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2005

Theingi. (2005). The study of joint-coupling in parallel manipulator design. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/6532>

<https://doi.org/10.32657/10356/6532>

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The Study of Joint-Coupling in Parallel Manipulator Design

Theingi

**SCHOOL OF MECHANICAL AND PRODUCTION ENGINEERING
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Thesis submitted to
Nanyang Technological University
in fulfillment of the requirements for the
Degree of Doctor of Philosophy

2005

Abstract

A parallel manipulator (PM) is a closed-loop kinematic mechanism whose end-effector is connected to the base by several kinematic chains. PMs have attractive features of high speed, high accuracy, high pay load ratio and low inertia. However, lack in workspace dimensions, more complex direct kinematics and singularities within the workspace make the applications of manipulators difficult. Singularity is the most serious among these drawbacks, because the manipulator loses or gains mobility and becomes uncontrollable in this condition. To overcome singularity conditions, the concept of *joint-coupling* (JC) is introduced and studied in this work. The idea behind JC is to couple the movement of several joints together with a single actuator by mechanical or electronic means while preserving the mobility of the manipulator. In this work, planar PMs with JCs are studied to illustrate the advantage of this approach for singularity management. In instantaneous kinematics of the PM with JC, the jacobian matrices of the manipulator become functions of coupling coefficients. Consequently, singularity manifolds (loci) of the manipulator can be fine-tuned by changing the values of coupling coefficients. The dynamics of PMs with JCs is formulated based on Lagrange formulation for the study of joint torque distribution in JCs. In selecting appropriate coupling coefficients, two methods are proposed: (1) the task-based approach, which uses kinematic property of the manipulator, (2) the motor torque limit approach, which requires the dynamic model of the manipulator. Simulation and experiment results indicate that the both methods are feasible. Two types of coupling devices are developed for experiments. A mechanical JC device is designed using a differential planetary gear mechanism with a fixed coupling ratio. A more flexible and adaptable electronic JC device is also designed based on an electronic gearing concept, which uses a microprocessor to synchronize the motors in the inputs joints at any arbitrary coupling ratio. Experimental work is conducted to verify the kinematic performance of the manipulator and to show the altering of the direct singularity. It is shown that PMs with JC can manage the singularity either mechanically or electronically. The JC concept can also be extend to spatial manipulators. The potential application of JC concept allows PMs to deal with the more dexterous tasks.

Acknowledgment

I would like to express my sincere thanks to my supervisors, Dr. Chen I-Ming and Dr. Li Chuan for their continuous guidance, understanding, infinite patience and supports during this research. Their expertise in technical background of this research have immensely assisted me towards the completion of this thesis.

My sincere gratitude is expressed to Prof. Jorge Angeles of McGill University, Canada for his valuable advices and guidance on this study. I am also thankful to Dr. Yeo Song Huat and Dr. Yang Guilin of SIMTech for their intellectual advice and timely encouragement. Their roles are important in guiding me in the fulfillment of this research work. I am also thankful to all the teachers in my life.

I am also very grateful to all my friends of our research group, Xing Shusong, Anjan Kumar Dash, Pham Huy Hoang, Jin Yan, Pham Cong Bang, Mustafa Shabbir Kurbanhusen, Lim Chee Kian and Yan Liang who have always shared with me good and bad feelings during my entire study. I would like to thank Dr. Gerald Seet, Mr. Lim Eng Cheng, Ms. Agnes Tan, Mr. You Kim San and Ms. Toh Yen Mei for providing me an excellent research environment at the Robotics Research Center (RRC).

The deepest gratitude is expressed to my parents, all my family members and husband for their highest kindness, continuous supporting and endless encouragement in my life.

Lastly, I would like to acknowledge the scholarship that I have received from Nanyang Technological University. The financial support of this project from the Ministry of Education, Singapore where Academic Research Fund RG 02/06 is also acknowledged.

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List of Notations

$\mathbf{A}_{J1}$	Forward Jacobian matrix of two-DOF planar manipulator
$\mathbf{A}_{J2}$	Forward Jacobian matrix of three-DOF planar manipulator
$\mathbf{a}_i$	Position vector of point A_i with respect to reference frame, O
$\mathbf{B}_{J1}$	Inverse Jacobian matrix of two-DOF planar manipulator
$\mathbf{B}_{J2}$	Inverse Jacobian matrix of three-DOF planar manipulator
$\mathbf{b}_i$	Position vector of joint B_i with respect to reference frame, O
$\mathbf{C}$	Coupling matrix
$\mathbf{c}_i$	Position vector of joint C_i with respect to reference frame, O
c_{ij}, c_i	Joint-coupling coefficients for($i = 1, 2...$ and $j = 1, 2$)
$Co(\mathbf{D})$	Convex hull of $\mathbf{D}$
$\mathbf{D}$	A set of given posture, $\{\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_k\}$
$\mathbf{D}$	Acceleration relation matrix of joint-coupling
$\mathbf{E}$	Rotational matrix
E	Energy consumption of manipulator
$\mathbf{e}_i$	Position vector of $\overrightarrow{C_iP}$ with respect to reference frame, O
$\mathbf{G}$	Inertial matrix in consideration of active joint velocity
$\mathbf{g}$	Gravitational acceleration
$\mathbf{H}$	Velocity relation matrix of joint-coupling
I_{ij}	Inertia at the center of link j from branch i
K_{ij}	Kinetic energy of link j from branch i
K_{mp}	Kinetic energy of moving platform
L	Lagrange equation

l_{ij}	Length of link j from branch i
$\mathbf{F}$	Inertial matrix in terms of joint rates $\dot{\theta}_{ij}$
$\mathbf{M}_{jc}$	Inertial matrix of in terms of joint-coupling rates ω_i
m_{ij}	Mass of link j from branch i
m_p	Mass of moving platform
$\mathbf{N}$	Coriolis matrix in terms of joint rates $\dot{\theta}_{ij}$
n	Velocity ratio of output velocities of electronic
O	Origin of reference frame of manipulators
$\mathbf{p}$	Position vector of end-effector with respect to reference frame O
$\dot{\mathbf{p}}$	End-effector velocity
$\ddot{\mathbf{p}}$	End-effector acceleration
P	A set of postures of trajectory
$\mathbf{q}_i$	Posture of end-effector
$\mathbb{R}$	Real number
R	Planetary differential gearbox ratio
$\mathbf{S}(g)$	Energy function
t_s	Sampling time
t_t	Total time
U	Potential energy
$\mathbf{V}$	Coriolis matrix in terms of joint rates $\dot{\theta}_{i1}$
$\mathbf{V}_{jc}$	Coriolis matrix in terms of joint-coupling rates ω_i
$\mathbf{v}_{ij}$	Translational velocity at center of link j on the branch i
w_i	Weight factor
$Z(f)$	Zero set
α_i	Joint-coupling coefficient ratio, $\frac{c_{i2}}{c_{i1}}$
Γ_i	Constraint equations
λ_i	Lagrange multiplier
ϕ	Angle of rotation of moving platform of three-DOF planar manipulator
$\dot{\Theta}$	Joint angular velocity matrix
Ω	Joint-coupling velocity matrix

$\dot{\Omega}$	Joint-coupling acceleration matrix
θ_{ij}	Displacement of joint j from branch i ($i = 1, 2, \dots$ and $j = 1, 2$)
$\dot{\theta}_{ij}$	Angular velocity of joint j from branch i ($i = 1, 2, \dots$ and $j = 1, 2$)
$\ddot{\theta}_{ij}$	Angular acceleration of joint j from branch i ($i = 1, 2, \dots$ and $j = 1, 2$)
τ_{ij}	Required torques of active joints
τ_i	Required torques of actuator of joint-coupling devices
ω_i	Input angular velocity of joint-coupling
$\dot{\omega}_i$	Input angular acceleration of joint-coupling

List of Abbreviation

CVT	Continuous Variable Transmission
CVTJC	Continuous Variable Transmission Joint-Coupling
DOF	Degree of Freedom
JC	Joint-Coupling
MAT	Move Absolute Time Based
MAV	Move Absolute Velocity Based
MRT	Move Relative Time Based
MRV	Move Relative Velocity Based
PM	Parallel Manipulator

Chapter 1

Introduction

Typically a PM is composed of several serial chains connecting the base to an end-effector (the moving platform). Each chain has subsets of active and passive joints. Parallel manipulators exhibit important characteristics of high stiffness, large payload capacity, less error accumulation, low inertia and high speed and high precision. Stewart platform [1] is probably the first PM reported in literature. A well known application of PMs is in flight simulators. In addition, milling machining centers (Giddings and Lewis, Ingersoll, Geodetics, Hitachi Seiki, Neos Robotics) also employ parallel mechanism design. However, there are several drawbacks of PMs such as lack in workspace dimensions, a more complex direct kinematics, singularities within the workspace and high coupling between the moving kinematic chains. The major concern in the design of PMs is the occurrence of kinematic singularities.

1.1 Singularity

Singularities are particular postures of the end-effector (the moving platform) in which PMs lose their inherent rigidity as the end-effector loses or gains degrees of freedom and becomes uncontrollable. In singularity poses, the manipulator will suffer from symptoms such as

Figure 1.1: 5-Axis Milling machine at Robotool, Denmark

decreased accuracy, large internal forces and loss of operational capacity. It is also known that the complex singularity manifold (loci) of a PM will further split its already limited workspace into several smaller regions. Consequently, singularity avoidance becomes an essential issue during the development and application of any kind of PMs, and has received great attention in PM literature.

Ma and Angeles [2] classified the singularities into three categories, namely *architecture* singularity, *configuration* singularity and *formulation* singularity. The architecture singularity refers to the singularity that is caused by the architecture of a PM. Such singularity exists for all configurations inside the entire or part of the workspace. Configuration singularity refers to the singularity that is caused by a particular configuration of a robot, while Formulation singularity refers to the singularity that is caused by failure of the kinematic model at a particular configuration. According to the above definitions of singularities, architecture singularity is a special case of the configuration singularity, while the formulation singularity is a trivial case because it is always avoidable by changing the kinematic model.

An important introduction to singularity of general closed-loop mechanisms is done by Gosselin and Angeles [3]. In this work, the configuration singularity is further classified into three main groups based on the properties of the Jacobian matrices. The instantaneous

relationship between the velocity of the moving platform $\dot{\mathbf{x}}$ and the velocities of the active joints $\dot{\theta}$ is given as

$$A\dot{\mathbf{x}} = B\dot{\theta}, \quad (1.1)$$

where A and B are the Jacobian matrices of the closed-loop mechanism. The first, second and third kinds of singularities occur respectively when matrix A is singular, matrix B is singular and both matrices A and B are singular. Tsai [4] termed these singularities as *forward*, *inverse* and *combined* singularities, respectively.

Numerous researchers, Sefrioui and Gosselin [5], Yang and Lee [6], Pennock and Kissner [7] have carried out extensive studies to determine the exact conditions of singularity postures in PMs. Liu *et al* [8] analyzed configuration space singularities by constructing a Morse function on an appropriately defined space. They varied the parameters of the manipulator to obtain insight on the configuration space singularities and to determine the design parameters of the manipulator. Parametrization singularities are then defined.

Recent approaches to closed-chain singularity analysis can be found in Tao [9], Agrawal [10], Shi and Fenton [11] and Kim, Chung [12] and Bhattacharya *et al* [13]. Mohammadi *et al* [14] developed a procedure to determine the instantaneous twist associated with the joints of a limb using the velocity vector-loop equation. A novel application of the Jacobian has been to analyze and synthesize under-actuated force generating mechanisms. Merlet [15] used Grassman geometry to identify the singular conditions for PMs. Hao and McCarthy [16], Notash [17] and others proposed singularity analysis for PMs based on line geometry.

Most of them involved the Jacobian based analysis, since the Jacobian matrix degenerates when the manipulator is in a singular position. Alici [18] reported the determination of singularity contour for five-bar planar PMs based on manipulator Jacobian matrices and links parameters. Wang [19] presented two types of singularity locus; the first type was obtained by finding the workspace limits of manipulators, while the second type was obtained through the solution of nonlinear algebraic equations based on velocity analysis. Yang and Chen [20, 21, 22] introduced the kinematics analysis of a three-DOF in

Figure 1.2: Two-DOF redundant manipulator at Hong Kong University, China

parallel-actuated manipulator using Product-of-Exponential Formula. Based on the concept of instantaneous centers, a simple and unified geometric method was proposed in their work. Among the singularity avoidance works, redundant actuation is most popular. Redundant PMs as shown in Fig. 1.2, whose number of actuators is greater than that of the task space coordinates were introduced by Gao and Liu [23]. They classified the configuration singularities based on the properties of the Jacobian matrices of the closed-loop mechanism relating the velocity vector of the end-effector with the vector of actuated joint rates. Park and Kim [24] presented a coordinate-invariant differential geometric analysis of singularities of closed-loop chains. In this geometric framework, both redundant and non-redundant systems, including the case of redundant actuation, were treated in a uniform manner. Because of redundancy actuation, the Jacobian was not a square matrix. The use of pseudo inverse was applied for redundancy resolution to PMs by Nenchev [25]. Doty, Melchiorri and Bonivento [26] focused on the difficulties in applying the pseudo inverse to manipulators problems and elaborated on the coordinate-dependent nature of the pseudo inverse and weighted pseudo inverse.

Matone and Roth [27] investigated the effect of different actuation schemes on singularities of PMs. They presented a framework on how to incorporate actuator location and to drive mechanism into the kinematic model of in-PMs by showing how to obtain the kinematic model in the joint space given the corresponding representation in the actuator space.

Jin and Chen [28] introduced the structural synthesis of Selectively Actuated PM (SA-PM)

Figure 1.3: Basic concept of joint-coupling using LEGO differential mechanisms

based on geometrical constraints from the line geometry. In their study, 6-DOF spatial motion depending on the types of the actuation (rotary or linear) can be chosen for the actuators arbitrarily. The manipulator architecture decouples the three-DOF translation and the three-DOF rotation of the end-effector for individual control. The singularity conditions of the SA-PM driven by different actuator motions are also studied.

O'Brien and Wen [29] investigated the use of joint-brakes on passive joints in the PM to avoid singularity. With additional actuators for redundant actuation and brakes on the passive joints, geometry of the manipulator singularity will differ from the original one. However, new manipulator singularity will arise in the course of these changes. Hence, in planning the manipulator tasks, it is necessary to determine which actuators or brakes to be active beforehand.

1.2 Concept of Joint-Coupling

In this study, *Joint-Coupling* (JC) is introduced to overcome singularity conditions of PMs. The idea behind joint-coupling is to couple the movement of several manipulator joints (active and passive) together with a single or multiple actuators. The joint-coupling can be realized in a simple manner using differential mechanism concept as shown in Fig. 1.3. A single input actuation is distributed to two or more neighboring joints through a differential

mechanism. Assuming that $\dot{\theta}$ and $\dot{\phi}$ are two adjacent joint rates of the manipulator, c_1 and c_2 are coupling coefficients of the two joints and ω is input active rate of the differential mechanism, the following relationship is obtained:

$$\omega = c_1\dot{\theta} + c_2\dot{\phi}. \quad (1.2)$$

The differential mechanism functions as an actuation power distribution mechanism in this closed-loop linkage in a sense that movement of the joints that include some active and passive ones are coordinated through the relationship given in Eq. (1.2). These joints are partially actuated depending on power distribution according to coupling coefficients. In this case, the mobility of manipulator remains unchanged after addition of this mechanism. With this differential mechanism, coupling coefficients c_1 and c_2 are incorporated into Jacobian matrices of the manipulator in instantaneous kinematics analysis. By changing values of the coefficients c_1 and c_2 , the geometry of manipulator singularities will change accordingly. Hence, it is possible to "engineer" the PM singularities without changing the dimension or location of the actuators.

1.3 Overview of Thesis

The main objectives of this research are to present and realize the concept of joint-coupling for PMs and to study how joint-coupling can be used for managing the singularity of a PM. This joint-coupling concept is focused through planar manipulators. The task encompasses the study of instantaneous kinematics, dynamic of PMs based on joint-coupling concept, designing and fabrication of joint-coupling devices and experimental validation. The remaining chapters of this work are organized as follows.

In Chapter 2, the kinematics of planar PMs is formulated. Inverse kinematics and direct kinematics of two-DOF and three-DOF planar manipulators are derived. Inverse kinematics and direct kinematics of manipulators are the same regardless of whether they have joint-coupling or not.

The crucial investigation on the singularity of planar PMs with joint-coupling is presented in Chapter 3. The Jacobians of PM with joint-coupling are formulated based on the instantaneous kinematics. It turns out that the direct kinematics Jacobian becomes a function of the coefficients, direct singularity manifold (loci) can be altered geometrically. However, the inverse kinematics Jacobian remains identical to the same manipulator without joint-coupling. Hence, inverse singularity manifolds (loci) cannot be changed, as geometrically the inverse singularity occurs at the boundary of the manipulator workspace which is a function of the geometric parameters of the manipulator and is independent of the addition of joint-coupling to the manipulator.

In Chapter 4, the dynamics of planar PMs with joint-coupling are investigated. Equations of motion for the manipulator are formulated using the Lagrangian approach. The purpose of deriving dynamic models of the manipulator is to control the manipulator and to design the joint-coupling parameters based on the dynamic behavior of the manipulator.

With adjustable joint-coupling coefficients, two methods: (1) the task-based approach, and (2) the motor limit torque approach to choose appropriate joint-coupling coefficients are proposed in Chapter 5. These approaches are based on the kinematics and dynamics of planar PMs. The task-based method can shape singularity manifolds within the workspace to make the desired region free from singularity, while the motor torque limit method can obtain singularity-free trajectory which consumes the lowest required motor torques. Both methods are verified through the simulation on two-DOF and three-DOF planar PMs with joint-coupling.

In Chapter 6, prototypes of joint-coupling devices are realized by mechanical and electronic means. A planetary differential mechanism are employed in the mechanical joint-coupling device prototype. This device can distribute the power from a single input axis to two output axes with fixed coupling coefficients. Electronic gearing is employed in the electronic joint-coupling device design. This device synchronizes the coupled joints through a microprocessor based motion control. Both joint-coupling device prototypes are deployed on a two-DOF planar PM platform for experimental tests.

In Chapter 7, an experimental work to verify that two-DOF planar PM with joint-coupling can alter the direct singularity. The motion control scheme is tested in physical experiments using Move Relative Velocity (MRV) based command from the *QuickControl* toolbox of *SiliverMax* motors which are used as input of the manipulator. Finally this work is concluded and future directions for this research are discussed in Chapter 8.

Chapter 2

Kinematics of Planar Parallel Manipulators

In this research, the concept of joint-coupling is demonstrated through two-DOF and three-DOF planar PMs due to their simplicity and well-behaved kinematic properties. The forward and inverse kinematics of two types of planar PMs are described as a basis for the study of the instantaneous kinematics and dynamics of planar PMs with joint-coupling.

2.1 Kinematics of Two-DOF Planar Parallel Manipulators

The two-DOF planar PM mentioned here has a five-bar linkage structure with all revolute joints as shown in Fig. 2.1. This is the simplest type of PM. In subsequent chapters, it will be used to study the joint-coupling. Because of its simplicity, it is often used as a benchmark for the study of PMs. Gao and Liu [23] reported the performance evaluation of two-DOF planar PMs of the same type. A geometric model of the configuration space is obtained based on link lengths of the manipulator. Cervantes-Sanchez [30] studied the

Figure 2.1: Kinematic diagram of a two-DOF planar PM

kinematic design of the 5R planar PM. By using the Cartesian coordinates of joint positions, the workspace and singularity analysis are reported. Kinematics of manipulators built with closed-loop mechanisms have been widely studied in literature [31, 32, 33, 34, 35, 26, 36, 37], with its Geometrical solutions to the kinematics of the PM [38, 39, 40, 23, 41]. Definition sets for the direct kinematics of PMs have been studied by Wenger and Chablat [42, 43]. In this study, geometrical approach is used to solve the manipulator kinematics.

As illustrated in Fig. 2.1, let point O be the origin of reference frame which is the center of joint A_1 and A_2 . A_1, A_2, B_1 and B_2 denotes the center of the revolute joints. The lengths of the links are represented by A_1A_2 , A_1B_1 , A_2B_2 , B_1P and B_2P . Point P denotes the end-effector of the manipulator. The orientation (or angle) of links are measured from the positive x -axis.

