Modeling multimodal travel corridor using differential complementarity system approach

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**MODELING MULTIMODAL TRAVEL
CORRIDOR USING DIFFERENTIAL
COMPLEMENTARITY SYSTEM APPROACH**

DU BO

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2015

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CORRIDOR USING DIFFERENTIAL
COMPLEMENTARITY SYSTEM APPROACH**

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SUMMARY

Many major cities are facing the traffic congestion problem in peak-hour commutes. Various mitigation measures have been proposed to deal with the traffic congestion, including road pricing, promoting public transport services, flexible work start time scheme, and integrated land use and transport planning. To evaluate the effectiveness of these measures and thus design the optimal management and control policies to contain the peak-hour traffic jam, the key lies in how to model and solve the travelers' choice behavior and therefore the equilibrium travel pattern with any given input of transportation management and control policies. In this study, a linear travel corridor with multimodal transport systems, i.e., highway and railway, was considered, which connected the residential area to the city center. Commuters were assumed to be located continuously along the travel corridor. They chose transportation mode and departure time to minimize their individual travel cost. Continuum modeling approach was applied to model the travel equilibrium, assuming continuous entry points to the transport systems along the corridor. Congestion effects on both of the highway and railway transport systems were captured. One could envisage that the travel choices among the travelers interacted with each other, affecting the flow dynamics of transport systems along the corridor. The major objective of this study was to develop modeling framework to model and solve the achieved spatial and dynamic equilibrium on the travel corridor. Or basically, given a certain input of transportation characteristics and management policies, the developed model must be able to fully describe how the commuters make choices in terms of travel mode and departure time, and the resultant flow and congestion dynamics that evolve on the corridor in a certain time horizon. To do so, this study applied the differential complementarity system modeling paradigm. Both static model solving only spatial equilibrium in a multimodal travel corridor and dynamic model capturing spatial and temporal equilibrium travel pattern simultaneously were studied. The static model was

applied in the specific context of park-and-ride services, while the dynamic one was used to design the optimal time and distance based road toll scheme.

In the static modeling effort, firstly a linear travel corridor with single park-and-ride service was considered. Commuters along travel corridor chose among three transport modes: auto, rail and park-and-ride. To make the model more realistic, travel time reliability was also incorporated in the multimodal choice. Commuters were assumed to be continuously distributed along the corridor to make travel mode choice based on the user equilibrium condition to complete their trips to the city center. The linear complementarity system approach was applied to develop the model to describe the spatial equilibrium travel pattern. The complementarity problem was solved by an equivalent transformation to a mixed integer linear programming. Then, the single park-and-ride assumption was relaxed in an extension work to consider continuous park-and-ride services and heterogeneous commuters along the corridor. Based on the well-studied complementarity theory, the proof of solution existence was also provided, and the travel time correlation among adjacent locations on highway was also considered. The developed model was converted to an equivalent linear complementarity problem and solved by the efficient Lemke's algorithm. The results of the model solution provided useful information for park-and-ride facility planning.

Moreover, the model was further extended to include dynamic travel equilibrium. Basically, the spatial dynamics of the traffic on both highway and railway systems in the travel corridor were modeled. In the literature, this problem was often referred as "corridor problem". However, the corridor problem was investigated by only analytical approach and its solution was not fully addressed. In this study, partial differential complementarity system approach was employed to model the corridor problem. Classical L-W-R flow theory was applied to describe the congestion flow dynamics. The model was also able to handle heterogeneous commuters. Efficient solution method was developed and solution existence proof was provided. With the aid of this modeling approach, this study further sought to find the optimal time and distance based road toll scheme to achieve best system performance. Besides, to highlight the effect of urban density distribution to the corridor transportation system performance, this study also aimed to find out the

optimal density distribution profile. The problem was formulated as a bi-level programming model, with the upper level optimizing the urban density distribution pattern, while the lower level model describing the optimal time-distance based road pricing problem, as well as the spatial and dynamic travel equilibrium. With a smooth relaxation scheme, the bi-level model was approximated by a relaxed nonlinear programming formulation and solved by an iterative algorithm. With a set of numerical experiments, optimal toll scheme and urban density distribution were obtained.

In the last scenario, the study focus was extended from linear travel corridor to a general network to figure out how to develop efficient solution algorithm to obtain the global optimum solution to the continuous network design problem. Using the generalized geometric programming method, both deterministic and stochastic user equilibrium based continuous network design problems were solved, which were approximated by a series of linear programming formulations via a condensation procedure so that the global approximation solution could be obtained.

NOTE

Chapter 2 and Chapter 3 include part of contents that has been published in the paper: Du, B. and Wang, Z.W. (2013) “Reliability-based modeling of park-and-ride service on a linear travel corridor”, Proceedings of Transportation Research Board 92nd Annual Meeting, Washington D.C., USA, 2013. Also on Journal of the Transportation Research Board (Network Modeling), Vol. 1, pp. 16-26.

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Chapter 2 and Chapter 6 include part of contents that has been accepted in the paper: Wang, Z.W. and Du, B. (2015) “Optimizing urban density along a linear travel corridor with time-distance toll scheme based on a continuous-time dynamic model”, The 21st International Symposium on Transportation and Traffic Theory, Kobe, Japan, 2015.

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LIST OF SYMBOLS AND ABBREVIATIONS

The following list of symbols and abbreviations is provided for ease of reference for those most frequently used in this thesis, therefore it is not exhaustive. Nonetheless all the symbols and abbreviations used in this thesis are defined at their first mentions in the main text.

Symbols

A	Set or arcs or links
C_{max}	Maximum road capacity
C_{min}	Minimum road capacity
$C_h(x)$	Generalized travel cost by auto from location x to CBD
$C_r(x)$	Generalized travel cost by railway from location x to CBD
$C_p(x)$	Generalized travel cost by P&R from location x to CBD
$C^*(x)$	Minimum generalized travel cost from location x to CBD
$C[f(x), x']$	Monomial approximation
$Cov(x, y)$	Covariance of x and y
$D(x)$	Variance of x
$E(x)$	Expectation x
k_t^x	Traffic density at location x and time t
$f_h(x)$	Operating cost for auto users from location x to CBD
f_h^0	Fixed part of the operating cost for auto users from location x to CBD
$f_r(x)$	Fare of railway services from location x to CBD

f_r^0	Fixed part of the railway fare
f_t^x	Traffic flow at location x and time t
g	Index of commuter groups
$\mathbb{G}$	Number of commuter groups
i	Index of corridor sections
L	Length of the corridor
P_h^w	Parking fee at workplace
$P_p(x_p)$	Parking fee at P&R facility location x_p
$q_h(x)$	Auto user density at location x
$q_p(x)$	Auto user density at location x
$q_r(x)$	Railway user density at location x
$q_0(x)$	Total travel demand density at location x
r_t^x	Departure rate at location x and time t
t	Index of time intervals
t^*	Preferred arrival time
$\bar{T}$	Length of time horizon
$T_b(x)$	Travel time budget for auto users from location x to CBD
$t_h(v_h(x))$	Travel time for auto users traversing unit distance at location x
t_h^{hh}	Access time for auto users from home to highway
t_h^{pw}	Access time for auto users from parking place to workplace
$T_h(x)$	Travel time for auto users from location x to CBD
t_h^0	Free flow travel time per unit of distance for auto users
t_p^{pr}	Transfer time for P&R users from parking place to train station nearby

t_r^{hr}	Access time for railway users from home to highway
t_r^{rw}	Egress time for railway users from train station to workplace
u_p	Inconvenience or disutility cost for P&R users
v	Total number of the corridor sections
V_r	Average travel speed of the train
$v_h(x)$	Traffic volume of auto users at location x
$v_r(x)$	Traffic volume of railway users at location x
W	Set of OD pairs
x_p	P&R location
$y_r(v_r(x))$	Crowding cost on train per unit distance at location x
$Y_r(x)$	Crowding cost for railway users traveling from location x to CBD
σ_T	Standard deviation of T
ε_v	The length of each equal section
ρ	Probability of a trip arrives within the travel time budget
τ	Time to cost coefficient or unit value of time
$\pi^{x,*}$	Minimum cost from location x to CBD in dynamic models
δ_{ap}^w	Link-path incidence factor

Abbreviations

BLP	Bi-level programming
BPR	Bureau of Public Roads
CBD	Central business district
CLT	Central limit theorem

CNDP	Continuous network design problem
CP	Complementarity problem
CTM	Cell transmission model
DAE	Differential algebraic equation
DCP	Differential complementarity problem
DCS	Differential complementarity system
DNDP	Discrete network design problem
DTA	Dynamic traffic assignment
DVI	Differential variational inequalities
DUE	Dynamic user equilibrium
ERP	Electronic road pricing
FIFO	First-in-first-out
GGP	Generalized geometric programming
GIS	Geographic information system
GP	Geometric programming
GPS	Global position system
IDUE	Instantaneous dynamic user equilibrium
IP	Integer programming
KKT	Karush–Kuhn–Tucker
LCP	Linear complementarity problem
LCS	Linear complementarity system
LP	Linear programming
L-W-R	Lighthill-Whitham-Richards
MIBLP	Mixed integer bilinear programming
MILP	Mixed integer linear programming

MINP	Mixed integer nonlinear programming
MNDP	Mixed network design problem
M-N	Merchant–Nemhauser
MP	Mathematical programming
MPCC	Mathematical programming with complementarity constraint
MPEC	Mathematical programming with equilibrium constraint
MSCP	Marginal social cost pricing
NCP	Nonlinear complementarity problem
NDP	Network design problem
NLP	Nonlinear programming
NP-hard	Non-deterministic polynomial-time hard
OD	Origin-destination
ODE	Ordinary differential equation
PDCS	Partial differential complementarity system
P&R	Park-and-ride
PUE	Probabilistic user equilibrium
SCDP	Single level continuously differentiable problem
SMA	Spatial moving average
SO	System optimum
SO-DTA	System-optimal dynamic traffic assignment
SUE	Stochastic user equilibrium
UE	User equilibrium
VI	Variational inequalities
VOT	Value of time

CHAPTER 1

INTRODUCTION

1.1 Research background

Since the user equilibrium principles of transportation network was proposed by Wardrop (1952) in 1952, a large number of research achievements have been obtained in this field and relevant extensions. For example, a comprehensive mathematical analysis of the equilibrium principle as well as its extensions, like elastic demand, trip distribution, etc. was shown in Sheffi (1985); various dynamic equilibrium models and extensions can be found in Peeta and Ziliaskopoulos (2001); Friesz et al. (2011a). Plenty of modeling methods have been applied to both static and dynamic user equilibrium, like variational inequalities, complementarity theory, etc. (Kinderlehrer and Stampacchia, 2000). As a special case of the general network, the static and dynamic equilibrium on a linear travel corridor or monocentric city is a simplified model description, but plays an important role in the improvement both on practical and theoretical aspects (Arnott and DePalma, 2011). On the practical aspect, travel corridor is widely constructed in metropolises as arterial road to connect suburbs and city center, which is very important and undertakes heavy traffic commute everyday. Therefore, it is imperative to study the corridor problem and provide useful information on the optimal design and management of corridor transportation system. On the theoretical aspect, studying of the corridor problem could contribute to several relevant issues, like bottleneck problem, dynamic traffic assignment, and so on. However, compared to the comprehensive achievements on general network equilibrium and bottleneck problem, only a few studies have been done on the corridor problem. Arnott and DePalma (2011) conducted a series of mathematical analysis on the corridor problem, however the preliminary results were based on strict assumptions and few numerical solutions were provided. Moreover, many corridor problem related issues, like toll scheme, urban density distribution, etc., are still lack of investigation.

Besides the simplified case, corridor problem, some key issues of the general network are still under research, such as the network design problem (Magnanti and Wong, 1984). Traditionally, the network design problem is usually modeled as a bi-level programming formulation and then solved by an approximation or heuristic method after transformation. Although a great deal of work has been done in network design problem, there is room for improvement with regard to the global optimum and efficient algorithms.

Traditional modeling tools, like variational inequalities, complementarity theory, fixed point theory, etc. have been widely applied to modeling the equilibrium in transportation research, as well as other engineering scopes, however, each method comes with its certain limitations. With the new scheme of differential complementarity system, a combination framework of several mathematical components, as well as geometric programming modeling approach, many untouched and difficult issues in transportation research, like the corridor problem and its extensions, network design problem, and so on, may achieve improvements to some extent.

1.2 Objectives

This research aims to achieve the following objectives:

- To make a comprehensive literature review on corridor problem along with its relevant issues and extensions;
- To apply the linear complementarity system approach to model and solve the spatial equilibrium travel pattern in a multimodal travel corridor;
- To apply the partial differential complementarity system approach to model and solve the spatial and dynamic equilibrium in a multimodal travel model;
- To develop a model to solve the optimal time-distance toll scheme and urban density distribution along a corridor and find some efficient solution methods to obtain numerical solutions;
- To apply the generalized geometric programming approach to model the deterministic and stochastic user equilibrium based continuous network design problem, and achieve the global optimal solution with efficient solution methods.

1.3 Outline of the research work

This thesis consists of eight chapters. Chapter 2 presents an extensive literature review on several subject matters relevant to this research, including static and dynamic user equilibrium, corridor problem, network design problem, and some relevant modeling and solution methods. A multimodal equilibrium along a travel corridor with single park-and-ride service is modeled as a linear complementarity system, and mathematical analysis and numerical examples are shown in Chapter 3. As an extension of the single park-and-ride corridor problem, a continuum equilibrium model with continuous park-and-ride services is developed in Chapter 4. To handle the corridor problem in a spatial and temporal dimensions, Chapter 5 develops a continuous-temporal-spatial dynamic equilibrium model on a linear corridor based on differential complementarity system approach. To achieve more meaningful applications in practical terms, a bi-level dynamic model along the corridor is developed to optimize the time-distance toll scheme and urban density along the corridor in Chapter 6. The corridor problem is a simplified and special case of the general network, as an extension, Chapter 7 applies a generalized geometric programming method to handle the continuous network design problem with promising numerical results. In the end, the conclusions and the recommendations for future studies are shown in Chapter 8.

CHAPTER 2

LITERATURE REVIEW

(Note: this chapter includes part of contents that has been published / submitted / presented in the papers / conferences:

Du, B. and Wang, Z.W. (2013) “Reliability-based modeling of park-and-ride service on a linear travel corridor”, Proceedings of Transportation Research Board 92nd Annual Meeting, Washington D.C., USA, 2013. Also on Journal of the Transportation Research Board (Network Modeling), Vol. 1, pp. 16-26.

Du, B. and Wang, Z.W. (2014) “Reliability-based continuum modeling of park-and-ride services with heterogeneous commuters - a linear complementarity system approach”, Transportation Research Part E, Vol. 71, pp. 58-81.

Du, B. and Wang, Z.W. (2014) “A global approximation model for the continuous network design problem with deterministic user equilibrium”, INFORMS 2014 Annual Meeting, San Francisco, USA, 2014.

Du, B. and Wang, Z.W. (2014) “An efficient solution method for the continuous network design problem with stochastic user equilibrium”, The 19th International Conference of Hong Kong Society for Transportation Studies, Hong Kong, China, 2014

Wang, Z.W. and Du, B. (2015) “Optimizing urban density along a linear travel corridor with time-distance toll scheme based on a continuous-time dynamic model”, The 21st International Symposium on Transportation and Traffic Theory, Kobe, Japan, 2015.

Wang, Z.W. and Du, B. “Continuum modeling of spatial and dynamic equilibrium in a linear travel corridor with heterogeneous commuters - a partial differential complementarity system approach”, Transportation Research Part B, under revision.)

2.1 Introduction

An extensive literature review on several subject matters relevant to this research is presented in this chapter, which mainly focuses on three topics: static user equilibrium along a linear travel corridor, dynamic corridor problem with extensions and network design problem. To cover these topics, this chapter opens with a short introduction and review on the traffic equilibrium both in static and dynamic cases, and then presents some reviews on the extensions of the traffic equilibrium, like travel time reliability, park-and-ride, road toll scheme, urban density, etc., which are relevant to the studies in subsequent chapters. Then a wide

range of previous studies regarding network design are reviewed, and some relevant modeling and solution methods are presented in the end.

2.2 Static user equilibrium and extensions

To my best knowledge, the first use of the term “equilibrium” in traffic pattern was by the economist F. H. Knight in Knight (1924). A short introduction of various equilibria is given in this section for better understanding of further chapters.

It is reasonable to assume that all travelers make route choices to minimize the corresponding travel times. An equilibrium status is achieved only when no traveler can reduce his/her travel time by unilaterally changing the route, and this definition is known as Wardrop’s first principle, namely deterministic user equilibrium (UE) condition (Wardrop, 1952). To make it distinct from dynamic user equilibrium in subsequent contents, UE is used to represent deterministic user equilibrium. Implicitly, UE condition implies that travelers have complete and accurate information on the whole traffic network to support their best choices and all the travelers are identical in their behaviors. However, there always exist some errors between the travel time travelers perceive and the actual travel time. To deal with this issue, a relaxation of UE condition was proposed. An equilibrium status, namely stochastic user equilibrium (SUE), is achieved when no traveler believes he/she can reduce the perceived travel time by unilaterally changing his/her route. Moreover, both UE and SUE conditions are in spatial dimension only, and they can be extended to two-dimension conditions, named dynamic user equilibrium (DUE), which handles the temporal-spatial equilibrium problem.

Equilibrium problem is one of the most well studied areas in traffic and transportation research, and plenty of formulations, mathematical properties and applications of traffic equilibrium models have been investigated with comprehensive achievements (Sheffi, 1985; Patriksson, 1994; Bell and Iida, 1997).

Besides various UE conditions, another commonly used traffic assignment condition is system optimum (SO), which requires the average travel time to be at a minimum at equilibrium. This is Wardrop’s second principle, which implies that individuals cooperate in choosing routes to make sure the whole traffic system is

efficient and optimal. Ideally, if the congestion effect is ignored, SO condition will generate identical result as UE condition.

2.2.1 Basic UE

After Wardrop (1952) proposed the UE principle in 1952, the first mathematical programming model of UE was formulated by Beckmann et al. (1956) in 1956. Before the basic formulation is discussed, some network notations throughout this thesis are presented first. The traffic network is represented by a directed graph with a set of nodes N , a set of links A and a set of origin-destination (OD) pairs W . In addition, q^w denotes the travel demand between OD pair w , $t_a(x_a)$ represents the link performance function (e.g. travel time) on link a , which is a function of the link flow x_a . Let f_r^w represent the flow on path r between OD pair w , then the UE traffic assignment pattern can be obtained by solving the mathematical program:

$$\begin{aligned}
 \min_x \quad & \sum_{a \in A} \int_0^{x_a} t_a(w) dw \\
 \text{s.t.} \quad & \sum_{r \in R^w} f_r^w = q^w, \quad \forall w \in W \\
 & x_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{ar}^w, \quad \forall a \in A \\
 & f_r^w \geq 0, \quad \forall r \in R^w, w \in W
 \end{aligned} \tag{2-1}$$

where R^w denotes the set of routes between OD pair w and δ_{ar}^w is the link-path incidence factor, which is 1 if link a is on path r between OD pair w , otherwise 0. Note that the objective function is regarded strictly as a mathematical function to solve the UE problem without any economic or behavioral interpretation. If the objective function is changed to

$$\min_x \quad \sum_{a \in A} x_a t_a(x_a) \tag{2-2}$$

then the problem [UE] (2-1) becomes a SO model.

To solve the UE problem, many approaches can be applied, including heuristic methods based on network loading mechanism, like capacity restraint methods, incremental assignment techniques, etc., and the convex combination method proposed by Frank and Wolfe in 1956, also known as Frank-Wolfe algorithm (Frank and Wolfe, 1956). After that, more and more achievements on the solution

methods and extensive applications of UE have been obtained. Smith (1979) gave a mathematical analysis of the conditions guaranteeing the existence, uniqueness and stability of UE. Dafermos (1980) reformulated the UE problem as a variational inequalities (VI) formulation with proof of the existence and uniqueness. Similarly, Vliet (1987) also showed that Frank-Wolfe algorithm could be well interpreted by VI theory. Based on finite-dimensional VI theory, Nagurney (2000) reformulated the UE conditions with analysis of the qualitative properties, and presented a modified projection method to solve the VI formulation. Later, as an extension of Nagurney (2000), Nagurney and Dong (2002) relaxed the travel demand to an elastic function depending on the disutility of all classes at all OD pairs, and solved the UE problem by reformulating it to a VI model. Patriksson (2004) analyzed the properties of UE and suggested an iterative algorithm to solve the equilibrium problem. To handle the inconsistency of the four-step model, Zhou et al. (2009) developed an integrated travel demand model based on mathematical programming and VI formulations to deal with trip generation, trip distribution, modal split and traffic assignment simultaneously.

2.2.2 Travel time reliability

Most existing studies assume the road travel time is deterministic, however, various events (unpredicted vehicle breakdowns, traffic accidents, random surrounding environment, traffic demand fluctuation, etc.) may occur on the transportation network and result in stochastic travel time, which has been found to have a significant impact on travelers' choice behaviors (Lam and Small, 2001; Lo and Tung, 2003; Li et al., 2010). Therefore, it is important to consider the impact of travel time uncertainty into the development of UE models, and travel time budget is the most commonly used concept to describe this uncertainty. Based on the assumption that travel time is susceptible to variation, travel time budget is expressed as a combination of expected travel time and additional travel time to hedge against travel time variability so that travelers can arrive at their destinations within a given on-time arrival probability, termed as travel time reliability in the literature (Chen et al., 2012).

To formulate travel time reliability, it is usually assumed that travel time is a combination of two parts: mean travel time determined by the commonly used Bureau of Public Roads (BPR) function plus a stochastic component $\varepsilon(x)$ with $E[\varepsilon(x)] = 0$ capturing the variability around the mean. Let $t(v(x))$ represent the travel time driving unit distance at location x , where x denotes a location point on a link and $v(x)$ represents the traffic volume at location x . For illustration purpose, travel time is supposed to be described by a first-order model as in D'Este (1987).

$$t(v(x)) = t^0 \left[1 + A \left(\frac{v(x)}{C_x} \right) \right] \quad (2-3)$$

where t^0 denotes the free flow travel time per unit of distance, C_x represents the link capacity at location x , and A is parameter. It is further assumed that the link capacity C_x at location x is the single source of travel time uncertainty and at each location the link capacity probability distribution is known and given. For simplicity, uniform distribution is adopted here for illustration purpose, as was done in Lo and Tung (2003). If the maximum and minimum capacities are denoted as C_{max} and C_{min} respectively, the uniform probability distribution function can be described as:

$$f(C_x) = \frac{1}{C_{max} - C_{min}}, \quad C_x \in [C_{min}, C_{max}] \quad (2-4)$$

where $C_{min} > 0$ and $C_{max} > 0$. Then the expectation and variance of the link capacity can be calculated as follows:

$$E(C_x) = \frac{C_{max} + C_{min}}{2} \quad (2-5)$$

$$D(C_x) = \frac{(C_{max} - C_{min})^2}{12} \quad (2-6)$$

Based on the assumption that the link capacity is independent of the traffic volume on it (Lo and Tung, 2003), the expectation and variance of travel time on unit distance at location x are derived as follows:

$$E(t(v(x))) = t^0 + t^0 A v(x) E(C_x^{-1}) = t^0 + \frac{t^0 A v(x) (\ln C_{max} - \ln C_{min})}{C_{max} - C_{min}} \quad (2-7)$$

$$\begin{aligned}
D(t(v(x))) &= t^{0^2} A^2 v(x)^2 D(C_x^{-1}) = t^{0^2} A^2 v(x)^2 \left(E(C_x^{-2}) - E(C_x^{-1})^2 \right) \\
&= t^{0^2} A^2 v(x)^2 \left[\frac{1}{C_{max} C_{min}} - \frac{(\ln C_{max} - \ln C_{min})^2}{(C_{max} - C_{min})^2} \right]
\end{aligned} \tag{2-8}$$

In order to analyze the travel time relationship between any two different locations on highway, the highway is divided into unit sections and assume that $T(x_i, x_j)$ is the travel time from section x_i to section x_j , therefore

$$T(x_i, x_j) = \sum_{k=i}^j t(v(x_k)) \tag{2-9}$$

then the mean and variance of $T(x_i, x_j)$ are, respectively:

$$E(T(x_i, x_j)) = \sum_{k=i}^j E(t(v(x_k))) = \sum_{k=i}^j \left[t^0 + \frac{t^0 A v(x) (\ln C_{max} - \ln C_{min})}{C_{max} - C_{min}} \right] \tag{2-10}$$

$$D(T(x_i, x_j)) = \sum_{k=i}^j D(t(v(x_k))) + 2 \sum_{a < b} Cov(t(v(x_a)), t(v(x_b))), \quad i \leq a < b \leq j \tag{2-11}$$

where the covariance part can be calculated as follows:

$$\begin{aligned}
&Cov(t(v(x_a)), t(v(x_b))) \\
&= Cov(t^0 A v(x_a) C_{x_a}^{-1}, t^0 A v(x_b) C_{x_b}^{-1}) = t^{0^2} A^2 v(x_a) v(x_b) Cov(C_{x_a}^{-1}, C_{x_b}^{-1}) \\
&= t^{0^2} A^2 v(x_a) v(x_b) \left[E(C_{x_a}^{-1} C_{x_b}^{-1}) - E(C_{x_a}^{-1}) E(C_{x_b}^{-1}) \right]
\end{aligned} \tag{2-12}$$

In practice, the joint probability distribution of road capacity between any two unit sections on highway can be obtained and calibrated from the real historical data. Therefore, the item $E(C_{x_a}^{-1} C_{x_b}^{-1})$ in Eq. (2-12) is readily calculated if the joint probability distribution function of stochastic link capacities is known. In most of the previous studies considering travel time reliability, the covariance part in Eq. (2-11) is ignored based on the assumption that the link capacity distributions are independent, which should be reasonable for relatively few network disruptions in the form of traffic incidents (Lo et al., 2006).

2.2.3 Travel time budget

When travel time is susceptible to variation, travelers would reserve some additional time to expected travel time to hedge against travel time variability. Here, the concept of travel time budget introduced in Lo et al. (2006) is adopted to represent the longer travel time reservation. Under the assumption of independent stochastic capacities, the travel time on highway between two locations follows a normal distribution according to the central limit theorem (CLT) as long as the Lindeberg's condition is satisfied. The travel time budget can be expressed as:

$$T_b = E(T) + \lambda \sigma_T \quad (2-13)$$

where T_b is travel time budget, T is the travel time on highway, $E(T)$ and σ_T are the expectation and standard deviation of T respectively, λ is a parameter related to the arrival punctuality requirement ρ , which can be written as

$$P\{T \leq T_b\} = P\{T \leq E(T) + \lambda \sigma_T\} = \rho \quad (2-14)$$

Therefore, for normal distributed travel time T , it is easy to obtain:

$$\lambda = \phi^{-1}(\rho) \quad (2-15)$$

2.2.4 Spatial equilibrium travel pattern in linear corridor

Usually the corridor problem is defined as a dynamic problem in the spatial and temporal dimensions. However, there exist several studies focusing on its static problem in the spatial dimension only, which is usually called a linear monocentric city problem, namely static corridor problem in this study. Here the definition of the static corridor problem is given first. A linear travel corridor connecting the suburbs to central business district (CBD) is subject to traffic congestion, and commuters with a given distribution of population along the corridor make multimodal choice or/and route choice under UE or SO conditions to travel to CBD at the end of the corridor, such kind of problem and its extensions are called static corridor problems.

Orthlieb and May (1976) applied a method to evaluate the traffic equilibrium, along with its influence on the control of a freeway corridor. Talvitie and Hasan (1980) proposed an equilibrium model of transportation corridor and applied it to a

computational example of a real case. Jehiel (1993) considered a traffic corridor with continuous entry points and a single exit point with mathematical analysis of UE, as well as SO on it. Wang et al. (2004a) studied the park-and-ride location and pricing issues based on a linear monocentric city with equilibrium multimodal choice. De Borger et al. (2007) investigated the pricing and investment strategies on a congested corridor. Larrain and Muñoz (2008) developed a UE model of a public transit corridor and provided mathematical analysis of the equilibrium existence on a simple case. Liu et al. (2009) proposed a continuum multimodal equilibrium model on a linear travel corridor with park-and-ride services, and provided detailed analysis of the park-and-ride properties. Li et al. (2012b) considered a linear travel corridor with many-to-one OD pattern and two alternative travel modes to explore the intermodal equilibrium using VI approach.

2.3 Dynamic user equilibrium and extensions

Dynamic traffic assignment (DTA) problem, determination of departure rate, departure time and route choice over a given temporal horizon, has gained considerable studies and achievements for decades. Comprehensive reviews of DTA models and extensions can be found in Peeta and Ziliaskopoulos (2001); Friesz et al. (2011a). With different objectives, DTA can be classified into two categories: DUE and system-optimal dynamic traffic assignment (SO-DTA). In this thesis, the focus is mainly put on the development of rigorous and analytical mathematical model of DUE problems, whose previous models were profoundly affected by the first, to my knowledge, SO-DTA model in discrete time formulated by Merchant and Nemhauser (1978a, b).

To understand the DUE literature better, a short introduction of the key points of the traffic flow theory is presented in the subsequent section first.

2.3.1 Traffic flow theory

In this section, some key points of traffic flow theory are introduced, which may be used in the following chapters. For more information on traffic flow theory, one can refer to Haberman (1977); Daganzo (1997); Gartner et al. (2001); Gazis (2002); Garavello and Piccoli (2006); Ortuzar and Willumsen (2011).

The general speed-flow relationship is usually presented as in Figure 2.1 (a), one can observe that as the traffic flow raises, speed decreases slowly at initial stage while the reduction rate increases as the traffic flow approaches road capacity. In practice, the speed-flow relationship is substituted by a travel time-flow relationship as in Figure 2.1 (b), which takes into account the congestion effects as a relationship among road capacity, free flow speed, traffic flow and travel time. In Figure 2.1 (c) the flow-density relationship shows that the traffic flow and density increase simultaneously at the first half stage, while the corresponding average traffic speed drops, shown in Figure 2.1 (d). The traffic flow reaches the maximum as the density increases to some optimum density as well as the corresponding optimum speed, after that it begins to decrease as the density increases and reaches zero flow at jam density with also zero traffic speed.

Figure 2.1 The fundamental diagrams of traffic flow relationships

(1) *L-W-R theory*

Based on the method of kinematic waves, the Lighthill-Whitham-Richards (L-W-R) theory of traffic flow was proposed by Lighthill and Whitham (1955); Richards (1956). The key postulate of the L-W-R theory is the relation between traffic flow and density (Newell, 1993a). At a macroscopic point of view, assume traffic flow f_x^t (the number of vehicles per hour passing location x at time t),

concentration/density k_x^t (the number of vehicles per km at location x at time t) and velocity u_x^t (the speed of vehicles passing location x at time t), along with their upper bound f^{max} , k^{max} and u^{max} respectively, then the flow-density relation is:

$$f_x^t = k_x^t u_x^t(k_x^t) \quad (2-16)$$

where the speed u_x^t is a function of the density k_x^t , $u_x^t = u_x^t(k_x^t)$. The most commonly used diagram to describe the velocity-density relation is Greenshields quadratic function (Greenshields, 1935) as Eq. (2-17), and its fundamental diagrams are shown in Figure 2.2.

$$u_x^t(k_x^t) = u^{max} \left(1 - \frac{k_x^t}{k^{max}}\right) \quad (2-17)$$

Figure 2.2 The fundamental diagrams of Greenshields function

The conservation/continuity equation is as follows:

$$\frac{\partial f_x^t}{\partial x} + \frac{\partial k_x^t}{\partial t} = 0 \quad (2-18)$$

which assumes that there is no entering and exiting traffic flow. If the entering traffic flow is considered, then the conservation equation can be expressed as:

$$\frac{\partial f_x^t}{\partial x} + \frac{\partial k_x^t}{\partial t} = r_x^t \quad (2-19)$$

where r_x^t denotes the entering flow at location x at time t .

As a simplification of the L-W-R theory, Newell (1993a, b, c) conducted a series of studies to evaluate the cumulative flow with rigorous mathematical analysis and

make some extensions of the traffic flow studies. Based on the theory of kinematic waves, Perakis and Roels (2006) proposed an analytical function of travel time and incorporated it into the DUE model. Kachani and Perakis (2006) proposed two fluid dynamics models to deal with travel time functions and dynamic pricing/inventory control respectively. Kachani and Perakis (2009) put forward a general and two approximation models of travel time based on fluid dynamics theory: the spillback polynomial and exponential models, and provided rigorous mathematical analysis.

(2) *M-N theory*

The Merchant–Nemhauser (M-N) model is a SO-DTA model in discrete-time scheme with exit-flow function assumption (Merchant and Nemhauser, 1978a, b). Given a traffic network with the set of nodes N and set of directed links A , $a \in A$ the study horizon is divided into v equal time intervals indexed by $t \in \{0, 1, \dots, v\}$ with equal time length $\Delta t = \frac{T}{v}$. h_a^t represents the number of vehicles entering link a at the time period t . x_a^t is the traffic volume on link a at the beginning of time period t , $e_a(x)$ denotes the exit flow from link a , $0 \leq e_a(x) \leq x$, which is a nondecreasing, continuous and concave function to represent the congestion. Then the state equation is as:

$$x_a^{t+1} = x_a^t - e_a(x_a^t) + h_a^t \quad (2-20)$$

As for the exit flow function, there are various forms can be applied to, for example,

$$e_a^t(x_a^t) = \min\{x_a^t, C_a^t\} \quad (2-21)$$

where C_a^t denotes the capacity of link a at time interval t . Let $O(q) = \{a \in A | a \text{ points out of node } q\}$, $I(q) = \{a \in A | a \text{ points into node } q\}$ and r_q^t denotes the external input at node q at the time interval t , then for each node q (not the sink node), the conservation equation holds at each time interval as:

$$\sum_{a \in O(q)} h_a^t = r_q^t + \sum_{a \in I(q)} e_a(x_a^t) \quad (2-22)$$

(3) *CTM theory*

The cell transmission model (CTM) proposed by Daganzo (1994, 1995) is one of the most commonly used method to describe the dynamic traffic flow over various transportation networks. Assume the road is divided into several sections with equal length, called cells, labeled as $i=1,2,\dots,I$; x_i^t denotes the number of vehicles at cell i at time t . To represent queue, assume C_i^t denotes the capacity of flow into cell i at time interval t , Q_i^t represents the maximum number of vehicles can be present at cell i at time interval t , and e_i^t represents the flow from last cell to cell i at time interval t , thus the cell flow propagation functions are as follows:

$$\begin{aligned} x_i^{t+1} &= x_i^t + e_i^t - e_{i+1}^t \\ e_i^t &= \min\{x_{i-1}^t, C_i^t, Q_i^t - x_i^t\} \end{aligned} \quad (2-23)$$

which mean the number of vehicles at cell i at time $t+1$ equals its occupancy at time t plus the inflow and minus the outflow. Based on the L-W-R theory, the flow-density relation could be $f = \min\{u^{\max}k^{\max}, f^{\max}, u^{\max}(k^{\max} - k)\}$, $0 \leq k \leq k^{\max}$, where $u^{\max}$ is the free flow speed, $k^{\max}$ is the jam/maximal density, and the maximal flow $f^{\max} \leq \frac{u^{\max}k^{\max}}{2}$. It can be derived that (2-23) is a discrete approximation to the L-W-R model (Daganzo, 1994), and the diagram is shown as follows:

Figure 2.3 The diagram of flow-density relationship for CTM

2.3.2 Dynamic user equilibrium

In the last several years, much effort has been expended to develop a theoretically sound formulation of DUE, which is also a canonical form acceptable to scholars

and practitioners alike (Friesz, 2010). For a comprehensive review of DUE, one can refer to Peeta and Ziliaskopoulos (2001); Friesz (2010).

Mahmassani and Herman (1984) investigated the DUE problem on departure time and route choice over a two-route network. Friesz et al. (1989) presented two continuous time DTA models, SO and DUE, based on optimal control theory. Wie et al. (1990) proposed a DUE model considering a multiple OD network, and discussed the optimality conditions with economic explanation based on the Pontryagin minimum principle. Friesz et al. (1993b) formulated the DUE problem as a VI formulation, which combined the route choice and departure time decision simultaneously. Friesz et al. (2001) and Friesz and Mookherjee (2006) formulated the DUE problem as differential variational inequalities models, which were equivalent to the formulation in Friesz et al. (1993b). Wie et al. (1995) applied the VI approach to formulate the DUE problem in discrete time scheme, whose solution could be guaranteed under some regularity conditions. To avoid the enumeration burden of large scale networks, Ran and Boyce (1996) and Ran et al. (1996) developed link-based VI models of the DUE problem. Ziliaskopoulos (2000) modeled the DTA with SO problem as a linear programming (LP) based on a CTM scheme, and discussed the marginal travel time, as well as the sufficient and necessary SO conditions. Later, Li et al. (2003) developed a Dantzig-Wolfe decomposition scheme to solve the LP model proposed in Ziliaskopoulos (2000), which was more efficient on networks. Using the VI approach, Lo and Szeto (2002) modeled the DUE problem based on an extension of CTM to a network version and used an alternating direction method to solve the VI formulations. As an extension of Lo and Szeto (2002), Szeto and Lo (2004) added route and departure time choice with elastic demand to the VI model, which was solved by a descent method with numerical studies. Under general assumptions, Lindsey (2004) discussed the existence, uniqueness and travel cost properties of DUE in a bottleneck model with heterogeneous users. Friesz and Mookherjee (2006) developed a fixed point algorithm to solve the DUE network flows based on VI formulations. Ban et al. (2008) modeled the DUE problem as a nonlinear complementarity problem via a discrete time scheme. Sumalee et al. (2011) extended the CTM scheme to a stochastic version to describe the traffic flow with stochastic demand and supply.

Five operational modes were discussed based on different congestion levels, and this stochastic CTM was tested on a corridor and an empirical study. Different from other stochastic traffic flow models considering random noise term to the deterministic equations, Jabari and Liu (2012) developed a new stochastic dynamic model, whose stochastic source came from driver gap choice, and this model could overcome the disadvantages of traditional models. Pang et al. (2012) applied the linear complementarity system approach to model a DUE problem with heterogeneous users in a single bottleneck, and solved it with Lemke's algorithm based on a time-stepping scheme. Ban et al. (2012a) applied the differential complementarity system method to formulate the DUE problem and provided some properties analysis. As a subsequent study of Ban et al. (2012a), Ban et al. (2012b) formulated the instantaneous DUE problem with multiple destinations with the differential complementarity system approach, and then it was solved via a time-stepping method. Han and Du (2012) applied the VI theory and the Lyapunov stability theory to analyze on the properties of a link based DTA model. Ukkusuri et al. (2012) modeled the CTM based DUE problem using a complementarity approach and discussed the existence of its solution.

In addition, in macroscopic modelling, first-in-first-out (FIFO) is commonly adopted although overtaking often happens in reality. Comprehensive discussion of FIFO could be found in the literature. Carey (1992) analyzed the nonconvexity in dynamic assignment with FIFO (or no passing) requirement and showed several ways to formulate FIFO by using discrete or integer and continuous variables. Xu et al. (1999) developed a formulation for the continuous dynamic network loading problem and gave sufficient conditions to satisfy the FIFO condition. Based on discrete-time scheme, Huang and Lam (2002) developed a DUE model to handle route and departure time choices simultaneously, in which the FIFO requirement at intersection was holden by formulating the link travel time properly. Carey et al. (2003) introduced a whole-link travel time model, which satisfied FIFO condition under mild assumptions. Later, Carey and Ge (2007) showed a new differencing method to guarantee the FIFO behavior, which was not always preserved in the continuous-time model proposed by Carey et al. (2003). Long et al. (2011) proposed three discretized travel time models based on cumulative flows, and

proper analysis were given to show that all the models satisfied the link FIFO condition. Carey et al. (2014) pointed out that the commonly used method to preserve FIFO did not fully ensure FIFO for the sequences of cells/links/routes, and developed three different levels of approximation models of FIFO with detailed analysis.

2.3.3 Dynamic corridor problem

When modeling the DUE problems, all sorts of models can be formulated by considering different assumptions, constraints, network topologies, flow propagation equations, and so on. Specifically, this study considers a simple network, a linear travel corridor connecting the suburbs to CBD, which is subject to traffic congestion, and commuters continuously distributed along the corridor choose the departure times to enter the corridor and go forward to the common destination, CBD, at the end of the corridor. Moreover, it is assumed that the distribution of population along the corridor is given exogenous, and all commuters have identical vehicles and common travel time function. In terms of the above description of the problem, it can be labeled as a corridor problem, which has simple model description but plays an important role in the improvement both on practical and theoretical aspects (Arnott and DePalma, 2011). Arnott and DePalma (2011) presented a clear definition of corridor problem, which is to determine equilibrium and optimum traffic flow patterns for continuous model and possible extensions based on the trip-timing condition, and then investigated the no-toll equilibrium of the basic corridor problem. The trip-time condition is consistent with the dynamic extension of Wardrop's UE principle, which guarantees that no commuter can reduce the travel cost by departing at a different time. Although they did not obtain a complete solution to the problem, they showed preliminary results, which provide substantial fundamentals for further studies. Later, a single entry corridor problem with L-W-R flow congestion and Greenshields relation was analyzed, which was a special case of the model considered in Newell (1988). This model was based on simplified assumptions so that quasi-analytic solutions for user optimum exist (DePalma and Arnott, 2012).

From another point of view, the corridor problem considered in this study also belongs to a morning commute problem with temporal-spatial dynamic equilibrium of rush-hour traffic congestion in a monocentric city. The morning commute problem was introduced by Vickrey (1969), which started work on single auto mode commuters driving on a single route with fixed road capacity and FIFO bottleneck to workplace at a preferred arrival time. From the early 1970s, most studies of congestion in an urban model were based on the method proposed by Mills (1972b); Solow (1972, 1973), which showed that the further away commuters live from CBD the earlier they depart, and all of the commuters arrive at CBD together. However, this method could not address the timing of work trips. As an extension of Vickrey (1969), Newell (1987) attached different values to queueing and schedule delays when considering travelers with different work starting times in the morning commute problem. The result showed that most travelers choose to arrive at workplace on time, and the queueing pattern was exceedingly different from the previous studies of identical travelers. Arnott et al. (1990a) investigated the commuters' route and departure time choice behavior on a simple network with user equilibrium and system optimum, and the efficiency of various tolls was analyzed as well. Liu and Nie (2011) extended the bottleneck model by considering simultaneous route, heterogeneous users, no-toll equilibrium, two system optima and related transportation economics analysis. Han et al. (2011) formulated a single OD DUE problem based on CTM as a complementarity problem, where two methods, maximum and average travel time, were used to estimate the travel time. Gonzales and Daganzo (2012) proposed a morning commute problem with auto and public transit modes, heterogeneity, and distributed demand. It showed that public transit was a Pareto improvement and it reduced the duration of the rush hours in UE status and the overall cost.

In general, DUE models, as well as dynamic corridor problems, are characterized with the description of dynamic interaction of the traffic flow and equilibrium conditions, accordingly, differential equations can be applied to describe the dynamic traffic conditions, such as traffic congestion and travel time on road; while complementarity conditions or VI take the advantage in modeling the equilibrium conditions. Therefore, an integrated modeling tool combining these two

components is quite important to develop the dynamic models, while differential complementarity system is such kind of modeling paradigm that can develop rigorous and analytical mathematical models for this class of dynamic problems. More introduction of differential complementarity system is given in Section 2.8.3.

2.4 Park-and-ride

In a multimodal traffic network, although competitions exist among different kinds of modes definitely, especially between public transit and private car or taxi, complementary and cooperative services are also possible and even preferred because cooperation results in the reduction of total travel cost, air pollution, energy consumption and improvement of efficiency. There are many sorts of complementarity and cooperation among different transportation modes, for example, in some cities with developed railway system, bus services along the railway line may be unsustainable due to the competition, however since not all the workplaces and residential places are along the railway line, people living or working far from the railway line desire some way to connect their places to the railway stations. In that case, feeder buses or shuttle buses are needed to operate in a complementary way to fill the gap, which run on the routes connecting railway stations to other places not close to the stations but with travel demand.

Another typical example of complementary service is park-and-ride (P&R). In many metropolises, the downtown streets are usually very crowded in the morning and evening peak hours. Although public transit, like railway, can guarantee commuters arrive at workplace on time at the expense of convenience, freedom and comfort to some extent, people owning private cars still prefer to drive if the congestion is affordable. However, traffic congestion often happens approaching to CBD, in that case, driving private car to a place not in congestion and then transferring to complete the journey by public transit could solve this problem if the parking fee there is more attractive than that in CBD, as well as the transfer cost is acceptable. Such combination travel mode is often called P&R.

As an useful travel demand management strategy, P&R has been widely used since 1930s (Noel, 1988; Verhoef et al., 1995), which has been demonstrated to be effective to reduce the traffic congestion and air pollution from the studies in United

Kingdom and Eastern Asia, like Hong Kong, Singapore, etc., especially for the increasing travel demand in the morning rush hours (Niblette and Palmer, 1993; Foo, 1997; Lam et al., 2001; Hounsell et al., 2011). The principle of P&R service is to encourage commuters to choose auto and public transit combination travel mode to reduce auto traffic approaching to CBD. This combined travel mode allows commuters to drive private car to the parking places provided by the P&R service, then take the public transit to the station near the workplace and walk to the workplace on foot in the end.

P&R service is a potential way to reduce travelers' cost. For example, in Figure 2.4, curves C_r and C_h represent the generalized cost by public transit users on railway and auto users on highway along the corridor respectively. It is assumed that the P&R facility is at location x_p , travelers living beyond x_p can drive to x_p and transfer to railway to complete the trip. In this case, the generalized cost can be reduced if the cost for P&R mode transfer is less than segment AB, which is the difference of generalized cost between auto users and public transit users.

Figure 2.4 Potentiality of reducing generalized cost by P&R

Many research works in the literature of P&R focus on the facility location problem. Horner and Grubestic (2001) proposed a flexible geographic information system (GIS)-based method to determine the potential locations of terminal P&R facilities along urban rail lines. Faghri et al. (2002) developed a hybrid knowledge-based expert system to assist in determining the optimal P&R location. Farhan and Murray (2005) proposed a GIS-based approach to delineate catchment areas, which accounted for P&R facility accessibility and user travel direction. Horner and

Groves (2007) developed a network flow-based model to optimize the P&R location so that the chance of removing users from the network was maximized. Farhan and Murray (2008) presented a multi-objective model to integrate three considerations simultaneously to site P&R facilities. Based on analytical formulations, Holguín-Veras et al. (2012) sought to find the optimal P&R location maximizing the potential market attracted by P&R, wherein a parabolic approximation was applied to the estimated catchment area for a given P&R site.

Furthermore, some other research works studied the formulation and analysis of network equilibrium models with P&R facilities. Elizabeth (1978) gave a mathematical analysis of a P&R scheme, coupled with a pedestrian precinct. Florian and Los (1979) described the multimodal choice as intermediate OD matrices combining an OD matrix from the trip origins to the P&R lots and an OD matrix from the P&R lots to the destinations, and proposed an entropy based trip distribution model with inequality constraints. Fernandez et al. (1994) presented three different approaches in the network equilibrium models with combined modes. Garcia and Marin (2002) focused on the optimization of parking investment and pricing decisions using a bi-level programming (BLP) model and it was solved through a simulated annealing algorithm. To maximize profit or minimize social cost during the morning peak hours, Wang et al. (2003) presented an equilibrium model with multimodal choice to achieve optimal P&R location and parking charge. Later, Wang et al. (2004a) developed multimodal equilibrium models with the objective of profit maximization or social cost minimization in a congested highway with congestion-free railway system, wherein the characteristics of the equilibrium patterns before and after P&R service introduced were compared. Garcia and Marin (2005) investigated the case of asymmetric costs for the network equilibrium with combined modes and formulated the model as a VI problem, which was solved by disaggregate simplicial decomposition algorithm. Similarly, Li et al. (2007) proposed a network equilibrium model to describe a multimodal transportation network with P&R services and elastic demand, which was equivalent to a VI problem as well. Later, Lam et al. (2007) also applied an equivalent VI approach to modeling a bimodal transport system with elastic demand, which was based on the super-network approach. As an extension of the study by Wang et al. (2004a), Liu

et al. (2009) relaxed two assumptions: railway was congestion-free and only one P&R facility was considered. A crowding cost function was added into the general cost of railway, and infinite P&R services were provided along the corridor instead. A deterministic continuum equilibrium model was developed to represent modal choice and P&R transfer behaviors, and the model was transformed to a discrete problem by a supernetwork approach in the end. Kepaptsoglou et al. (2010) offered a financial analysis model combined with a genetic algorithm for obtaining optimal pricing parameters, and applied the model to a shared-use, P&R facility of the Athens metro network in Greece. Wang and Du (2013) applied a linear complementarity system approach to model a railway and highway systems with a single P&R facility on a linear travel corridor, and the linear complementarity system formulation was transformed into a mixed integer linear programming (MILP) model to be solved.

Many remarkable achievements have been obtained in previous studies, but there are still many extensions that can be made in further study. The restrictive assumptions, like linear city corridor, uniform travel demand distribution, homogeneous commuters, deterministic modal choice, etc., can be relaxed. Models enabling the consideration of users' sensitivity to the costs by different travel modes need to be developed. Furthermore, reliability should be considered since both supply and demand cannot be perfectly known subject to stochastic variations in reality (Siu and Lo, 2008).

2.5 Optimal toll scheme

In major cities, most of commuters' work start-times fall into a small-interval time window and their departure choices concentrate in the peak hour and some major roads. This brings huge pressure to the transport system, to accommodate extremely high volume of demand in specific roads and peak hours. Various solution schemes have been proposed by policy makers and scholars to deal with the corresponding congestion problem. Robin (2006) conducted a wide-ranging literature review to find whether economists supported road pricing both in theory and practice, and road pricing had been unanimously regarded as an efficient management tool to reduce traffic congestion. As a management strategy, toll on road has been found to

be useful in many major cities, like Singapore and Beijing. In 1975, the first congestion pricing scheme of the world was carried out in Singapore. Not like traditional manual toll booth in the past, electronic toll system has been widely used worldwide, which makes toll strategy more flexible and reasonable, but more complex to design.

