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**SOIL WATER CHARACTERISTIC CURVE AND SHEAR
STRENGTH OF SOIL WITH BIMODAL
CHARACTERISTIC**

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**SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING
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SINGAPORE

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ABSTRACT

Many residual and colluvial soils show bimodal grain-size distribution (GSD). Some soils with bimodal characteristic of GSD result in bimodal characteristic of soil-water characteristic curve (SWCC), whereas others result in unimodal characteristic of SWCC. Modelling GSD and SWCC with a continuous function is advantageous. The GSD model can be used to classify the soil and predict the SWCC. On the other hand, the SWCC model can be used to obtain soil properties, such as: the air-entry value, residual water content and water-entry value. Many research works have been carried out on development of GSD and SWCC model. However, most of them were limited to soils with unimodal characteristics. The objective of this research is to study the mechanical behaviour of unsaturated soil, particularly the SWCC and shear strength of the unsaturated soil with bimodal characteristics. Several mathematical equations were proposed in this study, such as: the equations for best fitting the GSD, SWCC and shear strength of the unsaturated soil with bimodal characteristics, the equations for estimating the SWCC of soil with bimodal GSD and the equation for estimating the bimodal shear strength of the unsaturated soil. SWCC tests and unsaturated consolidated drained triaxial tests were carried out on nine different sand-kaolin mixtures which were compacted on dry density associated with wet of optimum (95% maximum dry density), optimum water content and dry of optimum (95% maximum dry density).

The SWCC test results showed that the specimen with bimodal GSD does not always produce bimodal SWCC, but they can also produce unimodal SWCC, depending on soil density, void ratio, water content and ratio of coarse to fine particles within soil specimen. The specimen with bimodal GSD and consists of high percentage of fine particles is likely to have unimodal SWCC. This happens since the percentages of small pores are much higher than those of big pores within this soil specimen. In addition, the saturated water content of this soil specimen will be higher than the saturated water content of the soil specimen with bimodal GSD, but consists of low percentage of fine particles. The specimen with bimodal GSD and high dry density will contribute in the low air-entry value (AEV) and the steep slope of SWCC.

The CD triaxial test results indicated that specimen with bimodal SWCC had a higher shear strength, a lower axial strain at failure and a higher stiffness than that with unimodal SWCC. The results from CD tests also showed that ϕ^b angle of specimen with bimodal SWCC at suction between AEV1 and AEV2 was less than ϕ^b angle at suction less than AEV1. However, the relationships between suction and shear strength at suction between AEV1 and AEV2 were still linear. This might be attributed to the reduction of area of water within macropores of specimen with bimodal characteristic at suction between AEV1 and AEV2. However, micropores within soil specimen with bimodal characteristic were still in saturated condition. At suction beyond AEV2, ϕ^b angle of specimen with bimodal SWCC started to decrease non-linearly. This happened since the area of water menisci within micropores started to decrease at suction beyond AEV2 due to the desaturation of micropores of soil specimen with bimodal SWCC.

The proposed GSD, SWCC and shear strength equations for soil with bimodal characteristics were evaluated with data from laboratory tests carried out in this study and published data from literatures. The proposed best fitting equations were shown to best fit GSD, SWCC and unsaturated shear strength of soil with bimodal characteristics successfully. In addition, the parameters in the proposed equations represent the variables of GSD, SWCC and unsaturated shear strength of soil. The proposed estimating equations were shown to estimate the SWCC of soil with bimodal GSD and to estimate the bimodal shear strength of unsaturated soil successfully.

TABLE OF CONTENTS

ACKNOWLEDGEMENT	iii
ABSTRACT	v
TABLE OF CONTENTS	vii
LIST OF TABLES	xi
LIST OF FIGURES	xvi
LIST OF SYMBOLS	xxx
Chapter 1 Introduction	1
1.1 Research Background	1
1.2 Objectives	3
1.3 Scope of Works	3
1.4 Methodology	4
1.5 Outline of Thesis	5
Chapter 2 Literature Review	7
2.1 Introduction	7
2.2 Unsaturated Soil Mechanics	7
2.3 Grain-size Distribution	13
2.3.1 Unimodal Grain-size Distribution Function	14
2.3.2 Bimodal Grain-size Distribution Function	15
2.4 Soil-water Characteristic Curve	16
2.4.1 Soil with Bimodal Soil-Water Characteristic Curve	19
2.4.2 Factors affecting Soil-Water Characteristic Curve	20
2.4.3 Measurement of Soil-Water Characteristic Curve	28
2.4.4 Unimodal Soil-Water Characteristic Curve Function	28
2.4.5 Bimodal Soil-Water Characteristic Curve Function	31
2.4.6 Estimation of Soil-Water Characteristic Curve	32
2.5 Shear Strength of Unsaturated Soil	35

2.5.1 Previous studies on shear strength characteristics of soils with unimodal soil-water characteristic curve	36
2.5.2 Previous studies on shear strength characteristic of soils with bimodal characteristic	40
2.5.3 Shear Strength Equations for Unsaturated Soils	42
2.5.4 Estimation of Shear Strength for Unsaturated Soils	45
2.6 Review of Laboratory Procedures for Measurement of Unsaturated Soil Properties.	49
2.6.1 Suction Controlled Technique	49
2.6.2 Shearing Stage in Triaxial Tests	51
2.7 Summary	51
 Chapter 3 Theoretical Model	 53
3.1 Introduction.....	53
3.2 Proposed Mathematical Model of Soil-Water Characteristic Curve	53
3.2.1 Proposed Equation of Soil-Water Characteristic Curve based on Lognormal Distribution Function	56
3.2.2 Proposed Equation of Soil-Water Characteristic Curve based on Exponential Function	61
3.3 Proposed Equation for Best Fitting Grain-size Distribution.....	63
3.3.1 Proposed Mathematical Model of Grain-Size Distribution Curve based on Lognormal Distribution Function	64
3.4 Proposed Equations for Shear Strength of Unsaturated Soil with Unimodal and Bimodal Characteristic.....	67
3.5 Summary of The Proposed Equations developed in This Study.....	75
 Chapter 4 Research Programme.....	 76
4.1 Introduction.....	76
4.2 Soil Design for Laboratory Experiments	77
4.3 Experimental Works	79
4.3.1 Index Property Tests	79
4.3.2 Standard Compaction Tests	79

4.3.3 Static Compaction Tests	80
4.3.4 Saturated Shear Strength Tests	83
4.3.5 Soil-Water Characteristic Curve Tests.....	86
0	
Unsaturated Shear Strength Tests	92
4.4 Selection of Published Equations for Evaluation of The Proposed Equations	97
4.5 Selection of Published Data for Evaluation of The Proposed Models.....	101
4.6 Criteria for Evaluation of the Proposed Models	101
 Chapter 5 Presentation and Results.....	 106
5.1 Introduction.....	106
5.2 Compaction Tests.....	106
5.2.1 Index Properties	107
5.3 Soil-Water Characteristic Curve	111
5.4 Shear Strength.....	132
5.5 Evaluation of The Proposed and Published Equation for Best Fitting Grain-Size Distribution using Experimental Data from Laboratory Tests carried out in This Study and Published Data from Literatures	151
5.6 Evaluation of the Proposed and Published Equations for Fitting Soil-Water Characteristic Curve using Experimental Data from Laboratory Tests carried out in This Study and using Published Data from Literatures	163
5.7 Evaluation of The Proposed Equation for Best Fitting Shear Strength of Unsaturated Soil using Experimental Data from Laboratory Tests carried out in This Study and using Published Data from Literatures	179
 Chapter 6 Discussion of Results	 184
6.1 Introduction.....	184
6.2 Mechanical Behavior of Soil with Bimodal Characteristic	184
6.3 Estimation of Soil-water Characteristic Curves and Shear Strength of Unsaturated Soil	197

6.4 Procedures for Esimating Shear Strength of Unsaturated Soil with Bimodal Characteristic	213
6.5 Evaluation of The Proposed Equations for Best Fitting Grain-Size Distribution, Soil-water Characteristic Curve and Unsaturated Shear Strength using Experimental Data from Laboratory Tests carried out in This Study and Published Data from Literatures.	217
6.6 Evaluation of The Proposed Equations for Estimating Soil-water Characteristic Curve using Experimental Data from Laboratory Tests carried out in This Study and Published Data from Literatures	222
6.7 Evaluation of The Proposed Equations for Estimating Bimodal Shear Strength using Experimental Data from Laboratory Tests carried out in This Study and Published Data from Literatures	234
 Chapter 7 Practical Significance of Research Findings.....	 239
7.1 Introduction.....	239
7.2 Case Study	240
 Chapter 8 Conclusions and Recommendations.....	 251
8.1 Conclusions.....	251
8.2 Publications from PhD Research works	254
8.3 Recommendations.....	255
 REFERENCES	 257
 APPENDICES	 275
Appendix A5.4.....	275
Appendix A7.1	295

LIST OF TABLES

Table 3-1 Summary of the published data from literatures used in the parameteric study	73
Table 3-2. SSENorm of the unsaturated shear strength of published data using Equation 3-36 with different parameters	74
Table 4-1. Soils used in the Laboratory Experiments	78
Table 4-2. Standards for Laboratory Tests	79
Table 4-3. Testing programme for sand-kaolin mixtures	91
Table 4-4. Published GSD equations for soil with bimodal characteristics	97
Table 4-5. Published SWCC equations for soil with unimodal and bimodal characteristics	99
Table 4-6. Published shear strength equations for unsaturated soil with unimodal and bimodal characteristics.....	100
Table 4-7. Soil data sets from published literatures used for evaluation of the proposed GSD and SWCC equation.....	102
Table 4-8. Soil data sets from published literatures used for evaluation of the proposed shear strength equation	103
Table 5-1. Maximum dry density and optimum water content of the sand-kaolin mixtures used in the laboratory experiments conducted in this study	107
Table 5-2. Summary of the GSD variables of the selected soils based on the bimodal GSD equation (Equation 3-21) proposed in this study	108
Table 5-3. Summary of index properties of soils M1 and M2.....	109
Table 5-4. Summary of index properties of soils M3 and M4.....	109
Table 5-5. Summary of index properties of soils M5 and M6.....	110

Table 5-6. Summary of index properties of soils M7 and M8.....	110
Table 5-7. Summary of index properties of soils M9	111
Table 5-8. SWCC fitting parameters of soils M1a, M1b, M1c, M3c, M4c, M5b, M5c and M6b with unimodal SWCC	130
Table 5-9. SWCC fitting parameters of soils M6c, M7b, M7c, M8b, M8c, M9a, M9b and M9c with unimodal SWCC.....	130
Table 5-10. Fitting parameters of the proposed SWCC equations for best fitting soil mixtures M2a, M2b, M2c, M3a and M3b with bimodal characteristic	131
Table 5-11. Fitting parameters of the proposed SWCC equations for best fitting soil mixtures M4a, M4b, M5a, M6a, M7a and M8a with bimodal characteristic	131
Table 5-12. Shear strength properties of specimens M2a, M2b, M3a, M3b, M4a and M4b with bimodal SWCC	143
Table 5-13. Shear strength properties of specimens M5a, M6a, M7b and M8a with bimodal SWCC	143
Table 5-14. Shear strength properties of specimens M2c, M3c, M4c, M5b M6b, M7b, M8b and M9a with unimodal SWCC	144
Table 5-15. Summary of parameters for best fitting bimodal GSD of soil data from experimental test carried out in this study using the proposed equation (Equation 3-21) developed in this study.....	154
Table 5-16. Summary of parameters for best fitting bimodal GSD of soil data from experimental test carried out in this study using Fredlund (2000) equation for best fitting bimodal GSD	154
Table 5-17. Summary of parameters for best fitting bimodal GSD of published data from literatures using the proposed equation (Equation 3-21) for best fitting bimodal GSD developed in this study.....	157
Table 5-18. Summary of parameters for best fitting bimodal GSD of published data from literatures based on the proposed equation (Equation 3-21) for best fitting bimodal GSD developed in this study.....	158

Table 5-19. Summary of parameters for best fitting bimodal GSD published data from literatures based on the Fredlund (2000) equation for best fitting bimodal GSD.....	158
Table 5-20. Summary of parameters for best fitting bimodal GSD of published data from literatures based on the Fredlund et al. (2000) equation for best fitting bimodal GSD..	159
Table 5-21. Summary of parameters for best fitting unimodal GSD from experimental test carried out in this study based on the proposed equation (Equation 3-24) for best fitting unimodal GSD developed in this study.....	160
Table 5-22. Summary of parameters for best fitting unimodal GSD of published data from literatures based on the proposed equation (Equation 3-24) for best fitting unimodal GSD developed in this study.....	161
Table 5-23. Best fitting parameters of proposed equation 3-15 for soil mixtures M2a, M2b, M2c, M3a and M3b with bimodal SWCC	166
Table 5-24. Best fitting parameters of proposed equation 3-15 for soil mixtures M4a, M4b, M5a, M6a, M7a and M8a with bimodal SWCC	167
Table 5-25. Best fitting parameters of proposed equation 3-20 for soil mixtures M2a, M2b, M2c, M3a and M3b with bimodal SWCC	167
Table 5-26. Best fitting parameters of proposed equation 3-20 for soil mixtures M4a, M4b, M5a, M6a, M7a and M8a with bimodal SWCC	168
Table 5-27. Best fitting parameters of Zhang and Chen (2005) equation for soil mixtures M2a, M2b, M2c, M3a and M3b with bimodal SWCC	168
Table 5-28. Best fitting parameters of Zhang and Chen (2005) equation for soil mixtures M4a, M4b, M5a, M6a, M7a and M8a with bimodal SWCC	169
Table 5-29. Best fitting parameters of proposed equation 3-15 for soil SB1, SB2, SB3, SB4, SB5, SB6, JF3 and JF13 with bimodal SWCC	171
Table 5-30. Best fitting parameters of proposed equation 3-20 for soil SB1, SB2, SB3, SB4, SB5, SB6, JF3 and JF13 with bimodal SWCC	172
Table 5-31. Best fitting parameters of Zhang and Chen (2005) equation for soil SB1, SB2, SB3, SB4, SB5, SB6, JF3 and JF13 with bimodal SWCC	172
Table 5-32. Best fitting parameters of proposed equation 3-11 for soil mixtures M3c, M4c, M5c, M6b, M6c, M7b, M7c, M8b and M8c with unimodal SWCC	175

Table 5-33. Best fitting parameters of proposed equation 3-11 for soil mixtures M3c, M4c, M5c, M6b, M6c, M7b, M7c, M8b and M8c with unimodal SWCC.....	176
Table 5-34. Best fitting parameters of proposed equation 3-19 for soil mixtures M3c, M4c, M5b, M5c and M6b with unimodal SWCC	176
Table 5-35. Best fitting parameters of proposed equation 3-19 for soil mixtures M6c, M7b, M7c, M8b and M8c with unimodal SWCC	176
Table 5-36. Best fitting parameters of Fredlund and Xing (1994) equation for soil mixtures M3c, M4c, M5b, M5c and M6b with unimodal SWCC.....	177
Table 5-37. Best fitting parameters of Fredlund and Xing (1994) equation for soil mixtures M6c, M7b, M7c, M8b and M8c with unimodal SWCC	177
Table 5-38. Best fitting parameters of proposed equation 3-11 for soil SB9, SB10, GM1b, GM2b, BTG8 and BTG14 with unimodal SWCC.....	178
Table 5-39. Best fitting parameters of proposed equation 3-19 for soil SB9, SB10, GM1b, GM2b, BTG8 and BTG14 with unimodal SWCC.....	179
Table 5-40. Best fitting parameters of Fredlund and Xing (1994) equation for soil SB9, SB10, GM1b, GM2b, BTG8 and BTG14 with unimodal SWCC	179
Table 5-41. Best fitting parameters of the proposed shear strength developed in this study for soils M2a, M3a, M4a, M6a and M8a	180
Table 5-42. Best fitting parameters of Lee et al. (2005) equation for soils M2a, M3a, M4a, M6a and M8a	181
Table 5-43. Best fitting parameters of the proposed shear strength developed in this study for soils from published data.....	186
Table 5-44. Best fitting parameters of Lee et al (2005) equation for soils from published data.....	186
Table 6-1. Characteristics of SWCC for compacted soils M1, M2, M3, M4 and M5....	190
Table 6-2. Characteristics of SWCC for compacted soils M6, M7, M8 and M9	191
Table 6-3. Soil data sets from published literatures used for development of procedure to predict SWCC from bimodal GSD	198
Table 6-4. Soil data sets from published literatures used for development of equation for fitting and estimating the unsaturated shear strength of soil.....	199

Table 6-5. Parameters of the proposed equation for estimating SWCC of soil M2a, M2b, M2c, M3a, M3b, M4a.....	225
Table 6-6. Parameters of the proposed equation for estimating SWCC of soil M4b, M5a, M6a, M7a, M8a.....	225
Table 6-7. Parameters of the proposed equation for estimating SWCC of soil SB1, SB2, SB3, SB4, SB5, SB6 , JF3 and JF13.....	227
Table 6-8. Parameters of the proposed equation for estimating SWCC of soil M3c, M4c, M5b, M5c and M6b	231
Table 6-9. Parameters of the proposed equation for estimating SWCC of soil M6c, M7b, M7c, M8b and M8c.....	232
Table 6-10. Parameters of the proposed equation for estimating SWCC of soil SB8, SB11, GM1b, GM2b, BTG8 and BTG14.....	232
Table 6-11. Parameters of the proposed equation for estimating bimodal shear strength of soil mixtures from laboratory tests carried out in this study.....	236
Table 6-12. Parameters of Goh et al (2010) equation for estimating bimodal shear strength of soil mixtures from laboratory tests carried out in this study	236
Table 6-13. Parameters of the proposed equation for estimating bimodal shear strength of soil data from literatures	239
Table 6-14. Parameters of Goh et al (2010) equation for estimating bimodal shear strength of soil data from literatures	239
Table 7-1. Best fitting parameters of the proposed bimodal SWCC equation for soils 1 to 5 with bimodal SWCC.....	241
Table 7-2. Best fitting parameters of the proposed unimodal SWCC equation for soils 1 to 5 with bimodal SWCC.....	242
Table 7-3. Details of numerical analyses carried out in this study	245
Table 7-4. Shear strength of soils 1 to 5	247

LIST OF FIGURES

Figure 2-1. Mechanism of rainfall-induced slope failure (modified from Rahardjo et al., 2007)	8
Figure 2-2. Capillary tube showing the air-water interfaces at different radius of curvature (modified from Janssen and Dempsey, 1980).....	10
Figure 2-3. Unsaturated soil element with continuous air phase (modified from Fredlund et al., 2012)	11
Figure 2-4. Schematic diagram of four-phase unsaturated soil system	11
Figure 2-5. Simplified phase diagram of unsaturated soil	12
Figure 2-6. Variables of a typical SWCC	17
Figure 2-7. Relationship between void ratio and air-entry value as studied by Tarantino et al. (2009).....	21
Figure 2-8. Drying SWCCs of clayey sand under different dry densities (Yang et al., 2004)	22
Figure 2-9. Drying SWCCs of Edosaki sand under different dry densities (Gallage and Uchimura, 2010)	22
Figure 2-10. SWCCs of soil specimens with (a) 25 kPa preconsolidation pressure and (b) 200 kPa preconsolidation pressure, which were compacted at different initial water contents (Vanapalli et al., 1996a)	23
Figure 2-11. Effect of stress state on SWCC (Ng and Pang, 2000).....	24
Figure 2-12. SWCCs of the granitic residual soil from Korea under different net confining pressure (Lee et al. , 2005)	24
Figure 2-13. Effect of net confining pressure on air-entry value of soil (Thu et al., 2007)	25
Figure 2-14. Effect of net confining pressure on dual porosity soil (Bagherieh et al., 2009)	25
Figure 2-15. Typical SWCC from different type of soil (Fredlund and Xing, 1994).....	26
Figure 2-16. Structure and pore-size distribution of soils with unimodal and bimodal ...	27

Figure 2-17. Variation of shear strengths and ϕ^b with respect to matric suction under, a. 25 kPa and b. 200 kPa, net normal stresses for compacted Indian Head glacial till specimen compacted with different initial water content (Vanapalli et al., 1996a)	38
Figure 2-18. Variation of shear strengths and ϕ' with respect to matric suction under various confining pressure for compacted granitic residual soil from Korea (Lee et al., 2005)	39
Figure 2-19. Effect of coarse fraction on saturated shear strength obtained from a. direct shear test and b. CU triaxial test (Irfan and Tang, 1993)	41
Figure 2-20. Variation of shear strengths with respect to matric suction under various net normal stress for granitic residual soil from Brazil (Coutinho et al., 2011)	42
Figure 2-21. SWCCs of soil specimens with (a) 25 kPa preconsolidation pressure and (b) 200 kPa preconsolidation pressure, which were compacted at different initial water contents (Vanapalli et al., 1996a)	50
Figure 3-1. Typical unimodal and bimodal SWCC with their corresponding variables ..	53
Figure 3-2. Proposed zone for SWCC with bimodal characteristic	55
Figure 3-3. Capillary pressure distribution curve	57
Figure 3-4. Lognormal distribution curve	57
Figure 3-5. Soil-water characteristic curve with bimodal characteristic	61
Figure 3-6. Graph of exponential function $f(x) = \exp(-x)$	62
Figure 3-7. Percentages of soil retained at different particle size	65
Figure 3-8. Grain-size distribution with unimodal characteristic	65
Figure 3-9. Grain-size distribution with bimodal characteristic	66
Figure 3-10. Extended Mohr-coulomb failure envelope for an unsaturated soil (after Fredlund and Rahardjo, 1993)	68
Figure 3-11. The proposed shear strength of an unsaturated soil with bimodal characteristic	70
Figure 3-12. Variation of k with respect to natural logarithmic of matric suction for different soils	72
Figure 3-13. Proposed equation for best fitting grain-size distribution of soil	75
Figure 3-14. Proposed equation for best fitting soil-water characteristic curve developed based on lognormal distribution function	75

Figure 3-15. Proposed equation for best fitting soil-water characteristic curve developed based on exponential function	76
Figure 3-16. Proposed equation for best fitting shear strength of the unsaturated soil	76
Figure 4-1. Typical compaction curve	80
Figure 4-2. Static compaction moulds and plugs that are used for soil specimen preparations in this study	81
Figure 4-3. (a) Removable disk; (b) Connection between two adjacent disks; (c) Stainless steel plug (modified from Ong, 1999)	81
Figure 4-4. Static compaction machine used for preparation of soil specimen used in this study	83
Figure 4-5. Schematic diagram of triaxial cell for saturated test	84
Figure 4-6. Schematic diagram of Tempe cell	86
Figure 4-7. Layout of saturation process of soil specimen prior to SWCC test	87
Figure 4-8. Layout of SWCC test at matric suction less than 10 kPa	88
Figure 4-9. SWCC test at low matric suction higher than 10 kPa	89
Figure 4-10. Schematic diagram of pressure plate	90
Figure 4-11. SWCC process using pressure plate test	90
Figure 4-12. Configuration of Unsaturated Triaxial Testing	92
Figure 4-13. Schematic Diagram of Modified Triaxial Apparatus for Unsaturated Triaxial Tests	93
Figure 5-1. Compaction curve of selected soils used in this study	106
Figure 5-2. Grain-size distribution of soil mixtures used in the laboratory tests	111
Figure 5-3. Soil-water characteristic curve of soil mixture M1a	112
Figure 5-4. Soil-water characteristic curve of soil mixture M1b	113
Figure 5-5. Soil-water characteristic curve of soil mixture M1c	113
Figure 5-6. Soil-water characteristic curve of soil mixture M2a	114
Figure 5-7. Soil-water characteristic curve of soil mixture M2b	115
Figure 5-8. Soil-water characteristic curve of soil mixture M2c	115
Figure 5-9. Soil-water characteristic curve of soil mixture M3a	116
Figure 5-10. Soil-water characteristic curve of soil mixture M3b	117
Figure 5-11. Soil-water characteristic curve of soil mixture M3c	117

Figure 5-12. Soil-water characteristic curve of soil mixture M4a	118
Figure 5-13. Soil-water characteristic curve of soil mixture M4b	119
Figure 5-14. Soil-water characteristic curve of soil mixture M4c	119
Figure 5-15. Soil-water characteristic curve of soil mixture M5a	120
Figure 5-16. Soil-water characteristic curve of soil mixture M5b	121
Figure 5-17. Soil-water characteristic curve of soil mixture M5c	121
Figure 5-18. Soil-water characteristic curve of soil mixture M6a	122
Figure 5-19. Soil-water characteristic curve of soil mixture M6b	123
Figure 5-20. Soil-water characteristic curve of soil mixture M6c	123
Figure 5-21. Soil-water characteristic curve of soil mixture M7a	124
Figure 5-22. Soil-water characteristic curve of soil mixture M7b	125
Figure 5-23. Soil-water characteristic curve of soil mixture M7c	125
Figure 5-24. Soil-water characteristic curve of soil mixture M8a	126
Figure 5-25. Soil-water characteristic curve of soil mixture M8b	127
Figure 5-26. Soil-water characteristic curve of soil mixture M8c	127
Figure 5-27. Soil-water characteristic curve of soil mixture M9a	128
Figure 5-28. Soil-water characteristic curve of soil mixture M9b	129
Figure 5-29. Soil-water characteristic curve of soil mixture M9c	130
Figure 5-30. Stress-strain curves for specimens a. M2 and b. M3 obtained from the saturated mutistage CU triaxial tests.....	132
Figure 5-31. Stress-strain curves for specimens a. M4 and b. M5 obtained from the saturated mutistage CU triaxial tests.....	133
Figure 5-32. Stress-strain curves for specimens a. M6 and b. M7 obtained from the saturated mutistage CU triaxial tests.....	133
Figure 5-33. Stress-strain curves for specimens a. M8 and b. M9 obtained from the saturated mutistage CU triaxial tests.....	133
Figure 5-34. a. Stress-strain curves and b. total volumetric change characteristics of specimen M2a at different matric suctions under a net confining pressure of 25 kPa ...	134
Figure 5-35. a. Stress-strain curves and b. total volumetric change characteristics of specimen M2b at different matric suctions under a net confining pressure of 25 kPa ...	134

Figure 5-36. a. Stress-strain curves and b. total volumetric change characteristics of specimen M2c at different matric suctions under a net confining pressure of 25 kPa ...	134
Figure 5-37. a. Stress-strain curves and b. total volumetric change characteristics of specimen M3a at different matric suctions under a net confining pressure of 25 kPa ...	135
Figure 5-38. a. Stress-strain curves and b. total volumetric change characteristics of specimen M3b at different matric suctions under a net confining pressure of 25 kPa ...	135
Figure 5-39. a. Stress-strain curves and b. total volumetric change characteristics of specimen M3c at different matric suctions under a net confining pressure of 25 kPa ...	135
Figure 5-40. a. Stress-strain curves and b. total volumetric change characteristics of specimen M4a at different matric suctions under a net confining pressure of 25 kPa ...	136
Figure 5-41. a. Stress-strain curves and b. total volumetric change characteristics of specimen M4b at different matric suctions under a net confining pressure of 25 kPa ...	136
Figure 5-42. a. Stress-strain curves and b. total volumetric change characteristics of specimen M4c at different matric suctions under a net confining pressure of 25 kPa ...	136
Figure 5-43. a. Stress-strain curves and b. total volumetric change characteristics of specimen M5a at different matric suctions under a net confining pressure of 25 kPa ...	137
Figure 5-44. a. Stress-strain curves and b. total volumetric change characteristics of specimen M5b at different matric suctions under a net confining pressure of 25 kPa ...	137
Figure 5-45. a. Stress-strain curves and b. total volumetric change characteristics of specimen M6a at different matric suctions under a net confining pressure of 25 kPa ...	137
Figure 5-46. a. Stress-strain curves and b. total volumetric change characteristics of specimen M6b at different matric suctions under a net confining pressure of 25 kPa ...	138
Figure 5-47. a. Stress-strain curves and b. total volumetric change characteristics of specimen M7a at different matric suctions under a net confining pressure of 25 kPa ...	138
Figure 5-48. a. Stress-strain curves and b. total volumetric change characteristics of specimen M7b at different matric suctions under a net confining pressure of 25 kPa ...	138
Figure 5-49. a. Stress-strain curves and b. total volumetric change characteristics of specimen M8a at different matric suctions under a net confining pressure of 25 kPa ...	139
Figure 5-50. a. Stress-strain curves and b. total volumetric change characteristics of specimen M8b at different matric suctions under a net confining pressure of 25 kPa ...	139

Figure 5-51. a. Stress-strain curves and b. total volumetric change characteristics of specimen M9a at different matric suctions under a net confining pressure of 25 kPa ...	139
Figure 5-52. Mohr-Coulomb failure envelope for specimens M2 obtained from multistage CU saturated triaxial tests	140
Figure 5-53. Mohr-Coulomb failure envelope for specimens M3 obtained from multistage CU saturated triaxial tests	140
Figure 5-54. Mohr-Coulomb failure envelope for specimens M4 obtained from multistage CU saturated triaxial tests	141
Figure 5-55. Mohr-Coulomb failure envelope for specimens M5 obtained from multistage CU saturated triaxial tests	141
Figure 5-56. Mohr-Coulomb failure envelope for specimens M6 obtained from multistage CU saturated triaxial tests	141
Figure 5-57. Mohr-Coulomb failure envelope for specimens M7 obtained from multistage CU saturated triaxial tests	142
Figure 5-58. Mohr-Coulomb failure envelope for specimens M8 obtained from multistage CU saturated triaxial tests	142
Figure 5-59. Mohr-Coulomb failure envelope for specimens M9 obtained from multistage CU saturated triaxial tests	142
Figure 5-60. Mohr circles and cohesion intercepts, C , of specimen M2a under a net confining pressure of 25 kPa.....	144
Figure 5-61. Mohr circles and cohesion intercepts, C , of specimen M2b under a net confining pressure of 25 kPa.....	144
Figure 5-62. Mohr circles and cohesion intercepts, C , of specimen M2c under a net confining pressure of 25 kPa.....	145
Figure 5-63. Mohr circles and cohesion intercepts, C , of specimen M3a under a net confining pressure of 25 kPa.....	145
Figure 5-63. Mohr circles and cohesion intercepts, C , of specimen M3a under a net confining pressure of 25 kPa.....	145
Figure 5-64. Mohr circles and cohesion intercepts, C , of specimen M3b under a net confining pressure of 25 kPa.....	145

Figure 5-65. Mohr circles and cohesion intercepts, C , of specimen M3c under a net confining pressure of 25 kPa.....	146
Figure 5-66. Mohr circles and cohesion intercepts, C , of specimen M4a under a net confining pressure of 25 kPa.....	146
Figure 5-67. Mohr circles and cohesion intercepts, C , of specimen M4b under a net confining pressure of 25 kPa.....	146
Figure 5-68. Mohr circles and cohesion intercepts, C , of specimen M4c under a net confining pressure of 25 kPa.....	147
Figure 5-69. Mohr circles and cohesion intercepts, C , of specimen M5a under a net confining pressure of 25 kPa.....	147
Figure 5-70. Mohr circles and cohesion intercepts, C , of specimen M5b under a net confining pressure of 25 kPa.....	147
Figure 5-71. Mohr circles and cohesion intercepts, C , of specimen M6a under a net confining pressure of 25 kPa.....	148
Figure 5-72. Mohr circles and cohesion intercepts, C , of specimen M6b under a net confining pressure of 25 kPa.....	148
Figure 5-73. Mohr circles and cohesion intercepts, C , of specimen M7a under a net confining pressure of 25 kPa.....	148
Figure 5-74. Mohr circles and cohesion intercepts, C , of specimen M7b under a net confining pressure of 25 kPa.....	149
Figure 5-75. Mohr circles and cohesion intercepts, C , of specimen M8a under a net confining pressure of 25 kPa.....	149
Figure 5-76. Mohr circles and cohesion intercepts, C , of specimen M8b under a net confining pressure of 25 kPa.....	149
Figure 5-77. Mohr circles and cohesion intercepts, C , of specimen M9a under a net confining pressure of 25 kPa.....	150
Figure 5-78. The unsaturated shear strength data of specimens a. M2 and b. M3	150
Figure 5-79. The unsaturated shear strength data of specimens a. M4 and b. M5	150
Figure 5-80. The unsaturated shear strength data of specimens a. M6 and b. M7	151
Figure 5-81. The unsaturated shear strength data of specimens a. M8 and b. M9	151

Figure 5-82. GSD of soil M1, M2, M3 best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD.....	153
Figure 5-83. GSD of soil M4, M5, M6 best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD.....	153
Figure 5-84. GSD of soil M7, M8, M9 best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD.....	153
Figure 5-85. GSD of soils 11491, 11492 and 11494 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD.....	155
Figure 5-86. GSD of soils SB3, JF3 and JF13 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD	156
Figure 5-87. GSD of soils BTG8, GM1b and GM2b (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD.....	156
Figure 5-88. GSD of soils GM3b, GRS and Doi Tung (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD	157
Figure 5-89. GSD of coarse sand and coarse kaolin best fitted with the proposed equation developed in this study and published equation for best fitting unimodal GSD	159
Figure 5-90. GSD of soils S4, S45 and S62 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting unimodal GSD	161
Figure 5-91. GSD of soils JF5, BTG4 and BTG20 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting unimodal GSD.....	162
Figure 5-92. GSD of soils SC17, SW1 and CH3 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting unimodal GSD.....	162
Figure 5-93. SWCC of soil mixtures M2a, M2b and M2c best fitted with the proposed SWCC equation developed in this study and published equation from literature	163

Figure 5-94. SWCC of soil mixtures M3a and M3b best fitted with the proposed SWCC equation developed in this study and published equation from literature	164
Figure 5-95. SWCC of soil mixtures M4a and M4b best fitted with the proposed SWCC equation developed in this study and published equation from literature	164
Figure 5-96. SWCC of soil mixtures M5a and M6a best fitted with the proposed SWCC equation developed in this study and published equation from literature	165
Figure 5-97. SWCC of soil mixtures M7a and M8a best fitted with the proposed SWCC equation developed in this study and published equation from literature	165
Figure 5-98. SWCC of soil SB1 and SB2 best fitted with the proposed SWCC equation developed in this study and published equation from literature	166
Figure 5-99. SWCC of soil SB3 and SB4 best fitted with the proposed SWCC equation developed in this study and published equation from literature	169
Figure 5-100. SWCC of soil SB5 and SB6 best fitted with the proposed SWCC equation developed in this study and published equation from literature	170
Figure 5-101. SWCC of soil JF3 and JF13 best fitted with the proposed SWCC equation developed in this study and published equation from literature	170
Figure 5-102. SWCC of soil mixtures M3c, M4c and M5b best fitted with the proposed SWCC equation developed in this study and published equation from literature	170
Figure 5-103. SWCC of soil mixtures M5c, M6b and M6c best fitted with the proposed SWCC equation developed in this study and published equation from literature	173
Figure 5-104. SWCC of soil mixtures M7b and M7c best fitted with the proposed SWCC equation developed in this study and published equation from literature	174
Figure 5-105. SWCC of soil mixtures M8b and M8c best fitted with the proposed SWCC equation developed in this study and published equation from literature	174
Figure 5-106. SWCC of soil SB9, SB10, GM1b best fitted with the proposed SWCC equation developed in this study and published equation from literature	177
Figure 5-107. SWCC of soil GM2b, BTG8, BTG14 best fitted with the proposed SWCC equation developed in this study and published equation from literature	178
Figure 5-108. Shear strength of soil mixtures M2a best fitted with the proposed shear strength equation developed in this study and published equation from literature.....	181

Figure 5-109. Shear strength of soil mixtures M3a best fitted with the proposed shear strength equation developed in this study and published equation from literature.....	182
Figure 5-110. Shear strength of soil mixtures M4a best fitted with the proposed shear strength equation developed in this study and published equation from literature.....	182
Figure 5-111. Shear strength of soil mixtures M6a best fitted with the proposed shear strength equation developed in this study and published equation from literature.....	183
Figure 5-112. Shear strength of soil mixtures M8a best fitted with the proposed shear strength equation developed in this study and published equation from literature.....	183
Figure 5-113. Shear strength of soil from Coutinho et al. (2011) best fitted with the proposed shear strength equation developed in this study and published equation from literature	184
Figure 5-114. Shear strength of soil from Jotisankasa et al. (2009) best fitted with the proposed shear strength equation developed in this study and published equation from literature	184
Figure 5-115. Shear strength of soil from Jotisankasa and Mairaing (2010) best fitted with the proposed shear strength equation developed in this study and published equation from literature	185
Figure 5-116. Shear strength of soil from Rahardjo et al. (2011) best fitted with the proposed shear strength equation developed in this study and published equation from literature	185
Figure 6-1. Relationship between fitting parameter ψ_a from the proposed equation for best fitting unimodal SWCC (Equation 3-11) and air-entry value of SWCC.....	186
Figure 6-2. Relationship between fitting parameter ψ_m from the proposed equation for best fitting unimodal SWCC (Equation 3-11) and the inflection point of SWCC.....	187
Figure 6-3. Relationship between fitting parameter ψ_{a1} from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and the first air-entry value of SWCC	188
Figure 6-4. Relationship between fitting parameter ψ_{a2} from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and the second air-entry value of SWCC.	188
Figure 6-5. Relationship between fitting parameter ψ_{m1} from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and the first inflection point of SWCC	189

Figure 6-6. Relationship between fitting parameter ψ_{m2} from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and the second inflection point of SWCC	190
Figure 6-7. Relationship between saturated volumetric water content (θ_s) and percentage of sand within soil mixture	193
Figure 6-8. Relationship between standard deviation within subcurve 1 (sd1) of GSD curve and percentage of sand within soil mixture	193
Figure 6-9. Relationship between standard deviation within subcurve 1 (s1) of SWCC and percentage of sand within soil mixture	194
Figure 6-10. Relationship between fitting parameter dm1, dm2 and ψ_{m1} , ψ_{m2} for soil with bimodal GSD and SWCC	200
Figure 6-11. Relationship between parameter u and coefficient of uniformity for coarse-grained soil with bimodal GSD and unimodal SWCC	201
Figure 6-12. Relationship between parameter u and dry density for fine-grained soil with bimodal GSD and unimodal SWCC	201
Figure 6-17. Relationship between ψ_a , ψ_m and Cu for coarse-grained soils with bimodal GSD and unimodal SWCC	206
Figure 6-16. Relationship between ψ_a , ψ_m , dm1, and dm2 for fine-grained soils with bimodal GSD and unimodal SWCC	206
Figure 6-18. Relationship between ψ_r and d10 for soils with bimodal GSD and unimodal SWCC	207
Figure 6-19. Relationship between s and dry density of fine-grained soils with bimodal GSD and unimodal SWCC	207
Figure 6-20 Relationship between s and coefficient uniformity of coarse-grained soils with bimodal GSD and unimodal SWCC	208
Figure 6-13. Relationship between parameter u1 and percentage of coarse particle for soil with bimodal GSD and bimodal SWCC	202
Figure 6-15. Relationship between parameter v1, v2 and saturated water content for soil with bimodal GSD and bimodal SWCC	203
Figure 6-14. Relationship between parameter u2 and maximum diameter of soil particle for soil with bimodal GSD and bimodal SWCC.....	202

Figure 6-21. Relationship between ψ_{a1} , ψ_m and d_{max1} for soils with bimodal GSD and SWCC	209
Figure 6-22. Relationship between ψ_{a1} , ψ_{a2} , ψ_{m1} and ψ_{m2} for soils with bimodal GSD and SWCC	209
Figure 6-23. Relationship between s_1 and coefficient uniformity of soils with bimodal GSD and SWCC	209
Figure 6-24. Relationship between s_2 and saturated water content of soils with bimodal GSD and SWCC	210
Figure 6-25. Relationship between θ_{r1} and saturated water content for soils with bimodal GSD and SWCC	210
Figure 6-26. Relationship between θ_{r2} and percentage of coarse particle for coarse-grained soils with bimodal GSD and SWCC.....	211
Figure 6-27. Relationship between θ_{r2} and saturated water content for fine-grained soils with bimodal GSD and SWCC	211
Figure 6-28. Relationship between ψ_r and percentage of fine particle for soils with bimodal GSD and SWCC	211
Figure 6-31. The relationship between b and liquid limit of soil in the selected data sets	214
Figure 6-32. The relationship between q and total density of soil in the selected data sets	215
Figure 6-33. The relationship between parameter f from unsaturated shear strength of soil and parameter s_2 from SWCC in the selected data sets.....	215
Figure 6-40. Comparison of the experimental data of soils M2a, M2b and M2c with the estimated SWCC using the proposed equation developed in this study.....	223
Figure 6-41. Comparison of the experimental data of soils M3a and M3b with the estimated SWCC using the proposed equation developed in this study.....	223
Figure 6-42. Comparison of the experimental data of soils M4a and M4b with the estimated SWCC using the proposed equation developed in this study.....	224
Figure 6-43. Comparison of the experimental data of soils M5a and M6a with the estimated SWCC using the proposed equation developed in this study.....	224

Figure 6-44. Comparison of the experimental data of soils M7a and M8a with the estimated SWCC using the proposed equation developed in this study	224
Figure 6-51. Comparison of the experimental data of soils M3c, M4c and M5b with the estimated SWCC using the proposed equation developed in this study	229
Figure 6-52. Comparison of the experimental data of soils M5c, M6b and M6c with the estimated SWCC using the proposed equation developed in this study	229
Figure 6-53. Comparison of the experimental data of soils M7b and M7c with the estimated SWCC using the proposed equation developed in this study	230
Figure 6-54. Comparison of the experimental data of soils M8b and M8c with the estimated SWCC using the proposed equation developed in this study	230
Figure 6-45. Comparison of the experimental data of soils SB1 and SB2 with the estimated SWCC using the proposed equation developed in this study	226
Figure 6-46. Comparison of the experimental data of soils SB3 and SB4 with the estimated SWCC using the proposed equation developed in this study	226
Figure 6-47. Comparison of the experimental data of soils SB5 and SB6 with the estimated SWCC using the proposed equation developed in this study	226
Figure 6-48. Comparison of the experimental data of soils JF3 and JF13 with the estimated SWCC using the proposed equation developed in this study	227
Figure 6-55. Comparison of the experimental data of soils SB9, SB11 and GM1b with the estimated SWCC using the proposed equation developed in this study	230
Figure 6-56. Comparison of the experimental data of soils GM1b, BTG8 and BTG14 with the estimated SWCC using the proposed equation developed in this study	231
Figure 6-59. Comparison of the experimental data of soil mixture M2a with the estimated shear strength	234
Figure 6-60. Comparison of the experimental data of soil mixture M3a with the estimated shear strength	235
Figure 6-61. Comparison of the experimental data of soil mixture M4a with the estimated shear strength	235
Figure 6-62. Comparison of the experimental data of soil mixture M6a with the estimated shear strength	235

Figure 6-63. Comparison of the experimental data of soil mixture M8a with the estimated shear strength	236
Figure 6-64. Comparison of the experimental data of soil from Coutinho et al. (2011) with the estimated shear strength.....	237
Figure 6-65. Comparison of the experimental data of soil from Jotisankasa et al. (2009) with the estimated shear strength.....	237
Figure 6-66. Comparison of the experimental data of soil from Jotisankasa and Mairaing (2010) with the estimated shear strength	238
Figure 6-67. Comparison of the experimental data of soil Rahardjo et al. (2011) with the estimated shear strength.....	238
Figure 6-68. RMSE of the proposed equation developed in this study for estimating bimodal shear strength of unsaturated soil.....	240
Figure 6-69. ARE of the proposed equation developed in this study for estimating bimodal shear strength of unsaturated soil.....	240
Figure 7-1. SWCC of soils 1, 2 and 3 fitted with unimodal and bimodal equations	240
Figure 7-2. SWCC of soils 4 and 5 fitted with unimodal and bimodal equations	241
Figure 7-3. Permeability functions of soils 1 modelled with the proposed bimodal and unimodal permeability equations.....	243
Figure 7-8. Slope model for seepage analyses of all cases in this study	247
Figure 7-9. Variations of factor of safety for soil modeled with (a) bimodal and (b) unimodal equations under 9 mm/h of rainfall.....	247
Figure 7-10. Variations of factor of safety for soil modeled with (a) bimodal and (b) unimodal equations under 22 mm/h of rainfall.....	248
Figure 7-11. Difference of factor of safety for soil modeled with bimodal and unimodal equations under (a) 9mm/h and (b) 22 mm/h of rainfall.....	250

LIST OF SYMBOLS

a	= fitting parameter corresponding to the soil suction at the inflection point and is closely related to the air entry value for the soil
AEV	= air-entry value
$AEV1$	= first air-entry value
$AEV2$	= second air-entry value
a_{gr}	= fitting parameter related to the inflection point of slope in the first subcurve of grain-size distribution
b	= parameter in the proposed shear strength equation developed in this study representing the ratio of ϕ^b/ϕ' at suction between $AEV1$ and $AEV2$
c	= cohesion intercept
c'	= effective cohesion
C_c	= coefficient of curvature
C_u	= coefficient of uniformity
c_{ult}	= cohesion intercept at maximum matric suction
d	= soil particle diameter
d_m	= parameter in the proposed grain-size distribution equation developed in this study representing the geometric mean of soil particle diameter
d_{max}	= parameter in the proposed grain-size distribution equation developed in this study representing the maximum soil particle diameter
d_{min}	= parameter in proposed grain-size distribution equation developed in this study representing the minimum soil particle diameter that can be measured using hydrometer analysis
d_r	= parameter in proposed grain-size distribution equation developed in this study representing the maximum particle soil diameter of fine particles
e	= void ratio
e	= natural number, 2.71828...
$erfc$	= the complimentary error function
f	= fitting parameter in the proposed shear strength equation developed in this study

$f(r)$	= pore-size distribution of soil
G_s	= specific gravity of soil
h_m	= capillary pressure head at the inflection point of soil-water characteristic curve
j_{gr}	= fitting parameter related to the inflection point of slope in the second subcurve of grain-size distribution
k	= parameter in the proposed shear strength equation developed in this study representing the ratio of ϕ^b/ϕ' at suction beyond AEV2
k_{gr}	= fitting parameter related to the steepest slope in the second subcurve of grain-size distribution
l_{gr}	= fitting parameter related to the shape of the second subcurve of grain-size distribution near the fines region
LL	= liquid limit of soil
m	= fitting parameter related to the curvature of the slope of soil-water characteristic curve
m_{gr}	= fitting parameter related to the shape of the first subcurve of grain-size distribution near the fines region
M_s	= mass of soil solids
M_w	= mass of water
n	= fitting parameter related to the maximum slope of soil-water characteristic curve
$n(i)$	= number of investigated data
n_{gr}	= fitting parameter related to the steepest slope in grain-size distribution curve
n_{pl}	= porosities of the components with the large pore series
n_{ps}	= porosities of the components with the small pore series
P	= cumulative grain-size distribution of soil
P_{at}	= atmospheric pressure
PI	= plasticity index of soil
PL	= plasticity limit of soil
p_l	= volumetric percentages of the component with the large pore series in the soil mass

p_s	= volumetric percentages of the component with the small pore series in the soil mass
r	= radius of soil pore
S	= degree of saturation of soil
s	= parameter in the proposed soil-water characteristic curve equation developed in this study representing the geometric standard deviation of soil-water characteristic curve
s_d	= parameter in the proposed grain-size distribution equation developed in this study representing the geometric standard deviation of grain-size distribution of soil
S_e	= effective degree of saturation of soil
SSE	= sum of square error
u	= constant related to the packing of particles and pores of soil
u_a	= pore air
u_w	= pore water
v	= constant related to the shapes of particles and pores of soil
V	= volume of soil
V_w	= volume of water
w	= gravimetric water content
W	= fitting parameter representing the percentage of soil particle within coarse- or fine-grained particles
Y_{li}	= data from laboratory test
Y_{mi}	= data from the proposed equation
q	= fitting parameter in the proposed shear strength equation developed in this study
α	= initial angle of shear strength with respect to matric suction
β_1	= peak value of shear strength contribution from matric suction
Δu	= pore-water pressure changes after the increment of confining pressure
$\Delta\sigma_3$	= increment of confining pressure
ϕ'	= effective angle of internal friction
ϕ^b	= angle indicating increase in shear strength due to increase in matric suction

κ	= fitting parameter
μ	= geometric mean of matric suction
Θ	= normalized volumetric water content
θ	= volumetric water content under consideration
θ_r	= residual volumetric water content
θ_s	= saturated volumetric water content
θ_w	= volumetric water content at any matric suction
ρ_d	= dry density
σ	= standard deviation
$\sigma_3 - u_w$	= net confining pressure
τ	= shear strength of soil
ψ	= matric suction under consideration
ψ_a	= parameter in the proposed soil-water characteristic curve equation developed in this study representing the air-entry value of soil
ψ_e	= air-entry value of soil
ψ_m	= parameter in the proposed soil-water characteristic curve equation developed in this study representing matric suction at the inflection point
ψ_r	= parameter in the proposed soil-water characteristic curve equation developed in this study representing matric suction corresponding to residual volumetric water content
ψ_w	= water-entry value of soil

CHAPTER 1 INTRODUCTION

1.1. Research Background

Global climate change results in an increase in intense rainfalls of a prolonged period around the world, especially in the tropical region (Kunkel, 2003; Mason et al., 1999). As a result, many slope failures are observed in many tropical areas that are covered by colluvial and residual soils. Such slope failures can pose potential danger to infrastructures and public safety. In addition, the cost for repairing these slopes can be quite expensive. Many research works have been carried out to study the characteristics of colluvial and residual soils in tropical area. The results indicated that most of residual and colluvial soils show the bimodal characteristics of their grain-size distribution (Miguel and Bonder, 2012; Li and Zhang, 2009; Feuerharmel et al., 2006; Zhang and Chen, 2005, Rahardjo et al. 2004). However, many engineers commonly modelled these soils using unimodal equations of soil-water characteristic curve and shear strength to simplify the numerical analyses of residual and colluvial soil slopes. As a result, the behaviour of water flow through residual and colluvial soils of bimodal characteristics may not be properly modelled numerically to represent the field conditions.

The studies by Fredlund et al. (2012), Ng et al. (2008), Li et al. (2005 and Rahardjo et al. (2001) show that rainfall-induced slope failures commonly occur in the unsaturated zone above groundwater table in many steep residual and colluvial soil slopes. Therefore, the mechanical behaviour of unsaturated soils with bimodal grain-size distribution will be investigated in this study. The unsaturated soils exhibit different behavior from the saturated soils because of the presence of negative pore-water pressure (matric suction) in the unsaturated soils. The matric suction varies according to the water content. The relationship between water content (volumetric water content or gravimetric water content or degree of saturation) and matric suction (matric potential or negative pressure head) is called by several names such as: 1) the soil-water characteristic curve (SWCC), 2) the water retention curve (WRC), or 3) the capillary pressure curve. In this report, the term SWCC is used to define a soil's ability to store and release water.

