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**NANYANG
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**HEAT FLUX AND ACOUSTIC POWER
IN A T-SHAPED THERMOACOUSTIC
SYSTEM**

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**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

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HEAT FLUX AND ACOUSTIC POWER IN A T-SHAPED THERMOACOUSTIC SYSTEM

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Declaration

Except where explicitly state, this dissertation is the result of my own work under the supervision of Prof. Dan Zhao and is not the outcome of work done in collaboration.

This dissertation is not substantially the same as any that I have submitted for a degree, diploma, or any other qualification at any other university. No part of my dissertation has already been or is being concurrently submitted for any such degree, diploma or any other qualification.

This dissertation contains approximately 8000 words, 5 tables, 47 figures.

Shihuai Li

27th January 2015

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Abstract

The present work considers a convection-driven T-shaped standing-wave thermoacoustic system. To gain insights on the conversion process of heat to sound and to study the nonlinear coupling between unsteady heat release and acoustic disturbances, thermodynamic analysis, numerical and experimental investigations are conducted. Three parameters are examined: (1) the inlet flow velocity, (2) heater temperature and (3) heat source location. Their effects on triggering limit cycle oscillations are first investigated in 2D numerical model. As each of the parameters is varied, the head-driven acoustic signature is found to change. The main nonlinearity is identified in the heat fluxes. To characterize the transient (growing) behavior of the pressure fluctuation, the thermoacoustic mode growth rate is defined and calculated. It is found that the growth rate decreases first and then ‘saturates’. Similar behavior is observed by examining the slope of Rayleigh index. Furthermore, the overall efficiency of converting the input thermal energy into acoustical energy is defined and calculated. It is found that the energy conversion efficiency can be increased by increasing the inlet flow velocity. To validate our numerical findings, a cylindrical T-shaped duct made of quartz-glass with a metal gauze attaching on top of a Bunsen burner is designed and tested. Supercritical bifurcation is observed. And the experimental measurements show a good agreement with the numerical results in terms of mode frequency, mode shape, sound pressure level and Hopf bifurcation behavior.

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List of Publications

1. Li Shihuai and Zhao Dan, *Heat flux and acoustic power in a convection driven T-shaped thermoacoustic system*, Energy Conversion and Management, 2013.
2. Dan Zhao, Shihuai Li, Zhiguo Zhang, Wenming Yang, *Transient Energy Growth Analysis of a standing wave thermoacoustic system*, 8th International Congress on Industrial and Applied Mathematics, Aug. 10-14, 2015, Beijing.
3. Xinyan Li, Dan Zhao, Shihuai Li, *Mean temperature effect on transient growth analysis of thermoacoustic oscillations*, 23rd International Conference on Noise and Fluctuations, June 2-6, 2015, Xi'An China.

List of Symbols

A	bottom branch cross-sectional area, m^2
A_l	Ar cross-sectional areas of left and right branches, m^2
A_f	flame surface area, m^2
A_s	heater surface area, m^2
A_0	initial amplitude, m/s
E_m	mechanical work done by the fluid supporting a normal
stress, J	
c	speed of sound in air, m/s
c_p, c_v	heat capacity ratio at constant pressure and volume, kJ/kg
K	
D	the diameter of the bottom stem, m
	generated sound power, J
E_a	acoustical energy, J
E_e	exchange rate from heat to sound, J
$G \quad \varepsilon_a$	exponential complex function
h	convection coefficient, W/m^2K
L_l, L_r	left and right branch length, m
L_f	the thermal length scale in the fluid, m
L_s	the thermal length scale in the heating band, m
M_w	molecular weight, kg/mol
$\mathbf{n}$	the unit vector of the direction normal to the enclosed
surface	
p	instantaneous pressure, Pa
p_i, p_o	inlet and outlet pressure, Pa
q	heat flux, W/m^2
Q	heat production per unit volume J/m^3

Q_u	unsteady heat release rate, kJ/s
Q_t	input heat power, Watt
R	gas constant, J/mol K
R	radius of the tube, m
R_u	universal gas constant, kJ/kmol K
R_a	normalized Rayleigh index
S	entropy per unit mole, J/mol K
s	specific entropy, J/kg K
S_u	flame speed, m/s
ε	viscous stress, Pa
T	Temperature, K
T	oscillation period, s
$\mathbf{u}(\mathbf{x}, t)$	velocity at a given point $\mathbf{x}$, m/s
v_i	inlet velocity in i direction, m/s
B	thermal and viscous damping, J
$\mathbf{x}$	coordinates of a given point
x	axial position, m
x_f	heat source location along x_2 , m
y	horizontal axial position, m
Greek symbols	
α	pressure fluctuation amplitude slope, Pa/s
δ	Kronecker delta function
Δq_r	the heat of reaction per unit mass of the mixture, J/kg
ΔT_s	the surface temperature difference, K
ΔT_f	the temperature difference between the fluid and the heater
surface, K	
γ	the ratio of specific heat
λ	conduction coefficient, W/m K
μ	viscosity, kg/m s

ω	oscillation frequency, rad/s
ρ	air density, kg/m^3
ρ_m	the fuel–air mixture density, kg/m^3
τ	the viscous stress tensor, N/m
ψ	energy conversion efficiency
σ	thermoacoustic oscillation growth rate, s^{-1}

Subscript

s	heating band surface
a	acoustic
o	branch outlet

Superscript

$-$	mean value
$\cdot$	time derivative
$'$	fluctuation part

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Chapter 1

Introduction

1.1 Background

To achieve low NO_x emission and increased fuel efficiency, lean premixed prevaporised (LPP) combustion technology is widely applied in both land-based gas turbine and aero-engines. However, this technology is often associated with combustion instabilities, which are characterized by large self-excited pressure oscillations may become so intense that they cause structural damage and costly mission failure.

Combustion instability is currently a major challenge for land-based gas turbine [1,2,3] and aero-engine manufactures [4,5]. It also occurs frequently in other types of combustion systems, including aero-engine afterburners [6,7,8], rocket motors [9], ramjets [10], boilers and furnace [11]. On a laboratory-scale, combustion instabilities can occur in Rijke tube [12,13], which is a simple opened vertical tube with a heat source in its lower half.

Considerable research has been carried out in both academic and industrial communities over recent decades to elucidate the processes responsible for these instabilities and to develop control approaches to suppress them. An overview of the physics of combustion instabilities and methods for alleviating is presented in the following.

Combustion instabilities are self-excited oscillations generated by the interaction between unsteady heat release and acoustic wave [14,15,16]. Under lean premixed conditions, any change in the flow rate of inlet air or fuel may cause unsteady heat release. Unsteady heat release is an efficient acoustic source [17,18], and generate acoustic waves. These pressure wave propagate within the combustor and partially reflect from boundaries to arrive back at the combustion zone, where they can cause

more unsteady heat release. This feedback can result in the oscillation amplitudes successively increasing, which is what is meant by a combustion instability. Eventually, some nonlinearity in the system will limit the amplitude of the oscillations. Whether or not an instability occurs depends on the nature of the coupling between the unsteady heat release and acoustic wave.

The first insight into the combustion instabilities was provided by Rayleigh [13] over a century ago. He stated that instability would occur when heat was added in phase with pressure, but would not occur if heat was added out of phase with pressure. Chu [19] analytically interpreted Rayleigh's description using a mathematical approach. He considered a simple example of a perfect gaseous reacting medium contained in a volume V and bounded by a surface S . For simplicity, it is assumed that there is no viscous force, mean flow and mean heat release. His result is shown in equation (1.1), where p denotes pressure, u the particle velocity, ρ the density, c the speed of sound and q the heat-release rate per unit volume. An overbar denotes a mean value and a prime a perturbation.

$$\frac{\partial}{\partial t} \int_V \left(\frac{\bar{\rho} u'^2}{2} + \frac{p'^2}{2\rho c^2} \right) dV = \int_V \left(\frac{\gamma-1}{\rho c^2} \right) p' q dV - \int_S p' \mathbf{u} \cdot \mathbf{n} dS \quad (1.1)$$

The term on the left-hand side (LHS) represents the rate of change of the total energy (sum of kinetic and potential energy) within the combustor V . The first term on the right-hand side (RHS) represents the exchange of energy between the combustion and acoustic waves; as noted by Rayleigh [13], whenever the pressure and rate of heat release have a component which is in phase (i.e. Their relative phase lies between -90° and $+90^\circ$), the acoustic energy tends to increase. The last term accounts for energy losses across the bounding surface S . If, when averaged over one oscillation cycle, the acoustic energy supplied by the combustion process is larger than that dissipated across boundaries, the energy in the system will increase and instability will occur. Thus the condition for instability is

$$\int_V \left(\frac{\gamma-1}{\rho c^2} \right) p' q dV > \int_S p' \mathbf{u} \cdot \mathbf{n} dS \quad (1.2)$$

Where the overbar denotes an average over one period of the acoustic oscillation. When instability occurs, the oscillation amplitudes initially increase exponentially. Eventually, they are limited by nonlinear effects and limit cycle oscillations then occur. The amplitude of the limit cycle oscillation is the one at which the terms in equation (1.2) become equal. Recent work has suggested that this may occur due to either phase-variation in the heat release response or heat release saturation effects [21].

