Construction and real-time programming of a bipedal robot

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CONSTRUCTION AND REAL -TIME PROGRAMMING OF A BIPEDAL ROBOT

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ABSTRACT

Bipedal robots have attracted much research interests in recent years. It is observed that most of these robots were using computer-based control systems. The hardware selected usually led to a high cost for the system. The possibility of developing bipedal robots with integrated position servos and compact micro-controllers was explored in this project. The objective of this thesis is to investigate and implement bipedal locomotion on these bipedal walking robots developed by the author. The scopes of this research include design and construction of functional bipedal robot prototypes, implementation of basic quasi-static locomotion, dynamic analysis of the bipedal robot based on simplified models and realization of dynamic walking on flat surfaces.

As the starting point of this research, a comprehensive survey was conducted on successful examples of bipedal robots such as ASIMO, Pino, QRIO and HRP-2, etc. To complete the literature review, recent progress in the research on bipedal dynamics and motion control was also studied extensively.

Throughout the research work, two functional prototypes of bipedal walking robots were designed and constructed. The problem of kinematics and inverse kinematics for these two robots was solved through combination of curve fitting and mapping. The quasi-static motions such as walking and turning were investigated to justify the validity of this approach. The issue of dynamic stability was thus attempted based on off-line calculation established from simplified dynamic models. Starting with the motion of rocking on a flat surface, the dynamic motions of walking and turning were consummated with manual tuning on the basis of theoretical analysis.

Both stable quasi-static and dynamic motions were realized in experiments. Effectiveness of the analytical establishments presented in this thesis is thereby justified. Based upon the stable basic dynamic motions developed in this project, the second prototype can be augmented to an adaptive platform when an appropriate feedback system and artificial intelligence are incorporated.

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LIST OF NOTATIONS

Notation	Description
CG	centre of gravity
$q = [q_1, \dots, q_n]$	vector of joint coordinate
$k = [k_1, \dots, k_n]$	geometric and dynamic parameter vector, dependent on second moment of inertia and the CG's local position of each link in the robot
ZMP	zero moment point
<i>pulse number</i>	time scale, one unit representing 0.02s
<i>pulsout unit</i>	pulse width unit used in Basic Stamp 2sx, one unit representing 0.8 μ s
<i>pulsout input</i>	width of a pulse specified in <i>pulsout unit</i>
q_A	input angle from the servo in a four-bar linkage coupling (Figure 4.3)
q_B	output angle of a four-bar linkage coupling (Figure 4.3)
PW_A	representation of q_A in the pulsout unit used in Basic Stamp 2sx
$M : A \rightarrow B$	a mapping from set A to set B
x_{ankle}, z_{ankle}	coordinates of the ankle joint relative to the hip (Figures 4.5 and 4.6)
PW_{thigh}	pulsout input to the servo actuating the thigh of a leg
PW_{calf}	pulsout input to the servo actuating the calf of leg
q_1, q_2	input angles to a leg mechanism (Figures 4.5 and 4.6)
q_a	angle required to keep the surface of sole parallel to the transverse plane of robot body (Figures 3.2, 4.5 and 4.6)
q_{LA}	angle between the left leg and the vertical axis in frontal plane of the left foot, equivalent to lateral angle of the left ankle joint (Figures 4.7 and 5.3)
q_{RA}	angle between the right leg and the vertical axis in frontal plane of the right foot, equivalent to lateral angle of the right ankle joint (Figures 4.7 and 5.3)
l_2	length of each leg seen in frontal plane of the corresponding foot (Figures 4.7 and 5.2)

h_O	height of the robot's CG when the robot is standing upright (Figure 4.7)
df_{LA}	offset angle at the servo driving the left ankle joint (Tables 4.1 and 4.2)
df_{RA}	offset angle at the servo driving the right ankle joint (Tables 4.1 and 4.2)
df_{LE}	offset angle at the servo driving the left hip joint (Tables 4.1 and 4.2)
df_{RE}	offset angle at the servo driving the right hip joint (Tables 4.1 and 4.2)
$d_{H_r-A_r}, d_{H_l-A_l}$	length of each leg seen in frontal plane of the corresponding foot (Figure 4.14)
x_r, z_r	coordinates of the right ankle joint relative in the right hip coordinate system (Figure 3.2)
x_l, z_l	coordinates of the right ankle joint relative in the left hip coordinate system (Figure 3.2)
v_{qx}	walking speed of the robot under quasi-static stability
β	motion constant of the gait for turning, equal to one quarter of the turning angle per motion cycle of turning
w	distance between the ZMP and the centre of stable region in frontal plane of a foot (Figure 5.1)
d	distance between the ZMP and the centre of stable region in lateral plane of a foot (Figure 5.1)
J_2	second moment of inertia of each leg on the lateral plane of the corresponding foot (Figure 5.2 and Eq. 5.2.1)
l_x	radius of gyration of the entire robot in its coronal plane
l_y	radius of gyration of the entire robot in its sagittal plane
$l_{H'_l-H'_r}, l_1$	length of the pelvis (Figure 5.2)
T_s	period of inverted pendulum motion, defined as the time spent by the inverted pendulum between rising from its initial position and falling back to the same position (Figure 5.6)
$q^*, q^\&$	motion constants specified for sideways motions of the robot (Eq. 5.3.20)
v^*	walking speed of the robot under dynamic stability

CHAPTER 1 INTRODUCTION

1.1 Background

On open and relatively gentle land terrain, wheeled and tracked mobile robots usually exhibit superior mobility to legged robots. Nonetheless, legged robots have the upper hand when maneuvering within confined and cluttered space or extremely rough terrain since they possess generally more degrees of freedom (DOF) and occupy smaller support areas. With these features, optimization in selection of footholds is implemented. Thereby exceptional adaptability to environments can be achieved by legged robots.

In the area of legged robots, bipedal robots attract much interest in research community. On account of their structural similarity to human beings, bipedal robots have high potential of human-like mobility, which allows them to respond to cluttered environments, especially those where humans live and work.

Quasi-static stability of a bipedal robot is usually achieved at the expense of motion speed. The robot is constrained to slow motions in which dynamics of the system can be ignored [1]. With increasing requirements in agility and mobility, the issue of dynamic stability signifies. Under dynamic motion, there are time intervals when the robot's projected centre of gravity onto the ground shifts out of the support areas. The control problem of a bipedal robot in this circumstance is significantly more involved compared to that for quasi-static stability. Whilst high mobility is in general essential to mobile robots for practical applications, extensive efforts on research of bipedal robots are devoted to the investigation and realization of dynamic bipedal locomotion by a large number of researchers.

Despite remarkable advancements in study of bipedal dynamics and control algorithms, the successful development of a bipedal robot still relies heavily on its hardware components such as actuators, sensors and controllers etc. Therefore, most of the programmes on bipedal robots entail large amounts of capital. However, there are

solutions with lower costs by resorting to commercially available components off the shelf. The research work carried out in this project was to explore and study dynamic behaviours of small-scale bipedal robots with platforms developed upon integrated servomotors and micro-controllers.

1.2 Objective and Scope

The objective of this thesis is to investigate and implement bipedal locomotion on the bipedal walking robots developed by the author (Figures 3.5 and 3.7). Two functional prototypes were constructed as the experimental platforms, where Futaba integrated servos [2] with built-in PD controller and Basic Stamp 2sx micro-controllers [3] were employed for actuation and control respectively.

Both prototypes were designed to have 6 DOFs on each leg, which amounts to 12 DOFs for each individual robot. Implementation of quasi-static locomotion on both robots is relatively straightforward since only geometrical analysis is required for quasi-static balancing, despite the fact that the real-time computation involved in some combined motions are too intensive to a compact array of Basic Stamp 2sx micro-controllers. For the dynamic regime, effectiveness of the dynamic analysis based on simplified physical models was examined on the first prototype that relied upon external power source. From the results obtained, improved dynamic locomotion with enhanced flexibility were realized on the second prototype which was tether-less with a payload of batteries as its power supply.

The scopes of this research are:

- Design and construction of functional bipedal robot prototypes
- Implementation of basic quasi-static locomotion
- Dynamic analysis of the bipedal robot based on simplified models
- Realization of dynamic walking on flat surfaces

1.3 Thesis Contributions

The contributions of this work are:

- 1) Design and construction of two 3D prototype bipedal walking robots
- 2) Accurate approximation methods for real-time calculation of inverse kinematics on a compact array of 16-bit micro-controllers made up by three Basic Stamp 2sx chips.
- 3) Implementation of quasi-static walking and turning on a flat and level surface
- 4) Analysis on dynamic behaviours of the bipedal robots based upon the linear inverted pendulum model and the four-bar linkage model
- 5) Implementation of dynamic walking and turning at various speeds on a flat and level surface
- 6) Realization of dynamic walking on ramps of different slope gradients
- 7) Teleoperation of the bipedal robots under dynamic walking and turning

1.4 Thesis Outline

Following the introduction in this chapter, the thesis proceeds as below:

Chapter 2 presents a literature review of the recent development in research on bipedal walking robots. In addition to a survey on successful bipedal robots, the issues of bipedal dynamics and motion control are also investigated.

Chapter 3 describes the mechanical development of both prototypes of bipedal robots in this research project.

Chapter 4 illustrates the identification of the system's static characteristics. In the synthesis of quasi-dynamic gaits for walking and turning on horizontal surfaces, the approximation approach is applied. The results of static testing on the actuators and the analysis on kinematics and inverse kinematics of the legs prepare the basics for this approach.

Chapter 5 provides the dynamic analysis on the second prototype of bipedal robot. The stability conditions for the robots are addressed. By the aid of simplified physical models, dynamic gaits for walking and turning are synthesized with respect to these conditions.

Chapter 6 presents conclusion of the entire project and suggests future developments.

At the back of this thesis is attached a CD-ROM, which records the movie clips demonstrating major parts of the research work accomplished in the project.

CHAPTER 2 LITERATURE REVIEW

Tremendous research endeavours were devoted to both powered and passive bipedal walking robots. Whilst passive robots generally do not contain any actuation, sensing or control systems, they are powered by gravity and dependent upon natural dynamic mechanisms to generate walking motions [4,5]. Since they have hardly any flexibility in their motions, this class of robots are more of academic interests in exploitation of natural dynamics [6,7]. On the other hand, powered bipedal robots are more capable of performing comprehensive motions. In recent years, prominent success has been achieved in paving their road towards practical application by researchers. Based on these observations, most of the efforts in this literature review were committed to powered bipedal walking robots and related issues on their dynamics and motion control.

2.1 Powered Bipedal Walking Robots

In recent years, a number of powered bipedal walking robots have been developed by various research groups throughout the world. Some representatives are described below following the time sequence of their debuts. From these successful examples, a glance at the latest advancement in research of bipedal dynamics and motion control theories is given.

Pino [8,9,10], which was developed under the Kitano Symbiotic Systems Project of the Japan Science and Technology Corporation, provides good interpretation to the idea of producing robots through low-cost off-the-shelf components (Figures 2.1 and 2.2). This robot possesses 26 DOFs that are actuated by the same number of Futaba integrated servomotors. The control system of robot consists of a host computer, which is composed of Pentium III 733 MHz processor and 512 Mb memory, and a controller for servomotors. Dynamic motions of the robot are controlled by utilizing direct kinematics and Jacobian matrices which denote velocity relationships between joint angles and positions and posture of each foot with respect to the coordinate frame attached to the robot body.

Figure 2.1: Whole View of Pino (Courtesy of Osaka University) [8]

Figure 2.2: Mechanical Structure of Pino (Courtesy of Osaka University) [8]

Spring Flamingo [11,12] (Figure 2.3) is a planar bipedal walking robot with 6 degrees of freedom developed at MIT Leg Laboratory by Jerry Pratt. Series elastic actuators [13,14] (Figure 2.4), which enabled force control, were utilized for the actuation of all degrees of freedom. Various walking algorithms were implemented on this robot to realize dynamic walking on flat surface and over ramps. Natural dynamics of the robot was also exploited to improve the efficiency of walking. This is achieved by adoption of the kneecaps and compliant ankles.

Figure 2.3: Whole View of Spring Flamingo (Courtesy of MIT Leg Laboratory) [12]

Figure 2.4: (A) Series Elastic Actuator (B) Schematic Diagram of the Actuator (Courtesy of MIT Leg Laboratory) [13,14]

Troody [15], developed by Massachusetts Institute of Technology Leg Laboratory, is a robotic troodon used for the study of biomechanical locomotion and robotics in general (Figure 2.5). This robot has 16 DOFs with onboard batteries and computers. It presented a good example for dynamic balancing and walking of bipedal robots. To improve the fluency and smoothness of walking motion, series elastic actuators [13,14] are also adopted on Troody. The control system employed is termed “P-Q control” which varies the forces applied to the robot feet based on the kinematical state of the robot

Figure 2.5: Troody (Courtesy of MIT Leg Laboratory) [15]

ASIMO [16,17], developed under the Honda Humanoid Project, has been one of the most successful bipedal robots hitherto (Figure 2.6). This robot features a total number of 26 DOFs. Predicted movement control is employed on ASIMO in addition to the earlier walking control technology which relied upon a number of stored walking patterns. The robot itself creates walking patterns in real-time and therefore achieves more intelligent and flexible walking.

Figure 2.6: ASIMO (Courtesy of HONDA) [16]

QRIO [18] (Figure 2.7) is an entertainment humanoid robot developed by Sony Corp. Intelligent Servo Actuator (ISA) is the drive system specially designed for the robot's joints and limbs. This system is able to produce varying levels of torque at varying rpm speeds and drives the joints to their target positions with precision and silence.

QRIO's underlying movement control method is based on Zero Moment Point (ZMP). The control system directs the robot's movements so that its ZMP always falls within its

zone of stability. Conditions of the walking surfaces are detected with four pressure sensors in the sole of each foot of QRIO. The holistic movement control system adjusts the attitude and walking pattern of the robot based on this information and the data collected from position sensors. The robot is thus enabled to negotiate uneven surfaces with disparities and slopes.

Figure 2.7: QRIO (Courtesy of SONY) [18]

HRP-2 [19,20] (Figure 2.8) is a humanoid robotics platform under the humanoid robotics project HRP, which was run by the Ministry of Economy, Trade and Industry of Japan. In addition to coping with uneven surfaces and narrow paths, this robot is able to get up by itself when tipped over safely. In order to realize these functions, two distinctive mechanisms are incorporated on the robot:

- Cantilever type structure in hip joint which helps the robot prevent tipping over and perform cross-legged walking

- A waist joint with 2 DOFs which assists the getting up motion besides improving the suppleness of the robot body and hence the smoothness of the walking gaits

Figure 2.8: HRP-2 and its Electrical System (Courtesy of National Institute of Advanced Industrial Science and Technology and Kawada Industries, Inc., Japan) [20]

HRP-2 has two CPU boards in its body. One of them is utilized for non-real-time control including the 3D object recognition system and the sound system, whilst the other governs the real-time control of whole body motion. Instead of following pre-defined walking gaits, HRP-2 generates its own gait patterns in real-time by manipulation of the position of its ZMP based on information from sensors [21, 22].

2.2 Review on Biped Dynamics

Based upon the current literatures on biped dynamics, three important aspects related to the research work in this thesis were studied: establishment of the problem of dynamic stability, overview of the zero moment point (ZMP) notion and mathematical formulation of biped dynamics.

2.2.1 Stability Issue

During locomotion, as the acceleration on each link of a bipedal walking robot is of small magnitude, the inertia forces are usually negligible. The problem of balancing for the robot thus approaches a static one. Since the robot is constantly in motion, the stability achieved under this circumstance is recognized as quasi-static one. Accelerations are generally neglected in the control during quasi-static locomotion. Whilst the motion of a bipedal robot involves continually changes in the velocities of its links, these changes generally increase with the velocities, which are in turn translated into the increases in accelerations. Therefore most of quasi-static motions are restrained to relatively low speeds. The maximum motion speed that a bipedal walking robot achieves before the accelerations become a pronounced factor in the stability issue is mainly dependent on the mechanism backlashes, structural elasticity, structural damping and dimensions of the robot, etc.

Dynamic stability emerges as an inevitable issue when manoeuvrability is coveted. While it appears somewhat difficult to find the formal definition of dynamic stability in literature, there are attempts to explore a general definition with the concepts in time-manifold control systems [23]. Nevertheless, for the bipedal robots developed in this project, dynamic stability is to be examined under the priori that the robot does not experience toppling during a finite time segment of dynamic movement. Here the toppling event is understood as the rotation of the entire or part of the robot about one edge of the supporting foot.

Control on torque/force output from the actuators [24] is generally required in addition to that over positions and velocities. The following expression is extensively applied in the dynamic modelling of bipedal robots with simple open kinematic chains [25]:

$$P = H(q, \dot{q}) \ddot{q} + h(q, \dot{q})$$

where $P = [P_1, \dots, P_n]$ is a vector of driving moments at the joints,

$H(q, \dot{q})$ is the $n \times n$ inertial matrix,

$h(q, \dot{q})$ is the $n \times 1$ vector of Coriolis, centrifugal and gravity forces,

$q = [q_1, \dots, q_n]$ is a vector of joint coordinates,

$k = [k_1, \dots, k_n]$ is a geometric and dynamic parameter vector dependent on the properties of each link in the kinematic chain.

2.2.2 Review of Zero Moment Point (ZMP) Notion

In the study of dynamic stability, the concept of zero moment point (ZMP) has been introduced as one of the most useful dynamic criteria for the analysis and synthesis of bipedal robot locomotion.

For bipedal walking robots, the joints formed by contact of their feet and the ground are the only points at which the mechanisms interact with the environment. The stability of a bipedal robot could be indicated by the ground reaction forces at these joints.

The pressure under the supporting foot can be replaced by the appropriate reaction force acting at a certain point of the foot. Since the sum of all moments of active forces with respect to this point is equal to zero, it is termed the zero-moment point (ZMP) [26].

For a bipedal robot consisting of n links, during locomotion the following active motion forces act on the body links (as shown in Figure 2.9):

$\vec{G}_i$ = Gravitation force of the i^{th} link acting at the mass centre C_i

$\vec{F}_i$ = Inertial force of the i^{th} link acting at the mass centre C_i

$\vec{M}_i$ = Moment of the inertial force of the i^{th} link for C_i

$\vec{R}$ = Resultant ground reaction force

The dynamic equilibrium during motion are expressed by the following equations in the reference coordinate system [26]:

$$\vec{R} + \sum_{i=1}^n (\vec{F}_i + \vec{G}_i) = 0 \quad (2.2.1)$$

$$\overrightarrow{OZMP} \times \vec{R} + \sum_{i=1}^n \overrightarrow{OC_i} \times (\vec{F_i} + \vec{G_i}) + \sum_{i=1}^n \vec{M_i} + \vec{M_{hZMP}} + \vec{M_{fZMP}} = 0 \quad (2.2.2)$$

where $\overrightarrow{OZMP}$ is the position vector from the origin of the reference frame to ZMP

$\overrightarrow{OC_i}$ is the position vector from the origin of the reference frame to C_i

$\vec{M_{hZMP}}$ is the moment about ZMP by vertical reaction forces

$\vec{M_{fZMP}}$ is the moment about ZMP by horizontal reaction forces (friction forces)

Figure 2.9: Active Motion Forces Acting on the Body Links

Based on the ZMP definition:

$$\vec{M_{hZMP}} = 0 \quad (2.2.3)$$

The ZMP coordinates in the horizontal ground plane are obtained by combining Eqs. (2.2.1), (2.2.2) and (2.2.3). To prevent the bipedal robot from toppling, which is generally

undesirable for the balancing of legged robots, the ZMP is always maintained in the supporting region during locomotion (Figure 2.10).

Figure 2.10: Possible Positions of ZMP: (A) ZMP is inside the supporting region (dynamically balanced), (B) ZMP is outside the supporting region (dynamically unbalanced)

2.2.3 Dynamics Formulation

Biped dynamics is usually rather complicated due to contacts and impact events occurring successively during the walking cycle. Analysis on dynamics of a bipedal robot is frequently based on some simplified models [11,27,28,29,30] including various constraints with the assumption of no slippage.

The dynamics formulation for bipedal robots proceeds as below [31]:

The general form of the dynamics of a robot subject to contact efforts can be written as:

$$M(q)\ddot{q} + N(q, \dot{q}) + G(q) = G + G_c \quad (2.2.4)$$

where,

- $q \in \mathbb{R}^n$ is the vector of configuration variables ($n-6=n'$ joint positions and 6 absolute position and orientation of the robot)

- $G = [t, 0]^T \in \mathfrak{R}^n$ is the vector of generalized efforts, where $t = WU \in \mathfrak{R}^{n'}$ is the joint torque vector, $U \in \mathfrak{R}^m$ is the vector of motor torques, W is the $n' \times m$ matrix of the motor actions on the joints ($m \leq n'$)
- M is the inertia matrix of $n \times n$ dimension
- N is the $n \times n$ matrix of the centrifugal, gyroscopic and Coriolis effects
- G is the generalized gravity forces vector of dimension n
- G_c are the torques generated by contact forces, like ground contacts

The geometrical constraints between robot joints can be expressed by the following equation:

$$f(q) = (f_n(q), f_t(q))^T = 0 \quad (2.2.5)$$

where $f_n(q)$ and $f_t(q)$ correspond to the vertical and horizontal projections of contact points respectively.

The associated contact torques due to ground efforts are expressed as:

$$\Gamma_c = C_n(q)^T I_n(q, \phi) + C_t(q)^T I_t(q, \phi) \quad (2.2.6)$$

where

- C_n and C_t are the Jacobian matrices of the constraints on the positions of contact points on the vertical plane and horizontal plane respectively
- I_n and I_t are the normal and lateral contact forces respectively

Unilateral constraints attest the fact that the points in contact cannot go under the ground:

$$F_n(q) \geq 0 \quad (2.2.7)$$

Complementarity conditions manifest that when a contact is broken, the corresponding normal force vanishes. The relation between normal support forces and linkage constraint are hence expressed as:

$$I_n^T(q, \phi) F_n(q) = 0, \quad I_n(q, \phi) \geq 0, \quad F_n(q) \geq 0 \quad (2.2.8)$$

When the condition of non-slippage is satisfied, the lateral displacements of the robots extremities in contact with the ground are null. The constraint on the lateral acceleration is therefore specified as:

$$C_i(q) + \dot{C}_i(q)\dot{q} = 0 \quad (2.2.9)$$

Formulation of the continuous dynamics with contacts, in which no discrete events (impact events etc.) are involved, is thereby completed through Eqs. (2.2.4) to (2.2.9).