SWCC is the most essential soil property in the unsaturated soil mechanics. The water flow and storage characteristics of an unsaturated soil are closely related to the

amount of water contained in the pores. The amount of water can be related to the negative pore-water pressure in the soil. Therefore, SWCC can be used to derive permeability function of soil. The SWCC has been used for characterizing the flow of water in unsaturated embankments and the stability of slope due to rainwater infiltration into slopes. SWCC is usually obtained by laboratory tests using one of the common test equipments such as Tempe cell and pressure plate apparatus (Fredlund et al., 2012; Rahardjo and Leong, 2003; Delage, 2004). However the high cost, the difficulty of the tests and time-consumption prevent the unsaturated soil technology from being applied in engineering practice. Therefore, several equations were proposed to estimate the mechanical behaviour of unsaturated soils using soil-water characteristic curve (SWCC). The behavior of water in soils (SWCC) depends on pore space geometry (pore-size distribution) of soil. Since pore-size distribution (PSD) is related to grain-size distribution (GSD), the SWCC can be easily estimated using GSD curve. Several equations have previously been proposed for the estimation of the SWCC (Chin et al., 2007; Hwang and Powers, 2003; Fredlund et al., 1997; Haverkamp and Parlange, 1986 and Arya and Paris, 1981). However, the existing published equations are limited to soils with unimodal characteristics and none of the existing published equations can be used to quantify the relationship between the SWCC and the structure of soil for soils with bimodal GSD. Previous research works also did not reveal whether soils with bimodal GSD will result in unimodal or bimodal SWCC. Therefore, it is necessary to have a new equation to estimate the SWCC of a soil with bimodal characteristics. The new equation must incorporate parameters that are able to represent the variables of SWCC closely and related to the physical properties of soil.

Shear strength is the most essential engineering properties in the design of geo-environmental and geotechnical structures such as retaining walls, embankment, highway, and soil cover system. Majority of these soil structures are observed in the unsaturated condition during their design life period. Therefore, the shear strength of the unsaturated soil is necessary in the design of geotechnical and geo-environmental structures. The shear strength of an unsaturated soil can be measured using modified triaxial cell apparatus. Laboratory tests for measurement of the shear strength of unsaturated soils are time consuming and require extensive laboratory equipments

(Fredlund et al, 2012; Goh et al., 2010; Lee et al., 2005; Vanapalli and Fredlund, 2000; Escario and Jucá, 1989 and Gan et al. 1988). Therefore, research works have been carried out to develop the methods to estimate the shear strength of the unsaturated soil (Goh et al., 2010; Lee et al., 2005; Vanapalli et al., 1996a, b; Fredlund et al., 1996; Khalili and Khabbaz, 1997; Oberg and Salfours, 1997). All methods for estimating the shear strength of the unsaturated soil use the SWCC data combined with the effective shear strength parameters (c' , ϕ') since the previous studies showed a strong relationship between water content and shear strength of a soil. The information about the distribution of the water in the liquid phase and the stress within the pore water which is derived from the SWCC are also incorporated in those methods. The existing published equations for best fitting and estimating shear strength of unsaturated soils from SWCC are limited to soil with unimodal soil-water characteristic curve. Therefore, it is necessary to develop new equations for best fitting and estimating the unsaturated shear strength of soils with bimodal soil-water characteristic curve. In addition, the effect of bimodal soil-water characteristic curve on the unsaturated shear strength of soils will also be studied.

1.2. Objectives

The main objectives of this research are to study the soil-water characteristic curve and unsaturated shear strength of soil with bimodal characteristic.

1.3. Scope of Works

In this research, the scopes of works include two main parts which are theoretical development and experimental programme. The theoretical development involves a comprehensive literature review, the development of necessary equations and comparative studies. The scopes of works carried out in this research are as follows:

- To study and review the general unsaturated soil behaviours as well as the mechanical behaviours of soils with bimodal characteristics from literatures.
- To study, review and evaluate the equations for best fitting and estimating soil-water characteristic curve and unsaturated shear strength of soil with bimodal characteristics.

- To develop formulations to best fit and to estimate bimodal soil-water characteristic curve.
- To develop formulations to best fit and to estimate the bimodal shear strength of the unsaturated soil.
- To compare the best fitted bimodal soil-water characteristic curve with the experimental data from this research as well as from literatures.
- To compare the estimated bimodal soil-water characteristic curve with the experimental data from this research as well as from literatures.
- To compare the best fitted bimodal shear strength of the unsaturated soil with the experimental data from this research as well as literatures.
- To compare the estimated bimodal unsaturated shear strength with the experimental data from this research as well as from literatures.
- To carry out seepage and slope stability analyses of soil with bimodal characteristics by incorporating the proposed equations developed in this study in the numerical analyses

1.4. Methodology

The research works mainly focused on the laboratory tests, development of soil-water characteristic curve equations and development of unsaturated shear strength equations for soil with bimodal characteristics. A comprehensive literature review of mechanical behaviour of unsaturated soil was carried out. Various soil-water characteristic curve and unsaturated shear strength equations from literatures were studied and reviewed to understand the limitations of these equations for implementation in soil with bimodal characteristic. All the controlling factors that affect the mechanical behaviour of unsaturated soils were studied. The equations for best fitting and estimating bimodal soil-water characteristic curves were developed and evaluated using soil data from laboratory tests carried out in this research and obtained from literatures. The equations for best fitting and estimating bimodal shear strength of the unsaturated soil were also developed and evaluated using experimental data from this research and from literatures. The proposed equations for soil-water characteristic curve and unsaturated shear strength for soil with bimodal characteristic were compared with various existing equations from

literatures. In order to obtain the bimodal characteristics of soil, the soil-water characteristic curve and unsaturated shear strength tests were performed from low suction (0.5 kPa) to high suction (1500 kPa) with narrow interval between different suction.

Reconstituted kaolin and standard sand were used in the experimental works carried out in this study to minimize the heterogeneity of soil specimen and to produce the bimodal grain-size distribution. Different designed sand-kaolin ratios were used in the experimental works carried out in this research to understand the suitable soil mixtures that could produce bimodal soil-water characteristic curve. Identical sand-kaolin specimens were prepared at the specified water content and dry density using the static compaction method. The index properties were obtained from basic property test and saturated triaxial tests. The soil-water characteristic curve tests of different soils were carried out using Tempe cell and pressure plate under zero net confining pressure. Unsaturated Consolidated Drained tests were conducted using the modified triaxial apparatus in order to investigate the characteristic of shear strength of soil with bimodal characteristics.

1.5. Outline of Thesis

This thesis consists of eight chapters. The content of each chapter is as follows:

Chapter 1 presents research background, objectives, scope of works, methodology and outline of thesis.

Chapter 2 presents reviews of unsaturated soil mechanics, grain-size distribution, soil-water characteristic curve, and unsaturated shear strength of soil with unimodal and bimodal characteristics. Various existing equations for soil-water characteristic curve and unsaturated shear strength are described. The brief reviews on soil-water characteristic curve and unsaturated shear strength tests are presented.

Chapter 3 presents the theoretical development of soil-water characteristic curve and shear strength of unsaturated soils which include the development of equations for best fitting bimodal grain-size distribution, development of equations for estimating soil-water

characteristic curve for soil with bimodal grain-size distribution, development of equations for best fitting bimodal soil-water characteristic curve and development of equations for best fitting and estimating shear strength of unsaturated soils with bimodal characteristics.

Chapter 4 presents the outline of research programme, criteria for soil selection, various basic property tests, soil preparation and saturated triaxial tests. The descriptions of soil-water characteristic curve and unsaturated shear strength test apparatuses are presented. The testing procedures and experimental programme are also included in this chapter.

Chapter 5 presents results from the laboratory tests carried out in this study on the selected soil. The results from basic property tests, soil-water characteristic curve tests and shear strength tests are also included in this chapter.

Chapter 6 contains discussions of the experimental results presented in chapter 5. The evaluation of the proposed equation using the experimental works carried out in this research and obtained from literatures are presented in this chapter. The comparisons between the proposed equations and various existing equations from the literatures are also presented in this chapter.

Chapter 7 presents the conclusions from this research. Further research recommendations are also given in this chapter.

CHAPTER 2 LITERATURE REVIEW

2.1. Introduction

In this chapter, reviews of the literatures related to this research are presented. First, a review of unsaturated soil mechanics is presented. The subsequent sections provide reviews on grain-size distribution (GSD) best fitting equations for soil with unimodal and bimodal characteristics, factors affecting soil-water characteristic curve (SWCC), SWCC best fitting equations for soil with unimodal and bimodal characteristics, SWCC estimation equations for soil with unimodal characteristic, shear strength characteristics of unsaturated soil, shear strength best fitting and estimation equations for soil with unimodal characteristics. Lastly, reviews of some laboratory testing for soil under unsaturated condition are presented.

2.2. Unsaturated Soil Mechanics

In recent decades, a global temperature increase produced rapid climate changes. Many parts of the world are experiencing more rainy days with higher rainfalls over a shorter time scale (Kumar et al., 2013). The increase in the rainfall amount caused the occurrences of many slope failures during rainfall especially in tropical countries like Singapore (Pitts, 1985; Chatterjea, 1989; Lim et al., 1996; Toll et al., 1999; Rahardjo et al., 2001). Most of the slope failures were observed in the residual soil with a steep slope angle (Krahn et al., 1989; Rahardjo et al., 1998; Ng et al., 2003, 2008 and Li et al., 2005). The steep residual soil slopes are usually accompanied with a significant thickness of the unsaturated zone above the groundwater table (Figure 2-1).

Geotechnical engineers commonly ignore the unsaturated zone (vadose zone) above groundwater table to avoid the complexity in numerical analyses for solving geotechnical problems. However, the unsaturated conditions are an important concern in many geotechnical designs, such as: slope stability and landfill cover system. The unsaturated zone above the groundwater table determines the unsaturated properties of soil and resulted in unsteady-state saturated/unsaturated flow conditions which result in the variations of pore-water pressures and shear strength of soil during dry and rainy periods (Fredlund et al., 2012; Fredlund and Rahardjo, 1993a). In other words, the

unsaturated conditions above the groundwater table have significant impact on the factor of safety of a slope. Therefore, the unsaturated soil mechanics should be incorporated in the geotechnical designs.

Figure 2-1. Mechanism of rainfall-induced slope failure (modified from Rahardjo et al., 2007)

The negative pore-water pressure in unsaturated soil is highly influenced by the flux boundary condition changes (i.e., infiltration, evaporation and transpiration) resulting from the variation in climatic conditions (Figure 2-1). On the other hand, the negative pore-water pressure contributes additional shear strength to the unsaturated soil. As water infiltrates into the slope, pore-water pressure in the slope increases (matric suction decreases), and the additional shear strength due to matric suction will decrease or even disappear, causing the slope to be more susceptible to failure. Evaporation and transpiration will restore the loss of matric suction in the slope and this climatic variation occurs at all times. A lot of geotechnical problems are attributed to the changes in the unsaturated zone above the groundwater table due to rainwater infiltration. Numerical analyses were conducted by Alonso et al. (2003) and Potts et al. (1997) to investigate the effect of rainfall on slope stability. The results indicated that rainwater infiltration caused the lateral displacements and heave of soil layer. Alonso et al. (2003) and Potts et al. (1997) also observed that the effect of rainwater infiltration on the pore-water pressure

distribution within soil layer could be quantified using the unsaturated soil mechanics principles.

Many geotechnical problems require the computation of evaporation, such as some slope stability analyses, design of embankment, waste cover systems and heave. Evaporation may result in the increase in the negative pore-water pressure within the soil layer and subsequently, increase the shear strength of the soil. Wilson et al. (1993) observe the performance of soil cover system within the unsaturated zone in a waste disposal site in Ontario, Canada by incorporating the effect of evaporation in the numerical analyses. Rykaart et al. (2001) carried out 3-D seepage analyses to investigate the effects of evaporation on pore-water pressure distribution within unsaturated zone of tailings impoundment. Gitirana et al. (2005, 2006) incorporated the effect of evaporation in the seepage analyses to obtain the pore-water pressure variations within the unsaturated zone of soil layers during rainfall.

The negative pore-water pressure in the unsaturated zone can also be expressed as matric potential or soil-water suction. The theoretical concept of soil suction was established in soil physics in the early 1900's (Edlefsen and Anderson, 1943). Soil suction (or total suction), commonly expressed in terms of relative humidity, is defined as the free energy of the soil water (Fredlund et al., 2012; Fredlund and Rahardjo, 1993a). The total suction of soil is divided into two components, osmotic suction and matric suction. The osmotic suction represents the suction that originates from dissolved solutes in the pore water of a soil. The change of salt concentration results in changes in osmotic suction of a soil. However, compared to matric suction, osmotic suction plays a less important role in soil suction since its changes are less significant, less frequent and slower than matric suction changes. Osmotic suction could be ignored in practice as long as the soil solution is dilute enough and the contained solutes do not affect the matric suction significantly (Hillel, 1998).

Matric suction in soil is defined as the negative pore-water pressure in the soil due to capillary effects which arise from the surface tension of water and its contact angle with the solid particles (Fredlund et al., 2012; Fredlund and Rahardjo, 1993a). The capillary phenomenon can be illustrated using the rise of water in the capillary tubes as shown in Figure 2-2. The pores in a soil are similar to a bundle of capillary tubes of

different sizes. The capillary effects cause water to rise above the water table in a capillary tube. The smaller the radius of a tube, the higher the water rise above the water table. The characteristics of the capillary effect in the soil pores are similar to those of in the water. The capillary of water has a negative pressure with respect to the air pressure (atmospheric pressure or $u_a = 0$). This condition causes the soil water to rise above the groundwater table. The matric suction in an unsaturated soil varies from zero (fully saturated) to 10^6 kPa. (Fredlund, 2006). Fredlund et al. (2012) and Fredlund and Rahardjo (1993a) defined matric suction as the difference between the pore-air pressure (u_a) and pore-water pressure (u_w) that act across the contractile skin. Since pore-air pressure (atmospheric pressure) is constant or assumed to be zero in the field, the value of matric suction is usually determined by the pore-water pressure.

Figure 2-2. Capillary tube showing the air-water interfaces at different radius of curvature (modified from Janssen and Dempsey, 1980)

An unsaturated soil is referred as four phase system, i.e. solid, water, air and contractile skin or air-water interface (Fredlund and Morgenstern, 1977). Contractile skin acts like an elastic membrane with the capability to exert tensile pull. This tensile pull is to balance the air pressure and the water pressure in the soil, as there is a difference

between the intermolecular force experienced by the molecules at the air-water interface and the force experienced by the molecules in the interior of the water (Townsend and Rice, 1991). Study by Terzaghi (1943) showed that contractile skin plays an important role in unsaturated soil behaviour. The idealized four phase system of unsaturated soil with a continuous air phase is shown in Figure 2-3. The properties of contractile skin is different from the properties of water. The molecular structure of contractile skin is similar to that of ice (Matsumoto and Kataoka, 1988). The contractile skin must be considered as independent phase when performing stress analysis of a multiphase continuum and the phenomenological behaviour of an unsaturated soil. The schematic phase diagram of four phase unsaturated soil is shown in Figure 2-4. The contractile skin can be assumed as part of the water phase without any significant error when considering volume-mass properties of the unsaturated soil (Fredlund, 2006). The simplified three phase diagram of unsaturated soil is shown in Figure 2-5.

Figure 2-3. Unsaturated soil element with continuous air phase (modified from Fredlund et al., 2012)

Figure 2-4. Schematic diagram of four-phase unsaturated soil system

Figure 2-5. Simplified phase diagram of unsaturated soil

The stress state variables of unsaturated soil were introduced for the first time by Biot (1941). Unlike saturated soil, the unsaturated soil behaviour must be described using two independent stress state variables as studied by Bishop and Donald (1961) and Bishop and Blight (1963). Further study by Fung (1977) indicated that the variables used for the description of a stress state should be independent of soil properties. However, the theoretical basis and justification of using two independent stress state variables for unsaturated soil were established for the first time by Fredlund and Morgenstern (1977). Their theories were developed based on the multiphase continuum mechanics and the incorporation of contractile skin which was considered as additional and independent phase of unsaturated soil. Fredlund and Morgenstern (1977) also used four phase system in unsaturated soil mechanics and assumed soil particles to be incompressible and chemically inert which were similar to the assumptions used in saturated soil mechanics (Terzaghi, 1943).

Fredlund and Morgenstern (1977) proposed three possible pairs of stress state variables to describe the mechanical behaviour of unsaturated soils, i.e. shear strength and volume change. The three pairs of stress state variables are as follows: $(\sigma - u_a)$ and $(u_a - u_w)$; $(\sigma - u_w)$ and $(u_a - u_w)$; $(\sigma - u_a)$ and $(\sigma - u_w)$, where σ is total stress, u_a is pore-air pressure and u_w is pore-water pressure. Fredlund and Rahardjo (1993a) evaluated the three pairs of these stress state variables and they concluded that the first pair of stress state variables is the most suitable and is always chosen for engineering application. These stress state variables form two independent stress tensors in a three-dimensional stress analysis.

2.3. Grain-size Distribution

Grain-size distribution (GSD) of a soil is defined as the relationship between the particle size and percent passing of soil particle (by mass). There are three general categories of GSD: well-graded soils, uniform soils and gap-graded soils. The physical attributes of the soils are described by the following variables: percentage of gravel (%gravel), percentage of sand (%sand), percentage of silt (%silt), percentage of clay (%clay), the diameter of soil particle corresponding to 10% finer in the GSD curve (d_{10}), the diameter of soil particle corresponding to 30% finer in the GSD curve (d_{30}), the diameter of soil particle corresponding to 60% finer in the GSD curve (d_{60}), coefficient uniformity of soil (C_u) and coefficient of curvature (C_c). The interpretation of the variables of the GSD is carried out manually using the GSD curve. The points on the GSD curve are obtained from sieve analysis and hydrometer analysis in accordance with ASTM D 422–63 (2002). The finest sieve used is sieve no 200 which allows particles finer than 0.075 mm to pass. Particles finer than 0.075 mm are classified as silt and clay particles and their points on the GSD curve are determined using a hydrometer analysis.

Many researchers developed a model to best fit the GSD data. Modeling GSD has two advantages, e.g. fundamental characterization of the soil using best-fitting parameters, the mathematical equation can be used as a basis for estimation of SWCC and the entire curve can be modeled as a continuous function using the mathematical equation. In general, the basic approaches adopted by researchers for developing the GSD equations are the use of parametric models for best fitting the distribution of soil particle (Gardner, 1956; Hagen et al., 1987; Buchan, 1989; Shiozawa & Campbell, 1991; Fredlund et al., 2000; Hwang & Powers, 2002). However, the existing best fitting GSD equations has several limitations. First, the model has difficulty in best fitting the GSD of soil at the extremes of the curve since it assumes that GSD is symmetric. Second, the model cannot be applied for gap-graded (bimodal) GSD. In reality, the GSD of soil is often non-symmetric. Fredlund et al. (2000) developed equations to represent GSD for a wide range of soils closely. The parameters in Fredlund et al. (2000) equations do not represent the variables of GSD of soil.

2.3.1. Unimodal Grain-size Distribution Function

Many researchers developed grain-size distribution (GSD) function to best fit the uniform and well-graded soils (unimodal GSD). Gardner (1956) and Kemper and Chepil (1965) used a two-parameter equation to best fit GSD data. However, the two-parameter equations could not be used to best fit the GSD data accurately especially at the extremes of the GSD curve (Hagen et al., 1987). Hwang and Powers (2003) developed a unimodal GSD function based on the assumption that particle diameter is lognormally distributed. One limitation associated with using this equation is the assumption that the GSD curve on a semilog scales is symmetric. In reality, the GSD is often nonsymmetrical. Therefore, the lognormal equation require modification to best fit GSD data accurately.

Fredlund et al. (2000) observed that the grain size distribution (GSD) curve has a shape similar to soil-water characteristic curve (SWCC). Therefore, they used Fredlund and Xing (1994) fitting equation for SWCC as a basis for the development of a unimodal GSD function. Since the GSD scale is a reverse of the SWCC scale, Fredlund et al. (2000) modified Fredlund and Xing (1994) equation as follow:

$$P = \left[\frac{1}{\ln \left(\exp(1) + \left(\frac{a_g}{d} \right)^{n_g} \right)^{m_g}} \right] \left[1 - \frac{\ln \left(1 + \frac{d_r}{d} \right)}{\ln \left(1 + \frac{d_r}{d_{\min}} \right)} \right]^7 \quad (2-1)$$

where:

P = cumulative particle size distribution

a_g = the inflection point

n_g = parameter related to the steepest slope in GSD curve

m_g = the shape of the GSD curve near the fines region

d_r = amount of fine particles

$d_{\min}$ = diameter of the minimum allowable particle size (mm)

Fredlund et al. (2000) function can be used to best fit GSD data more accurately than Hwang and Powers (2003) equation. Therefore, Fredlund et al. (2000) is commonly used to best fit GSD data of uniform and well-graded soils (unimodal GSD). However, this function can produce more than one combination of fitting parameters for one set of

GSD data. Therefore, it is necessary to develop a mathematical function that can best fit GSD data accurately with one set of fitting parameters (unique fitting parameters).

2.3.2. Bimodal Grain-size Distribution Function

The mathematical function for unimodal grain-size distribution (GSD) has limitation when it is applied to gap-graded (bimodal) soils. Therefore, Fredlund et al. (2000) develop the equation to best fit GSD data for gap-graded soil (bimodal GSD). The experimental data of a gap-graded (bimodal) soil normally show double humps in the GSD curve. These indicate that the particles are concentrated around two particle size ranges. Durner (1994) observed that a gap-graded soil is a combination of two or more separate soils. This type of soil allows for the application of more than one unimodal equation. Fredlund et al. (2000) proposed the mathematical function for bimodal GSD as follows:

$$P = \left\{ \begin{aligned} &w \left[\frac{1}{\left\{ \ln \left[\exp(1) + \left(\frac{a_{gr}}{d} \right)^{n_{gr}} \right] \right\}^{m_{gr}}} \right] + \\ &(1-w) \left[\frac{1}{\left\{ \ln \left[\exp(1) + \left(\frac{j_{gr}}{d} \right)^{k_{gr}} \right] \right\}^{l_{gr}}} \right] \end{aligned} \right\} \times \left\{ 1 - \frac{\ln \left(1 + \frac{d_r}{d} \right)}{\ln \left(1 + \frac{d_r}{d_{min}} \right)} \right\}^7 \quad (2-2)$$

where:

P = cumulative particle size distribution

a_{gr} = the inflection point of slope in the first subcurve

n_{gr} = parameter related to the steepest slope in the first subcurve

m_{gr} = the shape of the first subcurve near the fines region

j_{gr} = the inflection point of slope in the second subcurve

k_{gr} = parameter related to the steepest slope in the second subcurve

l_{gr} = the shape of the second subcurve near the fines region

d_f = amount of fine particles

$d_{\min}$ = diameter of the minimum allowable particle size (mm)

Fredlund et al. (2000) function can be used to best fit GSD data of gap-graded soil more accurately than Durner (1994) equation. Therefore, Fredlund et al. (2000) function is commonly used to best fit GSD data of gap-graded soil (bimodal GSD). However, similar to Fredlund et al. (2000) function for best fitting uniform and well-graded soils, Fredlund et al. (2000) function for best fitting gap-graded soil can produce more than one combination of fitting parameters for one set of GSD data. Therefore, it is necessary to develop a mathematical function that can best fit GSD data for gap-graded soil accurately with one unique set of fitting parameters. All best fitting parameters should represent the variables in GSD curve. This will help the engineers to easily characterize the variables of GSD of soil from the best fitting parameters.

In this study, the new equation for best fitting GSD of soil was developed. All best fitting parameters in the new GSD equations represented the variables in GSD curve of soil. The new GSD equation produced only one combination of best fitting parameters so that the best fitting parameters can be used to estimate the variables of SWCC.

2.4. Soil-water Characteristic Curve

Soil-water characteristic curve (SWCC) defines the relationship between water content and suction of the soil. Water content can be quantified in terms of gravimetric water content, w (ratio of mass of water, M_w , to mass of soil solids, M_s), or volumetric water content, θ_w (ratio of volume of water, V_w , to volume of soil, V). Suction can be either matric suction, $(u_a - u_w)$, or total suction, ψ . It was suggested that SWCC be used to represent the relationship between volumetric water content, θ_w , and matric suction, $(u_a - u_w)$ (Fredlund et al., 2001). Typical shape of SWCC is a sigmoidal curve and SWCC are commonly plotted on a logarithmic scale over the entire suction range.

A complete SWCC includes drying and wetting curves. At the same matric suction, the volumetric water content of the drying curve is higher than that of the wetting curve and this behaviour is called hysteresis. Hysteresis can be observed in SWCC due to the non-uniform distribution of pore sizes and the presence of air in the soil. Typical drying and wetting SWCCs are illustrated in Figure 2-6. The desaturation process of soil

was divided into three different stages in SWCC (boundary effect stage, transition stage and residual stage) by White et al. (1970) and Vanapalli et al. (1996b). As matric suction is increased from zero to the air-entry value, ψ_a , of the soil (within boundary effect stage), the volume water content, θ_w , of the soil is nearly constant since all the pores in the soil are still filled with water and the soil is assumed to be in a saturated condition. The air-entry value (bubbling pressure), ψ_a , is defined as the matric suction at which air first enters the largest pores of the soil during a drying process (Brooks and Corey, 1964, 1966). Transition stage starts from air-entry value point (AEV) to residual soil suction point (ψ_r). As the matric suction increases beyond the AEV, water content decreases to the residual water content, θ_r . The residual water content is the water content where a large suction change is required to remove additional water from the soil. Beyond residual water content (within residual stage), the soil pores are mainly occupied by air and the water phase becomes discontinuous. Under the capillary forces in soil pores created by the surface tension and the adsorption forces on the surfaces of clay particles and in the clay interlayer, water can be retained in soils up to a maximum suction of 10^6 kPa (Fredlund and Rahardjo, 1993a). This theory has been experimentally supported for a variety of soils (Croney and Coleman, 1961) and is supported by thermodynamic considerations (Richards, 1965).

Figure 2-6. Variables of a typical SWCC

Values of ψ_a were generally determined using the tangent method (as shown in Figure 2-6), as proposed by Brooks and Corey (1964). A consistent way to define the residual water content is also shown in Figure 2-6. A tangent line is drawn from the inflection point. The curve in the high-suction range can be approximated by another line. The residual water content, θ_r , can be approximated as the ordinate of the point at which the two lines intersect. The soil suction corresponding to the residual water content is called the residual soil suction, ψ_r . The water-entry value, ψ_w , on the wetting SWCC, is defined as the matric suction at which the water content of the soil starts to increase significantly during the wetting process (Yang et al. 2004). In general, drying and wetting SWCC tests are required to establish SWCC due to water desorption and water absorption of a soil. However, the measurement of wetting SWCC is tedious and time-consuming. Therefore, most of the existing SWCC data only include drying SWCC (Fredlund, 2006). In this research, the study also focused only in drying SWCC.

SWCC is similar to consolidation curve in saturated soil as studied by Fredlund and Rahardjo (1993a). In consolidation curve, the water content of soil decreases when a saturated soil is compressed and the effective stress of the soil increases. On the other hand, the water content of the soil increases or the soil swells when the effective stress of the soil decreases. The compression and swelling lines of consolidation curve are similar to the drying and wetting of SWCC, respectively. Blight (2007) compared the saturated consolidation curve and SWCC. The results showed that drying curve of SWCC changes in similar pattern with that of consolidation curve. Further analysis by Blight (2007) indicated that the drying curve of SWCC at low matric suction range is equal to the consolidation curve of clay at low stress range since the soil remained in saturated condition. In other word, the soil behaviour at matric suction lower than AEV is similar to the saturated soil behaviour as also suggested in various studies by Gan et al. (1988), Vanapalli et al. (1996b), Khalili and Khabbaz (1998) and Houston et al. (2008).

In SWCC test, the relationship between the radius of the pores which are filled with water and the soil suction follows Young-Laplace equation as studied by Brutsaert (1966). Therefore, SWCC can be used to define the pore-size distribution of soil. If the SWCC is integrated over the differential pore volume, the water content is equal to the volume of pores up to a radius r ,

$$w(r) = \int_0^r f(r) dr \quad (2-3)$$

where $f(r)$ = pore-size distribution of soil.

The characterization of pore-size distribution using SWCC data was also studied by Ragab et al. (1981), Klute (1986), Hajnos (1998), Aung et al. (2001), Hajnos et al. (2006), Rahardjo et al. (2012c). The water-air-solid contact angle is closely related to soil hydrophobicity. Soil Science Society of America (1997) defines soil hydrophobicity as the tendency for soil mass or soil particle to resist hydration. Soil Science Society of America (1997) also defines wettability as the ability of a liquid to spread over a material like soil. Non-wetting (drying) or wetting characteristic can be quantified by measuring the contact angle (the angle measured inside the liquid phase from the solid surface to a point tangent to the liquid-air interface). The contact angle varies from 0° (hydrophilic material) to 180° (completely non wettable, hydrophobic material). Normally, the drying contact angle is smaller than the wetting contact angle. Drying contact angles vary from 0° to 30° (Laroussi and DeBacker 1979), whereas the wetting angles contact angles in sands can be as high as 60 to 80° (Letey et al., 1962; Kumar and Malik, 1990).

2.4.1. Soil with Bimodal Soil-Water Characteristic Curve

Many residual and colluvial soils are gap-graded and show bimodal grain-size distribution (GSD) (Rahardjo et al., 2004; Zhang and Chen, 2005, Feuerharmel et al., 2006; Jotisankasa et al., 2009; Rahardjo et al., 2012c). Such bimodal feature of GSD has also been found in the materials within glacial tills, rockfill dams, debris flows, mudflows, solifluction sheets, and residual soil deposits that consist of a mixture of large (gravel or hard clay fragments) particles and a soft matrix of soil (clay, or silt-clay mixture) and have a distinct structure (Deere and Patton, 1971; Vallejo, 1979, 1989, 1997; Loganathan et al., 1992). Previous studies showed that bimodal SWCC is related to a bimodal pore-size distribution based on the capillary theory. Soils with bimodal SWCC consist of two levels of soil structure, i.e. microstructure and macrostructure (Burger and Shackelford, 2001; Stoicescu et al., 1999; Durner, 1994). Mitchell (1976) described macrostructure as the arrangement of the soil aggregates and microstructure as the elementary particle associations within the soil aggregates.

Durner (1994) observed that gap-graded soil (soil with bimodal GSD) is often related to dual porosity structure. However, no theoretical relationship has been developed between the bimodal GSD and the bimodal pore-size distribution up to date. Burger and Shackelford (2001) studied soil mixtures consisting of two different types of processed diatomaceous earth. The measured SWCCs for these soil mixtures showed clear bimodal characteristics. These bimodal SWCCs are considered to be indicators of dual-porosity structure (i.e. bimodal pore-size distribution). The typical dual-porosity structure within these soil mixtures was associated with the special structure of pellets within these soil mixtures. Feuerharmel et al. (2006) observed that SWCCs of majority colluvium soils showed clear bimodal characteristics that were related to their bimodal features of GSD. The bimodal pore-size distribution was also observed in other structured soils as studied by Alonso et al. (1987) and Carvalho et al. (2002).

Kutilek and Nielsen (1994) studied the presence of two peaks on the pore-size distribution function. They concluded that the bimodal behavior of the SWCC is resulted from the presence of two independent porous system within the domain of capillary pores. Those porous systems are: matrix (intra-aggregate, textural, micro) pores within soil aggregates and structural (inter-aggregate, macro) pores between the aggregates. The soil water hydrostatics in the matrix domain is affected by the arrangement of the soil skeleton. The connection between structural pores is dependent on the shape, size and stability of aggregates. The boundary between matrix and structural pores is called air-entry value of the matrix pores corresponds to residual water content of the structural pores (Kutilek, 2004).

2.4.2. Factors affecting Soil-Water Characteristic Curve

The shape of SWCC is affected by many factors, such as: soil structure, soil specimen preparation method, SWCC test method, soil types, soil state, initial water content, soil contents, mineralogy, GSD, pore-size distribution, dry density and stress state (history). All of these factors must be studied properly and fully understood before using SWCC data for describing unsaturated soil behaviour (unsaturated permeability functions, unsaturated shear strength and volume change). This study is required to avoid erroneous analyses that may lead to misunderstanding and wrong engineering solutions.

The effect of the initial void ratio or dry density on SWCC were studied by Aubertin et al. (1998), Kawai et al. (2000), Yang et al. (2004), Sun et al. (2006), Tarantino (2009), Malaya and Sreedeeep (2010) and Gallage and Uchimura (2010). All studies showed that the soil specimen with a lower initial dry density has a lower air-entry value and lower volumetric water content at a given matric suction than that with a higher dry density (Figure 2-7 to Figure 2-9). The soil with a lower dry density has a higher porosity, a higher void ratio and a larger average pore sizes than those with a higher dry density. Based on previous studies, it can be concluded that the ability of the soil with larger average pore sizes to retain water in the pores at higher matric suction is less than that with smaller average pore sizes since a higher suction is required for air to enter into the pores of the soil.

Figure 2-7. Relationship between void ratio and air-entry value as studied by Tarantino et al. (2009)

The effect of the initial water content on the shape of SWCC was studied by Elsbury et al. (1990), Vanapalli et al. (1996a), Vanapalli et al. (1999), Tarantino and Tombolato (2005), Birle et al., 2008, Malaya and Sreedeeep (2012). Previous studies by Vanapalli et al. (1999) indicated that specimens that were compacted at a higher initial water content (wet of optimum) have a higher air-entry value and a higher degree of saturation at a given matric suction than those at a lower initial water content (dry of optimum). The higher the initial water content of a soil, the steeper the SWCC of the soil will be (Figure 2-10). Vanapalli et al. (1996a) and Elsbury et al. (1990) observed that the

arrangement of soil particles in the wet of optimum is more homogeneous and well-arranged than that in the dry of optimum. They also found out that the SWCC behaviours of the soil compacted at wet of optimum and at dry of optimum are governed by the microstructure and macrostructure of the soil, respectively. As a result, the specimen at dry of optimum desaturates faster than that at wet of optimum. However, the effect of the initial water content becomes negligible at high matric suctions as observed by Tarantino and Tombolato (2005).

Figure 2-8. Drying SWCCs of clayey sand under different dry densities (Yang et al., 2004)

Figure 2-9. Drying SWCCs of Edosaki sand under different dry densities (Gallage and Uchimura, 2010)

The study by Birle et al. (2008) showed that the initial water content affects the SWCC with respect to the micro and macro fabric of the soil. The study by Malaya and Sreedeeep (2012) indicated that the initial water content has more significant impact on the shape of SWCC than density of soil.

Figure 2-10. SWCCs of soil specimens with (a) 25 kPa preconsolidation pressure and (b) 200 kPa preconsolidation pressure, which were compacted at different initial water contents (Vanapalli et al., 1996a)

The effect of stress state on SWCC was studied by Vanapalli et al. (1996b), Ng and Pang (2000), Lee et al. (2005) and Thu et al. (2007). Vanapalli et al. (1996b) investigated the effect of total stress on fine-grained soil and the results showed that the soil under a lower net normal stress has a lower air-entry value than that under a higher net normal stress (Figure 2-10). Ng and Pang (2000) studied the effect of stress state on the SWCC of undisturbed soils (completely decomposed volcanic soil). The results indicated that the specimen under a lower applied load has a gentler SWCC slope and a lower air-entry value than that under a higher applied load (Figure 2-11). Lee et al. (2005) investigated the effect of net confining pressure on SWCC of granitic residual soil in Korea. They found out that as the net confining pressure increased, the air-entry value increased linearly, the water content at a given matric suction increased and the slope of SWCC beyond the air-entry value became more flatten (Figure 2-12). Thu et al. (2007) also found out that the air-entry value increased with the increase in the net confining pressure as shown in Figure 2-13. In general, the findings of previous research works

indicated that the soil under a higher net confining pressure has ability to retain more water at a given matric suction as compared to that under a lower net confining pressure. This could be attributed to the fact that the soil under a higher net confining pressure has a lower void ratio with a smaller average pore sizes as compared to the soil under a lower net confining pressure.

Figure 2-11. Effect of stress state on SWCC (Ng and Pang, 2000)

Figure 2-12. SWCCs of the granitic residual soil from Korea under different net confining pressure (Lee et al. , 2005)

Figure 2-13. Effect of net confining pressure on air-entry value of soil (Thu et al., 2007)

The effect of stress state on soil with bimodal SWCC was investigated by Bagherieh et al. (2009). They found out that the bimodal SWCC of the investigated soil appeared to become unimodal under net confining pressure higher than 650 kPa (Figure 2-14). It could happen since the macro pores of soil have been closed under confining pressure of around 650 kPa, as shown by their consolidation data. This result also showed that the soil structure of the aggregated soil (dual porosity soil) approached that of the non-aggregated soil (single porosity soil).

Figure 2-14. Effect of net confining pressure on dual porosity soil (Bagherieh et al., 2009)

The effect of soil types and soil contents on SWCC were investigated by Fredlund and Xing (1994), Vanapalli et al. (1998), Vanapalli and Fredlund (2000), Yang et al. (2004), Lu et al. (2007), Rahardjo et al. (2010), Gallage and Uchimura (2010) and

Rahardjo et al. (2012b). They found out that the air-entry value, the slope and water content of SWCC change significantly from gravelly soil to clayey soil (Figure 2-15). In general, a soil with a lower fines content has a lower initial water content, a lower air-entry value, a lower water content at a given matric suction, a lower residual matric suction and a steeper slope of the SWCC than that with a higher fines content.

Figure 2-15. Typical SWCC from different type of soil (Fredlund and Xing, 1994)

The effects of soil types and soil contents on SWCC of dual porosity soil were investigated by Burger and Shackelford (2001), Zhang and Chen (2005), Jotisankasa et al. (2009), Rahardjo et al. (2012c) and Satyanaga et al. (2012). Burger and Shackelford (2001) and Zhang and Chen (2005) carried out SWCC tests on gap-graded soils. The results showed that bimodal SWCC can be found in gap-graded soils since the particle sizes of the coarse grains are far larger than the sizes of the fine grains. Therefore, the fine grains might not completely fill the pores formed by the coarse grains (Figure 2-16). As a result, two pore series were formed within soil specimen governed by the coarse particles (large pores) and the fine particles (small pores), respectively. Further studies by Jotisankasa et al. (2009), Rahardjo et al. (2012c) and Satyanaga et al. (2012) indicated that soils with bimodal GSD may exhibit unimodal SWCC. Rahardjo et al. (2012c) found that the presence of gravels of different particle sizes in the gap-graded soil did not affect the typical shape of the soil–water characteristic curves, which is unimodal. This happened since the large pore is filled with fine particles within the soil mixtures. Further

studies by Satyanaga et al. (2012) showed that soil structure (initial dry density, initial water content and compaction at dry or wet of optimum) and GSD of soil affected the unimodal or bimodal shape of SWCC for soils with bimodal GSD.

Figure 2-16. Structure and pore-size distribution of soils with unimodal and bimodal characteristics

The effect of high suction values on SWCC was studied by Fredlund et al. (1999). They found out that the behaviour of SWCC appears to be similar at high suctions (i.e. 20 to 300 MPa) regardless of the initial water content (i.e. dry of optimum, optimum and wet of optimum) and stress history. In other words, soil structure seems to have negligible influence on the behavior of SWCC in this high suction range. It could happen since water films at these suctions are so thin that all water is within the range of influence of osmotic and adsorptive fields.

In the compacted soils, the variation in the interlayer and the adsorbed water are minimal for matric suction less than 1500 kPa as studied by Cases et al. (1992) and Saiyouri et al. (2000). In this case, liquid transport is significant in this suction range, where capillary forces dominate. Hence, pore geometry controls the SWCC in this suction range. At suction beyond 1500 kPa, the relative humidity of the soil pores begins to decrease, and vapor flow becomes significant.

2.4.3. Measurement of Soil-Water Characteristic Curve

Fredlund (2006) observed that the measurement of SWCC plays an important role in applying unsaturated soil mechanics into geotechnical engineering practice. Controlled suctions are applied to a specimen in the SWCC tests using the axis translation technique and a pressure membrane apparatus (Richards, 1941; Hilf, 1956). Water content and volume changes are monitored by withdrawing the specimen from the cell, once equilibrium in suction is reached, the specimen is weighted and its dimension manually measured (Delage, 2002, Fredlund et al., 2012). Typical apparatuses for measuring SWCC are Tempe cells and pressure plate which can apply matric suction up to 100 kPa and 1500 kPa, respectively. A detailed description of testing procedures for measurement of SWCC is provided in the ASTM designation (ASTM D-6836-02, 2003).

2.4.4. Unimodal Soil-Water Characteristic Curve Function

Soil – water characteristic curve (SWCC) is an important property of unsaturated soils which can be used to solve many geotechnical problems. Research works by Yang et al. (2004) and Gasmo et al. (2000) showed that SWCC plays an important role in controlling infiltration characteristics of rainwater through soil layers. SWCC also affects the shear strength of soil under infiltration condition as studied by Melinda et al. (2004). Other research works showed that there is a relationship between SWCC and the properties of an unsaturated soil (Fredlund et al., 2012; Fredlund and Rahardjo, 1993a). For example, an indirect determination of the permeability function of unsaturated soil was established using the saturated coefficient of permeability and SWCC (Fredlund et al., 2012; Millington & Quirk, 1959; Marshall, 1958). Similar procedures have been suggested for determination of shear strength of an unsaturated soil (Fredlund and Rahardjo, 1993b; Fredlund et al., 1995). Since SWCC is used as the basis for the estimation of other unsaturated soil parameters, it is important to have a reasonably accurate characterization of SWCC.

Various equations have been proposed to represent unimodal SWCC (Pedroso et al., 2009, 2010; Gitirana and Fredlund, 2004; Kosugi, 1997, 1994; Fredlund and Xing, 1994; van Genuchten, 1980; Brooks and Corey, 1964; Gardner, 1958). One of the earliest equation for best fitting unimodal SWCC was developed by Gardner (1956). Originally,

this equation was proposed for modelling unimodal permeability function of soil. However, Sillers et al. (2001) studied that Gardner (1956) equation can also be used to best fit unimodal SWCC. Brooks and Corey (1964) divided their SWCC model into two parts. The first part, the water content of SWCC was assumed constant for the range of matric suction less than air-entry value. The second part, the SWCC was assumed to decrease exponentially and could be fitted using two fitting parameters. Sillers et al. (2001 and Lu and Likos (2004) observed that Brooks and Corey (1964) equation is not a continuous function since no inflection point is included in their equation. As a result, the performance of their equation is poor for a wide range of matric suction.

The first continuous function for fitting unimodal SWCC was proposed by Van Genuchten (1980). This equation provides a wide range of flexibility in fitting unimodal SWCC for different types of soil. Similar to Van Genuchten (1980) equation, Fredlund and Xing (1994) equation was also developed using a continuous function and applicable for best fitting a wide range of soil types. Leong and Rahardjo (1997) evaluated numerous SWCC equations and showed that all equations for fitting unimodal SWCC can be derived from a single generic form. It was concluded that Fredlund and Xing (1994) equation gave the best fit among other equations. The equation proposed by Fredlund and Xing (1994) can be written as follows:

$$\theta(\psi) = \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)} \right] \left\{ \frac{1}{\ln\left[e + \left(\frac{\psi}{a}\right)^n\right]} \right\}^m \quad (2-4)$$

where:

- e = base of natural logarithm;
- θ_s = saturated volumetric water content;
- ψ = soil suction;
- a = fitting parameter corresponding to the soil suction at the inflection point and is closely related to the air entry value for the soil;
- n = fitting parameter related to the maximum slope of the curve;
- m = fitting parameter related to the curvature of the slope;
- ψ_r = constant parameter used to adjust the lower portion of the curve;

Leong and Rahardjo (1997) recommended the use of a correction factor $C(\psi) = 1$ in the Fredlund and Xing (1994) equation for fitting unimodal SWCC with residual matric suction less than 1500 kPa. This would reduce the computation effort in determining the SWCC fitting parameter and result in the better performance of SWCC equation in fitting unimodal SWCC. Kosugi (1997) also developed the SWCC fitting equation by applying lognormal distribution laws to both soil pore-size distribution function and SWCC function with unimodal characteristics. However, this equation can not be used for best fitting a wide range of soils accurately. The equation developed by Kosugi (1994) is as follows:

$$S_e = 1 - \operatorname{erfc} \left(\frac{\ln \left(\frac{\psi}{\psi_m} \right)}{\sigma} \right) \quad (2-5)$$

where:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (2-6)$$

where:

- S_e = effective saturation
- θ_s = saturated volumetric water content
- θ_r = residual volumetric water content
- θ = volumetric water content under consideration
- ψ = matric suction under consideration
- ψ_m = matric suction at the inflection point or matric suction related to the diameter of pore with the highest number among other pores
- σ = standard deviation

Gitirana and Fredlund (2004) improved Fredlund and Xing (1994) equation in order to obtain the new SWCC equation that is able to best fit the experimental data of SWCC accurately. Gitirana and Fredlund (2004) differentiated the unimodal SWCC with one and two bending points. Since the equation is based on hyperbolas asymptotes, it becomes complicated and also requires additional parameter “a” with no physical

meaning. The equation for unimodal SWCC with one bending point as proposed by Gitirana and Fredlund (2004) is as follows:

$$S = \frac{\tan \theta (1 + r^2) \ln(\psi / \psi_b)}{(1 - r^2 \tan^2 \theta)} - \frac{(1 + \tan^2 \theta)}{(1 - r^2 \tan^2 \theta)} \sqrt{r^2 \ln^2(\psi / \psi_b) + \frac{a^2 (1 - r^2 \tan^2 \theta)}{(1 + \tan^2 \theta)} + 1} \quad (2-7)$$

Zhai and Rahardjo (2012) concluded that Fredlund and Xing (1994) equation gives the best fitting result for a wide range of soil over the entire range of matric suction. However, Zhai and Rahardjo (2012) found out that the fitting parameters in Fredlund and Xing (1994) equation may vary depending on the convergence procedure, number of iterations and the initial value of the fitting parameters. In addition, the fitting parameters do not represent the variables of SWCC. Zhai and Rahardjo (2012) proposed a mathematical equation to calculate air-entry value, residual matric suction and residual water content from Fredlund and Xing (1994) fitting parameters.

In this study, the new equation for best fitting unimodal SWCC was developed. The new unimodal SWCC equation produced a unique solution of best fitting parameter. In addition, the best fitting parameters can be used to represent the variables of unimodal SWCC.

2.4.5. Bimodal Soil-Water Characteristic Curve Function

Several models have been developed to represent bimodal SWCC. Durner (1994) extended the Van Genuchten (1980) unimodal equation to best fit the bimodal SWCC function by introducing factors for combining individual functions of unimodal SWCC. Burger and Shackelford (2001) combined the Brooks and Corey (1964), Van Genuchten (1980) and Fredlund and Xing (1994) SWCC functions to model the bimodal behavior of the peletized diatomaceous earth and sand-diatomaceous earth mixtures. Kutilek (2004) extended the unimodal Kosugi's equation to best fit the experimental data of bimodal SWCC by superimposing the SWCC function of matrix and structural pores. Among all the equations for bimodal SWCC, only Kutilek's equation that has physical meaning for its parameters related to the shape features of SWCC. However, his equation does not

provide a wide range of flexibility in fitting bimodal SWCC for different types of soil. Zhang and Chen (2005) modified Fredlund and Xing (1994) equation to best fit a bimodal SWCC by introducing the large and small-pore series component in the equation. However, similar as Fredlund and Xing equation, the fitting parameters in this equation are not related directly to the features of SWCC and may vary depending on the convergence procedure, number of iterations and the initial values of fitting parameters.

The equation developed by Zhang and Chen (2005) is as follow:

$$\theta(\psi) = p_l n_{pl} \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_{rl}}\right)}{\ln\left(1 + \frac{10^6}{\psi_{rl}}\right)} \right] \left\{ \frac{1}{\ln\left[e + \left(\frac{\psi}{a_l}\right)^{n_l}\right]} \right\} + p_s n_{ps} \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_{rs}}\right)}{\ln\left(1 + \frac{10^6}{\psi_{rs}}\right)} \right] \left\{ \frac{1}{\ln\left[e + \left(\frac{\psi}{a_s}\right)^{n_s}\right]} \right\}^{n_s} \quad (2-8)$$

where:

p_l = volumetric percentages of the component with the large-pore series in the soil mass

p_s = volumetric percentages of the component with the small-pore series in the soil mass

n_{pl} = porosities of the components with the large-pore series

n_{ps} = porosities of the components with the small-pore series

In this study, the new equation for best fitting bimodal SWCC was developed. The new bimodal SWCC equation produced a unique solution of best fitting parameter. In addition, the best fitting parameters can be used to represent the variables of bimodal SWCC.

2.4.6. Estimation of Soil-Water Characteristic Curve

Measurement of the SWCC in the laboratory and in the field is relatively time consuming and expensive. In several situations, it is very useful to estimate the SWCC, especially in the initial stages of a project when the available information is limited. Estimation of SWCC from simple geotechnical properties, such as GSD and porosity, is useful for analyzing and validating test results and also for evaluating the variations of SWCC to be expected in the field within a non homogenous soil. Many models have been proposed to

estimate the SWCC from soil properties, such as GSD. The predictive model of SWCC is known as Pedo-Transfer Functions (PTF). Bouma (1989) defined PTF as a function that relates SWCC variables to basic soil data such as the grain-size distribution or porosity and produces a soil property function. Huang et al. (2010) classified PTF into two groups:

1. Estimation of SWCC based on statistical relationship between soil properties and fitting parameters of SWCC (Gupta and Larson, 1979; Vereecken et al., 1989; Rawls and Brakensiek, 1989; Schaap and Leij, 1998; Fredlund et al., 2002 and Chin et al., 2010).

The soil properties include GSD and volume-mass properties.

2. Estimation of SWCC based on physic-empirical approach that converts GSD into pore-size distribution (PSD). Then PSD is used to develop the SWCC (Arya and Paris, 1981; Haverkamp and Parlange, 1986; Wagner and Ding, 1994; Rouault and Assouline, 1998; Haverkamp et al., 1999; Hwang et al., 2002 and Hwang and Powers, 2003).