As well as offering insight into the energy exchange that gives rise to and limits the growth of combustion instabilities, equation (1.2) also suggests ways in which combustion instabilities can be controlled. Either the energy source term, $\int_V (\gamma - 1) \bar{p}' \bar{q}' / (\bar{\rho} \cdot \bar{c}^2) dV$, must be reduced or the surface loss term, $\int_V \bar{p}' \bar{\mathbf{u}} dS$, must be increased.

1.2 Motivations

Since the combustion instability exists in various technological applications, it is essential to gain insights on the conversion process of heat to sound and to study the nonlinear coupling between unsteady heat release and acoustic disturbances. However, due to its nonlinearity and complexity, analytical calculation is limited to explain. Therefore, in the present investigation, Computational Fluid Dynamics (CFD) techniques are applied to simulate the combustion instabilities in a Rijke tube. Distinguishing from the classical straight Rijke tube which results in a single flow field with combustion and heating inside it, the newly designed tube, T-shaped tube, will result in a more complex flow field due to the T-shape bifurcation. Therefore, acoustic characteristics are expected to be different from the simple straight tube. The aim of this work for combustion instabilities in T-shaped tube is to experimentally and numerically study the flow and acoustic characteristics resulted from the combustion. By using commercial CFD software Fluent 14.0, details of the combustion instabilities in T-shaped tube can be captured.

1.3 Organizations

This report is organized in the following; Chapter 1 gives a brief description of combustion instability as well as the objectives. Chapter 2 is the literature review of the analytical, experimental and numerical investigations carried on thermoacoustic instability. In chapter 3, parametric studies are conducted to numerically investigate Inlet velocity effect, heater location effect and heat flux effect. Chapter 4 compares the experimental results with the numerical results. Chapter 5 is the conclusions of the entire work and future work recommendations.

Chapter 2

Literature Review

The heat-to-sound or visa versa conversion mechanism (also known as thermoacoustics) has fascinated scientists for many years now, due to its practical energy applications [21]. One of the typical types of combustion driven thermoacoustic system is Rijke tube combustor [25,26]. Named after its inventor Rijke [22], it is a straight vertical tube with a heat source enclosed in its lower half. It provides an elementary example of convection-driven thermoacoustic oscillations [22]. To elucidate the conversion process in Rijke-type thermoacoustic systems, extensive investigations have been conducted.

2.1 Theoretical investigations

Rayleigh [27] proposed a theory to qualitatively explain the development of self-excited oscillations in Rijke tube. The criterion stated the importance of phase shift between the unsteady heat release and pressure perturbation. When heat fluctuates in phase with the acoustic pressure, the self-amplification of acoustic fluctuations may lead to a limit cycle. Otherwise, damping occurs if they are out of phase.

The first attempt to quantitatively analyze the phenomena of the Rijke pipe was made by Lehmann [28]. He concluded that the intensity of the sound should increase indefinitely with an increase of convective velocity. However, this conclusion was not confirmed by later experimental results, which showed that the amplitude of oscillation has a maximum at a given convective flow and decreases for lower and higher velocities.

Neuringer and Hudson [29] modeled Rijke oscillations and concluded that the oscillations were dependent on a proper phasing of the energy transfer and on a frictional force between the heater and the gas flow. In addition, they thought the

oscillation is greatly dependent on the velocity gradient in the heater and turbulence. However, their results only partly agreed with the experiment results.

Carrier [30] also applied linear perturbation analysis to conservation equations to obtain a system of dynamic equations. He investigated the effect of heating band element to the acoustic velocity. The corresponding results suggest the minimum heating band width in triggering instability for a fixed temperature and mean flow velocity.

Similar to Carrier's model [30], Kwon and Lee [31] modeled fluctuating heat transfer from ribbons to flow with mean and oscillating parts. The fluid was considered as viscous but incompressible. With Reynolds number below the threshold, vortex shedding can be avoided thus a boundary layer is not yet formed. From their model, The authors calculated heat release rate and acoustic power generation. From this, they calculated stability limits for Rijke tubes in agreement with the experiments conducted by Katto and Sajiki [32]. The maximum thermoacoustic transformation efficiency was found to occur when a wire radius was on the order of the thermal boundary layer and mean flow velocity close to the thermal diffusion velocity.

The mode shape is mainly determined by temperature distribution inside the Rijke tube, thus thermal analysis is needed. Raun and Beckstead [25] developed a model which employed a non-uniform temperature profile at the certain points of the burner. Linear perturbation analysis was applied to the reduced equations to obtain differential equations describing acoustic oscillating. The obtained frequency and the acoustic mode shape agree well with the experiment results conducted by Braithewaite et al [33]. The authors stressed the importance accounting for temperature variation in the hot zone of the burner.

The transition to Rijke-type thermoacoustic oscillations is investigated by Matveev & Culick [34]. In their study, a quasi-steady fashion was adopted to find the instability boundary. The mathematical model developed here is based on a thermal analysis determining the temperature of the heater as well as the temperature of the flow inside the tube. The calculated result is compared with results of a modeling based on the common assumption of a constant temperature profile, which shows the

fact that non-uniformities of the temperature field significantly affect the stability boundaries. To predict the stability behaviors of a Rijke tube, theoretical modal stability analysis [35] is conducted using Green's function approach. It shows that the thermoacoustic frequency is oscillating with increased 'heater' location.

The limit cycle amplitude is mainly determined by nonlinear effects thus can not predicted by linear theory. Based on the experiment results, Heckle [36] developed an empirical model for the nonlinear behavior of boundary conditions and heat transfer. Together with the balance equations for the acoustic energy in the Rijke tube, Heckle's model successfully predicted the amplitudes of the limit cycle. In his analysis, flow reversal at the heat source and non-linear effects at the ends of the tube are important in limiting the amplitude.

A nonlinear theory to predict the limit cycle in combustion system is presented by Dowling [37]. The method based on the assumption of a nonlinear heat release rate in flame can lead to limit cycle. Compared with the experiment results, the predicted amplitude and spectrum showed good agreement. Also, the theory verified the validity of linear theory in predicting both the frequency of oscillation and mode shape.

More recently, Balasubramanian and Sujith [38] combined a heat release model suggested by Heckle with Galerkin method to solve the wave equation and investigated the non-normality and nonlinearity in a Rijke tube. They found the non-normality can cause a quick growth of oscillations even though the eigenvectors of the system could be decaying exponentially with time and the transient growth is capable of make large enough oscillations to trigger nonlinear driving.

For combustion driven oscillations, five main mechanisms needed to be considered, namely, pulsating feed supply, turbulence, pressure coupling, pulsating flame speed, and flame area variation. As described by Putnam [39] pressure oscillations within a combustion chamber may cause a pulsation in feed supply. If the variation in heat release caused by the pulsation is roughly in phase with the pressure oscillations (i.e. Rayleigh's criterion), then the oscillations may be amplified. Oscillations caused by this mechanism may be eliminated by using a large pressure drop to induce choked flow in the air or fuel supply. Even with choked feeds, however,

mixing and diffusion effects may still induce instability. Pulsating feed rate is responsible for the oscillations of some diffusion flames. Toong [40] discussed both linear and nonlinear coupling of a diffusion flame in a pipe. The pulsating feed mechanism may also be important for premixed flames. Putnam and Dennis [41] discussed the pulsating feed mechanism in connection with a premixed burner with deep ports.

Turbulence may occur when flow velocity is high, or when fuel injectors or bluff body flame holders are used. Turbulence patterns can affect fuel and oxidizer mixing and the resulting reaction rate. Fuel injectors may set up a periodic variation of pressure or velocity. This in turn may affect combustion behavior by changing the flow pattern or the breakup and atomization of fuel droplets. Tsuji and Takeno [42] used a premixed turbulent combustor to study the effect of equivalence ratio, temperature, pressure and injector configuration on high-frequency oscillations. Strahle [43] discussed theories of combustion-generated noise for several types of turbulent flames. Mugirdge [44] reported experimental and theoretical work on stability of turbulent flames in a burner. Valk [45] measured acoustic power and frequency change caused by turbulent premixed propane-air flames over a frequency range of 3-120 Hz.

Clavin et al. [46] and McIntosh [47] studied the pressure effects on free propagating premixed flames using multiple scale asymptotic analysis. They showed combustion rates increase as pressure increases. Consequently, a pressure oscillation in the combustion chamber leads to a variation in heat release rate which is nearly in phase with the pressure variation. Clavin showed that under certain conditions, the strength of such an instability can be sufficiently strong, if the flame travels a typical acoustical distance.

