(X_n)}$) equivalent to $\psi(X_n)$ beyond 100 kPa.

As previously stated, 145 soils in the SoilVision database met the above criteria. Forty (40) soils were then randomly selected from these 145 soils for use in the evaluation of the proposed estimation procedure. Table 4.3 shows the soil database used in this study. The procedure for estimating SWCC data points in a higher suction range, specifically beyond a suction of 100 kPa, as proposed in Section 3.4 was then applied to the selected soil database. The diameters corresponding to the available suction around 100 kPa (i.e., 90-110 kPa) and suction points beyond 100 kPa were computed according to Equation 3.23. The percent finer, PF, corresponding to those diameters was also determined from the GSD and multiplied by the saturated water content of the soil as a first estimation of water content. The first estimation of water content was then scaled by dividing it by the scaled factor, S_f .

Table 4.3- Selected soil database for evaluation of the proposed estimation procedure of SWCC data points in a high suction range

	Soil counter	Minimum diameter (mm)	PF of minimum diameter (%)	Maximum measured suction (kPa)	Maximum estimated suction (kPa)	Clay (%)	Silt (%)	Sand (%)
1	262	0.0028	0.120	513291.9	619.1	16.71	78.75	4.74
2	1446	0.002	0.520	1500.93	400.248	57.47	35.72	6.81
3	10760	0.002	0.318	981	981	42.23	25.4	32.243
4	10762	0.002	0.075	1500.93	500.31	12.55	21.05	66.18
5	10763	0.002	0.215	1500.93	500	25.75	18.79	55.37
6	10765	0.002	0.405	1500.93	500.31	41.55	13.79	44.65
7	10774	0.002	0.080	1471.5	392.4	11.61	11.19	77.05
8	10776	0.002	0.490	1471.5	392.4	53.94	9.32	36.29
9	10777	0.002	0.470	1471.5	392.4	50.1	10.36	39.36
10	10831	0.002	0.470	1471.5	294.3	66.54	29.53	3.89
11	10836	0.002	0.170	1554.885	619.011	20.35	66.33	13.32
12	10838	0.002	0.435	1554.885	619.011	66.54	29.53	3.89
13	10839	0.002	0.403	1554.885	619.011	47.83	42.5	9.66
14	10843	0.002	0.351	1554.885	619.011	41.37	52.32	6.3
15	10847	0.002	0.407	1554.885	619.011	57.46	38.21	4.3
16	10851	0.002	0.581	1554.885	619.011	70.97	28.32	0.71
17	10862	0.002	0.219	1554.885	619.011	25.37	64.93	9.7
18	10980	0.002	0.450	1471.5	784.8	52.6	32.29	14.84
19	10981	0.002	0.570	1471.5	784.8	69.35	27.92	2.72
20	10982	0.002	0.630	1471.5	784.8	76.74	20.52	2.68
21	11037	0.002	0.380	1471.5	294.3	47.62	51.28	1.1
22	11159	0.002	0.435	1500.93	500.31	62.37	34.89	2.73
23	11219	0.002	0.200	1013.373	567.509	30.04	69.78	0.17
24	11222	0.002	0.290	961.7724	961.772	34.62	65.35	0.03
25	11224	0.002	0.205	961.7724	961.772	43.79	56.21	0
26	11261	0.002	0.151	1471.5	618.03	19.03	80.3	0.67
27	11267	0.0014	0.137	163.827	163.827	18.59	79.51	1.9
28	11271	0.002	0.181	931.95	931.95	23.67	75.16	1.17
29	11405	0.002	0.070	1471.5	981	11.27	87.56	1.17
30	11409	0.002	0.473	1471.5	981	54.93	37	8.05
31	11410	0.002	0.544	1471.5	981	62.51	29.62	7.77
32	11474	0.0014	0.137	1471.5	1471.5	14.83	6.93	78.18
33	11477	0.002	0.011	1471.5	490.5	1.16	3.16	95.68
34	11496	0.0014	0.111	500	500	16.49	16.94	65.76
35	11497	0.0014	0.137	300	300	14.83	6.93	78.18
36	11525	0.0014	0.142	300	300	19	10.97	69.68
37	11537	0.0014	0.115	500	500	15.62	19.58	64.51
38	11540	0.0014	0.225	400	400	34.69	37.68	26.88
39	12217	0.001	0.397	324.319	324.319	52.58	46.6	0.82
40	18748	0.002	0.250	1000000	870.9634	32.76	22.46	43.45

The variability of the estimated SWCC data points versus the measured SWCC data points was computed by calculating the coefficient of determination, R^2 , according to Equation 4.4.

$$R^2 = 1 - \frac{SSE}{SST} \quad \text{Equation 4.4}$$

where:

$$SST = SSR + SSE$$

$$SSR = \sum_{i=1}^X (\hat{\theta}_{wi} - \bar{\theta}_{wi}) = 0$$

$$SSE = \sum_{i=1}^X (\theta_{wi} - \hat{\theta}_{wi}) = 0$$

where:

$\bar{\theta}_w$ is the average measured volumetric water content

$\hat{\theta}_w$ is the estimated volumetric water content

X is the number of data points

i is a counter

The reliability of the estimation procedure was investigated by computing the confidence interval of the observed error, $(\theta_{wi} - \hat{\theta}_{wi})$, using a normal distribution in accordance with Equation 4.5.

$$\mu(\theta_{wi} - \hat{\theta}_{wi}) \pm Z_{\alpha/2} \sigma(\theta_{wi} - \hat{\theta}_{wi}) \quad \text{Equation 4.5}$$

where:

$$\mu = \frac{1}{X} \sum_{i=1}^X \theta_{wi}$$

$$\sigma = \sqrt{\frac{1}{X} \sum_{i=1}^X (\theta_{wi} - \mu)^2}$$

α is the significance level

4.8 Experimental program

An experimental program consisting of two main tests, (1) soil-water characteristic curve tests and (2) saturated and unsaturated permeability tests was planned in this study. Different soils were used to conduct the experimental program. The results of the experimental program are used to validate the finding of the study.

4.8.1 Material selection and preparation

Identical soil specimens were needed in this experimental program to ensure that the results from the various soil specimens are reliable and reproducible. Therefore, man-made soil specimens were selected and used. The soil specimens must be homogeneous and uniform in composition and must have a similar initial dry density and initial water content in order to produce meaningful results for analyses and comparisons.

Soil mixtures of coarse kaolin and fine sand were selected for producing the soil specimens. Coarse kaolin, which is produced by Kaolin Malaysia SDN BHD (Malaysia), and Ottawa sand, which is furnished by the U.S. Silica Company, were selected and mixed to produce the soil specimens.

The kaolin-sand mixtures were selected to represent the two ends of the cohesive to non-cohesive spectrum of soils types, resulting in a comprehensive study on the parameters affecting the estimation of unsaturated permeability of soils. The soil mixtures were used to prepare the soil specimens for SWCC tests and permeability tests. Table 4.4 shows the selected kaolin-sand mixtures in this study.

Table 4.4-Selected kaolin-sand mixtures in this study

Soil name	Kaolin (%)	Sand (%)
100K0S	100	0
90K10S	90	10
80K20S	80	20
50K50S	50	50
20K80S	20	80
10K90S	10	90

4.8.2 Basic soil properties tests

Basic soil properties tests, including the Atterberg limit (i.e., liquid limit, plastic limit and plasticity index) tests, grain size analysis (i.e., dry sieving and hydrometer test) and specific gravity tests were performed on all soil mixtures. The tests were performed according to ASTM soil testing standards. Table 4.5 shows the corresponding standard for each test.

Table 4.5- Basic soil properties test standards

Test	Standard
Atterberg limit tests	ASTM-D4318
Grain size analysis	ASTM-6913
Specific gravity	ASTM-D854

4.8.2.1 Standard compaction test

The standard Proctor compaction test was performed according to ASTM D698 (1997) to determine the compaction curve for all the soil mixtures. The soils were mixed according to the designated percentage for each mixture. Distilled water was added to the soil mixtures at various water contents (i.e., a designated percentage of the dry mass of the mixture) and sealed in a zipped plastic bag, then left for two to three days in the humidity chamber for equalization. The moist soil mixtures were compacted at various water contents and their corresponding total densities were obtained. The relationship between water content and dry density for each soil mixture was plotted. The maximum dry density(ρ_{dmax}) and optimum water content, (w_{opt}) were obtained from the curve for all the soil mixtures. The soil specimens for SWCC and permeability tests were prepared by static compaction at the maximum dry density(ρ_{dmax}) and optimum water content(w_{opt}). A tolerance interval of $\pm 5\%$ of the maximum dry density was allowed in the specimen preparation. In order to prepare the soils for static compaction, the dry soil mixtures were mixed with their optimum water content(w_{opt}) and sealed in plastic bags, then left in the humidity chamber for two to three days for equalization. The water content was then measured and compared with the optimum water content(w_{opt}). If the measured water content was less or more than the optimum water content(w_{opt}), water was added or removed to the mixture, respectively, and the whole procedure was repeated until the

desired water content was achieved. The moist soil mixtures were then used for static compaction.

4.8.2.2 Static compaction

In order to prepare identical soil specimens that are homogenous and have a uniform density, the static compaction procedure described by Ong (1999) was followed. The compaction device consists of a stainless steel mould and two stainless steel plugs. The stainless steel mould is 200 mm in height and has two symmetrical parts that can be connected or disconnected by two Allen keys. The mould with a square-shaped cross-section has a cylindrical hole in the center. Figure 4.3 shows the mould and its plugs, which have a $70 \times 70 \text{ mm}^2$ cross-section with a 50 mm cylindrical hole at the center.

Figure 4.3-Static compaction mould and plugs

The stainless steel plugs consist of a number of removable 10 mm thick disks. The length of the plugs can be increased by adding disks or decreased by removing disks to prepare a soil specimen layer by layer. The disks can be attached to each other through the threaded screw that protrudes out from the center of the face of one disk into the threaded hole drilled into the center of the face of another disk. Each disk has two parts, namely a base part and a top part with 50 mm and 40 mm diameters, respectively, and the same thickness (i.e., 5 mm), resulting in a T-shape disk. The reason for the T-shape design is to

decrease the friction between the trapped soil particles in the gaps and along the wall of the mould. A compaction mould with an inner diameter of 50 mm is used to prepare the soil specimens for the SWCC, saturated and unsaturated permeability tests. The soil specimens are first compacted in three layers, each with a thickness of 10 mm. The mass of soil needed for each soil specimen is calculated based on the maximum dry density and divided into three parts. The soil is placed into the mould layer by layer and compacted at a slow displacement rate (i.e., 1 mm/min). The axial load is applied to the mould by a compression machine (see Figure 4.4).

Figure 4.4-Compression machine used for static compaction

After placing the soil in the mould, the top plug is inserted and the axial load is applied at a constant rate until the soil is compressed to a thickness of 10 mm. The mould is turned over after the layer of the soil specimen is successfully compacted so that the top plug is now at the bottom and the bottom plug is now at the top. The height of the top plug is then reduced by 10 mm by removing one disc. The compacted soil surface is scratched consistently before placing the soil for the next layer inside the mould; this ensures good

and consistent bonding between the layers. This procedure is repeated until the desired thickness is achieved. The soil specimen is then extruded using the same compression machine. However, after the soil specimens were used in the SWCC tests, they were found to be unbounded at different layers as they became drier. This discontinuity between the layers would result in inaccurate and unreliable results. Therefore, the soil specimens for SWCC and permeability tests were statistically compacted at one layer. Since the heights of the soil specimens were 30 mm, the uniformity was still satisfied.

4.8.3 Soil-water characteristic curve

As explained in Section 2.6, the measured SWCC data range is one of the parameters affecting the estimation of unsaturated permeability of soils. The range over which SWCC can be measured depends on the available equipment in the laboratory. In order to cover the designated measured SWCC data range in this study, as explained in Section 4.6, soil-water characteristic curve (SWCC) tests were performed using a Tempe pressure cell, 5-bar pressure plate, 15-bar pressure plate and saturated salt solutions. It should be noted that the SWCC tests were performed under zero net confining pressure. In the following sections, each equipment and testing procedure is explained in greater detail.

4.8.3.1 Tempe pressure cell

A Tempe cell was used in this study to measure the SWCC of the selected soil specimens up to a suction of 100 kPa (i.e., DR1). A Tempe cell consists of a 1-bar air-entry value ceramic disk, cell base, cell ring, cell cap, three O-rings and three screws. Figure 4.5a shows a schematic cross-section of an assembled Tempe cell, and parts of a disassembled Tempe cell is shown in Figure 4.5b.

Figure 4.5- Tempe cell: (a) schematic diagram of a Tempe cell cross-section; (b) disassembled parts of a Tempe cell

The soil specimen was prepared according to the procedure explained in Section 4.8.2.2 with a height of 30 mm and diameter of 50 mm. The mass of all necessary parts of the Tempe cell (i.e., cell base, cell ring, cell cap, O-rings and screws) was measured first. The soil specimen was then placed into the ring. The ring was placed on the cell base and the top cap was placed on top. The three parts were tightened together with three screws. After the assembly of the Tempe cell was complete, it was placed on a tripod stand and the soil specimen was saturated. The saturation was performed by connecting the bottom of the Tempe cell to a flexible tube that was connected to a burette filled with distilled and de-aired water. The level of water in the burette was slightly above the level of the ceramic disk. The water flowed to the bottom of the soil specimen through the pre-saturated ceramic disk (see Figure 4.6). It should be noted that the ceramic disk was saturated by submerging it into the distilled and de-aired water in a vacuum desiccator for a few days. The needed water for saturation was computed considering the volume-mass relationships. The mass of the Tempe cell was measured regularly to assure that the soil specimen was saturated. It should be noted that the water level in the burette was checked regularly to ensure there was no water loss due to evaporation.

The axis-translation technique (Hilf, 1956) as explained in Section 2.3.1, was used to apply matric suction to the soil specimen. The pore-water pressure inside the Tempe cell was considered to be zero due to the good contact between the soil specimen and the saturated ceramic disk. Air pressure was applied from the top of the Tempe cell to the soil specimen. The matric suction was equal to the difference between the air pressure and the equilibrated water pressure ($u_a - u_w$). Therefore, the matric suction of the soil was equal to the applied air pressure.

Figure 4.6-Assembled Tempe cell sitting on a tripod stand

The water was drained from the soil specimen by the application of matric suction. The mass of the Tempe cell was measured at regular time intervals and plotted against time. Once the Tempe cell reached equilibrium, the applied air pressure was increased to the next target value. Figure 4.7 shows a typical graph of water loss versus time at a specific suction value.

Figure 4.7-Typical graph of water loss versus time at an applied suction

Matric suction values of 10, 20, 30, 40, 50, 60, 70, 80 and 90 kPa were applied to the soil specimens. After the last target matric suction was applied and the equilibrium condition was met, the specimen was removed from the Tempe cell and transferred to the 5-bar pressure plate.

4.8.3.2 Automated Tempe cell set-up

As was explained in Section 4.8.3.1, a Tempe cell was used in this study to measure the SWCC of the selected soil specimens up to a suction of 100 kPa (i.e., DR1). Kaolin-sand mixtures were selected to represent the two ends of the cohesive to non-cohesive spectrum of soils types, as was explained in Section 4.8.1. The SWCC of the non-cohesive soil mixtures (i.e., 20K80S and 10K90S) was measured by the automated measurement system developed in this study, as shown in Figure 4.8. As the rate of water loss for sandy materials is quite fast due to applied matric suction, an automated Tempe cell set-up was developed to measure the SWCC of these soil mixtures up to a suction of 100 kPa (i.e., DR1). The automated Tempe cell set-up consists of four main components, namely a Tempe cell, a digital balance, a pressure gauge for air pressure control and a personal computer (PC). The digital balance was connected to the PC via the RS232 interface and the data was automatically recorded on the PC using software. The soil specimens for kaolin-sand mixtures of 20K80S and 10K90S were prepared according to the procedure explained in Section 4.8.2.2. The soil specimens were saturated according to the saturation process explained in Section 4.8.3.1. After the soil specimen achieved saturation conditions, it was placed on the digital balance where the outflow drained water was collected in a container on the balance, as shown in Figure 4.8. Matric suction was applied to the soil specimen by a pressure gauge that was connected to a manometer to control the application of very small matric suctions. The mass of drained water was recorded at 10 second intervals. Once the equilibrium condition was achieved, the mass of the Tempe cell was measured by another balance. To assure there was no evaporation, four containers filled with distilled water were attached to the inner walls of the balance breeze break to saturate the air and prevent evaporation.

Figure 4.8-Automated Tempe cell set-up developed in this study

4.8.3.3 Pressure plate

The pressure plate consists of a pressure chamber, a high air-entry value ceramic disk and a rubber membrane beneath the ceramic disk. The space between the ceramic disk and the rubber membrane serves as a water compartment. The water compartment is connected to a burette line open to the atmosphere. Figure 4.9 shows a schematic diagram of the 5-bar pressure plate apparatus set-up. Before starting the test, the ceramic disk was saturated by placing it in a desiccator filled with de-aired distilled water connected to a vacuum line for a few days. Once there was no sign of air bubbles on the ceramic disk, it was taken out and placed in the pressure chamber. The distilled water was poured on the surface of the ceramic disk and an air pressure below the air-entry value (i.e., 400 kPa for 5-bar and 1000 kPa for 15-bar) was applied to the ceramic disk. The de-aired distilled water flowed into the ceramic disk due to the pressure difference and flowed out from the pressure chamber. The procedure was repeated for a few times and the ceramic disk was considered saturated when no air bubbles were found in the water compartment. After saturation of the ceramic disk, the soil specimen was placed in the pressure plate.

Figure 4.9-Pressure plate set-up for SWCC test(from Samingan, 2001)

Suction was applied to the soil specimen by the axis-translation technique (Hilf, 1956), as explained in Section 2.3.1. The valve of the burette was open and exposed to the atmosphere and the level of the water was the same as the level of the ceramic disk.

Two types of pressure plates were used in this study, 5-bar and 15-bar, as shown in Figure 4.10.

Figure 4.10-Pressure plates used in this research: (a) 5-bar and (b) 15-bar

The 5-bar pressure plate had a ceramic disk with an air-entry value of 500 kPa and was used to measure the SWCC of soil specimens up to a suction of 500 kPa (i.e., DR2). The

15-bar pressure plate had a ceramic disk with an air-entry value of 1500 kPa and was used to measure the SWCC of soil specimens up to a suction of 1500 kPa (i.e., DR3).

After the equilibrium condition was achieved at the last target value of 90 kPa, the soil specimen was removed from the Tempe cell at 90 kPa matric suction (as explained in Section 4.8.3.1) and placed in the 5-bar pressure plate. An air pressure of 100 kPa was applied to the soil specimen and the water pressure was kept at zero (i.e., the level of the water in the connected burette was the same as the level of the ceramic disk). The mass of the soil specimen was measured daily and plotted versus time. Once the data reached the equilibrium condition, the air pressure was increased to the next target pressure. The soil specimen was moved to the 15-bar pressure plate after the equilibrium condition was achieved at the last target air pressure of 450 kPa. Once the soil specimen was transferred to the 15-bar pressure plate, an air pressure of 500 kPa was applied to the soil specimen and the same procedure as for the 5-bar pressure plate was followed for the target air pressures. After the equilibrium condition was achieved at the last target pressure of 1400 kPa, the soil specimen was dried in an oven for 24 hours and its initial dry mass was corrected according to the mass-volume relationship.

4.8.3.4 Salt solution

The salt solution method was used to measure SWCC at high suction values (i.e., DR4 and beyond). The theoretical basis of the salt solution method is related to the relative vapor pressure of water in a soil-water system. The relative humidity of the soil-water system is used to characterize the relative vapor pressure of water in equilibrium with the system. Therefore, by creating the relative humidity related to the concentration of a solution identical with the composition of the soil water, the suction can be established (Lang, 1967). Two relative humidities were created by preparing two different salt concentrations. Table 4.6 shows the salt solutions used in this study. The first salt solution was sodium chloride with a molality of 2 which was placed in a glass vacuum desiccator to create the vapor pressure equivalent to the required suction. The second salt solution was magnesium chloride with a molality of 15 which was placed in a glass vacuum desiccator to create the vapor pressure equivalent to the required suction.

Table 4.6-Salt solutions used in this study

Salt	Molality (m)	Measured RH (%)	Equivalent Suction (kPa)
NaCl	2	93.2	9507.03
MgCl ₂	15	33.7	146835.8

A relative humidity sensor was placed in the glass desiccator to ensure that the required relative humidity was achieved. The measured relative humidity was converted to equivalent suction using Equation 2.8. Soil specimen with a height of 10 mm were prepared according to Section 4.8.2.2 and, after saturation, placed into a ceramic crucible. The soil specimen in the ceramic crucible was then put in the desiccator. Figure 4.11 shows the soil specimens inside the glass desiccator. The soil specimen, together with the ceramic crucible, was weighed every week until there was no change in the mass of the soil specimen. Once the equilibrium condition was reached, the soil specimens were dried in an oven for 24 hours. The initial dry masses of the soil specimens were corrected according to the mass-volume relationship.

Figure 4.11-Salt solution apparatus

4.8.3.5 Testing program for SWCC measurements

As was explained in Section 4.8.3, the soil-water characteristic curve tests were conducted on selected soil mixtures using a Tempe cell, 5-bar pressure plate, 15-bar pressure plate and salt solutions.

Table 4.7 shows the selected soil mixtures and respective SWCC tests conducted in this study. The tests that were conducted for each soil mixture are shown as “Yes” in the table, while the tests that were not conducted are shown as “No”. For instance, Tempe cell tests were conducted for soil mixtures 100K0S, 90K10S, 80K20S and 50K50S, while Automated Tempe cell tests were conducted for 20K80S and 10K90S.

Table 4.7-Soil mixtures and their respective SWCC tests

Soil mixture	Tempe cell	Automated Tempe cell	5-bar pressure plate	15-bar pressure plate	Salt solution
100K0S	Yes	No	Yes	Yes	Yes
90K10S	Yes	No	Yes	Yes	Yes
80K20S	Yes	No	Yes	Yes	Yes
50K50S	Yes	No	Yes	Yes	Yes
20K80S	No	Yes	Yes	Yes	Yes
10K90S	No	Yes	Yes	Yes	No

Table 4.8, Table 4.9, Table 4.10 and show the applied suction for SWCC tests conducted in the Tempe cell, Automated Tempe cell and pressure plate tests.