Rawls and Brakensiek (1985) estimated the air-entry value, pore-size index and residual water content of SWCC for Brooks and Corey (1964) equation using basic soil properties. Fredlund et al. (2002) found that majority of the parameters were overestimated using Rawls and Brakensiek (1985) method. Vereecken et al. (1989) model was shown to be able to estimate SWCC with reasonable agreement using soil properties (GSD, dry density and carbon content). However, the model was mainly used in the agricultural discipline where organic soils were involved. Previous studies indicated that the estimation of SWCC using PTF no 1 required a wide range of soil database. The result is inaccurate if the estimated soil is outside the range of soil database. Huang et al. (2010) applied Fredlund et al. (2002) model into soil from the Loess Plateau, China. The results are slightly better as compared with those obtained from other models. However, this model has difficulty to control the packing porosity for particular types of soil. In addition, this model is not built based on physical characteristics of the soil.

Method no 2 was preferred since it uses the physical characteristics of soil for the estimation of SWCC. The first model from the second method was proposed by Arya and Paris (1981). They calculated volumetric water content of soil based on the relationship of pore-sizes and particles sizes. Pore radii were estimated based on the assumptions that all pores were in cylindrical and all particles were spherical in shape and pore radii were

converted to equivalent matric suction using capillary theory (Taylor, 1948). Wagner and Ding (1994) indicated that lognormal distributions were capable of fitting the central portion of the GSD of soil. Rouault and Assouline (1998) modelled the relationships between particle and pore size distributions of the multicomponent packed sphere case and they applied the model into soil. Haverkamp et al. (1999) and Hwang et al. (2002) improved the model and observed that there is non-linear relationship between soil particle and soil pore based on this model (Equation 2-9).

$$d = ur^v \quad (2-9)$$

where:

- u = constants related to the packing of particles and pores
- v = constants related to the shapes of particles and pores
- d = particle diameter
- r = pore diameter

Hwang and Powers (2003) applied Kosugi equation to GSD and SWCC and developed the new relationship between parameters in GSD function and SWCC function (Equations 2-10 and 2-11). Hwang and Powers (2003) equation does not provide a wide range of flexibility in predicting unimodal SWCC for different types of soil since their equation depends on constant value of parameters u and v.

$$\ln(h_m) = \ln(0.149) - \frac{\ln(d_m) - \ln(u)}{v} \quad (2-10)$$

$$\sigma = \frac{\sigma_d}{v} \quad (2-11)$$

where:

- h_m = capillary pressure head at the inflection point of SWCC (cm)
- d_m = geometric mean of particle diameter (mm)
- σ = geometric standard deviation of SWCC
- σ_d = geometric standard deviation of GSD

Fredlund et al. (1997 and 2002) proposed a model to estimate SWCC based on the capillary theory, GSD and the estimated initial packing factor of a soil. The initial packing factor which is the primary volume-mass variable used in the estimation of the SWCC is estimated using statistical method or the neural net. Fredlund et al. (1997)

defined the neural net as “an artificial-intelligence technique by which an algorithm is trained to respond to various input conditions”. Fredlund et al. (2002) showed that their model can be used to estimate SWCC of sand and silt successfully. However, the estimation of SWCC for clays, tills and loams were less accurate. This model was limited to estimate the SWCC that was initially prepared from a slurry paste and SWCC for soil with unimodal characteristics.

Numerous research works have been carried out to develop the equation or model to estimate SWCC of soils from soil properties and GSD. However, the existing equations are limited to soils with unimodal characteristics and none of the existing equations can be used to quantify the relationship between the SWCC and the soil structure for soils with bimodal GSD. Previous research works also did not reveal whether soils with bimodal GSD will result in unimodal or bimodal SWCC.

In this study, the relationship between variables of SWCC and GSD of soil with bimodal characteristics were investigated. Then, the new procedure for estimating SWCC for soil with bimodal GSD was established. The estimation of SWCC for soil with bimodal GSD were carried out using the proposed SWCC equation developed in this study.

2.5. Shear Strength of Unsaturated Soil

Many authors (Lumb, 1975; Fredlund, 1979; Brand, 1984; Chinniah et al., 1994; Fourie, 1996) suggested that conventional methods of slope stability analysis (using saturated soil mechanics), cannot be applied successfully for unsaturated soil slopes under heavy rainfall. It is necessary to incorporate negative pore-water pressure in the stability analysis of unsaturated soil slope. Constitutive equations, which are the relationships between shear strength or volume change with stress state variables, for unsaturated soils can be formulated as an extension of the constitutive equations for saturated soils. The constitutive equations for unsaturated soils show a smooth transition to the constitutive equations for saturated soils when degree of saturation approaches 100 % or matric suction goes to zero.

The failure envelope for unsaturated soil is normally described using Mohr circles in a three-dimensional form. The three-dimensional graph has the shear stress, τ , as the

ordinate and the two stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$, as the abscissas. This three-dimensional plot for unsaturated soil is an extension of the two dimensional Mohr-Coulomb failure envelope of saturated soil. For a saturated soil, the matric suction $(u_a - u_w)$ is zero and the pore-water pressure becomes equal to the pore-air pressure. The rate of change in shear strength with respect to net normal stress is expressed by ϕ' (effective friction angle) while the rate of change in shear strength with respect to matric suction is characterized by ϕ^b .

2.5.1. Previous studies on shear strength characteristics of soils with unimodal soil-water characteristic curve

Terzaghi (1943) introduced the effective stress principle to the classical soil mechanics and this theory is based on the assumption that soils are saturated. However, this theory cannot provide a complete understanding of shear strength behavior for unsaturated soils. With the renaissance of unsaturated soil mechanics at Imperial College, Bishop (1959) introduced a negative pore-water pressure (matric suction) parameter into the effective stress equation for saturated soils. Bishop (1959) equation incorporated the value of χ to represent the variation from a fully saturated condition to a totally dry condition. Studies by Aitchison (1961) and Blight (1967) indicated that the value of χ could be assumed to vary from one to zero. Further study by Coleman (1962) showed that χ is strongly related to the soil structure. However, later study by Bishop and Blight (1963), Burland (1965) and Matyas and Radhakrishna (1968) showed the use of single stress state variable in Bishop (1959) equation resulted in the limitation of this equation to model the behaviour of unsaturated soil.

Shear strength of unsaturated soil in term of the two independent stress state variables, net normal stress $(\sigma - u_a)$ and matric suction $(u_a - u_w)$, was proposed by Fredlund et al. (1978). Their equation was developed based on theory that an increase in matric suction causes an increase in shear strength as governed by ϕ^b . Initially, Fredlund et al. (1978) equation was assumed to be in a linear form. However, further studies showed that the variation of shear strength with respect to matric suction was found to be non-linear (Escario and Saez, 1986; Gan et al., 1988; Escario and Juca, 1989; Han et al., 1995; Vanapalli et al., 1996a; Khalili and Khabbaz, 1998; Houston et al., 2008; Sheng et al.,

2009; Goh et al., 2010). The value of ϕ^b is equal or close to ϕ' up to the air-entry value of the soil with unimodal SWCC. Beyond air-entry value, the soil starts to become unsaturated and consequently the shear strength envelope becomes non-linear. The variation of ϕ^b is related to the changes in the contact area between water menisci and soil particles when matric suction increases. Some studies indicated that ϕ^b can be larger than ϕ' as observed by Abramento and Carvalho (1989), Mesri and Abdel- Ghaffar (1995), Gan and Fredlund (1996), Vanapalli et al. (1996a), Melinda et al. (2004) and Noor and Anderson (2006).

Fredlund et al. (1987) reanalyzed the shear strength data from Satija (1978) research works. They replotted the non-linear failure envelopes on the shear strength data from the consolidated drained (CD) and constant water content (CW) triaxial tests of compacted Dhanauri clay. The cohesion intercept, c and ϕ^b , at zero matric suction were plotted using the c' and ϕ' obtained from the saturated triaxial tests. The results indicated a smooth transition between the saturated and unsaturated conditions. A series of multistage CD direct shear tests using the Indian Head glacial till at matric suction up to 500 kPa was carried out by Gan et al. (1988). They found out that ϕ^b is equal to ϕ' at low matric suctions (close to zero) and starts to decrease significantly in the matric suction range of 50-100 kPa and approximately equals to 7° at matric suctions higher than 300 kPa.

The comparison of the unsaturated shear strength of soil specimens compacted at wet of optimum, optimum water content and dry of optimum was carried out by Vanapalli et al. (1996a). The experimental results showed that the unsaturated shear strength of soil specimens at dry of optimum is the lower than that at optimum water content and at wet of optimum as shown in Figure 2-17. This happened since matric suction serves as the stress state variable, contributes to the shear strength of an unsaturated soil through the wetted surface of interaggregate contact points. Therefore, the specimens compacted at dry of optimum have a lower shear strength as less interaggregate contact area points are wetted than those at wet of optimum since the water content of the specimens compacted at dry of optimum is lower than those at wet of optimum. Vanapalli et al. (1996a) also found out that the rate of increase in the unsaturated shear strength of soil is related to the rate of desaturation characteristics of

the soil. They also observed that there is no different in c' and ϕ' of soil obtained from single stage direct shear test for different shearing displacement rates, different initial water contents and different initial densities.

Figure 2-17. Variation of shear strengths and ϕ^b with respect to matric suction under, a. 25 kPa and b. 200 kPa, net normal stresses for compacted Indian Head glacial till specimen compacted with different initial water content (Vanapalli et al., 1996a)

Rahardjo et al. (1995) conducted a series of multistage CD triaxial tests on the residual soil from the sedimentary Jurong Formation. The experimental results showed the variation of ϕ^b angle with matric suction where ϕ^b decreases significantly as the air-entry value is exceeded. The same results were also observed by Rahardjo et al. (2004) and Thu et al. (2006). In addition, the unsaturated shear strength of compacted residual soil from the sedimentary Jurong Formation and compacted kaolin from CD triaxial tests were found to be higher than those obtained from CW triaxial tests. Recent study by Goh et al. (2010) indicated that the soil specimens on the drying path of SWCC are considered to exhibit normally consolidated shearing behavior while those on the wetting path of SWCC are considered to exhibit overconsolidated shearing behavior.

Fredlund (2000) proposed to divide the cohesive strength of soil into two components, namely true and apparent cohesion. True cohesion come from bonding

between the soil particles such as cementation, electrostatic and electromagnetic attractions, and primary valence bonding when clays become over consolidated. Apparent cohesion is defined as the negative pore water pressure (matric suction) and negative excess pore water pressure due to expansion. The effect of net confining pressure on the shear strength of unsaturated soil was studied by Lee et al. (2005). They conducted a series of CD triaxial tests on a granitic residual soil from Korea. The results showed that the unsaturated shear strength of soil increases as the net confining pressures increases. This happened since the degree of saturation increases as the net confining pressure increases. As a result, the wettest contact area that matric suction acts at the soil surface is larger and the shear strength of soil is also higher. Lee et al. (2005) also observed that ϕ' of soil slightly increases as the net confining pressure increases as shown in Figure 2-18.

Figure 2-18. Variation of shear strengths and ϕ' with respect to matric suction under various confining pressure for compacted granitic residual soil from Korea (Lee et al., 2005)

Houston et al. (2008) studied the shear strength and volume change behaviour of compacted soil from four different type of soils, Price-Club soil (CL-ML), Sheely clay (CL), ASU east (SM) and Yuma sand (SP). The results showed that the soil tends to dilate more if it is sheared under a higher value of matric suction and lower net confining pressure. The dilative behaviour of soil is also affected by the plasticity index of soil. The

soil with high plasticity index tends to compress during shearing except at the highest values of suction.

2.5.2. Previous studies on shear strength characteristic of soils with bimodal characteristics

The study on shear strength of soil with bimodal characteristics is important since rainfall-induced slope failures often occurred in residual and colluvial soils where many of these soils are classified as gap-graded soil or soil with bimodal grain-size distribution (GSD) (Miguel and Bonder, 2012; Rahardjo et al., 2012c; Jotisankasa et al., 2009; Li and Zhang, 2009; Feuerharmel et al., 2006; Zhang and Chen, 2005, Rahardjo et al., 2004, 2008). Generally, soil with bimodal GSD is associated with bimodal SWCC (Li and Zhang, 2009; Zhang and Chen, 2005). A number of experiments have been carried out to study the shear strength of soils with bimodal GSD. However, most of them focused on saturated shear strength of soil (c' and ϕ'). Previous studies showed that the controlling parameter of the saturated shear strength of soils with bimodal GSD is percentage of granular or fines within the soil mixtures (Lupini et al., 1981; Shakoor and Cook, 1990; Georgiannou et al., 1990, Li and Zhang, 2009). Their experimental results showed that the shear strength (c' and ϕ') of soils increases with the increase in the percentage of granular material within soil mixtures.

Irfan and Tang (1993) studied the saturated shear strength of colluviums soil in Hong Kong. The experimental results indicated that the GSD of this soil has bimodal characteristic. Irfan and Tang (1993) also conducted some laboratory experiments to study the effect of the coarse fractions on the shear strength of colluviums soils. In their experiments, different proportions of granular materials were mixed with the fine-grained colluvium. The sizes of the granular materials varied from 5 to 6.3 mm. The fine-grained colluvium, which was sieved from a natural completely decomposed granitic soil from Mount Butler quarry in Hong Kong Island and having maximum grain sizes of 1.18 mm, was classified as silty sand and had clay content less than about 3%. Irfan and Tang (1993) found out that the shear strengths of the granite rock aggregate - fine colluvium mixtures obtained from the triaxial tests were much lower than those obtained from the direct shear tests (Figure 2-19). This happened since the weakest failure path will be

formed within the weaker fine colluvium in the triaxial tests. In the direct shear tests, the failure path is pre-determined and fixed along the box connection and the granite aggregate and the interlocking of the granite aggregate will result in a considerable increase of the measured shear strength. The back analyses of colluvial landslides indicated that the majority of shear strength parameters for normal stresses below 50 kPa are in the ranges of $c' = 5$ kPa to 12 kPa and $\phi' = 27^\circ$ to 42° .

Figure 2-19. Effect of coarse fraction on saturated shear strength obtained from a. direct shear test and b. CU triaxial test (Irfan and Tang, 1993)

Rojas (2008a, b) proposed a solid-porous model to simulate the structure of soil with bimodal characteristic to determine Bishop's parameter, χ . The model used the measurement data of soil grain size distributions, void ratios, and water retention curves of the soil. The shear strength tests results from Rojas (2008a, b) showed a unique failure line in the plots of the deviator stress versus equivalent stress obtained from series of tests conducted at different confining net stresses and suctions. The results confirmed that the proposed Rojas model is sufficient to estimate the shear strength of unsaturated soils with respect to the coupling of the hydraulic and mechanical behavior of soil. However, Rojas equation is not suitable for clayey soils.

Coutinho et al. (2011) conducted a series of unsaturated direct shear tests on granitic residual soils from Brazil. The experimental results showed the bimodal characteristics of GSD and SWCC. It was observed that the unsaturated shear strength of the investigated soils increased with the increase in net normal stress. Coutinho et al.

(2011) also found that the unsaturated shear strength of soils started to decrease for matric suctions higher than air-entry value of soil (Figure 2-20).

Figure 2-20. Variation of shear strengths with respect to matric suction under various net normal stress for granitic residual soil from Brazil (Coutinho et al., 2011)

2.5.3. Shear Strength Equations for Unsaturated Soils

An equation to describe the shear strength of unsaturated soils proposed by Fredlund et al (1978) is given in Equation 2-12. This equation was developed as an extension of the shear strength equation for saturated soil. Equation 2-12 can be used in both saturated and unsaturated soil mechanics and it shows a smooth transition between the shear strength of saturated and unsaturated soils. Under the saturated condition, the pore-water pressure is equal to the pore-air pressure and matric suction is zero. As a result, Equation 2-12 can be simplified into Equation 2-13.

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (2-12)$$

where:

τ = shear strength

c' = effective cohesion

ϕ' = effective angle of internal friction

ϕ^b = angle indicating increase in shear strength due to increase in matric suction

$$\tau = c' + (\sigma - u_w) \tan \phi' \quad (2-13)$$

Fredlund et al. (1978) assumed a constant value of ϕ^b for the entire matric suction of soil. However, further studies indicated that ϕ^b varies with suction. Shen and Yu

(1996) introduced parameter d in their hyperbolic equation (Equation 2-14) to describe the nonlinearity of the unsaturated shear strength of soil. Later, their equation was improved using additional parameter α to show the peak shear strength contribution from matric suction (Equation 2-15). A better equation to describe the unsaturated shear strength was proposed by Vanapalli et al. (1996b). Vanapalli et al. (1996b) proposed a nonlinear form of unsaturated shear strength equation using the entire SWCC (i.e. 0 to 10^6 kPa) and the saturated shear strength parameters (Equation 2-16).

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \left[\frac{1}{1 + (u_a - u_w)d} \right] \tan \phi' \quad (2-14)$$

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \left[\frac{1}{\cot \alpha + \frac{(u_a - u_w)}{\beta_1}} \right] \tan \phi' \quad (2-15)$$

where:

d = fitting parameter

α = initial angle of shear strength with respect to matric suction

β_1 = peak value of shear strength contribution from matric suction

$$\tau = c' + (\sigma - u_a) \tan \phi' + [(u_a - u_w) \Theta^\kappa] \tan \phi' \quad (2-16)$$

where:

$$\Theta = \frac{\theta_w}{\theta_s} \quad (2-17)$$

Θ = normalized volumetric water content

θ_s = saturated volumetric water content

θ_w = volumetric water content at any matric suction

κ = fitting parameter used for obtaining a best fit between the measured and estimated values

Later experimental studies of the unsaturated shear strength of soil showed that the unsaturated shear strength of soil is affected by net normal stress. Hence, Rassam and Williams (1999) suggested to incorporate the effect of net normal stress in the shear strength equation as shown in Equation 2-18.

$$\tau = c' + (\sigma - u_a) \tan \phi' + \psi \tan \phi' - (\psi - \psi_e)^\beta [\gamma + \lambda (\sigma - u_a)] \quad (2-18)$$

where:

ψ_e = air-entry value

β, γ, λ = fitting parameters

Lee et al. (2005) also developed the unsaturated shear strength equation to include the effect of net normal stress changes on the SWCC. Lee et al. (2005) combined Vanapalli et al. (1996b) and Rassam and Williams (1999) equations to obtain the air-entry value under various net normal stresses. The proposed shear strength equation by Lee et al. (2005) is shown in Equation 2-19.

$$\begin{aligned} \tau &= c' + [(\sigma - u_a) + (u_a - u_w)] \tan \phi' \text{ if } (u_a - u_w) \leq AEV \\ \tau &= c' + [(\sigma - u_a) + AEV] \tan \phi' + [(u_a - u_w) - AEV] \Theta^\kappa [1 + \lambda (\sigma - u_a)] \tan \phi' \\ &\text{if } (u_a - u_w) \geq AEV \end{aligned} \quad (2-19)$$

where:

λ = fitting parameter

AEV = air-entry value

Lee et al. (2005) indicated that ϕ^b is equal to ϕ' for the range of matric suction less than air-entry value and all matric suction contributes to the effective stress. Their study also showed that Equation 2-19 performed well in fitting the unsaturated shear strength data under various net normal stresses for low matric suction. This might be due to the fact that macropores of the soil that are dominant at low matric suction range of SWCC are sensitive to the changes in net normal stress. In addition, Lee et al. (2005) also found that the linear relationship between the degree of saturation and the applied net normal stress is only applicable to some soil types. Goh et al. (2010) studied the performance of several published equations in fitting ten sets of published unsaturated shear strength data of soil. They concluded that Vanapalli et al. (1996b) and Lee et al. (2005) equations performed the best among all shear strength equations in fitting the selected unsaturated shear strength data of soil. However, their studies were limited to soils with unimodal characteristic.

2.5.4. Estimation of Shear Strength for Unsaturated Soils

The measurement of the unsaturated shear strength is time consuming and the required apparatuses can be expensive. Therefore, several equations have been developed to estimate the unsaturated shear strength from SWCC and saturated shear strength of soil (c' and ϕ'). Vanapalli et al. (1996b) suggested a mathematical equation by normalizing volumetric water content to estimate the shear strength of the unsaturated soils for the range between zero to residual matric suction. The equation proposed by Vanapalli et al. (1996b) is shown in Equation 2-20 and can be written in terms of gravimetric water content, degree of saturation or volumetric water content.

$$\tau = c' + (\sigma - u_a) \tan \phi' + \left[(u_a - u_w) \left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r} \right) \right] \tan \phi' \quad (2-20)$$

where:

θ_r = residual volumetric water content

Garven and Vanapalli (2006) modified Vanapalli (1996b) fitting equation (Equation 2-16) for estimating the unsaturated shear strength of soil. They found that the fitting parameter, κ , is closely related to the plasticity index, PI, of soil. They suggested to use a polynomial relationship to estimate κ from PI as shown in Equation 2-21.

$$\kappa = -0.0016I_p^2 + 0.0975I_p + 1 \quad (2-21)$$

An equation for predicting the unsaturated shear strength for non-clayey soils, i.e. sand and silt, were developed by Oberg and Sallfors (1997). They found that the unsaturated shear strength of soil is closely related to the pore area of soil that is occupied by water. They used degree of saturation to predict the unsaturated shear strength of soil as shown in Equation 2-22.

$$\tau = c' + [\sigma - Su_w - (1 - S)u_a] \tan \phi' \quad (2-22)$$

where:

S = degree of saturation

Khalili and Khabbaz (1998) observed that the relationship between shear strength and matric suction is linear at matric suction range lower than air-entry value. They proposed to use Fredlund et al. (1978) equation (Equation 2-12) for predicting the unsaturated shear strength of soil up to the air-entry value. In addition, they developed a

mathematical equation (Equation 2-23) to predict the unsaturated shear strength of soil for all types of soil.

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w)(\lambda') \tan \phi' \quad (2-23)$$

where:

$$\lambda = \left\{ \frac{(u_a - u_w)}{AEV} \right\}^{-0.55} \quad (2-24)$$

AEV = air-entry value of soil

Bao et al. (1998) modified Vanapalli et al. (1996b) (Equation 2-20) to predict the shear strength of an unsaturated soil in the transition zone. They assumed a linear variation of SWCC in the transition zone and proposed an equation to predict the unsaturated shear strength of soil.

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \left[\frac{\log(u_a - u_w)_r - \log(u_a - u_w)}{\log(u_a - u_w)_r - AEV} \right] \tan \phi' \quad (2-25)$$

where:

$(u_a - u_w)_r$ = residual matric suction

Bao et al. (1998) assumed the shear strength of soil to be linearly proportional to matric suction, where $\tan \phi^b$ is equal to $\tan \phi'$, at matric suctions lower than air-entry value. At matric suction higher than air-entry value, the relationship between shear strength and matric suction is non-linear.

Villar et al. (2006) suggested a hyperbolic equation to predict the shear strength of an unsaturated soil. Villar (2006) proposed Equation 2-26 to calculate cohesion of soil.

$$c = c' + \frac{\psi}{a + b\psi} \quad (2-26)$$

where:

c = cohesion intercept

ψ = matric suction of the soil

a, b = curve-fitting parameter

The values of a and b can be obtained from the following procedure. When the soil is nearly saturated and matric suction approaches zero,

$$\left. \frac{dc}{d\psi} \right|_{\psi \rightarrow 0} = \frac{1}{a} = \tan \phi' \quad (2-27)$$

If matric suction of soil approaches infinity, the shear strength of soil is assumed to reach the ultimate value.

$$\lim_{\psi \rightarrow \infty} c = c_{ult} = c' + \frac{1}{b} \quad (2-28)$$

$$b = \frac{1}{c_{ult} - c'} \quad (2-29)$$

where:

c_{ult} = cohesion intercept at maximum matric suction

Vilar (2006) equation required the ultimate shear strength from air-dried specimen or the shear strength at any matric suction larger than the maximum matric suction as expected in the analysis and the saturated effective shear strength parameters to predict the shear strength of an unsaturated soil. Rassam and Cook (2002) modified Rassam and Williams (1999) fitting equation (Equation 2-18) to predict the shear strength of an unsaturated soil by incorporating a power additive technique in their shear strength equation. Rassam and Cook (2002) equation requires the shear strength at residual matric suction, the effective friction angle and SWCC to predict the shear strength of an unsaturated soil. In addition, ϕ^b value is assumed to be zero at the residual matric suction and the contribution of matric suction to the shear strength at residual matric suction is used in Rassam and Cook (2002) equation. Their equation is most suitable for coarse to medium grained soils whose residual matric suction could be obtained easily and reliably in laboratory.

Tekinsoy et al. (2004) proposed a mathematical equation to predict the unsaturated shear strength of fine-grained soil using an empirical-analytical logarithmic procedure. In Tekinsoy et al. (2004) equation (Equation 2-30), only a single value of matric suction (air-entry value) is needed to predict the shear strength of an unsaturated soil without using any other water content data.

$$\tau = c' + (\sigma - u_a) \tan \phi' + \tan \phi' (\psi_e + P_{at}) \ln \left[\frac{\psi + P_{at}}{P_{at}} \right] \quad (2-30)$$

where:

P_{at} = atmospheric pressure

Goh et al (2010) modified Vanapalli et al. (1996b) equation to predict the drying and wetting shear strength of an unsaturated soil. They assumed the shear strength of soil to be linearly proportional to matric suction, where $\tan \phi^b$ is equal to $\tan \phi'$, at matric suctions lower than air-entry value (Equation 2-31). At matric suctions higher than air-entry value, the relationship between shear strength and matric suction is non-linear where the calculated logarithm difference of matric suction and the air-entry value of soil increases nonlinearly when matric suction increases (Equation 2-32).

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \text{ if } (u_a - u_w) \leq AEV \quad (2-31)$$

where:

$$\phi^b = \phi'$$

$$\tau = c' + [(\sigma - u_a) + AEV] \tan \phi' + [(u_a - u_w) - AEV] b \Theta^\kappa \tan \phi' \text{ if } (u_a - u_w) > AEV \quad (2-32)$$

where:

$$\kappa = [\log(u_a - u_w) - \log AEV]^y \quad (2-33)$$

$$y_d = 0.502 \ln(PI + 2.7) - 0.387 \quad (2-34)$$

$$b_d = -0.245 \left\{ \ln[n_d(PI + 4.4)] \right\}^2 + 2.114 \left\{ \ln[n_d(PI + 4.4)] \right\} - 3.522 \quad (2-35)$$

Sheng et al. (2009 and 2011) studied different types of equation to estimate the shear strength of unsaturated soil. They concluded that all shear strength equations can be divided into two categories. The first shear strength equation was developed based on empirical relationship whereas the second one was developed based on elastoplastic constitutive model. The first category of shear strength equation was proposed by Fredlund et al. (1978), Vanapalli et al. (1996b), Oberg and Sallfors (1997), Bao et al. (1998), Khalli and Khabbaz (1998), Rassam and Williams (1999), Rassam and Cook (2002), Tekinsoy et al. (2004), Lee et al. (2005), Garven and Vanapalli (2006), Vilar (2006) and Goh et al. (2010). The second category of shear strength equation was

proposed by Bishop and Blight (1963), Alonso et al. (1990), Sun et al. (2000), Toll and Ong (2003) and Sheng et al. (2008).

The study by Sheng et al. (2009) also indicated that the shear strength equations using empirical relationship can be formulated in the form of Fredlund et al. (1978) equation whereas the shear strength equations using an elastoplastic constitutive model can be formulated in the form of Bishop and Blight (1963) equation. Sheng et al. (2009) found that the unsaturated shear strength of soil estimated using SWCC provide more reasonable prediction of shear strength compared to other shear strength equations.

Zhao et al. (2013) investigated the shear strength behaviour of soils with bimodal characteristics. They also developed model for estimating shear strength of soil with bimodal characteristics. However, their research works focused on coarse-grained soils.

In general, most of the published shear strength equations are limited to soils with unimodal SWCC. In this study, the new equation to best fit the bimodal shear strength of the unsaturated coarse-grained and fine-grained soils was developed. The relationships between the variables and fitting parameters of the new shear strength equation and the variables of SWCC and soil properties were investigated. Then, the procedure for estimating the bimodal shear strength was proposed.

2.6. Review of Laboratory Procedures for Measurement of Unsaturated Soil Properties

2.6.1. Suction Controlled Technique

The negative pore-water pressure in unsaturated soil causes the difficulties in the unsaturated soil testing when the negative pore-water pressure approaches -1 atm (i.e. zero absolute pressure) since water starts to cavitate when the water pressure approaches -1 atm (i.e. -101.3 kPa gauge). Fredlund and Rahardjo (1993) indicated that cavitation occurs when the air starts to fill the measuring system which results in the pore-water pressure and pore-water volume can not be controlled and measured accurately. Lu and Likos (2004) defined cavitation as the process of phase translation from liquid phase to vapour phase along a path of decreasing pressure (Figure 2-21). Cavitation could occur at a higher pressure under a higher temperature.

The cavitation of water in unsaturated soil testing can be prevented using axis translation technique which was developed by Hilf (1956). Hilf (1956) proposed to move the reference point, pore-air pressure, to a positive value to apply matric suction during unsaturated soil testing. In other words, the pore-water pressure is raised from negative to positive value to control matric suction up to a high value during SWCC and unsaturated shear strength tests. This technique has been applied successfully for measurement of SWCC, unsaturated permeability, unsaturated shear strength and volume change by many researchers (Bishop and Donald, 1961; Matyas and Radhakrishna, 1968; Fredlund et al., 1979; Gulhati and Satija, 1981; Josa et al., 1987; Gan et al., 1988; Alonso et al., 1990; Alonso and Delage, 1995; Wheeler and Sivakumar, 1995; Vanapalli et al., 1996b; Agus et al., 2003; Goh et al., 2010; Satyanaga et al., 2012). The application of axis translation technique in controlling suction is carried out by incorporating the high air-entry ceramic disk during unsaturated testing. The ceramic disk is required to separate the air and water phases and to allow the independent control of the water and air pressures up to the air-entry value of the ceramic disk (Fredlund et al., 2012; 1993a).

Figure 2-21. SWCCs of soil specimens with (a) 25 kPa preconsolidation pressure and (b) 200 kPa preconsolidation pressure, which were compacted at different initial water contents (Vanapalli et al., 1996a)

2.6.2. Shearing Stage in Triaxial Tests

In triaxial test, the shearing process is commonly performed using a constant strain rate. The suitable strain rate must be selected carefully prior to triaxial test in order to obtain accurate stress-strain results. For consolidated drained triaxial test, a low shearing rate is required to allow the dissipation of induced excess pore-water pressure due to the axial compression stress on soil specimen. In other words, the appropriate strain rate is necessary to maintain the drained condition of soil during shearing process (Lim et al., 1996).

A strain rate of 0.009 mm/min was selected by Miao et al. (2002) for measurements of the unsaturated shear strength of Nanyang expansive soil from Henan, Province, China using consolidated drained triaxial test. Lee et al. (2005) selected a strain rate of 0.006 mm/min to obtain the mechanical behaviour of the compacted Korea Weathered Granitic residual soil (silty sand) using consolidated drained triaxial test. A low strain rate of 0.0009 mm/min was normally selected by many researchers, i.e. Rahardjo et al. (1995), Lee et al. (1996), Rahardjo et al. (2004), Meilani et al. (2005), Thu et al. (2006) and Goh et al. (2010) for measurements of the unsaturated shear strength of residual and compacted soils using consolidated drained triaxial test. Their soils varied from silty sand to clay material.

2.7. Summary

- a. Previous studies indicated that most of residual and colluvial soils show the bimodal characteristics of their grain-size distributions. Some of these soils are associated with bimodal SWCC while others are associated with unimodal SWCC. Since majority of residual and colluvial soils are located in unsaturated zone above groundwater table, it is important to carry out a thorough study on the unsaturated properties of such soils with bimodal grain-size distribution. In this study, soil characterizations will be carried out to obtain the SWCC and shear strength of the unsaturated soil with bimodal GSD.
- b. SWCC is one of the most important properties in unsaturated soil. The shape and variables of SWCC depend on many factors, such as: grain-size distribution, soil

type, fine contents, initial water content, initial dry density and confining pressure of soil. Several equations have been developed by many researchers to best fit the SWCC. However, most of the equations were developed to best fit the SWCC with unimodal characteristics and the parameters in most of the equations do not represent the variables of SWCC. Therefore, there is a need to develop new SWCC equations to best fit the unimodal and bimodal SWCCs. The parameters of the proposed equations can be used to represent the variables of SWCC. In this study, new SWCC equations will be developed and the proposed SWCC equations shall be applicable for a wide range of soil types (coarse and fine-grained soils). In addition, the new equations for best fitting unimodal and bimodal GSD will also be proposed in this study.

- c. Several equations have been developed by many researchers to estimate the SWCC. However, majority of the equations were developed on the basis of soil with unimodal feature of GSD. In addition, the equation was only applicable for a particular type of soil. Therefore, there is a need to develop new equations to estimate the SWCC of soils with bimodal GSD. The parameters of the proposed equations shall be related to soil properties and variables of SWCC. In this study, the procedures and the equations to estimate the SWCC of soils with bimodal GSD for a wide range of soil type will be proposed. The proposed equations can also be used to determine whether soil with bimodal GSD will have a unimodal or bimodal SWCC.
- d. Previous studies indicated that the shear strength of soil with respect to suction was linear for suction less than air-entry value and non-linear for suction higher than air-entry value. Most of these equations were developed to explain the shear strength of unsaturated soil. The equations have been proved to be able to estimate the shear strength of unsaturated soil. However, previous studies were limited to soils with unimodal SWCC. Therefore, the equation to explain the shear strength of unsaturated soil with bimodal SWCC will be presented in this study. The parameter in the proposed shear strength equation can be related to SWCC and plasticity of soil in order to estimate the shear strength of unsaturated soil.

- e. Many experimental studies have been carried out to obtain the SWCC and shear strength data of unsaturated soil. However, only limited data available for soils with bimodal characteristics, especially bimodal shear strength data. In this study, properties of different type of soil with bimodal characteristic will be investigated.

CHAPTER 3 THEORETICAL MODEL

3.1. Introduction

In this chapter the theoretical developments are presented. The development of mathematical models for grain-size distribution (GSD), soil-water characteristic (SWCC) and unsaturated shear strength of soil with bimodal characteristics are presented.

3.2. Proposed Mathematical Model of Soil-Water Characteristic Curve

SWCC plays an important role in describing the engineering behaviour of unsaturated soil. SWCC is commonly defined as the relationship between volumetric water content, θ_w and matric suction, $u_a - u_w$ and this relationship is used throughout this research. SWCC can be obtained from laboratory tests or field measurements. Common terms used to represent the variables of unimodal SWCC are as follows: saturated volumetric water content (θ_s), air-entry value (AEV), ψ_a , residual suction, ψ_r , inflection point, ψ_m and residual volumetric water content (θ_r). For soil with bimodal SWCC, it is common to add subscript 1 and 2 to θ_s , ψ_a and ψ_m to represent saturated volumetric water content, air-entry value and inflection point of macropore (subcurve 1) and micropore (subcurve 2) of soil, respectively. These terms of SWCC variables were used throughout this research.

Figure 3-1. Typical unimodal and bimodal SWCC with their corresponding variables

Representation of SWCC variables can be achieved through fitting of SWCC data using a mathematical model. The air-entry value of soil is usually obtained from the intersection point between the tangent line at the beginning of SWCC from the saturated volumetric water content and the tangent line at the inflection point of SWCC (Figure 3-1). This technique has been widely used and adopted in various literatures, e.g. Fredlund and Xing (1994), Vanapalli et al. (1996b), Goh et al. (2010). The air-entry value obtained using this technique is considered as the “least possible” air-entry value and is used for the consistency in the data interpretation. In a bimodal SWCC, the first air-entry value of soil is commonly obtained from the intersection point between the tangent line at the beginning of SWCC from the saturated volumetric water content and the tangent line at the first inflection point of SWCC. The second air-entry value of soil with bimodal characteristic can be obtained from the intersection point between the tangent line at the beginning of the second subcurve, representing micropore of soil, and the tangent line at the second inflection point of SWCC (Figure 3-1).

Vanapalli et al. (1997b) suggested dividing SWCC with unimodal characteristics into three zones, i.e. boundary effect zone, transition zone and residual zone. In boundary effect zone, there is no significant change in volumetric water content as matric suction is increased from zero to air-entry value within soil with unimodal SWCC. This boundary effect zone is also observed within soil with bimodal SWCC (Figure 3-2). In transition zone, the volumetric water content decreases significantly and steadily to the residual water content for matric suctions between the air-entry value and the residual suction. This transition zone is also observed within soil with bimodal SWCC. However, the study by Delage (2002) indicated that water starts to drain out significantly from macropores for matric suctions between the first air-entry value and the second air-entry value of bimodal SWCC while the micropore is still in a saturated condition. Water in the micropores starts to drain out significantly for matric suctions between the second air-entry value and the residual suction of bimodal SWCC. Therefore, transition zone in a bimodal SWCC is divided into two zones. The zone between the first air-entry value and the second air-entry value is called macro transition whereas the zone between the second air-entry value and the residual suction is called micro transition. The residual volumetric water content is the water content at a residual state when the water phase is

discontinuous and does not exhibit significant change in volume with a further increase in matric suction.

Figure 3-2. Proposed zone for SWCC with bimodal characteristic

Two types of SWCC equations were proposed in this study. The first type of SWCC equation was developed based on lognormal distribution function whereas the other type of SWCC equation was developed based on exponential function. The first type of the proposed SWCC equation based on lognormal distribution function was developed by considering several factors, such as:

- The parameters in the proposed SWCC equation should represent the physical properties of the soil and variables of SWCC, such as: the air-entry value of soil, the inflection point of SWCC, residual suction and residual water content of soil.
- The proposed SWCC equation was developed to have a unique value for each SWCC parameter regardless the initial value used in the best fitting process of SWCC data.
- The water content of soil is set equal to zero when suction of soil reaches 10⁶ kPa (based on thermodynamic consideration as reported by Richards, 1965). Therefore, the correction function from Fredlund and Xing (1994) best fitting equation is adopted in the SWCC equation proposed in this study.

- d. The shape of SWCC is similar to lognormal distribution. Therefore, Kosugi (1994) equation is used as a basis for the development of the SWCC equation proposed in this study.
- e. Water content when soil is fully saturated (saturated water content, w_{sat}) must be measured in the laboratory.

The second type of the proposed SWCC equation based on exponential function was developed by considering the following factors:

- a. The proposed SWCC equation should consist of minimum number of best fitting parameters which are less than the number of best fitting parameters of the published SWCC equations from literatures.
- b. Similar to the first type of the proposed SWCC equations, the second type of the proposed SWCC equation was also developed to have a unique value for each SWCC parameter regardless the initial value used in the best fitting process of SWCC data.
- c. Similar to the first type of the proposed SWCC equations, the water content of soil in the second type of the proposed SWCC equation is set equal to zero when suction of soil reaches 10^6 kPa (based on thermodynamic consideration as reported by Richards, 1965).
- d. Water content when soil is fully saturated (saturated water content, w_{sat}) must be measured in the laboratory.

3.2.1. Proposed Equation of Soil-Water Characteristic Curve based on Lognormal Distribution Function

In this section, the development of a new equation for bimodal soil-water characteristic curve based on lognormal distribution function is described. The proposed SWCC equations based on lognormal distribution function has been published in Satyanaga et al. (2013). Figure 3-3 shows that the unimodal and bimodal capillary pressure distribution functions are similar to unimodal and bimodal lognormal distribution functions as shown in Figure 3-4. Therefore, the SWCC equation can be derived from lognormal distribution function (Equation 3-1) with some modifications for best fitting the SWCC accurately.

Figure 3-3. Capillary pressure distribution curve

Figure 3-4. Lognormal distribution curve

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}(\log x - \mu)^2\right) \quad (3-1)$$

where:

μ = mean

σ = standard deviation

For unimodal curve, Kosugi suggested Equation 3-2 for capillary pressure distribution function:

$$f(\psi) = \frac{(\theta_s - \theta_r)}{(\psi_a - \psi)\sigma\sqrt{2\pi}} \exp\left(-\frac{\left(\ln\left(\frac{\psi_a - \psi}{\psi_a - \psi_m}\right) - \mu\right)^2}{2\sigma^2}\right) \quad (3-2)$$

where:

σ = standard deviation of capillary pressure distribution function

θ_s = saturated volumetric water content

θ_r = residual volumetric water content

ψ_a = air-entry value

ψ_m = inflection point

The equation for fitting unimodal SWCC was obtained from integrating the capillary pressure distribution function. Capillary pressure distribution function is shown in Equation 3-3.

$$f(\psi) = d\theta / d\psi \quad (3-3)$$

Integrating Equation 3-3 yields Equation 3-4.

$$\theta(\psi) = \int_{-\infty}^{\psi} f(\psi) d\psi + \theta_r \quad (3-4)$$

Brooks and Corey suggested the relationship between effective saturation and volumetric water content as shown in Equation 3-5.

$$S_e(\psi) = \frac{\theta(\psi) - \theta_r}{\theta_s - \theta_r} \quad (3-5)$$

Substituting Equation 3-4 into Equation 3-5 produced Equation 3-6.

$$S_e(\psi) = \frac{1}{\theta_s - \theta_r} \int_{-\infty}^{\psi} f(\psi) d\psi \quad (3-6)$$

Integrating Equation 3-2 yielded Equation 3-7.

$$\int_{-\infty}^{\psi} f(\psi) d\psi = 1 - \operatorname{erfc} \left(- \frac{\ln \left(\frac{\psi_a - \psi}{\psi_a - \psi_m} \right)}{\sigma} \right) \quad (3-7)$$

where:

$$\operatorname{erfc} = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp \left(- \frac{x^2}{2} \right) dx \quad (3-8)$$

Substituting Equation 3-7 into Equation 3-6 produced Equation 3-9.

$$S_e(\psi) = \frac{1}{\theta_s - \theta_r} \left[1 - \operatorname{erfc} \left(-\frac{\ln \left(\frac{\psi_a - \psi}{\psi_a - \psi_m} \right)}{\sigma} \right) \right] \quad (3-9)$$

The proposed equation for fitting unimodal SWCC (Equation 3-10) was obtained by substituting Equation 3-9 into Equation 3-5.

$$\theta(\psi) = \left[\theta_r + \left\{ (\theta_s - \theta_r) \left(1 - \operatorname{erfc} \left(\frac{\ln \left(\frac{\psi_a - \psi}{\psi_a - \psi_m} \right)}{s} \right) \right) \right\} \right] \quad (3-10)$$

Based on thermodynamic considerations reported by Richards (1965), the water content of soil is equal to zero when suction of the soil reaches 10^6 kPa. Therefore, the correction function proposed by Fredlund and Xing (1994) was incorporated into Equation 3-10 to obtain the proposed equation for best fitting unimodal SWCC as shown in Equation 3-11.

$$\theta_w = \left[1 - \frac{\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{10^6}{\psi_r} \right)} \right] \left[\theta_r + \left\{ (\theta_s - \theta_r) \left(1 - (\beta) \operatorname{erfc} \left(\frac{\ln \left(\frac{\psi_a - \psi}{\psi_a - \psi_m} \right)}{s} \right) \right) \right\} \right] \quad (3-11)$$

where:

β = 0 when $\psi \leq \psi_a$; $\beta = 1$ when $\psi > \psi_a$

θ_w = calculated volumetric water content

θ_s = saturated volumetric water content

ψ = matric suction under consideration (kPa)

ψ_a = parameter represents the air-entry value of soil (kPa)

ψ_m = parameter represents the matric suction at the inflection point of SWCC (kPa)

s = parameter represents the geometric standard deviation of SWCC

$$s = (\theta_{s2}) \exp \sqrt{\frac{\sum_{i=1}^n \left(\ln \frac{\psi_i}{\mu} \right)^2}{n}} \quad (3-12)$$

μ = geometric mean of matric suction

$$\mu = \sqrt[n]{\psi_1 \psi_2 \dots \psi_n} \quad (3-13)$$

θ_r = parameter represents the residual volumetric water content

ψ_r = parameter represents the matric suction corresponding to residual volumetric water content (kPa)

$$\text{erfc} = \text{the complimentary error function} = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx \quad (3-14)$$

All parameters in the proposed SWCC equation for best fitting unimodal SWCC are related to the variables of SWCC as shown in Figure 3-1.

Figure 3-4 indicates that the bimodal capillary pressure distribution function is similar to bimodal lognormal distribution function. Therefore, the SWCC equation for best fitting SWCC with bimodal characteristic could be generated from the integration of the bimodal capillary pressure distribution function. The proposed equation for bimodal SWCC is shown in Equation 3-15.

$$\theta_w = \left(1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)} \right) \left[\theta_r + (\theta_{s1} - \theta_{s2}) \left(1 - (\beta_1) \text{erfc} \frac{\ln\left(\frac{\psi_{a1} - \psi}{\psi_{a1} - \psi_{m1}}\right)}{s_1} \right) \right] + (\theta_{s2} - \theta_r) \left(1 - (\beta_2) \text{erfc} \frac{\ln\left(\frac{\psi_{a2} - \psi}{\psi_{a2} - \psi_{m2}}\right)}{s_2} \right) \quad (3-15)$$

where:

β_1 = 0 when $\psi \leq \psi_{a1}$; $\beta_1 = 1$ when $\psi > \psi_{a1}$

β_2 = 0 when $\psi \leq \psi_{a2}$; $\beta_2 = 1$ when $\psi > \psi_{a2}$

θ_w = calculated volumetric water content

θ_s = saturated volumetric water content

ψ = matric suction under consideration (kPa)

ψ_a = parameter represents the air-entry value of soil (kPa)

ψ_m = parameter represents the matric suction at the inflection point of SWCC (kPa)

s = parameter represents the geometric standard deviation of SWCC

Subscripts 1 and 2 represent subcurve 1 (associated with macro pore) and subcurve 2 (associated with micro pore), respectively. All parameters of the proposed SWCC equation represent the variables of SWCC as shown in Figure 3-5.

Figure 3-5. Soil-water characteristic curve with bimodal characteristics

3.2.2. Proposed Equation of Soil-Water Characteristic Curve based on Exponential Function

A new equation of bimodal SWCC based on lognormal distribution function is proposed in Section 3.2.1. The proposed equation for best fitting unimodal SWCC based on lognormal distribution function (Equation 3-11) involves five parameters whereas the proposed equation for best fitting bimodal SWCC based on lognormal distribution function (Equation 3-15) involves nine parameters. Many parameters used in the proposed SWCC equations developed based on lognormal distribution function in Section 3.2.1 are very useful for representing the variables of SWCC. However, too many parameters used in the SWCC equation may cause difficulties during the convergence process for best fitting SWCC data. Therefore, another type of best fitting SWCC equation with few best fitting parameters is also proposed in this study.

The proposed SWCC equations with few best fitting parameters were developed based on the consideration that the shape of SWCC is also similar to the graph of the exponential function (Figure 3-6). The proposed equations were developed for best fitting bimodal and unimodal SWCC which consisted of six and three best fitting parameters,

respectively. The proposed SWCC equations based on exponential function were also developed to have a unique value for each SWCC parameter regardless the initial value used in the best fitting process of bimodal and unimodal SWCC data. However, the parameters in these equations can not be used to represent the SWCC variables.

Figure 3-6. Graph of exponential function $f(x) = \exp(-x)$

Figure 3-6 shows the values of the exponential function are within the range of [0, 1]. Therefore, the equation for SWCC can be expressed as:

$$\Theta = \exp[-F(\psi)] \quad (3-16)$$

where:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \text{normalized volumetric water content} \quad (3-17)$$

$F(\psi)$ = a function of matric suction. Parametric study was carried out to modify $F(\psi)$ for best fitting the SWCC data at matric suctions higher than air-entry value accurately. The parametric study showed that the exponential function could be modified into Equation 3-18 for best fitting unimodal SWCC.

$$F(\psi) = \begin{cases} 0 & \text{for } \psi \leq A \\ 5 \left(\frac{\psi - A}{R - A} \right) & \text{for } \psi > A \end{cases} \quad (3-18)$$

The thermodynamic theory was also applied to this equation where water content goes to zero at matric suction equal to 10^6 kPa. Therefore, the Fredlund and Xing correction factor was also incorporated in the proposed SWCC equation based on

exponential function. The proposed SWCC equation based on exponential function for best fitting unimodal SWCC is shown in Equation 3-19.

$$\theta(\psi) = \left[1 - \frac{\ln\left(1 + \frac{\psi}{D_v}\right)}{\ln\left(1 + \frac{10^6}{D_v}\right)} \right] \left\{ (\theta_s - R) \exp\left[-5\left(\frac{\psi - A}{R - A}\right)\right] + \theta_r \right\} \quad (3-19)$$

where:

θ_s = saturated volumetric water contents

D_v = residual volumetric water contents

A = air-entry value of soil

R = residual matric suction

For bimodal SWCC, a superposition of two exponential functions was made, and the proposed SWCC equation based on exponential function for best fitting bimodal SWCC has the following form:

$$\theta(\psi) = \left[1 - \frac{\ln\left(1 + \frac{\psi}{D_v}\right)}{\ln\left(1 + \frac{10^6}{D_v}\right)} \right] \left\{ (\theta_{s1} - \theta_{s2}) \exp\left[-5\left(\frac{\psi - A_1}{R_1 - A_1}\right)\right] + \right. \\ \left. (\theta_{s2} - R) \exp\left[-5\left(\frac{\psi - A_2}{R_2 - A_2}\right)\right] \theta_r \right\} \quad (3-20)$$

where:

θ_{s1} and θ_{s2} = saturated volumetric water contents in the first and second subcurves, respectively

D_v = residual volumetric water contents in the second subcurve

A_1 and A_2 = air-entry values of soil in the first and second subcurves, respectively

R_1 and R_2 = residual matric suctions in the first and second subcurves, respectively

3.3. Proposed Equation for Best Fitting Grain-size Distribution

The grain-size distribution (GSD) curve with unimodal characteristics is commonly associated with well-graded soil and uniform soil whereas the GSD curve with bimodal characteristics is associated with gap-graded soil. The interpretation of the variables of the GSD of soil (d_{10} , d_{30} , d_{60} , c_u , c_c , d_{max}) is carried out manually using the GSD curve.

However, this procedure is not always accurate and time consuming. Modeling GSD with a suitable equation provides solution for this problem. In addition, the mathematical equation of GSD can be used as a basis for estimation of SWCC and the entire GSD curve can be modeled as a continuous function using the mathematical equation. In this study, the equation for best fitting GSD of soil was developed based on lognormal distribution function. The proposed GSD equation was developed by modifying lognormal distribution function to be used as best fitting equation for GSD data. The best fitting parameters of this equation can be used to estimate the SWCC reasonably.