2.3 Motion Control

For dynamic stability, control synthesis is generally addressed in two steps: (a) synthesis of walking patterns, and (b) control of the robot along a reference trajectory, with on-line monitoring and modification of variables related to the walking stability.

2.3.1 Gait Synthesis

Biped locomotion exhibits the following characteristics that are essential during the gait synthesis [1]:

- Unpowered DOFs are formed by contacts of the feet of the biped robot with the ground. These DOFs are susceptible to disturbances and have crucial influence on stability of the system.
- The configuration of the legs' kinematic chain changes from the open to a closed one and back alternately, which divides a walking gait into single-support phase and double-support phase.
- The mechanism motion is periodical.

Reference trajectories are generated through gait pattern synthesis. These trajectories can generally be pursued in three directions: (i) anthropomorphic joint trajectories derived from human walking, (ii) torque-based trajectories, (iii) optimal trajectories computed off-line [31].

Besides the frequently practiced ZMP criterion [32], the problem of gait synthesis can be specified as a constrained optimisation problem [21,33]. The constraints include dynamic model of the robot, functional constraints and movement limitations. Functional constraints are: joint limits and actuator saturations in torque or force output. Movement limitations are usually recognized as space constraints. To improve adaptability of the dynamic gaits, the parametrization approach was also explored [34].

2.3.2 Control Synthesis

In general, the control structure applied on bipedal robots is of four levels [1] (Figure 2.11):

- 1) Highest control level: recognizes the obstacles in the operating space and the conditions under which a task is being performed, and makes the decisions on how the task imposed is to be accomplished.
- 2) Strategic control level: divides the imposed operation into elementary movements.
- 3) Tactical control level: distributes an elementary motion to the motion of each DOF of the robot.
- 4) Executive control level: executes the imposed motion of each DOF.

Figure 2.11: Hierarchical Architecture of Control Biped Locomotion Robots [1]

Whilst the two upper control levels are generally recognized as intelligence level, the lower two have essential effects on the stability of the robot, where trajectory modification and tracking are implemented. Except conventional PID controllers, other control schemes involving H_∞ controller, fuzzy logic [35], learning [36] or neural network [37] are also proposed to negotiate inaccuracies in dynamic modelling of the system and improve the robustness of control.

CHAPTER 3 MECHANICAL STRUCTURES

In this research project, the prototype bipedal robots were required to perform motions of walking and turning under both quasi-static and dynamic stability. This is implemented with a design possessing totally 12 DOFs. The distribution of these DOFs is illustrated in Figure 3.1:

Figure 3.1: Schematic Diagram of the Bipedal Robot Showing DOFs of the Joints for the Legs

3.1 List of Coordinate Systems

The coordinate systems used in this thesis include one global coordinate system and five local coordinate systems:

System $O - xyz$	global coordinate system, fixed to the ground
System $O_b - x_b y_b z_b$	body coordinate system, attached to the robot body at the centre of the pelvis
System $O_p - x_p y_p z_p$	right hip coordinate system, attached to right hip of the robot
System $O_q - x_q y_q z_q$	left hip coordinate system, attached to left hip of the robot
System $O_r - x_r y_r z_r$	right foot coordinate system, attached to right foot of the robot
System $O_l - x_l y_l z_l$	left foot coordinate system, attached to left foot of the robot

These coordinate systems are shown in Figure 3.2:

Figure 3.2: Coordinate Systems Used to Describe Motions of the Robot

As shown in Figure 3.2, when the bipedal robot is standing upright, all the coordinate systems are parallel to each other with the six z -axes pointing vertically upwards and the x -axes forwards.

Definitions of the planes [38] in the five local coordinate systems are listed below:

Plane $x_b - O_b - y_b$	transverse plane of the robot body
Plane $y_b - O_b - z_b$	coronal plane of the robot body
Plane $z_b - O_b - x_b$	sagittal plane of the robot body
Plane $x_p - O_p - y_p$	transverse plane of the right hip
Plane $y_p - O_p - z_p$	frontal plane of the right hip
Plane $z_p - O_p - x_p$	lateral plane of the right hip
Plane $x_q - O_q - y_q$	transverse plane of the left hip
Plane $y_q - O_q - z_q$	frontal plane of the left hip
Plane $z_q - O_q - x_q$	lateral plane of the left hip
Plane $x_r - O_r - y_r$	transverse plane of the right foot
Plane $y_r - O_r - z_r$	frontal plane of the right foot
Plane $z_r - O_r - x_r$	lateral plane of the right foot
Plane $x_l - O_l - y_l$	transverse plane of the left foot
Plane $y_l - O_l - z_l$	frontal plane of the left foot
Plane $z_l - O_l - x_l$	lateral plane of the left foot

3.2 Mechanical Design of Prototype I

Twelve Futaba S3003 Standard servomotors [2] are employed for actuation of Prototype I. All the joints of the robot are formed by interaction between brass shafts and aluminum forks.

3.2.1 Leg Structure

The structure of each leg is designed as a five-bar linkage (Figure 3.3). For this structure, $A-B-D$ form the calf with a fixed angle between Link AB and Link BD . Point A represents position of the ankle, which is the output of the five-bar linkage. The entire linkage is actuated at Joint E and Joint F , which are located at the hip.

Figure 3.3: Schematic Diagram of Leg Structure for Prototype I

The arguments for adoption of this arrangement are as follows:

- (a) Weight of the structure is supported by Link BC and Link DF . Based on analysis of the linkage, axial loads sustained by these two links would be dominant over bending loads (Appendix A.1). As the axial loads are far below the critical load for buckling, this arrangement provides higher rigidity.
- (b) The five-bar linkage forms a coupled system. The load on the system is evenly distributed to the two servomotors driving Joint E and Joint F respectively (Appendix A.2). In view of control, synchronization of the leg motors can be augmented since the loads and motor characteristics will be approximately the same for both motors.

- (c) Since its CG is offset from the hip joints, the robot body behaves as a flywheel with large inertia about these joints [11]. Following the analysis in Appendix A.2, this effect is much suppressed in a coupled system as five-bar linkage. Complexity in dynamic modeling of the biped robot can therefore be reduced.

With the above considerations, the following specifications were imposed on design of the five-bar linkage:

- Torque Amplification Ratio $\approx 1:1$
- Minimum $l_{A-EF} < 100\text{mm}$
- Maximum $l_{A-EF} > 250\text{mm}$
- Structural condition (referring to Condition A.1.8 in Appendix A.1)

where l_{A-EF} denotes the distance from Point A to the line along Link EF.

Based on these specifications and the factors of structural strength, manufacturability, motor dimensions, etc., dimensions of the entire leg were determined as below (referring to Figure 3.3):

$$\begin{array}{llll} l_{AB} = 125.4\text{mm} & l_{BC} = 130.0\text{mm} & l_{BD} = 25.5\text{mm} & l_{DF} = 130.0\text{mm} \\ l_{CE} = 22.4\text{mm} & l_{EF} = 13.0\text{mm} & b = 6.7^\circ & \end{array}$$

3.2.2 Actuation of the Leg Mechanism

Servomotors are coupled to the joints through four-bar linkages. The arrangement of these couplings for a single leg of the robot is illustrated Figure 3.4. The torques required to drive the joints varies with joint angles, the four-bar linkages are designed to be torque amplifiers with varying amplification ratios adapting to these changes in torque requirements.

Figure 3.4: Diagram Showing Actuation of the Leg Mechanism (Left Leg)
 (A) Side View (B) Front View

Range for the output angle of each four-bar linkage was specified with compromise between torque requirement and motion range of the corresponding joint:

Linkage A:	pronation and supination of ankle: $> 50^\circ$
Linkage B:	dorsiflexion and plantarflexion of ankle: $> 45^\circ$
Linkage C:	flexion and extension of knee: $> 30^\circ$
Linkage D:	flexion and extension of hip: $> 30^\circ$
Linkage E:	abduction and adduction of hip: $> 70^\circ$
Linkage H:	internal and external rotation of hip: $> 50^\circ$

Based upon the analysis on four-bar linkage, torque requirements on individual joints and geometries of the robot, lengths for the links in each linkage were determined and listed in Appendix A.3.

3.2.3 Final Design of Prototype I

Figure 3.5 shows the pictures of Prototype I under the unactuated state. The legs of the robot are connected through the pelvis, to which the circuit board is attached. The feet are of rectangular shape with a dimension of 90mm×50mm.

Figure 3.5: Pictures of Prototype I. (A) Front View (B) Side View

Some basic specifications of Prototype I are listed below:

Length of the pelvis: 110mm

Standing height: about 300mm ~ 500mm

Total weight: about 1.2kg

3.3 Mechanical Design of Prototype II

As a development platform for the dynamic modeling that will be discussed in Chapter 5, Prototype I is connected to an external power supply and most of its DOFs have limited working range. This constrains both the quasi-static and dynamic locomotion performed by the robot. In Prototype II, ten of the Futaba S3003 servomotors are replaced by the same number of Futaba S9450 servos [2] that have higher torque outputs. The two remaining Futaba S3003 servos control yaw motion of the legs. By virtue of the new actuators, Prototype II is capable of carrying its own on-board power supply and has larger working ranges for most of its joints. Thereby it is developed to be a more adaptive walking platform. Based on the above discussion, Prototype I is named “original model”, whilst Prototype II is identified as “higher motor-torque model”.

3.3.1 Leg Mechanism

Ball bearings are employed on Prototype II, which improve efficiency of the system at expense of the increase in sizes of the robot joints. To minimize the structural dimensions and weight, a serial chain mechanism is adopted in place of the five-bar linkage. The couplings between the servomotors and the joints are still achieved through four-bar linkages. Figure 3.6 depicts the mechanical structure of a single leg of Prototype II:

Figure 3.6: Leg Mechanism of Prototype II (Right Leg)
(A) Side View, (B) Front View

Range for the output angle of each four-bar linkage was specified as below and the linkage's final dimensions are listed in Appendix A.3:

Linkage A:	pronation and supination of ankle: $\approx 50^\circ$
Linkage B:	dorsiflexion and plantarflexion of ankle: $\approx 80^\circ$
Linkage C:	flexion and extension of knee: $\approx 80^\circ$
Linkage D:	flexion and extension of hip: $\approx 90^\circ$
Linkage E:	abduction and adduction of hip: $\approx 40^\circ$
Linkage H:	internal and external rotation of hip: $\approx 50^\circ$

3.3.2 Final Design of Prototype II

Two snapshots of Prototype II at its lowest height are shown in Figure 3.7:

Figure 3.7: Pictures of the Prototype II. (A) Front View (B) Side View

Length of the pelvis on Prototype II is shortened to 90 mm, on top of which are batteries and circuit board attached. Dimensions of the soles are 80 mm in length and 50 mm in width. Total weight of the robot is about 1.6 kg including the batteries.

3.4 Discussion

Acting as torque amplifiers in this application, four-bar linkages are more concise compared to other transmission methods such as gears and belts. With the involvement of four-bar linkages and five-bar linkages, which are mostly non-linear mechanisms, kinematics of these two bipedal walking robots is not expressed in transformation

matrices [39]. For the micro-controllers employed in both prototypes, this would not appear much as a disadvantage. Due to the relatively small memory and limited computation ability of these micro-controllers in exchange for small volume and low power consumption, calculation of kinematics would be reduced to simple arithmetic operations with minor errors negotiated by system compliance. This manipulation bypasses the calculation of trigonometric functions, which appear in most kinematics and extend the processing time in the micro-controllers beyond the real-time domain. Thereby the micro-controllers can be indiscriminate to kinematics of various complexities.

CHAPTER 4 STATIC AND KINEMATIC ANALYSES

Before the implementation of quasi-static motions, static analysis is conducted on both prototypes described in the previous chapter. The process is comprised of static testing of the servos, mapping of joint angles and positions of the feet to corresponding servo horn angles, and finally synthesis of quasi-static gaits.

4.1 Static Testing of Actuators

Futaba S3003 and S9450 integrated servomotors [2] are employed for actuation of the bipedal walking robots. The circuit diagram of the built-in controller for Futaba S3003 is shown in Figure 4.1:

Figure 4.1: Circuit Diagram of Futaba S3003 Controller (Courtesy of Lee Buse) [40]

This model of servomotors utilizes proportional control with position and velocity feedbacks. According to the characteristics of normal DC motors [41] and Figure 4.1, a simplified block diagram for the servo system of Futaba S3003 was derived (Figure 4.2), in which inductance of the DC motor was assumed negligible:

Figure 4.2: Simplified Block Diagram of Futaba S3003 (derived by the Author of This Thesis from Figure 4.1)

Similar control scheme is utilized in Futaba S9450 servos. Therefore, the block diagram shown in Figure 4.2 also applies to this model of servos. In static testing of the servos, the system constants determined are K_2 and K_4 :

From [42]:

For Futaba S3003: $K_{2(S3003)} \approx 1.06 \times 10^{-2} \text{ ms/deg}$

For Futaba S9450: $K_{2(S9450)} \approx 9.62 \times 10^{-3} \text{ ms/deg}$

K_4 was determined through experiment (Appendix B.1):

For Futaba S3003: $K_{4(S3003)} \approx 29.43 \text{ kg} \cdot \text{cm/ms}$

For Futaba S9450: $K_{4(S9450)} \approx 178.66 \text{ kg} \cdot \text{cm} / \text{ms}$

Since only PD control is adopted in the built-in controllers of Futaba S3003 and S9450 servomotors, steady state errors are introduced when the servos are subjected to external disturbance torques. In the synthesis of quasi-static gaits, these errors are estimated and compensated based on the values of K_2 and K_4 .

4.2 Analysis on Kinematical and Inverse Kinematics of Leg Mechanisms

Direct real-time computation of kinematics and inverse kinematics for the bipedal walking robots in this project is beyond the capability of a single Basic Stamp 2sx micro-controller. When parallel processing is practised, a bulky system composed of large numbers of micro-controllers would be needed to achieve the required accuracy. This is considered less efficient compared to a high-speed computer. The approach of approximation is employed by reducing the direct computations to basic arithmetic operations through a combination of curve fitting and look-up tables based on the results of off-line calculation.

4.2.1 Functions Applied in Curve Fitting

Computation undertaken in Basic Stamp 2sx micro-controllers is in nonnegative integers not exceeding 16 bits (0~65535). With this constraint, three types of functions are chosen for curve fitting involved in the reduction of calculation:

1) Linear: $P = a \cdot Q + b$

2) Parabolic: $P = a \cdot Q^2 + b \cdot Q + c$

3) Combination of linear and hyperbolic: $P = a + b \cdot Q + \frac{c}{d + Q}$

where: P, Q are the variables,

a, b, c and d are constants of the functions.

These three functions are all composed of basic arithmetic operations that can be completed in very short time by the micro-controllers. Linear and parabolic functions are mainly applied on curves with small curvatures, whilst the combination function is used for those with large curvatures at a portion of the curve. This is based on the observation that the first and second derivatives of the combination function have the shapes similar to those of the first and second derivatives of a monotonic curve with large curvatures. Determination of the constants in the first two types of functions is done using Microsoft Excel. In the establishment of the combination function, four points on a curve are substituted into the function and the constants are hence determined by solving simultaneously the four equations obtained.

4.2.2 Mapping of Joint Angles to Servo Angles

Each servomotor is coupled to its corresponding robot joint through a four-bar linkage. With reference to Figure 4.3, the angular position of the servo horn is related to the angle of robot joint by the following equation:

$$q_A = \cos^{-1} \frac{l_{AC}^2 + l_{AB}^2 - l_{BC}^2}{2 \cdot l_{AB} \cdot l_{AC}} + \cos^{-1} \frac{l_{AC}^2 + l_{AD}^2 - l_{CD}^2}{2 \cdot l_{AD} \cdot l_{AC}} \quad (4.2.1)$$

$$\text{where, } l_{AC} = \sqrt{l_{AB}^2 + l_{BC}^2 - 2 \cdot l_{AB} \cdot l_{BC} \cdot \cos q_B} \quad (4.2.2)$$

q_A is the angular position of the servo horn

q_B is the output angle, i.e. joint angle

Figure 4.3: Structure of Four-Bar Linkage

The calculation involved in Eqs. (4.2.1) and (4.2.2) is both time and memory consuming when conducted on a single Basic Stamp 2sx. Curve fitting is therefore utilized to alleviate the computation task.

In Basic Stamp 2sx, the servo horn angle is specified by a pulsout unit which represents $0.8\mu\text{s}$ per unit. Whence q_A is converted into a pulsout input specified in this unit via the servo constant K_2 determined in Section 4.1. The pulsout input is denoted by PW_A . As an example, the curve of PW_A versus q_B for Linkage A on the right leg of Prototype II (Figure 3.6) is plotted in the following graph:

Figure 4.4: PW_A vs. q_B for Linkage A on the Right Leg of Prototype II

Resolution of the joint angle, i.e. q_B , is set to 0.1° . The values of PW_A are integers between 800 and 2800. Errors of the curve fitting are specified to be within ± 2 , which could be translated into an error between $\pm 0.1^\circ$ for q_B . Based upon the observation on the first and second derivatives of the curve, the third type of function in Section 4.2.1 is selected for curve fitting. The curve is divided into two segments and approximated by the following equations:

$$PW_{A(1)} = a_1 + b_1 \cdot q_B + \frac{c_1}{d_1 + q_B}, \quad \text{when } q_B < q_{B(s)} \quad (4.2.3)$$

$$PW_{A(2)} = a_2 + b_2 \cdot q_B + \frac{c_2}{d_2 + q_B}, \quad \text{when } q_B > q_{B(s)} \quad (4.2.4)$$

where $a_1, a_2, b_1, b_2, c_1, c_2, d_1$ and d_2 are constants to be determined

$q_{B(s)}$ is the centre point of the motion range of the robot joint, i.e. $q_{B(s)} = 27^\circ$

To achieve the specified accuracy for Eqs. (4.2.3) and (4.2.4) within the 16-bit range, a_1 and a_2 are computed up to an accuracy of 1, b_1 and b_2 to 0.01, c_1, c_2, d_1 and d_2 to 0.1.

The results of curve fitting for the curve in Figure 4.4 are presented by:

$$PW_A = 1352 + 23.24q_B - 1653.2/(6.5 + q_B), \quad \text{when } q_B \leq 27$$

$$PW_A = 1238 + 22.44q_B + 3329.1/(64.8 - q_B), \quad \text{when } q_B > 27$$

These two equations are also plotted in Figure 4.4, from which it can be seen the difference between the actual curve and the curve-fitting result is so small that the two curves visually overlap each other. By the same token, mappings of the other robot joints to their respective servos are accomplished with similar considerations.

4.2.3 Mapping for Leg Mechanisms

The inverse kinematics involved in the leg structure is expressed as:

$$M : \begin{bmatrix} x_{ankle} \\ z_{ankle} \end{bmatrix} \rightarrow \begin{bmatrix} PW_{thigh} \\ PW_{calf} \\ q_a \end{bmatrix}$$

where

- x_{ankle} and z_{ankle} are position coordinates of the ankle relative to the hip coordinate system on the specific leg (Figure 1).
- PW_{thigh} is the pulsout input to the servo driving the thigh, which is the actuator for Linkage D shown in Figure 3.3 and 3.5.

- PW_{calf} is the pulsout input to the servo driving the calf, which is the actuator for Linkage C shown in Figure 3.3 and 3.5.
- q_a is the angle between the axis of the calf and $x_p(x_q)$ -axis, which will be also the angle required to keep the sole parallel to the robot's transverse plane (Figure 1).

Due to the intricacy in geometries of the five-bar linkage, the mapping for the leg mechanism of Prototype I is achieved with a combination of curve-fitting and look-up table based on off-line calculation. Whilst in Prototype II, the mapping is realized through direct geometric calculation with iterative and approximating methods since the serial mechanism involved requires less intensive computation. These two approaches are discussed respectively in Sections 4.2.3(A) and 4.2.3(B) as follows.

4.2.3(A) Prototype I

The geometrical relations existing in the leg mechanism of Prototype I are illustrated on the right figure. In Figure 4.5, point A represents the position of ankle and point F is taken as the origin of the hip coordinate system about which x_{ankle} and z_{ankle} are specified. Line E-F is along $x_p(x_q)$ -axis of the coordinate system. Line g-h is parallel to line E-F. q_a is the angle between link B-A and line g-h. After some geometrical manipulation, q_a is determined as:

$$q_a = \tan^{-1} \left(\frac{z_{ankle}}{x_{ankle}} \right) + b \quad (4.2.5)$$

Figure 4.5: Schematic Diagram of the Leg Mechanism of Prototype I

The joint angle at Joint F is determined as:

$$q_2 = \tan^{-1} \left(\frac{z_{ankle}}{x_{ankle}} \right) + \cos^{-1} \left(\frac{x_{ankle}^2 + z_{ankle}^2 + l_{DF}^2 - l_{AD}^2}{2 \cdot \sqrt{x_{ankle}^2 + z_{ankle}^2} \cdot l_{DF}} \right) \quad (4.2.6)$$

The joint angle at Joint E is determined as:

$$q_1 = \cos^{-1} \left(\frac{l_{BE}^2 + l_{EF}^2 - l_{BF}^2}{2 \cdot l_{BE} \cdot l_{EF}} \right) + \cos^{-1} \left(\frac{l_{BE}^2 + l_{CE}^2 - l_{BC}^2}{2 \cdot l_{BE} \cdot l_{CE}} \right) \quad (4.2.7)$$

where $l_{BF} = \sqrt{(x_{ankle} + l_{AB} \cdot \cos q_a)^2 + (z_{ankle} + l_{AB} \cdot \sin q_a)^2}$

$$l_{BE} = \sqrt{(x_{ankle} + l_{AB} \cdot \cos q_a + l_{EF})^2 + (z_{ankle} + l_{AB} \cdot \sin q_a)^2}$$

Referring to Figure 3.3, Joint E is actuated through Linkage C and Joint F through Linkage D . The servo horn angles in these linkages are therefore determined from q_1 and q_2 respectively through Eqs. (4.2.1) and (4.2.2) followed by conversion to the pulsout unit. PW_{thigh} and PW_{calf} are thus obtained as functions of x_{ankle} and z_{ankle} .