Table 4.8-Applied suction for Tempe cell test

Test	Applied Suction (kPa)									
Tempe cell	10	20	30	40	50	60	70	80	90	

Table 4.9-Applied suction for Automated Tempe cell test

Test	Applied Suction (kPa)									
Automated Tempe cell	0.5	0.8	1	1.5	2	2.5	3	3.5	4	
	4.5	5	6	7	8	9	10	12	15	
	20	25	30	40	50	60	70	80	90	

Table 4.10-Applied suction for pressure plate tests

Test-Continuous	Applied Suction (kPa)									
5-bar pressure plate	100	150	200	250	300	350	400	450	-	
15-bar pressure plate	500	600	700	800	900	1000	1100	1200	1400	

4.8.4 Permeability test

Saturated and unsaturated permeability tests were conducted for all of the selected soil mixtures in this study. In the following sections, the test method and equipment used in the saturated and unsaturated permeability tests are described.

4.8.4.1 Saturated permeability tests

Saturated permeability tests were conducted for all of the selected soil mixtures in this study. Table 4.11 shows the test methods used for each soil mixture.

Table 4.11-Saturated permeability tests conducted in the study

Soil mixture	Constant-head method	
	Flexible wall-triaxial permeameter	Rigid wall permeameter
100K0S	Yes	No
90K10S	Yes	No
80K20S	Yes	No
50K50S	Yes	No
20K80S	Yes	No
10K90S	Yes	No
0K100S	No	Yes

The saturated permeability tests were conducted using the constant-head method. As can be seen from the table, constant-head tests in the flexible wall permeameter were conducted for all soil mixtures. A constant-head test in the rigid wall permeameter was conducted for the 0K100S soil mixture. These tests are described in detail in the following sections.

Flexible wall permeability test

Constant-head permeability tests were conducted in the flexible-wall triaxial apparatus. The test procedure was performed according to D5084 (ASTM, 2003), as described in Section 2.4.4.1. The schematic layout of the triaxial apparatus is shown in Figure 4.12. The soil specimens were prepared according to the procedure explained in Section 4.8.2.2 with a diameter of 50 mm and a height of 30 mm.

Figure 4.12-Saturated triaxial apparatus

The soil specimen was prepared according to Section 4.8.2.2. The soil specimen was partially saturated by placing it on a saturated 5-bar high air-entry ceramic disk inside a pressure plate. The specimen was then set in the triaxial permeameter using a mould, rubber membrane and vacuum gauge. Incremental back pressure and cell pressure were applied to the soil specimen to achieve fully saturated conditions. The initial pore-water pressure of the soil specimen was measured by a pressure transducer connected to a valve at the bottom of the soil specimen (i.e., the valve was used as a flushing line). A cell pressure of 50 kPa (i.e., confining pressure, σ_3) was applied to the soil specimen through a valve connected to a digital pressure and volume controller (DPVC). The cell pressure caused pore-water pressure to build up in the soil specimen. The pore-water pressure was recorded and the pore-water pressure coefficient B value was then calculated. If the B value was less than 0.95 (Head, 1998), back pressure of 40 kPa, u_w , was applied through a valve at the top of the soil specimen connected to another DPVC. The back pressure was applied with a DPVC through a porous stone at the top of the soil specimen. The change in pore-water pressure was observed and, when it reached equilibrium, the cell pressure was increased by 50 kPa. The procedure was repeated until a B value of 0.95 or higher was achieved. The pore-water pressure coefficient, B, was calculated according to Equation 4.6.

$$B = \frac{\Delta u}{\Delta \sigma} \quad \text{Equation 4.6}$$

where:

Δu is the pore-water change after the increment of confining pressure and

$\Delta \sigma$ is the increment of confining pressure.

After saturation, the soil specimen was isotropically consolidated at a small designated net confining pressure ($\sigma_3 - u_w$) of 10 kPa. The confining pressure was kept small enough to simulate the zero net confining pressure. After consolidation, a hydraulic gradient was applied across the soil specimen to create an upward flow through the soil specimen. A pore-water pressure higher than the pore-water pressure of the soil was applied at the bottom of the soil specimen to achieve the desired hydraulic gradient. The inflow and outflow water volumes were measured by the two DVPCs used for application of back pressure and bottom pore-water pressure. The cumulative inflow and outflow water

volume rates, q (m^3/s), were plotted against time. Once the inflow water volume rate was equal to the outflow water volume rate, the steady state condition was achieved and the saturated permeability of the soil was calculated according to Equation 2.25.

Rigid wall permeability test

The constant-head method was conducted in a rigid wall permeameter for 0K100S soil, as shown in Figure 4.13. The test procedure was conducted according to ASTM D2434. The required mass of soil was computed according to its minimum void ratio and compacted into the permeameter cell. The soil was saturated by opening the valve connected to the bottom of the permeameter cell. The water flowed into the cell for a few days at a very slow rate. Once the water reached the top of the soil specimen and flowed out of the cell and the hydrostatic condition was achieved in the connected manometers, the soil specimen was considered to be fully saturated.

After saturation, different hydraulic gradients were applied to the soil, as measured by the manometers connected at three different levels of the permeameter cell. The outflow volume of water was collected in a cylinder for a specific period of time, as shown in Figure 4.13. The saturated permeability was computed according to Equation 2.25.

Figure 4.13-Rigid wall permeameter-constant head method

4.8.4.2 Unsaturated permeability test

4.8.4.2.1 Triaxial permeameter set-up

A triaxial apparatus similar to the triaxial apparatus for unsaturated soil testing described by Fredlund and Rahardjo (1993) was used to conduct the unsaturated permeability tests. The design of the triaxial apparatus used in this study was based on the modifications done by Goh et al. (2010). A schematic diagram of the triaxial permeameter is shown in Figure 4.14.

Figure 4.14-Schematic diagram of triaxial permeameter used in this study

As shown in Figure 4.14 the triaxial permeameter used in this study consists of a transparent triaxial cell, pedestal, ceramic disks, top cap, a diffused air volume indicator (DAVI), two digital pressure and volume controller (DPVC) for pore-water pressure, a digital pressure and volume controller (DPVC) for cell pressure (i.e., confining pressure), pore-air pressure control system, thermal sensor, four pressure transducers, data acquisition system and a personal computer.

Triaxial cell

A small transparent triaxial cell was used in this study. The maximum cell pressure that can be applied is 1700 kPa, which is much higher than the pressure required for this study. Figure 4.15 shows the triaxial cell along with the pressure transducers, top cap and pedestal.

Figure 4.15-Triaxial permeameter used in this study

Pressure transducers

Four pressure transducers were used in the set-up. One pressure transducer was attached to the valve for the cell pressure line (i.e., confining pressure), one pressure transducer was attached to the valve for the top flushing line to measure the pore-water pressure at the top of the soil specimen, one pressure transducer was attached to the valve for the bottom flushing line to measure the pore-water pressure at the bottom of the soil specimen and the last transducer was attached to the valve of the pore-air pressure line. The three pressure transducers were able to measure pressures up to 1000 kPa with an accuracy of ± 0.3 kPa and were well-calibrated within the range of 0-600 kPa with 20 kPa intervals.

Thermal sensor

A thermal sensor (LM35 from RS Components Pte.Ltd.) with an accuracy of $\pm 0.3^\circ \text{C}$ was attached to the outside of triaxial cell to monitor the ambient temperature. The thermal sensor was well-calibrated within the range of 20°C to 30°C .

Pedestal, Top cap and Cell base

As previously mentioned, a triaxial apparatus similar to the modified apparatus by Goh et al. (2010) was used to measure the unsaturated permeability of the soil mixtures in this study. The pedestal and top cap were made of stainless steel and aluminium, respectively. Spiral grooves were constructed inside the water compartments of the pedestal and top cap. Two water pressure outlets were placed at the base of the spiral grooves to apply pore-water pressure into the water compartment and, subsequently, into the soil specimen through the high air-entry ceramic disk. A protruding air pressure outlet was constructed on top of the grooves to apply pore-air pressure, which is passed through the ceramic disk and then distributed into the soil specimen through the porous metal. Therefore, the pedestal and the top cap each have three pressure outlets. Figure 4.16 and Figure 4.17 show the pedestal, top cap and high air-entry ceramic disks. The pressure outlets connect to the triaxial base through six different valves. The valves connecting the pedestal to the triaxial cell base are called the bottom pore-water pressure, bottom flushing line and bottom pore-air pressure valves. The valves connecting the top cap to the triaxial cell base are called the top pore-water pressure, top flushing line and top pore-air pressure valves. There is another water outlet in the cell base for connecting the cell pressure. Therefore, the cell base has seven outlets. Figure 4.18 shows the cell base and its seven valves.

Ceramic disk

Two types of high air-entry ceramic disk namely 1-bar and 5-bar, with a thickness of 7.14 mm were used in this study. The 1-bar ceramic disk was used to measure the unsaturated permeability of the soil mixtures up to a suction of 100 kPa, while the 5-bar ceramic disk was used to measure the unsaturated permeability of the soil mixtures up to a suction of 500 kPa. The ceramic disks were selected based on the applied matric suction and saturated permeability of the disks. The tops of the ceramic disks are grooved, as shown in Figure 4.16. An opening was made at the edge of the base of one of the grooves, as shown in Figure 4.18, to allow the passage of air from the protruding air pressure outlet of the pedestal through the ceramic disk. The opening of the ceramic disk was fitted with the protruding air pressure outlet from the spiral grooves. The grooves of the ceramic disk were filled with porous metal directly connected to the protruding air pressure outlet. The ceramic disk and porous metal provide a uniform distribution of water and air pressure to the soil specimen. Slow-setting epoxy glue was used to seal the ceramic disk to the modified pedestal along its circumference.

Figure 4.16-The top cap and pedestal with spiral grooves

Figure 4.17-The pedestal, ceramic disks and top cap

Figure 4.18-Cell base with seven outlets

Flushing system

The air diffused through the ceramic disk produces air bubbles in the water compartments due to the long duration of the test and high pore-air pressure in the soil specimen. Therefore, the triaxial permeameter was flushed regularly to remove air bubbles. Diffused air bubbles can affect the accuracy of the pore-water pressure and volume change measurements into and out from the soil specimen (Fredlund and Rahardjo, 1993). The flushing system included a Diffused Air Volume Indicator (DAVI) connected to the flushing lines. A pore-water pressure of 30 kPa was applied through the pore-water pressures lines and caused the air bubbles to be flushed out from the flushing lines to the DAVI. After flushing, all of the valves were closed.

Pore-water pressure control system

Two digital pressure and volume controllers (DPVC) manufactured by GDS Instruments Limited of England were used for the application of pore-water pressure to the soil specimen through the high air-entry ceramic disk. Another digital pressure and volume controller (DPVC) was used to apply cell pressure to the soil specimen during the test.

The volume change of the soil specimen was measured by the DPVC that supplied the cell pressure. The inflow and outflow water volume rates were also measured using the respective DPVCs that provided the pore-water pressure. In the original triaxial apparatus, the DPVCs were connected to the system via semi-rigid polyamide tubes with an 8 mm inner diameter, as shown in Figure 4.19. However, using this type of tubing resulted in leakage problems. Since the measurement of unsaturated permeability requires an accurate water volume change, the problem was solved by replacing the tubing system with steel tubes with an inner diameter of 1.75 mm, as shown in Figure 4.20.

Figure 4.19-Semi-rigid tubing system

Figure 4.20-Steel tubing system

Pore-air pressure control system

In order to create matric suction in the soil specimen, air pressure was applied to the soil specimen through the porous metal. The porous metal has a low AEV, which allows air to pass through and fill up the entire pore structure of the porous metal when its AEV is

exceeded. The porous metal is connected to the air pressure lines through the top cap and pedestal, and so is subsequently connected to the pore-air pressure control line and the air compressor. Therefore, the continuity of air voids in the soil specimen, porous metal, air pressure line and pore-air control line can be formed for controlling the pore-air pressure in the soil specimen. Filter paper was placed between the soil specimen and porous metal in order to prevent fine particles from being trapped in the pores of the porous metal.

Data acquisition system

An 8-channel data logger connected to a personal computer was used to computerize and automate the measurements of pore-water pressure, pore-air pressure, pore-water volume changes, total volume changes and temperature changes. The pressure transducers and thermal sensors were connected to the data logger, which was connected to a personal computer. The three DPVCs were directly connected to the personal computer for data collection and monitoring. The readings were taken in small time intervals using a Triax 4.0 data acquisition program (Toll, 1999).

Plumbing layout

The plumbing layout of the triaxial permeameter can be seen in Figure 4.21.

Figure 4.21- Schematic plumbing layout for the modified triaxial apparatus for unsaturated permeability

The layout includes the pore-air pressure lines, the pore-water pressure lines (back pressure line), the confining pressure line (cell pressure line) and the flushing lines. The electrical lines include the connections from the pressure transducer, thermal sensor and DPVCs to the data logger. The data logger collects all of the measurement signals and then sends them to the personal computer.

4.8.4.2.2 *Testing procedure*

Saturation

Before the start of the tests, saturation of the high air-entry ceramic disks was performed. The triaxial cell was filled with distilled de-aired water and a cell pressure of 400 kPa was applied. All the pressure lines were closed except the top and bottom flushing valves which connected to the DAVI. The distilled de-aired water flowed out from the cell to the DAVI through the high air-entry ceramic disks for 12 hours. The water compartments were flushed by applying 30 kPa of water pressure through the pore-water pressure valves and the trapped air bubbles were flushed out and collected in the DAVI. Flushing was performed every two hours to assure there were no air bubbles in the water compartments.

After the saturation of the ceramic disc was achieved, the saturated permeability was measured. To measure the saturated permeability of the ceramic disc, a cell pressure of 600 kPa was applied through the DVPC connected to the cell pressure line and the flushing line was opened. The outflow rate of water was measured by the DVPC connected to the cell pressure line. A graph of the outflow volume rate, q , was plotted against time. Once the outflow rate was constant, the saturated permeability of the ceramic disk was computed according to Equation 2.25.

The soil specimen was prepared according to Section 4.8.2.2 with a diameter of 50 mm and height of 30 mm and partially saturated by placing it on a saturated 5-bar high air-entry ceramic disk inside a pressure plate. The specimen was set in the triaxial permeameter using a mould, rubber membrane and vacuum gauge. The soil specimen was then saturated by applying incremental back pressure and cell pressure. The initial pore-water pressure of the soil specimen was measured by a pressure transducer connected to the bottom back pressure line. A cell pressure of 50 kPa (i.e., confining pressure, σ_3) was applied through the cell pressure line, which was connected to the DPVC. The cell

pressure caused pore-water pressure to build up in the specimen. The pore-water pressure was recorded and the pore-water pressure coefficient B value was then calculated according to Equation 4.6. If the B value was less than 0.95 (Head, 1998), a back pressure of 40 kPa, u_w , was applied through the back pressure lines connected to the two DPVCs. The back pressure was applied through the porous metal at the top and bottom of the soil specimen. The porous metal has a higher permeability than the high air-entry ceramic disk which shortens the saturation time. The change in pore-water pressure was observed and, when it reached equilibrium, the cell pressure was increased by 50 kPa. The procedure was repeated until a B value of 0.95 or higher was achieved. The net confining pressure ($\sigma_3 - u_w$) was kept small enough to simulate zero net confining pressure (i.e., 10 kPa) throughout the test.

Matric suction application (Drying)

The second step in the unsaturated permeability test is to apply matric suction to the soil specimen. In order to apply air pressure, the air pressure lines were connected to the air pressure supply and the pore-water pressure lines were connected to the DVPCs. The air pressure was kept constant during the test, while the pore-water pressure was decreased to the designated value. The pore-air pressure and pore-water pressures were measured by pressure transducers connected to their respective lines. The cell pressure was also kept constant in order to keep the net confining pressure constant. The water volume change and total volume change were measured by their respective DVPCs. The equilibrium condition was achieved when changes in water volume ceased. For subsequent increases in the matric suction, the pore-water pressure was decreased to the desired values.

Unsaturated permeability measurement

The last step in the unsaturated permeability test is to measure the unsaturated permeability. In order to create the designated hydraulic gradient to perform the unsaturated permeability test, two DVPCs were used to apply and control the pore-water pressure at the top and bottom of the soil specimen. Once the soil specimen reached the equilibrium condition under the designated matric suction, a hydraulic gradient was created by decreasing the pore-water pressure at the top of the soil specimen via the connected DVPC. As a result, an upward flow of water was created through the soil specimen. The inflow and outflow water volume rates from the top and bottom of the soil

specimen were continuously measured by the two DVPCs. A graph of the volume rate of flow, q (m^3/s), against time was then plotted from the beginning of the test for the top water outflow and bottom water inflow. When the inflow and outflow rates were approximately the same, a steady-state condition was achieved. The test was stopped after the water flowed at a constant rate for a given period of time.

The unsaturated permeability can be computed according to Equation 2.25. It should be noted that the impedance of the air-entry disks was taken into account. The soil and high air-entry disks were considered as a three-layered system (Samingan, 2001) (i.e., bottom disk+soil+top disk), as shown in Figure 4.22.

Figure 4.22-Three-layered soil system(from Samingan, 2001)

The average velocities of all three layers are equal due to the continuity requirement, which results in the following equation:

$$v_T = v_b = v_s = v_t \quad \text{Equation 4.7}$$

where:

v_T is the total average velocity

v_b is the average velocity of water through the bottom disk

v_s is the average velocity of water through the soil

v_t is the average velocity of water through the top disk

From Darcy's law and Equation 4.7, the following equation follows:

$$v_T = k_T i_T = k_b i_b = k_w i_s = k_t i_t \quad \text{Equation 4.8}$$

where:

k_T is the total unsaturated permeability

i_T is the total hydraulic gradient

k_b is the permeability of the bottom disk

i_b is the hydraulic gradient across the bottom disk

k_w is the permeability of the soil

i_s is the hydraulic gradient of the soil

k_t is the permeability of the top disk

i_t is the hydraulic gradient of the top disk

The hydraulic gradient of each layer can be computed from Equation 4.8 as follows:

$$i_t = \frac{k_T}{k_t} i_T; i_s = \frac{k_T}{k_w} i_T; i_b = \frac{k_T}{k_b} i_T \quad \text{Equation 4.9}$$

The total head loss is equal to the sum of the head loss over each layer according to the following equation:

$$h_w = h_{wb} + h_{ws} + h_{wt} = i_T L_T \quad \text{Equation 4.10}$$

where:

h_w is the total head loss

h_{wb} is the head loss through the bottom disk

h_{ws} is the head loss through the soil

h_{wt} is the head loss through the disk

L_T is the total length of the three layers

If Equation 4.9 is substituted into Equation 4.10, the following equation is obtained:

$$i(L_b + L_s + L_t) = i_b L_b + i_s L_s + i_t L_t = ki \left(\frac{L_t}{k_t} + \frac{L_s}{k_w} + \frac{L_b}{k_b} \right) \quad \text{Equation 4.11}$$

After rearranging Equation 4.11, the unsaturated permeability of the soil, k_w , can be computed according to Equation 4.12:

$$k_w = \frac{L_s}{\left(\frac{L}{k} - \left(\frac{L_t}{k_t} + \frac{L_b}{k_b} \right) \right)} \quad \text{Equation 4.12}$$

where:

L_s is the length of the soil specimen

L_t is the length of the top disk

L_b is the length of the bottom disk

The unsaturated permeability of the soil was computed according to Equation 4.12.

4.8.4.3 Testing program for permeability tests

A series of unsaturated permeability tests was conducted on selected soil mixtures in this study. As mentioned in Section 4.8.4.2.1, two different ceramic disk types were used in this study. Table 4.12 shows the designated matric suctions applied to the soil specimen to measure unsaturated permeability in the triaxial permeameter with 1-bar ceramic disk. The designed matric suctions applied to the soil specimen to measure unsaturated permeability in the triaxial permeameter with 5-bar ceramic disk is shown in Table 4.13.

Table 4.12-Modified triaxial permeameter used for unsaturated permeability measurement

Soil mixture	Triaxial permeameter (1-bar ceramic disk)			
	Matric suction			Net confining pressure
100K0S	30	50	90	10
90K10S	30	50	90	10
80K20S	30	50	90	10
50K50S	30	50	90	10
20K80S	30	50	90	10
10K90S	30	50	90	10

Table 4.13-Testing program for unsaturated permeability tests

Soil mixture	Triaxial permeameter (5-bar ceramic disk)			
	Matric suction			Net confining pressure
100K0S	200	300	400	10
90K10S	200	300	400	10
80K20S	200	300	400	10
50K50S	200	300	400	10
20K80S	200	300	400	10
10K90S	200	300	-	10

Chapter 5 Presentation of results

5.1 Introduction

This chapter presents the results of the study. First, the results of the evaluation of identified parameters affecting the estimation of unsaturated permeability of soils (i.e., k_r equations, SWCC equations and measured SWCC data range) are presented. The results of the proposed procedure for estimating SWCC data points in a high suction range are presented next. Lastly, the results of the experimental program conducted in this study (i.e., basic soil properties tests, SWCC tests and unsaturated permeability tests) are presented.

5.2 Estimation and evaluation of unsaturated permeability of soil by a matrix of unsaturated permeability models

As described in Section 4.2, a soil database consisting of 20 soils with both measured SWCC data and $k(\psi)$ data was selected to evaluate the effect of selected soil-water characteristic curve equations and relative permeability equations. The SWCC equations were used to best-fit the measured SWCC data according to the procedure described in Section 4.2. The unsaturated permeability of the soils was estimated using the proposed matrix of unsaturated permeability estimation models in accordance with the procedure in Section 4.2. The lower limit of unsaturated permeability (vapor permeability) was also computed according to the procedure explained in Section 4.2. Since four different SWCC equations were used in this study, four vapor permeability curves were obtained for each of the soils in the database. The results of the best-fit SWCCs and the estimated unsaturated permeability curves (i.e., shown as relative permeability) of Guelph loam and Mine tailings soils are presented in Figure 5.1 to Figure 5.4. The results for the rest of the soils from the database are presented in Appendix B.

Figure 5.1-Best-fit soil-water characteristic curves-Guelph loam (S1)

Figure 5.2-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Guelph loam (S1)

Figure 5.3-Best-fit soil-water characteristic curves-Mine tailings (S12)

Figure 5.4-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Mine tailings (S12)

Figure 5.1 shows the best-fit SWCCs for Guelph loam soil using the SWCC equations of Fredlund and Xing (1994), Fredlund and Xing (1994) with $C(\psi)=1$ as recommended by Leong and Rahardjo (1997), van Genuchten (1980) and van Genuchten (1980) with $m=1-1/n$. Figure 5.2 shows the estimated unsaturated permeability curves (shown as relative permeability) using the 12 unsaturated permeability estimation models in Table 3.2, namely FCM, FCM ($C(\psi)=1$), VCM, VCM ($m=1-1/n$), FMM, FMM ($C(\psi)=1$), VMM, VMM ($m=1-1/n$), FBM, FBM ($C(\psi)=1$), VBM and VBM ($m=1-1/n$). The estimated unsaturated permeability curves by the two existing models, F&X-1994 and VG-1980, are also shown in Figure 5.2. The relative vapor permeability curves for each of the best-fit SWCCs are also shown in Figure 5.2.