3.3.1. Proposed Mathematical Model of Grain-Size Distribution Curve based on Lognormal Distribution Function

The percentages of soil retained at different particle sizes are shown in Figure 3-7. It was observed that Figure 3-7 is similar to lognormal distribution function shown in Figure 3-4. The integration of the percentages of soil retained at different particle sizes is equivalent to the percentages of percent passing at different particles size (grain-size distribution curve). Therefore, the equation for best fitting GSD can be derived using lognormal distribution function (Equation 3-1). Since the shape of GSD curve is similar to that of SWCC, the characteristic of the proposed GSD equation is also similar to the proposed SWCC equation (Equation 3-15). However, the GSD is plotted in the reversed scale of the SWCC. Therefore, the correction factor in the proposed SWCC equation was modified and incorporated in the proposed GSD equation (see Equation 3-21). The proposed GSD equations for best fitting bimodal and unimodal GSD also consist of parameters which represent the variables in GSD of soil, i.e. maximum particle diameter of soil, standard deviation of GSD and percentage of particles within soil mixtures (Figure 3-8 and Figure 3-9).

Figure 3-7. Percentages of soil retained at different particle sizes

Figure 3-8. Grain-size distribution with unimodal characteristics

The proposed GSD equation for best fitting bimodal GSD is as follows:

$$P = \left[1 - \frac{\ln\left(1 + \frac{d_r}{d}\right)}{\ln\left(1 + \frac{d_r}{d_{\min}}\right)} \right] \left[W_1 \left(\operatorname{erfc} \frac{\ln\left(\frac{d_{\max 1} - d_{m1}}{d_{\max 1} - d}\right)}{s_{d1}} \right) + W_2 \left((\beta_1) + (\beta_2) \operatorname{erfc} \frac{\ln\left(\frac{d_{\max 2} - d_{m2}}{d_{\max 2} - d}\right)}{s_{d2}} \right) \right] \quad (3-21)$$

where:

$\beta_1 = 1$ when $d \leq d_{\max 2}$; $\beta_1 = 0$ when $d > d_{\max 2}$

$\beta_2 = 0$ when $d \leq d_{\max 2}$; $\beta_2 = 1$ when $d > d_{\max 2}$

P = cumulative grain-size distribution of soil

d = soil particle diameter under consideration (mm)

$d_{\max}$ = parameter represent the maximum diameter of soil particle (mm)

d_m = parameter represent the geometric mean of soil particle diameter (mm)

W = parameter represent the the percentage of soil particle within coarse- or fine-grained particles

s_d = parameter represent the geometric standard deviation of GSD curve

$d_{\min}$ = parameter represent the minimum particle size that can be measured using hydrometer analysis (typical value = 0.0001 mm)

d_r = parameter represent the maximum particle size of fine particles

Figure 3-9. Grain-size distribution with bimodal characteristic

$$s_d = (\%d_{0.001}) \exp \sqrt{\frac{\sum_{i=1}^n \left(\ln \frac{d_i}{d_m} \right)^2}{n}} \quad (3-22)$$

d_m = geometric mean of soil particle

$\%d_{0.001}$ = percentage of soil particle with diameter smaller than 0.001 mm

$$d_m = \sqrt[n]{d_1 d_2 \dots d_n} \quad (3-23)$$

Subscript 1 and 2 represent subcurves 1 and 2, respectively.

If GSD of soil has unimodal characteristics, W_2 is equal to zero, W_1 is equal to one and Equation 3-21 becomes:

$$P = 100 \left[1 - \frac{\ln\left(1 + \frac{d_r}{d}\right)}{\ln\left(1 + \frac{d_r}{d_{\min}}\right)} \right] \left[(\beta_1) + (\beta_2) \operatorname{erfc} \frac{\ln\left(\frac{d_{\max} - d_m}{d_{\max} - d}\right)}{s_d} \right] \quad (3-24)$$

where:

$$\beta_1 = 1 \text{ when } d \leq d_{\max}; \beta_1 = 0 \text{ when } d > d_{\max}$$

$$\beta_2 = 0 \text{ when } d \leq d_{\max}; \beta_2 = 1 \text{ when } d > d_{\max}$$

All parameters in the proposed GSD equation represent the variables of GSD curve as shown in Figure 3-8 and Figure 3-9.

3.4. Proposed Equations for Shear Strength of Unsaturated Soil with Unimodal and Bimodal Characteristic

The failure envelope for unsaturated soil can be established by plotting the Mohr circles in a three-dimensional form (Figure 3-10). This three-dimensional plot is an extension of the two dimensional Mohr-coulomb failure envelope of a saturated soil, with the additional third dimensional axis for matric suction, called the extended Mohr-Coulomb failure envelope. The frontal plane of the extended Mohr-Coulomb failure envelope is similar to the Mohr-Coulomb failure envelope for a saturated soil, with $(\sigma - u_a)$ axis becomes $(\sigma - u_w)$ axis, as u_w equals to u_a in a saturated soil. An increase in τ with respect to $(\sigma - u_a)$ is characterized by ϕ' whereas an increase in τ with respect to $(u_a - u_w)$ is expressed by ϕ^b . The shear strength of an unsaturated soil increases as the net normal stress or matric suction increases.

Figure 3-10. Extended Mohr-coulomb failure envelope for an unsaturated soil (after Fredlund and Rahardjo, 1993)

Numerous researchers (Fredlund et al., 1978; Shen and Yu, 1996; Vanapalli et al., 1996b; Rassam and Cook, 2002; Lee et al., 2005; Vilar, 2006 and Goh et al., 2010) have developed equations for fitting the laboratory data of the unsaturated shear strength of soil. However, all of them focused on the development of the equation for shear strength of soil with unimodal characteristics. Up to date, based on the author knowledge, no equation was developed for fitting the unsaturated shear strength of soil with bimodal characteristics. The research works on the shear strength of soil with bimodal SWCC were limited to shear strength of saturated soil (Li and Zhang, 2009; Irfan and Tang, 1993; Shakoore and Cook, 1990; Georgiannou et al., 1990, Lupini et al, 1981).

The information on the shear strength of soil with bimodal characteristics is crucial for practical purposes as many residual and colluvial soils are associated with bimodal characteristics. Since residual and colluvial soils are mainly found in tropical areas with high intensities of rainfall, a proper mathematical model are necessary to represent the unsaturated shear strength of these soils. The inappropriate unsaturated shear strength model may lead to the erroneous slope stability analyses. The time, costs and efforts needed to determine the unsaturated shear strength of soil with bimodal characteristics are excessive. Therefore, the new equation to estimate the bimodal shear

strength of the unsaturated soil is also required to reduce the required time in the unsaturated shear strength tests.

The proposed shear strength equation was developed with the consideration of a smooth transition of shear strength equation from bimodal characteristics into unimodal characteristics and a smooth transition from unsaturated shear strength to saturated shear strength. The proposed equation for fitting the unsaturated shear strength of soil should have the following criteria:

- a. To be able to best fit shear strengths for a wide range of soil type.
- b. To be able to best fit shear strengths of soil for matric suction up to residual suction value as obtained from SWCC.
- c. To be able to best fit shear strengths of soil with bimodal SWCC.
- d. To be able to best fit shear strengths of soil with unimodal SWCC.

The proposed general equation for best fitting the shear strength of unsaturated soil with unimodal and bimodal characteristics was developed based on Fredlund et al. (1978) (Equation 3-25). Therefore, the proposed shear strength equation can provide a smooth transition from unsaturated shear strength to saturated shear strength. During development of the proposed shear strength equation, Vanapalli et al.'s (1996b) physical model that identifies the three different saturation stages during the desaturation of a soil, i.e. boundary, transition and residual stages, is adopted (Figure 3-2).

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (3-25)$$

At matric suction lower than air-entry value (AEV), soil is in the boundary stage and assumed to be in saturated condition (Figure 3-2). Therefore, the assumptions of ϕ^b angle being equal to ϕ' at matric suctions lower than AEV (Figure 3-11) and all matric suctions that are in the range below the AEV contribute to the effective stress are incorporated in the proposed shear strength equation. In order to describe the condition in the boundary stage, Equation 3-25 can be rewritten into Equation 3-26.

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi' \quad (3-26)$$

For soil with bimodal characteristics, at matric suction higher than the first air-entry value (AEV1) and lower than the second air-entry value (AEV2), soil is in the transition stage 1 (Figure 3-2). At transition stage 1, the micropore and macropore of soil are assumed to be in saturated and unsaturated conditions, respectively. Therefore, the

relationship between shear strength and matric suction exhibits a linear behaviour similar to that in the boundary stage. However, the ϕ^b angle in the transition stage 1 is less than ϕ^b angle in the boundary stage (Figure 3-11). In order to describe the condition in the transition stage 1, Equation 3-26 can be written into Equation 3-27.

$$\tau = c' + (\sigma - u_a) \tan \phi' + AEV1 \tan \phi' + (u_a - u_w) \tan(b\phi') \quad (3-27)$$

where:

b = parameter representing the ratio of ϕ^b angle to ϕ' at the transition stage 1

At matric suction higher than AEV2, soil is in the transition stage 2 (Figure 3-2) where the relationship between shear strength and matric suction exhibits a non-linear behavior (Figure 3-11). Therefore, the ϕ^b angle at matric suction higher than AEV2 varies with matric suction (Figure 3-11). In order to describe the condition in the transition stage 2, Equation 3-27 can be written into Equation 3-28.

$$\tau = c' + (\sigma - u_a) \tan \phi' + AEV1 \tan \phi' + AEV2 \tan(b\phi') + (u_a - u_w) \tan(k\phi') \quad (3-28)$$

where:

k = parameter representing the ratio of ϕ^b angle to ϕ' at the transition stage 2

Figure 3-11. The proposed shear strength of an unsaturated soil with bimodal characteristics

One advantage of the proposed shear strength equation for unsaturated soil developed in this study is a smooth transition of shear strength of soil from bimodal

characteristics into unimodal characteristics and a smooth transition from unsaturated shear strength into saturated shear strength. Therefore, Equation 3-28 must be modified into Equation 3-29 in order to accommodate this smooth transition.

$$\tau = c' + (\sigma - u_a) \tan \phi' + AEV1 \tan \phi' + 2(u_a - u_w) \tan(b\phi') - AEV1 \tan(b\phi') - AEV2 \tan(b\phi') + AEV2 \tan(k\phi') - (u_a - u_w) \tan(k\phi') \quad (3-29)$$

Equation 3-29 can be rearranged into Equation 3-30.

$$\tau = c' + [(\sigma - u_a) + AEV1] \tan \phi' + [2(u_a - u_w) - AEV1 - AEV2] \tan(b\phi') + [AEV2 - (u_a - u_w)] \tan(k\phi') \quad (3-30)$$

Some constraints must be fulfilled in using Equation 3-30 for best fitting the shear strength of soil successfully.

- a. At matric suction lower than AEV1, b and k = 1. Therefore, Equation 3-30 becomes

$$\tau = c' + [(\sigma - u_a) + AEV1] \tan \phi' + [2(u_a - u_w) - AEV1 - AEV2] \tan(\phi') + [AEV2 - (u_a - u_w)] \tan(\phi') \quad (3-31)$$

or

$$\tau = c' + (\sigma - u_a) \tan \phi' + AEV1 \tan \phi' + 2(u_a - u_w) \tan(\phi') - AEV1 \tan(\phi') - AEV2 \tan(\phi') + AEV2 \tan(\phi') - (u_a - u_w) \tan(\phi') \quad (3-32)$$

or

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan(\phi') \quad (3-33)$$

- b. At matric suction between AEV1 and AEV2, k = b. Therefore, equation 3-30 becomes

$$\tau = c' + (\sigma - u_a) \tan \phi' + AEV1 \tan \phi' + 2(u_a - u_w) \tan(b\phi') - AEV1 \tan(b\phi') - AEV2 \tan(b\phi') + AEV2 \tan(b\phi') - (u_a - u_w) \tan(b\phi') \quad (3-34)$$

or

$$\tau = c' + (\sigma - u_a) \tan \phi' + AEV1 \tan \phi' + (u_a - u_w) \tan(b\phi') - AEV1 \tan(b\phi') \quad (3-35)$$

- c. At matric suction higher than AEV2, b and k = fitting parameter.

Constraints no a and b can be incorporated in the proposed equation (Equation 3-30) for fitting the shear strength of soil successfully. However, constraint no c requires

modification since the ϕ^b value for matric suction higher than AEV2 was found to be a non-constant value and vary with matric suction. Parametric study shows that the value of k varies with matric suction (Figure 3-12). It was observed that the calculated logarithm difference of matric suction and the AEV2 of a soil, $\log(u_a - u_w) - \log(\text{AEV2})$, varies with matric suction in a similar fashion (Figure 3-12). Therefore, the term of the logarithm difference of matric suction and the AEV2 of a soil was used in the development of the proposed equation for fitting shear strength of unsaturated soils with bimodal characteristics.

Figure 3-12. Variation of k with respect to natural logarithmic of matric suction for different soils

Parametric study was carried out to find the suitable parameters that can be used in the proposed general form of shear strength equation for relating $\log(u_a - u_w) - \log(\text{AEV2})$ to the variation of k . Five sets of published shear strength data as listed in Table 3-1 were used in the parametric study. Initially, seven hypothetical parameters were used in the proposed equation of k (Equation 3-36). Then, parametric study with a number of combinations for different number of parameters was carried out to obtain the least number of parameters in the equation that can provide the minimum normalized of sum square error (SSE_{norm}) for all selected data sets. Parameter b was used in Equation 3-36 to ensure the continuity of the unsaturated shear strength of soil at the last matric suction before AEV2 with that at the first matric suction after AEV2. Therefore,

parameter b is included in all combinations. All combinations that were used in the parametric study are shown in

Table 3-2.

$$k = b + d \left[a + q \left[p * \log \frac{(u_a - u_w)}{AEV2} \right]^f \right] \quad (3-36)$$

Based on the parametric study, the minimum normalized sum square error (SSE_{norm}) is observed if the unsaturated shear strength of soil with bimodal characteristics is fitted using Equation 3-30 with three parameters (b , d and f) as shown in combination no 3 in

Table 3-2. Therefore, the combinations that consist of three parameters (b , d and f) were incorporated in Equation 3-30 to best fit the unsaturated shear strength of soil with bimodal characteristic.

Table 3-1. Summary of the published data from literatures used in the parametric study

Properties	Adjari et al. (2009)	Coutinho et al. (2011)	Jotisankasa & Mairaing (2010) Omkoi	Jotisankasa & Mairaing (2010) Nakhomnayok	Rahardjo et al. (2011)
Dry density, ρ_d (Mg/m ³)	1.45	1.60	1.50	1.80	1.56
Water content, w (%)	15	12	12.5	11	9.5
Sand (%)	0	35	33.6	35.1	76.8
Silt (%)	48	26	40.1	40	20.2
Clay (%)	52	39	26.3	24.9	3
AEV1	1.5	0.75	4	3	0.80
AEV2	150	100	400	2000	50
c'	10	10	0	6.5	1
ϕ'	22	26	29	37	30
USCS classification	Silty Clay	Sandy Clay	Sandy Silt	Sandy Silt	Silty Sand

Table 3-2. SSENorm of the unsaturated shear strength of published data using Equation 3-36 with different parameters

Combination	Adjari et al. (2009)	Coutinho et al. (2011)	Jotisankasa & Mairaing (2010) Omkoi	Jotisankasa & Mairaing (2010) Nakhomnayok	Rahardjo et al. (2011)
1 (b,d,a,q,p,f)	0.134	0.251	0.064	0.094	0.031
2 (b,q,p,f)	0.294	0.346	0.175	0.206	0.137
3 (b,q,f)	0.002	0.008	0.006	0.004	0.001
4 (b,f)	0.381	0.421	0.390	0.357	0.294

As a result, the proposed equations for best fitting the unsaturated shear strength of soil with bimodal characteristics can be written as follows:

$$\tau = c' + [(\sigma - u_a) + AEV1] \tan \phi' + [2(u_a - u_w) - AEV1 - AEV2] \tan(b\phi') + [AEV2 - (u_a - u_w)] \tan(k\phi') \quad (3-37)$$

where:

b and k = 1 for $u_a - u_w < AEV1$

k = b for $AEV1 < u_a - u_w < AEV2$

$$k = b + q \left[\log \frac{(u_a - u_w)}{AEV2} \right]^f \quad \text{for } u_a - u_w > AEV2 \quad (3-38)$$

b, q and f = constant

For fitting the unimodal shear strength of soil, the following constraint must be followed in order to obtain the best fitting of the unsaturated shear strength of soil

- At matric suction lower than AEV, $b = k = 1$.
- At matric suction higher than or equal to AEV, $b = k = \text{fitting parameter}$.

As a result, the proposed equations for best fitting the unsaturated shear strength of soil with unimodal characteristics can be written as follows:

$$\tau = c' + [(\sigma - u_a) + AEV1] \tan \phi' + [2(u_a - u_w) - AEV1] \tan(b\phi') + (u_a - u_w) \tan(b\phi') \quad (3-39)$$

where:

b = 1 for $u_a - u_w < AEV$

$$b = \text{constant for } u_a - u_w > \text{AEV} \quad (3-40)$$

For fitting saturated soil successfully, the value of b and k in Equation 3-36 must be equal to 1.

3.5. Summary of The Proposed Equations developed in This Study

New grain-size distribution, soil-water characteristic curve and unsaturated shear strength equations for soils with bimodal characteristics have been proposed in Sections 3.2 to 3.4. The summary of the proposed equations developed in this study is presented in Figure 3-13 to Figure 3-16

Figure 3-13. Proposed equation for best fitting grain-size distribution of soil

Figure 3-14. Proposed equation for best fitting soil-water characteristic curve developed based on lognormal distribution function

Figure 3-15. Proposed equation for best fitting soil-water characteristic curve developed based on exponential function

Figure 3-16. Proposed equation for best fitting shear strength of the unsaturated soil

CHAPTER 4 RESEARCH PROGRAMME

4.1. Introduction

A comprehensive research programme was planned to study the soil-water characteristic curve (SWCC) and shear strength of unsaturated soil with bimodal characteristics. Different soil mixtures with different density and water content were used in the laboratory tests to obtain a better understanding on general behaviors of the unsaturated soil with bimodal characteristics. The results from laboratory tests were used to evaluate the proposed SWCC and shear strength equation developed in this study. In general, the research programme in this study consisted of the following parts:

- a. The soil mixtures for laboratory tests in this study were selected after reviewing several published literatures as described in Section 2.4.2. The criteria of selecting the soil mixtures in this study are presented in Section 4.2.
- b. The procedures of laboratory tests and the setup of equipments for laboratory tests are presented in Section 4.3.
- The laboratory tests include basic property tests and engineering property tests. The engineering property tests consisted of the saturated tests (saturated consolidated undrained (CU) triaxial tests) and the unsaturated tests (SWCC and unsaturated consolidated drained (CD) triaxial tests).
- c. In this study, the new grain-size distribution (GSD), SWCC and shear strength equations were developed to best fit the GSD, SWCC and shear strength of unsaturated soil with bimodal characteristics (Chapter 3). The new equations and procedures to estimate the SWCC and shear strength of unsaturated soil with bimodal characteristics were also proposed in this study (Chapter 6.3). The new equations proposed in this study were compared with various published equations using published data from literatures and experimental data from this study. The selected published equations of GSD, SWCC and shear strength of unsaturated soil for comparison with the equations proposed in this study are presented in Section 4.4. The selected published data for the evaluation of the proposed equations in this study are presented in Section 4.5.

- d. The criteria for the evaluation of the proposed equations in this study are presented in Section 4.6.
- e. Comprehensive analyses of the experimental results from GSD, SWCC and shear strength tests in this study are presented in Chapter 5.

4.2. Soil Design for Laboratory Experiments

In order to investigate the behavior of soils with bimodal characteristics, laboratory tests were conducted using identical specimens. The identical specimens were required in the laboratory tests to avoid the heterogeneity of the soil specimen for different tests, to ensure all results from different specimens can be related and comparable in order to establish the characteristics of the soil. The criteria for selecting the soil mixtures used in this study for SWCC and triaxial test were as follows:

- a. The soil specimen must be homogeneous and uniform in soil composition, initial water content and initial dry density to obtain meaningful results for the accuracy of the analyses.
- b. The soil specimen should have GSD curve with bimodal characteristics.
- c. The soil specimen should have a low residual matric suction, which is less than 1500 kPa to avoid the use of salt solution method in SWCC tests.
- d. The soil specimen should have SWCC with bimodal characteristics.
- e. The volume change of the soil specimen should not be too excessive and ideally should be an “inert” material which has non-expansive and non-collapsing characteristics.

Soil mixtures were used in the laboratory tests in order to obtain the identical soil specimens. In this study, soil mixtures with different percentages of medium sand and coarse kaolin were used. The coarse kaolin was manufactured by Kaolin Malaysia SDN BHD (Malaysia) and consisted of mainly silt particles (2 to 75 μm) with liquid limit of 68 %, plastic limit of 45 % and Plasticity index of 23 %. The medium sand, Ottawa sand (ASTM graded sand), was supplied by U.S. Silica company. The coarse kaolin was selected because it is an “inert” material that does not change volume excessively. Different mixtures of kaolin and sand were chosen to obtain soils with different characteristics. Different soil characteristics are required since the proposed model must

be applicable to a wide range of soils. Table 4-1 shows the types of soil and the quantity of soils used in the laboratory experiments.

Table 4-1. Soils used in the Laboratory Experiments

No	Soils	No of Specimens	Compacted at
1	100%Sand	1	$\rho_{\text{dry max}}$ and w_{opt}
2	90%Sand-10%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
3	80%Sand-20%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
4	70%Sand-30%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
5	60%Sand-40%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
6	50%Sand-50%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
7	40%Sand-60%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
8	30%Sand-20%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
9	20%Sand-10%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
10	10%Sand-90%Kaolin	3	Dry of optimum and Wet of optimum, $\rho_{\text{dry max}}$ and w_{opt}
11	100%Kaolin	1	$\rho_{\text{dry max}}$ and w_{opt}

4.3. Experimental Works

In this section, the procedures and set up of apparatuses for experimental works in laboratory are presented. Laboratory tests were performed on the soil mixtures which were used for the evaluation of the proposed equations. The laboratory tests in this study comprised of index property and engineering property tests in order to obtain the physical and mechanical behaviour of the investigated soils.

4.3.1. Index Property Tests

A series of index property tests were carried out to characterize the basic properties of soil. The index property tests included grain size analysis (dry sieving and hydrometer test), specific gravity tests and Atterberg limit (liquid limit and plastic limit) tests. All index property tests were conducted according to ASTM soil testing standard as shown in Table 4-2. The experimental results of grain-size distribution (GSD) were fitted using the proposed GSD equation as described in Section 3.3. Then the performance of the proposed GSD equation was compared with that of Fredlund et al. (2000) equation.

Table 4-2. Standards for Laboratory Tests

Laboratory Tests	ASTM standard
Specific gravity	D854-06E01
Grain-size distribution	D0422-63R07
Atterberg limit	D4318-05
Standard compaction	D0698-07E01
Consolidated undrained triaxial	D4767-04
Soil Classification (Unified Soil Classification System)	D2487-06E01

4.3.2. Standard Compaction Tests

Standard Proctor compaction tests were used to obtain the compaction curve of the investigated soils (Figure 4-1). Different water contents of the soil were used to obtain different total densities and to investigate the relationship between water content and dry density of the soil. The standard compaction tests follow the procedure given in ASTM D0698-07E01 (Table 4-2). In order to generate the specimen with unimodal and bimodal

characteristics, dry densities and water contents at dry of optimum, at wet of optimum, and at maximum dry density of soil mixture from different composition were selected.

Figure 4-1. Typical compaction curve

4.3.3. Static Compaction Tests

Identical specimens with the same initial condition are required for different type of test to fulfil the criteria for soil mixtures used in this study as described in Section 4.2. Therefore, static compaction was used to produce the specimens. The static compaction tests were conducted according to the procedure adopted from Ong (1999). Two sets of mould with two different inner diameters were used in the static compaction tests to obtain the soil specimen used in the laboratory tests. A mould of 50 mm in diameter was used to prepare the specimen for triaxial tests and a mould of 63.5 mm in diameter was used to prepare the specimen for SWCC tests. The static compaction mould with 50 mm in diameter is shown in Figure 4-2. Static compaction apparatus consists of a stainless steel mould and two stainless steel plugs. The compaction mould includes two symmetrical parts that can be connected and screwed together (Figure 4-3). The stainless steel plugs include nine removable disks with 10 mm thickness. The disks are connected to each other through the threaded screw that sticks out from the centre of the face of one disk with the threaded hole drilled into the centre of the face of another disk. The connection of the screw to the hole of the adjacent disk is illustrated in Figure 4-3b.

Figure 4-2. Static compaction moulds and plugs that are used for soil specimen preparations in this study

Figure 4-3. (a) Removable disk; (b) Connection between two adjacent disks; (c) Stainless steel plug (modified from Ong, 1999)

The length of steel plugs can be adjusted by adding or removing the disks accordingly to compact a specimen layer by layer. All disks are designed in a T-shape to reduce the friction between the trapped soil particles in the gaps and along the wall of mould. Each set of disk consists of two different diameters, where the top part has a slightly larger diameter with a 5 mm thickness and the bottom part has a slightly smaller diameter with a 5 mm thickness (Figure 4-3a).

The soil specimens for experimental works in this study were statically compacted to designated height with equal layers of 10 mm thickness. In this study, the soil specimen was compacted into 30 mm and 100 mm height for SWCC and shear strength tests, respectively. The soil specimens were produced from mixtures between coarse kaolin, different sizes of fine sands and water whose masses were calculated based on the selected ratio and mixed thoroughly and uniformly. The soil mixture was kept inside the sealed plastic bag and was left overnight in a humidity chamber to allow for equalization of water content of the soil mixture. The water content of the soil mixture was evaluated by collecting three small samples from the soil mixture inside the bag for the measurement of their water contents. The mass of soil mixture required for each layer was measured before placing it into the static compaction mould. Once the specimen was compacted successfully into the designated height, the mould was turned over so that the top plug would be at the bottom and the bottom plug would be at the top. Then, the height of the plug at the top was reduced 10 mm by removing one disk to allow space for the subsequent compaction layer.

Prior to placing the soil mixture inside the mould, the compacted soil surface was scratched consistently in order to ensure a good bonding between each soil layer. After placing the soil mixture into mould, the top plug was inserted into the mould and axial load was applied at a constant rate until the additional layer was compressed to 10 mm height. A fixed displacement rate of 1 mm/min was applied to the soil mixture using the compression machine (Figure 4-4) to prevent a high excess pore-water pressure built up and to obtain a uniform density throughout the whole specimen. The procedure was repeated until reaching the designated height of the specimen. Then the specimen was extruded from the mould using the same compression machine.

Upon completion of the static compaction process, the water content of the remaining soil mixture was evaluated by collecting three small samples from the remaining soil mixture inside the bag. The compacted soil was wrapped with transparent plastic and aluminium foil and kept inside humidity chamber. After verification of the designated water content, the total mass and volume of the compacted specimen was measured to calculate total density of the compacted specimen. Then, the initial dry density of the specimen can be evaluated and verified prior to SWCC and shear strength tests. The evaluation of the water content and dry density of the compacted soil was important to ensure the identical initial condition of the compacted soil for SWCC and shear strength test.

Figure 4-4. Static compaction machine used for preparation of soil specimen used in this study

4.3.4. Saturated Shear Strength Tests

Multistage saturated triaxial tests were performed using triaxial cell as shown in Figure 4-5. In this study, identical and homogenous statically compacted soil specimens of 50 mm in diameter and 100 mm in height were used for consolidated undrained (CU) triaxial

tests. The CU tests were selected since the soil mixtures consisted of silty material and the tests were carried out in accordance with ASTM standard as shown in Table 4-2. The multistage technique was selected to minimize the effect of variability of the soil specimen and to reduce the duration of the saturated triaxial tests as suggested by Rahardjo et al., 1995.

The saturation process of the soil specimen was performed by applying back pressure and cell pressure into triaxial cell in Figure 4-5. The cell pressure (confining pressure) was applied and controlled through valve A whereas back pressure (pore-water pressure) was applied and controlled through valve B. A pressure transducer was installed at valve D to monitor the pore-water pressure of the soil specimen. The pressure transducer was connected to data logger so the data could be recorded in real time. During saturation process, flushing of the pore-water pressure line was required to remove the air bubbles which disturbed the pressure transducer reading. The flushing process was conducted by supplying 30 kPa of de-aired distilled water pressure into valve C and the water with air bubbles flowed out from valve D.

Figure 4-5. Schematic diagram of triaxial cell for saturated test

After the soil specimen was fully saturated, it was isotropically consolidated to the designated net confining pressure, $(\sigma_3 - u_w)$. The pore-water pressure during consolidation process was monitored by the pressure transducer at valve D while the volume change of the specimen was measured by the volume change indicator. The consolidation process was considered complete after no excess pore-water pressure was observed in the soil specimen and the pore-water volume of the soil specimen had reached equilibrium.

After the consolidation process of the specimen to 25 kPa net confining pressure (first stage) was completed, the specimens was sheared by applying an axial load at a constant strain rate. During shearing, a constant slow strain rate was used in the saturated triaxial tests in order to maintain an essentially constant pore-water pressure within the soil specimen. The study by Goh (2012) indicated that a strain rate of 0.009 mm/min was used for CU triaxial tests on compacted sand-kaolin mixtures. Since a sand-kaolin mixture was also used in this research, the same strain rate of 0.009 mm/min was adopted for the CU triaxial tests in this research. During the shearing stage, valve A was opened and valve B was closed (Figure 4-5). The pore-water pressure was monitored using the pressure transducer at valve D, the pore-water volume change of the specimen was measured by the volume change indicator through valve B and the axial load and the axial displacement were measured through the load ring and dial gauges. Shearing of the first stage was terminated when failure was imminent. This could be observed from the deviator stress when it tends to reach a maximum value. Then, the specimen was unloaded and consolidated to a higher effective confining pressure (50 kPa) and the same procedures were repeated in the second stage of consolidation and shearing. In the final stage (effective confining pressure of 75 kPa) of shearing, the specimen was sheared until the deviator stress reached a constant value or a significant shear plane in the specimen was observed. The maximum axial strain was limited to 20 %. Then, the specimen was unloaded and all the pressures were released. The soil specimen was dismantled from the pedestal and the rubber membrane was removed after the water inside the cell was drained out. Subsequently, the final water content of the specimen was determined.

4.3.5. Soil-Water Characteristic Curve Tests

Soil-water characteristic curve (SWCC) is an important soil property of unsaturated soils. It relates water content to the matric suction ($u_a - u_w$) of the soil. The matric suction was applied to the specimen using axis translation technique as introduced by Hilf (1956). The magnitude of the matric suction was the difference between applied air pressure and water pressure in the specimen. In this study, SWCC was determined by combining the results from Tempe cell test and pressure plate test under zero net confining pressure. Tempe cell was used for measurement of SWCC up to 100 kPa of matric suction whereas the 5-bar and 15-bar pressure plates were used for measurement of SWCC up to 500 kPa and 1500 kPa of matric suction, respectively. Tempe cell consists of a one bar air-entry value ceramic disk, three O-rings, cell cap, cell base, cell ring and three screws. Figure 4-6 shows the schematic diagram of Tempe cell.

Figure 4-6. Schematic diagram of Tempe cell

Prior to the test, the ceramic disk of Tempe cell was saturated in a the vacuum chamber with de-aired distilled water. Soil specimens with 63.5 mm in diameter and 30 mm in height were prepared using the static compaction method as described in Section 4.3.3. Then, the specimen was cut into a specimen with 54 mm in diameter using the Tempe cell cutting mould. The cutting process was performed using the compression machine for static compaction test with a constant displacement rate of 1 mm/min in

order to minimize disturbance on the specimen due to the cutting process. Then, the specimen was extruded from the Tempe cell cutting mould, placed into the Tempe cell ring and the cell cap was tightened properly with the cell base.

The specimen in the Tempe cell was saturated from the bottom of the specimen through the pre-saturated ceramic disk in order to allow all the trapped air inside the soil specimen to flow upward and the specimen was saturated efficiently. The layout of the saturation process of soil specimen prior to SWCC test is shown in Figure 4-7. After the saturation of soil specimen in Tempe cell was completed, the specimen was weighed to determine the mass of water of the specimen. The diameter and height of the specimen were measured to determine the volume of specimen. Then, the saturated volumetric water content of the specimen could be calculated from the ratio between the volume of water and the total volume of the specimen prior to SWCC test.

Figure 4-7. Layout of saturation process of soil specimen prior to SWCC test

The matric suction was applied to the specimen in the Tempe cell from the lowest pressure of 0.5 kPa and subsequently increased to 1, 3, 5, 10, 20, 30, 40, 50, 70, 90 and 100 kPa. Controlling matric suction for SWCC test at matric suctions less than 10 kPa is

unattainable by using the conventional air compressor as its accuracy is about 1–5 kPa. For matric suctions less than 10 kPa, modification to the air pressure configuration was made by using two air pressure regulators A and B (Figure 4-8), regulator A is to control air pressure supply and regulator B is to control air release. A U-tube manometer was used to control water pressure head (Δh_a) (Yang et al. 2004; Indrawan et al. 2006). Desired values of matric suction were applied by regulating air pressure from the top of Tempe cell and regulating water pressure from the bottom of Tempe cell (Figure 4-8). Therefore, the matric suction is equal to the difference between the applied air pressure and water pressure. A burette was used to control water pressure supply (Δh_w) and to maintain the saturation condition of the ceramic disk during the SWCC test.

Figure 4-8. Layout of SWCC test at matric suction less than 10 kPa

For matric suctions higher than 10 kPa, the air pressure was applied and controlled only using the pressure regulator and pressure gauge without the manometer U-tube (Figure 4-9). When matric suction was applied, water from the soil specimen started to drain out through the saturated ceramic disk. The mass of the specimen was weighed daily and consistently in order to have a close monitoring of the equilibrium condition of the specimen. The equilibrium condition of the specimen can be checked by plotting weight of the Tempe cell (including soil specimen) with respect to time

(logarithmic scale). Once the equilibrium condition was reached, the subsequent matric suction was applied to soil specimen. The specimen was considered in equilibrium condition when the change of the specimen weight was negligible.

Figure 4-9. SWCC test at low matric suction higher than 10 kPa

The 15- bar pressure plate was used for the application of matric suctions between 100 kPa and 1500 kPa. The pressure plate consisted of a pressure chamber that contains a saturated high air-entry disk (Figure 4-10) and it was connected to a burette for measurement of water volume change and maintenance of degree of saturation of the disk (Figure 4-11). There must be a good contact between the soil specimen and the ceramic plate to ensure the water flow between the specimen and the plate is continuous. The same procedure of SWCC test in Tempe cell was also applied for the SWCC test in the pressure plate. The high air-entry disk and water reservoir (Figure 4-10) were flushed every time after the readings were taken to ensure that the disk and the water reservoir are always in a saturated condition.

Figure 4-10. Schematic diagram of pressure plate

Figure 4-11. SWCC process using pressure plate test

In this study, twenty seven (27) series of SWCC tests were carried out using nine (9) different sand-kaolin mixtures as presented in Table 4-3. The naming convention is adopted for each soil specimen as M.y.z, where the term, “M” indicated that the soil mixture was used in the experimental data in this study; “y” was associated with the percentage of sand and kaolin in each soil mixture; “z” was associated with the initial condition of the soil mixtures (compacted at dry of optimum, at wet of optimum and at maximum dry density and water content).

Table 4-3. Testing programme for sand-kaolin mixtures

No	Percentage of sand and kaolin	Compacted at	Soil name	No of specimen	No of test
1	90%Sand-10%Kaolin	Dry of optimum	M1a	1	1
2	90%Sand-10%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M1b	1	1
3	90%Sand-10%Kaolin	Wet of optimum	M1c	1	1
4	80%Sand-20%Kaolin	Dry of optimum	M2a	3	3
5	80%Sand-20%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M2b	3	3
6	80%Sand-20%Kaolin	Wet of optimum	M2c	3	3
7	70%Sand-30%Kaolin	Dry of optimum	M3a	3	3
8	70%Sand-30%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M3b	3	3
9	70%Sand-30%Kaolin	Wet of optimum	M3c	1	1
10	60%Sand-40%Kaolin	Dry of optimum	M4a	3	3
11	60%Sand-40%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M4b	3	3
12	60%Sand-40%Kaolin	Wet of optimum	M4c	1	1
13	50%Sand-50%Kaolin	Dry of optimum	M5a	3	3
14	50%Sand-50%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M5b	1	1
15	50%Sand-50%Kaolin	Wet of optimum	M5c	1	1
16	40%Sand-60%Kaolin	Dry of optimum	M6a	3	3
17	40%Sand-60%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M6b	1	1
18	40%Sand-60%Kaolin	Wet of optimum	M6c	1	1
19	30%Sand-20%Kaolin	Dry of optimum	M7a	3	3
20	30%Sand-20%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M7b	1	1
21	30%Sand-20%Kaolin	Wet of optimum	M7c	1	1
22	20%Sand-10%Kaolin	Dry of optimum	M8a	3	3
23	20%Sand-10%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M8b	1	1
24	20%Sand-10%Kaolin	Wet of optimum	M8c	1	1
25	10%Sand-90%Kaolin	Dry of optimum	M9a	1	1
26	10%Sand-90%Kaolin	$\rho_{\text{dry max}}$ and w_{opt}	M9b	1	1
27	10%Sand-90%Kaolin	Wet of optimum	M9c	1	1

4.3.6. Unsaturated Shear Strength Tests

The shear strength tests of soil under an unsaturated condition were carried out using modified triaxial apparatus (Figure 4-12). The modified triaxial equipment for unsaturated soil testing consisted of a transparent triaxial cell, high air-entry disk, constant rate compression machine, coarse corundum disk, two linear variable differential transformers (LVDT), a diffused air volume indicator (DAVI), a digital pressure and volume controller (DPVC-1) for pore-water pressure, a digital pressure and volume controller (DPVC-2) for confining pressure, pore-air pressure control system, three pressure transducers, data acquisition system and a personal computer. The axis translation technique was used in the unsaturated triaxial testing in order to prevent cavitation (at 100 kPa negative water pressure) (see Section 2.6.1) of water in the pore-water pressure line. The schematic diagram of the modified triaxial apparatus for unsaturated soil testing is shown in Figure 4-13. A transparent triaxial cell was used for unsaturated soil testing. The transparent cell wall was used in order to observe the deformation of the soil specimen. In this triaxial cell, the maximum cell pressure of 1700 kPa can be applied to the soil specimen. This maximum cell pressure was much higher than the pressure required (500 kPa) in the unsaturated triaxial testing in this study.

Figure 4-12. Configuration of Unsaturated Triaxial Testing

Figure 4-13. Schematic Diagram of Modified Triaxial Apparatus for Unsaturated Triaxial Tests

A 2-kN load with a constant strain rate during shearing process was applied using a 50 kN compression machine (Figure 4-12). This compression machine can be used to apply a load with the range of strain rate between 0.00001 mm/min to 9.9 mm/min. A slow and constant strain rate was applied to the specimen in order to prevent the development of excess pore-air and pore-water pressures during the shearing process. The study by Rahardjo et al. (2004) indicated that a strain rate of 0.0009 mm/min was found to be suitable for the CD triaxial tests on compacted residual soils from the sedimentary Jurong Formation in Singapore. The same strain rate of 0.0009 mm/min was also used by Thu et al. (2006) for the CD triaxial tests on compacted kaolin. Since the sand-kaolin mixture was also used in this research and it had similar properties as the soils used in both studies, the same strain rate of 0.0009 mm/min was adopted for the CD triaxial tests carried out in this research.

Prior to the unsaturated triaxial testing, the load ring LVDT was well-calibrated within the range of 0 to 2 mm (approximately 0 to 1.5 kN) with a 0.05 mm interval. The axial deformation of the soil specimen during the testing was measured using the LVDT that was attached to the loading ram as shown in Figure 4-12. The maximum displacement that can be measured using this LVDT was 25 mm. The sensitivity of the displacement was 0.01 mm. The LVDT was well-calibrated within the required range before the test started.

The modified triaxial apparatus include three pressure transducers that were used to measure the pore-water pressure, the pore-air pressure and the cell pressure. The accuracy of all transducers is about 0.3 kPa and the transducer can measure pressure up to 1000 kPa. All pressure transducers were well-calibrated within the range of 0 to 500 kPa with 20 kPa intervals.

During the testing, air can diffuse through the disks and turned into air bubbles in the water compartments due to the high pore-air pressure in the specimen and the long duration of the test. Air bubbles could affect the accuracy of the pore-water pressure and volume change measurements and impeded the water flow into and from the specimen (Ho and Fredlund, 1982). Therefore, the flushing system was required in the modified triaxial apparatus to remove the trapped air bubbles in the water compartments during the testing. During flushing, a 30 kPa of water pressure was applied to the water compartments and air bubbles were flushed out to the diffused air volume indicator (DAVI) which was kept open to atmospheric condition.

The pore-water pressure and confining pressure of the specimen were applied to the specimen using a digital pressure and volume controller which were manufactured by GDS Instruments Limited of England. During the testing, the water phase in the disks, soil specimen and the water compartments of the pedestal must be maintained in a continuous condition. In order to develop matric suction in the soil specimen, the air pressure was applied and distributed uniformly into the specimen from the top cap of the triaxial cell. The 5-bar air entry disk was placed below the soil specimen in order to maintain the matric suction in the soil specimen during the testing. Filter paper was placed in between the specimen and the top cap in order to prevent fine particles from being trapped inside the top cap.

The data acquisition system was used to measure the pore-water pressure, pore-air pressure, pore-water volume changes, total volume changes, displacement and the applied load continuously and automatically. All pressure transducers and LVDT were connected to the data logger that was connected to the personal computer whereas three DPVCs were directly connected to the personal computer for data collection and monitoring. All readings were recorded every 2 minutes using Triax 4.0 data acquisition

programme developed by Toll (1999). All data were transferred to personal computer and opened using Microsoft Excel for analysis.

Soil specimens of 50 mm in diameter and 100 mm in height were used for CD triaxial tests. The CD triaxial tests consisted of four stages: saturation, consolidation, matric suction equalization and shearing of soil specimens. In the beginning of the unsaturated triaxial testing, all specimens were saturated in order to have a uniform initial condition and to reduce the matric suction of the specimen to a low value. During saturation of the specimen, the back pressure was applied from the top of the specimen using DPVC in order to supply water and control the water pressure. The cell pressure (confining pressure) was applied to the specimen using another DPVC. The pore-water pressure of the specimen was measured by the pressure transducers, monitored continuously using data logger and recorded by Triax software automatically. The soil specimen was saturated by applying a cell pressure (σ_3) and a back pressure (u_w) to develop a net confining pressure ($\sigma_3 - u_w$) of 10 kPa until the pore-water pressure parameter B value was larger than 0.95 as proposed by Head (1986). Pore-water pressure parameter, B value is defined as the ratio of a change in pore-water pressure to a change in the confining pressure as described by the following equation

$$B = \frac{\Delta u}{\Delta \sigma_3} \quad (4-1)$$

where

Δu = Pore-water pressure changes after the increment of confining pressure

$\Delta \sigma_3$ = Increment of confining pressure

In the beginning of saturation process, 40 kPa of back pressure and 50 kPa of cell pressure were applied to the soil specimen. The cell pressure (confining pressure) was increased to 100 kPa after the equilibrium condition was reached. Then, the pore-water pressure of the specimen built up due to the increment of the confining pressure. After the pore-water pressure reached equilibrium, the pore-water pressure value was recorded and used for calculating the B value. Then, the back pressure was increased to 90 kPa if the B value was still lower than 0.95. These procedures were repeated until the B value was larger than 0.95. The total volume and pore-water volume changes of the soil specimen were measured by DPVC and recorded continuously using Triax programme.

After the saturation process was completed, the soil specimen was isotropically consolidated to the designated net confining pressure. During consolidation, the connection to the back pressure and cell pressure line remained the same as it was during the saturation process. The consolidation process was completed if there was no excess pore-water pressure and the pore-water volume reached equilibrium. Then, the air pressure lines were connected to the triaxial cell in order to apply matric suction on the soil specimen. Prior to applying the air pressure to the specimen, the pore-water pressure was first decreased to a designated pressure. The pore-air pressure was applied from the top of the specimen whereas the pore-water pressure was applied from the bottom of the specimen. The confining pressure was kept constant throughout the test as the pressure applied during consolidation. Equilibrium condition was reached when the excess pore-water pressures have been fully dissipated and the water volume change was less than 0.04% per day as suggested by Sivakumar and Wheeler (1993). Then, the soil specimen was sheared under a drained condition. During shearing stage, both pore-air and pore-water phases were in a drained condition and were maintained at the same pressures as the pressures before the shearing started.

The same procedures of matric suction equalization and shearing process were repeated on the specimen. The maximum axial strain during the shearing process was limited to 20 %. After the final shearing stage, all the pressures were released and the soil specimen was dismantled from the triaxial cell. Then, the final water content of the soil specimen was obtained. All the measurements, e.g. pore-air pressure, pore-water pressure, total and water volume changes, applied load and displacement were recorded automatically and continuously using data logger and Triax programme.

In this study, eleven (11) saturated shear strength tests and 11 unsaturated shear strength tests were carried out using seven (7) different sand-kaolin mixtures as presented in Table 4-3. The shear strength tests were only carried out on soil specimens with bimodal SWCC (M2a, M2b, M2c, M3a, M3b, M4a, M4b, M5a, M6a, M7a, M8a). Static compaction was used to produce the identical soil specimens for SWCC and shear strength tests of soil with bimodal SWCC. The initial water content and dry density of each soil specimen upon completion of static compaction tests were conducted according to procedure explained in Section 4.3.3.

4.4. Selection of Published Equations for Evaluation of The Proposed Equations

The proposed grain-size distribution (GSD), soil-water characteristic curve (SWCC) and shear strength equations in this study were compared with several published GSD, SWCC and shear strength equations from literatures. The GSD equation can be classified into fitting type for soils with unimodal and bimodal characteristics. The details of the published GSD equations for fitting soils with bimodal characteristics used in this study are shown in Table 4-4.

Table 4-4. Published GSD equations for soil with bimodal characteristics

Authors	Nature	GSD equations
Fredlund et al. (2000)	Fitting unimodal GSD	$P = \frac{1}{\ln \left[\exp(1) + \left(\frac{a_{gr}}{d} \right)^{n_{gr}} \right]^{m_{gr}}} \left[1 - \frac{\ln \left(1 + \frac{d_r}{d} \right)}{\ln \left(1 + \frac{d_r}{d_{min}} \right)} \right]^7 \quad 4-2$
Fredlund et al. (2000)	Fitting bimodal GSD	$P = \left\{ \frac{w \left[\frac{1}{\left\{ \ln \left[\exp(1) + \left(\frac{a_{gr}}{d} \right)^{n_{gr}} \right] \right\}^{m_{gr}}} \right]}{(1-w) \left[\frac{1}{\left\{ \ln \left[\exp(1) + \left(\frac{j_{gr}}{d} \right)^{k_{gr}} \right] \right\}^{l_{gr}}} \right]} \right\} \times \left\{ 1 - \frac{\ln \left(1 + \frac{d_r}{d} \right)}{\ln \left(1 + \frac{d_r}{d_{min}} \right)} \right\}^7 \quad 4-3$

Notes for equation in Table 4-4

d = corresponding particle diameter (mm)

$a, b, a_{gr}, n_{gr}, m_{gr}, j_{gr}, k_{gr}, l_{gr}$ = fitting parameters

d_r = amount of fine particles

d_{min} = the allowable minimum particle diameter

The SWCC equation can be classified into fitting and estimation types according to the nature of the unimodal or bimodal SWCC equation. The measurement of several points in SWCC are needed in the fitting type equation in order to estimate the volumetric water content at a particular matric suction or to estimate the air-entry value of the soil. In contrast, the SWCC data are not required in the estimation type equation. Grain-size distribution and other soil properties are used in the estimation type equations in order to estimate the unimodal or bimodal SWCC. Estimation type equations can be used to obtain a good first approximation of SWCC if the SWCC testing is not possible. The details of the published SWCC equations for fitting soils with unimodal and bimodal characteristics and for estimating SWCC of soils with unimodal GSD used in this study are shown in Table 4-5.

Similar to SWCC equations, the shear strength equations can also be classified into fitting and estimation type shear strength equations according to the nature of the shear strength equation. The measurement of the shear strength data of the unsaturated soil, e.g. peak shear strength or shear strength at different matric suctions, are needed in the fitting type equation in order to estimate the shear strength at a particular matric suction or to estimate the failure envelope. The SWCC data and soil properties are used in the estimation type equations to estimate the shear strength of the unsaturated soil. The details of the published shear strength equations for fitting soils with unimodal characteristics and for estimating the shear strength of the unsaturated soil with unimodal SWCC used in this study are shown in Table 4-6.

Table 4-5. Published SWCC equations for soil with unimodal and bimodal characteristics

Authors	Nature	SWCC equations
Fredlund and Xing (1994)	Fitting unimodal SWCC	$\theta(\psi) = \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)} \right] \left\{ \frac{\theta_s}{\ln\left[e + \left(\frac{\psi}{a}\right)^n\right]} \right\}^m$
Zhang and Chen (2005)	Fitting bimodal SWCC	$\theta(\psi) = \theta_{sl} \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_{rl}}\right)}{\ln\left(1 + \frac{10^6}{\psi_{rl}}\right)} \right] \left\{ \frac{1}{\ln\left[e + \left(\frac{\psi}{a_l}\right)^{nl}\right]} \right\}^{ml}$ $+ \theta_{ss} \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_{rs}}\right)}{\ln\left(1 + \frac{10^6}{\psi_{rs}}\right)} \right] \left\{ \frac{1}{\ln\left[e + \left(\frac{\psi}{a_s}\right)^{ns}\right]} \right\}^{ms}$

Notes for equation in Table 4-5

 θ_s = saturated volumetric water content

a, n, m = curve fitting parameters

 ψ_r = residual soil suction

subscript l & s in Zhang & Chen equation = large pore and small pore, respectively

Table 4-6. Published shear strength equations for unsaturated soil with unimodal and bimodal characteristics

Authors	Nature	SWCC equations
Fredlund et al. (1978)	Fitting shear strength of soil with unimodal SWCC	$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b$ 4-6
Vanapalli et al. (1996b)	Fitting shear strength of soil with unimodal SWCC	$\tau = c' + (\sigma - u_a) \tan \phi' + [(u_a - u_w) \Theta^\kappa] \tan \phi'$ 4-7 $\Theta = \frac{\theta_w}{\theta_s}$ 4-8
Goh et al. (2010)	Estimation of shear strength from unimodal SWCC	$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi'$ 4-9 <i>if</i> $(u_a - u_w) \leq AEV$ $\tau = c' + [(\sigma - u_a) + AEV] \tan \phi' + [(u_a - u_w) - AEV] b \Theta^\kappa \tan \phi'$ 4-10 <i>if</i> $(u_a - u_w) > AEV$ where: $\kappa = [\log(u_a - u_w) - \log AEV]^y$ 4-11 $y_d = 0.502 \ln(PI + 2.7) - 0.387$ 4-12 $b_d = -0.245 \left\{ \ln[n_d(PI + 4.4)] \right\}^2 + 2.114 \left\{ \ln[n_d(PI + 4.4)] \right\} - 3.522$ 4-13

Notes for equation in Table 4-6

 τ = shear strength c' = effective cohesion ϕ' = effective angle of internal friction ϕ^b = angle indicating increase in shear strength due to increase in matric suction

- Θ = normalized volumetric water content
 θ_s = saturated volumetric water content
 θ_w = volumetric water content at any matric suction
 κ = fitting parameter

4.5. Selection of Published Data for Evaluation of The Proposed Models

The proposed GSD and SWCC equations in this study were evaluated with the published data as shown in Table 4-7 whereas the shear strength equations proposed in this study were evaluated with the published data as shown in Table 4-8. The selected soils include the undisturbed soil, compacted residual soil and compacted soil soil mixture. The term "undisturbed soil" refers to residual soil that was sampled from the field from different locations in Hong Kong, Japan, Singapore, Thailand and Brazil. The term "compacted residual soil" refer to residual soil that was modified and compacted in the laboratory into soil specimen with cerntain dry density and water content. The term "compacted soil soil mixture" refers to different soil types that were mixed and compacted in the laboratory into soil specimen with certain dry density and water content.

