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A LAMB WAVE APPROACH FOR FATIGUE DAMAGE PROGNOSIS WITH PIEZOELECTRIC TRANSDUCERS

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Signature

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TABLE OF CONTENTS

	PAGE
ACKNOWLEDGEMENTS	i
TABLE OF CONTENTS	ii
SUMMARY	vii
LIST OF TABLES	ix
LIST OF FIGURES	x
LIST OF SYMBOLS	xiv
 CHAPTER 1 INTRODUCTION	 1
1.1 OVERVIEW OF STRUCTURAL DAMAGES AND FAILURES	1
1.2 STRUCTURAL HEALTH MONITORING AND DAMAGE PROGNOSIS	3
1.3 MOTIVATION FOR DAMAGE PROGNOSIS	5
1.4 RESEARCH OBJECTIVES	6
1.5 RESEARCH ORIGINALITY AND CONTRIBUTIONS	6
1.6 THESIS ORGANISATION	7
 CHAPTER 2 LITERATURE REVIEW ON LAMB WAVE IN STRUCTURAL HEALTH MONITORING (SHM) AND DAMAGE PROGNOSIS (DP)	 9
2.1 BACKGROUND	9
2.2 PIEZOELECTRICITY AND PIEZOELECTRIC MATERIALS	9
2.2.1 Piezoelectricity	9
2.2.2 Constitutive Relations	10
2.2.3 Piezoelectric Constants	12
2.2.4 Dielectric Constants	12
2.2.5 Piezoelectric Materials	12
2.2.5.1 Piezoelectric Ceramics	12
2.2.5.2 Piezoelectric Polymers	13
2.3 THEORY OF LAMB WAVES	14
2.3.1 Derivation of Lamb Waves Equations	14

2.3.2	Dispersion curves	17
2.4	LAMB WAVES IN STRUCTURAL HEALTH MONITORING	18
2.5	GENERATION OF LAMB WAVES WITH PIEZOELECTRIC CERAMICS	19
2.6	SIGNAL PROCESSING FOR FEATURE EXTRACTION	21
2.6.1	Cross Correlation	21
2.6.2	Hilbert Transform Envelope Detection	21
2.6.3	Spectrogram	23
2.7	IDENTIFYING DAMAGE WITH LAMB WAVES USING PZT	23
2.7.1	Disbonding and Delamination Detection	23
2.7.2	Damage Detection	24
2.7.3	Fatigue Crack Detection	25
2.8	ADVANTAGES OF LAMB WAVE INSPECTION TECHNIQUES	27
2.9	LIMITATIONS OF LAMB WAVE TECHNIQUE	27
2.10	FATIGUE AND CRACK GROWTH MODELS	27
2.10.1	Fatigue Damage	27
2.10.2	Linear Elastic Fracture Mechanics	28
2.10.3	Fatigue Crack Growth	30
2.11	DAMAGE PROGNOSIS	32
2.12	NEEDS FOR FURTHER RESEARCH IN LAMB WAVE TECHNIQUE FOR FATIGUE DAMAGE DETECTION AND PROGNOSIS	33
2.13	CONCLUDING REMARKS	33
 CHAPTER 3 MODELLING OF PZT TRANSDUCER RESPONSE IN A NARROW PLATE		 35
3.1	BACKGROUND	35
3.2	PZT TRANSDUCER RESPONSE IN AN INFINITE PLATE	35
3.3	REFLECTION OF LAMB WAVES	36
3.3.1	Normal incidence Lamb Waves	37
3.3.2	Obliquely incidence Lamb Waves	38
3.3.3	Numerical Simulation	39
3.3.3.1	Convergence Study	39

3.3.3.2	Reflection Coefficients	43
3.3.4	Experimental Study	43
3.4	PZT TRANSDUCER RESPONSE IN AN NARROW PLATE	47
3.4.1	Equivalent Imaginary Source	47
3.4.2	Analytical Formulation	52
3.4.3	Numerical Simulation	55
3.4.4	Experimental Study	56
3.4.5	Discussion	57
3.5	SUMMARY	58

CHAPTER 4 DETECTION AND MONITORING OF FATIGUE CRACK GROWTH IN NARROW PLATE

4.1	BACKGROUND	60
4.2	EFFECT OF WIDTH ON SENSOR RESPONSE	60
4.2.1	Numerical Study	60
4.2.2	Experimental Study	64
4.3	CRACK MONITORING	69
4.3.1	Equivalent Imaginary Source Model	69
4.3.2	Experimental Setup	72
4.3.3	Actuation Signal	73
4.3.4	Data Acquisition	74
4.3.5	Experimental Results	74
4.3.6	Damage Indexes	76
4.3.6.1	Variation in peak to peak amplitude	76
4.3.6.2	Variation in scatter energy	77
4.3.6.3	Variation in transient energy	79
4.3.6.4	Variation in time of flight	80
4.3.7	Discussions	81
4.4	SUMMARY	82

CHAPTER 5 UNCERTAINTY MANAGEMENT AND DAMAGE PROGNOSIS (DP)

5.1	BACKGROUND	84
5.2	DAMAGE PROGNOSIS	84
5.2.1	Linear Elastic Fracture Mechanics	84
5.2.2	Paris Equation	85
5.2.3	Remaining Life Estimation	86
5.2.4	Monte Carlo Simulation	86
5.3	EXPERIMENTAL STUDY	87
5.3.1	Uncertainty in Fatigue Properties	90
5.3.2	Uncertainty in Crack Length Estimation	92
5.3.3	Monte Carlo Convergence Test	94
5.3.4	Damage Prognosis	94
5.4	ITERATIVE FRAMEWORK	96
5.4.1	Damage Prognosis	98
5.5	CONCLUSION	100

CHAPTER 6 FATIGUE DAMAGE PROGNOSIS OF WELDED STEEL CONNECTIONS		101
6.1	BACKGROUND	101
6.2	FATIGUE DESIGN OF WELED STEEL SPECIMEN	101
6.3	FRACTURE ANALYSIS OF SEMI-ELLIPTICAL SURFACE CRACK	103
6.4	EXPERIMENTAL STUDY	104
6.4.1	Lamb wave propagation (LWP) Technique	107
6.4.1.1	Experimental Setup	107
6.4.1.2	Results	107
6.4.2	Electromechanical Impedance (EMI) Technique	111
6.4.2.1	Experimental Setup	111
6.4.2.2	Results	112
6.4.3	Discussions	114
6.5	CONCLUSIONS	119

CHAPTER 7 SUMMARY AND FUTURE WORK	
7.1 BACKGROUND	120
7.2 SUMMARY	120
7.3 RESEARCH ORIGINALITY AND CONTRIBUTIONS	122
7.4 FUTURE WORK	122
 AUTHOR'S PUBLICATIONS	 124
REFERENCES	R1
APPENDIX A	A1
APPENDIX B	B1

SUMMARY

Fatigue is a progressive and localised damage that occurs when a material is subjected to cyclic loading. Historical cases have shown that undetected fatigue cracks often leads to catastrophic failure, including the loss of lives and assets. It is therefore important to have a robust non-destructive evaluation (NDE) technique to detect and monitor these cracks. The field Structural Health Monitoring (SHM) evolved in search for a robust system for damage detection and monitoring. As SHM develops, the need to be able to detect and monitor a damage no longer suffices. Being able to predict the system's remaining useful life arises as the next big challenge. This leads to the conceptualisation of Damage Prognosis (DP).

This research aims to expand the present capabilities of the Lamb wave propagation (LWP) technique for structural health monitoring (SHM) and damage prognosis (DP) and to develop a DP model to predict the growth of fatigue crack and subsequently the remaining useful life of material or structure in a narrow plate environment. A novel concept of an equivalent imaginary source (EIS) is proposed to model the displacement and strain field of reflected Lamb waves. This EIS incorporates information on the reflection coefficient and phase delay of reflected waves. The concept of EIS is then incorporated into an existing model for voltage output of PZT transducers in a narrow plate. Using the extended model, a new damage index is proposed based on the change in peak amplitude of the sensor signals. Numerical simulations and experimental studies are conducted to verify the above model and damage index.

Fatigue is stochastic in nature and there are numerous uncertainties in terms of the materials and damage detection techniques. Monte Carlo simulation is adopted to manage the uncertainties. An iterative framework is proposed to estimate the remaining life of a component when information on the fatigue properties of the material is not available. The error in prognosis reduces by adopting the proposed iterative framework.

Damage prognosis is a multi-disciplinary and challenging problem. Thus, collective effort from the various field of research is required to solve this problem. The author hopes that the originality and contribution from this research will benefit the community and spur research in the field of damage prognosis.

LIST OF TABLES

	PAGE
Table 2-1: Matrix Notation (IEEE, 1987).....	10
Table 3-1: Material properties of 2024-T3	40
Table 3-2: TOA of A_0 wave for various element sizes	42
Table 4-1: Material properties	65
Table 4-2: Fatigue load cycles and crack lengths	73
Table 5-1 : Fatigue life of Specimens S1 - S6	88
Table 5-2: Paris coefficients of individual specimen.....	91
Table 5-3: Paris coefficients from the iterative framework	98
Table 5-4: Comparison of damage prognosis models	99
Table 6-1: Parameters for calculation of fatigue crack growth curve.....	116

LIST OF FIGURES

	PAGE
Figure 1-1 : (a) Fracture surface; & (b) Fatigue cracks of right wing rear spar (NSTB, 2005).....	2
Figure 1-2: (a) Sketch of bridge ; & (b) Point of failure initiation(NSTB, 1971)	3
Figure 1-3: Components and operations of typical SHM system (Boller, 2002)	4
Figure 1-4: Relationship between Usage Monitoring, Structural Health Monitoring, and Damage Prognosis (Farrar et al., 2003)	5
Figure 2-1: Axes notation of a typical piezoelectric material	11
Figure 2-2: PZT crystal structure	13
Figure 2-3: Lamb wave modes: (a) symmetric; (b) anti-symmetric	14
Figure 2-4: The isotropic plate for which dispersion relations are derived	15
Figure 2-5: Dispersion Curves for: (a) phase velocity ; & (b) group velocity.....	18
Figure 2-6: (a) Strain predicted based on Equation (2.31) ; (b) Experimental result of PZT response	20
Figure 2-7: Cross correlation of simulated tone-burst signal: (a) 3-count sine signal; (b) 3-count Hanning window smoothed signal	21
Figure 2-8: Envelope detection using Hilbert transform	22
Figure 2-9: Damage Index vs $\Delta\Omega$	26
Figure 2-10: Modes of crack surface displacement	29
Figure 2-11: Typical crack growth rate curve	31
Figure 2-12: Types of Failure Model: (a) curve fitting ; (b) group classification	32
Figure 3-1: Reflection coefficient of (a) normal incident S_0 wave and (b) normal incident A_0 wave (Gunawan and Hirose, 2007)	38
Figure 3-2: Phase and group velocities at 200kHz for 1mm thick 2024-T3 aluminium alloy	41
Figure 3-3: TOA of A_0 wave for various element sizes	42
Figure 3-4 : Layout of actuators and sensors.....	44
Figure 3-5 : Absolute reflection coefficient for reflected S_0 wave due to S_0 incident wave.....	45
Figure 3-6 : Phase delay for reflected S_0 wave due to S_0 incident wave	45

Figure 3-7 : Absolute reflection coefficient for reflected A_0 wave due to A_0 incident wave.....	46
Figure 3-8 : Phase delay for reflected A_0 wave due to A_0 incident wave.....	46
Figure 3-9 : Sensor signal at S2 before cutting of plate.....	47
Figure 3-10 : (a) Reflection form a source in a semi-infinite plate; (b) Propagation path from an equivalent imaginary source in an infinite plate	48
Figure 3-11 : 5-cycle Hanning windowed sine wave and its Fourier transform.....	49
Figure 3-12 : Comparison between EIS and numerical simulation for A_0 reflection	51
Figure 3-13 : Comparison between EIS and numerical simulation for S_0 reflection.....	51
Figure 3-14 : Sensor response for (a) direct S_0 ; (b) edge reflected S_0 ; (c) direct A_0 and (d) end reflected S_0	54
Figure 3-15 : Numerical Model in PZFlex	55
Figure 3-16 : Comparison between analytical model and numerical study for 400mm wide x 1mm thick 2024-T3 aluminium alloy plate	56
Figure 3-17 : Comparison between analytical model and experimental study for 400mm wide x 1mm thick 2024-T3 aluminium alloy plate	57
Figure 3-18 : Comparison between analytical model and experimental study for reduced width PZT.....	58
Figure 4-1: Schematic diagram of plate.....	61
Figure 4-2: Output voltage of PZT sensor from numerical simulation of a 400mm plate.....	62
Figure 4-3: Screenshots of the numerical simulation (out-of-plane displacement) at (a) 50 μ s ; (b) 100 μ s ; and (c) 130 μ s.....	64
Figure 4-4: Sensor signals at various widths	68
Figure 4-5: Variation in sensor signal for crack tip reflected Lamb wave using the EIS model	70
Figure 4-6: Variation in sensor signal for crack tip and edge reflected Lamb wave using the EIS model.....	70
Figure 4-7: Relationship between crack growth and proposed damage index	71
Figure 4-8: Dimensions of specimen and location of machined notch.....	72
Figure 4-9: Experimental Setup.....	73

Figure 4-10: Dispersion curves and frequency limits for diagnostic signal	74
Figure 4-11: Comparison of sensor signal at 5mm crack and 26.6mm crack	75
Figure 4-12: Correlation between crack growth and damage index based on change in peak to peak amplitude	77
Figure 4-13: Correlation between crack growth and damage index based on change in scattered energy	79
Figure 4-14: Correlation between crack growth and damage index based on change in transient energy.....	80
Figure 4-15: Correlation between crack growth and damage index based on change in time of flight	81
Figure 4-16: Comparison of damage indexes	82
Figure 5-2 : (a) Specimen size; (b) Experimental Setup.....	87
Figure 5-3 : Crack growth curve (Specimen S1 - S6).....	88
Figure 5-4 : Sensor response for baseline and after 180,000 load cycles	89
Figure 5-5 : Correlation between crack growth and damage index	90
Figure 5-6 : Variations in Paris coefficients	91
Figure 5-7 : Correlation between crack growth and damage index	92
Figure 5-8 : Estimated crack length against damage index	93
Figure 5-9 : Monte Carlo Convergence Test	94
Figure 5-10 : Monte Carlo simulation at 125,000 cycles for Specimen P2.....	95
Figure 5-11 : Estimated Remaining Fatigue Life for Specimen P2.....	96
Figure 5-12 : Algorithm for iterative framework.....	97
Figure 5-13 : Estimated remaining life using the iterative framework.....	99
Figure 6-1: Joint configuration for detail category E (DNV, 2013)	102
Figure 6-2: S-N curve for detail category E (DNV, 2013)	102
Figure 6-3: Schematic diagram of semi-elliptical surface crack configuration.....	103
Figure 6-4: Sketch of test specimen with piezoelectric transducers.....	106
Figure 6-5: Experimental setup.....	106
Figure 6-6: Sensor signal after 100,000 load cycles.....	108
Figure 6-7: Sensor signal after 160,000 load cycles.....	108
Figure 6-8: Sensor signal after 200,000 load cycles.....	109
Figure 6-9: Sensor signal after 290,000 load cycles.....	109

Figure 6-10: Correlation between damage index and number of cycles for LWP technique	110
Figure 6-11: Conductance signature after various loading cycles	112
Figure 6-12: Correlation between frequency shift and number of cycles for EMI technique	114
Figure 6-13: Normalised damage quantifier vs fatigue life for (a) EMI technique & (b) LWP technique	116
Figure 6-14: Experimental value for a/c	117
Figure 6-15: Surface crack length vs damage indicator for LWP technique	118
Figure 6-16: Surface crack length vs damage indicator for EMI technique	118

LIST OF SYMBOLS

a	Half-length of Piezoelectric Transducer
a_f	Fatigue crack length
a_{fcr}	Critical fatigue crack length
c	Transverse fatigue crack length
c_g	Group wave velocity
c_L	Longitudinal wave velocity
c_{ph}	Phase wave velocity
c_T	Transverse wave velocity
d	Half-thickness of plate
d_{kij}, d_{ikl}	Piezoelectric constants
ϵ_x	Displacement in x-direction
ϵ_0	Dielectric constant in vacuum
ϵ_{ik}^T	Dielectric permittivity tensor at constant stress
f	Frequency
f_0	Excitation frequency
h	Thickness of plate
λ	Lame's Constant
m	Paris coefficient
ω	Angular velocity
ρ	Density
s_{ijkl}^E	Elastic compliance tensor at constant electric field
σ	Far-field stress
σ_{min}	Minimum far-field stress
σ_{max}	Maximum far-field stress
t	Time
τ_0	Amplitude of harmonic shear stress
θ	Angle of incidence (or reflection)
φ	Signal phase

μ	Lame's Constant
u_x	Displacement in x -direction
u_y	Displacement in y -direction
u_z	Displacement in z -direction
ξ	Wave number
χ	Sensor signal waveform
Λ	Peak to peak amplitude
C	Paris coefficient
D_i	Displacement Vector in direction i
E_k	Electric Field Vector in direction k
$H(t)$	Hilbert transform
K	Stress intensity factor
K_c	Critical stress intensity factor
K_{th}	Stress intensity factor threshold
K_{min}	Minimum stress intensity factor
K_{max}	Maximum stress intensity factor
N_R	Remaining useful life
$P(t)$	Source Signal at time t
$P_{eq}(t)$	Equivalent Imaginary Source Signal at time t
R	Stress ratio
R_{xy}	Cross correlation
$\left R_{S_0}^{S_0} \right $	Absolute reflection coefficient of S_0 wave with S_0 incident wave
$\left R_{SH_0}^{S_0} \right $	Absolute reflection coefficient of SH_0 wave with S_0 incident wave
S_{ij}	Strain Tensor
T_{kl}	Stress Tensor
V_{in}	Input voltage
V_{out}	Sensor output (Voltage)
V_{edge}^S	Sensor output (Voltage) due to edge reflected symmetry mode

W	Width of plate
Y	Geometry factor

CHAPTER 1

INTRODUCTION

1.1 OVERVIEW OF STRUCTURAL DAMAGES AND FAILURES

By definition, structural damage is the deterioration in strength of a structure caused by external factors such as loading, environmental condition and human intervention (Yao, 1985). Damage can be in the form of cracks, corrosion, delamination and de-bonding. Failures may be described as any action leading to the inability of the structure to perform satisfactorily in its intended manner (Ugural and Fenster, 1995). Fracture is one of the mode of failures which often leads to catastrophic consequences. Other modes of failures include buckling, permanent and excessive deformation.

On 19th December 2005, an amphibious airplane (Chalk's Ocean Airways Flight 101) which took off from Fort Lauderdale, Florida, crashed into a shipping channel adjacent to the Port of Miami, Florida. The airplane heading to Bimini, Bahamas, has 18 passengers and 2 crew members on board. Shortly after take-off, the airplane's right wing separated during the flight and all 20 persons on board perished. The separation of the airplane's right wing resulted from the fracture of the right wing rear spar. The fracture was a result of the failure to identify and repair the fatigue cracks in the right wing rear spar (NSTB, 2007). These fatigue cracks (Figure 1-1) were not visible to the flight crew during the pre-flight inspection as they are skeletal structures beneath the "skin" of the wing.

*Figure 1-1 : (a) Fracture surface; & (b) Fatigue cracks of right wing rear spar
(NSTB, 2005)*

Prior to the 2005 Chalk's Ocean Airways disaster, American airlines flight 587 also experience in-flight separation of stabilizers in 2001 (NSTB, 2004). In 1988, Aloha airlines experience explosive decompression and structural failure (NSTB, 1989). For the above incidents, the existence of crack went undetected during pre-flight inspections.

Besides the aviation industry, civil engineering structures are also subjected to fracture failures. In 1967, the U.S. Highway 35 Bridge at Point Pleasant, West Virginia collapses, bringing down thirty-one vehicles and killing forty-six individuals (NSTB, 1971). The cause of the collapse is a fracture in the lower limb of the eye of an eyebar (Figure 1-2). The corrosion fatigue crack was undetected and allowed to develop to the critical crack size leading to fracture. During that era, there wasn't any inspection method to detect corrosion fatigue. Furthermore, the location of the corrosion fatigue is not easily accessible.

Figure 1-2: (a) Sketch of bridge ; & (b) Point of failure initiation(NSTB, 1971)

The above case studies of fracture failures reiterated the importance of crack detection and monitoring. Failure to detect these “invisible” cracks leads to undesirable consequences, which is often the loss of lives and assets. Non-destructive evaluation (NDE) techniques have proven to be able to detect damages but mostly localised and passive. The subject may have to be taken off service in order to conduct the NDE. The field of structural health monitoring (SHM) has evolved in response to the need for a rigorous and active damage detection and monitoring technique.