Toll pricing can be classified as first best toll pricing (FBTP) and second best toll pricing (SBTP), whereby the former considers all network links can be tolled while the latter assumes that tolls can only be imposed on parts of the links as a result of some constraints in practice. Early works on FBTP mainly focused on the application of the economic principle of marginal social cost pricing (MSCP). Moreover, toll pricing can also be divided into static/fixed toll and dynamic/time-varying toll. To my best knowledge, Pigou (1920) and Knight (1924) proposed the road pricing idea with MSCP application first, and then it was further studied by Beckmann et al. (1956); Walters (1959); Beckmann (1965); Vickrey (1969). Usually, the MSCP based toll strategy belongs to FBTP, and cordon-based pricing scheme is regarded as SBTP. Morrison (1986) investigated the theoretical and empirical work on road pricing based on a wide range of literature, including economic sources as well as engineering and planning aspects. Arnott et al. (1990a) studied various toll patterns for a simple network with parallel routes, and compared uniform and step tolls. The uniform toll only diverted auto drivers from one route to another without cutting their number, and step toll resulted in better efficiency by altering the departure times. Arnott et al. (1990b) presented a comprehensive economic analysis of a bottleneck model with road congestion in the morning peak hour to determine the coarse toll with optimal capacities. Verhoef et al. (1996) developed a two-route (one tolled, another untolled) based network simulation model to analyze the SBTP and demonstrated the effects of several cost parameters. Johansson (1997) applied MSCP to maximize the social benefit with optimal road charges, and presented the optimal road charges as a function of speed not traffic flow. Yang and Bell (1997) relaxed the travel demand as a function of the generalized travel cost and studied the network equilibrium with elastic demand and queue. As a result, a link toll pattern could be obtained to restrict the travel demand. Later, Yang and Huang (1998) explored the network equilibrium problem with

queue and delay based on the economic principle of marginal cost pricing, and presented some new properties of the marginal cost pricing. McDonald et al. (1999) presented a two-route based highway model and derived the second-best optimum, then they used several numerical results to demonstrate the differences between the SBTP and other toll schemes. May and Milne (2000) tested four kinds of road pricing schemes, cordon-based, distance-based, travel time-based and congestion time-based, on the network assignment models of Cambridge city. The test results showed that travel time-based toll outperformed the others on most indicators. Brotcorne et al. (2001) proposed a BLP model on a multi-commodity network with upper level to determine the optimal tolls on the arcs of the network and lower level to find the shortest path for each user. Verhoef (2002) built a generalized SBTP model based on a congested network and proposed an efficient algorithm to find the optimal SBTP scheme. Li (2002) conducted a relevant analysis of a congestion pricing scheme in practice with speed-flow relationship, the demand function and the generalized cost, and applied this analysis results to Singapore for demonstration. Yang and Zhang (2002) incorporated both social and spatial equity constraints into a bi-level toll design model and applied a simulated annealing method to solve this toll design problem. Yang et al. (2004) developed a BLP model to formulate the optimal entry-exit based toll design problem, which was transformed to an equivalent single level model and solved by an augmented Lagrangian algorithm. Chen and Bernstein (2004) conducted a SBTP study, in which a bi-level toll design model with heterogeneous users could be transformed to a single level nonlinear programming (NLP) model based on several assumptions. Friesz et al. (2004) proposed a theory of dynamic congestion pricing as a continuous-time optimal problem, which showed an analysis of the necessary conditions for the optimal congestion pricing. Based on FBTP framework, Yildirim and Hearn (2005) presented a variable demand traffic equilibrium model and toll pricing framework for traffic planners. Friesz et al. (2007) proposed a dynamic toll design problem with equilibrium constraints and showed a direct solution method for a small problem. Particularly, they gave detailed analysis of the UE with dynamic tolls and characteristics of efficient tolls. Zhang et al. (2008) applied a two-stage method to investigate the joint morning and evening commuting problem,

and optimized the time-varying road toll to improve the transportation system. One of the new findings on road toll showed that evening road toll affected both morning and evening travel pattern compared to the morning travel pattern was influenced by the morning toll only. Lu et al. (2008) developed a bi-criterion DUE model with time-varying toll scheme, which was solved by a simulation-based heuristic approach. Ban and Liu (2009) formulated the dynamic SBTP as a BLP model in discrete-time scheme, and solved it iteratively by converting it to a single level model using nonlinear complementarity formulation. Based on Singapore's electronic road pricing (ERP) traffic data, Xie and Olszewski (2011) developed a modeling method to predict short-term impacts of rate adjustment on peak hour traffic, and the experimental results proved that the proposed method was useful in short-term adjustment of rate. Meng and Liu (2011) presented a cordon-based toll scheme based on the Probit-based SUE constraints and used a trial-and-error method for solution. A numerical example of Singapore was used to demonstrate the validity of the model and efficiency of the algorithm. Based on a discrete time scheme, Doan et al. (2011) proposed a linear complementarity model to formulate the UE constraints with optimal pricing strategy and provided analytical discussion on the existence of the optimal toll. Ekström et al. (2012) applied a MILP model to approximate the toll design problem so that global optimal solution could be obtained. Both toll locations and levels would be determined through the model based on the principle of UE. Meng et al. (2012) focused on the optimal distance-based toll, which was determined by a positive and non-decreasing function of travel distance. This toll design problem was formulated as MILP model and solved by a hybrid genetic algorithm-cost averaging method. Based on binary Logit-based mode split model, Meng and Liu (2012) transformed a bi-modal transportation network with cordon-based toll scheme to a traffic assignment problem with elastic demand, and then transformed it again to a fixed-point model, which was solved by a convergent cost averaging method. To handle congestion pricing problem combined with DUE flow, Chung et al. (2012) adopted a robust optimization method to handle this SBTP problem and compared it with the other two alternative methods to show its efficiency. Li et al. (2012b) considered a continuum intermodal equilibrium model with optimal road toll pricing, which showed the toll scheme

influenced the multimodal split pattern and could be useful in the coordination of multimodal choice behavior. Ban et al. (2013) discussed the possibility of non-uniqueness of the UE solution by a SBTP model with a risk-neutral scheme and found that the optimal toll pricings were quite different correspondent to the non-unique UE solutions.

2.6 Optimal urban density

Urban transportation system is important urban infrastructure for any major city since transportation and land use affect each other in a reciprocal way that transportation brings accessibility and convenience to land use, and the increase of land utilization requires better transportation service. This relationship, as an ‘infinite shape’ cycle shown in Figure 2.5, is famous and important for both urban planners and transportation engineers (Emberger et al., 2008).

Figure 2.5 Land use & transportation ‘infinite shape’ interaction

It is ideal if the transportation system is optimal in social, environmental and financial dimensions. However, many major cities are facing the problem of traffic congestion and thus social loss, as well as the environmental issues. To address these problems, the key indeed lies in how the optimal urban policies are designed and implemented in an integrated way, with coordination across land use planning and urban transport management. Usually the urban form has a significant influence on transportation system, for example, Tong and Wong (1997) found that high density urban development appeared to have many transport advantages based on

the case study in Hong Kong, where high urban density was a leading factor of efficient public transit services. Kenworthy and Laube (1999) provided an insight into some summary findings from a large comparative international study of automobile dependence in 32 cities worldwide, and concluded that urban density was positively correlated with transportation system since higher urban density leads to higher level of public transit use while lower usage level of private car and lower total operating cost. Similarly, based on statistical analyses, Sinha (2003) found that higher level utilization of public transit was indeed associated with overall sustainability, and urban density was regarded as a key indicator of the level of automobile ownership and use, which was also related to transportation usage. Moreover, higher urban population density led to more transit boardings and less private car usage usually, and vice versa.

Traditionally, urban density distribution is regarded as given input for transportation planning. In practice, urban planners can apply appropriate policies, like spatially-variable excise subsidies to (or taxing on) housing and floor area ratio regulation, to achieve desired population allocation. The significant relationship between urban density and transport system optimization has received extensive research attentions in the literature. The approaches for combining land use and transportation models date back to early 1960s. In the early stage, two distinguished streams of methods were used to model the relationships between land use and transportation system: economic-behavioral based economic theories of utility/profit maximization, and mathematical programming/network equilibrium (Giuliano, 1989). It can be also summarized in another two categories: aggregate models and disaggregate models. Most of the earlier models, such as gravity model, entropy model, and so on, are aggregate models, while the majority of the subsequent models shift to disaggregate models. The fundamental difference between these two kinds of models is that the disaggregate models take into account the individual as the decision-making unit. Recently, the methods are divided into another two classes: the market simulation approach and the planned allocation approach (Yim et al., 2011).

As for the market simulation approach, the Lowry model is the initial transportation-land use model, which was developed in 1964 for the Pittsburgh region (Lowry, 1964). After that, a lot of studies have been done and

comprehensive literature reviews are available (Goldner, 1971; Wilson, 1971; Putman, 1980; Meyer and Miller, 1984; Clement, 1996; Wilson, 1998; Wong et al., 1998; Wegener, 2004b).

For the planned allocation approach, a large volume of research has been devoted to the investigation of combining land use and transportation problems, especially for the residential location and transportation problems. For elaborate literature review of land use–transportation combination studies, one can refer to Berechman and Small (1987); Giuliano (1989); Moore II and Kim (1995); Wegener (2004a). In early stage, Herbert and Stevens (1960) used the micro economic theory to develop a LP model to handle the residential location and transportation problems, after that many extensions appeared. Mills (1972a) put forward a LP model to analyze and determine the optimal amounts of resources to be devoted to the transportation system. In comparison to most of the previous studies based on aggregate choice formulation, Lerman (1976) proposed a disaggregate multinomial Logit-based model to deal with the joint choice of residence location and mode to work, and some improvements had been done to lessen the limitations of the original model (Williams, 1977; McFadden, 1978). Based on some assumptions and data from existing metropolitan areas of equivalent sizes, Kim (1979) presented a general equilibrium transportation demand model applied to hypothetical cities. Sequentially he developed a combined transportation-land use model considering the zonal travel demand endogenously determined, and another combined transportation-land use model to integrate transportation congestion cost and land use intensity explicitly (Kim, 1983, 1986). Empirical studies have confirmed that population density declines as distance to the urban center increases (Mills, 1970). The negative exponential distribution function is the most commonly used function to depict the population distribution, and Brueckner (1982) conducted a well investigation on its sufficient conditions. Another mathematical analysis of reformulating the population density functions and their applications can be found in Batty and Kim (1992). A combinatorial programming model of joint optimization of land use and transportation was proposed with different exact and heuristic methods being compared (Marc, 1978). After that, Marc (1979) investigated three kinds of design problems: network design problem, land use design problem and a

combination problem. A new improved continuous Logit-based model was proposed with a choice theory added in, thus the various continuous models could be unified to derive more general models (Ben-Akiva et al., 1985). Based on game theory, a bid-rent network equilibrium model was generated to formulate the relationship between transportation and residential location as an n-player non-cooperative game, and a path-based heuristic algorithm was proposed to solve the BLP (Chang and Mackett, 2006). As an extended work of Briceño et al. (2008), Bravo et al. (2010) put forward an integrated model to describe the combination of land use and transportation system, which could be formulated as a fixed-point problem. Yim et al. (2011) proposed a bi-level reliability-based land use and transportation combination model with OD demand following a certain distribution. This model was solved by a genetic algorithm (GA) with a simple numerical example as illustration.

2.7 Network design problem

Transportation network needs to be improved, addition of new links or expansion of existing links' capacities within the given budget, in order to satisfy the rapidly growing travel demand and deal with the congestion problem corresponding. In transportation network modeling and optimization literature, such an improvement is referred to as network design problem (NDP).

Generally, NDP, which is known as a non-deterministic polynomial-time hard (NP-hard) problem (Magnanti and Wong, 1984), determines the optimal decision variables, such as expansion of existing links' capacities, namely continuous network design problem (CNDP), addition of new links, formally named as discrete network design problem (DNDP), and both of them, called mixed network design problem (MNDP), within the investment budget so as to minimize or maximize a specific network performance index, like total travel time, generalized cost, total profit and reserve capacity (Yang and Bell, 1998a), while satisfying the SO criterion or accounting for the travelers' route choice behavior in a UE or SUE manner (Luathép et al., 2011). A great deal of work has been done in NDP, and a host of useful reviews on this problem are available (Magnanti and Wong, 1984; Yang and Bell, 1998b; Friesz and Shah, 2001; Meng et al., 2001).

Traditionally, the transportation problem is often formulated as a single level model, which could not reflect the interaction between the supply side and the demand side, as well as the behaviors of the demand side. Therefore, the NDP is often modeled as a BLP model according to its hierarchical characteristic, which is a special case of multilevel programming with a two-level structure: the transportation planner or operator wishes to achieve an optimal policy based on some control variables y , and users' response to these controls in the form of network flow x . The upper level problem is to determine the minimized value of a function of both y and x based on some constraints imposed, while the lower level problem focuses on finding the UE flow for x . Such kinds of problems can be found easily in transportation problems, including NDP, optimal toll pricing problem and network signal setting problem, and so on.

The general form of a BLP is as follows:

$$\begin{aligned}
 & \min_y f^u(x, y) \\
 \text{[Upper level]} \quad & s.t. \quad g_i^u(x, y) \geq a_i^u, i = 1, 2, \dots, n \\
 & \quad \quad h_j^u(x, y) = b_j^u, j = 1, 2, \dots, m
 \end{aligned} \tag{2-24}$$

where x is the solution of the following optimization problem:

$$\begin{aligned}
 & \min_x f^l(x, y) \\
 \text{[Lower level]} \quad & s.t. \quad g_i^l(x, y) \geq a_i^l, i = 1, 2, \dots, p \\
 & \quad \quad h_j^l(x, y) = b_j^l, j = 1, 2, \dots, q
 \end{aligned} \tag{2-25}$$

where y is fixed in this level.

The general properties of BLP models for NDP are shown in Figure 2.6, and a summary of previous studies in NDP is shown in Table 2.1.

Figure 2.6 General properties of BLP models for NDP

Table 2.1 A summary of NDP models

Year	1st author	Problem	Model	Algorithm	Upper level problem	Lower level problem
1975	Larry J. Leblanc	DNDP	BLP $\approx$ MINP	Branch-and-bound method	Min TC & budget constraint	UE
1979	Mustafa Abdulaal	CNDP	NLP	Powell's method & Hooke and Jeeves' method	Min TC+IC & budget constraint	SO
1979	George B. Dantzig	CNDP	NLP	Lagrange multiplier technique & decomposition procedure	Min TC+IC & budget constraint and link capacity bound	SO
1980	D.E. Boyce	DNDP + trip distribution	–	Tree-search algorithm; branch-and-bound method	–	–
1982	Hossain Poorzahedy	DNDP	BLP $\approx$ NLP	Two branch-and-backtrack based algorithms	Min TC & budget constraint and link capacity bound	UE
1986	Larry J. Leblanc	CNDP	BLP $\approx$ NLP	Frank-Wolfe approach	Min TC+IC & budget constraint	UE
1986	Patrice Marcotte	CNDP	BLP	Four heuristic procedures	Min TC	UE $\approx$ VI
1987	Chaisak Suwansirikul	CNDP	BLP	Equilibrium decomposed optimization heuristic method	Min TC+IC & link capacity bound	UE
1988	Omar Ben-Ayed	CNDP	BLP	–	Min TC+IC & link capacity bound	UE
1991	Mingyuan Chen	DNDP	BLP	Heuristic algorithm based on branch-and-bound method	Min TC+IC	SUE
1992	Terry L. Friesz	CNDP	MP	Simulated annealing algorithm	Min TC+IC	UE $\approx$ VI
1993	Terry L. Friesz	CNDP	Multi-objective MP	Simulated annealing algorithm & weighting method	Four objectives (TC, IC, total vehicle miles, total dwelling units) & budget constraint	UE
1994	Gary A. Davis	CNDP	NLP	Generalized reduced gradient method & sequential quadratic programming	Min TC+IC	SUE
1998	Rajendra S. Solanki	DNDP	MP	Decomposition with modified quasi-optimization	Min TC & budget constraint	–
1998	Hai Yang	NDP	–	–	Max Reserve capacity & budget constraint	UE / SUE
2001	Q. Meng	CNDP	BLP $\approx$ SCDP	Augmented Lagrangian method	Min TC+IC & link capacity bound	UE

2002	Q. Meng	CNDP	BLP	Penalty function combined with simulated annealing method	Optimal critical ratio interval & budget constraint	UE
2003	Hong K. Lo	CNDP with degradable link capacity	NLP	–	Max demand multiplier & budget constraint and reliability constraints	PUE
2003	Zvi Drezner	DNDP	MP	Descent algorithm, simulated annealing algorithm, tabu search and genetic algorithm	Min TC+IC	–
2005	Suh-Wen Chiou	CNDP	BLP	Gradient based descent approach	Min TC+IC	UE
2005	Ziyou Gao	DNDP	BLP	Support function based generalized benders decomposition method	Min TC & budget constraint	UE
2005	Hossain	DNDP	BLP	Ant system approach	Min TC & budget constraint	UE
2006	Poorzahedy Giulio Erberto Cantarella	MNDP with elastic demand	BLP	Heuristic multi-criteria technique based on genetic algorithm	Multi-objective & budget constraint	UE/SUE
2007	Hossain	DNDP	BLP	Hybrid metaheuristic algorithm	Min TC & budget constraint	UE
2007	Poorzahedy Richard D. Connors	CNDP with elastic demand	BLP	Link based sensitivity analysis & gradient based approach	Max network performance (revenue)	Probit SUE
2007	Magnus Josefsson	CNDP	BLP	Gradient projection algorithm	Min TC+IC & link capacity bound	UE
2008	Yikui Mo	MNDP	BLP	Genetic algorithm	Min TC+IC & link capacity bound	UE & trip distribution
2009	Sun Yang	MNDP	BLP	Immune clone annealing algorithm	Min TC & budget constraint	UE
2009	Satish V. Ukkusuri	CNDP with demand uncertainty	BLP $\approx$ FNDP	An interior point method based approach	Max PSCS & budget constraint and link capacity bound	UE $\approx$ NCP
2010	David Z. W. Wang	CNDP	BLP $\approx$ MLP	–	Min TC+IC & link capacity bound	UE
2011	Paramet Luathep	MNDP	BLP $\approx$ MLP	Algorithm based on cutting constraint method	Min TC & budget constraint and link capacity bound	UE $\approx$ VI

2011	Hamid Farvaresh	DNDP	BLP $\approx$ MILP	Branch-and-bound based method	Min TC & budget constraint	UE
2012	Changmin Li	CNDP	BLP $\approx$ NLP	Multicutting plane method	Min TC+IC & link capacity bound	UE
2013	Shuaian Wang	DNDP	BLP $\approx$ NLP	SO-relaxation method, UE-reduction method, dynamic outer-approximation scheme	Min TC & budget constraint	UE
2014	Guangmin Wang	CNDP with tradable credit scheme	BLP $\approx$ relaxed NLP	A relaxation algorithm	Min TC & budget constraint	UE
2014	Guangmin Wang	CNDP	NLP	Cutting constraint algorithm	Min TC & budget constraint	UE

(**Note:** ' $\approx$ ' – transformation to an equivalent approximation problem, Min – minimize, Max – maximize, TC – travel cost, IC – improvement cost, MP – mathematical programming, MINP – mixed integer nonlinear programming, PUE – probabilistic user equilibrium, SCDP – single level continuously differentiable problem, PESCS – present expected system consumer surplus)

A general summary of algorithms for NDP in previous studies is shown in Table 2.2 as follows.

Table 2.2 A general summary of algorithms for NDP in previous studies

NDP	Algorithm	References
CNDP	Powell's method & Hooke and Jeeves' method	(Abdulaal and LeBlanc, 1979)
	Simulated annealing algorithm	(Friesz et al., 1992), (Friesz et al., 1993a), (Meng and Yang, 2002)
	Equilibrium decomposed optimization	(Dantzig et al., 1979), (Suwansirikul et al., 1987)
	Gradient based method	(Davis, 1994), (Chiou, 2005), (Josefsson and Patriksson, 2007)
	Sensitivity analysis based method	(Sumalee et al., 2006), (Connors et al., 2007)
	Augmented Lagrangian method	(Meng et al., 2001)
	Interior point method based approach	(Ukkusuri and Patil, 2009)
	Path based mixed integer linear program	(Wang and Lo, 2010)
	Multicutting plane method	(Li et al., 2012a)
	Cut constraint algorithm	(Wang et al., 2014a)
	Relaxation scheme/algorithm	(Ban et al., 2006), (Wang et al., 2014b)
DNDP	Branch-and-bound based method	(Leblanc, 1975), (Boyce and Janson, 1980), (Poorzahedy and Turnquist, 1982), (Chen and Alfa, 1991), (Farvaresh and Sepehri, 2011), (Solanki et al., 1998)
	Decomposition with modified quasi-optimization	
	Descent algorithm, simulated annealing algorithm, tabu search, genetic algorithm	(Drezner and Wesolowsky, 2003)
	Ant system approach	(Poorzahedy and Abulghasemi, 2005)
	Support function based generalized benders decomposition method	(Gao et al., 2005)
	Hybrid metaheuristic algorithm	(Poorzahedy and Rouhani, 2007)
	SO-relaxation method, UE-reduction method, dynamic outer-approximation scheme	(Wang et al., 2013)
MNDP	Genetic algorithm based method	(Cantarella and Vitetta, 2006), (Mo et al., 2008)
	Immune clone annealing algorithm	(Sun et al., 2009)
	Cutting constraint algorithm based method	(Luathep et al., 2011)

2.7.1 Continuous network design problem

Recently, more and more attention has been devoted to CNDP, also named network capacity expansion problem, which is to determine a set of capacity enhancements (modeled as continuous variables) of existing links in a transportation network with the corresponding equilibrium flows to make the network performance optimal. CNDP is recognized as one of the most difficult and challenging problems in

transportation in terms of the fact that BLP model for CNDP is non-convex and non-differentiable, so that most of the methods are based on heuristic algorithms (Meng et al., 2001).

Mathematically, the common BLP model for CNDP under UE can be expressed as follows:

$$\begin{aligned}
 \text{[CNDP-Upper]} \quad & \min_y \tau \sum_{a \in A} t_a(x_a, y_a) \cdot x_a + \theta \sum_{a \in A} g_a(y_a) \\
 & s.t. \quad l_a \leq y_a \leq u_a, \quad \forall a \in A
 \end{aligned} \tag{2-26}$$

where x_a is UE flow (obtained from the lower level programming) on link a ; A is the set of links; y_a is the link capacity of link a after expansion, $\forall a \in A$; t_a and g_a are travel time and construction cost on link a respectively, $\forall a \in A$; l_a and u_a are lower and upper bounds of the capacity expansion on link a respectively, $\forall a \in A$; τ denotes value of time (VOT) parameter, and θ is the relative weight of total capacity enhancement cost in the objective function.

$$\begin{aligned}
 \text{[CNDP-Lower]} \quad & \min_x \sum_{a \in A} \int_0^{x_a} t_a(w, y_a) dw \\
 & s.t. \quad \sum_{r \in R^w} f_r^w = q^w, \quad \forall w \in W \\
 & \quad x_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{ar}^w, \quad \forall a \in A \\
 & \quad f_r^w \geq 0, \quad \forall r \in R^w, w \in W
 \end{aligned} \tag{2-27}$$

where f_r^w is the flow on path r between OD w ; W is the set of OD pairs; R^w is the set of routes between OD w , $\forall w \in W$; δ_{ar}^w is the link-path incidence factor, which is 1 if link a is on path r between OD pair w , otherwise 0, $\forall a \in A, r \in R^w, w \in W$, and q^w is the travel demand between OD pair w , $\forall w \in W$.

If the objective function in lower level problem is changed to

$$\min_x \sum_{a \in A} x_a t_a(x_a, y_a) \tag{2-28}$$

then the CNDP model under UE is changed to CNDP under SO.

To date, there are abundant models and algorithms that have been presented to deal with CNDP. Abdulaal and LeBlanc (1979) formulated the CNDP as a NLP problem, which could be converted to an unconstrained problem so that it could be solved by

direct search method, and they discussed the advantages and disadvantages of these methods as well. Dantzig et al. (1979) adopted Lagrange multiplier technique and decomposition procedure to handle CNDP with SO criterion. LeBlanc and Abdulaal (1984) compared the computational efficiency and results by solving two models of CNDP with user-optimum flow and SO flow respectively, and got the results of SO model as good as those from the user-optimum model. LeBlanc and Boyce (1986) proposed a piecewise linear BLP model for the NDP for middle and small size problem (fewer than 200 nodes), while for larger networks, an equivalent and approximating NLP was transformed from the BLP model and Frank-Wolfe method was adopted to solve this problem efficiently. Marcotte (1986) used four heuristic procedures to deal with the CNDP and gave detailed analysis. Suwansirikul et al. (1987) suggested an equilibrium decomposed optimization heuristic method to deal with the CNDP. Ben-Ayed et al. (1988) provided different formulations for CNDP with different investment function, such as linear, convex and/or concave function, and gave a more general representation of travel cost function. Friesz et al. (1992) constructed a mathematical programming with VI constraints to describe the CNDP, and proposed a simulated annealing algorithm to solve it. After that, Friesz et al. (1993a) used simulated annealing algorithm to solve a multi-objective model of CNDP. Davis (1994) analyzed that Logit-based SUE made the CNDP differentiable and tractable, and applied the generalized reduced gradient method and sequential quadratic programming to solve the CNDP with Logit-based SUE. Meng et al. (2001) created an equivalent single level continuously differentiable but still non-convex optimization formulation for the CNDP considering UE, and applied a locally convergent augmented Lagrangian method to solve this problem. Later on, Meng and Yang (2002) analyzed the benefit distribution and equity issue in the CNDPs, and used penalty function combined with a simulated annealing method to solve a BLP CNDP model with an equity constraint. Chiou (2005) exploited a gradient based descent method to solve CNDP with corresponding UE flows following Wardrop's first principle. Ban et al. (2006) proposed a general framework to describe CNDP as a mathematical programming with complementarity constraint (MPCC), which was converted to a single-level problem and solved by a relaxation scheme. Josefsson and Patriksson (2007) made a sensitivity analysis of separable

traffic equilibrium models, and used a gradient projection algorithm to solve the CNDP with DUE and budget constraint. The results showed that the sensitivity analysis was accurate and produced better solutions than previous heuristics. Based on sensitivity analysis, Connors et al. (2007) adopted a gradient based approach to solve the NDP with Probit-based SUE and elastic demand. Wang and Lo (2010) transformed the CNDP from BLP to MILP by linearization approximation so that the global optimum solution could be guaranteed, and this scheme was also applicable to other types of NDP. Li et al. (2012a) converted the CNDP to a sequence of single level NLP problems based on gap and penalty functions to obtain the global solution, and the NLP problem could be solved by a multicutting plane method. Different from the conventional homogeneous users assumption, Wang et al. (2014a) modeled the CNDP as a mathematical programming with equilibrium constraint (MPEC) and considered heterogeneous users with corresponding VOT. This problem was solved by a cut constraint algorithm and proved to be feasible via numerical experiments. Combined with tradable credit, Wang et al. (2014b) modeled the CNDP as a BLP problem with consideration of increasing road capacity and reducing traffic demand simultaneously, which was solved by a relaxation algorithm.

Although a large amount of studies have been done in CNDP, almost all the previous studies regarded the travel demand as fixed and the link capacity as stable. In real transportation system, there exists much uncertainty so that travel demand uncertainty, link capacity uncertainty and flexibility in NDP should be considered to describe the travel behavior in real and optimize the network reliability. Lo and Tung (2003) focused on modeling the network performance under degradable link capacities. As an extension, they also developed a CNDP model with degradable link capacities, and used the maximization of demand multiplier as the objective with reliability constraints. Ukkusuri and Patil (2009) formulated a flexible NDP model considering both demand uncertainty and elasticity with the objective to maximize the present expected system consumer surplus, and converted the UE constraint to an equivalent nonlinear complementarity condition. An interior point method based optimization solver KNITRO 5.0 was used to solve this problem.

2.7.2 Discrete network design problem

DNDP is concerned with adding indivisible links (modeled as integer variables) to existing transportation network under budget constraint and optimize the network performance with network equilibrium considered.

Mathematically, the common BLP model for DNDP under UE can be expressed as follows.

$$\begin{aligned}
 & \min_y \sum_{a \in A \cup \bar{A}} x_a t_a(x_a) \\
 \text{[DNDP-Upper]} \quad & s.t. \sum_{a \in \bar{A}} h_a \bar{y}_a \leq B, \quad \forall a \in \bar{A} \\
 & \bar{y}_a = 0 \text{ or } 1, \quad \forall a \in \bar{A}
 \end{aligned} \tag{2-29}$$

where A is the set of existing links which will not be modified; $\bar{A}$ is the set of new candidate links; h_a is the construction cost associated with new added link a , $\forall a \in \bar{A}$; $\bar{y}_a$ indicates whether link a will be constructed, which is 1 if link a is constructed, otherwise 0, $\forall a \in \bar{A}$; B is the total investment budget.

$$\begin{aligned}
 & \min_x \sum_{a \in A \cup \bar{A}} \int_0^{x_a} t_a(w) dw \\
 \text{[DNDP-Lower]} \quad & s.t. \sum_{r \in R^w} f_r^w = q^w, \quad \forall w \in W \\
 & x_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{ar}^w, \quad \forall a \in A \cup \bar{A} \\
 & f_r^w \geq 0, \quad \forall r \in R^w, w \in W \\
 & x_a \leq M \bar{y}_a, \quad \forall a \in \bar{A}
 \end{aligned} \tag{2-30}$$

where M is a large positive constant to make sure that no traffic flow exists on the link if it is not chosen to build. If the objective function in lower level problem is changed to

$$\min_x \sum_{a \in A \cup \bar{A}} x_a t_a(x_a) \tag{2-31}$$

then the DNDP model under UE is changed to DNDP under SO.

Compared to CNDP, less effort has been devoted to DNDP. Leblanc (1975) made a pioneer study of the DNDP by modeling it as a mixed integer nonlinear programming (MINP) to achieve the total congestion minimized, and adopted a branch-and-bound strategy to solve it. Boyce and Janson (1980) proposed three

DNDP formulations synthesizing NDP, trip distribution and traffic assignment to optimize discrete additions to the existing link capacities. In these problems, the first one considered variable trip distribution with fixed link travel costs, and the second problem was a DNDP combined with distribution and UE assignment, while the last one included trip distribution and SO assignment. The tree-search algorithm and branch-and-bound procedure were adopted to solve these problems. Poorzahedy and Turnquist (1982) used two branch-and-backtrack based algorithms to solve the DNDP by transforming it from BLP to a single level NLP. Chen and Alfa (1991) developed a heuristic method based on brand-and-bound method to solve DNDP considering SUE. Solanki et al. (1998) proposed an algorithm, a combination of decomposition technique and a variation of the modified quasi-optimization heuristic method, to deal with large DNDP by decomposing it in a sequence of small problems. Drezner and Wesolowsky (2003) compared four methods, descend algorithm, simulated annealing algorithm, tabu search and genetic algorithm, and found that genetic algorithm performed the best in the computational tests on DNDP. Based on the concept of support function in generalized benders decomposition method, Gao et al. (2005) transformed the upper level problem in DNDP to a usual NLP so that many optimization techniques could be used to solve it. Poorzahedy and Abulghasemi (2005) used ant system approach to solve DNDP with the Sioux Falls network for test, and got satisfactory results. Following the earlier effort, Poorzahedy and Rouhani (2007) hybridized seven heuristic algorithms and found that hybrids were more effective to solve DNDP than the original ant system. Farvaresh and Sepehri (2011) transformed the BLP model of DNDP to an equivalent approximated MILP formulation by complementarity conditions and linear approximation method. Wang et al. (2013) considered a multi-capacity DNDP model to optimize the addition of new links as well as their numbers of lanes. To solve this BLP model, they applied a SO-relaxation and a UE-reduction methods to analyze the properties of the model and solved it with a dynamic outer-approximation scheme.

2.7.3 Mixed network design problem

The MNDP combines CNDP and DNDP together and optimize the discrete and continuous decision variables simultaneously.

Mathematically, the common BLP model for MNDP under UE can be expressed as follows.

$$\begin{aligned}
 & \min_{y, \bar{y}} \sum_{a \in A} x_a t_a(x_a, y_a) + \sum_{a \in \bar{A}} x_a t_a(x_a) \\
 & s.t. \quad \sum_{a \in A} g_a(y_a) + \sum_{a \in \bar{A}} h_a \bar{y}_a \leq B \\
 & \quad l_a \leq y_a \leq u_a, \quad \forall a \in A \\
 & \quad \bar{y}_a = 0 \text{ or } 1, \quad \forall a \in \bar{A}
 \end{aligned} \tag{2-32}$$

$$\begin{aligned}
 & \min_x \sum_{a \in A} \int_0^{x_a} t_a(w, y_a) dw + \sum_{a \in \bar{A}} \int_0^{x_a} t_a(w) dw \\
 & s.t. \quad \sum_{r \in R^w} f_r^w = q^w, \quad \forall w \in W \\
 & \quad x_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{ar}^w, \quad \forall a \in A \cup \bar{A} \\
 & \quad f_r^w \geq 0, \quad \forall r \in R^w, w \in W \\
 & \quad x_a \leq M \bar{y}_a, \quad \forall a \in \bar{A}
 \end{aligned} \tag{2-33}$$

If the objective function in lower level problem is changed to

$$\min_x \sum_{a \in A} x_a t_a(x_a, y_a) + \sum_{a \in \bar{A}} x_a t_a(x_a) \tag{2-34}$$

then the MNDP under UE is changed to MNDP under SO.

By contrast, rare investigations have been done in MNDP. Cantarella and Vitetta (2006) analyzed the MNDP, including discrete and continuous decision variables, by a heuristic multi-criteria technique based on genetic algorithm, and considered different optimization objectives, elastic demand and peak periods. Based on genetic algorithm, Mo et al. (2008) gave a small example on MNDP. Sun et al. (2009) used an immune clone annealing algorithm, a combination of annealing tactic of simulated annealing algorithm and immune clone algorithm, to solve the BLP of MNDP. As an extension of Wang and Lo (2010), Luatkep et al. (2011) adopted a MILP formulation to approximate a MNDP so that a global optimum solution could be obtained, and the solution framework could be applicable to any type of NDP.

2.7.4 A short summary

In terms of the previous literature review on NDP, it can be concluded that most of the previous studies were focused on CNDP, while lesser attention was paid on DNDP, and rare work has been done on MNDP.

It is obvious that heuristic algorithms are the most commonly used methods on dealing with NDP in terms of the natural properties of the problem, however the global solution is hard to achieve. Although some algorithms could find reasonable local solutions, they sacrifice the efficiency and consume a lot of computational time. In future work, searching the global solution or solution near the global optimum and developing efficient algorithms are important issues.

No matter CNDP, DNDP or MNDP, it is often assumed that the travel demand is fixed, the network is stable, single user class, and so on. In reality, these restricted assumptions need to be relaxed to describe the NDP in a more realistic way. Multiple user classes, elastic travel demand, degradable link capacity, network reliability, and so on, should be considered into the NDP models.

2.8 Methodology

2.8.1 Variational inequalities

Variational inequalities (VI) provides us with a tool for formulating a variety of equilibrium problems and qualitatively analyzing the problems in terms of existence and uniqueness of solutions, and stability and sensitivity analysis (Kinderlehrer and Stampacchia, 2000). In general, VI is a commonly used method to handle equilibrium problem, and extensive studies have been done on modeling, solution methods and applications of VI formulations in last decades (Fisk and Boyce, 1983; Vliet, 1987; Nagurney, 1993; Bernstein, 1994; Kinderlehrer and Stampacchia, 2000; Nagurney, 2000; Nagurney and Dong, 2002).

Mathematically, the formal formulation of VI is described as below.

The VI problem $VI(F, K)$, is to determine a vector $x^* \in K \subset R^n$, such that

$$F(x^*)^T (x - x^*) \geq 0, \quad \forall x \in K \quad (2-35)$$

or equivalently,

$$\langle F(x^*)^T, (x - x^*) \rangle \geq 0, \quad \forall x \in K \quad (2-36)$$

where F is a given continuous function from K to R^n , K is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in n dimensional Euclidean space.

In geometric terms, the VI problem states that $F(x^*)^T$ is orthogonal to the feasible set K at the point x^* . This formulation plays an important role in unifying the description of equilibrium problems and other optimization problems.

For example, UE conditions are completely equivalent to VI (Smith, 1979; Vliet, 1987). UE mathematical programming model is shown in [UE] (2-1), and the equivalent VI problem for [UE] (2-1) is to find the equilibrium flow vector $f^* \in \Omega$ such that

$$[UE-VI] \quad \Omega = \left\{ x_a \left| \begin{array}{l} c(f^*)^T (f - f^*) \geq 0, \quad \forall f \in \Omega \\ \sum_{r \in R^w} f_r^w = q^w, \quad x_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{ar}, \quad f_r^w \geq 0 \\ \forall a \in A, \quad r \in R^w, \quad w \in W \end{array} \right. \right\} \quad (2-37)$$

where $c(f^*)$ is the route cost function, and Ω represents the feasible route set.

2.8.2 Complementarity problem

Nonlinear complementarity problem (NCP) was introduced by Cottle in his Ph.D. thesis in 1964 (Cottle, 1964), since then a great deal of research work has been done on this subject. Given a vector $q \in R^n$ and a matrix $M \in R^{n \times n}$, the NCP, written as $NCP(q, M)$, is to find a vector $x \in R^n$ satisfying the following conditions:

$$\begin{aligned} x &\geq 0 \\ f(x) = Mx + q &\geq 0 \\ x^T (Mx + q) &= 0 \end{aligned} \quad (2-38)$$

where $f(x)$ is a smooth mapping, M is the coefficient matrix of x , and q is the rest part not dependent on x , which is given or can be calculated directly (Facchinei and Pang, 2003). The above $NCP(q, M)$ can be expressed in a more succinct way as:

$$0 \leq x \perp Mx + q \geq 0 \quad (2-39)$$

where the notation “ $\perp$ ” means “perpendicular”. Moreover, if the function $f(x)$ is linear, then the $NCP(q, M)$ is called a linear complementarity problem (LCP), written as $LCP(q, M)$. As a special case of VI, complementarity problem (CP) has also been demonstrated to be a powerful tool to model and compute equilibrium, which is usually easier to solve than a general VI problem (Ban et al., 2008). For more research achievements on the formulations, solution methods and applications of CP, comprehensive studies can be referred to Harker and Pang (1990); Cottle et al. (1992); Ferris and Pang (1997); Murty (1997); Serali et al. (1998); Facchinei and Pang (2003); Yong (2010).

The UE problem [UE] (2-1) can also be transformed to an equivalent CP formulation as follows:

$$\begin{aligned}
 & 0 \leq f_r^w \perp c_r^w - \pi^w \geq 0, \quad \forall r \in R^w, w \in W \\
 & \sum_{r \in R^w} f_r^w = q^w, \quad \forall w \in W \\
 \text{[UE-CP]} \quad & x_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{ar}^w, \quad \forall a \in A \\
 & c_r^w = \sum_{a \in A} \delta_{ar}^w \cdot t_a(x_a), \quad \forall r \in R^w, w \in W
 \end{aligned} \tag{2-40}$$

where π^w represents the minimum travel cost between OD pair w , and c_r^w is the travel time on route r . If $c_r^w = \pi^w$, then $f_r^w \geq 0$; if $c_r^w > \pi^w$, then $f_r^w = 0$. That is, any path that carries flow has its travel time equal to the minimum OD travel time; while travel times on the paths not carrying flow are greater than or equal to the minimum OD travel time, which is exactly the UE principle (Wang and Lo, 2010).

Some fundamental definitions and theorems related to the solution properties are introduced as below (refer to Facchinei and Pang (2003) for complete theory).

Definition 2.1 A mapping $F : K \subseteq R^n \rightarrow R^n$ is said to be

- (1) *monotone on K* if $(F(x) - F(y))^T (x - y) \geq 0, \quad \forall x, y \in K$;
- (2) *strictly monotone on K* if $(F(x) - F(y))^T (x - y) > 0, \quad \forall x, y \in K$ and $x \neq y$;
- (3) *strongly monotone on K* if there exists a constant $c > 0$ such that

$$(F(x) - F(y))^T (x - y) \geq c \|x - y\|^2, \quad \forall x, y \in K.$$

Theorem 2.1 Let $F : K \rightarrow R^n$ be a continuous function on $K = R_+^n$, then if F is

- (1) monotone on K , then CP has a convex (possibly empty) solution set;
- (2) strictly monotone on K , then CP has at most one solution;
- (3) strongly monotone on K , then CP has a unique solution.

To solve CP, many methods can be applied to find the solution, like Lemke's method, nonsmooth Newton-type method, smooth Newton-type method, line search method, interior point method, merit function method, projection method, proximal point method, linearization method, and so on.

2.8.3 Differential complementarity system

Although extensive studies have been done to describe and solve the UE problem, a key drawback of the previous studies is the lack of a tractable analytical formulation, especially for time-dependent DUE. Differential variational inequalities (DVI), a combination of ordinary differential equation (ODE), differential algebraic equation (DAE), VI and CP, is a powerful unifying mathematical framework for a host of dynamic problems involving inequalities and discontinuities (Pang and Stewart, 2008). The applications of DVI include many time-dependent problems, such as continuous-time DUE problems. Using DVI approach, continuous-time models could be developed, whose continuity, differentiability, or high-order smoothness are absent in discrete-time models (Ban et al., 2012a).

The standard DVI comprises two major components: ODE and VI, and the formal definition is as below.

Let $f : R^{1+n+m} \rightarrow R^n$ and $F : R^{1+n+m} \rightarrow R^m$ be two continuous vector functions, K be a nonempty closed convex subset of R^m , $\Gamma : R^{2n} \rightarrow R^n$ be a boundary function, and $T > 0$ be a terminal time. Based on the functions f , F and Γ , the set K , and the scalar T , the DVI is to find time-dependent trajectories $x(t)$ and $u(t)$, which satisfy condition (2-41) in the weak sense of Carathéodory for $t \in [0, T]$ and the algebraic condition (2-42).

$$\begin{aligned} \dot{x}(t) &= f(t, x(t), u(t)) \\ u(t) &\in SOL(K, F(t, x(t), \cdot)) \end{aligned} \tag{2-41}$$

$$\Gamma(x(0), u(T)) = 0 \quad (2-42)$$

where $\dot{x} = \frac{dx}{dt}$ represents the time-derivative of a function $x(t)$, x is a continuous function and u is an integrable function on $[0, T]$. The differential equation requires to be satisfied for almost all t only. The membership for $u(t)$ means that for any continuous $\tilde{u}: [0, T] \rightarrow K$, therefore

$$\int_0^T (\tilde{u}(t) - u(t))^T F(t, x(t), u(t)) dt \geq 0 \quad (2-43)$$

$SOL(K, \phi)$ denotes the solution set of the VI problem $VI(K, \phi)$, which is described as below.

For a generic mapping $\phi: R^m \rightarrow R^m$ and a nonempty closed convex set K in R^m , the $VI(K, \phi)$ is to find a vector $u \in K$ such that

$$(u' - u)^T \phi(u) \geq 0, \quad \forall u' \in K \quad (2-44)$$

In the general framework above, both the initial state $x(0)$ and the terminal state $x(T)$ are unknown variables, which need to be calculated.

For the special case with K as a cone $C (C \equiv R_+^m)$, the differential complementarity system (DCS) is shown as follows:

$$\begin{aligned} \dot{x}(t) &= f(t, x(t), u(t)) \\ C \ni u(t) &\perp F(t, x(t), u(t)) \in C^* \end{aligned} \quad (2-45)$$

Moreover, a proper specialization of the DCS yields the linear complementarity system (LCS) with linear functions (Heemels et al., 2000). The LCS is a novel class of nonsmooth differential systems that have recently attracted a lot of research interests. In general, an LCS comprises a linear ODE with an algebraic variable as a solution to a finite-dimensional LCP, which in turn is parameterized by the state variable in the linear ODE. The LCS can be described as follows:

$$\begin{aligned} \dot{x}(t) &= p + Ax + Bu \\ 0 &\leq u \perp q + Cx + Du \geq 0 \end{aligned} \quad (2-46)$$

where $f(t, x(t), u(t)) \equiv p + Ax + Bu$ and $F(t, x(t), u(t)) \equiv q + Cx + Du$ are expressed as linear functions, p and q are n -vector and m -vector respectively. A , B , C and D are given matrices of order $n \times n$, $n \times m$, $m \times n$ and $m \times m$ respectively. The LCS provides a computational tractable framework for the rigorous treatment of the UE problem in continuous time, which makes a positive contribution pertaining to the application of LCS to dynamic traffic problems (Pang et al., 2012). Recently, some studies have been done in the formulation of dynamic equilibrium conditions for a single bottleneck model using DCS (Ramadurai et al., 2010; Ban et al., 2012a, b; Pang et al., 2012).

From a modeling point of view, continuous-time models have multiple advantages over discrete-time models; from an analytical point of view, continuous-time models provide a rigorous framework for the understanding of many important issues of traffic flows that are otherwise neglected in a discrete-time framework. Given the nature of the dynamic traffic problems, the DCS approach is a promising methodology to formulate more complex network level models and obtain analytical solution properties, and it also could be used to describe the problems in spatial dimension, such as basic UE, SUE, multimodal choice, and so on. Recently, the DCS approach has been extensively used in many different application contexts (Schumacher, 2004; Shen and Pang, 2005; Friesz et al., 2006; Camlibel, 2007; Camlibel et al., 2007; Han et al., 2009; Pang and Stewart, 2009; Ramadurai et al., 2010; Friesz et al., 2011b; Ban et al., 2012b).

2.8.4 Geometric programming

Currently, the geometric programming (GP) modeling approach has been proved to be solved efficiently and reliably by many methods, even for large scale problems, and GP has been used widely in engineering, like digital circuit design, resource allocation in communication and network systems, inventory control, etc. (Scott et al., 2004; Boyd et al., 2005; Chiang, 2005). For more information on GP and its extensions and applications, some useful studies can be referred to, such as Peterson (1978); Wang et al. (2004b); Liu (2006); Samanta and Majumder (2006); Boyd et al. (2007); Tsai et al. (2007). Although GP modeling approach has been well investigated rigorously and widely used in various application fields, it is rarely

applied to transportation related fields. To my best knowledge, no previous scholars applied GP to NDP related problems, and only a few works can be found on multimodal and trip distribution models in 1980s. Wong (1981) used GP to develop a primal-dual relationship between maximum likelihood and entropy maximization formulations of the trip distribution model. Marín (1988) analyzed a multimodal combined model using a more general GP framework.

A standard GP model is an optimization problem in a special form as follows:

$$\begin{aligned} \min_x & f_0(x) \\ \text{s.t.} & f_i(x) \leq 1, \quad i = 1, 2, \dots, m \\ & g_j(x) = 1, \quad j = 1, 2, \dots, p \end{aligned} \quad (2-47)$$

where g_j are monomial functions (in the form of $cx_1^{a_1}x_2^{a_2}\cdots x_n^{a_n}$ with $c > 0$ and $a_i \in R$, $i = 1, 2, \dots, n$), f_0 and f_i are posynomial functions (in the form of $\sum_{k=1}^K c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}}$ with $c_k > 0$ and $a_{ik} \in R$, $i = 1, 2, \dots, n$, $k = 1, 2, \dots, K$), and x is the optimization variable vector with the components $x_i > 0$.

Due to the strict format requirements of the standard GP model, for example, the form of equality constraints must be monomial, which is restrained in practical application. Some relaxation and extension of the standard GP can be made to develop a more general framework called generalized geometric programming (GGP) or extended GGP (usually more relaxations of rules than GGP), which has wider application scopes. GGP can be converted to equivalent GP using some transformation methods, and it can be solved reliably and efficiently as GP (Boyd et al., 2007).

Although it is restrictive to develop a GP/GGP model, it is usually very easy and efficient to solve. To solve a GP/GGP, it is usually converted to an equivalent nonlinear but convex optimization problem based on logarithmic transformation technique. Let $y_i = \log x_i$, minimize $\log f_0$ with inequality constraints $\log f_i < 0$ and equality constraints $\log g_i = 0$, thus the original GP/GGP model can be transformed as follows:

$$\begin{aligned}
& \min_y \log f_0(e^y) \\
& s.t. \log f_i(e^y) \leq 0, \quad i = 1, 2, \dots, m \\
& \quad \log g_j(e^y) = 0, \quad j = 1, 2, \dots, p
\end{aligned} \tag{2-48}$$

where vector $y = (y_1, y_2, \dots, y_n)$ is variable. This transformed model is convex, and global optimum solution is amenable.

2.8.5 Approximation technique

Generally speaking, the optimization methods can be divided into two categories: exact search methods and approximation search methods. The former are often employed when the problem can be formulated as a specific mathematical programming model, such as LP, integer programming (IP), MILP, and so on. Exact search methods can give detailed mathematical analysis, and often can achieve satisfactory and global optimal solutions at the cost of flexibility since a number of assumptions and restrictions are often added into the models.

Modern problems tend to be very intricate and related to analysis of large data sets. Even if an exact algorithm can be developed, its computational time or space complexity may turn out unacceptable. But in reality it is often sufficient to find an approximate or partial solution, which extends the set of techniques to cope with the problem. Therefore, approximation techniques are adopted more and more in large scale and complex problems.

For example, to solve the DCS problem in Section 2.8.3, two important features of the mathematical model need to be taken into account: time dependence related to the ODE, and nonlinearity referring to the input functions f , F and possibly Γ . Time-stepping schemes can be used to deal with the first issue by replacing the time derivative by a finite-difference quotient (Han et al., 2009), and linearization techniques are often adopted to handle the second issue.

A typical procedure of time-stepping method is shown as below.

Step 1: Divide the time interval $[0, T]$ into $N_h + 1$ equivalent subintervals as below, and each of the subinterval length $h > 0$, thus $(N_h + 1)h = T$.

$$0 = t_{h,0} < t_{h,1} < \dots < t_{h,N_h+1} = T \tag{2-49}$$

Step 2: Approximate the time derivative $\dot{x}$ by a backward finite-difference quotient:

$$\dot{x}(t_{h,i+1}) \approx \frac{x(t_{h,i+1}) - x(t_{h,i})}{h}, \quad i = 0, 1, \dots, N_h \quad (2-50)$$

Step 3: A scalar $\theta \in [0, 1]$ can be used here to distinguish an explicit ($\theta = 1$), and implicit ($\theta = 0$), or a semi-implicit ($\theta \in (0, 1)$) discretization of the ODE.

$$\begin{aligned} x_{h,i+1} &= x_{h,i} + hf(t_{h,i+1}, \theta x_{h,i} + (1-\theta)x_{h,i+1}, u_{h,i+1}) \\ u_{h,i+1} &\in SOL(K, F(t_{h,i+1}, \theta x_{h,i} + (1-\theta)x_{h,i+1}, \cdot)) \\ 0 &= \Gamma(x_{h,0}, x_{h,N_h+1}) \end{aligned} \quad (2-51)$$

In this case, the original DVI problem can be transformed to an equivalent CP or VI model. As for LCP, NCP and VI, a host of methods can be applied, like interior point method, projection method, smoothing Newton method, nonsmooth Newton method, Karush–Kuhn–Tucker (KKT) conditions based method, complementary pivot algorithm (e.g., Lemke’s algorithm), principal pivoting methods (e.g., Graves’ principal pivoting method), iterative methods, and so on (Murty, 1997; Heemels et al., 2000). However, most of these methods guarantee to find global optimum solution or feasible solution based on the requirement of the matrix M of Eq. (2-39) to be symmetric and/or copositive plus (Murty, 1997). For the nonlinear problem with matrix M in general form, the problem can be solved by reformulating into an equivalent NLP model so that the numerical solutions can be found.

2.9 Summary

This chapter presents a comprehensive literature review mainly on static and dynamic user equilibrium and one of its special case, corridor problem, as well as its relevant extensions. To solve equilibrium based corridor problem, some useful modeling and solution methods are introduced as well. Moreover, the network design problem is also reviewed. The following conclusions can be drawn based on the previous reviews.

- (1) Both static and dynamic UE have taken many researchers’ effort on them, and the most commonly used method to model the equilibrium problem are VI approach, complementarity theory and mathematical programming.

- (2) As a special and simplified case of general network equilibrium problem, both static and dynamic corridor problems attract some attention. However, compared to the comprehensive achievements on general network equilibrium problems, there are still plenty of work that needs to be done on the corridor problems.
- (3) Compared to the traditional VI and CP approaches, DCS is a new mathematical modeling method, but it has been found to be a useful and efficient tool with various applications. Many research achievements on DCS and relevant issues have been obtained in transportation engineering. This method is also supposed to be useful in dealing with corridor problem.
- (4) The network design problem is one of the most difficult problems in transportation research and application. A host of work have been done on its modeling and solution methods, however most of the algorithms are heuristic without rigorous mathematical analysis, and the solutions can only achieve local optimum or approximation optimum. A few methods and algorithms have been proposed to achieve the global optimum via some approximation technique, but the efficiency and convergence of the solution algorithms are still under investigation and improvement.

CHAPTER 3

ANALYSIS OF MULTIMODAL EQUILIBRIUM ON A TRAVEL CORRIDOR WITH SINGLE PARK-AND-RIDE SERVICE

(Note: this chapter includes part of contents that has been published in the paper:

Du, B. and Wang, Z.W. (2013) "Reliability-based modeling of park-and-ride service on a linear travel corridor", Proceedings of Transportation Research Board 92nd Annual Meeting, Washington D.C., USA, 2013. Also on Journal of the Transportation Research Board (Network Modeling), Vol. 1, pp. 16-26.)

3.1 Introduction

In this chapter, a linear travel corridor with highway and railway systems and a single P&R service facility is considered. Travelers are continuously distributed along the corridor but with only one identical trip destination, CBD, at the end of the corridor, and they make multimodal choice among auto, railway and P&R travel modes to fulfil their journeys based on the individual lowest generalized travel cost.