2.1.1 Inverse Kinematics

The inverse kinematics of the PM is to determine the actuator displacements based on the given end-effector positions. For the two-DOF planar PM, the position of the end-effector is given, which is defined with respect to reference frame O . Consider the left branch of the manipulator A_1B_1P . Let link lengths be $A_1B_1 = l_1, A_2B_2 = l_3, B_1P = l_2, B_2P = l_4, A_1A_2 =$

2a. The position of end-effector P can be expressed as:

$$\overrightarrow{A_1P} = \overrightarrow{A_1B_1} + \overrightarrow{B_1P} \quad (2.1)$$

The coordinate of point P is given as:

$$\mathbf{p} = (x_p, y_p) = (-a + l_1 \cos \theta_{11} + l_2 \cos \theta_{12}, l_1 \sin \theta_{11} + l_2 \sin \theta_{12}) \quad (2.2)$$

Eliminating angles θ_{12} , Eq. (2.3) is obtained:

$$a^2 + l_1^2 - l_2^2 + x_p^2 + y_p^2 + 2ay_p - 2y_pl_1 \sin \theta_{11} - 2l_1x_p \cos \theta_{11} - 2l_1a \cos \theta_{11} = 0. \quad (2.3)$$

Using the trigonometric identities, θ_{11} can be obtained in term of the end-effector position P as:

$$\theta_{11} = 2 \tan^{-1} \left(\frac{-b_1^2 \pm \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \right) \quad (2.4)$$

where

$$a_1 = a^2 + l_1^2 + x_p^2 + y_p^2 - l_2^2 + 2ax_p + 2l_1(x_p - a)$$

$$b_1 = -4y_pl_1$$

$$c_1 = a^2 + l_1^2 + x_p^2 + y_p^2 - l_2^2 + 2ax_p - 2l_1(x_p - a)$$

Similarly for the right branch, Eq. (2.5) can be expressed as:

$$\overrightarrow{A_2P} = \overrightarrow{A_2B_2} + \overrightarrow{B_2P}. \quad (2.5)$$

Hence, the coordinate of point P can be obtained:

$$\mathbf{p} = (x_p, y_p) = (a + l_3 \cos \theta_{21} + l_4 \cos \theta_{22}, l_3 \sin \theta_{21} + l_4 \sin \theta_{22}). \quad (2.6)$$

Eliminating angles θ_{22} , Eq. (2.7) is obtained:

$$a^2 + l_3^2 - l_4^2 + x_p^2 + y_p^2 - 2ax_p - 2y_pl_3 \sin \theta_{21} - 2l_3x_p \cos \theta_{21} + 2l_3a \cos \theta_{21} = 0. \quad (2.7)$$

Using the trigonometric identities, θ_{21} is obtained in term of the end-effector position P as

$$\theta_{21} = 2 \tan^{-1} \left(\frac{-b_2^2 \pm \sqrt{b_2^2 - 4a_2c_2}}{2a_2} \right) \quad (2.8)$$

where

$$a_2 = a^2 + l_3^2 + x_p^2 + y_p^2 - l_4^2 - 2ay_p + 2l_3(x_p + a)$$

$$b_2 = -4y_p l_3$$

$$c_2 = a^2 + l_3^2 + x_p^2 + y_p^2 - l_4^2 + 2ax_p + 2l_3(x_p + a)$$

The inverse kinematics of the manipulator have multiple solutions. From Eq. (2.4) and Eq. (2.8), there are four different inverse kinematic solutions to a given end-effector pose as illustrated in Fig. 2.2.

Figure 2.2: Multiple solutions to inverse kinematics of two-DOF planar PM

2.1.2 Direct Kinematics

The direct kinematics of a PM is to determine the location of the end-effector based on a set of given actuator displacements. Assuming that joint A_1 and A_2 are input joints and θ_{11} and θ_{21} are given, the positions of joint B_1 and B_2 are given as:

$$\mathbf{b}_1 = (x_{b1}, y_{b1}) = (-a + l_1 \cos \theta_{11}, l_1 \sin \theta_{11})$$

$$\mathbf{b}_2 = (x_{b2}, y_{b2}) = (a + l_3 \cos \theta_{21}, l_3 \sin \theta_{21})$$

Since the link lengths are known, the end-effector position can be determined by the following equations:

$$(x_p - x_{b1})^2 + (y_p - y_{b1})^2 - l_2^2 = 0 \quad (2.9)$$

$$(x_p - x_{b2})^2 + (y_p - y_{b2})^2 - l_4^2 = 0 \quad (2.10)$$

The following notations will be used for terms incurred in the solution of Eq. (2.9) and Eq. (2.10). Because of the quadric nature of Eq. (2.9) and Eq. (2.10) the position of the end-effector P has the following solution:

$$\mathbf{p} = (x_p, y_p) = \left(M \left(\frac{-b_3 \pm \sqrt{b_3^2 - 4a_3c_3}}{2a_3} \right) + F, \frac{-b_3 \pm \sqrt{b_3^2 - 4a_3c_3}}{2a_3} \right)$$

where:

$$F = \frac{S - R}{2(x_{b2} - x_{b1})}, \quad M = \frac{y_{b1} - y_{b2}}{x_{b2} - x_{b1}}, \quad R = (x_{b1})^2 + (y_{b1})^2, \quad S = (x_{b2})^2 + (y_{b2})^2$$

$$a_3 = M^2 + 1, \quad b_3 = 2(MF - Mx_{b1} - y_{b1}), \quad c_3 = F^2 - 2Fx_{b1} + R - l_2l_4$$

The two solutions to the end-effector pose can be illustrated in Fig. 2.3.

Figure 2.3: Two possible end-effector postures of two-DOF planar PM

2.2 Kinematics of Three-DOF Planar Parallel Manipulators

In addition to two-DOF planar PMs, a class of three-DOF planar PMs with revolute joints are also studied here as shown in Fig. 2.4. This type of planar PMs allow their end-effectors to produce complete three-DOF planar motions including translations in x - and y -directions and rotation about the z -axis. This three-DOF planar PM has three branches $A_1B_1C_1$, $A_2B_2C_2$ and $A_3B_3C_3$ which are usually designed symmetrically. All joints in the manipulator are revolute joints. The moving platform is defined by $C_1 C_2 C_3$ and connected with the three branches. The origin O of the reference is defined to be coincident with A_1 as shown in Fig. 2.4. The following notations will be used in this section.

l_{i1} = link length between joint A_i and B_i for i^{th} branch

Figure 2.4: Kinematic diagram of three-DOF planar PM

l_{i2} = link length between joint B_i and C_i for i^{th} branch

e_i = link length between joint C_i and P

m_i = length of triangles moving platform $C_1 C_2 C_3$

b = length of equilateral triangles $A_1 A_2 A_3$

γ_1 = the angle between position vectors $\overrightarrow{C_1C_2}$ and $\overrightarrow{C_1P}$

γ_2 = the angle between position vectors $\overrightarrow{C_2C_3}$ and $\overrightarrow{C_2P}$

γ_3 = the angle between position vectors $\overrightarrow{C_3C_1}$ and $\overrightarrow{C_3P}$

The location of the moving platform can be specified in term of the position vector of point C_1 and an orientation angle ϕ . Then

$$\mathbf{a}_1 = (x_{a1}, y_{a1}) = (0, 0)$$

$$\mathbf{a}_2 = (x_{a2}, y_{a2}) = (b, 0)$$

$$\mathbf{a}_3 = (x_{a3}, y_{a3}) = (b \cos \pi/3, b \sin \pi/3)$$

For branch i ($i = 1, 2, 3$), the position of the platform point C_i can be found by

$$\overrightarrow{A_iC_i} = \overrightarrow{A_iB_i} + \overrightarrow{B_iC_i}. \quad (2.11)$$

Figure 2.5: One branch of three-DOF planar PM

Thus, the position of the moving platform P becomes

$$\begin{aligned} x_p &= x_{ai} + l_{i1} \cos \theta_{i1} + l_{i2} \cos \theta_{i2} + e_i \cos(\phi + \gamma_i) \\ y_p &= y_{ai} + l_{i1} \sin \theta_{i1} + l_{i2} \sin \theta_{i2} + e_i \sin(\phi + \gamma_i) \quad \text{where } i = 1, 2, 3. \end{aligned}$$

2.2.1 Inverse Kinematics

The posture of the end-effector (the moving platform) for this manipulator is denoted as $(\mathbf{p}, \phi)$. Given a posture $(\mathbf{p}, \phi)$, the inverse kinematic of the manipulator is evaluated on a branch-by-branch basis. For the three-DOF planar PM, the coordinates of the middle joints is written as

$$\mathbf{b}_i = (x_{bi}, y_{bi}) = (x_{ai} + l_{i1} \cos \theta_{i1}, y_{ai} + l_{i1} \sin \theta_{i1}) \quad (2.12)$$

$$\mathbf{c}_i = (x_{ci}, y_{ci}) = (x_p - e_i \cos(\phi + \gamma_i), y_p - e_i \sin(\phi + \gamma_i)) \quad (2.13)$$

The link lengths l_{i2} is specified as

$$l_{i2}^2 = (x_{ci} - x_{bi})^2 + (y_{ci} - y_{bi})^2 \quad \text{where } i = 1, 2, 3. \quad (2.14)$$

Substituting Eq. (2.12) and Eq. (2.13) into Eq. (2.14), Eq. (2.15) is obtained as

$$\begin{aligned} &x_p^2 + y_p^2 + x_{ai}^2 + y_{ai}^2 - 2x_{ai}x_p - 2y_{ai}y_p + 2e_i\{x_{ai} \cos(\phi + \gamma_i) + y_{ai} \sin(\phi + \gamma_i) \\ &- x_p \cos(\phi + \gamma_i) - y_p \sin(\phi + \gamma_i)\} + 2l_{i1}x_{ai} \cos \theta_{i1} + 2l_{i1}y_{ai} \sin \theta_{i1} - 2l_{i1}x_p \cos \theta_{i1} \\ &- 2l_{i1}y_p \sin \theta_{i1} + 2e_i l_{i1}\{\cos(\phi + \gamma_i) \cos \theta_{i1} + \sin(\phi + \gamma_i) \sin \theta_{i1}\} + e_i^2 + l_{i1}^2 - l_{i2}^2 = 0. \end{aligned} \quad (2.15)$$

For branch i , the input angle θ_{i1} is determined using Eq. (2.15):

$$\theta_{i1} = \frac{-b_i \pm \sqrt{b_i^2 - 4a_i c_i}}{2a_i}. \quad (2.16)$$

where

$$a_i = 2l_{i1}x_{ai} - 2l_{i1}x_p + x_p^2 + y_p^2 + x_{ai}^2 + y_{ai}^2 - 2x_{ai}x_p - 2y_{ai}y_p + 2e_i\{l_{i1}\cos(\phi + \gamma_i) + x_{ai}\cos(\phi + \gamma_i) + y_{ai}\sin(\phi + \gamma_i) - x_p\cos(\phi + \gamma_i) - y_p\sin(\phi + \gamma_i)\} + e_i^2 + l_{i1}^2 - l_{i2}^2$$

$$b_i = 4l_{i1}y_{ai} - 4l_{i1}y_p + 4e_i l_{i1} \sin(\phi + \gamma_i)$$

$$c_i = x_p^2 + y_p^2 + x_{ai}^2 + y_{ai}^2 + l_{i1}^2 - l_{i2}^2 - 2\{x_{ai}x_p - y_{ai}y_p + e_i x_{ai} \cos(\phi + \gamma_i) + l_{i1}x_p - e_i x_p \cos(\phi + \gamma_i) + e_i y_{ai} \sin(\phi + \gamma_i) - e_i y_p \sin(\phi + \gamma_i) - e_i l_{i1} \cos(\phi + \gamma_i) - l_{i1}x_{ai}\} + e_i^2$$

Since each branch has two different solutions to a given posture, there are eight possible inverse kinematic solutions to the entire manipulator.

2.2.2 Direct Kinematics

A well-known feature of PMs is the existence of multiple solutions to the direct kinematics problem. Gosselin and Sefrioui [44] studied the polynomial solution for direct kinematic problem of PMs. The position coordinates of middle joints B_i and C_i are shown in Eq. (2.12) and Eq. (2.13). The lengths of C_1C_2 , C_2C_3 and C_3C_1 ($m_1 = m_2 = m_3 = m$) are assumed all identical. The link lengths l_{i2} is specified in Eq. (2.14). There are three equations for three branches, specifically,

$$l_{i2}^2 = (x_{ci} - x_{bi})^2 + (y_{ci} - y_{bi})^2 \quad (2.17)$$

$$l_{12}^2 = (x_{c1} - x_{b1})^2 + (y_{c1} - y_{b1})^2 \quad (2.18)$$

$$l_{22}^2 = x_{c1}^2 + y_{c1}^2 + Rx_{c1} + Sy_{c1} + Q \quad (2.19)$$

$$l_{32}^2 = x_{c1}^2 + y_{c1}^2 + Ux_{c1} + Vy_{c1} + W \quad (2.20)$$

where

$$x_{b1} = l_{11} \cos \theta_{11}$$

$$y_{b1} = l_{11} \sin \theta_{11}$$

$$x_{b2} = b + l_{21} \cos \theta_{21}$$

$$y_{b2} = l_{21} \sin \theta_{21}$$

$$x_{b3} = b \cos \pi/3 + l_{31} \cos \theta_{31}$$

$$y_{b3} = b \sin \pi/3 + l_{31} \sin \theta_{31}$$

$$x_{c2} = x_{c1} + m \cos \phi$$

$$y_{c2} = y_{c1} + m \sin \phi$$

$$x_{c3} = x_{c1} + m \cos \gamma + \phi$$

$$y_{c3} = y_{c1} + m \sin \gamma + \phi$$

$$R = 2m \cos \phi - 2l_{21} \cos \theta_{21} - 2b$$

$$S = 2m \sin(\Phi) - 2l_{21} \sin(\theta_{21})$$

$$Q = 2bl_{21} \cos(\theta_{21}) - 2bm \cos(\phi) - 2l_{21}m \cos(\theta_{21}) \cos(\phi) + -2l_{21}m \sin(\theta_{21}) \sin(\phi) \\ + l_{21}^2 + b^2 + m^2$$

$$U = -2b \cos(\pi/3) - 2l_{31} \cos(\theta_{31}) + 2m \cos(\gamma + \pi)$$

$$V = -2b \sin(\phi/3) - 2l_{31} \sin(\theta_{31}) + 2m \sin(\gamma + \phi)$$

$$W = 2bl_{31} \cos(\pi/3) \cos(\theta_{31}) - 2bm \cos(\pi/3) \cos(\gamma + \phi) - 2l_{31}m \cos(\theta_{31}) \cos(\gamma + \phi) \\ + 2bl_{31} \sin(\pi/3) \sin(\theta_{31}) - 2bm \sin(\pi/3) \sin(\gamma + \phi) - 2l_{31}m \sin(\theta_{31}) \sin(\gamma + \phi) \\ + l_{31}^2 + b^2 + m^2$$

Solving Eq. (2.17) to (2.20), the position of joint C_1 can be obtained as

$$C_1 = (x_{c1}, y_{c1}) = \left(\frac{VQ - SW}{RV - SU}, \frac{UQ - RW}{RV - SU} \right) \quad (2.21)$$

Then, the orientation ϕ of the platform can be solved by substituting Eq. (2.21) into Eq. (2.19).

2.3 Summary

In this chapter, inverse and direct kinematics of two-DOF and three-DOF planar PMs are derived. Multiple solutions exist in the inverse and direct kinematic of those planar PMs. These solutions serve as a basis for the study of singularity and dynamics of the PM with joint-coupling because the installation of joint-coupling devices into PMs will not affect their basic kinematic solutions.

Chapter 3

Singularity Analysis of Planar Parallel Manipulator with Joint-Coupling

In this chapter, the instantaneous kinematics of PMs with joint-coupling is formulated based on the vector method. It is demonstrated that direct singularities of a PM can be modified by introducing joint-coupling into two-DOF and three-DOF planar PMs without changing the manipulator architecture.

Joint-coupling is intended to synchronize motions of several joints in a PM to act as a group of active joints with the number of actuators smaller than the number of the coupled joints. Suppose that two joints of i^{th} branch of the PM are associated with variables $\dot{\theta}_i$ and $\dot{\phi}_i$ ($i = 1, 2, \dots, m$) where $2m \leq n$ and n is the total number of joints in the manipulator. In order to couple these two joints to form an active member with an input of ω_i (for $i = 1, 2, \dots, m$ where m is equal to the mobility of the PM) the relationship of coupled joints can be written as:

$$\omega_i = c_1 \dot{\theta}_i + c_2 \dot{\phi}_i \quad (3.1)$$

Figure 3.1: Kinematic diagram of two-DOF planar PM with joint-coupling

A generic repression of manipulators with joint-coupling has the following matrix form:

$$\mathbf{\Omega} = \mathbf{C} \dot{\mathbf{\Theta}} \quad (3.2)$$

where $\mathbf{C}$ is the $m \times 2m$ coupling matrix. The m -dimensional joint-coupling input vector $\mathbf{\Omega}$ and the $2m$ -dimensional coupled joint rate vector $\dot{\mathbf{\Theta}}$ are denoted as

$$\begin{aligned} \mathbf{\Omega} &= \begin{bmatrix} \omega_1, & \dots, & \omega_m \end{bmatrix}^T, \\ \dot{\mathbf{\Theta}} &= \begin{bmatrix} \dot{\theta}_1, \dot{\phi}_1, & \dots, & \dot{\theta}_m, \dot{\phi}_m \end{bmatrix}^T. \end{aligned}$$

Through the analysis of PMs with joint-coupling, it is found that the joint-coupling can provide a unified approach to analyze the singularities of PMs with different actuation schemes. The coupling coefficients of the joint-coupling can affect the conditions of direct singularities but not the conditions of inverse singularity. The inverse singularities occur naturally at the borders of the manipulator workspace that is restricted by the manipulator architecture. The coupling coefficients in this manner can shape direct singularity manifolds (loci) of the manipulator for the design of manipulator tasks.

3.1 Two-DOF Planar Parallel Manipulator

The joint-coupling concept on the two-DOF planar PM depicted in Fig. 3.1 is studied. This manipulator bears similar kinematic structure of the standard five-bar linkage mechanism. Joints A_i and B_i ($i = 1, 2$) are coupled and driven by actuator i from the input of joint-coupling. Hence, the manipulator can remain as a two-DOF system. For convenience, the origin O of the reference coordinate frame is set at the mid point of A_1A_2 and the x -axis is aligned with A_1A_2 . The position vectors of A_i, B_i , and P are denoted by $\mathbf{a}_i, \mathbf{b}_i$, and $\mathbf{p}$ respectively, as shown in Fig. 3.1. The orientations of link A_iB_i and B_iP are denoted by θ_{i1} and θ_{i2} respectively ($i = 1, 2$). The position vector $\mathbf{p}$ is given by

$$\mathbf{p} = \mathbf{a}_i + (\mathbf{b}_i - \mathbf{a}_i) + (\mathbf{p} - \mathbf{b}_i) \quad \text{where } i = 1, 2. \quad (3.3)$$

Differentiating Eq. (3.3) with respect to time, the relationship of all joint rates and the velocity of point P is obtained:

$$\dot{\mathbf{p}} = \dot{\theta}_{i1} \mathbf{E}(\mathbf{b}_i - \mathbf{a}_i) + \dot{\theta}_{i2} \mathbf{E}(\mathbf{p} - \mathbf{b}_i) \quad (3.4)$$

where $\mathbf{E} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is an orthogonal matrix rotating 2-dimensional vectors 90 degree counter clockwise. Pre-multiplying Eq. (3.4) with $(\mathbf{p} - \mathbf{b}_i)^T$, the joint rate $\dot{\theta}_{i2}$ can be eliminated:

$$(\mathbf{p} - \mathbf{b}_i)^T \dot{\mathbf{p}} = \dot{\theta}_{i1} (\mathbf{p} - \mathbf{b}_i)^T \mathbf{E}(\mathbf{b}_i - \mathbf{a}_i) \quad \text{where } i = 1, 2. \quad (3.5)$$

Re-writing Eq. (3.5) in matrix form, Eq. (3.6) can be obtained:

$$\mathbf{A}_1 \dot{\mathbf{p}} = \mathbf{B}_1 \dot{\Theta}_{i1} \quad i = 1, 2. \quad (3.6)$$

where

$$\dot{\Theta}_{i1} = \begin{bmatrix} \dot{\theta}_{11} \\ \dot{\theta}_{21} \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} (\mathbf{p} - \mathbf{b}_1)^T \\ (\mathbf{p} - \mathbf{b}_2)^T \end{bmatrix}$$

$$\mathbf{B}_1 = \text{diag} \left[(\mathbf{p} - \mathbf{b}_1)^T \mathbf{E}(\mathbf{b}_1 - \mathbf{a}_1), (\mathbf{p} - \mathbf{b}_2)^T \mathbf{E}(\mathbf{b}_2 - \mathbf{a}_2) \right]$$

Pre-multiplying Eq. (3.4) with $(\mathbf{b}_i - \mathbf{a}_i)^T$, the joint rate $\dot{\theta}_{i1}$ is eliminated:

$$(\mathbf{b}_i - \mathbf{a}_i)^T \dot{\mathbf{p}} = \dot{\theta}_{i2} (\mathbf{b}_i - \mathbf{a}_i)^T \mathbf{E}(\mathbf{p} - \mathbf{b}_i) \quad (3.7)$$

Re-writing Eq. (3.7) in matrix form, Eq. (3.8) is obtained as

$$\mathbf{A}_2 \dot{\mathbf{p}} = \mathbf{B}_2 \dot{\boldsymbol{\Theta}}_{i2} \quad (3.8)$$

where

$$\dot{\boldsymbol{\Theta}}_{i2} = \begin{bmatrix} \dot{\theta}_{12} \\ \dot{\theta}_{22} \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} (\mathbf{b}_1 - \mathbf{a}_1)^T \\ (\mathbf{b}_2 - \mathbf{a}_2)^T \end{bmatrix}$$

$$\mathbf{B}_2 = \text{diag} \left[(\mathbf{b}_1 - \mathbf{a}_1)^T \mathbf{E}(\mathbf{p} - \mathbf{b}_1), (\mathbf{b}_2 - \mathbf{a}_2)^T \mathbf{E}(\mathbf{p} - \mathbf{b}_2) \right]$$

According to the definition of joint-coupling of Eq. (3.1), the coupling of joints A_i and B_i ($i = 1, 2$) produce the following relationship:

$$\omega_i = c_{i1} \dot{\theta}_{i1} + c_{i2} \dot{\theta}_{i2} \quad \text{where } i = 1, 2. \quad (3.9)$$

The input rate ω_i at the joint-coupling i is the new active joint rates. Substituting Eq. (3.6) and Eq. (3.8) into Eq. (3.9), the relationship between the new active joint rates from the joint-coupling device and the velocity of point P is obtained as

$$\mathbf{A}_{J1} \dot{\mathbf{p}} = \mathbf{B}_{J1} \boldsymbol{\Omega} \quad (3.10)$$

where

$$\boldsymbol{\Omega} = \begin{bmatrix} \omega_1 & \omega_2 \end{bmatrix}^T$$

$$\mathbf{A}_{J1} = \begin{bmatrix} [c_{11}(\mathbf{p} - \mathbf{b}_1) + c_{12}(\mathbf{a}_1 - \mathbf{b}_1)]^T \\ [c_{21}(\mathbf{p} - \mathbf{b}_2) + c_{22}(\mathbf{a}_2 - \mathbf{b}_2)]^T \end{bmatrix}$$

$$\mathbf{B}_{J1} = \text{diag} \left[(\mathbf{p} - \mathbf{b}_1)^T \mathbf{E}(\mathbf{b}_1 - \mathbf{a}_1), (\mathbf{p} - \mathbf{b}_2)^T \mathbf{E}(\mathbf{b}_2 - \mathbf{a}_2) \right]$$

Matrices $\mathbf{A}_{J1}$ and $\mathbf{B}_{J1}$ are the forward and inverse Jacobians of the two-DOF planar PM under joint-coupling.