Kaskan [48] and Blackshear [49] showed that variation in flame speed is not a significant acoustic driving mechanism for free flames, i.e. flames where heat loss to a flame holder is absent. Instead, area variation associated with cellularity is the main source of acoustic driving in free flames. However, for premixed, flame holder stabilized flames, pulsating laminar flame speed is the main mechanism for acoustic

driving Recent work with large activation energy asymptotics has particular application to pulsating instability. McIntosh [50] showed that maximum driving of pulsating instability occurs when characteristic times associated with diffusive processes are on the same order as the period of the acoustic oscillation.

2.2 Numerical investigations

In light of the recent progress in computational fluid dynamics (CFD) techniques, more work for thermoacoustic instability based on CFD modeling are reported. Yan [51] used second-order accurate schemes to simulate two-dimensional self-excited oscillating flows in a Rijke tube. The combustion process was substituted by the presence of a heat source, in effect eliminating the instability caused by the heat transfer interaction between the flame stabilizer and the flame. A range of forcing frequencies of the heat source were chosen to investigate the its effect on flow pattern and the temperature distribution. The author stressed the strong coupling between the amplitude of the acoustic velocity and the heat transfer rate.

Another similar one-dimensional study using a heat source and heat sink pair was conducted by Ishii et al. [52]. A steady heat supply was maintained to the heat source and the heat flux into the fluid (assumed to be an ideal gas) was allowed to fluctuate because of the velocity variation. This variation was explained with a hypothesis that due to the steep temperature gradient between the heat source and the heat sink, thermal energy was transformed into mechanical energy (heatinduced vibration). Such a phenomena is different from thermoacoustic driving mechanism and hence the model cannot be of help in understanding combustion instabilities.

Behnam Entezam et al [53] developed a numerical model in a attempt to better understand the internal coupling among heat addition, pressure, and velocity oscillations inside a Rijke tube. Their study suggests the presence of a minimum threshold value for the heat power supplied below which no self-sustained acoustic oscillations may be possible. Besides, the acoustic pressure is found to lead thermal fluctuations by a 45-degree angle. This result may be used to specify the phase angle in Carvalho's analytical formulation, which predicted a value smaller than 90 degrees.

But one of the significant deficiency in their work is the absence of the comparison between the computed values with the experimentally obtained results.

Another similar two-dimensional study was conducted by Hantschk and Vortmeyer [54]. the authors applied commercial CFD software Fluent to simulate self-excited thermoacoustic instability in a Rijke tube. With an artificially imposed temperature gradient, the self-excited oscillation is initiated by an initial pressure disturbance at the boundary. In their work, the higher harmonics frequencies of the oscillations and the mode shapes agree well with the experimental results, which confirms the ability of Computation Fluid Dynamics (CFD) analysis in capturing the growth rate and frequency in details. However, No detailed analysis of the pressure power spectrum was made to investigate the nonlinear coupling between the unsteady heat release and the tube acoustics. Meanwhile, the authors mention the importance of modeling actual combustion systems in understanding thermoacoustic instabilities.

Instead of using heating bands or a artificially created temperature gradient, Chatterjee et al [55] has been simulated the reacting flow in Rijke type combustor by flame. A multi-passage honeycomb was included in the gird to capture the coupling mechanisms between acoustics and the pulsating flame. Although the fundamental frequency at which the instability occurs and its harmonics have been captured accurately and matches good with the experimental data, the model failed to predict the subharmonic response and the mode shapes exhibited by the flame.

Distinguishing from the previous work based on the traditional straight Rijke tube, A newly designed Y-shaped Rijke tube was adopted by Zhong Z and Zhao D [56] to investigate the effect of acoustical inlet on combustion instability. Two acoustical inlet conditions are considered as: (i) the mother tube as inlet and two bifurcation braches as outlet (ii) the mother tube and short branch as inlet while the long brach as outlet. Although the first case did not observed in their laboratorial work, the computed frequency and mode shape agrees well with experimental results. Furthermore, A higher harmonics was observed in the second case, which reveals that the multiple input design of Y-shaped Rijke tube can trigger higher mode to go unstable even they are usually thought to be prevented by large damping.

Schmidt, H, Jimenez, C [57] performed numerical studies of the response of lean methane flames using different reaction mechanisms. The author found that the global flame response obtained with both detailed chemistry (GRI3.0) and a reduced multi-step model lies slightly above the predictions of the analytical model, but is close to experimental results. Furthermore, an irreversible one-step Arrhenius reaction model was adopted and show the effect of the pressure dependence of the global reaction rate in the flame response. Thus they concluded that the current models have to be extended to capture the amplitude and phase results of the detailed mechanisms, and second that the correlation between the heat release and the measured OH* chemiluminescence should be studied deeper.

Distinguishing from these CFD studies, Subramanian et al [58] performed a bifurcation analysis of the instability in a Rijke tube by using the numerical continuation method which can identify the bifurcation of a nonlinear system of equations with one or more parameters varying. They used software called DDE-BIFTOOL to obtain the bifurcation plot of variations of heat power, damping coefficient and heat location for the different time lags and the linear and nonlinear stability boundary are also obtained for two parameters varying simultaneously. Their stability boundary and bifurcation behavior have the similar trends with the experimental data.

Selimefendigil and Oztop [59] employed a soft computing methods for thermoacoustic simulation. First, a dynamic model is constructed from the input-output data sets (velocity forcing - heat release) generated from the Burke-Schumann flame using Comsol. An efficient and cheap model of heat source is obtained using dynamic fuzzy identification. The full thermoacoustic system is simulated in a time domain with the Galerkin method using the identified heat source model. Finally, dynamic neural networks are utilized for obtaining a dynamic fit for a set of operating conditions for the acoustic velocity at the heater location. The overall agreement between the outputs of the soft computing tools (fuzzy and neural network tools) with the Comsol and Galerkin solver is found to be satisfactory.

2.3 Experimental investigations

Friedlander [60] conducted experiments to investigate the phenomenon of acoustic coupling in a horizontally placed Rijke tube with movable closed ends. They found that the maximum and minimal inlet flow velocity in initiating resonant conditions. The measured oscillating pressure amplitude showed good agreement with the predicted results from the theoretical model.

Marone and Tarakanovskii [61] observed phenomena associated with the placement of an extra gauze above the heated gauze in a Rijke tube. For the multiply-gauze system, instabilities could be triggered when the gauzes were placed in either half of the tube, which is different from the single-gauze system that the oscillation could not be observed when the gauze was placed in the upper part of the tube.

Collyer and Ayres [62] placed two wire spiral heaters at various locations in Rijke tubes. They found a heater at $3L/4$ suppressed the fundamental mode due to a heater at $L/4$. The second mode and its harmonics were excited by heaters at $L/8$ and $5L/8$. The third mode and its harmonics were excited by heaters at any two of the positions: $L/12$, $5L/12$ and $3L/4$. All of these effects agree with Rayleigh's criterion.

Katto and Sajiki [30] performed an extensive series of experiments in examining the dependence of thermoacoustic instability upon heater power, heater position and mean flow rate. They found that the external cooling, tube orientation was irrelevant to instability and the instability was amplified as the heater size was increased or as the heater location was approaching to one quarter above the inlet. Furthermore, the authors observed the minimum heater power required to drive oscillation occurred at intermediate flow velocity.

Madarame [63] Developed an equation relating growth rate in the Rijke tube to a dimensionless mean flow velocity. For a fixed heater location, the predicted optimum velocity is lower than the results obtained in the experiment. In his analysis, the disagreement was caused by the neglect of boundary layer effects around the heater. Furthermore, the author showed the existence of regions in the Rijke tube where the

Rayleigh criterion is satisfied simultaneously for both the fundamental and second modes. Specifically, he showed experimentally that both modes could be triggered with proper heater power and mass flow rate when the heater was placed at the $L/8$ positions. This result is in agreement with the previous result.

Heckl [64] used active feedback control to suppress oscillations in a Rijke tube. The air flow was created by suction. Pressure oscillation were measured by a microphone placed in the inlet side of the tube. The pressure signal was phase shifted, amplified and fed to a loudspeaker near the exit of the tube. The system reduced noise level from energy balance and eigenvalues methods, Heckl showed how the external sound source modifies the acoustic energy balance.

However, there were little discussions how these parameters were varied and how the order of variation influenced the stability boundaries. Experimental errors have not been reported in these works; and temperature measurements have been rarely accomplished; the exceptions are the recent papers by Finlinson et al. [65] and McQuay et al. [66] on Rijke burners, where only a few system conditions can be tested. All mentioned factors, not sufficiently studied in the previous papers on Rijke tubes, strongly affect the actual transition to instability and have to be accurately assessed.

Chapter 3

2D Numerical Investigation

3.1 Thermodynamic analysis

3.1.1 Heat driven acoustic model

Fig. 3.1 Schematic of a convection-driven T-shaped standing-wave thermoacoustic system.