4.6. Criteria for Evaluation of the Proposed Models

Statistical analyses were carried out to assess the evaluation of the proposed GSD, SWCC and shear strength equations. The goodness of fit of the proposed and the published unimodal and bimodal GSD, SWCC and shear strength equations with the experimental data was quantified with the R-squared (R^2). The value of R^2 equal to 1 indicates showing a good fitting between the data from the experimental tests and the proposed model.

Table 4-7. Soil data sets from published literatures used for evaluation of the proposed GSD and SWCC equation

Soil name	Sat w (%)	ρ_d (Mg/m ³)	Gravel (%)	Sand (%)	Silt (%)	Clay (%)	Remark*	Source
SB1	32.0	1.43	0	22.2	61.8	16.0	UDR	SoilVision (2002)
SB2	39.6	1.29	1.1	54.9	34.4	9.6	UDR	
SB3	25.5	1.31	1.6	56.0	33.4	10.6	UDR	
SB4	37.4	1.19	0	71.4	13.2	15.4	UDR	
SB5	44.5	1.18	0	29.5	45.2	25.3	UDR	
SB6	40.5	1.80	0	28.9	48.6	22.5	UDR	
SB9	21.7	1.80	76.6	23.4	76.6	0.0	CSM	
SB10	20.3	1.72	68.4	26.7	71.4	1.9	CSM	
S4	23.8	1.52	0	80	15	5	CR	
S45	28.2	1.59	0	50	20	30	CR	
S62	25.8	1.54	0	27	53	20	CR	
JF3	13.2	1.79	0	56.8	35.2	8.0	UDR	Rahardjo et al. (2004)
JF13	13.7	1.85	0	70.3	21.3	8.4	UDR	
JF25	3.4	2.27	0	78	14	8.0	UDR	
BTG4	35.7	1.14	0	33.8	39.4	26.9	UDR	
BTG8	32.8	1.71	0	30	60.2	9.8	UDR	
BTG20	27.2	1.57	0	80	16	4.0	UDR	
G-M1b	17.4	1.74	50	0	36.7	13.3	CSM	Rahardjo et al. (2012)
G-M2b	20.6	1.68	50	0	36.9	13.1	CSM	
G-M3b	23.1	1.62	50	0	31.1	18.9	CSM	
G-RS	30.1	1.44	5	36	39.9	19.1	CR	
Doi Tung	58.1	1.05	0	21	40.5	38.5	UDR	Jotisankasa et al. (2009)
SC17	20.3	1.72	0.8	72.4	10.1	16.7	UDR	Yang et al. (2004)
SW1	23.2	1.63	1.5	86.1	9.1	3.3	UDR	Alim et al. (2009)
CH3	38.9	1.94	0	43	45	12	CR	Chin et al. (2010)

*UDR = undisturbed residual soil; CR = Compacted residual soil; CSM = compacted soil from other soil mixtures

Table 4-8. Soil data sets from published literatures used for evaluation of the proposed shear strength equation

Soil name	Sat w (%)	ρ_d (Mg/m ³)	Gravel (%)	Sand (%)	Silt (%)	Clay (%)	Remark*	Source
Sandy Clay	25.7	1.60	0	35	26	39	UDR	Coutinho et al. (2011)
Clayey Silt	25.9	1.60	0	22	52	26	UDR	Jotisankasa et al. (2009)
Sandy Silt	26.1	1.80	0	31.4	40.1	26.3	UDR	Jotisankasa and Mairaing (2010)
Sandy Silt	34.7	1.50	0	40	57	3	UDR	Rahardjo et al. (2011)

*UDR = undisturbed residual soil; CR = Compacted residual soil; CSM = compacted soil from other soil mixtures

The calculation of R-squared is as follows:

$$R^2 = 1 - SSE \quad (4-14)$$

where:

$$SSE = \sum_{i=1}^n (y_{li} - y_{mi})^2 \quad (4-15)$$

SSE = Sum of Square Error

Y_{li} = Data from laboratory tests

Y_{mi} = Data from the proposed models

n = number of investigated data

The performance of the proposed models in estimating SWCC and shear strength of the unsaturated soil for soils with unimodal and bimodal characteristics were evaluated using two criteria. The first criterion was the root mean squared error (RMSE). The smaller the value of RMSE indicated better predictive capability of the proposed models. RMSE was defined as follows:

$$RMSE = \sqrt{\frac{SSE}{n}} \quad (4-16)$$

The second criterion was using average relative error (ARE). In this research, the performance of the proposed models to predict the SWCC successfully was considered good if ARE was less than or equal to 30%. ARE was defined as follows:

$$ARE = \frac{\sum_{i=1}^n \left| \frac{y_{li} - y_{mi}}{y_{li}} \right|}{n} \times 100 \quad (4-17)$$

CHAPTER 5 PRESENTATION AND RESULTS

5.1. Introduction

The results from the laboratory tests that were carried out in this study are presented in this chapter. The results of the compaction test are presented first. Subsequently, the results of the index properties, soil-water characteristic curve (SWCC), saturated consolidated undrained (CU) and unsaturated consolidated drained (CD) triaxial tests are presented.

5.2. Compaction Tests

The compaction curves showing the relationship between dry density (ρ_d) and water content (w) were obtained using the standard Proctor compaction tests. The compaction tests have been performed on nine soil mixtures with different percentages of sand and kaolin. Figure 5-1 shows the compaction curve of different percentages of sand and kaolin mixtures. The maximum dry density and optimum water content for each soil mixtures are presented in Table 5-1. Notations k and s in Table 5-1 stand for kaolin and sand, respectively.

Figure 5-1. Compaction curve of selected soils used in this study

Soil mixtures at dry density and water content of 95% of maximum dry density on the dry and wet of optimum as well as at the maximum dry density and optimum water content were selected as the initial conditions for SWCC and shear strength tests of soil mixtures with different percentages of sand and kaolin. As described in Section 4.2, the identical specimens with the same initial conditions for SWCC and shear strength tests were produced using static compaction test. Table 5-3 to Table 5-7 present the initial condition of each specimen used in the SWCC and shear strength tests. Notations a, b and c in Table 5-3 to Table 5-7 are associated with the initial condition of each specimen corresponding to 95% of maximum dry density on the dry of optimum, maximum dry density and 95% of maximum dry density on the wet of optimum, respectively.

Table 5-1. Maximum dry density and optimum water content of the sand-kaolin mixtures used in the laboratory experiments conducted in this study

Soils	Maximum dry density, ρ_{dry} (Mg/m³)	Optimum water content, w (%)
M1 (90s10k)	2.02	9.00
M2 (80S20K)	1.99	9.90
M3 (70S30K)	1.95	9.40
M4 (60S40K)	1.94	10.00
M5 (50S50K)	1.84	14.00
M6 (40S60K)	1.67	16.50
M7 (30S70K)	1.61	19.50
M8 (20S80K)	1.53	21.00
M9 (10S90K)	1.53	22.00

5.2.1. Index Properties

Index properties tests including specific gravity, Atterberg limits, grain-size distribution (GSD) and soil classification based on the Unified Soil Classification System were performed on the selected soils. Figure 5-2 shows GSD curve of the selected soils. The

GSD results from the laboratory experiments carried out in this study were fitted using the proposed model for unimodal and bimodal GSD (Equations 3-11 and 3-12). Table 5-2 shows the GSD variables of the selected soils investigated in this study. Table 5-3 shows the results of the index property tests for soils M1 and M2. Table 5-4 shows the results of the index property tests for soils M3 and M4. Table 5-5 shows the results of the index property tests for soils M5 and M6. Soils M1, M2, M3, M4 and M5 were classified as SM (silty sand), whereas soils M6, M7, M8 and M9 were classified as MH (sandy silt with high plasticity).

Table 5-2. Summary of the GSD variables of the selected soils based on the bimodal GSD equation (Equation 3-21) proposed in this study

Index Properties	M1	M2	M3	M4	M5	M6	M7	M8	M9
$d_{\max 1}$ (mm)	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
$d_{\max 2}$ (mm)	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
d_{m1} (mm)	0.706	0.706	0.706	0.706	0.706	0.706	0.706	0.706	0.706
d_{m2} (mm)	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
s_{d1}	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
s_{d2}	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86
W_1	0.90	0.79	0.70	0.58	0.49	0.45	0.35	0.15	0.05
W_2	0.10	0.21	0.30	0.42	0.51	0.65	0.75	0.85	0.95
D_{60}	0.53	0.706	0.702	0.701	0.70	0.68	0.005	0.050	0.013
D_{30}	0.46	0.695	0.60	0.013	0.011	0.01	0.009	0.013	0.007
D_{10}	0.40	0.009	0.007	0.004	0.003	0.004	0.002	0.002	0.003
C_u	1.33	78	100	175	233	170	25	12.5	4.33
C_c	0.99	76	73	0.06	0.06	0.04	0.81	0.85	1.26

Table 5-3. Summary of index properties of soils M1 and M2

Index Properties	M1a	M1b	M1c	M2a	M2b	M2c
Specific gravity, G _s	2.64	2.64	2.64	2.58	2.58	2.58
Liquid Limit, LL (%)	20.1	20.1	20.1	35.4	35.4	35.4
Plastic Limit, PL (%)	14.3	14.3	14.3	20.9	20.9	20.9
Plasticity Index, PI (%)	5.8	5.8	5.8	14.5	14.5	14.5
Void Ratio, e	0.38	0.31	0.38	0.36	0.30	0.36
Dry density, ρ_d (Mg/m ³)	1.92	2.02	1.92	1.89	1.99	1.89
Water content, w (%)	5.0	9.0	12.0	6.0	9.8	13.1
Sand (%)	90.0	90.0	90.0	80.0	80.0	80.0
Silt (%)	2.0	2.0	2.0	16.8	16.8	16.8
Clay (%)	8.0	8.0	8.0	3.2	3.2	3.2
Soil classification according to USCS	SM	SM	SM	SM	SM	SM

Note: a, b, c indicate the specimen was compacted at dry of optimum, optimum water content and at wet of optimum, respectively

Table 5-4. Summary of index properties of soils M3 and M4

Index Properties	M3a	M3b	M3c	M4a	M4b	M4c
Specific gravity, G _s	2.65	2.65	2.65	2.62	2.62	2.62
Liquid Limit, LL (%)	36.8	36.8	36.8	41.2	41.2	41.2
Plastic Limit, PL (%)	21.6	21.6	21.6	24.8	24.8	24.8
Plasticity Index, PI (%)	15.2	15.2	15.2	16.4	16.4	16.4
Void Ratio, e	0.43	0.36	0.43	0.42	0.35	0.42
Dry density, ρ_d (Mg/m ³)	1.85	1.95	1.85	1.84	1.94	1.84
Water content, w (%)	5.0	9.4	14.5	6.0	10.0	15.0
Sand (%)	70.0	70.0	70.0	60.0	60.0	60.0
Silt (%)	25.3	25.3	25.3	32.8	32.8	32.8
Clay (%)	4.7	4.7	4.7	7.2	7.2	7.2
Soil classification according to USCS	SM	SM	SM	SM	SM	SM

Note: a, b, c indicate the specimen was compacted at dry of optimum, optimum water content and at wet of optimum, respectively

Table 5-5. Summary of index properties of soils M5 and M6

Index Properties	M5a	M5b	M5c	M6a	M6b	M6c
Specific gravity, Gs	2.59	2.59	2.59	2.60	2.60	2.60
Liquid Limit, LL (%)	46.7	46.7	46.7	48.9	48.9	48.9
Plastic Limit, PL (%)	27.4	27.4	27.4	28.2	28.2	28.2
Plasticity Index, PI (%)	19.3	19.3	19.3	20.7	20.7	20.7
Void Ratio, e	0.48	0.41	0.48	0.64	0.56	0.64
Dry density, ρ_d (Mg/m ³)	1.75	1.84	1.75	1.59	1.67	1.59
Water content, w (%)	12.2	14.0	17.5	11.5	16.5	22.8
Sand (%)	50.0	50.0	50.0	40.0	40.0	40.0
Silt (%)	43.7	43.7	43.7	52.4	52.4	52.4
Clay (%)	6.3	6.3	6.3	7.6	7.6	7.6
Soil classification according to USCS	SM	SM	SM	MH	MH	MH

Note: a, b, c indicate the specimen was compacted at dry of optimum, optimum water content and at wet of optimum, respectively

Table 5-6. Summary of index properties of soils M7 and M8

Index Properties	M7a	M7b	M7c	M8a	M8b	M8c
Specific gravity, Gs	2.63	2.63	2.63	2.62	2.62	2.62
Liquid Limit, LL (%)	53.5	53.5	53.5	57.8	57.8	57.8
Plastic Limit, PL (%)	31.4	31.4	31.4	33.6	33.6	33.6
Plasticity Index, PI (%)	22.1	22.1	22.1	24.2	24.2	24.2
Void Ratio, e	0.72	0.63	0.72	0.81	0.71	0.81
Dry density, ρ_d (Mg/m ³)	1.53	1.61	1.53	1.45	1.53	1.45
Water content, w (%)	16.5	19.5	24.5	18.0	21.0	27.0
Sand (%)	30.0	30.0	30.0	20.0	20.0	20.0
Silt (%)	63.4	63.4	63.4	72.8	72.8	72.8
Clay (%)	6.6	6.6	6.6	7.2	7.2	7.2
Soil classification according to USCS	MH	MH	MH	MH	MH	MH

Note: a, b, c indicate the specimen was compacted at dry of optimum, optimum water content and at wet of optimum, respectively

Table 5-7. Summary of index properties of soils M9

Index Properties	M9a	M9b	M9c
Specific gravity, G_s	2.69	2.69	2.69
Liquid Limit, LL (%)	66.3	66.3	66.3
Plastic Limit, PL (%)	41.7	41.7	41.7
Plasticity Index, PI (%)	24.6	24.6	24.6
Void Ratio, e	0.86	0.76	0.86
Dry density, ρ_d (Mg/m^3)	1.45	1.53	1.45
Water content, w (%)	18.0	22.0	28.5
Sand (%)	10.0	10.0	10.0
Silt (%)	83.4	83.4	83.4
Clay (%)	6.6	6.6	6.6
Soil classification according to USCS	MH	MH	MH

Note: a, b, c indicate the specimen was compacted at dry of optimum, optimum water content and at wet of optimum, respectively

Figure 5-2. Grain-size distribution of soil mixtures used in the laboratory tests

5.3. Soil-Water Characteristic Curve

Series of soil-water characteristic curve (SWCC) tests were performed on twenty-seven (27) different sand-kaolin mixtures. The SWCC's obtained in this research are classified

as SWCCs under zero net confining pressure since the SWCC tests were carried out using Tempe cell and pressure plate. All SWCCs with unimodal characteristics were fitted using the unimodal (Equation 3-11) SWCC equations proposed in this study. All SWCCs with bimodal characteristics were fitted using the bimodal (Equation 3-15) SWCC equations proposed in this study.

The SWCC of the statically compacted specimen M1a is presented in Figure 5-3. Specimen M1a has bimodal characteristics of grain-size distribution and an initial density of 1.92 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of optimum. As shown in Figure 5-3, specimen M1a has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matrix suction which are 1 kPa and 800 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.192 and 0.02, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M1a are 1 kPa, 50 kPa and 5, respectively.

Figure 5-3. Soil-water characteristic curve of soil mixture M1a

The SWCC of the statically compacted specimen M1b is presented in Figure 5-4. Specimen M1b has bimodal characteristics of grain-size distribution and an initial density of 2.02 Mg/m^3 which corresponds to the maximum dry density. As shown in Figure 5-4, specimen M1b has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matrix suction which are 1 kPa and 7000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.222 and 0.02, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M1b are 1 kPa, 25 kPa and 6, respectively.

Figure 5-4. Soil-water characteristic curve of soil mixture M1b

The SWCC of the statically compacted specimen M1c is presented in Figure 5-5. Specimen M1c has bimodal characteristics of grain-size distribution and an initial density of 1.92 Mg/m^3 which corresponds to 95% of the maximum dry density on the wet of optimum. As shown in Figure 5-5, specimen M1c has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 1 kPa and 6000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.25 and 0.02, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M1c are 1 kPa, 60 kPa and 5, respectively.

Figure 5-5. Soil-water characteristic curve of soil mixture M1c

The SWCC of the statically compacted specimen M2a is presented in Figure 5-6. Specimen M2a has bimodal characteristics of grain-size distribution and an initial density of 1.89 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of

optimum. As shown in Figure 5-6, specimen M2a has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). As explained in Section 3.2.1, subscript 1 is associated with macropore and subscript 2 is associated with micropore. The AEV1 and AEV2 of specimen M2a are 20 kPa and 100 kPa, respectively. The residual matric suction of specimen M2a is 0.05. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M2a are 20 kPa, 25 kPa, 0.75, 100 kPa, 250 kPa and 2, respectively.

Figure 5-6. Soil-water characteristic curve of soil mixture M2a

The SWCC of the statically compacted specimen M2b is presented in Figure 5-7. Specimen M2b has bimodal characteristics of grain-size distribution and an initial density of 1.99 Mg/m^3 which corresponds to the maximum dry density. As shown in Figure 5-7, specimen M2b has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M2b are 20 kPa and 75 kPa, respectively. The residual matric suction of specimen M2b is 0.075. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M2b are 20 kPa, 25 kPa, 0.95, 75 kPa, 175 kPa and 2.5, respectively.

Figure 5-7. Soil-water characteristic curve of soil mixture M2b

The SWCC of the statically compacted specimen M2c is presented in Figure 5-8. Specimen M2c has bimodal characteristics of grain-size distribution and an initial density of 1.89 Mg/m^3 which corresponds to 95% of maximum dry density on the wet of optimum. As shown in Figure 5-8, specimen M2c has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M2c are 16 kPa and 65 kPa, respectively. The residual matrix suction of specimen M2c is 0.06. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M2c are 14 kPa, 18 kPa, 0.94, 65 kPa, 120 kPa and 2.0, respectively.

Figure 5-8. Soil-water characteristic curve of soil mixture M2c

The SWCC of the statically compacted specimen M3a is presented in Figure 5-9. Specimen M3a has bimodal characteristics of grain-size distribution and an initial density of 1.85 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of optimum. As shown in Figure 5-9, specimen M3a has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M3a are 6 kPa and 35 kPa, respectively. The residual matric suction of specimen M3a is 0.075. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M3a are 6 kPa, 10 kPa, 0.81, 35 kPa, 100 kPa and 1.5, respectively.

Figure 5-9. Soil-water characteristic curve of soil mixture M3a

The SWCC of the statically compacted specimen M3b is presented in Figure 5-10. Specimen M3b has bimodal characteristics of grain-size distribution and an initial density of 1.95 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-10, specimen M3b has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M3b are 2 kPa and 60 kPa, respectively. The residual matric suction of specimen M3b is 0.060. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M3b are 2 kPa, 7 kPa, 1.75, 60 kPa, 120 kPa and 2.0, respectively.

Figure 5-10. Soil-water characteristic curve of soil mixture M3b

The SWCC of the statically compacted specimen M3c is presented in Figure 5-11. Specimen M3c has bimodal characteristics of grain-size distribution and an initial density of 1.85 Mg/m³ which corresponds to 95% of maximum dry density on the wet of optimum. As shown in Figure 5-11, specimen M3c has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 7 kPa and 4000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_w) are 0.296 and 0.02, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M1b are 7 kPa, 150 kPa and 4, respectively.

Figure 5-11. Soil-water characteristic curve of soil mixture M3c

The SWCC of the statically compacted specimen M4a is presented in Figure 5-12. Specimen M4a has bimodal characteristics of grain-size distribution and an initial density

of 1.84 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of optimum. As shown in Figure 5-12, specimen M4a has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M4a are 3 kPa and 35 kPa, respectively. The residual matrix suction of specimen M4a is 0.030. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M4a are 3 kPa, 4 kPa, 0.36, 25 kPa, 100 kPa and 1.0, respectively.

Figure 5-12. Soil-water characteristic curve of soil mixture M4a

The SWCC of the statically compacted specimen M4b is presented in Figure 5-13. Specimen M4b has bimodal characteristics of grain-size distribution and an initial density of 1.94 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-13, specimen M4b has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M4b are 7 kPa and 40 kPa, respectively. The residual matrix suction of specimen M4b is 0.050. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M4b are 5 kPa, 8 kPa, 0.36, 40 kPa, 150 kPa and 1.25, respectively.

Figure 5-13. Soil-water characteristic curve of soil mixture M4b

The SWCC of the statically compacted specimen M4c is presented in Figure 5-14. Specimen M4c has bimodal characteristics of grain-size distribution and an initial density of 1.84 Mg/m³ which corresponds to 95% of maximum dry density on the wet of optimum. As shown in Figure 5-14, specimen M4c has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 10 kPa and 1000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_w) are 0.386 and 0.050, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M4c are 10 kPa, 100 kPa and 2.25, respectively.

Figure 5-14. Soil-water characteristic curve of soil mixture M4c

The SWCC of the statically compacted specimen M5a is presented in Figure 5-15. Specimen M5a has bimodal characteristics of grain-size distribution and an initial density of 1.75 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of optimum. As shown in Figure 5-15, specimen M5a has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M4a are 10 kPa and 30 kPa, respectively. The residual matric suction of specimen M4a is 0.020. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M5a are 10 kPa, 15 kPa, 0.195, 30 kPa, 120 kPa and 0.92, respectively.

Figure 5-15. Soil-water characteristic curve of soil mixture M5a

The SWCC of the statically compacted specimen M5b is presented in Figure 5-16. Specimen M5b has bimodal characteristics of grain-size distribution and an initial density of 1.94 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-16, specimen M5b has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 1 kPa and 9 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_w) are 0.405 and 0.02, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M5b are 2 kPa, 3 kPa and 1.8, respectively.

Figure 5-16. Soil-water characteristic curve of soil mixture M5b

The SWCC of the statically compacted specimen M5c is presented in Figure 5-17. Specimen M5c has bimodal characteristics of grain-size distribution and an initial density of 1.94 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-17, specimen M5c has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 1 kPa and 300 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.455 and 0.02, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M5c are 1 kPa, 21 kPa and 2.2, respectively.

Figure 5-17. Soil-water characteristic curve of soil mixture M5c

The SWCC of the statically compacted specimen M6a is presented in Figure 5-18. Specimen M6a has bimodal characteristics of grain-size distribution and an initial density

of 1.59 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of optimum. As shown in Figure 5-18, specimen M6a has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M6a are 10 kPa and 25 kPa, respectively. The residual matric suction of specimen M6a is 0.020. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M6a are 10 kPa, 7.5 kPa, 0.36, 25 kPa, 150 kPa and 1.00, respectively.

Figure 5-18. Soil-water characteristic curve of soil mixture M6a

The SWCC of the statically compacted specimen M6b is presented in Figure 5-19. Specimen M6b has bimodal characteristics of grain-size distribution and an initial density of 1.67 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-19, specimen M5b has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 3 kPa and 400 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_w) are 0.567 and 0.00, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M6b are 3 kPa, 30 kPa and 1.76, respectively.

Figure 5-19. Soil-water characteristic curve of soil mixture M6b

The SWCC of the statically compacted specimen M6c is presented in Figure 5-20. Specimen M6c has bimodal characteristics of grain-size distribution and an initial density of 1.94 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-20, specimen M6c has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 4.5 kPa and 400 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.636 and 0.0, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M6c are 4 kPa, 40 kPa and 1.8, respectively.

Figure 5-20. Soil-water characteristic curve of soil mixture M6c

The SWCC of the statically compacted specimen M7a is presented in Figure 5-21. Specimen M7a has bimodal characteristics of grain-size distribution and an initial density

of 1.53 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of optimum. As shown in Figure 5-21, specimen M7a has bimodal characteristics of its SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M7a are 1 kPa and 10 kPa, respectively. The residual matric suction of specimen M7a is 0.060. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M7a are 1 kPa, 1.5 kPa, 0.36, 10 kPa, 70 kPa and 2.00, respectively.

Figure 5-21. Soil-water characteristic curve of soil mixture M7a

The SWCC of the statically compacted specimen M7b is presented in Figure 5-22. Specimen M7b has bimodal characteristics of grain-size distribution and an initial density of 1.67 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-22, specimen M7b has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 5 kPa and 2000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_w) are 0.612 and 0.04, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M7b are 5 kPa, 80 kPa and 2.75, respectively.

Figure 5-22. Soil-water characteristic curve of soil mixture M7b

The SWCC of the statically compacted specimen M7c is presented in Figure 5-23. Specimen M7c has bimodal characteristics of grain-size distribution and an initial density of 1.53 Mg/m^3 which corresponds to 95% of maximum dry density on the wet of optimum. As shown in Figure 5-23, specimen M7c has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 8 kPa and 1000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.635 and 0.04, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M7c are 8 kPa, 130 kPa and 2.0, respectively.

Figure 5-23. Soil-water characteristic curve of soil mixture M7c

The SWCC of the statically compacted specimen M8a is presented in Figure 5-24. Specimen M8a has bimodal characteristics of grain-size distribution and an initial density of 1.45 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of optimum. As shown in Figure 5-24, specimen M8a has bimodal characteristics of SWCC with two air-entry values (AEV1 and AEV2), two inflection points (ψ_{m1} and ψ_{m2}) and two saturated volumetric water contents (θ_{s1} and θ_{s2}). The AEV1 and AEV2 of specimen M8a are 1 kPa and 6 kPa, respectively. The residual matrix suction of specimen M8a is 0.050. The fitting parameters of ψ_{a1} , ψ_{m1} , s_1 , ψ_{a2} , ψ_{m2} and s_2 (see Equation 3-15) for specimen M8a are 1 kPa, 1.1 kPa, 0.36, 6 kPa, 40 kPa and 2.50, respectively.

Figure 5-24. Soil-water characteristic curve of soil mixture M8a

The SWCC of the statically compacted specimen M8b is presented in Figure 5-25. Specimen M8b has bimodal characteristics of grain-size distribution and an initial density of 1.53 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-25, specimen M8b has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matrix suction which are 10 kPa and 1500 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_w) are 0.643 and 0.04, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M8b are 10 kPa, 200 kPa and 1.85, respectively.

Figure 5-25. Soil-water characteristic curve of soil mixture M8b

The SWCC of the statically compacted specimen M8c is presented in Figure 5-26. Specimen M8c has bimodal characteristics of grain-size distribution and an initial density of 1.45 Mg/m^3 which corresponds to 95% of maximum dry density on the wet of optimum. As shown in Figure 5-26, specimen M8c has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 20 kPa and 3000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.653 and 0.05, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M8c are 20 kPa, 300 kPa and 2.0, respectively.

Figure 5-26. Soil-water characteristic curve of soil mixture M8c

The SWCC of the statically compacted specimen M9a is presented in Figure 5-27. Specimen M9a has bimodal characteristics of grain-size distribution and an initial density of 1.45 Mg/m^3 which corresponds to 95% of maximum dry density on the dry of optimum. As shown in Figure 5-27, specimen M9a has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 100 kPa and 10000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.667 and 0.06, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M9a are 100 kPa, 1000 kPa and 2.50, respectively.

Figure 5-27. Soil-water characteristic curve of soil mixture M9a

The SWCC of the statically compacted specimen M9b is presented in Figure 5-27. Specimen M9b has bimodal characteristics of grain-size distribution and an initial density of 1.53 Mg/m^3 which corresponds to maximum dry density. As shown in Figure 5-27, specimen M9b has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 100 kPa and 6000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.704 and 0.06, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M9b are 100 kPa, 1000 kPa and 2.00, respectively.

Figure 5-28. Soil-water characteristic curve of soil mixture M9b

The SWCC of the statically compacted specimen M9c is presented in Figure 5-29. Specimen M9c has bimodal characteristics of grain-size distribution and an initial density of 1.45 Mg/m^3 which corresponds to 95% of maximum dry density on the wet of optimum. As shown in Figure 5-29, specimen M9c has unimodal characteristics of SWCC with one air-entry value (AEV) and one residual matric suction which are 100 kPa and 10000 kPa, respectively. The saturated volumetric water content (θ_s) and residual volumetric water content (θ_r) are 0.711 and 0.03, respectively. The fitting parameters of ψ_a , ψ_m and s (see Equation 3-11) for specimen M9c are 100 kPa, 1000 kPa and 2.50, respectively. Summary of variables and fitting parameters of SWCC with unimodal characteristic are shown in Table 5-8, Table 5-9 and Table 5-10. Summary of variables and fitting parameters of SWCC with bimodal characteristic are shown in Table 5-9.

Figure 5-29. Soil-water characteristic curve of soil mixture M9c

Table 5-8. SWCC fitting parameters of soils M1a, M1b, M1c, M3c, M4c, M5b, M5c and M6b with unimodal SWCC

Parameters	M1a	M1b	M1c	M3c	M4c	M5b	M5c	M6b
θ_s	0.182	0.222	0.25	0.296	0.386	0.405	0.455	0.568
ψ_a (kPa)	1	1	1	7	10	1	1	3
ψ_m (kPa)	50	25	6	150	100	3	21	30
s	5	6	5	4	2.25	1.8	2.2	1.76
ψ_r (kPa)	800	7000	6000	4000	1000	9	300	400
θ_r	0.02	0.02	0.02	0.02	0.05	0.02	0.02	0

Table 5-9. SWCC fitting parameters of soils M6c, M7b, M7c, M8b, M8c, M9a, M9b and M9c with unimodal SWCC

Parameters	M6c	M7b	M7c	M8b	M8c	M9a	M9b	M9c
θ_s	0.636	0.612	0.635	0.643	0.653	0.667	0.704	0.711
ψ_a (kPa)	4.5	5	8	10	20	100	100	100
ψ_m (kPa)	40	80	130	200	300	1000	600	1000
s	1.8	2.75	2	1.85	2	2.5	2	2.5
ψ_r (kPa)	400	2000	1000	1500	3000	10000	6000	100000
θ_r	0	0.04	0.04	0.04	0.05	0.06	0.06	0.03

Table 5-10. Fitting parameters of the proposed SWCC equations for best fitting soil mixtures M2a, M2b, M2c, M3a and M3b with bimodal characteristics

Parameters	M2a	M2b	M2c	M3a	M3b
θ_{s1}	0.203	0.267	0.316	0.352	0.339
ψ_{a1} (kPa)	20	20	14	6	2
ψ_{m1} (kPa)	25	25	18	10	7
s_1	0.75	0.95	0.94	0.81	1.75
θ_{s2}	0.175	0.225	0.27	0.29	0.28
ψ_{a2} (kPa)	100	75	65	35	60
ψ_{m2} (kPa)	250	175	120	100	120
s_2	2	2.5	2	1.5	2
ψ_r (kPa)	1000	1000	1000	350	600
θ_r	0.05	0.075	0.06	0.075	0.06
r^2	0.997	0.999	0.993	1.000	0.989

Table 5-11. Fitting parameters of the proposed SWCC equations for best fitting soil mixtures M4a, M4b, M5a, M6a, M7a and M8a with bimodal characteristics

Parameters	M4a	M4b	M5a	M6a	M7a	M8a
θ_{s1}	0.282	0.312	0.228	0.207	0.52	0.595
ψ_{a1} (kPa)	3	7	10	10	1	1
ψ_{m1} (kPa)	4	8	15	7.5	1.5	1.1
s_1	0.36	0.36	0.2	0.36	0.36	0.36
θ_{s2}	0.234	0.27	0.195	0.174	0.39	0.475
ψ_{a2} (kPa)	35	40	30	25	10	6
ψ_{m2} (kPa)	100	150	120	150	70	40
s_2	1	1.25	0.92	1	2	2.5
ψ_r (kPa)	500	500	300	500	1000	400
θ_r	0.03	0.05	0.02	0.03	0.06	0.05
r^2	0.990	0.963	0.991	1.000	0.992	0.983

5.4. Shear Strength

Series of multi-stage saturated consolidated undrained (CU) triaxial tests were carried out on the statically compacted specimens M2a, M2b, M2c, M3a, M3b, M3c, M4a, M4b, M4c, M5a, M5b, M6a, M6b, M7a, M7b, M8a, M8b and M9a. The saturated triaxial tests were conducted in the triaxial apparatus as described in Section 5.3.4 under different net confining pressures of 25 kPa, 50 kPa and 100 kPa. Figure 5-30 to Figure 5-33 show the stress-strain curves of specimens M2 to M9 as obtained from the multistage CU saturated triaxial tests whereas Figure A5.4-1 to Figure A5.4-4 in Appendix illustrate the variations in pore-water pressures during the shearing of specimens M2 to M9.

Series of unsaturated consolidated drained (CD) triaxial tests were carried out on the statically compacted sand-kaolin specimens using the modified triaxial apparatus. All specimens were initially saturated and consolidated following the procedures described in Section 4.3.6. Subsequently, the specimen was equalized to the designated matric suction through matric suction incremental and decremental steps. Finally, the specimen was sheared at a constant rate of 0.0009 mm/min until the defined failure condition occurred as explained in Section 4.3.6. The soil conditions of specimens M2 to M9 during the unsaturated CD triaxial tests are summarized in Table A5.4-1 to Table A5.4-18 in Appendix. The stress-strain curves and volume change characteristics of specimens M2 to M9 as obtained from the unsaturated CD triaxial tests are shown in Figure 5-34 to Figure 5-51.

Figure 5-30. Stress-strain curves for specimens a. M2 and b. M3 obtained from the saturated multistage CU triaxial tests

Figure 5-31. Stress-strain curves for specimens a. M4 and b. M5 obtained from the saturated multistage CU triaxial tests

Figure 5-32. Stress-strain curves for specimens a. M6 and b. M7 obtained from the saturated multistage CU triaxial tests

Figure 5-33. Stress-strain curves for specimens a. M8 and b. M9 obtained from the saturated multistage CU triaxial tests

Figure 5-34. a. Stress-strain curves and b. total volumetric change characteristics of specimen M2a at different matric suctions under a net confining pressure of 25 kPa

Figure 5-35. a. Stress-strain curves and b. total volumetric change characteristics of specimen M2b at different matric suctions under a net confining pressure of 25 kPa

Figure 5-36. a. Stress-strain curves and b. total volumetric change characteristics of specimen M2c at different matric suctions under a net confining pressure of 25 kPa

Figure 5-37. a. Stress-strain curves and b. total volumetric change characteristics of specimen M3a at different matric suctions under a net confining pressure of 25 kPa

Figure 5-38. a. Stress-strain curves and b. total volumetric change characteristics of specimen M3b at different matric suctions under a net confining pressure of 25 kPa

Figure 5-39. a. Stress-strain curves and b. total volumetric change characteristics of specimen M3c at different matric suctions under a net confining pressure of 25 kPa

Figure 5-40. a. Stress-strain curves and b. total volumetric change characteristics of specimen M4a at different matric suctions under a net confining pressure of 25 kPa

Figure 5-41. a. Stress-strain curves and b. total volumetric change characteristics of specimen M4b at different matric suctions under a net confining pressure of 25 kPa

Figure 5-42. a. Stress-strain curves and b. total volumetric change characteristics of specimen M4c at different matric suctions under a net confining pressure of 25 kPa

Figure 5-43. a. Stress-strain curves and b. total volumetric change characteristics of specimen M5a at different matric suctions under a net confining pressure of 25 kPa

Figure 5-44. a. Stress-strain curves and b. total volumetric change characteristics of specimen M5b at different matric suctions under a net confining pressure of 25 kPa

Figure 5-45. a. Stress-strain curves and b. total volumetric change characteristics of specimen M6a at different matric suctions under a net confining pressure of 25 kPa

Figure 5-46. a. Stress-strain curves and b. total volumetric change characteristics of specimen M6b at different matrix suctions under a net confining pressure of 25 kPa

Figure 5-47. a. Stress-strain curves and b. total volumetric change characteristics of specimen M7a at different matrix suctions under a net confining pressure of 25 kPa

Figure 5-48. a. Stress-strain curves and b. total volumetric change characteristics of specimen M7b at different matrix suctions under a net confining pressure of 25 kPa

Figure 5-49. a. Stress-strain curves and b. total volumetric change characteristics of specimen M8a at different matric suctions under a net confining pressure of 25 kPa

Figure 5-50. a. Stress-strain curves and b. total volumetric change characteristics of specimen M8b at different matric suctions under a net confining pressure of 25 kPa

Figure 5-51. a. Stress-strain curves and b. total volumetric change characteristics of specimen M9a at different matric suctions under a net confining pressure of 25 kPa

The results of CU saturated triaxial tests on the statically compacted specimens as carried out in this study were analyzed by drawing Mohr circles and failure envelopes. The Mohr circles of all the saturated triaxial test results from specimens M2 to M9 and the Mohr-Coulomb failure envelopes for specimens M2 to M9 are shown in Figure 5-52 to Figure 5-59. The c' varied from 3 to 18 kPa whereas the ϕ' varied from 23° to 43° . The lowest c' was observed in the specimen with the highest percentage of sand while the lowest ϕ' was observed in the specimen with the lowest percentage of sand.

Figure 5-52. Mohr-Coulomb failure envelope for specimens M2 obtained from multistage CU saturated triaxial tests

Figure 5-53. Mohr-Coulomb failure envelope for specimens M3 obtained from multistage CU saturated triaxial tests

Figure 5-54. Mohr-Coulomb failure envelope for specimens M4 obtained from multistage CU saturated triaxial tests

Figure 5-55. Mohr-Coulomb failure envelope for specimens M5 obtained from multistage CU saturated triaxial tests

Figure 5-56. Mohr-Coulomb failure envelope for specimens M6 obtained from multistage CU saturated triaxial tests

Figure 5-57. Mohr-Coulomb failure envelope for specimens M7 obtained from multistage CU saturated triaxial tests

Figure 5-58. Mohr-Coulomb failure envelope for specimens M8 obtained from multistage CU saturated triaxial tests

Figure 5-59. Mohr-Coulomb failure envelope for specimens M9 obtained from multistage CU saturated triaxial tests

The unsaturated shear strengths for specimens M2 to M9 were obtained from the multistage unsaturated CD triaxial tests. In order to obtain the cohesion intercepts of the specimens at different matric suctions, the extended Mohr-Coulomb failure envelope was constructed by drawing a straight line with its slope being the effective friction angle (ϕ') through the points of tangency of the Mohr circles. The interception of the failure envelope and shear stress axis is the cohesion intercept (c) (Figure 5-60 to Figure 5-77). The angle indicating the change in shear strength due to a change in matric suction (ϕ^b) was measured from the slope of the extended Mohr-Coulomb failure envelope (Figure 5-78 to Figure 5-81). The ϕ^b angles of specimens M2 to M9 are summarized in Table 5-12 to Table 5-14. The magnitudes of c' , ϕ' and ϕ^b for specimens M2 to M9 are summarized in Table 5-12 to Table 5-14.

Table 5-12. Shear strength properties of specimens M2a, M2b, M3a, M3b, M4a and M4b with bimodal SWCC

Properties	M2a	M2b	M2c	M3a	M3b
c' (kPa)	3	3	4	5	6
ϕ' ($^\circ$)	43	40	35	34	33
ϕ^b ($^\circ$) for $u_a - u_w \leq AEV_1$	43	40	35	34	33
ϕ^b ($^\circ$) for $AEV_1 < u_a - u_w \leq AEV_2$	38.7	36	31	29	28
ϕ^b ($^\circ$) for $u_a - u_w > AEV_2$	21	21	20	20	18

Table 5-13. Shear strength properties of specimens M5a, M6a, M7b and M8a with bimodal SWCC

Properties	M4a	M4b	M5a	M6a	M7a	M8a
c' (kPa)	8	7	9	12	14	16
ϕ' ($^\circ$)	31	29	26	26	26	25
ϕ^b ($^\circ$) for $u_a - u_w \leq AEV_1$	31	29	26	26	26	25
ϕ^b ($^\circ$) for $AEV_1 < u_a - u_w \leq AEV_2$	26	24	17	13	10	6
ϕ^b ($^\circ$) for $u_a - u_w > AEV_2$	17	16	12	7	8	5

Table 5-14. Shear strength properties of specimens M2c, M3c, M4c, M5b M6b, M7b, M8b and M9a with unimodal SWCC

Properties	M3c	M4c	M5b	M6b	M7b	M8b	M9a
c' (kPa)	5	9	8	10	12	15	18
ϕ' ($^{\circ}$)	31	28	27	25	26	23	23
ϕ^b ($^{\circ}$) for $u_a - u_w \leq \text{AEV}$	31	28	27	25	26	23	23
ϕ^b ($^{\circ}$) for $\text{AEV} < u_a - u_w \leq \text{AEV}$	30	25	24	20	12	12	10

Figure 5-60. Mohr circles and cohesion intercepts, C , of specimen M2a under a net confining pressure of 25 kPa

Figure 5-61. Mohr circles and cohesion intercepts, C , of specimen M2b under a net confining pressure of 25 kPa

Figure 5-62. Mohr circles and cohesion intercepts, C , of specimen M2c under a net confining pressure of 25 kPa

Figure 5-63. Mohr circles and cohesion intercepts, C , of specimen M3a under a net confining pressure of 25 kPa

Figure 5-64. Mohr circles and cohesion intercepts, C , of specimen M3b under a net confining pressure of 25 kPa

Figure 5-65. Mohr circles and cohesion intercepts, C , of specimen M3c under a net confining pressure of 25 kPa

Figure 5-66. Mohr circles and cohesion intercepts, C , of specimen M4a under a net confining pressure of 25 kPa

Figure 5-67. Mohr circles and cohesion intercepts, C , of specimen M4b under a net confining pressure of 25 kPa

Figure 5-68. Mohr circles and cohesion intercepts, C , of specimen M4c under a net confining pressure of 25 kPa

Figure 5-69. Mohr circles and cohesion intercepts, C , of specimen M5a under a net confining pressure of 25 kPa

Figure 5-70. Mohr circles and cohesion intercepts, C , of specimen M5b under a net confining pressure of 25 kPa

Figure 5-71. Mohr circles and cohesion intercepts, C , of specimen M6a under a net confining pressure of 25 kPa

Figure 5-72. Mohr circles and cohesion intercepts, C , of specimen M6b under a net confining pressure of 25 kPa

Figure 5-73. Mohr circles and cohesion intercepts, C , of specimen M7a under a net confining pressure of 25 kPa

Figure 5-74. Mohr circles and cohesion intercepts, C , of specimen M7b under a net confining pressure of 25 kPa

Figure 5-75. Mohr circles and cohesion intercepts, C , of specimen M8a under a net confining pressure of 25 kPa

Figure 5-76. Mohr circles and cohesion intercepts, C , of specimen M8b under a net confining pressure of 25 kPa

Figure 5-77. Mohr circles and cohesion intercepts, C , of specimen M9a under a net confining pressure of 25 kPa

Figure 5-78. The unsaturated shear strength data of specimens a. M2 and b. M3

Figure 5-79. The unsaturated shear strength data of specimens a. M4 and b. M5

Figure 5-80. The unsaturated shear strength data of specimens a. M6 and b. M7

Figure 5-81. The unsaturated shear strength data of specimens a. M8 and b. M9

5.5. Evaluation of The Proposed and Published Equation for Best Fitting Grain-Size Distribution using Experimental Data from Laboratory Tests carried out in This Study and Published Data from Literatures

In this section, the evaluation of the proposed equations for best fitting grain-size distribution (GSD) of soil with bimodal and unimodal characteristics as developed in this study are presented in detail. The performance of the proposed equations for best fitting GSD of soil was evaluated using experimental data from laboratory tests carried out in this study and published data from literatures. Then, the performance of the proposed equations was compared with the performance of published equations for best fitting

GSD of soil with bimodal and unimodal characteristics using the criteria explained in Section 4.6.

The GSDs of soils with bimodal characteristics obtained from laboratory tests carried out in this study are presented in Figure 5-82 to Figure 5-84. The GSDs of soils with bimodal characteristics obtained from literatures are presented in Figure 5-85 to Figure 5-88. Two distinct groups of soil particles size (called gap-graded soils) were observed in the GSD data in Figure 5-82 to Figure 5-88. Therefore, those GSD data were best fitted with the proposed equation for best fitting bimodal GSD (Equation 3-21) developed in this study and Fredlund et al. (2000) equation for best fitting bimodal GSD. The parameters of the proposed equation developed in this study for best fitting bimodal GSD from the experimental tests in this study are summarized in Table 5-15. The parameters of Fredlund (2000) equation for best fitting bimodal GSD are summarized in Table 5-16. The parameters of the proposed equation developed in this study for best fitting bimodal GSD from literatures are summarized in Table 5-17 and Table 5-18. The parameters of Fredlund (2000) equation for best fitting bimodal GSD from literatures are summarized in Table 5-19 and Table 5-20.

Table 5-15 indicates that the parameter d_{m1} for all specimens was 0.70 mm which corresponds to the geometric means of particle sizes within large particle and parameter d_{m2} for all specimens was 0.007 mm which corresponds to the geometric means of particle sizes within small particle as shown in Figure 5-82 to Figure 5-84. Parameter d_{max1} for all specimens was 2 mm which corresponds to maximum particles size within large particle and parameter d_{max2} for all specimens was 0.015 mm which corresponds to maximum particles size within small particle as shown in Figure 5-82 to Figure 5-84. The parameters d_{max1} , d_{max2} , d_{m1} and d_{m2} in Table 5-15 also shows that each specimen consisted of medium sand and silt as presented in Figure 5-82 to Figure 5-84. Parameter s_{d1} for all specimens was 0.007 which corresponds to the geometric standard deviation of particle size within large particle and parameter s_{d2} for all specimens was 0.86 which corresponds to the geometric standard deviation of particle size within small particle. The smaller value of parameter s_{d1} as compared with parameter s_{d2} indicated that the medium sand was distributed more uniformly than the silt within all specimens tested in this study (Table 5-15).

Figure 5-82. GSD of soil M1, M2, M3 best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD

Figure 5-83. GSD of soil M4, M5, M6 best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD

Figure 5-84. GSD of soil M7, M8, M9 best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD

Table 5-15. Summary of parameters for best fitting bimodal GSD of soil data from experimental test carried out in this study using the proposed equation (Equation 3-21) developed in this study

Parameter	M1	M2	M3	M4	M5	M6	M7	M8	M9
$d_{\max 1}$ (mm)	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
$d_{\max 2}$ (mm)	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
d_{m1} (mm)	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
d_{m2} (mm)	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
s_{d1}	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
s_{d2}	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86
W_1	0.90	0.79	0.70	0.58	0.49	0.40	0.30	0.19	0.10
W_2	0.10	0.21	0.30	0.42	0.51	0.60	0.70	0.81	0.90
$d_{\min}$ (mm)	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
d_r (mm)	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075

Table 5-16. Summary of parameters for best fitting bimodal GSD of soil data from experimental test carried out in this study using Fredlund (2000) equation for best fitting bimodal GSD

Parameter	M1	M2	M3	M4	M5	M6	M7	M8	M9
a_{bi}	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63
n_{bi}	100	100	100	100	100	100	100	100	100
m_{bi}	2.38	2.38	2.38	2.38	2.38	2.38	2.38	2.38	2.38
j_{bi}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
k_{bi}	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27
l_{bi}	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72
W	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
d_{rbi}	100	100	100	100	100	100	100	100	100
d_m	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Table 5-17 and Table 5-18 indicate that the parameter d_{m1} for specimens 11491, 11492, 11494, SB3, JF3, JF13, BTG8, GM1b, GM2b, GM3b, GRS and Doi Tung were 0.65, 0.55, 0.85, 2.35, 0.10, 0.30, 0.07, 15, 10, 4.5, 0.20 and 0.40 mm, respectively which were in accordance with the geometric means of particle sizes within large particle as shown in Figure 5-85 to Figure 5-88. Parameter d_{m2} for specimens 11491, 11492, 11494, SB3, JF3, JF13, BTG8, GM1b, GM2b, GM3b, GRS and Doi Tung were 0.004, 0.004, 0.01, 0.04, 0.003, 0.005, 0.008, 0.001, 0.001, 0.001, 0.003 and 0.001 mm, respectively which were in accordance with the geometric means of particle sizes within small particle as shown in Figure 5-85 to Figure 5-88. Parameter d_{max1} for specimens 11491, 11492, 11494, SB3, JF3, JF13, BTG8, GM1b, GM2b, GM3b, GRS and Doi Tung were 2, 2, 2, 4.75, 2, 2, 2, 30, 15, 15, 7 and 15 mm, respectively which were in accordance with maximum particles size within large particle as shown in Figure 5-85 to Figure 5-88. Parameter d_{max2} for specimens 11491, 11492, 11494, SB3, JF3, JF13, BTG8, GM1b, GM2b, GM3b, GRS and Doi Tung were 0.025, 0.35, 0.15, 2, 0.06, 0.2, 0.1, 0.25, 0.02, 0.03, 0.08 and 0.02 m, respectively which were in accordance with maximum particles size within small particle as shown in Figure 5-85 to Figure 5-88.

Figure 5-85. GSD of soils 11491, 11492 and 11494 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD

Figure 5-86. GSD of soils SB3, JF3 and JF13 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD

Figure 5-87. GSD of soils BTG8, GM1b and GM2b (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD

Parameter s_{d1} for specimens 11491, 11492, 11494, SB3, JF3, JF13, BTG8, GM1b, GM2b, GM3b, GRS and Doi Tung were 0.29, 0.06, 2.13, 0.49, 0.06, 0.18, 0.102, 0.05, 0.034, 0.046, 0.05 and 0.004, respectively which corresponds to the geometric standard deviation of particle size within large particle. Parameter s_{d2} for specimens 11491, 11492, 11494, SB3, JF3, JF13, BTG8, GM1b, GM2b, GM3b, GRS and Doi Tung were 0.5, 0.02, 3.95, 0.05, 0.02, 0.01, 0.05, 0.023, 0.3, 0.056, 0.072 and 0.14, respectively which corresponds to the geometric standard deviation of particle size within small particle.