1.2 STRUCTURAL HEALTH MONITORING AND DAMAGE PROGNOSIS

Structural Health Monitoring (SHM) has been defined in the literature as the “acquisition, validation and analysis of technical data to facilitate life-cycle management decisions” (Hall, 1999). The idea of SHM is illustrated in Figure 1-3. The typical components of a SHM system includes a computer, signal generator, signal amplifier, actuators, sensors and a signal acquisition unit. The signal generator will send the diagnostic signal to the actuator which transfers the signal mechanically to the structure. The mechanical response of the structure will propagate through the

structure and arrive at the sensor location. These mechanical responses will then be translated into data acquired by the signal acquisition unit. Any changes in the mechanical response of the structure will be reflected in the data acquired. Depending on the application, the amplifier can be used to amplify the diagnostic signal sent to the actuator or to amplify the response from the sensor.

Figure 1-3: Components and operations of typical SHM system (Boller, 2002)

Although SHM provides a solution to active monitoring and the current state of damage, it does not address any issues on the survivability of the structure. Therefore, the field of Damage Prognosis (DP) evolve to address the important question of survivability and remaining useful life. By definition, DP is the estimation of a system's remaining useful life. DP attempts to predict the remaining useful life of a system, by measuring the current state of the system and estimating the future loading environments (Farrar et al., 2003). The relationship between usage monitoring, structural health monitoring and damage prognosis is depicted in Figure 1-4.

Figure 1-4: Relationship between Usage Monitoring, Structural Health Monitoring, and Damage Prognosis (Farrar et al., 2003)

1.3 MOTIVATION FOR DAMAGE PROGNOSIS

The tremendous potential for life safety and economic benefit that DP can provide has sparked great interest in the research community. It has been shown in the previous section that undetected damage will lead to catastrophic failure and have undesirable consequences with possible loss of lives and assets. Therefore, the ability to predict the remaining useful life of a system, by measuring the current state of the system and estimating the future loading environments will definitely reduce the possibility of catastrophic failure due to undetected damages.

Being able to predict the system's remaining useful life also helps to prioritise the maintenance of a fleet of vehicles or an asset of infrastructures. This shift from schedule-based preventive maintenance to condition-based maintenance will

definitely reduce the time period for which the structures are taken offline for maintenance and correspondingly results in cost-savings. This shift also helps to reduce the labour requirements and relieve the pressure on limited resources.

For civil infrastructures, DP will provide the authorities with useful information for the decision-making process. For large-scale discrete event such as earthquakes, DP will provide a quick post-earthquake assessment of the structures and an estimate of the remaining useful life for life-cycle management decisions. The slow reoccupation process can have an extreme impact far beyond the reconstruction cost, especially for manufacturing facilities. For example, semi-conductor manufacturing facilities could lose up to \$10 million per hour when their plants closes down unexpectedly (Farrar et al., 2002)

In summary, DP can provide both life-safety and economic benefits to a wide variety of industries, by assessing the current state of health, estimating future loading environments and predicting the remaining useful life of the structure.

1.4 RESEARCH OBJECTIVES

The primary objective of this research is to investigate the use of Lamb wave propagation (LWP) technique to assess the current state of fatigue damage. This is pursued to enable the development of a damage prognosis model, by integrating the damage state and fatigue crack growth models with a correlation parameter, for constant amplitude loading (CAL).

1.5 RESEARCH ORIGINALITY AND CONTRIBUTIONS

This research programme aims to expand the present capabilities of the LWP technique for SHM and DP of structural systems. The original contributions of this research programme can be summarised as follow:

- (i) A novel concept of an equivalent imaginary source (EIS) is proposed to model the displacement and strain field of reflected Lamb waves. The EIS incorporates information on the reflection coefficient and phase delay of reflected waves;
- (ii) Existing model for infinite plate is extended to model the voltage output of PZT sensor in narrow plate using the EIS model;

- (iii) A new damage index is proposed based on the change in peak amplitudes of the of the sensor signals;
- (iv) A new iterative self-updating framework is proposed to predict the remaining fatigue life of narrow plate for constant amplitude loading;
- (v) The crack detection capabilities is extended to welded steel connections and a comparative study between EMI and Lamb wave technique is conducted experimentally.

The original work of this research programme have been presented in major conferences in the field of structural health monitoring. They have also been submitted to or published in international journals. The list of author's publications is attached at the end of this thesis.

1.6 THESIS ORGANISATION

This thesis consists of a total of seven chapters including this introductory chapter. Chapter 2 introduces the theory of Lamb waves. The developments of Lamb wave techniques in SHM and DP are also discussed. The theory of fatigue and estimation of the remaining fatigue life are also reviewed.

Chapter 3 presents work carried out by the author on the modelling of PZT transducer response in a narrow plate. The concept of an equivalent imaginary source is introduced. Numerical and experimental studies were carried out to verify the proposed model. Existing model for an infinite plate is extended to include the propose EIS model to predict the output voltage of the PZT sensor in a narrow plate environment.

The effects of a narrow plate on the sensor signal are presented in Chapter 4. Using the proposed EIS model, the signals due to crack propagation were estimated and investigated. Experimental study to monitor fatigue cracks were conducted and various damage indexes were investigated. Finally, a suitable damage index to quantify the crack growth in a narrow plate is proposed.

Chapter 5 presents the prognosis of fatigue crack in a narrow plate adopting the Monte Carlo simulation technique. An iterative self-updating framework is introduced to manage the uncertainties. The proposed iterative framework is capable

of estimating the remaining life of a system component even when the fatigue properties of the materials is not known *a priori*.

The monitoring of fatigue cracks is extended to welded steel specimens, non-load bearing welded joint in particular, and a comparison between the electromechanical impedance technique and the Lamb wave propagation technique is presented in Chapter 6.

A summary of work done and the future work to be carried out are presented in Chapter 7, follow by a comprehensive list of references and appendices.

CHAPTER 2

LITERATURE REVIEW ON LAMB WAVE IN STRUCTURAL HEALTH MONITORING (SHM) AND DAMAGE PROGNOSIS (DP)

2.1 BACKGROUND

This chapter provides some background information on piezoelectricity and the theory of Lamb waves. The development of Lamb waves in non-destructive evaluation (NDE) and, subsequently, Structural Health Monitoring (SHM) are reviewed. The advantages and limitations of Lamb wave techniques for SHM are also discussed. This is then followed by an introduction to linear elastic fracture mechanics (LEFM) and fatigue crack growth prior to the discussion on the damage prognosis.

2.2 PIEZOELECTRICITY AND PIEZOELECTRIC MATERIALS

2.2.1 Piezoelectricity

The application of pressure is often denoted by prefix “Piezo”, which was derived from the Greek word “*piezein*”. Therefore, the term piezoelectricity is usually used to describe electricity that is generated by the application of pressure on certain materials. In 1880, Pierre and Jacques Curie conducted studies on Quartz, tourmaline and Rochelle salt. They discover that electrical charge is generated by the crystal when pressures are applied on the crystals. This is the first discovery of the direct piezoelectric effect phenomenon. When the phenomenon is reversed, the material deforms upon the application of an electrical displacement between the surfaces of the material, it is known as the converse piezoelectric effect. This converse effect was deduced by Lipman from thermodynamics principles (Jordan and Ounaies, 2001).

2.2.2 Constitutive Relations

The derivation of the piezoelectric constitutive equations was based on linear theory of piezoelectricity, and is available in two forms. In the strain form, which is the common form, the strain tensor (S_{ij}) and the electric displacement vector (D_i) are written as linear functions of stress tensor (T_{kl}) and electric field vector (E_k), which are derived as (IEEE, 1987),

$$S_{ij} = s_{ijkl}^E T_{kl} + d_{kij} E_k \quad (2.1)$$

$$D_i = d_{ikl} T_{kl} + \epsilon_{ik}^T E_k \quad (2.2)$$

where s_{ijkl}^E is the elastic compliance tensor at a constant electric field, d_{kij} & d_{ikl} are the piezoelectric constants and ϵ_{ik}^T is the dielectric permittivity tensor at a constant stress. The converse piezoelectric effect is given by Equation (2.1) while the direct piezoelectric effect is given by Equation (2.2). Equation (2.1) is typically used in actuator applications where the application of electric field results in a strain field in the material. For sensor applications, where the application of stress field results in an electric charge output, Equation (2.2) is used.

The elastic and piezoelectric fields in Equations (2.1) and (2.2) are represented in tensor form. This can however be converted to matrix form, which is commonly used in the field. In order to do so, a compressed matrix notation is introduced to replace the tensor notation. This matrix notation consists of replacing ij or kl by p or q , where i, j, k, l , take the values of 1, 2, 3 and p, q , take the values 1, 2, 3, 4, 5, 6 according to Table 2-1.

Table 2-1: Matrix Notation (IEEE, 1987)

ij or kl	p or q
11	1
22	2
33	3
23 or 32	4
31 or 13	5
12 or 21	6

Upon replacement of the subscripts with reference to Table 2-1 to represent the matrix notation, Equation (2.1) and Equation (2.2) become,

$$\begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{21} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{31} & s_{32} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{Bmatrix} + \begin{bmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \\ d_{14} & d_{24} & d_{34} \\ d_{15} & d_{25} & d_{35} \\ d_{16} & d_{26} & d_{36} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (2.3)$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{Bmatrix} + \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (2.4)$$

And in compact form,

$$S_p = s_{pq}^E T_q + d_{kp} E_k \quad (2.5)$$

$$D_i = d_{iq} T_q + \epsilon_{ik}^T E_k \quad (2.6)$$

Typical shapes of piezoelectric transducers used in the field of SHM, are either a plate (square or rectangular) or a disc (circular or ring). Consider a piece of piezoelectric material, as shown in Figure 2-1, with the poling direction across the thickness denoted by 3-axis. The length and width directions are denoted by the 1-axis and 2-axis.

Figure 2-1: Axes notation of a typical piezoelectric material

It is a common practice in the industry for manufacturers to specify the values of the elastic and piezoelectric materials in compressed matrix notations, as described in Equation (2.5) and Equation (2.6). The typical parameters are the piezoelectric constants and the dielectric constants.

2.2.3 Piezoelectric Constants

Piezoelectric constants are one of the required parameters when considering the direct or converse piezoelectric effects. Piezoelectric constants are also known as piezoelectric charge coefficients. It describes the proportional correlation between displacement and stress field, or strain and electric field. Therefore, high piezoelectric constants are preferred due to the sensitivity to stress and electric field. Actuators and sensors will normally adopt piezoelectric materials with high piezoelectric constant due to this reason.

2.2.4 Dielectric Constants

The dielectric constant is another important property for sensor applications. It describes the charge stored in an electrode for a given voltage. The dielectric constant in vacuum, ϵ_0 , is 8.85×10^{-12} F/m. The relative dielectric constant (often referred to as “the dielectric constant”) is the ratio of ϵ and ϵ_0 , and is dimensionless.

2.2.5 Piezoelectric Materials

2.2.5.1 Piezoelectric Ceramics

The most common form of piezoelectric ceramics commercially available is the Lead Zirconate Titanate oxide or commonly known as PZT. As the name implies, PZT is composed of Lead (Pb), Zirconate (Zr) and Titanate (Ti) with the chemical composition of $\text{Pb}(\text{Zr}_{1-x}\text{Ti}_x)\text{O}_3$. PZT is a type of perovskite varieties in which the linear piezoelectric response dominates. At low electric fields, PZTs are well described by the linear piezoelectric equations described by Equation (2.1) and Equation (2.2). In the PZT unit cell, Lead, Pb^{2+} , occupies the corners, Zirconium/Titanium, $\text{Zr}^{4+}/\text{Ti}^{4+}$, occupies the octahedral voids and Oxygen, O^{2-} , occupies the faces (Figure 2-2).

Figure 2-2: PZT crystal structure

Typically, PZT is manufactured by heating a mixture of lead, zirconium, titanium oxide powders and dopants to around $800 - 1000^{\circ}C$ first to obtain a perovskite powder, which is then mixed with a binder and sintered into desired shape before electroding and poling to form the final product. As the result of the poling process, the material becomes piezoelectrically transversely isotropic in the plane normal to the poling direction, but remains mechanically isotropic (Sirohi and Chopra, 2000).

PZT is a versatile material that can be manufactured to any shapes and sizes. It is relatively cheaper and lighter compared to other active materials and has a wide range without serious non-linearity. They also exhibit high piezoelectric constant, due to which they can act as good actuators as well as good sensors.

Despite of the numerous advantages highlighted above, PZT does have its own set of limitations. As a ceramic, PZT is brittle and has low tensile strength. Due to its brittleness, PZT cannot withstand bending and exhibits poor conformability to curved surfaces.

2.2.5.2 Piezoelectric Polymers

The most common form of piezoelectric polymers used by the SHM community is the Polyvinylidene Fluoride (PVDF). For the PVDF to exhibit piezoelectric properties, the PVDF have to be stretched to several times the original length and a static electric field is applied across the thickness with temperature rising gradually from room temperature to $90^{\circ}C$ which is subsequently allow to cool slowly (Kawai, 1969).

As a thin film polymer, PVDF exhibits better ductility when compare to PZT which is brittle in nature. This gives PVDF an advantage over PZT when ductility is a concern for the sensor application.

2.3 THEORY OF LAMB WAVES

In 1917, Horace Lamb describe the phenomenon a guided propagation between two parallel free surfaces of a thin structure. In honour of his contributions in the field, these waves are named after him, and commonly known as Lamb waves.

At any frequency, there are two fundamental modes of Lamb waves which satisfy the wave equation and boundary conditions. These fundamental modes are the symmetric waves and the anti-symmetric waves, which satisfy the wave equation and the boundary conditions. For the symmetric mode, particle displacement is symmetrical about the mid-plane. While for the anti-symmetric modes, particle displacement is anti-symmetric about the mid-plane of the plate (Figure 2-3).

Figure 2-3: Lamb wave modes: (a) symmetric; (b) anti-symmetric

2.3.1 Derivation of Lamb Waves Equations

It is not the author's intention to reproduce the entire derivation of the Lamb wave equations. However, it'll be beneficial to highlight some key steps and assumptions in the derivation of the Lamb waves equations.

Consider straight-crested lamb waves propagating in an isotropic plate (Figure 2-4) of thickness ($h = 2d$). The governing equations of motion is given by,

$$\rho \ddot{\mathbf{u}} = (\lambda + \mu) \nabla \nabla \cdot \mathbf{u} + \mu \nabla^2 \mathbf{u} \quad (2.7)$$

where ρ is the density of the isotropic plate, $\mathbf{u}$ is the displacement vector, λ and μ are Lamé's constants for the isotropic plate and ∇ is the gradient operator.

Figure 2-4: The isotropic plate for which dispersion relations are derived

Using the Helmholtz decomposition,

$$\mathbf{u} = \nabla\Phi + \nabla \times \mathbf{H} \quad (2.8)$$

and the uniqueness condition,

$$\nabla \cdot \mathbf{H} = 0 \quad (2.9)$$

yields the wave equations for the scalar potential and the vector potential,

$$c_L^2 \nabla^2 \Phi = \nabla \ddot{\Phi} \quad (2.10)$$

$$c_T^2 \nabla^2 \mathbf{H} = \nabla \ddot{\mathbf{H}} \quad (2.11)$$

where $c_L = \sqrt{(\lambda + 2\mu)/\rho}$ and $c_T = \sqrt{\mu/\rho}$ are the longitudinal and transverse wave velocity. For straight-crested waves, the variations along the z -axis are negligible (or invariant), Equation (2.11) can be further reduced to,

$$c_T^2 \nabla^2 H_z = \nabla \ddot{H}_z \quad (2.12)$$

Assuming harmonic wave propagation along x -axis at angular frequency (ω), then the solutions will be of the form (assuming ξ is the wave number),

$$\Phi = \phi e^{i(\xi x - \omega t)} \quad (2.13)$$

$$H_z = \psi e^{i(\xi x - \omega t)} \quad (2.14)$$

These lead to the following differential equations for ϕ and ψ ,

$$\frac{d^2 \phi}{dy^2} + \left(\frac{\omega^2}{c_L^2} - \xi^2 \right) \phi = 0 \quad (2.15)$$

$$\frac{d^2\psi}{dy^2} + \left(\frac{\omega^2}{c_T^2} - \xi^2 \right) \psi = 0 \quad (2.16)$$

Introducing the notations,

$$p^2 = \frac{\omega^2}{c_L^2} - \xi^2 \quad ; \quad q^2 = \frac{\omega^2}{c_T^2} - \xi^2 \quad (2.17)$$

Equations (2.15) and (2.16) become,

$$\frac{d^2\phi}{dy^2} + p^2\phi = 0 \quad (2.18)$$

$$\frac{d^2\psi}{dy^2} + q^2\psi = 0 \quad (2.19)$$

The general solutions to Equations (2.18) and (2.19) are:

$$\phi = A \sin py + B \cos py \quad (2.20)$$

$$\psi = C \sin qy + D \cos qy \quad (2.21)$$

where A , B , C and D are constants. At the free surface ($y = \pm d$),

$$\sigma_{yy} = \sigma_{yx} = 0 \quad (2.22)$$

The traction terms in the Helmholtz components are,

$$\sigma_{yy} = (\lambda + \mu)\nabla^2\Phi - 2\mu \left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2 H_z}{\partial x \partial y} \right) \quad (2.23)$$

$$\sigma_{yx} = \mu \left(2 \frac{\partial^2\Phi}{\partial x \partial y} + \frac{\partial^2 H_z}{\partial y^2} - \frac{\partial^2 H_z}{\partial x^2} \right) \quad (2.24)$$

From Equations (2.13), (2.14) and (2.20) – (2.24),

$$\begin{bmatrix} -(\xi^2 - q^2) \cos pd & 2i\xi q \cos qd \\ -2i\xi p \sin pd & (\xi^2 - q^2) \sin qd \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.25)$$

$$\begin{bmatrix} -(\xi^2 - q^2) \sin pd & 2i\xi q \sin qd \\ -2i\xi p \cos pd & (\xi^2 - q^2) \cos qd \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.26)$$

Solution of the above homogeneous system of linear equations is only possible if the determinants of the two matrices vanish. These lead to the Rayleigh-Lamb equations for the plate, given as,

$$\frac{\tan pd}{\tan qd} = -\frac{(\xi^2 - q^2)^2}{4\xi^2 pq} \quad (2.27)$$

$$\frac{\tan pd}{\tan qd} = -\frac{4\xi^2 pq}{(\xi^2 - q^2)^2} \quad (2.28)$$

Equation (2.27) describes the symmetric solution of Lamb waves equations, while Equation (2.28) describes the anti-symmetric solution of Lamb waves equations.

2.3.2 Dispersion curves

The solution of the Rayleigh-Lamb equations shown above is not straightforward, since p and q also depends on ξ . Numerical methods to solve Equation (2.27) and equation (2.28) are outlined by Rose (1999). The solutions give frequency-wave number pairs corresponding to symmetric waves and anti-symmetric waves respectively. The phase velocity (c_{ph}) is given by,

$$c_{ph} = \frac{\omega}{\xi} \quad (2.29)$$

It is however more common to present the phase velocity as a function of the product of frequency (f) and half thickness (d). At a given fd product, several Lamb modes may exist. A plot of the phase velocity dispersion curves is shown in Figure 2-5(a). The group velocity of the various Lamb modes is another important property of Lamb waves. It provides information for the extraction of features like time-of-flight when examining the Lamb wave propagation. The group velocity (c_g) is given by,

$$c_g = c_{ph} - \lambda \frac{\partial c_{ph}}{\partial \lambda} \quad (2.30)$$

Figure 2-5: Dispersion Curves for: (a) phase velocity ; & (b) group velocity

2.4 LAMB WAVES IN STRUCTURAL HEALTH MONITORING

Adopting the theory of Lamb waves, Worlton (1961) conducted experiments to illustrate the use of lamb waves for non-destructive testing of thin structure. Many researchers recognises the potential of the Lamb waves approach to non-destructive testing and started working on the use of Lamb waves for non-destructive testing. Alleyne (1991) provided an extensive review of non-destructive testing using Rayleigh and Lamb waves in his PhD thesis.

Prior to the idea of using piezoelectric materials to generate Lamb waves, diagnostic Lamb waves used in NDE were generated by conventional Lamb wave probes, which include wedge and comb transducers. These probes are relatively heavy and expensive to be considered for deployment for NDE and SHM of large structures. This lead to the idea of using piezoelectric materials to generate the diagnostic Lamb waves.