Figure 5.3 and Figure 5.4 show the best-fit SWCCs and the estimated unsaturated permeability curves for Mine tailings soil, respectively.

As shown in Figure 5.1, the four best-fit SWCCs for Guelph loam soil were similar until the last measured data point after which they started to vary significantly. In contrast, the four best-fit SWCCs for Mine tailings soil were more or less similar in shape and did not vary significantly from each other (see Figure 5.3).

Figure 5.2 and Figure 5.4 show that the estimated unsaturated permeability curves from all 14 models varied for each soil. The lower limit of permeability obtained from four different best-fit SWCCs were approximately the same for each of the soils shown in Figure 5.2 and Figure 5.4. The values of the relative vapor permeability of Guelph loam and Mine tailings soil were around 5×10^{-9} (m/s) and 2×10^{-8} (m/s), respectively. The absolute value of vapor permeability for all the soils in the database was approximately the same as shown in Figure 5.5. It changed from as low as 2×10^{-15} (m/s) to as high as 3×10^{-14} (m/s) which was about one order of magnitude.

Figure 5.5-Lower limit of unsaturated permeability obtained for all the soils in database along with their saturated permeabilities

The root mean square error (RMSE) was used as a statistical measure to evaluate the fit of the estimation models to the measured data for the soil database as presented in Table 4.1. The RMSE was computed according to Equation 4.2 for all soils and the result are shown in Table 5.1. As shown in the table, each model resulted in a different RMSE value for each soil. However, the differences between some models were negligible. For instance, the FMM and FBM models resulted in 0.723 and 0.711 RMSE values, respectively, for Yolo light clay soil. On the other hand, the differences between some models were significant, such as the VG-1980 and FCM models resulting in 4.349 and 2.842 RMSE values for Live Oak soil, respectively.

Table 5.1-Computed RMSE values and their averages and standard deviations for the entire soil database

Model Soil Name	F&X- 1994	VG- 1980	FCM	FMM	FBM	FCM C(ψ)=1	FMM C(ψ)=1	FBM C(ψ)=1	VCM m=1-1/n	VMM m=1-1/n	VBM m=1-1/n	VCM	VMM	VBM	Ave.	SD.
Guelph loam	0.234	0.382	0.234	0.218	0.429	0.211	0.204	0.362	0.211	0.258	0.472	0.228	0.204	0.315	0.283	0.092
Beit Netofa clay	0.308	0.321	0.303	0.392	1.399	0.427	0.551	1.545	0.562	0.340	0.848	0.394	0.516	1.515	0.673	0.464
Yolo light clay	0.628	1.324	0.641	0.723	0.711	0.747	0.861	0.780	0.879	1.049	0.986	0.628	0.684	0.496	0.796	0.213
Touchet silt loam	0.170	0.406	0.178	0.123	0.594	0.182	0.178	0.360	0.252	0.255	0.277	0.166	0.154	0.353	0.260	0.130
Columbia sandy loam	0.084	1.185	0.072	0.096	0.256	0.070	0.080	0.269	0.796	0.824	0.154	1.513	1.546	0.496	0.532	0.547
Hygiene Sandstone	0.280	0.144	0.285	0.282	0.571	0.285	0.299	0.449	0.091	0.085	0.352	0.208	0.202	0.416	0.282	0.136
Superstition Sand	0.142	0.444	0.136	0.148	0.399	0.140	0.162	0.536	0.137	0.176	0.405	0.078	0.098	0.447	0.246	0.159
Silt loam	0.336	0.152	0.334	0.186	0.635	0.336	0.214	0.254	0.336	0.210	0.227	0.309	0.140	0.482	0.297	0.134
Wenatchee Clay	2.204	3.524	2.753	2.929	3.500	2.992	3.154	3.634	2.591	2.835	3.169	2.981	3.143	3.630	3.074	0.413
Live Oak	2.407	4.349	2.842	2.989	3.232	2.969	3.137	3.366	3.167	2.839	3.334	2.992	3.152	3.381	3.154	0.430
CDT	0.686	1.332	1.000	1.203	1.739	0.889	1.106	1.710	0.813	1.013	1.354	0.892	1.106	1.712	1.182	0.345
Mine Tailing	0.415	0.144	0.413	0.263	0.365	0.253	0.242	0.368	0.204	0.126	0.486	0.286	0.242	0.272	0.291	0.105
UP-1	1.045	0.176	1.637	1.747	2.762	1.596	1.712	2.734	0.609	0.287	0.934	1.552	1.677	2.707	1.512	0.845
UP-2	1.084	0.425	1.085	0.871	0.321	1.070	0.786	0.198	0.937	0.623	0.184	1.147	1.060	1.183	0.784	0.365
UP-3	1.348	0.367	1.766	1.970	3.308	1.786	1.191	3.315	0.133	0.297	1.384	1.755	1.962	3.296	1.706	1.059
UP-4	1.067	0.687	0.427	0.343	0.197	0.479	0.391	0.195	0.640	0.491	0.299	0.467	0.379	0.197	0.447	0.234
Weld silty clay	0.134	0.161	0.153	0.153	0.314	0.233	0.213	0.369	0.272	0.225	0.349	0.272	0.271	0.413	0.252	0.087
Fine sand	1.175	0.595	1.182	1.161	1.383	1.139	1.132	1.584	0.889	0.889	1.483	1.182	1.154	1.612	1.183	0.277
Volcanic sand	0.694	0.729	0.700	0.729	1.227	0.664	0.676	1.198	0.202	0.249	0.836	0.664	0.652	1.233	0.747	0.311
GE No.2 Sand	0.208	0.294	0.188	0.129	0.393	0.187	0.263	0.330	0.169	0.221	0.201	0.173	0.260	0.322	0.238	0.074
Average	0.732	0.857	0.816	0.833	1.187	0.833	0.828	1.178	0.694	0.665	0.887	0.894	0.930	1.224		
Standard Deviation	0.669	1.125	0.840	0.911	1.124	0.880	0.907	1.180	0.807	0.800	0.913	0.881	0.942	1.154		

As shown in the table, the average and standard deviation of the RMSE values were also computed for each soil and each estimation model. As shown, some of the soils had higher average RMSE values than other soils. This could be due to differences in the texture of the soils. For instance, the average RMSE value was 0.246 for Superstition sand (i.e., sandy soil) compared to 1.706 for UP-3 soil (i.e., clayey soil).

5.3 Evaluation of effect of k_r equation on estimation of unsaturated permeability

As described in Section 4.4, all of the unsaturated permeability estimation models in Table 3.2 were categorized into SWCC equation category to evaluate the effect of the

k_r equation. This resulted in four groups: Fredlund and Xing (1994), Fredlund and Xing (1994) with $C(\psi) = 1$, van Genuchten (1980) and van Genuchten (1980) with $m=1-1/n$ based models. This means that, for each group in this category, there are three estimation models based on the three relative permeability equations. For instance, in the van Genuchten (1980) with $m=1-1/n$ group, there are three estimation models, namely the VCM ($m=1-1/n$), VMM ($m=1-1/n$) and VBM ($m=1-1/n$) models, with van Genuchten (1980) as their SWCC equation.

The results for Guelph loam soil (i.e., S1) are shown in Figure 5.6 to Figure 5.9. Figure 5.6 shows the estimated unsaturated permeability curves for Fredlund and Xing (1994) based models (i.e., FMM, FCM and FBM models). As shown in the figure, the estimated unsaturated permeability curves had more or less the same shape and the variation between models was almost negligible. However, the estimated unsaturated permeability curve using the FBM model was slightly different at suction values below the air-entry value of the soil.

Figure 5.6-Estimated unsaturated permeabilities for Fredlund and Xing (1994) based models-Guelph loam (S1)

Figure 5.7 shows the estimated unsaturated permeability curves for Fredlund and Xing (1994) with $C(\psi)=1$ based models (i.e., FMM ($C(\psi)=1$), FCM ($C(\psi)=1$) and FBM ($C(\psi)=1$) models). As shown in the figure, the same behavior was observed for Fredlund and Xing (1994) with $C(\psi)=1$ based models. The estimated unsaturated permeability curves had more or less the same shape and the variation between models was almost negligible.

Figure 5.8 shows the estimated unsaturated permeability curves for van Genuchten (1980) based models (i.e., VMM, VCM and VBM models). As shown in the figure, the same behavior was observed for van Genuchten (1980) based models. The estimated unsaturated permeability curves had approximately the same shape and the variation between models was almost negligible. The FBM model had the same shape as the other two models below the air-entry value of the soil and had a slight variation above 100 kPa suction value.

Figure 5.7-Estimated unsaturated permeabilities for Fredlund and Xing (1994) with $C(\psi)=1$ based models-Guelph loam (S1)

Figure 5.8-Estimated unsaturated permeabilities for van Genuchten (1980) based models- Guelph loam (S1)

Figure 5.9-Estimated unsaturated permeabilities for van Genuchten (1980) with $m=1-1/n$ based models-Guelph loam (S1)

Figure 5.9 shows the estimated unsaturated permeability curves for van Genuchten (1980) with $m=1-1/n$ based models (i.e., VMM ($m=1-1/n$), VCM ($m=1-1/n$) and VBM ($m=1-1/n$) models). As shown in the figure, the estimated unsaturated permeability curves had more or less the same shape and the variation between models was almost negligible, similar to the other groups.

The characteristics shown in Figure 5.6 to Figure 5.9 were observed for all soils in the database, as shown in Appendix B.

5.4 Evaluation of effect of SWCC equation on estimation of unsaturated permeability

As described in Section 4.5, all of the unsaturated permeability estimation models in Table 3.2 were categorized into k_r equation category in order to study the effect of SWCC equations. This resulted in three groups: Childs Collis-George, Burdine and Mualem based models. This means that, for each group in this category, there are four estimation models based on the four SWCC equations. For instance, the Mualem group contains four estimation models, namely the FMM, FMM ($C(\psi)=1$), VMM and VMM ($m=1-1/n$) models.

The results for Guelph loam soil (i.e., S1) are shown in Figure 5.10 to Figure 5.12. Figure 5.10 shows the estimated unsaturated permeability curves for Childs Collis-George based models (i.e., FCM, FCM ($C(\psi)=1$), VCM and VCM ($m=1-1/n$) models).

Figure 5.11 shows the estimated unsaturated permeability curves for Mualem based models (i.e., FMM, FMM ($C(\psi)=1$), VMM and VMM ($m=1-1/n$) models).

Figure 5.12 shows the estimated unsaturated permeability curves for Burdine based models (i.e., FBM, FBM ($C(\psi)=1$), VBM and VBM ($m=1-1/n$) models). As shown in Figure 5.10 and Figure 5.11, the estimated unsaturated permeability curves for each group had more or less the same shape over the low suction range and started to vary as the suction increased. However, the estimated unsaturated permeability curves for Burdine based models, as shown in Figure 5.12, varied even in the low suction range, although the variation was marginal.

Figure 5.10-Estimated unsaturated permeabilities for Childs and Collis-George based models-Guelph loam (S1)

Figure 5.11-Estimated unsaturated permeabilities for Mualem based models-Guelph loam (S1)

Figure 5.12-Estimated unsaturated permeabilities for Burdine based models-Guelph loam (S1)

These patterns were observed for all soils in the database, as shown in Appendix B.

5.5 Evaluation of effect of measured SWCC data ranges on estimation of unsaturated permeability

The best-fit SWCCs were obtained according to the procedure explained in Section 4.6. As mentioned in that section, for each soil in the database, there were 16 (or 12) SWCCs. The unsaturated permeabilities were estimated according to the procedure also explained in Section 4.6. The results for UP-3 soil are shown in this section.

Figure 5.13, Figure 5.14, Figure 5.15 and Figure 5.16 show the SWCCs obtained by Fredlund and Xing (1994), Fredlund and Xing (1994) with $C(\psi)=1$, van Genuchten (1980) and van Genuchten (1980) with $m=1-1/n$ equations for different ranges of measured SWCC data, respectively.

Figure 5.13-Soil-water characteristic curves of UP-3-different measured data ranges-
Fredlund and Xing (1994)

Figure 5.14-Soil-water characteristic curves of UP-3-different measured data ranges-
Fredlund and Xing (1994) with $C(\psi)=1$

Figure 5.15-Soil-water characteristic curves of UP-3-different measured data ranges-van Genuchten (1980)

Figure 5.16-Soil-water characteristic curves of UP-3-different measured data ranges-van Genuchten (1980) with $m=1-1/n$

As shown in Figure 5.13, SWCCs resulting from DR1 and DR2 had similar shapes. The SWCCs from DR3 and DR4 had also more or less the same shape until the last measured data point at suction values of 1000 kPa and 10000 kPa, respectively. It seems that the shape of all four SWCCs started to vary after the last measured data point. However, the volumetric water content for all four best-fit SWCCs was equal to zero at a suction value of 106 kPa.

As shown in Figure 5.14, SWCCs obtained using Fredlund and Xing (1994) with $C(\psi)=1$ had more or less the same shape below a suction value of 100 kPa. The resulting SWCCs started to vary above the last measured data points at 100, 500, 1000 and 10000 kPa. This variation can clearly be seen in the volumetric water content at a suction value of 10^6 kPa, which was 0.1904, 0.1379, 0.0292 and 5.905×10^{-20} for SWCCs resulting from DR1, DR2, DR3 and DR4, respectively. The same pattern can be seen for SWCCs resulting from van Genuchten (1980), as shown in Figure 5.15.

As can be seen in Figure 5.16, SWCCs obtained by van Genuchten (1980) with $m=1-1/n$ using DR1, DR2 and DR3 had more or less the same shape until the last measured data point at 100, 500 and 1000 kPa, respectively. However, the SWCC resulting from DR3 did not perfectly fit the measured SWCC data at higher suctions. The poor fit of this equation to the measured data can clearly be seen in the resulting SWCC using DR4. This could be due to the lower flexibility of the equation (Leong and Rahardjo, 1997).

It seems that the different ranges of SWCC measured data could result in best-fit SWCCs that were significantly different from each other even when the same SWCC equation was used.

The best-fit SWCCs for UP-3 soil using the same set of measured SWCC data but different SWCC equations are shown in Figure 5.17, Figure 5.18, Figure 5.19 and Figure 5.20.

Figure 5.17-Soil-water characteristic curves of UP-3-different best-fit equations for DR1

Figure 5.18-Soil-water characteristic curves of UP-3-different best-fit equations for DR2

Figure 5.19-Soil-water characteristic curves of UP-3-different best-fit equations for DR3

Figure 5.20-Soil-water characteristic curves of UP-3-different best-fit equations for DR4

As shown in Figure 5.17, the four best-fit SWCCs obtained using the four SWCC equations for DR1 had the same shape until a suction value of 100 kPa, beyond which they started to vary. As shown in Figure 5.18, the best-fit SWCCs for DR2 had the same

characteristics as DR1; however, the variation between SWCCs was less than that for DR1. As shown in Figure 5.19, the best-fit SWCCs for DR3 showed the same behavior; however, the best-fit SWCCs for van Genuchten (1980) with $m=1-1/n$ did not perfectly fit the measured data at a suction value of 1500 kPa and started to have a different shape beyond suction values of 500 kPa. As shown in Figure 5.20, the best-fit SWCCs for DR4 had more or less the same shape over the full suction range (i.e., $0.1-10^6$ kPa), with the exception of the best-fit SWCC using the van Genuchten (1980) with $m=1-1/n$ equation.

It seems that the best-fit SWCCs had the same shape within the suction range where measured SWCC data were available, even when different SWCC best-fit equations were used. However, van Genuchten (1980) with $m=1-1/n$ did not show this characteristic, which could be due to the lower flexibility of the equation.

By comparing the best-fit SWCCs, as shown in Figure 5.13 to Figure 5.20, it seems that the effect of the measured SWCC data range was more significant on the best-fit soil-water characteristic curves than the effect of the selected SWCC equation itself. The same pattern was observed for all other soils in the database, as shown in Appendix C.

The estimated unsaturated permeability curves for Beit Netofa clay (BNC) soil obtained using all 12 models shown in Table 3.2 are presented in Figure 5.21 to Figure 5.32.

Figure 5.21-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by FCM

Figure 5.22-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by FMM

Figure 5.23-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by FBM

Figure 5.24-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by FCM $C(\psi)=1$

Figure 5.25-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by FMM $C(\psi)=1$

Figure 5.26-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by FBM $C(\psi)=1$

Figure 5.27-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by VCM

Figure 5.28-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by VMM

Figure 5.29-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by VBM

Figure 5.30-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by VCM $m=1-1/n$

Figure 5.31-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by VMM $m=1-1/n$

Figure 5.32-Estimated unsaturated permeability of BNC soil for DR1, DR2 and DR3 by VBM $m=1-1/n$

As can be seen from the figures, there are three different estimated unsaturated permeability curves for each estimation model. This means that different sets of measured SWCC data (i.e., DR1, DR2 and DR3) resulted in different relative permeability curves.

The root mean square error (RMSE) was computed according to Equation 4.2 for all estimated unsaturated permeability curves for all soils shown in Table 4.2. The resulting RMSEs of DR2 and DR3 are shown in Table 5.2.

Table 5.2-Computed RMSE for DR2 and DR3 of all soils

	UP-1	UP-2	UP-3	UP-4	BNC	WSC	LRC	
F&X-1994	1.607	2.476	1.745	1.984	0.702	10.404	9.547	DR2
VG-1980	2.053	1.479	3.161	2.015	1.540	14.109	17.070	
FCM	2.745	2.476	1.819	0.748	0.783	11.912	10.947	
FMM	3.121	2.027	2.075	0.595	1.324	13.010	14.720	
FBM	5.644	1.147	3.015	0.412	4.835	9.693	12.496	
FCM C(ψ)=1	2.311	2.426	2.331	0.916	2.755	13.053	13.371	
FMM C(ψ)=1	2.816	2.011	2.624	0.748	2.215	14.066	17.920	
FBM C(ψ)=1	5.292	1.595	3.393	0.412	0.729	16.371	13.871	
VCM m=1-1/n	0.575	2.244	2.173	0.916	2.593	10.318	12.671	
VMM m=1-1/n	1.232	1.795	2.441	0.702	1.891	11.567	13.421	
VBM m=1-1/n	3.496	1.579	3.235	0.473	0.756	13.721	13.246	
VCM	2.311	2.908	1.929	0.855	3.053	12.386	39.413	
VMM	3.039	3.124	2.161	0.687	3.026	13.441	40.763	
VBM	5.550	1.413	2.759	0.427	3.080	15.983	26.392	
	UP-1	UP-2	UP-3	UP-4	BNC	WSC	LRC	
F&X-1994	1.959	1.978	2.075	2.137	0.837	10.124	9.122	DR3
VG-1980	1.654	1.462	2.661	1.374	1.432	12.881	15.920	
FCM	2.487	1.828	2.624	0.855	0.810	11.890	11.197	
FMM	3.074	1.047	2.734	0.687	1.243	13.010	11.771	
FBM	5.092	1.246	3.784	0.397	4.781	15.617	13.021	
FCM C(ψ)=1	0.915	2.144	2.502	0.962	1.594	12.687	11.846	
FMM C(ψ)=1	1.607	1.861	2.661	0.763	2.053	13.614	12.421	
FBM C(ψ)=1	3.567	2.094	3.674	0.397	5.781	16.457	13.421	
VCM m=1-1/n	0.681	2.044	1.770	1.282	2.107	9.284	11.122	
VMM m=1-1/n	1.197	1.662	2.063	0.977	1.270	10.598	11.871	
VBM m=1-1/n	2.781	1.712	2.991	0.595	3.188	12.903	12.496	
VCM	0.681	2.044	2.600	1.282	2.107	12.666	11.122	
VMM	3.074	1.579	2.722	0.763	1.918	13.592	12.221	
VBM	5.092	1.064	3.760	0.397	5.673	16.478	13.296	

As shown in the table, when the measured SWCC data changed from DR2 to DR3, the minimum value of RMSE was obtained from a different unsaturated permeability estimation model. For instance, when DR2 was used to estimate the unsaturated permeability of BNC soil, F&X-1994 resulted in the lowest RMSE value and, therefore, gave the best prediction. On the other hand, when DR3 was used, FCM resulted in the lowest RMSE value and gave the best prediction. This characteristic was generally observed for all soils. It seems that for every measured SWCC data set, a different estimation model resulted in the minimum or maximum RMSE value and did not necessarily follow the same pattern. This behavior shows that SWCC measurement ranges can significantly influence conclusions about which estimation model gives the

best prediction and can greatly affect unsaturated permeability estimations. The computed RMSE values resulting from DR1 and DR4 are shown in Appendix C.

5.6 Evaluation of prediction of SWCC data beyond 100 kPa

As presented in Section 4.6, the measured SWCC data range can affect estimations of unsaturated permeability. Therefore, a procedure was proposed in Section 3.4 to estimate SWCC data points at higher suction ranges using grain sizedistribution data and measured SWCC data at a suction value of 100 kPa. The proposed method was evaluated for the selected soil database shown in Table 4.3 according to the procedure described in Section 4.7 and Section 3.4.

The coefficient of determination, R^2 , for the first estimate of SWCC data points using grain sizedistribution data (i.e., step 2 in Section 3.4) for all soils in the selected database was computed according to Equation 4.4. The measured volumetric water content versus first estimated volumetric water content data are shown in Figure 5.33. As shown in Figure 5.33, the computed R^2 was 0.665 for the first estimate of SWCC data points using grain size distribution data. The first estimate of SWCC data points was scaled by the scaled factor, S_f , as computed according to step 3 in the proposed procedure using the measured SWCC around 100 kPa (see Section 3.4).

The results of the measured SWCC data points versus the scaled estimated SWCC data points are shown in Figure 5.34. As shown in Figure 5.34, the coefficient of determination, R^2 , increased from 0.665 to 0.906. The 95% confidence interval of the observed errors, $(\theta_{wi} - \hat{\theta}_{wi})$, between the measured SWCC data points and scaled estimated data points was computed according to Equation 4.5 and is shown in Figure 5.34 by dashed lines. The 95% confidence interval of the observed error was found to be (-0.046, 0.083), as shown in Figure 5.34.

Figure 5.33-Estimated versus measured water content for selected soil database

Figure 5.34-Scaled estimated versus measured water content for selected soil database with 95% confidence intervals

The grain sizedistribution for soils 10982, 11537 and 18748 are shown in Figure 5.35, Figure 5.37 and Figure 5.39, respectively. The measured SWCC data points along with the scaled estimated SWCC data points for these three soils are shown in Figure 5.36, Figure 5.38 and Figure 5.40, respectively.