To shed light on thermoacoustic energy conversion, a general heat-driven acoustic wave equation is derived by conducting a generalized thermodynamic analysis [48]. A T-shaped thermoacoustic system is considered, as shown in Fig. 3.1. It consists of using multiple concentric constant-temperature rings as a heat source placed in the bottom stem of the tube. As the air is heated up, a buoyancy-driven convection flow with a low Mach number $\overline{M} \ll 1$ is generated. Thus the fluid considered is assumed to satisfy the perfect gas laws. The state equation of perfect gas [49] is given as

$$\frac{p}{\rho} = RT = \frac{R_u}{M_w} T = \frac{c^2}{\gamma} \quad (3.1)$$

where p is the pressure, q density, R is gas constant and T is the temperature, $R_u = 8.3143 \text{ kJ/kmol K}$ is the universal constant, $\gamma = 1.4$ is the ratio of specific heat, c is the sound speed, and M_w is the molecular weight. The pressure, velocity, density and temperature of the flow at a given location $\mathbf{x} = (x_1, x_2, x_3) = (x, y, z)$ and time consist of a mean part denoted by an overbar, and a fluctuating one denoted by a prime as given as

$$p(\mathbf{x}, t) = \bar{p}(\mathbf{x}) + p'(\mathbf{x}, t) \quad \mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{u}}(\mathbf{x}) + \mathbf{u}'(\mathbf{x}, t) \quad (3.2)$$

$$\rho(\mathbf{x}, t) = \bar{\rho}(\mathbf{x}) + \rho'(\mathbf{x}, t) \quad T(\mathbf{x}, t) = \bar{T}(\mathbf{x}) + T'(\mathbf{x}, t) \quad (3.3)$$

where velocity $\mathbf{u} = (\mathbf{x}, t) = (u_1, u_2, u_3)$.

If the internal energy of the fluid mixture by e , then the first law of thermodynamics considering energy conservation can be described by

$$\frac{de}{dt} + \frac{pd(\rho^{-1})}{dt} = T \frac{ds}{dt} \quad (3.4)$$

Now consider a control volume enclosing the heating rings, the density q varies through changes in both the pressure p and specific entropy s . Thus

$$\frac{d\rho}{dt} = \left. \frac{\partial \rho}{\partial p} \right|_s \frac{dp}{dt} + \left. \frac{\partial \rho}{\partial s} \right|_p \frac{ds}{dt} = \frac{1}{c^2} \frac{dp}{dt} + \left. \frac{\partial \rho}{\partial s} \right|_p \frac{ds}{dt} \quad (3.5)$$

For a perfect gas $\rho = p/RT = \mathcal{P}/c^2$, $\partial \rho / \partial s|_p = -pT(\gamma - 1)/c^2 = -\rho/c^2$ and so Eq. (5) can be rewritten as

$$\frac{d\rho}{dt} = \frac{1}{c^2} \frac{dp}{dt} - \frac{\rho(\gamma - 1)}{c^2} T \frac{ds}{dt} = \frac{1}{c^2} \frac{dp}{dt} - \frac{(\gamma - 1)}{c^2} Q \quad (3.6)$$

where $Q = \rho T ds$ is the heat addition per unit volume. And it consist of a mean and fluctuating part as

$$Q = \bar{Q} + Q' \quad (3.7)$$

The mass and momentum equations over the control volume, when linearized with respect to the perturbation quantities, can be written as

$$\frac{\partial \rho'}{\partial t} + \bar{\rho} \frac{\partial u_i'}{\partial x_i} = 0 \quad (3.8)$$

$$\bar{\rho} \frac{\partial u_i'}{\partial t} + \frac{\partial}{\partial x_i} (p' - S) = 0 \quad (3.9)$$

where S denotes the ‘friction’/damping (e.g. viscous stress) effect. By substituting linearized Eq. (3.6) into Eq. (3.8), multiplying Eq. (3.8) with $\bar{c}^2 \rho'$, Eq. (3.9) with u_i' and addition, an energy conservation equation is obtained namely,

$$\frac{\partial}{\partial t} \left[\frac{1}{2} + \frac{1}{2} \frac{\bar{c}^2 \rho'^2}{\bar{\rho}} \right] + \frac{\partial}{\partial x_i} (p' \delta_{ij} u_i') - u_i' \frac{\partial S}{\partial x_i} = \frac{(\gamma - 1)}{\gamma \bar{p}} p' Q' \quad (3.10)$$

Integrating Eq. (3.10) over the entire system leads to

$$\frac{\partial}{\partial t} \iiint \left[\frac{1}{2} \bar{\rho} u_i'^2 + \frac{1}{2} \frac{\bar{c}^2 \rho'^2}{\bar{\rho}} \right] dx_i = \iiint \frac{(\gamma - 1)}{\gamma \bar{p}} p' Q' dx_i - E_m - E_{tv} \quad (3.11)$$

Where

$$E_m = \iiint \frac{\partial}{\partial x_i} (p' \delta_{ij} u_i') dx_i \quad E_{tv} = \iiint -u_i' \frac{\partial S}{\partial x_i} dx_i \quad (3.12)$$

It can be seen from Eq. (3.11) that the term on the left hand side describes the classical acoustic disturbance energy E_a as defined as

$$E_a = \iiint \left[\frac{1}{2} \frac{\bar{c} \rho'^2}{\gamma \bar{\rho}} + \frac{1}{2} \bar{\rho} u_i'^2 \right] dx_i \quad (3.13)$$

The first term on right hand side of Eq. (3.13) describe the potential energy E_p .

The other term describes the kinetic energy i.e.

$$E_k = \bar{\rho} u_i'^2 / 2. \quad (3.14)$$

The first term on the right-hand-side of Eq. (3.13) as defined as E_p

$$E_e = \iiint \frac{(\gamma - 1)}{\gamma \bar{\rho}} p' Q' dx_i \quad (3.15)$$

describes the energy exchange between sound and head input. The second term E_m describes the mechanical work done by the fluid supporting an externally induced normal stress p' and moving with velocity u'_i . The last term E_{tv} is associated with thermal and viscous dissipation. Substituting Eqs. (3.13) and (3.15) into Eq. (3.11) and integrating it over one period leads to

$$\int_t^{t+T} E_e dt = E_a \Big|_t^{t+T} + \int_t^{t+T} [E_m + E_{tv}] dt \quad (3.16)$$

Eq. (3.16) reveals that the input thermal energy is partially converted into acoustical energy, partially dissipated by thermal and viscous effect, and partially used to produce mechanical work. When there is no heat addition, $Q = 0$ (isentropic), and the working fluid is quiescent (stagnant and uniform) and frictionless, Eq. (3.11) can be simplified into the classical acoustic energy balance equation:

$$\frac{\partial}{\partial t} E_a = -E_m = -\iint_A \mathbf{I} \cdot \mathbf{n} dA \quad (3.17)$$

the acoustic intensity is given as

$$\mathbf{I} = \rho' \mathbf{u}' \quad (3.18)$$

A is the surface enclosing the control volume with outer normal $\mathbf{n}$. Here the Gauss theorem is used to transform $\int_V (\nabla \cdot \mathbf{I}) dV$ into the surface integral $\int_V (\mathbf{I} \cdot \mathbf{n}) dA$.

And it can be used to calculate the acoustic power generated by heat addition.

To produce thermoacoustic oscillations (heat-driven sound), one of the necessary condition as suggested by Eq. (3.16) is

$$E_e > 0, \quad \text{equivalently} \quad \frac{-\pi}{2} < \angle p' Q' < \frac{\pi}{2} \quad (3.19)$$

that is to say p' and Q' are in phase. If $\angle |p' Q'| > 90^\circ$, i.e. p' and Q' are out of phase, then the energy exchange rate $E_e < 0$. And no sound will be produced, even with large amount of heat being added. This is known as ‘Rayleigh criterion’.

The other critical condition is that

$$\int_t^{t+T} E_e dt > \int_t^{t+T} [E_m + E_{iv}] dt \quad (3.20)$$

That is to say the heat exchange over one period must be greater than the sum of that dissipated due to thermal and viscous effects and that used to generate mechanical work. To optimize heat-to-sound conversion, it is necessary to maximize the difference between the terms on the left- and right-hand-side in Eq. (3.20). This can be achieved by maximizing E_e and minimizing E_{iv} .

By neglecting the damping term, substituting Eq. (3.6) into Eq.(3.8) differentiating Eq. (3.8) with respect to time t , taking divergence of Eq. (3.9), an inhomogeneous heat-driven wave equation is obtained namely,

$$\frac{1}{\bar{c}^2} \frac{\partial^2 p'}{\partial t^2} - \bar{\rho} \nabla \cdot \left(\frac{1}{\bar{\rho}} \bar{\rho}' \right) = \frac{\gamma - 1}{\bar{c}^2} \dot{Q}(t) \quad (3.21)$$

where overdot denotes the time derivative. Eq. (3.21) describes a general heat-driven acoustic wave equation. $\dot{Q}(t)$ describes the time rate of heat addition from a heat source. Typically, a flame or an electrical heater is used. The instantaneous heat release rate Q from such heat sources [25, 50, and 51] can be estimated as

$$Q(t) = \begin{cases} \rho_m S_u A_f \Delta q_r & \text{flame} \\ \lambda \partial T / \partial x A_s & \text{heater} \end{cases} \quad (3.22)$$

where ρ_m is the fuel–air mixture density, S_u is the flame speed, A_f is the flame surface area, and Δq_r is the heat of reaction per unit mass of the mixture, λ is the thermal conductivity, $\partial T / \partial x$ is the spatial temperature gradient between the heater and surrounding air, A_s is the surface area of the heater.