Keilers and Chang (1995) were among the first to explore the use of piezoelectric materials for delamination detection using Lamb waves techniques in SHM. In their experiments, piezoelectric ceramics were bonded to composite beams with and without artificial delaminations to measure the beam response. Their experiments showed that delaminations can be detected by a change in amplitude response measured using the piezoelectric sensors.

Besides Keilers and Chang (1995) other researchers including Giurgiutiu and Zagrai (2000); Lin and Yuan (2001); Giurgiutiu et al. (2002); Derriso et al. (2005) were also successful in using piezoelectric ceramics as actuators and sensors to detect damage in plates.

2.5 GENERATION OF LAMB WAVES WITH PIEZOELECTRIC CERAMICS

Consider a piezoelectric ceramic, or PZT, surface bonded to a structure. When the PZT is subjected to alternating electrical excitation, it undergoes oscillatory expansion and contractions. This oscillatory motion is transferred to the bonding layer, which transmits them to the structure's surface. In other words, the PZT generates Lamb waves in the structure by pinching the surface of the structure (Giurgiutiu, 2003).

Lin and Yuan (2001) proposed an analytical expression, that accounts for the interaction between an actuator and the wave propagation in the host structure, in the form of sensor voltage output in terms of a given excitation signal. The analytical expression is derived based on Mindlin plate theory to model the propagating waves and lamination plate theory to model the actuator and the structure under interrogation. It is noted that the proposed model is only applicable for pure bending motion, or the A_0 Lamb mode. And this model can only correctly predict the sensor response when the input frequency is lower than the cut-off frequency for higher modes generation.

Besides Lin and Yuan (2001), Giurgiutiu (2003) also studied the generation of Lamb waves in plates with PZT, by considering the excitation applied by the PZT through a surface stress. Assuming ideal bonding condition, the one dimensional strain wave and displacement wave solutions are derived using space-domain Fourier transform and plane strain conditions, and are given as (Giurgiutiu, 2003),

$$\begin{aligned} \epsilon_x(x, t) = & -i \frac{a\tau_0}{\mu} \sum_{\xi^S} \sin \xi^S a \frac{N_S(\xi^S)}{D'_S(\xi^S)} e^{i(\xi^S x - \omega t)} \\ & - i \frac{a\tau_0}{\mu} \sum_{\xi^A} \sin \xi^A a \frac{N_A(\xi^A)}{D'_A(\xi^A)} e^{i(\xi^A x - \omega t)} \end{aligned} \quad (2.31)$$

$$u_x(x, t) = -i \frac{a\tau_0}{\mu} \sum_{\xi^S} \frac{\sin \xi^S a}{\xi^S} \frac{N_S(\xi^S)}{D'_S(\xi^S)} e^{i(\xi^S x - \omega t)} - i \frac{a\tau_0}{\mu} \sum_{\xi^A} \frac{\sin \xi^A a}{\xi^A} \frac{N_A(\xi^A)}{D'_A(\xi^A)} e^{i(\xi^A x - \omega t)} \quad (2.32)$$

where a is the half-length of the PZT, τ_0 is the amplitude of the harmonic shear stress boundary excitation, μ is the Lamé constant, ω is the angular frequency, and

$$\begin{aligned} N_S &= \xi q (\xi^2 + q^2) \cos ph \cos qh \\ D_S &= (\xi^2 + q^2)^2 \cos ph \sin qh + 4\xi^2 pq \sin ph \cos qh \\ N_A &= \xi q (\xi^2 + q^2) \sin ph \sin qh, \\ D_A &= (\xi^2 + q^2)^2 \sin ph \cos qh + 4\xi^2 pq \cos ph \sin qh \end{aligned} \quad (2.33)$$

A plot of Equation (2.31) for a 1.6mm thick aluminium plate under the excitation of 7mm PZT is presented in Figure 2-6(a). The experimental result is illustrated in Figure 2-6(b). This study shows that frequencies can be found for which the response is dominated by certain preferentially excited modes (Giurgiutiu, 2003). In this particular case, 300kHz is the ‘sweet spot’ for S_0 mode excitation.

Figure 2-6: (a) Strain predicted based on Equation (2.31) ; (b) Experimental result of PZT response (Giurgiutiu, 2003)

The above analytical solutions are provided for one-dimensional case. To the author’s best knowledge, there isn’t any model available that can predict the sensor response in a 2-dimensional environment due to the complexity of the problem.

2.6 SIGNAL PROCESSING FOR FEATURE EXTRACTION

2.6.1 Cross Correlation

In signal processing, cross correlation is a measure of the difference between two signal waveforms. The cross correlation, $R_{xy}(t)$, of two signals, $x(t)$ and $y(t)$ is defined by (Yu et al., 2004),

$$R_{xy}(t) = \int_{-\infty}^{+\infty} x(\tau)y(\tau + t) d\tau \quad (2.34)$$

For discrete time signal, Equation (2.34) can be expressed as,

$$R_{XY}(m) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)y(n - m) \quad (2.35)$$

Yu et al. (2004) illustrated that by calculating the cross correlation of the actuation signal and the sensor signal, the time-of-flight can be clearly obtained at the peak of the cross correlation signal (Figure 2-7).

Figure 2-7: Cross correlation of simulated tone-burst signal: (a) 3-count sine signal; (b) 3-count Hanning window smoothed signal (Yu et al., 2004)

2.6.2 Hilbert Transform Envelope Detection

The Hilbert transform of a function, $f(x)$ is defined by,

$$H(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(x)}{t - x} dx \quad (2.36)$$

Hilbert transform can also be used to construct an “analytic” (complex) signal and is given as,

$$F(t) = f(t) + iH(t) \quad (2.37)$$

where the real part of the analytic signal, $f(t)$, is the real value input signal, and the imaginary part of the analytic signal, $H(t)$, is the Hilbert transform of the input signal. The Hilbert transform of the input signal is actually a copy of the input signal after a 90° phase shift (Figure 2-8). Thus, $H(t)$ has the same amplitude and frequency content as $f(t)$. Equation (2.37) can also be expressed as,

$$F(t) = A(t)e^{ip(t)} \quad (2.38)$$

where $A(t)$ is the amplitude of the analytic signal and $p(t)$ is the phase of the analytic signal. The magnitude of each complex signal has the same amplitude as the input signal, thus the magnitude of the analytic signal is the envelope of the input signal. And the amplitude of the analytic signal is given as,

$$A(t) = \sqrt{f(t)^2 + H(t)^2} \quad (2.39)$$

Figure 2-8: Envelope detection using Hilbert transform

Therefore, once the envelope is detected, the wave packets can easily be identified, and the time-of-flight or other relevant information can be extracted.

2.6.3 Spectrogram

Other than signal envelope, a spectrogram can also provide information on the time of flight for a wave packet. A spectrogram is a diagram showing the spectral amplitude distribution as a function of frequency and time. The spectrogram can be represented by a 3D plot, a surface plot or a contour plot. However, it was noted that the time and frequency resolution of the spectrograms depends greatly on the selection of the window type, window length and the window moving step. With smaller window size and moving step, more local details about the signal can be revealed. However, when the window size becomes too small, it will result in a broad frequency spectrum and calculation time will be longer (Giurgiutiu, 2008).

Wang and Chang (1999), Roh (1999) and Ihn and Chang (2004) had successfully used the spectrogram to identify the TOF for the wave packets in their studies.

2.7 IDENTIFYING DAMAGE WITH LAMB WAVES USING PZT

2.7.1 Disbonding and Delamination Detection

Keilers and Chang (1995) were among the first to explore the use of piezoelectric materials for delamination detection using Lamb wave techniques in SHM. In their experiments, piezoelectric ceramics were bonded to composite beams with and without artificial delaminations to measure the beam response. The detection of damage was deduced from the change in amplitude response of the structure.

Kim et al. (2007) conducted experiments on an aluminium plate with collocated PZT on both sides of the plate at the excitation location. They discovered that the ratio of the power spectrum densities at the peak frequencies of the symmetric and anti-symmetric modes has good correlation to the degree of delamination.

Yeum et al. (2014) proposes a reference free technique to detect delamination in composites. The study adopted the first A_0 wave reflected from the delamination as a quantifier. The reflected A_0 wave was extracted using the matching pursuit algorithm. This technique has been adopted under varying temperature and still yields good results.

2.7.2 Damage Detection

Roh (1999) investigated the use of Lamb waves to detect anomaly in plates. He found that the TOF in the scatter spectrogram provides information on the location, and the amplitude distribution in the scatter spectrogram provides information on the size of the anomaly.

Giurgiutiu (2003) developed an embedded ultrasonics structural radar (EUSR) to detect cracks. In his proof of concept experiment, the PZT phased array was placed in the centre of a thin aluminium plate. By sequentially firing the individual PZT at slightly different times, the ultrasonic wave front can be focused or steered in a specific direction. Once the beam steering and focusing was established, the crack can be detected using the pulse echo method.

Lu et al. (2006) successfully identified crack location with a network of piezoelectric sensors. The TOF was determined by using the cross correlation signal, and a triangulation approach based on TOF was applied to locate the crack position. Although this technique demonstrated the capability to identify and locate the crack, it is qualitative in nature, and does not provide any information relating to the crack length and orientation.

Yu et al. (2012) adopted the pitch catch technique to detect corrosion with a network of actuators and sensors. Statistical damage index definition and cross-time-frequency analysis were conducted to determine the size and evolution of the corrosion. The corrosion studied is idealised and simplified and the method has yet to be verified by experiments.

Lim et al. (2013) developed a reference free damage characterization technique for composites. The technique uses the acquired signal to estimate the size and location of damage without any comparison to previously acquired baseline. The A_0 mode is used as the key damage quantifier. The damage size is characterised based on a damage index and the location is derived based on the time of flight of the reflected A_0 mode from the damaged area.

Gorgin et al. (2014) developed damage size characterization algorithm using a network of actuators and sensors with a combination of pitch catch and pulse echo configurations. The A_0 mode is used as the diagnostic mode with reflection coefficient

increasing as corrosion depth increases. The effectiveness of the proposed algorithm was demonstrated by experiments conducted.

2.7.3 Fatigue Crack Detection

Grondel et al. (1999) conducted a study on riveted aluminium plate under fatigue. The parameter used to characterise crack growth is the time delay between the envelope derivative maxima of the time domain signals, which corresponds to the TOF of wave packets. However, this technique does not provide a correlation with the actual crack length. This parameter can only provide the stage of fatigue crack (i.e. microcrack or macrocrack).

Ihn and Chang (2004) developed a piezoelectric based in-situ diagnosis technique to monitor fatigue crack growth in metallic structure. A physics based damage index was developed by extracting features in sensor signals related to crack growth. The damage index (DI) is defined as the ratio of the scatter energy contained in the S_0 mode wave packet to the baseline energy contained in the S_0 mode wave packet, and is given by,

$$\text{Damage index (DI)} \equiv \left(\frac{\int_{t_i}^{t_f} |S_{sc}(\omega_0, t)|^2 dt}{\int_{t_i}^{t_f} |S_b(\omega_0, t)|^2 dt} \right)^\alpha \quad (2.40)$$

where S_{sc} is the time varying spectral amplitude of the scatter signal, S_b is the time varying spectral amplitude of the baseline signal, ω_0 is the driving frequency, t_f and t_i is the upper bound and lower bound of the S_0 mode in the time domain respectively, and α is the gain factor. Although Ihn and Chang (2004) illustrated that the damage index correlates well with a new parameter to represent crack growth (Figure 2-9), the construction of the time varying spectral amplitude is sensitive to the selection of the window type, window length and the window moving step for the short-time Fourier transform (STFT).

Figure 2-9: Damage Index vs $\Delta\Omega$

(Ihn and Chang, 2003)

Greve et al. (2007) investigated the feasibility of fatigue crack detection using a laboratory-sized plate girder. A damage index that measures the signal change, considering only the first S_0 wave packet, was proposed.

$$M = 1 - \frac{\sum_{i=m}^n f_i g_i}{\sqrt{\sum_{i=m}^n f_i^2 \sum_{i=m}^n g_i^2}} \quad (2.41)$$

where f_i and g_i are the i^{th} samples of two signals, and m and n have been chosen to include only the first S_0 wave packet. In the early stages of fatigue, the damage index could not be correlated to any physical appearance of fatigue crack. When the crack propagated from 7.5 mm to 11 mm, there was a decreasing trend when it was expected to increase. However, from 11 mm to 30 mm, the result was consistent with an increasing trend. They attributed the inconsistency to experimental error.

Kim and Sohn (2008) proposed a new damage detection technique that does not require comparison with baseline data. This reference-free technique employs two pairs of collocated PZTs placed on both sides of the plate to generate and measure Lamb waves. The trade-off for this baseline free technique is that the number of PZTs required is doubled and difficulty in instrumentation is increased.

He et al. (2013) proposed a multi-feature integrated method for fatigue crack detection and crack length estimation. Experiments was conducted on seven

specimens and it was found that the fatigue crack length has correlation with three features of the acquired signal, namely correlation coefficient, amplitude change and phase change. He et al. (2013) observed that a single feature cannot be used to quantify damage among different specimen thus propose a multi-feature integration method based on second-order multivariate regression analysis. The proposed method proves to be effective through experiments conducted on specimens obtained from different manufacturers and under different loading conditions.

2.8 ADVANTAGES OF LAMB WAVE INSPECTION TECHNIQUES

One key advantage of using Lamb waves for NDE or SHM of plate and pipe structures is the ability to interrogate a large area of the structure from only a few locations. Another advantage is the ability to ‘sense’ both sides, as well as the interior of the structure, from only one location on one side of the structure. The use of surface bonded or embedded sensors also translate to achieving consistent measurements as compared to the conventional large transducers. Monitoring of inaccessible area is also possible with the actuators and sensors integrated into the system during the construction phase.

2.9 LIMITATIONS OF LAMB WAVE TECHNIQUE

Despite of the numerous advantages of Lamb wave technique discussed above, the Lamb wave techniques still has a set of limitations. These limitations are generally due to the behaviour of wave propagation. The dispersive nature of Lamb waves and the existence of multiple Lamb modes in a single frequency complicate the signals acquired by the sensors and interfere with the diagnosis of damages. Therefore, these dispersive phenomena and the interaction of various modes should be understood for a good damage diagnosis.

2.10 FATIGUE AND CRACK GROWTH MODELS

2.10.1 Fatigue Damage

Fatigue is defined as progressive and localised damage that occurs when a material is subjected to cyclic loading. Fatigue damage, if undetected, can lead to serious consequences. Examples of catastrophic failures caused by fatigue damage

have been highlighted in Chapter 1. The fatigue process begins with the accumulation of damage at a localised region due to cyclic load. The localised accumulated damage eventually leads to the formation of cracks and their subsequent propagation. When the crack grows to a critical state, whereby the remaining net section is insufficient to carry the applied load, a sudden fracture occurs. Therefore, fatigue damage can be defined in 3 stages, namely, (1) crack initiation; (2) stable crack growth; and (3) final fracture. Cracks usually initiate at pre-existing highly stressed regions such as notches, pits and rivet holes.

Depending on the cyclic stress, a material may be subjected to either low-cycle fatigue or high-cycle fatigue. For low-cycle fatigue, typically less than 10^3 cycles, the cyclic load is usually larger than two-third of the yield load, and the deformation is in the plastic region. For high-cycle fatigue, typically greater than 10^3 , the bulk material response is elastic, and the cyclic load is usually less than two-third of the yield load.

2.10.2 Linear Elastic Fracture Mechanics

Fracture mechanics is concern with the analysis of materials and structures in the presence of flaws and cracks. This field of research was first pioneered by Griffith (1921), who formulated the concept that existing crack will propagate when the strain energy release by the crack growth is equal to the surface energy required to create new crack surfaces. His theory allows the estimation of the theoretical strength of brittle solids. However, Griffith's elastic stress field contains a singularity at the crack tip. Therefore, Irwin (1957) introduces a two-zone model, and considered inelastic effects to be confined to a small inner zone. A stress intensity parameter (K) was introduced, and by applying the assumption that the elastic K -field will not be influenced by the inelasticity in the inner zone, Irwin effectively related the energy release rate (G) to the crack tip K -field. These studies were based on elastic stress fields and demonstration of the equivalence of the energy release rate (G) and stress intensity (K) provided the basis for the development of Linear Elastic Fracture Mechanics (LEFM).

The stresses in the region of a crack tip may be categorised into three basic types, each associated with a mode of crack surface displacement (Figure 2-10). The first

mode, Mode I, is associated with the separation of crack faces in direction normal to the crack direction (i.e. Opening Mode). The second mode, Mode II, is associated with an in-plane shearing of the crack, where the shear load is applied in the direction of the crack (i.e. Sliding Mode). The last mode, Mode III, is associated with out-of-plane shearing of the crack, where the shear load is applied in the direction normal to the crack direction and parallel to the crack front (i.e. Tearing Mode).

Figure 2-10: Modes of crack surface displacement

(Janssen et al., 2004)

As Mode I is the predominant mode in many practical cases, discussions henceforth only relates to Mode I crack surface displacement. The stress intensity factor depends on the applied stress, crack size, and the geometry of the materials under consideration. In general, the stress intensity factor can be written as,

$$K = Y\sigma\sqrt{\pi a_f} \quad (2.42)$$

where Y is the geometry factor, σ is the nominal stress, or far field stress and a_f is the crack length. A summary of various established geometry factor (Janssen et al., 2004) is given in *Appendix B*.

Recall that LEFM is only applicable when the inelastic zone is small compared to the size of the crack. If this region of inelasticity is large before crack growth, then Elastic Plastic Fracture Mechanics (EPFM) must be adopted to analyse the

materials and structures. The author would like to highlight that EPFM is beyond the scope of this thesis.

2.10.3 Fatigue Crack Growth

LEFM can be used to describe fatigue crack growth and fracture behaviour of materials if the crack tip plasticity is small compared to the size of the crack. There are two main questions to address when assessing fatigue cracks. What is the maximum crack size (i.e. critical crack size, a_{fcr}) before fracture occurs? How long does it take for a crack to grow to the critical crack size? By applying LEFM (or EPFM) analysis, a_{cr} can be determined. With the knowledge of a fatigue crack growth curve, the time required for a given crack to grow to the critical crack size can be predicted. However, experimental determination of fatigue crack growth curve for every single type of component, loading condition and crack size, shape and orientation in a structure is impractical. Fortunately, one can use the correlation between fatigue crack growth rate, da_f/dN , and the stress intensity range, ΔK , for known material, to construct the fatigue crack growth curve.

A typical crack growth rate curve is shown in Figure 2-11, which divides the curve into 3 regions. Region I is the near threshold region, where a threshold stress intensity range, ΔK_{th} , is defined. For ΔK that falls below ΔK_{th} , cracks either propagate at extremely low rate or do not propagate at all. In Region III, the crack growth rate increases rapidly and terminates with fracture as the curve rises to an asymptote where the maximum stress intensity factor, K_{max} , equates the critical stress intensity factor, K_c . Region II defines a region whereby the crack growth rate is often some power function of ΔK , leading to a linear relationship between $\log(da_f/dN)$ and $\log(\Delta K)$. The most well known relationship is the Paris equation (Paris et al., 1961),

$$\frac{da_f}{dN} = C(\Delta K)^m \quad (2.43)$$

As Paris' equation (also known as Paris law) only describes the linear log-log (Region II) relationship between the crack growth rate and the stress intensity range, Forman et al. (1967) proposed an improved relationship that includes Region III of the crack growth curve and is given as,

$$\frac{da_f}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K} \quad (2.44)$$

where R is the stress ratio defined as $\sigma_{min}/\sigma_{max}$. Later, an equation which incorporates all of the features shown in Figure 2-11 has been used in the NASA/FLAGRO computer program for fatigue crack growth, and is given as (NASA, 1989),

$$\frac{da_f}{dN} = \frac{C(1-R)^n(\Delta K)^m(\Delta K - \Delta K_{th})^p}{[(1-R)K_c - \Delta K]^q} \quad (2.45)$$

Figure 2-11: Typical crack growth rate curve

For a given crack growth rate curve, it is possible to predict the crack growth, as long as the relation between a and ΔK is known for the component or structure. For constant amplitude loading, this can be done by integrating the crack growth rate ($da_f/dN - \Delta K$) curves to constructing the crack growth ($a_f - N$) curve.

2.11 DAMAGE PROGNOSIS

As defined in Chapter 1, damage prognosis (DP) is the estimation of a system's remaining useful life. DP attempts to predict the remaining useful life of a system, by measuring the current state of the system and estimating the future loading environments (Farrar et al., 2003). Therefore, a damage prognosis model can be developed in the form of,

$$N_R = f(P, L) \quad (2.46)$$

where N_R is the remaining useful life of the system, P consist of information relating to the current state of structural health (i.e. crack length) and L consist of information relating to the load environment. Depending on the physical observations, two main types of DP models may be developed either via the curve fitting technique or the group classification technique (Farrar et al., 2003).