For each x_{ankle} , curve fitting is applied to approximate the functions relating z_{ankle} to PW_{thigh} , PW_{calf} and q_a . Similar approach as that elucidated in Section 4.2.2 is used:

$$PW_{thigh} = a_{11} + b_{11} \cdot z_{ankle} + \frac{c_{11}}{d_{11} + z_{ankle}}, \text{ when } z_{ankle} \leq z_{thigh} \quad (4.2.8)$$

$$PW_{thigh} = a_{12} + b_{12} \cdot z_{ankle} + \frac{c_{12}}{d_{12} + z_{ankle}}, \text{ when } z_{ankle} > z_{thigh} \quad (4.2.9)$$

$$PW_{calf} = a_{21} + b_{21} \cdot z_{ankle} + \frac{c_{21}}{d_{21} + z_{ankle}}, \text{ when } z_{ankle} \leq z_{calf} \quad (4.2.10)$$

$$PW_{calf} = a_{22} + b_{22} \cdot z_{ankle} + \frac{c_{22}}{d_{22} + z_{ankle}}, \text{ when } z_{ankle} > z_{calf} \quad (4.2.11)$$

$$q_a = a_{31} + b_{31} \cdot z_{ankle} + c_{31} \cdot z_{ankle}^2, \quad \text{when } z_{ankle} \leq z_a \quad (4.2.12)$$

$$q_a = a_{32} + b_{32} \cdot z_{ankle} + c_{32} \cdot z_{ankle}^2, \quad \text{when } z_{ankle} > z_a \quad (4.2.13)$$

where a_{nm}, b_{nm}, c_{nm} ($n=1,2,3$ and $m=1,2$), d_{ef} ($e=1,2$ and $f=1,2$) and z_{thigh} , z_{cakf} , z_a are constants to be determined for different x_{ankle} .

At a fixed x_{ankle} , there corresponds one set of a_{nm}, b_{nm}, c_{nm} ($n=1,2,3$ and $m=1,2$) and d_{ef} ($e=1,2$ and $f=1,2$) to every set of z_{thigh} , z_{cakf} and z_a . The optimal values of z_{thigh} , z_{cakf} and z_a for each x_{ankle} are thus determined as the ones that produce the least errors in curve-fitting of Eqs. (4.2.8) to (4.2.13). The values of a_{nm}, b_{nm}, c_{nm} ($n=1,2,3$ and $m=1,2$), d_{ef} ($e=1,2$ and $f=1,2$) and z_{thigh} , z_{cakf} , z_a for each x_{ankle} , which has an interval of 1mm, are stored as look-up tables in the memory of Basic Stamp 2sx micro-controllers. These tables are generated by C programmes for both legs.

4.2.3(B) Prototype II

The geometries for a leg of Prototype II are presented in Figure 4.6. Point A is the position of ankle. The origin of the hip coordinate system is located at point C, which is the centre of the hip joint. Line g-h is parallel to $x_p(x_q)$ -axis. q_1 , q_2 and q_a are established as functions of x_{ankle} and z_{ankle} through direct calculation:

$$M : \begin{pmatrix} x_{ankle} \\ z_{ankle} \end{pmatrix} \rightarrow \begin{pmatrix} q_1 \\ q_2 \\ q_a \end{pmatrix}$$

Figure 4.6: Schematic Diagram of the Leg Mechanism of Prototype II

In Figure 4.6, l_0 is computed up to an accuracy of 0.1 by Newton-Raphson method:

$$1) \quad l_0 = a + b/10$$

where: a is an integer between 100 and 180; b is an integer between 0 and 9.

$$2) \quad a_n = a_{n-1} - (a_{n-1}^2 - x_{ankle}^2 - z_{ankle}^2) / (2a_{n-1}) \quad (4.2.14)$$

Eq. (4.2.14) generates a sequence of a_n . With a starting value of $a_0 = 100$, a_2 is accepted as the final value of a by observing that $a_n = a_{n-1}$ for $n \geq 3$ when the computation is conducted in Basic Stamp 2sx.

$$3) \quad b \text{ is whence determined from } a \text{ as:}$$

$$b = 5(x_{ankle}^2 + z_{ankle}^2 - a^2) / a$$

With the l_0 obtained above, some geometric calculation arrives at the following results:

$$q_1 = \cos^{-1} \frac{l_1^2 + l_2^2 - l_0^2}{2l_1l_2} \quad (4.2.15)$$

$$q_2 = 90^\circ + \tan^{-1} \frac{x_{ankle}}{z_{ankle}} - \cos^{-1} \frac{l_1^2 + l_2^2 - l_0^2}{2l_1l_0} \quad (4.2.16)$$

$$q_a = 90^\circ + \tan^{-1} \frac{x_{ankle}}{z_{ankle}} - \cos^{-1} \frac{l_1^2 + l_2^2 - l_0^2}{2l_2l_0} \quad (4.2.17)$$

In Eqs. (4.2.15) to (4.2.17), computation for the function of arctangent is performed by parabolic approximation, whilst that for the function of arccosine is through curve-fitting by the third type of function introduced in Section 4.2.1. PW_{thigh} and PW_{calf} are thereafter obtained from q_2 and q_1 respectively through the mapping for the four-bar linkage, which was demonstrated in Section 4.2.2.

4.3 Quasi-static Stability

Motions of the bipedal robots under quasi-static stability can be understood as those with inertia forces on each link much smaller than the reaction forces on the joints connected to that link. This usually limits the motions of the robot to low speed. CG of the robot

should be always kept within the supporting region, which is the polygon with the largest area among those formed by the contact points between the feet and the ground. Under this quasi-static stability, basic motions such as walking and turning were examined on both prototypes. The coordinate systems introduced in Figure 3.2 are utilized to describe motions of the robot.

4.3.1 Gait Synthesis for Walking with Quasi-static Stability

The gait synthesis for quasi-static walking is divided into sagittal and coronal planes of the robot. Translation of the robot body occurs in the sagittal plane, whilst transition between the single-support and double-support phases entails sideways motion of the robot in its coronal plane.

Figure 4.7 shows the sideways motion during which CG of the robot is shifted into the stable region of its left foot. The coordinate system attached to the left foot is chosen in describing the geometric relations.

Figure 4.7: Shifting of CG of the Robot Shown in the Frontal Plane of Left Foot

In Figure 4.7, w_0 is the width of stable region on each single foot and h_o is the height of the robot's CG from the ankle joint. By approximating the robot body as a single pelvis, the following relation can be obtained for Figure 4.7:

$$x_{O'} \approx \frac{l_{H_l-H_r}}{2} - h_o \cdot \sin q_{LA} \quad (4.3.1)$$

where: $l_{H_l-H_r}$ is the distance between the hip joints, i.e. length of the pelvis.

During the single support phase:
$$x_{O'} \leq \frac{1}{2} \cdot w_0 \quad (4.3.2)$$

Substituting Eq. (4.3.1) into Eq. (4.3.2):
$$q_{LA} = q_{RA} \geq \sin^{-1} \left(\frac{l_{H_l-H_r} - w_0}{2h_o} \right) \quad (4.3.3)$$

The standing height of the robot is specified by l_2 , which is defined as the length of each leg seen in frontal plane of the corresponding foot. The values of h_o at different l_2 are determined by experiment. The relationship between h_o and l_2 is established through linear approximation:

For Prototype I: $h_o \approx 0.6 \times l_2 \quad (4.3.4)$

For Prototype II: $h_o \approx 0.9 \cdot l_2 \quad (4.3.5)$

Eqs. (4.3.4) and (4.3.5) are substituted into Eq. (4.3.3) to obtain the minimum shift angle needed at the ankles to move CG of the robot into the stable region of the left foot:

For prototype I:
$$q_{LA} = q_{RA} \geq \sin^{-1} \left(\frac{l_{H_l-H_r} - w_0}{1.2 \times l_2} \right) \quad (4.3.6)$$

For prototype II:
$$q_{LA} = q_{RA} \geq \sin^{-1} \left(\frac{l_{H_l-H_r} - w_0}{1.8 \times l_2} \right) \quad (4.3.7)$$

With the presence of an external torque, the actual horn angles of servomotors at steady state always drift from the nominal angles specified by the control signals in order to provide necessary torque to counteract this external disturbance.

During the single-support phase, the right foot is lifted off the ground. For the servomotor driving the left ankle joint, i.e. Joint A_l , the actual horn angle f_{LA} offsets from the input angle in response to the gravitational force.

From Figure 4.2 and Figure 4.7, the offset angle is estimated to be:

$$df_{LA} \approx \frac{m \cdot g \cdot x_{O'}}{K_2 \cdot K_4 \cdot K_A} \quad (4.3.8)$$

where: m is the mass of the bipedal robot

K_A is the torque amplification ratio of four-bar linkage connecting the servo to the left ankle joint.

For both prototypes, the width of stable region for a single foot, w_0 , is taken to be 40mm based on a safety factor of 1.25. This value is used in the calculation from Eqs. (4.3.1) to (4.3.8). When CG of the robot is at the edge of the stable region:

$$x_{O'} = 20mm$$

$$\text{For Prototype I: } df_{LA} \approx 3.8^\circ \quad (4.3.9)$$

$$\text{For Prototype II: } df_{LA} \approx 1.8^\circ \quad (4.3.10)$$

Similar estimation is performed at the left hip joint. The offset angle of the servo driving this joint is denoted by df_{LE} :

$$\text{For Prototype I: } df_{LE} \approx 8.6^\circ \quad (4.3.11)$$

$$\text{For Prototype II: } df_{LE} \approx 4.7^\circ \quad (4.3.12)$$

For the right foot, which is the non-supporting foot, the disturbance torques created by gravity force are small. Therefore the offset angles at the servomotors actuating the right ankle joint and hip joint are negligible. Similar estimations are also conducted for the single-support phase with the right foot as the supporting foot. The actual offset angles at the servo horns are obtained through experiment (Appendix B.2). The results for different standing heights are summarized as follows:

For Prototype I:

Table 4.1: Actual Offset Angles at the Servos for Prototype I

Standing Height l_2	Left foot is the supporting foot		Right foot is the supporting foot	
	df_{LA}	df_{LE}	df_{RA}	df_{RE}
160mm	3.6°	8.3°	3.6°	8.2°
190mm	3.7°	8.4°	3.7°	8.4°

For Prototype II:

Table 4.2: Actual Offset Angles at the Servos for Prototype II

Standing Height l_2	Left foot is the supporting foot		Right foot is the supporting foot	
	df_{LA}	df_{LE}	df_{RA}	df_{RE}
130mm	1.8°	4.3°	1.7°	4.2°
170mm	1.8°	4.5°	1.8°	4.3°

From Tables 4.1 and 4.2, the actual offsets are observed to be practically unchanged with l_2 and estimations (4.3.9) to (4.3.12) are shown to be close to the experiment results. These offsets are therefore compensated for in gait synthesis by adding the estimated offsets to the desired angles.

Trajectory of each foot in lateral plane of the corresponding hip coordinate system is shown Figure 4.8. The trajectory is defined with respect to the specific hip coordinate system by the following equations:

$$x_{ankle}(t) = \begin{cases} -s_q & k \cdot t_q < t \leq k \cdot t_q + t_1 \\ -s_q + 8 \cdot v_{qx} \cdot (t - k \cdot t_q - t_1) & k \cdot t_q + t_1 < t \leq k \cdot t_q + t_2 \\ s_q & k \cdot t_q + t_2 < t \leq k \cdot t_q + t_3 \\ s_q - v_{qx} \cdot (t - k \cdot t_q - t_3) & k \cdot t_q + t_3 < t \leq (k+1) \cdot t_q \end{cases} \quad (4.3.13)$$

$$z_{ankle}(t) = \begin{cases} -l_2 + v_{qz} \cdot (t - k \cdot t_q) & k \cdot t_q < t \leq k \cdot t_q + t_1 \\ -l_2 + v_{qz} \cdot t_1 & k \cdot t_q + t_1 < t \leq k \cdot t_q + t_2 \\ -l_2 - v_{qz} \cdot (t - k \cdot t_q - t_1 - t_2) & k \cdot t_q + t_2 < t \leq k \cdot t_q + t_3 \\ -l_2 & k \cdot t_q + t_3 < t \leq (k+1) \cdot t_q \end{cases} \quad (4.3.14)$$

where

$x_{ankle}(t)$ and $z_{ankle}(t)$ are position coordinates of the foot at time t .

s_q , v_{qx1} , v_{qx2} and v_{qz} are constants of the trajectory.

t_q is the period of one walking cycle, which is defined as the time required for a foot to travel along the entire trajectory once.

t_1 , t_2 and t_3 are time constants of the trajectory and $t_1 < t_2 < t_3 < t_q$

k is a nonnegative integer, which represents the number of walking cycles completed.

Figure 4.8: Foot Trajectory for Quasi-static Walking (a) Trajectory of the Left Foot in Lateral Plane of the Left Hip Coordinate System (b) Trajectory of the Right Foot in Lateral Plane of the Right Hip Coordinate System

For the trajectory described by Eqs. (4.3.13) and (4.3.14) and Figure 4.8, the following relations are specified for the constants:

$$8 \cdot (t_2 - t_1) = t_q - t_3 \quad (4.3.15)$$

$$t_3 - t_2 = t_1 \quad (4.3.16)$$

$$t_q - t_3 = 4 \cdot t_3 \quad (4.3.17)$$

$$2s_q = v_{qx} \cdot (t_q - t_3) \quad (4.3.18)$$

The phase difference between the motions of left foot and right foot is $t_q / 2$. Therefore the trajectory of each foot is defined from Eqs. (4.3.13) and (4.3.14) as below:

$$x_{ankle(l)}(t) = x_{ankle}\left(t + t_q / 2\right) \quad z_{ankle(l)}(t) = z_{ankle}\left(t + t_q / 2\right)$$

$$x_{ankle(r)}(t) = x_{ankle}(t)$$

$$z_{ankle(r)}(t) = z_{ankle}(t)$$

where $x_{ankle(l)}(t)$ and $z_{ankle(l)}(t)$ are position coordinates of the left foot at time t .

$x_{ankle(r)}(t)$ and $z_{ankle(r)}(t)$ are position coordinates of the right foot at time t .

From Eqs. (4.3.15) to (4.3.18), the relation between v_{qx} , s_q and t_q is determined as below:

$$v_{qx} = 5 \cdot s_q / (2 \cdot t_q)$$

Therefore the walking speed, v_{qx} , is altered by changing the step size, which is equal to $2s_q$, or the walking frequency, which is equal to $1/t_q$.

Figure 4.9 illustrates motions of the robot legs during quasi-static walking:

Figure 4.9: Sequence of Leg Motions for Quasi-static Walking

For Prototype II, experiments on quasi-static walking were taken with different combinations of s_q and t_q . The results are listed in Table 4.3:

Table 4.3: Experiment Results on Walking Speed and Motion Parameters of s_q and t_q for Quasi-static Walking

Desired walking speed	5.0mm/s		5.0mm/s		7.5mm/s		7.5mm/s	
	s_q	t_q	s_q	t_q	s_q	t_q	s_q	t_q
Planned s_q and t_q	14.0mm	7.0s	16.0mm	8.0s	21.0mm	7.0s	24.0mm	8.0s
Actual s_q and t_q	12.5mm	7.0s	14.3mm	8.1s	19.8mm	7.1s	22.8mm	8.1s
Actual walking speed	4.5mm/s		4.4mm/s		7.0mm/s		7.0mm/s	

The errors in walking frequency, i.e. $1/t_q$, are originated from the accumulative variations in the actual computation time. As expected, these errors are small since the micro-controllers execute the programs at a very high speed. On the other hand, errors in the step sizes, i.e. $2s_q$, have the major effects on the discrepancies between the actual walking speeds and the desired ones. Based on examination of the actual robot structure, mechanical backlashes are identified as the main source of the errors in step sizes. As shown in Table 4.3, these errors are always within 3mm, which are much smaller than the step sizes used in the quasi-static walking motion.

4.3.2 Gait Synthesis for Turning with Quasi-static Stability

Yaw motions of the legs are required when the robot body is rotating. To achieve a stable transition for the robot's CG from one foot to the other, turning of the robot body is only executed during the single-support phases. The motion sequence for the gait of right turn is illustrated by Figure 4.10. In the figure, movements of the robot and its CG are shown on the horizontal plane of the global coordinate system (Figure 3.2) following the sequence from State (1) to State (11). The motions leading to the transition from a state to the ensuing one are differentiated by their key features.

Figure 4.10: Motion Sequence of the Gait for Quasi-static Turning Shown in the Horizontal Plane

State (1) → State (2): Switching the supporting foot

The motion of *switching the supporting foot* makes up the double-support phase, during which CG of the robot is shifted from the right foot to the left foot. Both hip joints move in the same direction at the same speed, whereby the robot body is maintained at constant attitude.

Geometries for the motion of *switching the supporting foot* are depicted in Figure 4.11. In the figure, ankle joints of the robot are attached to the ground surface through the feet, while the pelvis translates from the position of $H_l - H_r$ to that of $H'_l - H'_r$ along its own axis, which is also y_b -axis of the body coordinate system (Figure 3.2). The trajectories of both hip joints are defined from the motion of the pelvis.

Figure 4.11: Geometry for Motion of *Switching the Supporting Foot* Shown in the Horizontal Plane

The right hip joint shifts from H_r to H'_r along the axis of pelvis. This movement is decomposed into two orthogonal motions, which are identified as $H_r \rightarrow P_r$ and $P_r \rightarrow H'_r$. They are synthesised in the lateral and frontal planes of the right foot respectively.

Trajectory of the right hip joint in the frontal plane of the right foot is shown in Figure 4.12, where the same notations as those in Figure 4.12 are used.

Figure 4.12: Geometry in Frontal Plane of the Right Foot

In the frontal plane of the right foot, P_r and H'_r coincide. The motion of $H_r \rightarrow P_r$ is thus defined by the trajectory of the right hip in this plane. In the same plane, the distance between the right hip joint and the right ankle joint in the frontal plane of the right foot is designated as $d_{H_r-A_r}$. It is kept constant during the motion. Thereby in the frontal plane of the right foot, an arc is traced out by the right hip joint.

By maintaining a constant $d_{H_r-A_r}$, Trajectory of the right hip joint in lateral plane of the right foot follows the upper half of an eclipse. P_r and H_r coincide in lateral plane of the right foot. Thereby trajectory of the right hip in this plane specifies the motion of $P_r \rightarrow H_r'$. The geometries involved are illustrated in Figure 4.13.

Figure 4.13: Geometry in Lateral Plane of the Right Foot

The motions described in Figures 4.12 and 4.13 are executed simultaneously. The combined motion of the right hip joint is from H_r to H_r' along the straight line $H_r - H_r'$ in the x - O - y plane (with reference to Figure 3.2). From the geometrical analysis (Appendix C), the connection between the two sub-motions is established as below:

$$x_r^* - x_r = d_{H_r-A_r} \cdot (\sin q_{RA0} - \sin q_{RA}) \cdot \tan b_1 \quad (4.3.19)$$

where, b_1 , x_r^* and q_{RA0} are constants specified by State (1) and State (2).

Similar analysis are applied on the left leg, with the following modifications to the corresponding notations:

$$d_{H_r-A_r} \rightarrow d_{H_l-A_l}, q_{RA} \rightarrow q_{LA}, q_{RA0} \rightarrow q_{LA0} \text{ and } x_r \rightarrow x_l, x_r^* \rightarrow x_l^*.$$

The relation between the sub-motions $H_r \rightarrow P_r$ and $P_r \rightarrow H_r'$ is described by an equation similar to Eq. (4.3.19):

$$x_l - x_l^* = d_{H_l-A_l} \cdot (\sin q_{LA0} - \sin q_{LA}) \cdot \tan b_3 \quad (4.3.20)$$

where, b_3 , x_l^* and q_{LA0} are constants specified by State (1) and State (2).

For the pelvis to translate along its own axis in transverse plane of the robot with a constant attitude, the following conditions are imposed:

$$q_{RA} = q_{LA} = q \quad q_{RA0} = q_{LA0} = q_0 \quad b_1 = b_3 = b$$

$$d_{H_r-A_r} = d_{H_l-A_l} = l_2 \quad x_r^* = -x_l^* = x^*$$

where: q is the motion variable

b , q_0 and x^* are gait constants.

From the analysis elaborated in Appendix C, the following relations are derived for the motion from State (1) to State (2):

$$x_r \approx x^* - \frac{p \cdot l_2 \cdot b \cdot \sin q_0 \cdot (q_0 - q)}{180 \cdot q_0} \quad (4.3.21)$$

$$x_r = -x_l \quad (4.3.22)$$

where b is expressed in degrees.

For Eq. (4.3.21) to be valid, the following constraint is imposed on b which specifies the turning angle of the robot:

$$\text{For prototype I:} \quad b \leq 10^\circ \quad (4.3.23)$$

$$\text{For prototype II:} \quad b \leq 15^\circ \quad (4.3.24)$$

State (2) → State (3): Lifting the non-supporting foot

After CG of the robot is shifted into the stable region of left foot, the right foot is lifted off the ground and the left foot becomes the only supporting foot. The turning motion enters the single-support phase. This is understood as the motion of *lifting the non-supporting foot*, which is shown in Figure 4.10.

State (3) → State (4): Rotating the robot body

Four independent sub-motions are performed when the attitude of the robot changes from State (3) to State (4). These sub-motions are illustrated by Figures 4.14(A) to 4.14(D). The notations used in these figures conform to those in Figure 4.10. Since these four sub-motions are not geometrically inter-related, they are executed concurrently. During the entire motion of *rotating the robot body*, both the robot body and the right foot, which is the non-supporting foot, are maintained at constant heights from the ground surface.

Figure 4.14(A): Sub-motion (1) Shown in the Horizontal Plane

Figure 4.14(B): Sub-motion (2) Shown in the Horizontal Plane

Figure 4.14(C): Sub-motion (3) Shown in the Horizontal Plane

Figure 4.14(D): Sub-motion (4) Shown in the Horizontal Plane

State (4) → State (5): Turning the robot body

In the motion of *turning the robot body*, two sub-motions are performed simultaneously:

- (1) The pelvis continues to rotate about the left hip joint with an angle of b in the clockwise direction.
- (2) The right leg continues to rotate about the right hip joint with an angle of b in the clockwise direction.