Figure 5-88. GSD of soils GM3b, GRS and Doi Tung (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting bimodal GSD

Table 5-17. Summary of parameters for best fitting bimodal GSD of published data from literatures using the proposed equation (Equation 3-21) for best fitting bimodal GSD developed in this study

Variables	11491	11492	11494	SB3	JF3	JF13
$d_{\max 1}$ (mm)	2.00	9.50	0.875	4.75	2.00	2.00
$d_{\max 2}$ (mm)	0.025	0.35	0.15	2.00	0.06	0.20
d_{m1} (mm)	0.65	0.55	0.85	2.35	0.10	0.30
d_{m2} (mm)	0.004	0.004	0.010	0.040	0.003	0.005
s_{d1}	0.29	0.06	2.13	0.49	0.06	0.18
s_{d2}	0.50	0.02	3.95	0.05	0.02	0.01
W_1	0.27	0.60	0.53	0.35	0.86	0.86
W_2	0.73	0.40	0.47	0.65	0.14	0.14
$d_{\min}$ (mm)	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
d_r (mm)	0.075	0.075	0.075	0.075	0.075	0.075
R^2	0.998	0.996	0.999	0.997	0.999	0.999

Table 5-18. Summary of parameters for best fitting bimodal GSD of published data from literatures based on the proposed equation (Equation 3-21) for best fitting bimodal GSD developed in this study

Variables	BTG8	GM1b	GM2b	GM3b	GRS	Doi Tung
$d_{\max 1}$ (mm)	2.00	30	30	30	30	15
$d_{\max 2}$ (mm)	0.10	0.25	0.02	0.03	0.08	0.02
d_{m1} (mm)	0.07	15	10	4.5	0.20	0.40
d_{m2} (mm)	0.008	0.001	0.001	0.001	0.003	0.0001
s_{d1}	0.102	0.050	0.034	0.046	0.050	0.004
s_{d2}	0.051	0.023	0.30	0.056	0.072	0.140
W_1	0.42	0.5	0.49	0.49	0.60	0.11
W_2	0.58	0.5	0.51	0.51	0.40	0.89
$d_{\min}$ (mm)	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
d_r (mm)	0.075	0.075	0.075	0.075	0.075	0.075
R^2	0.992	0.997	0.996	0.998	0.998	0.999

Table 5-19. Summary of parameters for best fitting bimodal GSD published data from literatures based on the Fredlund (2000) equation for best fitting bimodal GSD

Variables	11491	11492	11494	SB3	JF3	JF13
a_{bi}	1.1	1.28	0.94	0.16	0.22	0.62
n_{bi}	3.69	3.23	40	2.59	3.97	3.52
m_{bi}	0.36	0.37	0.25	0.64	0.56	0.58
j_{bi}	0.006	0.011	0.14	2.64	0.82	0.85
k_{bi}	2.26	28.7	29	4.44	16	1.38
l_{bi}	7.6	13.9	14.7	1.92	32	0.84
W	0.57	0.93	0.80	0.68	0.99	1.00
d_{rbi}	70	272	272	1.23	15.5	5.0
d_m	0.001	0.001	0.001	0.001	0.001	0.001
R^2	0.998	0.996	0.999	0.997	0.999	0.999

Table 5-20. Summary of parameters for best fitting bimodal GSD of published data from literatures based on the Fredlund et al. (2000) equation for best fitting bimodal GSD

Variables	BTG8	GM1b	GM2b	GM3b	GRS	Doi Tung
a_{bi}	0.38	15.3	9.5	3.55	0.62	0.45
n_{bi}	2.61	9.9	30	6.24	1.67	43.4
m_{bi}	0.65	4.41	5.9	10.5	4.76	0.033
j_{bi}	0.001	0.01	0.01	0.06	0.01	0.45
k_{bi}	1.11	0.92	0.92	1.07	0.92	43.4
l_{bi}	24	1.75	1.56	0.44	1.83	0.033
W	0.35	0.49	0.49	0.49	0.32	0.49
d_{rbi}	0.001	1.28	1.00	0.98	1.22	25480
d_m	0.001	0.001	0.001	0.001	0.001	0.001
R^2	0.986	0.997	0.996	0.998	0.998	0.981

Figure 5-89. GSD of coarse sand and coarse kaolin best fitted with the proposed equation developed in this study and published equation for best fitting unimodal GSD

The GSD of soil with unimodal characteristics obtained from the experimental works in this study are presented in Figure 5-89. The GSD of soil data with unimodal characteristics from published literatures are presented in Figure 5-90 to Figure 5-92. Those GSD data were best fitted with the proposed equation for best fitting unimodal GSD (Equation 3-24) developed in this study. The parameters of the proposed equation

developed in this study for best fitting unimodal GSD of coarse sand and coarse kaolin are summarized in Table 5-21. The parameters of the proposed equation developed in this study for best fitting unimodal GSD from literatures are summarized in Table 5-22.

Table 5-21 indicates that the parameters d_m for coarse sand and coarse kaolin were 0.70 and 0.009 mm, respectively which correspond to the geometric means of particle sizes as shown in Figure 5-89. Parameter d_{max} for coarse sand and coarse kaolin were 2 and 0.06 mm which correspond to maximum particles size as shown in Figure 5-89. Parameter s_d for coarse sand and coarse kaolin were 0.10 and 0.075 which correspond to the geometric standard deviation of particle size. Table 5-22 indicates that the parameters d_m for soils S4, S45, S62, JF25, BTG4, BTG20, SC17, SW1 and CH3 were 0.15, 0.015, 0.025, 0.15, 0.025, 0.55, 0.5, 0.45 and 0.03 mm, respectively which correspond to the geometric means of particle sizes as shown in Figure 5-90 to Figure 5-92. Parameter d_{max} for soils S4, S45, S62, JF25, BTG4, BTG20, SC17, SW1 and CH3 were 2, 2, 2, 2, 2, 2, 2, 2 and 1 which correspond to maximum particles size as shown in Figure 5-90 to Figure 5-92. Parameter s_d for soils S4, S45, S62, JF25, BTG4, BTG20, SC17, SW1 and CH3 were 0.024, 0.19, 0.027, 0.102, 0.036, 0.137, 0.159, 0.04 and 0.03 which were in accordance with the geometric standard deviation of particle size.

Table 5-21. Summary of parameters for best fitting unimodal GSD from experimental test carried out in this study based on the proposed equation (Equation 3-24) for best fitting unimodal GSD developed in this study

Variables	Coarse sand	Coarse kaolin
d_{max} (mm)	1.18	0.075
d_m (mm)	0.70	0.009
s_d	0.10	0.075
d_{min} (mm)	0.0001	0.0007
d_r (mm)	0.075	0.003
R^2	0.999	0.999

Table 5-22. Summary of parameters for best fitting unimodal GSD of published data from literatures based on the proposed equation (Equation 3-24) for best fitting unimodal GSD developed in this study

Variables	S4	S45	S62	JF25	BTG4	BTG20	SC17	SW1	CH3
$d_{\max}$ (mm)	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	1.0
d_m (mm)	0.15	0.015	0.025	0.15	0.025	0.55	0.5	0.45	0.03
s_d	0.024	0.19	0.027	0.102	0.036	0.137	0.159	0.04	0.03
$d_{\min}$ (mm)	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
d_r (mm)	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075
R^2	1.00	1.00	0.998	1.00	0.999	1.00	1.00	0.999	0.999

Figure 5-90. GSD of soils S4, S45 and S62 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting unimodal GSD

Figure 5-91. GSD of soils JF5, BTG4 and BTG20 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting unimodal GSD

Figure 5-92. GSD of soils SC17, SW1 and CH3 (published data from literatures) best fitted with the proposed equation developed in this study and published equation for best fitting unimodal GSD

5.6. Evaluation of the Proposed and Published Equations for Fitting Soil-Water Characteristic Curve using Experimental Data from Laboratory Tests carried out in This Study and using Published Data from Literatures

In this section, the evaluation of the proposed equations for best fitting soil-water characteristic curve (SWCC) with bimodal (Equations 3-15 and 3-20) and unimodal (Equations 3-11 and 3-19) characteristics developed in this study are presented and discussed in detail. The performance of the proposed equations for best fitting SWCC were evaluated using experimental data from laboratory tests carried out in this study and published data from literatures. Then, the performance of the proposed equations was compared with the performance of published equations for best fitting SWCC with bimodal and unimodal characteristics using the criteria explained in Section 4.6.

Figure 5-93. SWCC of soil mixtures M2a, M2b and M2c best fitted with the proposed SWCC equation developed in this study and published equation from literature

The SWCC of soil with bimodal characteristics obtained from laboratory tests carried out in this study are presented in Figure 5-93 to Figure 5-97. The SWCC of soil with bimodal characteristics obtained from published literatures are presented in Figure 5-98 to Figure 5-101. Two slopes with different gradients were observed in the SWCC data in Figure 5-93 to Figure 5-97 and Figure 5-106 to Figure 5-107. Therefore, those SWCC data were best fitted with the proposed equations for best fitting bimodal SWCC (Equations 3-11 and 3-19) developed in this study and Zhang and Chen (2005) equation for best fitting bimodal SWCC. The parameters of the proposed equation (Equation 3-15)

developed in this study for best fitting bimodal SWCC from the laboratory tests carried out in this study are summarized in Table 5-23 and Table 5-24. The parameters of the proposed equation (Equation 3-20) developed in this study for best fitting bimodal SWCC from the laboratory tests carried out in this study are summarized in Table 5-25 and Table 5-26. The parameters of Zhang and Chen (2005) equation for best fitting bimodal SWCC are summarized in Table 5-27 and Table 5-28. The parameters of the proposed equation (Equation 3-15) developed in this study for best fitting bimodal SWCC from literatures are summarized in Table 5-29. The parameters of the proposed equation (Equation 3-20) developed in this study for best fitting bimodal SWCC from literatures are summarized in Table 5-30. The parameters of Zhang and Chen (2005) equation for best fitting bimodal SWCC from literatures are summarized in Table 5-31.

Figure 5-94. SWCC of soil mixtures M3a and M3b best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-95. SWCC of soil mixtures M4a and M4b best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-96. SWCC of soil mixtures M5a and M6a best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-97. SWCC of soil mixtures M7a and M8a best fitted with the proposed SWCC equation developed in this study and published equation from literature

Table 5-23 and Table 5-24 indicate that the parameter ψ_{a1} of the proposed bimodal SWCC equation (Equation 3-15) for soils M2a, M2b, M2c, M3a, M3b, M4a, M4b, M5a, M6a, M7a and M8a were 20, 20, 14, 6, 2, 3, 7, 10, 10, 1 and 1 kPa, respectively which correspond to the first air-entry value (AEV1) of these soils as shown in Figure 5-93 to Figure 5-97. Parameter ψ_{a2} for soils M2a, M2b, M2c, M3a, M3b, M4a, M4b, M5a, M6a, M7a and M8a were 100, 75, 65, 35, 60, 35, 40, 30, 25, 10 and 6 which correspond to the second air-entry value (AEV2) of these soils as shown in Figure 5-93 to Figure 5-97. Parameter ψ_r for soils M2a, M2b, M2c, M3a, M3b, M4a, M4b, M5a, M6a,

M7a and M8a were 1000, 1000, 350, 600, 500, 500, 300, 500, 1000 and 400 kPa, respectively which correspond to the residual suction of these soils as shown in Figure 5-93 to Figure 5-97. Parameter θ_r for soils M2a, M2b, M2c, M3a, M3b, M4a, M4b, M5a, M6a, M7a and M8a were 0.05, 0.075, 0.06, 0.075, 0.06, 0.03, 0.05, 0.02, 0.03, 0.06 and 0.05, respectively which correspond to the residual volumetric water content of these soils as shown in Figure 5-93 to Figure 5-97.

Table 5-23. Best fitting parameters of proposed equation 3-15 for soil mixtures M2a, M2b, M2c, M3a and M3b with bimodal SWCC

Parameters	M2a	M2b	M2c	M3a	M3b
θ_{s1}	0.203	0.267	0.316	0.352	0.339
ψ_{a1} (kPa)	20	20	14	6	2
ψ_{m1} (kPa)	25	25	18	10	7
s_1	0.75	0.95	0.94	0.81	1.75
θ_{s2}	0.175	0.225	0.27	0.29	0.28
ψ_{a2} (kPa)	100	75	65	35	60
ψ_{m2} (kPa)	250	175	120	100	120
s_2	2	2.5	2	1.5	2
ψ_r (kPa)	1000	1000	1000	350	600
θ_r	0.05	0.075	0.06	0.075	0.06
R^2	0.996	0.999	0.995	1.000	0.989

Figure 5-98. SWCC of soil SB1 and SB2 best fitted with the proposed SWCC equation developed in this study and published equation from literature

Table 5-24. Best fitting parameters of proposed equation 3-15 for soil mixtures M4a, M4b, M5a, M6a, M7a and M8a with bimodal SWCC

Parameters	M4a	M4b	M5a	M6a	M7a	M8a
θ_{s1}	0.282	0.312	0.228	0.207	0.52	0.595
ψ_{a1} (kPa)	3	7	10	10	1	1
ψ_{m1} (kPa)	4	8	15	7.5	1.5	1.1
s_1	0.36	0.36	0.2	0.36	0.36	0.36
θ_{s2}	0.234	0.27	0.195	0.174	0.39	0.475
ψ_{a2} (kPa)	35	40	30	25	10	6
ψ_{m2} (kPa)	100	150	120	150	70	40
s_2	1	1.25	0.92	1	2	2.5
ψ_r (kPa)	500	500	300	500	1000	400
θ_r	0.03	0.05	0.02	0.03	0.06	0.05
R^2	0.990	0.923	0.991	1.000	0.992	0.953

Table 5-25. Best fitting parameters of proposed equation 3-20 for soil mixtures M2a, M2b, M2c, M3a and M3b with bimodal SWCC

Parameters	M2a	M2b	M2c	M3a	M3b
θ_{s1}	0.203	0.267	0.316	0.352	0.339
A_1 (kPa)	20	15	14	6	2
R_1 (kPa)	25	330	328	32	30
θ_{s2}	0.175	0.176	0.176	0.29	0.283
A_2 (kPa)	100	75	65	35	60
R_2 (kPa)	1300	1243	1027	551	600
D	0.06	0.095	0.092	0.087	0.087
R^2	0.990	0.989	0.993	0.991	0.923

Table 5-26. Best fitting parameters of proposed equation 3-20 for soil mixtures M4a, M4b, M5a, M6a, M7a and M8a with bimodal SWCC

Parameters	M4a	M4b	M5a	M6a	M7a	M8a
θ_{s1}	0.282	0.312	0.228	0.207	0.520	0.595
A_1 (kPa)	3	5	10	10	1.1	1
R_1 (kPa)	5	14	32	20	2	2
θ_{s2}	0.234	0.270	0.195	0.174	0.390	0.480
A_2 (kPa)	25	40	30	25	11	8
R_2 (kPa)	500	706	750	794	450	185
D	0.05	0.064	0.020	0.047	0.080	0.122
R^2	0.977	0.979	0.983	0.994	0.987	0.930

Table 5-27. Best fitting parameters of Zhang and Chen (2005) equation for soil mixtures M2a, M2b, M2c, M3a and M3b with bimodal SWCC

Parameters	M2a	M2b	M2c	M3a	M3b
θ_{sl}	0.177	0.147	0.181	0.310	0.325
a_l	122	109	71	54	80
n_l	4.7	4.6	13	3.1	3.3
m_l	0.35	0.15	0.34	0.51	0.47
θ_{ss}	0.026	0.120	0.135	0.042	0.014
a_s	28	54	11	13	4.4
n_s	10	2	10	10	9.5
m_s	2.2	0.57	0.03	10	11
ψ_{rl} (kPa)	429	429	429	37	8
ψ_{rs} (kPa)	500	500	500	500	500
R^2	0.984	0.985	0.957	0.974	0.932

Table 5-28. Best fitting parameters of Zhang and Chen (2005) equation for soil mixtures M4a, M4b, M5a, M6a, M7a and M8a with bimodal SWCC

Parameters	M4a	M4b	M5a	M6a	M7a	M8a
θ_{sl}	0.244	0.274	0.196	0.173	0.421	0.532
a_l	68	75	84	62	33	11
n_l	5.3	3.3	3.5	3.4	1.1	1.0
m_l	0.51	0.69	1.0	0.58	1.27	1.1
θ_{ss}	0.038	0.038	0.031	0.034	0.099	0.062
a_s	1.8	6.8	16	12	1.5	2
n_s	9.1	10	10	10	11	10
m_s	11	2	1.5	1.2	11	10
ψ_{rl} (kPa)	21	429	429	429	213	213
ψ_{rs} (kPa)	500	500	500	500	500	500
R^2	0.942	0.953	0.925	0.943	0.924	0.921

Figure 5-99. SWCC of soil SB3 and SB4 best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-100. SWCC of soil SB5 and SB6 best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-101. SWCC of soil JF3 and JF13 best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-102. SWCC of soil mixtures M3c, M4c and M5b best fitted with the proposed SWCC equation developed in this study and published equation from literature

Table 5-29 indicate that the parameter ψ_{a1} for soils SB1, SB2, SB3, SB4, SB5, SB6, JF3, JF13 were 2, 2, 1, 1, 1, 1, 20 and 30 kPa, respectively which correspond to the first air-entry value (AEV1) of these soils as shown in Figure 5-98 to Figure 5-101. Parameter ψ_{a2} for soils SB1, SB2, SB3, SB4, SB5, SB6, JF3, JF13 were 30, 50, 10, 70, 20, 50, 200 and 200 kPa, respectively which correspond to the second air-entry value (AEV2) of these soils as shown in Figure 5-98 to Figure 5-101. Parameter ψ_r for soils SB1, SB2, SB3, SB4, SB5, SB6, JF3, JF13 were 1000, 9000, 974, 10000, 4000, 723, 4000 and 4000 kPa, respectively which correspond to the residual suction of these soils as shown in Figure 5-98 to Figure 5-101. Parameter θ_r for soils SB1, SB2, SB3, SB4, SB5, SB6, JF3, JF13 were 0.08, 0.02, 0.07, 0.05, 0.08, 0.15, 0.03 and 0.05 kPa, respectively which correspond to the residual volumetric water content of these soils as shown in Figure 5-98 to Figure 5-101.

Table 5-29. Best fitting parameters of proposed equation 3-15 for soil SB1, SB2, SB3, SB4, SB5, SB6, JF3 and JF13 with bimodal SWCC

Parameters	SB1	SB2	SB3	SB4	SB5	SB6	JF3	JF13
θ_{s1}	0.320	0.352	0.334	0.394	0.445	0.405	0.236	0.255
ψ_{a1} (kPa)	2.00	2.00	1.00	1.00	1.00	1.00	20	30
ψ_{m1} (kPa)	3.00	3.00	2.00	1.96	1.68	4.19	36	40
s_1	0.34	0.09	0.36	0.01	1.25	1.50	0.65	2.20
θ_{s2}	0.267	0.292	0.276	0.298	0.35	0.30	0.208	0.212
ψ_{a2} (kPa)	30	50	10	70	20	50	200	200
ψ_{m2} (kPa)	114	556	50	834	282	282	1045	1000
s_2	1.50	1.76	1.85	2.10	2.29	2.16	1.05	0.93
ψ_r (kPa)	1000	9000	974	10000	4000	723	4000	4000
θ_r	0.08	0.02	0.07	0.05	0.08	0.15	0.03	0.05
R^2	0.998	0.998	0.998	0.998	0.999	0.996	0.999	0.999

Table 5-30. Best fitting parameters of proposed equation 3-20 for soil SB1, SB2, SB3, SB4, SB5, SB6, JF3 and JF13 with bimodal SWCC

Parameters	SB1	SB2	SB3	SB4	SB5	SB6	JF3	JF13
θ_{sl}	0.320	0.352	0.334	0.394	0.445	0.405	0.236	0.255
A_1 (kPa)	2.0	2.0	1.0	1.0	1.0	1.0	20	30
R_1 (kPa)	9.4	9.4	6.0	2.0	6.0	28	50	100
θ_{s2}	0.267	0.292	0.276	0.298	0.350	0.300	0.208	0.212
A_2 (kPa)	30	40	10	70	20	40	200	200
R_2 (kPa)	571	2086	256	4000	600	569	7000	15000
D	0.108	0.100	0.107	0.100	0.200	0.215	0.000	0.000
R^2	0.997	0.985	0.978	0.992	0.996	0.997	0.992	0.998

Table 5-31. Best fitting parameters of Zhang and Chen (2005) equation for soil SB1, SB2, SB3, SB4, SB5, SB6, JF3 and JF13 with bimodal SWCC

Parameters	SB1	SB2	SB3	SB4	SB5	SB6	JF3	JF13
θ_{sl}	0.053	0.060	0.145	0.099	0.096	0.105	0.028	0.043
a_l	3.03	2.19	1.24	2.03	2.09	2.1	30	30
n_l	3.71	50	5.47	50	50	5.0	6.0	6.0
m_l	2.54	25	0.27	25	25	0.6	1.0	1.0
θ_{ss}	0.267	0.292	0.189	0.298	0.353	0.298	0.208	0.212
a_s	36	68	16	76	22	27	350	1000
n_s	6.0	24	14	24	24	2.78	14	5.0
m_s	0.06	0.01	0.03	0.01	0.01	0.05	0.03	0.30
ψ_{rl} (kPa)	200	200	200	201	201	201	200	200
ψ_{rs} (kPa)	500	502	502	502	501	501	502	502
R^2	0.998	0.998	0.998	0.999	0.998	0.993	0.999	0.997

The SWCC of soil with unimodal characteristics obtained from laboratory tests carried out in this study are presented in Figure 5-102 to Figure 5-105. The SWCC of soil with unimodal characteristics obtained from published literatures are presented in Figure 5-106 to Figure 5-107. Two slopes with different gradients were observed in the SWCC data in Figure 5-102 to Figure 5-107. Therefore, those SWCC data were best fitted with the proposed equations for best fitting unimodal SWCC (Equations 3-11 and 3-19) developed in this study and Fredlund and Xing (1994) equation for best fitting unimodal SWCC. The parameters of the proposed equation (Equation 3-11) developed in this study for best fitting unimodal SWCC from the laboratory tests carried out in this study are summarized in Table 5-32 and Table 5-33. The parameters of the proposed equation (Equation 3-19) developed in this study for best fitting unimodal SWCC from the laboratory tests carried out in this study are summarized in Table 5-34 and Table 5-35. The parameters of Fredlund and Xing (1994) equation for best fitting unimodal SWCC are summarized in Table 5-36 and Table 5-37. The parameters of the proposed equation (Equation 3-11) developed in this study for best fitting unimodal SWCC from literatures are summarized in Table 5-38. The parameters of the proposed equation (Equation 3-19) developed in this study for best fitting bimodal SWCC from literatures are summarized in Table 5-39. The parameters of Fredlund and Xing (1994) equation for best fitting unimodal SWCC from literatures are summarized in Table 5-40.

Figure 5-103. SWCC of soil mixtures M5c, M6b and M6c best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-104. SWCC of soil mixtures M7b and M7c best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-105. SWCC of soil mixtures M8b and M8c best fitted with the proposed SWCC equation developed in this study and published equation from literature

Table 5-32 and Table 5-33 indicate that the parameter ψ_a of the proposed unimodal SWCC equation (Equation 3-11) for soils M3c, M4c, M5b, M5c, M6b, M6c, M7b, M7c, M8b and M8c were 7, 10, 1, 1, 3, 5, 5, 8, 10 and 20 kPa, respectively which correspond to the air-entry value (AEV) of these soils as shown in Figure 5-102 to Figure 5-105. Parameter ψ_r for soils M3c, M4c, M5b, M5c, M6b, M6c, M7b, M7c, M8b, M8c were 4000, 1000, 9, 300, 400, 400, 2000, 1000, 1500 and 3000 kPa, respectively which were in accordance with the residual suction of these soils as shown in Figure 5-102 to

Figure 5-105. Parameter θ_r for soils M3c, M4c, M5b, M5c, M6b, M6c, M7b, M7c, M8b, M8c were 0.02, 0.05, 0.02, 0.02, 0, 0, 0.04, 0.04, 0.04 and 0.05, respectively which were in accordance with the residual volumetric water content of these soils as shown in Figure 5-102 to Figure 5-105.

Table 5-38 indicates that the parameter ψ_a of the proposed unimodal SWCC equation (Equation 3-11) for soils SB9, SB10, GM1b, GM2b, BTG8 and BTG14 were 10, 2, 10, 40, 100 and 100 kPa, respectively which were in accordance with the air-entry value (AEV) of these soils as shown in Figure 5-106 to Figure 5-107. Parameter ψ_r for soils SB9, SB10, GM1b, GM2b, BTG8 and BTG14 were 2000, 30, 2000, 100000, 4000 and 200 kPa, respectively which were in accordance with the residual suction of these soils as shown in Figure 5-106 to Figure 5-107. Parameter θ_r for soils SB9, SB10, GM1b, GM2b, BTG8 and BTG14 were 0.05, 0.005, 0.06, 0.06, 0.02 and 0.08, respectively which were in accordance with the residual volumetric water content of these soils as shown in Figure 5-106 to Figure 5-107.

Table 5-32. Best fitting parameters of proposed equation 3-11 for soil mixtures M3c, M4c, M5c, M6b, M6c, M7b, M7c, M8b and M8c with unimodal SWCC

Parameters	M3c	M4c	M5b	M5c	M6b
θ_s	0.296	0.386	0.405	0.455	0.568
ψ_a (kPa)	7	10	1	1	3
ψ_m (kPa)	150	100	3	21	30
s	4	2.25	1.8	2.2	1.76
ψ_r (kPa)	4000	1000	9	300	400
θ_r	0.02	0.05	0.02	0.02	0
R^2	0.991	0.999	0.999	0.999	0.999

Table 5-33. Best fitting parameters of proposed equation 3-11 for soil mixtures M3c, M4c, M5c, M6b, M6c, M7b, M7c, M8b and M8c with unimodal SWCC

Parameters	M6c	M7b	M7c	M8b	M8c
θ_s	0.636	0.612	0.635	0.643	0.653
ψ_a (kPa)	5	5	8	10	20
ψ_m (kPa)	40	80	130	200	300
s	1.8	2.75	2	1.85	2
ψ_r (kPa)	400	2000	1000	1500	3000
θ_r	0	0.04	0.04	0.04	0.05
R^2	0.998	0.999	0.999	0.999	0.999

Table 5-34. Best fitting parameters of proposed equation 3-19 for soil mixtures M3c, M4c, M5b, M5c and M6b with unimodal SWCC

Parameters	M3c	M4c	M5b	M5c	M6b
θ_s	0.296	0.386	0.405	0.455	0.568
A	8	10	1	1	3
R	554	1000	38	333	558
Dv	0.124	0.1	0.002	0.064	0.012
R^2	0.987	0.966	0.912	0.977	0.971

Table 5-35. Best fitting parameters of proposed equation 3-19 for soil mixtures M6c, M7b, M7c, M8b and M8c with unimodal SWCC

Parameters	M6c	M7b	M7c	M8b	M8c
θ_s	0.636	0.612	0.635	0.643	0.653
A	5	1	10	10	20
R	600	639	1523	2223	3000
Dv	0	0.174	0.113	0.1	0.16
R^2	0.967	0.903	0.990	0.895	0.947

Table 5-36. Best fitting parameters of Fredlund and Xing (1994) equation for soil mixtures M3c, M4c, M5b, M5c and M6b with unimodal SWCC

Parameters	M3c	M4c	M5b	M5c	M6b
θ_s	0.296	0.386	0.405	0.455	0.568
a	20	35	2.7	49	71
n	1.02	1.12	1.03	0.65	0.89
m	0.77	0.99	3.26	3.80	4.55
ψ_r	1500	1500	1500	1500	1500
R^2	0.989	0.975	0.934	0.983	0.982

Table 5-37. Best fitting parameters of Fredlund and Xing (1994) equation for soil mixtures M6c, M7b, M7c, M8b and M8c with unimodal SWCC

Parameters	M6c	M7b	M7c	M8b	M8c
θ_s	0.636	0.612	0.635	0.643	0.653
a	112	26	115	174	114
n	0.74	0.87	0.89	1.01	1.20
m	4.64	1.28	2.27	2.16	1.11
ψ_r	1500	1500	1500	1500	1500
R^2	0.974	0.938	0.995	0.916	0.967

Figure 5-106. SWCC of soil SB9, SB10, GM1b best fitted with the proposed SWCC equation developed in this study and published equation from literature

Figure 5-107. SWCC of soil GM2b, BTG8, BTG14 best fitted with the proposed SWCC equation developed in this study and published equation from literature

Table 5-38. Best fitting parameters of proposed equation 3-11 for soil SB9, SB10, GM1b, GM2b, BTG8 and BTG14 with unimodal SWCC

Parameters	SB9	SB10	GM1b	GM2b	BTG8	BTG14
θ_s	0.315	0.203	0.299	0.352	0.559	0.485
ψ_a (kPa)	10	2.0	10	40	100	100
ψ_m (kPa)	170	7.0	730	1300	830	2798
s	1.87	1.22	1.30	2.42	1.58	2.55
ψ_r (kPa)	2000	30	2000	100000	4000	2000
θ_r	0.05	0.005	0.06	0.06	0.02	0.08
r^2	0.999	0.999	0.999	1.00	1.00	1.00

Table 5-39. Best fitting parameters of proposed equation 3-19 for soil SB9, SB10, GM1b, GM2b, BTG8 and BTG14 with unimodal SWCC

Parameters	SB9	SB10	GM1b	GM2b	BTG8	BTG14
θ_s	0.315	0.203	0.229	0.352	0.559	0.485
A	10	2.0	20	40	80	100
R	2232	70	1000	5500	10000	15000
Dv	0.01	0.01	0.10	0.01	0.01	0.01
r^2	0.996	0.992	0.998	0.974	0.999	0.997

Table 5-40. Best fitting parameters of Fredlund and Xing (1994) equation for soil SB9, SB10, GM1b, GM2b, BTG8 and BTG14 with unimodal SWCC

Parameters	SB9	SB10	GM1b	GM2b	BTG8	BTG14
θ_s	0.316	0.203	0.299	0.354	0.559	0.485
a	500	5.28	46	443	500	1000
n	1.01	2.68	3.92	1.72	1.64	1.22
m	5.81	1.59	0.47	2.53	1.59	1.58
ψ_r	1500	1500	1500	1500	1500	1500
r^2	0.999	0.931	0.963	0.995	0.993	0.998

5.7. Evaluation of The Proposed Equation for Best Fitting Shear Strength of Unsaturated Soil using Experimental Data from Laboratory Tests carried out in This Study and using Published Data from Literatures

In this section, the evaluation of the proposed equations for best fitting the shear strength of an unsaturated soil with bimodal and unimodal soil-water characteristic curve (SWCC) (Equation 3-30) developed in this study are presented and discussed in detail. The performance of the proposed equations for best fitting SWCC was evaluated using experimental data from laboratory tests carried out in this study and published data from literatures. Then, the performance of the proposed equations was compared with the performance of published equations for best fitting SWCC with bimodal and unimodal characteristics using the criteria explained in Section 4.6.

The shear strengths of soil with bimodal characteristics obtained from laboratory tests carried out in this study are presented in Figure 5-108 to Figure 5-112. The shear strength of soil with bimodal characteristics obtained from published literatures are presented in Figure 5-113 to Figure 5-116. Those shear strength data were best fitted with the proposed equations for best fitting bimodal shear strength of unsaturated soil (Equation 3-37) developed in this study and Lee et al (2005) equation for best fitting bimodal shear strength of unsaturated soil. The parameters of the proposed shear strength equation developed in this study for best fitting bimodal shear strength from the laboratory tests carried out in this study are summarized in Table 5-41. The parameters of Lee et al (2005) equation for best fitting bimodal shear strength from the laboratory tests carried out in this study are summarized in Table 5-42. The parameters of the proposed shear strength equation developed in this study for best fitting bimodal shear strength from literatures are summarized in Table 5-43. The parameters of Lee et al (2005) equation for best fitting bimodal shear strength from literatures are summarized in Table 5-44.

Table 5-41. Best fitting parameters of the proposed shear strength developed in this study for soils M2a, M3a, M4a, M6a and M8a

Parameters	M2a	M3a	M4a	M6a	M8a
AEV1	15	6	3	4	0.8
AEV2	70	50	25	30	9
b	0.90	0.86	0.83	0.51	0.25
q	0.42	0.43	0.42	0.35	0.14
f	0.17	0.24	0.21	0.21	0.60
r^2	0.999	0.998	0.999	0.998	0.997

Table 5-41 indicate that the parameter b of the proposed shear strength equation for soils M2a, M3a, M4a, M6a and M8a were 0.9, 0.86, 0.83, 0.51 and 0.25, respectively which correspond to the ratio of ϕ^b/ϕ' of these soils for suction between AEV1 and AEV2 as shown in Figure 5-108 to Figure 5-112. Table 5-43 indicates that the parameter b of the proposed shear strength equation for soils from Coutinho et al. (2011), Jotisankasa et

al. (2009), Jotisankasa & Mairaing (2010) and Rahardjo et al. (2011) were 0.54, 0.32, 0.52 and 0.69, respectively which correspond to the ratio of ϕ^b/ϕ' of these soils for suction between AEV1 and AEV2 as shown in Figure 5-113 to Figure 5-116.

Table 5-42. Best fitting parameters of Lee et al. (2005) equation for soils M2a, M3a, M4a, M6a and M8a

Parameters	M2a	M3a	M4a	M6a	M8a
AEV	15	6	3	4	0.8
k	3.8	1.26	1.70	365	365
λ	1.0	1.0	1.0	1.0	1.0
r^2	0.951	0.898	0.610	0.657	0.869

Figure 5-108. Shear strength of soil mixtures M2a best fitted with the proposed shear strength equation developed in this study and published equation from literature

Figure 5-109. Shear strength of soil mixtures M3a best fitted with the proposed shear strength equation developed in this study and published equation from literature

Figure 5-110. Shear strength of soil mixtures M4a best fitted with the proposed shear strength equation developed in this study and published equation from literature

Figure 5-111. Shear strength of soil mixtures M6a best fitted with the proposed shear strength equation developed in this study and published equation from literature

Figure 5-112. Shear strength of soil mixtures M8a best fitted with the proposed shear strength equation developed in this study and published equation from literature

Figure 5-113. Shear strength of soil from Coutinho et al. (2011) best fitted with the proposed shear strength equation developed in this study and published equation from literature

Figure 5-114. Shear strength of soil from Jotisankasa et al. (2009) best fitted with the proposed shear strength equation developed in this study and published equation from literature

Figure 5-115. Shear strength of soil from Jotinsakasa and Mairaing (2010) best fitted with the proposed shear strength equation developed in this study and published equation from literature

Figure 5-116. Shear strength of soil from Rahardjo et al. (2011) best fitted with the proposed shear strength equation developed in this study and published equation from literature

Table 5-43. Best fitting parameters of the proposed shear strength developed in this study for soils from published data

Parameters	Coutinho et al. (2011)	Jotisankasa et al. (2009)	Jotisankasa & Mairaing (2010)	Rahardjo et al. (2011)
AEV1	0.75	0.80	4	5
AEV2	100	50	400	200
b	0.54	0.32	0.52	0.69
q	0.20	0.18	0.20	0.11
f	0.45	0.31	0.44	0.45
r^2	1.000	0.999	0.999	0.999

Table 5-44. Best fitting parameters of Lee et al (2005) equation for soils from published data

Parameters	Coutinho et al. (2011)	Jotisankasa et al. (2009)	Jotisankasa & Mairaing (2010)	Rahardjo et al. (2011)
AEV	0.75	0.80	4	5
k	1.5	100	4.4	0.3
λ	1.0	1.0	1.0	1.0
r^2	0.984	0.675	0.980	0.988

CHAPTER 6 DISCUSSION OF RESULTS

6.1. Introduction

The discussions on the results from the research works carried out in this study are presented in this chapter. Initially, the grain-size distribution and soil-water characteristic curve (SWCC) from experimental works performed in this study are discussed. Subsequently, the saturated and unsaturated shear strength from experimental works conducted in this study are discussed. The mechanical behaviours of soils with bimodal SWCC (dual-porosity soils) are explained in detail. Then, the proposed grain-size distribution (GSD), SWCC and shear strength equations developed in this study are verified and compared with various existing equations using experimental data obtained from this study as well as published data from literatures.

6.2. Mechanical Behavior of Soil with Bimodal Characteristics

The discussions on the characteristics of the sand-kaolin mixtures with unimodal and bimodal characteristics are presented in this section. As described in Section 4.2, nine soil mixtures with different percentages of kaolin and sand with different initial condition were tested in the laboratory. As described in Section 4.3.5, the SWCC tests were carried out on twenty-seven (27) different sand-kaolin mixtures, M1 to M9. Each composition of sand-kaolin mixture were statically compacted on three different conditions, soil mixtures associated with dry density and water content with respect to 95% of maximum dry density in the dry and wet of optimum as well as the maximum dry density and optimum water content of the compaction curve, to study the characteristics of bimodal and unimodal SWCC. It should be noted that only one test was carried out for every designated matric suctions due to the limited time and available equipments since the test requires a long time of testing. Therefore, all tests were conducted with good care and to the best knowledge of the author in order to produce reliable and good quality testing results. Some parts of the results and findings were published in Satyanaga et al. (2013).

As described in Section 0, the saturated CU and unsaturated CD triaxial tests were conducted on all (11 sets) sand-kaolin mixtures with bimodal SWCC and only some parts

(7 sets) of sand-kaolin mixtures with unimodal SWCC. It should be noted that multistage tests were carried out for each percentage of sand-kaolin mixtures with different initial condition due to the limited time and available equipments since the test requires a long time of testing and sophisticated equipments. Therefore, all tests were conducted with good care and to the best knowledge of the author in order to produce reliable and good quality testing results. Some parts of the results and findings were published in Satyanaga et al. (2014).

The compaction curves of sand-kaolin mixtures used in this study (Figure 5-1) indicate a wide range of maximum dry density (1.53 to 2.02 Mg/m³) and optimum water content (9.4 to 22.0 %). The soil mixture M9 with 10 % sand and 95 % kaolin has the lowest dry density and the highest optimum water content. Figure 5-1 shows that the maximum dry density increases and the optimum water content decreases with the increasing percentages of sand. This happens due to the fact that clay particles are lighter than sand particles. In addition the clay particles absorb water more than sand particles. The initial void ratio of soil mixtures increases with the higher percentage of clay particles since clay particles consist of a lot of small voids and the amount of void is much higher than the volume of soil solids.

The GSD data of soil mixtures M1 to M9 were best fitted using the proposed equation for fitting bimodal GSD curve (Equation 3-11) because two distinct groups of soil particles size (called gap-graded soils) were observed in the GSD data. The GSD parameters (Table 5-2) of each soil specimen were determined based on the proposed equation for GSD developed in this study. All specimens are classified as poorly-graded soils since coefficient of curvature (C_c) of all specimens are less than 1 or higher than 3. Table 5-2 indicates that the geometric means of particle sizes within large particle (medium sand) (dm_1) and small particle (silt) (dm_2) for all specimens were 0.706 mm and 0.007 mm, respectively. Maximum particles sizes within large particle (d_{max1}) and small particle (d_{max2}) for all specimens were 2 mm and 0.015 mm, respectively. The parameters d_{max1} , d_{max2} , dm_1 and dm_2 , shows that each specimen contained medium sand and silt. Geometric standard deviations of particle size within large particle (s_{d1}) and small particle (s_{d2}) were 0.007 and 0.86, respectively. The smaller value of parameter s_{d1} as compared with parameter s_{d2} indicated that medium sand was distributed more

uniformly than silt within all specimens tested in this study. Based on the above analyses, the parameters of the proposed GSD equation developed in this study were able to explain the characteristics of GSD curve of each specimen. In addition, the parameters of the proposed GSD equations (Equations 3-21 and 3-24) can also be used to calculate the variables of GSD curve, such as: D_{60} , D_{30} , D_{10} , C_u and C_c .

All SWCCs are presented in Figure 5-3 to Figure 5-29. The details of SWCC are given in Section 5-3. Results from specimens with unimodal SWCC were best fitted with the proposed equation developed in this study for unimodal SWCC (Equation 3-11). Results from specimens with bimodal SWCC were best fitted with the proposed equation developed in this study for bimodal SWCC (Equation 3-15). The relationships between parameters from the proposed equation for best fitting unimodal SWCC (Equation 3-11) and variables of SWCC that were obtained manually from the unimodal SWCC graph are presented in Figure 6-1 and Figure 6-2.

Figure 6-1. Relationship between fitting parameter ψ_a from the proposed equation for best fitting unimodal SWCC (Equation 3-11) and air-entry value of SWCC

Figure 6-1 shows that parameter ψ_a corresponded to the air-entry value of SWCC. Therefore, parameter ψ_a from the proposed equation can be used to represent the air-entry value of SWCC without the manual plotting of the SWCC graph. Figure 6-2 indicates that parameters ψ_m from the proposed equation corresponded to the inflection point of

SWCC. Therefore, the fitting parameter ψ_m of the proposed equation for best fitting unimodal SWCC can be used to represent the inflection point of SWCC without the manual plotting of the SWCC graph.

Figure 6-2. Relationship between fitting parameter ψ_m from the proposed equation for best fitting unimodal SWCC (Equation 3-11) and the inflection point of SWCC

Table 5-10 and Table 5-11 indicate that the standard deviations of SWCC associated with the macro pores were smaller than those with the micro pores of all specimens (Table 5-10 and Table 5-11), indicating the macro pores were distributed more uniformly than micro pore for all specimens tested in this study. It was mentioned in Section 5.2 that large particles (medium sand) were distributed more uniformly than small particles (silt). In other words, the distribution of soil particles within a soil specimen represents the distribution of soil pores within the soil specimen.

The relationships between fitting parameters from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and variables of SWCC that were obtained manually from the bimodal SWCC graph are presented in Figure 6-3 to Figure 6-6. Figure 6-3 shows that parameters ψ_{a1} obtained from the proposed equation for best fitting bimodal SWCC corresponded to the first air-entry value associated with the macro pores of specimen as obtained manually from the bimodal SWCC. Figure 6-4 indicates that parameters ψ_{a2} obtained from the proposed equation for best fitting bimodal SWCC corresponded to the second air-entry value associated with the micro pores of specimen

as obtained manually from the bimodal SWCC. Therefore, parameters ψ_{a1} and ψ_{a2} of the proposed equation for best fitting bimodal SWCC can be used to represent the first and second air-entry values of bimodal SWCC, respectively, without the manual plotting of the SWCC graph.

Figure 6-3. Relationship between fitting parameter ψ_{a1} from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and the first air-entry value of SWCC

Figure 6-4. Relationship between fitting parameter ψ_{a2} from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and the second air-entry value of SWCC

Figure 6-5 shows that parameters ψ_{m1} obtained from the proposed equation for best fitting bimodal SWCC corresponded to the first inflection point associated with the macro pores of specimen as obtained manually from the bimodal SWCC. Figure 6-6 indicates that parameters ψ_{m2} obtained from the proposed equation for best fitting bimodal SWCC corresponded to the second inflection point associated with the micro pores of specimen as obtained manually from the bimodal SWCC. Therefore, parameters ψ_{m1} and ψ_{m2} of the proposed equation for best fitting bimodal SWCC can be used to represent the first and second inflection points of bimodal SWCC, respectively, without the manual plotting of the SWCC graph. The above analyses indicate that parameters of the proposed SWCC equations developed in this study can be used to describe the variables and characteristics of SWCC.

Figure 6-5. Relationship between fitting parameter ψ_{m1} from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and the first inflection point of SWCC

Figure 6-6. Relationship between fitting parameter ψ_{m2} from the proposed equation for best fitting bimodal SWCC (Equation 3-15) and the second inflection point of SWCC

Table 6-1. Characteristics of SWCC for compacted soils M1, M2, M3, M4 and M5

Soils	Compacted	Characteristics of SWCC
M1a (90s10k)	at the dry of optimum	unimodal
M1b (90s10k)	at maximum dry density	unimodal
M1c (90s10k)	at the wet of optimum	unimodal
M2a (80S20K)	at the dry of optimum	bimodal
M2b (80S20K)	at maximum dry density	bimodal
M2c (80S20K)	at the wet of optimum	bimodal
M3a (70S30K)	at the dry of optimum	bimodal
M3b (70S30K)	at maximum dry density	bimodal
M3c (70S30K)	at the wet of optimum	unimodal
M4a (60S40K)	at the dry of optimum	bimodal
M4b (60S40K)	at maximum dry density	bimodal
M4c (60S40K)	at the wet of optimum	unimodal
M5a (50S50K)	at the dry of optimum	bimodal
M5b (50S50K)	at maximum dry density	unimodal
M5c (50S50K)	at the wet of optimum	unimodal

The characteristics of SWCC for compacted sand kaolin mixtures used in this study are summarized in Table 6-1 and Table 6-2. The bimodal characteristics of SWCC for coarse-grained soil (M2 to M4) were generated if the soil was compacted at the maximum dry density or at the dry of optimum. However, the bimodal characteristics of SWCC for the fine-grained soil (M5 to M8) were generated if the soil was compacted only at the dry of optimum. In other words, the distinctly large and small pore-size distributions that warrant a bimodal SWCC only occurred if coarse-grained or fine-grained soils were compacted at the dry of optimum (Table 6-1 and Table 6-2). For coarse-grained soils, the bimodal characteristics of SWCC can also be observed if the soil mixtures were compacted at the maximum dry density. As for fine-grained soils, the large pores between sand particles within the soil mixtures that were compacted at the maximum dry density, were fully occupied with fine particles (i.e. silt). As a result, the large pores between sand particles of different sizes did not have any effect on the conventional unimodal shape of the SWCC.

Table 6-2. Characteristics of SWCC for compacted soils M6, M7, M8 and M9

Soils	Compacted	Characteristics of SWCC
M6a (40s60k)	at the dry of optimum	bimodal
M6b (40s60k)	at maximum dry density	unimodal
M6c (40s60k)	at the wet of optimum	unimodal
M7a (30S70K)	at the dry of optimum	bimodal
M7b (30S70K)	at maximum dry density	unimodal
M7c (30S70K)	at the wet of optimum	unimodal
M8a (20S80K)	at the dry of optimum	bimodal
M8b (20S80K)	at maximum dry density	unimodal
M8c (20S80K)	at the wet of optimum	unimodal
M9a (10S90K)	at the dry of optimum	bimodal
M9b (10S90K)	at maximum dry density	unimodal
M9c (10S90K)	at the wet of optimum	unimodal

As presented in Table 6-1 and Table 6-2, the sand-kaolin mixtures with bimodal GSD, compacted at dry of optimum generally exhibit bimodal SWCC. This might happen due to the flocculated soil fabric of sand-kaolin mixtures compacted at dry of optimum (Holtz and Kovacs, 1981). During compaction, the large pores in between large particles of sand could not be filled with the small (fine) particles of kaolin. As a result, sand-kaolin mixtures compacted at dry of optimum would have bimodal pore-size distribution. During SWCC tests, relatively low suction values would be required to remove water from large pores (macropores), while high suction would be required to remove water from small pores (micropores).

Table 6-1 and Table 6-2 also show that the sand-kaolin mixtures with bimodal GSD, compacted at wet of optimum generally exhibit unimodal SWCC. This might happen due to the dispersed or more oriented soil fabric of sand-kaolin mixtures compacted at wet of optimum as compared to those compacted at dry of optimum (Holtz and Kovacs, 1981). During compaction, the fine particles from kaolin filled in the large pores in between sand particles. As a result, sand-kaolin mixtures compacted at wet of optimum would have unimodal pore-size distribution and unimodal SWCC.

Figure 6-7 presents the relationship between saturated volumetric water content (θ_s) and percentages of sand within soil mixture. It can be seen in Figure 6-7 that the saturated volumetric water contents of soil mixtures with a higher percentage of sand were lower than those with a lower percentage of sand. Figure 6-8 shows that the standard deviation of the first subcurve of GSD decreases with the increasing percentage of sand within soil mixture. This indicates that the first subcurve of GSD for soils with a high percentage of sand was steeper than those with a low percentage of sand. Figure 6-9 shows that the standard deviation of the first subcurve of SWCC decreases with the increasing percentage of sand within soil mixture. This indicates that the slopes of the first subcurves of SWCC for soil mixtures with a high percentage of sand were steeper than those with a low percentage of sand. It can be seen from Figure 6-8 and Figure 6-9 that the GSD of the soil mixtures governed the slope of the first subcurve of SWCC. The uniform GSD of a soil mixture generated a uniform pore-size distribution of the soil mixture, resulting in a steeper slope of the SWCC.

Figure 6-7. Relationship between saturated volumetric water content (θ_s) and percentage of sand within soil mixture

Figure 6-8. Relationship between standard deviation within subcurve 1 (sd_1) of GSD curve and percentage of sand within soil mixture

Figure 6-9. Relationship between standard deviation within subcurve 1 (s_1) of SWCC and percentage of sand within soil mixture

As presented in Section 5.4, the stress-strain characteristics of specimens M2 to M9 are presented in Figure 5-30 to Figure 5-51. In general, it was observed that the higher the matric suction within a soil specimen, the higher the peak deviatoric stress of the soil specimen. Figure 5-30 to Figure 5-51 also indicate that the shear strength of the unsaturated soil was higher than that of the saturated soil under the same confining pressure, indicating a higher stiffness of soil in unsaturated condition than in saturated condition. In other words, the matric suction in a soil plays an important role in the stiffness of the soil as well as in contributing additional strength to the soil.

It was noticed that the post-peak shear strengths of different specimens under saturated (Figure 5-30 to Figure 5-33) and unsaturated (Figure 5-34 to Figure 5-51) conditions were similar. The post-peak strain softening response was observed in all specimens with unimodal and bimodal characteristics. This happened since the behavior of specimens with bimodal characteristics at matric suctions beyond the second air-entry value was similar to those with unimodal characteristics since water already flowed out through the macro and micro pores of the soil specimens. In other words, the effect of the applied matric suction on the shear strength appeared to diminish after the post-peak strain softening when the shear plane has fully developed in the specimens with unimodal and bimodal characteristics.