As a useful method to reduce private car usage and corresponding congestion, P&R has attracted a host of studies in theory and applications in practice. One can refer to Section 2.4 for comprehensive literature review on P&R. This chapter contributes to the literature in two major aspects: Firstly, travel time reliability is considered in modeling the multimodal choice behavior. Empirical studies have suggested that travel time reliability plays an important role in travelers' route choice behavior (Lo et al., 2006). In this chapter, the highway is assumed to have stochastic capacity, therefore travel time is not deterministic but stochastic. Commuters using highway have to reserve a time budget to make sure not being late within certain level of requirement. Meanwhile, the travel time for rail transit service is assumed to be fixed. Higher level of travel time reliability is regarded as one of the advantages in favor of rail transit usage and taking into account this factor makes the model more

realistic. Secondly, linear complementarity system is adopted to formulate the multimodal choice behavior throughout the corridor. The linear complementarity system formulation belongs to a class of modeling paradigm of DVI, which was introduced by Pang and Stewart (Pang and Stewart, 2008) (Refer to Section 2.8.3 for more detailed introduction). The multimodal choice on each home location is modeled into a complementarity problem, while the spatial interaction of the travel costs between different locations can be characterized by ODEs. Therefore, it is natural to apply the DVI paradigm to model the spatial equilibrium travel pattern with multimodal choice on a linear traffic corridor. In the literature, Liu et al. (2009) proposed an infinite mathematical programming method to formulate the similar spatial equilibrium travel pattern problem. Nevertheless, as is well recognized, the optimization formulation of UE is not applicable when the travel modes have non-separable cost function. Meanwhile, the DVI modeling approach applied in this chapter is more general and is able to accommodate the situation with modal-specific cost.

3.2 Preliminaries on the corridor with multimodal choice

A corridor with P&R interchange service providing three travel modes: transit, auto and P&R is schematically described in Figure 3.1.

Figure 3.1 A corridor with multimodal choice

Let L represent the length of the corridor, Q denote the total number of commuters on this corridor, and CBD is at location L , the end of the corridor. It is assumed that all commuters are continuously distributed along the corridor and the density of travel demand at location x is $q_0(x)$, wherein x is the distance from the location to the boundary of the corridor and $L-x$ is the distance from the location to CBD. It is assumed that only one P&R facility is available at location x_p , which is the distance from the location to the boundary of the corridor, as shown in Figure 3.1.

3.2.1 Stochastic link capacity

Considering disruptions, like traffic accident, may happen on the transportation network, which result in reduction of the effective link capacities and the performance of the network, it is important to capture such kind of impact in order to improve the network performance or develop incident management strategies (Lo and Tung, 2003). Therefore, it is assumed that the link capacity is stochastic and the link capacity C_x at location x is the single source of uncertainty, which follows a specific probability distribution. For simplicity, uniform distribution is adopted here for illustration purpose. If the maximum capacity and minimum capacity are denoted as C_{max} and C_{min} respectively, the uniform probability distribution function can be described as follows:

$$f(C_x) = \frac{1}{C_{max} - C_{min}}, \quad C_x \in [C_{min}, C_{max}] \quad (3-1)$$

where $C_{min} > 0$ and $C_{max} > 0$. Then the expectation and variance of the link capacity can be calculated as follows:

$$E(C_x) = \frac{C_{max} + C_{min}}{2} \quad (3-2)$$

$$D(C_x) = \frac{(C_{max} - C_{min})^2}{12} \quad (3-3)$$

where C_x denotes the stochastic link capacity at location x , whose probability distribution is assumed to be given exogenously.

Let $t_h(v_h(x))$ represent the travel time traversing unit distance at location x , where the subscript h represents the highway system and $v_h(x)$ represents the traffic volume at location x , then the travel time can be calculated by BPR function:

$$t_h(v_h(x)) = t_h^0 \left[1 + A \left(\frac{v_h(x)}{C_x} \right)^B \right] \quad (3-4)$$

where t_h^0 represents the free flow travel time per unit of distance, A and B are calibrated parameters. As was done in D'Este (1987), the travel time is supposed to be fully described by a first-order model where $B = 1$.

The traffic volume at location x on highway is the accumulation of travelers choosing auto mode and P&R mode, if any, beyond location x , which can be described by

$$v_h(x) = \begin{cases} \int_0^x (q_h(w) + q_p(w)) dw, & x \in [0, x_p) \\ \int_0^x q_h(w) dw, & x \in [x_p, L] \end{cases} \quad (3-5)$$

where $q_h(w)$ and $q_p(w)$ represent the user densities for commuters choosing auto and P&R mode at location w respectively. Furthermore, it is reasonable to assume that no user will choose P&R mode between P&R facility location x_p and CBD, which can be described as:

$$\begin{cases} q_p(x) \geq 0, & x \in [0, x_p) \\ q_p(x) = 0, & x \in [x_p, L] \end{cases} \quad (3-6)$$

Then the travel time from location x to CBD by car can be expressed as:

$$T_h(x) = \begin{cases} \left[\int_x^{x_p} t_h^0 \left[1 + A \left(\frac{\int_0^z (q_h(w) + q_p(w)) dw}{C_z} \right) \right] dz \right. \\ \quad \left. + \int_{x_p}^L t_h^0 \left[1 + A \left(\frac{\int_0^z q_h(w) dw}{C_z} \right) \right] dz \right], & x \in [0, x_p) \\ \int_x^L t_h^0 \left[1 + A \left(\frac{\int_0^z q_h(w) dw}{C_z} \right) \right] dz, & x \in [x_p, L] \end{cases} \quad (3-7)$$

Assuming the link capacity is independent of the traffic volume on it (Lo and Tung, 2003), the expectation and variance of travel time through one unit distance at location x are derived as below.

$$E(t_h(v_h(x))) = t_h^0 + t_h^0 A v_h(x) E(C_x^{-1}) = t_h^0 + \frac{t_h^0 A v_h(x) (\ln C_{\max} - \ln C_{\min})}{C_{\max} - C_{\min}} \quad (3-8)$$

$$\begin{aligned} D(t_h(v_h(x))) &= t_h^{0^2} A^2 v_h(x)^2 D(C_x^{-1}) = t_h^{0^2} A^2 v_h(x)^2 (E(C_x^{-2}) - E(C_x^{-1})^2) \\ &= t_h^{0^2} A^2 v_h(x)^2 \left[\frac{1}{C_{\max} C_{\min}} - \frac{(\ln C_{\max} - \ln C_{\min})^2}{(C_{\max} - C_{\min})^2} \right] \end{aligned} \quad (3-9)$$

In order to analyze the travel time between any two locations on the corridor, highway is divided into n unit sections and $T_h(x_i, x_j)$ denotes the travel time from section x_i to section x_j , then

$$T_h(x_i, x_j) = \sum_{k=i}^j t_h(v_h(x_k)) \quad (3-10)$$

$$E(T_h(x_i, x_j)) = \sum_{k=i}^j E(t_h(v_h(x_k))) = \sum_{k=i}^j \left(t_h^0 + \frac{t_h^0 A v_h(x_k) (\ln C_{\max} - \ln C_{\min})}{C_{\max} - C_{\min}} \right) \quad (3-11)$$

$$D(T_h(x_i, x_j)) = \sum_{k=i}^j D(t_h(v_h(x_k))) + 2 \sum_{a < b} Cov(t_h(v_h(x_a)), t_h(v_h(x_b))), \quad i \leq a < b \leq j \quad (3-12)$$

where k, a, b, i, j are indices of the sections.

As shown in Eq. (3-12), when estimating the travel time variance between location x_i and x_j , two items need to be calculated; while the first item can be readily obtained from Eq. (3-9), the second one entails the covariance among the travel times traversing each unit distance on the highway between x_i and x_j , which stems from the correlated link capacity among neighboring highway sections. To calculate this part, it has:

$$\begin{aligned} Cov(t_h(v_h(x_a)), t_h(v_h(x_b))) &= Cov\left(t_h^0 + t_h^0 A v_h(x_a) E(C_{x_a}^{-1}), t_h^0 + t_h^0 A v_h(x_b) E(C_{x_b}^{-1})\right) \\ &= Cov\left(t_h^0 A v_h(x_a) C_{x_a}^{-1}, t_h^0 A v_h(x_b) C_{x_b}^{-1}\right) = t_h^{0^2} A^2 v_h(x_a) v_h(x_b) Cov(C_{x_a}^{-1}, C_{x_b}^{-1}) \\ &= t_h^{0^2} A^2 v_h(x_a) v_h(x_b) \left[E(C_{x_a}^{-1} C_{x_b}^{-1}) - E(C_{x_a}^{-1}) E(C_{x_b}^{-1}) \right] \end{aligned} \quad (3-13)$$

The item $E(C_{x_a}^{-1} C_{x_b}^{-1})$ in Eq. (3-13) can be computed if the joint probability distribution function of the stochastic capacities on any two link sections is known. In this chapter, the correlation, as well as the joint probability distribution of the stochastic link capacity variables between link sections, can be obtained from the historical data and is assumed exogenously given. When the correlation between link capacities of different sections is assumed to be negligible, the item of $E(C_{x_a}^{-1} C_{x_b}^{-1})$ in Eq. (3-13) will reduce to zero. This assumption is reasonable for highway system with relatively few network disruptions. In this chapter, the assumption of independent stochastic link capacity is used for illustration purpose, while the analysis process is still applicable to the situation without this assumption.

3.2.2 Travel time budget

Regardless of the unit travel time distribution, the travel time between two locations on highway follows a normal distribution according to the CLT as long as the Lindeberg's condition is satisfied. As was done in Lo et al. (2006), when travel time for auto users is considered to be stochastic, travelers would reserve some additional time to avoid late arrival, namely travel time budget. Usually the travel time budget can be expressed as

$$T_b = E(T) + \lambda \sigma_T \quad (3-14)$$

where T_b represents travel time budget, T is the travel time on highway, $E(T)$ and σ_T are the expectation and standard deviation of T respectively, λ is a parameter related to the punctuality probability requirement ρ , which can be mathematically described as follows.

$$P\{T \leq T_b\} = P\{T \leq E(T) + \lambda \sigma_T\} = \rho \quad (3-15)$$

Rearranging Eq. (3-15) to obtain that

$$P\left\{\frac{T - E(T)}{\sigma_T} \leq \lambda\right\} = \rho \quad (3-16)$$

Since $\frac{T - E(T)}{\sigma_T}$ is the standard normal variable, so the cumulative distribution function of the standard normal variable can be written as

$$\phi(\lambda) = \rho \quad (3-17)$$

Taking the inverse to derive that

$$\lambda = \phi^{-1}(\rho) \quad (3-18)$$

Accordingly, larger λ leads to higher punctuality requirement ρ , and vice versa. For example, in order to arrive within budget time reliability $\rho = 95\%$, the parameter should be set as $\lambda = 1.64$. Then the travel time budget expression is as

$$T_b = E(T) + 1.64 \sigma_T \quad (3-19)$$

which implies that travelers have 95% probability to arrive on time within travel time budget.

3.2.3 Generalized travel cost

Commuters make multimodal choice among auto, rail transit and P&R travel modes to minimize their generalized travel costs. As a result, UE status will be achieved with the multimodal choice pattern.

(1) Generalized travel cost by auto

The generalized travel cost by auto is comprised of three major components: travel time cost, operating cost and parking fee. The travel time cost includes two parts: access and egress time, and travel time budget $T_b(x)$, as defined in last section. The operating cost is assumed to be relevant to the travel distance.

The generalized travel cost by auto from location x to CBD is as follows:

$$C_h(x) = \tau(t_h^{hh} + t_h^{pw} + T_b(x)) + f_h(x) + P_h^w \quad (3-20)$$

where τ is the VOT, t_h^{hh} is the access time from home to highway, t_h^{pw} is the access time from parking place to workplace. The operating cost is assumed to be a linear function of the travel distance as $f_h(x) = f_h^0 + \gamma(L - x)$, where f_h^0 is constant, which may include fixed part of the tolls, and $\gamma(L - x)$ presents the variable part of the operating cost from location x to CBD, such as the fuel, insurance and variable parts of the highway tolls, etc., with γ as the operating cost per unit distance of traveling. P_h^w is the parking fee at CBD.

(2) Generalized travel cost by railway

Similar to auto users, the traffic volume of transit users at location x is calculated as Eq. (3-21).

$$v_r(x) = \begin{cases} \int_0^x q_r(w)dw, & x \in [0, x_p) \\ \int_0^x (q_r(w) + q_p(w))dw, & x \in [x_p, L] \end{cases} \quad (3-21)$$

where $q_r(w)$ is the user density for commuters choosing the public transit mode at location w . Crowding cost is considered here to reflect the loss of privacy and

comfort, decline of air quality and venture of pickpocket due to the increase of passengers on the train. Let $y_r(v_r(x))$ represent the crowding cost on the train per unit distance at location x , where $v_r(x)$ represents the number of passengers by railway at location x . As was done in Huang (2000), a simple linear function with respect to traffic volume on the train is applied to describe the crowding cost as:

$$y_r(v_r(x)) = \alpha + \beta v_r(x) \quad (3-22)$$

where α, β are the calibrated parameters. Therefore, the crowding cost for commuters traveling from location x to CBD should be

$$Y_r(x) = \int_x^L y_r(v_r(w)) dw \quad (3-23)$$

The crowding cost can be calculated by

$$Y_r(x) = \begin{cases} \int_x^{x_p} \left(\alpha + \beta \int_0^z q_r(w) dw \right) dz + \int_{x_p}^L \left[\alpha + \beta \int_0^z (q_r(w) + q_p(w)) dw \right] dz, & x \in [0, x_p) \\ \int_x^L \left[\alpha + \beta \int_0^z (q_r(w) + q_p(w)) dw \right] dz, & x \in [x_p, L] \end{cases} \quad (3-24)$$

The generalized travel cost by rail transit from location x to CBD is comprised of three major components: travel time cost, crowding cost, and service fare, which can be described as Eq. (3-25).

$$C_r(x) = \tau(t_r^{hr} + t_r^{rw} + \frac{L-x}{V_r}) + Y_r(x) + f_r(x) \quad (3-25)$$

where t_r^{hr} , t_r^{rw} represent the access and egress time, respectively; $\frac{L-x}{V_r}$ is the travel time by railway from location x to CBD, wherein V_r is the average travel speed of the train. $f_r(x) = f_r^0 + \kappa(L-x)$ is assumed to describe the fare structure of railway services from location x to CBD, wherein f_r^0 and $\kappa(L-x)$ represent the fixed and variable parts of the fare respectively, and κ is the railway fare per unit distance.

(3) Generalized travel cost by P&R

The generalized travel cost by P&R is comprised of three major parts: auto mode cost, P&R transfer cost and transit mode cost. Moreover, as stated in previous

content, it is assumed that no one will choose P&R between P&R facility location x_p and CBD.

$$C_p(x) = \begin{cases} \left(\tau t_h^{hh} + \tau (T_b(x) - T_b(x_p)) + f_h(x, x_p) + P_p(x_p) \right) \\ \quad + \tau t_p^{pr} + u_p + \tau \frac{L - x_p}{V_r} + Y_r(x_p) + f_r(x_p) + \tau t_r^{rw} \end{cases}, \quad x \in [0, x_p) \quad (3-26)$$

$$U, \quad x \in [x_p, L]$$

Here U is used to represent a large enough positive constant to describe the generalized travel cost as no one will choose P&R from location x_p to CBD. While at location $x \in [0, x_p)$, the generalized travel cost can be divided into three parts.

Auto mode cost: $\tau t_h^{hh} + \tau (T_b(x) - T_b(x_p)) + f_h(x, x_p)$

where $T_b(x) - T_b(x_p)$ and $f_h(x, x_p)$ are the travel time budget and operating cost from location x to P&R facility x_p respectively.

P&R transfer cost: $P_p(x_p) + \tau t_p^{pr} + u_p$

where $P_p(x_p)$ represents the parking fee at P&R facility location x_p , which can be calculated as $P_p(x_p) = p_h^w e^{-\frac{(L-x_p)^2}{2L}}$ (Liu et al., 2009). t_p^{pr} is the transfer time from parking place to train station nearby, and u_p is the inconvenience or disutility cost.

Transit mode cost: $\tau \frac{L - x_p}{V_r} + Y_r(x_p) + f_r(x_p) + \tau t_r^{rw}$

where $\frac{L - x_p}{V_r}$ is the travel time from P&R location x_p to CBD workplace, $Y_r(x_p)$ is the crowding cost from location x_p to CBD, and $f_r(x_p)$ is the service fare from location x_p to CBD.

As a general modeling framework of corridor problem, the typical daily commute cost adopted in most of the relevant studies is used to describe the generalized travel cost in this thesis. However, in some special cases, some other elements also need to be considered into the generalized travel cost. For example, the cost of owning a private car in Singapore is much more expensive than many other places, in this

case many auto users might not be quite sensitive to the daily commute cost, and they might not prefer to use P&R or railway although the generalized cost by auto is higher. To capture this characteristics and other special situations, the generalized travel cost function can be redesigned to reflect this insensitivity, or heterogeneous commuters with different personal characteristics can be applied to capture these issues. The most commonly used daily commute cost is adopted in this thesis to develop a general modeling framework, but there are no mathematical obstacles to redesign the generalized travel cost functions. In the subsequent chapters, heterogeneity is considered to capture the individual characteristics.

3.3 Model formulation

3.3.1 Spatial equilibrium with multimodal choice

UE status is achieved when no traveler can reduce the generalized travel cost by using other modes at any location. This spatial equilibrium problem is formulated by applying the complementarity theory as follows:

$$\begin{aligned}
 0 &\leq q_h(x) \perp C_h(x) - C^*(x) \geq 0 \\
 0 &\leq q_r(x) \perp C_r(x) - C^*(x) \geq 0 \\
 0 &\leq q_p(x) \perp C_p(x) - C^*(x) \geq 0 \\
 0 &\leq C^*(x) \perp q_h(x) + q_r(x) + q_p(x) - q_0(x) \geq 0 \\
 &x \in [0, L]
 \end{aligned} \tag{3-27}$$

where $C^*(x)$ represents the minimum generalized travel cost among the three modes at location x . The first three constraints in (3-27) make sure that at equilibrium, the individual travel cost by a mode at any location should be the minimum among the three modes if the mode is used at the location. The last complementarity constraint entails that total users by the three modes amounts to the given total travel demand density $q_0(x)$ at location x .

3.3.2 Linear complementarity system formulation

Now it is ready to formulate the spatial equilibrium travel pattern with multimodal choice into LCS (Refer to Section 2.8.3 for more introduction of LCS). Firstly, take second order derivative of travel time budget $T_b(x)$ with respect to x to get:

$$\frac{d^2 T_b(x)}{dx^2} = \begin{cases} -D \cdot (q_h(x) + q_p(x)), & x \in [0, x_p) \\ -D \cdot \left(q_h(x) - \int_0^{x_p} q_p(w) dw \right), & x = x_p \\ -D \cdot q_h(x), & x \in (x_p, L] \end{cases} \quad (3-28)$$

where D denotes the constant part in Eq. (3-28) as follows:

$$D = \frac{t_h^0 A (\ln C_{\max} - \ln C_{\min})}{C_{\max} - C_{\min}} + \lambda t_h^0 A \sqrt{\frac{1}{C_{\max} C_{\min}} - \left(\frac{\ln C_{\max} - \ln C_{\min}}{C_{\max} - C_{\min}} \right)^2} \quad (3-29)$$

Similarly, the second order derivative of the crowding cost by transit mode is obtained as:

$$\frac{d^2 Y_r(x)}{dx^2} = \begin{cases} -\beta \cdot q_r(x), & x \in [0, x_p) \\ -\beta \cdot \left(q_r(x) + \int_0^{x_p} q_p(w) dw \right), & x = x_p \\ -\beta \cdot (q_r(x) + q_p(x)), & x \in (x_p, L] \end{cases} \quad (3-30)$$

The ODEs above reflect the spatial interaction of the travel time and crowding cost along the corridor. Furthermore, initial conditions and boundary conditions of the ODEs (3-28) and (3-30) can be expressed as follows:

$$T_b(L) = 0, \quad Y_r(L) = 0 \quad (3-31)$$

$$\left. \frac{dT_b}{dx} \right|_{x=0} = t_h^0, \quad \left. \frac{dY_r}{dx} \right|_{x=0} = \alpha \quad (3-32)$$

The initial conditions ensure that the travel time and crowding cost at city center will be zero. The boundary conditions entails that no travel demand will be generated beyond the corridor boundary, i.e. $v_h(0) = v_r(0) = 0$.

Based on the above analysis, LCS can be used to describe the spatial equilibrium travel pattern with multimodal choice. The solution is to find $T_b(x)$, $Y_r(x)$, $q_h(x)$, $q_r(x)$, $q_p(x)$, $C^*(x)$ so that following conditions (3-33) and (3-34) are satisfied.

(A) For almost all $x \in (0, L]$

$$\begin{aligned} \frac{d^2 T_b(x)}{dx^2} &= \begin{cases} -D \cdot (q_h(x) + q_p(x)), & x \in [0, x_p) \\ -D \cdot (q_h(x) - \int_0^{x_p} q_p(w) dw), & x = x_p \\ -D \cdot q_h(x), & x \in (x_p, L] \end{cases} \\ \frac{d^2 Y_r(x)}{dx^2} &= \begin{cases} -\beta \cdot q_r(x), & x \in [0, x_p) \\ -\beta \cdot (q_r(x) + \int_0^{x_p} q_p(w) dw), & x = x_p \\ -\beta \cdot (q_r(x) + q_p(x)), & x \in (x_p, L] \end{cases} \end{aligned} \quad (3-33)$$

$$\begin{aligned} 0 &\leq q_h(x) \perp C_h(x) - C^*(x) \geq 0 \\ 0 &\leq q_r(x) \perp C_r(x) - C^*(x) \geq 0 \\ 0 &\leq q_p(x) \perp C_p(x) - C^*(x) \geq 0 \\ 0 &\leq C^*(x) \perp q_h(x) + q_r(x) + q_p(x) - q_0(x) \geq 0 \end{aligned}$$

(B) The initial and boundary conditions

$$\begin{aligned} T_b(L) &= 0, \quad Y_r(L) = 0 \\ \frac{dT_b}{dx} \Big|_{x=0} &= t_h^0, \quad \frac{dY_r}{dx} \Big|_{x=0} = \alpha \end{aligned} \quad (3-34)$$

For illustration purpose, $C_h(x)$, $C_r(x)$ and $C_p(x)$ are used in (3-33) to represent the generalized travel costs by auto, railway and P&R mode respectively, and the complete formulations are shown in Eqs. (3-20), (3-25) and (3-26) respectively.

3.3.3 Discrete approximation of the LCS model

The numerical solution to the formulated LCS (3-33) with (3-34) can be accomplished via a time-stepping scheme that solves a sequence of finite-dimensional discretized sub-problems. Specifically, the corridor is divided into equal length sub-intervals $\varepsilon_v = \frac{L}{v}$ by a positive integer v . Thus, the numerical solution to the LCS (3-33) with (3-34) can be obtained by computing the discretized scheme: $\{q_h^i\}, \{q_r^i\}, \{q_p^i\}, \{C^{*,i}\}, \{T_b^i\}, \{Y_r^i\}$, $i = 1, 2, \dots, v$. The boundary of the corridor and the CBD are assumed to be at dummy sections 0 and $v+1$ respectively. The sequence of sections from 1 to v represents the direction from the boundary of the corridor to CBD, and i^{x_p} represents the P&R facility location. For the commuters

choosing P&R mode, they are assumed to drive at least one section of distance before transferring to transit.

In the discretization scheme, the derivatives can be approximated by the forward difference quotient:

$$\frac{d^2 T_b(x)}{dx^2} \approx \frac{[T_b(x - \varepsilon_v) - T_b(x)] - [T_b(x) - T_b(x + \varepsilon_v)]}{\varepsilon_v^2} \quad (3-35)$$

$$\frac{d^2 Y_r(x)}{dx^2} \approx \frac{[Y_r(x - \varepsilon_v) - Y_r(x)] - [Y_r(x) - Y_r(x + \varepsilon_v)]}{\varepsilon_v^2} \quad (3-36)$$

Hence, the discretized approximation of condition (3-33) is:

$$\begin{aligned} \frac{T_b^{i+1} + T_b^{i-1} - 2T_b^i}{\varepsilon_v^2} &= \begin{cases} -D \cdot (q_h^i + q_p^i), & 1 \leq i < i^{x_p} \\ -D \cdot \left(q_h^i - \sum_{j=1}^{i^{x_p}} q_p^j \right), & i = i^{x_p} \\ -D \cdot q_h^i, & i^{x_p} < i \leq v \end{cases} \\ \frac{Y_r^{i+1} + Y_r^{i-1} - 2Y_r^i}{\varepsilon_v^2} &= \begin{cases} -\beta \cdot q_r^i, & 1 \leq i < i^{x_p} \\ -\beta \cdot \left(q_r^i + \sum_{j=1}^{i^{x_p}} q_p^j \right), & i = i^{x_p} \\ -\beta \cdot (q_r^i + q_p^i), & i^{x_p} < i \leq v \end{cases} \\ 0 \leq q_h^i \perp \tau t_h^{hh} + \tau t_h^{pw} + \tau T_b^i + f_h^0 + \gamma(v - i + 1)\varepsilon_v + P_h^w - C^{*,i} &\geq 0 \\ 0 \leq q_r^i \perp \tau t_r^{hr} + \tau t_r^{rw} + \tau \frac{(v - i + 1)\varepsilon_v}{V_r} + Y_r^i + f_r^0 + \kappa(v - i + 1)\varepsilon_v - C^{*,i} &\geq 0 \\ 0 \leq q_p^i \perp \begin{cases} \left(\tau t_h^{hh} + \tau (T_b^i - T_b^{i^{x_p}}) + f_h^0 + \gamma\varepsilon_v(i^{x_p} - i) + \right. \\ \left. p_h^w e^{\frac{(v - i^{x_p})^2}{2v}} + \tau t_p^{pr} + u_p + \tau \frac{(v - i^{x_p} + 1)\varepsilon_v}{V_r} + \right. \\ \left. Y_r^{i^{x_p}} + f_r^0 + \kappa(v - i^{x_p} + 1)\varepsilon_v + \tau t_r^{rw} - C^{*,i} \right) \\ U - C^{*,i} \geq 0, \end{cases} & 1 \leq i < i^{x_p} \\ & i^{x_p} \leq i \leq v \\ 0 \leq C^{*,i} \perp q_h^i + q_r^i + q_p^i - q_0^i &\geq 0 \\ & i = 1, 2, \dots, v \end{aligned} \quad (3-37)$$

where i^{x_p} represents the section where P&R facility is .

Let

$$\eta = \begin{cases} 1, & 1 \leq i < i^{x_p} \\ 0, & i^{x_p} < i \leq v \end{cases} \quad (3-38)$$

where η represents a binary variable. With this auxiliary parameter, the ODEs in (3-37) can be rewritten in a succinct form as follows:

$$\frac{T_b^{i+1} + T_b^{i-1} - 2T_b^i}{\varepsilon_v^2} = \begin{cases} -\mathbf{D} \cdot (q_h^i + \eta \cdot q_p^i), & i \neq i^{x_p} \\ -\mathbf{D} \cdot \left(q_h^i - \sum_{j=1}^{i^{x_p}} q_p^j \right), & i = i^{x_p} \end{cases} \quad (3-39)$$

$$\frac{Y_r^{i+1} + Y_r^{i-1} - 2Y_r^i}{\varepsilon_v^2} = \begin{cases} -\beta \cdot [q_r^i + (1-\eta)q_p^i], & i \neq i^{x_p} \\ -\beta \cdot \left(q_r^i + \sum_{j=1}^{i^{x_p}} q_p^j \right), & i = i^{x_p} \end{cases} \quad (3-40)$$

Note that Eqs. (3-39) and (3-40) can be converted into a LCP formulation in standard form, thus the following equivalent LCP formulation can be obtained, in which variables are $(q_h^i)_{i=1}^v, (q_r^i)_{i=1}^v, (q_p^i)_{i=1}^v, (C^{*,i})_{i=1}^v, (T_b^i)_{i=1}^v, (Y_r^i)_{i=1}^v$.

$$\begin{aligned} 0 \leq T_b^i \perp & \begin{cases} T_b^{i+1} + T_b^{i-1} - 2T_b^i + \varepsilon_v^2 \mathbf{D} \cdot (q_h^i + \eta \cdot q_p^i) \geq 0, & i \neq i^{x_p} \\ T_b^{i+1} + T_b^{i-1} - 2T_b^i + \varepsilon_v^2 \mathbf{D} \cdot \left(q_h^i - \sum_{j=1}^{i^{x_p}} q_p^j \right) \geq 0, & i = i^{x_p} \end{cases} \\ 0 \leq Y_r^i \perp & \begin{cases} Y_r^{i+1} + Y_r^{i-1} - 2Y_r^i + \varepsilon_v^2 \beta \cdot [q_r^i + (1-\eta)q_p^i] \geq 0, & i \neq i^{x_p} \\ Y_r^{i+1} + Y_r^{i-1} - 2Y_r^i + \varepsilon_v^2 \beta \cdot \left(q_r^i + \sum_{j=1}^{i^{x_p}} q_p^j \right) \geq 0, & i = i^{x_p} \end{cases} \\ 0 \leq q_h^i \perp & \tau t_h^{hh} + \tau t_h^{pw} + \tau T_b^i + f_h^0 + \gamma(v-i+1)\varepsilon_v + P_h^w - C^{*,i} \geq 0 \\ 0 \leq q_r^i \perp & \tau t_r^{hr} + \tau t_r^{rw} + \tau \frac{(v-i+1)\varepsilon_v}{V_r} + Y_r^i + f_r^0 + \kappa(v-i+1)\varepsilon_v - C^{*,i} \geq 0 \\ 0 \leq q_p^i \perp & \begin{cases} \left(\tau t_h^{hh} + \tau (T_b^i - T_b^{i^{x_p}}) + f_h^0 + \gamma\varepsilon_v(i^{x_p} - i) + \right. \\ \left. p_h^w e^{\frac{(v-i^{x_p})^2}{2v}} + \tau t_p^{pr} + u_p + \tau \frac{(v-i^{x_p}+1)\varepsilon_v}{V_r} + \right) \geq 0, & 1 \leq i < i^{x_p} \\ Y_r^{i^{x_p}} + f_r^0 + \kappa(v-i^{x_p}+1)\varepsilon_v + \tau t_r^{rw} - C^{*,i} \\ U - C^{*,i} \geq 0, & i^{x_p} \leq i \leq v \end{cases} \\ & 0 \leq C^{*,i} \perp q_h^i + q_r^i + q_p^i - q_0^i \geq 0 \\ & i = 1, 2, \dots, v \end{aligned} \quad (3-41)$$

The initial and boundary conditions (3-34) can be rewritten as Eqs. (3-42).

$$\begin{aligned} T_b^{v+1} &= 0, & Y_r^{v+1} &= 0 \\ T_b^0 - T_b^1 &= t_h^0 \varepsilon_v, & Y_r^0 - Y_r^1 &= \alpha \varepsilon_v \end{aligned} \quad (3-42)$$

Eqs. (3-42) can be substituted into (3-41) directly to simplify the formulation. The formulation LCP (3-41) can be expressed in a standard form as follows to find a vector $\mathbf{x}$ satisfying

$$0 \leq \mathbf{x} \perp M\mathbf{x} + \mathbf{s} \geq 0 \quad (3-43)$$

where

$$\mathbf{x} \triangleq \left[(q_h^i)_{i=1}^v \quad (q_r^i)_{i=1}^v \quad (q_p^i)_{i=1}^v \quad (C^{*,i})_{i=1}^v \quad (T_b^i)_{i=1}^v \quad (Y_r^i)_{i=1}^v \right]^T \quad (3-44)$$

$$M\mathbf{x} \triangleq \begin{bmatrix} (\tau T_b^i - C^{*,i})_{i=1}^v \\ (Y_r^i - C^{*,i})_{i=1}^v \\ \left\{ \left(\tau (T_b^i - T_b^{i^{x_p}}) + Y_r^{i^{x_p}} - C^{*,i} \right)_{i=1}^{i^{x_p}-1} \right. \\ \left. (-C^{*,i})_{i=i^{x_p}}^v \right. \\ (q_h^i + q_r^i + q_p^i)_{i=1}^v \\ \left\{ \begin{aligned} & (T_b^{i+1} + T_b^{i-1} - 2T_b^i + \varepsilon_v^2 D q_h^i + \varepsilon_v^2 D \eta q_p^i)_{i=1}^v, & i \neq i^{x_p} \\ & T_b^{i+1} + T_b^{i-1} - 2T_b^i + \varepsilon_v^2 D q_h^i - \varepsilon_v^2 D \sum_{j=1}^{i^{x_p}} q_p^j, & i = i^{x_p} \end{aligned} \right. \\ \left\{ \begin{aligned} & (Y_r^{i+1} + Y_r^{i-1} - 2Y_r^i + \varepsilon_v^2 \beta q_r^i + \varepsilon_v^2 \beta (1-\eta) q_p^i)_{i=1}^v, & i \neq i^{x_p} \\ & Y_r^{i+1} + Y_r^{i-1} - 2Y_r^i + \varepsilon_v^2 \beta q_r^i + \varepsilon_v^2 \beta \sum_{j=1}^{i^{x_p}} q_p^j, & i = i^{x_p} \end{aligned} \right. \end{bmatrix} \quad (3-45)$$

$$\mathbf{s} \triangleq \begin{bmatrix} (\tau t_h^{hh} + \tau t_h^{pw} + f_h^0 + \gamma(v-i+1)\varepsilon_v + P_h^w)_{i=1}^v \\ \left(\tau t_r^{hr} + \tau t_r^{rw} + \tau \frac{(v-i+1)\varepsilon_v}{V_r} + f_r^0 + \kappa(v-i+1)\varepsilon_v \right)_{i=1}^v \\ \left(\tau t_h^{hh} + f_h^0 + \gamma\varepsilon_v(i^{x_p} - i) + p_h^w e^{\frac{(v-i^{x_p})^2}{2v}} + \tau t_p^{pr} + u_p \right)_{i=1}^{i^{x_p}-1} \\ \left(+\tau \frac{(v-i^{x_p}+1)\varepsilon_v}{V_r} + f_r^0 + \kappa(v-i^{x_p}+1)\varepsilon_v + \tau t_r^{rw} \right)_{i=1}^{i^{x_p}-1} \\ (U)_{i=i^{x_p}}^v \\ (-q_0^i)_{i=1}^v \\ (0)_{i=1}^v \\ (0)_{i=1}^v \end{bmatrix} \quad (3-46)$$

3.3.4 Transformation of LCP to MILP

In this section, an equivalent MILP method is used to reformulate the LCP. Firstly, bilinear formulation approach is used to state the LCP as a mixed integer bilinear program (MIBLP) as follows:

$$\begin{aligned} \min_{x,z} \quad & z^T (Mx + s) + (e - z)^T x \\ \text{s.t.} \quad & x \geq 0, Mx + s \geq 0 \\ & z \text{ binary} \end{aligned} \quad (3-47)$$

where e presents the vector of n ones.

Proposition 3.1 *The solution of the proposed LCP (3-41) is equivalent to the solution of the MIBLP problem (3-47) with zero as minimal objective value.*

Proof. If x solves original LCP (3-41), define

$$z_i = \begin{cases} 0, & \text{if } M_i x + s_i > 0 \\ 1, & \text{if } x_i > 0 \end{cases} \quad (3-48)$$

then the x solving LCP (3-41) renders zero as objective value of the MIBLP problem (3-47). If x solves (3-47) with zero as minimal objective value, then $z^T (Mx + s) + (e - z)^T x = 0$. Since z is binary, either z^T or $(e - z)^T$ must be 0 and another one must be 1. In that case, either $Mx + s$ or x must be zero so that $z^T (Mx + s) + (e - z)^T x = 0$, then $x^T (Mx + s) = 0$ is also fulfilled, which is the solution to the LCP (3-41) as well. $\square$

Let M_{ij} denotes the (i, j) th element of M , (3-48) can be rewritten as follows:

$$\begin{aligned} \min_{x,z} \quad & s^T z + \sum_{i=1}^n \sum_{j=1}^n M_{ij} z_i x_j + (e - z)^T x \\ \text{s.t.} \quad & \sum_{j=1}^n M_{kj} x_j + s_k \geq 0, \quad k = 1, 2, \dots, n \\ & x \geq 0, z \text{ binary} \end{aligned} \quad (3-49)$$

Finally, reformulation-linearization technique is applied to covert the MIBLP into a MILP problem (Sherali et al., 1998). This method consists of two phases, the reformation phase and linearization phase. The reformation phase defines a set of nonnegative variable factors and then forms the products of these factors with

original constraints to generate various implied nonlinear restrictions. In the linearization phase, a variable substitution technique is used to linearize these nonlinear constraints. This method transforms the problem into a higher dimensional space problem such that its continuous relaxation approximates the closure of the convex hull of feasible solutions to the underlying mixed integer program (MIP) problem. Specifically, multiply constraints in (3-47) by $(1 - y_i)$ and y_i to reformulate the problem, and then linearize it by substituting $w_{ij} = y_i x_j$. Therefore, an equivalent MILP model can be obtained as follows:

$$\begin{aligned}
 \min_{x, z} \quad & s^T z + \sum_{i=1}^n \sum_{j=1}^n M_{ij} w_{ij} \\
 \text{s.t.} \quad & \sum_{j=1}^n M_{kj} w_{ij} + s_k y_i \geq 0, \quad \forall (i, k) \\
 & \sum_{j=1}^n M_{kj} x_j + s_k \geq \sum_{j=1}^n M_{kj} w_{ij} + s_k y_i, \quad \forall (i, k) \\
 & 0 \leq w_{ij} \leq x_j, \quad \forall (i, j) \quad \text{with} \quad w_{jj} = x_j, \quad \forall j \\
 & x \geq 0, \quad z \text{ binary}
 \end{aligned} \tag{3-50}$$

The proof of equivalence of MILP (3-50) with the MIBLP (3-47) can be referred to Sherali et al. (1998). To solve MILP (3-50), commercial software, like CPLEX, can be used to obtain the final solution.

3.4 Numerical results

In this section, a set of numerical results are presented on an example corridor. The input data for parameters are shown in Table 3.1 and the corridor is divided into 20 sections. The sequence of sections from 1 to 20 represents the direction from the boundary of the corridor to CBD, and the P&R facility location is given at section 11 ($i^{x_p} = 11$). For simplicity, assume each person corresponds to one vehicle on road. It is also assumed that P&R is not available between P&R facility location and CBD so that only commuters living between P&R facility and the corridor boundary can choose P&R. On each section the link capacity is stochastic and follows identical uniform distribution. Deterministic UE multimodal choice pattern is achieved on each section.

Table 3.1 List of parameter values and descriptions

Parameter	Value	Description
L	20	Length of the corridor (km)
v	20	Section number of the corridor
x_p	11	Location of the P&R facility
q_0	600	Commuter density (commuters/h/km)
τ	0.5	Time to cost coefficient (S\$/min)
ρ	95%	Probability of a trip arrives within the travel time budget
λ	1.64	Parameter related to ρ
A	1	BPR function parameter
t_h^0	0.8	BPR function parameter, free-flow travel time (min)
C_{max}	7000	Maximal link capacity (veh/h)
C_{min}	5000	Minimal link capacity (veh/h)
C	3000	Link capacity without considering the reliability (veh/h)
t_h^{hh}	2	Time from home to highway (min)
t_h^{pw}	2	Time from parking place to workplace (min)
f_h^0	2	Fixed part of the operating cost by auto (S\$/veh)
γ	0.02	Operating cost per unit distance by auto (S\$/km/veh)
p_h^w	7	Parking fee at workplace (S\$/veh)
t_r^{hr}	11	Time from home to railway station (min)
t_r^{rw}	5	Time from railway station to workplace (min)
f_r^0	0.5	Fixed part of the fare (S\$)
κ	0.07	Railway fare per unit distance (S\$/km)
V_r	0.7	Average speed of the train (km/min)
α	0	Parameter of the crowding cost
β	0.00003	Parameter of the crowding cost
t_p^{pr}	1	Time from P&R parking place to train station nearby (min)
u_p	0.2	Fixed transfer disutility

First of all, the modal split pattern at equilibrium status, shown in Table 3.2, is obtained for the scenario considering travel time reliability with probability requirement $\rho = 95\%$.

Table 3.2 shows that all of the three modes are used along the corridor, and railway has tremendous advantages at places close to CBD. On the contrary, P&R is preferred from the middle of the corridor to the boundary. Specifically, railway is chosen by commuters departing between locations 14 and 20; auto mode is preferred in the middle part of the corridor at locations 11 to 14, and P&R is the favorite modal choice from the corridor boundary to location 10, which means all

commuters living between locations 1 and 10 will drive to the P&R facility at location 11 and transfer to railway to complete the rest trips.

Table 3.2 Modal split pattern at equilibrium status with $\rho = 95\%$ (commuters/h/km)

Section	Auto user density	Railway user density	P&R user density
1	0	0	600
2	0	0	600
3	0	0	600
4	0	0	600
5	0	0	600
6	0	0	600
7	0	0	600
8	0	0	600
9	0	0	600
10	0	0	600
11	600	0	0
12	600	0	0
13	600	0	0
14	299.84	300.16	0
15	0	600	0
16	0	600	0
17	0	600	0
18	0	600	0
19	0	600	0
20	0	600	0

Figure 3.2 Generalized costs of three modes at equilibrium status with $\rho = 95\%$

Figure 3.2 depicts the generalized costs of three modes along the corridor. The minimum cost curve is a combination of three parts. From CBD to the boundary of the corridor, the generalized cost of railway is lower than other modes from location

14 to 20, and then it takes lowest generalized cost by auto mode at locations 11 to 14. From location 10 to the corridor boundary, P&R is preferred according to its lowest generalized cost. The generalized cost of P&R from location 11 to 20 is not available since no one will choose P&R between P&R facility location (section 11) and CBD as assumed in the previous content.

Next, a series of numerical results are shown to compare the modal split results based on deterministic link capacity (DLC) and stochastic link capacity (SLC). As is shown in Table 3.3, some of the auto users at locations 10 and 14 change to use P&R and railway respectively after considering SLC, because travel time budget needs to be considered on highway, which leads to higher generalized cost and less users by auto.

Table 3.3 Modal split patterns with DLC and SLC (commuters/h/km)

Section	DLC			SLC		
	Auto user density	Railway user density	P&R user density	Auto user density	Railway user density	P&R user density
1	0	0	600	0	0	600
2	0	0	600	0	0	600
3	0	0	600	0	0	600
4	0	0	600	0	0	600
5	0	0	600	0	0	600
6	0	0	600	0	0	600
7	0	0	600	0	0	600
8	0	0	600	0	0	600
9	0	0	600	0	0	600
10	421.32	0	178.68	0	0	600
11	600	0	0	600	0	0
12	600	0	0	600	0	0
13	600	0	0	600	0	0
14	600	0	0	299.84	300.16	0
15	0	600	0	0	600	0
16	0	600	0	0	600	0
17	0	600	0	0	600	0
18	0	600	0	0	600	0
19	0	600	0	0	600	0
20	0	600	0	0	600	0

Figure 3.3 illustrates the different generalized costs before and after considering SLC. It is obvious that all the generalized costs of three modes increase after considering SLC, as auto users reserve a large travel time budget for higher reliability, which results in higher generalized cost by auto; and it also changes the modal split pattern, wherein more people choose railway and P&R to take

advantage of punctuality of rail transit service, which results in higher crowding cost by rail transit, as well as higher generalized costs by railway and P&R.

Figure 3.3 Generalized costs at equilibrium status with DLC and SLC

(a) Travel time on highway (b) Crowding cost on railway
Figure 3.4 Travel time on highway and crowding cost on railway at equilibrium status with DLC and SLC

From Figure 3.4 it is easy to find that both travel time on highway and crowding cost on railway increase since auto users need to consider travel time budget, which results in higher generalized travel cost by auto. Therefore more commuters choose railway or P&R instead of auto mode, which brings higher railway usage, as well as higher crowding cost by railway.

All the previous numerical results are based on the P&R location given at section 11 ($i^{xp} = 11$). Now a simple computational example is given to find which location is

the best choice for P&R facility. To seek to this optimal P&R location, the P&R facility location is changing from locations 1 to 20, and the corresponding total generalized costs of the corridor at equilibrium status are calculated, as shown in Table 3.4. It is easy to understand that locating the P&R facility at different places will lead to different multimodal choice patterns. By varying the P&R location, it can provide general information to help find the optimal location that minimizes the total travel cost. From Table 3.4, one can find that the best location for P&R facility is location 10, on which minimum total generalized cost along the corridor is achieved at equilibrium status if P&R is located.

Table 3.4 Total generalized costs along the corridor at equilibrium status with different P&R locations considering SLC (\$ 1×10^5)

P&R facility location (section)	Total generalized cost
1	2.2014
2	2.1482
3	2.1832
4	2.1654
5	2.0369
6	2.0622
7	2.1772
8	2.1381
9	2.0888
10	1.6679
11	2.1814
12	2.1968
13	2.2059
14	2.2589
15	2.2048
16	2.2014
17	2.2382
18	2.2014
19	2.2236
20	2.2014

Figure 3.5 Shows the the computational times of CPLEX solver for different numbers of sections, wherein horizontal/X axis represents the number of sections dividing the corridor and vertical/Y axis is the computational time (seconds). The numerical results obtained in this chapter are based on a Dell Precision T3600 workstation (Intel Xeon CPU 3.2GHz, RAM 16GB, Windows 7 Professional 64-bit) using MATLAB R2012a and CPLEX Optimization Studio 12.4. It can be found that for small number of sections, the CPLEX solver could solve the MILP model efficiently. However, its running time increases greatly after the number of sections

exceeds 40, which reflects that MILP with CPLEX solver is not efficient for solving this problem in large scale.

Figure 3.5 Computational times with different numbers of sections

In this chapter, the LCP model is transformed to MILP, an equivalent and traditional mathematical programming model with objective function. However, LCP can be solved directly with some solution methods, like Lemke's algorithm. To demonstrate this, a simple comparison between CPLEX solver for MILP and Lemke's algorithm for LCP directly is shown in Table 3.5.

Table 3.5 Computational times for CPLEX solver and Lemke's algorithm (seconds)

Number of sections	CPLEX solver for MILP	Lemke's algorithm for LCP
10	8.6732	0.0073
15	38.2253	0.0085
20	126.2714	0.0086
25	238.9327	0.0110
30	1303.4846	0.0140
35	3127.3191	0.0170
40	4097.8804	0.0198
45	9402.2335	0.0243
50	23009.3921	0.0281

It is obviously that Lemke's algorithm is much more efficient than CPLEX solver at all different numbers of sections. Moreover, even the corridor is divided into 400 sections, the computational time for Lemke's algorithm is just 1.1492 seconds to obtain the optimal solution. The reason behind this is that the transformation of LCP to MILP introduces several new variables and makes the model formulation more

complex, therefore when the number of sections increases, the number of variables and constraints in numerical experiments grows rapidly, which leads to much higher computational time. In contrast, Lemke's algorithm is designed for LCP formulation directly, which avoids the transformation procedure and additional variables and constraints so that it can solve LCP efficiently. Therefore, Lemke's algorithm for LCP directly will be applied in the subsequent chapters instead of transforming LCP to MILP to solve.

3.5 Summary

In this chapter, a reliability-based modeling of park-and-ride service problem is considered. Travel time reliability is explicitly incorporated into the multimodal choice modeling. The spatial equilibrium travel pattern with multimodal choice is formulated into LCS, which belongs to the class of DVI modeling paradigm. Then it is transformed into an equivalent MILP by a discrete approximation method to be solved. A numerical example is conducted to compare the equilibrium patterns before and after the stochastic link capacity is considered, and the commuters' multimodal choice and P&R behaviors are characterized and analyzed. Numerical results show that public transit is preferred by the users living close to the CBD, while P&R mode is the prior choice for commuters far from CBD, and auto is also used around the middle part of the corridor.

It is proved in this chapter to be a successful attempt to apply the LCS modeling approach to model spatial equilibrium travel pattern with multimodal choice in a metropolitan area. However, the original model formulation needs to be converted from LCS to MILP to solve, which is a little complicated; moreover, additional parameters and variables are added through transformation, which also add some computational burdens. In addition, only static equilibrium pattern is described in this study, and dynamic elements should be introduced to model a dynamic spatial equilibrium with multiple transportation modes in later studies.

CHAPTER 4

CONTINUUM MODELING OF PARK-AND-RIDE SERVICES ON A TRAVEL CORRIDOR

(Note: this chapter includes part of contents that has been published in the paper:

Du, B. and Wang, Z.W. (2014) "Reliability-based continuum modeling of park-and-ride services with heterogeneous commuters - a linear complementarity system approach", Transportation Research Part E, Vol. 71, pp. 58-81.)

4.1 Introduction

As an extension of CHAPTER 3, this chapter carries the previous study steps forward with the following aspects of differences. First of all, infinite P&R services rather than a single P&R are considered, therefore a continuum modeling approach, rather than discrete modeling, is applied in this chapter to model traffic equilibrium. Discrete modeling approach is used for the detailed planning and analysis of transport system, however, during the initial phase of planning and modeling for regional study, there is usually insufficient data of the system for setting up a dense network for detailed analysis using the discrete modeling approach. In contrast, the continuum modeling approach adopted in this chapter is applied for the initial phase of planning and modeling of regional study, wherein more focuses are put on the general trend and pattern of the distribution and travel choices of users, and the corresponding responses to policy changes in the transport system at the macroscopic rather than detailed level. The continuum modeling approach is by no means a substitution for the discrete modeling approach, but can be complementary, especially for the macroscopic modeling of a very dense transport system (Ho and Wong, 2006; Ho et al., 2006). Second, commuter's heterogeneity is explicitly captured in this chapter, wherein commuters with general heterogeneity parameter assumptions are continuously distributed along the corridor. Third, in considering travel time uncertainty for auto users, the travel time correlations among adjacent highway segments are explicitly considered, while the stochastic travel time is

assumed to be independent in most of the previous studies and CHAPTER 3. Lastly, the modeled linear complementarity system is solved by applying the efficient Lemke's method; meanwhile, in CHAPTER 3, the formulated model is transformed into an MILP problem to be solved, which increases the computational loading significantly and is not suitable to large-size problem setting. Even more importantly, the proof of solution existence is also derived in this chapter to provide more rigorous mathematical analysis of the LCS formulation.

To sum up, this chapter considers a linear travel corridor with competitive railway and highway systems, continuous P&R services and heterogeneous commuters. Crowding cost on railway system is considered, and travel time reliability on highway is incorporated in this model to take into account the uncertain travel time, while the travel time by rail transit is assumed to be deterministic and fixed to the train service schedule. The equilibrium multimodal choice across the travel corridor with heterogeneous commuters is thus modeled into a LCS formulation, which is solved to obtain the achieved spatial equilibrium travel pattern in a linear travel corridor, and the resultant use intensity and non-use of each P&R facility reflect its location and charging characteristics. The model formulation and results are expected to provide useful information to the initial planning of P&R facilities in terms of optimal number, location and charging scheme when certain transportation management objectives are to be achieved.

This chapter contributes to the literature in the following aspects: firstly, heterogeneous commuters are explicitly considered in the continuum modeling. To my best knowledge, no previous studies provide rigorous mathematical formulation to clearly take into account heterogeneity assumptions when modeling and solving continuous spatial equilibrium travel pattern with multimodal choice. As a relevant study, Liu et al. (2009) proposed an infinite mathematical programming method in model formulation, however, the optimization formulation of the UE multimodal choice is not applicable for modeling multi-class equilibrium with heterogeneous commuters. In this chapter, LCS approach is applied. This novel formulation provides a formal and rigorous framework for the modeling and solution of the spatial equilibrium travel pattern with multimodal choice in a linear corridor, enabling the adoption of well-established complementarity theory and methods to

analyze and solve this problem. Most importantly, due to the application of LCS, this study offers a significant contribution to the existing literature that lacks rigorous formulation to explicitly consider commuter heterogeneity assumptions. Secondly, this chapter considers travel time uncertainty, as well as the correlation among adjacent locations on highway. In reality, higher level of travel time reliability is regarded as one of the advantages in favor of rail transit usage. Factoring in the travel time reliability makes the model formulation more realistic in describing the multimodal choice behavior in the transportation system. To further capture travel time correlation, a spatial moving average structure method is applied in depicting how the travel time variations are spatially correlated across the highway among adjacent road segments. Finally, comprehensive numerical studies are conducted to test the validity of the model in this chapter and potential applications are demonstrated. For example, the information of P&R users' transfer point choice is shown as it is significantly useful to decision makers on optimal planning of the P&R facilities; by assuming different urban density distribution profiles (uniform, linear and negative exponential density distribution, etc.), the model results can offer urban planners a better understanding of how urban development density pattern may affect the planning of transportation facilities (i.e., P&R facilities) and determine the best urban density distribution pattern required for ensuring optimal transportation system performance with P&R facilities. In a nutshell, the model formulation provides basic modeling framework and platform for many transportation and urban development planning analyses. With these contributions, this chapter is expected to serve as building block in providing more insights into the planning and layout of P&R facilities and the coordination of the development of multiple transportation modes, as well as the integrated land use and transportation planning.