Figure 2-12: Types of Failure Model: (a) curve fitting ; (b) group classification

(Farrar et al., 2003)

Kulkarni and Achenbach (2008) presented a two-stage approach to fatigue prognosis. In the first stage, a second harmonic amplitude has been chosen to monitor the pre-macrocrack damage accumulation process during fatigue. The probability of macrocrack initiation was estimated by Monte Carlo integration. In the second stage, Paris law was adopted as the crack growth model with the initial crack length as a random variable. Thus the probability of macrocrack initiation (from stage 1) could be carried forward to this stage. Assuming that the probability of detection (POD) of the inspection technique is known, the probability that there exists an undetected

critical crack (ie. $a_f > a_{fcr}$) can be determined. No attempt has been made to predict the remaining useful life of the specimen under study.

2.12 NEEDS FOR FURTHER RESEARCH IN LAMB WAVE TECHNIQUE FOR FATIGUE DAMAGE DETECTION AND PROGNOSIS

Although there are a few available literatures on fatigue damage detection, these literatures do not provide a good correlation between damage index and crack length for DP purpose. Therefore, there is a need for a damage index that can relate well with the fatigue crack length in order to conduct DP with more confidence.

In addition, these researchers worked on wide specimens where boundary reflections are not significant. Those works may be applicable to the aviation industry, where boundary reflections may not be significant in the sensor signal. However, many real-life civil infrastructures (e.g. bridges) are relatively long. To take advantage of the Lamb waves technique to sense a larger area with less sensors and to save on instrumentation cost, the sensors should be placed further apart, reducing the number of sensors required. Thus boundary reflections will have a significant role in the sensor signal, an issue which has not been accounted for in all of the above literatures. Therefore, there is a need for a damage index that accounts for the boundary reflections.

To the author's best knowledge, there is no literature that develops a DP model to predict the remaining useful life of a system under fatigue using the Lamb waves technique. Therefore, other than a suitable damage index to quantify the degree of fatigue damage, a DP model is also needed to predict the remaining useful life of the system. This is extremely useful when it comes to maintenance and resource planning.

2.13 CONCLUDING REMARKS

This chapter presents the developments of Lamb waves techniques for SHM, in particular, with the use of piezoelectric transducers. The needs for further research to develop a technique for fatigue damage prognosis with piezoelectric transducers are also highlighted.

The next chapter discusses some of the work that has been carried out by the author to study the feasibility of crack propagation monitoring with Lamb wave technique using piezoelectric transducers. This work serves as the foundation for the study of fatigue damage prognosis in the near future.

CHAPTER 3

MODELLING OF PZT TRANSDUCER RESPONSE IN A NARROW PLATE

3.1 BACKGROUND

This chapter presents a model to predict the output voltage of a PZT sensor in a narrow plate environment. Firstly, the PZT transducer's response in an infinite plate is reviewed. This is then followed by a review on the reflection of Lamb waves, in particular, obliquely incident Lamb waves. The concept of an equivalent imaginary source is introduced to model the response of a reflected Lamb wave. Numerical simulations are conducted to verify the applicability of the concept which is then extended to modelling of PZT transducer response in a narrow plate. The model is verified with numerical simulations and experiments.

3.2 PZT TRANSDUCER RESPONSE IN AN INFINITE PLATE

The response of a piezoelectric transducer (PZT or PWAS) is investigated and modelled by various researchers (Moulin et al., 2000; Lin and Yuan, 2001; Giurgiutiu, 2003; Raghavan and Cesnik, 2005; Giurgiutiu et al., 2012; Kamal et al., 2014). In recent years, there are various studies conducted by researchers to model the excitation, propagation and sensing of Lamb waves using surface bonded piezoelectric transducers. Moulin et al. (2000) presented a modelling approach that couples the normal mode expansion method with the finite element method (FEM) to predict the surface displacement of Lamb waves in a composite plate excited from a source. In that particular study, the source of the Lamb waves is either generated by a surface bonded or embedded piezoelectric transducer. The applicability of the model is validated with experimental study. Lin and Yuan (2001) presented a model to predict the response of the sensor in an infinite plate. The source is a transient flexural wave generated by a pair of collocated piezoelectric transducers (one on the top surface and one on the bottom surface of the plate), with out-of-phase excitation

with respect to each other. Mindlin plate theory is adopted to model the propagation of flexural waves. Experimental verification illustrated that the model is able to predict the sensor response when the excitation frequency is lower than the cut-off frequency of thickness shear waves. The cut-off frequency is defined as the frequency beyond which higher order Lamb modes are present in the medium. Giurgiutiu (2005) studied a one-dimensional plane harmonic Lamb wave propagation in an infinite plate. Assuming perfect bonding conditions, the interfacial shear is concentrated at the ends (i.e. pin-force model). The harmonic displacement and strain field is obtained by using the residual theorem to evaluate the integral of the inverse Fourier transform. This analytical model provides a predicted displacement in the direction of propagation at a distance from the source. Raghavan and Cesnik (2005) derived the sensor response of arbitrary shaped piezoelectric transducers in an infinite plate. It was illustrated that for a rectangular (or square) actuator, the far field displacements tend to a circular crested field with angular dependent amplitude. Giurgiutiu et al. (2012) uses multi-physics finite element method (MP-FEM) to predict the response of a PZT sensor. It was illustrated that the response agrees well with the analytical responses and the MP-FEM is capable of simulating both the S_0 and A_0 waves.

All the above work in the literature provides a model to predict either the displacement (or strain) fields or the voltage output of a sensor in an infinite plate. In other words, one can only predict the response of a Lamb wave that propagates directly from the source to the sensor location with the existing model. Therefore, the work in this chapter intends to extend the existing model to fill the gap by incorporating the reflection coefficients and phase information which will be discussed in the remaining section of this chapter.

3.3 REFLECTION OF LAMB WAVES

The reflection of Lamb waves was first investigated by Torvik (1967). To satisfy the stress free boundary condition of a free edge, a single incident mode will give rise to both propagating and non-propagating modes. The energy from the incident mode will be distributed between the propagating modes that are present at the particular frequency. Gregory and Gladwell (1983) then presented a study on the reflection of

Lamb waves on the fixed and free edge of a plate. The method of projection was proposed to model the reflection of symmetric Lamb wave at a free end and a fixed end. The results agrees well with Torvik (1967) at low frequency. Cho and Rose (1996) studied the mode conversion of reflection at the edge using the boundary element method. It was illustrated that this method has the advantages of application to elements of geometrical complexity. Similarly, the reflection coefficients are derived based on the principle of energy conservation. Morvan et al. (2003) also studied the reflection of Lamb waves at the free edge of the plate. The numerical and experimental studies conducted agrees well with the proposed theoretical model. The above studies only focus on normal incidence Lamb waves. For obliquely incidence Lamb waves, the first literature is by Gunawan and Hirose (2007). The reflection coefficients were obtained by a mode decomposition technique presented in their earlier paper (Gunawan and Hirose, 2004). Santhanam and Demirli (2013) presented an energy reflection coefficient for obliquely incidence Lamb waves by extending the collocation method presented by Morvan et al. (2003) for the normal incidence Lamb wave. From the above literatures, the reflection coefficients for low frequency (below the cut off frequency of higher order modes) is summarised in the following sections.

3.3.1 Normal incidence Lamb Waves

At a relatively low frequency, where only the two fundamental Lamb modes exist, the energy carried by the incident wave may be distributed between the two fundamental Lamb modes (Torvik, 1967). From the mode conversion study conducted by Cho and Rose (1996) on a semi-infinite plate with a plane symmetrical free edge, the A_0 incident wave is not converted to S_0 wave at low frequency. And the reflection coefficient of the A_0 reflected wave is 1.0. Thus, one can conclude that mode conversion between the two fundamental Lamb modes (at low frequency) only occur when the free edge is not plane and symmetrical about the mid-plane. Assuming the free edge is straight and symmetrical about the mid-plane, the energy reflection coefficient of S_0 incident wave is unity at frequency below the cut-off frequency of higher order modes (Figure 3-1(a)). Similarly, for A_0 incident wave, the energy reflection coefficient is unity as well (Figure 3-1(b)).

Figure 3-1: Reflection coefficient of (a) normal incident S_0 wave and (b) normal incident A_0 wave (Gunawan and Hirose, 2007)

3.3.2 Obliquely incidence Lamb Waves

Recall that the Rayleigh-Lamb equation is presented in Chapter 2 and is given as (Rose, 1999),

$$\frac{\tan(qd)}{\tan(pd)} = - \left[\frac{4\xi^2 pq}{(q^2 - k^2)^2} \right]^{\pm 1} \quad (3.1)$$

where $p^2 = \left(\frac{\omega}{c_L}\right)^2 - \xi^2$, $q^2 = \left(\frac{\omega}{c_T}\right)^2 - \xi^2$ and the superscript +1 and -1 represents the symmetric and antisymmetric modes respectively. Equation (3.1) will take real, imaginary and complex roots. The real roots (or wave number) represents the propagating modes while the complex and imaginary roots (or pure wave number) represents the non-propagating modes. At a given frequency, there is a finite number of propagating modes with real wave numbers and non-propagating modes with pure imaginary wave numbers, and an infinite number of inhomogeneous modes with complex wave number (Santhanam and Demirli, 2013). In theory, all these modes have to be considered in order to satisfy the stress free boundary condition at the free edge. In a study by Gunawan and Hirose (2007), an approximate solution for the reflection coefficient of obliquely incident S_0 wave is presented and is in excellent

agreement with the mode decomposition method proposed in their earlier work (Gunawan and Hirose, 2004). The approximated reflection coefficient is given as,

$$\left| R_{S_0}^{S_0} \right| \approx \left| \frac{\sin(2\theta_{S_0}) \sin(2\theta_{SH_0}) - \kappa^2 \cos(2\theta_{SH_0})}{\sin(2\theta_{S_0}) \sin(2\theta_{SH_0}) + \kappa^2 \cos(2\theta_{SH_0})} \right| \quad (3.2)$$

$$\left| R_{SH_0}^{S_0} \right| \approx \sqrt{1 - \left| R_{S_0}^{S_0} \right|^2}$$

where $\kappa = \xi_{SH_0}/\xi_{S_0}$, ξ_{SH_0} is the wave number for the fundamental shear horizontal mode and ξ_{S_0} is the wave number for the fundamental S_0 mode. The angle of incidences or reflection is represented by θ with wave modes as the subscript. The approximated reflection coefficient for obliquely incident S_0 wave is illustrated in Figure 3-5. The reflection of obliquely incident A_0 wave was investigated by Gunawan and Hirose (2007) and also Santhanam and Demirli (2013). Santhanam and Demirli (2013) adopted the collocation method proposed by Morvan et al. (2003) and extend it to oblique incidence Lamb waves. At low frequency, below the cut-off frequency of lowest order antisymmetric shear horizontal wave, the reflection coefficient of an obliquely incident A_0 wave is unity (Santhanam and Demirli, 2013). They were however unable to verify the propose method through experimental study. The phase information of the reflected wave, which is crucial for modelling of the reflected signal is not presented.

3.3.3 Numerical Simulation

3.3.3.1 Convergence Study

Prior to conducting a full simulation for the reflection of Lamb waves, a convergence study is conducted to select the optimum element size for further studies. In this study, various element size (0.83mm, 0.67mm, 0.5mm and 0.33mm) were used. The simulation is conducted using PZFlex, a commercially available software developed by Weidlinger Associates Inc. PZFlex is an explicit time domain solver which is suitable for wave propagation analysis.

Prior to the simulation, an appropriate plate type and size have to be determined. Since 2024-T3 Aluminium alloy (1mm thick) is readily available in the laboratory,

the author decided to use this material in the simulation. The material properties are tabulated in Table 3-1.

Table 3-1: Material properties of 2024-T3

Properties	Value
Young's Modulus [GPa]	73.10
Density [kg/m ³]	2,780
Poisson Ratio	0.33

To determine the appropriate plate size, the phase and group velocities of the Lamb waves have to be taken into consideration. This is to prevent interference with wave fields of the direct propagation path and any reflected waves from the edges and ends of the plate. To achieve this, the velocities of the phase and group velocity have to be computed using the Rayleigh Lamb equations as defined in Chapter 2. The actuation signal is a 5-cycle Hanning windowed tone burst. The advantages and rationale of using a 5-cycle Hanning windowed tone burst was presented in Chapter 2. At an actuation frequency of 200 kHz, the respective velocities are indicated in the dispersion curves as illustrated in Figure 3-2. From the velocities, the wavelength of the S_0 wave and the A_0 wave is determined as 27.2 mm and 6.6 mm respectively. Since the A_0 wave has a shorter wavelength, it is more sensitive to the selection of element size as compare to the S_0 wave. For the purpose of this convergence study, the author adopted the time of arrival (TOA) of the A_0 wave as the parameter of interest.

Figure 3-2: Phase and group velocities at 200kHz for 1mm thick 2024-T3 aluminium alloy

To prevent overlapping of signal, the minimum distance between the actuator and sensor is determined as 56mm. For better separation of the actuation signal and the signal at the sensor due to the fastest wave, the distance between the actuator and sensor used in the study is 200mm. Similarly, to prevent overlapping of the signal from the direct path and any reflection, the difference between these path lengths must be greater than 56mm. If the transducers are along the centreline of the plate, then the minimum width of the plate must be 160mm wide to prevent the interference of signals. For the purpose of this simulation, a plate 600 x 200 x 1mm thick plate is used. The TOA for the various element size is illustrated in Figure 3-3 and tabulated in Table 3-2.

It is apparent that the smaller the element size, the better is the simulation. However, computational time is another constraint for consideration. The element size of 0.33mm provide a good estimate of the wave propagation within a reasonable amount of computational time. By adopting the element size of 0.33mm, the author is modelling 20 elements within a wavelength which is more than the requirements of 12 elements per wavelength for reasonable results (PZFlez, 2009).

Figure 3-3: TOA of A_0 wave for various element sizes

Table 3-2: TOA of A_0 wave for various element sizes

Element Size	TOA (μ s)	Error
Theoretical Model	101.2	---
0.33 x 0.33	98.4	2.8%
0.5 x 0.5	98.0	3.2%
0.67 x 0.67	97.3	3.9%
0.83 x 0.83	96.7	4.4%

3.3.3.2 Reflection Coefficients

The numerical study on reflection coefficient is conducted using PZFlex. A 600mm x 400mm x 1mm thick plate is simulated in this study. A second simulation is conducted on a 600mm x 300mm x 1mm thick plate to simulate a cut along the edge of the plate. The plate is a 2024-T3 aluminium alloy. As the focus of this study is on the reflection of Lamb waves, the out of plane displacement is selected as the parameter of interest so as to eliminate the contribution of shear horizontal modes which only have in plane displacement. In this study, 19 sets of data were extracted to compute the reflection coefficient for oblique incident Lamb waves. The simulation results is plotted against the theoretical curve as shown in Figure 3-5 and Figure 3-7. The simulation results for the S_0 reflection coefficient (Figure 3-5 and Figure 3-6) agrees well with the approximate solution for S_0 reflection. The difference between the simulation results for the A_0 reflection coefficient and the theoretical curve (Figure 3-7 and Figure 3-8) may be a result of the element size used in the simulation. At the actuation frequency, the phase velocity of the A_0 Lamb wave is relatively slow and the wavelength is small. The current element size of 0.33mm has already satisfied the minimum requirement of 12 elements per wave cycle. Further reduction in element size will increase the computational time significant. It is therefore in the author's opinion that the element size of 0.33mm is good enough to illustrate the reflection coefficient of A_0 mode.

3.3.4 Experimental Study

Experiments were conducted to verify the approximate reflection coefficients. PZTs were bonded to a 2024-T3 aluminium plate as shown in Figure 3-4. The sensors are orientated in the direction of the propagation path in order to maximize the sensor response to S_0 and A_0 wave propagation, while minimizing the response from SH_0 wave. There is a set of reference sensor which the reference signal is acquired from, and another measurement set where the acquisition of the signal is acquired from. In order to determine the reflection coefficient and to exclude the direct path from the actuator to the measurement set, only the nett signal from the measurement set is

considered. The absolute reflection coefficient is then taken as the amplitude of the signal envelope of the measure set over the amplitude of the signal envelope of the reference set. The absolute reflection coefficient of the S_0 incident wave and A_0 incident wave, as a function of the angle of incidence, is illustrated in Figure 3-5 and Figure 3-7 respectively. For the S_0 incident wave, the experiment results agree well with approximate theoretical values. However, the authors are unable to actuate a pure A_0 incident wave and the signal thus consists of multiple modes. Figure 3-9 shows that other modes are present despite the author's effort in generating pure A_0 wave. The actuation of a pure A_0 incident wave requires precise collocation of the PZT actuators on both faces of the plate. The time lag between actuation signals to the top and bottom surface also affects the ability to generate pure A_0 incident wave. Therefore, at the TOA of the reflected A_0 wave, multiple mode coexists and the reflection coefficient obtained in this manner does not truly represent the reflection of A_0 incident wave. Nonetheless, the author agrees that the approximate solutions are sufficient to provide the reflection coefficient for S_0 incident wave and A_0 incident wave and to be included in the integrated model in the following section.

Figure 3-4 : Layout of actuators and sensors

Figure 3-5 : Absolute reflection coefficient of reflected S_0 wave due to S_0 incident wave

Figure 3-6 : Phase delay for reflected S_0 wave due to S_0 incident wave

Chapter 3: Modelling of PZT Transducer Response in Narrow Plate

Figure 3-7 : Absolute reflection coefficient of reflected A_0 wave due to A_0 incident wave

Figure 3-8 : Phase delay for reflected A_0 wave due to A_0 incident wave

Figure 3-9 : Sensor signal at S2 before cutting of plate

3.4 PZT TRANSDUCER RESPONSE IN AN NARROW PLATE

3.4.1 Equivalent Imaginary Source

Consider a semi-infinite plate with an excitation source as shown in Figure 3-10(a). Assuming plane waves, the propagation path of a single free edge reflected wave is shown by the solid line. Assuming all other factors remain unchanged, the displacement field (or strain field) at point B is assumed to be the same if the plane waves are generated by an equivalent imaginary source and propagating in the direction as illustrated by the dashed line in Figure 3-10(b). The source may be a harmonic or transient excitation signal. The equivalent imaginary source will incorporate the reflection coefficient as well as the phase delay information presented in the previous section. In general, the equivalent imaginary source may be defined as,

$$P_{eq}(t) = |R_i^j| P(t) e^{i\varphi} \quad (3.3)$$

where $|R_i^j|$ is the absolute reflection coefficient of mode j due to incident mode i , $P(t)$ is the source signal and φ is the phase. In SHM applications, the typical excitation signal frequently used within the community is Hanning windowed sine wave, which may be given as,

$$P(t) = 0.5 \left[1 - \cos \frac{2\pi f_0 t}{N} \right] \sin(2\pi f_0 t) \quad (3.4)$$

where f_0 is the excitation frequency and N is the number of cycles within the Hanning window. The number of cycles within the window typically ranges from 3 cycles to 7 cycles with 5 cycles being the norm. For this thesis, a 5-cycle Hanning windowed sine wave is used as the excitation signal as this gives a good balance between the time window of the excitation signal and the energy concentration for a small frequency bandwidth. For this signal, 99.7% of energy is within the main lobe and the first side lobe (Figure 3-11). It is thus sufficient to consider only this bandwidth for the equivalent imaginary source. The lower bound is defined by $f_0 - \frac{3f_0}{N}$ and the upper bound is defined by $f_0 + \frac{3f_0}{N}$.

Figure 3-10 : (a) Reflection from a source in a semi-infinite plate; (b) Propagation path from an equivalent imaginary source in an infinite plate

Figure 3-11 : 5-cycle Hanning windowed sine wave and its Fourier transform

At low frequency, the equivalent imaginary source for the reflected S_0 wave can be given by,

$$P_{eq}^S(t) = \begin{cases} 0.5 \left| R_{S_0}^{S_0} \right| \left[1 - \cos \frac{2\pi f_0 t}{N} \right] \sin(2\pi f_0 t) & , \text{for } \phi \text{ between } 60^\circ - 80^\circ \\ -0.5 \left| R_{S_0}^{S_0} \right| \left[1 - \cos \frac{2\pi f_0 t}{N} \right] \sin(2\pi f_0 t) & , \text{for all other angles} \end{cases} \quad (3.5)$$

where $\left| R_{S_0}^{S_0} \right|$ is the approximate reflection coefficient defined in Equation (3.2).