3.1.2 T-shaped thermoacoustic model

The T-shaped numerical configuration is generated with structural mesh in Pointwise [52], as shown in Fig. 3.2. Multiple con-centric rings with constant surface temperature are equally spaced along y-axis. They have a length of $L_h = 50mm$ and placed $150mm$ above the inlet cross section. Near the heat sources, finer gird (Hexahedra) is used. The geometric dimensions are $L_l = L_r = 500mm$, $L = 300mm$, $A_l = A_r = \pi R^2 = 1.6 \times 10^{-3} m^2$; $A = 2.0 \times 10^{-3} m^2$.

Fig. 3.2 Gird of a convection-driven T-shaped standing-wave thermoacoustic system.

Initially, a gird independence test is preformed to make sure the numerical result is independent of the mesh resolution. We start with a coarse gird which contains $4.5e4$ elements, then computations on consecutively refined gird ($7e4$, $11e4$) were carried. All of the scaled residual have reached a acceptable level after 2500 iterations, which demonstrating that the results presented in the paper are sufficiently accurate on the second gird level.

Fig. 3.3 Grid-independence study using three consecutively refined grid levels (a)
 4.5×10^4 (b) 7×10^4 (c) 11×10^4

Before starting the unsteady simulations to capture the thermoacoustic oscillations generated in the T-shaped system, a steady solution is first obtained in terms of the temperature and velocity contours, as shown in Fig. 3.3. The boundary conditions are set according to our experimental setup as shown later as: the bottom stem (mother tube) open end is pressure inlet with a gauge pressure 0.01 Pa, while the bifurcating branches open ends are pressure outlets $p_o' = 0 \text{ Pa}$. The heating band surface temperature is set to 1100 K, which is closed to the heat source temperature used in our experiment described later. The heat addition causes a convection flow moving towards the outlets.

Fig. 3.4 Temperature and velocity contours of the T-shaped thermoacoustic system.

To achieve the unsteady boundary condition, a second-order implicit formulation in time integration is performed based on the steady solution. A time step of $1 \times 10^{-4} \text{ s}$ was chosen to capture the time-varying pressure fluctuations. An initial disturbance is initially imposed to the pressure outlet of the right side branch for 10 time steps to

trigger the combustion instability.

The unsteady conservation equations of mass, momentum and energy with equation of state for ideal gas are solved iteratively for a laminar 2D flow as given as

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \quad (3.23a)$$

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_i} (-p \delta_{ij} + \tau_{ij}) \quad (3.23b)$$

$$\frac{\rho c_p}{\gamma} \left(\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} \right) + p \frac{\partial u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\lambda \frac{\partial T}{\partial x_j} \right) + \tau_{ij} \frac{\partial u_i}{\partial x_j} + Q \quad (3.23c)$$

where u_i the velocity component of $\mathbf{u}(\mathbf{x}, t)$ in i th direction, and τ_{ij} is the viscous stress tensor as given as

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2\mu}{3} \frac{\partial u_k}{\partial x_k} \delta_{ji} \quad (3.24)$$

δ_{ij} is the Kronecker delta function. When $i = j$, $\delta_{ij} = 1$. Otherwise $\delta_{ij} = 0$. μ is the dynamic viscosity, and λ is the coefficient of heat addition to the fluid particle. Q is the instantaneous heat production per unit volume from the constant-temperature heating bands. The heat transfer rate via conduction over the heating band can be shown as.

$$\dot{Q} = \lambda A_s \frac{\partial T_s}{\partial x_i} = \sum_{i=1}^N \lambda A_{s,i} \frac{\partial T_s}{\partial x_i} \quad (3.25)$$

where A_s is the total surface area of the heating bands. $A_{s,i}$ denotes the surface area of i th heating cell. As the surrounding fluid flows, convection heat transfer is expected. And it is balanced by the conduction heat transfer as given as

$$\dot{Q} = h A_s (T_f - T_s) = h A_s \Delta T_f = q A_s \quad (3.26)$$

where q is the heat flux. It can be shown that the convection heat transfer

coefficient h can be determined by using

$$h = \lambda \frac{\partial T_s / \partial x_i}{\Delta T_f} = F(Pr, Re) \quad (3.27)$$

It is known that the convection coefficient is also a function of Prandtl and Reynolds number, as shown in Eq. (3.27). For given characteristic thermal length scales L_f in the fluid and L_s in the heating band [53], nondimensional Prandtl Pr , Reynolds number Re , and Nusselt Nu can be shown as

$$Pr = \frac{c_p \mu}{\lambda} \quad (3.25)$$

$$Nu = \frac{h L_f}{\lambda} = \frac{\frac{\partial T_s}{\partial x_i} L_f}{\Delta T_f} \quad (3.26)$$

$$Re = \frac{\rho \bar{u}_i D}{\mu} \quad (3.27)$$

where λ is the thermal conductivity. In our work, Pr is approximately 0.86. Reynolds number is $1800 < 2300$ at the inlet of the bottom branch. Thus the inlet flow is a laminar one.

To solve these conservation equations, 2nd-order schemes are chosen for pressure interpolation and spatial discretization, while the SIMPLE algorithm is used for velocity–pressure coupling. To take the influence of temperature on the physical properties of air into account, the specific heat is given as,

$$c_p(T) = (\partial H / \partial T)_p \quad (3.28)$$

the viscosity is given as

$$\mu(T) = \mu_{ref} \frac{T_{ref} + c}{T + c} (T / T_{ref})^{3/2} \quad (3.31)$$

where c is Sutherland's constant and the thermal conductivity is given as

$$\lambda(T) = c_p(T)\mu(T)/\text{Pr} \quad (3.30)$$

where are modeled as temperature-dependent [54].

The following values at 300 K are applied: $c_p = 1.006 \text{ kJ/KgK}$,
 $\mu = 1.724 \times 10^{-5} \text{ kg/ms}$ and $\lambda = 0.0242 \text{ W/mK}$

3.2 Numerical result

To gain insights on the interaction between unsteady heat release and acoustics, parametric studies are conducted. This includes examining the effects of the inlet flow velocity, heat source location, and heater surface temperature on triggering thermoacoustic oscillations. These parameters are chosen, based on the facts: (1) varying the inlet velocity results in the changes of the convection coefficient $h = F(\text{Pr}, \text{Re})$ and so the heat flux q , (2) the heat source location play an important role in determining the coupling between p' and q' (i.e. the phase $\angle p'q'$) and so E_e' (3) the heater surface temperature is critical in determining the heat flux $q \propto \Delta T = T_s - T_f$. Each of the parameters is varied, while all the other parameters remain constant.

3.2.1 Inlet flow velocity effect

The effect of the inlet Flow Velocity at the bottom open end on the resulting thermoacoustic oscillations is first investigated. The inlet flow velocity can be varied by changing the bottom open end inlet pressure. Fig. 3.5 and Fig. 3.6 shows the variation of the pressure oscillation and heat fluxes, as the inlet velocity is set to 5 different values. It can be seen that when the inlet velocity is $v_i \leq 0.04 \text{ m/s}$, the initial disturbances decay. However, as the inlet velocity is increased, limit cycles occur. And the larger inlet velocity is, the larger the resulting pressure and heat flux oscillation amplitude is.

Fig. 3.5 Variation of pressure fluctuations with time (a) $v_i = 1.01m/s$ (b) $v_i = 0.81m/s$ (c) $v_i = 0.45m/s$ (d) $v_i = 0.21m/s$ (e) $v_i = 0.04m/s$, and $x_f/L = 0.5$,

$$T_s = 1100K$$

Fig. 3.6 Variation of heat flux fluctuations with time (a) $v_i = 1.01m/s$ (b) $v_i = 0.81m/s$ (c) $v_i = 0.45m/s$ (d) $v_i = 0.21m/s$ (e) $v_i = 0.04m/s$, and $x_f/L = 0.5$,

$$T_s = 1100K$$

In order to gain insight on the heat-to-sound energy conversion, the total acoustical energy can be estimated by using Eq. (3.12). Fig. 3.7(a) shows the variation of the acoustic energy with time, as the inlet velocity is set to 0.81 m/s. It can be seen that the acoustical power is increased with time by extracting more thermal energy. When a limit cycle occurs at $t \geq 1.0s$, the acoustic power keeps almost constant. This is due to the balanced periodic energy exchange between the heat and sound.