4.2 Model formulation

Consider a corridor with continuous entry points to the highway, railway, and continuous P&R transfer services. CBD is assumed to be the common destination and only exit point for all the commuters, as shown in Figure 4.1.

Figure 4.1 A corridor with multimodal choice

It is assumed that commuters along the corridor can choose either auto or railway to go to CBD directly, and one can also choose to drive private car and then transfer to railway at any location to complete the trip by using the P&R services. To maintain consistency, term the three travel modes as auto, railway and P&R mode, respectively. The multimodal choice formulation is based on the UE principle that a commuter would use one of the three travel modes if and only if the corresponding generalized travel cost is lower than the other two alternatives. L represents the length of the corridor, CBD is at location L , the end of the corridor, and x_p denotes the P&R facility location, which is the distance from the location to the boundary of the corridor. The P&R facilities are assumed to be distributed continuously along the corridor. It is also assumed that commuters are continuously distributed along the corridor with travel demand density of $q_0(x)$ at location x , where x is the distance from the location to the boundary of the corridor.

4.2.1 Preliminaries

(1) Travel time reliability considering correlation

Most existing studies assume deterministic auto travel time in modeling multimodal travel choices. However, various events may occur on the transportation network and result in stochastic travel time for auto users, which has been found to have a significant impact on travelers' choice behaviors (Lam and Small, 2001; Lo and Tung, 2003; Li et al., 2010). Therefore, it is important to consider the impact of travel time uncertainty into the modal choice equilibrium modeling. Travel time budget is the most commonly used concept to describe this uncertainty. Based on the assumption that travel time is susceptible to variation, travel time budget is expressed as a combination of expected travel time and additional travel time reservation to hedge against travel time variability so that travelers can arrive at their destinations within a given on-time arrival probability, termed as travel time

reliability in the literature (Chen et al., 2012). For more literature reviews on travel time reliability and travel time budget, one can refer to Section 2.2.2 and Section 2.2.3 respectively.

In considering travel time budget, most studies assume correlation independence among links (Lo et al., 2006), however, travel times are often spatially correlated across road links in reality (Chan et al., 2009; Ramezani and Geroliminis, 2012). For example, a traffic accident on downstream link may cause travel delay of the upstream links. For illustration purpose, travel time correlation is considered in this chapter in a simplified form based on a spatial moving average (SMA) structure (Jenelius and Koutsopoulos, 2013).

It is assumed that travel time on highway $t_h(x)$ (the subscript h represents the highway system) traversing unit distance at location x is a combination of two parts: mean travel time $\bar{t}_h(x)$ determined by the commonly used BPR function plus a stochastic component $\varphi(x)$ following a multivariate normal distribution according to a covariance structure based on SMA method. The mean travel time is further supposed to be fully described by a first-order model in Eq. (4-1), as was done in previous studies, like D'Este (1987).

$$\bar{t}_h(x) = t_h^0 \left[1 + A \left(\frac{v_h(x)}{C_x} \right) \right] \quad (4-1)$$

where t_h^0 denotes the free flow travel time on highway per unit of distance, $v_h(x)$ and C_x represent the highway traffic volume and road capacity at location x respectively, while both A and C_x are given parameters.

The stochastic component $\varphi(x)$ is designed to capture the travel time variability due to unobserved traffic conditions and traveler characteristics, then travel time function is $t_h(x) = \bar{t}_h(x) + \varphi(x)$. In the SMA model of Jenelius and Koutsopoulos (2013), $\varphi(x)$ is assumed to be a combination of an independent term $e(x)$ with zero mean and variance σ_x^2 , plus a linear combination of the independent components of the other locations $\varphi(x')$ as:

$$\varphi(x) = \varepsilon(x) + \psi \sum_{x' \neq x} w_{xx'} e(x') \quad (4-2)$$

wherein independence between different locations implies $E[\varphi(x)\varphi(x')] = 0$ for $x' \neq x$, parameter ψ is used to capture the overall magnitude of the covariance, $w_{xx'}$ denotes the level of influence from location x' on location x , and $w_{xx} = 0$ for all locations. In practice, σ_x^2 , ψ and $w_{xx'}$ are obtained based on observations, historical data, expert opinions and extensive experimentation with alternative specifications to capture the dependences between link travel times properly. Specifically, the observations may come from low frequency global position system (GPS) probe vehicles reporting data, like the time, positions, and so on, at certain intervals. The details of the derivation of $w_{xx'}$ can be referred to Jenelius and Koutsopoulos (2013).

The SMA model can be extended to include multiple weight matrices with each matrix representing a separate dimension of spatial dependence (Hepple, 2004). For simplicity, only one dimension of spatial dependence is considered and parameter $\mu_{xx'}$ is used in this chapter to represent $\psi w_{xx'}$ so that $\varphi(x)$ can be simplified as:

$$\varphi(x) = e(x) + \sum_{x' \neq x} \mu_{xx'} e(x') \quad (4-3)$$

In order to analyze the travel time correlations among different locations, a travel corridor is divided into several sections indexed by $1, 2, \dots, v$, and the sequence of the sections 1 to v is the direction from the boundary of the corridor to CBD. t_h^k is the travel time through section k , T_h^i is the travel time from section i to CBD. With the SMA model, the covariance between any two sections a and b is,

$$\text{Cov}(\varphi_a, \varphi_b) = E[\varphi_a \varphi_b] = \mu_{ba} \sigma_a^2 + \mu_{ab} \sigma_b^2 \quad (4-4)$$

The mean of travel time $T_h^i = \sum_{k=i}^v t_h^k$ is given by

$$E(T_h^i) = \sum_{k=i}^v E(t_h^k) = \sum_{k=i}^v \overline{t_h^k} \quad (4-5)$$

The variance of T_h^i is calculated from the variances and covariance of the traversed sections as

$$D(T_h^i) = D\left(\sum_{k=i}^v (\varphi_k)\right) = \sum_{k=i}^v \sigma_k^2 + 2 \sum_{a < b} Cov(\varphi_a, \varphi_b), \quad i \leq a < b \leq v \quad (4-6)$$

K-limited spatial dependence is considered here for term $Cov(\varphi_a, \varphi_b)$. As is indicated by empirical data, the travel time of a specific road section is affected much more by adjacent sections than the ones far away from it, therefore it is reasonable to assume that the correlations exist only between the adjacent segment travel times (Gajewski and Rilett, 2004; El Esawey and Sayed, 2011; Chen et al., 2012). Therefore, the segment travel time is assumed to be only spatially correlated with its adjacent sections within a local ‘impact area’, in which the topological distance (measured by number of sections) between any two sections is less than or equal to k , namely k-limited spatial dependence. As was done in the work of Nie and Wu (2009) and Nikolova (2009), only travel time correlations between the neighboring sections ($k=1$) are considered for illustration purpose, then Eq. (4-6) can be expressed as

$$D(T_h^i) = \sum_{k=i}^v \sigma_k^2 + 2 \sum_{k=i+1}^v Cov(\varphi_{k-1}, \varphi_k) = \sum_{k=i}^v \sigma_k^2 + 2 \sum_{k=i+1}^v (\mu_{k-1,k} \sigma_k^2 + \mu_{k,k-1} \sigma_{k-1}^2) \quad (4-7)$$

In general, Eqs. (4-4) and (4-7) are able to capture the interactions between any two locations on the corridor, including both directional influences from downstream and upstream. For illustration purpose, influence among traffic flow is often considered as unidirectional from downstream to upstream in many studies, so it is reasonable to assume that only downstream section affects the upstream section, that is $\mu_{k,k-1} = 0$, thus Eq. (4-7) can be reduced to

$$D(T_h^i) = \sum_{k=i}^v \sigma_k^2 + 2 \sum_{k=i+1}^v \mu_{k-1,k} \sigma_k^2 \quad (4-8)$$

Under the assumption of multivariate normal section travel time, travel time T_h^i follows a normal distribution. Therefore, travel time budget of T_h^i can be derived as

$$T^{b,i} = E(T_h^i) + \lambda \sigma_{T_h^i} \quad (4-9)$$

where $T^{b,i}$ is travel time budget from section i to CBD by auto, $\sigma_{T_h^i} = \sqrt{D(T_h^i)}$ is standard deviation of T_h^i , λ is a parameter related to the arrival punctuality requirement ρ , which can be written as

$$P\{T_h^i \leq T^{b,i}\} = P\{T_h^i \leq E(T_h^i) + \lambda \sigma_{T_h^i}\} = \rho \quad (4-10)$$

Therefore, for normal distributed travel time, it is easy to obtain:

$$\lambda = \phi^{-1}(\rho) \quad (4-11)$$

It should be noted that the concept of travel time budget is also applied in defining travel time of rail transit users. However, due to the rail transit operation with exclusive right of way, the travel time of rail users is assumed to be deterministic and fixed, i.e., the travel time budget is defined with $\rho=1.0$. The travel time for P&R users is a combination of the two components, which is also a travel time budget with ρ as specified for the highway component and $\rho=1.0$ for railway component.

(2) *Heterogeneous commuters*

Heterogeneity across commuters is considered in this chapter. A variety of characteristics can be used to define the heterogeneity among commuters. In this chapter, the focus is mainly put on the commuters' variations of two aspects: punctuality probability requirement ρ_g when defining the travel time budget and VOT τ_g when converting time to monetary cost. Punctuality probability requirement reflects traveler's risk aversion degree, and VOT depends on travelers' income level and trip characteristics. Therefore, the commuters can be classified into $\mathbb{G}$ groups with different heterogeneity parameters, indexed by $g = 1, 2, \dots, \mathbb{G}$.

4.2.2 Differential equations for spatial interactions

In this chapter, both travel time budget and crowding cost along the corridor are considered, and ODEs are used to describe their spatial interactions among different locations respectively. Based on the demonstration in Section 4.2.1, the travel time

budget $T_g^b(x)$ by auto from location x to CBD for commuter group g can be derived as follows.

$$T_g^b(x) = \int_x^L (t_h^0 + t_h^0 A v_h(w) C_w^{-1}) dw + \lambda_g \left(\int_x^L \sigma_w^2 dw + 2 \int_{x+\Delta x}^L \mu_{w-\Delta w, w} \sigma_w^2 dw \right)^{\frac{1}{2}} \quad (4-12)$$

In the above equation, the traffic volume $v_h(w)$ at location w on highway is comprised of two parts: the travelers choosing auto mode from the boundary of the corridor to location w , and the travelers choosing P&R mode with the entry points before location w while transfer points after location w . Therefore the total traffic volume of all commuter groups at location w on highway can be calculated as:

$$v_h(x) = \sum_{g=1}^G \left(\int_0^x q_g^h(w) dw + \int_0^x \int_x^L q_g^p(w, w_p) dw_p dw \right) \quad (4-13)$$

wherein $q_g^h(w)$ denotes the commuter density (number of commuters per unit distance) in group g choosing auto at location w , and $q_g^p(w, w_p)$ represents the density of P&R users in group g driving onto the highway at w and transfer at P&R location w_p to complete the rest of the trip by railway.

Now, taking second order derivative of travel time budget $T_g^b(x)$ with respect to location x , and the result is shown in Eq. (4-14).

$$\begin{aligned} \frac{d^2 T_g^b(x)}{dx^2} = & -\frac{t_h^0 A}{C_x} \sum_{g=1}^G \left(q_g^h(x) + \int_x^L q_g^p(x, w_p) dw_p - \int_0^x q_g^p(w, x) dw \right) \\ & - \frac{\lambda_g}{4} \left(\sigma_x^2 + 2 \mu_{x, x+\Delta x} \sigma_{x+\Delta x}^2 \right)^2 \left(\int_x^L \sigma_w^2 dw + 2 \int_{x+\Delta x}^L \mu_{w-\Delta w, w} \sigma_w^2 dw \right)^{-\frac{3}{2}} \end{aligned} \quad (4-14)$$

As for the crowding cost $Y_g^r(x)$ (the superscript r represents the railway system) by railway from location x to CBD for commuter group g , it quantifies the loss of privacy and comfort, decline of air quality and venture of pickpocket due to the increase of passengers on the train (Huang, 2000). Let $y_g^r(v_r(x)) = \alpha + \beta v_r(x)$ be the crowding cost for commuter group g on the train per unit distance at location x , where α, β are the calibrated parameters, and $v_r(x)$ denotes the total traffic volume

of all commuter groups by railway at location x , which can be calculated similarly as $v_h(x)$ as follows:

$$v_r(x) = \sum_{g=1}^G \left(\int_0^x q_g^r(w) dw + \int_0^x \int_w^x q_g^p(w, w_p) dw_p dw \right) \quad (4-15)$$

Wherein $q_g^r(w)$ denotes the commuter density (number of commuters per unit distance) in group g using railway at location w . Moreover, the crowding cost $Y_g^r(x)$ can be derived as:

$$Y_g^r(x) = \int_x^L \left[\alpha + \beta \sum_{g=1}^G \left(\int_0^z q_g^r(w) dw + \int_0^z \int_w^z q_g^p(w, w_p) dw_p dw \right) \right] dz \quad (4-16)$$

Similarly, taking second order derivative of crowding cost $Y_g^r(x)$, then

$$\frac{d^2 Y_g^r(x)}{dx^2} = -\beta \sum_{g=1}^G \left(q_g^r(x) + \int_0^x q_g^p(w, x) dw \right) \quad (4-17)$$

The above ODEs (4-14) and (4-17) reflect the spatial interactions of the travel time budgets and crowding costs for commuters departing from different locations along the corridor. For each $g \in \{1, \dots, G\}$, the initial and boundary conditions of Eq. (4-14) and Eq. (4-17) are shown as below.

$$T_g^b(x) \Big|_{x=L} = 0, \quad Y_g^r(x) \Big|_{x=L} = 0 \quad (4-18)$$

$$\frac{dT_g^b(x)}{dx} \Big|_{x=0} = t_h^0, \quad \frac{dY_g^r(x)}{dx} \Big|_{x=0} = \alpha \quad (4-19)$$

where the initial conditions (4-18) guarantee that the travel time budget and crowding cost at CBD will be zero, the boundary conditions (4-19) entail that travel demand beyond the corridor boundary will be zero, and no correlated influence will be considered beyond the both sides of the corridor.

4.2.3 Complementarity formulations for user equilibrium conditions

A set of complementarity formulations are used to model the equilibrium conditions considering heterogeneous commuters as follows.

$$\begin{aligned}
 0 &\leq q_g^h(x) \perp C_g^h(x) - C_g^*(x) \geq 0 \\
 0 &\leq q_g^r(x) \perp C_g^r(x) - C_g^*(x) \geq 0 \\
 0 &\leq q_g^p(x, x_p) \perp C_g^p(x, x_p) - C_g^*(x) \geq 0 \\
 0 &\leq C_g^*(x) \perp q_g^h(x) + q_g^r(x) + \int_x^L q_g^p(x, w_p) dw_p - q_g^0(x) \geq 0 \\
 &\forall x \in [0, L], \forall x_p \in (x, L], \forall g \in \{1, \dots, G\}
 \end{aligned} \tag{4-20}$$

where the first three conditions describe the UE principle that each commuter departing from home makes multimodal choice with the objective to minimize the generalized travel cost in each user group g , and the last condition guarantees that all the travel demand $q_g^0(x)$ (given in priori) in each user group g at location x along the corridor should make choice of travel modes. $C_g^*(x)$ denotes the equilibrium/minimum generalized travel cost for group g departing from location x . Moreover, the components of the generalized travel costs ($C_g^h(x)$, $C_g^r(x)$, $C_g^p(x, x_p)$) by three travel modes are given in the following.

First, the generalized travel cost by auto from location x to CBD for commuter group g is

$$C_g^h(x) = \tau_g \left(t_h^{hh} + t_h^{pw} + T_g^b(x) \right) + f_h(x) + R_h^w \tag{4-21}$$

wherein τ_g denotes the VOT for commuter group g , t_h^{hh} represents the access time from home to highway, and t_h^{pw} represents the egress time for auto users from parking place to workplace. $f_h(x) = f_h^0 + \gamma(L - x)$ is the operating cost on highway pertaining to travel distance from location x to CBD, which consists of two parts: f_h^0 is the fixed part of tolls, $\gamma(L - x)$ is the variable part, such as the variable part of highway tolls, fuel, etc., and γ is the operating cost per unit distance. R_h^w denotes the parking fee for auto users at CBD.

Second, the generalized travel cost by railway from location x to CBD for commuter group g is

$$C_g^r(x) = \tau_g \left(t_r^{hr} + t_r^{rw} + \frac{L - x}{V_r} \right) + Y_g^r(x) + f_r(x) \tag{4-22}$$

wherein t_r^{hr} and t_r^{rw} represent the access time for railway users from home to railway station and egress time from railway station to workplace respectively, $\frac{L-x}{V_r}$ denotes the travel time from location x to CBD by railway with the average train speed at V_r . As for the fare $f_r(x)$ by railway from location x to CBD, it is assumed to be a linear function with two parts: $f_r(x) = f_r^0 + \kappa(L-x)$, where f_r^0 is the fixed part, and κ is the fare per unit distance.

Third, the generalized travel cost by P&R for commuter group g is

$$\begin{aligned} C_g^p(x, x_p) = & \tau_g t_h^{hh} + \tau_g (T_g^b(x) - T_g^b(x_p)) + f_h(x, x_p) + R_p(x_p) \\ & + \tau_g t_p^{pr} + u_p + \tau_g \frac{L-x_p}{V_r} + Y_g^r(x_p) + f_r(x_p) + \tau_g t_r^{rw} \end{aligned} \quad (4-23)$$

wherein $T_g^b(x) - T_g^b(x_p)$ denotes the travel time budget from departure location x to P&R facility location x_p for commuter group g , $f_h(x, x_p)$ represents the operating cost from departure location x to P&R facility location x_p , $R_p(x_p) = R_h^w e^{-\frac{(L-x_p)^2}{2L}}$ denotes the parking fee at P&R facility location x_p for P&R users, which is assumed as a function of the P&R facility location (Liu et al., 2009). t_p^{pr} denotes the access time for P&R users from P&R facility location to railway station, and u_p denotes the inconvenience or disutility cost for P&R users.

4.2.4 Continuum equilibrium model with multimodal choice

Based on the above analysis, LCS approach can be used to describe the spatial equilibrium travel pattern with multimodal choice. The solution is to find $q_g^h(x)$, $q_g^r(x)$, $q_g^p(x, x_p)$, $C_g^*(x)$, $T_g^b(x)$, $Y_g^r(x)$ so that the following conditions (4-24) and (4-25) are satisfied.

(A) For almost all $x \in (0, L]$

$$\begin{aligned}
 \frac{d^2 T_g^b(x)}{dx^2} &= -\frac{t_h^0 A}{C_x} \sum_{g=1}^G \left(q_g^h(x) + \int_x^L q_g^p(x, w_p) dw_p - \int_0^x q_g^p(w, x) dw \right) \\
 &\quad - \frac{\lambda_g}{4} \left(\sigma_x^2 + 2\mu_{x, x+\Delta x} \sigma_{x+\Delta x}^2 \right)^2 \left(\int_x^L \sigma_w^2 dw + 2 \int_{x+\Delta x}^L \mu_{w-\Delta w, w} \sigma_w^2 dw \right)^{-\frac{3}{2}} \\
 \frac{d^2 Y_g^r(x)}{dx^2} &= -\beta \sum_{g=1}^G \left(q_g^r(x) + \int_0^x q_g^p(w, x) dw \right) \\
 0 &\leq q_g^h(x) \perp C_g^h(x) - C_g^*(x) \geq 0 \\
 0 &\leq q_g^r(x) \perp C_g^r(x) - C_g^*(x) \geq 0 \\
 0 &\leq q_g^p(x, x_p) \perp C_g^p(x, x_p) - C_g^*(x) \geq 0 \\
 0 &\leq C_g^*(x) \perp q_g^h(x) + q_g^r(x) + \int_x^L q_g^p(x, w_p) dw_p - q_g^0(x) \geq 0 \\
 &\quad \forall x_p \in (x, L], \forall g \in \{1, \dots, G\}
 \end{aligned} \tag{4-24}$$

(B) The initial and boundary conditions:

$$\begin{aligned}
 T_g^b(x) \Big|_{x=L} &= 0, \quad Y_g^r(x) \Big|_{x=L} = 0 \\
 \frac{dT_g^b(x)}{dx} \Big|_{x=0} &= t_h^0, \quad \frac{dY_g^r(x)}{dx} \Big|_{x=0} = \alpha \\
 &\quad \forall g \in \{1, \dots, G\}
 \end{aligned} \tag{4-25}$$

4.3 Solution

4.3.1 Discrete approximation of the continuum model

Based on a time-stepping scheme, the numerical solution of the formulated LCS model can be obtained by solving a sequence of finite-dimensional discretized sub-problems. In the discret scheme, the corridor is divided into v sections with the length of each section $\varepsilon_v = \frac{L}{v}$. The boundary of the corridor and the CBD are assumed to be at dummy sections 0 and $v+1$ respectively. $q_g^{p,ij}$ represents the density of P&R users in group g , who drive onto highway at section i and transfer to transit at section j after at least one section of distance ($1 \leq i < j \leq v$).

To present the overall discrete approximate equilibrium model with multimodal choice in a linear travel corridor, a list of notations used throughout this chapter is given first.

Table 4.1 List of notations and descriptions

Notation	Description
Indices	
i	Index of sections, $i \in \{1, \dots, v\}$
g	Index of commuter groups, $g \in \{1, \dots, G\}$
Variables	
$q_g^{h,i}$	The density of commuters in group g choosing auto mode at section i
$q_g^{r,i}$	The density of commuters in group g choosing rail mode at section i
$q_g^{p,ij}$	The density of commuters in group g driving from section i to j for P&R
$C_g^{*,i}$	Equilibrium cost for commuter group g from section i to CBD
$T_g^{b,i}$	Travel time budget for commuter group g from section i to CBD
$t_g^{b,i}$	Travel time on highway for user group g traversing through section i
$Y_g^{r,i}$	Crowding cost on rail for commuter group g from section i to CBD
$y_g^{r,i}$	Crowding cost on rail for commuter group g traversing through section i
Parameters	
L	Length of the corridor
v	Number of the corridor sections
G	Number of commuter groups
$q_g^{0,i}$	Total demand density of commuter group g at section i
τ_g	VOT for commuter group g
ρ_g	Punctuality probability requirement for commuter group g
λ_g	Parameter related to the reliability of punctual arrival for user group g
A	BPR function parameter
t_h^0	BPR function parameter, free-flow travel time
C_i	Road capacity of section i
t_h^{hh}	Time from home to highway
t_h^{pw}	Time from parking place to workplace
f_h^0	Fixed part of the operating cost by auto
γ	Operating cost per unit distance by auto
R_h^w	Parking fee at workplace
t_r^{hr}	Time from home to railway station
t_r^{rw}	Time from railway station to workplace
f_r^0	Fixed part of the railway fare
κ	Railway fare per unit distance
V_r	Average speed of the train
α, β	Parameters of the crowding cost
t_h^{pr}	Time from parking place at P&R location to train station nearby
u_p	Fixed transfer disutility cost
$\mu_{i,i-1}$	Level of influence from section $i-1$ on section i
σ_i^2	Variance of travel time at section i

In the discretized scheme, the derivative can be approximated by the forward difference quotient:

$$\frac{d^2 T_g^b(x)}{dx^2} \approx \frac{[T_g^b(x - \varepsilon_v) - T_g^b(x)] - [T_g^b(x) - T_g^b(x + \varepsilon_v)]}{\varepsilon_v^2} \quad (4-26)$$

$$\frac{d^2 Y_g^r(x)}{dx^2} \approx \frac{[Y_g^r(x - \varepsilon_v) - Y_g^r(x)] - [Y_g^r(x) - Y_g^r(x + \varepsilon_v)]}{\varepsilon_v^2} \quad (4-27)$$

Hence, the discretized approximation of the first two equations in condition (4-24) can be described as:

$$\begin{aligned} \frac{T_g^{b,i+1} + T_g^{b,i-1} - 2T_g^{b,i}}{\varepsilon_v^2} = & -\frac{t_h^0 A}{C_i} \sum_{g=1}^G (q_g^{h,i} + \sum_{j=i+1}^v q_g^{p,ij} - \sum_{j=1}^{i-1} q_g^{p,ji}) \\ & - \frac{\lambda_g}{4} \left(\sigma_i^2 + 2\mu_{i,i+1} \sigma_{i+1}^2 \right)^2 \left(\sum_{j=i}^v \sigma_j^2 + 2 \sum_{j=i+1}^v \mu_{j-1,j} \sigma_j^2 \right)^{-\frac{3}{2}} \end{aligned} \quad (4-28)$$

$$\frac{Y_g^{r,i+1} + Y_g^{r,i-1} - 2Y_g^{r,i}}{\varepsilon_v^2} = -\beta \sum_{g=1}^G (q_g^{r,i} + \sum_{j=1}^{i-1} q_g^{p,ji}) \quad (4-29)$$

Noting that $T_g^{b,i} > 0$, $Y_g^{r,i} > 0$, $i = 1, 2, \dots, v$, $g = 1, 2, \dots, G$, the discretized Eqs. (4-28) and (4-29) above can be converted into a LCP form. Furthermore, the initial and boundary conditions (4-25) can be rewritten in the following discretized form:

$$\begin{aligned} T_g^{b,v+1} = 0, \quad Y_g^{r,v+1} = 0, \quad \forall g \in \{1, \dots, G\} \\ T_g^{b,0} - T_g^{b,1} = t_h^0 \varepsilon_v, \quad Y_g^{r,0} - Y_g^{r,1} = \alpha \varepsilon_v, \quad \forall g \in \{1, \dots, G\} \end{aligned} \quad (4-30)$$

where $T_g^{b,0}$, $T_g^{b,v+1}$, $Y_g^{r,0}$ and $Y_g^{r,v+1}$ represent the travel times and crowding costs at CBD ($i = v + 1$) and corridor boundary ($i = 0$), which can be regarded as variables so that (4-30) can be transformed to equivalent LCP formulations in (4-31).

$$\begin{aligned} 0 \leq T_g^{b,v+1} \perp T_g^{b,v+1} + L \geq 0, \quad \forall g \in \{1, \dots, G\} \\ 0 \leq T_g^{b,0} \perp T_g^{b,0} - T_g^{b,1} - t_h^0 \varepsilon_v \geq 0, \quad \forall g \in \{1, \dots, G\} \\ 0 \leq Y_g^{r,v+1} \perp Y_g^{r,v+1} + L \geq 0, \quad \forall g \in \{1, \dots, G\} \\ 0 \leq Y_g^{r,0} \perp Y_g^{r,0} - Y_g^{r,1} - \alpha \varepsilon_v \geq 0, \quad \forall g \in \{1, \dots, G\} \end{aligned} \quad (4-31)$$

The equivalence between (4-30) and (4-31) is straightforward. For example, as for the first condition, $0 \leq T_g^{b,v+1} \perp T_g^{b,v+1} + L \geq 0$, in (4-31), it is obviously true that the right side $T_g^{b,v+1} + L > 0$, then it must have $T_g^{b,v+1} = 0$ based on the complementarity

condition, which is actually stated in (4-30). Similarly, the third condition in (4-31) is equivalent with $Y_g^{r,v+1} = 0$, which is identical with the equation in (4-30). In the same way, as for the second condition, $0 \leq T_g^{b,0} \perp T_g^{b,0} - T_g^{b,1} - t_h^0 \varepsilon_v \geq 0$, in (4-31), if $T_g^{b,0} = 0$, then the right side $-T_g^{b,1} - t_h^0 \varepsilon_v < 0$, which breaks the complementarity rule, thus it must have $T_g^{b,0} > 0$ and $T_g^{b,0} - T_g^{b,1} - t_h^0 \varepsilon_v = 0$. Similarly, the fourth condition in (4-31) is equivalent with $Y_g^{r,0} - Y_g^{r,1} - \alpha \varepsilon_v = 0$, which is identical with the last equation in (4-30).

In addition, assume that $t_g^{b,i} = T_g^{b,i} - T_g^{b,i+1}$, $y_g^{r,i} = Y_g^{r,i} - Y_g^{r,i+1}$ for $i = 1, 2, \dots, v$, $g = 1, 2, \dots, \mathbb{G}$, wherein $t_g^{b,i}$ and $y_g^{r,i}$ represent the travel time and crowding cost traversing through section i for commuter group g . Since the travel time $T_g^{b,i}$ and crowding cost $Y_g^{r,i}$ from section i to CBD should be a positive value, therefore the equations can be transformed to equivalent LCP conditions as below:

$$\begin{aligned} 0 &\leq T_g^{b,i} \perp T_g^{b,i} - T_g^{b,i+1} - t_g^{b,i} \geq 0 \\ 0 &\leq Y_g^{r,i} \perp Y_g^{r,i} - Y_g^{r,i+1} - y_g^{r,i} \geq 0 \end{aligned} \quad (4-32)$$

Therefore, the differential complementarity problem described in (4-24) and (4-25) can be converted into LCP formulation with a discretization scheme. Specifically, the following $LCP(s, M)$ with variables for each group $g \in \{1, \dots, \mathbb{G}\}$ can be obtained as:

$$x = \left\{ \left(q_g^{h,i} \right)_{i=1}^v, \left(q_g^{r,i} \right)_{i=1}^v, \left(q_g^{p,ij} \right)_{i=1}^v, \left(C_g^{*,i} \right)_{i=1}^v, \left(T_g^{b,i} \right)_{i=1}^v, \left(t_g^{b,i} \right)_{i=1}^v, T_g^{b,0}, T_g^{b,v+1}, \left(Y_g^{r,i} \right)_{i=1}^v, \left(y_g^{r,i} \right)_{i=1}^v, Y_g^{r,0}, Y_g^{r,v+1} \right\}_{g \in \mathbb{G}}.$$

$$\begin{aligned}
 & 0 \leq t_g^{b,i-1} \perp \left(t_g^{b,i-1} - t_g^{b,i} + \frac{\varepsilon_v^2 t_h^0 A}{C_i} \sum_{g=1}^G (q_g^{h,i} + \sum_{j=i+1}^v q_g^{p,ij} - \sum_{j=1}^{i-1} q_g^{p,ji}) \right. \\
 & \quad \left. + \frac{\lambda_g}{4} (\sigma_i^2 + 2\mu_{i,i+1} \sigma_{i+1}^2)^2 \left(\sum_{j=i}^v \sigma_j^2 + 2 \sum_{j=i+1}^v \mu_{j-1,j} \sigma_j^2 \right)^{\frac{3}{2}} \right) \geq 0 \\
 & 0 \leq y_g^{r,i-1} \perp y_g^{r,i-1} - y_g^{r,i} + \varepsilon_v^2 \beta \sum_{g=1}^G (q_g^{r,i} + \sum_{j=1}^{i-1} q_g^{p,ji}) \geq 0 \\
 & 0 \leq q_g^{h,i} \perp \tau_g t_h^{hh} + \tau_g t_h^{pw} + \tau_g T_g^{b,i} + f_h^0 + \gamma(v-i+1)\varepsilon_v + R_h^w - C_g^{*,i} \geq 0 \\
 & 0 \leq q_g^{r,i} \perp \tau_g t_r^{hr} + \tau_g t_r^{rw} + \tau_g \frac{(v-i+1)\varepsilon_v}{V_r} + Y_g^{r,i} + f_r^0 + \kappa(v-i+1)\varepsilon_v - C_g^{*,i} \geq 0 \\
 & 0 \leq q_g^{p,ij} \perp \left(\tau_g t_h^{hh} + \tau_g \sum_{m=i}^{j-1} t_g^{b,m} + f_h^0 + \gamma(j-i)\varepsilon_v + R_h^w e^{-\frac{(v-j)^2}{2v}} + \tau_g t_p^{pr} + u_p \right. \\
 & \quad \left. + \tau_g \frac{(v-j+1)\varepsilon_v}{V_r} + Y_g^{r,j} + f_r^0 + \kappa(v-j+1)\varepsilon_v + \tau_g t_r^{rw} - C_g^{*,i} \right) \geq 0 \quad (4-33) \\
 & 0 \leq C_g^{*,i} \perp q_g^{h,i} + q_g^{r,i} + \sum_{j=i+1}^v q_g^{p,ij} - q_g^{0,i} \geq 0 \\
 & 0 \leq T_g^{b,i} \perp T_g^{b,i} - T_g^{b,i+1} - t_g^{b,i} \geq 0 \\
 & 0 \leq Y_g^{r,i} \perp Y_g^{r,i} - Y_g^{r,i+1} - y_g^{r,i} \geq 0 \\
 & 0 \leq T_g^{b,v+1} \perp T_g^{b,v+1} + L \geq 0 \\
 & 0 \leq T_g^{b,0} \perp T_g^{b,0} - T_g^{b,1} - t_h^0 \varepsilon_v \geq 0 \\
 & 0 \leq Y_g^{r,v+1} \perp Y_g^{r,v+1} + L \geq 0 \\
 & 0 \leq Y_g^{r,0} \perp Y_g^{r,0} - Y_g^{r,1} - \alpha \varepsilon_v \geq 0 \\
 & \forall i \in \{1, \dots, v\}, \forall j \in \{i+1, \dots, v\}, \forall g \in \{1, \dots, G\}
 \end{aligned}$$

4.3.2 Existence of solution

In this section, the existence of solution to the proposed complementarity problem $LCP(s, M)$ (4-33) is provided, which is denoted by $SOL(s, M)$.

Definition 4.1 (Cottle et al., 1992). A matrix $M \in R^{n \times n}$ is said to be semimonotone if

$$[0 \neq x \geq 0] \Rightarrow [x_k > 0 \text{ and } (Mx)_k \geq 0 \text{ for some } k]$$

The class of such matrices is denoted E_0 , and its elements are called E_0 -matrices.

The following Lemma 4.1 describes the existence conditions for the solution set of $SOL(s, M)$.

Lemma 4.1 (Pang et al., 2012). Let M be a semimonotone matrix. If $d > 0$ is such that for every $\tau > 0$, $SOL(s + \tau d, M)$ is bounded, then the $LCP(s, M)$ has a solution that can be computed by Lemke's algorithm with d as the covering vector.

Applying Lemma 4.1 to the $LCP(s, M)$ (4-33), then it derives the following proposition.

Proposition 4.1 *For each integer $v=1,2,\dots$, the $LCP(s, M)$ (4-33) has a solution that can be computed by Lemke's algorithm with any positive vector as the covering vector.*

Proof. Refer to APPENDIX A.

The above proof verifies the existence of solution to the $LCP(s, M)$ (4-33) and ascertains that Lemke's algorithm can be applied to solve it. To solve LCP, many algorithms have been developed, among which the most well known method perhaps is the Lemke's algorithm. The Lemke's algorithm, also known as the complementary pivot algorithm, can solve LCP quickly and efficiently (Facchinei and Pang, 2003). In practice, many softwares and solvers, like PATH and KNITRO, can be used to solve the complementarity problem.

4.4 Numerical results

In this section, a set of numerical results are presented with an idealized linear corridor for two different scenarios: homogeneous and heterogeneous commuters. The corridor is partitioned into v discrete sections, wherein the sequence of the sections from 1 to v represents the direction from the boundary of the corridor to CBD, and user equilibrium multimodal choice is obtained on each section. For simplicity, assume each person corresponds to one vehicle on road.

4.4.1 Homogeneous case

The following parameter values, same as numerical settings in Section 3.4, are used as the input data: $L = 20\text{km}$, $v = 20$, $\tau = 0.5 \text{ S\$/min}$, $A = 1$, $t_h^0 = 0.8\text{min}$, $C = 5000 \text{ veh/h}$, $t_h^{hh} = 2 \text{ min}$, $t_h^{pw} = 2 \text{ min}$, $f_h^0 = 2 \text{ S\$/veh}$, $\gamma = 0.02 \text{ S\$/km/veh}$, $R_h^w = 7 \text{ S\$/veh}$, $t_r^{hr} = 11 \text{ min}$, $t_r^{rw} = 5 \text{ min}$, $f_r^0 = 0.5 \text{ S\$}$, $\kappa = 0.07 \text{ S\$/km}$, $V_r = 0.7 \text{ km/min}$, $\alpha = 0$, $\beta = 0.00003$, $t_h^{pr} = 1 \text{ min}$, $u_p = 0.2 \text{ S\$}$, $\rho = 95\%$, $\mu_{i+1,i} = 0.8$ for $i = 1, 2, \dots, v-1$, $\sigma_i^2 = 0.016 \text{ min}^2$ and $q^{0,i} = 600 \text{ commuters/h/km}$ for $i = 1, 2, \dots, v$.

To test the validity of the model formulation and solution method, firstly assume homogeneous commuters across the corridor and consider travel time reliability. Figure 4.2 depicts the generalized costs by three modes along the corridor at equilibrium status. From location 15 up to CBD, the generalized cost of railway is obviously lower than other two modes as it is immune from the traffic congestion in city center, wherein only the transit mode is used; between locations 11 and 14, direct auto mode has the lowest travel cost, as rail transit service becomes unfavorable due to the slow average travel speed in a longer traveling distance, while generalized travel cost of P&R mode is dominated by the high transfer cost. At location 10, the generalized costs by auto and P&R are identical, therefore both modes are used. Beyond location 10 until the corridor boundary, P&R mode is preferred as it takes full advantages of auto driving in suburban area and rail transit service in the city center.

Figure 4.2 Generalized costs of three modes at equilibrium status ($\rho = 95\%$)

The solved equilibrium travel pattern in stochastic travel time (STT) scenario is illustrated in Table 4.2, which is identical with the corresponding generalized costs of three modes in Figure 4.2. Moreover, Table 4.2 also shows the spatial equilibrium travel pattern of the scenario with deterministic travel time (DTT) on highway for comparison. One can also observe that P&R mode is more favorable from the corridor boundary to location 9 when travel time reliability is taken into

account, while it is the case from the corridor boundary to location 8 if no travel time reliability is considered. The reason behind this is that commuters using auto need to reserve higher value of travel time budget to hedge against the travel time variations on highway. Without considering the factor of travel time reliability, the model result of spatial equilibrium travel pattern will change in the way that more commuters outside the city center will choose auto usage.

Table 4.2 Modal split patterns with DTT and STT (commuters/h/km)

Section	DTT			STT ($\rho = 95\%$)		
	Auto user	Railway user	P&R user	Auto user	Railway user	P&R user
1	0	0	600	0	0	600
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8	0	0	600	0	0	600
9	321.2	0	278.8	0	0	600
10	600	0	0	92.5	0	507.5
11	600	0	0	600	0	0
12	600	0	0	600	0	0
13	600	0	0	600	0	0
14	600	0	0	600	0	0
15	0	600	0	0	600	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	0	600	0	0	600	0

Figure 4.3 Travel time on highway and crowding cost on railway at equilibrium status with DTT and STT

Figure 4.3 illustrates the different travel times on highway and crowding costs on railway respectively comparing the cases with DTT and STT. Both the travel time and crowding cost are higher if STT is considered. For travel time on highway, additional travel time is reserved to hedge against uncertainty; and for crowding cost on railway, more commuters switch their mode choices from auto to P&R after considering STT, which results in more railway passengers and thus higher crowding cost on railway.

4.4.2 Travel time reliability

A simple sensitivity analysis of the model result with respect to travel time reliability requirement is conducted in this section. The modal split results are plotted in Figure 4.4. The results show that commuters' punctuality requirements affect the spatial equilibrium travel pattern significantly. More auto users located beyond the city center towards the city boundary change to use P&R service when higher travel time reliability requirement is imposed, because by P&R users can enjoy both benefits and advantages of auto at suburb area and rail transit at city center area.

Figure 4.4 Modal split patterns with different travel time reliability requirements

Figure 4.5 Travel times on highway with different travel time reliability requirements

Figure 4.5 compares the travel times consumed on highway at equilibrium for different travel time reliability requirements. It shows that higher travel time reliability requirement leads to higher travel time on highway because commuters reserve more travel time budget to guarantee higher reliability. It is also easy to understand that in the case of DTT (without considering travel time reliability), no additional time is needed to reserve, therefore the travel time is the lowest compared to other cases with different travel time reliability requirements.

4.4.3 Travel time correlation

To analyze how the travel time correlation affects the modal split result at equilibrium status, a simple sensitivity analysis is conducted to compare the modal split results with different values of the correlation coefficient, and the result is shown in Figure 4.6. ‘cor’ denotes the abbreviation of ‘correlation’, which measures the degree of correlation/dependence between two road sections. The total modal split results along the corridor with different correlation coefficient values are shown, for example, at the value of ‘cor=0.8’, the total amount of auto users along the corridor is 2492.5, and 3600, 5907.5 for railway and P&R users respectively.

With different values of the correlation coefficient, the modal split results are similar in the pattern but a few differences at certain sections for auto and P&R

users. From locations 15 to 20, only railway is in use, and auto mode is the favorite between locations 10 and 14, while P&R mode is the commuters' choice from location 11 to the city boundary. At locations 10 and 11, the amounts of auto and P&R users switch complementarily corresponding to different correlation coefficient values. In Figure 4.6, one can find that the total amount of railway users remains constant, while the total number of auto users raises and the total number of P&R users drops when the value of the correlation coefficient drops from 1 to 0. This trend stems from the fact that higher travel time correlation leads to higher travel time budget and higher generalized cost by auto mode, so that more commuters choose P&R rather than auto to minimize their generalized travel costs.

Figure 4.6 Modal split results with different values of the correlation coefficient

4.4.4 P&R location choice

For the commuters who choose to use P&R facilities, the location choice of the transfer point is also modeled and solved in the developed model. Figure 4.7 demonstrates the trip assignments associated with transfer locations for all commuters using P&R services, in which the tail of the curve denotes the location P&R users depart from and the arrow represents the P&R location where the users transfer from driving to railway. It clearly indicates the P&R location choice for commuters originating from any location of the corridor. One can find that multiple choices occur in the solution (e.g., locations 3, 5). Furthermore, another interesting

finding is that commuters living further away from CBD tend to drive longer distance before transfer to railway. This can be interpreted by the fact that commuters located outside the city center will take full advantage of light highway traffic by driving long distance beyond the city center, while commuters located closer to city center have to make transfer earlier to fully benefit from the cost saving of riding on railway transit.

Figure 4.7 P&R users' choices of transfer point

Figure 4.8 Distribution of cumulative P&R users' choices of transfer point

Figure 4.8 depicts the distribution of cumulative P&R users' choices of transfer point, wherein horizontal/X axis represents the road section index and vertical/Y

axis is the cumulative density of P&R users' choices at this section. It can be found that the P&R users make the transfers between sections 7 and 12, and beyond this transfer interval, the P&R facilities are never used. Moreover, the mostly used transfer point is at section 11, and the amounts of P&R users decrease from section 11 towards both sides. This information is significantly useful for the transport planners to determine the optimal amount (how many), location (where) and capacity (size) of the P&R facilities. For example, in this case study, the P&R facilities should be built between sections 7 and 12, and the parking capacities should be determined based on the user densities shown on this figure.

4.4.5 Urban density distribution

This section aims to show how the model formulation can be used for integrating urban planning with transportation planning. As is well known, urban density distribution affects the transportation system performance and planning significantly. Therefore, it is imperative for the urban planners and transportation planners to understand how urban density affects the traffic pattern and what urban density distribution profile is preferred in terms of ensuring a transport system with best performance. The urban density distribution is relaxed as variant from being fixed and given, to find which density distribution profile can achieve a best transportation performance.

For simplicity, the road capacities of all corridor sections, as well as the commuter densities, are assumed to be identical as in the previous numerical experiments. For application in reality, it is easy to relax these assumptions in the real situation. Generally, road capacity is larger at city center area than at suburbs due to heavier traffic at city center; similarly, more residents stay close to city center than suburbs for convenience. Based on the 20 sections case, assume $C_i = 3000$ veh/h for $i = 1, 2, \dots, 6$, $C_i = 4000$ veh/h for $i = 7, 8, \dots, 14$ and $C_i = 5000$ veh/h for $i = 15, 16, \dots, 20$. Three cases of urban density distribution pattern are considered, i.e., uniformly, linear and negative exponential distribution, as schematically shown in Figure 4.9.

Figure 4.9 Three different distributions of urban density

To ascertain fair comparison, the total commuters along the corridor is kept identical for these three different distributions. Other than the uniform distribution as set in the previous numerical experiments, the linear distribution function is assumed to be $q_i^0 = q_1 + (i-1)\eta$ for $i=1,2,\dots,v$, where $\eta = 21.05$ is a parameter and $q_1 = 400$ commuters/h/km is the urban density at section 1. The negative exponential distribution function is assumed to be $q_i^0 = q_v e^{-\phi(v-i)\frac{L}{v}}$, where $\phi = 0.057$ is a parameter and $q_v = 1000$ commuters/h/km is the commuter density at section v .

The model results on equilibrium travel pattern with different urban density distributions are demonstrated in Table 4.3. One can find that different density distributions lead to different modal split travel patterns. More importantly, the total generalized cost of commuters in the case of negative exponential distribution is the lowest and the uniform distribution results in the highest total generalized cost. This has important policy implications for urban planners and transport planners in determining the optimal urban form and urban density distribution profile to ensure a more efficient transport system. From this numerical experiment, one can find that, with assumption of same total urban population, negative exponential distribution of urban density is more favorable than the other two alternatives in terms of ensuring better transportation system performance. With this model as the basic formulation framework and platform, one can easily further explore that what

parameter settings (ϕ and q_v) for negative exponential distribution should be to allow for best transportation performance.

Table 4.3 Modal split patterns with different commuter density distributions (commuters/h/km)

Section	Uniform dist.			Linear dist.			Negative exponential dist.		
	Auto user	Railway user	P&R user	Auto user	Railway user	P&R user	Auto user	Railway user	P&R user
1	0	0	600	0	0	400	0	0	319.8
2	0	0	600	0	0	421.1	0	0	339.6
3	0	0	600	0	0	442.1	0	0	360.6
4	0	0	600	0	0	463.2	0	0	382.9
5	0	0	600	0	0	484.2	0	0	406.6
6	0	0	600	0	0	505.3	0	0	431.7
7	0	0	600	0	0	526.3	0	0	458.4
8	0	0	600	0	0	547.4	0	0	486.8
9	0	0	600	0	0	568.4	0	0	516.9
10	0	0	600	0	0	589.5	0	0	548.8
11	0	0	600	0	0	610.5	0	0	582.7
12	294.4	0	305.6	62.4	0	569.2	18	0	600.8
13	600	0	0	652.6	0	0	657	0	0
14	562.5	37.5	0	669.4	4.3	0	697.7	0	0
15	0	600	0	0	694.7	0	0	740.8	0
16	0	600	0	0	715.8	0	0	786.6	0
17	0	600	0	0	736.8	0	0	835.3	0
18	0	600	0	0	757.9	0	0	886.9	0
19	0	600	0	0	778.9	0	0	941.8	0
20	0	600	0	0	800	0	0	1000	0
Total cost	214610 S\$			201590 S\$			192760 S\$

Figure 4.10 Minimum generalized costs with different commuter density distributions

Previous analyses have shown that negative exponential distribution leads to the lowest total generalized cost, and uniform distribution results in highest. Figure 4.10 compares the minimum equilibrium generalized costs at each section along the corridor with different urban density distributions. It can be found that between sections 15 to 20, all three cases have similar minimum generalized costs, while from section 14 to the boundary, the minimum generalized cost of negative exponential distribution is obviously the lowest at each location and the uniform distribution is the highest. This provides useful policy implications to urban planners that locating urban population more intensively in the city center makes the entire transportation system performs better.

4.4.6 Heterogeneous case

In this section, heterogeneous commuters is considered in the numerical study by segregating the commuters into different groups with different punctuality probability requirement ρ_g and VOT τ_g . For illustration purpose, suppose there are $G=4$ commuter groups with different parameter values of ρ_g and τ_g , such as:

$$g \in \{ \{ \rho_1 = 95\%, \tau_1 = 0.5 \}, \{ \rho_2 = 80\%, \tau_2 = 0.5 \}, \{ \rho_3 = 95\%, \tau_3 = 0.3 \}, \{ \rho_4 = 80\%, \tau_4 = 0.3 \} \},$$

and the travel demand density for each group is $q_g^{0,i} = 150$ for $i=1,2,\dots,v$, $g=1,2,\dots,G$. It should be noted that the heterogeneity parameter selection can be altered and the parameter setting is exogenously given. The sole purpose of this numerical study is to demonstrate the proposed model's capability of addressing heterogeneous commuters. All the other values of the parameters are kept identical with previous numerical settings.

Table 4.4 lists the modal split results among different commuter groups, as well as the total. In view of the space limit, the abbreviations 'A', 'R' and 'P' are used to represent 'Auto', 'Railway' and 'P&R' respectively in this table. Comparing group 1 ($\rho_1 = 95\%, \tau_1 = 0.5$) with group 2 ($\rho_2 = 80\%, \tau_2 = 0.5$), as well as group 3 ($\rho_3 = 95\%, \tau_3 = 0.3$) with group 4 ($\rho_4 = 80\%, \tau_4 = 0.3$), one can find that, for commuter groups with same VOT τ_g but different punctuality probability requirement ρ_g ,

more auto users switch to choose P&R mode with higher punctuality requirement. Similarly, from the cross-comparison of group 1 with group 3 and group 2 with group 4, one can see that commuter groups with lower VOT prefer railway because the generalized cost of railway transit service is characterized with larger time components, and therefore lower VOT leads to lower generalized cost and more users. Moreover, at low VOT $\tau_3, \tau_4 = 0.3$, although the punctuality probability requirements are different between groups 3 and 4, the modal split results are not quite different since lower VOT results in fewer contribution to the generalized cost, which affects the modal split result not significantly, while higher VOT will affect the modal split pattern more obviously, as the case of groups 1 and 2.

Table 4.4 Modal split patterns among different user groups (commuters/h/km)

Section	Group 1			Group 2			Group 3			Group 4		
	A	R	P	A	R	P	A	R	P	A	R	P
1	0	0	150	150	0	0	0	0	150	0	0	150
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
6	0	0	150	150	0	0	0	0	150	0	0	150
7	0	0	150	150	0	0	0	150	0	0	0	150
8	0	0	150	150	0	0	0	150	0	0	150	0
9	0	0	150	150	0	0	0	150	0	0	150	0
10	150	0	0	150	0	0	0	150	0	0	150	0
11	150	0	0	150	0	0	0	150	0	0	150	0
12	150	0	0	150	0	0	0	150	0	0	150	0
13	0	150	0	0	150	0	0	150	0	0	150	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	0	150	0	0	150	0	0	150	0	0	150	0

Figure 4.11 Minimum generalized cost among different commuter groups

For different commuter groups with heterogeneous parameters, the generalized costs at the achieved equilibrium status are different. The model result of the minimum generalized costs along the corridor among different commuter groups is shown in Figure 4.11. One can find that higher VOT leads to much higher minimum generalized cost on the basis of same punctuality probability requirement, meanwhile, with same VOT, higher punctuality probability requirement results in higher minimum generalized cost, but only marginally. It is not difficult to interpret this result as the changing of punctuality probability requirement only affects a few commuters who shift from one travel mode to another, and this change contributes insignificantly to the total generalized cost; while the varying of VOT influences most of the parts in the generalized cost functions in Eqs. (4-21) - (4-23), which leads to notable change of the minimum generalized cost.

These extensive numerical examples show that the proposed model is able to formulate and solve the problem with heterogeneous commuters, and one can apply this model to analyze the spatial equilibrium travel pattern along the travel corridor with multi-class commuters.