Figure 3.2: Inverse singularity postures of two-DOF planar PM

3.1.1 Inverse Singularity

Inverse singularity of PMs refers to special postures where the end-effector point P loses one or more degree of freedom instantaneously. From Eq. (3.6), Eq. (3.7) and Eq. (3.10), it is found that all inverse Jacobian are identical, i. e.,

$$\mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_{J1}$$

Hence, the determinant of these matrices are also equal:

$$\det(\mathbf{B}_1) = \det(\mathbf{B}_2) = \det(\mathbf{B}_{J1}) = [(\mathbf{p} - \mathbf{b}_1)^T \mathbf{E}(\mathbf{b}_1 - \mathbf{a}_1)][(\mathbf{p} - \mathbf{b}_2)^T \mathbf{E}(\mathbf{b}_2 - \mathbf{a}_2)] = 0. \quad (3.11)$$

From Eq. (3.11), it is apparent that inverse singularities occur when points A_1, B_1 and P (or A_2, B_2 and P) are aligned in a straight as shown in Fig. 3.2. This geometrical condition assures that neither joint-coupling nor a different actuation scheme will affect the inverse singularities of the manipulators.

Figure 3.3: Direct singularity postures of two-DOF planar PM

3.1.2 Direct Singularity

Direct singularity of a PM refers to special manipulator configurations in which operation point P can still move even though the active joints are all locked. Direct singularity occurs when the three joints B_1B_2, P are aligned as shown in Fig. 3.3. In other words, the direct singularity of the two-DOF planar PM without joint-coupling occurs when

$$\det(\mathbf{A}_1) = 0 \quad \text{or} \quad \det(\mathbf{A}_2) = 0 \quad (3.12)$$

For the two-DOF planar PM with joint-coupling, direct singularity occurs when

$$\det(\mathbf{A}_{J1}) = 0 \quad (3.13)$$

As indicated in Eq. (3.10), the matrix $\mathbf{A}_{J1}$ is a function of the joint-coupling coefficients, c_{ij} (for $i = 1, 2$ and $j = 1, 2$). Here one condition was observed that direct singularity $\det(\mathbf{A}_{J1}) = 0$ could be analyzed geometrically. Denote the row vectors of $\det(\mathbf{A}_{J1})$ as

$$\mathbf{r}_1 = [c_{11}(\mathbf{p} - \mathbf{b}_1) + c_{12}(\mathbf{a}_1 - \mathbf{b}_1)]$$

$$\mathbf{r}_2 = [c_{21}(\mathbf{p} - \mathbf{b}_2) + c_{22}(\mathbf{a}_2 - \mathbf{b}_2)]$$

where $c_{11}, c_{12}, c_{21}, c_{22} \in \mathbb{R}$ are defined as the coupling coefficients. Vector $\mathbf{r}_1$ represents a linear combination of vector $(\mathbf{p} - \mathbf{b}_1)$ and $(\mathbf{a}_1 - \mathbf{b}_1)$, while vector $\mathbf{r}_2$ is linear combination of vectors $(\mathbf{p} - \mathbf{b}_2)$ and $(\mathbf{a}_2 - \mathbf{b}_2)$. The direct-singularity condition $\det(\mathbf{A}_{J1}) = 0$ implies that the two row vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ are linearly dependent. This leads to an interesting geometric interpretation of the direct singularity under joint-coupling as shown in Fig. 3.4 when vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ are parallel.

Figure 3.4: Geometric interpretation of direct singularity of two-DOF planar PM with joint-coupling

i.e., $c_{11} = c_{12} = c_{21} = c_{22} = c$, then

$$\mathbf{r}_1 = [c(\mathbf{p} + \mathbf{a}_1 - 2\mathbf{b}_1)]$$

$$\mathbf{r}_2 = [c(\mathbf{p} + \mathbf{a}_2 - 2\mathbf{b}_2)]$$

The direct singularity then occurs when the diagonals at B_1 and B_2 of the parallelograms stemming from the dyads A_1B_1P and A_2B_2P are parallel. This geometric condition provides a quick way to inspect the direct singularity of the manipulator under joint-coupling.

Example 3.1

Since the Jacobian $\mathbf{A}_{J1}$ becomes a function of the coupling coefficients, the effect of joint-coupling can be studied by observing the evolution of the direct singularity loci under different coupling coefficients. The manipulator of Fig. 3.1 is used as an example to illustrate the effect of joint-coupling on direct and inverse singularity loci of PMs. Assume that the two branches of the manipulator are identical. The joint-coupling on the two branches are identical, (i.e., $c_{11} = c_{21} = c_1$ and $c_{12} = c_{22} = c_2$), the link lengths of the manipulators are given as: $A_1B_1 = A_2B_2 = 3$, $B_1P = B_2P = 5$, $A_1A_2 = 3$, in any units of length. For ease of comparison, the direct singularity loci, i.e., the contours of $\det(\mathbf{A}_{J1}) = 0$, corresponding to different set of coupling coefficients, are plotted in the θ_{11} - θ_{21} joint space.

The topology of the singularity loci varies drastically from one extreme case with joints B_1 and B_2 active (no joint-coupling: $c_1 = 0$; $c_2 = 1$) to the other extreme case with joints A_1 and A_2 active (no joint-coupling: $c_1 = 1$; $c_2 = 0$). Figure 3.5 represents various direct singularity loci of the manipulator. Similarly singularity manifolds in the task space can be determined as shown in Fig. 3.6 [45, 46]. Both singularity manifolds (in the joint space and the task space) can be used as a singularity catalog for inferring the direct singularity conditions of the PMs with joint-coupling.

Inverse singularity loci, i.e., the contours of $\det(\mathbf{B}_{J1}) = 0$ refer to special postures where the end-effector point P loses one dof. Neither the coupling coefficients come into play nor changing the location of the active joints will affect the validity of Eq. (3.11). Therefore, we can conclude that joint-coupling has no influence on the inverse singularities of the parallel manipulator. Therefore, it is clear to see that inverse singularity loci (shown in blue color in Fig. 3.6) does not depend on the changes of coupling-coefficients.

Figure 3.5: Direct singularity loci for different coupling coefficients in the joint space

Figure 3.6: Direct singularity loci for different coupling coefficients in the task space

in matrix form:

$$\mathbf{A}_1 \dot{\mathbf{P}} = \mathbf{B}_1 \dot{\boldsymbol{\Theta}}_{i1} \quad \text{where } i = 1, 2 \quad (3.17)$$

where

$$\dot{\mathbf{P}} = \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\theta}_3 \end{bmatrix}$$

$$\dot{\boldsymbol{\Theta}}_{i1} = \begin{bmatrix} \dot{\theta}_{11} \\ \dot{\theta}_{21} \\ \dot{\theta}_{31} \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} (\mathbf{c}_1 - \mathbf{b}_1)^T & (\mathbf{b}_1 - \mathbf{c}_1)^T E(\mathbf{p} - \mathbf{c}_1) \\ (\mathbf{c}_2 - \mathbf{b}_2)^T & (\mathbf{b}_2 - \mathbf{c}_2)^T E(\mathbf{p} - \mathbf{c}_2) \\ (\mathbf{c}_3 - \mathbf{b}_3)^T & (\mathbf{b}_3 - \mathbf{c}_3)^T E(\mathbf{p} - \mathbf{c}_3) \end{bmatrix}$$

$$\mathbf{B}_1 = \text{diag} \left[(\mathbf{c}_1 - \mathbf{b}_1)^T \mathbf{E}(\mathbf{b}_1 - \mathbf{a}_1), (\mathbf{c}_2 - \mathbf{b}_2)^T \mathbf{E}(\mathbf{b}_2 - \mathbf{a}_2), (\mathbf{c}_3 - \mathbf{b}_3)^T \mathbf{E}(\mathbf{b}_3 - \mathbf{a}_3) \right]$$

Pre-multiplying Eq. (3.15) with $(\mathbf{b}_i - \mathbf{a}_i)^T$, the joint rate $\dot{\theta}_{11}$ is eliminated.

$$(\mathbf{b}_i - \mathbf{a}_i)^T \dot{\mathbf{p}} - \dot{\theta}_{13} (\mathbf{b}_i - \mathbf{a}_i)^T \mathbf{E}(\mathbf{p} - \mathbf{c}_i) = \dot{\theta}_{12} (\mathbf{b}_i - \mathbf{a}_i)^T \mathbf{E}(\mathbf{c}_i - \mathbf{b}_i) \quad i = 1, 2, 3. \quad (3.18)$$

Re-writing Eq. (3.16) in matrix form,

$$\mathbf{A}_2 \dot{\mathbf{P}} = \mathbf{B}_2 \dot{\boldsymbol{\Theta}}_{i2} \quad \text{where } i = 1, 2 \quad (3.19)$$

where

$$\dot{\mathbf{P}} = \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\theta}_3 \end{bmatrix}$$

$$\dot{\boldsymbol{\Theta}}_{i2} = \begin{bmatrix} \dot{\theta}_{12} \\ \dot{\theta}_{22} \\ \dot{\theta}_{32} \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} (\mathbf{b}_1 - \mathbf{a}_1)^T & (\mathbf{a}_1 - \mathbf{b}_1)^T E(\mathbf{p} - \mathbf{c}_1) \\ (\mathbf{b}_2 - \mathbf{a}_2)^T & (\mathbf{a}_2 - \mathbf{b}_2)^T E(\mathbf{p} - \mathbf{c}_2) \\ (\mathbf{b}_3 - \mathbf{a}_3)^T & (\mathbf{a}_3 - \mathbf{b}_3)^T E(\mathbf{p} - \mathbf{c}_3) \end{bmatrix}$$

$$\mathbf{B}_2 = \text{diag} \left[-(\mathbf{c}_1 - \mathbf{b}_1)^T \mathbf{E}(\mathbf{b}_1 - \mathbf{a}_1), -(\mathbf{c}_2 - \mathbf{b}_2)^T \mathbf{E}(\mathbf{b}_2 - \mathbf{a}_2), -(\mathbf{c}_3 - \mathbf{b}_3)^T \mathbf{E}(\mathbf{b}_3 - \mathbf{a}_3) \right]$$

The coupling of joints A_i and B_i (for $i = 1, 2, 3$) produces the relationship:

$$\omega_i = \sum_{j=1}^2 c_{ij} \dot{\theta}_{ij}, \quad i = 1, 2, 3. \quad (3.21)$$

where ω_1 , ω_2 and ω_3 are the input angular velocities of three sets of joint-coupling. For illustration purpose, the same coupling coefficients are applied to all three sets of joint-coupling devices: Let $c_{11} = c_{21} = c_{31} = c_1$ and $c_{12} = c_{22} = c_{32} = c_2$. The similar approach for the two-DOF planar PM is used in this section. First, joint rates at B_i is eliminated and then the joint rates at A_i is eliminated. The relation between the joint rates at joint-coupling and the velocity of point P can be written as

$$\mathbf{A}_{J2} \dot{\mathbf{P}} = \mathbf{B}_{J2} \boldsymbol{\Omega} \quad (3.22)$$

where

$$\begin{aligned} \boldsymbol{\Omega} &= \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T \\ \mathbf{A}_{J2} &= \begin{bmatrix} (c_1(\mathbf{c}_1 - \mathbf{b}_1) + c_2(\mathbf{b}_1 - \mathbf{a}_1))^T, c_1(\mathbf{c}_1 - \mathbf{b}_1)^T \mathbf{E}(\mathbf{p} - \mathbf{c}_1) + c_2(\mathbf{b}_1 - \mathbf{a}_1)^T E(\mathbf{p} - \mathbf{c}_1) \\ (c_1(\mathbf{c}_2 - \mathbf{b}_2) + c_2(\mathbf{b}_2 - \mathbf{a}_2))^T, c_1(\mathbf{c}_2 - \mathbf{b}_2)^T \mathbf{E}(\mathbf{p} - \mathbf{c}_2) + c_2(\mathbf{b}_2 - \mathbf{a}_2)^T E(\mathbf{p} - \mathbf{c}_2) \\ (c_1(\mathbf{c}_3 - \mathbf{b}_3) + c_2(\mathbf{b}_3 - \mathbf{a}_3))^T, c_1(\mathbf{c}_3 - \mathbf{b}_3)^T \mathbf{E}(\mathbf{p} - \mathbf{c}_3) + c_2(\mathbf{b}_3 - \mathbf{a}_3)^T E(\mathbf{p} - \mathbf{c}_3) \end{bmatrix} \\ \mathbf{B}_{J2} &= \text{diag} \left[-(\mathbf{c}_1 - \mathbf{b}_1)^T \mathbf{E}(\mathbf{b}_1 - \mathbf{a}_1), -(\mathbf{c}_2 - \mathbf{b}_2)^T \mathbf{E}(\mathbf{b}_2 - \mathbf{a}_2), -(\mathbf{c}_3 - \mathbf{b}_3)^T \mathbf{E}(\mathbf{b}_3 - \mathbf{a}_3) \right] \end{aligned}$$

3.2.1 Inverse Singularity

Inverse singularities of the PM occur when $\det(\mathbf{B}_1) = \det(\mathbf{B}_2) = \det(\mathbf{B}_{J2}) = 0$. The determinants of the inverse Jacobians of the three-DOF planar PM with or without joint-coupling are identical. Consequently, an inverse singularity arises whenever any branch is in a fully stretched out or folded back configuration of the manipulator.

3.2.2 Direct Singularity

Direct singularities of the three-DOF planar PM occur when $\det(\mathbf{A}_{J2}) = 0$. In this singularity condition, the moving platform gains one or more degrees of freedom and cannot withstand any external moment about the center of the moving platform of the PM. The direct Jacobian $\mathbf{A}_{J2}$ becomes a function of the coupling coefficients, the effect of joint-coupling is studied by observing the evolution of the singularity loci under different coupling coefficients.

Example 3.2

The three-DOF PM shown in Fig. 3.7 is used as an illustration. Assuming that three branches of the manipulator are identical, the joint-coupling on all branches are identical as well, i.e., $c_{11} = c_{21} = c_{31} = c_1$; $c_{12} = c_{22} = c_{32} = c_2$. The following kinematic parameters are used.

$$A_1B_1 = A_2B_2 = A_3B_3 = 2$$

$$B_1C_1 = B_2C_2 = B_3C_3 = 3$$

$$C_1C_2 = C_2C_3 = C_3C_1 = 2.5$$

$$A_1A_2 = A_2A_3 = A_3A_1 = 5.$$

For ease of comparison, the direct singularity loci corresponding to different set of coupling coefficients, are plotted in the θ_{11} - θ_{21} - θ_{31} joint space. As the singularity loci of this manipulator are essentially 2-D manifolds in a 3-D joint space, sections of the 3-D joint space and the singularity loci are illustrated in the direction of θ_{11} (i.e., with θ_{11} fixed at different values). The topology of the singularity loci varies from one extreme case with joints B_1, B_2 and B_3 active (no joint-coupling: $c_1 = 0$; $c_2 = 1$) to the other extreme case with joints A_1, A_2 and A_3 active (no joint-coupling: $c_1 = 1$; $c_2 = 0$). Dramatic changes of the singularity loci with different coupling coefficients (c_1, c_2) at a fixed angle θ_{11} can be observed. The evolution of the direct singularity loci in the joint space and task space of the three-DOF manipulator under different coupling coefficients is shown in Fig. 3.8 to Fig. 3.11.

Figure 3.8: Singularity manifolds of three-DOF planar PM with joint-coupling

Figure 3.9: Singularity manifolds of three-DOF planar PM with joint-coupling ($\alpha = 5.0$)

Figure 3.10: Singularity manifolds of three-DOF planar PM with joint-coupling ($\alpha = 2.5$)

Figure 3.11: Singularity manifolds of three-DOF planar PM with joint-coupling ($\alpha = 1.25$)

3.3 Summary

Singularity analysis of planar PMs with joint-coupling has been introduced in this chapter. The instantaneous kinematics was employed for the derivation of the inverse and direct (forward) Jacobians of the planar PM with joint-coupling. Depending on which matrix is singular, the inverse and the direct singularity occur. The joint-coupling cannot affect on the inverse singularity conditions, which are in the boundaries of the workspace. However, the direct singularities of the manipulators depend on the coupling coefficients with the aid of two-DOF and three-DOF planar PM examples. Different singularity loci appeared based on applied coupled coefficients. Hence, it is possible to shape the singularity loci of the manipulator by only changing the coupling coefficients without altering the manipulator architecture. Extensive works on singularity analysis of planar PMs will be carried out to obtain appropriate coupling coefficients in the next chapter.

Chapter 4

Dynamic Analysis of Planar Parallel Manipulator with Joint-Coupling

Dynamics models plays an important role in the control of planar PMs in applications. In this chapter, the dynamics of planar PMs with joint-coupling is studied. In the manipulator with joint-coupling, the actuator torques are distributed among the coupled joints. Hence, the dynamic behavior of the manipulator and the trajectory planning will be different from those manipulators without joint-coupling. Furthermore, the coupling coefficients of the joint-coupling provide an additional way to adjust the dynamic performance of the manipulator. This will be seen in the next chapter as the adjustment of coupling coefficients for different task requirements. There are two major approaches to formulate the dynamics of planar PM, namely, Newton-Euler and Lagrangian formulations.

Do and Yang [47], Guglielmetti and Longchamp [48], Cheng [49], Lin and Song [50] introduced the Newton-Euler formulation for PMs. The Newton-Euler formulation requires the equations of motion to be obtained for each component of the manipulator, which generally leads to a large number of equations. The Lagrangian formulation is applied to PM dynamics by several researchers, for instance, Pfreundschuh *et al* [51], Kosuge *et al* [52], Pfreundschuh [53] and Stamper [54]. The Lagrangian formation eliminates unwanted reac-

tion forces and moments at the outset. However, due to the numerous constraints imposed by the closed-loop structure of the PM, deriving explicit equations of motion in terms of a set of independent generalized coordinates becomes a challenging task. Therefore, additional coordinates along with the set of Lagrangian multipliers are often introduced. The principle of virtual work and D'Alembert's principle are also applied to dynamic analysis of PMs in [55, 56]. The principle of virtual work determines states in static equilibrium by allowing infinitesimal displacements. One can equate the work done in Cartesian terms with the work done in joint space terms [57]. In a manipulator with joint-coupling, the number of input actuating scheme is set to be equal to the number of mobility of the manipulator, and generally there are more active joints than the mobility of the manipulator. These joints are either *partially actuated* or passive. The additional velocity constraints imposed by the joint-coupling devices can be described as

$$\omega_i = c_{i1}\dot{\theta}_i + c_{i2}\dot{\phi}_i. \quad (4.1)$$

Dynamic analysis for redundant actuation was introduced by Cheng *et al* [58], Park *et al* [59], Cheng [60] and Nakamura and Ghodoussi [61]. They analyzed the advantages of using redundancy in PMs and constructed the dynamical model of redundantly actuated closed-chain mechanisms by applying the Lagrange-D'Alembert formulations.

In this chapter, the dynamics of the planar PM with joint-coupling is formulated using the Lagrangian approach and the equation of motion of this type of manipulators is derived. The two-DOF planar PM is used as an example for the dynamic analysis of a manipulator with joint-coupling. Then the dynamic performance of the manipulator will be studied from the simulation examples.