State (5) → State (6): Lowering the non-supporting foot

After the robot has turned an angle of $2b$, the right foot is lowered and obtains contact with the ground. The single-support phase is transited to the double-support phase again.

State (6) → State (7) → State (8) → State (9) → State (10) → State (11):

The motion sequence from State (6) to State (11) is similar to that from State (1) to State (6). Nevertheless, in the motions from State (6) to State (11) the robot turns with the right foot as the supporting foot.

Trajectory of each foot in lateral plane of the corresponding hip coordinate system is shown in Figure 4.15, where the black dot in each trajectory represents the starting position of the corresponding foot for each turning cycle:

Figure 4.15: Foot Trajectory for Quasi-static Turning Shown in the Lateral Plane of Each Hip

The trajectories in Figure 4.15 are specified by the following equations:

$$x_{ankle(r)}(t) = -x_r = \begin{cases} v_{tx1} \cdot (t - k \cdot t_r) - x^* & k \cdot t_r < t \leq k \cdot t_r + t_2 \\ 0 & k \cdot t_r + t_2 < t \leq k \cdot t_r + t_3 \\ -v_{tx2} \cdot (t - k \cdot t_r - t_3) - x^* & k \cdot t_r + t_3 < t \leq k \cdot t_r + t_4 \\ -x^* & k \cdot t_r + t_4 < t \leq k \cdot t_r + t_6 \\ v_{tx1} \cdot (t - k \cdot t_r - t_6) - x^* & k \cdot t_r + t_6 < t \leq k \cdot t_r + t_7 \\ 0 & k \cdot t_r + t_7 < t \leq k \cdot t_r + t_8 \\ -v_{tx2} \cdot (t - k \cdot t_r - t_8) - x^* & k \cdot t_r + t_8 < t \leq k \cdot t_r + t_9 \\ -x^* & k \cdot t_r + t_9 < t \leq (k+1) \cdot t_r \end{cases} \quad (4.3.25)$$

$$z_{ankle(r)}(t) = \begin{cases} -l_2 & k \cdot t_r < t \leq k \cdot t_r + t_2 \\ -l_2 + v_{tz} \cdot (t - k \cdot t_r - t_2) & k \cdot t_r + t_2 < t \leq k \cdot t_r + t_3 \\ -l_2 + v_{tz} \cdot (t_3 - t_2) & k \cdot t_r + t_3 < t \leq k \cdot t_r + t_5 \\ -l_2 - v_{tz} \cdot (t - k \cdot t_r + t_2 - t_3 - t_5) & k \cdot t_r + t_5 < t \leq k \cdot t_r + t_6 \\ -l_2 & k \cdot t_r + t_6 < t \leq (k+1) \cdot t_r \end{cases} \quad (4.3.26)$$

$$x_{ankle(l)}(t) = -x_{ankle(r)}(t + t_8) \quad (4.3.27)$$

$$z_{ankle(l)}(t) = z_{ankle(r)}(t + t_8) \quad (4.3.28)$$

In Eqs. (4.3.25) to (4.3.28):

$x_{ankle(l)}(t)$ and $z_{ankle(l)}(t)$ are position coordinates of the left foot at time t .

$x_{ankle(r)}(t)$ and $z_{ankle(r)}(t)$ are position coordinates of the right foot at time t .

v_{tx1} , v_{tx2} and v_{tz} are constants of the trajectory.

t_2 to t_{10} is the time when the specific state in Figure 4.11 is reached in one turning cycle.

t_r is the period of one turning cycle.

k is a nonnegative integer, which represents the number of turning cycles completed.

For the trajectories described by Eqs. (4.3.25) to (4.3.28), the following relations are imposed on the constants:

$$v_{tx1} \cdot t_2 = v_{tx2} \cdot (t_4 - t_3) = x^* = l_2 \cdot \sin q_0 \cdot \tan b$$

$$t_3 - t_2 = t_6 - t_5 \quad t_7 - t_6 = t_2 \quad t_9 - t_8 = t_4 - t_3$$

When the robot body is rotating, it is always kept to rotate at a constant speed, w_t . The motion parameters for Eqs. (4.3.25) to (4.3.28) are further constrained by the following relation:

$$t_4 - t_3 = t_5 - t_4 = t_9 - t_8 = t_{10} - t_9 = b / w_t$$

For one complete cycle which is the motion sequence from State (1) to State (11), the robot body rotates an angle of $4b$. From condition (4.3.23) and (4.3.24), this value would be less than 40° for Prototype I and 60° for Prototype II.

The actual motions of quasi-static turning are examined for Prototype II by comparing the actual turning angles per cycle with the commanded ones. The experiments were conducted at $w_t = 10.0 \text{ deg/s}$ and the results are compiled as shown in Table 4.4:

Table 4.4: Comparison between the Desired and Actual Turning Angles For One Complete Motion Cycle

Turning direction viewed from on top	Clockwise		Counter-clockwise	
Period of turning cycle (t_r)	9.6s	11.2s	9.6s	11.2s
Actual period of turning cycle	9.6 s	11.2s	9.6s	11.3s
Desired turning angle per cycle	16.0°	32.0°	16.0°	32.0°
Actual turning angle per cycle	15.6°	31.8°	15.8°	31.3°

Similar to the motion of quasi-static walking, the actual period of turning cycle for turning with quasi-static stability is determined by execution time of the programs burnt into the micro-controllers. Therefore the errors in the periods of turning cycle are observed to be always minor. On the other hand, the possible main sources leading to the errors in turning angle per cycle are identified to be mechanical backlashes and errors in the calculation of inverse kinematics. As shown in Table 4.4, these errors are also satisfactorily compared to the desired turning angle per cycle.

4.3.3 Summary

In experiments both walking and turning motions under quasi-static stability are kept at low speeds. When the motions are paused and frozen at an arbitrary moment, no obvious jerks are observed and the robot balances itself at its final pose. This fact indicates that, during the locomotion, the robot is always balanced quasi-statically and its CG is always within the supporting region. The actual positions of the robot's CG are examined at discrete moments of one complete cycle of the motions through experiment (Appendix B.3). In the experiment, the positions of the CG's projection on the ground surface are measured in the local coordinate system attached to the corresponding supporting foot (referring to Figure 3.2). The origin of the coordinate system is placed at the centre of the foot.

For the motion of quasi-static walking, the measurements were done on Prototype II at a planned walking speed of 7.5 mm/s and the results are presented in Table 4.5:

Table 4.5: Positions of the CG of the Robot Relative to Its Supporting Feet at Discrete Moments of One Walking Cycle at the Commanded Walking Speed of 7.5 mm/s

Motion parameters	$s_q = 24\text{mm}$, $v_{qx} = 7.5\text{mm/s}$, $v_{qz} = 2.5\text{mm/s}$ $t_l = 0.4\text{s}$, $t_2 = 1.2\text{s}$, $t_3 = 1.6\text{s}$, $t_q = 8.0\text{s}$								
Supporting foot	Left foot				Right foot				Left foot
Time	0.0s 0	0.4s t_l	1.2s t_2	1.6s t_3	4.0s $t_q/2$	4.4s $t_q/2+t_l$	5.2s $t_q/2+t_l$	5.6s $t_q/2+t_l$	8.0s t_q
x_r (mm)	—	—	—	—	-9.5	-7.5	6.0	9.0	—
y_r (mm)	—	—	—	—	18.0	18.0	17.0	18.5	—
x_l (mm)	-10.0	-7.5	6.5	8.5	—	—	—	—	-10.0
y_l (mm)	-19.0	-18.0	-17.5	-20.0	—	—	—	—	-19.0

Similar measurements on Prototype II were also done for the motion of quasi-static turning with a planned turning angle of 32° per motion cycle. The results are listed in Table 4.6:

Table 4.6: Positions of the CG of the Robot Relative to Its Supporting Feet at Discrete Moments of One Turning Cycle at the Commanded Turning Angle of 32° Per Cycle

Motion parameters	$\omega_t = 10\text{deg/s}, \beta = 8^\circ, t_r = 11.2s$ $t_2 = 2.0s, t_3 = 3.0s, t_4 = 3.8s, t_5 = 4.6s, t_6 = 5.6s, t_7 = 7.6s, t_8 = 8.6s, t_9 = 9.4s, t_{10} = 10.2s$ $v_{lx1} = 2.9\text{mm/s}, v_{lx2} = 7.3\text{mm/s}, v_{qz} = 10.0\text{mm/s}$										
Supporting foot	Right foot	Left foot					Right foot				
State	1	2	3	4	5	6	7	8	9	10	11
x_r (mm)	-1.0	—	—	—	—	—	-7.0	-6.0	-6.0	-1.5	-1.0
y_r (mm)	18.5	—	—	—	—	—	18.5	18.0	18.0	17.0	18.5
x_l (mm)	—	8.0	8.0	6.5	-1.5	-1.5	—	—	—	—	—
y_l (mm)	—	-18.5	-18.5	-19.5	-19.0	-19.0	—	—	—	—	—

Though the experiment results shown in Tables 4.5 and 4.6 are accurate only up to $\pm 1\text{mm}$, they suffice to demonstrate the fact that CG of the robot is located well within the supporting region during the quasi-static motions of both walking and turning.

Based on Table 4.5, movement of the robot's CG on the ground surface during quasi-static walking is illustrated in Figure 4.16:

Figure 4.16: Projections of the CG and the Feet of the Robot on the Ground Surface during One Complete Cycle of Quasi-static Walking

Similarly, movement of the robot's CG on the horizontal plane during turning motion is depicted by Figure 4.17 based on the experiment results in Table 4.6:

Figure 4.17: Projections of the CG and the Feet on the Ground during the Motion of Quasi-static Turning

4.3.4 Discussion

The bipedal robot forms a closed-loop with the ground during the double-support phases. In this state, the entire system has only six DOF with 12 actuators. The coupling between the actuators and the corresponding robot joints is lack of reversibility due to the frictions and backlashes in the servomotors and four-bar linkages. Based on this observation, instead of freeing the redundant actuators, all the servos are actuated according to the geometrical relations between the robot joints. Minor errors emerging in synchronization of the servo motions are absorbed by system compliance of the robots. During the single-support phases, the bipedal robot regain all the 12 DOFs, whence those errors and discrepancies generated during the double-support phases are released and rectified.

CHAPTER 5 DYNAMIC ANALYSIS

Validity of the analysis on quasi-static motions is usually limited to low motion speeds. As the motion speed of a bipedal walking robot increases, disparities in accelerating /decelerating torques required from the servos are exacerbated. Under this circumstance, synchronization in motions of the integrated servomotors is hardly achievable by pure position control based on geometric calculation. The issue of stability will thus enter the dynamic regime. In this regime, three types of motions are investigated: rocking, walking and turning. The coordinate systems defined in Figure 3.2 are adopted in description of these motions.

5.1 Stability Conditions at the Foot

The forces and torques acting on a foot with active ankle joints are shown by Figure 5.1 in a coordinate system attached to the foot:

Figure 5.1: Free Body Diagram for the Foot (A) Frontal Plane
 (B) Lateral Plane (C) Transverse Plane

When there are no relative motions between the foot and the ground, i.e. no overturning or slipping of the foot occur, the following relations exist:

$$\begin{aligned}\sum F = 0: \quad & F_x = f_x \quad F_z = N \quad F_y = f_y \\ \sum T = 0: \quad & F_z \cdot w + F_y \cdot h = T_x \quad F_z \cdot d + F_x \cdot h = T_y \quad T_z = T_f\end{aligned}$$

To keep the foot from overturning in the frontal plane:

$$\left| \frac{T_x - F_y \cdot h}{F_z} \right| \leq \frac{w_0}{2} \quad (5.1.1)$$

To keep the foot from overturning in the lateral plane:

$$\left| \frac{T_y - F_x \cdot h}{F_z} \right| \leq \frac{d_0}{2} \quad (5.1.2)$$

To keep the foot from slipping relative to the ground:

$$\sqrt{F_x^2 + F_y^2} \leq \mu N \quad (5.1.3)$$

where μ is the coefficient of friction between the foot and the ground.

To keep the foot from rotating relative to the ground in the transverse plane:

$$|T_z| \leq \frac{|F_x \cdot w_0| + |F_y \cdot d_0|}{2} \quad (5.1.4)$$

Slipping and toppling of a bipedal robot relative to the ground, which occur at the interface between the feet and the ground surface, are usually undesirable. When there is slipping, the frictional force and torque exerted by the ground are unstable and hardly predictable or measurable on a ground surface with statistic roughness. For a bipedal robot subjected to toppling, the interaction between the supporting foot and the ground is on a line or a point. In this situation, control of the robot through both the ground forces and gravity force is no longer available. Swift and accurate adjustments of the CG position and robot posture are thus essential for the stability of the entire robot. Therefore, the dynamic control under slipping and toppling is generally unstable and

unreliable for a system without efficient sensors, which are usually unfeasible for a low-cost system.

For the control of the two prototype bipedal walking robots in this project, overturning of either foot about its own edge is to be avoided even though toppling of the robots might not occur necessarily. This is based on the consideration that when overturning of a foot happens, the number of links of the robot increases and an inactivated joint is formed at the interface between the foot and the ground surface. The implication of the additional link and inactive joint might destabilise the entire system by altering its controllability and observability. In practice, it is assumed that conditions (5.1.3) and (5.1.4) are always satisfied, i.e. no slipping of the foot exists. Conditions (5.1.1) and (5.1.2) are investigated based on this assumption.

5.2 Dynamic Models of the Bipedal Robots

Simplified models are employed in dynamic analysis on the robots and synthesis of the dynamic gaits. Analysis and synthesis of the bipedal motions are divided into double-support phases and single-support phases. Separate models are applied for these phases. As an exemplification, the dynamic models described in this section are established based on the structural characteristics of Prototype II.

5.2.1 Four-bar Linkage Model

During the double-support phases, the combined inertia force of the entire robot in its sagittal plane is kept much smaller than the supporting forces from the feet. Whilst CG of the robot is maintained near the centre of the stable region, it is therefore assumed that dynamic stability of the robot is always satisfied in the sagittal plane, which is verified in the actual motions (referring to the attached CD-ROM). With dynamics in the sagittal plane neglected, the bipedal robot is modelled by a planar four-bar linkage in the lateral plane (Figures 5.2 and 5.3), of which the fixed link (Link $A_l - A_r$) is parallel to the ground surface.

Figure 5.2: Planar Four-bar Linkage Model for the Robot

Figure 5.3: Free Body Diagram for Each Link of the Four-bar Linkage Model

In the four-bar linkage model, the entire structure is assumed to be driven by four rotational actuators located at the four joints respectively. It is assumed that all the three moving links are rigid bodies and links $H_r - A_r$ and $H_l - A_l$ have identical properties. The model parameters for Prototype II are found through measurements and experiments (Appendix B.4):

$$m_{H_r-A_r} \approx m_{H_l-A_l} = m_2 \approx 500 \text{ g} \quad m_{H_l-H_r} \approx 600 \text{ g} \quad J_{H_l-H_r} \approx 4.3 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$J_{H_l-A_l} \approx J_{H_r-A_r} = J_2 \approx 0.047 \cdot l_2^2 \quad (5.2.1)$$

$$l_0 \approx 0.8 \cdot l_2 \quad (5.2.2)$$

5.2.2 Linear Inverted Pendulum Model

For the single-support phases, linear inverted pendulum models with 3 DOFs are established. The model with the left foot as the supporting foot is shown in Figure 5.4. The coordinate system used for illustration of the model is the reference frame fixed to the ground with its z -axis vertically upwards and the y -axis aligned with the y_l -axis in Figure 4.7.

Figure 5.4: Linear Inverted Pendulum Model with Joint A_l as the Pivot

In this model, the left ankle joint is a pivot of 2 DOFs, which is driven by two rotational actuators in x -axis and y -axis. Other parts of the robot are modelled as an inverted pendulum with distributed mass supported by a linear actuator. A similar model is also established for the single-support phase during which the right foot is the supporting foot. Analysis on dynamics of these linear inverted pendulum models are decomposed into two orthogonal planes, which are y - O - z plane and z - O - x plane. F, T_l, a_l and l in Figure 5.4 are therefore projected onto the corresponding planes for investigation in dynamics of the model.

Variations in second moments of inertia of the robot are assumed to be small for different postures exhibited by the robot, which is verified by experiments. Constant values are thus adopted for these two model parameters (Appendix B.4):

For Prototype II: $J_x \approx 0.041 \text{ kg} \cdot \text{m}^2$ $J_y \approx 0.046 \text{ kg} \cdot \text{m}^2$

5.3 Rocking

The motion of rocking describes the reciprocating sway of the robot in its lateral plane (Figure 5.5), which involves the swift switching of the supporting foot between its left and right feet.

Figure 5.5: Motion Sequence for Rocking

Rocking is a planar dynamic motion in the robot's coronal plane while quasi-static stability is maintained in other directions. In Figure 5.5, the motions of (B)→(C) and (C)→(B) form the double-support phases, during which switching of the supporting foot occurs. The motions of (B)→(A)→(B) and (C)→(D)→(C) are the single-support phases.

5.3.1 Single-Support Phases

The linear inverted pendulum models with the variable l (Figure 5.4) fixed are employed for dynamic analysis of the single-support phases. The following figure illustrates the motion of (B)→(A)→(B) in Figure 5.5, during which the left foot is the supporting foot:

Figure 5.6: Motion Sequence of the Inverted Pendulum Shown in Frontal Plane of the Left Foot

In this planar motion, the following relations exist for the model depicted in Figure 5.4:

$$F_x = 0$$

$$T_{ly} = 0$$

$$a_{ly} = \text{constant}$$

$$l_{zx} = \text{constant}$$

where T_{ly} is the component of T_l in y -direction

where a_{ly} is the projection of a_l on z - O - x plane

where l_{zx} is the projection of l on z - O - x plane

Dynamic analysis is thus only conducted in y - O - z plane of the model and projection of the linear inverted pendulum model on this plane is shown in the following figure:

Figure 5.7: Planar Inverted Pendulum Model with Joint A_l as the Pivot

Applying condition (5.1.1) to the left foot with the forces and torque following the designated directions:

$$T_{lx} \leq F_y \cdot h + F_z \cdot \frac{w_0}{2} \quad (5.3.1)$$

Assuming a negligible foot thickness, i.e. $h \approx 0$, the following relation is arrived at for T_{lx} , whereby the distance in y -direction between the ZMP and the pivot point A_l is set to be w (Figure 5.1):

$$T_{lx} = F_z \cdot w \quad \text{where } w \leq \frac{w_0}{2} \quad (5.3.2)$$

In this equation, w can be specified as a function of time t , d (Figure 5.1) or a_{lx} etc, on which the trajectory of ZMP on y -axis is defined.

From the dynamic analysis on the linear inverted pendulum model (Appendix D.1), the following equation is established for the motion of sequence (a) to (g) in Figure 5.6:

$$\begin{aligned} & (l_{yz}^2 + l_x^2 - w \cdot l_{yz} \cdot \sin a_{lx}) \cdot \ddot{a}_{lx} - w \cdot l_{yz} \cdot \cos a_{lx} \cdot \ddot{a}_{lx}^2 + 2 \cdot (l_{yz} - w \cdot \sin a_{lx}) \cdot \dot{l}_{yz} \cdot \dot{a}_{lx} \\ & - g \cdot l_{yz} \cdot \sin a_{lx} + g \cdot w + w \cdot \dot{l}_{yz} \cdot \cos a_{lx} = 0 \end{aligned} \quad (5.3.3)$$

where, l_x is the radius of gyration in y - O - z plane, i.e. $J_x = ml_x^2$

When l_{yz} is kept constant: $\dot{l}_{yz} = 0$ and $\ddot{l}_{yz} = 0$, Eq. (5.3.3) is thus simplified to be:

$$(l_{yz}^2 + l_x^2 - w \cdot l_{yz} \cdot \sin a_{lx}) \cdot \ddot{a}_{lx} - w \cdot l_{yz} \cdot \cos a_{lx} \cdot \ddot{a}_{lx}^2 - g \cdot l_{yz} \cdot \sin a_{lx} + g \cdot w = 0 \quad (5.3.4)$$

In the above equation, w is remained constant for this motion, which places the desired location of ZMP at a fixed point. l_{yz} is determined from the double-support phase and related to $a_{lx(0)}$ (Section 5.3.2). It is kept constant during the single-support phase by freezing the posture of the robot body. The radius of gyration is found from J_x : $l_x \approx 0.16m$ for Prototype II.

The initial conditions for the solution of Eq. (5.3.4) are specified by State (a) in Figure 5.6, which is the initial state for the motion of this single-support phase:

$$a_{lx}(t=0) = a_{lx(0)} \quad \dot{a}_{lx}(t=0) = \dot{a}_{lx(0)}$$

Numerical solutions of Eq. (5.3.4) can be obtained for different initial conditions and different values of w . Nevertheless, in gait synthesis, the proper initial conditions are usually to be determined from other motion parameters. These motion parameters can be motion period T_s , which is defined as the time needed to complete the entire motion of (a) to (g) in Figure 5.6, and the minimal value of a_{lx} , etc. Approximate solutions are also explored (Appendix D.1), through which the required initial conditions are estimated based on the specified motion parameters.

Some of the numerical results for the solutions of Eq. (5.3.4) with the following initial conditions are presented in Figure 5.8:

$$a_{lx}(t=0) = a_{lx(0)} = 12.0^\circ \quad \dot{a}_{lx}(t=0) = \dot{a}_{lx(0)} = 45.8^\circ / s$$

For the solutions shown in Figure 5.8:

When $a_{lx(0)} = 12^\circ$: $l_{yz} \approx 0.16m$

The solutions for three different values of w are examined: $w=0mm$, $5mm$ and $8mm$

Figure 5.8: Examples of Numerical Solutions for Equation (5.3.4)

The results in Figure 5.8 imply that, for the single support phase in which the left foot is the supporting foot, the duration of this single support phase can be increased by placing the ZMP towards the right edge of the left foot. Torque T_{lx} is therefore required to keep the ZMP in position. Nevertheless, accurate control of torque output from the integrated servos is generally unavailable without implication of additional hardware. It is thus desired to set w to zero, whereby negligible torque output is required from the servo and the ZMP is positioned at the centre of the stable region. The servo driving this DOF is hence instructed to follow the motion of the inverted pendulum accordingly based on the off-line solution to the following equation, which is in fact the expression for Eq. (5.3.4) with $w = 0$:

$$(l_{yz}^2 + l_x^2) \cdot \ddot{\alpha}_{lx} - g \cdot l_{yz} \cdot \sin \alpha_{lx} = 0 \quad (5.3.5)$$

Similar analysis is applied to the motion of (C)→(D)→(C) in Figure 5.5, for which the model with the right ankle joint as the pivot is utilized.