4.5 Summary

In this chapter, a linear travel corridor with competitive railway and highway systems and multiple P&R services is considered, and the spatial equilibrium travel pattern with multimodal choice in the transportation system is formulated into a linear complementarity system, which is then transformed into a complementarity formulation by a discrete approximation method to be solved. The continuum model is able to incorporate commuter heterogeneity and travel time reliability, as well as travel time correlation among adjacent locations. The solution existence of the formulation is proven by using complementarity theory. A set of numerical examples are conducted to demonstrate the validity of the model formulation and its potential applications.

This model can be used for the initial phase of planning and modeling of regional transportation study, wherein there is usually insufficient data of the system for setting up a dense network for detailed analysis using the discrete modeling approach. During the initial phase of planning, the general trend and pattern of the

distribution and travel choices of users are the main focuses. Specifically, in this chapter, the model can be used to provide transportation planners with sufficient information on how to optimally deploy the P&R facilities and how to change policies to promote the P&R service usage. In the numerical studies, it is shown how the model results are used to determine the optimal number (how many), location (where) and capacity (size) of P&R facilities. Land use policies, like urban form and urban density distribution profiles, are also examined through the model results to provide policy indications on how to integrate urban development into P&R facilities planning. Numerical results also demonstrate the effect of different user groups on equilibrium travel pattern and travel cost; and the effect of taking into account travel time reliability on commuters' modal choice and spatial equilibrium travel pattern. The numerical results confirm the benefit of the developed model. In summary, the contributions of this chapter to the literature are in three aspects: heterogeneity is explicitly taken into account with rigorous mathematical formulation when modeling and solving continuous spatial equilibrium travel pattern with multimodal choice; travel time reliability, as well as the correlation among adjacent locations on highway, is considered to make the model more realistic; potential applications are demonstrated based on comprehensive numerical studies.

However, there are still some limitations. For example, in modeling travel time on highway system, a first order BPR function is assumed to simplify the problem. The consideration of travel time reliability on highway demands extensive data to describe the travel time correlation more accurately. The proposed model is also limited to static equilibrium, while dynamic elements of traffic flow dynamics and departure time choice are not incorporated yet. For large scale problem, Lemke's algorithm sometimes fails to find the optimal solution, therefore some other algorithms for large scale problem is the direction of further research.

CHAPTER 5

MODELING INSTANTANEOUS DYNAMIC USER EQUILIBRIUM ALONG A TRAVEL CORRIDOR

(Note: this chapter includes part of contents that has been submitted in the paper:

Wang, Z.W. and Du, B. "Continuum modeling of spatial and dynamic equilibrium in a linear travel corridor with heterogeneous commuters - a partial differential complementarity system approach", Transportation Research Part B, under revision.)

5.1 Introduction

Since Newell (1988) extended the bottleneck problem to a travel corridor case in terms of the L-W-R traffic flow regulation, many studies and achievements have been made on the dynamic corridor problem, which covers two major research fields in transport: the dynamics equilibrium and morning commute problems. Comprehensive literature reviews on these two fields can be referred to Section 2.3, among which, Arnott and DePalma (2011); DePalma and Arnott (2012) conducted a series of rigorous mathematical analysis, which contributed to this research field with solid foundation. However, there exist some restrictions and strict assumptions on the analytical results, for example when analyzing corridor problem, no late arrival is allowed, heterogeneous commuters are not considered, and the numerical results with Greenshields function are based on the population must be zero some finite distance before CBD, moreover, the solution cannot be guaranteed with arbitrary population density. All these assumptions and limitations deserve further improvements.

In this chapter, a dynamic equilibrium problem on a linear travel corridor with many-to-one OD pattern and heterogeneous commuters is considered. It is assumed that various travel demand distributions can be given exogenously, and all commuters make choice of the departure time considering minimizing travel cost to

go to CBD for work, which is the unique destination at the end of the corridor. Moreover, the preferred arrival time is given, and both early arrival and late arrival are allowed but with proper penalties. A special case of DVI is applied to model this corridor problem.

This chapter contributes to the literature in two major aspects. Firstly, heterogeneous commuters with different preferences and late arrival are considered in the corridor problem, which makes the model more reasonable and wider application in practice. Secondly, various travel demand distributions can be applied to the model, and meaningful numerical results can be obtained to demonstrate the validity of the model.

5.2 Model formulation

5.2.1 Equivalence among L-W-R, M-N and CTM models in discrete scheme

In this section, a short discussion is given first to prove that for a linear corridor problem, the continuous L-W-R flow-density relation can be transformed to an equivalent approximated M-N or CTM model in discrete scheme.

Corollary 5.1 *For a linear corridor problem, the flow propagation equations, both (2-20) in M-N model and (2-23) in CTM model, are equivalent discrete approximations of the continuous L-W-R flow-density relation (2-19).*

Proof. As a special topology structure of general traffic network, a linear corridor is an arterial road connected the suburb to city center. Therefore, in the discretized scheme as previous contents, the linear corridor can be regarded as a single road divided into a sequence of sections/cells with equal length, and no diverging or merging traffic flows need to be considered. In this case, the continuous L-W-R flow-density relation (2-19) can be approximated as follows.

$$\frac{\partial k}{\partial t} = \frac{k_{t+1}^i - k_t^i}{\Delta t} \quad (5-1)$$

$$\frac{\partial f}{\partial x} = \frac{f_t^i - f_t^{i-1}}{\Delta x} \quad (5-2)$$

Let x_t^i represent $\frac{k_t^i}{\Delta t}$, the number of vehicles in cell i at the beginning of the time interval t , e_t^i denote $\frac{f_t^i}{\Delta x}$, the number of vehicles flows out from cell i in time interval t , and r_t^i represent the number of departures at the beginning of the time interval t in cell i . In the discrete scheme above, Eq. (2-19) can be transformed to the flow propagation equation as follows.

$$x_{t+1}^i - x_t^i + e_t^i - e_t^{i-1} = r_t^i \quad (5-3)$$

Note that e_t^{i-1} can be regarded as the exit flow from cell $i-1$ in time interval t , which is also the inflow to cell i in time interval t , and the total number of vehicles entering cell i in time interval t includes the exit flow from last cell and new departures in this cell, denoted as h_t^i as follows.

$$h_t^i = r_t^i + e_t^{i-1} \quad (5-4)$$

Substitute Eq. (5-4) into Eq. (5-3) to get a simple case (no diverging or merging traffic flow) of the cell and node conservation equation (2-20) in the M-N model, which is an equivalent discrete approximation of Eq. (2-19), and the exit flow satisfies the exit flow function (2-21).

Moreover, if the external input flow r_t^i is not considered, and an additional constraint $Q_t^i - x_t^i$, the amount of empty space in cell i in time interval t , is added to the exit flow function (2-21), where Q_t^i represents the maximum number of vehicles can flow into cell i in time interval t , then Eq. (5-3) are described as

$$\begin{aligned} x_{t+1}^i &= x_t^i - e_t^i + e_t^{i-1} \\ e_t^{i-1} &= \min\{x_t^{i-1}, C_t^i, Q_t^i - x_t^i\} \end{aligned} \quad (5-5)$$

which are identical with (2-23) in the CTM model (note that in (2-23), e_t^i represents the flow from last cell into cell i at time interval t). This concludes the whole proof.

□

5.2.2 Continuum modeling of spatial and dynamic equilibrium travel pattern

In this section, an instantaneous dynamic user equilibrium (IDUE) problem on a linear travel corridor with many-to-one OD pattern and heterogeneous commuters is

considered. It is assumed that the distribution of the travel demand along the corridor is given but can be arbitrary, and all commuters make choice of the departure time considering minimizing instantaneous travel cost to go to CBD for work, which is the unique destination at the end of the corridor. Note that all the variables and parameters in this chapter are nonnegative. Suppose the study horizon is $\bar{T}$ and the length of corridor is L , then this problem can be formulated as a continuous-spatial-temporal based dynamic model.

For $\forall x \in [0, L]$, $\forall t \in [0, \bar{T}]$, the overall continuous IDUE model is as below:

$$\frac{\partial f_t^x}{\partial x} + \frac{\partial k_t^x}{\partial t} = r_t^x \quad (5-6)$$

$$f_t^x = k_t^x u_t^x(k_t^x) \quad (5-7)$$

$$[IDUE] \quad u_t^x(k_t^x) = -a_t^x k_t^x + b_t^x \quad (5-8)$$

$$\frac{\partial TT_t^x}{\partial x} = \frac{1}{u_t^x(k_t^x)} \quad (5-9)$$

$$\int_0^{\bar{T}} r_w^x dw = D_x \quad (5-10)$$

$$0 \leq r_t^x \perp \alpha TT_t^x + \beta \max(0, t^* - t - TT_t^x) + \gamma \max(0, t + TT_t^x - t^*) - \pi^{x,*} \geq 0 \quad (5-11)$$

where Eq. (5-6) is flow conservation equation (equation of continuity) with traffic density k_t^x (number of vehicle per unit distance), traffic flow f_t^x (number of vehicle per unit time) and departure rate r_t^x at location x and time t , which is the key postulate of L-W-R theory. Eqs. (5-7) and (5-8) denotes the flow-velocity-density relationship, and here for simplicity, the Greenshields function, $u(k) = u^{\max} (1 - \frac{k}{k^{\max}})$, is rewritten in a succinct form as: $u(k) = -ak + b$, where $a = u^{\max} / k^{\max} > 0$ and $b = u^{\max} > 0$. Eq. (5-9) reflects the spatial interaction of the travel time along the corridor, where TT_t^x represents the travel time for the commuters departing from location x at time t to CBD. Eq. (5-10) and Eq. (5-11) are DUE conditions, where Eq. (5-10) is the travel demand satisfaction condition and $\pi^{x,*}$ is the minimum cost, also the equilibrium cost, for commuters departing from location x , D_x is the travel demand at location x . Eq. (5-11) guarantees that whenever a departure happens, it generates the minimum cost. Moreover, $\max(0, t^* - t - TT_t^x)$ and $\max(0, t + TT_t^x - t^*)$ represent the penalties of early and late arrival respectively, and α, β, γ denote the unit VOTs for travel time, early arriving time and late arriving time respectively.

It is reasonable to assume that $\gamma > \alpha > \beta$. Moreover, considering that the population density, as well as the travel demand, usually declines as distance to CBD increases in reality, negative exponential function is applied here to describe the distribution of the travel demand as below, and many other distribution functions can also be applied in this research.

$$D^x = D^0 e^{-\phi(L-x)} \quad (5-12)$$

where D^0 is the travel demand at CBD, and ϕ is a parameter.

Substitute Eq. (5-8) into Eq. (5-7) and Eq. (5-9) respectively, and reformulate Eq. (5-7) into a complementarity condition as follows:

$$0 \leq k_t^x \perp -a_t^x (k_t^x)^2 + b_t^x k_t^x - f_t^x \geq 0 \quad (5-13)$$

It is easy to observe that if $k_t^x > 0$, then $f_t^x = -a_t^x (k_t^x)^2 + b_t^x k_t^x$ obviously; if $k_t^x = 0$, then the right side of (5-13) reduces to $-f_t^x \geq 0$, thus $f_t^x = 0$, so it still have $f_t^x = -a_t^x (k_t^x)^2 + b_t^x k_t^x$. Therefore, the complementarity condition (5-13) is equivalent to Eq. (5-7) combined with Eq. (5-8). Now, the original continuous instantaneous dynamic user equilibrium model can be expressed as a partial differential complementarity system (PDCS) as follows:

$$\frac{\partial f_t^x}{\partial x} + \frac{\partial k_t^x}{\partial t} = r_t^x \quad (5-14)$$

$$\frac{\partial TT_t^x}{\partial x} = \frac{1}{-a_t^x k_t^x + b_t^x} \quad (5-15)$$

$$[\text{PDCS}] \quad 0 \leq k_t^x \perp -a_t^x (k_t^x)^2 + b_t^x k_t^x - f_t^x \geq 0 \quad (5-16)$$

$$0 \leq \pi^{x,*} \perp \int_0^{\bar{T}} r_w^x dw - D^0 e^{-\phi(L-x)} \geq 0 \quad (5-17)$$

$$0 \leq r_t^x \perp \alpha TT_t^x + \beta \max(0, t^* - t - TT_t^x) + \gamma \max(0, t + TT_t^x - t^*) - \pi^{x,*} \geq 0 \quad (5-18)$$

5.2.3 Equivalent approximation complementarity problem

Due to the difficulty of solving partial differential equations (PDEs) in Eqs. (5-14) and (5-15), they are transformed to a set of equivalent equations, which are discrete approximation to the PDEs. By the discrete approximation method, the differential function in Eqs. (5-14) and (5-15), and $\max$ function in complementarity condition (5-18) can also be solved.

To present the equivalent discrete model, the variables and parameters used throughout this chapter are presented first for better understanding, and note that all variables in this chapter are nonnegative.

Table 5.1 List of notations and descriptions

Notation	Description
Indices	
t	Index of time intervals, $t \in \{0, 1, \dots, v\}$
i	Index of cells, $i \in \{1, 2, \dots, n\}$
g	Index of commuter groups, $g \in \mathbb{G} \triangleq \{1, \dots, G\}$
Variables	
$f_{t,g}^i$	Traffic flow from group g in cell i at the beginning of time interval t
$k_{t,g}^i$	Traffic density from group g in cell i in time interval t
$r_{t,g}^i$	Number of vehicles from group g departing from cell i at the beginning of time interval t
TT_t^i	Travel time for commuters departing in time interval t from cell i
$\pi_g^{i,*}$	Equilibrium cost for commuters from group g departing from cell i
$u_{t,g}^i$	Number of vehicles from group g entering cell i in time interval t
$\theta_{t,g}^i$	Auxiliary variables for travel cost computation for group g in cell i in time interval t
Parameters	
L	Length of the corridor
n	Total number of cells
v	Total number of time intervals
G	Total number of commuter groups
D_g^0	Total travel demand along the corridor for commuters from group g
D_g^i	Travel demand for commuters from group g departing from cell i
$q_{0,g}^i$	Initial traffic volume from group g in cell i
α_g	Unit cost of travel time for group g
β_g	Unit cost of arriving early to work for group g
γ_g	Unit cost of arriving late to work for group g
t_g^*	Preferred arrival time for group g
a_t^i	Parameter of the velocity-density function
b_t^i	Parameter to the velocity-density function
ϕ_g	Parameter of the negative exponential population density function for group g

In this section, the study horizon is divided into v subintervals indexed by

$t \in \{0, 1, \dots, v\}$ with equal time length $\Delta t = \frac{\bar{T}}{v}$, and the corridor is divided into n cells

indexed by $i \in \{1, 2, \dots, n\}$ with uniform length $\Delta x = \frac{L}{n}$. Heterogeneity across

commuters is considered in this study mainly on the commuting demand, preferred arrival time, unit cost of travel time and unit penalty costs of early and late arrivals. In particular, the commuters are classified into several groups indexed by $g \in \mathbb{G} \triangleq \{1, \dots, G\}$. Commuters can arrive at workplaces earlier or later than their preferred arrival time t_g^* subjected to different penalty costs, but they are supposed to depart no later than t_g^* . As shown in Figure 5.1, the sequence of cells 1 to n represents the direction from the boundary of the corridor to CBD, the distance between cell n and CBD is Δx , and all commuters along the corridor go to CBD (sink cell / cell $n+1$) for work.

Figure 5.1 A corridor with many-to-one OD pattern

(1) *Equivalent complementarity conditions*

Consider that heterogeneous commuters with different characteristics coexist along the corridor, and the heterogeneity refers to different VOTs for travel time, early arrival time and late arrival time, as well as various population density distributions, the L-W-R flow conservation Eq. (5-14) is not suitable to determine the individual flow for each commuter group. In the discretized scheme, in order to get the flow of each commuter group $f_{t,g}^i$ among the total exit flow $\sum_{g=1}^G f_{t,g}^i$, a proportional allocation procedure is used to determine the flow fraction as follows:

$$f_{t,g}^i = \begin{cases} \frac{\sum_{g=1}^G f_{t,g}^i \cdot \frac{k_{t,g}^i}{\sum_{g=1}^G k_{t,g}^i}}{\sum_{g=1}^G k_{t,g}^i}, & \text{if } \sum_{g=1}^G k_{t,g}^i > 0 \\ 0, & \text{otherwise} \end{cases} \quad (5-19)$$

In discrete scheme, the partial differential equation (5-14) can be expressed as $\frac{\partial k_t^x}{\partial t} = \frac{k_{t+1}^i - k_t^i}{\Delta t}$, $\frac{\partial f_t^x}{\partial x} = \frac{f_t^i - f_{t-1}^i}{\Delta x}$. Thus, for each group user g , Eq. (5-14) can be approximated in a discretized form as:

$$\frac{f_{t,g}^i - f_{t,g}^{i-1}}{\Delta x} + \frac{k_{t+1,g}^i - k_{t,g}^i}{\Delta t} = r_{t,g}^i \quad (5-20)$$

Here, $f_{t,g}^i$ could be treated as the traffic flow of group g in cell i in time interval t , and $k_{t,g}^i$ as the exit flow of group g from cell i to $i+1$ in time interval t .

As for condition (5-16) in equation form, it regulates the total flow in an aggregated form:

$$\sum_{g=1}^G f_{t,g}^i = -a_t^i \left(\sum_{g=1}^G k_{t,g}^i \right)^2 + b_t^i \sum_{g=1}^G k_{t,g}^i \quad (5-21)$$

Based on the relation (5-19), Eq. (5-21) can be converted to a simplified form as follows:

$$f_{t,g}^i = -a_t^i k_{t,g}^i \sum_{g=1}^G k_{t,g}^i + b_t^i k_{t,g}^i \quad (5-22)$$

In addition, Eq. (5-15) can also be converted to an equivalent complementarity condition as follows:

$$0 \leq TT_t^i \perp TT_t^i - TT_t^{i+1} - \frac{\Delta x}{-a_t^i \sum_{g=1}^G k_{t,g}^i + b_t^i} \geq 0 \quad (5-23)$$

If $TT_t^i > 0$, then the right side of (5-23) must be zero, thus

$$\frac{TT_t^i - TT_t^{i+1}}{\Delta x} = \frac{1}{-a_t^i \sum_{g=1}^G k_{t,g}^i + b_t^i}; \text{ otherwise if } TT_t^i = 0, \text{ then the right side of (5-23)}$$

reduces to $-TT_t^{i+1} - \frac{\Delta x}{-a_t^i \sum_{g=1}^G k_{t,g}^i + b_t^i}$. As stated in previous content, since all the

variables and parameters in this chapter are nonnegative, it is easy to derive that $-a_t^i (k_t^i)^2 + b_t^i k_t^i \geq 0$ from complementarity condition (5-16). Therefore it has

$TT_t^{i+1} \geq 0$ and $\frac{\Delta x}{-a_t^i \sum_{g=1}^G k_{t,g}^i + b_t^i} > 0$, thus the right side of (5-23) is

$-TT_t^{i+1} - \frac{\Delta x}{-a_t^i \sum_{g=1}^G k_{t,g}^i + b_t^i} < 0$, which breaks the complementarity rule, therefore it must

have $TT_t^i > 0$ and $\frac{TT_t^i - TT_t^{i+1}}{\Delta x} = \frac{1}{-a_t^i \sum_{g=1}^G k_{t,g}^i + b_t^i}$, which is equivalent to the travel time

function (5-15).

The complementarity condition (5-17) in a discrete form is shown as below.

$$0 \leq \pi_g^{i,*} \perp \sum_{t=0}^{v-1} r_{t,g}^i - D_g^0 e^{-\phi_g(n-i)} \geq 0 \quad (5-24)$$

Moreover, complementarity conditions also can be used to represent *max* function by adding a nonnegative auxiliary variable $\theta_{t,g}^i$, then (5-18) can be expressed as follows:

$$0 \leq r_{t,g}^i \perp \alpha_g TT_t^i + \beta_g \theta_{t,g}^i + \gamma_g [\theta_{t,g}^i - (t_g^* - t - TT_t^i)] - \pi_g^{i,*} \geq 0 \quad (5-25)$$

$$0 \leq \theta_{t,g}^i \perp \theta_{t,g}^i - (t_g^* - t - TT_t^i) \geq 0 \quad (5-26)$$

(2) The overall complementarity model

Similarly as was done in Han et al. (2011), traffic flow $f_{t,g}^i$ in cell i at the beginning of time interval t can be written as an implicit function of departure rates $\mathbf{r} = (r_{t,g}^i)_{t=1,\dots,v, i=1,\dots,n, g=1,\dots,G}$, denoted by $f_{t,g}^i(\mathbf{r})$. The overall [PDCS] model with many-to-one OD pattern on a linear travel corridor can be presented in standard complementarity problem (CP) form, $CP(q, M)$, to find a vector of variables $\mathbf{z} = \{r_{t,g}^i, k_{t,g}^i, \theta_{t,g}^i, TT_t^i, \pi_g^{i,*} |_{t=0,1,\dots,v-1, i=1,\dots,n; g=1,\dots,G}\}$ to satisfy

$$0 \leq \mathbf{z} \perp F(\mathbf{z}) \geq 0 \quad (5-27)$$

Specifically, the [PDCS] model can be transformed to a CP formulation as follows:

$$\begin{aligned} & 0 \leq TT_t^i \perp TT_t^i - TT_t^{i+1} - \frac{\Delta x}{-a_t^i \sum_{g=1}^G k_{t,g}^i + b_t^i} \geq 0 \\ & 0 \leq k_{t,g}^i \perp -a_t^i k_{t,g}^i \sum_{g=1}^G k_{t,g}^i + b_t^i k_{t,g}^i - f_{t,g}^i(r_{t,g}^i) \geq 0 \\ & 0 \leq \pi_g^{i,*} \perp \sum_{t=0}^{v-1} r_{t,g}^i - D_g^0 e^{-\phi_g(n-i)} \geq 0 \\ & 0 \leq r_{t,g}^i \perp (\alpha_g + \gamma_g) TT_t^i + (\beta_g + \gamma_g) \theta_{t,g}^i - \pi_g^{i,*} + \gamma_g (t - t_g^*) \geq 0 \\ & 0 \leq \theta_{t,g}^i \perp \theta_{t,g}^i + TT_t^i + t - t_g^* \geq 0 \\ & \forall i = 1, \dots, n, t = 0, 1, \dots, v-1, g = 1, \dots, G \end{aligned} \quad [\text{CP}] \quad (5-28)$$

where the initial traffic volume in cell i at the beginning of time $t=0$ is $k_{0,g}^i$, which is given, and the boundary condition, $TT_t^{n+1}=0$, guarantees that travel time at CBD is zero.

5.2.4 Solution existence of the formulations

Theorem 5.1 (Facchinei and Pang, 2003) *Let K be a closed convex cone in R^n ; Let F be a continuous map from K into R^n . If there exists a copositive matrix $E \in R^{n \times n}$ on K such that (K, E) is an R_0 pair and the union $\bigcup_{\tau > 0} SOL(K, F + \tau E)$ is bounded, then the $CP(K, F)$ has a solution.*

The proof of Theorem 5.1 can be referred to Facchinei and Pang (2003).

Lemma 5.1 as below is given to describe the solution existence of [CP] (5-28) according to Theorem 5.1, which states that the original CP has a solution if the solutions are uniformly bounded.

Lemma 5.1 *Let $\Phi: R^n \rightarrow R^n$ be a continuous function. If there exists a constant $d > 0$ such that all the solutions of CP: $0 \leq z \perp F(z) + \tau z \geq 0, \forall \tau > 0$ satisfy $\|z\| \leq d$, then the CP (5-28): $0 \leq z \perp F(z) \geq 0$ has a solution.*

Proof. Refer to APPENDIX B.

So far the solution existence of [CP] (5-28) has been given, however, it is difficult to show the solution uniqueness. If $F(z)$ is a strongly monotone and continuous function on $\mathcal{Z}$, then a unique solution is guaranteed. Such uniqueness is not hard to prove for simple CP models, but it is usually difficult to prove the monotonicity and continuity properties for complex models.

5.3 Numerical results

In this section, a set of numerical results are presented with an idealized linear corridor for three different scenarios: homogeneous commuters without late arrival, homogeneous commuters with late arrival and heterogeneous commuters. A corridor with length of 10km is adopted here for illustration, which connects the city center with suburbs. The corridor is divided into 10 sections, and the sequence of sections 1 to 10 represents the direction from the boundary of the corridor to

CBD. The study time horizon is set between 8:00am to 8:40am, which is divided into 100 intervals with each time interval of 0.4min. The free flow speed is set at 60km/h and the maximum road capacity is given at 150 veh/min.

5.3.1 Scenario I: no late arrival

In this scenario, homogeneous users are assumed uniformly distributed along the corridor with travel demand 150 veh at each section, and the preferred arrival time is set at 8:32am, the 80th time interval. No late arrival is allowed for all commuters along the corridor, and the unit VOT for travel time and early arrival time are set at S\$1 and S\$0.05 respectively. Figure 5.2 and Table 5.2 show the departure rate over time at each location along the corridor. Generally, from the boundary of the corridor to CBD, the commuters living further away from CBD set out earlier than the ones living closer to CBD since more travel time is consumed to achieve the workplace, and the departure time range decreases from the boundary to CBD. Another interesting finding is that, except the last section 10, the departure rate at each location is constant at most of the departure time range (shaded area in Table 5.2), mainly at the middle of the departure time range rather than the head and tail. This result is partially coincident with the conclusion in Arnott and DePalma (2011) that the departure rate is constant at each location along the corridor based on Greenshields relation and zero population some finite distance before CBD.

Figure 5.2 Departure rate over time at each location

Table 5.2 Departure rate over time at each location

Time	Location									
	1	2	3	4	5	6	7	8	9	10
23	0	0	0	0	0	0	0	0	0	0
24	4.651	0	0	0	0	0	0	0	0	0
25	4.659	14.262	0	0	0	0	0	0	0	0
26	2.930	3.222	12.597	0	0	0	0	0	0	0
27	2.930	3.230	6.993	5.387	0	0	0	0	0	0
28	2.930	3.230	3.097	8.930	0	0	0	0	0	0
29	2.930	3.230	3.097	8.607	0	0	0	0	0	0
30	2.930	3.230	3.097	3.156	5.754	0	0	0	0	0
31	2.930	3.230	3.097	3.147	5.435	0	0	0	0	0
32	2.930	3.230	3.097	3.147	5.303	0	0	0	0	0
33	2.930	3.230	3.097	3.147	5.231	0	0	0	0	0
34	2.930	3.230	3.097	3.147	3.084	2.263	0	0	0	0
35	2.930	3.230	3.097	3.147	2.886	2.433	0	0	0	0
36	2.930	3.230	3.097	3.147	2.886	2.558	0	0	0	0
37	2.930	3.230	3.097	3.147	2.886	2.863	0	0	0	0
38	2.930	3.230	3.097	3.147	2.886	3.355	0	0	0	0
39	2.930	3.230	3.097	3.147	2.886	4.021	0	0	0	0
40	2.930	3.230	3.097	3.147	2.886	4.828	0	0	0	0
41	2.930	3.230	3.097	3.147	2.886	5.056	0.711	0	0	0
42	2.930	3.230	3.097	3.147	2.886	3.620	3.162	0	0	0
43	2.930	3.230	3.097	3.147	2.886	3.620	4.090	0	0	0
44	2.930	3.230	3.097	3.147	2.886	3.620	4.997	0	0	0
45	2.930	3.230	3.097	3.147	2.886	3.620	5.867	0	0	0
46	2.930	3.230	3.097	3.147	2.886	3.620	6.708	0	0	0
47	2.930	3.230	3.097	3.147	2.886	3.620	3.860	3.928	0	0
48	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.664	0	0
49	2.930	3.230	3.097	3.147	2.886	3.620	3.860	5.412	0	0
50	2.930	3.230	3.097	3.147	2.886	3.620	3.860	6.216	0	0
51	2.930	3.230	3.097	3.147	2.886	3.620	3.860	7.054	0	0
52	2.930	3.230	3.097	3.147	2.886	3.620	3.860	7.895	0	0
53	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	4.724	0
54	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	5.399	0
55	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	6.300	0
56	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	7.224	0
57	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	8.100	0
58	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	8.906	0
59	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	9.634	0
60	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	10.288	0
61	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	5.790	6.018
62	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	5.790	7.206
63	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	5.790	8.121
64	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	5.790	8.809
65	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	5.790	9.310
66	2.930	3.230	3.097	3.147	2.886	3.620	3.860	4.427	5.790	9.656
67	20.575	3.244	3.097	3.147	2.886	3.620	3.860	4.427	5.790	9.873
68	0	0	6.513	3.147	2.886	3.620	3.860	4.427	5.790	9.982
69	0	0	0	4.348	2.886	3.620	3.860	4.427	5.790	10.002
70	0	0	0	0	2.147	3.620	3.860	4.427	5.790	9.947
71	0	0	0	0	22.024	3.620	3.860	4.427	5.790	9.829
72	0	0	0	0	0	14.022	3.860	4.427	5.790	9.660
73	0	0	0	0	0	0	24.103	26.289	10.730	17.868
74	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	8.571
76	0	0	0	0	0	0	0	0	9.215	8.617
77	0	0	0	0	0	0	0	0	0	6.529
78	0	0	0	0	0	0	0	0	0	0
79	0	0	0	0	0	0	0	0	0	0
$80(t^*)$	0	0	0	0	0	0	0	0	0	0

Figure 5.3 illustrates the cumulative departure over time at each location along the corridor. For people living close to the boundary of the corridor, the first departure happens earlier than those living close to CBD, so does the last departure. Moreover, from the boundary of the corridor to CBD, the departure window becomes narrower.

Figure 5.3 Cumulative departure at each location over time along the corridor

Figure 5.4 Traffic density at each location over time along the corridor

Figure 5.4 and Figure 5.5 illustrate the traffic density and flow changing over time at each location along the corridor respectively. In a general trend, as time elapses at each location along the corridor, both traffic density and flow increase at beginning and then keep constant for a certain time range due to the constant departure rate as showed at previous result, after that both traffic density and flow drop to zero as

flow propagates downstream towards to CBD. It is easy to observe that the peaks of density and flow are higher at locations closer to CBD than those at further distances to CBD since flow propagates from upstream to downstream, therefore upstream traffic volume cumulates at downstream to a higher peak. Also based on the flow propagation rule, both density and flow raise from zero earlier at locations further distance to CBD than those closer to CBD, and drop to zero earlier as well.

Figure 5.5 Traffic flow at each location over time along the corridor

Figure 5.6 shows the travel time at each location over time along the corridor, and one can observe that at each location travel time raises as departure happens and then keeps a constant growth rate to a peak, after that it drops when commuters complete the trips to CBD.

Figure 5.6 Traffic time at each location over time along the corridor

Next, a comparison of the departure sets with different unit VOTs of travel time and early arrival time is shown in Figure 5.7. Departure set depicts the trajectory of the first and last departure time at each location along the corridor. For comparison, the unit VOT of travel time is kept unchanged at S\$1, while the unit VOT of early arrival time varies case by case, and their ratios are shown in the figure for better understanding.

Figure 5.7 Departure sets along the corridor with different ratios β/α

In Figure 5.7, ‘width’ at location 1 denotes the departure time range, and the number shows how many time intervals the time range covers, for example, at the first case $\beta/\alpha = 0.03$, the departure time range covers 58 time intervals at location 1. To illustrate how the ratio β/α affects the departure set, the unit early arrival penalty is varying to compare the different departure patterns. It is easy to find that as the ratio β/α raises, the width of departure set becomes narrower. Moreover, as the ratio β/α raises, the upper boundary trajectory (graphed with solid line) varies slightly, but the lower boundary trajectory (graphed with dashed line) raises higher and higher obviously, which means the first departure time is postponed after the unit early arrival penalty raises. It is reasonable to achieve this result since

commuters would like to depart earlier to avoid traffic congestion and minimize the travel cost if the unit early arrival penalty is low, while the unit early arrival penalty raises, earlier departure may cause higher cost, which makes less earlier departure and the lower boundary trajectory raising in Figure 5.7.

5.3.2 Scenario II: with late arrival

In this scenario, unit early arrival penalty is set at a higher value at S\$0.02, and late arrival is allowed to extend the no late corridor problem, and the unit VOT of late arrival time is set at S\$ 1.5. Moreover, negative exponential distribution function is considered here with the parameter $\phi = 0.1$, $D^0 = 150$, and travel demand drops from CBD to the corridor boundary. The equilibrium results over time at each location along the corridor are shown in Table 5.3. Compared to the scenario I with no late and uniform travel demand distribution scenario, the departure set is narrower and the departure time range is shorter at all locations along the corridor. There are two reasons to lead to this result. First of all, higher unit early arrival penalty and late arrival penalty allowed force the commuters to depart neither too early nor too late, which cut the departure time range. Second, negative exponential distribution leads to less travel demands at locations further away from CBD than those closer to CBD, which brings the locations close to CBD higher risk to generate congestion, corresponding with slow flow propagation, than those further away from CBD.

Table 5.3 Departure rate over time at each location

Time	Location									
	1	2	3	4	5	6	7	8	9	10
68	0	0	0	0	0	0	0	0	0	0
69	60.985	0	0	0	0	0	0	0	0	0
70	0	67.399	0.120	74.955	0	0	0	0	0	0
71	0	0	74.367	7.367	65.937	0	0	0	0	0
72	0	0	0	0	25.042	47.655	0	0	0	0
73	0	0	0	0	0	52.893	51.000	0	0	0
74	0	0	0	0	0	0	60.123	51.000	0	0
75	0	0	0	0	0	0	0	26.430	0	0
76	0	0	0	0	0	0	0	45.380	38.558	0
77	0	0	0	0	0	0	0	0	97.167	48.972
78	0	0	0	0	0	0	0	0	0	101.028
79	0	0	0	0	0	0	0	0	0	0
80(t^*)	0	0	0	0	0	0	0	0	0	0

Next, in order to figure out how travel demand distribution affects the departure pattern, different values of parameter ϕ are set as in Figure 5.8 for comparison.

Figure 5.8 Different travel demand distributions

Figure 5.9 Minimum costs with different travel demand distributions

Figure 5.9 compares the equilibrium costs along the corridor with different travel demand distributions. As the value of parameter ϕ raises, the travel demand at last section (section 10) keeps constant ($D^0 = 150$) but other travel demands drop along the corridor. It is easy to observe that as the value of ϕ raises, it results in lower travel demands along the corridor and corresponding lower equilibrium costs at all locations along the corridor.

Figure 5.10 compares the departure sets with different travel demands. As the value of ϕ raises, travel demand drops at all locations along the corridor, and the corresponding equilibrium departure set becomes narrower.

Figure 5.10 Departure sets with different travel demand distributions

Figure 5.11 Different types of travel demand distribution

To make a more reasonable analysis on the travel demand distribution, three different distribution types (uniform, liner and negative exponential distribution shown in Figure 5.11) with same total travel demand along the corridor is considered for further comparison. For simplicity, as in previous content, negative exponential distribution with parameter $\phi = 0.1$ and $D^0 = 150$ is still adopted here, which results in total travel demand 996.380 along the corridor. Therefore, for

uniform distribution, the number of commuters at each location is 99.638; for linear distribution $D^i = ai + b$, the parameters are set as $a = 17.2069$ and $b = 5$.

Figure 5.12 Minimum costs with different travel demand distributions

It is easy to observe that the negative exponential distribution results in lower equilibrium cost at each location than the other two distribution types in Figure 5.12. Compared to the uniform distribution, the linear distribution results in lower minimum cost at locations from the boundary to the middle of the corridor, while higher at the rest locations along the corridor. This result shows that negative exponential distribution is more reasonable and economical for commuters along the corridor. This result is identical with the static case in Section 4.4.5.

The departure sets and departure rates corresponding to different distribution types are shown in Figure 5.13 and Table 5.4 respectively. One can find that negative exponential distribution results in narrower departure set than the other two distribution types. Moreover, commuters distributed in exponential form usually depart later than the commuters with the other two types of distribution.

Figure 5.13 Departure sets with different distribution types

Table 5.4 Departure rates with different distribution types

	Time	Location									
		1	2	3	4	5	6	7	8	9	10
U n i f o r m	65	0	0	0	0	0	0	0	0	0	0
	66	0	0	49.650	60.381	0	0	0	0	0	0
	67	0	0	0	14.180	54.739	0	0	0	0	0
	68	0	0	0	0	0.994	51.000	0	0	0	0
	69	99.638	99.638	7.789	12.600	3.638	0	50.994	0	0	0
	70	0	0	42.199	12.477	3.756	0	38.046	0	0	0
	71	0	0	0	0	36.511	48.638	7.588	38.560	0	0
	72	0	0	0	0	0	0	0	28.825	0	0
	73	0	0	0	0	0	0	0	0.696	36.924	0
	74	0	0	0	0	0	0	0	31.557	29.594	0
	75	0	0	0	0	0	0	0	0	24.040	0
	76	0	0	0	0	0	0	0	0	9.081	38.894
L i n e a r	77	0	0	0	0	0	0	0	0	0	33.960
	78	0	0	0	0	0	0	0	0	0	26.785
	79	0	0	0	0	0	0	0	0	0	0
	66	0	0	0	0	0	0	0	0	0	0
	67	0	0	0	0	20.179	66.489	0	0	0	0
	68	0	0	0	0	0	14.290	53.071	0	0	0
	69	22.206	0	0	0	4.804	0	16.630	51.030	0	0
	70	0	39.413	0	0	4.831	0	0	16.205	51.014	0
	71	0	0	56.620	73.828	4.830	26.821	0	0	33.778	0
	72	0	0	0	0	56.390	0.640	0	0	18.671	43.942
	73	0	0	0	0	0	0	0	0	0	39.160
	74	0	0	0	0	0	0	55.746	0	0	34.760
E x p o n e n t i a l	75	0	0	0	0	0	0	0	75.419	7.522	30.473
	76	0	0	0	0	0	0	0	0	48.875	25.419
	77	0	0	0	0	0	0	0	0	0	3.314
	78	0	0	0	0	0	0	0	0	0	0
	68	0	0	0	0	0	0	0	0	0	0
	69	60.985	0	0	0	0	0	0	0	0	0
	70	0	67.399	0.120	74.955	0	0	0	0	0	0
	71	0	0	74.367	7.367	65.937	0	0	0	0	0
	72	0	0	0	0	25.042	47.655	0	0	0	0
	73	0	0	0	0	0	52.893	51.000	0	0	0
	74	0	0	0	0	0	0	60.123	51.000	0	0
	75	0	0	0	0	0	0	0	26.430	0	0
	76	0	0	0	0	0	0	0	45.380	38.558	0
	77	0	0	0	0	0	0	0	0	97.167	48.972
	78	0	0	0	0	0	0	0	0	0	101.028
	79	0	0	0	0	0	0	0	0	0	0

Moreover, it can be found in Figure 5.14 that uniform distribution results in earlier first departure at locations close to the boundary, while linear distribution results in earlier first departure at locations close to CBD. Negative exponential distribution generally leads to later first and last departures along the corridor.

Figure 5.14 Cumulative departures with different distribution types

5.3.3 Scenario III: heterogeneous commuters

In this scenario, the studying time horizon is divided into 40 time intervals and heterogeneity is considered with two group commuters for illustration. Users in group 1 have lower unit VOTs, higher travel demand and later preferred arrival time than those in group 2. The heterogeneous parameter values are given as:

$$\alpha_1 = 1 \text{ S\$ / min}, \beta_1 = 0.1 \text{ S\$ / min}, \gamma_1 = 1.6 \text{ S\$ / min}, D_1^0 = 80 \text{ veh}, \varphi_1 = 0.1, t_1^* = 36,$$

$$\alpha_2 = 1.3 \text{ S\$ / min}, \beta_2 = 0.3 \text{ S\$ / min}, \gamma_2 = 2.2 \text{ S\$ / min}, D_2^0 = 70 \text{ veh}, \varphi_2 = 0.06, t_2^* = 30.$$

Figure 5.15 Minimum costs along the corridor with different user groups

Figure 5.15 compares the minimum costs of two group commuters along the corridor at equilibrium status. It is obvious that commuters from group 2 pay more costs since they have higher unit VOTs and earlier preferred arrival time.

Table 5.5 Departure rates with different user groups

Group	Time	Location									
		1	2	3	4	5	6	7	8	9	10
Group 1 $t_1^* = 36$	10	0	0	0	0	0	0	0	0	0	0
	11	0.371	2.093	9.958	19.229	0	0	0	0	0	0
	12	5.424	8.169	10.061	4.181	19.157	2.250	0	0	0	0
	13	0	0	0	1.313	3.756	17.038	0	0	0	0
	14	0	0	0	2.726	0	4.697	16.027	0	0	0
	15	0	0	1.884	3.906	5.964	0	7.722	6.767	0	0
	16	0	0	3.126	4.183	0	4.968	3.138	11.272	0	0
	17	26.730	25.683	3.773	4.182	5.960	0	2.275	10.712	0	0
	18	0	0	10.923	4.185	0	3.805	4.256	6.110	6.547	0
	19	0	0	0	0	13.683	0	2.676	4.211	10.231	0
	20	0	0	0	0	0	20.866	2.981	3.238	13.771	0
	21	0	0	0	0	0	0	20.191	0	10.520	10.615
	22	0	0	0	0	0	0	0	0	3.294	15.363
	23	0	0	0	0	0	0	0	23.186	4.803	19.067
	24	0	0	0	0	0	0	0	0	23.220	20.340
	25	0	0	0	0	0	0	0	0	0	14.614
	26	0	0	0	0	0	0	0	0	0	0.001
	27	0	0	0	0	0	0	0	0	0	0
Group 2 $t_2^* = 30$	17	0	0	0	0	0	0	0	0	0	0
	18	40.792	0.001	26.426	0	0	0	0	0	0	0
	19	0	43.314	9.859	22.789	0	0	0	0	0	0
	20	0	0	9.708	0.034	12.462	0	0	0	0	0
	21	0	0	0	26.015	0	12.271	0	0	0	0
	22	0	0	0	0	39.395	3.225	29.993	0	0	0
	23	0	0	0	0	0	39.568	6.170	0	0	0
	24	0	0	0	0	0	0	22.306	0	0	0
	25	0	0	0	0	0	0	0	62.084	24.740	0
	26	0	0	0	0	0	0	0	0	41.183	27.335
	27	0	0	0	0	0	0	0	0	0	7.774
	28	0	0	0	0	0	0	0	0	0	34.891
	29	0	0	0	0	0	0	0	0	0	0

Figure 5.16 Departure rates with different user groups

The departure patterns of these two group users are shown in Table 5.5 and Figure 5.16. One can note that although group 1 users have later preferred arrival time, the first departure happens at each location earlier than those in group 2, and the departure time range is also wider. It is because that group 1 commuters have lower unit VOTs, therefore they may choose early or late arrival to avoid traffic congestion due to the lower penalty costs.

Figure 5.17 Departure sets with different user groups

In Figure 5.17, the dashed line represents the first departure at each location while the solid line represents the last departure. One can find that users in group 1 with higher unit VOTs result in a narrower departure set than those in group 2.

Figure 5.18 Cumulative departures with different user groups

Figure 5.18 shows the cumulative departures over time at each location with different user groups. People living closer to the boundary usually depart earlier due to more time consumption, and the total departure decreases from CBD to the boundary since negative exponential distribution function is applied here to reflect high density at CBD area while low density at suburb area, and many other distribution functions can also be applied here.

5.4 Summary

In this chapter, a linear traffic corridor with many-to-one OD pattern and heterogeneous commuters is considered to formulate a dynamic equilibrium model based on the PDCS approach. To solve the PDCS formulation numerically, an equivalent discrete scheme is applied to transform the PDCS model to a CP formulation. Solution existence is proved and a series of numerical studies are conducted to verify the validity of the model. It can be found that the values of unit VOTs affect the equilibrium departure pattern significantly, and the travel demand distribution has influence on the equilibrium costs along the corridor. The numerical results show the validity of the model and reflect some properties of the corridor problem. In further study, real time dynamic user equilibrium should be formulated instead of instantaneous equilibrium, and multimodal choice should be added to make the model more meaningful and wider application in practice.

CHAPTER 6

OPTIMIZING TIME-DISTANCE TOLL ALONG A TRAVEL CORRIDOR BASED ON A CONTINUOUS DYNAMIC MODEL

(Note: this chapter includes part of contents that has been accepted in the conference:

Wang, Z. W. and Du, B. “Optimizing urban density along a linear travel corridor with time-distance toll scheme based on a continuous-time dynamic model”, *The 21st International Symposium on Transportation and Traffic Theory*, Kobe, Japan, 2015.)

6.1 Introduction

Due to limited land resources and prohibitively huge financial investments, many major cities cannot afford to build up more and more new transportation infrastructures. The ideal way is to best plan and manage the existing urban transportation infrastructure systems. Urban form and spatial distribution of urban density are the key issues of urban planning and have significant effects on achieving efficient urban transportation system. For a closed city with fixed population or population growth, various density spatial distribution profiles can result in the same total population realization, as shown schematically in Figure 6.1.

Figure 6.1 Various urban density profiles

It is indicative for the urban planners to ask the question that what type of urban distribution profile is more favorable to ensure optimal transport system. Basically, should the urban density in the city center be relatively high? If so, how high the urban density should be? And how the urban density should be distributed? Many big cities, like Singapore, are expecting rapid population growth in a near future. Singapore has adopted a multi-pronged approach, including integration of land use and transport planning, to meet travelers' needs (Ibrahim, 2003). It is necessary for the urban planners to understand how to plan and allocate the new population growth in a closed city while making it possible to ensure optimal transport system. This is vital for any growing big city, especially in Asia.

On the other hand, toll strategy has been theoretically proved to be an efficient management measure to ensure smooth and less congested commute traffic. In practice, many large cities, including Singapore, has implemented road pricing in urban area to reduce traffic congestion. In practice, cordon-based pricing scheme is the most commonly used method, which conducts the pay-per-entry basis charge, regardless of the travel distance or time in the pricing cordons, but this toll-charge method is not quite fair and reasonable (Richardson and Bae, 2008). In Singapore, ERP has been implemented for years and proved to be efficient in containing traffic congestion. The new generation, ERP II based on GPS technology, enables time-distance tolling scheme in the city to make more effective road traffic management (Ohno et al., 2007). The new generation of ERP enables much more flexible road tolling, with more variables in time and spatial dimensions to be determined. For example, the potential charging area boundary is flexible and continuous, rather than discrete gantry points in current system. Specifically, the charging area has continuous potential charging boundary for next generation ERP rather than discrete one in the current ERP. Since the next generation of ERP is more flexible with more parameters to be determined, how to find out the optimal design of pricing strategy remains an open issue. This chapter will take advantage of the DCS modeling approach considering both dynamic and spatial equilibrium travel pattern to investigate how to design optimal toll charge that varies across time and places. The derived first-best time-distance tolling rate is schematically shown in Figure 6.2, wherein the optimal road usage charging rate per unit distance is continuously

distributed across the time and space dimensions. The practical charging rate scheme could be a stepwise approximation to this continuous result.

Figure 6.2 Continuous time-distance tolling rate

On the theoretical side, the fundamental problem, sometimes termed as “corridor problem”, describing the temporal and spatial traffic dynamics along a travel corridor, has been a stumbling block to the development of theoretical literature on both transportation modeling and urban economics for a long time. Previous works mainly focused on the temporal departure choices, i.e., a bottleneck problem, without considering the spatial equilibrium. Though stimulating and interesting, these preliminary results are not applicable to depict the temporal and spatial equilibrium simultaneously. In this chapter, explicit traffic flow dynamics is taken into account. It is supposed to take a significant step forward in examining the simultaneous temporal and spatial dynamics of commute traffic and contribute to the theoretical development on both transportation modeling and urban economics. The model developed in this study can be applied as the basic analysis tool to examine many other transportation and urban economics related problems.

Specifically, the PDCS approach is applied to describe the flow dynamics and the equilibrium travel choices as well, wherein complementarity formulation describes the trip-timing conditions that assumes travelers minimize individual travel cost through multimodal and departure time choices, and partial differential equations capture the traffic flow dynamics and the travel time interactions. In this way, the time and distance based toll scheme can be derived to achieve system optimal travel pattern. Then, the assumption of exogenous population density distribution is

relaxed to examine how the population density profile affects the multimodal transportation system performance. A mathematical programming problem is formulated to find the optimal urban development density distribution with associated time-distance toll scheme. In all, this chapter applies rigorous mathematical programming approach to investigate the important integrated urban development and transportation planning problem that what the optimal urban population distribution should be so as to achieve best transportation system performance. This chapter also successfully incorporates the time and distance based toll scheme in the model formulation to consider the optimal pricing strategy for the transportation system. The developed model formulation framework is supposed to be a fundamental building block to model and solve many related problems in urban and transportation planning.

6.2 Model formulation

In this chapter, a linear travel corridor with both highway and railway transportation systems is considered. Commuters are located along the corridor continuously and travel to the CBD for work. They choose travel modes and departure times to minimize their travel costs. The study horizon of the peak hours and the length of corridor are assumed to be $\bar{T}$ and L respectively.

6.2.1 Spatial and dynamic equilibrium in the multimodal corridor

With given population distribution profile and road toll scheme along the corridor, the achieved spatial and dynamic equilibrium travel pattern is determined through the UE conditions, which can be written in a form of CP constraints as follows:

$$\int_0^{\bar{T}} (q_{h,x}^w + q_{r,x}^w) dw = D_x \quad (6-1)$$

$$0 \leq q_{h,x}^t \perp \alpha TT_{h,x}^t + \beta \max(0, t^* - t - TT_{h,x}^t) + \gamma \max(0, t + TT_{h,x}^t - t^*) + p_{h,x}^t - \pi_x^* \geq 0 \quad (6-2)$$

$$0 \leq q_{r,x}^t \perp \alpha TT_{r,x}^t + \beta \max(0, t^* - t - TT_{r,x}^t) + \gamma \max(0, t + TT_{r,x}^t - t^*) + s_{r,x} + c_{r,x}^t - \pi_x^* \geq 0 \quad (6-3)$$

$$\forall x \in [0, L], t \in [0, \bar{T}] \quad (6-4)$$

Eq. (6-1) ensures that the total number of trips across the study horizon at any location amounts to the given travel demand, wherein $q_{h,x}^t$ and $q_{r,x}^t$ denote the endogenous departure rates of auto and railway users, respectively, at location x and time t . It should be noted that x here represents the distance from the location

to the city boundary, and CBD is at location L , the end of the corridor. D_x represents the travel demand at location x . For illustration purpose, assume D_x equals the product of trip generation rate g_x and the population rate Q_x at location x . Trip generation rate is regarded as a given parameter, while the negative exponential function $Q_x = Q_L e^{-\varphi(L-x)}$ is used to describe the population density distribution, which is determined by two parameters Q_L and φ . Q_L denotes the population density at CBD, and φ reflects the changing rate of exponent value. Thus it is easy to derive the travel demand distribution function $D_x = g_x Q_L e^{-\varphi(L-x)}$. Many other distribution functions can also be applied with no theoretical obstacles.