Figure 5.35-Grain sizedistribution for soil number 10892

Figure 5.36-Measured and estimated soil-water characteristic curve for soil number 10982

Figure 5.37-Grain sizedistribution for soil number 11537

Figure 5.38-Measured and estimated soil-water characteristic curve for soil number 11537

Figure 5.39-Grain sizedistribution for soil number 18748

Figure 5.40-Measured and estimated soil-water characteristic curve for soil number 18748

The proposed method was also evaluated for the soil mixtures (see Table 4.4) used in the experimental program of this study according to the procedure described in Section 3.4.

The scaled estimated water content versus measured water content is shown in Figure 5.41 for the soil mixtures used in this study. As shown in the figure, the coefficient of determination, R^2 , increased from 0.906 to 0.913 and the estimated SWCC data points fell within the 95% confidence interval computed for the soil database. As presented in Figure 5.41, the 95% confidence interval of the observed error, $(\theta_{wi} - \hat{\theta}_{wi})$, decreased to (-0.031, 0.083) from (-0.046, 0.083).

Figure 5.41-Scaled estimated versus measured water content for soil mixtures used in this study along with the selected soil database

It should be noted that the estimated SWCC data points might contain error from the actual SWCC measurements shown here. Therefore, only two data points above a suction of 100 kPa should be selected to complement the actual measured SWCC data points of soil up to a suction of 100 kPa. As described in Section 4.2, the best-fit exercise of SWCCs is obtained by minimizing the residual sum of squares (SSE) according to Equation 4.1. If more than two estimated SWCC data points in the high suction range are

selected, these estimated points tend to influence the best-fitting procedure. Since these estimated SWCC data points tend to have some errors from the actual measurements, they could reduce the accuracy of the best-fit SWCC to the actual SWCC data below 100 kPa.

5.7 Experimental program

The results of the experimental program conducted in this study are presented in this section. First, the results of the basic soil properties tests are presented. The results of the SWCC tests are then presented and the results of the unsaturated permeability tests are presented last.

5.7.1 Basic soil properties

Six different kaolin-sand mixtures—100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S—were used in this study. Table 5.3 summarizes all the basic soil properties test results.

Table 5.3-Soil properties of kaolin-sand mixtures

Soil properties of soil mixtures	100K0S	90K10S	80K20S	50K50S	20K80S	10K90S
Maximum dry density, ρ_{dmax} (Mg/m ³)	1.40	1.54	1.6	1.75	1.94	1.84
Optimum water content, w_{opt} (%)	23.9	21.8	18.9	12.5	9.2	8.3
Specific gravity, G_s	2.67	2.67	2.67	2.67	2.66	2.66
Liquid limit (LL)	50.20	45.88	41.19	27.68	15.86	15.69
Plastic limit (PL)	30.29	26.76	23.9	19.44	N.A.	N.A.
Plasticity index (PI)	19.91	19.12	17.29	8.24	N.A.	N.A.
Sand (4.75~0.075) mm (%)	0	9.99	19.97	49.93	79.89	89.87
Silt (0.075~0.002) mm (%)	90.63	82.00	72.91	45.62	18.33	9.24
Clay (<0.002) mm (%)	9.37	8.01	7.12	4.45	1.78	0.89
D_{60} (mm)	0.0187	0.02	0.024	0.32	0.41	0.42
D_{30} (mm)	0.009	0.0115	0.014	0.02	0.28	0.32
D_{10} (mm)	0.0025	0.005	0.003	0.005	0.015	0.072
C_u (D_{60}/D_{10})	7.48	4	8	16	27.3	5.83
C_c ($D_{30}^2/D_{60}D_{10}$)	1.73	1.32	2.72	0.25	12.75	3.39
USCS Symbol	MH	ML	CL	SC	SM	SP
Saturated permeability, k_s (m/s)	1.87×10^{-8}	8.02×10^{-8}	1.67×10^{-7}	2.7×10^{-7}	8.80×10^{-7}	1.27×10^{-5}

The grain sizedistributions of all soil mixtures are shown in Figure 5.42. The grain sizedistribution of Ottawa sand is also shown in the figure. As shown in the figure, the soil mixtures represent silty to sandy soils (i.e., cohesive to non-cohesive spectrum of soil types).

The specific gravity was 2.67 for soil mixtures 100K0S, 90K10S, 80K20S and 50K50S and 2.66 for soil mixtures 20K80S and 10K90S, as shown in Table 5.3.

Figure 5.42-Grain sizedistribution of kaolin-sand mixtures

The plasticity indices of soil mixtures 100K0S, 90K10S, 80K20S and 50K50S were 19.9, 19.1, 17.3 and 8.2, respectively. Soil mixtures 100K0S, 90K10S, 80K20S and 50K50S were classified as silt with high plasticity (MH), silt with low plasticity (ML), clay with low plasticity (CL) and clay with low plasticity (CL), respectively, according to the Unified Soil Classification System (ASTM D2487-93), as shown in Figure 5.43. Soil mixtures 20K80S and 10K90S were classified as silty sand (SM) and poorly graded sand (SP), respectively, according to the Unified Soil Classification System (ASTM D2487-93).

Figure 5.43-Classification of 100K0S, 90K10S, 80K20S and 50K50S soil mixtures in the Unified Soil Classification System (USCS)

The compaction curve, which shows the relationship between the dry density(ρ_d) and water content was obtained for all soil mixtures using the standard Proctor test described in Section 4.8.2.1. The results for soil mixtures 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S are shown in Figure 5.44.

The maximum dry density (ρ_{dmax}) of 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S soil mixtures was 1.40 Mg/m³, 1.54 Mg/m³, 1.60 Mg/m³, 1.75 Mg/m³, 1.94 Mg/m³ and 1.84 Mg/m³, respectively. The optimum water content (w_{opt}) of 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S soil mixtures was 23.9%, 21.8%, 18.9%, 12.5%, 9.2% and 8.3%, respectively.

All soil specimens used for SWCC and permeability tests were prepared at the maximum dry density (ρ_{dmax}) and optimum water content (w_{opt}), by static compaction as described in Section 4.8.2.2. A $\pm 5\%$ tolerance was allowed for the maximum dry density (ρ_{dmax}). Table 5.3 summarizes the soil properties of all soil mixtures used in the experimental program. The saturated permeability of all soil mixtures is also shown in the table.

Figure 5.44-Compaction curves of kaolin-sand mixtures

5.7.2 Soil-water characteristic curve tests

Soil-water characteristic curve tests were conducted for all statically compacted 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S soil mixtures used in this study. The SWCC tests were conducted using a Tempe cell, 5-bar pressure plates, 15-bar pressure plates and salt solutions as described in Section 4.8.3. The experimental results for soil mixtures 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S are shown here. The measured gravimetric water content of the soil mixtures was converted to volumetric water content using the volume-mass relationship. It should be noted that the volume change of the soil mixtures 100K0S, 90K10S, 80K20S and 50K50S was taken into account by using the results from the shrinkage test. The procedure for conducting the shrinkage test is presented in Appendix D. The SWCCs are also shown in terms of degree of saturation for the soil mixtures 100K0S, 90K10S, 80K20S and 50K50S.

Figure 5.45 and Figure 5.46 show the measured SWCC of the 100K0S soil mixture in terms of volumetric water content and degree of saturation versus suction, respectively.

Figure 5.45-SWCC of 100K0S soil mixture in terms of volumetric water content versus suction

Figure 5.46-SWCC of 100K0S soil mixture in terms of degree of saturation versus suction

The saturated volumetric water content of the 100K0S soil was 0.497 as shown in Figure 5.45. The air-entry value of the soil was obtained from the degree of saturation versus suction curve (i.e., Figure 5.46) by graphical method. The AEV was 40.4 kPa as shown in Figure 5.46. It should be noted that the value of 21.8 kPa was obtained for the AEV when

the SWCC in terms of volumetric water content versus suction was used. The difference in the AEV shows the importance of considering the volume change of the soil for interpreting the AEV of the soil (Fredlund and Huston, 2013). The 100K0S soil started to desaturate at a noticeable rate after the AEV and the rate of desaturation decreased around a suction value of 900 kPa when the volumetric water content value was 0.1 and the degree of saturation was 0.206. The result of volumetric water content versus suction was later used for estimation of unsaturated permeability of the 100K0S soil.

Figure 5.47 and Figure 5.48 show the measured SWCC of the 90K10S soil mixture in terms of volumetric water content and degree of saturation versus suction, respectively. The saturated volumetric water content of the soil was 0.462 as shown in Figure 5.47. The air-entry value of the soil was obtained from the degree of saturation versus suction curve (i.e., Figure 5.48) by graphical method. The AEV of the soil was 37.8 kPa as shown in Figure 5.48. The value of 26.8 kPa was obtained for the AEV when the SWCC in terms of volumetric water content was used which was different from that obtained from the degree of saturation curve. The 90K10S soil started to desaturate at a noticeable rate after the AE and the rate of desaturation decreased around a suction value of 1200 kPa when the volumetric water content value was 0.086.

Figure 5.47-SWCC of 90K10S soil mixture in terms of volumetric water content versus suction

Figure 5.48-SWCC of 90K10S soil mixture in terms of degree of saturation versus suction

Figure 5.49 and Figure 5.50 show the measured SWCC of the 80K20S soil mixture in terms of volumetric water content and degree of saturation versus suction, respectively.

The saturated volumetric water content of the soil was 0.415 as shown in Figure 5.49. The air-entry value of the soil was obtained from the degree of saturation versus suction curve (i.e., Figure 5.50) by graphical method. The AEV of the soil was 31.8 kPa as shown in Figure 5.50. The value of 24.2 kPa was obtained for the AEV when the SWCC in terms of volumetric water content was used which was different from that obtained from the degree of saturation curve. The 80K20S soil started to desaturate at a noticeable rate after the AEV and the rate of desaturation decreased around a suction value of 1200 kPa when the volumetric water content value was 0.076.

Figure 5.51 and Figure 5.52 show the measured SWCC of the 50K50S soil mixture in terms of volumetric water content and degree of saturation versus suction, respectively. The saturated volumetric water content of the soil was 0.367 as shown in Figure 5.51. The 50K50S soil started to desaturate before a suction value of 10 kPa at a slow rate, as shown in both figures. However, as there were no measurements before 10 kPa, the exact suction value when the soil started to desaturate cannot be determined. When the graphical method was used to obtain the air-entry value of the soil from the degree of saturation versus suction curve (i.e., Figure 5.52), two distinct points could be obtained for the AEV of the soil 50K50S: 4.6 kPa and 24.2 kPa. This characteristic of the 50K50S soil could be considered as bimodal characteristic. It should be noted that the values of 5.4 kPa and 24.2 kPa were obtained when the SWCC in terms of volumetric water content was used. The AEVs obtained from both figures were marginally different at first point and the same at the second point. This means that volume change of the 50K50S soil was not significant and did not affect the interpretation of the AEV of the soil. The rate of desaturation decreased around a suction value of 700 kPa when the volumetric water content value was 0.075.

Figure 5.49-SWCC of 80K20S soil mixture in terms of volumetric water content versus suction

Figure 5.50-SWCC of 80K20S soil mixture in terms of degree of saturation versus suction

Figure 5.51-SWCC of 50K50S soil mixture in terms of volumetric water content versus suction

Figure 5.52-SWCC of 50K50S soil mixture in terms of degree of saturation versus suction

The measured SWCC of the 20K80S soil mixture is shown in Figure 5.53. The saturated volumetric water content of the soil was 0.238. The SWCC of the 20K80S soil was measured using an automated Tempe cell in a suction range of 0.1-90 kPa, as described in Section 4.8.3.2. The soil started to desaturate at a slow rate up to a suction value of 60 kPa. The rate of desaturation increased after a suction value of 60 kPa up to 500 kPa. It can be seen from the figure that the 20K80S soil had a bimodal characteristic.

The measured SWCC of the 10K90S soil mixture is shown in Figure 5.54. The saturated volumetric water content of the soil was 0.219. The SWCC of the 10K90S soil was measured using an automated Tempe cell in a suction range of 0.1-90 kPa, as described in Section 4.8.3.2. The soil started to desaturate at a noticeable rate after a suction value of 1.5 kPa up to a suction value of 3 kPa. The volumetric water content of the soil was 0.151 at a suction value of 3 kPa. The rate of desaturation decreased after a suction value of 3 kPa up to 20 kPa when the volumetric water content of the soil was 0.121. The rate of desaturation increased again after a suction value of 20 kPa. It can be seen from the figure that the 10K90S soil had a bimodal characteristic.

Figure 5.53-SWCC of 20K80S soil mixture in terms of volumetric water content versus suction

Figure 5.54-SWCC of 10K90S soil mixture in terms of volumetric water content versus suction

The soil-water characteristics of all the soil mixtures are shown together in Figure 5.55.

Figure 5.55-SWCC of all the soil mixtures in terms of volumetric water content versus suction

As shown in the figure, the higher the sand percentage, the lower the saturated volumetric water content. The suction value when the soil mixture started to desaturate (i.e., the air-entry value of the soil) decreased with an increase in the sand percentage. The SWCCs of 100K0S and 90K10S soil mixtures showed more or less the same characteristic.

5.7.3 Permeability test

The saturated permeability of all soil mixtures was measured according to the procedure described in Section 4.8.4.1 using a saturated triaxial permeameter. The results of the tests are shown in Table 5.3. The saturated permeability of soil mixture 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S was 1.87×10^{-8} (m/s), 8.02×10^{-8} (m/s), 1.67×10^{-7} (m/s), 2.7×10^{-7} (m/s), 8.80×10^{-7} (m/s) and 1.27×10^{-5} (m/s), respectively. As shown, the saturated permeability of the soil mixture increased as the sand percentage increased. The saturated permeability of the soil mixtures varied within three orders of magnitude and could represent the soils ranging from sandy to silty, as shown in Figure 5.56.

Figure 5.56-Measured saturated permeability of all soil mixtures used in this study

The range of measured saturated permeability of the soils used in this study fell within the category of mixture of sand, silt and clay according to Casagrande (1938) as shown in Figure 5.57.

Figure 5.57-The range of saturated permeability, k_s (adapted from Casagrande, 1938 after Holts and Kovacs, 1981)

Before measuring the unsaturated permeability of the soil mixtures in the triaxial permeameter, the saturated permeability of the high-air entry ceramic disk was measured by applying cell pressure to the ceramic disk and measuring the outflow volume rate of water (described in greater detail in Section 4.8.4.2.2).

The saturated permeability of the top and bottom 1-bar ceramic disks was 1.86×10^{-8} (m/s) and 2.07×10^{-8} (m/s), respectively. The outflow volume rates for the top and bottom ceramic disks are shown in Figure 5.58 and Figure 5.59. The unsaturated permeability of the soil mixtures was measured at suction values of 30, 50 and 90 kPa using the 1-bar ceramic disk.

The saturated permeability of the top and bottom 5-bar ceramic disks was 4.73×10^{-10} (m/s) and 4.68×10^{-10} (m/s), respectively. The outflow volume rates for the top and bottom ceramic disks are shown in Figure 5.60 and Figure 5.61. The unsaturated permeability of the soil mixtures was measured at suction values of 200, 300 and 400 kPa using the 5-bar ceramic disk.

Figure 5.58-Outflow volume rate of water through 1-bar ceramic disk (Top)

Figure 5.59-Outflow volume rate of water through 1-bar ceramic disk (Bottom)

Figure 5.60-Outflow volume rate of water through 5-bar ceramic disk (Top)

Figure 5.61-Outflow volume rate of water through 5-bar ceramic disk (Bottom)

After saturation of the ceramic disks inside the triaxial permeameter was complete, the statically compacted soil specimen was set into the triaxial permeameter and saturated

according to the procedure described in Section 4.8.4.2.2. Matric suction was applied to the soil specimen by controlling the pore-water pressure and air pressure. The change in water volume draining out of the specimen through the ceramic disks was monitored. Once the soil specimen reached the equilibrium condition (i.e., no water flowing out of the soil specimen), the unsaturated permeability was measured at that matric suction. Figure 5.62 shows the volume of water drained from the 100K0S soil mixture at an applied suction of 200 kPa.

Figure 5.62-Outflow water volume of the 100K0S soil specimen at a suction of 200 kPa

The unsaturated permeability was measured by applying a hydraulic gradient across the soil specimen with two DVPCs according to the procedure described in Section 4.8.4.2.2. Hydraulic gradient ranging from 10 to as high as 70 was applied across the soil specimen. Agus et al. (2003) reported that unsaturated permeability is not sensitive to the magnitude of hydraulic gradient especially at high matric suction values. Once the outflow volume rate and inflow volume rate of water were approximately equal, the soil specimen was considered to have reached the steady-state condition (see Figure 5.63). The unsaturated permeability was then computed according to Darcy's equation (i.e., Equation 2.25) considering the soil specimen as a three-layered system, as described in Section 4.8.4.2.2.

Figure 5.63-Steady-state flow condition of soil mixture 100K0S at an applied matrix suction of 200 kPa and a hydraulic gradient of 27 in triaxial permeameter with 5-bar ceramic disk

Figure 5.63 shows the steady-state condition for the 100K0S soil mixture at a suction value of 200 kPa. The 100K0S soil reached steady-state condition approximately after 65 hours. Figure 5.64 shows the steady-state condition for the 90K10S soil mixture at a suction value of 30 kPa. The 90K10S soil reached steady-state condition after approximately 40 hours. As shown in the figure, a shorter time was required in the triaxial permeameter with the 1-bar ceramic disk to reach the steady-state flow condition. This was due to the higher saturated permeability of the 1-bar ceramic disk (i.e., 1.86×10^{-8} (m/s)) as compared to that of the 5-bar ceramic disk (i.e., 4.68×10^{-10} (m/s)). In addition, the unsaturated permeability of 90K10S soil at the suction value of 30 kPa (i.e., 1.69×10^{-8} (m/s)) was higher than the unsaturated permeability of the 100K0S soil at the suction value of 200 kPa (i.e., 9.16×10^{-11} (m/s)).

Figure 5.64-Steady-state flow condition of soil mixture 90K10S at an applied matric suction of 30 kPa and a hydraulic gradient of 12

The steady-state flow condition of 90K10S soil mixture at other suction values is shown in AppendixE.

The results of direct measurement of the unsaturated permeability tests are shown in Figure 5.65 for soil mixtures 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S. The unsaturated permeability data, (k_w) are plotted on a semilog plot versus suction. As shown in the figure, the unsaturated permeability decreased significantly as the suction applied to the soil increased.

Figure 5.65-Unsaturated permeability of all the soil mixtures used in this study

The permeability of 100K0S soil mixture that was classified as MH (i.e., silt with high plasticity) varied from 1.78×10^{-8} (m/s) (i.e., fully saturated) to 1.89×10^{-11} (m/s) (i.e., a suction value of 400 kPa). The permeability of 90K10S soil mixture that was classified as ML (i.e., silt with low plasticity) varied from 8.02×10^{-8} (m/s) (i.e., fully saturated) to 7.05×10^{-11} (m/s) (i.e., a suction value of 400 kPa). The permeability of 80K20S soil mixture that was classified as CL (i.e., clay with low plasticity) varied from 1.67×10^{-7} (m/s) (i.e., fully saturated) to 1.60×10^{-11} (m/s) (i.e., a suction value of 400 kPa). The permeability of 50K50S soil mixture that was classified as CL (i.e., clay with low plasticity) varied from 2.7×10^{-7} (m/s) (i.e., fully saturated) to 6.19×10^{-12} (m/s) (i.e., a suction value of 400 kPa). The permeability of 20K80S soil mixture that was classified as SM (i.e., poorly graded sand) varied from 8.8×10^{-7} (m/s) (i.e., fully saturated) to 9.68×10^{-11} (m/s) (i.e., a suction value of 400 kPa).

¹² (m/s) (i.e., a suction value of 400 kPa). The permeability of 10K90S soil mixture that was classified as SM (i.e., poorly graded sand) varied from 1.27×10^{-5} (m/s) (i.e., fully saturated) to 2.16×10^{-12} (m/s) (i.e., a suction value of 300 kPa).

It can be seen that the reduction in the unsaturated permeability compared to the saturated permeability became more significant as the sand percentage of the soil mixture increased. The value of reduction in the unsaturated permeability compared to the saturated permeability was 2.97, 3.06, 4.02, 4.64, 4.96 and 6.77 orders of magnitude for 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S soil mixtures, respectively.

As shown in Figure 5.65, the results indicated that the triaxial permeameter with the 1-bar ceramic disk was able to measure unsaturated permeability in the range of 1.75×10^{-8} to 7.20×10^{-11} (m/s). The triaxial permeameter with the 5-bar ceramic disk was able to measure unsaturated permeability in the range of 4.39×10^{-10} to 2.16×10^{-12} (m/s).

It should be noted that the maximum unsaturated permeability that can be obtained using the triaxial permeameter is limited by the saturated permeability of the high-air entry ceramic disk. ASTM D5084 specified that the saturated permeability of high-air entry ceramic disk must be at least two orders higher than the permeability of the soil specimen being measured so the analysis can be performed without considering the impedance of the high-air entry ceramic disk. However, the impedance of the ceramic disks was taken into account as described in Section 4.8.4.2.2. Therefore, the maximum unsaturated permeability that can be obtained from the triaxial permeameter with 1-bar and 5-bar ceramic disk in this study was limited to 1.86×10^{-8} (m/s) and 4.68×10^{-10} (m/s), respectively. On the other hand, the minimum unsaturated permeability that can be obtained using the triaxial permeameter is limited by the air-entry value of the ceramic disk. The maximum suction value that can be applied in the triaxial permeameter with 1-bar and 5-bar ceramic disk in this study was limited to 100 kPa and 500 kPa. If the applied suction is higher than this limit value, air will enter the ceramic disk and the water compartments in the top cap and pedestal, resulting in unreliable measurements. This limit value corresponds to the maximum suction that can be applied to the soil specimen or the minimum unsaturated permeability of the soil that can be measured.

Therefore, measurement of unsaturated permeability near saturation (i.e., near the air-entry value of the soil) for the 50K50S, 20K80S and 10K90S soil mixtures was not

reliable since their saturated permeabilities (i.e., 2.7×10^{-7} (m/s), 8.8×10^{-7} (m/s) and 1.27×10^{-5} (m/s), respectively) were higher than the saturated permeability of the 1-bar ceramic disk (i.e., 1.86×10^{-8} (m/s)).