4.1 Formulation of Lagrangian Equations

There are two kinds of dynamic formulations for a PM based on the task requirements. The first is forward dynamics in which the motor torques are given in order to find the position, velocity and acceleration of the end-effector. The second is inverse dynamics in

which the position, velocity and acceleration of the end-effector are given in order to find the required motor torques. For a PM, the Lagrangian equation can be written as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_{ij}}\right) - \frac{\partial L}{\partial \mathbf{q}_{ij}} = \sum \lambda_i \frac{\partial \Gamma_i}{\partial \mathbf{q}_{ij}} + \tau_{ij} \quad \text{for } i = 1, 2; j = 1, 2 \quad (4.2)$$

where L is Lagrange, $\mathbf{q}$ is generalized coordinates, λ_i is the Lagrangian multiplier, Γ_i represents the constraints, and τ is an external force or the torque requirement in the generalized coordinate. Therefore, the formulation requires a set of constraint equations derived from the kinematics of the manipulator. In the two-DOF planar manipulator shown in Fig. 4.1, m sets of joint-coupling (where $m = 2$) are employed at m branches and each set of joint-coupling couples n , (where $n = 2$ for 2-DOF manipulator) joints together as an active joint group. Thus, the number of all partially actuated joints in the manipulator with joint-coupling become $m \times n$. In Eq. (4.2), Lagrange L is defined as the difference between the kinetic energy K and the potential energy U .

$$L = K - U. \quad (4.3)$$

Figure 4.1: Dynamic model of two-DOF planar PM with joint-coupling

4.2 Formulation of Equation of Motion

In this section, the direct (forward) dynamic problem of the two-DOF planar PM with joint-coupling is highlighted. The equation of motion of the two-DOF planar PM with joint-coupling is formulated.

4.2.1 Kinetic Energy

The moment of inertia of link j on the branch i is denoted as I_{ij} . Assuming that the mass m_{ij} of link j on branch i is at the center point of the link, m_p is defined as the mass of the end-effector (moving platform) and $\mathbf{v}_{ij}$ is translational velocity at center of link j on the branch i . The kinematic energy can be written as

$$K_{ij} = \frac{1}{2}m_{ij}\mathbf{v}_{ij}\mathbf{v}_{ij}^T + \frac{1}{2}I_{ij}\dot{\theta}_{ij}^2 \quad \text{where } i = 1, 2 \text{ and } j = 1, 2 \quad (4.4)$$

Specifically

$$\begin{aligned} K_p &= \frac{1}{2}m_p(\dot{x}_p^2 + \dot{y}_p^2) \\ K_{11} &= \frac{1}{2}(I_{11} + 0.25m_{11}l_{11}^2)\dot{\theta}_{11}^2, \\ K_{12} &= \frac{1}{2}m_{12}l_{11}^2\dot{\theta}_{11}^2 + \frac{1}{2}m_{12}(I_{12} + 0.25m_{12}l_{12}^2)\dot{\theta}_{12}^2 + \frac{1}{2}m_{12}l_{11}l_{12}\cos(\theta_{11} - \theta_{12})\dot{\theta}_{11}\dot{\theta}_{12} \\ K_{21} &= \frac{1}{2}(I_{21} + 0.25m_{21}l_{21}^2)\dot{\theta}_{21}^2, \\ K_{22} &= \frac{1}{2}m_{22}l_{21}^2\dot{\theta}_{21}^2 + \frac{1}{2}m_{22}(I_{22} + 0.25m_{22}l_{22}^2)\dot{\theta}_{22}^2 + \frac{1}{2}m_{22}l_{21}l_{22}\cos(\theta_{21} - \theta_{22})\dot{\theta}_{21}\dot{\theta}_{22} \end{aligned}$$

4.2.2 Potential Energy

Assume that the potential energy of the end-effector (moving platform) U_p and the potential energy of link j on branch i is U_{ij} . The total potential energy U of the planar PM is the

summation of potential energies from individual links and the end-effector, given as

$$U = U_p + \sum_{j=1}^2 \sum_{i=1}^2 U_{ij} \quad (4.5)$$

$$U_p = m_p \mathbf{g} p_z \quad (4.6)$$

$$U_{ij} = m_{ij} \mathbf{g} p_z \quad \text{for } i = 1, 2; j = 1, 2$$

where $\mathbf{g}$ is the gravitational acceleration and p_z is the distance change along the end-effector of the manipulator along z -direction. Assuming that the planar PM moves along a plane perpendicular to the direction of the gravitational acceleration, then there is no distance change in z -direction during the motion of the planar PM. Hence,

$$U_p = 0 \quad \text{and} \quad U_{ij} = 0 \quad \text{for } i = 1, 2; j = 1, 2 \quad (4.7)$$

4.2.3 Lagrangian Formulation

In this study, Lagrange's formulation is employed by introducing two redundant coordinates, x_p and y_p i.e., the coordinate of the end-effector P , in order to find the Lagrangian multipliers, λ_i ($i = 1, 2$). Due to the absence of potential energy U , the Lagrangian formulation Eq. (4.3) becomes

$$L = K_p + K_{11} + K_{12} + K_{21} + K_{22}. \quad (4.8)$$

Taking the derivatives of Lagrangian function with respect to generalized coordinates, the left hand side of Eq. (4.2) becomes

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_p} \right) - \frac{\partial L}{\partial x_p} = m_p \ddot{x}_p \quad (4.9)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}_p} \right) - \frac{\partial L}{\partial y_p} = m_p \ddot{y}_p \quad (4.10)$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_{11}} \right) - \frac{\partial L}{\partial \theta_{11}} &= (I_{11} + 0.25m_{11}l_{11}^2 + m_{12}l_{12}^2)\ddot{\theta}_{11} + (I_{12} + 0.5m_{12}l_{11}l_{22} \cos(\theta_{11} - \theta_{12}))\ddot{\theta}_{12} \\ &\quad - 0.5m_{12}l_{11}l_{12} \sin(\theta_{11} - \theta_{12})\dot{\theta}_{12}^2 \end{aligned} \quad (4.11)$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_{12}} \right) - \frac{\partial L}{\partial \theta_{12}} &= (0.5m_{12}l_{11}l_{12} \cos(\theta_{11} - \theta_{12}))\ddot{\theta}_{11} + (I_{12} + 0.25m_{12}l_{12}^2)\ddot{\theta}_{12} \\ &\quad - 0.5m_{12}l_{11}l_{12} \sin(\theta_{11} - \theta_{12})\dot{\theta}_{11}^2 \end{aligned} \quad (4.12)$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_{21}}\right) - \frac{\partial L}{\partial \theta_{21}} &= (I_{21} + 0.25m_{21}l_{21}^2 + m_{22}l_{22}^2)\ddot{\theta}_{21} + (I_{22} + 0.5m_{22}l_{21}l_{22}\cos(\theta_{21} - \theta_{22}))\ddot{\theta}_{22} \\ &\quad - 0.5m_{22}l_{21}l_{22}\sin(\theta_{21} - \theta_{22})\dot{\theta}_{22}^2 \end{aligned} \quad (4.13)$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_{22}}\right) - \frac{\partial L}{\partial \theta_{22}} &= (0.5m_{22}l_{21}l_{22}\cos(\theta_{21} - \theta_{22}))\ddot{\theta}_{21} + (I_{22} + 0.25m_{22}l_{22}^2)\ddot{\theta}_{22} \\ &\quad - 0.5m_{22}l_{21}l_{22}\sin(\theta_{21} - \theta_{22})\dot{\theta}_{21}^2 \end{aligned} \quad (4.14)$$

Generally the dynamic analysis can be accomplished by using two generalized coordinates as there are only two actuator inputs. However, joint-coupling results in more active joints than the actuators, hence this formulation requires two more constraint equations. Kinematic constraint equations Γ_i are applied as:

$$\begin{aligned} \Gamma_i &= (x_p - x_{bi})^2 + (y_p - y_{bi})^2 - l_{i2}^2 \\ &= (x_p + a - l_{i1}\cos\theta_{i1})^2 + (y_p - l_{i1}\sin\theta_{i1})^2 - l_{i2}^2 \\ &= 0 \quad \text{for } i = 1, 2 \end{aligned} \quad (4.15)$$

Taking the partial derivatives of the constraint function Γ_i from Eq. (4.15) with respect to the generalized coordinates, we have

$$\begin{aligned} \frac{\partial \Gamma_1}{\partial x_p} &= 2a + 2x_p - 2l_{11}\cos\theta_{11} \\ \frac{\partial \Gamma_2}{\partial x_p} &= -2a + 2x_p - 2l_{21}\cos\theta_{21} \\ \frac{\partial \Gamma_1}{\partial y_p} &= 2y_p - 2l_{11}\sin\theta_{11} \\ \frac{\partial \Gamma_2}{\partial y_p} &= 2y_p - 2l_{21}\sin\theta_{21} \\ \frac{\partial \Gamma_1}{\partial \theta_{11}} &= -2l_{11}y_p\cos\theta_{11} + 2al_{11}\sin\theta_{11} + 2l_{11}x_p\sin\theta_{11} \\ \frac{\partial \Gamma_2}{\partial \theta_{11}} &= 0 \\ \frac{\partial \Gamma_1}{\partial \theta_{12}} &= 0, \quad \frac{\partial \Gamma_2}{\partial \theta_{12}} = 0 \\ \frac{\partial \Gamma_1}{\partial \theta_{21}} &= 0, \quad \frac{\partial \Gamma_2}{\partial \theta_{21}} = -2l_{21}y_p\cos\theta_{21} - 2al_{21}\sin\theta_{21} + 2l_{11}x_p\sin\theta_{21} \\ \frac{\partial \Gamma_1}{\partial \theta_{22}} &= 0, \quad \frac{\partial \Gamma_2}{\partial \theta_{22}} = 0 \end{aligned}$$

Assume that there is no external force exerted on the end-effector as indicated in Eq. (4.2). The Lagrangian multipliers λ_i in terms of joint angles, the position and acceleration the

end-effector which are given for forward dynamic case can be obtained as

$$\lambda_1 = \frac{\ddot{y}_p m_p S_2 - \ddot{x}_p m_p S_4}{S_2 S_3 - S_1 S_4} \quad (4.16)$$

$$\lambda_2 = \frac{\ddot{x}_p m_p S_3 - \ddot{y}_p m_p S_1}{S_2 S_3 - S_1 S_4} \quad (4.17)$$

where

$$S_1 = 2a + 2x_p - 2l_{11} \cos \theta_{11}$$

$$S_2 = -2a + 2x_p - 2l_{21} \cos \theta_{21}$$

$$S_3 = 2y_p - 2l_{11} \sin \theta_{11}$$

$$S_4 = 2y_p - 2l_{21} \sin \theta_{21}.$$

Once the Lagrange multipliers are found, the required motor torques for each active joint can be determined from Eq. (4.11) through Eq. (4.14).

$$\begin{aligned} \tau_{11} = & (I_{11} + 0.25m_{11}l_{11}^2 + m_{12}l_{12}^2)\ddot{\theta}_{11} + (I_{12} + 0.5m_{12}l_{11}l_{22} \cos(\theta_{11} - \theta_{12}))\ddot{\theta}_{12} - 0.5m_{12}l_{11}l_{12} \\ & \sin(\theta_{11} - \theta_{12})\dot{\theta}_{12}^2 + \lambda_1(-2l_{11}y_p \cos \theta_{11} + 2al_{11} \sin \theta_{11} + 2l_{11}x_p \sin \theta_{11}) \end{aligned} \quad (4.18)$$

$$\tau_{12} = 0.5m_{12}l_{11}l_{12}(\cos(\theta_{11} - \theta_{12})\ddot{\theta}_{11} - \sin(\theta_{11} - \theta_{12})\dot{\theta}_{11}^2) + (I_{12} + 0.25m_{12}l_{12}^2)\ddot{\theta}_{12} \quad (4.19)$$

$$\begin{aligned} \tau_{21} = & (I_{21} + 0.25m_{21}l_{21}^2 + m_{22}l_{22}^2)\ddot{\theta}_{21} + (I_{22} + 0.5m_{22}l_{21}l_{22} \cos(\theta_{21} - \theta_{22}))\ddot{\theta}_{22} - 0.5m_{22}l_{21}l_{22} \\ & \sin(\theta_{21} - \theta_{22})\dot{\theta}_{22}^2 + \lambda_2(-2l_{11}y_p \cos \theta_{11} + 2al_{11} \sin \theta_{11} + 2l_{11}x_p \sin \theta_{11}) \end{aligned} \quad (4.20)$$

$$\tau_{22} = 0.5m_{22}l_{21}l_{22}(\cos(\theta_{21} - \theta_{22})\ddot{\theta}_{21} - \sin(\theta_{21} - \theta_{22})\dot{\theta}_{21}^2) + (I_{22} + 0.25m_{22}l_{22}^2)\ddot{\theta}_{22} \quad (4.21)$$

Since joint-coupling devices are employed in the manipulator, the manipulator inputs become the input of joint-coupling ω_i (for $i = 1, 2$). In order to show Eq. (4.18) to Eq. (4.21) in terms of ω_i and $\dot{\omega}_i$ of the joint-coupling devices, the velocity and acceleration relationships between the coupled joints and input motors are needed to be determined.

4.2.4 Velocity and Acceleration Analysis

From the vector loop equations, kinematic constraints of the two-DOF planar manipulator can be written as

$$l_{11} \cos \theta_{11} + l_{12} \cos \theta_{12} - l_{21} \cos \theta_{21} - l_{22} \cos \theta_{22} - 2a = 0 \quad (4.22)$$

$$l_{11} \sin \theta_{11} + l_{12} \sin \theta_{12} - l_{21} \sin \theta_{21} - l_{22} \sin \theta_{22} = 0 \quad (4.23)$$

Differentiating Eq. (4.22) and Eq. (4.23) with respect to time, the instantaneous relationship becomes

$$-l_{11} \sin \theta_{11} \dot{\theta}_{11} - l_{12} \sin \theta_{12} \dot{\theta}_{12} + l_{21} \sin \theta_{21} \dot{\theta}_{21} + l_{22} \sin \theta_{22} \dot{\theta}_{22} = 0 \quad (4.24)$$

$$l_{11} \cos \theta_{11} \dot{\theta}_{11} + l_{12} \cos \theta_{12} \dot{\theta}_{12} - l_{21} \cos \theta_{21} \dot{\theta}_{21} - l_{22} \cos \theta_{22} \dot{\theta}_{22} = 0. \quad (4.25)$$

Differentiating Eq. (4.24) and Eq. (4.25) with respect to time again, the joint acceleration relationship is obtained as

$$\begin{aligned} l_{11} \sin \theta_{11} \ddot{\theta}_{11} + l_{12} \sin \theta_{12} \ddot{\theta}_{12} - l_{11} \cos \theta_{11} \dot{\theta}_{11} + l_{12} \cos \theta_{12} \dot{\theta}_{12} - l_{21} \sin \theta_{21} \ddot{\theta}_{21} - l_{22} \sin \theta_{22} \ddot{\theta}_{22} \\ + l_{21} \cos \theta_{21} \dot{\theta}_{21} - l_{22} \cos \theta_{22} \dot{\theta}_{22} = 0 \end{aligned} \quad (4.26)$$

$$\begin{aligned} l_{11} \cos \theta_{11} \ddot{\theta}_{11} + l_{12} \cos \theta_{12} \ddot{\theta}_{12} - l_{11} \sin \theta_{11} \dot{\theta}_{11} + l_{12} \sin \theta_{12} \dot{\theta}_{12} - l_{21} \cos \theta_{21} \ddot{\theta}_{21} - l_{22} \cos \theta_{22} \ddot{\theta}_{22} \\ + l_{21} \sin \theta_{21} \dot{\theta}_{21} - l_{22} \sin \theta_{22} \dot{\theta}_{22} = 0 \end{aligned} \quad (4.27)$$

Hence, $\dot{\theta}_{i2}$ can be written in terms of the base angle rate $\dot{\theta}_{i1}$ (for $i = 1, 2$):

$$\dot{\theta}_{12} = Q_{11} \dot{\theta}_{11} + Q_{12} \dot{\theta}_{21} \quad (4.28)$$

$$\dot{\theta}_{22} = Q_{21} \dot{\theta}_{11} + Q_{22} \dot{\theta}_{21} \quad (4.29)$$

Similarly, $\ddot{\theta}_{i2}$ can be written in terms of the base angle acceleration $\ddot{\theta}_{i1}$ (for $i = 1, 2$):

$$\ddot{\theta}_{12} = R_{11} \ddot{\theta}_{11} + R_{12} \ddot{\theta}_{21} + R_{13} \quad (4.30)$$

$$\ddot{\theta}_{22} = R_{21} \ddot{\theta}_{11} + R_{22} \ddot{\theta}_{21} + R_{23} \quad (4.31)$$

The complete expressions of Q_{ij} , G_{ij} , R_{ij} , N_{ij} and F_{ij} are provided in Appendix A. Applying joint-coupling effect to the manipulator, the input-output velocity and acceleration relationships of joint-coupling can be written as:

$$\omega_1 = c_{11} \dot{\theta}_{11} + c_{12} \dot{\theta}_{12} \quad (4.32)$$

$$\omega_2 = c_{21} \dot{\theta}_{21} + c_{22} \dot{\theta}_2 \quad (4.33)$$

$$\dot{\omega}_1 = c_{11} \ddot{\theta}_{11} + c_{12} \ddot{\theta}_{12} \quad (4.34)$$

$$\dot{\omega}_2 = c_{21} \ddot{\theta}_{21} + c_{22} \ddot{\theta}_{22} \quad (4.35)$$

Combining Eqs. (4.28), (4.29), (4.32) and (4.33) together, the velocity relationship of the joint-coupling of the manipulator becomes

$$\dot{\Theta} = \mathbf{H}^{-1} \mathbf{\Omega} \quad (4.36)$$

where $\mathbf{H}$ is a 2×2 velocity relation matrix of joint-coupling,

$$\mathbf{H} = \begin{bmatrix} (c_{11} + c_{12}Q_{11}) & c_{12}Q_{12} \\ (c_{21} + c_{22}Q_{21}) & c_{22}Q_{22} \end{bmatrix}, \quad \dot{\boldsymbol{\Theta}} = \begin{bmatrix} \dot{\theta}_{11} \\ \dot{\theta}_{21} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

Similarly, combining Eqs. (4.30),(4.31),(4.34) and (4.35) together, the acceleration relationship of the joint-coupling of the manipulator can be obtained as

$$\ddot{\boldsymbol{\Theta}} = \mathbf{D}^{-1}\mathbf{W} \quad (4.37)$$

where $\mathbf{D}$ is a 2×2 acceleration relation matrix of joint-coupling,

$$\mathbf{D} = \begin{bmatrix} (c_{11} + c_{12}R_{11}) & c_{12}R_{12} \\ (c_{21} + c_{22}R_{21}) & c_{22}R_{22} \end{bmatrix}, \quad \ddot{\boldsymbol{\Theta}} = \begin{bmatrix} \ddot{\theta}_{11} \\ \ddot{\theta}_{21} \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} \dot{\omega}_1 - c_{12}R_{13} \\ \dot{\omega}_2 - c_{22}R_{23} \end{bmatrix}$$

Substituting Eq. (4.28)~(4.31) into Eq. (4.18)~(4.21), equations of motion in terms of $\dot{\theta}_{i1}$ and $\ddot{\theta}_{i1}$ ($i = 1, 2$) can be obtained as:

$$\boldsymbol{\tau}_{ij} = \mathbf{F}(\theta_{ij})\ddot{\boldsymbol{\Theta}} + \mathbf{N}(\dot{\theta}_{ij}, \theta_{ij}) \quad (4.38)$$

$$\text{where } \ddot{\boldsymbol{\Theta}} = \begin{bmatrix} \ddot{\theta}_{11} \\ \ddot{\theta}_{21} \end{bmatrix}, \quad \boldsymbol{\tau}_{ij} = \begin{bmatrix} \tau_{11} \\ \tau_{12} \\ \tau_{21} \\ \tau_{22} \end{bmatrix}, \quad \mathbf{F}(\theta_{ij}) = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \\ F_{31} & F_{32} \\ F_{41} & F_{42} \end{bmatrix}, \quad \mathbf{N}(\dot{\theta}_{ij}, \theta_{ij}) = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix}$$

Now required joint torques are shown in term of the manipulator input angular velocities ω_i and accelerations $\dot{\omega}_i$ by substituting Eqs. (4.36) and Eq. (4.37) into Eq. (4.38).

$$\boldsymbol{\tau}_{ij} = \mathbf{F}(\theta_{ij})\mathbf{D}^{-1}\mathbf{W} + \mathbf{N}(\dot{\theta}_{ij}, \theta_{ij}) \quad (4.39)$$