Conditions (6-2) and (6-3) entail the UE conditions, i.e., if there is a positive departure rate, the travel cost is minimized with the equilibrium cost π_x^* for commuters departing from location x ; otherwise, the travel cost must be larger than or equal to the minimum cost π_x^* . The total travel cost includes the travel time, scheduling delay and monetary cost for auto users, while crowding effect is also considered for rail transit users. In conditions (6-2) and (6-3), t^* denotes the preferred arrival time, $TT_{h,x}^t$ represents the travel time for auto users departing from location x at time t to CBD. $p_{h,x}^t$ denotes the road toll for auto users departing at time t from location x to CBD, which varies with time and travel distance. Specifically, $p_{h,x}^t$ can be expressed through an integral function, $p_{h,x}^t = \int_x^L \bar{p}_{h,w}^t dw$, where $\bar{p}_{h,w}^t$ denotes the toll charging rate per unit distance varying with time t and location x . $TT_{r,x}^t$ represents the travel time for railway users departing from location x at time t to CBD, which can be calculated as a linear function $TT_{r,x}^t = (L-x)/V_r$, where V_r is the fixed average train speed. $s_{r,x}$ denotes the fare by railway from location x to CBD, which is assumed to be a linear function with two parts: $s_{r,x} = s_r^0 + \kappa(L-x)$, where s_r^0 is the fixed part, and κ is the fare per unit distance. $c_{r,x}^t$ represents the crowding cost for railway users departing at time t from location x to CBD. A linear crowding cost function with respect to the in-vehicle passenger traffic is applied, as was done in Huang (2000). Specifically, the crowding cost for

rail users departing at time t from location x to CBD can be calculated as

$$c_{r,x}^t = \int_x^L \left(c_0 + \lambda \int_0^w q_{r,y}^t dy \right) dw, \text{ where } c_0 \text{ and } \lambda \text{ are given calibrated parameters.}$$

Moreover, $\max(0, t^* - t - TT_{h,x}^t)$ and $\max(0, t + TT_{h,x}^t - t^*)$ in condition (6-2), as well as those $\max$ functions in condition (6-3), are used to represent the penalties of early and late arrival respectively. α , β and γ denote the unit VOTs of travel time, early arrival delay and late arrival delay respectively, and it is reasonable to assume that $\gamma > \alpha > \beta$. The $\max$ functions in conditions (6-2) and (6-3) can also be expressed in complementarity form as follows:

$$\begin{aligned} 0 &\leq \theta_{h,x}^t \perp \theta_{h,x}^t - (t^* - t - TT_{h,x}^t) \geq 0 \\ 0 &\leq \theta_{r,x}^t \perp \theta_{r,x}^t - (t^* - t - TT_{r,x}^t) \geq 0 \end{aligned} \quad (6-5)$$

6.2.2 Traffic flow propagation on highway

Classical flow congestion is assumed on highway of the travel corridor. The flow dynamics is described by applying the L-W-R traffic flow theory and the Greenshields function.

$$\frac{\partial f_{h,x}^t}{\partial x} + \frac{\partial k_{h,x}^t}{\partial t} = q_{h,x}^t \quad (6-6)$$

$$f_{h,x}^t = k_{h,x}^t u_{h,x}^t(k_{h,x}^t) \quad (6-7)$$

$$u_{h,x}^t(k_{h,x}^t) = a_x^t k_{h,x}^t + b_x^t \quad (6-8)$$

$$\frac{dTT_{h,x}^t}{dx} = \frac{1}{u_{h,x}^t(k_{h,x}^t)} \quad (6-9)$$

Eq. (6-6) entails the flow conservation (equation of continuity) for auto traffic, with $k_{h,x}^t$ representing the traffic density, $f_{h,x}^t$ the traffic flow and $q_{h,x}^t$ the departure rate (entry flow rate into the highway) at location x and time t . Eqs. (6-7) and (6-8) denote the flow-velocity-density relationship, and the Greenshields model is employed to depict the density-velocity relation, wherein $a_x^t = -\frac{u^{\max}}{k^{\max}}$ and $b_x^t = u^{\max}$.

Eq. (6-9) reflects the relationship between the travel time of travelers departing from two neighboring locations, i.e., $TT_{h,x}^t - TT_{h,x+dx}^t = \frac{dx}{u_{h,x}^t(k_{h,x}^t)}$. Eq. (6-8) can be substituted into Eq. (6-7) and Eq. (6-9) to simplify the model form.

6.2.3 The overall model

The assumption of given toll scheme and fixed population distribution is relaxed, and a bi-level continuous model is developed, wherein the upper level optimizes the urban density distribution and road toll, while the lower level model describes the spatial and dynamic equilibrium travel pattern in the multimodal travel corridor. Such a bi-level programming model can be equivalently formulated as a MPEC or MPCC model. In this chapter, vector $\mathbf{z}$ is used to represent the variables as: $\mathbf{z} = \{Q_L, \varphi, q_{r,x}^t, q_{h,x}^t, TT_{r,x}^t, TT_{h,x}^t, f_{h,x}^t, k_{h,x}^t, \theta_{h,x}^t, \bar{p}_{h,x}^t, \pi_x^*\}$. Thus, for $\forall x \in [0, L], \forall t \in [0, \bar{T}]$, the overall MPCC model is given as follows:

$$\begin{aligned}
 \min_{\mathbf{z}}: & \int_0^L \int_0^{\bar{T}} (q_{h,x}^t TT_{h,x}^t + q_{r,x}^t TT_{r,x}^t) dt dx \\
 \text{s.t.} & \int_0^L Q_L e^{-\varphi(L-x)} dx = Q_{total} \\
 & \frac{\partial f_{h,x}^t}{\partial x} + \frac{\partial k_{h,x}^t}{\partial t} = q_{h,x}^t \\
 & f_{h,x}^t = a_x^t (k_{h,x}^t)^2 + b_x^t k_{h,x}^t \\
 & \frac{dTT_{h,x}^t}{dx} = \frac{1}{a_x^t k_{h,x}^t + b_x^t} \\
 & TT_{r,x}^t = (L-x) / V_r \\
 & \int_0^{\bar{T}} (q_{h,x}^w + q_{r,x}^w) dw = g_x Q_L e^{-\varphi(L-x)} \\
 & 0 \leq \theta_{h,x}^t \perp \theta_{h,x}^t - (t^* - t - TT_{h,x}^t) \geq 0 \\
 & 0 \leq q_{h,x}^t \perp \alpha TT_{h,x}^t + \beta \theta_{h,x}^t + \gamma [\theta_{h,x}^t - (t^* - t - TT_{h,x}^t)] + \int_x^L \bar{p}_{h,w}^t dw - \pi_x^* \geq 0 \\
 & 0 \leq \theta_{r,x}^t \perp \theta_{r,x}^t - (t^* - t - TT_{r,x}^t) \geq 0 \\
 & 0 \leq q_{r,x}^t \perp \left(\alpha TT_{r,x}^t + \beta \theta_{r,x}^t + \gamma [\theta_{r,x}^t - (t^* - t - TT_{r,x}^t)] + \right. \\
 & \quad \left. s_r^0 + \kappa(L-x) + \int_x^L (c_0 + \lambda \int_0^w q_{r,y}^t dy) dw - \pi_x^* \right) \geq 0
 \end{aligned} \tag{6-10}$$

where the objective is to minimize the total travel time along the corridor from the perspective of transport planners. The first constraint assumes that a certain total population across the corridor is maintained, while the population density distribution varies after the two parameters Q_L and φ . An optimal population density distribution profile will be derived to understand how urban density regulation policy should be imposed to ensure that optimal system performance can be achieved. Q_{total} represents the total population to be accommodated along the corridor, which is given exogenously.

6.3 Solution approach

This section demonstrates the numerical approach for solving the continuous-spatial-temporal MPEC formulation (6-10) shown above. The continuous model is firstly transformed to a discrete problem by applying a discretization scheme.

The corridor is divided into several equal sections, $i=1,2,\dots,N$ with each section length as $\varepsilon = \frac{L}{N}$, and equal time intervals $t=0,1,\dots,M$ with each time interval length as $\omega = \frac{\bar{T}}{M}$, thus the variable vector $\mathbf{z}$ in discrete form is given as:

$\mathbf{z} = \{Q_L, \varphi, q'_{r,i}, q'_{h,i}, TT'_{r,i}, TT'_{h,i}, f'_{h,i}, k'_{h,i}, \theta'_{h,i}, \bar{p}'_{h,i}, \pi_i^* \mid i=1,2,\dots,N, t=0,1,\dots,M\}$. Suppose no traffic exists on the corridor at initial time $t=0$ and railway operates at a constant average speed. Furthermore, it is assumed that the total travel demand along the corridor as well as the upper bound of the toll are given exogenously. Note that all the variables are nonnegative in this chapter.

Based on the continuous MPCC model (6-10) and proper assumptions above, an equivalent MPCC approximation model in discrete scheme is shown as below:

$$\begin{aligned}
 & \min_{\mathbf{z} \geq 0} : \sum_{i=1}^N \sum_{t=0}^M (q'_{h,i} TT'_{h,i} + q'_{r,i} TT'_{r,i}) \\
 & \text{s.t.} \quad \left\{ \begin{aligned} & \sum_{i=1}^N Q_N e^{-\varphi(N-i)} = Q_{total} \\ & \frac{f'_{h,i} - f'_{h,i-1}}{\varepsilon} + \frac{k'_{h,i+1} - k'_{h,i}}{\omega} = q'_{h,i} \\ & f'_{h,i} = a_i^t (k'_{h,i})^2 + b_i^t k'_{h,i} \\ & TT'_{h,i} - TT'_{h,i+1} = \varepsilon / (a_i^t k'_{h,i} + b_i^t) \\ & TT'_{r,i} = (N-i+1)\varepsilon / V_r \\ & \sum_{t=0}^M (q'_{h,i} + q'_{r,i}) = g_i Q_N e^{-\varphi(N-i)} \end{aligned} \right. \quad (6-11) \\
 & \text{[MPCC]} \quad \left\{ \begin{aligned} & 0 \leq \theta'_{h,i} \perp \theta'_{h,i} - (t^* - t - TT'_{h,i}) \geq 0 \\ & 0 \leq q'_{h,i} \perp \alpha TT'_{h,i} + \beta \theta'_{h,i} + \gamma [\theta'_{h,i} - (t^* - t - TT'_{h,i})] + \sum_{j=i}^N \varepsilon \bar{p}'_{h,j} - \pi_i^* \geq 0 \\ & 0 \leq \theta'_{r,i} \perp \theta'_{r,i} - (t^* - t - TT'_{r,i}) \geq 0 \\ & 0 \leq q'_{r,i} \perp \left(\alpha TT'_{r,i} + \beta \theta'_{r,i} + \gamma [\theta'_{r,i} - (t^* - t - TT'_{r,i})] + s_r^0 + \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q'_{r,m} \right) - \pi_i^* \right) \geq 0 \end{aligned} \right. \quad (B)
 \end{aligned}$$

The constraints in formulation [MPCC] (6-11) consists of two groups: equality conditions in (A) and complementarity conditions in (B), and the latter is difficult to solve directly based on its immediate form as in (B). However, the [MPCC] (6-11) can be solved after it is converted to an equivalent single level NLP model via some reformulation techniques (Ferris et al., 2002) and relaxation scheme. Under certain conditions, such a relaxation scheme can guarantee to generate an optimal solution of the original MPCC model (Ban et al., 2006).

6.3.1 Equivalent NLP model for MPCC

The most straightforward way to convert the MPCC problem to a single level standard NLP model is to use a series of equivalent inequality and equality functions to represent the complementarity conditions. Therefore, the [MPCC] (6-11) above can be readily transformed to a NLP model precisely as follows:

$$\begin{aligned}
 & \min_{z \geq 0}: \sum_{i=1}^N \sum_{t=0}^M (q_{h,i}^t TT_{h,i}^t + q_{r,i}^t TT_{r,i}^t) \\
 & \text{s.t. (A) in [MPCC] (6-11)} \\
 \text{[NLP]} \quad & \left\{ \begin{aligned}
 & \theta_{h,i}^t - (t^* - t - TT_{h,i}^t) \geq 0 \\
 & \theta_{h,i}^t (\theta_{h,i}^t + t + TT_{h,i}^t - t^*) = 0 \\
 & \alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma [\theta_{h,i}^t - (t^* - t - TT_{h,i}^t)] + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \geq 0 \\
 & q_{h,i}^t \left[\alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma (\theta_{h,i}^t + t + TT_{h,i}^t - t^*) + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \right] = 0 \\
 & \theta_{r,i}^t - (t^* - t - TT_{r,i}^t) \geq 0 \\
 & \theta_{r,i}^t (\theta_{r,i}^t + t + TT_{r,i}^t - t^*) = 0 \\
 & \left(\alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma [\theta_{r,i}^t - (t^* - t - TT_{r,i}^t)] + s_r^0 + \right. \\
 & \quad \left. \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \right) \geq 0 \\
 & q_{r,i}^t \left[\alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma (\theta_{r,i}^t + t + TT_{r,i}^t - t^*) + s_r^0 + \right. \\
 & \quad \left. \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \right] = 0
 \end{aligned} \right. \quad (6-12)
 \end{aligned}$$

wherein the part (A) in [MPCC] (6-11) keeps unchanged, and the complementarity conditions in (B) of [MPCC] (6-11) are replaced with equivalent series of equations and inequalities in (B) of [NLP] (6-12) following the concept of CP, i.e., $0 \leq x \perp f(x) \geq 0 \Leftrightarrow x \geq 0, f(x) \geq 0, xf(x) = 0$.

This [NLP] model (6-12) is non-convex with poor mathematical properties and the Mangasarian Fromovitz Constraint Qualification (MFCQ) cannot hold (Luo et al., 1996). It is rather difficult to solve this NLP formulation directly. The most commonly used method to handle this kind of problem is to introduce an auxiliary parameter $\mu > 0$, which constructs a relaxed complementarity slackness conditions of the exact ones in [NLP] (6-12). In this case, MFCQ can be guaranteed and it can be solved by existing NLP solution algorithms. Specifically, the following inequality constraints in (6-13) are used to replace the corresponding equality constraints in [NLP] (6-12) and generate a new NLP formulation, named [NLP] $_{\mu}$.

$$\begin{aligned}
 & \min_{z \geq 0}: \sum_{i=1}^N \sum_{t=0}^M (q_{h,i}^t TT_{h,i}^t + q_{r,i}^t TT_{r,i}^t) \\
 & \text{s.t. (A) in [MPCC] (6-11)} \\
 & \left. \begin{aligned}
 & \theta_{h,i}^t - (t^* - t - TT_{h,i}^t) \geq 0 \\
 & \theta_{h,i}^t (\theta_{h,i}^t + t + TT_{h,i}^t - t^*) \leq \mu \\
 & \alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma [\theta_{h,i}^t - (t^* - t - TT_{h,i}^t)] + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \geq 0 \\
 & q_{h,i}^t \left[\alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma (\theta_{h,i}^t + t + TT_{h,i}^t - t^*) + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \right] \leq \mu \\
 & \theta_{r,i}^t - (t^* - t - TT_{r,i}^t) \geq 0 \\
 & \theta_{r,i}^t (\theta_{r,i}^t + t + TT_{r,i}^t - t^*) \leq \mu \\
 & \left(\alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma [\theta_{r,i}^t - (t^* - t - TT_{r,i}^t)] + s_r^0 + \right. \\
 & \quad \left. \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \right) \geq 0 \\
 & q_{r,i}^t \left[\alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma (\theta_{r,i}^t + t + TT_{r,i}^t - t^*) + s_r^0 + \right. \\
 & \quad \left. \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \right] \leq \mu
 \end{aligned} \right\} \quad \text{(B)} \quad (6-13)
 \end{aligned}$$

The equation constraint sets in (B) of [NLP] (6-12) are replaced with inequality conditions by introducing new auxiliary parameter μ in (B) of [NLP] $_{\mu}$ (6-13). Based on an iteration procedure, a sequence of problems [NLP] $_{\mu}$ (6-13) will be solved repeatedly by driving the value of μ to zero progressively. One can refer to Scheel and Scholtes (2000); Ferris and Kanzow (2002); Ban et al. (2006) to obtain more applications of this approach.

6.3.2 A smooth relaxation scheme

$[NLP_\mu]$ (6-13) contains a number of complicated constraints. It can be further transformed to a smooth function using the approximation function proposed initially in Facchinei et al. (1999).

Definition 1 (Ferris and Kanzow, 2002) *A function $\Phi: \mathbb{R}^2 \rightarrow \mathbb{R}$ is called an NCP-function if it satisfies the condition*

$$\Phi(x, y) = 0 \Leftrightarrow x \geq 0, y \geq 0, xy = 0 \quad (6-14)$$

The most popular NCP-functions include minimum function $\Phi_M(x, y) = \min\{x, y\}$ and Fischer-Burmeister (FB) function $\Phi_{FB}(x, y) = \sqrt{x^2 + y^2} - x - y$ (Fischer, 1992). Although the FB function is convex and differentiable on $\mathbb{R}^2$, both of the above NCP-functions are nonsmooth. To get a smooth approximation of the FB function, an approximate smooth FB (S-FB) function is adopted as follows:

$$\Phi_{S-FB}(x, y, \delta) = \sqrt{x^2 + y^2 + \delta^2} - x - y \quad (6-15)$$

by introducing an auxiliary parameter $\delta > 0$, which is differentiable. Therefore, the nonsmooth problem is transferred to a smooth problem with the auxiliary parameter δ , except when $\delta = 0$. The different feasible sets of the standard complementarity constraints and the approximate S-FB function $\Phi_{S-FB}(x, y, \delta)$ are shown schematically in Figure 6.3.

Figure 6.3 The feasible sets of standard complementarity constraints and $\Phi_{S-FB}(x, y, \delta)$

Proposition 6.1 (Facchinei et al., 1999) For each $\delta \in \mathbb{R}$, $\Phi_{S-FB}(x, y, \delta) = 0$ if and only if $x \geq 0, y \geq 0, \delta = \sqrt{2xy}$ for $\delta \geq 0$, which is a parameter converging to zero.

Proposition 6.2 If $\delta > 0$, then $\Phi_{S-FB}(x, y, \delta)$ satisfies the inequality $|\Phi_{S-FB}(x, y, \delta) - \Phi_{FB}(x, y)| \leq \delta$ for all $(x, y) \in \mathbb{R}^2$.

Proof. In a similar way as the proof of Lemma 3.7 in Kanzow (1997), for any $(x, y) \in \mathbb{R}^2$,

$$|\Phi_{S-FB}(x, y, \delta) - \Phi_{FB}(x, y)| = \left| \sqrt{x^2 + y^2 + \delta^2} - \sqrt{x^2 + y^2} \right| = \frac{\delta^2}{\left| \sqrt{x^2 + y^2 + \delta^2} + \sqrt{x^2 + y^2} \right|} \leq \delta$$

Note that this proposition still holds at $\delta = 0$. $\square$

In terms of the S-FB function (6-15) above, $\Phi_{S-FB}(\mathbf{x}, \mathbf{y}, \delta)$ is used to represent the vectors $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ and $\mathbf{y} = \{y_1, y_2, \dots, y_n\}$ as follows:

$$\Phi_{S-FB}(\mathbf{x}, \mathbf{y}, \delta) \equiv \begin{pmatrix} \Phi_{S-FB}(x_1, y_1, \delta) \\ \vdots \\ \Phi_{S-FB}(x_n, y_n, \delta) \end{pmatrix} \quad (6-16)$$

Then a relaxation scheme is proposed as follows:

$$\|\Phi_{S-FB}(\mathbf{x}, \mathbf{y}, \delta)\|_2 \leq \mu \quad (6-17)$$

Consequently, an approximated smooth model based on the relaxation scheme (6-17) for the original formulation [MPCC] (6-11) is proposed as follows:

$$\begin{aligned} \min_{\mathbf{z}}: & \sum_{i=1}^N \sum_{t=0}^M (q_{h,i}^t TT_{h,i}^t + q_{r,i}^t TT_{r,i}^t) \\ \text{s.t.} & \quad (\text{A}) \text{ in [MPCC] (6-11)} \end{aligned}$$

$$[\text{R-NLP}] \quad (\text{B}) \left\{ \left\| \Phi_{S-FB} \left(\begin{Bmatrix} \theta_{h,i}^t \\ q_{h,i}^t \\ \theta_{r,i}^t \\ q_{r,i}^t \end{Bmatrix}, \begin{Bmatrix} \theta_{h,i}^t - (t^* - t - TT_{h,i}^t) \\ \left(\alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma [\theta_{h,i}^t - (t^* - t - TT_{h,i}^t)] + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \right) \\ \theta_{r,i}^t - (t^* - t - TT_{r,i}^t) \\ \left(\alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma [\theta_{r,i}^t - (t^* - t - TT_{r,i}^t)] + s_r^0 \right) + \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \end{Bmatrix} \right\}, \delta \right\|_2 \leq \mu \right. \quad (6-18)$$

One can find that when the two parameters δ and μ approach zero, the above inequality constraints in reformulated [R-NLP] (6-18) can be used to approximate the complementarity conditions in [MPCC] (6-11). There are three major advantages for the reformulated [R-NLP] (6-18). Firstly, it is a smooth approximation of the original model; secondly, it transforms multiple inequality and equality conditions in [NLP] (6-12) and [NLP] _{μ} (6-13) to a single nonlinear inequality constraint as (B) in [R-NLP] (6-18). Moreover, this formulation can be solved by many existing efficient NLP solution methods, such as sequential quadratic programming (SQP), penalty interior point algorithm (PIPA), and so on.

6.3.3 Iterative algorithm

To solve the proposed [R-NLP] (6-18), an iterative algorithm can be applied to solve it repeatedly with the values of parameters δ and μ reduced gradually toward zero. Generally, the iterative algorithm is illustrated as follows, also shown in Figure 6.4.

Step 1 Initialization

Initialize the parameters δ and μ with the iteration number as superscript, $\delta^0 > 0$, $\mu^0 > 0$; set the update parameter $0 < \rho < 1$, current iteration number $k = 0$, and the termination conditions $\sigma_z > 0$ or $\sigma_f > 0$.

Step 2 Major iteration

Solve the current problem [R-NLP] (6-18) with δ^k and μ^k to obtain solution z^k and objective value f^k .

Step 3 Termination check

Check $\frac{\|z^k - z^{k-1}\|}{\|z^{k-1}\|}$ or $\frac{\|f^k - f^{k-1}\|}{\|f^{k-1}\|}$, if $\frac{\|z^k - z^{k-1}\|}{\|z^{k-1}\|} \leq \sigma_z$ or $\frac{\|f^k - f^{k-1}\|}{\|f^{k-1}\|} \leq \sigma_f$, then stop and go to Step 4; otherwise, update the parameters $\delta^{k+1} = \rho\delta^k$, $\mu^{k+1} = \rho\mu^k$, $k = k + 1$, then go to Step 2.

Step 4 Solution report

Current solution is optimal approximation solution.

Figure 6.4 Flow chart of the iterative solution algorithm for R-NLP

6.4 Numerical results

In this section, case studies are conducted based on the context of a travel corridor with length of 15.8km, which is divided into 30 sections (the sequence of cells 1 to 30 is the direction from the boundary of the corridor to CBD). The total population along the corridor is given as 25000 persons, and trip generation rate is assumed to be 0.1 along the corridor. For simplicity, assume each person corresponds to one vehicle, thus the total travel demand is 2500 veh. The maximum auto speed is 60 km/h, the maximum capacity density is 145 veh/min, the average railway speed is 36km/h, and the parameters of crowding cost are set as $c_0 = 0$, $\lambda = 0.002$. The parameters of railway fare are set as $s_r^0 = 0.6\text{S\$}$ and $\kappa = 0.06\text{S\$}/\text{km}$. The unit VOTs of travel time, early penalty and late penalty are $\alpha = 1\text{S\$}/\text{min}$, $\beta = 0.4\text{S\$}/\text{min}$, $\gamma = 1.4\text{S\$}/\text{min}$. The time horizon is 7:00am-8:00am, which is divided into small equal intervals with each length 1 min, and the preferred arrival time is 7:45am. The iteration parameters are set as $\delta^0 = 0.01$, $\mu^0 = 0.01$, $\sigma_z = 1 \times 10^{-4}$, $\rho = 0.1$.

Solving the model developed in this chapter to obtain the optimal population density distribution with the two parameters: $Q_{30} = 1511.1$ and $\varphi = 0.047$.

Table 6.1 Departure rate of auto users

Time	Location (1-15)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7:01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:02	0.70	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:03	1.20	0.91	0.87	0.64	0	0	0	0	0	0	0	0	0	0	0
7:04	1.20	0.64	0.49	0.27	0	0	0	0	0	0	0	0	0	0	0
7:05	1.19	1.16	0.85	0.51	0.19	0	0	0	0	0	0	0	0	0	0
7:06	1.07	0.91	0.87	0.65	0.38	0	0	0	0	0	0	0	0	0	0
7:07	0.71	0.73	0.62	0.49	0.43	0	0	0	0	0	0	0	0	0	0
7:08	1.25	1.90	2.84	3.36	3.64	3.65	3.08	2.76	1.98	0	0	0	0	0	0
7:09	1.90	1.99	2.07	2.18	2.12	1.98	2.10	2.35	1.36	0	0	0	0	0	0
7:10	1.83	2.18	2.33	2.28	2.10	1.88	1.80	1.79	1.91	1.41	0	0	0	0	0
7:11	1.83	2.14	2.37	2.45	2.36	2.19	1.91	1.89	2.00	1.52	4.28	1.48	0	0	0
7:12	1.83	2.14	2.34	2.43	2.43	2.50	2.36	1.86	6.35	1.55	2.37	5.69	0	0	0
7:13	1.82	2.14	2.35	2.42	2.39	2.47	2.58	2.26	1.22	1.31	1.61	0.00	0	0	0
7:14	1.82	2.11	2.33	2.43	2.40	2.40	2.51	2.38	1.19	1.14	1.65	1.93	0	0	0
7:15	1.80	2.07	2.25	2.33	2.35	2.44	2.52	2.28	1.21	1.13	1.48	1.94	0	0	0
7:16	1.79	2.03	2.16	2.18	2.15	2.24	2.53	2.27	1.13	1.13	1.42	1.91	0.48	0	0
7:17	1.77	1.96	2.06	2.04	1.95	1.92	2.00	1.98	1.20	1.08	1.31	1.71	0.59	0	0
7:18	1.76	1.88	1.92	1.87	1.76	1.70	1.71	1.73	1.16	1.05	1.24	1.53	0.93	0	0
7:19	1.76	1.85	1.83	1.72	1.59	1.51	1.49	1.53	6.49	1.10	1.21	1.32	0.89	0	0
7:20	1.76	1.85	1.81	1.67	1.50	1.41	1.37	1.31	1.21	1.15	1.18	1.08	1.23	0.61	0.59
7:21	1.75	1.84	1.80	1.66	1.49	1.40	1.40	1.41	1.28	1.05	1.16	1.01	1.31	0.54	0.46
7:22	1.73	1.80	1.77	1.63	1.47	1.39	1.43	1.56	1.62	1.25	1.14	0.85	1.61	0.42	0.22
7:23	1.70	1.74	1.69	1.56	1.40	1.31	1.36	1.51	1.75	9.24	10.89	3.60	1.21	0.28	0.03
7:24	1.66	1.66	1.59	1.45	1.29	1.20	1.23	1.37	1.64	9.48	8.26	3.57	1.76	1.04	0.24

7:25	1.56	1.52	1.43	1.28	1.13	1.04	1.03	1.09	1.30	5.08	2.95	3.84	2.21	1.93	1.01
7:26	1.23	1.18	1.13	1.01	0.88	1.01	0.79	0.86	1.16	1.87	8.32	15.61	10.88	26.01	
7:27	0.53	0.42	0.48	0.43	0.75	0.38	0.49	0.38	0.42	0.47	0.85	1.38	1.69	0.88	0.44
7:28	0	0.27	0.32	0.26	0.63	0.24	0.44	0.51	0.58	0.63	0.72	0.96	0.92	2.13	0.40
7:29	0	0	0	0	0	0.38	0.30	0.34	0.77	1.04	1.19	1.05	0.88	1.92	3.76
7:30	0	0	0	0	0	0.58	0.19	0.26	0.22	1.16	0.32	0.38	0.60	1.51	1.01
7:31	0	0	0	0	0	0	0	0.78	0.23	0.18	0	0	0	0	0.36
7:32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Time	Location (16-30)														
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
7:20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:21	0	1.73	0	0	0	0	0	0	0	0	0	0	0	0	0
7:22	0.39	1.30	0	0	0	0	0	0	0	0	0	0	0	0	0
7:23	0.52	1.22	0	0	0	0	0	0	0	0	0	0	0	0	0
7:24	0.93	2.19	0	0	0	0	0	0	0	0	0	0	0	0	0
7:25	1.01	1.13	1.90	0	0	0	0	0	0	0	0	0	0	0	0
7:26	27.36	2.71	2.44	0	0	0	0	0	0	0	0	0	0	0	0
7:27	0	0.73	2.35	0	0	0	0	0	0	0	0	0	0	0	0
7:28	0	0.13	0.69	1.70	0.79	0	0	0	0	0	0	0	0	0	0
7:29	2.78	3.20	1.81	3.01	3.64	0	0	0	0	0	0	0	0	0	0
7:30	2.33	3.38	2.37	1.11	2.24	1.88	2.99	0	0	0	0	0	0	0	0
7:31	0.32	2.97	15.15	1.11	1.03	2.30	3.82	0	0	0	0	0	0	0	0
7:32	1.27	0.22	1.74	0.23	0.18	0.58	2.83	0	0	0	0	0	0	0	0
7:33	0.15	0.04	0.68	0.29	0.79	0.84	4.98	0.58	9.40	0	0	0	0	0	0
7:34	0	0.11	0.17	0.49	1.48	2.85	2.40	0.60	2.53	20.47	0	0	0	0	0
7:35	0	0.23	0.21	0.42	2.04	0.09	2.24	0	0	0.71	6.64	0	0	0	0

7:36	0	0	1.25	1.13	0.41	0.10	3.10	0	0	0	0	11.59	0	0	0	0
7:37	0	0	0	0.65	0	1.29	0.34	0	0	0	0	0	5.25	1.41	0	0
7:38	0	0	0	0	0	0	0.74	0	0	0	0	0	0	8.47	0	0
7:39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.40	0
7:40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10.73	0
7:41	0	0	0	0	0	0	0	0	2.12	0.12	0	0	0	0.12	9.09	0
7:42	0	0	0	0	0	0	0	0	0.71	1.20	3.30	0.14	0.28	4.52	0.79	0
7:43	0	0	0	0	0	0	0	0	0	0	3.43	2.19	0.55	3.77	5.02	0
7:44	0	0	0	0	0	0	0	0	0	0	0	0	12.48	0.23	0	0
7:45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6.2 Departure rate of railway users

Time	Location (1-15)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7:18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:19	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:20	0	0.03	0.43	0.29	0	0	0	0	0	0	0	0	0	0	0
7:21	0	0	0	3.55	2.29	0.50	0	0	0	0	0	0	0	0	0
7:22	0	0	0	0	6.09	4.00	0.86	0	0	0	0	0	0	0	0
7:23	0	0	0	0	0	7.72	5.41	1.66	0	0	0	0	0	0	0
7:24	0	0	0	0	0	0	8.91	6.85	2.95	0	0	0	0	0	0
7:25	0	0	0	0	0	0	0	9.38	8.32	4.35	0	0	0	0	0
7:26	0	0	0	0	0	0	0	0	6.49	8.69	5.80	1.67	0	0	0
7:27	0	0	0	0	0	0	0	0	0	2.19	9.50	8.24	4.55	0	0
7:28	0	0	0	0	0	0	0	0	0	0	0	10.88	10.88	7.72	3.49
7:29	0	0	0	0	0	0	0	0	0	0	0	0	21.12	10.71	6.62

[illegible]

Departure rates of auto and railway users at equilibrium are shown in Table 6.1 and Table 6.2 respectively. One can find the information about travelers' travel mode and departure time choice. It is straightforward to observe that auto mode is preferred by travelers living far from the city center while rail service is favored close to the city center. For the auto users living further toward city boundary, they need to depart earlier but have more flexible departure time choices, i.e., there is a larger departure time window interval.

The travel costs incurred by auto users departing at different time and different locations along the corridor are shown in Figure 6.5. It can be found that for users living close to city boundary, minimum travel cost occurs within a longer time interval, which means auto users departing from this location have more flexible departure time choices, for example, auto users departing from location 5 could depart between 7:05am and 7:28am coming with the same minimum travel cost. However, the auto users living closer to CBD have a rather narrower departure time window, for example, at location 30, auto users can only depart at 7:39am or 7:45am to ensure minimum travel cost.

Figure 6.5 Travel cost of auto mode along the corridor

Similarly, the different travel costs for railway users departing at different time and different locations along the corridor are shown in Figure 6.6. For the railway users along the corridor, travelers depart earlier for those living close to city boundary, and multiple departure time choices exist along the corridor.

Figure 6.6 Travel cost of rail mode along the corridor

The optimum toll scheme changing after time and distance along the corridor is shown in Table 6.3. One can find that the road toll scheme is determined by the congestion flow dynamics evolving on the highway. At locations far from the city center, the toll is raised at early time intervals to affect travelers' multimodal and departure time choices. With evolving congestion dynamics, tolls are imposed at locations toward the city center. It can be also be found that at some locations, different toll rates are derived within the time horizon to avoid congestion incurred by travelers' departure time choices.

Table 6.3 Time-distance based toll scheme

Time	Location (1-15)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7:01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:02	0.843	0.880	0.429	0	0	0	0	0	0	0	0	0	0	0	0
7:03	0.840	0.877	0.488	0.493	0.107	0	0	0	0	0	0	0	0	0	0
7:04	0.840	0.876	0.486	0.513	0.466	0.266	0	0	0	0	0	0	0	0	0
7:05	0.840	0.875	0.485	0.511	0.839	0.602	0	0	0	0	0	0	0	0	0
7:06	0.840	0.876	0.485	0.511	0.161	0.151	0.147	0.144	0	0	0	0	0	0	0
7:07	0.257	0.247	0.244	0.241	0.239	0.237	0.235	0.233	0	0	0	0	0	0	0
7:08	0.842	0.879	0.490	0.517	0.878	0.878	0.889	0.899	0	0	0	0	0	0	0
7:09	0.838	0.872	0.480	0.505	0.867	0.867	0.880	0.898	0	0	0	0	0	0	0
7:10	0.839	0.871	0.478	0.501	0.861	0.861	0.869	0.891	0.116	0.098	0	0	0	0	0
7:11	0.839	0.872	0.478	0.499	0.858	0.856	0.870	0.899	0.900	0.900	0	0	0	0	0
7:12	0.839	0.872	0.478	0.500	0.857	0.852	0.859	0.893	0.900	0.900	0	0	0	0	0
7:13	0.839	0.872	0.478	0.500	0.857	0.852	0.855	0.870	0.896	0.900	0.153	0.122	0	0	0
7:14	0.839	0.872	0.478	0.500	0.858	0.853	0.857	0.872	0.897	0.900	0.886	0.899	0.069	0.053	0.048
7:15	0.839	0.872	0.478	0.500	0.857	0.853	0.858	0.875	0.898	0.900	0.884	0.200	0.176	0.158	0.144
7:16	0.839	0.872	0.478	0.500	0.858	0.852	0.857	0.877	0.899	0.900	0.890	0.253	0.225	0.204	0.188
7:17	0.839	0.872	0.479	0.501	0.860	0.855	0.855	0.877	0.899	0.900	0.893	0.312	0.275	0.250	0.231
7:18	0.839	0.872	0.479	0.502	0.860	0.858	0.864	0.874	0.899	0.900	0.893	0.378	0.324	0.294	0.273
7:19	0.839	0.872	0.480	0.503	0.862	0.858	0.865	0.890	0.898	0.900	0.890	0.437	0.369	0.335	0.313
7:20	0.839	0.872	0.480	0.504	0.864	0.861	0.866	0.883	0.899	0.900	0.893	0.899	0.837	0.709	0.831
7:21	0.839	0.872	0.480	0.504	0.864	0.863	0.872	0.897	0.900	0.900	0.898	0.899	0.832	0.706	0.830
7:22	0.839	0.873	0.480	0.504	0.864	0.862	0.885	0.900	0.900	0.900	0.900	0.900	0.845	0.701	0.826
7:23	0.839	0.873	0.480	0.504	0.864	0.862	0.889	0.900	0.900	0.900	0.900	0.900	0.871	0.700	0.824
7:24	0.839	0.873	0.481	0.505	0.865	0.863	0.871	0.889	0.897	0.900	0.866	0.899	0.860	0.525	0.502

7:25	0.839	0.873	0.481	0.505	0.866	0.864	0.873	0.898	0.900	0.900	0.814	0.864	0.819	0.735	0.835
7:26	0.839	0.874	0.482	0.506	0.867	0.866	0.884	0.900	0.900	0.900	0.900	0.814	0.758	0.658	0.869
7:27	0.840	0.875	0.484	0.508	0.869	0.872	0.899	0.900	0.900	0.900	0.900	0.900	0.597	0.553	0.467
7:28	0.152	0.032	0.316	0.682	0.875	0.892	0.900	0.900	0.900	0.900	0.900	0.900	0.899	0.476	0.614
7:29	0	0	0.061	0.064	0.816	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.757	0.600
7:30	0	0	0	0	0.061	0.221	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.775	0.851
7:31	0	0	0	0	0	0.062	0.060	0.347	0.900	0.896	0.896	0.896	0.896	0.854	0.863
7:32	0	0	0	0	0	0	0	0.070	0.067	0.046	0.870	0.866	0.864	0.863	0.883
7:33	0	0	0	0	0	0	0	0	0	0	0.061	0.621	0.843	0.840	0.898
7:34	0	0	0	0	0	0	0	0	0	0	0	0.061	0.034	0.566	0.900
7:35	0	0	0	0	0	0	0	0	0	0	0	0	0	0.116	0.112
7:36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Location (16-30)															
Time	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
7:13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:14	0.046	0.045	0.044	0	0	0	0	0	0	0	0	0	0	0	0
7:15	0.133	0.124	0.118	0.114	0.112	0	0	0	0	0	0	0	0	0	0
7:16	0.176	0.162	0.153	0.146	0.142	0	0	0	0	0	0	0	0	0	0
7:17	0.216	0.201	0.189	0.180	0.173	0.169	0.166	0	0	0	0	0	0	0	0
7:18	0.256	0.238	0.224	0.214	0.206	0.199	0.194	0.191	0.189	0	0	0	0	0	0
7:19	0.296	0.276	0.260	0.249	0.239	0.232	0.225	0.221	0.218	0.043	0.056	0	0	0	0
7:20	0.867	0.153	0.125	0.114	0.107	0.102	0.099	0.097	0.095	0.095	0.102	0.094	0.087	0	0
7:21	0.866	0.226	0.187	0.170	0.158	0.150	0.143	0.139	0.136	0.134	0.133	0.132	0.132	0.132	0
7:22	0.863	0.274	0.243	0.226	0.213	0.203	0.195	0.189	0.185	0.181	0.179	0.178	0.177	0.147	0.177
7:23	0.859	0.346	0.302	0.282	0.266	0.254	0.244	0.238	0.232	0.228	0.225	0.223	0.222	0.201	0.221
7:24	0.491	0.454	0.421	0.400	0.384	0.371	0.358	0.350	0.344	0.339	0.335	0.333	0.331	0.330	0.329

7:25	0.866	0.899	0.497	0.419	0.351	0.321	0.289	0.277	0.267	0.261	0.258	0.255	0.253	0.265	0.252
7:26	0.876	0.899	0.474	0.456	0.437	0.412	0.378	0.362	0.347	0.339	0.334	0.331	0.329	0.331	0.327
7:27	0.802	0.899	0.519	0.502	0.493	0.486	0.449	0.434	0.415	0.402	0.396	0.392	0.389	0.388	0.387
7:28	0.692	0.900	0.900	0.900	0.900	0.900	0.779	0.778	0.172	0.151	0.144	0.141	0.139	0.139	0.138
7:29	0.632	0.791	0.900	0.900	0.900	0.900	0.414	0.410	0.376	0.354	0.343	0.335	0.332	0.330	0.329
7:30	0.606	0.653	0.827	0.900	0.900	0.900	0.854	0.854	0.742	0.257	0.237	0.227	0.222	0.220	0.219
7:31	0.894	0.622	0.702	0.900	0.815	0.900	0.862	0.862	0.798	0.362	0.336	0.320	0.314	0.310	0.308
7:32	0.900	0.900	0.523	0.900	0.729	0.886	0.894	0.882	0.821	0.419	0.412	0.396	0.390	0.386	0.384
7:33	0.900	0.900	0.900	0.900	0.825	0.900	0.900	0.900	0.867	0.768	0.768	0.294	0.286	0.280	0.278
7:34	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.859	0.819	0.815	0.373	0.371	0.365	0.363
7:35	0	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.805	0.432	0.427	0.423	0.419
7:36	0	0.122	0.060	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.473	0.461	0.456	0.453
7:37	0	0.065	0	0	0.646	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.148	0.144
7:38	0	0	0	0	0.115	0.089	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.300	0.299
7:39	0	0	0	0	0	0	0.081	0.093	0.651	0.900	0.900	0.900	0.900	0.900	0.312
7:40	0	0	0	0	0	0	0	0	0.093	0.791	0.900	0.900	0.900	0.900	0.442
7:41	0	0	0	0	0	0	0	0	0	0	0	0.263	0.900	0.900	0.553
7:42	0	0	0	0	0	0	0	0	0	0	0	0	0	0.900	0.900
7:43	0	0	0	0	0	0	0	0	0	0	0	0	0	0.172	0.900
7:44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.620
7:45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.356
7:46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.042
7:47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6.4 Comparison of different distributions of density and toll schemes

Location	(I) Toll and exponential density			(II) Uniform density			(III) Fixed toll		
	Auto	Railway	Equilibrium cost	Auto	Railway	Equilibrium cost	Auto	Railway	Equilibrium cost
1	39.14	0.03	27.81	69.90	13.43	28.97	35.20	0.44	23.67
2	41.01	0.03	27.05	63.58	19.75	28.33	37.10	0.44	23.34
3	42.56	0.43	26.27	63.00	20.33	27.53	39.09	0.44	23.06
4	41.20	3.84	25.70	58.29	25.05	26.74	41.29	0.35	22.83
5	38.80	8.39	25.11	57.18	26.15	25.93	43.48	0.37	22.55
6	37.22	12.22	24.33	49.85	33.48	25.13	45.80	0.37	21.88
7	36.62	15.17	23.54	52.02	31.31	24.33	48.29	0.34	21.61
8	36.38	17.88	22.76	56.39	26.95	23.51	50.85	0.36	21.11
9	39.09	17.76	21.96	59.71	23.63	22.71	39.13	14.81	20.92
10	44.32	15.24	21.15	51.86	31.47	21.86	36.13	20.67	20.58
11	47.09	15.30	20.31	44.85	38.48	21.02	30.44	29.38	19.91
12	44.57	20.79	19.50	28.51	54.82	20.24	28.57	34.43	19.23
13	31.93	36.55	18.68	22.05	61.28	19.47	30.96	35.39	18.70
14	22.13	49.61	17.89	18.06	65.28	18.58	2.99	66.88	18.03
15	34.54	40.62	17.17	10.15	73.18	17.81	73.08	0.50	17.53
16	37.06	41.68	16.38	7.51	75.82	16.92	0.24	77.25	16.25
17	21.30	61.20	15.58	4.61	78.73	16.04	2.69	78.93	15.64
18	30.77	55.66	14.75	3.85	79.48	15.11	0.09	85.87	14.72
19	10.15	80.40	13.91	9.56	73.77	14.02	32.04	58.48	14.33
20	12.61	82.25	12.92	17.85	65.49	13.13	0.06	95.27	12.70
21	9.93	89.46	12.07	15.29	68.04	11.95	99.76	0.64	11.58
22	23.44	80.68	11.13	11.35	71.98	10.86	41.99	63.75	10.42
23	1.18	107.90	10.02	0.04	83.29	9.31	0.06	111.29	9.48
24	11.93	102.35	9.14	3.63	79.71	8.59	7.53	109.74	8.98
25	24.01	95.72	8.20	15.25	68.08	7.72	28.84	94.67	8.20

26	7.96	117.47	7.01	0.22	83.11	6.43	30.66	99.41	7.35
27	18.33	113.08	5.90	15.25	68.08	5.48	11.28	125.70	6.46
28	7.58	130.10	4.66	11.68	71.65	4.36	33.09	111.17	5.39
29	23.54	120.70	3.64	20.21	63.13	3.34	29.45	122.48	3.98
30	36.33	114.78	2.46	36.00	47.34	2.23	56.51	103.49	2.84
Sum	852.72	1647.28		877.72	1622.28		956.70	1543.30	
Total time			2.18×10 ⁴			2.85×10 ⁴			2.29×10 ⁴
Total cost			3.32×10 ⁴			4.15×10 ⁴			3.16×10 ⁴

To demonstrate the effects of toll scheme and urban density distribution on the performance of the multimodal transportation system, the previous equilibrium results based on both optimal toll scheme and urban density distribution is compared with the other two cases, wherein toll is charged with fixed rate at fixed location (toll collected by physical gantries) across the time horizon, and a certain uniform urban density distribution is assumed. The comparison results are shown in Table 6.4. For illustration purpose, the case with both toll and negative exponential density distribution is labeled as case (I), and the cases with uniform density and fixed toll are labeled as cases (II) and (III) respectively. Here the fixed toll is assumed to be paid at the fixed section 20. In this model, the objective is to minimize the total travel time along the corridor, which is shown in the second row from the bottom of Table 6.4. One can observe that case (I) results in lowest total travel time while uniform density case brings in highest. The gap between cases (I) and (II) is more prominent than that between cases (I) and (III), which implies that urban density distribution, as an urban planning factor, affects the total travel time of the transportation system more significantly.

Next, a simple sensitivity analysis is made with respect to the population density distribution to further illustrate how it affects the transportation system performance along the corridor. Assume the total population and trip generation rate along the corridor are fixed at 25000 persons and 0.1 respectively as stated in previous content, and the population density still follows the negative exponential distribution with highest density at the last section Q_{30} . Therefore, the total travel demand along the corridor is 2500 veh. Different values of Q_{30} are considered for numerical experiments to check how urban density affects the transportation system performance. When the density at city center increases, i.e., more population resides near CBD, more travelers choose rail transit and the total travel time of the entire system reduces. However, high density at city center raises the rail transit congestion effects and brings more auto users. The traffic congestion propagates on highway and imposes the auto users along the corridor longer travel time despite departure time choice.

Figure 6.7 Total travel times with different density distributions

Figure 6.7 shows the total travel times along the corridor with different density distributions, and the optimal density occurs at $Q_{30} = 1511.1$. The obtained optimal urban population density would provide urban planners a guideline for integrating the planning of urban development with best urban transport system performance, which in practice can be realized by implementing urban policies, like floor area ratio regulation and spatially-variable excise subsidies (or taxes) on housing.

6.5 Summary

This chapter investigates the problem on how the urban density distribution may affect the performance of the transportation system in a multimodal travel corridor and what the best density distribution profile is if optimal transportation system performance is desired. The time and distance based toll scheme is also considered. A mathematical programming approach is applied, wherein the flow congestion dynamics and trip timing equilibrium conditions are fully captured. The model formulation framework developed in this chapter is assumed to be able to contribute on modeling and solving more related research problems on integrated urban development and transportation planning. Despite this, there are several limitations. For example, some other realistic distribution patterns would be considered in the future study; more realistic and sophisticated traffic flow theory could be applied in the future work.

CHAPTER 7

A GLOBAL OPTIMUM APPROXIMATION MODEL FOR CONTINUOUS NETWORK DESIGN PROBLEM BASED ON GENERALIZED GEOMETRIC PROGRAMMING

(Note: this chapter includes part of contents that has been presented in the conferences:

Du, B. and Wang, Z.W. (2014) "An efficient solution method for the continuous network design problem with stochastic user equilibrium", The 19th International Conference of Hong Kong Society for Transportation Studies, Hong Kong, China, 2014.

Du, B. and Wang, Z.W. (2014) "A global approximation model for the continuous network design problem with deterministic user equilibrium", INFORMS 2014 Annual Meeting, San Francisco, USA, 2014.)

7.1 Introduction

In previous contents, it is seen that many studies have been done on the linear travel corridor. This chapter extends the research focus from linear corridor to a general network, and mainly aims to develop an efficient method to solve one of the most difficult problems in transportation fields, CNDP.

In order to satisfy the growing travel demand and reduce the traffic congestion, the CNDP is often proposed to optimize the road network performance by expanding some existing links' capacities. CNDP is recognized as one of the most difficult and challenging problems in transportation in terms of the fact that it is non-convex and non-differentiable, therefore most of the solution methods are based on heuristic algorithms (Meng et al., 2001). One can refer to Section 2.7.1 for more literature review on CNDP.

Although comprehensive models and solution methods have been developed to handle CNDP, there are still several key issues under investigation. Most of the previous studies consider travelers' behavior following deterministic UE and limited attention is paid to SUE model, which is more reasonable than deterministic UE in realistic application. The main obstacles to solve SUE based CNDP stem

from its nonconvex property and the Logit function. Moreover, most of the existing solution methods can only handle CNDP with either UE or SUE constraints, but not both. Based on the GGP framework, this chapter develops a GGP model to find an approximated global optimal solution to CNDP, which is applicable to both deterministic and stochastic UE cases.

7.2 Model formulation

7.2.1 Generalized geometric programming

Currently, the GP modeling approach has been proved to be solved efficiently and reliably by many methods, even for large scale problems, and GP has been used widely in engineering, like digital circuit design, resource allocation in communication and network systems, inventory control, etc. (Scott et al., 2004; Boyd et al., 2005; Chiang, 2005). For more information on GP and its extensions and applications, some useful studies can be referred to, such as Peterson (1978); Wang et al. (2004b); Liu (2006); Samanta and Majumder (2006); Boyd et al. (2007); Tsai et al. (2007). Although GP modeling approach has been well investigated rigorously and widely used to various application fields, it is rarely applied to transportation area.

A standard GP model is an optimization problem in a special form as introduced previously in (2-47), and its commonly used solution method is stated in (2-48).

In this chapter, in attempts to solve CNDP, an extended GGP model in a special form (7-1) is given as below, which is applicable to both UE and SUE based CNDP.

$$\begin{aligned}
 & \min x_0 \\
 & s.t. \quad \frac{f_0(x)}{g_0(x) + x_0} \leq 1 \\
 & \quad \frac{f_i(x)}{g_i(x)} \leq 1, \quad i = 1, 2, \dots, M \\
 & \quad \frac{f_j(x)}{g_j(x)} = 1, \quad j = 1, 2, \dots, N \\
 & \quad 0 < x_k^{LB} \leq x_k, \quad k = 1, 2, \dots, K
 \end{aligned} \tag{7-1}$$

where x_k^{LB} is lower bounds of variable x_k , $k = 1, 2, \dots, K$, f and g are posynomial functions.

7.2.2 GGP based CNDP model with UE constraints

In this section, CNDP with UE constraints is formulated as a single-level MPCC model as below (Wang and Lo, 2010).

$$\begin{aligned}
 \min \quad & Z = \sum_{w \in W} q^w \pi^w + \lambda \sum_{a \in A} g_a(y_a) \\
 \text{s.t.} \quad & \underline{y}_a \leq y_a - y_a^0 \leq \overline{y}_a, \quad \forall a \in A \\
 & \sum_{p \in P} f_p^w = q^w, \quad \forall w \in W \\
 & C_p^w - \pi^w \geq 0, \quad \forall w \in W, \forall p \in P \\
 & f_p^w (C_p^w - \pi^w) = 0, \quad \forall w \in W, \forall p \in P \\
 & C_p^w = \sum_{a \in A} \delta_{ap}^w t_a(x_a, y_a), \quad \forall w \in W, \forall p \in P \\
 & x_a = \sum_{w \in W} \sum_{p \in P} \delta_{ap}^w f_p^w, \quad \forall a \in A \\
 & t_a(x_a, y_a) = \alpha_a + \beta_a x_a^4 y_a^{-4}, \quad \forall a \in A
 \end{aligned} \tag{7-2}$$

where x_a is traffic flow on arc a , y_a^0 and y_a are capacity before and after enhancement on arc a respectively, $\underline{y}_a$ and $\overline{y}_a$ are lower and upper bound of capacity enhancement on arc a respectively, $t_a(x_a, y_a)$ is travel time through arc a , which is a function of x_a and y_a , α_a and β_a are parameters of travel time function, $g_a(y_a)$ is construction cost of arc capacity enhancement, which is a function of y_a , $\forall a \in A$, A is the set of arcs. f_p^w and C_p^w are traffic flow and total travel time on path P between OD pair w , $\forall w \in W$, W is the set of OD pairs, $\forall p \in P$, P is the set of paths. q^w and π^w are the travel demand and equilibrium cost between OD pair w respectively, $\forall w \in W$, δ_{ap}^w is the link-path incidence factor, which is 1 if arc a is on path p between OD pair w , otherwise 0, $\forall a \in A, \forall w \in W, \forall p \in P$. λ is the relative weight of total capacity enhancement cost in the objective function.