Figure 5-30 to Figure 5-51 also show that specimens compacted at the dry of optimum have a steeper stress-strain curve than those compacted at the optimum water content and at the wet of optimum indicating a higher stiffness of the specimens compacted at the dry of optimum than those compacted at the optimum water content and at the wet of optimum. In other words, specimens with bimodal characteristics are stiffer than those with unimodal characteristics. The water volume change measurements from saturated and unsaturated triaxial tests (Figure 5-30 to Figure 5-51) indicate that the volumes of water in all specimens decreased during shearing process. However, the rates of water volume change varied for different soil mixtures and different water contents. The changes of water volume in specimens with a high percentage of silt were higher than those in specimens with a low percentage of silt. This occurred since specimens with a high percentage of silt had higher volumetric water content and had a tendency to shrink more than those with a low percentage of silt.

Figure 5-34 to Figure 5-51 show that the rates of water volume change of specimens compacted at the dry of optimum were lower than those of specimens compacted at the wet of optimum. This was attributed to the flocculated structure of soil particles within specimens compacted at the dry of optimum. As a result, water was difficult to flow within soil pores of specimens compacted at dry of optimum (with bimodal characteristics) during the shearing process of unsaturated triaxial tests.

Figure 5-52 to Figure 5-59 indicate that effective cohesions of specimens compacted at the dry of optimum (specimens with bimodal SWCC) were lower than those compacted at the wet of optimum (specimens with unimodal SWCC). On the other hand, the effective friction angles of specimens compacted at the dry of optimum were higher than those compacted at the wet of optimum. This might happen since the silt particles were trapped inside the micropores within the soil specimens compacted at the dry of optimum whereas the silt particles were distributed uniformly within the soil specimens compacted at the wet of optimum. In other words, specimens with bimodal characteristics had lower effective cohesion and higher effective friction angle than those with unimodal characteristic subjected to the similar grain-size distribution of both soils.

Figure 5-60 to Figure 5-77 show that ϕ^b angle obtained from unsaturated triaxial test was equal to ϕ' of each specimen at matric suctions less than or equal to the air-entry

value of soil with unimodal characteristics. Similar behavior was observed for soil with bimodal characteristics whereby ϕ^b angle was equal to ϕ' at matric suctions less than or equal to the first air-entry value. ϕ^b angle started to decrease at matric suctions higher than the air-entry value for soils with unimodal characteristics. Similar behavior was also observed for soils with bimodal characteristics where ϕ^b angle started to decrease at matric suctions higher than the first air-entry value. However, ϕ^b angle of soils with unimodal characteristics varied with suction at suctions higher than the air-entry value whereas ϕ^b angle of soils with bimodal characteristics was constant at suctions higher than the first air-entry value and less than the second air-entry value. ϕ^b angle of soil with bimodal characteristics started to vary with suction at suctions higher than the second air-entry value. This might be attributed to the reduction of area of water (water menisci area which were in contact with soil particles) within macropores of soil specimen with bimodal characteristics at suctions higher than the first air-entry value. However, micropores within the soil specimen with bimodal characteristics were still in a saturated condition. As a result, the rate of decrease of ϕ^b angle was constant at suctions between the first and the second air-entry value. As suctions went beyond the second air-entry value, the area of water menisci within micropores started to decrease due to the desaturation of micropores of soil specimen with bimodal characteristics. Hence, the ϕ^b angle decreased non-linearly at suctions beyond the second air-entry value.

Figure 5-78 to Figure 5-81 indicate that the shear strengths of specimen under a saturated condition at the dry of optimum were higher than those at the optimum water content and at the wet of optimum. Similar behavior was also observed on the shear strength of specimen under unsaturated condition indicating that the unsaturated shear strength of specimen with bimodal characteristics was higher than the unimodal characteristic with respect to the same grain-size distribution and the same soil suction. This is attributed to the flocculated structure of specimen with bimodal characteristics. The flocculated structure of specimen with bimodal characteristics offers greater resistance to compression than the dispersed structure of the specimen with unimodal characteristic (Rakesh et al., 2010; Horpibulsuk et al., 2013). Figure 5-78 to Figure 5-81 also indicate that ϕ^b angle of soil specimen with bimodal characteristics decreased with the decrease of ϕ' for soil mixtures with different grain-size distribution. This might occur

due to the reduction of density of soil specimen with respect to the increase of fines content within soil specimen. In another word, the unsaturated shear strength of soil specimen with bimodal characteristics decreased with the increase of fines content.

6.3. Estimation of Soil-water Characteristic Curves and Shear Strength of Unsaturated Soil

Measurement of the SWCC in the laboratory and in the field is relatively time consuming and expensive. Therefore, it is very useful to estimate the SWCC, especially in the initial stages of a project when the available information is limited. In this section, procedures for the estimation of soil-water characteristic (SWCC) and unsaturated shear strength of soils with bimodal characteristics are presented. In the development of procedures for estimating SWCC for soils with bimodal GSD, the proposed mathematical equations of bimodal SWCC (Equation 3-11), unimodal SWCC (Equation 3-15) and bimodal GSD (Equation 3-21) developed in this study were used. A total of 26 data sets (Table 6-3) from SoilVision database (SoilVision, 2004), Rahardjo et al. (2004) and Jotisankasa et al. (2009) was used to establish the relationship between index properties, grain-size distribution (GSD) and SWCC of soil with bimodal GSD.

SoilVision database contains information such as grain-size distribution, dry density, porosity, specific gravity, saturated water content, saturated and unsaturated coefficient of permeability. However, there is no information associated with plasticity of soil (i.e., LL and PL). Therefore, plasticity of soil was not considered in the development of procedures for estimating SWCC of soil with bimodal GSD in this study. Parametric studies carried out to understand the relationship between SWCC and the unsaturated shear strength of soil with bimodal characteristics are also described in this chapter. A total of 5 data sets (Table 6-4) from Jotisankasa et al. (2009), Jotisankasa & Mairaing (2010), Adjari et al. (2010), Coutinho et al. (2011) and Rahardjo et al. (2012) was used to develop procedures to estimate the unsaturated shear strength of soil with bimodal characteristics.

Table 6-3. Soil data sets from published literatures used for development of procedure to predict SWCC from bimodal GSD

Soil name	Sat w (%)	ρ_d (Mg/m ³)	Gravel (%)	Sand (%)	Silt (%)	Clay (%)	Source
10843	30.07	1.50	0.82	0	13.50	56.60	SoilVision (2002)
10861	27.19	1.47	0.84	0	3.40	84.40	SoilVision (2002)
10862	18.80	1.61	0.63	0	15.40	67.60	SoilVision (2002)
10863	26.76	1.45	0.84	0	16.00	64.30	SoilVision (2002)
11049	37.65	1.30	1.03	0	7.60	70.40	SoilVision (2002)
11050	14.59	1.43	0.85	0	7.40	66.70	SoilVision (2002)
11148	14.41	1.50	0.84	0	4.40	57.50	SoilVision (2002)
11149	16.08	1.83	0.82	0	4.70	56.60	SoilVision (2002)
11372	18.86	1.49	0.81	0	17.50	56.90	SoilVision (2002)
11491	32.02	1.43	0	22.2	61.8	16.0	SoilVision (2002)
11492	39.6	1.29	1.1	54.9	34.4	9.6	SoilVision (2002)
11493	25.52	1.31	1.6	56.0	31.8	10.6	SoilVision (2002)
11494	37.4	1.19	0	71.4	13.2	15.4	SoilVision (2002)
11498	44.54	1.17	0	29.5	45.2	25.3	SoilVision (2002)
11520	30.57	1.46	0	53.0	37.3	9.7	SoilVision (2002)
11538	45.54	1.80	0	28.9	48.5	22.6	SoilVision (2002)
11586	21.67	1.80	76.6	23.4	0.0	0.0	SoilVision (2002)
12462	25.07	1.61	2.6	66.3	27.6	3.5	SoilVision (2002)
18719	20.32	1.72	68.4	26.7	3.0	1.9	SoilVision (2002)
Nakhonnayok	33.00	1.41	6.20	21.40	42.40	36.20	Jotisankasa et al. (2009)
Omkoï	60.25	1.80	3.00	30.50	43.89	25.61	Jotisankasa et al. (2009)
BTG-14	24.45	2.00	0.00	63.30	26.50	10.20	Rahardjo et al. (2004)
GM1a	18.33	1.74	50.00	0.00	36.70	13.30	Rahardjo et al. (2012)
GM2a	22.26	1.68	50.00	0.00	36.90	13.10	Rahardjo et al. (2012)
GM3a	26.48	1.62	50.00	0.00	31.10	18.90	Rahardjo et al. (2012)
GRSa	32.08	1.44	5.00	36.00	39.90	19.10	Rahardjo et al. (2012)

Table 6-4. Soil data sets from published literatures used for development of equation for fitting and estimating the unsaturated shear strength of soil

Soil name	Sat w (%)	ρ_d (Mg/m³)	Gravel (%)	Sand (%)	Silt (%)	Clay (%)	Source
Silty Clay	51.6	1.45	0	20	30	50	Adjari et al. (2010)
Sandy Clay	31.3	1.50	0	24	5	71	Coutinho et al. (2011)
Clayey Silt	25.9	1.60	0	22	52	26	Jotisankasa et al. (2009)
Sandy Silt	26.1	1.80	0	31.4	40.1	26.3	Jotisankasa & Mairaing (2010)
Silty Sand	20.2	1.56	0	73.5	20.2	3	Rahardjo et al. (2011)

The proposed bimodal GSD equation developed in this study was used to best fit the published bimodal GSD data from literatures whereas the proposed bimodal SWCC equation developed in this study was used to best fit the published bimodal SWCC data from literatures. Then, the relationship between parameter d_m in the proposed GSD equation and parameter ψ_m in the proposed SWCC equation was drawn (Figure 6-10). The term "coarse" in Figure 6-10 is defined as soil particle with diameter larger than 0.075 mm whereas, the term "fine" in Figure 6-10 is defined as soil particle with diameter equal to or less than 0.075 mm. Figure 6-10 shows a non-linear relationship between parameters d_{m1} and ψ_{m1} as well as parameters d_{m2} and ψ_{m2} for soils with bimodal GSD and bimodal SWCC. Therefore, Rouault and Assouline (1998) theory was incorporated in the development of procedures for the estimation of SWCC for soil with bimodal GSD.

Figure 6-10. Relationship between fitting parameter d_{m1} , d_{m2} and ψ_{m1} , ψ_{m2} for soil with bimodal GSD and SWCC

Rouault and Assouline (1998) indicated that the relationship between diameter of soil particle (d) and radius of soil pore (r) is non-linear as shown in Equation 6-1.

$$d = ur^v \quad (6-1)$$

where: u and v are parameters related to particle shape and soil packing, respectively.

In this study, parameters u and v were proposed to be functions of soil properties. The statistical analyses were carried out to find the suitable relationship between parameters u , v and soil properties. The results indicated that parameter u for coarse-grained soils with bimodal GSD and unimodal SWCC is a function of coefficient of uniformity of soil as shown in Equation 6-2 and Figure 6-11. On the other hand, parameter u for fine-grained soils with bimodal GSD and unimodal SWCC is a function of dry density as shown in Equation 6-3 and Figure 6-12.

Figure 6-11. Relationship between parameter u and coefficient of uniformity for coarse-grained soil with bimodal GSD and unimodal SWCC

$$u(\text{coarse}) = 0.028c_u + 9.21 \quad (6-2)$$

$$u(\text{fines}) = -1.031\rho_d + 19.53 \quad (6-3)$$

where:

$u(\text{coarse})$ = parameter related to shape of soil particle with diameter larger than 0.075 mm

$u(\text{fines})$ = parameter related to shape of soil particle with diameter equal to or less than 0.075 mm

c_u = d_{60}/d_{10} = coefficient uniformity of soil

d_{60} = diameter of soil particle corresponding to 60 % finer in the GSD of soil

d_{10} = diameter of soil particle corresponding to 10 % finer in the GSD of soil

ρ_d = dry density of soil

Figure 6-12. Relationship between parameter u and dry density for fine-grained soil with bimodal GSD and unimodal SWCC

The results of statistical analyses also showed that parameters u_1 for soils with bimodal GSD and bimodal SWCC is a function of percentage of coarse particle (%coarse) as shown in Equation 6-4 and Figure 6-13 whereas parameters u_2 is a function of maximum soil particle diameter within subcurve 2 of bimodal GSD as shown in Equation 6-5 and Figure 6-14, respectively. On the other hand, parameters v_1 and v_2 for soils with bimodal GSD and bimodal SWCC are functions of saturated water content as shown in Equation 6-6, Equation 6-7 and Figure 5-14.

Figure 6-13. Relationship between parameter u_1 and percentage of coarse particle for soil with bimodal GSD and bimodal SWCC

Figure 6-14. Relationship between parameter u_2 and maximum diameter of soil particle for soil with bimodal GSD and bimodal SWCC

$$u_1 = 0.039 \exp^{(0.088\%coarse)} \quad (6-4)$$

$$u_2 = 2.102 \ln(d_{\max 2}) + 10.65 \quad (6-5)$$

$$v_1 = 0.701 \exp^{(0.011 w_{sat})} \quad (6-6)$$

$$v_2 = 0.851 \exp^{(0.003 w_{sat})} \quad (6-7)$$

where:

%coarse = percentage of coarse particle within soil specimen

d_{max2} = maximum soil particle diameter within subcurve 2 of bimodal GSD

w_{sat} = saturated water content

Figure 6-15. Relationship between parameter v_1 , v_2 and saturated water content for soil with bimodal GSD and bimodal SWCC

Soil pore radius in Equation 6-1 can be related to pressure head using Young-Laplace equation as shown in Equation 6-2.

$$h = \frac{2\phi \cos \theta}{\rho_w g r} = \frac{A}{r} \quad (6-8)$$

where:

r = pore radius

ϕ = surface tension between water and air

θ = contact angle

ρ_w = density of water

g = acceleration of gravity

A = $(2 \phi \cos \theta) / (\rho_w g r)$

Brutsaert (1966) proposed to use value of $A = 0.149 \text{ cm}^2$ for air-water-soil system. Therefore, Equation 6-2 can be rewritten as follows:

$$\ln h = \ln \frac{0.149}{r} \quad (6-9)$$

Substituting Equation 6-1 into Equation 6-9 yields:

$$\ln h = \ln 0.149 - \ln \left(\frac{d}{u} \right)^{1/v} \quad (6-10)$$

Equation 6-10 can also be written as follow:

$$\ln h = \ln 0.149 - \left(\frac{\ln d - \ln u}{v} \right) \text{ or} \quad (6-11)$$

$$h = \exp \left[\ln 0.149 - \left(\frac{\ln d - \ln u}{v} \right) \right] \quad (6-12)$$

In order to obtain the corresponding matric suction, Equation 6-12 can be rearranged as follows:

$$\psi = \frac{\exp \left[\ln 0.149 - \left(\frac{\ln d - \ln u}{v} \right) \right]}{10} \quad (6-13)$$

where: ψ is in kPa and d is in cm

Since soil particle diameter is commonly measured in term of mm unit, Equation 6-13 can be written as follows:

$$\psi = \frac{\exp \left[\ln 0.149 - \left(\frac{\ln \left(\frac{d}{10} \right) - \ln u}{v} \right) \right]}{10} \quad (6-14)$$

Rearranging Equation 6-14 yields Equation 6-15 that can be used to estimate parameter ψ_m in the proposed SWCC equation developed in this study.

$$\psi_m = 0.1 \left[\exp \left(\ln (0.149) - \frac{\ln \left(\frac{0.1d_m}{u} \right)}{v} \right) \right] \quad (6-15)$$

Previous studies showed that soils with bimodal GSD are usually associated with bimodal SWCC as observed by Durner (1994), Stoicescu et al. (1999), Burger and Shackelford (2001) and Zhang and Chen (2005). However, some researchers also found that soils with bimodal GSD may produce unimodal SWCC as observed by Jotisankasa et al. (2009), Rahardjo et al. (2012c) and Satyanaga et al. (2012). Therefore, parametric studies on soil database in Table 6-3 were carried out to develop the criteria for the SWCC characteristics for soils with bimodal GSD. The results showed that the unimodal or bimodal characteristics of SWCC depend on: saturated water content, dry density and coefficient uniformity of soil. If $(\rho_d > 1.44 \text{ Mg/m}^3)$ and $(18\% \leq w_{\text{sat}} \leq 33\%)$, the characteristic of SWCC for soil with bimodal GSD is unimodal. If $(\rho_d \leq 1.44 \text{ Mg/m}^3)$ for any saturated water content or $(\rho_d > 1.44 \text{ Mg/m}^3)$ for $(w_{\text{sat}} > 33\% \text{ and } w_{\text{sat}} < 18\%)$, the characteristic of SWCC for soil with bimodal GSD is bimodal.

The statistical analyses were also carried out to study the relationship between other parameters in the proposed unimodal SWCC equation developed in this study, i.e. ψ_{aev} , ψ_r , s and θ_r , with soil properties and parameters in the proposed bimodal GSD equation developed in this study. The results showed that ψ_a of coarse-grained soil with bimodal GSD and unimodal SWCC is a function of ψ_m , d_{m1} , and d_{m2} as shown in Equation 6-16 and Figure 6-16 whereas ψ_a of fine-grained soil with bimodal GSD and unimodal SWCC is a function of ψ_m and c_u as shown in Equation 6-17 and Figure 6-17.

$$\psi_a(\text{coarse}) = \frac{\psi_m}{0.429(Cu^{0.613})} \quad (6-16)$$

$$\psi_a(\text{fines}) = \frac{\psi_m}{3.167 \left(\left(\frac{d_{m2}}{d_{m1}} \right)^{-0.5} \right)} \quad (6-17)$$

where:

d_{m1} = parameter representing the geometric mean of soil particle diameter within subcurve 1 of bimodal GSD

d_{m2} = parameter representing the geometric mean of soil particle diameter within subcurve 2 of bimodal GSD

Figure 6-16. Relationship between ψ_a , ψ_m , d_{m1} , and d_{m2} for fine-grained soils with bimodal GSD and unimodal SWCC

Figure 6-17. Relationship between ψ_a , ψ_m and C_u for coarse-grained soils with bimodal GSD and unimodal SWCC

Parameter ψ_r for soils with bimodal GSD and unimodal SWCC is a function of diameter of soil particle corresponding to 10% finer in the GSD of the soil as shown in Equation 6-18, Equation 6-19 and Figure 6-18. Parameter s for coarse-grained soil with bimodal GSD and unimodal SWCC is a function of dry density of soil as shown in Equation 6-20 and Figure 6-19 whereas parameter s for fine-grained soil with bimodal GSD and unimodal SWCC is a function of coefficient uniformity of soil as shown in Equation 6-21 and Figure 6-20.

$$\psi_r(\text{coarse}) = 0.512(d_{10}^{-1.42}) \quad (6-18)$$

$$\psi_r(\text{fines}) = 0.771(d_{10}^{-1.53}) \quad (6-19)$$

$$s(\text{coarse}) = -0.471\rho_d + 9.716 \quad (6-20)$$

$$s(\text{fines}) = 9.89(Cu^{-0.17}) \quad (6-21)$$

Figure 6-18. Relationship between ψ_r and d_{10} for soils with bimodal GSD and unimodal SWCC

Figure 6-19. Relationship between s and dry density of fine-grained soils with bimodal GSD and unimodal SWCC

Figure 6-20 Relationship between s and coefficient uniformity of coarse-grained soils with bimodal GSD and unimodal SWCC

Figure 6-21 to Figure 6-28 present the relationships between other parameters in the proposed bimodal SWCC equation developed in this study, i.e. ψ_{a1} , ψ_{a2} , θ_{r1} , θ_{r2} , s_1 , s_2 and ψ_r , with soil properties and parameters in the proposed bimodal GSD equation developed in this study. The parameter ψ_{a1} for soils with bimodal GSD and bimodal SWCC is a function of ψ_{m1} and maximum soil particle diameter in subcurve 1 in GSD of soil as shown in Equation 6-22 and Figure 6-21. The parameter ψ_{a2} for soils with bimodal GSD and bimodal SWCC is a function of ψ_{a1} , ψ_{m1} and ψ_{m2} as shown in Equation 6-23 and Figure 6-22. The parameter s_1 for soils with bimodal GSD and bimodal SWCC is a function of c_u as shown in Equation 6-24, Equation 6-25 and Figure 6-23 whereas the parameter s_2 for soils with bimodal GSD and bimodal SWCC is a function of w_{sat} as shown in Equation 6-26 and Figure 6-24. The parameter θ_{r1} for soils with bimodal GSD and bimodal SWCC is a function of w_{sat} (Figure 6-25). The parameter θ_{r2} for coarse-grained soils with bimodal GSD and bimodal SWCC is a function of %coarse (Figure 6-26) whereas the parameter θ_{r2} for fine-grained soils with bimodal GSD and bimodal SWCC is a function of w_{sat} (Figure 6-27). The parameter ψ_r for soils with bimodal GSD and bimodal SWCC is a function of %fine (Figure 6-28).

Figure 6-21. Relationship between ψ_{a1} , ψ_m and d_{max1} for soils with bimodal GSD and SWCC

Figure 6-22. Relationship between ψ_{a1} , ψ_{a2} , ψ_{m1} and ψ_{m2} for soils with bimodal GSD and SWCC

Figure 6-23. Relationship between s_1 and coefficient uniformity of soils with bimodal GSD and SWCC

Figure 6-24. Relationship between s_2 and saturated water content of soils with bimodal GSD and SWCC

$$\psi_{a1} = \psi_{m1} / (1.114 \exp^{(0.083d_{max1})}) \quad (6-22)$$

$$\psi_{a2} = \psi_{a1} \exp \left[0.905 (Ln(\psi_{m2} / \psi_{m1}))^{0.704} \right] \quad (6-23)$$

$$s_1 = 2.063 \exp^{(-0.01c_u)} \quad (6-24)$$

$$s_1 = 0.077c_u - 2.29 \quad (6-25)$$

$$s_2 = 0.594 \exp^{[0.034w_{sat}]} \quad (6-26)$$

Figure 6-25. Relationship between θ_{r1} and saturated water content for soils with bimodal GSD and SWCC

Figure 6-26. Relationship between θ_{r2} and percentage of coarse particle for coarse-grained soils with bimodal GSD and SWCC

Figure 6-27. Relationship between θ_{r2} and saturated water content for fine-grained soils with bimodal GSD and SWCC

Figure 6-28. Relationship between ψ_r and percentage of fine particle for soils with bimodal GSD and SWCC

$$\theta_{r1} = 0.0078w_{sat} + 0.008 \quad (6-27)$$

$$\theta_{r2} = 0.002\%coarse - 0.088 \quad (6-28)$$

$$\theta_{r2} = 0.0092w_{sat} - 0.2935 \quad (6-29)$$

$$\psi_r = 3991(\%fine)^{-0.3594} \quad (6-30)$$

Based on the relationship between soil properties, parameters in the GSD equation proposed and parameters in the SWCC equation developed in this study, the equations and procedures to estimate the SWCC were established. The procedures to estimate the bimodal and unimodal SWCC for soils with bimodal GSD are summarized in Figure 6-29 and Figure 6-30, respectively. The computer codes for the computation of the estimated bimodal and unimodal SWCC are presented in Appendix A7.1-19 and Appendix A7.1-20. The computer codes were written using Visual Basic platform. After the parameters in the proposed SWCC equations are estimated, the SWCC for soils with bimodal GSD can be established. Then, the estimated SWCC may be used to describe other properties of unsaturated soil, such as: permeability function and unsaturated shear strength.

Figure 6-29. Procedures to estimate unimodal SWCC for soil with bimodal GSD

Figure 6-30. Procedures to estimate bimodal SWCC for soil with bimodal GSD

6.4. Procedures for Estimating Shear Strength of Unsaturated Soil with Bimodal Characteristics

The measurement of the unsaturated shear strength is time consuming and the apparatuses can be expensive. Therefore, it is useful to estimate the shear strength of unsaturated soil, especially in the initial stages of a project when the available information is limited. In this section, the equations for estimating the shear strength for unsaturated soils with bimodal SWCC are described and explained in detail. The proposed equations for estimating the parameters b , q and f in the proposed shear strength equation for soils with bimodal SWCC were developed based on the following criteria:

- The proposed equation must be able to provide the minimum SSEnorm for all selected data sets
- The proposed equation must be applicable over the entire matric suction range up to 10^6 kPa.
- The proposed equation must be applicable for all soil types in the selected data sets

Trial and errors as well as best-fit analyses were performed to relate parameters b , d and f with the soil properties. The parameter b was found to be closely related to liquid

limit of soil in selected data sets in natural logarithm (ln) form as shown in Figure 6-31. As a result, the parameter b can be related to liquid limit using the following relationship:

$$b = 5 \exp(-0.047LL) \quad (6-31)$$

where:

b = the parameter in the proposed unsaturated shear strength equation (Equation 3-36)

LL = Liquid limit of soil (%)

Figure 6-31. The relationship between b and liquid limit of soil in the selected data sets

The parameter q was found to be closely related to total density of soil (ρ) in the selected data sets in linear form as shown in Figure 6-32. As a result, the parameter q can be related to ρ using the following relationship:

$$q = 0.752\rho - 1.12 \quad (6-32)$$

where:

q = the parameter in the proposed unsaturated shear strength equation (Equation 3-36)

ρ = total density of soil (Mg/m^3)

Figure 6-32. The relationship between q and total density of soil in the selected data sets

The parameter f was also found to be exponentially related to standard deviation of second subcurve of SWCC (parameter s_2) in the selected data sets as shown in Figure 6-33. As a result, the parameter f can be related to parameter s_2 using the following relationship:

$$f = 0.088 \exp(0.83s_2) \quad (6-33)$$

where:

f = the parameter in the proposed unsaturated shear strength equation (Equation 3-64)

s_2 = parameter in SWCC represents standard deviation of second subcurve

Figure 6-33. The relationship between parameter f from unsaturated shear strength of soil and parameter s_2 from SWCC in the selected data sets

As a result, the proposed equations for predicting the unsaturated shear strength of soils with bimodal SWCC are as follows:

$$\tau = c' + \left[(\sigma - u_a) + AEV1 \right] \tan \phi' + \left[2(u_a - u_w) - AEV1 - AEV2 \right] \tan(b\phi') + \left[AEV2 - (u_a - u_w) \right] \tan(k\phi') \quad (6-34)$$

where:

b and $k = 1$ for $u_a - u_w < AEV1$

$k = b$ for $AEV1 < u_a - u_w < AEV2$

$$k = b + q \left[\log \frac{(u_a - u_w)}{AEV2} \right]^f \quad \text{for } u_a - u_w > AEV2$$

$$b = 5 \exp(-0.047LL)$$

$$q = 0.752\rho - 1.12$$

$$f = 0.088 \exp(0.83s_2)$$

As presented in Sections 5.3 and 6.2, there are several conditions in soil that need to be fulfilled in order to exhibit bimodal SWCC. The conditions of soil to exhibit bimodal SWCC as follows:

- Soil with bimodal GSD
- Compacted coarse-grained soil that is compacted at the dry of optimum or maximum dry density of the compaction curve
- Compacted fine-grained soil that is compacted only at the dry of optimum
- Undisturbed soil with the initial dry density less than 1.44 Mg/m^3 for any saturated water content
- Undisturbed soil with the initial dry density higher than 1.4 Mg/m^3 for saturated water content higher than 33 % or less than 18 %

As presented in Sections 5.4, all soils (either fine-grained or coarse-grained soils) with bimodal SWCC will exhibit bimodal shear strength either the soil is classified as compacted soil or undisturbed soil. However, these findings are limited to soils used in this study.

6.5. Evaluation of The Proposed Equations for Best Fitting Grain-Size Distribution, Soil-water Characteristic Curve and Unsaturated Shear Strength using Experimental Data from Laboratory Tests carried out in This Study and Published Data from Literatures

The evaluation of the proposed equations for best fitting grain-size distribution (GSD) of soils with bimodal and unimodal characteristics developed in this study has been presented in Section 5.5. The performances of the proposed equations for best fitting GSD of soil have been evaluated using experimental data from laboratory tests carried out in this study and published data from literatures. Then, the performances of the proposed equations have been compared with the performance of published equations for best fitting GSD of soil with bimodal and unimodal characteristics using the criteria explained in Section 4.6.

Figure 6-34 and Figure 6-35 show that the R^2 of the proposed GSD equation developed in this study were close to one, indicating a good performance of the proposed GSD equations developed in this study in best fitting the GSD data from the experimental works carried out in this study and published data from literatures. Figure 6-34 and Figure 6-35 also show that the proposed GSD equations developed in this study were better than Fredlund et al. (2000) equation in best fitting the GSD curve of soil data used in this study. In addition, the proposed GSD equations developed in this study have additional benefit as compared to Fredlund et al. (2000) equation. The parameters of the proposed GSD equations developed in this study can be used to describe the variables of GSD curve of soil data used in this study, such as: maximum particle size (diameter), geometric means of particle size and geometric standard deviations of particle size (see Section 6.2).

Figure 6-34. R^2 of the equation developed in this study and Fredlund et al. (2000) equation for best fitting unimodal GSD

The evaluation of the proposed equations for best fitting SWCC with bimodal and unimodal characteristics developed in this study has been presented in Section 5.6. The performances of the proposed equations for best fitting SWCC have been evaluated using experimental data from laboratory tests carried out in this study and published data from literatures. Then, the performances of the proposed equations have been compared with the performance of published equations for best fitting SWCC with bimodal and unimodal characteristics using the criteria explained in Section 4.6.

Figure 6-35. R^2 of the equation developed in this study and Fredlund et al. (2000) equation for best fitting bimodal GSD

Figure 6-36 and Figure 6-37 show that the R^2 values of the proposed SWCC equation developed in this study were close to one indicating a good performance of the proposed SWCC equations developed in this study in best fitting the SWCC data from the experimental works carried out in this study and published data from literatures. Figure 6-36 show that the proposed unimodal SWCC equation developed in this study was better than Fredlund & Xing (1994) equation in best fitting the SWC of soil data used in this study. In addition, the proposed unimodal SWCC equation developed in this study has additional benefit as compared to Fredlund & Xing (1994) equation. The parameters of the proposed unimodal SWCC equation developed in this study can be used to represent the variables of SWCC of soil data used in this study, such as: air-entry value, inflection point of SWCC, residual suction and residual water content (see Figure 6-1 and Figure 6-2).

Figure 6-37 show that the proposed bimodal SWCC equations developed in this study was better than Fredlund & Xing (1994) equation in best fitting the SWCC of soil data used in this study. In addition, the proposed bimodal SWCC equation developed in this study has additional benefit as compared to Zhang & Chen (2005) equation. The parameters of the proposed bimodal SWCC equation developed in this study can be used to represent the variables of SWCC of soil data used in this study, such as: first and second air-entry value, first and second inflection point of SWCC, residual suction and residual water content (see Figure 6-3 to Figure 6-6).

Figure 6-36. R^2 of the equation developed in this study and Fredlund & Xing (1994) equation for best fitting unimodal SWCC

Figure 6-37. R^2 of the equation developed in this study and Zhang & Chen (2005) equation for best fitting bimodal SWCC

The evaluation of the proposed equations for best fitting the bimodal shear strength developed in this study has been presented in Section 5.7. The performances of the proposed equations for best fitting the bimodal shear strength have been evaluated using experimental data from laboratory tests carried out in this study and published data from literatures. Then, the performances of the proposed equations have been compared with the performance of published equations for best fitting the bimodal shear strength using the criteria explained in Section 4.6.

Figure 6-38 shows that the R^2 of the proposed shear strength equation developed in this study were close to one indicating a good performance of the proposed shear strength equation developed in this study in best fitting the bimodal shear strength data obtained from the experimental works carried out in this study and published data from literatures. Figure 6-38 also show that the proposed shear strength equation developed in this study was better than Lee et al. (2005) equation in best fitting the bimodal shear strength of soil data used in this study. In addition, the parameter b of the proposed shear strength equation developed in this study can be used to describe the ratio of ϕ^b/ϕ' of soil with bimodal characteristics (see Figure 6-39).

Figure 6-38. R^2 of the proposed equation developed in this study and Lee et al. (2005) equation for best fitting bimodal shear strength of soil

Figure 6-39. Relationship between ratio of ϕ^b / ϕ' at suctions between AEV1 and AEV2 and parameter b of the proposed shear strength equation developed in this study

6.6. Evaluation of The Proposed Equations for Estimating Soil-water Characteristic Curve using Experimental Data from Laboratory Tests carried out in This Study and Published Data from Literatures

In this section, the evaluation of the proposed equations developed in this study for estimating soil-water characteristic curve (SWCC) for soil with bimodal grain-size distribution (GSD) (Equations in Section 6.3) are presented and discussed in detail. The estimated bimodal and unimodal SWCC were compared with experimental data of soil with bimodal GSD from laboratory tests carried out in this study and published data from literatures. The performance of the proposed equations for estimating SWCC were evaluated using the criteria explained in Section 4.6.

The comparisons of bimodal SWCC from laboratory tests carried out in this study with the estimated bimodal SWCC using the proposed estimating equation developed in this study (Section 4.1) are presented in Figure 6-40 to Figure 6-44. The parameters of the proposed equation developed in this study for estimating bimodal SWCC for soil with bimodal GSD are summarized in Table 6-5 and Table 6-6. The comparisons of bimodal SWCC from published literatures with the estimated bimodal SWCC using the proposed estimating equation developed in this study (Section 4.1) are presented in Figure 6-45 to

Figure 6-48. The parameters of the proposed equation developed in this study for estimating bimodal SWCC for soil with bimodal GSD are summarized in Table 6-7.

Figure 6-40. Comparison of the experimental data of soils M2a, M2b and M2c with the estimated SWCC using the proposed equation developed in this study

Figure 6-41. Comparison of the experimental data of soils M3a and M3b with the estimated SWCC using the proposed equation developed in this study

Figure 6-42. Comparison of the experimental data of soils M4a and M4b with the estimated SWCC using the proposed equation developed in this study

Figure 6-43. Comparison of the experimental data of soils M5a and M6a with the estimated SWCC using the proposed equation developed in this study

Figure 6-44. Comparison of the experimental data of soils M7a and M8a with the estimated SWCC using the proposed equation developed in this study

Table 6-5. Parameters of the proposed equation for estimating SWCC of soil M2a, M2b, M2c, M3a, M3b, M4a

Parameters	M2a	M2b	M2c	M3a	M3b	M4a
θ_{s1}	0.203	0.267	0.316	0.352	0.339	0.282
ψ_{a1} (kPa)	20	20	15	6	7	3
ψ_{m1} (kPa)	29	24	19	10	10	4
s_1	0.94	0.94	0.94	0.81	0.64	0.36
θ_{s2}	0.173	0.224	0.261	0.289	0.280	0.234
ψ_{a2} (kPa)	100	75	60	40	40	35
ψ_{m2} (kPa)	225	116	106	100	105	108
s_2	2	2	1.5	1.1	1.5	1
ψ_r (kPa)	1000	500	500	500	600	600
θ_r	0.075	0.01	0.10	0.09	0.08	0.06
RMSE	0.005	0.008	0.009	0.010	0.011	0.007
ARE	3.2	2.9	3.7	8.7	4.4	7.3

Table 6-6. Parameters of the proposed equation for estimating SWCC of soil M4b, M5a, M6a, M7a, M8a

Parameters	M4b	M5a	M6a	M7a	M8a
θ_{s1}	0.312	0.228	0.207	0.520	0.595
ψ_{a1} (kPa)	3	4	10	1	0.6
ψ_{m1} (kPa)	4	11	5	1.5	1
s_1	0.36	0.20	0.36	0.36	0.36
θ_{s2}	0.270	0.191	0.174	0.418	0.475
ψ_{a2} (kPa)	40	30	25	10	8
ψ_{m2} (kPa)	108	117	117	68	57
s_2	1	0.92	0.92	1.9	2.4
ψ_r (kPa)	300	300	500	1000	300
θ_r	0.07	0.02	0.03	0.06	0.10
RMSE	0.015	0.008	0.014	0.019	0.027
ARE	5.1	2.6	5.9	5.8	8.7

Figure 6-45. Comparison of the experimental data of soils SB1 and SB2 with the estimated SWCC using the proposed equation developed in this study

Figure 6-46. Comparison of the experimental data of soils SB3 and SB4 with the estimated SWCC using the proposed equation developed in this study

Figure 6-47. Comparison of the experimental data of soils SB5 and SB6 with the estimated SWCC using the proposed equation developed in this study

Figure 6-48. Comparison of the experimental data of soils JF3 and JF13 with the estimated SWCC using the proposed equation developed in this study

Table 6-7. Parameters of the proposed equation for estimating SWCC of soil SB1, SB2, SB3, SB4, SB5, SB6, JF3 and JF13

Parameters	SB1	SB2	SB3	SB4	SB5	SB6	JF3	JF13
θ_{s1}	0.320	0.352	0.334	0.394	0.445	0.405	0.236	0.255
ψ_{a1} (kPa)	1	1	1	2	2	2	20	30
ψ_{m1} (kPa)	2	1.3	2	3.3	3.5	4.3	28	41
s_1	1.5	0.4	0.4	0.01	2.5	0.9	1.3	0.7
θ_{s2}	0.258	0.283	0.276	0.315	0.355	0.324	0.198	0.213
ψ_{a2} (kPa)	20	10	20	50	20	40	300	250
ψ_{m2} (kPa)	156	556	50	833	143	362	1045	667
s_2	1.8	2	1.8	2.3	2.7	2.4	0.9	0.9
ψ_r (kPa)	1000	6000	1500	5000	4000	10000	5000	2000
θ_r	0.08	0.05	0.08	0.04	0.10	0.09	0.04	0.08
RMSE	0.030	0.026	0.003	0.090	0.024	0.040	0.008	0.011
ARE	6.1	6.2	1.1	5.5	5.1	3.2	3.5	4.8

Figure 6-49 shows that the values of Root Mean Square Error (RMSE) of the proposed equation for estimating bimodal SWCC developed in this study varied between 0.005 and 0.027 (close to zero). Figure 6-50 shows that the Average Relative Error

(ARE) of the proposed equation for estimating bimodal SWCC of soil data used in this study varied between 2.6 and 8.7. As explained in Section 5, the small value of RMSE (close to zero) indicated a good estimation capability of the proposed equation in estimating bimodal SWCC for soils with bimodal GSD. In addition, the overall values of ARE were less than 10, indicating the proposed equation was able to estimate the bimodal SWCC for soils with bimodal GSD successfully.

Figure 6-49. RMSE of the proposed equation developed in this study for estimating bimodal SWCC of soil data used in this study

Figure 6-50. ARE of the proposed equation developed in this study for estimating bimodal SWCC of soil data used in this study

The comparisons of unimodal SWCC from laboratory tests carried out in this study with the estimated unimodal SWCC using the proposed estimating equation developed in this study (Section 4.1) are presented in Figure 6-51 to Figure 6-54. The parameters of the proposed equation developed in this study for estimating unimodal SWCC for soil with bimodal GSD are summarized in Table 6-8 and Table 6-9. The comparisons of unimodal SWCC from published literatures with the estimated unimodal SWCC using the proposed estimating equation developed in this study (Section 4.1) are presented in Figure 6-55 and Figure 6-56.

Figure 6-51. Comparison of the experimental data of soils M3c, M4c and M5b with the estimated SWCC using the proposed equation developed in this study

Figure 6-52. Comparison of the experimental data of soils M5c, M6b and M6c with the estimated SWCC using the proposed equation developed in this study

Figure 6-53. Comparison of the experimental data of soils M7b and M7c with the estimated SWCC using the proposed equation developed in this study

Figure 6-54. Comparison of the experimental data of soils M8b and M8c with the estimated SWCC using the proposed equation developed in this study

Figure 6-55. Comparison of the experimental data of soils SB9, SB11 and GM1b with the estimated SWCC using the proposed equation developed in this study

Figure 6-56. Comparison of the experimental data of soils GM1b, BTG8 and BTG14 with the estimated SWCC using the proposed equation developed in this study

Table 6-8. Parameters of the proposed equation for estimating SWCC of soil M3c, M4c, M5b, M5c and M6b

Parameters	M3c	M4c	M5b	M5c	M6b
θ_s	0.296	0.386	0.405	0.455	0.580
ψ_a (kPa)	20	20	1	2	3
ψ_m (kPa)	160	245	2.9	21	22
s	4.7	4.1	1.1	1.5	1.9
ψ_r (kPa)	4000	1000	9	200	300
θ_r	0.02	0.10	0.02	0.00	0.00
RMSE	0.019	0.023	0.015	0.029	0.028
ARE	6.7	9.6	8.5	9.4	9.8

Figure 6-57 shows that the values of Root Mean Square Error (RMSE) of the proposed equation for estimating bimodal SWCC developed in this study varied between 0.004 and 0.042 (close to zero). Figure 6-58 shows that the Average Relative Error (ARE) of the proposed equation for estimating bimodal SWCC from laboratory tests carried out in this study varied between 6.7 and 9.8. As explained in Section 5, the small value of RMSE (close to zero) indicated a good estimation capability of the proposed

equation in estimating unimodal SWCC for soils with bimodal GSD. In addition, the overall values of ARE were less than 10, indicating the proposed equation was able to estimate the unimodal SWCC for soils with bimodal GSD successfully.

Table 6-9. Parameters of the proposed equation for estimating SWCC of soil M6c, M7b, M7c, M8b and M8c

Parameters	M6c	M7b	M7c	M8b	M8c
θ_s	0.636	0.612	0.635	0.643	0.653
ψ_a (kPa)	3	2	3	10	15
ψ_m (kPa)	40	80	131	234	347
s	2.2	2.1	2.5	2.5	2.9
ψ_r (kPa)	1000	200	300	2000	3000
θ_r	0.00	0.00	0.00	0.08	0.10
RMSE	0.025	0.024	0.026	0.026	0.033
ARE	8.0	8.5	7.7	7.8	5.5

Table 6-10. Parameters of the proposed equation for estimating SWCC of soil SB8, SB11, GM1b, GM2b, BTG8 and BTG14

Parameters	SB8	SB11	GM1b	GM2b	BTG8	BTG14
θ_s	0.448	0.204	0.303	0.346	0.560	0.49
ψ_a (kPa)	800	1	70	60	100	150
ψ_m (kPa)	11316	0.57	1107	1069	1098	1851
s	5.1	5.7	1.9	2.1	1.6	3.9
ψ_r (kPa)	10000	300	20000	20000	100000	20000
θ_r	0.1	0.0	0.0	0.0	0.0	0.08
RMSE	0.008	0.017	0.004	0.006	0.021	0.042
ARE	1.8	9.8	1.1	1.3	4.5	9.5

Figure 6-57. RMSE of the proposed equation developed in this study for estimating unimodal SWCC of soil data used in this study

Figure 6-58. ARE of the proposed equation developed in this study for estimating unimodal SWCC of soil data used in this study

6.7. Evaluation of The Proposed Equations for Estimating Bimodal Shear Strength using Experimental Data from Laboratory Tests carried out in This Study and Published Data from Literatures

In this section, the evaluation of the proposed equations developed in this study for estimating shear strength of soil with bimodal characteristics are presented and discussed in detail. The estimated bimodal shear strengths were compared with the experimental data of soil with bimodal characteristics from laboratory tests carried out in this study and published data from literatures. The performances of the proposed equations for estimating bimodal shear strength of unsaturated soil were evaluated using the criteria explained in Section 4.6.

The comparisons of bimodal shear strength from laboratory tests carried out in this study with the estimated bimodal shear strength using the proposed estimating equation developed in this study are presented in Figure 6-59 to Figure 6-63. The parameters of the proposed equation developed in this study for estimating bimodal shear strength of unsaturated soil are summarized in Table 6-11. The parameters of Goh et al. (2010) equation for estimating bimodal shear strength of unsaturated soil are summarized in Table 6-12.

Figure 6-59. Comparison of the experimental data of soil mixture M2a with the estimated shear strength

Figure 6-60. Comparison of the experimental data of soil mixture M3a with the estimated shear strength

Figure 6-61. Comparison of the experimental data of soil mixture M4a with the estimated shear strength

Figure 6-62. Comparison of the experimental data of soil mixture M6a with the estimated shear strength

Figure 6-63. Comparison of the experimental data of soil mixture M8a with the estimated shear strength

Table 6-11. Parameters of the proposed equation for estimating bimodal shear strength of soil mixtures from laboratory tests carried out in this study

Parameters	M2a	M3a	M4a	M6a	M8a
b	0.90	0.89	0.72	0.57	0.23
d	0.41	0.42	0.34	0.43	0.10
f	0.18	0.23	0.21	0.19	0.66
RMSE	0.024	0.040	0.050	0.052	0.056
ARE	0.50	3.44	4.77	3.90	3.67

Table 6-12. Parameters of Goh et al (2010) equation for estimating bimodal shear strength of soil mixtures from laboratory tests carried out in this study

Parameters	M2a	M3a	M4a	M6a	M8a
y_d	0.68	0.65	0.72	1.15	1.37
b_d	0.41	0.32	0.72	0.46	0.80
RMSE	0.170	0.157	0.197	0.347	0.370
ARE	12.6	12.3	17.1	30.4	33.3

The comparisons of the bimodal shear strength of soil data from literatures with the estimated bimodal shear strength using the proposed estimating equation developed in

this study are presented in Figure 6-64 to Figure 6-67. The parameters of the proposed equation developed in this study for estimating bimodal shear strength of unsaturated soil are summarized in Table 6-13. The parameters of Goh et al. (2010) equation for estimating bimodal shear strength of unsaturated soil are summarized in Table 6-14.

Figure 6-64. Comparison of the experimental data of soil from Coutinho et al. (2011) with the estimated shear strength

Figure 6-65. Comparison of the experimental data of soil from Jotisankasa et al. (2009) with the estimated shear strength

Figure 6-66. Comparison of the experimental data of soil from Jotisankasa and Mairaing (2010) with the estimated shear strength

Figure 6-67. Comparison of the experimental data of soil Rahardjo et al. (2011) with the estimated shear strength

Table 6-13. Parameters of the proposed equation for estimating bimodal shear strength of soil data from literatures

Parameters	Coutinho et al. (2011)	Jotisankasa et al. (2009)	Jotisankasa & Mairaing (2010)	Rahardjo et al. (2011)
b	0.57	0.33	0.49	0.69
q	0.23	0.21	0.15	0.14
f	0.46	0.31	0.46	0.46
RMSE	0.04	0.09	0.13	0.02
ARE	3.45	7.89	9.20	1.52

Table 6-14. Parameters of Goh et al (2010) equation for estimating bimodal shear strength of soil data from literatures

Parameters	Coutinho et al. (2011)	Jotisankasa et al. (2009)	Jotisankasa & Mairaing (2010)	Rahardjo et al. (2011)
y_d	1.03	0.58	1.43	0.49
b_d	1.18	1.24	0.98	0.69
RMSE	0.36	0.84	0.27	0.53
ARE	28.05	60.29	24.24	44.22

Figure 6-68 and Figure 6-69 show that the values of Root Mean Square Error (RMSE) and the Average Relative Error (ARE) of the proposed equation for estimating bimodal shear strength developed in this study were less than Goh et al. (2010) equation. These results indicated the proposed shear strength equation developed in this study performed better than Goh et al. (2010) equation. As explained in Section 5, the small value of RMSE of the proposed shear strength equation, varying between 0.024 and 0.056 (close to zero), indicated a good estimation capability of the proposed equation in estimating the bimodal shear strength of unsaturated soil. The overall values of ARE of the proposed shear strength equation (varying between 0.50 and 4.77) were less than 10

indicating the proposed equation was able to estimate the the bimodal shear strength of the unsaturated soil successfully.

Figure 6-68. RMSE of the proposed equation developed in this study for estimating bimodal shear strength of unsaturated soil (Note: COU = soil from Coutinho et al. (2011), Jot1 = soil from Jotisankasa et al. (2009), Jot2 = soil from Jotisankasa & Mairaing (2010), Rah1 = soil from Rahardjo et al. (2011))

Figure 6-69. ARE of the proposed equation developed in this study for estimating bimodal shear strength of unsaturated soil (Note: COU = soil from Coutinho et al. (2011), Jot1 = soil from Jotisankasa et al. (2009), Jot2 = soil from Jotisankasa & Mairaing (2010), Rah1 = soil from Rahardjo et al. (2011))

CHAPTER 7 PRACTICAL SIGNIFICANCE OF RESEARCH FINDINGS

7.1. Introduction

Series of soil data from laboratory tests carried out in this study, i.e. soil-water characteristic curve (SWCC) and unsaturated shear strength tests, contribute significantly to the unsaturated soil mechanics database and literature for both researchers and practicing engineers. The findings from the laboratory tests will help engineers to understand and estimate the behaviours of soil with bimodal characteristics under unsaturated conditions. The observed behaviours of SWCC and the unsaturated shear strength for soils with bimodal grain-size distribution are important for practicing engineers. The development of equation for best fitting the bimodal and unimodal SWCC can assist the engineers to use the suitable SWCC parameters for unsaturated soil functions in the implementation of unsaturated soil mechanics in geotechnical design. The findings on the bimodal shear strength of the unsaturated soil are important for engineers in designing and modelling geotechnical structures. The understanding of the differences between the bimodal and unimodal SWCC and the unsaturated shear strength of soil helps the engineers to use the appropriate soil parameters for solving geotechnical problems such as slope stability.

The proposed shear strength equation developed in this research can be used to best fit the bimodal and unimodal shear strength of unsaturated soil. The behaviours of the unsaturated shear strengths of soil with bimodal characteristics for suctions between the first and the second air-entry values of soil are different with those for suctions higher than the second air-entry value of soil. The proposed shear strength equation may also be used by engineers to estimate the bimodal shear strength of soil, which is crucial for newly constructed slopes and other geotechnical structures during rainfalls.

In this section, case study is presented to show the role of the proposed bimodal SWCC and shear strength equations on the slope stability analyses of soils with bimodal characteristics. This case study has been published in Satyanaga et al. (2014). The SWCC and permeability function of the investigated materials are best fitted using the proposed bimodal equation and compared with those best fitted using the unimodal equation. The

hydraulic properties of soil best fitted with the unimodal and bimodal equations are incorporated into seepage analyses to obtain the pore-water pressure profiles during and after rainfalls. Then, the pore-water pressure contours for each time increment are used in stability analyses to calculate the variation of factor of safety during and after rainfall.