Chapter 6 Discussion of results

6.1 Introduction

In this chapter, discussions of the results obtained in the study are presented. First, the fit of the matrix of unsaturated permeability estimation models to the soil database is discussed. Next, the effects of the SWCC and k_r equations are discussed independent of the soil database. Subsequently, the sensitivity of the SWCC and k_r equations to the SWCC data range is discussed. The effect of the SWCC data range on unsaturated permeability estimation for the soil database is presented afterwards. Finally, the validation of the experimental results of this study is discussed

6.2 Estimation and evaluation of unsaturated permeability of soil by a matrix of unsaturated permeability models

The root mean square errors (RMSE) for the entire soil database were computed and the results are presented in Section 5.2 (see Table 5.1). The RMSE values show the deviation between the measured unsaturated permeability and estimated values. The average and standard deviation of the RMSE values were computed and are shown in Table 5.1 as well. An overall comparison of the RMSE values for all 14 estimation models (matrix of unsaturated permeability estimation models) suggests that the VMM $m=1-1/n$, VCM $m=1-1/n$ and F&X-1994 models resulted in the lowest average and standard deviation of RMSE values (i.e., Ave=0.665 and SD=0.8, Ave=0.694 and SD=0.807 and Ave=0.732 and SD=0.669, respectively) for the soil database used in this study. On the other hand, the Burdine based estimation models (i.e., FBM, FBM $C(\psi) = 1$, VBM and VBM $m=1-1/n$) resulted in the highest RMSE values for the 16 soils. The average and standard deviation of the RMSE values for these models were the highest, with the exception of the VBM model (i.e., Ave=1.187 and SD=1.124, Ave=1.178 and SD=1.18, Ave=1.887 and SD=0.913, Ave=1.224 and SD=1.154 for FBM, FBM $C(\psi) = 1$ and VBM and VBM $m=1-1/n$, respectively).

The high RMSE values of the Burdine based models were due to underestimation of the unsaturated permeability values at a relatively lower suction range and overestimation at a higher suction range. This behavior is shown in Figure 6.1a for Silt loam soil.

Figure 6.1-Estimated unsaturated permeability curves by Burdine based models with and without tortuosity for Silt loam soil

As mentioned in Section 3.3.1, no tortuosity factor was considered in the estimation models combined in this study. However, as shown in Figure 6.1, considering a tortuosity factor of $S_e^2 = \left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r} \right)^2$ in the Burdine based estimation models would result in a poorer estimation. The computed RMSE values increase and it appears that considering a tortuosity factor is a modification in the wrong direction. However, considering a

tortuosity factor with a negative value would result in an improved estimation. As mentioned in Section 2.4.5.4, some negative values have been reported for the tortuosity factor (e.g., Kosugi, 1999; Schaap and Leij, 2000), which physically means that the flow paths are straighter than straight (Hunt et al., 2013). Brutsaert (1966) reported that the tortuosity factor was introduced in parallel models to compensate for overestimation. However, considering a tortuosity factor of $S_e^2 = \left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r}\right)^2$ did not result in any improvement in this study. Therefore, models developed based on the Burdine relative permeability equation are not necessarily good estimation models. van Genuchten (1980) reported that his closed form equation developed based on the Burdine relative permeability model was in less agreement with the measured data even though the tortuosity factor was considered in his model. Mualem (1976) also reported that Burdine based models were in less agreement with experimental values.

It should be noted that the RMSE values show how good the estimated unsaturated permeabilities fit the experimental data within a limited suction range where experimental data were available. However, the performance of the estimation models could not be evaluated for the entire suction range due to the limited laboratory measurement data available in the literature.

It seems that, based solely on RMSE values, the conclusion of what model gives the best or worst estimation is soil database dependent and may vary with different databases. Therefore, it is important to evaluate the variation between the estimation models independent from the soil database and to consider the potential effective parameters identified in this study, such as the SWCC equation, k_r equation and measured SWCC data range.

6.3 Evaluation of effect of SWCC equation and k_r equation on estimation of unsaturated permeability

As described in Section 5.3 and Section 5.4, the permeability estimation models were categorized into a SWCC equation category (to study the effect of k_r equations) or a k_r equation category (to study the effect of SWCC equations). The SWCC equation category contained four groups, namely the Fredlund and Xing (1994), Fredlund and Xing (1994) with $C(\psi) = 1$, van Genuchten (1980) and van Genuchten (1980) with $m=1-1/n$ based

models. The k_r equation category contained three groups, namely the Childs Collis-George, Burdine and Mualem based models.

The permeability values estimated by the estimation models in each group of the respective category were then compared. For instance, the permeability values estimated by the FCM, FMM and FBM models in the Fredlund and Xing (1994) group in the SWCC category were compared. To evaluate the effect of the SWCC equation and k_r equation independent from the measured unsaturated permeability values, the values estimated by the various models were compared. The comparison was done by computing the logarithmic difference between the maximum and minimum permeability values estimated by the models at the last measured suction value of the SWCC data point according to Equation 4.3. For example, the last measured SWCC data point for Guelph loam soil (i.e., S1) was at a suction value of 95.559 kPa, so all of the difference indices were computed and compared at this suction. The results for all of the soils are presented in Table 6.1. A smaller value for the difference index meant that the models of the respective group estimated more or less the same relative permeability values, while a larger value meant the models of the respective group estimated different relative permeability values.

Table 6.1-Computed difference index at a suction value of the last measured SWCC data point for entire soil database

	Soils	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	Average	Standard Deviation
SWCC Category	Fredlund and Xing 1994	0.115	1.643	0.130	0.163	0.119	0.142	0.444	0.661	0.832	0.413	0.717	0.362	0.865	0.223	0.774	0.295	0.769	0.582	1.330	0.254	0.542	0.418
	Fredlund and Xing 1994, $C(\psi)=1$	0.287	1.546	0.356	0.676	0.328	0.561	1.099	0.207	0.727	0.581	0.807	0.665	0.804	0.260	0.713	0.277	0.784	1.154	1.416	0.656	0.695	0.377
	van Genuchten 1980	0.395	1.601	0.449	0.861	0.825	0.944	2.387	0.676	0.739	0.556	0.904	0.960	0.909	0.769	0.763	0.293	0.632	1.247	1.972	0.654	0.927	0.521
	van Genuchten 1980 $m=1-1/n$	0.527	1.534	0.547	1.544	0.650	1.704	2.832	0.219	0.614	0.249	0.611	0.699	1.042	3.220	3.377	0.195	1.209	1.727	3.954	0.664	1.356	1.139
k_r Category	Mualem	0.938	0.619	1.000	1.678	0.931	2.236	3.529	1.656	0.221	0.390	0.132	0.725	1.557	3.519	1.058	0.067	1.201	1.337	5.232	0.372	1.420	1.323
	Childs and Collis-George	0.732	0.678	0.853	1.617	0.874	2.177	3.484	1.524	0.306	0.359	0.258	0.653	1.692	0.617	2.021	0.111	1.102	1.277	5.044	0.299	1.284	1.209
	Burdine	0.509	0.690	0.775	0.796	0.435	1.038	1.677	0.921	0.420	0.050	0.336	0.916	1.514	1.428	1.990	0.193	0.624	0.470	2.514	0.296	0.880	0.645

Table 6.2-Computed index at a suction value of the last measured SWCC data for entire soil database-No Burdine

	Soils	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	Average	Standard Deviation
SWCC Category	Fredlund and Xing 1994	0.115	0.312	0.082	0.034	0.022	0.002	0.059	0.223	0.183	0.150	0.250	0.182	0.231	0.188	0.402	0.118	0.070	0.019	0.094	0.175	0.146	0.105
	Fredlund and Xing 1994, $C(\psi)=1$	0.136	0.305	0.131	0.038	0.005	0.055	0.074	0.186	0.165	0.145	0.289	0.185	0.153	0.260	0.414	0.130	0.020	0.002	0.058	0.248	0.150	0.111
	van Genuchten 1980	0.210	0.301	0.073	0.023	0.061	0.019	0.073	0.355	0.164	0.141	0.383	0.060	0.249	0.089	0.409	0.126	0.004	0.054	0.028	0.241	0.153	0.129
	van Genuchten 1980 $m=1-1/n$	0.321	0.365	0.229	0.067	0.051	0.054	0.103	0.219	0.252	0.177	0.348	0.253	0.365	3.220	3.377	0.174	0.103	0.080	0.094	0.125	0.499	0.963
K_r Category	Mualem	0.938	0.619	1.000	1.678	0.931	2.236	3.529	1.656	0.221	0.390	0.132	0.725	1.557	3.519	1.058	0.067	1.201	1.337	5.232	0.372	1.420	1.323
	Childs and Collis-George	0.732	0.678	0.853	1.617	0.874	2.177	3.484	1.524	0.306	0.359	0.258	0.653	1.692	0.617	2.021	0.111	1.102	1.277	5.044	0.299	1.284	1.209

As presented in Table 6.1, the average of the difference indices for the SWCC category (i.e., effect of k_r equations) was smaller than that of the k_r category (i.e., effect of SWCC equations). However, there were two exceptions: one in the SWCC category for van Genuchten (1980) $m=1-1/n$ based models with an average of 1.356, which was comparable to the k_r category averages, and another in the k_r category for Burdine based models with an average of 0.880, which was comparable to the SWCC category averages. When Burdine based models were excluded from the analyses of the difference indices, the pattern of the results became quite clear, as shown in Table 6.2. As Table 6.2 shows, the difference indices were 0.146, 0.150, 0.153 and 0.499 for the SWCC category, which were much smaller than 1.420 and 1.282 for the k_r category. It can be observed that the SWCC equation had a more significant effect on the estimation of permeability of unsaturated soil than the relative permeability equation.

It can also be seen from Table 6.2 that the difference index for the van Genuchten $m=1-1/n$ based models decreased from 1.356 to 0.499 when Burdine based models were excluded and the results became consistent with the other SWCC category models (although slightly higher, due to the lower flexibility of this equation compared to other SWCC equations). As explained earlier in Section 6.2, of all the models, the Burdine based models resulted in the highest average and standard deviation of the RMSE values for the soil database in this study. Mualem (1976) and van Genuchten (1980) also reported that Burdine based models were in lower agreement with the measured data. The Burdine (1953) model was developed based on the assumption that soil consists of a number of parallel portions of capillary tubes each with a uniform pore-size to the flow direction. In contrast, the Childs and Collis-George (1950) and Mualem (1976) models were developed by incorporating a random distribution of pore-sizes in the direction of flow, which was done by the “cutting and rejoining” concept (Brutsaert, 1966). The lower accuracy of the Burdine based models might be due to the simplified assumption inherent to the model. The series obtained from the combination of the SWCC equations and Burdine relative permeability equation (Equation 3.17) is much simpler than that obtained from the Childs and Collis-George (Equation 3.16) and Mualem (Equation 3.18) relative permeability equations. Therefore, Burdine based models are unable to execute an accurate integration over the entire suction range. This is demonstrated later in Section 6.5 of the discussion showing the lower sensitivity of Burdine based models to changes in

the shape of SWCCs. Therefore, with the exception of Section 6.5, Burdine based models were not studied in detail for the sake of clarity in the discussion.

In order to illustrate the effect of SWCC equations on the estimation of permeability of unsaturated soils, the results for Guelph loam soil (i.e., S1) using Mualem based models (i.e., FMM, FMM $C(\psi)=1$, VG-1980 and VMM) are shown in Figure 6.2. As shown in Figure 6.2b, the four estimated relative permeability curves using the Mualem based models varied although the same k_r equation was used. The four best-fit SWCCs, as shown in Figure 6.2a, had more or less the same shape until the last measured SWCC data point and then started to vary significantly from each other. It appears that, for soils whose best-fit SWCCs vary significantly, the estimated relative permeability curves will also vary significantly. This means that the SWCC equation plays an important role in the estimation of unsaturated permeability of soil. This behavior was observed for all of the soils in the k_r category, as indicated by the larger values of the difference indices in Table 6.2.

In order to illustrate the effect of k_r equations on the estimation of unsaturated permeability of soils, the results for Guelph loam soil (i.e., S1) using Fredlund and Xing (1994) based models (i.e., FCM, FMM) are shown in Figure 6.3. As can be seen from the figure, the estimated relative permeability curves had more or less the same shape and the variation between models was almost negligible even though two different relative permeability equations were used. With the exception of Burdine based models, it appears that the relative permeability equation plays a less important role in the estimation of permeability of unsaturated soil. This behavior was observed for all of the soils in the SWCC category, as indicated by the smaller values of the difference indices in Table 6.2. Therefore, if different SWCC best-fit equations are used, the resulting relative permeability curves will have different shapes even if the same relative permeability equation is used in developing the estimation model. On the other hand, if the same SWCC best-fit equation is used (or if the SWCC curves are quite similar), the resulting relative permeability curves will have marginal variation even if different k_r equations are used in the development of the estimation model.

Figure 6.2-Typical effect of SWCC equations on the estimation of permeability: (a) Guelph loam-SWCCs, (b) Guelph loam-Mualem based models

Figure 6.3-Typical effect of relative permeability equations on the estimation of permeability: (a) Guelph loam-Fredlund and Xing (1994) SWCC, (b) Guelph loam-Fredlund and Xing based models

The difference indices were also computed at a suction value of one log cycle after the last measured SWCC data point (see Figure 6.2 and Figure 6.3). The results are shown in Table 6.3.

Table 6.3-Computed index at a suction value of 1 log cycle after the last measured SWCC data point for the entire soil database-No Burdine

		Soils	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	Average	Standard Deviation
SWCC Category	Fredlund and Xing 1994		0.111	0.317	0.087	0.016	0.079	0.061	0.035	0.199	0.210	0.148	0.262	0.168	0.289	0.225	0.482	0.119	0.150	0.032	0.145	0.101	0.162	0.114
	Fredlund and Xing 1994, $C(\psi)=1$		0.117	0.313	0.116	0.089	0.082	0.113	0.102	0.156	0.207	0.777	0.379	0.168	0.333	0.288	0.501	0.134	0.071	0.063	0.086	0.286	0.219	0.179
	van Genuchten 1980		0.137	0.316	0.073	0.023	0.066	0.014	0.068	0.373	0.215	0.161	0.387	0.061	0.307	0.118	0.485	0.134	0.002	0.053	0.026	0.281	0.165	0.145
	van Genuchten 1980 $m=1-1/n$		0.229	0.372	0.229	0.067	0.055	0.044	0.096	0.221	0.254	0.131	0.258	0.254	0.397	1.125	1.053	0.178	0.099	0.079	0.089	0.128	0.268	0.298
k_r Category	Mualem		2.658	0.839	2.494	11.658	24.574	19.510	10.433	11.527	0.636	0.629	2.129	1.708	3.328	1.893	8.756	0.261	6.938	9.325	13.236	78.477	10.550	17.349
	Childs and Collis-George		2.540	0.898	2.352	11.575	24.429	19.405	10.371	11.352	0.675	0.568	2.000	1.623	3.392	0.993	9.308	0.305	6.690	9.213	13.002	78.292	10.449	17.332

It can be seen from the table that the increase in the average value for the k_r category was quite significant compared to that for the SWCC category. For instance, the average value for Fredlund and Xing (1994) based models increased from 0.146 to 0.162 in the SWCC category, while the average value for Mualem based models increased from 1.420 to 10.550 in the k_r category (it should be noted that the average value for the Burdine based models increased from 0.880 to 5.339, which was consistent with the results of the study). From the results presented in Table 6.3, it can be concluded that if the same SWCC equation and different k_r equations are used for the estimation of unsaturated permeability in seepage analyses, the results of the analyses will have small variation even at the extrapolated suction range. This means that a change of k_r model may not noticeably change the results, even if the permeability is estimated beyond the measured SWCC data. On the other hand, if different SWCC equations and the same k_r equation are used, the results of the analyses will be significantly different, especially at the extrapolated region. This means that a change of SWCC model may significantly change the results, especially if the permeability is estimated beyond the measured SWCC data. Therefore, it can be concluded that the SWCC equation has a more significant effect on the estimation of permeability of unsaturated soil than the relative permeability equation.

6.4 Sensitivity of soil-water characteristic curve equations to measured SWCC data range

As discussed in Section 6.3, the SWCC equation had a significant effect on the estimation of unsaturated permeability of soils. In addition, the results presented in Section 5.5, showed that, despite using the same SWCC equation, different measured SWCC data ranges will result in different best-fit SWCCs. Therefore, it is important to investigate the sensitivity of all four SWCC equations used in this study to the range of SWCC measurements. The sensitivity of a SWCC equation to the range of SWCC measurements shows the importance of having measured SWCC data from a wide suction range. This means that a less sensitive SWCC best-fit equation has less variation for the different ranges of SWCC measurements and will be less affected by incomplete measured SWCC data, while a more sensitive SWCC equation has much variation for the measured SWCC data ranges and will be more affected by incomplete measured SWCC data. In order to investigate the sensitivity of the SWCC equations to the range of SWCC measurements, a variation parameter was defined according to Equation 6.1:

$$Variation = \sqrt{\frac{\sum_i^n (\theta_{wi(DR1)} - \theta_{wi(DR4)})^2}{\sum_i^n (\theta_{wi(DR4)})^2}} \quad \text{Equation 6.1}$$

where

θ_{wi} is the computed volumetric water content at a suction value of ψ_i by any best-fit SWCC equation

As defined in Equation 6.1, the computed volumetric water content values obtained from the best-fit SWCC equation for the DR1 (i.e., 0.1-100 kPa), DR2 (i.e., 0.1-500 kPa), DR3 (i.e., 0.1-1500 kPa) and DR4 (i.e., 0.1-10000 kPa) sets of SWCC data were normalized relative to the computed values obtained from the best-fit SWCC equation for the most complete set of SWCC data. DR4 was the most complete set of SWCC data for three of the soils in the database and DR3 was the most complete set of SWCC data for four of the soils in the database. The best-fit SWCC obtained from the most complete data range DR4 (or DR3) had a perfect fit to the full range of measured SWCC data with a high coefficient of determination and could represent SWCC measured data for normalization. The variation parameters were computed according to Equation 6.1 and the results are shown in Figure 6.4 for the entire soil database. For every best-fit SWCC equation, variation parameters were plotted versus different sets of measured SWCC data (i.e., DR1, DR2, DR3 and DR4).

Figure 6.4-Computed variation for SWCC data ranges for all the soils

As can be seen from the figures, the value of the variation parameter changed from one set of measured SWCC data to the other for all best-fit SWCC equations and for the entire soil database. In general, the variation parameter decreased as the measured SWCC data set increased for all the soils. However, there were two exemptions: one, the variation parameter of UP-1 soil resulting from DR3 that increased compared to that of DR2 and the variation parameter of Live oak red clay resulting from DR2 that increased compared to that of DR1. The reason behind this behavior was due to the bimodal characteristic of measured SWCC of these two soils as shown in Appendix C (Figures C.1-C.8 and Figures C.38-C.44). For instance, the measured SWCC data of UP-1 soil started to flatten at suction values of 1000 kPa and 1400 kPa. By increasing the suction to values beyond 1500 kPa, the volumetric water content decreased significantly which caused a bimodal characteristic for this soil. Therefore, the best-fit SWCCs for DR3 were affected by two data points at 1000 kPa and 1400 kPa which caused the curves to flatten at these suction values. This characteristic increased the variation parameters of DR3 as compared to those of DR2. The minimum values of the variation parameters were obtained from Fredlund and Xing (1994) for all soils in the database and the equation had the least sensitivity to the range of SWCC measurements. The maximum values of the variation parameters were obtained from van Genuchten (1980) for UP-1, UP-2, UP-3, Beit Netofa Clay and Live Oak Red Clay soils and from Fredlund and Xing (1994) $C(\psi)=1$ for UP-4 and Wenatchee Silty Clay soils. It appears that van Genuchten (1980) had the most sensitivity to the measured SWCC data ranges. The sensitivity of van Genuchten (1980) $m=1-1/n$ was lower than the sensitivity of van Genuchten (1980), which was probably due to the lower flexibility of the equation. The reason for the low sensitivity of the Fredlund and Xing best-fit SWCC equation to the measured SWCC data ranges is due to the correction factor in the Fredlund and Xing (1994) equation $C(\psi) = \left(1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)} \right)$ that forces the equation to equal zero at a suction value of 10^6 kPa as supported by thermodynamic consideration (Richards, 1965). This correction factor acts as an additional data point and, therefore, reduces significant variation of the SWCC at the high suction range of the best-fit curve. For the most sensitive best-fit SWCC equations, namely van Genuchten (1980) and Fredlund and Xing (1994) with correction factor, $C(\psi)=1$, the equations were not constrained at the high suction range, causing the best-fit SWCCs to vary significantly, as reflected by the high variation parameter. Therefore, it

can be concluded that the Fredlund and Xing (1994) equation is the most suitable SWCC equation in terms of sensitivity to limited SWCC data range.

6.5 Sensitivity of relative permeability equations to measured SWCC data range

Similar to Section 6.4, it is important to evaluate the sensitivity of the relative permeability equations to the measured SWCC data range. The relative permeability equation integrates the best-fit soil-water characteristic curve to estimate the permeability function of an unsaturated soil. The sensitivity of an integration equation (i.e., relative permeability equation) shows how accurately the equation reflects the different best-fit soil-water characteristic curves. Therefore, being more sensitive to different best-fit SWCCs means that the performance of the integration equation is better and the integration equation is more capable of reflecting the best-fit SWCC. As discussed in previous sections, different SWCC measurement ranges resulted in different best-fit SWCCs. Therefore, to investigate the sensitivity of the relative permeability equations to the SWCC measurement ranges, a variation parameter is defined according to Equation 6.2:

$$Variation = \sqrt{\frac{\sum_i^n (\log(k_{ri(DR1)}) - \log(k_{ri(DR4)}))^2}{\sum_i^n (\log(k_{ri(DR4)}))^2}} \quad \text{Equation 6.2}$$

As defined in Equation 6.2, the computed relative permeability values obtained from the best-fit SWCCs using the measured SWCC data sets of DR1 to DR4 were normalized relative to the computed relative permeability values obtained from best-fit SWCC equations using the most complete set of SWCC data. DR4 was the most complete set of SWCC data for four of the soils in the soil database and DR3 was the most complete set of SWCC data for three of the soils in the soil database. The variation parameters were computed for all of the estimation models and the results for UP-3 soil are shown in Figure 6.5. The variation parameters are plotted versus different sets of measured SWCC data (i.e., DR1, DR2, DR3 and DR4).

Figure 6.5-Computed variation parameters for UP-3 soil using (a) Fredlund and Xing (1994) based models, (b) Fredlund and Xing (1994) $C(\psi)=1$ based models, (c) van Genuchten (1980) based models and (d) van Genuchten (1980) $m=1-1/n$ based models

As shown in the figure, the variation parameter changed for different SWCC measurement ranges for all models. The changes in the Burdine based models were smaller than the changes in the Childs Collis-George and Mualem based models. This means that Burdine based models are less capable of reflecting changes in SWCC curves and that the sensitivity of the Childs Collis-George and Mualem based models was larger than that of the Burdine based models. As illustrated in Figure 6.5, the differences between the Childs Collis-George and Mualem based models were quite small. Therefore, it can be concluded that the models based on the Childs Collis-George and Mualem equations performed better at estimating the permeability of unsaturated soils and are almost similar in terms of their sensitivity to the measured SWCC data range. The same behavior was observed for all other soils. The variation parameters for van Genuchten (1980) $m=1-1/n$ based models for all soils are shown for in Appendix C.