In consideration of all the equations in model (7-2) are multinomial functions, standard GP approach cannot be applied directly, however, the CNDP-UE model (7-2) can be transformed to an equivalent GGP based model in the form of (7-1). Recalling the GGP model (7-1) requires that all variables positive, while the arc traffic volume x_a , as well as path traffic volume f_p^w , might be zero if the corresponding arcs or paths are not chosen in the equilibrium pattern. In order to

satisfy the requirement of positive variables, $f_p^{w'} = f_p^w + M_p^w$ is used instead of f_p^w in (7-2), where M_p^w is a positive constant. Similarly, $x_a' = x_a + M_a$ is substituted for x_a with a positive constant $M_a = \sum_{w \in W} \sum_{p \in P} \delta_{ap}^w M_p^w$. The construction cost $g_a(y_a)$ is considered as a linear function $g_a(y_a) = d_a(y_a - y_a^0)$ as in previous studies (Friesz et al., 1992; Meng et al., 2001; Wang and Lo, 2010), but nonlinear function is also accepted. By adding a new additional variable Z , an equivalent GGP based CNDP-UE model is given as below.

$$\begin{aligned}
 & \min Z \\
 & s.t. \quad \frac{\sum_{w \in W} q^w \pi^w + \lambda \sum_{a \in A} d_a y_a}{\lambda \sum_{a \in A} d_a y_a^0 + Z} \leq 1 \\
 & \quad \frac{1}{y_a + y_a^0} y_a \leq 1, \quad \forall a \in A \\
 & \quad \frac{1}{q^w + \sum_{p \in P} M_p^w} \sum_{p \in P} f_p^{w'} = 1, \quad \forall w \in W \\
 & \quad \pi^w C_p^{w-1} \leq 1, \quad \forall w \in W, \forall p \in P \\
 & \quad \frac{f_p^{w'} C_p^w + M_p^w \pi^w}{f_p^{w'} \pi^w + M_p^w C_p^w} = 1, \quad \forall w \in W, \forall p \in P \\
 & \quad C_p^{w-1} \sum_{a \in A} \delta_{ap}^w t_a = 1, \quad \forall w \in W, \forall p \in P \\
 & \quad \frac{\sum_{w \in W} \sum_{p \in P} \delta_{ap}^w f_p^{w'} + M_a}{\sum_{w \in W} \sum_{p \in P} \delta_{ap}^w M_p^w + x_a'} = 1, \quad \forall a \in A \\
 & \quad \frac{\beta_a x_a'^4 y_a^{-4} + 6\beta_a M_a^2 x_a'^2 y_a^{-4} + \beta_a M_a^4 y_a^{-4} + \alpha_a}{4\beta_a M_a x_a'^3 y_a^{-4} + 4\beta_a M_a^3 x_a' y_a^{-4} + t_a} = 1, \quad \forall a \in A
 \end{aligned} \tag{7-3}$$

7.2.3 GGP based CNDP model with SUE constraints

In terms of the previous notations, the model for CNDP under SUE can be expressed as below (Daganzo, 1982; Davis, 1994).

$$\begin{aligned}
\min \quad & Z = \sum_{a \in A} t_a(x_a, y_a) \cdot x_a + \lambda \sum_{a \in A} g_a(y_a) \\
s.t. \quad & \underline{y}_a \leq y_a - y_a^0 \leq \overline{y}_a, \quad \forall a \in A \\
& x_a = \sum_{w \in W} q^w \sum_{p \in P} \delta_{ap}^w \rho_p^w(C_p^w), \quad \forall a \in A \\
& C_p^w = \sum_{a \in A} \delta_{ap}^w t_a(x_a, y_a), \quad \forall w \in W, \forall p \in P \\
& t_a(x_a, y_a) = \alpha_a + \beta_a x_a^4 y_a^{-4}, \quad \forall a \in A \\
& \rho_p^w(C_p^w) = \frac{e^{-\theta C_p^w}}{\sum_{p' \in P} e^{-\theta C_{p'}^w}}, \quad \forall w \in W, \forall p \in P
\end{aligned} \tag{7-4}$$

where ρ_p^w is the probability of choosing path p between OD pair w , which is a function of C_p^w , and θ is a parameter of the Logit model. In a similar way, this CNDP-SUE model can also be transformed to an equivalent GGP based formulation. However, the Logit function cannot be addressed directly by GGP, therefore it must be transformed to posynomial form. Here an additional new variable u_a is used to represent $e^{-\theta t_a} = e^{-\theta(\alpha_a + \beta_a x_a^4 y_a^{-4})}$, thus

$$\log u_a = -\theta(\alpha_a + \beta_a x_a^4 y_a^{-4}), \quad \forall a \in A \tag{7-5}$$

$$e^{-\theta C_p^w} = e^{-\theta \sum_{a \in A} \delta_{ap}^w t_a} = \prod_{a \in A} u_a^{\delta_{ap}^w} \tag{7-6}$$

$$\rho_p^w(C_p^w) = \frac{\prod_{a \in A} u_a^{\delta_{ap}^w}}{\sum_{p' \in P} \prod_{a \in A} u_a^{\delta_{ap'}^w}}, \quad \forall w \in W, \forall p \in P \tag{7-7}$$

$$x_a = \sum_{w \in W} q^w \sum_{p \in P} \delta_{ap}^w \frac{\prod_{a \in A} u_a^{\delta_{ap}^w}}{\sum_{p' \in P} \prod_{a \in A} u_a^{\delta_{ap'}^w}}, \quad \forall a \in A \tag{7-8}$$

To handle logarithm function in Eq. (7-5), an approximation is used here as described in Boyd et al. (2007).

$$\log u_a \approx W(u_a^{\frac{1}{W}} - 1) \tag{7-9}$$

where W is a large positive constant, then Eq. (7-5) can be approximated as follows:

$$W(u_a^{\frac{1}{W}} - 1) = -\theta(\alpha_a + \beta_a x_a^4 y_a^{-4}) \tag{7-10}$$

To sum up, by adding a new additional variable z , the GGP based CNDP-SUE model can be formulated as follows:

$$\begin{aligned}
 & \min Z \\
 & s.t. \quad \frac{\sum_{a \in A} \alpha_a x_a + \sum_{a \in A} \beta_a x_a^5 y_a^{-4} + \lambda \sum_{a \in A} d_a y_a^2 + \lambda \sum_{a \in A} d_a y_a^{0^2}}{2\lambda \sum_{a \in A} d_a y_a^0 y_a + Z} \leq 1 \\
 & \quad \frac{1}{y_a + y_a^0} y_a \leq 1, \quad \forall a \in A \\
 & \quad \frac{\sum_{w \in W} q^w \sum_{p \in P} \delta_{ap}^w \prod_{a \in A} u_a^{\delta_{ap}^w}}{x_a \sum_{p \in P} \prod_{a \in A} u_a^{\delta_{ap}^w}} = 1, \quad \forall a \in A \\
 & \quad \frac{W}{W - \theta \alpha_a} u_a^{\frac{1}{W}} + \frac{\theta \beta_a}{W - \theta \alpha_a} x_a^4 y_a^{-4} = 1, \quad \forall a \in A
 \end{aligned} \tag{7-11}$$

7.3 Solution algorithm

A condensation procedure based on successive monomial approximation technique is introduced first, which is useful to handle the GGP model. The strength of this successive monomial approximation technique is that it can handle posynomial equality constraints in an efficient way, which makes it applicable to a wide application fields, even large scale problems with posynomial equality constraints (Burns, 1987).

Given a posynomial function $f(x) = \sum_{i=1}^N h_i(x)$, where $h_i(x)$ is monomial, the condensation process is defined as

$$C[f(x), x'] = \prod_{i=1}^N \left[\frac{h_i(x)}{\delta_i} \right]^{\delta_i} \tag{7-12}$$

where $\delta_i = \frac{h_i(x')}{f(x')}$, $i = 1, 2, \dots, N$, and $C[f(x), x']$ is a monomial approximation of $f(x)$

such that $C[f(x), x']|_{x=x'} = f(x')$.

Based on the successive monomial approximation technique, the GGP model (7-1) can be converted to standard GP form so that solutions can be found.

The general procedure to solve the GGP model (7-1) is stated as follows:

[Solution procedure]

Step 1. Choose initial values of the variables as x' and apply the condensation procedure (7-12) to the denominators in the constraints of model (7-1), thus all of the constraints should be posynomial functions.

Step 2. Set new variables $x'' = x'$ initially and reapply the condensation procedure to the posynomial constraints, thus all of the constraints should be monomial functions as below.

$$\begin{aligned}
 & \min x_0 \\
 & s.t. \quad C_i \prod_{k=1}^K x_k^{A_{ik}} \leq 1, \quad i = 0, 1, \dots, M \\
 & \quad \quad C_j \prod_{k=1}^K x_k^{A_{jk}} = 1, \quad j = 1, 2, \dots, N \\
 & \quad \quad 0 < x_k^{LB} \leq x_k, \quad k = 0, 1, \dots, K
 \end{aligned} \tag{7-13}$$

Step 3. Set new variables $y_k = \log \left[\frac{x_k}{x_k^{LB}} \right]$, $k = 0, 1, \dots, K$, and transform the model (7-13) into a LP model as follows:

$$\begin{aligned}
 & \min y_0 \\
 & s.t. \quad \log \left[C_i \prod_{k=0}^K (x_k^{LB})^{A_{ik}} \right] \leq \sum_{k=0}^K (-A_{ik} y_k), \quad i = 0, 1, \dots, M \\
 & \quad \quad \log \left[C_j \prod_{k=0}^K (x_k^{LB})^{A_{jk}} \right] = \sum_{k=0}^K (-A_{jk} y_k), \quad j = 1, 2, \dots, N \\
 & \quad \quad 0 \leq y_k, \quad k = 0, 1, \dots, K
 \end{aligned} \tag{7-14}$$

Step 4. Solve the LP model (7-14) to obtain the solution y'' and corresponding $x_k'' = x_k^{LB} e^{y_k''}$, $k = 0, 1, \dots, K$.

Step 5. Find the most violated inequality posynomial constraints in Step 1 based on the evaluation at x'' .

$$V = \max \left(\left. \frac{f_0(x)}{C[g_0(x) + x_0, x']} \right|_{x=x''}, \left. \frac{f_i(x)}{C[g_i(x), x']} \right|_{x=x''}, \quad i = 1, 2, \dots, M \right) \tag{7-15}$$

Let R represent a small positive tolerance, if $V > 1 + R$, then recondense the most violated posynomial constraint using the solution x'' and transform it to LP form

based on Step 3, and add this linear constraint to the current LP model, then return to Step 4. Mathematically, this procedure is actually an implementation of the Kelly cutting plane procedure for solving convex problem (Kelley, 1960). Based on certain conditions, this repeated process results in a sequence of GP solutions converging to a Kuhn-Tucker optimum solution (Avriel and Williams, 1970).

If $V \leq 1+R$, check the convergence criterion by calculating the difference D between x' and x'' .

$$D = \sum_{k=0}^K \left(\frac{x_k'' - x_k'}{x_k'} \right)^2 \quad (7-16)$$

If D is not small enough, then set $x' = x''$ and return to Step 1, otherwise end the procedure with x' as the solution to the GGP model (7-1).

Proposition 7.1 *The logarithm of the condensation on x' of a posynomial function is equivalent to the linear part of the corresponding Taylor's series expansion, and the above [Solution procedure] is equivalent to a combination of solving cutting plane problems in log space for the inequality constraints and using Newton-Raphson method in log space to handle equality constraints (Ypma, 1995).*

Proof. Refer to Burns (1987) for complete proof since the proof contents are identical. $\square$

Based on the [Solution procedure] above, the GGP based CNDP-UE model (7-3) and CNDP-SUE model (7-11) can be solved with approximated global optimums.

7.4 Numerical experiments

7.4.1 Case I: GGP based CNDP with UE

For comparison, a benchmark network adopted by Suwansirikul et al. (1987); Friesz et al. (1992); Meng et al. (2001); Chiou (2005); Wang and Lo (2010) with same data input is applied in this section, as shown in Figure 7.1. The travel demand from nodes 1 to 6 is 5, and the one from 6 to 1 is 10, the upper bound of each link capacity expansion is 10, and the other values of parameters are shown in Table 7.1.

Figure 7.1 16 arcs network

Table 7.1 Data input for the 16 arcs network

Arc a	α_a	β_a	y_a^0	d_a
1	1.00	10.00	3.00	2.00
2	2.00	5.00	10.00	3.00
3	3.00	3.00	9.00	5.00
4	4.00	20.00	4.00	4.00
5	5.00	50.00	3.00	9.00
6	2.00	20.00	2.00	1.00
7	1.00	10.00	1.00	4.00
8	1.00	1.00	10.00	3.00
9	2.00	8.00	45.00	2.00
10	3.00	3.00	3.00	5.00
11	9.00	2.00	2.00	6.00
12	4.00	10.00	6.00	8.00
13	4.00	25.00	44.00	5.00
14	2.00	33.00	20.00	3.00
15	5.00	5.00	1.00	6.00
16	6.00	1.00	4.50	1.00

Table 7.2 Numerical results with different approaches

Arcs	MINOS	H-J	EDO	IOA	SA	MILP	GGP
y1							
y2							
y3		1.20	0.13				
y4							
y5							
y6	6.58	3.00	6.26	6.95	3.16	4.41	3.40
y7							
y8							
y9							
y10							
y11							
y12							
y13							
y14							
y15	7.01	3.00	0.13	5.66			
y16	0.22	2.80	6.26	1.79	6.72	7.70	5.04
Total Cost	211.25	215.08	201.84	210.86	198.10	197.88	194.22

The comparison among different approaches is shown in Table 7.2, which shows that the GGP based model and algorithm achieve better solution result than the other methods.

7.4.2 Case II: GGP based CNDP with SUE

A test network with 6 nodes, 7 links and 4 O-D pairs used in Davis (1994), shown in Figure 7.2, is adopted here to demonstrate the validity of the GGP based CNDP model with SUE constraints. The input data and the local solutions of the network achieved in Davis (1994) are shown in Table 7.3 and Table 7.4 respectively.

Figure 7.2 7 arcs network

Table 7.3 The O-D demands of the 7 arcs network

w	O-D pair	q^w
1	O1-D1	500
2	O1-D2	400
3	O2-D1	400
4	O2-D2	600

Table 7.4 Data input and numerical results of the 7 arcs network

Arc a	α_a	β_a	y_a^0	d_a	y	
					SQP	GRG2
1	1.00	0.15	200	3.00	227.80	227.80
2	0.25	0.0375	300	2.00	307.15	307.15
3	0.25	0.0375	700	1.00	708.20	708.14
4	0.25	0.0375	300	2.00	302.53	302.53
5	0.25	0.0375	300	2.00	318.95	318.94
6	0.25	0.0375	300	2.00	318.95	318.94
7	1.00	0.15	200	3.00	220.78	220.77
Total cost					3992.7	3992.7
CPU secondes					6.1	7.0

Since the values of the parameter λ and θ in model (7-4) are not specified in Davis (1994), it is not possible to compare the result directly. However, numerical experiment can be conducted to verify the validity of the model and solution method. In this section, set $\lambda=0.01$ and $\theta=1$, and assume the tolerance criteria $R=1\times 10^{-5}$ and convergence criteria $D=1\times 10^{-4}$. The numerical results of GGP model are shown in Table 7.5 as below.

Table 7.5 Numerical results of GGP model

Link	Capacity before expansion	Capacity after expansion	SUE flow
1	200	241.8	296.3
2	300	345.6	603.7
3	700	768.4	1253.1
4	300	345.6	603.7
5	300	357.9	649.4
6	300	357.8	649.4
7	200	261.1	350.6
Total cost		3568.2	
CPU secondes		5.6963	
Convergence		$D = 7.0731 \times 10^{-5}$	

7.5 Summary

The GP method provides an alternative approach to CNDP and other transportation related research fields with its computational efficiency. In this chapter two GGP based CNDP model with UE and SUE are developed respectively. Although these GGP models require strict form and various constraints, which is difficult to solve directly, by applying a successive monomial approximation technique and logarithmic transformation technique, the GGP models are transformed to LP formulations, which can be solved with global optimum easily. Numerical experiments of both UE and SUE cases are conducted to testify the validity of the GGP based models and solution method, and promising results are obtained. These models can be applied to large scale CNDP problems and other optimization problems of similar nature. Although it is very efficient to solve GGP after transformation, it also has some disadvantages. As any Newton-like method, smooth convergence cannot be guaranteed for all starting points. With more research effort, GGP can be explored with more capabilities to handle more transportation related problems.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Summary and conclusions

Using the well developed complementarity theory and new rising powerful modeling paradigm of DCS, a series of studies on the corridor problem in both static and dynamic scenarios were conducted to analyze the interactions between equilibrium patterns with urban densities and toll scheme respectively. In addition, by using GGP method, one of the most difficult issues in transportation, CNDP, was solved with promising results.

The following conclusions can be made from the studies:

- (1) DCS, or a more general paradigm DVI, showed good ability and flexibility in modeling both static and dynamic multimodal corridor problems. This modeling framework not only fully described the commuter's choice behavior but also captured the resultant dynamic flow and congestion effect. Moreover, heterogeneity, travel time reliability, and so on, can also be considered in the model.
- (2) By applying the DCS modeling approach, some properties of the rigorous mathematical formulation could be derived, such as the solution existence, formulation characteristics, and so on.
- (3) This modeling framework was quite adaptable to develop continuous-time models, whose continuity, differentiability, or high-order smoothness are absent in discrete-time models.
- (4) The static scenarios showed that P&R was preferred for commuters living close to the boundary of the corridor, railway was favorite for users living close to CBD, and auto mode was used in the middle of the corridor.
- (5) No matter single P&R or continuous P&R facilities along the corridor, there existed optimal amount and location of the P&R facility to optimize the equilibrium pattern and corresponding generalized travel cost.

- (6) Various distributions of the population density were applicable to the models in this study, which showed obvious influence on the equilibrium results. Among uniform, linear and negative exponential population distribution, the last one usually led to the best equilibrium results.
- (7) Higher travel time reliability usually resulted in higher travel time, high generalized travel cost, as well as less mode usage for auto commuters.
- (8) Heterogeneous commuters with different characteristics showed different equilibrium results. For example, commuters with higher unit VOTs often had higher generalized travel cost; commuters with later preferred arrival time often resulted in later departure time range.
- (9) When considering no late arrival, the departure set varied with different ratios of unit early arrival penalty and unit travel time cost. The departure set along the corridor and the departure width at first location became narrow when the ratio increased.
- (10) When considering various distributions of the population density, like uniform, linear and negative exponential, the departure set varied with different distributions. Uniform distribution resulted in widest departure set, while negative exponential distribution led to narrowest. As for the negative exponential distribution, the departure set became narrower as the value of the parameter ϕ became larger.
- (11) Optimal toll scheme and population density distribution were obtained simultaneously based on dynamic equilibrium constraints, which provided useful information for transport and urban planners.
- (12) Comparison between fixed and time-distance based toll scheme showed that the latter scheme led to more reasonable highway usage and less travel time.
- (13) Although it was strict to formulate GGP model, it was quite efficient to solve, and global approximation solution could be guaranteed. For both UE and SUE based CNDPs, the promising results proved the validity of the GGP model and the efficiency of the solution method.

8.2 Recommendations for future work

The present work attempted to investigate the corridor problem with its extensions both in spatial and spatial-temporal dimensions, as well as UE and SUE based CNDPs. Some recommendations for further studies are proposed as follows.

- (1) In the modeling of P&R services along linear travel corridor, a first order BPR function was used to simplify the problem, which can be relaxed to a more reasonable and higher order function.
- (2) In the modeling of P&R services along linear travel corridor, the solution algorithms were efficient for small and middle size problems, but for large size problem, more efficient and robust algorithms should be developed.
- (3) In the modeling of P&R services along linear travel corridor, the P&R services were assumed to be single or continuously distributed along the corridor, while the practical situation may be more than one but absolutely not possible to be continuously available. Therefore, although it is not a key issue in this thesis, determining the optimal amount and locations of the P&R facilities along the corridor is an important issue in future work. Moreover, limited capacity of P&R facility should be considered to make the model more reasonable and applicable in reality.
- (4) When handling the corridor problem in spatial and temporal dimensions, the equilibrium was achieved based on instantaneous generalized travel cost. More accurate dynamic model should be developed to capture the real generalized travel cost.
- (5) Multimodal choice and toll strategy can be considered in the corridor problem in spatial-temporal dimensions, which should make the model more meaningful in practical application.
- (6) Linear corridor is just a simplified form of the general traffic network, extension from corridor to network scope is a big step to move forward.
- (7) Although promising results were obtained on both DUE and SUE based CNDPs, it still needs to test large networks.
- (8) GGP modeling approach has been proved to be efficient in dealing with CNDP, which should be extended from CNDP to DNDP and MNDP to enlarge its application scope.

REFERENCES

- Abdulaal, M. and LeBlanc, L.J. (1979) "Continuous equilibrium network design models", Transportation Research Part B: Methodological, Vol. 13, No. 1, pp. 19-32.
- Arnott, R., de Palma, A. and Lindsey, R. (1990a) "Departure time and route choice for the morning commute", Transportation Research Part B: Methodological, Vol. 24, No. 3, pp. 209-228.
- Arnott, R., de Palma, A. and Lindsey, R. (1990b) "Economics of a bottleneck", Journal of Urban Economics, Vol. 27, No. 1, pp. 111-130.
- Arnott, R. and DePalma, E. (2011) "The corridor problem: preliminary results on the no-toll equilibrium", Transportation Research Part B: Methodological, Vol. 45, No. 5, pp. 743-768.
- Avriel, M. and Williams, A. (1970) "Complementary geometric programming", SIAM Journal on Applied Mathematics, Vol. 19, No. 1, pp. 125-141.
- Ban, X.G., Ferris, M.C., Tang, L. and Lu, S. (2013) "Risk-neutral second best toll pricing", Transportation Research Part B: Methodological, Vol. 48, No., pp. 67-87.
- Ban, X.G. and Liu, H.X. (2009) "A link-node discrete-time dynamic second best toll pricing model with a relaxation solution algorithm", Networks and Spatial Economics, Vol. 9, No. 2, pp. 243-267.
- Ban, X.G., Liu, H.X., Ferris, M.C. and Ran, B. (2006) "A general MPCC model and its solution algorithm for continuous network design problem", Mathematical and Computer Modelling, Vol. 43, No. 5-6, pp. 493-505.
- Ban, X.G., Liu, H.X., Ferris, M.C. and Ran, B. (2008) "A link-node complementarity model and solution algorithm for dynamic user equilibria with exact flow propagations", Transportation Research Part B: Methodological, Vol. 42, No. 9, pp. 823-842.
- Ban, X.G., Pang, J.S., Liu, H.X. and Ma, R. (2012a) "Continuous-time point-queue models in dynamic network loading", Transportation Research Part B: Methodological, Vol. 46, No. 3, pp. 360-380.
- Ban, X.G., Pang, J.S., Liu, H.X. and Ma, R. (2012b) "Modeling and solving continuous-time instantaneous dynamic user equilibria: a differential complementarity systems approach", Transportation Research Part B: Methodological, Vol. 46, No. 3, pp. 389-408.
- Batty, M. and Kim, K.S. (1992) "Form follows function: reformulating urban population density functions", Urban Studies, Vol. 29, No. 7, pp. 1043-1069.
- Beckmann, M.J. (1965) "On optimal tolls for highways, tunnels and bridges. In vehicular traffic science", Proceedings of the Third International Symposium on the Theory of Traffic Flow, Vol., No., pp. 331-341.
- Beckmann, M.J., McGuire, C.B. and Winsten, C.B. (1956) Studies in the Economics of Transportation. Yale University Press, New Haven.
- Bell, M.G.H. and Iida, Y. (1997) Transportation Network Analysis. J. Wiley.

- Ben-Akiva, M., Litinas, N. and Tsunokawa, K. (1985) "Continuous spatial choice: the continuous logit model and distributions of trips and urban densities", Transportation Research Part A: General, Vol. 19, No. 2, pp. 119-154.
- Ben-Ayed, O., Boyce, D.E. and Blair Iii, C.E. (1988) "A general bilevel linear programming formulation of the network design problem", Transportation Research Part B: Methodological, Vol. 22, No. 4, pp. 311-318.
- Berechman, J. and Small, K.A. (1987) Modeling Land Use and Transportation: An Interpretive Review for Growth Areas, Working Paper, Institute of Transportation Studies, University of California, Irvine.
- Bernstein, D. (1994) "Network economics: a variational inequality approach", Transportation Science, Vol. 28, No. 4, pp. 356-358.
- Boyce, D.E. and Janson, B.N. (1980) "A discrete transportation network design problem with combined trip distribution and assignment", Transportation Research Part B: Methodological, Vol. 14, No. 1-2, pp. 147-154.
- Boyd, S.P., Kim, S.J., Patil, D.D. and Horowitz, M.A. (2005) "Digital circuit optimization via geometric programming", Operations Research, Vol. 53, No., pp. 899-932.
- Boyd, S.P., Kim, S.J., Vandenberghe, L. and Hassibi, A. (2007) "A tutorial on geometric programming", Optimization and Engineering, Vol. 8, No. 1, pp. 67-127.
- Bravo, M., Briceno, L., Cominetti, R., Cores, C.E. and Martinez, F. (2010) "An integrated behavioral model of the land-use and transport systems with network congestion and location externalities", Transportation Research Part B: Methodological, Vol. 44, No. 4, pp. 584-596.
- Briceño, L., Cominetti, R., Cortés, C. and Martínez, F.J. (2008) "An integrated behavioral model of land-use and transport system: an extended network equilibrium approach", Networks and Spatial Economics, Vol. 8, No. 2-3, pp. 201-224.
- Brotcorne, L., Labbé, M., Marcotte, P. and Savard, G. (2001) "A bilevel model for toll optimization on a multicommodity transportation network", Transportation Science, Vol. 35, No. 4, pp. 345.
- Brueckner, J.K. (1982) "A note on sufficient conditions for negative exponential population densities", Journal of Regional Science, Vol. 22, No. 3, pp. 353-359.
- Burns, S.A. (1987) "Generalized geometric programming with many equality constraints", International Journal for Numerical Methods in Engineering, Vol. 24, No. 4, pp. 725-741.
- Camlibel, M.K. (2007) "Popov–Belevitch–Hautus type controllability tests for linear complementarity systems", Systems & Control Letters, Vol. 56, No. 5, pp. 381-387.
- Camlibel, M.K., Pang, J.S. and Shen, J.L. (2007) "Lyapunov stability of complementarity and extended systems", SIAM Journal on Optimization, Vol. 17, No. 4, pp. 1056-1101.

- Cantarella, G.E. and Vitetta, A. (2006) "The multi-criteria road network design problem in an urban area", Transportation, Vol. 33, No. 6, pp. 567-588.
- Carey, M. (1992) "Nonconvexity of the dynamic traffic assignment problem", Transportation Research Part B: Methodological, Vol. 26, No. 2, pp. 127-133.
- Carey, M., Bar-Gera, H., Watling, D. and Balijepalli, C. (2014) "Implementing first-in–first-out in the cell transmission model for networks", Transportation Research Part B: Methodological, Vol. 65, No. 0, pp. 105-118.
- Carey, M. and Ge, Y.E. (2007) "Retaining desirable properties in discretising a travel-time model", Transportation Research Part B: Methodological, Vol. 41, No. 5, pp. 540-553.
- Carey, M., Ge, Y.E. and McCartney, M. (2003) "A whole-link travel-time model with desirable properties", Transportation Science, Vol. 37, No. 1, pp. 83.
- Chan, K.S., Lam, W.H.K. and Tam, M.L. (2009) "Real-time estimation of arterial travel times with spatial travel time covariance relationships", Transportation Research Record: Journal of the Transportation Research Board, Vol. 2121, No., pp. 102-109.
- Chang, J.S. and Mackett, R.L. (2006) "A bi-level model of the relationship between transport and residential location", Transportation Research Part B: Methodological, Vol. 40, No. 2, pp. 123-146.
- Chen, B.Y., Lam, W.H.K., Sumalee, A. and Li, Z.L. (2012) "Reliable shortest path finding in stochastic networks with spatial correlated link travel times", International Journal of Geographical Information Science, Vol. 26, No. 2, pp. 365-386.
- Chen, M. and Alfa, A.S. (1991) "A network design algorithm using a stochastic incremental traffic assignment approach", Transportation Science, Vol. 25, No. 3, pp. 215.
- Chen, M. and Bernstein, D.H. (2004) "Solving the toll design problem with multiple user groups", Transportation Research Part B: Methodological, Vol. 38, No. 1, pp. 61-79.
- Chiang, M. (2005) "Geometric programming for communication systems", Communications and Information Theory, Vol. 2, No. 1/2, pp. 1-154.
- Chiou, S.W. (2005) "Bilevel programming for the continuous transport network design problem", Transportation Research Part B: Methodological, Vol. 39, No. 4, pp. 361-383.
- Chung, B.D., Yao, T., Friesz, T.L. and Liu, H. (2012) "Dynamic congestion pricing with demand uncertainty: A robust optimization approach", Transportation Research Part B: Methodological, Vol. 46, No. 10, pp. 1504-1518.
- Clement, L. (1996) Review of Existing Landuse Transport Models. Centre d'études sur les réseaux, les transports, l'urbanisme et les constructions publiques.
- Connors, R.D., Sumalee, A. and Watling, D.P. (2007) "Sensitivity analysis of the variable demand probit stochastic user equilibrium with multiple user-classes", Transportation Research Part B: Methodological, Vol. 41, No. 6, pp. 593-615.

- Cottle, R.W. (1964) Nonlinear Programs with Positively Bounded Jacobians, Ph.D. Thesis, Department of Mathematics, University of California, Berkeley.
- Cottle, R.W., Pang, J.S. and Stone, R.E. (1992) The Linear Complementarity Problem. Academic Press, Cambridge.
- D'Este, G. (1987) "Trip assignment to radial major roads", Transportation Research Part B: Methodological, Vol. 21, No. 6, pp. 433-442.
- Dafermos, S. (1980) "Traffic equilibrium and variational inequalities", Transportation Science, Vol. 14, No. 1, pp. 42-54.
- Daganzo, C. (1997) Fundamentals of Transportation and Traffic Operations. Pergamon.
- Daganzo, C.F. (1982) "Unconstrained extremal formulation of some transportation equilibrium problems", Transportation Science, Vol. 16, No. 3, pp. 332.
- Daganzo, C.F. (1994) "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory", Transportation Research Part B: Methodological, Vol. 28, No. 4, pp. 269-287.
- Daganzo, C.F. (1995) "The cell transmission model, part II: Network traffic", Transportation Research Part B: Methodological, Vol. 29, No. 2, pp. 79-93.
- Dantzig, G.B., Harvey, R.P., Lansdowne, Z.F., Robinson, D.W. and Maier, S.F. (1979) "Formulating and solving the network design problem by decomposition", Transportation Research Part B: Methodological, Vol. 13, No. 1, pp. 5-17.
- Davis, G.A. (1994) "Exact local solution of the continuous network design problem via stochastic user equilibrium assignment", Transportation Research Part B: Methodological, Vol. 28, No. 1, pp. 61-75.
- De Borger, B., Dunkerley, F. and Proost, S. (2007) "Strategic investment and pricing decisions in a congested transport corridor", Journal of Urban Economics, Vol. 62, No. 2, pp. 294-316.
- DePalma, E. and Arnott, R. (2012) "Morning commute in a single-entry traffic corridor with no late arrivals", Transportation Research Part B: Methodological, Vol. 46, No. 1, pp. 1-29.
- Doan, K., Ukkusuri, S. and Han, L. (2011) "On the existence of pricing strategies in the discrete time heterogeneous single bottleneck model", Transportation Research Part B: Methodological, Vol. 45, No. 9, pp. 1483-1500.
- Drezner, Z. and Wesolowsky, G.O. (2003) "Network design: selection and design of links and facility location", Transportation Research Part A: Policy and Practice, Vol. 37, No. 3, pp. 241-256.
- Ekström, J., Sumalee, A. and Lo, H.K. (2012) "Optimizing toll locations and levels using a mixed integer linear approximation approach", Transportation Research Part B: Methodological, Vol. 46, No. 7, pp. 834-854.
- El Esawey, M. and Sayed, T. (2011) "Travel time estimation in urban networks using limited probes data", Canadian Journal of Civil Engineering, Vol. 38, No. 3, pp. 305-318.

- Elizabeth, C. (1978) "A mathematical analysis of a park-and-ride scheme", Transportation Research, Vol. 12, No. 4, pp. 247-252.
- Emberger, G., Shepherd, S.P. and May, A.D. (2008) "The effects of appraisal methodology on the specification of optimal urban transport strategies", International Journal of Sustainable Transportation, Vol. 2, No. 1, pp. 58-75.
- Facchinei, F., Jiang, H. and Qi, L. (1999) "A smoothing method for mathematical programs with equilibrium constraints", Mathematical Programming, Vol. 85, No. 1, pp. 107-134.
- Facchinei, F. and Pang, J.S. (2003) Finite Dimensional Variational Inequalities and Complementarity Problems. Springer-Verlag New York, Inc.
- Faghri, A., Lang, A., Hamad, K. and Henck, H. (2002) "Integrated knowledge-based geographic information system for determining optimal location of park-and-ride facilities", Journal of Urban Planning and Development, Vol. 128, No. 1, pp. 18-41.
- Farhan, B. and Murray, A.T. (2005) "A GIS-based approach for delineating market areas for park and ride facilities", Transactions in GIS, Vol. 9, No. 2, pp. 91-108.
- Farhan, B. and Murray, A.T. (2008) "Siting park-and-ride facilities using a multi-objective spatial optimization model", Computers & Operations Research, Vol. 35, No. 2, pp. 445-456.
- Farvaresh, H. and Sepehri, M.M. (2011) "A single-level mixed integer linear formulation for a bi-level discrete network design problem", Transportation Research Part E: Logistics and Transportation Review, Vol. 47, No. 5, pp. 623-640.
- Fernandez, E., De Cea, J., Florian, M. and Cabrera, E. (1994) "Network equilibrium models with combined modes", Transportation Science, Vol. 28, No. 3, pp. 182.
- Ferris, M.C., Dirkse, S.P. and Meeraus, A. (2002) "Mathematical programs with equilibrium constraints: automatic reformulation and solution via constrained optimization", in: Kehoe, T., Srinivasan, T., Whalley, J. (Eds.), Frontiers in Applied General Equilibrium Modeling. Cambridge University Press, USA, pp. 67-93.
- Ferris, M.C. and Kanzow, C. (2002) "Complementarity and related problems: a survey", Handbook of Applied Optimization, pp. 514-530.
- Ferris, M.C. and Pang, J.S. (1997) "Engineering and economic applications of complementarity problems", SIAM Review, Vol. 39, No. 4, pp. 669-713.
- Fischer, A. (1992) "A special newton-type optimization method", Optimization, Vol. 24, No. 3-4, pp. 269-284.
- Fisk, C.S. and Boyce, D.E. (1983) "Alternative variational inequality formulations of the network equilibrium-travel choice problem", Transportation Science, Vol. 17, No. 4, pp. 454.
- Florian, M. and Los, M. (1979) "Determining intermediate origin-destination matrices for the analysis of composite mode trips", Transportation Research Part B-Methodological, Vol. 13, No. 2, pp. 91-103.
- Foo, T.S. (1997) "Experiences from Singapore's park-and-ride scheme (1975-1996)", Habitat International, Vol. 21, No. 4, pp. 427-443.

- Frank, M. and Wolfe, P. (1956) "An algorithm for quadratic programming", Naval Research Logistics Quarterly, Vol. 3, No. 1-2, pp. 95-110.
- Friesz, T., Bernstein, D. and Kydes, N. (2004) "Dynamic congestion pricing in disequilibrium", Networks and Spatial Economics, Vol. 4, No. 2, pp. 181-202.
- Friesz, T., Bernstein, D., Suo, Z. and Tobin, R. (2001) "Dynamic network user equilibrium with state-dependent time lags", Networks and Spatial Economics, Vol. 1, No. 3-4, pp. 319-347.
- Friesz, T., Kwon, C. and Mookherjee, R. (2007) "A computable theory of dynamic congestion pricing", in: Allsop, R., Bell, M., Benjamin, H. (Eds.), Transportation and Traffic Theory 2007. Elsevier, pp. 1-26.
- Friesz, T.L. (2010) Dynamic Optimization and Differential Games. Springer, New York.
- Friesz, T.L., Anandalingam, G., Mehta, N.J., Nam, K., Shah, S.J. and Tobin, R.L. (1993a) "The multiobjective equilibrium network design problem revisited: A simulated annealing approach", European Journal of Operational Research, Vol. 65, No. 1, pp. 44-57.
- Friesz, T.L., Bernstein, D., Smith, T.E., Tobin, R.L. and Wie, B.W. (1993b) "A variational inequality formulation of the dynamic network user equilibrium problem", Operations Research, Vol. 41, No. 1, pp. 179-191.
- Friesz, T.L., Hsun-jung, C., Mehta, N.J., Tobin, R.L. and Anandalingam, G. (1992) "A simulated annealing approach to the network design problem with variational inequality constraints", Transportation Science, Vol. 26, No. 1, pp. 18.
- Friesz, T.L., Kim, T., Kwon, C. and Rigdon, M.A. (2011a) "Approximate network loading and dual-time-scale dynamic user equilibrium", Transportation Research Part B: Methodological, Vol. 45, No. 1, pp. 176-207.
- Friesz, T.L., Lee, I. and Lin, C.C. (2011b) "Competition and disruption in a dynamic urban supply chain", Transportation Research Part B: Methodological, Vol. 45, No. 8, pp. 1212-1231.
- Friesz, T.L., Luque, J., Tobin, R.L. and Wie, B.W. (1989) "Dynamic network traffic assignment considered as a continuous time optimal control problem", Operations Research, Vol. 37, No. 6, pp. 893-901.
- Friesz, T.L. and Mookherjee, R. (2006) "Solving the dynamic network user equilibrium problem with state-dependent time shifts", Transportation Research Part B: Methodological, Vol. 40, No. 3, pp. 207-229.
- Friesz, T.L., Rigdon, M.A. and Mookherjee, R. (2006) "Differential variational inequalities and shipper dynamic oligopolistic network competition", Transportation Research Part B: Methodological, Vol. 40, No. 6, pp. 480-503.
- Friesz, T.L. and Shah, S. (2001) "An overview of nontraditional formulations of static and dynamic equilibrium network design", Transportation Research Part B: Methodological, Vol. 35, No. 1, pp. 5-21.

- Gajewski, B.J. and Rilett, L.R. (2004) "Estimating link travel time correlation: an application of Bayesian smoothing splines", Journal of transportation and statistics, Vol. 7, No. 23, pp. 53-70.
- Gao, Z., Wu, J. and Sun, H. (2005) "Solution algorithm for the bi-level discrete network design problem", Transportation Research Part B: Methodological, Vol. 39, No. 6, pp. 479-495.
- Garavello, M. and Piccoli, B. (2006) Traffic Flow on Networks: Conservation Laws Models. American Institute of Mathematical Sciences.
- Garcia, R. and Marin, A. (2002) "Parking capacity and pricing in park'n ride trips: a continuous equilibrium network design problem", Annals of Operations Research, Vol. 116, No., pp. 153-178.
- Garcia, R. and Marin, A. (2005) "Network equilibrium with combined modes: models and solution algorithms", Transportation Research Part B: Methodological, Vol. 39, No. 3, pp. 223-254.
- Gartner, N.H., Messer, C.J. and Rathi, A.K. (2001) Traffic Flow Theory: A State-of-the-Art Report. Committee on Traffic Flow Theory and Characteristics (AHB45), USA.
- Gazis, D.C. (2002) Traffic Theory. Springer.
- Giuliano, G. (1989) "Research policy and review 27. New directions for understanding transportation and land use", Environment and Planning A, Vol. 21, No. 2, pp. 145-159.
- Goldner, W. (1971) "The Lowry model heritage", Journal of the American Institute of Planners, Vol. 37, No., pp. 100-110.
- Gonzales, E.J. and Daganzo, C.F. (2012) "Morning commute with competing modes and distributed demand: User equilibrium, system optimum, and pricing", Transportation Research Part B: Methodological, Vol. 46, No. 10, pp. 1519-1534.
- Greenshields, B.D. (1935) "A study of traffic capacity", Proceedings Highway Research Record, Vol. 14, No., pp. 448-477.
- Haberman, R. (1977) Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow. Society for Industrial and Applied Mathematics.
- Han, L.S., Alok, T., Kanat, C.M. and Shi, P.J. (2009) "Convergence of time-stepping schemes for passive and extended linear complementarity systems", SIAM Journal on Numerical Analysis, Vol. 47, No. 5, pp. 3768-3796.
- Han, L.S. and Du, L.L. (2012) "On a link-based day-to-day traffic assignment model", Transportation Research Part B: Methodological, Vol. 46, No. 1, pp. 72-84.
- Han, L.S., Ukkusuri, S. and Doan, K. (2011) "Complementarity formulations for the cell transmission model based dynamic user equilibrium with departure time choice, elastic demand and user heterogeneity", Transportation Research Part B: Methodological, Vol. 45, No. 10, pp. 1749-1767.
- Harker, P.T. and Pang, J.S. (1990) "Finite-dimensional variational inequality and nonlinear complementarity problems: a survey of theory, algorithms and applications", Mathematical Programming, Vol. 48, No. 1-3, pp. 161-220.

- Heemels, W.P.M.H., Schumacher, J.M. and Weiland, S. (2000) "Linear complementarity systems", SIAM Journal on Applied Mathematics, Vol. 60, No. 4, pp. 1234.
- Hepple, L.W. (2004) "Bayesian model choice in spatial econometrics", Advances in Econometrics, Vol. 18, No., pp. 101-126.
- Herbert, J.D. and Stevens, B.H. (1960) "A model for the distribution of residential activity in urban areas", Journal of Regional Science, Vol. 2, No. 2, pp. 21-36.
- Ho, H.W. and Wong, S.C. (2006) "A review of the two-dimensional continuum modeling approach to transportation problems", Journal of Transportation Systems Engineering and Information Technology, Vol. 6, No. 6, pp. 53-72.
- Ho, H.W., Wong, S.C. and Loo, B.P.Y. (2006) "Combined distribution and assignment model for a continuum traffic equilibrium problem with multiple user classes", Transportation Research Part B: Methodological, Vol. 40, No. 8, pp. 633-650.
- Holguín-Veras, J., Yushimito, W.F., Aros-Vera, F. and Reilly, J. (2012) "User rationality and optimal park-and-ride location under potential demand maximization", Transportation Research Part B: Methodological, Vol. 46, No. 8, pp. 949-970.
- Horner, M.W. and Groves, S. (2007) "Network flow-based strategies for identifying rail park-and-ride facility locations", Socio-Economic Planning Sciences, Vol. 41, No. 3, pp. 255-268.
- Horner, M.W. and Grubestic, T.H. (2001) "A GIS-based planning approach to locating urban rail terminals", Transportation, Vol. 28, No. 1, pp. 55-77.
- Hounsell, N., Shrestha, B. and Piao, J.N. (2011) "Enhancing park and ride with access control: a case study of Southampton", Transport Policy, Vol. 18, No. 1, pp. 194-203.
- Huang, H.-J. and Lam, W.H.K. (2002) "Modeling and solving the dynamic user equilibrium route and departure time choice problem in network with queues", Transportation Research Part B: Methodological, Vol. 36, No. 3, pp. 253-273.
- Huang, H.J. (2000) "Fares and tolls in a competitive system with transit and highway: the case with two groups of commuters", Transportation Research Part E: Logistics and Transportation Review, Vol. 36, No. 4, pp. 267-284.
- Ibrahim, M.F. (2003) "Improvements and integration of a public transport system: the case of Singapore", Cities, Vol. 20, No. 3, pp. 205-216.
- Jabari, S.E. and Liu, H.X. (2012) "A stochastic model of traffic flow: Theoretical foundations", Transportation Research Part B: Methodological, Vol. 46, No. 1, pp. 156-174.
- Jehiel, P. (1993) "Equilibrium on a traffic corridor with several congested modes", Transportation Science, Vol. 27, No. 1, pp. 16.
- Jenelius, E. and Koutsopoulos, H.N. (2013) "Travel time estimation for urban road networks using low frequency probe vehicle data", Transportation Research Part B: Methodological, Vol. 53, No., pp. 64-81.

- Johansson, O. (1997) "Optimal road-pricing: Simultaneous treatment of time losses, increased fuel consumption, and emissions", Transportation Research Part D: Transport and Environment, Vol. 2, No. 2, pp. 77-87.
- Josefsson, M. and Patriksson, M. (2007) "Sensitivity analysis of separable traffic equilibrium equilibria with application to bilevel optimization in network design", Transportation Research Part B: Methodological, Vol. 41, No. 1, pp. 4-31.
- Kachani, S. and Perakis, G. (2006) "Fluid dynamics models and their applications in transportation and pricing", European Journal of Operational Research, Vol. 170, No. 2, pp. 496-517.
- Kachani, S. and Perakis, G. (2009) "A dynamic travel time model for spillback", Networks and Spatial Economics, Vol. 9, No. 4, pp. 595-618.
- Kanzow, C. (1997) "A new approach to continuation methods for complementarity problems with uniform P-functions", Operations Research Letters, Vol. 20, No. 2, pp. 85-92.
- Kelley, J.E. (1960) "The cutting-plane method for solving convex programs", Journal of the Society for Industrial and Applied Mathematics, Vol. 8, No. 4, pp. 703-712.
- Kenworthy, J.R. and Laube, F.B. (1999) "Patterns of automobile dependence in cities: an international overview of key physical and economic dimensions with some implications for urban policy", Transportation Research Part A: Policy and Practice, Vol. 33, No. 7-8, pp. 691-723.
- Kepaptsoglou, K., Karlaftis, M.G. and Li, Z. (2010) "Optimizing pricing policies in park-and-ride facilities: A model and decision support system with application", Journal of Transportation Systems Engineering and Information Technology, Vol. 10, No. 5, pp. 53-65.
- Kim, T.J. (1979) "Alternative transportation modes in an urban land use model: A general equilibrium approach", Journal of Urban Economics, Vol. 6, No. 2, pp. 197-215.
- Kim, T.J. (1983) "A combined land use-transportation model when zonal travel demand is endogenously determined", Transportation Research Part B: Methodological, Vol. 17, No. 6, pp. 449-462.
- Kim, T.J. (1986) "Modeling the density variations of urban land uses with transportation network congestion", Journal of Urban Economics, Vol. 19, No. 3, pp. 264-276.
- Kinderlehrer, D. and Stampacchia, G. (2000) An Introduction to Variational Inequalities and Their Applications. Society for Industrial and Applied Mathematics.
- Knight, F.H. (1924) "Some fallacies in the interpretation of social cost", The Quarterly Journal of Economics, Vol. 38, No. 4, pp. 582-606.
- Lam, T.C. and Small, K.A. (2001) "The value of time and reliability: measurement from a value pricing experiment", Transportation Research Part E: Logistics and Transportation Review, Vol. 37, No. 2-3, pp. 231-251.

- Lam, W.H.K., Holyoak, N.M. and Lo, H.P. (2001) "How park-and-ride schemes can be successful in eastern Asia", Journal of Urban Planning and Development-Asce, Vol. 127, No. 2, pp. 63-78.
- Lam, W.H.K., Li, Z.C., Wong, S.C. and Zhu, D.L. (2007) "Modeling an elastic-demand bimodal transport network with park-and-ride trips", Tsinghua Science and Technology, Vol. 12, No. 2, pp. 158-166.
- Larrain, H. and Muñoz, J. (2008) "Public transit corridor assignment assuming congestion due to passenger boarding and alighting", Networks and Spatial Economics, Vol. 8, No. 2-3, pp. 241-256.
- Leblanc, L.J. (1975) "An algorithm for the discrete network design problem", Transportation Science, Vol. 9, No. 3, pp. 183.
- LeBlanc, L.J. and Abdulaal, M. (1984) "A comparison of user-optimum versus system-optimum traffic assignment in transportation network design", Transportation Research Part B: Methodological, Vol. 18, No. 2, pp. 115-121.
- LeBlanc, L.J. and Boyce, D.E. (1986) "A bilevel programming algorithm for exact solution of the network design problem with user-optimal flows", Transportation Research Part B: Methodological, Vol. 20, No. 3, pp. 259-265.
- Lerman, S.R. (1976) The Use of Joint Disaggregate Choice Models for Residential Location Decisions, Working Paper, Center for Transportation Studies, Massachusetts Institute of Technology, Cambridge, MA.
- Li, C.M., Yang, H., Zhu, D.L. and Meng, Q. (2012a) "A global optimization method for continuous network design problems", Transportation Research Part B: Methodological, Vol. 46, No. 9, pp. 1144-1158.
- Li, M.Z.F. (2002) "The role of speed-flow relationship in congestion pricing implementation with an application to Singapore", Transportation Research Part B: Methodological, Vol. 36, No. 8, pp. 731-754.
- Li, Y., Waller, S.T. and Ziliaskopoulos, T. (2003) "A decomposition scheme for system optimal dynamic traffic assignment models", Networks and Spatial Economics, Vol. 3, No. 4, pp. 441-455.
- Li, Z.C., Huang, H.J. and Lam, W.H.K. (2010) "Modelling heterogeneous drivers' responses to route guidance and parking information systems in stochastic and time-dependent networks", Transportmetrica, Vol. 8, No. 2, pp. 105-129.
- Li, Z.C., Lam, W.H.K., Wong, S., Zhu, D.L. and Huang, H.J. (2007) "Modeling park-and-ride services in a multimodal transport network with elastic demand", Transportation Research Record: Journal of the Transportation Research Board, Vol. 1994, No., pp. 101-109.
- Li, Z.C., Lam, W.H.K. and Wong, S.C. (2012b) "Modeling intermodal equilibrium for bimodal transportation system design problems in a linear monocentric city", Transportation Research Part B: Methodological, Vol. 46, No. 1, pp. 30-49.
- Lighthill, M.J. and Whitham, G.B. (1955) "On kinematic waves. II. A theory of traffic flow on long crowded roads", Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, Vol. 229, No. 1178, pp. 317-345.

- Lindsey, R. (2004) "Existence, uniqueness, and trip cost function properties of user equilibrium in the bottleneck model with multiple user classes", Transportation Science, Vol. 38, No. 3, pp. 293-314.
- Liu, S.T. (2006) "A geometric programming approach to profit maximization", Applied Mathematics and Computation, Vol. 182, No. 2, pp. 1093-1097.
- Liu, T.L., Huang, H.J., Yang, H. and Zhang, X.N. (2009) "Continuum modeling of park-and-ride services in a linear monocentric city with deterministic mode choice", Transportation Research Part B: Methodological, Vol. 43, No. 6, pp. 692-707.
- Liu, Y. and Nie, Y. (2011) "Morning commute problem considering route choice, user heterogeneity and alternative system optima", Transportation Research Part B: Methodological, Vol. 45, No. 4, pp. 619-642.
- Lo, H.K., Luo, X.W. and Siu, B.W.Y. (2006) "Degradable transport network: travel time budget of travelers with heterogeneous risk aversion", Transportation Research Part B: Methodological, Vol. 40, No. 9, pp. 792-806.
- Lo, H.K. and Szeto, W.Y. (2002) "A cell-based variational inequality formulation of the dynamic user optimal assignment problem", Transportation Research Part B: Methodological, Vol. 36, No. 5, pp. 421-443.
- Lo, H.K. and Tung, Y.K. (2003) "Network with degradable links: capacity analysis and design", Transportation Research Part B: Methodological, Vol. 37, No. 4, pp. 345-363.
- Long, J., Gao, Z. and Szeto, W.Y. (2011) "Discretised link travel time models based on cumulative flows: Formulations and properties", Transportation Research Part B: Methodological, Vol. 45, No. 1, pp. 232-254.
- Lowry, I.S. (1964) A Model of Metropolis. Rand Corporation.
- Lu, C.C., Mahmassani, H.S. and Zhou, X.S. (2008) "A bi-criterion dynamic user equilibrium traffic assignment model and solution algorithm for evaluating dynamic road pricing strategies", Transportation Research Part C: Emerging Technologies, Vol. 16, No. 4, pp. 371-389.
- Luathap, P., Sumalee, A., Lam, W.H.K., Li, Z.C. and Lo, H.K. (2011) "Global optimization method for mixed transportation network design problem: A mixed-integer linear programming approach", Transportation Research Part B: Methodological, Vol. 45, No. 5, pp. 808-827.
- Luo, Z.Q., Pang, J.S. and Ralph, D. (1996) Mathematical Programs with Equilibrium Constraints. Cambridge University Press.
- Magnanti, T.L. and Wong, R.T. (1984) "Network design and transportation planning: models and algorithms", Transportation Science, Vol. 18, No. 1, pp. 1.
- Mahmassani, H. and Herman, R. (1984) "Dynamic user equilibrium departure time and route choice on idealized traffic arterials", Transportation Science, Vol. 18, No. 4, pp. 362.
- Marc, L. (1978) "Simultaneous optimization of land use and transportation: A synthesis of the quadratic assignment problem and the optimal network problem", Regional Science and Urban Economics, Vol. 8, No. 1, pp. 21-42.