Re-arranging Eq. (4.39), the equation of motion becomes

$$\boldsymbol{\tau}_{ij} = \mathbf{Z}(c_{ij}, \theta_{ij})\dot{\boldsymbol{\Omega}} + \mathbf{V}(c_{ij}, \omega_i, \theta_{ij}) \quad (4.40)$$

$$\text{where } \boldsymbol{\tau}_{ij} = \begin{bmatrix} \tau_{11} \\ \tau_{12} \\ \tau_{21} \\ \tau_{22} \end{bmatrix}, \quad \dot{\boldsymbol{\Omega}} = \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix}, \quad \mathbf{Z}(c_{ij}, \theta_{ij}) = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \\ Z_{31} & Z_{32} \\ Z_{41} & Z_{42} \end{bmatrix}, \quad \mathbf{V}(c_{ij}, \omega_i, \theta_{ij}) = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\begin{aligned}
Z_{11} &= \frac{1}{r} \{ (-c_{21} - c_{22}Q_{21})R_{12}G_{12} + c_{22}Q_{22}(G_{11} + R_{11}G_{12}) \} \\
Z_{12} &= \frac{1}{r} \{ (c_{11} + c_{12}Q_{11})R_{12}G_{12} - c_{12}Q_{12}(G_{11} + R_{11}G_{12}) \} \\
Z_{21} &= \frac{1}{r} \{ (-c_{21} - c_{22}Q_{21})R_{12}G_{22} + c_{22}Q_{22}(G_{21} + R_{11}G_{22}) \} \\
Z_{22} &= \frac{1}{r} \{ (c_{11} + c_{12}Q_{11})R_{12}G_{22} - c_{12}Q_{12}(G_{21} + R_{11}G_{22}) \} \\
Z_{31} &= \frac{1}{r} \{ c_{22}Q_{22}R_{21}G_{32} + (-c_{21} - c_{22}Q_{21})(G_{31} + R_{22}G_{32}) \} \\
Z_{32} &= \frac{1}{r} \{ -c_{12}Q_{12}R_{21}G_{32} + (-c_{11} + c_{12}Q_{11})(G_{31} + R_{22}G_{32}) \} \\
Z_{41} &= \frac{1}{r} \{ c_{22}Q_{22}R_{21}G_{42} + (-c_{21} - c_{22}Q_{21})(G_{41} + R_{22}G_{42}) \} \\
Z_{42} &= \frac{1}{r} \{ -c_{12}Q_{12}R_{21}G_{42} + (-c_{11} + c_{12}Q_{11})(G_{41} + R_{22}G_{42}) \} \\
V_1 &= \frac{1}{r} \{ [(-c_{21} - c_{22}Q_{21})R_{12}G_{12} + c_{22}Q_{22}(G_{11} + R_{11}G_{12})](-c_{12}R_{13}) + \{ R_{12}G_{12} \\
&\quad (c_{11} + c_{12}Q_{11}) - c_{12}Q_{12}(G_{11} + R_{11}G_{12}) \}(-c_{22}R_{23}) + G_{14} + G_{13}(Q_{11}\dot{\theta}_{11} + Q_{12}\dot{\theta}_{12}^2) \} \\
V_2 &= \frac{1}{r} \{ [(c_{11} + c_{12}Q_{11})R_{12}G_{22} - c_{12}Q_{12}(G_{21} + R_{11}G_{22})](-c_{12}R_{13}) \\
&\quad + \{ (c_{11} + c_{12}Q_{11})R_{12}G_{22} - c_{12}Q_{12}(G_{21} + R_{11}G_{22}) \}(-c_{22}R_{23}) + G_{23}\dot{\theta}_{11}^2 \} \\
V_3 &= \frac{1}{r} \{ [c_{22}Q_{22}R_{21}G_{32} + (-c_{21} - c_{22}Q_{21})(G_{31} + R_{22}G_{32})](-c_{12}R_{13}) - \{ c_{12}Q_{12}R_{21}G_{32} \\
&\quad - (-c_{11} + c_{12}Q_{11})(G_{31} + R_{22}G_{32}) \}(-c_{22}R_{23}) + G_{34} + G_{33}(Q_{21}\dot{\theta}_{21} + Q_{22}\dot{\theta}_{22}^2) \} \\
V_4 &= \frac{1}{r} \{ [c_{22}Q_{22}R_{21}G_{42} + (-c_{21} - c_{22}Q_{21})(G_{41} + R_{22}G_{42})](-c_{12}R_{13}) \\
&\quad + \{ -c_{12}Q_{12}R_{21}G_{42} + (-c_{11} + c_{12}Q_{11})(G_{41} + R_{22}G_{42}) \}(-c_{22}R_{23}) + G_{43}\dot{\theta}_{21}^2 \} \\
r &= c_{11}c_{22}Q_{22} + c_{12}c_{22}Q_{11}Q_{22} - c_{12}c_{21}Q_{12} - c_{12}c_{22}Q_{12}Q_{21}
\end{aligned}$$

4.2.5 Equation of Motion

The joint-coupling device is mounted on each branch of the manipulator. Thus, the required torques on the input actuator of the joint-coupling device are determined by

$$\tau_1 \omega_1 = \tau_{11} \dot{\theta}_{11} + \tau_{12} \dot{\theta}_{12} \quad (4.41)$$

$$\tau_2 \omega_2 = \tau_{21} \dot{\theta}_{21} + \tau_{22} \dot{\theta}_{22} \quad (4.42)$$

Substituting $\dot{\theta}_{12}$ and $\dot{\theta}_{22}$, Eq. (4.41) and Eq. (4.42) can be written as

$$\begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \tau_{11} + Q_{11}\tau_{12} & Q_{12}\tau_{12} \\ Q_{21}\tau_{22} & \tau_{21} + Q_{22}\tau_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_{11} \\ \dot{\theta}_{21} \end{bmatrix} \quad (4.43)$$

Substituting Eq. (4.36) into Eq. (4.43), the required torques of joint-coupling device are obtained.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega_1} & 0 \\ 0 & \frac{1}{\omega_2} \end{bmatrix} \begin{bmatrix} \tau_{11} + Q_{11}\tau_{12} & Q_{12}\tau_{12} \\ Q_{21}\tau_{22} & \tau_{21} + Q_{22}\tau_{22} \end{bmatrix} \mathbf{H}^{-1} \boldsymbol{\Omega} \quad (4.44)$$

This resulting equation of motion is in general form for the two-DOF planar PM with joint-coupling devices:

$$\boldsymbol{\tau}_{jc} = \mathbf{M}_{jc}(c_{ij}, \theta_{ij}) \dot{\boldsymbol{\Omega}} + \mathbf{V}_{jc}(c_{ij}, \theta_{ij}, \omega_i) \quad (4.45)$$

where the 2×2 Inertia matrix $\mathbf{M}_{jc}(c_{ij}, \theta_{ij})$ and the 2×1 Coriolis matrix $\mathbf{V}_{jc}(c_1, c_2, \omega_i, \theta_{ij})$ are defined as

$$\mathbf{M}_{jc}(c_1, c_2, \theta_{ij}) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad \mathbf{V}_{jc}(c_1, c_2, \omega_i, \theta_{ij}) = \begin{bmatrix} M_{13} \\ M_{23} \end{bmatrix}, \quad \dot{\boldsymbol{\Omega}} = \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix}$$

with the elements in the matrices as

$$\begin{aligned} M_{11} &= c_{11}Z_{11} + c_{11}Z_{21}Q_{11} + c_{21}Z_{21}Q_{12} + \frac{\omega_2}{\omega_1}(c_{12}Z_{11} + c_{12}Z_{21}Q_{11} + c_{22}Z_{21}Q_{12}) \\ M_{12} &= c_{11}Z_{12} + c_{11}Z_{22}Q_{11} + c_{21}Z_{22}Q_{12} + \frac{\omega_2}{\omega_1}(c_{12}Z_{12} + c_{12}Z_{22}Q_{11} + c_{22}Z_{22}Q_{12}) \\ M_{21} &= c_{22}Z_{31} + c_{12}Z_{41}Q_{21} + c_{22}Z_{41}Q_{22} + \frac{\omega_1}{\omega_2}(c_{21}Z_{31} + c_{11}Z_{41}Q_{21} + c_{21}Z_{41}Q_{22}) \\ M_{22} &= c_{22}Z_{32} + c_{12}Z_{42}Q_{21} + c_{22}Z_{42}Q_{22} + \frac{\omega_1}{\omega_2}(c_{21}Z_{32} + c_{11}Z_{412}Q_{21} + c_{21}Z_{42}Q_{22}) \\ M_{13} &= c_{11}V_1 + c_{11}Q_{11}V_2 + c_{21}Q_{12}V_2 + \frac{\omega_2}{\omega_1}(c_{12}V_1 + c_{12}Q_{11}V_2 + c_{22}Q_{12}V_2) \\ M_{23} &= \frac{\omega_1}{\omega_2}(c_{21}V_3 + c_{11}Q_{21}V_4 + c_{21}Q_{22}V_4) + (c_{22}V_3 + c_{12}Q_{21}V_4 + c_{22}Q_{22}V_4) \end{aligned}$$

4.3 Dynamic Simulation of Planar Parallel Manipulators with Joint-Coupling

Based on the equations of motion obtained from the previous section, the required joint-coupling torques for the manipulator can be computed with a given trajectory. It is shown that the required motor torque input to the joint-coupling device variate on the selection of coupling coefficients. Due to the limitation of the workspace of the manipulator, a trajectory is always chosen to be within the reachable workspace. However, there are complex singularity manifolds within the reachable workspace. The actuator torques becomes infinitely large when the manipulator reaches singularity conditions.

As shown in Chapter 3, by changing the coupling coefficients of joint-coupling devices, the geometry of the direct singularity loci can be altered. Hence, given the required tasks or the trajectory, it is possible to avoid these singularities by changing the coupling coefficients. In terms of the manipulator dynamics, the required torque from the joint-coupling device can be fine-tuned according to the coefficients based on a given trajectory or a task.

The trajectory generation for the robot manipulator has been studied extensively. Given an initial pose $\mathbf{p}_i$ and an end pose $\mathbf{p}_f$, it is possible to produce a smooth trajectory based on spline functions. To make a smooth motion, at least four constrains on $\mathbf{p}(t)$ are needed where $\mathbf{p}(t) = (x_p, y_p)$ is the trajectory of the end-effector. These four constraints can be satisfied by a third order polynomial equation [62, 63, 57]. Two constraints on the functional values can be taken from the given initial and final values. Another two constraints can be obtained by setting initial and final velocities to zero. A vector cubic polynomial has the form

$$\mathbf{p}(t) = \mathbf{a}_0 + t\mathbf{a}_1 + t^2\mathbf{a}_2 + t^3\mathbf{a}_3, \quad (4.46)$$

and the end-effector velocity and acceleration along the trajectory are clearly as follows

$$\dot{\mathbf{p}}(t) = \mathbf{a}_1 + 2t\mathbf{a}_2 + 3t^2\mathbf{a}_3, \quad (4.47)$$

$$\ddot{\mathbf{p}}(t) = 2\mathbf{a}_2 + 6t\mathbf{a}_3. \quad (4.48)$$

Substituting four constraints into Eq. (4.46) and Eq. (4.47), it yields for four vector coefficients ($\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$) unknowns.

$$\mathbf{p}_i = \mathbf{a}_0,$$

$$\mathbf{p}_f = \mathbf{a}_0 + t_f \mathbf{a}_1 + t_f^2 \mathbf{a}_2 + t_f^3 \mathbf{a}_3,$$

$$\dot{\mathbf{p}}_i = \mathbf{a}_1$$

$$\dot{\mathbf{p}}_f = \mathbf{a}_1 + 2t_f \mathbf{a}_2 + 3t_f^2 \mathbf{a}_3.$$

Now the cubic polynomial that connects any initial and final postures with desired velocities is applied for the trajectory-tracking. Desired end-effector velocities at the via points are mapped to the Jacobian of the manipulator to determine joint rates. The Jacobian can be assigned joint rates according to the joint-coupling coefficients of the manipulator. After knowing joint rates, the joint accelerations can be found.

Consider the two-DOF planar PM with joint-coupling shown in Fig. 4.1. The dynamic parameters of manipulator are given in Table. 4.1. The desired postures of the end-effector, velocity and acceleration are given in Table. 4.2.

Table 4.1: Dynamic parameters of two-DOF planar PM with joint-coupling

Link	mass (Kg)	length (m)	Inertia($Kg.m^2$)
A_1B_1	10	0.356	1.581
A_2B_2	10	0.356	1.581
B_1P	10	0.60	2.090
B_2P	10	0.60	2.090
A_1A_2	—	0.67	—

Table 4.2: Desired end-effector's positions, velocities and accelerations

—	$\mathbf{p}_1$	$\mathbf{p}_2$	$\mathbf{p}_3$	$\mathbf{p}_4$	$\mathbf{p}_5$	$\mathbf{p}_6$	$\mathbf{p}_7$
$x_p(m)$	0.14	0.075	0.0	−0.05	−0.093	−0.21	−0.376
$y_p(m)$	0.38	0.295	0.135	0.187	0.216	0.3	0.475
$\dot{\mathbf{p}} (m/s)$	0.0	0.07071	0.1224	0.1414	0.1224	0.07071	0.0
$\ddot{\mathbf{p}} (m/s^2)$	0.0	3.139	1.271	0.209	0.236	0.114	0.0

Figure 4.2: Postures of manipulator within trajectory

Figure 4.3: Trajectory in task space of manipulator without joint-coupling

Figure 4.4: Desired velocity and acceleration of end-effector

Example 4.1

Since Eq. (4.45) is a function of the coupling coefficients, the required input torques can be regulated by coupling coefficients. Different sets of coupling coefficients are applied to the trajectory shown in Fig. 4.3. For convenience, let the coupling ratio, $\frac{c_2}{c_1}$ be equal to α . There are two direct singularity postures at point 2 and 5 from Fig. 4.2 for the two-DOF planar PM. These conditions occur at 5 sec and 20sec of the sampling time. The required torques are computed according to different coupling coefficient ratio α . Figure 4.5 represents the singularity manifolds of the manipulator with $\alpha = 0.5$, 0.75 and $\alpha = 2.5$ respectively. Figure. 4.6 and 4.7 show their required torques of input actuators for the same trajectory-tracking. Results show the effectiveness of joint-coupling coefficients ratios on the dynamic model of the manipulator.

Figure 4.5: Singularity manifolds of manipulator with joint-coupling

Figure 4.6: Required input torques of Joint-Coupling

Figure 4.7: Required input torques of Joint-Coupling 2

Example 4.2

This example compares the required torques of the manipulator with and without joint-coupling. The end-effector pose at point 2 and 5 in Fig. ?? represents the direct singularity of the two-DOF planar PM. The end-effector of the manipulator without joint-coupling (i.e., $c_1 = 1, c_2 = 0$) cannot be controlled when the end-effector reaches point 2 and 5. Thus, the required torques for that pose rise to $800Nm$ and $-25Nm$. Using a proper joint-coupling coefficient ratio, less input torques are required than that of the manipulator without joint-coupling. The joint-coupling torques are $0.18 Nm$ and $0.62 Nm$ for the manipulator with $\alpha = 2.5$ because these positions are no longer a direct singularity at points 2 and 5. Hence, the required torques retain within a certain limit. The comparisons of required torques between the manipulators with and without joint-coupling are shown in Fig. 4.8 and Fig. 4.9.

Figure 4.8: Required input torque of joint A_1 and A_2 ($\alpha = 0$)

Figure 4.9: Required input torques of joint-coupling 1 and 2 ($\alpha = 2.5$)

4.4 Summary

In this chapter, the dynamic model for planar manipulators with joint-coupling has been studied. Equations of motion of the manipulator with joint-coupling are formulated using Lagrangian method. In order to find the input torques at the joint-coupling device, trajectories are generated with different coupling coefficient ratios on the two-DOF planar PM. Example 4.1 shows that required torques are dependent on coupling coefficient ratios. Example 4.2 displays the comparison of input torques between the manipulators with and without joint-coupling devices. It is possible to reduce the input torques by altering direct singularity in workspace using an appropriate joint-coupling ratio. Therefore, it will be formulated as an optimization problem to obtain appropriate coefficients based on the given trajectory and the motor limit torque in the next chapter.

Chapter 5

Singularity Management Using Joint-Coupling

From Chapter 3, it can be seen that by adding joint-coupling to the PM, it is now possible to change the singularity loci of the manipulator through the coupling coefficients. In this chapter two approaches are investigated to "engineer" the manipulator singularities according to different task requirements. This kind of approach is termed as *singularity management* because the architecture of the manipulator remains the same while the coupling coefficients are changed.

Two approaches for singularity management, namely, the task-oriented approach and the motor limit torque approach are proposed. The task-oriented method is based on the manipulator kinematics. Given a set of task points, this method employs an optimization algorithm to search for the singularity-free region enclosing all the task points in the joint or task space. The maximum torque method is originated from the dynamics of the manipulator. Given the fact that every actuator has a limit on the torque it can produce, this method determines the coupling coefficients by minimizing the power consumption of the prescribed trajectory set by the motors torque limits. Feasibility study of the managing manipulator singularities is presented using simulation examples.

5.1 Task-Oriented Singularity Management

The manipulator tasks can be given as the positions of the end-effector or the joint displacement of the manipulator. Here, the examples of two-DOF and three-DOF planar manipulators are used to show both approaches.

5.1.1 Task-Oriented Singularity Management in Joint Space

If a set of manipulator postures (in the joint space) is given, it is possible to adjust the values of the coupling coefficients and confine the manipulator postures within a certain region of the joint space to ensure that all given postures lie in a region free from direct singularities. The two-DOF manipulator of Fig. 3.1 is used to illustrate this concept. The singularity manifolds (loci) need to be constructed based on the two independent joint or input variables due to the mobility of the system. For convenience, the two variables are chosen to be the rotation of angles θ_{11} and θ_{21} . Defining the ratio of coupling coefficient as

$$\begin{aligned}\alpha_1 &= \frac{c_{12}}{c_{11}} \\ \alpha_2 &= \frac{c_{22}}{c_{21}} \quad \text{where } \alpha_1, \alpha_2 \in \mathbb{R}.\end{aligned}$$

The direct singularity condition can be written as

$$\det(\mathbf{A}_{J1}) = f(\theta_{11}, \theta_{21}, \alpha_1, \alpha_2) = 0 \quad (5.1)$$

$$\text{or more precisely, } \det(\mathbf{A}_{J1}) = f_{\alpha_1, \alpha_2}(\theta_{11}, \theta_{21}) = 0. \quad (5.2)$$

Eq. (5.2) indicates that the direct singularity loci are homotopy of ratios of the coupling coefficients α_1 and α_2 as illustrated in Fig. 5.1. Once a set of coupling parameters is chosen, the ratios of the parameters and, subsequently, the singularity loci are determined. Note that singularity loci of Eq. (5.2) are given in terms of the trigonometric functions of θ_{11} and θ_{21} . Substituting the half-angle transformation,

$$\sin \theta_i = \frac{2t_i}{1+t_i^2} \quad \text{and} \quad \cos \theta_i = \frac{1-t_i^2}{1+t_i^2} \quad \text{where} \quad t_i = \tan \frac{\theta_i}{2} \quad \text{and} \quad i = 1, 2$$

Figure 5.1: Homotopy of direct singularity loci and convex hull of task postures

into Eq. (5.2), a polynomial equation of the direct singularity loci written in t_1 and t_2 is obtained. For convenience, this equation can be written as:

$$f_{\alpha_1, \alpha_2}(t_1, t_2) = 0. \quad (5.3)$$

Without loss of generality, let $\mathbf{q} = (t_1, t_2)^T$ be the posture of the two-DOF planar PM. Note that $\mathbf{q}$ represents the postures of link A_1B_1 and A_2B_2 . The position of the end-effector can be obtained through direct kinematics. Let $D = \{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \dots, \mathbf{q}_k\}$ represents a set of planar PM postures. The ultimate goal is to determine appropriate ratios of the coupling coefficients α_1 and α_2 such that all given postures are within a singularity-free region of the singularity loci, i.e.,

$$f_{\alpha_1, \alpha_2}(D) < 0 \quad \text{or} \quad f_{\alpha_1, \alpha_2}(D) > 0 \quad (5.4)$$

Because of the enclosure property of Eq. (5.4), the computation of selecting appropriate coupling coefficients as an optimization problem can be formulated. Given a set of manipulator postures D , a singularity locus that can minimally contain D can be found. Denote the set of roots of the singularity locus $f(\mathbf{q}) = 0$ by $Z(f)$ i.e., $Z = \{\mathbf{q} : f(\mathbf{q}) = 0\}$.

Note that the arguments of function f are dropped for convenience. Considering point $\mathbf{q}$ and its normal projection $\hat{\mathbf{q}}$ onto the zero set Z , the first order series expression of f in the

neighborhood of points $\mathbf{q}$ [64], the distance from $\mathbf{q}$ to the singular locus can be written as

$$\text{dist}(\mathbf{q}, Z(f))^2 = \|\mathbf{q} - \mathbf{q}\|^2 \simeq \frac{f(\mathbf{q})^2}{\|\nabla f(\mathbf{q})\|^2}. \quad (5.5)$$

In order to find a singularity locus $f = 0$ to minimally contain the set of postures D , a weighted mean-square distance from D to the singularity loci Z defined below can be used as a measure

$$\text{dist}(D, Z(f))^2 = \frac{1}{k} \sum_{i=1}^k w_i \frac{f(\mathbf{q}_i)^2}{\|\nabla f(\mathbf{q}_i)\|^2} \quad (5.6)$$

where w_i are the weight factors. Given the posture set $D = \{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \dots, \mathbf{q}_k\}$, the above expression becomes a function of the coupling ratios α_1 and α_2 . Hence, this expression can be minimized with respect to α_1 and α_2 . Because of Eq. (5.6) is a distance function, this minimization can only find a singularity loci very close to the isolated manipulator postures. It does not guarantee that all postures are contained in one connected region of the workspace. Thus, additional conditions ensuring that Eq. (5.4) holds is required. Denoting $Co(D)$ as the convex hull of D , a necessary condition for Eq. (5.4) to hold is:

$$f_{\alpha_1, \alpha_2}(Co(D)) < 0 \quad \text{or} \quad f_{\alpha_1, \alpha_2}(Co(D)) > 0 \quad (5.7)$$

Because the zero set Z may not be simply connected, the condition of Eq. (5.7) ensures that the set of task postures D is contained in a connected region of Z as shown in Fig. 5.1. Let points $\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_l$ ($l < k$) be the vertices of the convex hull $Co(D)$ in sequence. Condition Eq. (5.7) can be interpreted as l inequalities, i.e.,

$$g_i = f_{\alpha_1, \alpha_2}((1 - \beta_i)\mathbf{q}_i + \beta_i\mathbf{q}_{i+1}) < 0 \quad (\text{or } > 0) \quad (5.8)$$

$$0 \leq \beta_i \leq 1 \quad (i = 1, 2, \dots, l) \quad \text{and} \quad \mathbf{q}_{l+1} = \mathbf{q}_1$$

Combining Eq. (5.7) and Eq. (5.8), a constrained nonlinear optimization problem is obtained for the determination of the coupling coefficients, namely,

$$\text{Minimize} \quad F(\alpha_1, \alpha_2, \beta_1, \dots, \beta_l) = \text{dist}(D, Z(f))^2 = \frac{1}{k} \sum_{i=1}^k \frac{f(\mathbf{q}_i)^2}{\|\nabla f(\mathbf{q}_i)\|^2} \quad (5.9)$$

$$\text{Subject to} \quad g_i = f_{\alpha_1, \alpha_2}((1 - \beta_i)\mathbf{q}_i + \beta_i\mathbf{q}_{i+1}) < 0, \quad (\text{or } > 0)$$

$$0 \leq \beta_i \leq 1 \quad (i = 1, 2, \dots, l)$$

The solution to problem Eq. (5.9) will lead to a set of coupling coefficients that allows the manipulator to pass through a given set of postures without encountering direct singularities. To ensure that the manipulator postures lie far enough from the singularity locus, a convex hull slightly larger than $Co(D)$ is constructed and used as constraints for the nonlinear optimization problem.