6.6 Effect of measured SWCC data ranges on unsaturated permeability estimation models

Based on the discussions in the above sections and the results presented in Section 5.5, it was determined that the measured SWCC data range has a significant effect on the estimation of unsaturated permeability of soils. To illustrate this effect, the results for Beit Netofa clay (BNC) soil presented in Section 5.5 are rearranged based on the measured SWCC data range. The estimated unsaturated permeability curves using best-fit SWCCs by DR1 (i.e., 0.1-100 kPa) and DR3 (i.e., 0.1-1500 kPa) for eight of the estimation

models (excluding Burdine based models) from the matrix of unsaturated permeability estimation models along with the existing models F&X-1994 and VG-1980 are shown in Figure 6.6 and Figure 6.7, respectively. By comparing Figure 6.6 and Figure 6.7, it is obvious that measured SWCC data are the most important factor affecting the estimation of unsaturated permeability of soils. Figure 6.7 shows that the variation between all 10 estimation models was relatively small (i.e., within one order of magnitude). The FMM with $C(\psi)=1$ model was the lower limit of all the estimation models, while the VCM $m=1-1/n$ model was the upper limit of all the estimation models for BNC soil.

Figure 6.6-Estimated unsaturated permeability curves using best-fit SWCCs by DR1-Beit
Netofa clay (S2)

Figure 6.7-Estimated unsaturated permeability curves using best-fit SWCCs by DR3-Beit
Netofa clay (S2)

Figure 6.8 shows that the best-fit SWCCs for BNC soil using DR3 were close to each other. However, slight variation can be observed at a higher suction range where

measured SWCC data were not available and this fact contributed to variation in the estimated unsaturated permeability curves, as shown in Figure 6.7. Perhaps if more measured SWCC data were available at the higher suction range (i.e., DR4), the variation between the estimation models would be further reduced.

Figure 6.8-Best-fit soil-water characteristic curves using DR3-Beit Netofa clay (S2)

In fact, for all 20 soils used in this study (shown in Table 4.1), the variation between estimated unsaturated permeability curves was quite small when the best-fit SWCCs were similar over the entire suction range. For instance, Figure 6.9 shows the SWCCs for Mine tailing soil, which were similar to each other, and this similarity resulted in slight variation in the estimated unsaturated permeability, as shown in Figure 6.10. Therefore, it can be concluded that the measured SWCC data range is the most important factor affecting the estimation of unsaturated permeability of soil and that if the best-fit SWCCs are close to each other over the entire suction range, the variation between the estimated unsaturated permeability curves will be quite small.

Figure 6.9-Best-fit soil-water characteristic curves-Mine tailing (S12)

Figure 6.10-Estimated unsaturated permeability curves-Mine tailings (S12)

As described in Section 4.8.3, SWCC was measured over a wide suction range for 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S soil mixtures in this study to eliminate the effect of SWCC data range and the results are discussed here.

Figure 6.11 shows DR1 (0.1-100 kPa) measured SWCC data for the 90K10S soil mixture. The DR1 measured SWCC data were best-fit using the four SWCC equations used in this study according to the procedure explained in Section 4.2. The full measured SWCC data (0.1-146835.8 kPa) for 90K10S soil is shown in Figure 6.12. The full measured SWCC data were best-fit using the four SWCC equations used in this study as shown in the figure. By comparing Figure 6.11 and Figure 6.12, it can be seen that the best-fit SWCCs obtained using the full measurement of SWCC data were similar over the entire suction range while the best-fit SWCCs obtained using only the measured data for DR1 had the same shape until a suction value of 100 kPa, beyond which they started to vary significantly. This is similar to the conclusion obtained for the soils from the database.

The estimated unsaturated permeability curves by the matrix of unsaturated permeability estimation models using the best-fit SWCCs by only DR1 measured SWCC data along with the directly measured unsaturated permeability data of 90K10S soil are shown in Figure 6.13. The estimated unsaturated permeability curves by the matrix of unsaturated permeability estimation models using the best-fit SWCCs by the full measurement of SWCC data along with the directly measured unsaturated permeability data of 90K10S soil are shown in Figure 6.14. By comparing Figure 6.13 and Figure 6.14, it can be seen that the variation between the estimated unsaturated permeability curves was quite small when the best-fit SWCCs using the full measurement of SWCC data were used compared to that when only the best-fit SWCCs using DR1 measured SWCC data were used. If only the best-fit SWCCs using the DR1 measured SWCC data were used to estimate the unsaturated permeability curves, the variation between estimated curves increased significantly beyond 100 kPa (i.e., the last measured SWCC data point in DR1). The maximum variation between the estimation models was 3.53 (in orders of magnitude). This value was computed by considering the maximum and minimum estimated unsaturated permeability values at every suction value for the entire suction range above the relative vapor permeability. On the other hand, when the full measurement of SWCC data was used to estimate the unsaturated permeability curves, the variation between estimated curves was significantly reduced and it resulted in a very narrow band-width of 0.92 (in orders of magnitude), which is less than one order of magnitude.

Figure 6.11-Measured SWCC data and best-fit soil-water characteristic curves for 90K10S soil (DR1 SWCC measurement range)

Figure 6.12-Measured SWCC data and best-fit soil-water characteristic curves for 90K10S soil (full SWCC measurement range)

Figure 6.13-Directly measured unsaturated permeability data and estimated unsaturated permeability curves by a matrix of unsaturated permeability models using DR1 SWCC measurement range-90K10S soil mixture

Figure 6.14-Directly measured unsaturated permeability data and estimated unsaturated permeability curves by a matrix of unsaturated permeability models using full SWCC measurement range-90K10S soil mixture

Comparing Figure 6.13 and Figure 6.14, it is obvious that the effect of measured SWCC data for a limited suction range is the dominant factor causing the significant variation between the estimation models.

Figure 6.15 shows the four best-fit SWCCs for the 10K90S soil mixture obtained using only the measured SWCC data for DR1 (0.1-100 kPa). Figure 6.15 shows that the SWCCs obtained using the Fredlund and Xing (1994) with $C(\psi)=1$, van Genuchten (1980) and van Genuchten (1980) with $m=1-1/n$ equations had the same shape until a suction value of 100 kPa, beyond which they started to vary significantly. The SWCC obtained using the Fredlund and Xing (1994) equation had a different fit to the measured data than the other three SWCCs and a different shape over the entire suction range. This characteristic could be due to the bimodal behavior of the 10K90S soil mixture.

The full measurement of SWCC data for 10K90S soil is shown in Figure 6.16. It should be noted that the SWCC of 10K90S soil was measured up to DR3 (i.e., 0.1-1500 kPa) and was not measured beyond this value as the volumetric water content of the soil was relatively small of 0.028. The measured SWCC data of DR3 were best-fit using the four SWCC equations used in this study, as shown in the figure. Figure 6.16 shows that the best-fit SWCCs obtained using the different SWCC equations did not have a perfect fit to the measured data. However, the Fredlund and Xing (1994), Fredlund and Xing (1994) with $C(\psi)=1$ and van Genuchten (1980) SWCC equations showed a similar fit to the measured SWCC data, resulting in SWCCs that were similar over the entire suction range. On the other hand, the van Genuchten (1980) with $m=1-1/n$ equation did not fit the data perfectly and its shape was slightly different from the other three SWCCs.

However, by comparing Figure 6.15 and Figure 6.16 it is obvious that the measured SWCC data range greatly affected the best-fit SWCCs, which is in accordance with the findings of this study.

Figure 6.15-Measured SWCC data and best-fit soil-water characteristic curves for 10K90S soil (DR1 SWCC measurement range)

Figure 6.16-Measured SWCC data and best-fit soil-water characteristic curves for 10K90S soil (DR3 SWCC measurement range)

Figure 6.17-Directly measured unsaturated permeability data and estimated unsaturated permeability curves by a matrix of unsaturated permeability models using DR1 SWCC measurement range-10K90S soil mixture

Figure 6.18-Directly measured unsaturated permeability data and estimated unsaturated permeability curves by a matrix of unsaturated permeability models using full SWCC measurement range-10K90S soil mixture

The estimated unsaturated permeability curves by the matrix of unsaturated permeability estimation models using the best-fit SWCCs by only the measured SWCC data for DR1 along with the directly measured unsaturated permeability data of 10K90S soil are shown in Figure 6.17. The estimated unsaturated permeability curves by the matrix of unsaturated permeability estimation models using the best-fit SWCCs by the DR3 measured SWCC data range along with the directly measured unsaturated permeability data of 10K90S soil are shown in Figure 6.18. By comparing Figure 6.17 and Figure 6.18, it can be seen that the variation between estimated curves was small when the best-fit SWCCs using DR3 measured SWCC data were used compared to that of when the best-fit SWCCs using only the measured SWCC data of DR1 were used. When the best-fit SWCCs using only the measured SWCC data of DR1 were used to estimate the unsaturated permeability curves, the maximum variation between the estimation models was 3.38 (in orders of magnitude). On the other hand, when the best-fit SWCCs using DR3 measured SWCC data were used to estimate the unsaturated permeability curves, the variation between estimated curves was significantly reduced and resulted in a very narrow band-width of 1.19 (in orders of magnitude).

Comparing Figure 6.17 and Figure 6.18, it is obvious that the effect of measured SWCC data for a limited suction range is the dominant factor causing the significant variation between the estimation models.

The deviation between the directly measured unsaturated permeabilities of 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S soil mixtures using the modified triaxial permeameter in this study and the estimated unsaturated permeabilities for all the soil mixtures was computed by the root mean square error (RMSE) according to Equation 4.2. The RMSEs were computed for the estimated unsaturated permeabilities using the best-fit SWCCs by the full measurement of SWCC data and DR1 measured SWCC data and the results are shown in Table 6.4. The average and standard deviation of RMSE values of all the soil mixtures are also shown in Table 6.4. The average of the RMSEs of all the soils and all the models was 2.107 and 1.036 for DR1 and the full measurement of SWCC data, respectively. It means that the unsaturated permeability estimation models had a better fit to the directly measured unsaturated permeability data when the best-fit SWCCs using the full measured SWCC data were used, as shown by the lower values of RMSEs (i.e., average of RMSEs for all the soils and all the models was 1.036).

Table 6.4-Computed RMSE values for soil mixtures of this study-full measured SWCC and DR1 measured SWCC

	Model	F&X-1994	VG-1980	FCM	FMM	FCM $C(\psi)=1$	FMM $C(\psi)=1$	VCM $m=1-1/n$	VMM $m=1-1/n$	VCM	VMM
	Soil Mixture										
DR1	100K0S	0.987	3.115	1.126	1.973	0.852	1.069	1.923	2.252	14.199	14.707
	90K10S	0.636	4.985	0.629	0.649	0.964	0.997	3.450	3.693	2.481	2.684
	80K20S	1.063	2.448	1.000	0.673	1.048	0.840	1.441	1.614	8.073	8.559
	50K50S	1.124	1.520	0.693	0.861	0.646	0.954	0.683	0.788	0.681	0.929
	20K80S	0.613	3.396	0.818	1.332	0.868	1.450	0.831	1.393	0.750	1.172
	10K90S	2.765	3.571	1.443	1.038	0.745	0.918	1.524	1.952	0.429	0.432
	Average	1.198	3.172	0.952	1.088	0.854	1.038	1.642	1.949	4.435	4.747
	Standard Deviation	0.798	1.162	0.304	0.502	0.145	0.216	0.998	0.990	5.588	5.726
Full	100K0S	0.388	1.091	0.392	1.035	0.426	1.185	0.333	0.691	0.384	0.542
	90K10S	0.650	0.531	0.641	0.433	0.575	0.508	0.747	0.310	0.850	0.423
	80K20S	1.151	0.600	1.075	0.447	1.087	0.450	1.501	0.870	1.497	0.864
	50K50S	1.637	1.208	1.475	0.692	1.390	0.623	2.216	1.585	1.384	0.622
	20K80S	1.901	2.243	1.178	0.626	1.195	0.627	3.395	2.672	1.183	0.604
	10K90S	2.624	1.257	1.303	0.908	1.312	0.901	1.023	0.628	1.239	0.832
	Average	1.392	1.155	1.011	0.690	0.997	0.715	1.536	1.126	1.089	0.648
	Standard Deviation	0.831	0.616	0.413	0.243	0.401	0.277	1.118	0.868	0.409	0.170

The lowest average RMSE values were 0.648 and 0.690 resulting from VMM and FMM estimation models when the best-fit SWCCs using the full measurement of SWCC data were used. However, when the best-fit SWCCs using only DR1 measured SWCC data were used, the average RMSE values of these estimation models increased from 0.648 to 4.747 for VMM and from 0.690 to 1.088 for FMM. This behavior shows the significant sensitivity of VMM estimation model to the limited measured SWCC data and less sensitivity of FMM model to the limited measured SWCC data. Therefore, it can be concluded that FMM model was the best unsaturated permeability estimation model.

6.7 Estimation of unsaturated permeability using the measured SWCC data of DR1 in combination with predicted SWCC data beyond 100 kPa

Based on the discussion in 6.6, it was concluded the effect of measured SWCC data for a limited suction range is the dominant factor causing the significant variation between the estimation models. Therefore, it is important to have measured SWCC data for a wider

suction range. However, the measurement of SWCC over the entire suction range requires a long testing time that may not be realistic for practical purposes. In addition, expensive equipment are required to perform such complete measurements which may not be available in all the laboratories. Therefore, as described in Section 3.4, a procedure was proposed to estimate SWCC data points over a wider suction range using the grain size distribution data and measured SWCC data point at 100 kPa suction.

Figure 6.19 shows the results of the best-fit SWCCs for the 90K10S soil mixture using the measured data of DR1 and the estimated SWCC data points using the proposed procedure in Section 3.4. The four best-fit SWCCs were quite close to each other as shown in Figure 6.19. The variation between the four best-fit SWCCs was significantly reduced when both the measured data for DR1 and the estimated SWCC data points were used compared to that shown in Figure 6.11, which only used the measured SWCC data for DR1.

The estimated unsaturated permeability curves for the 90K10S soil mixture using the best-fit SWCCs by the measured SWCC data for DR1 and the estimated SWCC data points are shown in Figure 6.20. It can be seen from the figure that the variation between the estimated unsaturated permeability curves was significantly lower than that shown in Figure 6.13 where only the measured SWCC data from DR1 was used. The maximum variation between estimated unsaturated permeability curves was 0.90 (in orders of magnitude), which is the same as the band-width obtained using the SWCC measured data over the full suction range.

Figure 6.19-Best-fit soil-water characteristic curves for 90K10S soil (DR1 SWCC measurement range & estimated SWCC data points)

Figure 6.20-Directly measured unsaturated permeability data and estimated unsaturated permeability curves by a matrix of unsaturated permeability models using DR1 SWCC measurement range & estimated SWCC data points-90K10S soil mixture

Figure 6.21-Best-fit soil-water characteristic curves for 10K90S soil (DR1 SWCC measurement range & estimated SWCC data points)

Figure 6.22-Directly measured unsaturated permeability data and estimated unsaturated permeability curves by a matrix of unsaturated permeability models using DR1 SWCC measurement range & estimated SWCC data points-10K90S soil mixture

The procedure proposed in Section 3.4 to estimate SWCC data points over a wider suction range was used for the 10K90S soil. The results of the best-fit SWCCs using the proposed procedure with the measured data for DR1 and the estimated SWCC data points are shown in Figure 6.21. As can be seen from the figure, the variation between the four best-fit SWCCs was significantly lower than that in Figure 6.15 where only the measured SWCC data for DR1 was used. The estimated unsaturated permeability curves for the 10K90S soil mixture using the best-fit SWCCs by the measured SWCC data for DR1 and the estimated SWCC data points are shown in Figure 6.22. It can be seen from the figure that the variation between the different models was significantly lower than that in Figure 6.17 where only the measured SWCC data for DR1 was used. A band-width of 1.8 (in orders of magnitude) was obtained which is smaller than the band-width using the measured SWCC data for DR1 (i.e., 3.38) and comparable to the SWCC measured data over the full suction range (i.e., 1.19). It can also be seen from Figure 6.22 that the slightly larger variation of the unsaturated permeability estimation using DR1 and estimated SWCC data is due to variation in VG-1980 where the SWCC model has less flexibility to best-fit the SWCC measured data. The SWCCs estimated at higher suction values using the procedure defined in Section 3.4 in combination with the SWCC measured data for DR1 had successfully eliminated the variation in SWCC models and unsaturated permeability estimation due to the incomplete measured SWCC data.

The deviation between the directly measured unsaturated permeabilities of 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S soil mixtures using the modified triaxial permeameter in this study and the estimated unsaturated permeabilities for all the soil mixtures was computed by the root mean square error (RMSE) according to Equation 4.2. The RMSEs were computed for the estimated unsaturated permeabilities using the best-fit SWCCs by DR1 measured SWCC data in combination with the estimated SWCC data points and the results are shown in Table 6.5. The average and standard deviation of RMSE values of all the soils is also shown in Table 6.5. The average of the RMSEs of all the soils and all the models was 0.984 which was similar to that of full measured SWCC data (i.e., 1.036).

The lowest average RMSE value was 0.671 resulting from FMM estimation model which was more or less the same to that obtained when the full measurement of SWCC data was used (i.e., 0.690). The RMSE value resulting from FMM estimation model reduced from

1.088 (when only the best-fit SWCCs using DR1 were used) to 0.671 when the proposed method for the estimation of SWCC data points beyond 100 kPa was used. Therefore, it can be concluded that the proposed method to estimate the SWCC data points beyond 100 kPa successfully eliminated the variation due to the limited data range and provided a good fit to the directly measured unsaturated permeability data.

Table 6.5-Computed RMSE values for soil mixtures of this study- DR1 measured SWCC data in combination with the estimated SWCC data points

DR1 and Estimated points	Model	F&X-1994	VG-1980	FCM	FMM	FCM $C(\psi)=1$	FMM $C(\psi)=1$	VCM $m=1-1/n$	VMM $m=1-1/n$	VCM	VMM
	Soil Mixture										
	100K0S	0.670	1.056	0.663	1.140	0.666	0.881	0.676	0.637	0.614	0.692
	90K10S	0.698	0.530	0.696	0.454	0.663	0.462	0.736	0.301	0.871	0.534
	80K20S	1.142	0.598	1.066	0.441	1.144	0.449	1.489	0.864	1.525	0.933
	50K50S	1.208	1.277	0.737	0.694	0.756	0.669	2.281	1.654	0.716	0.742
	20K80S	2.106	1.840	1.392	0.696	1.382	0.694	3.047	2.304	1.357	0.693
	10K90S	1.009	2.217	0.766	0.602	0.804	1.019	0.605	0.416	1.024	1.019
	Average	1.139	1.253	0.887	0.671	0.903	0.696	1.472	1.029	1.018	0.769
	Standard Deviation	0.523	0.673	0.287	0.255	0.294	0.226	1.006	0.788	0.360	0.177

6.8 Comparison between direct measurement of unsaturated permeability data and estimated unsaturated permeability data

As discussed in Section 6.6, FMM was found to be the least sensitive to the limited SWCC data range and the best model to fit the directly measured unsaturated permeability data of 100K0S, 90K10S, 80K20S, 50K50S, 20K80S and 10K90S soil mixtures using the modified triaxial permeameter. Figure 6.23 shows the directly measured unsaturated permeability data versus the estimated unsaturated permeability data using the best-fit SWCCs by the full measurement of SWCC data for FMM estimation model for all the soil mixtures used in this study.

Figure 6.23-Directly measured unsaturated permeability data versus estimated unsaturated permeability by FMM for the soil mixtures used in this study-Full measured SWCC data

The coefficient of determination, R^2 was computed according to Equation 4.4 for all the directly measured unsaturated permeability data of all the soil. The value of R^2 was 0.854 and shows that FMM model was able to reasonably fit the directly measured data. The 95% confidence interval of the observed errors, $(k_{wi} - \hat{k}_{wi})$, between the directly measured unsaturated permeability data and the estimated unsaturated permeability data was computed according to Equation 4.5 and is shown in Figure 6.23 by dashed lines. The 95% confidence interval of the observed error was found to be $(-0.614, 0.597)$ that created a range of 1.21 in orders of magnitude.

Figure 6.24 shows the directly measured unsaturated permeability data versus the estimated unsaturated permeability data using the best-fit SWCCs by DR1 measured SWCC data in combination with the estimated SWCC data points for FMM estimation model for all the soil mixtures used in this study

Figure 6.24-Directly measured unsaturated permeability data versus estimated unsaturated permeability by FMM for the soil mixtures used in this study-DR1 measured SWCC data in combination with estimated SWCC data points

The coefficient of determination, R^2 was computed according to Equation 4.4 for all the directly measured unsaturated permeability data of all the soil. The value of R^2 was 0.920, indicating that FMM model was able to reasonably fit the directly measured data. The 95% confidence interval of the observed errors, $(k_{wi} - \hat{k}_{wi})$, between the directly measured unsaturated permeability data and the estimated unsaturated permeability data was computed according to Equation 4.5 and is shown in Figure 6.24 by dashed lines. The 95% confidence interval of the observed error was found to be $(-0.535, 0.635)$ that created a range of 1.17 in orders of magnitude.

Although the direct measurement of unsaturated permeability by trained professional and sophisticated equipment could provide the most reliable results, however, this is an expensive and time-consuming process. For instance, the time required for sample preparation and direct measurement of unsaturated permeability of 6 data points for a

typical soil mixture in the modified triaxial permeameter used in this study, was 6-8 weeks. However it is not possible to obtain the full unsaturated permeability curve based on limited data points. It is still required to conduct the full SWCC measurement to estimate the unsaturated permeability curve. Table 6.6 summarizes the approximate duration and lists of equipment required for each approach namely: 1) direct measurement of unsaturated permeability, k_w 2) full SWCC measurement and estimation of unsaturated permeability 3) SWCC measurement up to 100 kPa, grain-size distribution measurement for estimation of SWCC data point above 100 kPa and estimation of unsaturated permeability.

Table 6.6-Time and cost estimation for three different approaches to obtain unsaturated permeability of soils

Measurements	Approach	Duration (Weeks)	Equipment
Direct measurement of k_w and full SWCC measurement	1-Measurement of k_w 2-Full SWCC measurement* 3-Estimation of k_r by estimation models	6-8 + 10-12	Triaxial cell Pressure transducers Digital volume and pressure controllers Data acquisition system Tempe cell 5-bar pressure plate 15-bar pressure plate Salt solution system
Full SWCC measurement & k_s measurement	1-Full SWCC measurement 2-Estimation of k_r by estimation models	10-12	Tempe cell 5-bar pressure plate 15-bar pressure plate Salt solution system
DR1 (0.1-100 kPa) SWCC measurement & grain-size distribution measurement & k_s measurement	1-DR1 SWCC measurement 2-Grain-size distribution measurement 3-Estimation of SWCC data points beyond 100 kPa 4-Estimation of k_r by estimation models	1-2	Tempe cell Sieve Hydrometer

As shown in Table 6.6, for the second approach, even though the full measurement of SWCC data needs less expensive equipment as compared to that of direct measurement of unsaturated permeability, the full measurement of SWCC data still requires a substantial testing time. On the other hand, third approach requires both shorter testing time and much cheaper equipment which are easily available in most of geotechnical laboratories. From the results presented in Figure 6.24, it can be concluded that the third approach

proposed in this study can provide a reliable estimation of unsaturated permeability in a reasonable amount of testing time using inexpensive equipment.