- Marc, L. (1979) "A discrete-convex programming approach to the simultaneous optimization of land use and transportation", Transportation Research Part B: Methodological, Vol. 13, No. 1, pp. 33-48.
- Marcotte, P. (1986) "Network design problem with congestion effects: a case of bilevel programming", Mathematical Programming, Vol. 34, No. 2, pp. 142-162.
- Marín, A.G. (1988) "A multimodal combined model analyzed via generalized geometric programming", Transportation Research Part B: Methodological, Vol. 22, No. 5, pp. 391-397.
- May, A.D. and Milne, D.S. (2000) "Effects of alternative road pricing systems on network performance", Transportation Research Part A: Policy and Practice, Vol. 34, No. 6, pp. 407-436.
- McDonald, J., d'Ouille, E. and Liu, L. (1999) "Urban highway congestion: an analysis of second-best tolls", Economics of Urban Highway Congestion and Pricing. Springer US, pp. 67-83.
- McFadden, D. (1978) "Modelling the choice of residential location", Spatial Interaction Theory and Planning Models, Vol. 673, No., pp. 75-96.
- Meng, Q. and Liu, Z. (2011) "Trial-and-error method for congestion pricing scheme under side-constrained probit-based stochastic user equilibrium conditions", Transportation, Vol. 38, No. 5, pp. 819-843.
- Meng, Q. and Liu, Z. (2012) "Impact analysis of cordon-based congestion pricing on mode-split for a bimodal transportation network", Transportation Research Part C: Emerging Technologies, Vol. 21, No. 1, pp. 134-147.
- Meng, Q., Liu, Z. and Wang, S. (2012) "Optimal distance tolls under congestion pricing and continuously distributed value of time", Transportation Research Part E: Logistics and Transportation Review, Vol. 48, No. 5, pp. 937-957.
- Meng, Q. and Yang, H. (2002) "Benefit distribution and equity in road network design", Transportation Research Part B: Methodological, Vol. 36, No. 1, pp. 19-35.
- Meng, Q., Yang, H. and Bell, M.G.H. (2001) "An equivalent continuously differentiable model and a locally convergent algorithm for the continuous network design problem", Transportation Research Part B: Methodological, Vol. 35, No. 1, pp. 83-105.
- Merchant, D.K. and Nemhauser, G.L. (1978a) "A model and an algorithm for the dynamic traffic assignment problems", Transportation Science, Vol. 12, No. 3, pp. 183.
- Merchant, D.K. and Nemhauser, G.L. (1978b) "Optimality conditions for a dynamic traffic assignment model", Transportation Science, Vol. 12, No. 3, pp. 200.
- Meyer, M.D. and Miller, E.J. (1984) Urban Transportation Planning: A Decision-Oriented Approach. McGraw-Hill, New York.
- Mills, E.S. (1970) "Urban density functions", Urban Studies, Vol. 7, No. 1, pp. 5-20.
- Mills, E.S. (1972a) "Markets and efficient resource allocation in urban areas", Swedish Journal of Economics, Vol. 74, No. 1, pp. 100-113.

- Mills, E.S. (1972b) Studies in the Structure of the Urban Economy. Johns Hopkins University. Press, Baltimore.
- Mo, Y., Cheng, F. and Yan, K. (2008) "Optimization method of mixed network design problem based on genetic algorithm", Eighth International Conference of Chinese Logistics and Transportation Professionals, Vol., No., pp. 4452-4459.
- Moore II, J.E. and Kim, T.J. (1995) "Mills' urban system models: perspective and template for LUTE (Land Use/Transport/Environment) applications", Computers, Environment and Urban Systems, Vol. 19, No. 4, pp. 207-225.
- Morrison, S.A. (1986) "A survey of road pricing", Transportation Research Part A: General, Vol. 20, No. 2, pp. 87-97.
- Murty, K.G. (1997) Linear Complementarity, Linear and Nonlinear Programming. Internet Edition.
- Nagurney, A. (1993) Network Economics: A Variational Inequality Approach. Kluwer Academic Publishers Group.
- Nagurney, A. (2000) "A multiclass, multicriteria traffic network equilibrium model", Mathematical and Computer Modelling, Vol. 32, No. 3-4, pp. 393-411.
- Nagurney, A. and Dong, J. (2002) "A multiclass, multicriteria traffic network equilibrium model with elastic demand", Transportation Research Part B: Methodological, Vol. 36, No. 5, pp. 445-469.
- Newell, G.F. (1987) "The morning vomute for nonidentical travelers", Transportation Science, Vol. 21, No. 2, pp. 74.
- Newell, G.F. (1988) "Traffic flow for the morning commute", Transportation Science, Vol. 22, No. 1, pp. 47.
- Newell, G.F. (1993a) "A simplified theory of kinematic waves in highway traffic, part I: General theory", Transportation Research Part B: Methodological, Vol. 27, No. 4, pp. 281-287.
- Newell, G.F. (1993b) "A simplified theory of kinematic waves in highway traffic, part II: Queueing at freeway bottlenecks", Transportation Research Part B: Methodological, Vol. 27, No. 4, pp. 289-303.
- Newell, G.F. (1993c) "A simplified theory of kinematic waves in highway traffic, part III: Multi-destination flows", Transportation Research Part B: Methodological, Vol. 27, No. 4, pp. 305-313.
- Niblette, R. and Palmer, D.J. (1993) "Park-and-ride in London and the South East", Journal of Institution of Highway and Transportation, Vol. 40, No. 2, pp. 4-10.
- Nie, Y.M. and Wu, X. (2009) "Reliable a priori shortest path problem with limited spatial and temporal dependencies", in: Lam, W.H.K., Wong, S.C., Lo, H.K. (Eds.), Transportation and Traffic Theory 2009: Golden Jubilee. Springer US, pp. 169-195.
- Nikolova, E.V. (2009) "Strategic Algorithms", Department of Electrical Engineering and Computer Science. Massachusetts Institute of Technology.
- Noel, E.C. (1988) "Park - and - ride: alive, well, and expanding in the United States", Journal of Urban Planning and Development, Vol. 114, No. 1, pp. 2.

- Ohno, H., Suzuki, T., Yamaguchi, Y., Okamoto, S. and Iizuka, K. (2007) "Development of the next generation road pricing system with GPS technology", Technical Review, Vol. 44, No. 2, pp. 1-5.
- Orthlieb, M. and May, A. (1976) "Evaluation of freeway corridor assignment equilibria", Traffic Equilibrium Methods. Springer Berlin Heidelberg, pp. 298-321.
- Ortuzar, J.d.D. and Willumsen, L.G. (2011) Modelling Transport. Wiley.
- Pang, J.S., Han, L.S., Ramadurai, G. and Ukkusuri, S.V. (2012) "A continuous-time linear complementarity system for dynamic user equilibria in single bottleneck traffic flows", Mathematical Programming, Vol. 133, No. 1-2, pp. 437-460.
- Pang, J.S. and Stewart, D. (2008) "Differential variational inequalities", Mathematical Programming, Vol. 113, No. 2, pp. 345-424.
- Pang, J.S. and Stewart, D. (2009) "Solution dependence on initial conditions in differential variational inequalities", Mathematical Programming, Vol. 116, No. 1-2, pp. 429-460.
- Patriksson, M. (1994) The Traffic Assignment Problem: Models and Methods. VSP.
- Patriksson, M. (2004) "Algorithms for computing traffic equilibria", Networks and Spatial Economics, Vol. 4, No. 1, pp. 23-38.
- Peeta, S. and Ziliaskopoulos, A. (2001) "Foundations of dynamic traffic assignment: the past, the present and the future", Networks and Spatial Economics, Vol. 1, No. 3-4, pp. 233-265.
- Perakis, G. and Roels, G. (2006) "An analytical model for traffic delays and the dynamic user equilibrium problem", Operations Research, Vol. 54, No. 6, pp. 1151-1171.
- Peterson, E.L. (1978) "Optimality conditions in generalized geometric programming", Journal of Optimization Theory and Applications, Vol. 26, No. 1, pp. 3-13.
- Pigou, A.C. (1920) The Economics of Welfare. Mac Millan, New York.
- Poorzahedy, H. and Abulghasemi, F. (2005) "Application of ant system to network design problem", Transportation, Vol. 32, No. 3, pp. 251-273.
- Poorzahedy, H. and Rouhani, O.M. (2007) "Hybrid meta-heuristic algorithms for solving network design problem", European Journal of Operational Research, Vol. 182, No. 2, pp. 578-596.
- Poorzahedy, H. and Turnquist, M.A. (1982) "Approximate algorithms for the discrete network design problem", Transportation Research Part B: Methodological, Vol. 16, No. 1, pp. 45-55.
- Putman, S.H. (1980) "Calibrating urban residential location models: 3. Empirical results for non-US cities", Environment and Planning A, Vol. 12, No., pp. 813-827.
- Ramadurai, G., Ukkusuri, S.V., Zhao, J.Y. and Pang, J.S. (2010) "Linear complementarity formulation for single bottleneck model with heterogeneous commuters", Transportation Research Part B: Methodological, Vol. 44, No. 2, pp. 193-214.

- Ramezani, M. and Geroliminis, N. (2012) "On the estimation of arterial route travel time distribution with Markov chains", Transportation Research Part B: Methodological, Vol. 46, No. 10, pp. 1576-1590.
- Ran, B. and Boyce, D.E. (1996) "A link-based variational inequality formulation of ideal dynamic user-optimal route choice problem", Transportation Research Part C: Emerging Technologies, Vol. 4, No. 1, pp. 1-12.
- Ran, B., Hall, R.W. and Boyce, D.E. (1996) "A link-based variational inequality model for dynamic departure time/route choice", Transportation Research Part B: Methodological, Vol. 30, No. 1, pp. 31-46.
- Richards, P.I. (1956) "Shock waves on the highway", Operations Research, Vol. 4, No. 1, pp. 42-51.
- Richardson, H.W. and Bae, C.H.C. (2008) Road Congestion Pricing in Europe: Implications for the United States. Edward Elgar Publishing, Incorporated.
- Robin, L. (2006) "Do economists reach a conclusion on road pricing? The intellectual history of an idea", Econ Journal Watch, Vol. 3, No. 2, pp. 292-379.
- Samanta, B. and Majumder, S.K. (2006) "Entropy based transportation problem use geometric programming method", Applied Mathematics and Computation, Vol. 179, No. 2, pp. 431-439.
- Scheel, H. and Scholtes, S. (2000) "Mathematical programs with complementarity constraints: stationarity, optimality, and sensitivity", Mathematics of Operations Research, Vol. 25, No. 1, pp. 1-22.
- Schumacher, J.M. (2004) "Complementarity systems in optimization", Mathematical Programming, Vol. 101, No. 1, pp. 263-295.
- Scott, C.H., Jefferson, T.R. and Jorjani, S. (2004) "Duals for classical inventory models via generalized geometric programming", Journal of Applied Mathematics and Decision Sciences, Vol. 8, No. 3, pp. 191-200.
- Sheffi, Y. (1985) Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods. Prentice Hall.
- Shen, J.L. and Pang, J.S. (2005) "Linear complementarity systems: zeno states", SIAM Journal on Control and Optimization, Vol. 44, No. 3, pp. 1040-1066.
- Sherali, H.D., Krishnamurthy, R.S. and Al-Khayyal, F.A. (1998) "Enumeration approach for linear complementarity problems based on a reformulation-linearization technique", Journal of Optimization Theory and Applications, Vol. 99, No. 2, pp. 481-507.
- Sinha, K.C. (2003) "Sustainability and urban public transportation", Journal of Transportation Engineering, Vol. 129, No. 4, pp. 331.
- Siu, B.W.Y. and Lo, H.K. (2008) "Doubly uncertain transportation network: Degradable capacity and stochastic demand", European Journal of Operational Research, Vol. 191, No. 1, pp. 166-181.
- Smith, M.J. (1979) "The existence, uniqueness and stability of traffic equilibria", Transportation Research Part B: Methodological, Vol. 13, No. 4, pp. 295-304.

- Solanki, R.S., Gorti, J.K. and Southworth, F. (1998) "Using decomposition in large-scale highway network design with a quasi-optimization heuristic", Transportation Research Part B: Methodological, Vol. 32, No. 2, pp. 127-140.
- Solow, R.M. (1972) "Congestion and the use of land in transportation", Swedish Journal of Economics, Vol. 74, No., pp. 602-618.
- Solow, R.M. (1973) "Congestion and the use of land for streets", Bell Journal of Economics and Management Science, Vol. 4, No., pp. 602-618.
- Sumalee, A., Watling, D.P. and Nakayama, S. (2006) "Reliable network design problem: case with uncertain demand and total travel time reliability", Transportation Research Record, Vol. 1964, No., pp. 81-90.
- Sumalee, A., Zhong, R.X., Pan, T.L. and Szeto, W.Y. (2011) "Stochastic cell transmission model (SCTM): A stochastic dynamic traffic model for traffic state surveillance and assignment", Transportation Research Part B: Methodological, Vol. 45, No. 3, pp. 507-533.
- Sun, Y., Song, R., He, S. and Chen, Q. (2009) "Mixed transportation network design based on immune clone annealing algorithm", Journal of Transportation Systems Engineering and Information Technology, Vol. 9, No. 3, pp. 103-108.
- Suwansirikul, C., Friesz, T.L. and Tobin, R.L. (1987) "Equilibrium decomposed optimization: a heuristic for the continuous equilibrium network design problems", Transportation Science, Vol. 21, No. 4, pp. 254.
- Szeto, W.Y. and Lo, H.K. (2004) "A cell-based simultaneous route and departure time choice model with elastic demand", Transportation Research Part B: Methodological, Vol. 38, No. 7, pp. 593-612.
- Talvitie, A. and Hasan, I. (1980) "An equilibrium model system for transportation corridors and its application", Transportation Research Part B: Methodological, Vol. 14, No. 1-2, pp. 165-174.
- Tong, C.O. and Wong, S.C. (1997) "The advantages of a high density, mixed land use, linear urban development", Transportation, Vol. 24, No. 3, pp. 295-307.
- Tsai, J.F., Lin, M.H. and Hu, Y.C. (2007) "On generalized geometric programming problems with non-positive variables", European Journal of Operational Research, Vol. 178, No. 1, pp. 10-19.
- Ukkusuri, S.V., Han, L. and Doan, K. (2012) "Dynamic user equilibrium with a path based cell transmission model for general traffic networks", Transportation Research Part B: Methodological, Vol. 46, No. 10, pp. 1657-1684.
- Ukkusuri, S.V. and Patil, G. (2009) "Multi-period transportation network design under demand uncertainty", Transportation Research Part B: Methodological, Vol. 43, No. 6, pp. 625-642.
- Verhoef, E., Nijkamp, P. and Rietveld, P. (1995) "The economics of regulatory parking policies: The (IM)possibilities of parking policies in traffic regulation", Transportation Research Part A: Policy and Practice, Vol. 29, No. 2, pp. 141-156.
- Verhoef, E., Nijkamp, P. and Rietveld, P. (1996) "Second-best congestion pricing: the case of an untolled alternative", Journal of Urban Economics, Vol. 40, No. 3, pp. 279-302.

- Verhoef, E.T. (2002) "Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal toll levels and toll points", Transportation Research Part B: Methodological, Vol. 36, No. 8, pp. 707-729.
- Vickrey, W.S. (1969) "Congestion theory and transport investment", The American Economic Review, Vol. 59, No. 2, pp. 251-260.
- Vliet, D.V. (1987) "The Frank-Wolfe algorithm for equilibrium traffic assignment viewed as a variational inequality", Transportation Research Part B: Methodological, Vol. 21, No. 1, pp. 87-89.
- Walters, A.A. (1959) The Theory and Measurement of Private and Social Cost of Highway Congestion. University of Birmingham, Faculty of Commerce and Social Science.
- Wang, D. and Du, B. (2013) "Reliability-based modeling of park-and-ride service on linear travel corridor", Transportation Research Record: Journal of the Transportation Research Board, Vol. 2333, No., pp. 16-26.
- Wang, D.Z.W. and Lo, H.K. (2010) "Global optimum of the linearized network design problem with equilibrium flows", Transportation Research Part B: Methodological, Vol. 44, No. 4, pp. 482-492.
- Wang, G.M., Gao, Z.Y. and Xu, M. (2014a) "An MPEC formulation and its cutting constraint algorithm for continuous network design problem with multi-user classes", Applied Mathematical Modelling, Vol. 38, No. 5–6, pp. 1846-1858.
- Wang, G.M., Gao, Z.Y., Xu, M. and Sun, H.J. (2014b) "Models and a relaxation algorithm for continuous network design problem with a tradable credit scheme and equity constraints", Computers & Operations Research, Vol. 41, No. 0, pp. 252-261.
- Wang, J.Y.T., Yang, H. and Lindsey, R. (2003) "Modelling park-and-ride service in a linear monocentric city", Journal of the Eastern Asia Society for Transportation Studies, Vol. 5, No., pp. 1377-1392.
- Wang, J.Y.T., Yang, H. and Lindsey, R. (2004a) "Locating and pricing park-and-ride facilities in a linear monocentric city with deterministic mode choice", Transportation Research Part B: Methodological, Vol. 38, No. 8, pp. 709-731.
- Wang, S.A., Meng, Q. and Yang, H. (2013) "Global optimization methods for the discrete network design problem", Transportation Research Part B: Methodological, Vol. 50, No., pp. 42-60.
- Wang, Y.J., Zhang, K.C. and Gao, Y.L. (2004b) "Global optimization of generalized geometric programming", Computers & Mathematics with Applications, Vol. 48, No. 10–11, pp. 1505-1516.
- Wardrop, J.G. (1952) "Some theoretical aspects of road traffic research", Proceedings of the Institution of Civil Engineers, Part II. Institution of Civil Engineers, pp. 325-362.
- Wegener, M. (2004a) "Chapter 9 Overview of land-use transport models in Handbook of Transport Geography and Spatial Systems", in: Hensher, D.A., Button, K. (Eds.), Handbook of Transport Geography and Spatial Systems. Pergamon, Kidlington, UK, pp. 127-146.

- Wegener, M. (2004b) "Overview of land use transport models", Handbook of Transport Geography and Spatial.
- Wie, B.W., Friesz, T.L. and Tobin, R.L. (1990) "Dynamic user optimal traffic assignment on congested multidestination networks", Transportation Research Part B: Methodological, Vol. 24, No. 6, pp. 431-442.
- Wie, B.W., Tobin, R.L., Friesz, T.L. and Bernstein, D. (1995) "A discrete time, nested cost operator approach to the dynamic network user equilibrium problem", Transportation Science, Vol. 29, No. 1, pp. 79.
- Williams, H.C.W.L. (1977) "On the formation of travel demand models and economic evaluation measures of user benefit", Environment and Planning A, Vol. 9, No. 3, pp. 285-344.
- Wilson, A.G. (1971) "A family of spatial interaction models, and associated developments", Environment and Planning, Vol. 3, No., pp. 1-32.
- Wilson, A.G. (1998) "Land-use/transport interaction models: past and future", Journal of Transport Economics and Policy, Vol. 32, No. 1, pp. 3-26.
- Wong, C.K., Wong, S.C. and Tong, C.O. (1998) "A new methodology for calibrating the Lowry model", Journal of Urban Planning and Development, Vol. 124, No., pp. 72-91.
- Wong, D.S. (1981) "Maximum likelihood, entropy maximization, and the geometric programming approaches to the calibration of trip distribution models", Transportation Research Part B: Methodological, Vol. 15, No. 5, pp. 329-343.
- Xie, L. and Olszewski, P. (2011) "Modelling the effects of road pricing on traffic using ERP traffic data", Transportation Research Part A: Policy and Practice, Vol. 45, No. 6, pp. 512-522.
- Xu, Y.W., Wu, J.H., Florian, M., Marcotte, P. and Zhu, D.L. (1999) "Advances in the continuous dynamic network loading problem", Transportation Science, Vol. 33, No. 4, pp. 341.
- Yang, H. and Bell, M.G.H. (1997) "Traffic restraint, road pricing and network equilibrium", Transportation Research Part B: Methodological, Vol. 31, No. 4, pp. 303-314.
- Yang, H. and Bell, M.G.H. (1998a) "A capacity paradox in network design and how to avoid it", Transportation Research Part A: Policy and Practice, Vol. 32, No. 7, pp. 539-545.
- Yang, H. and Bell, M.G.H. (1998b) "Models and algorithms for road network design: a review and some new developments", Transport Reviews, Vol. 18, No., pp. 257-278.
- Yang, H. and Huang, H.J. (1998) "Principle of marginal-cost pricing: how does it work in a general road network?", Transportation Research Part A: Policy and Practice, Vol. 32, No. 1, pp. 45-54.
- Yang, H. and Zhang, X. (2002) "Multiclass network toll design problem with social and spatial equity constraints", Journal of Transportation Engineering, Vol. 128, No. 5, pp. 420-428.

- Yang, H., Zhang, X. and Meng, Q. (2004) "Modeling private highways in networks with entry-exit based toll charges", Transportation Research Part B: Methodological, Vol. 38, No. 3, pp. 191-213.
- Yildirim, M.B. and Hearn, D.W. (2005) "A first best toll pricing framework for variable demand traffic assignment problems", Transportation Research Part B: Methodological, Vol. 39, No. 8, pp. 659-678.
- Yim, K.K.W., Wong, S.C., Chen, A., Wong, C.K. and Lam, W.H.K. (2011) "A reliability-based land use and transportation optimization model", Transportation Research Part C: Emerging Technologies, Vol. 19, No. 2, pp. 351-362.
- Yong, L. (2010) "Nonlinear complementarity problem and solution methods", in: Wang, F., Deng, H., Gao, Y., Lei, J. (Eds.), Artificial Intelligence and Computational Intelligence. Springer Berlin Heidelberg, pp. 461-469.
- Ypma, T.J. (1995) "Historical development of the Newton-Raphson method", SIAM Review, Vol. 37, No. 4, pp. 531-551.
- Zhang, X.N., Huang, H.J. and Zhang, H.M. (2008) "Integrated daily commuting patterns and optimal road tolls and parking fees in a linear city", Transportation Research Part B: Methodological, Vol. 42, No. 1, pp. 38-56.
- Zhou, Z., Chen, A. and Wong, S.C. (2009) "Alternative formulations of a combined trip generation, trip distribution, modal split, and trip assignment model", European Journal of Operational Research, Vol. 198, No. 1, pp. 129-138.
- Ziliaskopoulos, A.K. (2000) "A linear programming model for the single destination system optimum dynamic traffic assignment problem", Transportation Science, Vol. 34, No. 1, pp. 37.

APPENDIX A Proof of Proposition 4.1 in Section 4.3.2

Proof. Firstly, prove that the matrix M_g in $LCP(s, M)$ (4-33) is semimonotone according to Definition 4.1.

For each group $g = 1, 2, \dots, G$, fix an integer $v > 0$. To show the semimonotonicity of M_g in $LCP(s, M)$ (4-33), let $0 \neq x \geq 0$ be given, then examine each component to the vector x .

If $C_g^{*,i} > 0$ for some $g \in G$ and some $i \in \{1, \dots, v\}$, then $q_g^{h,i} + q_g^{r,i} + \sum_{j=i+1}^v q_g^{p,ij} \geq 0$, thus the

semimonotonicity of M_g follows in this case. Therefore, assume $C_g^{*,i} = 0$ for all

$g \in G$ and all $i \in \{1, \dots, v\}$. If $q_g^{h,i} > 0$ for some $g \in G$ and some $i \in \{1, \dots, v\}$, then the

corresponding component in $M_g x_g$ is $\tau_g T_g^{b,i} \geq 0$, hence the semimonotonicity of M_g

follows in this case. Therefore, assume $q_g^{h,i} = 0$ for all $g \in G$ and all $i \in \{1, \dots, v\}$. If

$q_g^{r,i} > 0$ for some $g \in G$ and some $i \in \{1, \dots, v\}$, then the corresponding component in

$M_g x_g$ is $Y_g^{r,i} \geq 0$, thus the semimonotonicity of M_g follows in this case. Hence

assume $q_g^{r,i} = 0$ for all $g \in G$ and all $i \in \{1, \dots, v\}$. If $q_g^{p,ij} > 0$ for some $g \in G$,

$i \in \{1, \dots, v\}$ and $j \in \{i+1, \dots, v\}$, then the corresponding component in $M_g x_g$ is

$\tau_g \sum_{m=i}^{j-1} t_g^{b,m} + Y_g^{r,j} \geq 0$, thus the semimonotonicity of M_g follows in this case. Therefore,

assume $q_g^{p,ij} = 0$ for all $g \in G$, $i \in \{1, \dots, v\}$ and $j \in \{i+1, \dots, v\}$. If $T_g^{b,v+1} > 0$ for some

$g \in G$, then it has the corresponding component in $M_g x_g$ is $T_g^{b,v+1} > 0$, hence assume

$T_g^{b,v+1} = 0$ for all $g \in G$. If $Y_g^{r,v+1} > 0$ for some $g \in G$, then the corresponding

component in $M_g x_g$ is $Y_g^{r,v+1} > 0$, hence assume $Y_g^{r,v+1} = 0$ for all $g \in G$. If $t_g^{b,i-1} > 0$

for some $g \in G$ and some $i \in \{1, \dots, v\}$, take $i_* - 1$ to be the first index such that

$t_g^{b,i-1} > 0$ for g_* , then the element corresponding to $t_{g_*}^{b,i_*-1}$ in $M_g x_g$ is simply $t_{g_*}^{b,i_*-1}$,

hence assume that $t_g^{b,i-1} = 0$ for all $g \in G$ and all $i \in \{1, \dots, v\}$. If $y_g^{r,i-1} > 0$ for some

$g \in G$ and some $i \in \{1, \dots, v\}$, take $i_* - 1$ to be the first index such that $y_{g_*}^{r,i-1} > 0$ for g_* ,

then the element corresponding to $y_{g_*}^{r,i_*-1} > 0$ in $M_g x_g$ is simply $y_{g_*}^{r,i_*-1}$, hence assume that $y_g^{r,i-1} = 0$ for all $g \in \mathbb{G}$ and $i \in \{1, \dots, v\}$. If $T_g^{b,i} > 0$ for some $g \in \mathbb{G}$ and some $i \in \{1, \dots, v\}$, take i_* to be the first index such that $T_g^{b,i_*} > 0$ for g_* , then the element corresponding to $T_{g_*}^{b,i_*}$ in $M_g x_g$ is simply $T_{g_*}^{b,i_*}$. Therefore, assume that $T_g^{b,i} = 0$ for all $g \in \mathbb{G}$ and all $i \in \{1, \dots, v\}$. If $Y_g^{r,i} > 0$ for some $g \in \mathbb{G}$ and some $i \in \{1, \dots, v\}$, take i_* to be the first index such that $Y_g^{r,i_*} > 0$ for g_* , then the element corresponding to $Y_{g_*}^{r,i_*}$ in $M_g x_g$ is simply $Y_{g_*}^{r,i_*}$. Therefore, assume that $Y_g^{r,i} = 0$ for all $g \in \mathbb{G}$ and $i \in \{1, \dots, v\}$. If $T_g^{b,0} > 0$ for some $g \in \mathbb{G}$, then the corresponding component in $M_g x_g$ is $T_g^{b,0} > 0$, hence assume $T_g^{b,0} = 0$ for all $g \in \mathbb{G}$. Since x is nonzero, it must have $Y_g^{r,0} > 0$ for some $g \in \mathbb{G}$, and the semimonotonicity of M_g follows readily too.

To complete the proof, the boundedness of $SOL(s + \tau d, M)$ is discussed as follows.

The vector I of all ones is used as the covering vector, and a similar proof applies to any positive vector. Suppose that for some scalar $\theta > 0$, the $LCP(s + \theta I, M)$ has a sequence of solutions:

$$\left\{ \begin{array}{l} (q_g^{h,i,k})_{i=1}^v, (q_g^{r,i,k})_{i=1}^v, (q_g^{p,ij,k})_{i=1}^v, (C_g^{*,i,k})_{i=1}^v, (T_g^{b,i,k})_{i=1}^v, \\ (t_g^{b,i,k})_{i=1}^v, T_g^{b,0,k}, T_g^{b,v+1,k}, (Y_g^{r,i,k})_{i=1}^v, (y_g^{r,i,k})_{i=1}^v, Y_g^{r,0,k}, Y_g^{r,v+1,k} \end{array} \right\}_{g \in \mathbb{G}}$$

which satisfy the following complementarity conditions:

$$\begin{aligned}
0 \leq t_g^{b,i-1,k} \perp & \left[t_g^{b,i-1,k} - t_g^{b,i,k} + \frac{\varepsilon_v^2 t_h^0 A}{C_i} \sum_{g=1}^G (q_g^{h,i,k} + \sum_{j=i+1}^v q_g^{p,ij,k} - \sum_{j=1}^{i-1} q_g^{p,ji,k}) \right. \\
& \left. + \frac{\lambda_g}{4} (\sigma_i^2 + 2\mu_{i,i+1} \sigma_{i+1}^2)^2 \left(\sum_{j=i}^v \sigma_j^2 + 2 \sum_{j=i+1}^v \mu_{j-1,j} \sigma_j^2 \right)^{-\frac{3}{2}} \right] + \theta \geq 0 \\
0 \leq y_g^{r,i-1,k} \perp & y_g^{r,i-1,k} - y_g^{r,i,k} + \varepsilon_v^2 \beta \sum_{g=1}^G (q_g^{r,i,k} + \sum_{j=1}^{i-1} q_g^{p,ji,k}) + \theta \geq 0 \\
0 \leq q_g^{h,i,k} \perp & \tau_g t_h^{hh} + \tau_g t_h^{pw} + \tau_g T_g^{b,i,k} + f_h^0 + \gamma(v-i+1)\varepsilon_v + R_h^w - C_g^{*,i,k} + \theta \geq 0 \\
0 \leq q_g^{r,i,k} \perp & \tau_g t_r^{hr} + \tau_g t_r^{rw} + \tau_g \frac{(v-i+1)\varepsilon_v}{V_r} + Y_g^{r,i,k} + f_r^0 + \kappa(v-i+1)\varepsilon_v - C_g^{*,i,k} + \theta \geq 0 \\
0 \leq q_g^{p,ij,k} \perp & \left[\tau_g t_h^{hh} + \tau_g \sum_{m=i}^{j-1} t_g^{b,m,k} + f_h^0 + \gamma(j-i)\varepsilon_v + R_h^w e^{\frac{(v-j)^2}{2v}} + \tau_g t_p^{pr} + u_p \right. \\
& \left. + \tau_g \frac{(v-j+1)\varepsilon_v}{V_r} + Y_g^{r,j,k} + f_r^0 + \kappa(v-j+1)\varepsilon_v + \tau_g t_r^{rw} - C_g^{*,i,k} \right] + \theta \geq 0 \quad (\text{A-1}) \\
0 \leq C_g^{*,i,k} \perp & q_g^{h,i,k} + q_g^{r,i,k} + \sum_{j=i+1}^v q_g^{p,ij,k} - q_g^{0,i} + \theta \geq 0 \\
0 \leq T_g^{b,i,k} \perp & T_g^{b,i,k} - T_g^{b,i+1,k} - t_g^{b,i,k} + \theta \geq 0 \\
0 \leq Y_g^{r,i,k} \perp & Y_g^{r,i,k} - Y_g^{r,i+1,k} - y_g^{r,i,k} + \theta \geq 0 \\
0 \leq T_g^{b,v+1,k} \perp & T_g^{b,v+1,k} + L + \theta \geq 0 \\
0 \leq T_g^{b,0,k} \perp & T_g^{b,0,k} - T_g^{b,1,k} - t_h^0 \varepsilon_v + \theta \geq 0 \\
0 \leq Y_g^{r,v+1,k} \perp & Y_g^{r,v+1,k} + L + \theta \geq 0 \\
0 \leq Y_g^{r,0,k} \perp & Y_g^{r,0,k} - Y_g^{r,1,k} - \alpha \varepsilon_v + \theta \geq 0 \\
\forall i \in \{1, \dots, v\}, \forall j \in \{i+1, \dots, v\}, \forall g \in \{1, \dots, G\}
\end{aligned}$$

Then the desired boundedness of these solutions is proved as follows:

Boundedness of $T_g^{b,v+1,k}$. Suppose that this is not true for some g_* , then assume without loss of generality that

$$\lim_{k \rightarrow \infty} T_{g_*}^{b,v+1,k} = \infty \quad (\text{A-2})$$

and $T_{g_*}^{b,v+1,k} > 0$ for all $k=1,2,\dots,\infty$. It follows that

$$T_{g_*}^{b,v+1,k} + L + \theta = 0 \quad (\text{A-3})$$

for all $k=1,2,\dots,\infty$. It can be deduced that $T_{g_*}^{b,v+1,k} = -L - \theta < -L$. This is a contradiction, so the boundedness of $T_g^{b,v+1,k}$ follows.

Boundedness of $Y_g^{r,v+1,k}$. Similar as the above proof of **Boundedness of $T_g^{b,v+1,k}$** .

Boundedness of $q_g^{h,i,k}$. Suppose that this is not true for some g_* and i_* , then assume without loss of generality that

$$\lim_{k \rightarrow \infty} q_{g_*}^{h,i_*,k} = \infty \quad (\text{A-4})$$

and $q_{g_*}^{h,i_*,k} > 0$ for all $k = 1, 2, \dots, \infty$. It follows that

$$\tau_{g_*} t_h^{hh} + \tau_{g_*} t_h^{pw} + \tau_{g_*} T_{g_*}^{b,i_*,k} + f_h^0 + \gamma(v - i_* + 1)\varepsilon_v + R_h^w - C_{g_*}^{*,i_*,k} + \theta = 0 \quad (\text{A-5})$$

for all $k = 1, 2, \dots, \infty$. By the nonnegativity of $T_{g_*}^{b,i_*,k}$, it can be deduced that

$$C_{g_*}^{*,i_*,k} \geq \tau_{g_*} t_h^{hh} + \tau_{g_*} t_h^{pw} + f_h^0 + \gamma(v - i_* + 1)\varepsilon_v + R_h^w + \theta > 0 \quad (\text{A-6})$$

On the other hand, it can also be derived that

$$q_{g_*}^{h,i_*,k} + q_{g_*}^{r,i_*,k} + \sum_{j=i_*+1}^v q_{g_*}^{p,i_*,j,k} - q_{g_*}^{0,i_*} + \theta > 0 \quad (\text{A-7})$$

for all k sufficiently large, then the corresponding component in $M_g x_g$ is $C_{g_*}^{*,i_*,k} = 0$.

This is a contradiction, so the boundedness of $q_g^{h,i,k}$ follows.

Boundedness of $q_g^{r,i,k}$. Similar as the above proof of **Boundedness of $q_g^{h,i,k}$** .

Boundedness of $q_g^{p,ij,k}$. Similar as the above proof of **Boundedness of $q_g^{h,i,k}$** .

Boundedness of $t_g^{b,i-1,k}$. Let $i_* - 1 \in \{0, \dots, v\}$ be the smallest index such that $t_{g_*}^{b,i_*-1,k}$ is unbounded, then assume that

$$\lim_{k \rightarrow \infty} t_{g_*}^{b,i_*-1,k} = \infty \quad (\text{A-8})$$

and $t_{g_*}^{b,i_*-1,k} > 0$ for all $k = 1, 2, \dots, \infty$. Thus,

$$\begin{aligned} & t_{g_*}^{b,i_*-1,k} - t_{g_*}^{b,i_*,k} + \frac{\varepsilon_v^2 t_h^0 A}{C_x} \sum_{g_*=1}^G \left(q_{g_*}^{h,i_*,k} + \sum_{j=i_*+1}^v q_{g_*}^{p,i_*,j,k} - \sum_{j=1}^{i_*-1} q_{g_*}^{p,j,i_*,k} \right) \\ & + \frac{\lambda_g}{4} \left(\sigma_{i_*}^2 + 2\mu_{i_*,i_*+1} \sigma_{i_*+1}^2 \right)^2 \left(\sum_{j=i_*}^v \sigma_j^2 + 2 \sum_{j=i_*+1}^v \mu_{j-1,j} \sigma_j^2 \right)^{-\frac{3}{2}} + \theta = 0 \end{aligned} \quad (\text{A-9})$$

for all $k = 1, 2, \dots, \infty$. This implies that

$$t_{g_*}^{b,i_*-1,k} - t_{g_*}^{b,i_*,k} < \frac{\varepsilon_v^2 t_h^0 A}{C_x} \sum_{g_*=1}^G \sum_{j=1}^{i_*-1} q_{g_*}^{p,j,i_*,k} \quad (\text{A-10})$$

for all $k=1,2,\dots,\infty$. Due to the boundedness of $q_{g_*}^{p,i_*,k}$, it can be concluded that $t_{g_*}^{b,i_*,k}$ is also unbounded. This contradicts the definition of index $i_* - 1$.

Boundedness of $y_g^{r,i-1,k}$. Let $i_* - 1 \in \{0, \dots, v\}$ be the smallest index such that $y_{g_*}^{r,i_*-1,k}$ is unbounded, then assume that

$$\lim_{k \rightarrow \infty} y_{g_*}^{r,i_*-1,k} = \infty \quad (\text{A-11})$$

and $y_{g_*}^{r,i_*-1,k} > 0$ for all $k=1,2,\dots,\infty$. Thus,

$$y_{g_*}^{r,i_*-1,k} - y_{g_*}^{r,i_*,k} + \varepsilon_v^2 \beta \sum_{g_*=1}^G \left(q_{g_*}^{r,i_*,k} + \sum_{j=1}^{i_*-1} q_{g_*}^{p,j,i_*,k} \right) + \theta = 0 \quad (\text{A-12})$$

for all $k=1,2,\dots,\infty$. This implies that

$$y_{g_*}^{r,i_*-1,k} - y_{g_*}^{r,i_*,k} < -\varepsilon_v^2 \beta \sum_{g_*=1}^G \left(q_{g_*}^{r,i_*,k} + \sum_{j=1}^{i_*-1} q_{g_*}^{p,j,i_*,k} \right) \quad (\text{A-13})$$

for all $k=1,2,\dots,\infty$. Due to the boundedness of $q_{g_*}^{r,i_*,k}$ and $q_{g_*}^{p,i_*,k}$, it can be concluded that $y_{g_*}^{r,i_*,k}$ is also unbounded. This contradicts the definition of index $i_* - 1$.

Boundedness of $T_g^{b,i,k}$. Similar as the above proof of **Boundedness of $t_g^{b,i-1,k}$** .

Boundedness of $Y_g^{r,i,k}$. Similar as the above proof of **Boundedness of $t_g^{b,i-1,k}$** .

Boundedness of $T_g^{b,0,k}$. Suppose that this is not true for some g_* , then assume without loss of generality that

$$\lim_{k \rightarrow \infty} T_{g_*}^{b,0,k} = \infty \quad (\text{A-14})$$

and $T_{g_*}^{b,0,k} > 0$ for all $k=1,2,\dots,\infty$. It follows that

$$T_{g_*}^{b,0,k} - T_{g_*}^{b,1,k} - t_h^0 \varepsilon_v + \theta = 0 \quad (\text{A-15})$$

for all $k=1,2,\dots,\infty$. It can be deduced that $T_{g^*}^{b,0,k} = T_{g^*}^{b,1,k} + t_h^0 \varepsilon_v - \theta < T_{g^*}^{b,1,k} + t_h^0 \varepsilon_v$ for all $k=1,2,\dots,\infty$. This is a contradiction due to the boundedness of $T_{g^*}^{b,1,k}$, so the boundedness of $T_g^{b,0,k}$ follows.

Boundedness of $Y_g^{r,0,k}$. Similar as the above proof of **Boundedness of $T_g^{b,0,k}$** .

Boundedness of $C_g^{*,i,k}$. According to (A-1),

$$0 \leq q_g^{h,i,k} \perp \tau_g t_h^{hh} + \tau_g t_h^{pw} + \tau_g T_g^{b,i,k} + f_h^0 + \gamma(v-i+1)\varepsilon_v + R_h^w - C_g^{*,i,k} + \theta \geq 0 \quad (\text{A-16})$$

therefore, for some g^* and i_* , the following inequality holds,

$$\tau_{g^*} t_h^{hh} + \tau_{g^*} t_h^{pw} + \tau_{g^*} T_{g^*}^{b,i_*,k} + f_h^0 + \gamma(v-i_*+1)\varepsilon_v + R_h^w - C_{g^*}^{*,i_*,k} + \theta \geq 0 \quad (\text{A-17})$$

Since $\tau_{g^*} T_{g^*}^{b,i_*,k}$ is bounded, so the boundedness of $C_g^{*,i,k}$ follows. This concludes the whole proof. $\square$

APPENDIX B Proof of Lemma 5.1 in Section 5.2.4

Proof. First of all, the flow propagation functions, Eq. (5-19) with Eq. (5-20), are proved to be continuous in $r_{t,g}^i$, $\forall t=0,1,\dots,v-1$, $i=1,\dots,n$; $g=1,\dots,G$. As a linear equation, Eq. (5-20) is continuous obviously, furthermore, it can also show that $f_{t,g}^i$ is continuous in $k_{t,g}^i$ when $\sum_{g=1}^G k_{t,g}^i > 0$ and $\sum_{g=1}^G f_{t,g}^i > 0$, $\forall t=0,1,\dots,v-1$; $i=1,\dots,n$; $g=1,\dots,G$. Since

$$0 \leq \sum_{g=1}^G f_{t,g}^i \frac{k_{t,g}^i}{\sum_{g=1}^G k_{t,g}^i} \leq \sum_{g=1}^G f_{t,g}^i, \quad \forall t=0,1,\dots,v-1; i=1,\dots,n; g=1,\dots,G \quad (\text{B-1})$$

it is easy to get that

$$\lim_{\left(\sum_{g=1}^G f_{t,g}^i \rightarrow 0\right)} \sum_{g=1}^G f_{t,g}^i \frac{k_{t,g}^i}{\sum_{g=1}^G k_{t,g}^i} = 0, \quad \forall t=0,1,\dots,v-1; i=1,\dots,n; g=1,\dots,G \quad (\text{B-2})$$

So the continuity of the flow propagation functions holds.

Now it is ready to prove the uniform boundedness of the solutions to [CP] (5-28).

Suppose there exists a sequence of positive scalars $\{\lambda^l\}_{l=1}^\infty$ and a sequence of vectors

$\{TT^l, k^l, \pi^l, r^l, \theta^l\}_{l=1}^\infty$ satisfying for each $l=1,\dots,\infty$.

$$0 \leq TT_t^{i,l} \perp TT_t^{i,l} - TT_t^{i+1,l} - \frac{\Delta x}{-a_t^i \sum_{g=1}^G k_{t,g}^i + b_t^i} + \lambda^l TT_t^{i,l} \geq 0 \quad (\text{B-3})$$

$$0 \leq k_{t,g}^{i,l} \perp -a_t^i k_{t,g}^i \sum_{g=1}^G k_{t,g}^i + b_t^i k_{t,g}^i - f_{t,g}^{i,l}(r_{t,g}^{i,l}) + \lambda^l k_{t,g}^{i,l} \geq 0 \quad (\text{B-4})$$

$$0 \leq \pi_g^{i*,l} \perp \sum_{t=0}^{v-1} r_{t,g}^{i,l} - D_g^0 e^{-\phi_g(n-i)} + \lambda^l \pi_g^{i*,l} \geq 0 \quad (\text{B-5})$$

$$0 \leq r_{t,g}^{i,l} \perp (\alpha_g + \gamma_g) TT_t^{i,l} + (\beta_g + \gamma_g) \theta_{t,g}^{i,l} - \pi_g^{i*,l} + \gamma_g(t - t_g^*) + \lambda^l r_{t,g}^{i,l} \geq 0 \quad (\text{B-6})$$

$$0 \leq \theta_{t,g}^{i,l} \perp \theta_{t,g}^{i,l} + TT_t^{i,l} + t - t_g^* + \lambda^l \theta_{t,g}^{i,l} \geq 0 \quad (\text{B-7})$$

$$\forall t=0,1,\dots,v-1, i=1,\dots,n, g=1,\dots,G \quad (\text{B-8})$$

Boundedness of $\{(k_{t,g}^{i,l})_{t=0,1,\dots,v-1; i=1,\dots,n; g=1,\dots,G}\}_{l=1}^\infty$

Suppose that this is not true for some $t \in \{0, 1, \dots, v-1\}$, $i \in \{1, \dots, n\}$, $g \in \{1, \dots, G\}$, then assume without loss of generality that

$$\lim_{l \rightarrow \infty} k_{t,g}^{i,l} = \infty \quad (\text{B-9})$$

and $k_{t,g}^{i,l} > 0$ for all $l = 1, \dots, \infty$. It follows from (B-4) that

$$-a_t^i k_{t,g}^{i,l} \sum_{g=1}^G k_{t,g}^{i,l} + b_t^i k_{t,g}^{i,l} - f_{t,g}^{i,l}(r_{t,g}^{i,l}) + \lambda^l k_{t,g}^{i,l} = 0 \quad (\text{B-10})$$

for all $l = 1, \dots, \infty$. This implies that

$$k_{t,g}^{i,l} = \frac{f_{t,g}^{i,l}(r_{t,g}^{i,l})}{-a_t^i \sum_{g=1}^G k_{t,g}^{i,l} + b_t^i + \lambda^l} \quad (\text{B-11})$$

for all $l = 1, \dots, \infty$. Due to $\lim_{l \rightarrow \infty} k_{t,g}^{i,l} = \infty$ and $k_{t,g}^{i,l} > 0$, it must have

$$\lim_{l \rightarrow \infty} \left(-a_t^i \sum_{g=1}^G k_{t,g}^{i,l} + b_t^i + \lambda^l \right) < 0, \text{ therefore } \lim_{l \rightarrow \infty} k_{t,g}^{i,l} < 0. \text{ This is a contradiction, so the}$$

boundedness of $\{(k_{t,g}^{i,l})_{t=0,1,\dots,v-1; i=1,\dots,n; g=1,\dots,G}\}_{l=1}^{\infty}$ follows.

Boundedness of $\{(TT_t^{i,l})_{t=0,1,\dots,v-1; i=1,\dots,n}\}_{l=1}^{\infty}$

Let $i_* \in \{1, \dots, n\}$ be the biggest i (note that cell 1 to cell n is the direction from the corridor boundary to CBD and $TT_t^1 > TT_t^2 > \dots > TT_t^n$, $\forall t = 0, 1, \dots, v-1$) such that $TT_t^{i,l}$ is unbounded, then assume that

$$\lim_{l \rightarrow \infty} TT_t^{i_*,l} = \infty \quad (\text{B-12})$$

and $TT_t^{i_*,l} > 0$ for all $l = 1, \dots, \infty$. Thus,

$$TT_t^{i_*,l} - TT_t^{i_*+1,l} - \frac{\Delta x}{-a_t^{i_*} \sum_{g=1}^G k_{t,g}^{i_*} + b_t^{i_*}} + \lambda^l TT_t^{i_*,l} = 0 \quad (\text{B-13})$$

for all $l = 1, \dots, \infty$. This implies that

$$(1 + \lambda^l) TT_t^{i_*,l} - TT_t^{i_*+1,l} = \frac{\Delta x}{-a_t^{i_*} \sum_{g=1}^G k_{t,g}^{i_*} + b_t^{i_*}} \quad (\text{B-14})$$

for all $l=1, \dots, \infty$. Due to the boundedness of $k_{t,g}^{i^*}$, it can be concluded that $TT_t^{i^*+1,l}$ is also bounded. This contradicts the definition of index i^* .

Boundedness of $\{(\pi_g^{i^*,l})_{i=1, \dots, n; g=1, \dots, G}\}_{l=1}^\infty$

Suppose that this is not true for some $i \in \{1, \dots, n\}$ and $g \in \{1, \dots, G\}$, then assume without loss of generality that

$$\lim_{l \rightarrow \infty} \pi_g^{i^*,l} = \infty \quad (\text{B-15})$$

and $\pi_g^{i^*,l} > 0$ for all $l=1, \dots, \infty$. It follows from (B-5) that

$$\sum_{t=0}^{v-1} r_{t,g}^{i,l} - D_g^0 e^{-\phi_g(n-i)} + \lambda^l \pi_g^{i^*,l} = 0 \quad (\text{B-16})$$

for all $l=1, \dots, \infty$. It suffices to deduce that $\pi_g^{i^*,l} = \frac{D_g^0 e^{-\phi_g(n-i)} - \sum_{t=0}^{v-1} r_{t,g}^{i,l}}{\lambda^l} \leq \frac{D_g^0}{\lambda^l}$. This is a contradiction, so the boundedness of $\{(\pi_g^{i^*,l})_{i=1, \dots, n; g=1, \dots, G}\}_{l=1}^\infty$ follows.

Boundedness of $\{(r_{t,g}^{i,l})_{t=0,1, \dots, v-1; i=1, \dots, n; g=1, \dots, G}\}_{l=1}^\infty$

Suppose that this is not true for some $t \in \{0, 1, \dots, v-1\}$, $i \in \{1, \dots, n\}$, $g \in \{1, \dots, G\}$, then assume without loss of generality that

$$\lim_{l \rightarrow \infty} r_{t,g}^{i,l} = \infty \quad (\text{B-17})$$

and $r_{t,g}^{i,l} > 0$ for all $l=1, \dots, \infty$. It follows from (B-6) that

$$(\alpha_g + \gamma_g) TT_t^{i,l} + (\beta_g + \gamma_g) \theta_{t,g}^{i,l} - \pi_g^{i^*,l} + \gamma_g (t - t_g^*) + \lambda^l r_{t,g}^{i,l} = 0 \quad (\text{B-18})$$

for all $l=1, \dots, \infty$. It suffices to deduce that

$$r_{t,g}^{i,l} = \frac{\gamma_g (t_g^* - t) + \pi_g^{i^*,l} - (\alpha_g + \gamma_g) TT_t^{i,l} - (\beta_g + \gamma_g) \theta_{t,g}^{i,l}}{\lambda^l} \leq \frac{\gamma_g t_g^* + \pi_g^{i^*,l}}{\lambda^l} \quad (\text{B-19})$$

Note that $\pi_g^{i^*,l}$ has been proved to be bounded, thus this is a contradiction, so the boundedness of $\{(r_{t,g}^{i,l})_{t=0,1, \dots, v-1; i=1, \dots, n; g=1, \dots, G}\}_{l=1}^\infty$ follows.

Boundedness of $\{(\theta_{t,g}^{i,l})_{t=0,1, \dots, v-1; i=1, \dots, n; g=1, \dots, G}\}_{l=1}^\infty$

Suppose that this is not true for some $t \in \{0, 1, \dots, v-1\}$, $i \in \{1, \dots, n\}$, $g \in \{1, \dots, G\}$, then assume without loss of generality that

$$\lim_{l \rightarrow \infty} \theta_{t,g}^{i,l} = \infty \quad (\text{B-20})$$

and $\theta_{t,g}^{i,l} > 0$ for all $l = 1, \dots, \infty$. It follows from (B-7) that

$$\theta_{t,g}^{i,l} + TT_t^{i,l} + t - t_g^* + \lambda^l \theta_{t,g}^{i,l} = 0 \quad (\text{B-21})$$

for all $l = 1, \dots, \infty$. It suffices to deduce that $\theta_{t,g}^{i,l} = \frac{t_g^* - TT_t^{i,l} - t}{(1 + \lambda^l)} \leq \frac{t_g^*}{(1 + \lambda^l)}$. This is a contradiction, so the boundedness of $\{(\theta_{t,g}^{i,l})_{t=0,1,\dots,v-1; i=1,\dots,n; g=1,\dots,G}\}_{l=1}^{\infty}$ follows. This concludes the whole proof. $\square$