Similarly, the equivalent imaginary source for reflected A_0 wave, at low frequency, can be given by,

$$P_{eq}^A(t) = -0.5 \left[1 - \cos \frac{2\pi f_0 t}{N} \right] \cos(2\pi f_0 t) \quad , \text{for all angles} \quad (3.6)$$

The far-field out of plane displacement for a square PZT actuator has been presented by Raghavan and Cesnik (2005). For a transient signal, the out of plane displacement for A_0 mode can be given by,

$$\begin{aligned}
u_y^A(x, y, t) = & -i \frac{\tau_0}{\pi \mu} \\
& \times \sum_{f=f_0-\frac{3f_0}{N}}^{f=f_0+\frac{3f_0}{N}} \left\{ \frac{T_A}{\xi_A D'_A} \sqrt{\frac{2\pi}{\xi_A r}} \frac{\sin(\xi_A a \cos \phi) \sin(\xi_A a \sin \phi)}{\sin \phi \cos \phi} \right. \\
& \times \tilde{P}_{eq}^A 2\pi \Delta f e^{-i(\xi_A r - \frac{\pi}{4} - 2\pi f t)} \left. \right\} \quad (3.7)
\end{aligned}$$

where τ_0 is the amplitude of the shear stress due to the actuation of the PZT, μ is the Lamé's constant, $T_A = 2\xi_A^2 pq \cos pd \sin qd + \xi_A^2 (\xi_A^2 - q^2) \sin pd \cos qd$, $D'_A = dD_A/d\xi_A$ where $D_A = (\xi_A^2 - q^2)^2 \sin pd \cos qd + 4\xi_A^2 pq \cos pd \sin qd$, r is the propagation distance, ϕ is the approach angle of the propagating wave and $\tilde{P}_{eq}^A$ is the temporal Fourier transform of the equivalent imaginary source for A_0 mode. Similarly, the out of plane displacement for S_0 mode can be given by,

$$\begin{aligned}
u_y^S(x, y, t) = & -i \frac{\tau_0}{\pi \mu} \\
& \times \sum_{f=f_0-\frac{3f_0}{N}}^{f=f_0+\frac{3f_0}{N}} \left\{ \frac{T_S}{\xi_S D'_S} \sqrt{\frac{2\pi}{\xi_S r}} \frac{\sin(\xi_S a \cos \phi) \sin(\xi_S a \sin \phi)}{\sin \phi \cos \phi} \right. \\
& \times \tilde{P}_{eq}^S 2\pi \Delta f e^{-i(\xi_S r - \frac{\pi}{4} - 2\pi f t)} \left. \right\} \quad (3.8)
\end{aligned}$$

where $T_S = 2\xi_S^2 pq \sin pd \cos qd + \xi_S^2 (\xi_S^2 - q^2) \cos pd \sin qd$ and $D'_S = dD_S/d\xi_S$

where $D_S = (\xi_S^2 - q^2)^2 \cos pd \sin qd + 4\xi_S^2 pq \sin pd \cos qd$.

Numerical simulations are conducted using PZFlex to verify that the reflected Lamb waves can be modelled as a wave propagating from an imaginary source in an infinite plate. Similar to the previous section, only the out of plane displacement is used for comparison in order to capture only the Lamb modes.

Using the equivalent imaginary source for the reflected S_0 wave defined in Equation (3.5), the out of plane displacement can be computed using Equation (3.8). Similarly, the out of plane displacement for the reflected A_0 wave can be computed using Equation (3.7) with the equivalent imaginary source defined by Equation (3.6).

Chapter 3: Modelling of PZT Transducer Response in Narrow Plate

The comparison between the response from an equivalent imaginary source and the reflected signal from the numerical simulation of an A_0 wave and a S_0 wave is illustrated in Figure 3-12 and Figure 3-13 respectively.

Figure 3-12 : Comparison between EIS and numerical simulation for A_0 reflection

Figure 3-13 : Comparison between EIS and numerical simulation for S_0 reflection

Figure 3-12 and Figure 3-13 above illustrates that the displacement field for reflected Lamb waves can be modelled as Lamb waves propagating in an infinite plate from an equivalent imaginary source. The model can then be extended to predict the response of a PZT transducers from a reflected Lamb waves.

3.4.2 Analytical Formulation

In a narrow plate, reflections from the sides are apparent in the sensor signal. To account for these signals and possible interference (constructive or destructive) between the guided wave modes, it is proposed to account for these reflected signals by integrating the reflection coefficient and phase information from the previous sections in the sensor response model. By adopting the concept of an equivalent imaginary source, the output voltage due to edge reflected S_0 wave can be written as,

$$V_{edge}^S(x, y, t) = -i \frac{\tau_0 Y_c^{11} h_c g_{31}}{\mu a^2 (1 - \nu)} \times \sum_{f=f_0 - \frac{3f_0}{N}}^{f=f_0 + \frac{3f_0}{N}} \left\{ \frac{N_S}{\xi_S^2 D'_S} \sqrt{\frac{2\pi}{\xi_S r}} \frac{\sin^2(\xi_S a \cos \phi) \sin^2(\xi_S a \sin \phi)}{\sin^2 2\phi} \right. \quad (3.9)$$

$$\left. \times \tilde{P}_{eq} \Delta \omega e^{-i\left(\xi_S r - \frac{\pi}{4} - 2\pi f t\right)} \right\}$$

where τ_0 is the amplitude of the shear stress due to the actuation of the PZT, Y_c^{11} is the Young's modulus of the PZT sensor in the x-direction, h_c is the thickness of the PZT sensor, g_{31} is the piezoelectric constant of the PZT sensor, a is half-length of the PZT sensor, μ is the Lamé's constant, ν is the Poisson's ratio of the PZT sensor, f_0 is the actuation frequency, N is the number of cycles in the actuation frequency, $N_S = \xi_S q (\xi_S^2 + q^2) \cos pd \cos qd$, ξ_S is the wavenumber of the S_0 wave, D'_S is the derivative of D_S where $D_S = (\xi_S^2 - q^2)^2 \cos pd \sin qd + 4\xi_S^2 pq \sin pd \cos qd$, r is the propagation distance between the actuator and the sensor, ϕ is the approach angle of the propagating wave, $\tilde{P}_{eq}$ is the temporal Fourier transform of the equivalent imaginary source and $\Delta \omega$ is the step size of the angular frequency. In a similar manner, the output voltage due to a edge reflected A_0 wave can be written as,

$$\begin{aligned}
V_{edge}^A(x, y, t) = & -i \frac{\tau_0 Y_c^{11} h_c g_{31}}{\mu a^2 (1 - \nu)} \\
& \times \sum_{f=f_0 - \frac{3f_0}{N}}^{f=f_0 + \frac{3f_0}{N}} \left\{ \frac{N_A}{\xi_A^2 D'_A} \sqrt{\frac{2\pi}{\xi_A r}} \frac{\sin^2(\xi_A a \cos \phi) \sin^2(\xi_A a \sin \phi)}{\sin^2 2\phi} \right. \\
& \left. \times \tilde{P}_{eq} \Delta \omega e^{-i\left(\xi_A r - \frac{\pi}{4} - 2\pi f t\right)} \right\} \quad (3.10)
\end{aligned}$$

where $N_A = \xi_A q (\xi_A^2 + q^2) \sin pd \sin qd$ and D'_A is the derivative of D_A where $D_A = (\xi_A^2 - q^2)^2 \sin pd \cos qd + 4\xi_A^2 pq \cos pd \sin qd$.

The same can be done for end reflections. The response of each individual wave packet is shown in Figure 3-14 below.

By summing all possible reflections within the time window of interest, one can predict the response of the sensor in a narrow plate. The total response is given as,

$$V_{out} = V_{direct}^S + V_{edge}^S + V_{end}^S + V_{direct}^A + V_{edge}^A + V_{end}^A + V_{edge}^{SH} \quad (3.11)$$

The predicted sensor response for a 600mm long by 400mm wide by 1mm thick 2024-T3 Aluminium alloy plate is shown in Figure 3-16 and Figure 3-17.

Chapter 3: Modelling of PZT Transducer Response in Narrow Plate

Figure 3-14 : Sensor response for (a) direct S_0 ; (b) edge reflected S_0 ; (c) direct A_0 and (d) end reflected S_0

4.3 Numerical Simulation

Numerical simulation is conducted using a commercial finite element analysis (FEA) package, PZFlex. PZFlex is an explicit time domain solver which is suitable for wave propagation analysis. In order to have a smooth wave profile, there must be at least 12 elements within a wave cycle (PZFlex, 2009). A 600mm x 400mm x 1mm thick plate is simulated in this study. The plate is a 2024-T3 aluminium alloy. The actuator and sensor is bonded along the centreline with 200mm spacing between them. The end distance for the actuator and the sensor are 200mm (Figure 3-15). The dimensions for the transducers are 10mm x 10mm x 0.3mm thick. A bonding layer of 0.03mm is also modelled in the study. The actuation signal used in this study is a 200 kHz 5-cycle Hanning window toneburst. Since the actuation is only on the top surface, both S_0 and A_0 is assumed to be generated at the actuator location. The simulated response is plotted in Figure 3-16 for comparison. There is good agreement between the model and the numerical simulation.

Figure 3-15 : Numerical Model in PZFlex

3.4.4 Experimental Study

An experiment study is conducted on a 2024-T3 aluminium alloy plate. The dimensions of the plate are 600mm (length) by 400mm (width) by 1mm (thickness). A pair of PZT transducers are surface bonded to the aluminium plate along the centre line, spaced at 200mm apart. One PZT will act as the actuator while the other is the sensor. As in the numerical simulation, the actuation signal is a 200 kHz 5-cycle Hanning window tone burst. The actuation signal is sent through an arbitrary function generator (Tabor Electronics WW1071) to the PZT actuator and the surface strain field is translated to voltage through the PZT and recorded by an oscilloscope (Yokogawa DL1620). The sensor signal is plotted in Figure 3-17 for easy comparison.

Figure 3-16 : Comparison between analytical model and numerical study for 400mm wide x 1mm thick 2024-T3 aluminium alloy plate

Figure 3-17 : Comparison between analytical model and experimental study for 400mm wide x 1mm thick 2024-T3 aluminium alloy plate

3.4.5 Discussion

From Figure 3-16 and Figure 3-17 above, it can be seen that there is excellent agreement between the analytical, numerical and experimental results for the direct path S_0 wave. Subsequent wave packets in the numerical and experimental study also agrees well with the analytical model in terms of the time of flight. However, it is observed that the time window for each wave packet appears to be wider in the numerical and experimental study. In the analytical model, the PZT will be subjected to oscillatory expansion and contraction in the presence of alternating voltage applied across the electrodes. This oscillatory motion will stop immediately when the alternating voltage is removed. Therefore, there is no strain transfer to the plate immediately after the alternating voltage is removed. However, in the numerical and experimental study, there are residual vibrations after the alternating voltage is removed, which is then transferred to the plate through the bonding layer, resulting in a wave packet with a wider time window at the tail end.

Another observation is the difference in the relative amplitude between the S_0 and A_0 wave. The relative amplitude between the S_0 and A_0 wave depends on the frequency and size of PZT (Giurgiutiu, 2003; Raghavan and Cesnik, 2005). In the modelling of the sensor response, the bonded PZT can be assumed to have an effective width size smaller than its physical size. Therefore, the model can be calibrated to match the numerical or experimental results by adjusting the effective width of the PZT.

Figure 3-18 : Comparison between analytical model and experimental study for reduced width PZT

This illustrates that the model can be calibrated to suit the on-site condition once the baseline sensor response is obtained on the field. The calibrated model can then be used for further parametric study or crack monitoring applications.

3.5 SUMMARY

The existing model for PZT transducer response in an infinite plate is extended to include reflections from the sides and the ends. Numerical simulations and experimental studies are conducted to verify the proposed model. There is close

Chapter 3: Modelling of PZT Transducer Response in Narrow Plate

agreement between the proposed model and the numerical simulations as well as experimental studies. The proposed model can be applied to LWP based SHM to provide a prediction of the sensor response of a narrow plate. For crack monitoring applications, the model can also be used to derive a relationship between the crack length and any proposed damage index.

CHAPTER 4

DETECTION AND MONITORING OF FATIGUE CRACK GROWTH IN NARROW PLATE

4.1 BACKGROUND

This chapter presents the study on the effect of width on the PZT sensor signal. Numerical and experimental studies on the effect of plate width on the sensor signal are conducted and presented. The equivalent imaginary source model present in Chapter 3 is adopted to study the relationship between crack length and damage indicators. Experimental study is conducted to determine a suitable damage index for crack monitoring in a narrow plate. To achieve this, various damage indices based on the concepts of energy reduction are studied and presented.

4.2 EFFECT OF WIDTH ON SENSOR RESPONSE

The response for a PZT transducers in an infinite plate was reviewed in Chapter 2 and the concept of an equivalent imaginary source is introduced in the preceding Chapter to model the output voltage of a reflected Lamb waves. Numerical and experimental studies are conducted to illustrate the complexity and the interference of signal (constructive or destructive) for different plate widths.

4.2.1 Numerical Study

In the numerical study to illustrate the effect of width on the output voltage of the PZT sensor, various plate widths were considered. The simulation is conducted using PZFlex, a commercially available software developed by Weidlinger Associates Inc. The advantages of using PZFlex for wave propagation study were discussed in Chapter 3. The study started with a 400 mm wide plate width and gradually reducing to 200 mm width in steps of 50mm. The material of the plate is a 2024-T3 aluminium alloy. The length of the plate is 600 mm with a thickness of 1mm.

Two numbers of PIC 151 are spaced as 200 mm apart along the centreline. A schematic diagram of the plate is illustrated in Figure 4-1.

Figure 4-1: Schematic diagram of plate

The same modelling technique as the preceding Chapter is adopted. Figure 4-2 shows the output voltage from the simulation of a 400 mm width plate. The dispersion curves for fundamental Lamb waves travelling in a 1mm thick 2024-T3 aluminium alloy plate is illustrated in Figure 3-2. From the group velocities, the S_0 wave is identified as the faster wave and will thus arrive at the sensor first. The time of flight can be determined as 36.9 μs for a S_0 wave propagating a distance of 200 mm. With a 5-cycle Hanning windowed actuation of 200 kHz, the time of arrival is computed to be 49.4 μs . This coincides with the first wave packet in the sensor signal. In a similar manner, the time of arrival of A_0 wave propagating a distance of 200 mm is computed as 101.4 μs . For edge reflected S_0 wave, the time of arrival for propagating a distance of 447 mm is computed as 95 μs . Therefore, the second and third wave packet is the edge reflected S_0 wave and direct propagation A_0 wave respectively. In the preceding Chapter, it was shown that for an obliquely incident S_0 wave, mode

Chapter 4: Detection & Monitoring of Fatigue Crack Growth in Narrow Plate

conversion occurs, and both S_0 wave and SH_0 wave will be reflected off the edge of the plate. Using the relationship defined in Equation (3.2), the TOA for the SH_0 wave is computed as 123.8 μs . A normal incident S_0 wave will be reflected as S_0 wave with no mode conversion at low frequency. The time of arrival of the end reflected S_0 wave propagating a distance of 600 mm is computed as 123.2 μs . The last wave packet in Figure 4-2 is therefore identified as a combination of the end reflected S_0 wave and edge reflected SH_0 wave.

Figure 4-2: Output voltage of PZT sensor from numerical simulation of a 400mm plate

In order to further justify the identification of wave packets illustrated in Figure 4-2, a few screenshots from the simulation is presented in Figure 4-3. Figure 4-3(a) shows the screenshot at 50 μs . It is obvious that at this time step, only the S_0 mode direct path reached the sensor, thus the first wave packet of the sensor signal is the S_0 direct path. At 100 μs (Figure 4-3 (b)), the sensor “received” both the A_0 mode direct path as well as the S_0 edge reflection. The reflection from both ends reached the sensor at approximately 130 μs (Figure 4-3 (c)). The reflection of the SH_0 wave cannot be capture since the SH_0 wave only have in-plane displacement and no out-of-plane displacement. The difference in wave length can also be used to identify the wave

Chapter 4: Detection & Monitoring of Fatigue Crack Growth in Narrow Plate

nodes from the simulation. The S_0 wave being the faster wave has longer wave length while the A_0 wave being the slower wave has shorter wave length.

Figure 4-3: Screenshots of the numerical simulation (out-of-plane displacement) at (a) $50\mu\text{s}$; (b) $100\mu\text{s}$; and (c) $130\mu\text{s}$

The numerical simulation is then repeated for various plate width and the sensor signals are illustrated in Figure 4-4 . The numerical simulation results shows that it is in close agreement with respect to the TOA of the wave packets. It is also noted that as width reduces, the second wave packet, which contains the edge reflection S_0 mode and the direct A_0 mode, arrives at the sensor location earlier (i.e. shorter TOA). Therefore, any damages or crack along the edge is likely to affect this wave packet.

4.2.2 Experimental Study

An experimental study is conducted to investigate the effect of reflection on the sensor signal waveform. For equal comparison, the dimensions and material of the plate used in the experimental study is the same as the numerical study present above. The material of the plate used in this study is the 2024-T3 aluminium alloy. The thickness of the plate is 1mm. The initial size of the plate is 600mm long by 400mm wide. The width of the plate was gradually reduced from 400mm to 200mm in steps of 50mm. The piezoceramics (PZT) transducers used in this study are PIC-151 PZTs from PI Ceramics. The transducers (one as actuator and the other as sensor) are

bonded onto the plate along the centreline, spaced at 200mm apart, as shown in Figure 4-1. The adhesive used is RS-850-940 two part epoxy from RS Components. The properties of the plate and PZTs are shown in Table 4-1.

Table 4-1: Material properties

<i>2024-T3 Aluminium Alloy</i>		
Modulus of Elasticity	(GPa)	73.1
Density	(kg/m ³)	2,780
Poisson Ratio		0.33
<i>PIC-151Piezoceramics</i>		
Density	(g/cm ³)	7.8
Electric Permittivity		2400
Piezoelectric charge constant, d31	(C/N)	-210 x 10 ⁻¹²
Piezoelectric charge constant, d33	(C/N)	500 x 10 ⁻¹²
Elastic Constant, s11	(m ² /N)	15 x 10 ⁻¹²
Elastic Constant, s33	(m ² /N)	19 x 10 ⁻¹²

The common practice in the SHM community is to use a Hanning-windowed tone burst as the diagnostic signal. The main reason is the concentration of energy at the actuation frequency with minimum side loops. In this study, a 5-cycle Hanning-windowed tone burst is adopted as the actuation signal. The equation for the actuation signal is given as,

$$V_{in} = \frac{1}{2} \sin(2\pi ft) \left[1 - \cos\left(\frac{2\pi ft}{N}\right) \right] \quad (4.1)$$

where f is the actuation frequency, t is the time and N is the number of cycles. Ideally, the actuation frequency should be lower than the cut-off frequency of the higher order Lamb modes (i.e. S_1 and A_1 mode) so as to reduce the complication of signal waveform from higher order modes. The upper limit of the working frequency range is thus determined. The actuation frequency should also be higher than the overlapping frequency to prevent overlapping of the first wave packet with the electro-magnetic interference signal, which forms the lower limit of the working

frequency range. For the material and the configuration used in the study, the lower and the upper limit of the working frequency range is determined as 140 kHz and 1.6 MHz respectively. An actuation frequency of 200 kHz is adopted in this study. At this actuation frequency, there is only two fundamental Lamb modes (S_0 and A_0) and overlapping of the first wave packet with the electro-magnetic interference is prevented.

A personal laptop is used to control the arbitrary function generator (Tabor Electronics WW5061) via the ArbConnection 4.3 user interface. A 10 volt actuation signal is sent to the actuator, which “pinched” the surface of the plate and sent waves propagating in the plate. The sensor then “received” these surface strains and converted them into electrical displacement via the direct piezoelectric effects of the PZT. The voltage, as a result of the electrical displacement, is then displayed on the digital oscilloscope (Yokogawa DL1620). The signal is then acquired by the personal laptop via Yokogawa Xviewer software. In this study, the 32-cycle averaging method is adopted to reduce the random noise. The averaging is computed using the built-in function of the digital oscilloscope (Yokogawa, 2009)

The sensor signal waveform is acquired at each width of interest and shown in Figure 4-4. It is observed that, as the width is reduced from 400mm to 200mm, the time of flight (TOF) for some wave packets reduced too. This is due to the shortening of the wave propagation path resulted from the reduction of the plate width. The propagation length for a 400mm width and a 200mm width plate is 447mm and 283mm respectively. This translates to 95 μ s and 65 μ s respectively, which is illustrated as a continuous blue line in Figure 4-4. There are also two wave packets that are not affected by the reduction in plate width. By working out the TOF for the S_0 direct propagation path (i.e. 200mm) and the S_0 end reflection path (i.e. 600mm), it coincides with the measured TOF of two unaffected wave packets. Hence, two wave packets with TOF at 50 μ s and 130 μ s are identified to be the S_0 direct wave packet and the S_0 end reflection wave packet, respectively.