Chapter 7 Conclusions and recommendations

7.1 Introduction

This chapter presents conclusions drawn from this study. Recommendations for future work are subsequently presented.

7.2 Conclusions

The conclusions drawn from this study are as follows:

1. Twenty sets of published data, which included measured SWCC data and measured $k(\psi)$ data, were collated from the literature. The root mean square error (RMSE) was computed for every soil in the database for the matrix of unsaturated permeability estimation models (i.e., 12 unsaturated permeability estimation models and the two existing models F&X-1994 and VG-1980). Comparisons between the average RMSE values for all estimation models on an overall basis suggest that VMM $m=1-1/n$, VCM $m=1-1/n$ and F&X-1994 models result in the lowest average and standard deviation RMSE values for all models for the selected soil database used in this study. On the other hand, the Burdine based estimation models (i.e., FBM, FBM $C(\psi) = 1$, VBM and VBM $m=1-1/n$) gave the highest RMSE values for 16 of the soils.
2. The conclusion of which model offers the best or worst estimation, based only on RMSE values, is soil database dependent and varies for different databases. Therefore, variation between all the estimation models was studied independently from the soil database by considering the potential effecting parameters identified in this study, such as the SWCC and k_r equations and measured SWCC data range, .
3. From the evaluation of the effect of the SWCC and k_r equations, it was concluded that if different SWCC best-fit equations were used, the resulting relative permeability curves would have different shapes even if the same relative permeability equation was used in developing the estimation model. On the other hand, if the same SWCC best-fit equation was used (or the SWCC curves were quite similar to each other), the resulting relative permeability curves would have marginal

variation even if different k_r equations were used in the development of the estimation model.

4. From the discussion, it can be concluded that the measured SWCC data range is the most important factor affecting the estimation of unsaturated permeability of soil. For a particular soil, if the best-fit SWCCs are close to each other over the entire suction range, the variation between the estimated unsaturated permeability curves will be quite small.
5. Of the SWCC best-fit models, F&X (1994) had the least sensitivity to an incomplete SWCC data range. The reason for the low sensitivity of the Fredlund and Xing best-fit SWCC equation (1994) to the measured SWCC data ranges is due to the correction factor, $C(\psi) = \left(1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)} \right)$, which forces the equation to be zero at a suction value of 10^6 kPa. Therefore, it can be concluded that the Fredlund and Xing (1994) equation is the most suitable SWCC equation in terms of sensitivity to an incomplete SWCC data range.
6. Based on the evaluation of the soil database and study of the sensitivity of relative permeability equations, it was concluded that Burdine based models are in less agreement with the experimental results and are less capable of reflecting changes in the SWCC curve. The sensitivity of the Childs Collis-George and Mualem based models was greater than that of the Burdine based models. This means that these two models can better reflect variation in the SWCC curves. The differences between the Childs Collis-George and Mualem based models were marginal. Therefore, it can be concluded that models based on the Childs Collis-George and Mualem equations are better at estimating the permeability of unsaturated soils and are almost similar in terms of sensitivity to the measured SWCC data range.
7. Based on the experimental results of the directly measured unsaturated permeability data and the estimated unsaturated permeability data, it was concluded that the SWCC should be measured over a wide range of suctions in order to obtain reliable estimation of unsaturated permeability of soils.
8. Based on RMSE values comparing the directly measured unsaturated permeability data with the estimation models for the kaolin-sand mixtures used in this study and sensitivity of the estimation models to the limited measured SWCC data, it can be concluded that FMM model was the best unsaturated permeability estimation model.

9. The estimated SWCC at a higher suction by the procedure proposed in this study (using the grain size distribution and SWCC measured data of DR1) successfully eliminated the variation in best-fit SWCCs and estimated unsaturated permeabilities due to the limited measured SWCC data. The proposed method also provided a good fit to the directly measured unsaturated permeability data. Therefore, this method can be used to eliminate the need for SWCC measurement over a wide suction range. This approach can provide a reliable estimation of unsaturated permeability in a reasonable amount of time with inexpensive equipment.

7.3 Recommendations

Based on the finding of this study and the available literature, the following studies are recommended:

1. The proposed matrix of unsaturated permeability estimation models can be further verified for natural soils considering the complete measurement of SWCC and estimation of SWCC by using the proposed procedure in this study.
2. The volume change of soil that potentially affects SWCC can be considered as another affecting parameter in the estimation of unsaturated permeability of soils.
3. The measurement of unsaturated permeability near saturation (near the air entry value of soil), which was not conducted in this study, should be considered.
4. The findings of this study can be applied to geotechnical problems, such as seepage and slope stability analyses.

7.4 Publications

A. Rahimi., H. Rahardjo, E.-C. Leong,. (2014). "Effect of range of soil-water characteristic curve measurements on estimation of permeability function." Under secon revision Engineering Geology

A. Rahimi., H. Rahardjo, E.-C. Leong,. (2014). Underlying parameters affecting estimation of unsaturated permeability of soils. Unsaturated Soils: Research & Applicatins. Sydney.1073-1077.

A. Rahimi., H. Rahardjo, E.-C. Leong,. (2014). "Effect of soil-water characteristic curve and relative permeability equations on estimation of unsaturated permeability of soils." To be submitted

References

Ahuja, L., J. Naney, Williams, R. D. (1985). "Estimating soil water characteristics from simpler properties or limited data." Soil Science Society of America Journal**49**(5): 1100-1105.

Arya, L. M. and J. F. Paris (1981). "A Physicoempirical Model To Predict The Soil Moisture Characteristic From Particle-size Distribution And Bulk Density Data." Soil Sci. Soc. Am. J.**45**(6): 1023-1030.

ASTM (1997), "Test Method for Permeability of Granular Soils (Constant Head)" Annual books of ASTM Standard D2434-68**4.8(4)**: 202-206 West Conshohocken, Philadelphia.

ASTM (1997), "Standard Classification of Soils for Engineering Purposes (Unified Soil Classification System)", Annual books of ASTM Standard D2487-93**4.8(4)**: 217-227 West Conshohocken, Philadelphia.

ASTM (1997), "Test Method for Laboratory Compaction Characteristic of Soil Using Standard Effort", Annual Books of ASTM Standard D698 **4.8(4)**: 77-87 West Conshohocken, Philadelphia.

ASTM (2003). "Determination of the Soil-water Characteristic Curve for Desorption Using a Hanging Column, Pressure Extractor, Chilled Mirror Hydrometer, and/or Centrifuge." Annual Books of ASTM Standard D6836-02 West Conshohocken, Philadelphia.

Assouline, S. (2001). "A model for soil relative hydraulic conductivity based on the water retention characteristic curve." Water Resources Research**37**: 265-272.

Assouline, S. (2004). "Correction to "A model for soil relative hydraulic conductivity based on the water retention characteristic curve"." Water Resources**40**(2): W02901.

Aubertin, M., M.Mbonimpa, B. Bussiere, R. P. Chapuis, (2003). "A model to predict the water retention curve from basic geotechnical properties." Canadian Geotechnical Journal**40**(6): 1104-1122.

Averjanov, S. F. (1950). " About permeability of subsurface soils in case of incomplete saturation." English Collection**7**: 19-21.

Azam, S., M. Ito, F.Khan, (2013). "Influence of cracks on soil water characteristic curve." Advances in Unsaturated Soils: 217.

Agus, S. S., E. C. Leong, H. Rahardjo. (2005). "Estimating permeability functions of Singapore residual soils." Engineering Geology**78**(1-2): 119-133.

Barbour, S. L. (1998). "The soil-water characteristic curve : a historical perspective." Canadian Geotechnical Journal**35**: 873-894.

Barden, L., Pavlakis, G., 1971. "Air and water permeability of compacted unsaturated soil." Journal of Soil Science**22**: 302-318.

Baver, L. D., W. H. Gardner, et al. (1972). Soil physics / [by] L. D. Baver, Walter H. Gardner [and] Wilford R. Gardner, New York : Wiley, [1972] 4th ed.

Blake, F. C. (1922). "The resistance of packing to fluid flow." Transactions of the American Institute of Chemical Engineers**14**: 415-421.

Bouma, J. (1989). Using soil survey data for quantitative land evaluation. In: Advance in soil science, B.A. Stewart (ed.). Springer-Verlag New York. Vol. 9 (1989) 225-239.

Brooks, R. H. and A. T. Corey (1964). "Hydraulic properties of porous media." American Society of Agricultural Engineers -- Transactions**7**(1): 26-28.

Bruce, R. and A. Klute (1956). "The measurement of soil moisture diffusivity." Soil Science Society of America Journal**20**(4): 458-462.

Brutsaert, W. (1966). "Probability Laws for Pore-Size Distributions." Soil Science**101**(2): 85-92.

Buckingham, E. (1907). "Studies of the movement of soil moisture." USDA Bureau of Soils Bulletin No. 38.

Burdine, N. T. (1953). "Relative permeability calculations from pore-size distribution data." American Institute of Mining, Metallurgical, and Petroleum Engineers**198**: 71-78.

Burdine, N. T., L. S. Gournay, et al. (1950). "Pore Size Distribution of Petroleum Reservoir Rocks." Journal of Petroleum Technology**2**(7): 195-204.

Carman, P. C. (1937). "Fluid Flow Through Granular Beds." Transactions - Institution of Chemical Engineers**15**: 150-166.

Carman, P. C. (1956). Flow of gases through porous media. London, Butterworths Scientific Publications

Childs, E. C. (1969). An introduction to the physical basis of soil water phenomena. New York, Wiley Interscience.

Childs, E. C. and N. Collis-George (1950). "The permeability of porous materials." Proceedings of the Royal Society of London, Series A (Mathematical and Physical Sciences)**201**: 392-404.

Chin, K.-B., E.-C. Leong, H. Rahardjo, (2010). "A simplified method to estimate the soil-water characteristic curve." Canadian Geotechnical Journal**47**(12): 1382-1400.

Cihan, A., J. S. Tyner, E. Perfect, (2009). "Predicting relative permeability from water retention: A direct approach based on fractal geometry." Water Resources Research**45**(4).

Croney, D. and J. Coleman (1961). "Pore pressure and suction in soil."

Darcy, H. (1856). Les fontaines publiques de la ville de Dijon. Paris: Dalmont.

Ebrahimi-B, N., G. F. N. Gitirana Jr., D. G. Fredlund, M. D. Fredlund, L. Samarasekara, (2004). A lower limit for the water permeability coefficient. Proceedings of the fifty seventh canadian geotechnical conference, Quebec. **1**: 12-19.

Edlefsen, N. E. and A. B. C. Anderson (1943). "Thermodynamics of Soil Moisture." Hilgardia**15**: 31-298.

Elrick, D. E. and D. H. Bowman (1964). " Note on an Improved Apparatus for Soil-moisture Flow Measurement." Soil Science Society of America Proceeding**28**: 450-453.

Fair, G. M. and L. P. Hatch (1933). "Fundamental factors governing the streamline flow of water through sand." Journal of the American Water Works Association**25**: 1551-11565.

Falkovich, G. (2011). Fluid mechanics [electronic resource] : a short course for physicists / Gregory Falkovich, Cambridge ; New York : Cambridge University Press, 2011.

Fatt, I. and H. Dykstra (1951). "Relative permeability studies." American Institute of Mining, Metallurgical, and Petroleum Engineers**192**: 249-255.

Fleureau, J. and S. Taibi (1994). "A new apparatus for the measurement of water–air permeabilities." First international congress on environmental geotechnics, July.

Fredlund, D. G. (2000). "The 1999 R.M. Hardy Lecture: The implementation of unsaturated soil mechanics into geotechnical engineering." Canadian Geotechnical Journal**37**: 963–986.

Fredlund, D. G. (2002). Use of soil-water characteristic curve in the implementation of unsaturated soil mechanics. Third International Conference on Unsaturated Soils. Pecife, Brazil.

Fredlund, D. G. (2006). "Unsaturated Soil Mechanics in Engineering Practice." Journal of Geotechnical & Geoenvironmental Engineering**132**(3): 286-321.

Fredlund, D. and S. Houston (2013). "Interpretation of soil-water characteristic curves when volume change occurs as soil suction is changed." Advances in unsaturated soils. CRC Press, Boca Raton, FL: 15-31.

Fredlund, D. G. and N. R. Morgenstern (1977). "Stress state variables for unsaturated soils." Journal of Geotechnical and Geoenvironmental Engineering**103**(GT5): 447-466.

Fredlund, D. G. and H. Rahardjo (1993). Soil mechanics for unsaturated soils / D.G. Fredlund, H. Rahardjo, New York : Wiley, 1993.

Fredlund, D. G., H. Rahardjo, Fredlund, M. D. (2012). Unsaturated soil mechanics in engineering practice, Wiley.

Fredlund, D. G. and A. Xing (1994). "Equations for the soil-water characteristic curve." Canadian Geotechnical Journal**31**(4): 521-532.

Fredlund, D. G., A. Xing, S. Huang (1994). "Predicting the permeability function for unsaturated soils using the soil-water characteristic curve." Canadian Geotechnical Journal**31**(4): 533-546.

Fredlund, M. D., G. W. Wilson, D. G. Fredlund (2002). "Use of the grain sizedistribution for estimation of the soil-water characteristic curve." Canadian Geotechnical Journal**39**(5): 1103-1117.

Gardner, W. R. (1956). "Calculation of Capillary Conductivity from Pressure Plate Outflow Data1." Soil Sci. Soc. Am. J.**20**(3): 317-320.

Gardner, W. R. (1958). Mathematics of isothermal water conduction in unsaturated soils. 40 International Symposuim on Physio-Chemical Phenomenon in Soils. Washington D. C., Highway Research Board: 78-87.

Garg, A., A. Garg, K. Tai, S. Barontini, A. Stokes (2014). "A Computational Intelligence-Based Genetic Programming Approach for the Simulation of Soil Water Retention Curves." Transport in Porous Media**103**(3): 497-513.

Gates, J. I. and W. Tempelaar Lietz (1950). Relative Permeabilities of California Cores by the Capillary - Pressure Method. Drilling and Production Practice, American Petroleum Institute: 285-298.

Guan, G., H. Rahardjo, E. C. Leong (2010). "Shear Strength Equations for Unsaturated Soil under Drying and Wetting." Journal of Geotechnical and Geoenvironmental Engineering**136**(4): 594-606.

Gupta, S. C. and W. E. Larson (1979). "Estimating soil water retention characteristics from particle size distribution, organic matter percent, and bulk density." Water Resour. Res.**15**(6): 1633-1635.

- Hamilton, D., G. Gloeckler, S. M. Krimigis, L. J. Lanzerotti (1981). "Composition of nonthermal ions in the Jovian magnetosphere." Journal of Geophysical Research: Space Physics (1978–2012)**86**(A10): 8301-8318.
- Head, K. H. (1998). Manual of soil laboratory testing: Effective stress tests, John Wiley & Sons.
- Hilf, J. W. (1956). An Investigation of Pore-water Pressure in Compacted Cohesive Soils. Doctor of Philosophy Dissertation, Tech. Memo. No. 654, U. S. Bureau of Reclamation, Department of the Interior, Design and Construction Division
- Hillel, D. (1982). Introduction to soil physics / Daniel Hillel, New York : Academic Press, 1982.
- Holtz, R. D. and W. D. Kovacs (1981). An introduction to geotechnical engineering.
- Huang, S., Fredlund, D.G., Barbour, S.L., 1998. "Measurement of the coefficient of permeability for a deformable unsaturated soil using a triaxial permeameter." Canadian Geotechnical Journal**35**: 426-432.
- Hunt, A., R. Ewing, R. Horton (2013). "What's Wrong with Soil Physics?" Soil Science Society of America Journal**77**(6): 1877-1887.
- Hunt, A. G. and G. W. Gee (2002). "Water-retention of fractal soil models using continuum percolation theory." Vadose Zone Journal**1**(2): 252-260.
- Irmay, S. (1954). "On the hydraulic conductivity of unsaturated soils." Trans. American Geophysical Union**35**: 463-467.
- Israelachvili, J. N. (1992). Intermolecular and surface forces / Jacob N. Israelachvili, London : Academic Press, 1992. 2nd ed.
- Iyer, K., S. Jayanth, S. Gurnani, D. Singh (2013). "Influence of Initial Water Content and Specimen Thickness on the SWCC of Fine-Grained Soils." International Journal of Geomechanics**13**(6): 894-899.

Jaafar, R. and W. J. Likos (2011). "Estimating water retention characteristics of sands from grain size distribution using idealized packing conditions." Geotechnical Testing Journal**34**(5).

Johari, A., G. Habibagahi, A. Ghahremani (2006). "Prediction of soil–water characteristic curve using genetic programming." Journal of Geotechnical and Geoenvironmental Engineering**132**(5): 661-665.

Klute, A. (1965). Laboratory measurement of hydraulic conductivity of unsaturated soils. In Methods of soil analysis Madison. Wis. , Monograph 9, Part 1, American Society of Agronomy.

Klute, A. (1972). "The Determination of the Hydraulic Conductivity and Diffusivity of Unsaturated Soils1." Soil Science**113**(4): 264-276.

Klute, A. and C. Dirksen (1986). "Hydraulic Conductivity and Diffusivity: Laboratory Methods." Methods of Soil Analysis: Part 1—Physical and Mineralogical Methodssssabookseries(methodsofsoilan1): 687-734.

Krisdani, H., H. Rahardjo, E.C. Leong (2009). "Use of instantaneous profile and statistical methods to determine permeability functions of unsaturated soils." Canadian Geotechnical Journal**46**(7): 869-874.

Kool, J., J. Parker, M. Th van Genuchten (1985). "Determining soil hydraulic properties from one-step outflow experiments by parameter estimation: I. Theory and numerical studies." Soil Science Society of America Journal**49**(6): 1348-1354.

Kosugi, K. (1999). "General model for unsaturated hydraulic conductivity for soils with lognormal pore-size distribution." Soil Science Society of America Journal**63**(2): 270-277.

Kovács, G. (1981). Seepage hydraulics / by György Kovács ; [translated by Katalin Kovács], Amsterdam ; New York : Elsevier Scientific Pub. Co. : distribution for the U.S.A. and Canada, Elsevier/North-Holland, 1981.

Kunze, R. J., G. Uehara, K. Graham (1968). "Factors Important In The Calculation Of Hydraulic Conductivity." Soil Sci. Soc. Am. J.**32**(6): 760-765.

Kozeny, J. (1927). "Ueber kapillare Leitung des Wassers im Boden." Sitzungsberichte Wiener Akademie**136**(2a): 271–306.

Lambe, T. W. and R. V. Whitman (1979). Soil mechanics, SI version, Wiley.

Lang, A. (1967). "Osmotic coefficients and water potentials of sodium chloride solutions from 0 to 40 C." Australian Journal of Chemistry**20**(9): 2017-2023.

Lee, I.-M., S.-G. Sung, G.-C. Cho (2005). "Effect of stress state on the unsaturated shear strength of a weathered granite." Canadian Geotechnical Journal**42**(2): 624-631.

Leong, E. C. and H. Rahardjo (1997). "Review of soil-water characteristic curve equations." Journal of Geotechnical and Geoenvironmental Engineering**123**(12): 1106-1117.

Li, X., L. M. Zhang, D. G. Fredlund (2009). "Wetting front advancing column test for measuring unsaturated hydraulic conductivity." Canadian Geotechnical Journal**46**(12): 1431-1445.

Lu, N. and W. J. Likos (2004). Unsaturated soil mechanics / Ning Lu and William J. Likos, Hoboken, N.J. : J. Wiley, c2004.

Marshall, T. J. and J. W. Holmes (1988). Soil physics / T.J. Marshall, J.W. Holmes, Cambridge [England] ; New York : Cambridge University Press, 1988. 2nd ed.

Massey, B. S. and A. J. Ward-Smith (2012). Mechanics of fluids / Bernard S. Massey ; revised by John Ward-Smith, Abingdon, Oxon ; New York, NY : Spon Press, 2012. 9th ed.

McKee, C. R. and A. C. Bumb (1984). The importance of unsaturated flow parameters in designing a hazardous waste site. In Hazardous Waste and Environmental Emergencies, Hazardous Materials Control Research Institute National Conference. Houston, Tex, Hazardous Materials Control Research Institute, Silver Spring: 50–58.

McKee, C. R. and A. C. Bumb (1987). Flow-testing coalbed methane production wells in the presence of water and gas. Society of Petroleum Engineers (SPE) Formation Evaluation. Richardson, Tex.: 599–608.

Meerdink, J., C. Benson, M. V. Khire (1996). "Unsaturated hydraulic conductivity of two compacted barrier soils." Journal of Geotechnical Engineering**122**(7): 565-576.

Miller, E. E. and R. D. Miller (1955a). "Theory Of Capillary Flow: I. Practical Implications." Soil Sci. Soc. Am. J.**19**(3): 267-271.

Miller, R. D. and E. E. Miller (1955b). "Theory Of Capillary Flow: Ii. Experimental Information." Soil Sci. Soc. Am. J.**19**(3): 271-275.

Millington, R. J. and J. P. Quirk (1961). "Permeability of porous solids." Transactions of the Faraday Society**57**: 1200-1207.

Mohammadi, M. H. and M. Vanclooster (2011). "Predicting the soil moisture characteristic curve from particle size distribution with a simple conceptual model." Vadose Zone Journal**10**(2): 594-602.

Moore, R. E. (1939). Water conduction from shallow water tables. *Hilgardia*. **12**: 383-426.

Mualem, Y. (1976). "New model for predicting the hydraulic conductivity of unsaturated porous media." Water Resources Research**12**(3): 513-522.

Mualem, Y. (1986). "Hydraulic conductivity of unsaturated soils: predictions and formulas " Methods of soil analysis, part 1, physical and mineralogical methods (2nd edition), American Society of Agronomy, Agronomy Monographs 9(1)**5**(1).

Mualem, Y. and G. Dagan (1978). "Hydraulic Conductivity of Soils: Unified Approach to the Statistical Models." Soil Sci. Soc. Am. J.**42**(3): 392-395.

Ng, C. W. W. and A. K. Leung (2011). "Measurements of Drying and Wetting Permeability Functions Using a New Stress-Controllable Soil Column." Journal of Geotechnical and Geoenvironmental Engineering**138**(1): 58-68.