Table 5.1: Given postures of the manipulator

—	(θ_1, θ_2) rad	(t_1, t_2)
$\mathbf{q}_1$	(1.4, 5.0)	(0.842, -0.75)
$\mathbf{q}_2$	(2.08, 4.0)	(1.7, -2.185)
$\mathbf{q}_3$	(2.9, 4.5)	(8.23, -0.25)
$\mathbf{q}_4$	(2.8, 5.8)	(5.8, -1.238)

Figure 5.2: Task postures

Example 5.1

To illustrate the above concept, the two-DOF planar PM is considered again. Without loss of generality, let the coupling coefficients of the two branches of the manipulator be the same, i.e., $\alpha_1 = \alpha_2 = \alpha$. With the link lengths defined in Chapter 3, the direct singularity loci can be written in terms of t as: $f_\alpha(t_1, t_2) = 0$. Four manipulator configurations are given in Table. 5.1 in θ -domain and t -domain respectively. Fig. 5.2 illustrates the physical locations of those postures. It can be seen that the manipulator will encounter a singularity

between the configurations $\mathbf{q}_1$ and $\mathbf{q}_2$ if joints B_1 and B_2 are used as active joints without joint-coupling, i.e., if the coupling coefficients are $(c_1, c_2) = (0, 1)$. From the illustration of the singularity loci of this case in the θ -domain and t -domain of the configuration spaces, it can be seen that the configuration $\mathbf{q}_1$ lie in a region separated from the other configurations by singularity locus.

Now an appropriate set of coupling parameters is found to avoid this direct singularity. Based on the configurations listed in Table. 5.1, the objective function can be obtained using Eq. (5.9). The weight factors are chosen to be $w_1 = w_2 = w_3 = w_4 = 1$. The convex hull of the configurations in t -domain (obtained manually for this example) is the quadrilateral formed by $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ and $\mathbf{q}_4$. Hence, there are in total four inequality constraints in the form of Eq. (5.9).

Using the nonlinear optimization toolbox of *MATLAB*, the nonlinear programming problem of Eq. (5.9) can be solved. The result of the optimization gives $\alpha = 0.267$. Based on the fact that $c_1 + c_2 = 1$ as indicated in Fig. 3.1, the optimized coupling coefficients are obtained as $(c_1, c_2) = (0.789, 0.211)$. Figure 5.4 shows the direct singularity loci with optimized coupling coefficients in the θ and t -joint spaces respectively. The result indicates that the configurations are indeed enclosed in a region of the singularity loci with the same sign. Hence, there will not be singularity for the manipulator when carrying out the tasks. The distance function from the given configurations to the singularity loci is also minimized during the optimization [65, 66].

5.1.2 Task-Oriented Singularity Management in Task Space

Singularity management can work in the manipulator task space as well. The three-DOF planar PM of Fig. 3.7 is used as an example to illustrate singularity management in the task space. First, singularity loci are constructed in the joint space of the manipulator in a manner similar to the previous section. The variables to be the base joint angles θ_{11} , θ_{21} and θ_{31} are chosen.

Figure 5.3: The locations of manipulator postures in singularity loci (without joint-coupling): $(c_1, c_2) = (0, 1)$

Figure 5.4: The locations of manipulator postures in singularity loci (with joint-coupling): $(c_1, c_2) = (0.789, 0.211)$

Define

$$\alpha_1 = \frac{c_{12}}{c_{11}}, \alpha_2 = \frac{c_{22}}{c_{21}}, \alpha_3 = \frac{c_{32}}{c_{31}} \quad \text{where } \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}.$$

The direct singularity condition can be written as:

$$\det(\mathbf{A}_{J2}) = f(\theta_{11}, \theta_{21}, \theta_{31}, \alpha_1, \alpha_2, \alpha_3) = 0, \quad (5.10)$$

$$\text{or } \det(\mathbf{A}_{J2}) = f_{\alpha_1, \alpha_2, \alpha_3}(\theta_{11}, \theta_{21}, \theta_{31}) = 0. \quad (5.11)$$

The form of Eq. (5.10) suggests that ratios of the coupling coefficients α_1 , α_2 and α_3 are parameters of the singularity locus f defined in the θ_{11} - θ_{21} - θ_{31} joint space. The ratios α_1 , α_2 and α_3 define a class of homotopy of the direct singularity loci. Using direct kinematic transformation, singularity loci of Eq. (5.10) can be written in the task space of the end-effector of the manipulator as

$$\det(\mathbf{A}_{J2}) = f_{\alpha_1, \alpha_2, \alpha_3}(\mathbf{p}, \phi) = 0 \quad \text{where } \mathbf{p} = [x_p \ y_p]^T. \quad (5.12)$$

Without a loss of generality, $\mathbf{q}$ can be denoted as $[\mathbf{p}^T, \phi]^T$ the posture (or the task point) of the moving platform of the manipulator. Let $D = \{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \dots, \mathbf{q}_k\}$ be the set of task points that the moving platform is desired to reach. It is necessary to determine appropriate ratios of the coupling coefficients, α_1 , α_2 and α_3 such that all given task points are within a singularity-free region of the singularity loci, i.e.,

$$f_{\alpha_1, \alpha_2, \alpha_3}(D) < 0 \quad \text{or} \quad f_{\alpha_1, \alpha_2, \alpha_3}(D) > 0. \quad (5.13)$$

A singularity locus that can minimally contain D is required to be found. The set of roots of the singularity locus is denoted as $f(\mathbf{q}) = 0$ by $Z(f)$, i.e., $Z = \{\mathbf{q} : f(\mathbf{q}) = 0\}$. Point $\mathbf{q}$ and its normal projection $\hat{\mathbf{q}}$ onto the zero set Z are considered. Using the first-order Taylor series expansion of f in the neighborhood of point $\mathbf{q}$ [67], the distance from $\mathbf{q}$ to the singular locus can be similarly written like Eqs. (5.5), (5.6) and (5.7). For this study, a constrained nonlinear optimization problem is required, namely,

$$\text{Minimize } F(\alpha_1, \alpha_2, \alpha_3) = \text{dist}(D, Z(f))^2 = \frac{1}{k} \sum_{i=1}^k w_i \frac{f(\mathbf{q}_i)^2}{\|\nabla f(\mathbf{q}_i)\|^2} \quad (5.14)$$

$$\begin{aligned} \text{Subject to } g_i &= f_{\alpha_1, \alpha_2, \alpha_3}((1 - \beta_i)\mathbf{q}_i + \beta_i\mathbf{q}_{i+1}) < 0, \quad (\text{or } > 0) \\ 0 &\leq \beta_i \leq 1 \quad (i = 1, 2, \dots, l) \end{aligned}$$

The solution to the optimization problem of Eq. (5.14) leads to a set of coupling coefficients that allows the manipulator to pass through the given set of tasks points without encountering direct singularity loci.

Example 5.2

Considering the three-DOF planar PM as shown in Fig. 3.7, without loss of generality, let the joint-coupling coefficients of all branches of the manipulator be the same, i.e., $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$. Based on Chapter 3, the direct singularity loci of the manipulator can be written as: $f_\alpha(\mathbf{p}, \phi) = 0$.

Figure 5.5: Postures of the moving platform of three-DOF planar PM with joint-coupling

Eight task points and the corresponding postures of the manipulator are given in Table. 5.2 and illustrated in Fig. 5.5. Figure 5.6(a) illustrates the section view along ϕ -direction of the workspace and direct singularity loci of the manipulator *without* joint-coupling (driven by joints A_1 , A_2 and A_3).

Table 5.2: Task points of the moving platform

task	1	2	3	4	5	6	7	8
$x_p(m)$	0.15	0.35	0.15	0.15	0.15	0.15	0.35	0.35
$y_p(m)$	0.0	0.0	0.10	0.10	0.10	0.0	0.0	0.10
$\phi(rad)$	-1.0	-1.0	-1.0	-1.0	-0.75	-0.75	-0.75	-0.75

Figure 5.6: Direct singularity loci in task space of three-DOF planar PM (a)without joint-coupling (b)with joint-coupling

The convex hull of the eight task points forms a rectangular polytope in the task space, which is rendered as shaded rectangles in the sectional view of the workspace in Fig. 5.6(a). Apparently, the manipulator will encounter singularity as it moves along the convex hull formed by the task points. Now an appropriate set of coupling parameters is found to avoid these direct singularities. Based on the given task points, the objective function can be obtained using Eq. (5.6). The weight factors are chosen to be

$$w_1 = w_2 = \dots = w_8 = 1$$

The convex hull of the task points is the polytope formed by $\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_8$. Using a nonlinear optimization toolbox in *MATLAB*, the nonlinear optimization problem Eq. (5.14) can be solved. The result obtained from *MATLAB* is $\alpha = 0.8394$. Substituting the optimal coupling coefficient into Eq. (5.13), it can be observed from Fig. 5.6(b) that the direct singularity loci in the workspace do not intersect with the convex hull formed by the task points.

The result indicates that the task points are enclosed in a singularity-free region. Based on Example 5.1 and 5.2, it can be concluded that the task-oriented singularity management method satisfies the requirement obtaining singularity-free regions based on the prescribed task [68].

5.2 Singularity Management Based on Motor Limit Torque

In literature, several researchers, [69, 70, 71] studied on torque optimization. Gravity balancing methods are used in industrial machines to decrease the required actuator efforts during motion. They include counterweights, springs and auxiliary parallelograms that determine the center of mass. Reaction less methods is used to normally obtain the lower inertia of the parallel mechanisms.

Among these, Gosselin [69] introduced reactionless design for manipulators using balancing

equations, which are derived by imposing that the center of mass of the mechanism is fixed and the total angular momentum is constant with respect to a fixed point. This optimization is performed to determine the counter weights and counter rotations based on the balancing condition. When the manipulator moves close to the singular postures, the torque exerted by the actuators becomes very large and may eventually exceed the limit that the actuator can generate. Hence, joint-coupling provides a way to adjust the torque output of the "actuated" joints so that it will keep the actuator torque within the design limits.

Given a set of the end-effector postures along the desired trajectory, the power consumption of actuators can be obtained based on the dynamic model of the manipulator derived in Chapter 4. Because of joint-coupling, the total power consumption along the desired trajectory becomes a function of the coupling coefficients. With a maximum limit on the motor torque output, it is now possible to find an appropriate set of coupling coefficients that minimize the total power consumption.

Let $\mathbf{p} = (x_p, y_p)^T$ be the position of the end-effector of the two-DOF planar PM with joint-coupling. The displacement of joints corresponding to $\mathbf{p}$ can be obtained from the inverse kinematics. Assuming that there are k points along the trajectory of the end-effector, let τ_i be the torque output of joint-coupling device i and τ_{max} be the maximum torque output of the actuator. Now an optimization problem can be defined where the torque limit τ_{max} can be used as a constraint in the optimization problem, i.e.,

$$\tau_i \leq \tau_{max}. \quad (5.15)$$

The total power consumption of the manipulator at this point can be written as:

$$\mathbf{E} = \sum_{i=1}^2 \tau_i \omega_i. \quad (5.16)$$

where ω_i is angular velocity of the joint-coupling input i . Adding up the power consumption at all k positions along the trajectory, the total power consumption along the given trajectory can be obtained. Thus, the singularity management problem based on average

power consumption can be formulated as an optimization problem.

$$\text{Minimize} \quad E_{c_1, c_2} = \sum_{i=1}^2 \frac{(\tau_i \omega_i) \Delta t}{T_{total}} \quad (5.17)$$

$$\text{Subject to} \quad \tau_i \leq \tau_{max} \quad \text{where} \quad i = 1, 2$$

where Δt = sampling time, T_{total} = total time taken

The input torque of the joint-coupling device i from each branch. The solutions to problem Eq. (5.17) is a set of coupling coefficients that minimize the power consumption along the desired trajectory.

Example 5.3

To illustrate the above concept, consider the two-DOF planar PM with joint-coupling in Fig. 3.1. The following parameters are given:

Initial posture of the end-effector : $\mathbf{p}_i = (0.17, 0.7238)$

Final posture of the end-effector : $\mathbf{p}_f = (-0.42, 0.3)$

Ramp velocity of joint-coupling : $V_r = 0.0862 \text{ rad/sec}$

Ramp acceleration of joint-coupling : $A_r = 0.1724 \text{ rad/sec}^2$

Sampling time : $t_s = 0.5 \text{ sec}$

Sampling steps : $k = 10$

Total time : $t_t = 5 \text{ sec}$

Maximum motor torque $\tau_{max} = 5 \text{ Nm}$

Now an appropriate set of coupling coefficients based on maximum motor torques can be found. The objective functions E can be obtained using Eq. (5.17). There are in total of $2k$ constraints in the form of Eq. (5.15). The required torques for a two-DOF planar PM *without* joint-coupling to go through this trajectory are shown in Fig. 5.8. The active joints are θ_{11} and θ_{21} as one can observe that when the manipulator go across the singularity, the torque value becomes very high.

Table 5.3: Dynamic parameters of two-DOF planar PM with joint-coupling

Link	mass(Kg)	length(m)	Inertia(Kg.m ²)
A_1B_1	10	0.356	1.581
A_2B_2	10	0.356	1.581
B_1P	10	0.60	2.090
B_2P	10	0.60	2.090
A_1A_2	—	0.67	—

Using the optimization toolbox *fminimax* of MATLAB which applies Quasi-Newton algorithm, the optimization problem can be solved. The result obtained from *MATLAB* is $c_1 = 8.229$ and $c_2 = 21.668$ which means $\alpha = 2.633$.

Figure 5.7 compares the trajectory of the manipulator and singularity loci of the manipulator with and without joint-coupling. It clearly shows that the optimized coupling coefficients can ensure that the manipulator trajectory is free from the singularity. In Fig. 5.8, the required torques at 3 sec and 4.5 sec become as high as -27 Nm and 27 Nm while the manipulator passing through direct singularity conditions. Figure 5.9 indicates that the torque produced by the joint-coupling devices in which the obtained coupling coefficient ratio is used does not exceed the maximum torque limit by altering singularities. This example shows that the joint-coupling effect can manage the actuator torques and avoid direct singularity at the same time.

Figure 5.7: Trajectory and singularity manifolds of two-DOF planar PM with joint-coupling

Figure 5.8: Required torques of two-DOF planar PM without joint-coupling

Figure 5.9: Required torques of two-DOF planar PM with joint-coupling

5.3 Summary

This chapter described methodologies for managing singularities of planar PMs with joint-coupling based on specific task requirements and motor torque limit. In task-oriented method, singularity-free regions can be constructed for required operation in both joint space and the task space. Example 5.1 and 5.2 illustrate this approach based on a non-linear constrained optimization. In motor limit torque approach, motor maximum torque are set as the constraints. The motor power consumption along a given trajectory is

set as the objective function for optimization. Example 5.3 shows that minimal power consumption is used along the trajectory with appropriate coupling coefficients obtained from the optimization. Hence, the maximum torque method is also a feasible way to manage manipulator singularity on PM with joint-coupling.

Chapter 6

Construction of Joint-Coupling Devices

It was demonstrated in Chapter 1 that a simple differential mechanism can achieve the joint-coupling in a parallel manipulator built from LEGO components as shown in Fig. 1.3. In this chapter, the design and implementation of joint-coupling on a physical parallel manipulator will be discussed. In this research, a modular reconfigurable robotic system is used as an experimental platform. The modular reconfigurable robot system consists of a large inventory of standard actuator modules so that they can be assembled into robots with different degrees of freedom and structures. The work here is to develop a joint-coupling module compatible with these existing modules so as to facilitate the construction of two-DOF and three-DOF planar parallel manipulators for validating the joint-coupling concept and properties mentioned in the previous chapters.

Various design concepts for a joint-coupling module actuators are developed here. Two design strategies, namely the mechanical joint-coupling and an electronic joint-coupling are employed in the final prototype development. The mechanical joint-coupling module consists of a planetary differential mechanism, which has a fixed coupling ratio. The electronic joint-coupling module is designed based on electronic gearing of multiple intelligent

motors. This design allows the coupling ratio to be varied based on the user input and task requirements. The construction of the two devices are described. They are used in the experiment study of the joint-coupling effect stated in the next chapter.

6.1 Joint-Coupling Testbed and Design Requirement

To investigate the usage and control of parallel manipulators with joint-coupling, a modular reconfigurable planar parallel manipulator system is employed as the basic experimental platform. This system consists of an inventory of standardized active and passive joints and customized links, connectors and moving platforms [72, 73, 74, 75] as shown in Fig. 6.1. The manipulator can be quickly constructed with arbitrary configurations like the three-DOF planar parallel manipulator or the six-DOF three-legged spatial parallel manipulator shown in Fig. 6.2. Due to the use of standard active and passive modules, the system integration and programming effort can be simplified. The customizable links and connectors allow the engineers to fine-tune the manipulator parameters for performance at minimal cost and time. As the ordinary inventory of the modular manipulator system does not con-

Figure 6.1: Customizable links, connectors and modules

tain such joint-coupling devices, this research effort is to develop a joint-coupling device that is modular and reusable. The basic requirements of such a joint-coupling module for modular manipulators are:

Figure 6.2: Reconfigurable parallel manipulator platforms

1. A stand-alone module with the actuator, control and communication capabilities.
2. The dimensions and the inter-module connecting interface must be compatible with the original modular components.
3. The module should be reusable for the two-DOF and three-DOF planar parallel manipulators.

6.2 Concept Development

To meet the third requirement mentioned in Section 6.1, a device with one input actuator and two coupled output joints is sufficient as indicated in Fig. 3.1 and Fig. 3.7. One coupling device can be installed on each branch of the manipulator to keep the mobility of the manipulator same as the original one.

As the design of the joint-coupling device involves the selection of appropriate joint-coupling ratios or joint-coupling coefficients, all the possible design strategies for the joint-coupling device are displayed in Table. 6.1. Basically joint-coupling by mechanically and electronic means can be realized. Based on the tunability of the coupling ratio, it can be further divided into devices with a fixed or a continuous changeable coupling ratio. A mechanical joint-coupling device requires different types of transmission mechanisms to coordinate the

Table 6.1: Concept development for joint-coupling

	Fixed Coupling Ratio	Variable Coupling Ratio
Mechanical	Differential mechanism <ul style="list-style-type: none"> • easy to design fabrication • not adaptable • more weight module • not compact 	Continuous variable transmission mechanism <ul style="list-style-type: none"> • complex in structure • limited range of ratio • more weight • not compact
Electronic	Electronic gearing <ul style="list-style-type: none"> • easy to design • less cost-effect • light weight • easy to control 	Electronic gearing <ul style="list-style-type: none"> • adaptable for different task • cost-effect • easy to program and control • compact design

motion of couple joints physically. According to Table. 6.1, there are two possible ways to realize a mechanical joint-coupling device depending on the selection of coupling ratios.

(A) Using differential mechanism with fixed coupling ratio

In this case, planetary differential gears or differential drives can be employed to achieve this purpose. The joint-coupling module has fixed coupling ratio due to the use of geared mechanism. The advantage of this approach is easy to design and fabricate as a stand alone module used on the modular robot platform. However, the size of the coupling device will be considerably large in order to hold all transmission mechanisms. The main drawback of this design approach is that an exact coupling coefficients is needed for the particular joint-coupling module as it is not possible to change the coupling coefficients once the module is fabricated.

(B) Continuous variable transmission mechanism with stepped or continuously variable coupling ratios

Since the direct singularity manifolds can be altered by changing the coupling coefficients continuously, the continuous variable transmission mechanism (CVT) between two joints to synchronize joint motions mechanically is considered. Based on the mechanical principle, there will be no delay or deviations in transmission except ignoring lags due to flexibility of mechanical parts such as belts and chains in a CVT design. However, the CVT type coupling mechanisms also have drawbacks. In order to fit into standard module size, the diameter of the pulleys in CVT must be small in size and the range of transmission will be limited. In addition to CVT mechanism, addition mechanism is needed to achieve the coupling effect. Thus, the entire construction will be either too large or limited due to the requirement on the size of the module. The weight of such a joint-coupling module will be also heavy due to the complexity of the entire design, and hence increases the inertia of the parallel manipulator.