Due to system uncertainties and imperfection in modelling, actual motions of the robot deviate from the calculated ones. Besides minor manual adjustments on the actual motions about the calculation results, this deviation is mainly compensated within the servo systems by a large proportional gain in the servo controllers (Section 4.1).

5.3.2 Double-Support Phases

For the double-support phases, the four-bar linkage model is applied in dynamic analysis. The motion of (B)→(C) in Figure 5.5 is elaborated in Figure 5.9:

Figure 5.9: Motion Sequence of the Double-Support Phase Shown in Coronal Plane of the Robot Body

When the distance between the ankle joints is equal to length of the pelvis, the following relation exists for the four-bar linkage model of Figures 5.2 and 5.3:

$$q_{LA} = q_{RA} = q_l \quad (5.3.6)$$

In dynamic analysis for this double-support phase (Appendix D.2), the equation describing the motion of (a) to (d) in Figure 5.9 is derived as below:

$$J_{\Sigma} \cdot \ddot{q}_l - I_{\Sigma} \cdot g \cdot \sin q_l + T_{\Sigma} = 0 \quad (5.3.7)$$

where: $T_{\Sigma} = T_{A_l} + T_{H_r} - T_{H_l} - T_{A_r}$

$$J_{\Sigma} = 2 \cdot J_2 + 2 \cdot m_2 \cdot l_0^2 + m_{H_l-H_r} \cdot l_2^2$$

$$I_{\Sigma} = m_{H_l-H_r} \cdot l_2 + 2 \cdot m_2 \cdot l_0$$

T_{A_r} , T_{A_l} , T_{H_r} and T_{H_l} are independent torque outputs from the four actuators in the model, which are specified as functions of time or q_l , etc. They produce the combined torque output of T_Σ . Condition (5.1.1) is applied on both feet for examination of the stability of the feet. Based on the assumption $h \approx 0$, the following constraints on the independent torque outputs are obtained from the dynamic analysis on the four-bar linkage model (Appendix D.2):

$$\left| I_\Sigma \cdot \left(\sin q_l \cdot \frac{T_\Sigma + I_\Sigma \cdot g \cdot \sin q_l}{2J_\Sigma} + \cos q_l \cdot \ddot{q}_l \right) - \left(\frac{m_{H_l-H_r}}{2} + m_2 \right) \cdot g + \frac{2(T_{H_r} - T_{H_l})}{l_1} \right| \geq \left| \frac{2T_{A_r}}{w_0} \right| \quad (5.3.8)$$

$$\left| I_\Sigma \cdot \left(\sin q_l \cdot \frac{T_\Sigma + I_\Sigma \cdot g \cdot \sin q_l}{2J_\Sigma} + \cos q_l \cdot \ddot{q}_l \right) - \left(\frac{m_{H_l-H_r}}{2} + m_2 \right) \cdot g - \frac{2(T_{H_r} - T_{H_l})}{l_1} \right| \geq \left| \frac{2T_{A_r}}{w_0} \right| \quad (5.3.9)$$

The initial conditions for the motion are determined by State (a) in Figure 5.9: $q_l(t=0) = q_{l(0)}$; $\ddot{q}_l(t=0) = \ddot{q}_{l(0)}$. Numerical solutions of Eq. (5.3.7) can be obtained when T_Σ , $q_{l(0)}$ and $\ddot{q}_{l(0)}$ are specified. Similar analysis is also conducted for the double-support phase of (C)→(B) in Figure 5.5.

5.3.3 Transitions between Single-Support and Double-Support Phases

Kinematical relations are established between the single-support and double-support phases during the phase transitions. The final state of a double-support phase determines the initial conditions for the motion of the following single-support phase. On the other hand, the initial conditions for this double-support phase are also defined from the final state of the preceding single-support phase.

The following figure shows the final state of the double-support phase (C)→(B) in Figure 5.5, from which the initial conditions for the ensuing single-support phase, i.e. motion of (B)→(A)→(B), are determined.

Figure 5.10: Geometry for Final State of Double-Support Phase (C)→(B)

At the end of this double-support phase, $q_l = q_{l(1)}$. The following relations are thus obtained from Figure 5.2 and Figure 5.10:

$$l_{yz} = \sqrt{l_1^2/4 - l_1 \cdot h_o \cdot \sin q_{l(1)} + h_o^2} \quad (5.3.10)$$

$$a_{lx} = \tan^{-1} \frac{(l_1/2) - h_o \cdot \sin q_{l(1)}}{h_o \cdot \cos q_{l(1)}} \quad (5.3.11)$$

CG of the bipedal robot is shifting towards the left foot. At the moment just before the double-support phase ends, $\dot{q}_l = \dot{q}_{l(1)}$. The velocity of CG is derived as below:

$$\dot{v} = h_o \cdot \dot{q}_{l(1)} \angle q_{l(1)} \quad (5.3.12)$$

where the angle between the direction of the vector and y_l -axis is specified in the counter-clockwise sense.

Based on continuity in the angular position of CG, one of the initial conditions for the single-support phase is determined from Eq. (5.3.11) as:

$$a_{lx(0)} = \tan^{-1} \frac{(l_1/2) - h_o \cdot \sin q_{l(1)}}{h_o \cdot \cos q_{l(1)}} \quad (5.3.13)$$

When the single-support phase is initiated by lifting the right foot, it is assumed that no slipping of the supporting foot (left foot) occurs during the event. At this instant, the component of $\dot{v}$ along the line A_l-O is absorbed by the system through its elasticity and backlash, whilst the perpendicular component is preserved. The velocity of CG at this moment is hence found from Eqs. (5.3.11) and (5.3.12) as:

$$\dot{v}_{s(0)} = h_o \cdot \dot{q}_{l(1)} \cdot \cos(a_{lx(0)} + q_{l(1)}) \angle -a_{lx(0)} \quad (5.3.14)$$

The other initial condition is obtained from Eqs. (5.3.10) and (5.3.14):

$$\dot{a}_{lx(0)} = -\frac{h_o \cdot \dot{q}_{l(1)} \cdot \cos(a_{lx(0)} + q_{l(1)})}{l_{yz}} \quad (5.3.15)$$

Determination of the initial conditions for the single-support phase of (B)→(A)→(B) is therefore completed by Eqs. (5.3.13) and (5.3.15) from $q_{l(1)}$ and $\dot{q}_{l(1)}$.

In Figure 5.5, when the motion for the double-support phase of (B)→(C) starts, its initial conditions are defined by the final state of the single-support phase (B)→(A)→(B) in which $a_{lx} = a_{lx(1)}$ and $\dot{a}_{lx} = \dot{a}_{lx(1)}$.

During the transition, the four-bar linkage model for the bipedal robot assumes the same posture as that in Figure 5.10 again. The initial condition of q_l for the double-support phase of (B)→(C) is obtained from geometrical analysis based on continuity in the angular position of CG:

$$q_{l(0)} = \cos^{-1}(l_{yz} \cdot \cos a_{lx(1)} / h_o) \quad (5.3.16)$$

At the instant right before the motion enters the double-support phase, the velocity of CG is expressed as:

$$\dot{v} = l_{yz} \cdot \dot{a}_{lx(1)} \angle (180^\circ - a_{lx(1)}) \quad (5.3.17)$$

When no slippage of either foot occurs during the phase transition, abrupt change in the velocity of CG is experienced again by the system. At the moment when the double-support phase is initiated with landing of the right foot, the velocity of CG is found from Eqs. (5.3.16) and (5.3.17) as:

$$\dot{v}_{d(0)}^y = l_{yz} \cdot \dot{\alpha}_{lx(1)} \cdot \cos(a_{lx(1)} + q_{l(0)}) \angle (180^\circ + q_{l(0)}) \quad (5.3.18)$$

The other initial condition for the motion of (B)→(C) is thus obtained as:

$$\dot{q}_{l(0)}^{\&} = -\frac{l_{yz} \cdot \dot{\alpha}_{lx(1)} \cdot \cos(a_{lx(1)} + q_{l(0)})}{h_o} \quad (5.3.19)$$

Similar analysis applies for the motion of (C)→(D)→(C)→(B) with the following modifications in the corresponding notations:

$$\begin{aligned} q_l &\rightarrow q_r, \quad q_{l(0)} \rightarrow q_{r(0)}, \quad q_{l(1)} \rightarrow q_{r(1)}, \quad \dot{q}_{l(0)}^{\&} \rightarrow \dot{q}_{r(0)}^{\&}, \quad \dot{q}_{l(1)}^{\&} \rightarrow \dot{q}_{r(1)}^{\&} \\ a_{lx} &\rightarrow a_{rx}, \quad a_{lx(0)} \rightarrow a_{rx(0)}, \quad a_{lx(1)} \rightarrow a_{rx(1)}, \quad \dot{\alpha}_{lx(0)} \rightarrow \dot{\alpha}_{rx(0)}, \quad \dot{\alpha}_{lx(1)} \rightarrow \dot{\alpha}_{rx(1)} \end{aligned}$$

5.3.4 Synthesis of Rocking Motion on Horizontal Surfaces

To uniquely define a motion of rocking, the initial conditions for the single-support and double-support phases are determined, based on which unique solutions for Eqs. (5.3.5) and (5.3.7) are established.

The initial conditions and final states of the single-support and double-support phases are recognized as the motion constants. Rocking on a horizontal surface is a cyclic motion symmetrical about the saggital plane of the bipedal robot. Based on this fact, the following relations are determined for the motion constants in addition to those derived in the previous section:

$$\begin{aligned} a_{lx(0)} &= a_{lx(1)} = a_{rx(0)} = a_{rx(1)} & \dot{\alpha}_{lx(0)} &= -\dot{\alpha}_{lx(1)} = \dot{\alpha}_{rx(0)} = -\dot{\alpha}_{rx(1)} \\ q_{l(0)} &= q_{l(1)} = q_{r(0)} = q_{r(1)} & \dot{q}_{l(1)}^{\&} &= \dot{q}_{r(1)}^{\&} \\ \dot{q}_{l(0)}^{\&} &= \dot{q}_{r(0)}^{\&} = -\dot{q}_{l(1)}^{\&} \cdot \cos^2(a_{lx(0)} + q_{l(1)}) \end{aligned}$$

Therefore two constants need to be specified for the entire motion of rocking to be defined uniquely:

$$q_{l(0)} = q_{l(1)} = q_{r(0)} = q_{r(1)} = q^* \quad \dot{q}_{l(1)} = \dot{q}_{r(1)} = \dot{q}^* \quad (5.3.20)$$

During each double-support phase, the increase in total energy of the bipedal robot is expressed as:

$$\Delta E = \frac{1}{2} \cdot \dot{q}^{*2} \cdot l_2^2 \cdot \left\{ 1 - \cos^4 \left[\tan^{-1} \frac{(l_1/2) - h_o \cdot \sin q^*}{h_o \cdot \cos q^*} + q^* \right] \right\} \quad (5.3.21)$$

For a fixed standing height, which corresponds to fixed l_2 and l_0 , ΔE is found to be practically independent of q^* when q^* is small:

$$\Delta E \approx C^* \cdot \dot{q}^{*2} \quad \text{where } C^* \text{ is a constant determined by } l_2$$

The average value of the combined torque required providing this amount of energy during the double-support phase is thus estimated as:

$$T_{\Sigma(average)} \approx C^* \cdot \dot{q}^{*2} / (2q^*) \quad (5.3.22)$$

From numerical results of Eq. (5.3.22), it is predicted that this torque output can be generated by negligible drift of the servo horns from their desired angles, since the integrated servos have a large proportional gain in their controllers. Therefore, the synthesis for the motions of double-support phases is primarily dictated by conditions (5.3.8) and (5.3.9).

Motions of rocking with different standing heights and motion periods were examined. Eqs. (5.3.5) and (5.3.7) are found to be satisfactorily accurate description of the actual motions of the bipedal robot. No slipping or spinning of the robot relative to the ground were observed in the experiments. This fact justifies the assumption that conditions (5.1.3) and (5.1.4) are always satisfied.

Figure 5.11 shows the profiles of the frontal angles at both ankle joints during one complete cycle of a specific motion of rocking which is synthesised at $l_2 \equiv 160\text{mm}$:

Figure 5.11: Frontal Angles of Ankle Joints in One Cycle of Rocking Motion

Since the integrated servos are controlled by series of pulses at a frequency of 50 Hz, the pulse number is used in the above figure as the time scale. The motion has a period of 72 units, which is equivalent to 1.44 s. In Figure 5.11, Phase I is the double-support phase during which the robot body is shifting leftward and Phase II is the single-support phase when the left foot is the supporting foot. Phases III and IV are the corresponding phases of the other half cycle respectively. For this specific motion of rocking, each single-support phase spans 0.48 s, whilst each of the double-support phases consumes 0.24 s.

5.4 Straight Walking

Synthesis of the gaits for dynamic walking is decomposed into those on sagittal and coronal planes of the robot respectively. Dynamic analysis for the motions in the coronal

plane is similar to that for the motion of rocking. Since it is assumed that dynamic stability in the sagittal plane of the robot is always satisfied during the double-support phase (Section 5.2.1), dynamics of the robot in this plane is investigated only for the single-support phases.

5.4.1 Walking on Horizontal Surfaces

In synthesis of the gaits for dynamic walking on horizontal surfaces, the sideways motion of the robot is designed to be essentially the same as that for the motion of rocking. The gait described in Figure 5.11 is adopted for regular dynamic walking of the robot on horizontal surfaces.

In the sagittal plane, the robot body is kept moving forward or backward with nearly constant horizontal speeds. Half cycle of the motion in this plane is shown in Figure 5.11. The robot body moves forward along the path $a \rightarrow b \rightarrow c \rightarrow d$ in the figure. From point a to point b is the double-support phase, whilst the single-support phase is from point b to point d during which the left foot is the supporting foot.

Figure 5.12: Motion Sequence of Dynamic Walking Shown in the Sagittal Plane

When the horizontal speed of the robot body is specified to be v^* , the following relations are established based on the sideways motion of the robot:

$$x_1 = x_2 = x_3 = 0.24 \cdot v^* \quad s_1 = s_2 = x_1 + x_2 + x_3 = 0.72 \cdot v^* \quad (5.4.1)$$

From the dynamic analysis for the linear inverted pendulum model in z - O - x plane (Appendix D.1), the following equations are derived at the left ankle joint for the single-support phase:

$$l_{zx} = \sqrt{l_{yz}^2 \cdot \cos^2 a_{lx} + (0.24 - t)^2 \cdot v^{*2}} \quad (5.4.2)$$

$$a_{ly} = \tan^{-1} \left[\frac{(0.24 - t) \cdot v^*}{l_{yz} \cdot \cos a_{lx}} \right] \quad (5.4.3)$$

$$T_{ly} = m \cdot \left[(l_{zx}^2 + l_y^2) \cdot \ddot{a}_{ly} + 2 \cdot \dot{a}_{ly} \cdot \dot{l}_{zx} \cdot l_{zx} - g \cdot l_{zx} \cdot \sin a_{ly} \right] \quad (5.4.4)$$

where l_y is the radius of gyration in y - O - z plane, i.e. $J_y = ml_y^2$.

T_{ly} is thus calculated from the numerical solution of a_{lx} with Eq. (5.4.4). The ZMP is maintained near the centre of the stable region on the left foot by keeping $|T_{ly}|$ small. Again, due to a large proportional gain in its controller, the servo coupled to this DOF is instructed to follow the profile of a_{ly} , whilst any drift of the actual motion from the theoretical one is compensated and corrected spontaneously by the servo system.

It is found from Eq. (5.4.4) that the maximal speed of the robot body, $v_{\max}^*$, is limited by the maximal value of $|T_{ly}|$. For the motion of regular dynamic walking, $v_{\max}^*$ is predicted to be about 0.11m/s (Appendix D.1). This value is close to the one found through experiments, which is about 0.09m/s. As v^* exceeds this value, stability of the dynamic motion is perturbed by slipping and overturning of the feet until finally toppling of the entire robot is observed. To ensure stable dynamic motions, the commanded walking speeds of the robot are always capped to be not more than 75mm/s and the regular commanded walking speed is 50mm/s.

Figure 5.13 illustrates the trajectory of each foot in lateral plane of the corresponding hip coordinate system during the motion elucidated by Figure 5.12:

Figure 5.13: Trajectories of the Feet Shown in the Sagittal Plane of Robot Body

The trajectories in the above figure follow the sequence $a \rightarrow b \rightarrow c \rightarrow d$. The dashed lines in the figure represent the trajectories for the other half cycle of the motion. In Figure 5.13, height of both triangles, h^* , is kept constant, while the lengths of the bottoms are varied according to the walking speed of the robot. The trajectories the figure are specified by the following equations:

$$x_{ankle(r)}(n) = \begin{cases} -v_d^* \cdot (n - 72 \cdot k + 12) & 72 \cdot k < n \leq 72 \cdot k + 12 \\ 2 \cdot v_d^* \cdot (n - 72 \cdot k - 24) & 72 \cdot k + 12 < n \leq 72 \cdot k + 36 \\ -v_d^* \cdot (n - 72 \cdot k - 60) & 72 \cdot k + 36 < n \leq 72 \cdot (k + 1) \end{cases} \quad (5.4.5)$$

$$z_{ankle(r)}(n) = \begin{cases} -l_2 & 72 \cdot k < n \leq 72 \cdot k + 12 \\ -l_2 + h^* \cdot (n - 72 \cdot k - 12)/12 & 72 \cdot k + 12 < n \leq 72 \cdot k + 24 \\ -l_2 + 3 \cdot h^* - h^* \cdot (n - 72 \cdot k)/12 & 72 \cdot k + 24 < n \leq 72 \cdot k + 36 \\ -l_2 & 72 \cdot k + 36 < n \leq 72 \cdot (k + 1) \end{cases} \quad (5.4.6)$$

$$x_{ankle(l)}(n) = x_{ankle(r)}(n + 36) \quad (5.4.7)$$

$$z_{ankle(l)}(n) = z_{ankle(r)}(n + 36) \quad (5.4.8)$$

where k is a nonnegative integer representing the number of walking cycles completed

n is a nonnegative integer specifying the time with pulse number

v_d^* is v^* specified in the unit of mm per pulse number

For a motion of dynamic walking with frequency f and step size s^* , the walking speed of the robot, v^* , is expressed as:

$$v^* = s^* \cdot f \quad \text{where } s^* = s_1 + s_2 \text{ as in Figure 5.12.} \quad (5.4.9)$$

Therefore the walking speed of the bipedal robot can be altered by changing either the frequency or step size. Nevertheless, to achieve stable and smooth sideway motions at a certain frequency, minor manual adjustments are usually necessary in addition to the mathematical formulation. Thus the sideway motion for regular dynamic walking is fixed to the one specified at the end of Section 5.3.4, whilst the walking speed is adjusted by solely varying the step size. Since the sideway motion has a period of 1.44s, Eq. (5.4.9) will be essentially the same as Eq. (5.4.1).

The motion sequence of the gait for dynamic walking is shown in the following figure:

Figure 5.14: Motion Sequence of the Robot for Dynamic Walking Shown in the Sagittal Plane

The dynamic motions at different commanded walking speeds were tested. The experimental results are tabulated in Table 5.1:

Table 5.1: Actual Walking Speeds of Prototype II for the Dynamic Motion of Walking on Horizontal Surfaces

Walking direction	Forward			Backward		
Commanded speed	25.0mm/s	50.0mm/s	75.0mm/s	25.0mm/s	50.0mm/s	75.0mm/s
Actual speed	23.6mm/s	47.1mm/s	68.8mm/s	22.1mm/s	46.8mm/s	66.7mm/s

Table 5.1 exhibits satisfactory experimental results of the dynamic walking motions in which errors of the actual walking speeds are mostly less than 10% of the commanded ones. Compared to the results in Table 4.3, the bipedal robot is observed to walk at much higher speeds under dynamic stability. Similar to the quasi-static motions, walking frequency of the robot for dynamic motions is accurately controlled by the micro-controllers. Nevertheless, in addition to mechanical backlashes, un-modelled dynamics in the robot's sagittal plane is also recognized to be the other main source causing the errors in actual walking speeds.

The actual sideway motion of Prototype II was investigated by examining snapshots of the robot at discrete moments of the motion (Appendix B.5). The actual frontal angles of both ankle joints were measured (Figure 5.15) on these snapshots and compared with those planned in Section 5.3.4. The measurements were done on the walking motion at the regular commanded walking speed, i.e. 50mm/s, and the results are recorded in Table 5.2.

Figure 5.15: (A) A Snapshot of Prototype II during the Double-support Phase
(B) A Snapshot of Prototype II during the Single-support Phase

Table 5.2 Actual Frontal Angles of Both Ankle Joints During One Cycle of the Dynamic Motion of Walking on a Horizontal Surface at the Commanded Speed of 50mm/s

Motion phase	Time	Planned θ_l	Actual θ_l	Planned θ_r	Actual θ_r
Double-support phase	0.00 sec	-5.0°	-4.8°	5.0°	5.0°
	0.08 sec	-2.5°	-2.4°	2.5°	2.6°
	0.16 sec	0.7°	1.0°	-0.7°	-0.8°
	0.24 sec	5.0°	5.0°	-5.0°	-4.7°
Single-support phase with the left foot as the supporting foot	0.32 sec	8.3°	7.8°	-5.0°	-4.8°
	0.40 sec	10.0°	8.8°	-5.0°	-4.8°
	0.48 sec	10.5°	9.5°	-5.0°	-4.8°
	0.56 sec	10.0°	9.2°	-5.0°	-4.8°
	0.64 sec	8.2°	7.6°	-5.0°	-4.8°
Double-support phase	0.72 sec	5.0°	5.2°	-5.0°	-4.8°
	0.80 sec	2.5°	2.8°	-2.5°	-2.5°
	0.88 sec	-0.7°	-0.5°	0.7°	1.0°
	0.96 sec	-5.0°	-4.9°	5.0°	5.2°
Single-support phase with the right foot as the supporting foot	1.04 sec	-5.0°	-4.9°	8.3°	7.9°
	1.12 sec	-5.0°	-4.9°	10.0°	8.8°
	1.20 sec	-5.0°	-4.9°	10.5°	9.7°
	1.28 sec	-5.0°	-4.9°	10.0°	9.1°
	1.36 sec	-5.0°	-4.9°	8.2°	8.0°
	1.44 sec	-5.0°	-4.9°	5.0°	5.0°

As shown in Table 5.2, both ankles follow the planned motions well during the double-support phases. This is expected since during the double-support phases, the dynamic motion in the coronal plane of the robot is driven by four motors (referring to Section 5.3.2). The actual horn angle of each individual servo only needs to deviate from the commanded one by a small amount to provide enough combined torque output, which forces the robot to follow the planned motion. On the other hand, larger errors appear in the motion of the ankle joint at the supporting foot during the single-support phases. Dynamics involved in the posture change of the robot during dynamic locomotion is not modelled in the linear inverted pendulum models of the single-support phase. This defect

contributes to most of the modelling errors. However, conformity of the actual sideway motion to the planned one is not jeopardized based on the result in Table 5.2 and the observation that in experiments, sideway motion of the robot is always repeatable and stable for a walking speed less than 80mm/s.