7.2. Case Study

Five (5) independent sets of soil data from literatures were used in this study. Soils 1, 2, 3, 4 and 5 were obtained from Coutinho et al. (2011); Jotisankasa and Mairaing (2010), Jotisankasa et al. (2009), Rahardjo et al. (2011) and Rahardjo et al. (2012b), respectively. The SWCC of each soil is presented in Figure 7-1 and Figure 7-2. Each SWCC is fitted with the proposed bimodal and unimodal SWCC equations developed in this study. The variables of SWCC from each soil are presented in Table 7-1 and Table 7-2. Figure 7-1 and Figure 7-2 also present the values of root square mean error (RSME) and average relative error (ARE) where lower values of RSME and ARE indicate better performance of the unimodal or bimodal equation in fitting the SWCC of soils 1 to 5. Table 2 shows that soils 1 to 5 have higher values of $RSME_u$ and ARE_u as compared to values of $RSME_b$ and ARE_b , indicating that the SWCCs of soils 1 to 5 were bimodal and these soils can be classified as soils with bimodal characteristics

Figure 7-1. SWCC of soils 1, 2 and 3 fitted with unimodal and bimodal equations

Figure 7-2. SWCC of soils 4 and 5 fitted with unimodal and bimodal equations

Table 7-1. Best fitting parameters of the proposed bimodal SWCC equation for soils 1 to 5 with bimodal SWCC

Parameters	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5
θ_{s1}	0.411	0.473	0.414	0.316	0.52
ψ_{a1} (kPa)	1	3	0.8	0.8	5
ψ_{m1} (kPa)	8.2	20	3	2	9
s_1	1.25	2.8	0.9	1.5	0.7
θ_{s2}	0.248	0.257	0.221	0.125	0.11
ψ_{a2} (kPa)	100	20000	50	50	200
ψ_{m2} (kPa)	5000	40000	500	500	400
s_2	2	1.2	1.5	1.5	2
ψ_r (kPa)	20000	100000	5000	1000	1000
θ_r	0.03	0.05	0	0.02	0.04
$RMSE_b$	0.006	0.012	0.009	0.13	0.005
ARE_b	1.68	2.88	2.9	4.17	1.54

Note: subscript b refers to the proposed bimodal SWCC equation

Table 7-2. Best fitting parameters of the proposed unimodal SWCC equation for soils 1 to 5 with bimodal SWCC

Parameters	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5
θ_s	0.411	0.473	0.414	0.316	0.52
ψ_a (kPa)	1.5	3	1	1	5
ψ_m (kPa)	184	200	10	9.5	10
s	4	4.5	2	5	3
ψ_r (kPa)	0.1	10000	100	1000	300
θ_r	0.05	0.12	0.13	0	0
RMSE _u	0.015	0.04	0.032	0.023	0.03
ARE _u	4.92	9.32	11.5	12.3	24.4

Note: subscript u refers to the proposed unimodal SWCC equation

The permeability function of each soil was calculated using the statistical method (Childs and Collis-George, 1950). The calculated permeability function was generated as data points. These data points need to be best fitted with a continuous function which could be used as an input for seepage analyses. For this purpose, a new equation was proposed to best fit the bimodal and unimodal permeability functions of the soil. The equations for fitting unimodal and bimodal permeability functions were developed based on the proposed SWCC equations developed in this study since permeability function and SWCC have similar shapes (Fredlund and Rahardjo, 1993). The proposed equations for fitting bimodal and unimodal permeability functions are shown in Equations 8-1 and 8-2.

$$k_w = (k_{s1} - k_{s2}) \left(1 - \operatorname{erfc} \frac{\ln \left(\frac{\psi_{a1} - k_w}{k_{a1} - k_{m1}} \right)}{S_{k1}} \right) + (k_{s2}) \left(1 - \operatorname{erfc} \frac{\ln \left(\frac{\psi_{a2} - k_w}{k_{a2} - k_{m2}} \right)}{S_{k2}} \right) \quad 7-1$$

$$k_w = k_s \left(1 - \operatorname{erfc} \frac{\ln \left(\frac{\psi_a - k_w}{k_a - k_m} \right)}{S_k} \right) \quad 7-2$$

where:

k_w = calculated permeability for different suction

k_s = saturated permeability (measured in the laboratory)

ψ = matric suction under consideration

ψ_a = parameter represents the air-entry value of soil

k_m = parameter represents the matric suction at the inflection point of permeability function

s_k = parameter represents the geometric standard deviation of permeability function

Subscripts 1 and 2 represent subcurves 1 (associated with macro pores) and 2 (associated with micro pores), respectively.

Figure 7-3 to Figure 7-7 indicate that the shape of permeability function (k_w) of each soil fitted with unimodal and bimodal permeability equations were similar to that of SWCC fitted with unimodal and bimodal SWCC equations, respectively. The permeability function of all soils investigated in this study started to decrease at the air-entry value of the respective soils. Figure 7-3 to Figure 7-7 also indicate that the differences between the permeability function fitted with bimodal and unimodal equations varied for different soil types.

Figure 7-3. Permeability functions of soils 1 modelled with the proposed bimodal and unimodal permeability equations

Figure 7-4. Permeability functions of soils 2 modelled with the proposed bimodal and unimodal permeability equations

Figure 7-5. Permeability functions of soils 3 modelled with the proposed bimodal and unimodal permeability equations

Figure 7-6. Permeability functions of soils 4 modelled with the proposed bimodal and unimodal permeability equations

Figure 7-7. Permeability functions of soils 5 modelled with the proposed bimodal and unimodal permeability equations

Four cases of numerical analyses (cases 1 to 4) were performed on each soil investigated in this study. Transient seepage analyses were conducted using the finite element software Seep/W (Geoslope International Ltd., 2007a) and slope stability analyses were conducted using Slope/W (Geoslope International Ltd., 2007b). Two different rainfall intensities i.e. 9 mm/h and 22.2 mm/h, were used in the seepage analyses to study the effect of low and high rainfall intensities on slope of soils with bimodal characteristics. Simplified slope profiles with homogeneous soil (same SWCC and permeability function for all the elements) were used in all cases. The details of each numerical analysis are summarized in Table 7-3.

Table 7-3. Details of numerical analyses carried out in this study

Description	SWCC equation	k_w equation	Rainfall intensity (mm/h)	Duration (hours)
Case 1	Bimodal	Bimodal	9	24
Case 2	Unimodal	Unimodal	9	24
Case 3	Bimodal	Bimodal	22	24
Case 4	Unimodal	Unimodal	22	24

In case 1, the SWCC and permeability function of each soil used in the seepage analyses were fitted using the bimodal equation. A 9 mm/h of rainfall for 24 hours duration was applied to the surface of slope with respect to different types of soil (Soils 1 to 5). In case 2, the SWCC and permeability function of each soil used in the seepage analyses were fitted using the unimodal equation. A 9 mm/h of rainfall for 24 hours duration was applied to the surface of slope with respect to different types of soil (Soils 1 to 5). In case 3, the SWCC and permeability function of each soil used in the seepage analyses were fitted using the bimodal equation. A 22 mm/h of rainfall for 24 hours duration was applied to the surface of slope with respect to different types of soil (Soils 1 to 5). In case 4, the SWCC and permeability function of each soil used in the seepage analyses were fitted using the unimodal equation. A 22 mm/h of rainfall for 24 hours duration was applied to the surface of slope with respect to different types of soil (Soils 1 to 5).

The initial condition for all cases was taken as hydrostatic pore-water pressure conditions. The finite element model and the boundary conditions for all cases were similar as shown in Figure 7-8. The initial position of groundwater table (6 m and 1 m depths below the crest and the toe of the slope, respectively) as well as the height (15 m) and the angle (27°) of the slope for all cases in this study were obtained from Rahardjo et al. (2010) to represent typical position of groundwater table and slope geometry of residual soil slopes in Singapore. Stability analyses of different types of soil slope were carried out by incorporating the pore-water pressures from different time increments as obtained from seepage analyses. The factors of safety at different time increments were calculated using Bishop's simplified method of slices. The saturated and unsaturated shear strength of soils investigated in this study as shown in Table 7-4 were used in Slope/W for the stability analyses involving different types of soil.

Figure 7-9 and Figure 7-10 show the variation of factor of safety of different types of soil under rainfall intensities of 9 mm/h and 22 mm/h, respectively. Figure 7-9 shows that the variations in factor of safety with respect to time during and after rainfall of 9 mm/h for soils fitted with bimodal and unimodal equations were similar. These could be attributed to the low rainfall intensity on the surface of the slope, resulting in small changes in pore-water pressures near the ground surface. Hence, the differences in the

variations of factor of safety for soils fitted with bimodal and unimodal equations were very small in this case.

Figure 7-8. Slope model for seepage analyses of all cases in this study

Table 7-4. Shear strength of soils 1 to 5

Description	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5
Effective cohesion, c' (kPa)	10	7	2	1	0
Effective friction angle, ϕ' ($^{\circ}$)	26	37	32	30	33
AEV1 (kPa)	1.5	3	1	1	4
ϕ^b ($^{\circ}$) for $0 \leq (u_a - u_w) \leq \text{AEV1}$	26	37	32	30	33
ϕ^b ($^{\circ}$) for $(u_a - u_w) > \text{AEV1}$	22	29	28	28	22

Figure 7-9. Variations of factor of safety for soil modeled with (a) bimodal and (b) unimodal equations under 9 mm/h of rainfall

Figure 7-10 indicates that factor of safety of slope of soil 4 decreased more significantly during rainfall of 22 mm/h as compared to that of other soil slopes. These happened since the permeability functions of soil 4 was higher than those of other soils, resulting in the higher water flow within this soil layer (soil 4) as compared to other soil layers.

(a)

(b)

Figure 7-10. Variations of factor of safety for soil modeled with (a) bimodal and (b) unimodal equations under 22 mm/h of rainfall

Figure 7-10 also shows that the variations of factor of safety with respect to time during and after rainfall of 22 mm/h for soil 2 fitted with bimodal equations were higher than those fitted with unimodal equations. These could happen since the permeability functions of soil 2 fitted with bimodal equations were higher than those fitted with unimodal equations, resulting in the higher water flow within soil 2 fitted with bimodal equations as compared to soil 2 fitted with unimodal equations. On the other hand, the variations of factor of safety with respect to time during and after rainfall of 22 mm/h for soils 1, 3 to 5 fitted with bimodal equations were lower than those fitted with unimodal equations. This was attributed to the lower permeability functions of soil 1, 3 to 5 fitted with bimodal equations than those fitted with unimodal equations, resulting in the slower rate of water flow within soils 1, 3 to 5 fitted with bimodal equations as compared to those of soils 1, 3 to 5 fitted with unimodal equations.

The differences in the variations of factor of safety (ΔFoS) for stability analyses under 9 and 22 mm/h between soils fitted with bimodal and unimodal equations are presented in Figure 7-11. The negative value indicates that the factor of safety for soils fitted with bimodal equation were lower than those fitted with unimodal equation. Figure 7-11a shows that the variations of factor of safety for soils fitted with bimodal and unimodal equations were similar for stability analyses under 9 mm/h of rainfall. Figure 7-11b indicates that the factor of safety for soils fitted with unimodal equation were much higher than those fitted with bimodal equation for stability analyses under 22 mm/h of rainfall. Figure 7-11b also shows that soil 1 with the lowest ARE_u of its SWCC among all soils had the lowest percentages of difference in the variations of factor of safety. On the other hand, soil 5 with the highest ARE_u of its SWCC among all soils had the highest percentage of difference in the variations of factor of safety.

The results of case study indicated that the variations in factor of safety for SWCC and permeability function fitted with the bimodal equation can be lower or higher than those for SWCC and permeability function with the unimodal equation. The highest percentage of differences in the factor of safety between SWCC fitted with bimodal and unimodal equations were observed to occur at the critical factor of safety of the slope. The percentage differences in factor of safety increase with the increasing average relative error of the unimodal SWCC. In conclusion, it is important to use the proposed

bimodal equation for soil with bimodal characteristics to obtain the appropriate factor of safety for particular slope.

(a)

(b)

Figure 7-11. Difference of factor of safety for soil modeled with bimodal and unimodal equations under (a) 9mm/h and (b) 22 mm/h of rainfall

CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

8.1. Conclusions

The conclusions from research works in this study are as follows:

- a. The experimental results show that soils with bimodal grain-size distribution (GSD) do not always produce bimodal soil-water characteristic curve (SWCC), but they can also produce unimodal SWCC, depending on the structure of the soil. The soil specimen with bimodal GSD compacted at the dry density and water content associated with the dry of optimum will result in the bimodal SWCC. On the other hand, the soil specimen with bimodal GSD compacted at the dry density and water content associated with the wet of optimum will result in the unimodal SWCC. The soil with bimodal GSD with high percentage of fine particles will have lower ratio of AEV_2/AEV_1 as compared to that with low percentage of fine particles. As a result, the soil with bimodal GSD with high percentage of fine particles is likely to have unimodal SWCC due to the dominance of small pores within this soil. This could be due to the fact that the pores between coarse particles are filled with fine particles.
- b. The results from the SWCC tests indicated that soil properties, i.e. soil density, void ratio, water content and GSD of soil affected the shape and variables of SWCC. The soil with high dry density will result in the low air-entry value (AEV) of the soil and the steep slope of the SWCC. The soil with a gentle slope of GSD will result in a gentle slope of the SWCC. The soil with high percentage of coarse particles will result in the low saturated water content of the SWCC.
- c. The equations for best fitting the GSD of soils with bimodal and unimodal characteristics had been proposed and developed in this study. The proposed GSD equations had been evaluated using soil data from laboratory tests carried out in this study and literatures. The proposed equations were able to best fit GSD of soil successfully since R^2 of the proposed equations are close to 1. In addition, the parameters of the proposed GSD equation can be used to represent the physical properties of soil and shape of GSD, such as majority of soil particle size within

the soil specimen, maximum diameter of soil particle and the percentage of coarse or fine particle within soil specimen.

- d. The equations for best fitting the SWCC with bimodal and unimodal characteristics had been proposed and developed in this study. The proposed SWCC equations had been evaluated using soil data from laboratory tests carried out in this study and literatures. The proposed equations were able to best fit SWCC successfully since R^2 of the proposed equations were close to 1. In addition, the parameters of the SWCC equation can be used to represent the variables of SWCC, such as the air-entry value of soil, matric suction at the inflection point of the SWCC, and the residual volumetric water content and matric suction related to the residual volumetric water content.
- e. The equations and procedures for estimating SWCC for soils with bimodal grain-size distribution had been proposed and developed in this study. The estimated SWCC had been evaluated using soil data from laboratory tests carried out in this study and literatures. The proposed equations were able to estimate SWCC successfully since RMSE of the proposed equations were very small and ARE of the proposed equations are less than 10 %.
- f. Results of series of unsaturated consolidated drained (CD) triaxial tests showed that soil specimens compacted at dry of optimum (soils with bimodal SWCC) had a higher volumetric water content for suctions higher than the air-entry value (AEV) as compared to those compacted at wet of optimum (soils with unimodal SWCC). These results indicated that a larger wetted area of soil was observed within soil specimen with bimodal SWCC as compared to that with unimodal SWCC. As a result, the shear strength of soil with bimodal SWCC was higher than that of soil with unimodal SWCC.
- g. It was observed that soils with bimodal SWCC had a steeper stress-strain curve than those with unimodal SWCC indicating a higher stiffness of soils with bimodal SWCC than those with unimodal SWCC. The similarity of behaviour of specimens with bimodal SWCC at suction beyond AEV₂ with those of specimens with unimodal SWCC occurred since water already flowed through the macro and

micro pores of soil specimens with bimodal and unimodal SWCC at suction beyond AEV2.

- h. The rates of water volume change soils with bimodal SWCC were lower than those with unimodal SWCC. This could be happened since the soil particles within soils with bimodal SWCC were more disoriented as compared to those within soils with unimodal SWCC. As a result, it was difficult for water to flow within soil pores of soils with bimodal SWCC.
- i. The effective cohesion of soil with bimodal SWCC was lower than that with unimodal SWCC whereas the effective friction angle of soil with bimodal SWCC was higher than that with unimodal SWCC. This could happen since the fine particles were trapped inside the micropores within the soil with bimodal SWCC whereas the fine particles were distributed uniformly within the soil with unimodal SWCC.
- j. Similar behavior was observed for soils with bimodal characteristics where ϕ^b angle was equal to ϕ' at matric suctions less than or equal to the first air-entry value. However, ϕ^b angle of soils with unimodal SWCC varied with suction at suctions higher than air-entry value whereas ϕ^b angle of soils with bimodal SWCC was constant at suctions higher than AEV1 and less than AEV2. The ϕ^b angle of soils with bimodal SWCC started to decrease constantly at suctions higher than AEV1. This might be attributed to the reduction of area of water within the macropores of soil specimens with bimodal characteristics at suctions higher than AEV1. However, micropores within soil specimens with bimodal characteristics were still in saturated conditions. As suction went beyond the second air-entry value, the area of water menisci within micropores started to decrease due to the desaturation of micropores of soil specimen with bimodal SWCC. As a result, ϕ^b angle of soils with bimodal SWCC started to vary with suction at suctions higher than AEV2.
- k. The equations for best fitting the bimodal shear strength of the unsaturated soil had been proposed and developed in this study. The proposed shear strength equations had been evaluated using soil data from laboratory tests carried out in this study and literatures. The proposed equations were able to best fit shear

strength successfully since R^2 of the proposed equations were close to 1. In addition, the parameter b of the shear strength equation can be used to represent the ratio of ϕ^b/ϕ' .

1. The equations and procedures for estimating the bimodal shear strength of the unsaturated soil had been proposed and developed in this study. The estimated shear strength had been evaluated using soil data from laboratory tests carried out in this study and literatures. The proposed equations were able to estimate the bimodal shear strength successfully since RMSE of the proposed equations were very small and ARE of the proposed equations are less than 10 %.

8.2. Publications from PhD Research works

1. Satyanaga, A., Rahardjo, H., Leong, E.C., Chia, Y.H. and Wang, J.-Y. (2014). "Effect of Dual-Porosity Material on Stability of Slope during Rainfall." Proceedings of the International Conference on Unsaturated Soils, UNSAT 2014, 2-4 July, Sydney.
2. Rahardjo, H., Leong, E.C., Satyanaga, A., Ng, Y.S., Tan, H.T. and Hua, C.J. (2014). "Rainfall-induced Slope Failures and Preventive Measures in Singapore." Geotechnical Engineering Monograph. NTU-HDB Research Collaboration Project, Nanyang Technological University, Singapore, 84 pp (ISBN 978-981-07-9250-3).
3. Satyanaga, A., Rahardjo, H., Leong, E.C. and Wang, J.-Y. (2013). "Water Characteristic Curve of Soil with Bimodal Grain-size Distribution". Computer and Geotechnics, Vol. 48, pp. 51-61.
4. Rahardjo, H., Satyanaga, A., Leong, E.C. and Ng, Y.S. (2012). "Variability of Residual Soil Properties". Journal of Engineering Geology, Vol. 141-142, No. 124-140
5. Rahardjo, H., Satyanaga, A., D'Amore, G.A.R. and Leong, E.C. (2012). "Soil-water Characteristic Curves of Gap-graded Soils". Journal of Engineering Geology, Vol. 125, pp. 102-107.

6. Priono, Rahardjo, H. and Satyanaga, A. (2013). "Relationship between Pore Size Distribution and Shear Strength of Soil." Proceedings of the URECA@NTU 2011-12, Nanyang Technological University, Singapore, August, pp. 2095-2100.
7. Priono, Rahardjo, H. and Satyanaga, A. (2012) "Mathematical Modeling of Grain Size Distribution, Soil-Water Characteristic Curve, Pore Size Distribution and Shear Strength of Soil." Proceedings of the URECA@NTU 2010-11, Nanyang Technological University, Singapore, April, pp 276-281.
8. Hoach, T.K., Rahardjo, H. and Satyanaga, A. (2012) "Equations for Bimodal Grain Size Distribution and Soil-Water Characteristic Curve." Proceedings of the URECA@NTU 2010-11, Nanyang Technological University, Singapore, April, pp 213-218.
9. Rahardjo, H., Satyanaga, A., Leong, E.C. and Ng, Y.S. (2010). "Effects of groundwater table position and soil properties on stability of slope during rainfall". Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 136, No.11, pp. 1555–1564.

8.3. Recommendations

The recommendation for future studies of this research can be divided into two categories namely laboratory works and theoretical development.

Laboratory works

The analyses of slope stability during rainfall encourage engineers to incorporate the SWCC and shear strength under wetting condition. Therefore, the laboratory tests of SWCC and shear strength for soils with bimodal characteristics should be further investigated and studied. Apparatuses for rapid measurement of wetting SWCC and shear strength with accurate measurements of volume change of soil with bimodal characteristics should be developed and studied.

This study focused on the mechanical behavior of soil with bimodal grain-size distribution. Further investigation on the effect of chemical composition and particle shape on the behaviour of soil with bimodal grain-size distribution should be carried out.

Theoretical Development

The SWCC and shear strength equations for best fitting and estimating the bimodal SWCC and bimodal shear strength of the unsaturated soil may be further modified by incorporating the effects of net normal stress on soil. In addition, the SWCC and shear strength equations for estimating the wetting bimodal SWCC and shear strength should be developed and studied.

Hysteresis index should be developed to quantify the magnitude of hysteresis of soils with bimodal SWCC. It is expected that the hysteresis index can be estimated from soil properties to produce a good first approximation of the magnitude of hysteresis for soils with bimodal SWCC.

Further study should be carried out on the development of mathematical equation for modeling the permeability function of soil with bimodal SWCC. This study will be useful to investigate the effect of permeability function on the stability of slopes with soils of bimodal characteristics.

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APPENDICES

This section contains all secondary information for this research, i.e. figures and tables, which are relatively less important as compared to those included in the main test. A standard symbol is adopted for all figures and tables in the appendix to ease the way of referring in the main content of the report. The standard symbol adopted for figure and table in appendix as Figure Aa.b-c and Table Aa.b-c where the term A refers to chapter, the term a.b means the section numbers and the term c indicated the figure or table number. For example, "Figure A6.4.1" is referred to the first figure in Appendix used for Section 6.4 (Chapter 6) while "Table A6.4.1" in the Appendix is referred to the first table in Appendix used for Section 6.4 (Chapter 6).

Appendix A5.4

Figure A5.4-1. Variation of pore-water pressures during shearing of the saturated mutistage CU triaxial tests specimen a. M2 and b. M3

Figure A5.4-2. Variation of pore-water pressures during shearing of the saturated mutistage CU triaxial tests specimen a. M4 and b. M5

Figure A5.4-3. Variation of pore-water pressures during shearing of the saturated mutistage CU triaxial tests specimen a. M6 and b. M7

Figure A5.4-4. Variation of pore-water pressures during shearing of the saturated mutistage CU triaxial tests specimen a. M8 and b. M9

Table A5.4-1. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M2a under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.365	6	42.40
After saturation	0.365	14.10	99.64
After consolidation	0.360	13.82	99.15
After 10 kPa matric suction equalization	0.358	13.77	99.16
End of shearing under 10 kPa matric suction	0.356	13.62	98.83
After 20 kPa matric suction equalization	0.354	13.39	97.54
End of shearing under 20 kPa matric suction	0.351	13.07	96.13
After 30 kPa matric suction equalization	0.348	12.14	89.99
End of shearing under 30 kPa matric suction	0.344	11.76	88.20
After 50 kPa matric suction equalization	0.341	11.63	88.10
End of shearing under 50 kPa matric suction	0.338	11.28	86.13
After 70 kPa matric suction equalization	0.335	11.17	85.99
End of shearing under 70 kPa matric suction	0.331	10.73	83.62
After 100 kPa matric suction equalization	0.328	10.61	83.37
End of shearing under 100 kPa matric suction	0.326	10.39	82.32
After 200 kPa matric suction equalization	0.323	7.56	60.40
End of shearing under 200 kPa matric suction	0.320	7.13	57.45
After 400 kPa matric suction equalization	0.320	5.60	45.12
End of shearing under 400 kPa matric suction	0.320	5.13	41.34

Table A5.4-2. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M2b under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.296	9.80	85.28
After saturation	0.296	11.40	99.20
After consolidation	0.291	11.23	99.46
After 10 kPa matric suction equalization	0.290	11.07	98.48
End of shearing under 10 kPa matric suction	0.287	10.69	95.96
After 20 kPa matric suction equalization	0.286	10.51	94.77
End of shearing under 20 kPa matric suction	0.283	10.07	91.83
After 30 kPa matric suction equalization	0.280	10.43	95.99
End of shearing under 30 kPa matric suction	0.276	10.16	94.82
After 50 kPa matric suction equalization	0.273	10.08	95.18
End of shearing under 50 kPa matric suction	0.271	9.61	91.61
After 70 kPa matric suction equalization	0.268	9.54	91.82
End of shearing under 70 kPa matric suction	0.264	9.13	89.16
After 100 kPa matric suction equalization	0.262	8.29	81.76
End of shearing under 100 kPa matric suction	0.259	7.81	77.79
After 200 kPa matric suction equalization	0.256	7.27	73.14
End of shearing under 200 kPa matric suction	0.254	6.79	69.01
After 400 kPa matric suction equalization	0.254	6.57	66.77
End of shearing under 400 kPa matric suction	0.254	6.06	61.59

Table A5.4-3. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M2c under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.365	13.10	92.58
After saturation	0.432	16.70	99.83
After consolidation	0.426	16.46	99.69
After 10 kPa matric suction equalization	0.425	16.18	98.32
End of shearing under 10 kPa matric suction	0.422	15.74	96.29
After 20 kPa matric suction equalization	0.420	14.56	89.37
End of shearing under 20 kPa matric suction	0.417	13.97	86.48
After 30 kPa matric suction equalization	0.414	13.84	86.26
End of shearing under 30 kPa matric suction	0.410	13.31	83.81
After 50 kPa matric suction equalization	0.406	13.29	84.42
End of shearing under 50 kPa matric suction	0.403	12.77	81.68
After 70 kPa matric suction equalization	0.401	11.16	71.89
End of shearing under 70 kPa matric suction	0.396	10.68	69.53
After 100 kPa matric suction equalization	0.393	9.76	64.00
End of shearing under 100 kPa matric suction	0.391	9.22	60.90
After 200 kPa matric suction equalization	0.388	8.09	53.82
End of shearing under 200 kPa matric suction	0.385	7.63	51.14
After 400 kPa matric suction equalization	0.385	7.09	47.52
End of shearing under 400 kPa matric suction	0.385	6.64	44.50

Table A5.4-4. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M3a under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.432	5.00	30.64
After saturation	0.502	18.90	99.73
After consolidation	0.496	18.54	98.99
After 10 kPa matric suction equalization	0.495	15.37	82.31
End of shearing under 10 kPa matric suction	0.492	14.88	80.16
After 20 kPa matric suction equalization	0.490	14.73	79.60
End of shearing under 20 kPa matric suction	0.487	14.26	77.64
After 30 kPa matric suction equalization	0.484	14.18	77.68
End of shearing under 30 kPa matric suction	0.479	13.64	75.42
After 50 kPa matric suction equalization	0.476	13.43	74.84
End of shearing under 50 kPa matric suction	0.473	12.94	72.56
After 70 kPa matric suction equalization	0.470	12.27	69.24
End of shearing under 70 kPa matric suction	0.465	11.81	67.28
After 100 kPa matric suction equalization	0.462	11.09	63.58
End of shearing under 100 kPa matric suction	0.459	10.64	61.40
After 200 kPa matric suction equalization	0.456	9.35	54.31
End of shearing under 200 kPa matric suction	0.453	8.82	51.56
After 400 kPa matric suction equalization	0.449	7.76	45.82
End of shearing under 400 kPa matric suction	0.445	7.14	42.51

Table A5.4-5. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M3b under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.359	9.40	69.39
After saturation	0.425	16.02	99.85
After consolidation	0.420	15.53	98.08
After 10 kPa matric suction equalization	0.418	14.28	90.49
End of shearing under 10 kPa matric suction	0.415	13.85	88.36
After 20 kPa matric suction equalization	0.414	13.76	88.08
End of shearing under 20 kPa matric suction	0.410	13.31	85.93
After 30 kPa matric suction equalization	0.408	13.22	85.94
End of shearing under 30 kPa matric suction	0.403	12.86	84.48
After 50 kPa matric suction equalization	0.400	12.69	84.09
End of shearing under 50 kPa matric suction	0.397	12.17	81.22
After 70 kPa matric suction equalization	0.394	11.34	76.22
End of shearing under 70 kPa matric suction	0.390	10.82	73.51
After 100 kPa matric suction equalization	0.387	10.29	70.42
End of shearing under 100 kPa matric suction	0.384	9.74	67.15
After 200 kPa matric suction equalization	0.382	8.25	57.29
End of shearing under 200 kPa matric suction	0.379	7.79	54.50
After 400 kPa matric suction equalization	0.375	6.74	47.69
End of shearing under 400 kPa matric suction	0.371	6.23	44.50

Table A5.4-6. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M3c under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.432	14.50	88.86
After saturation	0.502	18.90	99.73
After consolidation	0.496	18.47	98.61
After 10 kPa matric suction equalization	0.495	17.62	94.36
End of shearing under 10 kPa matric suction	0.492	17.13	92.29
After 20 kPa matric suction equalization	0.490	16.19	87.49
End of shearing under 20 kPa matric suction	0.487	15.64	85.16
After 30 kPa matric suction equalization	0.484	14.71	80.59
End of shearing under 30 kPa matric suction	0.479	14.28	78.96
After 50 kPa matric suction equalization	0.476	13.76	76.68
End of shearing under 50 kPa matric suction	0.473	13.25	74.30
After 70 kPa matric suction equalization	0.470	12.43	70.14
End of shearing under 70 kPa matric suction	0.465	11.87	67.62
After 100 kPa matric suction equalization	0.462	10.94	62.72
End of shearing under 100 kPa matric suction	0.459	10.46	60.36
After 200 kPa matric suction equalization	0.456	9.63	55.93
End of shearing under 200 kPa matric suction	0.453	9.28	54.25
After 400 kPa matric suction equalization	0.449	7.54	44.52
End of shearing under 400 kPa matric suction	0.445	6.92	41.20

Table A5.4-7. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M4a under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.424	6.00	37.08
After saturation	0.493	18.76	99.64
After consolidation	0.487	18.26	98.15
After 10 kPa matric suction equalization	0.486	16.28	87.77
End of shearing under 10 kPa matric suction	0.483	15.84	85.92
After 20 kPa matric suction equalization	0.482	15.76	85.75
End of shearing under 20 kPa matric suction	0.478	15.37	84.27
After 30 kPa matric suction equalization	0.475	13.71	75.64
End of shearing under 30 kPa matric suction	0.470	13.42	74.73
After 50 kPa matric suction equalization	0.467	12.81	71.90
End of shearing under 50 kPa matric suction	0.464	12.39	69.99
After 70 kPa matric suction equalization	0.461	11.83	67.25
End of shearing under 70 kPa matric suction	0.456	11.32	64.98
After 100 kPa matric suction equalization	0.454	10.67	61.64
End of shearing under 100 kPa matric suction	0.451	10.13	58.91
After 200 kPa matric suction equalization	0.448	9.27	54.26
End of shearing under 200 kPa matric suction	0.445	8.76	51.62
After 400 kPa matric suction equalization	0.440	6.65	39.58
End of shearing under 400 kPa matric suction	0.437	6.27	37.63

Table A5.4-8. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M4b under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.351	10.00	74.75
After saturation	0.416	15.85	99.75
After consolidation	0.411	15.37	98.03
After 10 kPa matric suction equalization	0.409	13.86	88.71
End of shearing under 10 kPa matric suction	0.407	13.41	86.42
After 20 kPa matric suction equalization	0.405	13.34	86.26
End of shearing under 20 kPa matric suction	0.402	12.91	84.21
After 30 kPa matric suction equalization	0.399	12.84	84.34
End of shearing under 30 kPa matric suction	0.395	12.46	82.71
After 50 kPa matric suction equalization	0.391	11.24	75.28
End of shearing under 50 kPa matric suction	0.388	10.86	73.26
After 70 kPa matric suction equalization	0.386	10.13	68.83
End of shearing under 70 kPa matric suction	0.381	9.72	66.77
After 100 kPa matric suction equalization	0.379	9.09	62.91
End of shearing under 100 kPa matric suction	0.376	8.67	60.45
After 200 kPa matric suction equalization	0.373	7.76	54.51
End of shearing under 200 kPa matric suction	0.370	7.23	51.17
After 400 kPa matric suction equalization	0.366	6.49	46.46
End of shearing under 400 kPa matric suction	0.362	5.92	42.79

Table A5.4-9. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M4c under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.424	15.00	92.71
After saturation	0.493	18.77	99.69
After consolidation	0.487	18.43	99.06
After 10 kPa matric suction equalization	0.486	18.22	98.23
End of shearing under 10 kPa matric suction	0.483	17.76	96.34
After 20 kPa matric suction equalization	0.482	15.94	86.73
End of shearing under 20 kPa matric suction	0.478	15.52	85.09
After 30 kPa matric suction equalization	0.475	14.71	81.15
End of shearing under 30 kPa matric suction	0.470	14.39	80.14
After 50 kPa matric suction equalization	0.467	13.24	74.31
End of shearing under 50 kPa matric suction	0.464	12.76	72.08
After 70 kPa matric suction equalization	0.461	11.95	67.93
End of shearing under 70 kPa matric suction	0.456	11.42	65.55
After 100 kPa matric suction equalization	0.454	10.21	58.99
End of shearing under 100 kPa matric suction	0.451	9.76	56.76
After 200 kPa matric suction equalization	0.448	8.19	47.94
End of shearing under 200 kPa matric suction	0.445	7.67	45.19
After 400 kPa matric suction equalization	0.440	6.35	37.79
End of shearing under 400 kPa matric suction	0.437	5.86	35.17

Table A5.4-10. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M5a under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	S_r (%)
Before saturation	0.480	12.20	65.83
After saturation	0.552	21.30	99.92
After consolidation	0.546	21.06	99.89
After 10 kPa matric suction equalization	0.544	20.97	99.75
End of shearing under 10 kPa matric suction	0.541	20.48	97.97
After 20 kPa matric suction equalization	0.540	17.94	86.06
End of shearing under 20 kPa matric suction	0.536	17.53	84.70
After 30 kPa matric suction equalization	0.533	17.41	84.60
End of shearing under 30 kPa matric suction	0.528	16.97	83.18
After 50 kPa matric suction equalization	0.525	14.79	73.03
End of shearing under 50 kPa matric suction	0.521	14.28	70.92
After 70 kPa matric suction equalization	0.518	13.53	67.59
End of shearing under 70 kPa matric suction	0.514	13.06	65.83
After 100 kPa matric suction equalization	0.511	11.47	58.16
End of shearing under 100 kPa matric suction	0.508	10.95	55.86
After 200 kPa matric suction equalization	0.505	9.01	46.24
End of shearing under 200 kPa matric suction	0.502	8.68	44.82
After 400 kPa matric suction equalization	0.497	7.73	40.29
End of shearing under 400 kPa matric suction	0.493	7.24	38.03

Table A5.4-11. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M5b under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	Sr (%)
Before saturation	0.408	14.00	88.96
After saturation	0.476	18.38	99.97
After consolidation	0.470	18.15	99.93
After 10 kPa matric suction equalization	0.469	16.74	92.45
End of shearing under 10 kPa matric suction	0.466	16.28	90.48
After 20 kPa matric suction equalization	0.465	15.52	86.52
End of shearing under 20 kPa matric suction	0.461	15.07	84.68
After 30 kPa matric suction equalization	0.458	14.36	81.20
End of shearing under 30 kPa matric suction	0.454	13.84	79.02
After 50 kPa matric suction equalization	0.450	13.17	75.80
End of shearing under 50 kPa matric suction	0.447	12.62	73.11
After 70 kPa matric suction equalization	0.444	11.85	69.10
End of shearing under 70 kPa matric suction	0.440	11.37	66.96
After 100 kPa matric suction equalization	0.437	10.46	62.01
End of shearing under 100 kPa matric suction	0.434	9.83	58.67
After 200 kPa matric suction equalization	0.431	9.17	55.10
End of shearing under 200 kPa matric suction	0.428	8.54	51.67
After 400 kPa matric suction equalization	0.424	7.29	44.56
End of shearing under 400 kPa matric suction	0.420	6.79	41.86

Table A5.4-12. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M6a under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	S_r (%)
Before saturation	0.635	11.50	47.07
After saturation	0.715	27.48	99.94
After consolidation	0.708	26.83	98.50
After 10 kPa matric suction equalization	0.706	26.67	98.15
End of shearing under 10 kPa matric suction	0.703	26.14	96.66
After 20 kPa matric suction equalization	0.701	21.49	79.66
End of shearing under 20 kPa matric suction	0.697	20.94	78.09
After 30 kPa matric suction equalization	0.694	20.48	76.75
End of shearing under 30 kPa matric suction	0.689	19.93	75.24
After 50 kPa matric suction equalization	0.684	19.93	75.24
End of shearing under 50 kPa matric suction	0.681	15.21	58.07
After 70 kPa matric suction equalization	0.678	14.19	54.44
End of shearing under 70 kPa matric suction	0.673	13.64	52.73
After 100 kPa matric suction equalization	0.669	12.35	47.98
End of shearing under 100 kPa matric suction	0.666	11.83	46.20
After 200 kPa matric suction equalization	0.662	10.19	40.00
End of shearing under 200 kPa matric suction	0.659	9.67	38.15
After 400 kPa matric suction equalization	0.654	8.58	34.11
End of shearing under 400 kPa matric suction	0.650	8.09	32.37

Table A5.4-13. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M6b under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	S_r (%)
Before saturation	0.557	16.50	77.04
After saturation	0.633	24.26	99.69
After consolidation	0.626	23.87	99.09
After 10 kPa matric suction equalization	0.625	21.91	91.18
End of shearing under 10 kPa matric suction	0.622	18.07	75.59
After 20 kPa matric suction equalization	0.620	17.53	73.53
End of shearing under 20 kPa matric suction	0.616	16.98	71.69
After 30 kPa matric suction equalization	0.613	15.56	66.04
End of shearing under 30 kPa matric suction	0.608	15.12	64.68
After 50 kPa matric suction equalization	0.604	13.86	59.69
End of shearing under 50 kPa matric suction	0.601	13.37	57.89
After 70 kPa matric suction equalization	0.597	11.73	51.06
End of shearing under 70 kPa matric suction	0.592	11.25	49.37
After 100 kPa matric suction equalization	0.589	10.19	44.96
End of shearing under 100 kPa matric suction	0.586	9.68	42.95
After 200 kPa matric suction equalization	0.583	8.29	36.98
End of shearing under 200 kPa matric suction	0.580	7.71	34.59
After 400 kPa matric suction equalization	0.575	6.23	28.18
End of shearing under 400 kPa matric suction	0.571	5.84	26.61

Table A5.4-14. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M7a under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	S_r (%)
Before saturation	0.719	16.50	60.36
After saturation	0.803	30.43	99.70
After consolidation	0.796	25.74	85.08
After 10 kPa matric suction equalization	0.794	22.82	75.60
End of shearing under 10 kPa matric suction	0.790	22.46	74.74
After 20 kPa matric suction equalization	0.789	20.37	67.94
End of shearing under 20 kPa matric suction	0.784	19.85	66.58
After 30 kPa matric suction equalization	0.781	18.51	62.37
End of shearing under 30 kPa matric suction	0.775	18.09	61.38
After 50 kPa matric suction equalization	0.771	16.73	57.09
End of shearing under 50 kPa matric suction	0.767	16.24	55.68
After 70 kPa matric suction equalization	0.764	14.51	49.98
End of shearing under 70 kPa matric suction	0.758	14.06	48.77
After 100 kPa matric suction equalization	0.755	12.62	43.98
End of shearing under 100 kPa matric suction	0.751	12.13	42.47
After 200 kPa matric suction equalization	0.748	10.37	36.48
End of shearing under 200 kPa matric suction	0.744	9.84	34.78
After 400 kPa matric suction equalization	0.739	7.85	27.95
End of shearing under 400 kPa matric suction	0.734	7.36	26.36

Table A5.4-15. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M7b under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	S_r (%)
Before saturation	0.634	19.50	80.95
After saturation	0.713	27.08	99.87
After consolidation	0.706	25.74	95.83
After 10 kPa matric suction equalization	0.705	23.67	88.34
End of shearing under 10 kPa matric suction	0.701	23.15	86.81
After 20 kPa matric suction equalization	0.700	21.26	79.92
End of shearing under 20 kPa matric suction	0.695	20.79	78.63
After 30 kPa matric suction equalization	0.692	19.37	73.61
End of shearing under 30 kPa matric suction	0.687	18.86	72.21
After 50 kPa matric suction equalization	0.683	17.29	66.61
End of shearing under 50 kPa matric suction	0.679	16.75	64.85
After 70 kPa matric suction equalization	0.676	15.34	59.69
End of shearing under 70 kPa matric suction	0.671	14.87	58.29
After 100 kPa matric suction equalization	0.667	13.69	53.94
End of shearing under 100 kPa matric suction	0.664	13.14	52.04
After 200 kPa matric suction equalization	0.661	11.28	44.90
End of shearing under 200 kPa matric suction	0.657	10.85	43.41
After 400 kPa matric suction equalization	0.652	8.69	35.04
End of shearing under 400 kPa matric suction	0.648	8.13	33.00

Table A5.4-16. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M8a under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	S_r (%)
Before saturation	0.807	18.00	58.45
After saturation	0.895	34.08	99.77
After consolidation	0.888	33.17	97.92
After 10 kPa matric suction equalization	0.886	26.83	79.37
End of shearing under 10 kPa matric suction	0.882	26.39	78.40
After 20 kPa matric suction equalization	0.880	23.51	69.99
End of shearing under 20 kPa matric suction	0.875	23.08	69.08
After 30 kPa matric suction equalization	0.872	20.64	62.04
End of shearing under 30 kPa matric suction	0.866	20.17	61.02
After 50 kPa matric suction equalization	0.861	18.52	56.34
End of shearing under 50 kPa matric suction	0.858	18.13	55.39
After 70 kPa matric suction equalization	0.854	16.75	51.40
End of shearing under 70 kPa matric suction	0.848	16.29	50.32
After 100 kPa matric suction equalization	0.844	15.34	47.59
End of shearing under 100 kPa matric suction	0.841	14.86	46.31
After 200 kPa matric suction equalization	0.837	12.57	39.35
End of shearing under 200 kPa matric suction	0.833	12.06	37.92
After 400 kPa matric suction equalization	0.828	9.61	30.42
End of shearing under 400 kPa matric suction	0.823	9.13	29.07

Table A5.4-17. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M8b under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	S_r (%)
Before saturation	0.712	21.00	77.23
After saturation	0.796	30.36	99.95
After consolidation	0.789	30.05	99.81
After 10 kPa matric suction equalization	0.787	29.87	99.43
End of shearing under 10 kPa matric suction	0.783	29.46	98.51
After 20 kPa matric suction equalization	0.782	26.72	89.55
End of shearing under 20 kPa matric suction	0.777	26.41	89.02
After 30 kPa matric suction equalization	0.774	23.54	79.71
End of shearing under 30 kPa matric suction	0.768	23.08	78.69
After 50 kPa matric suction equalization	0.764	20.63	70.75
End of shearing under 50 kPa matric suction	0.760	20.17	69.49
After 70 kPa matric suction equalization	0.757	18.54	64.18
End of shearing under 70 kPa matric suction	0.752	18.06	62.96
After 100 kPa matric suction equalization	0.748	16.82	58.91
End of shearing under 100 kPa matric suction	0.744	16.49	58.03
After 200 kPa matric suction equalization	0.741	14.37	50.82
End of shearing under 200 kPa matric suction	0.737	13.82	49.11
After 400 kPa matric suction equalization	0.732	11.29	40.41
End of shearing under 400 kPa matric suction	0.728	10.71	38.57

Table A5.4-18. Void ratio (e), water contents (w) and degree of saturation at different stages of specimens M9a under a net confining pressure of 25 kPa from the CD triaxial tests

Stress state	e	w (%)	S_r (%)
Before saturation	0.855	18.00	56.62
After saturation	0.946	35.13	99.94
After consolidation	0.938	34.64	99.35
After 10 kPa matric suction equalization	0.936	34.27	98.49
End of shearing under 10 kPa matric suction	0.932	33.76	97.42
After 20 kPa matric suction equalization	0.930	33.19	95.98
End of shearing under 20 kPa matric suction	0.925	32.64	94.88
After 30 kPa matric suction equalization	0.922	32.38	94.51
End of shearing under 30 kPa matric suction	0.916	31.85	93.55
After 50 kPa matric suction equalization	0.911	31.46	92.89
End of shearing under 50 kPa matric suction	0.907	30.97	91.83
After 70 kPa matric suction equalization	0.903	30.63	91.21
End of shearing under 70 kPa matric suction	0.898	30.12	90.27
After 100 kPa matric suction equalization	0.894	29.85	89.85
End of shearing under 100 kPa matric suction	0.890	29.37	88.78
After 200 kPa matric suction equalization	0.886	25.46	77.30
End of shearing under 200 kPa matric suction	0.882	24.83	75.71
After 400 kPa matric suction equalization	0.876	16.71	51.29
End of shearing under 400 kPa matric suction	0.872	16.25	50.15

Appendix A7.1

Table A7.1-19. Computer Codes for The Computation of Parameters in The Estimated Bimodal SWCC

```

Public Function sat_vol(satw, rd) 'to calculate saturated volumetric water content
    sat_vol = satw * rd / 1000
End Function
Public Function inflec1(satw, P200, dm1) 'to calculate suction corresponding to
inflection point of first subcurve
    u1 = 0.039 * Exp(0.088 * (100 - P200))
    v1 = 0.701 * Exp(0.011 * satw)
    a1 = -1.9038 - ((Application.WorksheetFunction.Ln(0.1 * dm1 / u1)) / v1)
    inflec1 = 0.1 * Exp(a1)
End Function
Public Function AEV1(ym1, dmax1) 'to calculate the first air-entry value
    AEV1 = ym1 / (1.114 * Exp(0.083 * dmax1))
End Function
Public Function stdev1(D10, D60, P200) 'to calculate standard deviation of first
subcurve
    cu = D60 / D10
    If P200 > 50 Then
        stdev1 = (0.077 * cu) - 2.29
    Else
        stdev1 = 2.063 * Exp(-0.01 * cu)
    End If
End Function
Public Function sat_vol2(satw, rd) 'to calculate saturated volumetric water content of
second subcurve corresponding to saturated water content within micropore of
soil
    sat_vol2 = ((0.0078 * satw) + 0.008) * rd / 10
End Function
Public Function inflec2(satw, dm2, dmax2) 'to calculate suction corresponding to
inflection point in the second subcurve
    u2 = (2.102 * Application.WorksheetFunction.Ln(dmax2)) + 10.65
    v2 = 0.851 * Exp(0.003 * satw)
    a2 = -1.9038 - ((Application.WorksheetFunction.Ln(0.1 * dm2 / u2)) / v2)
    inflec2 = 0.1 * Exp(a2)
End Function
Public Function AEV2(ya1, ym1, ym2) 'to calculate second air-entry value
    AEV2 = ya1 * Exp(0.905 * (Application.WorksheetFunction.Ln(ym2 / ym1) ^ 0.704))
End Function

```

Table A7.1-19. (cont.). Computer Codes for The Computation of Parameters in The
Estimated Bimodal SWCC

```

Public Function stdev2(satw) 'to calculate standard deviation of second subcurve
    stdev2 = 0.594 * Exp(0.034 * satw)
End Function
Public Function res_suc(P200) 'to calculate residual suction of bimodal SWCC
    res_suc = 3391 * (P200 ^ -0.36)
End Function
Public Function res_vol(P200, satw, rd) 'to calculate residual volumetric water content
of bimodal SWCC
    If P200 > 50 Then
        res_vol = ((0.0092 * satw) - 0.294) * rd / 10
    Else
        res_vol = ((0.002 * (100 - P200)) - 0.088) * rd / 10
    End If
End Function
Public Function volwat(x, yr, qr, qs1, qs2, ya1, ym1, s1, ya2, ym2, s2) 'to calculate
volumetric water content for every suction
    c1 = Application.WorksheetFunction.Ln(1 + (x / yr))
    c2 = Application.WorksheetFunction.Ln(1 + (1000000 / yr))
    cr = 1 - (c1 / c2)
    If x > ya1 Then
        slope1a = Application.WorksheetFunction.Ln((ya1 - x) / (ya1 - ym1)) / s1
        slope1b = 1 - Application.WorksheetFunction.NormSDist(slope1a)
    Else
        slope1b = 1
    End If
    If x > ya2 Then
        slope2a = Application.WorksheetFunction.Ln((ya2 - x) / (ya2 - ym2)) / s2
        slope2b = 1 - Application.WorksheetFunction.NormSDist(slope2a)
    Else
        slope2b = 1
    End If
    volwat = cr * (qr + ((qs1 - qs2) * slope1b) + ((qs2 - qr) * slope2b))
End Function

```

Table A7.1-20. Computer Codes for The Computation of Parameters in The Estimated Unimodal SWCC

```

Public Function sat_vol(satw, rd) 'to calculate saturated volumetric water content
    sat_vol = satw * rd / 1000
End Function
Public Function inflec(D10, D60, P200, rd, dm1) 'to calculate suction corresponding to
inflection point
    cu = D60 / D10
    If P200 > 50 Then
        u = (-1.031 * rd / 10) + 19.53
    Else
        u = (0.028 * cu) + 9.21
    End If
    v = 0.5
    a = -1.9038 - ((Application.WorksheetFunction.Ln(0.1 * dm1 / u)) / v)
    inflec = 0.1 * Exp(a)
End Function
Public Function AEV(D10, D60, P200, ym, dm1, dm2) 'to calculate the air-entry value
    cu = D60 / D10
    If P200 > 50 Then
        AEV = ym / (3.167 * ((dm2/dm1)^2))
    Else
        AEV = ym / (0.429 * (cu^0.613))
    End If
End Function
Public Function stdev(rd, D10, D60, P200) 'to calculate standard deviation
    cu = D60 / D10
    If P200 > 50 Then
        stdev = (-0.471 * rd / 10) + 9.716
    Else
        stdev = 9.89 * (cu ^ -0.17)
    End If
End Function
Public Function res_vol(P200, D10) 'to calculate residual volumetric water content    If
    P200 > 50 Then
        res_vol = 0.771 * (D10 ^ -1.53)
    Else
        res_vol = 0.512 * (D10 ^ -1.42)
    End If
End Function

```

Table A7.1-20. (cont.). Computer Codes for The Computation of Parameters in The
Estimated Unimodal SWCC

```
Public Function volwat(x, yr, qr, qs, ya, ym, s) 'to calculate volumetric water content for  
every suction  
    c1 = Application.WorksheetFunction.Ln(1 + (x / yr))  
    c2 = Application.WorksheetFunction.Ln(1 + (1000000 / yr))  
    cr = 1 - (c1 / c2)  
    slopea = Application.WorksheetFunction.Ln((ya - x) / (ya - ym)) / s  
    If x > ya Then  
        slopeb = 1 - Application.WorksheetFunction.NormSDist(slopea)  
    Else  
        slopeb = 1  
    End If  
    volwat = cr * (qr + ((qs - qr) * slopeb))  
End Function
```