Ng, C. W. W. and Y. W. Pang (2000). "Influence of stress state on soil-water characteristics and slope stability." Journal of Geotechnical and Geoenvironmental Engineering**126**(2): 157-166.

Olson, R. E. and D. E. Daniel (1981). "Measurement of the Hydraulic Conductivity of Fine-Grained Soils." Permeability and Groundwater Contaminant Transport. ASTM STP 746: 18-64.

Ong, B. H. (1999). Shear Strength and Volume Change of Unsaturated Soil. Master of Engineering Thesis, Nanyang Technological University.

Pachepsky, Y. A., D. Timlin, G. Y. Varallyay (1996). "Artificial neural networks to estimate soil water retention from easily measurable data." Soil Science Society of America Journal**60**(3): 727-733.

Peters, A. and W. Durner (2008). "A simple model for describing hydraulic conductivity in unsaturated porous media accounting for film and capillary flow." Water Resources Research**44**(11).

Prunty, L. (2003). "Soil water retention and conductivity when vapor flow is important." Journal of Irrigation and Drainage Engineering**129**(3): 201-207.

Purcell, W. R. (1949). "Capillary pressures—their measurement using mercury and the calculation of permeability therefrom." Trans. American Institute of Mining, Metallurgical, and Petroleum Engineers**186**: 39-46.

Richards, B. G. (1965). "Measurement of the free energy of soil moisture by the psychrometric technique using thermistors" In Moisture equilibria and moisture changes in soils beneath covered areas. Edited by G.D. Aitchis: 39-46.

Rassam, D. W. and D. J. Williams (1999). "Unsaturated Hydraulic Conductivity of Mine Tailings under Wetting and Drying Conditions." Geotechnical Testing Journal**22**(2): 144-152.

Richards, L. A. (1931). "Capillary conduction of liquids through porous mediums." Physics (American Institute of Physics; American Physical Society; Society of Rheology)**1**(5): 318-333.

Richards, S. J. and L. V. Weeks (1953). "Capillary conductivity values from moisture yield and tension measurements on soil columns." Soil Science Society of America Journal**17**(3): 206-209.

Rowe, P. P. (1960). An equation for unsaturated flow based upon the Darcy equation and an analogy of the Poiseuille equation Hanford Atomic Products Operation. .

Saito, H., J. Šimůnek, B. P. Mohanty (2006). "Numerical Analysis of Coupled Water, Vapor, and Heat Transport in the Vadose Zone." Vadose Zone J.**5**(2): 784-800.

Samingan, A. S. (2001). Hydraulic properties of residual soils. Master of Engineering, Nanyang Technological University.

Samingan, A. S., E.-C. Leong, H. Rahardjo (2003). "A flexible wall permeameter for measurements of water and air coefficients of permeability of residual soils." Canadian Geotechnical Journal**40**(3): 559-574.

Saxton, K., W. J. Rawls, J. S. Romberger, R. I. Papendick (1986). "Estimating generalized soil-water characteristics from texture." Soil Science Society of America Journal**50**(4): 1031-1036.

Schaap, M. G. and F. J. Leij (2000). "Improved prediction of unsaturated hydraulic conductivity with the Mualem-van Genuchten model." Soil Science Society of America Journal**64**(3): 843-851.

Sillers, W. S., D. G. Fredlund, N. Zakerzadeh (2001). "Mathematical attributes of some soil-water characteristic curve models." Geotechnical and Geological Engineering**19**(3-4): 243-283.

Šimůnek, J., R. Angulo-Jaramillo, M. G. Schaap, J. P. Vandervaere, M. T. van Genuchten (1998). "Using an inverse method to estimate the hydraulic properties of crusted soils from tension-disk infiltrometer data." Geoderma**86**(1): 61-81.

Smettem, K. and P. Gregory (1996). "The relation between soil water retention and particle size distribution parameters for some predominantly sandy Western Australian soils." Soil Research**34**(5): 695-708.

SoilVision Systems Ltd. (2006), "SoilVision: a knowledge based database system for unsaturated-saturated soil properties (computer programme), version 4.0." SoilVision Systems Ltd., Saskatoon, Canada.

Terzaghi, K. (1943). Theoretical soil mechanics, New York : J. Wiley and sons, inc.: London, Chapman and Hall, limited, [1943].

Thu, T. M., H. Rahardjo, E. C. Leong (2007). "Soil-water characteristic curve and consolidation behavior for a compacted silt." Canadian Geotechnical Journal**44**(3): 266-275.

To-Viet, N., T.-K. Min, H. Shin (2013). "Using inverse analysis to estimate hydraulic properties of unsaturated sand from one-dimensional outflow experiments." Engineering Geology**164**: 163-171.

Tuller, M. and D. Or (2001). "Hydraulic conductivity of variably saturated porous media: Film and corner flow in angular pore space." Water Resources Research**37**(5): 1257-1276.

Valiantzas, J. D. (2011). "Combined Brooks-Corey/Burdine and van Genuchten/Mualem Closed-Form Model for Improving Prediction of Unsaturated Conductivity." Journal of Irrigation & Drainage Engineering**137**(4): 223-233.

van Genuchten, M. T. (1980). "Closed-form equation for predicting the hydraulic conductivity of unsaturated soils." Soil Science Society of America Journal**44**(5): 892-898.

van Genuchten, M. T. and D. Nielsen (1985). "On describing and predicting the hydraulic properties of unsaturated soils." Ann. Geophys**3**(5): 615-628.

Vanapalli, S., W. Sillers, M. D. Fredlund (1998). The meaning and relevance of residual state to unsaturated soils. 51st Canadian geotechnical conference, Citeseer.

Vanapalli, S. K., D. G. Fredlund, D. E. Pufahl (1999). "The influence of soil structure and stress history on the soil-water characteristics of a compacted till." Geotechnique**49**(2): 143-159.

Vanapalli, S. K., D. G. Fredlund, D. E. Pufahl, A. W. Clifton (1996). "Model for the prediction of shear strength with respect to soil suction." Canadian Geotechnical Journal**33**(3): 379-392.

Wang, Y. H. and D. G. Fredlund (2003). Towards a better understanding of the role of the contractile skin. Second Asian Conference on Unsaturated soils. Osaka, Japan.

Wösten, J., Y. A. Pachepsky, W. J. Rawls (2001). "Pedotransfer functions: bridging the gap between available basic soil data and missing soil hydraulic characteristics." Journal of hydrology**251**(3): 123-150.

Wyllie, M. R. J. and G. H. F. Gardner (1958). "The generalized Kozeny-Carman equation." World Oil**146**(4): 121-128.

Wyllie, M. R. J. and M. B. Spangler (1952). "Application of electrical resistivity measurements to problem of fluid flow in porous media." American Association of Petroleum Geologists -- Bulletin**36**(2): 359-403.

Yang, H., H. Rahardjo, E. C. Leong, D. G. Fredlund (2004). "Factors affecting drying and wetting soil-water characteristic curves of sandy soils." Canadian Geotechnical Journal**41**(5): 908-920.

Yuster, S. T. (1951). Theoretical considerations of multiphase flow in idealized capillary systems. Proceedings of the 3rd World Petroleum Congress.

Zapata, C. E. (1999). Uncertainty in soil-water-characteristic curve and impacts on unsaturated shear strength predictions, Arizona State University.

Appendix A

The integration was solved according to following numerical procedure:

$$a = \ln(\psi_{aev})$$

$$b = \ln(1000000)$$

$$a = y_1 < y_2 < \dots < y_n < y_{N+1} = b$$

$$\Delta y = \frac{b - a}{N}$$

The denominator of equation was evaluated as follows:

$$\int_{\ln(\psi_{aev})}^b \frac{\theta(e^y) - \theta_s}{e^y} \theta'(e^y) dy \approx \Delta y \sum_{i=1}^N \frac{\theta(e^y) - \theta_s}{e^y} \theta'(e^y)$$

The derivative of soil-water characteristic curve was described as follow:

$$\theta'(\psi) = \frac{-1}{(C_r + \psi) \ln \left[1 + \frac{1000000}{C_r} \right]} \frac{\theta_s}{\left[\ln \left(e + \left(\frac{\psi}{a} \right)^n \right) \right]^m} - C(\psi) \frac{\theta_s}{\left[\ln \left(e + \left(\frac{\psi}{a} \right)^n \right) \right]^{m+1}} \frac{mn \left(\frac{\psi}{a} \right)^{n-1}}{a \left[e + \left(\frac{\psi}{a} \right)^n \right]}$$

The numerator of equation was evaluated as follows:

$$\int_{\ln(\psi)}^b \frac{\theta(e^y) - \theta_\psi}{e^y} \theta'(e^y) dy \approx \Delta y \sum_{i=1}^N \frac{\theta(e^y) - \theta_\psi}{e^y} \theta'(e^y)$$

Therefore, the following equation shows the numerical integration solution for Fredlund and Xing relative hydraulic conductivity equation.

$$K_r(\psi) = \sum_{i=j}^N \frac{\theta(e^y) - \theta_\psi}{e^y} \theta'(e^y) \bigg/ \sum_{i=1}^N \frac{\theta(e^y) - \theta_s}{e^y} \theta'(e^y)$$

It should be noted that in this study, N was selected to be equal to 100 and integration was performed for the entire soil database.

Appendix B

Soil-water characteristic curves and estimated unsaturated permeabilities for soil database
(Table 4.1)

Figure B. 1-Best-fit soil-water characteristic curves-Beit Netofa Clay (S2)

Figure B. 2-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models- Beit Netofa Clay (S2)

Figure B. 3-Best-fit soil-water characteristic curves-Yolo light Clay (S3)

Figure B. 4-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models- Yolo light Clay (S3)

Figure B. 5-Best-fit soil-water characteristic curves-Touchet silt loam (S4)

Figure B. 6-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Touchet silt loam (S4)

Figure B. 7-Best-fit soil-water characteristic curves-Columbia Sandy loam (S5)

Figure B. 8-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Columbia Sandy loam (S5)

Figure B. 9-Best-fit soil-water characteristic curves-Hygiene Sandstone (S6)

Figure B. 10-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Hygiene Sandstone (S6)

Figure B. 11-Best-fit soil-water characteristic curves-Superstition Sand (S7)

Figure B. 12-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Superstition Sand (S7)

Figure B. 13-Best-fit soil-water characteristic curves-Silt loam (S8)

Figure B. 14-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Silt loam (S8)

Figure B. 15-Best-fit soil-water characteristic curves-Wenatchee Silty Clay (S9)

Figure B. 16-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Wenatchee Silty Clay (S9)

Figure B. 17-Best-fit soil-water characteristic curves-Live Oak Red Clay (S10)

Figure B. 18-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Live Oak Red Clay (S10)

Figure B. 19-Best-fit soil-water characteristic curves-CDT (S11)

Figure B. 20-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-CDT (S11)

Figure B. 21-Best-fit soil-water characteristic curves-UP-1 (S13)

Figure B. 22-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-UP-1 (S13)

Figure B. 23-Best-fit soil-water characteristic curves-UP-2 (S14)

Figure B. 24-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-UP-2 (S14)

Figure B. 25-Best-fit soil-water characteristic curves-UP-3 (S15)

Figure B. 26-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-UP-3 (S15)

Figure B. 27-Best-fit soil-water characteristic curves-UP-4 (S16)

Figure B. 28-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-UP-4 (S16)

Figure B. 29-Best-fit soil-water characteristic curves-Weld Silty Clay (S17)

Figure B. 30-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Weld Silty Clay (S17)

Figure B. 31-Best-fit soil-water characteristic curves-Fine Sand (S18)

Figure B. 32-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Fine Sand (S18)

Figure B. 33-Best-fit soil-water characteristic curves-Volcanic Sand (S19)

Figure B. 34-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-Volcanic Sand (S19)

Figure B. 35-Best-fit soil-water characteristic curves-G.E. No.2 Sand (S20)

Figure B. 36-Estimated unsaturated permeability curves by a matrix of unsaturated permeability models-G.E. No.2 Sand (S20)

Appendix C

Results related to effect of measured SWCC data ranges

Figure C. 1-Soil-water characteristic curves of UP-1-Different measured data ranges-
Fredlund and Xing (1994)

Figure C. 2-Soil-water characteristic curves of UP-1-Different measured data ranges-
Fredlund and Xing (1994) with $C(\psi)=1$

Figure C. 3-Soil-water characteristic curves of UP-1-Different measured data ranges-van Genuchten (1980)

Figure C. 4-Soil-water characteristic curves of UP-1-Different measured data ranges-van Genuchten (1980) with $m=1-1/n$

Figure C. 5-Soil-water characteristic curves of UP-1-Different best-fit equations for DR1

Figure C. 6-Soil-water characteristic curves of UP-1-Different best-fit equations for DR2

Figure C. 7-Soil-water characteristic curves of UP-1-Different best-fit equations for DR3

Figure C. 8-Soil-water characteristic curves of UP-1-Different best-fit equations for DR4

Figure C. 9-Soil-water characteristic curves of UP-2-Different measured data ranges-
Fredlund and Xing (1994)

Figure C. 10-Soil-water characteristic curves of UP-2-Different measured data ranges-
Fredlund and Xing (1994) with $C(\psi)=1$

Figure C. 11-Soil-water characteristic curves of UP-2-Different measured data ranges-van Genuchten (1980)

Figure C. 12-Soil-water characteristic curves of UP-2-Different measured data ranges-van Genuchten (1980) with $m=1-1/n$

Figure C. 13-Soil-water characteristic curves of UP-2-Different best-fit equations for DR1

Figure C. 14-Soil-water characteristic curves of UP-2-Different best-fit equations for DR2

Figure C. 15-Soil-water characteristic curves of UP-2-Different best-fit equations for DR3

Figure C. 16-Soil-water characteristic curves of UP-2-Different best-fit equations for DR4

Figure C. 17-Soil-water characteristic curves of UP-4-Different measured data ranges-
Fredlund and Xing (1994)

Figure C. 18-Soil-water characteristic curves of UP-4-Different measured data ranges-
Fredlund and Xing (1994) with $C(\psi)=1$

Figure C. 19-Soil-water characteristic curves of UP-4-Different measured data ranges-van Genuchten (1980)

Figure C. 20-Soil-water characteristic curves of UP-4-Different measured data ranges-van Genuchten (1980) with $m=1-1/n$

Figure C. 21-Soil-water characteristic curves of UP-4-Different best-fit equations for DR1

Figure C. 22-Soil-water characteristic curves of UP-4-Different best-fit equations for DR2

Figure C. 23-Soil-water characteristic curves of UP-4-Different best-fit equations for DR3

Figure C. 24-Soil-water characteristic curves of BNC-Different measured data ranges-
Fredlund and Xing (1994)

Figure C. 25-Soil-water characteristic curves of BNC-Different measured data ranges-Fredlund and Xing (1994) with $C(\psi)=1$

Figure C. 26-Soil-water characteristic curves of BNC-Different measured data ranges-van Genuchten (1980)

Figure C. 27-Soil-water characteristic curves of BNC-Different measured data ranges-van Genuchten (1980) with $m=1-1/n$

Figure C. 28-Soil-water characteristic curves of BNC-Different best-fit equations for DR1

Figure C. 29-Soil-water characteristic curves of BNC-Different best-fit equations for DR2

Figure C. 30-Soil-water characteristic curves of BNC-Different best-fit equations for DR3

Figure C. 31-Soil-water characteristic curves of WSC-Different measured data ranges-Fredlund and Xing (1994)

Figure C. 32-Soil-water characteristic curves of WSC-Different measured data ranges-Fredlund and Xing (1994) with $C(\psi)=1$

Figure C. 33-Soil-water characteristic curves of WSC-Different measured data ranges-van Genuchten (1980)

Figure C. 34-Soil-water characteristic curves of WSC-Different measured data ranges-van Genuchten (1980) with $m=1-1/n$

Figure C. 35-Soil-water characteristic curves of WSC-Different best-fit equations for DR1

Figure C. 36-Soil-water characteristic curves of WSC-Different best-fit equations for DR2

Figure C. 37-Soil-water characteristic curves of WSC-Different best-fit equations for DR3

Figure C. 38-Soil-water characteristic curves of LRC-Different measured data ranges-Fredlund and Xing (1994)

Figure C. 39-Soil-water characteristic curves of LRC-Different measured data ranges-Fredlund and Xing (1994) with $C(\psi)=1$

Figure C. 40-Soil-water characteristic curves of LRC-Different measured data ranges-van Genuchten (1980)

Figure C. 41-Soil-water characteristic curves of LRC-Different measured data ranges-van Genuchten (1980) with $m=1-1/n$

Figure C. 42-Soil-water characteristic curves of LRC-Different best-fit equations for DR1

Figure C. 43-Soil-water characteristic curves of LRC-Different best-fit equations for DR2

Figure C. 44-Soil-water characteristic curves of LRC-Different best-fit equations for DR3

Table C. 1-Computed RMSE for DR1 for all soils

	UP-1	UP-2	UP-3	UP-4	BNC	WSC	LRC	DR1
F&X-1994	1.431	2.816	4.683	1.390	0.778	10.129	9.352	
VG-1980	2.692	2.167	7.454	3.319	2.172	8.887	16.451	
FCM	1.844	2.816	5.305	0.446	1.113	10.511	11.314	
FMM	2.487	2.157	5.801	0.414	1.685	11.538	11.928	
FBM	4.681	0.556	7.792	0.407	5.443	13.655	13.009	
FCM C(ψ)=1	1.223	2.812	5.327	0.750	2.231	13.705	12.560	
FMM C(ψ)=1	1.811	2.223	5.752	0.874	2.260	14.920	13.257	
FBM C(ψ)=1	3.164	1.112	6.093	0.721	2.830	15.889	13.612	
VCM m=1-1/n	1.007	3.042	6.069	0.528	2.037	15.332	11.489	
VMM m=1-1/n	1.693	2.472	6.637	0.580	1.972	16.677	12.199	
VBM m=1-1/n	3.851	1.392	6.654	0.582	2.057	16.701	12.739	
VCM	0.558	3.042	5.424	0.541	2.271	10.855	30.383	
VMM	0.904	2.472	5.858	0.539	2.219	11.752	31.668	
VBM	1.995	1.392	5.955	0.460	2.773	12.789	22.031	

Table C. 2-Computed RMSE for DR4-UP-1, UP-2 and UP-3 soils

	UP-1	UP-2	UP-3	DR4
F&X-1994	2.304	2.422	3.062	
VG-1980	0.393	0.948	0.822	
FCM	3.590	2.427	3.994	
FMM	3.850	1.946	4.442	
FBM	6.123	0.696	7.415	
FCM C(ψ)=1	3.580	2.393	3.993	
FMM C(ψ)=1	3.840	1.757	4.441	
FBM C(ψ)=1	6.119	0.443	7.414	
VCM m=1-1/n	1.358	2.097	0.299	
VMM m=1-1/n	0.642	1.395	0.664	
VBM m=1-1/n	2.086	0.411	3.095	
VCM	3.477	2.564	3.926	
VMM	3.759	2.371	4.386	
VBM	6.052	2.647	7.370

Figure C. 45-Computed variation parameters for vanGenuchten (1980) $m=1-1/n$ based models (a) UP-2 (b) UP-3 (c) UP-4 (d) Beit Netofa Clay (e) Wenatchee Silty Clay (f) Live Oak Red Clay

Appendix D

Shrinkage test

The shrinkage limit of a soil is defined as the water content corresponding to a saturated specimen at the void ratio achieved (minimum volume) upon drying to zero water content (Fredlund and Houston, 2013). Fredlund and Houston (2013) recommended that the entire shrinkage curve, the plot of void ratio versus gravimetric water content, from an initially saturated soil condition to completely oven-dry conditions is important for the interpretation of SWCC data. Therefore, the shrinkage curve test was conducted for 100K0S, 90K10S, 80K20S and 50K50S soil mixtures in this study. The test procedure was according to that recommended by Fredlund and Houston (2013).

Soil specimen with a height of 30 mm and a diameter of 50 mm was prepared according to Section 4.8.2.2 and, after saturation, placed on a digital balance. The dimensions of the soil specimens were small enough such that cracking of the soil did not occur during the drying process. The specimen was dried through evaporation to the atmosphere. A digital micrometer was used for the measurement of the volume at different stages of drying as shown in Figure D. 1.

Figure D. 1-Shrinkage specimen, digital micrometer and digital balance used for measurement of diameter, thickness and weight of the shrinkage specimen

The mass and volume of soil specimen was measured every two to three hours. The height and diameter of the soil specimen were measured at eight different locations on the specimens as shown in Figure D. 1. The resulting shrinkage curves of the soil mixtures were then best-fit using the equation by Fredlund et al., (1996, 2002) as follows:

$$e(w) = a_{sh} \left[\frac{w^{c_{sh}}}{b_{sh}^{c_{sh}}} + 1 \right]^{\left[\frac{1}{c_{sh}} \right]} \quad \text{Equation D. 1}$$

where:

a_{sh} is the minimum void ratio (e_{min})

b_{sh} is slope of the line of tangency, (e.g., $= e / w$ when drying from saturated conditions)

c_{sh} is curvature of the shrinkage curve

w is gravimetric water content.

The volumetric water content and degree of saturation of the soil mixtures were then obtained using the best-fit shrinkage curve according to and respectively.

$$\theta_w = \frac{w_\psi G_s}{1 + e(w)} \quad \text{Equation D.2}$$

where:

w_ψ is the gravimetric water content versus suction

$$S = \frac{w_\psi G_s}{e(w)} \quad \text{Equation D.3}$$

The results of shrinkage curve for 100K0S, 90K10S, 80K20S and 50K50S soil mixtures are shown in Figure D.2, Figure D.3, Figure D.4 and Figure D.5 respectively.

Figure D.2-Shrinkage curve of 100K0S soil mixture

Figure D.3-Shrinkage curve of 90K10S soil mixture

Figure D.4-Shrinkage curve of 80K20S soil mixture

Figure D.5-Shrinkage curve of 50K50S soil mixture

Appendix E

Steady-state flow condition of soil mixture 90K10S at different matrix suctions

Figure E. 1-Steady-state flow condition of soil mixture 90K10S at an applied matrix suction of 50 kPa and a hydraulic gradient of 14

Figure E. 2-Steady-state flow condition of soil mixture 90K10S at an applied matrix suction of 90 kPa and a hydraulic gradient of 18

Figure E. 3-Steady-state flow condition of soil mixture 90K10S at an applied matric suction of 200 kPa and a hydraulic gradient of 32

Figure E. 4-Steady-state flow condition of soil mixture 90K10S at an applied matric suction of 300 kPa and a hydraulic gradient of 32

Figure E. 5-Steady-state flow condition of soil mixture 90K10S at an applied matric suction of 400 kPa and a hydraulic gradient of 32