(C) Electronic gearing with a fixed coupling ratio

Electronic coupling means that both joints in a coupling device are replaced with actuators, which are driven in a coordinated fashion through a single motion controller. Based on Table. 6.1, it is possible to design a joint-coupling device with electronic gearing with either fixed or continuously changing coupling ratio. Generally, electronic gearing is the profile generator of one axis receiving its command via the output of the other axis. It means that two or more controlled motors produce a gearing mechanism by synchronizing their speed as the mover (Master) and the follower (Slave). The design of the electronic gearing system with a fixed coupling ration can obtain optimal results because the motion controller can be designed with a specific goal. Hence, its performance can be stable. However as the electronic gearing is flexible, it is not cost effective to use it as a joint-coupling device with only one fixed ratio.

(D) Electronic gearing with continuously variable coupling ratios

Variation of the coupling ratio by electronic gearing can be achieved through software control of the motion controller. Hence, the flexibility of using this manipulator for different task under singularity-free conditions can be easily achieved. It is also cost-effective because

of this flexibility. Without a mechanical transmission mechanism, the design of joint-coupling module can be compact. Furthermore, the orientation of the coupling axes is no longer restricted by the mechanical device to be in parallel or orthogonal. It is possible to design coupling devices for spatial parallel manipulators based on the electronic gearing.

After the investigation of the above joint-coupling design concept, a mechanical joint-coupling device with a fixed coupling ratio is chosen as the first prototype for implementation on the modular platform due to its simplicity in design and quick development time for concept verification. An electronic joint-coupling device with variable coupling ratio is also chosen to be the subsequent prototype in order to investigate the singularity management concept in full.

6.3 Mechanical Joint-Coupling Device

As all the revolute joint axes in the planar parallel manipulator are parallel in this prototype, a planetary differential gear mechanism is employed as the core of the joint-coupling module. The ordinary differential transmission mechanism used in the automobiles is not feasible because their input and output axes are perpendicular to each other. Figure 6.3 represents the schematic diagram of the mechanical joint-coupling module. The working principle of the module is as follows. A DC servo motor is mounted on the input shaft of the joint-coupling device forming a complete joint-coupling module. The input motion is transmitted from Sun Gear $S1$ to Planet Gear $P1$. Planet Gear $P2$ is rigidly coupled with $P1$. Therefore, $P1$ and $P2$ are rotated as a single body. Then, the power transmission path is divided into two: one to the carrier which becomes Output 1; the other to Sun Gear $S2$, which is coupled with SA and eventually to Output 2. Output 1 and 2 lead to the two coupled joints in the manipulator. To reduce the inertia effect during the manipulator movement, the joint-coupling module is designed as a base unit with the input actuator rigidly mounted on the base and the revolute axes of Input and Output 1 aligned. As Output 2 is situated at the distal end of the module, a belt drive was installed to transmit

Figure 6.3: Design of mechanical joint-coupling module

power from Sun Gear $S2$ to the shaft of Output 2 through Gear SA and SB .

6.3.1 Determination of Coupling Ratio

Based on the planetary differential mechanism, the relationship of the pulley diameters and the coupling ratio can be derived. From Fig. 6.3, input actuator motion ω_1 rotate Sun Gear $S1$ with

$$N_{S1} = \omega_1 \quad (6.1)$$

The planetary differential gear box ratio R can be written as input-output relation

$$R = \frac{N_{S1} - N_{P1}}{N_{S2} - N_{P2}} \quad (6.2)$$

where

N_{S1} = angular velocity of Sun gear $S1$

N_{S2} = angular velocity of Sun gear $S2$

N_{P1} = angular velocity of Planet gear $P1$

N_{P2} = angular velocity of Planet gear $P2$

Since $P1$ and $P2$ rotated as a single body, they deliver the rotation to the carrier, which

is Output 1 of the joint-coupling device. Subsequently $S2$ becomes the second output of a planetary gear system. The notation of the velocity ratio of the gears can be re-written based on the two-DOF planar parallel manipulator as

$$R = \frac{\omega_i - N_{P1}}{N_{S2} - N_{P2}} \quad (6.3)$$

The joint-coupling device allows two adjacent joints to move together. Since input angular velocity ω_1 and Planet Gear $P1$ rotate in the opposite direction, the output 1 (Carrier) will rotate in opposite direction with respect to ω_1 . If we assume that ω_1 as positive, Eq. (6.3) can be re-written as

$$R = \frac{\omega_i + \dot{\theta}_{i1}}{N_{S2} + \dot{\theta}_{i1}} \quad (6.4)$$

It can also be expressed as $N_{S2} = N_{SA}$ and $N_{SB} = \dot{\theta}_{i2}$ from Fig. 6.3. A belt drive is used to transmit to the second coupled joint, which is joint B_i in Fig. 3.1. The speed ratio in the timing belt drive is

$$\frac{\dot{\theta}_{i2} - \dot{\theta}_{i1}}{N_{S2}} = \frac{n_{A1}}{n_{B1}} \quad (6.5)$$

where

n_{A1} = Number of teeth of gear SA

n_{B1} = Number of teeth of gear SB

Substituting Eq. (6.5) into Eq. (6.3),

$$(R(1 - \frac{n_{A1}}{n_{B1}}) - 1)\dot{\theta}_{i1} + (R\frac{n_{A1}}{n_{B1}})\dot{\theta}_{i2} = \omega_i \quad (6.6)$$

From Eq. (6.6), the coupling coefficient ratio c_{i1} and c_{i2} can be expressed based on the gear ratios as

$$c_{i1} = R(1 - \frac{n_{A1}}{n_{B1}}) - 1 \quad (6.7)$$

$$c_{i2} = R\frac{n_{A1}}{n_{B1}} \quad (6.8)$$

In our module, $n_{A1} = 20$ teeth and $n_{B1} = 40$ teeth and the planetary differential gear ratio $R = 10$ are used in order to achieve $c_{i1} = 4$ and $c_{i2} = 5$.

6.3.2 Prototype of Mechanical Joint-Coupling Module

A *SilverMax* servomotor shown in Fig. 6.4 is used as the input actuation of the joint-coupling device. The specification of the motor is listed in Table. 6.2. The motor is placed in the base module of the mechanical joint-coupling module shown in Fig. 6.6.

Figure 6.4: *SilverMax* servomotor

Table 6.2: *SilverMax* motor specifications

Maximum Speed (<i>rpm</i>)	4000
Optimal speed(<i>rpm</i>)	2500
Torque at optimal speed(<i>Nm</i>)	0.28
Peak Power(<i>Watt</i>)	72
Size(<i>mm</i>)	42 × 48 × 84

The initial design of the gearbox is intended to use standard spur gears. However, to achieve the high accuracy and light weight, a standard planetary gearbox is used for this application. *Ondrives* gearbox is selected because of its high efficiency and small size. It is mounted on the top module (the first output module) of the mechanical joint-coupling shown in Fig. 6.6.

Table 6.3: *Ondeives* gearbox specifications

Gearbox ratio(<i>rpm</i>)	10 : 1
Efficiency(%)	92
Rated torque(<i>Nm</i>)	2.5
Weight(<i>gm</i>)	140
Head length(<i>mm</i>)	40

A timing belt is used to transfer motion from joint-coupling input to the second output passing through pulleys *SA* and *SB* as shown in Fig. 6.3. Assembly drawing of the mechanical joint-coupling module is shown in Fig. 6.5. The part list is provided in Table. 6.4. A complete mechanical joint-coupling module is illustrated in Fig. 6.6.

Figure 6.5: Assembly of mechanical joint-coupling device

Table 6.4: Parts list of mechanical joint-coupling device

No	Part Name	No	Part Name
1	Gear top cover	10	Connecting link
2	Gear housing	11	Belt drive
3	Gear box	12	Connecting link
4	Gear bottom cover	13	Right angle connector
5	Motor bearing housing	14	Encoder bearing housing
6	Motor housing	15	Encoder shaft
7	Motor	16	Encoder housing
8	Motor bottom cover	17	Encoder bottom cover
9	Base connector

Figure 6.6: Prototype of the mechanical joint-coupling module

Table 6.5: Specifications of mechanical joint-coupling module

Dimensions of(motor unit)	$90 \times 90 \times 110\text{mm}$
Dimensions of(JC housing)	$90 \times 90 \times 90\text{mm}$
Dimensions of(tubular link)	$\phi 70 \times 260\text{mm}$
Weight	9kg(excluding motor unit)
coupling coefficients	$(c_{11}, c_{12}) = (4, 5)$

6.3.3 Test on Mechanical Joint-Coupling Module

Several tests were carried out on the prototype to verify the performance of mechanical joint-coupling device using the differential mechanism. These tests showed how to achieve two coupled joint outputs to two adjacent joints and also investigated the torque sufficiency of the motors. The actuator for the input *SilverMax*TM has the ability to communicate via the RS-232 serial interface to the host PC. This interface transmits two commands for motion control and supervision at 1 MBit/sec.

The host PC is a Pentium III, 500 mHz computer system with Windows XP. A software package *QuickControl*TM designed as a programming, diagnostic and prototyping tool for *SilverMax*TM motors is used. The software uses the PC's RS-232 serial port to send commands to *SilverMax*TM motors. As shown in Fig. 6.4, the SilverMax servomotors are connected to QCI breakout module, which is connected to any I/O devices and the power supply. The breakout module allows easy access to the serial, I/O, and power lines of the servomotor. The PC running *QuickControl* is used to program and setup the *SilverMax*. Using the Control Panel shown in Fig. 6.7, a control command can be selected to control the motor among four kinds of commands such as (1)MAT: move absolute time based, (2)MAV: move absolute velocity based, (3)MRT: move relative time based and (4)MRV: move relative velocity based. Running on Test Move 1 or 2, the device status: *target position*, *velocity*, *position error* and *applied torque* of the motor can be obtained. The position, velocity and acceleration profiles can be obtained from the Strip Chart from the control panel shown in Fig. 6.7.

Figure 6.7: Control panel of SilverMax servomotor

Experiment 1

This experiment is conducted on the relation between input and output of the mechanical joint-coupling device. The performance of individual output axes can be determined first. The connecting link is fixed and the motor is activated to rotate the distal output axes only.

Fixing the connecting link means $\dot{\theta}_{11} = 0$,
the relationship, $c_{11}\dot{\theta}_{11} + c_{12}\dot{\theta}_{12} = \omega_1$ becomes $c_{12}\dot{\theta}_{12} = \omega_1$.

Thus, the second joint rotates at $\frac{1}{c_{12}}$ time of input rotation ω_1 . Then the platform of distal joint is fixed by means of stoppers as shown in Fig. 6.10. It can be clearly seen in the figure where the arrows indicate the screw holes that can be used as stoppers to prevent rotation. Similarly, the rotation of $\dot{\theta}_{11}$ becomes $\frac{1}{c_{11}}$ time of the input rotation ω_1 .

For example, input velocity of the mechanical joint-coupling is given as 44000 *cps* and acceleration is desired as 628571 *cps/s* within 1000 *msec* time interval. Results are shown in Fig. 6.8.

Figure 6.8: Input and output velocity of the mechanical joint-coupling (test-1)

Figure 6.9: Input and output velocity of the mechanical joint-coupling (test-2)

Then, input velocity of the mechanical is given as 127500 c and distance is desired as 127500 cps within 1000 $msec$ time interval. Results are shown in Fig. 6.9.

In order to check the applied torque of the device, the speed of the motor is variate from 50 rpm to 100 rpm. The torque produced by the motor is also found out to be sufficient for the application when the generated torque to carry a load of 20 N at different speeds is in the range of 12% to 15% of the maximum torque (100%) at the respective speeds.

6.4 Electronic Joint-Coupling Device

As mentioned earlier, electronic joint-coupling can be used to achieve continuously variable coupling coefficients c_{i1} and c_{i2} through electronic gearing. In the electronic gearing, both

Figure 6.10: Position to hold stoppers in module

joints in the coupling device are replaced with actuators, which are driven in a coordinated fashion through a single motion controller. Thus, there is no physical input axis and the input joint rate ω_i is issued from the controller to the couple actuator. Only one velocity command is required to control the two motors of the couple joints based on the coupling coefficients. Thus, the mobility of the manipulator can be maintained as the original one.

In electronic gearing, two or more controlled motors act as a gearing mechanism by synchronizing their speed as the mover (Master) and the follower (Slave). In other words, the master motor is driven with the control system as a complete closed-loop system while the other motor (slave) use the master's encoder signal as driving command to follow the motion according to gearing ratio provided by the control system. The gearing ratio can be achieved by scaling, multiplying or dividing the incoming signals or outgoing signals. Therefore, by relating the motions of two servo motors according to the relationship of joint-coupling, the electronic joint-coupling can be achieved.

6.4.1 Principle of Electronic Joint-Coupling Device

The principle of electronics gearing, configuration and programming vary according to the different integrated motor systems. As the basic principle of electronic gearing, the slave motor requires command signals (encoder signals) from the master motor through

Figure 6.11: Working principle of Electronic Gearing

IO ports. The *SilverMax* motor has seven IO ports for high-speed IO functions: external encoder input, internal encoder output and scaled internal encoder (module) output. The working principle of electronic gearing of *SilverMax* motors is shown in Fig. 6.11. The master 1 receive motion profile from the quick control and the encoder data of motor 1 is monitored. This data is sent to the servo controller of motor 2 and feedback to motor 1. Simultaneously encoder data of motor 2 is sent back to the servo controller of motor 2. The velocity reading of motor 1 and motor 2 are changing according to the given speed ratio, of the output shaft of the joint-coupling device n .

$$\text{where } n = \frac{\dot{\theta}_{i1}}{\dot{\theta}_{i2}} \quad (6.9)$$

When the input velocity is given to the master (motor 1), the slave (motor 2) will follow in order to keep desired coupling coefficients and joint-coupling inputs.

The main advantages using electronic gearing in the joint-coupling device are:

- (1) The speed of the slave motor can be modulated according to the master motor and can be controlled according to coupling coefficients precisely by a second loop feedback.

(2) With the help of servo motor accuracy, performance of electronic gearing can be enhanced.

(3) Unlike continuous variable transmission mechanism, it is able to change the coupling ratio arbitrarily and randomly.

The disadvantages of electronic gearing are the processing time and response delay time of the servo mechanism. There is a small time lag to make the system to settle down. The delay time depends on the microprocessor and the control mechanism . It can be eliminated with today's technology advancement.

Figure 6.12: Assembly of base modules of electronic joint-coupling device

6.4.2 Prototype of Electronic Joint-Coupling Module

The electronic joint-coupling module mainly consists of master and slave motors, harmonic drives to transmit power and standardized housing modules. The assembly of a single motor unit is shown in Fig. 6.12 and 6.13. The part list of the motor is in Table. 6.6.

Figure 6.13: Assembly of top and base modules of electronic joint-coupling device

SilverMax motors used in the mechanical joint-coupling module described in section 6.3.2 are employed here. Since compact design and high gear ratio are required, harmonic drives

Table 6.6: Parts list of electronic joint-coupling module

No	Part Name	No	Part Name
1	Screw side inserts	10	Motor shaft adaptor
2	Screw side cover	11	Harmonic drive components
3	Housing of base cube	12	Harmonic drive housing
4	Motor	13	Assembled harmonic drive on base cube
5	Nut side cover	14	Screw side inserts
6	Nut side inserts	15	Screw side cover
7	Assembled base cube	16	Housing of upper cube
8	Bearing	17	Nut side cover
9	Bearing mounting ring	18	Nut side inserts

are used as the transmission system in the motor module. The specification of the harmonic drive are listed in Table. 6.7.

The design of the joint-coupling module structure satisfies the following requirements:

- It provides support and accommodation for motors, harmonic drives and the electronic

Table 6.7: Specifications of Harmonic drive

Gear ratio	100
Rated Torque(Nm)	24
Limit for average torque(Nm)	39
Weight (kg)	0.15
Limit for repeated torque (Nm)	54
Moment of inertia ($Kg.cm^2$)	0.079
Max. input speed (rpm)	10000(Oil Lub.)

Figure 6.14: Fabricated modules for electronic joint-coupling

interface

- It produces 1-DOF rotation output between the upper and lower cubical housing.
- The motor is compatible with other units of the modular platform for reconfiguration.

The specification of the electronic joint-coupling module is shown in Table. 6.8. The motor modules connected through interface to perform electronic gearing are shown in Fig. 6.15. The completed joint-coupling module (with rigid connector) is shown in Fig. 6.16.

Figure 6.15: Prototype of electronic joint-coupling module

Table 6.8: Specifications of electronic joint-coupling module

Dimensions of(motor unit)	$70 \times 70 \times 140mm$
Dimensions of(JC housing)	$70 \times 70 \times 70mm$
Dimensions of(tubular link)	$\phi 40 \times 220mm$
Weight	2 kg (excluding motor unit)
coupling coefficients	continuously variable

6.4.3 Tests on Electronic Joint-Coupling Module

One electronic joint-coupling module is built and tested to be incorporated into the two-DOF planar parallel manipulator. The goals of the experiments are to monitor the actual running angular position of the joints (master as well as slave) simultaneously and to compare with the working principle of the desire joint-coupling mechanism. Three tests with different conditions are done to judge the achievement of the electronic gearing performance.

DC Power supply of 48V is used to provide electrical power for the motors. The host PC (QuickControl) downloads the motion control program for the motors. Two *SilverMax* RS-232 Cables are used to communicate between motors and the host PC. The electronic interface PCB is used to interface IO channels for electronic gearing. The PCB was designed

and developed for IO channels interface of *SilverMax* motors for the project. The other requirements, the control panel and control commands are the same as indicated in Section 6.4.3 for mechanical joint-coupling module motors. The IO ports from the motors are connected to electronic interface PCB. The IO channels are connected according to electronic gearing programming mode. The electronic connection is shown in Fig. 6.16.

Figure 6.16: Experiment setup for electronic joint-coupling module

Experiment 2

This experiment is conducted on the electronic gearing of two motors with speed ratio of $n = 1.0$. Result shown in Fig. 6.17 indicates that the speed of the master joint and slave joint are almost identical with the ratio of 1. The slave joint responded with a small time delay (40msec) from the master joint. There is a small overshoot on slave response when it is trying to synchronize with the master joint. This can be fine-tuned with the servo parameters of the slave joint.

Experiment 3

This experiment is conducted on the electronic gearing of two motors with speed ratio of $n = 0.5$. Result shown in Fig. 6.18 indicates that the slave joint approximately followed the master joint with velocity ratio of 0.5. It showed that the exact differential ratio is achievable. Similar to the previous experiment result, the slave joint responded with a delay time about 40ms compared with the master joint. There is still overshoot in the

Figure 6.17: Coupled joints velocities profile (for $n = 1.0$)

slave joint.

Figure 6.18: Coupled joints velocities profile (for $n = 0.5$)

Experiment 4

This experiment is conducted on the electronic gearing of two motors with variable speed ratios $n = 0.5, 2, -0.5$. Test results shows that the electronic gearing works well as the slave joint can follow the variable speed change in the master joint. The slave joint responded with a delay about 40ms compared with the master joint. In the electronic joint-coupling, the speed ratio of the couple joints change according to the coupling formula and the

kinematics of the manipulator. This experiment demonstrated that electronic gearing on the couple joint can achieve such kind of variable speed ratio changes. Therefore, it can be concluded that the electrical gearing approach is a feasible and practical way to achieve continuous variable joint-coupling. The performance of the joint-coupling module will be more reliable with only actuators and electronic motion controllers without additional mechanical transmission components.

Figure 6.19: Continuous variable joint velocity profile (for $n = 0.5, 2, -0.5$)

6.5 Summary

In this chapter, the design of joint-coupling devices for parallel manipulators is elaborated. Prototypes of joint-coupling devices based on mechanical and electronic means are developed. The mechanical joint-coupling using fixed coupling ratio on a differential planetary mechanism is adopted. For electronic joint-coupling device, an electronic gearing system, which can variate the coupling ratio is developed. All module are deployed on a reconfigurable parallel manipulator system for further a experimental study. Experiments on each prototype are conducted to verify the concept of joint-coupling. In the next chapter, experimental work on the two-DOF planar parallel manipulator with the joint-coupling module will be described for the overall investigation of the joint-coupling concept.

Chapter 7

Experimental Study on Joint-Coupling

As a follow-up to Chapter 6, where the realization of joint-coupling devices has been studied, the experimental work on the prototype of the two-DOF planar PM with mechanical joint-coupling is carried out. The advantage of using joint-coupling is to alter the direct singularity within the reachable workspace. To verify the concept of joint-coupling devices, experimental work on the two-DOF planar PM with mechanical joint-coupling module is conducted. Generally speaking, the position control of PMs can provide high positioning accuracy and hence, the speed and smoothness of manipulator motion do not play major roles. On the other hand, velocity control can provide smooth operation for trajectory-tracking and also high positioning accuracy. Therefore, the velocity control has been also proposed for high speed PMs like Delta [76], Hexa [77] and other applications [78].