Fig. 3.7 Variation of acoustic power (a), heat input (b) instantaneous velocity and pressure fluctuation (c) and the overall energy efficiency (d)

If we define the energy conversion efficiency ψ of converting input heat power Q_t into output acoustical power E_a as

$$\psi = \frac{E_a}{Q_t} = \frac{\iint_A p'(\mathbf{v} \cdot \mathbf{n}) dA}{\sum_{i=1}^3 2\pi R_i H_s q'} \quad (3.26)$$

then the fraction of heat input being converted into acoustical energy is shown in Fig. 3.7(d), as denoted by the solid line. Here p' denotes the pressure fluctuation, A is the cross-sectional area of the tube, h_s is the heating band height, $\mathbf{v}$ is the velocity fluctuation, R_i denotes the radius of the i th heating ring, and q' is the heat flux fluctuation. It can be seen that about 0.79% of total heat addition is converted

into acoustical energy, which is much efficient than 0.28% when the inlet velocity is set to 0.21 m/s, as denoted by the dot line. Thus the increased pressure fluctuation amplitude with the inlet velocity, as shown in Fig. 3.7(a) indicates the intensified energy conversion between the heat and sound.

Flow reversal [25] would occur when the velocity perturbations exceed the mean. This is accompanied by large fluctuations in heat release rate. Fig. 3.8 describes one period of velocity perturbation near the heater. After reaches its maximum, velocity in y direction will gradually decay to zero, as shown in Fig. 3.8(b). At the time during a cycle when the minimum velocity in the tube center is reached. It is seen that vortex shedding is formed between two neighboring heating bands. Finally, the fluid will reverse its direction, thus causing the previously-heated gas flow back to the heating bands, and receives more energy.

The variation of the instantaneous velocity between two neighboring heating bands is shown in Fig. 3.7(c). The dash line denotes the mean flow speed. It can be seen that the fluctuating part is larger than the mean part. Furthermore, the presence of the spurs in the velocity indicates the flow reversal and nonlinearity of the thermoacoustic system. Similar nonlinearity behavior is observed in the thermal power added to the system, as shown in Fig. 3.7(b). However, this is different from the pressure fluctuations, as denoted by the dash line.

Fig. 3.8 velocity contour near the heater surface over one period of the limit cycle

As the inlet flow velocity is changed, Nusselt number is found to vary as shown in Fig. 3.9. It can be seen that with inlet flow velocity increased, the mean Nu number is increased. This reveals the dominant role of convection over the conduction. Furthermore, compared with the steady case (corresponding to no thermoacoustic oscillations present) as denoted by the dash line, the presence of thermoacoustic oscillations results in significant increase of heat transfer, which has the potential for energy efficient drying.

Fig. 3.9 Variation of Nusselt number with normalized time t/T_0 ($T_0 = 2\pi/\omega_0$)

3.2.2 Heater location effect

Fig. 3.10 Variation of pressure fluctuations with time, heater is set to 4 different location (a) $x_f/L = 4/6$ (b) $x_f/L = 3/6$ (c) $x_f/L = 2/6$ (d) $x_f/L = 1/6$, and

$$T_s = 1100K, \quad p_o = 0pa, \quad p_i = 0.3pa$$

Fig. 3.11 Variation of heat flux fluctuations with time, heater is set to 4 different location (a) $x_f/L = 4/6$ (b) $x_f/L = 3/6$ (c) $x_f/L = 2/6$ (d) $x_f/L = 1/6$, and

$$T_s = 1100K, \quad p_o = 0pa, \quad p_i = 0.3pa$$

The effect of the heater location on the resulting thermoacoustic oscillations is then investigated. Fig. 3.10 and Fig. 3.11 shows the variation of the pressure and heat flux fluctuations, as the heater is placed at 4 different locations. It can be seen that when $x_f/L = 1/6$, initial flow disturbances decay. However, as x_f/L is increased, the amplitudes of the pressure oscillation and heat flux are increased, and limit cycles eventually occur. This is due to the intensified energy conversion from heat to sound.

Fig. 3.12 Compared heater location contours in T-shaped Rijke tube

As shown in Fig. 3.12, When the $x_f/L > 1/6$, the temperature gradient will distribute become chaos in distributing inside all of the tube. It is also a symptom to indicated that the different between combustion stability and instability in temperature performance.

In addition, the growing of the pressure oscillations at $0 \leq t < 1.0$ as shown in Fig. 3.10 can be approximated by using the real part of a complex exponential function.

$$G(\omega, \sigma) = R\{A_0 e^{i\omega x + \sigma t}\} \quad (3.27)$$

where A_0 denotes the initial amplitude at $t = 0$, x denotes the oscillation mode frequency and r describes the mode growth rate. The thick dash line in Fig. 3.7(a) describes $R\{26.67e^{j2\pi 216t + 4.287t}\}$. Comparing it with the amplification of the pressure oscillations as $x_f/L = 4/6$ reveals that the exponential function can approximate the growing process very well when $t < 0.55s$. However, if the growth rate α keeps constant, the approximation at $t > 0.55s$ does not agree with the pressure fluctuation growing. This indicates that the growth rate α is varied during the amplification process. It is worth noting that the heat flux amplification cannot be simply approximated by using an exponential function due to its inherent nonlinearity

as discuss later.

Fig. 3.13 Variation of the mode growth rate σ as heater is placed at 4 locations (a) Rayleigh index between $q(t)/\bar{q}$ and $p(x_f, t)/\bar{p}$, as $x_f/L = 3/6$ (b).

The time-varying growth rate of the dominant mode of the pressure oscillations is shown in Fig. 3.13(a). It can be seen that the growth rate is $\sigma = \tan \alpha < 0$ and the pressure fluctuation decays at $x_f/L = 1/6$, since σ is negative (anti-clockwise). However, when the heater is moved to $x_f/L \geq 2/6$, the mode growth rate $\sigma = \tan \alpha > 0$ and the pressure oscillations are amplified. Furthermore,

$\sigma(x_f/L=2/6) \leq \sigma(x_f/L=3/6) \leq \sigma(x_f/L=4/6)$. The larger σ mean that initial triggering disturbances will grow into a limit cycle more quickly. Furthermore, $\sigma \rightarrow 0$ with $t \rightarrow \infty$, and the mode growth rate is saturated.

Rayleigh index R_A is generally used as a critical indicator of the coupling between heat and sound [25, 27 and 51]. It describes the energy exchange intensity between unsteady heat release and the pressure oscillations. If $R_A > 0$, the acoustical energy in the present system is increased and tends to drive the unstable process into a limit cycle. However, a negative R_A indicates a ‘destruction’ interaction between unsteady heat release and pressure oscillations and so the thermoacoustic oscillations decay with time. Thus it would be interesting to check the Rayleigh index R_A . Following the work [46], we defined a normalized Rayleigh index R_A as given as

$$R_A(t) = \iint \sigma(\mathbf{x} - \mathbf{x}_f) d\mathbf{x} \int_0^t \frac{p'(\omega, \psi) q'(\omega, \psi) d\psi}{\sqrt{\int_0^{2\pi\psi} p'^2(\omega, \psi) d\psi} \sqrt{\int_0^{2\pi\psi} q'^2(\omega, \psi) d\psi}} \quad (3.28)$$

Fig. 3.13(b) shows the variation of the Rayleigh index $R_A(t)$ with time, as $x_f/L=3/6$. It can be seen that when initial perturbation grows into a limit cycle, Rayleigh index R_A is positive, keeps increasing and finally ‘saturated’ at approximately 0.90. This verifies our numerical model and explains why self-sustained thermoacoustic oscillations occur in the T-shaped system.

Fig. 3.14 Spectra of the heat flux (a) and the pressure fluctuation (b), as

$$T_s = 1100K, \quad p_i = 0.2pa \quad \text{and} \quad x_f/L = 3/6.$$

Fig. 3.14 shows the time–frequency analysis of the heat flux $q'(t)$ and the pressure oscillations $p'(t)$. It can be seen that the spectra of the heat flux consists of one dominant frequency at approximately 220 Hz and two harmonics with large amplitude. However, the harmonics are negligible in the pressure spectra. This confirms that acoustic waves can be approximated by using linearization analysis. However, nonlinearity is mainly arising from the unsteady heat release.

3.2.3 Heat flux effect

As shown in Eq. (3.13), the heat flux $q(t) \propto (T_s - T_f)$ plays an important role in generating thermoacoustic oscillations. To investigate its effect, the heater temperature T_s is set to 5 different values and so the mean heat addition $\bar{q}$. Fig. 3.11 illustrates the variations of the heat flux $q'(t)$ and pressure fluctuations $p'(t)$, as the heating bands are set to different temperatures. It can be seen that the mean heat flux is increased with increased heater temperature. Furthermore, the instantaneous heat flux $q(t)$ and pressure fluctuations $p(t)$ decay, as the heater temperature is lower than $T_s \leq 700\text{K}$. However, as the temperature is above 800K , as show in Fig. 3.15(b)–(d), the nonlinear thermoacoustic oscillations as indicated by the circle-shaped diagrams are generated. And the amplitudes of the heat flux and the pressure fluctuations are increased with the heater temperature. Moreover, the main nonlinearity is identified in the rate of heat release, as revealed by comparing the phase diagrams of the heat fluxes (pomelo-like) and pressure fluctuation (circle-shaped).