It is also observed that for narrow plates, there may be constructive or destructive interference between the S_0 direct wave packet and the S_0 edge reflected wave packet. When the plate width is 200mm, the edge reflected wave packet started to interfere

with the first wave packet (S_0 wave packet). This needs to be taken into consideration for the damage detection and monitoring algorithm.

Figure 4-4: Sensor signals at various widths

4.3 CRACK MONITORING

To provide a better understanding the effects of crack growth in a narrow plate, the equivalent imaginary source model is adopted to identify the relationship between a proposed damage indicators with crack growth. Experimental studies are conducted to establish and verify the relationship between the proposed damage indicators and crack growth. The crack of interest is a fatigue crack. The details of the experiments, results and analysis are presented in the following sections.

4.3.1 Equivalent Imaginary Source Model

The equivalent imaginary source (EIS) model proposed in Chapter 3 is adopted in this study to identify a crack monitoring parameter. Consider a 6mm thick 6061-T6 aluminium specimen with an initial crack length of 5 mm as shown in Figure 4-8. The actuation signal is a 200 kHz 5-cycle Hanning windowed tone burst. Assuming the crack tip remains plane and parallel to the edge, the EIS model may be applied. As the crack propagates, the propagation angle, the angle of reflection changes, the reflection coefficient and path length of the edge reflected waves from the crack tip changes. Considering only the edge reflected waves from the crack tip, the changes in the sensor signal is illustrated in Figure 4-5.

This trend satisfies the theory of reflection coefficient presented in Chapter 3. At this range of reflection angle, the reflection coefficient increase as reflection angle increase. This explains why the amplitude of the signal increase as crack length increase.

However, for a specimen geometry shown in Figure 4-8, the reflected waves from the opposite of the crack will also need to be considered as there is interference of signal. Considering the interference, the changes in the sensor signal from the two reflected paths is illustrated in Figure 4-6.

Figure 4-5: Variation in sensor signal for crack tip reflected Lamb wave using the EIS model

Figure 4-6: Variation in sensor signal for crack tip and edge reflected Lamb wave using the EIS model

From the above observations, a damage index based on the variation in amplitude is proposed as a parameter to correlate with the crack growth. The damage index is proposed as,

$$(DI_{pp})_i = \frac{\Lambda_0 - \Lambda_i}{\Lambda_0} \quad (4.2)$$

where Λ is the peak-to-peak amplitude in voltage and the subscript 0 represents the pristine state and the subscript i represents the current state of measurement. The relationship between the crack growth and the damage index is illustrated in Figure 4-7.

Figure 4-7: Relationship between crack growth and proposed damage index

From above, it can be seen that there is a linear relationship between the crack growth and the proposed damage index based on the variation in peak to peak amplitude for the given geometry. This damage index, along with other damage indexes will be studied and compared in the experimental study presented in the following sections.

4.3.2 Experimental Setup

A 5mm notch is created on a 300 x 76 x 6 mm thick 6061-T6 aluminium specimen (Figure 4-8) to act as a stress concentrator for fatigue crack initiation and propagation. The cyclic tensile load is applied to the specimen to induce fatigue crack. In this study, the minimum and maximum stress are 117.76 MPa and 147.59 MPa respectively, with a load ratio of 0.8. At this load range, the change in stress intensity for a 5mm notch is $4.29 \text{ MPa}\cdot\text{m}^{1/2}$, which is well above the threshold stress intensity to ensure that crack will initiate and propagates.

Figure 4-8: Dimensions of specimen and location of machined notch

The experimental set up is similar to that described in the previous section. The sensor signal is acquired after the stipulated numbers of cyclic load, and the corresponding physical crack length is also recorded. The stipulated number of cycles and the crack length are tabulated in Table 4-2.

Chapter 4: Detection & Monitoring of Fatigue Crack Growth in Narrow Plate

*Figure 4-9: Experimental Setup***Table 4-2: Fatigue load cycles and crack lengths**

Fatigue Load Cycles (x 1000)	Crack length (mm)
0	5.0
75	6.2
125	8.6
160	10.7
190	13.9
230	22.7
233	25.3
235	26.6
237	29.5

4.3.3 Actuation Signal

The actuation signal used in this study is a 5-cycle Hanning windowed tone burst. The advantages of using this waveform is briefly discussed in Section 3.4.1. For a 6061-T6 Aluminium alloy, the dispersions curves are plotted and illustrated in Figure 4-10. To prevent the overlapping of sensor signal with the electromagnetic interference, the minimum frequency is 190 kHz. The cut-off frequency for higher

order Lamb modes is determined as 300 kHz, beyond which higher order Lamb modes will be present. The author had chosen to use 200 kHz as it is near the lower bound where separation between the S_0 Lamb wave and A_0 Lamb wave is more significant.

Figure 4-10: Dispersion curves and frequency limits for diagnostic signal

4.3.4 Data Acquisition

The electrical displacement, which is being captured as the output voltage is acquired using a digital oscilloscope (Yokogawa DL1620). The cycle averaging method is used to eliminate or reduce the random noise that is present in the signal. A 32 cycle averaging is performed using the built-in algorithm on the digital oscilloscope before the final reading is acquired (Yokogawa, 2009). The acquisition is conducted using a personal laptop via the Yokogawa XViewer software.

4.3.5 Experimental Results

Figure 4-11 shows the sensor signals prior to any loading and after 235,000 of load cycles. At 235,000 cycles, the crack length is 26.6mm. At 200 kHz, the velocity

for the fundamental Lamb waves determined from the dispersion curves are 5,218m/s (S_0 phase), 4,852m/s (S_0 group), 2,418m/s (A_0 phase) and 3,029m/s (A_0 group). By computing the TOF for S_0 direct path ($\approx 31\mu\text{s}$) and S_0 reflected path ($\approx 35\mu\text{s}$), it is determined that the first wave packet consisted of both the S_0 direct path and S_0 reflected path. The A_0 mode came after the first wave packet. The TOF for A_0 direct path is approximately $50\mu\text{s}$ and the A_0 reflected path is approximately $55\mu\text{s}$. The S_0 end reflection path worked out to be $62\mu\text{s}$. With this information, the Lamb wave modes of each individual wave packet is identified. In general, as the crack propagates, the amplitude of all the wave packets within the time window reduces.

Figure 4-11: Comparison of sensor signal at 5mm crack and 26.6mm crack

There are various feature extraction techniques available in the literature to characterize the damage. A few popular features include the time-of-flight (TOF) (Giurgiutiu et al., 2002; Yu et al., 2004) and transient energy (Ihn and Chang, 2004). The time of flight technique is difficult to apply in this scenario due to the inference of two or more wave modes within a wave packet. In this study, only the damage index based on the concept of variation in scattered wave energy is presented.

4.3.6 Damage Indexes

From the experimental results, it can be observed that the amplitude of the signal decreases as the crack length increases. As crack length increases, the scattering behavior changes and the amount of transient energy of the wave propagating from the actuator to the sensor also changes. And as crack increases, the propagation path length of the reflected wave from the tip of the crack shortens, suggesting a change in the time of flight. All the above parameters will be explored in the following sections to identify a suitable damage index for the monitoring of crack.

4.3.6.1 Variation in peak to peak amplitude

As the signal amplitude reduces in relation to crack growth, it is proposed to use the peak to peak amplitude as a parameter to quantify the crack growth. Only the first wave packet is considered in this scenario since it contains the edge reflected Lamb waves. The proposed damage index using the peak to peak amplitude as a quantifying parameter is given as,

$$(DI_{pp})_i = \frac{\Lambda_0 - \Lambda_i}{\Lambda_0} \quad (4.3)$$

where Λ is the peak-to-peak amplitude in voltage and the subscript 0 represents the pristine state and the subscript i represents the current state of measurement. The relationship between the crack growth and the damage index is illustrated in Figure 4-12.

Figure 4-12: Correlation between crack growth and damage index based on change in peak to peak amplitude

4.3.6.2 Variation in scatter energy

Since the voltage of the signal is an interpretation of the surface strain at the sensor, one may use the square of the voltage amplitude as an indication of energy (Ihn and Chang, 2004). Thus, the transient energy within a time window may be written as,

$$E = \int_{t_i}^{t_f} \chi^2 dt \quad (4.4)$$

where χ is the amplitude of the sensor signal. From Figure 4-11, it is apparent that the amplitude reduces as the crack propagates. By subtracting the baseline signal from the signal at each individual damage state, one can obtain scattered waves due to crack growth only. Thus, one may use the variation in the scattered wave energy as a damage indicator, which may be written as,

$$(DI_{sc})_i = \frac{\int_{t_i}^{t_f} (\chi_0 - \chi_i)^2 dt}{\int_{t_i}^{t_f} \chi_0^2 dt} \quad (4.5)$$

where t_i and t_f are the start and end of the time window, and χ_0 and χ_i are the amplitude of the sensor signal at baseline and damage state, respectively. The selection of the time window may have influence on the sensitivity of the damage index.

The edge reflected wave packet may include more information about the crack especially when the crack is located close to the edge. So, it is useful to include the effects of edge reflection in the damage quantification process. This would mean that the time window has to include the wave packet of the edge reflection. According to the experimental and numerical studies in the previous sections, the edge reflected wave packet may interfere with the direct wave for narrow specimen. From the dispersion curves, the TOF for the individual wave mode may be computed. Assuming a non-dispersive S_0 wave, the time window for the direct propagation wave packet is determined as 31 μ s to 56 μ s. The window for the edge reflected wave packet is determined as 35 μ s to 60 μ s. Although the time window of 31 μ s to 56 μ s is predominately the direct S_0 wave packet, it also contained part of the edge reflected S_0 wave packet. The same applies to the time window of 35 μ s to 60 μ s. Decomposition of the sensor signal into the individual wave mode is difficult and is not the authors' intention to do so. Therefore, the window of 31 μ s to 60 μ s, which encompass both direct and edge reflected S_0 wave comparison, is adopted in this study. The correlation between the damage index defined in Equation (4.5) and the crack growth is illustrated in Figure 4-13.

Figure 4-13: Correlation between crack growth and damage index based on change in scattered energy

From Figure 4-13, it is apparent that the proposed time window that contained the edge reflected S_0 wave packet has a good correlation with the crack growth with a R-square value of 0.94. This observation enhances the concept that edge reflected S_0 waves should be included in the damage quantification and is sensitive for cracks originating from the edge.

4.3.6.3 Variation in transient energy

Still on the concept of energy reduction, this focus on the reduction in energy arriving at the sensor location. For consistency, the same time window (31 μ s to 60 μ s) is used to compute the damage index given as,

$$(DI_{tr})_i = \frac{\int_{t_i}^{t_f} \chi_0^2 - \chi_i^2 dt}{\int_{t_i}^{t_f} \chi_0^2 dt} \quad (4.6)$$

Figure 4-14: Correlation between crack growth and damage index based on change in transient energy

Although this damage index is quite similar to the previous section, it is different in that this measure the reduction of energy arriving at the sensor location while the previous one measures the change in scattering energy originating from the crack source.

4.3.6.4 Variation in time of flight

The variation in time of flight (TOF) is also consider in this study. The proposed damage index using the TOF as a parameter is given as,

$$(DI_{TOF})_i = TOF_0 - TOF_i \quad (4.7)$$

where TOF is the time of flight of the first wave packet measures from the peak of the actuation signal envelope to the peak of the first wave packet of the sensor signal envelope.

Figure 4-15: Correlation between crack growth and damage index based on change in time of flight

It is apparent from Figure 4-15 that the TOF is not a suitable parameter to quantify the crack growth as the data are sparsely scattered and does not give a clear relationship between the proposed damage index and the crack growth. In theory, when the path length reduces, the TOF reduces. However, the first wave packet in the signal encompasses both the direct path and edge reflected S_0 waves where interference occurs. Interference could be either constructive or destructive. The peak of the combined envelope, which was used to determine the TOF, does correlates well with the crack growth due to the interference. Therefore, the TOF parameter is not a good indicator of crack growth.

4.3.7 Discussions

From the above sections, it is obvious that the time of flight is not a good indicator of crack growth and thus will not be discussed further. To have a better comparison between the proposed damage indexes, the damage indexes were normalized and plotted on the same chart as illustrated in Figure 4-16. Their

respective linear fitting curve is also plotted on the figure. Among the three damage indexes compared, the transient energy damage index is more sensitive to crack growth while the scattered energy plot is less sensitive. Although the transient energy damage index is more sensitive, the data are more scattered with a R2-value of 0.913. In comparison, the peak to peak amplitude provides a better correlation with reasonable sensitivity to the crack growth. There is also less data processing required for this damage index giving it an advantage for large scale implementation.

Figure 4-16: Comparison of damage indexes

4.4 SUMMARY

The effect of width on the response of the piezoelectric sensor signal is presented. Numerical and experimental study illustrated that as the width of a plate reduces edge reflected waves and mode conversions begin to complicate the sensor signal. The EIS model is a useful tool to study the relationship between proposed damage indicators

Chapter 4: Detection & Monitoring of Fatigue Crack Growth in Narrow Plate

and the crack growth for a given geometry. Experimental study was carried out to verify the relationship between the damage indicators and crack growth. Four different damage indexes were explored and compared. In conclusion, the peak to peak amplitude damage index gives a better correlation with the crack length.

CHAPTER 5

UNCERTAINTY MANAGEMENT AND DAMAGE PROGNOSIS (DP)

5.1 BACKGROUND

The uncertainties in predicting the remaining fatigue life will be discussed. Fatigue is stochastic in nature and there are numerous uncertainties in terms of the materials and damage detection techniques. This chapter focus on handling the uncertainties in material fatigue properties and the uncertainties in the damage detection technique, in particular, the Lamb wave detection technique. An iterative framework is proposed to estimate the remaining life of a component when information on the fatigue properties of the material is not available.

5.2 DAMAGE PROGNOSIS

5.2.1 Linear Elastic Fracture Mechanics

Fracture mechanics is the field of mechanics for the study of crack propagation in solid materials. Fracture mechanics is generally divided into two schools, namely the Linear Elastic Fracture Mechanics (LEFM) and the Elastic-Plastic Fracture Mechanics (EPFM). As the name implies, LEFM is developed based on the theory of linear elasticity and EPFM is based on a combination of elastic and plasticity theories. For a crack with a small plastic zone near the crack tip, LEFM is adequate to model the crack growth. For the purpose of this study, LEFM will be adequate and only the LEFM will be discussed from this point forward.

LEFM was first presented by Griffith (1921) who formulated the concept that existing crack will propagate when the strain energy release by the crack growth is equal to the surface energy required to create new crack surfaces. His theory allows the estimation of the theoretical strength of brittle solids. However, Griffith's elastic stress field contains a singularity at the crack tip. Therefore, Irwin (1957) introduces a two-zone model, and considered inelastic effects to be confined to a small inner

zone. A stress intensity parameter (K) was introduced, and by applying the assumption that the elastic K -field will not be influenced by the inelasticity in the inner zone, Irwin effectively related the energy release rate (G) to the crack tip K -field. These studies were based on elastic stress fields and the demonstration of the equivalence of the energy release rate (G) and stress intensity (K) provided the basis for the development of Linear Elastic Fracture Mechanics (LEFM).

5.2.2 Paris Equation

The Paris equation is briefly explained in Chapter 2. Recall that a typical crack growth rate curve is shown in Figure 2-11, which divides the curve into 3 regions. Region I is the near threshold region, where a threshold stress intensity range, ΔK_{th} , is defined. For ΔK that falls below ΔK_{th} , cracks either propagate at an extremely low rate or do not propagate at all. In Region III, the crack growth rate increases rapidly and terminates with fracture as the curve rises to an asymptote where the maximum stress intensity factor, K_{max} , equates the critical stress intensity factor, K_c . Region II defines a region whereby the crack growth rate is often some power function of ΔK , leading to a linear relationship between $\log(da_f/dN)$ and $\log(\Delta K)$. The most well-known relationship is the Paris equation (Paris et al., 1961),

$$\frac{da_f}{dN} = C(\Delta K)^m \quad (5.1)$$

The term C and m in Equation (5.1) is known as the Paris coefficients and is a function of the material properties when the stress ratio is zero. For stress ratio other than zero, where the mean stress effect is applicable, the Paris equation can be modified using the empirical Walkers relationship where,

$$C' = \frac{C}{(1 - R)^{m(1-\lambda)}} \quad (5.2)$$

where R is the stress ratio and λ is a material constant. The typical value of λ for metallic components is 0.5. However, it can also range from 0.3 to 1.0. Therefore, for a non-zero stress ratio with mean stress effects, the crack growth can be defined by,

$$\frac{da}{dN} = C'(\Delta K)^m \quad (5.3)$$

5.2.3 Remaining Life Estimation

From LEFM, the critical crack length, a_{fcr} , of the component under a constant stress ratio can be computed when the critical stress intensity factor is known. The remaining useful life can then be computed by integrating the crack growth curve from the current crack length to the critical crack length and can be given by,

$$N_R = \int_{a_{fn}}^{a_{fcr}} \frac{1}{C'(\Delta K)^m} da \quad (5.4)$$

From Equation (5.4), it can be seen that the remaining fatigue life is a function of the Paris coefficients, the stress ratio, the geometry of the components and the current crack length of the specimen. Assuming a constant load ratio and the same component is used, the only parameters that affect the estimation of the remaining fatigue life are the Paris coefficients and the current crack length. Therefore, only the uncertainties in Paris coefficients and the crack length estimation will be discussed from this point forward.

5.2.4 Monte Carlo Simulation

Monte Carlo simulation is a numerical method that uses random variables (from a known distribution) in a proposed model and repeating a specified number of trials to obtain a distribution of possible outcomes.

In engineering, Monte Carlo simulation is normally conducted for probabilistic analysis or sensitivity analysis. For example, there are 100 random variables for a 100 trials Monte Carlo simulation which will yield 100 outcomes. The probability of a required outcome can then be derived from the simulations. Numerous researchers have successfully apply Monte Carlo simulation to estimate the remaining life of a system under fatigue (Laz and Hillberry, 1998; Farrar et al., 2004; Liu and Mahadevan, 2009; Sankararaman et al., 2011; Gobbato et al., 2012; Ling and Mahadevan, 2012).

5.3 EXPERIMENTAL STUDY

Two sets of experimental study are conducted on 6061-T6 aluminum alloy specimens. The size of the specimens is 300mm x 76mm x 6mm thick. A 5mm notch is created on one side of the specimen by means of electronic discharge machining (Figure 5-1(a)). The first set consists of six specimens (specimen S1-S6), used to determine the Paris coefficients for the material and load ratio of interest. The second set consists of 2 specimens (P1 and P2) with a pair of surface bonded PZTs each. One PZT will act as the actuator while the other as the sensor. The PZTs used in this study are PIC-151 from PI ceramics (PI Ceramic, 2013).

Figure 5-1 : (a) Specimen size; (b) Experimental Setup

For the first set, only a dynamic testing rig is used to apply the cyclic load to the specimen. For the second set, the experimental setup consists of an arbitrary function generator, a digital oscilloscope, a dynamic testing rig and a personal computer (Figure 5-1(b)). A cyclic load range of 117.76 MPa to 147.59 MPa with a load ratio of 0.8 is adopted for this study. This will allow ΔK to be just beyond the threshold

value for crack to initiate and propagate. The crack growth curve for specimens S1-S6 is illustrated in Figure 5-2, and their respective fatigue life tabulated in Table 5-1.