Movement of the robot's CG during the actual dynamic walking motion was studied through experiments. In line with the investigation of the sideway motion, this study was also done on the walking motion with the regular commanded walking speed of 50mm/s. The robot is frozen to its actual poses at discrete moments of the motion based on its snapshot at each specific moment. The position of the robot's CG on the ground surface for a specific pose is measured with the experiment setup described in Appendix B.3. The measurement results are listed in Table 5.3:

Table 5.3: Positions of the CG of at Discrete Moments of the Dynamic Walking Motion at the Commanded Walking Speed of 50mm/s

Motion phase	Time	x_r (mm)	y_r (mm)	x_l (mm)	y_l (mm)
Double-support phase	0.00 sec	11.0	38.5	—	—
	0.08 sec	13.5	41.0	—	—
	0.16 sec	—	—	-15.0	-43.5
	0.24 sec	—	—	-10.5	-37.5
Single-support phase with the left foot as the supporting foot	0.32 sec	—	—	-5.5	-31.0
	0.40 sec	—	—	-3.0	-27.5
	0.48 sec	—	—	-0.5	-26.0
	0.56 sec	—	—	3.5	-27.0
	0.64 sec	—	—	5.5	-32.0
Double-support phase	0.72 sec	—	—	10.0	-36.0
	0.80 sec	—	—	13.0	-42.5
	0.88 sec	-16.0	42.0	—	—
	0.96 sec	-11.0	36.5	—	—
Single-support phase with the right foot as the supporting foot	1.04 sec	-6.5	31.0	—	—
	1.12 sec	-3.0	26.0	—	—
	1.20 sec	-0.5	24.5	—	—
	1.28 sec	2.5	26.0	—	—
	1.36 sec	5.0	31.0	—	—
	1.44 sec	10.5	38.0	—	—

Similar to those for the quasi-static motions, the results in Table 5.3 are accurate only up to $\pm 1\text{mm}$. They still provide good understanding on the movement of the robot's CG during the motion of walking under dynamic stability, though. Each foot has a width of 50mm. Comparing this to the results in Table 5.3, it is found that the robot's CG is mostly outside the supporting region during the single-supporting phases. This is in contrary to the quasi-static walking.

Based on Table 5.3, Figure 5.16 is drawn for illustration of the movement of CG of the robot corresponding to the motion sequence depicted in Figure 5.14:

Figure 5.16 Movement of CG and Feet Shown on the Horizontal Plane

From the experimental results in Tables 5.1 to 5.3, it is revealed that implementation of the simplified dynamic models discussed in Section 5.2 does generate satisfactory results for synthesis of the dynamic motion of walking on horizontal surfaces.

5.4.2 Traversing a Ramp

While travelling across a ramp, the sideways motion of the bipedal robot is no longer symmetrical with respect to its sagittal plane. To utilize the sideways motion developed for dynamic walking on horizontal surfaces, a bias in the frontal angles of the ankle joints is added as correction to the motion.

Figure 5.17 shows the stances of the bipedal robot on a ramp during the double-support phases. The angle of slope is specified as j for the ramp.

Figure 5.17: Stances of the Bipedal Robot on a Ramp

In the Figure 5.17, the following relations are established as part of the initial conditions for the single-support phases:

$$a_{lx(0)} = \tan^{-1} \frac{(l_1/2) - h_o \cdot \sin q_{l(1)}}{h_o \cdot \cos q_{l(1)}} + j \quad (5.4.10)$$

$$a_{rx(0)} = \tan^{-1} \frac{(l_1/2) - h_o \cdot \sin q_{r(1)}}{h_o \cdot \cos q_{r(1)}} - j \quad (5.4.11)$$

To utilize the sideway motion for walking on horizontal surfaces in the situation of traversing ramps, the following conditions are imposed:

$$q_{r(1)} + q_{l(1)} = 2q^* \quad \dot{q}_{r(1)} = \dot{q}_{l(1)} = \dot{q}^*$$

where, q^* and $\dot{q}^*$ are the same as those specified in Eq. (5.3.20).

Hence when a bias angle of dq is superimposed to the frontal angles of both ankle joints, Eqs. (5.4.12) and (5.4.13) assume the following forms:

$$a_{lx(0)} = \tan^{-1} \frac{(l_1/2) - h_o \cdot \sin(q^* + dq)}{h_o \cdot \cos(q^* + dq)} + j \quad (5.4.14)$$

$$a_{rx(0)} = \tan^{-1} \frac{(l_1/2) - h_o \cdot \sin(q^* - dq)}{h_o \cdot \cos(q^* - dq)} - j \quad (5.4.15)$$

The other initial conditions for the single-support phases are:

$$\text{For Figure 5.16(A): } \dot{a}_{lx(0)} = - \frac{h_o \cdot \dot{q} \cdot \cos(a_{lx(0)} + q^* + dq)}{l_{yz}} \quad (5.4.16)$$

$$\text{For Figure 5.16(B): } \dot{a}_{rx(0)} = - \frac{h_o \cdot \dot{q} \cdot \cos(a_{rx(0)} + q^* - dq)}{l_{yz}} \quad (5.4.17)$$

Initial conditions (5.4.14) to (5.4.17) are applied to Eq. (5.3.5) and its counterpart for the right ankle joint to solve for the motions of the single-support phases. To obtain motions with the same motion period T_s (Section 5.3.1), the relation between j and dq is established numerically and linearised to the following expression:

$$dq \approx 1.08j \quad (5.4.18)$$

During the double-support phases, dq is also superimposed to the corresponding portion of the original gait. The dynamic analysis for the double-support phases is similar to that discussed in Section 5.3.2 to Section 5.3.4, except for that in conditions (5.3.8) and (5.3.9) w_0 is replaced by $w_0 \cdot \cos j$. For $|j| < 18^\circ$, these two conditions are always satisfied by the modified gait, which is also justified in experiments.

Eq. (5.4.18) is implemented in the traversing motions on ramps with different slope angles. The actual walking speeds were recorded and compared with the planned speeds in Tables 5.4 and 5.5:

Table 5.4: Actual Walking Speeds of Prototype II When Traversing Ramps Slanting Rightward with Different Angles

Rightward slanted slope: the robot's right foot is lower than its left foot when standing on the slope								
Slope angle (degree)	6°				12°			
Walking direction	Forward		Backward		Forward		Backward	
Desired speed (mm/s)	25.0	50.0	25.0	50.0	25.0	50.0	25.0	50.0
Actual speed (mm/s)	23.8	46.9	23.4	47.2	23.6	46.6	22.9	46.4

Table 5.5: Actual Walking Speeds of Prototype II When Traversing Ramps Slanting Leftward with Different Angles

Leftward slanted slope: the robot's left foot is lower than its right foot when standing on the slope								
Slope angle (degree)	6°				12°			
Walking direction	Forward		Backward		Forward		Backward	
Desired speed (mm/s)	25.0	50.0	25.0	50.0	25.0	50.0	25.0	50.0
Actual speed (mm/s)	24.0	46.4	23.7	46.8	23.3	46.3	23.2	46.2

Similar to the dynamic motion of walking on horizontal surfaces, the actual traversing speeds across a ramp are close to the planned ones. At the same time, the actual traversing motions were also observed to be stable and repeatable, as recorded in the attached CD-ROM (referring to the file index in the CD-ROM). The bias specified by Eq. (5.4.18) is therefore accepted a satisfactorily accurate correction to the motion of dynamic walking when the bipedal robot is traversing a ramp.

5.4.3 Walking up/down Slopes

When the bipedal robot is walking up a ramp, the motion of dynamic walking is modified from that on the horizontal surfaces by superposition of an upward velocity to the original gait. The modified motion is illustrated in Figure 5.18:

Figure 5.18: Motion Sequence of the Robot for Walking up a Ramp Shown in the Sagittal Plane

Superposition of the upward velocity u^* is achieved through the following modifications to Eq. (5.4.6), which is one of the equations describing the original foot trajectories:

$$z_{ankle(r)}(n) = \begin{cases} -l_2 - \Delta h \cdot (n - 72 \cdot k - 12) / 48 & 72 \cdot k < n \leq 72 \cdot k + 12 \\ -l_2 + (2 \cdot h^* + \Delta h) \cdot (n - 72 \cdot k - 12) / 24 & 72 \cdot k + 12 < n \leq 72 \cdot k + 24 \\ 2 \cdot h^* + (\Delta h - 2 \cdot h^*) \cdot (n - 72 \cdot k - 12) / 24 - l_2 & 72 \cdot k + 24 < n \leq 72 \cdot k + 36 \\ -l_2 - \Delta h \cdot (n - 72 \cdot k - 84) / 48 & 72 \cdot k + 36 < n \leq 72 \cdot (k + 1) \end{cases} \quad (5.4.19)$$

$$\text{where } \Delta h = 0.96 \cdot v^* \cdot \tan j / \cos q^* \quad (5.4.20)$$

j is slope angle of the ramp as shown in Figure 5.17

v^* is the horizontal speed of the robot body as specified in Section 5.4.1

The new foot trajectories are thereby defined by Eqs. (5.4.5), (5.4.19), (5.4.7) and (5.4.8). With the modification introduced through Eq. (5.4.19), the movements of both feet are allowed to adapt to the slanted surface of a ramp. Figure 5.19 depicts these modified trajectories in the lateral plane of the hip coordinate systems.

Figure 5.19: Modified Trajectories of the Feet Shown in Lateral Planes of the Hip Coordinate Systems

At the ankle joints, a bias angle equal to φ is also added to θ_a (Figure 4.5 and 4.6) to compensate for the change in terrain.

During each of the double-support phases, the change in l_2 (Figure 5.2) is:

$$\Delta l_2 = 0.24 \cdot v^* \cdot \tan j / \cos q^* \quad (5.4.21)$$

For $v^* < 0.05 \text{ m/s}$ and $j < 15^\circ$: $\Delta l_2 < 3.2 \text{ mm}$ and $u^* < 0.014 \text{ m/s}$

These two values are much smaller than l_2 and the velocity of robot's CG respectively. Their effects on the motions of double-support phases are assumed to be negligible. Therefore the control on frontal angles of the ankle joints is unchanged for the double-support phases.

During the single-support phase when the left foot is the supporting foot, the entire robot is modelled as a linear inverted pendulum with the following characteristics:

$$\mathcal{K}_{yz} = v^* \cdot \tan j / \cos q^* \quad (5.4.22)$$

$$l_{yz} = v^* \cdot \tan j \cdot t / \cos q^* + l_{yz}^* \quad (5.4.23)$$

where, $l_{yz}^* = 164.3mm$ and $q^* = 5^\circ$ for the original gait.

The differential equation describing the modified motion is derived by inserting Eqs. (5.4.22) and (5.4.23) into Eq. (5.3.3) with $w = 0$. The numerical solution obtained for $v^* = 0.025m/s$ is depicted in Figure 5.20:

Figure 5.20: Comparison on Solutions for Modified Gait and Original Gait

Based on the numerical results, the sideways motion for the modified gait deviates from the one for regular dynamic walking by a difference less than 0.5° . Since it is predicted that this difference can be neutralized by the large proportional gain of the servo system, control on the servo driving this DOF of the left ankle joint is parallel to that for regular dynamic walking. Similar considerations are also applied to the single-support phase in which the right foot is the supporting foot.

When the bipedal robot is walking up a ramp, the motion gait is modified in the sagittal plane as specified by Figure 5.19, whilst the sideways motion for regular dynamic walking is retained. For the situation of travelling down slope, a similar approach is employed. Effectiveness of the proposed modification to the gait is justified by experiments for $j < 15^\circ$ and $v^* \leq 0.05m/s$. The experimental results are tabulated as in Table 5.6:

Table 5.6: Actual Speeds of Prototype II along the Slope Surface while Ascending/descending a Slope

Slope angle (degree)	6°				12°			
Walking direction	Upslope		Downslope		Upslope		Downslope	
Desired horizontal speed (mm/s)	25.0	50.0	25.0	50.0	25.0	50.0	25.0	50.0
Planned speed along the slope (mm/s)	25.1	50.2	25.1	50.2	25.6	51.1	25.6	51.1
Actual speed along the slope (mm/s)	23.5	46.8	24.0	47.2	23.1	46.4	23.8	47.4

In Table 5.6, the planned speed along the slope is specified from the desired horizontal speed and the slope angle:

$$\text{Planned speed along the slope} = \frac{\text{Desired horizontal speed}}{\text{Cosine of the slope angle}}$$

As shown in Table 5.6, the walking speeds of the robot measured along the slope surface deviates from the planned ones by less than 10%. On the other hand, repeatability and stability of the actual ascending/descending motions are also confirmed through experiments similar to those recorded in the attached CD-ROM (referring to the file index in the CD-ROM). Implementation of the modified foot trajectories in Figure 5.19 is therefore justified for the dynamic motions of walking up/down slopes.

5.5 Turning on Horizontal Surfaces

The gait for dynamic turning on horizontal surfaces is similar to the one under quasi-static stability except for that the motion enters the single-support phases before CG of the bipedal robot is shifted completely into the supporting foot.

On the horizontal plane, the pelvis is seen to travel along its own axis during the double-support phases. Figure 5.21 portrays one of these motions, during which CG of the robot is shifted towards the left foot.

Figure 5.21: Movement of CG Observed on the Horizontal Plane

In Figure 5.21, which is actually reproduced from Figure 4.11, $l-l'$ and $r-r'$ are along the longitudinal axes of the left foot and right foot respectively, whilst H_l-P_l and H_r-P_r are along the traversal axes of the feet. Since CG of the robot is approximately at the same height of the hip joints, it is practically seen to travel along the axis of the pelvis with velocity $\dot{v}_O$. This velocity is decomposed into two orthogonal components $\dot{v}_1$ and $\dot{v}_2$. $\dot{v}_1$ is along $r-r'$ and $\dot{v}_2$ is along $l-l'$.

In Figure 5.22, projection of the motion described by Figure 5.21 is shown in frontal plane of the left foot. The following expressions are obtained from Figure 5.21 and 5.22:

$$v_1 = l_2 \cdot \dot{q}_l \cos q_l \quad (5.4.24)$$

$$v_2 = v_1 \cdot \tan b \quad (5.4.25)$$

$$a_{lx} = \tan^{-1} \frac{(l_1 \cdot \cos b/2) - h_O \cdot \sin q_{l(1)}}{h_O \cdot \cos q_{l(1)}}$$

Figure 5.22: Movement of CG Observed in $x_l - O_l - z_l$ Plane

When $b < 15^\circ$, a_{lx} is practically the same as that in the double-support phase of the gait for dynamic walking (Figure 5.10). Therefore, the dynamic analysis assisted with the four-bar linkage model still applies for the projection of the motion on lateral plane of the left foot. Similar considerations are also adapted to the right foot. Hence the control on the longitudinal axis of each individual ankle joint during the double-support phases conforms to that for regular dynamic

walking on horizontal surfaces. On the other hand, the gait specified in the sagittal plane of each foot is defined to comply with Eqs. (4.3.21), (4.3.22), (5.4.24) and (5.4.25).

During the single-support phase, yaw motion in the hip joints introduces further dynamic disturbance to the motion of linear inverted pendulum model. As a rough estimation, when the left foot is the supporting foot, the additional angular velocity of the inverted pendulum generated by this disturbance is calculated approximately as:

$$d\mathcal{A}_{\text{LX}} \approx \frac{l_1 \cdot b \cdot \sin[b - (2 \cdot b \cdot t/T_s)]}{T_s \cdot l_{\text{vz}}} \quad (5.4.26)$$

For $b \leq 15^\circ$, the values of $d\mathbf{\hat{a}}_K$ calculated from Eq. (5.4.26) are compared with $\mathbf{\hat{a}}_K$. The result for $b = 15^\circ$ is depicted in Figure 5.23, in which the curve of $d\mathbf{\hat{a}}_K/\mathbf{\hat{a}}_K$ is plotted versus time. The maximum occurs at the midpoint of the single-support phase. This corresponds to the moment at which both $d\mathbf{\hat{a}}_K$ and $\mathbf{\hat{a}}_K$ approach zero. The value of $d\mathbf{\hat{a}}_K/\mathbf{\hat{a}}_K$ at this point is obtained with L'Hospital's Rule from Eqs. (5.4.26) and (D.1.9):

$$\lim_{t \rightarrow T_s/2} \frac{d\mathcal{A}_{\text{IX}}}{\mathcal{A}_{\text{IX}}} = \frac{d\mathcal{A}_{\text{IX}}/dt}{\mathcal{A}_{\text{IX}}} \bigg|_{t=T_s/2} \approx 0.118$$

$d\alpha_k$ is thereby found to be always much smaller than α_k . Based on estimation presented in the figure, it is predicted that the perturbation to the motion caused by rotation of the pelvis can be negotiated by the servo controller and system compliance of the entire robot. In practice, from the experiment results it was observed that rotation of the pelvis had negligible effect on stability of the motion. For the single-support phase during which the right foot is the supporting foot, similar result can be obtained.

Figure 5.23: $d\alpha_k/\alpha_k$ vs. Time during the Single-Support Phase

As a result of the above discussion on both the single-support and double-support phases, the series of control commands for the frontal angle of each ankle joint in regular dynamic walking are also applied for the motion of dynamic turning. In the lateral plane of each hip, the foot trajectory is exactly the same as that defined in Section 4.3.3. The turning speed of the robot can be specified as 4β per cycle following Section 4.3.3 or 2.78β per second since one motion cycle spans 1.44 s. Experiments were done for different values of β , i.e. turning speed, and the results are recorded in Table 5.7:

Table 5.7: Comparison between Planned and Actual Turning Speeds for the Motion of Dynamic Turning

Turning direction	Turning left			Turning right		
Planned β (deg)	4.0	8.0	12.0	4.0	8.0	12.0
Commanded turning speed (deg/s)	11.1	22.2	33.3	11.1	22.2	33.3
Actual β (deg)	4.0	7.9	12.1	3.8	7.8	11.9
Actual turning speed (deg/s)	11.2	22.0	33.7	10.8	21.8	33.1

Stability and repeatability of the actual turning motions were also verified in the experiments (referring to the attached CD-ROM). Similar to the locomotion discussed in the previous sections, the period of one motion cycle for dynamic turning is accurately controlled to be 1.44 seconds throughout the experiments. From examination of the recorded turning motions, durations of the single-support and double-support phases are observed to be around 0.46 sec and 0.26 sec respectively. These two values only deviate from the planned ones by less than two pulse cycles, i.e. 2×0.02 sec, of the input signal to the servos. On the other hand, the actual turning angle per motion cycle, i.e. $4b$, is also close to the planned ones, as indicated in Table 5.7. Based on these facts, the motion of dynamic turning discussed in this section is adopted in the regular locomotion for the robot to maneuver on horizontal surfaces, in combination with the walking motion described in Section 5.4.1.

5.6 Discussion

During locomotion, the inertial forces of the bipedal robot are expressed as below:

$$F_{in(x)} = Ma_x \quad F_{in(y)} = Ma_y \quad F_{in(z)} = Ma_z$$

where $F_{in(x)}$, $F_{in(y)}$ and $F_{in(z)}$ are the inertial forces in the x , y and z directions of the global coordinate system respectively

a_x , a_y and a_z are the accelerations of the robot's CG in the x , y and z directions of the global coordinate system respectively

In the sagittal plane of the robot, the inertial force is: $F_{sag} = \sqrt{F_{in(x)}^2 + F_{in(z)}^2}$

In the coronal plane of the robot, the inertial force is: $F_{cor} = \sqrt{F_{in(y)}^2 + F_{in(z)}^2}$

Based on the dynamic analysis developed in Appendices D.1 and D.2, these two forces are estimated for the dynamic motions synthesized in this chapter:

$$F_{sag} < 0.15Mg \quad F_{cor(max)} \approx 0.41Mg$$

It is seen from the above estimations that the inertial force in the coronal plane can reach a notable fraction of the gravity force. This implies that in this plane, dynamic forces comparable effects on the dynamic motions as the static force. In the sagittal plane, the inertial force is always much smaller, thereby the dynamic forces in this plane can be neglected without significant detriment to the stability of the dynamic motions. This consideration is reflected in the synthesis of dynamic motions presented in the previous sections and it is seen to be reasonable based upon experiments on the dynamic motions.

CHAPTER 6 CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

Both prototypes of bipedal walking robots built during this research project are open-loop systems composed of integrated servomotors and 16-bit micro-controllers. Prototype I constructed in first year of the project was primarily used as an experimental tool to examine the effectiveness of gait synthesis based on mathematical approximation and simplified dynamic models. In second year, Prototype II was developed, on which augmented mechanical design and dynamic models were applied. By virtue of more efficient servomotors and a compact control system formed by Basic Stamp 2sx micro-controllers, Prototype II is enabled to perform as a tetherless walking platform with high capacity batteries as its on-board power supply.

To implement real-time control through Basic Stamp 2sx micro-controllers, the primary approach is to reduce the tedious and lengthy mathematical relations emerging in the conventional calculation of inverse kinematics and dynamics to basic arithmetic computations. The minor errors introduced during this process of simplification are negotiated by system compliance of the robots.