Fig. 3.15 Phase-space plots for the pressure oscillations and heat fluxes (a) phase diagram between $\dot{q}(t)$ and $p'(t)$, $\diamond$ denotes $T_s = 700K$, and $\circ$ denotes $T_s = 600K$, (b) phase diagram between $\dot{q}(t)$ and $p'(t)$ $T_s = 800K$ (c) phase diagram between $\dot{p}(t)$ and $p'(t)$ and $-\circ-$ denotes $T_s = 900K$ and $-\times-$ denotes $T_s = 1000K$

In addition, the growth rate of the dominant thermoacoustic mode is calculated and shown in Table 3-1. The negative mode the heater temperature is set to different values. This might be due to the variation of the sound speed $c \propto \sqrt{T}$.

Heater temperature (K)	600	700	800	900	1000
Mode growth rate	$-17.67 \rightarrow 0$	$-9.00 \rightarrow 0$	$0.73 \rightarrow 0$	$1.33 \rightarrow 0$	$1.86 \rightarrow 0$
Mode frequency (Hz)	NA	NA	214.4	216.38	218.82

Table 3-1. Measured growth rate and frequency of the dominant thermoacoustic mode, as $x_f/L = 3/6$ and $p_i = 0.1\text{pa}$.

Chapter 4

Experiment Investigation

4.1 Experimental Setup

In order to validate our numerical findings, a T-shaped tube with a premixed laminar flame confined is designed and tested. It is made of quartz glass. And it has a mother tube (bottom stem), which splits into two daughter tubes (i.e. horizontal branches) with equal lengths, see Fig. 4.1. T-shaped junctions protruding from both bifurcating branches are designed for placing microphones. To measure pressure fluctuations along the setup, two arrays of B&K microphones (Type 4794) are implemented: each array is involved with 3 microphones. The geometry, dimensions of the setup and the flow parameters are shown in Table 4-1.

Geometry	Value
Left branch length, L_l (m)	0.5
Right branch length, L_r (m)	0.3
Bottom branch length, L (m)	0.3
Cross-sectional area of left branch, A_l (m ²)	0.0016
Cross-sectional area of right branch, A_r (m ²)	0.0016
Cross-sectional area of bottom branch, A (m ²)	0.002
Inlet temperature, T_u (K)	298
Inlet and outlet mean pressure, $\bar{p}_u = \bar{p}_d$ (pa)	1.01325e5
Fuel,	Propane
Lower Heating value, $\bar{Q}$ (MJ/Kg)	46.35

Table 4-1 Geometry and flow parameters of the T-shaped thermoacoustic system

Fig. 4.1 Experimental setup of T-shaped thermoacoustic system with a fine metal gauze attached on top of a Bunsen burner.

4.2 Experimental result

4.2.1 Parametric measurement

Experimental measurements are conducted in two sets, corresponding to our numerical simulations. One set of measurements are performed by placing the flame at 4 different locations, as shown in Fig. 4.2(a) in time domain. Fig. 4.2(b)–(e) shows the frequency spectra of measured and predicted pressure fluctuations. It can be seen from Fig. 4.2(a) that thermoacoustic oscillations could be generated, depending on the flame location. Correspondingly, the mean flow velocity resulting from natural convection at the inlet of the bottom stem is increased from 0.04 m/s, 0.10 m/s, 0.17 m/s to 0.25 m/s as measured by using a thermal anemometer TA5, as the flame location is varied from 1/6 to 4/6. When a limit cycle is produced, the maximum pressure oscillation can reach about 136 dB, as shown in Fig. 4.2(b). The dominant

mode is at around 216 Hz and it has multiple harmonics (indicating nonlinearity) as shown in the graphs (c)–(e). Comparison of the experimental measurements and the numerical predictions shows good qualitative agreement in terms of the sound pressure level (SPL) and the mode frequencies.

Fig. 4.2 Measured pressure oscillations, as the fuel flow rate is set to 230 m/s , and the heat source is placed at 4 different locations (a) time-domain measurement, (b)-(e) frequency spectra as heater is placed at 4 locations

Fig. 4.3 Temperature contour measurements by using an infrared camera, as the heat source is placed at 4 different locations

The temperature along the T-shaped system is measured by using an infrared camera [24,26]. It captures the quartz tube surface temperature spectrum. However, it can represent the ‘hot’ status of the sound waves inside each branch. The camera is set to operating range of 0-500° Celsius (corresponding to 273–773 K). Fig. 4.3 shows the measured temperature contour. It can be seen that there is a local maximum temperature at the junction section. And the temperature is decreased along the axis of the horizontal branch. The measured temperature contour (mushroom-shaped) is similar as our numerical prediction as shown in the top graph of Fig. 3.2.

To gain insight on the thermoacoustic oscillations, the mode shapes along the horizontal branches are measured, as shown in Fig. 4.4(a). It can be seen that pressure nodes are located at each ends. And the maximum pressure fluctuation occurs at approximately 0.4 m away from the open ends. Comparing the measured mode shape with the numerical predicted one, good agreement is obtained. Fig. 4.4(b) shows the mode shape along the bottom stem and the right branch.

Fig. 4.4 Comparison of the measure and predicted mode shapes.

The total length is corresponding to half of wave length of the thermoacoustic oscillations at approximately 220 Hz. It is worth noting that the mode shapes are measured by using the classical two-microphone technique [54]. The sound speed is obtained by using the mean value of the local temperature measurements. And the mean flow speed is neglected.

The other set of experiments is to investigate the effect of heat flux. This is achieved by varying the fuel flow rate. Fig. 4.5 shows the measured pressure oscillations, as the fuel flow rate is set to 4 different values. It can be seen that when the fuel flow rate is lower than 150 cc/min, no limit cycle is generated. This is most likely due to the fact that the thermal energy input is not larger than that dissipated due to viscous and thermal effects. The temperature contour measurement as shown in Fig. 4.6 is similar to those as shown in Fig. 4.3. However, the local maximum temperature at the junction section is decreased from 711.75 K to 685.55 K, as the fuel flow rate is increased. This is most likely due to the intensified heat-to-sound conversion (see Fig. 4.7).

Fig. 4.5 Measured pressure oscillations, as the fuel flow rate is set to 4 different values: (a) 150 cc/min, (b) 200 cc/min, (c) 230 cc/min and (d) 260 cc/min.

Fig. 4.6 Temperature contour measurement by using an infrared thermal imaging camera, as the fuel flow rate is set to 4 different values.

4.2.2 Supercritical hopf bifurcation

A bifurcation diagram in terms of the pressure fluctuation amplitude can be obtained by varying the heater location gradually while all the other parameters remain constant. The nonlinear behavior around the Hopf bifurcating point x_{fc} is characterized by a gradually increase of the amplitude as $x_f > x_{fc}$. For $x_f < x_{fc}$, all perturbations imposed on the thermoacoustic system tend to decay to zero.

In general, the measured results are in good agreement with the numerical predictions, in terms of the mode shapes, mode frequencies and the supercritical Hopf bifurcation diagram. This reveal that the numerical model developed can be used to study the onset of thermoacoustic oscillations and facilitate the design of convection-driven standing-wave heat engine systems, which might be utilized for energy harvesting or drying.

Fig. 4.7 Hopf bifurcation diagram of measured and numerical predicted thermoacoustic oscillations, as the fuel flow rate is set to 300 cc/min.

Chapter 5

3D Numerical Study of Premixed Flame-sustained Thermoacoustic instability

In this chapter, 3D numerical investigation is conducted to study nonlinear thermoacoustic instability of a premixed flame in a T-shaped combustor. The geometry of the three-dimensional T-shaped thermoacoustic system is shown in Fig. 5.1. The burner consists of a mother tube joint with two daughter tubes. A fuel injector is placed 100mm away from the inlet cross-section. The geometric dimensions are $L_l = L_r = 500mm$, $L = 300mm$, $A_l = A_r = \pi r^2 = 1.6 \times 10^{-3} m^2$, $A = 2.0 \times 10^{-3} m^2$, $A_f = 9.80 \times 10^{-6} m^2$.

Taking advantage of the geometric symmetry, only one half of the burner is modeled. This also significantly reduces the computational cost. To increase the computational accuracy, high quality structural grids are generated with ICEM CFD 15.0 , finer grid is used near the fuel injector as shown in Fig. 5.2.

Fig. 5.1 Schematic of a convection-driven T-shaped standing-wave thermoacoustic system with a premixed burner confined.

Fig 5.2. Mesh grid of a convection-driven T-shaped standing-wave thermoacoustic system with a fuel injector confined.