Figure 5-2 : Crack growth curve (Specimen S1 - S6)

Table 5-1 : Fatigue life of Specimens S1 - S6

Specimen	Fatigue Life (cycles)
S1	318,435
S2	296,201
S3	241,592
S4	276,557
S5	268,713
S6	375,176

In the second set of experiment, one specimen (Specimen P1) is used to determine the correlation factor for the specimen of interest. A typical sensor response is illustrated in Figure 5-3. The fatigue crack length increases as the number of loading cycle increases. As the crack length increases, the back scattering of Lamb

waves increases, thus the amplitude of the response at the sensor decreases. The specimen is a narrow plate and the first wave packet consisted of the direct and edge reflected symmetric Lamb waves. Therefore, the damage index adopted in this study is the change of the peak to peak amplitude defined as,

$$DI_i = \frac{\Lambda_0 - \Lambda_i}{\Lambda_0} \quad (5.5)$$

where the subscript 0 represents the pristine state and the subscript i represents the current state of measurement. Λ is the peak-to-peak amplitude in voltage. A plot of the crack growth against the damage index for Specimen P1 is illustrated in Figure 5-4

Figure 5-3 : Sensor response for baseline and after 180,000 load cycles

Figure 5-4 : Correlation between crack growth and damage index

5.3.1 Uncertainty in Fatigue Properties

From the first set of experiment, it is clear that there are uncertainties in the fatigue properties of the aluminum alloy tested (Table 5-2). The average values of the Paris coefficient m and $\log C$ are 2.7 and -9.35 respectively. The solid line in Figure 5-5 represents the Paris coefficient. The dashed lines in the figure indicate the upper bound and lower bound. Therefore, any combination of m and $\log C$ values should produce a linear line within the boundaries. In order to ensure that the random variables generated in the Monte Carlo simulation fall within the upper and lower bounds, standard deviations of 0.066 and 0.033 were used for the m value and $\log C$ value respectively. This corresponds to a 99.7% confidence level, assuming normal distribution.

Table 5-2: Paris coefficients of individual specimen

Paris Coefficients	Specimen						mean
	S1	S2	S3	S4	S5	S6	
m	2.36	2.16	2.75	3.16	2.75	3.04	2.70
$\log C$	-9.02	-8.85	-9.39	-9.80	-9.38	-9.73	-9.36

Figure 5-5 : Variations in Paris coefficients

5.3.2 Uncertainty in Crack Length Estimation

In the second set of experiment, one specimen (Specimen P1) is used to determine the correlation factor for the specimen of interest. The damage index adopted in this study is the change of the peak to peak amplitude defined as,

$$DI_i = \frac{\Lambda_0 - \Lambda_i}{\Lambda_0} \quad (5.6)$$

where the subscript 0 represents the pristine state and the subscript i represents the current state of measurement, Λ is the peak-to-peak amplitude in voltage. A plot of the crack growth against the damage index for Specimen P1 is illustrated in Figure 5-6.

Figure 5-6 : Correlation between crack growth and damage index

From the experimental results, the correlation factor is determined as the rate of change in crack growth with respect to damage index (defined by the solid line). Using least square linear curve fitting, the correlation factor is computed as 50 with a R^2 -value of 0.98. Although the data fits the linear relationship well, there still exist some variations. To account for these variations, and assuming normal distribution

with 99.7% confidence limit, the upper and lower bounds of the estimate are identified and illustrated as dashed lines in Figure 5-6.

The crack length can be estimated using the following linear relationship,

$$a_{fi} = \Phi DI_i + a_{f0} \quad (5.7)$$

where the subscript 0 represents the pristine state and the subscript i represents the current state of measurement, and Φ is the correlation factor. Using Equation (5.7), the crack length for specimen P2 can be estimated and is illustrated in Figure 5-7. The actual crack length at the respective state of measurement is plotted in Figure 5-7 for easy reference. It is noted that other than a damage index at 75,000 cycles that exhibits a large magnitude, the other predictions are well within the predicted range. This is attributed to the “settling-in” period where initial changes are high and later stabilizes over time. The 75,000-cycles data will not be considered in subsequent analysis.

Figure 5-7 : Estimated crack length against damage index

5.3.3 Monte Carlo Convergence Test

Prior to conducting the full Monte Carlo simulation for a damage prognosis, a convergence test is conducted to determine the suitable number of trials for the simulation. The number of trials tested range from 10 to 6000 trials. The variations are large when the number of trials are small. As the number of trials increases, the mean fatigue life tends to converge to a straight line. It is noted that beyond 2000 trials, the variations in the mean fatigue life is insignificant. Therefore, 2000 trials will be used for subsequent Monte Carlo simulations in this thesis. A typical Matlab script for Monte Carlo simulation use to generate the fatigue crack growth curves is shown in Appendix A.

Figure 5-8 : Monte Carlo Convergence Test

5.3.4 Damage Prognosis

The Paris crack growth model is adopted for the estimation of the remaining life of the specimen. To account for the variation in the material properties and the errors in the crack length estimation, Monte Carlo simulation is conducted for 2000 trials. The Paris coefficients for the material of interest (6061-T6 aluminum alloy) are determined as 2.7 and -9.35 for the m and $\log C$ values respectively. In order to ensure that the random variables generated in the Monte Carlo simulation fall with the upper and lower bounds, standard deviation of 0.066 and 0.033 were used for m value and $\log C$ value

respectively. This corresponds to a 99.7% confidence level, assuming normal distribution.

For this material, the critical stress intensity factor K_c is determined as $159 \text{ MPa}\cdot\text{m}^{1/2}$ and the respective critical crack length a_{fc} is calculated as 39.8mm . From the Monte Carlo simulation, different fatigue lives are simulated based on the random variation in the material properties and the errors in crack length estimation. As an illustration, the Monte Carlo simulation at 125,000 loading cycles is shown in Figure 5-9. The mean is then used as the estimated fatigue life with an upper bound and lower bound defined by the maximum and minimum of data. The estimated remaining life, along with the upper and lower bounds for each damage index are illustrated in Figure 5-10. The actual remaining life is also plotted on the same figure, and it shows that the estimated remaining life is in close agreement with the actual remaining life within the upper and lower limits.

Figure 5-9 : Monte Carlo simulation at 125,000 cycles for Specimen P2

Figure 5-10 : Estimated Remaining Fatigue Life for Specimen P2

From Figure 5-10, it can be seen that although the estimated remaining life using the mean value of the Monte Carlo simulation provides a good estimate, there are still occasions when the remaining life is overestimated (i.e. longer than the actual remaining life) which could be dangerous. For such cases, using the lower bound as the estimated remaining life provides a safe and conservative estimate.

5.4 ITERATIVE FRAMEWORK

The prognosis proposed in the previous section requires material properties and uncertainties to be known *a priori*. However, for a material with unknown properties, the Monte Carlo simulation procedure can still be used with slight changes in the way the uncertainty in fatigue properties is handled. Here, the fatigue material properties are updated continuously using historical data points. Since the rate of crack growth curve requires a minimum of two points to determine the Paris coefficients, this will implied that a minimum of two historical data is required before the proposed iterative framework can be applied. The algorithm of the proposed iterative framework is illustrated in Figure 5-11.

Figure 5-11 : Algorithm for an iterative framework

For illustration purposes, another sample is prepared from a different batch of aluminium specimen. Thus the properties might be slightly different. Instead of finding out the fatigue properties from a set of training data, this framework utilises the historical data points to determine the fatigue properties of materials. The fatigue

properties will be determined from N historical data points, and applied to the Monte Carlo simulation to predict the remaining life of the specimen.

5.4.1 Damage Prognosis

Applying the proposed iterative framework to the same specimen as Section 5.3.2. (Specimen P2), one can obtain the updated Paris coefficients (Table 5-3) and the estimated remaining life for the specimen (Figure 5-12).

Table 5-3: Paris coefficients from the iterative framework

Acquisition Step	Damage Index	Paris Coefficient	
		m	Log C
160,000	0.1746	4.919	-11.043
190,000	0.2001	2.675	-9.243
210,000	0.2397	2.598	-9.185
230,000	0.3217	2.575	-9.169
233,000	0.3386	3.164	-9.673
235,000	0.3539	3.529	-9.987
237,000	0.4226	3.947	-10.355

It is noted that if only the minimum of two historical data points were used, the variation between the estimated and actual remaining life is significant. This is because the Paris coefficients derived from the two data points may be vague and not representative of the material. For better estimation, the author proposed to limit the minimum historical data points to three in order to start the iterative framework. Similar to the previous section, the estimated remaining life using the mean value of the Monte Carlo simulation provides a good estimate. Table 5-4 shows the comparison between the damage prognosis models presented above. Generally, the iterative framework model provides a better prognosis with less occurrence of over-estimation which could be dangerous.

Figure 5-12 : Estimated remaining life using the iterative framework

Table 5-4: Comparison of damage prognosis models

Actual Remaining Life	Damage Prognosis			
	Without iterative framework		With iterative framework	
	Prognosis	Error	Prognosis	Error
241,592	-	-	-	-
116,592	141,441	-24,849	-	-
81,592	96,743	-15,151	-	-
51,592	75,451	-23,859	36,343	15,249
31,592	18,716	12,876	29,237	2,355
11,592	20,373	-8,781	16,803	-5,211
8,592	17,794	-9,202	9,183	-591
6,592	12,429	-5,837	5,930	662
4,592	3,256	1,336	2,121	2,471

5.5 CONCLUSION

This chapter illustrates the applicability of a proposed iterative framework for the prognosis of single-edge notched specimens. In order to use the framework for prognosis, there should be at least two historical data available. The other limitation is that it can only provide a good estimate if the crack growth is in a steady phase (or in the Paris zone). If the crack is still in the crack nucleation phase, under-estimation of the remaining life prevails. Nonetheless, during the steady crack growth phase, the estimation is good and the actual remaining life is within the lower and upper bounds of the estimation. In conclusion, the proposed iterative framework can be used to estimate the remaining life of a system component even when the fatigue properties of the materials is not known *a priori*.

CHAPTER 6

FATIGUE DAMAGE PROGNOSIS OF WELDED STEEL CONNECTIONS

6.1 BACKGROUND

This chapter focuses on the fatigue crack detection and monitoring of welded steel connections, with non-load bearing welded joints in particular. The current fatigue design procedure of a welded specimen is first review followed by a review of fracture analysis of semi-elliptical surface crack using linear elastic fracture mechanics. Experimental study is conducted on a laboratory sized welded steel specimen. Two promising techniques are adopted to investigate the feasibility of detecting and monitoring the fatigue cracks. A comparison between two techniques, namely the Lamb wave propagation (LWP) technique and the electromechanical impedance (EMI) technique is presented.

6.2 FATIGUE DESIGN OF WELED STEEL SPECIMEN

At present, the civil and offshore industry has been using design codes (BSI, 2005; DNV, 2013) to determine the fatigue life of the structures. These codes usually adopt the strain-based approach where numerous empirical formulas are provided with S-N curves. Most codes are similar in nature, therefore it'll be sufficient to review one of the available codes. The author has chosen to review the DNV (2013) as it has the latest revision. For the purpose of this study, only the information relevant to the proposed joint configuration is review. A non-load bearing welded specimen (Figure 6-1) is classified as detail category E in the code. The S-N curve for detail category E is illustrated in Figure 6-2. From the S-N curve, the engineer can determine the fatigue life of the connection for a given stress range. This design S-N curve was derived based on mean minus two standard deviations from the numerous experimental data for the detail categories in the code. This translates to 97.7% confidence limit. This would mean that there is 97.7% chance that the actual fatigue

life will be greater than the design fatigue life. However, this difference cannot be quantified. In this aspect, the design may not be optimised. For example, the structure may have reached the design life, but there could still be a long time before the structure fracture. On the other hand, high confidence limit is necessary for critical structures. The chances of the structure failing during its service lifetime must be very low. Therefore, there are opportunities to study the optimisation of design while keeping the required level of confidence. This can be possibly achieve using state-of-the-art crack detection and monitoring techniques.

The detail category is given for edge distance ≥ 10 mm.

For edge distance < 10 mm, the detail category

Figure 6-1: Joint configuration for detail category E (DNV, 2013)

Figure 6-2: S-N curve for detail category E (DNV, 2013)

6.3 FRACTURE ANALYSIS OF SEMI-ELLIPTICAL SURFACE CRACK

Fracture analysis of semi-elliptical surface crack was investigated and reported by Newman Jr (1973). For completeness, the concept and equations used are summarised in this section. Consider a semi-elliptical surface crack as shown in Figure 6-3, where the crack length is $2c$, crack depth is a and the plate width and thickness is W and t respectively.

Figure 6-3: Schematic diagram of semi-elliptical surface crack configuration

The stress intensity factor for the semi-elliptical surface crack configuration as shown in Figure 6-3 can be given as (Newman Jr, 1973),

$$K = \sigma \sqrt{\pi \frac{a_f}{Q}} M_e \quad (6.1)$$

where σ is the far field stress, a is the crack depth and

$$\begin{aligned} Q &= 1 + 1.147 \left(\frac{a_f}{c} \right)^{1.64} \quad \text{for} \quad \frac{a_f}{c} \leq 1.0 \\ Q &= 1 + 1.147 \left(\frac{c}{a_f} \right)^{1.64} \quad \text{for} \quad \frac{a_f}{c} > 1.0 \end{aligned} \quad (6.2)$$

and the elastic magnification factor is given by,

$$M_e = \left[M_1 + \left(\sqrt{Q \frac{c}{a_f}} - M_1 \right) \left(\frac{a_f}{h} \right)^p \right] \sqrt{\sec \left[\frac{\pi c}{W} \left(\frac{a_f}{h} \right) \right]} \quad (6.3)$$

where

$$p = 2 + 8 \left(\frac{a_f}{c} \right)^3 \quad (6.4)$$

and M_1 is the front-face correction term given by,

$$\begin{aligned} M_1 &= \sqrt{\frac{c}{a_f}} \left(1 + 0.03 \frac{c}{a_f} \right) \quad \text{for} \quad \frac{a_f}{c} > 1.0 \\ M_1 &= 1.13 - 0.1 \left(\frac{a_f}{c} \right) \quad \text{for} \quad 0.02 \leq \frac{a_f}{c} \leq 1.0 \end{aligned} \quad (6.5)$$

W and h are the width and thickness of the plate respectively.

As the crack propagates, there will be a point in time when the semi-elliptical surface crack propagates through the thickness of the plate and “breakthrough”. It is assumed that once a breakthrough occurs, the crack will propagate as through thickness crack. The stress intensity factor for the through thickness crack can be given as (Newman Jr, 1973),

$$K = \sigma \sqrt{\pi c \sec \left(\frac{\pi c}{W} \right)} \quad (6.6)$$

The stress intensity factors will form the basis of calculation to determine the fatigue crack growth curve and the remaining life of the structure.

6.4 EXPERIMENTAL STUDY

In this chapter, experimental study is conducted to investigate the applicability of the damage indicator proposed in the preceding chapters. The subject material used in this study is structural steel which is different from the preceding chapters. The configuration also differs from the single-edge notched specimens used in the preceding experiments. The configuration used in this experiment relates better to the configurations used in practice. The details of the non-load carrying welded specimen is illustrated in Figure 6-4. This configuration corresponds to connection detail category E in the (DNV, 2013). PIC151 piezoelectric ceramics (PZTs) from PI ceramics are surface bonded to the Grade 43 (S275) steel specimen using a two part industrial epoxy. Since the objective of the experiment is to study the feasibility of

using LWP technique for monitoring of fatigue cracks under this configuration and to compare EMI and LWP technique for the detection and monitoring of fatigue cracks, the experiments are planned such that data acquisition by both techniques can be carried out after a stipulated number of loading cycles. The experimental setup is depicted in Figure 6-5. The left side of Figure 6-5 shows the equipment for EMI technique, while the right side of Figure 6-5 shows the equipment required for LWP technique.

The specimen shown in Figure 6-4 is subjected to cyclic axial load to initiate and propagate the fatigue crack in the specimen. Through a series of trials, the fatigue life of the specimen will be within the reasonable limit for laboratory based experiments when the cyclic load ranges from 10% to 80% of yield stress. In this study, the mean stress is approximately 45% of the yield stress, which is 123.75 MPa. The applied maximum and minimum stress is 225 MPa and 22.5 MPa respectively. This translates to an amplitude of 101.25 MPa with a load ratio of 0.1. In order to simulate the specimen under working condition, all data acquisition will be conducted at mean stress, whereby the specimen is held in the machine at the mean load. The first acquisition is conducted at this state prior to any cyclic load to provide a “baseline” for the study. The cyclic load is applied using a dynamic testing rig to initiate and propagate the fatigue crack. Traditional NDE methods, in particular the dye penetration technique, is use to identify and quantify the fatigue crack. Dyes were applied to both welds as well as the underside to identify surface cracks and through thickness cracks.

Figure 6-4: Sketch of test specimen with piezoelectric transducers

Figure 6-5: Experimental setup

6.4.1 Lamb wave propagation (LWP) Technique

6.4.1.1 Experimental Setup

The actuation signal is generated using an arbitrary function generator (Tarbor Electronics WW5061) to send alternating voltage across the PZT. The oscillations caused by the converse piezoelectric effect is transferred to the structural steel specimen through an epoxy layer. A digital oscilloscope (Yokogawa DL1620) is used to acquire the sensor signal. In order to reduce the random noise which is present in the signal, a 64-cycles averaging acquisition mode is adopted.

The actuation signal used is a 5-cycle Hanning windowed sine burst. In the author's opinion, the actuation signal used is appropriate for the short distance between the actuator and the sensor. Hanning windowed sine wave also has the advantages of concentrating a large amount of energy at the actuation frequency.

From the dispersion curves, the frequencies for higher order Lamb waves can be determine. In order to minimize the complications from higher order Lamb waves, the actuation frequency should be lower than the cut-off frequency for higher order Lamb waves. In this experiment, the actuation frequency is selected at 200 kHz. At this frequency, neither overlapping of electro-magnetic interference nor presence of higher order Lamb waves will occur.

6.4.1.2 Results

Figure 6-6 to Figure 6-9 shows the Lamb wave response at selected load cycles. In general, as load cycle increases, the amplitude of the sensor response decreases.

Figure 6-6: Sensor signal after 100,000 load cycles

Figure 6-7: Sensor signal after 160,000 load cycles

Figure 6-8: Sensor signal after 200,000 load cycles

Figure 6-9: Sensor signal after 290,000 load cycles

Due to the configuration of the specimen, it is difficult to decompose the signal into their individual components. However, from the dispersion curves, one can identify the fastest wave as the fundamental symmetric (S_0) wave. Therefore, the first wave packet in the signal (approx. 40-70 μ s) comprises of the S_0 wave along with other reflections and mode conversions from the edge and the welds (Lim et al., 2011). It is not the aim of this study to identify these wave packets. It is however the intention of the author to use the information to relate to the identification and monitoring of fatigue crack.

In Chapter 4, it is illustrated that the through thickness crack propagating from a single edge notched specimen can be quantified using a damage indicator based on the peak to peak amplitude of the first wave packet, which comprises of the edge reflected Lamb waves. For a non-load bearing welded specimen, the crack will normally initiate from the weld toe, starting from a semi-elliptical surface crack and propagate along the crack front. Therefore, the damage indicator based on the peak to peak amplitude of the first wave packet may not be applicable for such instance. The energy approach where the variation in energy within a stipulated time window may be a better option. The damage index based on variation in transient energy is given in Equation (4.6). In this study, the time window adopted is 38 μs to 130 μs . The results is illustrated in Figure 6-10.

Figure 6-10: Correlation between damage index and number of cycles for LWP technique

During the first 40,000 cycles, there is a sharp increase in the damage index. This is attributed to the “settling-in” effect where the system is self-calibrating and adjusting to a steady state. Ryles et al. (2008) have made the same observation in their study on an aluminum specimen. After the “settling-in” period, the damage index stabilises over the next 60,000 cycles. However, due to the sensitivity to noise, this damage index could fluctuate even though no damage is present. To account for this fluctuation, a repeatability study was carried out at 40,000 cycles. Sixteen readings were acquired and the mean and standard deviation are determined as 0.18 and 0.044 respectively. Assuming a normal distribution and a 95% confidence limit, the upper bound is determined as 0.25. In other words, if the data point falls below the upper bound (indicated by the dashed line), we are 95% confident that no crack has occurred. Therefore, this upper bound defines the “undamaged-zone”. Here, the first point beyond the undamaged zone is at 140,000 cycles. This implies that crack could have initiated between 120,000 cycles and 140,000 cycles. Using this crack detection algorithm, crack is assumed to have occurred at 120,000 cycles. Beyond 140,000 cycles, there is still some fluctuation of data before it eventually increases with load cycles. This observation reflects the fatigue crack growth correctly since the rate of change increases with load cycles, as defined by the Paris equation.

6.4.2 Electromechanical Impedance (EMI) Technique

6.4.2.1 Experimental Setup

An impedance analyzer (Wayne Kerr 6420) is used to send an alternating voltage across the PZT which then transmit a harmonic sine wave to the host structure. At the same instance, the impedance analyzer is also used to acquire the steady state response in the form of complex electrical admittance. The bonding layer and ambient temperature may contaminate the signature for frequency beyond 100kHz (Yang et al., 2008; Yang et al., 2008). Therefore, the frequency range selected for this study is limited to 100 kHz. A frequency range between 45 kHz to 55 kHz is adopted in this study since there are two prominent peaks located within this range. The resolution used for the frequency sweep is 0.01 kHz.