Motions of walking and turning under quasi-static stability and dynamic stability were investigated. In the quasi-static regime, gaits were synthesised mainly from inversed kinematics and geometrical analysis. For the dynamic gaits, motions of the robots were analysed through simplified dynamic models. The synthesis of gaits was thus devolved to motion planning of the specific joints of the robots contained in these models. After minor manual adjustments to the synthesised gaits, both stable quasi-static and dynamic motions are realized in experiments.

As an experimental tool to examine the synthesized dynamic gaits, Prototype I did not possess the ability of on-board variation in walking speed and turning speed. On the other hand, Prototype II was developed to be the platform of an autonomous bipedal robot.

With improvements in both mechanical structure and control system, control upon the speeds of both walking and turning was enabled on this prototype. In addition to mobility on horizontal surfaces, Prototype II was also programmed to adapt its walking motions to ramps of different slope angles.

A microcomputer control module (Figure 6.1) for basic locomotion of dynamic walking and turning was developed for Prototype II. Through this module, movements of the bipedal robot on horizontal surfaces were controlled by additional micro-controllers implementing artificial intelligence or RF remote control.

Figure 6.1: Schematic Diagram of the Control Module for Basic Locomotion on Horizontal Surfaces

In the control module, Basic Stamp 2sxs (1) to (3) perform calculation on inverse kinematics, whilst Basic Stamp 2sx (4) determines the motions and trajectories based on the value of control variable received. The control variable has a length of 1 byte with the following specification on each bit (Bit7 is not used and always set to 0):

Bit6: selection of motion (Bit6=1: walking; Bit6=0: turning)

Bit5: direction of walking/turning (Bit5=1: forward/right turn; Bit5=0: backward/left turn)

Bit0→ Bit4: speed of walking/turning

Two examples are listed below for elaboration:

- To command the robot to walk forward at a desired speed of 0.5 mm per pulse number, the control variable is specified as 01100101.
- To command the robot to turn right at a desired speed of 10 degrees per half turning cycle, the control variable is specified as 00101100.

The maximum commanded walking speed of Prototype II was limited to 75 mm/s, whilst the highest commanded turning speed was about 16.67 deg/s. Mobility of the robot on horizontal flat surfaces was examined under teleoperation by different users. Stable dynamic locomotion such as walking and turning was achieved in experiments.

6.2 Future work

Prototype II is susceptible to uncertainties in terrain. To enhance stability and adaptability of the bipedal dynamic motions, incorporation of a sensory system composed of force/pressure sensors [43,44] and gyroscope/inclinometer is recommended. Dynamic gaits synthesised off-line can thus be corrected and modified on-board based on the information from this feedback system.

Translation of the bipedal robot in any specified direction was attempted. Nevertheless, fully stable dynamic motions were not achieved. Knowledge on the location of ZMP is

required to maintain the specified stability on both feet (Section 5.1). This can be achieved through calculation based on information from force/pressure feedback [43].

Dynamic turning of the robot on a ramp is hardly stable or repeatable without knowing the actual posture of the robot relative to the terrain. Therefore involvement of gyroscope /inclinometer and force/pressure sensors is generally inevitable for this motion to be executed with dynamic stability.

The two dynamic motions discussed above are to be combined with those developed in previous chapters of this thesis. The bipedal robot will be thus rendered the mobility on most flat terrain.

As Prototype II is developed as a highly stable and adaptable system, different behaviour patterns and artificial intelligence are to be implemented with the assistance of more sophisticated sensory system. The robot can therefore manoeuvre in more complex environments.

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APPENDICES

A.1 Force Analysis on Five-bar Linkage

Force diagram for the five-bar linkage shown in Figure 3.3 is generated in the following figure. When the ankle is supposed to be actuated at Point A so that Link EF is parallel to the ground surface, the positions of Joint B and Joint D are considered as fixed relative to the ground. As Joint E and Joint F are actuated, C–D–E–F forms a rigid polygon.

Figure A.1: Force Diagram of the Five-Bar Linkage

In the force diagram:

Point B is chosen as the origin of the coordinate system. When the mass of the linkage is assumed negligible, the gravity force is acting through the center of Link EF. For quasi-static motion, acceleration is neglected. The static analysis proceeds as following:

Along the x-direction:

$$\sum F_X = F_{DX} - F_B \cdot \cos q_1 = 0 \quad (\text{A.1.1})$$

Along the y-direction:

$$\sum F_Y = F_{DY} + F_B \cdot \sin q_1 - mg = 0 \quad (\text{A.1.2})$$

Summing the moments about point B:

$$\sum M_B = F_{DY} \cdot x_D - F_{DX} \cdot y_D - mg \cdot (x_D - l_{DF} \cdot \cos q_2 - \frac{1}{2} l_{EF}) = 0 \quad (\text{A.1.3})$$

Combining Eqs. (A.1.1), (A.1.2) and (A.1.3):

$$F_B = \frac{mg(l_{DF} \cdot \cos q_2 + \frac{1}{2}l_{EF})}{x_D \cdot \sin q_1 + y_D \cdot \cos q_1} \quad (\text{A.1.4})$$

$$F_{DX} = \frac{mg \cos q_1 \cdot (l_{DF} \cdot \cos q_2 + \frac{1}{2}l_{EF})}{x_D \cdot \sin q_1 + y_D \cdot \cos q_1} \quad (\text{A.1.5})$$

$$F_{DY} = mg - \frac{mg \sin q_1 \cdot (l_{DF} \cdot \cos q_2 + \frac{1}{2}l_{EF})}{x_D \cdot \sin q_1 + y_D \cdot \cos q_1} \quad (\text{A.1.6})$$

The direction of the combined force at Joint D is hence determined as:

$$\angle \vec{F}_D = \tan^{-1} \frac{F_{DY}}{F_{DX}} \quad (\text{A.1.7})$$

In the actual design, the following structural condition is applied:

$$\begin{cases} l_{BC} \gg l_{BD} \\ l_{BC} \gg l_{CE} \\ l_{BC} \gg l_{EF} \\ l_{BC} \approx l_{DF} \end{cases} \quad (\text{A.1.8})$$

When q_1 and q_2 are not near 0° or 90° , the following relation can be obtained from Eqs. (A.1.5), (A.1.6), (A.1.7) and (A.1.8):

$$\tan \angle \vec{F}_D \approx \frac{x_D \cdot \sin q_1 + y_D \cdot \cos q_1 - l_{DF} \cdot \cos q_2 \cdot \sin q_1}{l_{DF} \cdot \cos q_1 \cdot \cos q_2} \quad (\text{A.1.9})$$

When Link DF has a dominant axial load, $\vec{F}_D$ will be almost acting along Link DF:

$$\tan \angle \vec{F}_D \approx -\tan q_2 \quad (\text{A.1.10})$$

Combining Eqs. (A.1.9) and (A.1.10):

$$\frac{x_D \cdot \sin q_1 + y_D \cos q_1}{l_{DF}} \approx \sin(q_1 - q_2) \quad (\text{A.1.11})$$

From condition (A.1.8), there is: $q_1 \approx q_2$ (A.1.12)

Based on condition (A.1.8) and Eq. (A.1.12), Eq. (A.1.11) is always true when q_1 and q_2 are not close to 0° or 90° . A dominant axial load on Link DF can be achieved with this arrangement of five-bar linkage. When the axial loads are far lower than the critical loads for buckling, this five-bar linkage provides a rigid structure for the legs of the bipedal robot.

A.2 Torque Analysis on Five-bar Linkage

The five-bar linkage shown in Figure 3.3 functions as a coupled system. Under quasi-static condition, loads on the system are evenly distributed to the motors actuating Joint E and Joint F.

Referring to Figure A.1:

The static torque required at Joint E is:

$$T_E = -(F_B \cdot \cos q_1 \cdot y_E + F_B \cdot \sin q_1 \cdot x_E) \quad (\text{A.2.1})$$

Substituting Eq. (A.1.4) into Eq. (A.2.1):

$$T_E = -\frac{mg(x_E \cdot \sin q_1 + y_E \cdot \cos q_1)(l_{DF} \cdot \cos q_2 + \frac{1}{2}l_{EF})}{x_D \cdot \sin q_1 + y_D \cdot \cos q_1} \quad (\text{A.2.2})$$

The static torque required at Joint F is:

$$T_F = F_{DY} \cdot l_{DF} \cos q_2 + F_{DX} \cdot l_{DF} \sin q_2 \quad (\text{A.2.3})$$

Substituting Eqs. (A.1.5) and (A.1.6) into Eq. (A.2.3):

$$T_F = \frac{mg \cdot l_{DF} \cdot \sin(q_2 - q_1)(l_{DF} \cdot \cos q_2 + \frac{1}{2}l_{EF})}{x_D \cdot \sin q_1 + y_D \cdot \cos q_1} + mgl_{DF} \cos q_2 \quad (\text{A.2.4})$$

From condition (A.1.8):

$$\begin{cases} T_E \approx -\frac{mgl_{DF} \cos q_2}{x_D \cdot \sin q_1 + y_D \cdot \cos q_1} \cdot (x_E \cdot \sin q_1 + y_E \cdot \cos q_1) \\ T_F \approx \frac{mgl_{DF} \cos q_2}{x_D \cdot \sin q_1 + y_D \cdot \cos q_2} \cdot [l_{DF} \cdot \sin(q_2 - q_1) + x_D \cdot \sin q_1 + y_D \cdot \cos q_1] \end{cases} \quad (\text{A.2.5})$$

When $|T_E| \approx |T_F|$:

$$x_E \sin q_1 + y_E \cos q_1 \approx l_{DF} \sin(q_2 - q_1) + x_D \sin q_1 + y_D \cos q_1 \quad (\text{A.2.6})$$

Rearranging Eq. (A.2.6):

$$(x_E - x_D) \cdot \sin q_1 + (y_E - y_D) \cdot \cos q_1 \approx l_{DF} \sin(q_2 - q_1) \quad (\text{A.2.7})$$

From condition (A.1.8)

$$\begin{cases} l_{DE} \approx l_{DF} \\ x_E - x_D \approx -l_{DE} \cdot \cos q_2 \\ y_E - y_D \approx l_{DE} \cdot \sin q_2 \end{cases} \quad (\text{A.2.8})$$

Substituting Eqs. (A.2.8) into Eq. (A.2.7), it is seen that Eq. (A.2.7), therefore Eqs. (A.2.6) and (A.2.5), are always true when q_1 and q_2 are not close to 0° or 90° . With this revelation, the five-bar linkage adopted for the design of leg has the property that loads are evenly distributed between Joint E and Joint F.

Since the attitude of the robot body is controlled by two servos, higher torque is therefore required to rotate the robot body about either Joint E or Joint F. This helps to reduce the flywheel effect discussed in Section 3.1.1.

A.3 Final Dimensions of Four-Bar Linkages

The following diagram shows the structure of the four-bar linkages used for actuation of both prototype bipedal robots:

Figure A.2: Structure of Four-Bar Linkage

Dimensions of the four-bar linkages on Prototype I are determined as below (with reference to Figure 3.4):

Table A.1: Dimensions of Four-Bar Linkages for Actuation of Prototype I

Linkage	L1 (mm)	L2 (mm)	L3 (mm)	L4 (mm)
Linkage A	7.4	24.0	14.7	27.1
Linkage B	7.4	50.0	19.9	60.2
Linkage C	7.4	50.0	27.0	49.9
Linkage D	7.4	50.0	27.0	49.7
Linkage E	7.4	75.0	17.0	65.7
Linkage H	7.4	34.5	17.3	47.1

Dimensions of the four-bar linkages on Prototype II are listed in Table A.2 (with reference to Figure 3.6):

Table A.2: Dimensions of Four-Bar Linkages for Actuation of Prototype II

Linkage	L1 (mm)	L2 (mm)	L3 (mm)	L4 (mm)
<i>Linkage A</i>	7.4	30.0	15.0	31.1
<i>Linkage B</i>	7.4	59.0	11.8	59.0
<i>Linkage C</i>	7.4	51.0	12.0	51.0
<i>Linkage D</i>	7.4	26.0	10.0	26.4
<i>Linkage E</i>	7.4	66.0	17.0	67.7
<i>Linkage H</i>	7.4	40.0	14.8	43.2

B.1 Experiment for Static Testing of Servomotors

Figure B.1: Apparatus for Determination of K_4 for servomotor

The value of K_4 in Figure 4.2 was determined by experiment. The set-up for this experiment is shown Figure B.1, in which the tested servomotor is attached onto a fixture. It was then subjected to an external torque T_d provided by a weight hung at the end of an aluminium arm. Actual angular position of the servo horn therefore deviated from the nominal one specified by the pulse input. This deviation was measured with the protractor. K_4 was thus calculated as below:

$$\text{At steady state: } \Delta q = \frac{T_d}{K_2 \cdot K_4}, \text{ therefore: } K_4 = \frac{T_d}{K_2 \cdot \Delta q} \quad (\text{B.1.1})$$

where, Δq is the deviation in angular position of the servo horn.

Assuming that the mass of the weight is m and the length of the arm is l_a :

$$T_d = mg \cdot \sin(q_n + \Delta q) \quad (\text{B.1.2})$$

where, θ_n is the nominal angular position specified by the pulse input to the servo.

K_4 was therefore determined by substituting Eq. (B.1.2) into (B.1.1):

$$K_4 = \frac{mg \cdot \sin(q_n + \Delta q)}{K_2 \cdot \Delta q} \quad (\text{B.1.3})$$

B.2 Experiment for Determination of Offset Angles at Servomotors

Figure B.2: Experiment Set-up for Determination of Offset Angles at Servomotor

The actual offset angles of the servos are determined by experiment. As shown in Figure B.2, the edge of the support is placed along the boundary of the stable region of the supporting foot. The left ankle joint, Joint A_l , and the left hip joint, Joint H_l (referring to Figure 4.7) are adjusted to the point that the pelvis is along the horizontal direction and the robot is just able to balance itself on the support. At this state, CG of the robot is along the vertical line X-Y, i.e. projection of the CG falls right on the edge of the stable region of the supporting foot.

The joint angle of Joint A_l , q_{LA} in Figure 4.7, is measured with protractor, whilst the joint angle of Joint H_l is equal to q_{LA} when the pelvis is horizontal. From these two angles, the actual servo angles are calculated. By comparing the actual servo angles to the nominal inputs to these servos, the actual offset angles, i.e. df_{LA} and df_{LE} , are obtained. The actual values of df_{RA} and df_{RE} are obtained by a similar approach with the right foot placed on top of the support.

The experiment is conducted for different standing heights and the results show that the actual offsets are practically unchanged with the height.

B.3 Experiment for Determination of the Position of CG

Figure B.3: Experiment Set-up for Determination of the Position of CG
 (A) Measurement of x_l (B) Measurement of y_l

The position of the robot's CG is measured relative to the local coordinate system attached to the main supporting foot that carries the major portion of the robot weight. In Figure B.3, the left foot is the main supporting foot. Its sole is replaced by a much larger metal plate. As shown in the figure, the robot is frozen at a certain pose and the position of its CG is measured relative to system $O_l - x_l y_l z_l$ which is attached the left foot (referring to Figure 3.2).

To determine the x_l coordinate of the CG, the robot is shifted along the x_l -axis on the support until it is just able to balance about the edge of the support. In this situation, CG of the robot is along the vertical line $Z-Z'$ which passes through the edge of the support. The value of x_l is thereby determined as the distance between the edge of the support and the origin O_l . The y_l coordinate of the CG is measured by the same token.

When the right foot is the main supporting foot, similar measurements for the x_r and y_r coordinates are performed. Since the measurements are done mainly with hand and naked eyes, the accuracy of the experimental results would be only up to $\pm 1\text{mm}$.

B.4 Experiment for Determination of Second Moment of Inertia

Figure B.4: Pendulum with Second Moment of Inertia

The examined mechanical structure was modelled as a pendulum with distributed mass (Figure B.2). The frequency of this pendulum can be found as follows:

$$f \approx 2p \cdot \sqrt{\frac{ml^2 + J}{mgl}} \quad (\text{B.4.1})$$

For the bipedal robots of this project, the examined mechanical structure was attached to a fixture and released to swing freely about one of the joints on the structure. By measuring the frequency of pendular motion, the second moment of inertia of the structure was obtained from Eq. (B.4.1):

$$J \approx \frac{mglf^2}{4p^2} - ml^2 \quad (\text{B.4.2})$$

B.5 Snapshots of Prototype II at Discrete Moments of the Motion of Dynamic Walking

0.00 sec

0.08 sec

0.16 sec

0.24 sec

0.32 sec

0.40 sec

0.48 sec

0.56 sec

0.64 sec

0.72 sec

0.80 sec

0.88 sec

0.96 sec

1.04 sec

1.12 sec

1.20 sec

1.28 sec

1.36 sec

1.44 sec

C Geometrical Analysis for Turning Motion

The general form of the motion from State (1) to State (2) in Figure 4.11 is expressed by the following diagram:

Figure C.1: Geometry for Positions of Hip Joints and Ankles

In the above figure:

$P_l - H_l$ is parallel to $P_l' - H_r$ and $P_r - H_r$ is parallel to $P_r' - H_l$. Since the motion of the body frame is only translation, polygon $H_l H_r H_r' H_l'$ is a parallelogram, which implies that $H_l - H_l'$ and $H_r - H_r'$ are parallel to each other and of the same length. Therefore $\Delta H_l' P_l' H_l$, $\Delta H_l' P_l H_l$, $\Delta H_r' P_r H_r$ and $\Delta H_r' P_l' H_r$ are congruent triangles. The following relations can be obtained from this fact:

$$d_{H_l - P_l} = d_{H_r - P_r} \quad d_{P_l - H_l'} = d_{P_r - H_r'}$$

$$b_3 = b_4 = \frac{g_2}{2} = \frac{g_1}{2} = \frac{b_1 + b_2}{2}$$

$$d_{P_r - H_l'} = d_{H_r - P_r} \cdot \tan b_4 = d_{H_r - P_r} \cdot \tan \frac{b_1 + b_2}{2} \quad (C.1)$$

$$d_{P_l - H_l'} = d_{H_l - P_l} \cdot \tan b_3 = d_{H_l - P_l} \cdot \tan \frac{b_1 + b_2}{2} \quad (C.2)$$

$$d_{H_r - H_r'} = \frac{d_{H_r - P_r}}{\cos b_4} = \frac{d_{H_r - P_r}}{\cos \frac{b_1 + b_2}{2}} \quad (C.3)$$

$$d_{H_l-H_l'} = \frac{d_{H_l-P_l}}{\cos b_3} = \frac{d_{H_l-P_l}}{\cos\left(\frac{b_1+b_2}{2}\right)} \quad (C.4)$$

In the actual gait design, to keep the pelvis parallel to the ground surface, the following conditions were applied:

$$q_{RA} = q_{LA} = q \quad (C.5)$$

$$q_{RA0} = q_{LA0} = q_0 \quad \text{where } q_0 \text{ is a constant} \quad (C.6)$$

Referring to Figure 4.13(a), the follow relations are obtained from Eqs. (C.1), (C.3), (C.5) and (C.6) at an intermediate instant between State (1) and State (2):

$$d_{H_r-P_r} = d_{H_r-A_r} \cdot (\sin q_0 - \sin q) \quad (C.7)$$

$$d_{P_r-H_r'} = d_{H_r-A_r} \cdot \tan\left(\frac{b_1+b_2}{2}\right) \cdot (\sin q_0 - \sin q) \quad (C.8)$$

$$d_{H_r-H_r'} = \frac{d_{H_r-A_r} \cdot (\sin q_0 - \sin q)}{\cos\left(\frac{b_1+b_2}{2}\right)} \quad (C.9)$$

As $d_{H_l-A_l} = d_{H_r-A_r}$, similar relations hold for the left leg:

$$d_{H_l-P_l} = d_{H_l-A_l} \cdot (\sin q_0 - \sin q) \quad (C.10)$$

$$d_{P_l-H_l'} = d_{H_l-A_l} \cdot \tan\left(\frac{b_1+b_2}{2}\right) \cdot (\sin q_0 - \sin q) \quad (C.11)$$

$$d_{H_l-H_l'} = \frac{d_{H_l-A_l} \cdot (\sin q_0 - \sin q)}{\cos\left(\frac{b_1+b_2}{2}\right)} \quad (C.12)$$

As shown in Figure 4.13(b), at State (2): $q = -q_0$

Eqs. (C.7) to (C.12) are thereby reduced to the following forms:

$$d_{H_r-P_r} = 2 \cdot d_{H_r-A_r} \cdot \sin q_0 \quad (C.13)$$

$$d_{P_r-H_r'} = 2 \cdot d_{H_r-A_r} \cdot \tan\left(\frac{b_1 + b_2}{2}\right) \cdot \sin q_0 \quad (C.14)$$

$$d_{H_r-H_r'} = \frac{2 \cdot d_{H_r-A_r} \cdot \sin q_0}{\cos\left(\frac{b_1 + b_2}{2}\right)} \quad (C.15)$$

$$d_{H_l-P_l} = 2 \cdot d_{H_l-A_l} \cdot \sin q_0 \quad (C.16)$$

$$d_{P_l-H_l'} = 2 \cdot d_{H_l-A_l} \cdot \tan\left(\frac{b_1 + b_2}{2}\right) \cdot \sin q_0 \quad (C.17)$$

$$d_{H_l-H_l'} = \frac{2 \cdot d_{H_l-A_l} \cdot \sin q_0}{\cos\left(\frac{b_1 + b_2}{2}\right)}$$

When $|q|$ was small: $\sin q \approx \frac{p \cdot q}{180}$ (C.17)

In the actual gait: $|q| \leq 20^\circ$

Therefore, Eqs. (C.8) and (C.11) can be approximated by the following linear relations with minor errors negotiated through system compliance:

$$d_{P_r-H_r'} = \frac{d_{H_r-A_r} \cdot \tan\left(\frac{b_1 + b_2}{2}\right) \cdot \sin q_0 \cdot (q_0 - q)}{q_0} \quad (C.18)$$

$$d_{P_l-H_l'} = \frac{d_{H_l-A_l} \cdot \tan\left(\frac{b_1 + b_2}{2}\right) \cdot \sin q_0 \cdot (q_0 - q)}{q_0} \quad (C.19)$$

For $b_1 + b_2 \leq 30^\circ$: (C.20)

$$\tan\left(\frac{b_1 + b_2}{2}\right) \approx \frac{b_1 + b_2}{2} \cdot \frac{p}{180} \quad (C.21)$$

Therefore Eqs. (C.18) and (C.19) are further reduced to the following forms:

$$d_{P_r-H_r} = \frac{p \cdot d_{H_r-A_r} \cdot (b_1 + b_2) \cdot \sin q_0 \cdot (q_0 - q)}{360 \cdot q_0} \quad (C.22)$$

$$d_{P_l-H_l} = \frac{p \cdot d_{H_l-A_l} \cdot (b_1 + b_2) \cdot \sin q_0 \cdot (q_0 - q)}{360 \cdot q_0} \quad (C.23)$$

where both b_1 and b_2 are in degrees.