Before conducting unsteady simulations to study the onset of thermoacoustic oscillations generated in T-shaped system, a steady solution is first obtained in terms of the temperature, velocity, heat of reaction and propane mass fraction contours, as shown in Fig. 5.3 to Fig. 5.6. The boundary conditions are set according to our experimental setup as shown later as: the air inlet open end is pressure inlet with a gauge pressure 0 Pa,

while the bifurcating branches ends are assumed as pressure outlet with a gauge pressure 0 Pa. At the fuel nozzle a constant mass flow of 2.6190×10^{-5} kg/s of mixture gas containing 50% of propane enters the computational domain. With a lower heating value 46.35MJ/kg for propane, the overall heat release rate is 300W. The heat addition causes a free convection flow moving towards the outlets. Combustion is assumed to be a single-step reaction according to the following molar stoichiometric reaction equation for gaseous propane as fuel:

Fig. 5.3. Temperature contour of the T-shaped thermoacoustic system with a premixed turbulent flame confined.

Fig. 5.4. Velocity contour of the T-shaped thermoacoustic system with a premixed turbulent flame confined.

Fig. 5.5. Heat of reaction contour of the T-shaped thermoacoustic system with a premixed turbulent flame confined.

Fig. 5.6. Propane mass fraction contour of the T-shaped thermoacoustic system with a premixed turbulent flame confined.

The conservation equation of mass, momentum, species of reactive and energy are solved by using Fluent 15.0 coupled solver with the equation of state for ideal gas.

Conservation of mass:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \quad (5.1)$$

Conservation of momentum:

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_i} + g \quad (5.2)$$

Where.

$$\tau_{ji} = \eta \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \eta \frac{\partial u_k}{\partial x_k} \delta_{ji} \quad (5.3)$$

Conservation of species:

$$\frac{\partial \rho Y_n}{\partial t} + \frac{\partial (\rho u_i Y_n)}{\partial x_i} + \frac{\partial J_{n,i}}{\partial x_i} = \omega_n \quad (5.4)$$

Where ω_n is the mass rate of creation or depletion of species n by chemical reaction. Y_n denotes the mass fraction of n species. And $J_{n,i}$ refers to the diffusion mass flux of species n in the direction of coordinate x_i .

Conservation of energy:

$$\begin{aligned} & \frac{\partial(\rho h)}{\partial t} + \frac{\partial(\rho u_i h)}{\partial x_i} - \frac{1}{\rho} \left(\frac{\partial(\rho p)}{\partial t} + \frac{\partial(\rho u_i p)}{\partial x_i} \right) \\ &= \frac{\partial}{\partial x_i} \left(\lambda \frac{\partial T}{\partial x_i} \right) + \rho_i g_i u_i + \tau_{ji} \frac{\partial u_j}{\partial x_i} - \frac{\partial}{\partial x_i} \left(\sum_{n=1}^N (h_n J_{n,i}) \right) - \sum_{n=1}^N (\omega_n \Delta h_{f,n}^o) \end{aligned} \quad (5.5)$$

With the specific enthalpy h of the mixture, the source term $\omega_n \Delta h_{f,n}^o$ in Eq. (5.5) describes energy released by chemical reaction with the specific standard heat of formation $h_{f,n}^o$ for species n.

To solve these conservation equations, 2nd-order schemes are chosen for pressure interpolation and spatial discretization, while the Coupled algorithm is used for velocity-pressure coupling due to a strong coupling between these equations.

Fig. 5.7 shows the onset of the nonlinear limit cycle oscillations within the T-shaped burner, at $y=0.125\text{m}$. The amplitude of flow perturbations initially grows quickly. However, it will not increase any further at $t>1.8\text{ s}$. This constant-amplitude oscillations are known as combustion instability. The generated oscillation frequency is about 240 Hz; accordingly, it corresponds to the first harmonic eigenfrequency of the burner. Fig. 5.6 shows the corresponding sound pressure power spectrum. It is seen that when a limit cycle is produced, the maximum pressure oscillation can reach about 143 dB. Fig. 5.9 shows the variation of pressure profiles during one period of oscillation. It can be seen that there are two pressure nodes at the combustor ends. They are corresponding to the acoustically open boundary condition.

Fig. 5.7 Variation of pressure with time

Fig. 5.8 Pressure power spectrum

Fig. 5.9 Pressure profiles during one period.

Fig. 5.10 Variation of acoustic power (a), heat input (b), instantaneous velocity and pressure fluctuation (c) and the overall energy efficiency (d).

In order to gain insight on the heat-to-sound energy conversion, the total acoustical energy can be estimated by using Eq. (5.6). Fig 5.10(a) shows the variation of the acoustic energy with time. It can be seen that the acoustical power is increased with time by extracting more thermal energy. When a limit cycle occurs at $t \geq 1.5s$, the acoustic power keeps almost constant. This is due to the balanced periodic energy exchange between the heat and sound.

If we define the energy conversion efficiency ψ of converting input heat power Q_t into output acoustical power E_a as

$$\psi = \frac{E_a}{Q_t} = \frac{\iint_A p'(\mathbf{v} \cdot \mathbf{n}) dA}{Q_t} \quad (5.6)$$

then the fraction of heat input being converted into acoustical energy is shown in Fig. 5.10(d), as denoted by the solid line. Here p' denotes the pressure fluctuation, A is the cross-sectional area of the tube, h_s is the heating band height, $\mathbf{v}$ is the velocity fluctuation, and q' is the heat flux fluctuation. It can be seen that about 1.2% of total heat addition is converted into acoustical energy.

The variation of the instantaneous velocity near the flame area is shown in Fig. 5.10(c). The dash line denotes the mean flow speed. It can be seen that the fluctuating part is larger than the mean part. Flow reversal would occur when the velocity perturbations exceed the mean. Fig. 5.11 (a)-(h) shows one period of flow reversal. This is accompanied by large fluctuations in heat release rate. As the flow rate fluctuations occur in the reaction zone, the flame will be supplied with fluctuating amounts of combustible mixture and therefore change the local equivalence ratios. As a result, it will lead to fluctuating conversion rates within the flame, and thus to fluctuating amounts of heat released by combustion per unit of time. Fig 5.12 (a)-(h) shows propane mass fraction contour over one period of the limit cycle. It can be seen the mass fraction of propane is fluctuating near the fuel nozzle, thus caused the bottom of flame is also changing with time, as shown in Fig 5.13(a)-(h). Since any pulsating release of thermal energy acts like a source of sound, feedback into the sound field will be established, thus creating the closed loop necessary for self-excitation.

Fig. 5.11 Velocity contour over one period of the limit cycle

Fig. 5.12 Propane mass fraction contour over one period of the limit cycle

Fig. 5.13 Temperature contour over one period of the limit cycle

Chapter 6

Conclusions and Future Work

In summary, a nonlinear T-shaped standing-wave thermoacoustic system is numerically and experimentally investigated to gain insights on the energy conversion process from heat to sound. The physical processes of the generation and propagation of sound waves by inputting thermal energy is mathematically illustrated by performing a generalized thermodynamic analysis. Three key parameters are identified and examined: (1) inlet flow velocity, (2) the heater location, and (3) heater surface temperature. Their effects on triggering limit cycle oscillations are first investigated in 2D numerical model by varying one parameter, while all the other parameters remain constant. The main nonlinearity is identified in the heat fluxes.

To characterize the transient (growing) behavior of the pressure fluctuation, the thermoacoustic mode growth rate is defined and calculated. It is found that the growth rate decreases first and then ‘saturates’. Similar behavior is observed in examining the slope of Rayleigh index, which is widely used as an onset/triggering indicator to characterize the phasing between unsteady heat release and sound. The transient behavior of heat flux cannot be simply described by a growth rate due to its nonlinearity. In addition, the overall efficiency of converting the input heat power into acoustic power is estimated and compared. It is found that the energy conversion efficiency can be increased by increasing the inlet flow velocity.

To validate our numerical findings, a cylindrical T-shaped duct made of quartz-glass with a metal gauze attaching on top of a Bunsen burner is designed and tested. To monitor the temperature and acoustic fields in the bifurcating branches, an infrared thermal imaging camera and two arrays of microphones are used. As a premixed laminar flame is placed in the bottom branch of the tube, self-sustained thermoacoustic oscillations might be generated, depending on the 3 identified critical parameters. It is shown that Hopf bifurcation is associated with the thermoacoustic system.

Finally, 3D numerical simulations are conducted to study the nonlinear limit cycle oscillations of a premixed turbulent flame. The simulations reveal that unsteady heat release from the flame is an efficient sound source. Flow reversal is successfully predicted to occur during part of the limit cycle oscillation and the flame then moves upstream of the flame holder.

In the future, laser-involved experiments need to be conducted to validate the numerical simulation results. Since the limit cycle thermoacoustic oscillations are detrimental to the engine systems, control approaches need to be developed to mitigate the oscillations. This involves designing and implementing either active or passive control approaches. This means that further study is needed.

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