6.4.2.2 Results

In this study, PZT2 is used as the transducer for the EMI technique. The conductance for the PZT transducers is illustrated in Figure 6-11. It can be observed that as the number of loading cycle increases, the location of the peaks shifts leftwards. This observation is in agreement with observations by other researchers (Lim and Soh, 2010; Lim and Soh, 2011). The shift of the peaks implies changes in the resonance frequency (in which leftward shifts implies reduction in resonance frequency). The presence of damage reduces the stiffness of the structure resulting in a reduction of resonance frequency. Therefore, if the resonance frequency can be a damage quantifier, the shift of peaks in conductance signatures (which relates to the resonance frequency) can be used as a damage quantifier.

Figure 6-11: Conductance signature after various loading cycles

The shift in frequency is plotted against the number of loading cycles and is illustrated in Figure 6-12. Similar to the LWP technique, a “settling-in” period is observed in the first 40,000 load cycles. Thereafter it stabilized and remain constant for the next 60,000 load cycles. During this first 100,000 load cycles, no physical

cracks were detected using the dye penetration technique. Therefore, the resonance frequency remains unaffected, and the frequency of the peak conductance signature remains unchanged. The peak started to shift leftwards after 120,000 load cycles, implying a change in the mechanical response of the structure, which can be interpreted as the presence of damage in the structure. The damage could have initiated between 100,000 and 120,000 load cycles. Conservatively, one can assume that the damage could have initiated after the last data acquisition (i.e. 100,000 load cycles).

The peaks gradually shift leftwards after 120,000 load cycles. The shift is almost linear with respect to the load cycles. However, from the fatigue crack growth curve, one can observe that the crack growth is not a linear relationship with the loading cycles. For a semi-elliptical surface crack which eventually propagates through the thickness and breakthrough, the rate of crack growth increases with the increase in load cycles.

Figure 6-12: Correlation between frequency shift and number of cycles for EMI technique

6.4.3 Discussions

From the experimental results of the LWP and EMI technique, a steady state damage indicator can be observed after the “settling-in” period and before the onset of fatigue crack. This steady state can then be considered as the baseline where the system is still in a healthy state. After removing the “settling-in” period and normalizing the data for comparison of the technique, the relationship between the normalized damage quantifier and the fatigue life is illustrated in Figure 6-13.

Prior to the breakthrough of the fatigue crack, the crack is assumed to be a semi-elliptical surface crack. Fracture analysis of semi-elliptical surface cracks was investigated and reported by Newman Jr (1973) and revisited in Section 6.3. By substituting the stress intensity factors into the Paris equation and integrating over the

range from the initial crack size till the critical crack width, the fatigue crack growth curve can be determined. The parameters used in the calculations are summarised in Table 6-1. However, the C -value presented in Table 6-1 is for a stress ratio of 0. In this study, the stress ratio is 0.1. Thus, the C -value has to be modified to reflect the correct stress ratio. The Walker equation (Stephend et al., 2001) is adopted to modify the C -value. After modification, the C -value, is determined as 1.495×10^{-10} m/cycle. From the fractured section (Figure 6-14), the a/c ratio can be determined as 0.67.

In order to determine the crack growth curve, there must be an initial crack size to start with. In this study, the initial crack size is assumed to be 1mm. Once the crack growth curve is determined, the theoretical crack growth curve can be plotted against the experimental values for comparison. This is done by matching the theoretical fatigue life with the experimental value (293,802 cycles). The number of fatigue cycles when the surface crack is 1mm (crack initiation) can be extracted from the fatigue crack growth curve (99,540 cycles). The normalized theoretical crack growth curve is illustrated in Figure 6-13 for easy comparison.

Figure 6-13: Normalised damage quantifier vs fatigue life for (a) EMI technique & (b) LWP technique

Table 6-1: Parameters for calculation of fatigue crack growth curve

Parameter	Value	Reference
$C (R=0)$	0.660×10^{-8} in/cycle [1.356×10^{-10} m/cycle]	Boyer and Metals (1985)
m	2.25	Boyer and Metals (1985)
a_0	0.34mm	Exp. [Figure 6-14]
c_0	0.50mm	---

Figure 6-14: Experimental value for a/c

Due to the sensitivity to noise and the nature of the LWP technique, it is concluded that the LWP technique is not sensitive to detect the crack initiation. However, when the crack starts propagating at a steady state (i.e. Paris law) the LWP technique has a better agreement with the trend of the theoretical crack growth curve. The correlation between the theoretical surface crack length and the damage indicator is illustrated in Figure 6-15. The change in damage index is taken with reference from the mean value (i.e. 0.18).

The EMI technique has proven its capability to detect incipient damages. The crack is detected after 100,000 load cycles, which is very close to the theoretical prediction of 99,540 load cycles. In this aspect (detection of crack initiation), the EMI technique is superior as compared to the LWP technique. However, beyond 100,000 load cycles, the EMI technique may not reflect the trend of crack growth well enough as illustrated in Figure 6-16. The shift in frequency (x-axis) is taken with respect to the undamaged state (0.05kHz).

Figure 6-15: Surface crack length vs damage indicator for LWP technique

Figure 6-16: Surface crack length vs damage indicator for EMI technique

By comparing Figure 6-15 and Figure 6-16, it is obvious that the LWP technique has a better correlation with the surface crack length. Therefore, the LWP technique may be a better candidate for fatigue crack growth monitoring.

Recall that in Section 6.2, the S-N curve for this connection detail is presented. Based on the design procedure given in (DNV, 2013), for a stress range of 202.5 MPa, the fatigue life is determined as 123,232 load cycles. However, the specimen in this study only fractures at 293,802 load cycles (138% difference). This further reiterate the fact that the design based on codes may not be optimized and efficient.

6.5 CONCLUSIONS

This chapter investigate the feasibility of using state-of-the-art techniques, electromechanical technique and Lamb wave propagation technique in particular, to detect and monitor fatigue crack in a welded steel specimen. An experimental study was conducted on a laboratory sized welded steel connection. The specimen is subjected to cyclic tensile load. After stipulated load cycles, data were acquired for both EMI and LWP techniques.

The EMI technique provides an early indication of incipient damage (i.e. the onset of fatigue crack) while the LWP technique provides a better correlation to the surface crack length. Therefore, for a robust fatigue crack detection and monitoring system, an integrated or hybrid EMI/LWP technique may be investigated.

CHAPTER 7

SUMMARY AND FUTURE WORK

7.1 BACKGROUND

This chapter concludes the work done by the author in the area of structural health monitoring (SHM) and damage prognosis (DP), in particular, the Lamb wave propagation technique. The assumptions and limitations is highlighted and improvements to be done in the future is discussed.

7.2 SUMMARY

The model for PZT transducers in a narrow plate is developed in Chapter 3. The concept of an equivalent imaginary source (EIS) is introduced to model the response of reflected Lamb waves. In the development of the model, plane harmonic wave is assumed. The amplitude and phase of reflected Lamb waves and SH waves are incorporated into the model to predict the response of PZT transducers in a narrow plate environment. Experimental and numerical studies were conducted to verify the proposed EIS model. The experimental and numerical results agree well with the proposed model at low frequencies, below the cut-off frequency of higher order Lamb modes.

The work carried out in Chapter 3 is used to support the work presented in Chapter 4. In Chapter 4, the effect of width reduction on sensor signal is presented. As width reduces, the edge reflected Lamb wave approaches the direct path Lamb waves and interference occurs. This highlighted that the first wave packet in the sensor signal of a narrow plate consists of both the direct path and edge reflected Lamb waves. The EIS model proposed in Chapter 3 is applied to study the monitoring of crack. The assumption of the crack tip remains plane and parallel to the edge prevails in order for the model to be applicable. For the geometry considered, there exist a linear relationship between the crack growth and the damage indicator based on the amplitude of the signal. Experimental studies were conducted to verify the

relationship. In the experimental study, various damage indexes were investigated and compared. The damage indexes investigated includes variation in peak-to-peak amplitude, variation in scattered energy, variation in transient energy and variation in time of flight. The damage index based on variation in peak-to-peak amplitude is proposed as a suitable damage indicator due to better curve fitting and reasonable sensitivity to crack growth.

The work from Chapter 3 and 4 is extended to the prognosis of fatigue crack presented in Chapter 5. Using the proposed damage index based on variation in peak-to-peak amplitude, an iterative framework is proposed for the prognosis of single-edge notched specimens. Monte Carlo simulation is carried out to address the uncertainty in the materials and the crack monitoring technique. The advantages of the iterative framework is that the fatigue properties of the materials need not be known *a priori*. In theory, only a minimum of two historical data is required to start the process. However, the Paris coefficient derived from the two data points may be vague and not representative of the materials which lead significant differences between the prognosis and the actual remaining life. The author proposed a minimum of three data points to start the iterative process. It is illustrated that the iterative framework can be used to estimate the remaining life of the system and the error reduces as the number of historical data increases.

Chapter 6 presents an extension of the Lamb wave propagation technique to monitor fatigue crack in a welded steel specimen. A comparison between the electromechanical impedance technique and the Lamb wave propagation technique is carried out. It is well known that EMI has superior capabilities in detection of incipient crack, thus from the experiments, the EMI technique is superior during the crack initiation phase. However, during the steady crack growth phase, the Lamb wave technique provides a more reliable correlation with the crack length. Therefore, it can be concluded that Lamb wave technique is suitable for crack monitoring during the steady crack growth phase while EMI is suitable for the crack initiation phase.

7.3 RESEARCH ORIGINALITY AND CONTRIBUTIONS

The original contributions of this thesis is presented in Section 1.5. The work presented in Chapter 3 to Chapter 6 is summarised above and the original contributions are reiterated below:

- (vi) A novel concept of an equivalent imaginary source (EIS) is proposed to model the displacement and strain field of reflected Lamb waves. The EIS incorporates information on the reflection coefficient and phase delay of reflected waves. The work presented in Chapter 3 illustrates that the concept of EIS model is viable as it agrees well with numerical simulations and experimental results;
- (vii) Existing model for infinite plate is extended to model the voltage output of PZT sensor in narrow plate using the EIS model. The work presented in Chapter 3 illustrates that the EIS model agrees well with numerical simulations and experimental results;
- (viii) A new damage index based on the variation in the peak-to-peak amplitude is proposed among the four damage indexes were investigated and presented in Chapter 4;
- (ix) A new iterative self-updating framework is proposed to predict the remaining fatigue life of narrow plate for constant amplitude loading. The work presented in Chapter 5 illustrates that the iterative self-updating framework can predict the remaining life well;
- (x) The crack detection capability is extended to welded steel connections and a comparative study between the EMI and Lamb wave technique is conducted experimentally. From the comparative study presented in Chapter 6, it can be concluded that Lamb wave technique is superior for crack monitoring in the steady crack growth phase.

7.4 FUTURE WORK

The work presented in this thesis covers mainly the Lamb wave propagation technique and its ability to monitor cracks, in particular fatigue cracks. The

experimental work conducted focuses on a narrow plate in a controlled laboratory environment. Further experiments may be conducted on working structures in the real-world environment to verify the applicability of the work presented in this thesis.

The actuation frequency adopted in this these are relatively low (below the cut-off frequency of the higher order Lamb modes) in order to reduce the complication of signals by higher order Lamb modes. Although, the frequency presented in this thesis is adequate for most practical applications, higher frequency can also be investigated to explore the feasibility for crack monitoring.

In the study of fatigue crack monitoring, only the case of a single-edge notch specimen with constant amplitude loading is considered. The single-edge notch specimen is the simplest geometry with established data for laboratory implementation. The author understands that constant amplitude loading may not be realistic in the real world environment, but it serves as a first step to provide an understanding of the problem with one factor of uncertainty (variation in loading conditions) under control. For future work, the author proposed to extend the iterative model to other geometry and variable amplitude loading condition to study its applicability.

AUTHOR'S PUBLICATIONS

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SUBMITTED MANUSCRIPTS

Lim, S. I., Liu, Y. and Soh, C. K. "Modelling of Sensor Response of Piezoelectric Transducers in a Narrow Plate", Journal of Intelligent Material Systems and Structures.

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APPENDIX A

MatLab Scripts

a) Matlab script for Calculation of Lamb Wave Phase and Group Velocities

```

% ***** MATLAB Script for Calculation of Lamb Wave Dispersion Curves *****
% ***** Author: Nanyang Technological University Library *****
% ***** Date: 2020-01-01 *****
% ***** Version: 1.0 *****
% ***** Description: This script calculates the phase and group velocities of Lamb waves for a given material and frequency range.
% ***** The results are displayed in a table and saved to a CSV file. *****

close all
clear

% ----- Material Properties -----
E = 68.95; % Young's Modulus (GPa)
nu = 0.33; % Poisson's ratio
rho = 2715; % Density (kg/m3)

% ----- Longitudinal & Transverse wave speed -----
lamda = (E*nu)/((1-2*nu)*(1+nu));
mu = E/(2*(1+nu));
cL = sqrt((lamda+2*mu)/rho)*1000; % Longitudinal wave speed (mm/us)
cT = sqrt((mu/rho)*1000); % Transverse wave speed (mm/us)

% ----- Phase Velocity Calculation -----
p12 = w^2./cL^2 - 1./cp^2;
q12 = w^2./cT^2 - 1./cp^2;
tan(q12)/q12 + 4*p12*(tan(p12)/p12)/((w/cp)^2 - q12)^2; % (symmetric)
q12*tan(q12) + 4*p12*(q12*tan(p12)/4*p12)/((w/cp)^2 - q12)^2; % (anti-symmetric)

fd_range = 0.1:0.1:5;
len = length(fd_range);
for f_id = 1:len
    fd = fd_range(f_id);
    d = 3;
    f = fd/d;
    w = 2*pi*f;
    cp = 0.1;
    p = w*sqrt((1/cL^2 - 1/cp^2));
    q = w*sqrt((1/cT^2 - 1/cp^2));
    s_ph1 = tan(q*d)/q + 4*p*(w/cp)^2*tan(p*d)/((w/cp)^2 - q^2)^2;
    a_ph1 = q*tan(q*d) + (tan(p*d)*((w/cp)^2 - q^2)^2)/(4*p*(w/cp)^2);
    sym_id = 1;
    anti_id = 1;
    for cp = 0.2:0.0003:20
        p = w*sqrt((1/cL^2 - 1/cp^2));
        q = w*sqrt((1/cT^2 - 1/cp^2));
        s_ph2 = tan(q*d)/q + 4*p*(w/cp)^2*tan(p*d)/((w/cp)^2 - q^2)^2;
        a_ph2 = q*tan(q*d) + (tan(p*d)*((w/cp)^2 - q^2)^2)/(4*p*(w/cp)^2);
        if s_ph1*s_ph2 < 0 & s_ph2 - s_ph1 > 0
            sym(f_id, sym_id) = cp;
            sym_id = sym_id + 1;
        end
        if a_ph1*a_ph2 < 0 & a_ph2 - a_ph1 > 0
            anti(f_id, anti_id) = cp;
            anti_id = anti_id + 1;
        end
        s_ph1 = s_ph2;
        a_ph1 = a_ph2;
    end
end

% ----- Group Velocity Calculation -----
% group speed, cg = cp^2/(cp-id*d*cp)/d(f_id)

[ms,ns] = size(diff(sym));
sym_grp = (sym(1:end-1,:).^2)./(sym(1:end-1,:)-([1:len-1]'*ones(1,ns)).*(diff(sym)));
[ma,na] = size(diff(anti));
anti_grp = (anti(1:end-1,:).^2)./(anti(1:end-1,:)-([1:len-1]'*ones(1,na)).*(diff(anti)));

% ----- Write to CSV File -----
dlmwrite('sym_ph.csv',sym);
dlmwrite('anti_ph.csv',anti);

```



```
dlmwrite('sym_grp.csv',sym_grp);
dlmwrite('anti_grp.csv',anti_grp);
```

b) Matlab script for Hilbert transform envelope detection

```
%----- Hilbert Transform Envelope Detection -----
%----- Read CSV File -----
close all
clear

Signal = csvread('signal.csv');
[m,n] = size(Signal);

%----- Perform HT -----
for col = 1:n;
    signal = Signal(1:m,col);
    signal_h = hilbert(signal); % find the hilbert transform of the measured signal
    envelope_est = sqrt(signal_h.*conj(signal_h)); % square the hilbert transform and take the +ve square root to get the estimated envelope
    Envelope(1:m,col) = envelope_est;
end

%----- Write to CSV File -----
csvwrite('signal-env.csv',Envelope);
```

c) Matlab script for Monte Carlo simulations

```
%----- Monte Carlo Simulation for Fatigue Crack Growth Curve -----
%----- Input Parameters -----
close all
clear

Sy = 276; %Yield Strength (MPa)
Smax =147.59; %Max Stress (MPa)
Smin =117.76; %Min Stress (MPa)
K1c = 39; %Plane strain fracture toughness (MPa-m1/2)
W = 0.076; %Plate width (m)
t = 0.006; %Plate thickness (m)
a0 = 0.00505; %Initial crack length
acr = 0.0398;
DI = 0.128194605312618; %Damage Index
nrun = 2000; %No. of trials
phil = 149.17; %phi
C = 10^(-9.35);
M = 2.7;

%----- Generate NS samples from Normal Distribution -----
A = (phil*DI)/1000+a0+(randn(nrun,1)*0.00269);
logC = (randn(nrun,1)*0.033)-9.35;
C = 10.^logC;
M = (randn(nrun,1)*0.066)+2.7;

%----- Initialise N_index and a_index -----
Nind = 0;
aind = 1;

%----- Start Monte Carlo Simulation with Looping -----
for NS = 1:nrun;
    a = A(NS);
    if a < a0
        a = a0;
    end
    if a > acr
        a = acr;
    end
    c = C(NS,1);
```

```

m = M(NS,1);
Kc = K1c*sqrt(1+(1.4/t^2)*(K1c/Sy)^4);
Kmax = (1.122-0.231*(a/W)+10.550*(a/W)^2-
21.710*(a/W)^3+30.382*(a/W)^4)*Smax*sqrt(pi*a);
N = 235000;
da = 0.0001;
ind = 1;
Na(ind,NS+Nind) = N;
Na(ind,NS+aind) = a;
--loop
while Kmax < Kc
    Kmax = (1.122-0.231*(a/W)+10.550*(a/W)^2-
21.710*(a/W)^3+30.382*(a/W)^4)*Smax*sqrt(pi*a);
    dK = (1.122-0.231*(a/W)+10.550*(a/W)^2-21.710*(a/W)^3+30.382*(a/W)^4)*(Smax-
Smin)*sqrt(pi*a);
    dN = da/(c*dK^m);
    N = N + dN;
    a = a + da;
    ind = ind+1;
    Na(ind,NS+Nind) = N;
    Na(ind,NS+aind) = a;
end
Nind = Nind+1;
aind = aind+1;
plot(Na(1:ind,NS*2-1),Na(1:ind,NS*2),'-b');
hold all
end
----- Create a vector for Fatigue life -----
len = size(Na,1);
Nf(1,1) = max(Na(1:len,1));
for rind = 2:nrun
    Nf(rind,1) = max(Na(1:len,rind*2-1));
end
----- Calculate summary statistics -----
Nf_mean = mean(Nf);
Nf_std = std(Nf);
Nf_median = median(Nf);

```

APPENDIX B

List of Geometry Factors for Calculation of Stress

Intensity Factor

Recall that the stress intensity factor is given as,

$$K = Y\sigma\sqrt{\pi a}$$

where Y is the geometry factor, σ is the nominal stress, or far field stress and a is the crack length. A number of well known and commonly used stress intensity factors can be found in most Fracture Mechanics textbooks. A summary of various established geometry factor is given below (Janssen et al, 2004).

Table B.1 : List of Geometry Factors

Description	Geometry Factors (Y)
Infinite plate	1
Semi-infinite plate with edge crack	1.12

Table B.1 : List of Geometry Factors (Continued)

Description	Geometry Factors (Y)
<p>Single edge notched specimen</p>	$1.122 - 0.231 \left(\frac{a}{W} \right) + 10.550 \left(\frac{a}{W} \right)^2 - 21.710 \left(\frac{a}{W} \right)^3 + 30.382 \left(\frac{a}{W} \right)^4$
<p>Double edge notched specimen</p>	$\frac{1.122 - 1.122 \left(\frac{a}{W} \right) - 0.822 \left(\frac{a}{W} \right)^2 + 3.768 \left(\frac{a}{W} \right)^3 - 3.040 \left(\frac{a}{W} \right)^4}{\sqrt{1 - \frac{2a}{W}}}$
<p>Centre cracked finite plate</p>	$1 + 0.256 \left(\frac{a}{W} \right) - 1.152 \left(\frac{a}{W} \right)^2 + 12.200 \left(\frac{a}{W} \right)^3$