In this chapter, an experiment is studied based on the velocity (MRV) command from the SilverMax control panel for desired trajectory-tracking. We focus on altering direct singularities according to desired specifications of the trajectory. The kinematic model formulated in Chapter 2 and the dynamic model from Chapter 4 are applied in this chapter. The results verify our proposed kinematic model performance and show that the end-

Figure 7.1: Prototype of two-DOF planar PM with mechanical joint-coupling

effector of the two-DOF planar PM with joint-coupling can overcome the direct singularity postures.

7.1 Description of Move Relative Velocity Based Command

We apply two *SilivreMax* motors as inputs of the two-DOF planar PM. *QuickControlTM* is a software package designed as a programming, diagnostic and prototyping tool for *SilverMaxTM* motors. From this package, the *Move Relative Velocity* based (MRV) command is used for the trajectory-tracking in the manipulator. MRV command initiates a move relative to the current target position. Relative distances are based on the incremental encoder resident in the motor. The move profile uses input velocities and accelerations.

Inputs for MRV command are shown in Fig. 7.2 as an example. Relative distance is an encoder count value that the motor will move from its current position. Giving a positive value will cause the motor move in a positive count direction. Giving a negative value will cause the motor to move in a negative count direction. The polarity distance number

Figure 7.2: Move Relative Velocity Based command example

always determines the direction that the motor will turn. The distance units are in encoder counts. An encoder in the motor that provides 4000 counts per revolution. Time-based moves will attempt to make the desired move within the given time. Velocity based moves use acceleration and velocity parameters to accomplish the motion profile. If the velocity value needed to make the move exceeds the maximum permissible limit, the move will not be executed and an error code will be set in the Foreground Command Error. Foreground Command Error is generated when a command cannot be executed due to parameters that are out of range. Example commands of *QuickControl* for Move Relative Velocity Based (MRV) is shown in Fig. 7.2. The main menu of *QuickControl* toolbox is illustrated in Fig. 7.3.

7.2 Trajectory Computation

In MRV control scheme, the mid-instance of every sampling time can be found for each interpolation to maintain that the ramp velocity ($\dot{\theta}_r$) is the average velocity for that period. This will be carried out the following steps.

- (1). Knowing input angular velocities ω_1 and ω_2 and coupling coefficients c_1 and c_2 , the coupled joint rates $\dot{\theta}_{11}$, $\dot{\theta}_{12}$, $\dot{\theta}_{21}$ and $\dot{\theta}_{22}$ can be found by applying Eq. (4.36). Meanwhile,

Figure 7.3: Main menu of *QuickControl* toolbox

$\ddot{\theta}_{11}$, $\ddot{\theta}_{12}$, $\ddot{\theta}_{21}$ and $\ddot{\theta}_{22}$ can be obtained using Eq. (4.37) and given angular accelerations $\dot{\omega}_1$ and $\dot{\omega}_2$.

(2). The joints moves at the rate of $\dot{\theta}_k$ for a sampling period t_s to each joint displacement θ_{k+1} . The desired joint displacement is calculated by $\theta_{k+1} = \theta_k + \dot{\theta}_k \cdot t_s$.

(3). Find the elapsed time in the accelerated zone as $t_a = \dot{\theta}_r / \ddot{\theta}_r$.

(4). Find the number of interpolations in accelerated and decelerated zones as $step_{acce} = t_a / t_s$.

(5). Since the time taken along trajectory is desired as $t_t = 2t_a + t_v$, the total time for velocity constant zone t_v can be obtained.

(6). Calculate the displacement covered entire trajectory of the actuator, d from the relation of : $t_v = (d - \ddot{\theta}_r \cdot t_a^2) / \dot{\theta}_r$

- (7). Find the number of interpolations in the constant velocity zone as $step_{velo} = t_v/t_s$.
- (8). Cover along the trajectory with $k = 0, 1, 2, 3 \dots$ for $step_{acce}$, $step_{velo}$ and $step_{deacce}$ representing the k^{th} interpolation in each of the zone and calculate $t_k = (2k + 1)/2$ at the mid instance of every interpolation.
- (9). Solve the direct kinematics to find the end-effector position $(x_p, y_p)_{k+1}$ after knowing the joint postures θ_{k+1} in every instance.
- (10). Calculate $\mathbf{A}_{J1}$ and $\mathbf{B}_{J1}$ from Eq. (3.10) using θ_{k+1} related to the joint positions of the manipulator.
- (11). Calculate $\dot{\mathbf{p}}$, the velocity of end-effector of the manipulator through Eq. (3.10) to verify the desired input velocities.
- (12). Calculate the required torques of joint-coupling devices, τ_1 and τ_2 through Eq. (4.45) to compare with experimental results and the theoretical ones.

Here the dynamic model of the planar PM in Chapter 4 is used for the experiment. The mechanical joint-coupling module prototype presented in Chapter 6 with fixed coefficients ($c_1 = 4$ and $c_2 = 5$) is applied to illustrate the joint-coupling concept.

7.3 Experimental Study

To verify the velocity based motion control and its usefulness in trajectory-tracking, the experiment is conducted on the two-DOF planar PM with mechanical joint-coupling module. We use the same experimental setup mentioned in Section 6.4.3 for trajectory-tracking.

We desire the angular displacement of joint A_1 as 8.7 rad and joint A_2 as 2.55 rad in positive direction and as 1.39 rad in negative direction. As mentioned in Section 7.2, the joint rate of $\dot{\theta}_k$ can be calculated using $\dot{\theta}_k = \frac{\theta_{k+1} - \theta_k}{t_s}$. Knowing $\dot{\theta}_k$, the required input

velocity of joint-coupling can be determined using Eq. (4.36). In this experiment, the following data is prescribed for desired conditions.

Prescribed trajectory data:

Joint coupling coefficient $(c_1, c_2) = (4, 5)$

Total angular displacement of joint-coupling 1 = 4940 *counts*

Total angular displacement of joint-coupling 2 = 1623 *counts* in positive direction

Total angular displacement of joint-coupling 2 = -900 *counts* in negative direction

Ramp velocity of joint-coupling 1, $\omega_1 = 0.5 \text{ rad/sec}$ (or) 512.67 *cps*

Ramp velocity of joint-coupling 2, $\omega_2 = 0.5 \text{ rad/sec}$ for positive direction (or) 512.67 *cps*

Ramp velocity of joint-coupling 2, $\omega_2 = -0.25 \text{ rad/sec}$ for negative direction (or) -356 *cps*

Acceleration of joint-coupling 1, $\dot{\omega}_1 = 1 \text{ rad/sec}^2$ (or) 1424 *cps/s*

Acceleration of joint-coupling 2, $\dot{\omega}_2 = 1 \text{ rad/sec}^2$ (or) 1424 *cps/s* in positive direction

Acceleration of joint-coupling 2, $\dot{\omega}_2 = 0.5 \text{ rad/sec}^2$ (or) -712 *cps/s* in negative direction

Initial posture of the joints $(\theta_{11}, \theta_{21}) = (2.44, 3.2) \text{ rad}$

Total time taken $t_t = 22 \text{ sec}$

Total time taken for entire trajectory $t_s = 0.50 \text{ sec}$.

The desired joint-coupling input velocity profile is shown in Fig. 7.4. Results obtained from experiment are shown in Fig. 7.5. These figures show the presence of noise in the encoder reading.

Figure 7.4: Desired angular velocity of joint-coupling device 1 and 2

A technique based on controlled-continuity splines [79] is used to eliminate the noise. In

Figure 7.5: Angular velocity of joint-coupling device 1 and 2 from experiment

this smoothing technique, let $S(g)$ be an energy functional typically represented by the basic generalized spline stabilizers [80], which measures the smoothness of function $g(x)$. A function $g(x)$ with a lot of discontinuities generate high energy, $S(g)$. A smooth function will produce a low value for $S(g)$. A generalized spline as a smoothing function $f(x)$ is defined as

$$\text{minimize } \int_{x_0}^{x_n} S(g)dx \quad (7.1)$$

among all functions $g(x)$ such that

$$\sum_{i=0}^n \left(\frac{g(x_i) - y_i}{\delta y_i} \right)^2 = 0, \quad g \in U^2[x_0, x_n] \quad (7.2)$$

Where $\{(x_i, y_i), i = 0, \dots, n\}$ is set of data points and U^2 is the space of functions which are continuous and have continuous first and second derivatives. By using this technique, the noise can be eliminated. Figure 7.6 shows the velocity of joint-coupling device 1 and 2 after smoothing.

Figure 7.7 shows the desired joint displacement and the obtained displacement from experimental results. The purpose of this trajectory-tracking is to verify that the manipulator can operate in the singularity-free joint space using coupling coefficients $c_1 = 4$ and $c_2 = 5$. The comparison of singularity manifolds of the manipulator with and without joint-coupling ($c_1 = 1$ and $c_2 = 0$) is shown in Fig. 7.8. Figure 7.9 represents the end-effector movement along the trajectory-tracking. Velocities of joints and required torques are calculated based on experimental results and shown in Fig. 7.10 and Fig. 7.11.

Figure 7.6: Angular velocity of joint-coupling device 1 and 2 after smoothing

Figure 7.7: Displacement of joints

Figure 7.8: Trajectory and singularity manifolds in joint-space

7.3.1 Results and Discussion

The *SilverMax* motor encoder reading supports the velocity of the input motor of the joint-coupling device. These readings show the presence of noise in the results. The time taken for each command to be sent out from the host PC to the actuator module is found to be within 40 to 50 milliseconds. This delay time introduces the error in the end-effector motion because no information is available from the motors during this time. There is some position error in the experiment in which the trajectory of the planar PM is very close to the workspace boundary (i.e., inverse singularity posture). Due to the inertial effect and closeness to the workspace boundary, the end-effector goes beyond the desired final position. The error between the simulation and experimental velocity and torque result is acceptable. It is clearly seen that the end-effector of planar PM with joint-coupling can overcome the direct singularity posture of the two-DOF planar PM without joint-coupling as shown in Fig. 7.11 using proper torque inputs.

Figure 7.9: End-effector displacement from trajectory-tracking

Figure 7.10: Comparison of joint velocities

Figure 7.11: Comparison of required torques of joint-coupling

7.4 Summary

In this chapter, the experiment on the two-DOF planar PM with the mechanical joint-coupling module has been presented. This prototype uses *SilverMax* servomotors as input actuation. *SilverMax* servomotors are controlled by *QuickControl* software package on the host PC. There are motion control commands based on time and velocity in this software. Motion Relative Velocity Based (MRV) commands in this software are used to control the manipulator trajectory. The position and velocity profiles can be obtained from the encoder reading of the motors. Motors respond with delay time of 40 to 50 ms, which is acceptable. Through this example, it can be verified that the two-DOF planar PM with mechanical joint-coupling indeed can manage this singularity of the manipulator in reality.

Chapter 8

Conclusions

In this work, the necessary issues of using joint-coupling in PMs have been addressed. The issues include kinematics and dynamics, singularity analysis, singularity management in manipulators with joint-coupling. Design and implementation of joint-coupling device prototypes and experiments on the prototypes are also carried out. Specific work are summarized as follows.

Instantaneous Kinematics and Singularity Analysis of Joint-Coupling

The instantaneous kinematics of planar PMs with joint-coupling are derived based on the vector method. The Jacobian matrices of planar two-DOF and three-DOF PMs are obtained from the instantaneous kinematics. The direct Jacobian matrix of the manipulator is a function of the coupling coefficients. Hence, changing the value of coupling coefficients can alter the direct singularity manifolds of the planar PMs. However, the inverse Jacobian matrix is independent of the coupling coefficients and thus inverse singularities of planar PMs cannot be changed. This result tallies with current understanding of the occurrence of inverse singularity at the workspace boundary that is defined by the link parameters of the planar PMs.

Dynamic Model of Planar PM with Joint-Coupling

Lagrangian formulation for generating the equations of motion of planar PMs with joint-coupling is proposed. Besides the kinematic constraints on the planar PM structure, joint-coupling introduces new constraints on the generalized coordinates (in this case, all the joint angles in the planar PM) in the Lagrangian equations. The equation of motion of the manipulators then can be written in terms of the input from the joint-coupling devices.

Singularity Management Using Joint-Coupling

One major goal of introducing joint-coupling in a planar PM is to deal with manipulators singularities. As indicated in Chapter 3, the coupling coefficients appear in the direct Jacobian matrix of the manipulators, it is now possible to "engineer" the singularity by giving different values of the coupling coefficients while the architecture of the manipulators, (i.e., the link parameters) remains unchanged. Here we proposed task-oriented singularity management method for choosing the appropriate coupling coefficients based on the given task. This method is formulated as a nonlinear optimization problem to minimize the distance from the given task point to the singularity manifolds. Examples are provided to show the feasibility of this method for two-DOF and three-DOF planar PMs. Extension of this method to spatial PMs is also possible. Other than changing the singularities using the kinematic properties, the method of limiting manipulators torques at the joint-coupling input is proposed. Based on a prescribed trajectory, this method minimizes the total manipulator power (or power at the actuators) allow the trajectory, which requires the dynamic model of the planar PM with joint-coupling, and also keeps the torque input from the joint-coupling module within the maximum value. This optimization method can determine appropriate joint-coupling coefficient ratios which produce minimal actuator power and also avoid direct singularities. Simulation results show the desired end-effector path can pass through the direct singularities of planar PMs without joint-coupling. It can also regulate the input actuator torque within a maximum limit.

Development of Joint-Coupling Device Prototypes

The joint-coupling device is realized in two manners: mechanical joint-coupling, in which planetary differential mechanisms are employed for a 1-input 2-output joint-coupling module, and electronic joint-coupling, in which both joints in the coupling device are replaced with integrated actuators, which are driven in a coordinate fashion through a single motion controller. The prototype of joint-coupling devices are designed based on a modular robot platform that can be easily reconfigured. The mechanical joint-coupling device can only provide a fixed coupling ratio because of the use of gearing mechanisms. Electronic joint-coupling can give continuously variable coupling ratio by using a microprocessor control for the two motors to obtain synchronized motion. Both mechanical and electronic prototypes are developed for verification. Although many new concepts can be examined using computer simulations, experiments on a real manipulator provide more valuable and reliable knowledge of its functions applicability in practical applications.

8.1 Future Work

This work is a starting point to address the joint-coupling concept in a planar PM. There are still quite a few issues which remain unsolved.

A. Kinematics and Singularity Analysis on Spatial Parallel Manipulator

Extension of current kinematics to spatial PMs is possible. By using electronic joint-coupling, single input multi-output (more than two) synchronized motion control scheme can be an interesting area for singularity management. Geometrically axes of the coupled joints are no longer required to be in parallel or intersecting. Hence, it is quite easy to design spatial PMs with joint-coupling.

B. Singularity Management

The proposed singularity management methods are based on kinematics and dynamics.

task-oriented and the motor limit torque. Though both approaches can determine appropriate coupling coefficient ratio, extension to optimal time based and manipulator-architecture based approaches are also possible. It is also possible to find optimal coupling design in order to obtain the largest singularity-free region within the reachable workspace of the manipulators.

The ultimate goal is to determine the direct singularity within the reachable workspace and to operate the end-effector in the singularity-free region for all kinds of tasks. In addition, joint-coupling can be extended to all type of actuators, revolute or linear types. In the research of PMs, this thesis highlighted a different way to avoid direct singularities within the workspace and the jointspace of the planar PMs with the introduction of joint-coupling. Several interesting problems in joint-coupling and solutions have been illustrated; experimental works have been carried out on the prototype of joint-coupling devices. The thesis showed that the use of joint-coupling in PMs is very promising in the near future.

Appendix A

Expressions in Equations of Motion

$$G_{11} = I_{11} + 0.25m_{11}l_{11}^2 + m_{12}l_{12}^2$$

$$G_{12} = I_{12} + 0.5m_{12}l_{11}l_{22}\cos(\theta_{11} - \theta_{12})$$

$$G_{13} = -0.5m_{12}l_{11}l_{12}\sin(\theta_{11} - \theta_{12})\dot{\theta}_{12}^2 + \lambda_1(2al_{11}\sin\theta_{11} + 2l_{11}X_p\sin\theta_{11} - 2l_{11}Y_p\cos\theta_{11})$$

$$G_{21} = 0.5m_{12}l_{11}l_{12}\cos(\theta_{11} - \theta_{12})$$

$$G_{22} = I_{12} + 0.25m_{12}l_{12}^2$$

$$G_{23} = -0.5m_{12}l_{11}l_{12}\sin(\theta_{11} - \theta_{12})\dot{\theta}_{11}^2$$

$$G_{31} = I_{21} + 0.25m_{21}l_{21}^2 + m_{22}l_{22}^2$$

$$G_{32} = I_{22} + 0.5m_{22}l_{21}l_{22}\cos(\theta_{21} - \theta_{22})$$

$$G_{33} = -0.5m_{22}l_{21}l_{22}\sin(\theta_{21} - \theta_{22})\dot{\theta}_{22}^2 + \lambda_2(2al_{21}\sin\theta_{21} + 2l_{21}X_p\sin\theta_{21} - 2l_{21}Y_p\cos\theta_{21})$$

$$G_{41} = 0.5m_{22}l_{21}l_{22}\cos(\theta_{21} - \theta_{22})$$

$$G_{42} = I_{22} + 0.25m_{22}l_{22}^2$$

$$G_{43} = -0.5m_{22}l_{21}l_{22}\sin(\theta_{21} - \theta_{22})\dot{\theta}_{21}^2$$

$$F_{11} = G_{11} + G_{12}R_{11}$$

$$F_{12} = G_{12}R_{12}$$

$$F_{21} = G_{21} + G_{22}R_{11}$$

$$F_{22} = G_{22}R_{12}$$

$$\begin{aligned}
F_{31} &= G_{32}R_{21} \\
F_{32} &= G_{31} + G_{32}R_{22} \\
F_{41} &= G_{42}R_{21} \\
F_{42} &= G_{41} + G_{42}R_{22} \\
N_1 &= G_{14} + G_{13}(Q_{11}\dot{\theta}_{11} + Q_{12}\dot{\theta}_{12})^2 \\
N_2 &= G_{23}\dot{\theta}_{11}^2 \\
N_3 &= G_{34} + G_{33}(Q_{21}\dot{\theta}_{21} + Q_{22}\dot{\theta}_{22})^2 \\
N_4 &= G_{43}\dot{\theta}_{21}^2 \\
Q_{11} &= -\frac{l_{11}\sin(\theta_{11} - \theta_{22})}{l_{12}\sin(\theta_{12} - \theta_{22})} \\
Q_{12} &= -\frac{l_{11}\sin(\theta_{22} - \theta_{21})}{l_{12}\sin(\theta_{12} - \theta_{22})} \\
Q_{21} &= -\frac{l_{11}\sin(\theta_{11} - \theta_{12})}{l_{12}\sin(\theta_{12} - \theta_{22})} \\
Q_{22} &= -\frac{l_{11}\sin(\theta_{12} - \theta_{21})}{l_{12}\sin(\theta_{12} - \theta_{22})} \\
R_{11} &= -\frac{l_{11}l_{12}\sin(\theta_{11} - \theta_{22})\sin(\theta_{12} - \theta_{22})}{l_{12}^2\sin^2(\theta_{12} - \theta_{22})} \\
R_{12} &= -\frac{l_{11}l_{12}\sin(\theta_{22} - \theta_{21})\sin(\theta_{12} - \theta_{22})}{l_{12}^2\sin^2(\theta_{12} - \theta_{22})} \\
R_{13} &= \frac{1}{l_{12}^2\sin^2(\theta_{12} - \theta_{22})}[\{l_{12}\sin(\theta_{12} - \theta_{22})\}[-l_{11}\cos(\theta_{11} - \theta_{22})\dot{\theta}_{11}(\dot{\theta}_{11} - \dot{\theta}_{22}) \\
&\quad - l_{11}\cos(\theta_{22} - \theta_{21})\dot{\theta}_{21}(\dot{\theta}_{22} - \dot{\theta}_{21})] - l_{12}\cos(\theta_{12} - \theta_{22})(\dot{\theta}_{12} - \dot{\theta}_{22})l_{11}\sin(\theta_{11} - \theta_{22})\dot{\theta}_{11} \\
&\quad + l_{11}\sin(\theta_{22} - \theta_{21})\dot{\theta}_{21}] \\
R_{21} &= -\frac{l_{21}l_{22}\sin(\theta_{11} - \theta_{12})\sin(\theta_{12} - \theta_{22})}{l_{22}^2\sin^2(\theta_{12} - \theta_{22})} \\
R_{22} &= -\frac{l_{21}l_{22}\sin(\theta_{12} - \theta_{21})\sin(\theta_{12} - \theta_{22})}{l_{22}^2\sin^2(\theta_{12} - \theta_{22})} \\
R_{23} &= \frac{1}{l_{22}^2\sin^2(\theta_{12} - \theta_{22})}[\{l_{22}\sin(\theta_{12} - \theta_{22})\}[-l_{21}\cos(\theta_{11} - \theta_{22})\dot{\theta}_{11}(\dot{\theta}_{11} - \dot{\theta}_{12}) \\
&\quad - l_{21}\cos(\theta_{12} - \theta_{21})\dot{\theta}_{21}(\dot{\theta}_{12} - \dot{\theta}_{21})] - l_{22}\cos(\theta_{12} - \theta_{22})(\dot{\theta}_{12} - \dot{\theta}_{22})l_{21}\sin(\theta_{11} - \theta_{22})\dot{\theta}_{11} \\
&\quad + l_{21}\sin(\theta_{22} - \theta_{21})\dot{\theta}_{21}]
\end{aligned}$$

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