When $b_1 = b_2$, which is adopted in the actual gait, Figure C.1 is reduced to Figure 4.12. For this arrangement, the robot body travels along the axis of the pelvis, which is the y_b axis of the body coordinate system:

$$b_1 = b_2 = b_3 = b_4 = b \quad (C.24)$$

Substituting Eq. (C.24) into condition (C.20) and Eqs. (C.22) and (C.23):

$$b_1 = b_2 = b \leq 15^\circ \quad (C.25)$$

$$d_{P_r-H_r} = \frac{p \cdot d_{H_r-A_r} \cdot b \cdot \sin q_0 \cdot (q_0 - q)}{180 \cdot q_0} \quad (C.26)$$

$$d_{P_l-H_l} = \frac{p \cdot d_{H_l-A_l} \cdot b \cdot \sin q_0 \cdot (q_0 - q)}{180 \cdot q_0} \quad (C.27)$$

D.1 Dynamic Analysis on the Linear Inverted Pendulum Model

The 3-D linear inverted pendulum model introduced in Section 5.2.2 is reproduced in the following figure:

Figure D.1: Linear Inverted Pendulum Model with Joint A_l as the Pivot

The model shown above is established for the single-support phase in which the left foot is the supporting foot. The dynamic analysis on this model is separated into two orthogonal planes: z - O - x plane and y - O - z plane. The projection of 3-D inverted pendulum model onto each individual plane defines a planar model on that plane. The planar linear inverted pendulum on y - O - z plane is shown in Figure D.2 as an exemplification. The following assumptions are made upon this planar model:

- J_x is constant during the motion irrespective of the robot postures. Therefore, $\frac{dJ_x}{dl_{yz}} \approx 0$. This assumption is justified through experiments by measurements of J_x at different robot postures.
- The linear actuator is located close to the pivot and has negligible mass and dimensions.

Figure D.2: Projection of the Linear Inverted Pendulum Model on the Lateral Plane

Based on the above assumptions, the following expressions are obtained by applying the equilibrium conditions to the inverted pendulum in Figure D.2:

$$F_{3y} = M \cdot a_y \quad (\text{D.1.1})$$

$$M \cdot g - F_{3z} = M \cdot a_z \quad (\text{D.1.2})$$

$$M \cdot g \cdot l_{yz} \cdot \sin a - T_{lx} = \frac{d}{dt} \left[\frac{1}{2} \cdot M \cdot (l_{yz}^2 + l_x^2) \cdot \dot{a}_{lx}^2 \right] \quad (\text{D.1.3})$$

Some kinematical formulation arrives at the results below:

$$\cancel{r}_{yz} \cdot \sin a_{lx} + 2 \cdot \cancel{r}_{yz} \cdot \cancel{a}_{lx} \cdot \cos a_{lx} + l_{yz} \cdot \cancel{a}_{lx} \cdot \cos a_{lx} - l_{yz} \cdot \cancel{a}_{lx}^2 \cdot \sin a_{lx} = a_y \quad (\text{D.1.4})$$

$$l_{v_z} \cdot \cos a_{lx} - 2 \cdot l_{v_z} \cdot a_{lx} \cdot \sin a_{lx} - l_{v_z} \cdot a_{lx}^2 \cdot \sin a_{lx} - l_{v_z} \cdot a_{lx}^2 \cdot \cos a_{lx} = a_z \quad (\text{D.1.5})$$

When a torque of T_{lx} is applied at the pivot, the position of ZMP is specified as below:

$$w = -T_{lx}/F_{3z} \quad (\text{D.1.6})$$

where w is the location of ZMP with respect to the pivot along the y -axis (Figure 5.1).

Combining Eqs. (D.1.1) to (D.1.6), the following differential equation is established for description of the pendular motion:

$$\begin{aligned} & (l_{yz}^2 + l_x^2 - w \cdot l_{yz} \cdot \sin a_{lx}) \cdot \ddot{a}_{lx} - w \cdot l_{yz} \cdot \cos a_{lx} \cdot \dot{a}_{lx}^2 + 2 \cdot (l_{yz} - w \cdot \sin a_{lx}) \cdot \dot{l}_{yz} \cdot \dot{a}_{lx} \\ & - g \cdot l_{yz} \cdot \sin a_{lx} + g \cdot w + w \cdot \dot{l}_{yz} \cdot \cos a_{lx} = 0 \end{aligned} \quad (D.1.7)$$

The above equation is reproduced in Chapter 5 as Eq. (5.3.3). Similar result is achieved for the planar model in z - O - x plane:

$$\begin{aligned} & (l_{zx}^2 + l_y^2 - d \cdot l_{zx} \cdot \sin a_{ly}) \cdot \ddot{a}_{ly} - d \cdot l_{zx} \cdot \cos a_{ly} \cdot \dot{a}_{ly}^2 + 2 \cdot (l_{zx} - d \cdot \sin a_{ly}) \cdot \dot{l}_{zx} \cdot \dot{a}_{ly} \\ & - g \cdot l_{zx} \cdot \sin a_{ly} + g \cdot d + d \cdot \dot{l}_{zx} \cdot \cos a_{ly} = 0 \end{aligned} \quad (D.1.8)$$

where, a_{ly} is the projection of a_l on z - O - x plane

T_{ly} is the component of T_l in y -direction

l_{zx} is the projection of l on z - O - x plane

d is the location of ZMP with respect to the pivot along the x -axis (Figure 5.1)

In Section 5.3.1, $l_{yz} = \text{constant}$ and $w = \text{constant}$. Eq. (D.1.7) is thus simplified to be:

$$(l_{yz}^2 + l_x^2 - w \cdot l_{yz} \cdot \sin a_{lx}) \cdot \ddot{a}_{lx} - w \cdot l_{yz} \cdot \cos a_{lx} \cdot \dot{a}_{lx}^2 - g \cdot l_{yz} \cdot \sin a_{lx} + g \cdot w = 0 \quad (D.1.9)$$

The following initial conditions are specified for the above differential equation:

$$a_{lx}(t=0) = a_{lx(0)} \quad \dot{a}_{lx}(t=0) = \dot{a}_{lx(0)} \quad (D.1.10)$$

Therefore establishment of the solution for Eqs. (D.1.9) and (D.1.10) proceeds as below:

$$\begin{aligned} & (l_{yz}^2 + l_x^2 - w \cdot l_{yz} \cdot \sin a_{lx}) \cdot \frac{d\ddot{a}_{lx}}{da} \cdot \dot{a}_{lx} - w \cdot l_{yz} \cdot \cos a_{lx} \cdot \dot{a}_{lx}^2 - g \cdot l_{yz} \cdot \sin a_{lx} + g \cdot w = 0 \\ & \frac{l_{yz}^2 + l_x^2 - w \cdot l_{yz} \cdot \sin a_{lx}}{2} \cdot \frac{d\dot{a}_{lx}^2}{da} - w \cdot l_{yz} \cdot \cos a_{lx} \cdot \dot{a}_{lx}^2 - g \cdot l_{yz} \cdot \sin a_{lx} + g \cdot w = 0 \\ & \frac{d\dot{a}_{lx}^2}{da} - \frac{2 \cdot w \cdot l_{yz} \cdot \cos a_{lx}}{l_{yz}^2 + l_x^2 - w \cdot l_{yz} \cdot \sin a_{lx}} \cdot \dot{a}_{lx}^2 = \frac{2 \cdot g \cdot (l_{yz} \cdot \sin a_{lx} - w)}{l_{yz}^2 + l_x^2 - w \cdot l_{yz} \cdot \sin a_{lx}} \end{aligned}$$

$$\begin{aligned} \ddot{a}_{lx} = & g \cdot \frac{4 \cdot l_{yz} \cdot (l_x^2 + l_{yz}^2 + w) \cdot (\cos a_{lx(0)} - \cos a_{lx}) + w \cdot (4 \cdot l_x^2 + 6 \cdot l_{yz}^2) \cdot (a_{lx(0)} - a_{lx})}{2 \cdot (l_x^2 + l_{yz}^2 - w \cdot l_{yz} \cdot \sin a_{lx})^2} \\ & + g \cdot \frac{w \cdot l_{yz}^2 \cdot (\sin 2a_{lx} - \sin 2a_{lx(0)})}{2 \cdot (l_x^2 + l_{yz}^2 - w \cdot l_{yz} \cdot \sin a_{lx})^2} + \ddot{a}_{lx(0)}^2 \end{aligned} \quad (D.1.11)$$

For $w=0$, the torque applied in x -axis of the left ankle joint is zero and ZMP is located at the centre of the left foot. In such a situation, Eq. (D.1.11) assumes the following form:

$$\ddot{a}_{lx} = -\sqrt{\frac{2 \cdot g \cdot l_{yz} \cdot \cos a_{lx(0)} + (l_x^2 + l_{yz}^2) \cdot \ddot{a}_{lx(0)}^2}{l_x^2 + l_{yz}^2}} - 2 \cdot g \cdot l_{yz} \cdot \cos a_{lx} \quad (D.1.12)$$

This expression is valid in the first half of the single-support phase until $\ddot{a}_{lx}$ reaches zero.

Eq. (D.1.12) is transformed to the following form:

$$\ddot{a}_{lx} = -\sqrt{\frac{4 \cdot g \cdot l_{yz} \cdot \cos^2 \frac{a_{lx(0)}}{2} + (l_x^2 + l_{yz}^2) \cdot \ddot{a}_{lx(0)}^2}{l_x^2 + l_{yz}^2}} - 4 \cdot g \cdot l_{yz} \cdot \cos^2 \frac{a_{lx}}{2} \quad (D.1.13)$$

The following substitutions are applied to the constants appearing in the above equation:

$$a^2 = \frac{4 \cdot g \cdot l_{yz}}{l_x^2 + l_{yz}^2} \quad b^2 = \frac{4 \cdot g \cdot l_{yz} \cdot \cos^2 \frac{a_{lx(0)}}{2} + (l_x^2 + l_{yz}^2) \cdot \ddot{a}_{lx(0)}^2}{4 \cdot g \cdot l_{yz}}$$

A refined expression of Eq. (D.1.13) is thus achieved as below:

$$\ddot{a}_{lx} = -a \cdot \sqrt{b^2 - \cos^2 \frac{a_{lx}}{2}} \quad (D.1.14)$$

The solution of Eq. (D.1.14) is therefore approached as below:

$$\left(b^2 - \cos^2 \frac{a_{lx}}{2} \right)^{-1/2} \cdot da_{lx} = -a \cdot dt$$

$$\left[\left(1 - \cos^2 \frac{a_{lx}}{2} \right) \left(b^2 - \cos^2 \frac{a_{lx}}{2} \right) \right]^{-1/2} \cdot \left(\sin \frac{a_{lx}}{2} \right) \cdot d \left(\frac{a_{lx}}{2} \right) = -\frac{a}{2} \cdot dt$$

$$\left[\left(1 - \cos^2 \frac{a_{lx}}{2} \right) \left(b^2 - \cos^2 \frac{a_{lx}}{2} \right) \right]^{-1/2} \cdot d \left(\cos \frac{a_{lx}}{2} \right) = \frac{a}{2} \cdot dt \quad (D.1.15)$$

By integration, the solution is obtained from Eq. (D.1.15) with the initial condition of Eq. (D.1.10):

$$\text{cd}^{-1} \left[\frac{\cos \frac{a_{lx(0)}}{2}}{b}, b \right] - \text{cd}^{-1} \left[\frac{\cos \frac{a_{lx}}{2}}{b}, b \right] = \frac{a \cdot t}{2} \quad (D.1.16)$$

where cd is defined in terms of elliptic functions as: $\text{cd } u = \frac{\text{cn } u}{\text{dn } u}$ [45]

The time at which $\mathcal{A}_{lx} = 0$ can be obtained from Eqs. (D.1.14) and (D.1.16). This time is equal to half of the motion period T_s (Section 5.3.1). The solution of Eq. (D.1.11) for the other half of the single-support phase is the mirror image of Eq. (D.1.16) about the line of $t = T_s/2$. The complete solution for Eqs. (D.1.9) and (D.1.10) is therefore symmetrical about $t = T_s/2$.

When $w \neq 0$, it is usually not possible to solve Eq. (D.1.11) explicitly. Approximate solution is pursued as below:

The right-hand side of Eq. (D.1.11) is expanded into Taylor series about $a'_{lx} = 16^\circ$ up to the second order:

$$\mathcal{A}_{lx}^2 \approx a_0 + a_1 \cdot (a_{lx} - a'_{lx}) + a_2 \cdot (a_{lx} - a'_{lx})^2 \quad (D.1.17)$$

where, a_0, a_1, a_2 are constants determined from $a_{lx(0)}$ and $\mathcal{A}_{lx(0)}$.

Before $\mathcal{A}_{lx} = 0$, Eq. (D.1.17) is essentially of the following form:

$$\frac{da_{lx}}{\sqrt{a_0 + a_1 \cdot (a_{lx} - a'_{lx}) + a_2 \cdot (a_{lx} - a'_{lx})^2}} \approx -dt \quad (D.1.18)$$

Eq. (D.1.18) is whence solved explicitly by direct integration:

$$\frac{1}{\sqrt{a_2}} \cdot \ln \frac{C}{2 \cdot a_2 \cdot (a_{lx} - a'_{lx}) + a_1 + 2 \cdot \sqrt{a_2} \cdot \sqrt{a_0 + a_1 \cdot (a_{lx} - a'_{lx}) + a_2 \cdot (a_{lx} - a'_{lx})^2}} \approx t \quad (D.1.19)$$

$$\text{where } C = 2 \cdot a_2 \cdot (a_{lx(0)} - a'_{lx}) + a_1 + 2 \cdot \sqrt{a_2} \cdot \sqrt{a_0 + a_1 \cdot (a_{lx(0)} - a'_{lx}) + a_2 \cdot (a_{lx(0)} - a'_{lx})^2}$$

The other half of solution for Eq. (D.1.17) is again obtained by symmetry about the axis of $t = T_s/2$. By comparison to the numerical results obtained with software, this approach is justified to be an accurate approximation.

Based on the specified values for T_s and the minimum value of a_{lx} (Section 5.3.1), $a_{lx(0)}$ and $a'_{lx(0)}$ can be determined from Eq. (D.1.11) and Eqs. (D.1.16) or (D.1.19).

The motion of the 3-D linear inverted pendulum model in z - O - x plane is described by Eq. (D.1.8). The motion in this plane is coupled to that in y - O - z plane through the variable l_{zx} . For the gait defined in Section 5.4.1, the following kinematical relation is obtained:

$$l_{zx} = \sqrt{l_{yz}^2 \cdot \cos^2 a_{lx} + v^{*2} \cdot (t - 0.48)^2} \quad (D.1.20)$$

l_{zx} and $\dot{l}_{zx}$ can therefore determined from Eq. (D.1.20) with the solution of a_{lx} . By substituting l_{zx} , $\dot{l}_{zx}$ and $\ddot{l}_{zx}$ into Eq. (D.1.8), the deviation of ZMP from the centre of left foot in x -axis is estimated. Using a safety factor of 1.25, the length of stable region along x -axis is 72mm. Substituting this value into Eq. (D.1.8), the maximum walking speed of the robot was predicted to be about 0.11mm/s.

For the single-support phase with the right foot as the supporting foot, similar dynamic model and analysis apply.

D.2 Dynamic Analysis on the Four-bar Linkage Model

The free body diagram for the four-bar linkage model discussed in Chapter 5 is reproduced in the following figure:

Figure D.3: Free Body Diagram for Each Link of the Four-bar Linkage Model

From the assumptions placed upon the four-bar linkage model, the following kinematical relations are observed:

$$q_2 = q_3 = q_l \quad \ddot{q}_{l1} = \ddot{q}_{l1} = 0$$

$$a_{2y} = a_{3y} = a_{Ly} = -l_0 \cdot \ddot{q}_{l1} \cos q_l + l_0 \cdot \dot{q}_{l1}^2 \sin q_l \quad (D.2.1)$$

$$a_{2z} = a_{3z} = a_{Lz} = l_0 \cdot \ddot{q}_{l1} \sin q_l + l_0 \cdot \dot{q}_{l1}^2 \cos q_l \quad (D.2.2)$$

$$a_{1y} = -l_2 \cdot \ddot{q}_{l1} \cos q_l + l_2 \cdot \dot{q}_{l1}^2 \sin q_l \quad (D.2.3)$$

$$a_{1z} = l_2 \cdot \ddot{q}_l \sin q_l + l_2 \cdot \dot{q}_l^2 \cos q_l \quad (\text{D.2.4})$$

The dynamic equations for each link are listed below following the notations in Figure 5.2, Figure D.3 and Section 5.2.1:

For Link $H_l - H_r$:

$$F_{1y} - F_{2y} = m_{H_l-H_r} \cdot a_{1y} \quad (\text{D.2.5a})$$

$$m_{H_l-H_r} \cdot g - F_{1z} - F_{2z} = m_{H_l-H_r} \cdot a_{1z} \quad (\text{D.2.5b})$$

$$2 \cdot T_{H_l} \cdot g + F_{1z} \cdot l_1 = 2 \cdot T_{H_r} \cdot g + F_{2z} \cdot l_1 \quad (\text{D.2.5c})$$

For Link $H_l - A_l$:

$$F_{3y} - F_{1y} = m_2 \cdot a_{2y} \quad (\text{D.2.6a})$$

$$m_2 \cdot g + F_{1z} - F_{3z} = m_2 \cdot a_{2z} \quad (\text{D.2.6b})$$

$$T_{H_l} - T_{A_l} + F_{1z} \cdot l_2 \cdot \sin q_l + F_{1y} \cdot l_2 \cdot \cos q_l + m_2 \cdot g \cdot l_0 \cdot \sin q_l = (m_2 \cdot l_0^2 + J_2) \cdot \ddot{q}_l \quad (\text{D.2.6c})$$

For Link $H_r - A_r$:

$$F_{2y} + F_{4y} = m_2 \cdot a_{3y} \quad (\text{D.2.7a})$$

$$m_2 \cdot g + F_{2z} - F_{4z} = m_2 \cdot a_{3z} \quad (\text{D.2.7b})$$

$$T_{A_r} - T_{H_r} + F_{2z} \cdot l_2 \cdot \sin q_l - F_{2y} \cdot l_2 \cdot \cos q_l + m_2 \cdot g \cdot l_0 \cdot \sin q_l = (m_2 \cdot l_0^2 + J_2) \cdot \ddot{q}_l \quad (\text{D.2.7c})$$

Some manipulation upon the above equations leads to the following result:

$$J_\Sigma \cdot \ddot{q}_l - I_\Sigma \cdot g \cdot \sin q_l + T_\Sigma = 0 \quad (\text{D.2.8})$$

where:

$$T_\Sigma = T_{A_l} + T_{H_r} - T_{H_l} - T_{A_r}$$

$$J_\Sigma = 2 \cdot J_2 + 2 \cdot m_2 \cdot l_0^2 + m_{H_l-H_r} \cdot l_2^2$$

$$I_\Sigma = m_{H_l-H_r} \cdot l_2 + 2 \cdot m_2 \cdot l_0$$

The series of equations from Eq. (D.2.5a) to Eq. (D.2.7c) consists of essentially 13 unknowns with nine equations. Therefore Eq. (D.2.8) is solved uniquely with the initial conditions after the actuating torques T_{A_l} , T_{H_r} , T_{H_l} and T_{A_r} are specified.

The following results are derived from Eqs. (D.2.5b), (D.2.5c), (D.2.6b), (D.2.7b) and (D.2.8):

$$F_{3z} = I_{\Sigma} \cdot \left(\sin q_l \cdot \frac{T_{\Sigma} + I_{\Sigma} \cdot g \cdot \sin q_l}{2J_{\Sigma}} + \cos q_l \cdot \ddot{q}_l \right) - \left(\frac{m_{H_l-H_r}}{2} + m_2 \right) \cdot g + \frac{2(T_{H_r} - T_{H_l})}{l_1} \quad (\text{D.2.9a})$$

$$F_{4z} = I_{\Sigma} \cdot \left(\sin q_l \cdot \frac{T_{\Sigma} + I_{\Sigma} \cdot g \cdot \sin q_l}{2J_{\Sigma}} + \cos q_l \cdot \ddot{q}_l \right) - \left(\frac{m_{H_l-H_r}}{2} + m_2 \right) \cdot g - \frac{2(T_{H_r} - T_{H_l})}{l_1} \quad (\text{D.2.9b})$$

The stability conditions are obtained by applying Eqs. (D.2.9a) and (D.2.9b) to Eq. (5.1.1) as below:

$$\left| I_{\Sigma} \cdot \left(\sin q_l \cdot \frac{T_{\Sigma} + I_{\Sigma} \cdot g \cdot \sin q_l}{2J_{\Sigma}} + \cos q_l \cdot \ddot{q}_l \right) - \left(\frac{m_{H_l-H_r}}{2} + m_2 \right) \cdot g + \frac{2(T_{H_r} - T_{H_l})}{l_1} \right| \geq \left| \frac{2T_{A_r}}{w_0} \right| \quad (\text{D.2.10a})$$

$$\left| I_{\Sigma} \cdot \left(\sin q_l \cdot \frac{T_{\Sigma} + I_{\Sigma} \cdot g \cdot \sin q_l}{2J_{\Sigma}} + \cos q_l \cdot \ddot{q}_l \right) - \left(\frac{m_{H_l-H_r}}{2} + m_2 \right) \cdot g - \frac{2(T_{H_r} - T_{H_l})}{l_1} \right| \geq \left| \frac{2T_{A_l}}{w_0} \right| \quad (\text{D.2.10b})$$